This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

**Usage guidelines**

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

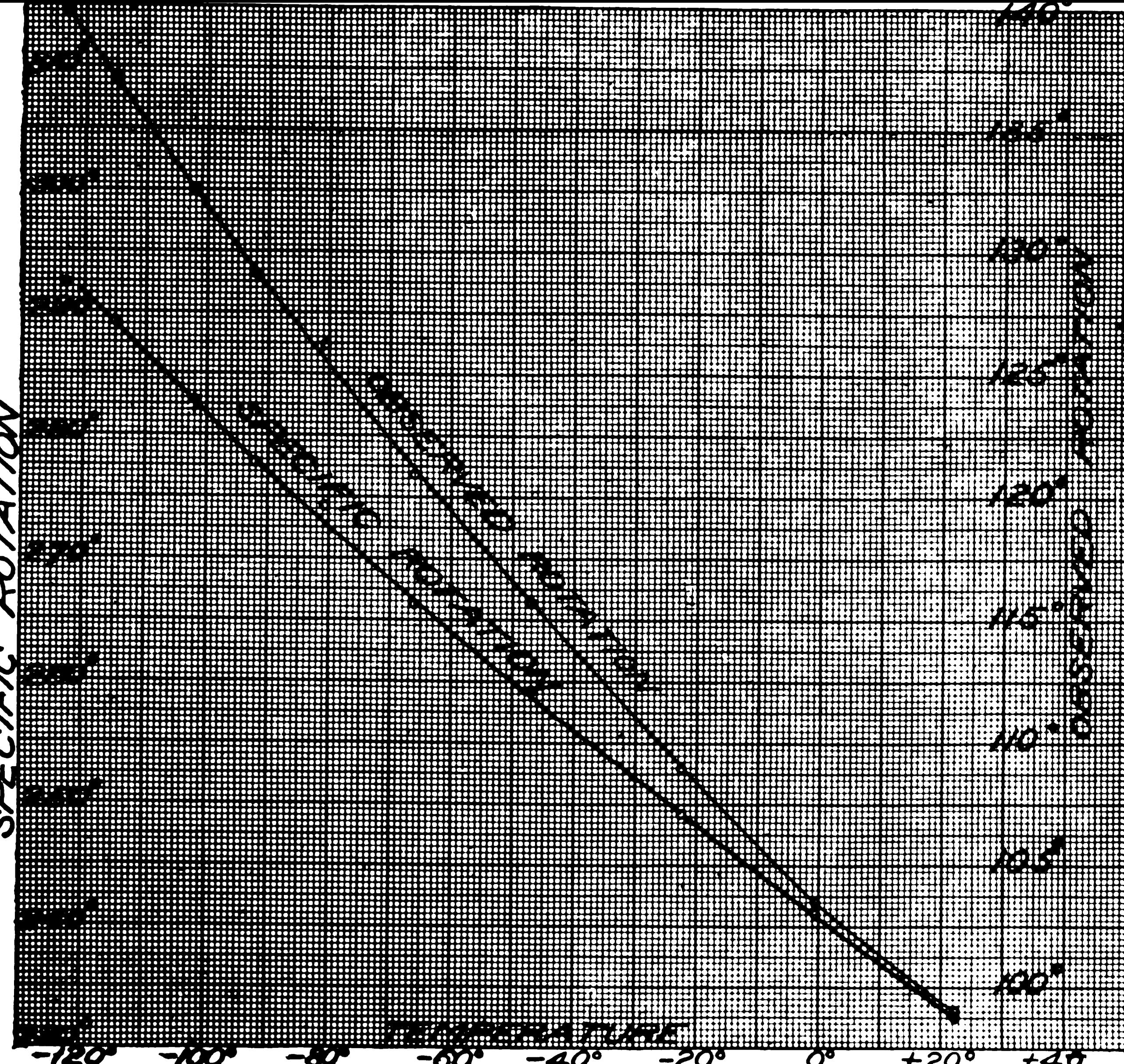
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

**About Google Book Search**

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`



# Physical review

American Institute of Physics,
American Physical Society, Cornell University


One of the Books of
Hark! the world so loud, and these, the movers of the world, so still.
George W. Pierce

# THE

# PHYSICAL REVIEW

*A JOURNAL OF EXPERIMENTAL AND*
*THEORETICAL PHYSICS*

CONDUCTED WITH THE CO-OPERATION
OF THE
AMERICAN PHYSICAL SOCIETY
BY
EDWARD L. NICHOLS, ERNEST MERRITT,
AND FREDERICK BEDELL

VOL. XXX

THE MACMILLAN COMPANY
NEW YORK & LONDON
BERLIN: MAYER AND MUELLER
1910

PRESS OF
THE NEW ERA PRINTING COMPANY
LANCASTER, PA.

# CONTENTS OF VOLUME XXX.

## CLXV. JANUARY, 1910.

## CLXVI. FEBRUARY, 1910.

CLXVII. MARCH, 1910.

CLXVIII. APRIL, 1910.

CLXIX. MAY, 1910.

CLXX. JUNE, 1910.

*Volume XXX.* *January, 1910.* *Number 1.*

THE

PHYSICAL REVIEW.

---

# THE VARIATION OF THE HALL EFFECT IN METALS WITH CHANGE OF TEMPERATURE.

BY ALPHEUS W. SMITH.

## INTRODUCTION.

IN 1879 Professor Hall[1] discovered that when a thin sheet of metal which is carrying a current of electricity is brought into a magnetic field, so that the lines of force are perpendicular to the plane of the plate, there is set up in the plate an electromotive force which is at right angles to that which is driving the current. This effect which is known as the Hall effect, has been the subject of a large amount of experimental and theoretical work. In later years several other phenomena which are related to it have been discovered and studied by other investigators. A fairly complete list of titles of articles on the Hall effect and allied phenomena has been prepared by McKay[2] and a review of the more important papers dealing with these phenomena has been recently published by Zahn.[3] It does not seem desirable at this time to review again at any length the work which has been done in this region.

One phase of the subject which seems to have been inadequately handled either theoretically or experimentally is the variation of the Hall effect with change of temperature. So far as the author is aware the temperature coefficient of this effect is known in the case of the non-magnetic metals only for bismuth, antimony and tellurium. The temperature coefficient of the effect in bismuth and antimony

---

[1] Amer. Jour. Math. (2), pp. 287–292, 1879.

[2] Proc. Amer. Acad. of Arts and Sci. (41), pp. 385–395, 1906.

[3] Jahr. der Radioaktivität und Elektronik (5), p. 166, 1908.

has been studied between — 186° C. and the melting point of the metal. The effect is found to decrease with rising temperature and within the error of observation to vanish at the melting point of the metal in question. It seems that the presence of the Hall effect in metals in the liquid state cannot be regarded as definitely proved.[1] In tellurium v. Ettingshausen and Nernst[2] found a decrease with rising temperature. Clough and Hall[3] failed to find any certain change in copper and phosphor-bronze between 20° and 300° C.

The behavior of the Hall effect in the magnetic metals is more interesting than its behavior in the non-magnetic, but the influence of temperature on the effect in the former has not been systematically studied. The temperature coefficient of the Hall effect in iron and cobalt is known only in the neighborhood of room temperature and the effect increases with rising temperature. Clough and Hall[3] studied nickel and steel for certain temperatures between 20° and 300° C. but for only two magnetic fields — 1,500 and 3,000 c.g.s. units. These investigators showed that in nickel the effect increases rapidly with rising temperature, reaches a maximum at about 290° C. and then decreases very rapidly. In steel the effect continues to increase with rise of temperature over the interval of temperature used in their experiments. Zahn and Schmidt[4] found that between — 186° and 23° C. the Heusler alloys behave with regard to temperature like the magnetic metals.

The attempts of Riecke,[5] Drude,[6] Lorentz,[7] Thomson[8] and others to account for the Hall effect and allied phenomena on the basis of the electron theory have emphasized the importance of a more detailed study of these effects, for any theory which can adequately account for them will be a large contribution to the electron theory of metals and will do much to explain the relations which exist between heat, electricity and magnetism. In view of the importance

---

[1] Zahn, ibid., p. 183.

[2] Acad. Wiss., Wien, Anz., pp. 173–174, 1886.

[3] Proc. Amer. Acad. of Arts and Sci. (28), pp. 189–199, 1892–3.

[4] Vehrh. d. D. Physik. Ges. (9), pp. 98–108, 1907.

[5] Ann. d. Phys. (66), p. 353, 1898.

[6] Ann. d. Phys. (1), p. 563, and (3), p. 369, 1900.

[7] Kon. Acad. Wet. Amst. (7), pp. 438, 585 and 684, 1905.

[8] Congres International de Physique (Paris), (3), p. 138, 1900. Corpuscular Theory of Matter, p. 99.

which such data might have in the electron theory, it seemed worth while to make a systematic study of a number of metals over as large a range of temperature as possible and over as large a range of magnetic fields as possible. It was especially interesting to examine the magnetic metals at temperatures above their respective critical temperatures in order to compare their behavior at such temperatures with the behavior of the non-magnetic metals. The desirability of data in this region has been recently pointed out by Zahn.[1] Furthermore, according to a suggestion made by J. J. Thomson[2] the reversal of the Hall effect in iron may be explained by supposing that the field actually acting on the free electrons is not only the impressed external field but also the field due to the orbital motion of the electrons in the metal. Under the action of the external field these electrons give fields which are in the opposite direction to the external field. The Hall effect would, therefore, consist of two terms of opposite signs. At temperatures above the critical temperature it seemed that the field due to the motion of the electrons in the molecules might be expected to nearly vanish. At such temperatures the Hall effect would consist of but one term and according to the electron theory would have the direction of the effect in bismuth. To test this theory was one of the purposes of this investigation.

Part I. of this paper concerns itself with the Hall effect in certain non-magnetic metals and silicon. Part II. deals with the magnetic metals.

## PART I. NON-MAGNETIC METALS AND SILICON.

### *Method and Apparatus.*

The method of observation was similar to that employed by other observers of this effect. The metals to be investigated were rolled to the desired thickness and then from the sheet were cut plates similar in form to those originally used by Professor Hall. These plates were in the form of rectangles 2.8 cm. long and 1.5 cm. wide. The thickness of the plates was determined from a knowledge of the density, area and weight of the plate. Strips of

---

[1] Ibid., p. 180.

[2] Ibid., p. 70.

copper 1.5 mm. in thickness were soldered along the ends of the plates. To these copper strips, which served as electrodes, were soldered the lead wires *p* and *p′* (Fig. 1) in such a way that the lines of flow were as nearly as possible parallel to the edges of the plate. From the middle of each side of the plate projected a narrow arm to which was soldered at the points *s* and *s′* respectively (Fig. 1) the copper lead wires which lead to the galvanometer on which the effect was to be observed. By filing these arms near the edge of the plate they could be shifted until they were nearly on the same equipotential line. In the cases of aluminum, gold and zinc the contacts for the lead wires at *p* and *p′* and those at *s* and *s′* could not be made by soldering. In these cases the ends of the plates were clamped between strips of copper and the lead wires for the current were soldered to these strips. The contacts for the wires to the galvanometer were made in a similar way by joining these wires by means of copper clamps to the arms at *s* and *s′*. After the lead wires had been connected to the plate in the way indicated above, the plate was mounted on a small rectangular piece

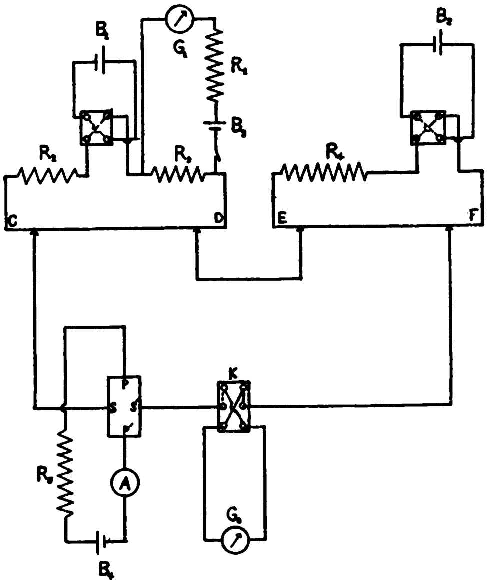


Fig. 1.

of hard fiber. It was held in position on the fiber by covering the plate and the fiber with paraffin. The plate thus mounted could be fastened in the plate-holder, which was made for this purpose and brought into the desired position in the magnetic field.

For the measurement of the electromotive force generated by the magnetic field the ordinary potentiometer method was used. The arrangement of the apparatus is shown diagramatically in Fig. 1, where *pp′* is the plate to be investigated. *EF* is an auxiliary potentiometer by means of which any lack of equipotentiality between *s* and *s′* and any thermo-electromotive forces in the circuit could be compensated. *CD* is the potentiometer for measuring the Hall electromotive force; $B_1$ is a storage battery of 2 volts; $B_3$, a standard cadmium cell of 1.0186 volts; $G_1$, a d'Arsonval galvanometer; $R_3$, a standard resistance of 900 ohms; $R_2$, the auxiliary resistance which could be adjusted in the usual way to make the drop of potential over $R_3$ equal to 1.0186 volts; $B_4$, a battery of 4 volts which sends the current through the plate *pp′*; $R_5$, an auxiliary resistance and $G_2$, a sensitive galvanometer of the Thomson type. On the potentiometer a copper slide wire with a resistance of .0004506 ohm per cm. was used.

When the plates had been placed in the proper position between the poles of the magnet, the position of the slider on the auxiliary potentiometer was adjusted until there was no deflection in the galvanometer $G_2$ on closing the key *K*. The magnet was then excited and the difference of potential between *s* and *s′* measured in the usual way on the potentiometer. In order to obtain one value of the Hall electromotive force plotted in the following curves, four readings were usually taken. With the current flowing from top to bottom of the plate and the magnetic field from left to right, one reading was made. Keeping the direction of the current fixed, the direction of the magnetic field was reversed and a second reading was made. The same process was repeated with the current flowing from bottom to top of the plate. The mean of these four observations was taken as the value of the Hall electromotive force. In some cases where the experimental difficulties were greater than usual, eight readings in all were taken and the mean of these has been plotted.

For the production of the magnetic fields a powerful electro-magnet was constructed. The pole-pieces were 10 cm. in diameter and tapered to 4 cm. at the air-gap. The exciting coils consisted of 2,850 turns of copper wire which could carry as much as 40 amperes. There was an arrangement for circulating water between the coils and the iron-cores. Some of the heat generated by the exciting current was thus carried away and the iron-cores kept at nearly constant temperature. At room temperature the length of the air-gap amounted to .9 cm. In the work at the temperature of liquid air the air-gap had to be increased to about 2.5 cm. to allow the introduction of the vessel containing the liquid air. The magnetic fields were measured ballistically with a flip-coil and a standard solenoid.

For the work at low temperatures it was necessary to provide for the liquid air a vessel which could be placed between the poles of the magnet. To use an ordinary Dewar flask for this purpose required that the pole-pieces of the magnet be very far apart and the magnetic fields correspondingly weak. A vessel somewhat similar to that used by van Everdingen[1] was finally adopted. A cross-section of this vessel together with a cross-section of the pole-pieces is shown in Fig. 2. It was a double-walled paper vessel with wooden top and bottom. Heavy drawing paper was used for the walls and the joints were rendered liquid-tight by means of glue. The horizontal section of that part of the vessel which was not between poles of the magnet was an ellipse, the major axis of which was 10.5 cm. and the minor axis, 7.0 cm. The part between the poles of the magnet was about 2.5 cm. thick, 10.5 cm. long and 7.0 cm. wide. The outer wall of the vessel had no bottom and when the top of the vessel had been completely closed by packing with cotton, the cold vapors from the air in the

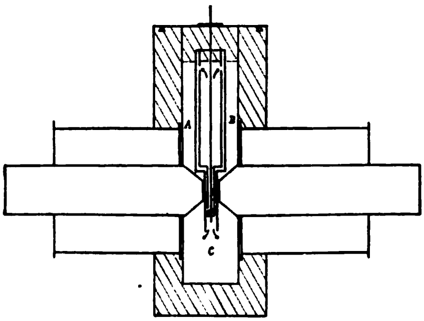


Fig. 2.

---

[1] Roy. Acad., Amsterdam, Proc. (3), 177–195, 1900.

vessel passed out through the openings at the top of the inner wall, down through the annular space between the walls and then out into the space $ABC$ (Fig. 2), which was enclosed in a wooden box surrounded with cotton to make the insulation as good as possible. From the space $ABC$ the cold vapor could come into the room. The circulation of cold vapor prevented to a large measure the evaporation of the air in the vessel, so that it was only necessary to refill with liquid air about once in forty-five minutes. A float served to indicate the height of the liquid air. At its top the vessel was fastened to a heavy brass plate which moved along suitable ways so that the vessel could be carried out of the magnetic field and the flip-coil which was mounted on the same brass plate could be carried into position for measuring the magnetic fields. The relative dimensions of the vessel and the magnet are indicated in Fig. 2, which has been drawn to scale. Before the mouth of the vessel was packed with cotton, the plate-holder which carried the plate to be studied was lowered into the liquid air and suitably clamped to the plate which carried the vessel. The proper distance to which the plate was to be lowered in order to bring it in the center of the magnetic field was determined before the vessel was placed between the poles of the magnet.

The temperature of the liquid air was not determined but was taken roughly as — 190° C. There seemed to be little point in knowing it with greater accuracy. The other temperatures were determined with a mercury-in-glass thermometer in the neighborhood of the plate.

### *Purity of the Metals.*

The values of the Hall effect are known to be to a large degree a function of the purity of the metals, so that it is desirable to use in these experiments, metals which are as pure as possible. The platinum, silver and palladium were obtained from Kahlbaum. The copper, zinc and gold came from Eimer and Amend and were marked "chemically pure." The aluminum was of unknown origin and concerning its purity nothing can be said.

### *Results.*

The results of these observations are plotted in Figs. 3 to 9 inclusive. In these figures the magnetic field in absolute units is plotted for abscissæ and the Hall electromotive force in absolute units, for ordinates. It is seen from these figures that in the metals studied in this paper the Hall electromotive force is proportional to the magnetic field at the temperature of liquid air as well as at room temperatures. Previous observers have shown that in the neighborhood of room temperature the relation between *E*, the Hall

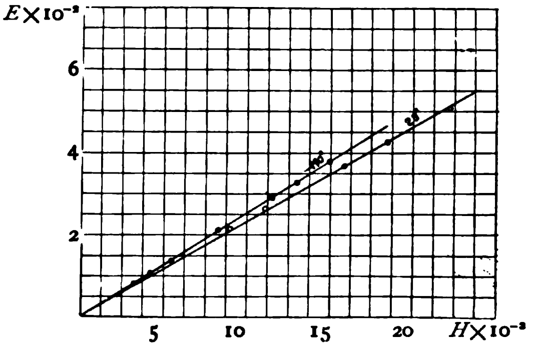


Fig. 3. Platinum.

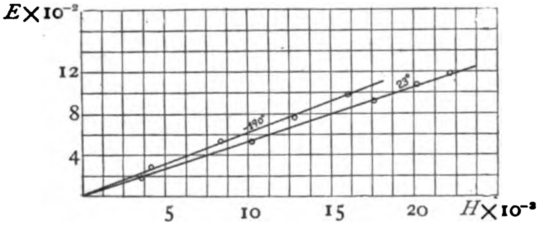


Fig. 4. Copper.

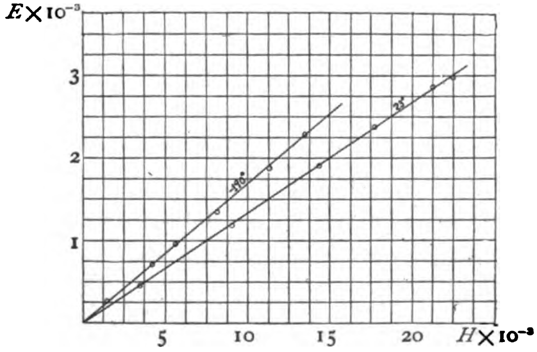


Fig. 5. Palladium.

electromotive force, $i$ the current in the plate, $H$ the magnetic field and $d$ the thickness of the plate is expressed by the equation, $E = R \cdot Hi/d$, where $R$ is a factor of proportionality which is independent of the electromotive force, the magnetic field, the current and the thickness of the plate. By means of this equation $R$ has been calculated for the plates at room temperature and at the temperature of liquid air. In these calculations the current, the electromotive force and the magnetic field have been expressed in absolute units and the thickness of the plate in centimeters. The values of $R$ thus obtained are given in Table I. The (+) sign in the fifth

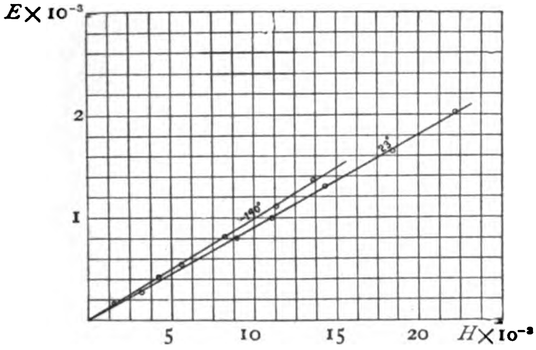


Fig. 6. Silver.

TABLE I.

| Metal. | Thickness, cm. × 10⁴. | Current, c.g.s. | Temp., ° C. | $R \times 10^4$ | $\Delta R/\Delta t \times 10^7$ |
|---|---|---|---|---|---|
| Platinum. | 9.3 | .104 | −190 | − 2.22 | − 0.9 |
| | | | 23 | − 2.02 | |
| Copper. | 27.1 | .265 | −190 | − 6.5 | − 5.1 |
| | | | 23 | − 5.4 | |
| Palladium. | 6.65 | .104 | −190 | −10.9 | −11.0 |
| | | .103 | 23 | − 8.6 | |
| Silver. | 9.5 | .102 | −190 | − 9.2 | − 4 |
| | | | 23 | − 8.4 | |
| Gold. | 37.7 | .265 | −190 | − 7.25 | − 0.9 |
| | | | 25 | − 7.04 | |
| Zinc. | 28.7 | .265 | −190 | +10.9 | −20 |
| | | | 25 | + 6.3 | |
| Aluminum. | 23.0 | .265 | −190 | − 1.92 | + 6 |
| | | | 25 | − 3.20 | |

column means that the effect has the direction of the effect in iron and the (—) that it has the direction of the effect in bismuth. In the last column of this table is given the average change in $R$ per degree between the temperature of liquid air and room temperature. The (+) in this column means that the magnitude of $R$ increases with rising temperature and the (—) that it decreases with rising temperature. In silver, gold, platinum, palladium, zinc and copper the effect increases as the temperature is decreased from room temperature to that of liquid air. This increase is greatest in zinc where it amounts to about 70 per cent. of the value at room temperature;

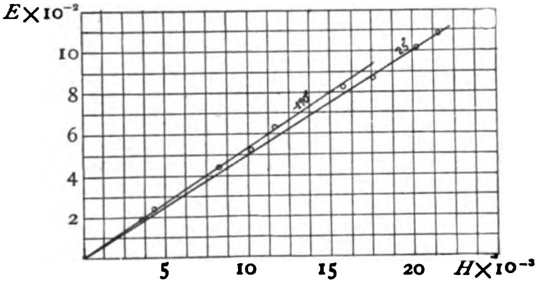


Fig. 7. Gold.

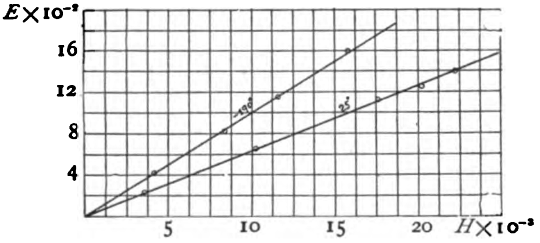


Fig. 8. Zinc.

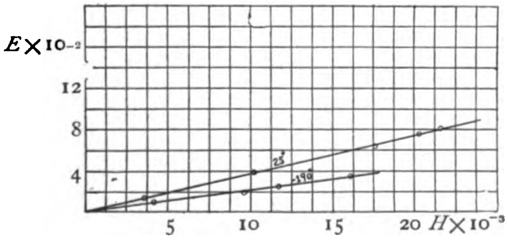


Fig. 9. Aluminum.

and least in gold, where it is only about 3 per cent. of the value at room temperature. In the specimen of aluminum studied in this paper a lowering of the temperature causes a decrease in the Hall effect. This is the only one of the non-magnetic metals in which such a decrease was found. How much influence the impurities of the aluminum had on this result it is impossible to say.

For purposes of comparison some of the values of $R$ obtained by previous observers are given in Table II. It will be seen that in the purer and better defined metals they agree well with the values obtained in this work.

Table II.

| **Metal.** | $R \times 10^4$ | **Observer.** |
|---|---|---|
| Gold | $-6.6$ | Hall |
| | $-7.1$ | Ettingshausen and Nernst |
| | $-7.04$ | Smith |
| Copper | $-5.4$ | Zahn |
| | $-5.2$ | Ettingshausen and Nernst |
| | $-5.4$ | Smith |
| Platinum | $-1.27$ | Zahn |
| | $-2.4$ | Ettingshausen and Nernst |
| | $-2.02$ | Smith |
| Silver | $-8.97$ | Zahn |
| | $-8.3$ | Ettingshausen and Nernst |
| | $-8.4$ | Smith |

## *Silicon.*

The Hall effect has been studied by Miss Wick[1] in two specimens of silicon. One of these specimens contained 95 per cent. of silicon; the other contained 99 per cent. The direction of the effect in each of these specimens was the same as the direction of the effect in bismuth. The magnitude of the effect was greater in the specimen which contained 99 per cent. of silicon than in the specimen which contained 95 per cent. She also found that the prediction of Beattie[2] concerning the relation between the thermo-electromotive forces and the rotary coefficients of the Hall effect was verified for silicon. Beattie pointed out that metals and alloys which occupy extreme positions in the thermo-electric series will occupy similar

---

[1] Phys. Rev. (27), p. 76, 1908.

[2] Roy. Soc. Edinb. Proc. (20), p. 481, and (21) p. 146.

positions in a series of rotary coefficients. Miss Wick found that silicon occupied in the thermo-electric series a position above that of bismuth and that in a series of rotary coefficients, the rotary coefficient of silicon also stood above that of bismuth. Mr. C. I. Zimmermann who was working in the physical laboratory of this university on the electric properties of silicon found that in most of the specimens which he studied, the current flowed from copper to silicon across the hot junction of a copper-silicon couple. As these specimens which were known to be very pure, gave thermo-electromotive forces in the direction opposite to that given by Miss Wick, it seemed of interest to examine the Hall effect in some of them in order to see whether a reversal of the thermo-electromotive force is accompanied by a reversal of the Hall effect. Since Miss Wick did not determine the relation between the Hall electromotive force and the magnetic field, it seemed worth while to determine this relation.

By means of a wire-saw fed with carborundum powder the plates were cut from irregular pieces of silicon which had crystallized from a reservoir of molten silicon. With a rotating disc on which carborundum powder was used for an abrasive, the plates were ground down to the desired thickness. The contacts for the wires which lead from the plate to the battery were made by copper-plating the ends of the plate for a width of .4 cm. and then soldering the copper lead wires to these regions. The contacts for the wires which lead to the galvanometer were made in a similar way by copper-plating a small region in the middle of the edge of the plate and then soldering fine lead wires to these copper-plated regions, which had been filed down to make them as small as possible.

Five plates were used in these experiments. They were cut from three different specimens of silicon. These specimens will be referred to as specimens *A*, *B* and *C*. Analyses of these specimens were kindly made by Mr. F. J. Tone, of the Carborundum Company, of Niagara Falls, for which kindness the author is very grateful. The analyses of the specimens gave the following results.

| | SiC | Fe | Al | Si |
|---|---|---|---|---|
| Specimen *A* | Trace | .41% | .17% | 99.4% |
| Specimen *B* | Trace | .32 | .14 | 99.5 |
| Specimen *C* | .85 | 1.89 | .73 | 96.5 |

Plates I., II. and III. were made from specimen *A*; Plate IV. from *B*; and Plate V. from *C*. Plates I. and II. were from a slice of silicon which was cut from that part of specimen *A* which had been first to crystallize. A second slice parallel to the first was also cut from specimen *A* and from it was made Plate III. Plates I., II., III. and IV. were very homogeneous and free from perforations of any sort. Plate V. contained two small pin-holes but since they were covered by the copper-plating, they could have little influence on the distribution of the current in the plate. This plate was too small to allow a satisfactory determination of the Hall effect and was used only to get the order of magnitude and the direction of the effect.

Table III. contains the values of *R* for Plates I., II., III. and IV. In Fig. 10 have been plotted the curves showing the relation between the Hall electromotive force and the magnetic field. The dotted curve is for Plate I. and the continuous curve for Plate IV. Similar curves were obtained for Plates II. and III. but since they differed in no way from the curves for Plates I. and IV. they have not been included in this paper. The direction of the Hall effect in

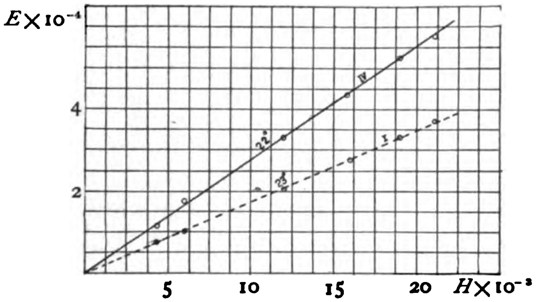


Fig. 10. Silicon.

TABLE III.

*Silicon.*

| Plate. | Thickness, cm. | Current, c.g.s. | Temperature, C. | *R* |
|---|---|---|---|---|
| I. | .244 | .0102 | 23° | 41.0 |
| II. | .179 | .0103 | 22° | 48.7 |
| III. | .189 | .0102 | 22° | 59.8 |
| IV. | .153 | .0102 | 22° | 40.0 |

each of these plates was opposite to the direction of the effect in bismuth. Each of these plates was found to have a thermo-electromotive force which made the current flow from copper to silicon across the hot junction of the copper-silicon couple. In Plate V. the Hall electromotive force was found to have the same direction as that electromotive force in bismuth. The rotary coefficient could not be accurately determined in this plate but its order of magnitude was about 20, where all of the quantities involved in it are measured in the same units in which they were measured in the other plates. The thermo-electromotive force in this plate had the opposite direction to that in Plates I., II., III. and IV. The results for Plate V. agree completely with the results obtained by Miss Wick both with regard to the direction of the Hall effect and with regard to the direction of the thermo-electromotive force. On the other hand in the other plates both the Hall electromotive forces and the thermo-electromotive forces are in directions opposite to their respective directions in Plate V. It seems, therefore, that in some specimens of silicon the Hall effect has the direction of the effect in bismuth and in other specimens it has the direction of the effect in iron. In some specimens the current flows from copper to silicon across the hot junction of a copper-silicon couple and in other specimens the current flows in the opposite direction. It is worthy of note that the direction of the Hall effect in the purer specimens is the direction of the effect in iron. From the study made in this paper it is impossible to say that the reversal of the effect is due to the presence of impurity in the silicon. Whatever be the cause of the reversal it appears that a reversal of the thermo-electromotive force is accompanied by a reversal of the Hall effect.

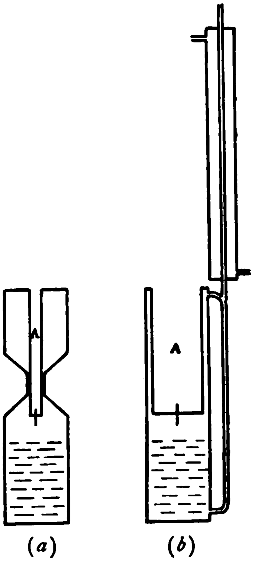
From Fig. 10 where the curves showing the relation between the Hall electromotive forces and the magnetic fields are plotted it will be seen that within the range of these observations the Hall electromotive force is proportional to the magnetic field.

## PART II. MAGNETIC METALS.

### *Method and Apparatus.*

The method of observation in the work at room temperature and at the temperature of liquid air was precisely the same as the method employed in Part I. of this paper on non-magnetic metals. For temperatures above room temperature some additional apparatus had to be constructed to secure the desired temperatures and to overcome experimental difficulties.

In order to obtain the three temperatures 100°, 184° and 300° C. a vapor-bath of the form indicated in Fig. 11*a* and 11*b* was constructed. Fig. 11*a* shows a section perpendicular to the axis of the pole-pieces and Fig. 11*b* a section by a plane determined by the axis of the pole-pieces and the vertical. The vessel was made of brass and the joints were brazed. It was provided with the ordinary form of metal condenser. The space *A* was 18 cm. long, 7 cm. wide, and 2 cm. thick at the top and 1 cm. thick at the bottom where it was between the pole-pieces. Into this space the plate-holder could be lowered and fastened precisely as in the liquid air vessel. The mouth of the space *A* was packed with asbestos to prevent the circulation of the air. The liquids used in the bath were water, aniline and diphenylamine boiling under atmospheric pressure.



(*a*) (*b*)

Fig. 11.

For the realization of temperatures between 300° and 900° C. an electric furnace was constructed. The heating coil was made of nichrome wire wound on a copper box, 18 cm. long, 9 cm. wide and 1 cm. thick. Before winding with wire the box was covered with mica for purposes of insulation. After the heating coil had been wound on the box it was covered with a paste made of water-glass and magnesium oxide. The copper box and the heating coil were placed inside of a brass-jacket which was 25 cm. long, 12 cm. wide and 2.5 cm. thick. The space between the heating coil and the jacket was filled with infusorial earth to prevent as far as possible the loss of heat. The parts of the walls of the jacket which were opposite the pole-pieces were made of mica to reduce the flow of heat into the pole-pieces with the resulting cooling of that part of the furnace between the pole-pieces.

Since neither the copper box nor the nichrome wire would stand temperatures above 900° C. the electric furnace had to be somewhat modified for the temperatures between 900° and 1100° C. A box 20 cm. long, 9 cm. wide and 1 cm. thick was built up out of plates made by baking magnesium oxide mixed with a small amount of aluminum oxide in a graphite mould which was heated in an electric furnace to about 1600° C. The thickness of these plates was about 2 mm. On the outside of this box platinum foil, which served as the heating coil, was wound. The box with the heating coil was then placed in a brass box, 25 cm. long, 12 cm. wide and 3.5 cm. thick. The space between the brass box and the heating coil was filled with infusorial earth which gave satisfactory insulation.

For temperatures between 100° and 400° C. the connections for the lead wires from the battery to the plate and those from the galvanometer to the plate were made by means of copper clamps to which were hard-soldered the lead wires. These clamps were mounted at the desired distances on a sheet of brass from which they were insulated by a sheet of mica. After the plate had been put in position and clamped between the electrodes, the plate and electrodes were covered with sheets of mica and all held in position by a thin sheet of brass fastened by means of screws to the brass plate carrying the electrodes. This clamping of the plates between sheets of mica prevented them from being pulled out of position by the magnetic field.

At temperatures above 400° C. provision had to be made against the oxidation of the plate. During the work at temperatures less than 900° the plates and electrodes were mounted inside of a thin copper box which was nearly air-tight. A photograph of that part of the box which contained the plate and the electrodes is shown in Fig. 12. The plate and the electrodes were insulated from the box by sheets of mica, and the lead wires by small magnesia tubing. Copper lead wires were used with this box. For temperatures above 900° C. a nickel box and nickel electrodes were substituted for the copper box and electrodes and the copper lead wires were replaced by platinum wires. In other respects the arrangement was precisely the same as with the copper box. Sheets of mica were placed underneath and on top of the plate in such a way that after the two halves of the box had been fastened together with screws, the magnetic field could not produce a displacement of the plate. The interior of this box was connected by means of a tube of small bore to a hydrogen generator. The hydrogen which had been generated in the usual way was allowed to flow through the space containing the plate and the electrodes for some time before heating and for the entire time the plates were above 300° C. To remove the oxygen and any water vapor from the hydrogen it was made to flow through sulphuric acid, over heated copper oxide and through phosphorous pentoxide before entering the box. By this method it was possible to prevent almost entirely the oxidation of the plates.

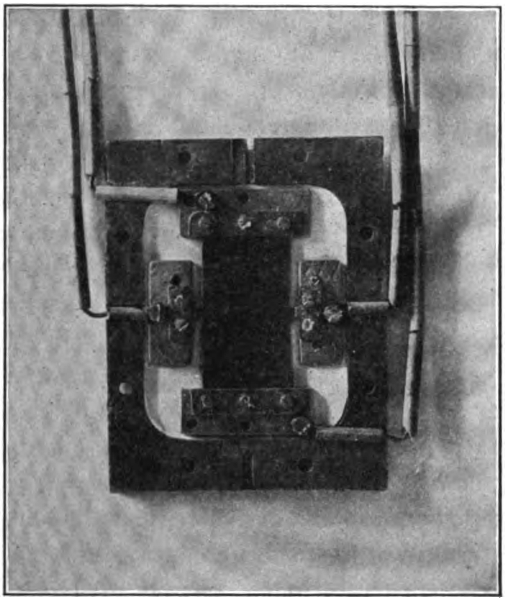


Fig. 12.

### *Measurement of Temperatures.*

The temperatures up to and including 300° C. were determined with the mercury-in-glass thermometer referred to in Part I. of this paper and those above 300° were determined by means of a platinum-platinum-iridium thermal couple which had been calibrated over the interval of temperature used in these experiments. The temperatures used for this calibration were the boiling point of water, the boiling point of sulphur and the melting point of gold.

### *Purity of the Metals.*

The cobalt, nickel and iron were obtained from Kahlbaum. The electrolytic iron was kindly furnished by Professor Watts, of the electrochemical department of the University of Wisconsin. It was prepared according to the method of Burgess and Hambuechen[1] and was known to be very pure. After having been deposited it had been annealed by heating in a hydrogen bath to about 1200° C. For purposes of comparison a plate of nickel furnished by H. J. Slaker, of the United States Mint, was also examined.

In the work at the temperature of liquid air it was necessary to use thinner plates than had been used at ordinary temperatures, because the effect becomes so small in liquid air that it can no longer be observed with accuracy in any except very thin plates. Certain accidents to the plates in the course of the experiments made it necessary to work on more than one plate of the same metal. At some of the high temperatures it was also desirable to make the observations on thinner plates than had been used at lower temperatures in order to overcome some of the error due to thermo-electromotive forces in the circuit. All of the plates for any particular metal were cut from the same sheet of metal which had been treated in precisely the same way and was originally not more than 8 or 10 cm. square. The purity of the plates for any particular metal must have been nearly the same. All of the work on the electrolytic iron was done on the same plate but in some preliminary experiments before satisfactory means for preventing oxidation had been devised, it became somewhat oxidized and in removing this layer of oxide the thickness was reduced from .0148 cm. to .0133 cm.

### *Results.*

The results for the magnetic metals have been plotted in Figs. 13–19 inclusive. Each value plotted in these curves is the mean of at least four observations taken in the way indicated in Part I.

---

[1] Iron and Steel Mag. (8), p. 48, 1904.

In these figures the Hall electromotive forces in absolute units have been plotted for ordinates and the magnetic fields for abscissæ. The current in the plate for which all of these curves have been plotted was 1.04 amperes. Where the current in the plate when the observations were made differed slightly from this value, suitable correction was made on the basis that the Hall electromotive force is proportional to the current.

In plotting these curves showing the relation between the Hall electromotive force and the magnetic field, it was necessary to take account of the fact that, for reasons given above, all of the observations for a particular metal were not made on the same plate but on plates of different thicknesses. Previous observers have shown that the Hall electromotive force for a particular magnetic field and for a given current is inversely proportional to the thickness of the plate. If the Hall electromotive force has been observed in a plate of known thickness for a given magnetic field and current, it is possible by means of this relation to calculate the electromotive force which would have been observed in any other plate of the same metal for the same magnetic field and current. In this way the electromotive forces where necessary have been calculated so that the values plotted in Fig. 13 are for a plate of nickel .0185 cm. thick; in Fig. 14 and Fig. 15 for a plate of Kahlbaum iron .0089 cm. thick; in Fig. 16 and Fig. 17 for a plate of electrolytic iron .0148 cm. thick; and in Fig. 18 and Fig. 19 for a plate of cobalt .0213 cm. thick.

### *Nickel.*

From Fig. 13 where the curves showing the relation between the Hall electromotive force and the magnetic fields are plotted for Kahlbaum nickel for various temperatures, it will be seen that for temperatures not greater than 385° C. the electromotive force is at first proportional to the magnetic field and that when the field is increased beyond a certain value which is a function of the temperature, a sort of saturation occurs after which a further increase in the field produces a small but perfectly definite increase in the Hall electromotive force. As the temperature is raised from — 190° to 385° C., the magnitude of the field necessary to produce saturation becomes less and less. At the former temperature it requires about

8,000 c.g.s. units for saturation; at the latter about 1,000 c.g.s. units. For all temperatures not greater than 355° C. the slopes of the curves beyond saturation is nearly the same, although their slopes before saturation are quite different. The magnitude of the effect at the temperature of liquid air before saturation is about one twenty-third of its magnitude under the same conditions at 300° C. and the magnitude at 410° is about one ninth of that at 300° C. For fields of equal intensity the Hall electromotive force at 355° is greater than that at 300° until the field has the value of approximately 3,750 c.g.s. units but for fields greater than 3,750 units the effect is greater at 300° than at 355° C. For fields above 2,000 c.g.s. units the effect is much smaller at 385° than at 300° C. At the former temperature when the magnetic field exceeds 2,000 c.g.s. units, that part of the curve showing the relation between the Hall electromotive force and the magnetic field is no longer nearly horizontal but has been rotated through a certain angle. This rotation is seen more clearly in the curve for 395° C. When the temperature has been raised to 410° all sign of change of slope in the curves

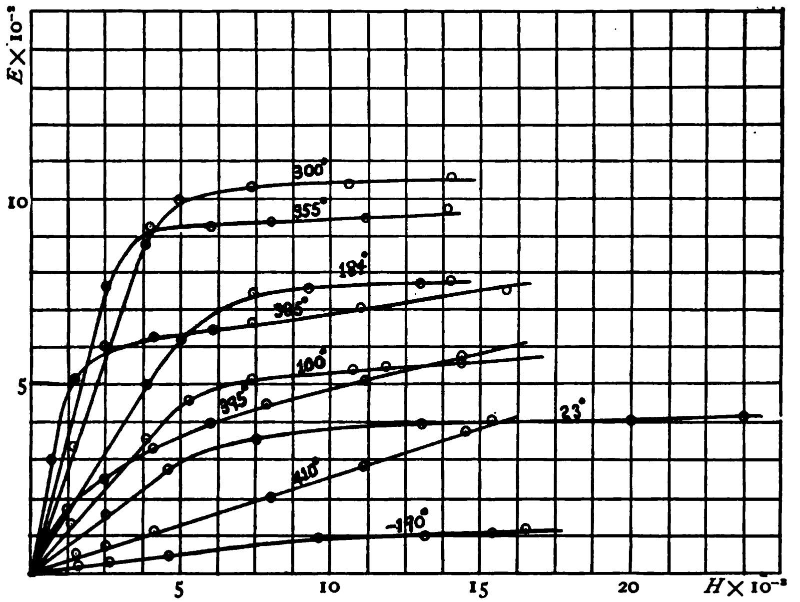


Fig. 13. Nickel.

has disappeared and the effect is proportional to the magnetic field for all fields used in these experiments. When the critical temperature has been passed, a further increase of temperature causes a decrease in the magnitude of the effect. At these temperatures nickel behaves like non-magnetic metals.

The results for the nickel from the United States Mint agreed except for small differences in magnitude with the results for the Kahlbaum nickel. Since these results brought out no new points, it has not seemed worth while to include them in this paper.

### *Kahlbaum Iron.*

In Figs. 14 and 15 are plotted for the Kahlbaum iron the curves analogous to those plotted in Fig. 13 for nickel. Because the effect in iron varies between such wide limits for the temperatures used in these experiments it was necessary to plot the curves on two different scales. Fig. 14 contains the curves for temperatures between — 190° and 300° inclusive and Fig. 15 the curves between 300° and 900° on a different scale. The value of the effect at — 190° is only about one four-hundredth of the value at 735° and the value at 900° is approximately one thirty-fifth of the value at 735° C. The fields

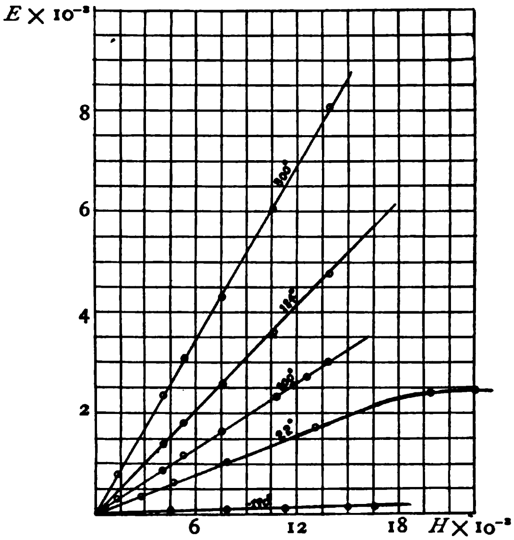


Fig. 14. Kahlbaum iron.

required for saturation are so large that at temperatures less than 545° C. it was only possible to reach saturation at room temperature, where the distance between the pole-pieces being small, the intensity of the magnetic field could be made as much as 22,000 c.g.s. units. When the temperature had been raised to 545° C., the point at which saturation occurred had shifted toward lower fields, so that it appeared in the neighborhood of fields with an intensity of about 13,000 c.g.s. units. Further increase of temperature caused saturation to shift still more toward lower fields and at 735° C. it occurred when the field was about 10,000 c.g.s. units. The shift between 735° and 796° C. is small and nearly within the error of observation. In another plate of Kahlbaum iron this shift for the corresponding range of temperature was somewhat more certain, so that there can be no doubt of a shift toward lower fields for this interval of temperature. It is impossible to say that the shift between 22° and 545° C. is continuous, because the fields necessary for saturation at intermediate temperatures could not be realized under the conditions of this experiment. From later considerations it will appear probable that there

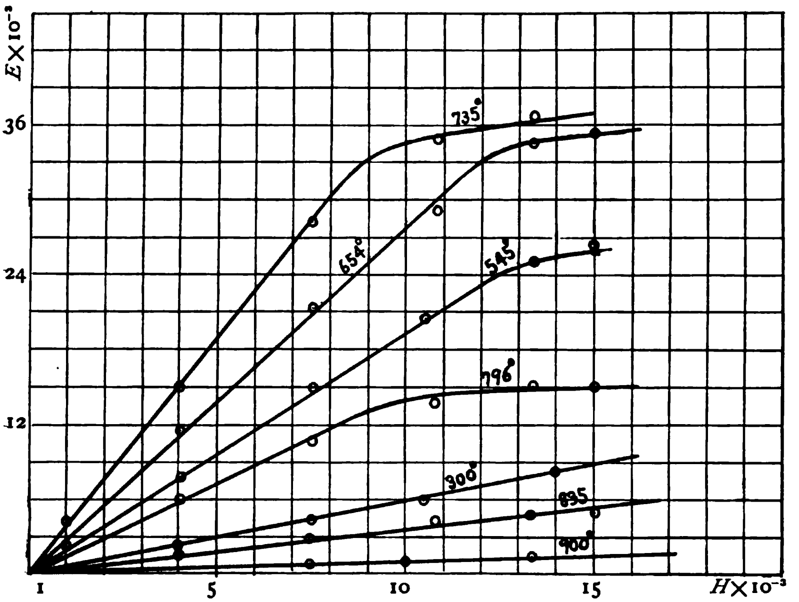


Fig. 15. Kahlbaum iron.

is a continuous shift between 22° and 796° C. For temperatures somewhat above 800° the Hall electromotive force is proportional to the magnetic field, for the range of fields used in these experiments. Just as in the case of nickel the temperature coefficient of the effect reverses sign when the critical temperature is crossed. The direction of the effect on the two sides of the critical temperature is the same and opposite to the direction of the effect in bismuth. At 1000° C., the highest temperature at which the iron was studied, the effect had become so small and the thermo-electromotive forces so large that it was impossible to determine the magnitude of the effect with any certainty but its direction was the direction of the effect at the other temperatures. There is no reversal of the direction of the effect of iron between — 190° and 1000° C.

### *Electrolytic Iron.*

The curves in Fig. 16 and Fig. 17 for electrolytic iron have the same form as the corresponding curves for Kahlbaum iron. No observations were made on this plate at the temperature of liquid air. The shifting of the bends in these curves is in every way

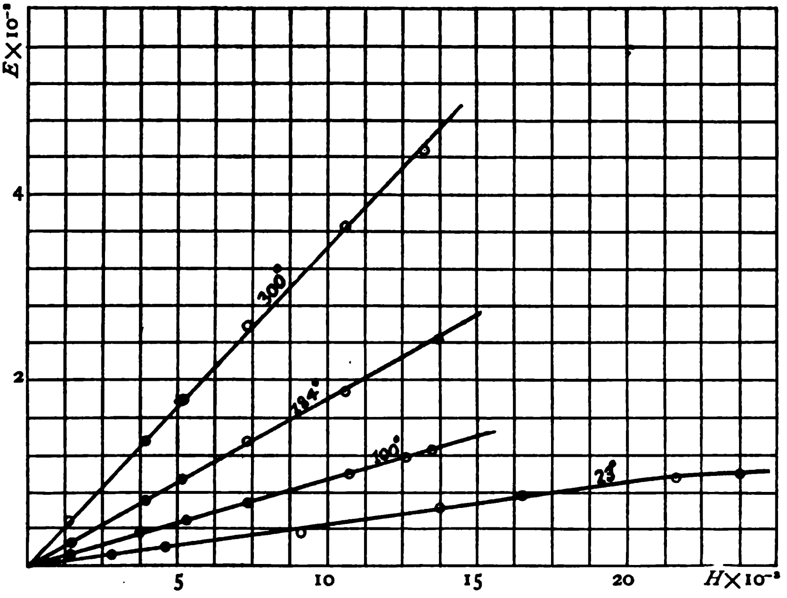


Fig. 16. Electrolytic iron.

similar to the shifting of the bends in the curves for the Kahlbaum iron. In addition to confirming the results on the Kahlbaum iron, these data show that the magnitude of the effect at room temperature is much smaller in electrolytic iron than in Kahlbaum iron.

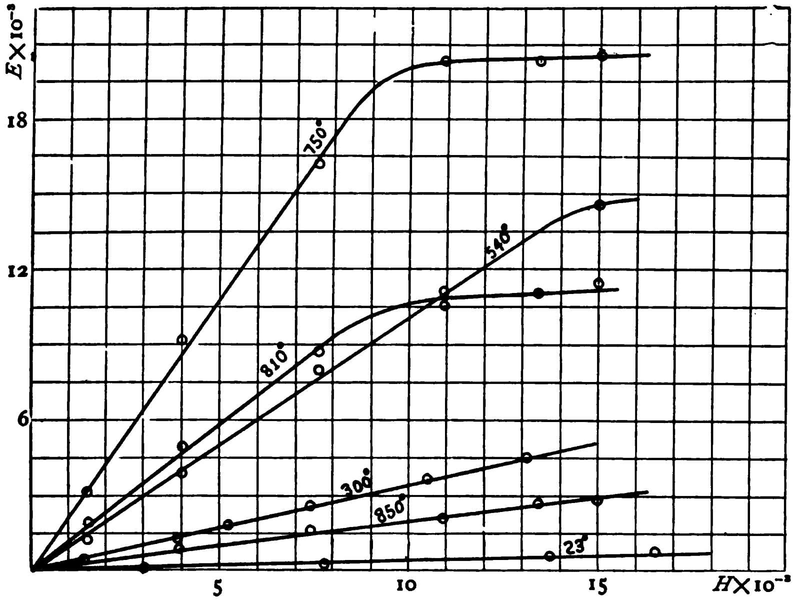


Fig. 17. Electrolytic iron.

On the other hand, the rate at which the effect increases with rising temperature between 23° and 780° C. is greater in the electrolytic iron than in the Kahlbaum iron, so that at the critical temperature the effect in the former is nearly equal to the effect in the latter. Since the electrolytic iron was much purer than the Kahlbaum iron, these results suggest that the purest iron has the least Hall effect at ordinary temperatures and the greatest temperature coefficient of the effect.

### *Cobalt.*

The curves for cobalt in Figs. 18 and 19 do not differ essentially in form from those for iron and nickel in the preceding figures. For fields less than those necessary to produce saturation the magnitude of the effect at — 190° is not more than three fourths of one per cent. of its magnitude at 960° and the magnitude at 1,100° is about one fourth of the magnitude at 960° C. For temperatures above the critical temperature of cobalt it was not possible to get satisfactory curves showing the relation between the Hall electromotive forces and the magnetic fields. The thermo-electromotive forces become so troublesome at these temperatures and the variation of the effect with the temperature is so rapid that accurate observations were quite impossible. Moreover, the highest fields to be obtained for temperatures above 900° C. only amounted to 9,300

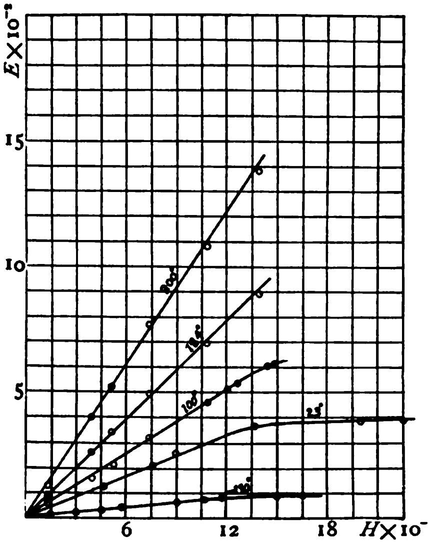


Fig. 18. Cobalt.

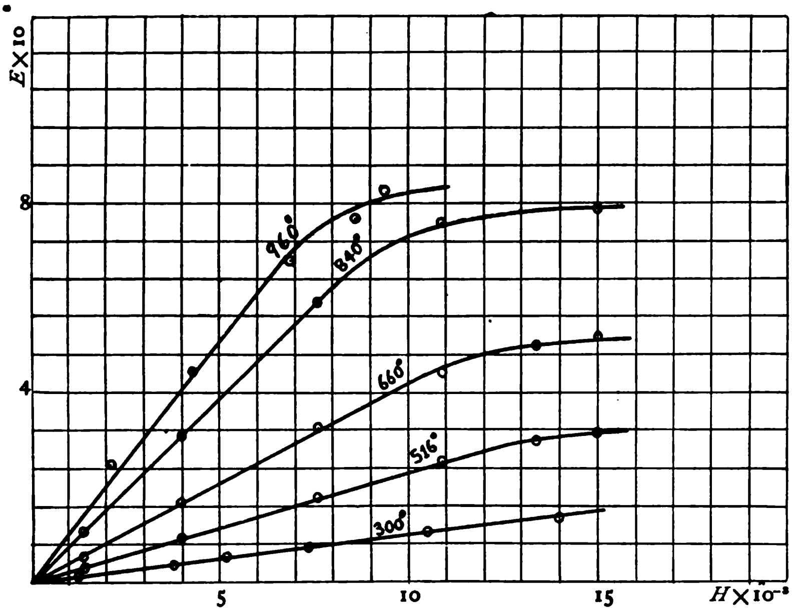


Fig. 19. Cobalt.

c.g.s. units, because the pole-pieces had to be 3.5 cm. apart in order to allow the introduction of the electric furnace at these temperatures. The range of magnetic fields was, therefore, much less than it had been in the other experiments. It was, however, possible to obtain two approximate values, one at 1030° and the other at 1100° C. The magnetic field had an intensity of 8,600 c.g.s. units. These values have been plotted in Fig. 23. Although they are approximate, large errors in them would not in any important way affect the character of the curve in that figure. At either of these temperatures the Hall electromotive force seemed to be proportional to the magnetic field but the range of fields was so small and the observations so inexact that not much importance could be attached to these results. It is to be noted that saturation in cobalt occurs at fields which are more intense than those necessary for saturation in nickel and less intense than those for saturation in iron. It will appear later that there is a relation between the maximum value of the intensity of magnetization and the fields necessary for saturation, and from this relation the fact noted above will find explanation. As the temperature of the cobalt is raised from — 190° to 300° C., the saturation points shift toward higher fields. A further increase of temperature causes the saturation points to shift back to lower fields. The behavior of cobalt in this respect is to be contrasted with that of iron and nickel in which an increase of temperature seems to cause a continuous shift of the saturation points toward lower fields. The explanation of this difference in behavior will be presented in a later paragraph. The curves for cobalt like those for iron and nickel become nearly horizontal after saturation has been reached.

According to Kundt[1] the Hall electromotive force in the magnetic metals is proportional to the intensity of magnetization and not to the magnetic field. Saturation would occur in the curves plotted in this paper, when the intensity of magnetization had reached its maximum value. The work of Honda and Schimizu[2] has shown that in the case of iron and nickel, the maximum value of the intensity of magnetization is greatest at the temperature of liquid air and with

[1] Wied. Ann. (49), pp. 257–271, 1893.

[2] Phil. Mag. (6), [10], pp. 548 and 642, 1905.

rising temperature decreases to almost zero at a temperature somewhat above the critical temperature. The rate of decrease is a function of the temperature and is largest for temperatures which are in the neighborhood of the critical temperature. In the case of cobalt, however, the variation of the maximum value of the intensity of magnetization with the temperature is quite different from what it is in iron and nickel. From its value at — 186° C. the maximum value of the intensity of magnetization in cobalt increases with rise of temperatue to a maximum at 300° C., then decreases with further rise of temperature in a way similar to that in which it is found to behave in nickel and iron.

If the Hall electromotive force is proportional to the intensity of magnetization, the bends in the curves should occur when the metals are saturated. If $4\pi$ is taken as the demagnetizing factor for a thin plate, $H'$, the magnetic field inside of the plate for a particular temperature is given by the equation $H' = H - 4\pi I$, where $H$ is the field in the air-gap before the plate was introduced and $I$ the intensity of magnetization in the plate. If by increasing $H$, $H'$ is increased in such a value that $I$ becomes a maximum, any further increase in $H$ will not increase the Hall electromotive force. Let $I_m$ denote the value of the intensity of magnetization when the metal is saturated. Since $H'$ is small in comparison with $H$ and $4\pi I_m$, at saturation $4\pi I_m = H$ approximately. The values of $I_m$ were not known for the plates used in these experiments but taking the values from the data of Honda and Schimizu it is found that the bends in the curves occur roughly, when $H = 4\pi I_m$. The fact that $I_m$ is greatest for iron and least for nickel, offers the explanation for the fact that the fields necessary for saturation are greatest in iron and least in nickel. The fact that with increasing fields the Hall electromotive force continues to increase after the plate is in condition here referred to as saturation, may be due to the fact that the metal never becomes completely saturated or it may be that the Hall electromotive force is not exactly proportional to the intensity of magnetization.

Since according to the data of Honda and Schimizu the maximum value of the intensity of magnetization decreases from liquid air to the critical temperature in the case of iron and nickel, the bends in these curves with rising temperature should shift continuously toward lower fields, for $4\pi I_m$ is less for high than for low temperatures. It has been seen that this is precisely what happens in the case of nickel. In the case of iron it was seen that there is also a shift toward lower fields with rising temperature. For the reasons given above it was not possible to determine whether the bends shift continuously between 22° and 545° toward lower fields. Such a shift is, however, very probable. It has already been pointed out that the bends in the curves for cobalt instead of shifting continuously toward lower fields with rising temperature, shift toward higher fields as the temperature rises from — 190° to 300° and then shift toward lower fields with further rise of temperature. It will be recalled that the maximum value of the intensity of magnetization was found to increase up to 300° C. and then to decrease. It seems that an increase in the maximum value of the intensity of magnetization causes an increase in the field necessary to produce saturation and that a decrease in the former causes a decrease in the latter. The temperature at which the bend in cobalt occurs at the highest field is about 300° C. which is nearly the temperature at which the maximum value of the intensity of magnetization has its largest value.

### *Rotary Coefficients.*

It has been shown that in the non-magnetic metals the relation between $E$, the Hall electromotive force; $i$, the current in the plate; $H$, the magnetic field; and $d$, the thickness of the plate, is expressed by the equation $E = R \cdot Hi/d$ where $R$ is a factor of proportionality independent of the current, the thickness of the plate, the magnetic field and the electromotive force. This equation holds for the magnetic metals, until the field has reached the value at which saturation begins to appear. For fields greater than these the equation no longer holds, although the curves after saturation seem to be straight lines with slopes very different from the slopes before saturation.

The values of $R$ calculated from this equation are given in Table IV. for Kahlbaum nickel; in Table V. for Kahlbaum iron; in Table VI. for electrolytic iron; and in Table VII. for cobalt. The units in which current, electromotive force, magnetic field and thickness of the plate have been expressed are those used for the corre-

## Table IV.

*Nickel.*

| Thickness, cm. × 10³. | Current, c.g.s. | Temperature, C. | $R \times 10^3$ | $\Delta R/\Delta t \times 10^5$ |
|---|---|---|---|---|
| 2.87 | .108 | − 190° | 1.85 | + 5 |
| 18.5 | .103 | 23° | 12.7 | 5 |
| 18.5 | .104 | 100° | 16.3 | 8 |
| 2.87 | .103 | 184° | 23.4 | 15 |
| 2.59 | .104 | 300° | 40.8 | 28 |
| 2.59 | .104 | 355° | 56.0 | — |
| 2.59 | .103 | 385° | 58.9 | − 400 |
| 2.59 | .104 | 395° | 19.2 | − 100 |
| 2.59 | .104 | 410° | 4.5 | − 3 |
| 2.59 | .104 | 546° | 0.7 | |

sponding quantities in Part I. of this paper. The last column of each of these tables contains the average change in $R$ per degree over the successive intervals of temperature, which are given in the third column of the corresponding table. The first column gives the thickness of the plate and the second column the current in the plate.

## Table V.

*Kahlbaum Iron.*

| Thickness, cm. × 10³. | Current, c.g.s. | Temperature, C. | $R \times 10^3$ | $\Delta R/\Delta t \times 10^4$ |
|---|---|---|---|---|
| 3.32 | .103 | − 190° | 0.75 | + 0.5 |
| 8.9 | .105 | 22° | 11.0 | 1.0 |
| 8.9 | .104 | 100° | 18.9 | 1.2 |
| 3.32 | .104 | 184° | 29.2 | 1.8 |
| 3.32 | .105 | 300° | 50.4 | 4.4 |
| 9.04 | .105 | 545° | 167 | 7.0 |
| 9.04 | .105 | 654° | 243 | 9.3 |
| 9.04 | .105 | 735° | 319 | − 60 |
| 9.04 | .105 | 796° | 120 | − 23 |
| 9.04 | .105 | 835° | 32.8 | − 3.7 |
| 9.04 | .105 | 900° | 9.0 | |

The relation between $R$, this factor of proportionality, and $T$, the absolute temperature of the plate, is brought out in Fig. 20 for Kahlbaum nickel; in Fig. 21 for Kahlbaum iron; in Fig. 22 for electrolytic iron; and in Fig. 23 for cobalt. In these figures the absolute temperatures of the plate are plotted for abscissæ and the

## Table VI.

*Electrolytic Iron.*

| Thickness, cm. $\times 10^2$. | Current, c.g.s. | Temperature, C. | $R \times 10^3$ | $\Delta R/\Delta t \times 10^4$ |
|---|---|---|---|---|
| 1.48 | .105 | 23° | 6.6 | + 0.9 |
| 1.48 | .103 | 100 | 13.5 | 1.3 |
| 1.48 | .104 | 184 | 24.7 | 2.1 |
| 1.48 | .104 | 300 | 49.3 | 3.7 |
| 1.33 | .104 | 540 | 139 | 7.3 |
| 1.33 | .104 | 664 | 229 | 8.5 |
| 1.33 | .104 | 750 | 302 | − 47.0 |
| 1.33 | .104 | 810 | 162 | − 33.0 |
| 1.33 | .104 | 850 | 30 | |

## Table VII.

*Cobalt.*

| Thickness, cm. $\times 10^3$. | Current, c.g.s. | Temperature, C. | $R \times 10^3$ | $\Delta R/\Delta t \times 10^5$ |
|---|---|---|---|---|
| 6.34 | .103 | − 190° | 1.51 | + 2.0 |
| 21.3 | .103 | 23 | 5.88 | 3.6 |
| 21.3 | .104 | 100 | 8.65 | 5.9 |
| 6.34 | .104 | 184 | 13.6 | 6.6 |
| 6.34 | .104 | 300 | 21.2 | 12.5 |
| 6.34 | .105 | 516 | 48.3 | 28.5 |
| 6.34 | .104 | 660 | 89.4 | 38.0 |
| 6.34 | .104 | 840 | 158 | 35 |
| 21.3 | .105 | 960 | 200 | −200 |
| 21.3 | .105 | 1,030 | 170 | −130 |
| 21.3 | .105 | 1,100 | 60 | |

values of $R$ for ordinates. It will be seen from these curves that $R$ increases with rise of temperature until the critical temperature of the metal is reached and that the rate of increase of $R$ also increases up to the critical temperature. In going above the critical temperature a few degrees the value of $R$ drops suddenly to a small fraction of its value at the critical temperature. In the case of iron and nickel $R$ continues to decrease more and more slowly with further rise of temperature and the curve seems to approach asymptotically the axis of temperature. In case of cobalt it was not possible to follow the curve far enough to determine whether it also approaches asymptotically the axis of temperature. The reason for not studying the cobalt above 1100° C. was the fact that at temperatures above 1100° C. the mica used for insulation almost fused and became a sufficiently good conductor to make further observations impossible.

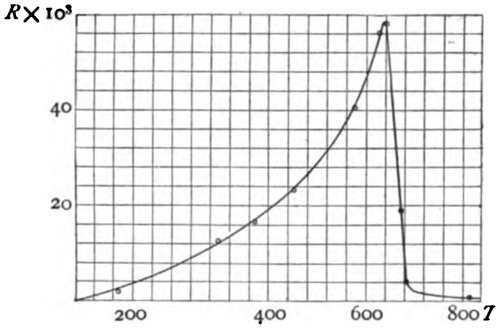


Fig. 20. Nickel.

The curve given in Fig. 20 is similar to that given by Clough and Hall,[1] although the temperature at which the decrease in $R$ begins is higher in the curve given in this paper than in their curve. This discrepancy may be attributed to difference in purity of the nickel in the two cases.

### *Reversal of the Hall Effect in Iron.*

In the discussion of the reasons for the reversal of the Hall effect in iron and its failure to be always proportional to the magnetic field J. J. Thomson[2] has pointed out that in addition to the effect of the magnetic force on the electrons, while they are moving over their free paths, one must take into account the magnetic force which acts on the electrons when collisions between electrons and molecules occur. Thomson states this possible explanation of the reversed effect in iron as follows: "Imagine a body whose molecules are little magnets. Then if the body is placed in a magnetic field so that the lines of force are vertical and downwards, the molecules will arrange them-

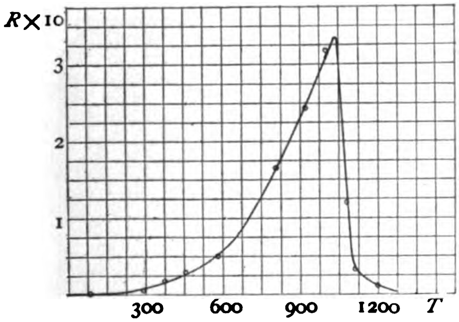


Fig. 21. Kahlbaum iron.

---

[1] Ibid., pp. 189–199.

[2] Corpuscular Theory of Matter, p. 70.

selves so that their axes tend to be vertical, the negative poles being at the top, the positive poles at the bottom. Then close to the magnet, in the region between its poles, the lines of force due to the magnet will be in the opposite direction to those due to the magnetic field and the intensity of the magnetic force close to the magnet may be very much greater than that of the external field. In this case when a corpuscle came into collision with a molecule the velocity would be rotated in the opposite direction to its rotation by the magnetic field before collision with the magnet — while it was moving over its free path. In this case the Hall effect would consist of two terms, one arising from the mean free path, the other from the collisions and these would be of opposite signs."

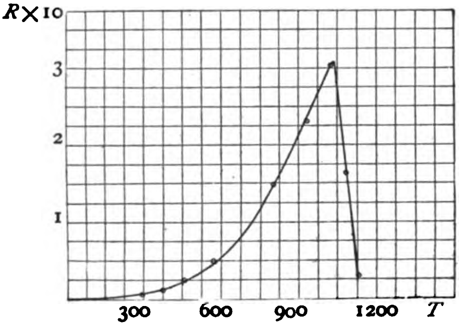


Fig. 22. Electrolytic iron.

If this be the correct explanation of the reversed effect in iron, it seemed that when the temperature of the iron had been raised to the critical temperature, the term in the Hall effect arising from the collision of electrons with magnetic molecules might be expected to drop out. If one assumes with Thomson only free negative electrons in the metal, the direction of the Hall effect arising from the action of the magnetic field on the electrons while they are moving over their free paths must be the same in all metals. According to this theory the Hall effect might be expected to

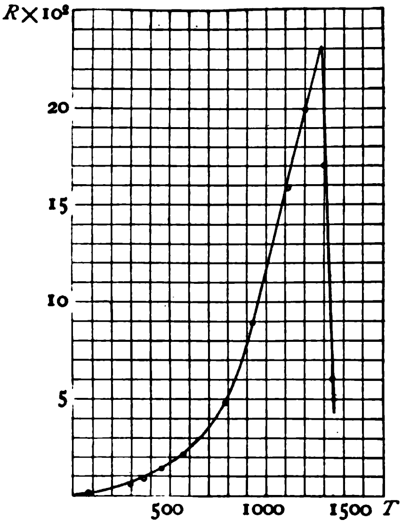


Fig. 23. Cobalt.

change sign at some temperature above the critical temperature and to have the direction of the effect in bismuth. From Fig. 21 it is evident that, although there is a sudden change in the magnitude of the effect in passing through the critical temperature, there is no indication of a change in the direction of the effect. The permeability of iron at 1000° C. is of the order of magnitude of 1.2. It seems that any term in the Hall effect depending on the magnetic properties of the iron ought to have nearly dropped out at that temperature. After passing the critical temperature a few degrees the influence of temperature on the effect in iron is of the same nature as its influence in the non-magnetic metals and there is no better reason for expecting a change of sign in iron with further rise of temperature than in a non-magnetic metal like platinum or copper. In view of the behavior of iron at temperatures above the critical temperature it does not seem probable that this explanation can account for the reversed effect in iron.

The effect in cobalt behaves in essentially the same way as the effect in iron but, for the reasons given above, observations were not made at temperatures as far above the critical temperature as in the case of iron, so that the conclusions for cobalt are less certain than they are for iron.

## SUMMARY.

The chief results of this paper are:

1. The temperature coefficient of the Hall effect in gold, zinc, platinum, silver and aluminum has for the first time been determined with certainty and it has been found that with the exception of aluminum there is in each of these cases a decrease in the effect as the temperature is raised from — 190° to about 22°. In the case of aluminum there is an increase, instead of a decrease.

2. The Hall electromotive force in each of these metals at the temperature of liquid air has been found to be proportional to the magnetic field.

3. The Hall effect in silicon has been found to have sometimes the direction of the effect in iron and sometimes the direction of the effect in bismuth. The purer specimens gave the effect in the direction of that in iron.

4. A reversal of the thermo-electromotive force in silicon has been found to be associated with a reversal of the Hall effect.

5. The Hall electromotive force in silicon has been shown to be proportional to the magnetic field.

6. The relation between the Hall electromotive force and the magnetic field has been examined in iron, nickel and cobalt for a number of temperatures between — 190° and 1100° C. For a given magnetic field the Hall effect in these metals has been found to increase with rising temperature until the critical temperature of the metal in question has been reached. The rate of increase becomes the greater the nearer the critical temperature is approached. In passing the critical temperature the effect sinks rapidly to a small fraction of its original value and then in the case of iron and nickel, and probably in cobalt, decreases slowly with further rise of temperature.

7. In the magnetic metals for any particular temperature the Hall electromotive force is proportional to the magnetic field, until the maximum intensity of magnetization in the plate has been reached. After this condition which has been called saturation has been reached, the curves showing the relation between the Hall electromotive force and the magnetic fields are nearly parallel to the axis of magnetic fields.

8. With increasing temperature the fields necessary to produce saturation become less in nickel and iron. In cobalt, however, the fields for saturation become greater with rising temperature until 300° is reached and with further rise of temperature the fields for saturation become less.

9. In nickel, iron and cobalt at temperatures a few degrees above the critical temperature the Hall electromotive force is proportional to the magnetic field over its range in these experiments.

10. It has been pointed out that the explanation of Thomson for the reversal of the Hall effect in iron is improbable in view of the fact that the effect in iron does not reverse sign at temperatures above the critical temperature where it has lost nearly all of its magnetic properties.

A similar investigation of several series of alloys is already in progress and the results will be presented in a subsequent paper.

Physical Laboratory of University of Wisconsin,
July 19, 1909.

# ON THE PENETRATING RADIATION AT THE SURFACE OF THE EARTH.[1]

By G. A. Cline.

## 1. Introduction.

DURING the last few years a number of investigators have made a study of the penetrating radiation which is known to be present at the surface of the earth with a view to locating its source or origin. The presence of radioactive substances in the soil and of radioactive emanations in the atmosphere suggest both the earth and the atmosphere as sources for part at least of this radiation. It is possible too for such a penetrating radiation as that present at the earth to have its origin in the sun or other celestial bodies.

It is known besides that the amount of radioactive emanation present in the atmosphere at any time is largely dependent upon the barometric changes which have taken place for a short time previously, and upon the precipitation which has occurred recently in the locality concerned. If then the penetrating radiation comes largely from the atmosphere it would follow from the above considerations that frequent though perhaps irregular changes should occur in the intensity of the radiation at any particular locality.

If the sun, however, contributed the major portion of the penetrating radiation we should then expect to find regular daily variations in its intensity. On the other hand if the greater part of the penetrating radiation has its origin in radioactive substances in the soil and rocks, we should expect to find but little, if any, variation in its intensity throughout the day or even from day to day in any particular region.

One of the first to note a diurnal change in the ionization of air contained in closed metallic cylinders was J. J. Borgmann.[2] His

---

[1] Communicated by Professor J. C. McLennan, and read before the Royal Society of Canada on May 26, 1909.

[2] Science Abstracts, 1905, No. 1580.

experiments were carried out in the neighborhood of St. Petersburg. Among other results he found that the ionization of the air in cylinders of copper, aluminium, brass, iron, zinc, tin, or lead, increased for some days after being enclosed, and finally reached a limit. This limit he found to vary from day to day and even during a single day. The minimum of conductivity noted by him was most frequently observed about three o'clock p. m.

Mach and Rimmer [1] while investigating the emanation content of the atmosphere at Vienna also made some measurements on the ionization of air confined in closed metallic vessels and from their measurements they were led to conclude that the penetrating radiation was more intense in the mornings and in the evenings than at noonday.

Amongst others Wood and Campbell,[2] at Cambridge, England, made an extensive examination of the penetrating radiation for a daily variation. In their experiments they made measurements on the ionization in different gases contained in closed metallic vessels of 6,000 c.c. capacity, and they used a sensitive quadrant electrometer as the measuring instrument.

From their experiments they showed that a periodic variation occurred in the ionization of the enclosed gases having two maxima and two minima each twenty-four hours. They also showed that the periodic variation was independent of the nature of the enclosed gas, and that the periodicity was independent of the metal of which the enclosing receiver was made. Their ionization curves representing these periodic changes were found too to be approximately the same as the curves representing the variations of atmospheric potential. As to the magnitude of the changes noted these experimenters found on the average that the variations in the ionization which took place in any one day amounted to about 12 per cent. of the whole.

McKeon,[3] of Washington, U. S. A., made an attempt to study the radiation by examining the variations in the potential assumed by an insulated metal cylinder placed within and completely sur-

---

[1] Phys. Zeit., 7, p. 617, Sept. 15, 1906.

[2] Phil. Mag., Feb., 1907.

[3] PHYS REV., 1907.

rounded by a second metallic cylinder 120 cm. long and 20 cm. in diameter. In his observations he noted a double daily variation in the potential of the insulated cylinder. The effect measured in his experiments, however, does not appear to be exclusively dependent upon the intensity of the penetrating radiation.

Strong,[1] of Baltimore, also made a study of this penetrating radiation using a small enclosed gold-leaf electroscope as the measuring instrument. In his experiments he observed the loss of charge from this instrument in a variety of places including (1) a room in the Physical Laboratory at Johns Hopkins University, (2) a cistern filled with rain water, the electroscope being placed at the center and (3) a room in the open country in the State of Pennsylvania near Mechanicsburg. Strong from his measurements drew the conclusion that the atmosphere contributed by far the major portion of the penetrating radiation. He found the intensity of the radiation greater in summer than in winter, and he too noted a double diurnal period in the ionization in his electroscope. Precipitation of rain or snow always produced a drop in the intensity of the radiation. But all changes in intensity were eliminated when he surrounded his electroscope with thick lead and iron screens. Strong's measurements are rather remarkable for the extremely wide variations which they indicated and it is difficult to account for them. On January 30, 1907, for example, he observed ionizations at various times of the day represented by 12, 82, 100, 77, on an arbitrary scale, and again on September 3, 1909, ionizations were observed represented by 25, 18, 50 and 155, 150, 42, 10, 15 on the same scale. Variations so extensive as these do not appear to have been observed by any other investigators and they seem to point to some very special and exceptional local conditions.

Some experiments were also made recently on this phenomenon by D. Pacini, at Sestola, in Italy.[2] This investigator used an aluminium-leaf electroscope and studied the ionization in air enclosed in large zinc receivers. He too found daily maxima and minima values in the ionization. His minimum observations ranged from 8, 9, 10 to 12 ions per c.c. per second while his maximum observa-

---

[1] Phys. Rev., July, 1908.

[2] Rend. Acc. Lincei, 18, 123–129, 1909.

tions extended in some cases to as high as 30 ions per c.c. per second. His ionization values present a double daily period with two maxima at two to three o'clock and nine to ten o'clock, and two minima from seven to eight o'clock and from twelve to one o'clock.

Wulf[1] too who devised a new type of electrometer, the movable system of which consists of a double conducting quartz thread, has applied this instrument also to the investigation of the penetrating radiation. His experiments were carried out at Valkenburg in Holland, both on the surface of the earth and in the chalk quarries in the neighborhood of that municipality. He too found a parallelism between the intensity of the penetrating radiation and that of the atmospheric potential, maximum values being obtained for both phenomena in the summer about eight or nine in the morning and evening and minima about noon and midnight. In winter the midday minimum was only slightly marked.

The amplitude of the morning variations was found to be about 16 per cent., while that of the afternoon ones was about 10 per cent. of the total ionization. The mean ionization corresponded to from 25 to 30 ions per c.c. per second. The ionizations observed in the measurements in the chalk quarries were found to be only about 42 per cent. of those obtained on the surface of the ground.

In this connection it will be recalled that Elster and Geitel[2] observed a fall of 28 per cent. in the conductivity of air enclosed in an aluminium receiver when the apparatus was taken from the surface down to the bottom of a rock salt mine. It will be remembered too that C. S. Wright in his experiments at Toronto found the conductivity of air confined in closed metallic vessels in experiments on the ice of Lake Ontario to be about 46 per cent. less than in similar ones made on the lawn in the neighborhood of the physical laboratory. It will be recalled too that the ionization values obtained by Wright on the sand bars on the island near Toronto were but slightly higher than those obtained by him on the ice. The experiments at Toronto coupled with those at Valkenburg, and with those of Elster and Geitel, would seem to show that the earth and not the atmosphere is the source of the penetrating radiation and that certain

---

[1] Phys. Zeit., 10, 1909, 152–157.

[2] Phys. Zeit., No. 1, 1905, p. 733.

waters, soils and salt deposits are comparatively free from radioactive substances, and can therefore act as efficient screens. It also seems evident from these experiments that the penetrating radiation, in some localities at least, does not come from a very great depth in the earth's crust.

The experiments made at Sestola, however, seem to point to the atmosphere as the source of the radiation.

By an inspection of the data furnished it will be seen that very little connection exists between the times of the maxima and minima obtained in the different localities mentioned. It seems evident, too, after surveying all the recorded observations that the variations noted must be ascribed to changes in atmospheric conditions rather than to solar influences. No such regularity occurs in the variations as one should expect if the penetrating radiation had its origin in the sun.

In the course of his experiments at Toronto, C. S. Wright on several occasions made observations on the conductivity of air enclosed in metallic receivers over periods of time extending in some cases up to six or seven hours. Two typical sets of observations are given in the following table and from the readings recorded

Table I.

| Time. | | $q_0$=Number of Ions per c.c. per sec. Reduced to 0° C. |
|---|---|---|
| December 24, 1907. | 10:23 a. m. | 22.48 |
| | 10:43 | 22.51 |
| | 11:10 | 22.54 |
| | 11:32 | 22.45 |
| | 11:55 | 21.74 |
| | 12:15 p. m. | 22.73 |
| | 12:40 | 21.96 |
| | 1:00 | 22.94 |
| December 26, 1907. | 9:35 a. m. | 22.44 |
| | 9:58 | 21.86 |
| | 10:20 | 21.87 |
| | 10:45 | 22.25 |
| | 11:05 | 22.96 |
| | 11:28 | 22.79 |
| | 11:50 | 22.53 |
| | 3:55 p. m. | 22.65 |
| | 4:15 | 22.54 |
| | 4:40 | 22.94 |
| | Mean | 22.45 |

it will be seen that there is no evidence of any appreciable regular variation in conductivity, and it is to be noted also that the extreme values obtained for the conductivity did not differ from the mean by more than 3 per cent. of the latter.

It would seem from these observations, therefore, that very little diurnal variation exists in the penetrating radiation at Toronto. Wright's observations, however, were extremely limited in number and it was decided to extend them, in order to ascertain whether it would be necessary to modify in any way the conclusions which might be drawn from them. The following paper contains an account of these observations and it will seen from the results that they point quite definitely to the absence of any regular diurnal variations.

## 2. Apparatus.

The measuring instrument used was the latest type of Wilson gold-leaf electroscope, and the arrangement of apparatus adopted is shown in Fig. 1. The case of the electroscope was joined to earth and the leaf system to an electrode which passed up into the ionizing chamber *R*. This chamber was insulated from the electroscope case, and could be charged as desired by means of the battery *B* to any selected potential. Attached also to the leaf system was the inner tube of the sliding condenser *C*, for details of which the reader is referred to the paper by C. S. Wright mentioned above. In the measurements to be described the receiver was charged negatively to about 180 volts, which was found sufficient to insure a saturation current. The small quartz Leyden jar *Q* was kept at a negative potential of approximately 50 volts, and the slide tube

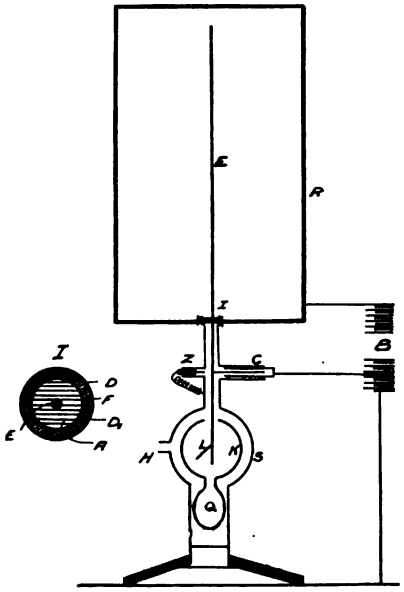


Fig. 1.

condenser *C* at varying negative voltages depending on the sensibility desired.

By moving this condenser any charge acquired by the gold leaf system through the conductivity of the air enclosed in *R* could be annulled. In the experiments the condenser was always moved over a standard distance and the time was taken for the conduction current to annul the deflection of the gold leaf produced by the displacement of the sliding condenser.

The charge annulled per unit voltage applied to the compensator tube was .00501 e.s.u., a number which was determined by using the auxiliary parallel plate condenser supplied with the instrument.

Assuming the charge carried by an ion to be $3.4 \times 10^{-10}$ e.s.u., it follows when the volume of the receiver is known and also the time required for the conduction current to annul a given quantity of electricity, that the number of ions per c.c. per second "$q$" in the receiver *R* can be readily found.

### 3. Experiments, Series I.

In this series of experiments the receiver *R* was made of sheet zinc. The potentials used were obtained from a set of small dry cells which remained constant over the whole range of measurements.

The receiver *R*, whose capacity was 31,180 c.c., was kept at a constant potential of 184.5 volts, throughout, the Leyden jar *Q* at 51.5 volts and the sliding condenser *C* at 13.2 volts. In this series of measurements the receiver was not hermetically sealed, so that barometric changes were necessarily followed by changes in the air content of the receiver.

The readings were taken visually and were commenced by a few preliminary observations over periods of from four to six hours' duration selected from different parts of the day. These were afterwards followed by observations taken continuously over a twenty-four hour period.

In the measurements for the shorter periods the apparatus was set up in a room in the Physical Laboratory and rested on a solid stone table which constituted the sill of one of the windows in the room.

The first observations were taken during the evening of November 25, 1908, and are shown in Fig. 2. During the readings the barometer was practically stationary and stood at 753 mm. The mean value of the readings it will be seen was 15.95 ions. In no case was there a deviation from the mean value of greater than 3 per cent., and the curve shows no evidence of any marked variations which might be ascribed to changes in the radiations which contributed to the conductivity of the enclosed air.

Fig. 2.
November 25, 1908; barometer, 753.

Similar sets of observations were taken on the afternoon and even-

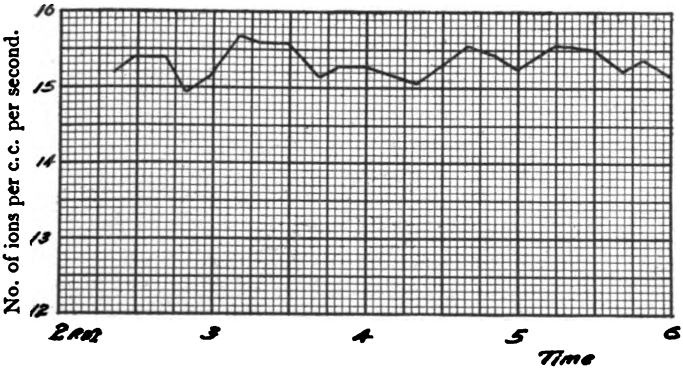


Fig. 3.
November 27, 1908; barometer, 760.

ing of November 27, 1908, and are illustrated by the curves shown in Figs. 3 and 4. Here again it will be seen that the extreme deviation from the mean value did not exceed 3.5 per cent. Moreover, the variations which did occur represent practically the limits of accuracy with which the readings could be taken by the measuring instrument.

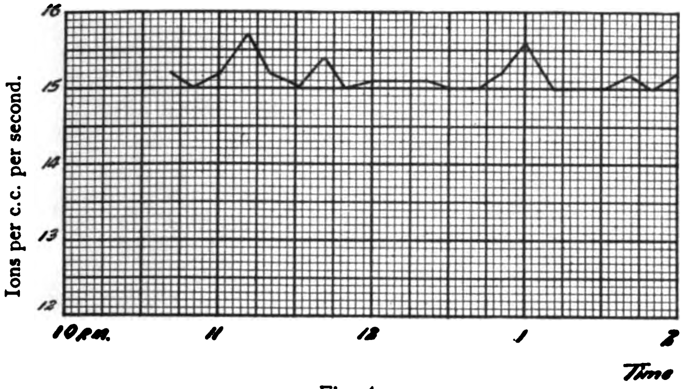


Fig. 4.
November 27 and 28, 1908 ; barometer, 762.4.

Again, sets of observations were taken on the afternoons of December 3, 4 and 7, the results being shown in Figs. 5, 6 and 7.

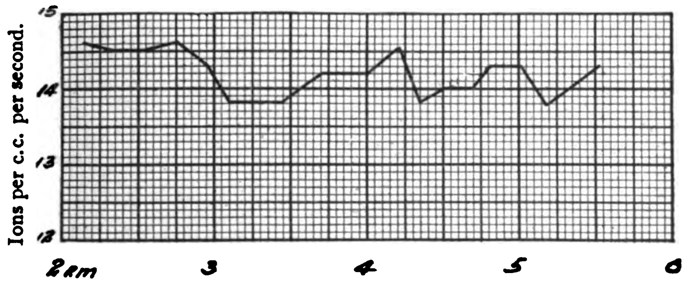


Fig. 5.
December 3, 1908 ; barometer, 762.5 to 761.6.

From these it will be seen that in one case the deviations from the mean were not greater than 1.5 per cent., while in the other two they did not exceed 3 per cent.

Further the curves shown in Figs. 2, 3, 4, 5, 6 and 7 indicate, it will be seen, but little variation during the periods of taking the readings. The readings from day to day, however, showed considerable variation but these were no doubt due to variations in the amount of radioactive emanation present in the atmosphere. As the receiver was not hermetically sealed the free interchange of air which this condition permitted could easily account for the differences noted.

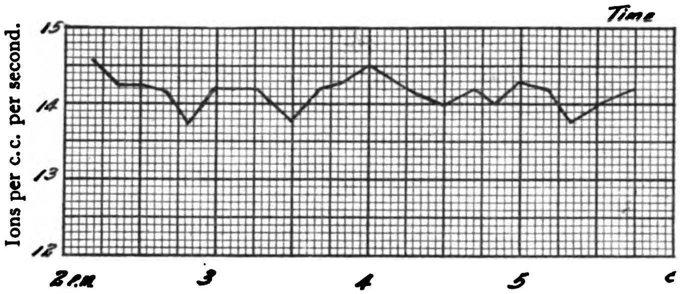


Fig. 6.
December 4, 1908; barometer, 747.2.

A point of interest in connection with these observations and one which has been observed by other investigators was that the ionization was greater when the barometric pressure was low than when it was high. A falling barometer would promote the effusion of gases occluded in the soil, and this process of effusion would consequently result in an increase in the amount of emanation present in the air and so account for the higher conductivity.

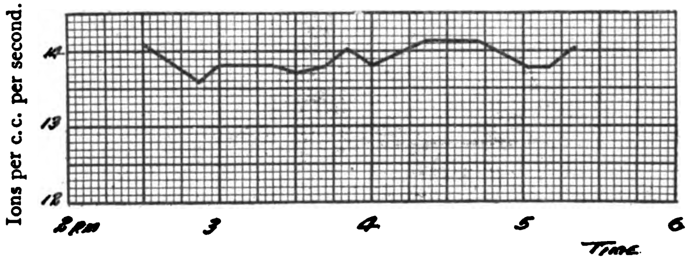


Fig. 7.
December 7, 1908; barometer, 747.7.

It was also noted in these experiments that the conductivity of the air in the cylinder was less for corresponding barometric pressures when the ground was frozen and covered with snow than when the temperature was above freezing point and the ground bare.

After completing the preliminary series of measurements just described it was decided to undertake a series extending over a twenty-four hour period. The readings were taken on December 18 and 19, 1908, in the library of the Physical Laboratory. This was a large well ventilated room facing the southeast and in it the apparatus was set up on a table close to one of the windows. During this set of observations the barometer remained fairly steady, the maximum change being from 746.3 to 749.5. The readings are shown in Fig. 8.

From an inspection of these readings and the curve, it will be seen that there is no evidence of a regular diurnal variation in the ionization. The extreme deviations in the readings from the mean taken throughout the whole period were not greater than 3 per cent., and as the instrument could not be used under the conditions of the experiment to give a greater accuracy than 2 or 3 per cent. one is forced to conclude from these observations that at Toronto there are no daily variations in the intensity of the penetrating radiation greater in magnitude than such variations as came within the limits of sensibility of the measuring apparatus.

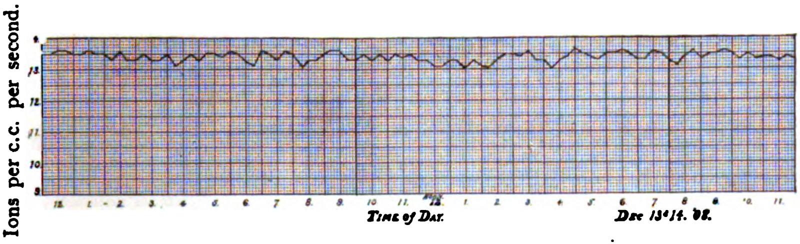


Fig. 8.
Open zinc receiver.

In the experiments which have been described no special precautions were taken to have the cylinder hermetically sealed. At a number of joints minute openings were left unclosed and so there must necessarily have been a constant interchange between the air inside the cylinder and that outside. It is worthy of note to observe that even with this interchange only very small variations in the conductivity of the air occurred during the twenty-four hour period.

## 4. EXPERIMENTS, SERIES II.

In order to submit the question of a diurnal change in the intensity of the penetrating radiation to a further test a second set of readings was taken over a period of twenty-four hours. In these a cylinder of galvanized iron was used of capacity 29,950 c.c. The electrode in this case was divided into sections, the one portion extending from the gold leaf to the top of the electroscope and the other being supported by insulating materials in position in the cylinder. The insulated electrode in the receiver was protected by an earthed guard tube which was also sealed in position and finally all the joints of the cylinder were made air tight by means of either solder or marine glue.

When the cylinder was placed in position above the electroscope a small spring made metallic contact between the two sections of the electrode.

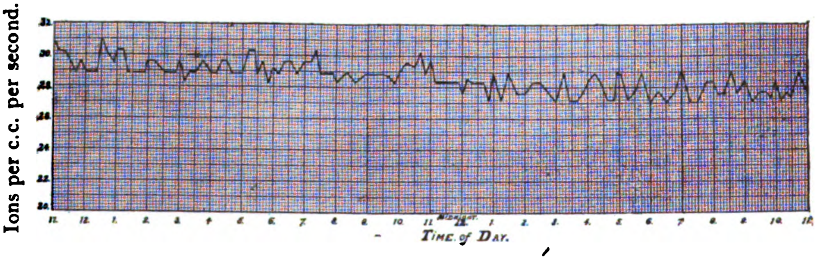


Fig. 9.
Closed galvanized iron receiver.

A small glass U-tube mercury manometer was also sealed into the side of the receiver to give indication of any air leak in the vessel.

In preparing for the observations care was taken to thoroughly clean the inside of the receiver and freshly filtered air was introduced into it to a pressure slightly greater than atmospheric pressure. This slight excess of the inside pressure over that of the atmosphere outside produced a difference of level in the two arms of the manometer. From the manner in which this difference of level followed the changes in the barometer it was soon seen that the receiver was airtight, and therefore in the condition desired for the observations.

When this point was made certain the readings were commenced.

These were taken on December 18 and 19, 1908, and are shown in Fig. 9.

This curve also, it will be seen, shows no regular periodic variation. Further the greatest deviation from the mean value was not more than 4 per cent. The capacity of the measuring system was greater in this set of measurements than in the previous one but the same voltage was applied to the sliding condenser as in the previous measurements. The result of this change was to reduce the sensibility of the apparatus slightly and this would account for the 4 per cent. variations in the readings apart altogether from any variation in the conductivity.

A point of special interest in connection with the readings is the gradual drop which they indicate in the conductivity of the enclosed air. This drop, which has been observed by others in similar measurements, was no doubt due to the gradual removal by the field of the suspension particles designated by Langevin[1] as "large ions" and first investigated by him. When the air was introduced into the receiver it was passed through a tube filled with cotton wool. This filter it was thought would remove all the dust and fine particles in suspension, but it was found after the completion of the measurements [illegible] when air filtered in the same way was [illegible]ed into an expansion cloud chamber of the C. T. R. Wilson type that it still contained suspension particles in abundance which acted as cloud nuclei. During these measurements the barometer only changed from 744 to 752 mm.

On account of the higher radioactivity possessed by the walls of this receiver the conductivity of the air in this case was about double its value in the early measurements. This enabled the writer to take a greater number of observations in a given time, but on account of the failure to increase the voltage of the condenser tube it did not contribute anything to the sensitiveness with which the readings were taken.

## 5. Experiments, Series III.

The experiments which have been described up to the present were all conducted indoors and as the walls of the physics building

---

[1] Comptes Rendus, 140, pp. 232, 1905.

very probably reduced in some degree the intensity of the penetrating radiation it was thought best to complete the series of observations by placing the receiver of the apparatus outside and so exposed directly to the atmosphere.

To accomplish this the electroscope was taken to the attic of the building and set up beneath a small opening in the roof. The receiver was placed on the roof over this opening on insulating supports and the electrode which it carried was joined to that of the leaf system by the insertion of a light metal rod 1 2/3 meters in length. A guard tube was also added to protect this added length to the electrode.

In setting up the apparatus this time a lining of thick sheet zinc was inserted which had been carefully abraded with fine glass paper and then thoroughly washed in turn with dilute hydrochloric acid, ammonia, methyl alcohol and distilled water. Fresh filtered air was drawn into the receiver and it was then hermetically sealed. Readings were taken immediately afterwards. In this case also it was noticed that the conductivity of the air gradually decreased and ultimately assumed a steady value. In the initial measurements the conductivity corresponded to as high as 19 ions per c.c. per second, but in the steady state it approximated to the generation of only 11.0 ions per c.c. per second.

This low value illustrates the great differences which exist in the radioactivity of different metals. With the galvanized iron cylinder alone it will be remembered that the conductivity corresponded to the production of about 35 ions per c.c. per second. The insertion of the zinc lining, it will be seen, therefore cut off by far the greater portion of the radiation from the galvanized iron. The reduction in the conductivity of the enclosed air necessarily made that portion contributed by the penetrating radiation of greater relative importance and it was thought that a more severe test would thus be available for the detection of diurnal variations. It was found however that the insertion of the long electrode greatly reduced the sensibility of the electroscope. By the addition of this the capacity of the instrument was nearly trebled, and consequently, although the displacement of the gold leaf for a given movement of the condenser tube could be increased by adding to the voltage of the latter still this device had the defect of adding to the time of taking a reading, and as it was desirable to have the readings taken as frequently as possible it was found necessary, therefore, to apply only a moderate voltage to the condenser. For this purpose a potential of 21 volts was maintained on the condenser tube, and this permitted readings to be taken, though with diminished accuracy, every 15 or 20 minutes.

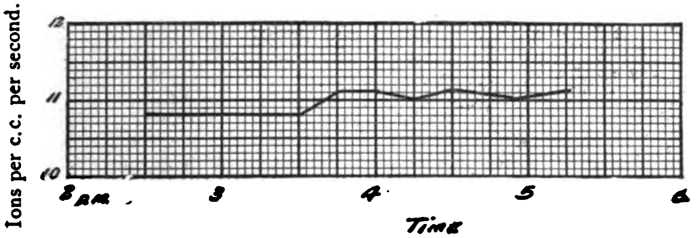


Fig. 10.
March 15.

With the apparatus set up in the manner just described sets of readings were taken during the afternoons of March 15, 22 and 26, 1909, the results being shown in Figs. 10, 11 and 12.

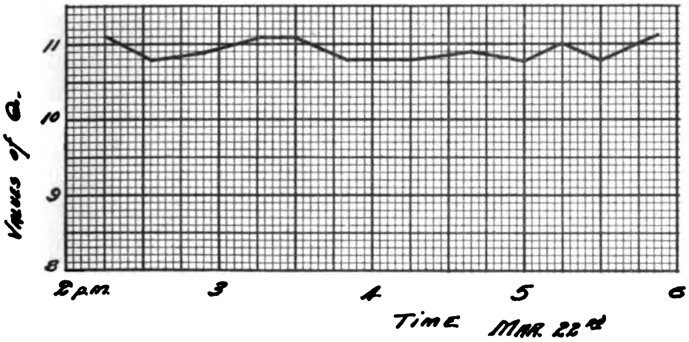


Fig. 11.

The readings for the three afternoons, it will be seen, approximated to 11 ions per c.c. per second.

By comparing the three curves it will be seen that although the readings were not uniformly regular still no periodic daily variation was brought into evidence. On March 28 and 29 and again on April 4 and 5, 1909, readings were again taken with this apparatus continuously over periods of twenty-four hours, as shown in Figs.

13 and 14. These readings it will be seen are far from being so uniform as those of the earlier observations. The variations from the mean it will be seen are very considerable, and are attributable,

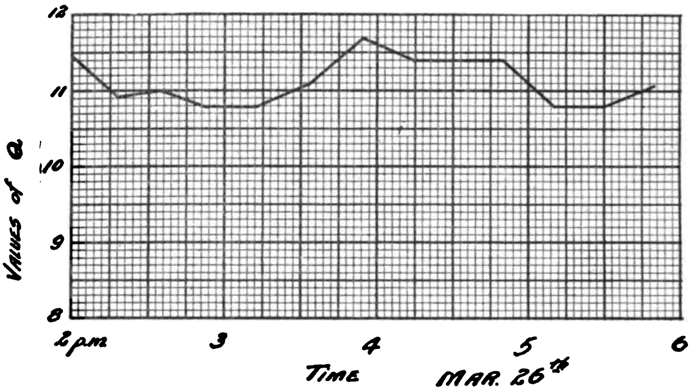


Fig. 12.

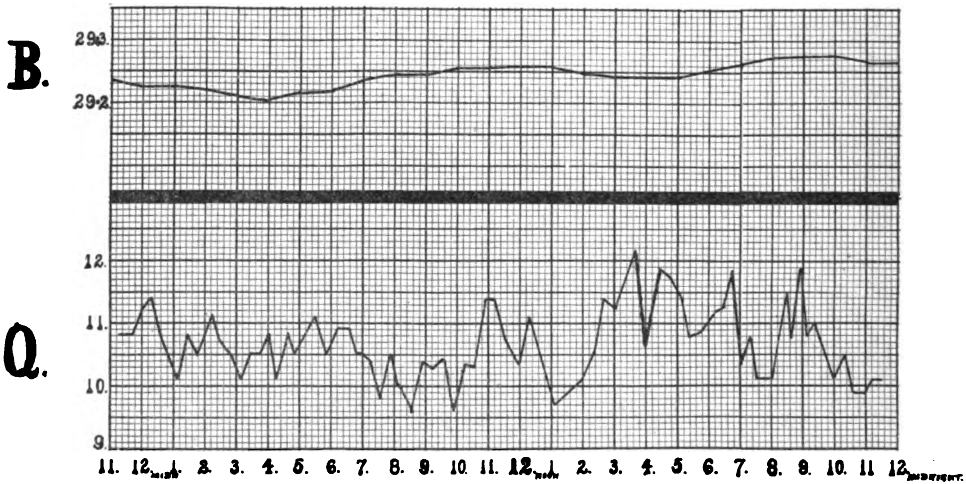


Fig. 13.

in the judgment of the writer, to the lack of sensitiveness in the instrument and the consequent difficulty in taking the readings, rather than to any variations in external influences.

In Fig. 13 the barometric curve is drawn for the corresponding twenty-four hour period. From an inspection of the two curves there does not appear to be any connection between the changes in conductivity and the changes in atmospheric pressure as indicated by the barometric readings.

In order to see whether a combination of the curves shown in Figs. 13 and 14 would give any indication of a pronounced maximum and minimum conductivity the two curves were compounded by taking the mean of the readings for the same time of the day. The individual curves are shown overlapping in the upper portion of Fig. 15, and the compound curve is shown at the bottom of the same figure. From the figure, however, it is impossible to draw the conclusion that any maximum or minimum conductivity was associated with any particular hours of the day.

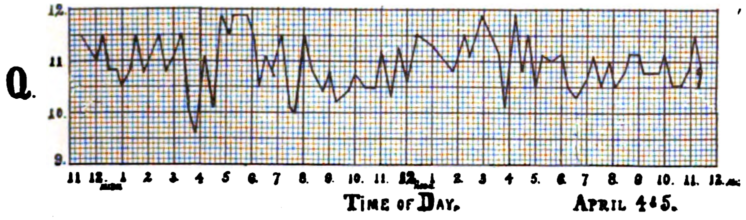


Fig. 14.

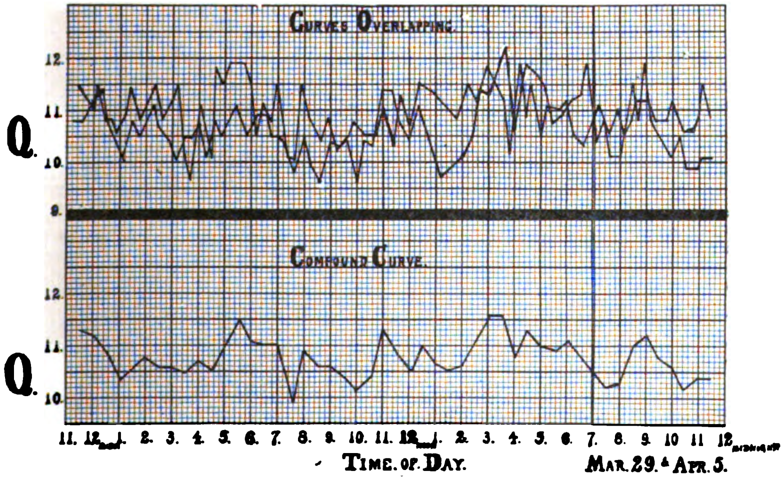


Fig. 15.

While the lack of sensibility in the measuring instruments prevented these later observations from leading to as satisfactory conclusions as might be desirable, still they agree with the earlier ones in failing to point to any variation of a regular diurnal character, and in this they seem to show that the conditions at Toronto are somewhat different from those which prevail in a number of other localities where similar observations have been made.

The writer had hoped to continue the investigation out in the open country with the electroscope arranged as adjusted for maximum sensibility, and possibly too with auxiliary apparatus attached for taking the readings automatically in place of taking them visually, but time has not permitted and the further investigation of this point has been of necessity deferred.

## 6. SUMMARY.

In summarizing the results of the investigation the following are the chief points which have been noted.

(*a*) With an open receiver. (1) No daily regular maxima or minima conductivities were observed. (2) Changes in conductivity occurred from day to day which seemed to be directly connected with concurrent barometric changes. (3) The conductivity was found to be slightly less when the ground was frozen and covered with snow, than when it was bare and the temperature was above freezing point.

(*b*) With a closed receiver. (1) Larger variations in the conductivity were observed but no regular diurnal maxima or minima values were noted. The larger variations in the conductivity were attributed to a lack in the sensitiveness of the measuring electroscope. (2) Different metals which were used in the construction of the receiver were found to possess different activities, zinc being very low. (3) Atmospheric air even when well filtered through cotton wool was found to contain many suspension particles.

As a general result of the investigation it would appear that the soil contributes by far the greater proportion of the penetrating radiation present at the earth's surface at Toronto and, by comparison, any that may have its source in the atmosphere or in the sun may be considered to be negligible in amount.

In closing I wish to express my gratitude to Professor McLennan for his kindness throughout the research, for suggestions and for assistance in overcoming mechanical difficulties. My best thanks are also due Mr. E. F. Burton for his kindness in taking a number of the readings.

# THE VARIATION WITH VELOCITY OF $e/m$ FOR CATHODE RAYS.

By C. A. Proctor.

THE experimental investigation of the variation with velocity of electromagnetic mass has been carried out with great care by Kaufmann[1] and Bücherer[2] with the aid of the $\beta$ rays from radium. The conclusions from these two researches are not in accord, and while the weight of evidence since the recent publication of Bücherer's results is certainly on the side of the Lorentz-Einstein rather than that of the Abraham theory, it is important that these results should be checked by different observers and different methods. There are two reasons at least why it appears the best check would be furnished by measurements on cathode rays in a very high vacuum. The range of velocities covered by the $\beta$ rays is from about four to nine tenths of the velocity of light. Experiments with cathode rays would cover quite a different range — probably from about one to six tenths of the velocity of light. Furthermore a series of simultaneous measurements of discharge potentials, electric deflection and magnetic deflection furnishes a double check on the accuracy of the theoretical formulæ, since the longitudinal as well as the transverse inertia of the electrons here comes into play. This phase of the question has been discussed in some detail by Planck.[3] It is in this respect that the discharge tube has a distinct advantage over radium rays, since in observations upon the latter the only quantities measured are the two deflections and in consequence the transverse mass alone is involved.

Such a series of measurements has been carried out by H. Starke,[4] but the range of potentials was too small or the experimental error

---

[1] Kaufmann, Ann. d. Phys., 19, p. 487, 1906.

[2] Bücherer, Ann. d. Phys., 29, p. 589, 1909.

[3] Planck, Verh. der D. Phys. Ges., 8, p. 418, 1906.

[4] Starke, Verh. der D. Phys. Ges., 8, p. 418, 1906.

too large for the results to be of value as evidence in favor of one theory or the other. The work of Classen[1] and Besthemeyer[2] also touches upon this question, but in neither case does the experimenter consider that any conclusive results have been reached.

It was in the hope of obtaining measurements over a sufficient range and of sufficient accuracy to be admissible as evidence on this matter that the work about to be described was undertaken. The results so far obtained are based on measurements of the two deflections alone, and cover a range of velocities only from twelve to forty-three hundredths of the velocity of light. Nevertheless they seem to be of sufficient interest to warrant their publication at this time especially as they are in much closer agreement with the Abraham than with the Lorentz theory.

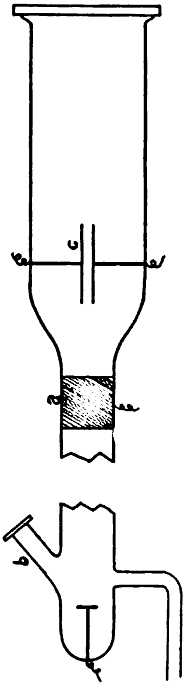


Fig. 1.

That the discharge in a cathode tube may take place in the highest possible vacuum, whatever the discharge potential, a condition clearly desirable if not essential, it is necessary to excite it by external means. Two such means suggest themselves — ultra-violet light and the Wehnelt cathode. The former was tried by the writer. The discharge tube was provided with a short branch (*b*), Fig. 1, closed by a quartz window. Through this window the aluminum cathode was illuminated by a powerful oscillatory spark between zinc electrodes. While this spark was rich in ultra-violet light the discharge produced by it was insufficient to cause the screen in the tube to fluoresce even at high discharge potentials. Careful cleaning of the cathode and modifications of the spark circuit failed to change this condition. In view of the results obtained by Lenard this was most surprising, and the writer has been entirely at a loss to account for it. In subsequent work a Wehnelt cathode will be employed as experiment shows that there

---

[1] Classen, Verh. der D. Phys. Ges.

[2] Besthemeyer, Ann. d. Phys., 22, p. 429, 1907.

is no difficulty in obtaining a sufficient discharge from it. In the measurements so far made the potential was controlled largely by the degree of vacuum in the tube. The effect of this on the results will be considered later.

To obtain potential measurements of sufficient accuracy to be of any value, a voltometer which can be read quickly and whose readings can be relied on to a fraction of one per cent. is essential. None of the various arrangements of apparatus so far tried by the writer has met these requirements at high potentials, so that only readings of magnetic and electric deflections have as yet been obtained. A new voltometer of the general design of that used by Müller[1] is now under construction in the shop of Ryerson laboratory. Judging by Müller's results this should meet the needs of the experiment.

The arrangement of the discharge tube is shown in the accompanying sketch. The anode (Fig. 1) is a brass cylinder closed at one end except for a slit 0.5 mm. wide and 1.5 cm. long. The condenser (*c*) consists of two brass plates 4.2 × 2.4 cm. and 0.47 cm. apart. The end of the tube opposite the cathode is closed by a piece of plate glass coated on the inner side with calcium tungstate and cemented on.

The length of the condenser, distance from anode to screen, and distance from condenser to screen which enter into the computation of the electric field integral were measured with a steel scale graduated in hundredths of an inch. The distance between the condenser plates was measured by means of a cathetometer. From the dimensions so obtained the electric field integral

$$\int_a^w \int_a^x F dx \cdot dx$$

was computed with the aid of Maxwell's formula. The value obtained was held under suspicion, however, as the proximity of the walls of the tube, shielded by grounded tinfoil, to the end of the condenser next the anode rendered questionable the applicability of the formula. This suspicion was confirmed by the fact that the values of $e/m$ at low velocities computed from this value of the field integral

---

[1] Ann. d. Phys. (28), 3, p. 591, 1909.

were impossibly large. It was accordingly thought best to determine this quantity as follows. A series of forty readings were taken of the magnetic deflection and discharge potential at about 8,000 volts. At this value the potential, furnished by a twenty-four plate static machine, could be held very constant and measured with considerable accuracy with a Braun 10,000-volt electrometer. From these readings the values of $e/m$ and the velocity were obtained in the usual way and the value of $e/m$ for zero velocity computed with the aid of the Abraham formula. The value obtained was $1.859 \times 10^7$. This agrees within the limits of experimental error with that of Simon[1] but not with the more recently published results of Classen, Besthemeyer and Bücherer. It should be noted however that a very considerable change in this value would not appreciably affect the conclusions regarding relative values of $e/m$ at different velocities. From this value of $e/m$ and a series of readings of the electric and magnetic deflections also taken at discharge potentials of about 8,000 volts the value of the electric field integral was computed. The value thus obtained was about 10 per cent. less than that given by Maxwell's formula and was used in all subsequent computations. The justification for this method of determining the field integral is found in the work of Seitz,[2] who has shown that it is correct at the potentials here employed.

The magnetic field was obtained by means of a solenoid 24 cm. in diameter made in two sections each 48 cm. long and separated by a gap of 3 cm. to allow of the introduction of the discharge tube. This was mounted with its axis east and west and the discharge tube was so placed that the undeflected rays cut this axis at right angles. As the discharge tube was horizontal there was a considerable lateral deflection of the rays at low potentials due to the vertical component of the earth's field. This was compensated by a large horizontal coil immediately beneath the tube. The form of the magnetic field, proper position of the tube with reference to the solenoid, and correction for finite curvature of the magnetically deflected rays were determined by the methods given in detail by S. Simon.[3] The absolute value of the field on the axis of the solenoid

---

[1] S. Simon, Ann. d. Phys., 69, p. 589, 1899.

[2] W. Seitz, Ann. d. Phys., 8, p. 233, 1902.

[3] S. Simon, *l. c.*

was determined as follows: A magnetometer with a heavy phosphor-bronze suspension was mounted so that the magnet was on the axis of the solenoid where the field is sensibly uniform, and adjusted until the magnet was perpendicular to this axis. This condition was judged to be fulfilled when the deflections on opposite sides of the zero were the same for any current within the possible range. The mean deflection for a known current was then determined from ten separate readings. This work was then repeated substituting for the solenoid employed in the experiment another solenoid whose field could be computed from its known dimensions. This reference solenoid was made, and has been repeatedly used, for the determination of the absolute value of the ohm. As in that work it gives results correct to within 0.1 per cent. there can be no doubt that its computed field is very little in error. Currents were measured with an "American" ammeter which was calibrated throughout its range by comparison with a Kelvin balance whose accuracy had been recently checked with the silver voltameter. Two determinations of the field made as above with different magnetometers and widely different currents gave values differing by 0.3 per cent. and their mean was taken as the value of the field.

The potential difference in the plates of the condenser was obtained from a battery of small storage cells. The values used ranged from 70 to 800 volts. This potential was measured with a 320-volt Kelvin multicellular voltometer, calibrated by comparison with a 300-volt Weston direct current instrument. The Weston instrument itself was sent to the Bureau of Standards for calibration. A resistance of 10,000 ohms was placed in series with the battery and condenser. The potential was found to remain sensibly constant during any series of readings. Its value was therefore taken only at the beginning and end of each series and the mean used. These readings were commonly taken while the discharge was passing through the tube. Repeated tests showed, however, that the readings were the same when the discharge was not passing.

The general procedure in taking readings was as follows: The condenser potential and current in the solenoid were so adjusted that the deflection due to either was from one to two centimeters at the particular discharge potential employed. Readings of the electric and magnetic deflections were then taken alternately until a series usually consisting of five of each was obtained. The deflections were measured with a cathetometer which could be set on the fluorescent spot with an error not greater than 0.02 mm. The current in the solenoid was read with each of the magnetic deflections. The means of the currents and of each set of deflections were then taken. Readings were taken in this way to eliminate as far as possible the effect of the unsteadiness in the discharge potential which could not be held perfectly constant. The variations were commonly of the order of 1 per cent. The deflection per ampere in the solenoid and per 100 volts difference of potential on the condenser were then computed.

Forty-four pairs of magnetic and electric deflections obtained as above were for comparison with the theoretical formulæ divided into ten groups according to magnitude and the mean of each group taken. From the magnetic deflections the value of $e/m$ for zero velocity, and the constants of the apparatus, the electric deflections to be expected on the Lorentz-Einstein and the Abraham theories were computed.

The reduction of observations was made by this method for the sake of comparing results with those of Kaufmann, to whom it is due. The method is in brief as follows:

If $z =$ magnetic deflection corrected for finite curvature,
$y =$ electric deflection,
$M =$ magnetic field integral,
$E =$ electric field integral,
$e =$ charge on electrons,
$m =$ mass of electrons,
$\beta =$ velocity of electron divided by velocity of light,
$c =$ velocity of light,

we have from the ordinary theory

$$z = \frac{eM}{m\beta c^2},$$

$$y = \frac{eE}{m\beta^2 c^2}.$$

If we set $m = m_0\varphi(\beta)$ where $m_0$ is the mass of the electron at zero velocity and $\varphi(\beta)$ expresses the dependence of mass upon velocity we have

$$z = \frac{e}{m^0}\frac{M}{c}\frac{1}{\beta\varphi(\beta)},$$

$$y = \frac{e}{m_0}\frac{E}{c^2}\frac{1}{\beta^2\varphi(\beta)}.$$

If we know $e/m_0$, $z$ and the form of the function $\varphi(\beta)$ we can compute $y$. For the details of this computation the reader is referred to Kaufmann's paper. For the Lorentz-Einstein theory we have

$$\varphi(\beta) = (1 - \beta^2)^{-\frac{1}{2}}.$$[1]

For the Abraham theory we have

$$\varphi(\beta) = \frac{3}{4}\frac{1}{\beta^2}\left\{\frac{1+\beta^2}{2\beta}\log\frac{1+\beta}{1-\beta} - 1\right\}.$$[1]

Table I. shows the results of computation compared with observed deflections. There is close agreement between the writer's results and those of Kaufmann, the variations of computed from observed values being in the same direction in both cases and about twice as great for the Lorentz formula as for the Abraham. On the other hand, both are quite different from those given by the recent experiments of Bücherer, who finds complete agreement between experiment and

TABLE I.

| $\beta$ | Magnetic Defn. | Elect. Defn. Observed. | Elect. Defn. Lorentz. | Per Cent. | Elect. Defn. Abraham. | Per Cent. |
|---|---|---|---|---|---|---|
| .432 | 2.678 | 0.1467 | 0.1511 | 3.0 | 0.1486 | 1.3 |
| .408 | 2.866 | 0.1665 | 0.1712 | 2.8 | 0.1686 | 1.2 |
| .387 | 3.022 | 0.1849 | 0.1887 | 1.9 | 0.1862 | 0.7 |
| .341 | 3.485 | 0.2423 | 0.2465 | 1.7 | 0.2438 | 0.6 |
| .285 | 4.205 | 0.3499 | 0.3525 | 0.7 | 0.3499 | 0.0 |
| .229 | 5.353 | 0.5545 | 0.5657 | 2.0 | 0.5614 | 1.2 |
| .187 | 6.535 | 0.8306 | 0.8313 | 0.1 | 0.8296 | −0.1 |
| .157 | 7.857 | 1.188 | 1.195 | 0.6 | 1.194 | 0.5 |
| .140 | 8.749 | 1.483 | 1.478 | −0.3 | 1.476 | −0.5 |
| .123 | 10.043 | 1.932 | 1.943 | 0.6 | 1.943 | 0.6 |

---

[1] Abraham, Theorie d. Elect., Vol. II., pp. 191 and 203.

the Lorentz formula. A careful consideration of possible sources of error fails to reveal any that could account for the large discrepancy between the results and those of Bücherer, unless it be the residual gas in the dischage tube. As the discharge potential was controlled to a large extent by the degree of vacuum in the tube there was a considerable amount of gas present at the lower potentials. As has been pointed out by J. J. Thomson and others it is a matter of uncertainty how much this may affect the values of $e/m$ obtained. In his discussion of Kaufmann's results Planck[1] has shown that some modification of the theory of the experiment is necessary, as the value of the velocity of the electrons computed from the apparatus constants and the smallest deflections is greater than the velocity of light, and that they conform more nearly to the Lorentz than to the Abraham formula if the assumption is made that the electric field integral is modified by the presence of residual gas. An attempt to apply this to the writer's results leads to difficulty, for the effect in question is a reduction of the electric field integral and we should expect this reduction to increase with the amount of gas present. In the present work then this effect should be greatest at the low discharge potentials and diminish progressively as the potential increased. If we took account of this in computing the results the apparent variation of $e/m$ with velocity would become smaller. As it is already too small to fit either theory it is evident that this explanation in its present form does not suffice.

Another method of exhibiting the results is shown in Table II. From the observed deflections and constants of the apparatus $\beta$ and $e/m$ are computed. These appear in columns one and two of the

TABLE II.

| $\beta$ | Magnetic Defn. | Elect. Defn. Lorentz. | Elect. Defn. Abraham. | $\beta$ | Magnetic Defn. | Elect. Defn. Lorentz. | Elect. Defn. Abraham. |
|---|---|---|---|---|---|---|---|
| .432 | 1.738 | 1.927 | 1.886 | .229 | 1.838 | 1.888 | 1.877 |
| .408 | 1.755 | 1.922 | 1.885 | .187 | 1.828 | 1.861 | 1.854 |
| .387 | 1.757 | 1.905 | 1.873 | .157 | 1.848 | 1.871 | 1.866 |
| .341 | 1.783 | 1.896 | 1.872 | .140 | 1.835 | 1.853 | 1.849 |
| .285 | 1.796 | 1.875 | 1.858 | .123 | 1.856 | 1.871 | 1.867 |

---

[1] Verh. d. El. Phys. Gesch., 9, p. 301, 1907.

table. From them are computed the value of $e/m_0$ by the Lorentz and by the Abraham formulæ. These are shown in columns three and four. If either formula exactly represented the observations $e/m_0$ should be constant. The values computed from the Abraham formula are perhaps constant to within the limits of observational errror, but those computed from the Lorentz formula certainly are not.

Fig. 2 shows the observed values of $e/m$ as a function of $\beta$, and the values computed from the two formulæ in question.

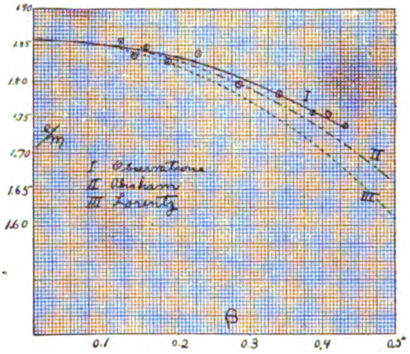


Fig. 2.

In conclusion I wish to express my great indebtedness to Professor Millikan to whose suggestion this work is due, and who has during its progress been ever ready with friendly interest and helpful advice.

July 13, 1909.

# THE FATIGUE OF METALS SUBJECTED TO THE ROENTGEN RADIATION.

By R. C. Gowdy.

PROFESSOR L. T. MORE[1] has found that the secondary Roentgen radiation from metals is not independent of the duration of the exposure, but that the metals suffer a fatigue comparable to that produced by ultra-violet light.

His results show that the radiations from old surfaces of iron, lead, nickel and zinc, exposed to air, depend on the duration of the exposure, while the radiations from aluminium and copper are nearly constant.

Freshly polished surfaces of iron and lead show an initial rise in sensitiveness.

The effects when the metals are surrounded by an atmosphere of coal gas do not differ from those obtained in air. An inert gas, like hydrogen, seems to reduce the fatigue, while the presence of ozone increases it, though not in any way comparable to the extent noted by Hallwachs[2] for the ultra-violet radiation.

In conclusion the writer states that his results indicate that the fatigue is caused by "changes in the gas in or on the plates, rather than any changes in the metals themselves."

Following the suggestion of Professor More, this work has been carried on for the purpose of verifying his results and continuing the investigation, particular attention being given to the effect of oxidation on the behavior of the secondary radiation.

The first measurements of fatigue in this paper were made by the method described in the earlier paper by Professor More. A brief account of the method will be sufficient here. For more detailed description the reader is referred to the above mentioned work.

---

[1] More, Phil. Mag., XIII., 1907, p. 708.

[2] Hallwachs, Sitzungsber. der Ges. der Wissensch. zu Leipzig, LVIII., 1906, p. 341.

The Roentgen radiation is allowed to fall upon two similar plates, and their secondary radiations are measured and compared. If then one of these plates, which we will call the "standard," be shielded from the radiation, and the other, known as the "trial" plate, be exposed for a length of time, a second comparison of the radiations may be expected to show any change in sensitiveness of the "trial" plate, due to its prolonged exposure to the X-rays.

This method, being comparative, eliminates to a great extent the variations in the working of the coil and bulb from one set of observations to the next, but assumes that the primary radiation is constant or varies regularly during each particular set of measurements. Even with the most careful manipulation, irregularities in the action of the coil and bulb introduce considerable error, in some cases masking the effects sought.

If the radiation from both plates could be measured at the same time, and by the same instrument, the effect of the irregularities above noted would be nearly eliminated. A differential method was therefore devised, which accomplished practically this result.

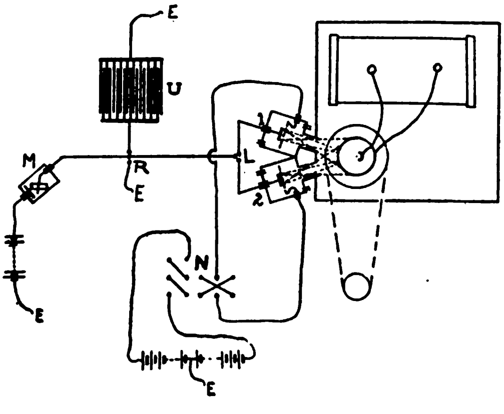


Fig. 1.

Fig. 1 represents diagrammatically the arrangement of the new apparatus.

The ionization chambers, 1 and 2, are placed so that the radiation from the X-ray bulb passes through the perforated aluminium electrodes and falls normally on the plates to be tested. As will be shown later, the radiation from aluminium is constant, and therefore the secondary radiation from this source will not add a confusing effect. The aluminium electrodes can be charged with electricity of like or unlike sign by means of the switches *N*, and are kept at potentials sufficient to produce saturation. The test plates themselves formed the insulated electrodes, and the ionization currents were measured by the inclined Wilson electroscope *M*. The spring mercury switches *L* allowed either or both plates to be connected to the electroscope. The switches *R* connected a variable condenser *U* in parallel with the electroscope to reduce its sensibility, and provided a means of earthing the electroscope, condenser or plates.

The observations are made in the following manner. The electrode in chamber 1 is charged positively, and that in chamber 2, negatively. If the two plates are giving off the same amount of radiation, the electroscope will show no deflection. The plates, in spite of precautions taken, may not be equally sensitive to the primary radiation. In this case an equality of the secondary radiation may still be brought about, in general, by turning the X-ray tube about a vertical axis, until the inequality of the primary radiation falling on the two plates compensates for the difference in their sensitivenesses.

Having obtained this balance in the radiations, the "standard" plate in 1 is shielded from the X-rays. Now if any change in the sensitiveness of the trial plate takes place, when the two plates are again connected to the electroscope, a deflection will be obtained, due to the current produced by the secondary radiation from the "trial" plate being either greater or less than that from the "standard" plate.

Let us denote the original conditions by:

$c_1$ = current in chamber 1,
$c_2$ = current in chamber 2,
$t_1$ = time required for given deflection with 1,
$t_2$ = time required for given deflection with 2,
$s_1$ = sensitiveness of plate 1,
$s_2$ = sensitiveness of plate 2,

$k_1 =$ radiation falling on plate 1,
$k_2 =$ radiation falling on plate 2,

the units being chosen so that we may write

$$c_1 = k_1 s_1 = 1/t_1, \tag{1}$$

$$c_2 = k_2 s_2 = 1/t_2. \tag{2}$$

Now, after a certain length of time, during which the "trial" plate has been exposed, and the "standard" shielded from the X-rays, any or all of these quantities may have changed and we may denote the new values by the same letters primed.

The change in sensitiveness of the "trial" plate is given by

$$s_2' - s_2 = 1/k_2' t_2' - 1/k_2 t_2 \tag{3}$$

and the proportional change by

$$\frac{s_2' - s_2}{s_2} = \frac{k_2 t_2}{k_2' t_2'} - 1. \tag{4}$$

By preliminary test it was found that, if the test plates be of the same material, and have the same kind of surface, the balance once obtained is not disturbed by changes in the working of the coil and bulb. We may therefore assume that the distribution of the rays about the tube does not vary with the working of the coil, and we may write

$$\frac{k_1}{k_1'} = \frac{k_2}{k_2'}.$$

It was also found by trial that the sensitiveness of the "standard" plate varied but little with the intensity of the rays provided their character did not change. If the tube be kept at the same degree of hardness throughout we may then take

$$s_1 = s_1'.$$

When the radiations from the two plates are balanced, we have the condition

$$t_1 = t_2.$$

Noting these relations, equation (4) becomes

$$\frac{s_2' - s_2}{s_2} = \frac{t_1' - t_2'}{t_2'}. \tag{5}$$

But the resultant current due to the two plates working oppositely to each other is

$$c_1' - c_2' = \frac{t_2' - t_1'}{t_1' t_2'} = \frac{1}{t_d}, \qquad (6)$$

where $t_d$ is the time required for the gold leaf to move through the given deflection when both plates are connected to the electroscope at the same time.

From this it is evident that

$$\frac{s_2' - s_2}{s_2} = \frac{-t_1'}{t_d} = -\text{(fatigue)}. \qquad (7)$$

We have tacitly assumed all the foregoing equations to be simultaneous, a condition impossible in practice, since readings must necessarily be taken consecutively. The error thus introduced will be that due to variations in the sensibilities, caused by changes in the character of the rays. But it will be seen from equation (7) that proportional variations in radiating power will not affect the value of the result. This requirement of proportional variation may be fulfilled by having both plates of the same metal, and their surfaces in the same condition.

An instrument sufficiently sensitive to measure $t_d$ would be too sensitive to use for measuring $t_1'$, since it is very small compared to $t_d$. To obviate this difficulty the condenser $U$ is connected to the system when measuring $t_1'$, thereby reducing the rate at which the potential will rise. If $Q$ denote the total capacity of the system and condenser, and $q$ the capacity of the system exclusive of the condenser, then we have

$$\frac{s_2' - s_2}{s_2} = -\frac{T_1' q}{t_d Q}, \qquad (8)$$

where $T_1'$ is the time required for the given deflection, when the condenser is connected and the "standard" plate alone is radiating.

The meaning of the minus sign in equations (7) and (8) is, that if the resultant deflection, after the "trial" plate has changed its sensitiveness, be in the same direction as the deflection produced by the "standard" plate alone, then the change in the "trial" plate has been a *decrease* in sensitiveness or a *positive* fatigue, and vice versa. In practice, both $t_d$ and $T_1'$ were measured for deflections in the same direction and of equal magnitude, the polarity of the electrode in 1 being changed while measuring $T_1'$ when a negative fatigue was encountered.

The variable air condenser consisted of nine stationary and eight sliding plates, each 17.6 cm. square, separated by air spaces of one centimeter each. The stationary plates were carried on sulphur pillars, and all corners carefully rounded so that leakage from even so large a surface was negligible. The condenser was free from residual charge, and all parts were nickel-plated and highly polished to prevent contact electromotive forces. For convenience, the factor $q/Q$ was determined for various positions of the movable plates and the results plotted in a curve (Fig. 2), having for ordinates the values of $q/Q$, and for abscissæ the linear distance which the movable plates were withdrawn from their position of maximum capacity. In nearly all cases the condenser was used at maximum capacity, which gave the value $q/Q = 0.088$. By withdrawing the movable plates this ratio could be increased to $q/Q = 0.487$. The (calculated) maximum value of $Q$ was approximately 350 c.g.s. units.

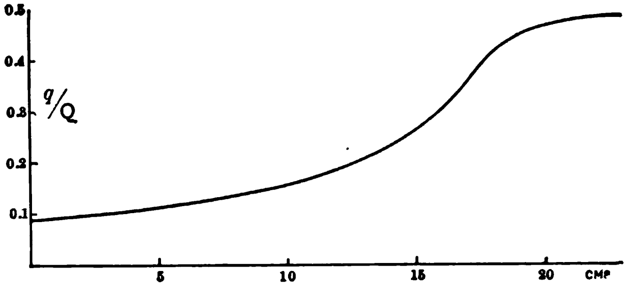


Fig. 2.

The results, following in this paper, have been obtained by this differential method, with the exception of a very few which have been noted.

## METALS AND OXIDES IN AIR.

In Table I. are given the length of exposures and the corresponding values of the radiation from freshly polished surfaces of aluminium, copper, zinc, tin, silver, lead and iron, immersed in air. The

### TABLE I.

| Exposure. | Aluminium | Copper. | Zinc. | Tin. | Silver. | Lead. | Iron. |
|---|---|---|---|---|---|---|---|
| 0 hrs. | 100[1] | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| 1 hr. | 100 | 99.8 | 100.0 | 100.0 | 99.3 | 100.0 | 101 |
| 2 hrs. | 100 | 100.0 | 99.7 | 99.0 | 99.3 | — | 101 |
| 3 hrs. | 100 | 99.9 | 99.7 | 98.5 | 98.9 | 97.1 | 104 |
| 4 hrs. | — | — | 99.4 | 98.1 | — | — | 102 |
| 5 hrs. | — | — | 99.3 | 98.4 | 99.1 | 93.9 | 99 |
| 6 hrs. | — | — | — | — | 98.6 | 94.1 | — |
| 7 hrs. | — | — | — | — | 98.5 | — | — |

surfaces of the metals in these experiments were polished on the finest grades of crocus cloth obtainable, and were tested as soon as they became cool after polishing. The surfaces thus obtained, while not of mirror brightness in the case of the softer metals, were clean, and quite smooth and uniform.

Iron alone shows an increase in radiation, reaching a maximum in three hours. Silver, lead and tin do not show this initial rise, the silver beginning to fatigue during the first hour, and lead and tin not until the second. Aluminium and copper show no change in radiation and zinc but little.

In Tables II., III. and IV. are given the results obtained with oxidized surfaces of the same metals, the radiation from the polished metals also being given for the purpose of comparison.

The coating of magnetic oxide was prepared by heating the polished metal plate to a temperature of about 700° C., a thin even bluish black coating being thus obtained. The irregularities shown

### TABLE II.

| Exposure. | Freshly Polished Iron. | Magnetic Oxide. | | Ferric Oxide. |
|---|---|---|---|---|
| 0 hrs. | 100 | 100 | | 100 |
| | | Two samples. | | |
| 1 hr. | 101 | 99 | 101 | 101 |
| 2 hrs. | 101 | 101 | — | 101 |
| 3 hrs. | 104 | 99 | 101 | 101 |
| 4 hrs. | 102 | 100 | 100 | 100 |
| 5 hrs. | 99 | — | — | 100 |

---

[1] To avoid confusion and make the nature of the results more apparent, *irregular* variations of less than 0.2 per cent. have been omitted from columns marked with the asterisk.

### Table III.

| Exposure. | Freshly Polished Copper. | Cuprous Oxide. | Cupric Oxide. | |
|---|---|---|---|---|
| 0 hrs. | 100.0 | 100* | 100 | |
| | | | Two samples. | |
| 1 hr. | 99.8 | 100 | 99 | 101 |
| 2 hrs. | 100.0 | 100 | 99 | 101 |
| 3 hrs. | 99.9 | 100 | 100 | 99 |
| 4 hrs. | 100.0 | — | 101 | 101 |
| 5 hrs. | — | — | 100 | 99 |
| 6 hrs. | — | — | 100 | 101 |

### Table IV.

| Exposure. | Freshly Polished Tin. | Tin Surface Two Weeks Old. | "Tin Oxide." | Stannic Oxide. |
|---|---|---|---|---|
| 0 hrs. | 100.0 | 100.0 | 100* | 100* |
| 1 hr. | 100.0 | — | 100 | 100 |
| 2 hrs. | 99.0 | 100.0 | 100 | 100 |
| 3 hrs. | 98.5 | — | 100 | 100 |
| 4 hrs. | 98.1 | 98.2 | 100 | 100 |
| 5 hrs. | 98.4 | 97.8 | 100 | 100 |

for this and the ferric oxide in Table II. are probably due to experimental errors involved in the first method, used for these determinations. The magnetic oxide may be said to show no decisive change during a four-hour exposure.

The ferric oxide coating was obtained by covering the plate with dilute hydrochloric acid, forming chloride of iron on the surface, and the ferric hydroxide, eventually produced by the addition of a few drops of ammonia, was allowed to dry. The plate was then set away for a week to allow further oxidation, after which the ammonium chloride was expelled by heating the plate. The well rusted surface thus obtained also shows no pronounced change during five hours exposure.

It will be noticed that the change in sensibility, exhibited by the polished metal, is not shown by either oxide. Both of these oxides are quite stable in air, while the metal itself is very liable to rust.

Table III. shows the behavior of the two oxides of copper.

The cuprous oxide was formed by gently heating the polished copper plate, care being taken that the temperature did not become sufficiently high to form the cupric oxide completely. Such a coating will, at best, probably contain a considerable quantity of cupric oxide.

The cupric oxide was formed by heating the plate to dull redness in an alcohol (sulphur-free) flame. Neither of these oxides shows any fatigue in air.

These three surfaces are stable in air, and none shows variation in radiating power beyond that of experimental error.

The results of tests on two oxides of tin are given in Table IV.

The coating called "tin oxide" is of undetermined composition, but it is probably a lower oxide, or a mixture of oxides, containing less oxygen than the stannic oxide. It was formed by heating a polished tin plate to a temperature of 170° C. for several hours, and appeared as an olive to a yellowish tarnish on the surface of the metal. As might be expected from its method of formation, this surface appears to be quite stable in air at ordinary temperatures, and undergoes no further change.

The stannic oxide was prepared by igniting the nitrate formed on the surface of the metal by coating it with nitric acid.

Neither oxide shows any fatigue and the surface of the polished metal allowed to age in air for two weeks does not behave differently from the freshly polished surface.

A thin coating of a stable oxide has completely stopped the fatigue experienced by the fresh surface of the metal not thus protected.

Table V. shows the radiation from zinc, a thick and a thin layer of the oxide, ZnO.

TABLE V.

TABLE VI.

| Exposure. | Freshly Polished Zinc. | Zinc Oxide, Thick Layer. | Zinc Oxide, Thin Layer. | Freshly Polished Silver. | Tarnished Silver. |
|---|---|---|---|---|---|
| 0 hrs. | 100.0 | 100* | 100.0 | 100.0 | 100* |
| 1 hr. | 100.0 | 100 | 100.7 | 99.3 | 100 |
| 2 hrs. | 99.7 | 100 | 100.2 | 99.3 | 100 |
| 3 hrs. | 99.7 | 100 | 99.5 | 98.9 | 100 |
| 4 hrs. | 99.4 | 100 | 100.3 | — | 100 |
| 5 hrs. | 99.3 | — | 100.7 | 99.1 | 100 |
| 6 hrs. | — | — | 99.8 | — | — |

The thick layer of the oxide was formed by igniting the nitrate, as with tin. The thin layer was the result of slow oxidation in air, allowed to proceed for a year and a half.

The irregularities of the figures for the thin layer are probably due to experimental error since but one of the surfaces was obtainable and a fresh surface of the metal had to be used for a "standard." As has been pointed out this may cause some error. No decisive change is shown by either surface. This oxide is quite stable in air.

The radiation from a freshly polished silver plate, and one covered with a dark tarnish, acquired by a month's exposure to the air in the laboratory, are given in Table VI. These results show a slight fatigue for the polished silver and none whatever for the tarnished surface.

Whatever may have been the nature of this tarnish, the fact of its formation indicates that it was a more stable surface, chemically, than the silver. The slight fatigue shown by the fresh metal is stopped by this protecting tarnish.

In addition an aluminium oxide surface, prepared by the ignition of the nitrate, showed no perceptible change during three hours exposure.

A coating of lead peroxide, prepared by electrolysis, did not vary appreciably during four hours exposure.

While five out of the seven metals tested, exhibit variation in radiating power, of the thirteen oxidized surfaces formed on these metals, *none shows fatigue.*

If oxidation does affect the secondary radiation, we might expect an increased effect if an oxidizing atmosphere, such as ozone, be used in place of air.

## METALS AND OXIDES IN OZONE.

The results thus far obtained for some of the metals and their oxides, when exposed in ozone, are given in Tables VII.–XIV. inclusive.

Table VII. shows the fatigue of copper, cuprous oxide and cupric oxide, and it will be observed that the fatigue decreases as the oxidation increases. The fatigue occasioned in the freshly polished copper and cuprous oxide in two hours requires about three and one half times as long to be produced in the higher oxide.

These plates on examination after exposure showed decided changes in the character of the surface. The polished copper had become dull and had ken on the appearance of old copper surface exposed to air for some time. The cuprous oxide had changed from a reddish-brown to about the same appearance as the copper had after exposure. The cupric oxide, originally black, became dark reddish-brown after seven hours in ozone.

Table VII.

Table VIII.

| **Exposure.** | **Freshly Polished Copper.** | **Cuprous Oxide, $\mathrm{CuO}$.** | **Cupric Oxide, $\mathrm{CuO}$.** | **Freshly Polished Iron.** | **Magnetic Oxide, $\mathrm{FeO}$.** | **Ferric Oxide, $\mathrm{FeO}$.** |
|---|---|---|---|---|---|---|
| 0 hrs. | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100* |
| ⅓–½ hr. | — | — | — | 98.6 | 97.3 | 100 |
| 1 hr. | 96.3 | 97.0 | 97.8 | 96.1 | 94.1 | 100 |
| 2 hrs. | 93.3 | 93.5 | 97.3 | 94.6 | 91± | 100 |
| 3 hrs. | 91+ | — | 96.5 | 94.1 | 90.4 | 100 |
| 4 hrs. | — | — | 95.4 | — | — | — |
| 5 hrs. | — | — | 95.9 | — | — | — |
| 7 hrs. | — | — | 93.4 | — | — | — |

The radiations from iron, magnetic oxide and ferric oxide surfaces are given in Table VIII., but show in this case the opposite effect with the lower oxide, that it fatigues about three times as rapidly as the freshly polished surface. Both the iron and the magnetic oxide surfaces showed that further oxidation had taken place; the polished metal had become dull and the blue-black magnetic oxide had taken on a reddish-brown cast. It is probable that the film of higher oxide, formed by the ozone on the magnetic oxide surface, was much thicker than that on the iron, which may account for the greater fatigue. The ferric oxide showed no fatigue nor was its appearance changed by the exposure.

Table IX., giving the results for tin, the "tin oxide" before described, and stannic oxide, shows again a more rapid fatigue for the lower oxide than the pure metal, and no change whatever for the stannic oxide. An inspection of the polished metal and the "tin oxide" surfaces after the test showed the formation of stannic oxide by the ozone, the film formed on the "tin oxide" being appreciably thicker than that on the tin. The stannic oxide surface showed no change in appearance.

The fatigue of tin seems to have the peculiarity of being slow in starting under any conditions, not appearing until the third hour in ozone, while the other metals experience from one to four per cent. fatigue during the first hour of exposure in this gas.

The fatigue for the fresh surface of zinc, and for a surface covered with a layer of zinc oxide, when exposed in ozone, are given in Table X.

TABLE IX.

| Exposure. | Freshly Polished Tin. | "Tin Oxide." | Stannic Oxide, SnO. |
|---|---|---|---|
| 0 hrs. | 100.0 | 100.0 | 100* |
| ½ hr. | — | 99.1 | — |
| 1 hr. | 100.0 | — | — |
| 2 hrs. | 100.0 | 97.9 | 00 |
| 3 hrs. | 98.4 | 97.3 | 100 |
| 4 hrs. | 99.2 | 95.6 | 100 |
| 5 hrs. | 96.8 | 94.2 | — |
| 6 hrs. | 94.1 | — | — |

TABLE X.

| Exposure. | Freshly Polished Zinc. | Zinc Oxide, ZnO. |
|---|---|---|
| 0 hrs. | 100.0 | 100* |
| ½ hr. | 98.2 | — |
| 1 hr. | 96.6 | 100 |
| 2 hrs. | 95.2 | 100 |
| 3 hrs. | — | 100 |
| 4 hrs. | — | — |
| 5 hrs. | — | — |
| 6 hrs. | — | — |

The radiation from the freshly polished surface diminishes rapidly, while that from the zinc oxide surface remains constant.

The results with a freshly polished surface of silver, and a surface coated with silver oxide, are given in Table XI.

Table XII. shows the behavior of aluminium and aluminium oxide surfaces, exposed in ozone.

The silver and aluminium surfaces fatigue, while their oxides show no appreciable variation.

The radiations from lead and lead peroxide are given in Table XIII.

The lead peroxide fatigues at about the same rate as the freshly polished surface during the first hour, and then remains nearly constant. This coating of lead peroxide was porous and undoubtedly allowed the ozone to penetrate to the metal itself, where perhaps some slight further oxidation took place.

All the metallic surfaces after exposure showed evidence of oxidation, the lead especially being badly tarnished.

By comparison of Tables IX., XI. and XIII. with Table I. it will be seen that the fatigue of tin, silver and lead progresses at about twice the rate in ozone that it does in air. Comparison of Tables X. and I. shows that zinc fatigues about fifteen times as fast in ozone as in air.

TABLE XI.

| Exposure. | Freshly Polished Silver. | Silver Oxide, AgO. |
|---|---|---|
| 0 hrs. | 100.0 | 100* |
| ½ hr. | 98.8 | — |
| 1 hr. | 97.7 | 100 |
| 2 hrs. | 97.5 | 100 |
| 3 hrs. | 98.1 | 100 |
| 4 hrs. | 97.5 | — |
| 5 hrs. | 97.0 | — |
| 6 hrs. | 96.9 | — |

TABLE XII.

| Exposure. | Freshly Polished Aluminium. | Aluminium Oxide, AlO. |
|---|---|---|
| 0 hrs. | 100.0 | 100* |
| ⅓ hr. | 97.8 | 100 |
| 1 hr. | 95.7 | 100 |
| 2 hrs. | 95.7 | 100 |

Platinum and palladium plates were exposed in ozone, and as will be seen from Table XIV., their radiations remain constant within the limits of experimental error. These two metals are unaffected by ozone.

TABLE XIII.

| Exposure. | Freshly Polished Lead. | Lead Peroxide, PbO. |
|---|---|---|
| 0 hrs. | 100.0 | 100.0 |
| ⅓ hr. | 98.6 | — |
| 1 hr. | 97.7 | 97.1 |
| 2 hrs. | 95.1 | 97.3 |
| 3 hrs. | 93 ± | 97.0 |

TABLE XIV.

| Exposure. | Platinum. | Palladium. |
|---|---|---|
| 0 hrs. | 100* | 100* |
| 1 hr. | 100 | 100 |
| 2 hrs. | 100 | 100 |
| 3 hrs. | 100 | 100 |

Lastly, a few experiments were tried to determine the radiating powers of these oxidized surfaces compared to the polished metals, and the results for copper and iron and their oxides are given below.

| | | | |
|---|---|---|---|
| Cu | 100 | Fe | 100 |
| $Cu_2O$ | 95 | $Fe_3O_4$ | 94 |
| CuO | 92 | $Fe_2O_3$ | 84 |

The cuprous oxide and ferro-ferric oxide films were very thin, perhaps not over a few wave-lengths of light in thickness, and yet their effect on the secondary radiation is very marked. The same qualitative relations hold for the tin oxide surfaces, and in fact it may be said, that in every case in our experience the radiation decreases as oxidation progresses.

During the course of the experiments a tin and a silver plate accidentally became amalgamated, and I was surprised to note that these mercury-coated surfaces gave much more radiation than the pure metals. Quantitative measurements of this increase in sensitiveness have not yet been made.

The seven (oxidizable) metals, and three lower oxides tested, *all* showed fatigue when exposed in ozone. Of the seven highest oxides, but two showed *any* fatigue and that relatively small. The sensitiveness of the two metals unaffected by oxygen, platinum and palladium, remains constant.

The fatigue suffered by the cupric oxide was probably due to the further oxidation of sub-layers of the lower oxide which we might expect to be present. The change in appearance of the surface indicates some chemical reaction or physical change, which might produce a variation in the radiating power.

The behavior of the lead peroxide may possibly be explained on a similar basis.

## SUMMARY.

Freshly polished surfaces of lead, tin, silver and iron show fatigue in air, while those of aluminium, copper and zinc experience little or no change in their secondary radiation.

None of the oxidized surfaces shows any fatigue in air.

All the metals tried, except platinum and palladium, gave fatigue in ozone, with accompanying change in appearance due to oxidation.

All the lower oxides tested fatigue in ozone.

None of the highest oxides, with the exception of cupric oxide and lead peroxide, shows any change in sensitiveness.

Metals not subject to oxidation do not fatigue in ozone.

The radiating power of surfaces decreases as oxidation progresses.

The results obtained by Professor More have been confirmed with the exception of those for unpolished zinc and freshly polished lead in air, although in either case it would be difficult to say whether or nor the surfaces of the plates were in the same condition in the two experiments.

## CONCLUSION.

It appears from these results that there is a close relation between the behavior of the surface when exposed to X-rays and its "chemical stability."

The theory that fatigue is due to great absorptive power of ozone, when absorbed or adsorbed by the metal, is evidently not applicable to this case, since it does not account for the absence of fatigue from a chemically stable surface.

The results are also against disintegration theories, for particularly in the ozone experiments there were just as many atoms to disintegrate after oxidation as before, in fact a number of oxygen atoms have been added. If a density law of absorption holds, the oxides would probably have less absorbing power than the metal itself, and the decrease in radiation cannot be accounted for by assuming increased absorption of the radiation coming from the body of the metal. This limits the secondary radiation mainly to a surface effect.

The explanation of fatigue upon a basis of chemical change would lead us to expect that the more stable a surface the less the fatigue. This order is followed, without exception, in the results given.

In conclusion, I wish to extend to Professor More my most sincere thanks and appreciation, for his suggestions and direction of this work. Thanks are also due to a grant from the Bache Fund of the National Academy of Sciences which greatly facilitated the work.

UNIVERSITY OF CINCINNATI,
May, 1909.

# THE ROTATORY POWER OF LIMONENE AT LOW TEMPERATURES.[1]

By F. A. Molby.

THE work described in this paper is a continuation of the study of the effect of low temperatures upon the optically active substances, the first results having been published in the January number of the Physical Review of the present year. The former paper contained a description of the apparatus used in the study of quartz. The apparatus used in the study of limonene is essentially the same, namely, a Lippich triple field polariscope of most recent and improved construction (Schmidt & Haensch), graduated in circular degrees and hundredths of a degree, and an iron-constantan thermo-couple, with potentiometer, for measuring temperatures. The range of temperature through which the limonene has been studied is from 20° to — 130°. For the most part the observations end at — 120°.

The limonene studied was procured through the Fritzsche Brothers of New York. It was ordered especially for this work and bears Schimmel and Company's (of Leipsig, Germany) mark "limonene pure." Occasion is here taken to thank the firm for furnishing the special order free of charge. The sample was secured from the manufacturers in July, 1908, and the package was kept unopened until March of this year. It is hoped that by keeping it thus and by using much care to prevent contamination and to prevent oxidation by exposure to the air, the results will be faultless so far as the purity of the specimen can enter into them.

As considerable difficulty was encountered in obtaining a satisfactory tube for observations, due to the very great solvent power of the liquid on all rubber washers or organic seals tried for making joints, the tube used will be described here with a view of a

---

[1] The substance of this paper was in part given at the April meeting of the Physical Society in Washington.

vertical section through the tube and cooling device. The illustration is drawn accurately to scale.

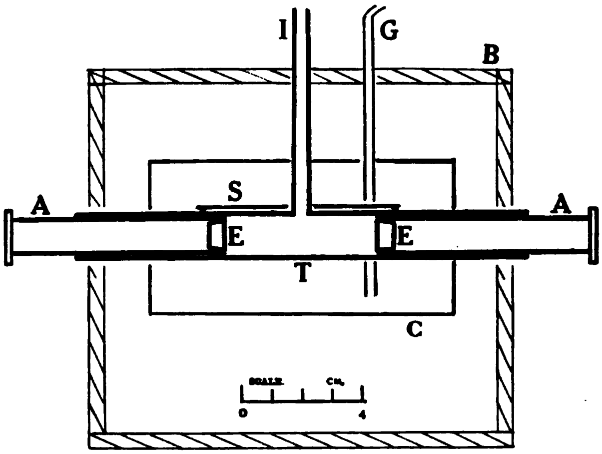


Fig. 1.

*T* is a glass tube 15 cm. long and 1.4 cm. inside diameter. *AA* are two glass air cells 8 cm. long and 1.1 cm. inside diameter. *E E* are of plate glass 8 mm. thick and ground into *AA*, intended for making vacuum cells. *I* is an intake tube through which the thermo-junction is inserted. *C* is a tin can for cold air space around *T*. *S* is a sheet copper saddle leaving 2 mm. air space around *T*. *G* is a glass tube extending into the bottom of the can *C*.

As the limonene soon dissolved out any tallow or wax seal on *EE* and passed through as a vapor, the idea of exhausting the air from the tubes *AA* had to be abandoned. Black paper tubes were fitted inside *AA* with a thin space between them and the glass for phosphorus pentoxide. The outside diameter of the air cells is just small enough to allow them to be slipped inside of *T* so that the joints between *T* and *AA* are 5 cm. from the cooled liquid, and a seal of water glass forms insoluble joints which are not exposed to extreme cooling. The length of tube left between *AA* for the specimen to be observed is 4.97 cm. The wooden box *B* serves as a holder for the tube and for a form into which wool is packed for insulating the tube from the exterior. The inside wall of the box is lined with bright tin to help insulate the tube and cooler.

The glass tube *G* is, during a set of readings, connected to a Dewar flask of liquid air and the cooling of the specimen is accomplished by blowing liquid air over in small quantities. The key of the potentiometer is kept closed with one junction of the thermo-couple in the limonene while the other is in cracked ice (but protected by a thin glass tube), so that small changes of temperature are at once apparent and by careful watching the temperature of the specimen can be held at any temperature with not more than one half of one degree variation from the mean temperature desired. The copper saddle is for the purpose of securing uniform temperature along the specimen, which is very much aided by the fact that the liquid air evaporates at once on reaching the bottom of the tube *G* and causes a good circulation of the air surrounding the tube *T*. The thermo-couple responds so quickly to changes made by the cold air supplied that there is apparently a good thermal conductivity in the limonene and the results also tend to show that the temperature distribution is good.

The calibration of the thermo-couple is somewhat different from that in the work with quartz. The usual second degree equation did not give a curve passing through all of the observed points, so a third degree equation of the form

$$E = at + bt^2 + ct^3$$

was assumed, and the constants were found from the values observed between ice and steam, between ice and $CO_2$ ether paste, and between ice and liquid air (by analysis of air). From these constants intermediate E.M.F.'s were computed and a smooth curve drawn through the plotted points. The table follows as used.

TABLE I.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $T$ | −192° | −182° | −160° | −140° | −130° | −120° | −100° |
| $E \times 10^{-6}$ | −810 | −782 | −714 | −644 | −606 | −567 | −484 |
| $T$ | − 78.3 | − 60 | − 40 | − 20 | 0 | 30 | 99.2 |
| $E \times 10^{-6}$ | −388 | −302 | −205 | −104 | 0 | 160 | 538 |

Observations have been made with several wave-lengths of light. With any one wave-length the observations extended over four or five hours. For the most part ten settings of the polariscope were made for any one temperature of the limonene while the temperature was held practically constant for fifteen or twenty minutes. The temperature was then lowered as many degrees as desired and a number of settings made for the new temperature; the process was then repeated at 15 to 20 degree intervals until a series for the one wave-length was completed. In further work some method might be used by which observations can be made on the several wave-lengths one after the other at a single cooling of the substance. The wave-length 589.2 $\mu\mu$ was obtained by using a sodium light from a double Bunsen burner with a solution of potassium bichromate in the small cell at the front of the polarization apparatus. The wave-length 670.8 $\mu\mu$ was obtained by holding an asbestos sponge in the lower part of the Bunsen flame, the sponge being saturated with lithium chloride; the bichromate solution was removed from the cell in front of the polarizer and two thicknesses of color screens were put in place of the cell, the screens having been first tested with a direct vision spectroscope to see that only the red light of the lithium was transmitted. The wave-length 435.9 $\mu\mu$ was obtained by means of a spectroscope in connection with a Cooper-Hewitt mercury arc lamp; the slit of the instrument was opened quite wide and the images of the slit were formed on a black paper screen with a slit only wide enough to transmit one image at a time. The polarizer was placed about 20 cm. from the slit so that the light diverging from the image illuminated the field of the polariscope satisfactorily. A methyl violet screen was placed between the polariscope and the image to absorb stray light that otherwise came from the other colors of the arc spectrum. The wave-length 546.1 $\mu\mu$ was obtained in the same way as the wave-length 435.9 except that the slit of the telescope was made much narrower and the distance from the image of the slit to the polarizer was somewhat greater. The wave-length 491.6 was too dim to be of any use as obtained in the above manner from the mercury arc. A combination of two screens, an alkali blue screen and a yellow colored one found in a collection of colored screens, was found to absorb completely all wave-lengths of the mercury arc spectrum except 491.6 $\mu\mu$. A condenser system similar to that used on an arc lantern was placed between the mercury arc and the polarizer to make stronger illumination of the polariscope field, while the screens formed a window through a box used for shielding the apparatus and observer from the light of the arc. The several wave-lengths used were carefully identified by a calibrated spectrophotometer, and the purity of the light was in each case tried by the direct vision spectroscope.

Throughout the work the same waterglass joints have been in use so that the tube length is the same for all. The polarization angle of the polarizer, *i. e.*, the angle between the principal plane of the large polarizer and the principal plane of the small polarizers, has been the same.

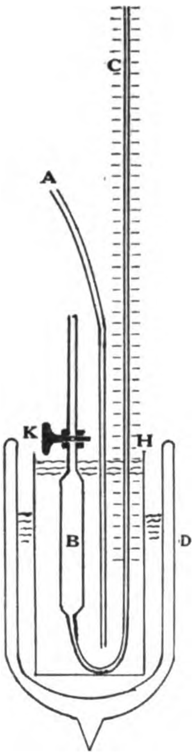


Fig. 2.

In dealing with the temperature effect upon the optical activity of a liquid substance, the change in density of the liquid will increase the number of active molecules in a given linear distance if the liquid becomes more dense or decrease the number if the liquid becomes less dense. As the change in density of the limonene with temperature is such as to produce in part the increased optical activity of the substance observed upon cooling the limonene, the following approximately correct density determinations were made through the range of temperatures used.

The absolute density of the limonene at 20° was found by weighing a 25 c.c. specific gravity flask on a chemical balance. The net weight of distilled water in the flask was 24.917 grams weight in air. The net weight of the limonene in the flask was 21.222 grams weight. The ratio of the two weights gives the relative density of the two materials, which multiplied by the density of the distilled water at 20° gives the absolute density of the limonene to be .850 at 20° C. In order to find the density of the limonene at other temperatures the expansion of a given volume was observed between —115° C. and 30° C. while it warmed up through about four hours of time. The apparatus used for the purpose is shown in vertical section in Fig. 2. A stop-cock *K* was sealed into a glass bulb *B* and to the other end of the bulb was sealed a long tube of small cross-section. This tube *C* was bent as shown in the figure so that the lower part of the U-shaped portion was 48 cm. from the open end of the tube *C*. The cross-section of the tube *C* was .085 sq. cm. With the tube *B* inverted the limonene was forced through the stop-cock into the bulb *B*. The bulb *B* was then inserted in the tin can *H* and the can *H* was filled with petroleum ether, thus surrounding the bulb up to about 2 cm. of the stop-cock. The can *H* was placed in a Dewar cylinder *D* and surrounded with petroleum ether and liquid air. A millimeter scale was attached to the long tube *C* so that the height of the liquid could be read at any temperature. On cooling the limonene the stop-cock was opened and the tube *C* filled to a point opposite the letter *B* in the figure. The petroleum ether was continually stirred by a stream of air forced to the bottom of the can *H* through the tube *A* leading from a tank of compressed air. The top of the whole was covered by loose wool. The temperature of the bath was measured by the thermo-junction inserted in the petroleum ether. Frequent excursions made through the bath with the junction showed the tempera-

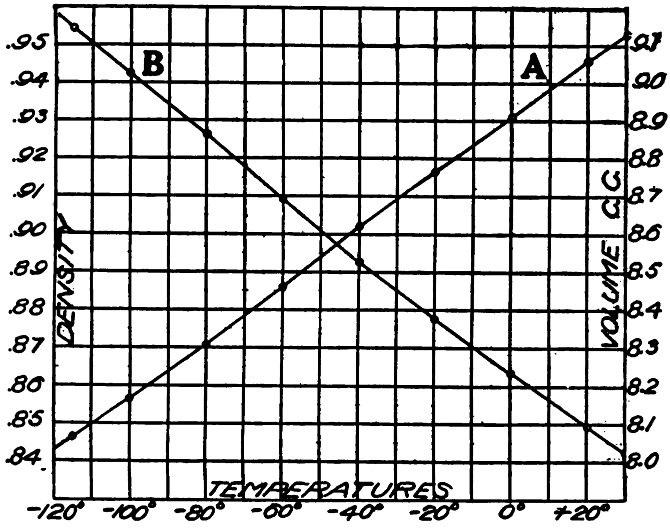


Fig. 3.

ture to be fairly uniform. The volume of the tube *B* and the stem *C* was calibrated later with mercury measured out from pipettes of 5 c.c. capacity and small cross-section so that volumes could be read with reasonable accuracy to .01 c.c.

Curve *A* in Fig. 3 shows the volume of limonene in the apparatus at the various temperatures. The apparent change of volume is somewhat less. A simple calculation using three times the linear coefficient of expansion of glass[1] as volume coefficient gives the volume of glass at – 120° to be less than at 30° by the amount 3 × .000008 × 150 × 8 c.c. or .0288 c.c., while the apparent changes as measured in the tube *C* could be read accurately to .02 × .089 c.c. or about .0018 c.c. The change of volume for the same range of temperature was from the curve *A* equal to 1.1 c.c. so that we have the relative expansion of the limonene about forty times larger than that of glass. If we assume the curve *A* to be a straight line the mean coefficient of expansion of the limonene between 30° and – 120° is

$$B = \frac{9.13 - 8.03}{9.06 \times 150} = .00081,$$

9.06 being the volume at 20° C.

In my experiments I have found the liquid to solidify at – 130° to – 135° C. Its boiling point is 175 or 176 as given by various authorities. It is seen to have a coefficient of expansion several times larger than that of mercury and it is suggested that the substance possesses some if not all of the properties needed to make thermometers with a range of two hundred and eighty or possibly three hundred degrees.

Fig. 3, curve *B*, shows the density of the liquid over the range of temperature used. The density at any temperature *t* was found by the formula

$$D_t = \frac{D_{20^\circ} V_{20^\circ}}{V_t}.$$

$D_{20}$ represents the density at twenty degrees and $V_{20}$ represents the volume at twenty degrees and $V_t$ represents the corrected volume

---

[1] H. G. Dorsey, Phys. Rev., Vol. 25, p. 98.

of the limonene at any temperature $t$ as read from the curve $A$, Fig. 3.

### TABLE II.

*Limonene.*

**Wave-length 435.9 $\mu\mu$.**

| Temp. | Obs. $R$. | $[\alpha]_t$ |
|---|---|---|
| 22°.2 | 98°.0 | 232°.5 |
| − 0.4 | 103.5 | 241.0 |
| − 22.5 | 109.0 | 249.3 |
| − 47.0 | 115.7 | 259.3 |
| − 66.0 | 120.9 | 266.2 |
| − 80.6 | 126.2 | 274.3 |
| − 92.0 | 129.1 | 277.8 |
| −102.0 | 132.5 | 282.7 |
| −115.0 | 137.1 | 289.3 |
| −123.0 | 139.7 | 292.9 |

**Wave-length 491.6 $\mu\mu$.**

| Temp. | Obs. $R$. | $[\alpha]_t$ |
|---|---|---|
| 22°.0 | 73°.6 | 174°.6 |
| − 12.2 | 79.9 | 184.3 |
| − 30.4 | 83.1 | 188.4 |
| − 51.0 | 87.1 | 194.5 |
| − 74.5 | 92.0 | 201.1 |
| − 85.0 | 94.3 | 204.1 |
| −104.0 | 98.9 | 210.7 |
| −120.8 | 102.2 | 214.6 |

**Wave-length 546.1 $\mu\mu$.**

| Temp. | Obs. $R$. | $[\alpha]_t$ |
|---|---|---|
| 20°.5 | 57°.2 | 135°.7 |
| − 0.5 | 60.2 | 140.2 |
| − 20.5 | 63.0 | 144.5 |
| − 40.8 | 66.0 | 148.8 |
| − 54.0 | 68.3 | 152.1 |
| − 67.0 | 70.5 | 155.0 |
| − 79.7 | 73.1 | 159.0 |
| − 89.7 | 75.3 | 163.0 |
| −116.4 | 81.6 | 172.0 |
| −124.5 | 84.2 | 176.0 |

**Wave-length 589.2 $\mu\mu$.**

| Temp. | Obs. $R$. | $[\alpha]_t$ |
|---|---|---|
| 22°.2 | 48°.8 | 115°.9 |
| .0 | 51.5 | 120.0 |
| − 18.0 | 54.0 | 124.0 |
| − 64.0 | 59.9 | 132.1 |
| − 76.4 | 61.9 | 135.0 |
| − 89.0 | 63.8 | 137.7 |
| −103.7 | 66.4 | 141.4 |
| −114.0 | 67.8 | 143.2 |
| −118.0 | 68.3 | 143.8 |
| −123.0 | 69.3 | 145.4 |
| −125.7 | 69.9 | 146.4 |
| −128.0 | 70.0 | 146.3 |

**Wave-length 670.8 $\mu\mu$.**

| Temp. | Obs. $R$. | $[\alpha]_t$ |
|---|---|---|
| 21°.0 | 35°.9 | 85°.1 |
| 4.0 | 37.3 | 87.1 |
| − 9.0 | 38.6 | 89.2 |
| − 24.2 | 39.9 | 91.2 |
| − 41.0 | 41.5 | 93.6 |
| − 61.0 | 43.6 | 96.5 |

**Wave-length 670.8 $\mu\mu$.**

| Temp. | Obs. $R$. | $[\alpha]_t$ |
|---|---|---|
| − 82°.0 | 46°.3 | 100°.4 |
| −102.0 | 49.4 | 105.6 |
| −108.0 | 50.2 | 106.5 |
| −120.7 | 52.1 | 109.4 |
| −123.5 | 52.2 | 109.5 |

The specific rotation of a pure liquid active substance is defined by the equation

$$[\alpha]_t = \frac{(\alpha)_t}{L_t D_t},$$

where $[a]_t$ is the specific rotation, $(a)_t$ is the observed angle of rotation of the plane of polarization, $D_t$ is the density of the liquid at the temperature $t$ and $L_t$ is the length of the column of liquid at the temperature $t$ in decimeters of length.

Table II. gives the observed rotations for the various wave-lengths at the various temperatures, temperature being read from the curve of E.M.F.'s described in connection with Table I. The third column under each wave-length gives the specific rotation of the liquid as computed by use of the formula above. The density of the liquid for use in the formula was read from the curve $B$ in Fig. 3.

The curves in Figs. 4, 5, 6, 7 and 8 are plotted from the values

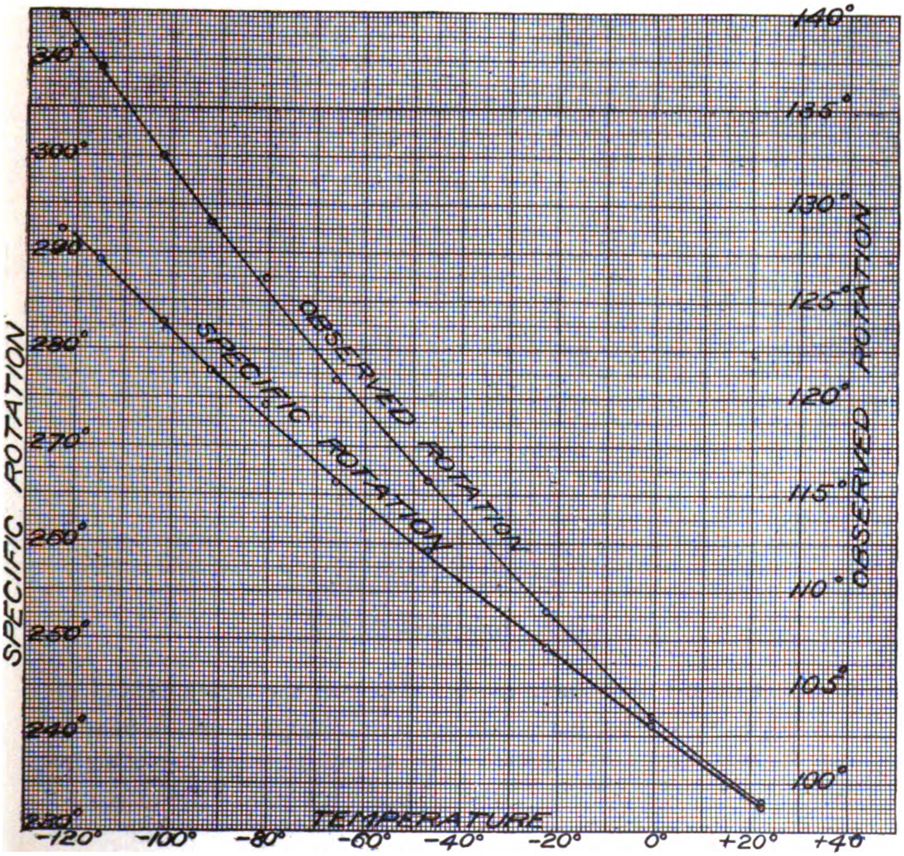


Fig. 4. Wave-length 435.9 $\mu\mu$.

given in Table II. The upper curve in each of the five figures shows the observed rotation for a column of the liquid whose length was 4.97 cm. at 20°, and which changed but little on cooling as the tube was made of glass. The rotations corresponding to the upper curve in each of the five figures are given on the right-hand margin. The rotations corresponding to the lower curve in each figure are given on the left-hand margin. The abscissæ are the temperatures at which observations were made. The two curves are thus given in order to show what part of the observed change in optical activity may be explained by the presence of an increased number of active molecules in a given linear distance, and what part of the increased rotation is due to the increased activity of the individual molecules when they are subjected to a loss of temperature which means, of

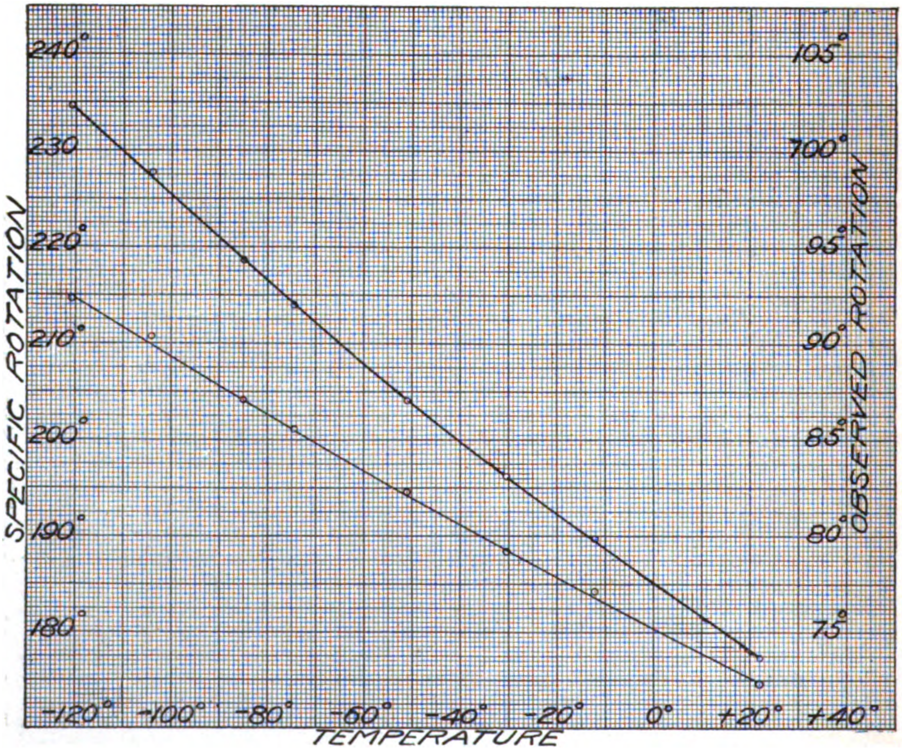


Fig. 5. Wave-length 491.6 $\mu\mu$.

course, a loss of molecular energy so far as molecular vibration due to temperature is concerned. The Fresnel explanation of optical rotation, which looks upon the active substance as being capable

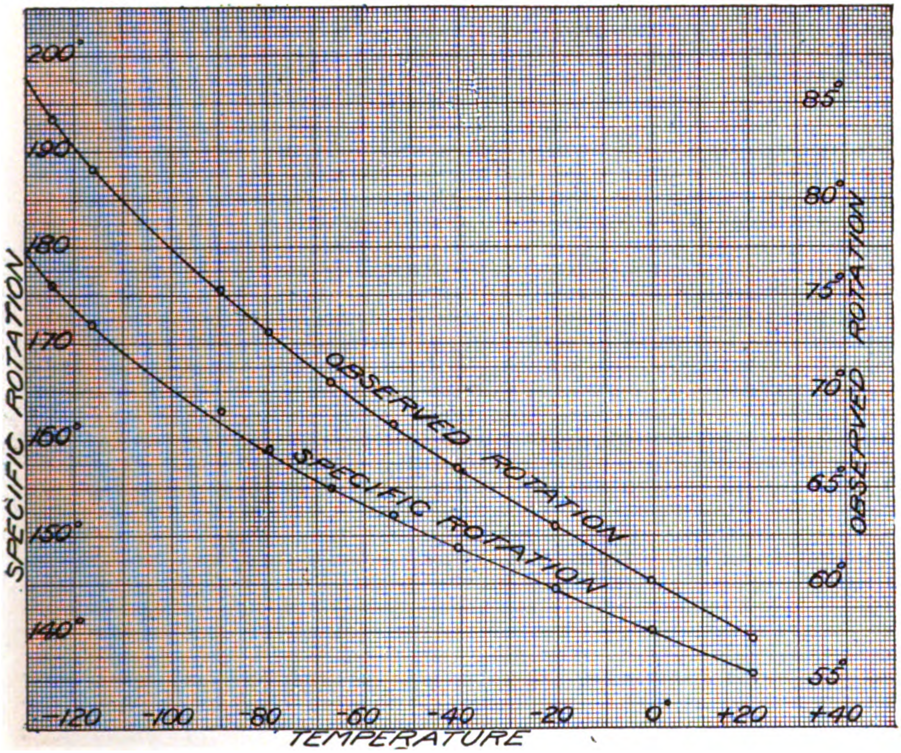


Fig. 6. Wave-length 546.1 $\mu\mu$.

of splitting a beam of plane polarized light into two circularly polarized beams having oppositely directed vibrations which unite to form a plane polarized beam again upon their emergence from the active substance but polarized in a different plane because of the different velocities with which the two beams travel through the substance, is made to explain the phenomena dealt with in this paper by simply saying that the refractive index of the substance, being different at room temperature, for the two circularly polarized beams, is rendered still more different by diminishing the temperature of the substance. Reference to the curve shows that no one of the specific rotation curves for any of the wave-lengths is strictly or even approximately a straight line function of the temperature. Based upon this fact, and also referring to the curve published in the January number of the Physical Review showing that for quartz the rotation temperature curve is not a straight line, the following conclusion is offered. [It should be noted that for quartz the rotation is diminished, not increased, by cooling, but that the flexure of the curve in question is in the same direction as that for the limonene curves.]

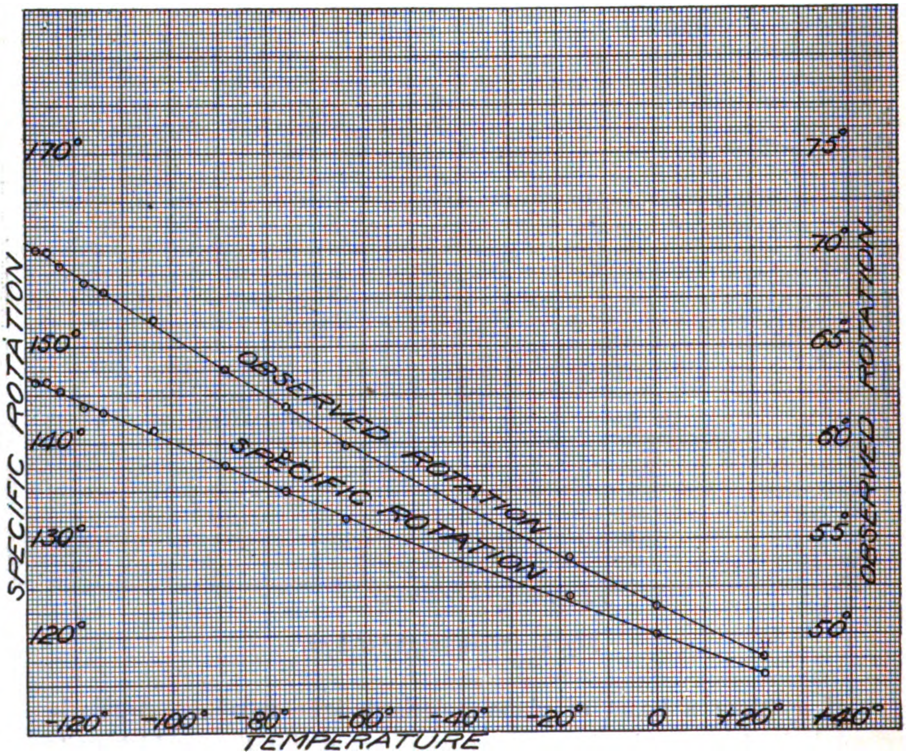


Fig. 7. Wave-length 589.2 $\mu\mu$.

*The indices of the substance for the two circularly polarized beams do not continue to bear the same ratio to each other on cooling the substance.* For the amount of rotation produced is proportional to the length of the linear path through the substance. From this it appears that the amount of rotation is proportional to the difference of phase brought about between the two beams during their passage through the substance. But the phase difference brought about is also proportional to the difference between the indices of refraction for the respective beams. Hence if the index for the one circularly polarized beam bore a constant ratio to the index for the other circularly polarized beam we would obtain a straight line when plotting temperatures and specific rotation for the substance. On the other hand, *the indices of refraction of the substance for the two circularly polarized beams are not increased by the same absolute quantity on cooling the substance*, for if they were the curve between temperatures and specific rotation for a given wave-length would be

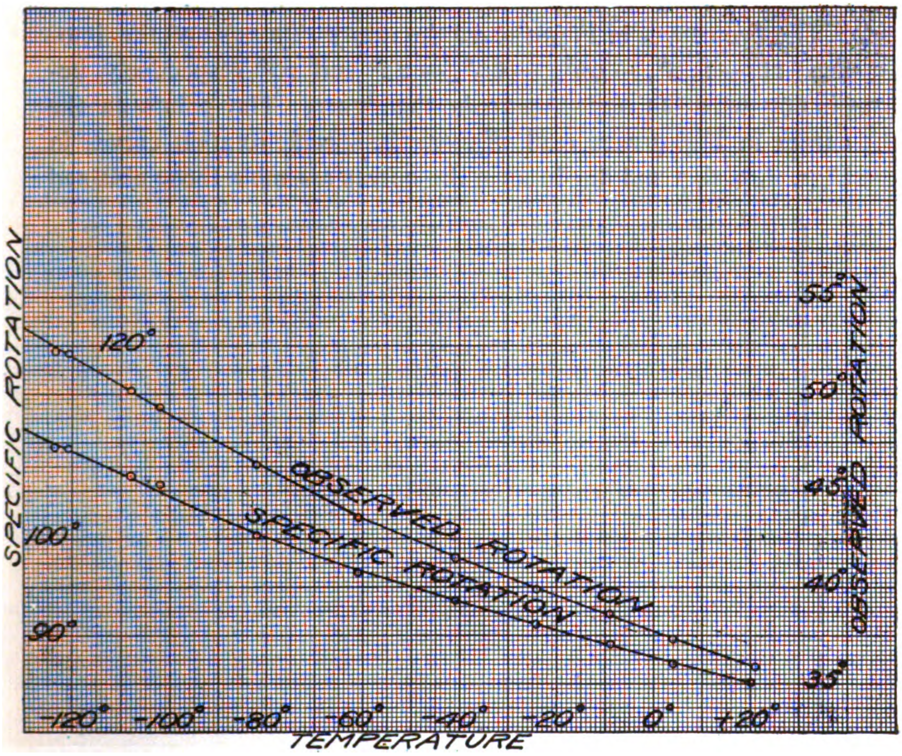


Fig. 8. Wave-length 670.8 $\mu\mu$.

a straight line parallel to the axis of temperature, *i. e.*, a change in temperature would produce no change in the specific rotation of the substance.

The five curves in Fig. 9 show the ***rotary dispersion*** of the limonene at − 120°, − 90°, − 60°, − 20° and 20° respectively, beginning with the upper curve in the figure. The points on the curves in Fig. 9 were read off from the specific rotation curves in Figs. 4, 5, 6, 7 and 8. In form the curves resemble those obtained

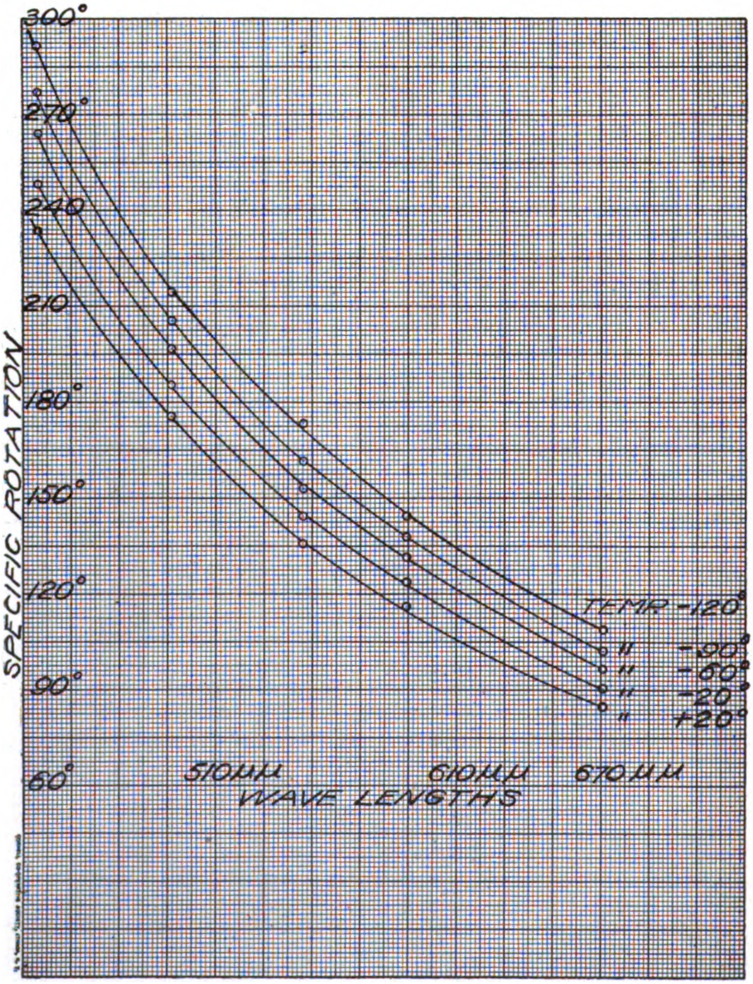


**Fig. 9.** **Rotary dispersion.**

by Mr. P. G. Nutting[1] for several of the substances used by him in his work on ultra-violet rotary dispersion. In comparing them, however, they should be compared only in that part of the spectrum covered in my work. On page 8 of his article is given a curve of rotary dispersion of limonene which he stated was not of the form that should be expected and which is quite dissimilar to the curves here published.

The work will be continued with other of the substances which admit of low temperature study.

Cornell University, June, 1909.

---

[1] Phys. Rev., Vol. 18, July, 1903.

# THE ABSORPTION OF LIMONENE AT LOW TEMPERATURES.

By F. A. Molby and R. C. Gibbs.

IN the previous article Mr. Molby has recorded the effect of low temperatures on the rotatory power of limonene. In obtaining the measurements here described he and Mr. Gibbs used the apparatus previously set up and calibrated by the latter for use in his spectrophotometric studies.[1]

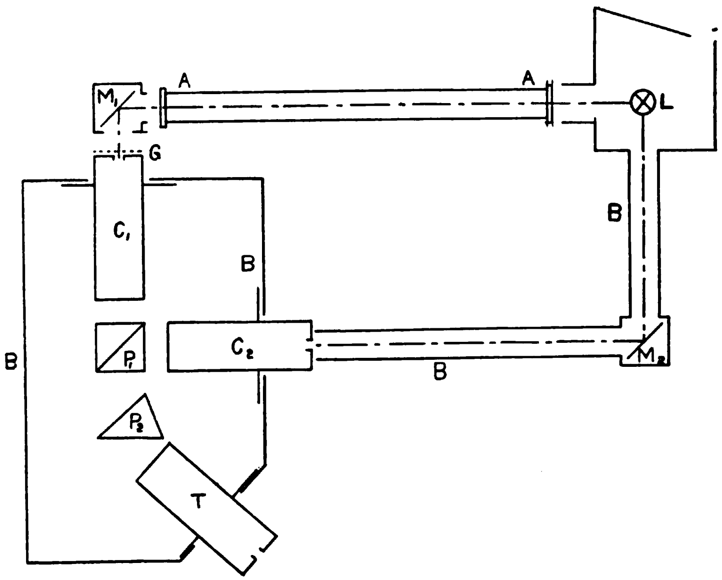


Fig. 1.

Fig. 1 shows the arrangement of the apparatus. The tube containing the limonene is represented by *AA* (the detailed construction and method of mounting this tube are shown in the previous article). Light from an enclosed acetylene flame *L* passed through this tube and was reflected by a glass mirror $M_1$ through the slit of one of the collimators of a Lummer-Brodhun spectrophotometer shown in Fig.

---

[1] Phys. Rev., Vol. XXVIII., May, 1909.

1. A piece of ground glass $G$ was securely fastened to the front of this collimator in order to secure a uniform field. Light from the same flame was reflected by a block of magnesium carbonate $M_2$ through the slit of the other collimator, thus precluding the discrepancies of separate transmission and comparison sources.

A series of readings throughout the spectrum was made with the empty tube in place; the limonene was then poured into the tube and six sets of measurements were made through the spectrum, the first with the temperature of the limonene at 22°, then at − 4°, −30°, − 60°, − 90°, and at − 125° C. Each observer made independent readings for every point measured and the mean of the two observations was used. The curves in Fig. 2 show the ratio of transmitted

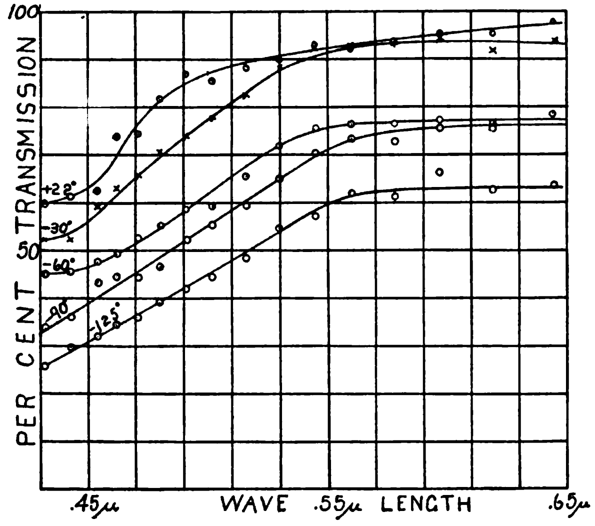


Fig. 2. Showing the percentage transmission at the various temperatures.

light to incident light for all the above-mentioned temperatures except − 4°, which curve, if plotted, would lie between the curves for 22° and − 30°. It has been, therefore, omitted to avoid confusion with these curves. The curves in Fig. 3 show the coefficients of absorption of limonene for the various parts of the spectrum at the extreme temperatures used. These coefficients were obtained from the data of the corresponding curves in Fig. 2, by the use of the equation

$$\beta = \frac{\log I - \log T}{x},$$

where $\beta$ is the coefficient of absorption, $I$ the intensity of incident light, $T$ the intensity of transmitted light, and $x$ the thickness in centimeters of the limonene traversed by the light.

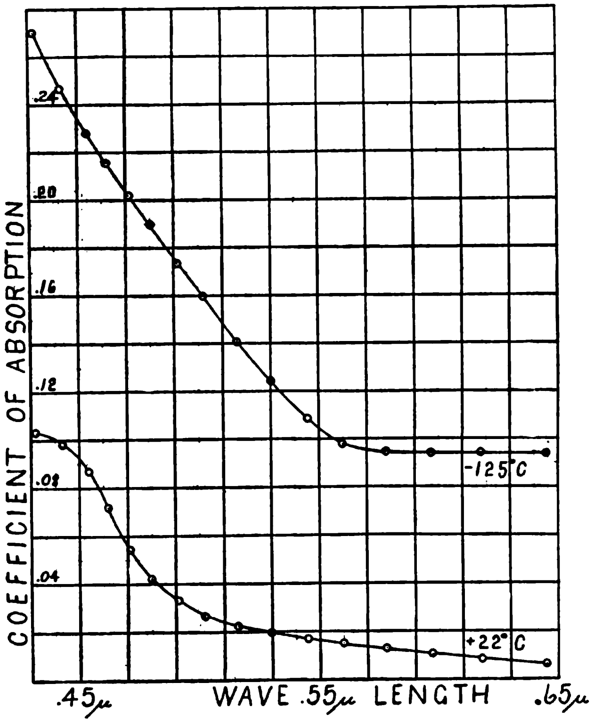


Fig. 3. Showing coefficient of absorption, $\beta$, for 22° and — 125° C.

The curves in Fig. 2 show a considerable increase in the absorption of limonene as the temperature is decreased and that the rate of increase per degree is much larger at the lower temperatures. These curves as also those in Fig. 3 show that the absorption band is considerably broadened in the direction of the longer wave-lengths when the temperature is reduced to — 125° C.

It should be mentioned that all surfaces on which the collection of frost or moisture would result in incorrect readings for the transmitted light, were carefully watched and neither frost or moisture was detected when the above-described measurements were made.

In view of the more or less general law that absorption decreases on lowering the temperature, it seemed advisable to measure the light transmitted through the empty tube at temperatures below that of the room, in order to determine whether the decrease in transmission was to be accounted for in whole or in part by some change in the tube itself. After proper precautions had been taken to eliminate the collection of frost or moisture on the inside surface of the ends of the tube, no material change in the transmission through the empty tube was observed throughout the entire range of temperature above mentioned.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY.

# POLARIZATION OF RÖNTGEN RAYS.

BY WM. R. HAM.

THE ether pulse theory of Röntgen rays would lead us to expect a greater intensity of the rays in a direction at right angles to the kathode stream than in the direction of that stream. All efforts which have thus far been made to detect any such difference by direct observation on the primary rays have given negative results. The latest attempt was made by Haga.[1]

Barkla, however, has concluded from observations on secondary X-rays, that the primary rays are partially polarized, and has investigated qualitatively the effect upon this polarization of variation in the hardness of the tube used. His results have been verified by Haga.[2]

At the Christmas meeting of the American Physical Society in 1907 at Chicago, the author showed that the intensity around certain X-ray tubes varied in such a manner as to indicate some polarization, the intensity being measured in these experiments by the rate of discharge of electroscopes.

Since that time the work on the polarization of X-rays has been carried on continuously with apparatus modified from time to time as sources of error became apparent.

The aim of the investigation has been to answer the following questions:

1. Can a polarization be shown to exist by direct observation on the primary beam itself?

2. If so, is the intensity a maximum in the plane normal to the kathode ray stream?

3. Does the intensity diminish symmetrically on either side of this plane?

---

[1] H. Haga, Ann. d. Physik, 23, 3, pp. 439–444, 1907. Since this paper was written my attention has been called to some very recent work by Herwig (Ann. d. Physik, 29, 7, pp. 398–400, 1909) upon a carbon target, which perhaps constitutes an exception to the above statement.

[2] C. G. Barkla, Phil. Trans., A., 204, p. 467.

4. What effects do changes of potential and of gas pressure have upon the polarization?

5. What effects do absorbing sheets of different materials have upon the polarization?

These questions will be discussed in Part I.

In the course of the investigations it has also been found possible to make accurate determinations of:

6. The coefficients of absorption of certain materials for rays of a given hardness.

7. The variation of these coefficients with the hardness of the tube.

8. The mean depth in the target at which the rays originate.

9. The variation of this mean depth with hardness.

The last four points will be discussed in Part II.

## Arrangement of Apparatus.

The method employed for measuring and controlling potentials was one which was being used in the laboratory by Professor Millikan and which he had found to yield satisfactory results.

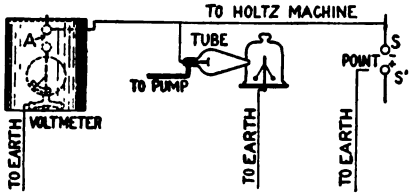


Fig. 1.

In Fig. 1, *SS′* are terminals of a twelve-plate Holtz machine, motor driven. Near the positive terminal is a single point connected to earth. The negative terminal is connected directly to the tube and to *A*.

*A* is an air condenser in series with a Braun voltmeter. The voltmeter reads direct to 10,000 volts. With the air condenser in series, the reading must be multiplied by a constant varying from 6.0 to 6.4 according to the length of the gap which was used. This combination was calibrated by a transformer having a secondary of twelve divisions, a Kelvin E.S. voltmeter being connected across one of these divisions. The calibration was repeated with an absolute electrometer of large size. The arrangement was found quite satisfactory, the readings of the potential between limits of 9,000 and 30,000 volts being probably correct to 2 per cent., and relatively correct to 1 per cent. The tube is placed between the static machine and the voltmeter, its distance from either being about 2 meters.

## Construction of Tubes.

The two types of tubes principally used are illustrated in the diagrams of Fig. 2 (*a*, *b* and *c*).

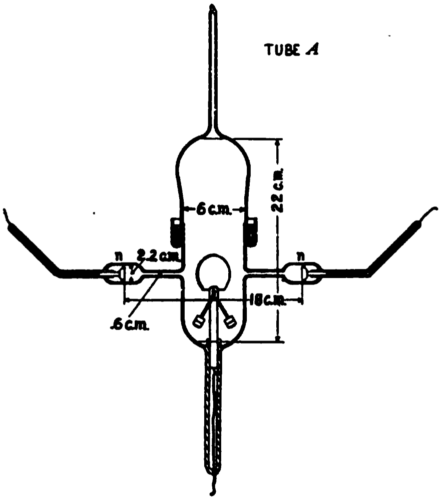


Fig. 2*a*.

The following points are to be noted:

1. It is necessary that the rays emerging from the tube in different directions pass through glass walls of exactly the same thickness. On measuring the thickness of the glass in different parts of the common commercial Rōntgen ray tubes, wide variations were found, so wide in fact as to make untrustworthy any comparison of intensities in various directions. It was also found impossible to correct for the absorption of the glass in such tubes.

Two types of tubes, illustrated by *A* and *B*, were used to eliminate this source of error. *A* is a tube constructed from a selected glass cylinder of very uniform thickness; *B*, a tube with windows *a*, *b*, *c*, *d*, *e*, cut from a given selected sheet of glass of uniform thickness.

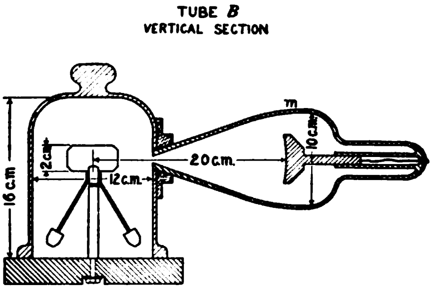


Fig. 2*b*.

2. Secondary rays from the walls of the tube were found to influence the readings of the electroscopes. According to rough measurements, the intensity of these secondary rays, as indicated by their ionizing power, amounted in certain cases to as much as 5 per cent. of the total intensity. This source of

error was almost entirely eliminated in tubes of both types by placing the kathode in side tubes, *m* and *n*, which were screened from the electroscopes by lead plates.

3. Since the Röntgen rays originate within the target, they undergo a certain amount of absorption within the target itself, and in general the amount of this absorption varies with the angle of emergence. This source of error was eliminated by making the target magnetically adjustable and always placing the normal so as to bisect the angle between the two measuring electroscopes and the target. The absorption within the target was thus necessarily the same for the two beams which were being compared.

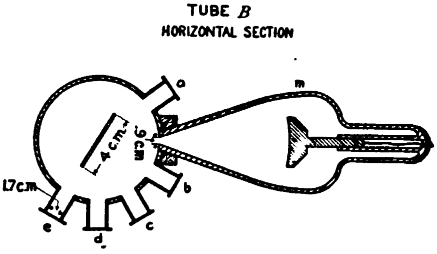
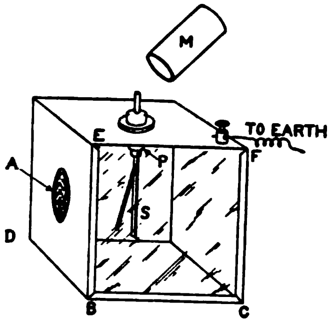


Fig. 2*c*.



BC = BD = BE = 10 CM.
A, ARPERTURE OF CM DIAM. COVERED WITH RICE PAPER
P, AMBER PLUG HOLDING SUPPORTS FOR LEAF
BCFE, GLASS FRONT (SILVERED)
M, BRASS CAP

Fig. 3.

## PROCEDURE.

In general, two electroscopes of the form shown in Fig. 3 were used.

These electroscopes were charged always to the same potential (about 480 volts) by means of a storage battery, and read in the usual way by micrometer microscopes. Their relative rates of discharge were obtained, in the earlier work, by placing them side by side at a considerable distance from an ordinary Röntgen ray tube, taking a discharge, then interchanging their positions and discharging again. Later it was found quite as satisfactory to place one on either side of tube *B* at the same angle to the kathode stream, the target being perpendicular to that stream. This gave a perfectly symmetrical arrangement of apparatus so that the rates of discharge had to be proportional to the discharge constants of the two electroscopes. As a check, an interchange of position was also made, but it was found to be unnecessary. Both methods give consistent determinations of the ratio of the discharge constants of two electroscopes.

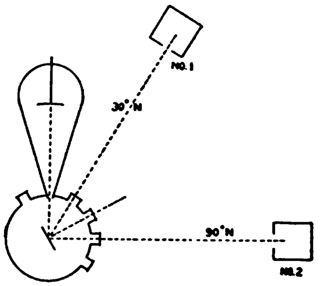


Fig. 4.

In the following discussion the angles mentioned will always be those formed between the line joining the electroscope and the target and the line joining the kathode and the target. In these experiments the kathode stream was directed from west to east; hence the letters N. and S. will be used to show on which side of the kathode stream the electroscope was placed. Thus, 30° N. will indicate a point on the north side of the tube, the angle between the kathode stream and the line from this point to the target being 30°.

## PART I.

1. To determine whether there is a polarization two methods were used.

1st. (See Fig. 4.)

Electroscope No. 1 at 30° N.
Electroscope No. 2 at 90° N.
Normal to target at 60° N.

With various potentials the following results were obtained, using tube *B*.

TABLE I.

| Discharge of No. 2 at 90° N. | Discharge of No. 1 at 30° N. | Per Cent. Polarization. | P.D. |
|---|---|---|---|
| 16.6 | 13.5 | 18.7 | 11,000 |
| 14.95 | 13.0 | 13.0 | 16,000 |
| 15.7 | 13.7 | 13.0 | 16,200 |
| 26.5 | 22.3 | 16.0 | 13,000 |
| 16.8 | 14.0 | 16.8 | 13,000 |

In this case the ratio of the discharge constants of the electroscopes, which will hereafter be denoted simply by (No. 1/No. 2) was 1.185. The actual discharges of electroscope No. 2 have been multiplied by this ratio to obtain the values given in the first column.

The "per cent. polarization" is obtained by dividing the numbers in the second column by those in the first, multiplying by 100 and then subtracting from 100.

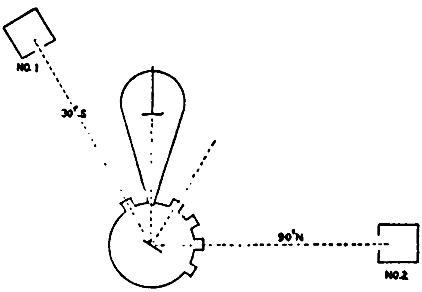


Fig. 5.

2d. (See Fig. 5.)
Electroscope No. 1 at 30° S.
Electroscope No. 2 at 90° N.
Normal to target at 30° N.

### READINGS.

### TABLE II.

| Discharge of No. 2 at 90° N. | Discharge of No. 1 at 30° S. | Per Cent. Polarization. | P.D. |
|---|---|---|---|
| 46.9 | 41.3 | 12.0 | 18,000 |
| 46.0 | 40.0 | 13.0 | 16,100 |
| 32.8 | 26.7 | 18.6 | 11,700 |
| 34.1 | 28.1 | 17.8 | 12,000 |
| 54.8 | 49.6 | 10.2 | 22,000 |
| 40.5 | 33.8 | 16.5 | 13.000 |

It is to be especially noted that the change in intensity between the 90° and 30° points is completely independent of the position of the normal to the target.

*The direction of maximum intensity of the rays has therefore no connection with the angle of incidence of the kathode stream upon the target.*

2 and 3. The questions as to whether there was a maximum at 90° and whether the change in intensity was symmetrical with respect to the 90° point, were investigated at the same time. Having determined the relative intensity at 90° and 30° in the preceding experiments, the following arrangement was used to compare the intensities in the 60° N. and 120° N. positions, first with each other and then with that in the 90° N. position.

1st. (See Fig. 6.)
Electroscope No. 1 at 30° S.
Electroscope No. 2 at 60° N.
Normal to target at 15° N.

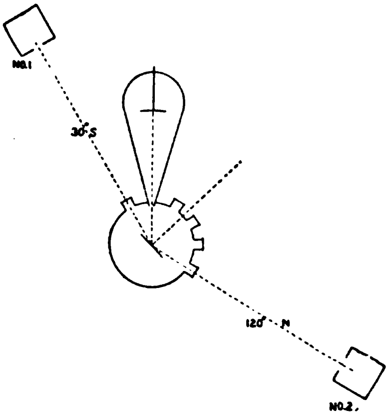


Fig. 6.

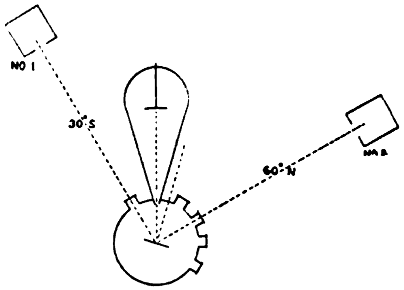


Fig. 7.

2d. (See Fig. 7.)
Electroscope No. 1 at 30° S.
Electroscope No. 2 at 120° N.
Normal to target at 45° N.

RESULTS.

TABLE III.

| Discharge of No. 1 at 30° S. | Discharge of No. 2 at 120° N. | Per Cent. Decrease from No. 2 to No. 1. | P.D. |
|---|---|---|---|
| 29.1 | 33.9 | 14.1 | 12,000 |
| 25.8 | 30.5 | 14.3 | 12,500 |
| 27.1 | 30.5 | 11.2 | 15,600 |
| 33.7 | 37.6 | 10.5 | 16,200 |
| 28.4 | 33.2 at 60° N. | 14.5 | 11,600 |
| 26.4 | 30.6 " " " | 13.8 | 12,200 |
| 35.5 | 39.7 " " " | 10.5 | 15,000 |
| 32.6 | 36.3 " " " | 10.1 | 15,200 |

Comparing the discharges taken at the same P.D. in the upper and lower halves of the above table, it is to be noticed that there is approximate symmetry, and referring to Table II. it will be seen that, between potentials of 16,000 and 12,000 volts, the intensity at

90° N. is greater than at either 60° N. or 120° N. by from 3 per cent. to 4 per cent.

4. To determine the effect of change of potential and gas pressure, the arrangement shown in Fig. 5 was used. The pressure was varied by means of the pump, and the P.D. varied accordingly. The results are represented graphically in plot A (Fig. *7a*). By per cent. polarization is meant here the per cent. change in intensity between 90° and 30°. In obtaining the data for plot A a great number of readings were taken, those in Tables I. and II. being merely samples. The procedure was to obtain the polarization with as great accuracy as possible at a P.D. at which the conditions were found most favorable for consistent results, and then to vary the potential. The discharge was very steady at a P.D. of 12,000 volts, and consequently about forty readings extending over several months were taken at this P.D. The mean polarization at 12,000 volts is slightly less than 18 per cent. The consistency of the results is indicated by the position of the points in the graph. (See Fig. *7a.*)

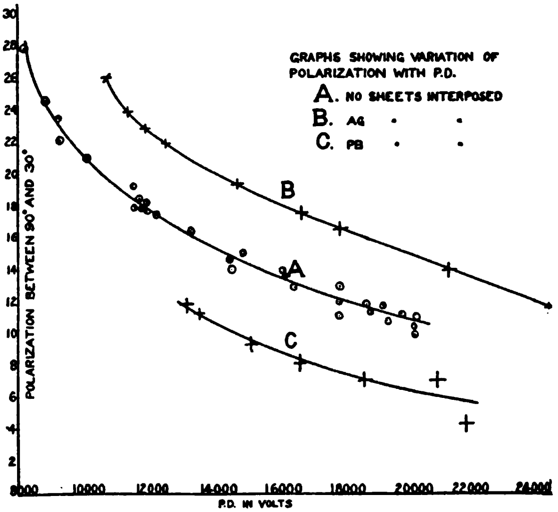


Fig. *7a.*

5. To investigate the effect of absorbing sheets on the polarization, thin sheets of various materials were placed over the windows through which the rays passed to electroscopes No. 1 and No. 2, the

arrangement of apparatus and method of observation being the same as in 4. The sheets placed over the windows were as nearly equal in thickness as possible, but to eliminate the slight irregularities in thickness which were always found to exist, the sheets were interchanged and the mean readings used.

Table IV. shows a typical set of readings taken in order to obtain the relative rate of discharge of electroscope No. 1 with respect to No. 2. This relative rate is indicated in the tables as ratio No. 1/No. 2. This constant is of course changed whenever the electroscopes are changed or a leaf altered.

### Table IV.

| Discharge of No. 1 at 30° S. | Discharge of No. 2 at 30° N. | P.D. | Ratio, $\frac{\text{No. 1}}{\text{No. 2}}$. |
|---|---|---|---|
| 55.4 | 45.4 | 13,200 | 1.220 |
| 52.8 | 43.0 | 13,200 | 1.227 |
| 65.2 | 53.8 | 15,000 | 1.230 |
| 67.5 | 55.5 | 14,400 | 1.217 |
| 67.0 | 55.3 | 13,800 | 1.212 |
| 38.0 | 31.1 | 12,000 | 1.223 |
| 41.5 | 33.9 | 11,700 | 1.225 |
| | | | Mean, 1.222 |

The data obtained upon the effect of absorbing sheets on the polarization is shown in Tables V., VI., VII. and VIII., and in plots B and C (Fig. 7 *a*).

### Table V.

*Effect of interposing Ag sheets .100 mm. thick in path of beams.*

| Ratio, $\frac{\text{No. 1}}{\text{No. 2}} = 1.22$ | Discharge of No. 1 at 30° S. | Discharge of No. 2 at 90° N. | No. 2 Times Ratio. | P.D. | Per Cent. Polarization. |
|---|---|---|---|---|---|
| Sheets interposed. | 39.7 | 45.9 | | 12,600 | |
| Sheets interchanged. | 44.9 | 42.4 | | 12,600 | |
| Total discharge. | 84.4 | 88.3 | 107.7 | 12,600 | 21.8 |
| Sheets removed. | 62.8 | 58.6 | 71.6 | 16,000 | 13.6 |
| Last reading reduced by plot A.[1] | | | | 12,600 | 16.8 |

*Using Ag sheets .030 mm. thick. New electroscopes.*

| Ratio, $\frac{\text{No. 1}}{\text{No. 2}} = 1.155$ | | | | | |
|---|---|---|---|---|---|
| Sheets interposed. | 38.3 | 42.3 | | 12,000 | |
| Sheets interchanged. | 38.0 | 44.1 | | 12,000 | |
| Total discharge. | 76.3 | 86.4 | 100.0 | 12,000 | 23.7 |
| Sheets removed. | 82.3 | 86.0 | 99.9 | 12,000 | 17.6 |

*New leaf on electroscope No. 2. Same sheets as before.*

| Ratio, $\frac{\text{No. 1}}{\text{No. 2}} = 1.20$ | | | | | |
|---|---|---|---|---|---|
| Sheets interposed. | 45.6 | 49.2 | | 12,000 | |
| Sheets interchanged. | 45.9 | 49.0 | | 12,000 | |
| Total discharge. | 91.5 | 98.2 | 118.0 | 12,000 | 22.5 |
| Sheets removed. | 53.1 | 53.5 | 62.2 | 12,000 | 17.3 |
| Sheets interposed. | 46.0 | 49.2 | | 12,000 | |
| Sheets interchanged. | 43.6 | 49.1 | | 12,000 | |
| Total discharge. | 89.6 | 98.3 | 118.0 | 12,000 | 23.4 |

*Using Ag sheets .067 mm. thick.*

| | | | | | |
|---|---|---|---|---|---|
| Sheets interposed. | 53.9 | 58.0 | | 12,000 | |
| Sheets interchanged. | 56.1 | 61.8 | | 12,000 | |
| Total discharge. | 110.0 | 119.8 | 143.8 | 12,000 | 23.5 |
| Sheets removed. | 62.3 | 63.4 | 76.0 | 12,000 | 18.0 |

Increase in polarization caused by Ag sheets .100 mm. thick, 5.0 per cent.
Increase in polarization caused by Ag sheets .030 mm. thick, 5.4 per cent.
Increase in polarization caused by Ag sheets .067 mm. thick, 5.5 per cent.

## TABLE VI.

*Effect of interposing Pb sheets .038 mm. thick.*

| Ratio, $\frac{\text{No. 1}}{\text{No. 2}} = 1.12$ | Discharge of No. 1 at 30° S. | Discharge of No. 2 at 90° N. | Discharge of No. 2 Times Ratio. | P.D. | Per Cent. Polarization. |
|---|---|---|---|---|---|
| Without sheets. | 50.2 | 53.4 | 59.6 | 13,800 | 15.8 |
| Sheets interposed. | 42.0 | 38.3 | | 13,800 | |
| Sheets interchanged. | 40.4 | 45.3 | | 13,800 | |
| Mean of last two observations. | 41.2 | 41.8 | 46.8 | 13,800 | 11.9 |
| Sheets removed. | 48.2 | 51.2 | 57.3 | 13,800 | 15.9 |

*Using Pb sheets .025 mm. thick.*

| | | | | | |
|---|---|---|---|---|---|
| Sheets interposed. | 40.0 | 43.1 | | 14,100 | |
| Sheets interchanged. | 36.3 | 34.4 | | 14,100 | |
| Mean of last two observations. | 38.2 | 38.7 | 43.4 | 14,100 | 11.6 |
| Sheets removed. | 61.6 | 63.9 | 71.6 | 14,400 | 14.0 |
| Last reading reduced by plot A. | | | | 14,100 | 14.3 |

Decrease in polarization caused by Pb sheets .038 mm. thick, 4.0 per cent.
Decrease in polarization caused by Pb sheets .025 mm. thick, 2.7 per cent.

---

[1] By "reading reduced" is meant that the per cent. polarization at a given P.D. is reduced by means of plot A to a value corresponding to some other P.D., *i. e.*, the last reading in the above table was taken at a P.D. of 16,000 volts. The polarization obtained must be reduced to a value corresponding to 12,600 volts before comparisons can be made with the preceding data.

Plot C shows the effect of changing the P.D. when sheets of lead .05 mm. thick are interposed in the path of the beams to the two electroscopes. The probable error is rather large, due to the slow discharge of the electroscopes and the necessity of making considerable correction for natural leak.

### TABLE VII.

*Effect of interposing Sn sheets of .015 mm. thickness.*

| Ratio, $\frac{\text{No. 1}}{\text{No. 2}} = 1.144$ | Discharge of No. 1 at 30° S. | Discharge of No. 2 at 90° N. | Discharge of No. 2 Times Ratio. | P.D. | Per Cent. Polarization. |
|---|---|---|---|---|---|
| Sheets interposed. | 48.9 | 52.3 | | 12,600 | |
| Sheets interchanged. | 50.0 | 53.4 | | 12,600 | |
| Total discharge. | 98.9 | 105.7 | 120.9 | 12,600 | 18.2 |
| Four discharges, sheets removed. | 193.3 | 203.8 | 233.0 | 12,600 | 17.0 |
| *Sheets increased to .045 mm. thickness.* | | | | | |
| Sheets interposed. | 46.9 | 51.8 | | 14,100 | |
| Sheets interchanged. | 45.3 | 47.6 | | 14,100 | |
| Total discharge. | 92.2 | 99.4 | 113.7 | 14,100 | 19.0 |

In the first of the above cases, an increase in polarization of 1.2 per cent. is shown when the two windows leading to the electroscopes are covered with tin of .015 mm. thickness. On making the thickness of tin three times as great, the increase became about 3.9 per cent. This was found to increase but slightly with the further addition of sheets. With sheets of .060 mm. thickness, the increase seemed to coincide with that caused by silver, and no change was observed with a greater thickness. With the thickness greater than .045, however, the results became less reliable, because of the long time required for a discharge.

In none of the above cases is there a change which is larger than the experimental error.

From the four preceding tables it is seen that, of the substances tried, tin, silver and lead caused a change in the polarization. The maximum effect of tin and silver, as well as the change in polarization with variation of P.D., is shown in plot B, and the effect of lead sheets of .05 mm. thickness in plot C. The effect on the polarization of increasing the thickness of the absorbing sheets is shown in Tables V., VI. and VII.

## Table VIII.

*Effect of interposing paper* 1.0 *mm. thick.*

| Ratio, $\frac{\text{No. 1}}{\text{No. 2}} = 1.22$ | Discharge of No. 1 at 30° S. | Discharge of No. 2 at 90° N. | Discharge of No. 2 Times Ratio. | P.D. | Per Cent. Polarization. |
|---|---|---|---|---|---|
| Sheets interposed. | 26.8 | 26.0 | | 12,600 | |
| Sheets interchanged. | 30.0 | 31.3 | | 12,600 | |
| Weighted total discharge | 53.0 | 53.4 | 65.1 | 12,600 | 18.7 |
| *Effect of interposing paper* 3.0 *mm. thick.* | | | | | |
| Sheets interposed. | 44.7 | 44.7 | | 12,000 | |
| Sheets interchanged. | 34.0 | 33.7 | | 12,000 | |
| Weighted total discharge. | 68.0 | 67.7 | 82.4 | 12,000 | 17.5 |
| *Effect of interposing glass sheets respectively* .772 *and* .762 *mm. thick.* | | | | | |
| Sheets interposed. | 41.6 | 41.8 | | 12,000 | |
| Sheets interchanged. | 44.4 | 44.1 | | 12,000 | |
| Total discharge. | 86.0 | 85.9 | 104.8 | 12,000 | 18.0 |
| Sheets removed. | 44.0 | 43.7 | 53.3 | 12,000 | 17.9 |
| *Effect of interposing Al sheets* .04 *mm. thick.* | | | | | |
| Ratio, $\frac{\text{No. 1}}{\text{No. 2}} = 1.12$ | | | | | |
| Sheets interposed. | 51.8 | 55.0 | | 13,800 | |
| Sheets interchanged. | 52.1 | 55.5 | | 13,200 | |
| Mean of last two observations. | 51.9 | 55.3 | 61.9 | 13,500 | 16.2 |
| Sheets removed. | 48.5 | 51.4 | | 12,600 | |
| " " | 51.1 | 53.3 | | 12,300 | |
| " " | 53.0 | 56.4 | | 12,600 | |
| " " | 44.6 | 46.5 | | 12,600 | |
| Total discharge. | 194.2 | 207.6 | 232.5 | 12,500 | 16.5 |
| Last reading reduced by plot A. | | | | 13,500 | 16.0 |

Two remarkable results are noticed in these tables: (1) that lead has an effect on the polarization opposite to that of tin and silver, (2) that in the cases of tin and silver, a maximum effect is obtained at a given thickness of sheets and no further increase of polarization is produced by the use of thicker sheets.

No limit to the decrease in the polarization produced by lead was reached.

## Discussion of Results.

The results of 1, 2, 3 and 4, made by direct observation on the primary rays, confirm completely the conclusions regarding the polarization of the primary rays arrived at by Barkla, and recently confirmed by Haga, from a study of observations on the secondary rays. They indicate that Röntgen rays consist, in part at least, of ether pulses such as would be caused by accelerations given to kathode ray particles on entering a target.

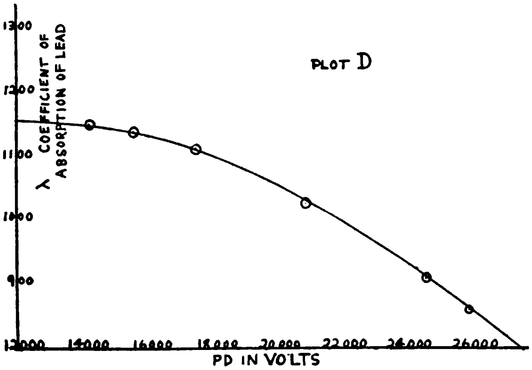


Fig. 7*b*.

In explanation of the fact that the polarization is decreased when the rays pass through lead and increased when they pass through tin and silver, and of the further fact that the polarization cannot be increased beyond a certain limit by using increasing thicknesses of silver and tin, the following suggestions are offered.

Secondary rays are set up when Röntgen rays strike or pass through a metal plate.[1] Since the rays under consideration have originated within the target and in emerging pass through its upper layers, it is to be expected that these rays would consist of a mixture of primary and secondary rays.

According to the work of Barkla[2] the secondary rays from most

---

[1] G. C. Barkla, Phil. Mag., 11, pp. 812–828, 1906.

[2] Barkla and Sadler, Phil. Mag., 14, pp. 408–422, 1907. G. C. Barkla, Phil. Mag., 16, pp. 550–684, 1908.

metals except tin and silver, have the same intensity in all directions, *i. e.*, are unpolarized. Therefore if the secondary rays from the lead target emerge from the tube and cause ionization in the electroscopes, the polarization of the total beam from the target will be less than if secondary lead rays were not present. In case the windows are covered with sheets of any material which has a different absorption coefficient for the two types of rays, a change in the amount of polarization between 90° and 30° will take place.

Secondary rays are particularly penetrating for the same substance as that of the plate in which they are generated.[1] Hence, if they exist outside the tube, it is not surprising that lead, of which all the targets used in these experiments are made, absorbs the secondary rays less strongly than the primary, and hence that lead decreases the polarization, the primary rays alone being originally polarized.

Since paper, glass and aluminium apparently do not change the polarization, it is probable that these substances, at least at the P.D. at which the experiments were made, have about the same absorption for secondary and primary rays from lead.

Silver and tin seem to have a much greater absorption for the secondary rays from lead than for the primary, and if this be true, an upper limit to the polarization would be reached when the secondary rays are practically all absorbed. The ratio of the intensity of these secondary rays to the primary rays at a given P.D. could then be calculated if it be assumed that the secondary rays are completely unpolarized. Thus let

$I_{p90}$ = intensity of primary at 90°,
$I_{p30}$ = " " " " 30°,
$I_s$ = " " secondary.

Let

$$\frac{I_{p90}}{I_{p30}} = a.$$

Let

$$\frac{I_{p90} + I_s}{I_{p30} + I_s} = b.$$

---

[1] G. W. C. Kaye, Roy. Soc. Phil. Trans., Ser. A, 209, pp. 123–151, 1908. G. W. C. Kaye, Cambridge Phil. Soc. Proc., 14, pp. 236–245, 1907. See also footnote 2.

Solving

$$I_s = I_{p90}\left(\frac{a-b}{a(b-1)}\right).$$

If plot B represents the condition in which the secondary rays have all been absorbed, $a$ can be found at once for any P.D. from plot B; $b$ can of course be found from plot A.

In the specific case when P.D. = 12,000 volts,

$$a \doteqdot 1.30, \quad b \doteqdot 1.22$$
$$\therefore I_s = .27 I_{p90}$$

When P.D. = 22,000 volts,

$$a \doteqdot 1.16, \quad b \doteqdot 1.11$$
$$\therefore I_s = .40 I_{p90}$$

On the above hypothesis, then, an idea is obtained of the relative intensity at 90° of the primary and secondary radiation emerging from a tube provided with a lead target.

Unfortunately it was not possible to obtain reliable determinations of the P.D. beyond the points indicated in the plots, owing to the discharge through the tube becoming unsteady. But by using an induction coil, when the tube was hard the polarization was reduced nearly to zero. The curves must come very near the axis at a P.D. probably between 40,000 and 45,000 volts, which means that the X-rays originating under a P.D. larger than 45,000 volts probably show no polarization.

## SUMMARY OF RESULTS IN PART I.

1. A polarization exists and is independent of the position of the normal to the target, as shown by comparing the results of the two methods used in 1.

2 and 3. The maximum intensity is, within experimental error, in the plane through the target normal to the kathode ray stream, and the intensity falls off symmetrically on either side of this plane.

4. The polarization is affected by the P.D. alone and increases with a decrease of P.D., as is to be expected from the ether pulse theory.

5. The interposition of sheets of silver and tin causes an increase in polarization which reaches the same upper limit for both metals. Lead decreases the polarization but no lower limit was observed. Aluminium, paper and glass cause very slight change if any in the polarization.

All the observed results are completely explained if it be assumed that the ordinary X-ray beam consists of a mixture of primary and secondary rays originating in the target, and that the secondary rays are completely unpolarized.

## PART II.

## 6 AND 7. ABSORPTION COEFFICIENTS AND VARIATION OF THESE COEFFICIENTS WITH THE P.D.

### (*a*) *The Coefficient of Absorption of Lead.*

The following arrangement was used in determining the coefficients of absorption of lead and tin. (See Fig. 8.)

Electroscope No. 1 at 30° S.
" " 2 " 30° N.
Normal to target at 0°.

Sheets of the substance under investigation were placed over one of the windows, and the per cent. decrease in discharge rate of the corresponding electroscope was taken as a measure of the absorption.

The thickness was determined by weighing the sheets, measuring the area and assuming the density to be that found in tables. Since relative intensity only is measured, density is not an important point provided it is the same for the sheets used in a given experiment. The sheets were cut from the same large sheet. In the case of lead the sheets were rolled from the same strip of lead of which the target was made.

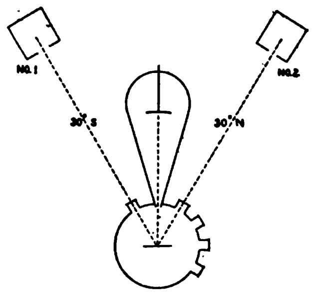


Fig. 8.

Table IX. gives both the coefficient of absorption at a given P.D. as calculated from sheets of lead of various thicknesses and the change in coefficient of absorption with change of P.D. In this table $\lambda x$ is obtained from the equation $I = I_0e^{-\lambda x}$ where $I_0$ is the intensity of the beam before, and $I$ after the sheet under consideration is interposed, and where $x$ is the thickness of the sheet in centimeters.

### TABLE IX.

*Absorption of Pb. Ratio,* $\frac{No.\ 2}{No.\ 1} = 1.096$. *Lead sheet placed over window leading to No. 2.*

| No. 1 at 30° S. | No. 1 Times Ratio. | No. 2 at 30° N. | Thickness of Pb in cms. | Per Cent. Remaining After Absorption. | P.D. | $\lambda X$ | $\lambda$ |
|---|---|---|---|---|---|---|---|
| 113.2 | 124.0 | 8.8 | .00238 | 7.1 | 14,300 | 2.65 | 1120 |
| 104.0 | 114.0 | 21.3 | .00147 | 18.7 | 13,700 | 1.675 | 1139 |
| 102.8 | 112.7 | 50.5 | .00070 | 44.9 | 14,300 | .800 | 1140 |
| 107.6 | 117.9 | 65.9 | .00051 | 56.0 | 14,300 | .580 | 1137 |

*Sheet transferred to window leading to No. 1. Ratio,* $\frac{No.\ 2}{No.\ 1} = 1.100$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 19.5 | 21.6 | 61.5 | .000909 | 35.1 | 14,300 | 1.047 | 1152 |
| 20.5 | 22.7 | 64.3 | .000909 | 35.3 | 15,600 | 1.040 | 1144 |
| 22.2 | 24.6 | 67.3 | .000909 | 36.4 | 17,500 | 1.010 | 1116 |
| 22.6 | 25.0 | 63.8 | .000909 | 39.2 | 21,000 | .935 | 1029 |
| 18.5 | 20.4 | 47.6 | .000909 | 43.7 | 24,800 | .828 | 911 |
| 24.6 | 27.2 | 59.8 | .000909 | 43.8 | 24,800 | .825 | 909 |
| 22.7 | 25.1 | 55.0 | .000909 | 45.7 | 26,000 | .782 | 860 |

The coefficient $\lambda$ seems to be very nearly constant at a given P.D., $\lambda$ perhaps decreasing slightly in the case of the thickest sheet used. At first sight this may seem to be inconsistent with the hypothesis, set up in Part I., of complex radiations emitted by the lead target. The absorption coefficients of lead, however, appear to be so nearly the same for the two radiations that in order to show an appreciable decrease in polarization in the experiments of Part I., it was necessary to use lead sheets of greater thickness than any of these in the tables.

The values of the coefficient of absorption for lead at various potentials are shown graphically in plot D.

### (*b*) *The Coefficient of Absorption of Tin.*

In a similar way the following data with regard to the coefficient of absorption of tin was obtained.

TABLE X.

*Absorption of Sn. Ratio,* $\frac{No.\ 2}{No.\ 1} = 1.100$

| No. 1 at 30° S. | No. 1 Times Ratio. | No. 2 at 30° N. | Thickness of Sn in cms. | Per Cent. Remaining After Absorption. | P.D. | $\lambda X$ | $\lambda$ |
|---|---|---|---|---|---|---|---|
| 59.6 | 65.6 | 39.0 | .00126 | 59.5 | 15,600 | .519 | 412 |
| 110.4 | 121.4 | 43.5 | .00259 | 35.9 | 15,600 | 1.030 | 395 |
| 112.3 | 123.5 | 27.0 | .00399 | 21.9 | 15,600 | 1.519 | 381 |
| 177.4 | 195.1 | 27.5 | .00521 | 14.1 | 15,600 | 1.959 | 376 |

It appears from this table that tin differs from lead in not showing a constant coefficient of absorption at a given P.D.

Seitz[1] has found that beyond a given thickness (.0177 mm.) tin shows a constant coefficient of absorption. The above data are not sufficiently extended to enable one to draw that conclusion, but it shows clearly that the change in coefficient grows rapidly smaller as thicker sheets are used. This agrees with the results obtained in Part I. as to the effect of Sn on the polarization.

### (c) *The Coefficient of Absorption of Silver.*

The absorption of a given sheet of silver .030 mm. thick at 30° S. was compared with its absorption at 90° N. The arrangement is the same as in Fig. 5. The following table gives the results.

TABLE XI.

*Absorption of Ag. Ratio,* $\frac{No.\ 2}{No.\ 1} = 1.295.$ *Ag sheet .030 mm. thick placed over window leading to No. 2.*

| No. 1 at 30° S. | No. 1 Times Ratio. | No. 2 at 90° N. | Per Cent. Remaining After Absorption. | P.D. |
|---|---|---|---|---|
| 101.1 | 130.8 | 31.7 | 24.2 | 14,000 |
| *Sheet transferred to window leading to No. 1.* | | | | |
| 22.7 | 29.4 | 131.5 | 22.4 | 14,000 |

The fact that the absorption is greater at 30° S. than at 90° N. fully confirms the results obtained in Part I. in regard to the effect of silver sheets on the polarization.

---

[1] W. Seitz, Ann. d. Physik, 27, 2, pp. 301–310, Oct. 15, 1908.

## 8. Determination of the Mean Depth at Which X-Rays Originate.

In determining the polarization in Part I. it will be remembered that the effect of absorption within the target was eliminated by placing the normal to the target midway between the two electroscopes.

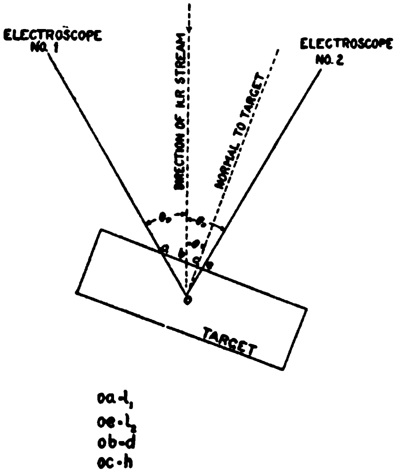


Fig. 9.

If the normal to the target is not midway between the electroscopes, the rays from a given point in the target have to pass through more lead in going to one electroscope than to the other. (See Fig. 9.)

Let *o* (Fig. 9) be the source of some ether pulse.

Let $l_1 - l_2 = x$, the excess of lead, which the pulse has to traverse in passing to electroscope No. 1, over that which it traverses in passing to No. 2.

Let *d* be the distance from *o* to the point at which the kathode ray particle enters the target.

Let *h* be the perpendicular distance from *o* to the surface of the target.

Let $\theta_0$ be the angle which the lines from the electroscopes to the target make with the kathode ray stream.

Let $\theta$ be the angle which the normal to the target makes with the kathode ray stream.

Then

$$x = h\left(\frac{1}{\cos(\theta_0 + \theta)} - \frac{1}{\cos(\theta_0 - \theta)}\right).$$

But

$$h = d\cos\theta,$$

$$\therefore x = d\cos\theta\left(\frac{1}{\cos(\theta_0 + \theta)} - \frac{1}{\cos(\theta_0 - \theta)}\right).$$

Consider two positions, $\theta_1$ and $\theta_2$ of the normal to the target such that for the first $\theta = \theta_1$, for the second $\theta = \theta_2$.

Then

$$\frac{x_1}{x_2}=\frac{d_1\cos\theta_1\left(\frac{1}{\cos(\theta_0+\theta_1)}-\frac{1}{\cos(\theta_0-\theta_1)}\right)}{d_2\cos\theta_2\left(\frac{1}{\cos(\theta_0+\theta_2)}-\frac{1}{\cos(\theta_0-\theta_2)}\right)}.$$

For brevity let the above be written:

$$(1)\qquad \frac{x_1}{x_2}=\frac{d_1\cdot F(\theta_0,\theta_1)}{d_2\cdot F(\theta_0,\theta_2)}.$$

Now let the intensity of the ray at the electroscope No. 2 be $I_0$, and assume that the intensity at No. 1 is $I_0e^{-\lambda x}$, where $x=l_1-l_2$ and $\lambda$ may be called the apparent coefficient of absorption at a given value of $x$.

Then $I_0e^{-\lambda x}/I_0=e^{-\lambda x}=a$ where $a$ is some constant given by the discharges obtained in the electroscopes.

Then $\lambda x=\log 1/a=A$, another constant.

Then, for particular values of $\theta$,

$$\lambda_1x_1=A_1,\quad \lambda_2x_2=A_2,\quad \text{etc.}$$

Or

$$(2)\qquad \frac{x_1}{x_2}=\frac{\lambda_2}{\lambda_1}\cdot\frac{A_1}{A_2}.$$

A comparison of 1 and 2 shows that

$$(3)\qquad \frac{d_1\cdot F(\theta_0,\theta_1)}{d_2\cdot F(\theta_0,\theta_2)}=\frac{\lambda_2}{\lambda_1}\cdot\frac{A_1}{A_2}.$$

Table XII. shows that experimentally

$$\frac{F(\theta_0,\theta_1)}{F(\theta_0,\theta_2)}=\frac{A_1}{A_2},$$

$$\therefore\frac{d_1}{d_2}=\frac{\lambda_2}{\lambda_1}\quad\text{or}\quad d_1\lambda_1=d_2\lambda_2,\ \text{etc.}$$

Outside the tube, $\lambda$ for lead is found to be a constant and the above indicates that $\lambda$ inside the tube is a constant also unless $d$, the mean depth of penetration of the kathode ray stream in the target, varies in such a way as to make the above relation hold when the target is turned through various angles. The last is very improbable. Indeed it would have been very reasonable to assume at the start that $d$ was a constant. Hence the conclusion is reached that $\lambda$ within the tube is constant.

Further, since the value of $\lambda$ for lead is a constant when found by direct observation outside the tube, and appears to be so within the target, it will be assumed that the value of $\lambda$ is the same in both cases.

The following table gives one of the sets of readings taken in the determination of the absorption within the target.

TABLE XII.

$\theta_0 = 30°$. *Ratio*, $\frac{No.\ 1}{No.\ 2} = .910$

| No. 1 at 30° S. | No. 2 at 30° N. | No. 2 Times Ratio. | $\theta$ Position of Normal to Target. | P.D. | $F(\theta_0, \theta)$ | $a$ Per Cent. | Mean $a$ Reduced to 14,000 Volts. See Topic 9. | |
|---|---|---|---|---|---|---|---|---|
| 55.8 | 60.0 | 54.6 | $\theta_1$ 15° S. | 15,600 | .366 | 97.8 | | |
| 51.7 | 55.5 | 50.4 | 15° S. | 16,500 | .366 | 97.7 | | |
| 52.4 | 58.8 | 53.5 | 15° N. | 16,500 | .366 | 97.9 | $a_1$ | $A_1$ |
| 50.9 | 56.9 | 51.8 | 15° N. | 14,000 | .366 | 98.6 | 98.3 % | .017 |
| 48.2 | 55.4 | 50.4 | $\theta_2$ 30° N. | 14,000 | .866 | 95.6 | | |
| 46.8 | 53.7 | 48.9 | 30° N. | 14,300 | .866 | 95.7 | | |
| 49.7 | 52.8 | 48.0 | 30° S. | 14,600 | .866 | 96.1 | $a_2$ | $A_2$ |
| 55.0 | 58.0 | 52.8 | 30° S. | 14,000 | .866 | 96.0 | 95.9 % | .042 |
| 49.3 | 49.0 | 44.6 | $\theta_3$ 45° S. | 15,000 | 2.00 | 90.5 | | |
| 51.4 | 51.0 | 46.6 | 45° S. | 14,300 | 2.00 | 90.7 | $a_3$ | $A_3$ |
| 45.5 | 55.9 | 50.9 | 45° N. | 14,600 | 2.00 | 89.4 | 90.9 % | .095 |

From the above table

$$\frac{F(\theta_0, \theta_1)}{F(\theta_0, \theta_2)} = .424, \quad \frac{A_1}{A_2} = .40$$

$$\frac{F(\theta_0, \theta_1)}{F(\theta_0, \theta_3)} = .183, \quad \frac{A_1}{A_3} = .18$$

$$\frac{F(\theta_0, \theta_2)}{F(\theta_0, \theta_3)} = .433, \quad \frac{A_2}{A_3} = .44$$

These results furnish excellent confirmation of the assertion made concerning equation (3).

The calculation of $d$ is then made as follows:
From Table XII. it will be seen that when

$$\theta = 45^\circ,\ A_3 = .095 = \lambda x_3,$$

$$\lambda \text{ from plot D} \doteqdot 1150,$$

$$\therefore x_3 \doteqdot \frac{.095}{1150} = .000083 \text{ cm.},$$

$$\therefore d = .000041 \text{ cm.}$$

Further, when $\theta = 30^\circ$, $x_2 = .000037$ cm., $d = .000043$ cm.; and when $\theta = 15^\circ$, $x_1 = .000015$ cm., $d = .000041$ cm.

Or, mean $d \doteqdot 4.2 \times 10^{-5}$ cm. at 14,000 volts.

## 9. Variation of Mean Depth with P.D.

To obtain the variation of this mean depth with the P.D. the apparatus was adjusted once for all, with the target turned at any convenient angle, and discharges of the electroscopes were taken at various potentials.

The following table illustrates the results obtained.

Table XIII.

$\theta_0 = 30^\circ$. *Ratio*, $\frac{No.\ 1}{No.\ 2} = .905$

| No. 1 at 30° S. | No. 2 at 30° N. | No. 2 Times Ratio. | $\theta$ Position of Normal to Target. | P.D. | $F(\theta_0, \theta)$ | Mean $a$. | Mean $A$. | Mean $x$. | Mean $d$. |
|---|---|---|---|---|---|---|---|---|---|
| 55.0 | 64.2 | 58.1 | 30° N. | 21,000 | .866 | 94.4% | .056 | $\frac{.056}{1025} = 5.5 \times 10^{-5}$ cm. | $6.4 \times 10^{-5}$ |
| 55.6 | 58.1 | 52.6 | 30° S. | 21,600 | .866 | | | | |
| | | | | 21,300 | .866 | | | | |

This table gives the depth at 21,300 volts. The depth at 14,000 volts is given in the preceding table. The only difference in the two tables is in the P.D.

The effect on the computed value of the mean depth, of covering the windows leading to the electroscopes with similar silver sheets, is shown in the following table.

### TABLE XIV.

$\theta_0 = 30°$. *Ratio*, $\frac{No.\ 1}{No.\ 2} = .895$. *Thickness of Ag. sheets* .030 *mm.*

| No. 1 at 30° S. | No. 2 at 30° N. | No. 2 Times Ratio. | $\theta$ Position of Normal to Target. | P.D. | $F(\theta_0, \theta)$ | Mean *e*. | Mean *A*. | | Mean *d*. |
|---|---|---|---|---|---|---|---|---|---|
| 55.0 | 63.9 | 57.2 | 30° N. | 15,500 | 8.66 | | | | |
| 43.1 | 50.1 | 45.6 | 30° N. | 16,200 | 8.66 | | | $x =$ | |
| 51.7 | 56.0 | 50.1 | 30° S. | 15,900 | 8.66 | | .042<br>at | $\frac{.042}{1140} =$<br>$3.7 \times 10^{-5}$ | $4.3 \times 10^{-5}$<br>cm. |
| 51.7 | 55.8 | 49.9 | 30° S. | 15,900 | 8.66 | 95.9 | 15,900 | cm. | |
| 50.2 | 53.9 | 48.1 | 30° S. | 15,900 | 8.66 | | | | |
| 49.0 | 51.8 | 46.4 | 30° S. | 22,200 | 8.66 | | | | |
| 49.5 | 58.0 | 51.9 | 30° N. | 21,000 | 8.66 | | | $x =$ | |
| 45.3 | 53.3 | 47.7 | 30° N. | 22,200 | 8.66 | 94.9 | .052<br>at | $\frac{.052}{1020} =$<br>$5.1 \times 10^{-5}$ | $5.9 \times 10^{-5}$<br>cm. |
| 49.7 | 58.1 | 52.2 | 30° N. | 21,000 | 8.66 | | 21,500 | cm. | |
| 44.9 | 52.8 | 47.3 | 30° N. | 22,200 | 8.66 | | | | |

From Tables XII. and XIII.

$$\frac{4.2 \times 10^{-5}}{6.4 \times 10^{-5}} = .65, \qquad \frac{14,000}{21,300} = .658$$

And from Table XIV.

$$\frac{4.3 \times 10^{-5}}{5.9 \times 10^{-5}} = .73, \qquad \frac{15,900}{21,500} = .740$$

The difference between the ratio of the depths and the ratio of the potentials is not greater than 2 per cent. in either case.

The last three tables lead to the conclusion that, within experimental error, *the mean depth in the target at which the Röntgen rays originate is proportional to the P.D.*

Further, it is to be noted that the interposition of silver sheets lessens the value ot this mean depth, at a given P.D., as is shown by a comparison of Table XIV. with XII. and XIII.

## Discussion of Part II.

Nearly all the data in Part II. was obtained with a view to finding the mean depth in the target at which Röntgen rays originate. Incidentally some of the conclusions of Part I. were confirmed. Thus it was found by direct experiment that the coefficients of absorption of silver, tin and lead do not depend upon the P.D. alone, but decrease if the thickness is increased, the change taking place, however, very slowly in the case of lead. Furthermore, the absorption of a silver sheet at 90° was found to be less than at 30°.

The most important results of Part II., however, are (1) the proof that the mean depth in the target at which Röntgen rays, produced at a given P.D., originate, is a perfectly definite quantity and quite independent of the angle at which the kathode stream strikes the target; (2) the determination of the relation between this depth and the potential; and (3) the determination of this mean depth when the rays under investigation have passed through absorbing silver sheets.

If the hypothesis be adopted that the X-ray beam consists of a mixture of primary and secondary rays, the latter of which are more readily absorbed by silver than are the former, it is easy to see why the mean depth decreases when the measurements upon it are made through silver sheets. For since the primary X-rays are much more penetrating than the kathode rays which give rise to them, the secondary rays produced by these primary X-rays will obviously come from greater depths than do the primary rays themselves; and hence when the secondary rays from the target are absorbed, the mean depth of the emergent rays is smaller than before they were absorbed.

These considerations make possible the calculation of the depth within a lead target at which complete scattering of the entering kathode rays occurs. Thus it was shown in Part I. that the polarization is practically zero above about 45,000 volts. If then the secondary rays are absorbed by silver sheets and measurements or calculations made of the mean depth at which the primary rays originate for any P.D. above that at which the polarization of the primary has become inappreciable, that depth is itself the depth at which complete scattering[1] occurs for this P.D. For, a completely unpolarized beam is one in which the number of pulses produced by the kathode beam before complete scattering has occurred is negligible in comparison with those produced after complete scattering. Hence, for potentials at which there is no polarization of the primary, the mean depth of emergence of the primary is the depth of complete scattering. It will be seen from Table XIV. that when the secondary rays were absorbed by silver the $d$ corresponding to a potential of 21,500 volts, for example, is $5.9 \times 10^{-5}$ cm. Since $d$ is proportional to the P.D. at say 90,000 volts (which is far above the potential at which there is polarization) $d$ would be

$$5.9 \times 10^{-5} \times \frac{90,000}{21,500} = 2.5 \times 10^{-4} \text{ cm.}$$

Crowther[2] has determined this thickness for complete scattering in the case of $\beta$ rays and from his results the depth for complete scattering in gold at a velocity of $1.6 \times 10^{10}$ (which corresponds to a P.D. of 90,000 volts according to Lorentz's formula) is found to be $2.0 \times 10^{-4}$ cm.

That gold and lead should have the depth for complete scattering at a given P.D. so nearly the same is not surprising in view of their similarity in atomic weights. At any rate the comparison makes it evident that the two entirely different methods give results of the same order of magnitude.

### Summary of Results in Part II.

6. The coefficient of absorption for lead is a constant, within experimental error, at a P.D. of 14,000 volts. The coefficient of absorption of tin decreases slightly as thicker sheets are used in determining it. Silver shows a different absorption at 90° from that shown at 30°.

7. The coefficient of absorption decreases as the hardness of the tube increases.

---

[1] By the depth of complete scattering is meant the depth at which on the average, as many of the kathode particles which have entered the target are suffering accelerations in one direction as in any other.

[2] Scattering of $\beta$ Rays by Matter, J. A. Crowther, Roy. Soc. Proc., Ser. A, 80, pp. 186–206, March 5, 1908.

8 and 9. The mean depth at which the Röntgen rays originate in a lead target is proportional to the P.D. and at a P.D. of 21,500 volts is $5.9 \times 10^{-5}$ cm. which is of the order of the wave-length of sodium light.

The thickness of lead required to produce complete scattering of an X-ray beam is proportional to the P.D. and at a P.D. of 90,000 volts is $2.5 \times 10^{-4}$ cm.

In conclusion the author wishes to express his thanks to Professor Michelson and the staff of the Ryerson Laboratory, particularly to Professor Millikan, under whose direction and with whose continual assistance this research has been conducted.

UNIVERSITY OF CHICAGO,
July 31, 1909.

# PROCEEDINGS

# OF THE

# AMERICAN PHYSICAL SOCIETY.

## MINUTES OF THE FORTY-NINTH MEETING.

A REGULAR meeting of the Physical Society was held in the new Physical Laboratory of the University of Illinois, Urbana, Illinois, on Saturday, November 27, 1909. President Henry Crew presided.

The following papers were presented:

1. Preparation and Properties of the Heusler Alloys. A. A. Knowlton.

2. Magnetic Properties of Heusler Alloys. A. A. Knowlton and O. G. Clifford.

3. The Magnetic Properties of the Heusler Alloys. E. B. Stephenson.

4. The Effect of Temperature on the Magnetic Properties of Electrolytic Iron. Earle M. Terry.

5. The Point Discharge in Air for Pressures Greater than Atmospheric. O. A. Gage.

6. On the Mechanical Equivalent of Heat by a Porous Plug Method. J. R. Roebuck.

7. The Elastic Properties of Platinum-Iridium Wire. Karl E. Guthe.

8. An Apparatus for Measuring Sound. F. R. Watson.

9. Polarization of Cadmium Cells. R. R. Ramsey.

10. A Method for Determining the Optical Constants of Metals Applicable to Measurements in the Infra-red. L. R. Ingersoll.

11. The Absolute Values of the Moments of Elementary Magnets. Jakob Kunz.

12. An Apparatus for Studying Moment of Inertia. C. M. Smith.

13. Some Curious Phenomena Observed in Connection with Melde's Experiment. J. S. Stokes.

14. "Porous Plug" and "Free Expansion" Effects under varying Pressure. A. G. Worthing.

15. The Absorption of X-Rays an Additive Property. R. A. Millikan and E. J. Moore.

16. A Comparison of the Echelon and Diffraction Gratings. H. B. Lemon.

17. The Value of $e$ by Wilson's Method. L. BEGEMAN.
18. The Flow of Energy in an Interference Field. MAX MASON.
19. The Stark Effect with Canal Rays. G. S. FULCHER.

ERNEST MERRITT,
*Secretary.*

## PREPARATION AND PROPERTIES OF THE HEUSLER ALLOYS.[1]

BY A. A. KNOWLTON.

THE fact that practically all workers who had studied these alloys found occasion to complain of the mechanical qualities of their specimens led the writer to undertake a study of the precautions necessary in order to reduce these difficulties to a minimum. The essential features of the treatment finally adopted were: prolonged heating, the use of nearly silica-free crucibles, casting in hot molds and annealing to a temperature of 250° C. The prolonged heating is necessary when preparing the Cu-Mn alloys in order to insure a uniform mixture. Casting and remelting somewhat aids the process. The bath should be in the molten state for at least two hours for satisfactory results. Graphite crucibles must be used because of the rapid action of manganese upon silica. Even with the best graphite crucibles from one tenth to one per cent. of Si is found in the alloy. The hardness and brittleness is very greatly increased if the Si present amounts to more than one half per cent. A considerable advantage in this respect might be gained by the use of magnesium oxide crucibles. The castings were made in carbon moulds heated to a temperature somewhat above the melting point of the alloys and were packed in asbestos so as to cool to 250° C. in about an hour. From that temperature they were chilled by quenching. This procedure was adopted because it was found that annealing greatly improved the mechanical qualities while the magnetic qualities of specimens annealed to temperatures below 200° were usually very poor. Rods 14 cm. long and 8 mm. in diameter made in this way were of good quality both mechanically and magnetically.

From a very large number of castings made, fourteen were selected for examination. The electrical resistance, temperature coefficient of resistance, magnetic properties and change of magnetic properties with heating were studied and compared with the result of a microscopic study of the crystalline structure.

Previous observers have reported specimens which showed strongly marked minima in the resistance temperature curves. None of the writer's specimens showed any such effect. The curves are in all cases

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

straight lines differing from those for the pure metals only in their slope, and in the fact that at high temperatures a large permanent decrease in resistance takes place as in many other alloys. This change, which may amount to as much as 40 per cent. of the original value, takes place slowly at ordinary temperature and is always accompanied by an improvement in the crystalline structure. Any change which is injurious to the crystalline structure causes a marked increase in the resistance. The magnetization reached a saturation value of about 400 c.g.s in the best specimens. The field required for saturation varied considerably with different specimens. No simple relation seems to hold between the proportions of any two constituents and the magnetic properties. The part played by the copper in the alloy appears to be much more important than has been generally assumed. Both the maximum induction and the transformation temperature are greatly affected by the per cent. of copper present. In fact the transformation temperature appears to be almost wholly dependent upon the ratio of copper to manganese. By varying this proportion simply by the addition of copper it was possible to make specimens with transformation points from 0° C. to 300° C.

The results of the heat treatments are substantially the same as have been found by other observers. Transformation from the magnetic to the non-magnetic state extends over a range of about 50° C. in all cases, although the absolute value of this temperature depends upon the composition of the specimen. Annealing at the lower limit of transformation is injurious to the magnetic quality. Annealing at the upper limit improves the magnetic quality. Since the lower and upper limits for average specimens are 220° and 270° this explains the good results obtained by quenching from 250°.

Microscopically the alloy is found to contain three sorts of crystals sharply differentiated from one another. One type is seemingly unaffected by the reagent (a mixture of HCl and $FeCl_3$) and retains its polished surface, one is colored nearly black and the third which appears to partake of the nature of a matrix is yellowish in color. The unaffected crystals are those which possess the magnetic properties as shown by the fact that no specimens from which they were lacking were found to be magnetic while whenever they were present a fair estimate of the value of I at saturation could be made by visual inspection. The area covered by the unaffected crystals was measured in a considerable number of cases and compared with the magnetization. Within the limits of error of the measurement the saturation value of I is directly proportional to the area of the crystals of this sort.

Specimen no. 14, Cu 72.2 per cent., Mn 17.4 per cent., Al 10.3 per cent., forms an exception to the above rule. On the basis of unaffected crystal area, its saturation value of I should be about 225 c.g.s. As a matter of fact the specimen appears not to be appreciably magnetic until cooled to 0° C. This seems to indicate that while the structure of the crystals and the magnetic quality are very closely related, the crystal is not itself the magnetic unit. The difference whatever it may be between no. 14, or in fact any other specimen, in the magnetic state and the non-magnetic state would appear to be due to exceedingly slight differences in molecular grouping since very careful experiments on a number of other specimens have failed to detect any phenomena of the nature of recalesence. It appears certain that the ferro-magnetism of the Heusler alloys depends upon molecular grouping and is therefore a crystaline rather than a molecular property.

PHYSICAL LABORATORY,
UNIVERSITY OF UTAH,
November 6, 1909.

## MAGNETIC PROPERTIES OF HEUSLER ALLOYS.[1]

BY A. A. KNOWLTON AND O. C. CLIFFORD.

DURING the work with these alloys carried on by one of the writers, it became apparent that a further investigation of the dependence of properties on composition was desirable and also that in spite of the extensive work done by others there were many points in relation to hysteresis losses still little understood. For testing a considerable number of rings, mostly of low aluminum content were prepared. These rings were prepared and cast with the precautions outlined in a previous paper except that in some cases the rings were quenched from a high temperature in order to investigate the effect of this procedure. As was to be expected, the rings so treated proved very unsatisfactory from a mechanical standpoint and in most cases broke while being ground. The rings were ground to uniform dimensions, wound with 864 turns of no. 22 copper wire and a secondary of 100 turns and tested by the step by step method. For heating purposes a bath containing about a gallon and a half of gas engine cylinder oil with mechanical stirring device was heated over a gas flame. Seven specimens running from 67.8 per cent. Cu, 27.8 per cent. Mn, 4.4 per cent. Al to 60.3 per cent. Cu, 25.4 per cent. Mn, 14.3 per cent. Al, were tested. Hysteresis curves were taken at 0°, room temperatures, and at several selected temperatures up to transformation. At room temperature the specimens showed wide variations in quality. Ring number one which was cast from scrap many times remelted was by far the best. Since there does not appear to be adequate cause for this either in composition or in heat treatment it is probably largely due

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

to a very perfect mixing of the constituents. On gradually heating this specimen two temperatures (125° and 170°) were found at which very marked increase in induction took place. When annealed from 125° the increase in induction is permanent and is accompanied by a marked decrease in hysteresis. Annealing from 170°, on the contrary, does not leave the specimen in as good a condition as the original as to either maximum induction or hysteresis loss. Annealing from 223° at which temperature it was nearly non-magnetic greatly decreased the induction and the hysteresis losses became relatively enormous. On the contrary quenching from temperatures from 220 to 260 degrees restored the specimen in large measure to its original condition. Other specimens behaved in a similar manner although none of the others seemed as sensitive to heat treatment as no. 1. A curve showing the extent of recovery on chilling from various temperatures indicates a maximum at 243°. After quenching the specimens again showed marked improvement at the same temperatures as before. No such effect occurred after annealing from the neighborhood of 200°. The transformation points were not changed by any of the heat treatments and appeared to depend wholly upon composition as previously found by one of the writers.

While taking the original curves on no. 1, it was noticed that on removal of the magnetizing field, a sharp click was produced, distinctly audible at a distance of ten or fifteen feet. All other specimens were found to give the same result in a degree depending on their magnetic softness. When no. 1 was hardened by annealing, this effect could be observed only by placing the ring to the ear. No sound appears to be produced when the field is applied, wherein it is different from the analogous phenomena in the case of iron.

The microscopic examination previously made by one of us shows that the magnetic quality of these alloys is closely associated with a particular type of crystal which is however itself magnetic only under certain temperature conditions and which probably contains in its structure molecular groups which are the magnetic units. If in addition these crystals contain a matrix of two components one of which softens at 120° and remains soft on annealing while the other softens at 170° but hardens on annealing it seems possible to explain the observed effects as the result of mechanical constraints imposed by the matrix upon the magnetic group.

Physical Laboratory,
University of Utah,
November 6, 1909.

## MAGNETIC PROPERTIES OF HEUSLER ALLOYS.[1]

BY E. B. STEPHENSON.

A SERIES of alloys with varied percentage composition was made in the university foundry and tests were made of the magnetic properties by the magnetometric method. These properties varied with the temperatute at which they were determined and with the previous heat treatment. It was found that when heated to a particular temperature, about 650° C., and allowed to cool in air, the alloys were almost non-magnetic. By use of a method similar to the Frankenheimer method for the calibration of thermocouples, cooling curves were run, and it was found that a transformation point occurs at about the same temperature, accompanied by an evolution of heat.

The cooling curves were those characteristic of solid solutions.

Photo-micrographs were taken of specimens quenched from various temperatures, and additional data were collected in regard to several physical properties.

## THE EFFECT OF TEMPERATURE ON THE MAGNETIC PROPERTIES OF ELECTROLYTIC IRON.[1]

BY EARLE M. TERRY.

THIS paper is a continuation of some work, a preliminary report of which was presented to the Physical Society at the Evanston meeting in November, 1908. As stated there, the purpose was to test Burgess electrolytic iron, which has been shown to be very pure, at temperatures ranging from that of liquid air up to the magnetic transformation point, with a view to determining to what extent the facts already discovered are characteristic of iron itself and what are the effects of impurities.

The following are the principal characteristics of electrolytic iron which have been established by this investigation:

1. Freshly prepared iron is very hard magnetically.
2. Different samples show marked dissimilarities at low fields which disappear upon heating to 1000° C.
3. At high fields, all samples are quite similar, the values obtained for susceptibility being intermediate between those of Leich and Schild.
4. Plunging into liquid air produces no permanent hardening.
5. The retentivity has a maximum in the neighborhood of room temperature.

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

6. Marked softening magnetically occurs at the hydrogen transformation points (300° C. and 500° C.) as shown by depressions in the curves for the variation of hysteresis loss per cycle and field for maximum permeability with temperature. These depressions disappear after annealing.

7. Ferro-magnetism reappears on cooling at the same temperature at which it disappears on heating. This temperature is 785° C.

8. The depression in the permeability-temperature curves obtained by Morris for Swedish iron does not exist for electrolytic iron.

9. The best temperature for annealing is 1100° C.

10. Although when properly annealed it has a lower coercive force and a higher maximum permeability than Swedish iron, its high retentivity causes a larger hysteresis loss.

## AN APPARATUS FOR MEASURING SOUND.[1]

BY F. R. WATSON.

THE apparatus was designed to be used in an investigation of the acoustics of an auditorium, and for that reason it was found desirable to have an instrument that was portable and easily adjusted. A suitable apparatus was set up consisting of a telephone receiver connected to a Duddell thermogalvanometer.

To augment feeble sounds, the principle of resonance was used. Thus the source of sound was adjusted to give a maximum effect with the telephone plate used. Further augmentation of the effect was secured by placing a resonator of suitable dimensions over the telephone plate, by inserting a condenser in the circuit and by adjusting the distance between the telephone plate and the magnet.

By means of this apparatus maxima and minima of sound were easily detected in a small laboratory, and a series of standing waves near a wall measured. The apparatus is not merely a detector of sound, but gives quantitative measurements that may be repeated.

## POLARIZATION OF CADMIUM CELLS.[1]

BY R. R. RAMSEY.

CELLS of the H form were polarized by short-circuiting or by connecting them to an outside E.M.F. until their voltage was very small, 0.1 volt say. By means of a siphon filled with cadmium sulphate solution connection was made to unpolarized cells and it was found that the E.M.F. between the cadmium amalgam terminal of the polarized and the mercury terminal of the unpolarized cell was practically normal

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

while the E.M.F. between the cadmium amalgam terminal of the unpolarized cell and the mercury terminal of the polarized cell was the same as the E.M.F. of the polarized cell.

A cell was short-circuited for some days and part of the mercury was removed with a pipette washed, filtered through a pin hole, and made the mercury terminal of new cell from which the mercurous sulphate was omitted. The E.M.F. of this cell was small and after standing seven months is less than 0.1 volt. The original cell has recovered to practically the normal value.

## ON THE MECHANICAL EQUIVALENT OF HEAT BY A POROUS PLUG METHOD.[1]

BY J. R. ROEBUCK.

A NEW determination of the mechanical equivalent of heat is being carried out by forcing water through a porous plug and reading the drop in pressure and the resulting difference in temperature. To this must be added the temperature drop on free expansion between these pressure limits. This will then give the equivalent in terms of the heat capacity of water and the weight of a mercury column.

The porous plug is of an entirely new form devised by Burnett and Roebuck and described by them in detail in a paper now in course of publication in THE PHYSICAL REVIEW. A porous porcelain cylinder with one hemispherical end is surrounded outside by the fluid under the higher pressure. The fluid passes through the wall and escapes from the open end. A platinum thermometer is placed with its bulb entirely surrounded by the porous wall and the stem surrounded by the escaping fluid. Under these conditions heat flow outward is entirely eliminated by the liquid flow inward and the only radiation and conduction errors are those from the support of the porous plug. These will affect the fluid almost entirely after it has passed the thermometer. The initial temperature is read by a second thermometer placed in the flowing liquid just before entering the plug. The temperature of the entering fluid is fixed by passage through a long coiled copper tube which with the plug carrying tube is immersed in a large electrically heated thermostat provided with specially delicate regulation.

The water under pressure is supplied by a special six plunger pump and the maximum pressure contemplated is 100 atmos. The pressure is regulated by a rotating piston which, by its rise or fall, introduces or cuts out resistance from the armature circuit of the motor driving the pump. For the final pressure reading an open U tube mercury manom-

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

eter after the Kamerlingh Onnes pattern was constructed. This is a series of glass U tubes partly filled with mercury and the pressure is transmitted from one lower mercury surface to the next upper by means of compressed air. The eight tubes will read about 35 atmos. pressure differences.

The apparatus is all assembled and tested and preliminary readings are in progress.

Physics Department,
University of Wisconsin.

## A Method of Determining the Optical Constants of Metals Applicable to Measurements in the Infra-red Spectrum.[1]

By L. R. Ingersoll.

THE ordinary way of determining the refractive index and absorption index of metals is to find the incidence for which circularly polarized light is reduced to plane polarized as well as the azimuth of polarization of the latter. In the present method the process is reversed by determining the angle of incidence and plane of polarization — principal incidence and principal azimuth respectively — for which plane polarized light is reduced to circular. The circularly polarized light is recognized as such by allowing it to pass through a large Wollaston double image prism. Two "sections" at right angles to each other are thus taken of the beam and when these are equal, for all azimuths of the Wollaston, the polarization of the beam must be circular.

The instrumental arrangement is very similar to that already used by the writer in work on magnetic rotation.[2] The two beams from the Wollaston are analyzed with the aid of a sensitive spectrobolometric apparatus and equality of intensity is very accurately determined. The principal azimuths and incidences and hence the optical constants may be determined in this way for a range of spectrum from the *D* lines to $\lambda = 2.2\,\mu$ with an accuracy at least equal to that of the best visual measurements.

Preliminary measurements have been made on steel, nickel, and cobalt. These show refractive indices at wave-length $2\,\mu$, nearly twice as large as for the *D* lines and do not indicate any considerable increase after this wave-length. Reflecting powers computed from these optical constants are in excellent agreement with the measurements of Hagen and Rubens, especially for the longer wave-lengths.

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

[2] See Phys. Rev., Dec., 1906, and Phil. Mag., Jan., 1906, and July, 1909.

## THE ABSOLUTE VALUE OF THE MOMENTS OF ELEMENTARY MAGNETS.[1]

By Jakob Kunz.

THE moments of the molecular magnets play a fundamental rôle in the theory of magnetism. The electron theory of ferromagnetism due to Professor P. Weiss allowsthe determination of the moments of the elementary magnets of iron, nickel and magnetite. It appears that the molecular magnet of iron contains two atoms, that of nickel six, while each molecule $Fe_3O_4$ of magnetite constitutes an elementary magnet. From the moments of the molecular magnets the number of atoms of hydrogen per unit volume can be derived, and the value thus determined coincides with that deduced from radioactive phenomena by Rutherford.

## "THE VALUE OF *e* BY WILSON'S METHOD."[1]

By L. Begeman.

THE article contains three sets of data. The first set contains data of observations taken on clouds composed of singly charged droplets. The second set contains data of observations on layers in the clouds composed of doubly charged droplets and the third set deals with multiply charged droplets.

There are 28 separate determinations of single charges made with fields of different intensity. The average of all the determinations of the first set is $4.67 \times 10^{-10}$ E.S. units for the charge of *e*; the second set $4.69 \times 10^{-10}$ E.S. units and the third $4.65 \times 10^{-10}$.

The article contains also data to show that the errors due to evaporation in the clouds are within the limits of other experimental errors.

## THE ABSORPTION OF X-RAYS AN ADDITIVE PROPERTY.[1]

By R. A. Millikan and E. J. Moore.

THE object of this investigation was to make a careful test of the recently widely circulated assertion[2] that a body becomes more transparent to the X-ray from a given source when it is simultaneously traversed by a transverse X-ray beam from a second source.

The observations presented in this paper show conclusively that the ionizing effects which an X-ray beam can produce after passing through

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909. The paper is published in full in the Journal of the American Medical Association, Vol. LIII., p. 1026, September 25, 1909.

[2] A New Method of Taking X-Ray Pictures, Jour. Am. Med. Assn., April 17, 1909, LII., p. 1246.

an absorbing body are altogether independent of whether or not the body is being simultaneously traversed by other X-ray beams.

## THE ELASTIC BEHAVIOR OF PLATINUM-IRIDIUM WIRE.[1]

BY K. E. GUTHE AND L. P. SIEG.

THE elastic behavior of platinum-iridium wires was studied by the method of torsional vibrations. The vibrating system was set swinging with large amplitudes and allowed to die down. Amplitudes and periods were observed and the conditions of the experiments varied in several ways. The following is the summary of the results.

1. Platinum-iridium wires containing over 30 per cent. iridium show striking peculiarities in their elastic behavior which become the more marked the higher the percentage of iridium.

2. For small amplitudes the period of torsional vibration is nearly proportional to the amplitude; but for larger amplitudes the rate of increase of the period becomes smaller and for very large amplitudes the periods tend towards a constant value.

3. The logarithmic decrement increases with the amplitude, reaches a maximum, after which it decreases again and probably reaches a minimum for very large amplitudes. The maximum is the more pronounced the larger the original distortion.

4. If the wire has been allowed to rest some time before the torsional vibrations are produced the total number of vibrations necessary to bring the system to rest is the smaller the larger the initial amplitude.

5. When tested by the static method the wires closely obey Hooke's law. The torsional moment calculated from the static experiment is larger than the largest torsional moment found by the dynamic method.

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

*Volume XXX.* *February, 1910.* *Number 2.*

THE

PHYSICAL REVIEW.

# THE EFFECT OF TEMPERATURE UPON THE MAGNETIC PROPERTIES OF ELECTROLYTIC IRON.

By Earle M. Terry.

## Introduction.

ALTHOUGH a large amount of study has been devoted to the magnetic properties of iron as a function of temperature, in most cases the purity of the iron has been considered only in a general way, many researches indicating little more concerning its chemical composition than might be inferred from its commercial name. Since the magnetic properties of iron depend to so great an extent upon the presence of certain impurities, notably carbon, silicon, and manganese, and in Burgess electrolytic iron we have an iron of extraordinary purity, it seemed worth while to repeat and extend some of the earlier experiments with a view to determining to what extent the observed phenomena are characteristic of iron itself, and to correlate them with metallurgical changes. To bring to mind the principal facts already discovered, a brief summary of the previous work will be given, the experiments being reviewed in chronological order.

That temperature has a marked influence upon the magnetic properties of iron has been known from the time of Gilbert (1540–1603); who discovered that a needle, when heated red hot, is no longer attracted by the lodestone, but regains this property when the temperature has fallen. This fact was also noted by Canton and studied qualitatively by Saussure. The first, however, to attempt quantitative measurements was Coulomb (1736–1806), who observed the period of vibration of a heated magnetic needle in a field of known intensity.

At the beginning of the nineteenth century, the increasing refinement of instruments for studying the earth's field made it necessary to know the influence of temperature upon the deflections of the magnetometer needle. The first careful experiments were those of Christie who found that the highest temperature to which a magnet can be heated without sensible loss of magnetism is about 100° F., and that the diminution of magnetic moment is not a linear function of the temperature, its rate of change increasing with the temperature. He also found that the moment of a magnet, when placed in a freezing mixture, is increased, returning to its original value when room temperature is restored. The work of Hansteen, Kupffer, Lamont, and others, which followed shortly after, also dealt with permanent magnets.

Rowland,[1] in the years 1873–74, carried out a long series of experiments on iron, nickel, and cobalt, experimenting at two different temperatures, 5° C. and 230° C., in the latter part of his work. It is a curious fact that he missed what would have been a most interesting point, for the magnetic properties of nickel begin to change very rapidly just above the highest temperature he used. He was able to determine, however, that the susceptibility of iron and nickel for small fields is greater at high temperatures than low, while for large fields, just the reverse is true. The fields employed were comparatively small, and insufficient, in the case of cobalt, to reverse the sign of the variation of susceptibility with temperature, which he found to be always positive.

The first attempt to investigate the magnetic properties of iron throughout the ferro-magnetic range of temperature was in 1880, by Baur,[2] of Zurich. Placing a bar of iron, while at a white heat, within a double solenoid, one coil serving to produce a magnetic field, the other measuring the changes in the induced magnetism, he studied the iron as it cooled. Besides confirming the results of Rowland, he found, for small fields, that the rate of increase of permeability is greater for high temperatures than low, while as a critical temperature is approached, the permeability decreases very rapidly for all fields, becoming practically unity.

During the next ten years, the matter was taken up by Perkins,[3]

---

[1] Phil. Mag., vol. 48, p. 321, 1874.

[2] Wied. Ann., vol. 11, p. 394, 1880.

[3] Sill. Journ., vol. 30, p. 218, 1885.

J. Trowbridge,[1] Berson,[2] Tomlinson,[3] and Hopkinson;[4] by far the most important work being done by the latter. Hopkinson used the ring method of Rowland, insulating the windings from each other by asbestos, and estimating temperatures by means of a platinum thermometer. He determined a very complete temperature series of magnetization curves for wrought iron, mild steel, hard steel, nickel, and cobalt. In the case of wrought iron he found that for small fields, *e.g.*, .3 dyne, the temperature at which the rapid rise of permeability begins is about 600° C., that it reaches a maximum at 775° C., and falls abruptly to unity at 785° C., while for large fields, the permeability remained practically constant to 600 C., then fell rapidly to unity. Mild steel showed essentially the same characteristics, except that the permeability had a smaller maximum, becoming unity at 735° C., while for hard steel, the decrease in the maximum was still more pronounced, magnetism disappearing at 690° C. Returning from higher temperatures, for wrought iron, magnetism reappeared at practically the same temperature at which it disappeared, but the steels showed a marked temperature lag which increased with the carbon content.

Between the years 1890 and 1900 a great deal of work was done, and it will be possible here to briefly point out only the additions which were made to the results obtained by Hopkinson. We find a long list of names the most important of which are the following: Du Bois,[5] Wilde,[6] Rücher,[7] Le Chatelier,[8] Curie,[9] Fleming and Dewar,[10] Morris,[11] Fromme,[12] Guilliaume,[13] Dumont,[14] Dumas,[15] Os-

---

[1] Proc. Am. Acad., p. 462, 1885.
[2] Ann. de Chim. et Phys., vol. 8, p. 433, 1886.
[3] Phil. Mag., vol. 25, p. 372, 1888.
[4] Proc. Roy. Soc., vol. 45, p. 318, 1889.
[5] Phil. Mag., vol. 29, p. 293, 1890.
[6] Proc. Roy. Soc., vol. 50, p. 109, 1891.
[7] Lum. Elect., vol. 46, p. 37, 1892.
[8] Compt. Rend., vol. 119, p. 272, 1894.
[9] Journ. de Phsy., vol. 5, p. 289, 1895.
[10] Proc. Roy. Soc., vol. 60, p. 83, 1896.
[11] Phil. Mag., vol. 44, p. 213, 1897.
[12] Wied. Ann., vol. 61, p. 55, 1897.
[13] Journ. de Phys., vol. 7, p. 262, 1898.
[14] Compt. Rend., vol. 126, p. 741, 1898.
[15] Compt. Rend., vol. 129, p. 42, 1899.

mond,[1] and Wills.[2] The physical transformations which take place in iron were worked out with great detail during this decade by the metallurgists Osmond, Howe, Roberts-Austin, Roozeboom and others, a brief review of whose work will be given later.

Of those mentioned above, Curie did perhaps the most important work. He studied not only iron, nickel and cobalt, but para- and diamagnetic substances, covering fields from 90 to 1,350, and temperatures from 15° C. to 1400° C. He found the magnetic transformation point for iron to be 760° C., somewhat lower than Hopkinson's value, and showed that the susceptibility does not become zero at this temperature, as previously supposed, but remains positive, and though small, decreases rapidly up to 950° C., then slowly to 1280° C., when an abrupt increase indicated another transformation point. The transformation near 750° C., was found to be not an abrupt, but a gradual change, depending upon the field, the curves showing the variation of susceptibility with temperature being similar in shape to Amagat's curves for the density of carbon dioxide near the critical point.

With the advent of liquid air, Fleming and Dewar, in 1896, studied the changes which are produced in the magnetic qualities of Swedish iron at low temperatures. In annealed specimens, they found the permeability for all fields less at low temperatures, while for unannealed and hardened iron, it increased as the temperature was lowered, the effect being greater with hardened iron. The hysteresis loss was independent of the temperature. More recently, Honda and Shimizu[3] found for Swedish iron cooled in liquid air that the permeability and hysteresis loss decreased for small fields, but increased for large. These results have been confirmed by Waggoner[4] for low carbon iron.

During the following year, Morris repeated Hopkinson's experiments, improving upon certain details, and found that for Swedish iron, after careful annealing, the permeability-temperature curves show a distinct minimum near 550° C. which was not found for

[1] Compt. Rend., vol. 128, p. 1513, 1899.

[2] Phil. Mag., vol. 50, p. 1, 1900.

[3] Phil. Mag., vol. 10, p. 548, 1905.

[4] PHYS. REV., Vol. XXVIII., p. 393, 1909.

charcoal iron. His principal contribution was the determination of the hysteresis loss as a function of temperature. Kunz,[1] three years earlier, had suggested a linear relationship, but Morris' results show distinct departures from a straight line, there being a well marked depression at 425° C.

In 1900, Wills studied the effect of temperature on the field which gives the maximum permeability, called by Ewing the point of separation between the second and third stages of magnetization. Working with wrought iron and alloys of aluminum, nickel, and manganese, he found that the maximum permeability occurs at lower fields as the temperature is raised, the relation being nearly linear. With alloys, as the relative amount of iron is decreased, the difference of temperature between the disappearance and reappearance of magnetism is increased. This point has been carefully studied for nickel steels by Osmond, Guilliaume, Dumont, and others who have given the complete curves for all percentages of alloy.

## ELECTROLYTIC IRON.

After the work of Wills in 1900, the question of the effects of temperature on the magnetic properties of iron seems to have been regarded as settled, attention having been turned for the most part toward alloys. However, the recent perfection, by Prof. C. F. Burgess,[2] of the University of Wisconsin, of an electrolytic process for the production of very pure iron in large quantities has afforded an opportunity to study further the magnetic properties of this element.

Owing to the difficulties of production, electrolytic iron has hitherto been very little studied magnetically, the work thus far having been confined to room temperatures. It is the purpose of this investigation to determine the magnetic constants of this pure iron at various temperatures, ranging from that of liquid air to the temperature of magnetic transformation, with a view to determining, as far as possible, to what extent the phenomena already observed are characteristic of iron itself, and to point out the effects of impurities.

[1] Electrotech. Zeitsch., 1894, p. 196.

[2] Iron and Steel Mag., vol. 8, p. 48, 1904.

Beetz[1] and Holz[2] speak of "galvanic" iron as being very hard and having a large coercive force, while Kramer[3] says that iron deposited from a solution of ferrous chloride is so soft that it can be cut with a knife. Leick[4] has studied iron deposited from three different solutions: iron sulphate, iron chloride, and iron ammonium sulphate. He concludes that the magnetic properties are practically independent of the salt from which the iron is deposited, that the coercive force is large and the susceptibility small. He gave, however, a very large value for the maximum induction. Schild[5] has recently made a careful study of the magnetic properties of electrolytic iron at room temperature, and obtained values for susceptibility consistently lower than those of Leick. He criticizes the latter's work rather severely, and attributes the discrepancy to the small amount of material used (.004 gr. to .06 gr.). The largest specimen tested by Schild, however, weighed only 3.8 gr.

A detailed account of the production of Burgess electrolytic iron would be beyond the limits of this paper. Those interested in the process are referred to Professor Burgess' original articles. Suffice it here to say that the iron used was twice deposited from an electrolyte consisting of ferrous and ammonium sulphates, the current density being about six amperes per square foot, and requiring about four weeks to produce a plate one inch thick. Plates have been made weighing seventy-five pounds. Chemical analyses have shown that this iron is very pure. In fact the impurities are so slight that special methods have had to be devised to determine them. The average of several analyses is given below and may be taken as a fair indication of the purity.

| Sulphur. | Silicon. | Phosphorus. | Manganese. | Carbon. | Hydrogen. |
|---|---|---|---|---|---|
| ·001 per cent. | .003 per cent. | .004 per cent. | None | .012 per cent. | .072 per cent. |

It will be noted that the amount of hydrogen present greatly exceeds that of all the other impurities combined, and it is to the presence of this element that the characteristic properties of electrolytic iron are usually ascribed.

[1] Phil. Mag., vol. 20, p. 458, 1860.
[2] Pogg. Ann., vol. 151, p. 69, 1874.
[3] Dingl. Polyt. Journ., p. 111, 1861.
[4] Wied. Ann., vol. 58, p. 691, 1896.
[5] Ann. der Phys., vol. 25, p. 594. 1908.

Burgess and Taylor[1] made a preliminary study of single refined iron about two years ago, and found that it is both magnetically and mechanically very hard. During the past year Burgess and Aston have been making an extensive study of the magnetic properties of alloys of this iron, but their results have not been published.

## Method of Testing.

Of the various methods that have been used for magnetic testing, the Rowland ring seemed best suited for the present work. Since it gives a continuous magnetic circuit without a joint of any kind, no correction for demagnetizing effect is necessary, and with a fairly uniform winding of the magnetizing coil, it may be safely assumed that the flux crossing each section of the ring is the same, and that the leakage is negligible. If a D'Arsonval galvanometer is used, this method is independent of the earth's field or any other extraneous effects. It is, however, subject to three sources of error:

I. The field is not uniform across the specimen, and further, the mean field is not the field at the center of the section calculated from the ampere turns.

II. Since permeability is a function of field strength, the mean flux density, calculated from the deflections of the galvanometer, is not the flux density at the center of the section.

III. As pointed out by Ewing[2] and Lord Rayleigh,[3] for small fields and fields for which the differential permeability $dB/dH$ is large, iron changes its magnetic state slowly, sometimes requiring several seconds or even minutes. Since the ballistic method requires that the total change shall take place before the galvanometer swings appreciably from its zero position, a serious error is introduced in testing iron possessing magnetic viscosity.

The first two errors may be reduced to negligible magnitudes by choosing the width of the ring small in comparison to its diameter. Lloyd[4] has shown, for a ring with rectangular cross-section, that

$$\frac{H_0}{H_R} = \frac{1}{2p} \log \frac{1+p}{1-p}$$

---

[1] Proc. Am. Inst. of Elect. Engrs., May, 1906.

[2] Mag. Ind. in Iron and Other Metal, p. 122.

[3] Proc. Roy. Soc., June 20, 1889.

[4] Bull. Bur. Stds., vol. 5, p. 435.

where

$H_0$ = average value of field.

$H_R$ = field at mean radius of ring.

$p$ = ratio of radial width to mean diameter of ring.

The value of "$p$" for the rings which I have used in this work was approximately .084. Substituting this value in the above formula we find

$$\frac{H_0}{H_R} = 1.0024.$$

The calculation of the error mentioned under II. is very complex and has never been handled mathematically. Richter[1] has shown, however, assuming that the permeability is constant across the ring, and that the hysteresis loss varies as the $m$th power of the flux density, that

$$\frac{W_0}{W} = (2p)^{1-m}\left(\log\frac{1+p}{1-p}\right)^m \frac{2-m}{(1+p)^{2-m}-(1-p)^{2-m}}$$

where

$W_0$ = hysteresis loss for uniform flux density,

$W$ = hysteresis loss for actual flux density.

Taking $m = 2$ for unannealed specimens and $m = 1.6$ for annealed, we find 1.0024 and 1.0021 respectively for $W_0/W$.

The effects of magnetic viscosity are nearly eliminated by a method suggested by Taylor.[2] The essential feature of this modification is that all observations are referred to one of the retentivity points of the curve. For instance, to locate the point representing positive saturation, subject the specimen to maximum field; then close the secondary circuit through the galvanometer, and break the primary. The deflection of the galvanometer measures the difference of induction between the saturation and retentivity points. Points on the curve between these two are obtained in a similar manner, the iron having been first carried to saturation and the induction reduced to the desired value without breaking the magnetizing current. For points between retentivity and negative saturation, the iron is first taken to positive saturation as before,

---

[1] Electrotech. Zeitsch., vol. 24, p. 710, 1903.

[2] PHYS. REV., Vol. XXIII., p. 95, 1906.

and at once returned to retentivity by breaking the primary current. The secondary is then closed and the deflection taken when the proper magnetizing current has been applied in the opposite direction. The calculation of the induction in terms of the field, as is ordinarily given, is then merely a matter of changing the origin by the amount of the retentivity, for computing which the necessary data are at hand. The advantages of this method are as follows:

I. Observations on any one point may be repeated as many times as desired.

II. The magnetizing current need be applied for only a few seconds, thus reducing its heating effect.

III. The errors due to magnetic viscosity, while not entirely eliminated, cease to be cumulative as in Rowland's original method, and are reduced to negligible quantities.

## Experimental Details.

The work may be divided into three parts:

I. To test the iron magnetically at different temperatures.

II. To find the best temperature for annealing.

III. To investigate the relation between crystalline structure and magnetic properties.

The procedure in the first part was to run hysteresis and magnetization curves at various intervals from the temperature of liquid air up to that at which iron ceases to be ferro-magnetic. For the lowest temperatures, the rings, after being wound, were dipped for an instant in melted paraffine, then plunged into liquid air, and tested when the temperature had become uniform. To obtain temperatures intermediate between those of liquid air and the room, the following method, which is due to Knipp,[1] was used. A hollow copper sphere, with walls about .8 cm. in thickness, was mounted inside a cylindrical Dewar flask as shown in Fig. 1. To the lower side of the sphere was attached a copper spindle which dipped into liquid air. The Dewar flask was placed upon a

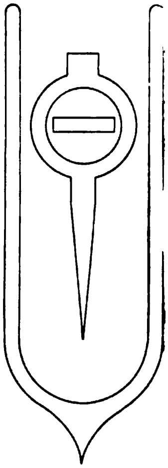


Fig. 1.

---

[1] Phys. Rev., Vol. XV., p. 125.

platform which could be raised or lowered with a slow motion, while the sphere remained fixed. By properly adjusting the depth of the spindle in the liquid air, any desired temperature could be very constantly maintained. While there was an appreciable temperature gradient from top to bottom of the enclosure, the fall of temperature across the ring, which was placed horizontally, never exceeded two degrees.

For the production of high temperatures, an electric furnace of the Heræus type was used. This consisted of a porcelain tube 35 cm. long, 5 cm. internal diameter, wound with platinum foil 2.5 cm. in width. Outside of this, leaving an air space of .7 cm. was another tube surrounded by magnesia packing, the whole being mounted in upright asbestos boards fastened to a slate base. A current of 15 amperes gave a temperature of 1200° C.

For the measurement of temperature, two thermo-elements whose E.M.F.'s were determined by means of a specially constructed low resistance potentiometer, and an optical pyrometer were used. One of the couples, a copper-constantin junction, used for temperatures below that of the room, was standardized at the temperature of freshly prepared liquid air, — 188° C., the melting point of ice, and the boiling point of water. Its indications are believed to be correct to .5° C. The other thermo-element was a platinum-platiniridium junction and was used from room temperature up to 1000° C. In its standardization the following temperatures were used: melting point of sulphur, 444.6° C.; melting point of antimony, 631° C.; and melting point of silver, 962° C. It is believed to be correct to 1° C. for the lower range, and to 2° C. for the upper. The optical pyrometer was used above 1000° C. It was checked against a standard couple and its indications may be relied upon to 5° C.

Making the rings proved to be a matter of some difficulty, since freshly deposited iron is so hard and brittle that it cannot be cut with a tool. The rings had to be cut from the deposited plate by means of a rotating tube fed with emery dust, after which they were mounted in a lathe and ground into shape, the process requiring from two to three days for each ring. The dimensions of the rings were, roughly, as follows: external diameter, 3.5 cm., internal diameter, 3 cm.; height, .4 cm.

One of the chief difficulties encountered was the problem of insulation between primary and secondary windings, since most substances which are good insulators at ordinary temperatures begin to conduct when heated, and a very slight leak is sufficient to give the galvanometer an uncertain drift, rendering accurate work impossible. The method finally adopted was to use rings cut from thin magnesia plates prepared by roasting the oxide between two plates of graphite in an electric furnace at a temperature of about 1700° C. In order to insure insulation between turns, a paste, made of fine magnesia powder and alcohol, was worked into the spaces, which, when dry, could be kept solid by careful handling.

Another problem was the protection of the rings against oxidation. Since the magnetic properties of iron oxide are very different from those of metallic iron, and the cross-section of the specimen was always small, even a thin film would result in serious error. The first attempt to overcome this was to fill the furnace with an inert gas; but it was difficult to close the ends sufficiently tight, and even then the water of crystallization of the mica, upon which the primary was wound, given up on heating, produced considerable oxidation. This difficulty was finally met by plating the rings electrolytically with copper. It was found that with a plate .7 mm. in thickness, a ring could be heated several times in perfect safety.

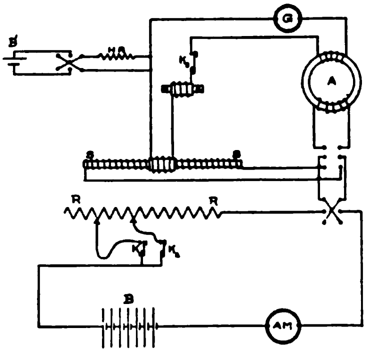


Fig. 2.
Electrica connections.

The electrical connections are shown in Fig. 2.

*A* = ring under test.

*RR* = control rheostat for primary circuit.

*SS* = standard solenoid for calibrating galvanometer.

*NS* = magnet for galvanometer control.

*B'* = battery for compensating thermals in galvanometer circuit.

The primary consisted usually of fifty turns of No. 18 copper wire, while the secondary was a three strand cable of No. 40 platinum wire, from twelve to fifteen turns being used, depending upon the cross-section of the ring. The galvanometer was of the D'Arsonval type, having a period of fifteen seconds, a resistance of seven ohms, and when closed through the secondary circuit, was nearly aperiodic. With a scale distance of two meters, its calibration curve, for deflections up to 20 cm., departed from a straight line by less than .5 per cent. The resistance of the magnesium oxide being very high at 800° C., the low resistance of the secondary circuit rendered errors due to leakage between turns extremely small.

## Results.

Ten rings in all have been studied, being cut from two different plates which I shall call *A* and *B*. The iron in these two plates was quite different, *A* having a finer crystalline structure and being much harder than *B*, both mechanically and magnetically. Rings cut from the same plate were quite similar, the slight discrepancies being doubtless due to the difference in depth in the plate from which they were taken. Since the deposition of a single plate is a matter of several weeks, the unavoidable changes in acidity of bath and strength of current would necessarily result in non-uniformity of deposit. The results obtained are summed up in the following tables and curves.

Table I.

*Plate A—Unannealed. Permeability as a Function of the Temperature for Different Fields.*

| Temp. | Field Strength. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | .5 | 1 | 2 | 3 | 4 | 5 | 10 | 20 | 30 | 40 | 100 |
| 25 | 160 | 170 | 200 | 240 | 295 | 400 | 638 | 540 | 440 | 360 | 174 |
| 97 | 200 | 200 | 250 | 280 | 338 | 430 | 680 | 560 | 443 | 362 | 173 |
| 205 | 260 | 250 | 275 | 326 | 390 | 510 | 740 | 580 | 450 | 358 | 172 |
| 295 | 340 | 350 | 380 | 433 | 500 | 640 | 790 | 590 | 448 | 358 | 172 |
| 400 | 370 | 380 | 424 | 500 | 638 | 820 | 896 | 592 | 445 | 358 | 177 |
| 505 | 440 | 470 | 570 | 756 | 1,150 | 1,340 | 1,020 | 650 | 470 | 375 | 170 |
| 595 | 700 | 780 | 1,050 | 1,800 | 1,820 | 1,670 | 1,090 | 640 | 465 | 362 | 155 |
| 655 | 900 | 1,180 | 2,500 | 2,430 | 2,130 | 1,860 | 1,120 | 635 | 441 | 338 | 146 |
| 700 | 2,400 | 4,200 | 3,500 | 2,690 | 2,200 | 1,890 | 1,080 | 580 | 398 | 302 | 121 |
| 750 | 5,600 | 4,400 | 2,870 | 2,130 | 1,700 | 1,420 | 760 | 390 | 262 | 197 | 95 |
| 765 | 7,200 | 4,700 | 2,670 | 1,900 | 1,480 | 1,220 | 650 | 336 | 226 | 170 | 70 |
| 780 | 4,800 | 2,800 | 1,550 | 1,080 | 850 | 690 | 360 | 180 | 120 | 92 | 55 |

### TABLE II.

*Plate A—Unannealed. Variation of Hysteresis Constants with Temperature.*

| Temperature. | Energy Loss in Ergs. | Maximum Induction $H=140$. | Retentivity. | Coercive Force. |
|---|---|---|---|---|
| 25 | 41,400 | 18,200 | 9,560 | 7.53 |
| 97 | 37,200 | 18,100 | 9,300 | 7.30 |
| 205 | 32,700 | 18,100 | 8,700 | 6.50 |
| 295 | 27,700 | 18,050 | 8,050 | 5.55 |
| 400 | 20,600 | 17,800 | 7,800 | 5.00 |
| 505 | 14,800 | 17,600 | 7,050 | 3.20 |
| 595 | 8,800 | 15,700 | 7,030 | 2.04 |
| 655 | 5,180 | 13,900 | 6,040 | 1.42 |
| 700 | 3,590 | 12,300 | 4,600 | 1.01 |
| 750 | 1,980 | 9,000 | 2,700 | .35 |
| 765 | 920 | 7,260 | 2,300 | .22 |

### TABLE III.

*Plate A—Annealed at* 800° *C. Permeability as a Function of Temperature at Different Fields.*

| Temp. | Field Strength. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | .2 | .5 | 1 | 1.5 | 2 | 3 | 10 | 60 |
| −190 | 60 | 90 | 100 | 140 | 190 | 430 | 1,130 | 286 |
| −121 | 65 | 120 | 250 | 250 | 700 | 1,000 | 1,200 | 284 |
| − 60 | 70 | 190 | 400 | 400 | 1,100 | 1,800 | 1,300 | 283 |
| 23 | 80 | 260 | 500 | 870 | 1,900 | 2,600 | 1,400 | 281 |
| 102 | 100 | 400 | 800 | 2,200 | 3,150 | 3,130 | 1,420 | 280 |
| 203 | 160 | 600 | 1,200 | 4,000 | 4,350 | 3,630 | 1,430 | 275 |
| 308 | 200 | 800 | 1,900 | 5,400 | 4,850 | 3,760 | 1,440 | 270 |
| 403 | 260 | 1,200 | 5,500 | 5,740 | 4,900 | 3,780 | 1,360 | 270 |
| 498 | 425 | 2,600 | 7,000 | 6,140 | 5,160 | 3,830 | 1,320 | 266 |
| 547 | 900 | 4,000 | 7,000 | 6,000 | 5,050 | 3,710 | 1,290 | 265 |
| 596 | 1,200 | 4,400 | 6,800 | 5,300 | 4,950 | 3,610 | 1,290 | 255 |
| 642 | 1,800 | 5,600 | 6,600 | 5,740 | 4,800 | 3,500 | 1,240 | 242 |
| 688 | 2,600 | 6,400 | 6,300 | 5,540 | 5,500 | 3,230 | 1,170 | 220 |
| 724 | 3,200 | 8,400 | 6,100 | 4,800 | 4,050 | 2,950 | 1,040 | 186 |
| 752 | 5,000 | 9,200 | 6,100 | 4,600 | 3,650 | 2,600 | 880 | 153 |
| 781 | 2,400 | 4,200 | 2,800 | 2,140 | 1,700 | 1,260 | 430 | 78 |

In addition to those already given, permeability-field strength curves at different temperatures were plotted for each sample, but these have not been reproduced. The field which gives maximum permeability was determined from them, and its variation with temperature is given in Table VII. and plotted in Fig. 12.

### Table IV.

*Plate A—Annealed at 800° C. Variation of Hysteresis Constants with Temperature.*

| Temperature. | Ergs per Cycle. | Maximum Induction $H = 90$. | Retentivity. | Coercive Force. |
|---|---|---|---|---|
| − 190 | 26,900 | 17,600 | 11,300 | 4.00 |
| − 120 | 24,150 | 17,600 | 12,300 | 3.55 |
| − 60 | 21,200 | 17,500 | 13,200 | 2.97 |
| 23 | 17,490 | 17,400 | 13,600 | 2.31 |
| 102 | 14,320 | 16,900 | 13,440 | 1.93 |
| 203 | 9,670 | 16,150 | 12,920 | 1.48 |
| 308 | 7,630 | 15,960 | 12,400 | 1.39 |
| 402 | 6,990 | 15,900 | 11,400 | 1.32 |
| 498 | 5,080 | 15,840 | 9,930 | .87 |
| 550 | 4,260 | 15,640 | 9,340 | .86 |
| 596 | 3,420 | 14,880 | 8,290 | .83 |
| 640 | 3,040 | 14,390 | 7,860 | .75 |
| 685 | 2,640 | 13,050 | 6,800 | .60 |
| 724 | 1,610 | 11,090 | 5,280 | .44 |
| 752 | 1,020 | 8,760 | 3,470 | .25 |
| 768 | 564 | 5,140 | 1,030 | .14 |

### Table V.

*Plate B—Unannealed. Permeability as a Function of Temperature for Different Fields.*

| Temp. | Field Strength. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | .2 | .5 | 1 | 2 | 4 | 7 | 10 | 100 |
| − 190 | 150 | 200 | 300 | 350 | 550 | 760 | 750 | 177 |
| − 120 | 200 | 240 | 350 | 425 | 675 | 840 | 823 | 177 |
| − 61 | 280 | 360 | 400 | 520 | 850 | 960 | 885 | 177 |
| 23 | 350 | 440 | 520 | 740 | 1,125 | 1,130 | 980 | 178 |
| 97 | 400 | 480 | 640 | 850 | 1,340 | 1,185 | 984 | 177 |
| 195 | 550 | 520 | 750 | 1,025 | 1,410 | 1,200 | 984 | 176 |
| 297 | 600 | 600 | 800 | 1,075 | 1,440 | 1,200 | 995 | 176 |
| 392 | 800 | 700 | 900 | 1,150 | 1,460 | 1,200 | 1,000 | 176 |
| 496 | 900 | 800 | 1,000 | 1,350 | 1,500 | 1,230 | 1,015 | 172 |
| 550 | 1,050 | 1,100 | 1,260 | 1,650 | 1,700 | 1,310 | 1,050 | 170 |
| 605 | 1,200 | 1,300 | 1,500 | 2,200 | 1,770 | 1,340 | 1,070 | 156 |
| 655 | 1,500 | 2,000 | 2,900 | 3,325 | 2,450 | 1,610 | 1,280 | 143 |
| 704 | 2,500 | 3,600 | 4,700 | 3,750 | 2,370 | 1,550 | 1,150 | 122 |
| 740 | 8,500 | 7,200 | 5,800 | 3,750 | 2,170 | 1,340 | 955 | 102 |
| 762 | 10,000 | 7,600 | 4,300 | 3,150 | 1,740 | 1,020 | 716 | 72 |
| 775 | 10,500 | 6,200 | 4,000 | 2,400 | 1,340 | 810 | 560 | 58 |

To study the effect of annealing at temperatures higher than 800° C., the following method was used. A cylindrical box with a tight fitting cover was turned from a bar of Swedish iron in which the ring to be tested was packed in finely powdered electrolytic iron. The ring could then be heated to any desired temperature with practically no oxidation, and at the same time be free from contamination with carbon. When the desired temperature was

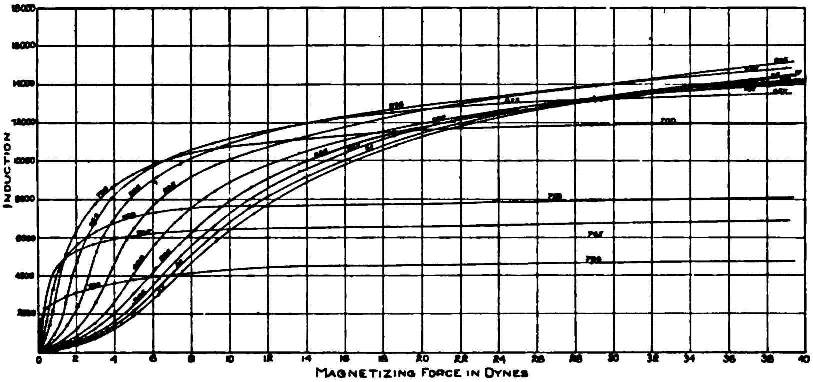


Fig. 3. Magnetization curves at various temperatures — Plate A — unannealed.

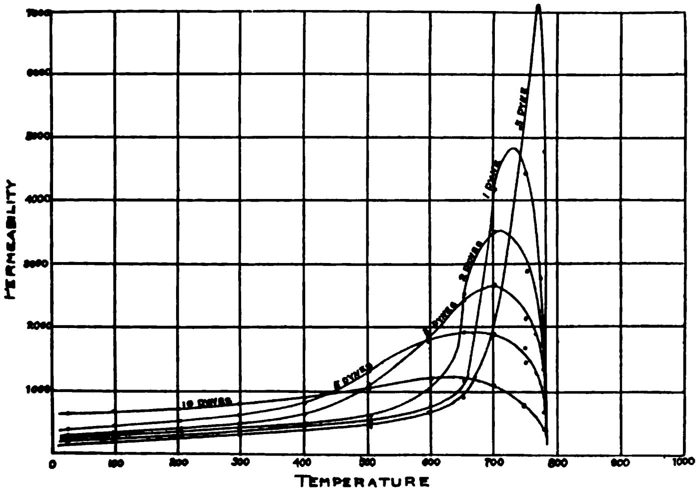


Fig. 4. Variation of permeability with temperature — Plate A — unannealed.

## TABLE VI.

*Plate B—Unannealed. Variation of Hysteresis Constants with Temperature.*

| Temperature. | Ergs per Cycle. | Maximum Induction $H = 100$. | Retentivity. | Coercive Force. |
|---|---|---|---|---|
| −190 | 24,800 | 17,700 | 7,000 | 5.16 |
| −120 | 24,000 | 17,750 | 7,350 | 4.58 |
| − 61 | 22,900 | 17,750 | 7,750 | 4.43 |
| 23 | 21,300 | 17,800 | 7,940 | 3.84 |
| 97 | 18,760 | 17,700 | 8,040 | 3.23 |
| 195 | 16,000 | 17,600 | 7,600 | 2.93 |
| 297 | 15,230 | 17,600 | 7,100 | 2.62 |
| 392 | 13,400 | 17,600 | 6,430 | 2.50 |
| 496 | 8,730 | 17,200 | 5,850 | 1.99 |
| 550 | 6,820 | 17,000 | 5,700 | 1.60 |
| 605 | 5,660 | 15,600 | 5,400 | 1.29 |
| 655 | 3,190 | 14,300 | 5,100 | .90 |
| 704 | 1,890 | 12,200 | 4,870 | .53 |
| 740 | 890 | 10,200 | 4,000 | .29 |
| 762 | 413 | 7,200 | 2,580 | .15 |

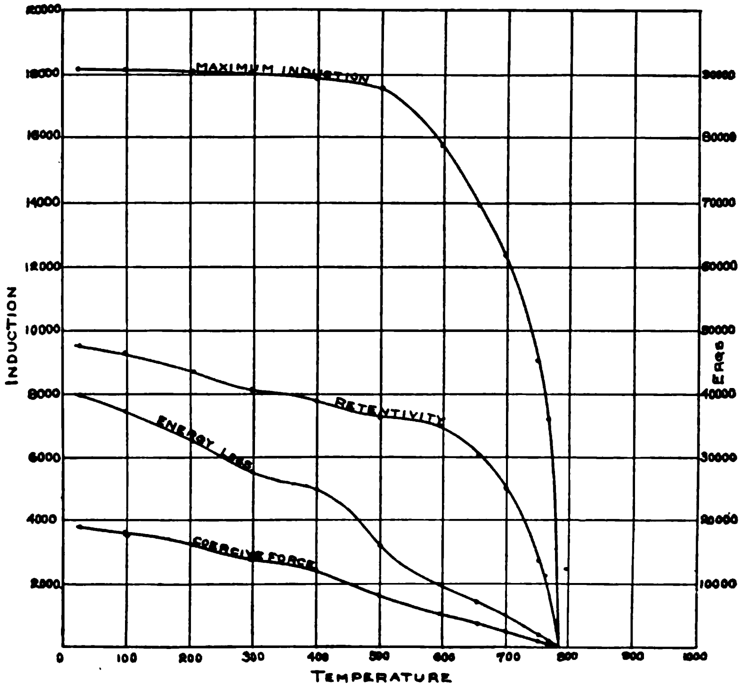


Fig. 5. Variation of hysteresis constants with temperature — Plate A — unannealed.

reached, the heating current was gradually reduced, care being taken that the rate of cooling should be, as nearly as possible, the same in each case. The time of cooling was about twenty-four hours. After each heating, the ring was unpacked, rewound, and tested. The results are shown in Table VIII. For the sake of comparison, I have added in the last line of this table some results obtained on a sample of Swedish iron annealed at 1150° C. Experi-

## TABLE VII.

*Variation of the Field Strength for Maximum Permeability with Temperature.*

| **Plate A.** | | | | **Plate B.** | |
|---|---|---|---|---|---|
| **Unannealed.** | | **Annealed at 800° C.** | | **Unannealed.** | |
| **Temp.** | **Field.** | **Temp.** | **Field.** | **Temp.** | **Field.** |
| 25 | 12.2 | −190 | 6.9 | −190 | 10.1 |
| 97 | 11.3 | −121 | 5.4 | −121 | 7.7 |
| 205 | 10.3 | − 60 | 4.2 | − 61 | 6.8 |
| 295 | 9.3 | 23 | 2.9 | 23 | 5.9 |
| 400 | 6.2 | 102 | 2.4 | 97 | 4.8 |
| 505 | 5.0 | 203 | 1.8 | 195 | 4.7 |
| 595 | 2.9 | 308 | 1.45 | 297 | 4.1 |
| 655 | 2.2 | 403 | 1.25 | 392 | 3.7 |
| 700 | 1.5 | 498 | 1.00 | 496 | 3.3 |
| 750 | .6 | 547 | .90 | 550 | 2.6 |
| 765 | .33 | 596 | .85 | 605 | 2.4 |
| | | 642 | .80 | 655 | 1.5 |
| | | 688 | .75 | 704 | 1.2 |
| | | 724 | .44 | 740 | .3 |

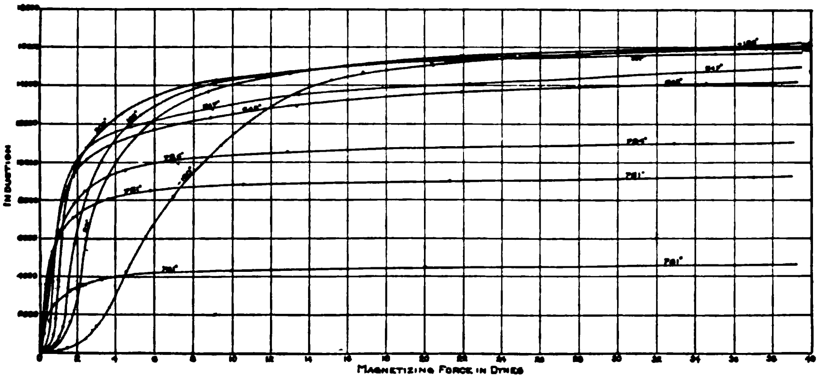


Fig. 6. Magnetization curves at various temperatures — Plate A — annealed at 800° C.

ments were performed on rings from plate *B* only, since it had previously been determined, that after annealing at 1000° C., all of the iron prepared by this process becomes practically identical. The field in these tests was carried to 80 dynes.

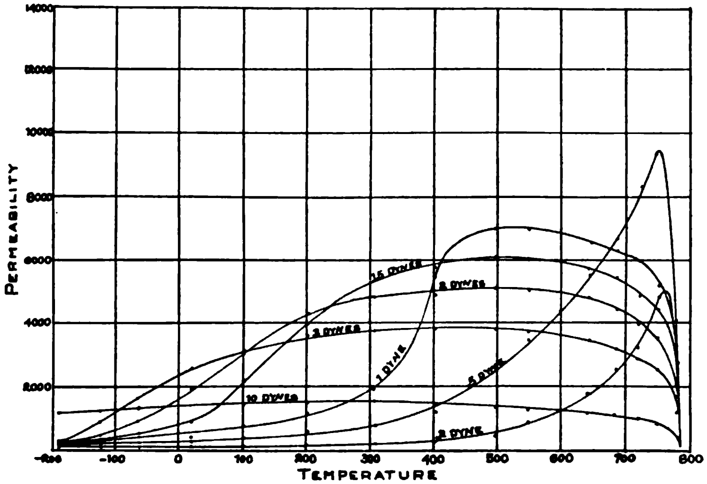


Fig. 7. Variation of permeability with temperature — Plate A — annealed at 800° C

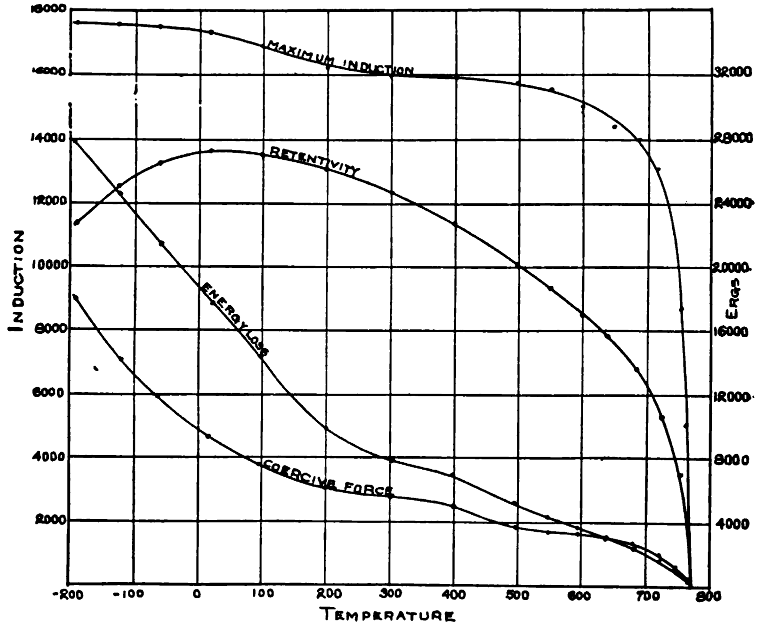


Fig. 8. Variation of hysteresis constants with temperature — Plate A — annealed at 800° C.

## TABLE VIII.

*Hysteresis Constants after Annealing at Various Temperatures.*

| Temperature. | Energy Loss. | Retentivity. | Coercive Force. | Maximum Induction. | Maximum Permeability. |
|---|---|---|---|---|---|
| 23° C. | 21,300 | 7,940 | 3.80 | 17,100 | 1,040 |
| 780 | 11,280 | 8,450 | 1.30 | 17,300 | 3,070 |
| 1000 | 5,060 | 14,100 | .75 | 17,280 | 9,080 |
| 1100 | 4,900 | 12,800 | .53 | 17,600 | 11,000 |
| 1200 | 5,600 | 13,000 | .85 | 17,400 | 8,750 |
| 1300 | 7,160 | 13,800 | .97 | 17,400 | 7,120 |
| 1150° C. | 3,060 | 9,000 | .54 | 17,160 | 8,500 |

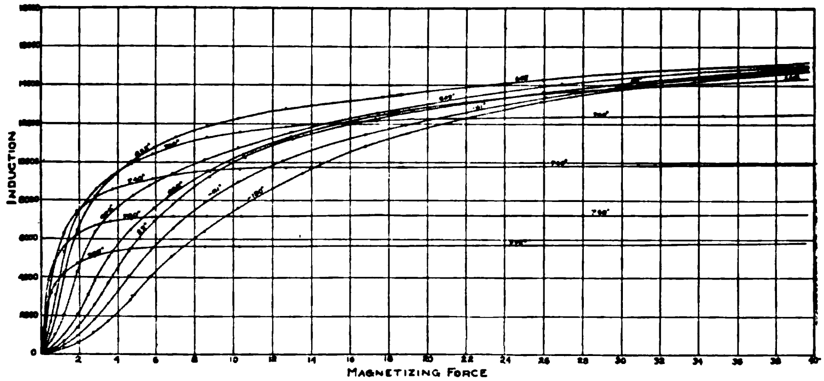


Fig. 9. Magnetization curves at various temperatures — Plate B — unannealed.

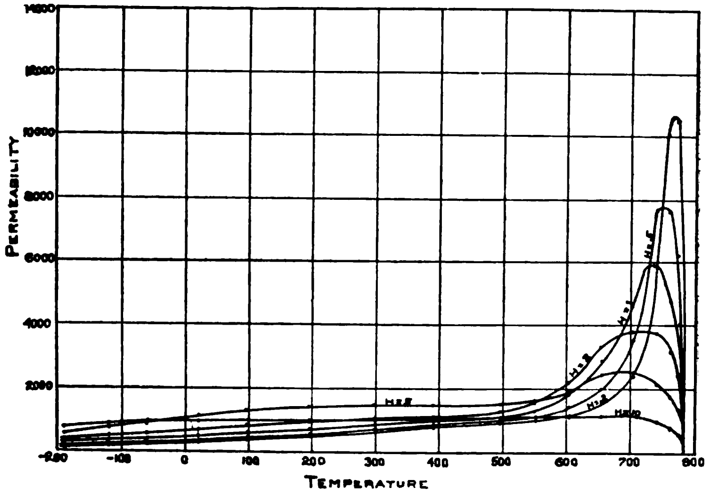


Fig. 10. Variation of permeability with temperature — Plate B — unannealed.

In connection with these last experiments, an attempt was made to study the relation between crystalline structure and magnetic properties. After each test, the ring was unwound and polished on one face. This surface was then etched with picric acid and

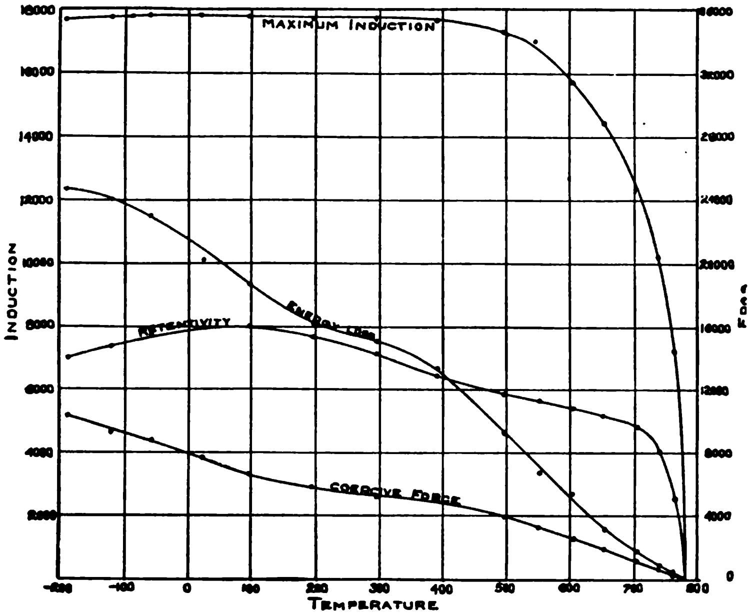


Fig. 11. Variation of hysteresis constants with temperature — Plate B — unannealed.

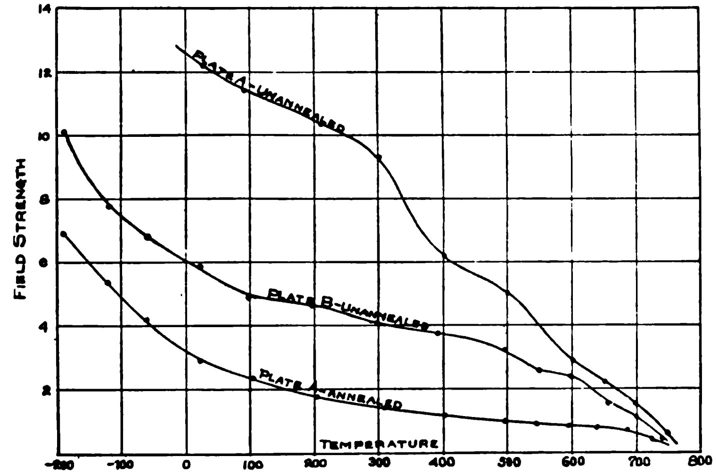


Fig. 12. Field strength for maximum permeability.

photographed microscopically, the magnification in each case being 60 diameters. The changes which were produced by the different heat treatments in the size of the crystals are shown in Fig. 13.

## Discussion of Results.

Before entering upon a discussion of these results, it will be well to make a brief digression on the temperature changes in the physical state of iron, upon which its magnetic properties so largely depend. According to Osmond, iron presents itself in three allotropic phases: Alpha, Beta and Gamma. Alpha-iron is weak, ductile, ferromagnetic, stable below 785° C., and is characteristic of wrought iron and low carbon steels. It crystallizes in cubes. Beta-iron is weakly paramagnetic, stable between 785° C. and 890° C., and is probably hard and brittle. It is characteristic of certain self-hardening steels, also of normal carbon steel when hardened by sudden cooling, and like Alpha-iron, crystallizes in the cubic system. Gamma-iron is stable above 890° C., is paramagnetic, and characteristic of nickel and manganese steels. It is probably hard, but ductile, while its crystals are cubic with octahedral modification.

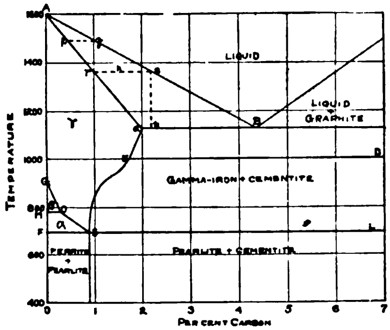


Fig. 14.

The relations between these phases are best shown by means of the Roozeboom[1] diagram of Fig. 14, where the lines separating the different regions indicate the temperatures at which the transformations take place, when different amounts of carbon are present. Although the significance of this diagram is quite apparent, perhaps the field *AaB* requires special explanation, since it represents a mixture of the solid and liquid phases. The line *Aa* gives the greatest amount of carbon a solid solution may contain, while *AB* shows the least amount for a liquid; and any point, such as *K*, representing a given carbon content at a particular temperature, divides a horizontal line drawn through it into parts proportional to

---

[1] Metallographist, Vol. III., p. 293, 1900.

the relative amounts of the liquid and solid phases. As a molten mass of iron, containing one per cent. of carbon, for instance, is cooled, the point $p$ represents the composition of the first crystals that are formed. As the temperature falls, the remaining liquid, being enriched in carbon, follows along the line $AS$ until the temperature $r$ has been reached, when the whole mass is solidified, the process having covered the temperature interval $qr$. The point $B$ is called the *eutectic* point, and represents the combination which gives an abrupt solidification to a mixture of pure iron and graphite crystals.

The line $SED$ represents the formation of a definite compound $Fe_3C$, called cementite, while $FL$ gives the transformation known as "recalescence" which consists of two simultaneous changes, the separation of the alloy of iron and carbon into alternate layers of ferrite, or pure iron, and cementite, and the transformation of the ferrite to alpha-iron. The magnitude of the recalescence increases with the carbon content. The line $MOSE$ is of special interest in this paper since it corresponds to the disappearance of ferro-magnetism.

Roberts-Austin[1] has studied the transformation points of electrolytic iron deposited from a solution of ferrous chloride by heating the specimen and determining its cooling curve. Besides the normal transformation points at 895° C. and 785° C., he found three others, namely, at 1130° C., 475° C., and 270° C. The first of these had already been noted by Ball, and is of no particular interest in connection with this paper. The second and third points he attributed to occluded hydrogen, and suggested that the one at 475° C. may be due to the separation of a hydride of iron from the solid solution, while the one at 270° C. is the corresponding eutectic point. He has further shown that by repeated heating in a vacuum, pumping out the liberated hydrogen, these points vanish. By making the iron the cathode of an electrolytic cell, it may again be partially saturated with hydrogen, when the point at 475° C. will reappear. These points are of special interest, for as will be pointed out later, certain changes in the magnetic properties have been found to occur near these temperatures. It is an interesting fact that the recales-

---

[1] Metallographist, Vol. II., p. 187, 1899.

cence point, which is due to the presence of carbon, was found to be absent. Its absence in Burgess iron also has been shown in this laboratory.

Returning now to a consideration of the results which I have obtained, one of the first conclusions is that freshly deposited electrolytic iron is very hard magnetically, which agrees with the work of previous investigators. It is to be noted, however, that the magnetic properties of different samples are not at all uniform, as a glance at any of the corresponding curves for rings from plates *A* and *B* will show. In all respects, mechanically as well as magnetically, *B* is much softer than *A*, although they were prepared, as far as I am aware, under exactly similar conditions. This is not in agreement with the results of Leick, who found a marked uniformity in all samples, whether deposited from a chloride, sulphate, or ammonium sulphate solution. My results show, however, that these differences exist for small fields only, vanishing for fields of 100 dynes. I have found that after annealing at 1000° C., they be come practically identical for all fields. Some samples of single refined iron were much harder than either of these, one having a coercive force of 20 dynes. In this connection it would seem that Schild's criticism of Leick's results is not entirely justified. As pointed out previously, Leick obtained results consistently higher than Schild's, obtaining for a field of 125 dynes, an induction of over 19,000, while Schild obtained only 15,770. I have obtained 18,600.

Plunging a ring in liquid air and leaving it for several hours produced no measureable permanent hardening. As the temperature is reduced below that of the room, the permeability decreases for small fields, but increases for large fields, this effect being greater after annealing. The hysteresis loss continually increases as the temperature is lowered, the rate of increase being greater after annealing. This is entirely at variance with Fleming and Dewar's results on transformer iron, for they found no change in hysteresis loss at the temperature of liquid air. They also found the permeability to increase for all fields in the case of unannealed and hardened iron, which is again contrary to my results. A very curious effect is the decrease of the retentivity for low temperatures, giving a maximum at about 50° C. A careful search through the literature has revealed no mention of such a phenomenon, and it does not exist for several samples of commercial iron which I have tested. It is believed to be characteristic of electrolytic iron.

The temperature of transformation from the ferro- to the para-magnetic state has been found to be 785° C., which is in good agreement with the best data available on the temperature of transition from the alpha to the beta state. Hopkinson has obtained the following values:

| Sample. | Carbon Content. | Temp. of Transf. |
|---|---|---|
| Soft iron, | .01 per cent. | 786° C. |
| Mild steel, | .126 " | 734° C. |
| Hard steel, | .962 " | 695° C. |

Morris gives 780° C. for a sample containing .08 per cent. carbon. Referring to the diagram of Fig. 14, these values are in accord with what we should expect from metallurgical considerations. Other experimenters have given values differing somewhat from the above, but unfortunately they have not included chemical analyses, hence their results are of little value for comparison. If the temperature were raised a few degrees above this critical value and allowed to fall slowly, magnetism always reappeared at the same temperature at which it disappeared. This seems to be characteristic of the purity of the metal. Hopkinson states that for his purest metal, the difference in the temperatures corresponding to the disappearance and reappearance of magnetism was about 3° C., while for hard steel, containing a relatively large per cent. of carbon, it was about 30° C. Wills has also confirmed this point. The presence of another metal increases this effect in a very marked degree. Osmond has given the complete curves for nickel steels, an extraordinary case being that of the eutectic alloy containing 23 per cent. nickel, which remains non-magnetic until cooled to — 20° C.

The change in the hysteresis loss with temperature shows some interesting variations. Kunz[1] concluded from his experiments on soft iron that hysteresis loss is expressible as a linear function of the temperature, while Morris' curves show distinct departures from a straight line. An inspection of curves 5, 8 and 11, shows marked

[1] Electrotech. Zeitsch., p. 196, 1894.

depressions just after the temperatures 300° C. and 500° C. have been passed. These are in accord with the hydrogen points of Roberts-Austin, already mentioned. It is worthy of note that sample *A*, after annealing, shows this effect to a less extent than before, a large portion of the occluded hydrogen having been driven off in the process.

Perhaps a still more striking proof of the existence of these points is given in the curves of Fig. 12, where the field which gives maximum permeability is shown as a function of the temperature. As stated before, this point of maximum permeability was chosen by Ewing to mark the change from the second to the third stages of magnetization, and the field at which it occurs furnishes a convenient index to the softness of the iron. It will be noticed that in sample *A*, very marked depressions appear in this curve at the temperatures in question before annealing, but that after annealing, they can barely be detected. The changes in the magnetic properties at these temperatures can also be traced in the curves for coercive force and retentivity. Sample *B*, being much softer, and probably containing less hydrogen, shows this effect less. The corresponding curves of Wills for wrought iron and alloys show no such abrupt changes.

The variation of permeability with temperature is shown in Figs. 4, 7 and 10. Probably the most striking effect here is the sudden rise in permeability for low fields just before the transformation to beta iron takes place. The highest value of permeability which I have obtained is 11,000, occurring at a field of .5 dyne. Morris obtained a maximum of 14,600 for $H = .2$ in the case of Swedish iron, while Wills gives 17,200 for wrought iron for $H = .172$. Although this last value seems large, the comparison shows that electrolytic iron is not so soft as some commercial irons. The chief respect in which my permeability curves differ from those of Morris is in the matter of regularity. It will be noted that nowhere in my curves, barring slight increases at the hydrogen points, do any irregularities appear, and this is believed to be characteristic of the purity of the metal. In marked contrast to this Morris found, for wrought iron, a distinct depression in the temperature-permeability curves near 550° C., which could be traced in all except those for the largest fields.

It is well-known to metallurgists that the transition from alpha to beta iron is not abrupt, but covers a considerable temperature interval, the temperature at which the transition begins being greatly lowered by the presence of impurities. This has been shown independently by Arnold[1] and Howe[2] by experiments on the tempering of steel. The depression in Morris' permeability curves, which has just been mentioned, has been interpreted by Osmond[3] as making the lower limit of this transition. In view of the fact that I have found no indication of such an effect in electrolytic iron, it would seem as though this depression cannot be characteristic of iron itself in changing from the alpha to the beta state, but must be due to the formation of a molecular combination of the iron with some impurity.

The effects of annealing at low temperatures are clearly shown by a comparison of Figs. 4 and 7, where it is seen that the maximum permeability is increased about 50 per cent. by heating to 800° C., while for small fields the increase is many fold. Table VIII. shows the results of heating to higher temperatures. From this it is seen that the iron is in its best state magnetically when annealed at 1100° C. Further heating decreases the magnetic quality very decidedly. Comparing electrolytic iron with Swedish iron we see that while it has a slightly lower coercive force and a higher maximum permeability, its high retentivity gives it an energy loss nearly 60 per cent. greater.

Stansfield[4] has studied the physical changes that take place in iron when heated to temperatures that are high but insufficient to melt it. He recognizes two stages, (*a*) overheated, (*b*) burned. The first refers to cases in which the temperature has not been carried above the line *Aa* in the Roozeboom diagram. Although the point of incipient fusion has not been reached, there has been a marked diminution in the closeness with which the crystals are held together. He has given no lower limit for this stage, but from my results it would appear to be just above 1100° C. The iron can

---

[1] Engineering, July 9, 1897.

[2] Journal of the Iron and Steel Inst., 1895, Part II., p. 256.

[3] Metallographist, Vol. II., p. 187. 1897.

[4] Journ. of the Iron and Steel Inst., vol. 2, p. 433, 1903.

practically be restored by annealing at a lower temperature. The second stage refers to cases in which the temperature has been taken above the line *Aa*. A highly carburized liquid is here formed which accumulates in the intercrystalline joints, and if the metal is held at this temperature too long this is forced out by the carbon-monoxide gas which is formed, leaving interstices in which oxidation may take place. If there has been no oxidation the metal may be restored by reheating to a temperature a little above that represented by the line *GOS*, and cooling slowly, giving the carbon time to diffuse through the mass of the metal.

## SUMMARY.

The following are the principal characteristics of Burgess electrolytic iron which have been established by this investigation:

1. Freshly prepared iron is very hard magnetically.

2. Different samples show marked dissimilarities at low fields which disappear upon annealing at 1000° C.

3. For large fields, all samples are quite similar, the values obtained for susceptibility being intermediate between those of Leick and Schild.

4. Plunging into liquid air produces no permanent hardening.

5. The retentivity has a maximum in the neighborhood of room temperature.

6. Marked softening magnetically occurs at the hydrogen transformation points.

7. Ferro-magnetism reappears on cooling at the same temperature at which it disappears on heating. This temperature is 785° C.

8. The depression in the permeability temperature curves obtained by Morris does not exist for this iron.

9. The best temperature for annealing is 1100° C.

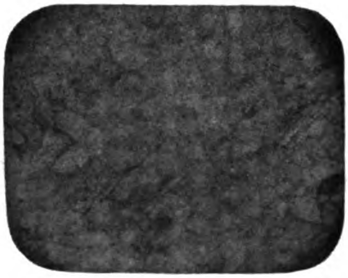
10. Although when properly annealed it has a lower coercive force and a higher maximum permeability than Swedish iron, its high retentivity causes a large hysteresis loss.

Thus far the iron has been studied up to 785° C. Owing to the limitations of the method, it has not been possible to extend the work up into the region of paramagnetism. It is my intention, however, with differently designed apparatus, to investigate this hitherto little studied field, in which I think there is reason to expect even more marked characteristics of this very pure iron.

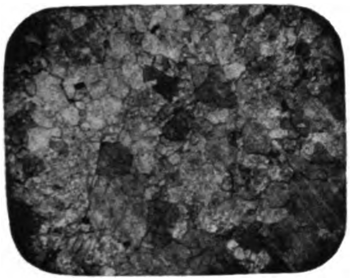
In conclusion I wish to express my gratitude to Prof. C. E. Mendenhall for many valuable suggestions throughout the progress of this work, to Prof. C. F. Burgess for his generous supply of electrolytic iron, and to Prof. O. P. Watts for assistance in making the microphotographs.

Physical Laboratory,
University of Wisconsin,
August 7, 1909.

1 ORIGINAL DEPOSIT



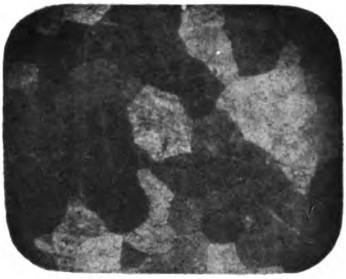
2 ANNEALED AT 800°C



3 ANNEALED AT 1000°C

4 ANNEALED AT 1100°C

5 ANNEALED AT 1200°C



6 ANNEALED AT 1300°C

EARLE M. TERRY.

**Micro-photographs showing** crystalline structure.

# A STUDY OF THE WEHNELT ELECTROLYTIC INTERRUPTER.

By Karl Taylor Compton.

THE study of electrolysis has led to many unlooked for discoveries, but probably few if any of these were more strange than when M. Violle passed a large current through an electrolytic cell with small wire electrodes, for the purpose of studying their polarization, and was startled by a regular succession of sharp explosions. Although in reporting this he ascribes the noise to interruptions of the current, the discovery was not put to practical use until seven years later. In 1899 Dr. Wehnelt conceived the idea of utilizing this remarkable phenomenon in an electrolytic device, to replace the clumsy mechanical circuit breaker of the Ruhmkorff coil, and successfully used a coil so equipped in wireless telegraphy. The result of his experimenting is the Wehnelt interrupter. It will be our purpose, first to study the conditions which affect the action of this instrument; and second, to use the results of this investigation in formulating a theory explaining its action.

## Part I. The Action of the Interrupter.

The interrupter consists essentially of a lead plate as the cathode, and as the anode, a platinum wire sealed in a glass tube so as to leave from .5 mm. to 2 cm. in contact with the $H_2SO_4$ electrolyte. Other materials may be used, but the above are probably the best. In the following pages the points designated as 1, 2 and 3 are .38 mm. in diameter, and 2.5, 5 and 9.5 mm. long respectively. Points 4 and 5 are 2.15 mm. in diameter and 3.5 and 7.5 mm. long.

If a gradually increasing battery current is sent through this apparatus, the first effect is simply ordinary electrolysis, with unusually high polarization of the wire anode. Then when the current reaches a certain value, the polarization increases greatly and the current almost ceases; the wire becomes white hot and hisses in the liquid. Further increase of the applied E.M.F. only increases the polarization. These two stages we shall call the electrolytic stage and the fatigue stage, so called since it occurs when the instrument is overloaded. If a dynamo current is used, and better if an inductive coil is introduced into the circuit, these same two stages exist, but between them is introduced a third stage, that in which the interruptions occur, and which we shall call the interruption stage.

In the interruption stage, several noteworthy phenomena occur. The strident thumping noise due to the explosive interruptions is almost deafening, and the violence of the action may be felt by putting a finger into the electrolyte. At each interruption a violet light surrounds the anode, although the wire itself is not heated hot enough to produce light. The bubbles of gas are thrown off violently, and on bursting give off fumes of $SO_2$ and $H_2SO_4$ together with $O_2$ and ozone. During all this the electrolyte becomes warm. If the current becomes too large, the interruptions become irregular and soon the fatigue stage sets in. With but little trial, one finds that the size of the current and of the anode point, the concentration of the solution, the temperature and pressure, the inductance and capacity in the circuit, all affect the interruptions. Their various effects will be discussed in detail, and curves plotted to illustrate the relations which exist. These curves will be only a few selected from quite an extensive set of tables and curves, all agreeing in form and significance.

To illustrate the change in conditions as we pass from the electrolytic to the interruption stage, an ammeter was placed in the circuit with the interrupter and inductance coil, and a voltmeter, placed across the interrupter, read the drop across the instrument. Of course the readings in the interruption stage represent average values — the square root of the mean square — and $R$ is found by dividing $E$ by $I$. These specifications apply throughout the article. An A.C. wattmeter was placed to read the power consumed in the interrupter, and these readings were a constant check on the values of $E$ and $I$.

Evidently a marked change occurs at the points marked with circles. (See Fig. 1.) These are the points corresponding to the instant when the interruptions first occur — and this point will be called the lower limit. Below this point on curve 1, if we subtract from the total E.M.F. the drop due to polarization, which Koch and Wüllner found to approach 3.79 volts in the electrolytic stage,[1] we see that the remaining volts are proportional to the current, *i. e.*, the electrolyte obeys Ohm's law. Here too, on curve 2, the fact that the resistance decreases with increasing current is due to the fact that this counter E.M.F. becomes a less and less influential factor in the total resistance. Curve 3 shows that above the lower limit, the potential drop is proportional to the resistance, minus a certain constant resistance which is an inherent property of the electrolyte and the electrodes. As the current is increased in the interruption stage, the interruptions become more and more rapid and irregular. Curves 1 and 2 show that there is an upper, as well as a lower limit, and if one exceeds this, the fatigue stage sets in. Having taken this general survey of the action, we shall now look more closely at the conditions existing in particular stages of the interruption.

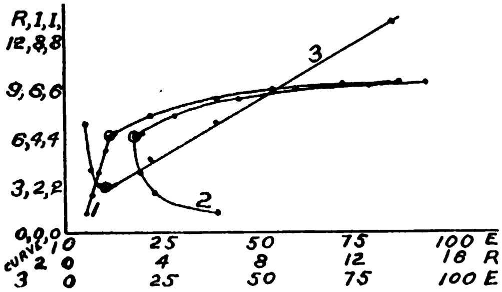


Fig. 1. 20 per cent. $H_2SO_4$. $L = .04$ henry. Point 2. 20° C. Curve 1 shows the variation of $E$ with $I$; curve 2 shows the variation of $R$ with $I$, and curve 3 shows the variation of $R$ with $E$ — all as the current is raised from a low to a high value.

### 1. *The Lower Limit.*

The values of $E$, $I$ and $R$ in this section are those values at which the interruption stage first begins. It is found that, for a given set of conditions, this stage begins at a very definite point. From the regularity of the curves in Fig. 2 it is evident that there are very regular relations existing at the lower limit.

Plainly, the larger the point, the greater the voltage required to

---

[1] Polarization of Small Points, Annalen der Physik und Chemie, No. 3, p. 502, 1892.

produce interruptions. Each curve resembles an hyperbola and has as asymptotes the $X$ axis and a line $A$ parallel to the $Y$ axis. We infer then that the total E.M.F. may be resolved into three parts: (1) that necessary to overcome the counter E.M.F.; (2) that additional which would be required to interrupt in a 100 per cent. solution; (3) that to overcome the extra resistance due to dilution. The first and second of these are constant for a given point and represent the values from the origin to the line $A$. The third part varies nearly inversely as the concentration, as we should expect. It is in part (2) where the additional E.M.F. is required for larger points.

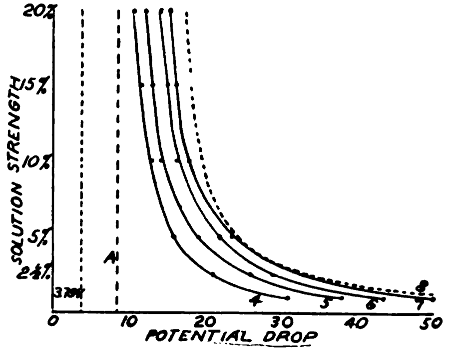


Fig. 2. $L = .04$ henry. 20° C. Curves 4–7 show the total E.M.F. required to produce interruptions, each curve representing a given anode point used in electrolytes of different concentrations. The curves are for points having surfaces of 3.26, 11.45, 30.12 and 54.02 $\text{mm.}^2$ respectively. Curve 8 is an equilateral hyperbola to show the departure of the curves from true hyperbolic form.

The curves in Fig. 3 approximate sections of hyperbolæ, and indeed we should expect this, for as additional surface is equivalent to other circuits in parallel, the resistance would vary inversely as the surface, provided we can consider practically the total resistance as very close to the anode point. This assumption is shown to be true later. This interesting fact, however, is noticed. Whereas points, 1, 2 and 3 lie on one curve, points 4 and 5 in each case seem to lie on another. The explanation suggests itself as follows: Points 1, 2 and 3 are of wire .38 mm. in diameter, while points 4 and 5 are 2.15 mm. in diameter. Another point, 6, has the same length as 4, and the same diameter as 1, 2 and 3.

Let the inner and outer circles represent cross sections of points 6 and 4 respectively. Then the difference in the resistance between these two points is seen to be the resistance of the included electrolyte, which the current from 6 must traverse, but not that from 4.

Then, by considering additional length as so much parallel circuit, we can reduce the resistances of points 4 and 5 to that value which they would have were they .38 mm. in diameter. Making these reductions and placing their values on the curves they are seen to fall exactly in line with points 1, 2 and 3, at the positions 4′ and 5′.

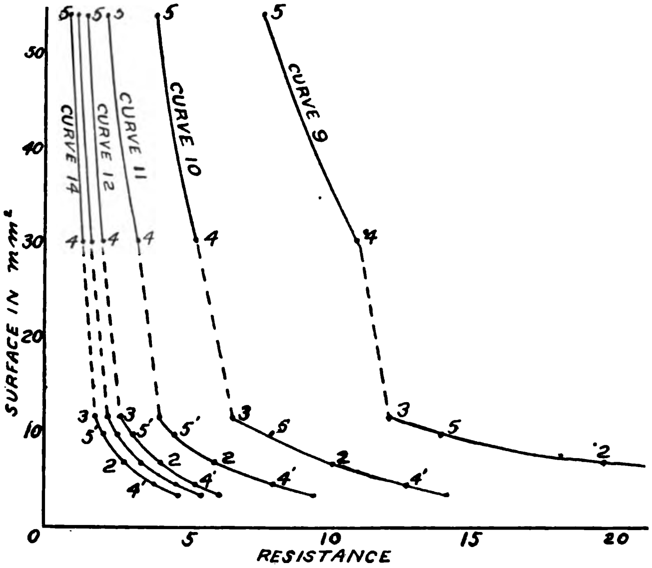


Fig. 3. 20° C. $L = .04$ henry. These curves show how the resistance of the interrupter at the lowest current which will produce interruptions in each case, varies with the surface of the anode point. Curves 9, 10, 11, 12, 13 and 14 show this for solutions of 1, 2.5, 5, 10 and 20 per cent. concentration.

By experiment it is found that the stronger the solution the larger the current and the less the E.M.F. required to interrupt. This is often of practical value, for the current may be cut down, using the same anode point, by using a weaker solution and a higher E.M.F. The question as to whether the product of volts × amperes remains a constant is answered by Fig. 4.

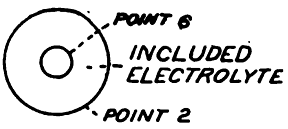


Fig. 3a.

Evidently slightly greater power is required to interrupt in weak solutions. This is probably due to the greater amount of energy required to overcome the electrolytic resistance outside the immediate vicinity of the anode point, leaving it possible that the energy actually used in producing the interruption at the anode is constant for a given point. If the interruptions are produced by vaporization of the electrolyte touching the anode point, the energy necessary to interrupt in a given solution would be proportional to the amount of electrolyte to be vaporized, that is, to the surface of the anode point. This view is borne out by Fig. 5.

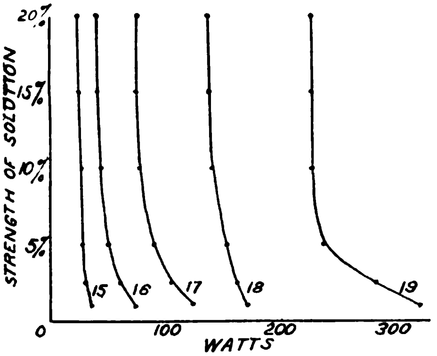


Fig. 4. $L = .04$ henry. 20° C. Points 1, 2, 3, 4 and 5. Curves 15, 16, 17, 18 and 19 show the relation of the watts necessary to produce interruptions to the strength of the electrolyte for points 1, 2, 3, 4 and 5.

Although we might expect that the distance between the anode point and the cathode plate would affect the resistance of the instrument, we find that this has practically no influence; and this still further proves that almost the whole electrolytic resistance is due to the counter E.M.F. and the resistance of those layers of liquid immediately surrounding the anode, where the current's path is very small. This, together with the fact that the watts consumed are proportional to the anode surface, has important bearing on the theory of the interrupter.

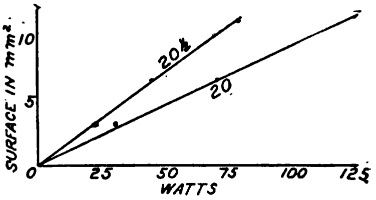


Fig. 5. 1 per cent. $H_2SO_4$. 20° C. $L = .04$ henry. Curves 20 and 20½ show that for 1 per cent. and 20 per cent. solutions, the watts consumed at the lower limit are proportional to the surfaces of the anode points. This is found to be true only when the anode points are of the same cross-section.

### 2. *The Upper Limit.*

A glance at Fig. 1 shows that there is an upper limit, beyond which further increase of E.M.F. produces no further increase of current. This upper limit is found to be larger for larger anode points and for more concentrated solutions. For a given solution, the upper limiting current is proportional to the surfaces of the anode points. For instance, for a 20 per cent. $H_2SO_4$ solution with points 2 and 3 the surfaces are in the ratio of 4.3 to 6.7, or .63; while the upper limiting currents are as 4 is to 6.4, or .63.

### 3. *The Frequency of Interruption.*

In the practical use of the interrupter, the two vital points to be attained are high frequency and sharpness of interruption. The ordinary hammer circuit breaker never produces more than 200 interruptions per second, and with a large coil does not make more than 25 to 30. With the Wehnelt interrupter, the frequency with very small points may reach 1,500 to 2,000, and as high as 200 with large currents. In the following measurements two methods for determining the frequency were used: (1) determination by the pitch of the sound produced by the interrupter; and (2) calculation by the three voltmeter method.[1] This latter method strictly applies only to harmonic E.M.F.'s, but by comparison of the two methods and by calculation of the resistance of the inductive coil, it was proved to be quite accurate in measuring the frequency.

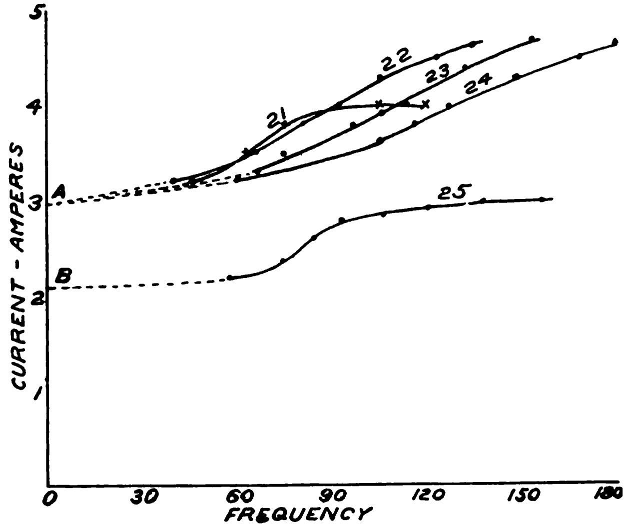


Fig. 6. 20° C. Point 2. These curves show the increase of frequency with increasing current. Curves 21, 22, 23 and 24 are taken using a 10 per cent. $H_2SO_4$ solution and with .01, .013, .023, .043 henry inductance in the circuit, respectively. Curve 25 is taken in a 1 per cent. solution with .043 henry.

---

[1] Nichols' Laboratory Manual of Physics and Applied Electricity, Vol. II., p. 124.

Low frequency readings were not taken owing to the increased inaccuracy at low frequency in the methods employed. Apparently, however, the first four curves produced meet at a common point *A*, indicating that the lower limit is independent of the inductance, provided there is enough inductance to produce interruptions, as is found to be true. Furthermore, by referring to previous readings, it is found that points *A* and *B* represent the lower limiting current under the same conditions. The bend in each curve marks the upper limiting current, which is larger for large inductances. Curves 24 and 25 show that it takes more current to produce a given frequency with strong than with weak solutions — a point of practical value. The frequency may also be varied by varying the self-induction in the circuit — the larger the self-induction the lower the frequency.

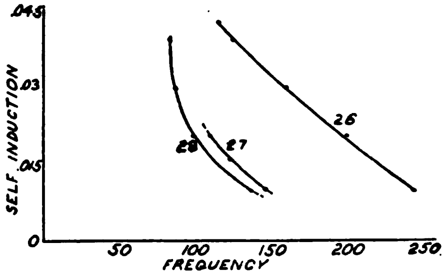


Fig. 7. 20° C. Point 2. Curves 26, 27 and 28 show the variation of the frequency with the self-induction — curve 26 with a 1 per cent. solution and a constant current of 2.7 amperes; curve 27 with a 20 per cent. solution and a current of 5.6 amperes; curve 28 with a 20 per cent. solution and a current of 5.0 amperes.

However, the self-induction must be of a sufficient value or there will be no interruptions at all. There is also an upper limit to the usable self-induction. In curve 27, when the inductance was made higher than at point *a*, the fatigue stage set in.

Finally the greatest variation in frequency may be obtained by regulating the size of the anode points. In using very small points it was found best to bend the glass tube through 180° so that the point might extend vertically upward into the solution. This partially prevents the bubbles of gas liberated from catching at the point and insulating it.

### 4. *The Influence of Temperature on the Interruptions.*

The fact that heat influences the lower limit and the frequency can be explained only by supposing that the resistance of the electrolyte is changed, or that the electrolyte just surrounding the anode point is vaporized. By experiment, however, it is proved at the lower limit that although the required current and E.M.F. decrease rapidly with increasing temperature of the electrolyte, as shown by curves 29 and 30, yet their ratio and hence the resistance remains practically constant. So we conclude that the reason that it takes less energy to interrupt at high temperatures is that the electrolyte is already partially heated toward vaporization.

Curves 31 and 32 illustrate the increase of frequency produced by increasing the temperature of the electrolyte. In this connection, and leading likewise to the conclusion that liquid is vaporized during interruptions, is the fact that increasing the pressure on the electrolyte increases the current through the interrupter, decreases the frequency, and raises the lower limiting current.

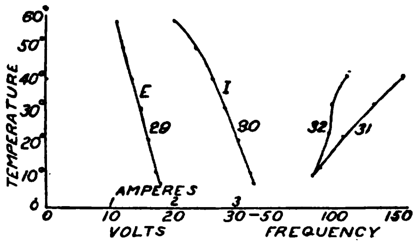


Fig. 8. Point 2. $L = .023$ henry. 5 per cent. $H_2SO_4$ solution. Curves 29 and 30 show the decrease in the number of volts and amperes required to interrupt as the temperature rises. Curves 31 and 32 show the effect of temperature on the frequency; curve 31 with a constant current of 4.4 amperes and curve 32 at a constant E.M.F. of 32 volts.

## 5. *The Effect of Capacity on the Interruptions.*

If a condenser is connected across an interrupter, as is done with the hammer circuit breaker, instead of bettering its action, it softens it, and if the capacity is large, stops it entirely, bringing on the fatigue stage. This effect is important in the theory of the interrupter and will be explained under that head.

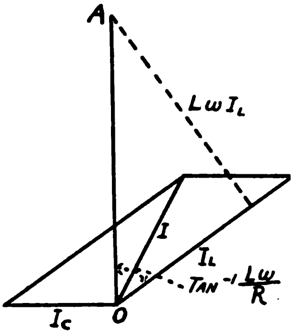


Fig. 9.

If the condenser is put in parallel with the inductance, the sound is much less violent, but the frequency is changed but slightly. This effect is explained thus:[1]

Let $\overline{OA}$ represent the impressed E.M.F. The current in the

---

[1] Bedell and Crehore's Alternating Currents, p. 309.

condenser branch $Ic$ is 90° in advance of $\overline{OA}$ and is small, equal to $CE\omega$. The current in the inductive branch $I_L$ lags behind $\overline{OA}$ by an angle $\tan^{-1} L\omega/R$ and is equal to $E/L\omega$. The actual current is their geometric sum $I$. Thus the effect is to diminish both the current and the effect of the inductance. As the former of these tends to lower the frequency and the latter to increase it, we see why the frequency suffers but little change; but both tend to soften the sound.

### 6. *Interrupters in Parallel.*

If two anode points are connected in parallel as in Fig. 9*a* rather surprising results are obtained. Instead of the smaller interrupting first and more rapidly than the large one, we find that the two act simultaneously as a unit. As to the lower limit, the watts consumed, the frequency, and in every other respect, any number of points connected thus act precisely like a single point with a surface equal to the sum of the surfaces of the points used. Moreover the watts consumed in each branch are such as would make it interrupt by itself, proving again that the watts consumed are proportional to the surfaces of the points.

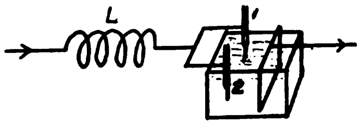


Fig. 9a.

If, however, an additional inductance is introduced in one branch, and another in the other, in general the points do not act in unison, and the interruptions are very irregular. By adjusting the inductances the points may be made to act with the same frequency, and may even be made to act in unison when one lags a whole interruption behind the other. The following table shows the watts required to work points 1 and 2 simultaneously, and those required to work the same points with the same frequency, but not in unison.

| Case 1, Points 1 and 2. | | | Case 2. | | | |
|---|---|---|---|---|---|---|
| I. | E. | W. | Point. | I. | E. | W. |
| 7.2 | 16 | 115.2 | 1 | 1.4 | 8 | 11.2 |
| | | | 2 | 5 | 12 | 60 |
| | | | | | Total watts, | 71.2 |

Evidently in case 2 when one point interrupts the other gets the whole current and so these points will act on less power than in case 1, where the current is always divided between the two.

### 7. *The Action of the Interrupter with Alternating Currents.*

Thus far we have considered only the use of direct currents, and in that direction which makes the wire points the anode. If the current is reversed there are no interruptions at all, the electrolytic stage passing directly into the fatigue stage as the current is increased. Thus it seems necessary that O and H both be liberated at the point, — a fact made use of later.

Using an alternating current through an interrupter, we find as before a lower and an upper limit. Temperature, the size of the point, and the concentration of the solution all affect the lower limit as in the case of the direct currents. But there is this important difference that none of these quantities nor the self-induction nor the capacity affects the frequency. This depends only on, and is constantly equal to, the frequency of the alternating current. This phenomenon was investigated by Professor Bailey, of the University of Michigan, and he obtained the following curves for the current and voltage cycles.

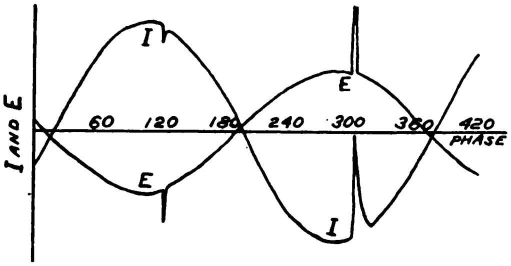


Fig. 10.

In Fig. 10 the current and voltage curves are purposely placed on opposite sides of the $x$-axis. Plainly in one half of the cycle there is a very slight interruption, while there is a complete interruption in the other half, which is that half making the wire point the anode. Although it might be expected that in some cases two interruptions would occur in the same half cycle, this has never been found to be true. This being so, we should not expect the interrupter to be as effective on A.C. as on D.C. circuits, and this is true. For instance, with a current of four amperes through the interrupter, an induction coil gave a 4½-inch spark on the A.C. circuit, and a 6½-inch spark on the D.C. circuit. Mr. Waggoner, of Case School of Applied Science, has shown that by using wire points for both anode and cathode, two interruptions a cycle may be obtained. It was found that if a D.C. ammeter is placed on an A.C. circuit in which an interrupter is working, a slight deflection occurs in the direction making the wire point the cathode. This is due to the partial breaking down of the opposite half cycle by the interruption.

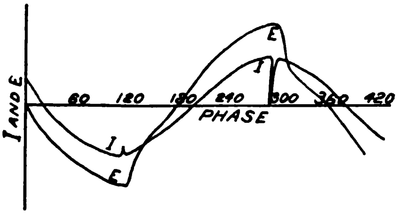


Fig. 11.

Another very important curve by Professor Bailey is shown in Fig. 11.

In this figure it is plainly shown that the current lags behind the impressed E.M.F. as we should expect from the self-induction of the circuit. But at the instant of the interruption the current surges ahead so that at this point the two are in phase. The only thing that can produce this neutralization of the self-induction is a capacity, manifesting itself at the instant of interruption. As the capacity just neutralizes the self-induction it must adapt itself to the value $C = 1/L\omega^2$ from the equation

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}}.$$

The important thing to be noted is that at the instant of interruption, and then only, there is a capacity effect in the interrupter.

### 8. *Nature of the Light Surrounding the Anode.*

The light seems to be of the nature of a discharge of electricity through gases, and this is rendered more probable by the fact that the gases will be present partly in an ionized condition, owing to dissociation and electrolysis, and possible heat developed at the anode. The spectrum of the light is, as we should suppose from the above, a line spectrum. The positions of the lines are marked by their positions on the scale, and their intensity by their relative lengths. The shading represents a glow as from a continuous spectrum.

From this we learn that O and H are both involved in the formation of light. There is also evidence of a spark, since we have plainly the sodium spectrum from the glass, and also a faint glow reinforced in the neighborhood of the platinum lines. This must be due to the spark from the platinum, since Professor McClenahan proved, by using fusible alloys for points, that in the interruption stage the temperature never rose above 168° C., and hence not high enough to produce incandescence. Thus we learn that there is a spark discharge through O and H in the gaseous condition at the instant of interruption.

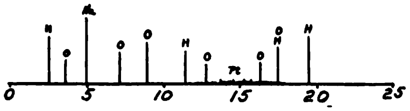


Fig. 12.

### 9. *Nature of the Interruptions.*

In this section we shall go more fully than we have gone so far into several points concerning the nature of the interruptions. We have seen that the interruptions meet the requirements of frequency and suddenness. It is still a question whether the make or break is the more sudden. The fact that an induction coil used with the Wehnelt interrupter gives definitely directional sparks, and that an X-ray tube shows very little back action proves that the make and break are not of equal suddenness; while by examining the windings of the coil, it is evident that the break is more sudden than the make.

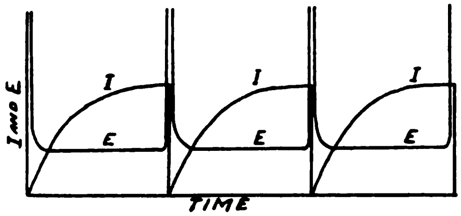


Fig. 13.

This conclusion is verified by curves taken on the oscillograph. The curve shown in Fig. 13 was taken by Professor Blondel.

The next three curves, Fig. 13*a*, were taken by the writer.

These curves show the wonderful regularity of action of the interrupter under good conditions, and render it very evident that the break is more sudden than the make. The significance of these curves will be discussed in Part II.

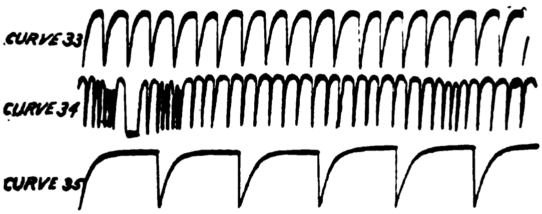


Fig. 13a.

## Part II. The Theory of the Interrupter.

We now have material enough collected to formulate a theory explaining the action of the interrupter, and to test this theory in explaining phenomena hitherto left unexplained. Concerning the cause of the interruption opinion has been very much divided. An article in the *Scientific American Supplement*, No. 1223, says:

"We shall not undertake to give an explanation of the theoretical operation of this curious apparatus, a point on which opinions are as yet much divided. . . . We have here therefore a very complex phenomenon in which the condenser of variable capacity and the self induction of the circuit play the leading parts. The heating of the wire has no direct action, as was at first thought."

Professor Thompson says: "The cause of this remarkable phenomenon is the rhythmic sealing and unsealing of the anode by the liberated gas."

Elihu Thompson considers that the current across the gas film is like an electric arc, and that the heat of the arc decomposes the $\overline{OH}$ and this insulating layer stops the current. Then these gases reunite with an explosion, and the current is again made.

Armagnat proposes the theory that the variable high resistance at the points liberates energy which vaporizes the adjacent liquid and thus insulates the point. Then the vapor condenses and the current is once more set up.

Hill gives the theory that the function of the self-induction is to set up a back action liberating H, and that thus an explosive mixture is formed.

Professor Blondel says: "The energy of the inductance coil charges the variable capacity to a high potential. When this discharges, it drives the gas away and again lets the liquid back to the wire."

Many writers consider the cause to be intimately connected with resonance effects, since, as we have seen, there seem to be certain relations between the self-induction and the capacity.

This last is, we believe, an unnecessary complication, and not borne out by the facts. For instance it cannot explain the fact that temperature, pressure, current or applied E.M.F. affect the frequency and the limiting current. Moreover it is difficult to see how the resonance effect and the electrical oscillations can occur consistently, since it is possible to obtain a wide range of frequency with the same value of inductance. Even granting all this, resonance cannot explain the violence of the interruption action.

Hill's theory of back action is also rather far fetched, we believe. The oscillograph records show no indication whatever of any reverse current. Besides, if the current is already broken, no back current can flow, and if it is not already broken, there is no tendency for it to flow. Thompson and Armagnat in their theories leave no place for the action of the self-induction, which is certainly a factor. Blondel does not refer to any explosion, but simply to the electrical discharge, as the thing which drives the gas away from the point; neither does he explain the action of temperature. None of these theories, we believe, presents the whole truth.

We wish to propose the following theory of the action of the Wehnelt interrupter.

The electrolyte immediately surrounding the wire anode is vaporized not by the heat developed in overcoming the ohmic resistance of the platinum, but in overcoming the transitional resistance between the wire and the liquid, and the resistance of the electrolyte very close to the point. As to the former, it is well known that high resistance is offered to the passage of electricity from a smooth metal surface to a fluid; and as to the latter, we saw, in sec. 1, that almost the whole electrolytic resistance is in the immediate neighborhood of the point. As this greatly increased resistance increases the amount of heat evolved for the instant, a portion of the vapor is dissociated into O and H and thus an insulating sheath of an explosive mixture is formed around the point, stopping the current. This gaseous sheath is the dielectric and the point and the liquid are the plates of an electrolytic condenser. This condenser is charged with the electricity given back to the circuit at the break by the inductive coil, and, as its capacity is small, its potential will be high enough to break down the dielectric by a spark, thus exploding the gaseous mixture and driving away the remaining gas from around the point and permitting the liquid again to make contact. This theory does not interfere with the progress of ordinary electrolytic action, which takes place between the make and break of the circuit. We will now proceed to make a series of deductions from the above theory, and test our own conclusions by fact.

There will be a definite amount of liquid around a given point, and a definite amount of current will be required to vaporize it. Also the larger the point, the greater the amount of liquid to be vaporized, and the energy required to vaporize it will be proportionally greater. This explains the fact that under definite conditions there is a definite lower limiting current, and that the watts consumed at this limiting point are proportional to the surface of the anode point.

The function of the self-induction in the circuit is to charge the electrolytic condenser at the break. Without it the circuit remains permanently broken, which means that the fatigue stage sets in. Why is not the make instantaneous, and does not the current rise at once to its full value? Because of the self-induction in the circuit, and the greater the self-induction the more slowly will the current rise to its maximum value. The times required to make or break will be proportional to the times required for the current to rise and fall to the maximum and minimum values, and these times will be proportional to the time constants of the circuits, *i. e.*, to $L/R$. Thus the frequency will vary inversely as the self-induction, provided the interruption takes place by the time the current reaches its maximum. If this condition is not fulfilled, the relation will not be exact. The first statement agrees with the facts previously stated in sec. 3. Furthermore, since the time required for the make is proportional to $L/R$, and as $R$ is comparatively small, being the resistance of the circuit, the time required for the make is comparatively large. But at the break $R$ includes the exceedingly large resistance of the insulating gas sheath, and hence the break is almost instantaneous. These facts agree with those noted in sec. 9. As the time of interruption is proportional to $L/R$, we should get a higher frequency with a high resistance in a 220-volt circuit than with a low resistance and a 110-volt circuit. This is found to be true.

We have seen that enough energy must be given off to vaporize the liquid around the wire point. If the liquid is already hot, less current will be required and the speed with which the vaporization can take place will be increased. This corresponds to the facts that the lower limiting current decreases and the frequency increases with the rise of temperature.

If the self-induction has a value of .02 henry and the resistance is 10 ohms, the time constant is .002. That is, in .002 second the current will rise to 1/2.718 of its maximum value. It would, therefore, reach nearly its maximum value in .005 second. The break is instantaneous. Why then is not the frequency 5,000 as soon as the lower limit is reached? Because, although the maximum current is large enough to vaporize the liquid, it is not large enough to do it instantaneously. Some time is required to vaporize it, although, as we said before, this time is less if the liquid is already hot. These statements receive additional proof from two observations. First, solutions of low specific heat give a higher frequency than those of a high specific heat. Second, stirring the electrolyte decreases the frequency, and may even stop the interruptions entirely by sweeping away all the liquid before it has had time to vaporize. Thus the fact that time is required to vaporize the liquid accounts for the fact that the interval between the interruptions is greater than the sum of the times required for the make and break. This explains also the fact stated above that the frequency does not always vary exactly inversely as the self-induction. The time required for the make does vary as the self-induction, but any additional time required to vaporize the liquid is independent of the self-induction.

If we place a condenser in parallel with the interrupter, as in sec. 5, we have the following explanation of the facts noted there. The extra current due to the self-induction, which charges the electrolytic condenser at the break, is now split up and part of it flows into the parallel condenser. This is equivalent to increasing the capacity of the electrolytic condenser, and consequently lowering the potential. Thus the potential may be so lowered as to prevent the spark. Then the point would remain insulated, and the fatigue stage would set in. This is exactly what happens.

Why is it not possible, by decreasing the self-induction or the size of the point, or by increasing the current indefinitely, to produce an infinitely high frequency? The self-induction cannot be made less than the amount necessary to charge the electrolytic condenser to the sparking point, and as long as that much is present, the frequency is limited. The current cannot be increased indefinitely because if it becomes too large it heats the anode point above 200° C. and then the spheroidal state causes the fatigue stage to set in. Even below this value, the workable size of the current is limited by the fact that when gas is evolved in too large quantities it interferes with the liquid coming back to the anode point after interruption, and thus the interruptions are irregular. This is shown by oscillograph record no. 34 in sec. 9, which was put in partially to illustrate this fact. Lastly the point cannot be made indefinitely small since with very small points, the surface tension of the tiny bubbles prevents their escape from the point, although this defect may be partially remedied by bending the glass tube so that the wire tip extends upward instead of downward from the glass.

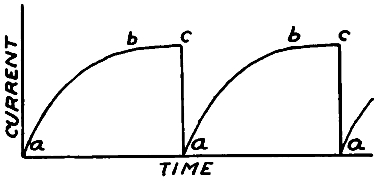


Fig. 14.

In conclusion, further to establish this theory, we shall construct a current curve to fit the theory and compare it with the oscillograph records in sec. 9. See Fig. 14. The curves are logarithmic, due to the inductance of the circuit. The section $\overline{ab}$ represents the make, the section $\overline{bc}$ represents the nearly constant current, while the liquid is vaporizing; and the section $\overline{ca}$ represents the break.

Following out the theory we make the following observations: (*a*) The higher the temperature of the electrolyte the shorter $\overline{bc}$. (*b*) The greater the self-induction, the longer and less erect the part $\overline{ab}$. (*c*) The larger the point, the larger the dimensions of the whole figure. (*d*) At excessive current, the portion $\overline{bc}$ may entirely disappear, and the break even occur before $b$ is reached. The figures become irregular, and some sections may be skipped. (*e*) The higher the conductivity of the solution, the larger the whole figure and the more erect $\overline{ab}$. (*f*) The greater the specific heat of the solution, the longer $\overline{bc}$. (*g*) The greater the self-induction in the circuit, the more regular the interruptions.

The majority of these deductions are borne out by the oscillograph records in sec. 9. Thus while the above theory is simple, we believe it is adequate to explain the main features of the action of the Wehnelt interrupter.

In closing we wish to express our thanks to Professor Henry Dates, of Case School of Applied Science, for the use of the oscillograph of that institution, and to Dr. John Whitmore, of the University of Wooster, for his suggestion and direction throughout the investigation.

Physical Laboratory,
University of Wooster.

# MAGNETIZATION CURVES FOR A SAMPLE OF IRON WIRE ONE MIL IN DIAMETER.

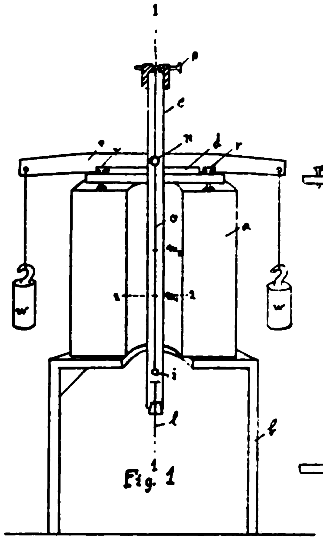
BY J. A. VEAZEY.

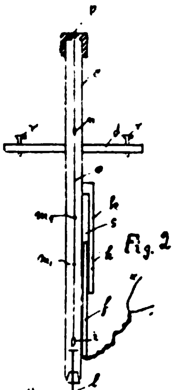
THE following experiment was suggested by Professor W. S. Franklin; and to him the author is greatly indebted for many valuable ideas, and to Professor J. H. Wily for assistance in working out details of mechanical construction. The object of the experiment was to obtain a number of magnetization curves for a sample of mil iron wire, and from the curves to determine something in regard to the relative magnitude of the molecules, or groups of molecules constituting the elementary magnets of which a large magnet has been thought to consist. If the magnetization curves show changes indicative of sudden changes of magnetic flux in the wire, the inference is that the groups are of the order of one mil in magnitude. If the magnetization curves are smooth, the inference is that the groups are less than one mil in order of magnitude.

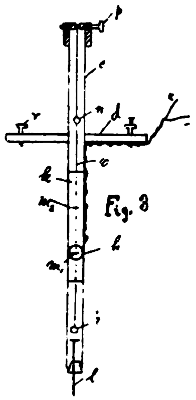
The method of testing the iron wire is a modification of what Ewing calls the magnetometric method. The changes of magnetic flux in a mil iron wire are so small with magnetizing fields of moderate intensity that the ballistic method of detecting them was not practicable.

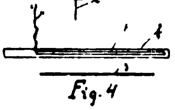
The apparatus used is a modification of that employed by Ewing. Referring to the drawings, Fig. 1 is a front elevation of the special form of magnetometer used in the test, one half of the iron shield *a* being cut away. Fig. 2 is a section of the suspension case *c* taken on the line 1–1, Fig. 1. Fig. 3 is a rear view of the suspension case *c* showing the coil *h* and the lead wires used in determining the strength of the controlling field at the small magnet $m_1$. Fig. 4 is a view of wooden slide *f* which carries the magnetizing coil *t*. The glass tube *g* carrying the sample of wire to be tested is also shown in this figure. The tube *g* is small enough to slide into the coil *t*.

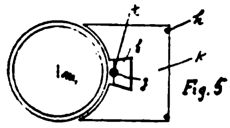
Fig. 5 is a horizontal section of the case *c* taken on the line 2–2, Fig. 1. Fig. 6 is a general view of the electrical connections used during the test. The letter *B* refers to a 120-volt storage battery,

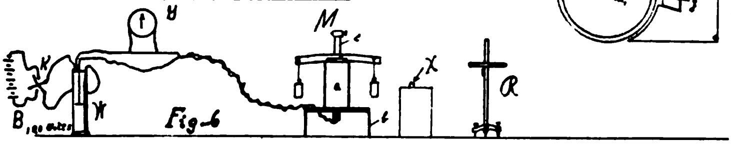












*W* to a water rheostat, *G* to a galvanometer standardized to read values of current, *M* to special form of magnetometer, *X* to a large steel magnet producing a directing field at the small magnet $m_1$, and *R* to a telescope and scale.

Referring to the apparatus in detail, like letters of reference refer to like parts in the several figures. The letter *a* refers to a well-annealed iron spool 20 cm. outside diameter, 10 cm. inside diameter, and 30 cm. high wound full of well-annealed iron wire. The spool is used as a magnetic shield to shield the suspended magnets $m_1$ and $m_2$ as far as possible from the effects of a fluctuating earth field. The letter *b* refers to a wooden bench which has a large hole in it to permit of the free adjustment of the apparatus suspended in the shield *a*. The letter *c* designates a glass tube containing an astatic system of magnets $m_1$ and $m_2$ fastened to a long, fine glass rod *o* carrying the mirror *n* at one end and a thin mica vane *i* at the other end. The function of the mica vane was to help make the suspended system "dead beat." The rod *o* was suspended from the brass pin *p* by a single fiber of silk. A cork was fitted into the lower end of the tube, and through it extended a copper rod *l* to the upper end of which a thin copper disk was soldered. When the magnetometer was not in use, the copper rod was pushed up until the disk at its end formed a support for the delicate glass rod *o*. One silk fiber was used to suspend the magnetometer needle. The first fiber used lasted during the subsequent trials. The letter *d* refers to a wooden disk through the center of which passes the glass tube *c*. This tube is wedged fast and the wedges are glued in place. The disk *d* is fitted with leveling screws *rr*. The letter *e* designates a yoke which passes across the top of the disk *d* and supports the weights *ww*. The object of the yoke is to keep the delicate suspension inside the case *c* from being too much shaken while the disk *d* is being leveled. The letter *f*, Fig. 4 and Fig. 2, is a wooden slide which slides with sufficient friction to hold it where put in groove *s*. This wooden slide carries the magnetizing coil *t*. The letter *g* represents a fine glass tube containing the sample of annealed mil iron wire to be tested. This tube is small enough to slide into the

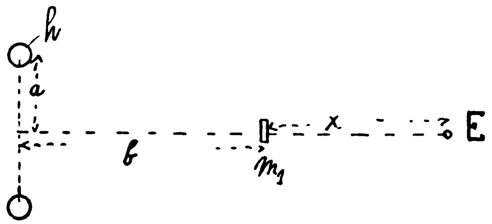


Fig. 7.

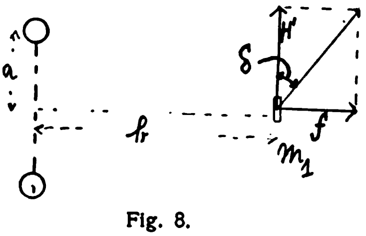


Fig. 8.

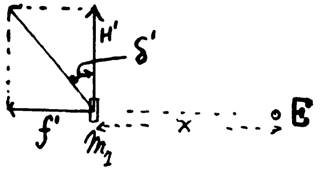


Fig. 9.

magnetizing coil *t* whose inside diameter is .05 cm. The letter *h* refers to a coil of wire consisting of a single turn 2 cm. in diameter and whose plane was 2.5 cm. from the center of the magnet $m_1$. This coil was used to measure the value of the directing field at the small magnet $m_1$. The letter *k* refers to a piece of wood bound to the glass tube *c* which contains the groove *s*. The letters $m_1$ and $m_2$ refer to two small magnets, each .3 cm. long, made of glass hardened hair spring. They form an astatic system and are fixed to the glass rod *o*, 8 cm. apart. The magnetizing coil *t* was made of silk insulated copper wire No. 36 wound 50 turns per centimeter on a glass tube of .05 cm. bore.

## CALCULATIONS.

Let Fig. 7 be a view of a horizontal section through the standardizing coil *h*, the small magnet $m_1$, and the upper end *E* of the sample of mil wire under test.

Let *M* equal the strength of the induced pole at *E*.

Let *l* equal the length of the sample of iron wire.

Let *f* (Fig. 8) equal the field at $m_1$ produced by current *i* in coil *h*.

$$f = \frac{\pi a^2 n i}{5(b^2 + a^2)^{\frac{3}{2}}},$$

where *i* is expressed in amperes and *n* equals one turn in coil *h*.

Let *f′* (Fig. 9) equal the field produced at $m_1$ by the induced pole *M*.

$$f' = M\left[\frac{1}{x^2} - \frac{x}{(x^2 + l^2)^{\frac{3}{2}}}\right]. \tag{2}$$

$$H = \frac{4\pi z I}{10}, \tag{3}$$

where *H* is the magnetizing field in the coil *t* of *z* turns per centimeter produced by the current *I* expressed in amperes.

$$H' = \frac{f}{\tan \delta} \tag{4}$$

where *H′* is the directing field produced at the magnet $m_1$ by the permanent magnet *X*, Fig. 6.

$$(5) \qquad \frac{f'}{H'} = \tan \delta', \quad \text{or} \quad f' = H' \tan \delta'.$$

Substituting the value of $f'$ from (5) in (2)

$$(6) \qquad M = \frac{H' \tan \delta'}{\frac{1}{x^2} - \frac{x}{(x^2 + l^2)^{\frac{3}{2}}}}.$$

Let $s =$ section of mil iron wire, or 1 circular mil, or $5.07 \times 10^{-6}$ cm.$^2$

$$(7) \qquad B = \frac{4\pi M}{s} + H.$$

where $B$ is the flux density, in lines per square centimeter.
From (6)

$$(8) \qquad B = \frac{4\pi H' \tan \delta'}{\left[\frac{1}{x^2} - \frac{x}{(x^2 + l^2)^{\frac{3}{2}}}\right] s} + H.$$

## OBSERVATIONS.

The following observations were taken to determine the value of the directing field $H'$ at the magnet $m_1$.

| Current in Coil $h$, Amperes. | Scale of Magnetometer. | Current in Coil $h$, Amperes. | Scale of Magnetometer. |
|---|---|---|---|
| 0.125 | 13.25 | −.0125 | 18.90 |
| 0.312 | 9.80 | −.0312 | 23.96 |

The distance from the scale of the magnetometer to the mirror of the magnetometer suspension . . . . . . . . . . . . . . . . . . . . . . . . . = 95 cm.
Radius $a$ of coil $h$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . = 1 cm.
Turns $n$ in coil $h$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . = 1 turn.
Distance $b$ from plane of coil $h$ to magnet $m_1$ . . . . . . . . . . . . . = 2.5 cm.
Distance $x$ from end $E$ of iron sample to magnet $m_1$ . . . . . . . . . . = 1.7 cm.
Length $l$ of sample of mil iron wire under test . . . . . . . . . . . . . . = 6.0 cm.
Number of turns per centimeter $z$ of magnetizing coil $t$ . . . . . . . . . = 50 turns.

Using the first set of observed values:
From (4)

$$H' = \frac{f}{\tan \delta}.$$

From (1) by substitution $f = .0322 \times .0125 = .0004$ gauss.

$$\tan 4\delta = \frac{\text{diff. scale readings}}{95},$$

$$= \frac{18.9 - 13.25}{95} = .0595,$$

$$4\delta = 3^\circ\ 24',$$

$$\tan \delta = .0148,$$

$$H' = \frac{.0004}{.0148} = .0272 \text{ gauss.}$$

Substituting in equation (8)

$$B = 198{,}600 \tan \delta' + 62.83 I.$$

Using the second set of observed values and employing the same method of finding the constant multiplier of $\tan \delta'$ we have

$$B = 196{,}000 \tan \delta' + 62.83 I.$$

Taking the average of the constant multipliers of $\tan \delta'$ we have

$$B = 197{,}000 \tan \delta' + 62.83 I.$$

This is the form of equation used in determining the values of $B$ the flux density in the sample of mil iron wire. $H$ is determined from equation (3) and equals $62.83I$, where $I$ is the current in the coil $t$, expressed in amperes.

## ERRORS.

1. The magnetizing field $H$ is somewhat less than $4\pi zI/10$ because of the demagnetizing action of the rod upon itself. Ewing has shown that this error is negligibly small when the rod is six hundred diameters long. The specimen used was six centimeters long and one thousandth of an inch in diameter. Hence the error in this experiment due to the demagnetizing action of the induced poles may be neglected.

2. The field due to the current in the magnetizing coil acts directly upon the suspended magnet and produces some deflection, whereas equation (5) assumes that the deflection is due entirely to the field

which emanates from the magnet poles of the rod. Referring to Figs. 2 and 4 it will be seen that by means of the slide *f* the end of the magnetizing coil *t* can be pushed up or down until the magnetic field produced by a current in the coil has an effect upon $m_2$ equal and opposite to its effect upon $m_1$. Before sliding the glass rod with the sample of iron wire into the magnetizing coil *t*, the zero reading of the magnetometer was taken. A current was sent through the coil *t* and the slide *f* was moved to such a position as gave no deflection of the magnetometer needle when the current was reversed in direction.

No attempt was made to compensate for errors due to the distribution of the induced poles at the ends of the sample. Extreme accuracy was not attempted and the calculations were all made with the slide-rule where possible. The form of the magnetization curve was the thing sought for rather than the actual values of the flux in the wire. The readings were taken after the glass tube *g* had been slid into the magnetizing coil *t* until the upper end of the sample of iron wire was opposite the magnet $m_1$. The effect of the induced poles in the iron sample would be small at the magnet $m_2$ and the effect upon the two magnets was considered as a single effect at the magnet $m_1$. This would not alter the smoothness or jaggedness of the magnetization curve, but would change the numerical value of the flux density.

| *H* | *B* | *H* | *B* |
|---|---|---|---|
| 8.1 | 14,458 | −2.80 | 13,093 |
| 7.6 | 14,657 | −3.10 | 12,803 |
| 7.1 | 14,907 | −3.45 | 12,403 |
| 6.3 | 14,655 | −3.70 | 11,354 |
| 5.5 | 14,655 | −4.00 | 5,864 |
| 4.6 | 14,655 | −4.30 | 867 |
| 3.9 | 15,504 | −4.70 | − 3,275 |
| 3.1 | 14,643 | −5.10 | − 5,685 |
| 2.4 | 14,642 | −5.40 | − 6,306 |
| 1.57 | 13,772 | −5.66 | − 6,885 |
| 0.50 | 14,300 | −5.96 | − 8,436 |
| 0.00 | 14,200 | −6.28 | − 9,816 |
| −0.50 | 14,000 | −7.10 | −10,807 |
| −1.63 | 13,500 | −8.63 | −13,550 |
| −2.39 | 13,252 | | |

The apparatus was found so delicate that the pounding of steam pipes, walking on floors overhead, or the moving of machinery in distant parts of the building caused the magnetometer to be in a state of constant vibration through small limits.

The night was found the best time for taking observations, and the following values of $B$ and $H$ were calculated from a set of reading taken between the hours of eleven and twelve o'clock P. M.

By placing a light supplied with alternating current under the glass tube $c$, the position of the astatic system of magnets $m_2$ and $m_1$ could be directly observed by looking in at the top of the tube $c$, which was left open for this purpose.

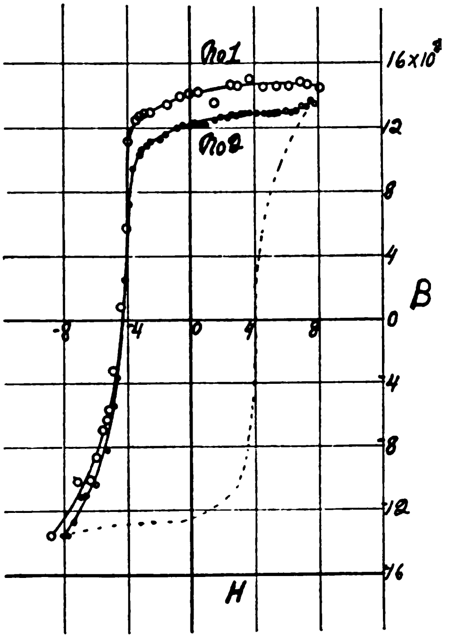


Fig. 10.

Curve number 1, in Fig. 10, is that which corresponds to this series of values of $B$ and $H$.

The last trial was begun at two o'clock A. M. At this time the vibrations of the building were reduced to a minimum and the following values of $B$ and $H$ were calculated from a set of readings taken between the hours of two and four o'clock A. M.

| *H* | *B* | *H* | *B* |
|---|---|---|---|
| 7.86 | 13,538 | 0.31 | 12,340 |
| 7.73 | 13,628 | 0.00 | 12,230 |
| 7.42 | 13,387 | −0.31 | 12,230 |
| 7.10 | 13,387 | −0.69 | 12,050 |
| 6.66 | 13,207 | −1.57 | 11,690 |
| 6.28 | 13,206 | −1.95 | 11,458 |
| 6.03 | 13,146 | −2.39 | 11,248 |
| 5.66 | 13,146 | −2.70 | 11,007 |
| 5.28 | 13,145 | −3.08 | 10,757 |
| 4.90 | 13,145 | −3.26 | 10,377 |
| 4.65 | 13,145 | −3.70 | 9,566 |
| 4.14 | 13,084 | −3.84 | 7,406 |
| 3.89 | 13,029 | −4.14 | 2,399 |
| 3.70 | 13,029 | −4.65 | − 3,767 |
| 3.26 | 12,873 | −4.90 | − 5,655 |
| 3.08 | 12,873 | −5.28 | − 8,145 |
| 2.70 | 12,872 | −6.03 | −10,226 |
| 2.39 | 12,732 | −6.34 | −10,996 |
| 1.95 | 12,702 | −6.91 | −11,007 |
| 1.57 | 12,625 | −7.42 | −12,692 |
| 0.88 | 12,341 | −7.73 | −13,498 |
| 0.57 | 12,340 | −7.86 | −13,498 |

Curve number 2 is the curve corresponding to this set of values of $B$ and $H$.

The last set of observations was taken under the most favorable conditions and were in sufficient number to locate the $B$ and $H$ curve with greater certainty than in any previous trial. The suspension system in the glass tube *c* was observed at frequent intervals during the test, and moved without constraint due to floating hairs or interfering dust particles.

The points locating the last curve are distributed in such a manner as to force the conclusion that the curve is perfectly smooth and that the elementary magnets of whatever kind they may be are less than one mil in order of magnitude.

PHYSICAL LABORATORY,
LEHIGH UNIVERSITY.

# AN IMPROVED FORM OF VOLUMENOMETER.

By John Zeleny and L. W. McKeehan.

In the usual form of volumenometer,[1] one of the mercury columns is open to the atmosphere, and the pressure of the air inside the instrument is obtained from simultaneous readings of the barometer and the difference in the height of the two mercury columns. Determinations of volume, and more especially those where the volume of the substance is only a minor fraction of its exterior bulk, are subject to a considerable error caused by the fluctuations in the atmospheric pressure, owing to the attendant variations in the volume of the enclosed air and the changes occurring in the relative states of the two mercury meniscuses.

Moreover, with substances in the form of extremely fine powders, the canals between the particles are so minute that the gas passes through them but slowly when the differences in pressure are small. An equilibrium of pressure between the air among the particles and that outside is established, therefore, only after an appreciable time, and when the outside pressure is undergoing variations a complete equilibrium is never attained.

These difficulties are obviated very simply by replacing the open tube of the instrument with a baromanometer, thus making the pressure readings independent of the pressure of the atmosphere.

The accompanying figure gives a diagram of a form of such an improved volumenometer which has been found very serviceable in making a number of density determinations, for which it was specially devised.

The cylindrical brass vessel *A*, used for holding the substance whose volume is to be determined, is shown enlarged at the side of the figure. This vessel screws into the brass piece *B*, which is rigidly attached to the frame supporting the instrument. The surfaces of contact between *A* and *B* were turned true in a lathe, and

---

[1] Regnault, Ann. de Chem. et Phys. (3), XIV., 1845.

the thinnest coating of vacuum wax is sufficient to ensure a perfectly air-tight joint. The hole at *C* permits the use of a lever bar for tightening and loosening the cylinder. A groove *D* was cut across the threads at one place to serve as an outlet for the air while the cylinder is being screwed into position.

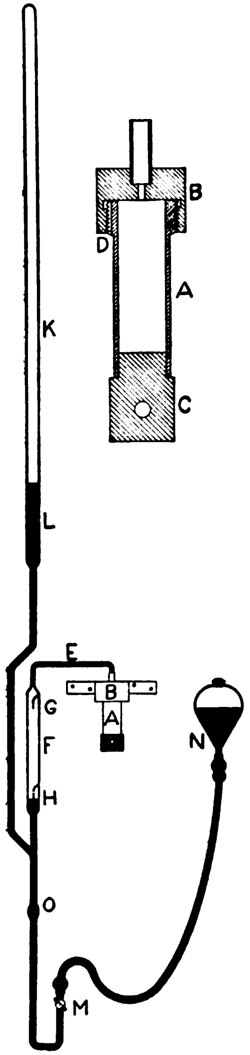


Fig. 1.

The glass tube *E* leads from *A* to the cylindrical vessel *F*, in which the two projecting points *G* and *H* mark off a definite volume. The baromanometer *K*, which is the new feature of the apparatus, is of the same diameter (18 mm.) as the tube *F*. The air had been removed from the portion of *K* above the air trap *L*, by boiling the mercury, so that the difference between the two surfaces in *F* and *K* gives the true pressure in *F* and *A*.

The level of the mercury in *F* can be changed by opening the stop-cock *M* and raising or lowering the mercury reservoir *N*. The trap *O* catches any air-bubble that may be carried in by the flowing mercury. The taking of readings was facilitated by etching the ends of the points *G* and *H* to prevent troublesome reflections, and by placing a paper with inclined lines ruled on it, back of the tubes to establish better the top outlines of the mercury meniscuses. The mercury was not brought into actual contact with the points, as even a light contact is sufficient to produce an appreciable depression. A small gap was left and measured and a correction applied to the volume involved.

The volumenometer was used in a constant temperature room and an accuracy of one half of one per cent. was obtained even when the volume measured was less than one fifth of the volume of the cylinder *A*.

The essential parts of the apparatus should be screened against the radiation from lights and the observer, and if used in an ordinary room the whole should be enclosed in a tight box with a stirrer working inside to equalize the temperature between the top and bottom.

By unscrewing the cylinder $A$, the instrument is always available for use as a barometer.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
October 21, 1909.

# ELECTRIC WAVE RECEIVERS.

By Harold W. Webb and L. E. Woodman.

THE study of short electric waves has been difficult because of the small and variable emission of the vibrators employed and the lack of sufficiently sensitive receivers. The improvement of the latter, together with the construction and tests of new forms, was made the object of extended study, the results of which are briefly described in this paper.

As will be shown later, the quantitative measurement of the currents set up in a resonator by incident waves, less than 10 cm. in length, can be best made by using the heat effects. Hence, in any method employed we should have a resonator of the constricted type, that is, one at the center of which is a large resistance with a small heat capacity. For the determination of the temperature increase, all of the methods well known in the study of thermal radiation have been employed. The familiar Klemencic receiver[1] and the bolometric receiver[2] depend upon the thermo-electric effect and the temperature coefficient of resistance, respectively. The Boy's radiomicrometer was adapted to the study of electric waves by G. W. Pierce,[3] in his combined Klemencic receiver and D'Arsonval galvanometer, while the "radiometer receiver" of G. F. Hull,[4] who placed the vane of a radiometer opposite the gap of a Righi resonator, corresponds to the Nichols' radiometer.[5]

The Klemencic receiver has already been treated in an earlier paper,[6] in which a method for greatly increasing its sensitiveness was described. The bolometric receiver was not studied at any great length, as its high resistance and the indirect methods of determining the changes of resistance were very inconvenient. The

---

[1] I. Klemencic, Wied. Ann., 45, 78, 1892.

[2] Rubens and Ritter, Wied. Ann., 40, 58, 1890.

[3] G. W. Pierce, Phil. Mag., 1, 179, 1901.

[4] G. F. Hull, Science, N. S., 16, p. 175, 1902.

[5] E. F. Nichols, Phys. Rev., IV., 298, 1897.

[6] Webb and Woodman, Phys. Rev., XXIX., No. 2, 94, 1909.

use of a differential galvanometer seems to be the best method with this receiver. For long waves the bolometer has the advantage of great sensitiveness.[1]

The receiver devised by Hull[2] was experimented with at great length in this investigation. By the introduction of different constrictions, the attempt was made to increase its sensitiveness, but it was not found possible to exceed that of an ordinary Klemencic receiver (thermo-couple .025 mm. in diameter, with galvanometer, sensitiveness $4 \times 10^{-9}$ volt). Furthermore, the unavoidable dissymmetry of the apparatus caused great unsteadiness, which with the lack of proportionality between the deflections and the incident energy made this construction very unsatisfactory.

A new form of receiver depending upon "radiometric" action was therefore devised, which took full advantage of the heat developed. The improvement consisted in making the resonator a part of the suspended system, and heating a metal radiometer vane directly by the current. The construction shown in Fig. 1 (*a, b, c*) was adopted, in order to reduce the moment of inertia as much as

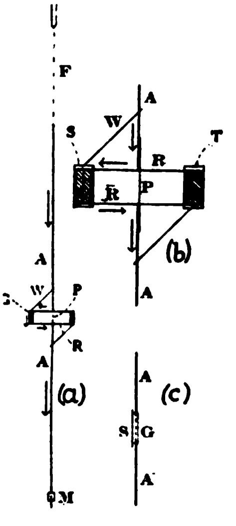


Fig. 1.

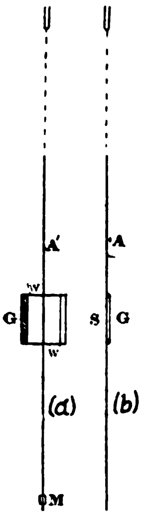


Fig. 2.

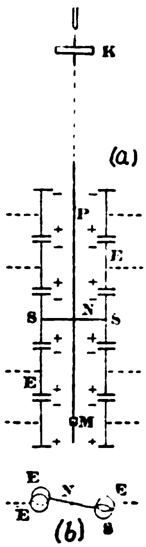


Fig. 3.

---

[1] W. P. White, Phys. Rev., XXV., 138, 1907.

[2] G. F. Hull, *l. c.*

possible, its value in a well built receiver being less than $4 \times 10^{-5}$ gr. cm.[2] The straight resonator antennæ $A$ are of wire, 0.12 mm. in diameter, each 2.6 cm. in length. A small glass rod $P$, cemented with shellac, holds them rigid in almost perfect alignment, with their inner ends 0.15 cm. apart. The two cross wires $R$ are .037 mm. in diameter and 6 mm. in length, soldered at right angles to the inner ends of the antennæ. Metallic connection is made between the two parts of the resonator by a strip of gold foil $G$, 0.7 mm. in width and .0001 mm. in thickness (the thinnest commercial foil), soldered to the ends of the cross wires with soft solder. The system was suspended by a fine quartz fiber $F$, in a closed vessel (2.5 cm. in diameter), connected with a vacuum pump. The gas pressure in the surrounding vessel was varied from .04 to .07 mm. of mercury during the experiments.

The current flow in the resonator during one half of the oscillation is shown in the figure by arrows. The effect of the current is to heat the gold, and the consequent gas force, which is cut off on one side by the light mica shield $S$, supported within a few tenths of a millimeter of the foil by the glass whip $W$, causes a rotation of the system, measured by the image of an illuminated scale in the small plane mirror $M$. For the sake of symmetry, a similar strip of gold foil, $D$, not soldered to the cross wires but insulated with shellac, and a second mica shield $T$, similar to $S$, was fixed to the opposite side of the suspension. Thus, the system was indifferent to ordinary heat radiation and perfectly steady. To avoid the effects of jarring, the apparatus was set upon a heavy block of stone resting on rubber stoppers.

The sensitiveness of the receiver was increased about six times by the use in place of the gold foil of a platinum strip, .0004 mm. in thickness and .01 mm. in width, obtained by rolling out a silver wire with a platinum core (.002 mm. in diameter) to a width equal to five times its initial diameter, the silver being later dissolved off with nitric acid. This also increased the resistance of the constriction from four to thirty ohms. This receiver was compared with a Klemencic having a soldered iron-constantan thermo-element (.025 mm. in diameter — galvanometer sensitiveness $4 \times 10^{-9}$ volt). The radiometric was approximately forty times as sensitive as the thermo-electric receiver, as measured in mm. deflection. While this advantage is reduced to approximately two times, if we consider the improved Klemencics already referred to,[1] the actual gain in sensitiveness is much larger. For the true test depends not only upon the ratio of the actual deflections, but also upon the inverse ratio of the variations of the zero reading. A deflection of 10 divisions, which, on account of a steady zero, can be read accurately to one tenth of a division, is equivalent to a deflection of 100 divisions, in which the error due to a shifting zero is one division. Now the galvanometers used with the Klemencic receiver, even though they be carefully shielded, vary in their zero reading by at least one half of a division, while a carefully constructed radiometer suspension is steady within one tenth of a division. Again the "line effect," as we shall call the disturbances in the receivers, resulting from the currents set up in the lead wires by the varying static charges on the induction coil and vibrator system, causes a zero error in the Klemencic (also in the bolometric type), which in some cases is very large. This further reduces its true sensitiveness. From these considerations we may conclude that the new radiometric receiver is more than ten times as sensitive as the best thermo-electric couple.

The effect of varying the gas pressure was investigated.[2] The sensitiveness was a maximum for pressures between .04 and .07 mm. of mercury. No advantage could be observed in the use of a mica wall placed 1 mm. from the vane, and as it tended to make the suspension unsteady it was removed.

The advantages of the radiometric receiver are:

1. Increased sensitiveness.
2. Steady zero reading.
3. Freedom from "line effect" and similar disturbances.
4. Simplicity: it is self-contained, having no lead wires or other metallic connections. Being independent of very sensitive galvanometers, its use is possible where such instruments are not readily available.

On the other hand, its disadvantages are such that it is not convenient for many kinds of measurements. The difficulty of altering

---

[1] Webb and Woodman, *l. c.*

[2] E. F. Nichols, *l. c.*

its length and of adjusting its period,[1] the necessity of using the receiver in a vertical position, and of supporting it so as to be free from jarring, and the increased difficulty of construction, make it impossible to use in many cases. The peculiar form of the constriction affects the period in an unknown way, but this difficulty can be avoided by an alteration in the construction.

For very short waves the radiometric receiver, while less sensitive (in divisions deflection) than the thermo-electric,[2] is of great advantage. The form which is then used is very simple and easy to construct. It consists (Fig. 2, *a*, *b*) in a resonator formed by a strip of very thin gold leaf *G*, stretched between two glass whips *W*, cemented at right angles to the glass suspension axis *A*. Behind the gold leaf, attached to the opposite side of the whips, is a mica shield *S* (Fig. 2, *b*), cutting off the gas action on the rear. A similar mica vane on the opposite side of the system is used to balance the resonator. The zero reading of this receiver was very steady, and the small deflections obtained with the short waves could be read to one tenth of a division. On the other hand the thermo-electric receiver with which it was compared, was subject to large disturbances,[3] which could not be entirely eliminated. The resulting error was considerable and hence the less sensitive radiometric receiver was of great advantage. The absence of lead wires is also an advantage with the short wave-lengths.

Two other types of receiver have been used for measuring the energy in electric wave experiments, the coherer and the mechanical receivers. The coherer, with[4] or without[5] resonating antennæ, has already been considered.[6] It has the advantage of great sensitiveness, but its variableness and the impossibility of using it for accurate quantitative measurements makes its usefulness very limited. The mechanical receivers, depending upon the electrostatic and electrodynamic forces produced by the currents in resonators, were first used by Hertz[7] with waves on wires, and by Bjerknes[8] for free

---

[1] Willard and Woodman, Phys. Rev., XVIII., 5, 1904.

[2] P. Lebedew, Wied. Ann., 56, 3, 1895.

[3] P. Lebedew, *l. c.*

[4] G. V. Maclean, Phil. Mag., 48, 115, 1899.

[5] J. G. Bose, Phil. Mag., 43, 60, 1897.

[6] Webb and Woodman, *l. c.*

[7] H. Hertz, Wied. Ann., 42, 411, 1891.

[8] V. Bjerknes, Wied. Ann., 44, 76, 1891.

radiation of wave-length 50 cm. Their use has also been suggested[1] for very short waves, and some experiments were tried with them during these investigations. Fig. 3 (*a*, *b*) shows an apparatus which was constructed to measure the attractive forces between the opposite charges on the opposite ends of two similar resonators, due to the currents set up by an incident wave-train. Each of the ten dumb-bell resonators, *E* and *S*, consisted of a fine wire, .037 mm. in diameter and 6 mm. in length, connecting the centers of two parallel circular tin-foil disks, radius 1 mm., in the manner shown. One pair *S* was fastened by the glass cross-arm *N* to the glass suspension axis *P*, which was hung on a fine quartz fiber, so that these resonators swung in a horizontal plane between the stationary resonators *E*, with a clearance of only 0.5 mm. By means of the magnet *K*, on the torsion head, the suspended resonators were displaced 0.8 mm. from their symmetrical position between the others (see Fig. 3, *b*), and the electrostatic force acting to draw them back, when they were set into electric oscillation by the electric waves, was observed by means of the mirror *M*. Although energy, sufficient to give several thousand divisions deflection with a similarly placed Klemencic receiver, was incident upon this system, the force was too small for detection.

This was due largely to the size of the resonators, which were too small to take up sufficient energy. Furthermore, it can be shown that the efficiency of this mechanical receiver decreases much more rapidly, with decreasing wave-length, than that of a receiver depending upon the heat developed, of a Klemencic, for example. This is due to the high frequency of the oscillations, which correspond to a wave-length of about 10 cm. If the varying charge on the end of a resonator is $E \sin nt$ ($n = 2\pi \times$ frequency), the current between the ends is given by

$$C = (2\pi \times 3 \times 10^{10} \div \lambda) E \cos nt.$$

Now the force acting on the system is intermittent, being produced by about 150 wave-trains per second, each approximately equivalent to three undamped oscillations. The average value of the force is therefore equal to the sum of these impulses for one second; or

$$F = 150\{(3 \times \lambda) \div (3 \times 10^{10})\} K \frac{n}{2\pi} \int_0^{\frac{2\pi}{n}} (E \sin nt)^2 dt$$

---

[1] E. F. Nichols, paper read before the International Electrical Congress of St. Louis, 1904.

$$= 150\{(3 \times \lambda) \div (3 \times 10^{10})\}\frac{KE^2}{2} = 7.5 \times 10^{-9}KE^2\lambda,$$

where $K$ is a constant depending on the dimensions of the system. The current for a given force is, therefore, $C =$ const. $\div \lambda^{\frac{3}{2}}$, so that for the same deflection the current in the resonators, the constants of the system remaining the same, must increase as the wave-length is diminished according to this relation.

On the other hand, in the case of the temperature receivers, the current necessary for a given deflection is $C = M \div \sqrt{t}$, where $M$ is a constant depending upon the resistance and other constants of the system (the same for all sizes of receiver of the same type), and $t$, the time during which the current is actually flowing in an interval of one second, given, as above, by

$$t = (3 \times 150 \times \lambda) \div (3 \times 10^{10}).$$

Hence $C =$ const. $\div \lambda^{\frac{1}{2}}$, so that for a given deflection the current in the resonator is inversely proportional to the square root of the wave-length. Thus as the wave-length is decreased the temperature receivers decrease in sensitiveness much more slowly than the mechanical system, the ratio of the deflection given by the former to that given by the latter for constant current varying inversely as the square of the wave-length; *i. e.*,

$$\frac{\text{Deflection: temperature receiver}}{\text{Deflection: mechanical receiver}} = \frac{\text{constant}}{\lambda^2}.$$

Hence we find that for short waves the former are still very sensitive while the latter are not available. An approximate computation showed that a current of at least .04 ampere would have been necessary in the mechanical system described to obtain an observable effect. With much longer waves and more suitable resonators the effect should be measurable, and this type of receiver capable of being used to measure the incident energy.

In conclusion, we wish to thank Professor G. F. Hull for his kindness in loaning us apparatus, and especially to acknowledge our indebtedness to Professor E. F. Nichols, at whose suggestion and under whose guidance this work was done.

PHŒNIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY,
December, 1908.

# THE WAVE-LENGTH AND OVERTONES OF A LINEAR ELECTRICAL OSCILLATOR.

By James E. Ives.

THE experiments described in this paper were undertaken to determine the wave-length of the electric waves emitted by linear oscillators. By a linear oscillator is meant an oscillator of cylindrical form, the diameter being small compared with the length.

Two theories exist to-day as to the wave-length of such an oscillator. One theory, due to Abraham,[1] reaches the conclusion that the wave-length is equal to about twice the length, and the other, due to Macdonald,[2] that it is equal to about two and a half times the length of the linear oscillator. The first theory is supported by Rayleigh,[3] Kiebitz[4] and others; the second by Pollock,[5] Cole[6] and others. The subject is one of great theoretical interest, and of practical importance.

The oscillator used in these experiments consisted of two cylinders of brass supported by a hollow cylinder of hard rubber. The arrangement is shown in Fig. 1. In the figure, *rr* is the cross-section of the hard rubber cylinder; *bb*, of the brass cylinders, and *c* the cross-section of a cork. Three oscillators were used having total lengths of 5, 7.5 and 10 cm. respectively. The diameter was the same in each case, and equal to .25 cm. The error in the measurement of the dimensions in any case was less than 2 per cent. The hard rubber cylinder was 3.85 cm. (1 ½ inches) long, and had an

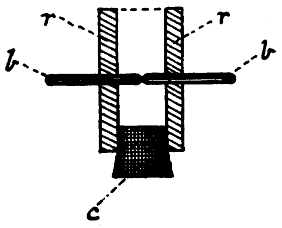


Fig. 1.

---

[1] Ann. Phys., 66, 1898, pp. 435–472.

[2] Electric Waves, p. 111.

[3] Phil. Mag., 8, 1904, pp. 105–107.

[4] Ann. Phys., 5, 1901, pp. 872–904.

[5] Phil. Mag., 7, 1904, pp. 635–655.

[6] Phys. Rev., 20, 1905, pp. 268–272.

outer diameter of 2.26 cm. (⅞ inch) and an inner diameter of 1.26 cm. (½ inch). The lower end of the cylinder was plugged with a cork *c*, and it was filled to two thirds of its depth with kerosene oil.

The oscillator was charged with an induction coil operated by a rotating mercury interrupter. The charge was led to the brass cylinders by two brass wires bent at their ends into elbows. The lead sparks were about 3 mm. long. The length of the spark gap between the brass cylinders was about .015 cm.

The receiver was made of two No. 29 (Brown and Sharpe gauge) copper wires, .0285 cm. in diameter, connected by a thermo-electric junction of iron and constantan wires .00208 cm. in diameter. The portion of the receiver consisting of the fine wire was about 4 mm. long, and had a resistance of about 6 ohms. The receiver and its lead wires were fastened with sealing wax to a small wooden board, 4 cm. long by 1 cm. wide. The receiver is shown in Fig. 2. In the figure, $R_1$ and $R_2$ are the receiver wires, and $L_1$ and $L_2$, the lead wires. The lead wires passed through two holes in the board and were led straight back horizontally through two small glass tubes which supported the board.

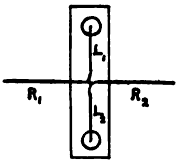


Fig. 2.

Both oscillator and receiver were usually placed in the foci of cylindrical parabolic mirrors made of sheet zinc. The mirrors had a focal length of 7.5 cm.; were 60 cm. long, 64 cm. high and 31 cm. deep. The oscillator and receiver were used in a horizontal position, and the measurements made when they were about a meter apart, and 154 cm. above the floor of the room. The room was 6 meters long, 3.5 meters wide and 3.75 meters high.

The galvanometer used to detect the thermo-electric current was constructed in the shop of the Physical Laboratory of the University of Cincinnati, from specifications given by Coblentz in the Bulletin of the Bureau of Standards (vol. 4, pp. 424–435, Figs. 5 and 6). It is a four-coil Thomson galvanometer with small coils, and a very light astatic suspension made of ten magnets each 1.1 mm. long, five above and five below. The whole suspension weighed only about 7 milligrams. The resistance of each coil was about 18.3 ohms. Since it was used with all four coils in parallel the resultant resistance was about 4.7 ohms. As used in these experiments it had a sensitiveness (figure of merit) of about $2.5 \times 10^{-8}$ amperes for 1 mm. of deflection at a scale distance of 1 meter, with a period of about 4 seconds. It was calibrated, and the deflections were found to be proportional to the current.

The resonant length of the receiver was found by cutting off the No. 29 copper wire of which it was made, piece by piece, with a pair of scissors, and noting the corresponding galvanometer readings. These readings when plotted gave a resonance curve from which the resonance length could be obtained. Taking the receiver of this length, the common wave-length of both was found by the interferometer method used by Hull,[1] Williard and Woodman,[2] Blake and Fountain,[3] and others.

The results obtained may be grouped under the following heads.

1. Harmonic overtones in the receiver.
2. Sources of error in the interferometer method.
3. Effect of the medium surrounding the spark gap.
4. Influence of the diameter of the receiver upon its resonant length.
5. The resonant length of a linear receiver with a thermo-electric junction.
6. The most probable value of the wave-length of the linear oscillator.

1. *Harmonic Overtones in the Receiver.* — Fig. 3 shows the resonance curve for the 5 cm. oscillator. This experiment was repeated a number of times, and all the curves agree closely, so that there can be no question as to their accuracy. They were made with the sender and receiver in the foci of the parabolic mirrors. The curve shows three well-marked maxima for lengths of 4.4, 13.8 and 25.7 cm., respectively. The 4.4 cm. maximum shows what the length of the receiver must be in order that its natural period shall be the same as that of the sender. The 13.8 cm. maximum gives its length when its natural period is three times, and the 25.7 cm. maximum when its natural period is five times that of the sender.

---

[1] PHYS. REV., 5, 1897, pp. 231–246.

[2] PHYS. REV., 18, 1904, pp. 1–22.

[3] PHYS. REV., 23, 1906, pp. 258–264.

Abraham[1] states that a linear conductor may vibrate in a free manner with its fundamental frequency or with any whole multiple of this frequency. The fundamental vibration is said to be of the first order; a vibration having twice the frequency of the fundamental, of the second order; three times that of the fundamental, of the third order, and so on. A first order vibration is the fundamental vibration; second order, the first overtone; third order, the second overtone, and so on. In general, for any linear oscillator of circular cross-section, the wave-length $\lambda_n$ for an oscillation of the $n$th order, is given according to Abraham's theory, by

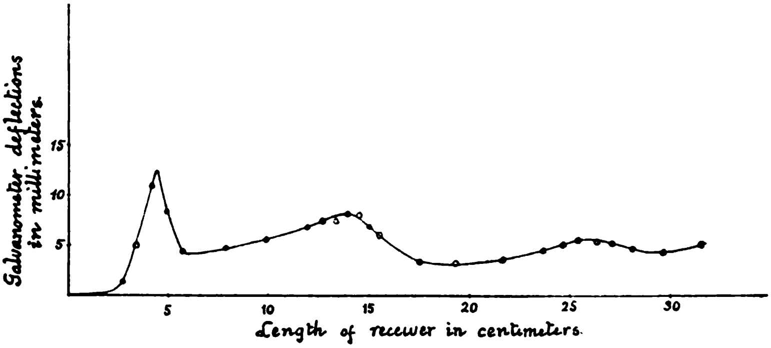


Fig. 3. Resonance curve for the 5 cm. oscillator.

$$\lambda_n = \frac{2l}{n}\left(1 + c_n\varepsilon^2\right), \tag{1}$$

where

$l =$ length of the oscillator,

$n =$ order of the oscillation,

$$c_n = \frac{4.8 + 2\log_e n}{n},$$

$$\varepsilon = \frac{1}{4\log_e \frac{l}{b}},$$

and

$b =$ radius of the cross-section of the oscillator.

As the order of the overtone increases, the correction to be applied to determine the wave-length of this overtone decreases. From

---

[1] *l. c.*, pp. 433–472.

Fig. 3 it is evident that when the receiver is 25.7 cm. long, it is vibrating with an overtone of the fifth order, and that this overtone has the same frequency as the oscillator. The wave-length of the oscillator is then given by formula (1). For this case $n = 5$, and $c_n\epsilon^2$ is equal to .00178. This is a small quantity with respect to 1, and may be neglected. We then have, for our experiment,

$$\lambda_n = \frac{2 \times 25.7 \times 1}{5} = 10.3 \text{ cm.}$$

Of course the accuracy of this result is diminished by the fact that the receiver is not made of wire of the same diameter throughout its length, since it has the fine wire forming the thermal junction at its middle portion. This part of the receiver has a greater resistance than the rest of it, and this will tend to make its length less than it would be if it were constructed entirely of the copper wire. This agrees with the fact that the first maximum occurs at 4.4 cm. instead of at 5.1 cm. The higher, however, the order of the overtone used, the smaller will be the error due to this source. In this particular case the wave-length obtained in this way agrees very closely with that obtained by the interferometer method. It also agrees with that calculated from Abraham's formula given by equation (1). This experiment, assuming that Abraham's theory is correct, suggests a method of determining the wave-length of an oscillator from the overtones set up by it in a linear receiver. Such a determination would not depend, in any way, upon the phenomena of interference in free space, as do most of the methods now in use. This method might appropriately be called the method of harmonic overtones.

2. *Sources of Error in the Interferometer Method of Determining the Wave-length.* — The plane mirrors of the interferometer were made of well-seasoned wooden boards, 122 cm. wide by 121 cm. high, covered with tin foil. The grating was composed of No. 32 (Brown and Sharp gauge) copper wires (.020 cm. in diameter) spaced 5 cm. apart, supported by a wooden frame 100 cm. × 100 cm.

Fig. 4 shows the interference curve obtained for the 5 cm. sender when the receiver was tuned to it, namely, when it was 4.4 cm. long. The mirrors, in this case, were probably equidistant from the grating

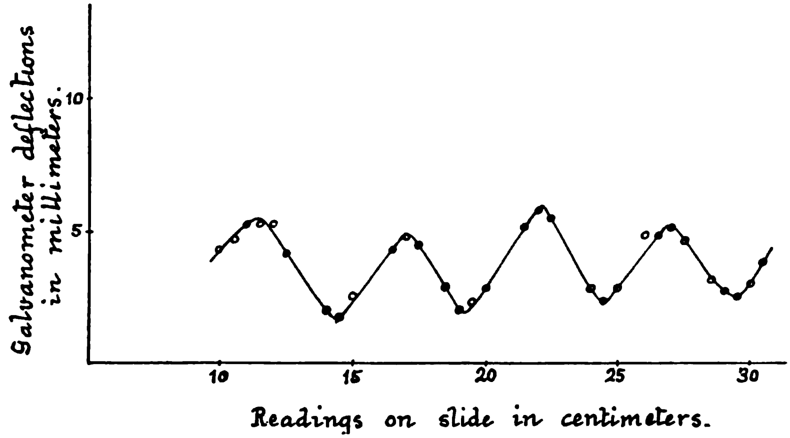


Fig. 4. Interferometer curve for the 5 cm. oscillator.

when the reading on the slide was 27 cm. This curve gives a mean wave-length of 10.2 cm.

Curves *a* and *b* of Fig. 5 show the interference curves for the same oscillator when the receiver was 10 and 30 cm. long, respectively.

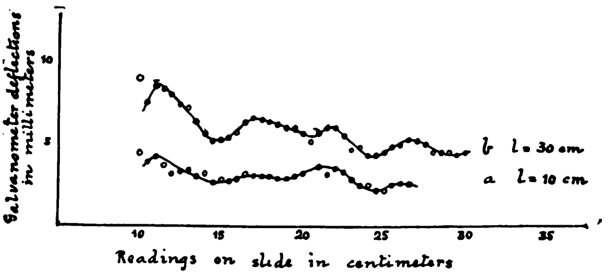


Fig. 5.

They show that if the receiver is not tuned to the sender, the maxima and minima become indistinct, and the determination of the wave-length of the sender becomes indefinite.

To investigate this effect of the length of the receiver upon the apparent wave-length of the sender, I made a series of determinations of the wave-length of the 5 cm. sender, using receivers of different lengths. As in the preceding experiments both the sender and receiver were placed in the foci of the parabolic mirrors. The results are shown in Fig. 6. It will be seen that the apparent wave-length varies all the way from 4.3 cm. to 10 cm. The fact the position of the principal maximum varies from 27.5 cm. to 29.0 cm. is due to the accidental shifting of the position of the grating. Curves 1–3, 4–8 and 9–11 were taken at different times. If they had been taken, one after another, without any shifting of the grating, the principal maximum would, of course, always have occurred at the same reading on the slide.

These curves show that the apparent wave-length of a linear oscillator as determined by the interferometer, with a linear receiver, depends, in general, upon the length of the receiver. If the receiver is too long, the value obtained for the wave-length will be too great. If the receiver is too short, the value obtained will be too small.

One theory that may be offered to explain these results is that of multiple resonance due to Sarasin and De la Rive, which assumes that an oscillator gives out waves of several frequencies and that a res-

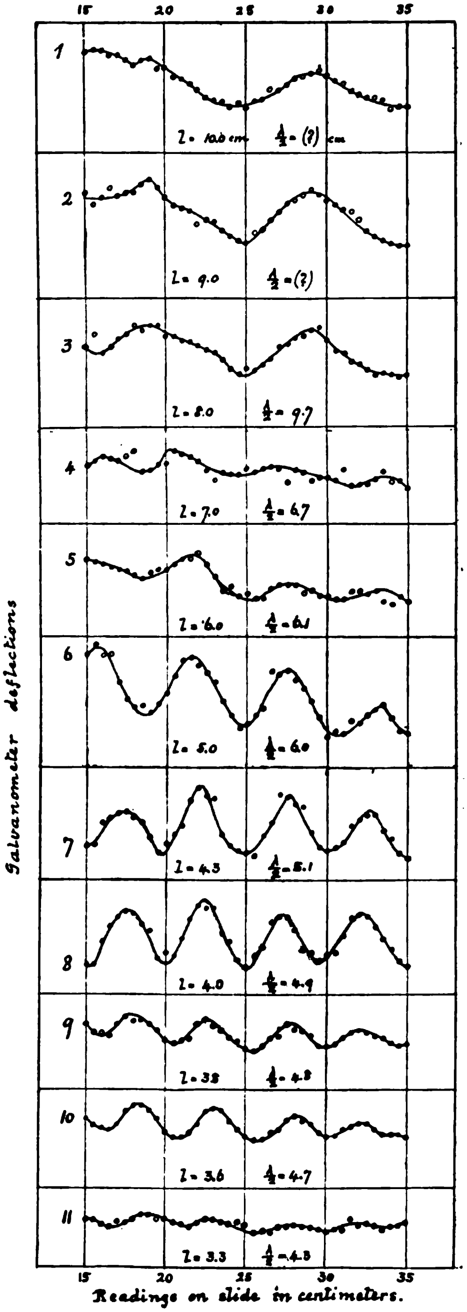


Fig. 6. Interferometer curves for the 5 cm. oscillator.

onator will respond most strongly to those of its own frequency. This theory does not appear to me to be admissible in the present case, since all experimental evidence goes to show that a *linear* oscillator has a definite period and frequency for its fundamental vibration.[1] Also if the linear oscillator emitted a radiation that was sufficiently complex to account for the curves of Fig. 6, it would not be possible to obtain the sharp tuning for the fundamental shown in Fig 3. Also such a phenomenon, even if it exists, must have a very limited range of frequencies.

It appears to me that the results may be explained if we assume that the receiver, being itself a linear oscillator, radiates energy in the form of waves. These secondary waves will, of course, have the wave-length of the receiver. They will go from the receiver to the grating, where the energy will divide, part of it going through the grating to the fixed mirror and part of it being reflected from the grating to the movable mirror. These two wave trains will be reflected from the mirrors and come together again at the receiver, and there will be produced maxima and minima for these secondary waves of the same nature as those produced by the primary waves coming direct from the sender. Each set of maxima and minima will produce its own effect upon the receiver. If the secondary waves got back to the receiver before the primary waves ceased to excite it, we would have to take into account the difference in phase of those two sets of impulses. But since, in these experiments, the distance from the receiver to the fixed mirror and back again was about 260 cm., the wave-length was about 10 cm., and the sender was strongly damped, it is probable that the oscillations in the receiver due to the primary waves had entirely died out before the secondary wave arrived. Lindman[2] found for a linear oscillator, geometrically similar to mine (25.1 cm. long, and 1.5 cm. in diameter), a logarithmic decrement of 1.4, which means that the waves had fallen to about one sixteenth of their original strength in three oscillations. If the effect upon the galvanometer of each train of waves could be plotted separately we should have, for each,

---

[1] See Bjerknes, Ann. Phys., 44, 1891, pp. 92–101, and 54, 1895, pp. 58–63; Kiebitz, *l. c.;* also Winkelmann's Handbuch, 2d edition, Bd. V., p. 662.

[2] Ann. Phys., 7, 1902, p. 842.

a damped harmonic curve. What we actually get is a curve which is the sum of two harmonic curves of different frequencies. Both curves must have a maximum value for that reading on the slide when the mirrors are equidistant from the grating. When the receiver is exactly in tune with the sender, the two curves will coincide, and we get a harmonic curve for the sum of the two. In the present experiment when the receiver is 10 cm. long its wave-length is about twice that of the sender, and the maxima due to it should occur about 10 cm. apart, and would therefore coincide with alternate primary maxima. This is what is actually shown in curve *a* of Fig. 5. The evident fact that curve *a* is made up of two harmonic curves having maxima 5 and 10 cm. apart, respectively, could not be explained by the hypothesis of multiple resonance.

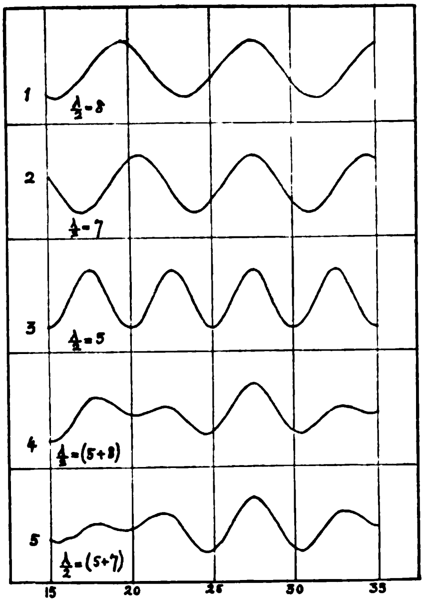


Fig. 7. Calculated curves.

That a linear receiver does send out waves of its own has been proved by Lindman[1] in his experiments on the stationary waves produced by reflection from a plane mirror.

To test the reasonableness of my theory, and to see if two harmonic curves of such wave-lengths as those of my experiment, when added together would give the sort of curve that I had obtained, I plotted three harmonic curves of wave-lengths 8, 7 and 5 cm., respectively, and then added together the ordinates of the 8 and 5 cm. curves. I thus obtained a resultant curve which should be approximately of the same form as I should expect to get with a sender giving out a wave-length of 5 cm., and a receiver giving out a wave-length of 8 cm. The same thing was done for the 7 and 5 cm. curves. All five curves are shown in Fig. 7. It was assumed that the maxima of the two component curves, in each

---

[1] *l. c.*, pp. 824–850.

case, coincided at 27.5 cm. It will be seen that curves 4 and 5 of Fig. 7 correspond fairly well to curves 4 and 5 of Fig. 6, if allowance is made for the decrease in amplitude as the slide readings increase, and for other disturbing causes which undoubtedly exist. Since curves 4 and 5 of Fig. 6 were made with receivers 7 and 6 cm. long, respectively, I should expect the length of the waves given out by these receivers to be about 8 and 7 cm., respectively, or of the same magnitude as those of the calculated curves.

3. *The Effect of the Medium Surrounding the Spark Gap.* — As already stated, I used a hard rubber cylinder to support the oscillator. Since the inductivity of hard rubber is much greater than that of air, and the hard rubber formed a closed circuit for some of the lines of electric force, the question arose as to the effect of the hard rubber support upon the wave-length.

To determine this I supported the cylinders as shown in Fig. 8 by two thin pieces of cork inserted in the ends of a glass tube. In the figure, *bb* are the cylinders; *cc*, the cork, and *gg*, the glass tube. The glass tube was, of course, filled with kerosene oil. The tuning curve was then obtained, giving the maxima for the fundamental and the first overtone. This curve did not differ appreciably from that shown in Fig. 3. We may therefore assume that the hard rubber support has no more effect upon the wave-length than that exerted by a support made of a piece of thin glass tubing with cork ends. As there is no closed circuit through the glass in this case, and it is thin, the results show that the effect of the hard rubber can be neglected.

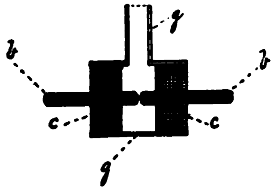


Fig. 8.

The effect of the kerosene oil surrounding the spark gap still remains undetermined. This can only be found conclusively by measuring the wave-length of a linear oscillator, first sparking in air and then in oil. Since, however, the inductivity of kerosene oil is only a little over 2, the wave-length varies as the square root of the inductivity, and the kerosene oil is only present around the middle portion of the oscillator, while the potential loops are at its ends, we may I think assume that the effect of the kerosene oil upon the wave-length must be small.

4. *The Influence of the Diameter of the Linear Receiver upon its Resonant Length.* — The measurements of wave-length described in this paper were finally performed with a receiver made of No. 29 copper wire, but a tuning curve was also obtained, with the 5 cm. sender, for a receiver constructed of two brass cylinders having the same diameter as those of the sender. The length of this receiver was varied by slipping over the cylinders sleeves of brass or tin foil. In this way the resonance curve shown in Fig. 9 was obtained. From the curve, we see that the receiver was in resonance with the fundamental of the sender when it was 3.5 cm. long, and with the first overtone when 12.2 cm. long. We must conclude, therefore, that if the receiver is made of brass cylinders .25 cm. in diameter, its resonant length will be less than if it is made of copperwires .0285 cm. in diameter.

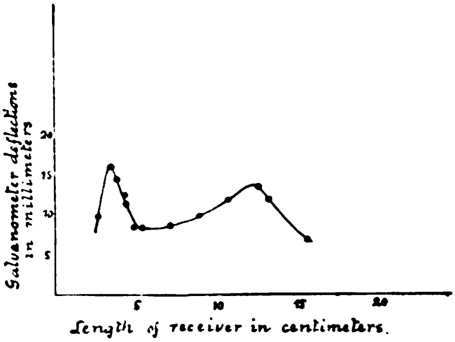


Fig. 9.

5. *The Resonant Length of a Linear Receiver with a Thermo-electric Junction.* — The results described above seem to show that for a linear thermo-electric receiver the tuned length of the receiver will be *less* than one half of the wave-length. This is probably due to the constriction at its middle portion through the use of the fine wire for the thermo-electric junction. The fine wire will increase the ohmic resistance of the receiver very considerably. It will also increase its inductance, perhaps faster than its capacity decreases. If the ohmic resistance or the impedance of an oscillating system is increased, its period will be increased.

My results are not in accordance with those obtained by Cole[1] who used a linear sender of the same dimensions as those of my 5 cm. sender, and a linear receiver having the same diameter as the sender. He found the resonant length to be 6.2 cm., whereas I found a value of 3.5 cm., under the same conditions (see Fig. 9).

---

[1] *l. c.*, p. 269.

He finds the resonant length of the receiver to be considerably greater than that of the 5 cm. sender (20 per cent. greater), whereas I find it to be considerably less. On the other hand, my results agree with those obtained by Blake and Fountain.[1] They found for a wave-length of 9.9 cm., and a receiver made of two strips of copper 2 mm. broad, a resonant length of 4 cm.

My results as to resonance length and wave-length are also in good general agreement with those obtained by Lindman[2] for his oscillators and linear receivers.

6. *The Most Probable Value of the Wave-length of the Linear Oscillator.* — The experiments described, so far, were all made with the 5 cm. sender. As already stated, three senders were used, 5, 7.5 and 10 cm. long, respectively. The interferometer curve shown in Fig. 4 gives a wave-length of 10.2 cm. for the 5 cm. oscillator. There is no reason to doubt the accuracy of this value. The receiver was placed in the focal line of the large parabolic mirror whose focal length was 7.5 cm., and was therefore in the neighborhood of a loop of electric force if stationary waves were formed in front of the mirror. The measurements shown in Figs. 5 and 6 also point to a wave-length for the 5 cm. sender approximately equal to 10.2 cm. My results, however, do not agree with those of Cole,[3] who has obtained a wave-length of 15 cm. for an oscillator of the same dimensions.

The measurement of the wave-length of the 10 cm. oscillator presented the same difficulties as that of the 5 cm. oscillator, the apparent wave-length depending upon the length of the receiver. In this case, also, for some unknown reason, it was more difficult to get a definite value for the resonant length of the receiver. Thinking that the parabolic mirrors might, in some way, influence the resonance curve, they were removed and both the resonant length, and the wave-length, determined without them. Seven values for the resonant length were obtained in this way, namely, 9.8, 10.6, 9.7, 9.1, 9.4, 9.4 and 10.2 cm. The fourth, fifth and sixth of these values I consider to be the most reliable. The

[1] *l. c.*, pp. 258, 261.

[2] *l. c.*, p. 833.

[3] Phys. Rev., 23, 1906, p. 243.

mean of these three values gives 9.3 cm. for the resonant length. Using a receiver of this length I obtained two interference curves which gave a mean value for the wave-length of 20.8 cm. One of these is curve 4 of Fig. 10. When the mirrors were not used, a cylindrical lens of kerosene, in a glass bottle 12.7 cm. in diameter, was placed in front of the sender to concentrate the energy upon the receiver. Interferometer measurements made both with and without the mirrors seemed to show that the presence of the mirrors did not affect the value of the wave-length. The interference curves obtained for the 10 cm. oscillator, using a linear receiver of different lengths, both with and without the parabolic mirrors are shown in Fig. 10. Calculating the wave-length of the 10 cm. oscillator from Abraham's formula we get 20.36 cm.; which differs from the value obtained by experiment by about 2 per cent.

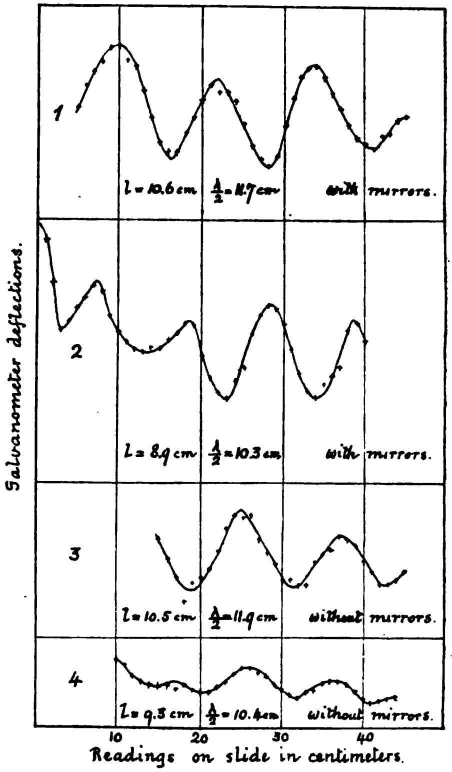


Fig. 10. Interferometer curves for 10 cm. oscillator with linear receiver.

As it was found that the apparent wave-length depended upon the length of the receiver, and as it was believed that this was due, as explained above, to secondary radiation from the receiver itself, I sought, when measuring the wave-length of the 10 cm. oscillator, to construct a receiver which would not itself radiate waves, by making it of a *closed loop* of No. 29 wire. One of these loops is shown in Fig. 11. In the figure, *abcd* is the rectangular loop of No. 29 wire, and *t* is the thermo-electric junction. These loops consisted of from 1 to 10 turns, were of dimensions varying from 4 cm.

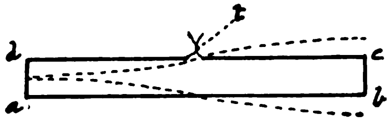


Fig. 11.

in width and 5 cm. in length to 1 cm. in width and 11 cm. in length, and were placed either horizontally or vertically, usually horizontally.

In all cases it was found that the apparent wave-length as measured with these loops depended upon their dimensions, and particularly upon their length, that is, upon that dimension which was parallel to the electric force.

The interference curves obtained when the loops were used in the focus of the parabolic mirror are shown in Fig. 12. Unless otherwise stated the loop consisted of a single turn, and was placed in a horizontal plane. The dimensions of the loop are given for each curve. 11 × 1 means that the loop was 11 cm. long and 1 cm. wide. For instance curve 3 was obtained with a loop 9 cm. long and 1 cm. wide, which is the loop shown in Fig. 11. The remarkable distortion of these curves led me to obtain curves for the same loops without the mirrors. The results

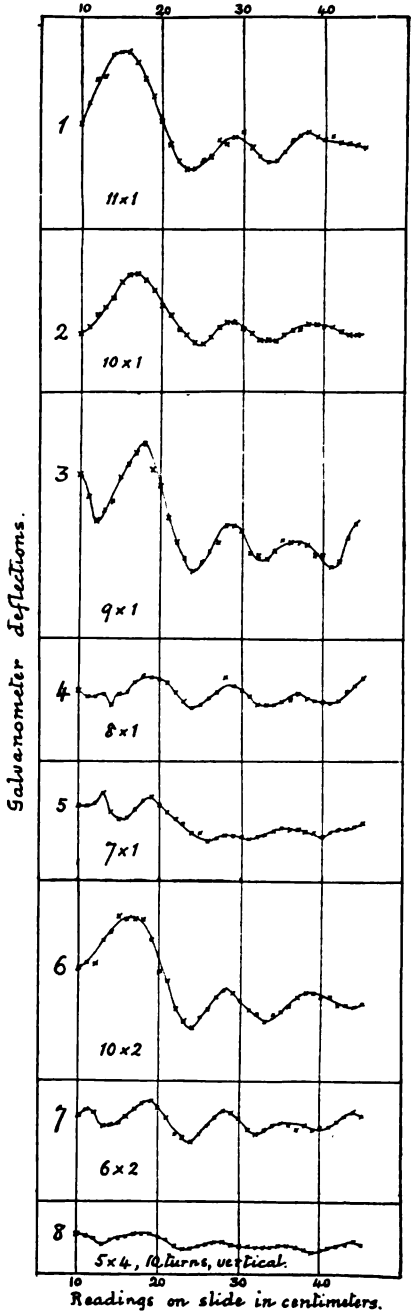


Fig. 12. Interferometer curves for 10 cm. oscillator with loop receiver and mirrors.

are shown in Fig. 13. The distortion has disappeared, but the effect of the dimensions of the loop still remains. The distortion of the curves of Fig. 12 must therefore be due in some way to the presence of one or both of the parabolic mirrors, probably to that of the receiving mirror.

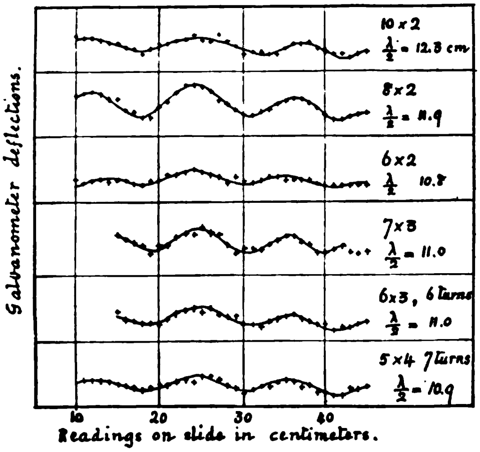


Fig. 13. Interferometer curves for 10 cm. oscillator with loop receiver without mirrors.

If my theory as to the cause of the influence of the length of the receiver upon the apparent wave-length of the sender is correct, these curves show that a *closed loop*, under proper conditions, is a good radiator of electric waves. This is opposed to the view put forward by Fleming[1] in his theory of the closed circuit oscillator, for he says in his paper entitled "On the Elementary Theory of Electric Oscillators," that "the closed circuit is an exceedingly poor radiator compared with the open oscillator." It is, however, I think, evident that if the length of the loop receiver is great with respect to its breadth, if the electric force is parallel to its length, and if its own natural period does not differ greatly from that of the sender, it will be set into electric oscillation with loops of electric force at its ends, and nodes at its two middle points as shown by the dotted lines in Fig. 10. Each half of the loop will then radiate waves as though the other half did not exist. If the loop is long and narrow the two waves will be practically in the same phase, and will reinforce each other. This is the same thing as the loop antenna, which is now sometimes used in wireless telegraphy, taken together with its image in the earth. It is to be observed that the amount of energy that will be radiated from a closed circuit will depend upon the manner in which the circuit is constructed, and upon the way in which it is excited. Fleming in his original

---

[1] Electrician, 59, 1907, pp. 978, 979.

derivation of the theory of the closed oscillator assumes that it is rectangular and made up of four Hertz oscillators. In this case there will be four loops of electric force in the circuit. But this is only a special case of a closed circuit. A closed circuit can be constructed in an infinite number of ways. It can, for instance, be built up of *two* Hertz oscillators, or it can contain a condenser. The last case has been experimented upon by Fleming,[1] and he finds that the radiation is extremely small. The first case is the one occurring in my experiment, and in this case there is undoubtedly a large radiation. Again, the manner in which any circuit will vibrate will depend upon the manner in which it is excited. For instance, a *square* rectangular circuit if excited by waves whose electric field is parallel to one of its sides, will under proper conditions vibrate so that there will be only two loops of electric force present in the circuit. In this case it will radiate much energy. A *nearly closed circuit* is yet another type. It has been found by Lindman[2] and others to radiate very little energy. In this type there are still free ends, but they are bent together so that they nearly touch each other.

An attempt was made to determine the wave-length of the 7.5 cm. oscillator, but for some unknown reason it was found to be very difficult to determine what was the proper resonant length of the receiver. Several sets of resonance readings were taken both with and without the parabolic mirrors, but no definite value was obtained. The wave-length of the 7.5 cm. oscillator was consequently not determined.

## SUMMARY.

1. By shortening the length of a linear receiver a few millimeters at a time harmonic overtones are made apparent. The overtones of the highest order can be used to determine the wave-length of the sender.

2. The interferometer method can only be used with a linear receiver, which is not dead-beat, when it is closely tuned to the sender.

3. A hard rubber cylinder supporting a linear oscillator at its middle portion does not appreciably affect its wave-length.

---

[1] Phil. Mag., 14, 1907, pp. 677–697.

[2] *Loc. cit.*

4. If the diameter of a linear receiver is considerably increased its length must be decreased in order that it may remain in tune with the sender.

5. The resonant length of a linear thermo-electric receiver is less than that of the oscillator if the oscillator is linear.

6. The wave-length of a linear oscillator 5 cm. long and .25 cm. in diameter, sparking in kerosene oil, and supported by a hard rubber cylinder was found to be 10.2 cm.; that of an oscillator 10 cm. long and of the same diameter, under the same conditions, 20.8 cm. These results agree with Abraham's theory rather than with Macdonald's.

7. Under proper conditions a closed circuit may radiate as much energy as an open circuit of the same linear dimensions. This is opposed to the general proposition made by Fleming.

In conclusion I wish to express my indebtedness to Mr. W. W. Coblentz, of the Bureau of Standards in Washington, for his kind advice in the construction of my galvanometer; and to acknowledge the aid received from apparatus and supplies purchased out of a grant from the Bache Fund of the National Academy of Sciences.

UNIVERSITY OF CINCINNATI,
June, 1909.

# THE EXPANSION OF FUSED QUARTZ AT HIGH TEMPERATURES.

By H. M. RANDALL.

QUARTZ glass is being used at a rapidly increasing rate in many lines of scientific work and, as its low rate of expansion is the property, in many cases, which especially recommends its use, an exact knowledge of its coefficient of expansion is highly to be desired. Scheel[1] and Chappuis[2] have determined the expansion of fused quartz from 0° to 100° C. While Scheel and Dorsey[3] have extended the investigation in the direction of low temperatures to approximately that of liquid air. In the region of high temperatures the results of Le Chatelier[4] and Callendar[5] cannot be considered very accurate. This is especially true in regard to the result of Le Chatelier. Holborn and Henning,[6] using the method developed by Holborn and Day[7] in their work on the high temperature gas thermometer, have made what appears to be as careful a study of the expansion of fused quartz as that method will permit.

In 1906 H. D. Minchin,[8] working in this laboratory, obtained and published a value for the mean coefficient of expansion of fused quartz between room temperatures and 1000° C. Certain errors have since been found in his work and as the methods now available insure more accurate results than those employed by him it has seemed advisable to repeat the entire work, using the same specimen of fused quartz.

The general method is that employed by Minchin and first de-

---

[1] Scheel, Wiss. Abhand. d. Phys.-Tech. Reich., Band IV., s. 54, 1904.

[2] Chappuis, Verh. Naturf. ges. Basel., 16, s. 173, 1903.

[3] Dorsey, PHYS. REV., p. 101, Aug., 1907.

[4] Le Chatelier, Comp. Rend., 130, p. 1703, 1900.

[5] Callendar, Chemical News, 83, p. 151, 1901.

[6] Holborn and Henning, Ann. der Phys., p. 446, 1903.

[7] Holborn and Day, Ann. der Phys., p. 505, 1900.

[8] Minchin, PHYS. REV., p. 1, Jan., 1906.

veloped by Fizeau[1] in which the expansion of quite small objects can be accurately measured by the shift of a set of interference fringes. This method has been very frequently used, generally for but moderate ranges of temperature, but work in this laboratory has shown that it is equally applicable to ranges of 500° or 1000° C. In the work on fused quartz at low temperatures by Scheel, Chappuis and Dorsey this method was adopted. The general nature of the interference system employed in this method has been so frequently described[2] that the briefest description of the particular system here used will be quite ample. Upon a bed plate of fused quartz, 34 mm. in diameter and 15 mm. thick, with the upper surface plane polished and the lower ground rough, there is placed a fused quartz ring, 9.901 mm. long and 25 mm. internal and 30 mm. external diameter. Upon the ring rests a second fused quartz plate, the cover plate, 1 cm. thick and 34 mm. in diameter with both faces plane polished and inclined to one another at an angle of about 1′, At the center of the under surface of the cover plate is cut a fine circle 2 mm. in diameter to act as a fixed reference point. The ring, which is the piece of quartz whose coefficient of expansion is to be determined, was cut from a plate of quartz whose surfaces were accurately plane polished and slightly inclined to one another. Each end of this ring is so ground away as to leave three equidistant feet, the surfaces of these feet being thus parts of the original plane surfaces of the plate from which the ring was cut. The inclination of these surfaces was such that, with this ring mounted between the cover and bed plates, and with the proper illumination, there is produced a set of interference fringes due to the combination of the light reflected from the upper surface of the bed plate and the lower surface of the cover plate. All three pieces of this interference system are beautiful specimens of Heræus fused quartz, very clear and quite free from bubbles; they were obtained through the agency of Carl Zeiss, Jena.

The coefficient of expansion of the ring is expressed by the relation

---

[1] Fizeau, Ann. de Chem. et de Phys., 2, p. 143, 1864; 8, p. 335, 1866.

[2] Pulfrich, Zeitschr. für Krystall., 31, p. 372, 1899. Randall, PHYS. REV., p. 10. Jan., 1905. Ayres, PHYS. REV., p. 38. Jan., 1905, Scheel, Ann. der Phys., p. 837, 1902. Dorsey, PHYS. REV., Aug., 1907.

$$\alpha = \frac{f\frac{\lambda}{2}}{L(t_2 - t_1)},$$

where $f$ is the number of the bands of the interference system passing the reference circle on the cover plate during the temperature interval $(t_2 - t_1)$, $\lambda$ the wave-length of the light used and $L$ the length of the ring. As the value of $\lambda$, $.54623\,\mu$ for the Hg green, has been determined with great accuracy by Fabry and Perot and $L$ is easily measured to approximately .001 mm. by a Geneva society spherometer reading directly to .001 mm., it becomes evident that the exact determination of $f$ and $(t_2 - t_1)$ present the chief experimental difficulties. With a Pulfrich interferometer[1] the proper illumination of the quartz interference system by a mercury vapor lamp is readily obtained and the measurement of the shift of the interference fringes easily possible to 0.02 of a band width, corresponding to a separation of the cover and bed plates of 0.01 of a wave-length.

With this instrument available the problem is largely reduced to the designing of an oven which will uniformly heat the entire quartz system for any length of time at any temperature under such conditions that the system and the thermo-couple used to measure the temperature be at the same temperature. Also the oven must permit the quartz system to be so mounted that no part of the shift of the interference fringes measured by the interferometer be due to other cause than the expansion of the ring separating the reflecting surfaces of the cover and bed plates. With these aims in view the oven and high temperature regulator to be described in the following pages have been constructed.

## THE HIGH TEMPERATURE OVEN.

The quartz system, shown in Fig. 1, at $A$, has been mounted upon an earthenware platform securely adjusted to the upper end of a vertical tube of Berlin porcelain $B$, 100 cm. long and 2 cm. in diameter. With this length of tube enough projects from the oven to permit clamping the tube very rigidly by wooden blocks $w$, and

---

[1] Pulfrich, Zeitschrift für Instrumentenkunde, 1893, p. 365; 1898, p. 261; Randall, PHYS. REV., p. 10, 1905.

bolts, not shown in Fig. 1, to a well dried pine plank, 2 by 10 inches, which in turn is very firmly clamped to a stone pier. Throughout the length of this porcelain tube runs an exhaust tube *X*, 3 mm. in bore, the purpose of which is to connect the interior of the oven with the air pump. The upper end of this tube is of fused quartz and is surrounded by asbestos wool to prevent direct radiation to the lower end of tube *B*. Holes not shown in the figure are bored through the earthenware platform in such a way as to connect freely the end of the exhaust tube with the interior of the oven. The lower end of the exhaust tube is of glass and passes through a rubber stopper fitting tightly into the end of the porcelain tube, bends upward and is sealed to the tube coming from a Geryk oil pump; a McLeod gauge is also connected to this same pump tube. [This latter tube is so long and slender that no vibrations are communicated to the oven from the pump, which is motor driven.] The rubber joint at the end of the porcelain tube is made air tight by being immersed in a mercury bath.

Over this tube *B* is slipped the larger porcelain tube *C*, 6 cm. in external diameter and 5

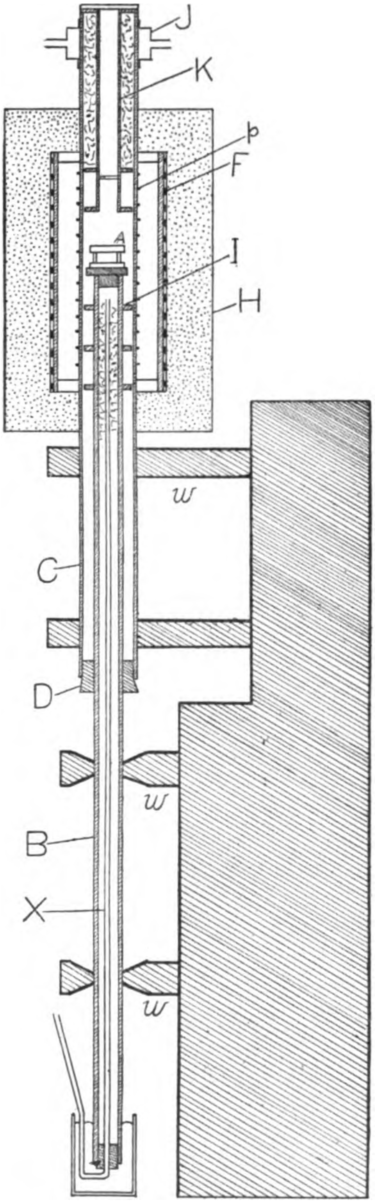


Fig. 1.

cm. internal. Its length, 70 cm., is again sufficient for a very rigid clamping of the lower end, which remains cool, to the pine plank. When adjusted and clamped this tube is without points of contact with tube *B* except at the rubber stopper *D*. The heating coil, consisting of heavy platinum ribbon, is not wound directly on the outside of this larger tube *C* but upon eight porcelain rods, .5 cm. in diameter and 20 cm. long, mounted symmetrically around the tube at a distance of 1.5 cm. from it. This method of mounting is better shown in Fig. 2 where the rods are shown at *E* and the ribbon at *F*, Fig. 2 being a horizontal cross-section of the oven through its center, the outer insulation being omitted. The ribbon is also shown in Fig. 1 by the heavy dots. Around the outside of these rods is wound sheet asbestos, the outer layers being soaked with sodium silicate; a firm cylindrical casing is thus formed around the heating coil but not touching it except at the contact with the rods. An outer casing *H*, Fig. 1, of sheet iron, at a uniform distance of 5 cm. from the inner one encloses a space which is filled with infusorial earth as an insulating material. The upper end of tube *C* is provided with a water cooling jacket *J*.

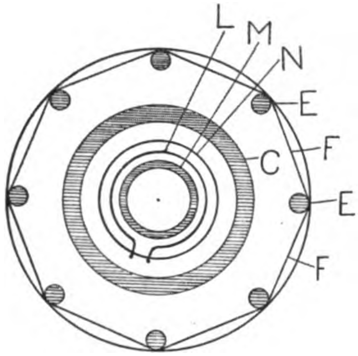


Fig. 2.

To prevent direct radiation from the quartz system, mounted at the center of the heating chamber, to the colder parts of the tube *C*, porcelain diaphragms *I* are mounted upon the tube *B* below the systems; these just escape contact with the tube *C*. Above, an observational porcelain tube *K* is fitted at its lower end with two similar diaphragms and above these the space between tubes *K* and *C* is filled with asbestos wool. There is a mica disk mounted in *K* at about the level of the top of the heating coil and the entire upper end of *C* is closed with a circular plate glass disk. To render the joints at the ends of the tubes air tight they are painted with alternate layers of Le Page glue and beeswax. The hot wax being applied when the outer surface of the glue has just ceased being sticky. With the occasional addition of a coat of glue and wax this has been very effective, and vacua of a pressure, on the average, of .05 mm. have been maintained.

## Temperature Regulation.

Automatic devices are now almost universally used to maintain at constant temperatures baths whose temperatures do not differ much from room temperatures. For high temperatures, constancy of current or hand regulation has generally sufficed. In this work, as the voltage of the power circuit from which the heating current was drawn fluctuated considerably, the first method was impossible and the second so tedious that some automatic regulator seemed necessary. Any self-registering temperature devise could be readily converted into an automatic temperature regulator. An arrangement due to Darwin[1] for use at ordinary temperature would be equally effective at high temperatures. The regulator used in this work has already been briefly described.[2] Its principal recommendation is its simplicity though it possesses other points of some merit. In Fig. 2 the heating coil *F* is shown suspended in air and any change in the current through it will very quickly make itself felt in the 1.5 cm. air layer surrounding the tube *C*, *i. e.*, the tube to be heated is surrounded by a bath of very small thermal capacity, as but a thin outer layer of the tube can follow the temperature changes, its thermal conductivity being poor. The scheme of the regulation consists in keeping the temperature of this air bath fluctuating about some definite temperature so rapidly that no changes are transmitted through the thick-walled tube *C*. To detect the changes in the temperature of the air bath a fine platinum wire *p*, Fig. 1 (not shown in Fig. 2), is wound non-inductively on the outside of tube *C*, and forms one arm of a Wheatstone bridge. The bridge having been adjusted for equilibrium for a given resistance of the coil *p*, the galvanometer in the bridge circuit stands at its zero position when the temperature of the oven is such as to give the coil *p* this resistance and any variation in temperature of the air bath surrounding the tube *C* will be indicated by the movement from its

---

[1] Astrophys. Jour., 20, p. 347, 1904.

[2] Phys. Rev., p. 142, Feb., 1909.

position of rest of a light aluminum boom *S*, Fig. 3, with which the galvanometer coil is provided. This boom moves between two spring jaws *T*, 10 cm. long and 1 cm. wide, which open and close regularly; this motion being caused by the slow rotation of the drum *V*, driven by a motor at low speed, a reducing gear being employed. This drum is provided with small elevations *Z*, which cause the periodic separation of the jaws. In Fig. 3 the upper figure shows the jaws closed and the lower the same jaws opened by the elevations *Z*. The ends of the jaws are furnished with platinum contact strips mounted in hard rubber blocks, the lower jaw having a single strip whose length is the entire width of the jaw and the upper jaw two strips, in line with one another, separated by a small space. A side view of this arrangement is shown in Fig. 3, and an end view in Fig. 4. Lead wires soldered to each of these strips connect with a cell and a modified relay. If, for example, with the cooling of the oven, the boom *S* moves to the left of its position of equilibrium at the center of the jaws, as shown in Fig. 4, the relay operates to short circuit part of the resistance in the heating circuit, aud this resistance remains cut out, though the jaws may open and close any number of times, until the larger current heats the air bath above the desired temperature and the boom moves to the right side of the center when the relay throws the resistance into circuit again. Thus, though, with the motor speed usually adopted, the jaws open and close regularly every ten seconds,

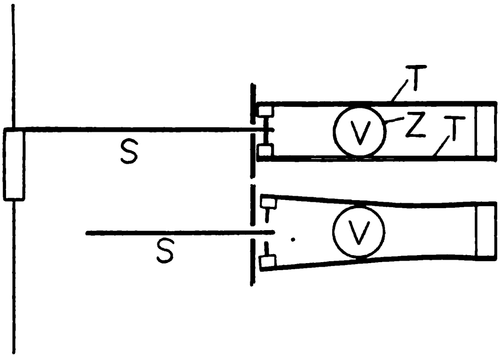


Fig. 3.

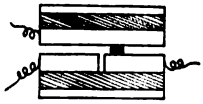


Fig. 4.

the relay so works that the regulating resistance in the heating coil is thrown out or in only with the movement of the boom from one side of the zero position to the other. This method of regulating has maintained the temperature of the interior of the oven when the quartz is mounted constant for periods as long as eight hours and at temperatures lying between room temperature and 1000° C.; no changes occurring which a Pt, Pt-Rh couple connected to a potentiometer of the Lindeck and Rothe type would detect.

Some points in favor of this regulator may be enumerated: (1) It is mechanically very simple. (2) Contact with the moving boom is very sure, the pressure of the jaws on the boom being governed by the stiffness of the springs. (3) When the jaws close upon the boom it is neither lifted nor lowered, drawn in or out, *i. e.*, it suffers no displacement with accompanying strain upon the suspensions and therefore much lighter suspensions can be used than where the boom is depressed to make contact. There is thus a gain in sensitiveness. (4) No great care has to be exercised in determining the size of the heating and cooling currents. As a matter of fact the heating current has usually been about double the cooling current. The ratio of the heating effects, 4 to 1, is sufficiently large to take care of any change in room temperature or any change in heat absorption of the insulation as equilibrium is established. This advantage is due more properly to the small heat capacity of the air layer surrounding the tube *C*, within which the temperature is to be maintained constant and to the non-conducting nature of the walls of the tube, as wide variations in the temperature outside the latter, if sufficiently frequent, produced no variation within. (5) The scheme is quite pliable. A thermo-junction with the galvanometer in circuit may be used in place of the platinum coil, its voltage being balanced against the fall of potential due to a constant current over a fixed resistance, or a couple in a vapor bath balanced against the couple in the oven, the galvanometer of the regulating device being in circuit.

To work satisfactorily the galvanometer must maintain a fixed zero. The lack of strain upon the suspensions when contact is made with the boom is a great aid to this. As a further protection against shift the supports of the suspensions are borne by two phosphor bronze rods; thus no strain is put on the suspensions with change in room temperature.

## TEMPERATURE MEASUREMENTS.

A platinum, platinum-rhodium couple was used to measure the temperature. This couple, made of Heræus wire, was provided with a Reichsanstalt certificate. It was mounted so as to encircle the quartz ring at its mid-height, the couple being about 2 mm. from the ring. This method of mounting is shown in Fig. 2, where *L* is the bead of the couple and *m* the cross-section of the quartz ring, while the circle *n* represents the outer edge of the circular quartz bed plate upon which the ring rests. The cover plate, not shown, which is mounted on top of the ring, extends beyond the ring in a similar way so that the couple encircling the ring is midway between the two plates. The couple, bending downward over the edge of the bed plate, on the opposite side of the ring from the bead, runs down the tube *B*, and the wires pass through the rubber stopper *D* at points about 1 cm. apart. As the Le Page's glue used at this point was found to be somewhat conducting, beeswax alone was used around the wires. Careful testing showed that the wires were insulated from each other at this place. The ends of the couple, soldered to copper leads, were immersed in a suitable ice bath and the voltage measured by means of a Siemens and Halske potentiometer after the design of Lindeck and Rothe.[1]

The method suffices for temperatures above 300° C., but for lower temperatures is not sufficiently accurate. To determine the initial temperature, which results from allowing the oven to stand over night, the cold junctions of the couple are placed in an oil bath at or very near the temperature of the oven; a galvanometer in the circuit gives deflections very approximately proportional to the difference in temperature between the oven and oil bath whose temperature is determined by a mercury thermometer. The deflection denoting this temperature difference is but a few mm. at the most. The cold junctions are then placed in another oil bath whose temperature is a degree or so different from the first one and the galvanometer deflection noted. The oven being at a very constant tem-

---

[1] Zeitschr. für Instr., 20, p. 285, 1900.

perature the data thus obtained are sufficient to determine the temperature of the oven with an accuracy considerably beyond that required, as the D'Arsonval galvanometer used gave a deflection of 30 mm. per degree.

To determine the temperature of the oven in the neighborhood of 80° C., the oven was maintained at a constant temperature for about two hours by means of the regulator just described and then the cold junctions, in a tube filled with oil, were placed in a water bath whose temperature was adjusted to very nearly equal that of the oven, this temperature being maintained by the usual mercury thermostat and stirrer. When a condition of constancy had been attained the deflection of the galvanometer, a few mm., divided by the deflection per degree, gave the difference in temperature of the oven and water bath; this latter being again determined by a calibrated thermometer in the same oil tube as the cold junctions.

For temperatures near 200° C., naphthalin was boiled in a closed tube, the pressure within which could be varied at will. The cold junctions, fastened to the bulb of a standard mercury thermometer, were mounted in a thin glass tube extending into this vapor bath. After the oven had been kept at a constant temperature for a sufficient time the pressure on the boiling naphthalin was varied till the galvanometer in circuit showed no deflection or but a few mm. The deflection translated into degrees gave the difference in temperature between the oven and the vapor bath. This latter as before was given by the standard thermometer, the stem of which was completely immersed in the vapor. For temperatures in the neighborhood of 400° a sulphur vapor bath under variable pressures was used in place of the naphthalin bath. The standard thermometer used in the vapor baths read to 550° in degrees and was calibrated at the Reichsanstalt in 1897 and at the Bureau of Standards in 1906, the thermometer not having been used since the latter calibration.

It has been shown[1] that couples used above 1000° C. very readily become contaminated under certain conditions with resulting change in their calibration. The couple employed has been heated only a few times above 1000° and then in an electrically heated furnace

---

[1] Phys. Rev., XXIII., p. 449, 1906. Am. Jour. Sci., Vol. XXVI., p. 405, 1908. Ann. der Phys., p. 538, 1900.

with no known source of contamination present. While there was good reason to think therefore that the original Reichsanstalt calibration still held yet as White has shown that the iridium contained as an impurity in the platinum coils of the furnace is sometimes sufficient to cause an appreciable contamination, certain tests were made with the couple still undisturbed in its original mounting immediately at the close of the expansion measurements. These tests involved a direct comparison of the couple with a standand couple of the Department of Chemistry kindly loaned by Professor Campbell, a determination of the melting point of gold by the wire method, the pure gold wire having been made for the Department of Chemistry by Prof. G. A. Hulett, and finally a comparison with the high temperature mercury standard, making use of the sulphur vapor bath previously described. The temperatures determined by the three methods differed but little from those registered by the couple employed and as these differences were both positive and negative there seemed every reason for considering the original calibration still correct. It was accordingly used.

The potentiometer was itself tested for any changes resulting from aging or usage but none were found. If it be assumed then that the measurements made upon the couple correctly determine its temperature, there still remains the question as to whether the quartz ring is of the same temperature as the encircling couple. Their nearness together would favor such an equality of temperature, and, moreover, the design of the oven is such that, when a state of equilibrium has been attained after an hour's maintenance at a constant temperature, the inner chamber containing the quartz system must approximate rather closely a black body chamber.

## Measurement of Expansion.

As has been said earlier, the shift of the fringe system can be measured, with the aid of the Pulfrich interferometer, with an accuracy to .02 of a band width, corresponding to an expansion of the ring of .01 of the wave-length of the light used. It is essential to ascertain if any of this fringe displacement is due to any other cause than the expansion of the ring. With crystalline or fused quartz rings, one would have the right to expect that the forward shift of the fringes resulting from a rise in temperature would be exactly equalled by the return shift when the system cools to the initial temperature, as quartz shows practically no "after-effect." Scheel[1] has found that the two shifts are frequently not the same and has overcome the difficulty by weighting the cover plate by a heavy ring, having attributed the slight irregularity which occasionally occurs in the shifts to the presence of an air film remaining between the points of contact. Moreover it has been the general experience of those who have worked with these interference systems at high temperatures that, when first put together with all surfaces carefully cleaned, they slide easily upon each other while, after a heating to a high temperature, the surfaces cling and grate as they are made to slide upon one another; the clinging sometimes being so marked that it is possible to lift the entire interference system by the cover plate alone. If clinging air films are largely responsible for these effects then the mounting of the quartz system in a high vacuum accompanied by heating to a high temperature should largely remove all such irregularities of shift as have been observed. This treatment will also obviate making a correction to the observed shift of fringes resulting from a change in the index of refraction of air with change in temperature and pressure. This is an important consideration in the present instance, as the correction would be of the same order of magnitude as the shift due to expansion. This is due to the extreme smallness of the expansion of the fused quartz and the thickness of the air layer, between the reflecting surfaces. The effectiveness of thus working in a vacuum is clearly shown by the data collected in Table I., where the forward shift of fringes on heating as shown in the second column is practically equalled by the return shift on cooling as shown by column three. The difference between these two shifts, contained in the fourth column, is in only one instance larger than what was to be expected from the accuracy with which the shift could be measured, *i. e.*, .02 of a band width. While in each case the shift between room temperature and the upper temperature was the one observed it was corrected to correspond to the shift between an initial temperature of 16° C., and the upper temperature, use being made of the value found by

---

[1] Wiss. Abhand. der Phys.-Tech. Reich., Band IV., Heft 1, 1904, s. 41.

Scheel for the coefficient of expansion of fused quartz between 16° C. and 100° C. Twenty-four hours were generally required by the oven to cool to room temperature after a heating, and during this time things occasionally happened to the oven which made a determination of the return shift of questionable value, and in such cases no attempt was made to determine it; if, for example, it was found that the oven had lost its vacuum appreciably on standing it was necessary to apply a coating of wax and glue before pumping out. It was not thought advisable to connect the readings on the fringe system made after such an operation with those made before it. This and similar causes resulted in cutting down the observations on the return shift to about half those made upon heating. When, however, there was no known disturbance of the oven during cooling a determination of the return shift was always made and all such values are found in Table I.

TABLE I.

*Initial and Final Temperature* 16° *C.*

| Upper Temperature. | Shift on Heating. | Return Shift on Cooling. | Difference. |
|---|---|---|---|
| 81° | 1.002 | 1.010 | −.008 |
| 300 | 5.602 | 5.627 | −.025 |
| 304 | 5.659 | 5.658 | +.001 |
| 425 | 8.385 | 8.410 | −.025 |
| 467 | 9.416 | 9.442 | −.026 |
| 533 | 10.699 | 10.694 | +.005 |
| 539 | 10.668 | 10.731 | −.063 |
| 702 | 13.978 | 13.973 | +.005 |

The shifts between room temperature and 16° are computed by means of Scheel's values, and are combined with the observed shifts to give the shifts of the above table.

While the close agreement between the forward and backward shifts may be fairly taken to indicate that small erratic changes in the fringe shift have been avoided there still remains a possible source of error in the tilting of the interference system arising from unequal expansions in its supports. When only moderate temperature ranges are employed this is not a serious matter but with the large ranges here used the unequal expansions of the different parts of the system supporting the quartz interference system may be so great that the light reflected from the interference surfaces no longer returns to the interferometer; and a readjustment of the interferometer by means of its leveling screws becomes necessary. It has been noticed however that tilting the interferometer when the interfering surfaces are a cm. or so apart produces a slight shift of the fringe system with respect to the reference circle; the same shift being observed also when the interference system is tilted and the interferometer is stationary. The tilting, to produce measurable effects, has to be such that the brilliancy of the fringe system changes appreciably. By mounting the interference system on top of a single porcelain tube, *i. e.*, tube *B*, this source of error seems to have been avoided, as the fringe system maintained a very constant brilliancy and no adjustment of the interferometer was ever necessary, except in the neighborhood of 1100° C., which latter fact will be considered more fully later.

## METHOD OF PROCEDURE.

The entire apparatus having been put in adjustment the oven was evacuated and the temperature raised to 700° C. approximately. Having been maintained at this temperature an hour or so, the pump working during a larger part of the time, the oven was allowed to cool over night and the work taken up the following day when it had cooled to room temperature, as at this time the pressure in the oven was apt to be a few centimeters due to a slight leakage, the pump was run till the pressure was reduced to about .05 mm. and it was maintained at this value during the subsequent measurements. The temperature of the oven was determined, by the methods already described, both before and after the positions of the bands of the fringe system with respect to the reference circle were measured by means of the micrometer eye-piece of the interferometer. The initial temperature and fringe position having been determined the pump is stopped and the heating current applied. As the temperature rises the passages of the bands of the fringe system past the reference circle are counted and their times of passage recorded. These periods are so regular that there is practically no opportunity for error in the count. When the desired temperature, as shown by the thermo-couple, has been attained, the

Wheatstone bridge in the automatic temperature regulating devise is balanced and the oven maintained at a constant temperature for an hour or so. When trials, separated by suitable time intervals, show that the fringe system has also become stationary, the pump is set in action and the pressure reduced to its initial value in case there has been a slight leakage. Temperature measurements and readings upon the fringe system are again made. By combining the results of the initial and final readings on the fringe system with the number of bands counted as passing the reference circle, the shift may be measured with an accuracy of .02 of a band width as before mentioned. When time permitted the temperature of the oven was increased by several successive steps of about 100° C. each, the foregoing operations being repeated in each case.

## THE RESULTS OBTAINED.

In all, thirty-six separate determinations of the coefficient of expansion of the fused quartz were obtained. In Table II. the entire number of determinations will be found, arranged according to increasing temperatures, the values of $a$ are the mean coefficients of expansion between 16° C. and the upper temperature opposite which they stand in the table. These values have been multiplied by $10^8$. These same results are shown graphically in Fig. 5, where the abscissæ are the upper temperatures and the ordinates the values of $a \times 10^8$. Circles crossed by bars indicate two identical values.

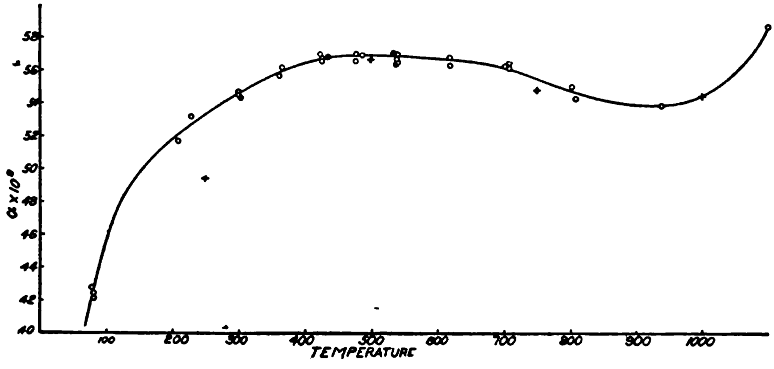


Fig. 5.

Very few values lie off a regular curve by amounts larger than were to be expected from the accuracy with which the measurements of temperature and fringe shift could be made; most of the values lie well within the limits set by these measurements.

Considering the small value of the coefficient, it increases rapidly with rise in temperature, up to about 350° C. From this point the increase is slower to a rather well defined maximum value of $56.8 \times 10^{-8}$ in the neighborhood of 500°; the values then decrease to a minimum of $53.6 \times 10^{-8}$ at 950°, this low value being followed by a marked increase to $58.10^{-8}$ at 1100° C. It is quite unfortunate that more determinations could not be made in this neighborhood. The phenomena attending the determination at 939° were normal in all respects, and from this temperature a regular heating to 1100° produced a very regular shift of fringes which was counted and timed with extreme care. The temperature was then maintained constant and the customary drawing made of the fringe

### Table II.

*Initial Temperature in Each Case* 16° *C.*

| Higher Temperature. | $a \times 10^{-8}$ | Date. | Higher Temperature. | $a \times 10^{-8}$ | Date. |
|---|---|---|---|---|---|
| 78.93° | 42.78 | January 16, 1909 | 487.0° | 56.87 | January 28, 1909 |
| 81.66 | 42.10 | " 18, " | 533.0 | 57.11 | " 8, " |
| 81.66 | 42.42 | " 19, " | 533.0 | 57.09 | " 9, " |
| 209.9 | 51.62 | " 24, " | 538.5 | 56.31 | Sept. 19, 1908 |
| 229.4 | 53.17 | " 24, " | 538.5 | 56.64 | " 20, " |
| 300.0 | 54.45 | Sept. 16, 1908 | 539.5 | 56.94 | " 18, " |
| 300.0 | 54.64 | " 17, " | 540.0 | 56.43 | " 21, " |
| 303.7 | 54.25 | " 18, " | 618.3 | 56.70 | " 18, " |
| 303.7 | 54.25 | " 19, " | 618.8 | 56.22 | " 21, " |
| 362.0 | 55.61 | January 16, 1909 | 702.0 | 56.19 | January 8, 1909 |
| 365.8 | 56.13 | " 24, " | 702.0 | 56.19 | " 9, " |
| 425.3 | 56.53 | Sept. 16, 1908 | 708.4 | 56.11 | Sept. 21, 1908 |
| 425.3 | 56.66 | " 17, " | 708.9 | 56.38 | " 18, " |
| 423.1 | 56.94 | " 18, " | 802.6 | 54.91 | Feb. 4, 1909 |
| 434.0 | 56.77 | January 30, 1909 | 808.0 | 54.20 | Sept. 21, 1908 |
| 435.0 | 56.79 | " 30, " | 939.0 | 53.74 | Feb. 4, 1909 |
| 476.5 | 56.51 | " 11, " | 1100.0 | 58.5 | " 4, " |
| 476.5 | 56.96 | " 12, " | | | |

system with respect to the reference circle, when the shift had practically stopped. As has been previously mentioned it was customary to hold the temperature constant for from one to two hours till all change in length of the quartz had ceased. As time passed in this case the fringe gradually faded out and within half an hour had disappeared. There was therefore no opportunity to take readings upon the fringe system with the micrometer but the drawing just mentioned showed that the line of the system nearest the reference circle, whose position is determined by the micrometer readings, was very accurately tangent to the circle. As it happened several preceding determinations had just this position of the fringe system, and in assuming that the values there obtained would apply in this case an error larger than one tenth of a band width would be quite impossible. As the total shift from 16° to 1100° is 22.9 bands the value of the coefficient obtained may claim a considerable degree of accuracy. It should be regarded as a small value if anything, as maintenance at a constant temperature generally results in a slight increase in shift.

Cooling to 950° did not cause a return of the interference fringes but a readjustment of the interferometer by means of its leveling screws did, showing that there had been, between 939° and 1100°, such an irregular expansion either in the porcelain tube *B* or the earthenware plate upon which the interference system rested, that the light, reflected from the interfering surface, no longer returned to the interferometer. Upon reheating to 1100° the fringe system was maintained as sharp and distinct as at lower temperatures by occasional adjustment of the interferometer. No accurate determination of $a$ was now possible as the position of the bands had been lost during the disappearance of the fringes, but two interesting phenomena manifested themselves. One was that there was a continuous though slow shift of the fringes, at constant temperature, in a direction to indicate a lengthening of the quartz ring. This shift was observed for an hour after the shift would normally have stopped and it had shown no signs of being completed when the second phenomenon made further observations impossible. This effect was the unequal expansions of the fused quartz cover and bed plates by which the reflecting surfaces ceased to be plane.

The interference bands became very distorted being no longer parallel to one another or equidistant. At the same time the mica disk mounted in the viewing tube at the top of the oven became completely opaque over half its surface and further work had to be abandoned. When cool the quartz plates showed the same distorted fringes. This property of the fused quartz, manifesting itself at 1100°, seems to mark the upper limit to which this method is applicable unless some other transparent substance, capable of withstanding higher temperatures without showing this unequal expansion, is found. It is possible that fused quartz plates having once experienced this effect would not again show it after repolishing.

## COMPARISON OF RESULTS WITH THOSE OF OTHER OBSERVERS.

While the apparatus here employed has been designed for work at high temperatures and therefore many refinements of method available at lower temperatures not possible here, it still seemed desirable to make a determination of the value of the coefficient in the region covered by Chappuis and Scheel. The three values, $42.78 \times 10^{-8}$, $42.10 \times 10^{-8}$ and $42.42 \times 10^{-8}$ obtained for the temperature range 16° to 80° agree very satisfactorily with one another. Their mean is $42.43 \times 10^{-8}$. The formula of Chappuis, $l = l_0(1 + 0.385 \times 10^{-6}t + 0.00115 \times 10^{-6}t^2)$, gives as the mean coefficient between 0° and 80°, $47.7 \times 10^{-8}$, while that of Scheel, $l = l_0(1 + 0.322 \times 10^{-6}t + 0.00147 \times 10^{-6}t^2)$, gives $43.9 \times 10^{-8}$. The value here obtained, when reduced to the same temperature interval as those of Chappuis and Scheel, possesses a value which is about as much smaller than Scheel's as Chappuis's is larger. Moreover the very rapid increase in the rate of expansion between 16° and 350° is in agreement with the results of Scheel and Dorsey for low temperatures.

Callendar states that the rate of expansion of fused quartz is quite regular to 1000° C., that above this temperature there is a marked increase in the rate of expansion and that if the temperature be held constant at any temperature above 1000° the quartz continues to slowly expand and that upon cooling after such an expansion it does not recover its initial length but remains somewhat longer.

The results here obtained show that the rate of expansion is not quite regular up to 1000° C. but in other respects there is an entire agreement and the value found by Callendar for the mean coefficient of expansion between 0° and 1000° C., *i. e.*, $59 \times 10^{-8}$, is not greatly different from $58.5 \times 10^{-8}$, the value obtained at 1100° in this work, the difference possibly arising from the different methods of measuring temperature, Callander using for this purpose the expansion of the platinum tube containing the quartz rod.

To facilitate comparison with results of Holborn and Henning a table of their results is here given.

| $\theta_1$ | $\theta_2$ | $\lambda_s$ | $\theta_1$ | $\theta_2$ | $\lambda_s$ |
|---|---|---|---|---|---|
| 17° | 250 | 54 | 500° | 750 | 68 |
| | | 57.5 | | | 62 |
| | | 54 | | | 60 |
| | | 52 | | | 62.5 |
| 25 | 500 | 79.5 | 16 | 750 | 192.5 |
| | | 74 | 750 | 1000 | 66 |
| | | 77 | | | 61 |

The numbers in column $\lambda_s$ measure the expansion of the quartz rod in $\mu$ [.001 mm.] for the temperature intervals $\theta_2 - \theta_1$. A comparison of the results for any temperature range shows quite marked variations in the measured expansions, amounting to 10 per cent. in two cases. As the corresponding variations in the present method are generally but a fraction of 1 per cent. the superiority of the interference method becomes manifest especially where the expansions to be measured are small as in this instance. The data contained in the above table give, as the mean expansion of the rod between 0° and 1000°, .262 mm. This value in connection with the length, 486.9 mm., gives 0.00000054 as the mean coefficient between 0° and 1000° C. This value falls directly upon the curve here obtained and shows a very remarkable agreement between the results obtained in the two investigations. This close agreement is still more strikingly shown when use is made of the data contained in the second table given by Holborn and Henning. In this table the expansions of the rod, computed by means of the mean coefficient, $54 \times 10^{-8}$, are compared with the measured expansions for given temperature intervals; Table II. is as follows:

| $\theta_1$ | $\theta_2$ | Observed Expansion. | Comp. Expansion. | Diff. |
|---|---|---|---|---|
| 0 | 250 | 59 | 65.5 | −6.5 |
| | 500 | 136 | 131 | +5.0 |
| | 750 | 198 | 196.5 | +1.5 |
| | 1,000 | 262 | 262 | 0 |

If the mean coefficient of expansion between 16° and the temperatures of the above table be computed, using Scheel's value to change from 0° to 16°, the results obtained, shown by the crosses on the plot, again lie very near the curve, the value at 250° being the only exception. This remarkably close agreement of the mean values obtained in the two investigations strongly indicates that the variations occurring in each method are due to accidental errors of observation and not to systematic errors, and that an average of several determinations may be assumed to give the correct value quite closely. Confidence in the result is certainly increased when such radically different methods yield such concordant values.

## SUMMARY.

1. Description of an oven and regulator for maintaining constant, temperatures lying between room temperature and 1000° C.

2. The interferometer method has been shown applicable to 1100° C. and to yield much more concordant results than any other method hitherto used. It can be used at still higher temperature if suitable reflecting surfaces can be found.

3. The mean coefficient of expansion between 0° and 1000° of Holborn and Henning has been confirmed and in addition an accurate statement of the mean coefficient between 16° and any temperature to 1000° is now possible. These values are contained in the following table and were read directly off the curve.

| $\theta_1$ | $\theta_2$ | $\alpha \times 10^8$ | $\theta_1$ | $\theta_2$ | $\alpha \times 10^8$ |
|---|---|---|---|---|---|
| 16 | 200 | 51.8 | 16 | 700 | 56.0 |
| | 300 | 54.6 | | 800 | 54.6 |
| | 400 | 56.4 | | 900 | 53.8 |
| | 500 | 56.8 | | 1,000 | 54.3 |
| | 600 | 56.6 | | 1,100 | 58.5 |

# EXPERIMENTS ON THE POULSEN ARC.

BY WILLIAM E. STORY, JR.

## INTRODUCTION.

DUDDELL[1] gives as the condition for the existence of an oscillatory current in a condenser circuit shunting an arc

$$\frac{dV}{dA} < 0 \quad \text{and} \quad \left|\frac{dV}{dA}\right| > R,$$

where $dA$ is the change of current between the terminals in an infinitesimal time, $dV$ the corresponding change in potential and $R$ the resistance of the shunt circuit.

The analytical deduction of these conditions has been carried out by M. Janet[2] on the supposition that (1) the current in the condenser circuit is of the sine form, that (2) the resistance of the main circuit is large as compared to that of the shunt circuit, and that (3) the frequency of the oscillating current is

$$n = \frac{1}{2\pi\sqrt{LC}},$$

depending only on the capacity and inductance.

The more accurate formula is

$$n = \frac{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}{2\pi}.$$

Corbino[3] says the current in the capacity circuit is not sinusoidal and he gives the equation for it

$$L\frac{di}{dt} + \left(r - \frac{b}{(i - i_2)^2}\right)\frac{di_2}{dt} + \frac{1}{C} = 0,$$

---

[1] W. Duddell, "On Rapid Vibrations in the Current through the Direct Current Arc," Journ. Inst. Elect. Eng., Vol. 30, p. 232, 1900.

[2] P. Janet, "Duddell's Musical Arc." Comptes Rendus, Vol. 134, p. 821, 1902.

[3] Corbino, Atti dell' Assoc. Elletr. Ital., Vol. 7, pp. 369 and 597; or Sci. Abstr., 1904, Vol. 7A, p. 537.

$i_2$ being the current in the shunt and $i$ the current in the main circuit.

The preliminary experiments in the present work gave rise to resonance curves so similar to those obtained in the case of sinusoidal vibrations, that an effort has been made to show that the oscillations in the Poulsen arc are of the sine form, at least for the potentials and currents considered.

---

In the present experiments the variable inductance and capacity were connected to the arc terminals by heavy copper wires. The antenna when directly coupled was attached to the same terminal of the inductance as the capacity, and when it was magnetically coupled the primary was inserted between the inductance and capacity, as usual. The resistance of the leads was .038 ohm and that of the variable inductance .028 ohm for each turn. A hot wire galvanometer calibrated by a steady current was inserted at *A* (Fig. 1).

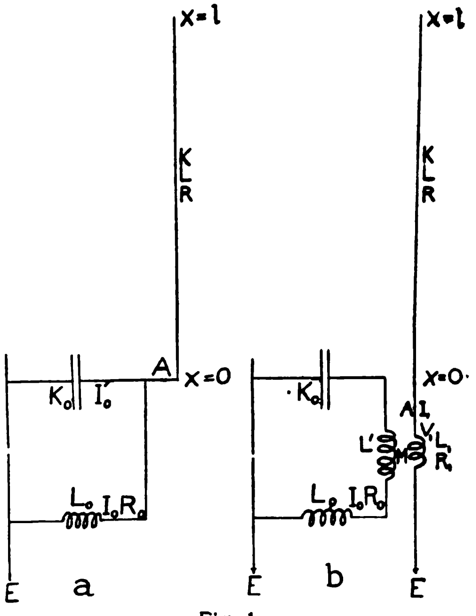


Fig. 1.

### APPARATUS.

The main current was obtained from a storage battery of slightly over 120 volts. This current was regulated by 5 rheostats and a variable graphite resistance in series,—in all about 35 ohms.

The inductance was one made by Dr. R. A. Fessenden, consisting of two cylinders side by side, one ebonite and the other metal, which revolving together by a connecting cog-wheel wound a .5 mm. wire from a spiral cut in the metal cylinder in a corresponding spiral on the ebonite to increase the inductance. The pitch of the spiral was .79 mm., and its external diameter 10.17 cm. The inductance for various numbers of turns of wire was calculated from a formula given by J. G. Coffin,[1]

$$L = 4\pi a n^2 \left\{ \left(N - \frac{1}{2}\right) + \frac{b^2}{32a^2}\left(N + \frac{1}{4}\right) - \frac{1}{1024}\frac{b^4}{a^4}\left(N - \frac{2}{3}\right) + \frac{10}{131072}\frac{b^6}{a^6}\left(N - \frac{109}{120}\right) \right\},$$

where $a$ is the radius and $b$ the length of the winding, $n$ the whole number of turns of wire and $N = \log 8a/b$.

Coffin shows[2] that for any frequency the inductance is

$$L = \left[ 1 + 1.25 \frac{2d}{3r_1} \left( \frac{3 \sinh x - \sin x}{x \cosh x - \cos x} \right) \right] L_\infty = yL_\infty,$$

where $L_\infty$ is the inductance for an infinite frequency, $r_1$ the inner radius of the coil, $d$ the diameter of the wire and $x = 4\pi d\sqrt{\sigma n}$, $\sigma$ being the specific conductivity and $n$ the frequency. The values of $y$ for the variable inductance and for the long coil are shown in the accompanying table:

| Frequency. | Coil. | $y$ |
|---|---|---|
| $370 \times 10^3$ | Variable inductance. | 1.0027 |
| 370 | Long coil. | 1.0064 |
| 137 | Variable inductance. | 1.0044 |
| 137 | Long coil. | 1.0105 |

The condenser was made in the workshop at Clark University. It consisted of 16 brass semicircular plates 19.5 cm. in diameter, fastened rigidly one above another 1.8 cm. apart by a brass rod at the center of the circular edge. Half way between each pair of these plates a plate 15.5 cm. in diameter of a similar tier, fastened together by a rod at the center of the circle, could be introduced by turning the rod. These rods projected through the top of a glass jar filled with kerosene and served as terminals. The angle of the plates was measured on a circular scale fastened to the top of the jar.

The condenser was standardized by running an alternating

---

[1] J. G. Coffin, "Absolute Standards of Inductance," Bull. Bureau of Standards, Vol. 2, No. 1, p. 136, 1906.

[2] J. G. Coffin, "The Influence of Frequency upon the Self-Inductance of Coils," Proc. Amer. Acad., Vol. 41, p. 789, 1906.

current of frequency 1,000 through it shunted with a standard and with variable resistances in series with each. The terminals of a telephone receiver were connected to the two points between the resistances and capacities, forming a modified Wheatstone bridge.

If there is no current through the telephone, then

$$\frac{R_1}{R_2}=\frac{C}{C_1} \quad \text{and} \quad C=\frac{R_1C_1}{R_2}.$$

If $R_1$ and $R_2$ were large, the capacity could be determined to within ½ per cent.

The standard capacity was the one used by Professor Webster[1] in his measurement of electrical oscillations, consisting of two circular steel plates 50.0 cm. in diameter and 1.69 cm. in thickness, held apart by glass cylinders. From the formula given by Kirchhoff

$$K=\frac{r^2}{4d}+\frac{r}{4\pi}\left\{1+\log_e\frac{16\pi r(d+b)}{d^2}+\frac{b}{d}\log_e\frac{b+d}{b}\right\},$$

the capacity is found to be 494.4 cm. if $d$ is .332 cm.

The graph of the capacity of the variable condenser in centimeters per degree from the zero position proved an approximately straight line passing through the point 150°, 916 cm. and meeting the axis of capacities at + 100 cm.

In order to determine what effect the frequency had on the capacity of the condenser, it was also measured by a potentiometer method at a frequency of 50. Under this condition the capacity was found to be 708 cm. at 0 and 1,696 cm. at 180 degrees of the scale. Since the slope of the line is approximately the same by both methods, the excessive value obtained with a low period is probably due to an absorption in the dielectric.

Accordingly, a third determination was made at the actual period of most of the experiments. The standard capacity was inserted in the oscillating system alone and the inductance changed till the system was in resonance with the long coil. The standard was then replaced by the variable condenser and the position of this adjusted until resonance was obtained. Positions of the variable capacity were thus compared with the standard plates held apart by five

---

[1] A. G. Webster, "An Experimental Determination of the Period of Electrical Oscillations," Phys. Rev., Vol. VI., p. 297, 1898.

sets of cylinders, the lengths in cm. (*d*) of which are given below with the corresponding capacity in centimeters as calculated.

|  | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| *d* | .195 | .332 | .486 | .623 | 1.000 |
| *K* | 827.0 | 494.4 | 343.9 | 272.2 | 175.9 |

As the arc could not be satisfactorily maintained with a capacity as low as 272.2 cm. only the first three of these values could be directly transferred to the variable capacity. To these values corresponded scale readings of 128.4, 69.0, and 43.1.

To obtain other values, the standards with their equivalent known values of the variable condenser in parallel, were balanced in the same way against the variable alone, and in this way the following table obtained :

|  | *K* | Degrees Added. | Total Cap. | Degrees to Balance. |
|---|---|---|---|---|
| 1 | 494.4 | 69.0 | 988.8 | 155.7 |
| 2 | 343.9 | 43.1 | 687.8 | 104.6 |
| 3 | 343.9 | 104.6 | 1,031.7 | 164.2 |
| 4 | 175.9 | 32.8 | 475 | 64.0 |
| 5 | 175.9 | 64.0 | 651 | 96.3 |
| 6 | 175.9 | 96.3 | 827 | 128.4 |
| 7 | 175.9 | 128.4 | 1,003 | 160.5 |

Of these 4, 5 and 6 were found by the reverse process.

In addition to these, and as a test of them, balance points were similarly obtained as follows :

|  | *K* | Degrees Added. | Degrees to Balance. |
|---|---|---|---|
| 1 | 494.4 | 0 | 87.0 |
| 2 | 494.4 | 30 | 117.7 |
| 3 | 494.4 | 60 | 148.6 |
| 4 | 494.4 | 90 | about 180 |
| 5 | 343.9 | 0 | 63.0 |
| 6 | 343.9 | 30 | 91.0 |
| 7 | 343.9 | 60 | 121.1 |
| 8 | 343.9 | 90 | 152.3 |
| 9 | 272.2 | 0 | 51.0 |
| 10 | 272.2 | 30 | 78.8 |
| 11 | 272.2 | 60 | 110 |
| 12 | 272.2 | 90 | 138 |
| 13 | 272.2 | 120 | 165 |

From these three sets of values the final plot of the capacity was obtained as shown in Fig. 2.

From the shape of the variable condenser we should expect the plot of its capacity to be very nearly a straight line, and as the readings all lie near such a line the points of this line were taken as the true values of the capacity.

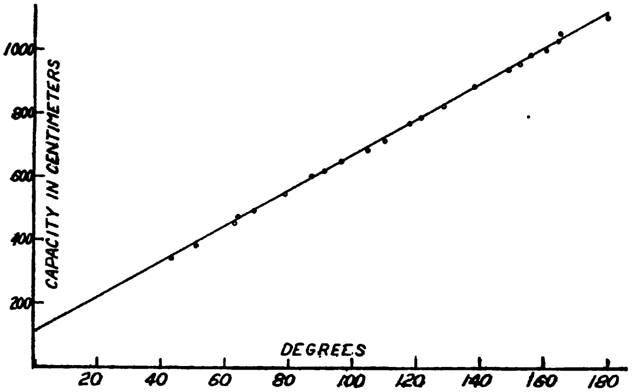


Fig. 2.

Fleming[1] has given a formula for the velocity of electric waves along a close wire helix whose diameter is small in comparison with its length, and the resistance and dielectric conductance small in comparison with its inductance:

$$W = \frac{3 \times 10^{10}}{\sqrt{LC}},$$

where $L$ and $C$ are the inductance and capacity in centimeters per unit length of the coil.

The approximate formula for the inductance for low frequencies in centimeters of a coil of great length as compared to its diameter is

$$L_{c_0} = \pi^2 D^2 m^2 l,$$

where $D$ is the mean diameter of the spiral, $l$ the length and $m$ the number of turns per centimeter.

In the present case the coil or imitation antenna consisted of enameled wire on a wooden core made of four strips fastened together

---

[1] J. A. Fleming, "Electric Waves along Spiral Wires," Phil. Mag., Vol. 8, 1904.

to prevent warping. The dimensions were as follows: $D = 4.285$, $l = 199.7$, $m = 14.94$. From these values $L_{c0} = 8.078 \times 10^6$ and for infinite frequencies, since the diameter of the wire is .067 cm., $L_{c_\infty} = 7.828 \times 10^6$. From this we get the inductance at the actual frequencies 370,000 and 137,000 as $L_{c_1} = 7.878 \times 10^6$ and $L_{c_2} = 7.910 \times 10^6$.

With an earth wire 5 cm. from this coil its capacity $C_c$, as determined by an electrometer method described in a paper by J. C. Hubbard,[1] was 43.5 cm.

From these values of the capacity and inductance, $W_{c_1} = 3.237 \times 10^6$ and $W_{c_2} = 3.229 \times 10^6$ centimeters per second.

Now, since $n = W/\lambda$, to find $n$ we have to measure the wave-length $\lambda$ along the coil. Fleming shows (*loc. cit.*) that in the case of the fundamental the true quarter wave-length is given by $3\lambda_1/2$, where $\lambda_1$ is the half wave-length of the first harmonic. This $\lambda_1$ was determined with the aid of a Geissler tube filled with rarefied alcohol vapor and presented to the coil at a node, to be 134 cm., therefore $\lambda = 804$ cm. and accordingly $n_{c_1} = 4.03 \times 10^5$ and $n_{c_2} = 4.02 \times 10^5$. The difference between these is beyond the limit of accuracy in the measurement of $\lambda$, so a common value $4.03 \times 10^5$ was used in both cases and will be designated by $n_1$.

The arc (Fig. 3) was a modified form of that used by Poulsen. The anode $A$ was a copper plug .5 cm. long and .5 cm. in diameter screwed into the cap of a brass tube $T$ 1.2 cm. in diameter into which water was slowly flowing. The kathode $K$ was carbon 1.6 cm. in diameter and was slowly revolved by a small motor. The carbon made one revolution in about two minutes. The arc proper was surrounded by a brass box $B$ fitting between the magnetizing coils $M$, insulated by a wood fiber plug $P$ from the anode and made tight around the kathode with a stuffing box $S$. A highly tempered steel knife $N$ was fastened rigidly to the interior of this gas box in such a way that a spiral spring behind the kathode pressed it firmly against the knife. A tube 3.7 cm. in diameter and 10 cm. long screwed into a hole in the front of the box ended in a glass window which allowed the light egress. A current of

---

[1] J. C. Hubbard, "On the Conditions for Sparking at the Break of an Inductive Circuit," PHYS. REV., Vol. XXII., p. 129, 1906.

ordinary illuminating gas was passed through the box continually during the experiment and a water jacket prevented excessive heating. The coils consisted of 1,500 turns of heavily insulated wire giving a field sufficiently strong to localize the arc. This apparatus also was constructed in the shop.

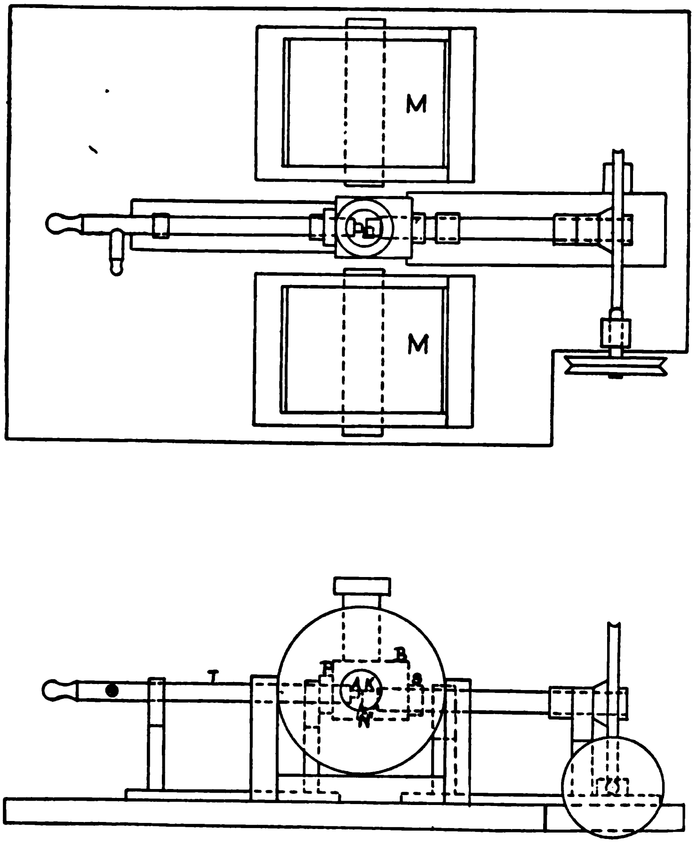


Fig. 3.

In making the photographs the light from the arc passed through the window in the case, through a photographic lens of focal length 24 cm., and was reflected by a Foucault revolving mirror[1] to a focus on the plate. The air blast was furnished by a fan blower driven by a three quarter horse power electric motor. In the pres-

---

[1] C. A. Saunders, PHYS. REV., 1896, p. 81.

ent experiments the mirror was run up to a maximum speed of a little over 100 turns a second. The speed was regulated by changing the blast from the fan until the image in the mirror of an electrically driven tuning fork of frequency approximately 100, vibrating parallel to the axis of the mirror, appeared stationary. This state is reached whenever the time of rotation of the mirror is commensurate with the time of vibration of the tuning fork, at least for rapid vibrations; but the images for rotations of 100, 50, $33\frac{1}{3}$, 25, 20, $16\frac{2}{3}$ were so much stronger that there was no difficulty in distinguishing these from the others. To determine which velocity any of these stationary positions of the fork represented, an exposure of the plate was made for the next above and the next below. Then the ratios of the distances between scintillations determined the actual speed of the mirror. When compared with a standard, the fork's frequency was found to be 99.0.

It was found that dropping the plate past the flash would not give a sufficiently long exposure to affect the plate appreciably. Accordingly it was put into an ordinary camera having the lens, shutter and lens carrier replaced by a shutter so arranged that a long slit in the plane of the reflected light could be opened and closed by pulling and releasing a string. This exposed the stationary plate for perhaps a fifth of a second. Of course, if the total distance traversed by the image in a half revolution of the mirror were an integral multiple of the distance between two successive scintillations, the shutter could be kept open any length of time without the markings overlapping. As a matter of fact, however, an exposure of from $\frac{1}{10}$ to 1 second generally gave a measurable plate. The shutter was held rigidly but the back of the camera could be raised and lowered, thus allowing several photographs to be made on the same plate.

The formula for the frequency is

$$n = \frac{4\pi rt}{d},$$

where $n$ is the frequency, $t$ the number of turns per second, $r$ the distance of the plate from the mirror and $d$ the length of the markings.

## THEORY OF THE EXPERIMENT.

The theory for the current in two directly coupled systems has been given by Abraham.[1] The theory as here presented is that given by Professor Webster in his lectures on electricity during the present year.

If $R_0$, $L_0$, $K_0$ are the resistance, inductance and capacity of the shunt across the arc and if $I_0$ is the current through the inductance, we have the well-known equations

$$L_0\frac{dI_0}{dt}+R_0I_0+V_0=0, \tag{1}$$

$$I_0'=K_0\frac{dV_0}{dt}, \tag{2}$$

where $I_0'$ is the current through the capacity.

If $I_1$ (Fig. 1, *a*) is the current in any uniform conductor or antenna of length $l$, capacity $K$, inductance $L$ and resistance $R$ (all for one centimeter of length along the axis) at the end joined to the arc circuit, then

$$I_0=I_1+I_0'; \tag{3}$$

that is,

$$\frac{dI_0}{dt}=\frac{dI_1}{dt}+K_0\frac{d^2V_0}{dt^2}. \tag{4}$$

If $x$ is the distance along the conductor from the same end,

$$K_0\frac{d^2V_0}{dt^2}=K_0\left(\frac{\partial^2V}{\partial t^2}\right)_{x=0}, \tag{5}$$

where $V$ is the potential at the point $x$ corresponding to a current $I$.

We have then, as the end conditions for the conductor,

$$L_0\frac{dI_1}{dt}+L_0K_0\frac{d^2V_0}{dt^2}+R_0I_1+R_0K_0\frac{dV_0}{dt}+V_0=0 \tag{6}$$

for $x=0$, and

$$I_2=0 \tag{7}$$

for $x=l$.

---

[1] M. Abraham, "Zur drahtosen Telegraphie," Phys. Zeitschr., Bd. 5, p. 174, 1904.

If in the telegrapher's equation

$$KL\frac{\partial^2 V}{\partial t^2} + KR\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} \tag{8}$$

we put

$$V = e^{\lambda t}u(x) \tag{9}$$

and divide through by $e^{\lambda t}$, we have

$$(KL\lambda^2 + KR\lambda)u(x) = \frac{d^2u(x)}{dx^2} = -k^2u(x), \tag{10}$$

where

$$-k^2 = KL\lambda^2 + KR\lambda, \tag{11}$$

or

$$\frac{d^2u(x)}{dx^2} + k^2u(x) = 0; \tag{12}$$

that is,

$$u = A\cos kx + B\sin kx, \tag{13}$$

where $A$ and $B$ are arbitrary constants. Now, along any such conductor we have

$$L\frac{\partial I}{\partial t} + RI = -\frac{\partial V}{\partial x}; \tag{14}$$

and, if we put

$$I = e^{\lambda t}w(x),$$

we get, by substituting the value of $V$ from (9) and (13),

$$w(x) = \frac{k(A\sin kx - B\cos kx)}{L\lambda + R}. \tag{15}$$

When $x = l$,

$$I_2 = e^{\lambda t}w(l) = 0,$$

or

$$\frac{B}{A} = \tan kl. \tag{16}$$

When $x = 0$, by substituting $V_0$ and $I_1$ in (6), we have

$$(L_0K_0\lambda^2 + R_0K_0\lambda + 1)u(0) + (L_0\lambda + R_0)w(0) = 0; \tag{17}$$

so that, by (13) for $x = 0$ and (16),

$$\frac{(L\lambda + R)(L_0K_0\lambda^2 + R_0K_0\lambda + 1)}{L_0\lambda + R_0} = k\tan kl. \tag{18}$$

So far the solution is entirely general. Now neglect the resistances $R$ and $R_0$, since they are very small as compared with $L\lambda$ and $L_0\lambda$ $(1/10^4)$.

Then we get from (11)

$$(19) \qquad -k^2 = KL\lambda^2,$$

so that $\lambda$ is a pure imaginary; that is, there is no damping factor. If we put

$$(20) \qquad p^2 = -\lambda^2,$$

we get from (18), (19) and (20)

$$(21) \qquad \frac{L}{L_0}(1 - K_0L_0p^2) = \sqrt{KL}p \tan (\sqrt{KL}pl);$$

or, if we let

$$(22) \qquad z = \sqrt{KL}pl,$$

we get finally

$$(23) \qquad \tan z = \frac{lL}{L_0}\left(\frac{1}{z} - \frac{K_0L_0}{KL}\frac{z}{l^2}\right) = \frac{lL}{L_0z} - \frac{K_0}{lK}z$$

and

$$(24) \qquad L_0 = \frac{KLl^2}{z(K_0z + Kl \tan z)}.$$

In Fig. 4, $L_0$ is plotted as ordinate and $z$ as abscissa in the neighborhood of $z = \pi/2$. $K_0$ being $= 1122$, $Kl = 43.5$ and $Ll = 7.878 \times 10^6$.

Since, for the fundamental of the long coil alone

$$p = \frac{2\pi W}{4l} = \frac{\pi}{2l\sqrt{KL}} \text{ within } \tfrac{1}{2} \text{ of 1 per cent.},$$

from (22) the period of the whole system will be equal to the fundamental of the coil alone, if $z = \pi/2$; and then by (24) $L_0 = 0$. We should then have complete resonance and a correspondingly enormous amount of energy taken from the capacity circuit. In like manner we get complete resonance if $z = 3\pi/2$, $5\pi/2$, etc., corresponding to the first, second, etc., harmonics of the coil; and in these cases $L_0$ will be 0.

Now since for changes of $L_0$, $n = p/2\pi$ is by (22) a constant multiple $C_1$ of $z$, the graph of $L_0$, $n$ is similar to Fig. 4. So that theoretically for every value of $L_0$ we have an infinite number of different frequencies — for the fundamental and harmonics. Also, any frequency once obtained cannot by any change of $L_0$, be continuously changed by an amount greater than $\pi C_1$, nor can we find more than one resonance point, though we may with a large value of $L_0$, obtain a frequency near a second resonance. For $z < \pi/2$, $z$ decreases as $L_0$ increases and the curve approaches $z = 0$ as an asymptote: that is, we have a steady falling off of the frequency with an increase of inductance.

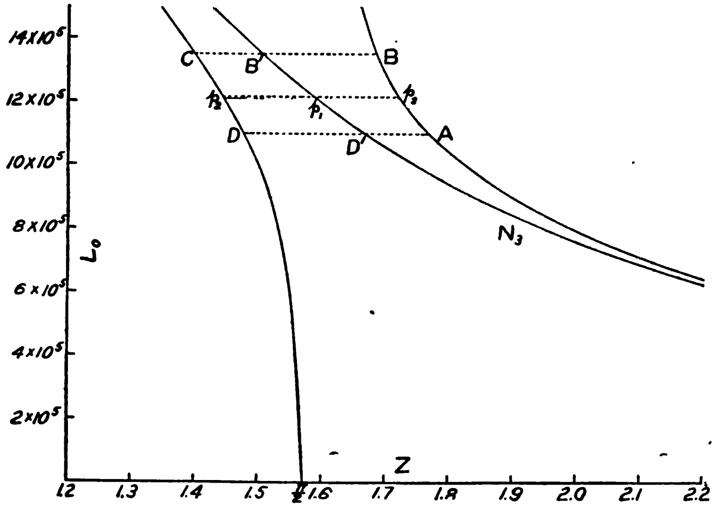


Fig. 4.

As a matter of fact, if the inductance is decreased from a large value (say $2 \times 10^6$), the current into the antenna increases until a point corresponding to a value of $z$ a little less than $\pi/2$ is reached. So much energy is then taken out of the arc that it is momentarily extinguished; but the point on the neighboring branch of the curve having the same inductance corresponds to a value of $z$ far from $\pi/2$. Accordingly the frequency springs over to this branch of the curve, so to speak. This change is of course accompanied by a sudden drop in the current entering the coil. If the inductance is still further decreased the frequency travels down this second branch to a point near the next point of resonance, that is, near the period corresponding to the first harmonic of the coil, and then goes through the same procedure as before. If, however, we increase the inductance when the first point of discontinuity is reached, the frequency

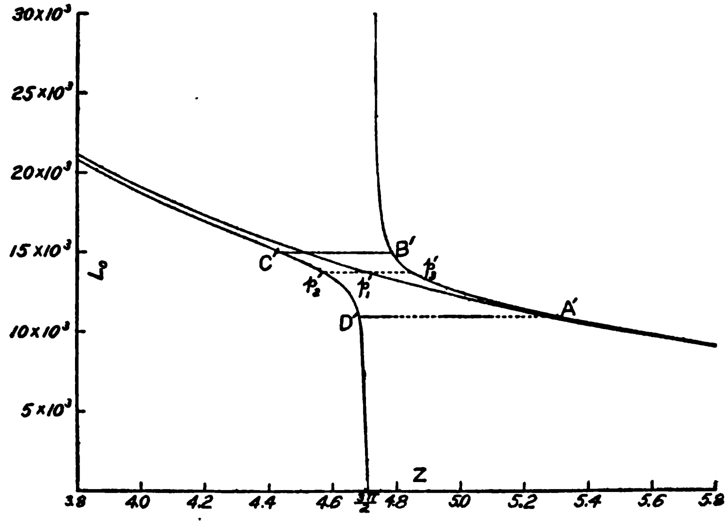


Fig. 4a.

will not jump back to the first branch directly, but will remain on the second branch until such a value of *z* is reached that the energy given the coil is approximately the same as at the first break. The

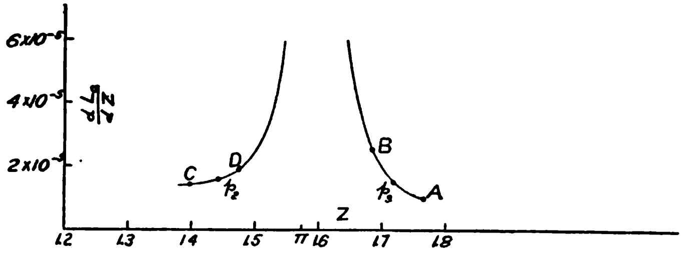


Fig. 4b.

breaking inductance will accordingly be higher for the increasing inductance than for the decreasing.

## INDUCTIVE COUPLING.

If $I_0$ is the current, $R_0$ the resistance, $L_0$ the inductance, $K_0$ the capacity and $V_0$ the potential difference between the condenser plates, — all in the condenser circuit; if $I_1$ is the current, $R_1$ the resistance, $L_1$ the self-inductance of the coil at the bottom of the antenna and $V_1$ the potential at the upper end of this coil; and if $M$ is the mutual inductance between the two circuits; then

$$(25) \qquad I_0 = K_0 \frac{dV_0}{dt},$$

$$(26) \qquad L_0 \frac{dI_0}{dt} + M \frac{dI_1}{dt} + R_0 I_0 + V_0 = 0,$$

$$(27) \qquad M \frac{dI_0}{dt} + L_1 \frac{dI_1}{dt} + R_1 I_1 + V_1 = 0,$$

from which are obtained

$$(28) \qquad K_0 \left( L_0 \frac{d^2 V_0}{dt^2} + R_0 \frac{dV_0}{dt} \right) + M \frac{dI_1}{dt} + V_0 = 0,$$

$$(29) \qquad K_0 M \frac{d^2 V_0}{dt^2} + L_1 \frac{dI_1}{dt} + R_1 I_1 + V_1 = 0.$$

If $L$, $K$ and $R$ are the inductance, capacity and resistance per unit length of the antenna, and $V$ and $I$ the potential and current at the point whose distance from the lower end is $x$,

$$(30) \qquad L \frac{\partial I}{\partial t} + RI = - \frac{\partial V}{\partial x},$$

$$(31) \qquad - \frac{\partial I}{\partial x} = K \frac{\partial V}{\partial t},$$

from which follows

$$(32) \qquad L \frac{\partial^2 V}{\partial t^2} + R \frac{\partial V}{\partial t} = \frac{1}{K} \frac{\partial^2 V}{\partial x^2}.$$

Assuming the oscillations to be sinusoidal, that is,

$$(33) \qquad V = u e^{\lambda}, \quad I = w e^{\lambda t},$$

$$(34) \qquad u = A \cos kx + B \sin kx,$$

and, from (30), (33) and (34),

$$(35) \qquad w = \frac{k}{L\lambda + R} (A \sin kx - B \cos kx),$$

we have, for $x = 0$, from (28) and (29),

$$E(K_0L_0\lambda^2 + K_0R_0\lambda + 1) - \frac{Mk\lambda}{L\lambda + R}B = 0, \tag{36}$$

$$EK_0M\lambda^2 + A - \frac{(L_1\lambda + R_1)k}{L\lambda + R}B = 0, \tag{37}$$

where $E$ is the constant factor of $V_0$, and, for $x = l$, because then $I = 0$,

$$\frac{B}{A} = \tan kl. \tag{38}$$

Let $z = kl$; then, from (36), (37) and (38),

$$z \tan z = \frac{(L_c\lambda + R_c)(K_0L_0\lambda^2 + K_0R_0\lambda + 1)}{(L_1\lambda + R_1)(K_0L_0\lambda^2 + K_0R_0\lambda + 1) - K_0M^2\lambda^3}, \tag{39}$$

where $L_c = lL$ and $R_c = lR$. Since $R$ is small in comparison with $L\lambda$ (1/1000), we may neglect it and obtain, from (11),

$$-k^2 = KL\lambda^2. \tag{40}$$

Let

$$p = -i\lambda, \tag{41}$$

then

$$KLp^2 = k^2, \tag{42}$$

$$z \tan z = \frac{L_c\left(1 - \frac{K_0L_0}{K_cL_c}z^2\right)}{L_1 - \frac{K_0}{K_cL_c}(L_0L_1 - M^2)z^2}. \tag{43}$$

With the present coupling, $M^2$ is only about 1 per cent. of $L_0L_1$ and $K_0/K_cL_c \cdot (L_0L_1 - M^2)z^2$ only 4 per cent. of $L_1$; so we may neglect $M$ in determining $z$ and get

$$z \tan z = \frac{L_c}{L_1} \tag{44}$$

and since $L_1 = 7.86 \times 10^6$, $z = 1.429$, or $4.307$, etc. A variation of 10 per cent. in the value of $L_c/L_1$ will make a difference of less than 1 per cent. in the value of $z$. The frequency is given by (42)

$$n = \frac{z}{2\pi\sqrt{K_cL_c}}. \tag{45}$$

## Results.

To determine whether the quantity $K_0 L_0$ is or is not a constant for a fixed value of the frequency, the condenser was set at a series of values and the values of the inductance that gave resonance with the long coil attached inductively were observed. This inductance could be easily determined to one part in three hundred. From these values the quantities $C$ and $L'$ in the equation

$$(L_0' + L')K_0 = C$$

were determined by the method of least squares for the least mean error of $K_0 L_0$ to be

$$C = 1.445 \times 10^8,$$
$$L' = 4.1 \times 10^3.$$

This $L'$ is the inductance in the primary outside of the variable inductance $L_0'$. Even the second figure of this quantity is doubtful, but as $L'$ is less than one per cent. of $L$ this inaccuracy is of no consequence. $A$ (Fig. 5) is a graph of the curve $L_0$ and $K_0$. The points from which it was calculated are shown.

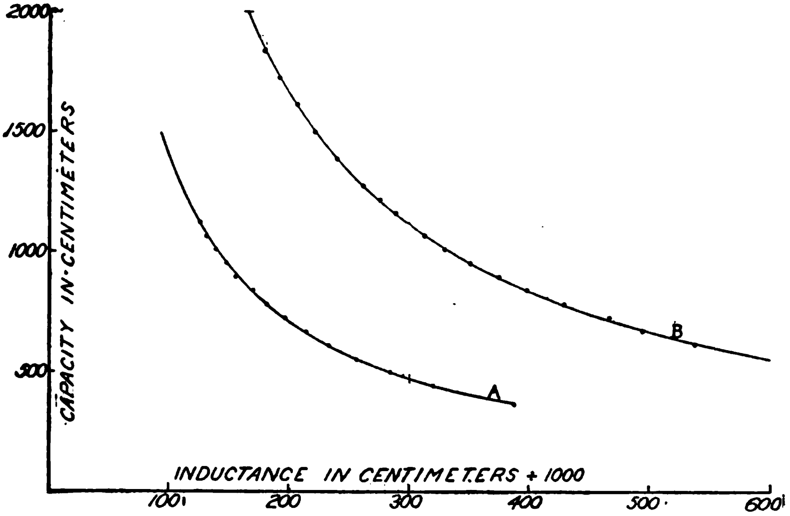


Fig. 5.

A similar curve $B$ (Fig. 5) was plotted for the capacity circuit tuned to an antenna, about 81 meters long fastened to the upper end of the inductance $L_1$. This antenna consisted of a single copper wire run out through the window to the top of a neighboring building where it was connected to a porcelain insulator strung on a loop of wire about a meter long. In this case

$$C = 3.364 \times 10^8,$$
$$L' = 3.4 \times 10^3,$$

where again the value of $L'$ is very uncertain on account of the method of calculating the curve.

In both of these cases the slight variations from the curve seem due to momentary conditions rather than to any tendency to depart from the hyperbolic form.

During the experiments the main current varied from 1.5 to 2.5 amperes and the potential drop across the arc from 35 to 60 volts. Neither of these variations appreciably affected the value of $L_0$ for resonance with the long coil. Accordingly, the variable capacity was set at its largest value (1,122 cm.) and photographs were made for known inductances. The results are tabulated below. The arc length was about 1.5 mm. during all the experiments.

| Plate. | Tuned to | Coupling. | Inductance. $L_0' + L'$ | $s$ | $n$ as Determined by | | | | Remarks. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Coil Meas. $n_1$ | Theory. $n_2$ | $\frac{3\times10^{10}}{2\pi\sqrt{K_0L_0}}$ $n_3$ | Mirror. $n_4$ | |
| 1<br>2 | Fundamental. | Inductive. | 1.440× $10^5$ | 1.429 | 403,000 | 367,000 | 376,000 | 357,000 | Geissler tube attached. |
| 12 | " | " | 1.411 " | 1.429 | 403,000 | 367,000 | 372,000 | 342,000 | " |
| 22 | " | " | 1.290 " | 1.429 | 403,000 | 367,000 | 397,000 | 376,000 | |
| 23*a* | " | " | 1.276 " | 1.429 | 403,000 | 367,000 | 399,000 | 381,000 | |
| 23*b* | — | " | 1.276 " | — | — | — | 399,000 | 389,000 | |
| 18 | Fundamental. | Direct. | $L_0$<br>1.140× $10^5$ | 1.465 | 403,000 | 376,000 | 427,000 | 381,000 | |
| 24 | " | " | 1.130 " | 1.753 | 403,000 | 450,000 | 424,000 | 454,000 | |
| 6<br>7 | 3×fund. wave-length. | " | 10.50 " | .535 | 403,000 | 137,000 | 139,000 | 136,000 | " |
| 26<br>27 | — | None. | 1.336 " | — | — | — | 390,000 | 396,000 | |

The frequency is given as calculated from the dimensions of the capacity circuit ($n_3$) as well as from the above theory. The value

$n_2$ under "theory" is obtained by multiplying the natural period ($n_1$) by $2z/\pi$. If the quarter wave-length were just the length of the coil, we could calculate $n_1$ directly from (22), page 247, and (45), page 251. $n_4$ is the frequency as measured by the mirror. All the inductances have been corrected for the frequency.

In plates 1, 2 and 12 a Geissler tube at the upper end of the long coil added a capacity sufficient to change the period quite decidedly. Naturally then, $n_4$ is much less than $n_1$, and $n_3$ is below its value for resonance with the long coil alone. The low values in 12 are probably due to a change in the position of the tube.

In plates 22 and 23*a* the Geissler tube was moved about 2 mm. away from the coil, where it did not appreciably affect the frequency, but where it still glowed for inductances near resonance. $n_3$ and $n_4$ have risen about 5 per cent. $n_1$ and $n_3$ are both greater than $n_2$ and $n_4$, that is, the period of the system is less than the period of either part, in spite of the looseness of the coupling.

Plates 18 and 24 show the different frequencies that may be obtained with the same value of the inductance in the case of direct coupling; the one for decreasing inductance and the other for increasing. In this case the values of $z$ were taken directly from the $L_0 - z$ curve (Fig. 4) and $n_2$ differs from $n_4$ by less than 1 per cent. These two values of $n_2$ for nearly the same value of $n_3$ differ widely from each other. This difference is perhaps the cause of the change often observed in the product $K_0L_0$ for the same frequency.

It was found that if, when the long coil was directly coupled to the arc circuit, the inductance $L_0$ was increased to about 9 times its value for resonance, the Geissler tube glowed for this region also, though not as brightly as before. The coil then responds to an oscillation having a quarter wave-length equal to three times the length of the coil: that is, the wave would receive the negative pulse only upon its second arrival at the capacity circuit. Since there is a large change of inductance at the circuit end of the coil we might expect a sufficiently strong reflection there to make it respond to a wave-length of about twelve times its own length. Of course this response cannot be nearly as large as in the case of the fundamental. Plates 6 and 7 were made for the inductance that gave the largest galvanometer reading. The value of $z < \pi/2$ which corresponds to this value of $L_0$ in Fig. 4 is given under $z$. Of course $n_4$ should be smaller than $n_2$ because of the capacity added by the Geissler tube and not considered in the theory.

Plates 26 and 27 were made with no coupling. Here the value of the frequency as calculated from the formula

$$n_3 = \frac{3 \times 10^{10}}{2\pi\sqrt{K_0 L_0}}$$

agrees with the observed value within 1.5 per cent. These two plates were made to determine the effect of the potential drop over the arc. For differences of potential from 35 to 60 volts no appreciable change took place.

In every case (except those with the Geissler tube attached) the theoretical frequency was a little less than that observed by the mirror. This seems to show that either the mirror was not spinning as fast as supposed — that is, that the standard tuning fork was a little under pitch — or else that the frequency $n_1$ as calculated from the coil was really greater than 403,000. This latter seems the more probable source of error, since a difference of 2 cm. in the position of the node of potential as determined by the Geissler tube would be more than enough to account for this discrepancy. This undervaluation of $W$ might account for the difference between $n_2$ and $n_4$. Accordingly it seems reasonable to regard the discrepancy as due to errors in the measurements rather than to a fault in the theoretical assumptions.

The measurements of plate No. 26 are given below as a specimen.

### PLATE 26.

Exposure 2, 33 volts drop, length of 70 oscillations 5.115 cm., $d = .0745$
" 10, 60 " " " " 20 " 1.491 " $d = .0745$
" 11, 55 " " " " 30 " 2.206 " $d = .0735$
$t = 49.5$, $r = 45.3$.

Below are given some specimen resonance curves. In Fig. 6 inductance in centimeters is plotted against current in amperes, and in the others the galvanometer deflections in centimeters are plotted as ordinates and the turns of the variable inductance in the condenser circuit as abscissæ, the capacity being 1,122 cm.

When the coil is directly coupled as in Fig. 6, the small rise corresponding to the first harmonic is noticeably double pointed only when the arc is running its best, as the energy of the vibrations at this period is very small. The *fundamental*, however, shows the effect of the double branch very strikingly. As the inductance $L_0$ was increased from 0 the galvanometer deflection increased rapidly near the resonance point and then suddenly dropped to a smaller value. If the inductance was decreased at the point *C*, instead of being increased, the deflection rose along the line *CD* to approximately the same height as before, then again dropped and a decrease of inductance then caused the curve to travel back along its former path. These two branches were nearly symmetrical with respect to an ordinate drawn through *P*. The portions *PB* and *PD* exhibited all the properties of unstable motion. That is, if in any way the capacity of the coil was momentarily increased — by a person passing close to it, for example — the reading of the galvanometer, if on *PD*, would rise to *D*, fall to *A* and rise again, but on the lower branch *AP*. These points *B* and *D* evidently represented those values of the current in the coil that drew out enough energy from the circuit to destroy the arc momentarily. For if the energy in the arc circuit was decreased, the heights of these maxima were diminished. A Geissler tube held at the end of the coil became warm, and in this case also the maxima were lessened. Continuing on out we get another rise in the curve at 112 turns, shown at the right in Fig. 6. As might be expected this curve has a simple rounded top and no double values.

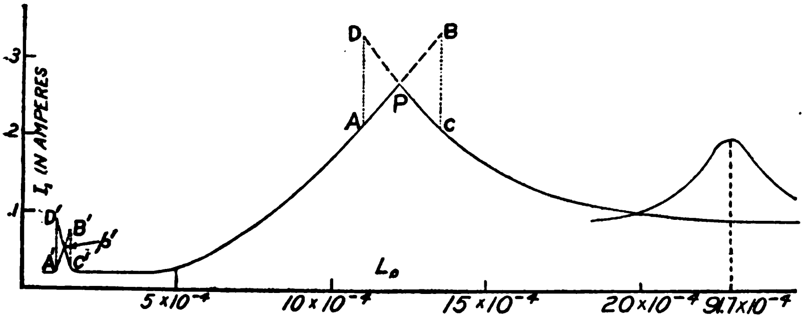


Fig. 6.

If we equate the expression for $n_2$ to that for $n_3$, that is, if we neglect the effect of the coil on the frequency of the primary, we get the equation

$$L_0 z^2 = \frac{K_c L_c}{K_0} = 3.054 \times 10^5,$$

the same equation as obtained from (24) by considering $Kl = 0$: that is, by regarding the coil as unable to take any current from the circuit. This is then the equation of the primary alone for changes of $L_0$ and $z$. The plot of this equation has been drawn in in Fig. 4, and the corresponding points of Fig. 6 designated.

The difference between the value of $z$ for this curve $N_3$ and for the actual curve, for the same value of $L_0$ is proportional to the effect of the antenna on the frequency in the primary. We should then naturally expect the values of $z$ for the breaking points $D$ and $B$ to differ from the values of $z$ at the points $D'$ and $B'$ where these inductances are cut by the $N_3$ curve, by approximately the same amount. As, however, the arc persists longer at some times than at others, we cannot expect the break to come always at the same value of the current entering the coil. But at the point $p$ there is the same tendency to stay on one branch as on the other. Accordingly, that point of the curve $N_3$ having the inductance of the point $p$ should have a value of $z$ half way between those of the two branches of the true curve. As measured from Fig. 4 the value of

$$\frac{p_2 p_1}{p_1 p_3} = 1.07.$$

This is as close to unity as we could expect, considering the changes in the strength of the oscillations as mentioned above.

Since in Fig. 4 the slope of the curve is approximately the same at $p_2$ and $p_3$, and, since the slope of the left branch immediately below $p_2$ increases with decrease of $L_0$ about as fast as the slope of the other branch just above $p_3$ increases with increase of $L_0$, we should expect the two branches of Fig. 6 to be nearly symmetrical just above the point $p$. This proves to be the case. The graph of the tangent is shown in Fig. 4*b*.

From equation (24) we see that $z$ for the branch $AB$ (Fig. 4) must always be greater than the resonance point $\pi/2$ by a finite quantity $z' = -K_c/K_0 \cdot \tan z - \pi/2$, depending only on the capacities. That is, the current into the antenna will never become infinite for the fundamental on this branch, but will approach a finite value as the inductance is increased. Instead then of the two branches of Fig. 6 rising symmetrically, we should expect the branch $CD$ to run up asymptotically to the axis of ordinates as $L_0$ decreases to 0, and as $L_0$ increases indefinitely, the branch $AB$ to turn after a time and approach a large value of the current as an asymptote. In order to show this property of the $AB$ branch we must make $z' + \pi/2$ greater than $z$ at the breaking point $B$ — in this case 1.687; or the value of $K_c/K_0$ would have to be as great as $1/5.084$. As this ratio could not be obtained in the case in hand without diminishing $K_0$ to 220 cm. — below the amount necessary to maintain the arc — no such asymptote could be found. The smaller $K_c/K_0$, the nearer the curve approaches the curve $N_3$ except at the resonance points, and when $K_c/K_0 = 0$ the curve coincides with $N_3$ as already stated.

The region of the first harmonic is shown in Fig. 4*a* on a different scale. Here the curvature is much sharper than before, and since the curve is also a great deal flatter, a small change of inductance near the first harmonic will make a very much larger change in $z$. That is, the resonance curve should be steeper for the first harmonic than for the fundamental. Fig. 6 shows this to be so. In this case also the point of intersection of the two branches should have a value of $z$ about half way between those of the curve for like inductance. Here

$$\frac{p_2'p_1'}{p_1'p_3'} = 1.0.$$

The branches above $p'$ are as before.

If we start with a very large value of $L_0$, the coil decreases the natural period of the condenser circuit by a greater and greater amount as we decrease $L_0$ until the curve breaks, and from the break to the point $z = \pi$ the coil accelerates the primary. When $z = \pi$ the curve $N_3$ crosses the actual curve. This corresponds to a wave-length of twice the coil length and in such a case the current into the coil ($I_1$) is zero. For decrease of $L_0$ this procedure is duplicated again and again for each addition of $\pi$ to the value of $z$.

Fig. 7 shows the resonance curve for the coil coupled inductively — the maxima being very sharply defined because of the loose couplings. The second harmonic was detected but not measured. No maxima beyond the fundamental were observed in this case.

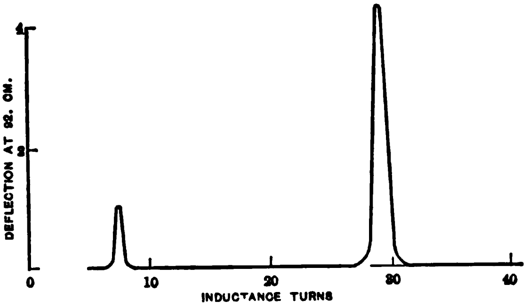


Fig. 7.

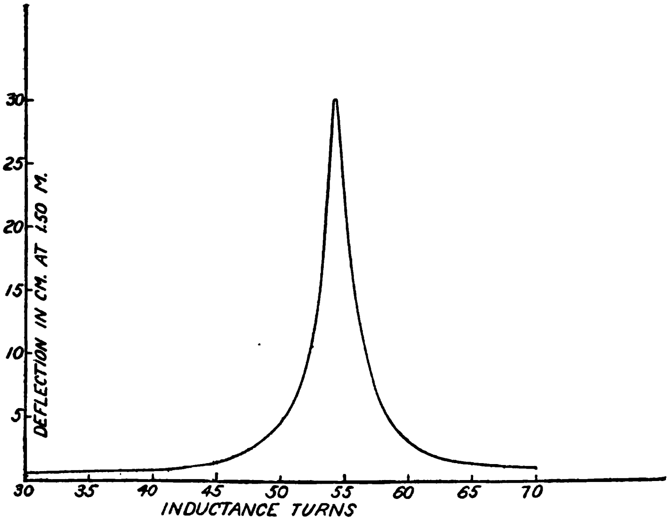


Fig. 8.

Fig. 8 is the resonance curve for the long wire antenna fastened to the upper end of $L_1$. No maxima could be detected excepting for the fundamental.

Fig. 9 is the plot of the inductance and current when a small capacity is inserted between the galvanometer and the condenser circuit, thus constituting electrostatic instead of electromagnetic couplings, the distance between plates being 0.5, 1, 2, 4 and 8 cm., respectively, for curves *I, II, III, IV* and *V*. The double maxima are very strongly brought out in this case.

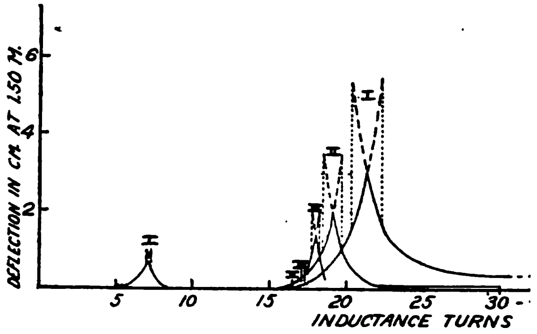


Fig. 9.

## SUMMARY.

First, the curves between $L'_0 + L'$ and $K_0$ for resonance with the coil and antenna seem to show definitely that, for these frequencies and currents, the product of the inductance and capacity is a constant.

Second, within the limits of error of the experiment, the current in the Poulsen arc as here used is of the sine form,—a simple harmonic vibration, otherwise the above theory would not have been confirmed.

Third, there is good reason to believe that, in a Poulsen arc circuit unattached to any other system, the frequency is given by the ordinary formula

$$n = \frac{3 \times 10^{10}}{2\pi\sqrt{KL}}.$$

In conclusion, I wish to thank Professor Webster for great kindness and assistance during the course of the work, and the Trustees of Clark University for the means of carrying it out.

## NOTE.

The above experiments were completed over a year ago, but owing to the author's absence in Europe the publication of the results has been delayed. Meanwhile a paper by Mr. G. W. Nasmyth[1] has appeared, in which he gives a formula for the frequency of the oscillating arc

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{\left(R - \frac{c + ld}{A}\right)^2}{4L^2}},$$

where $A$ is the current through the arc, $l$ the arc length and $c$ and $d$ constants depending on the electrodes used. When $l = 0$ the second term under the radical is supposed negligible in comparison with the first and we get the equation used in the present work. Since $l$ is never 0, $n$ will be less than the frequency given by the simple formula

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}.$$

This probably accounts for the difference between $n_3$ and $n_4$ in the inductively coupled cases.

---

[1] George W. Nasmyth, "The Frequency of the Singing Arc," PHYS. REV., 27, p. 117, 1908.

# PROCEEDINGS

OF THE

# AMERICAN PHYSICAL SOCIETY.

## MINUTES OF THE FIFTIETH MEETING.

THE annual meeting of the Physical Society was held in Boston, Mass., beginning on Tuesday morning, December 28, 1909, and ending Thursday evening, December 30, 1909. The sessions were held in Room 22, Walker Building, Massachusetts Institute of Technology, except on Wednesday, December 29, when the society met at Cambridge in the Jefferson Physical Laboratory of Harvard University.

All the sessions were joint sessions with Section B of the American Association for the Advancement of Science, and during the afternoon session of Tuesday, December 28, Section A of the American Association also met with these two organizations. On this occasion the presiding officers were Vice-President L. A. Bauer of Section B and Vice-President E. W. Brown of Section B. For the other sessions either President Crew of the Physical Society or Vice-President Bauer of Section B acted as presiding officer.

On Wednesday, December 29, tellers being duly appointed, the ballots for the annual election of officers were counted. The vote resulted in the election of the following officers for the year 1910:

*President*, Henry Crew; *Vice-President*, W. F. Magie; *Secretary*, Ernest Merritt; *Treasurer*, J. S. Ames; *Members of the Council*, Karl E. Guthe, C. A. Skinner.

At the joint session Tuesday afternoon, January 28, the program was as follows:

Address of the retiring Vice-President of Section B, Karl Guthe: "Some Reforms needed in the Teaching of Physics."

On the Determination of Latitude and Longitude in a Balloon. C. RUNGE.

The Ruling of Diffraction Gratings. A. A. MICHELSON.

On Certain Physical Hypotheses Sufficient to Explain an Anomaly in the Moon's Motion. E. W. BROWN.

Following is a list of the other papers presented at the meeting:

1. Temperature Coefficient of Electrical Resistance. Tungsten, Molybdenum, Nickel and Nichrome. A. A. SOMERVILLE.

2. The Flow of a Gas through a Capillary Tube. WILLARD J. FISHER.

3. Effect of Surface Tension upon a Falling Jet of Water. F. R. WATSON.

4. The Variation of the Hall Effect in Metals with Change of Temperature. ALPHEUS W. SMITH.

5. The Effect of Pressure on the Electrolytic Rectifier. A. P. CARMAN and G. J. BALZER.

6. The Analysis of the Principal Mercury Lines by Diffraction Gratings, and a Comparison with the Results Obtained by Other Methods. HENRY G. GALE and HARVEY B. LEMON.

7. The Spectra of Some Gases in the Region of Extremely Short Wave-length. THEODORE LYMAN.

8. The Variation of the Hall Effect with the Temperature in the Case of the Principal Magnetic Metals. THOMAS C. MCKAY.

9. The Rectifying Effect in Point and Plane Discharge. ROBERT F. EARHART and CHAS. H. LAKE.

10. Photographic Photometry, and Some Interesting Photographic Phenomena. CHARLES F. BRUSH.

11. Note on "Changes in Density of the Ether, and Some Optical Effects of Changes in Ether Density." CHARLES F. BRUSH.

12. The Tone Quality of the Flute. D. C. MILLER.

13. An Instrument for Projecting and Recording Sound Waves. D. C. MILLER.

14. The Magnetic Measurements on Board the "Carnegie" in 1909. L. A. BAUER.

15. The Relativity Dilemma. D. F. COMSTOCK.

16. "Bound Mass" and the Fitzgerald-Lorentz Contraction. WILL C. BAKER.

17. Physical Properties of Binary Liquid Mixtures. J. C. HUBBARD.

18. On the Use of Polar Coördinates in Thermodynamics. J. C. HUBBARD.

19. The Theory of the Vibration Galvanometer. F. WENNER.

20. Coefficients of Linear Expansion at Low Temperatures. H. G. DORSEY.

21. The Freezing of Mercury at High Pressures. P. W. BRIDGEMAN.

22. Phenomena of Spark Discharge through Wire Conductors. FRANCIS E. NIPHER.

23. Some Minute Phenomena of Electrolysis. H. W. MORSE.

24. A New Method of Measurement of Small Angles. C. W. CHAMBERLAIN.

25. The Photographic Evidence for Dispersion of Light in Space: — Is it a purely Photographic Phenomenon? H. E. IVES.

26. On the Secondary $\beta$ Radiation from Solids, Solutions, and Liquids. S. J. ALLEN.

27. The Effect of Filter Paper upon the Mass and Form of the Deposit in the Silver Coulometer. E. B. ROSA, G. V. VINAL, and A. S. MCDANIEL.

28. Experiments in Impact Excitation with the Lepel Singing Arc. GEORGE NASMYTH.

29. On the Coefficients of Diffusion of the Emanation and the Active Deposit Particles of Actinium. J. C. MCLENNAN.

30. On the Relative Numbers of Positive and Negative Ions present in Atmospheric Air. A. THOMSON.

31. Note on the Cause of the Discrepancy between the Observed and Calculated Temperatures after Expansion in the Space between the Plates of a Wilson Expansion Apparatus. R. A. MILLIKAN, E. K. CHAPMAN, and H. W. MOODY.

32. Some New Values of the Positive Potentials assumed by Metals under the Influence of Ultra-violet Light. R. A. MILLIKAN.

33. The Second Order Effect of Ether Drift on the Intensity of Radiation. A. TROWBRIDGE and C. E. MENDENHALL.

34. The Rotary Dispersion of Quartz at — 190° C. and Observations at other Temperatures. F. A. MOLBY.

35. The Pyrheliometric Scale and the Solar Constant. C. G. ABBOT.

36. Single-line Series in the Spectra of Ca and Sr. F. A. SAUNDERS.

36. The Relative Motion of the Earth and the Ether. H. A. WILSON.

37. A Study of the Multiple Reflection of Short Electric Waves between two Reflecting Surfaces. L. E. WOODMAN AND H. W. WEBB.

38. A Hot Air Engine Indicator Diagram. A. G. WEBSTER.

39. The Nitrogen Thermometer from Zinc to Palladium. A. L. DAY and R. R. SOSMAN.

40. On Calcium Clouds in Space. DR. STIFER (presented by Percival Lowell).

41. The Second Postulate of Relativity. R. C. TOLMAN.

42. The Terminal Velocity of Fall of Small Spheres in Air. (By title.) JOHN ZELENY and L. W. MCKEEHAN.

43. The Present State of our Knowledge concerning Permanent Magnetism. (By title.) A. A. Knowlton.

44. The Heat of Dilution of Aqueous Salt Solution. (By title.) F. L. BISHOP.

45. Uranous and Uranyl Bands. A Very Fine Band Absorption Solution Spectrum. (By title.) W. W. STRONG.

46. Insulation of Observatory Domes for Protecting Telescopes and other Apparatus against Extremes of Heat and Cold. (By title.) DAVID TODD.

47. On the Free Vibrations of a Lecher System. (By title.) F. C. BLAKE and CHAS. SHEARD.

48. Thunderstorm Electricity. (By title.) W. W. Strong.

Ernest Merritt,
*Secretary.*

## Report of Treasurer, December 1, 1909.

### Receipts.

| | |
|---|---|
| William Hallock, Treasurer | $ 144 00 |
| Dues for 1908 and before | 29 50 |
| Dues for 1909 | 2,957 45 |
| Payments for Section "B," Science Abstracts | 48 00 |
| Payments for Section "B," for 1910 | 2 00 |
| Life Membership | 50 00 |
| Entrance Fees | 45 00 |
| Interest on Deposit | 11 11 |
| Deposit in Bank, January 1, 1909 | 114 44 |
| Total | $3,401 50 |

### Disbursements.

| | |
|---|---|
| Allowance to Office of Secretary | $ 91 67 |
| Allowance to Office of Treasurer | 50 00 |
| Subscriptions to Physical Review | 1,554 00 |
| Subscriptions to Science Abstracts "A" | 768 00 |
| Subscriptions to Science Abstracts "B" | 50 00 |
| Stationery and Printing, Orders of Secretary | 190 98 |
| Stationery and Printing, Orders of Treasurer | 50 95 |
| Stationery and Printing, A. A. A. S. | 4 00 |
| Mileage of Secretary | 80 27 |
| Mileage of President | 88 38 |
| Incidentals, Office of Treasurer | 9 48 |
| Incidentals, Office of Secretary | 12 50 |
| Deposit in Bank, December 1, 1909 | 451 27 |
| Total | $3,401 50 |

J. S. Ames,
*Treasurer.*

We certify that we have audited these accounts and that we have found them correct.

J. B. Whitehead,
December 15, 1909.
Wm. J. A. Bliss,
December 16, 1909.

## THE POROUS-PLUG AND FREE-EXPANSION EFFECTS IN AIR AT VARYING PRESSURES.[1]

BY A. G. WORTHING.

A STUDY of the high values of the ratio of the two heat capacities of air obtained by Witkowski and by Koch has led to the derivation of the formula

$$\gamma = 1 + \frac{P}{c_v}\frac{\partial v}{\partial \theta} + \frac{\eta \frac{\partial P}{\partial \theta}}{1 - \eta \frac{\partial P}{\partial \theta}}, \tag{I.}$$

where $\eta$ is the free-expansion effect. This differs from the common expression for $\gamma$ derived for a perfect gas by the addition of the last term of the right-hand member of the equation. The above expression combined with another thermodynamic formula gives

$$\eta = \frac{\theta - P\frac{\partial \theta}{\partial P}}{\frac{\gamma}{\gamma - 1}\,\theta\frac{\partial P}{\partial \theta} - P}. \tag{II.}$$

The combination of two well-known formulæ gives

$$\mu = \frac{\gamma - 1}{\gamma}\,\frac{\theta - v\frac{\partial \theta}{\partial v}}{\theta\frac{\partial P}{\partial \theta}}, \tag{III .}$$

where $\mu$ is the porous-plug effect. Witkowski's values for the product $Pv$ for air at various pressures and temperatures and Koch's data on $\gamma$ when combined by means of (II.) and (III.) give values of $\mu$ and $\eta$ for varying pressures at 0° C. and at $-79.4°$ C. The maximum value of $\mu$ obtained thus for air at 0° C. is about .85 of the value obtained from the experimental results of Joule and Kelvin. The curves obtained show that $\mu$ and $\eta$ decrease with increasing pressure after a certain pressure is reached, a result which is to be expected from a consideration of (II.) and (III.). Theoretically, (II.) and (III.) also shows that, beginning with small pressures, $\mu$ and $\eta$ approach zero as the pressure approaches zero. This fact should have an important bearing on the determination of the relations existing between gaseous thermometric scales and the absolute scale of temperature.

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

## A NEGLECTED TYPE OF RELATIVITY.[1]

BY D. F. COMSTOCK.

THE negative result of all experiments to detect the earth's motion through space by its effect on terrestrial phenomena positively requires explanation, and the modern principle of relativity is generally conceded to be the most successful attempt to explain so far made. The principle has, however, led to great complexity in detail and its implications have not yet been completely worked out.

It does not seem to have been generally noticed in this connection that if the velocity of light is assumed to depend on the velocity of the source, in the same way in which the velocity of a bullet depends upon the velocity of the gun, then we have a simple type of relativity which in many ways presents a striking contrast to the principle now in vogue.

Such a light-velocity is not obviously consistent with the conception of light as a wave motion in a semi-material medium, but if there is anything that modern discoveries in fundamental physics have shown, it is that we must not reason too much from analogy or expect to find the most fundamental of all transformations of energy to resemble those transformations which we are familiar with in complex material systems.

The assumption that the velocity of light depends on that of the source has, so far as the author is aware, never been properly examined. This is strange, but is probably explainable as a natural result of the complete trust which has been put for years in the conception of an ether.

In the paper of which this is an abstract, the astronomical consequences of the assumption that the velocity of light depends on the source-velocity, in the same way that the velocity of a bullet depends on that of the gun, has been partially developed. It is found that a change in the line-of-sight-component of a star's velocity causes the time interval between any two events which happen on the star to be different from the *apparent* time interval between the same events as observed from the earth. If the star is very distant and changes its velocity as in the case of a binary star, it is even possible that two events may appear to happen in their reverse order.

In the case of spectroscopic binaries certain characteristic peculiarities should appear in the spectrum, peculiarities which apparently do not exist. The investigation is only just begun but so far the indications are rather against the existence of the effect.

---

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

## TEMPERATURE COEFFICIENTS OF ELECTRICAL RESISTANCE.[1]

By A. A. Somerville.

THE range of temperature through which the work extends is from 0° C. to about 1060° C.; limits well within the range of the platinum resistance thermometer. The temperatures are determined by means of such a thermometer calibrated at melting ice, steam and the melting points of copper in air and in nitrogen. It is believed that at points near 1000° C. readings are correct to within one or two degrees. Temperatures are automatically recorded by a Callendar recorder and Callendar's platinum resistance curve is taken as a standard and the coefficients of other metals referred to that as a standard.

Specimens are placed in a quartz tube similar to the thermometer container and the two placed inside a porcelain tube which is the core of a resistance furnace. Using a dial bridge the resistance of the unknown is measured to the fourth decimal place.

Four materials have been studied: nickel and nichrome, two good furnace wires inasmuch as they do not oxidize readily and tungsten and molybdenum, which must be protected from oxygen when heated. The nickel and nichrome curves are very similar in shape, but the coefficient of the former is twenty times that of the latter. The coefficients in either case are relatively small between 400° C. and 1000° C. as compared with the coefficients outside these limits. The tungsten and molybdenum curves are slightly convex toward the temperature axis. The coefficients are large: 1 ohm of molybdenum at 0° C. measuring 5.7 ohms at 1000° C. and 1 ohm of tungsten at 0° C. measuring 6.9 ohms at 1000° C.

It is intended to extend the work to a large number of substances.

## THE SPECTRA OF SOME GASES IN THE REGION OF EXTREMELY SHORT WAVE-LENGTH.[1]

By Theodore Lyman.

THE paper deals with experiments on the spectra of oxygen, nitrogen, carbon monoxide, carbon dioxide, helium and argon. The results confirm the statements which the writer has already made on this subject: Oxygen, nitrogen and helium give no spectra in the region between $\lambda 1850$ and $\lambda 1250$ of sufficient strength to be observed with the writer's spectroscope.

Carbon monoxide and carbon dioxide give the same spectrum. The greater part of the present paper is devoted to the measurement of the bands of this spectrum. The laws of Deslandres are applied to the data

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28-31, 1909.

with the result that the carbon bands in the region of the extreme ultra-violet appear to form a continuation of the scheme proposed by Deslandres for the bands between $\lambda 2000$ and $\lambda 3000$.

It is hoped that the values of the wave-lengths given in this paper may prove of use to those working in this region.

## THE FLOW OF A GAS THROUGH A CAPILLARY TUBE.[1]

BY WILLARD J. FISHER.

IN a previous article[2] there was proposed as a second approximation for the equation representing the flow of a gas in a capillary tube the equation

$$F=\frac{\pi a^4}{8\eta l p^{\frac{1}{n}}\left(1+\frac{1}{n}\right)}\left(1+\frac{4\zeta}{a}\right)\left(p_1^{1+\frac{1}{n}}-p_2^{1+\frac{1}{n}}\right)$$

which is based upon the equation of expansion

$$p=k\rho^n;$$

and it was shown that $n$ must lie between the limits 1 and $\gamma = C_p/C_v$. If $n = 1$, the equation reduces to the well-known form of O. E. Meyer.

Assuming that all the quantities entering into the equation have been experimentally determined, with the exception of $\eta$ and $n$, then $\eta$ is a function of $n$, and

$$\frac{\partial\eta}{\partial n}=\text{const.}\times\frac{-1}{(n+1)^2}\left[\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}\left(\frac{1+1/n}{n}p_2+p_1\right)-p_2\right]$$

if $p$ is taken equal to $p_2$. This is negative for all the possible values of $n$, hence $\eta$ diminishes with increasing $n$, and vice versa.

In the use of O. E. Meyer's equation $n$ is taken equal to 1, while its true value is certainly greater. Hence the deduced values of $\eta$ are certainly too large, and the value of $C/K$ too small.

Further, gases with large values of $\gamma$, like helium or air, allow $n$ a wider range of variation than those with a small value, like $CH_3Cl$; hence it is probable that $n - 1$ is actually larger for high values of $\gamma$ than for small; and the computed values of $C/K$ based upon $n = 1$ would then be progressively too small as $\gamma$ increases — which is apparently the fact.

It is not probable that $n$ is actually constant throughout a capillary tube. The expansion is probably nearly isothermal in the neighborhood of the walls, and less so in the central stream, so that $n$ is a function of position.

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

[2] PHYS. REV., 29, p. 147, 1909.

## EFFECT OF SURFACE TENSION UPON A FALLING JET OF WATER.[1]

BY F. R. WATSON.

IF a stream of water flows from the lower end of a vertical tube the cross-section of the stream gets less and less due to the contractive force of surface tension until finally the stream is broken into drops. If, however, an obstacle is placed in the path of the water near its exit from the tube the form of the stream is changed. It now appears as a series of beads of water connected together, the size of the beads diminishing from the top down.

Although no mathematical solution is offered at this time, a simple explanation may not be out of place. The obstacle may be thought of as sending a train of waves *upward* along the surface of the jet, in the same way that a vertical fishline in a moving stream sends ripple waves against the current. The wave form is stationary, and indicates that the velocity of the ripple wave is equal and opposite to that of the current at every point. In the case of the vertical jet the velocity of the water increases as it falls, so that the wave-length gets shorter and shorter the further the stream falls.

The jets were produced by allowing water to fall from the end of a siphon on to a glass sphere. It is necessary that the flow be symmetrical on the sphere, otherwise the jet will be pulled away from the vertical. The head of water pressure is kept constant by having the water supply in a dish of large diameter which is continuously replenished by a regulated stream of water. The difference in levels between the water in the supply dish and the end of the siphon was less than a centimeter.

## SINGLE-LINE SERIES IN THE SPECTRA OF CA AND SR.[1]

BY F. A. SAUNDERS.

THE "combination principle" discovered by Ritz leads us to expect to find single-line series in the spectra of Ca and Sr, related to the strong ultra-violet series announced a year ago.[2] If we take the number of waves per cm. in vacuo for each line of the two ultra-violet series and subtract from each value a constant (21849 for Ca, 20149 for Sr), we obtain the wave-numbers of two new series, whose approximate wave-lengths are: Ca, 5042, 4527, 4240, 4059, 3946; Sr, 4755, 4481, 4313, 4203. These lines are all faint and hazy; the last ones are particularly difficult objects, and were found only by reducing the pressure about the arc. The aspect of the lines leads one to classify these

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

[2] PHYS. REV., XXVIII., 152, 1909.

series as of the "second subordinate" type. The agreement between calculated and observed wave-lengths is within 0.1 A.U. for the first two lines; 0.5 A.U. for all.

The blue and violet regions of these spectra contain other single lines, most of which are seen to belong to companion series, running to the same end as the above. The approximate wave-lengths are: Ca, 4878, 4355, 4109, [3973], 3890; Sr, 5156, 4678, 4406, 4254. The Ca line at 3973 was not observed on account of the nearness of the very much stronger triplet-line 3973.89. According to the aspect of the lines, these series are of the "first subordinate" type. They are very diffuse unless the pressure about the arc is greatly reduced. They show a large "pressure shift"; that is, their density maximum with arc in air is not in the same place as when the arc is under reduced pressure (measured with reference to unaffected lines). All these series are "arc" lines, diminished in strength by reduction of pressure and appearing with reduced intensity in the spark.

The new "first subordinate" series can be satisfactorily represented by three constant formulæ almost of the Ritz type. The ultra-violet series and the "second subordinate" series (which have the same formula except for the end-constant) are more difficult, but can be represented by four-constant formulæ. The two latter might, perhaps, at first sight be regarded as a single series of very wide pairs; this seems impossible, however, when the utterly different appearance of the lines is considered, the ultra-violet ones being strong and heavily reversed; the others weak and not reversed. Their relations, doubtless very intimate, will be shown by means of their formulæ.

## COEFFICIENTS OF LINEAR EXPANSION AT LOW TEMPERATURES.[1]

By Herbert G. Dorsey.

THIS is a continuation of work already published.[2] The specimen to be examined is in the form of a hollow cylinder and its changes in length are measured while in vacuo by means of interference bands of green mercury light. The coefficients are measured for 40° intervals between + 20° and − 180° C.

Curves were given for eight specimens of iron-carbon alloys ranging from 0.058 per cent. carbon to 1.38 per cent. At 0° C. the coefficient decreases steadily from $1150 \times 10^{-8}$ for 0.058 per cent. carbon to a minimum of $1055 \times 10^{-8}$ for about 1.25 per cent. carbon and then increases. At lower temperatures no such regularity exists but at − 140° the coeffi-

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

[2] Phys. Rev., Vol. XXV., p. 88, 1907, and Vol. XXVII., p. 1, 1908.

cients have decreased to $600 \times 10^{-8}$ or less. The average coefficient between + 20° and — 180° is nearly a straight line between $930 \times 10^{-8}$ for carbonless iron and $840 \times 10^{-8}$ for 1.4 per cent. carbon, by extrapolation.

Curves and data were also given for fused silica. Besides the many remarkable properties of this material, a thick rod of it if not annealed is in a state of strain and a diamond scratch will produce a crack clear around the rod. Three specimens were tested, both in the annealed condition and after being quenched in water from a yellow heat. Curves plotted from the average values in the two states are nearly parallel, the curve of the quenched values being about $60 \times 10^{-9}$ below that of the annealed. The actual values of the coefficients being $+393 \times 10^{-9}$ at 0° and $-661 \times 10^{-9}$ at — 160° for the annealed while corresponding values for the quenched are $+297 \times 10^{-9}$ and $-715 \times 10^{-9}$.

## SOME MINUTE PHENOMENA OF ELECTROLYSIS.[1]

By H. W. MORSE.

IF pure water is electrolyzed between silver electrodes of small area at a voltage of 1.5 to 2.5 volts no gas appears at the electrodes. Instead a cloud is sent out from the anode and this moves along the current lines to the kathode, where metallic silver begins to separate. Very slight quantities of electrolytes prevent the phenomenon completely, as low as 0.0003 normal for some salts. The cloud appears under higher power as made up of approximately spherical particles, probably of silver oxide, in rapid Brownian motion. On coagulation, the particles collect in bunches which are frequently symmetric along three axes.

## THE PHOTOGRAPHIC EVIDENCE FOR DISPERSION OF LIGHT IN SPACE — IS IT A PURELY PHOTOGRAPHIC PHENOMENON?[1]

By HERBERT E. IVES.

M. TIKHOFF[2] found on star photographs taken through colored glasses that the faint stars were more distinct on the negatives taken with light of longer wave-length. He argued that this proved dispersion or scattering of light in space. The faint stars are presumably, on the whole, more distant, and their light would be more subject to scattering. The scattering would be greater the shorter the wave-length.

There is involved in this argument the assumption that the scale of gradation of the photographic plate is the same for all wave-lengths.

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

[2] Comptes Rendus, 148, p. 267, 1909.

According to Abney and Eder this is not true. According to Leimbach it is true for normal exposure and the one mode of development used in his investigation.

Two phenomena of the photographic plate have been investigated with this particular question (space dispersion) in view. First, is the fact that a "red" image has been observed to develop up more slowly than a "blue," though giving the same final density. Second, in a plate sensitized by bathing the sensitive layer is very thin. Either of these conditions might result in "red" and "blue" plates being exposed in different parts of their characteristic curves, or having different characteristic curves, although developed in the same way to the same density.

Simultaneous red and blue exposures of gradually increasing length were made on three types of plates. First, an isochromatic plate, not very sensitive to red, in which the sensitizer was incorporated in the emulsion. Second, a plate sensitized to red by bathing with pinacyanol. Third, a Cramer "spectrum" plate, sensitizer in the emulsion, quite sensitive to red. Long and short development were tried with all.

The results demonstrated that under laboratory conditions, where there is no question of space dispersion, either one of three conditions may be exhibited by the photographic plate. The scale of gradation may be steeper for red, for blue, or the same for each. The determining factor is apparently the relative thickness of sensitive film. Where it is necessary to depend on long development to obtain the full density of the red image a thicker layer of emulsion is used than for the blue, owing to the opacity of the silver bromide to blue light. Hence, it is possible to have a longer range of gradation in the red image. When the red sensitive layer is thin, as in the bathed plate, the blue image is thicker and possesses (on long development) a greater range of densities. When the red sensitive layer is thick, and its effective sensitiveness near that of the blue, both images may be of nearly the same thickness and gradation.

It is, therefore, possible to obtain, on bathed plates, from purely photographic causes, the phenomenon observed by Tikhof. The photographic evidence for dispersion in space is, therefore, inconclusive, unless it be first proved by laboratory test that the plates as used cannot themselves be responsible for the effect observed.

## THE ROTATORY DISPERSION OF QUARTZ AT — 190° AND OBSERVATIONS AT OTHER TEMPERATURES.[1]

BY F. A. MOLBY.

THE work described inthis paper is an extension of the work with rotatory quartz at low temperature, the method of which was

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

described in the Physical Review, January, 1909. An effort has been made to obtain direct measurements with accuracy; the Lippich polariscope has been used, with several light sources covering the range of the visible spectrum. Two plates of L and two of R quartz, aggregating 13.258 mm. have been studied by the writer. The measurements made at room temperature agree closely with the published values for the specific rotation of quartz, the L quartz also agreeing with the R quartz to one part in 2,000 at all temperatures used. The writer finds that on reducing the temperature of the quartz from 24° to — 188° the rotation is diminished by the proportionate parts, 10/400, 10/427, 10/424, 10/427, 10/429 and 10/429 respectively for the wave-lengths 435.9, 491.6, 546.1, 579, 589.25 and 670.8 $\mu\mu$. Becquerel reports[1] that by a photographic method he found a decrease on cooling from room to liquid air temperature, of approximately 1/43 for various wave-lengths. Curves showing the way in which the rotation depends upon temperature over the range of 212° C. were shown. Curves showing the relation of rotatory power to temperature for the volatile oil, limonene, were also shown. Similarities are noted in the effect of temperature upon the rotatory power of the two substances, though cooling has the effect of increasing, not diminishing, the rotatory power of the limonene.

## The Theory of the Vibration Galvanometer.[2]

By F. Wenner.

A SOLUTION of the fundamental galvanometer equation is obtained giving the amplitude of the vibration in terms of the amplitude and frequency of the current and the intrinsic constant of the instrument.

Attention is called to the electromotive force developed by the relative motion between the magnet and winding of a galvanometer, and it is pointed out that the energy available for producing a deflection is the time integral of the product of the current and the generated or back electromotive force. In the vibration galvanometer the power available for maintaining the vibration is the product of the current and back electromotive force into the cosine of their phase angle. This back electromotive force is expressed in terms of the amplitude and frequency of the vibration and one of the intrinsic constants.

Taking for the current the vector sum of the impressed and back electromotive forces divided by the resistance of the galvanometer circuit, a relation is obtained giving the amplitude of the vibration in terms of

---

[1] Comptes Rendus, December 14, 1908.

[2] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28-31, 1909.

the effective value and frequency of the impressed electromotive force and the intrinsic constants of the instrument.[1]

## THE RECTIFYING EFFECT IN POINT AND PLANE DISCHARGE.[2]

BY ROBT. F. EARHART AND CHAS. H. LAKE.

THE authors have sought to make quantitative measurements on the rectifying effect produced in the point to plane discharge.

A discharge chamber into which various gases could be introduced was used. Suitable connections with air-pump and McLeod gauge permitted accurate measurements of pressure within the chamber.

The electrodes consisted of a pointed Pt wire and a brass disk 3.8 cm. in diameter. The distance between the electrodes was varied from 2.5 mm. to 2 cm.

With point negative a lower potential difference sufficed to produce a discharge than when the point was made positive.

Families of curves show graphically the discharge potential under various conditions of distance, pressure and polarity.

Measurements of the potential difference to produce discharge were made with a Weston voltmeter. The results show that within the limits of the experiment the most favorable conditions for rectification in air occurs at a pressure of about 1 mm. Hg and a distance of 2 cm. between the electrodes.

## THE FREEZING OF MERCURY AT HIGH PRESSURES.[2]

BY P. W. BRIDGMAN.

WHEN a metal solidifies it suffers in general a discontinuous change in its electrical resistance. Mercury is enclosed in a fine glass capillary and the resistance measured at various temperatures as a function of the pressure. Freezing is indicated by a sudden decrease in the resistance to about one quarter of its value for the liquid. In this way the freezing curve has been followed from — 15° C. to + 21.5° C., the pressure required to produce solidification at 21.5° being almost exactly 12,000 kgm. per $cm^2$. Furthermore, over this range the curve is linear with a probable error of less than $\frac{1}{10}$ per cent., the line passing through the freezing point at atmospheric pressure. The increment of pressure per degree rise of the freezing point is 198.7 kgm./$cm^2$. The value calculated from Clapeyron's equation is 197, taking as the heat of freez-

---

[1] The theory of the vibration galvanometer is discussed fully in a paper being published in Vol. 6 of the Bulletin of the Bureau of Standards.

[2] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

ing 2.82 cal., and as the change of volume on freezing 0.00260. The agreement is within the errors of the determination of these last two quantities.

THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

## THE EFFECT OF PRESSURE ON THE ELECTROLYTIC RECTIFIER.[1]

BY A. P. CARMAN AND G. J. BALZER.

A LARGE number of oscillograph curves were photographed of the alternating current through different electrolytic rectifiers at pressures from 15 centimeters of mercury up to 21 atmospheres and at different temperatures. These curves show that at the higher pressures the rectifying property decreased so that at 21 atmospheres a cell with aluminum-carbon electrodes and alum for electrolyte showed about half rectification. The property of rectification was regained when the pressure was reduced. Increase of temperature is also shown to act similarly to increase of pressure, but the cell does not recover its rectifying properties when the temperature is lowered again. Observations were also made on the deposits formed on the electrode. The bearings on the theories advanced were briefly discussed. Slides of the oscillograms and readings of currents and electromotive forces were shown.

## ON THE SECONDARY $\beta$ RADIATION FROM SOLIDS, SOLUTIONS AND LIQUIDS.[1]

BY S. J. ALLEN.

THIS paper is a continuation of the work published under the same title in the PHYSICAL REVIEW for September, 1909. The main facts brought out in that research may be summarized as follows:

1. *Solids.* — The results obtained for the pure elements agree in general with those obtained by other investigators, and show that the secondary radiation is some function of the atomic weights, increasing with increase of atomic weight. The penetrating power of the rays does not differ very much for the different atoms, but was nearly a constant for the angle of incidence used (about 70°).

The secondary rays from salts, and compound substances, such as sugar, do not in general bear any definite relation to the sum of the atomic weights (molecular weights), but the presence of an atom of high atomic weight can always be seen by an increase in the radiation.

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

2. *Aqueous Solutions.* — The secondary rays from solutions increase with the concentration of the solute and for light atoms and small concentration the increase is nearly proportional to the concentration, but for high concentrations there is a tendency towards saturation. If several solutions of equal concentrations and containing a common constituent (*e. g.*, the chlorides) are taken then the radiation increases very rapidly with the atomic weight of the other constituent. If substances such as sugar, tartaric acid, acetic acid, which have the same intrinsic radiation as the solvent, are dissolved, the radiation keeps constant for any concentration.

3. *Pure Liquids.* — The secondary radiations from pure liquids are in general very complex and surprising. No prediction can be made beforehand from a knowledge of the secondary radiation of the constituents just what the radiation from the compound will prove to be. It certainly is not a function of the molecular weight. The various compounds of C, H and O differ considerably in radiating power. There is a large group of liquids having a radiation about equal to that of alcohol, and a group like the esters and amylene which has a much lower radiation.

The presence of an atom of high atomic weight generally manifests itself by an increase of radiation though there are several remarkable exceptions. The group of halides (compounds of Cl, Br and I) show the greatest effects. Propyl bromide has a radiation of 230, while propyl chloride only has a radiation of 80. On the other hand, ethylene bromide has a radiation of 245, while ethylene chloride jumps to 284. Here the substitution of ethylene for the propyl constituent causes an enormous increase for the chloride but not for the bromide, which is certainly not due to the atoms alone. This phenomenon is characteristic of all the results, though not to such a great extent.

It seems very evident from this research that in a compound containing several atoms the secondary radiation is not a function of the molecular weight, but that there are in many cases other effects entering which may be so great as to almost entirely dominate the final result. It may be that the way in which the atoms are bound together to form molecules or the aggregation of the molecules themselves may have large influences on the radiation.

The results obtained by the author in the paper referred to above were only for one angle of incidence. Since McClelland has shown that the angle of incidence has a large effect in determining the amount of secondary radiation the author has made a fresh set of observations at different angles which are given in the present paper. Also at the same time the decrease of radiation caused by passing through a definite thickness of tinfoil was measured.

The apparatus used and the methods employed are the same as before.

The radium was placed in a block of lead which could be placed so that the primary $\beta$ rays fell upon the substance to be tested at any desired angle.

Only those substances were tested which gave marked effects, viz.: water, alcohol, esters, the halides, saturated solutions of KI, $CdI_2$, $Pb(NO_3)_2$ and $CuCl_2$, and some of the metals. A few of the results for solids, liquids and solutions are expressed in the following table. The figures there given are in arbitrary units; (*a*) representing those without, and (*b*) those through .001 mm. of tinfoil.

| Substance. | Radiation. | | | | | | % Unabsorbed Rays. | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0° | | 30° | | 50° | | 0° | 30° | 50° |
| | (*a*) | (*b*) | (*a*) | (*b*) | (*a*) | (*b*) | | | |
| Lead. | 600 | 400 | 665 | 440 | 688 | 490 | 66.7 | 66.2 | 71.0 |
| Aluminum. | 205 | 115 | 260 | 153 | 370 | 248 | 56.1 | 58.8 | 67.2 |
| Paper. | 134 | 64 | 180 | 100 | 292 | 203 | 47.7 | 55.5 | 69.5 |
| Potassium iodide. | 315 | 192 | 381 | 231 | 471 | 312 | 61.0 | 60.7 | 66.4 |
| Copper chloride. | 190 | 114 | 259 | 133 | 357 | 229 | 60.0 | 53.2 | 65.0 |
| Ethyl iodide. | 481 | 307 | 541 | — | 600 | 397 | 64.0 | — | 66.2 |
| Tetra brom. acetylene. | 388 | 257 | 481 | 281 | 530 | 347 | 66.0 | 58.6 | 65.8 |
| Ethylene chloride. | 358 | 228 | 441 | 258 | 503 | 333 | 63.7 | 58.5 | 66.2 |
| Ethylene bromide. | 323 | 207 | 382 | 223 | 493 | — | 64.0 | 58.4 | — |
| Chloroform. | 262 | 154 | 304 | 182 | 419 | 273 | 58.6 | 60.2 | 65.9 |
| Water. | 114 | 69 | 170 | 96 | 280 | 182 | 60.5 | 56.5 | 67.5 |
| Esters. | 74 | 46 | 150 | 77 | 235 | 154 | 62.0 | 51.0 | 65.6 |

We see at once from an examination of these results that the radiation from the heavy atoms does not increase very much with the angle of incidence. Lead only increases by about 15 per cent. when the angle is changed from 0° to 50°. On the other hand the radiation from the light atoms increases very rapidly when the angle is increased. From 0° to 50° paper increases by 118 per cent. The same is true for the solutions and liquids. The esters, from 0° to 50° increase by about 217 per cent.

These results agree with those obtained by McClelland for solids. It is also seen that the relative order of the substances at one angle of incidence is the same as at any other angle. Any two substances which have the same radiation at one angle will have an equal radiation at any other angle of incidence. Ethylene chloride shows anomalous results at all angles of incidence. The angle of incidence does not change the relative order of the radiating power but only the magnitude.

The penetrating power of the rays from heavy atoms at normal incidence is greater than for the light atoms, but at a large angle of incidence the penetrating power of the rays is practically independent of the atomic weight. It seems almost certain that the secondary rays are due to a scattering of the primary rays and not to a disintegration of the atom. Also, no simple hypothesis, which assumes that the radiation is a function of the atomic weights is alone sufficient to account for the observed effects in the case of compounds.

UNIVERSITY OF CINCINNATI.

## URANOUS AND URANYL BANDS. A VERY FINE BAND ABSORPTION SOLUTION SPECTRUM.[1]

BY W. W. STRONG.

UP to the present time practically no work has been done upon the absorption spectra of uranous salts. A great many of the uranous salts in solution show the uranyl bands much more sharply defined than corresponding uranyl solutions. Besides these uranyl bands there are several other bands in the green, yellow and red which never appear in uranyl solutions. Whether the uranyl bands come from any unreduced uranyl salts or from the uranous salt is uncertain. In general all uranous salts show bands very similiar to the uranyl bands and in the same region of the spectrum. These bands are affected in the same way by chemical reagents as the uranyl bands.

Uranous acetate, bromide, chloride and sulphate have been investigated spectroscopically in various solvents. The spectra are very much affected by the solvents and by the presence of free acid and the salts of other elements than uranium. The absorption spectra of various salts in the same solvent are in general very much the same, but the absorption spectra of the same salt in different solvents is very different. In many cases there are distinct water, glycerol, alcohol or acetone bands and these bands coexist in mixtures of these solvents. A change of percentage of any solvent causes the characteristic bands of that solvent to change in intensity. Usually this change in intensity is not accompanied by any change in wave-length. On the other hand the general effect of the presence of free acid or of foreign salts is to change the wave-length of the bands as well as their intensities. Quite a large number of new bands have been discovered.

When a uranous or uranyl salt is changed to another salt there is usually a very marked change in the positions and relative intensities of the uranyl bands. A very fine example is the case of uranyl nitrate in water to which hydrochloric acid is gradually added. The change of

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

wave-length of the *d* nitrate band is quite large and the change of wave-length is a gradual one. This is an interesting example where chemical changes in solution can be followed spectroscopically and this method opens up a new field for studying these changes.

Accidentally it was found that by adding sulphuric acid to a nitric acid solution, the absorption spectra of the solution consisted of large numbers of very fine bands. These bands appear to be exactly the same as the absorption bands of gaseous nitrous oxide.

By the study of the uranyl bands when a solvent is gradually replaced by another solvent or when a salt is gradually changed into another salt some very interesting relationships have been discovered between the uranyl bands. These will be published in detail.

New and quite fine bands have been discovered in the absorption spectrum of uranous chloride in acetone to which some free HCl has been added. The uranyl bands are broken into several components — *e. g.*, *c* λ 4605, λ 4550; *d* λ 4470 (weak), λ 4430 (strong), λ 4385 (weak); *e* λ 4340 (weak), λ 4290 (strong), λ 4250 (weak); *f* λ 4205 (weak), λ 4160 (strong), and λ 4120 (weak). For the uranous chloride solution in acetone the uranyl bands are affected in the same way by the addition of HCl as are the uranyl bands of uranyl chloride. Very fine acetone bands (from 10 to 20 å.ü. wide) are at λλ 6780, 6740, 6690, 6625, 6600, 6555, 6490, 6470, 6040, 6000, 5960, 5910, 5220, 5210 and 5195.

The effect of free HCl, $ZnCl_2$, $AlCl_3$ and $CaCl_2$ is to cause the uranyl and uranous bands to shift towards the red. Free nitric acid causes the uranyl nitrate bands to shift towards the violet.

The various uranous salts in water show a wide absorption band at about λ 6500 and one at λ 6750 about 30 å.ü. wide. As the amount of uranous chloride is increased these bands broaden into a single band. Glycerol solutions give a strong diffuse band in this region but in no case do two bands ever appear. In marked contrast to these solvents, methyl and ethyl alcohols and acetone give transmission throughout this region. Similar changes occur in other parts of the spectrum. In general the glycerol and water spectra are much alike, as are the methyl and ethyl alcohol spectra. But these two types of spectra differ much from each other, and differ very markedly from the absorption spectra of an acetone solution. The absorption bands of an acetone solution are usually much sharper than that of any other solvent.

(A report upon the work on absorption spectra being continued by Professor Jones and myself with the aid of a grant from the Carnegie Institution of Washington.)

## THE HEAT OF DILUTION OF AQUEOUS SALT SOLUTION.[1]

By F. L. Bishop.

THIS is a continuation of the work discussed in a previous paper.[2] The same apparatus was used, but certain modifications were necessary because the substances used in the present research, *i. e.*, zinc sulphate, copper sulphate, cadmium sulphate and zinc chloride evolve heat upon dilution.

These modifications were as follows:

1. The calorimeter was surrounded by a single coil of iron pipe about 0.8 cm. in diameter. This coil was perforated with a large number of holes 1 mm. in diameter. Connection was made to a supply of compressed air which could be maintained at any desired temperature.
2. A Beckmann thermometer was used which read directly to 1/500 degree C.
3. The surface of the calorimeter in contact with the solutions was gold plated. This was in turn covered with a thin coating of wax before each run.

The procedure in making a set of readings was as follows:

The water was placed in the outer calorimeter and the solution in the inner. The solution was then run until a state of constant temperature was reached when the liquids were mixed and the resulting temperature observed. The cold air was then turned into the cooling coil and the temperature of the solution reduced to that of the liquids before mixing. The apparatus was then run until the temperature remained constant when the electric current was turned into the heating coil and the change in temperature resulting from diluting the solution was reproduced.

This method of determining the heat of dilution entirely eliminates the errors due to radiation and to thermometers. It avoids among other things the determination of the specific heat of the solutions.

The results indicate that with certain exceptions the heat of dilution is a linear function of the concentration.

## EXPERIMENTS IN IMPACT EXCITATION WITH THE LEPEL SINGING ARC.[1]

By G. W. Nasmyth.

THE Lepel singing arc, which differs from the Duddell and Poulsen arcs chiefly in the use of very short arc lengths, obtained by placing a few sheets of paper between flat metal electrodes, affords a means of pro-

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

[2] Phys. Rev., 26, 169, 1908.

ducing high frequency oscillations of considerable power which are only slightly damped. Its origin goes back to the discovery by Wien in 1906 that for small spark-lengths of about 0.15 mm. a third maximum appears between the two resonance frequencies usually met with in coupled circuits. This third maximum is higher than the other two, and its frequency corresponds to the natural frequency of the antenna, or secondary of the coupled circuits. According to the theory of impact excitation ("Stosserregung") the third wave is explained by the rapid recovery of the resistance of short gaps, so that the oscillations in the primary are extinguished after they have given up the greater part of their energy to the secondary circuit. The oscillations in the secondary then continue as if it were an isolated circuit, with its own natural frequency and damping factor.

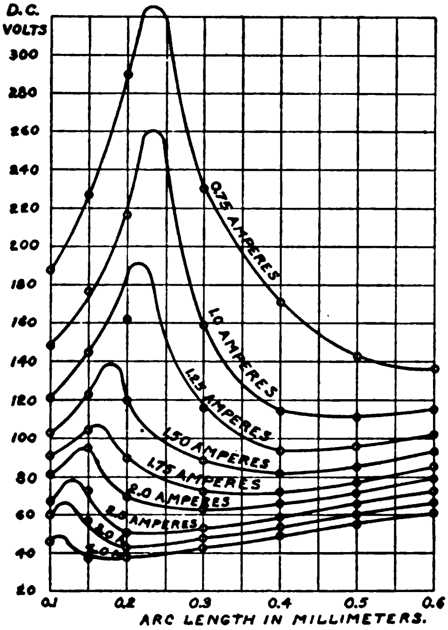


Fig. 1.

The author's experiments indicate that the phenomena cannot be completely explained by the rapid recovery of resistance in the arc due to its short length alone. The form of the curves shown in Fig. 1, which give the potential difference indicated by a d.c. voltmeter across the arc when the current is kept constant and the arc length varied, demonstrates the presence of a resonance effect. The capacity of the arc, which at short lengths is about equal to that of a Leyden jar, makes it possible to have oscillations set up through the choke coils and dynamo in the main circuit, and the relation of the frequency of these oscillations to that of the oscillation circuit determines the number and character of the discharges through the arc. Curves similar to those of Fig. 1 can be obtained at longer arc lengths if a variable capacity is placed in parallel with the arc, and it may be possible in this way to obtain impact excitation effects with spark gaps of the usual lengths, and with the Duddell singing arc.

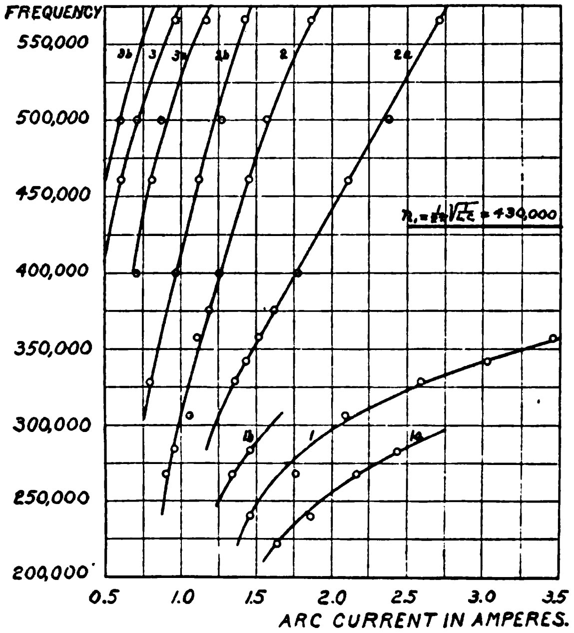


Fig. 2.

Although the Lepel arc is in some respects similar to a spark gap, it resembles the singing arc more closely in the large number of harmonics present, and in its frequency characteristics. Fig. 2 shows that the frequency, as determined by a loosely coupled resonance circuit, increases with the arc current; it also decreases with increasing arc length, in accordance with the equation for the frequency of the singing arc.[2] The arc length was 0.15 mm. using three sheets of W. S. & B. Paragon paper, and the electrodes were copper plates six inches in diameter. The primary capacity was 0.0094 m.f., inductance 14,100 cm. and the applied E.M.F. 500 volts d.c.

[2] PHYS. REV., 37, 2, p. 139, 1908.

## PHYSICAL PROPERTIES OF BINARY LIQUID MIXTURES.[1]

BY J. C. HUBBARD.

MEASUREMENTS have been made of the densities and indices of refraction of six binary mixtures with, in each case, from ten to twenty different percentages of composition. The densities have been measured at two temperatures; the indices of refraction for one temperature and for four wave-lengths (lines $C$, $D$, $F$, $G'$). For the purpose of securing a broader basis of comparison mixtures were chosen for which the vapor pressures had already been carefully studied by Zawidzki,[2] viz., carbon disulphide and acetone, carbon disulphide and methylal, acetone and chloroform, ethyl iodide and ethyl acetate, carbon tetrachloride and benzol, acetic acid and benzol. The substances were purified by Mr. C. W. Bacon, of the Department of Chemistry of Clark University.

The following are some of the conclusions resulting from the work:

1. The deviations of specific volume from the additive law are greater in absolute value at the higher temperature.

2. Deviations from the additive law for specific volumes and for vapor pressures are of the same sign.

3. Deviations of the refractivity are of either sign, and may increase or decrease with the wave-length.

4. The Pulfrich[3] "constant" $c$ in the equation

$$(N - N_0)/N = c(D - D_0)/D$$

may vary within wide limits.

5. There is evidence supporting the hypothesis of Dolezalek[4] that the deviation may be explained by the dissociation in the mixture of molecules which are associated in the pure state.

CLARK COLLEGE.

## ON THE USE OF POLAR COÖRDINATES IN THERMODYNAMICS.[1]

BY J. C. HUBBARD.

LET the absolute temperature $T$ be plotted as radius vector and let the entropy $S - S_0$ of the system with respect to that of the standard state be represented by the angular displacement of the radius vector from its position of reference $S_0$.

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

[2] Zawidzki, Zs. Phys. Chem., 35, p. 129, 1900.

[3] Pulfrich, Zs. Phys. Chem., 4, p. 561, 1889.

[4] Dolezalek, Zs. Phys. Chem., 64, p. 727, 1908.

In a reversible transformation

$$dQ = T\cdot dS.$$

Quantity of heat is therefore represented by an arc. In the transformation $AC$ (see figure)

$$Q = \int T\cdot dS = \overline{T}(S - S_0), \text{ where } \overline{T} = \frac{\int T\cdot dS}{S - S_0}.$$

It is clear that the value of the integral depends upon the path and that $dQ$ is not a perfect differential.

In a reversible isothermal transformation we have $Q/T = S - S_0$. Arc divided by the radius is an angle. In a positive transformation the entropy increases, the radius vector turning in the anti-clockwise direction. The reversible adiabatic transformation is isentropic. Adiabatic lines are therefore radii. The Carnot cycle is therefore represented by $ABCDA$ and it is visually evident that, since $Q_1/T_1 = Q_2/T_2 = Q_3/T_3$, as the temperature of the refrigerator is made lower and lower the quantity of heat given to it is less and less and at the absolute zero the heat would all be converted into work.

The unavailable energy at $T_1$ with reference to the auxiliary medium at $T_2$ is, in a reversible process, represented by the line $AF = Q_2 = (T_2/T_1)Q_1$. As $T_2$ is lowered $Q_2$ and $AF$ diminish.

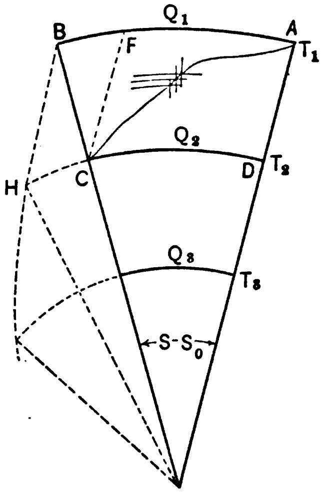


Fig. 1.

The available energy is represented by $FB$, or

$$Q_1 - Q_2 = Q_1\left(1 - \frac{T_2}{T_1}\right),$$

which is greater as $T_2$ becomes less.

As an example of an irreversible process we may consider heat conduction, in which $Q_1$ remains constant and is completely unavailable. As the radius is diminished the arc remains constant, the entropy increasing without limit as $T_2$ approaches zero. The path of the transformation is the hyperbolic spiral $Q = ST =$ const.

In any irreversible process the increase of unavailable energy calls for an increase of entropy with respect to the auxiliary medium at $T_2$.

The main advantage of the use of polar coördinates in thermodynamics lies in the fact that work and heat may be represented by lines and the results of a process are more easy of comprehension than when they are represented by areas.

CLARK COLLEGE.

## NOTE ON THE CAUSE OF THE DISCREPANCY BETWEEN THE OBSERVED AND THE CALCULATED TEMPERATURES AFTER EXPANSION IN THE SPACE BETWEEN THE PLATES OF A WILSON EXPANSION APPARATUS.[1]

BY R. A. MILLIKAN, E. K. CHAPMAN AND H. W. MOODY.

IN a paper presented to the Physical Society at Princeton on October 23, one of the authors showed that the actual fall of temperature between electrodes 5 mm. apart in a Wilson expansion apparatus was not more than 0.8° C. under conditions in which the calculated equilibrium temperature after condensation was more than 12° below the initial temperature.

The cause of this discrepancy has been definitely located in the very rapid inflow of heat from the metal electrodes into the space between them.

In the middle of a two-liter flask one third full of water was placed a thermo couple made of iron and copper wires, 1 mil. (.025 mm.) in diameter. Metal plates 3 cm. in diameter were placed on either side of this copper-iron junction and were so supported that their distances from the couple might be increased or decreased at will. When the distance from each plate to the couple was 3 cm. the thermal current through the galvanometer produced by an expansion of 25 cm. of mercury was sufficient to cause a galvanometer deflection of 90 scale divisions. As the distance from the couple to the plates was decreased this deflection, under otherwise identical conditions, gradually diminished until, when the distance from each plate to the couple was 3 mm., the deflection had fallen to only 10 mm. In other words the fall in temperature as measured by the couple had been reduced by the approach of the metal plates to one ninth its original value. When the plates were again removed the deflections increased regularly and at the original distance had regained their original value. These observations were repeated a large number of times with uniformly consistent results. It is perhaps worthy of note that when a water surface was made to approach the couple under conditions essentially the same as those used in the approach of the metal surfaces, the diminution in the deflection of the galvanometer was very

---

[1] Abstract of a paper presented at the Boston Meeting of the Physical Society, December 28–31, 1909.

slight. In one experiment the level of the water in the jar was raised from a distance of 10 cm. below the couple to a distance of only 2 mm. below the couple without causing a change in the galvanometer deflection of more than 15 mm. out of a total of 95 mm.

## SOME NEW VALUES OF THE POSITIVE POTENTIALS ASSUMED BY METALS IN A HIGH VACUUM UNDER THE INFLUENCE OF ULTRA-VIOLET LIGHT.[1]

By R. A. Millikan.

All previous observations upon the positive potentials assumed by metals under the influence of ultra-violet light have given values falling between 0 and 5 volts. The observations herewith reported were made upon five different metals, four of which had been kept for three years in the best possible vacuum obtainable with a mercury pump without the aid of a charcoal bulb and liquid air. The average pressure within the vacuum chamber, as measured by a McLeod gauge, was between .00001 and .000001 mm. of mercury. Observations taken upon these same four metals three years ago when they were first inserted in the vacuum chamber gave values of the positive potentials varying between 0 and 1.34 volts. The values now obtained with the same source as that originally used are from ten to thirty times larger. Thus, the original value observed in the case of silver was 1.34 volts; the largest value observed in these later experiments is 10.5 volts. The original value in the case of zinc was .2 volt; a value 30 times larger, namely, 6.6 volts has recently been obtained. The original value in the case of iron was 1.2 volts; it is now 15.4 volts. Copper showed at first a potential of 1.1 volts; it now shows a potential of more than 20 volts.

These increases have been brought about in part, at least, if not wholly, by prolonged and powerful illumination of the surfaces of the metals with ultra-violet light.

The fifth metal experimented upon has been aluminum. It was placed six months ago in the best vacuum obtainable by Dewar's method. Under powerful illumination continued intermittently for three weeks its positive potential has gradually been pushed up from an initial value of .25 volt to a present value of 12.5 volts. The increase pertains not merely to the maximum value of the positive potential but also to the rate at which positive potential is gained at any potential between zero and the maximum. Curves showing the distribution of the velocities of the escaping electrons will be published later.

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

A plausible explanation of this change in the apparent velocity of proection of electrons from metals is found in the assumption that there exists at the surface of every metal, even in a good vacuum, a gaseous film which gives rise to an electrical double layer of negative sign on the outside, positive on the inside. Electrons liberated by the ultra-violet light are retarded in passing out through this film and escape with a velocity of but 2 or 3 volts at most. Prolonged illumination with ultra-violet light drives out this gas and breaks up the double layer. The electrons then escape with their initial velocities of projection. Experiments are in progress to test more fully this hypothesis, and in particular to determine the effect of wave-length including those of the Schumann region upon these high positive potentials.

The phenomena herewith reported were first observed in the case of silver in January, 1908, and publication until they could be checked with other metals and in other tubes.

UNIVERSITY OF CHICAGO.

## ON THE RELATIVE NUMBERS OF POSITIVE AND NEGATIVE IONS PRESENT IN ATMOSPHERIC AIR.[1]

BY MR. A. THOMSON, B.A.
(PRESENTED BY PROF. J. C. MCLENNAN.)

IN this paper the author gave an account of a series of observations on the electrical conductivity of atmospheric air under electrical fields similar to those which obtain under natural conditions. The measurements were carried out with a Wilson electrometer, the use of which rendered it possible to study the conductivity of the air in its undisturbed state.

Observations were made with positive and negative fields on the natural conductivity of the air and also on atmospheric air ionized by gamma rays from radium of varying intensities.

In these experiments it was found that the positive current was generally greater than the negative one. Variations in the size of the plate electrode in the Wilson instrument did not appear to exert any marked effect on the ratio of the two current intensities. With high fields, however, the positive and negative currents more nearly approached equality than when lower ones were used.

Measurements made when slight air currents such as draughts existed in the observation room always showed an increase in ratio of the positive to the negative current. In some cases the ratio of the two currents in the presence of air currents was as high as 1.3 to 1.

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28-31, 1909.

A lighted Bunsen flame placed in the neighborhood of the air ionized by gamma rays always produced a marked diminution in both the positive and the negative currents. The presence of such a flame in the room was also found to increase the ratio of the positive to the negative current.

Differences in the positive and negative currents such as those noted above when observed in connection with measurements on the conductivity of air under natural conditions have in the past been taken as indicating an excess of positive ions in the atmosphere but since in the experiments described above equal quantities of positive and negative ions were produced by the gamma rays it is clear that the difference in the two currents must be ascribed not so much to differences in the number of ions of each kind present, but rather to differences in the mobilities of the positive and negative ions, and to differences in their rates of diffusion.

The results confirm the observations made by Eve[1] with an Ebert apparatus, and they indicate that the readings of the Wilson type of electrometer as well as those of the Ebert apparatus may be misleading if they are taken as indicating a large excess of positive over negative ions in the air.

## URANOUS AND URANYL BANDS. A FINE BAND SOLUTION SPECTRA.[2]

By W. W. Strong.

Up to the present very little spectroscopic work has been done with uranous salts. In the present work uranous acetate, bromide, chloride and sulphate have been studied in various solvents and the absorption spectra photographed. Most of the uranous salts show the uranyl bands. The spectra of the different salts in the same solvent are very similar whereas the spectra of the same salt in different solvents is entirely different. In many cases there are distinct water, glycerol, alcohol or acetone bands and these bands coexist in mixtures of the solvents. These solvent bands do not shift as the amount of the particular solvent is changed but only change in intensity. For example the various uranous salts in water show a wide band at $\lambda$ 6500 and a characteristic water band at $\lambda$ 6750, 30 a.u. wide. Glycerol solutions give a wide band in this region but in no case do two bands appear. Alcohol and acetone solutions do not have any bands at all in this region. There are probably no other salts whose spectra are more affected by physical and chemical conditions than those of the uranous salts.

Characteristic solvent bands indicate a very definite relationship between salt and solvent.

---

[1] Nature, March 11, 1909.

[2] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

On the other hand there are bands that gradually shift in position as the solvent is changed. The shifts are usually very different for neighboring bands. The effect of the solvent is probably very different here from what it is in the case of characteristic solvent bands.

A large number of new bands and new groups of bands have been discovered. In some cases these bands are quite fine, and by using certain acids the uranyl bands have been broken up into narrow component bands, a resolution that has heretofore only been accomplished by very low temperatures.

Free HCl, $ZnCl_2$, $AlCl_3$ and $CaCl_2$ cause the uranous and uranyl bands (in general) to shift to the red. Nitric acid on the other hand causes a shift towards the violet.

A change of a uranous or uranyl salt to another salt is usually accompanied by a very marked change in the positions and intensities of the uranyl bands. A good example is the change of uranyl nitrate to uranyl chloride in aqueous solution. The *d* nitrate band is shifted many Ångström units while the neighboring *c* band is but very slightly changed. This spectroscopic method opens up a new method of studying such changes.

Accidentally it was found that by adding sulphuric acid to nitric acid, the absorption spectra of the solution consisted of hundreds of fine lines. This spectrum is probably that of nitrous oxide.

By the study of the uranyl and uranous bands when a solvent is replaced by another solvent or when a salt is transformed into another salt very interesting relationships between the bands are brought out. A detailed study of these relations has been made.

## THUNDERSTORM ELECTRICITY.[1]

BY W. W. STRONG.

IN a recent discussion Simpson[2] has shown that for three reasons the Wilson-Gerdien theory is incapable of explaining the origin of thunderstorm electricity. The most important difficulty is that the rate of ionization of the air is not great enough to account for the rapid production of lightning flashes.

During the past summer it was found that the rate of leak of closed electroscopes was increased many times by removing them from the warm surface of the ground to a cool cave. This effect took place only when the weather was very warm.

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

[2] Phil. Mag., April, 1909.

The effect was found for some eight electroscopes, one of which was arranged so that the electrode above the sulphur insulation was at a higher potential than the gold leaf. This insured that the effect was not due to a leak over the insulation. The effect was not due to a penetrating radiation and can hardly have been due to the formation of an active deposit on the walls of the electroscopes.

If this increased rate of leak was due to an increased ionization in the electroscopes, then this change is very similar to that which would take place when surface air is rapidly carried into upper regions of the atmosphere during thunderstorms. Such a large increase in ionization would remove one of the serious difficulties of the Wilson-Gerdien theory.

Recent work by Simpson indicates that the breaking of rain drops probably causes a considerable part of the separation of positive and negative electricity during thunderstorms. But it also seems possible that the condensation upon negative ions may also cause some separation of electricity. These ions may carry several charges and thus cause a greater separation. Simpson only considers negative ions to carry a single charge. It would be interesting to know whether the supersaturation necessary for condensation on negative ions is dependent on the size of the charge they carry.

## The Second Postulate of Relativity.[1]

By Richard C. Tolman.

THE remarkable conclusions drawn from the theory of relativity are caused by the peculiar nature of the second postulate of relativity. This postulate may be derived, by combining the hitherto unquestioned first postulate of relativity with the principle that the velocity of light is independent of its source.

The alternative hypothesis, that the velocity of light and its source are additive, would lead to none of the complications of the theory of relativity. It is shown, however, that this new hypothesis as to the velocity of light would not lead to exactly the same results for the Doppler effect as the older and more usual hypothesis as to the velocity of light. An experiment is also described which indicated that the velocity of light from the approaching and receding limbs of the sun was the same.

Finally, a method is developed for obtaining all the more important conclusions of the theory of relativity based merely on the first postulate of relativity and the results of Kauffmann-Bucherer experiment without making any use of the second postulate of relativity.

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

## BOOKS RECEIVED.

*Elements of Physics.* By HENRY CREW and FRANKLIN T. JONES. Pp. xi + 435. New York, The Macmillan Co., 1909.

*Light.* By RICHARD C. MACLAURIN. Pp. ix + 251. New York, The Columbia University Press, 1909.

*Radiation, Light, and Illumination.* A Series of Lectures by CHARLES P. STEINMETZ; edited by J. L. HAYDEN. Pp. xii + 305. New York, McGraw-Hill Book Company, 1909.

*Text-Book of Physical Chemistry.* By ARTHUR W. EWELL. Pp. ix + 370. Philadelphia, P. Blakiston's Son and Company, 1909.

*Electric Waves.* By W. S. FRANKLIN. Pp. vii + 315. New York, The Macmillan Company, 1909.

*Dynamos and Motors.* By WM. S. FRANKLIN and WILLIAM ESTY. Pp. viii + 489. New York, The Macmillan Company, 1909.

*Dynamo Laboratory Manual.* Vol. I., Direct Current Studies and Tests. By WM. S. FRANKLIN and WILLIAM ESTY. Pp. viii + 152. New York, The Macmillan Co., 1909.

*The Alternating Current Commutator Motor and the Leakage of Induction Motors.* By RUDOLF GOLDSCHMIDT. Pp. viii + 209. New York, D. Van Nostrand Company, 1909.

*The Evolution of Worlds.* By PERCIVAL LOWELL. Pp. xiii + 262. New York, The Macmillan Company, 1909.

*Volume XXX.* *March, 1910.* *Number 3.*

# THE
# PHYSICAL REVIEW.

---

## IONIZATION PRODUCED BY ENTLADUNGSSTRAHLEN AND EXPERIMENTS BEARING ON THE NATURE OF THE RADIATION.

By Elizabeth R. Laird.

A SHORT time ago the author reported some experiments on Entladungsstrahlen [1] which were largely confirmatory of work done by Wiedemann [2] and by Hoffmann.[3] The results showed that a solid salt solution, $CaSO_4 + 2$ per cent. $MnSO_4$, is made thermoluminescent by the action of a radiation from the spark of a Wimshurst machine or induction coil, that the effect increases with increased capacity or potential difference, that the radiation is propagated in straight lines, as evidenced by shadows, that it is stopped by quartz and other solid substances, that it is not affected by a strong air blast, that it is somewhat more absorbed by carbon dioxide than by air. These experiments have been carried further and experiments have also been made showing that the same agent produces marked ionization in the air, and in other gases and vapors. For convenience' sake the experiments have been grouped in sections.

### I. Experiments on the Thermoluminescent Effect.

The general method was the same as that described previously.

1. By using a spark 1.5 cm. long and placing pieces of mica as indicated in the diagram (Fig. 1), and the salt below, it was shown

---

[1] Phys. Rev., March, 1909, p. 225.

[2] Zeitschr. f. Electrochem, p. 159, 1895.

[3] Wiedemann, Annalen, LX., p. 269, 1897.

that the radiation comes from the whole length of the spark and cannot be due to metallic dust driven from the electrodes, again confirming Hoffmann's work.

2. The spark arranged vertically produced a very sharp shadow of the lower electrode on a plate of salt placed some centimeters below.

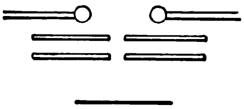


Fig. 1.

3. The effect was not cut off if the salt was placed in a metal box with a metal gauze cover, all of which is earth connected. Hence the effect is not due to direct electrical induction.

4. Aluminum .0004 cm. thick, very thin glass, very thin mica cut off the effect completely.

5. The only solid substance thus far found which transmits the effect is thin celluloid in the form of films made by letting fall a drop of a solution of celluloid in amyl acetate on water. These films when dry transmit the radiation. Placed under the spark of a Wimshurst machine they are easily broken, and in breaking become thicker, so that with this arrangement it appeared as if they too cut off the effect. When however the spark *B* obtained from Leyden jars arranged as in the diagram (Fig. 2), where the spark gap *A* is connected to an induction coil, is used as source of radiation, it is found that one half, or one third, or sometimes a smaller fraction of the effect is transmitted. The length of the spark gap *B* was usually 1 mm.

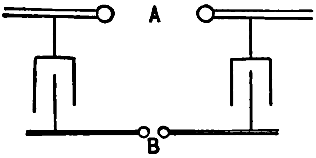


Fig. 2.

6. As these films were so easily broken by the spark, it seemed possible that the thermoluminescent effect might be due to a mechanical pressure wave. It was found however that thin paper or goldbeater's skin stretched over a frame did not transmit the effect. Also if the spark gap was placed around the bend of a tube, the ends of which were closed as indicated in Fig. 3 no effect was produced in 15 minutes exposure on salt placed at the lower opening, although 1 minute direct radiation at the same distance produced some effect. It appeared also that direct mechanical tapping on the salt did not produce thermoluminescence.

## II. General Observations on the Ionization Effect.

The ionization produced by this radiation from the spark was observed with the apparatus which is illustrated in Fig. 4. An ionization chamber 5 cm. deep, 8 cm. in diameter, with a window 3.2 cm. in diameter, was provided with a wire electrode connected directly to an ordinary type gold-leaf electroscope. The window was covered with a celluloid film and protected by metal gauze. The gauze and vessel were earthed and the electroscope charged. The leak was observed through a reading microscope. The rate of leak varied under different conditions from 5 to 2,000 scale divisions per minute. At a window distance of 2 cm. from the spark the leak in air was about 20 scale divisions in 20 seconds. It varied with the film used and the functioning of the interrupter. The insulation leak was usually not greater than 0.2 scale division per minute, in some instances it was 1 scale division a minute, only on two or three occasions did it rise higher. To aid in estimating the sensitiveness of the apparatus and magnitude of the results, a thick layer of uranium oxide 9 cm.$^2$ in extent, was placed on the bottom of the chamber; the rate of leak was 25 scale divisions per minute.

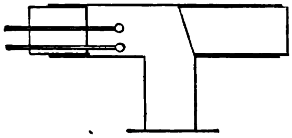


Fig. 3.

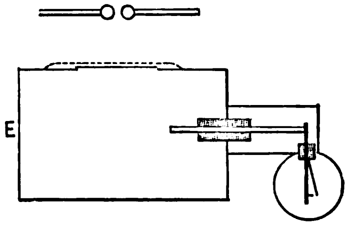


Fig. 4.

## III. Experiments Showing that the Thermoluminescence and Ionization are Due to the Same Cause.

1. As already mentioned, it is known that increasing the capacity parallel to the spark gap increases the thermoluminescent effect. An experiment was made in which one half of a salt plate was exposed to the spark and the rate of leak in the ionization chamber was simultaneously observed when the source of radiation was the spark of the induction coil used without capacity. Then the other

half of the salt was exposed, and the leak observed when the source was the secondary spark from the Leyden jars. The rates of leak were in the ratio of 1 : 8, the times of exposure 2 minutes and 30 seconds respectively. The half of the salt given the shorter exposure gave a brighter luminescence when heated. If the luminescence had been exactly proportional to the ionization it should have been twice as bright.

2. Both effects are stopped by thin mica, and aluminum, but are transmitted considerable distances through air, and as already stated are transmitted by celluloid films.

3. Both effects are stopped by quartz. In the case of thermoluminescence under conditions when an exposure of 30 seconds gave quite appreciable thermoluminescence, an exposure up to 15 minutes gave no effect when the salts were well heated previously and covered with quartz 0.6 mm. thick. With longer exposures of 25 or 30 minutes some slight thermoluminescence was difficult to avoid, but it seems probable that the slight effect sometimes produced is caused by known light and not by Entladungsstrahlen. In the case of ionization when the bottom of the chamber was not cleaned, results such as follow were obtained, the leak per minute being given.

TABLE I.

| Insulation Leak. | Leak (Computed) with Film. | Leak with Quartz. |
|---|---|---|
| 0 | 100 | 1 |
| .8 | 75 | .8 |
| .5 | 1600 | 1 |

4. Both effects are produced by a radiation travelling in straight lines from the spark. The existence of distinct shadows cast on the thermoluminescent salts has already been referred to. To test the same for the ionization effect the apparatus was arranged so that the spark was 2 cm. above the window of the ionization chamber, and a piece of mica of such a size as just to place the window in shadow was put 1 cm. above it. There was no film over the window. The following results were obtained.

| | |
|---|---|
| Insulation leak | .8 in 1 minute. |
| Leak, mica in position | 1 " 1 " |
| | 2 " 1 " |
| Leak, mica moved slightly | 4 " 1 " |
| Leak, mica carefully replaced | 1 " 1 " |

5. Neither effect is altered by changing from brass to aluminum electrodes at the spark gap.

In the case of thermoluminescence, one half of a salt plate was exposed to the spark between brass electrodes, the other half to the same between aluminum electrodes for the same time. The times used varied from 20 seconds to 15 minutes. No perceptible difference was seen in the luminiscence of the two halves.

In the case of ionization the average of a number of experiments with brass wire electrodes was a leak of 18 scale divisions a minute, with aluminum 17 scale divisions a minute. The difference is not as great as between individual experiments with the same electrodes.

6. Neither effect is altered by a strong air blast placed between the spark and the salt or ionization chamber. In the actual experiment on ionization a slight increase was observed, probably due to the blowing away of absorbing vapors.

7. The cause of both effects is absorbed at least approximately in the same degree by air.

In one experiment on thermoluminescence it appeared that about one half the radiation was absorbed by 6 cm. of air, since at a distance of 9 cm. with nine times the exposure the luminescence appeared only about half as bright as at 3 cm. In another case it appeared as if 2 cm. absorbed a large proportion.

Experiments on ionization gave the leak at a window distance of 2.9 cm. as 50 scale divisions in 6 seconds, and at 6.5 cm. as 50 scale divisions in 70 seconds, indicating an absorption of somewhat over 50 per cent. in 3.6 cm. The absorbing air in this case may have had a trace of turpentine vapor in it.

8. The celluloid films absorb approximately the same proportion whether estimated by the thermoluminescent or the ionization effect. In the first case it was attempted to estimate the proportion directly, also by observing the times during which the thermoluminescence under the film and outside lasted, also by trying to expose one part of the salt plate directly for a time which would give about the same luminosity as a longer exposure through the film. In various cases it was estimated that the film transmitted one fourth to one half the effect.

On a number of different occasions during the experiments on ionization the electroscope leak was observed with the window covered and uncovered. The times given for the gold-leaf to pass over the same number of scale divisions are given below.

Table II.

| With Film. | Without Film. |
|---|---|
| 70 seconds. | 25 seconds. |
| 33 " | 10 " |
| 28 " | 15 " |
| 35 " | 10 " |
| 60 " | 10 " |

It will be seen that the ratios vary from $\frac{1}{6}$ to $\frac{1}{2}$ approximately.

9. Neither effect is produced largely by the radiation from the carbon arc.

A salt plate exposed 10 minutes at 6.5 cm. distance was not made thermoluminescent. When the window of the ionization chamber was placed 5 cm. from the arc the known effect of ultra-violet light outbalanced any other, as shown by observing the leaks with a piece of quartz 1 mm. thick, and a celluloid film placed alternately over the window. The ratios were as follows in different experiments $\frac{30}{26}$, $\frac{30}{30}$, $\frac{25}{27}$, $\frac{13}{14}$.

## IV. Experiments Showing that the Ionization Takes Place in the Air or Gas in the Chamber, and is not an Effect of the Metal Electrode or Walls of the Chamber, nor of the Diffusion into the Chamber of Ions from Outside.

1. The leaks whether the electrode is positively or negatively charged are approximately equal if the bottom of the ionization chamber is oxidized, or in such condition that no leak is observed when the window is covered by quartz. The time to pass over 20 scale divisions from 50–30 is given in a variety of cases taken on different days in different conditions, sometimes, with vapors in the chamber. The last two numbers are times taken to pass over 50 divisions.

When, as happened in some later experiments, the bottom of the vessel was cleaned, the leak with the electrode positively charged was greater, but the ratios became appreciably equal if the amount through quartz was subtracted. For example in one case the insulation leak, also the leak through quartz, with the electrode negatively charged was 1 scale division in 60 seconds; the leak through a film was 47–30 divisions in 90 seconds, but with the wire positively charged the same leak occurred in 72 seconds, and the leak through quartz was 4 divisions in 60 seconds. Subtracting the rate of leak through quartz from that through the film it becomes 10.1 and 10.3 scale divisions a minute in the two cases.

Table III.

| **Positively Charged.** | **Negatively Charged.** | **Positively Charged.** | **Negatively Charged.** |
|---|---|---|---|
| 18 seconds | 16 seconds | 15 seconds | 15 seconds |
| 10 " | 10 " | 14 " | 15 " |
| 3 " | 3.2 " | 3.5 " | 3.5 " |
| 16 " | 14 " | 14 " | 13 " |

2. No difference was observed whether a copper gauze electrode or a brass plate electrode of the same size was used. The same leak was obtained when the electrode was placed so as to be entirely shielded from the light of the spark.

3. The films used as windows are not specially photoelectric. Quartz placed over the film cut off the effect just as it did when the film was not there. A film spread over the bottom of the vessels did not increase the effect. A film attached to the electrode, and so to the electroscope, so that the light of the spark would fall on it, acquired no charge when the window was shielded in the usual way, and if initially charged either positively or negatively lost the charge completely.

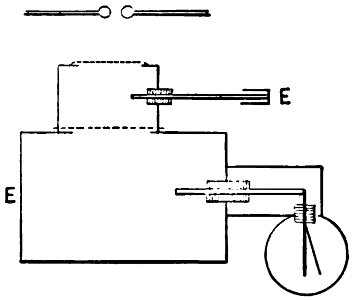


Fig. 5.

4. As already described in III., 6, air blown across the top of the window does not diminish the effect, also the ionization remains as large if the ions are removed above the window in the following way.

The main ionization chamber was placed with its window 6.5 cm.

from the spark. Above it was placed a second similar to the first but smaller, with a window protected by gauze, and an insulated wire electrode connected to a small additional capacity (Fig. 5).

The leak in the lower chamber was then observed when the upper electrode was charged and uncharged. Since the current in the gas appeared to be a saturation current, stopping abruptly when the coil was stopped, the same would be true in the upper chamber. The upper electrode was also connected directly to the electroscope to make sure that the capacity used was such as to keep the electrode charged during the course of an experiment. Below are some of the results.

Table IV.

| Insulation Leak. | Leak, Upper Electrode Uncharged. | | Leak, Upper Electrode Charged. | |
|---|---|---|---|---|
| | Scale Divisions. | Time. | Scale Divisions. | Time. |
| .5 | 50–43 | 60 seconds | 49–41 | 60 seconds |
| | | | 50–43 | 60 " |
| .5 | 50–41 | 115 " | 50–41 | 120 " |
| | 49.5–39 | 125 " | 49.5–39 | 120 " |

The rate of leak is the same in the two cases, hence the ionization observed in the lower chamber is produced there.

5. The same result follows from the fact that the ionization observed depends very largely on the vapor or gas in the ionization chamber. It is difficult to obtain accurate values of the relative ionization produced in different cases, as the celluloid films permit some vapors to pass through, and the effects are greatly influenced by this fact. The substances that show a large ionization in most cases appear also to absorb strongly, and so the actual ionization observed would not be that produced by an incident radiation equal to that in case the air were the only absorbing medium. Another factor rendering difficult the comparison is that very small amounts of vapor change appreciably the ionization, and it was difficult to know when all traces of other vapors were removed. Although the results can be regarded as at most approximations, they are nevertheless given as being interesting.

*Illuminating Gas* (Coal Gas). — Without moving the ionization chamber gas was led in to fill it, and the window was covered with a film. The smell of the gas could be detected above the chamber. The leak changed from 40 scale divisions in 18 seconds to 10 sc. div. in 30 secs., but it was found that fanning above the window increased the leak, and putting in less gas to begin with, and fanning the top, the leak rose to 50 sc. div. in 5 secs., 4½ times as large as the leak in air, repeating this several times the largest ratio obtained was 12 times the leak in air. These results are explained if one supposes that the gas absorbs the radiation strongly so that when there is some gas between the window of the chamber and the spark, the radiation entering the chamber is much less intense than when air is the medium, but at the same time the radiation produces more ions in coal gas than in air. It is hoped to repeat these and other similar experiments using a current of air to keep the upper space fairly free of foreign vapors and gases.

*Carbon Dioxide.* — Solid carbon dioxide was put into the chamber and allowed to partly evaporate before readings were taken. The reading fell at first from 20 sc. div. in 20 secs. to 4 in 60. Fanning the top it rose to 10 sc. div. in 25 secs., and in no case was a value greater than one half that in air obtained.

*Turpentine.* — It was found incidentally that the presence in the ionization chamber of soft wax, that had been worked by the hands, increased appreciably the leak. This led to trying the effect of placing a few drops of turpentine in the chamber. Very small glass receptacles which could be introduced without disturbing the apparatus appreciably were used for this and other liquids. From 20 sc. div. in 20 secs. the leak rose to 50 in 1.6 secs., making the rate about 30 times that in air. In one case a ratio considerably higher was obtained but more frequently a ratio of about 30.

*Ether.* — A small amount of ether increased the leak from 30 sc. div. in 30 secs. to 50 sc. div. in 3 secs. or 16 times. Letting a drop fall on top of the chamber the effect diminished to 5 in 25 secs.

*Ammonia.* — With liquid in the chamber the leak rose from 20 sc. div. in 51 secs. to 20 sc. div. in 9 secs. Removing the liquid, and introducing fresh air above the window, it rose to 43 sc. div. in 5 secs., or 22 times that in air.

*Alcohol.* — It was very difficult to remove the traces of ammonia in the chamber, but with the introduction of alcohol the leak rose from 30 sc. div. in 37 secs. to 30 sc. div. in 12½ secs. or increased three times.

*Methyl Iodide.* — When some drops were put in the chamber the leak increased from 30 sc. div. in 17 secs. to 30 in 7 secs. It was observed however that there was a distinct smell of the vapor above the window. Taking the liquid out of the chamber, leaving vapor in, and fanning fresh air on top, the leak rose to 50 sc. div. in 1.8 secs. or 16 times that in air, which is probably still less than the true value.

*Hydrogen Peroxide.* — A few drops of a 20 per cent. solution of hydrogen peroxide were placed in the chamber. The window being 6 cm. from the spark the leak rose from 3 sc. div. in 60 secs. to 7 sc. div. in 60 secs.

It seems rather evident from the experiments that the amount of ionization is not directly proportional to the absorption by the vapor or gas of the radiation, as for instance a larger ionization could not be obtained with carbon dioxide than with air. It is evident also that it is not at all proportional to the density. It is hoped to make experiments on the absorption of vapors for this radiation.

## V. EXPERIMENTS INDICATING THAT THE IONIZATION OBSERVED ORDINARILY IN THE NEIGHBORHOOD OF A SPARK IS ALMOST ENTIRELY DUE TO THIS RADIATION.

Experiments have already been described (IV., 4) showing that the ionization observed in the chamber when the window is covered with a film is not due to any diffusion of ions through the film, but that the ions are produced in the chamber itself. Also the experiment showing that a piece of mica casts a shadow in which few ions are found, would show that the ions ordinarily found in an air space some distance from a spark do not reach there by slow diffusion. The following experiment makes the point more clear. The ratio of the leak in the ionization chamber with the window film covered, to that with the window uncovered, varied from one sixth to one half. Putting first one film, then two, then three over the window, readings such as the following were obtained.

1 film 10 scale divisions in 60 seconds.
2 films 5.6 " " " 120 "
3 films 3.5 " " " 120 "

This shows that the ratio of the effect traversing three films to that through two, likewise that through two to that through one, is of the same order as the ratio of that through one film to that through the uncovered window. This is most simply explained if one supposes that each film absorbs a certain fraction of the incident radiation which without any film would therefore produce in the chamber the ionization observed.

## VI. EXPERIMENTS ON REFLECTION.

Since the radiation producing the two effects studied travels in straight lines from all parts of the spark, casting shadows identical in position with those cast by the light of the spark, and since ultra-violet light is known to produce special luminescent effects and according to Lenard[1] to ionize a gas, it was sought to determine if this particular radiation were a kind of light by seeing if it could be reflected. That it is not light of wave-length greater than 1,000 Angström units seems plain from the fact that it is transmitted through at least 10 cm. of air. Such a layer of air would not be much more transparent than quartz in any region above λ 1750, and absorbs completely in the region below that photographed by Lyman.

Hoffmann tried to reflect the Entladungsstrahlen but obtained no result, although at the same distance bright luminescence, he states, would have been produced by direct radiation. It seemed quite possible however that there might be reflection, and yet that the amount reflected would be only a fraction of the incident radiation. Using thermoluminescence as a detector of the radiation, a polished silver surface, made by polishing the back of a plate glass mirror from which the shellac had been removed, was used as a reflector. The first arrangement was as indicated in Fig. 6, where *B* is the spark gap, *M* the mirror, *S* the salt, and *A* a piece of mica placed to shield the salt from direct radiation. When one minute would have produced some luminescence for the same direct distance through air, in a large number of trials only once did there seem to be any reflection for

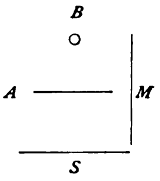


Fig. 6.

---

[1] Annalen der Physik, IV., 1, p. 486, 1900.

15 minutes exposure. Putting a willemite screen in the same position as the salt, however, it was apparent that the fluorescence was much less than would have been produced at the same distance without reflection.

Another arrangement of the apparatus was then made, which permitted the distances used to be made smaller. The salt was placed at one side a few millimeters above the spark, thus placing it in shadow with respect to the direct light. The mirror was placed above and as close as possible. Still up to twelve minutes exposure no effect was observed, at fifteen minutes in one case a faint light lasting about one second was seen, while the luminescence produced by direct radiation of five minutes lasted during one hundred seconds. In the hope of making it easier to detect any faint reflection a piece of paper with a rectangular hole in it was placed over the mirror, so that the more intense patch of light reflected, as indicated by the effect on a willemite screen, was sharply bounded. Exposures of twenty-five to thirty minutes were given. Unfortunately it was found that in several of the experiments made in this way the salts used were more than ordinarily affected by light. In the other trials either there was no certainty of any thermo-luminescence, or the faint luminescence seen had not the shape of the rectangular patch as expected. In the cases where more light was seen and an effect could also be seen under quartz, the light patch was fairly rectangular. On account of the difficulty in keeping salts that would not be affected by light in this long exposure so near to the spark, this particular method was abandoned for the time being in the hope that more would be obtained by the ionization method.

To test for reflection by this method the ionization chamber was placed somewhat to one side of the spark, and the window was screened from direct radiation by a piece of mica. The mirror was placed above the spark, with its reflecting surface downwards and slightly slanted so as to throw a good light on the window. The insulation leak was taken, then the leak with the coil working without mirror, and with mirror alternately. A steady leak of .4 or .5 scale division per minute was obtained with the mirror in position, when the leak without the mirror but with the coil working, measured over 4 or 5 minute intervals was not greater than .1 scale division per minute. It was found, however, that as nearly as could be estimated the same leak was obtained if the shellacked end of the mirror was used or if the glass surface was used as reflector. It was not easy to measure the distance through air which the radiation had to travel, but at approximately the same distance the leak was 20 scale divisions a minute. Leaving the chamber in position and removing the mica screen the leak was 46 scale divisions a minute. This would mean that a fraction not greater than one fortieth of the radiation was being reflected. The amount was so small absolutely, that although the leak was regular it seemed not quite evident that it was due to a special sort of Entladungsstrahlen. When, however, the effect of vapors on the amount of ionization was discovered, it seemed easy to test, as the presence of these vapors did not increase any leak due to ordinary light, and would not increase a leak due to the diffusion of ions.

Accordingly an experiment was made in which the leak without mirror was first taken, which was small as before, then with mirror in position, when a leak of 1.8 scale divisions per minute was obtained. Then turpentine was put in and the leak went up to 30 scale divisions a minute at once, and later, with some rearrangement, to 150 scale divisions. This seemed to make it clear that the leak observed was due to a special radiation, and again quartz cut off the leak.

To find the approximate ratio between the amount of ionization obtained in this way and from the direct radiation an experiment was made with considerably less turpentine vapor present. The leak with current and without mirror was 1 scale division per minute, with mirror 22 scale divisions per minute, and at the same distance through air the rate was 500 scale divisions per minute. This would indicate the fraction reflected as about one twenty-fifth. With the same arrangement the angle of the mirror was altered to throw less light on the window of the ionization chamber. The leak fell to 6 scale divisions per minute. This appeared to indicate that the ionization was due to a direct reflection.

In order to test this further, experiments were made as follows, The mirror placed horizontally, was moved from a position where no light was reflected, to one where the window was partly in light, and then in full light. The movement was through 1 cm. each time. The insulation and direct leak being 1.4 scale divisions per minute, the readings were 1.4, 7.8 and 29 scale divisions per minute. Again with positions of the mirror edge 0 cm., .5 cm., 1.3 cm., 1.5 cm., from the spark, the following rates of leak were obtained, 102, 107, 15, 10 scale divisions per minute. In this case the film was found broken, and the direct leak without mirror was 5 scale divisions per minute. At the position 1.5 cm. the window appeared in shadow, and at .5 cm. in full light. In another experiment the mirror was slanted so that a smaller motion would change from the position of full illumination of the window to that of little or none. In this case with a motion of a few millimeters each time towards a position of no reflection, the rates of leak were as follows: 11, 6, 0, 2 scale divisions per minute. The leak did not fall to zero at exactly the point expected but did so nearly so, that the radiation appeared as if directly reflected.

On the other hand it has been already mentioned that different surfaces reflect about equally well. With the mirror horizontal a number of different surfaces were placed on it, and the rates of leak were observed as follows.

| | |
|---|---|
| Glass | 18 scale divisions per minute. |
| Cardboard | 12 " " " " |
| Paper | 20 " " " " |
| Copper, unpolished | 20 " " " " |
| Lead | 23 " " " " |

These results are rather remarkable, and the writer hopes to experiment further to test if the apparently reflected radiation is of the same quality as the incident or if it is a kind of secondary radiation. These experiments explain the difficulty experienced in obtaining a reflection when the thermoluminescent method was used, as the paper used to cover all but a portion of the mirror would reflect almost equally with the mirror itself. A general luminescence should have been looked for and not one bearing a resemblance to the rectangular patch of light seen on the salt. It is worth while, perhaps, to note that the light producing fluorescence in willemite is reflected quite differently from paper and silver, as in the above case the fluorescence on the willemite screen was rectangular, corresponding to reflection from the mirror itself.

## VI. SOME ADDITIONAL EXPERIMENTS ON IONIZATION.

The effect of turpentine vapor on the ionization was so great, that it was thought it might be due to some chemical action and be connected with the action of turpentine vapor on ozone. That the ionization is not directly due to the formation of ozone was shown by the following experiment. A piece of potassium iodide starch paper was placed in the chamber rather close to the film and exposed to the spark a certain time. Then another piece was exposed in the same position but with quartz over the window instead of the celluloid film. The paper showed slightly more darkening under the quartz than under the film.

It was found also that whether an appreciable leak was observed with a quartz window, or not, it was not increased by the presence of vapors. Thus on one occasion the leak through quartz was 5 scale divisions in 60 seconds, adding turpentine the leak was 50 scale divisions in 3½ seconds through a film, but 4 scale divisions in 60 seconds through quartz. With methyl iodide vapor, when the leak with a film window was 50 scale divisions in 1.8 seconds, with a quartz window it was 1 scale division per minute.

It was further found that with a stream of coal gas flowing through the chamber, in which was a little turpentine, an ionization effect was still observed, the leak being 8 scale divisions in 60 seconds. That it was not larger is evidently due to the fact that the air above the film was so impregnated with coal gas that a large portion of the radiation was absorbed before reaching the chamber.

Attempts were made to see if soap films could be made which would transmit this radiation. A leak of from .5 to 1 scale division per minute was obtained with the window covered with a film showing colors, made from a solution to which a little glycerine had been added, but this leak was not increased by the presence of turpentine vapor.

From the preceding experiments it is evident that there is a radiation from the spark in air, at atmospheric pressure, which produces thermoluminescence in certain salts, and ionizes the gas through which it passes, and that it is not mechanical in its nature such as particles driven from the electrodes or a pressure wave. That it is not of the nature of $\alpha$ or $\beta$ rays, or neutral pairs shot off from the spark appears evident when it is considered that this radiation passes through 10 cm. of air, and yet is stopped by very thin aluminum. This is quite contrary to the behavior of material particles moving with high velocities. It is evidently not light of wave-length greater than 1,000 Angström units, the limit of the spectrum photographed by Lyman, since in the known regions quartz is everywhere comparable in transparency with air.

The question whether this radiation may be light of extremely short wave-length, or soft Röntgen rays, is exceedingly interesting. Lenard found that ultra-violet light in the region studied by Schumann and by Lyman would produce ionization in a gas, and it would seem plausible that light of still shorter wave-length should act more vigorously. Since in the known region of the spectrum small amounts of substances sometimes affect the transmission enormously, one need not be surprised at the large effect of various vapors on the ionization if light were the cause. The existence of shadows and of reflection would also be directly explained. On the other hand, the fact that the results are independent of the material of the electrodes, when all known light effects, including the ionization observed by Lenard, vary greatly with different materials is a reason for thinking the effect more electrical in its origin, in the sense of being independent of the particular atom. Theory leads one to expect a sort of soft Röntgen radiation from a spark due to the accelerated or retarded motions of ions, and this explanation of Entladungsstrahlen has already been given.[1] Such radiation would probably be increased in intensity by additional capacity, it also would travel in straight lines, one would expect it to produce ionization. It might seem remarkable that the ionization produced by such a radiation should vary so greatly with different vapors. Experiments show, however, that the ionization produced by X-rays from a soft tube is not at all proportional to the density. Strutt[2] gives the ionization in methyl iodide as 72

---

[1] J. J. Thomson, Conduction of Electricity through Gases, p. 603.

[2] Proc. Roy. Soc., 72, p. 209, 1903.

times that in air at the same pressure. The radiation in question would be softer than that from any tube; it is probably absorbed 80 per cent. by 10 cm. of air, as compared with one per cent. for radiation from a tube; and it might reasonably be concluded that the ionization produced by it would bear little relation to the density of the substance traversed. If the penetrating power and lack of reflection in X-rays is due to the thinness of the pulse, here, where the change in the motion of the ions is less rapid, the pulse would be less thin, the properties would be somewhat altered, and an explanation of the apparent reflection and the absorption by thin aluminum and other substances might be found.

Facts bearing on the penetrating power of the radiation might enable one to distinguish between the hypothesis of a light radiation and a soft Röntgen radiation, as the cause of the effects noted. Light of a given wave-length would have a constant coefficient of absorption whatever its source, whereas Röntgen rays produced under various conditions, have different penetrating powers. Sir J. J. Thomson[1] found some years ago that ionization was produced near the spark, but since at atmospheric pressure the ionization chamber had to be placed within 1 cm. of the spark for an effect, it did not seem clear that the ionization observed was due to a special radiation, and there seemed a possibility that the same observed at lower pressures was due to known ultra-violet light. When one adds however to these experiments the writer's observation that with a small induction coil a layer of 2 cm. of air produced a large absorption, and that using a larger coil with capacity, 5 cm. produced perhaps no more, and when one notes that Hoffmann with a very large Wimshurst machine observed still less absorption, one is led to think that there is a radiation here of different penetrating power depending on the circumstances of production. To these facts relating to penetrating power should be added the observation o Hoffmann, confirmed at a pressure of about 2 mm. by the writer, that with diminishing pressure the radiation gains the power of penetrating and affecting a photographic plate. It is not yet proved beyond doubt that it is the same quality of radiation in the two cases, but that seems most probable. In that case the properties

---

[1] Proc. Camb. Phil. Soc., X., p. 74, 1899.

of "Entladungsstrahlen" would ally them very closely to soft Röntgen rays.

In conclusion the writer desires to express her thanks to Prof. Sir J. J. Thomson who made it possible for her to carry out this investigation at the Cavendish Laboratory.

Mount Holyoke College.

# RELATION BETWEEN TEMPERATURE OF ELECTRODES AND VOLTAGE OF THE ELECTRIC ARC.

BY C. D. CHILD.

THE theory of the electric arc, as at present held, leads us to expect that with cooled electrodes a higher potential difference will be needed to maintain the arc than with electrodes not cooled, but the experimental evidence to a large extent favors the opposite view. It, therefore, seemed worth the trouble to review what has been done on the subject and to repeat some of the experiments.

*Résumé of Previous Experiments.* — It may first of all be stated that phenomena which bear indirectly on this subject lead us to expect that the arc will be maintained more easily when the terminals are hot than when they are cooled. For example, if the terminals of the arc are separated for some distance when it is first started, the arc is more apt to go out, than it is, if the terminals are allowed to become hot before being separated, as if heating the terminals allows the current to flow more easily.

Again it is much more difficult to maintain an arc in hydrogen than in air, and this has generally been explained as being due to the fact that hydrogen cools the electrodes more rapidly than air, as if cooling the electrodes made necessary a greater potential difference.

When we come to the more direct evidence we find that many of the earlier experiments are of little value.

Such for example as that of de la Rive[1] who attempted to heat the arc by placing an alcohol lamp under it. There would be three results produced by this experiment other than the one which he had in mind. First, the flame would probably cool the carbons of the arc instead of heating them. Secondly, the flame would blow

---

[1] Archives de l'Electricité, 1, 262, 1841.

the arc out of its original shape, and thus cause a greater drop in potential through it. Lastly, the gases of the flame would change the character of the arc and this might have a very appreciable effect on the voltage.

Tommasi[1] formed an arc between copper tubes, and found that when they were cooled by passing water through them, the arc was very unsteady and the luminosity became less. This is one of the most satisfactory methods of examining the question, but his results are not as definite as is desired.

Cross and Shepard[2] endeavored to heat the electrodes by protecting them from convection currents. This method is more satisfactory than some, and yet it is conceivable that the protection would alter the character of the gas sufficiently to change the potential difference.

They cooled the anode by surrounding it with a water jacket. This method would be entirely satisfactory providing no water vapor was allowed to pass into the arc. This experiment has been repeated by myself and will be discussed further in connection with that work.

Lecher[3] used three methods for changing the temperature of the electrodes. The first was to warm the electrode with a gas lamp, which we have already seen is open to serious objection. The second method was to wrap fine copper wire about the electrodes, his idea being that the wire would cool the carbon, since it had greater thermal conductivity. It is not, however, self evident that it would do this, for even if the copper had infinite conductivity, it would not cool the carbon unless it was able to get rid of the heat at the surface faster than the carbon does, and one cannot be certain that it would do this. There is also the possibility that the copper would be heated sufficiently to send some of its vapor into the arc, and this would, no doubt, decrease the resistance of the arc.

The third method which he used was to cool the electrode by immersing it in a mercury bath, which was in turn kept cool by running water. This would apparently be a suitable method, providing that the mercury vapor did not pass into the arc. If it should, it would be expected to lower the resistance of the arc.

[1] C. R., 93, 716, 1881.

[2] Proc. Amer. Acad. Sc., 22, 227, 1885.

[3] Wien. Ber., 95, II., 992, 1887.

This experiment was also repeated by myself and will be further considered in connection with the account of that work.

The experiment of Hertzfeld[1] where he blew cold $CO_2$ into the arc was valueless, for the reason that the change in the character of the gases and the distortion of the arc by blowing into it would produce far greater effects than those produced by cooling the electrodes.

Schultze[2] used the same method as that of Tommasi. He experimented with the arc between copper electrodes which could be cooled by passing water through them. He found that cooling the cathode raised the potential difference about one volt, cooling the anode raised it about four volts, and cooling both electrodes about six volts. Where an oxide is formed by the action of the arc this effect was not found. Even with iron the effect could not be shown. This was partly due to the smaller heat conductivity of the iron.

Mitkiewitz[3] found that a current of 2 amperes could be passed through an arc with a potential difference of only 2 volts if the anode was heated in an oxy-hydrogen flame. But it is a question whether this is an arc at all and not merely the discharge produced by a hot flame.

Stark and Cassuto[4] cooled the electrodes by placing a small quantity of the substance to be examined on the top of a water cooled jacket. They were unable to determine with certainty any change when carbon electrodes were used. With copper electrodes they found the voltage much lower when the arc was several millimeters long and no change when it was 2 mm. long. From this they concluded that with still shorter lengths the voltage would be higher with hot electrodes and that the sum of the anode and cathode drops is greater with hot electrodes than with cold ones.

Malcolm and Simon[5] using copper electrodes and small currents with arcs 4 mm. long found that the cooled electrodes required higher voltages. With currents varying from .4 to 1 ampere the voltage was from 20 to 5 volts higher when the anode was cool

---

[1] Wied. Ann., 62, 442, 1897.

[2] Ann. d. Phys., 12, 837, 1903.

[3] Beib., 29, 739, 1904.

[4] Phys. Z. S., 5, 267, 1904.

[5] Phys. Z. S., 8, 478, 1907.

than when both electrodes were hot, and 40 to 10 volts higher when the cathode was cooled.

Of the experiments, therefore, which are least open to objection there are two sets, and these two arrive at directly opposite conclusions. Cross and Shepard using carbon electrodes came to the conclusion that cooling the electrodes lowered the required voltage, while Schultze, Stark and Cassuto, and Simon and Malcolm using copper electrodes reached the conclusion that cooling the electrodes raises the required voltage. It therefore seemed worth while to repeat some of these experiments and to find the cause of the discrepancy.

*Electrodes Cooled by Water-jacket.* — The experiment of Cross and Shepard was first repeated. One of the electrodes was surrounded by a copper jacket through which water could be passed. This was made to fit tightly around the carbon, so that no steam could get into the arc. The top of the carbon extended between 1 and 2 mm. above the top of the jacket which was made out of sheet copper, so that the water would come as near as possible to the top of anode. The carbon was 12 mm. in diameter.

The arc was started without any water in the jacket and allowed to run as long as possible without overheating the apparatus. It was then cooled by passing water through the jacket. The potential difference at the terminals of the arc was taken before the anode was cooled and again immediately afterwards. The water was then allowed to flow out of the jacket and the potential difference again noted.

When the cathode was thus cooled there was an increase in the potential difference of 2 or 3 volts. When the temperature increased on drawing off the water there was a decrease in voltage of about 1 volt. It was to be expected that the decrease would not be as large as the increase, since the potential difference would gradually increase due to the burning away of the electrodes.

This change was found to be practically the same for currents varying from 8 to 20 amperes and for lengths varying from 1 to 5 mm., as long as the arc was quiet. With the hissing arc it was difficult to detect any change when the jacket was filled with water.

When the anode was cooled, no definite change could be detected, but this may have been due to the difficulty of performing the experiment properly. The anode was so hot that it began to melt the jacket if the arc ran for more than a few seconds without being cooled. This may be remedied by raising the carbons further above the jacket, but then the water has no effect on the temperature of the carbon.

Practically the same effects were observed with solid and with cored carbons, but with a mineralized carbon, such as is used in the "flaming arc," no effect could be definitely determined.

These results are opposite to those found by Cross and Shepard, and to find the cause of the difference the experiment was again performed, but with the water vapor passing freely into the arc. For this purpose the electrode was placed in a water jacket having no top, the arc being just above the surface of the water when the jacket was filled. Under these conditions it was found that filling the jacket with water always caused the arc to go into the hissing form, and with this change there was, of course, a large decrease in the voltage. If the arc was hissing before the water was passed into the jacket, cooling the anode apparently had no effect. It is possible that the decrease in voltage which Cross and Shepard found was due to the change to the hissing form of the arc and not to the change in the temperature of the electrodes.

This same experiment was then repeated, using mercury instead of water to cool the electrode as was done by Lecher. A jacket with open top was used so that the vapor of the mercury could pass into the arc freely. It was found that there was a decrease in the potential difference of 10 or 15 volts, when the mercury reached the hot part of the anode.

That this was due to the passing into the arc of mercury fumes was shown by the fact that the mercury lines began to appear in the spectrum of the arc even before the decrease in the potential difference, and became very prominent at the time of this decrease. It was also shown by placing a jacket about the lower carbon which was covered with sheet iron. The carbon was fitted tightly into the opening in the sheet iron. With this arrangement the effect was the same as when the carbon had been cooled with the water jacket, namely, there was a slight increase in voltage when the arc was cooled and a slight decrease when it was allowed to rise in temperature.

*Hollow Carbons Cooled by Flow of Water.* — As far as I have been able to learn, the method employed by Tommasi and by Schultze with copper electrodes has not been used with carbon electrodes. It, therefore, seemed worth while to try their method with such electrodes. The arrangement of apparatus is shown in Fig. 1. The arc is formed between a cored or solid carbon *B* and a hollow carbon *C* which was connected to a water faucet so that it could be cooled by running water through it. *V* is a Weston voltmeter for measuring the potential difference between the carbons. *A* is a Weston ammeter for measuring the current. *D* is a 110-volt dynamo. *R* is a variable resistance.

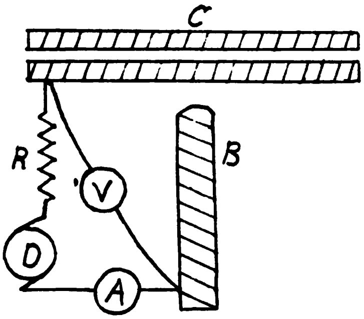


Fig. 1.

*Effect of Cooling Cathode.* — In the first experiments the hollow carbon was used as cathode. It was found that the voltage always rose when the cathode was cooled. On stopping the flow of water the voltage diminished, but did not come entirely back to its original value, due to the burning away of the carbons. The change in voltage occurred, as nearly as one could tell, at the same time as the change in the luminosity of the cathode.

In the first experiments the outer diameter of *C* was 12.5 mm., the inner 4 mm. The diameter of carbon *A* was 10 mm. The potential difference was approximately 55 volts and the current 8 amperes. In this case the increase in potential difference varied from 3 to 6 volts. The average of ten readings was 4.5 volts. The average of ten readings for the decrease when the flow of water had stopped was 1.8 volts.

The change in the voltage appeared to be the same for currents varying from 6 to 15 amperes, but increased somewhat as the length of the arc was increased from 2 to 10 mm.

*Effect of Cooling Anode.* — The change in potential difference is not so great when *C* is used for the anode. It is also more difficult in that case to make satisfactory readings. The arc does not become steady until a crater is formed on the anode and by the time this is done a hole is apt to be burnt through the carbon, so that water flows directly into the arc. But as near as could be determined the potential difference increased about 2 volts on cooling the anode, decreasing nearly the same amount on allowing it again to become heated.

The variation in the voltage appeared to be the same for currents varying from 6 to 15 amperes, but increased somewhat as the length of the arc was increased from 2 to 10 mm.

With the hissing arc the effect was much smaller and so irregular as to make accurate measurements impossible. There was apparently an increase of 1 or 2 volts on cooling the cathode and a decrease of something less than 1 volt on allowing it to again become heated.

With thicker carbons the effect was, of course, smaller. Thus with carbons having an outside diameter of 16 mm. and an inside diameter of 4 mm. the largest increase was 2 volts. With an outside diameter of only 10 mm. and the same inside diameter the effect was somewhat greater than that given above.

Objection may perhaps be raised against this method due to the fact that the carbon was porous. It is conceivable that steam passes through the carbon and changes the arc by the introduction of a different gas. To test this, steam instead of water was passed through the hollow carbon. This was under a pressure of 5 cm. of water, and might have been expected to have a greater effect than the water, if the effect was due to the passing of the steam through the carbon; first, because the steam was under pressure, while the water was not, and secondly, because the carbon was red hot with the steam and it is more porous when hot than when cold. But it was found that the passing of the steam had no appreciable effect on the arc.

The second test was made by inserting a hollow copper tube within the carbon rod. This would keep any water vapor from coming into contact with the carbon, but it also protected the carbon from the cooling effect of the water. However, there was an increase in voltage each time that the carbon was cooled. This was approximately 2 volts.

*Effect of Heating Electrodes.* — A second method of studying this problem is to heat the electrode $C$ by passing an electric current through it. For this purpose a small carbon 3.3 mm. in diameter was used. It was found, however, that when the current was sent through this carbon, the magnetic field about it deflected the arc. This caused the voltage to be higher and also made the arc unstable, so that any effect produced by heating the carbon was masked by the other change.

Pressures of the surrounding gas as low as .1 mm. were also tried with this method, and while the magnetic effect was then less troublesome, it was still impossible to determine whether there was any effect due to the heating of the electrode or not.

*Change in the Cathode and Anode Drops.* — In the explanation of the arc commonly given today[1] it is claimed that the fundamental difference between the arc and other forms of discharge through gases is that in the arc the hot cathode allows the electricity to pass from it with great ease. To do this the cathode must have a high temperature and to produce the high temperature there must be an expenditure of electrical energy in its neighborhood. The rate at which this is done equals the cathode drop times the current. It is natural to expect that if the cathode is cooled artificially, it will require a greater expenditure of energy to maintain the needed temperature, and to do this with a given current the cathode drop must be greater. Similarly with any method of heating we should expect the cathode drop to be less.

To test this view, the cathode drop was examined by inserting a carbon pencil 1.7 mm. in diameter between the electrodes of the arc and connecting this through a voltmeter to the cathode. The arc was approximately 4 mm. long and the exploring electrode was placed as near the middle of the arc as possible. The potential difference measured in this way included both the cathode drop and half of the fall of potential through the vapor of the arc, but the method answers for getting an approximation to the cathode drop. Contrary to what had been expected, it was found that there was but a small change in the value of the cathode drop when the cathode was cooled. The anode drop was then examined and it was

---

[1] Phys. Rev., 19, 126, 1904.

found that there was here a large change. In one case the total potential difference changed from 41 to 47 volts, the cathode drop from 6 to 7 volts and the anode drop from 35 to 40 volts. It is evident from this that the change in potential produced by cooling the cathode does not all occur near the cathode. On the contrary it would appear that the change in different parts of the arc is roughly proportional to the potential existing before the cathode was cooled.

In order to test this view a shunt was placed about the arc and a point on this which had the same potential as that of the exploring pencil was connected to it through a voltmeter. While this connection was made, the cathode was again cooled. At times when this was done the voltmeter would give no reading. At other times there was a small reading in a direction indicating that the ratio of the increase to the original value was slightly greater in the cathode drop than in the remaining part of the arc, but it could be said that the change in the different parts of the arc is nearly proportional to the original potential.

While this was contrary to what I had expected, it is not difficult to explain the phenomena observed. It was thought that extra energy would be developed near the cathode, so that the temperature in its immediate neighborhood would be the same as before. It would appear that what really happens is that the bright spot on the cathode is not as hot as before and that fewer ions pass into the vapor of the arc and consequently its conductivity is less.

This view is also in agreement with the fact mentioned above that the change in potential was found to be greater when the arc was long than when it was short. It apparently follows from this that the anode drop itself depends on the ions produced at some distance from it. There are other facts which would indicate the same. For example the anode drop becomes very much smaller when a salt is placed in the cathode, or when the cathode is a metal instead of a carbon.

*Effect of Heating Anode in Mercury Arc.* — Stark, Retchinsky, and Schaposnikoff[1] found that with a carbon anode in a mercury arc the anode drop was much smaller when the anode was cold than when hot. It varied in their experiment from 12.4 volts when

[1] Ann. d. Phys. 18, 247, 1905.

the carbon was bright red to 4.7 volts, when it was non-luminous. In the experiment described above there was but a very small change produced by cooling the anode. Because of the difference in these results, their experiment was repeated with the conditions as nearly as possible the same as those described by them.

While a small increase in the anode drop was at times found to accompany a rise in temperature of the anode, it was one of only two or three volts, and it was not possible to show definitely that it was due to the increase in temperature and not to an increase in the pressure of the gas. Ordinarily a carbon rod when heated in a vacuum gives out a large quantity of gas and absorbs some of this again on cooling. Under certain conditions a slight change in the pressure of the gas will produce a large change in the value of the anode drop, and I was not able to find any change in the anode drop when there was no change in the pressure.

This, however, would not explain the results of Stark, Retchinsky and Schaposnikoff for in their experiments the tube containing the arc was not connected to the pump after the arc was started. There was a rise in the pressure of the mercury vapor and the increased amount of vapor *cooled off* the anode so that it became non-luminous. So that with the increase in the amount of pressure there was a decrease in the temperature of the electrode and a decrease in the anode drop. As far as the relations between the pressure and the anode drop, and between the pressure and the temperature of the anode are concerned, this was exactly opposite to that found by myself. I cannot but feel that it would be well if their experiment could be again repeated and more exact measurements made of the various related quantities.

*Summary.* — A few of the experiments which have been tried by others for the purpose of finding the effect on the electric arc produced by cooling the electrodes were repeated with some modifications and it was shown that cooling the cathode of a carbon arc raised the voltage several volts. There was a smaller increase in the voltage when the anode was cooled.

The anode and cathode drops were measured and both were found to increase when the cathode was cooled, the principal increase being in the anode drop. The increase in the potential in different parts of the arc was approximately proportional to the potential existing before the cathode was cooled.

Part of this work was done in the physical laboratory of Cornell University, and I desire to express my thanks to Professor Nichols for the privilege which was given me of working in that laboratory.

COLGATE UNIVERSITY,
October, 1909.

# NOTE ON WATER OF CRYSTALLIZATION.

BY W. W. COBLENTZ.

IN previous papers[1] on this subject the writer has shown that in substances containing "*water of crystallization*," or "*water of solid solution*" the absorption bands coincide with those of water in its free, liquid state. On the other hand, substances which contain "*water of constitution*" (*i. e.*, the constituents which form water when the substance is heated to a high temperature) do not show absorption bands of water, except hydroxyl groups, which show a band at 3 $\mu$.

The spectroradiometric analysis does not appear to distinguish between water of crystallization and water of solid solution. The absorption bands are identical in position and perhaps in intensity. There are, however, other physical and chemical tests which show a marked distinction between these two forms.

In the process of crystallizing water may enter the substance in varying proportions; but there is a fixed proportion corresponding to a definite compound that is homogeneous and has definite physical properties.[2] On dehydration, the substance becomes inhomogeneous — copper sulphate being a good example. The water escapes at one or more fixed temperature points (and continues to go off at such fixed temperatures until all water is given off) with a sudden change in its properties. On the other hand, in a substance containing water of solid solution or dissolved water, the water is given off gradually throughout a range of temperatures, the physical properties varying with the change in composition. Since there is no loss in homogeneity, and since the water is lost gradually throughout a range of temperatures it cannot be chemically combined. The phenomenon is supposed to be molecular, while in

---

[1] Coblentz, PHYS. REV., Vol. 20, p. 252, 1905; Vol. 23, p. 125, 1906. Jahrbuch der Radioaktivitat und Elektronik, III Band, Heft 4. Carnegie Publication, No. 65.

[2] See a clear and concise summary of a recent paper on this subject by Zambonini, given by Professor Hillebrand in Chemical Abstracts, 3, p. 412, 1909.

the crystallization process the behavior would be atomic in its action in order to form a true compound. This leaves an outstanding difference in the interpretation of the various observations which remains unexplained; for the spectroradiometric observations indicate that, both in compounds with water of crystallization and in those of solid solution, the water is present (*i. e.*, has the same physical properties to heat waves) in the same condition as water in its free, liquid state. Selenite, for example, shows the composite spectra of anhydrite ($CaSO_4$) and of water ($H_2O$); similarly for opal (quartz + water) which is a solid solution. It has been found that the absorption spectrum of a compound is not the composite of the spectra of the constituent elements (*e. g.*, the spectrum of HCl is not the composite of the absorption spectra of H and of Cl, etc.). How then are we to interpret observations on crystallization products? These differences need not disconcert us, for the spectroradiometric test is continually finding new phenomena needing explanation, the most important, perhaps, being the question of density. Ångström[1] has shown that water vapor is more transparent than the liquid phase, when in such layers that the thickness is inversely proportional to the density of the existing phase. A familiar example is (atmospheric) water vapor which transmits the sun's radiation[2] to $11\,\mu$ while a layer of water about two centimeters thick, which is equivalent to the water vapor in the atmosphere, absorbs everything beyond $1.2\,\mu$. Recently he has shown[3] that the absorption of the same quantity of $CO_2$ varies with the pressure, and that if, when under reduced pressure, an inert gas (*e. g.*, hydrogen) is introduced in such quantities that the original pressure is restored, then the absorption bands of the $CO_2$ increase to their original intensity. These facts are of interest in connection with the experimental data on tremolite given on a subsequent page. In previous work the writer failed to detect water in amphiboles which generally contain a few per cent. of dissolved water. Through the kindness of Dr. E. T. Allen, of the Geophysical Laboratory, an opportunity was granted to examine several samples of tremolites in which he found

---

[1] Ångström, Ann. der Phys. (3), 39, p. 267, 1890; (4), 6, p. 163, 1901.

[2] See Carnegie Publication, No. 97, p. 143.

[3] Ångström, Arkiv for Matematik Astronomi och Fysik, 4, No. 3, 1908. Eva von Bahr, Ann. der Phys. (4), 29, p. 780, 1909.

about 2 per cent. of water[1] which is present in solid solution. The sections of the mineral were chosen of sufficient thickness to insure detecting the water, provided its absorption is as great as in the free liquid phase, but as will be noticed presently no water bands could be detected. Its behavior is therefore somewhat like the highly attenuated water vapor in our atmosphere. The bolographs made by Langley[2] show these water bands to be complex and probably shifted slightly towards the short wave-lengths. They show wide absorption bands of water at .92 μ and 1.1 μ which in the liquid are not measurable until the layer has attained a length of about a meter, as shown by Abney and by Julius.

In the present spectroradiometric examination of minerals, a bolometer and fluorite prism were used. The dispersion is much larger than in previous work, so that the absorption bands are much wider than previously found. (See selenite, Fig. 2, and the same specimen described in Carnegie Publication 65, Fig. 2.)

WATER ($H_2O$).

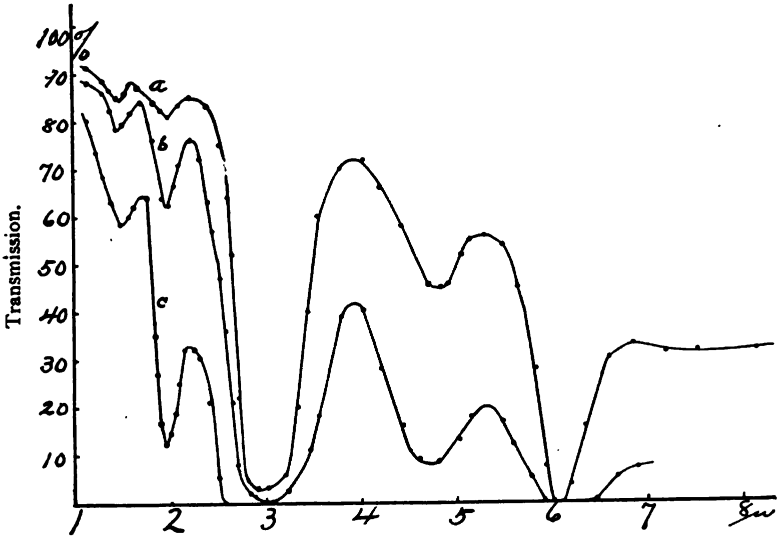


Fig. 2. Water. Curve a; $t = .014$ mm. Curve b; $t = .038$ mm. Curve c; $t = .31$ mm.

---

[1] Allen and Clement, Amer. Jour. of Sci., Vol. 26, p. 101, 1908.

[2] Annals Astrophys. Obs., Vol. 1.

In Fig. 1 are given the transmission spectra of pure distilled water, held between two clear plates of fluorite. The thicknesses of the layers of water (accurately determined by placing different thicknesses of tinfoil between the plates) was as follows, viz: Curve $a$ = .014 mm., curve $b$ = .038 mm. and curve $c$ = .31 mm. of water. For thinner layers see Carnegie Publication No. 65 and the PHYSICAL REVIEW. The important absorption bands occur at 1.48, 1.95, 3.0, 4.7 and 6.1 μ respectively.

### SELENITE ($CaSO_4 + 2H_2O$).

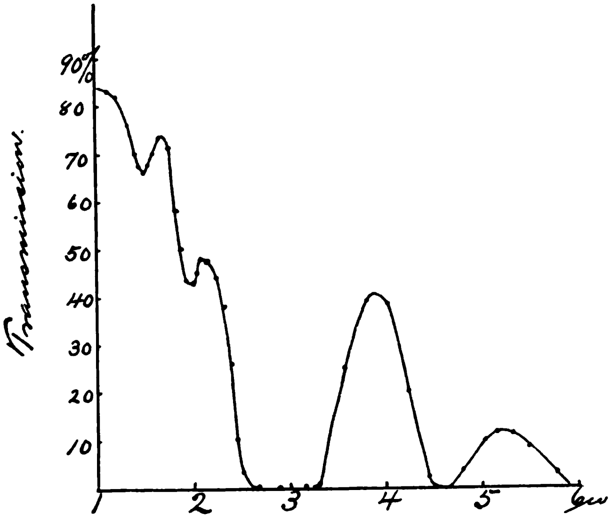


Fig. 2. Selenite. $t$ = .648 mm.; water = .318 mm.

The transmission spectrum of a specimen of selenite 0.648 mm. in thickness, is given in Fig. 2. Since 21 per cent. of selenite is water, the thickness of the water if free would be 0.318 mm. (sp. gr. of selenite is 2.33). The specimen is transparent to 6 μ, while a layer of water 0.31 mm. is already opaque at 2.5 μ. The density of the water in selenite is so reduced that the absorption is only about one sixth to one tenth as great, see Fig. 1, curve $b$. For other thinner specimens see previous papers just quoted.

## TREMOLITE, CaMg$(SiO_3)_4$.

The specimen of tremolite was quite transparent. The transmission of a section 0.23 mm. in thickness is given in curve *a*, Fig. 3. The equivalent layer of water would be 0.0138 mm. It is evident from this and from the transmission of a layer of water .014 mm. (Fig. 1), that the water in this mineral behaves like a highly attenuated vapor. No absorption bands of water are visible, the small bands which are present being common to silicates. In curve *b* is shown the transmission of a dehydrated specimen of tremolite .015 mm. in thickness. It was an opaque white mass and the increase in transmission with wave-length is no doubt due to the decrease in the scattering of the incident energy.

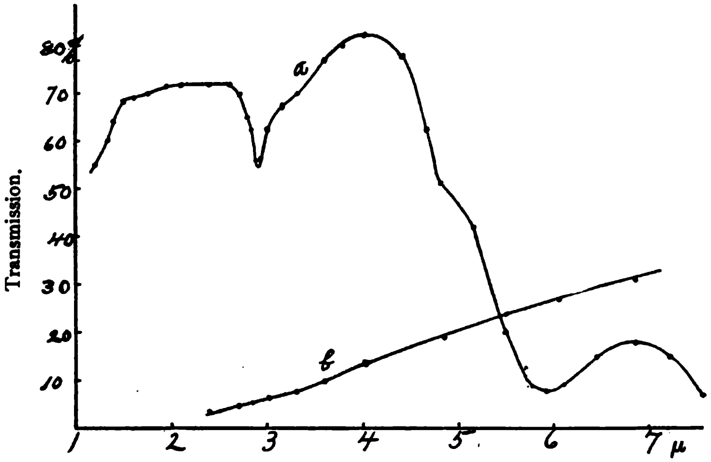


Fig. 3. Tremolite. $t = .23$ mm. Curve b, dehydrated specimen, $t = .015$ mm.

## MUSCOVITE MICA, $H_2KAl_3(SiO_4)_3$.

The transmission spectra are given in Fig. 4. Curve *a* was obtained with a rock salt prism, while curves *b* and *c* were observed with a fluorite prism. The thickness was .04 mm. Curve *b* shows the transmission of the specimen after dehydration. The absorption bands are common to silicates; and none are in common with those of water. The band of selective reflection beyond 9 μ does not seem to be affected by the constitutional water. The other bands disappear on dehydration, as they should, since on dehydration a new compound is formed — or rather, as in this case, only the constituent oxides remain. The layer of mica is too thin to show the weak absorption band of silica at 2.9 $\mu$.

In conclusion it may be added that in substances (whether solid solutions or crystals) containing 7 per cent. of water the water bands were easily detected spectroradiometrically. But when the water content falls as low as 2 to 3 per cent. of the total weight of the substance, then the conditions which obtain in water vapor are found, and in the thin sections investigated the water bands are not detectable. The question then remains unexplained why a crystal containing water, and a solid solution of the same, show the absorption bands of water when other tests indicate that the crystal is a definite compound instead of discrete molecules of the substance cemented together with a definite amount of water.

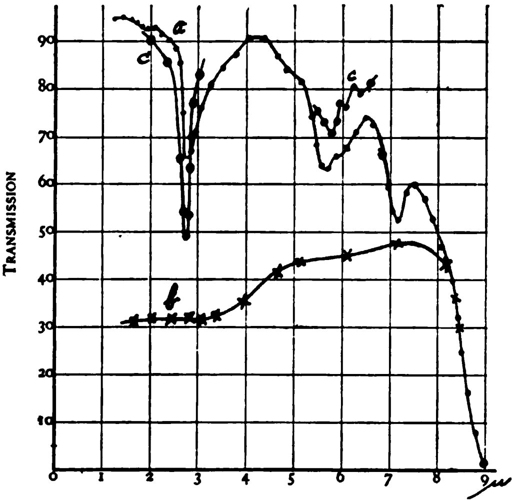


Fig. 4. Muscovite mica. Curve b, the same after dehydration; $t = .04$ mm.

Washington, D. C.,
August 21, 1909.

# STUDIES IN LUMINESCENCE.

By E. L. Nichols and Ernest Merritt.

## XI. The Distribution of Energy in Fluorescence Spectra.[1]

THE energy of most continuous spectra is too feeble to permit of accurate measurements excepting in the infra-red and the longer wave-lengths of the visible spectrum, although we have a few determinations of the energy of the visible spectrum of the acetylene flame by G. W. Stewart[2] and by Coblentz that extend beyond the green. The direct measurement of the energy of even the brightest of fluorescence spectra, which are very small in intensity as compared with those of our ordinary artificial light sources, is therefore impracticable. We have shown however in previous papers of this series that it is possible to make quantitative spectrophotometric comparisons between fluorescence spectra and the spectrum of a standard such as the acetylene flame. If the distribution of energy of the source used for comparison be known, it is therefore easy to compute that of the fluorescence spectrum.

We have adopted this method in determining the energy distribution in the fluorescence spectra of fluorescein, eosin, and resorufin, with the results recorded below. The experimental work naturally falls under three heads: (1) The determination of the energy distribution in the standard source; (2) the spectrophotometric comparison of the fluorescence spectrum with the standard; (3) the measurement of the absorption of the fluorescent liquid, in order that the observed curve of fluorescence may be used to compute the typical curve.

### *Determination of the Distribution of Energy in the Spectrum of the Comparison Flame.*

The comparison source used in the experiments to be described in this paper was an ordinary flat flame from an acetylene burner,

---

[1] A portion of the apparatus used in this investigation was purchased under a grant from the Carnegie Institution.

[2] G. W. Stewart, Phys. Rev., XVI., p. 123.

in front of which, at a distance of 1.4 cm., was mounted a metal screen having a circular hole 0.6 cm. in diameter, so as to cut off the light from all but the brighter central portions of the flame. This flame with its diaphragm was mounted in a metal box having a circular window opposite the diaphragm. The box was fastened to a base fitted to slide along a straight metal track. This track was mounted horizontally in the same vertical plane as the axis of one of the collimators of a Lummer-Brodhun spectrophotometer, and at such a height that the axis of the collimator extended would pass through the window in the box and through the center of the diaphragm to the flame itself.

In front of the slit of the collimator was mounted a sheet of clear white glass, the surface of which had been sufficiently roughened by grinding with powdered carborundum so that at whatever distance the flame might be placed the contrast field of the spectrophotometer would be of uniform brightness throughout. The loss of light by the interposition of the ground glass was found to be about 40 per cent. Its transmission throughout the range of wave-lengths used in our measurements was not measurably selective.

To determine the distribution of energy in the spectrum of the light received from the comparison flame after passing through the ground glass and the optical parts of the spectrophotometer, this spectrum was carefully compared wave-length by wave-length with the light received through the other collimator of the instrument from a black body of known temperature.

Fig. 78.

The black body, Fig. 78, consisted of a tube of Acheson graphite about 50 cm. long, of 1.7 cm. bore and 4.0 cm. external diameter. In the middle this tube was turned down for about 20 cm. until the thickness of the walls was reduced to about 0.4 cm. and the thin-walled cylindrical chamber thus formed was heated by means of an alternating electric current furnished by a step-down transformer, of whose secondary circuit it formed the principal part.

The ends of the cylindrical body were graphite plugs, each with an axial hole one cm. in diameter. Through one of these passed a tube of fused quartz containing a platinum-rhodium-platinum thermo-junction of wires which had been calibrated at the Bureau of Standards. This junction received radiation from the surrounding walls and could be pushed in and out at will so as to ascertain the range of temperatures within the black body. Through the opening in the other plug and through corresponding openings in diaphragms located nearer the ends of the graphite tube, light from the incandescent top of the quartz tube reached the spectrophotometer.

To reduce heat losses and prevent the too rapid oxidation of the graphite, the tube was embedded to a depth of about 8 cm. in a mass of powdered magnesite, which was contained in a hollow cylinder of magnesium oxide and asbestos such as is used for the packing of large steam pipes.

When the primary circuit was supplied with 80 amperes at 110 volts the temperature of this improvised furnace, as indicated by the thermo-junction, rose slowly to nearly 1500° C. at which temperature it remained with little change for a considerable time. Temperatures were determined in the usual way by means of a potentiometer and cadmium cell.

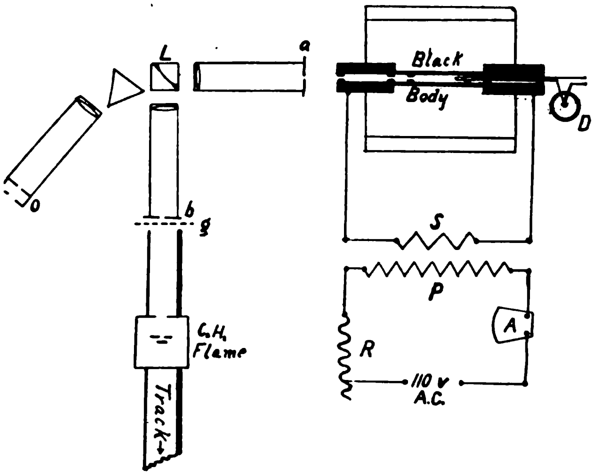


Fig. 79.

The arrangement of the apparatus is shown in Fig. 79, in which *R* is an adjustable resistance, *A* is an a.c. ammeter, *P* is the primary coil of transformer, $S$ is the secondary coil, $D$ is a Dewar flask with the cold junction in ice, $a$ and $b$ are the collimator slits of the spectrophotometer, $L$ is the Lummer-Brodhun prism of the spectrophotometer, $O$ is the observing telescope of the spectrophotometer, $g$ is the ground glass in front of slit $b$.

The slits $a$ and $b$ were set once for all to convenient widths, $b$ being 30 divisions = 0.6 mm. in width, and $a$ 0.06 mm. The adjustable diaphragm in $O$ was of the same width as slit $b$.

In this determination one observer made settings for wave-length and watched the contrast fields of the spectrophotometer, while another recorded the positions of the comparison flame when, for each region of the spectrum, equality had been reached. In the meantime a third observer followed the changes of temperature with the potentiometer and reported the E.M.F. of the thermo-junction for each setting of the spectrophotometer. Readings were begun when a temperature of 1410° (absolute) was reached. Subsequently the current was slightly reduced and further sets of readings were made throughout the spectrum.

From these data the distribution of energy in the spectrum of the comparison flame was computed.

Wien's equation

$$I = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}}$$

was taken as giving the energy in the region of the spectrum at which measurements were made. The accepted value, for an ideal black body, of the constant $C_2$ (*i. e.*, $C_2$, = 14,500) was assumed to be applicable to the present case, and the quantity $C_2/\lambda T$ was calculated from the readings of wave-length and temperature. Since relative values only were desired, the constant $C_1$ was given a value convenient for purposes of computation.

The two slits of the spectrophotometer were maintained at a constant width throughout, and the distance ($d$) of the comparison flame was varied until the intensities of the spectra were equal The energy of any given region of wave-length $\lambda$ was therefore proportional to the ratio $I_\lambda/(1/d^2)$, where $I_\lambda$ was the energy of the corresponding region of spectrum of the light from the black body, computed as above.

Observations were made for twelve regions lying between .477 μ and .656 μ, and in the course of the determination the spectrum was traversed four times. Good agreement existed between the various readings in each region with the exception of the observations immediately following changes made in the resistance of the

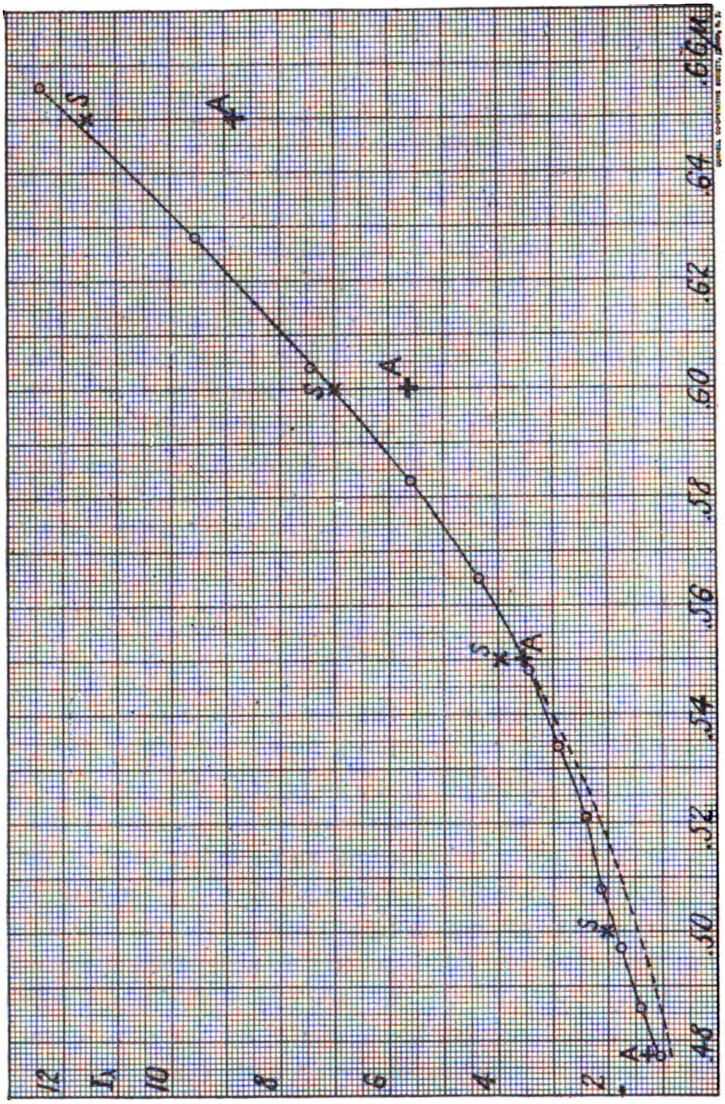


Fig. 80. Distribution of energy in the spectrum of the acetylene flame. The points marked *S* correspond to Stewarts' direct measurements with the radiometer. The points marked *A* are derived from Ångstrom's measurements of the Hefner Lamp. The dotted line is computed from Wien's equation.

primary circuit. These were rejected and all other readings were used in the computations and averaged for each wave-length separately. The results are given in Table I. and are shown graphically in Fig. 80.

TABLE I.

| Wave-length. | $I_\lambda/(1/d^2)$ | Wave-length. | $I_\lambda/(1/d^2)$ |
|---|---|---|---|
| .656 | 248.0 | .534 | 58.70 |
| .628 | 189.0 | .521 | 48.50 |
| .604 | 149.2 | .508 | 42.50 |
| .583 | 112.9 | .497 | 37.73 |
| .565 | 87.0 | .486 | 29.33 |
| .548 | 69.2 | .477 | 21.25 |

Wien's equation has been employed, although in quite a different way, by Knut Ångström[1] for the determination of the distribution of energy in the spectrum of the flame of the Hefner lamp and it is of considerable interest to compare his determination, which involved neither the direct measurement of temperatures nor observations upon a black body, with the results of the experiment just described.

TABLE II.

*Spectrophotometric comparison of the acetylene comparison flame viewed through the circular diaphragm and ground glass, with flame of a Hefner standard lamp.*

| $\lambda$ | $C_2H_2$/Hefner. | $\lambda$ | $C_2H_2$/Hefner. |
|---|---|---|---|
| .656 $\mu$ | .916 | .534 $\mu$ | 1.32 |
| .628 | .950 | .521 | 1.43 |
| .604 | .990 | .508 | 1.53 |
| .590 | 1.00 | .495 | 1.69 |
| .583 | 1.04 | .487 | 1.83 |
| .565 | 1.11, 1.15 | .477 | 1.90 |
| .548 | 1.25 | | |

For this purpose we made a careful spectrophotometric comparison between our acetylene standard and a Hefner flame (see Table II. and Fig. 81). From Ångström's curve of the distribution of energy in the spectrum of the Hefner standard we then computed the relative intensities of our comparison flame for several regions lying between .656 $\mu$ and .483 $\mu$. The points so determined are shown in Fig. 80 by crosses marked "A."

---

[1] Ångström, Nova Acta Upsaliensis, III., Vol. XXII., 1904.

Several points determined by Stewart by direct measurement with the radiometer are also shown in Fig. 80, being marked "S."

It is a matter of some interest to determine to what extent the visible radiation from the acetylene flame corresponds to the visible radiation from a black body. We find that the curve computed from Wien's equation is practically identical with our experimental curve from 0.56 μ to 0.65 μ. But for wave-lengths less than 0.56 μ the curve based upon Wien's equation (shown by the broken line in Fig. 80) deviates considerably from that determined by experiment. The acetylene flame appears to possess a band of abnormally high radiating power in the region lying between .55 μ and the violet end of the spectrum.

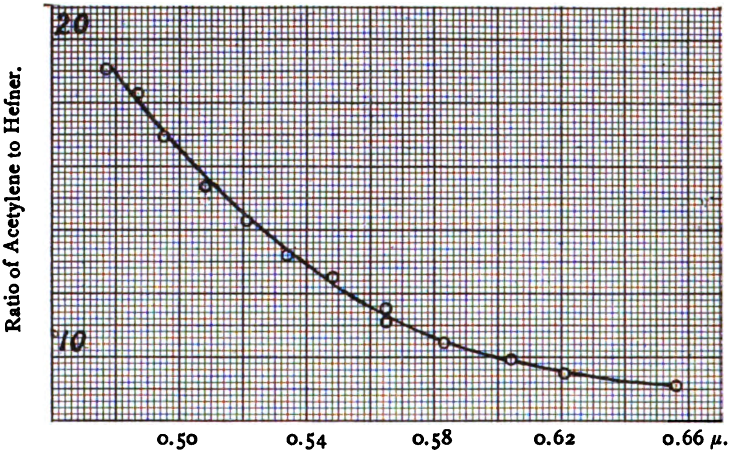


Fig. 81.

### *Comparison of the Fluorescence Spectra with the Spectrum of the Standard Acetylene Flame.*

Before making the final spectrophotometric determinations from which the distribution of energy in the spectrum of various fluorescence spectra were to be computed, a careful study was made of the effect of slit-width upon the form of the observed curves.

In the ordinary use of the Lummer-Brodhun spectrophotometer slit *a* (Fig. 79) would be of constant width and slit *b* would be varied. The accuracy of the screw of the latter was therefore tested by mounting the slit, which had been removed from the instrument, in the field of a projecting lantern and measuring the width of the slit-image, focused upon a horizontal millimeter scale at a distance of about eight meters. Variations from constancy of the ratio of widths to micrometer readings were found negligible for a range of two turns of the screw. Studies of the brightness of the spectrum obtained when this slit was used showed, however, marked deviations from the expected proportionality between slit-width and intensity, especially for widths of less than .01 cm.

It was therefore decided to avoid changes in slit width by using the method of comparison used in determining the distribution of energy in the spectrum of the comparison flame.

In this method the two slits of the spectrophotometer remain unchanged in width, and equality of brightness is obtained for each region of the spectrum by moving the comparison flame along a bar or track parallel to the axis of the collimator. With the system of screens which we employed to exclude stray light, the law of inverse squares was found to hold for the entire range of distances used in our experiments.

The three substances selected for measurement were fluorescein in aqueous solution slightly alkaline, eosine in alcohol, and resorufin in alcohol.

The solutions in each case were as dilute as was found practicable so as to reduce the correction for absorption to a minimum.

In the determination of the fluorescence curves the solution was placed in rectangular cell of white glass (*f*, Fig. 82) and was ex-

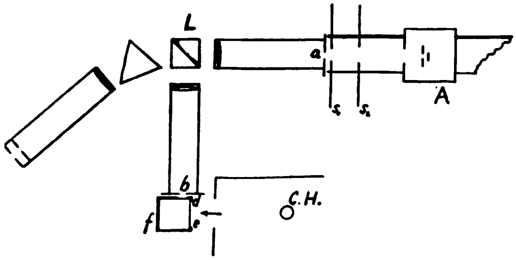


Fig. 82.

cited by the light from a Cooper-Hewitt mercury lamp, *C. H.* The tube of this lamp was vertical and mounted at a distance of about

30 cm. from the wall of the cell. Only those portions of the tube were used which were nearly in the same horizontal plane as the cell. The beam of exciting light entered the cell, *f*, Fig. 82, in the direction of the arrow at right angles to the axis of the collimator.

The cell was enclosed within a metal box with black, matte, oxidized surfaces, and having only the broad rectangular opening *de* for the admission of the exciting light and a narrow, vertical slit-like aperture opposite the slit *b* through which the fluorescence was viewed.

The selection of the mercury arc, with its almost complete absence of light in the region occupied by the fluorescence bands to be measured, afforded further protection against stray light. In the study of the fluorescein solution the additional precaution was taken of inserting a cell of ammonio-sulphate of copper in water between the lamp and the fluorescent liquid, thus cutting off the yellow and green lines of the arc almost completely. The mercury lamp was fed from a storage battery of 120 volts with suitable resistance in series, and under these conditions it furnished an exciting light of unexpected constancy — surpassing in this respect any other source of suitable character and sufficient intensity with which we have had experience.

The arrangement of the apparatus for determining the fluorescence spectra is shown in Fig. 82. The plan is similar to that used in comparing the acetylene flame with the black body. But the photometer track carrying the flame was mounted in line with collimator *a*, while the fluorescence cell was placed in front of slit *b*. In the figure, *A* is the comparison flame; $s_1$ and $s_2$ are screens to prevent stray light from entering slit *a*; *f* is the cell of fluorescent liquid, and *C. H.* is the Cooper-Hewitt mercury arc lamp.

The procedure was as follows:

Slits *a* and *b* were set at equal widths of 50 divisions = .05 cm. The comparison flame was then moved up to a point on the track just in front of the aperture in screen $s_2$. The observing telescope was set for that region of the spectrum corresponding with the maximum of the fluorescence band to be measured. The fluorescent solution in cell *f* was diluted until its spectrum for that region was slightly stronger than the corresponding region in the spectrum of the comparison flame. By slightly shifting the observing telescope in either direction two places could now be found, lying a

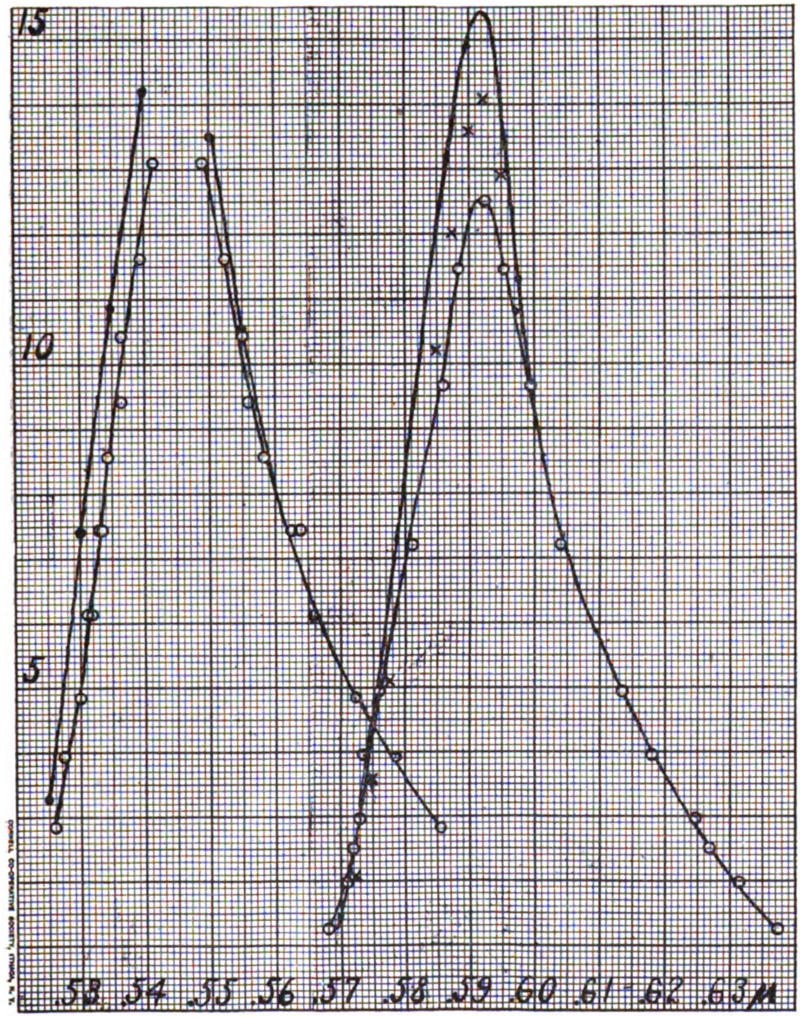


Fig. 83. Eosin (to the left) and Resorufin. The observed points are marked by circles.

short distance from the crest of the fluorescence band, at which the two spectra were of equal brightness. The circle readings of these positions were noted, and the position of the comparison flame was read upon the scale of the photometer track. The flame was then moved to a slightly greater distance from slit *a*, and two new positions were formed for the observing telescope, corresponding to

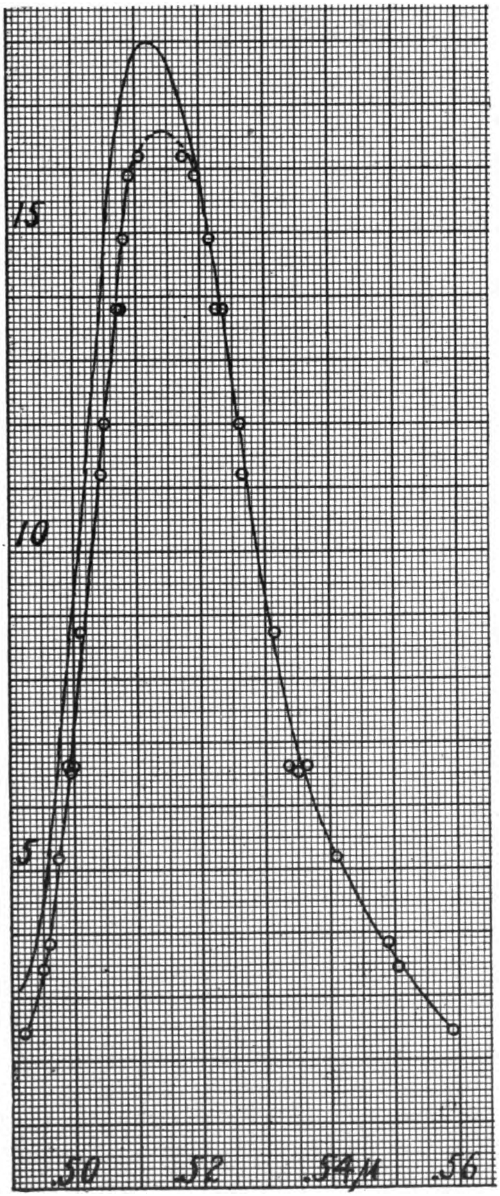


Fig. 84. Fluorescein. The observed points are marked by circles.

points of equal brightness of the band further from the crest. In this way the entire band was explored several times, and from these sets of readings the intensity of the band at various wave-lengths was computed in terms of the corresponding intensities of the spectrum of the acetylene flame.

From these values the observed curves in Figs. 83 and 84 were plotted. The observed points are indicated by circles. In the case of eosin the wave-length of the crest is nearly the same as that of the green mercury line. Measurements in the region of the crest were therefore rendered uncertain by the presence of stray light from the green line. For this reason all the measurements in this region have been discarded. Observations near the crest of the fluorescein curve were also somewhat discordant, so that we do not regard this part of the curve as determined with much accuracy.

To obtain from the observed curves the distribution of energy in the fluorescence spectra it was necessary to make corrections for slit-width and absorption, and to multiply the ordinates of each corrected curve by the ordinates of the same wave-length in the curve giving the distribution of energy in the spectrum of the acetylene flame.

### *The Correction for Slit-width.*

In the spectrophotometric comparison of sources of light having continuous spectra and nearly the same luminosity curves the correction for slit-width disappears; but in the case of spectra consisting of narrow bands this is far from being the case. The slit-width correction used in the determination of the energy curves of incandescent solids does not apply, partly because it is the luminosity of the rays rather than their energy that is important, and partly because the distribution of luminosity in *two* sources has to be considered. The slit correction applicable to the Lummer-Brodhun spectrophotometer may be derived as follows: Let the luminosity curve of the source $S_1$, in front of the slit $A$, have the equation

$$L_1 = f(\lambda)$$

when the distance of the source is such as to give the standard intensity, which we shall call unity. If the distance is varied so that the intensity becomes $i$, then

$$L_1 = if(\lambda),$$

$\lambda$ being the wave-length.

The luminosity of the source $S_2$ in front of the slit *B* will then be given by the equation

$$L_2 = rf(\lambda),$$

where *r* is the ratio of the energy of $S_2$ at the wave-length $\lambda$ to the energy of $S_1$ at the same wave-length. *r* is itself a function of $\lambda$ unless two sources are identical in quality.

Images of the slit *A* are found in the focal plane of the telescope for each wave-length of the spectrum of $S_1$. If the spectrum is continuous, we therefore have a series of overlapping images, forming a spectrum of greater or less impurity according to the width of the slit *A*.

The light reaching the eye from the slit *A* will depend upon the width of *A*, being proportional to this width if other conditions remain constant. But it will also depend upon the width of the aperture, *C*, at the principal focus of the telescope, through which aperture the light used in making settings must pass. If the spectrophotometer is used without an eye-piece, so as to obtain the benefit of the contrast field formed by the Lummer-Brodhun cube, all the light passing through *C* is used in illuminating the field, and the color is that resulting from mixing all the wave-lengths present. When the instrument is set to a match we therefore have equality between the total luminosity of the rays passing through *C* from *A*, and the total luminosity of the rays passing through *C* from *B*.

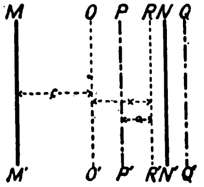


Fig. 85.

An expression for the total luminosity of the rays from *A* may be found as follows:

Let *MN* be the aperture at the focus of the telescope. It will be convenient to express the width $2c$ of this aperture in terms of wave-length. $2c$ is therefore not a constant, even if the actual width of the aperture is invariable, but depends upon the dispersion in the region of the spectrum where the observations are made. The widths of the slits *A* and *B*, denoted by $2a$ and $2b$ respectively, will also be expressed in terms of wave-length. We shall consider first the case were $a < c$.

Let the center of *C* ($OO'$ in the diagram) correspond to the wave-length $\lambda$. That image of the slit *A* which is formed by light of wave-length $\lambda$ will have its center coincident with the center of the aperture. The image formed by light of wave-length $\lambda + x$ will be displaced by the distance $x$, so that its central line falls at $RR'$. For values of $x$ lying between $+(c - a)$ and $-(c - a)$ all of the light forming the image of $a$ will pass through the aperture. The total luminosity reaching the eye from such images will therefore be

$$L_1 = \frac{2am}{d_1^2}\int_{-(c-a)}^{+(c-a)} f(\lambda + x)dx = \frac{2am}{d_1^2}\int_0^{c-a}\{f(\lambda + x) + f(\lambda - x)\}dx,$$

where $d_1$ is the distance of the source $S_1$ from the slit. $m$ is a factor, depending upon the dispersion of the prism, such that $ma$ is proportional to the width of the slit in scale divisions. It is clear that $2am/d^2$ measures the intensity of the light entering the slit, the intensity being unity when the source is at unit distance and the slit width one division.

For values of $x$ lying between $c - a$ and $c + a$ only a part of the image will be transmitted, the transmitted fraction being

$$1 - \frac{x + a - c}{2a}.$$

The total transmitted luminosity from these images that are partially transmitted will therefore be

$$\frac{2am}{d^2}\int_{c-a}^{c+a}\left(1 - \frac{x + a - c}{2a}\right)\{f(\lambda + x) + f(\lambda - x)\}dx$$

$$= \frac{2am}{d^2}\int\left\{\frac{a + c}{2a} - \frac{x}{2a}\right\}\{f(\lambda + x) + f(\lambda - x)\}dx.$$

Upon expanding $f(\lambda + x)$ and $f(\lambda - x)$ we have

$$\begin{aligned} f(\lambda + x) + f(\lambda - x) &= f(\lambda) + xf'(\lambda) + \frac{x^2f''}{2}(\lambda) + \cdots \\ &\quad + f(\lambda) - xf'(\lambda) + \frac{x^2f''}{2}(\lambda) - \cdots \\ &= 2f(\lambda) + x^2f''(\lambda) + \cdots. \end{aligned}$$

For any ordinary case the terms in higher powers of $x$ may be neglected.

The two integrals therefore become

$$\int_0^{c-a}\{f(\lambda+x)+f(\lambda-x)\}dx=\int_0^{c-a}[2f(\lambda)+x^2f''(\lambda)]dx$$

$$=2(c-a)f+\frac{(c-a)^3}{3}f''$$

and

$$\int_{c-a}^{c+a}\left[\frac{c+a}{2a}-\frac{x}{2a}\right][2f+x^2f'']dx=\frac{c+a}{2a}\left[4af+\frac{(c+a)^3-(c-a)^3}{3}f''\right.$$

$$\left.-\frac{1}{2a}\left[\{(c+a)^2-(c-a)^2\}f+\frac{(c+a)^4-(c-a)^4}{4}f''\right]\right.,$$

where $f$ and $f''$ are written for $f(\lambda)$ and $f''(\lambda)$ respectively.

Upon adding the two integrals and introducing the factor $am/d_1^2$ we have

$$L_1=\frac{2ma}{d_1^2}\left[2cf+\frac{a(\overline{c-a}^2+2c^2)}{3}f''\right].$$

In case $a>c$ the corresponding expression is found to be

$$L_1=\frac{2ma}{d_1^2}\left[2cf+\frac{c(a^2+c^2)}{3}f''\right].$$

In any case therefore we have

$$L_1=\frac{2ma}{d_1^2}(2cf+k_1f'')$$

where $k_1$ takes the value $c(c^2+a^2)/3$ when $a>c$, $a(\overline{c-a}^2+2c^2)/3$ when $a<c$, and $2a^3/3$ when $c=a$.

Similarly for that portion of the field which is illuminated by $S_2$

$$L_2=\frac{2mb}{d_2^2}\left[2crf+k_2\left(\frac{\partial}{\partial\lambda}\right)^2(rf)\right].$$

When the two fields are set to equality we have therefore

$$\frac{2ma}{d_1}\left[2cf+k_1f''\right]=\frac{2mb}{d_2^2}\left[2crf+k_2\frac{\partial^2(rf)}{\partial\lambda^2}\right].$$

Expanding $\partial^2(rf)/\partial\lambda^2$ and remembering that the second term in the bracket is in each case small we have:

$$\frac{d_2^2}{d_1^2}\frac{a}{b}\left[1+\frac{k_1 f''}{2cf}\right]=r\left[1+\frac{k_2}{2crf}(r''f+2r'f'+rf'')\right],$$

$$r=\frac{d_1^2}{d_2^2}\frac{a}{b}\left[1+\frac{k_1-k_2}{2c}\frac{f''}{f}-\frac{k_2}{c}\frac{r'f'}{rf}-\frac{k_2}{2c}\frac{r''}{r}\right].$$

An approximate value of $r$, usually a close approximation, is obtained by neglecting all terms after the first. This approximate value having been plotted as a function of $\lambda$, $r'$ and $r''$ may be determined; and since $f'$ and $f''$ may be obtained from the luminosity curve of the source $A$, the correction terms in the above expression can readily be computed.

In the experiments described in this paper the conditions were so chosen as to make $a = b = c$. In this case the expression for $r$ becomes

$$r=\frac{d_1^2}{d_2^2}\left[1-\frac{a^2}{3}\cdot\frac{r''}{r}-\frac{2a^2}{3}\frac{r'}{r}\frac{f'}{f}\right].$$

If $\Delta r$ is the increase in $r$ which results from changing the wave-length from $\lambda$ to $\lambda + 2a$, and if $\Delta f$ is the corresponding increase in $f$, we have approximately:

$$r' = \Delta r/2a, \quad f' = \Delta f/2a.$$

For purposes of graphical computation we may conveniently express the approximate value of $r''$ in terms of the distance $DC$, Fig. 86. If this distance be called $\delta r$ it may be readily shown that $r'' = 2\delta/a^2$,

$$\therefore r=\frac{d_1^2}{d_2^2}\left[1-\frac{2}{3}\frac{\delta r}{r}-\frac{1}{6}\frac{\Delta r}{r}\frac{\Delta f}{f}\right].$$

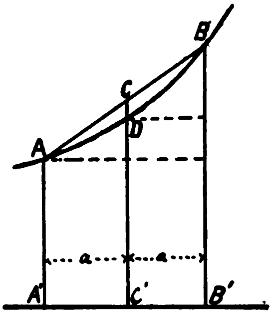


Fig. 86.

In computing these correction terms the luminosity curve given by Tufts was used.[1] The values of $\Delta r$ and $\delta r$ were taken from the curve showing the relation between $\lambda$ and the approximate value of $r$.

---

[1] Tufts' curve was obtained for an incandescent light. But the difference between the color of the acetylene flame and that of the glow lamp in the region considered is so small that the error introduced is quite inappreciable.

The correction depending upon $\delta r$ is important only when the curvature of the $r$ curve is considerable. In the present instance this correction is large for regions near the crest of the curve, but insignificant elsewhere. The importance of the correction is well shown in the curves for resorufin (Fig. 83) where the points marked with crosses show the curve after the slit correction has been applied. In the case of eosin and fluorescein the region near the crest was uncertain for the reasons already mentioned ($p$), so that the slit correction has not been applied in this region.

The second correction term, depending upon the product $\Delta r \Delta f$, is negligible near the crest of the luminosity curve of the source, where $\Delta f$ is small. The term is thus of no significance in the case of eosin and fluorescein. Even with resorufin, where the whole luminescence spectrum lies well to infra side of the crest of the luminosity curve, this correction is important only on the steep side of the curve. The result of applying the correction is shown by the points marked with crosses.

### *The Correction for Absorption.*

The fluorescent light which enters the slit of the spectrophotometer in the foregoing experiments comes from a layer of liquid in the fluorescence cell having a thickness determined by the opening *de*, Fig. 82, through which the exciting light enters the cell. This layer, which is to be considered as uniformly fluorescent, extends from $x = x_1$ to $x = x_2$, where $x$ is the distance from any point from which fluorescent light emanates to the wall of the cell in the direction towards the slit of the spectrophotometer.

Let the intensity of light, of any wave-length $\lambda$, sent towards the slit from unit layer be $i$, and from a layer of thickness $dx$, be $idx$. If the coefficient of absorption of the liquid be $\alpha$ the light reaching the slit from layer $dx$ is $idxe^{-\alpha x}$ and the total intensity of light reaching the slit from the whole of the excited layer is

$$I = i\int_{x_1}^{x_2} e^{-\alpha x} dx$$

$$= \frac{i}{\alpha}\left[e^{-\alpha x_1} - e^{-\alpha x_2}\right].$$

In the cell used in these determinations $x_2$ was 36.5 mm. and $x_1$ was 2.5 mm.

The coefficient $\alpha$ was determined by measuring the transmission of a cell containing a sufficient thickness of the fluorescent liquid to reduce the intensity of light of the wave-length of the middle of the absorption band to about 40 per cent. The correction for the losses of light in the glass and the solvent was determined by measuring transmission through the same cell when filled with absolute alcohol (or in the case of fluorescein with water).

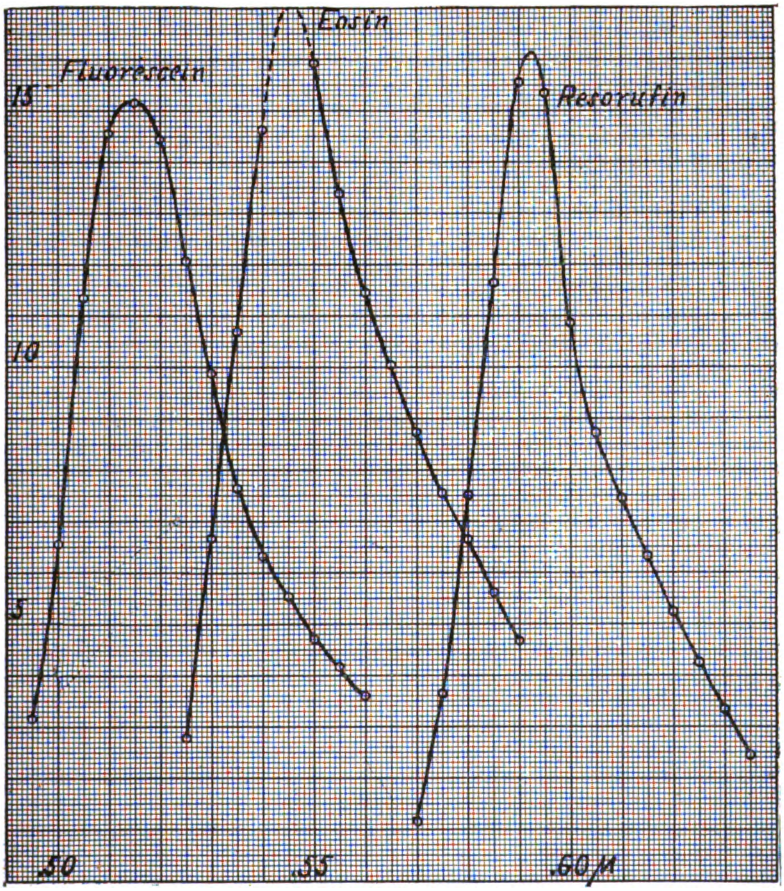


Fig. 87.

The correction for absorption, like that for slit-width, is of importance only in certain regions. The form of the fluorescence curves after the application of both corrections is shown in Figs. 83 and 84.

## *The Energy Curves of Fluorescence.*

From the data corrected in the manner indicated in the two preceding sections curves for the distribution of energy in the fluorescence spectrum of the three substances under investigation were derived by multiplying various ordinates of the corrected curves of Figs. 83 and 84 by the ordinate of same wave-length in the energy curve of the acetylene standard (Fig. 80).

The resulting curves are given in Fig. 87. They suggest by their form a close resemblance between black body radiation and fluorescence, excepting that the range of wave-lengths in the latter case is much smaller. Attempts to find some simply modified form of Wien's equation which will represent the results have thus far been unsuccessful.

*Physics Laboratory of Cornell University,*
*April, 1909.*

# ON THE PROBABILITY OF AN INHERENT RELATION OF THE ELECTRICAL RESISTANCE AND THE HARDNESS OF STEEL.

By Carl Barus.

1. *Introduction.* — Some time ago[1] Professor Strouhal and the writer brought forward a relation between the thermo-electric power and the specific resistance of steel varying with temper from hard to soft. More than 90 states of temper were examined, the rods being of the same kind of Stubb's steel throughout, tempered, etc., with scrupulous care. This relation, as shown in the annexed figure (*a* thermoelectric power in micro-volts relatively to pure silver, *s* specific resistance in microhms per c.c., at 0° C.), was remarkable in the first place for the enormous interval between the extreme values of the variables (change of electrical resistance being over three fold from soft to hard, for instance) and in the second place, from the fact that the coördinates in question determine a straight line. This is not true (*l. c.*) for other alloys. Moreover for the case of such large variations as those in question for steel, one can hardly speak of an initial tangent.

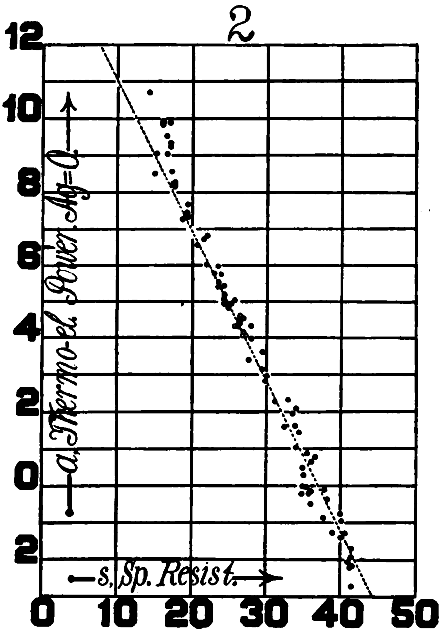


Fig. 1.

---

[1] Wied. Annalen, VII., 1879, p. 383; XI., 1880, p. 930. Also Bulletin No. 14, U. S. Geological Survey, 1885, chapter 2.

The constants of the line for steel were found to be, if

$$a = m + ns,$$

$$m = 15.176, \quad n = -.4123,$$

all data being expressed in c.g.s. units and referred to pure silver and 0° C. Hence $m - a$, which we called thermo-electric hardness, is proportional to $s$ and is virtually referred to a standard steel rod whose specific resistance is zero. This eliminates the silver.

2. *Case of Two Phases.*—The two functions, $a$ and $s$, of the temper of steel between the glass hard and soft states, however sensitive for discrimination, have thus far been regarded as a mere index to that quality. It does not seem improbable that they are more than this, if we regard tempered steel as a mixture of hard and soft molecules, sufficiently intimate to give but little variation of the elastic constants, while the metal passes from hard to soft. I have already made such a supposition in an earlier paper[1] with advantage.

In such a case, let $q$ be the number of hard molecules and $1 - q$ the number of soft molecules per sq. cm. of any interface within the metal. The passage from hard to soft is thus a change of $q$ from 1 to 0. Hence for conduction normal to the interface we may write

$$\frac{1}{s} = \frac{q}{s_1} + \frac{1-q}{s_2}, \tag{1}$$

and

$$\frac{a}{s} = q\frac{a_1}{s_1} + (1-q)\frac{a_2}{s_2}, \tag{2}$$

where $a_1$, $s_1$ denote the thermo-electric power and electrical resistance of extremely hard steel, $a_2$, $s_2$ the same variables for extremely soft steel, while $a$, $s$, refer to any intermediate states of temper. Hence if equations (1) and (2) are to be compatible

$$a = \frac{a_2 s_1 - a_1 s_2}{s_1 - s_2} + \frac{a_1 - a_2}{s_1 - s_2}s. \tag{3}$$

In other words, the relation of $a$ and $s$ is inherently linear as it was actually found to be for steel, but not generally for alloys.

---

[1] PHYS. REV., Vol. XXIX., 1909, pp. 516–524. The substanc e of this and of the above paper were given at Clark University, September 8.

3. *Case of More than Two Phases.* — It is clear moreover that the linear relation will in general cease to hold, if there are more than two kinds of molecules present. In case of three groups for instance

$$\frac{1}{s} = \frac{q_1}{s_1} + \frac{q_2}{s_2} + \frac{1 - q_1 - q_2}{s_3}, \tag{4}$$

$$\frac{a}{s} = q_1 \frac{a_1}{s_1} + q_2 \frac{a_2}{s_2} + (1 - q_1 - q_2) \frac{a_3}{s_3}, \tag{5}$$

whence

$$a = \frac{a_2 s_3 - a_3 s_2}{s_3 - s_2} + \left( \frac{a_3 - a_2}{s_3 - s_2} + \frac{q_1}{s_1} \frac{s_3(a_1 - a_2) + s_1(a_2 - a_3) + s_2(a_3 - a_1)}{s_3 - s_1} \right) s, \tag{6}$$

which reduces to the preceding equation (3) when $q_2 = 0$. In general, however, since $q$ is essentially variable with $s$, it follows that $a$ is no longer linear with respect to $s$. This is the probable case for alloys. To use equation (6), however, some relation between the three groups of molecules present in the ratio of $q_1 : q_2 : 1 - q_1 - q_2$ would have to be supplied, and it would be interesting to endeavor to derive this from investigations with alloys. If there are more phases than three the case would be correspondingly involved.

4. *Conclusion.* — It appears therefore probable that the electrical resistance or the thermo-electric hardness of steel, is more than a mere empiric expression of its state of temper. We may write for $r$ the ratio of the number of hard to the number of soft molecules per c.c. of tempered steel

$$r = \frac{q}{1 - q} = \frac{1/s_2 - 1/s}{1/s - 1/s_1} = \frac{\lambda_2 - \lambda}{\lambda - \lambda_1},$$

*i. e.*, the position which the electrical conduction $\lambda$ takes between the conductions $\lambda_2$ and $\lambda_1$ of extremely soft and extremely hard steel.

From this point of view, moreover, the phenomenon of temper as exhibited in the preceding paper (*l. c.*) becomes an admirable test of a law adequately broad to express dissociation in solid media, subject to time and temperature.

Brown University,
Providence, R. I.

# ON THE THERMODYNAMICS OF BLACK RADIATION.

By Paul Saurel.

In a recent memoir,[1] Planck has obtained formulas which express the energy, the entropy, the electromagnetic momentum and the pressure of a moving hollow space, which is the seat of black radiation, in terms of its volume, temperature and velocity. Two distinct demonstrations are given by Planck. The first consists in combining with the general principles of thermodynamics some formulas that are due to Mosengeil, while the second is an illustration of the fruitfulness of the principle of relativity. In view of the interest which attaches to these formulas it may not be out of place to indicate how simply they can be obtained by combining the general principles of thermodynamics with a single result of electromagnetic theory.

Before considering a moving hollow space let us first consider a hollow space which is at rest, and let us denote the energy, the entropy, the volume, the temperature and the pressure of the black radiation which fills this space by $\varepsilon$, $\eta$, $v$, $t$, $p$ respectively. Then if the general principles of thermodynamics apply to this system, we may write the well known equation

$$d\varepsilon = t d\eta - p dv. \tag{1}$$

But we can go further than this and write

$$\varepsilon = t\eta - pv. \tag{2}$$

To justify this equation we observe that the energy must be a homogeneous function of the first degree of the entropy and the volume and that consequently

$$\varepsilon = \eta \frac{\partial \varepsilon}{\partial \eta} + v \frac{\partial \varepsilon}{\partial v}. \tag{3}$$

---

[1] M. Planck, Zur Dynamik bewegter Systeme, Berliner Berichte, p. 542, 1907; Ann. d. Phys., vol. 26, p. 1, 1908.

If we now make use of the fact that the partial derivatives which appear in this last equation are by equation 1 respectively equal to $t$ and $-p$ we get at once equation 2.

By differentiating equation 2 and by making use of equation 1, we get

$$0 = \eta dt - v dp, \tag{4}$$

and from this it is easy to show that $p$ is a function of $t$. Thus we may write

$$p = f(t). \tag{5}$$

Moreover, from equations 4 and 5 we get

$$\frac{\eta}{v} = \frac{dp}{dt} = f'(t), \tag{6}$$

and finally from equations 2, 5 and 6 it follows that

$$\frac{\varepsilon}{v} = tf'(t) - f(t). \tag{7}$$

Thus far we have used only the general thermodynamic principles. Let us now introduce into our reasoning one result of electromagnetic theory, namely, the fact that the radiation pressure is equal to one third of the specific energy:

$$p = \frac{1}{3}\frac{\varepsilon}{v}. \tag{8}$$

From equations 5, 7 and 8 we get

$$tf'(t) - f(t) = 3f(t), \tag{9}$$

or

$$\frac{f'(t)}{f(t)} = \frac{4}{t}, \tag{10}$$

or finally

$$f(t) = \frac{a}{3} t^4, \tag{11}$$

where $a$ is a constant. If we insert this value of $f(t)$ in equations 5, 6, 7 we get at once the classic formulas for a system at rest

$$p = \frac{a}{3} t^4, \tag{12}$$

$$\eta = \frac{4a}{3} t^3 v, \tag{13}$$

$$\varepsilon = at^4 v. \tag{14}$$

Let us now consider a hollow space moving with the velocity $q$ and let $g$ denote its electromagnetic momentum. Our thermodynamic equation 1 no longer applies; as Planck has shown we must complete it by adding a term $qdg$ corresponding to the work done in adding momentum to the system. Accordingly we write

$$d\varepsilon = td\eta - pdv + qdg. \tag{15}$$

As before we can go further and write

$$\varepsilon = t\eta - pv + qg. \tag{16}$$

For since the energy must be a homogeneous function of the first degree of the entropy, volume and momentum, we may write

$$\varepsilon = \eta \frac{\partial \varepsilon}{\partial \eta} + v \frac{\partial \varepsilon}{\partial v} + g \frac{\partial \varepsilon}{\partial g}. \tag{17}$$

If we notice that the partial derivatives in this equation are, in virtue of equation 15, respectively equal to $t$, $-p$ and $q$ we get at once equation 16.

By differentiating equation 16 and by making use of equation 15 we get

$$0 = \eta dt - vdp + gdq, \tag{18}$$

and from this it is easy to show that $p$ is a function of $t$ and $q$. Thus we may write

$$p = f(t, q). \tag{19}$$

Moreover, if we write equation 18 in the form

$$dp = \frac{\eta}{v} dt + \frac{g}{v} dq, \tag{20}$$

it follows that

$$\frac{\eta}{v} = \frac{\partial p}{\partial t} = \frac{\partial f}{\partial t}, \tag{21}$$

$$\frac{g}{v} = \frac{\partial p}{\partial q} = \frac{\partial f}{\partial q}. \tag{22}$$

Finally, from equations 16, 19, 21 and 22 we get

$$\frac{\varepsilon}{v} = t\frac{\partial f}{\partial t} + q\frac{\partial f}{\partial q} - f. \tag{23}$$

Thus far we have used only general thermodynamic principles. Let us now introduce into our reasoning a theorem taken from electromagnetic theory. The theorem that seems best adapted to our purpose is due to Planck and is expressed by the equation

$$g = \frac{q}{c^2}(\varepsilon + pv), \tag{24}$$

where $c$ represents the velocity of light. A simple demonstration of this formula has been given by Hasenöhrl.[1]

From equations 19, 22, 23 and 24 we obtain

$$\frac{\partial f}{\partial q} = \frac{q}{c^2}\left(t\frac{\partial f}{\partial t} + q\frac{\partial f}{\partial q}\right), \tag{25}$$

or

$$\frac{1 - \frac{q^2}{c^2}}{\frac{q}{c^2}}\frac{\partial f}{\partial q} = t\frac{\partial f}{\partial t}. \tag{26}$$

Let us now introduce a new variable $s$ defined by the equation

$$s = \left(1 - \frac{q^2}{c^2}\right)^{\frac{1}{2}}. \tag{27}$$

Then

$$\frac{\partial f}{\partial q} = \frac{\partial f}{\partial s}\frac{ds}{dq} = -\frac{\partial f}{\partial s}\frac{\frac{q}{c^2}}{\left(1 - \frac{q^2}{c^2}\right)^{\frac{1}{2}}}, \tag{28}$$

and equation 26 becomes

$$s\frac{\partial f}{\partial s} + t\frac{\partial f}{\partial t} = 0. \tag{29}$$

---

[1] F. Hasenöhrl, Zur Thermodynamik bewegter Systeme, Wiener Berichte, vol. 116, 2a, p. 1400, 1907. Cf. M. Abraham, Elektromagnetische Theorie der Strahlung, 2d edition, p. 349, 1908.

From this equation it follows that $f$ must be a homogeneous function of degree zero in $t$ and $s$; in other words, $f$ is a function of the ratio $t/s$. Accordingly, equation 19 becomes

$$p = f\left(\frac{t}{\sqrt{1-\frac{q^2}{c^2}}}\right). \tag{30}$$

If we observe that for $q = 0$ this equation must reduce to equation 12, it follows that

$$p = \frac{a}{3}\frac{t^4}{\left(1-\frac{q^2}{c^2}\right)^2}, \tag{31}$$

and equations 21, 22 and 23 enable us to write at once

$$\eta = \frac{4a}{3}\frac{t^3 v}{\left(1-\frac{q^2}{c^2}\right)^2}, \tag{32}$$

$$g = \frac{4a}{3c^2}t^4 v\frac{q}{\left(1-\frac{q^2}{c^2}\right)^3}, \tag{33}$$

$$\epsilon = \frac{a}{3}t^4 v\frac{3+\frac{q^2}{c^2}}{\left(1-\frac{q^2}{c^2}\right)^3}. \tag{34}$$

Equations 31, 32, 33 and 34 are Planck's equations.

Finally, we observe that since equation 15 can be written in the form

$$d(t\eta + qg - \epsilon) = \eta dt + pdv + gdq, \tag{35}$$

it follows that if we can express the function $t\eta + qg - \epsilon$ in terms of $t$, $v$, $q$, its partial derivatives will give us at once $\eta$, $p$, $g$. This function $t\eta + qg - \epsilon$ is called by Planck the kinetic potential of the system. If we denote the kinetic potential by $h$ and if we make use of equations 16 and 31 we get at once

$$h = t\eta + qg - \varepsilon = pv \tag{36}$$

or

$$h = \frac{a}{3}\frac{t^4 v}{\left(1 - \frac{q^2}{c^2}\right)^2}. \tag{37}$$

This equation 37 is also due to Planck. It is easy to verify that

$$\frac{\partial h}{\partial t} = \eta, \quad \frac{\partial h}{\partial v} = p, \quad \frac{\partial h}{\partial q} = g. \tag{38}$$

New York,
November 20, 1909.

# ON WIEN'S DISPLACEMENT LAW.

By Paul Saurel.

In a previous note we have shown how readily the general principles of thermodynamics lend themselves to the treatment of the problem of a moving hollow space which is the seat of black radiation. In the present note we shall show that the same method can be used to establish Wien's displacement law.

If we consider a hollow space which is the seat of black radiation and if we denote by $\varepsilon$, $\eta$, $v$, $t$, $p$ the energy, entropy, volume, temperature and pressure of the system, we know that we may write the fundamental thermodynamic equation

$$d\varepsilon = t d\eta - p dv.$$

If, however, we suppose that the radiation within the hollow space instead of containing waves of all possible frequencies, contains only waves whose frequencies lie between zero and a certain upper limit $\nu$, then the above equation no longer holds; we must complete it by adding a term which shall correspond to the change in the energy of the system due to a change in the upper limit of frequencies. We are thus led to assume the equation

$$d\varepsilon = t d\eta - p dv + \nu dr, \qquad (1)$$

where $\nu dr$ represents the change in the energy of the system when, at constant entropy and constant volume, the upper limit of frequencies is varied. The new variable $r$ which we have thus introduced may be called the radiation of the system.

If now we assume that the energy of the system is a homogeneous function of the first degree of the entropy $\eta$, the volume $v$ and the radiation $r$, we may write

$$\varepsilon = \eta \frac{\partial \varepsilon}{\partial \eta} + v \frac{\partial \varepsilon}{\partial v} + r \frac{\partial \varepsilon}{\partial r}. \qquad (2)$$

From equation 1, however, it follows that the partial derivatives in equation 2 are respectively equal to $t$, $-p$ and $\nu$. Accordingly, equation 2 becomes

$$\epsilon = t\eta - pv + \nu r. \tag{3}$$

If we differentiate equation 3 and make use of equation 1 we obtain at once the equation

$$0 = \eta dt - vdp + rd\nu. \tag{4}$$

From this equation it is easy to show that $p$ must be a function of $t$ and $\nu$. Accordingly, we may write

$$p = f(t, \nu). \tag{5}$$

Moreover, since equation 4 can be written in the form

$$dp = \frac{\eta}{v}dt + \frac{r}{v}d\nu, \tag{6}$$

it follows that

$$\frac{\eta}{v} = \frac{\partial f}{\partial t}, \tag{7}$$

$$\frac{r}{v} = \frac{\partial f}{\partial \nu}. \tag{8}$$

Finally, from equations 3, 5, 7 and 8 we get

$$\frac{\epsilon}{v} = t\frac{\partial f}{\partial t} + \nu\frac{\partial f}{\partial \nu} - f. \tag{9}$$

Let us now introduce the fact that the radiation pressure is equal to one third of the specific energy. Then from the equation

$$p = \frac{1}{3}\frac{\epsilon}{v} \tag{10}$$

and the equations 5 and 9 we get at once the equation

$$t\frac{\partial f}{\partial t} + \nu\frac{\partial f}{\partial \nu} = 4f. \tag{11}$$

Thus $f$ must be a homogeneous function of the fourth degree in $t$

and $\nu$. Accordingly, we may change our notation and replace $f(\nu, t)$ by the expression $\nu^4 f(\nu/t)$. Equations 5, 7, 8 and 10 then become

$$p = \nu^4 f\left(\frac{\nu}{t}\right), \tag{12}$$

$$\frac{\eta}{\nu} = \frac{\partial}{\partial t}\left[\nu^4 f\left(\frac{\nu}{t}\right)\right], \tag{13}$$

$$\frac{r}{v} = \frac{\partial}{\partial \nu}\left[\nu^4 f\left(\frac{\nu}{t}\right)\right], \tag{14}$$

$$\frac{\varepsilon}{v} = 3\nu^4 f\left(\frac{\nu}{t}\right). \tag{15}$$

Wien's displacement law now follows at once. From equation 15 we get

$$\frac{\partial}{\partial \nu}\left(\frac{\varepsilon}{v}\right) = \frac{\partial}{\partial \nu}\left[3\nu^4 f\left(\frac{\nu}{t}\right)\right], \tag{16}$$

and if we carry out the indicated differentiation we shall find without difficulty that the right-hand side of the equation is equal to the product of $\nu^3$ by a function of $\nu/t$. Thus we may write

$$\frac{\partial}{\partial \nu}\left(\frac{\varepsilon}{v}\right) = \nu^3 F\left(\frac{\nu}{t}\right). \tag{17}$$

This equation is one of the forms of Wien's law.

NEW YORK,
December 13, 1909.

# THE ABSOLUTE VALUES OF THE MOMENTS OF THE ELEMENTARY MAGNETS OF IRON, NICKEL AND MAGNETITE.

By Jakob Kunz.

THE electron theory of magnetism in its present shape based upon the views of Ampere is due to Larmor, M. P. Langevin and P. Weiss. The molecules of a paramagnetic gas under the action of a magnetic field tend to arrange themselves in the direction of the magnetic force, while they are continually deflected by their mutual collisions. If there were no thermal agitation, the maximum intensity of magnetization would be equal to $I_m = N_1 m$, where $N_1$ represents the number of molecules per unit volume and $m$ the resulting magnetic moment of each molecule. The intensity of magnetization $I$ at a temperature $T$ and under the influence of a magnetic field $H$ is given by the formula

$$\frac{I}{I_m} = \frac{\cosh a}{\sinh a} - \frac{1}{a}, \tag{1}$$

$$a = \frac{mH}{rT}.$$

$T$ is the absolute temperature and $rTN = p$. $N$ is the number of molecules of a gas in unit volume under the pressure $p$ and at the temperature $T$. Professor P. Weiss expanded the expression (1) by assuming that every molecule of a ferromagnetic substance is acted upon by the surrounding molecules which produce a uniform field $H_m = Ny$, proportional to the intensity of magnetization at the point considered and coinciding in direction with the direction of the magnetization. This internal or intrinsic field has been termed molecular field. It must account for the strong intensity of magnetization in ferromagnetic bodies by means of the laws of paramagnetic bodies in the same way as the internal pressure added to the external pressure accounts for the high density of liquids. An important analogy has indeed been drawn by P. Curie between, on the one side, the simple law of expansion of a gaseous substance at high temperature and the sudden change which it undergoes on lowering the temperature beyond a critical point so that the mutual attractions of the molecules come into play and produce the liquid state; and on the other side the simple law of magnetization of a substance like iron or nickel at high temperatures and the sudden change which it undergoes when the temperature is lowered beyond the point at which the material passes into its strongly magnetic or ferromagnetic condition. P. Weiss[1] assumes that besides the actions governed by the molecular field the rotations of the molecules in ferromagnetic bodies are as free as the molecules of a gas. Let us suppose that the internal field $H_m = NI$ acts alone without the aid of an external field. The parameter $a$ assumes then the value

$$a = \frac{mNI}{rT}$$

or

$$I = \frac{arT}{mN}. \tag{2}$$

The equation (1) is represented by the curve $OA$ of Fig. 1; the relation (2) is represented by the straight line $OA$. The intensity of magnetization must conform to the two curves, *i. e.*, for a given temperature $T$ the point of intersection $A$ must have an ordinate equal to the ratio $I/I_m$. Of course the origin of the coördinates $O$ also satisfies the same condition, but it can be easily shown that the point $A$ corresponds to the stable equilibrium of magnetization. As the paramagnetic susceptibility is very small, enormous magnetic fields would be required to increase this spontaneous magnetization, which is due to the internal field. Reassuming the mechanical analogy the increase of density of a liquid requires external pressures that are incomparably greater than those by means of which the density of a gas is changed. We arrive then at the conclusion that for the absolute temperature $T$ the ordinate $AA'$ represents the saturation value of the intensity of magnetization; or a ferro-

---

[1] P. Weiss, L'hypothèse du champ moléculaire et la propriété ferromagnétique, Journal de physique, tome VI., 1907, p. 661.

magnetic substance is saturated without the least external field. Though this inference drawn from the theory seems in contradiction with all experimental facts observed, yet it agrees perfectly with the phenomena of magnetization observed in crystals and especially in those of the normal pyrrhotite. This crystal has indeed the very valuable property of a magnetic plane, in which all the elementary magnets are situated. And in this magnet plane appears to be a direction in which saturation takes place almost by itself, at least under very small fields, while a field of about 7,300 units is required to bring about saturation in a perpendicular direction. The supposition of a molecular field accounts indeed, very satisfactorily for the laws that govern the magnetization of the normal pyrrhotite as a function of the external field. The intensity of magnetization as a function of the temperature is a very complicated phenomenon, varying from one substance to the other, and varying also with the magnetic field for the same substance and it appears doubtful whether the most simple hypothesis of the uniform molecular magnetic field is able to account for all the phenomena observed. In the case of magnetite, however, P. Weiss has shown that the theoretical curve coincides exactly with the experimental curve between the temperatures: $-79$ and $+587°$. On the assumption that a piece of ordinary iron is composed of small crystals having the property of a magnetic plane P. Weiss has shown in addition that the loops of hysteresis of annealed iron can be given a theoretical interpretation.

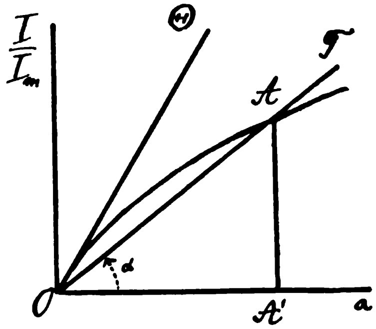


Fig. 1.

In order to determine the absolute values of the internal magnetic field, we have to examine the magnetic properties of the ferromagnetic substances in the neighborhood of the point where the spontaneous magnetization disappears. Thus iron loses the spontaneous magnetization at a temperature of 756° C. Between this point and 920°, the iron has still a considerable susceptibility, the magnetism appearing however only under the combined action of the external and internal field. In this region we have

$$a = \frac{m(H_e + NI)}{rT},$$

(4) $$T = \frac{m(H_e + NI)}{ar},$$

$$\frac{I}{I_m} = \text{cotgh}\, a - \frac{1}{a} = \sum (-1)^{n-1} \cdot 2^{2n} . B_{2n-1} a^{2n-1}/(2n)!$$

$$= \frac{1}{3} a - \frac{2}{90} a^3 + \frac{4}{45 \cdot 42} a^5 - \cdots .$$

$B_1, B_2, B_3 \cdots$ are the numbers of Euler and Bernoulli.

As long as we consider only the beginning of the curve $OB$ of Fig. 1, whose tangent through the origin corresponds to the temperature $\theta = 756 + 273 = 1029$, we may consider only the first term on the right-hand side of the last equation.

$$\frac{I}{I_m} = \frac{a}{3}.$$

Up to the temperature $\theta$ we have

$$a = \frac{mNI}{rT}.$$

Replacing in this expression $I$ by $I_m \cdot a/3$ and $T$ by $\theta$ we get

$$a = \frac{mNI_m a}{3r\theta},$$

hence

(3) $$\theta = \frac{mNI_m}{3r}.$$

Dividing (4) by (3) we find

$$\frac{T}{\theta} = \frac{3H_e}{aNI_m} + \frac{3}{a} \frac{I}{I_m}$$

or

$$\frac{T}{\theta} = \frac{3}{a} \frac{H_e}{NI_m} + 1,$$

$$\frac{T-\theta}{\theta}=\frac{H_e}{NI},$$

$$(T-\theta)I=\frac{\theta H_e}{N}.$$

This equation represents an equilateral hyperbola and allows us to determine the coefficient $N$. It is however more convenient to proceed in the following way: The coefficient of specific magnetization $\chi$ for paramagnetic bodies is inversely proportional to the absolute temperature, according to Curie's law.

$$C=\chi T=T\frac{k}{d}.$$

$k$ is the paramagnetic susceptibility and $d$ the density of the substance.

At a temperature a little above $\theta$ the ferromagnetic properties appear only if an external field $H_e$ coöperates with the molecular field. The apparent susceptibility given by the experiment is equal to

$$k'=\frac{I}{H_e}.$$

The real susceptibility however is equal to

$$k=\frac{I}{H_e+NI}, \quad k'>k,$$

$$\frac{1}{k'}=\frac{H_e}{I}; \quad \frac{1}{k}=\frac{H_e+NI}{I},$$

$$\frac{1}{k}=\frac{1}{k'}+N.$$

If we approach $\theta$ from lower temperatures $H_e$ is insignificant in comparison with $NI$, therefore we have for $T=\theta$

$$k=\frac{I}{NI}=\frac{1}{N}.$$

Curie's law applied for the temperature $\theta$ gives the expression

$$\frac{k}{d}\theta=C,$$

$$\theta=\frac{d\cdot C}{k}=d\cdot C\cdot N.$$

At a temperature $T$ above $\theta$, where the iron appears to be paramagnetic, we have

$$C = \frac{T \cdot k}{d},$$

$$\frac{1}{k} = \frac{T}{C \cdot d}.$$

Now we find

$$\frac{1}{k} = \frac{1}{k'} + \frac{\theta}{Cd} = \frac{T}{Cd},$$

$$C = \frac{k'(T - \theta)}{d}.$$

Observing corresponding values ot $k'$ and $T$ we find $C$ and from $\theta = d \cdot C \cdot N$ we find $N$. In this way P. Weiss[1] found for

iron : $N = 3{,}850$, $H_m = NI = 6{,}560{,}000$,
nickel : $N = 12{,}700$, $H_m = NI = 6{,}350{,}000$,
magnetite : $N = 33{,}200$, $H_m = NI = 14{,}300{,}000$.

If the molecular magnets act upon one another with magnetic forces of this enormous amount, then the potential energy due to the molecular magnetic field must have considerable values.

$$E = -\tfrac{1}{2}IH, \quad H = NI,$$

$$E = -\tfrac{1}{2}NI^2.$$

In the state of stable equilibrium this energy must be a minimum, or negative. If the temperature of a ferromagnetic body is raised from a lower value to a point $\theta$ where the spontaneous ferromagnetism disappears, an amount of energy equal to $\frac{1}{2}NI^2$ will gradually be absorbed, therefore the specific heat appears to be greater than it would be without the increase of the ferromagnetic energy. In the case of iron we have $\frac{1}{2}NI^2 = 5.58 \cdot 10^9$ ergs per cm.$^3$ $= 7.08 \cdot 10^8$ ergs per gram.

The amount of heat absorbed is equal to

$$q_m = \frac{7.08 \cdot 10^8}{4.19 \cdot 10^7} = 16.8 \text{ calories.}$$

---

[1] P. Weiss et P. Beck, Chaleur spécific et champ moléculaire des substances ferromagnétiques, Journal de physique, tome 7, p. 249, 1908.

Now we have

$$\int_T^\theta c_m dt = q_m,$$

$$c_m = \frac{dq_m}{dt} = \frac{1}{2}\frac{NdI^2}{yd},$$

$$y = 4.19 \cdot 10^7, \quad d = \text{density}.$$

As the intensity of magnetization is known as a function of the temperature, we can determine $dI^2/dt$ and calculate cm. for each temperature. This has been done by P. Weiss and Beck and they found a very satisfactory agreement between the calculated apparent specific heat and the changes that occur in the specific heat with increasing temperature, up to $\theta$. The apparent specific heat $c_m$ due to the disappearing molecular magnetic field in the neighborhood of $\theta = 756$ for iron is equal to 0.136 and the abrupt change of specific heat at the same point is equal to 0.112. Thus the theory of the intrinsic magnetic field agrees at least in four different directions very well with the phenomena observed.

I am going to draw from the above theory a further conclusion, which will prove to be in perfect agreement with the fundamental quantities of nature.

The electron theory of ferromagnetism gives us a means of determining the moment $m$ of the molecular magnets of those substances whose internal magnetic field has been determined. We can apply several equations, but there is one expression which gives a very simple way to determine $m$. Let us apply the equation

$$a = \frac{mNI}{rT},$$

for the absolute temperature $\theta$ where the relation holds

$$\frac{I}{I_m} = \frac{a}{3}.$$

We find

$$a = \frac{mNI_m a}{3rT},$$

$$m = \frac{3r\theta}{NI_m}$$

$\theta$ is the temperature where the ferromagnetism disappears, $r$ is given by the equation of gases

$$rN_1T = p,$$
$$p = 1.01 \cdot 10^6, \quad T = 273, \quad N_1 = 2.7 \cdot 10^{19},$$
$$r = 1.36 \cdot 10^{-16}.$$

$I_m$ is the saturation value of the intensity of magnetization at the absolute zero of temperature. $I_m$ has to be determined from the above theory by means of the value $I$, intensity of magnetization for the case of saturation at the temperature $t°$.

In the case of iron we have

$$I = 1{,}850, \quad I_m = 2{,}120, \quad N = 3{,}850, \quad \theta = 756 + 273 = 1{,}029,$$
$$m = 5.15 \cdot 10^{-20} \text{ absolute electromagnetic units.}$$

Let $N_1$ be the number of molecular magnets in unit volume at the absolute zero, then we have:

$$N_1 m = I_m = 2{,}120,$$
$$N_1 = 4.12 \cdot 10^{22}.$$

If this number $N_1$ of the elementary magnets is at the same time the number of molecules of iron, and if the mass of 1 molecule of iron is equal to $\mu$ we have:

$$N_1\mu = \delta = 7.86,$$

where $\delta$ is the density of iron at the absolute zero.

$$\mu = \frac{\delta}{N_1} = 1.91 \cdot 10^{-22} \text{ gr.}$$

If the molecule of iron consists of two atoms, it will be 111.8 times heavier than the atom of hydrogen, and the mass of the atom of hydrogen $\mu_H$ will be equal to

$$\mu_H = \frac{1.91 \cdot 10^{-22}}{111.8} = 1.66 \cdot 10^{-24} \text{ gr.}$$

A recent value of this quantity has been determined by Rutherford who found

$$\mu_H = 1.61 \cdot 10^{-24} \text{ gr.}$$

from radioactive phenomena. The coincidence of the values of 1 atom of hydrogen deduced from so very different phenomena as those of radioactivity and ferromagnetism by very different considerations, is a strong evidence in favor of the theories involved. It follows immediately that the molecular magnet of iron contains two atoms. In the case of magnetite we find the following numbers:

$$N = 33{,}200,$$
$$\delta = 5.2,$$
$$I = 430, \quad I_m = 490.$$

According to Curie $\theta = 536 + 273 = 809$.
According to Weiss $\theta = 589 + 273 = 862$.

$$m = 2.02 \cdot 10^{-20},$$
$$N_1 m = I_m = 490,$$
$$N_1 = 2.42 \cdot 10^{22}.$$

If the mass of each molecule is equal to $\mu$ we have

$$N_1 \mu = 5.2,$$
$$\mu = 2.15 \cdot 10^{-22},$$
$$Fe_3O_4 = 231.7,$$

accordingly

$$\mu_H = \frac{2.15 \cdot 10^{-22}}{231.7} = 0.93 \cdot 10^{-24} \text{ gr.};$$

a value that agrees pretty well with that derived from the kinetic theory of gases.

Taking for $\theta$ Weiss' value I find for $\mu_H$

$$\mu_H = 0.99 \cdot 10^{-24}.$$

In the case of nickel I find the following numbers:

$$N = 12{,}700, \quad I = 500, \quad I_m = 570, \quad \theta = 376 + 273 = 649,$$
$$m = 3.65 \cdot 10^{-20},$$
$$N_1 m = I_m,$$
$$N_1 = 1.56 \cdot 10^{22},$$
$$N_1 \mu = \delta = 8.8 \quad \mu = 5.63 \cdot 10^{-22}.$$

The atomic weight of nickel is equal to 58.7, hence, assuming that each molecular magnet contains two atoms, we find

$$\mu_H = \frac{\mu}{117.4} = 4.8 \cdot 10^{-24},$$

a value that is exactly three times bigger than that given by Rutherford. As the degree of accuracy is the same in the case of iron, nickel and magnetite we have to admit that the molecular magnet of nickel is made up of 6 atoms.

The quantities considered are given in the following table:

| **Substance.** | $I_{t=20^\circ}$ | $I_m$ | $\Theta$ | $N$ | $NI=H_m$ | $m 10^{20}$ | $\mu_H \cdot 10^{24}$ |
|---|---|---|---|---|---|---|---|
| Fe | 1,860 | 2,120 | 756 | 3,850 | 6,560,000 | 5.15 | 1.66 |
| $Fe_3O_4$ | 430 | 490 | 536? | 33,200 | 14,300,000 | 2.02 | 0.93 |
| Ni | 500 | 570 | 376 | 12,700 | 6,350,000 | 3.65 | 4.8 |

As the ratio of the density of nickel and iron 1.12 is nearly equal to the ratio of the atomic weights 1.05 of the two meatls, the number of molecules per unit volume must be the same for both metals, assuming that each molecule contains two atoms. And as the moment of the molecular magnets of nickel is only 29 per cent. smaller than that of iron we should expect that the intensity of magnetization of nickel varies only about 30 per cent. from that of iron, while in fact the magnetization of iron is 3.7 times superior to that of nickel; this consideration indicates again, that either the molecular magnet of nickel contains 6 atoms or only every third molecule is an elementary magnet. This fundamental difference in the molecular magnets of iron and nickel must be taken into account, when some very interesting differences in the magnetic behavior of the two metals have to be explained. Thus the first layer of electrolytically deposited nickel is stronger magnetically than the subsequent layers, while for iron the opposite statement is true. In addition a longitudinal compression decreases the magnetization of iron and increases that of nickel, while a longitudinal pull increases the magnetization of iron and decreases that of nickel.

The discussion of the molecular magnetic field in terms of the electron theory offers some difficulties, which, however, are not of such a nature as to involve contradictions to other theories. If $4.12 \cdot 10^{22}$ molecules of iron are contained in unit volume, the radius of one molecule cannot be much bigger than $1.5 \cdot 10^{-8}$ cm. $= r$.

We may try to determine the velocity $v$ of an electron in its orbit by means of the equation

$$\frac{mv^2}{r} = \frac{e^2}{r^2},$$

$m$ being the mass and $e$ the charge of an electron.

$$m = 8.77 \cdot 10^{-28}, \quad e = 4.65 \cdot 10^{-10},$$

$$v = 1.3 \cdot 10^8 = \frac{2\pi r}{T} = 2\pi r n_1.$$

$n_1$ is the frequency of the electron.

$$n_1 = 1.4 \cdot 10^{15},$$

a frequency bigger than that of sodium light $5.1 \cdot 10^{14}$.

The magnetic moment $m_1$ due to the revolution of

$$1 \text{ electron} = \Phi i = \pi r^2 \frac{e}{T} = \pi r^2 e n_1 = 1.54 \cdot 10^{-20}.$$

Thus one electron alone, revolving with high frequency, is able to produce a magnetic moment of the right order of magnitude. The magnetic force however under the given circumstances in the center of the electronic orbit is equal to

$$h_1 = \frac{2\pi i}{r} = \frac{2\pi e n_1}{r} = 8{,}800,$$

a value about 1,000 times smaller than the molecular magnetic field. If each electron would produce a field of this magnitude, we can see that even in this case of a rather weak elementary field a large number of surrounding molecules would contribute to the molecular field of the central molecule. Let $a$ be the distance of a point on the axis of the orbit of the electron from the center of the orbit, then the magnetic force in that point $a$ will be equal to

$$h_a = \frac{2\pi i r^2}{\sqrt{a^2 + r^2}^3} = \frac{h r^3}{\sqrt{a^2 + r^2}^3} = h \left(\frac{r}{a}\right)^3,$$

if we put approximately $a = 10r$,

$$h_a = 8.8.$$

Thus even in a distance $a = 20r$ the magnetic field will still be appreciable, and the sphere of activity will still be larger if the magnetic field, due to one molecule, is superior to that of 8,800 units. If the frequency $n_1 = 1.4 \cdot 10^{15}$ would be the real one, the iron should emit even at the absolute zero of temperature a radiation of a frequency higher than that of sodium light. We have therefore to admit that the frequency of the electrons producing the magnetic properties is much smaller. If we assume $n_2 = 0.5 \cdot 10^{13}$ corresponding to a wave-length of ultra-red light $\lambda = 6 \cdot 10^{-3}$ cm., we find

$$h = 31\,; \quad m = 5.5 \cdot 10^{-23},$$

values that are too small.

If there were $N$ revolving electrons in a molecule contributing to the magnetic moment $m$ would be equal to

$$m = r^2 \pi enN,$$

$$h = \frac{2\pi}{r} enN.$$

URBANA, ILLINOIS,
October 7, 1909.

# ON CONTACT DIFFERENCE OF POTENTIAL IN A MAGNETIC FIELD.

By E. P. Adams.

ABOUT two years ago the writer made some experiments to determine whether the contact difference of potential between pairs of metals is changed in the presence of a magnetic field. The line of reasoning which led to the thought that such an effect might exist will first be given.

It seems well established that the contact difference of potential between metals in air is a chemical effect, the result of differential chemical action between the metals and condensed air films on their surfaces. At any rate it seems certain that these air films are essential in order that the contact difference of potential may be developed. There is another way of looking at the subject which may be worth some attention. The fundamental thing to be explained is that the potential of an insulated metal differs from that of the surrounding air; and that this difference of potential varies from metal to metal. Let us assume that each unit area of the surface of the metals emits in unit time $N$ electrically charged particles, each of mass $m$ and charge $e$. The metal is further supposed to be coated with a thin film of condensed air, or other substance, with the very high transverse electric resistance $R$ per unit surface. The charged particles must be supposed to be emitted from the metal with an initial velocity sufficient to carry them through the film. A difference of potential $V$ is thus set up between the two sides of the film. In the steady state the electric current due to the flow of charged particles from the metal through the film is balanced by the conduction current, the result of the difference of potential $V$ and the conductivity $1/R$ of the film. Therefore,

$$Ne = V/R$$

or

$$V = NeR.$$

Both $N$ and $R$ may be supposed to vary with the metal. Thus each insulated metal will be at a definite potential above or below that of the surrounding air, and this is required according to our present views concerning the contact difference of potential between metals.

Suppose next that the metal in the form of a disc is placed in a magnetic field parallel to its plane. The charged particles will tend to describe circular paths whose radius of curvature is given by

$$\frac{mv^2}{\rho} = Hev$$

or

$$\rho = \frac{mv}{He},$$

$v$ being the velocity of the particle and $H$ the magnetic force. If this radius of curvature is small enough some of the particles will be prevented from getting through the film; the difference of potential between the two sides of the film will be diminished, and thus the contact difference of potential between a pair of metals, which is all that can be measured, will be changed.

The results of the experiments, which will be described, were decisively negative. The sensitiveness of the method employed was such that a change of 0.001 volt in the contact difference of potential could have been observed. The matter was therefore dropped as it was thought that negative results of an experiment of this kind were not of sufficient value to justify their publication.

Very recently, however, M. V. Posejpal[1] has published the results of some experiments, a part of which bear upon this same problem. Contrary to the results of the writer, M. Posejpal finds that the contact difference of potential between iron and zinc is changed in a magnetic field; since the method he employed is quite different from that of the writer it may be worth while now briefly to describe the experiments of the latter in order that a comparison of results may be made.

The compensation method devised by Lord Kelvin was employed. In the accompanying diagram $A$ and $B$ represent the two metallic discs the contact difference of potential between which is

---

[1] Annales de Chimie et de Physique, August, 1909.

to be measured. These are screwed on to the brass ends of the hard rubber insulators *R*, *R*, which are carried on brass rods, and these in turn freely slide through fixed brass tubes, as shown. The lower rod is fixed in position, its height being adjusted by means of a set screw. The limits of motion of the upper rod are controlled by the adjustable collars *I*, *I*.

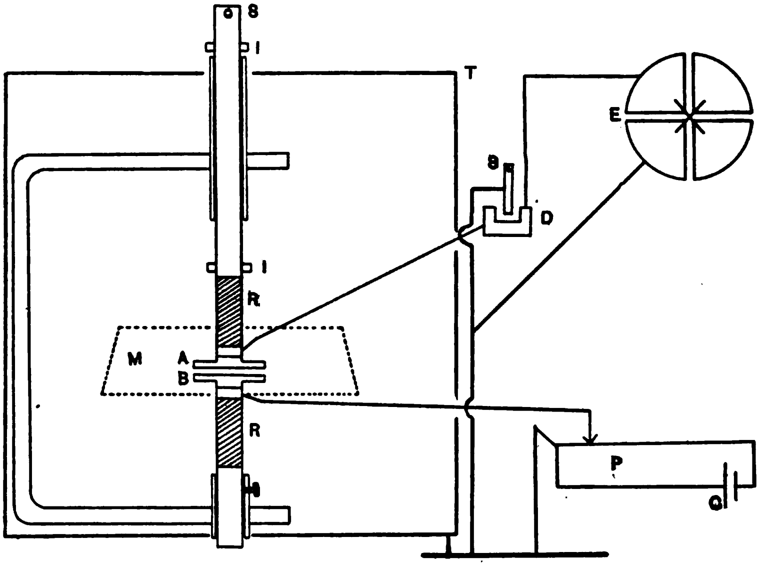


Fig. 1.

The upper disc is permanently connected to one pair of quadrants of the electrometer, *E*. By means of the mercury key *D*, this pair of quadrants may be connected to earth. When the rod *S* is raised the earth connection is broken. The other pair of quadrants is connected to earth. The lower disc *B* is connected to the slider of the potentiometer *P*, so that it may be kept at any desired potential. Strings are attached to *S*, *S* by means of which the earth connection of the first pair of quadrants may be broken and the upper disc raised at the observing telescope.

The determination of the contact difference of potential is made as follows: the electrometer quadrants being earthed, the two discs are brought close together, but not touching. Electrically they are of course in contact. The earth connection is then broken and the upper disc raised. A deflection of the electrometer needle, in general, results. The potential of the lower disc is then changed until on repetition of the above process no deflection of the electrometer results. The potential applied to the lower disc is then the contact difference of potential between the two discs.

An estimate can easily be formed of the sensitiveness of this method. Let the capacity of the condenser formed by the two discs when close together be $c$ and when separated, $c'$. If the lower disc be kept at zero potential by connecting it to earth the quantity of electricity set free on raising the upper disc is $V(c - c')$, $V$ being the contact difference of potential. This quantity divides between the electrometer and disc condenser, so that $V(c - c') = v(C + c')$, where $v$ is the potential to which the insulated quadrants are raised. $c'$ is small compared to $C$ and to $c$, so that $v = Vc/C$. The discs were 19 millimeters in diameter; their distance when near together was approximately 0.2 mm., and when separated something over 1 cm. The capacity of the electrometer and connections was approximately 90. Hence considering the disc condenser as a simple parallel plate condenser, we find $v = V/8$. The electrometer had a sensitiveness of about 8,000 divisions per volt. Hence a deflection of one scale-division corresponds to a contact difference of potential of 0.001 volt. Direct experiment showed that when the contact difference of potential between the two discs was balanced by the potential applied to the lower a change of 0.001 volt in the latter produced a distinct deflection.

The pole-pieces of the electro-magnet, one of which is shown in section at $M$, projected through holes cut into opposite sides of a zinc box $T$, which completely shielded the interior. As a further precaution against outside electrostatic disturbances the pole-pieces themselves were earthed. The electrometer connections were run through earthed brass tubes.

The electrometer used was of the Dolezalek type; the needle suspension was a platinum wire, 0.0004 inch in diameter through which the needle was kept at a potential of about 80 volts by means of a battery of small storage cells. A Wolff potentiometer with five dials and a total resistance of 15,000 ohms was employed. The current for it was supplied by a cadmium storage cell for which the writer is indebted to Professor Hulett.[1] The electromotive force of this cell was tested from time to time by comparison with a standard cell and proved to be remarkably constant.

The discs used were made of iron, zinc, and gold- and platinum-plated brass. After the discs were cleaned and screwed into position the contact difference between them changed rapidly; but after the course of an hour or so it became so constant that no change could be detected during a period of ten to fifteen minutes. During this stage the contact difference was measured, then the magnetic field was applied and the contact difference again measured, and a final measurement was made after the magnetizing current was broken. Many repetitions of this were made, using the combinations gold-zinc, platinum-zinc, and gold-iron, but in no case was the slightest effect of the magnetic field observed. The magnetic force was measured by a calibrated bismuth spiral and was approximately 4,800 electromagnetic units.

The conclusion thus followed: the effect of a uniform magnetic field of 4,800 units, if any, on the contact difference of potential between pairs of these metals is certainly less than 0.001 volt.

The experiments of M. Posejpal already referred to differ in at least three important respects from those just described. In the first place, he used a direct deflection method, instead of a null method, which necessarily involved greater possibility of error. In the second place the two discs were allowed to come into actual contact with each other before separation. It seems possible that the observed effects resulted from this actual contact. The contact difference of potential between metals is extremely sensitive to changes in their surfaces. If one of the discs is allowed to fall on the other considerable changes in the contact difference are almost certain to result. The writer has observed that if the initial distance between the two discs was made so small that on dropping the upper disc it touched the lower one, due to a lack of rigidity in the support, a distinct change in the contact difference was found on the next measurement. M. Posejpal found that the contact difference was by no means constant and sought to get correct values by tak-

---

[1] Phys. Rev., 1908, 27, p. 33.

ing the average of many results. Now when a magnetic field is applied this mechanical effect would be altered, particularly in M. Posejpal's experiments since he used only the combination zinc-iron. The iron, unless very firmly held, would be certain to move in the magnetic field, and thus a secondary effect of the magnetic field on the contact difference of potential might be expected to result. Furthermore, when two unlike metals are separated from contact one would expect a production of frictional electricity, which of course would be added to the electricity set free by the true contact difference of potential. In the magnetic field, due to mechanical action on the iron, the amount of frictional electricity would be altered, and thus a change in the apparent contact difference of potential would result.

The third difference between the two methods lies in the position of the two discs with respect to the magnetic field. In M. Posejpal's experiment the lower disc remained in the region of strong magnetic force, while the upper disc was pulled into a region of comparatively weak magnetic force. In the writer's experiment, the pole faces of the magnet were of such a size that both discs remained in a fairly uniform field even after the upper disc was raised. Whether an explanation of the results obtained lies in this fact or not can be tested only by experiment.

Unfortunately, M. Posejpal does not state the strength of the magnetic field he employed, nor the diameter of the discs. But the large effects he obtained, combined with the comparative insensitiveness of his method (his electrometer gave only 60 divisions for a volt), make it improbable that his positive results were due to using a magnetic field of much greater strength than that used by the writer.

PRINCETON, N. J.

# THE EFFECT OF TEMPERATURE ON FLUORESCENCE AND ABSORPTION.

By R. C. Gibbs.

## II. Fluorescence and Absorption of Canary Glass at Low Temperatures.[1]

THE study of the fluorescence and absorption of canary glass at low temperatures is a continuation of a similar investigation made with the same specimen at high temperatures.[2]

The substances used in making the glass are: $SiO_2$ 70 per cent., $K_2O$ 24 per cent., CaO 6 per cent. to which 2.5 to 3.0 per cent. sodium uranate is added.

The specimen, ground to the shape of a rectangular plinth 3.37 cm. by 2.88 cm. by .74 cm., was mounted in the same iron case as was used for the high temperatures, the plan, front elevation and end elevation of which are shown in Fig. 9. The heavy broken lines indicate the position of the glass. The measurements were made with a Lummer and Brodhun spectrophotometer which together with the remaining apparatus is shown in Fig. 10.

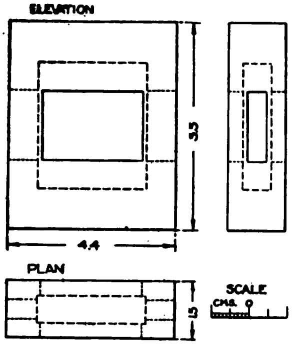


Fig. 9.

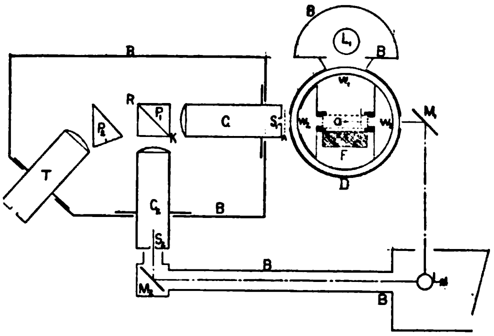


Fig. 10.

---

[1] A summary of the work described in this paper was presented to the British Association for the Advancement of Science at its Winnipeg meeting August, 1909.

[2] Gibbs, R. C., Phys. Rev., Vol. XXVIII., No. 5, p. 361.

The iron case the corners of which are indicated by the heavy marks and which contained the specimen of glass *G* was securely fastened to a fiber rod *F* and the whole rigidly supported inside of an upright unsilvered cylindrical Dewar bulb *D*. Cardboard covered with black paper and fastened to the iron case extended to the walls of the Dewar as shown by four light lines. The heavy lines inside and outside the bulb represent black paper so arranged as to leave three windows $W_1$, $W_2$ and $W_3$ through which light may pass in or out of the bulb.

To excite fluorescence a Cooper-Hewitt mercury lamp $L_1$ was used, the light from which passed through window $W_1$. The fluorescent light passing through window $W_2$ fell upon slit $S_1$ of collimator $C_1$. For a standard of comparison light from an acetylene flame $L_2$ was reflected by a block of magnesium carbonate $M_2$ upon slit $S_2$ of collimator $C_2$. The lines marked *B* represent screens which for the most part were covered with black paper.

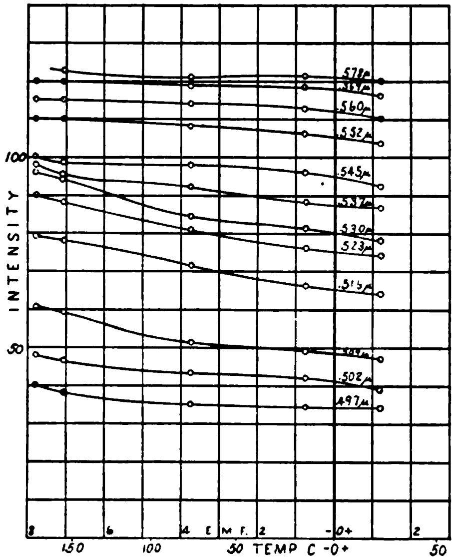


Fig. 11. Showing effect of temperature on transmission intensity for various wave-lengths.

For determining the absorption, light from $L_2$ was reflected through $W_3$ by a glass mirror $M_1$. The transmitted light fell upon slit $S_1$. For the purpose of securing a uniform field a ground glass *A* was inserted in front of the slit. For a comparison light the same arrangement as in the case of the fluorescence was used. To obtain the incident light a set of readings was made with the glass *G* removed.

In all the measurements slit $S_1$ was kept at a constant width of .5 mm. and readings were taken by varying the slit $S_2$ until the intensities of light from the two sources were equal.

The upper part of the Dewar bulb was packed with wool and cooling was secured by pumping liquid air into the bottom of the bulb a little at a time. In this way the temperature was kept from varying more than 2° C. during any particular set of observations through the spectrum. A much greater uniformity could be secured by this method if required. The temperature was measured by means of an iron-constantan thermo-junction and a potentiometer.

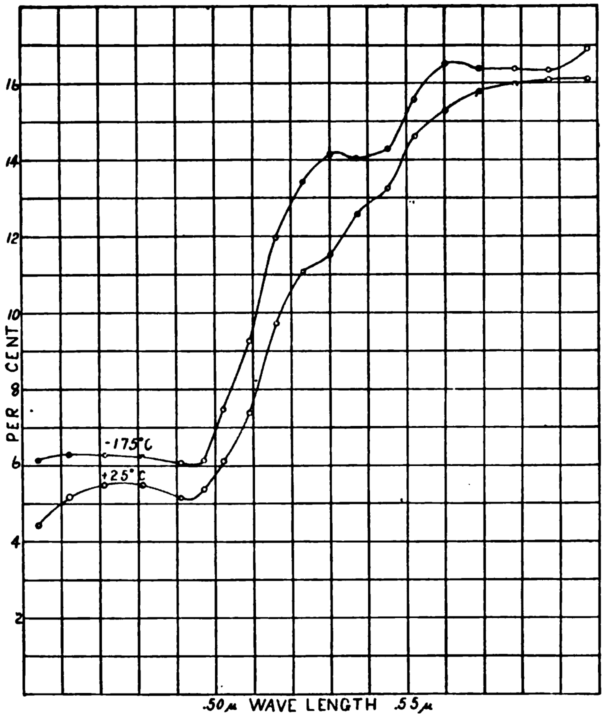


**Fig. 12. Showing percentage transmission at + 25° C. and — 175° C.**

The equation for the E.M.F. of the junction was determined by reference to three fairly well separated fixed temperatures. The E.M.F. for several other temperatures were determined from this equation and a curve plotted which checked very well with other observed values of E.M.F. and known temperatures.

During all of the observations one face of the glass was visible and it was carefully watched for any sign of moisture or frost on its surface. In order to eliminate the formation of moisture on the outside walls of the bulb through which light passed, heating coils were placed in contact with the blub below each of the three windows $W_1$, $W_2$ and $W_3$ so that the glass in those regions was kept well above the dew point.

The transmission intensities for the more important wave-lengths are shown in Fig. 11. The increase due to a lowering of temperature is general throughout the spectrum, the rate of increase being largest on the boundary of the absorption band, *i. e.*, from .502 μ to about .537 μ as may be seen by reference to Fig. 12.

From the curves in Fig. 11 and others not there shown, the transmission intensities were read off at temperature intervals of 40 degrees from 25° C. to — 175° C.

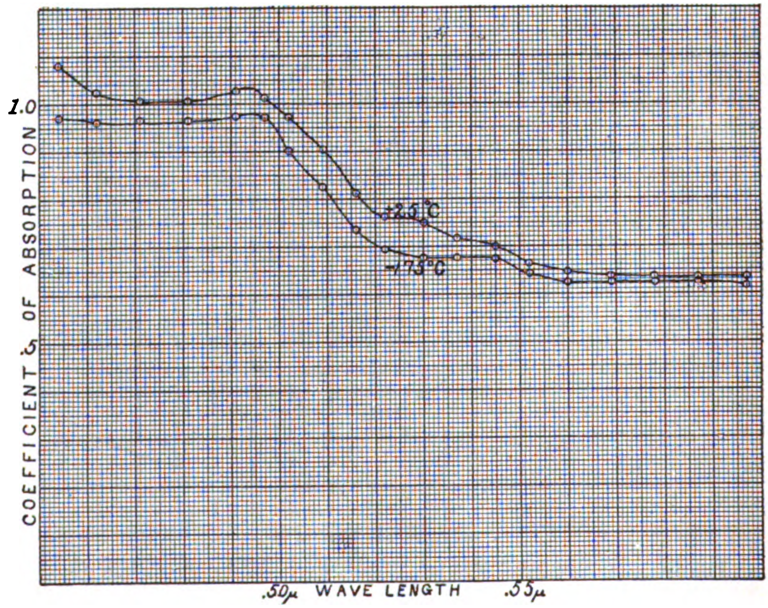


**Fig. 13.** Showing the coefficient of absorption at + 25° C. and — 175° C.

The percentage transmission, *i. e.*, the ratio of the incident light to the graphically determined transmitted light is shown in Fig. 12 for the highest and lowest temperatures, from which it may be seen that lowering the temperature produces a slight shift of the boundary of the main absorption band toward the shorter wave-lengths. It is to be noticed that the absorption is rather large in all parts of the spectrum and that the rather large scale to which these curves are plotted somewhat exaggerates their irregularities. This is especially true for the curve at — 175° between .53 $\mu$ and .55 $\mu$. That the presence of this small absorption band is however not due to error in measurements at this temperature is evidenced by the fact that similar curves for the intermediate temperatures show its gradual development.

In Fig. 13 is shown the effect of a decrease of temperature on the coefficient of absorption ($\beta$) as computed from the following formula.

$$\beta = \frac{\log I - \log T}{x}$$

wherein $I$ represents incident light, $T$ transmitted light and $x$ the thickness through which the light passes.

The observed fluorescence intensities for the majority of the wave-lengths measured are shown in Fig. 14. From these curves were read off the intensities for the same temperatures as in the case of transmission. In order to obtain the values for the lowest temperature these curves had to be slightly exterpolated but with the two lowest observed temperatures rather close together such a procedure seemed to be justified. From these values of fluorescence and the coefficients of absorption the fluorescence corrected for absorption was computed according to a formula derived in a previous article to which a reference was made at the

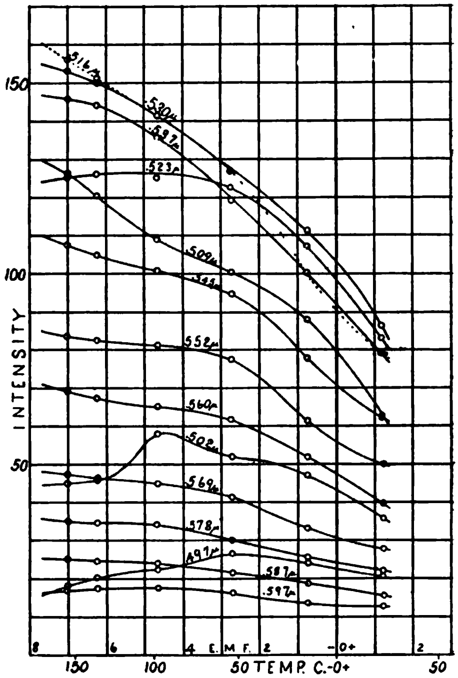


Fig. 14. Showing effect of temperature on fluorescence intensity for various wave-lengths.

beginning of this paper. The results obtained from such computations are seen in the typical fluorescence curves shown in Fig. 15.

The rather broad fluorescence band at room temperature seems to show a slight indication of being made up of two overlapping bands

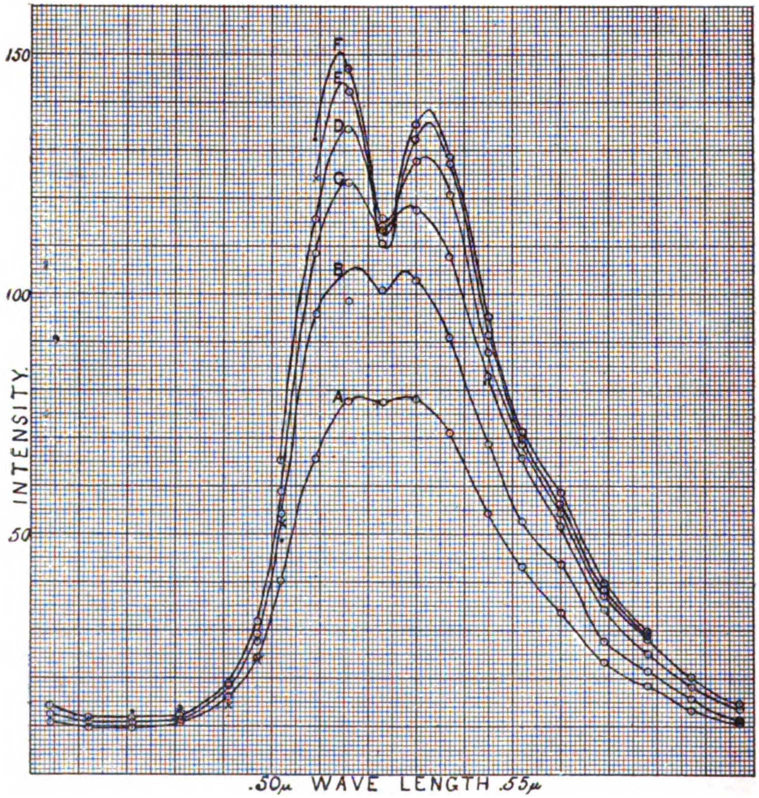


Fig. 15. Fluorescence corrected for absorption.
Curve *A*, + 25° ; *B*, – 15° ; *C*, — 55° ; *D*, — 95° ; *E*, — 135° ; *F*, —175 ° C.

with their maxima very near together. The band shades off a little more gradually on the side of the longer wave-lengths. On cooling the two bands develop very distinctly, the intensity increases, for the most part, throughout the fluorescent region and the curves become very much steeper especially on the side of the shorter wave-lengths. The two maxima gradually separate, one shifting toward the longer wave-lengths and the other in the opposite direction. The minimum which develops between the two bands increases at first and then gradually decreases. The order of increasing values of the intensity for that region is as follows: Curve *A*, + 25°; *B*, − 15°; *F*, − 175°; *E*, − 135°; *C*, − 55°; *D*, − 95° C.

On the side of the shorter wave-lengths the curves (Fig. 15) for the three lowest temperatures are not completed because they would cross the curves already drawn and make it hard to distinguish one from another. Some of the points for these curves are included, and by referring to the various marks used in plotting the last point through which each of these curves is drawn, one may readily see approximately their position.

The general increase in fluorescence intensity with decreasing temperature agrees with the results secured for increasing temperatures above that of the room. The typical fluorescence at room temperature obtained in connection with the higher temperatures, observations for which were made a year before, differs somewhat from that shown here. The curve for the former shows a maximum at .537 $\mu$ with a somewhat irregular decrease toward the violet end. There are two possible explanations for this difference. Probably the glass is not strictly homogeneous and in all likelihood the slit of the spectrophotometer was not placed exactly opposite the same part of the glass in the two sets of observations. A much more probable explanation is that the glass has gradually recovered from the effect of heating. It was noted in the previous article on high temperatures that the intensity of fluorescence at room temperature was larger a day or so after heating when under excitation, than before but that a general recovery resulted if allowed to stand. Two or three weeks seemed to be sufficient for the greater part of the spectrum. The shape of the crest of the room temperature curve shown here agrees more closely with that obtained before the glass was heated than with any of the others obtained a relatively short time after heating. It may be well to add here that this specimen did not show phosphorescence.

The bands which develop on cooling with maxima at .514 $\mu$ and .533 $\mu$ respectively, showing an increase in intensity of about 100 per cent., seem to be the same bands which were practically destroyed by a temperature of 350° C. The fact that there is no evidence at the lower temperatures of a band at about .545 $\mu$ which persisted at the higher temperatures together with the position and development of the two bands at the lower temperatures strengthens the assumption that the band at .454 $\mu$ was slightly increased, if not entirely developed, by heating. The latter suggestion is supported by the fact that the fluorescence at room temperature measured shortly after cooling showed very little variation from that measured at the same temperature a few days before cooling.

## SUMMARY.

The essential results of the above recorded observations may be stated as follows:

1. Decrease of temperature produces a decrease in the absorption throughout that region of the spectrum measured and a corresponding narrowing of the absorption band, a result consistent with the effect of increasing temperatures.
2. The main portion of the rather broad fluorescence band in the yellow-green portion of the spectrum increases very decidedly on cooling and breaks up into two narrow overlapping bands very close together.
3. On the side of the shorter wave-lengths the fluorescence intensity decreases at the extreme low temperatures thus narrowing the band on that side.
4. The fluorescence bands which develop on cooling appear to be the same ones which were practically destroyed at the higher temperatures and the band which persisted at the higher temperatures has been destroyed by cooling or else did not exist at room temperature.
5. The fluorescence of this specimen evidently depends, to a more or less small extent, upon the previous heat treatment and the period of recovery to normal fluorescence may extend over a considerable time.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY.

# SOME PHOTO-ELECTRIC PROPERTIES OF THE ALKALI METALS.

## III. THE DEPENDENCE OF PHOTO-ELECTRIC CURRENT ON THE WAVE-LENGTH OF THE INCIDENT LIGHT.[1]

BY F. K. RICHTMYER.

SOON after the discovery that the alkali metals were photo-active in the visible spectrum Elster and Geitel[2] published some brief observations showing the sensibility of these metals for different colors. These data however were qualitative rather than quantitative, and as the exact nature of the relation between wave-length and photo-electric current would not only be of value in the theory of photo-electric phenomena, but would also help in correlating it with other phenomena, the writer has attempted the determination of this relation for sodium — which has a maximum of sensibility in the visible spectrum.

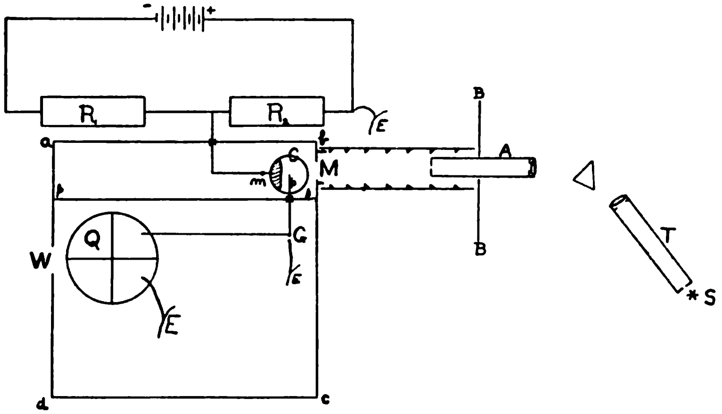


Fig. 1.

The apparatus used has already been described in the first article

---

[1] Paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

[2] Ann. der. Ph., Vol. 52, 1894, p. 433.

of this series,[1] the only modification being that the source $S$ was replaced by a spectrometer, whose movable telescope arm $T$ (see Fig. 1) carried the source of light, the spectrum of which was to be thrown on the sensitive surface of the photo-electric cell, through the fixed collimator tube $A$. The spectrometer was calibrated by the lines from a mercury vapor lamp, the slit of the collimator tube $A$ being, for this purpose, replaced by an eye-piece. The prism gave a spectrum about eight degrees in width. The widths of the two slits were equal and represented about 10 minutes of arc.

As before described,[1] the platinum wire leading to the sensitive surface was charged to such a negative potential as to reduce the leakage current, through the walls of the cell, to zero, thus making the photo-electric surface at zero potential with respect to the receiving Pt wire $p$. Under these conditions the rate of deflection of the electrometer as measured by a chronograph was proportional to the photo-electric current. Care was taken to adjust the screening so that there was no discharge when the source $S$ was cut off.

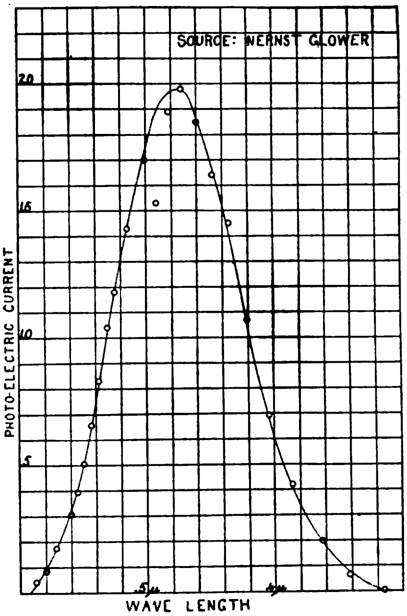


Fig. 2.

A Nernst glower was at first used as a source at $S$. Although this furnished a very intense light the glowers did not last long enough on direct current for making a good set of observations, and no convenient means of regulation was at hand when the glowers were used on alternating current. However, as the Nernst glower formed a very satisfactory source, one run was made and is recorded here in Fig. 2. The ordinates are the observed photo-

---

[1] Phys. Rev., Vol. XXIX., 1909, pp. 71–80.

electric currents expressed as the rate of deflection of the electrometer, in mm./sec. No correction has been made for the variable dispersion of the prism or for the energy distribution in the spectrum of the Nernst glower.

As this curve is not very smooth, probably due to fluctuations in the alternating current voltage, it was thought best to substitute an acetylene flame. This had a twofold advantage. Not only was it of constant intrinsic brightness but also the energy distribution was known, having been recently determined by Professors Nichols and Merritt.[1]

In order to make the results useful for comparison two corrections were made: First, due to the variable dispersion of the prism, it was necessary to reduce the observed photo-electric currents to what would have been obtained had the spectrometer slit included a constant number of wave-lengths at each observation. The slit width was such as to make it possible to do this by graphical means from the slope of the calibration curve of the spectrometer. Second, the final relation between wave-length and photo-electric current should show the current that would be produced by the same amount of energy at each wave-length. This correction was made by dividing the observed currents by the relative energy for each wave-length in the spectrum of acetylene. As the above curve of Nichols and Merritt for the energy distribution extends only as far as .48 $\mu$, while the observations in this experiment extend to .42 $\mu$, the following method of extrapolation was used.[2]

Wien's equation for the energy distribution in the spectrum of a black body

$$E = c\lambda^{-5}\cdot e^{\frac{-\beta}{\lambda T}}$$

may be written in the form

$$\frac{1}{\lambda} = -\frac{T}{\beta}\log_e E\lambda^5 + \frac{T}{\beta}\log c$$

which is the form of a straight line, where $1/\lambda$ and $\log_e E\lambda^5$ are the

---

[1] Phys. Rev., Vol. XXX., p. 328.

[2] This method is perhaps not altogether justified since we are not sure that acetylene follows Wien's law of radiation exactly. It is used here, however, with the hope of checking it up later.

variables. These values were determined from the above mentioned energy curve, and plotted as a (nearly) straight line in Fig. 3. See data Table I.

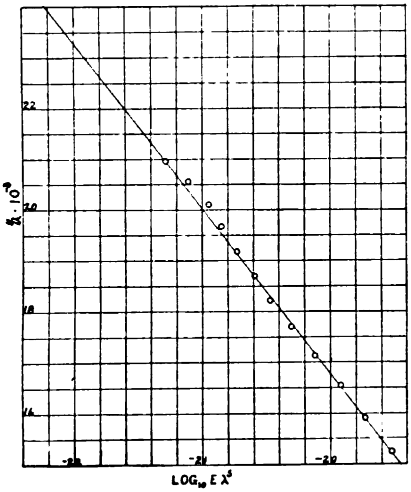


Fig. 3.

Table I.

| $\lambda \cdot 10^4$ | Relative Energy. | $\mathrm{Log}_{10}\, E\lambda^5$ | $\frac{1}{\lambda} \cdot 10^{-1}$ |
|---|---|---|---|
| .656 | 24.76 | −20.4782 | 1,525 |
| .628 | 18.86 | .2655 | 1,591 |
| .604 | 14.88 | .0776 | 1,654 |
| .583 | 11.28 | −21.8808 | 1,713 |
| .565 | 8.65 | .6970 | 1,769 |
| .548 | 6.86 | .5303 | 1,822 |
| .534 | 5.90 | .4083 | 1,870 |
| .522 | 4.82 | .2690 | 1,918 |
| .509 | 4.20 | .1547 | 1,967 |
| .498 | 3.70 | .0522 | 2,010 |
| .487 | 2.84 | −22.8888 | 2,055 |
| .477 | 2.10 | 7.147 | 2,096 |

This line was extended as far as necessary toward shorter wavelengths and from it, by computation in reverse order, a new energy distribution curve obtained, extending to .42 $\mu$. (See Fig. 4, *III.*) The column labeled energy in data Table II. is obtained directly from this curve. And the observed photo-electric currents, after making the correction for variable dispersion are divided by these energy values. The resulting currents are those which would have been obtained from a normal spectrum containing the same amount of energy for each wave length. The results are shown in data Table II., and graphically in Fig. 4, curve *I* being the observed curve, and *II* the curve after making both the above corrections.

## Table II.

| $\lambda \cdot 10^4$ | Slope of Calibration Curve. | Observed Rate of Deflection, mm./sec. | Rate of Deflection Corrected for Dispersion. | Relative Energy. | Final Corrected Photo-electric Current. |
|---|---|---|---|---|---|
| .685 | 2.90 | .0252 | .0087 | 28.0 | .0031 |
| .658 | 2.49 | .0491 | .0197 | 24.6 | .0080 |
| .635 | 2.15 | .151 | .0703 | 19.8 | .0355 |
| .614 | 1.89 | .311 | .1645 | 16.1 | .1022 |
| .596 | 1.69 | .660 | .390 | 13.1 | .298 |
| .579 | 1.51 | 1.012 | .671 | 10.7 | .627 |
| .565 | 1.38 | 1.351 | .980 | 8.95 | 1.095 |
| .552 | 1.26 | 1.620 | 1.285 | 7.50 | 1.71 |
| .540 | 1.16 | 1.840 | 1.588 | 6.27 | 2.53 |
| .528 | 1.07 | 1.928 | 1.800 | 5.24 | 3.44 |
| .518 | .99 | 1.960 | 1.980 | 4.42 | 4.48 |
| .508 | .92 | 1.94 | 2.11 | 3.72 | 5.67 |
| .499 | .86 | 1.82 | 2.12 | 3.18 | 6.77 |
| .491 | .81 | 1.69 | 2.09 | 2.74 | 7.63 |
| .483 | .76 | 1.49 | 1.97 | 2.35 | 8.49 |
| .475 | .71 | 1.37 | 1.92 | 1.99 | 9.66 |
| .462 | .64 | .955 | 1.49 | 1.49 | 10.00 |
| .450 | .57 | .640 | 1.12 | 1.13 | 9.90 |
| .440 | .52 | .394 | .758 | .89 | 8.52 |
| .429 | .47 | .224 | .477 | .67 | 7.12 |
| .420 | .44 | .106 | .241 | .52 | 4.64 |

Column 2: Determined graphically from calibration curve of spectrometer.
" 4: Column 3 divided by column 2.
" 6: Column 4 divided by column 5, with decimal point shifted.

The significance of the exact shape and location of this curve for sodium is not at present apparent. The maximum occurs at about $\lambda = .46\,\mu$ and yet $\lambda = .46\,\mu$ does not seem to bear any unique relation to any of the known properties of sodium. Still some relation must exist. There must be some reason why a sodium surface shows a maximum at one wave-length, and rubidium, for example, at another. If such a relation could be established it would doubtless do much to explain not only photo-electric phenomena but also many questions connected with the structure of matter.

Whatever theory of the photo-electric discharge we accept the shape of the above curve is very strongly suggestive of resonance in some form or other and that the expulsion of the electron is due to selective absorption of light. If such is the case the free period of the electron in the sodium atom in its solid state should be determined by the period of vibration for $\lambda = .46\,\mu$ and metallic sodium ought to show selective absorption for this wave-length. But since the curve extends over a comparatively large range of frequencies, the question might be raised whether the frequencies of the electrons are correspondingly distributed, or whether all the electrons have the same frequency and the effect of light other than $\lambda = .46\,\mu$ is due to vibrations of a semi-forced nature. Further experiments along this line are in progress.

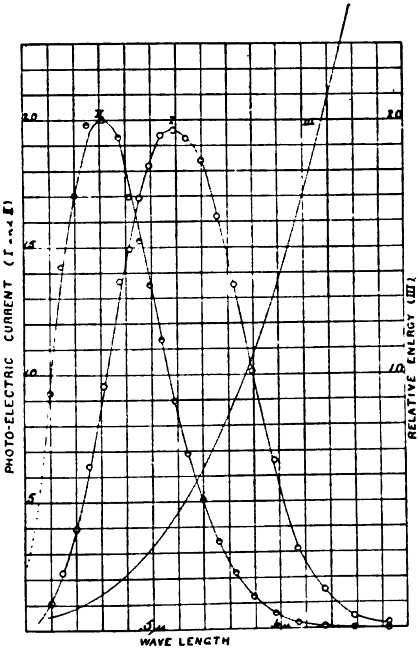


Fig. 4.

Apropos of the present discussion as to whether the energy of the expelled electron comes from an atomic explosion under the influence of light or from the absorption of energy from the incident light,[1] it might be of interest to consider the energy changes involved. Unfortunately data are available for only the roughest kind of an approximation, but the magnitude of the quantities concerned can at least be shown.

In a recent experiment by the writer[2] a sodium surface containing

---

[1] See article by Jakob Kunz, Phys. Rev., XXIX., p. 212.

[2] Phys. Rev., Vol. XXIX., p. 212.

10 square centimeters was exposed to the radiation from a 2,000 candle-power arc at a distance of something less than 1.5 meters. If we assume the reflecting power of sodium as given by Drude (for white light?) to be 99.7 per cent. we obtain, after making due allowance for other absorptions and reflections, that approximately 400 ergs per second of light energy are absorbed by the sodium surface. Comparing the above distribution curve with the visibility curve we might conclude that 100 ergs per second of this energy fall within those wave-lengths which are capable of producing the discharge.

The photo-electric current resulting was $25 \cdot 10^{-9}$ amperes, corresponding to $25 \cdot 10^{10}$ electrons per second if we accept the value of $e$ as $10^{-20}$. In the article cited Kunz has found the initial velocity of the electrons from a sodium-potassium alloy to be $6 \cdot 10^{7}$ cm./sec. for $\lambda = .42\ \mu$. The velocity of the electrons from the above sodium surface can hardly be more than $10 \cdot 10^{7}$ cm./sec., giving as the kinetic energy per electron $3 \cdot 10^{-12}$ ergs, where $m$ is taken as $6 \cdot 10^{-28}$ grams. This makes the total kinetic energy of the expelled electrons .75 erg per second, a quantity which so far as its magnitude is concerned could easily be supplied from the 100 ergs per second of absorbed energy.

Since the above was prepared for publication, Hallwachs[1] has published some data showing the "specific photo-electric activity" for potassium from .2 $\mu$ to .6 $\mu$, using the lines of the mercury arc in quartz as the exciting source. Excluding the existence of secondary maxima, his data, when plotted as a smooth curve, take the form shown in Fig. 5, *I*. It was suggested to the writer that the radical difference between this curve and Fig. 4, *II*, above, might have been in part due to the use of a flint glass prism, which undoubtedly did absorb some of the energy in the blue and violet region. It did not seem possible, however, that this absorption could have been sufficient to change Fig. 4, *II*, over into the form shown by the investigations of Hallwachs.

It is to be regretted that measurements of the energy distribution in the spectrum of acetylene in the neighborhood of .4 $\mu$ are almost impossible. However, to test in a qualitative way the influence of absorption, the flint glass prism was replaced by a carbon bisulphide

---

[1] Ann. d. Ph., XXX., p. 593.

prism, which is especially transparent to the blue and violet and the measurements repeated. The results are shown in data Table III., and graphically in Fig. 5, curves *II*, *III*, and *IV*. Curve *II* is plotted from the observed data as recorded in Table III., without making any corrections. The ordinates, as in Fig. 4, are rates of deflection of the electrometer in mm. per second. It is to be noted that the maximum ordinate is only a trifle over one tenth of the corresponding ordinate in Fig. 4, *II*. This is due to smaller light intensity. A slight correction to curve *II* is necessary on account of the rapid change of the index of refraction of $CS_2$ with change of temperature. Fig. 5, *III*, is the result of this correction, the points being omitted to avoid confusion. Finally, after reducing to a normal spectrum with equal energy distribution, we get Fig. 5, *IV*.

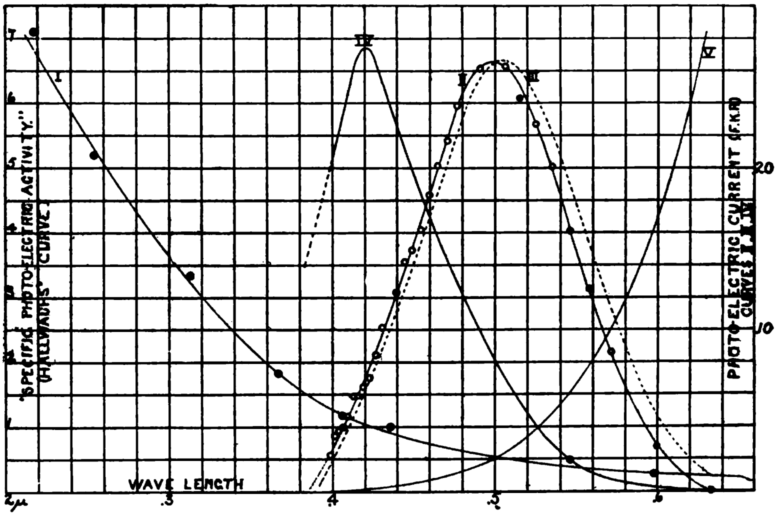


Fig. 5.

The maximum persists although, as might be expected, it is shifted somewhat toward shorter wave-lengths. Still it seems sufficiently prominent to call it a primary rather than a secondary maximum, the exact location of the crest being somewhat in doubt. In other words the use of the flint glass prism seems to have shifted the curve rather than changed its form.

TABLE III.

| Wave-length. | Observed Photo-electric Currents. | Wave-length. | Observed Photo-electric Currents. |
|---|---|---|---|
| .6330 μ | .0000 mm./sec. | .4492 μ | .1492 mm./sec. |
| .5980 | .0274 | .4442 | .1420 |
| .5712 | .0858 | .4396 | .1230 |
| .5581 | .1251 | .4306 | .1016 |
| .5460 | .1610 | .4265 | .0843 |
| .5353 | .2004 | .4226 | .0703 |
| .5250 | .2270 | .4190 | .0643 |
| .5152 | .2428 | .4158 | .0594 |
| .5064 | .2625 | .4124 | .0587 |
| .4908 | .2612 | .4092 | .0463 |
| .4770 | .2380 | .4064 | .0398 |
| .4708 | .2165 | .4038 | .0374 |
| .4650 | .2015 | .4013 | .0342 |
| .4598 | .1835 | .3984 | .0222 |
| .4542 | .1619 | | |

In conclusion, the extreme sensibility of the photo-electric cell, even at zero potential, may be mentioned. In these later experiments, the shape and size of the $CS_2$ prism was such as to necessitate cutting down the beam of light through it to a few millimeters diameter. The light emerging from the collimator *A* (see Fig. 1), when allowed to fall on a piece of white cardboard, was scarcely visible even in a dark room. And yet the photo-electric currents produced by it were measured without much difficulty. This is more strikingly illustrated by reference to Fig. 5, *V*, which shows the relative energy for each wave-length falling on the cell, taking into account the variable dispersion of the prism.

CORNELL UNIVERSITY,
October, 1909.

# SOME PHOTO-ELECTRIC PROPERTIES OF THE ALKALI METALS.

## IV. LABORATORY APPLICATIONS.[1]

BY F. K. RICHTMYER.

WHILE performing some photo-electric experiments the writer was greatly impressed with the possibilities of putting this phenomenon to various laboratory uses where optical measurements by eye are either extremely difficult or altogether impossible. For example as a photometric device for monochromatic light a sensitive sodium surface excels the eye in several respects:

1. In a properly constructed cell there is absolutely no fatigue.

2. The greatest sensibility is in the blue and violet region, where the eye is least sensitive.

3. The magnitude of the photo-electric current, *i. e.*, the sensibility of the device, can be increased by (*a*) increasing the area of the sodium surface, (*b*) increasing the area of the photometric field, and (*c*) decreasing the distance of the field from the sensitive surface; whereas the action of the eye depends simply on the intensity of light in the photometric field. In a photometer little gain in sensibility results from increasing the size of field, or, within limits, of moving the field nearer the eye.

A case in point is the measurement of decay of phosphorescence. A sample of Balmain's paint on a visiting card was exposed to sunlight for thirty seconds, and then placed over a window about eight cm. in front of a sodium surface, so that 10 or 12 square centimeters of the card were exposed. At first the rate of deflection of the electrometer[2] was too rapid for accurate measurement. After about six minutes, however, observations were begun and the rate of

---

[1] Paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

[2] For description of apparatus and adjustments see the first paper in this series, PHYS. REV., Vol. XXIX., pp. 71–80.

charge taken by chronograph about every four minutes, continuing for one hour after exposure. The results are shown in data table and graphically in Fig. 1 (*I*), in which the ordinates, while photoelectric currents, are proportional to the intensity of light emitted by the phosphorescing paint.

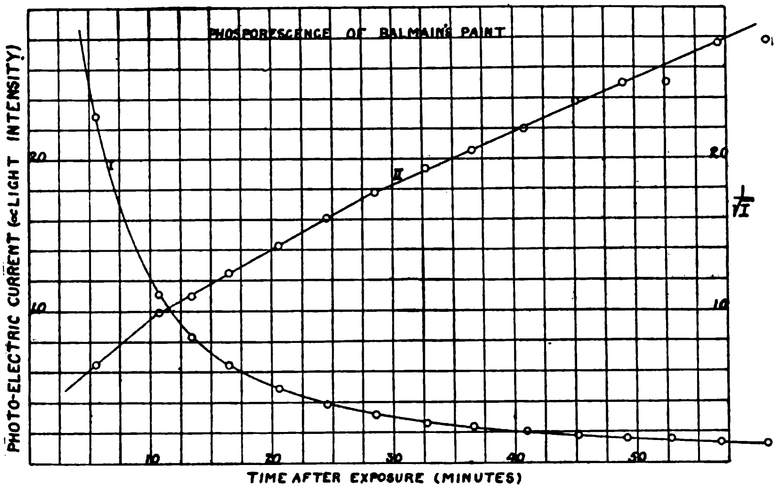


Fig. 1.

TABLE I.

| Time after Exposure. | Rate of Charge of Electrometer, Proportional to Intensity of Light. | $\frac{1}{\sqrt{I}}$ | Time after Exposure. | Rate of Charge of Electrometer, Proportional to Intensity of Light. | $\frac{1}{\sqrt{I}}$ |
|---|---|---|---|---|---|
| 5.53 min. | 2.29 mm./sec. | .65 | 36.65 min. | .236 mm./sec. | 2.06 |
| 10.68 | 1.11 | .95 | 40.95 | .206 | 2.20 |
| 13.40 | .829 | 1.10 | 45.18 | .176 | 2.38 |
| 16.53 | .641 | 1.25 | 49.15 | .159 | 2.50 |
| 20.60 | .489 | 1.43 | 52.83 | .158 | 2.51 |
| 24.67 | .385 | 1.61 | 56.88 | .131 | 2.76 |
| 28.73 | .316 | 1.78 | 60.7 | .130 | 2.77 |
| 32.75 | .259 | 1.96 | | | |

At the end of one hour the substance was just visible. Accurate measurements by the eye would have been practically impossible, whereas the electrometer was being deflected at a fairly measurable rate — over .1 mm./sec. And this rate of deflection could have been increased many times had the exposed area of paint, and its distance from the cell been sufficiently altered. For example, a card 10 cm. square, placed 6 cm. from the cell would have increased the rate of deflection twenty times or more, *i. e.*, to 2 mm./sec. at the end of one hour, which would make observations possible for several hours.

When the above data are plotted in the usual manner, with $1/\sqrt{I}$ against time the curve shown in Fig. 1 (*II*) results. Except in a general way no great amount of importance should be attached to the compound linear character of this curve, as no special care was taken to eliminate or correct for the leakage currents (previously mentioned[1]) through the glass walls of the cell, the object being simply to test the cell as a photometric device and not the study of phosphorescence.

Another possible application results from the fact that the total quantity of electricity discharged from the sensitive surface, and therefore the permanent deflection given to the electrometer, depends on the integral $I \cdot dt$, where $I$ represents the light intensity over an interval of time $dt$. That is, such a combination of cell and electrometer is to light measurements what the electrolytic cell is to the measurement of quantity of electricity. In this way the study of short decay phosphorescence might be possible. The writer has also made measurements of the relative total quantity of light passing through a photographic shutter set at speeds of .01, .02 and .04 sec. For this purpose the closed shutter, mounted in front of the cell, was illuminated by a beam of parallel light. The "exposure" was then made and the resulting deflection of the electrometer noted. Unfortunately these data were not preserved and it must suffice to say that in the case of .01 sec. the deflection was 20 or 30 mm.

Although possibly a little troublesome to work with, nevertheless it would seem that this phenomenon is capable of more extended laboratory use than has heretofore been made of it.

Cornell University,
October, 1909.

---

[1] Phys. Rev., Vol. XXIX., pp. 71–80.

# THE TEMPERATURE OF THE DIVIDED BUNSEN FLAME.

By Daniel W. Shea.

IN the course of an experimental study of the electrical character of the source of light radiation made with a divided Bunsen flame, in this laboratory, by Dr. T. P. Irving,[1] it became desirable to know the temperature of the different parts of the flame, when in its simple state and when colored by burning in it various metal vapors. As the literature on this flame, so far as we could learn, contained no temperature records of service to us, I measured the temperature at numerous points. The measurements, though made only for the size and form of burner described below, yielded some results interesting in themselves and of importance in connection with Dr. Irving's work. The following pages contain a brief account of the apparatus and measurements, and some discussion of the results.

## I. The Apparatus.

The burner used to separate the cones of the Bunsen flame consisted of a brass tube *a*, Fig. 1, 24 cm. long and 1.4 cm. wide inside, and a glass tube *b* shaped like that employed by Fredenhagen,[2] 15 cm. long, 4.8 cm. wide inside at the bottom and 2.7 cm. inside at the top. The glass tube was fitted over the brass tube by means of a bored plug of rubber, 3 cm. thick, inserted 1 cm. into the glass tube. By sliding this plug up or down on the brass tube the distance betweeen the bases of the two cones could be varied at pleasure.

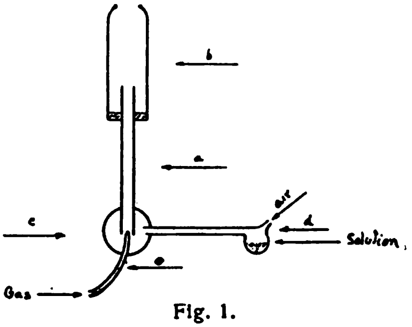


Fig. 1.

---

[1] Phys. Rev., XXIX., p. 243, 1909.

[2] Physik. Zeitschr., VIII., p. 404, 1907.

The device for mixing the gas, air and metal vapor for the burner consisted of a glass bulb *c*, Fig. 1, of 6 cm. internal diameter, bored at the top for receiving the lower end of the brass tube *a* of the burner, at the side for receiving the neck of a small retort *d* and at the bottom for the introduction of the gas tube *e*. The tube *e* was of glass with an internal diameter of 1.5 cm., but drawn out at the upper end so as to leave an opening only .08 cm. in diameter for the issuing of the gas. This drawn-out end extended up into the brass tube *a* a few millimeters. The retort *d* contained the solution for producing the metal vapor. Through its tubulure the necessary air was admitted. The tubes *a* and *e* and the neck of the retort were sealed hermetically with wax into the glass bulb.

This combination of burner and mixing device gave a steady divided Bunsen flame when the flow of air and gas was properly regulated.

The metal vapor was introduced by the Beckmann[1] method, which consists in putting an acid solution of a salt of the metal, together with some pieces of coppered zinc, into the retort, and allowing the air entering through the loosely stoppered tubulure to carry the metal vapor through the bulb *c* up into the tube *a* of the burner.

The thermometer was an Heraeus Pt-PtRh thermo-element connected with a Weston millivoltmeter. The wires were protected by 12-inch lengths of fire-clay tube near the junction, and their terminals were kept at the temperature of melting ice. The junction was globular, about 1 mm. in diameter. It was glowed by electric current to expel any occluded gases before beginning the measurements.

## II. The Temperature Measurements.

These were made by inserting the junction of the thermo-element into the various parts of the outer cone, and through the outer cone, into the parts of the inner cone, and into the space between the two cones, reading the millivoltmeter as soon as steady temperature was indicated. The outer cone is so stable that its burning is not interfered with by thrusting through it the protected thermo-element. The inner cone is more sensitive to disturbance than is the outer

---

[1] Zeitsch. für Angewandte Chemie, p. 561, 1907.

cone, but it is not so sensitive as to make difficult the measurements of its temperature. Several observations were made for each part studied, and the mean of the millivoltmeter readings for each were converted into degrees centigrade by means of the Reichsanstalt certificate accompanying the thermo-element.

The divided flame in its simple state, that is, in the form of two conical shells of burning illuminating gas, was studied first, and for the distances 11, 8, 6 and 0 cm., respectively, between the bases of the cones. Then studies were made for the same distances when the divided flame was colored by burning in it the vapors of Na, Rb, Cs, Ca, Sr and Ba, produced from the solution of their respective chloride in the retort *d*. The outer conical shell in all cases rested directly upon the top of the glass tube *b*; the inner conical shell directly upon the top of the brass tube *a*, excepting for the distance near zero, when this shell shifted to the top of the glass tube *b*. The gas was taken from the gas mains of Washington.

The height of the outer cone was about 14 cm. and that of the inner cone about 1.75 cm. for all values of the distance between the bases of the cones.

### 1. *Temperature of the Outer Cone.*

Measurements were made at several points in the conical shell of flame at its base, then at several points in the shell at its intersection with a horizontal plane about 2 cm. above the base, again at the intersection with a plane about 4 cm. above the base, finally at the apex of the cone, 14 cm. above the base. The mean results are given in centigrade degrees in the Tables I.–IV. The grayish color

TABLE I.

*Distance between the Bases of the Cones,* 11 *cm.*

| | At Base. | At 2 cm. | At 4 cm. | At Apex. |
|---|---|---|---|---|
| Simple flame | 563.7 | 743.8 | 823.4 | 884.1 |
| Colored by Na | 699.1 | 793.3 | 893.1 | 904.3 |
| Rb | 647.8 | 788.4 | 881.8 | 906.1 |
| Cs | 629.9 | 768.2 | 828.1 | 901.8 |
| Ca | 623.6 | 753.7 | 830.1 | 884.1 |
| Sr | 563.7 | 703.9 | 803.6 | 893.1 |
| Ba | 647.8 | 783.2 | 846.2 | 897.4 |

of the outer cone of the simple flame changes to that characteristic of the metal when a metal vapor is introduced. There was so little change in the temperature from about 4 cm. to the apex, and that so regular that measurements made between those points are omitted from the tables.

### TABLE II.

*Distance between the Bases of the Cones, 8 cm.*

| | At Base. | At 2 cm. | At 4 cm. | At Apex. |
|---|---|---|---|---|
| Simple flame | 614.9 | 782.6 | 829.3 | 891.5 |
| Colored by Rb | 716.1 | 801.8 | 884.1 | 910.4 |
| Ca | 648.3 | 770.3 | 890.1 | 919.1 |

Measurements for the simple flame and for Rb and Ca only are given in Table II. and the two following tables, since they are sufficient to show how the temperature varies with variation in distance.

### TABLE III.

*Distance between the Bases of the Cones, 6 cm.*

| | At Base. | At 2 cm. | At 4 cm. | At Apex. |
|---|---|---|---|---|
| Simple flame | 653.6 | 793.6 | 890.4 | 939.9 |
| Colored by Rb | 751.8 | 873.2 | 947.9 | 966.7 |
| Ca | 716.9 | 844.5 | 927.8 | 965.2 |

### TABLE IV.

*Distance between the Bases of the Cones, 0 cm.*

| | At Base. | At 2 cm. | At 4 cm. | At Apex. |
|---|---|---|---|---|
| Simple flame | 704.0 | 837.7 | 937.3 | 968.2 |
| Colored by Rb | 730.4 | 969.8 | 993.1 | 990.0 |
| Ca | 880.1 | 973.6 | 1,013.0 | 1,002.5 |

### 2. *Temperature of the Inner Cone.*

Measurements were made, as for the outer cone, at the base, at about 0.5 cm., 0.9 cm., 1.3 cm., and at the apex. The results are given in degrees centigrade in Tables V.–VIII. The results for Rb and Ca only will be given in Tables VI.–VIII. The inner cone retains always its greenish color. A solid colored flame burns just above it, characteristic of the metal introduced.

## TABLE V.

*Distance between the Bases of the Cones,* 11 *cm.*

| | At Base. | At 0.5 cm. | At 0.9 cm. | At 1.3 cm. | At Apex. |
|---|---|---|---|---|---|
| Simple flame | 996.5 | 1,024.6 | 1,045.2 | 1,058.9 | 990.0 |
| Colored by Na | 1,000.0 | 1,048.1 | 1,090.0 | 1,085.0 | 1,000.0 |
| Rb | 988.6 | 1,009.9 | 1,048.1 | 1,023.0 | 988.6 |
| Cs | 1,015.8 | 1,034.3 | 1,056.6 | 1,083.3 | 997.4 |
| Ca | 987.3 | 1,012.7 | 1,021.4 | 1,029.3 | 980.0 |
| Sr | 990.0 | 1,006.4 | 1,042.9 | 1,053.2 | 980.0 |
| Ba | 988.6 | 1,017.0 | 1,042.2 | 1,047.0 | 997.4 |

## TABLE VI.

*Distance between the Bases of the Cones,* 8 *cm.*

| | At Base. | At 0.5 cm. | At 0.9 cm. | At 1.3 cm. | At Apex. |
|---|---|---|---|---|---|
| Simple flame | 998.9 | 989.6 | 1,049.9 | 1,057.2 | 993.7 |
| Colored by Rb | 997.4 | 1,028.9 | 1,047.6 | 1,038.1 | 988.6 |
| Ca | 1,000.0 | 1,051.3 | 1,085.0 | 1,050.0 | 988.6 |

## TABLE VII.

*Distance between the Bases of the Cones,* 6 *cm.*

| | At Base. | At 0.5 cm. | At 0.9 cm. | At 1.3 cm. | At Apex. |
|---|---|---|---|---|---|
| Simple flame | 996.2 | 1,000.7 | 1,011.1 | 1,027.9 | 980.0 |
| Colored by Rb | 995.2 | 1,017.3 | 1,023.8 | 1,034.1 | 988.6 |
| Ca | 1,006.0 | 1,047.1 | 1,062.7 | 1,037.3 | 980.0 |

## TABLE VIII.

*Distance between the Bases of the Cones,* 0 *cm.*

| | At Base. | At 0.5 cm. | At 0.9 cm. | At 1.3 cm. | At Apex. |
|---|---|---|---|---|---|
| Simple flame | 732.5 | 821.3 | 950.9 | 973.2 | 998.6 |
| Colored by Rb | 730.4 | 839.8 | 965.2 | 992.0 | 1,013.3 |
| Ca | 880.1 | 830.6 | 958.9 | 987.7 | 1,008.4 |

## 3. *Temperature Along the Axis of the Burner.*

Measurements were made from the apex of the inner cone, at intervals of 1 cm., up along the axis of the burner, to the base of the upper cone, and at the center of the axis of the upper cone. The results are given in centigrade degrees in Tables IX.–XII. The results for Rb and Ca only will be given in Tables X.–XII.

The part of the axis just above the apex of the inner cone is, for the colored flames, in the solid flame of burning metal vapor which rests on this cone, and extends above its apex. The length of the flame is about the same for the various elements, Sr excepted, and for the various distances between the bases of the cones.

The length of these flames of burning metal vapor, measured from the tip of the cone, were about as follows:

Na-flame, 8.0 cm.; Rb-flame, 9.5 cm.; Cs-flame, 8.5 cm.; Ca-flame, 8.0 cm.; Sr-flame, 3.0 cm.; Ba-flame, 8.5 cm.

### TABLE IX.

*Distance between the Bases of the Cones*, 11 *cm.*

| | Distance in cm. above Apex of Inner Cone. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 16 |
| Simple flame | 990.0 | 965.9 | 935.7 | 910.4 | 880.1 | 840.2 | 788.4 | 752.7 | 717.0 | 703.5 | 793.3 |
| Colored by Na | 1,000.0 | 972.7 | 972.7 | 951.4 | 937.8 | 909.1 | 882.0 | 893.1 | 788.4 | 727.0 | 793.2 |
| Rb | 988.9 | 972.7 | 965.9 | 953.9 | 935.7 | 910.4 | 884.1 | 860.1 | 832.5 | 727.0 | 788.4 |
| Cs | 997.4 | 972.7 | 965.9 | 953.9 | 932.2 | 901.8 | 867.8 | 832.5 | 793.3 | 735.3 | 815.2 |
| Ca | 980.0 | 972.7 | 965.9 | 953.9 | 927.8 | 884.1 | 860.1 | 832.5 | 793.2 | 744.3 | 788.4 |
| Sr | 980.0 | 972.7 | 965.9 | 910.4 | 860.1 | 823.2 | 806.3 | 793.3 | 788.4 | 735.3 | 727.0 |
| Ba | 977.4 | 972.7 | 965.9 | 953.9 | 945.4 | 927.8 | 893.1 | 867.8 | 814.9 | 727.0 | 762.6 |

The point on the axis at the distance 9 cm. above the apex of the inner cone coincides with the center of the base of the outer cone; the point at distance 16 cm. with the central point of the axis of the outer cone.

### TABLE X.

*Distance between the Bases of the Cones*, 8 *cm.*

| | Distance in cm. above Apex of Inner Cone. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 13 |
| Simple flame | 978.5 | 967.3 | 950.1 | 931.2 | 926.0 | 874.6 | 761.5 | 803.9 |
| Colored by Rb | 988.6 | 972.7 | 965.9 | 953.9 | 935.7 | 901.8 | 788.4 | 806.3 |
| Ca | 980.0 | 972.7 | 965.9 | 953.9 | 927.8 | 884.1 | 746.3 | 806.3 |

The point on the axis at 6 cm. above apex of the inner cone coincides with the center of the base of the outer cone; the point at 13 cm. with the central point of the axis of the outer cone.

### TABLE XI.

*Distance between the Bases of the Cones*, 6 *cm.*

| | Distance in cm. above Apex of Inner Cone. | | | | | |
|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 11 |
| Simple flame | 980.0 | 965.9 | 884.6 | 840.2 | 788.4 | 832.5 |
| Colored by Rb | 988.6 | 972.7 | 953.9 | 932.2 | 822.3 | 849.6 |
| Ca | 980.0 | 972.7 | 965.9 | 945.6 | 840.2 | 867.8 |

The point on the axis at 4 cm. above apex of inner cone coincides with the center of the base of the outer cone; the point at 11 cm. with the central point of the axis of the outer cone.

### TABLE XII.

*Distance between the Bases of the Cones*, 0 *cm.*

| | Distance in cm. above Apex of Inner Cone. | | | |
|---|---|---|---|---|
| | −1.75 | −1 | 0 | 5.25 |
| Simple flame | 247.9 | 471.8 | 984.1 | 1,061.6 |
| Colored by Rb | 258.5 | 498.7 | 997.4 | 1,070.9 |
| Ca | 250.0 | 483.2 | 972.7 | 1,013.5 |

The point on the axis at − 1.75 cm. from the apex of the inner cone coincides with the center of the common base of the two cones; the point at 5.25 cm. coincides with the center of the axis of the outer cone, and is in the metal flames.

Measurements of the temperature of the solid flame of burning metal vapor which rests on the inner cone were made at various distances from the axis, and they indicated a slight falling off of the temperature as the distance from the axis increased. Thus for the Sr-flame, at a point opposite the apex of the inner cone and just within the surface of the flame, the temperature was 972.7 when it was 980.8 at the axis as given in Table IX.; and it was 935.7 at a point opposite that on the axis 1 cm. above apex at which axial point the temperature was then 972.7, as given in Table IX.

### 4. *Temperature at Points Outside the Axis.*

Measurements were made at numerous points other than those given in the preceding subsections. Tables XIII.–XV. contain the results for points at about 0.5 cm. distant from the inner surface of the glass tube *b* and from the top of the plug to the top of the glass tube.

### TABLE XIII.

*Distance between the Bases of the Cones,* 11 *cm.*

| | Distance in cm. above Top of Plug. | | | | |
|---|---|---|---|---|---|
| | 0 | 3 | 5 | 9 | 14 |
| Simple flame | 288.5 | 450.9 | 569.1 | 631.8 | 653.7 |
| Colored by Na | 258.5 | 425.8 | 471.8 | 499.0 | 554.6 |
| Rb | 444.5 | 544.5 | 644.5 | 699.1 | 752.7 |
| Cs | 288.5 | 471.8 | 554.6 | 635.8 | 725.4 |
| Ca | 415.8 | 453.7 | 528.2 | 663.7 | 790.0 |
| Sr | 288.5 | 480.9 | 571.8 | 644.5 | 663.7 |
| Ba | 329.8 | 429.8 | 499.0 | 553.6 | 635.8 |

The point 5 cm. above top of plug is opposite apex of inner cone; the point 14 cm. is at the base of the outer cone.

### TABLE XIV.

*Distance between Bases of the Cones,* 8 *cm.*

| | Distance in cm. above Top of Plug. | | | | |
|---|---|---|---|---|---|
| | 0 | 3 | 5 | 8 | 14 |
| Simple flame | 361.4 | 560.1 | 703.8 | 719.3 | 767.5 |
| Colored by Rb | 338.2 | 535,4 | 671.2 | 725.4 | 788.4 |
| Ca | 355.7 | 544.5 | 671.2 | 780.0 | 788.4 |

The point 8 cm. is opposite the apex of the inner cone; the point 14 cm. is at the base of the outer cone.

### TABLE XV.

*Distance between Bases of the Cones,* 6 *cm.*

| | Distance in cm. above Top of Plug. | | | | |
|---|---|---|---|---|---|
| | 0 | 3 | 5 | 10 | 14 |
| Simple flame | 379.1 | 583.2 | 698.6 | 825.4 | 819.9 |
| Colored by Rb | 355.7 | 562.7 | 663.7 | 814.9 | 822.3 |
| Ca | 385.5 | 599.0 | 725.4 | 822.3 | 840.2 |

The point 10 cm. is opposite the apex of the inner cone; the point 14 cm. is at the base of the outer cone.

## III. Analysis of the Results.

Tables I.–IV. show that: (1) The temperature of the outer conical shell in all cases increases rapidly from the base to about two thirds way up the cone, then falls more rapidly to the apex, so that, if represented graphically, taking temperatures as ordinates and distances above the base as the abscissæ the curve is as shown in Fig. 2, *a*; (2) the temperature of the outer cone is somewhat greater for the colored flame than for the simple flame, the differences being greater near the base, where they average, for Table I., about 75°, while near the apex they average only 30°; (3) the temperature of all parts of the outer cone for all cases is greater the shorter the distance between it and the inner cone. If the temperature for a given part of a given outer cone be plotted taking the temperature as ordinates and the distance between the bases of the cones as

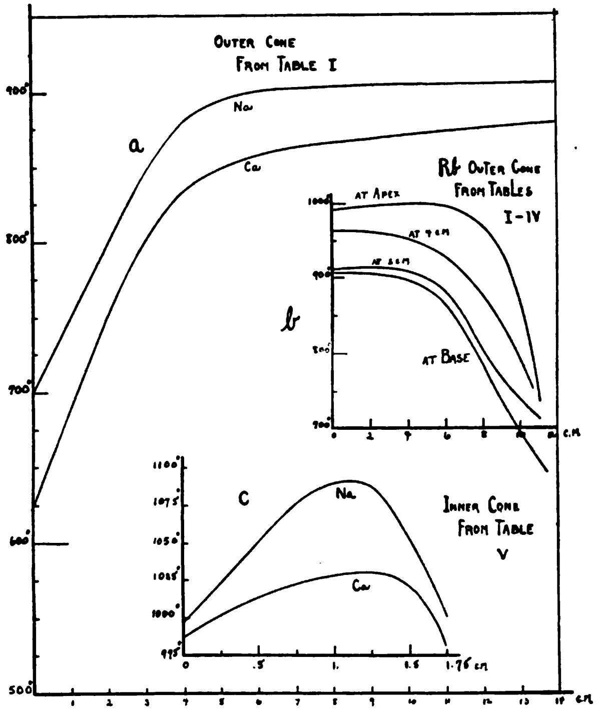


Fig. 2.

abscissæ, the curve is as represented in Fig. 2, *b*, which shows the curves for Rb at apex, at 4 cm. point, at 2 cm. point and at the base; (4) the temperature of the outer cone seems to be somewhat greater for the elements of the first Mendeléeff group than for those of the second group, when the distance between the base of the cone is considerable; the reverse is the case when the distance becomes small.

Tables V.–VIII. show that: (1) The temperature of the inner conical shell for the simple and the colored flames and for distances between the bases of the cones not near zero increases from the base to a maximum value in a zone about 3 or 4 mm. broad, just above the middle of the cone, then decreases to a value at the apex which is about the same as at the base. These maxima for Table V. average about 70° higher than the minima. Near the zero of distance between the bases of the cones, the temperature distribution for the inner cone becomes similar to that of the outer cone, and the values are smaller than for longer distances. Fig. 2, *c*, shows the temperature curve for the inner cone when the distances between the cones are large; (2) the temperature of corresponding parts of the inner cone for the simple flame and for all the colored flames has practically the same value for all distances between the bases of the cones. As noted elsewhere in this paper, the characteristic greenish color of the inner cone for the simple flame remains unchanged through the burning of metal vapor, while that of the outer cone is changed, and a characteristic solid flame of the metal vapor appears between the two cones.

A comparison of Tables I.–IV. with V.–VIII. shows that the temperature of the inner cone is higher than that of the outer cone until the distance between the bases of the cones is nearly zero. Then the outer cone becomes the hotter, and the hottest point of the flame, which is then the ordinary Bunsen flame, is above the apex of the inner cone and near the center of the outer cone. The highest temperature found there, 1070.9°, occurred when burning Rb vapor. The highest temperature of the flame of an ordinary Bunsen burner using the same gas was found at about the same distance above the inner cone as in the case just mentioned, and it had the value 1186.6°, the mean of a large number of observations. The temperature near the base of the same flame measured 361.3° and was the lowest temperature found in that flame. A comparison of all the Tables I.–XV. shows that the highest temperature attained in the divided Bunsen flame is that of the zone of the inner cone just above its middle. The highest temperature found there, 1090.0°, occurred at the time the flame was burning Na-vapor (see Table V.). In Fig. 3, *a*, the two cones and the flame of burning metal vapor are shown, and in Fig. 3, *b*, this zone of highest temperature is indicated by the heavy parts of the lines.

Tables IX.–XII. show that: (1) The temperature along the axis of the burner decreases regularly up to the base of the upper cone, then increases to the apex of the upper cone; (2) the temperature is about the same for the corresponding points of the first three centimeters of this axis for all the colored flames, and is only slightly larger in the average than that for the simple flame. At points higher up the temperature differs considerably for the different flames, and averages for the colored flames considerably higher than for the simple flame, until the base of the outer cone has been reached; (3) the temperature for the simple and for the colored flames remains about the same for each corresponding point of the axis below the outer cone, as the distance between the bases of the cones is shortened, but it increases rather rapidly in the outer cone; (4) the temperatures for the elements of the first Mendeléeff group average about the same as for the elements of the second group.

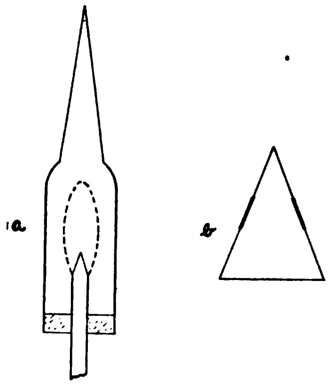


Fig. 3.

Tables XIII. to XV. show: (1) At points in the space outside the inner cone, and outside the flame between the cones, the temperature increases considerably as the distance between the bases of the cones decreases; (2) for any given point it is practically the same for both simple and colored divided Bunsen flames.

## IV. SUMMARY.

The most interesting results of the measurements are these: (1) The highest temperature in a divided Bunsen flame is confined sharply to a small zonal segment of the inner conical shell, no matter what substances are used to color the flame, nor what the distance is between the bases of the cones, provided that that distance is at least a few centimeters; (2) the temperature of the inner conical shell is not changed by varying its distance from the outer conical shell, provided that the distance is more than two or three centimeters, nor by introducing through it metal vapors that burn in a flame above it and in the outer cone; (3) the temperature distribution of the inner conical shell changes to one that increases from base to apex when the distance between the cones approaches zero; (4) the temperature of the flames of metal vapor burning just above the inner cone is the same for all the vapors used, excepting near the tips of these flames, and it is not changed by varying the distance between the cones; (5) the temperature of the outer conical shell increases from the base to the apex and it is always greatest at the apex; (6) the temperature of the outer conical shell increases when its distance from the inner cone decreases; also when it burns metal vapors introduced through the inner cone.

## V. RELATION TO IRVING'S RESULTS.

Irving found the negative electrical wandering of the inner cone stronger than that of any other part of the divided Bunsen flame, so strong that it "was torn off the brass tube on the side towards the negative plate" even by weak fields. The temperature measurements show that this inner cone is much the hottest part of the divided Bunsen flame. There are no distortions of the inner cone, in an electrical field, definite enough in location to say that they are at the hottest zone. The wandering was about the same for all distances between the bases of the cones. The temperature of the inner cone is also the same for all distances between the cones, if the distances are not too small. The magnitude of the negative wandering of the flames of metal vapor burning just above the inner cone Irving found to be the same for all the metals and for all distances between the cones. The temperature of these flames is the same for all the metals and for all distances between the cones. The electrical wandering of the flames towards the negative plate is less violent than is that of the inner cone. The temperature of these flames is less than that of the inner cone.

Irving found no wandering of the simple outer cone, nor for this cone colored with Li, but he found wandering towards the negative plate, when the cone was colored by the other metals, ranging from 9° for Na to 45° for Ca and Ba. The temperature of the simple outer cone is less than that of the inner cone and of the flame, for like conditions, ranging from about 100° to 500° cooler for different parts. Its temperature is higher when burning metal vapor than when in the simple state.

In Irving's experiments it was found that the outer cone wanders as soon as the metal flame burning above the inner cone reaches it, on decreasing the distance between the cones. Table IV. shows that the outer cone is then at its highest temperature.

The foregoing comparisons would indicate that the electrical wandering of the parts of the Bunsen flame is greatest for the hottest parts. This does not hold, however, for the metal vapor before it enters the inner cone. Its temperature there is about that of the room, yet it is even more electro-positive than is the hot inner cone. Further it does not hold in all respects for outer cones when compared with one another, for, although the temperatures given in Table I. for Ca, Sr and Ba are in the same order, excepting at the apex of the cone, as that given by Irving for the respective electrical wanderings, yet the temperatures for Na, Rb and Cs, excepting at the apex, are in an order the reverse of that given by him for the electrical wanderings. Futher the temperature given for Rb and Ca in Tables II. and III. are in an order the reverse of that for the electrical wanderings, while the temperatures given for the same elements in Table IV. are in the same order as that of the wanderings. Nevertheless, the parts having the higher temperatures show the larger electrical wanderings for any given divided Bunsen flame, whether it be a simple flame, or one colored by any one of the elements studied.

The correspondence of magnitudes of electrical wanderings and magnitudes of respective temperatures may be mere coincidences, not due to any relationship of the two phenomena. But it seems probable that the changes in magnitudes occur simultaneously in the same direction necessarily. Irving offers an explanation of the decrease in the magnitude of the electrical wandering on passing from the inner to the outer cone, by supposing that the negative electrons from the cones neutralize the positive metal ions in the formation of oxides, which takes place chiefly in the flame above the inner cone. The formation of oxides would also explain the observed differences in the temperatures of the inner cone, metal flame and outer cone, if we suppose, as we must, that the parts are hottest where the oxidation is most extensive. The oxidation in the metal flame above the inner cone takes place to such an extent for all the metals studied that the flame is equally positive for all of them. At the same time the extent of oxidation is such that the flames are equally hot for all these metals.

To Dr. Irving and Mr. Louis H. Crook, I acknowledge my obligation for valuable assistance.

Physical Laboratory,
The Catholic University of America,
October 20, 1909.

# PROCEEDINGS

# OF THE

# AMERICAN PHYSICAL SOCIETY.

## THE RELATIVE MOTION OF THE EARTH AND THE ETHER.[1]

By H. A. Wilson.

IT is shown in this paper that the aberration of light and the negative results of the experiments made to detect the relative motion of the earth and the ether can be explained without supposing that bodies change in length when they are set in motion relatively to the ether. The theory proposed is a modification of Sir G. G. Stokes' theory according to which the ether is regarded as behaving for differentially irrotational motions like an incompressible perfect fluid. The ether near an atom is supposed to be in irrotational motion as though the atom contained a point source of ether and an equal point sink so that the ether going into the atom at the sink comes out again at the source. The atoms consequently behave like hydrodynamical doublets. If preferred each atom may be supposed to contain many such doublets instead of only one. It is well known that sources and sinks in hydrodynamics are analogous to positive and negative charges in electrostatics. Thus matter containing hydrodynamical ether doublets is analogous to matter containing electrical doublets. High specific inductive capacity can be explained by supposing matter to contain electrical doublets which are polarized by an electric field and in the same way matter containing ether doublets will possess an analogous hydrodynamical property which may be called etherial permeability. The stream lines near a piece of matter placed in a stream of ether will be similar to the lines of electric force near a similar piece of insulator placed in a similar electric field provided the specific inductive capacity of the insulator is equal to the etherial permeability of the matter. The velocity potential of the etherial motion corresponds with the electric potential in the corresponding electrostatic case and so is a single valued function both inside and outside matter.

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

It is easy to prove that the supposed motion of the ether will not have any first order optical effects provided the influence of relative motion between matter and ether on the velocity of light in the matter postulated by Fresnel and verified by Fizeau is assumed.[1] The phenomena of aberration will therefore be the same as though the ether were at rest and the spectroscopic determination of star motion in the line of sight will also be unaffected by the supposed irrotational motion.

If we regard the matter forming the bulk of the earth as having a high etherial permeability the stream lines at the earth's surface will be nearly vertical (where the surface is horizontal) so that in experiments like Michelson and Morley's in which the apparatus is turned about a vertical axis a negative result will be obtained. A space inside matter will be partly screened off from the etherial stream just as the interior of a piece of iron is screened off from a magnetic field.

The rotation of a piece of matter about an axis of symmetry will not set the ether near it in motion. This agrees with Sir O. Lodge's negative result.

The theory described appears to offer a simple explanation of all available facts so that some new crucial experiment appears to be necessary to decide between it and the theory of a stagnant ether which is unmoved by moving matter.

It is necessary to suppose that the density of the ether is very small because otherwise there would be appreciable forces acting on pieces of matter placed in the etherial stream. If all matter has a high etherial permeability it may be extremely difficult to detect the supposed motion even by experiments capable of detecting second order effects because the etherial stream will be partly screened off by the stands supporting the apparatus.

It seems possible that optical and electrical phenomena depend on rotational motion and elasticity of the ether and so are very little influenced by irrotational motions. If the ether is really stagnant it is difficult to see how any phenomena can take place in it.

McGill University,
Montreal, Canada.

## The Nitrogen Thermometer from Zinc to Palladium.[2]

By Arthur L. Day and Robert B. Sosman.

The investigation of the fundamental high temperature scale with the nitrogen thermometer, which was begun in the Geophysical Laboratory in 1904 and upon which a preliminary report was given to

---

[1] See for example Æther and Matter, by Sir J. Larmor.

[2] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

the Physical Society at the Washington meeting in 1907[1] is now completed up to the temperature of melting palladium. A description of the apparatus is contained in a more extensive preliminary report published in the American Journal of Science in November, 1908.[2] The investigation has yielded an instrument: (1) in which the sensitiveness can be varied within considerable limits; (2) in which no zero change greater than the errors of observation occurs up to temperatures of 1550°; (3) in which all the well-known errors of earlier gas thermometry are reduced to about one fourth of their former magnitude; (4) in which the metal of the bulb (80 parts platinum, 20 parts rhodium) has very little contaminating effect upon the thermoelement used to transfer its temperatures to the standard melting points. The melting points, determined with this apparatus, of the pure substances commonly used as standards in high temperature measurements, are contained in the following table, together with the electromotive forces in microvolts of the standard (Pt 90, Rh 10) thermoelement.

| | $t$ | | $E$ |
|---|---|---|---|
| Cadmium . . . . . . | 320.0 | ± 0.3° | 2504 |
| Zinc . . . . . . . . . | 418.2 | ± 0.3 | 3429 |
| (Interpolation Point) . | 524.9 | | 4470 |
| Antimony . . . . . . | 629.2 | ± 0.5 | 5530 |
| (Interpolation Point) . | 854.1 | | 7929 |
| Silver . . . . . . . . | 960.0 | ± 0.7 | 9113 |
| Gold . . . . . . . . | 1062.4 | ± 0.8 | 10295 |
| Copper . . . . . | 1082.6 | ± 0.8 | 10534 |
| (Interpolation Point) . | 1206.4 | | 12000 |
| (Interpolation Point) . | 1298.5 | | 13100 |
| Diopside . . . . . . . . | 1391.2 | ± 1.5 | 14228 |
| Nickel . . . . . . . . | 1452.3 | ± 2.0 | 14977 |
| Cobalt. . . . . . . | 1489.8 | ± 2.0 | 15439 |
| Palladium . . . . . . . | 1549.2 | ± 2.0 | 16143 |

A quadratic equation of the form $E = -a + bt + ct^2$ passed through the melting points of zinc, antimony and copper, reproduces all temperatures within this region with an accuracy equal to that of the absolute measurement. A similar curve passed through copper, diopside and palladium reproduces the upper region equally well. To write an interpolation formula for the entire curve requires a cubic of the form $E = -a + bt + ct^2 - dt^3$, and is much more cumbersome to handle. The extrapolation upward of the zinc-antimony-copper curve (which has been common practice for several years) gives a temperature 19° too low at the melting-point of palladium.

---

[1] J. K. Clement, PHYS. REV., 24, 531, 1907.

[2] Day and Clement, Am. Jour. Sci. (4), 26, 405, 1908.

There appears to be no prohibitive difficulty in the way of continuing the scale beyond 1550° if necessity arises. The greatest existing uncertainty is in the distribution of temperature over the surface of the bulb which amounts to about 2° at 1500°.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
WASHINGTON, D. C., December, 1909.

*Volume XXX.* *April, 1910.* *Number 4.*

# THE PHYSICAL REVIEW.

## THE EFFECT OF CHANGES IN THE PRESSURE AND TEMPERATURE OF GASES UPON THE MOBILITY OF THE NEGATIVE IONS PRODUCED BY ULTRA-VIOLET LIGHT.

By Alois F. Kovarik.

### § 1. Introduction.

THE experiments to be described in this paper had for their object the investigation of the changes produced in the ionic velocity by variations in the pressure and temperature of the gas. The ions used were the negative ones which are produced by ultra-violet light falling upon a negatively charged metal surface.

It is known that at very low pressures these ions are corpuscular in size, and if, when the gas is at atmospheric pressure, they consist of clusters of molecules about charged nuclei, as is generally believed, then evidently as the pressure is reduced the cluster must eventually fail to form. A study of the ionic velocity at different pressures should reveal whether a change in the size of the ion actually takes place, and if it does, whether the cluster gets smaller gradually as the pressure is diminished or whether the change in size is more or less abrupt.

Again, the size of the ion may depend upon the temperature, and a determination of the ionic velocity at different temperatures should throw some light upon this point.

In the present investigation the ionic velocity was determined in air and carbon dioxid for pressures between 8.8 mm. and ·760 mm. of mercury, and in air at atmospheric pressure for temperatures between 84.5 and 698 degrees absolute.

Some results[1] on the effects of pressure and temperature upon the ionic velocity had been published already when these experiments were begun two years ago, and other papers[2] have appeared since, but the present results extend the ranges of the variations heretofore published and are not in complete agreement with some of the previous observations.

The subject matter will be treated under the following headings:

§ 2. Method.
§ 3. Pressure apparatus.
§ 4. Precautions aad corrections observed in the experiments.
§ 5. First pressure experiments in dry air and in dry carbon dioxid.
§ 6. Final experiments in dry air and in dry carbon dioxid.
§ 7. Temperature experiments.
§ 8. Summary and conclusion.

## § 2. METHOD.

The method used for determining the mobility of the ions is a modification of that used by Rutherford.[3] One of two parallel plates is illuminated by the ultra-violet light and charged with an alternating potential, and the electrical charge carried by the ions to the second plate is observed for different values of the alternating potential.

Let $t$ be the half-period of the alternating field, $u$ the mobility of the ions (*i. e.*, their velocity in a field of one volt per centimeter), $P$ the potential difference between the plates, and $d$ their distance apart. Then the space the ions will travel from the illuminated plate in the time $t$ is

$$s = \frac{P}{d} ut.$$

While $P$ is so small that $s$ is less than $d$, no ions starting from the illuminated plate can get to the second plate before their motion is reversed by the reversal of the field. It is only after the potential

---

[1] Rutherford, E., Proc. Camb. Phil. Soc., Vol. 9. p. 401, 1898. Langevin, P., Ann. de Chimie et de Physique, 28, p. 289, 1903. Phillips, P., Proc. Roy. Soc., A, p. 167, 1906.

[2] Blanc, A., J. de Phys. (4), 7, p. 825, 1908. Wellisch, E. M., Proc. Camb. Phil. Soc., Vol. 15, p. 1, 1908; Phil. Trans., A, Vol. 209, pp. 249–279, 1909.

[3] Loc. cit.

has reached some value $P_0$, which makes $s$ equal to $d$, that some of the ions which were produced at the beginning of an alternation can get across to the second plate before the field is reversed. As the potential is increased further, the quantity of electricity reaching the second plate should for a time increase regularly with the increase of potential, and finally when practically all of the ions produced by the light are carried to the second plate, any further increase in the potential should not alter the charge received.

If, therefore, values proportional to the charge received are plotted against the corresponding values of $P$, the curve should start as a straight line from the point $P_0$ on the axis of $P$ and finally become horizontal. The experimental curve does not actually start as a straight line from $P_0$, for reasons given later, but it has a well defined straight portion which produced intersects the axis of potentials at $P_0$.

Knowing the value of $P_0$, the rate of alternation and the distance between the plates, the value of the mobility can be obtained from the relation

$$u = \frac{d^2}{P_0 t} = \frac{2nd^2}{P_0},$$

where $n$ represents the number of alternations of the field per second.

## § 3. Pressure Apparatus.

The vessel in which the ionic velocity was measured at different pressures (see Fig. 1.) was made of a brass tube 10 cm. long and 10 cm. in diameter. The rear of the vessel was permanently closed by a heavy brass plate $P$ having in its center an ebonite plug $E$ forced into a brass collar. A brass rod $R$, holding the illuminated plate and strong enough not to bend appreciably when in a horizontal position, was screwed into the ebonite plug and had a wire leading to the source of potential. The zinc plate $Z$ exposed to the light was 8 cm. in diameter, turned to a true plane and polished with a buffer and vienna lime. It had a ball and socket arrangement which permitted its plane to be adjusted by means of three srcews whose points rested against its rear face and passed through another plate rigidly fixed to a strong brass tube $T$ which fitted over the rod $R$

to which it was firmly screwed when the zinc plate was set for any required distance from the opposite parallel plate. The fore part of the vessel was closed by a removable heavy brass plate having a circular groove cut in it to fit the cylinder. This cover could be brought tight against the cylinder by means of thumb screws *SS* fitted to four rods passing through the cover and extending to the rear plate on the outside of the vessel. The second of the parallel plates was a gauze *G* placed flush with the inner surface of the cover and had a ring of air one millimeter wide around it. The cover thus served as a guard ring. The gauze was made of No. 40 brass wire, two wires to a millimeter running in one direction only. It was held in position by a screw passing through an ebonite plug which was forced into a brass collar in the cover. A wire soldered to the screw led to the electroscope. A window 3.6 cm. in diameter was cut centrally with respect to the parallel plates and was closed by a quartz plate *Q* held in position by paraffin. The groove and all points of juncture were paraffined.

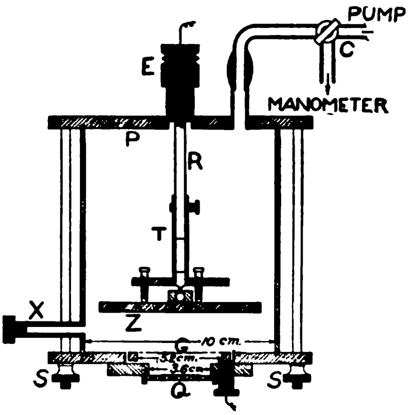


Fig 1.

A three-way stop-cock *C* was sealed with wax into the rear plate. One of its arms led to the manometers and another to the pump and source of supply of gases. An exit tube *X* was also provided in the fore part of the vessel. A diagram of the various connections is given in Fig. 2.

The mercury manometers used were the open air manometer *A* and the baromanometer *B*, the latter being used for pressures less than 150 millimeters. Readings were made by means of a cathetometer.

A C. T. R. Wilson's inclined electroscope *W* was used for the measurement of electric charges. An electroscope was chosen in preference to a quadrant electrometer because the inappreciable inertia of the leaf permits quick repetition of observations. The plate of the electroscope was connected to a series of 91 small lead accumulators *D*, the other pole of the series being connected to a slide movable over a high resistance *R* through which a current was continually flowing from a large chloride accumulator *F*, the positive pole of which was earthed. In this manner the zero could

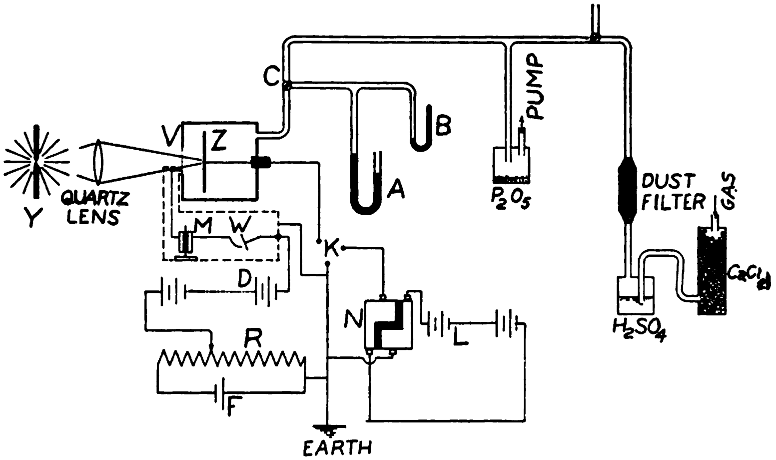


Fig. 2.

be easily kept fixed if for any reason the potential of the plate changed. The leaf was connected through a McLennan key *M* to the gauze of the testing vessel *V*. The sensitiveness of the electroscope was 17.5 divisions in the eye-piece for one volt, but only 5 divisions, corresponding to 0.35 volt, were used, as the sensitiveness over this distance was found convenient and also uniform.

The source of potential was a series of small lead accumulators *L*. The alternation of the field was brought about by a secohmmeter *N*. A triple key *K* was placed between the secohmmeter and the zinc plate, the third connection being to earth.

In the first experiments, the source of light *Y* was a spark between two zinc electrodes connected to the terminals of an induction coil. In the course of the investigation it was found that when synchronism existed between the alternation of the field and the intermittence of the light, the observations could not be depended upon. An arc between iron electrodes was consequently substituted for the spark. The light from the arc was generally not as steady as from the spark, but by giving it sufficient attention it could be kept fairly constant.

### § 4. PRECAUTIONS AND CORRECTIONS OBSERVED IN THE EXPERIMENTS.

1. A small portion of the light incident on the zinc plate strikes the walls of the vessel which are charged alternately positively and negatively by induction. Some of the ions thus produced reach the gauze. Again a certain number of the ions produced at the zinc diffuse toward the gauze. In both of these cases, the gauze will receive a charge even when the potential difference between the parallel plates is less than $P_0$. Evidently, the experimental curve showing the relation between the current and the potentials will begin with values of $P$ smaller than $P_0$. The quantity of electricity due to these ions is generally very small compared with that due to the ions coming directly from the zinc plate and for this reason the straight line portion of the experimental curve obtained with potentials immediately above $P_0$ is readily separated from the curve below $P_0$.

It was also observed that when the zinc plate became coated with a film of oxide or of moisture, the gold leaf showed a charge opposite in sign to that which it received when the zinc plate acted photo-electrically. This is due to the production of negative ions at the surface of the gauze by the light reflected from the zinc plate and also by that directly incident upon it, thus charging the gauze to a positive potential. When the zinc is polished, the effect of the gauze in sending out ions toward the zinc on alternate intervals is to decrease the effect of the zinc by a certain per cent. The effect of this in the curve is to decrease each ordinate by a fixed per cent. of itself, which alters the slope of the straight line portion of the curve without altering the intersection with the axis of potentials, *i. e.*, $P_0$ is unaffected by this phenomenon. This point was carefully tested by coating the gauze and the interior of the vessel with lamp black.

2. It was observed that with a slow rate of alternation of the field — *i. e.*, when the time required by the ions to pass between the plates was large — the velocity obtained was smaller than with a

high rate of alternation. At atmospheric pressure the difference between the values obtained is quite small but at low pressures it is very pronounced. This behavior is probably due to the diffusion of the ions to the sides.[1] It is however possible that if the ions are complex, they require a time comparable to that used in the experiments to grow to the final size, in which case also the value of the velocity with the long time of passage between the plates should be smaller than with the short time. When the values of the velocity were plotted against the time required for the ions to cross the field, the distribution was found to be linear and by producing the straight line thus obtained until it intersected the ordinate at the time $T = 0$, a value of the velocity was found which in each case was used as the desired velocity. It was therefore necessary to obtain values for the mobility using different rates of alternation and hence different values of $T$.

3. The use of a guard ring, small distances between the plates and a large zinc plate insured a uniform field. The ions were made to move, in most of the experiments, in the central portion, by having only a small central area of the zinc illuminated. This was brought about by a quartz lens in front of the apparatus.

No tests were made on the non-uniformity of the distribution of potential between the plates.[2] Experiments were, however, performed with different intensities of light but the value of the velocity was found to be unaffected by such changes. No correction was, therefore, made in the calculation.

4. The source of light is, of course, subject to changes in intensity but these changes were greatly obviated by proper attention. However, the readings were taken in such a manner as to minimize any errors arising from any changes in the intensity occurring during an experiment. Sometimes readings were taken with increasing potentials and then decreasing, the mean for any potential being used as the final value. Generally, however, some high potential was chosen and readings taken for this potential were considered as control readings. The control readings preceded and followed

---

[1] See also J. Zeleny, Phil. Trans., A, 195, p. 193, 1900.

[2] Zeleny, J., Phil. Mag., 46, p. 120, 1898. Childs, Wied. Ann., 65, p. 152, 1898. Schweidler, quoted by Prof. Thomson, Cond. of Elect., p. 260. Buisson, H., C. R., CXXVII., p. 224, 1898.

each individual reading for any potential. Then by interpolation all the readings were reduced to some uniform value of the control reading. In this manner any gradual changes in the intensity of the light during any one reading could be readily corrected. This plan of taking readings also corrects for the photo-electric fatigue when this is small; and it was found to be small, after the plate had been exposed to the light for some time. Tests on this point were made in air and in carbon dioxid, at atmospheric pressure and at low pressure.

5. On account of electrostatic induction the potential difference between the parallel plates is less than the potential of the zinc plate. The amount of the diminution can be readily found experimentally. This it shown by a specific example. The gauze at zero potential was insulated from the earth. The zinc plate then received a charge from 8 cells equivalent to 16.6 volts, and the leaf of the electroscope became deflected 8.8 divisions equivalent to 0.558 volt. The induction was, therefore, $0.558/16.6 = 3.37$ per cent. Using 22 cells $= 45.6$ volts, the deflection was 32 divisions $= 1.55$ volts, and the induction was $1.55/45.6 = 3.4$ per cent. Consequently, the value of $P_0$ obtained from the curve had to be diminished by 3.4 per cent., the distance between the plates in this case being 19.34 mm.

It is evident that on account of this induction the leaf will vibrate with the alternation of the field. The amount of this vibration depends on the induction, rate of alternation and the inertia of the leaf. In the present investigation it was found that for the potentials and the rate of alternation used, the vibration of the leaf could hardly be noticed. Exceptions noted are the ones with a very low rate of alternation and when the plates were very close together; but even in these cases, readings were easily taken as the vibration of the leaf did not exceed one half of a scale division. In these cases the mean position of the leaf had to be estimated.

6. The method used demands an equality of time for the opposite directions of the field. This depends on the secohmmeter itself and on the uniformity in the rate of rotation. If the brushes were not placed correctly or if they formed a poor contact with the segments, the leaf of the electroscope showed a "kick" to one side at the

moment when the zinc received the alternating charge. The "kick" is due to unequal induction. It was also noticed at times that the contact of the brushes with the segments produced a difference of potential which was also registered by a "kick" of the leaf. The former was remedied by careful adjustment of the brushes and the latter by cleaning the brushes and the segments, and rubbing both with paraffin oil,[1] carefully removing all excess.

It is here supposed that the field was fully established as soon as the brush came in contact with the segment of the secohmmeter. A water resistance always kept between the brush and the zinc plate was taken out and no difference was observed in the values of the mobility. Presumably then, the field establishes itself practically instantly through the resistance.

The alternating potential through the brushes was taken on a static voltmeter on different occasions and was found to correspond to the reading from a current voltmeter applied directly to the cells.

7. When there is an excess of ions of one kind in a gas, a force is exerted on the gas in these regions, which will produce a motion of the gas when the ions are in motion. A current of gas of this kind has been observed by Zeleny.[2] The motion of the gas is, however, very small in comparison with the velocity of the ions and the correction for it may be left out of account.

Convection currents due to difference of temperature in various parts of the field undoubtedly existed in a few of the temperature experiments. These will be noted in their proper place.

8. The reliability of the method used was tested by varying the distance between the plates, by varying the intensity of the incident light, and by changing the apparatus itself. In view of the fact that the values of the mobility of the ions in air at atmospheric pressure were all concordant under these various changes, it is believed that the method used is accurate.

9. In the experiments with dry gases the air was slowly passed into the exhausted vessel through a series of purifying and drying agents arranged as follows: potassium permanganate with dilute sulphuric acid, concentrated potassium hydroxid, calcium chloride,

---

[1] Allen, H. N., The Electrician (London), Vol. XXXIX., p. 379, 1897.

[2] Zeleny, J., Proc. Camb. Phil. Soc., X., p. 14, 1898.

concentrated sulphuric acid, and cotton; the carbon dioxid passed through the same series with the hydroxid omitted. The vessel was refilled with the gas 5 to 10 times before readings were taken at any pressure.

10. It hardly seems necessary to remark that due precautions were taken to carefully screen those portions of the apparatus requiring shielding from outside electrostatic disturbances.

## § 5. FIRST PRESSURE EXPERIMENTS.

*Dry Air.* — In the first experiments a spark was used as the source of light, the intensity of which varied very little. Only one set of readings was taken to obtain the value of the mobility. A set illustrating the readings and the calculation is given below.

### TABLE I.

Pressure = Barometric reading = 734.0 mm.
Temperature = Room temperature = 24°.0 C.
5,000 alternations of the field in 139.0 sec. (beginning),
139.2 sec. (end),
139.1 sec. (mean),
or 35.95 alternations per second.
100 cells = 215.8 volts.
Distance between the parallel plates = 19.34 mm.

| Potential of the Plate. | Time to Deflect Leaf 25th to 30th Division. | | Reciprocal of the Time × $10^4$. |
|---|---|---|---|
| 60 cells | 485.0 | 485.0 sec. | 21 |
| 70 | 90.0 | | |
| | 90.0 | 90.0 | 111 |
| 80 | 44.2 | | |
| | 42.8 | | |
| | 45.0 | 44.0 | 227 |
| 90 | 29.4 | | |
| | 31.6 | | |
| | 30.8 | 30.6 | 327 |
| 100 | 24.0 | | |
| | 24.0 | | |
| | 24.0 | 24.0 | 417 |
| 95 | 27.0 | | |
| | 27.0 | | |
| | 27.0 | 27.0 | 370 |
| 85 | 36.2 | | |
| | 37.2 | 36.7 | 272 |
| 75 | 60.4 | | |
| | 60.4 | 64.0 | 165 |
| 65 | 166.2 | 166.2 | 60 |

The reciprocal of the time is plotted against the potential in Fig. 3. The straight line portion of the curve intersects the axis of the

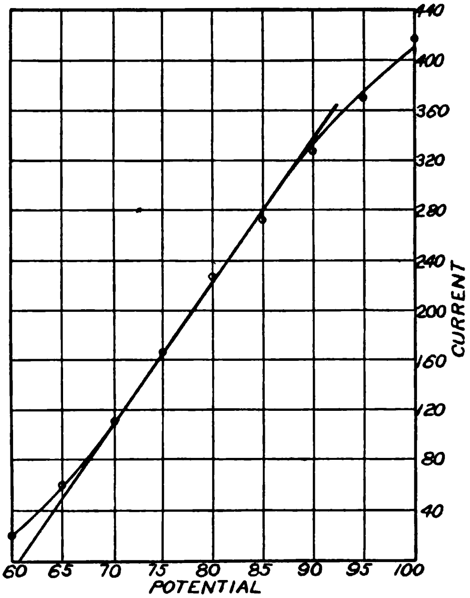


Fig. 3.

potentials at $P_0 = 60.7$ cells = 131.0 volts. The correction for the induction, as explained in § 4, 5, is 3.4 per cent. when $d = 19.34$ mm. and is, therefore, 4.5 volts. Hence,

$$P_0 = 131.0 - 4.5 = 126.5 \text{ volts.}$$

Substituting in the equation

$$u = \frac{2nd^2}{P_0},$$

we get

$$u = \frac{2 \times 35.95 \times 1.934 \times 1.934}{126.5}$$

$$= 2.13 \text{ cm. per sec. per volt per centimeter.}$$

The mobility will in each case be reduced to what it would be at a pressure of 760 mm., on the supposition that the mobilities at two pressures are inversely proportional to the pressures. The mobility at 760 mm. pressure thus obtained will be designated by $v$. In the above example

$$v = \frac{u \times p}{760} = \frac{2.13 \times 734}{760} = 2.06 \text{ cm. per sec.}$$

Table II. gives the collected data for various pressures for dry air when $d = 19.25$ mm.

TABLE II.

*Dry Air. Light from Spark Used.* $d = 19.25$ *mm.*

| $p$ | $t$ | $T$ | $n$ | $P_0$ | $u$ | $v$ |
|---|---|---|---|---|---|---|
| 746.0 | 20.0° C. | 0.0129 | 38.7 | 133.4 | 2.15 | 2.11 |
| 743.4 | 19.0 | .0100 | 49.5 | 180.4 | 2.03 | 1.98 |
| 735.0 | 20.0 | .0078 | 64.0 | 224.0 | 2.12 | 2.05 |
| 736.0 | 20.0 | .0125 | 40.2 | 143.5 | 2.06 | 2.00 |
| 734.0 | 24.0 | .0139 | 35.95 | 123.7 | 2.16 | 2.08 |
| 510.5 | 20.0 | .0080 | 62.5 | 146.6 | 3.16 | 2.12 |
| 408.1 | 19.0 | .0080 | 62.0 | 121.3 | 3.78 | 2.03 |
| 268.5 | 20.0 | .0079 | 63.5 | 78.4 | 6.00 | 2.12 |
| 207.9 | 20.0 | .0083 | 60.6 | 61.0 | 7.35 | 2.01 |
| 180.4 | 24.0 | .0079 | 63.7 | 48.3 | 9.75 | 2.31 |
| 169.3 | 19.0 | .0081 | 61.9 | 45.0 | 10.4 | 2.32 |
| 166.0 | 25.0 | .0079 | 63.5 | 45.6 | 10.3 | 2.25 |
| 140.2 | 20.0 | .0079 | 62.8 | 36.3 | 12.8 | 2.36 |
| 90.3 | 20.0 | .0084 | 59.3 | 20.34 | 21.6 | 2.57 |
| 64.0 | 20.0 | .0104 | 48.0 | 10.88 | 32.7 | 2.76 |
| 53.9 | 21.0 | .0080 | 62.3 | 9.64 | 48.0 | 3.40 |
| 36.0 | 19.5 | .0079 | 63.5 | 4.59 | 102.4 | 4.85 |
| 27.7 | 21.0 | .0080 | 62.3 | 3.47 | 133.0 | 4.84 |
| 11.4 | 20.0 | .0079 | 63.3 | 1.16 | 404.0 | 6.07 |

The results of this table are shown graphically in Fig. 4. The curve shows that $v$ is independent of $p$ between the pressures 760 mm. and about 200 mm. At 200 mm. the curve rises somewhat, and at 100 mm. it passes through a pronounced bend. This rather sudden change in the mobility $v$ indicates that near this pressure there is a rapid change in the mass of the ion, *i. e.*, the number of molecules which are supposed to be gathered about a central charge to form the ion rapidly diminishes on the average ion as the pressure is reduced below 100 mm.

*Carbon Dioxid.* — Readings similar to those for air were also taken for carbon dioxid. A set of these is given in Table III.

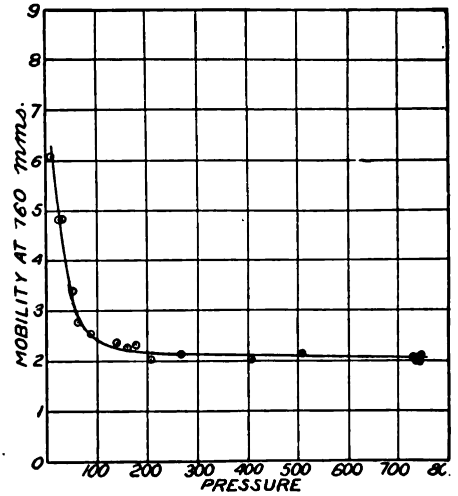


Fig. 4.

Table III.

*Dry Carbon Dioxid. Light from Spark Used. d* = 19.25 *mm.*

| $p$ | $t$ | $n$ | $P_0$ | $u$ | $v$ |
|---|---|---|---|---|---|
| 739.0 | 20°.0 | 18.3 | 148.2 | 0.91 | 0.88 |
| 755.0 | 21.0 | 19.0 | 149.0 | 0.94 | 0.93 |
| 755.0 | 20.0 | 19.0 | 144.0 | 0.98 | 0.97 |
| 754.0 | 20.0 | 23.2 | 188.0 | 0.91 | 0.90 |
| 748.0 | 20.0 | 18.8 | 144.0 | 0.97 | 0.95 |
| 748.0 | 21.0 | 32.3 | 253.6 | 0.95 | 0.93 |
| 447.6 | 21.0 | 23.4 | 121.2 | 1.43 | 0.84 |
| 402.0 | 20.0 | 49.1 | 177.0 | 2.05 | 1.09 |
| 323.0 | 21.0 | 35.2 | 115.6 | 2.26 | 0.96 |
| 299.7 | 20.0 | 49.0 | 128.6 | 2.82 | 1.11 |
| 266.6 | 22.0 | 47.9 | 132.7 | 2.67 | 0.94 |
| 187.0 | 21.0 | 35.3 | 53.4 | 4.90 | 1.21 |
| 150.0 | 22.0 | 47.8 | 54.8 | 6.45 | 1.27 |
| 118.0 | 20.0 | 61.7 | 50.8 | 9.00 | 1.40 |
| 102.1 | 22.0 | 47.7 | 32.4 | 10.9 | 1.46 |
| 77.1 | 19.0 | 47.7 | 22.5 | 20.7 | 2.10 |
| 62.5 | 22.0 | 62.9 | 11.92 | 29.6 | 2.43 |
| 58.2 | 20.0 | 63.0 | 12.55 | 37.2 | 2.85 |
| 38.2 | 20.0 | 62.9 | 4.06 | 114.2 | 5.75 |
| 25.4 | 22.0 | 47.7 | 1.34 | 264.0 | 8.80 |

The values of $v$ are plotted against $p$ in Fig. 5. It will be noticed that here also there is a sudden change in the mobility $v$.

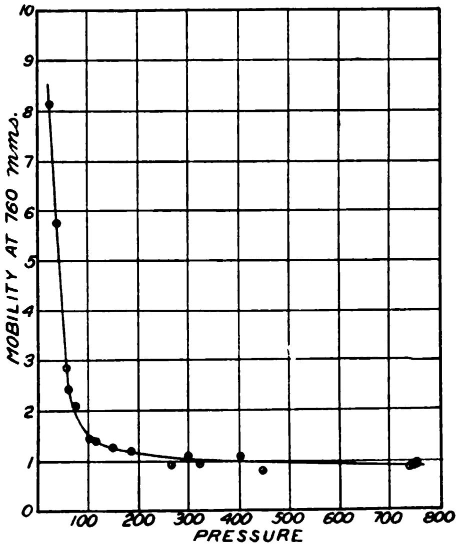


Fig. 5.

## § 6. FINAL PRESSURE EXPERIMENTS.

*Dry Air.* — In the final experiments all the precautions noted in § 4 were carefully observed. An arc between iron electrodes was used as the source of light in all of these experiments. The rate of alternation of the field was changed a sufficient number of times in each pressure experiment to permit the determination of the value of the mobility at $T = 0$, vide § 4, 2. An illustration of the general method of taking the readings with the arc as the source of light is given in Table IV($a$).

The readings in Table IV($a$) were next reduced and corrected for the changes in the intensity of the arc, vide § 4, 4. These results are tabulated in Table IV($b$), 2,400 being taken as the current corresponding to the highest potential, namely 15 cells.

The curve showing the relation between the potential and the current is given in Fig. 6. The straight line portion of the curve produced intersects the axis of the potentials at 8.68 cells. The correction for induction is 3.4 per cent. The value of $P_0$ is therefore 8.38 cells or 17.27 volts.

### TABLE IV(*a*).

*Dry Air. Fe Arc Light.* $d = 19.34$ *mm.*

Baromanometer reading = 97.8 mm.
20 cells = 41.2 volts. Temperature = 24°.0 C.
5000 alternations of the field in 104.4 sec. (beginning),
104.4 sec. (end),
104.4 sec. (mean),
or $n = 47.8$ per second.

| Potential of the Plate. | Time to Deflect Leaf 25th to 30th Division. | | Reciprocal of the Time $\times 10^4$. |
|---|---|---|---|
| 15 cells | 4.4 | | |
| | 4.6 | | |
| | 4.6 | | |
| | 4.6 | | |
| | 4.4 | | |
| | 4.2 | 4.47 | 2,237 |
| 10 | 20.2 | | |
| | 18.6 | | |
| | 18.0 | | |
| | 20.4 | 19.3 | 518 |
| 15 | 4.4 | | |
| | 4.0 | | |
| | 4.4 | | |
| | 4.2 | 4.25 | 2,353 |
| 12 | 9.4 | | |
| | 9.2 | | |
| | 8.0 | | |
| | 7.4 | 8.50 | 1,176 |
| 12 | 6.6 | | |
| | 6.8 | | |
| | 6.8 | 6.73 | 1,486 |
| 15 | 3.8 | | |
| | 3.8 | 3.8 | 2,632 |
| 8 | 44.6 | | |
| | 42.4 | 43.5 | 230 |
| 15 | 3.8 | | |
| | 4.0 | | |
| | 3.6 | 3.8 | 2,632 |
| 7 | 98.0 | 98.0 | 102 |
| 15 | 3.8 | | |
| | 3.8 | 3.8 | 2,632 |
| 9 | 24.6 | | |
| | 23.8 | 24.2 | 413 |
| 15 | 3.6 | | |
| | 3.6 | | |
| | 3.6 | 3.6 | 2,778 |
| 11 | 8.8 | | |
| | 9.2 | | |
| | 9.4 | | |
| | 10.0 | 9.35 | 1,070 |
| 15 | 4.2 | | |
| | 4.0 | | |
| | 3.8 | | |
| | 3.8 | | |
| | 4.0 | 3.96 | 2,525 |
| 13 | 5.8 | | |
| | 5.6 | | |
| | 5.6 | 5.67 | 1,764 |
| 15 | 4.0 | | |
| | 4.0 | | |
| | 4.0 | 4.0 | 2,500 |
| 10 | 18.0 | | |
| | 16.8 | | |
| | 17.0 | 17.3 | 578 |
| 15 | 4.0 | | |
| | 4.0 | 4.0 | 2,500 |
| 8 | 53.0 | | |
| | 59.0 | 56.0 | 179 |
| 15 | 4.4 | | |
| | 4.4 | 4.4 | 2,273 |
| 12 | 8.4 | | |
| | 8.0 | | |
| | 7.8 | 8.07 | 1,239 |
| 15 | 4.4 | | |
| | 4.4 | 4.4 | 2,273 |
| 14 | 5.0 | | |
| | 5.0 | 5.0 | 2,000 |
| 15 | 4.6 | | |
| | 4.6 | | |
| | 4.4 | | |
| | 4.6 | 4.55 | 2,198 |

### TABLE IV(*b*).

| Alternating Potential. | | Current in Arbitrary Units. |
|---|---|---|
| 15 cells. | | 2,400 |
| 10 | | 542 |
| 12 | 1,132 | 1,281 |
| 12 | 1,430 | |
| 8 | | 210 |
| 7 | | 93 |
| 9 | | 362 |
| 11 | | 969 |
| 13 | | 1,688 |
| 10 | | 555 |
| 8 | | 180 |
| 12 | | 1,309 |
| 14 | | 2,146 |

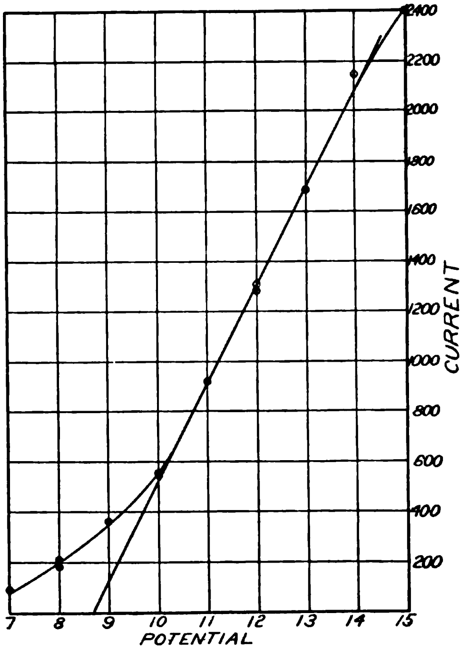


Fig. 6.

Now,

$$u = \frac{2nd^2}{P_0}$$

$$= 20.7 \text{ cm. per sec.,}$$

and

$$v = \frac{u \times p}{760}$$

$$= \frac{20.7 \times 97.8}{760},$$

$$= 2.67 \text{ cm. per sec.}$$

The time for the ions to cross the field is

$$T = \frac{1}{2} \times \frac{1}{5,000} \times 104.4$$

$$= 0.01044 \text{ second.}$$

In a similar manner, the values of the mobility for other rates of alternation of the field were obtained using the same air pressure; and these are given in Table V.

TABLE V.

*Dry Air.* $p = 97.8$ *mm.* $t = 24°.0$ *C.* *Arc.*

| $T$ | $u$ | $v$ |
|---|---|---|
| 0.0076 | 22.4 | 2.88 |
| .0104 | 20.8 | 2.68 |
| .0148 | 19.2 | 2.47 |
| .0209 | 17.3 | 2.23 |
| .0266 | 16.4 | 2.11 |

A set of results for dry air at atmospheric pressure and room temperature is given in Table VI.

TABLE VI.

*Dry Air.* $d = 19.235$ *mm.* *Arc Light.*

| $T$ | $n$ | $p$ | $t$ | $P_0$ | $u$ | $v$ |
|---|---|---|---|---|---|---|
| 0.0077 | 64.5 | 739.0 | 19°.0 | 229.0 | 2.08 | 2.02 |
| .0077 | 64.5 | 754.0 | 22.5 | 230.8 | 2.08 | 2.06 |
| .0104 | 47.9 | 739.0 | 19.0 | 174.0 | 2.04 | 1.98 |
| .0126 | 39.56 | 739.0 | 23.0 | 134.0 | 2.18 | 2.12 |
| .0130 | 38.5 | 738.0 | 23.0 | 140.0 | 2.04 | 1.98 |
| .0149 | 33.67 | 739.0 | 24.0 | 116.4 | 2.12 | 2.06 |
| .0183 | 27.3 | 739.0 | 25.0 | 97.0 | 2.08 | 2.02 |
| .0275 | 18.2 | 742.2 | 22.0 | 65.4 | 2.06 | 2.01 |
| .0282 | 17.76 | 739.0 | 25.0 | 63.0 | 2.08 | 2.02 |
| .0294 | 17.01 | 739.0 | 25.0 | 60.7 | 2.07 | 2.02 |
| .0328 | 15.30 | 742.0 | 23.0 | 52.3 | 2.16 | 2.10 |
| .0354 | 14.12 | 754.0 | 22.0 | 54.0 | 1.94 | 1.92 |
| .0401 | 12.47 | 742.2 | 23.0 | 43.7 | 2.11 | 2.06 |
| .0459 | 10.92 | 742.0 | 23.0 | 39.3 | 2.06 | 2.01 |

The graphical representation of the values of $v$ for the various values of $T$ is shown in Fig. 7. The curve in this case is a horizontal straight line which intersects the ordinate for $T = 0$ at a point $v = 2.04$, which is the value taken for the mobility.

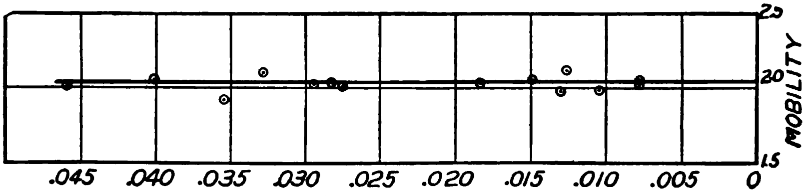


Fig. 7.

A similar set of readings at atmospheric pressure was taken when the distance between the plates was 6.854 mm. In this set the illuminated area of the zinc plate was nearly the size of the gauze This may account for the greater slope of the curve on account of diffusion to the sides, vide § 4, 2. The readings are given in Table VII. and the corresponding curve is given in Fig. 8. The value of the mobility obtained from this set is 2.05 cm. per second.

TABLE VII.

*Dry Air.* $d = 6.854$ *mm.* *Arc Light.*

| $T$ | $n$ | $p$ | $t$ | $P_0$ | $u$ | $v$ |
|---|---|---|---|---|---|---|
| 0.00776 | 64.4 | 738.0 | 23.0 | 28.28 | 2.14 | 2.07 |
| .00795 | 62.8 | 725.0 | 22.0 | 27.2 | 2.16 | 2.07 |
| .00795 | 62.9 | 722.0 | 24.0 | 27.7 | 2.13 | 2.02 |
| .0090 | 55.56 | 738.0 | 23.5 | 24.8 | 2.10 | 2.05 |
| .0103 | 48.5 | 722.0 | 24.0 | 21.7 | 2.10 | 2.00 |
| .0117 | 44.7 | 722.0 | 24.0 | 20.2 | 2.08 | 1.98 |
| .0135 | 37.1 | 722.0 | 25.0 | 16.37 | 2.13 | 2.03 |
| .0146 | 34.13 | 738.0 | 23.5 | 15.67 | 2.05 | 1.99 |
| .0172 | 29.0 | 735.0 | 24.0 | 14.09 | 1.94 | 1.88 |
| .0188 | 26.6 | 738.0 | 23.0 | 12.76 | 1.96 | 1.90 |
| .0203 | 24.63 | 738.0 | 23.0 | 11.70 | 1.98 | 1.92 |
| .0230 | 21.7 | 738.0 | 23.0 | 10.44 | 1.95 | 1.90 |
| .0233 | 21.46 | 738.0 | 23.0 | 10.30 | 1.96 | 1.90 |
| .0266 | 18.8 | 738.0 | 23.5 | 9.21 | 1.92 | 1.86 |
| .0253 | 19.76 | 735.0 | 24.0 | 9.23 | 2.01 | 1.94 |
| .0280 | 17.83 | 738.0 | 23.0 | 7.66 | 2.18 | 2.12 |
| .0343 | 14.58 | 722.0 | 24.0 | 7.16 | 1.92 | 1.82 |
| .0362 | 13.8 | 735.0 | 24.0 | 6.88 | 1.89 | 1.82 |
| .0404 | 21.39 | 738.0 | 23.0 | 5.77 | 2.02 | 1.95 |

Table VIII. gives the collected values of the mobility at atmospheric pressure, reduced to 760.0 mm., obtained under various conditions. In numbers 1 and 2 the apparatus used was that employed in the pressure experiments while in the numbers 3, 4 and 5, the apparatus used was the one constructed for the temperature experiments. In number 3 the gauze and the interior of the vessel were covered with lamp black whereby the effect of the reflected light was reduced to a minimum. Zinc, platinum and brass were used as metals for the illuminated plate. It will be noticed from the table that the change of metal had no effect on the velocity. This was observed by Rutherford.[1] The agreement in the values of the mobility for the different distances between the parallel plates adds weight to the accuracy of the method.

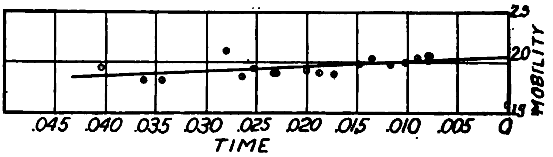


Fig. 8.

## Table VIII.

*Dry Air. Atmospheric Pressure. Room Temperature.*

| No. | *d* | Metal. | *v* at 760 mm. | *t* | Remarks. |
|---|---|---|---|---|---|
| 1 | 19.34 mm. | Zinc | 2.04 | 20°.0 | Pressure apparatus. |
| 2 | 6.854 | Zinc | 2.05 | 23.5 | |
| 3 | 18.70 | Platinum | 2.05 | 21.0 | Temperature apparatus. |
| 4 | 18.70 | Brass | 2.05 | 24.0 | |
| 5 | 9.93 | Brass | 2.02 | 20.0 | |
| | | | 2.04 = mean | | |

The experiments with the variation of the intensity of light were performed with a fixed distance of 19.24 mm. between the plates and a fixed rate of alternation of 64.5 per second. The intensity of the light was changed in two ways, namely, either by screening off some of the light incident on the quartz lens or by changing the position of the arc relative to the apparatus. With weak intensities,

---

[1] Rutherford, E., Proc. Camb. Phil. Soc., IX., p. 401, 1898.

the motion of the gold-leaf of the electroscope was slow and fewer divisions on the scale were taken than generally (the scale divisions having a uniform value in the portion of the scale used), and with strong intensities an air condenser was put in parallel with the gauze to decrease the motion of the leaf. The intensity was calculated from the effect it produced and the one with the smallest effect was taken as unity. In order to compare the intensities, readings in each case were taken for a potential difference of 160 cells. Table IX. gives the results of the experiments. It will be noticed that the variation of the intensity has no effect on the value of the mobility. In numbers 6 and 7 the arc was close to the apparatus and the whole of the quartz plate was illuminated.

TABLE IX.

*Dry Air.* $d = 19.24$ *mm.* $n = 64.5$ *Alternations per Second.* *Potential* $= 160$ *Cells.*
*Arc Used.* $p = 730.0$ *mm.* $t = 22°.0$ *C.*

| No. | Deflection. | Time for the Deflection. | Capacity in E.S. Units. | Intensity of Light. | Mobility at 760 mm. |
|---|---|---|---|---|---|
| 1 | 25–28 div. | 34.0 sec. | 30 | 1.00 | 2.05 |
| 2 | 25–27 | 17.1 | 30 | 1.30 | 2.05 |
| 3 | 25–28 | 19.0 | 30 | 1.79 | 2.04 |
| 4 | 25–27 | 10.0 | 30 | 2.27 | 2.05 |
| 5 | 25–30 | 25.9 | 68 | 4.96 | 2.04 |
| 6 | 25–30 | 11.55 | 57 | 9.30 | 2.02 |
| 7 | 25–30 | 11.7 | 295 | 47.5 | 2.02 |

The intensity generally used corresponds to that in number 4.

*Change of Pressure in Air.*—The values of the mobility at different pressures were obtained in the same manner as those obtained at atmospheric pressure.

The set of results given in Table X. is one of a number of such sets that were obtained and it was chosen because in this set the conditions of illumination were uniform, for which reason a comparison of the inclination curves for the different pressures can be made (vide Fig. 10). The column designated "tan $a$" represents the inclination of the straight line which determined the value of $v$ at $T = 0$, and is the ratio of the increase of the mobility divided by the change of time to cross the field.

## TABLE X.

*Dry Air. $d = 19.34$ mm. Arc Light.*

| $p$ | $t$ | $u$ | $v$ | tan $\alpha$ |
|---|---|---|---|---|
| 750.0 mm. | 23°.0 C. | 2.07 | 2.04 | 1.0 |
| 547.4 | 21.5 | 2.79 | 2.01 | 2.0 |
| 316.0 | 22.0. | 4.93 | 2.05 | 2.8 |
| 215.3 | 24.0 | 7.49 | 2.12 | 11.8 |
| 160.0 | 19.5 | 10.9 | 2.30 | 19.6 |
| 125.0 | 23.0 | 15.2 | 2.50 | 27.1 |
| 97.8 | 24.0 | 24.6 | 3.16 | 37.6 |
| 77.0 | 23.0 | 42.9 | 4.34 | 62.5 |
| 51.8 | 21.0 | 90.0 | 6.14 | 116 |
| 34.0 | 24.0 | 180.0 | 8.05 | 178 |
| 16.0 | 24.0 | 470.0 | 9.90 | 230 |
| 8.8 | 26.5 | 1,800.0 | 20.8 | 614 |

The relation between the reduced mobility $v$ and the pressure is shown in Fig. 9. From 760 mm. to 200 mm. the curve is linear and nearly parallel to the axis of pressures, which indicates that the mobility $u$ varies inversely with the pressure. At 160 mm. the

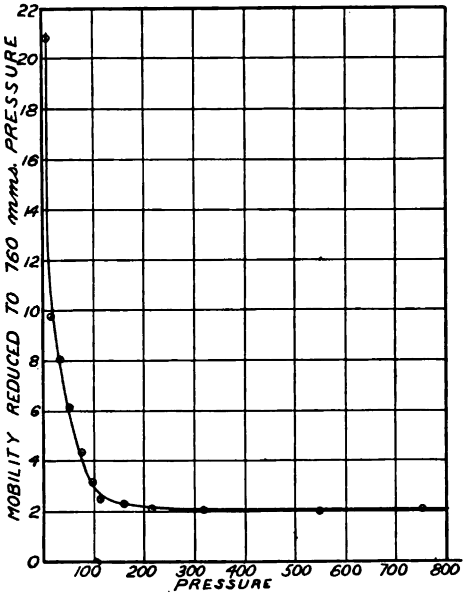


Fig. 9.

value $v$ is 12 per cent. higher than at atmospheric pressure, at 125 mm. it is 22 per cent. higher, at 100 mm. it is 55 per cent. higher, at 77 mm. it is more than double the value at atmospheric pressure, and at 8.8 mm. it is ten times that value. The curve shows a marked bend at a pressure of about 110 mm. and below this pressure it rises rapidly. The mobility below this pressure changes far more rapidly than the inverse-pressure law indicates. The comparatively sudden change in the velocity at 110 mm. pressure is very suggestive of a sudden change in the constitution of the ion, as if, at this pressure, the complex structure suddenly disintegrated. As there is a sudden change in the mobility of the ions there should be a sudden change in their coefficient of diffusion. This phase of the problem was not studied, but the curve of Fig. 11, in which "tan $a$" is plotted against the pressure, may point in that direction.

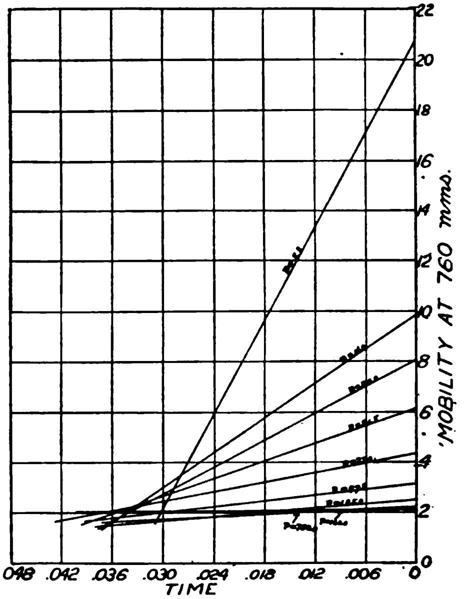


Fig. 10.

*Dry Carbon Dioxid.* — Experiments on the velocity of the negative ions in carbon dioxid at various pressures were performed in the same manner as the experiments on the velocity in air. Ordinary liquid carbon dioxid was used and on testing it by the absorption method, it was found to be 99.6 per cent. pure. Several curves were obtained for the relation between the mobility reduced to 760 mm. and the pressure, all having the characteristic bend at low pressures noticed in the curves for air. All but one of the curves were like the one shown as curve *I* in Fig. 12, having the bend at about 150 mm. and rising much more suddenly than was the case with air. In the exceptional case noted, shown as curve *II*, Fig. 12, the bend is at a much lower pressure and the curve otherwise also resembles more closely the one for air. The reason for this is not known unless it be due to some impurities, for the gas used in this set was the last supply in the tank and its purity was unfortunately not tested. The results corresponding to the curves *I* and *II*, Fig. 12, are given in the table below.

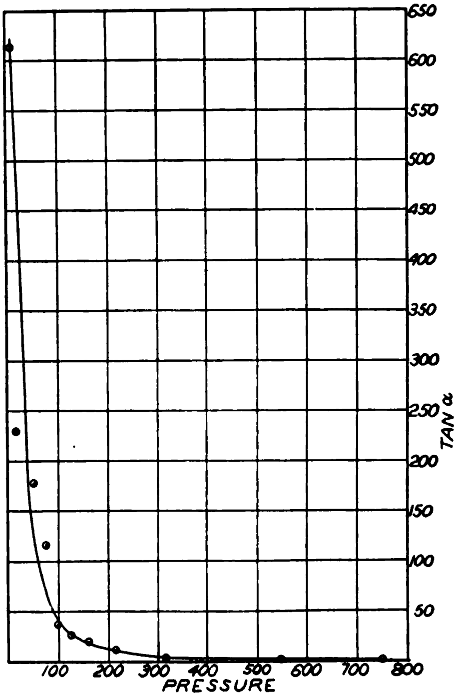


Fig. 11.

For the curve *I* the per cent. of increase above that at atmospheric pressure is 80 per cent. at 154 mm., 170 per cent. at 118 mm.

and 600 per cent. at 88 mm., while for the curve *II* it is 24 per cent. at 176.6 mm., 50 per cent. at 87 mm. and 700 per cent. at 17 mm. The per cent. of increase in the value of $v$ with the diminution of pressure over the value at atmospheric pressure is greater throughout in the case of carbon dioxid than in the case of air.

Table XI.

*Dry Carbon Dioxid. Arc Light.* $d = 19.34$ *mm.*

*Curve I.*

| $p$ | $t$ | $u$ | $v$ |
|---|---|---|---|
| 738.0 mm. | 22°.0 C. | 1.04 | 1.01 |
| 362.0 | 21.5 | 2.22 | 1.06 |
| 257.0 | 22.5 | 3.72 | 1.26 |
| 197.3 | 21.0 | 4.65 | 1.20 |
| 154.0 | 23.0 | 8.93 | 1.81 |
| 118.0 | 23.0 | 17.7 | 2.74 |
| 118.0 | 23.0 | 17.4 | 2.69 |
| 88.5 | 24.0 | 57.5 | 6.70 |
| 66.9 | 24.0 | 222.0 | 19.6 |

*Curve II.*

| $p$ | $t$ | $u$ | $v$ |
|---|---|---|---|
| 738.0 | 22.0 | 1.04 | 1.01 |
| 501.2 | 20.5 | 1.66 | 1.09 |
| 362.0 | 20.0 | 2.20 | 1.05 |
| 176.6 | 21.0 | 5.37 | 1.25 |
| 139.0 | 21.0 | 7.43 | 1.36 |
| 109.5 | 21.0 | 9.56 | 1.38 |
| 87.0 | 21.0 | 13.2 | 1.51 |
| 52.4 | 20.0 | 33.5 | 2.31 |
| 20.0 | 21.0 | 100.5 | 3.70 |
| 17.1 | 21.0 | 366.0 | 8.24 |

*Experiments with Moist Gases.* — It was observed by Zeleny[1] that the presence of moisture in gases decreases the mobility of the negative ions. Professor Zeleny in his paper expresses the view that the molecules of aqueous vapor collect upon the negative ion which, thus loaded, necessarily moves slower. Similar observations on the decrease of the mobility of the ions in vapors of ether and alcohol

---

[1] Zeleny, J., Phil. Trans., A, 195, p. 193, 1900.

were made by Rutherford,[1] and recently Blanc[2] reported similar results.

Experiments with moist gases were performed at atmospheric pressure and room temperature. The gases, air and carbon dioxid, were saturated with aqueous vapor by passing them through a flask containing boiling water and then through a flask containing water at room temperature. A difficulty in carrying out the experiments was at first encountered for zinc covered with a layer of moisture will not act photo-electrically. By increasing the intensity of the light, it was noticed that the gauze produced enough photo-electric effect (vide § 4, 1) to make it possible to take readings. The leaf of the electroscope became in these cases, of course, charged positively. This action of the brass wire gauze seemed strange in view of the fact that metals covered with moisture are reported to cease acting photo-electrically, as indeed zinc does. It may, therefore, be of interest to note that by covering the zinc with a sheet of brass, the photo-electric effect in gases saturated with water vapor appeared the same as in dry gases. The readings, however, were taken by

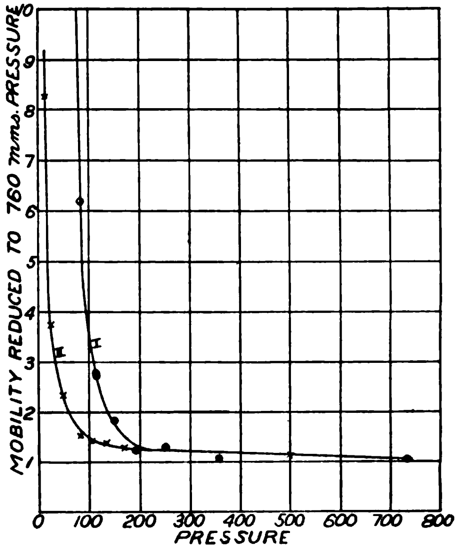


Fig. 12.

---

[1] Rutherford, E., Phil. Mag., VI., 2, p. 210, 1901.

[2] Blanc, A., J. de Physique, 4, 7, p. 825, 1908.

utilizing the effect from the gauze. The results for moist air are given in Table XII. and for moist carbon dioxid in Table XIII. The linear distribution of the mobilities gives a horizontal line, which in the case of moist air intersects the ordinate for $T = 0$ at $v = 1.64$ cm. per sec. and in the case of moist carbon dioxid at $v = 0.806$ cm. per sec. It will thus be noticed that the velocity of the negative ions both in moist air and in moist carbon dioxid is 20 per cent. below the value in dry air and in dry carbon dioxid, respectively.

TABLE XII.

*Moist Air. Arc Light. Gauze = Source of Negative Ions. Pressure = 745.4 mm. Temperature = 26°.0 C.*

| $T$ | $u$ | $v$ |
|---|---|---|
| 0.0076 | 1.66 | 1.63 |
| .0098 | 1.60 | 1.57 |
| .0139 | 1.72 | 1.68 |
| .0173 | 1.72 | 1.68 |
| .0212 | 1.77 | 1.74 |
| .0267 | 1.55 | 1.52 |

TABLE XIII.

*Moist Carbon Dioxid. Arc Light. Gauze = Source of Negative Ions. Pressure = 740.0 mm. Temperature = 25°.0 C.*

| $T$ | $u$ | $v$ |
|---|---|---|
| 0.0270 | 0.824 | 0.802 |
| .0218 | .832 | .800 |
| .0175 | .816 | .795 |
| .0138 | .850 | .828 |

## § 7. CHANGE OF TEMPERATURE EXPERIMENTS.

*High Temperature.*—The apparatus used in the final temperature experiments, both at high and low temperatures, is shown in Fig. 13. The gauze *G* was fastened by its wires to a piece of tubing *T*, 3 cm. in diameter, which was held in position by 3 quartz rods *Q* from brass blocks adjustable on the front face plate of the apparatus. The gauze was set so as to be in the plane of the inner surface of the plate, and it had a ring of air 1 mm. wide around it. The plate to be illuminated *P* was held in position by 3 glass rods *R* passing through cleaved collars. The front plate screwed into a brass cylinder *B*. The whole apparatus could be readily placed in position in the oven by an iron rod *A* screwed into the front plate. The heating electric oven had a porcelain cylinder 600 mm. long and 80 mm. in diameter. The heating current was generally an alternating current. The interior of the porcelain cylinder was lined with a sheet of brass connected to earth so as to avoid induction effects from the heating current which were very marked when the screen was omitted.

The apparatus was placed in the middle of the oven. Several centrally perforated discs of asbestos were placed inside the oven at both ends of the apparatus and at some distance from each other, so as to break up any air currents inside the oven and to prevent changes of temperature.

The ends of the porcelain cylinder were closed quite tightly by means of brass covers, allowing only a small leak for the equalization of the pressure with the outside. One of these covers was provided with a quartz plate to allow transmission of the light, and an ebonite plug (quartz and glass tubes were also used instead of the ebonite plug in some experiments) for the wire leading to the electroscope; the other cover had a glass window (used for viewing the interior) and two tubes used respectively for a wire leading to the secohmmeter and for a clay tube containing the thermo-couple used for measuring the temperature. The junction was placed in contact with the rear central portion of the illuminated plate.

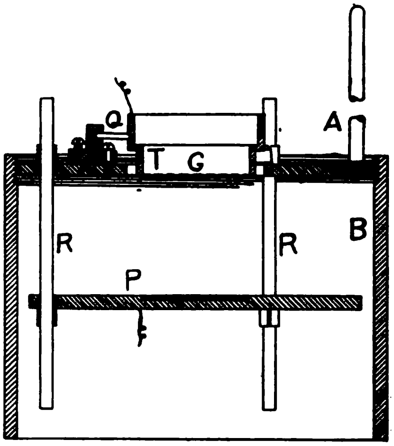


Fig. 13.

Observations were not carried above 425° C. because of a leak through the quartz rods at higher temperatures. The leak of an electric charge from the electroscope through the quartz rods was studied and found to increase with the increase of temperature, becoming very fast indeed at about 425° C. It was found to obey an exponential law, and for this reason it was utilized in taking a few readings at high temperatures by the constant deflection method.

The readings so taken give the values in numbers 1, 2 and 9, Table XIV. The remaining readings were taken by the rate method, as before.

*Low Temperatures.* — The apparatus was placed in a tin can through whose cover all the wires passed out. Between the apparatus and the cover were several discs of paper to break up currents of air. Phosphoric anhydride was placed at the bottom of the can and also in a ring at about the middle portion of the can. A tube through the cover and leading to the bottom of the can was also provided for passing dry air into the can previous to any experiment. The quartz plate in the cover of the can was kept dry by blowing dry air over it continually during an experiment.

The low temperatures were obtained in no. 18 (Table XIV.) by packing the can in a freezing mixture of ice and salt; in no. 23 by immersion into liquid air; in nos. 19–22 by lowering the liquid air flask so that the apparatus was above the liquid air but the bottom of the can still remained in the liquid air.[1] The lowering or raising of the liquid air flask could be done by very small amounts and the temperature could be kept constant within 5° C. However, the temperature at the upper plate differed from the temperature at the lower plate by as much as 12° C. and it is possible that some convection currents existed in these experiments. Such currents would move upwards in the center of the apparatus and would bring the ions to the gauze sooner than would otherwise be the case. This was somewhat apparent from a small downward slope in the curve for the different values of $T$ in the experiments given by nos. 19–22. The temperature was measured by two thermo-junctions of iron and nickel wires; one of them rested on the top of the upper portion of the apparatus while the other rested against the side of the apparatus at about the position of the illuminated plate. The mean value of the two temperature readings was used as the temperature of the gas in the experiment. The calibration at low temperatures was done by means of liquid air, the liquid air temperature was obtained by the density method[2] which gave — 193°.4 C., while a pentane thermometer read in the same air — 193°.0 C.; intermediate calibration

---

[1] Compare Zeleny, J., PHYS. REV., XXIV., p. 42.

[2] U. Behn und F. Kiebitz, Ann. d. Phys., 4 S., 12, p. 421, 1903.

points were obtained by placing the pentane thermometer and the junction in a tube of pentane and raising the tube out of the liquid air to different points, the pentane being constantly stirred.

The illumination was brought about by reflecting the light by means of a polished brass plate downward into the apparatus. The results for all the temperatures tried are given in Table XIV.

TABLE XIV.

*Dry Air.*

| No. | Absolute Temperature. | $p$ | $u$ | $v$ | $V$ | $d$ | Plate. |
|---|---|---|---|---|---|---|---|
| 1 | 698° | 744.0 | 5.72 | 5.60 | 2.19 | 18.70 mm. | Brass. |
| 2 | 648 | 745.0 | 5.60 | 5.50 | 2.32 | " | Brass. |
| 3 | 570 | 746.0 | 4.57 | 4.48 | 2.15 | " | Platinum. |
| 4 | 540 | 746.0 | 4.43 | 4.35 | 2.20 | " | Platinum. |
| 5 | 503 | 741.0 | 4.12 | 4.02 | 2.18 | " | Platinum. |
| 6 | 468 | 732.0 | 4.08 | 3.94 | 2.30 | " | Brass. |
| 7 | 463 | 746.0 | 3.80 | 3.73 | 2.20 | " | Platinum. |
| 8 | 428 | 734.0 | 3.82 | 3.68 | 2.35 | " | Brass. |
| 9 | 416 | 742.0 | 3.46 | 3.38 | 2.27 | " | Brass. |
| 10 | 409 | 738.4 | 3.42 | 3.32 | 2.22 | " | Platinum. |
| 11 | 388 | 733.0 | 3.02 | 2.92 | 2.05 | " | Platinum. |
| 12 | 378 | 742.0 | 2.84 | 2.77 | 2.00 | " | Platinum. |
| 13 | 360 | 744.0 | 2.69 | 2.64 | 2.00 | " | Platinum. |
| 14 | 340 | 731.0 | 2.72 | 2.62 | 2.10 | " | Brass. |
| 15 | 335 | 737.0 | 2.58 | 2.50 | 2.04 | " | Platinum. |
| 16 | 300 | 732.6 | 2.18 | 2.10 | 1.91 | " | Brass. |
| 17 | 296 | 738.0 | 2.10 | 2.05 | 1.88 | " | Brass. |
| 18 | 268 | 738.0 | 1.853 | 1.80 | 1.83 | 9.93 | Brass. |
| 19 | 237 | 738.0 | 1.588 | 1.54 | 1.77 | " | Brass. |
| 20 | 202 | 740.2 | 1.160 | 1.13 | 1.53 | " | Brass. |
| 21 | 180 | 738.0 | 0.896 | 0.87 | 1.32 | " | Brass. |
| 22 | 149 | 741.2 | 0.638 | 0.622 | 1.14 | " | Brass. |
| 23 | 84.5 | 746.0 | 0.226 | 0.222 | 0.717 | " | Brass. |

The mobility reduced to normal pressure is plotted against the temperature in Fig. 14. It will be noticed that the points are very nearly on a straight line which intersects the temperature axis at about 80° absolute. Phillips (loc. cit.) using a shorter range, obtained a less inclined straight line.

The above curve does not give direct information about any changes that may take place in the ion with change of temperature, for, although the values were all obtained at atmospheric pressure, the density of the gas was markedly different at the different temperatures. The mobility $v$ for each temperature was reduced therefore to the density at 0° C., on the supposition that the mobility varies inversely as the density of the gas. The values thus obtained are given in the above table in the column marked $V$, and they are plotted against their temperatures in Fig. 15. From 700 down to 400 degrees absolute there is no change in the value of $V$, but below 400 it diminishes until at 84.5 degrees absolute it is only one third of the value at the higher temperatures. This decrease in the mobility of the ions indicates that at the lower temperatures the cluster of molecules about the ion becomes larger on the average as the temperature is lowered.

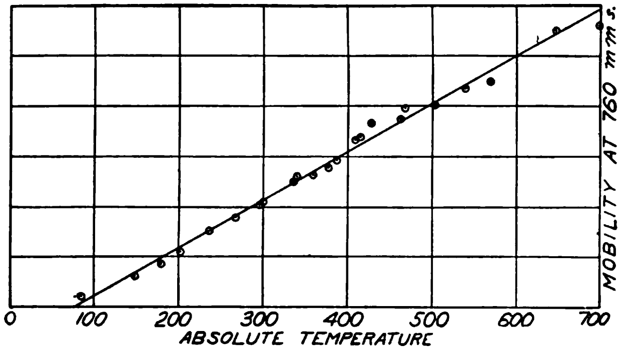


Fig. 14.

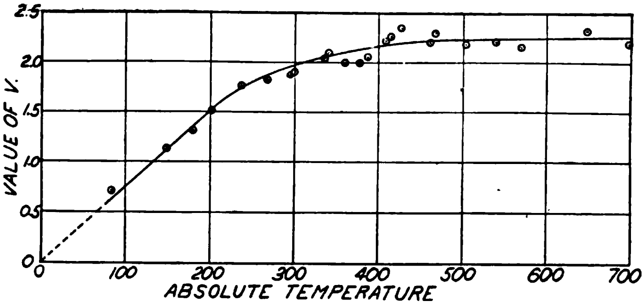


Fig. 15.

## § 8. Summary and Conclusion.

1. At atmospheric pressure and room temperature the mobility of the ions produced by ultra-violet light, when reduced to 760

mm. pressure, was found to be 2.04 cm. per sec. per volt per cm. for dry air, and 1.01 cm. per sec. for dry carbon dioxid. In air saturated with water vapor at 26°.0 C. the reduced value was 1.64 cm. per sec. and in carbon dioxid saturated with water vapor at 25°.0 C. the reduced value was 0.806 cm. per sec.

2. The mobility of the negative ions in air at ordinary room conditions is the same whether the metal illuminated by the ultra-violet light is zinc, platinum or brass.

3. With changing pressure the mobility of the ions, in both dry air and dry carbon dioxid, varies inversely with the pressure from 760 mm. down to about 200 mm., so that the product of the mobility by the pressure is constant.

4. In dry air below 200 mm. the product of the mobility by the pressure increases as the pressure is reduced, the increase being slow at first but very sudden at about 100 mm. so that at 8.8 mm. the product was found to be ten times as great as that at atmospheric pressure. This result indicates that below 200 mm. the size of the ion is increasingly smaller as the pressure is reduced.

5. In dry carbon dioxid below 200 mm. there is likewise a sudden increase in the product of the mobility by the pressure, but this occurs at a somewhat higher pressure than in air.

6. With a change of temperature from 84.5 to 698.0 degrees absolute, the mobility of the negative ions in dry air at atmospheric pressure was found to increase uniformly with rise of temperature. When reduced to a common gas density, the value obtained for the mobility remains unchanged from 700° to 400° absolute, but below that temperature it diminishes until at the temperature of liquid air it is only about one third of its value at the higher temperatures. In this region the cluster of molecules forming the ion increases in size, therefore, as the temperature is lowered.

In conclusion, I desire to express my deep gratitude to Professor John Zeleny, who suggested this investigation and under whose supervision these experiments were carried out, for the valuable advice from him, and to Dean Frederick S. Jones for the kindly interest he has shown in my work.

Physical Laboratory,
University of Minnesota,
August 23, 1909.

# SCATTERED LIGHT IN SPECTROPHOTOMETRY AND A NEW FORM OF SPECTROPHOTOMETER.

By Herbert E. Ives.

RECENTLY, in making some spectrophotometric measurements of artificial light sources in terms of daylight, the writer's attention was called to the importance of scattered light as a source of error in spectrophotometry. Scattered light is practically impossible to avoid in optical instruments. For this reason it is surprising that it receives next to no attention in the design or the published descriptions of the manipulation of spectrophotometers. In such instruments as the Brace and Lummer-Brodhun diaphragms in the collimator and telescope tubes constitute the sum of the precautions taken to avoid stray light. Yet under certain circumstances, to be outlined below, such precautions are insufficient; in fact, only by the use of other forms of instruments can the stray light error be entirely eliminated.

The first essential of a spectrophotometer is complete symmetry; the light from the two sources must be subject to identical treatment in its passage through the instrument to the eye. This symmetry must hold not only when the two sources are identical, but when they are different in special character. It is in this latter requirement that some of the present forms are deficient. Scattered light, when present, is present usually in different quantity and quality in the fields due to the two sources. This deficiency becomes apparent usually only on measuring light sources as different in character as daylight and the yellow artificial illuminants, in which case considerable errors are possible.

The source of this scattered light, its amount in different types of spectrophotometers, and the means to obviate it, form the subject of the present note.

In a spectrophotometer with a perfect system of diaphragms the remaining scattered light — visible as an illumination of the instrument field if monochromatic light is used and the eye is observing at another wave-length — is due to light from the lens and prism surfaces, either from dust and scratches or from "ghosts." The peculiarity of this light (and also scattered light due to imperfect diaphragms) is that it is proportional not to the intensity of the wave-length under observation but to the total amount of light of all colors striking the scattering surface. When making observations at the ends of the spectrum, for instance, what one observes, even in the newest and best instruments, will be the deep red or blue superposed on a background of gray due to scattering. The older the instrument, the more dust or film on lens surfaces, or the more numerous the lens ghosts, the larger the proportion of the total light is this scattered background. This is especially noticeable in a spectrophotometer if a large quantity of light is used in an attempt to secure measurements in the fainter portions of the spectrum.

Now this scattered light can or cannot cause errors, depending on the following principle: If the scattered light is spread equally over the two photometric fields to be compared, it is harmless; if it is present on one side alone it results in error. In the former case a loss of photometric sensibility results which is not desirable, but is far less undesirable than the error which arises in the latter case from comparing the colored light of one side with the colored light plus scattered light of the other.

In undertaking the measurements above referred to the writer had available a Lummer-Brodhun spectrophotometer and a spectrometer converted into a spectrophotometer by the use of a right angle prism device at the slit. The chief difference in the instruments — which are representatives of two usual types of spectrophotometers — is that in the Lummer-Brodhun the spectra are focused on the eye; hence the prism face is seen of one color. In the other instrument two adjacent spectra are viewed by an eyepiece. In the Lummer-Brodhun instrument the division of the beams of light from the two sources is effected by the use of two collimators, and the photometric field is a Lummer-Brodhun prism between the collimator and the dispersing prism. In the other instrument the two beams of light are divided at the slit, but pass over identical paths through one collimator to the image plane of the telescope.

Were there no other factors to be considered than ease of observation the Lummer-Brodhun instrument would have been chosen, because the large monochromatic surface and extended dividing lines which are observed in that type permit of more accurate settings than does the narrow strip from the adjacent spectra on which comparisons are made in the adapted spectrometer. When, however, observations were made on two sources with a steep spectral gradient with respect to each other, for instance a light weak in red and strong in blue, against one strong in red and weak in blue, the following condition was found: At the red end of the spectrum it was necessary to admit so much of the light of the red deficient source that a mist of gray light was caused on the corresponding field by the large amount of the green and blue admitted. At the blue end the scattered light was in the field corresponding to the other source. At each end of the spectrum, therefore, one of the fields was grayer than the other and an intensity match would not be a true one. This grayness, it should be noted, may easily be confused with the lack of color match caused by very wide slits, with sources of steep spectral gradient. The effect of scattered light of this kind, present on one side of the instrument and not on the other, is in general to make the two sources measured appear nearer alike in character. Anomalous results at the ends of the spectrum may also be expected.

Attempts to eliminate the unsymmetrical illumination on the two sides met with partial success. By having the collimator lenses and Lummer-Brodhun prism scrupulously clean and by providing additional diaphragms outside the slits, large enough to place all the interior diaphragms in shadow, considerable improvement was produced. By using colored glasses of tints to absorb the portion of the spectrum not being observed (this, of course, changed the values of the readings) the improvement was still greater, and for most purposes this method would probably be perfectly satisfactory. It was, however, decided to use the instrument giving two juxtaposed spectra because the scattered light trouble appeared to be entirely absent. Scattered light was present; but with this important distinction, that the light forming the two spectra, in its passage to the image had filled the whole surfaces of all the lenses and prisms, hence the scattered light was spread equally over both images and the dividing line, thereby merely resulting in a loss of sensibility. Interposition of a colored glass at the ends of the spectrum made no difference in the values of the readings. There was, therefore, no uncertainty as to whether the disturbing factor had been entirely eliminated or in the periods between observations had changed in amount.

As mentioned above, the spectrophotometer in which two adjacent spectra are viewed is not sensitive because the dividing edge for any strip of the spectra such as could be compared, is extremely narrow. On the other hand, in existing designs of spectrophotometers in which the prism face is observed by monochromatic light the two beams of light usually pass through two separate collimators. In all, the prism, biprism or other device forming the photometric field, is a material object. It therefore separates not only the direct but the scattered light of each half of the field from the other half. In this class belong the Lummer-Brodhun and the Brace instruments. The Koenig spectrophotometer which separates the light partly at the slit and partly by a biprism, after passage through the single collimator, lies between the two classes.[1]

In an attempt to secure the advantage of both styles of spectrophotometer a new design has recently been constructed. The principles of this are: (1) That the light from both sources should go through one collimator and dispersive system in such manner that all surfaces which could cause scattering would receive light from both sources equally. (2) That the photometric plane should be a virtual one in space, through which the background of scattering could be seen behind each field equally, no matter from which side the scattered light originated.

*A*, Fig. 1, is a Hilger constant deviation wave-length spectrometer. At 1 is placed any convenient device for forming a divided photometric field, such as a plaster-of-paris wedge, or a Lummer-Brodhun contrast prism. As set up a large and small right angle prism were cemented together, the face *b* being entirely silvered,

---

[1] The writer has had no opportunity to work with a Koenig spectrophotometer, but study of its construction would indicate it to be practically free from the possibility of scattered light asymmetry.

the face *a* silvered half way up. 6 and 7 are the two sources to be compared, which must be extended, preferably white surfaces illuminated by the lights compared. At 2 (the slit) is a convex lens,

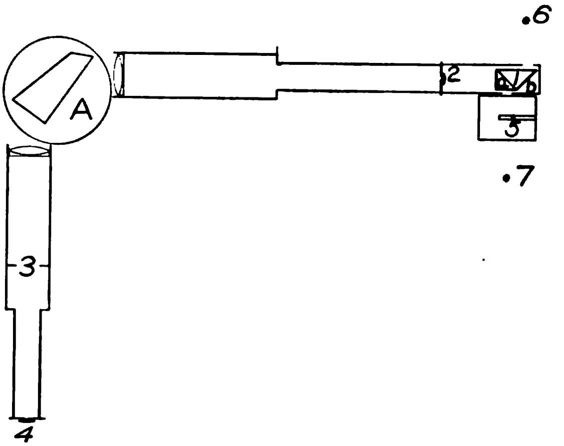


Fig. 1.

where distance and focal length are such that it forms, with the collimator and telescope lenses, an image of the prism face *a* at some place in the telescope tube 3, as distant as possible from the lenses and prism of the instrument. 5 is a Brodhun sector disc for reducing the intensity of one source by a measurable amount. At 4 is a spectacle lens to focus the image at 3.

Now, if the dividing line at *a* is perpendicular to the length of the slit at 2 a sharp image of it will be formed at 3 with any slit width practicable to work with. (Because of diffraction, lines parallel to the slit lose their sharpness with a very narrow slit.) Further, if the slit of the spectrophotometer be made long (as long as the diameter of the collimator lens for the best effect) the beam of light coming from any part of *a* is spread over a large area of lenses and prisms, and if dust or scratches be present to cause scattering, the scattered light is seen as a background to the image at 3. By properly choosing the length of slit and place of forming the image, the condition found in the second type of instrument described above is obtained, namely, the scattered light from all the material surfaces of the instrument is spread equally over the two photometric fields at their dividing line.[1]

Severe tests of this instrument, such as directly viewing incandescent lamp filaments and an acetylene flame, have proved it to have the characteristics for which it was designed, not only when all surfaces are perfectly clean and new, but when purposely covered with dust, to imitate the conditions which might occur after long use.

In working with this spectrophotometer the substitution method was always used, to avoid any errors due to selective absorption in the Brodhun sector, or silvered prism surfaces. This should, of course, always be done in spectrophotometry where accuracy is desired. A possible objection that may be raised to the instrument —that it is impossible to vary the relative intensities by changing two slit openings—may be dismissed on similar ground. The range of slit openings necessary to make matches with sources very different in color is altogether too great for accuracy unless the dispersion is large.

The objections may be made to the discussion above that measurements should never be made with instruments not in perfect condition, and that the errors indicated are small except where the light sources measured are so different that one is less concerned with absolute accuracy than with the relative order of magnitude of the two. The objections are perfectly valid. Nevertheless, under certain conditions, such as the use of portable instruments for daylight measurements, laboratory conditions of cleanliness and ability to shield an instrument from light cannot be obtained. When so used, an instrument which loses its symmetry with the presence of scattered light may give results in error by an indefinite and often large amount. Prudence would indicate therefore that when very differently colored sources are compared under other than the most favorable laboratory conditions an instrument should be used which preserves its symmetry. Or if a form is used in which scattered light destroys the symmetry, the instrument should

---

[1] In practice it proved well to have the photometric dividing surface quite near the front slit, using a short focused lens. Otherwise the Brodhun sector with its rather small angular opening, cuts the field of vision.

have its diaphragms looked to with especial care, and, for safety, colored glasses should be used as indicated above.

The object of this note has been to call attention to the fact that certain spectrophotometers which are symmetrical when the two sources observed are the same in spectral character, may become unsymmetrical when sources different in character are compared, if scattered light is present. The two types of instrument in which this defect is and is not present, are indicated. An instrument possessing the large monochromatic field of the Lummer-Brodhun and Brace instruments, but which does not become unsymmetrical in the presence of stray light, is described.[1]

PHYSICAL LABORATORY OF THE NATIONAL ELECTRIC LAMP ASSOCIATION,
CLEVELAND, O.

---

[1] Since the construction of the above instrument a description of a similar instrument is given by M. J. Thovert in the Journal de Physique for November, 1909. It is described as a convenient means of altering an ordinary spectroscope to a spectrophotometer and dispensing with the double collimator of the Lummer-Brodhun instrument while obtaining a similar field. No discussion is given, however, of its behavior toward scattered light or the arrangement of image, slit length, etc., to avoid scattered light effects.

# THE ELECTRICAL CONDUCTIVITY OF FLUORESCENT ANTHRACENE VAPOR.

BY H. E. HOWE.

THE following paper is an account of an attempt to determine whether fluorescence has any effect on the electrical conductivity of anthracene vapor. Although the results of the experiment were negative, it being impossible to detect any conductivity of the vapor either unilluminated or fluorescent, the accuracy of the work was such as to set a rather definite upper limit for any effect that may be present, and the results are presented with the hope that some experimenter may be able to apply an improvement of the method to the same substance or, preferably, to others that might be expected to show the effect sought for.

The idea of testing the effect of fluorescence on the conductivity of a vapor was suggested to the writer by the paper of Nichols and Merritt[1] in which they reported an increase of 1.1 per cent. in the conductivity of an alcoholic solution of eosin and smaller changes for solutions of fluorescein, rhodamin, cyanin and naphthalin roth, the effect being ascribed to a dissociation of the X-ray type. The final form of the experiment, which was the result of several preliminary attempts, was carried out under the guidance of Professors Nichols and Merritt, and the writer wishes to thank them here for their many very useful suggestions and for their patient encouragement in the face of difficulties.

The expectation of a change in conductivity during fluorescence is based on the theory that fluorescence is a dissociation phenomenon. Whether this dissociation is such as to produce any such effect is as yet undetermined. Some of the theories and experiments that are for or against the assumption of a change will be briefly mentioned.

In the work of Nichols and Merritt with solutions, a Wheatstone-

---

[1] Nichols and Merritt, PHYS. REV., Vol. 19, p. 296, 1904.

bridge method was used and difficulties were encountered due to heating on illumination and to the difficulty of eliminating disturbances due to polarization. Since the appearance of their paper the experiment has been repeated by Camichel,[1] who found no effect that could not be accounted for by heating. Regner[2] set 0.1 per cent. as the upper limit of change for solutions of eosin and fluorescein. More recently Hodge[3] and Goldman[4] have shown separately that the effect found by Nichols and Merritt was due to an electromotive force produced by light at the electrodes of the cell, and that no measurable change of conductivity was present.

Little work has been done on vapors. Henry[5] could find no conduction of iodine vapor due to light. On the other hand, J. J. Thomson[6] states, without giving reference, that light increases the conductivity of sodium vapor. Wiedemann and Schmidt,[7] in their paper in which they propose the dissociation theory, say that this does not call for any ionization in the case of fluorescent vapors.

The desirability of anthracene vapor for this experiment is seen from the following considerations:

1. The vapor is strongly fluorescent.

2. It has banded absorption and fluorescent spectra[8] lying in the region 320–400 $\mu\mu$, light of these wave-lengths being transmitted by the glass used in this experiment. Stark,[9] in his work on band spectra, has concluded that fluorescence and photoelectric properties are intimately associated with absorption in bands shaded toward the red, and that the "carriers" of the band spectra are the negative electrons, which are pulled out of their normal position by the incident light, and upon their return to the position of equilibrium liberate the energy of the fluorescent light.

3. If the separation of the electron is complete, photoelectric properties would be exhibited. It is thus that Stark and Steubing

---

[1] Camichel, Jour. de Phys., 4, p. 873, 1905.

[2] Regner, Phys. Zeit., 4, p. 862, 1903.

[3] Hodge, Phys. Rev., 28, p. 25, 1908.

[4] Goldman, Ann. der Phys., 27, p. 332, 1908.

[5] Henry, Proc. Camb. Soc., 9, p. 319, 1897.

[6] Thomson, Cond. through Gases, p. 213.

[7] Wiedemann and Schmidt, Ann. der Phys., 56, p. 201, 1895.

[8] Elston, Astr. Jour., 25, p. 155, 1907.

[9] Stark, Phys. Zeit., 8, p. 81, 1907.

explain the fluorescence of solid anthracene.[1] After investigating a number of substances, they say: "It is highly probable that for organic substances, at least, the photoelectric effect and fluorescence are intimately connected with each other." Hence it would seem that the vapor of anthracene would be as likely as any vapor to possess photoelectric properties.

## APPARATUS.

The glass tube used to contain the anthracene is shown in Fig. 1. The larger part of the tube was 15 cm. long and 2.5 cm. in diameter. The electrodes between which the current passes consisted of an outer aluminum cylinder, to which connection was made by sealing a platinum wire through the glass at *M*, and an inner brass wire supported by a quartz tube *Q* and visible through the slit in the outer electrode.

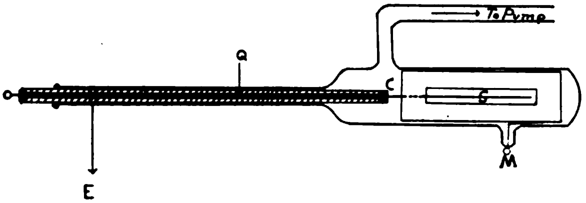


Fig. 1.

The quartz tube served as insulation for the brass wire electrode. Further protection was afforded by a deposit of silver on the inside of the glass tube around the quartz, this silver layer extending around to the outside of the open end where it was wrapped with tinfoil and grounded to form a guard-ring. Thus no charge could leak over the glass surface from the outer electrode.

The openings at the outer end of the quartz tube were sealed with de Kochtinsky cement. This would not stand heating, and the furnace was arranged to enclose only the large part of the tube (see dotted outline in Fig. 2). It was necessary to have the quartz tube fit snugly in place and to provide a tight-fitting cap *C* at the inner end in order to prevent the sublimation of the anthracene to the colder parts of the tube.

The tube, after being charged with anthracene, was exhausted to

---

[1] Stark and Steubing, Phys. Zeit., 9, p. 481, 1908. Also, Pochettino, Nuo. Cim., 15, p. 171, 1906.

a pressure of a few tenths of a millimeter, sealed off, and heated electrically. Light from a carbon arc *A* was converged by the lens *L*, and could be passed into the furnace through a mica window or could be cut off by the screen *S*. When the light was on, a bright cone of violet fluorescence could be seen in the center of the tube.

The outer electrode was charged from a storage battery *B*, and the rate at which the inner electrode acquired a charge was measured by means of a sensitive Dolezalek electrometer, which was enclosed in a wire cage to protect it from the electrostatic disturbances. The end of the anthracene tube projected through a hole in this cage, making the protection of the inner electrode practically perfect.

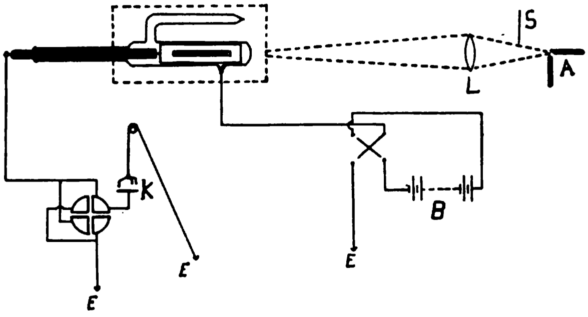


Fig. 2.

The electrometer needle was charged to a constant potential (in most of the work this was 60 volts), one pair of quadrants was grounded and the other pair connected to the wire electrode. This pair could be grounded or insulated at will be means of the key *K*. When *K* is raised the motion of the needle is noted by observing on a ground-glass scale the image of a lamp filament formed by the small concave electrometer mirror. The current can be calculated from the known constants of the electrometer.

If $V_q$ be the potential of the charging quadrants;
$K$ be the deflection when $V_q$ is one volt;
$C$ be the capacity of the charging system;
$Q$, the charge; $D$, the deflection; $I$, the current; then

$$D = K \times V_q \quad \text{and} \quad Q = C V_q.$$
$$I = dQ/dt = C \times d\,V/dt = K \times C \times dD/dt$$

on the assumptions that $K$ is the same for the moving needle as for the needle at rest and that $C$ does not change with the position of the needle. Such assumptions are allowable for slow motion and small deflections.

## OBSERVATIONS AND RESULTS.

$K = 375$ mm. on a scale one meter distant, when the potential of the needle was 60 volts.

$C$ was determined by dividing the charge with a coaxial cylinder condenser of 30 cm. capacity, and was found to be 45 cm. or $5 \times 10^{-5}$ microfarads.

A charging rate of 1 division per second could easily be observed if definite. This would mean a current $1.3 \times 10^{-13}$ amperes. The insulation was such that the rate of leak when charged to 50 div. deflection was about 10 div. per minute, or one sixth the rate mentioned as easily measurable for charging.

At the low pressure used, anthracene vaporizes sufficiently to show fluorescence between 200° and 250° C. "90 per cent. sublimed" anthracene was used without attempt at purification as it was easily obtainable and the presence of impurities in the material does not seem to affect the fluorescence of the vapor. No attempt was made to shield the electrodes from the light scattered by the end of the tube, nor to use monochromatic light.

In liquids there is an electrode photoelectric effect, dependent upon which of the electrodes is illuminated. The possibility of the existence of such an effect in the vapor was not considered in this experiment.

In most of the work, the outer electrode was charged to 120 volts from an ordinary storage battery. $K$ was raised and the rate of motion noted for the unilluminated tube. This was repeated with the light on.

When the tube was newly made up the electrometer remained quiet under all conditions, except for an occasional drift of not more than 1 division in 5 seconds. In the earlier work, after a few heatings, the electrometer began showing a tendency to rise rather quickly to a more or less definite deflection, regardless of conditions of potential or illumination. This peculiar effect varied from run to run and became very great in a case or two when the cement with which the tube was sealed cracked and admitted air. A brown deposit was then left in the tube, apparently the result of a combination of the oxygen of the air with the anthracene.

A possible explanation of this attainment of a steady deflection even when the outer electrode was earthed is that the product of the decomposition of the anthracene is deposited on the quartz tube separating the silver guard-ring from the inner electrode and forms with these two metals a sort of voltaic cell. This "cell" had a very high temperature coefficient and in some cases the E.M.F. was so great as to throw the spot of light off the scale.

After several trials it was found possible, by taking great care to prevent the entrance of air, to get almost entirely rid of this effect. When this was possible the electrometer behaved normally.

No ordinary conduction was found. If the light had any effect it was too small to be detected. Thinking that the vapor under the action of the light might be almost ionized, an attempt was made to help out the ionization by exposing the vapor to the action of a small sample of radium bromide. The ionizing rays from this substance did not pass through the glass sufficiently to be of any use.

Higher potential was obtained by making up a set of small lead-sulphuric acid storage cells in test-tubes. This battery, when put in series with the one already at hand, gave from 360 to 540 volts, depending upon the condition of the small cells. The sensibility was increased by charging the electrometer needle to 120 volts. With the greater sensitiveness and the unsteady potential furnished by the cells, the electrometer wandered more or less in the neighborhood of the zero, but still did not indicate any steady rate of charging such as would have been evident had there been any considerable conductivity.

In the freshly exhausted tube the pressure was so low that the application of 400 volts caused a continuous luminous discharge through the tube, but this soon disappeared as the pressure rose due to the vaporization of the anthracene.

The number of trials was great enough to leave no doubt as to the behavior of the vapor under the various conditions mentioned.

## CONCLUSIONS.

1. The conductivity of anthracene vapor at the temperature and density used in this experiment is too small to measure in the manner described.

2. The effect of fluorescence, if any exists, is too small to be found in this way.

The rate of leak of the electrometer system, while small, might mask a very slight effect. This matter, as well as the effect of pressure and temperature and of screening off the white light from the carbons of the arc, might be made the subject of any further investigation with this or another fluorescent vapor.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY.

# THE RECTIFYING EFFECT IN POINT AND PLANE DISCHARGE.

By R. F. Earhart and Chas. H. Lake.

IN this investigation the authors have sought to secure quantitative measurements on the rectifying effect between an electrified point and a plane.

It has been known for some time that when an electrified point is separated from a plane positively electrified a sufficient potential difference may be attained to produce a discharge, and if the point is positive the potential difference required for the discharge will be greater than if the point is negatively electrified.

In "Conduction of Electricity through Gases" J. J. Thomson quotes measurements from observations made by several experimenters, but as the minimum potential varies with the sharpness of the point used it is very hard to make comparisons except with different pressures and different distances of the electrodes when the same point is used. Tamm[1] found that the minimum potential required to cause a perceptible leak between a point and a plane several centimeters removed, at atmospheric pressure, was 2,140 volts when the point was electrified negatively and 3,760 volts when the point was positive.

Results of observations made by Röntgen,[2] Precht,[3] Gorton and Warburg[4] are also recorded. These observations were made at pressures ranging between atmospheric pressure and a pressure of 10 centimeters of mercury, for a number of gases including air. In air at atmospheric pressure Precht found the minimum potential necessary to produce a discharge for point positive to be 2,730 volts and for point negative 2,050 volts.

In a previous paper published in the *Philosophical Magazine*,

---

[1] Tamm, Ann. der Phys., VI., p. 259, 1901.

[2] Röntgen, Göttingen Nach., 1878, p. 390.

[3] Precht, Wied. Ann., XLIX., p. 150, 1893.

[4] Gorton and Warburg, Ann. der Phys., 1905.

July, 1908, the results were given for a series of observations on the "Discharge from an Electrified Point and the Nature of the Discharge occurring through very small Distances."[1] The distances ranged from one to one hundred wave-lengths of sodium light and the ionizing potential for a point and a plane was found to be about 338 volts in air. The critical distance was greater for a point and plane than for two plane electrodes. In view of these results it seemed advisable to extend the work, using the luminous discharge and a greater range of distances and pressures.

In Fig. 1 is shown a cross-section of the discharge chamber. The electrodes consisted of a plane brass plate $B$ 3.5 cm. in diameter, which was fastened to a plug of red fiber $D$, and a small platinum wire slightly rounded at the point and firmly set in red fiber so that the end would be just even with the plane surface of the fiber. As shown in the figure the pieces of fiber were turned to fit the glass tube $EE$ and further served to confine the gas in the chamber $C$. The piece of fiber in which the wire electrode was set was bored with a small hole $K$ for the removal of the gas. For the purpose of making the joints air tight the Khotinski cement was used. The distances between the electrodes were determined by fiber rings carefully ground until they were accurate to one one-hundredth of a millimeter. The chamber was then connected in series with a drying chamber which had a capacity of about one and a half liters, a McLeod gauge and an air pump. A constant supply of phosphor pentoxide was kept in the drying chamber. A battery of 600 storage cells was used as a source to obtain the potential differences required in the work and a carefully calibrated Weston voltmeter with a suitable multiplier was used to measure them.

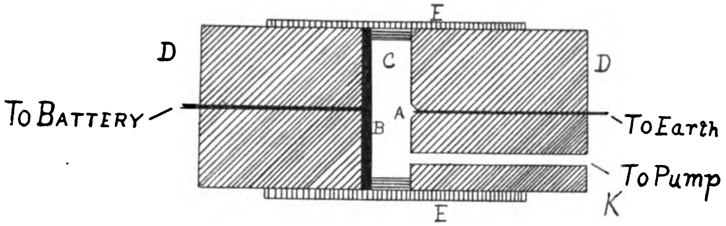


Fig. 1.

---

[1] Earhart, Phil. Mag., July, 1908.

The diagram in Fig. 2 shows the arrangement of the apparatus. One terminal of the battery is earthed direct; the other is connected through a liquid resistance *R* to one electrode of the discharge chamber and the voltmeter which is in parallel with the discharge chamber. The other electrode of the discharge chamber and the other terminal of the voltmeter are connected to earth. With this arrangement it is quite easy to make the point or the plate electrode either positive or negative by interchanging the earth and battery connections of the chamber.

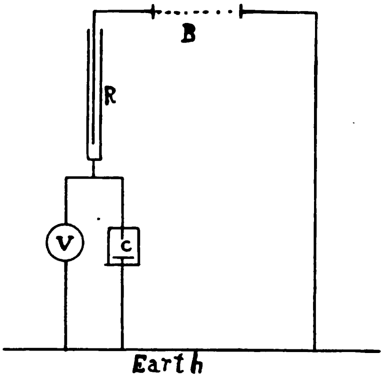


Fig. 2.

In making the measurements the potential was increased by small steps and sufficient time allowed to ascertain whether or not a discharge would occur before it was increased. The passage of the discharge could be readily detected by a sudden dropping off in the

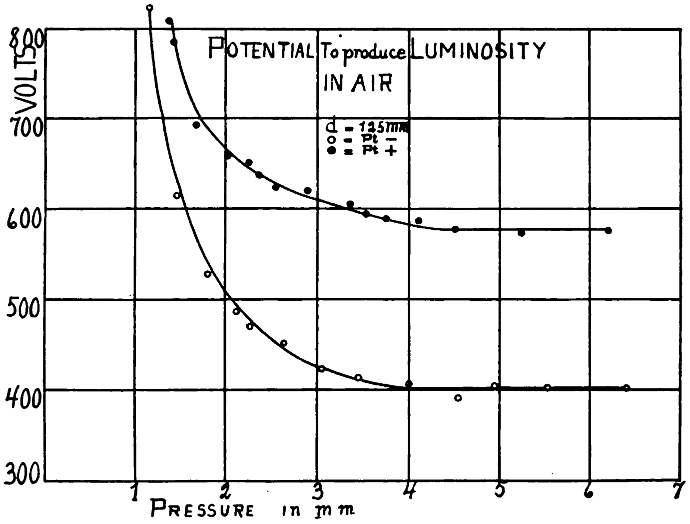


Fig. 3.

voltmeter reading and the cessation of the discharge could likewise be noted by a sudden increase in the voltmeter reading. In order to insure a greater degree of certainty as to the beginning of the

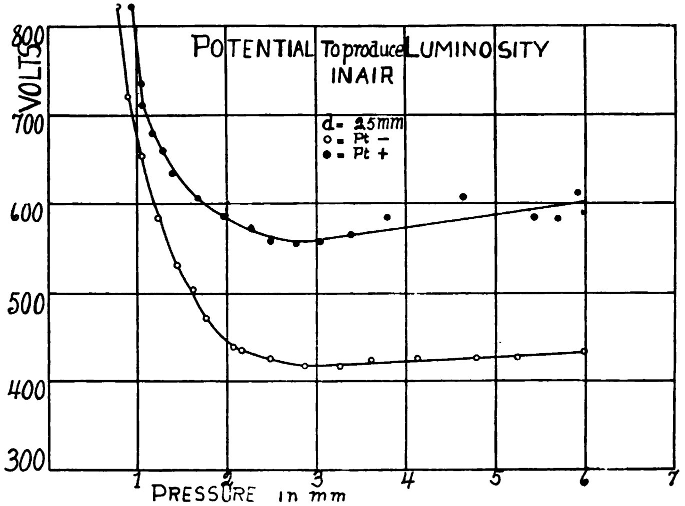


Fig. 4.

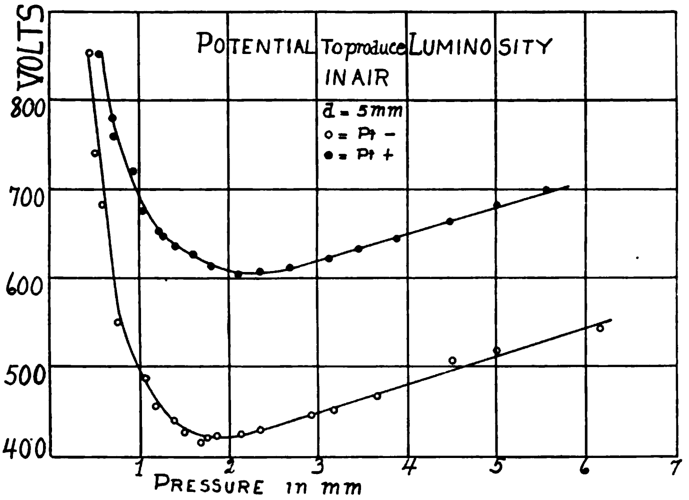


Fig. 5.

luminous discharge a small hole was bored in each of the washers employed to separate the electrodes so that the discharge could be seen. However this precaution was not necessary, the kick of the voltmeter being sufficiently reliable.

Since the luminous discharge presupposes a supply of ions a discharge was produced to create them and then after several minutes the voltage was increased by small steps in the manner previously described until the discharge again passed. After each observa-

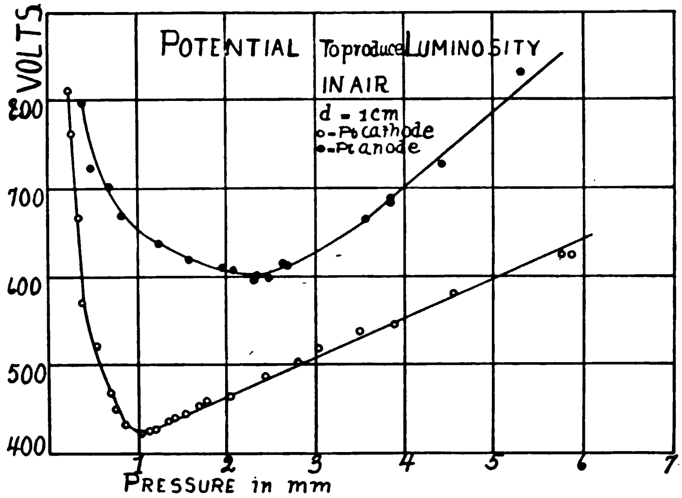


Fig. 6.

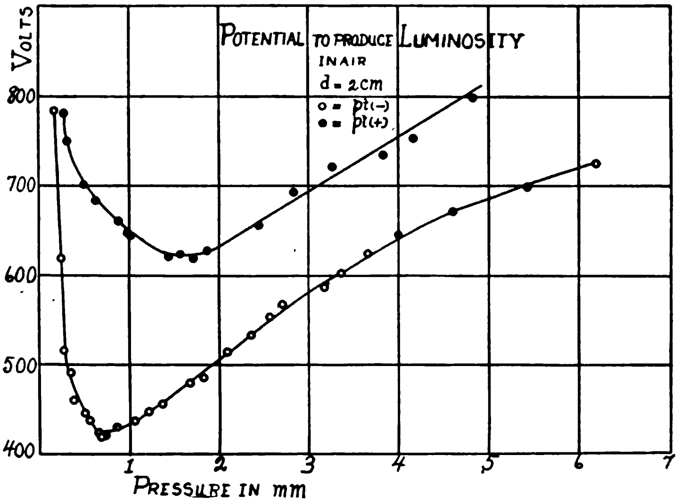


Fig. 7.

tion sufficient time was allowed to elapse for the gas to reach its normal condition before another reading was taken.

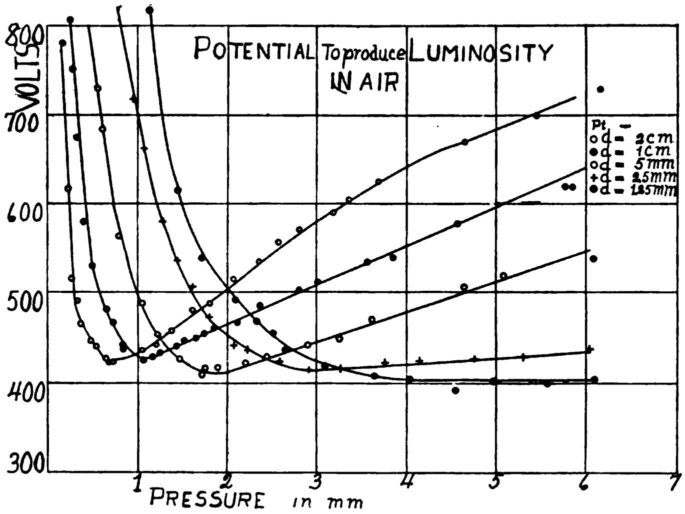


Fig. 8.

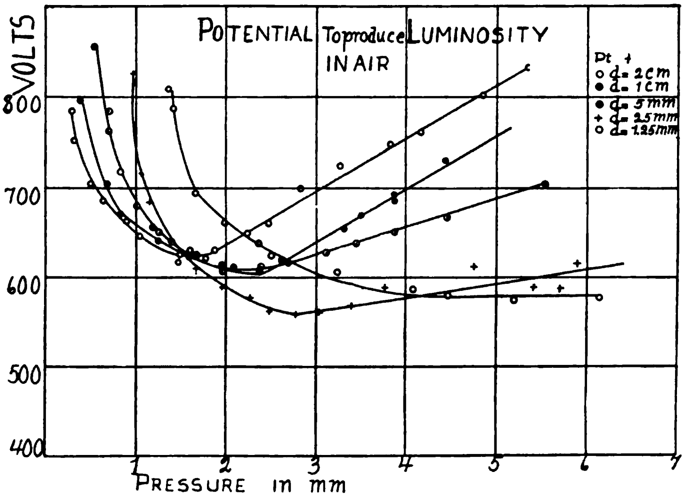


Fig. 9.

The delay in the passage of the discharge which has been noted by a number of investigators and carefully studied by Warburg[1]

---

[1] Warburg, Ann. der Phys., 62, p. 385.

was observed. As Carr[1] has noted, the lag was greater near the critical pressure. It was also greater below the critical pressure than above it.

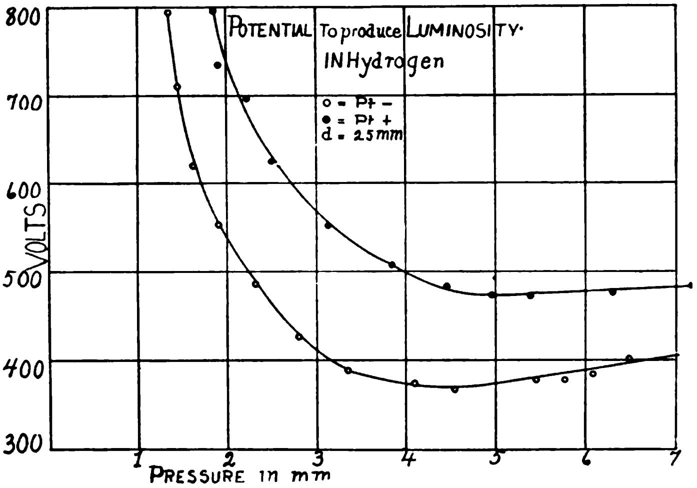


Fig. 10.

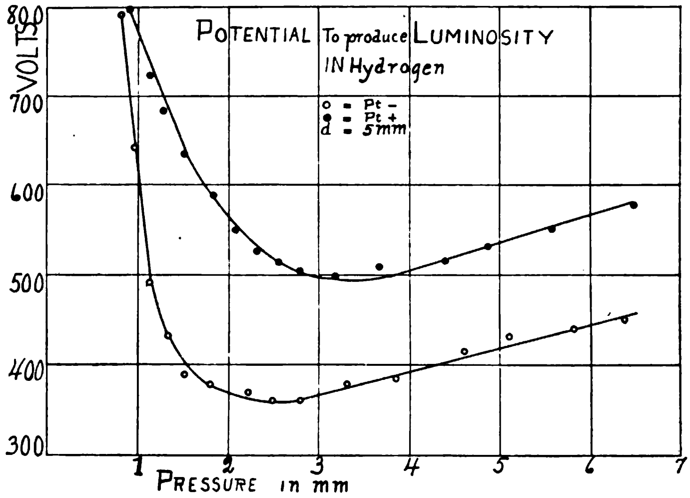


Fig. 11.

In beginning a series of readings in air the whole apparatus was exhausted to a pressure of about one millimeter of mercury and

---

[1] Carr, Phil. Trans. Roy. Soc., CCI., p. 403.

then refilled by allowing the air to pass in slowly through a tube packed with phosphoric pentoxide. The apparatus was then exhausted to a pressure of 10 mm. of mercury and allowed to stand over night, the air in the chamber being in constant contact with the phosphoric pentoxide in the drying chamber.

The measurements in the air were made at five different distances of the electrodes, 1.25, 2.5, 5, 10 and 20 mm., and extending over a range of pressure from 8 mm. to 0.27 mm. of mercury. The results of these measurements are shown graphically in Figs. 3, 4, 5, 6 and 7. Each curve shown is made from one continuous set of readings and is not a selection from several sets, although three series of readings were made for each distance and the average one was selected.

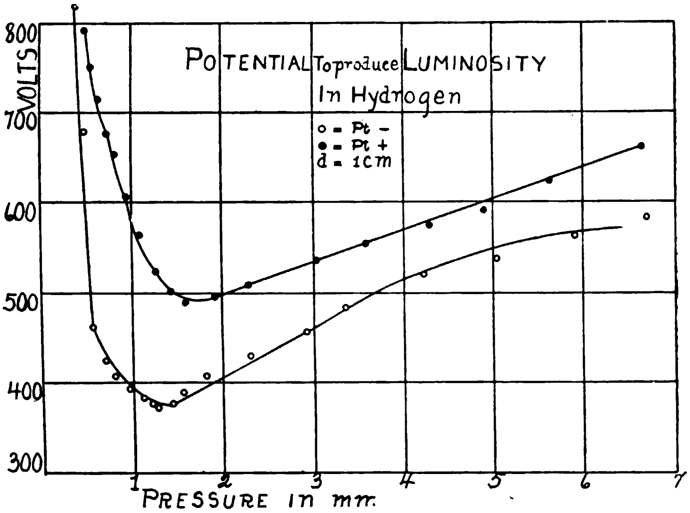


Fig. 12.

It will be seen that the curves for point negative are similar to those obtained by Carr and other experimenters for parallel plates, but with point positive the voltage required for a discharge at any certain pressure is higher than the voltage required at the corresponding pressure for point negative and the critical pressure for point positive is higher than for point negative. The potentials where rectification is possible obviously lie between the lines representing the discharge for point positive and for point negative. Theoretically for an alternating potential within this region discharge would proceed only in one direction. It should be noted that within the limits of the experiment the most favorable region for this effect lies near the critical pressure and for the longer spark gap. For the purpose of comparison all the curves made with point negative in air are shown in Fig. 8, while Fig. 9 shows a similar set of curves for point positive. Carr found the minimum potential required for a discharge between two parallel plates to be independent of the distance which separated them. In general the potential required for discharge depended only on the mass of gas between the electrodes, that is, Paschen's Law. In the case of discharge between parallel plates the entire mass of gas without doubt is affected. In the case of a point and a plane it is quite improbable that the entire quantity of gas within the chamber functions when the discharge takes place. Our lack of knowledge on the distribution of the discharge from a pointed conductor renders it impossible to say just what portion of the gas within the chamber functions; however it is not improbable that Paschen's Law holds in this case also. The minimum potentials as shown on the different curves are somewhat more consistent for point negative than for point positive. The

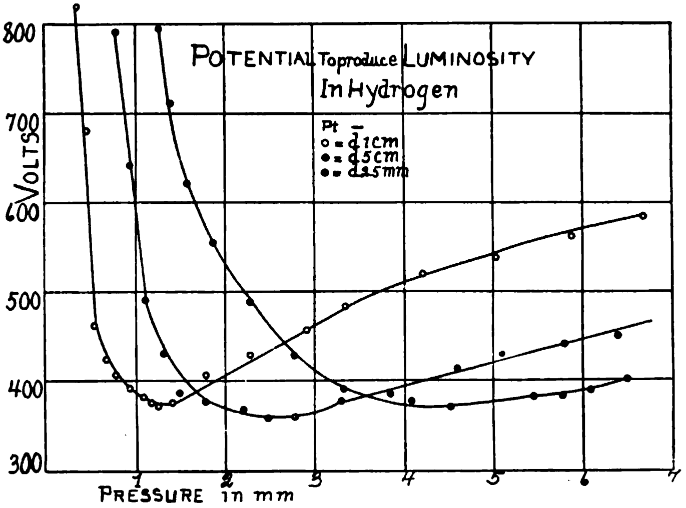


Fig. 13.

critical pressure varies with the distance of the electrodes in every case, but for any certain separation of the electrodes the critical pressure for the negatively electrified point is less than when the point is electrified positively.

In order to further test the laws governing such discharges it seemed advisable to carry out the experiment using another gas. Figs. 10, 11, 12, 13 and 14 show graphically the results obtained with hydrogen. The apparatus used was the same as in the experiments with air. The hydrogen was prepared in a Kipp apparatus by the action of sulphuric acid on chemically pure zinc and after passing through solutions of potassium permanganate and pyrogallic acid was dried by being passed through tubes packed with phosphoric pentoxide. While in this way absolutely pure hydrogen cannot be procured it is thought that the small amount of impurity did not seriously affect the results.

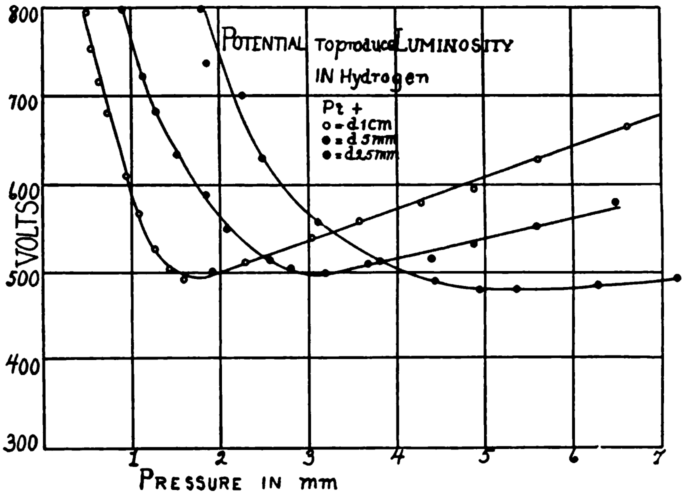


Fig. 14.

Before making the measurements with hydrogen the apparatus was exhausted of air to a pressure of about .5 mm. of mercury and then filled with hydrogen to atmospheric pressure. It was then exhausted and refilled four times to make certain that all air was removed after which the gas was allowed to stand for several hours in contact with phosphor pentoxide at a pressure of about 10 mm. of mercury before any readings were attempted.

The results obtained for hydrogen are quite similar to those obtained in air. The differences being just such differences as would be expected; the minimum potential required for the discharge being considerably less than for air and occurring at higher pressures.

These experiments have suggested several phases of this problem and it is hoped to make some considerable extension of it in the near future.

Physical Laboratory,
Ohio State University, Columbus, Ohio.

# AN APPARATUS FOR MEASURING SOUND.

By F. R. Watson.

THE apparatus which is described here was devised to measure quantitatively the intensity of sound in auditoriums. It was desired to have the instrument that received the sound impression both portable and easily adjusted. The apparatus which was developed to fulfill these conditions consists essentially of a telephone receiver connected in series with a thermo-galvanometer. Feeble sounds were augmented by altering the apparatus so as to make it resonant. Thus the source of sound was adjusted until a maximum deflection of the galvanometer was obtained. A resonator was then mounted on the telephone plate and adjusted to give a second maximum deflection. The electric circuit was also tuned by inserting a condenser and altering the capacity to get a third maximum. Finally, the distance between the telephone plate and

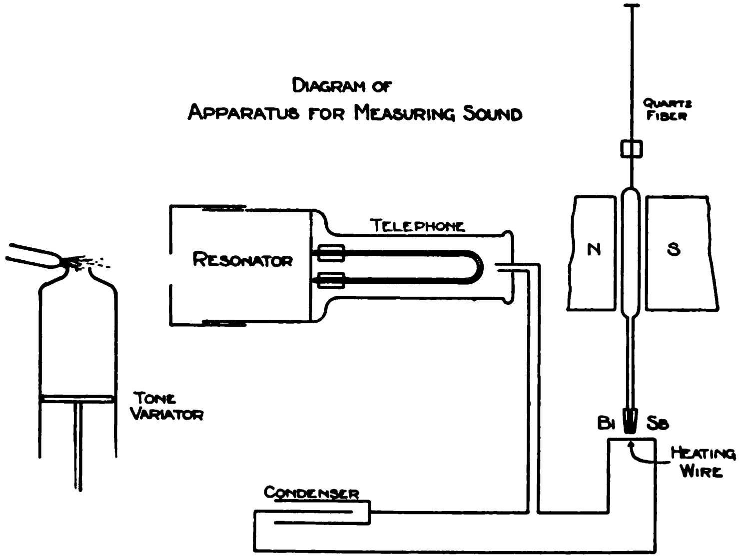


Fig. 1.

the telephone magnet was adjusted for the greatest effect. Fig. 1 shows diagrammatically the apparatus used.

The Duddell thermo-galvanometer[1] used was found especially suited for this work, as it gave a steady deflection for the alternating currents set up by the telephone. Some idea of magnitude of the current used is given in the following data. Seventy microamperes caused a deflection of one centimeter when the scale was 100 cm. from the galvanometer. A current of 350 microamperes gave 25 cm. It is thus seen that the deflection varies as the *square* of the current; a fact that should be held in mind in making the apparatus sensitive. Thus an increase in the intensity of the source of sound magnifies the deflection of the galvanometer according to the square.

A tone variator was used as the source of sound. It consists of a hollow brass cylinder open at the top and closed at the bottom by an adjustable piston. The telephone receiver is one of an ordinary bipolar type. The resonator is a hollow brass cylinder, the adjustable cap fitting smoothly and closely over the base piece. The effect of adjusting the different parts of the apparatus is shown in the following tables.

TABLE I.

*Effect of Heaters of Different Resistances.*

| | | | |
|---|---|---|---|
| Resistance in ohms, | 94.8 | 47.3 | 3.68 |
| Deflection in cm., | 9.0 | 4.0 | 0.5 |

TABLE II.

*Effect of Resistance added in Circuit.*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Resistance in ohms, | 0.0 | 1.0 | 5 | 20 | 40 | 100 | 200 | 300 |
| Deflection in cm., | 11.0 | 11.0 | 11+ | 10 | 9+ | 6.5 | 4.5 | 3+ |

Tables I. and II. show that the best effect is obtained with a heater resistance of about 100 ohms in the thermo-galvanometer.

TABLE III.

*Effect of Varying Pitch of Source of Sound.*

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Pitch, | 500 | 600 | 650 | 660 | 670 | 680 | 690 | 700 | 710 | 750 | 770 |
| Deflection, | .4 | 1.3 | 3.3 | 3.9 | 4.6 | 5.7 | 6.2 | 7 | 7− | 6.7 | 5.6 |

It is seen that the maximum deflection is reached for a pitch of 700 vibrations. To augment the effect, a wooden organ pipe giving a more intense sound was used as the source of sound. It was first tuned in unison with the tone variator.

---

[1] W. Duddell, Phil. Mag., 8, 91, 1904.

### TABLE IV.

*Effect of a More Intense Source of Sound.*

| Source of Sound. | Distance from Telephone. | Deflection. |
|---|---|---|
| Tone variator, | 2 cm. | 7 cm. |
| Organ pipe, | 2 cm. | off scale |
| Organ pipe, | 5 cm. | 10 cm. |

### TABLE V.

*Effect of Inserting Capacity in the Telephone Circuit.*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Capacity, | 0 | .2 | .4 | .7 | .9 | 1.0 | 1.2 | 1.5 | 1.7 |
| Deflection, | 10 | 1.7 | 6.4 | 13 | 14.5 | 15 | 14.5 | 14 | 13.3 |

The maximum deflection occurs for a capacity of one microfarad. To get a still greater effect, a resonator was then screwed over the telephone plate.

### TABLE VI.

*Effect of Resonator.*

| Instrument. | Distance from Horn. | Deflection. |
|---|---|---|
| Telephone without resonator, | 10 cm. | 0.5 cm. |
| Telephone with resonator, | 10 cm. | off scale |

### TABLE VII.

*Effect of Different Telephone Plates.*

| Diameter. | Thickness. | Distance from Horn. | Deflection. |
|---|---|---|---|
| 5.3 cm. | .0335 cm. | 15 cm. | 4.8 cm. |
| 5.2 cm. | .0195 cm. | 20 cm. | 20.0 cm. |

The increased effect noted in Table VII. was probably not due to the difference in the thickness of the plates. The thinner plate was drawn closer to the telephone magnet and thus increased the magnetic field and the induced currents.

The apparatus was finally assembled as follows: An organ pipe having a pitch of about 700 vibrations a second was blown by a constant air pressure. The sound was received by a telephone receiver, having a plate 5.2 cm. in diameter and .0205 cm. thick. The brass resonator on the telephone plate was 4.9 cm. long, 5 cm. diameter, with a circular aperture 1.9 cm. in diameter. The telephone was connected in series with a condenser of one microfarad capacity, and with a thermo-galvanometer having a heating wire of 94.8 ohms resistance. Maxima and minima of sound were easily detected in a small laboratory. The apparatus gives quantitative measurements that may be repeated.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS.

# SOME ELECTRICAL PROPERTIES OF SELENIUM.

BY LOUISE S. MCDOWELL.

## II. THE RECOVERY FROM EXCITATION BY X-RAYS.

SINCE the experiments of Pfund[1] and Berndt[2] have conclusively disproved the theory of Bidwell that the increased conductivity of selenium under the action of light is due to the formation of conducting selenides, the most commonly accepted theory has been that of "allotropic dissociation," first proposed by Siemens.[3] According to the present form of the theory the effect of heat is to change the selenium from the amorphous to the crystalline form and further to a second crystalline form which alone is conducting. Following the notation of Schrott and Marc we may call the non-conducting crystalline form $A$ and the conducting, $B$. Sensitive selenium consists of a solid solution of $B$ in $A$. According to Marc[4] the effect of light is merely to alter the equilibrium in favor of $B$, but according to Hesehus[5] and Schrott[6] the effect of light is to dissociate the $B$ dissolved in $A$, thus producing free ions which in the dark recombine. It is the latter form of the theory which we shall consider.

In an earlier paper the writer described some experiments dealing with the rate of recovery of the normal resistance of a selenium cell after exposure to light. When the recovery curves were plotted with the reciprocal square roots of the changes in conductivity as ordinates and the times as abscissæ, the form of the curves was found to be exactly similar to the curves obtained by Nichols and Merritt[7] for the decay of phosphorescence. Their method of plot-

---

[1] A. H. Pfund, Phil. Mag., Vol. 7, p. 26, 1904.

[2] G. W. Berndt, Phys. Zeit., Vol. 5, p. 121.

[3] Siemens, Phil. Mag., Vol. 50, p. 416.

[4] R. Marc, Zeit. für Anorgan. Chem., Vol. 50, p. 446, Vol. 53, p. 298.

[5] N. A. Hesehus, Phys. Zeit., Vol. 7, p. 163.

[6] Schrott, Akad. Wiss. Wien, Sitzungsber., Vol. 115, 2a, p. 1081.

[7] Nichols and Merritt, PHYS. REV., Vol. 22, p. 279, 1906.

ting rested upon the assumption that the intensity of phosphorescence is proportional to the number of recombinations per second. The hypothesis of Rutherford[1] that the number of recombinations is proportional to the number of collisions and hence to the number of positive and of negative ions leads to the relation $I = k\alpha n^2 = k\alpha/(1/n_0 + \alpha t)^2$, where $n$ equals the number of positive (or negative) ions present per c.c. at any time $t$. This expression may be written in the form $1/\sqrt{I} = a + bt$, which indicates a linear relation between $I^{-\frac{1}{2}}$ and $t$. Experiments showed the curve to consist of three parts—a straight line for the early portion of the decay, a second part nearly straight but of radically different slope and bending somewhat towards the axis of abscissæ, the two connected by a sharp curve.

Merritt[2] has shown that this deviation from the straight line form indicated by the simple theory may be accounted for by an irregular distribution of the active substance, by diffusion, or by ionic grouping, and that any one of these factors is sufficient to explain the form of the curve. Since the recovery curves for selenium are of exactly the same form as the decay curves for phosphorescence, it seemed worth while to determine what part, if any, diffusion plays in determining the form of the curve. Selenium in the sensitive form is known to be almost entirely opaque to light,[3] so that any increase in the number of free ions in the interior must be due to diffusion from the surface layers. On the contrary, selenium is relatively transparent to X-rays. If diffusion is an important factor in determining the bend towards the axis of abscissæ, the recovery curves for X-ray excitation should approach more nearly the straight line form.

The cells used were four in number, all of the Bidwell type: cell $A$, the Max Kohl cell previously used, with a dark resistance of approximately 400,000 ohms; cell III., a home-made cell several months old, the dark resistance of which had risen gradually to approximately 450,000 ohms, cells XIV. and XX., two home-made cells with resistances, respectively, of 70,000 and 180,000 ohms.

---

[1] Rutherford, Phil. Mag., Vol. 44, p. 422.

[2] Merritt, Phys. Rev., Vol. 27, p. 367, 1908.

[3] Pfund. Phys. Zeit., Vol. 10, p. 340, 1909.

The cells were tested with the fluoroscope and found to be only partially transparent so that the spaces between the wires were but faintly discernible. When the hand was held behind the cell the shadow of the cell was roughly half that of the bones of the hand, whereas the mica form cast but a slight shadow.

The arrangement of apparatus was as follows:

The X-ray tube was placed about 12 cm. above the cell, enclosed in black paper. No precautions were taken to insure constancy of temperature or to eliminate moisture both because neither factor seemed to exert any noticeable disturbing influence and because the short duration of the experiments insured reasonable constancy of conditions. As a current source a single storage cell was used.

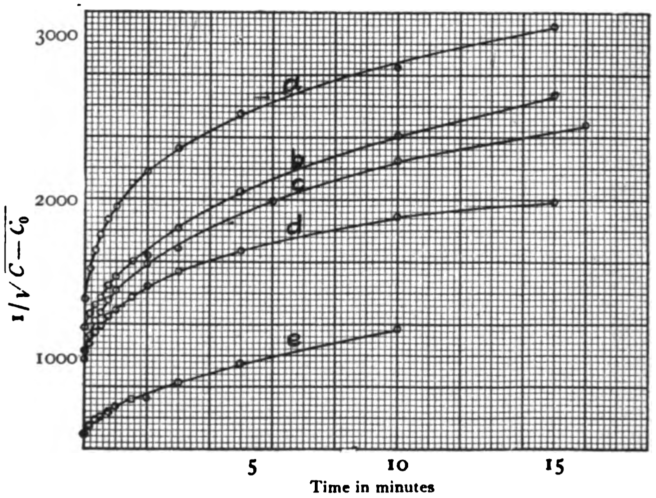


Fig. 1. Recovery after a two-minute excitation by X-rays. *a*, Cell III., initial dark resistance, 472,000; *b*, Cell XX., 181,500; *c*, Cell III., 446,000; *d*, Cell *A*, 414,000; *e*, Cell XIV., 70,000.

The resistance was measured, as in the earlier experiments, by making the cell one arm of a Wheatstone bridge, balanced for a known resistance approximately equal to that of the cell, and calibrated by the substitution for the cell of known resistances. Deflections were read by means of a beam of light reflected from the galvanometer mirror to a graduated scale. The galvanometer used was a Sullivan galvanometer with a straight suspension to give an extremely short period. The method followed was to excite the cell to saturation, a process requiring approximately two minutes, then to shut off the X-rays and at short intervals during the recovery to note the resistance of the cell.

The curves were found to be exactly similar in form to the curves for recovery from light (see Figs. 1 and 2). No lessening of the bend towards the axis of abscissæ was perceptible. Hence it seems reasonable to conclude that the deviation from the straight line form is not to be explained by diffusion of ions from the surface layer.

No saturation curves were taken for the reason that the X-rays were not sufficiently constant. During the earlier part of the two minutes they increased somewhat in intensity and towards the end sometimes diminished slightly because of the increased resistance of the tube, indicated by a sparking across the air gap and a faint lighting of the auxiliary tube. So far as could be judged, however, the saturation was approached somewhat more suddenly than in

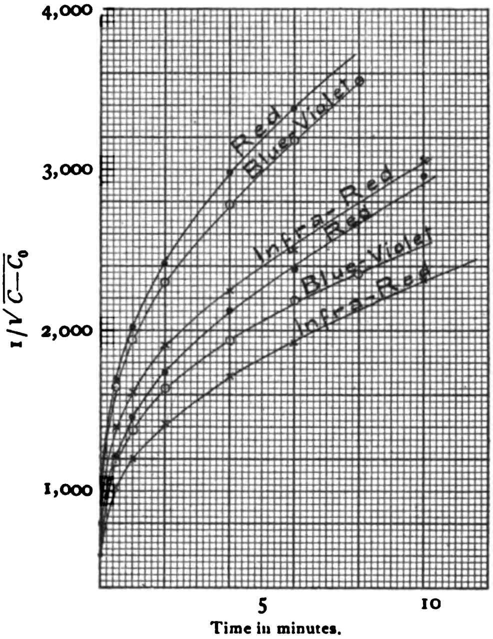


Fig. 2. Variation in rate of recovery with change in wave-length of light. Cell *A*.

the case of the light, with less of the long-continued, slight drifting of the galvanometer noticeable there.

While no special experiments were performed to compare the rate of recovery from excitation by light and by X-rays, a comparison of the curves of Figs. 1 and 2 shows that for cell *A* the recovery after a two-minute excitation by X-rays is much slower than after a five-second excitation by light even when the initial effect produced is the same. If it can be shown that the form of the recovery curve depends only upon the initial change in conductivity produced and not at all upon the time of excitation, then we may justly conclude that the rate of recovery from X-ray excitation is slower than from light. A study of the curves of Figs. 3 and 4, obtained in the earlier experiments, indicates a strong probability that such is the case. These curves were taken for the same cell and show the effect respectively of intensity and of time of excitation upon the

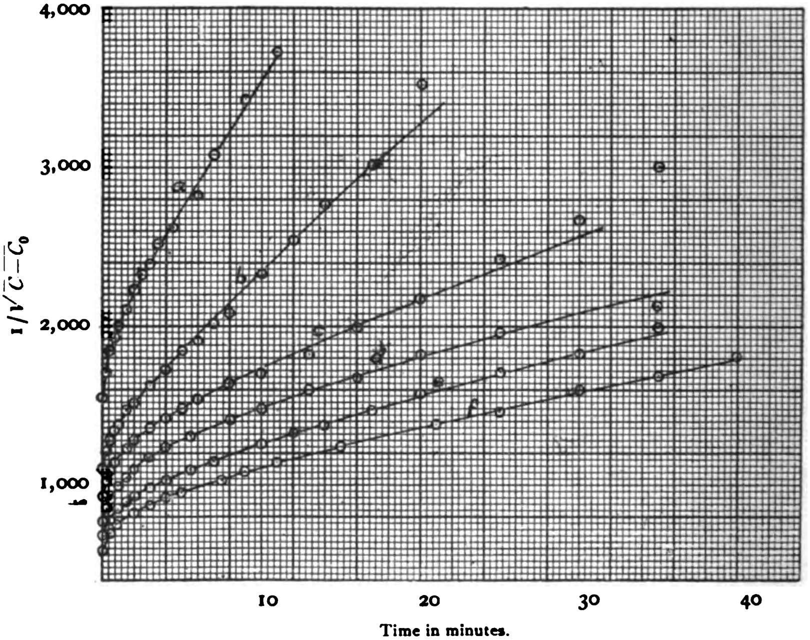


Fig. 3. Variation in the rate of recovery with intensity of excitation. Cell I. Twenty-second excitation by red light. Initial dark resistances : *a*, 53,920; *b*, 51,040; *c*, 51,800; *d*, 52,070; *e*, 55,160; *f*, 54,010.

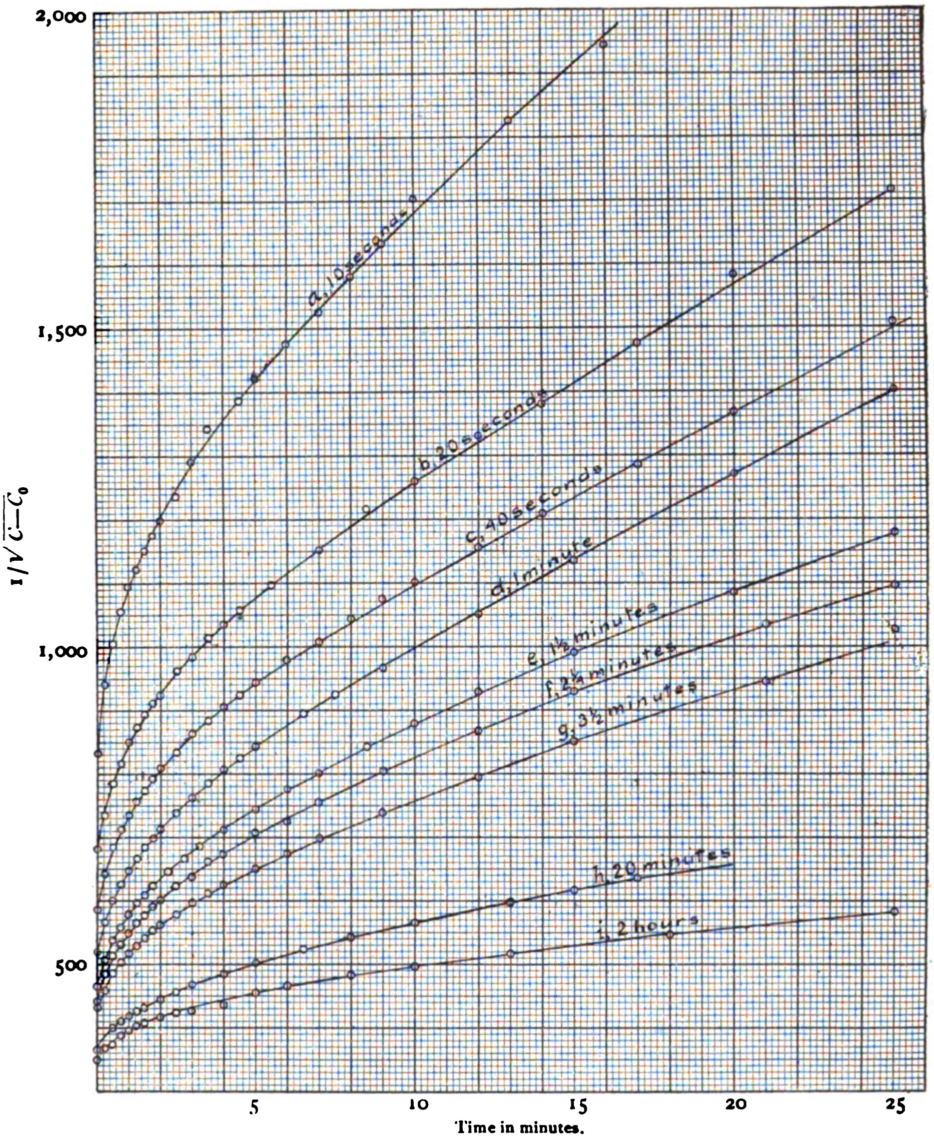


**Fig. 4.** Variation in rate of recovery with time of excitation. Cell I. Red light excitation. Initial dark resistances: *a*, 49,760; *b*, 55,160; *c*, 52,810; *d*, 52,870; *e*, 52,350; *f*, 53,120; *g*, 52,380; *h*, 51,480; *i*, 52,200.

recovery curve. In Fig. 5 are placed together curve *f*, Fig. 3, and curve *c*, Fig. 4, which chance to have approximately the same initial ordinate. Although produced by excitations of twenty and forty seconds respectively, these curves very nearly coincide. On the assumption that time was without influence on the form of the curve there was calculated by means of auxiliary curves (for method see the earlier article[1]) the curve intermediate between *a* and *b*, Fig. 4, the initial ordinate of which was equal to that of curve *d*, Fig. 3. This derived curve was found to coincide approximately

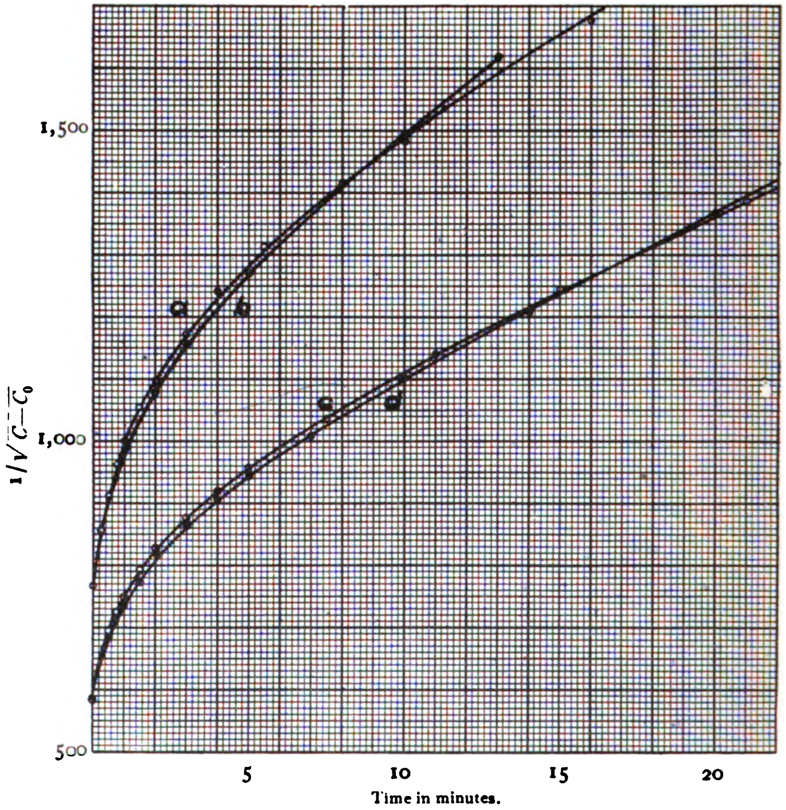


Fig. 5. Curves showing that the rate of recovery is independent of the time required to produce the initial change in conductivity. Cell *I*. *a*, Curve *d*, Fig. 3, time of excitation 20 seconds; *b*, derived curve intermediate between *a* and *b*, Fig. 4; *c*, curve *f*, Fig. 3, 20 seconds; d, curve *c*, Fig. 4, 40 seconds.

---

[1] Phys. Rev., Vol. 29, p. 1.

with the curve experimentally determined. Although this evidence is hardly sufficient to be conclusive, it indicates a decided probability that the form of the curve is independent of the time required to produce the initial change of conductivity. This result in itself suggests that diffusion is not an important factor in determining the form of the curve. If it were, the longer excitation would be followed by a slower recovery, since in that case the ions would be diffused through a larger volume. The conclusion that the rate of recovery from X-ray excitation is slower than from light excitation is in exact agreement with the work of Perreau.[1]

The results of the investigation may be briefly summarized as follows:

1. The forms of the recovery curve for X-ray excitation and for light excitation are identical.
2. The form of the recovery curve for excitation by light depends only upon the magnitude of the initial change in conductivity and not at all upon the time required to produce that change.
3. The rate of recovery from excitation by X-rays is slower than from excitation by light.
4. Hence diffusion is probably not an important factor in producing the deviation from the straight line of the selenium recovery curves.

These experiments were carried on during the summer of 1909 in the Physics Laboratory of Cornell University, and in conclusion the writer wishes to express her gratitude to Professor Merritt at whose suggestion the work was undertaken.

PHYSICS LABORATORY,
WELLESLEY COLLEGE.

---

[1] M Perreau, Comptes Rendus, Vol. 129, p. 956.

# TYPICAL CASES OF SECONDARY RADIATION EXCITED BY URANIUM-X.

BY W. B. HUFF.

AMONG recent papers dealing with the various aspects of secondary radiation, several describe in some detail the effects obtained by using Ur-X as the source of the primary. An extremely suggestive article by Bragg[1] calls attention to the errors introduced by using the usual ionization method of comparing radiations which are not only markedly different in intensity, but which have not produced the entire ionization each is capable of effecting.

A somewhat earlier paper by Schmidt[2] gives an account of his work with the secondary from the $\beta$- and $\gamma$-rays of Ur-X. In his experiments the radio-active material was placed just above the ionization chamber of an electroscope. The resulting effect was due, in the main, to the primary radiation. A metal plate was then placed directly over the Ur-X and the secondary radiation from this added its ionization to that produced by the primary. The method is therefore a differential one.

This increase of ionization due to the secondary from thick plates of various metals was found to follow the order of atomic weights. Schmidt also studied thin plates as reflectors and as absorbers.

The present investigation arose from some apparently quite anomalous results which were obtained during a study of the radiation from single crystals of uranium nitrate. During the earlier part of the work, a method was used which, as it happened, was essentially that of Schmidt, and a considerable part of his observations was verified. However, a further consideration of the experimental problem brought into sharper relief certain difficulties inherent in this method. The secondary radiations obtained may vary in ion-

---

[1] Phil. Mag., Oct., 1907.

[2] Ann. d. Phys., 23, 1907.

izing effect (as measured in a small chamber) from about 70 per cent. of the primary down to only a small fraction of this. Further, the secondary itself varies in intensity and amount with the reflecting substance, and may be absorbed by thin plates quite differently from the primary. It is also necessary to cut out the strongly ionizing $\alpha$-rays, and about 0.10 mm. of aluminium foil must be used for this purpose. This shield not only modifies the radiations, but also adds to their ionizations an additional one due to their emergence from the aluminium.

It seemed desirable therefore to choose a method which would not necessitate such differential measurement, and which would allow a study of the secondary $\beta$-rays modified as little as possible by shielding.

## APPARATUS.

The plan finally adopted was to use the thinnest possible leaf of aluminium as the top of a small, cylindrical gold-leaf electroscope of very small capacity. The Ur-X was placed in a shallow circular groove in a brass plate nearly in the plane of the aluminium leaf and surrounding it. The thickness of plate and electroscope case was such that only the very penetrating $\gamma$-rays added their effect to the natural leak of the instrument.

The plate giving off the secondary to be studied was placed at a little distance above the Ur-X. Thus a considerable part of the secondary radiation from this plate passed down through the aluminium leaf into the electroscope.

The case of the electroscope used for most of the observations was of cylindrical brass tubing 4.7 cm. inside diameter and 14 cm. high, built up of accurately fitted sections so that the total height could be varied and the effect of the change of wall area noted. The insulation of gold-leaf and standard was of sulphur or of quartz. These were about equally good if the air within was kept dry. A sulphur condenser could be used to decrease the rate of leak. The aluminium leaf forming the top of the case was as nearly free from holes as possible and was held taut by a brass ring grooved to fit the vertical wall of the case. A circular area of the leaf 4 cm. in diameter was exposed to radiation from the reflecting plate. The case was closed and waxed, and the inside air kept dry with sulphuric acid at the bottom of the chamber.

The charging and reading were done in the usual manner, ten scale-divisions of the reading microscope corresponding to a movement of the gold-leaf through 1 mm. At 200 volts the natural leak, after standing and drying, was about three scale-divisions per hour. This varied somewhat from day to day as well as at different times of the day, but such variations could usually be taken into account in so far as they were important.

A second brass plate on top of the electroscope had a circular opening 4.2 cm. in diameter and concentric with the 4 cm. hole exposing the aluminium leaf, thus forming a shoulder for the support of discs used in studying absorption. Outside this 4.2 cm. hole and concentric with it was a circular groove, 1 mm. deep, 1 cm. wide and 5 cm. inside diameter, which contained the Ur-X. A slender brass tripod on this second ring supported the reflecting plate. Its height could be varied, thus varying the air-path and the angle of the secondary beam. The aluminium leaf was about 5 mm. below the Ur-X and was thus shielded from all direct radiation.

The reflectors were discs about 8 cm. in diameter and rested, polished face down, on the thin ring of the tripod. The latter insured a constant area of the reflector exposed to primary radiation and protected the edge of the disc. Most of the measurements were taken with the disc about 1 cm. from the Ur-X.

The Ur-X was prepared from the nitrate by the usual method of separating from an ether solution by water and then by successive rapid crystallizations and subsequent filtering through the oxide which formed on standing. The result was a fine, grayish powder which did not collect moisture when exposed to the atmosphere.

A very thin layer was distributed and kept as nearly uniform as possible over the bottom of the groove. Its absorption was neglected. The Ur-X was covered with 0.10 mm. of aluminium, waxed down to the edges of the groove. This cut out the $\alpha$-rays and in some degree modified the penetrating radiation. Some observations taken without this screen will be noted later.

## Sources of Error.

It is obvious that the above arrangement does not give parallel secondary rays; nor are these completely absorbed in the air of the ionizing chamber. As a result, the measurements may be affected by the walls of the electroscope. Yet any method of getting parallel rays by increasing distances, or by using a bundle of tubes or slits, would bring in either a very considerable air-absorption, or an increase of surface, either of which might seriously modify the radiation before it entered the ionizing chamber.

Except for very weak ionizations, variations in the rate of natural leak were not of great importance. It is a much more difficult matter to avoid disturbances of the gold-leaf by air-currents arising from temperature changes in the electroscope. Unless the case is carefully shielded, and readings taken only after the observer has been for some time at the microscope, such errors may reach 10 per cent.

In changing reflectors, especially when the ionizations are weak, a considerable time must be allowed for the ionization to become steady. The time taken for the gold-leaf to pass over 10 scale-divisions varied from 5 minutes to several hours. Except when the variation in the rate of natural leak became comparable with the ionization, a set of readings under given conditions usually agreed to well within 1 per cent. The effects from a thick plate of iron and one of nickel could be distinguished readily.

It is evident also that whatever shallow vessel contains the radio-active material, the secondary from the bottom will be added to the primary. Thus a lead vessel would add something like 70 per cent., much of this increase being very penetrating. Aluminium would add much less and the increase would be softer. In either case, the effect on the result from the reflecting plate may become considerable, especially in a study of the absorption of such superposed radiations by very thin plates.

There will also be what may be styled multiple-reflections between the top of the electroscope and the plate above it. These were judged to be small when electroscope and plate were not less than 1 cm. apart, since covering the exposed brass of the top of the electroscope with lead made little difference in the readings. But when reflecting plate and radio-active material are very close together, as in the differential method first used, it is possible that very penetrating rays may give higher order effects which would become important.

To study possible tertiary effects from the inner walls of the electroscope, a second instrument was constructed. Its case was 6.5 cm. high and 12 cm. in diameter. The bottom could be of thick brass, or of aluminium foil 0.01 mm. thick. The ratios of ionizations from various pairs of reflectors such as lead and aluminium, bismuth and magnesium, did not differ materially from values obtained by using the taller and narrower type of instrument.

## Experiments.

The values of the ratio of the secondary from a thick plate of lead to that from one of aluminium was found to depend on the distance of the reflecting plate from the Ur-X, as shown by the following numbers.

| Distance. | Ratio. |
|---|---|
| 4.0 cm. | 3.2 |
| 1.0 " | 2.5 |
| 0.3 " | 2.0 |

Difference in air-path and change in possible angles of incidence of the primary upon the plate as well as of the secondary upon the aluminium leaf, contribute to this variation.

Such results show at once the difficulty of doing more than arranging the elementary substances according to the ionization produced by their secondary radiation. Schmidt found that the order is that of atomic weights. This result was easily verified for fifteen elements, though the ratios for each element to some one of them differed considerably (and not all in the same direction) from those calculated from Schmidt's results. However, it may be added that S and As fall into place. Alloys of Cd and Bi containing but a few per cent. of one constituent were only slightly different from the pure metals as reflectors. Accordingly a sodium alloy of mercury was compared with bismuth. The result placed mercury roughly in the position which would be given by its atomic weight.

Using thick plates of lead and aluminium as typical examples of reflectors of high and low atomic weights, and a thick plate of tin as intermediate between these extremes, curves were obtained for the absorption of the secondary by thin plates, the same three metals being used as typical absorbers. Considerable variation was observed in different samples of lead foil. That finally used was nearly pure. The reflecting tin plate was cast from pure tin. The absorbing tin leaf was marked pure and was of nearly uniform thickness. The aluminium was the commercial article.

To study absorption, circular discs of the foil were cut, fitting into the 4.2 cm. hole above the aluminium leaf. These discs were made as nearly plane as possible and lay almost in the plane of the Ur-X, to minimize $\gamma$-ray effects.

In the final reductions of readings, the decay of the Ur-X was taken into account. The ratio of the effect from lead to that from aluminium remained nearly constant throughout the time each sample was used.

Corrections for leak were also made, and for change of this leak due to decay of the Ur-X. Allowance was also made for the secondary given off from the tripod. The effect of the tripod alone, *i. e.*, carrying no reflecting plate, was about equal to the natural leak.

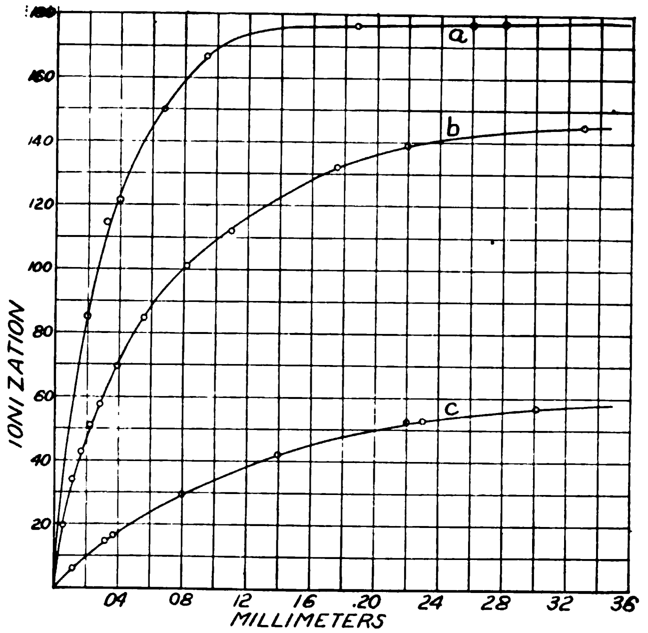


Fig. 1.

The amount and character of the returned radiation depends also on the thickness of the reflecting discs. Comparison curves are given for the three metals used.

The curves for reflection and absorption are drawn with thicknesses of metal in hundredths of 1 mm. as abscissas, and ionization proportional to scale-divisions per minute as ordinates. The fact that the ratio of the ionization is nearly constant and independent of decay of the radio-active material was used in reducing the ordinates to a common scale.

Fig. 1 shows the increase of ionization arising from increase of secondary as the thickness of the reflecting metal plates above the Ur-X is increased. The curves for Pb, Sn and Al are shown by *a*, *b*, *c*, respectively.

In Fig. 2 the curves *a*, *b*, *c* show the absorption of the secondary from a thick plate of lead when it passes through Pb, Sn or Al.

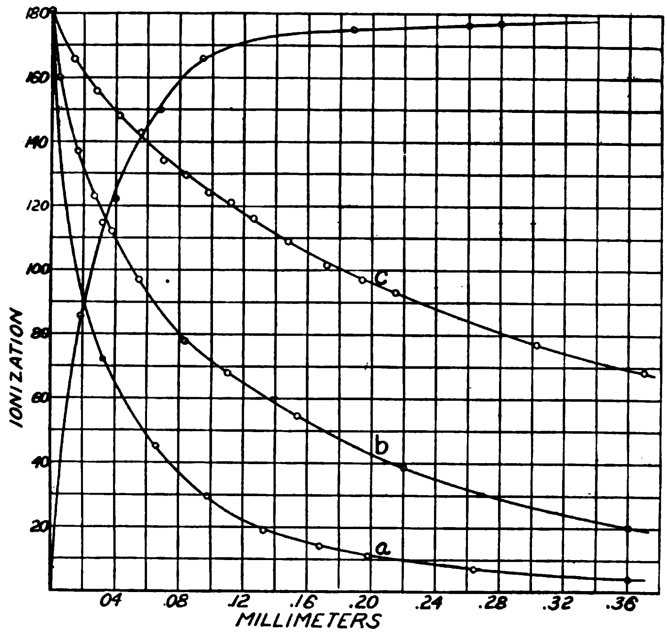


Fig. 2.

The curve *a*, Fig. 1, is given for comparison. Fig. 3 corresponds to Fig. 2, except that the secondary comes from aluminium.

Except for thin layers, the absorption by aluminium is almost linear, and continues so up to thicknesses of about 1 mm. The absorption by lead is more nearly exponential at first, but for both lead and tin the effect becomes roughly linear for the more penetrating rays. Secondary $\gamma$ radiation, since it would not be much affected by plates thick enough to absorb most of the secondary $\beta$-rays, would become of importance in studying absorption by any but comparatively thin sheets.

The reflection and absorption curves for lead and for aluminium cut at a point corresponding to about half the maximum ionization. Thus the ionization from a thick plate of each metal is reduced to half value by an absorbing layer of the same metal which is approximately equal in thickness to a plate returning half the primary.

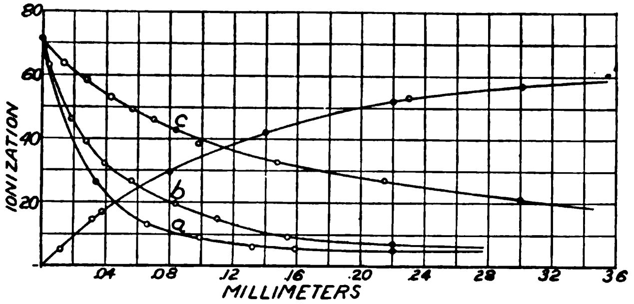


Fig. 3.

An attempt was also made to study the secondary radiation from extremely thin layers of gold and of aluminium. The effect of a single gold leaf could be detected easily, and each successive addition up to fifteen leaves gave an increase of ionization. The curve thus obtained was, however, more irregular than inequalities of distance between leaves and Ur-X, or than variations in the thickness of the leaves (obtained by weighing, measuring, and assuming the density) could account for. It was further noted that, with a steady electroscope and under almost the same weather conditions, measurements of ionization from the same group of leaves and made between the same hours of successive days gave decidedly smaller results; never larger ones. How far this may have been due to shutting the inner leaves partially away from the air remains to be determined. It is also probable that for such thin reflectors the results are complicated by secondary from the air above them.

The thinnest sheet of lead which could be got gave a large percentage of the effect obtainable from a thick plate. Therefore the first part of the reflection curve typical of elements of high atomic weight cannot be said to be based on accurate data. But it may be said that the addition of successive small thicknesses to a very thin reflecting layer increases the ionization fairly regularly.

It was conceivable that the reflected radiations from metals of nearly the same atomic weights might be differently absorbed by plates of the same material as one of the reflectors. Various combinations were tried, including Pb and Bi, Zn and Cu, Fe and Ni, and Al and Mg. The ratios of ionizations produced after partial absorption of the secondary by discs of various thicknesses did not differ markedly from those calculated for ionization when the secondary passed through only the aluminium leaf.

Throughout the preceding experiments the Ur-X was covered so as to cut out the $\alpha$-rays. This shield of course modified the radiation passing through it and consequently also the return radiation from the reflecting plate. With each plate at the same distance from the Ur-X, the ratio for lead to aluminium with the shield was 2.54; without it, 2.39. Curves were drawn showing the absorption by aluminium of the secondary from lead and from aluminium, the Ur-X being uncovered. The curve for lead was fairly normal until the secondary radiation had passed through about 0.08 mm. of the foil. Then there was a sudden decrease in the ionization, the curve afterward becoming normal again. A similar discontinuity, though much less marked, was observed in the curve for the secondary from aluminium when this passed through about 0.04 mm. of aluminium foil. Such sudden changes in the absorption curves may be accounted for by scattered $\alpha$-rays, or by a soft and fairly homogeneous secondary due to them.

In conclusion it may be said that the experimental results of this direct method of comparing the secondary radiations are in agreement with Schmidt's statement that the intensities of secondary radiation excited by $\beta$-rays of Ur-X are in the order of atomic weights for elementary substances. Although the order seems to be definite, the total ionization effects vary greatly with the arrangement of apparatus.

The reflection and absorption seem not to follow any simple law.

The curves bring out clearly characteristic differences for three typical elements when acting as reflectors and as absorbers.

Nothing in the nature of selective absorption was detected. This may be held to indicate that the secondary is largely returned or scattered primary, rather than independent, though characteristic, radiation emitted merely as a result of incident primary.

Questions as to the reflection from extremely thin plates, and as to the absorption of radiation from plates so thin as to give only a part of the total secondary which may be obtained from thick plates, are at present under examination.

Physical Laboratory,
Bryn Mawr College.

# AN AUTOMATIC CIRCULAR DIVIDING ENGINE.

By W. R. Stamper.

## With Calibration of the Worm and Gear.

By Elmer E. Hall.

HAVING felt the need of a reliable circular dividing engine, the department of physics of the University of California decided, some five years ago, to build such an instrument and it devolved upon the writer as mechanician in the department of physics to design and build the instrument. Being unable to get any literature on the subject of dividing engines either from libraries or from factories, after having constructed one of my own peculiar design I have decided to publish an account of its construction in detail, with drawings and photographs, so that those interested may derive some benefit, if possible, from my experience.

The gear wheel *A* (see sectional drawing, Fig. 1) weighs 150 pounds and was made by the Brown and Sharpe Manufacturing Co., of Providence, Rhode Island. The wheel is made of soft cast iron, suitably braced and designed. It is 24 inches in diameter, 1 ¼ inches thick at the teeth, with a taper hole in the hub. The hub is 4 inches long, with a two-inch hole through it, the taper being one half inch per foot. This hole is not the running bearing, the gear wheel being made rigid to the main spindle *D*, by means of the nut *B*, which is threaded on the top of the spindle. On the top face of the gear, near the edge, there is a rough, coarse scale cut in the cast iron and divided into degrees for convenience in starting, rough dividing and calibration. There are 360 teeth in the gear wheel, thus dividing the wheel into degrees, the teeth being of standard worm pitch.

The main spindle *D* is a piece of box-annealed machinery steel and was well seasoned before the final finishing was done. The spindle has a taper hole extending all the way through, which receives the taper spindle *J*, upon which all work to be divided is

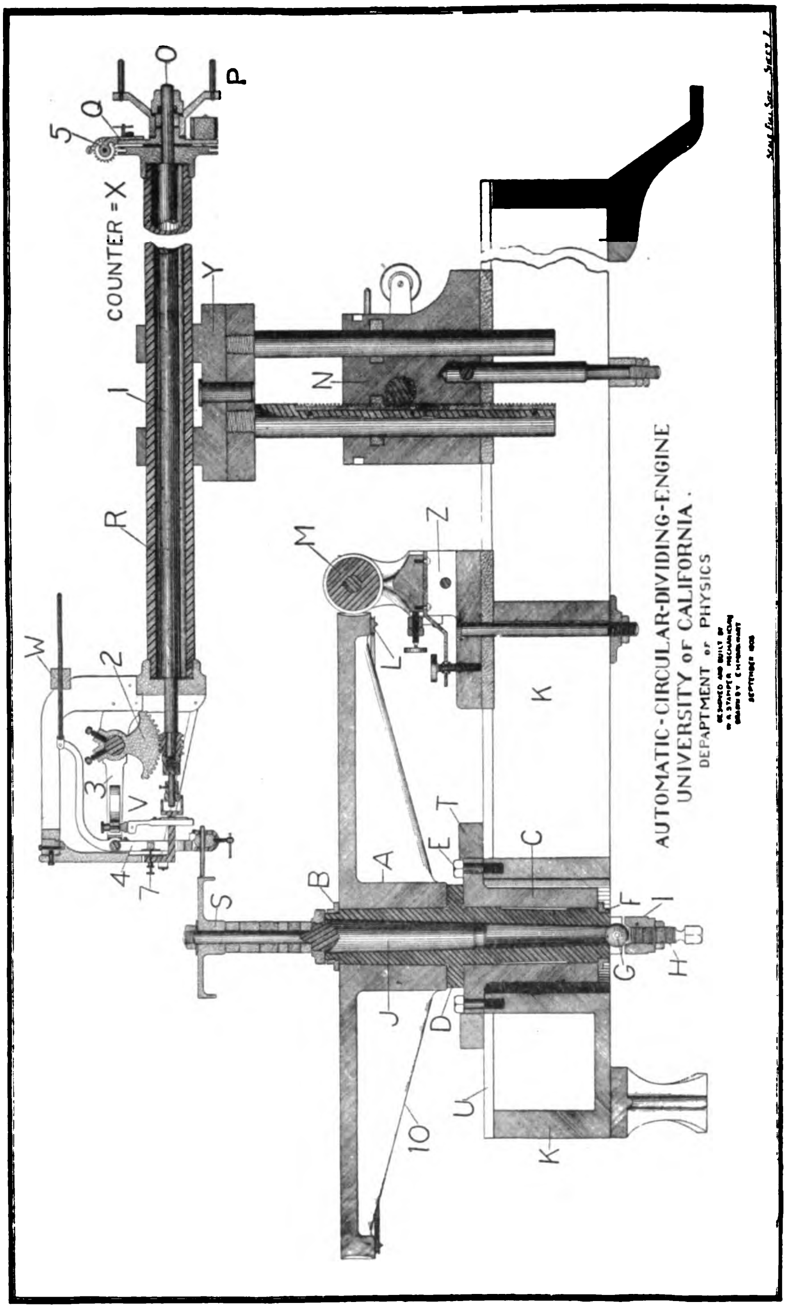

COUNTER = X
O
P
Q
5
Y
I
R
W
2
3
V
4
7
S
B
A
E
T
C
F
I
G
H
J
D
U
K
10
M
Z
L
K
N
AUTOMATIC-CIRCULAR-DIVIDING-ENGINE
UNIVERSITY of CALIFORNIA.
DEPARTMENT of PHYSICS

Fig. 1.

placed. This taper hole is 1¼ inches in diameter at the upper end and three fourths inch in diameter at the lower end, the taper being five eighths of an inch per foot. The main bearing of the spindle has a taper of one half inch per foot and is 5 inches long and 2½ inches in diameter at the large end. The spindle revolves in a cast-iron collar or bearing *C*, which was also treated so as not to change its form in the future. The flange of the collar or bearing *C* was scraped and carefully fitted on the bed *K* before the cap screws *EE* were screwed in place. I consider this bearing one of the most essential parts of the dividing engine, and I was extremely careful to secure a good fit. The lapping of the taper hole was done with the aid of Arkansas stone dust. The Arkansas stone dust, being readily dissolved in benzine, can be easily and thoroughly cleaned out when the abrasion is done. I am using the best watch oil for the lubrication, it being light and not gummy. A critic may say that this bearing should be much longer. Perhaps so, but I doubt if it would be any improvement in accuracy, provided the short bearing is accurately made. However, if the scope of the machinery had permitted, I would have made the bearing somewhat longer, but on the same general plan. To take the weight off the bearing I use, instead of the double-ended male center used by most of the factories, the one inch hardened steel ball *G*. If there be any eccentricity in the rotation, the double-ended male center rod would have a tendency to raise the gear, while the steel ball would not, there being a hardened surface on the end of the adjusting screw *H*, for the ball to rest on. This is, I believe, the better method.

The adjustment of the bearing is made, assuming that the bearing is in its working position free from dust and oiled with good watch oil, by applying a spring balance to the periphery of the gear wheel. I find it takes a pull of from five to six pounds weight to start the wheel. I then screw up the screw *H* until about 149 of the 150 pounds are lifted, allowing a free running bearing with a film of oil about one ten-thousandth of an inch thick, which is sufficient for lubricating and retaining a good working bearing. I then found that it took a pull of about 1½ to 2 pounds weight to start the gear wheel. After starting, it took practically nothing to keep it going.

However in actual work the intervals during which the wheel is moving are so short that it is only the starting torque that is important. Therefore the weight must be taken off the taper bearing in order that there may not be too much strain on the worm in driving the gear wheel, for much strain would mean inaccuracy. An added load of 145 pounds placed on the gear wheel did not appreciably change the pull required to start the wheel.

At first I applied what I called a balancing device, consisting of a cord which passed around the sheave *L*, underneath the gear wheel, and then to a pulley on the wall, from which it passed over a second pulley on the ceiling, extending downward to receive a weight nearly sufficient to supply the starting force of two pounds. It was found that this balancing device rendered the engine too sensitive to dust, and the unequal distribution of oil on the gear made the readings vary from 10 to 15 seconds. The tangential pull of two pounds which must be supplied by the worm seems to be better than reducing this pull by balancing, provided the friction head *O*, on the worm shaft, remains constant in its rotation.

The worm and shaft *M* were also made by the Brown and Sharpe Manufacturing Co., to the same degree of accuracy as the gear, and are mounted on a suitable bearing or support, *Z*. It has a thrust bearing, stops, and all required adjustments, and is provided with rachet and automatic friction head not shown in Fig. 1. In addition to the friction head on the driving shaft of the worm there is a second friction head *O*, shown in Fig. 3, on the shaft *P*, controlling the cutting device. These friction heads have wooden surfaces, endwise of the grain, rubbing against cast iron. The greatest care possible was taken to get constant friction. The wood used was teak, which furnishes its own lubrication, it being naturally an oily wood. The wear is taken up by a spiral spring as shown in the figure. The two friction heads are run from the same source, one friction head waiting while the other catches up, and vice versa.

The tail stock *N*, Fig. 1, is movable either way along the bed *K*, and is provided with tighteners similar to the ordinary lathe. The support *Y* is provided with a rack and pinion adjustment, by means of which it is adjustable up and down to suit the class of work. There are also tighteners provided to keep it in the required position after it has been moved. The hollow steel shaft *R* passes through the support *Y* and is also provided with tighteners, so that by the adjustments of the tail stock it is possible to divide a circle of any size up to five and one half feet in diameter. *X*, *8* and *10* show the positions, respectively, of the counter, stop and vibration support seen in Fig. 5.

The marking head *V*, which is rigidly attached to the end of the hollow shaft *R*, is operated by means of the shaft *1*, and by the rotation of the friction head which also propells the part *Q*. The slotted disc, shown in Fig. 2, on the piece *Q* strikes against stops placed at the required distance apart. On the piece *P*, Fig. 1, there are three stops, as seen in Fig. 2, long, medium and short. When a long line is made, the piece *Q* will rotate and allow the deep slot in the slotted disc to pass over the two shorter stops and stop at the longer one. Part *Q* is then rotated back until the star or escapement wheel *5* strikes a hook or stop, shown in Fig. 2, which rotates disc *5* one tooth, and also rotates the slotted disc, so that when *Q* is again rotated toward the stop it strikes a section of the slotted disc where there is no slot, and therefore makes a short line. This operation is continued, enabling one to produce graduations with any required lengths of lines.

The worm *9* is keyed to the shaft *1*, so that when shaft *1* is rotated, it rotates gear segment *2*, moving the arm *3*, lifting the part *4*, and thereby causing the knife or cutter *6* either to cut or return upon the periphery of the wheel *S*. The counterpoise *W* is adjustable so as to make the line as fine as desired. The screw *7* provides adjustment of the knife up or down.

The cutting head *V* is provided with cams, screws, lifts and all necessary adjustments, as may be seen from the several figures. In Fig. 1 the head is set to divide the periphery of the wheel *S*. In Fig 2 it is set to divide the top surface near the edge as an ordinary spectrometer scale is divided. It can be seen from the drawings that there are only a few parts to be interchanged in shifting from the top surface to the periphery, or the reverse. This head may seem complicated, but a study of the drawings will show that it is extremely simple.

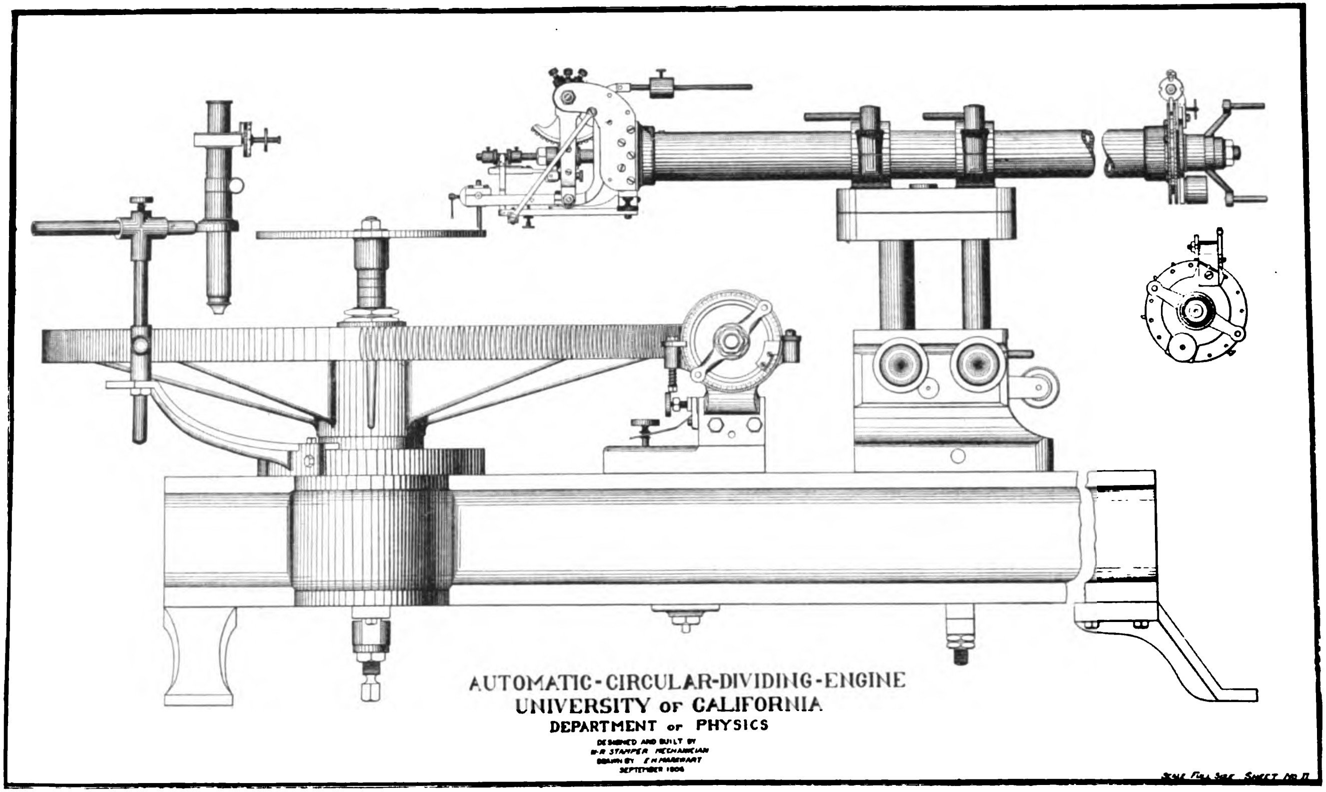

AUTOMATIC-CIRCULAR-DIVIDING-ENGINE
UNIVERSITY OF CALIFORNIA
DEPARTMENT OF PHYSICS
DESIGNED AND BUILT BY
W. R. STAMPER MECHANICIAN
DRAWN BY E. H. MARGWART
SEPTEMBER 1906
SCALE FULL SIZE SHEET NO. II

Fig. 2.

The dividing engine is driven by a ½-H.P. 104-volt self-starting A.C. Wagner motor, making 1,800 revolutions per minute. Owing to the fact that we were unable to get a slow running automatic self-starting A.C. motor it was necessary to gear down so that a motor speed of 1,800 revolutions per minute would make one line in about two seconds. That is why the apparently complicated device shown in Fig. 3 is used to run the engine. This automatic device is propelled by the motor by means of the steel worm *A*, Fig. 3, which engages worm wheel *B*, the motion being communicated by gearing to the change gear *C*, and then down the shaft to the gear box *J*, which is driven by the pawl *F*. When the arm *E* is in an upward position, the tapered cone *V*, which is feathered on the shaft *P*, slides downward forcing the driving arm or pawl *F* to be engaged in the toothed head *G*, turning the shell *J*, and also the shaft *P*, as *F* is rotating The rotating switch *W* changes the current from one of the electromagnets *D* to the other. As the pawl *F* rotates, a position is reached when the rotating switch shifts the current to the lower magnet, thus pulling the reversing arm *E* downward, engaging the arm *I* in the toothed band *H*, which is rigidly around the shell *J*, and releasing the pawl *F*. This forces the shaft to rotate in the opposite direction. The switch *W* can be changed to reverse the motion at any number of revolutions, or a part of a revolution, as desired. The shell or gear box *J* contains four bevelled gears, two tight ones and two loose ones, the loose ones being in a horizontal position and not fastened to their shaft. They only act as intermediates and transmit the power to the other two which are fastened to the shaft. When pawl *F* is engaged the whole case *J* is revolved and these gears are not turning, but are revolving with *J*. When arm *I* is engaged, and pawl *F* disengaged, the case *J* stands still and the gearing rotates. By this means I obtain the reversing movement which enables the dividing engine to give any kind of reverse movement needed.

When the dividing engine is ready to start, the operator throws the main switch *T*, which makes all electrical connections, including the direct current to run the electromagnets. Closing the switch *T* pulls up the rod *U*, allowing it to slip into the collar *X*, the thumb screw *Z* being released. The motor and the reversing parts are now running but the engine is not. Wishing to start the engine, the operator raises the clutch arm *L*, at a certain stroke, forward or backward, allowing the trigger *M* to be engaged or cocked, the clutch being now in and the engine running. The thumb screw *Z* is now tightened, making the device ready for the automatic stopping. Having cut the required number of graduations on the scale, the counter or register, automatically makes electrical connection, exciting the electromagnet *R*, Fig. 3, thereby disengaging trigger *Q* and opening the main switch *T*. At the same time the clutch *O* is thrown out by the lever *L* and the entire apparatus is stopped. The opening of the clutch *O* stops the engine immediately while the motor may make a few revolutions, but all electrical connections are broken, and the knife *6* on the dividing head is left in a raised position. The writer intends, when time permits, to construct another automatic device to be run by compressed air, which will do away with many of the parts and eliminate the noisy hum of the motor.

The counter has an important and extremely necessary part to play. Every time a line is made the counter registers one, enabling the operator to know just how many lines have been made. The setting of the counter is done by subtracting the number of lines to be cut from the limit of the counter. The limit of the counter used is 10,000 so if it is desired to cut 360 lines the counter would be set at 9,640, and when zero comes in sight the machine stops automatically and the scale is cut.

The Brown and Sharpe Manufacturing Co. deserve great praise for their interest and their workmanship on the worm and gear of this machine. They stated in a letter to the department of physics that they could cut a gear that would be accurate to within five seconds of arc and it would appear from the calibration described below that, after proper lapping has been done, their statement will have been fulfilled.

In conclusion I wish to thank Professor Slate and the other members of the department of physics for their kind interest and valuable suggestions during the progress of the work.

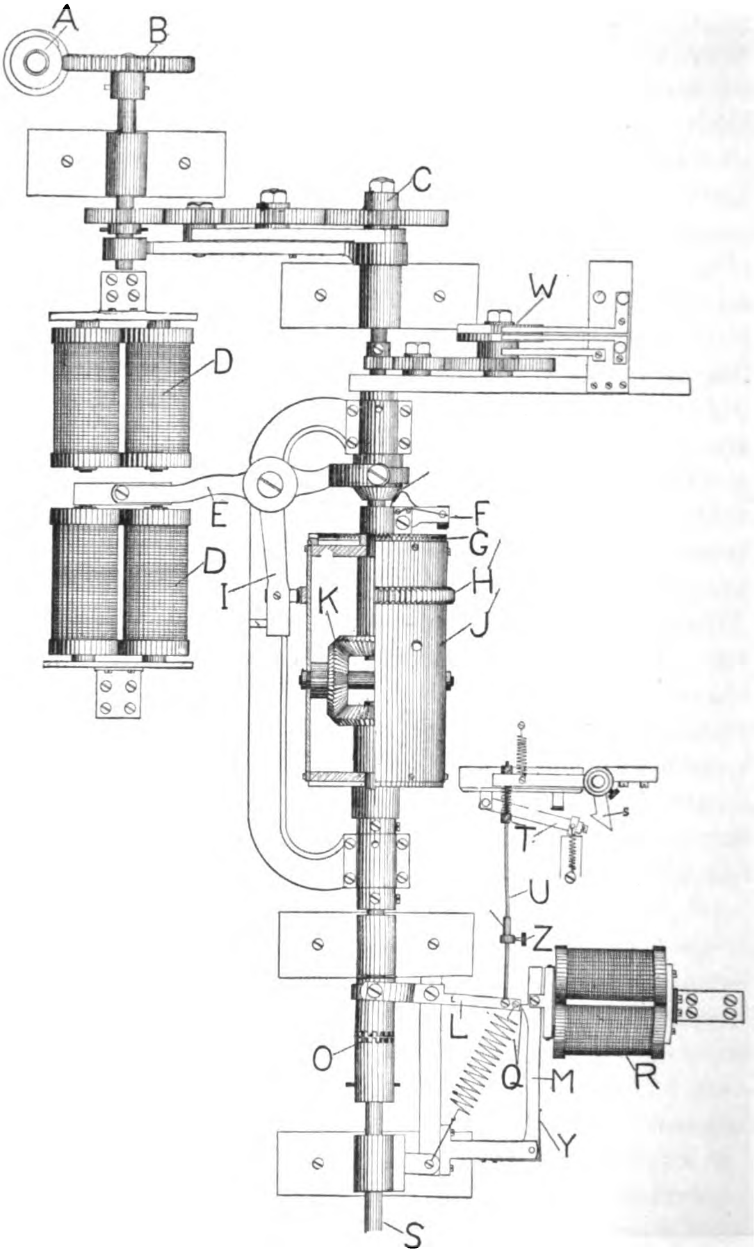


Fig. 3.

## CALIBRATION.

After the automatic dividing engine, described above, had been completed, and a few circles divided, it seemed desirable to calibrate the instrument, not merely to ascertain the errors of graduation but more especially as an aid in reducing these to a minimum by the further lapping of the gear wheel. The details of the calibration were carried out by Mr. T. A. Davidson, with the aid of Mr. Stamper and under the direction of the writer.

The method used was an optical one, a mirror being mounted on the axis of the main gear wheel in which the image of a fixed scale was observed through a fixed telescope. The scale was strongly illuminated, and placed about 20 meters from the dividing engine. The telescope system, placed close to the engine, consisted of an Edelmann telescope with an objective of 50 cm. focal length, the eye-piece of the telescope being replaced by a compound microscope. This arrangement gave clear definition and good magnification, enabling the scale to be read to 0.1 mm., and the aberration errors were negligible. The mirror was of speculum metal, finished by Brashear, and mounted in a wooden clamp which fitted into a brass collar attached to the shaft of the gear wheel. The mirror could thus be rotated around the shaft but would remain in the same position relative to the gear wheel during the taking of a reading. A small rod inserted horizontally in the clamp could be operated by means of a lever, provided with stops, thus enabling the observer to rotate the mirror back to the zero position without leaving his station at the telescope. The lever was arranged so as to move out of the way during the taking of a reading.

During the preliminary work several minor defects and peculiarities were noted and their disturbing effects in large part remedied. For example, the gear wheel was found to be very slightly eccentric. The introduction of a strip of platinum foil on one side between shaft and gear wheel caused a shifting of about one four-thousandth of an inch and almost completely remedied the defect. At first about 80 per cent. of the force moment necessary to start the gear wheel was supplied by an external weight system, the balance of the force moment being supplied by the worm. Consistent readings could not be obtained, owing to the unequal thickness of the oil film on different parts of the gear and on the same part for succeeding revolutions. After a fresh application of oil successive readings might differ by as much as 15 seconds. Minute dust particles floating in the air and lodging on the gear or worm also caused variations of about the same magnitude. The suspended weight system was removed, allowing the worm to supply the entire force moment necessary to start the gear, and it was found that the force required was sufficient to reduce the film of oil between the bearing parts of the worm and gear to a uniform thickness as was evidenced by the consistent readings obtained. The oil used on the gear was "three in one," and, after the fresh application of oil, the gear was turned by the worm for several revolutions to secure as uniform a distribution of oil as possible. In actual work the engine with the circle which is being cut is protected by a glass covering case, thus reducing dangers from dust to a minimum. It was found moreover that the dust seemed to squeeze out from between the bearing surfaces with the oil, thus giving little trouble. Minute particles of lint floating in the air are more troublesome, since they neither crush or squeeze out with the oil. The tangential push exerted by the worm upon the gear in starting is about two pounds weight. Changes in temperature due to the presence of the operator and the small changes in temperature which occur from day to day produce only a negligible effect.

In the preparation of the calibration curve shown in Fig. 6 the initial readings were taken with two telescope systems, similar to the one described, placed 90° apart. First the reading of the scale was taken in one telescope, then the gear and mirror were rotated 45° and the reading taken in the second telescope. This was repeated several times around the circle, the average reading for each point being taken. It was by means of the eight points of the curve thus determined that the eccentric error of the gear was located and corrected. One telescope was now moved to a position 30° from the other telescope, which remained fixed throughout the entire calibration, and the two 15° points between each of the 45° points were located. The second telescope was again moved to a position 10° from the fixed telescope and the 5° points carefully located. The 1° points were then located using only one telescope.

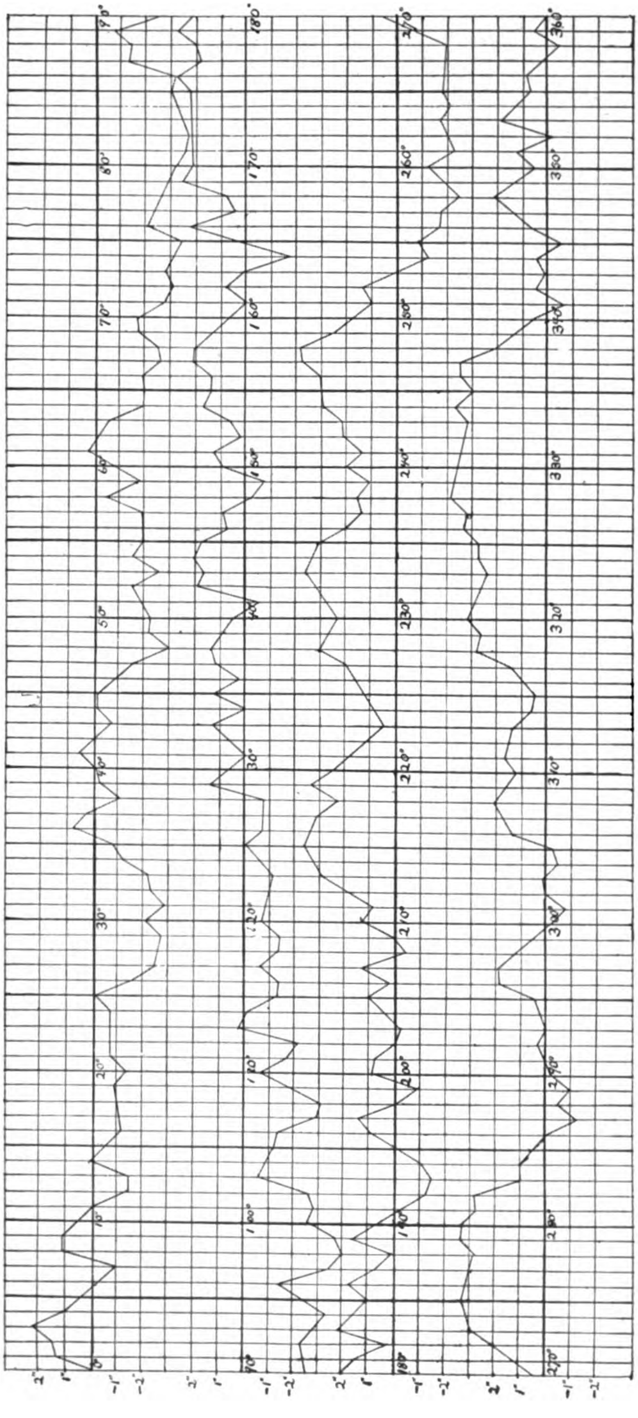

0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
120°
130°
140°
150°
160°
170°
180°
190°
200°
210°
220°
230°
240°
250°
260°
270°
280°
290°
300°
310°
320°
330°
340°
350°
360°
2"
1"
−1"
−2"

**Fig. 4.**

In the curve of Fig. 4, abscissas denote degrees corresponding to divisions on the top face of the gear wheel and the ordinates give errors in seconds of arc. The maximum error is seen to be about 8 seconds. In graduating a circle, fractions of a degree are obtained by means of a ratchet wheel. A ratchet wheel having $\frac{1}{6}$° notches was used and the calibration curve found to agree, within observational errors, with the curve obtained for the degree divisions shown in Fig. 5. In addition to the errors due to the worm and gear there appeared the periodic errors due to the ratchet. The calibration curve for this particular ratchet wheel when in actual use is given in Fig. 5. The error is something less than one second. When using this ratchet the complete calibration curve would be given by adding the curve of Fig. 5 to each degree of the curve of Fig. 6, the two curves being reduced, of course, to the same scale. This ratchet wheel was first cut on a milling machine, the finishing being done by trial so that when in actual use with the worm and gear a degree would be divided closely into six equal parts. No circle has been cut since the errors detected by the calibration have been corrected, hence it is not known whether or not the calibration curve for a circle divided on the machine agrees with the calibration curve of the worm and gear.

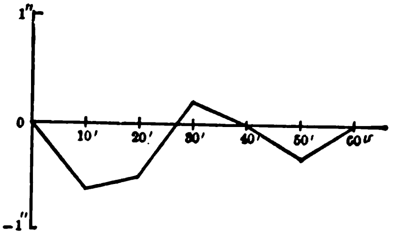


Fig. 5.

Readings for the $\frac{1}{6}$° and for the 1° were also made when the machine was running automatically. The readings differ slightly from those made by hand, being on the whole more constant. This arises from the fact that with the hand turning, the force with which the wheel strikes the stop varies somewhat, while with the automatic turning the friction heads maintain uniform action.

When furnishing the worm and gear the Brown and Sharpe Manufacturing Co. advised the department of physics to continue the lapping for at least three days using an abrasive material which they supplied. They assured the department that if this were done the error would not exceed five seconds of arc. Very little lapping has as yet been done. It is now proposed to lay off a calibration curve to proper scale on the face of the gear wheel, then during the process of lapping to increase the friction at the divisions denoted by the high places in the curve, the worm and gear being run by a motor. Arkansas stone dust will be used as abrasive material for this final lapping. It is hoped that by this systematic lapping the error may be reduced to at least one half of its present value.

University of California,
Department of Physics,
July 10, 1909.

# COEFFICIENT OF EXPANSION OF NICKEL NEAR ITS CRITICAL TEMPERATURE.

By Walter F. Colby.

THE coefficient of expansion of nickel at ordinary temperatures was investigated by Fizeau in 1869. He used an interference method in which the expansion of the object was measured by the movement of a set of interference fringes. His method[1] of determining this displacement made possible the measurement of an expansion in the object under examination, of .02 wave-length of the light used. The delicacy of measurement was such that the coefficient of expansion of objects of but 1 cm. length could be determined with a high degree of accuracy. The improvements which have been made upon the method of Fizeau, notably by Benoit,[2] Abbe[3] and Pulfrich,[4] have been largely in the direction of increasing the ease and accuracy with which the displacement of the interference fringes can be observed and measured. In the Pulfrich interferometer, the instrument used in this investigation, we have undoubtedly the highest type of apparatus used for this purpose. In certain other methods which have been employed it has been necessary to use specimens 50 cm. long to reduce the percentage of error in measuring changes of length due to expansion. This greatly increases the difficulty in establishing and maintaining a uniform temperature, and in consequence only values of the mean coefficient over long ranges of temperature have been possible. In the present study it was desired so to refine the temperature measurements that ranges of five degrees might be used through a special region, *i. e.*, that of the critical temperature. The Pulfrich interferometer is extremely well adapted to this, allowing, as it does, high accuracy of measurements with small specimens.

---

[1] Ann. d. Chemie de Phys., 2, p. 143, 1864; 8, p. 335, 1866.

[2] Trav. et Mem. du Bureau Int. Poids et Mes., I., 1181; VI., 1888.

[3] Wied. Ann., 38, p. 453, 1889.

[4] Zeitschr. für Instr., 1893, pp. 365, 401, 435.

Le Chatelier in 1888 published values of this coefficient for high temperatures, but not until recently were any precise measurements made in the critical region.

In 1901, Holborn and Day[1] studied the expansion of metal rods, 50 cm. long in an electrically heated porcelain tube. Readings were taken on scratches near the ends of the rods and the temperatures measured by means of a thermal couple placed at the middle. The results below 300° were expressed as a quadratic function of the temperature. Above 300° this function no longer held, the linear term becoming smaller and the quadratic larger. The temperature throughout the bar was far from uniform, and the method therefore could throw no light on the nature of the change at 300°.

Harrison[2] in 1904 made a similar investigation, using a nickel wire 60 cm. long, kept under tension by an adjustable spring. Lengths were measured by telescopes 10 cm. apart at the mid portion of the wire. He found an anomalous expansion at 370°, the critical temperature of his specimen. Between this temperature and 380° the wire experienced an elongation of about .05 per cent., whereas in the preceding ten degrees the elongation had been only .002 per cent. Above 380° the rate became .0019 per cent. per 10°. The discrepancy between these results and those of Holborn and Day may be explained by assuming that the elongation of the wire, due to the decrease of the elastic constant with rising temperature[3] increased the absolute value of the coefficient. Temperatures were in this instance measured by the resistance of 20 cm. of the nickel wire itself, the calibration of which is not described in the article. The temperature recorded was therefore the mean throughout this length. The wire was placed centrally in a groove cut in a mahogany board and covered with a strip of wood with mica windows. Losses by unequal radiation and conduction through the leads were evidently considered too great for uniformity of temperature as results were computed only for fifty degree ranges.

Randall[4] in a recent paper has described his work in this field. He worked with a solid nickel cylinder 1 cm. long and 2 cm. in

[1] Ann. d. Phys., 4, 104.

[2] Phil. Mag., June, 1904.

[3] Pogg. Ann., 145, p. 147, 1872.

[4] PHYS. REV., February, 1905.

diameter, employing an interference method and using the Pulfrich interferometer to measure the displacement of the fringe system. Temperature measurements were made with mercury thermometers, and although uniformity of temperature seemed certain, stem corrections may easily have introduced errors of such magnitude that measurement of temperature differences for intervals of less than twenty degrees became doubtful. With small ranges a slight constant error in each of the two temperatures, whose difference constituted the temperature interval, would not have materially altered the results. These stem corrections however were dependent on the temperature gradient of the oven packing, which in turn depended on the thermal history both of the oven and the room. Thus no error could be considered constant. Even these small ranges of twenty degrees may mask an anomaly. This specimen lost its magnetism at about 310°. The results showed no such anomaly at the critical point as those of Harrison. The coefficient increased steadily till about 275° and from there to 400° remained almost constant.

The present work grew out of this disagreement and takes as its problem the further refinement of temperature measurements and the investigation of other specimens of nickel. Three specimens have been used, which for convenience will be designated by the letters A, B and C. The work was begun in 1907 and preliminary tests made with specimen A. These were repeated in 1908 with slight modifications, and the same tests applied to B and C.

## MAGNETIC MEASUREMENTS.

The first part of the problem consisted in finding definitely the temperature region in which magnetizability disappeared for the different specimens. A and B, being annular in shape, were wound ringwise with a primary and secondary of copper wire insulated with asbestos paste. A thermal junction of iron constantan encircled the nickel in direct contact with it, and was firmly fixed by the insulation. The whole was placed in an electric oven having an inner chamber of porcelain surrounded by 10 cm. of mineral wool. A current of one ampere was passed through the primary and the effect of its reversal noted by a ballistic galvanometer in series with the secondary. Temperatures were measured by means of a Siemens and Halske potentiometer after the design of Lindeck and Rothe.[1] The preliminary tests were made first with temperature steadily rising to points somewhat above critical temperature and then steadily falling to room temperature, to ascertain the type of the curve. This excluded possibilities of overlooking an abrupt anomaly which might easily fall between two stationary temperatures. It gave, of course, no truly corresponding values of temperature and deflection, and, plotting these quantities as coördinates, the resulting curves for heating and cooling naturally included a considerable area, since the change of temperature of the interior of the nickel lagged behind that of the thermal junction. In the final tests each temperature was maintained from twenty to thirty minutes, allowing, it was thought, ample time for such thermal equilibrium as this test demanded. If the change of temperature was not great, the galvanometer deflection gradually increased during the first five minutes of the new temperature but was afterwards constant. Even in the steep portions of the curve the deflections gave no evidence of an unstable state. Although the included area was here greatly reduced, it did not disappear, showing a slight temperature hysteresis.

Fig. 3 (p. 518) shows the results obtained with specimen A. The loss of magnetic properties occurred here in the region between 370° and 380° and at temperatures just below 370° a decided rise in permeability was found both on heating and cooling. No such maximum occurred in the curve for specimen B (Fig. 4) although the method and strength of field were the same. Here the phenomenon took place at a very much lower temperature, between 280° and 310°, showing a marked difference in purity or in structure of the nickel. Specimen C was a cylinder 8 mm. in length and 2 cm. in diameter. The ballistic method was again used but the cylinder was placed inside a glass tube on which a primary and secondary were wound for a distance of about 4 cm. The sensitiveness was so much reduced that it was necessary to use a current of six amperes to obtain reliable results. Except in magnitude, the curve (Fig. 5) did not differ from that of specimen B, the region centering about 312°.

---

[1] Zeitschr. für Instr., 20, p. 285, 1900.

## Optical System.

Since the method used in this investigation for measuring the changes in length caused by expansion, has been fully described,[1] it will be necessary only to outline it briefly and to denote adaptations to this problem. It consists fundamentally in the production of interference fringes by two plane reflecting surfaces, opposite and slightly inclined to one another, whose distance apart changes with the length of the specimen. The bands are viewed and their displacement measured by means of a Pulfrich interferometer (Fig. 1). Light from the source of illumination is focused on a totally reflecting prism *p*, placed at the focus of the objective *F*. Approximately parallel rays emerge from the objective and are reflected downwards by another totally reflecting prism at the end of the interferometer tube. These rays are reflected from the optical surfaces, reënter the objective and after passing the free half of the telescope at prism *p*, and a second lens *C*, are brought to a focus at an adjustable horizontal slit *D*, immediately in front of two vertical cross hairs. Adjustment of lenses *C* and *E* brings the cross hairs and the images of the fringes in sharp outline. These are viewed through the direct vision spectroscope *G* and the horizontal images of the slit thus appear in the spectral colors. The images of the interference bands are visible in each of the slit images and can be made vertical by rotating the Dove's prism *H*. A micrometer attachment makes it possible to move the cross hairs parallel to themselves and thus to measure the distance of any interference band from a fiducial mark in the field.

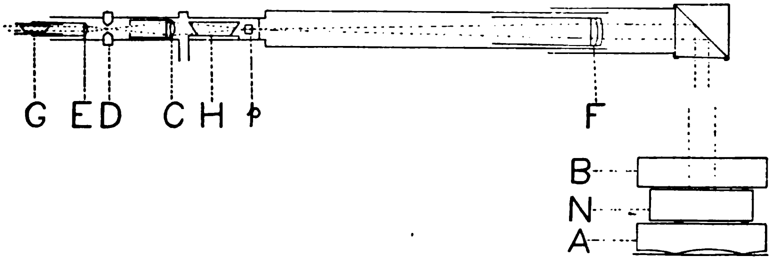


Fig. 1.

---

[1] Pulfrich, Zeitschr. für Instr., 1893, pp. 365, 401, 435. Randall, Phys. Rev., February, 1905.

Specimen A was in the form of a ring 8.2 mm. long, 5 cm. internal diameter and 4 cm. external diameter. Specimen B, also a ring, was 2.3 cm. long, 2 cm. internal diameter and 3 cm. external diameter. Both ends of these rings had been ground away so as to form three equidistant feet. The optical system for these tests was formed by placing the nickel ring *N* on a quartz bed plate *A*, with upper surface plane polished and lower surface ground that it might give no disturbing reflections. The ring was surmounted by a similar plate *B*, both sides of which were plane polished. On the lower side was etched a small circle which was used as a point of reference in the measurements. To avoid reflections from the upper surface this cover plate was ground slightly wedge-shaped. For the preliminary tests this lower surface of the upper plate and the upper surface of the lower plate were used as reflecting surfaces, but their distance was so great that the clearness of the bands was not sufficient for the best results. For the final tests, accordingly, this air layer was reduced to a thickness of approximately 1 mm. by placing a fused quartz cylinder, with upper surface plane polished and lower surface ground so as to form three feet, on the bed plate *A* (Fig. 1) inside the nickel ring *N*. The coefficient of expansion of the quartz is so small in comparison with that of the nickel that when added to it it does not affect the first two significant figures. Any error in the value of the quartz coefficient or in the length of the quartz specimen would therefore appear outside the limits of error of the nickel. The increased distinctness of the bands doubtless more than compensated for the error introduced by this new material. Specimen *C*, being a circular cylinder, 1 cm. long and 2 cm. in diameter, was ground similarly to the quartz cylinder just described and placed inside a fused quartz ring. Since the surface of the nickel did not preserve its polish at high temperatures, it was surmounted by a thin fused quartz disk. This arrangement left an air layer of 0.2 mm. between the bottom of the cover plate and the upper surface of the quartz disk, producing a very satisfactory fringe system.

## ILLUMINATION.

Illumination was produced by an Arons mercury lamp. The light was very brilliant and steady, and for the thin air layers used in these tests, the green ($\lambda = 546\ \mu\mu$) and two yellow bands ($\lambda_1 = 578.8\ \mu\mu$, $\lambda_2 = 576.8\ \mu\mu$) were sharply defined. These two yellow lines of the spectrum were not competely separated by the direct vision spectroscope of the interferometer but overlapped for a considerable part of their width. Consequently the dark interference bands which crossed them transversely were at times so completely superposed that settings were made on their common dark portion and the mean wave-length used in computations. Later in the same series the relative shift of these dark bands made it necessary to use the "upper yellow" alone.

## THE OVEN.

The oven (Fig. 2) consisted of a vertical cylindrical iron frame formed by two rings 15 cm. in diameter connected by rods 35 cm. long. This, being well insulated with asbestos, was wound lengthwise with german silver wire *R* and surrounded by packing 8 cm. thick of alternate layers of asbestos and infusorial earth. This shell rested on a thick pad of asbestos through the center of which rose two porcelain rods, supports for the optical system. The top consisted likewise of an asbestos pad resting on a porcelain lid. An asbestos chimney *A*, fitted transversely with four squares of plate glass, penetrated the cover. All joints were closely packed with infusorial earth. The whole rested on a platform supported from the asphalt floor by steel rods clamped to the brick pier. The porcelain rods *P*, entering from below, supported a cylindrical brass chamber *C*, coated with asbestos and fitted at the top with a glass window. A brass disk *D*, resting on three leveling screws in the bottom, formed a base for the optical system. Two platinum thermometers *T* were firmly attached to the porcelain rods. The bulb of one was introduced into the inner chamber and the other placed in the air chamber outside in the same horizontal plane as the optical system. Since the heating coil was suspended in the outer chamber and free to air currents, it seemed evident that the temperature of that chamber would be almost immediately affected by any adjustment of the heating current, whereas the inner chamber because of its thermal capacity and asbestos coating would lag behind those changes and tend to remain constant, especially if the fluctuations in the temperature of the surrounding air were somewhat rapid. The winding of the heating coil was such that it produced very slight effect on the magnetic field at the center of the oven. A part of the heating current, however, was passed through a coil wound about the outside of the oven and so adjusted that no meas-

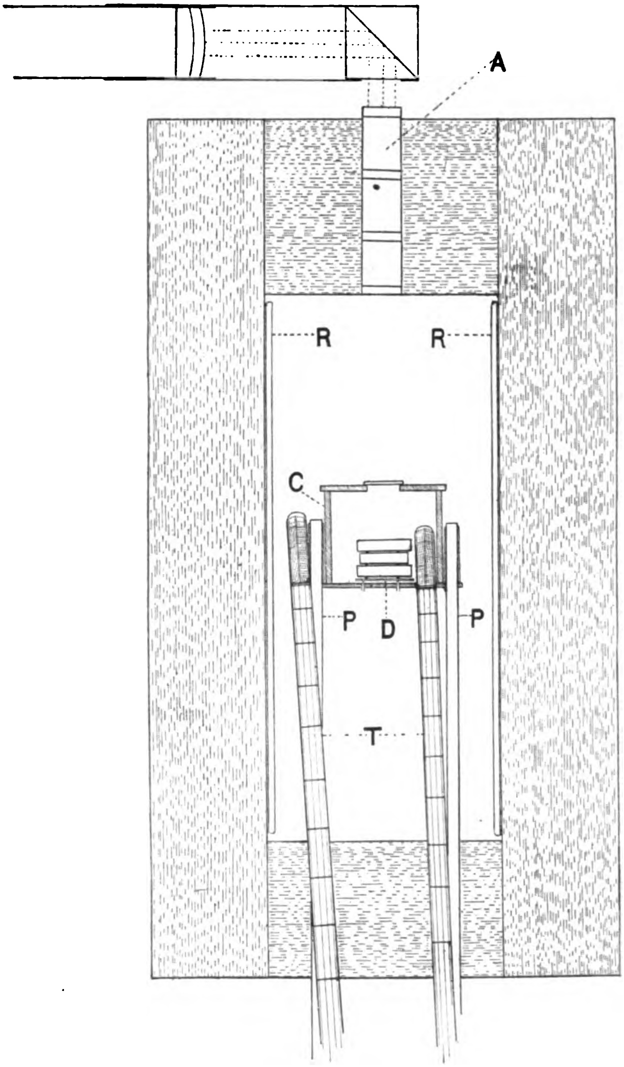


Fig. 2.

urable magnetic effect could be detected in the vicinity of the specimen.

## THERMOMETERS.

The two thermometers were made by winding on mica vanes, fine platinum wires and gold soldering the terminals to heavy platinum leads, according to the description given by Callendar.[1] To eliminate the resistance of these leads a loop of similar wire identical in length was mounted close and parallel to them. The whole was fitted in a glass tube with the bulb end sealed. Circular mica disks through which the four leads passed were fitted at frequent intervals in the stem both to hold the leads in position and prevent air currents. The resistances of the thermometers were measured by means of a Carey-Foster slide wire bridge, the thermometer being used as an extension of the wire on one side and a resistance box and the dummy leads on the other. The lead resistance was thus automatically eliminated. The box and bridge wire were calibrated to six decimal places by comparison with standard resistances in the laboratory. One millimeter of the wire had a resistance of .000718 ohm and readings could be estimated to tenths of millimeters. The thermometers were calibrated by measuring their resistances in ice, steam, naphthaline vapor and sulphur vapor baths. The results were expressed by means of the parabolic formula given by Callendar, viz.,

$$t = 100\frac{R_t - R_0}{R_{100} - R_0} + d\left(\frac{t}{100} - 1\right)\frac{t}{100}.$$

$R_0$ for thermometer I., 39.59842 ohms.
$d$ " " I., 1.554
$R$ " " II., 40.82634
$d$ " " II., 1.568

These values give a change of approximately one ohm for each ten degrees or .007° for each millimeter on the bridge wire. The galvanometer used was of such sensibility that the current passing through the bridge could be made small enough to eliminate heating effect in the thermometer. To guard against thermal effects

---

[1] Phil. Mag., 47, 191.

the wire and contact key of the bridge were of manganin, as were all resistance coils so that the entire circuit, with the exception of the platinum thermometers, was composed of manganin and copper. The junctions of the platinum leads with the copper leads connecting the thermometer with the bridge, were placed near one another to insure, if possible, the same temperature for all the junctions. These measures seem to have been sufficiently effectual, since no disturbing irregularities were experienced during the progress of the work.

### SPHEROMETER.

A spherometer was used to measure the length of the rings and cylinders. It was made by the Geneva Society and read directly to .001 mm.

### METHOD OF PROCEDURE.

To produce satisfactory fringes the lower surface of the cover plate and the upper surface of the bed plate must be inclined to one another at the proper angle. This angle was found by trial. The nickel rings were so adjusted by careful grindings of their feet, that, when placed between the cover and bed plates, fringes of the desired width were obtained. A fringe width of approximately one hundred scale divisions, as measured by the micrometer adjustment at the eye piece of the interferometer, was found to give most satisfactory results. It was feared there might be some irregularities due to air films or particles of dust between the points of contact of plates and rings. After the optical system had once been heated to 500° the band width was practically constant and without tilt, and on examination it even required some pressure of the fingers to separate the parts; moreover, while the application of pressure to the cover plate before heating produced a noticeable movement of the fringes, after being heated to 500°, a weight of 100 to 200 gms. on the cover plate produced no measurable effect. The oven was therefore heated to 500° before beginning each series, that this apparently more permanent condition might be attained.

The optical arrangements being thus in order, the oven was raised to the desired temperature by frequent adjustment of the heating current. To accomplish this the bridge was connected with the thermometer in the outer chamber and set at the desired reading, the galvanometer being kept at the zero point by manipulation of a variable resistance in the heating circuit. For the first reading this was continued from three to five hours, for the later readings from two to three hours. Maximum variation in the outer chamber was .04°, in the inner chamber about .01°. The temperature of the inner chamber was recorded both before and after and sometimes in the midst of the reading taken on the fringe system with the interferometer. The temperature of the outer chamber was then changed and the number of bands passing the reference circle noted. Readings were taken on the green and yellow bands and Abbe's[1] computation used as a check on the number of bands counted during the shift. If $\delta_2$ be the distance from the zero point to the nearest band in fractions of a band width at the higher temperature, $\delta_1$ a similar quantity at the lower temperature and $M$ the whole number of bands passed, then the apparent shift $t_1 = M + \delta_2 - \delta_1$.

At each interferometer reading a barometer reading was also taken and Pulfrich's[2] correction applied for change of wave-length in the air layer due to change in density. This correction is

$$K = d(t_2 - t_1)\frac{b_1}{760}\cdot\frac{1}{1 + at_1}\,\frac{1}{1 + at_2}\left[\frac{2(N - 1)a}{\lambda}\right]$$

$$- d(b_2 - b_1)\frac{1}{1 + at_2}\left[2\,\frac{N - 1}{\lambda}\,\frac{1}{760}\right].$$

Where $d$ is the thickness of air layer, $t_2$ the higher temperature, $t_1$ the lower temperature, $b_2$ and $b_1$ the corresponding barometric readings, $a$ the coefficient of expansion of air and $n$ the index of refraction of air under standard conditions. $K$ is here expressed in terms of band widths. The first term, a correction for the increased temperature, is always positive. The second, a correction for change in barometric pressure, is negative when $(b_2 - b_1)$ is positive, that is, when the greater pressure occurs at the higher temperature. If the apparent shift $t_1$ be taken as positive for increasing and negative for decreasing thickness of air film, the absolute value of true shift $f = f_1 + k$. The change of length is $f \cdot \lambda/2$.

---

[1] Wied. Ann., 38, 1889, p. 473.

[2] Zeitschr. für Instr., p. 437, 1893.

## DEGREE OF ACCURACY.

The formula expressing the value of the coefficient of expansion is:

$$a = \frac{f\frac{\lambda}{2}}{L(t_2 - t_1)} + \frac{la'}{L}.$$

Where $l$ and $a'$ denote respectively the length and coefficient of expansion of the fused quartz cylinder or ring and $L$ the length of the nickel specimen. The value of $a'$ was taken from recent, unpublished work of Randall's in which he expresses the true coefficient by the parabolic formula:

$$a \cdot 10^8 = 41.94 + .156\theta - .000225\theta^2,$$

giving an almost constant value through this region of $68 \times 10^{-8}$.

The value of $f$, the number of displaced fringes, is dependent on two measurements each of which can be determined within a probable error of .01 band, giving a probable error for $f$ of ± .015. The temperature interval $(t_2 - t_1)$ is likewise dependent on two measurements. The sensitiveness of the bridge made readings of the platinum thermometers definite to half millimeters on the bridge wire or .0035°. Variations in these readings, however, might be due to two causes, actual variation in the inner chamber and irregularities of the bridge. Two or three readings were taken for each determination of the band position, these determinations occupying approximately fifteen minutes. These temperature readings were usually very much in accord, and in fully seven eighths of the observations did not vary more than .01°. In a few cases, irregularities in the heating current which could not be properly compensated during the band determination, caused fluctuations in the inner chamber as great as .07°. For the greatest of these variations, the temperature was maintained an hour longer and the reading repeated. The probable error for the majority of the temperature readings was computed from the sets of temperature readings taken during a determination of the band position and found to be ± .0075°, giving for $(t_2 - t_1)$ a probable error of ± .010°. The length of the nickel $L$ may be determined within a mean error of ± .001 mm.

For specimen A the shift for a change of temperature of ten degrees was about five bands. The error in $\alpha$ due to $f$ for this interval amounts to $4.9 \times 10^{-8}$; due to error in $L$, $.2 \times 10^{-8}$; due to error in $(t_2 - t_1)$, $1.6 \times 10^{-8}$; giving a probable error in $\alpha$ due to the three quantities of $\pm 10.2 \times 10^{-8}$. Similarly for specimen B. Its length was 23.01 mm. and the shift for ten degrees, 14 bands. The error in $\alpha$ for 10° intervals was $\pm 2.4 \times 10^{-8}$, for 5° intervals $\pm 4.8 \times 10^{-8}$. Specimen C was 7.88 mm. long and gave for 10° a shift of 5 bands. 10° intervals, accordingly, gave results within a probable error of $\pm 5.45 \times 10^{-8}$ and 5° intervals $\pm 10.9 \times 10^{-8}$.

In series II. of specimen A, two successive values of $\alpha$ lie considerably outside this probable error. It was not possible to trace the cause of this but since the first is too high and the second too low, it does not seem unreasonable that a mistake was made in reading the mid temperature on which they both depended. It was possible to pass a consistent curve through the majority of the values of $\alpha$. In fact only six of the results were distant from these curves by an amount exceeding their respective mean errors.

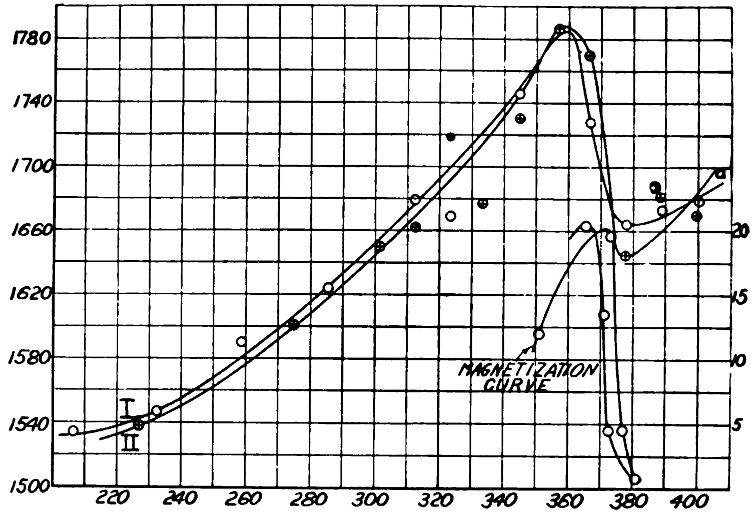


Fig. 3. Specimen A.

The results for the three specimens are shown in Figures 3, 4, and 5. During the first series, with specimen C, corrosion of the bridge wire so reduced the sensitiveness of the temperature measurements that all results for less than 20-degree intervals were discarded. A thorough cleaning of the bridge restored accuracy for series II. As had been found in the preliminary tests, series II. could not be combined with series I., the value of the coefficient being dependent on the thermal history of the specimen. The values of the coefficient have been plotted as ordinates, with mean temperatures as abscissas, and the resulting curves show a striking similarity for the three specimens.

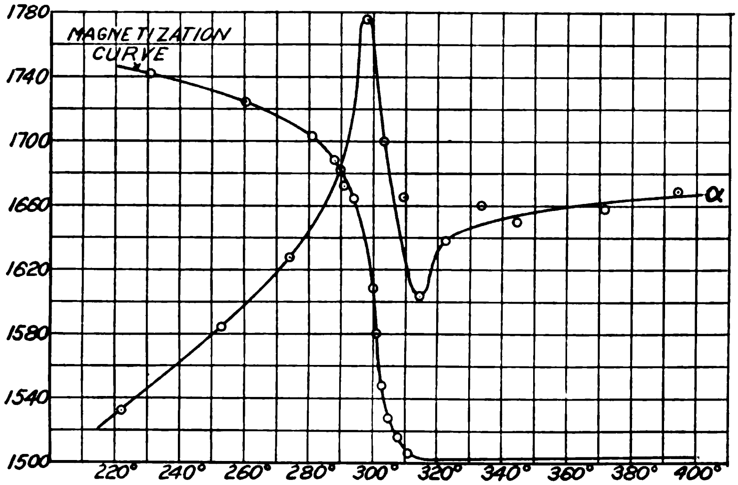


Fig. 4. Specimen B.

Specimen A is electrolytically prepared, and although a quantitative analysis has not yet been secured, it may be presumed to be unusually free from impurities. A qualitative test shows only a trace of cobalt. Its critical point lies 60° higher than the others and coincides very closely with the anomaly in expansion. This coincidence is equally good for the other two specimens, although the anomaly itself varies in magnitude. Specimen B was cut from a rolled nickel rod obtained from the Westphalian Nickel Works, and judging from its color on oxidation, contains far more iron than A and doubtless other impurities. Specimen C is the one used by Randall and was prepared by Zeiss of Jena. Its color on oxidation is similar to that of B, but the anomaly is far less marked.

For the sake of comparison with Randall's curve, computations of $\alpha$ were made for 20-degree intervals and the resulting curve plotted. With one or two exceptions, the values did not vary from his outside the limits of error.

No results have been given from the preliminary work with specimen A in 1907, but at least a brief mention of them should be made. The oven and optical system in this series differed in no way from those of 1908, but instead of the platinum thermometers, two groups of seven copper advance thermal junctions were used. One group was placed at equal intervals about the nickel in the inner chamber and the other similarly in the outer chamber. A Wolff potentiometer was used to measure their E.M.F. Although their sensitiveness and quick response was very satisfactory, the copper oxidized so readily at the high temperature and they soon became so fragile, that it was impossible to work with them. The observations in this work were unfortunately always discontinued at the close of each day and the isolated groups of about five readings, although consistent with each other, could never be satisfactorily combined with those of the following day. As mentioned above, this was due to the well-known change in the value of the coefficient of expansion of metals at a given temperature after the metal has been heated. The type of the curves which the readings suggested, however, agreed with that found in the later work, showing a decrease in a special region and an increase on either side.

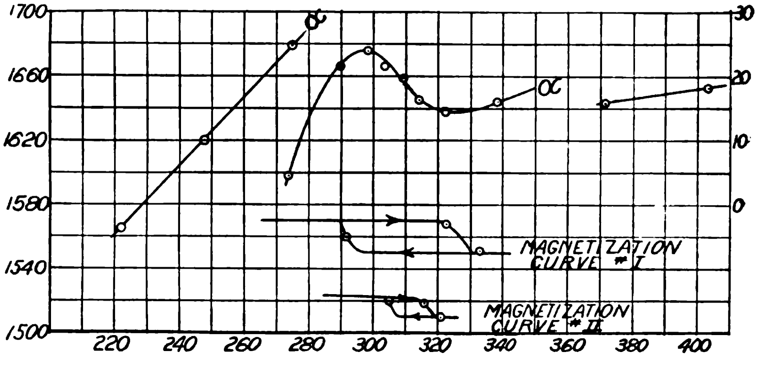


Fig. 5. Specimen C.

In conclusion the results may be summarized as follows:

1. The electrolytically prepared specimen shows a high value of permeability just below its critical temperature. This phenomenon is absent in the other specimens and is possibly due to the greater purity or crystalline structure of specimen A.

2. For the first sample, the critical temperature is in the neighborhood of 370° and, as in the case of iron, is lowered by impurities.

3. The absolute value of the expansion coefficient is in part dependent on the history of the specimen.

4. Through a continuous heating the relative values show an increase at the beginning of the region followed by a decrease.

5. Before and after the critical regions, the coefficient increases at a regular rate.

The writer wishes to express his gratitude to Professor Randall both for his aid in the final determinations and his invaluable suggestions throughout the work.

# NOTE ON THE RELATION BETWEEN THE TEMPERATURE AND THE RESISTANCE OF NICKEL.

BY C. F. MARVIN.

THE splendid work of Callendar confirmed and extended by many others has placed in the hands of physicists that admirable and indispensable tool, the platinum resistance thermometer.

All this work demonstrates that the relation between the temperature and the resistance of platinum wires can be represented over a very wide range of temperature by a parabolic curve, such as given by an equation of this form:

$$R = R_0 + \frac{R_{100} - R_0}{100}\left\{T - \delta\left[\left(\frac{T}{100}\right)^2 - \left(\frac{T}{100}\right)\right]\right\} \qquad (1)$$

or in simpler form:

$$R = R_0 + aT - bT^2. \qquad (2)$$

This equation is due to Callendar who has pointed out that the constant $\delta$ depends only on the quality, or purity, of the particular sample of platinum employed. $R_0$ and $R_{100}$ are the resistances of the thermometer coil at 0° and 100° C., respectively.

Probably no other metal can compete with platinum for use at high temperatures, but it is believed the striking advantages in nickel wire for the construction of resistance thermometers over a considerable range of moderate temperatures is not generally recognized.

Nickel exhibits an inversion point between 350° and 400° C., and sufficient data to determine its suitability for resistance thermometers at very low temperatures are not at present available. Its greater resistance, greater change of resistance with temperature, and its small cost as compared with platinum, all render nickel better than platinum for thermometric purposes over a considerable range of temperatures. While it may possibly be difficult to procure nickel wires of a uniformly high purity, yet the maximum purity seems unessential since what passes as commercially pure nickel seems to possess the advantages now in mind. Callendar has already shown that the temperature resistance curve of platinum is a parabola. At least three points must be located to define the parabolic curve.

The nickel resistance curve seems to be very accurately represented by a logarithmic curve of this form:

$$\text{Log } R = a + mT. \qquad (3)$$

*Table of Observed and Calculated Resistances of Nickel-Resistance Thermometers.*

| A | | | | B | | | | C | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Tem. °C. | Resistance. | | Dif. Ob. — Cal. %. | Tem. °C. | Resistance. | | Dif. Ob. — Cal. %. | Tem. °C. | Resistance. | | Dif. Ob. — Cal. %. |
| | Obsd. | Calc. | | | Obsd. | Calc. | | | Obsd. | Calc. | |
| −25 | 11.030 | 11.036 | −.05 | −27.7 | 70.72 | 70.79 | −.10 | | | | |
| −20 | 11.250 | 11.254 | −.04 | −27.6 | 70.74 | 70.82 | −.11 | 0[1] | 8.42 | 9.10 | − 7.5 |
| −15 | 11.475 | 11.477 | −.02 | −24.6 | 71.64 | 71.71 | −.10 | 100 | 12.66 | 12.73 | − 0.5 |
| −10 | 11.700 | 11.703 | −.02 | −23.6 | 72.34 | 72.02 | +.45 | 200 | 17.97 | 17.80 | + 0.9 |
| − 5 | 11.935 | 11.935 | ±.00 | −23.6 | 71.92 | 72.02 | −.14 | 300 | 24.95 | 24.90 | + 0.2 |
| 0 | 12.173 | 12.173 | ±.00 | −18.4 | 73.57 | 73.60 | −.04 | 325 | 27.06 | 27.08 | − 0.1 |
| + 5 | 12.420 | 12.413 | +.06 | −12.9 | 75.39 | 75.32 | +.09 | 350 | 29.33 | 29.45 | − 0.4 |
| 10 | 12.660 | 12.659 | +.01 | − 7.04 | 77.29 | 77.20 | +.12 | 375 | 31.98 | 32.02 | − 0.1 |
| 15 | 12.920 | 12.908 | +.10 | − 5.95 | 77.65 | 77.55 | +.13 | 400[1] | 33.67 | 34.84 | − 3.5 |
| 20 | 13.173 | 13.163 | +.08 | + 8.4 | 82.42 | 82.36 | +.06 | 425[1] | 34.70 | 37.87 | − 8.4 |
| 25 | 13.435 | 13.423 | +.09 | 8.8 | 82.54 | 82.50 | +.05 | 450[1] | 35.63 | 41.19 | −15.6 |
| 30 | 13.700 | 13.689 | +.08 | 9.0 | 82.61 | 82.57 | +.05 | | | | |
| 35 | 13.965 | 13.959 | +.04 | 9.2 | 82.70 | 82.64 | +.07 | | | | |
| 40 | 14.240 | 14.235 | +.04 | 11.38 | 83.44 | 83.40 | +.05 | | | | |
| 45 | 14.520 | 14.516 | +.03 | 23.76 | 87.70 | 87.84 | −.16 | | | | |
| 50 | 14.800 | 14.802 | −.01 | 24.05 | 87.82 | 87.95 | −.15 | | | | |
| 55 | 15.080 | 15.093 | −.09 | | | | | | | | |
| 60 | 15.385 | 15.393 | −.05 | | | | | | | | |
| 65 | 15.690 | 15.697 | −.05 | | | | | | | | |
| 70 | 16.000 | 16.007 | −.04 | | | | | | | | |
| 75 | 16.320 | 16.323 | −.02 | | | | | | | | |

[1] Not included in computation of equation.

A = Thermometer made by Leeds & Northrup for Weather Bureau for the measurement of solar radiation:

$$\text{Equation: Log } R = 1.08539 + .001699t. \qquad (4)$$

B = Data supplied by Leeds & Northrup.

$$\text{Equation: Log } R = 1.90045 + .001818t. \qquad (5)$$

C = Data supplied by Bureau of Standards on a single specimen of wire regarded as impure:

$$\text{Equation: Log } R = 0.96145 + .00145t. \qquad (6)$$

Only two points are required to locate such a curve. The coefficient *m* depends upon the quality, that is, the purity, and possibly some other of the physical properties and conditions of the metal.

The data at present available to the writer in support of the logarithmic curve is conceded to be too scanty and inadequate to demonstrate anything like a general law, but the conformity is sufficient for a great many purposes, and is shown in the following table:

In the two thermometers from Leeds & Northrup the conformity to the logarithmic equation is really very close, although no great range of temperature is embraced. The residuals are of about the same order of magnitude as the probable accuracy of the resistance measurements, viz: one tenth of one per cent.

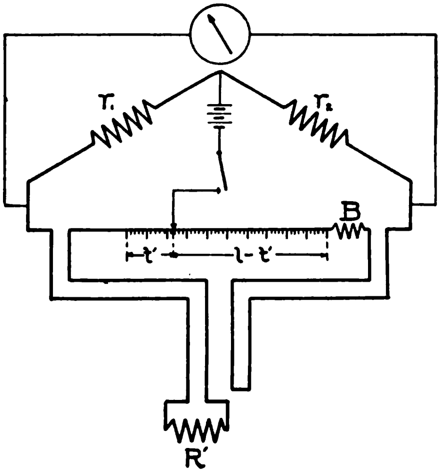


Fig. 1. Theoretical diagram of Wheatstone's bridge.

The data from the Bureau of Standards are not so well fitted by a logarithmic curve. The point at zero degrees, especially, is discordant, but from 100° to 375° the conformity is much closer.

Admitting that the nickel wire temperature resistance curve is approximately logarithmic, it remains to point out that resistance readings of a nickel thermometer on the usual form of slide-wire bridge, or equivalent, having a scale of equal parts, may also be made to give gas scale temperatures with a very considerable accuracy.

The two diagrams, Figs. 1 and 2, show two forms of bridge connections commonly employed in this kind of work. We consider only in each case a scale of equal parts. Any scale of unequal parts is an undesirable, if not an impracticable or impossible, thing.

The relation between resistance and scale reading in the Fig. 1 arrangement is strictly linear, and therefore the scale readings show temperatures on the gas scale only when the resistance of the thermometer varies according to a strictly linear law. This arrangement is used extensively with the platinum thermometer, and the scale readings are the so-called platinum temperatures.

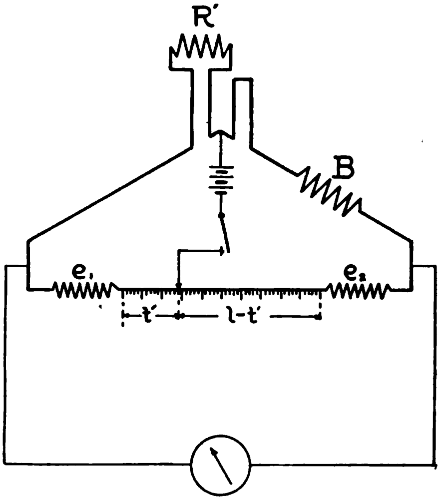


Fig. 2. Theoretical diagram of Wheatstone's bridge.

In the arrangement of Fig. 2 the relation between the scale readings $t'$ and the resistance $R'$ is given by the equation:

$$R' = B\frac{e + t'}{e + l - t'} \quad \text{when} \quad e_1 = e_2. \tag{7}$$

The curve represented by this equation between $R'$ and $t'$ is concave upward as is also the logarithmic curve representing nickel resistance, viz:

$$\text{Log } R = a + mt. \tag{3}$$

I now wish to show that a section of the $R't'$ curve, representing a temperature range of say 50° to 100° will conform with extreme closeness to the logarithmic curve over the same range; that is, the $t'$ scale of the bridge is then accurately the gas temperature scale.

Suppose, for example, we want the extreme range of the bridge scale to embrace 100 degrees of temperature, say from $t_1$ to $t_2$. Now, the $R't'$ curve can be made to intersect the $Rt$ logarithmic curve in two points determined by the two constants, $B$ and $e$ ($l$ = the resistance of the whole bridge wire, is determined by the range of temperature embraced, viz: 100 degrees in this case). The two curves will conform most closely over the whole range when the points of intersection lie at certain particular points between the two extremes $t_1$ and $t_2$ as at $PQ$, Fig. 3.

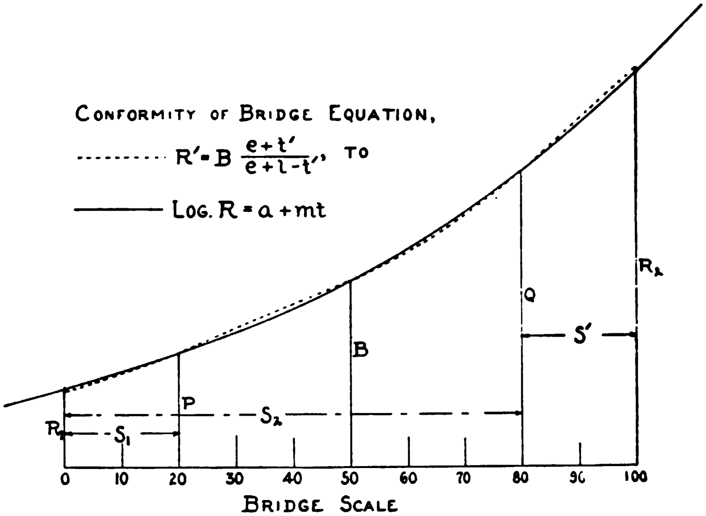


Fig. 3.

It is quite sufficient to select the two points, $PQ$, arbitrarily for example, at say $\frac{1}{4}l$, and $\frac{3}{4}l$. We can even pass the $R't'$ curve through the points $t_1$ and $t_2$, or through $t_1$ and the middle point $m$, or through $m$ and $t_2$. The deviation of the curves one from the other is smaller, as a rule, than the ordinary errors of observation, except in work of the highest precision. The general equations for computing the values of $B$ and $e$ giving two intersections intermediate between the extremities of the range comprised by a bridge scale are:

$$B^2 = PQ,^1 \tag{8}$$

$$e = \frac{P(l - s_1) - BS_1}{B - P}. \tag{9}$$

In expressing these equations it is assumed for simplification that $S_1$ and $S_2$ are symmetrically located on the bridge scale so that $S_1 + S_2 = l$ in all cases.

A single example will illustrate the magnitude of the deviations between the two curves, and, in order to bring this out clearly it has been necessary to carry out the computation to six figures, which, of course, represents a much higher order of precision than obtains in ordinary work.

TABLE II.

*Table of Corrections to Reduce Bridge Scale Readings, $t'$, to Gas Scale Temperatures, $t$, with a Nickel-resistance Thermometer:*

*Equation:* $\log R = 1.0859391 + 0.00169881t$. $S_1 = 20°$, $S_2 = 80°$, $l = 100°$.

| **Bridge Scale $t'$** | 0° | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° | 100° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Corrections to gas scale $t$ | +.10 | +.04 | −.00 | −.01 | −.01 | .00 | +.01 | +.01 | .00 | −.04 | −.10 |

Suppose we have fixed upon values of $B$ and $e$ in the equation

$$R' = B\frac{e + t'}{e + l - t'}. \tag{10}$$

Such that the $R't'$ curve intersects the logarithmic curve at two points, $PQ$. We may now imagine the $R't'$ curve to slide upward or downward along the logarithmic curve, and it will still conform closely to the logarithmic curve. This means that by increasing or diminishing the resistance $B$ we can shift the whole thermometric scale to a higher or lower range of temperatures, while still retaining the value of $e_1 = e_2$ on the bridge, and without changing the deviations of the bridge scale from the temperature scale.

---

[1] It is a fundamental property of the logarithmic curve that the ratio of any two ordinates, $P \div R_1$ for example, is the same as the ratio, $R_2 \div Q$, of any two other ordinates which are separated by the same horizontal space; that is, when $S_1 = S'$. As a corollary of this proposition, the middle ordinate between two ordinates is the square-root of their product. Therefore, in equation (8), $B = \sqrt{PQ}$ is a point on the logarithmic curve. Consequently, the $R't'$ curve must always intersect the logarithmic curve in the middle point as well as at $P$ and $Q$. The product $PQ$ is a constant for every pair of ordinates of which $B$ is the middle ordinate.

The data whose examination has called forth the results herein presented are admitted to be open to uncertainty, but it seems that easily available samples of nickel wire show a temperature resistance curve very nearly represented by the logarithmic equation, or, what is nearly the same thing, by the equation (10).

It is therefore worth while to bring out the useful mathematical relations existing between the logarithmic and the bridge scale equations. The matter is of the greatest practical utility, not only in scientific investigations, but in a large field of commercial work demanding a more or less accurate knowledge of moderate temperatures.

In view of the foregoing, we hope those in a position to do so may investigate more fully the variations of resistance of nickel with temperature, and especially the determination of the relation between the composition of the material employed and the character of the resistance curve.

The practical application of all this theory is very simple. The maker of bridges sets up the arrangement shown in Fig. 2. The resistance $B$ must be accurately adjusted to equal the resistance the thermometer will have when its temperature is that represented by the middle point of the bridge scale. The resistances $e_1$ and $e_2$ must be accurately *equal*, but the exact value is not of much importance. If $e$ is a little greater or smaller it simply shifts the $P$ and $Q$ points of intersection either nearer to or farther from the extremities of the bridge scale. Having approximated the $e$ coils it remains only to "point" the bridge scale for graduation into equal parts. Put into the bridge any convenient known resistance equal to the thermometer resistance at some known temperature, preferably near the $P$ and $Q$ points of the range. The point at which the bridge balances corrected for the small errors in table II is the particular temperature point of the bridge scale, and nothing more is necessary than to run a scale of equal subdivisions through this point and the middle point of the bridge. The careful man will probably prefer to locate another point of the scale near the upper temperature limit. Any inequalities of resistance of the bridge wire introduce errors, as is well understood, but which can often be neglected or corrected for by calibration if necessary.

# ON A RADIAL FLOW POROUS PLUG AND CALORIMETER.

By E. S. Burnett and J. R. Roebuck.

IN any investigation involving the accurate determination of the thermal properties of fluids, the freedom of the experimental apparatus and methods from inherent contributory errors of more than negligible value is of the utmost importance. Where, as is usually the case, the fluid under investigation must pass through several parts of the same or successive pieces of apparatus whose temperatures may differ, the external variants can seldom be under proper control nor can their disturbing influence be sufficiently determined. Undesired and frequently indeterminate interchange of heat by conduction and radiation between the fluid and its surroundings and between the temperature-measuring devices and their surroundings, affect the temperature so measured to an uncertain extent. In the ordinary form of porous plug and fluid calorimeter where the flow is continuous and linear, the most serious errors arise from the complicated radiation and conduction losses unavoidably involved with any temperature gradient between the points at which the temperatures are measured. It is the purpose of this note to describe an experimental arrangement in which these errors are avoided by the use of a radial fluid flow and accompanying radial temperature gradient whose inherent freedom from all detrimental heat interchange is demonstrated in detail below.

For several years past Mr. Burnett has been engaged in an investigation of the Joule-Thompson effect in various gases, and much of the time has been spent in overcoming experimental difficulties of the disturbing nature above indicated. Dr. Roebuck has more recently undertaken some work elsewhere noticed[1] involving apparatus of a similar nature. During one of the many discussions of our respective researches we evolved a scheme of apparatus and procedure which are almost ideal and of wide application.

---

[1] Phys. Rev., Vol. 30, p. 129.

The fluid under investigation is brought to any desired initial temperature, and maintained there by passing through a coil immersed in a bath controlled by a suitable thermostat. After passing through the coil its temperature is measured by a thermometer immersed in the fluid; under these conditions the thermometer, fluid and walls being continuously at the same temperature, radiation and conduction errors are avoided.

The next part of the apparatus which may be described as the "radial flow" portion constitutes the essential improvement. It is situated in the bath with adequate thermal insulation therefrom. Figs. 1 and 2 represent diagrammatically the principal features of two cases. In the porous plug form this consists of a round-ended porous tube of suitable non-conducting material (*e. g.*, porcelain). In calorimetry it consists of two concentric opaque tubes offering however practically no resistance to the flow of the fluid which is electrically heated as it crosses the space between them. The fluid is let toward the outer surface of these non-conducting tubes, flows radially through them, gathers within, and is led away through a passage preferably insulated from the bath. The second temperature is measured within the porous tube by a thermometer whose stem is thus protected by the outflowing fluid.

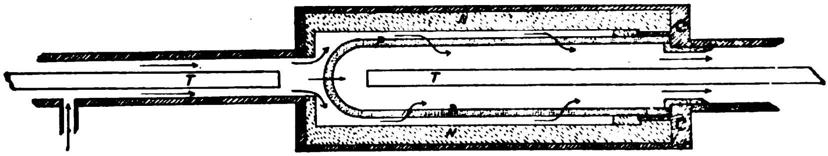


Fig. 1.

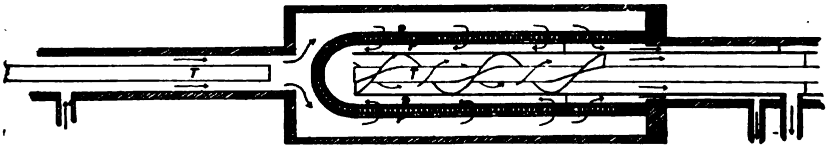


Fig. 2.

*C*, metal container; *N*, insulating material; *P*, porous plugs; *I*, thermometers.

Two cases are to be distinguished, first when the fluid passes through the porous partition with a rise, and second with a fall of temperature. In the first case the conduction of heat by the porous walls is opposed to the flow of fluid and only very small flows would be necessary to completely offset this minute conduction; as a consequence both outer and inner surfaces of the plug would be exactly at the temperature of the fluid and thus both radiation and conduction losses would be entirely avoided. In the second case the conduction takes place in the same direction as the flow, but conduction from the outside can only carry heat from the fluid entering to the fluid leaving the walls and therefore cannot influence the thermometer readings. Its total effect will be to shift slightly the temperature of the plug surfaces from that of the passing fluid, and so might give rise to appreciable radiation errors. The minute conductivity of porous material and the intimate contact of the fluid with the wall render this error negligible in most cases. Insulation of the porous tube from its container in the bath by suitable nonconducting material, confining the fluid flow to the region adjacent to the porous wall, will minimize any possible radiation loss from the plug, or other interchange of heat after its initial temperature has been measured.

In both cases the support of the porous tube should be arranged to prevent radiation to or from the second thermometer. When used as a porous plug the effect of axial conduction along the porous wall from its support is without influence on the second thermometer reading because the fluid which passes close to the support does not come in contact with the bulb of the thermometer. When used as a calorimeter the fluid must be mixed before passing the thermometer. The end effects near the tube support can be eliminated by a guard ring arrangement. This can be accomplished by respective division of the heat supply and the fluid flow as it passes through the porous walls and leading the divided flows away independently.

Either apparatus may be so designed as to be compact, self-contained, yet flexible, and adapted to a wide range of temperature and pressure without departing from the essential features of merit.

Physics Laboratory,
University of Wisconsin, December 7, 1909.

# PROCEEDINGS

# OF THE

# AMERICAN PHYSICAL SOCIETY.

## MINUTES OF THE FIFTY-FIRST MEETING.

A REGULAR meeting of the Physical Society was held in the Physical Laboratory of the College of the City of New York on Saturday, March 5, 1910, President Henry Crew presiding. The following papers were presented:

Presidential Address: Physics and Metaphysics. HENRY CREW.

1. On the Thermodynamics of Black Radiation. PAUL SAUREL.
2. The Recovery of Selenium from X-Ray Excitation. LOUISE S. McDOWELL.
3. The Clock as a Measure of Time and of Space. G. N. LEWIS.
4. The Second Law of Thermodynamics. W. S. FRANKLIN.
5. Heat Transfer due to Steam Condensation. S. LEROY BROWN.
6. Experimental Demonstration of Vortex Rings in Fluids. E. F. NORTHRUP.
7. The Heat Liberated by the Absorption of Electrons by Metals. O. W. RICHARDSON and H. L. COOKE.
8. Hydrodynamical Theory of an Electron in an Elastic Medium. J. G. COFFIN.
9. On Wien's Displacement Law. PAUL SAUREL.
10. Mechanical Model of an Electron. J. G. COFFIN.
11. A Commerical Type of Ammeter for the Precise Measurement of Alternating Currents Greater than 500 Amperes. E. F. NORTHRUP.

## TEMPERATURE COEFFICIENTS OF ELECTRICAL RESISTANCE.[1]

BY ALBERT A. SOMERVILLE.

THE range of temperature through which the work extends is from 0° C. to about 1060° C. — limits well within the range of the platinum resistance thermometer. Callendar's[2] temperature resist-

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society.

[2] Phil. Trans., Vol. 178.

ance curve for platinum is taken as a standard and temperatures are measured in terms of this.

The heating device consists of an electrical resistance furnace, made by wrapping nichrome ribbon wire around a porcelain tube having an inner diameter of 3 cm. and walls 1 cm. thick. This is enclosed in a second porcelain tube and a coating of asbestos about 6 cm. thick. The total length of the furnace is 50 cm. Its resistance when cold is 5 ohms. When connected to a 55 volt circuit a temperature of almost 1200° C. is attained in an hour. It can be held constant at any temperature by varying the current. Usually about three hours' time is taken to run through a temperature change of 1000° C.

The platinum thermometer is calibrated at the temperatures of melting ice, steam and the melting points of copper in air and in nitrogen. Frequently when making runs the copper lead wires are melted off in order to verify the temperature reading at that point.

The thermometer is connected with a Callendar automatic temperature recorder, consisting essentially of a slide wire bridge, the galvanometer when deflected closing the circuit through electromagnets which draw the sliding contact along to a new balance. The chronograph attachment draws a time-resistance curve from which may be read a time-temperature curve. By placing a resistance in series with the dummy leads the instrument may be brought within the range of the temperature to be measured and by putting a resistance in parallel with the slide wire the sensibility may be varied as desired.

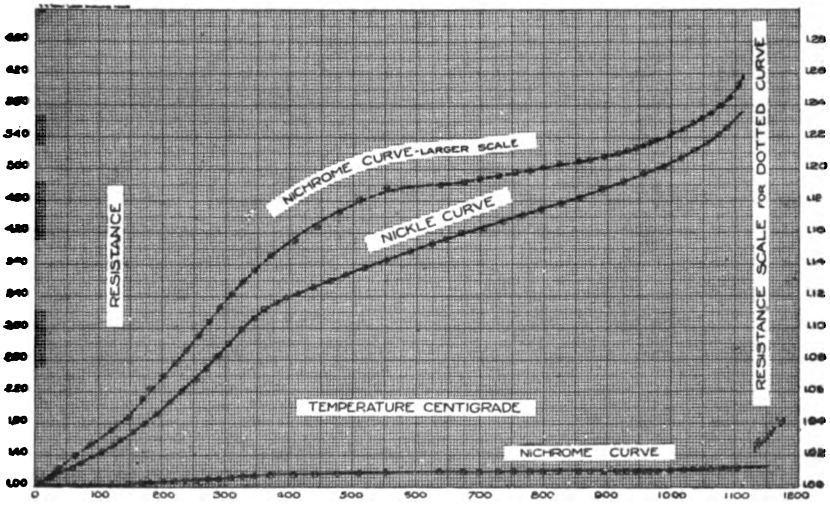


Fig 1.

Two materials tested were nickel and nichrome, the latter being an alloy made by Driver Harris Wire Company, Harrison, N. J., which in the form of a ribbon is well adapted to furnace manufacture. It stands a temperature of not less than 1200° C. and does not oxidize readily.

Both of these materials were made up in the form of resistance thermometers, that being a convenient form in which to test them. The resistance was measured by a dial bridge to the fourth decimal place. The temperatures were indicated by the platinum thermometer in connection with the Callendar recorder. The temperature-resistance curves for nickel and nichrome are similar in general form and that a peculiar one, but the coefficient of the former, is about twenty times that of the latter. These wires could be drawn and sufficient lengths used in a compact form so as to measure half an ohm at 0° C.

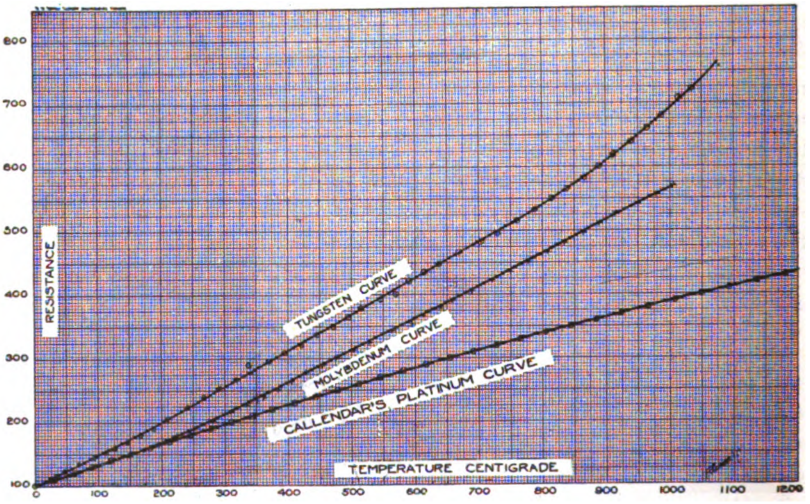


Fig. 2.

Other specimens tested were tungsten and molybdenum, furnished by General Electric Company, Harrison, N. J. These were in the form of hairpin filaments about .3 mm. in diameter and 20 cm. in length. These oxidize quickly at a rather low temperature. Copper or platinum lead wires were fused onto these filaments, using an arc in an atmosphere of hydrogen. They must also be kept in a non-oxidizing atmosphere while in the furnace. This is accomplished by passing a small stream of nitrogen through the quartz containing tube while it is being heated.

It is expected to extend the work to other materials.

Physical Laboratory,
Cornell University.

*Volume XXX.* *May, 1910.* *Number 5.*

THE

PHYSICAL REVIEW.

---

# THE TERMINAL VELOCITY OF FALL OF SMALL SPHERES IN AIR.

BY JOHN ZELENY AND L. W. McKEEHAN.

## § 1. INTRODUCTION.

THE resistance which a fluid offers to a body moving through it increases with the speed, and hence a body falling freely through a fluid attains a limiting velocity called the terminal velocity of fall, when the resistances to motion become equal to the body's resultant weight.

Stokes[1] obtained the expression $6\pi\mu a V$ for the resistance experienced by a spherical body, when all other resistances are negligible compared to that due to the viscosity of the fluid, and when no slipping is supposed to occur at the surface of separation. In this expression, $\mu$ is the coefficient of viscosity of the fluid, $a$ the radius of the sphere, and $V$ its velocity through the fluid. The terminal velocity of a freely falling sphere, obtained by putting the above value of the resistance equal to the resultant weight of the sphere, is

$$V = \frac{2}{9}\frac{ga^2(\sigma - \rho)}{\mu},$$

$\sigma$ being the density of the sphere, and $\rho$ that of the fluid. The exclusion of all but viscous resistance restricts the applicability of the formula to very minute spheres, the condition to be fulfilled being that the radius of the sphere must be small compared to $\mu / V\rho$.

In recent years Stokes' formula has been used in important meas-

---

[1] G. G. Stokes, Mathematical and Physical Papers, Vol. III., p. 59.

urements for determining the size of minute spheres from their rate of fall in air, and it is the object of the experiments to be described, to test the validity of the formula when air is the fluid. Such a test requires the measurement of the radius, the density, and the terminal velocity of the spheres used, the density and viscosity of air being known. The necessity of using spheres of microscopic size gives rise to the chief difficulties of the experiments. The first of these is to obtain spheres which are sufficiently small for the purpose.

## § 2. EXPERIMENTS WITH SPORES.

The microscopic spores produced by certain plants are quite spherical in some species, and so were chosen as suitable for testing Stokes' formula. The choice is limited, however, to such spores as not only approximate to a spherical shape but also are obtainable free from other material in a sufficient quantity to permit an accurate determination of density to be made. The three kinds of spores selected were those of lycopodium, lycoperdon, and polytrichum.

Lycopodium was used first because it is obtainable as a commercial powder, very free from other material. The individual spores are only roughly spherical, their shape being shown in the microphotographs[1] (Plate, Figs. 1 and 2). Fig. 1 shows that the outer cell walls are free from any hair-like projections.

All the lycoperdon spores used were obtained from one large, well-seasoned, Colorado puff-ball. The spores were gotten practically free from impurities by rubbing masses of the puff-ball on a cloth sieve. These spores are, on the whole, very spherical, and are among the smallest known. A group of them, magnified 900 diameters, is shown (Plate, Fig. 3).

The polytrichum spores (Plate, Fig. 4) were obtained from some freshly gathered plants.[2] The ferns were dried, and the spores collected by cutting open the capsules and shaking them on a sieve. The smoothness of the walls of the spores is to be noticed in the microphotograph. The spores were not perfectly seasoned when

---

[1] All of the microphotographs shown were kindly made for us by Prof. E. W. D. Holway.

[2] Kindly gathered for us by Prof. C. O. Rosendahl.

used, since, several months after the close of the experiments with them, changes in the shape of some were observable.

The question will now be considered with what accuracy it is possible by the aid of Stokes' formula to calculate the terminal velocity of fall of particles which deviate somewhat from the spherical shape. When such a particle is taken to be equivalent to a sphere having a diameter equal to the average diameter of the particle, the calculated terminal velocity may be in error for two reasons; the calculated volume may differ from the true volume, and the calculated resistance to motion may differ from the true resistance. The magnitude of these errors may be learned by considering a body of some regular shape other than spherical. As an extreme case take an ellipsoid[1] whose semi-axes are 4, 5 and 6. The volume of the assumed equivalent sphere (radius 5) is four per cent. greater than the true volume. The resistance to motion in the direction of the shortest axis is equivalent to that of a sphere of radius 5.2 instead of 5, and for a motion in the direction of the longest axis the resistance is equal to that of a sphere of radius 4.8.

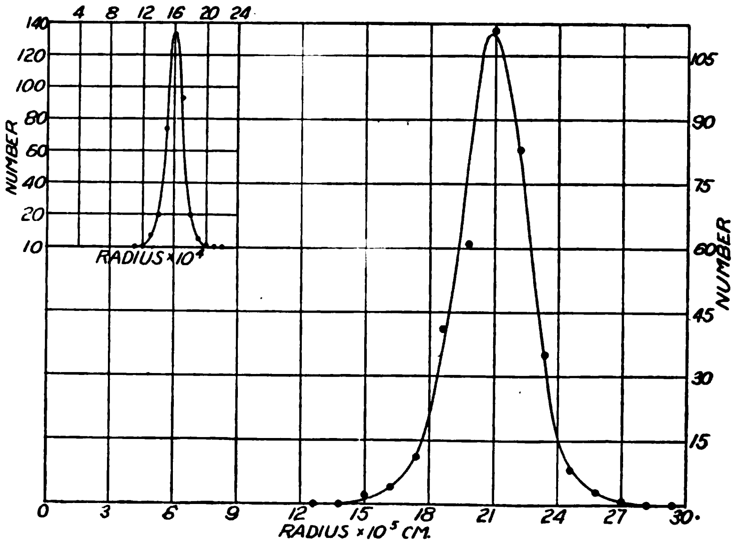


Fig. 1.

---

[1] For the equations to be used see Lamb, Hydrodynamics, p 534.

Taking into account the differences due to both the volume and the resistance to motion, the terminal velocity of the ellipsoid, calculated on the supposition that it is equivalent to a sphere having a radius the average of the three axes, is eight per cent. larger than the velocity calculated for the ellipsoid when moving in the direction of its shortest axis, and the same as that of the ellipsoid moving in the direction of its longest axis. Now the shape of the spores used is much more nearly spherical than that of the supposed ellipsoid, so that the error involved in assuming them spheres cannot be, in any case, more than a minor fraction of that found for the ellipsoid moving in the direction of its shortest axis. Since the spores are not homogeneous the axial direction in which they fall depends partly upon the position of the oil globules and other bodies within the walls. The arrangement of some of these bodies is seen in the microphotograph of polytrichum. Thus in a group of spores some may fall in the direction of the shortest, and others in the direction of the longest axis. It must be noted, however, that here, as in the development of Stokes' formula, the absence of eddy motion is presupposed.

### (*a*) *Measurement of Size.*

The diameters of the spores were measured with an ocular micrometer. A field of particles was traversed, and as the cross-hair moved across the field the diameter of each particle was determined in turn, in its accidental orientation. In this way the diameters of some 300 to 400 particles were obtained for each kind of spore. With the smallest spores magnifications of over 1,000 diameters were used.

Lycopodium and lycoperdon were measured dry. The latter and polytrichum were measured when immersed in lactic acid for better definition. The results by the two methods for lycoperdon were in agreement, though it appears from the work of Buller,[1] since published, that in some cases of this kind differences in size due to mounting have been observed.

The individual spores vary considerably in size, and the curve

---

[1] A. H. R. Buller, Researches on Fungi, 1909. An account is also given here of the author's experiments on the velocity of fall of various spores.

in Fig. 1 shows in what proportion the different sizes occur in the case of lycoperdon. The number of particles per .000012 cm. variation in radius is plotted as the ordinate of the curve at the center of the range indicated. The curve closely resembles a probability curve. The corresponding curves for lycopodium and polytrichum are quite similar to the one for lycoperdon; that for lycopodium is shown on a reduced scale in the upper left-hand corner of the figure.

The values obtained for the average radius of the three kinds of spores are as follows:

| | |
|---|---|
| Lycopodium | .00158 cm. |
| Polytrichum | .000478 cm. |
| Lycoperdon | .000209 cm. |

### (*b*) *Measurement of Density.*

The average density of the spores was obtained by determining the volume of a known mass of the substance by means of a volumenometer. The usual form of Regnault's volumenometer was at first used, but later an improved form was constructed having a baromanometer for measuring the pressures, thus making these independent of the fluctuations of the atmosphere. By using this instrument in a constant temperature room, results in which much more confidence could be placed were obtained with less labor.[1] The determinations with the two instruments, both of which were used for two of the substances, were, however, in good agreement. The volume of the substance measured varied from 3 c.c. to 10 c.c., and the following were the densities obtained:

| | |
|---|---|
| Lycopodium | 1.175 |
| Lycoperdon | 1.44 |
| Polytrichum | 1.53 |

### (*c*) *Terminal Velocity of Lycopodium.*

Since only the average densities and diameters of the spores were determined it became necessary to obtain their average terminal velocities. The apparatus used for lycopodium is shown in Fig. 2.

The lycopodium was started on its fall in the box *A*, and after passing through the tube *B*, fell upon a moving strip of black paper *CC*. This strip of paper, held tight by the weights *WW*, was moved

---

[1] J. Zeleny and L. W. McKeehan, Phys. Rev., XXX., p. 189, 1910.

uniformly by the chronograph drum *D*. A piece of white paper, pasted at one side of the black paper strip, received the time record from the two pens *PP′*. One of these marked the seconds from the clock, and the other recorded the moment when the powder was released. The portion of the strip on which the powder fell was surrounded by the metal case *EE* to shield it from air currents. Two metal rods *FF′* held the paper up against the lower end of the fall tube. This portion is shown enlarged at the lower part of the figure. The fall tube consisted of the two concentric tubes *GH*, the outer of which protected the inner from temperature changes. The length of both tubes was usually about 71 cm.; the diameter of the outer one was 3.5 cm., and that of the inner was varied. The lower part of the latter was closed by a cap having a small

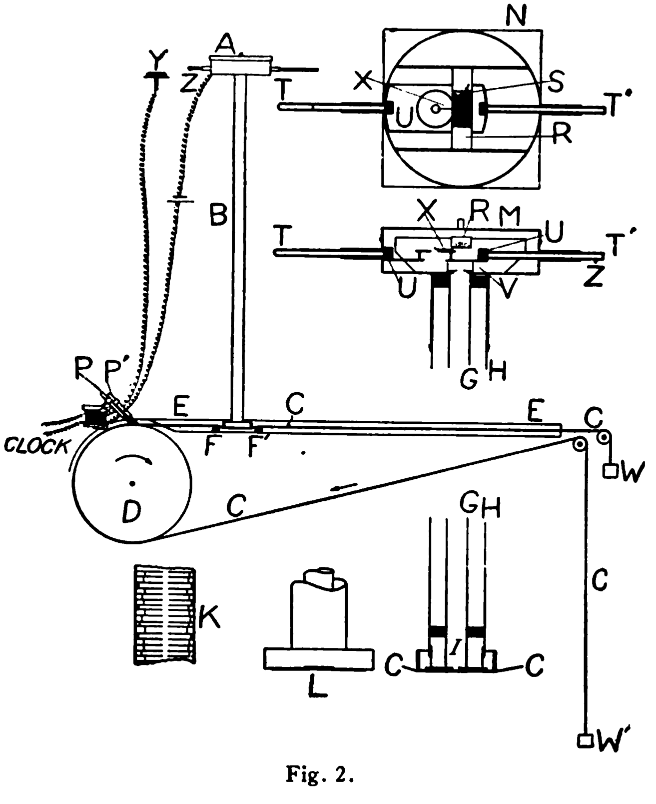


Fig. 2.

hole *I*, this permitting only such particles to pass through onto the paper *CC* as had travelled through the central portion of the tube, and so had suffered no retardation from its sides. The adjacent drawing, representing the tube turned 90° on its axis, shows the shallow opening *L*, through which the particles that had fallen onto the paper could pass without being rubbed off by the metal parts above them. The details of the box *A* from which the powder was started are shown in elevation at *M* and in plan at *N*, the position shown being 180° from that at *A*. The lycopodium powder was placed above the center of the fall tube in the metal tray *R*, the bottom of which was made of a tightly stretched silk cloth *S*. The rods *TT′*, which were movable in the tubes shown, carried the flat slider *U*. This slider just grazed the top of the short cylinder *V*, which had a perforated diaphragm in it. There was a hole in the center of the slider above which was a short cylinder, closed by a lid with a small hole in its center. Projecting over this hole in the lid and fastened to the edge of the lid itself, was the flexible pin-point *X*. When the slider was pushed to the right (in the enlarged figure) the pin would scratch lightly on the bottom of the cloth, allowing some of the lycopodium particles to shift into the fall tube through the small hole in the slider, when the latter was in its central position. The point *Z* at this instant made an electric contact with *Y*, actuating the pen *P*. The motion of the slider past its central position again closed the top of the fall tube. A rubber band drawn around the edge of the lid of the box *A*, and oil in the slide tubes, made the upper portion of the apparatus air-tight and prevented the passage of a convection current up or down the tube.

The method of conducting an experiment was as follows. After the paper strip *CC* was set into motion by the drum *D*, the slider was pushed by the rod *T*, freeing some of the particles. The motion of the paper was allowed to continue until previous experience had taught that all of the particles had reached the bottom. The drum was then stopped, the part of the paper carrying the record was cut off from the rest, placed on a smooth surface, and carefully covered with a strip of glass to prevent any subsequent displacement of the particles. These particles could easily be seen against the black paper with a low power microscope, and the time taken for each particle to fall was determined from the time record at the side. The average time taken to fall the known length of tube gave the average terminal velocity. The number of particles obtained on the paper at any trial depended upon how hard the point $X$ scratched the cloth. Only light scratches were used, as otherwise many thousands of particles fell to the paper. In the preliminary experiments several other methods of starting the particles were tried, but the one described above was finally adopted as the best.

The fall tube was modified in a number of ways. Inner tubes of different diameters were used, ranging from 3 mm. to 25 mm. Again, the outer tube was filled with centrally perforated paper discs, separated by paper rings as indicated by the section $K$ in Fig. 2. This arrangement was designed to prevent air circulation in the tube and yet diminish any effect which might exist in the small fall tubes due to the proximity of the wall of the tube. The height of fall of the particles was varied by employing fall tubes of lengths 71 cm. and 211 cm. As the results show, none of these variations had any marked effect on the value obtained for the terminal velocity of the particles.

The particles of lycopodium adhere to each other quite readily, so that in addition to the single particles a number of groups of particles would start to fall at the same time. The largest of these would reach the bottom first and then would come the smaller groups, ending in threes and twos, and finally single particles. It was attempted to avoid the liberation of large groups by scratching the cloth very lightly.

Fig. 3 shows an example of the distribution of the particles on the paper strip in respect to time of fall, the number falling in any second being represented by the ordinate at its end. Curve $A$ shows the distribution of the single particles, curve $B$, the doubles, and curve $C$, the triples. The average velocity of the singles obtained from this set of particles is 1.76 cm./sec., that of the doubles is 2.40 cm./sec. The doubles thus fall 1.37 times as fast as the singles. An unsymmetrical form of distribution curve for the time of fall is to be expected, although the size distribution curve is symmetrical (see Fig. 1), because the time of fall of a particle should vary inversely as the square of its radius. Thus three particles whose sizes are in the ratio of 2/3: 1:4/3 should have times of fall in the ratio 9/4: 1:9/16, a series which is no longer arithmetical. The extreme times of fall of the particles represented by the curve *A* of Fig. 3 are in about the ratio to be expected from the distribution of sizes (Fig. 1), but the fact that the number of particles to the right of the maximum in Fig. 3 is greater than that to the left indicates that the smaller particles are, on the whole, the denser ones. A summary of the results obtained for the terminal velocity of fall of lycopodium is given in Table I (p. 544). The results obtained under the various conditions are seen to be in satisfactory agreement.

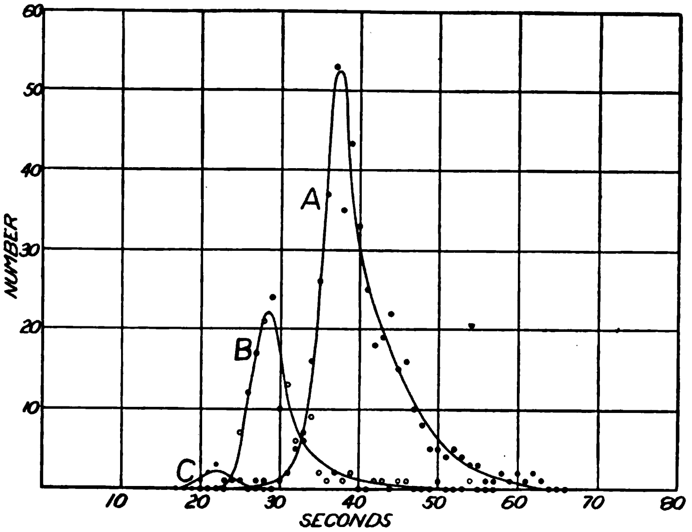


Fig. 3. Temperature 21°.1 C. Length of tube 71.5 cm. Average time, single particles 40.7 sec. Terminal velocity 1.76 cm./sec. Average time, doubles 29.8 sec. Terminal velocity 2.40 cm./sec.

### (*d*) *Terminal Velocity of Lycoperdon.*

The minuteness of the lycoperdon particles made it impossible to employ for them the apparatus which was used to determine the velocity of fall of lycopodium, as it was almost impossible to find the particles after they had fallen, with the high power microscope required to see them. A new apparatus was constructed, which is shown in Fig. 4 (p. 545).

TABLE I.

*Lycopodium.*

| Length of Tube. | Diameter of Tube. | Number of Particles. | Terminal Velocity of Fall. |
|---|---|---|---|
| 71.5 cm. | .32 cm.[1] | 26 | 1.72 cm./sec. |
| 71.5 | .32[1] | 18 | 1.77 |
| 71.5 | .32 | 162 | 1.70 |
| 71 5 | .32 | 133 | 1.70 |
| 71.5 | .64 | 30 | 1.71 |
| 71.5 | .64 | 170 | 1.73 |
| 71.5 | 1.27[2] | 354 | 1.76 |
| 71.5 | 1.27[2] | 431 | 1.76 |
| 71.5 | 1.27[2] | 327 | 1.78 |
| 71.5 | 1.27[2] | 267 | 1.74 |
| 71.5 | 1.27[2] | 359 | 1.80 |
| 71.5 | 1.27 | 66 | 1.76 |
| 71.5 | 1.27 | 60 | 1.74 |
| 210.9 | 1.27 | 462 | 1.82 |
| 71.5 | 2.54 | 34 | 1.74 |
| 71.5 | 2.54 | 138 | 1.80 |
| 71.5 | 2.54 | 52 | 1.78 |

Average temperature 22°.4 C. Average terminal velocity 1.76 cm./sec.

The upper portion *A*, which contained the arrangement for starting the particles on their fall was the same as that used in the other apparatus, and is shown in detail in Fig. 2. From *A* the particles fell through the innermost of the four concentric brass tubes *B*, the total height of fall being usually 31.2 cm. At the bottom of the fall tube the particles fell upon the revolving plate glass disc *C*, 12.7 cm. in diameter. This disc was cemented to the brass plate *D*, to which the axis of rotation was attached. The grooved wheel *E* was fastened to the same axis and was kept in rotation by a belt from a chronograph motor, through a speed reducer. The time of one revolution was about twenty minutes. A paper disc was attached to the top of the wheel *E*, and a stationary pen (not shown), actuated by a clock, marked second intervals upon the paper. The plate glass disc was completely enclosed in a metal box *F*, so that the whole space inside the apparatus was practically air-tight. Opposite the fall tube *B*, the box *F* had an opening for observing the particles with the microscope *M*. A section of this part is

---

[1] Size of apertures in paper discs .35 cm. apart.

[2] Size of apertures in paper discs .64 cm. apart.

shown enlarged at the lower right-hand corner of the figure. The opening in the cover of the box was closed by a thin microscope cover-glass *G*. The upper surface of the glass disc *C* revolved very close to this cover-glass, enabling a high power objective *M* to be used. Opposite the upper opening there was a corresponding window in the lower wall of *F*, which was closed by the glass plate *K*.

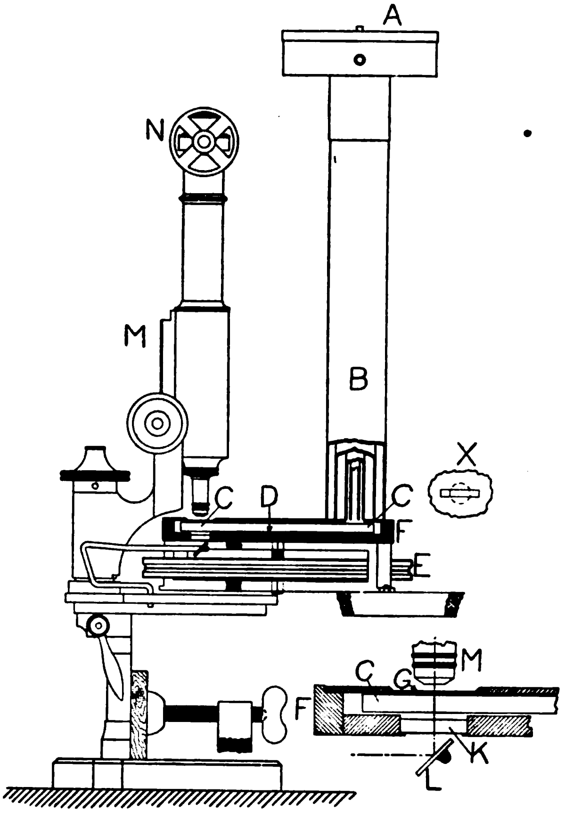


Fig. 4.

Light was reflected up from the mirror *L* to illuminate the field of the microscope. As the plate was slowly revolved the particles on its surface came into the clear microscope field where they could be seen very clearly, and their diameter could be measured, if desired, with the micrometer eyepiece *N*. The motion of the glass disc was so true that as it revolved its surface could readily be kept in focus. An additional window, which is not shown, closed by a removable cover, was provided in the top of the box *F* for removing the particles from the glass disc at the end of any experiment. The opening at the lower end of the fall tube was closed with a diaphragm having a slot 3 mm. wide (shown at *X*). This not only permitted a more exact determination of the time of fall, but also prevented particles which fell along the sides of the fall tube from alighting among those which had fallen through the center of the tube.

Inasmuch as the time required for some of the smallest particles to fall the distance of 31.2 cm. was 15 minutes, great precautions were taken to guard against convection currents in the fall tube.

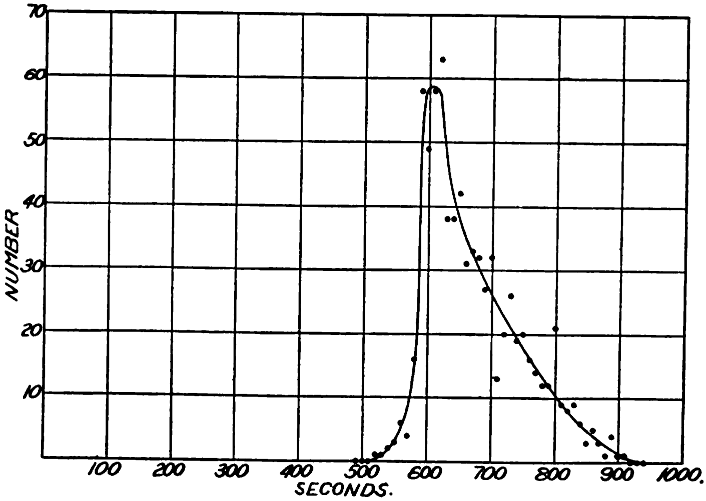


Fig. 5. Temperature 20°.8 C. Length of tube 31.2 cm. Average time of fall 678 sec. Terminal velocity .0460 cm./sec.

The apparatus was placed in a constant temperature room, and in no case were the particles started on their fall until several hours after the adjustments had been made. The operator entered the room only long enough to begin the experiment. After starting the chronograph motor, disconnected from the running gear, the particles were released, and at the same instant the running gear was closed, setting the disc into slow rotation. The time-recording pen thus began to trace its record at the instant the particles started to fall.

After the particles had fallen, the disc was turned to a determined position which placed that part of it under the microscope which in the initial part of the experiment had been under the center of the fall tube. A stationary index point was then placed opposite the beginning of the time record upon the paper. When the disc was now turned until a particle appeared in the center of the microscope field, the index showed the time it had taken that particle to fall. Starting with the initial position of the disc, this was slowly turned and the time taken for the different particles found. Then the microscope was moved radially the width of one field and the process repeated until the time of fall of all the particles on the disc had been determined. An illustration of the nature of the results obtained with lycoperdon is shown in Fig. 5.

The time interval was divided into ten-second spaces, and the number of particles found in each of these intervals is plotted opposite the end of the interval. The curve representing the distribution on the disc is similar to that obtained for lycopodium. There is the same indication here, too, that the smaller particles are, on the whole, denser than the larger ones. The height of fall was 31.2 cm. in all but two experiments in which it was changed to 10.4 cm. without, however, altering the value of the velocity obtained. In some of the experiments a tube of radium was placed in the starting box to discharge any electrification which the particles might get from the cloth in passing through it, but this too had no effect upon the value obtained for the terminal velocity. A summary of the results obtained for lycoperdon is given in Table II.

TABLE II.

*Lycoperdon.*

| Diameter of Tube. | Number of Particles. | Terminal Velocity. |
|---|---|---|
| 1.27 | 280 | .0463 cm./sec. |
| 1.27 | 757 | .0462 |
| .64 | 713 | .0473 |
| .64 | 104 | .0480 |
| .64 | 75 | .0470 |
| .64 | 75 | .0475 |
| .64 | 216 | .0457 |
| .64 | 652 | .0460 |
| .64 | 292 | .0465 |
| .64 | 186 | .0468 |

Length of tube 31.2 cm. Average temperature 20°.5 C. Average terminal velocity .0467 cm./sec.

The velocity of lycoperdon is so small that it was thought possible to determine it by observation of the fall across the field of a microscope. A square glass cell 1 mm. in cross-section and 8 cm. deep was made and set up in front of a horizontal microscope. A piece of the puff-ball was placed in the opening at the top so that tapping the cell started a shower of spores down the narrow channel. The lower portion of the space was quite free from air currents, as the individual particles descended in straight lines. No Brownian movements could be observed, nor was any rotation of the single particles noticed, though there was rotation of some of the doubles. The time required for the particles to move a distance of 2 mm. was taken with a stop-watch, and an average velocity of .05 cm./sec. was obtained. This value agrees roughly with the result of the more accurate method. The method of direct observation is, however, unsatisfactory for two reasons. The measurement of the time for the particles moving rapidly across the field is difficult, and it is impossible by timing individual particles chosen at random out of a group which started to fall at the same time, to obtain the average velocity of the group, the fraction of the total number that have any observed velocity being unknown.

### (*e*) *Terminal Velocity of Polytrichum.*

The experiments with polytrichum were carried out in the same way as those with lycoperdon. The curves showing the distribution of the particles as to time of fall are similar to those for lycoperdon, except that the maxima are not as sharply marked. A summary of the results obtained with a fall tube .64 cm. in diameter and a height of fall of 31.2 cm., is given in Table III.

TABLE III.

*Polytrichum.*

| Number of Particles. | Terminal Velocity. |
|---|---|
| 35 | .221 cm./sec. |
| 15 | .229 |
| 1551 | .238 |
| 1674 | .218 |
| 2337 | .238 |
| 832 | .250 |
| 2653 | .226 |
| 668 | .215 |

Average temperature 25°.3 C. Average terminal velocity .229 cm./sec.

### (*f*) *Comparison with Theory.*

In Table IV., a summary of the various results for spores is given, and the observed values of the terminal velocities of fall are compared with those calculated by the formula

$$V = \frac{2}{9}\,\frac{ga^2(\sigma - \rho)}{\mu}.$$

The value of $\mu$ used in the calculations ($\mu = .000191$ at 18°·C.) is that recently obtained by Zemplèn,[1] which is somewhat larger than that heretofore accepted.

TABLE IV.

*Comparison of Observed and Calculated Velocities for Spores.*

| Name. | Radius. | Density. | $\frac{aV\rho}{\mu}$ | Average Temp. | Average Vel. Observed. | Velocity Calculated. | Ratio $\frac{V\text{ calc.}}{V\text{ obs.}}$ |
|---|---|---|---|---|---|---|---|
| Lycopodium | .00158 | 1.175 | .02 | 22°.4 | 1.76 | 3.40 | 1.93 |
| Polytrichum | .000478 | 1.53 | .0007 | 25°.3 | .229 | .396 | 1.73 |
| Lycoperdon | .000209 | 1.44 | .00007 | 20°.5 | .0467 | .0718 | 1.52 |

The ratio of $a$ to $\mu/V\rho$ is given in the fourth column to show that the spheres fulfill the size criterion for the applicability of the formula, which is that "$a$" should be small compared to $\mu/V\rho$.[2] It is seen that the calculated velocity is in each case much larger than the value observed, the difference being greater for the larger particles.

## § 3. EXPERIMENTS WITH ARTIFICIAL SPHERES.

### (*a*) *Black Wax.*

Because of the discrepancies found in the case of spores between the observed and calculated values of the terminal velocity, attention was next given to the making of artificial spheres. It was found that paraffin wax when melted could be broken up by an atomizer into minute spherical drops which retained their shape on solidifying, although the attendant contraction left the surface of the spheres somewhat rough. A hard, brittle, black wax, of unknown composition, was next utilized and proved to possess admirable

---

[1] Gy. Zemplèn, Annalen der Physik, Vol. 29, p. 869, 1909.

[2] Rayleigh, Phil. Mag., V., 36, p. 365, 1893.

qualities for the purpose in hand. The perfect spheres obtained (Plate, Fig. 5) have a very smooth surface, and could be made in all sizes desired. The method of making and collecting the spheres was as follows. A piece of the wax was melted in a beaker (*A*, Fig. 6), and heated considerably above the melting point (to about 150° C.) in a sand-bath *B*. The atomizer *C* was made of two glass tubes, the size and adjustment of whose openings had much to do with the size of the spheres obtained. Unless the temperature of the vertical tube was maintained sufficiently high by an occasional application of a flame, the blast of air would cool the wax so that it would be drawn out into long fibers, or perhaps close the end of the tube completely. The atomized wax was blown by the blast itself into the end of a long inclined glass tube *E* about 3 cm. in diameter and 150 cm. long. As the air passed up this tube many of the suspended spheres settled on the wall, while the very finest ones issued from the upper end as a thin smoke. Some of these could be caught in a loose plug of cotton placed in the upper end of the tube, but nearly all of them were too small for accurate measurement. To collect the particles, the glass tube *E* was held vertically over a paper and tapped. An enormous number of perfect spheres of a large range of diameters was thus obtained.

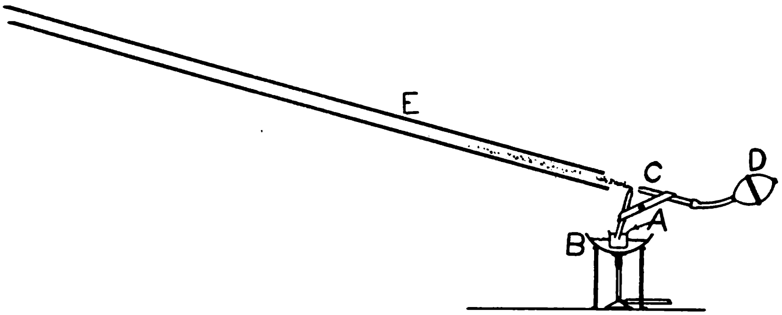


Fig. 6.

The density of the wax was found, by weighing in water, to be 1.058 at 20° C.

The apparatus of Fig. 4 was used for determining the terminal velocity of fall, a supply of the collected spheres being placed in the trough in the starting box (see *R*, Fig. 2). The only difference between the conduct of the experiment with the new material and with the spores was that now it was necessary to measure the diameter of each particle when found on the rotating disc, in addition to getting its time of fall. The particles are somewhat transparent, and the surface does not reflect so well but that a sharply outlined image is obtained, permitting accurate readings to be taken. Some of the particles were found to contain one or more air bubbles, and all particles where these constituted an appreciable part of the

### Table V.

*Black Wax.*

| Radius of Sphere. | Time of Fall Observed. | Time of Fall Calculated. | Radius of Sphere. | Time of Fall Observed. | Time of Fall Calculated. |
|---|---|---|---|---|---|
| cm. | sec. | sec. | cm. | sec. | sec. |
| .00366 | 3.4±6 | 1.9 | .00113 | 21.1 | 20.2 |
| .00218 | 6.3 | 5.4 | .00105 | 22.5 | 23.4 |
| .00217 | 6.6 | 5.5 | .00087 | 32.6 | 34.0 |
| .00211 | 6.2 | 5.8 | .00075 | 47.3 | 45.8 |
| .00205 | 6.9 | 6.2 | .00070 | 54.4 | 52.6 |
| .00197 | 6.4 | 6.7 | .00069 | 54.9 | 54.0 |
| .00195 | 7.9 | 6.8 | .00067 | 61.9 | 57.4 |
| .00194 | 7.1 | 6.9 | .00067 | 62.7 | 57.4 |
| .00191 | 8.5 | 7.1 | .00066 | 57.1 | 59.1 |
| .00181 | 10.1 | 7.9 | .00064 | 60.6 | 62.9 |
| .00178 | 8.0 | 8.2 | .00059 | 67.8 | 74.0 |
| .00172 | 8.2 | 8.7 | .00058 | 75.8 | 76.6 |
| .00169 | 8.8 | 9.0 | .00054 | 82.5 | 88.4 |
| .00167 | 9.9 | 9.3 | .00050 | 97.3 | 103.2 |
| .00165 | 10.6 | 9.5 | .00049 | 105 | 107 |
| .00165 | 11.1 | 9.5 | .00049 | 113 | 107 |
| .00156 | 11.4 | 10.6 | .00048 | 116 | 112 |
| .00151 | 12.2 | 11.3 | .00046 | 120 | 122 |
| .00147 | 11.4 | 11.9 | .00045 | 118 | 127 |
| .00146 | 12.4 | 12.1 | .00044 | 152 | 133 |
| .00145 | 13.4 | 12.2 | .00042 | 135 | 146 |
| .00142 | 13.3 | 12.8 | .00041 | 144 | 153 |
| .00130 | 15.6 | 15.2 | .00039 | 166 | 169 |
| .00130 | 15.7 | 15.2 | .00038 | 184 | 178 |
| .00124 | 16.8 | 16.8 | .00036 | 191 | 199 |
| .00121 | 17.4 | 17.6 | .00035 | 166 | 211 |
| .00119 | 13.0 | 18.2 | .00035 | 227 | 211 |
| .00119 | 16.6 | 18.2 | .00035 | 231 | 211 |
| .00117 | 13.6 | 18.8 | | | |

Average temperature 20°.0 C. Length of fall 31.2 cm. Diameter of tube .64 cm.

volume were discarded, together with those to which any dust was found adhering. Owing to the width of the slit at the bottom of the fall tube (see $X$, Fig. 4) the error in the time of fall of any one particle may be as much as ± .6 sec. Table V. gives a summary of the results obtained for all of the particles in four different experiments.

The values given in the third column were calculated by the aid of Stokes' formula, using .000192 for the value of $\mu$ at 20° C. The limiting size to which the formula may be applied is found by putting

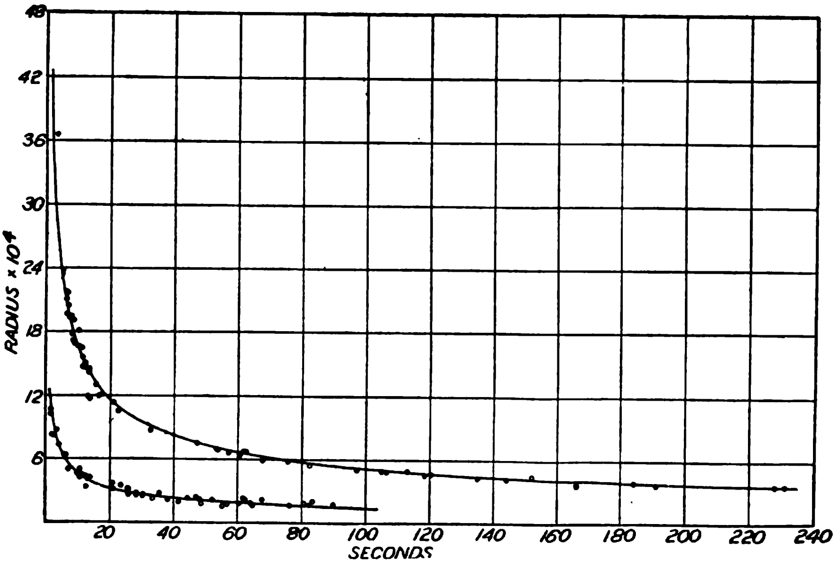


Fig. 7.

$a V \rho / \mu = .1$[1] (calling .1 a small quantity). This corresponds to a radius of approximately .0022 cm. All of the particles except the first one are within this limit. The results given in Table V. are shown graphically in the upper curve of Fig. 7, the various circles representing the individual observations, and the continuous curve, the theoretical values. The deviation of a few of the individuals from the theoretical curve is large, but on the average the deviation is less than one half of one per cent. The results covering such a large range of sizes show quite conclusively the applicability of Stokes' formula to these spheres.

---

[1] See reference to Rayleigh above.

Some of the black wax particles are so large that it is necessary to calculate the error that may arise because of the time taken for the particle to attain the terminal velocity. The equation of motion for the falling sphere is

$$\frac{dV}{dt} + kV = g,$$

where

$$k = \frac{6\pi\mu a}{\frac{4}{3}\pi a^3(\sigma - \rho)} = \frac{9\mu}{2a^2(\sigma - \rho)},$$

whence

$$V = \frac{g}{k}(1 - \varepsilon^{-kt}),$$

where $g/k$ is the terminal velocity. We can find how long a time elapses before the velocity of the sphere is within one per cent. of the terminal velocity, by finding the value of $t$ for which $\varepsilon^{-kt}$ is equal to .01.

For the largest sphere to which Stokes' formula is applicable

$$a = .0022 \text{ cm.},$$

and therefore

$$k = 165,$$

so that $t = .03$ sec. This is less than .6 per cent. of the total time of fall, hence the error introduced by assuming that the terminal velocity is instantly attained is still less than this, and may safely be neglected.

### (*b*) *Mercury.*

Some experiments were next performed with minute mercury drops which were also made by means of an atomizer. The time of fall and the diameter of the drops were determined with the same apparatus in the same way as for the black wax. In the measurement of diameter, however, a serious difficulty arose because of inability to get well-defined images of the drops, owing to the high reflecting power of the mercury. The best system of illumination under the restricted conditions was found to be a side illumination together with a faint illumination from below. Nevertheless, the errors of the measurements were quite large, especially so, because in order that Stokes' formula may be applied to such a dense substance extremely minute drops have to be used.

The results obtained for the mercury drops in several experiments, together with theoretical values, are given in Table VI.

### Table VI.

*Mercury.*

| Radius of Sphere. | Time of Fall Observed. | Time of Fall Calculated. | Radius of Sphere. | Time of Fall Observed. | Time of Fall Calculated. |
|---|---|---|---|---|---|
| cm. | sec. | sec. | cm. | sec. | sec. |
| .001075 | 1.6±6 | 1.8 | .000274 | 25.8 | 27.0 |
| .001030 | 1.6 | 1.9 | .000273 | 28.3 | 27.2 |
| .000880 | 3.2 | 2.6 | .000267 | 25.8 | 28.5 |
| .000830 | 1.9 | 2.9 | .000267 | 30.3 | 28.5 |
| .000826 | 2.5 | 3.0 | .000254 | 28.3 | 31.5 |
| .000736 | 4.0 | 3.7 | .000252 | 30.2 | 32.0 |
| .000640 | 5.7 | 4.9 | .000239 | 47.0 | 35.5 |
| .000640 | 4.7 | 4.9 | .000238 | 47.1 | 35.8 |
| .000508 | 10.6 | 7.9 | .000234 | 61.8 | 37.1 |
| .000506 | 7.0 | 7.9 | .000233 | 44.4 | 37.4 |
| .000465 | 9.8 | 9.4 | .000225 | 33.4 | 40.1 |
| .000449 | 11.3 | 10.0 | .000223 | 62.3 | 40.8 |
| .000437 | 12.4 | 10.6 | .000222 | 48.1 | 41.2 |
| .000428 | 10.5 | 11.1 | .000216 | 38.0 | 43.5 |
| .000428 | 10.8 | 11.1 | .000215 | 52.1 | 43.9 |
| .000422 | 13.8 | 11.4 | .000205 | 83.6 | 48.3 |
| .000413 | 13.0 | 11.9 | .000194 | 41.5 | 53.9 |
| .000370 | 20.8 | 14.8 | .000188 | 80.7 | 57.5 |
| .000349 | 23.3 | 16.6 | .000186 | 64.1 | 58.6 |
| .000338 | 12.5 | 17.7 | .000185 | 67.6 | 59.4 |
| .000324 | 25.6 | 19.3 | .000181 | 48.7 | 62.0 |
| .000324 | 25.6 | 19.3 | .000179 | 58.3 | 63.4 |
| .000322 | 25.5 | 19.6 | .000177 | 56.8 | 64.8 |
| .000318 | 20.9 | 20.1 | .000174 | 89.9 | 67.0 |
| .000314 | 25.8 | 20.6 | .000170 | 82.3 | 70.4 |
| .000314 | 21.0 | 20.6 | .000165 | 76.3 | 74.7 |
| .000309 | 20.7 | 21.3 | .000164 | 64.7 | 75.6 |
| .000277 | 35.4 | 26.5 | .000159 | 55.2 | 80.4 |
| .000275 | 28.3 | 26.8 | | | |

Average temperature 20°.3 C. Length of fall 31.2 cm. Diameter of tube .64 cm.

These observations are represented graphically in the lower curve of Fig. 7, where the continuous line again shows the theoretical values. It is seen that the individual observations show very large deviations, but nevertheless the results in general cluster about the line representing the theoretical values. For a mercury drop of radius equal to .001 cm. the ratio $aV\rho/\mu = .107$, and hence all drops smaller than this are within the range of applicability of Stokes' formula. The time required for a drop of the size stated to attain a velocity within 1 per cent. of its terminal velocity is .08 sec., which is 4 per cent. of the total time of fall in the apparatus used.

### Table VII.

*Paraffin.*

| Radius of Sphere. | Time of Fall Observed. | Time of Fall Calculated. | Radius of Sphere. | Time of Fall Observed. | Time of Fall Calculated. |
|---|---|---|---|---|---|
| cm. | sec. | sec. | cm. | sec. | sec. |
| .00415 | 1.5±6 | 1.8 | .00214 | 6.7 | 6.6 |
| .00372 | 3.2 | 2.2 | .00206 | 7.3 | 7.2 |
| .00369 | 2.5 | 2.2 | .00202 | 8.55 | 7.4 |
| .00354 | 3.4 | 2.4 | .00200 | 7.43 | 7.6 |
| .00343 | 3.75 | 2.6 | .00200 | 7.4 | 7.6 |
| .00328 | 2.65 | 2.8 | .00180 | 8.7 | 9.4 |
| .00327 | 3.8 | 2.8 | .00179 | 9.05 | 9.5 |
| .00324 | 4.1 | 2.9 | .001725 | 10.95 | 10.2 |
| .00317 | 3.5 | 3.0 | .001645 | 10.05 | 11.2 |
| .00313 | 3.3 | 3.1 | .00163 | 10.6 | 11.4 |
| .00309 | 3.2 | 3.2 | .00161 | 11.5 | 11.7 |
| .00307 | 4.15 | 3.2 | .00157 | 12.5 | 12.3 |
| .00302 | 3.9 | 3.3 | .001554 | 12.1 | 12.5 |
| .00300 | 3.0 | 3.4 | .001535 | 11.3 | 12.9 |
| .002785 | 3.8 | 3.9 | .00152 | 14.2 | 13.1 |
| .00278 | 3.85 | 3.9 | .00142 | 15.2 | 15.0 |
| .00278 | 4.3 | 3.9 | .00140 | 15.6 | 15.5 |
| .00274 | 5.3 | 4.0 | .00135 | 18.8 | 16.6 |
| .00272 | 5.3 | 4.1 | .00123 | 18.1 | 20.0 |
| .00267 | 4.75 | 4.3 | .00122 | 22.5 | 20.4 |
| .00260 | 3.75 | 4.5 | .00117 | 21.3 | 22.2 |
| .00256 | 4.9 | 4.6 | .00116 | 20.7 | 22.5 |
| .00256 | 1.8 | 4.6 | .00116 | 20.2 | 22.5 |
| .00255 | 4.8 | 4.7 | .001145 | 24.6 | 23.2 |
| .00247 | 5.3 | 5.0 | .00109 | 27.3 | 25.5 |
| .00247 | 5.9 | 5.0 | .00106 | 25.7 | 27.0 |
| .00244 | 6.7 | 5.1 | .001035 | 29.6 | 28.3 |
| .00242 | 5.1 | 5.2 | .000965 | 30.4 | 32.5 |
| .00240 | 5.85 | 5.3 | .000862 | 45.7 | 40.7 |
| .00230 | 5.25 | 5.7 | .00085 | 38.6 | 41.9 |
| .00223 | 6.6 | 6.1 | .000665 | 77.7 | 68.4 |
| .00221 | 6.3 | 6.2 | .000579 | 92.1 | 90.4 |
| .00220 | 6.5 | 6.3 | .000527 | 106.0 | 109.0 |
| .00216 | 7.3 | 6.5 | .000500 | 123.2 | 121.4 |

Average temperature 20°.3 C. Length of fall 31.2 cm. Diameter of tube .64 cm.

### (*c*) *Paraffin.*

The spherical particles made with the atomizer from paraffin wax differ from those of black wax and mercury in not having perfectly smooth walls. The contraction of the paraffin on cooling leaves the walls quite rough and even more or less jagged in some cases (Plate, Fig. 6). The terminal velocity of a number of these spheres was determined to see if this roughness has any effect upon their rate of fall. The density of the paraffin was obtained in bulk by weighing in water and found to be .906. In determining their time of fall, the spheres were not placed inside the trough *R* (Fig. 2), because of their adhesive property. Instead, the stream of atomized particles was allowed to impinge directly upon the under side of the trough, to which they adhered. The starting pin-point simply dislodged them from this position. The results are given in Table VII., together with values calculated by the formula.

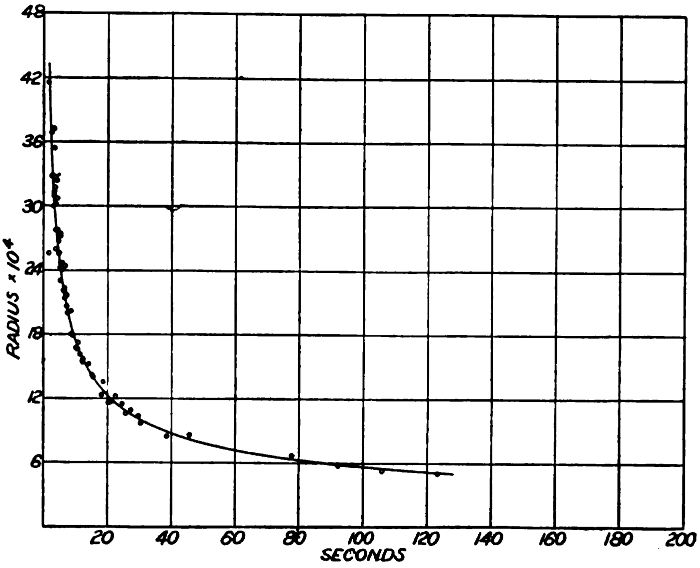


Fig. 8.

The limit of size for the applicability of the formula is found to be .0024 cm., so that many of the larger particles in this table are outside the limit. For these the actual resistance to motion should be appreciably greater than that taken into account in calculation, and the results show that this is the case. The average deviation from the theoretical values for the particles within the limit is less than 1 per cent. The results are represented graphically in Fig. 8, where again the continuous curve gives the values by Stokes' formula.

It appears from a comparison of these results with those for the black wax spheres, that roughness of surface of the order of that found in these paraffin droplets is without effect upon the terminal velocity of fall, and hence that it does not start eddy motion for the velocities used.

## § 4. DISCUSSION OF RESULTS.

Since the terminal velocity of the artificial spheres agrees so well with the theoretical formula, the discrepancy observed with spores calls for some special explanation. How large this discrepancy is can best be seen by referring to the time of fall distribution curves for lycopodium (Fig. 3) and lycoperdon (Fig. 5). The theoretical time of fall of the average lycopodium particle is 21 sec., whereas the curve shows that not even one of the fastest of the particles had fallen within that time. The theoretical time of fall of the average lycoperdon particle is 434 sec., whereas the fastest one out of the whole group took 520 sec. to fall, the others varying in order up to 910 sec. The agreement among the separate determinations of the terminal velocity for the same substance under various conditions precludes the possibility of any large error entering into the results for this quantity. As further evidence one of the experiments with mercury drops will be cited, in which a few black wax particles and a considerable number of lycopodium particles fell in the tube at the same time. The times of fall of the wax spheres agreed closely with the calculated times, and are, indeed, included under the results given for black wax. The average velocity of the 44 lycopodium particles was 1.73 cm./sec., which agrees well with the results obtained previously with the other apparatus. Of the three kinds of particles falling here under exactly identical conditions the wax and mercury spheres had velocities agreeing with Stokes' formula, while the lycopodium fell with barely more than half of the theoretical velocity. The discrepancy in the case of the latter cannot therefore be due to convection currents, or other errors in the determination of the velocity of fall.

It does not seem possible to attribute the difference between theory and experiment to errors in the measurements of the diameters of the spores, since that would require this measurement to be about 40 per cent. too large in the case of lycopodium, and 24 per cent. too large for lycoperdon. In these two cases, at least, the spores were assuredly of the same size in the fall experiments as when measured. They were perfectly seasoned and hence undergoing no change, and they were measured dry just as they were used in the velocity determinations. Different magnifying powers were used which required separate calibration of the ocular micrometer.

In the density determinations, the question arises whether the volume obtained with the aid of the volumenometer may not be in error, because of gases which might be liberated from the surface or interior of the spores when the pressure is reduced. An experiment was carried out with glass, the surface of which is known to condense gases. The density of a sample was obtained first by weighing in water, next by means of the volumenometer when in large pieces, and finally by the latter method after it had been powdered as finely as possible in a mortar. The three results were in good agreement. Still, the behavior of spores in this respect may differ from that of glass. A possible substitute method of getting the density of the spores is by finding the density of a solution in which they will just sink. This method is obviously open to the objection that osmosis produces changes in the spores when put into the liquid, so that the density obtained for the wet spores is not their density when dry. Nevertheless, some experiments were carried out by observing the motion of individual spores under a microscope when suspended in a solution, and also by observing them in bulk in test-tubes. The unreliability of the method was at once shown by the fact that the spores behaved differently in different kinds of solutions of the same density. Thus lycoperdon sank in a saturated water solution of NaCl whose density was 1.201, while it rose in a solution of $CaCl_2$ of density 1.200, also sinking, however, in a solution of the latter substance whose density was 1.163. The density of lycopodium determined by this method was found to be 1.09 ± 1 when a calcium chloride solution was used, and 1.13 ± 2 when a sodium chloride solution was used. With the volumenometer it was found to be 1.175, and to explain the discrepancy between the observed and calculated values of the terminal velocity the density would have to be but .61.

The effect of the shape of the spores alone remains to be considered. Evidence has been given already (§2) showing that, in the calculations, a nearly spherical particle may be represented without a large error by a sphere having a diameter equal to the average diameter of the particle. There is one respect, however, in which this equivalence may not hold. This is in the tendency to the production of eddy motion. The conditions for a stable motion are not well understood, but in the case of the sphere the perfect symmetry of the stream lines may well represent the greatest stability in the motion. The symmetry of these lines in the rear of the moving body would not be influenced appreciably by small roughnesses on the surface. Dissymmetry in the stream lines, caused by an imperfectly spherical body, must be accompanied by forces in the fluid at right angles to the direction of motion, and these may suffice to cause eddies in the rear of the body for velocities much smaller than are necessary to produce them in the case of the sphere. Some explanation of this character must at present be regarded as the most probable one, none other appearing adequate to explain the large difference between the observed and calculated terminal velocities of the spores used.

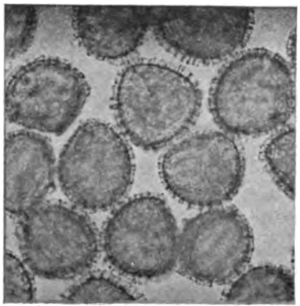
## § 5. SUMMARY.

The average size, density, and terminal velocity of fall in air of the approximately spherical spores of lycopodium, lycoperdon and polytrichum were obtained experimentally. The observed values of the terminal velocity were much smaller than the corresponding values calculated by Stokes' formula.

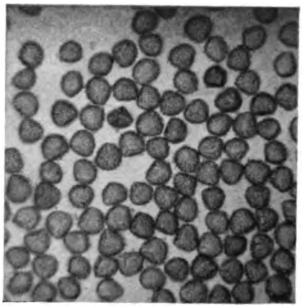
Similar experiments performed with artificial spheres made from a black wax, from mercury, and from paraffin gave results for the terminal velocity, over a large range of sizes, which were in good agreement with the theoretical formula, proving its applicability under the conditions used.

No satisfactory explanation for the discrepancies in the case of spores is apparent. It may be that turbulent motion of the fluid is started more readily by a nearly spherical body than by one which is perfectly spherical.

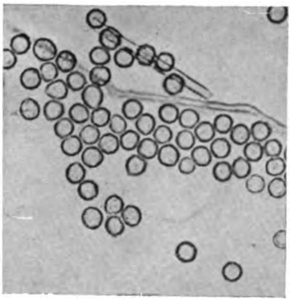
PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA, December 2, 1909.



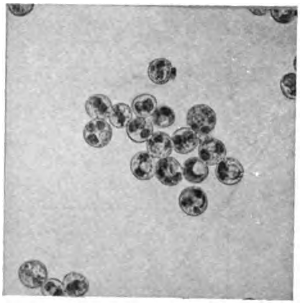
1. Lycopodium x500



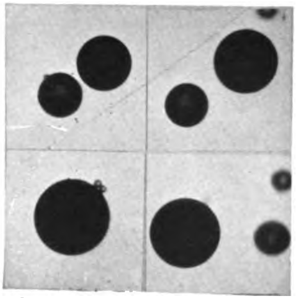
2. Lycopodium x125



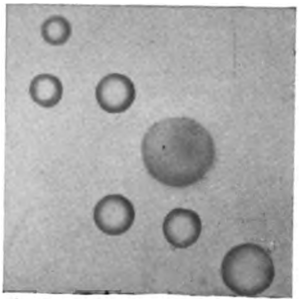
3 Lycoperdon x900



4. Polytrichum x500



5. Black Wax x500



6. Paraffin x500

PLATE I. ZELENY AND McKEEHAN.

# A STUDY OF THE MULTIPLE REFLECTIONS OF SHORT ELECTRIC WAVES BETWEEN TWO OR MORE REFLECTING SURFACES.

By L. E. Woodman and H. W. Webb.

## I. Introduction.

THE phenomenon of "multiple reflections" between two parallel reflecting surfaces is well known, especially in its application to the optics of thin films, but its application to electric waves has been investigated only within the last few years. Several investigators[1] have worked with resonators on glass plates or with two or more screens of resonators placed parallel to each other and perpendicular to the direction of propagation and have evidently not considered the effect upon their results due to the multiple reflections between the various reflecting surfaces. On the other hand Blake and Fountain[2] while working with resonators pasted on glass plates encountered the peculiar phenomenon of a larger per cent. of incident energy transmitted and a smaller per cent. reflected when the resonators were of certain lengths than was transmitted or reflected by the bare glass. Blake and Fountain attributed the phenomenon to a change of refractive index due to change of responsiveness of the medium as required by the dispersion theory. Cartmel,[3] on the other hand, gave a different explanation based on the consideration of multiple reflections between the two surfaces of the glass plate. At bottom the two explanations are but different aspects of the same fundamental phenomenon, *i. e.*, phase change and change of velocity due to the varying response of the changed character of the medium. Blair,[4] following Stokes and Fresnel, derived the formulæ for the change of phase and also

[1] Garbasso and Aschkinass, Ann. der Physik, Vol. 53, p. 534, 1894. Aschkinass and Schaefer, Ann. der Physik, Vol. 5, p. 489, 1901.

[2] Blake and Fountain, Phys. Rev., Vol. 23, p. 257, 1906.

[3] W. B. Cartmel, Phys. Rev., Vol. 25, p. 64, 1907.

[4] W. R. Blair, Phys. Rev., Vol. 26, p. 76, 1908.

the intensity of the transmitted and reflected energy when electric waves are transmitted through thin plates. He considers the observed variations in intensity of both the transmitted and reflected energy in accord with the formulæ, but his study of the energy relations was very limited as he was more interested in the phase relations upon reflection.

About two years ago the writers began investigating the absorption of electric waves by several resonator screens in succession. Eleven screens were made by pasting tin-foil resonators on sheets of straw-board. These were placed one behind the other in a holder about half way between the vibrator and receiver. The screens were parallel to one another and inclined to the direction of propagation, making an incidence angle of 22.5°. The distance between them was approximately 5 cm. Measurements were taken of the energy transmitted by one screen, two screens, and so on, up to eleven. The energy fell off gradually for the first three screens but four screens were found to transmit about three per cent. more than three. For the next three screens, *i. e.*, the fifth, sixth and seventh, the energy again fell off gradually, but on placing the eighth screen in the holder no diminution in transmission was observed. On the contrary a slightly increased transmission was noted. When the distance between the screens was varied, these peculiar breaks in the orderly course of the absorption curve also varied, occurring for instance upon the addition of the fifth and tenth screens, or the third, sixth and ninth. Thus instead of building up a more or less homogeneous medium for electric waves, *i. e.*, one for which transmission varied regularly with the thickness, each screen of resonators acted like a new reflecting surface, and between each two screens "multiple reflections" took place, giving anomalous results which depended upon the combination of amplitudes and phase relations of the transmitted and reflected beams. The results were further complicated by the absorption of the medium. These phenomena, first observed in the attempt to solve a different problem, appeared sufficiently new and important to justify further study, so that the writers turned aside from their original purpose to seek a solution of these interesting interference phenomena complicated by resonance.

## 2. Apparatus and Methods of Measurement.

*Vibrator.*—The vibrator used was a special form of Righi vibrator,[1] chosen after several months' work with vibrators of the same general type, but of quite different construction. It was designed to meet the following needs: a vibrator which could be easily cleaned, easily and accurately reset at the focus of the collecting mirror after repolishing, and one which would have the smallest possible amount of metal and dielectric coming within the aperture of the mirror. The essential details of construction are shown in Fig. 1. Two steel balls *B*, 0.95 cm. in diameter, rested in sockets accurately ground in the upper ends of two glass tubes *TT′*, of one centimeter outside diameter. Below the aperture of the mirror *M*, these glass tubes were rigidly fastened to a plank by means of the hard rubber blocks *C* and about 10 cm. below the steel balls they were further stiffened by means of the hard rubber turn-buckle *A*. The screw

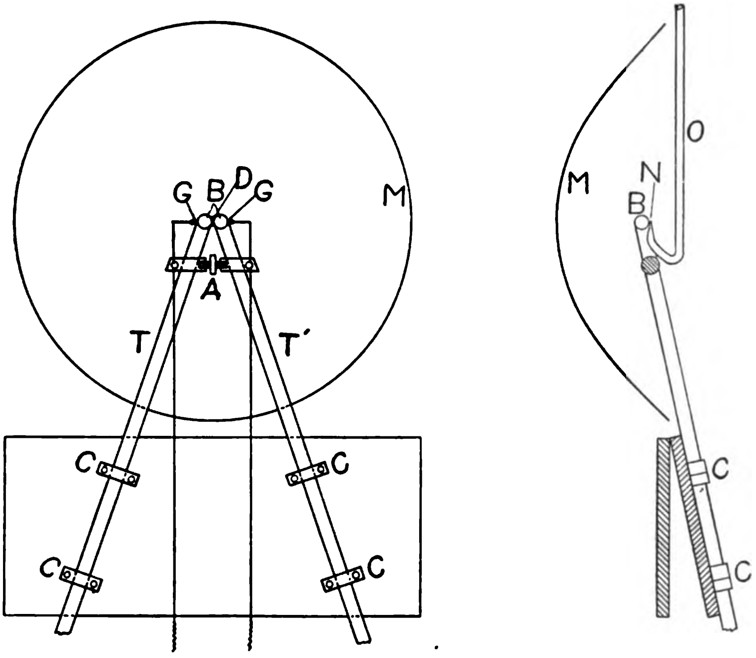


Fig. 1.

---

[1] Webb and Woodman, Phys. Rev., Vol. 29, p. 89, 1909.

of the turn-buckle was finely threaded and made a very satisfactory arrangement for adjusting the oil-gap. The turn-buckle also served as a convenient place for supporting the lead wires from the induction coil, which were bent at right angles just before coming to the air gaps *G* and terminated in small balls of copper made by fusing the ends of the wires in a Bunsen flame. The air gaps were about 3 mm. long and the spark took place along the axis of the vibrator. The source of energy was a 10-in. induction coil, with interrupter and condenser removed, connected to a 110-volt 60-cycle alternating current circuit. The current in the primary was usually kept at approximately seven amperes. The oil was fed through a glass tube *O*, ending in a capillary nozzle *N* directly in front of and slightly above the centers of the balls. By properly adjusting the flow of oil, a "film" *D* of any desired size would collect between the balls, the surplus of oil flowing down on the outside of the nozzle and dropping into a receptacle from the lowest point of the feed tube. The oil current served to dislodge any carbon bridges which might form due to the decomposing action of the spark as well as to keep a constant supply of fresh oil in the gap. As long as the film was kept constant and the length of gap small no trouble was experienced with the passing of the spark around the film instead of through it. Paraffin oil was used throughout these experiments.

The vibrator was set as nearly as possible at the focus of the parabolic mirror and then the final adjustments were made by setting up a bare receiver (one with no collecting mirror) 360 cm. from the vibrator and so adjusting the position of the vibrator that the distribution of energy in the reflected beam was symmetrical about the axis of the mirror in both the vertical and horizontal planes. This adjustment was tested from time to time, but it was not found necessary to reset the vibrator during a period of three months. In order to clean the vibrator it was only necessary to remove the balls, polish them and replace them in their sockets again, all of which could be done in less than five minutes without disturbing either the oil feed or the position of the vibrator with reference to the focus. This could be done at any time during a set of readings, as changing the vibrator usually made very little difference in the ratios of the galvanometer throws, especially when working with tuned receivers. Such a vibrator as this gives a much more symmetrical beam than does a vibrator having an enclosed oil receptacle. This is probably due to the small amount of dielectric in contact with the balls, as well as the small amount of metal and dielectric in the aperture of the mirror. The symmetry of the beam from a vibrator of this type will be more fully discussed under the section dealing with the parabolic mirrors.

*Receiver.*—Both tuned and non-selective[1] receivers of the Klemencic type were used. In all receivers the wings were made of copper foil 0.15 mm. thick and 3 mm. wide and were fastened securely to a wooden base 12 mm. wide by means of wax, the space between them being 1 mm. The galvanometer lead wires were soldered to one side of the inner ends of the copper strips, while directly opposite to them on the upper side of these strips were soldered the fine wires of the thermo-junction. For the less sensitive receivers the thermo-couple was made of commercial iron and constantan wires 0.04 mm. and 0.025 mm. in diameter respectively, while for the more sensitive receivers platinum and constantan wires approximately 0.0045 mm. in diameter were used. The fine wires were soldered to the ends of the copper strips with a free end about a millimeter in length extending beyond the end of the strips. Under a binocular microscope these free ends were looped around each other with a spring contact and then either welded by means of an electric spark or soldered. The resistance of the thermo-junction was carefully determined before and after each set of observations and the sensitiveness was assumed to remain constant as long as there was no change in resistance.

In order to eliminate as far as possible the error due to the deterioration[2] of the vibrator and the consequent change in the energy emitted, a check receiver[3] was used, constructed as nearly like the main one as possible and the readings from both receivers were taken simultaneously. The ratio of the two galvanometer throws was always taken as proportional to the energy received by the main receiver.

---

[1] Webb and Woodman, *l. c.*

[2] Blake and Fountain, *l. c.*

[3] Klemencic and Czermak, Wied. Ann., Vol. 50, p. 175.

Several tuning curves were taken, some of which are plotted in Fig. 2. The total lengths of the main receivers are plotted as abscissae and the corresponding ratios of the galvanometer throws as ordinates. For convenience in plotting most of the ordinates have been reduced so as to make 100 the highest point on the curve. $L_{\text{I}}$ shows a maximum at 23 mm., with an indication of a second maximum at 27 mm. In $L_{\text{II}}$ the maximum occurs at 24 mm. while in $L_{\text{III}}$ it comes at 25 mm. $L_{\text{IV}}$ shows a decided maximum at 27 mm., $L_{\text{V}}$ and $L_{\text{VI}}$ have somewhat broader maxima, approximately at 24 mm. in $L_{\text{V}}$ and at 27 mm. in $L_{\text{VI}}$. From a study of ten or more such curves the conclusion is drawn that the vibrator emits a short spectrum, and that whether the maximum energy is obtained for a receiver length of 23 mm. or 27 mm. depends upon the characteristics of the receiver. This coincides well with the conclusion reached by Willard and Woodman,[1] that the form of the energy curves depends upon both receiver and vibrator. All of the receivers used were as nearly alike as possible, yet the distribution of the dielectric and the distance apart of the copper strips may have varied sufficiently to slightly change the resonance length of receivers of which the total length was only 25 mm. The fact that the vibrator cannot be considered a source of monochromatic radiation will be of considerable importance when we come to the discussion of the interference curves. Twenty-four mm. was chosen as the length of receiver to use in all work involving the use of a tuned receiver. The non-selective receivers were 310 mm. long, or approximately thirteen times the length of the tuned receiver.

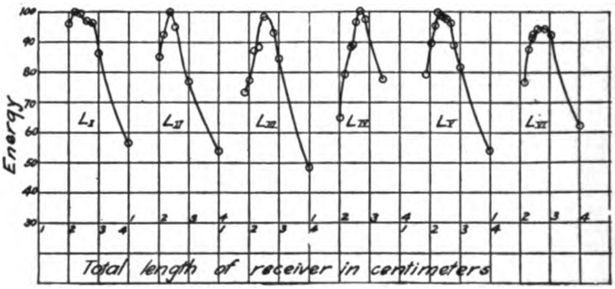


Fig. 2.

---

[1] Willard and Woodman, PHYS. REV., Vol. 18, p. 3, 1904.

*Parabolic Mirrors.*[1] — The vibrator and receiver were each mounted at the focus of a copper parabolic mirror (paraboloid of revolution), silver-plated on the reflecting surface. Each mirror had an aperture of 56 cm., a depth along the axis of 15.2 cm. and a focal length of 12.2 cm. As most of the results in this paper are independent of any effect the mirrors may have had in determining the wave-length, no attempt was made to avoid "mirror action"[2] in the selection of these mirrors. Each mirror was mounted with its optic axis parallel to the floor and could be brought into proper alignment by rotation about either a vertical or a horizontal axis.

As the object in using these mirrors was to obtain a parallel beam of energy, it seemed worth while to test the beam for parallelism. This was attempted in three ways. The first consisted in comparing the actual distribution of energy in a plane perpendicular to the optic axis with that calculated on the assumption that the vibrator represented a point source placed at the focus of the mirror. The second consisted in comparing the distribution of energy perpendicular to the optic axis at a distance of 240 cm. from the mirror with the corresponding distribution at a distance of 360 cm. In a parallel beam the distribution at the two distances should be the same, provided the distances are sufficiently large to disregard the direct radiation. The third method consisted in taking readings with the main receiver placed at successive distances from the mirror along the optic axis. Unless the beam was diverging or the energy absorbed by the medium through which the waves passed, these readings ought to remain constant. In all three cases the distribution of energy was measured with a bare receiver (one with no collecting mirror), tuned to the incident energy and mounted in such a way that readings could be taken well outside the limits of the beam either horizontally or vertically, always starting from the center of the beam and taking the readings first on one side of the center and then on the other.

The theoretical distribution of energy from a point source placed at the focus of a paraboloid of revolution may be derived as follows:

---

[1] The mirrors were head-light mirrors, bought from the Adams and Westlake Company of Philadelphia.

[2] Webb and Woodman, *l. c.*

Let $OP$, Fig. 3, be a section of the paraboloid having its focus at $F$. About $F$ as a center construct a sphere of any radius $r$.

Let $E_s$ equal the energy per sq. cm. passing through the surface of the sphere. This will be a constant for a sphere of given radius. The total energy passing through the annulus on the sphere of width $d\theta$ and situated at an angle $\theta$ from the axis of the mirror is $2\pi r^2 \sin\theta d\theta E_s$. This same energy is contained in the hollow cylinder of thickness $AB$. The area of the cross-section of the hollow cylinder is $2\pi y dy = A_1$.

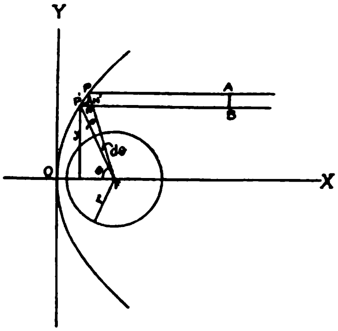


Fig. 3.

But $y = \rho \sin\theta$ where $\rho$ is the radius vector of the paraboloid and $dy = AB = \rho d\theta$. Hence $A_1 = 2\pi y dy = 2\pi\rho^2 \sin\theta d\theta$.

Let $E_p$ be the energy per sq. cm. passing through the plane $AB$. Then

$$E_p 2\pi\rho^2 \sin\theta d\theta = E_s 2\pi r^2 \sin\theta d\theta,$$

or

$$E_p = \frac{r^2}{\rho^2} E_s = \frac{r^2 E_s}{4p^2}(1 + \cos\theta)^2 = K(1 + \cos\theta)^2,$$

where $p$ is the focal length of the parabola and $K$ is a constant.

The theoretical curve can now be plotted by plotting as abscissæ the distances from the axis of the mirror perpendicular to the direction of propagation and for ordinates the intensities for the corresponding values of $\theta$. Thus for abscissæ

$$\rho \sin\theta = \frac{2p \sin\theta}{1 + \cos\theta} = K' \frac{\sin\theta}{1 + \cos\theta}$$

and for ordinates

$$E_p = K(1 + \cos\theta)^2.$$

For convenience in plotting $E_p$ is taken as proportional to

$$\frac{(1 + \cos\theta)^2}{4}.$$

The theoretical values are represented by the solid dots in Fig. 4, while the experimental values are represented by circles. Curves $V_{\mathrm{I}}$, $V_{\mathrm{II}}$, $H_{\mathrm{I}}$ and $H_{\mathrm{II}}$ represent the actual distribution of energy in the reflected beam. The abscissæ represent distances on either side of the center of the beam (the axis of the mirror), and the ordinates represent the energy expressed in per cent. of the energy at the center of the beam. It was found by trial that a metal screen having a circular opening 57 cm. in diameter placed 110 cm. from the vibrator cut off some of the direct radiation and hence gave a much more sharply defined *outside limit* to the beam. Most of the curves show that when the receiver had been moved 30 cm. from the center of the beam, which corresponds very closely with the half aperture of the mirror, the energy had decreased to approximately ten per cent. of its value at the center of the beam. For distances greater than 30 cm. the energy decreased very rapidly and at 40 cm. from the center of the beam barely five per cent. of the maximum energy could be detected.

$V_{\mathrm{I}}$ represents the vertical distribution at a distance of 360 cm. from the vibrator and $H_{\mathrm{I}}$ the horizontal distribution for the same distance. $V_{\mathrm{II}}$ and $H_{\mathrm{II}}$ are the corresponding distributions at a distance of 240 cm. from the vibrator. $V_{\mathrm{I}}$ and $H_{\mathrm{II}}$ correspond very closely with the theoretical distribution in a parallel beam. $V_{\mathrm{II}}$ and $H_{\mathrm{I}}$ do not agree so closely with the theoretical distribution, but even in these cases the agreement is remarkably close when one takes into consideration the fact that the vibrator is not a point source, as was assumed in working out the theoretical curve, and also the fact that the vibrator itself, together with its mountings, tends to disturb the symmetry of the beam. But perhaps the better test is to compare $V_{\mathrm{I}}$ with $V_{\mathrm{II}}$ and $H_{\mathrm{I}}$ with $H_{\mathrm{II}}$. Such a comparison shows that the beam is slightly diverging, and also that the divergence is slightly greater in the horizontal plane than in the vertical. This difference in divergence in two planes at right angles to each other is probably due to the lack of symmetry of the vibrator, one distribution being measured along a line parallel to the direction of the electric force and the other along a line perpendicular to the electric force. The fact that the receiver is nearly ten times as long as it is wide would also tend to give different results as it is moved parallel to itself either horizontally or vertically across the beam.

Curve *A*, Fig. 4, shows the results obtained by moving the bare receiver along the axis of the mirror in the direction of propagation of the incident energy. Readings were taken at every ten centimeters between the two points used in determining curves $V_{II}$, $V_I$, etc., *i. e.*, between 240 cm. and 360 cm. from the vibrator. Distances from the vibrator are plotted as abscissæ and galvanometer ratios, which are proportional to the energy, are plotted as ordinates. In moving the receiver a distance of 120 cm. towards the vibrator the energy nearly doubles, which would also indicate a diverging beam. The curve corresponds very closely to the distribution of energy in a beam diverging from a point source 110 cm. back of the vibrator, the intensity of the energy decreasing according to the inverse square law. If this inverse square relation were rigidly true, curve *A* could not be a straight line as plotted, but in any case such a limited portion of an inverse square curve taken at so great a distance from the origin would be very nearly a straight

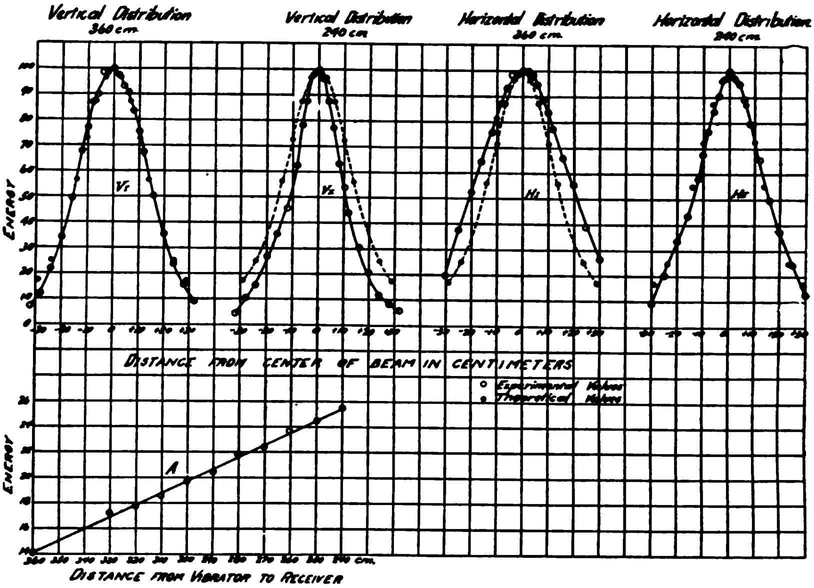


Fig. 4.

line. The angle representing the total divergence of the beam is approximately ten degrees, in which case the error would be very slight in considering a portion of the wave surface 60 cm. long taken at a distance of 350 cm. from the source as a plane wave front. An attempt was made to vary the divergence of the beam by moving the vibrator on either side of the focus along the optic axis of the mirror, but practically no change was found in the distribution of energy unless the vibrator was moved a centimeter or more from the focus, and as such a change seemed too large for continued use the vibrator was finally set back at the geometrical focus and kept there for the remainder of the work.

The three tests for parallelism showed that the beam was slightly diverging and yet nearly enough parallel to introduce no serious error in the experimental results which follow.

*Galvanometers.*—The galvanometer used for measuring the energy received by the check receiver was of the du Bois-Rubens two-coil type, triply iron-clad. Previous investigators[1] had found it necessary to increase the shielding from magnetic disturbances by adding additional armor. This reduced the throw due to outside disturbances to so small a quantity that it was disregarded. The sensitiveness of this galvanometer was $4 \times 10^{-9}$ volts. For measuring the energy of the main receiver was used a galvanometer of the type designed by Nichols and Williams,[2] with a sensitiveness of $2 \times 10^{-9}$ volts. The half-period of each galvanometer was 2.8 seconds.

### 3. Interference Curves with Tuned Receivers.

Airy[3] has shown that the intensity of the light transmitted and reflected by thin plates or films may be computed from the following formulæ:

$$I_T = \frac{a^2(1 - b^2)^2}{1 - 2b^2 \cos \delta + b^4}$$

for the transmitted energy, and

$$I_R = \frac{4a^2b^2 \sin^2 \frac{1}{2}\delta}{1 - 2b^2 \cos \delta + b^4}$$

---

[1] Blake and Fountain, *l. c.*

[2] Nichols and Williams, Phys. Rev., Vol. 27, p. 250, 1908.

[3] Sir G. Airy, Trans. Camb. Phil. Soc., Vol. 4, p. 419, 1830.

for the reflected energy, in which $a$ is the amplitude of the incident ray, $b$ the reflection coefficient when the reflection takes place in the rarer medium and $\delta$ is the phase retardation represented by the equation

$$\delta = \frac{2\pi}{\lambda} 2e \cos \omega,$$

$\lambda$ being the wave-length in the material of the film, $e$ the thickness of the film and $\omega$ the angle of refraction. These formulæ take no account of a possible phase change due to reflection or refraction and were therefore not directly applicable to the present problem. Professor R. C. Maclaurin assisted the writers in working out the following relations which more nearly express the conditions of the present investigation.

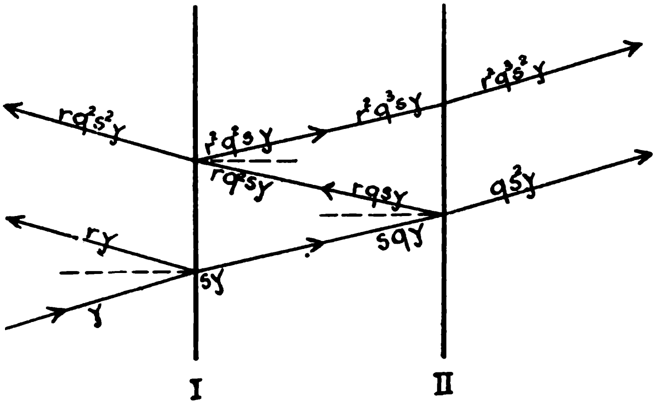


Fig. 5.

Let $I$ and $II$, Fig. 5, be two screens of resonators placed parallel to each other a known distance apart. An incident ray $y$ will give rise to multiple reflections between the screens and the first two transmitted and reflected rays will be as shown. The total energy in either the transmitted or the reflected beam will be equal to the sum of all these rays that emerge after one or more reflections or refractions, taking into account the amplitude and the phase of each ray at the time when the addition takes place. For the sake of showing the separate rays the figure has been drawn with the incident ray making a small angle of incidence, but in the derivation of the expressions for the intensities as well as in the experimental work the incident ray was normal to the reflecting surface.

We can represent the incident wave by the expression

$$y = A \cos 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right) = A \cos \theta,$$

equals the real part of $Ae^{i\theta}$; or, $y =$ the real part of $e^{i\theta}$, since we can assume the amplitude of the incident wave as unity.

Let $r = Be^{i\psi}$ be a complex number such that when it is multiplied by the expression for the incident wave it will give the amplitude and also the phase of the reflected wave. In case there is no absorption $B$ is the amplitude of the first reflected wave.

Let $s = Ce^{i\phi}$ be a similar complex multiplier for the transmitted wave. In case there is no absorption in the medium and no loss of energy at the reflecting surface, $B^2 + C^2 = 1$.

Let $q = e^{-Kx}\,e^{i\alpha}$ be a complex quantity such that when multiplied by the expression for the transmitted beam at one surface, it gives the value of the transmitted beam just previous to the next reflection or refraction. $K$ is the damping factor to take account of the loss of energy due to heating the resonators, scattering, etc., $x$ is the distance between the screens, $\alpha$ is the phase change due to passing a distance $x$ through the medium between the screens.

If $y$ represents the incident wave,

$ry$ will represent the first reflected wave,
$sy$ will represent the first refracted wave at surface $I$,
$qsy$ will represent the first refracted wave at surface $II$,
$qs^2y$ will represent the first transmitted wave,
and so on.

The total transmitted wave will be represented by

$$y_t = qs^2y[1 + r^2q^2 + (r^2q^2)^2 + \cdots] = \frac{qs^2y}{1 - r^2q^2}.$$

Substituting the values given above for $q$, $s$, $y$ and $r$, and writing $2\psi + 2\alpha = \delta$ we find

$$\frac{qs^2y}{1 - r^2q^2} = \frac{e^{-Kx}\,C^2 e^{i\alpha} e^{i(\theta+2\phi)}}{1 - B^2 e^{-2Kx} e^{i\delta}} = Re^{i\beta},$$

where $R$ is the amplitude of the transmitted wave and $\beta$ is the phase angle.

The value of $R$ is found to be

$$R = \frac{C^2 e^{-Kx}}{\sqrt{1 - 2B^2 e^{-2Kx} \cos \delta + B^4 e^{-4Kx}}}.$$

Therefore, since the intensity is proportional to the square of the amplitude,

$$I_t = K' R^2 = K' \frac{C^4 e^{-2Kx}}{1 - 2B^2 e^{-2Kx} \cos \delta + B^4 e^{-4Kx}}.$$

This reduces to the same form as Airy's formula if we put $K = 0$ and remember that $1 - B^2 = C^2$ on the assumption that the reflected energy plus the refracted energy is equal to the incident energy.

The expression for the transmitted energy will be nearly a maximum when

$$\delta = 2n\pi$$

and nearly a minimum when

$$\delta = (2n + 1)\pi,$$

In the expression

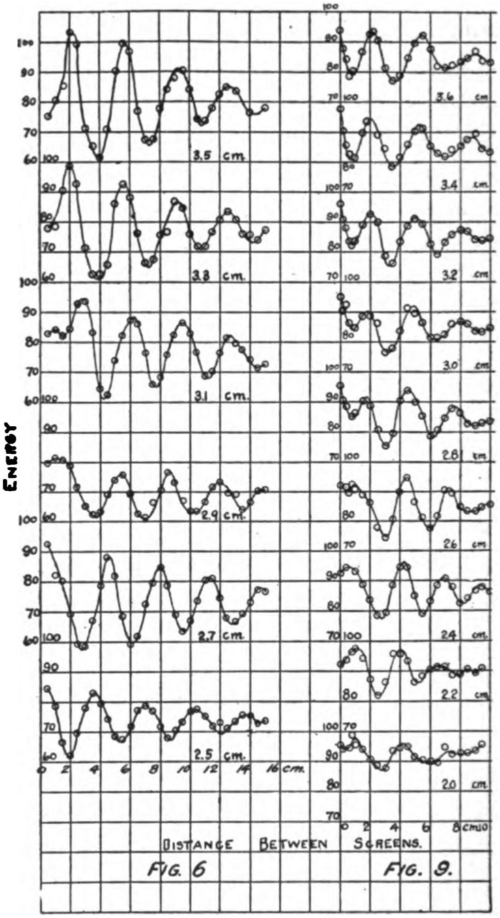
$$\delta = 2\psi + 2\alpha,$$

$2\psi$ represents the phase change due to reflection and would remain constant for any one wave-length and any given distribution of resonators on the resonator reflectors. $\alpha$, on the other hand, is the change of phase due to the passage of the wave through the distance between the screens. If now the distance between the screens is varied continuously through several wave-lengths, $\alpha$ and hence $\delta$ will take all possible values between zero and $2n\pi$, and the energy transmitted by the two screens will increase and decrease through a regular succession of maxima and minima.

This was tested in the following way. Half way between the vibrator and the main receiver there was placed a frame for holding the screens of resonators perpendicular to the direction of propagation. The first screen was fixed 180 cm. from the vibrator, and the second was so mounted that its distance from the first could be varied through a range of 20 cm. or more. The screens of resonators were made as nearly alike as possible. The resonators were pasted on straw-board nailed to wooden frames 86 cm. square, and consisted of strips of tin foil 2 mm. wide, and 3.5 cm. long, placed 1 cm. apart "end on," and 4 cm. apart "side on." On each screen were 18 columns and 21 rows.

The check receiver was 2.4 cm. long and was placed about 30 cm. directly in front of the vibrator. Owing to the lack of sensitiveness of the check and the lack of planeness of the reflecting screens, the effect of the reflected energy on the check was so small that it was disregarded. It was found necessary to change the position of the check in some later work with more sensitive receivers. The main receiver was also 2.4 cm. long and was mounted in the center of the beam 360 cm. from the vibrator. No collecting mirror was used. In order to further eliminate the errors due to change in ratios as the oil gap lengthened, alternate readings were taken on some standard condition and the condition under investigation. The average of the readings for any given distance apart of the screens was divided by the average of all the readings on the standard condition taken just before and just after the readings on the point in question. Inasmuch as the standard condition was a single screen, this quotient would represent the per cent. of the energy transmitted by the first screen alone that was transmitted by both screens.

The results with tuned receivers are plotted in Fig. 6. The distances between the screens are plotted as abscissæ. The ordinates marked 100 would mean that the same amount of energy was transmitted by two screens as by one, whereas the ordinate marked 60 would mean that two screens transmitted sixty per cent. of the energy transmitted by one alone. Six curves were taken, showing the results obtained with resonators of lengths 3.5 cm., 3.3 cm., etc., to 2.5 cm., the length for resonance being 2.9 cm. The curves all show very decided maxima and minima, which for the most part fall at regular intervals, the average distance between successive maxima or minima being 3.5 cm. This agrees fairly closely with the half wave-length of the incident energy, which is probably either 3.1 cm. or 3.2 cm. If the vibrator was emitting anything like monochromatic radiation we would expect the distance between the successive maxima or minima to equal the half wave-length in air, but when we remember that the wave-length determination is at best only an average of a very complex emission, and that the interference curves are taken under conditions where we get interference between certain groups of wave-lengths at a time depending on the distance between the reflecting surfaces, similar to the interference of white light when falling on a film of oil of varying thickness, it does not seem so remarkable that the half wave-length does not check more closely than ten per cent.



Figs. 6 and 9.

It is interesting to note in Fig. 6 the change of curvature within the first two centimeters as the length of the resonators is changed. The first two curves start out with a very decided upward slope, while the last two have a downward slope in the same region, the change apparently occurring at or near the resonance length of the resonators. We may regard this merely as a shift of the maxima and minima to the right or the left. The position of the first maximum is undoubtedly determined in part by the phase change due to reflection. Although the curves do not admit of accurate numerical calculations of this phase change, it seems justifiable to conclude that the change in position of the first maximum indicates that the phase change due to reflection is not a constant for all lengths of resonators. On the contrary, it takes different values as the length of the resonators is varied, at least when the lengths are taken near the resonating length.

The change in the per cent. of energy transmitted by two screens becomes less and less as the distance between the reflecting surfaces increases. By the time the distance between the screens has become equal to two wave-lengths, the per cent. of energy transmitted by two screens has become nearly constant, being about seventy or eighty per cent. of the energy transmitted by the single screen. This apparent decrease in the energy is probably due partly to the complexity of the radiation and partly to the loss of energy by absorption in the resonators themselves as well as by scattering.

## 4. INTERFERENCE CURVES WITH NON-SELECTIVE RECEIVERS.

(*a*) *By transmission.*—The arrangement of the apparatus for the interference curves obtained by transmission through two screens of resonators with non-selective receivers is given in Fig. 7. $M_1$ and $M_2$ are the parabolic mirrors, the vibrator, $V$, being at the focus of $M_1$ and the main receiver, $MR$, at the focus of $M_2$. The resonator screens were placed at $S$, halfway between the vibrator and the Boltzmann mirrors, $BM$. The energy transmitted by the resonator screens was reflected from the Boltzmann mirrors in the direction of $M_2$, the angle between the incident and reflected rays being 20°. $S_2S_2$ show the position of the screen used to cut off part of the direct radiation and so make the distribution of energy in the reflected beam correspond more closely to that in a parallel beam. At $S_1$ was placed a small zinc screen covering up the lower portion of the aperture of $M_1$. It was found that with sensitive receivers the energy of the check would increase nearly twenty per cent. due to the energy reflected back from the resonator screens. As the focus of the parabolic mirror was inside the plane of the aperture no position for the check could be found where it would not receive this reflected energy and still receive direct radiation from the vibrator. By placing the screen at $S_1$ a position could be found for

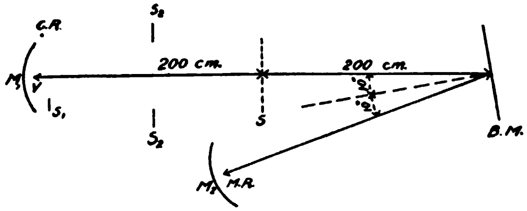


Fig. 7.

the check receiver, $CR$, where the readings of the check did not vary when readings were taken with and without the screen $S$ in position.

The Boltzmann mirrors were placed in the position indicated, in order that wave-length determinations could be made using the energy which had been transmitted by the screens. For the interference curves the two mirrors were set in the same plane and used simply as a metal reflector. Each of the mirrors was made of plate glass, 76 cm. wide and 38 cm. high, covered with tin foil. They were mounted on vertical wooden frames with horizontal arms which in turn were made to slide on cross pieces on a rigid wooden frame. A more detailed description of the Boltzmann mirrors may be found in an earlier paper by the writers.[1]

The resonators were pasted on tracing cloth which had been moistened and stretched as tightly as possible over an artist's frame for stretching canvas. Two screens of resonators were pasted, each containing eighteen columns and twenty-nine rows. The resonators were 3.6 cm. long at the start, 3.0 cm. apart "side on" and 1.0

---

[1] Webb and Woodman, *l. c.*

cm. apart "end on." The resonators were shortened 0.2 cm. each time until the final length was 2.0 cm.

The non-selective receiver was chosen for this set of interference curves on account of the complex character of the energy given out by the vibrator. In fact the expression "tuned receiver" has very little meaning when used in connection with energy that has been transmitted through one or more screens of resonators. A receiver that is in tune with the incident energy without the screen of resonators is no longer in tune with the energy that has come through the screen. And in work of this kind where several different lengths of resonators are used a receiver of fixed length cannot possibly be in tune with the energy transmitted through the various lengths of resonators. This conclusion is based partly on a study of tuning curves taken with resonator screens placed in the path of the beam, but largely upon a study of the wave-length curves to be given later.

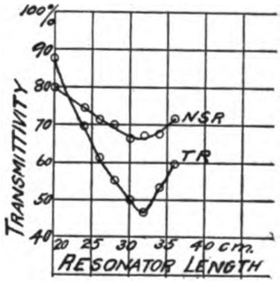


Fig. 8.

A comparison of the results obtained with the two types of receivers is given by the curves in Fig. 8. The abscissæ represent the lengths of resonators in centimeters and the ordinates represent the per cent. of the incident energy which is transmitted by the screens on which were various lengths of resonators. The curve marked *TR* gives the results obtained with a tuned receiver, while the curve marked *NSR* gives the results with the non-selective receiver. The most striking difference is the flatness of the curve taken with the non-selective receiver in comparison with the sharp minimum of the curve taken with the selective receiver. The latter very closely resembles the curves obtained by Blake and Fountain,[1] and would give 3.2 cm. as the length of resonator for a minimum of transmission and a maximum of reflection. While the curve taken with the non-selective receiver is too flat to admit of very great accuracy in locating the minimum of transmission, yet it seems to agree very closely with that obtained with the tuned receiver.

---

[1] Blake and Fountain, *l. c.*

Either receiver would measure the same per cent. of the incident energy when this energy has been transmitted through resonators 2.2 cm. long. Undoubtedly the curves would also have crossed for some length of resonator longer than the resonator length, but no attempt was made to find this length. From a study of such curves as these it seems to the writers that for work of this kind the non-selective receiver has very marked advantages over the tuned receiver, expecially when one is dealing with energy as complicated as that emitted by a Righi vibrator. The chief objection to the use of the non-selective receiver lies in the fact that the ratios of the energies measured by the two receivers decrease very rapidly as the oil gap becomes foul. This difficulty was overcome in a very satisfactory manner by adopting some standard condition with which readings could be taken as often as desired. For most of the work five readings were taken on the standard condition and four on the condition under investigation, always beginning and ending with the standard condition. The five readings on the standard condition were then averaged and also the four on the condition under investigation. Finally the ratio of the two averages was taken for the basis of comparison as well as for use in plotting the curves. These ratios usually agreed within two per cent., in which case only two sets of observations were taken for any given set of conditions, but in case the disagreement was more than two per cent. three and sometimes four sets of observations were taken.

The transmission interference curves taken with the non-selective receivers are given in Fig. 9. For the purpose of comparison they are plotted to the same scale as those obtained with the tuned receivers, the numbers just below each curve being the resonator length in each case. Most of the curves show well-defined maxima and minima, the average distance between two successive maxima or minima being 3.2 cm., which is a very close agreement with the half wave-length as determined with non-selective receivers. The total change in energy between the highest maximum and the lowest minimum is usually less than twenty per cent. of the energy transmitted through one screen, whereas with the tuned receiver it sometimes amounted to as much as forty per cent. The energy approaches a constant value much more rapidly than with the tuned receivers, so that it was useless to increase the distance between the screens beyond 10 cm. The last curve taken with a resonator length of 2.0 cm. is very flat and the maxima and minima disappear when the distance between the screens is only six centimeters. For the first seven curves the constant value towards which the energy approaches was approximately 85 per cent. of the energy transmitted by the single screen. However, when the resonators are 2.2 cm. long, this constant value is 90 per cent., and for resonators 2.0 cm. long it is approximately 92 per cent. This means in the last curve that the two screens placed six centimeters or more apart will transmit 92 per cent. of the energy that would be transmitted by a single screen, whereas, if the resonators were 2.6 cm. long, the same two screens placed 10 cm. or more apart would transmit 85 per cent. of this energy. A comparison of the positions of any particular maximum or minimum, like the second or third maximum for instance, in the curves taken in order down the page shows a gradual shifting of the position of the maximum towards the left. This shift is probably due to the fact that the change of phase due to the reflection gradually changes as the resonators are shortened. In some of the curves certain irregularities occur within the first centimeter or two. These are probably due partly to experimental errors and partly to other causes for which no adequate explanation can be given.

One set of readings was taken using three screens of resonators. The two screens nearest the vibrator were kept at the constant distance apart of 4.5 cm. This distance was chosen because with it the energy transmitted by the two screens was neither a maximum nor a minimum, in fact it corresponded to a point on the axis of the interference curve. The resonators on each of the screens were 3.6 cm. long and were in all respects like the screens already described for the work with the non-selective receivers. The distance between the second and third screens was increased from zero to six centimeters, usually adding 0.5 cm. each time, and the energy transmitted by the three screens determined. The energy passed through a series of maxima and minima as for the two screens. Two successive maxima or minima were found to be 3.2 cm. apart, and the total change in energy amounted to about fifteen per cent. of the energy transmitted through the two screens, which was taken as the standard for comparison.

(*b*) *By Reflection.*—The arrangement of the apparatus for the work by reflection was very similar to that ordinarily used for wave-length determinations with the interferometer.[1] The separating surface[2] consisted of a wooden frame 101 cm. by 96.5 cm., on which were stretched strands of no. 34 copper wire spaced one centimeter apart. The frame was mounted with the wires horizontal, or parallel to the electric force. The energy reflected from this wire grid agreed with the transmitted energy within less than two per cent. The movable mirror was a piece of plate glass 76 cm. by 77 cm. covered with tin foil. It was mounted in the same position as that occupied by the resonator screen in the work on transmission, and could be replaced at any time by the resonator screen. The distance between the vibrator and the movable mirror was 200 cm. while the distance between the vibrator and the center of the wire grid was 95 cm. The fixed mirror was also of plate glass covered with tin foil, 76 cm. square, and was placed 105 cm. from the center of the wire grid. The distance between the main receiver and the center of the wire grid was 150 cm.

For determining the interference curves the fixed mirror was replaced by a large metal screen supported in such a way that the energy falling upon it was deflected to one side and produced no effect upon the receiver. The movable mirror was removed and the two tracing cloth screens already described were put in its place. The position of the check was not changed from that shown in Fig. 7. The curves marked $R$ in Fig. 10 show the results obtained by reflection. The screen nearest the vibrator was kept in the same position throughout the observations and the energy reflected from it was used as a standard condition. Four readings were taken with two screens a known distance apart alternating with five readings on the standard condition, always beginning and ending with the standard condition. Then the readings of the energy reflected from one screen and those of the energy reflected from two screens were averaged separately and the latter average divided by the

---

[1] Willard and Woodman, *l. c.* Blake and Fountain, *l. c.*

[2] G. F. Hull, PHYS. REV., Vol. 5, p. 231, 1897.

former. These ratios are plotted as the ordinates in Fig. 10, and the distances between the screens are plotted as abscissæ. For the purpose of comparison, the corresponding curves by transmission are plotted just below the curves by reflection. The two curves, *i. e.*, either reflection curve with its corresponding transmission curve, are not plotted to the same scale on the *Y*-axis and no attempt was made to compare the absolute energy in the two beams. The same scale is used throughout for the abscissæ, so that the curves can be compared with reference to the position of the maxima and minima. Such a comparison shows that aside from a few irregularities within the first two centimeters, the corresponding curves agree very closely with each other, the maxima on the reflection curve

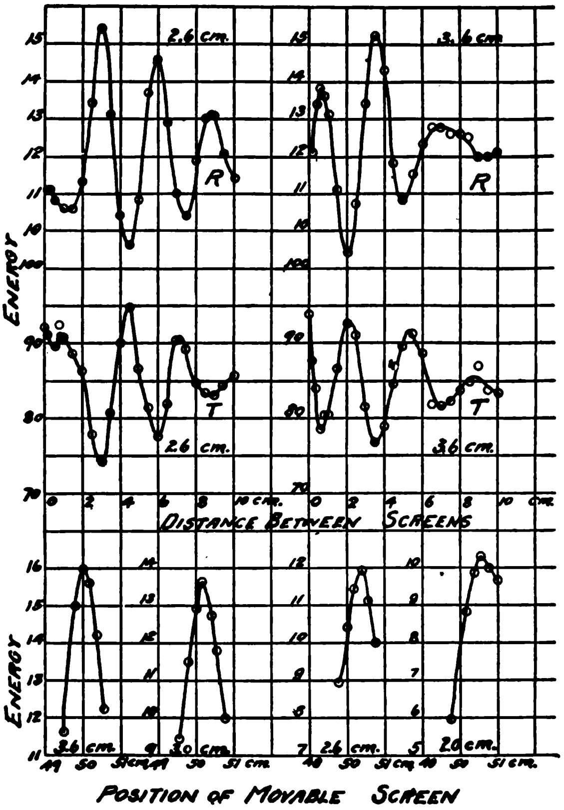


Fig. 10.

falling at the same points as the minima on the transmission curve and vice versa. This also agrees well with the deductions which can be drawn from Airy's formulæ for the intensity of the reflected and transmitted energy. The resonators used in determining the two curves plotted were 2.6 cm. and 3.6 cm. long respectively.

The phase change due to reflection was studied as follows: The fixed mirror was replaced in its proper position and one of the screens of resonators was used in place of the movable mirror. Readings were taken around what would correspond to the center maximum on the ordinary wave-length curve as determined with the interferometer. If there is no phase change due to reflection the position of the center maximum should not change whether the readings are taken with the movable mirror in that arm of the interferometer, or with the screen of resonators substituted in place of it. Furthermore the position of the center maximum should not change for different lengths of resonators on the screen. The results of these tests are given in the four curves at the bottom of Fig. 10, from which it is seen that there is a gradual shifting of the center maximum as the resonators are shortened. By shortening the resonators from 3.6 cm. to 2.0 cm. the position of the center maximum has been shifted 6 mm., which would correspond to the phase change produced in the wave while going a distance of 1.2 cm. This corresponds very well with the variation of the phase change due to reflection as shown in the interference curves obtained by transmission. See Fig. 9. The position of the center maximum for resonators 2.0 cm. long was just the same as for the movable mirror, which would seem to indicate that the phase change due to reflection for resonators of this length was zero. The whole question of phase change upon reflection should be more carefully investigated than the writers have attempted to do in this work.

## 5. Wave-length Determinations.

Another factor that must be taken into consideration when sending the complex radiation from a Righi vibrator through the screens of resonators is the *selective* reflection which takes place, especially if the resonators are near the resonating length. The phenomena which take place as the length of the resonators is gradually shortened through the resonating length may be easily understood, at least qualitatively, from a consideration of Fig. 11. The continuous curve represents an assumed distribution of energy in the short spectrum of a Righi vibrator, the wave-lengths being plotted as abscissæ and the corresponding intensities as ordinates. The dotted curves represent the assumed transmission curves of a screen of resonators for five different lengths of resonators, the curve *C* corresponding to the resonating length. The same coordinates are used for these curves as for the continuous curve. When the resonators are of the length corresponding to curve *A* they will transmit the energy from the vibrator with practically no change of wave-length, by which is meant the wave-length as determined with the Boltzmann mirrors or the interferometer. But when the resonators are of the length corresponding to *B* they will transmit much more of the shorter wave-lengths than of the longer and the resultant wave-length as measured by the Boltzmann mirrors and a nonselective receiver will be shorter than that measured without the screen. When the resonators correspond to *C* they will transmit the shorter and the longer wave-lengths of the spectrum nearly equally but the intermediate wave-lengths with much less intensity. Now the wave-length determination will no longer give a smooth curve, especially around the second maxima. We would expect that such a combination of wave-lengths as this would give a very broad second maximum on either side of the center maximum. But when the resonators correspond to *D* they will transmit more of the longer wave-lengths than of the shorter and the resultant wave-length will accordingly be longer than that obtained without the screen. Resonators of the length corresponding to *E* transmit all of the wave-lengths represented in the characteristic curve of the vibrator with practically no change in relative intensity.

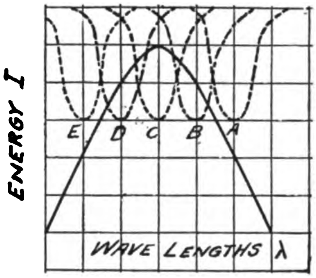


Fig. 11.

That this reasoning is correct, at least qualitatively, is clearly shown by the results plotted in Fig. 12. The arrangement of the apparatus was the same as that shown in Fig. 7, and used for the interference curves by transmission with non-selective receivers. For each length of resonators used in obtaining the interference curves a wave-length curve was taken by means of the Boltzmann mirrors. The non-selective receivers were also used for the wave-length determinations, thus doing away with the many errors[1] introduced by the use of selective receivers in wave-length work. For the curves given in Fig. 12 the energy was transmitted through one screen of resonators. The distances between the two Boltzmann mirrors are plotted as abscissæ and the corresponding intensities as ordinates. The readings were taken with reference to some position of the mirrors taken as a standard, a reading being taken for the standard position, then on the point in question, and finally

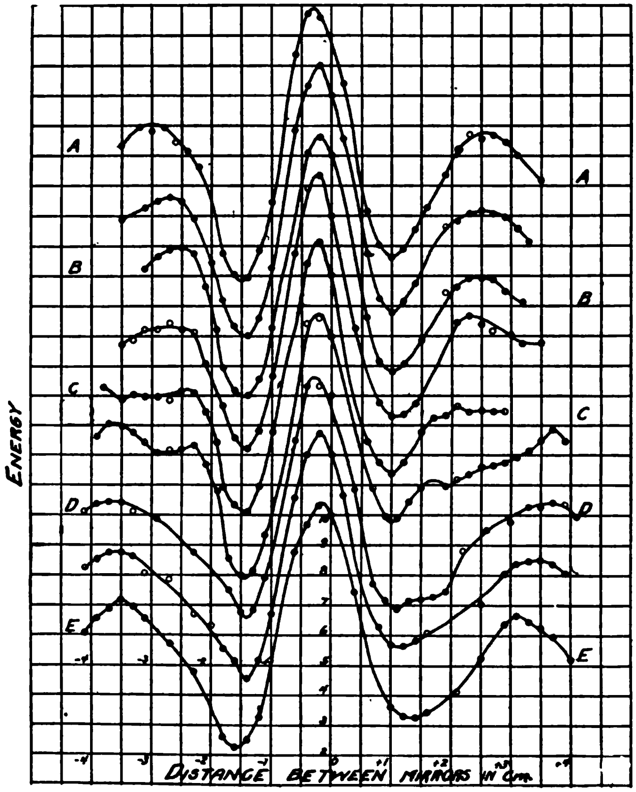


Fig. 12.

---

[1] Webb and Woodman, *l. c.*

on the standard again. The standard position was usually taken as near the center of a maximum or a minimum as possible, and then the ratio of the readings on the point in question to the average of the readings on the standard taken just before and after the point in question was computed. Then these ratios were multiplied by the proper factor to bring them into the proper relation to the center maximum, so that, as plotted, any ordinate represents the energy for the corresponding abscissa with reference to the energy received by the receivers for zero difference in path.

One great error in work of this kind arises from the fact that the characteristic curve of the emission of the vibrator changes as the spark gap deteriorates. This changes the selective action of resonators of a given length during a long set of observations and tends to make the wave-length curves irregular. For this reason the writers have made no attempt to use the curves in Fig. 12 for other than qualitative comparisons. The action of the receiver mirror as well as the comparatively large angle of incidence at the Boltzmann mirrors also tend to make the results unreliable for quantitative measurements, but do not make them any the less valid for the qualitative comparison intended by the writers. The curves in Fig. 12 are lettered to correspond at least roughly to the dotted curves in Fig. 11. The shortening of the wave-length as the resonators are shortened is very clearly marked in the first three curves. The next two show a flattening of the second maxima with the maximum energy on the side of the shorter wave-lengths, while the next two show the same flattening of the second maxima with the maximum energy on the side of the longer wave-lengths. In the last two the second maxima have become much sharper and indicate a wave-length longer than the true wave-length of the vibrator and gradually decreasing to the true wave-length.

Applying the same reasoning to the reflected energy one would expect that the wave-length of the reflected energy would be shorter than the true wave-length of the vibrator for the same length of resonators that gave a lengthening of the transmitted wave-length. This was tested by means of the interferometer arrangement already described in connection with the interference curves by reflection and found to be the case. The screen of resonators was used as the movable mirror in the interferometer and the wave-length curves taken in the usual way. For resonators 3.6 cm. long the wave-length by reflection was lengthened about ten per cent. of the true wave-length, while for resonators 2.0 cm. long it was shortened about the same amount. The curves are not given as they are very similar to those of Fig. 12.

## 6. RESONATORS ON GLASS.

Further data bearing upon multiple reflections was obtained by substituting a large piece of plate glass for the tracing cloth screens. The two reflecting surfaces in this case were the two surfaces of the glass plate. The glass surfaces themselves reflected a large per cent. of the incident energy, but the reflection coefficients of one or both surfaces could be varied by the addition of resonators of different lengths. The plate was 96 cm. by 122 cm. and was placed in the beam perpendicular to the direction of propagation. The vibrator and receiver were placed 360 cm. apart and the plate was arranged on a carriage so that readings could be taken with the plate in and out of the beam alternately. The readings without the plate in the beam were used as a standard, so that the ratios of the readings "in" to the readings "out" give us the transmittivity of the plate under the given condition. The readings were taken with both tuned and non-selective receivers.

The results are plotted in Fig. 13. Resonator lengths are plotted as abscissæ and the transmittivity as ordinates. Curves *I* and *II* were taken with a glass plate 0.8 cm. thick, curve *I* being taken with resonators pasted on only the surface of the glass towards the vibrator, and curve *II* with the resonators on both surfaces, those on one surface being directly opposite those on the other. In curve *I* the reflection coefficient for one surface of the glass was constant, while that for the other surface varied as the resonators were shortened. In curve *II* the reflection coefficients for both surfaces were changed whenever the resonators were changed. The resonators were 3.0 cm. long at the start, 0.5 cm. apart end on, and 3.0 cm. apart side on, and there were thirty rows and twenty-three columns. The curves marked *NSR* were taken with the non-selective receiver, those marked *TR* with the tuned receiver. The curves with the non-selective receiver tell us very little about the length of resonators for maximum resonance. Apparently the minima fall at the same places as those for the tuned receiver. With the tuned receivers the minima of transmission at the resonance length fall at 1.5 cm. for both curves *I* and *II*, showing that the addition of the resonators to the other surface of the glass did not change the length for resonance. It did slightly change the form of the curve, but more particularly the per cent. of the original energy that was transmitted. The transmittivity is in general less for the plate having the resonators on both sides. This may be due in part to increased scattering, but is more probably due to the different combinations of reflection coefficients and phase changes due to reflection in the two cases. It is interesting to note that for resonators approximately 1.0 cm. long the transmittivity is the same for either type of receiver.

Fig. 13.

Curve *III* was taken with a plate of glass 0.6 cm. thick. The resonators were 6.0 cm. long at the start, 3.0 cm. apart side on and 1.0 cm. end on, and there were thirty rows and twelve columns. Very little need be said about the curve taken with the non-selective receiver, as the total change in the transmittivity was so slight that it made accurate work with such a receiver very difficult. The curve is published merely to show that the change in the total energy is very much less marked than the change for a particular group of wave-lengths in resonance with a tuned receiver.

If we compare curve *I* with curve *III*, we see that the transmittivity for bare glass depends upon the thickness of the glass. For the plate 0.6 cm. thick the transmittivity was 51 per cent. of the incident energy, while for the plate 0.8 cm. thick the transmittivity was 59 per cent., using the tuned receiver curves for the basis of comparison. This agrees well with what we might expect from the results of the work on the transmission of two screens of resonators on tracing cloth. If we should measure the transmittivity of a large number of glass plates of different thicknesses, we would undoubtedly find that the transmittivity would pass through a succession of maxima and minima, only of course in this case the results might be further complicated by the possible absorption of the glass. In all probability the thicker of the two plates used absorbed more than the thinner plate and yet it transmitted 8 per cent. more of the incident energy than the thinner plate.

There is no evidence of *extra-transmission* in either curves *I* or *II*, unless it be for resonators less than 1.0 cm. long. For resonators 0.5 cm. in length the transmittivity is apparently one or two per cent. higher than for bare glass. Of course it is difficult to measure to this degree of accuracy and the apparent increase in transmittivity may be due to experimental error, but the readings were repeated several times and the higher transmittivity always came when the resonators were on the glass. The fact that it occurs in both curves *I* and *II* makes it seem more probable that it is a case of *extra-transmission*. In curve *III* the transmittivity for the plate when the resonators were 2.3 cm. long was equal to that for the bare glass, which is only a special case of *extra-transmission*. As this curve was taken with the same plate and with the same distribution of resonators as that used by Blake and Fountain[1] it might be interesting to compare the curve with the tuned receiver with the corresponding curve (curve *T*, Fig. 9) in their paper. It must be borne in mind that their results were obtained with 10.0 cm. waves, while the wave-length used by the writers was only 6.0 cm. approximately. The length for resonance was 2.55 cm. for the 10 cm. waves and 1.5 for the 6.0 cm. waves. They found the greatest extra-

---

[1] Blake and Fountain, *l. c.*

transmission when the resonators were twice the resonating length, while the length for *equal transmission* in curve *III* is approximately 1.5 times the resonating length. This seems to strengthen the idea of Blake and Fountain that the *extra-transmission* does not always come for lengths twice the resonating length but may come for "any length greater than the fundamental length." The writers are of the opinion that it may also occur for lengths shorter than the resonating length. In fact in several of the curves published by Blake and Fountain the transmittivity is the same as that for bare glass when the resonators were still a centimeter or more in length, and it is an interesting conjecture as to what form the curves would have taken if the readings had been taken for lengths shorter than one centimeter. Comparing the two curves further we notice that the change in transmittivity is very much less for the 6.0 cm. waves than for the 10.0 cm. waves, varying from 45 per cent. at the minimum to 51 per cent. at the highest point for the 6.0 cm. waves and from 38 per cent. to 76 per cent. for the 10.0 cm. waves. Also the transmittivity of the bare glass was different for the different wave-lengths, being 51 per cent. for the 6.0 cm. waves and 60 per cent. for the 10.0 cm. waves. That this was not due entirely to a greater absorption of the shorter wave-lengths is well brought out in the following paragraph.

Using the adjustable system of rod vibrators described in another paper by the writers[1] and for which the wave-lengths were fairly accurately determined, the transmittivity of a plate of glass was measured for six different wave-lengths. For this purpose the plate of glass 0.8 cm. thick was used with no resonators on either side. It was mounted perpendicular to the direction of propagation of the incident energy and could be easily moved in or out of the beam. The energy without the plate in the beam was again used as a standard, the readings being taken alternately "out" and "in," always beginning and ending a set of readings with the standard condition. Readings were taken with both the tuned and non-selective receivers, the length of the check receiver as well as that of the main being changed each time to correspond to the wave-length used. The length of the non-selective receiver, was kept constant at 60.0 cm. The results are given in the accompanying table.

---

[1] Webb and Woodman, *l c.*

| Total Length of Vibrator. | Receiver Length (Tuned Receiver). | Approximate Wave-Length. | Per Cent. Transmitted. | |
|---|---|---|---|---|
| | | | Tuned Receiver. | Non-selective Receiver. |
| 6.55 cm. | 6.60 cm. | 15.7 cm. | 56.8 per cent. | 68.2 per cent. |
| 5.05 | 5.05 | 12.1 | 64.5 | 64.0 |
| 3.78 | 3.80 | 9.1 | 53.7 | 55.0 |
| 3.21 | 3.20 | 7.7 | 41.0 | 53.4 |
| 2.50 | 2.50 | 6.0 | 50.5 | 61.7 |
| 2.00 | 2.00 | 4.8 | 69.0 | — |

It will be seen that for the tuned receiver the transmittivity varies from 41 per cent. to 69 per cent. for different wave-lengths. If the change had always been either an increase or a decrease, it might have been ascribed to the varying absorption by the glass of different wave-lengths. But if it had been due to absorption we would expect to get the greater absorption and hence the smaller transmittivity for the shorter wave-lengths, but this is clearly not the case. The fact, too, that the thickness of the plate is, even for the shortest wave-length, only one sixth of a wave-length makes the explanation based upon absorption seem very improbable. As one would expect, the variation in the transmittivity for the non-selective receiver is much less than for the tuned receiver. Here, too, the transmittivity decreases for the first four wave-lengths tried and then increases again as the wave-lengths are made still shorter. These results all show that the transmittivity of a plate of glass of given thickness and dielectric constant passes through a series of maxima and minima for varying wave-lengths, so that in specifying the transmittivity of a given plate of glass it is always necessary to specify for what wave-length the transmittivity is found.

## 7. CONCLUSIONS.

As a result of the present study it appears:

1. That a Righi vibrator of small dimensions, mounted with no oil holder surrounding the balls and with as little dielectric as pos-posible in front of the aperture of the mirror and placed at the focus of a paraboloid of revolution acts very nearly like a point source and gives very nearly a parallel beam of energy.

2. That the tuning curves as well as the wave-length curves show that the radiation from a Righi vibrator consists of a short spectrum

of wave-lengths and that the maximum on the tuning curves is governed within the limits of this spectrum by the peculiarities of the receiver.

3. That the non-selective receiver is far superior for all work involving the determination of wave-lengths or the tuning of resonators pasted on screens and placed in the path of the incident energy.

4. That the intensity of the transmitted and reflected energy passes through a series of maxima and mimina as the distance between the reflecting surfaces is increased through a small number of wave-lengths. This agrees well with the theory as worked out for optics.

5. That a screen of resonators acts like a selectively reflecting surface for waves very nearly in resonance with the resonators. This was tested by determining the wave-length of the transmitted beam as well as of the reflected beam. These were found to check at least qualitatively with what one would expect from a consideration of an assumed emission curve for the vibrator combined with an assumed transmission curve for the screen of resonators.

6. That with a tuned receiver *extra-transmission* may be expected for plates of glass of the proper thickness and of the proper dielectric constant and for wave-lengths within proper limits.

7. That the transmittivity of a glass plate of a given thickness and given dielectric constant depends upon the wave-length of the incident energy.

8. That the change of phase due to reflection is not a constant but varies with the length of the resonators pasted on the reflecting surface.

In conclusion the writers wish to thank Mr. H. W. Farwell and Dr. L. B. Morse for kindly assistance in taking some of the observations. They also wish to acknowledge their indebtedness to Professor M. I. Pupin who gave the services of his assistant, Mr. W. E. Cushman, during an illness of one of the writers. They are indebted further to Professor E. F. Nichols who suggested the problem and many details of the method for solving it.

Phœnix Physical Laboratories,
Columbia University, February, 1909.

# THE ABSORPTION OF SODIUM VAPOR.

BY GEO. B. CLINKSCALES.

## HISTORICAL.

THE fluted absorption spectrum of sodium vapor was first observed by Roscoe and Schuster in 1874. Subsequent investigations were made by Liveing and Dewar in connection with their work on the reversal of lines of metallic vapors, and also by Wood. Nine bands were photographed in the green and some others were observed visually in the red region. These bands they found consisted of a great number of fine lines, more or less regularly spaced along the region of absorption.

The fluorescence of the same vapor, excited by white light, was studied by Wiedemann and Schmidt[1] in 1896.

In 1904 Wood and Moore[2] made an investigation of both the absorption and fluorescence, endeavoring to find how the two were related, also to determine the effect of changing the wave-length of the exciting light. They proved conclusively that the two are in every way complementary, *i. e.*, an absorption band in one corresponds in position to a fluorescent band in the other. The presence of hydrogen was found to magnify greatly the appearance of the fluorescence, and to cause its disappearance when the pressure was above a few centimeters. Since these phenomena are so closely related it is perfectly natural to suppose that a study of the effects of pressure upon absorption will throw some light upon the mechanism of fluorescence.

Quite recently A. Dufour[3] has found that the absorption of bromine vapor and some other gases is greatly modified by the presence of some chemically inert gas above atmospheric pressure, and although fifty or more lines were found to shift, others were not.

---

[1] Wied. Ann., VII., 447, 1896.

[2] Phil. Mag. (6), 1904.

[3] Compt. Rendus, 145, 757–758, 1907.

Dufour investigated separately, and under the same conditions the effect of hydrogen, CO and air, all producing the same effect. This points to a purely mechanical effect as distinguished from a chemical one.

Wood in 1906[1] found that the absorption spectrum of sodium vapor is fluted in the presence of hydrogen at atmospheric pressure and above, but that at very low pressures the fluted form entirely disappears. These observations were made under comparatively low dispersion; and at the suggestion of Professor Wood the investigation to be described was begun in order to study the same phenomena with apparatus having higher dispersion.

The effects which will be considered in this paper should not be confused with the usual "pressure shifts" studied by Humphreys. It is probable that they are more nearly analogous to the phenomena described by Wood in the case of mixtures of mercury vapor with hydrogen, nitrogen, helium and other chemically inert gases in varying proportions.[2]

In this paper it was shown that the shape and even the position of an absorption band is greatly changed by the presence of a foreign gas at low pressures. It was first noticed when the vapor was in the presence of hydrogen, and thinking that it was due to some sort of chemical combination, he repeated the same experiment with nitrogen and helium. The effect was the same. There is much evidence brought out in this investigation that certain of the lines of sodium behave in a similar manner. The pressures are in most cases between 1 mm. and 760 mm., far below the pressure at which any true "pressure-shift" can be observed.

## OBJECT OF THE RESEARCH.

1. To observe and record the fine structure of the absorption spectrum of sodium vapor.

2. To determine the influence of variation in pressure upon the fluted absorption spectra.

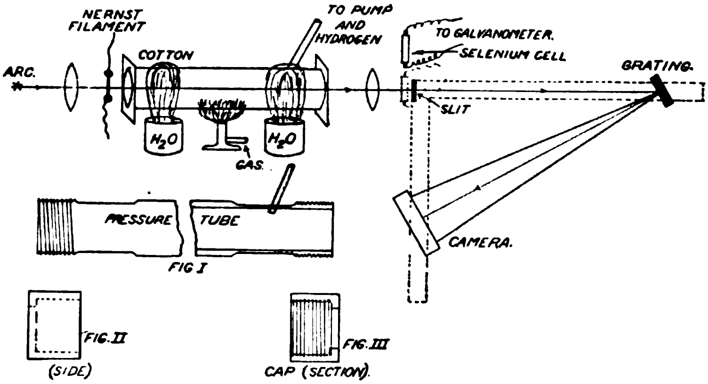
---

[1] Phil. Mag. (6), 12, 1906.

[2] Astrophys. Jl., XXVI., No. 1, July, 1907.

## METHOD OF INVESTIGATION AND RESULTS.

*Apparatus.*—In brief the apparatus consisted of a piece of seamless steel tubing $1\frac{1}{4}$ inches in diameter, 18 inches in length and wall $\frac{1}{16}$ inch in thickness, closed with glass plates hermetically sealed with sealing wax. About an inch from one end of this tube a hole was drilled into which was brazed a piece of $\frac{1}{4}$-inch brass tubing, through which the tube was exhausted. A small pan was placed in the central portion of the tube in which the sodium was confined. This kept the molten metal from running to the ends. This tube was employed for all of the work up to atmospheric pressure. For pressure higher, another one was constructed of the same type, except that the walls were much thicker, and the glass windows held on by threaded caps. It is shown in Fig. (1) of the diagram.



Figs. 1, 2, 3.

The internal diameter is $1\frac{1}{8}$ inches, thickness $\frac{1}{8}$ of an inch. Each end was threaded to receive the caps, which held the glass windows. This tube similar to one used by Wood.[1]

Just inside of the threaded portion on each end was turned down in a lathe to a thickness of $\frac{1}{32}$ of an inch, around which were wrapped the cotton pads. These were necessarily larger than those used on the low pressure tube, on account of the high temperature to which the tube must be raised to obtain the fluted effect at the higher pressures. This left 6 inches of the middle for heating area which

---

[1] Phil. Mag. (6), 1904.

would withstand the high pressure when the tube was at a bright red heat.

The caps for the ends were made of cast brass, turned out and threaded to fit rather tightly, a flange being left to hold the glass windows which were made with a special tool so that the fit would be close. They were taken from a piece of plate glass $\frac{3}{16}$ inch thick. Black sealing wax was used on account of its comparatively high melting point and hardness when cool.

As a source of light, the carbon arc was employed throughout, on account of its great brilliancy, care being exercised to keep the image of the positive pole on the slit.

In order to maintain a definite density of vapor it was necessary to keep the temperature of the tube constant. This was accomplished by using an ordinary Bunsen burner supplied with a modified type of fish tail tip. The electric furnace was first employed, but on account of lack of resistance, properly divided, a small enough increase or decrease of current could not be obtained. Besides, the furnace is very slow in reaching a steady state, requiring from one half to one hour at low temperature. It gives much more uniform layer of vapor, as the heating is on all sides. When great density is desired the use of Bunsen burners is more satisfactory, distillation to the colder parts of the tube taking place much less rapidly. To keep air currents from blowing the flame from the tube, and to give a more uniform temperature on all sides the flame was enclosed in an asbestos sheath, with a tin cap placed on the tube just above it.

A part of the work which did not require high dispersion was done on the 14-ft. concave grating. Later on the 21-ft. concave grating was employed, as the other proved unequal to the task of showing the small details of the spectrum.

The hydrogen used was produced by action of hydrochloric acid on zinc. Great trouble was at first experienced in high pressure work by the formation of fog in the tube, which made the atmosphere inside absolutely opaque for one or two hours. It was found however, that this fogging was the direct effect of impurities in the hydrogen. After elimination of these by passage through well-known solutions, little fog was observed.

## METHODS FOR KEEPING CONSTANT DENSITY.

To determine when the density was at a certain stage, one of two methods was employed.

The first was used when taking a series of exposures in which the pressure was varied from 1 mm. to 760 mm. It was desirable to keep the actual absorption unchanged, noting the effect upon the flutings as a whole. Since the formation of flutings is due to the loss of only a few lines in the heads, it is natural to assume that, if the total absorption is unchanged, so is the density of the vapor, regardless of the change from bands into flutings due to variations in pressure of the gas. In order to determine the total absorption by the vapor, a selenium cell — kindly provided by Dr. Pfund — a Nernst filament and a sensitive galvanometer were used. In regulating the density of the vapor the light from the filament was passed through the tube and allowed to fall upon the selenium cell. The galvanometer deflection was noted; the filament and cell were removed, and the absorption spectrum photographed. If, now, this same density is desired for a different pressure of the gas, the filament and cell are replaced and the temperature of the tube varied until the galvanometer deflection is the same as before. In regulating the density for a certain state of absorption the Nernst filament was placed behind the tube so that when the galvanometer became steady, the filament and cell were removed, and the arc replaced. A photograph is taken, and for a change of pressure the above is repeated. A different deflection would mean simply a different density.

The increase of absorption upon increase of pressure of the foreign gas is due to the fact that the gas prevents the rapid diffusion of the metallic vapor, with the result that the partial pressure of the sodium is increased; in other words, more vapor is present in unit volume. It is therefore necessary to cool down the tube in order to reduce the absorption.

The second method for keeping the density constant was employed when working on the *D* lines. It consisted simply in focusing the *D* lines of the third order on a piece of ground glass, while the *D* lines of the second order were being photographed. Their widths were marked on the ground glass when the pressure was

1 mm., and it is assumed that this width of the absorption line marks a definite density of the vapor. Therefore, if a constant density is desired during any series of observations, all that is necessary is to maintain this width constant by slight changes in the temperature of the tube.

## ABSORPTION SPECTRUM UNDER HIGH DISPERSION AT 1 MM. PRESSURE.

The small pan was filled with metallic sodium and pushed to the middle of the tube, and the glass plate cemented to the ends. The tube was then exhausted and heated until the absorption lines appeared in the green and red. The *D* lines at this stage were about 1 Angström unit in width. The density was increased by gradually raising the temperature, and the absorption spectrum photographed at five different stages. The width of the *D*-line absorption region at the fifth stage was about 20 Angström units. The photographs show that the lines are of unequal intensity. In the first spectrum, which was taken with vapor of considerable density, certain groups of absorption lines have fused together, forming what appear to be broad bands with sharp edges. The sharpness and narrowness of the transmitted regions between the absorption lines is especially noticeable, the bright lines seeming quite as narrow as the iron lines. The general appearance is indeed quite similar to that of an emission spectrum. There is, even in this case, a slight suggestion of a grouping of the lines into bands. As we shall see later on, the admixture of hydrogen or some other inert gas brings out the fluted bands very distinctly by altering the relative intensity of the absorption lines. Figs. 1 and 2, Plate I., represent the green of the second order and Fig. 1 the red in the first order enlarged six times. Broken down under such high resolving power they are found to consist of an immense number of fine lines, extending from λ 6500 to λ 4500. They appear very sharp in the blue and blue-green regions, but as we approach the greenish-yellow they become less sharp, assuming a rather hazy form when λ 5150 is crossed. Although the dispersion is quite sufficient for studying individual lines, some of them still present a very complex form. This is seen in Fig. 4, Plate I., which also represents a portion of the original negative enlarged nearly six times. The groups at $\lambda$ 4615.5, 4624.5 and many appear as if composed of several narrower ones superimposed.

The red of the second order could be studied only visually owing to the mounting of the grating. It presented an even more beautiful appearance than the green and blue. The broad dark lines seen with lower dispersion are easily resolved, many of them being composed of from ten to fifteen very fine hair-like lines. These are spaced in irregular groups, one or two broader ones being placed in between. They are much finer than those in the green and are in much closer proximity. These were at their best when the *D* lines were just fusing together.

This fine line absorption spectrum of sodium vapor is not confined to the red and green, but immediately on each side of the absorbed *D*-line region the lines show nearly as fine and numerous as in the red. A rough estimate of the total number of absorption lines given by sodium vapor, at low pressures, is about 8000 or 10,000. No doubt higher dispersion will reveal many more.

Although the red could not be photographed in the second order on account of the limit to the movement of the 21-ft. grating, a good idea could be gotten from photographs taken in the first order. They appear very much as do those in the green.

Wratten and Wainwright plates (panchromatic) were used and found very satisfactory in this region.

## EFFECT OF INTRODUCING HYDROGEN.

1. The appearance of the flutings as a whole as the pressure is increased from 1 mm. to 11 atmospheres.
2. The effect upon individual lines and groups of lines.

## FLUTING WITH PRESSURE 1 MM. TO 760 MM.

In studying the appearance of the flutings as the pressure of the gas changes, a series of photographs were made varying the pressure from 1 mm. to 760 mm., by seven steps, keeping the actual degree of absorption the same. This was accomplished by the method already described with the selenium cell and sensitive galvanometer. Two sets of photographs were taken, each with a definite density of vapor. The principal set was with a density such that the flutings were just entering their best stage, and when the central ones in the green were partially absorbed. All show the same thing, viz., the extent to which the bands become fluted varies directly with the pressure up to atmospheric, which seems to be a critical value; as the appearance remains practically the same, for an increase of pressure amounting to several atmospheres.

The photographs at 1 mm. pressure, taken in the first order with the 14-ft. grating, presents a series of irregularly spaced fine lines from λ 4600 to λ 5200, from which point they appear more regular, breaking up into a group of three small bands between λ 5100 and λ 5136, six even more regular ones between λ 5180 and λ 5220, and three others between λ 5240 and λ 5270. All of these lines, except those included in the three named sets of bands, show a tendency toward fluting, as soon as the pressure is increased above a few centimeters, the fine lines composing the heads of the flutings dropping out or becoming diminished in intensity. Points which at 1 mm. show as absorption lines gradually become replaced by transmitted regions, smoothing out the irregularities, causing the spectrum as a whole to acquire a uniform fluted appearance.

The three sets of small bands between λ 5200 and λ 5300 simply grow more feeble, until at 60 cm. pressure they fade almost completely, the spectrum becoming nearly continuous. The series of fine lines between λ 4700 and λ 4800 not only flute regularly, but begin to fade away, leaving no trace of lines below λ 4730.

The second series of photographs with greater absorption shows the same general effect, except that the extreme ends of the green absorption region between λ 4700 and λ 4800; and between λ 5100 and λ 5320, come out more strongly owing to the greater absorption. The group at λ 5200 was still visible at 760 mm. This was also true on the violet side, *i. e.*, the fine line flutings pushing out further into the shorter wave-lengths. The central large flutings between λ 4850 and λ 4950 become much weaker, as the pressure is increased. In order therefore to photograph them and the more intense bands at the same time a mechancial screen was employed. This was placed in front of the camera box, and operated by a small motor. It consists of a piece of cardboard with a section cut out about the length of the portion of the spectrum needing more exposure. This, when moved back and forward in front of the plate shades in the brighter portion of the field into the darker. Absorbing screens were tried but with little success. All possess too much general absorption and further, no combination could be found that would give a narrow transmitted band in the portion of the spectrum desired.

In the red at the lowest pressure there appears between λ 6000, and λ 6100, beginning indistinctly at the former, a series of eight bands or semi-flutings, which aside from becoming a little more distinct, undergo little change as the pressure is increased. These correspond to the invariable bands observed in the greenish-yellow region. Reaching atmospheric pressure there are eight much longer ones which appear between λ 6100 and λ 6500; being, however, more sharply defined.

Close examination of the photograph shows a slight lack of definition at 76 cm. compared to that at 2 mm. This will be discussed under head of "Effect upon individual Lines," but it is mentioned here as a forerunner to the discussion of the fluting under high pressure.

## EFFECT OF VARIATIONS IN PRESSURE UPON FLUTING.
## ONE TO TEN ATMOSPHERES.

Series of exposures were made with the pressure varying from one to ten atmospheres, increasing from one and one-half to two atmospheres each time. The result is shown in Fig. 2, Plate II. No special trouble was experienced in getting the spectrum to flute up to three atmospheres. But above this it required two good Bunsen burners, with strong blast, well inclosed, to bring the flutings out distinctly. This made the tube a bright cherry-red color, and after reaching 10 atmospheres, fear of breakdown limited a further increase of pressure.

There was a very faintly fluted spectrum obtained at the last named pressure, but in the process of printing this is lost, and therefore is not shown here, but it affords a good indication of what might be expected at higher pressures.

The spectrum at atmospheric pressure as was mentioned above shows a lack of sharpness. This is also true at higher pressures. At nine atmospheres there remains only a well fluted continuous spectrum. This is perfectly distinct at this pressure, although at ten atmospheres scarcely any trace of it appears.

The behavior in the red was observed visually, under the same pressures and similar changes noticed, except that the flutings and lines disappear at much lower pressures than do those in the green. Owing to the fact that the vapor was made under such high pressure and consequently so dense, very little light could be gotten through the tube above five atmospheres, and photographs could not be obtained in the second order. Exposures in the first required from three fourths to one and one half hours, and the red region, even with a comparatively wide slit could scarcely be seen, only presenting a weak continuous appearance. At ten atmospheres the absorption is very general in its nature, practically all of the red-yellow and blue-green is gone, leaving the violet much stronger than the rest.

## Effect of Pressure upon Individual Lines and Groups of Lines.

In this portion of the investigation the second order of the 21-ft. grating was used where possible.

The tube being cleaned out and a lump of sodium inserted, it was exhausted to a pressure of 1 mm. An exposure was made when the absorption lines were most numerous and the definition greatest. Then hydrogen was admitted to atmospheric pressure, the tube being left connected to the gas supply. This was done to make sure that the pressure in the tube did not rise on account of the property possessed by metallic sodium when heated, to occlude great quantities of hydrogen.

A photograph was taken when the conditions of absorption were as near as possible the same as at the lower pressure. The result is shown in Figs. 1 and 2 of Plate I., Fig. 4 being given to show the width of the absorbed region at the *D* lines, at density used. The heads of the flutings can now be measured accurately and are given below. With lower dispersion they appear as "single line" heads, and were measured as such by Moore, but, as can be seen from the data, they consist of one, two and sometimes three lines, some of them having no definite head. The form of these heads is given along with that produced at 760 mm., so that some idea can be gotten of the phenomena taking place.

The lack of definition mentioned in the preceding part is seen to be much more pronounced in the green than in the red, especially at the longer wave-lengths. Instead there is more actual absorption at 760 mm., many of the strongest bright lines being cut out completely, and feebler ones making their appearance. These remain sharp compared with the yellow, green and blue absorption lines near λ 5700 and λ 4600 respectively. The intermediate regions between λ 4650 and λ 4920 show only the fluting effect. The two spectra have nearly the same intensity at this point, but at λ 4600 comparatively little light is absorbed.

### Appearance of Heads of Flutings in Green. Wave-lengths Accurate to .01 Angström Unit.

| | Vacuum of 1 mm. p. | Atmospheric Pressure. |
|---|---|---|
| 1. | 4766.71 | |
| | 66.97 doublet. | Fuses together, no shift. |
| | Slightly broken effect at<br>4782.92 and<br>4792.95 | |
| 2. | 4819.75 heavy single line. | Spreads some and shifts to 4819.85 |
| | Very irregular fluting. | |
| 3. | 4837.15 | |
| | 37.31 doublet. | 1 Shifts. 4837.26 |
| | | 2 Does not shift. 4837.31 |
| 4. | 4865.15 very heavy. | |
| | 4865.30 weak. | Breaks into triplet.<br>4865.04<br>65.30<br>65.50 |
| | Consists of a series of irregular lines. | Breaks into series small doublets increasing in size to head. |
| 5. | 4894.48 heavy single. | Shifts to 4894.37 |
| 5. | Semi break at 4911.40 | Breaks into symmetrical doublet.<br>Resembling Zeeman effect. 4911.84 and 20.1<br>The whole fluting consists of irregular groups of doublets and triplets. |
| 6. | 4932.41 | |
| | .54 doublet. | Fuse into cone at 4932.38 |

| | | |
|---|---|---|
| 7. | No definite lines. | Transformed into hazy triplet at<br>4962.04<br>.23<br>.47<br>Lines irregular all through |
| 8. | 5000.94 heavy.<br>1.12 weak. | Forms hazy doublet<br>4000.86<br>5001.10<br>1 shifts. 2 widens toward violet.<br>Very irregular series shown in Fig. 2, Plate III. |
| 9. | 5039.92<br>40.08 triplet.<br>.18 | Contracts to close doublet almost inseparable at 5039.98 |
| 10. | Is weak and hard to notice<br>5079.65<br>5076.89 doublet. | Remains doublet, both shifting symmetrically to<br>5079.56<br>5079.89 |

There are besides these the seven small bands, or groups of lines which produce a sort of "rolling" effect, and appear like this:

These are found in the blue between λ 4703.46 and λ 4810.16. They are not apparent at vacuum, but are brought out at 76 cm. The wave-lengths are given below:

4703.46
4704.78
4716.10
4727.89
4741.59
4767.22
4810.11

The bands found in the yellow green, and unaffected at atmospheric pressure are given below:

5100.5
5112.6
5124.7
5136.8
5184.1
5196.2
5208.3
5223.5
5240.1
5252.1
5264.2
5276.3

These bands in the violet are clearly noticeable, at eleven atmospheres, while those in the yellow green stand fairly well up to six atmospheres.

**Flutings in the Red and Orange.**

5997.82
6019.23
6042.05
6080.40
6099.64
6119.12
6141.01
6168.00

Well prounced at vacuum, only slightly brought out at pressure.

| | |
|---|---|
| 6179.23 | Slightly visible at vacuum but brought out clearly by pressure. |
| 6185.06 | |
| 6219.87 | |
| 6276.29 | |
| 6329.44 | |
| 6373.88 | |
| 6417.50 | |
| 6464.62 | |
| 6512.36 | Not at all visible at vacuum, but strongly fluted by pressure. |
| 6559.64 | |
| 6610.30 | |

In the orange the two spectra are about equal intensity, and the lines are fused together, but as the pressure is raised to 760 mm. the intensity is much less, and the fluting is more pronounced.

One of the chief characteristics of the absorption spectrum is the doublet formation observed at atmospheric pressure. A small part of this can be seen very clearly in Fig. 3, Plate I., between $\lambda$ 5001 and $\lambda$ 5025, which represents a portion of the original negative enlarged about two and one half times. In the red part of the spectrum the reverse is true, that is, this doublet formation is more evident at 1 mm. pressure, than at 760 mm. Between wave-lengths $\lambda$ 5900 and $\lambda$ 6200 this is very common, especially near $\lambda$ 6100.

This characteristic of doubling by the lines extends up to 6600 and appears only at very low pressures, disappearing at atmospheric pressure.

In the enlarged portions represented by Figs. 1 and 2, Plate I., Fig. 3 shows at 760 mm. absolutely no regularity, whereas at 1 mm. the groups between $\lambda$ 5001 and $\lambda$ 5025, break up or are transformed into the regular series shown.

Many of the lines are displaced as much as .15 A.U. for a change of 760 mm. One line out of a group will move to the red, the others remaining in this same position.

Nearly all of those which shift at all none are in the direction of longer wave-length, although many are displaced toward the violet. This apparent change of position does not differ in amount between wave-lengths $\lambda$ 4600 and $\lambda$ 5100, the average being about .11 of an Angström unit. At $\lambda$ 5006.2 can be plainly seen the difference in the absorption at 760 and 2 mm. respectively.

Judging from the green absorption region one would immediately look for similar changes in the red. But, in fact, although all of the lines were examined, there can be noticed only a few instances of unsymmetrical broadening or apparent shifts.

In view of the fact that so many changes occur in the absorption spectrum of sodium vapor, in the presence of hydrogen as the pressure is changed from 1 mm. to 760 mm., some sort of chemical combination seems altogether possible. Consequently to decide this point the hydrogen was replaced by nitrogen. A fresh piece of sodium was used and the tube cleaned before taking a photograph. The tube while heated was exhausted; air was allowed to enter the tube, was again exhausted, etc., several times, each time allowing air to enter the tube at atmospheric pressure. On account of the great affinity of sodium for oxygen little else remained in the tube except nitrogen. A photograph was obtained under the same conditions as with hydrogen, and the two spectra showed exactly the same results line for line.

## Study of the D Lines.

The effect of pressure upon the *D* lines is much different from that on the other absorption lines of sodium. Photographs were taken with pressure varying from 1 mm. to 12 atmospheres, the density being kept constant by the method already described. Except for a slight decrease in sharpness no change in the spectra was observed as the pressure was changed from 1 mm. to 760 mm. This can be seen only when one of the *D* lines is about 2 A.U. in width. Any greater density causes the absorbed space to become so broad and hazy that this effect can not be detected, and with very low density, even at the highest pressures, the *D* lines should remain very fine and sharp.

Fig. 3, Plate II., shows the effect of high pressure. There seems to be an unsymmetrical broadening, which extends to the longer wave-length. It can be best described in this way. If we heat up the tube at 760 mm. pressure until *D* is about 2 Angström units in width, then remove the burner, and allow the tube to cool, the two *D* lines of course will be seen to die away symmetrically, getting narrower until they disappear. If, now, we raise the pressure up to 10 atmospheres, and again observe the decay of the lines, the fine *D*'s will be seen just on the edge of the shadow which vanishes some time before the *D* lines themselves. It is much easier to observe this change than to photograph it, but an excellent idea can be gotten from the figure on Plate II.

This effect is very much the same as that observed by Julius in working with anomalous dispersion of sodium vapor, where he causes unsymmetrical broadening of the *D* lines, either one or both, by merely changing locality of heating. The source of heat was obtained by an electric current passing through two nickel wires running parallel through the tubes containing the vapor.

The heating in the present instance was a small flame under the tube. By using an electric the point could be immediately decided. But for lack of time the author would have tried this.

## SUMMARY.

The results of the investigation may be summed up as follows:

1. Under high dispersion the absorption spectrum of sodium vapor in the region investigated is found to consist of approximately 10,000 lines, the region of absorption extending mainly from λ 4500 to λ 6700, which was as far as results were obtained.

2. The bands can be divided into two classes: first, those in which the extent of fluting varies directly as the pressure is increased from 1 mm. to 760 mm., and second those which are not affected much by pressure.

3. As the pressure is increased to 8 atmospheres, the absorption spectrum becomes continuous in appearance, the fluting disappearing almost completely at 10 atmospheres.

4. The transformations of the lines which occur as the pressure is raised from 1 mm. to 760 mm. may be briefly stated as follows:

(*a*) Many of the lines between λ 4600 and λ 5100 shift their positions as much as .15 A.U. caused either by unsymmetrical broadening or by the pressure of the foreign gas.

No such effect is observed in the red part of the spectrum.

(*b*) A single line is changed into a doublet, and a doublet into a triplet, although the reverse is more often true.

(*c*) The whole spectrum is characterized more or less by a doublet formation, which is more noticeable in the shorter wave lengths at 760 mm., and in the longer wave-lengths at low pressures.

PHYSICAL REVIEW CLXIX
May, 1910

To face page 608

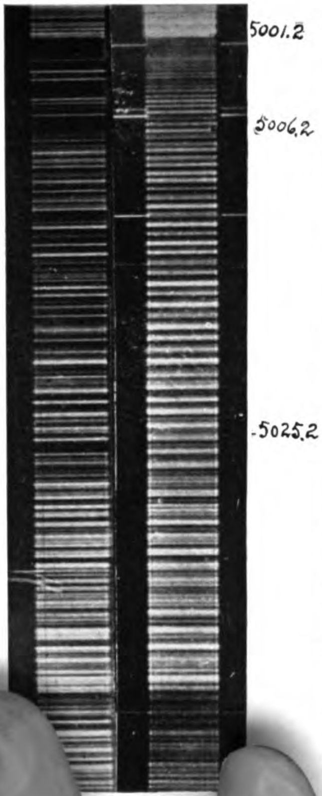


Fig III

Fig II

Fig I

IV

NKSCAL

Sodium

PHYSICAL REVIEW CLXIX
May, 1910

To face page 608

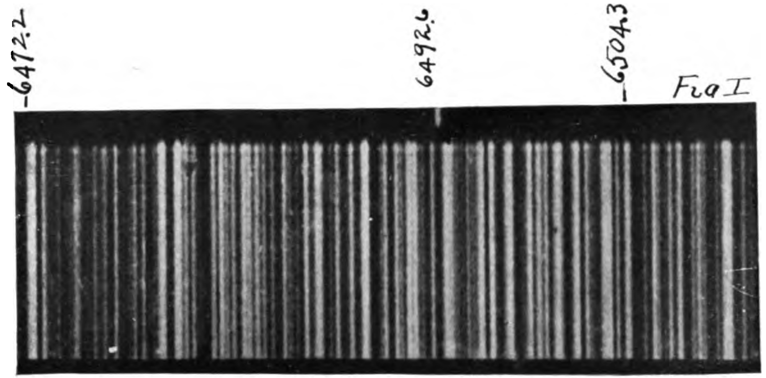


Fig I

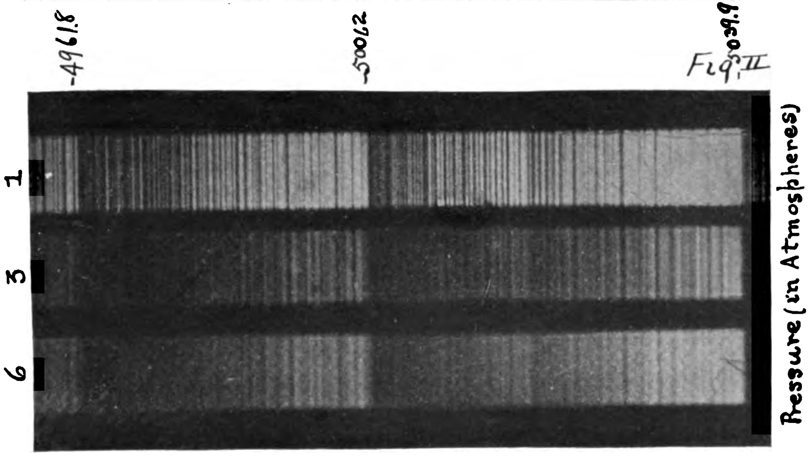


Fig II

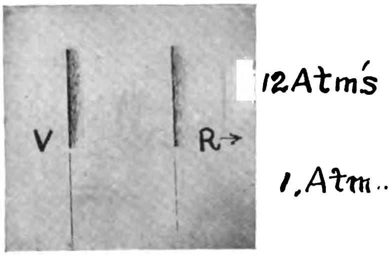


Fig III

PLATE II. CLINKSCALES.

The Absorption of Sodium Vapor.

(*d*) The *D* lines, when *D* is about one Angström unit in width, are unsymmetrically broadened nearly 5 A.U. under 12 atmospheres pressure, but at extremely low pressure, remains very sharp and shows no broadening.

5. No difference can be noticed in the absorption spectra for the different gases used.

In conclusion I wish to acknowledge my great indebtedness to Professor Wood, who suggested the work and under whose direction it was carried out.

JOHNS HOPKINS UNIVERSITY,
June, 1908.

# THE ELASTIC PROPERTIES OF PLATINUM-IRIDIUM WIRES.[1]

BY K. E. GUTHE AND L. P. SIEG.

1. About two years ago one of the authors[2] found that platinum-iridium wires show a remarkable deviation from the elastic behavior of ordinary wires when used as suspensions for experiments on torsional vibrations. This peculiarity was especially pronounced in a wire containing 40 per cent. of iridium.. It was shown that for the amplitudes employed — ten degrees and less — the torsional period of cylinders suspended by this wire decreased proportionally to the amplitude, the decrease amounting to nearly two per cent.; and further, that the exceptionally large logarithmic decrement also is, for small amplitudes, a linear function of the amplitude. With different moments of inertia the logarithmic decrement remained unchanged and the various curves, obtained by plotting periods and amplitudes coincided, after proper scale reduction was made. In a wire containing 25 per cent. of iridium these peculiarities were less pronounced, but still noticeable.

The present investigation was undertaken in order to extend the observations to larger amplitudes and to study the influence of different percentages of iridium upon the elastic properties of platinum-iridium wires. We are under obligation to Dr. Heraeus, of Hanau, for preparing the alloys and drawing the wires needed in our experiments.

2. *Apparatus.*—The ends of the wire were soldered in brass cylinders one of which served as the supporting rod and was clamped in a wall bracket. The lower cylinder formed a part of the vibrating system; it was accurately turned to a diameter of about 6 mm.,

---

[1] This investigation was carried out with the aid of a grant from the Elizabeth Thompson Science Fund.

[2] K. E. Guthe, Proc. Iowa Ac. Sci., 15, p. 147, 1908; abstract in PHYS. REV., 26, p. 201, 1908.

leaving at its lower end a flat disk of about 9 mm. diameter thus forming a support for a number of hollow cylinders which fitted snugly over the supporting pin. By the use of different hollow cylinders the moment of inertia of the moving system could be varied from 366 g.-cm$^2$ to 1344.5 g.-cm$^2$. At its lower extremity the pin carried two small mirrors with their backs glued together. An optical test showed that they were parallel to within a few seconds.

The wire was suspended with the axis of the vibrating cylinder in the center of a semicircular scale of 149.2 cm. diameter. The mirror reflected upon the scale the image of a hole with a fine crosswire placed in front of an arc light. A lens between mirror and crosswire focused the image of the latter upon the scale. An angular displacement of one degree of the mirror produced thus a displacement of 2.6 cm. of the image. This arrangement of the apparatus allowed to read amplitudes of the vibrating system from 765 to 675, 405 to 315 and 90 to 0 degrees. In later experiments the half circle was replaced by a complete circle so that practically the whole series of amplitudes could be observed.

3. *Method.*—The passage of the image of the crosswire through the zero point was recorded by an observer upon a chronograph which latter was also provided with means of marking double seconds from an astronomical chronometer. The intervals between successive coincidences were calculated from the chronograph record. When the amplitude had died down to a few degrees the eye and ear method was used for the determination of coincidences and the calculation of the average half periods.

It was impossible to obtain by this subjective method a very accurate determination of the time between two successive passages of the crosswire through the zero. It was therefore necessary to calculate the period from longer intervals, usually, except for the first 20 swings, from 40 vibrations, each corresponding to a half period. In the following tables and figures the values given under "period" refer without exception to half periods, expressed in seconds. The curve for the period when plotted as a function of the time is for large amplitudes nearly parallel to the $X$ axis, but soon bends downward and forms a curve convex towards the origin. The periods calculated in the manner described will therefore, for the greater part of the curve, be slightly larger than the actual periods. By taking different intervals, for example, of 20, 30 and 40 vibrations we assured ourselves that the error thus introduced did not exceed the error introduced by our subjective method of recording the passage of the crosswire, which is about one one-thousandth of a second when the period is calculated from 40 swings. The accuracy of the values for the first 20 periods is necessarily smaller since no such large intervals could be chosen for their calculation.

The amplitudes were plotted on cross-section paper and, unless otherwise stated, the amplitudes given in the tables are the values read off from this curve as corresponding to a given average period.

4. *Period and Amplitude.* — We began our experiments with the finest 40 per cent. platinum-iridium wire received from Heraeus. Its length was 80.4 cm., and its diameter 0.194 mm. Since former experiments had shown that previous heat treatment greatly influences the elastic properties, and since we wished to have all wires in the same condition, we annealed the wire under the load which it was to carry, by sending a current of 2.0 amperes through it. This raised its temperature to a bright yellow heat and straightened it. Even at these high temperatures it can still support a considerable load.

Our observations corroborate fully the results obtained in the earlier experiments, the increase in period, for small amplitudes, being proportional to the amplitude. However, with larger amplitudes, the rate of increase becomes smaller and finally the period approaches a constant value, though even with amplitudes of 700 degrees there is still a slight increase. A large number of experiments were made, varying such conditions as load, moments of inertia and temperature. In all cases the curves show the same characteristic form. The data under A in Table I., plotted in Fig. 1, are taken from one of these experiments. The first column contains the number of half swings, the second the amplitude in degrees, the third the half period in seconds. It is seen that starting with an amplitude of 767 degrees the decrease of the period reaches the enormous value of 7 per cent., of which six sevenths fall upon the interval from 250 to 0 degrees. For the last 60 degrees the decrease is proportional to the amplitude.

5. *Change of Length.* — We next shortened the wire to three quarters of its original length. This was done for the double purpose of testing if the peculiarities might possibly be due to some lack of homogeneity in the wire, and of obtaining still larger torsional strains.

### Table I.

40 *Per Cent. Platinum-iridium Wire No.* 2.

*A. Jan.* 30; *Length* 804 *mm.; Moment of Inertia* 366 *g.-cm*$^2$; *Temp.* 14°.3 *C.*
*B. Apr.* 19; *Length* 602 *mm.; Moment of Inertia* 838.5; *Temp.* 21°.0 *C.*
*C. May* 7; *Length* 403 *mm. Moment of Inertia* 1344.5; *Temp.* 21°.5 *C.*

| No. of Swings. | *A* | | *B* | | *C* | |
|---|---|---|---|---|---|---|
| | Amplitude. | Period. | Amplitude. | Period. | Amplitude. | Period. |
| 0 | 767 | 4.788 | 786 | 6.435 | 765 | 6.459 |
| 2 | 700 | .783 | 732 | .432 | 725.4 | .458 |
| 4 | 640 | .778 | 687 | .427 | 692 | .456 |
| 6 | 565 | .770 | 649 | .423 | 651 | .455 |
| 8 | 506 | .762 | 614 | .420 | 630 | .453 |
| 10 | 443 | .753 | 580 | .417 | 597 | .452 |
| 12 | 388 | .744 | 548 | .413 | 570 | .451 |
| 14 | 340 | .731 | 516 | .409 | 544 | .450 |
| 16 | 297 | .715 | 486 | .405 | 517 | .449 |
| 18 | 260 | .695 | 454 | .402 | 494 | .447 |
| 20 | 228 | .673 | 426 | .396 | 470 | .445 |
| 25 | 159 | .616 | 358 | .385 | 421.4 | .440 |
| 30 | 116 | .573 | 298 | .370 | 372 | .435 |
| 35 | 84 | .543 | 243 | .353 | 326 | .429 |
| 40 | 65 | .521 | 197 | .334 | 284 | .422 |
| 45 | 52 | .503 | 157 | .308 | 246 | .414 |
| 50 | 44 | .487 | 122 | .281 | 212 | .405 |
| 60 | 32.6 | .466 | 74 | .226 | 152 | .385 |
| 70 | 25.3 | .455 | 47 | .174 | 103 | .351 |
| 80 | 20.2 | .447 | 32 | .134 | 67 | .316 |
| 90 | 16.6 | .440 | 24 | .105 | 46 | .278 |
| 100 | 13.8 | .434 | 19 | .086 | 31.4 | .244 |
| 120 | 10.1 | .428 | 12 | .062 | 17.1 | .193 |
| 140 | 7.4 | .426 | 8 | .036 | 10.4 | .161 |
| 160 | 5.8 | .424 | | | 6.9 | .140 |

The results are given in Table I. under B and plotted in Fig. 1. The period closely corresponds to what would be expected for an ordinary wire, considering the change in length and torsional moment. The conclusion is therefore justified that the wire had a uniform structure throughout its length. As we expected, the period did not drop nearly as fast at the beginning as it did with the longer wire. A decided falling off in period did not occur until the amplitude had decreased to about 500 degrees. The final decrease of period is much more rapid than with the longer wire.

When the wire was shortened to 403 mm., half the original length, the influence of the length upon the variation of the period becomes still more marked. For the largest amplitudes the period is practically constant, and the final drop very rapid. The results are given under C in Fig. 1 and Table I. For comparison the scale for the period of A in the figure has been chosen one and one third times larger than that for B and C.

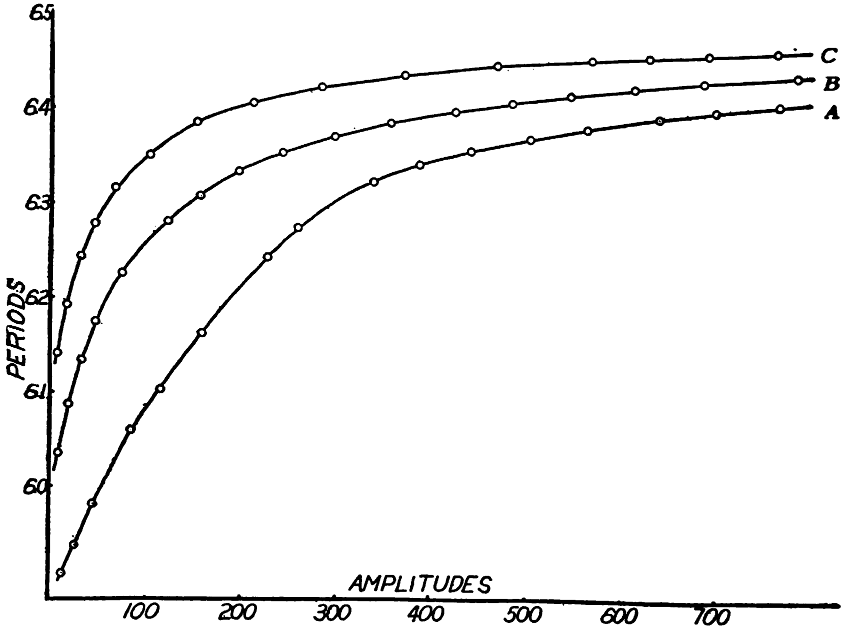


Fig. 1.

6. *Change of Moment of Inertia.*—Varying the moment of inertia of the moving system did not change the character of the curves. It became, however, soon apparent that the phenomenon was greatly influenced by the conditions under which the vibrations were produced. We could not in general obtain Guthe's results with small amplitudes, namely, that all curves coincide when plotted on

corresponding scales. Only in exceptional cases when the conditions of temperature and initial amplitude must have been exactly alike, a close agreement between the curves was found. Table II. is an example of such a case. Only the amplitudes and periods are given. The scale for the periods in the experiment of March 6 was reduced to that of March 13 by multiplying the periods by 1.692.

### TABLE II.

40 *per cent. Platinum-iridium Wire No.* 2, *Length* 804 *mm.*
*A. March* 13 ; *Moment of Inertia* 1344.5 *g.-cm*$^2$ ; *Temp.*= 21°.1 *C.*
*B. March* 6 ; *Moment of Inertia* 473.7 *g.-cm*$^2$ ; *Temp.*= 23°.0 *C.*

| No. of Swings. | *A* |  | *B* |  |  |
|---|---|---|---|---|---|
|  | Amplitude. | Period. | Amplitude. | Period. | Reduced Period. |
| 0 | 742 | 9.267 | 730 | 5.480 | 9.273 |
| 2 | 688 | .264 | 670 | .477 | .267 |
| 4 | 635 | .260 | 623 | .475 | .264 |
| 6 | 585 | .254 | 572 | .471 | .257 |
| 8 | 540 | .245 | 523 | .466 | .249 |
| 10 | 492 | .235 | 474 | .459 | .237 |
| 12 | 453 | .225 | 423 | .451 | .223 |
| 14 | 415 | .214 | 379 | .440 | .205 |
| 16 | 380 | .198 | 343 | .427 | .182 |
| 18 | 346 | .182 | 308 | .415 | .162 |
| 20 | 313 | .165 | 277 | .403 | .142 |
| 25 | 241 | .124 | 209 | .374 | .093 |
| 30 | 187 | .082 | 158 | .346 | .046 |
| 35 | 143 | .037 | 118 | .320 | .001 |
| 40 | 110 | 8.994 | 90 | .295 | 8.960 |
| 45 | 83 | .953 | 72 | .273 | .921 |
| 50 | 68 | .915 | 57 | .253 | .888 |
| 60 | 42 | .860 | 38 | .224 | .839 |
| 70 | 32 | .822 | 28 | .205 | .806 |
| 80 | 24 | .798 | 21 | .193 | .786 |
| 90 | 19.3 | .783 | 15 | .185 | .773 |
| 100 | 15.3 | .768 | 13 | .177 | .759 |
| 120 | 10.1 | .746 | 8 | .161 | .732 |

In most cases such close agreement could not be obtained. The slope of the curve does not alone depend upon the initial amplitude but also upon whether or not the wire had been kept for some time in a twisted condition before being released or had been kept swinging continuously before the experiment was started.

7. *Influence of the Initial Amplitude.*—For this part of our investigation we made use of the old specimen of platinum-iridium wire which had not been vibrated for considerable time. This had been obtained from Heraeus some years ago; it was marked 40 per cent. platinum-iridium, had a length of 402 mm. and a diameter of 0.192 mm., *i. e.*, the same diameter as the finest wire, more recently secured.

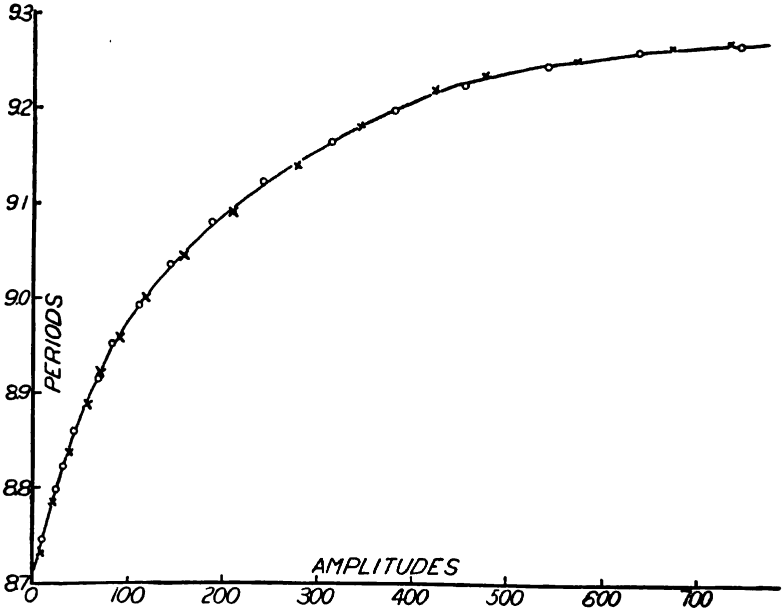


Fig. 2.

The wire was heated under a load which it was to carry, 240 g., the moment of inertia of the suspended system being 979.5 g.-cm$^2$. After being placed in position the wire was allowed to rest for some days. The first set of readings was taken with a small initial amplitude, 21 degrees; after which the wire was allowed to rest a day. Then the initial amplitude was increased to 47.3 degrees, and again a day's rest given; this process was continued, the amplitude being increased each day. The results are found in Tables III.–VIII., where not alone the periods and corresponding amplitudes are given (in columns 4 and 5) but also the observed amplitudes and the observed times of passage of the image of the crosswire through

### TABLE III.

40 *Per Cent. Platinum-iridium Wire No.* 1; *Length* 40.2 *cm. Maximum Amplitude* 21 *Degrees; Temperature* 22°.3 *C.*

| No. | Amplitude. | Coincidences. | Period. | Corr. Ampl. | Log. Dec. |
|---|---|---|---|---|---|
| 0 | 14.3 | 0′ 22″.66 | | | |
| 10 | 13.5 | 1 14 .81 | | | .00227 |
| 20 | 12.85 | 2 6 .97 | 5.214 | 12.77 | |
| 30 | 12.05 | 2 59 .01 | | | 222 |
| 40 | 11.6 | 3 51 .20 | 5.212 | 11.6 | |
| 50 | 10.95 | 4 43 .25 | | | 216 |
| 60 | 10.5 | 5 35 .37 | 5.213 | 10.7 | |
| 70 | 10.2 | 6 27 .53 | | | 194 |
| 80 | 9.6 | 7 19 .62 | 5.211 | 9.6 | |
| 90 | 9.2 | 8 11 .76 | | | 189 |
| 100 | 8.8 | 9 4 .00 | 5.213 | 8.8 | |
| 110 | 8.5 | 9 56 .00 | | | 175 |
| 120 | 8.12 | 10 48 .10 | 5.210 | 8.12 | |
| 130 | 7.85 | 11 40 .12 | | | 167 |
| 140 | 7.52 | 12 32 .38 | 5.207 | 7.55 | |
| 150 | 7.25 | 13 24 .39 | | | 180 |
| 160 | 6.92 | 14 16 .45 | 5.209 | 6.92 | |
| 170 | 6.60 | 15 8 .61 | | | 136 |
| 180 | 6.50 | 16 0 .54 | 5.206 | 6.50 | |
| 190 | 6.30 | 16 52 .78 | | | 120 |
| 200 | 6.15 | 17 44 .85 | 5.205 | 6.15 | |
| 210 | 6.00 | 18 36 .88 | | | 90 |
| 220 | 5.90 | 19 28 .82 | 5.204 | 6.00 | |
| 230 | 5.85 | 20 20 .94 | | | 75 |
| 240 | 5.70 | 21 12 .85 | | 5.55 | |

the zero point (under coincidences in column 3). These tables show the very interesting fact that the wire makes a larger number of vibrations when started with a small amplitude than when started with a larger amplitude. This is brought out clearly in Fig. 3, in which the zero points from which the number of swings are counted are placed the farther towards the right the larger the initial amplitude.

In each successive series the wire makes about 50 vibrations less in reaching an amplitude of five degrees. Thus it takes only 125 half vibrations to fall from 364.3 degrees to 5 degrees while in the first case 200 swings are required to cover the interval from 14.3 to 6.15 degrees. If we had chosen a still smaller amplitude than five degrees for our comparison the difference would be still greater, judging from the slope of the curves at the small amplitudes.

All six curves may be made to coincide if the scale for the vibration number is reduced. This is shown in Fig. 4. The values of the amplitudes from Table VIII. are plotted as ordinates, the number of half vibrations as abscissæ. Then for the maximum recorded amplitude in each set of observations the corresponding position in the diagram was found, and from this point on the amplitudes were plotted by reducing the scale of the number of vibrations of each set, multiplying by a constant reduction factor. These factors are 9.3/20, 7.225/20, 6.15/20, 4.75/20 and 2.915/20 respectively for the series beginning with amplitudes of 117, 86.4, 47 and 22.3 degrees.

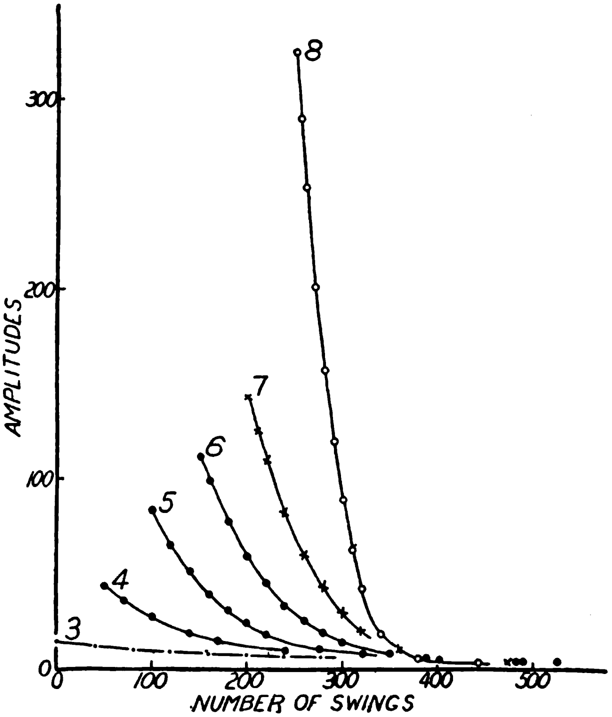


Fig. 3.

It seems to be a curious coincidence that the reduction factor in the first case, namely, 9.3/20 = 2.15 is almost exactly the ratio of the two maximum amplitudes, 364.3/168.9 = 2.16. In all other cases the reduction factors are smaller than the ratio of the maximum amplitudes, the difference increasing as we proceed towards the series with smallest amplitude.

As could be expected the curves for the period as functions of the amplitude is greatly dependent upon the initial amplitude, the change in the period being the smaller the smaller the initial amplitude. Fig. 5 shows this very clearly.

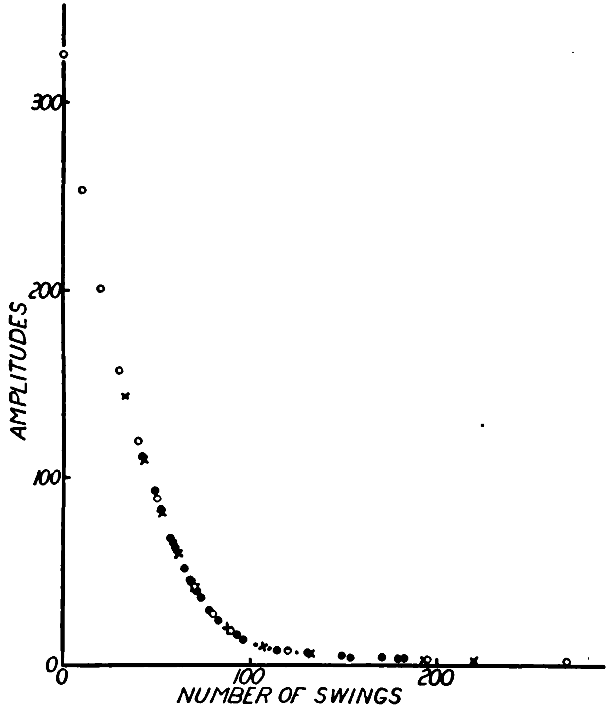


Fig. 4.

8. *The Logarithmic Decrement.*—Though the logarithmic decrement loses all significance in elastic phenomena of this kind, it gives a very interesting curve for the various cases considered. From the results discussed in the last paragraph it follows immediately that the logarithmic decrement will be the smaller the smaller the initial amplitude, though in all cases the mathematical expression for its dependence on the amplitude must be the same except for a numerical factor. When plotted as a function of the amplitude the decrement starts as a straight line for small amplitudes, but

reaches a maximum for an amplitude which is the farther towards the left the larger the initial amplitude; this maximum becoming most pronounced in short wires. Beyond this maximum the decrement decreases at first rapidly, then slower, and probably reaches a minimum for large amplitudes. In Fig. 6 the logarithmic decrement is plotted as a function of the amplitude for three series, (*a*) A in Table I., (*b*) C in Table I., and (*c*) Table VIII.

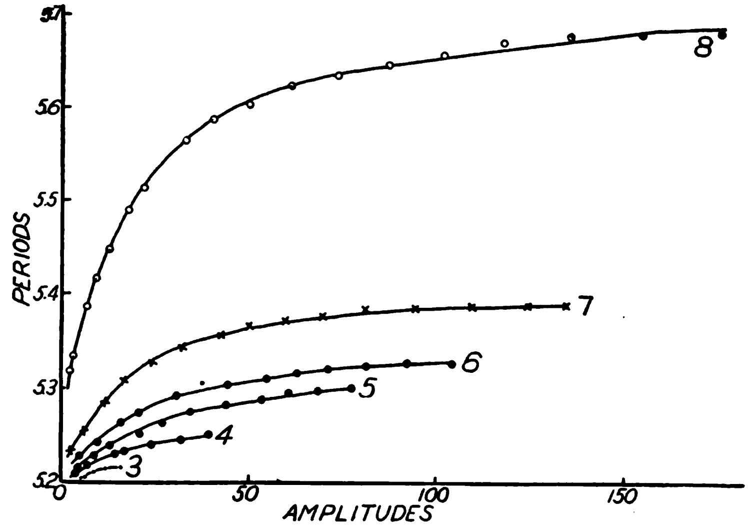


Fig. 5.

The curves show a great similarity to those of magnetic permeability of iron as a function of the magnetizing field intensity. Bouasse[1] who, for many years, has studied the elastic behavior of ordinary wires has obtained very similar curves to ours in the case of an iron wire. He also found a minimum for large amplitudes.

9. *Hooke's Law.*—The preceding results suggest that the wire may have been strained beyond its elastic limit or that the elastic after-effect may play an important rôle in the phenomenon. The resting point of the wire remained, however, remarkably constant during all of our experiments and previous investigation by one of us showed that, when tested by the ordinary method, the elastic after-effect in these wires is quite small, indeed, smaller than for

---

[1] Bouasse and Carrière, Ann. de Chim. et Phys., 14, p. 190, 1908.

steel. These tests were made by holding the suspended cylinder in a fixed position twisting the torsion head 360 degrees, holding it there for two or three minutes, turning the torsion head back and determining as soon as possible afterwards—about half a minute—the new resting point. No displacement greater than a degree was observed, the after-effect slowly disappearing in the usual manner.

We repeated an experiment mentioned in the earlier publication, namely, a test of Hooke's law for these wires. The wire was connected by a brass pin to a steel wire for which the period had previously been determined and found constant. Both wires were soldered into the pin, which carried a mirror. The platinum-iridium wire formed the upper part of the system and was clamped in a torsion head; the steel wire carried at its lower end a load, a part of which was a thick brass plate fitting a groove in a holder, fastened to the floor. This prevented a twisting of the lower end of the system, but allowed the plate to slide up and down in the groove when the length of the wire changed due to the twist. The load

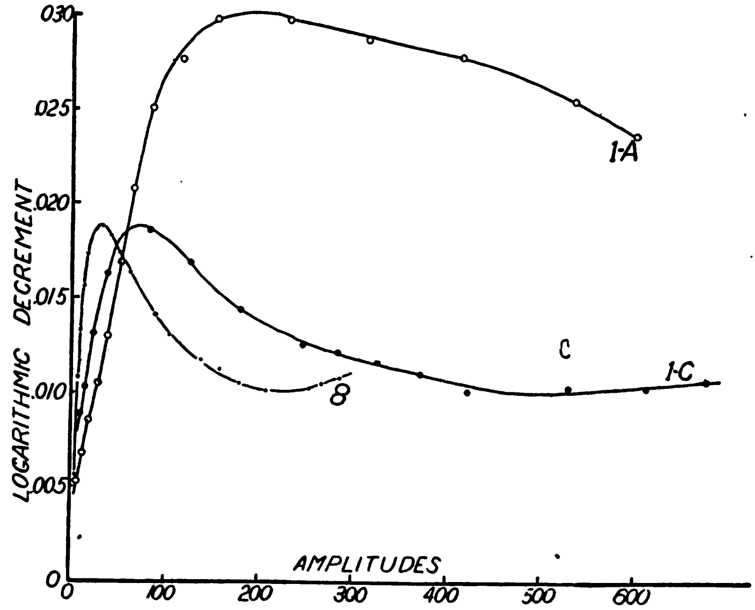


Fig. 6.

## TABLE IV.

*Same Wire as in Table III. Maximum Amplitude* 47.3 *Degrees;*
*Temperature* 24°.9 *C.*

| No. | Amplitude. | Coincidences. | Period. | Corr. Ampl. | Log. Dec. |
|---|---|---|---|---|---|
| 0 | 43.6 | 0′ 6″.51 | | | |
| 10 | 39.2 | 0 59 .03 | 5.250 | 39.0 | .00458 |
| 20 | 35.3 | 1 51 .51 | 5.248 | 35.1 | 447 |
| 30 | 31.9 | 2 44 .00 | 5.245 | 31.8 | 434 |
| 40 | 28.9 | 3 36 .44 | 5.244 | 28.8 | 436 |
| 50 | 26.1 | 4 28 .83 | 5.241 | 26.1 | 417 |
| 60 | 23.85 | 5 21 .27 | 5.239 | 23.8 | 401 |
| 70 | 21.7 | 6 13 .65 | 5.237 | 21.6 | 393 |
| 80 | 19.9 | 7 6 .00 | 5.234 | 19.8 | 370 |
| 90 | 18.3 | 7 58 .30 | 5.234 | 18.25 | 368 |
| 100 | 16.8 | 8 50 .61 | 5.232 | 16.7 | 361 |
| 110 | 15.5 | 9 43 .00 | 5.228 | 15.5 | 327 |
| 120 | 14.45 | 10 35 .37 | 5.228 | 14.4 | 308 |
| 156 | | | 5.221 | 11.1 | 328 |
| 191 | 8.61 | 16 46 .00 | | | |
| 209 | | | 5.222 | 7.95 | 219 |
| 227 | 7.22 | 19 54 .00 | | | |
| 250 | | | 5.217 | 6.50 | 188 |
| 273 | 5.94 | 23 54 .00 | | | |
| 306 | | | 5.212 | 5.32 | 155 |
| 339 | 4.71 | 29 38 .00 | | | |
| 358 | | | 5.211 | 4.46 | 134 |
| 377 | 4.20 | 32 56 .00 | | | |
| 401 | | | 5.208 | 3.94 | 122 |
| 425 | 3.70 | 37 6. 00 | | | |
| 449 | | | 5.208 | 3.55 | 107 |
| 473 | 3.40 | 41 16 .00 | | | |

on the wire remained therefore practically constant. A damping arrangement, immersed in oil, was connected to the brass pin between the two wires. After the zero point had been accurately fixed the torsion head was twisted by steps of 50 degrees, observing at the same time the deflection of the mirror. The torsion head was twisted 450 degrees either way and several complete cycles observed. If Hooke's law holds the deflection of the mirror must be proportional to the twist of the steel wire. Fig. 7 shows that this is the case. The deflections of the mirror are plotted as a function of the actual twist of the platinum-iridium wire, *i. e.*, the angular displacement of the torsion head minus that of the mirror. There is a slight hysteresis effect. We had to wait nearly half a minute after applying a new twist before the system came to rest. In another experiment we waited three minutes between successive observations, but this had no effect upon the area included between the ascending and descending branches of the curve.

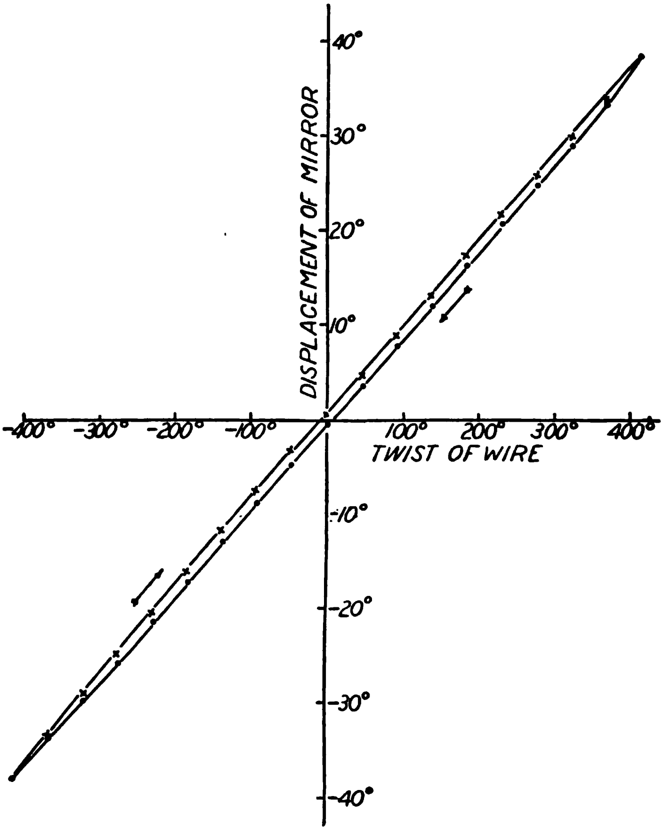


Fig. 7.

It is apparent that for slowly applied twists Hooke's law holds to the same degree of accuracy as for any other wire with small elastic after-effect. This does not exclude the possibility of an elastic after-effect being present in the torsional vibrations of our wire; but if that were the case it must disappear very rapidly. We intend to take up experiments in this direction.

## TABLE V.

*Same Wire as in Table III. Maximum Amplitude* 86.4 *Degrees ; Temperature* 24°.1 *C.*

| No. | Amplitude. | Coincidences. | Period. | Corr. Ampl. | Log. Dec. |
|---|---|---|---|---|---|
| 0 | 83.1 | 0′ 30″.36 | | | |
| 5 | 78.1 | 0 56 .81 | 5.302 | 77.6 | .00545 |
| 10 | 73.3 | 1 23 .38 | 5.299 | 73.0 | |
| 15 | 68.8 | 1 49 .83 | 5.298 | 68.5 | 542 |
| 20 | 64.7 | 2 16 .34 | 5.295 | 64.4 | |
| 25 | 60.7 | 2 42 .75 | 5.293 | 60.4 | 550 |
| 30 | 57.0 | 3 9 .29 | 5.290 | 56.6 | |
| 35 | 53.4 | 3 35 .65 | 5.288 | 53.2 | 562 |
| 40 | 50.2 | 4 2 .14 | 5.285 | 49.8 | 571 |
| 50 | 44.0 | 4 54 .96 | 5.282 | 43.8 | 556 |
| 60 | 38.6 | 5 47 .71 | 5.278 | 38.5 | 533 |
| 70 | 34.0 | 6 40 .55 | 5.275 | 34.2 | 509 |
| 80 | 30.2 | 7 32 .25 | 5.270 | 30.4 | 517 |
| 90 | 26.9 | 8 25 .94 | 5.262 | 26.8 | 527 |
| 100 | 23.8 | 9 18 .48 | 5.255 | 23.8 | 512 |
| 110 | 21.1 | 10 11 .01 | 5.250 | 20.9 | 508 |
| 120 | 18.8 | 11 3 .44 | 5.247 | 18.7 | |
| 149 | | | 5.238 | 12.8 | 512 |
| 177 | 9.6 | 16 2 .00 | | | |
| 199 | | | 5.227 | 8.35 | 319 |
| 221 | 6.95 | 19 52 .00 | | | |
| 239 | | | 5.222 | 6.39 | 253 |
| 257 | 5.67 | 23 0 .00 | | | |
| 280 | | | 5.217 | 5.17 | 202 |
| 302 | 4.6 | 27 0 .00 | | | |
| 344 | | | 5.214 | 4.2 | 85 |
| 386 | 3.93 | 34 18 .00 | | | |

Knowing the torsional period of the iron wire with a known moment of inertia, and taking into account the relative angular twists of the two wires, the torsional moment of our wire and the half period with a given moment of inertia may be calculated assuming the elastic laws to hold. Thus we calculated for the case tabulated under 1C a period of 5.375 seconds. The smallest actual period is 6.14 seconds.

10. *Influence of the Amount of Iridium.*—The earlier experiments indicated a large influence of the chemical composition of the wire. This became very apparent in the following series of experiments. We had received three wires, all marked 40 per cent., of different

## TABLE VI.

*Same Wire as in Table III. Maximum Amplitude* 117 *Degrees; Temperature* 24°.5 *C.*

| No. | Amplitude. | Coincidences. | Period. | Corr. Ampl. | Log. Dec. |
|---|---|---|---|---|---|
| 0 | 111.3 | 0′ 53″.71 | | | |
| 5 | 104.7 | 1 20 .29 | 5.327 | 104.3 | .00526 |
| 10 | 98.6 | 1 46 .98 | 5.328 | 98.2 | |
| 15 | 92.6 | 2 13 .59 | 5.327 | 92.2 | 538 |
| 20 | 87.1 | 2 40 .27 | 5.325 | 86.5 | |
| 25 | 81.6 | 3 6 .84 | 5.324 | 81.1 | 565 |
| 30 | 76.5 | 3 33 .52 | 5.325 | 76.0 | |
| 35 | 71.5 | 4 0 .13 | 5.321 | 71.0 | 569 |
| 40 | 67.1 | 4 26 .71 | 5.319 | 66.7 | |
| 45 | 62.7 | 4 53 .25 | 5.317 | 62.3 | 618 |
| 50 | 58.2 | 5 19 .98 | 5.314 | 58.0 | |
| 55 | 54.6 | 5 46 .42 | 5.310 | 54.3 | 565 |
| 60 | 51.1 | 6 13 .03 | 5.309 | 50.7 | |
| 65 | 47.6 | 6 39 .53 | 5.309 | 47.2 | 620 |
| 70 | 44.3 | 7 6 .09 | 5.303 | 44.0 | |
| 75 | 41.4 | 7 32 .54 | 5.303 | 41.0 | 643 |
| 80 | 38.2 | 7 59 .06 | 5.299 | 38.0 | |
| 85 | 35.4 | 8 25 .60 | 5.298 | 35.1 | 649 |
| 90 | 32.9 | 8 52 .08 | 5.294 | 32.7 | |
| 95 | 30.5 | 9 18 .54 | 5.291 | 30.3 | 647 |
| 100 | 28.35 | 9 45 .00 | 5.289 | 28.1 | |
| 110 | 24.3 | 10 37 .84 | 5.279 | 24.0 | 666 |
| 120 | 20.9 | 11 30 .61 | 5.273 | 20.7 | 647 |
| 130 | 18.5 | 12 23 .25 | 5.269 | 17.8 | 626 |
| 140 | 15.7 | 13 15 .88 | 5.263 | 15.6 | 577 |
| 150 | 13.75 | 14 8 .60 | 5.257 | 13.6 | 566 |
| 160 | 12.1 | 15 1 .12 | | | |
| 180 | | | 5.241 | 9.48 | 566 |
| 201 | 7.25 | 18 36 .00 | | | |
| 227 | | | 5.231 | 6.15 | 316 |
| 253 | 5.00 | 23 8 .00 | | | |
| 275 | | | 5.227 | 4.42 | 267 |
| 297 | 3.84 | 26 58 .00 | | | |
| 315 | | | 5.222 | 3.55 | 203 |
| 333 | 3.26 | 30 6 .00 | | | |
| 356 | | | 5.217 | 3.02 | 154 |
| 379 | 2.78 | 34 6 .00 | | | |

diameters. The experiments described under paragraphs 4, 5 and 6 were made with the finest one of these three. The following gives the results with the others.

(*a*) Wire No. 3, marked 40 per cent iridium, diameter 0.306 mm.

## Table VII.

*Same Wire as in Table III. Maximum Amplitude* 168.9 *Degrees; Temperature* 25°.6 *C.*

| No. | Amplitude. | Coincidences. | Period. | Corr. Amp. | Log. Dec. |
|---|---|---|---|---|---|
| 0 | 143.3 | 0′ 58″.60 | | | |
| 2 | 139.6 | 1 9 .39 | | | |
| 4 | 135.9 | 1 20 .19 | 5.390 | 134.9 | |
| 6 | 132.4 | 1 30 .94 | 5.393 | 131.5 | |
| 8 | 129.0 | 1 41 .75 | 5.393 | 128.1 | |
| 10 | 125.4 | 1 52 .45 | 5.390 | 124.5 | |
| 15 | 117.0 | 2 19 .58 | 5.391 | 116.1 | .00601 |
| 20 | 109.2 | 2 46 .41 | 5.388 | 108.5 | |
| 25 | 101.9 | 3 13 .51 | 5.390 | 101.1 | 610 |
| 30 | 94.9 | 3 40 .34 | 5.387 | 94.2 | |
| 35 | 88.2 | 4 7 .38 | 5.386 | 87.5 | 656 |
| 40 | 81.6 | 4 34 .19 | 5.385 | 81.0 | |
| 45 | 75.8 | 5 1 .17 | 5.379 | 75.2 | 647 |
| 50 | 70.3 | 5 27 .94 | 5.378 | 69.8 | |
| 55 | 65.0 | 5 54 .95 | 5.374 | 64.5 | 703 |
| 60 | 59.8 | 6 21 .80 | 5.373 | 59.3 | |
| 65 | 55.0 | 6 48 .67 | 5.368 | 54.5 | 734 |
| 70 | 50.5 | 7 15 .47 | 5.366 | 50.0 | |
| 75 | 46.2 | 7 42 .34 | 5.361 | 45.8 | 770 |
| 80 | 42.3 | 8 9 .11 | 5.355 | 42.0 | |
| 85 | 38.3 | 8 35 .89 | 5.353 | 38.0 | 810 |
| 90 | 35.1 | 9 2 .60 | 5.348 | 34.8 | |
| 95 | 32.0 | 9 29 .42 | 5.344 | 31.7 | 829 |
| 100 | 29.0 | 9 56 .00 | 5.339 | 28.7 | 843 |
| 110 | 23.8 | 10 49 .37 | 5.327 | 23.6 | 867 |
| 120 | 19.5 | 11 42 .65 | 5.318 | 19.3 | 850 |
| 130 | 16.1 | 12 35 .69 | 5.308 | 16.0 | 823 |
| 140 | 13.35 | 13 28 .73 | 5.295 | 13.2 | 729 |
| 150 | 11.3 | 14 21 .67 | 5.285 | 11.2 | 727 |
| 160 | 9.55 | 15 14 .45 | 5.279 | 9.50 | 670 |
| 170 | 8.3 | 16 7 .09 | 5.272 | 8.25 | 628 |
| 180 | 7.8 | 16 59 .89 | | 7.8 | |
| 191 | | | 5.254 | 5.96 | 552 |
| 212 | 4.76 | 19 48 .00 | | | |
| 244 | | | 5.250 | 3.88 | 313 |
| 276 | 3.0 | 25 24 .00 | | | |
| 305 | | | 5.241 | 2.62 | 208 |
| 334 | 2.23 | 30 28 .00 | | | |
| 368 | | | 5.235 | 2.00 | 148 |
| 402 | 1.77 | 36 24 .00 | | | |

## TABLE VIII.

*Same Wire as in Table III. Maximum Amplitude* 364.3 *Degrees; Temperature* 24°.6 *C.*

| No. | Amplitude. | Coincidences. | Period. | Corr. Ampl. | Log. Dec. |
|---|---|---|---|---|---|
| 0 | 325.3 | 0′ 54″.40 | | | |
| 1 | 316.8 | | | | |
| 2 | 308.8 | 1 5 .76 | 5.690 | 305 | |
| 3 | 300.7 | 1 11 .33 | | | |
| 4 | 293.4 | 1 17 .16 | 5.689 | 290 | |
| 5 | 286.1 | 1 22 .70 | | | 01081 |
| 6 | 279.4 | 1 28 .53 | 5.687 | 275.8 | .01070 |
| 7 | 272.6 | 1 33 .97 | | | |
| 8 | 266.6 | 1 39 .79 | 5.687 | 262.8 | .01058 |
| 9 | 259.6 | 1 45 .47 | | | |
| 10 | 253.6 | 1 51 .25 | 5.687 | 250.4 | .01030 |
| 12 | 242.0 | 2 2 .65 | 5.688 | 329.0 | .01031 |
| 14 | 230.3 | 2 14 .00 | 5.688 | | |
| 16 | 220.2 | 2 25 .40 | 5.685 | | |
| 18 | 210.0 | 2 36 .70 | 5.687 | 207.6 | .01022 |
| 20 | 200.6 | 2 48 .14 | 5.685 | 198.2 | .01020 |
| 25 | 178.3 | | 5.682 | 176.0 | .01058 |
| 30 | 157.0 | 3 44 .95 | 5.681 | 154.8 | .01135 |
| 35 | 137.4 | | 5.679 | 135.4 | .01178 |
| 40 | 119.4 | 4 41 .81 | 5.672 | 117.8 | .01742 |
| 45 | 103.1 | 5 9 .88 | 5.659 | 101.8 | .01315 |
| 50 | 88.2 | 5 38 .46 | 5.647 | 87.0 | .01411 |
| 55 | 74.5 | 6 6 .64 | 5.637 | 73.3 | .01531 |
| 60 | 62.0 | 6 35 .02 | 5.625 | 61.0 | .01646 |
| 65 | 51.0 | 7 2 .98 | 5.603 | 50.0 | .01743 |
| 70 | 41.5 | 7 31 .11 | 5.589 | 40.8 | .01832 |
| 75 | 33.45 | 7 58 .81 | 5.565 | 32.7 | .01883 |
| 80 | 26.9 | 8 26 .83 | 5.534 | 26.5 | .01859 |
| 85 | 21.8 | 8 54 .02 | 5.515 | 21.3 | .01818 |
| 90 | 17.7 | 9 22 .00 | 5.490 | 17.3 | .01726 |
| 95 | 14.65 | 9 49 .23 | 5.471 | 14.4 | .01563 |
| 100 | 12.35 | 10 16 .36 | 5.447 | 12.2 | .01517 |
| 105 | 10.33 | 10 43 .58 | 5.439 | 10.15 | .01331 |
| 110 | 9.09 | 11 10 .69 | 5.416 | 8.9 | .01137 |
| 115 | 7.95 | 11 37 .64 | 5.403 | 7.8 | .01073 |
| 120 | 7.0 | 12 4 .71 | 5.393 | 6.9 | .01080 |
| 125 | 6.2 | 12 31 .59 | 5.386 | 6.1 | .00992 |
| 130 | 5.65 | 12 58 .65 | 5.377 | 5.55 | .00848 |
| 163 | | | 5.338 | 4.00 | .00561 |
| 195 | 2.44 | 18 46 .00 | | | |
| 210 | | | 5.333 | 2.22 | .00288 |
| 225 | 2.00 | 21 26 .00 | | | |
| 247 | | | 5.318 | 1.80 | .00220 |
| 269 | 1.60 | 25 20 .00 | | | |

TABLE IX.

| Twist of Torsion Head. | Twist of Mirror. | Differences. | Twist of Torsion Head. | Twist of Mirror. | Differences. |
|---|---|---|---|---|---|
| +450 | +38.16 | +411.84 | −450 | −38.06 | −411.94 |
| 400 | 33.52 | 366.48 | 400 | 33.51 | 366.49 |
| 350 | 29.19 | 320.81 | 350 | 29.10 | 320.9 |
| 300 | 24.94 | 275.06 | 300 | 24.86 | 275.14 |
| 250 | 20.76 | 229.24 | 250 | 20.50 | 229.5 |
| 200 | 16.35 | 183.65 | 200 | 16.13 | 183.87 |
| 150 | 12.13 | 137.87 | 150 | 11.81 | 138.19 |
| 100 | 7.85 | 92.15 | 100 | 7.65 | 92.35 |
| 50 | 3.55 | 46.45 | 50 | 3.54 | 46.46 |
| 0 | − 0.51 | − 0.51 | 0 | + 0.52 | + 0.52 |
| − 50 | 4.68 | 45.32 | + 50 | 4.60 | 45.40 |
| 100 | 8.85 | 91.15 | 100 | 8.81 | 91.19 |
| 150 | 13.08 | 136.92 | 150 | 13.19 | 136.81 |
| 200 | 17.30 | 182.70 | 200 | 17.48 | 182.52 |
| 250 | 21.50 | 228.50 | 250 | 21.74 | 228.26 |
| 300 | 25.93 | 274.07 | 300 | 25.98 | 274.02 |
| 350 | 29.87 | 320.13 | 350 | 30.0 | 320.0 |
| 400 | 33.95 | 366.05 | 400 | 34.09 | 365.91 |
| 450 | 38.06 | 411.94 | 450 | 38.16 | 411.84 |

This wire was treated in the same manner as the thinner one, *i. e.*, it was heated under load by a current sufficiently large to raise its temperature to a bright yellow heat. Much to our surprise the wire showed only a small decrease in period, starting with 3.336 seconds at 752 degrees, decreasing to 3.330, at 150 degrees, from which point on the decrease was more rapid, down to 3.289 sec. at an amplitude of 2 degrees. The whole change was only 1.5 per cent. as compared with 7 per cent. to 10 per cent. in the thinner wire.

We drew this wire to a smaller diameter without changing its elastic behavior. Before every experiment the wire was again annealed. The results obtained, after the diameter was reduced to 0.25 mm., are given in Table X., under B. The length of the wire was 82.9 cm. In order to allow an easy comparison between all tests made in this part of our investigation all observations were reduced to what they would have been if the wire had had a length of 100 cm. and the moment of inertia such a value so as to make the smallest period just 5 seconds. This was done by multiplying the amplitudes by the ratio of 100 to the length of the wire in centimeters, and the period by such a factor as to make the period at zero amplitude 5 seconds. The values under A in Table X. are calculated from the experiment tabulated under 1C, in Table I.

Table X.

| *A* | | | *B* | | |
|---|---|---|---|---|---|
| **No.** | **Amplitude.** | **Period.** | **No.** | **Amplitude.** | **Period.** |
| 0 | 923 | .350 | 11 | 936 | 5.070 |
| 2 | 856 | .349 | 21 | 904 | .069 |
| 4 | 790 | .346 | 40 | 847 | .069 |
| 6 | 728 | .343 | 58 | 796 | .070 |
| 8 | 672 | .338 | 97 | (750) | .070 |
| 10 | 612 | .332 | 179 | 632 | .072 |
| 15 | 491 | .315 | 199 | 474 | .072 |
| 20 | 389 | .291 | 239 | 405 | .070 |
| 25 | 300 | .268 | 279 | 344 | .070 |
| 30 | 233 | .244 | 350 | (251) | .067 |
| 35 | 178 | .218 | 374 | (225) | .066 |
| 40 | 137 | .193 | 437 | (162) | .062 |
| 45 | 103 | .169 | 472 | (127) | .059 |
| 50 | 84.6 | .146 | 502 | 98.2 | .055 |
| 60 | 52.2 | .116 | 541 | 73.2 | .051 |
| 70 | 39.8 | .094 | 580 | 54.3 | .044 |
| 80 | 29.8 | .080 | 625 | 38.8 | .038 |
| 90 | 24.0 | .072 | 686 | 26.05 | .028 |
| 100 | 19.0 | .063 | 766 | 16.47 | .021 |
| 110 | 15.4 | .055 | 820 | 12.73 | .018 |
| 120 | 12.6 | .050 | 1001 | 6.14 | .010 |
| 130 | 10.3 | .040 | 1150 | 3.78 | .006 |
| 140 | 8.5 | .035 | 1461 | 1.86 | .001 |
| 150 | 6.97 | .030 | | | |

(*b*) Wire No. 4, marked 40 per cent. iridium, diameter 0.247 mm. This wire was used in the form in which it was received, *i. e.*, not annealed. Now the characteristic peculiarities were more pronounced than with the preceding wire, though considerably less than with the finest wire. After annealing the changes of the period with amplitude were even smaller than with wire No. 3. Table XI. contains the results, C for the unannealed, C′ for the annealed wire.

(*c*) 35 per cent. platinum-iridium wire. The wire had a diameter of 0.20 mm.; it was used as well in the unannealed as in the annealed state. The period remained almost constant from an amplitude of 700 degrees down to 100 degrees, after which a small decrease of period with amplitude can be observed, less pronounced in the annealed than in the unannealed state. In both cases it is very small. Table XII. contains the results for the unannealed (D) and for the annealed (D′) wire. Fig. 8 contains the curves for the data given in Tables X.–XII.

The influence of the amount of iridium in the wire is apparent. The 35 per cent. wire shows the characteristic properties to a slight degree. They become more pronounced as the percentage of iridium increases. The great difference between the thicker wires and the thinner one in our set of 40 per cent. wires, and the great similarity between the thinner one and the one originally used by Guthe led us to the belief that the latter two were originally made of the same alloy, while the thicker ones were made of another mixture containing a smaller relative amount of iridium. Upon inquiry Dr. Heraeus stated that this might be possible, but that no record was kept about the wires. It seems very difficult to draw wires containing more than 40 per cent. of iridium since the material becomes

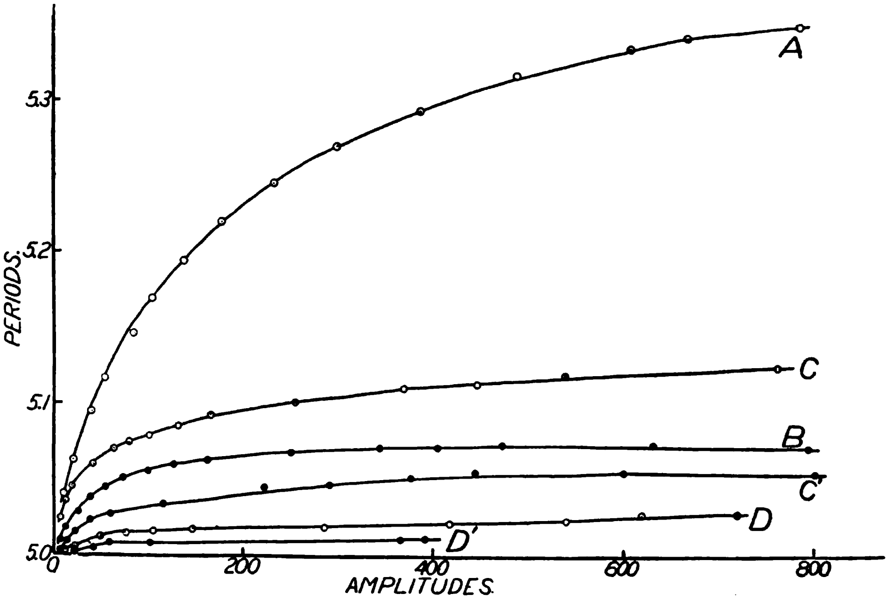


Fig. 8.

TABLE XI.

| *C* | | | *C′* | | |
|---|---|---|---|---|---|
| **No.** | **Amplitude.** | **Period.** | **No.** | **Amplitude.** | **Period.** |
| 20 | 784 | 5.123 | 21 | 858 | 5.053 |
| 60 | 540 | .117 | 36 | 801 | .054 |
| 80 | 447 | .112 | 100 | (600) | .054 |
| 100 | 370 | .109 | 160 | 445 | .054 |
| 120 | (310) | .105 | 180 | 420 | .054 |
| 140 | (255) | .100 | 219 | 378 | .050 |
| 160 | (207) | .096 | 273 | (291) | .046 |
| 180 | (165) | .092 | 321 | (223) | .044 |
| 200 | (131) | .084 | 405 | (116) | .033 |
| 220 | 100.1 | .078 | 490 | 5.59 | .027 |
| 240 | 79.0 | .074 | 530 | 47.19 | .025 |
| 260 | 63.7 | .070 | 570 | 38.13 | .023 |
| 280 | 51.4 | .065 | 607 | 31.44 | .019 |
| 300 | 41.4 | .059 | 683 | 22.21 | .015 |
| 340 | 28.2 | .053 | 743 | 17.66 | .011 |
| 380 | 18.9 | .045 | 825 | 13.10 | .009 |
| 400 | 16.2 | .042 | 926 | 9.70 | .006 |
| 436 | 12.4 | .036 | 1028 | 7.46 | .002 |
| 475 | 9.13 | .026 | 1138 | 5.99 | .002 |
| 530 | 6.22 | .024 | 1411 | 3.48 | .001 |

extremely brittle. The wires are very hard. Successive holes in the draw plate must not differ more than 0.01 mm. and the wire must be annealed after each drawing. We hope to continue our investigation with a new set of wires just received and extend the investigation also to other brittle wires. At the present state of our knowledge about the wires it seems to us too early to discuss the theory of the phenomena observed, or to propose an empirical formula which will more or less accurately fit our curves. Such a formula will be of value only after we have learned how to control the conditions sufficiently so that we may at will reproduce any given curve.

The large logarithmic decrement points towards an energy loss by internal friction. It should, however, be kept in mind that one of us has pointed out that a carbon filament has a very large decrement but that the latter is perfectly constant. In one of the experiments with a carbon filament the amplitude decreased to one tenth of its original value in 176 swings. The elastic after-effect and its

TABLE XII.

| D | | | D′ | | |
|---|---|---|---|---|---|
| **No.** | **Amplitude.** | **Period.** | **No.** | **Amplitude.** | **Period.** |
| 20 | 939 | 5.027 | 20 | 392 | 5.011 |
| 40 | 862 | .026 | 40 | 379 | .011 |
| 80 | (720) | .027 | 60 | 365 | .010 |
| 120 | (620) | .026 | 810 | 101.3 | .008 |
| 160 | 541 | .022 | 1300 | 58.4 | .008 |
| 200 | 475 | .021 | 1620 | 42.0 | .004 |
| 240 | 419 | .020 | 2110 | 27.9 | .002 |
| 280 | 373 | .018 | 2751 | 17.5 | .002 |
| 350 | (287) | .018 | 3558 | 12.3 | .001 |
| 420 | (230) | .017 | 4215 | 8.5 | .001 |
| 564 | (147) | .017 | | | |
| 700 | 105 | .016 | | | |
| 820 | 76.9 | .014 | | | |
| 1000 | 49.67 | .012 | | | |
| 1112 | 38.55 | .007 | | | |
| 1393 | 21.48 | .005 | | | |
| 1573 | 15.65 | .004 | | | |
| 1775 | 11.58 | .003 | | | |
| 2256 | 6.58 | .001 | | | |

possible influence upon the period has been discussed above. The behavior of platinum-iridium wire may not be exceptional. Bouasse's work and that of many others point towards a more or less marked deviation from the properties of a perfectly elastic body in all cases; but so far only very small deviations were known. The peculiarities of the platinum-iridium wire are easily studied and may possibly facilitate the understanding of the actual conditions of elastic deformations of ordinary wires.

## SUMMARY.

1. Platinum-iridium wires containing over 30 per cent. iridium show striking peculiarities in their elastic behavior which become the more marked the higher the percentage of iridium.

2. For small amplitudes the period of torsional vibration is nearly proportional to the amplitude; but for larger amplitudes the rate of increase of the period becomes smaller and for very large amplitudes the period tends towards a constant value.

3. The logarithmic decrement increases with the amplitude,

reaches a maximum, after which it again decreases and probably reaches a minimum for very large amplitudes. The maximum is the more pronounced the larger the original distortion.

4. If the wire has been allowed to rest some time before the torsional vibrations are produced the total number of vibrations necessary to bring the system to rest is the smaller the larger the initial amplitude.

5. When tested by the static method the wires closely obey Hooke's law. The torsional moment calculated from the static experiment is larger than the largest torsional moment found by the dynamic method.

# "BOUND MASS" AND THE FITZGERALD LORENTZ CONTRACTION.[1]

BY WILL. C. BAKER.

FROM the theory that regards electromagnetic momentum as being due to a mass of ether entrained or "bound" by the moving Faraday tubes it follows that the component momentum (per unit volume) in the direction of the motion of the charged body is given by $4\pi\mu N^2 \sin^2\theta\omega$ and that the component perpendicular to this is $4\pi\mu N^2 \sin\theta\cos\theta\omega$ and is directed towards the line along which the body is moving. In these expressions $N$ is the polarization and $\theta$ is the angle between the direction of polarization and that of the velocity $\omega$.[2] In considering the momentum of the entire system we need only consider the component parallel to the direction of motion as the summation of the other component must vanish by symmetry. The present note records a simple method of calculating from this idea expressions for both the transverse and longitudinal masses of the system on the supposition that the whole mass is electrical.

It has been shown that as a charged particle attains a steady velocity, a given tube will leave the position occupied by it (relatively to the body) when at rest and will move up towards the equatorial plane, *i. e.*, the plane through the particle normal to the direction of motion.[3] In the case of a body moving along the $Z$ axis with a speed $\omega$ the transformation from the configuration that obtains at rest, to that obtaining during steady motion is given (relatively to axes through the particle and moving with it) by

$$x' = x, \quad y' = y, \quad z'\frac{V}{(V^2 - \omega^2)^{\frac{1}{2}}} = z,$$

---

[1] A paper read at the Boston meeting of the American Physical Society, Dec. 29, 1909.

[2] J. J. Thomson, Electricity and Matter, p. 37.

[3] Heaviside, Phil. Mag., April, 1889. See also J. J. Thomson, Recent Researches, p. 19.

where the unprimed letters refer to a particular part of a tube when at rest and the primed letters refer to the same part of the same tube when in steady motion. $V$ of course is $3 \times 10^{10}$. This transformation is evidently that of the FitzGerald-Lorentz contraction,[1] for where $\beta = \omega / V$

$$\frac{V}{(V^2 - \omega^2)^{\frac{1}{2}}} = \frac{1}{(1 - \beta^2)^{\frac{1}{2}}}.$$

Taking now these two ideas and regarding the mass of the whole system as being due to the effective part of the "bound" ether, *i. e.*, assuming the whole effective mass as electrical, the Lorentz expression for the ratio of the transverse masses at different velocities follows almost immediately.

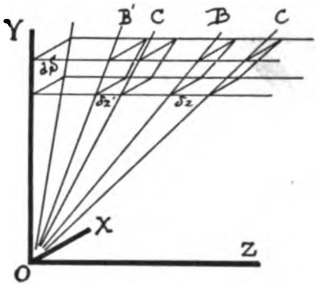


Fig. 1.

Consider axes as shown in the figure, moving with the charged particle as origin. Let $BOC$ represent the postion of an elementary cone of $n$ tubes when the system is moving with a *very small velocity* $\omega_0$. As the velocity increases to a large value, $\omega'$, these same tubes will draw up into a position such as $B'OC'$, the change being given by

$$x' = x, \quad y' = y, \quad z' = z(1 - \beta^2)^{\frac{1}{2}}.$$

Now the volumes cut out of these cones by a column parallel to the $Z$ axis and of constant section $dS$ will be "corresponding volumes," *i. e.*, the tubes that occupy one when at rest will occupy the other during the steady motion we are considering. In virtue of the contraction we know that a length $\delta z$ will shorten to $\delta z(1 - \beta^2)^{\frac{1}{2}}$ also that the elemenatry volumes $\delta v$ and $\delta v'$ will be given by

$$\delta v = \delta x \delta y \delta z \quad \text{and} \quad \delta v' = \delta x \delta y \delta z (1 - \beta^2)^{\frac{1}{2}}.$$

---

[1] It is to be noted that this transformation is a direct result of Maxwell's equations. It gives the contraction for the field of the particle independently of the considerations on which the "Fitzgerald Lorentz assumption" is usually based, and it is equally independent of any relativity principle,

The polarizations in the two cases are

$$N = \frac{n}{\delta x \delta z \sin\theta} \quad\text{and}\quad N' = \frac{n}{\delta x \delta z (1-\beta^2)^{\frac{1}{2}} \sin\theta'}.$$

Therefore the $Z$-components of the momenta *per unit volume* are $(G_z = 4\pi\mu N^2 \sin^2\theta\omega)$:

$$G_z = 4\pi\mu \frac{n^2}{\delta x^2 \delta z^2}\omega_0 \quad\text{and}\quad G_z' = 4\pi\mu \frac{n^2}{\delta x^2 \delta z^2 (1-\beta^2)}\omega',$$

and the momenta *in the corresponding volumes* will be

$$g_z = 4\pi\mu \frac{n^2\omega_0}{\delta x^2 \delta z^2}\delta x \delta y \delta z \quad\text{and}\quad g_z' = 4\pi\mu \frac{n^2\omega'}{\delta x^2 \delta z^2 (1-\beta^2)}\delta x \delta y \delta z (1-\beta^2)^{\frac{1}{2}}.$$

Now the transverse mass of a system is given by dividing its (longitudinal) momentum by its (longitudinal) velocity,[1] so it follows at once from the values of $g$ and $g'$ that the transverse masses of the corresponding volumes and (as the ratio is the same for all such pairs) the transverse masses of the entire corresponding systems are in the ratio

$$\frac{m'}{m_0} = (1-\beta^2)^{-\frac{1}{2}}.$$

This is precisely the relation given by Lorentz and verified experimentally by Bucherer[2] in the case of the high velocity particles from radium.

Again, if a force act on the system so as to increase $\omega$, it produces

---

[1] For let the system be moving with a velocity $v$ through a field of $H$ lines. It will suffer in time $\delta t$ a change of direction $\delta\theta$. Let the (longitudinal) momentum be $G$. Now equating impulses

$$Hev\delta t = G\delta\theta;$$

but, where $\rho$ is the radius of curvature of the path,

$$\delta\theta = \frac{\delta s}{\rho} = \frac{v\delta t}{\rho}.$$

Again $v^2/\rho$ is the acceleration normal to the path, so if $m_t$ be put for the *transverse* mass we obtain

$$\frac{m_t v^2}{\rho} = Hve.$$

and on comparing these expressions for $He\rho$ it follows that $M = G/v$. See J J Thomson, Conduction of Electricity through Gases, p. 652.

[2] Annalen der Physik, No. 3, 1909, p. 525.

a change of momentum at a rate given by

$$F = \frac{dG}{dt} = \frac{dG}{d\omega} \cdot \frac{d\omega}{dt} = \frac{1}{V} \frac{dG}{d\beta} \cdot \frac{d\omega}{dt},$$

for $\beta = \omega / V$. Hence the longitudinal mass is

$$m_2' = \frac{1}{V} \cdot \frac{dg_2'}{d\beta}.^{1}$$

Applying this to our expressions for $g_2$ and $g_2'$, we find that the longitudinal masses are in the ratio

$$\frac{m_2'}{m_2^0} = \frac{1}{(1-\beta^2)^{\frac{3}{2}}}.$$

This is also the relation obtained by Lorentz from the integration of the energy flux.

It is to be noted that the above given expressions refer to the electrical part of the mass only; but this from Bucherer's work would seem to be the whole mass. Also it must be remembered that the Fitzgerald Lorentz contraction — in the above — applies to the tubes of force, *i. e.*, to the field of the charged particle and so through this to actual systems built up of such particles. These considerations show that the question of the deformability of the electron itself tends to pass away and we are led to the conception of the electron advocated by Sir J. J. Thomson; *i. e.*, a simple point charge, the whole inertia of which is due to the entrained ether.[2]

Physical Laboratory, School of Mining,
Queen's University, Kingston, Ont., December 9, 1909.

---

[1] H. A. Lorentz, Theory of Electrons, p. 37.

[2] Conduction through Gases, pp. 654–655.

# THE SECONDARY RADIATION PRODUCED BY THE BETA RAYS OF RADIUM.

By W. H. Bragg.

In the September number of the Physical Review Mr. S. J. Allen discusses the secondary radiation due to the $\beta$ rays of radium, and in the introduction to his paper he describes the two theories of secondary radiation due to $\gamma$ and $x$ rays, which, he says, "are at the present time in prominence."

For one of these I am responsible, and therefore I would ask you leave to correct Mr. Allen's statement of it. On page 178 Mr. Allen writes as follows: "He assumes the beam of $\gamma$ or $x$ rays to be made up of a number of electric doublets travelling at high speed, and each doublet as composed of a negative and positive electron of the same charge and mass . . .," and again, "Bragg has to assume the existence of a positive electron of the same mass as the negative." Mr. Allen subsequently argues that on such a theory the positive electron should be as capable of isolation as the negative; and that since it has not been isolated in the same way, the theory is not justifiable.

The criticism might have been just if I had made any such statements as those with which Mr. Allen credits me. But I have always held that the positive differed from the negative in more than sign. In my first attempt to frame a corpuscular theory of $x$ and $\gamma$ rays I considered the possibility of a negative electron being associated with an $\alpha$ particle, but subsequent experiments led me to suppose the mass of the positive to be small compared to that of the negative.[1] This is in accordance, I think, with general ideas. If the negative electron derives its mass from the concentration of its charge on a minute surface of about $10^{-13}$ cm. diameter, and the positive is more diffused, so to speak, it seems reasonable to expect such a difference in mass. The fact that no positive has been iso-

---

[1] See, for example, Phil. Mag., VI., 16, p. 938.

lated of less than atomic mass is no argument against such a view, for such a positive may contain numbers of negatives all fitted, except one or two, with their positive counterparts. To conclude, my hypothesis is not therefore "contrary to the known experimental facts"; on the other hand, I think it correlates them simply and fairly completely.

I venture to think that Mr. Allen is also in error in another of the main arguments of his paper. "If the radiation from a pure element," he says on page 188, "is a function of the atomic weight, we should expect as a first approximation the radiation of a salt to be a function of the sum of the atomic weights of its chemical constituents, that is, of its molecular weight." Surely we should expect the opposite conclusion from the given premiss. A bundle of incident rays is limited in quantity; those which are scattered by, for example, the Ba in $BaCl_2$ will often be unavailable for scattering by the Cl; the acts of scattering are, to a considerable extent, mutually exclusive; it is not right to add and subtract, as Mr. Allen does, the effects of the various atoms in the molecules. Many of the results which Mr. Allen classifies as anomalous or inconsistent are quite normal when truly considered in relation to the theory that a $\beta$ particle acts upon an atom in a way which is independent of the neighborhood or association of other atoms. The amount of secondary radiation will not depend on molecular grouping at all, but rather in a general way on the relative numbers of large and small atoms present. It is not wrong that a sugar solution should give much the same amount of secondary radiation as water, nor that LiCl should give more than $Li_2CO_3$: nor that $CH_2O_2$ should give more than $H_2O$.

Nevertheless, there are certain results of Mr. Allen's work which cannot be explained in this way. The most notable is the comparison of the properties of ethylene chloride and ethylene bromide, which shows that the former gives more secondary radiation than the latter, and possesses three times the absorbing power even when the absorption coefficients are measured in terms of thickness and not of weight. This is very remarkable, and at my request certain students of this university, Messrs. Armin, Dixon and Morton, have repeated the experiments. The liquids were prepared for them by Prof. Julius Cohen, and they used two methods, one the same as Mr. Allen's, the other different in that the liquid was always enclosed between two thin Al sheets so that no vapor could complicate the results. Their measurements on both methods are quite regular and consistent with each other and they find that ethylene chloride does not absorb three times as much as ethylene bromide, but less, distance for distance. Weight for weight the absorption coefficients are practically the same, as would be expected on the theory that atoms act on $\beta$ rays independently of each other.

They also find that the secondary radiation from ethylene chloride is less than that from ethylene bromide, as also would be expected.

I am at a loss to explain this complete divergence from Mr. Allen's results; I cannot but think that in some way he has made a mistake. I should like to add a few words in relation to the principles of secondary radiation to which Mr. Allen makes frequent reference in his paper. When any one of what may be called the radiant entities, $\alpha$, $\beta$, $\gamma$, $x$ or cathode rays passes through an atom, the chance of its being deflected in a given direction is a function of several variables, but for a ray of specified nature and quality falling on a given atom the chance may be represented by the length of a radius vector drawn in that direction. One of the main purposes of present experiment is to determine the curves which are the loci of the extremities of these radii vectores in the multitude of various cases. Madsen has made approximations to some determinations of this kind for $\beta$ rays.[1] In the case of $\beta$ rays and a heavy atom like lead, the atom is nearly centric to the curve; in the case of a light atom like aluminium it is very excentric, as might reasonably be expected. This is quite sufficient to account for the observed dependence of secondary radiation on the angle of incidence, and there is no need at present to suppose a true secondary radiation, or a "specular reflection"; indeed this last is inconsistent with the hypothesis that the $\beta$ ray acts only on one atom at a time.

THE UNIVERSITY OF LEEDS.

---

[1] Brisbane Report, January, 1909; Phil. Mag., December, 1909.

# A CONVENIENT FORM OF QUARTZ TUBE MERCURY LAMP.

By Chas. T. Knipp.

THE mercury arc in a quartz tube forms a convenient source of ultra-violet light. Several forms of tubes are in general use, prominent among them being the tubes made by Heræus and described by Küch and Retschinsky.[1]

Three years ago the writer had occasion to design and set up a mercury arc-in-quartz tube to be used by the biological department in some work upon bacteria. Fig. 1 shows the tube and manner of mounting. The diameter of the main branch $AB$ is 25 mm., and the length of the straight portion 15 cm. It had, as used in Fig. 1, three outlets. The tubes $AM_1$ and $BM_2$ are 20 cm. long and 4 mm. internal diameter, while the exhaust nipple $mn$ was somewhat smaller in diameter. The tube was hung on two asbestos covered pegs mounted on a suitable support. Glass extensions $M_1E_1$ and $M_2E_2$ were added of sufficient lengths to support the barometric columns $AE_1$ and $BE_2$ respectively. Mercury seals insured tight joints at $M_1$ and $M_2$. The vessels $E_1$ and $E_2$ served also as the leading in terminals to the arc. After exhausting the tube it was sealed off beyond $M_3$. The arc was started in the usual way—by lifting $E_2$ slightly until the mercury would flow from $B$ down to $A$. The mercury levels at $A$ and $B$ were adjusted readily by raising or lowering $E_1$ and $E_2$.

This arc was suited only for low voltages and hence gave out comparatively weak ultra-violet radiations. Any attempt to increase the energy consumption would, because of the vapor pressure, depress the mercury electrodes at $A$ and $B$. In order to obtain a more intense light the shallow vessels $E_1$ and $E_2$ were replaced by narrow tubes (about 1.2 cm. in diameter) extending some distance up the barometric legs. This method automatically kept the mer-

---

[1] Ann. d. Phys., 20, p. 563, 1906.

cury surfaces at $A$ and $B$ at practically constant levels, and admitted of a small range of light intensity. However, another difficulty was encountered, the increased temperature of the arc interfered with the seal at $M_3$. In the attempt to obtain a light source of much greater intensity the manner of setting up the tube was entirely changed recently.

In the new arrangement the exhaust nipple $mn$ in Fig. 1 was fused off near the main tube, and instead connection was made below the mercury seal $M_1$ as shown in Fig. 2. This exhaust tube should join the barometric leg at a point $p$ far enough down to admit of lowering the seal $M_1$. A valve protected by a mercury seal is placed at $V_3$. The lower end of the leg is shaped in the form of an S having a valve $V_1$ in its outer right-hand limb and a no. 24 Pt terminal wire sealed into a projection from its middle limb. In the figure the terminal $E_1$ is shown in the plane of the paper, while in actual construction it is more conveniently joined to the front. This allows the S to be made more compactly than shown. The opening $t_1$ to the projection carrying the terminal $E_1$ should be rather large, otherwise some difficulty may be experienced in filling it with mercury. The length of the leg $M_1E_1$ must be such as to support a barometric column *below* the point $p$ and thus not interfere with the process of exhaustion.

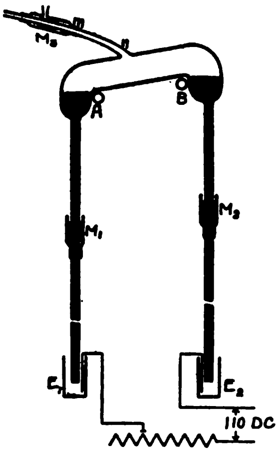


Fig. 1.

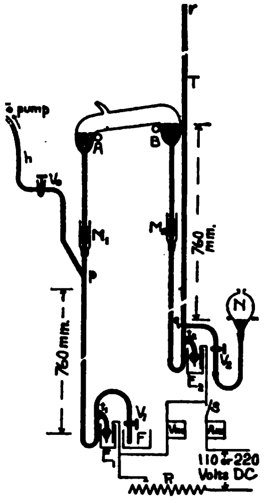


Fig. 2.

The leg $M_2E_2$ as will be seen by the figure is similar to the other with the following exceptions: The point $q$ is 760 mm. below the mercury level at $B$. At $q$ is joined a manometer tube about 150 cm. long and 3 mm. internal bore with its upper end $r$ open. A mercury reservoir $N$ is joined by a short rubber tube to the outlet below the valve $V_1$ as shown in the figure.

To operate: Fill the cups shown at the terminals $E_1$ and $E_2$ with mercury and connect the same through a suitable resistance $R$ and switch $S$ to a 110-volt, or in case very intense light is desired, 220-volt D.C. source. Include an ammeter and voltmeter in the circuit. Now close the valve $V_1$ and open the valves $V_2$ and $V_3$. Let the outlet below $V_1$ dip into a vessel $F$ containing mercury. Pour mercury into the reservoir $N$ until the mercury rises above the valve $V_2$ and then close it. Close $r$ with a rubber tube and pinch-cock, and then start the pump. After a few strokes close $V_3$ and open the valves $V_1$ and $V_2$ slightly. This serves to fill the projections at $E_1$ and $E_2$. Repeat the operation if necessary. Now open $V_1$ and $V_2$, pour a considerable quantity of mercury into $N$, open the pinch-cock at $r$, start the pump and then open the valve $V_3$. After continued pumping the mercury should stand below $p$ in the leg $AE_1$ and at $B$ in the leg $BE_2$. When the tube $AB$, upon sparking, is found evacuated sufficiently (and it is well to let the apparatus stand over night before sealing off) close $V_1$ and pour mercury slowly into $N$. The mercury will overflow at $B$ and fill the leg $AE_1$ rising in both branches and "seal off" the system from the pump. When the mercury has reached $h$ in the exhaust tube close $V_3$, and when it has reached $A$ in the main tube close $V_2$. The quartz tube is now ready to start the arc. Close the switch $S$ and open the valve $V_2$ only long enough to let a stream of mercury flow from $B$ to $A$. If properly exhausted the tube should light up at once. As the arc burns the current fails off in value. By keeping the current constant—increasing the impressed voltage—the arc will exhibit the characteristics as described by Küch and Retschinsky for increasing vapor pressures. The pressure is read by the manometer $qr$ and may exceed with safety one atmosphere.

The quartz tube $AB$ may have almost any shape which the needs of the investigation may require, keeping in mind the manner of starting the arc. The electrode *B* should be comparatively large in area so that a slight depression of the mercury will correspond to a considerable height in the manometer tube *qr*. Either *A* or *B* may be the anode. As *A* fills up, by condensation and also by overflow from *B*, it may be lowered at will by opening $V_1$ for a moment. In case the vacuum falls the tube can be exhausted readily again. The valves should be worked dry to avoid contaminating the mercury. To give added strength to the joints at $M_1$ and $M_2$, the pure gum tubing used should be wound over with one layer of adhesive tape which in turn should be wrapped with strong thread. The conduction of heat from the electrodes to these joints is not sufficient to injure them even for continued running at high pressures.

Some of the advantages of this form of quartz tube mercury lamp are: Simplicity of the quartz tube; ease of starting the arc and adjustability of the mercury surfaces at *A* and *B*; apparatus may be taken apart readily for purposes of cleaning or for introducing different materials into the arc; when properly mounted it is quite portable.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS

# NOTE ON THE REFLECTING POWER OF TANTALUM, TUNGSTEN, AND MOLYBDENUM.

By W. W. Coblentz.

To many of us it seems sufficient to explain the high luminous efficiency of the new, metal filament, incandescent lamps by saying that is is due to selective emission. The question of the region of the spectrum in which this selectivity occurs seems to be overlooked. In a recent number of the Bulletin of the Bureau of Standards, the writer has given reasons for believing that in these metals the selectivity must fall in the region of the spectrum of wave-lengths less than 1.5 to $2\mu$ and especially in the visible and ultra-violet; for a low reflectivity (and hence high emissivity) in the visible and ultra-violet is a characteristic of pure metals. In other words, since the reflectivity of metals is a smooth and continuous function of the wave-length in the infra-red, there is no reason for expecting these unexamined metals (tungsten, tantalum, etc.) to behave otherwise. Their spectral energy curves must therefore be smooth and continuous in the infra-red (no indentations or protuberances) and experimental curves showing such indentations are to be regarded with suspicion as to the accuracy of the observations.

It is difficult to show experimentally this selective emission in other than "colored metals" (*e. g.*, gold and copper) by means of emission spectra, and since the emissivity in the short wave-lengths appears to have only a small temperature coefficient it is possible to gain some knowledge of the spectral emissivity of the metal at high temperatures by determining its reflectivity at low temperatures. It is therefore proposed to study the reflecting power of the various metals used in incandescent lamps, and as a preliminary the following observations are presented.

To obtain the reflectivity, a vacuum bolometer, mirror spectrometer and fluorite prism are used. The metal, in the form of a plane surface, is compared with pure silver of known reflectivity, from which the absolute reflecting power of the unknown metal is readily obtained.

The sample of tantalum (dimensions, 10 × 15 × 1 mm.) was the purest obtainable, and in spite of the excellent polish it shows an excessively low reflectivity throughout the spectrum, as compared with pure metals. The curve in Fig. 1 represents three series of observations, using different adjustments and an angle of incidence on the mirrors of 12 to 15°, the observations usually agreeing to 1 part in 300. In the visible spectrum the reflectivity is only 10 per cent., which gradually increases to 63 per cent. at $10\mu$. There is no indication of an approach to constant reflectivity in the infra-red such as obtains in pure metals. In this respect the tantalum now obtainable is not unlike steel, except that the latter has a higher reflectivity (60 per cent. in visible to 95 per cent. at $12\mu$) throughout the spectrum. The reflectivity is very closely the same as natural graphite (dotted curve, Fig. 1) which increases from a reflectivity of 30 per cent. at $1\mu$ to 58 per cent. at $10\mu$. The reflection spectrum of tantalum has the characteristic of an impure metal or an alloy. To the eye, the sample appears flawless, so that the low reflectivity in the short wave-lengths cannot be explained on the hypothesis of roughness and scattering of the light. A rough comparison was made with a small spectrophotometer in the visible and violet, which showed that the reflectivity of tantalum continues to decrease toward the violet but not very rapidly as compared with silver.

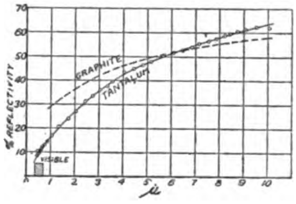


Fig. 1.

The low reflectivity in the visible spectrum tends to produce a high emissivity, but contrary to the properties of pure metals, *e. g.*, platinum, the reflectivity continues low (emissivity high) in the infra-red, which tends to lower the luminous efficiency. Omitting the question of operating temperatures, the reflectivity data give us some clue to the anomalous position of tantalum in the scale of luminous efficiencies of metals. Incandescent lamps of tungsten and osmium have an efficiency of about 1.25 w.p.c. while for no particular reason tantalum must be classified with graphitized carbon with an efficiency of about 2 to 2.5 w.p.c.

The foregoing data on the reflectivity of tantalum and graphite show that this classification is consistent. The radiation constants of these two substances are almost the same (total radiation; tantalum proportional to the 5.3 power of temp., graphitized filament proportional to the 5.1 power of temp.) and the manner in which they differ is in the right direction, since graphite is a non-metal. Hence, while it was somewhat contrary to expectation to find such a low reflectivity in the infra-red, for tantalum, this is not inconsistent with other data. Since the reflectivity curve is uniform it is evident that the spectral energy curve must also be smooth and free from such indentations as have been found in the emission spectra of non-metals.

The reflecting power of tungsten and of molybdenum is entirely different from tantalum. These two metals have a low reflectivity (50 p.c. and 48 p.c. respectively) in the yellow, which rises abruptly to 85 p.c. at $2\mu$, beyond which point the reflecting power increases gradually to 96 p.c. at $10\mu$. This is a characteristic of pure metals.

Since molybdenum and tungsten have almost the same reflectivity throughout the spectrum their luminous efficiencies must be practically the same, when operated at the same temperature; and since the former is tough and ductile, while the latter is brittle, it would be advantageous to use the former as a lamp filament, provided certain other physical defects can be overcome.

WASHINGTON, D. C.,
March 1, 1910.

# THE CATHODE EQUILIBRIUM IN THE WESTON STANDARD CELL.

BY G. A. HULETT.

IN previous articles[1] the cathode systems of standard cells have been studied from the chemical and electrochemical standpoints. The cathode system of the cadmium or Weston cell is made from a saturated solution of cadmium sulphate to which is added an excess of cadmium sulphate crystals, mercurous sulphate and mercury. In this system it was found that the depolarizer, mercurous sulphate, was hydrolyzed by the cadmium sulphate solution but the reaction proved to be so very slow that it was not possible to bring the system to equilibrium by rotating it at 25°. The various facts obtained in these experiments were best explained by assuming that this hydrolysis proceeded more rapidly at the surface of the mercury than at any other point in the system. This assumption has been objected to by Dr. F. A. Wolff as not in accord with some of his experiments,[2] and, as this assumption is also a part of the explanation of the slow changes which have been observed in the Weston standard cells, it seemed to be desirable to get more evidence on the point.

In the previous work a tube, 32 × 150 mm., was charged with mercury, mercurous sulphate, cadmium sulphate and solution in about the proportions used in standard cells. It was so arranged that the anode leg could be removed, the tube closed and rotated horizonally in a constant temperature bath and whenever desired the anode was adjusted and the value of the cell determined.[3] It was found that the rotation of the cathode system of this cell increased its E.M.F. and in most experiments by about two millivolts. This increase in the value of the cell indicated an increase in the concentration of the mercury in the solution of the cathode

---

[1] Zeit. Phys. Chem., 49, 483. Trans. Amer. Electrochemical Soc., 5, 59. PHYS. REV., 22, 321; 23, 166; 25, 16 and 27, 337.

[2] PHYS. REV., 28, 310.

[3] PHYS. REV., 27, 346.

system, a conclusion which has been fully substantiated by analyses of these solutions.[1] When the system was allowed to stand, after rotation, the E.M.F. was found to slowly decrease, but there was not a corresponding decrease in the mercury concentration in the solution above the paste or in the paste, so that it was evident that some change had taken place which decreased the mercury concentration in the solution which formed the surface layer between the paste and the mercury electrode. This conclusion was arrived at so many times and under such varying conditions that there could be no question as to facts. At first it seemed that there must be some secondary reaction which caused the decrease at the boundary,[2] but when more information was obtained it was seen that this change could be readily explained as the result of an increase of the rate of hydrolysis at the mercury surface, for that would mean an increase in the basic salt and acid at the expense of the normal salt, a condition which would decrease the mercury concentration in this surface next to the electrode and it is the mercury concentration in this layer alone which determines the cathode potential of the cell. It was also noted that the acid formed by the hydrolysis could diffuse out to regions where there was little or no acid and thus the reaction would proceed until nothing but the basic salt was left in the surface next to the mercury cathode. The basic salt has only a very small solubility and such a condition would cause the E.M.F. of the cell to decrease to an abnormally low value, and this is exactly what has happened in a number of cases[3] and with some of our own cells.

Now the question as to whether the rate of hydrolysis of mercurous sulphate is affected by the presence or absence of mercury in this system can be readily tested as follows: If two rotation experiments are carried out, alike in all respects except mercury is absent from one tube, there should be a difference in the rate and, since none of these systems can be brought to equilibrium in any experimental time, there should also be a difference in the amount of the hydrolysis in the two cases.

---

[1] Phys. Rev., 27, 348.

[2] Phys. Rev., 25, 27.

[3] Philos. Trans., A, 207, 400.

The same apparatus[1] which had served for the previous work was used for this test. The cathode tube was charged with the normal amount of mercurous sulphate, cadmium sulphate and saturated solution but precautions were taken to exclude mercury. The mercurous sulphate was prepared chemically.[2] It was white and crystalline and easily freed from mercury by decantation, using a 1 to 6 sulphuric acid; it was then washed with alcohol and a cadmium sulphate solution. The tube was rotated as previously described; from time to time some of the clear liquid was removed and tested on a clean mercury surface against a cadmium amalgam, thus forming with it a standard cell.[3] In this way the following results were obtained:

| Time of Rotation. | E.M.F. |
|---|---|
| 23½ hours. | 1.01800 |
| 3 days. | 50 |
| 7 | 66 |
| 12 | 70 |

On continuing the rotation no further change was observed, while in the previous experiments which were like this one, except that mercury was present, values ranging about 1.020 volts were obtained before a steady state was reached. It was of interest now to determine the effect of adding mercury to this particular system, accordingly 20 c.c. of the clear liquid was removed and an equal volume of mercury added. An *increase* in mercury concentration was at once indicated by the E.M.F. measurements as the following results show:

| Time of Rotation after Adding the Mercury. | E.M.F. |
|---|---|
| 7 hours. | 1.01895 |
| 2 days. | 1.01925 |
| 9 | 1.01945 |
| 18 | 1.01955 |

A second experiment was tried in which electrolytic mercurous sulphate was used and before mercury was added to the cathode system the following results were obtained:

---

[1] PHYS. REV., 27, 346.

[2] PHYS. REV., 127, 345.

[3] PHYS. REV., 25, 21.

| Time of Rotation. | E.M.F. |
|---|---|
| 3½ hours. | 1.01825 |
| 2 days. | 68 |
| 5 | 80 |
| 10 | 90 |
| 22 | 1.01910 |

Mercury was now added to the system and the rotation continued.

| Time of Rotation. | E.M.F. |
|---|---|
| 2 days. | 1.01924 |
| 9 | 50 |

It was not possible to remove the mercury from the electrolytic mercury as completely as in the case of the chemically prepared depolarizer, but the effect of the presence of an excess of mercury was still distinctly measurable. The increase in the E.M.F. of the cells show a corresponding increase in the mercury concentration of the cathode system as has been abundantly shown so these experiments give good evidence of the effect of mercury in increasing the rate of hydrolysis of mercurous sulphate in a cadmium sulphate solution.

In the rotation experiments the cathode tube with the adjustable anode contained materials in the same proportions as an ordinary Weston standard cell and they have shown the same values as the regular cells made from the same materials. When, however, the cathode was rotated the possible changes were made to take place more rapidly and could then be readily studied. These changes are such as would take place in any Weston standard cell when the cathode system is similarly treated, for example a cell (W. 141) which was made in December, 1906, and has not varied over .00002 V., from 1.01834 at 25° for three years, was opened and the cathode paste carefully but thoroughly stirred. The E.M.F. immediately increased to 1.01881, then it decreased rapidly at first and in two weeks was down to 1.01837 and still going down. All the time this cell had been constantly at 25°. Another cell (W. 38) made in May, 1906, was opened in February, 1907, and the paste stirred. It did not go so high and its value December 25, 1909, was 1.01813, so we get a variety of results from this experiment as might be expected from our present knowledge of the hydrolysis which takes place.

Two cells which had fallen to very low values A17, made in November, 1905, showed the value of 1.01212 on December 25, 1909, and A51 made in July, 1906, was 1.01794, December 25, 1909. These cells were opened and an adjustable anode showed that the anodes of these cells were normal and that the changes had been exclusively in the cathode system. When these cathode systems were stirred the values 1.01922 and 1.02310 respectively were registered and these cells are now showing decreasing values. This is further evidence that in the ordinary cell the surface layer of the electrolyte surrounding the mercury cathode has a distinctly lower mercury concentration than the rest of the electrolyte in the paste and above and that there must have been an increased rate of reaction next to the mercury.

No such changes as have been observed could occur if this system was in equilibrium for the possible disturbances due to surface tension or oxidation have been shown to be without effect in the experiments as carried out[1]. On the other hand, hydrolysis has been shown to take place in this system and it offers an explanation of the observed changes. The hydrolysis, with which we are concerned here, is an interaction between mercurous sulphate and water, which results in the formation of a basic salt and acid according to the following reaction: $2\ Hg_2SO_4 + H_2O = Hg_2(OH)_2Hg_2SO_4 + H_2SO_4$. The paste is made by mixing solid mercurous sulphate, cadmium sulphate and a saturated solution of cadmium sulphate. The mercurous sulphate very rapidly saturates the solution as has been shown, even when large crystals are used,[2] but the hydrolysis is an exceptionally slow reaction and equilibrium is not reached until a certain concentration of sulphuric acid is formed, the equilibrium concentration, and not until this point is reached are further changes excluded. The basic salt formed remains in solution at first and increases the concentration of the mercury and this continues until the solution is saturated in respect to the basic salt which may then appear as a solid phase. When the basic salt appears as a solid phase, the increase in mercury concentration ceases and begins to decrease as the hydrolysis furnishes an increasing acid concen-

---

[1] Phys. Rev., 25, 21.

[2] Phys. Rev., 27, 360.

tration and the decreasing mercury concentration continues until the acid concentration has reached .08 molecular weight in a liter.[1] Then, and not until then, is this system in equilibrium. The situation may best be illustrated by the following diagram which gives the concentrations of mercurous mercury in a saturated solution of cadmium sulphate as a function of the varying concentration of sulphuric acid, all at 25°.

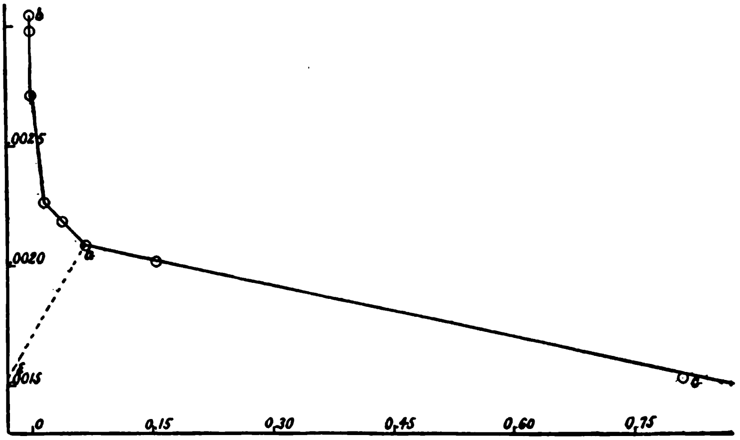


Fig. 1.

Starting at the point *c*, where the cadmium sulphate solution contains .810 molecular weight of sulphuric acid to a liter, there was found to be .001524 molecular weight of mercurous sulphate in solution when the system was in equilibrium.[2] On decreasing the acid concentration the mercurous sulphate concentration increased uniformly to the point *a* where the acid concentration was .08 molecular weight in a liter. This line *ac* is an equilibrium curve for this system and represents the solubility of mercurous sulphate in a saturated solution of cadmium sulphate with varying concentrations of sulphuric acid. This curve extends further to the right but stops at *a*, along this line the system is made up of mercury and the saturated solution with crystallized cadmium sulphate and normal mercurous sulphate as solid phases. The basic salt cannot

---

[1] PHYS. REV., 27, 353.

[2] PHYS. REV., 27, 349.

exist in this region if the system is in equilibrium. At the point *a* a distinct break occurs in the solubility curve, for on continuing to decrease the acid concentration the curve *ab* was obtained. A break of this kind in a solubility curve shows that some change has taken place in the nature or number of the solid phases. The change in this system was found to be due to the appearance, at this point, of the basic mercurous sulphate so the point *a* gives the acid concentration at which there may exist in equilibrium, mercurous sulphate, basic mercurous sulphate, cadmium sulphate crystals, mercury, saturated solution and vapor. Now these phases may all result from the four components, cadmium sulphate crystals, mercurous sulphate, water and mercury, therefore at the point *a* we are dealing with a nonvariant system and in order to proceed along the equilibrium curve *ac* one phase must disappear and it is the basic salt in this case. Likewise, on taking the system to the left of the point *a*, where the acid concentrations are less than .08, the normal salt must *disappear* if the system is in equilibrium. Now in all the experiments made with the system in this region, the normal salt was always present in large excess, so the curve *ab* could not have been an equilibrium curve, but represented the results obtained by rotating such systems until no further changes were observed. If these systems had come to equilibrium they would all have arrived at the point *a*, since there was always more than enough normal mercurous sulphate present to generate the necessary .08 molecular concentration of sulphuric acid The reason they did not come to equilibrium is due to the slow rate of hydrolysis. When we do have the system in equilibrium in this region—from zero to .08 molecular concentration of sulphuric acid—only the basic mercurous sulphate can be present, and since this has only a small solubility the equilibrium curve here would be something like the dotted line *ae* which would represent the solubility of basic mercurous sulphate in a saturated cadmium sulphate solution with varying concentrations of sulphuric acid.

If we cannot bring the cathode system of the Weston standard cell to equilibrium by rotating it, there is of course no chance of attaining equilibrium in the cell where the conditions are most unfavorable, but these unfavorable conditions bring about, in a relatively short time, a "steady state" which is not noticeably affected by ordinary temperature changes, and the result is that most of the Weston standard cells are serviceable for years. They suggest under-cooled liquids or supersaturated solutions, as now and then a cell will slowly decrease to low and indefinite values, while others made at the same time and in the same way do not. In order to furnish a reliable basis of electromotive force it is necessary to set up these cells at regular intervals and reject those found to be decreasing to low values. When certain precautions are taken the Weston standard cell has a very satisfactory reproducibility, but the construction of these and Clark cells may best be treated in a separate article.

PRINCETON, January, 1910.

# PROCEEDINGS

# OF THE

# AMERICAN PHYSICAL SOCIETY.

## MINUTES OF THE FIFTY-SECOND MEETING.

THE spring meeting of the Physical Society was held in Washington, D. C., on Friday, April 22, and Saturday, April 23, 1910.

At the Friday afternoon session, which was held in the Main Lecture Room of George Washington University, President Henry Crew presided.

The Saturday sessions were held at the Bureau of Standards, Vice-President W. F. Magie presiding during the morning, and Ernest Merritt acting as temporary chairman during the afternoon.

The following papers were presented:

Notes on Potentiometer Design. FRANK WENNER.

The Relation between Zero Shift and Size of Wire in the Moving Coil Galvanometer. WALTER P. WHITE.

A Graphic Recorder for Cooling Curves. C. B. THWING.

The Platinum-Rhodium Thermoelement from 0° to 1755°. ARTHUR L. DAY and ROBERT B. SOSMAN.

Preliminary Report on the Temperature Coefficient of Resistivity of Copper. J. H. DELLINGER.

A Test Object for Photometry by Visual Acuity. HERBERT E. IVES.

The Selective Radiation of Platinum. E. P. HYDE.

Spectral Energy Curves. (By title.) W. W. COBLENTZ.

A Quantitative Study of the Emission of Hydrogen, Argon, and Helium in the Visible Spectrum. P. G. NUTTING and ORIN TUGMAN.

Notes on the Physics of Meteor Crater, Arizona. W. F. MAGIE.

A Modified Method for the Determination of Relative Wave-Lengths, Especially Adapted to the Establishment of Secondary Spectroscopic Standards. I. G. PRIEST.

On the Reflecting Power of Ice in the Extreme Infra-red Spectrum. A. TROWBRIDGE and B. J. SPENCE.

The Selective Reflection of Oxides. L. B. MORSE.

Some Phosphorescent Salts of Sodium with Cadmium. C. W. WAGGONER.

The Distribution of Energy in the Spectrum of Sidot Blende. (By title.) C. A. Pierce.

Further Experiments on Fluorescence Absorption. E. L. Nichols and Ernest Merritt.

The Effect of Temperature on the Ionization of a Gas. J. Harry Clo.

The Isolation of an Ion and a Precision Measurement of its Charge. R. A. Millikan.

A New Form of Rotating Sectored Disk for Spectro-Photometric Measurements. E. P. Hyde.

Neutral Contacts and Switches. Walter P. White.

A New Form of Surface Bolometer. E. H. Northrup.

The Inductance of a Metal Tube Bent into the Form of a Circular Ring. F. W. Grover.

Absolute Formula for the Mutual Induction of Coaxial Cylindrical Current Sheets. G. R. Olshausen.

The Reliability of Magnetic Measurements on Rods. Chas. W. Burrows.

Some Investigations on Mica Condensers. H. L. Curtis.

A Condition Limiting the Precision Attainable by Increasing the "Order," in the "Method of Diameters" for Wave-Lengths. I. G. Priest.

A Method for Measuring and Establishing the Ten-to-One Ratio of Resistance. Frank Wenner.

A Direct Reading Candle Power Scale for Photometers. G. W. Middlekauff.

Ernest Merritt, *Secretary*.

## THE USE OF THE ELECTRICAL RESISTANCE THERMOMETER IN APPARATUS FOR THE ABSOLUTE MEASUREMENT OF SOLAR RADIATION.[1]

By C. F. Marvin.

POUILLET'S actinometer, although greatly improved by a long list of physicists has, as yet, failed to give us a satisfactory absolute measurement of solar radiations. This is mostly because mercurial thermometers are employed. The problem is a problem in calorimetry. The use of electrical resistance thermometers removes serious difficulties almost completely, as, for example, bad heat conductivity and losses along the thermometer stem, heterogeneous material of different specific heats, and the uncertain relation between temperature change shown by thermometer and true change in temperature of calorimeter. A calorimeter,

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 21–24, 1909.

or "block," for absorbing solar radiation can be constructed of thin metallic ribbon which is at once a thermometric bulb and a calorimeter. Its specific heat measured with great accuracy *in situ* by sending through the ribbon measured electric currents and observing the heating effects. The details of a pyrheliometer constructed on this plan with accessory apparatus were shown by lantern slides.

## NOTE ON THE RELATION BETWEEN THE RESISTANCE OF NICKEL AND THE TEMPERATURE.[1]

BY C. F. MARVIN.

THE use of nickel in the construction of resistance thermometers is not recognized as completely as its merits and advantages warrant. It cannot compete with platinum at high temperatures, but apparently nickel is as good, if not better than platinum at lower ranges of temperature, say below 350° C. The data for several samples of nickel, although admittedly scanty, show that the curve of variation of resistance with temperature is strictly logarithmic, viz.: $\log r = a + mt$. Hence two accurate measurements of resistance at two temperatures suffice to fix the curve. This law of resistance change is of peculiar advantage in resistance measurements with a slide-wire bridge, since readings of the slider on a scale of equal parts can be made to accurately represent temperature over a wider range than otherwise and troublesome computations are avoided. The residuals between the observed and computed values in the data so far available do not exceed 0.1 per cent.

## THE EFFECT OF FILTER PAPER UPON THE MASS AND FORM OF THE DEPOSIT IN THE SILVER COULOMETER.[1]

BY E. B. ROSA, G. W. VINAL AND A. S. MCDANIEL.

THE Rayleigh coulometer, which has been widely used and many times investigated in several different countries, has generally been found to yield a heavier deposit of silver than the Kohlrausch or Richards forms in which filter paper is not employed. The difference has been attributed to secondary reactions at the anode, in which a complex ion is formed, and this complex ion being brought to the cathode by convection currents, has been supposed to yield a larger quantity of silver for each univalent charge. We have shown that the trouble with the Rayleigh coulometer is not due to anode products, but to the reduction of neutral silver nitrate by furfuraldehyde resulting from the hydrolysis of the cellulose of the filter paper, the heavier deposit resulting from

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 21-24, 1909.

a certain quantity of colloidal silver that is produced and deposited together with that carried by the current. The colloidal silver not only increases the mass of the deposit by an appreciable amount, but changes the form of the deposit, as seen by the unaided eye and also as seen by a microscope.

A large number of experiments have been made to test the various questions involved, and many photographs aud microphotographs made of the silver deposits.

## SOME CURIOUS PHENOMENA OBSERVED IN CONNECTION WITH MELDE'S EXPERIMENT.[1]

By J. S. Stokes.

To one prong of an electrically maintained tuning fork, vibrating in a horizontal plane, at a rate about one hundred times per second was attached a linen thread. This was drawn out at right angles to the prong, passed over a small brass pulley and a weight attached a few inches below the pulley.

When adjustment swere such that the string was vibrating in segments with sharp nodes, the pulley was seen to rotate slowly with its upper edge moving toward the prong.

A little beeswax was now applied to the string in contact with the pulley and the latter now rotated in the opposite direction. A drop of kerosene was then applied to the string in the groove of the pulley and the direction of rotation was again reversed.

The fork was now slipped toward the pulley until the overhanging portion of the string vibrated as a whole, when the speed of rotation of the pulley was increased to more than one hundred revolutions per minute.

The explanations offered for these phenomena were as follows :

When the prong strikes toward the pulley, the slackening of the tension in the string at the prong, sends a wave form along the string, which on reaching the pulley tends to throw the string outward clear of the pulley and to allow the weight to drop without turning the pulley. When the prong strikes backward, the inertia of the weight and the tightening of the string create pressure on the pulley, and the consequent friction rotates it toward the fork.

When wax is applied to the string, the latter clings to the pulley and the descending weight rotates it outwards and the string now seems to exert less friction on the pulley on the backward stroke than on the forward. This is probably analogous to the slipping of the belt on a flywheel of an engine when the wheel is started too suddenly.

---

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

When the oil is applied to the string, conditions return to what they were at the start and the wheel reverses direction of motion.

No explanation was offered for the increased speed of the pulley when the overhanging vertical portion of the string was vibrating and the horizontal part seemed not to be doing so. The plane of vibration of this vertical portion has been observed to rotate at about one half the speed of the pulley, without any rotation of the weight it supports. This is no doubt due to a transfer of the twist in the string, along the string.

For the successful performance of the experiment certain conditions must be realized: viz., the pulley must turn easily on its pivots as the torque obtained is very small. The groove must be well polished. The string must not be too large, about no. 30 linen thread works well. The weight must not hang more than a few inches below the pulley. The tension in the string must be such that the string vibrates in two or more segments. The string vibrating as a whole seems to rotate the pulley always in the same direction. Too much wax must not be used on the string. The string can be rubbed with a cloth to remove excess of wax. Also the groove of the pulley. As far as the demonstrator has been able to ascertain, these phenomena have not been hitherto observed or described by any one.

## INSULATION OF OBSERVATORY DOMES FOR PROTECTING TELESCOPES AND OTHER PHYSICAL APPARATUS AGAINST UNNECESSARY EXTREMES OF HEAT AND COLD.[1]

By David Todd.

PRIOR to constructing the domes of the new Observatory at Amherst, thermometric experiments were made with a variety of insulating materials to ascertain which was best adaptable to the necessary conditions. Accordingly the 20-foot dome is insulated with about 3 inches of granulated cork, and the 35-foot dome with 1½ inches of hair felt. The base of each dome is further protected by a sliding diaphragm of felt. This simple arrangement affords great protection to the instruments, reducing the range of temperature variation, and also protecting them in large measure against dust and condensation.

## NOTE ON THE COEFFICIENTS OF DIFFUSION OF THE EMANATION AND THE ACTIVE DEPOSIT PARTICLES OF ACTINIUM.[1]

By Prof. J. C. McLennan.

IN this note attention is drawn to some results recently published by W. T. Kennedy on "The Active Deposits of Actinium." In his experiments it was shown that a maximum activity existed in different gases

---

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

at a definite distance from the salt and the position of this maximum was found to vary with the pressure at which the active deposits were investigated.

The variation in the position of this maximum was ascribed to diffusion, and the relative coefficients of diffusion for this maximum effect in the gases air, hydrogen and carbon dioxide were reduced.

It is now pointed out that these relative coefficients of diffusion are practically identical with those found by Russ for the diffusion of the emanation from actinium. The effect observed by Kennedy, however, involved the diffusion not only of the emanation but also of the active deposit particles, and the results go to show that the active deposit particles and those of the emanation possess the same diffusion coefficient.

In the recent work of Wellish[1] and in that of Franck[2] also it has been shown that ions produced from different substances and differing as widely as 100 per cent. in their mass possess the same mobilities in a gas such as hydrogen. In particular it has been shown that ions produced by Röntgen rays in methyl iodide and also the particles which constitute the active substance thorium D in hydrogen possess the mobilities of hydrogen ions themselves.

This result is in accordance with a theory of ionization recently brought forward by Sutherland[3] in which he draws the conclusion that a viscosity arising from the electric polarization of the neighboring molecules by an ion must exist and exert a predominant influence in determining mobilities.

According to Sutherland's theory this electric viscosity would come into operation in the diffusion of ions also, and it would explain the close agreement found by Kennedy between the coefficients of diffusion of the particles of the active deposits from actinium and that of the particles constituting the emanation of this substance.

## A QUANTITATIVE STUDY OF THE EMISSION OF HYDROGEN, ARGON AND HELIUM IN THE VISIBLE SPECTRUM.[4]

### BY P. G. NUTTING AND ORIN TUGMAN.

AN ideal gas spectrum would consist of but a single line whose intensity would vary in direct proportion to the internal energy of the gas. Ordinary lines are known to diverge both ways from this linear relation. Primary lines increase rapidly at first and more slowly with intense excitation, secondary lines increase slowly at first and very rapidly later. Some lines (*e. g.*, the blue spectrum of argon) have been supposed to appear only when the discharge through the gas is extremely

---

[1] Proc. Roy. Soc., A., 557, July 31, 1909.

[2] Verh. der Deut. Phys. Ges., No. 19, 3397, 1909.

[3] Phil. Mag., September, 1909.

[4] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22–23, 1910.

violent. The spectra chosen for study exhibit lines and groups of lines of both these types.

The method of study was to excite the gases in large Plücker tubes by sending a known alternating current through them. A spectrophotometric comparison with a reference tube held at constant current (or pressure) then gave line intensity as a function of current (or pressure). Mean potential gradients were determined subsequently. A special form of tube with porcelain capillary gave more constant results than all-glass tubes and carried twenty times the current (half an ampere) that these would safely carry.

In the hydrogen spectrum three different portions of the primary spectrum, red λ611, green λ545 and blue λ460 gave the same variation with current

$$\frac{P}{P_{20}} = 1.042\,\frac{i}{i_{20}} - .0432\left(\frac{i}{i_{20}}\right)^2 + .00098\left(\frac{i}{i_{20}}\right)^2.$$

The three lines, $\alpha$, $\beta$, $\gamma$, studied of the secondary spectrum were found to bear a simple relation $S = P^m$ to the primary where $m_\alpha = 1.67$, $m_\beta = 1.35$, $m_\gamma = 1.14 \pm .02$. $m$ as a function of wave-length may be closely represented by $m - 1 = 681(1/366 - 1/\lambda)$ indicating very slight variability out toward the head of the Balmer series.

Intensity at constant current (20 m.a.) and variable gas density gave a pronounced maximum near 1 mm.

A series spark with condenser gave, with low current, relative intensities of $\alpha$, $\beta$, $\gamma$ and $P$, such as were obtained with 250 m.a. of single phase alternating current.

In the argon spectrum the lines studied were red λ696, orange λ603, green λ556, blue λ454 and violet λ420 in the primary spectrum and the groups at λ443 and λ481 in the secondary or spark spectrum. Each of these departs widely from a linear relation to current. The secondary lines were followed from 450 m.a. down to 100 m.a., and the extrapolated curves indicate that they persist down to zero current.

The pressure at which the intensity is a maximum is 0.7 for the primary lines and under 0.2 for the secondary. Series spark and condenser gave relative line intensities such as were produced with about 200 m.a. of steady current.

In the helium spectrum no new lines could be brought out with either the heaviest currents or the most violent disruptive discharge available. In all cases the ratio of line intensity to current steadily decreases with increase of current. The curves for the bright red, yellow, blue and violet lines lie close together, but the bright green line lies much higher. The pressure at which the line intensity is a maximum varies from 4 mm. for the extreme red line λ706 to 2 mm. for the blue λ447. The faint green line λ504 apparently has no maximum, its intensity increasing more and more rapidly with decrease in pressure down to 0.5 mm.

*Volume XXX.* *June, 1910.* *Number 6*

THE

PHYSICAL REVIEW.

---

# STUDIES IN THERMO-LUMINESCENCE.

## III. THE DISTRIBUTION OF ENERGY IN THE LUMINESCENCE SPECTRUM OF SIDOT BLENDE.[1]

BY C. A. PIERCE.

### FLUORESCENCE AT ROOM TEMPERATURE.

ABOUT two years ago an attempt was made by the author to study the energy distribution in spectra of fluorescent and phosphorescent powders by means of photographic methods. After a little success, the experiment was dropped due to the lack of a photographic plate sensitive over the region of fluorescence of the powder that was being experimented upon. At the beginning of the present college year, interest was again stimulated by the publication of an article by Ives and Coblentz, on the distribution of energy in the light of the firefly, in which the spectrum of the firefly's light was photographed. At about the same time, Professor E. L. Nichols suggested that Cramer's "trichromatic plates" might be sensitive to the portion of the spectrum included by the fluorescence light of Sidot blende. The plates were tried and proved to be suitable.

The method used in the study of the energy distribution consisted of photographing on the same plate one spectrum of the fluorescence light and four spectra of the light of an acetylene flame. Different intensities of acetylene were used for the four spectra, while the time

---

[1] The fluorescent powder that was used in these experiments was "Emanations-pulver." The same sample of powder was used that had been used earlier in some experiments described in THE PHYSICAL REVIEW, Vol. XXVI., No. 4, April, 1908.

of exposure for the five spectra was the same. Knowing the distribution of energy in the acetylene flame,[1] the distribution in the fluorescence light was obtained by photometric comparisons of the spectra on the plate.

During this set of experiments the fluorescent powder was contained in a square dish made of platinum foil. The foil was connected to and supported by copper leads so that the temperature of the powder could be raised and controlled by passing electric current through the foil. The temperature was measured, if different from room temperature, by means of a copper-constantine thermo-couple placed in the midst of the powder. The blue lines of a mercury-arc lamp were used to excite the powder. The various intensities of acetylene light were obtained by screening off all but the center of an acetylene flame and moving the flame with the screen to different distances from the slab of magnesia which reflected the light into the slit of the spectrum camera. The camera consisted of a direct-vision spectroscope set so that the spectrum could be focused on a sensitive plate held in a plate holder.

The plates were handled and developed in complete darkness. The developer was freshly mixed from a stock solution and of a standard strength, and the length of development was timed, being for most of the plates eight minutes. The temperature was brought to 20° at the beginning of development and the plates were rocked mechanically during the development. The plates were fixed, at first, for thirty minutes, after washing in three separate waters, and the temperature of the fixing bath was brought to 20° at the beginning of the fixing. Later not so much care was used in the fixing because it was found to be inadvisable to intercompare spectra on different plates. After fixing, the plates were washed in running water for about thirty minutes, then thoroughly rubbed by hand and dried. Each plate was dried in such a position that the last portion to drain was a part upon which no measurements were to be made.

The distribution of denseness in any spectrum on a plate was obtained by pushing the plate past a brightly illuminated slit placed

---

[1] On the Distribution of Energy in the Visible Spectrum, by E. L. Nichols, PHYS. REV., Vol. XXI., p. 147, Sept., 1905.

squarely across the spectrum, and measuring the transmitted light by means of a Lummer-Brodhun photometer. The source of light to illuminate the slit and the standard light consisted of two carbon filament electric lamps with frosted bulbs. The light back of the plate was concentrated on the slit by a reflector. The current supplied to the lamps was held practically constant. It was found by actual test that a change of five volts was necessary to vary the relative intensities of the lamps enough to affect the settings of the photometer appreciably. The lamps were run at six per cent. above normal voltage to increase the candlepower and were lighted only long enough to make the measurements.

The wave-length measurements were made by calibrating in wave-lengths the screw which pushed the plate past the slit on the photometer bar. A zero wave-length was obtained on the plate by photographing the three blue lines of the mercury arc superimposed on the different spectra. These lines were in an entirely different region from that occupied by the spectrum of the fluorescence light. The calibration of the screw was made by photographing the entire visible spectrum of the mercury arc with the spectrum camera and setting one line after the other directly in front of the slit on the photometer bar. This slit was always opened just as wide as the width of the lines in the spectra and was placed parallel to these lines. The standard lamp was screened off by means of a variable slit so that the photometer could be used on whatever part of the bar that was desirable.

Fig. 41 shows the curves obtained by plotting the light transmitted through the spectra on one of the plates which had been exposed as described above. Each exposure was 60 minutes long and was made with the slit opened 40 units. Curves 1, 2, 3 and 4 represent the spectra of acetylene light at intensities 32, 16, 8 and 4 respectively. Curve 7 represents the spectrum of the fluorescence light. Curve 6 represents the distribution of energy in the acetylene flame and is assumed to correspond to curve 3. Then the energy curve corresponding to curve 2 will have ordinates equal to 16/8 times the corresponding ordinates of curve 6, etc. Wherever curve 7 crosses curve 3, the energy for that wave-length is represented by the ordinate on curve 6 for that wave-length. Wherever curve 7 crosses curve 2, the energy for that wave-length is 16/8 times the ordinate of curve 6 for that wave-length, etc.

Since the curves intersect at only a few points, it is necessary to get other points on the desired energy curve by interpolation. This is done systematically and as follows: Suppose wave-length $\mu_1$ is under consideration. A curve is plotted with the intensities of acetylene, 4, 8, 16 and 32, as abscissæ and the intersections of the vertical at $\mu_1$ with the curves 1, 2, 3 and 4 as ordinates. From this curve is picked off the intensity of acetylene that would have coincided with curve 7 at wave-length $\mu_1$. Fig. 42 shows six curves drawn in connection with Fig. 41. Curves 1, 2, 3, 4, 5 and 6 correspond to wave-lengths $\mu = 0.50$, 0.51, 0.52, 0.53, 0.54 and 0.55 respectively. The circle on each curve shows the point picked off.[1] On curve 6, Fig. 42, 23.5 is the abscissæ

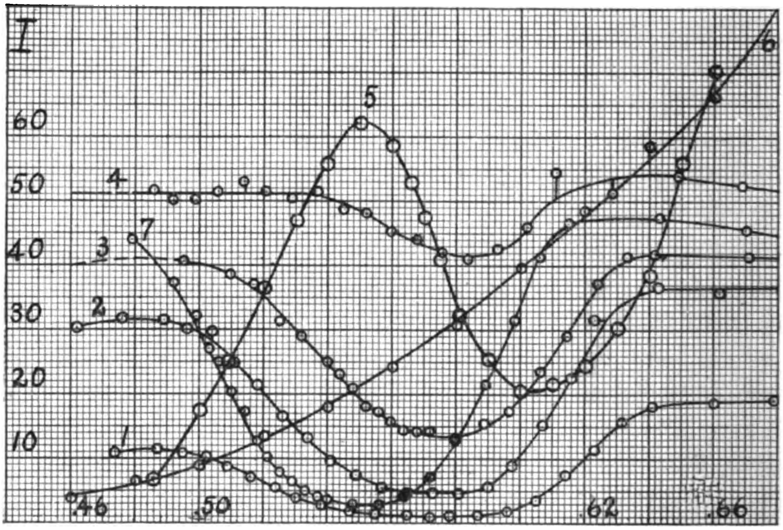


Fig. 41.

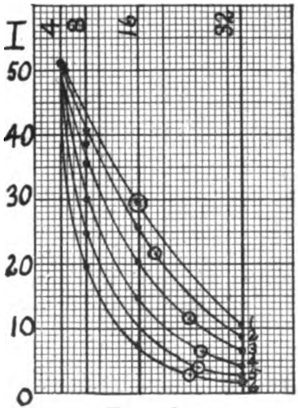


Fig. 42.

---

[1] The curves shown in this article do not exactly agree with the original curves from which the calculations were made due to inaccuracies in inking in the lines.

corresponding to 2.8, the ordinate on curve 7, Fig. 41, at $\mu = 0.55$. Hence the ordinate at $\mu = 0.55$ on curve 5, Fig. 41, which shows the distribution of energy of the fluorescence light is equal to 23.5/8 times 21.3, the ordinate on curve 6, Fig. 41, at $\mu = 0.55$.

Curve 5, Fig. 41, shows the energy curve of fluorescence as computed from the other curves in Fig. 41. The curve has a well-defined maximum at $\mu = 0.55$, a well-defined minimum between $\mu = 0.46$ and $\mu = 0.48$, and another minimum at $\mu = 0.60$. Later it will be seen that the shape of the curve for wave-lengths longer than $\mu = 0.60$ is uncertain.

Fig. 43 shows another set of curves corresponding to those in Fig. 41. The length of exposure was 30 minutes for each spectrum

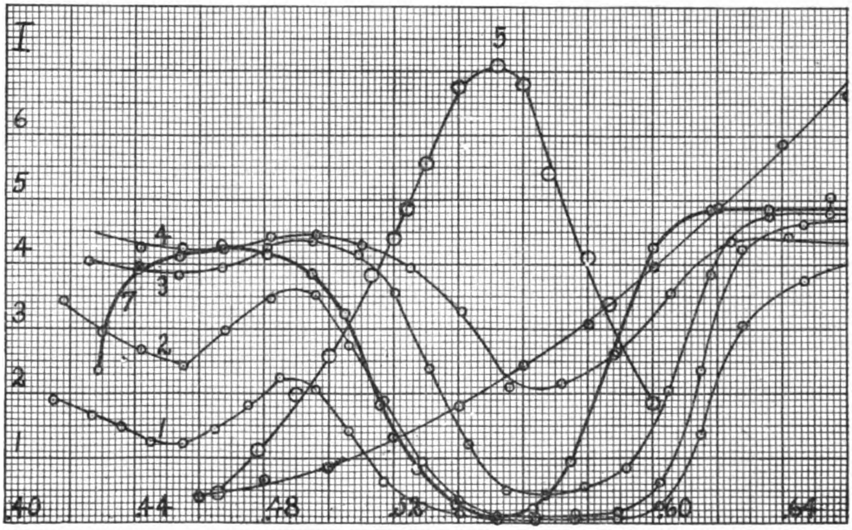


Fig. 43.

with a slit width of 20 units. The intensities of acetylene flame were 32, 20, 12 and 6 for curves 1, 2, 3 and 4 respectively. Curve 6 is assumed to represent the energy distribution in curve 4. Curve 5, which shows the distribution of energy in the fluorescence light, is similar to the corresponding curve 5, in Fig. 41. This shows that the curve is not materially influenced by two factors, length of exposure and width of slit in the spectroscope.

The Sidot blende was excited by the three blue lines of the mercury-arc spectrum. While these lines are in a different region of the spectrum from that occupied by the fluorescence band, yet there was considerable halation shown on the plates about the three lines and the halation might extend far enough to change the shape of the curves in Figs. 41 and 43 in the region of smaller wave-lengths. To test this point, a set of curves were taken in which one curve represents the light of the three lines of the mercury arc reflected from a slab of magnesia, which was assumed to be as good a reflector as the Sidot blende. Fig. 44 shows the curves. Curves 1, 2, 3 and 4 correspond to different intensities of acetylene light

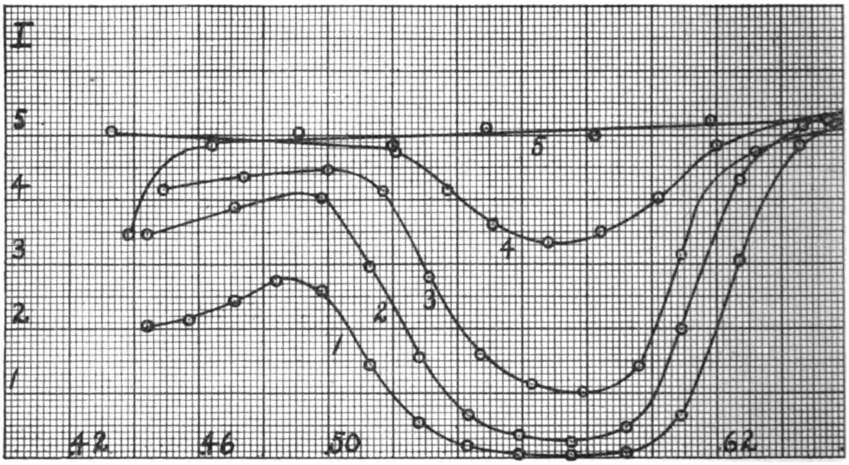


Fig. 44.

and 5 is the curve of reflected light. It is seen that halation of the mercury lines plays no important part for wave-lengths longer than $\mu = 0.46$, *i. e.*, no error is introduced in the curves marked 5 in Figs. 41 and 43. The slit opening in the spectroscope was 20 units and the length of exposure was 30 minutes in Fig. 44.

A set of curves, Fig. 45, was taken with the conditions so arranged that very intense negatives were obtained, in an endeavor to obtain the shape of the energy curve for wave-lengths longer than $\mu = 0.60$. The curves are marked similarly to those in the previous figures. Curve 5 shows the energy distribution of the fluorescence light. The maximum is shifted a little from $\mu = 0.55$

but otherwise it corresponds very well to curves 5 in Figs. 41 and 43. In Fig. 45, the data are inaccurate for computing curve 5 for wave-lengths greater than $\mu = 0.60$. Inspection of curve 4 shows that it crosses curves 1 and 2 in the region of longer wave-lengths. The same general departure is found in Fig. 43. This inaccuracy is probably due to a reversal in the negative though none is evident to the eye. It was found to be possible to take negatives which showed a decided reversal. The conditions were not found, however, under which a negative would surely show no reversal. If curves 3 and 4, Fig. 45, were where they apparently ought to be, with reference to curves 1 and 2, then curve 5 would rise for wave-lengths longer than $\mu = 0.60$, just as in Fig. 41. It is clear, as stated before, that no evidence is obtainable from these curves as

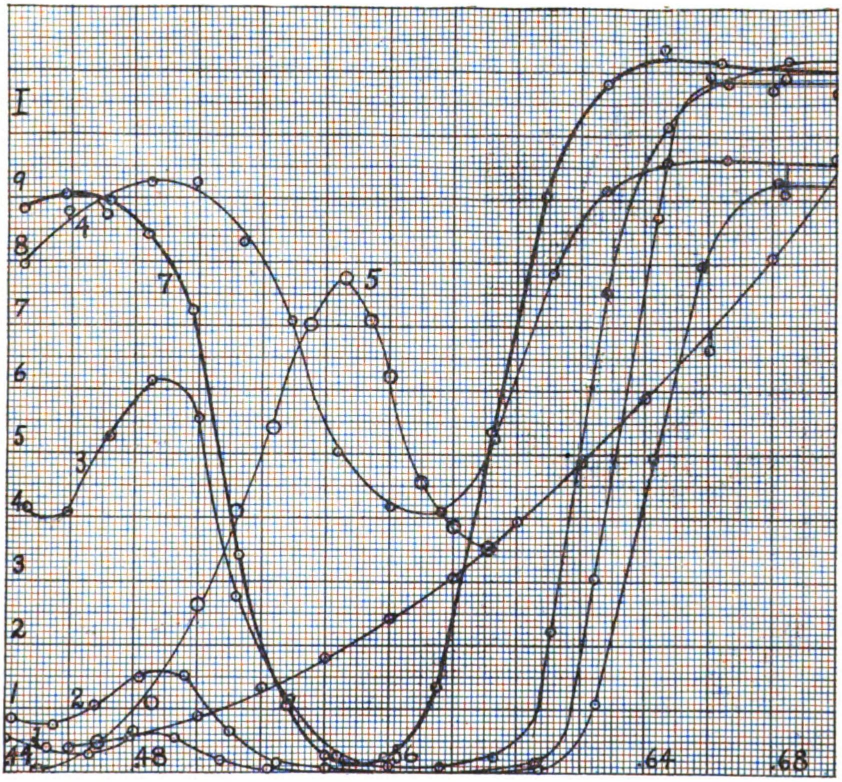


Fig. 45.

to the shape of the energy curve of the fluorescence light for wavelengths greater than $\mu = 0.60$. Many more curves were taken than these shown, but no additional evidence was obtained, and the curves here reproduced are typical of all.

## PHOSPHORESCENCE AT ROOM TEMPERATURE.

In order to obtain the distribution of energy in the phosphorescence light, it was necessary to add to the apparatus already described a shutter which would close the spectrum camera and excite the powder; then shut off the exciting light and open the camera. The shutter was operated by a constant speed motor. Levers, springs and triggers were so arranged that the shutter operated in a small part of a second. The length of excitation and decay could be increased or decreased together by changing the speed of the motor, while either the length of excitation or the length of decay could be varied alone by rearrangement of parts of the controlling devices. The total time of exposure of the photographic plate to the phosphorescence light was made equal to the length of exposure of the acetylene spectra.

With the apparatus described above, the photographic negative of phosphorescence was obtained by light which varied in intensity, for the phosphorescence decayed after the exciting light was closed off in the manner typical to phosphorescent powders. However, if an energy curve is obtained similar to that for fluorescence, it is nearly certain, not only that the energy curve does not change with decay but also that it does not change when the exciting light is shut off.

Fig. 46 shows the energy distribution in the phosphorescence light of Sidot blende. The powder was excited each time for $8\frac{3}{4}$ seconds and allowed to decay $10\frac{1}{3}$ seconds. The powder was not exposed to infra-red before each excitation. The effective length of exposure of the photographic plate was 60 minutes. Curves 1, 2, 3 and 4 show the transmitted light of the photographic spectra corresponding to intensities of acetylene equal to 6, 4, 2 and 1 respectively. Curve 5′ shows the transmitted light of the phosphorescence spectrum. Curve 5 shows the energy curve for the phosphorescence light.

It is seen that in general curve 5 corresponds to the similar curves for fluorescence light. That is, there is apparently no difference between fluorescence light and phosphorescence light immediately after excitation in the case of Sidot blende. Further experiments are needed to show whether any change occurs in the energy curve during decay and data for this are being obtained by the author at the present time.

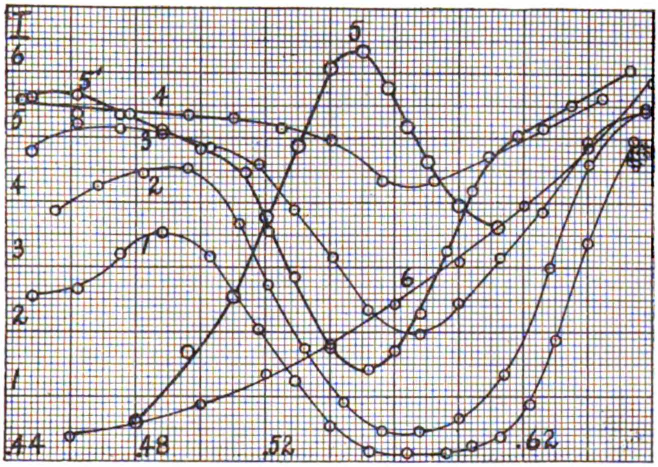


Fig. 46.

## Discussion of Method and Results.

The chemical change made in a photographic plate depends upon the wave-length and intensity of the incident light and upon the length of exposure. If several spectra are photographed upon the same plate and the length of exposure is the same for all the spectra, then the denseness of the different negatives at any wave-length will depend only upon intensity of incident light at that wave-length, assuming a small opening of the slit to the spectroscope, uniform development, non-halation, etc. Since intensity is proportional to the rate at which energy is received from the source, the denseness of the different negatives at any wave-length depends upon the energy received from the source at that wave-length. By photographing several negatives of one source of light of known energy distribution at different distances from the spectroscope, curves can be drawn showing the relation between energy received at any wave-length and denseness of negative. If a spectrum of another light of unknown energy distribution is photographed on the same plate, its energy distribution can be obtained by comparison of the denseness of its negative with the calibrated densities. The densities can be compared by means of the light transmitted through the negatives at different wave-lengths.

If the slit opening in the spectroscope is not very small, the denseness of the negative at any wave-length will not depend alone upon the energy received at that wave-length, but will also depend upon the energy received at wave-lengths differing but little from the wave-length in question. Consequently a small dimple in a curve might be obliterated by a wide slit opening. The effect of slit opening can be tested by using different openings, finally using the one which experience dictates.

The effect of halation is more troublesome than slit opening, and was present more or less in all of the negatives. The effect of halation is to broaden the band of energy distribution, but not to change the position of the maximum point of the band. It is believed that halation in the present experiments was not of sufficient effect to cause any serious error in the curves.

The plates were developed immediately after the exposures, which were made one directly after the other, so any error due to creeping of the chemical reactions in the film was eliminated as far as possible.

The photographic measurements were difficult to make due to the large differences in amount of light transmitted in different parts of the negatives. At the beginning of the experiments, four settings of the photometer were made for each point measured. Later two settings were found to be sufficient, one approaching uniformity of illumination from each direction. The average of the two values calculated from the two settings was taken as the true ratio of standard and transmitted lights. Remeasurement of a set of curves never proved any exceptional accuracy, but always gave the same type of energy curve with the maximum at about the same wave-length. Consequently the measurements can be said to be substantiately correct.

The results of these experiments show that the energy curve of the fluorescence light of Sidot blende consists of a band extending from about $\mu = 0.46$ to $\mu = 0.60$, having a maximum at $\mu = 0.55$. There may be another band situated in the region of longer wavelengths. Furthermore, the energy distribution in the fluorescence light and the phosphorescence light immediately after excitation is the same.

Further experiments are now being carried on by the writer to find whether any change in the energy curve takes place during decay, or is brought about by the influence of infra-red rays or by other means.

Physical Laboratory, Cornell University,
March 21, 1910.

# ON THE EXTRA TRANSMISSION OF ELECTRIC WAVES.[1]

BY F. C. BLAKE.

IN the May number of the PHYSICAL REVIEW of 1907 appeared a criticism by Dr. Clemens Schaefer[2] of the work of Blake and Fountain[3] on the transmission of electric waves through resonator gratings. Dr. Schaefer takes these authors to task for overlooking an important article by him,[4] and with good right. On behalf of Dr. Fountain and myself I here express regret that this oversight occurred and cheerfully acknowledge any priority that Dr. Schaefer's article contains.[5]

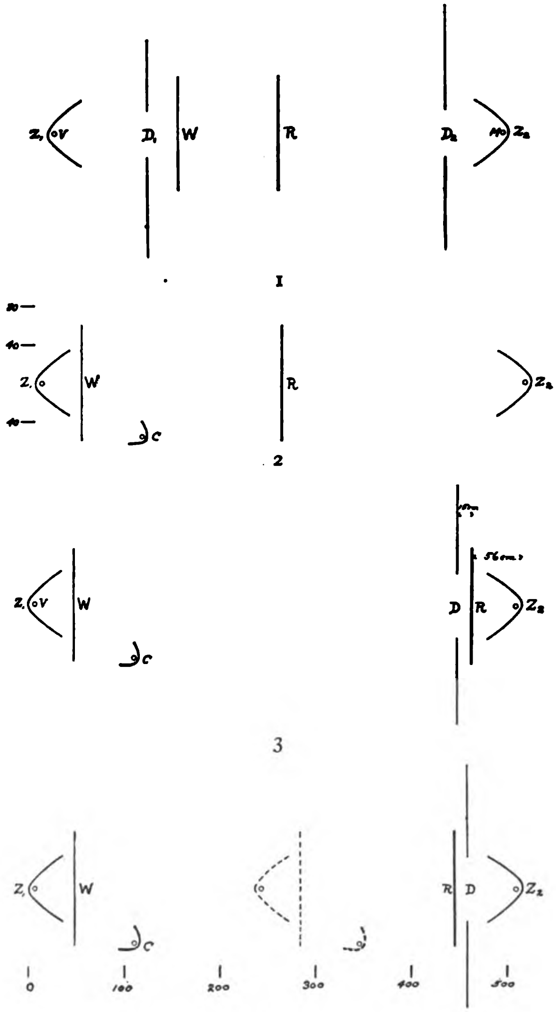
Before replying to Mr. Schaefer, I wanted to perform some experiments, and it is only lately that I have been able to complete them. In one part of his criticism, he claims to have repeated some of our experiments, the only change he made being the insertion of two diaphragms. With this change he was unable to verify our results on extra transmission. I felt that the use of diaphragms was unwarranted on the ground of the enormity of the diffraction effects thereby introduced, but it plainly was a question for further experimentation to decide, hence the delay in my reply.

It seems best before taking up the various points in Schaefer's criticism, to describe the new experiments I have performed bearing upon the phenomenon of extra transmission. The apparatus used was identical with that used in the previous work,[6] with the exception of the resonator grating and its supporting framework. Columbia University kindly donated the parabolic mirrors, the vibrator

---

[1] Read before Section B, A. A. A. S., December, 1908.
[2] PHYS. REV., XXIV., p. 421, 1907.
[3] PHYS. REV., XXIII., p. 257, 1906.
[4] Ann. d. Phys., Vol. 16, p. 106, 1905.
[5] By letter at the time I made such acknowledgment.
[6] Blake and Fountain, loc. cit.

and the receivers and I herewith express my best thanks to Professor Hallock and the department of physics there. The arrangement of apparatus is shown in Figs. 1–4, drawn to scale. $Z_1$, $Z_2$ are the



4

Figs. 1–4.

zinc mirrors, $D_1$ and $D_2$ are diaphragms of galvanized iron of variable aperture, $W$ is a rectangular wire grid 120 cm. on a side, with the distance between the wires 2.5 mm. The wires, No. 36 B. & S., were strung vertically, at right angles to the electric force of the oncoming wave. This grid was used to make sure that the wave was plane-polarized, although I am convinced as before[1] that its use is unnecessary provided one is careful that the axes of vibrator and receiver are in the focal lines of their respective mirrors. *C* and *M* are the check and main receivers and *V* the vibrator. *R* is the resonator grating—a sheet of plate glass 122 × 96.5 cm. carrying tin-foil strips whose distribution was varied. Instead of *R*, a bare sheet of plate glass *B* could be inserted at will. The average thickness of the bare glass was 7.3 mm., that of the resonator glass 6.8 mm., a difference sufficiently great to make the ratio $R/B$ for transmission, with both glasses bare 1.022, obtained by a long series of alternate readings. In all the curves and results of this paper this ratio has been taken into account.

One of the galvanometers used was a du Bois-Rubens type, the other a Broca instrument. They were given the same period and were interchanged in the receiver circuits whenever it was desirable. The former had a sensitiveness roughly of $2.5 \times 10^{-9}$, the latter, $10^{-8}$, for a period of 4 seconds. The period was varied from time to time, according to the energy available, the time for a complete swing being varied from 4 to 6.5 seconds.

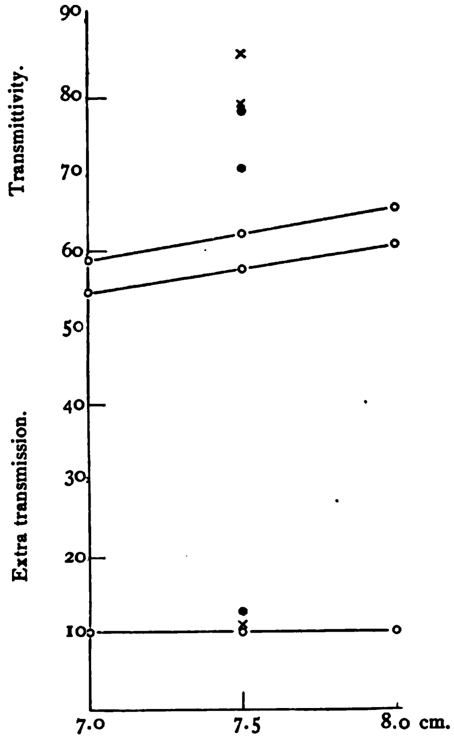
Since Schaefer had insisted upon the use of diaphragms in such work on electric waves, the first thing I did was to introduce two such diaphragms each 8 ft. 6 in. square, using a resonator system identical with Table I., Fig. 9, Blake and Fountain, except that the resonators were 5 instead of 6 cm. long. Fig. 5 shows the results obtained. It was found impossible under the working conditions at first obtainable to get sufficient energy through two apertures 32 cm. vertical by 24 cm. horizontal to make the results trustworthy, so apertures 47 × 38 were chosen. The points shown are the mean of two sets of readings taken thus: *ARBRBRA* (*A* = air = free radiation, *R* = resonator glass, *B* = bare glass). These apertures fulfill the condition imposed by Schaefer, that they be smaller than the mirror apertures, 68.8 × 60.5 cm.[2] Larger aper-

---

[1] Blake and Fountain, loc. cit., p. 266.

[2] Apparently the figures 70 × 63 cm. given on page 259, Blake and Fountain, are slightly too large. This error in measurement is of course unimportant.

tures were then used and finally both diaphragms were removed. Extra transmission is plainly present in all cases. It is affected, though not seriously, by the size of the diaphragm apertures. It should be noted that the energy at the main receiver was greater for the apertures 70 × 60 than for the diaphragms wholly removed. This point will be discussed later.



Position of vibrator [$Z_1V$, Fig. 1].

Fig. 5. Resonator grating, 13 columns of 39 each. Length of resonator, 5 cm.; width 0.2 cm. Distance between resonators, side-on, 2.8 cm., end-on, 1 cm. For the points marked by crosses the apertures $D_1$ and $D_2$ were 68.8 cm. × 60.5 cm.; circles, 47 × 38 cm.; dots, both diaphragms removed.

Schaefer explains extra transmission on the ground of the divergent properties of the waves employed by us, although he doesn't state why the waves sent out by a vibrator and collected by a receiver each of which is placed accurately within 0.5 mm. in the focal line of a parabolic mirror of 7.5 cm. focus should be divergent.

I cannot conclude by a study of his criticism that he refers to the well-known divergent property of electric waves in a direction parallel to the focal line of the parabolic mirrors. This latter property will also be discussed later. To test this explanation the vibrator was purposely moved from the focal line in both directions, the main receiver being left undisturbed at 7.5 cm. With apertures 47 × 38, the transmittivity for each glass separately (*i. e.*, the ratio between the energy at the main receiver through each glass compared to that through air) was increased in going from a divergent to a convergent wave but the extra transmission remained constant (Fig. 5).

It is well known that a vibrator formed of spherical balls shows greater irregularity in its action after a few readings than one having flat parallel surfaces at the spark gap. Accordingly a vibrator was made by boring the two ⅜-inch balls and inserting steel cylinders, threaded and slightly conical, and projecting beyond the brass about one millimeter. The area of each of the cylinders at the spark gap was roughly 0.2 sq. mm. It was assumed that the length of the wave emitted by such a vibrator was not materially changed from 10 cm., but as I couldn't be sure of this it was thought better to replace the resonator system by a system consisting of strips 90 cm. long by 0.2 cm. wide, with a side-on distance of 2.8 cm.[1] Such a distribution should be independent of small variations in the wave-length. Moreover, it had been found[2] to give a larger extra transmission than that given by a system of double the resonance-length. Throughout the rest of this paper this latter system was used.

The results obtained with such a system are shown in Fig. 6. The arrangement of apparatus is shown in dotted outline in the figure, the location of the resonator system being varied between the diaphragms. The apertures 32 × 24 allowed so little energy to pass through that for them it was thought better to lay more stress on the extra transmission than on the transmittivity. Accordingly twenty-five readings were taken alternately thus: *BR*-

---

[1] The side-on distance used by Blake and Fountain was 2.8 cm., not 3.0 cm. as Dr. Schaefer understood. This would of course affect his arguments not at all.

[2] See Fig. 11, Blake and Fountain, page 273.

*BRBRB*, etc., for each of the points shown. To obtain an idea of the constancy of the readings and of the accuracy of the work the figures are given for the position of the resonator system nearest the main receiver. Ratio $M/C$ for $B$'s, 1.152, 1.168, 1.175, 1.089, 1.145, 1.147, 1.158, 1.121, 1.137, 1.112, 1.093, 1.118, 1.130. Ratio $M/C$ for $R$'s, 1.226, 1.229, 1.237, 1.187, 1.256, 1.262, 1.258, 1.274, 1.208, 1.211, 1.226, 1.229. Mean $R$, 1.234; mean $B$, 1.134. $R/B$ = 1.088; extra transmission reduced, 6.5 per cent. For apertures 47 × 38, the extra transmission as well as the transmittivity seemed more or less independent of the location of the resonator system. For apertures the size of those of the parabolic mirrors the extra transmission was larger when the resonator system was near the

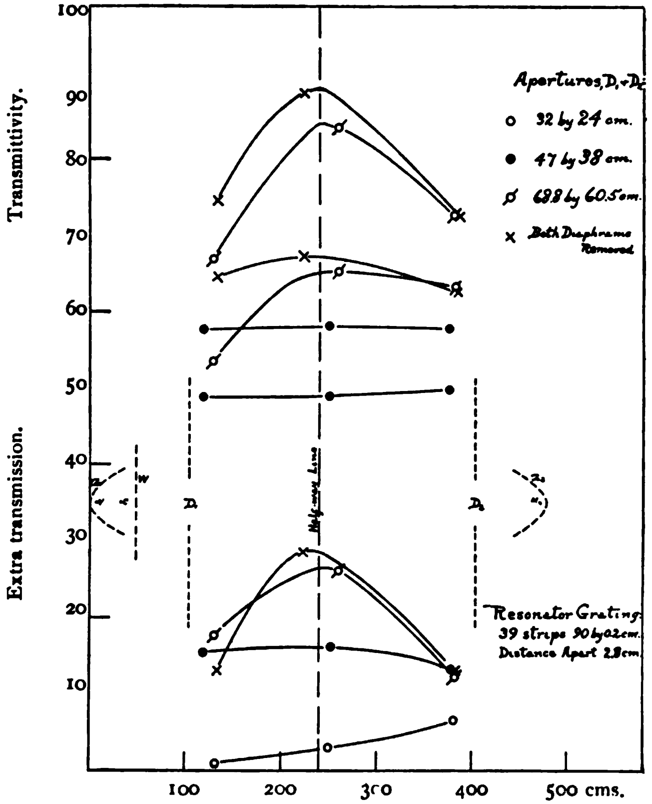


Fig. 6. In the curves in the upper part of the figure the lower curve of each pair refers to bare glass, the upper curve to the resonator glass.

position half-way between vibrator and main receiver than for either of the two other positions. This increase in the extra transmission in the mean position was augmented when the diaphragms were removed, and so a more careful study was made of it. It should be noted from Fig. 6 how, for apertures 47 × 38 or larger, the extra transmission was practically constant at 13 per cent. for the positions of the resonator system nearest the main receiver. The fact that for apertures 68.8 × 60.5 cm. the transmittivity for the bare glass was a function of the location of the resonator system with respect to the diaphragms points to diffraction effects around the glass. I think that these diffraction effects, as will be seen later, occur around those edges of the glass which are at right angles to the electric force.

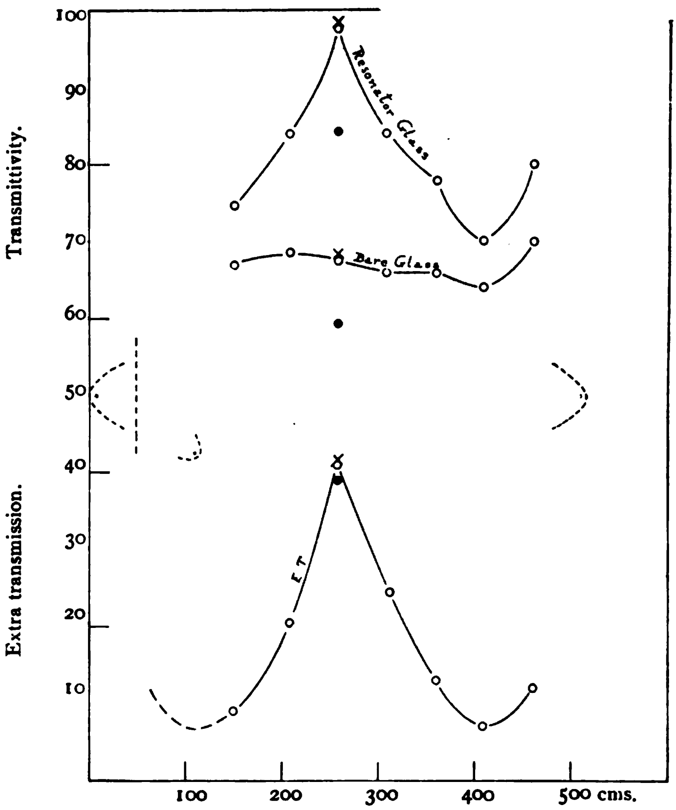


Fig. 7. Both diaphragms removed.

After changing the location of the check receiver to the position shown in Fig. 2 and in dotted outline in Fig. 7, the curves of Fig. 7 were taken with both diaphragms removed. These curves were taken before the great importance of *extremely* accurate adjustment was realized for this position of the resonator system half-way between the two parabolic reflectors. Each point of the curves was determined from the following series, *AARBRBRBRAA*, and so could not be much in error. However, the extra transmission curve of Fig. 7 shows the half-way position to be a critical one, one where it is very necessary when diaphragms are not used to have the straight line joining the vibrator and main receiver accurately divide the 90 cm. of the strip lengths into two equal parts. To illustrate, the points (circles o) of Fig. 7 having been taken with the resonator grating of 39 strips the top eight and the lower eight strips were removed, leaving 23 strips extending over 66.2 cm. space in a vertical direction, so that in a direction at right angles to the electric force the resonator grating was now smaller than the aperture (68.8 cm.) of the parabolic mirrors. With no great care being used in pushing the resonator plate into the path of the wave the black circles of Fig. 7 were obtained. A sideward adjustment of 2.3 cm. (determined by the use of plumb-lines at the centers of the vibrator, resonator-plate and main-receiver) gave the crosses shown in the figure. Repetition of the adjustment and non-adjustment confirmed this. To show clearly the need for this adjustment the actual figures are given in Table I. Fig. 7 shows clearly that when the proper care was used to have the straight line connecting the vibrator and main receiver pass through the center of the resonator system, the transmittivity was unaffected by the removal of the eight strips that extended above and below the apertures of the parabolic mirrors. Indeed, although without this adjustment the transmittivity for both the resonator and the bare glass is affected, the extra transmission is but slightly changed.

I can not ascertain from Schaefer's discussion of our experimental arrangements whether in his criticism of the resonator system's having a larger area than the aperture of the parabolic reflectors his objection refers so much (if at all) to the direction parallel to the focal line as to the direction at right angles to such line. If one

TABLE I.

| Imperfect Adjustment (Displaced 2.3 cm.). | | | | | Plumb-line Adjustment. | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Medium Inserted.** | **Ratio $M/C$.** | **Means.** | **Ratios (Per Cent.).** | **Extra Transmission (Reduced).** | **Medium Inserted.** | **Ratio $M/C$.** | **Means.** | **Ratios (Per Cent.).** | **Extra Transmission (Reduced).** |
| *A* | 1.243 | *A*1.238 | | | *A* | 1.234 | *A*1.248 | | |
| *A* | 1.213 | | *R/A* | | *A* | 1.261 | | *R/A* | |
| *R* | 1.062 | | 84.2 | | *R* | 1.214 | | 98.4 | |
| *B* | 0.748 | *R*1.042 | | | *B* | 0.853 | R1.227 | | |
| *R* | 1.030 | | *B/A* | | *R* | 1.237 | | *B/A* | |
| *B* | 0.719 | | 59.4 | | *B* | 0.846 | | 68.2 | |
| *R* | 1.030 | *B*0.735 | | | *R* | 1.229 | *B*0.850 | | |
| *B* | 0.738 | | *R/B* | | *B* | 0.850 | | *R/B* | |
| *R* | 1.045 | | 141.8 | 38.8 per | *R* | 1.227 | | 144.4 | 41.3 per |
| *A* | 1.246 | | | cent. | *A* | 1.230 | | | cent. |
| *A* | 1.248 | | | | *A* | 1.267 | | | |

is to limit the resonator system in the former direction to dimensions less than the mirror apertures in this direction it becomes at once important to ask oneself to what extent this dimension of the parabolic mirrors plays a part in his experiments. In other words, if the focal line be thought of as limited in its length by the two parallel bounding planes of the parabolic mirrors, what, expressed in wave-lengths, should be its length. The question is a fair one and has, so far as I know, never been investigated in electric-wave work. Although I think it is of some importance, yet I do not believe Schaefer had it in mind in expressing his objections. If he did, he might well have asked himself ought not he and Aschkinass and all the other investigators in this field to have employed paraboloidal rather than cylindrically parabolic mirrors?

The importance of side-adjustment was investigated more fully in Fig. 8. With the same distances and apparatus as in Fig. 7 (except that the bare glass alone was used) the central position was carefully determined by the use of plumb-lines in the manner already described. To eliminate vibrator-deterioration this central position was used as a check position after determining any two successive points of curve *B*, Fig. 8. The energy at the main receiver through the bare glass in the central position was chosen as 100. When the bare glass plate was 27 cm. out of center enough energy got around it by free radiation to give 100 at the main receiver again. The slope of curve *B* at the central position toward either side shows the need for accurate adjustment. Then the apparatus was changed to an exact duplication of the arrangement employed by Blake and Fountain for transmission (*l. c.*, Fig. 4). Employing the resonator glass alone a similar curve, *R*, Fig. 8, was obtained. However, it has far less slope than *B*, showing the smaller need for accurate side-adjustment as the resonator system is moved from the position half-way between the parabolic mirrors. Then although the vibrator was pretty well worn and only a few readings were taken it was thought worth while to see to what extent the extra transmission was affected in the latter position by any lack of side-adjustment. Curve *ET*, Fig. 8, shows that it is practically unaffected, just as curve *ET*, Fig. 7, does for the half-way position. The transmittivity curves of Fig. 7 are subject to some correction due to lack of plumb-line side-adjustment. However, as I have just shown, curve *ET* of the same figure is free from this objection

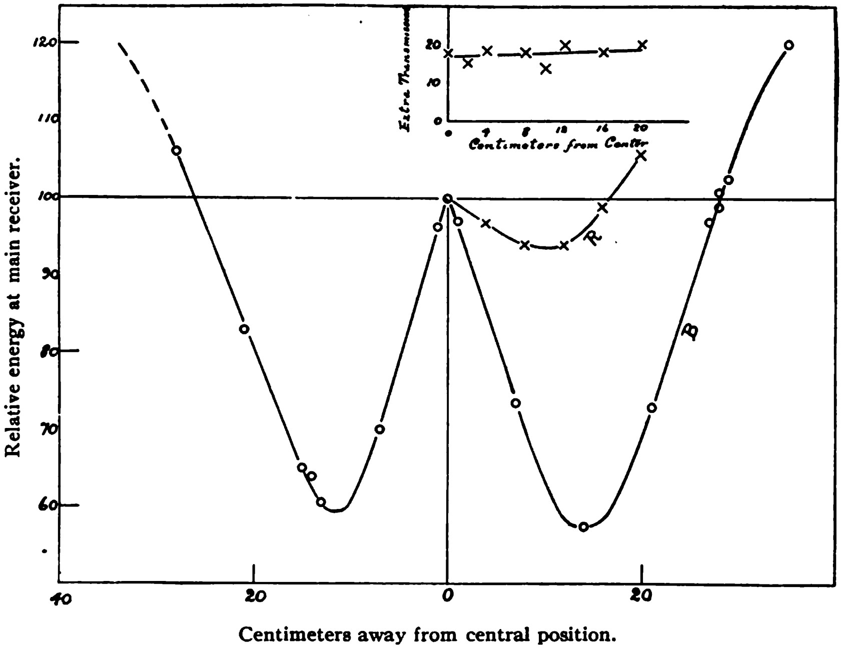


Fig. 8.

largely, since care was always used to have the bare glass plate duplicate *exactly* the position of the resonator plate in taking a set of alternate readings. The minimum point at 408 cm. in curve *ET* is undoubtedly present. At the half-way position the extra transmission is a maximum and is seen to be a very large effect, so large that for 23 strips 90 cm. long and 2.8 cm. apart the resonator glass comes within 2 per cent. of giving the full free radiation energy at the main receiver! Undoubtedly diffraction effects play some part at the main receiver and it is very probable that the amount of diffracted energy as well as its phase is a function of the index of refraction of the intervening obstacle. If this is so it is not easy if at all possible to separate diffraction and extra transmission phenomena.

Since, as Fig. 7 shows, the extra transmission is so large in the half-way position, the cause for this is naturally to be sought. The phenomenon may be said to be a sort of *lens action* of the resonator system. The front of the wave as it strikes the resonator system may be taken as cylindrical approximately, with the generating line of the cylinder at right angles to the tin-foil strips. Thus the plane system of long strips takes these cylindrical waves and converges them, still kept cylindrical, upon the main receiver mirror just as a cylindrical lens converges the waves from a line source of light. As I have said I cannot determine from Dr. Schaefer's criticism that it is *this* divergent property of electric waves from cylindrically parabolic reflectors that he refers to. Moreover, if this property be an error it is an error that is common to the work of all the previous investigators. For I doubt if Dr. Schaefer is able to tell us what the width of an aperture in a screen should be in a direction parallel to the vibrator axis to eliminate this property. For instance, had the focal line of each of my parabolic mirrors been 100 cm. in length, and thereby for the arrangement represented in Fig. 7 the condition imposed by Schaefer being fulfilled that the resonator system be smaller in area than the mirror apertures, I believe the curves of Fig. 7 would not have been materially changed.

If the *lens-action* of the resonator system is the proper explanation for this very large increase in the extra transmission when vibrator and receiver are equally distant from the system, I see no reason why this lens-action should not be present if one were to employ resonator strips in air instead of on glass. I mean to test this point as soon as may be feasible and to see how, starting with long strips, the lens-action if present is affected by successive shortenings of the strips. Probably there is a certain angle formed by the line joining vibrator to receiver and the line joining vibrator to either end of that particular strip in the same horizontal plane as the vibrator, where the effect is a maximum.

On account of the wave-front being cylindrical in form and the lens action of the resonator grating being thus for certain positions of the grating superimposed upon the true effect of extra transmission it seemed best to place the resonator grating as near as possible to the main receiver and by means of a single diaphragm of aperture 68.5 × 16 cm. to allow so small a portion of the wave-front through (except for diffraction effects due to the presence of the diaphragm) that the transmitted portion could be said to be plane in both directions of the aperture. This being done as shown in Fig. 3, the effect of changing the vibrator from its normal position was tried. In this work the usual series *AARBRBRBRAA* was taken for each position of the vibrator. Fig. 9 (circles o) shows the results. The extra transmission was about twelve per cent. (the vibrator was made of spherical balls without points) and was slightly larger for a convergent wave than for a divergent one. Then the effect of changing the width of the aperture was tried. Fig. 9 (crosses ×) gives the results. The extra transmission was constant at 11 per cent. though the transmittivity of both glasses varied. The relative energy at the main receiver for this last case as the width of the aperture was varied is plotted in curve *B*, Fig. 10. This curve is a free radiation curve through air, the energy at the main receiver for a width of aperture 32 cm. being taken as 100. The interposition of either the bare or resonator glass changed the character of the curve only for the larger apertures (see curve *B′*—here of course the energy through the bare or resonator glass for diaphragm aperture 32 was taken as 100). This curve was plotted incidentally from the results obtained from Fig. 9 (crosses ×) and as only four points were taken its character was not wholly determined. The fact that for an aperture 61 cm. the energy at the main receiver was only 77 per cent. of that for an aperture 32 cm. showed the presence of diffraction effects, as was to be expected. Other diffraction curves are shown in curves *A* and *C*, Fig. 10, *A* being taken with a diaphragm placed 15 cm. in front of the resonator position, Fig. 2. *C* was taken as shown in Fig. 4 in dotted outline. For all three curves of Fig. 10 the length of aperture was 68.5 cm., the width alone being varied. *C* shows a diffraction maximum at 39 cm. and *A* at 63 cm.

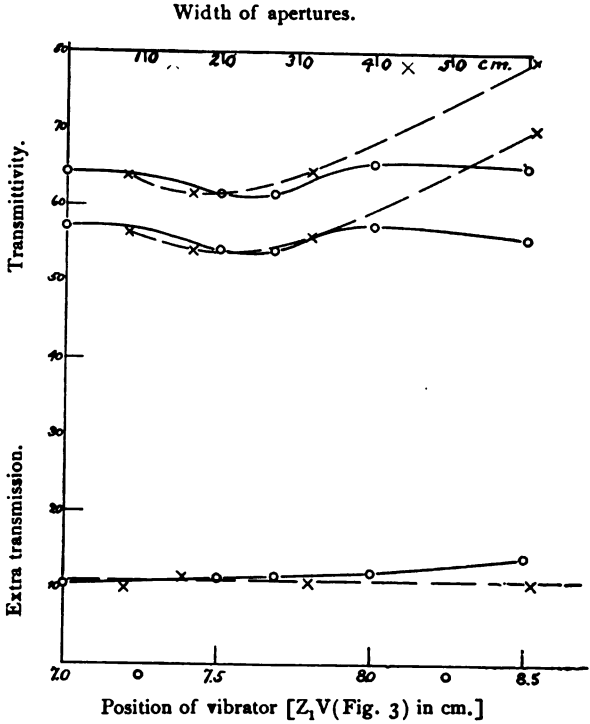


Fig. 9. For arrangement of apparatus see Fig. 3. Length (vertical) of aperture, 68.5 cm.

Now theory shows[1] that if *a* is the distance of the vibrator and *b* that of the main receiver from the screen and $\delta$ the width of aperture then maxima occur when

$$\frac{\delta^2}{2\lambda}\frac{a+b}{ab}=\frac{3}{2}+4h$$

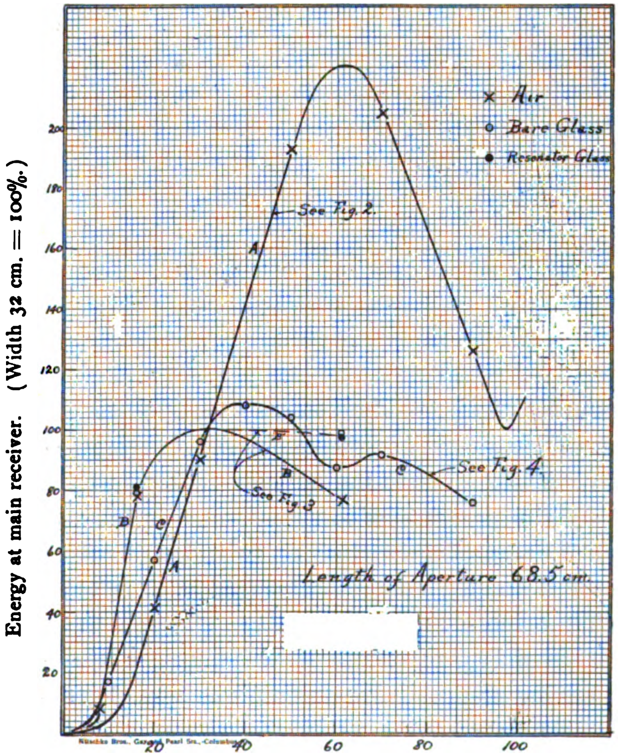
---

[1] See, for example, Winkelmann's Handbuch der Physik, 2d edition, Vol. 6, p. 1060.

and minima when

$$\frac{\delta^2}{2\lambda}\frac{a+b}{ab}=\tfrac{7}{2}+4h,$$

where $h$ is a whole number. Of course both $a$ and $b$ are to have 15 cm. (twice the focal distance) added to them. Doing this and making $h = 0$, the figures for curve $A$ are $a = 250$ cm., $b = 280$ cm., $\lambda = 10$ cm., whence $\delta_{\text{max.}} = 62.9$ cm., $\delta_{\text{min.}} = 96.1$ cm. For curve $C$, $a = 231$ cm., $b = 64$ cm. and $\delta_{\text{max.}} = 38.8$ cm., $\delta_{\text{min.}} = 59.2$ cm. There is thus for the maxima entire agreement between theory and experiment. Making the same calculation for curve $B$ with $a = 451$, $b = 79$, $\delta_{\text{max.}}$ comes out 44.9 and $\delta_{\text{min.}} = 68.6$ cm. The maximum for $B$ could easily be at 44.9 instead of at 32 as



Width of aperture in cm.

Fig. 10.

shown, since no points between 32 and 61 were determined. For *A* and *B* the minima are not yet reached in the curves. For *C* the theoretical minimum at 59.2 cm., though small, plainly shows itself. For *A* I have extrapolated the minimum at 96 cm.

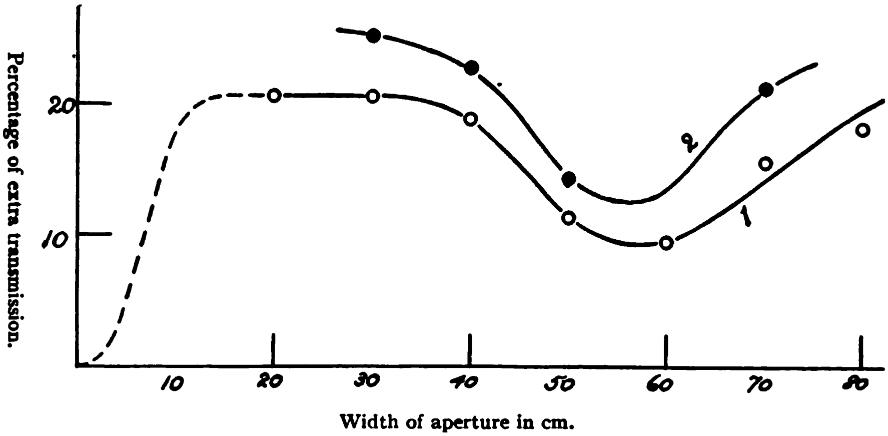


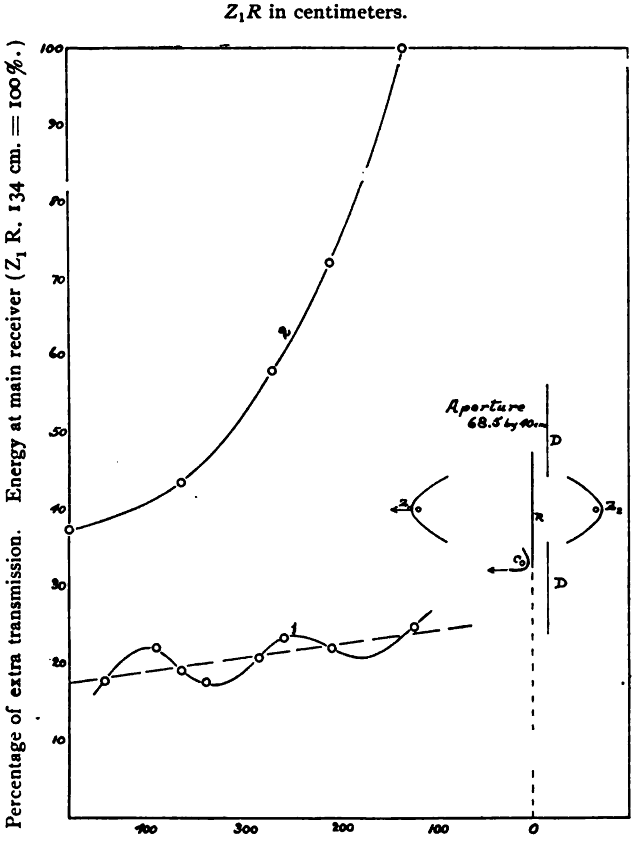
Fig. 11.

These curves of Fig. 10 show how strongly diffraction effects are present at the main receiver when a diaphragm is employed. Moreover, curve *B′* shows that the amount of diffracted energy is influenced by the presence of the resonator or bare glass, and in general in different degrees by the two glasses. Vice versa, the presence of diffracted energy influences in different degrees the energy that reaches the main receiver through the two glasses, as the curves of Fig. 11 clearly show. Curve 1 was taken with a vibrator having conical points about one mm. long, the parallel faces of the points having a cross-section of about 0.2 sq. mm. Each point on the curve was determined as the mean of the following series, *RBRBRBRBR*, and vibrator irregularity was eliminated by increasing the width of aperture by successive steps to the maximum width and then returning successively by steps to the minimum width. Relative distances for the apparatus are shown in Fig. 4 (vibrator mirror dotted), and plumb-line adjustment was used throughout. Curve 2 was taken under identical conditions except that the vibrator had 3 mm. points. The extra transmission is greater for the 3 mm. vibrator than for the 1 mm. vibrator, to be explained probably by the change in the length of the wave emitted. Fig. 11 shows plainly the influence of diffraction on extra transmission. Curve *C*, Fig. 10, is the diffraction curve obtained from the measurements taken for curve 1, Fig. 11. It is very significant that the minimum extra transmission occurs at 59 cm., the exact location of the diffraction band in curve *C*, Fig. 10. Weak though this band is, it has a very large effect on extra transmission.

If, then, diaphragms are to be employed what should be their aperture width? A study of the curves of Fig. 10 shows that, in order to be as free as possible from the influence of diffraction, an aperture width should be chosen which falls on the first straight line part of that diffraction curve which corresponds to the momentary arrangement of apparatus. Naturally, one would choose if possible the width such that the energy would be equal to that corresponding to the entire absence of diffraction bands. For instance in Fig. 10, curve *A*, the ordinate for minimum is 100, for maximum 220. This makes the ordinate corresponding to the asymptotic point of Cornu's spiral 151, thus approximately giving for the diffractionless width of aperture 42 cm. But this width for another arrangement of apparatus may not be justifiable at all (*e. g.*, that for curve *C*). On such reasoning one is permitted from curve *C*, Fig. 10, and hence in Fig. 11 to choose width 30 cm. Thus for a vibrator having 1 mm. points the true extra transmission is 20 per cent., for a 3 mm. vibrator 25 per cent. For a spherical ball vibrator without points the true extra transmission was not obtained in the way indicated above, but judging from the results of Fig. 9 it is distinctly smaller, say 12 per cent. Now the curves of Fig. 10 were taken with a 1 mm. point vibrator and the diffraction bands were calculated for $\lambda = 10$ cm., the wave-length determined by Blake and Fountain for a spherical ball vibrator; moreover, the observed and calculated diffraction bands were found to agree exactly. And yet the point vibrator gives greater extra transmission than the spherical ball vibrator of the same wave-length. Possibly the point vibrator throws a larger per cent. of its radiated energy into the equatorial region than does the spherical ball vibrator and hence the greater extra transmission. It is to be borne in mind, of course, that these values for the true extra transmission are given as correct only for the particular resonator system used in this paper. Fig. 11, curve 1, shows the same extra transmission for width 20 as for width 30 cm. It would seem that for smaller widths the extra transmission remains constant at 20 per cent. so long as one remains on the straight line part of the diffraction curve, *i. e.*, down to 13 cm. width. Below that it probably bends rapidly toward zero. I have thus extrapolated the curve in Fig. 11.

Keeping the width of aperture constant at 40 cm. I then tried to see the effect on extra transmission of moving the vibrator away from the resonator system. In doing this the relative distance between vibrator and check receiver was kept constant. Here to eliminate vibrator irregularity the vibrator was moved away and then nearer again by successive steps. The usual series of nine alternate readings were taken for each position. Fig. 12, curve 1, shows the extra transmission curve, the energy curve being curve 2. The sinusoidal nature of curve 1 is very marked, and I thought it could be explained by giving successive values to $h$ in the diffraction formulæ above. But I haven't been able to make the figures fit the formulæ. However, it should be remembered that the diffraction system is a compound system consisting of the resonator or bare glass and the diaphragm.

From the first it was thought that in general the use of diaphragms was not justifiable and so with the arrangement of apparatus shown in Fig. 3, the following variation was tried. Diaphragm aperture 47 × 38 cm., extra transmission 17 per cent.; aperture 32 × 24 cm., extra transmission 9 per cent.; relative energy at main receiver, 3 to 1. Then a second screen was inserted 120 cm. in front of vibrator, aperture 32 × 24 cm. for both screens. Extra transmission 5 per cent.; relative energy at main receiver for one screen (the one near the receiver) as compared with that for two screens, 3 to 1. Thus one ninth as much energy gets to the main receiver with two screens of aperture 32 × 24 as with one of aperture 47 × 38. Moreover, the second screen cuts the extra transmission almost in two. All this shows, it seems to me, that the relative energy diffracted out into regions other than the main direction of the wave is much larger for the smaller aperture than for the large one, and hence the extra transmission is cut down. Each new aperture radically changes the character of the wave-front, diverting more and more of the energy from the equatorial path and the amount diverted would be affected differently by the resonator



$Z_1R$ in centimeters.

Fig. 12.

and bare glasses, hence the lowering of the extra transmission by using a second screen. Moreover, changing the length of the gap ought to introduce diffraction bands just as changing the width has been seen to do.

I contend, after these experiments, that in general in electric wave work of the sort here described, the use of diaphragms is *not justifiable.* The employment of two diaphragms is *never* justified, and if a single one is used, it should be placed as near to the main receiver mirror as possible and the width of its aperture should be such that diffraction bands are entirely absent at the receiver. If one uses cylindrically parabolic mirrors, the aperture *length,* should in my opinion, be at least equal to the height of the mirror aperture, although a small change of the length either way is probably immaterial. I have already explained how, in the most crucial position of the resonator system (the half-way position, Fig. 7) it was immaterial whether one employed a resonator system whose *length* was greater than or equal to the height of the mirror apertures. Doubtless the effect measured at the main receiver is dependent upon the *width* of the resonator system and its relative distance from the receiver. But so long as cylindrically parabolic mirrors are employed I see no criterion by which one may determine what the proper width of the resonator system is to be. To my mind it is meaningless to say that the width should be less than the width (dimension parallel to the focal line) of the parabolic mirrors.

In my opinion, for transmission work in electric waves it is far safer to employ no diaphragms but to have the transmission system as near as possible to the main receiver mirror and of such a *width* that it may properly be said to be infinite, thus preventing diffraction past the edges affecting the receiver. This could best be done by having the system practically touching the mirror and yet having the parabolic mirrors shallow.

Dr. Schaefer's objections consisted of two. He denied the existence of the phenomenon of extra transmission because he could not find it when he used diaphragms. And for good reasons. The apertures he employed were too small. The effect was there to the extent of 5 per cent. even with his arrangement, though it was not surprising he didn't find it. Had he studied the effect of the size of aperture he might have seen where the trouble lay. Nor is it surprising that Aschkinass and Schaefer[1] didn't find the phenomenon, although they just missed it in their curve 4, Fig. 2,

---

[1] Aschkinass and Schaefer, Ann. d. Phys., V., p. 489, 1901.

for a length 2 cm., twice the resonance length. Had they used an aperture in their diaphragm somewhat larger than 20 cm. on a side they doubtless would have found it. For the other curves the number of resonators used of double the resonance length was entirely too small and the side-on distance between them too great to show the effect. Nor does the fact that they didn't find it presuppose, as Schaefer argues, "large errors of observation" on their part.

Dr. Schaefer's second objection to our work lay in this, that when we changed resonator gratings, holding vibrator and receiver constant, we didn't change all distances and dimensions proportionately. He maintains, following the analogies[1] of optics, that in work with a resonator grating one can obtain correct results only when all dimensions of the grating are changed by the same relative amounts, using a constant vibrator and receiver. Before I can answer this objection I shall have to discuss the results that Schaefer obtained in his work[2] with resonator systems. With a constant resonator grating by means of a variable vibrator and receiver Schaefer determined the maximum absorption of a single column grating. By repeating this with a grating differing from the first only in that the resonators were closer together, he observed that the absorption maximum was displaced toward the smaller wave-lengths. By considering two adjacent resonators Schaefer explains this result theoretically as follows. Since the two resonators are entirely identical, if we represent the resistance, self-induction, mutual induction and capacity of each of the resonators by $w$, $L_{11}$, $L_{12}$, $C$ respectively, these differential equations hold:

$$\frac{d^2 i_1}{dt^2} + \frac{w}{L_{11}} \frac{di_1}{dt} + \frac{i_1}{L_{11}C} = -\frac{L_{12}}{L_{11}} \frac{d^2 i_2}{dt^2},$$

$$\frac{d^2 i_2}{dt^2} + \frac{w}{L_{11}} \frac{di_2}{dt} + \frac{i_2}{L_{11}C} = -\frac{L_{12}}{L_{11}} \frac{d^2 i_1}{dt^2},$$

where $i_1$ and $i_2$ are the currents in the two resonators at the time $t$. Neglecting the resistance term the general integral is of the form

---

[1] For a view upon the closeness of these analogies see Webb and Woodman, PHYS. REV., Vol. XXIX., p. 90, 1909.

[2] Clemens Schaefer, loc. cit.

$$i_1 = A \cos 2\pi \left( \frac{t}{T_1} + \delta_1 \right) + B \cos 2\pi \left( \frac{t}{T_2} + \delta_2 \right),$$

$$i_2 = A \cos 2\pi \left( \frac{t}{T_1} + \delta_1 \right) - B \cos 2\pi \left( \frac{t}{T_2} + \delta_2 \right),$$

where $A$, $B$, $\delta_1$, $\delta_2$ are constants of integration and

$$T_1 = 2\pi \sqrt{(L_{11} + L_{12})C}, \quad T_2 = 2\pi \sqrt{(L_{11} - L_{12})C}.$$

Schaefer then says that since the two resonators are wholly identical and since, for both, the initial conditions are the same it follows that $i_1 = i_2$, that is, that $B = 0$ and hence that

$$i_1 = i_2 = A \cos 2\pi \left( \frac{t}{T_1} + \delta_1 \right).$$

Thus the free period of vibration of a resonator

$$T_0 = 2\pi \sqrt{L_{11} C_0},$$

is altered by the presence of the second resonator to the period $T_1$ above. Schaefer says further, "In distinguishing between $T_1$ and $T_2$ it must not be overlooked that the capacity $C_0$ is changed by the approach of the second resonator." By writing $i_1 = i_2$ he thus arbitrarily drops $T_2$, the second possible[1] period of vibration, and so *is forced* to explain the displacement of the absorption maximum upon the approach of the second resonator by the lowering of the capacity, that is, "as if $L_{12} = 0$." No wonder Schaefer found that in each of the twenty gratings that he worked with the influence of the capacity factor far exceeded that of the mutual induction! Surely, in considering the effect upon the absorption maximum of the approach of a second resonator, one cannot rightly do else than say that $i_2$ in the differential equations above is the current in the second resonator due to the impressed electromotive force in the first resonator. It is true, of course, that the total current in each resonator is the same, for their mutual effects are the same and both are subject to the same external force of the wave. But

---

[1] See A. Oberbeck. Ann. d. Phys., Vol. 55, p. 624, 1895, or Fleming, "Principles of Electric Wave Telegraphy," 1st edition, p. 209.

it is the mutual effects *alone* (in general *both* of capacity and induction) that enter into the question of the displacing of the absorption maximum.

Now it is well known[1] that the effective inductance of a circuit is *decreased* by the presence of a second circuit, and for currents of high frequency the amount of the effective inductance is $L_{11}-L_{12}^2/L_{11}$. That is, instead of Dr. Schaefer's value for $T_1$ above, the correct value should be

$$T_1 = 2\pi\sqrt{\left(L_{11}-\frac{L_{12}^2}{L_{11}}\right)C},$$

which shows that for two resonators approaching each other side-on, *both* the inductance and capacity tend to decrease the period. Now the fact that Curve $A$, Fig. 11, for Blake and Fountain was a straight line showed that the capacity of a resonator was practically unchanged by the *end-on* approach of a second resonator up to 5 mm. from the first. Accordingly we argued that for *side-on* approach of two resonators, since in our experiments they were never closer than 1 cm., any change in period must be attributed to inductance and not to capacity changes. By false theoretical considerations Schaefer was forced to explain the facts of experiment (which I do not at all deny) by capacity changes alone. I agree with him that for end-on approach one may safely take $L_{12}=0$ and hence any change of period must be due to increase of capacity. If this is so I see no better way of trying to determine to what extent change of period is due to inductance and to what extent to capacity for side-on approach of resonators than by first determining within what limits changes of distance in end-on approach do not affect the capacity. This is what we did in curve $A$, Fig. 11, although I am free to say that we did not try this out in any thorough manner.

Only one other point in Schaefer's criticism needs to be mentioned. Is it necessary, as he insists, for satisfactory conclusions to be drawn, that in changing from one grating to another all dimensions and distances be changed relatively the same amount?

---

[1] See Maxwell, Phil. Trans., 1865. See also Lord Rayleigh, Phil. Mag., Vol. 21, p. 375, 1886.

I do not believe it is. Certainly one cannot stick too close to optical analogies in determining what are proper methods of experimentation. For, so long as optical resonators are in general very small compared with the wave-lengths they emit, while electrical resonators are comparable in size to the wave emitted, the analogies are certain to break down in many vital points just as Schaefer himself found to be the case in comparing the theory developed by Planck[1] with experiment.

In this connection Schaefer invites comparison between the curves of Blake and Fountain with those of Aschkinass and Schaefer as well as with his own. He insists that by keeping the same relative distances for all the gratings he and Aschkinass obtained relatively simple relations where we obtained complicated ones. Remembering that Blake and Fountain did not change the dielectric, what could be more simple than to expect minimum absorption for twice the resonance length? And yet for only two of the four curves does Fig. 2 of the work of Aschkinass and Schaefer show this minimum correctly. One has only to compare the character of their curve 2 with that of curve 3 for resonator lengths greater than 4 cm. to see how simple (?) the relation between them is. I venture the assertion that if Aschkinass and Schaefer had for all their gratings kept their resonators constant at 2 mm. width and the side-on distance between the resonators constant at 6 or 8 cm., then, starting with an end-on distance great enough so that capacity changes would not enter (say 2 cm.), had they proceeded to change the length only of their resonators they would have obtained the same ratio $L_1$ that their Table II. shows and far more satisfactory and comparable curves than their Fig. 2 shows.

Late as is the date, I believe that I have now satisfactorily refuted Schaefer's objections. It seems certain that the work of Blake and Fountain must stand practically as they left it. It remains only to say a word about Cartmel's paper.[2] Cartmel has endeavored to explain extra transmission on the ground that the phase relation between the energy reflected from the front and

---

[1] Planck, Sitzungsber. d. k. Akad. d. Wissensch. zu Berlin, Vol. I., p. 470, 1902; loc. cit., p. 480, 1903.

[2] Cartmel, Phys. Rev., Vol. XXV., p. 64, 1907.

back faces of the glass is affected by the presence of the resonators. I am unable at the present time to throw any light on the question as to which is the better explanation, Cartmel's or ours. Certainly it couldn't account for the *lens-action* of the resonator system. At most it could account for what I (as well as Cartmel) have called *true extra transmission*. But it seems no simpler than ours.[1]

I cannot close this paper without seconding the appeal made by Webb and Woodman[2] for a more systematic study of apparatus and conditions in the field of short electric waves. Certainly there have been great confusion and contradiction of experimental data and results. May we hope that in the future experimenters in this field will not take so much for granted as they have in the past.

To several of the teaching staff of the department I owe my best thanks for their help in various ways. Especially do I wish to thank Professor Kester, now of the University of Kansas, for much aid in taking the observations and for many valuable suggestions and criticisms.

OHIO STATE UNIVERSITY,
COLUMBUS, OHIO.

---

[1] Possibly Woodman and Webb (see PHYSICAL REVIEW, Vol. XXX., page 561, 1910) are right in saying that phase change and change in velocity necessarily accompany each other, and hence that the two explanations are the same. Perhaps further work will give us clearer notions on the *modus operandi* of what we term "change of phase."

[2] Webb and Woodman, loc. cit.

# MAGNETOSTRICTION IN IRON-CARBON ALLOYS.

BY HERBERT G. DORSEY.

SINCE Joule's discovery in 1842 that iron changes its dimensions when magnetized, much experimental work has been done on this interesting phenomenon. But of the numerous articles published I have found none which gives a complete chemical analysis of an iron specimen.

The excellent series of steel rods of known analysis used by Dr. C. W. Waggoner[1] being available it seemed desirable to make tests on their changes in length in a magnetic field.

After studying the various magnifying devices used by different workers and as a result of a few preliminary trials the method used by Guthe and Austin[2] in their work on magnetostriction in Heusler alloys was employed with but few changes.

Fig. 1 gives a general view of the entire apparatus with details of the magnifier. The steel rod shown in black was soldered into the ends of brass rods *B* and *B′* and the latter was clamped rigidly at one end of the frame of brass tubing. At the other end of this frame was attached the wooden base of the magnifier. This consisted of a glass plate freely movable on roller bearings of needles. Two glass plates were fixed at right angles in a block of wood and the third larger plate was held to the vertical needles by a rubber band *R*. The two proximate vertical surfaces were ground together with fine emery and this seemed essential to prevent slipping in the greater changes of length. The rubber band not only holds the plate in place but presses it lightly against the rod *B* so that any motion of *B* is communicated to the glass plate. To one of the vertical needles was fastened a glass pointer *P*, which was made by drawing a glass tube very small, bending at right angles near the larger end and fastening the point of the needle into the tube with

---

[1] PHYS. REV., Vol. 28, p. 393, 1909.

[2] Bul. Bur. Stds., Vol. 2, p. 297, 1906.

sealing wax. Opposite this needle in a hole in the wooden block was placed a glass tube bent at right angles, the other end being drawn down to a point. This glass arm being thus pivoted near the rotating needle gives a method of quick rough adjustment without changing much the sensibility. To the ends of the glass arm and pointer were fastened two silk fibers which supported the

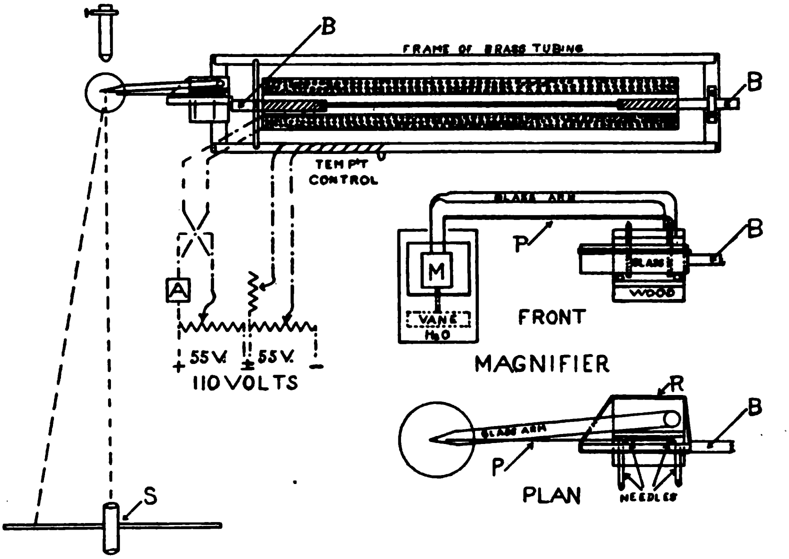


Fig. 1.

tiny mirror $M$. The mirror carried a small glass vane under water which rendered it nearly although not quite aperiodic. Mirror, vane and water were in a brass tube with glass window. The deflections of the mirror were observed by the telescope and scale.

If $a$ = diameter of the needle = 0.0652 cm.
$b$ = distance between threads = from 0.08 to 0.2 cm.
$c$ = scale deflection.
$d$ = distance scale to mirror, usually about 120 cm.
$e$ = length of glass pointer = 6.90 cm.
$l$ = length of rod tested,

then

$$\frac{\Delta l}{l} = \frac{abc}{2del}.$$

Scale deflections could be easily read to 0.1 cm. so that the smallest change in length which could be easily detected was

$$\frac{abc}{2de} = \frac{.0652 \times .08 \times .01}{2 \times 120 \times 6.90} = 3.15 \times 10^{-7} \text{ cm.}$$

At first a number twelve needle of 0.035 cm. diameter was used and a pointer 15 cm. long, but the magnifier was so much more sensitive than was necessary for such long rods that the larger needle and shorter pointer were chosen. With all of its high magnification, the parts were all so light that there seemed to be no lost motion of any sort and there was no appreciable trouble from vibrations. The entire apparatus was tested with a brass rod in the place of the iron rod and absolutely no effect was observable when the strongest magnetizing current was applied.

## Temperature Compensation.

With a magnification of 300,000 a change in temperature of one degree for a 40 cm. rod would produce a deflection of more than a hundred scale divisions so that efforts to maintain a constant temperature were soon abandoned and compensation was employed. On the side tubes of the brass frame a few turns of fine insulated advance wire were wound, through which was sent a small current which could be closely adjusted by means of the shunt and rheostat shown in Fig. 1. This current warmed the brass frame and by its expansion the magnifier was moved forward an amount equal to the increase in length of the steel rod due to temperature changes. The changes in length due to magnetism are so nearly instantaneous and the magnifier responds so quickly that no trouble was experienced in getting readings before temperature produced any effect. In any case the temperature changes produce slow drifts while the magnetism changes are sudden, so that there is no difficulty in distinguishing between the two. With the harder steels the warming due to hysteresis of a single reversal of a current of one ampere could be detected.

## Magnetization.

The magnetizing coil contained 2,240 turns of number 10 copper wire wound on a brass tube of 1.1 cm. external diameter and 70 cm. length. An alternate current of about 50 amperes was sent through it a few minutes while containing an iron core as a test for short circuits, for if there were any the short-circuited turns would get hotter than the rest of the coil. The heating was uniform, indicating no short circuits, but the coil length contracted permanently to 69.5 cm. The calculated value of the field strength is 40.5 gausses per ampere. This was checked by a small exploring coil, the same one described later, in two other long slim solenoids and the calculated values for all three differed by less than one per cent. The coil was then explored with the test coil and for a distance of over 40 cm. the field varied by not more than 1.3 per cent.

Current was taken from a motor generator set having nearly perfect automatic regulation of field excitation. To one side of the 55-55-110-volt circuit was shunted a tin resistance frame and from this 28 wires were connected to a dial switch so that currents through the magnetizing coil could be varied by steps from 0.05 to 42.5 amperes. Currents were measured by a Weston milli-voltmeter with two shunts, one for currents 0 to 5 amperes, the other 0 to 50 amperes..

All values of field strength $H$ and magnetic intensity $I$, unless otherwise noted, are corrected for the demagnetizing effect of the ends of the rods. These corrections for $H$ vary from 15 per cent. for $A3$ to more than 100 per cent. for $PI$. The values of $m$ = length/diameter are given in Table X. Magnetization curves were taken by the ballistic method. The ballistic coil already mentioned as a test coil consisted of 185 turns of number 40 wire wound upon a brass tube of 0.80 cm. external diameter. The coil length was 3.1 cm. The galvanometer was of the D'Arsonval type of medium period and large damping. It was calibrated by means of the test coil used in two different long solenoids with several different currents measured by the same ammeter used in the rest of the work. A line was plotted from these different values and the galvanometer constant determined from the slope of the curve. It was also checked by a standard cell and condenser, proper corrections for logarithmic decrement being made for all measurements.

In taking data for the magnetization curves the specimen was first demagnetized by reversals and then the desired current reversed at least 25 times before the galvanometer throw was recorded and a second reading taken as a check. The current was then increased, 25 more reversals made and the throw recorded and this process repeated until the entire range was covered. With the harder steels 35 to 40 reversals were necessary. This approximately gives what C. W. Burrows[1] calls a normal magnetization curve.

In order to find the correction to be applied to the values of $H$ for the length of ballistic coil used a magnetization curve for $P1$ was made by the magnetometer method, and corrections applied for the demagnetizing effects of the rod ends according to the method of C. R. Mann.[2] The corrected curve was then plotted along with the curve made by the ballistic method and their differences found, from the average of which it appeared that if Mann's values of the correction factor $N$ were divided by 1.66 the corrected values of $I$ could be found up to $I = 800$. Above this value a sliding scale was used, and it is believed that the corrected values are not far from the truth. The values of $N$ for the different specimens are given in Table X. This method of making corrections appears to be reliable for it is checked by an entirely different method. In Fig. 7, where the uncorrected susceptibility of $D1$ and $D2$ are plotted as a function of diameter divided by length these curves cross the $Y$ axis at $K = 141$ and $115$ respectively, which would represent the values in rings or rods without poles, and the values by correction are 139 and 113 respectively. The magnetic tests were made after the magnetostriction tests. To find values of $I$ and $H$ corresponding to values of current used in the magnetostriction tests large curves were plotted for each specimen and values taken from them.

The value of $I$, the intensity of magenetism, is given by the formula

$$I = \frac{\frac{\varphi}{n} - AH'}{4\pi\alpha - AN},$$

---

[1] Bul. Bur. Stds., Vol. 4, p. 205, 1908.

[2] PHYS. REV., Vol. 3, p. 367, 1895.

in which $2\varphi$ is the change in flux for a reversal of the current, $n$ is the number of turns of the ballistic coil, $= 185$, $A$ is the area of the ballistic coil, $= 0.503$ sq. cm., $\alpha =$ area of the specimen, $N =$ the correction factor, and $H' =$ uncorrected field strength $= 40.5 \times$ current in amperes.

$$2\varphi = q_0\delta(1 + \lambda/2)R \times 10^8,$$

in which $q_0 =$ the quantity constant of the galvanometer $= 100.7 \times 10^{-9}$, $\delta =$ the throw of the galvanometer, $\lambda =$ the logarithmic decrement and $R =$ the total resistance in the circuit. For any given specimen the above formula reduces to

$$I = a\delta(1 + \lambda/2)R - b \times \text{current},$$

where $a$ and $b$ are constants.

## SPECIMENS.

The rods were first tested in the following condition as received from Dr. Waggoner. They were about 40 cm. long and about .5 cm. in diameter, the ends being drilled longitudinally for about .5 cm. for lathe centers when they were turned down from the original rods. They were then annealed at 1000° for two hours and his magnetic tests made at room and liquid air temperatures. The chemical analysis is here repeated as given in his paper.

*Chemical Analysis.*

| Mark. | C | P | Si | Mn | S |
|---|---|---|---|---|---|
| P.I. | .058 | Trace | .008 | .071 | — |
| A1 | .60 | .013 | .15 | .14 | .012 |
| A2 | .74 | .012 | .16 | .14 | 013 |
| A3 | .89 | .010 | .19 | .155 | .013 |
| A4 | .98 | .012 | .16 | .15 | .013 |
| A5 | 1.18 | .012 | .14 | .14 | .013 |
| A55 | 1.26 | .012 | .16 | .17 | .014 |
| A6 | 1.37 | .011 | .19 | .16 | .012 |

## MAGNETOSTRICTION TESTS.

The following procedure was taken in all measurements on magnetostriction. The specimen being placed in the coil and adjustments made was demagnetized by reversals. The desired current applied and deflection and current noted. Specimen again demagnetized and the same current applied in the opposite direction and deflection noted. This was then repeated, thus giving four readings for each step of the current. The deflections given in the following tables are the average of four measurements, each taken with the current in a direction opposite to that used in the preceding measurement. The last three scale deflections are exceptions and are the average of only two measurements on account of excessive heating of the higher currents.

### TABLE I.

*PI*, 0.058 *Per Cent. Carbon.* $b=0.2892$ *cm.*, $d=120.5$ *cm.*, $l=39.5$ *cm.*

| $H$ | $I$ | $\Delta l/l$ | $H$ | $I$ | $\Delta l/l$ |
|---|---|---|---|---|---|
| 1.96 | 270 | $.17\times10^{-6}$ | 63.9 | 1,308 | $3.24\times10^{-6}$ |
| 2.93 | 486 | .37 | 73.0 | 1,322 | 3.10 |
| 4.33 | 712 | .83 | 93.7 | 1,350 | 2.76 |
| 5.00 | 800 | 1.26 | 142.5 | 1,414 | 1.81 |
| 6.49 | 924 | 1.64 | 214. | 1,486 | .32 |
| 7.80 | 1,022 | 2.04 | 242. | 1,511 | − .26 |
| 11.00 | 1,115 | 2.50 | 300. | 1,545 | −1.55 |
| 14.0 | 1,163 | 2.84 | 352. | 1,572 | −2.57 |
| 18.4 | 1,202 | 3.13 | 493. | 1,611 | −4.88 |
| 24.0 | 1,230 | 3.30 | 580. | 1,624 | −5.57 |
| 29.2 | 1,244 | 3.39 | 757. | 1,643 | −6.58 |
| 36.8 | 1,260 | 3.45 | 1,090. | 1,660 | −7.41 |
| 44.3 | 1,278 | 3.45 | 1,580. | 1,675 | −8.05 |
| 52.0 | 1,292 | 3.39 | | | |

### TABLE II.

*A*1, 0.60 *Per Cent. Carbon.* $b=0.1568$ *cm.*, $d=120.5$ *cm.*, $l=39.5$ *cm.*

| $H$ | $I$ | $\Delta l/l$ | $H$ | $I$ | $\Delta l/l$ |
|---|---|---|---|---|---|
| 4.7 | 263 | $.14\times10^{-6}$ | 77.6 | 1,184 | $1.78\times10^{-6}$ |
| 6.8 | 413 | .28 | 110. | 1,240 | 1.41 |
| 8.1 | 490 | .43 | 157. | 1,298 | .74 |
| 9.6 | 572 | .56 | 194. | 1,334 | .16 |
| 11.4 | 653 | .71 | 214. | 1,344 | − .11 |
| 14.0 | 760 | .89 | 239. | 1,370 | − .50 |
| 17.7 | 870 | 1.07 | 272. | 1,395 | −1.03 |
| 24.0 | 972 | 1.45 | 353. | 1,434 | −2.30 |
| 30.0 | 1,022 | 1.54 | 428. | 1,472 | −3.17 |
| 36.4 | 1,058 | 1.67 | 583. | 1,520 | −4.76 |
| 43.0 | 1,090 | 1.68 | 770. | 1,550 | −5.80 |
| 49.6 | 1,120 | 1.70 | 1,190. | 1,595 | −6.63 |
| 59.5 | 1,150 | 1.73 | 1,685. | 1,640 | −7.35 |

## TABLE III.

*A2*, 0.74 *Per Cent. Carbon.* $b=0.1205$ *cm.*, $d=121$ *cm.*, $l=39.6$ *cm.*

| $H$ | $I$ | $\Delta l/l$ | $H$ | $I$ | $\Delta l/l$ |
|---|---|---|---|---|---|
| 5.0 | 180 | $.02\times 10^{-6}$ | 61.5 | 1,162 | $1.40\times 10^{-6}$ |
| 6.4 | 280 | .07 | 70.1 | 1,180 | 1.39 |
| 8.0 | 386 | .15 | 90.1 | 1,215 | 1.28 |
| 9.0 | 450 | .20 | 138. | 1,280 | .76 |
| 10.5 | 526 | .27 | 205. | 1,346 | − .12 |
| 13.4 | 650 | .40 | 230. | 1,365 | − .44 |
| 15.7 | 732 | .56 | 290. | 1,404 | −1.24 |
| 20.5 | 840 | .77 | 342. | 1,430 | −1.90 |
| 25.8 | 922 | .95 | 490. | 1,482 | −3.40 |
| 30.1 | 974 | 1.06 | 575. | 1,503 | −3.94 |
| 37.5 | 1,044 | 1.19 | 760. | 1,532 | −4.85 |
| 43.5 | 1,076 | 1.31 | 1,110. | 1,571 | −5.55 |
| 50.9 | 1,127 | 1.38 | 1,660. | 1,618 | −6.11 |

## TABLE IV.

*A3*, 0.89 *Per Cent. Carbon.* $b=0.1385$ *cm.*, $d=120$ *cm.*, $l=39.4$ *cm.*

| $H$ | $I$ | $l/l$ | $H$ | $I$ | $\Delta l/l$ |
|---|---|---|---|---|---|
| 9.0 | 228 | $.01\times 10^{-6}$ | 73.5 | 1,130 | $.68\times 10^{-6}$ |
| 10.5 | 290 | .03 | 90.0 | 1,175 | .62 |
| 12.5 | 375 | .04 | 138. | 1,268 | .29 |
| 16.6 | 545 | .07 | 207. | 1,333 | .37 |
| 17.3 | 552 | .14 | 230. | 1,350 | − .68 |
| 22.5 | 700 | .24 | 290. | 1,385 | −1.37 |
| 28.0 | 790 | .32 | 347. | 1,410 | −1.86 |
| 32.5 | 850 | .40 | 482. | 1,456 | −3.07 |
| 39.5 | 928 | .49 | 568. | 1,474 | −3.64 |
| 46.5 | 988 | .56 | 760. | 1,510 | −4.52 |
| 54.0 | 1,036 | .62 | 1,100. | 1,550 | −5.45 |
| 65.0 | 1,095 | .67 | 1,635. | 1,615 | −6.11 |

In Fig. 2 the percentage change in length is plotted as a function of $H$ for the first five rods of the series. The others are not plotted as they are so close to *A*2 that plotting them in this figure would only tend to produce confusion.

Fig. 3 shows the relation between magnetostriction and the intensity of magnetization.

Data for residual magnetostriction curves are plotted for specimens *A*4 and *A*6 in Fig. 4. To get these a reading of the telescope

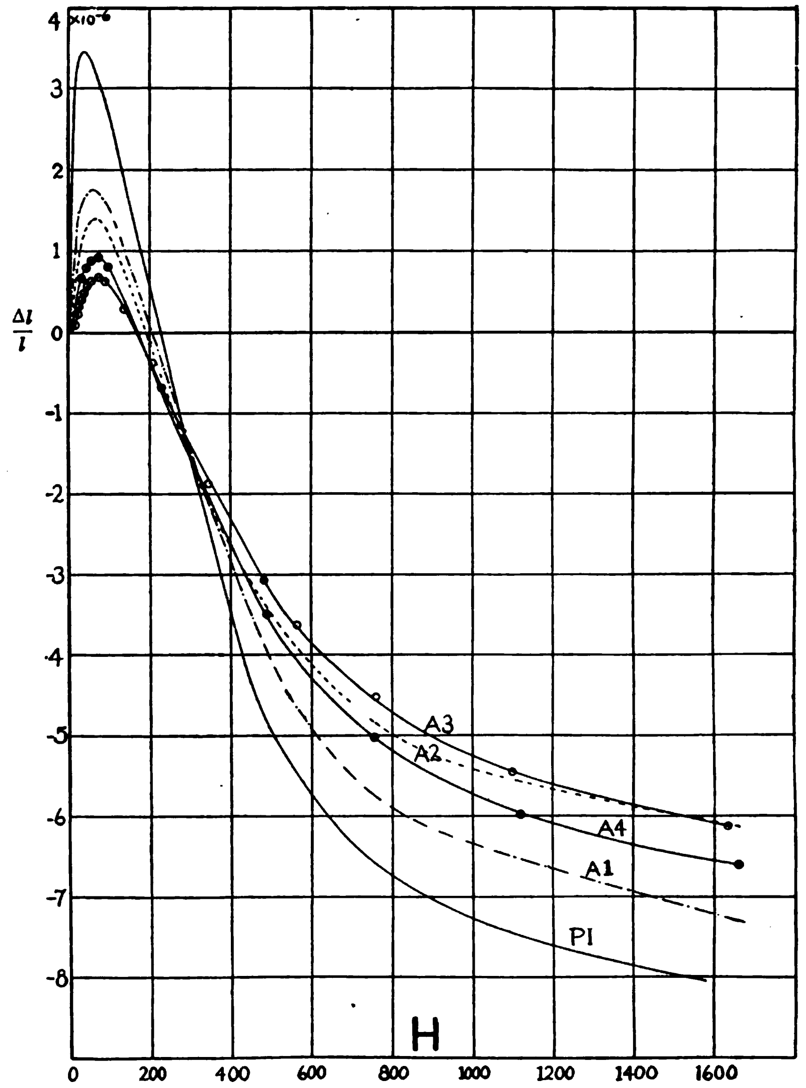


**Fig. 2.**

**Magnetostriction as a function of field strength.**

was made after the circuit was broken and as the deflection is then in the opposite direction as when the circuit is closed deflections for residual are recorded with an opposite sign. In the specimens magnetically soft scarcely any residual effect could be found but those which are harder remain longer after being magnetized unless the actual field strength reaches about 400 or more.

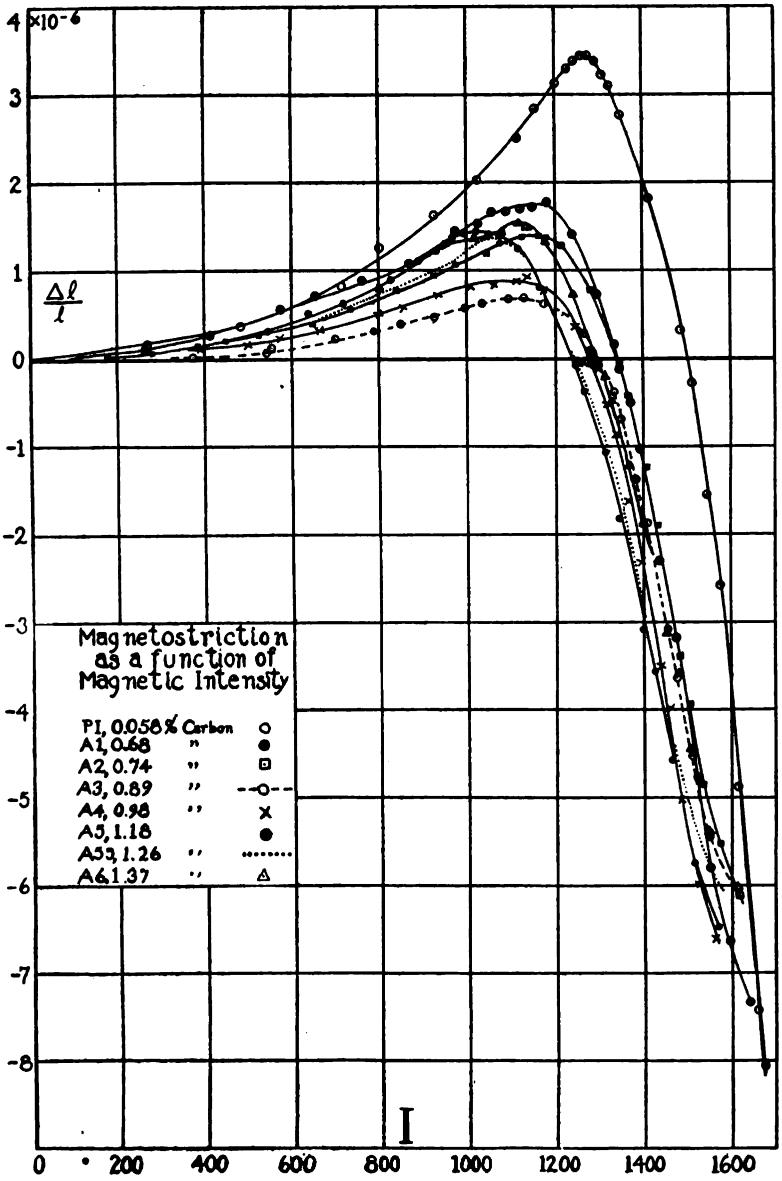


Fig. 3.

## TABLE V.

A4, 0.98 *Per Cent. Carbon.* $b=0.1568$ *cm.*, $d=120.5$ *cm.*, $l=39.5$ *cm.*

| $H$ | $I$ | $\Delta l/l$ | Residual. | |
|---|---|---|---|---|
| | | | Deflec. | $\Delta l/l$ |
| 8.6 | 244 | $.02\times10^{-6}$ | | |
| 9.8 | 300 | .05 | | |
| 11.7 | 390 | .11 | | |
| 14.4 | 500 | .17 | − .5 | $.09\times10^{-6}$ |
| 16.4 | 575 | .25 | − .7 | .14 |
| 19.5 | 665 | .34 | − .8 | .22 |
| 26.2 | 800 | .51 | − 1.7 | .25 |
| 30.0 | 854 | .59 | − 1.9 | .30 |
| 36.3 | 932 | .73 | − 2.5 | .34 |
| 44.5 | 1,008 | .81 | − 3.0 | .34 |
| 52.3 | 1,060 | .84 | − 3.5 | .30 |
| 63.4 | 1,110 | .89 | − 3.3 | .34 |
| 71.7 | 1,137 | .92 | − 3.6 | .34 |
| 92.0 | 1,173 | .76 | − 3.0 | .30 |
| 138. | 1,245 | .36 | − .8 | .23 |
| 211. | 1,315 | − .53 | + 5.1 | .26 |
| 235. | 1,335 | − .86 | + 6.8 | .20 |
| 290. | 1,367 | −1.62 | +11.8 | .19 |
| 350. | 1,395 | −2.33 | +16.0 | .15 |
| 490. | 1,440 | −3.50 | +23.4 | .14 |
| 578. | 1,460 | −3.98 | | |
| 758. | 1,488 | −5.01 | | |
| 1,120. | 1,528 | −6.98 | | |
| 1,660. | 1,562 | −6.60 | | |

## TABLE VI.

A5, 1.18 *Per Cent. Carbon.* $b=0.1036$ *cm.*, $d=120.5$ *cm.*, $l=39.6$ *cm.*

| $H$ | $I$ | $\Delta l/l$ | $H$ | $I$ | $\Delta l/l$ |
|---|---|---|---|---|---|
| 7.30 | 270 | $.06\times10^{-6}$ | 74.5 | 1,073 | $1.40\times10^{-6}$ |
| 8.50 | 345 | .13 | 98. | 1,116 | 1.25 |
| 9.51 | 422 | .20 | 143. | 1,167 | .77 |
| 11.5 | 542 | .32 | 215. | 1,248 | − .07 |
| 13.5 | 638 | .51 | 238. | 1,267 | − .37 |
| 16.0 | 718 | .64 | 300. | 1,314 | −1.06 |
| 21.0 | 824 | .90 | 352. | 1,344 | −1.82 |
| 26.2 | 890 | 1.11 | 495. | 1,400 | −3.08 |
| 31.6 | 930 | 1.23 | 580. | 1,425 | −3.56 |
| 38.0 | 964 | 1.36 | 780. | 1,463 | −4.56 |
| 45.8 | 996 | 1.43 | 1,147. | 1,513 | −5.75 |
| 53.7 | 1,020 | 1.45 | 1,680. | 1,570 | −6.48 |
| 65.6 | 1,050 | 1.43 | | | |

## Table VII.

*A55, 1.26 Per Cent. Carbon.* $b=0.1568$ *cm.,* $d=120$ *cm.,* $l=39.5$ *cm.*

| *H* | *I* | $\Delta l/l$ | *H* | *I* | $\Delta l/l$ |
|---|---|---|---|---|---|
| 7.73 | 195 | $.03\times10^{-6}$ | 78.0 | 1,107 | $1.26\times10^{-6}$ |
| 8.75 | 275 | .06 | 100. | 1,140 | 1.15 |
| 10.0 | 370 | .11 | 148. | 1,193 | .59 |
| 11.8 | 465 | .20 | 212. | 1,265 | − .23 |
| 14.7 | 595 | .36 | 238. | 1,285 | − .50 |
| 17.0 | 678 | .48 | 295. | 1,322 | −1.14 |
| 22.0 | 790 | .72 | 353. | 1,352 | −1.73 |
| 27.2 | 867 | .87 | 490. | 1,400 | −3.02 |
| 32.6 | 928 | 1.00 | 580. | 1,425 | −3.43 |
| 39.3 | 980 | 1.12 | 775. | 1,460 | −4.27 |
| 46.0 | 1,018 | 1.17 | 1,120. | 1,508 | −5.14 |
| 53.8 | 1,050 | 1.37 | 1,660. | 1,575 | −6.01 |
| 63.7 | 1,080 | 1.30 | | | |

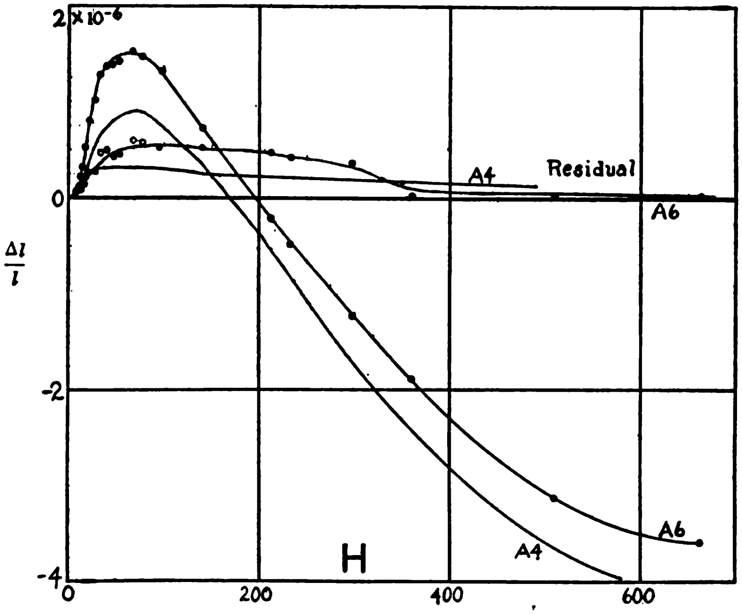


Fig. 4.

### In what Portion of a Rod is the Change in Length Greatest?

Since iron gets longer for weak intensities of magnetization and shorter for stronger intensities in might be asked if all parts of a

## TABLE VIII.

*A6*, 1.37 *Per Cent. Carbon.* $b=0.1399$ *cm.*, $d=119.5$ *cm.*, $l=39.4$ *cm.*

| $H$ | $I$ | $\Delta l/l$ | Residual. | |
|---|---|---|---|---|
| | | | Deflec. | $\Delta l/l$ |
| 8.8 | 270 | $.06\times10^{-6}$ | $-$ .2 | $.03\times10^{-6}$ |
| 10.4 | 368 | .11 | $-$ .6 | .03 |
| 12.3 | 476 | .21 | $-$ .9 | .08 |
| 15.0 | 600 | .37 | $-$ 1.5 | .15 |
| 16.9 | 674 | .53 | $-$ 2.3 | .21 |
| 21.5 | 800 | .80 | $-$ 3.5 | .31 |
| 27.0 | 898 | 1.01 | $-$ 4.9 | .32 |
| 32.5 | 950 | 1.28 | $-$ 5.8 | .46 |
| 40.0 | 1,012 | 1.37 | $-$ 6.2 | .50 |
| 46.5 | 1,050 | 1.39 | $-$ 6.8 | .44 |
| 54.1 | 1,080 | 1.42 | $-$ 6.9 | .45 |
| 66.0 | 1,115 | 1.54 | $-$ 6.7 | .60 |
| 75.6 | 1,136 | 1.49 | $-$ 6.4 | .59 |
| 96.5 | 1,180 | 1.32 | $-$ 5.7 | .52 |
| 140. | 1,240 | .72 | $-$ 1.4 | .52 |
| 212. | 1,313 | $-$ .20 | + 4.9 | .49 |
| 233. | 1,330 | $-$ .48 | + 6.4 | .42 |
| 298. | 1,368 | $-$1.21 | +11.2 | .36 |
| 360. | 1,400 | $-$1.88 | +13.5 | .01 |
| 510. | 1,450 | $-$3.12 | +22.0 | .03 |
| 662. | 1,480 | $-$3.59 | +25.7 | .01 |
| 780. | 1,502 | $-$4.45 | | |
| 1,114. | 1,545 | $-$5.40 | | |
| 1,700. | 1,610 | $-$6.04 | | |

rod are changing by the same amount when magnetized. To investigate this question several determinations were made by soldering slotted tubes on *PI* at equal distances from the center so as to leave the rod its original length and yet test the change in length for different portions of the rod. Only enough points were determined to find the maximum elongation although the maximum current was turned on each time to see that the retractions of the rod were about the same no difference what portion was tested.

After these tests were made a ballistic coil of a single layer was wrapped over one half of the rod and leads taken off at different portions so as to have the equivalent of several test coils of different lengths on the rod. It was assumed that the magnetism would be symmetrically distributed on each side of the rod center. A normal magnetization curve was then taken, the galvanometer being switched to the different test coils for each point. The maximum susceptibilities without any correction for end effects were thus determined for each of the different coils. These values are given in Table IX. and plotted in Fig. 5. It will be seen that the elongation and apparent susceptibility increase regularly as the center of the rod is approached although there is a sudden decrease in both near the end of the rod. As would be expected, the different points of maximum elongation and points of maximum apparent susceptibility occurred at the same values of $H$ as they did when the full rod length was tested, viz., $H = 44.3$ for magnetostriction and $H = 2.93$ for susceptibility.

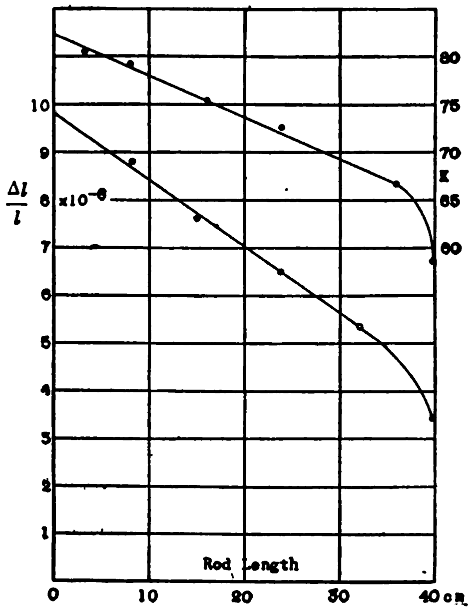


Fig. 5.

Maximum percentage elongation and maximum apparent susceptibility of *PI* as functions of the length of rod tested.

## EFFECT OF LENGTH OF ROD.

In order to preserve the rods of the series another rod was taken for this test. It was of soft iron and cut to the same size as the others and annealed. It was called *D*1. After tests were made on the full length portions were cut from the end after each test so that eight different lengths were used. Values are given in Table IX. and in Fig. 6 maximum apparent susceptibilities and maximum

### TABLE IX.

*P1, Full Length, First Condition. $\Delta l/l$ and Uncorrected Susceptibility Tested in Different Portions of the Rod.*

| Length. | $\Delta l/l$ Max. | Coil Length. | $k$ Max. (n.c.). |
|---|---|---|---|
| 39.8 cm. | $3.45 \times 10^{-6}$ | 39.8 cm. | 58.6 |
| 32.0 | 5.35 | 35.9 | 66.6 |
| 23.9 | 6.50 | 23.8 | 72.6 |
| 15.0 | 7.62 | 16.0 | 75.5 |
| 8.1 | 8.80 | 8.0 | 79.2 |
| 7.9 | 9.15 | 3.1 | 80.5 |

*D1, Soft Iron, No Analysis. Annealed and then Cut to Various Lengths. Diameter = 0.58 cm. Corrected $k$ = 139.5.*

| Length. | $\frac{\text{Diameter}}{\text{Length}}$ | $\Delta l/l$ Max. | $k$ Max. (n.c.). |
|---|---|---|---|
| 39.8 cm. | .0146 | $4.78 \times 10^{-6}$ | 75.1 |
| 35.0 | .0166 | 4.08 | 65.5 |
| 30.0 | .0193 | 3.93 | 54.3 |
| 25.0 | .0232 | 2.23 | 42.8 |
| 22.0 | .0263 | 1.87 | 38.6 |
| 20.0 | .0290 | 3.19 | 30.8 |
| 12.0 | .0474 | 2.33 | |
| 4.8 | .121 | 1.90 | |
| ∞ | 0 from curve | 5.80 | 141. |

*D2, Soft Steel, No Analysis. Eight Pieces Cut from the Same Rod. Ends Threaded and then Annealed in Nitrogen. Diameter = 0.638 cm. Corrected $k$ = 113.*

| Length. | $\frac{\text{Diameter}}{\text{Length}}$ | $\Delta l/l$ Max. | $k$ Max. (n.c.). |
|---|---|---|---|
| 40 | .0160 | 3.10 | 62.9 |
| 35 | .0182 | 3.32 | 54.7 |
| 30 | .0213 | 2.90 | 43.9 |
| 25 | .0255 | 2.85 | 34.4 |
| 20 | .0319 | 2.78 | 24.4 |
| 15 | .0425 | 2.70 | 16.1 |
| 10 | .0638 | 2.40 | 8.7 |
| 5 | .128 | 2.35 | 3.15 |
| ∞ | 0 from curve | 3.80 | 115. |

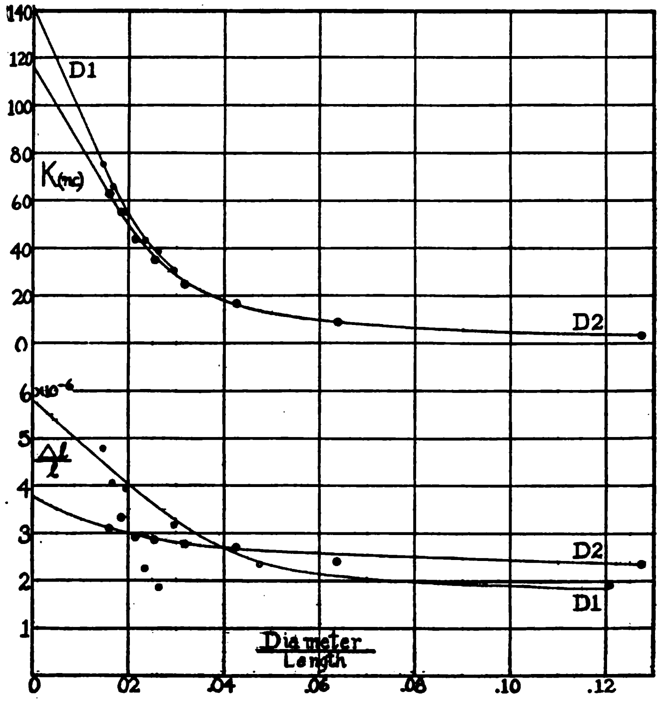


Fig. 6.

percentage elongations are plotted as functions of diameter divided by length. It will be noticed that two points for magnetostriction are far from the curve, but these are thought to be due to accidenta- conditions of the iron rather than to errors of measurements. All though carefully taken at the time they could not be checked afterwards as the rod had been made shorter. The rods were each time soldered to the brass end rods and the temperature reached in soldering might have been sufficient to have altered the condition. To avoid this another series of rods was made from a single long rod of commerical soft steel called *D2*. The ends were threaded so as to avoid soldering and then all were heated to 1000°C. in an atmosphere of nitrogen and furnace cooled. Each rod was then tested as before.

It should be noted that for the shorter rod lengths it required a much higher apparent field strength, uncorrected for end effects, to produce the maximum elongations. Thus for $D1$ full length maximum elongation occurred at apparent $H = 60$ while for the 4.8 cm. length it required apparent $H = 145$. For $D2$ the corresponding values are 54 and 243. These results quite confirm those of Sidney Lochner,[1] who showed that a short thick rod apparently expands for all fields only because the actual field strength is small due to the demagnetizing effects of the ends.

The values obtained for $D1$ and $D2$ are given in Table IX. and curves in Fig. 6. If these curves are projected back to cut the $Y$ axis we have values for rods of infinite length as compared to their diameters, comparable to a ring without poles. As before mentioned the values of susceptibility check almost exactly with those obtained by making corrections for end effects. From similar reasoning then, a steel of the quality of $D1$ in the shape of a ring would have a maximum percentage elongation along the circumference of $5.8 \times 10^{-6}$ and for steel of the quality of $D2$ it would be $3.8 \times 10^{-6}$.

## MODULUS OF ELASTICITY.

Having found a close relationship between maximum elongation and maximum susceptibility it was thought desirable to see if there is any relation to the modulus of elasticity. This was tested by means of an Olsen testing machine and a mirror extensometer reading to 1/40,000 inch. The values are given in dynes per square cm. in Table X. and plotted as a function of the carbon content in Fig. 7. These values are in good general agreement with those found by Benedicks.[2]

It will be noticed that there is a general tendency for the curve to slope downwards for the higher per cent. carbon. Also that the retraction curve for a value of $H = 1,500$ has a general slope upwards so that roughly we can say that the amount of shortening in a strong field varies directly as the modulus of elasticity, a result exactly opposite to what I expected to find.

## EFFECT OF HEAT TREATMENT.

To remove any possible hardening effect due to stretching during the modulus tests the specimens were annealed at about 800° and

---

[1] Phil. Mag., Vol. 36, p. 504, 1897.

[2] Reserches Physiques et physico chemiques sur l'acier au carbone. Carl Benedicks, Upsala, 1904.

furnace cooled. Curves were again taken which had the same general shape as those already given but there was a notable change in the maximum values for *A*55 and *A*6 both for magnetostriction and susceptibility. Both values had increased. *PI* had also increased very materially for magnetostriction with apparently no change in susceptibility, although only a rough test was made of it

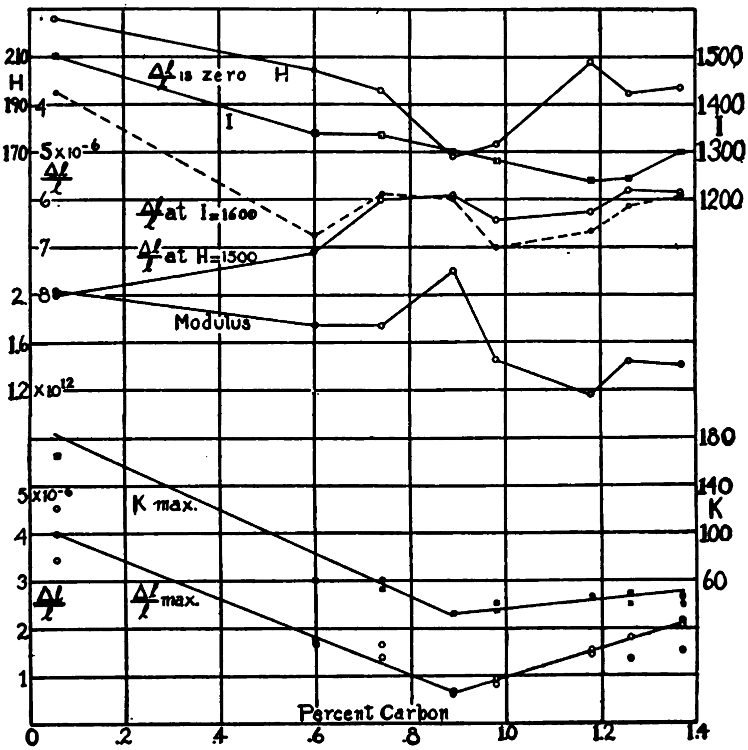


Fig. 7.

for this specimen. *PI*, *A*3 and *A*6 were then heated to about 1000° and quenched in water and tests made. *PI* and *A*6 were again annealed at 1000° and furnace cooled and *D*1 had been carried through the same process before being cut down. These values are all given in Table X. and plotted in Fig. 5 as a function of the per cent. carbon. The new values for *A*55 and *A*6 point to a probability that they were not entirely annealed in their first condition, and with the exception of *PI* and *D*1 it may be said that quenching or sudden cooling lowers the amount of maximum elongation while annealing raises it. This is in accordance with Joule's third law, that "the elongation is for the same intensity of magnetism proportional to the softness of the metal, greatest in iron, least in hard steel." *PI* and *D*1 which were soft iron both confirm and deny this general truth. For each in one so-called annealed condition gave values higher than in the quenched state. But, on the other hand, each in a quenched condition gave values higher than in one so-called annealed condition. I think this simply points to the fact that annealing in soft iron at least is not definitely stated until the exact temperature is stated, how long it is held at that temperature and how long a time is required in cooling. Some physical chemists claim that 5,000 hours at 1000° is necessary to anneal iron-carbon alloys.

This anomalous case for soft iron has also been reported by Shelford Bidwell[1] who found one soft iron ring which in the "annealed" condition contracted for all values of field strength. He considered this a rare specimen, and was unwilling to quench it for fear he could not reproduce the unusual condition.

### TABLE X.

*Collected Data for the Different Specimens.*

| | *m* | *N* | $E \times 10^{12}$ | First Condition. | | Annealed. | | Quenched. | | Annealed. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $\Delta l/l$ max. | *k* max. | $\Delta l/l$ max. | *k* max. | $\Delta l/l$ max. | *k* max. | $\Delta l/l$ max. | *k* max. |
| *PI* | 66 | .00663 | 2.02 | 3.45 | 166. | 4.52 | | 3.49 | 90.3 | 3.59 | 222. |
| *A*1 | 73 | .00542 | 1.75 | 1.78 | 60.6 | 1.70 | | | | | |
| *A*2 | 65 | .00680 | 1.75 | 1.40 | 52.4 | 1.68 | 60.8 | | | | |
| *A*3 | 74 | .00530 | 2.20 | .68 | 32.8 | .63 | | .26 | 10.9 | | |
| *A*4 | 64 | .00693 | 1.45 | .92 | 35.1 | .81 | 42.3 | | | | |
| *A*5 | 66 | .00663 | 1.16 | 1.45 | 47.3 | 1.54 | | | | | |
| *A*55 | 70 | .00595 | 1.44 | 1.37 | 40.5 | 1.80 | 49.8 | | | | |
| *A*6 | 68 | .00608 | 1.41 | 1.54 | 40.0 | 2.08 | 46.4 | 1.21 | 20.9 | 2.15 | 46.9 |
| *D*1 | 69 | .00603 | | | | 2.78 | 154. | 3.55 | 91.8 | 4.78 | 139.5 |

## ACCURACY.

It is believed that errors in all measurements of length were not greater than 0.3 per cent. and there were five such measurements

---

[1] Proc. Roy. Soc., Vol. 56, p. 94, 1894.

entering into the computations. Single readings of deflections may have been in error by from 0.5 per cent. to 3 per cent., depending upon the amount of deflection, but as these were mostly the average of four readings the error might be considered 1.5 per cent. Combining by square root of sum of squares gives 1.6 per cent. for measurements of magnetostriction. In the magnetic work currents may have been in error by 1 per cent., magnetic deflections on the average by 0.4 per cent., areas of rods and test coil by 0.6 per cent. each, resistances by 0.2 per cent., magnetic constants by 0.3 per cent. and field strength variation 1.3 per cent., giving a possible total error of 1.8 per cent. which is about the same as the discrepancy between the two different methods of arriving at the maximum susceptibilities of $D1$ and $D2$.

## CONCLUSIONS.

From these experiments upon this series of iron-carbon alloys the following conclusions may be drawn:

1. The maximum elongation decreases with the carbon content to 0.9 per cent. carbon and then increases and may be represented by the equation

$$(\Delta + 4C - 4.2)(\Delta - 3.1C + 2.13) = 0,$$

in which $\Delta = \Delta l/(l \times 10^8)$ and $C$ = per cent. carbon. Likewise the maximum susceptibility may be represented by the equation

$$(K + 185C - 194)(K - 40C + 4) = 0,$$

in which $K$ = maximum susceptibility and $C$ = per cent. carbon.

2. In a somewhat similar manner but not so definitely does the value of $H$ at which the rods retract to their original length drop to a minimum value at about 0.9 per cent. carbon.

3. $\Delta l/l$ is practically the same, at a value of $-1.4 \times 10^{-6}$ for about $H = 300$.

4. Up to field strengths of about 1,600 the rods were still contracting at individual uniform rates. But for a field strength of 1,500 it may be said in a general way that the greater the carbon content the less the retraction. Also, the modulus of elasticity decreases with carbon. Or, the amount of shortening in strong fields varies directly as the modulus.

5. Steels slowly cooled have greater elongations and susceptibilities than when quenched. Accidental exceptions for soft iron. Magnetostriction depends upon the previous history of the specimen.

6. The percentage elongation in the middle of a 40-cm. rod of soft iron is nearly three times as much as for the entire rod.

7. Rods of different lengths of the same quality iron give different values, therefore

8. Absolute values of magnetostriction as determined by different workers cannot be comparable with each other unless experiments are made with rings or else some method of correction is devised.

## BIBLIOGRAPHY.

1847. J. P. Joule, Phil. Mag., Vol. 30, p. 76, 225.
1848. M. G. Wertheim, Ann. de Chim. et de Phys., Vol. 23, p. 302.
1849. A. de La Rive, Ann. de Chim. et de Phys., Vol. 26, p. 158.
1866. W. Beetz, Pogg. Ann., Vol. 128, p. 193.
1873. A. M. Mayer, Phil. Mag., Vol. 45, p. 350; Vol. 46, p. 177.
1874. W. F. Barrett, Phil. Mag., Vol. 47, p. 51.
1879. A. Righi, Mem. di Bolog., Vol. 10, p. 407.
1882. W. F. Barrett, Nature, Vol. 26, p. 585.
1885. G. Kirchhoff, Wied. Ann., Vol. 24, p. 52.
1885. S. Bidwell, Proc. Roy. Soc., Vol. 38, p. 265.
1886. S. Bidwell, Proc. Roy. Soc., Vol. 40, p. 109, 257.
1888. S. Bidwell, Proc. Roy. Soc., Vol. 43, p. 406.
1888. S. Bidwell, Phil. Trans., Vol. 179A, p. 205.
1890. S. Bidwell, Proc. Roy. Soc., Vol. 47, p. 469.
1890. Cantone, Mem. del Acc. Lincei.
1890. C. Chree, Phil. Trans., Vol. 181A, p. 329.
1891. G. C. Knott and A. Shand, Proc. Roy. Soc. Edin., Vol. 18, p. 315.
1892. G. C. Knott and A. Shand, Proc. Roy. Soc. Edin., Vol. 19, pp. 85, 249.
1892. S. Bidwell, Proc. Roy. Soc., Vol. 51, p. 495.
1892. A. Berget, Compt. Rend., Vol. 115, p. 722.
1893. S. Lochner, Phil. Mag., Vol. 36, p. 504.
1893. G. C. Knott and A. Shand, Proc. Roy. Soc. Edin., Vol. 20, pp. 290, 334.
1894. S. Bidwell, Proc. Roy. Soc., Vol. 55, p. 228, Vol. 56, p. 94.
1894. H. Nagaoka, Wied. Ann., Vol. 53, p. 481.
1894. H. Nagaoka, Phil. Mag., Vol. 37, p. 131.
1894. G. C. Knott, Phil. Mag., Vol. 37, p. 141.
1895. E. T. Jones, Phil. Mag., Vol. 39, p. 254.
1895. Bock, Wied. Ann., Vol. 54, p. 442.
1895. Bachmetjew, Wien. Ber., Vol. 104, II., p. 71.
1895. B. Rosing, Jour. Rus. Phys. Chem. Soc., Vol. 26, p. 253.

1895. L. T. More, Phil. Mag., Vol. 40, p. 345.
1895. L. T. More, Phys. Rev., Vol. 3, p. 210.
1896. E. F. Gallaudet, Thesis, Johns Hopkins Univ.
1896. H. Nagaoka and E. T. Jones, Phil. Mag., Vol. 40, p. 454.
1896. E. T. Jones, Phil. Mag., Vol. 41, p. 153.
1897. G. Klingenberg, Thesis, Rostock Univ.
1897. B. B. Bracket, Phil. Mag., Vol. 44, p. 122.
1897. B. B. Bracket, Phys. Rev., Vol. 5, p. 257.
1897. E. T. Jones, Proc. Roy. Soc., Vol. 63, p. 44.
1897. E. T. Jones, Phil. Trans., Vol. 189A, p. 189.
1898. H. Nagaoka and K. Honda, Phil. Mag., Vol. 46, p. 261.
1898. J. S. Stevens, Phys. Rev., Vol. 7, p. 19.
1898. E. Rhoades, Phys. Rev., Vol. 7, p. 65.
1899. G. A. Shakespear, Phil. Mag., Vol. 47, p. 539.
1900. L. W. Austin, Phil. Trans., Vol. 194A, p. 201.
1900. L. W. Austin, Phys. Rev., Vol. 10, p. 180.
1900. J. S. Stevens, Phys. Rev, Vol. 11, p. 95.
1901. Carl Barus, Phys. Rev., Vol. 13, p. 283.
1902. S. Sano, Phys. Rev., Vol. 14, p. 158.
1902. K. Honda and S. Shimizu, Phil. Mag., Vol. 4, p. 338.
1902. W. E. Williams, Phil. Mag., Vol. 4, p. 430.
1902. E. Rhoades, Phys. Rev., Vol. 15, p. 321.
1903. E. Van Aubel, Phys. Rev., Vol. 16, p. 60.
1903. K. Honda and S. Shimizu, Phil. Mag., Vol. 6, p. 392.
1903. S. Bidwell, Proc. Roy. Soc., Vol. 73, p. 413.
1904. L. W. Austin, Vern. d. Deu. Phys. Ges., Vol. 6, p. 21.
1904. S. Bidwell, Proc. Roy. Soc., Vol. 74, p. 60.
1905. K. Honda and S. Shimizu, Phil. Mag., Vol. 10, p. 548, 642.
1906. K. E. Guthe and L. W. Austin, Bul. Bur. Stds., Vol. 2, p. 297.
1906. D. O. S. Davies, Nature, Vol. 75, p. 102.
1907. J. C. McLennan, Phys. Rev., Vol. 24, p. 449.
1908. L. Tieri, Acad. Lincei, Atti, 17, p. 204.

Physical Laboratory,
Cornell University.

# THE POINT DISCHARGE IN AIR FOR PRESSURES GREATER THAN ATMOSPHERIC.

BY O. AMSDEN GAGE.

THE effect of changes in pressure upon the ionization of gases has been quite extensively studied at pressures less than an atmosphere, but not so much attention has been paid to the higher values. The first workers in this field studied the change in spark potential difference and a number of such papers[1] has been published. The results obtained are most contradictory except that the relation between potential difference and gas pressure is a linear one up to 8 or 10 atmospheres. Above that, nothing consistent has been found.

Recently there has appeared a series[2] of papers on the effect of increased pressure upon ionization due to radium; all showing that, up to at least 17 atmospheres, the leakage current increases; in some cases the relation is practically linear. W. Wilson[3] has studied the increase in natural ionization of a closed vessel and also finds this linear relation for pressures greater than 4 atmospheres.

In planning the present work, the intention was to examine the change in disruptive discharge with pressure, but it was found desirable to use instead the point discharge studying the relation between current and potential difference. This particular form of discharge has been carefully investigated by a number of observers, but only Tamm[4] and Ewers[5] have studied to any extent the effect of pressure, and that for values less than one atmosphere. Instead

---

[1] Wolf, Wied. Ann., 37, p. 306, 1889. De Hemptine, Bull. de Belg., p. 603, 1903. Cassinto and Occhiatini, Rend. R. Acc. dei Linc., 15, p. 715, 1906. J. J. Thomson, Cambridge Phil. Trans., 6, p. 326. Ryan, Sibley Journal, 1904. C. E. Guye and H. Guye, Phys. Zeitsch., Jan. 15, 1906, p. 62.

[2] Rothé, Comptes Rendus, 147, p. 1279, 1908. Erickson, PHYS. REV., 27, p. 473, 1908. Laby and Kaye, Phil. Mag., 6, 16, p. 216, 1908.

[3] W. Wilson, Phil. Mag., 6, 17, p. 216.

[4] Tamm, Annalen, 6, p. 259, 1901.

[5] Ewers, Annalen, 17, p. 781, 1905.

of using for electrodes a point and plane, as practically all the workers have done previously, a fine wire and concentric cylinder were employed similar to the apparatus used by Almy.[1]

The luminous effect of the true point discharge is always confined to the point or its immediate neighborhood. This is a region of ionization in which ions of both signs are present. Those of the same sign as the point are repelled, pass out into the non-luminous portion and carry the current from the point to the other electrode, but it is generally assumed that their energy is too small to produce ions by collision. In the present work, the wire was covered uniformly with a blue glow when it was charged positively, but when negative in sign the discharge seemed to be localized in spots which were in constant motion. Between these spots could be seen a faint glow which covered more or less of the remainder of the wire.

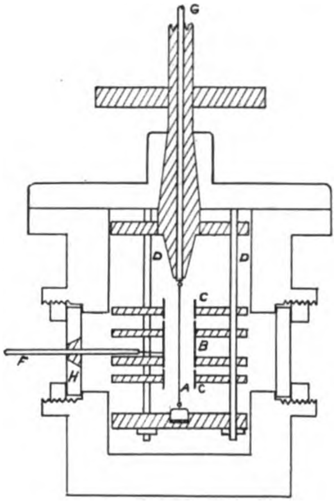


Fig. 1.

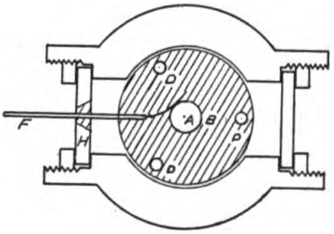


Fig. 2.

## APPARATUS.

The arrangement of apparatus was comparatively simple. A carefully insulated wire was attached to one terminal of an influence machine whose other terminal was grounded. This wire formed one electrode and was placed in the center of a cylinder which

---

[1] Almy, Amer. J. Sci., 4, 12, p. 175, 1901.

formed the other, and which was connected to earth through a galvanometer that measured the leakage current. The form of the discharge chamber and the arrangement of the electrodes in it are given in Figs. 1 and 2, which show vertical and horizontal sections.

This chamber was made of brass with walls 1.3 cm. thick; the interior had a depth of 12 cm. and a diameter of 6.7 cm. It was closed with a cast-iron lid which was fastened to it by twelve bolts. Through this lid and carefully insulated from it passed a steel rod *G* and joined to this rod was the wire *A* which served as one of the electrodes. The cylindrical electrode *B*, 2 cm. long, was held in place by discs of hard rubber which were attached to the lid by the steel rods *D*. In order to obtain an uniform field two guard cylinders *C* were used. These were connected to the grounded walls of the chamber.[1] The insulated rod *F*, tipped with a german silver wire served as a connection between the cylinder *B* and the galvanometer.

The influence machine was the Toepler-Holtz type, electric-driven and provided with plates of mica three feet in diameter. It was capable of maintaining a difference of potential of approximately 100,000 volts and could furnish a current of at least 600 or 700 microamperes. The potential was regulated by means of a brush discharge occurring between the terminals of the machine and was measured by two Kelvin instruments, a vertical electrostatic voltmeter and an electrostatic balance whose respective ranges were from 1,000 to 16,000 and from 10,000 to 100,000 volts.[2] The gal-

---

[1] The actual distance between *C* and *B* seemed to have very little effect upon the current flowing, provided the gap between them was comparatively small. During the experiment no attempt was made to keep the distance uniform, but it was usually in the neighborhood of 0.02 cm.

[2] No accurate calibration of these instruments was attempted, but, as a check, the spark potential differences between spheres 2 cm. in diameter were determined for varying lengths. The results obtained by the two instruments, when plotted, gave a smooth curve with no discontinuity where the change was made. The results were then compared with those given by Heydweiller (Wied. Ann., 48, p. 220, 1893) and were found to be in fair agreement up to 26,000 volts. As far as this reading, the Kelvin instruments gave higher values with a maximum variation of 5 per cent. and an average of 3 per cent. For higher values, much lower results were obtained, but subsequent work made it quite certain that the instrument was more nearly correct than these readings showed, and the lack of agreement was probably due to a different arrangement of apparatus.

vanometer was of the d'Arsonval type, its maximum sensitiveness being $4.10^{-8}$ amperes per millimeter deflection for the scale distance employed. A shunt box was arranged so that currents up to 800 microamperes could be measured. The gas pressures were determined by a Schaeffer and Budenberg standard gauge, whose range was 100 atmospheres and it was not checked in any way. The instruments were grouped together in such a way that all the variables could be controlled and the readings taken by one observer.

## Variations in the Discharge.

In reading over the literature of the subject, frequent references are made to the discordant results obtained, and how almost imperceptible changes in the conditions would cause most decided variations in the results, especially with the negative discharge. Naturally many such effects were met with in the course of the present work.

*Size of Cylinders and Wire.*—The first irregularity encountered was that due to the size of the cylinder and wire. With a phosphor-bronze wire, number 40, and a cylinder with an internal diameter of 0.8 cm., the currents obtained for various voltages did not follow any definite law and would not repeat themselves. When a wire of the same material but with a radius of 0.0018 cm. was used, the positive discharge gave concordant results, but the negative one was irregular. When a wire of supposedly the same diameter but made of platinum was introduced, both forms of discharge became regular, *i. e.*, if the values of the current and the corresponding differences of potential were plotted, a smooth curve was obtained. When the cylinder was replaced by another with a diameter of 1.59 cm., regular results were obtained even when the radii of the wires were varied from 0.037 to 0.001 cm.[1] A possible explanation of these results may be found in the work of Frank[2] who obtained

---

[1] The size of the wire seemed to have little effect upon the current-potential-difference curves, though, for small currents, the values arranged themselves according to the diameter, the smallest wire giving the greatest current. In passing from a number 40 wire to one having 0.037 cm. for a radius, there was a decided decrease throughout the entire range. Precht (Wied. Ann., 49, p. 150, 1893) found that, for small currents, the sharper the point the greater the current flowing, but for large currents all points had about the same effect.

[2] Frank, Wied. Ann., 21, p. 972, 1906.

evidence that ionization by collision was not confined to the immediate neighborhood of the point but occurred at some distance from it. He used a wire and cylinder whose radii were 0.005 and 0.7 cm. respectively and computed that with an electric field of 7,000 volts per centimeter the negative carriers would have suffcient energy to ionize air under atmospheric pressure at a distance of 0.64 cm. from the wire. The positive could do this only at a distance of 0.56 cm., even when the electric force was increased to 10,000 volts per cm. Thus, for the first cylinder employed in this work, the carriers, acted upon by 6,500 volts per cm., would probably reach the cylinder with sufficient energy to ionize the gas and thus change the form of the discharge. As can be seen in the case of the two wires which were supposedly of the same size but probably differed somewhat, it is evident that the dimensions of the apparatus had been unintentionally chosen so that the resulting discharge was on the boundary between a pure point discharge and some other form.

*Temperature.*—Readings obtained from day to day would rarely agree, and so some data were taken to see if different temperatures have any effect. The air in the room was cooled from 19° C. down to 13° C. and then allowed to warm slowly. Observations were made by keeping the machine at a constant voltage, and by taking the readings of the current for the different temperatures. One such series is given.

*Positive Discharge. Potential Difference* 4.14 *K.V.*

| | | | | | | |
|---|---|---|---|---|---|---|
| Temperature, | 12°.3 | 13°.3 | 14°.9 | 17°.4 | 18°.5 | 19°.2 C. |
| Deflection of galvanometer, | 6 | 5.83 | 5.75 | 5.86 | 5.75 | 5.75 cm. |

This would indicate that the effect was very slight, if any, and that the increase in temperature caused a decrease in current.

This effect of temperature has been investigated more or less and the results show a decidedly complicated relation but, in most cases, the effect is small. Warburg[1] examined the discharge in oxygen and hydrogen at room temperatures and at 175° C. The presence of ozone complicated the former, but with the latter the current was slightly decreased with increase of temperature for the

---

[1] Warburg, Drude Ann., 2, p. 295, 1900.

negative discharge. The positive was affected only to a small extent; with small differences of potential, a decrease; with large differences, an increase, with rise in temperature. Ewers[1] found that apparently the only effect temperature has is to vary the value of the minimum potential. Thus if the differences between the actual potentials and the minimum potential are considered, temperature has no effect upon the current flowing. He states that the change in minimum potential depends upon a number of factors—gas used, its pressure, etc., and there seems to be no general relation. His range in temperature was from that of the room to that of liquid air. Pringsheim investigated the effect up to 600° C. and states that a change in temperature from 20° to 230° C. caused an increase in the negative discharge of about 10 per cent., and the positive discharge showed a similar change, though much smaller. From these results it seems certain that slight changes in the temperature of the gas could not be the cause of any decided variations.

*Disruptive Discharge.*—It was very early discovered that the potential which could be applied in the case of the positive discharge was limited for high pressures because values of potential were quickly reached which would cause the passage of a spark. On the other hand, no trouble of this kind occurred with the negative discharge. This effect is clearly seen in the following data:

*Positive Discharge.*

| Pressure. | Potential Difference. | |
|---|---|---|
| 37 Atm. | 27.5 K.V. | Disruptive discharge. |
| 29 " | 24–25 " | " " |
| 21 " | 23 " | " " |
| 12 " | 24 " | No disruptive discharge. |
| 9 " | 33 " | " " " |

*Negative Discharge.*

| | | |
|---|---|---|
| 37 Atm. | 30 K. | No disruptive discharge. |
| 29 " | 26 " | " " " |

When the positive would stand such high potentials without a spark passing, as it did at 9 and 13 atmospheres, the point discharge was always well established and comparatively large cur-

---

[1] Loc. cit.

[2] Pringsheim, Annalen, 24, p. 145, 1907.

rents were flowing. The most plausible explanation of this is found in the ease with which the negative ions move. With the wire negative, the electrons would be repelled and, on account of their small inertia, would move rapidly away, leaving a preponderance of positive ions. Thus the decrease in the electric field would be even more abrupt than in the undisturbed state, and so the ionization would be limited to the neighborhood of the wire. This would aid in the establishing of the pure point discharge. In the case of the slow-moving positive ions, the effect would not be so pronounced and conditions similar to the ordinary spark discharge would arise.

*Breaking of the Wire.*—Considerable trouble was experienced in preventing the wire from breaking. This was due in part to the occasional passage of the disruptive discharge and also to the vibrations which the wire made when the current was passing. The vibrations were very rapid and seemed to be uniformly distributed in all planes. This tended to weaken the wire at its points of attachment. A wire large enough to eliminate these vibrations was tried, but it raised the potential necessary to give the required currents to such high values that it had to be discarded.[1] Finally platinum wire was used exclusively and was mounted by means of rings as shown in Fig. 1. If only moderate currents were allowed to flow, breakages were very infrequent.

*Changes in the Surfaces of the Electrodes.*—The cylinders were made of brass and, of course, tarnished somewhat after they had been in the discharge chamber for some time. Later they were silvered and then did not tarnish so quickly, but usually showed a mottled appearance. In three different cases, at atmospheric pressure, it was found that the polished cylinders gave values which were slightly lower than those given by the tarnished ones. Individual values could be found that would give the opposite of this, but when comparatively large currents were used and the discharge appeared to be normal, this result was obtained.

---

[1] Although the machine could furnish 100,000 volts, values much above 25,000 could not be used without remodeling the entire arrangement of apparatus because of the great leakage which occurred from all points of the system at these high potentials.

Fig. 3 gives a curve which shows the effect upon the discharge when the wire becomes corroded. In this case number 40 phosphor-bronze wire was used and when taken from the container was found to be covered by quite a thick layer of oxide which would obviously roughen the wire. Fig. 4 would indicate that the minimum potential was alone affected.

*Source of Electricity.*—Since the static machine furnished a constant current, it was thought that possibly the machine did not supply, at all times, sufficient current for the point discharge since the machine had to maintain as well a brush discharge used as a potential regulator. This was tested by placing a high resistance between the terminals of the machine and connecting the electrodes in parallel with a part of this resistance. Then the total current furnished by the machine was varied by changing the speed at which it was run. The maximum current used in the discharge chamber during this test was 50 microamperes and the total current furnished was varied from 150 to 300 microamperes without any effect upon the curves obtained. Consequently it is safe to conclude that the machine was capable of furnishing all the current that the discharge required. Pringsheim[1] found that the same variations were present whether he used a static machine or a high potential storage battery.

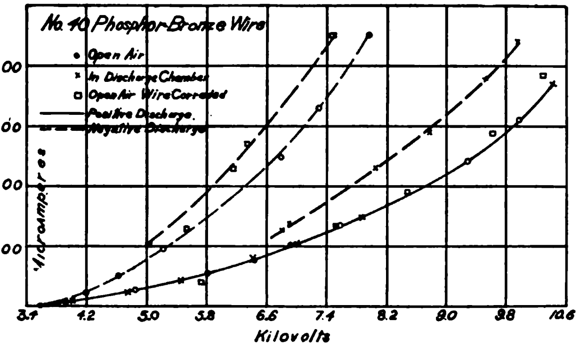


Fig. 3.

*Discharge Chamber.*—When the electrodes were introduced into

---

[1] Loc. cit.

the discharge chamber, the value of the current for a given difference of potential gradually fell for both forms of discharge. To see if this effect was due to the shielding of the electrodes, they were left in open air but were surrounded by a grounded cylinder of sheet iron, open at both ends. The current was the same as in open air. Similar results were obtained when the discharge took place in the chamber without the disc *H* (Fig. 1) in place. As soon as the chamber was completely closed, the decrease mentioned above appeared. If the disc were removed after the gas had carried the current for some time with the disc in place, values could be obtained which would lie between the two sets just described. The insulation of the rod *F* was tested and found to be good, so that could not be the reason for this variation.

It has been the experience of a number of experimenters who have worked with electrical discharges where it is necessary to enclose the electrodes, that similar phenomena frequently appear and no very good explanation could be given. Thus Tamm[1] and Zeleny[2] found that a glass vessel gave this effect most markedly, whereas vessels of cast iron or brass showed either no effect at all or else it was greatly diminished. Zeleny and Almy[3] overcame the difficulty by keeping fresh air circulating through the discharge chamber. Warburg[4] found that by a careful purifying of the gases these changes could be eliminated.

It was found that this effect could be observed with both positive and negative discharges, though occasionally the former would be unaffected as shown in Figs. 3 and 4. This would tend to indicate the effect of impurities, for ozone was always produced and it has a greater effect upon the negative than upon the positive discharge. Some attempt was made to find the dependence of this decay upon time and, as the effect was more noticeable at high pressures than at low, currents given by a definite potential at 28 atmospheres were observed every minute until a number of readings had been obtained. They did not seem to follow any definite law but gave

---

[1] Loc. cit.

[2] Zeleny, PHYS. REV., 25, p. 305.

[3] Loc. cit.

[4] Warburg, Drude Ann., 2, p. 295, 1900.

most irregular results, even occasionally increasing for a reading or two instead of decreasing. They apparently follow to a limited extent the form $i = Ke^{-at}$ but the variations were so great that it could not be said definitely that this law held.

*The Gas.*—The entire work was with air which was first passed through the purifiers of the liquid air machine and so contained very little carbon dioxide or water vapor, but some oil vapor may have been present. This air was stored under high pressure in a steel cylinder and a large quantity was obtained at one time. Thus, in taking most of the data during August the same air was used throughout, therefore the variations could not be due to the differences in the chemical constituents. A few runs were made with carbon dioxide, but it was found even more unsatisfactory than air.

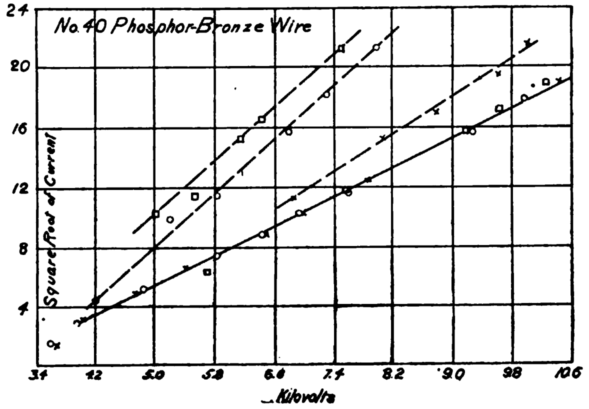


Fig. 4.

When the gas was left in the chamber for several hours without a discharge passing, it was found that the current would be less than that obtained from fresh gas, but tended to increase as the discharge was allowed to continue. This could not be due to temperature change, for fresh gas and gas which had been in the container for about fifteen minutes gave identical results. After observing this an attempt was made to get consistent results by leaving the gas in the chamber for a number of hours but without success.

Possibly the room in which the experiments were carried on may have had something to do with the trouble, for it was in the basement and had no outside windows, though there were large doors on two sides of it which opened into light airy rooms and one of these doors was open constantly. Practically all the data were taken between April and August and most of the observations were made during July and August. It was found that if the discharge took place in the open air during the spring, fairly consistent results were obtained but, when the same thing was tried in the summer, nothing of any value resulted.

Some of these variations may be due to unstable conditions in the discharge itself. For example, Chattock[1] found that, in hydrogen, the values of the velocities differed among themselves by amounts so large that they could not be attributed to experimental errors. The same thing was noticed in air, but not to so great an extent. Evidently there are a number of factors which contribute to these variations, and this fact makes it difficult to say that a particular result is due to one of these factors and not to another. Thus the lessening of the discharge on July 27 was attributed to the polished surface of the cylinder. This explanation looks plausible; but, on the other hand, no explanation can be given for the sudden increase on August 12. Thus much of the preceding discussion can be considered only as plausible explanations rather than definite effects of specific causes.

## RESULTS.

The work was almost entirely limited to finding the relation between current and impressed difference of potential, and curves of the form shown in Fig. 3 were always obtained. In Fig. 4 these same data are plotted with differences of potential as abscissæ and square roots of current as ordinates. The points determine more or less definitely a series of straight lines. This result is interesting, for Thomson[2] has developed, for the particular electrodes used in this work, an equation which would be represented by a similar

---

[1] Thomson, Conduction of Electricity through Gases, 2d ed., p. 504.

[2] Chattock, Phil. Mag., V., 48, p. 401, 1899. Chattock, Walker and Dixon, Phil. Mag., VI., 1, p. 79, 1901.

curve. It has been found that, at atmospheric pressure, this relation between current and difference of potential holds in almost every case. The exceptions are—for the negative discharge when out of the container, though the data in Fig. 4 follow this equation; also when a small static machine was used, in which case irregular results were obtained for the small currents used. As a result of the almost universal agreement, it is safe to say that Thomson's equation represents the general relation between current and potential difference when the current is larger than 10 microamperes, so in plotting the curves the square roots of the currents were always used as ordinates.

As the results for small currents did not fit the equation, the various approximations made in its development were investigated to see if they would have any effect. This departure is noticeable for currents of several microamperes and the terms neglected when such currents were used would introduce an error of less than 1 per cent.; hence their omission cannot explain the variation.

Thomson's complete equation is of the form

$$i = \frac{K}{2b^2}(V - V_0 - \alpha)^2.$$

$k$ = the velocity of the ions under unit electric force.

$a$ = radius of the wire.

$b$ = radius of the cylinder.

$V_0$ = minimum potential difference for disruptive discharge.

$$\alpha = \tfrac{1}{2}\sqrt{C}\log\frac{2C}{\frac{i}{K}a^2}\cdot$$

$C$ is a constant of integration and cannot be larger than $i/K \cdot a^2$.

By assuming the critical spark length as 0.001 cm. and the maximum intensity as 100 electrostatic units, which is really less than its true value, the numerical values of the constants can be computed for the apparatus used. The two forms of discharge give the following:

$$i^{\frac{1}{2}} = 17.3(V - 1.43) \text{ positive.}$$

$$i^{\frac{1}{2}} = 20.5(V - 1.45) \text{ negative.}$$

$\alpha$ is a function of the current and thus varies considerably. In the positive discharge, for 0.1 microampere it is equal to 1.21; for 400 microamperes, it is 1.65. In the above equations, the average values are used. The experimental values of these constants were determined for three sets of curves obtained with the same sized wire and cylinder and are as follows:

$$i^{\frac{1}{2}} = 38.7(V - 8.72) \text{ positive.}$$

$$i^{\frac{1}{2}} = 45.7(V - 8.32) \text{ negative.}$$

$$i^{\frac{1}{2}} = 45.2(V - 8.28) \text{ negative.}$$

The agreement between the two sets is not good and the discrepancies are too great to be attributed to experimental errors. Of course the electric intensity used is lower than its true value, so possibly the agreement for $\alpha$ should be better than is indicated; but no error of that kind can account for the lack of agreement in the case of the other constant. This divergence may de due to lack of symmetry in the apparatus, so that the discharge might be concentrated along certain radii or radial planes instead of being uniformly distributed. The equation is also developed on the assumption of only one kind of ions being present in the non-luminous portion, while Wilson[1] and Campbell[2] in their work on the mass of these ions have shown that ions of both signs may be present.

The work of Frank[3] offers the best explanation for the variation in the slopes of the two sets of curves. He found that if the velocity of the ion due to unit electric intensity is determined in the neighborhood of the small electrode, the values varied most decidedly among themselves and were much larger than those determined by other workers. On the other hand, when he measured this quantity at some distance from the point, he obtained results in close agreement. This quantity is assumed a constant in Thomson's equation.

Figs. 5, 6, 7 and 8 show the effect upon the point discharge of an increase in gas pressure. The values plotted in Fig. 5 are the average of a number of runs with the wire charged negatively.

---

[1] C. T. R. Wilson, Phil. Trans., A, Vol. CXCII., p. 403, 1899.

[2] Campbell, Phil. Mag., 6, 6, p. 618, 1903.

[3] Loc. cit.

As was to be expected, the increase in pressure caused a great decrease in current, which might in part be attributed to the decrease in velocity of the ions as shown by the change in the slope of the curves, and also to the much higher value of the minimum potential.

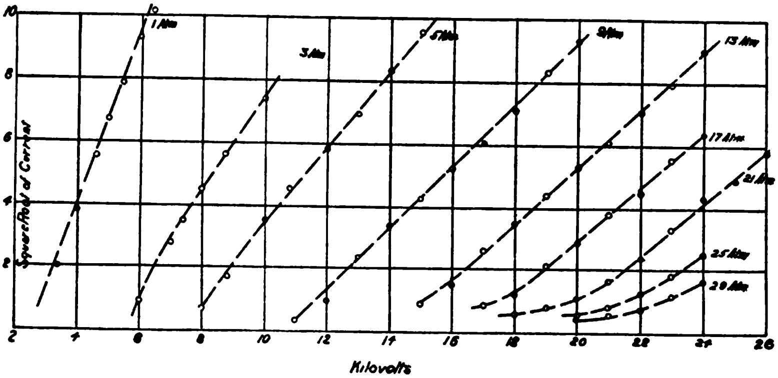


Fig. 5.

In Fig. 6 are plotted the values obtained with a phosphor-bronze wire using currents up to 400 microamperes. The irregularities seen in the last two points on the curve at 9 atmospheres are probably due to inaccuracies in the voltmeter readings. These curves show very noticeably a variation from the straight line for values less

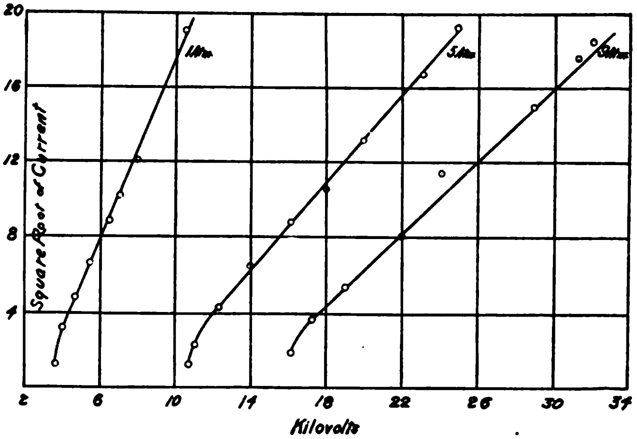


Fig. 6.

than 10 microamperes[1] and this same variation can be seen in curve 7 which indicates the effect of pressure upon both forms of discharge.[2]

It has been found that these small currents satisfy much better an equation suggested by Warburg of the form

$$i = AV(V - M).$$

$A$ is a constant and $M$ represents the minimum potential. Most observers have found that this equation seems to be followed quite generally when the point and plane are used as the electrodes.

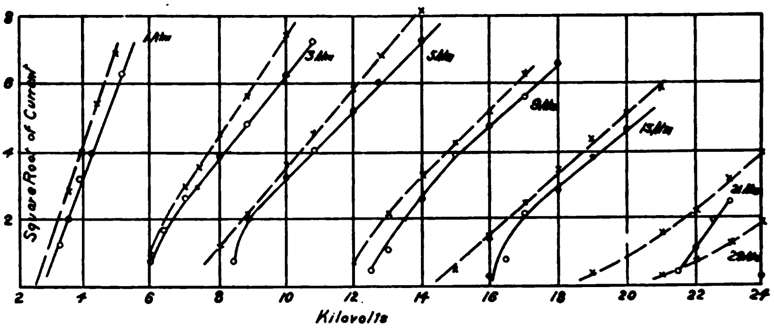


Fig. 7.

No explanation can be given why, in this work, the change from one equation to the other should appear for the small currents.

At a pressure of 17 atmospheres or more, the sign of this curvature reverses and the curves approach the $X$-axis more slowly than they should. This can be seen in Fig. 7, but is especially noticeable in Fig. 8. The data for this last set of curves were taken on August 12 and no reason is known for the unusual departure from the normal form. The general form of these curves is similar to those obtained when some agent is used to ionize a gas and the electric field between the electrodes is increased to such a strength that ionization by collision occurs. Hence it is natural to suppose that

---

[1] Curves of the same form have been obtained when the data published by other observers have been plotted with the square root of current as ordinates. Especially good examples were in the work of Tamm (loc. cit.), Zeleny (loc. cit.) and Sieveking (Ann., 1, p. 299, 1900) all of whom used the point and plane electrodes.

[2] In all the subsequent curves, the negative discharge will be represented by dotted lines and the positive, by full lines.

some other ionizing agent was present, which furnishes ions for these small currents and that the electric field had a very small part in producing them. This view is strengthened by the fact that pressures of 60 or 70 atmospheres could not entirely stop the negative current produced by a potential difference of 25,000 volts. The ions producing the current could hardly have been formed by collision, and the fact that it remained practically constant throughout wide ranges of pressures would indicate a saturation current.

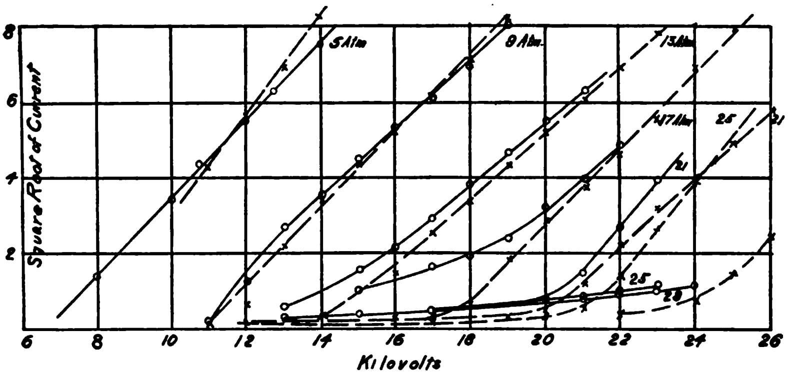


Fig. 8.

The work of Rothé and others mentioned in the early part of this paper would indicate that this effect should be more noticeable at high pressures than at low ones.

As a result of these curvatures, the minimum potential, which is characteristic of the point discharge, was not sharply defined. Other observers have usually found that the discharge began at a certain potential and then stopped at a lower one. They also mentioned that the current did not die away gradually but fell abruptly to zero. Most of them were working with galvanometers about 100 times as sensitive as the one used in this work; and the big influence machine did not work satisfactorily when run slowly enough to give the low potentials needed. Consequently none of these effects were noticed. At the high gas pressures it looks as though the minimum potential depends upon the sensitiveness of the galvanometer used, and if the true point discharge is concealed by some other discharge, then the minimum potential can have but little significance.

According to Thomson's equation, the ratio of the slopes of the curves for the different discharges should be the square root of the velocities of the two kinds of ions. Trying this for the constants in the equations given on page 732, they give as the ratio 1.37, whereas Chattock found it to be 1.36. This close agreement is a matter of chance, for other curves give values varying from 1.22 to 1.48. Though two cylinders of unequal size were used, it was impossible to see if the slopes of the lines varied inversely as the radii because the results with the small cylinder were so irregular.

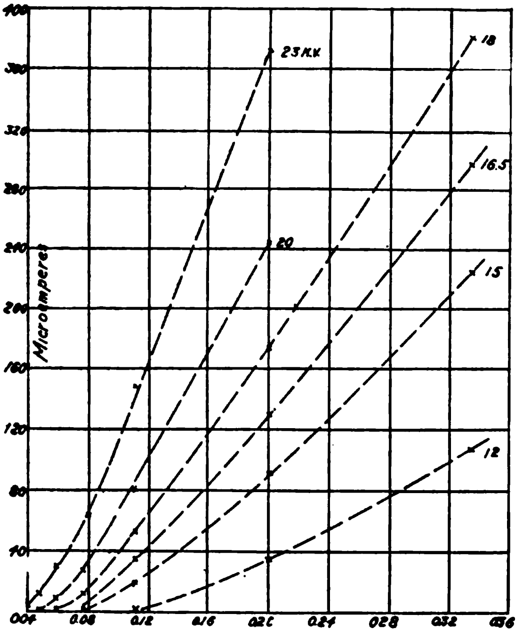


Fig. 9.

If the assumption is made that the velocity of the ions is inversely proportional to the gas pressure, then certain relations may be deduced from the equation. Thus if the voltage is kept constant, then the current must vary inversely with the pressure. This holds for large currents and rather high voltages, as shown in Fig. 9. To obtain this curve, the lines in Fig. 5 had to be exterpolated, but no values greater than 400 microamperes were used, and Fig. 6

shows that the point discharge still holds for that current. In a similar way the equation says that, for constant current, the voltage must increase as the square root of the pressure and Figs. 10 and 11 show that this condition is satisfied for large currents. Fig. 11 also indicates that the size of the wire has a very decided effect upon these curves. These figures tend to confirm the idea that the velocity of the ions vary inversely as the pressure.

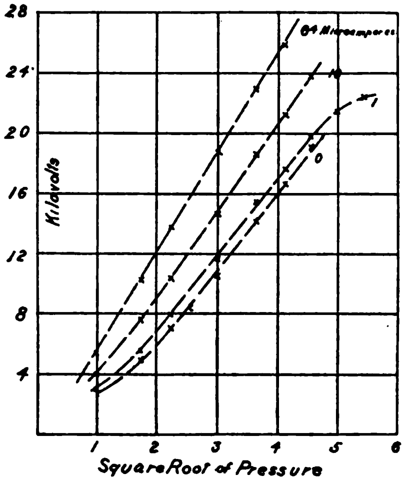


Fig. 10.

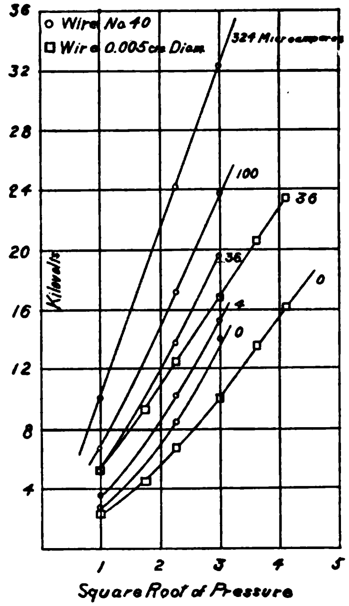


Fig. 11.

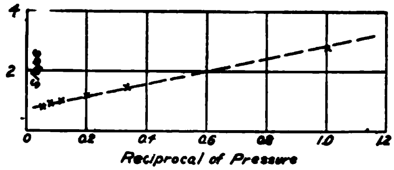


Fig. 12.

If the slopes of the lines given in Fig. 5 are plotted as ordinates and the reciprocals of pressure as abscissæ, a straight line results as shown in Fig. 12. According to Thomson's equation, this would mean that the velocity varies inversely as the square of the pressure, which is antagonistic to the results given in Figs. 9, 10 and 11, but it is in approximate agreement with Tamm's[1] work, for he found that the following equation held for pressures less than one atmosphere:

$$i_x = i_{76}\left\{\frac{76}{x} - \sqrt[3]{\frac{v}{10^5}}\log\frac{76}{x}\right\}^2.$$

## Conclusions.

1. The relation between current and impressed potential difference follows the equation

$$i^{\frac{1}{2}} = A(V - M)$$

for currents varying from 10 to 400 microamperes. For currents less than 10 microamperes, the results are irregular.

2. Pressure has a very great influence upon the true point discharge, due both to an increase in minimum potential and to a decrease in the velocity of the ions. The curves would indicate that the velocities of the positive and the negative ions become more nearly equal at high pressures, due probably to the loading down of the electron by neutral molecules.

3. It was found that the slopes of these straight lines vary inversely as the gas pressure; that, for a given current, the corresponding voltages vary as the square root of the pressures; that the current given by a constant difference of potential vary inversely as the pressure. These last two relations support the view that the velocity of the ions decrease directly as the pressure, while the first one indicates that the velocity varies inversely as the square of the pressure. From the present work there is no means of telling which is the correct relation.

4. These relationships were found to hold up to 17 atmospheres, but above that currents large enough could not be obtained by the present arrangement of apparatus to test them.

5. At the high pressures, phenomena were observed which might be explained by the presence of an ionizing agent, other than the electric field, which tended to mask the true point discharge.

---

[1] Loc. cit.

The experimental work was done in the Physical Laboratory of Cornell University during the spring and summer of 1907. The author wishes to express his appreciation of the advice and suggestions offered by the various members of the department, especially Professors Merritt and Shearer.

UNIVERSITY OF WISCONSIN,
MADISON, WIS.

# PHYSICAL PROPERTIES OF BINARY LIQUID MIXTURES.

BY J. C. HUBBARD.

## INTRODUCTION.

THE study of a solution, a salt in water, for example, is in general, complicated by electrolytic dissociation, and by the probable formation of hydrates. These two effects may more or less effectually mask other effects, which, if they exist, would cause apparent deviations from a theory of solutions which takes only the two mentioned into account. There are, however, certain binary liquid mixtures the physical properties of which present such simple and small deviations from the properties of the pure substances added together, that it may be hoped by an investigation of them to be able to trace minutely the condition of each component in the mixture, thereby gaining new insight into some of the second order effects in solutions and also, possibly, learning something of the mechanism of the liquid state itself.

For the study here summarized are chosen such simple properties as could be measured with great precision, as density and index of refraction; and, in order to secure as wide a basis of comparison as possible, it seemed best to choose for study those pairs of liquids for which the partial and total vapor pressures have already been so carefully investigated by Zawidzki.[1]

The specific volumes are given for the temperatures at which he measured the vapor pressures. In addition are given the specific volumes at 25° C. and the indices of refraction at the same temperature for the lines *C*, *D*, *F* and *G′*, thus securing data for determining the mean temperature coefficient of expansion, and for the dispersion. It is unfortunate that the indices of refraction could not also have

---

[1] J. von Zawidzki, Über die Dampfdrucke binärer Flussigkeitsgemische, Zs. phys. Chem., 35, p. 129, 1900. See also Rosanoff and Easley, Amer. Chem. Soc. Jour., XXXI., No. 9, p. 953, 1909; Zs. für phys. Chem., p. 641, 1910.

been measured for the same temperature as the vapor pressures, but, on account of the great volatility of most of the substances, it was found that such determinations required facilities not available.

The methods of obtaining results such as are given here call for nothing original in principle, but it is hoped that the procedure evolved has produced results of greater accuracy than hitherto obtained for such mixtures.

## APPARATUS.

On account of the excessive volatility of the substances studied, and of the difference, in general, of composition of liquid and vapor, it is very difficult to prevent changes of composition through evaporation. Each point of the procedure was carefully examined, and the process was finally reduced to a certain rigid, mechanical routine, which, though arbitrarily varied in detail, is capable of giving results usually reproducible to from two to five parts in one hundred thousand, depending upon the volatility of the substances.

After much preliminary work in measuring specific volumes, involving a trial of specially designed pyknometers, it was found that the most reliable results were to be secured by the use of the common specific gravity bottle with graduated neck and solid glass stopper. The bottles were each of 10 c.c. capacity; the necks were about 4 cm. in length, graduated in millimeters and of about 1.5 mm. internal diameter. The glass stoppers were ground until no loss by evaporation of any of the liquids contained in the bottles could be detected in an interval of several hours. The volume of each bottle was calibrated for about twenty points along the scale at both temperatures at which the density was to be measured. It was found that where the temperature changes to which the bottles were subjected varied in a regular cycle the calibration values could always be reproduced to $2 \times 10^{-4}$ cm.$^3$, or to two parts in one hundred thousand of the capacity of the bottle. When the bottle was to be used at a new temperature it was recalibrated. Much of interest came of the calibration of these bottles. The calibration curves finally settled down into a state very nearly independent of temperature changes between 0° and 60° C.

Two thermostats, each of five gallons capacity, regulated and heated electrically, were used in connection with the specific volume measurements. One of these was kept at 25°.00 ± .002 C. throughout the series, and the other at either 35°.17 ± .001 or 50° ± .001 to agree with Zawidzki's temperatures as the case required. The thermometers read to .001° and were compared from time to time with a standard (Baudin, calibrated at the International Bureau of Weights and Measures) loaned by Professor Webster.

The pyknometer bulbs were held in the thermostats by means of wire stages. The stopper, with a few millimeters of neck of a bulb, was allowed to protrude from the water, and over this was placed a heating coil which kept the stopper and the upper part of the neck several degrees warmer than the water in the thermostat. This was found necessary to prevent the condensation of liquid mixture in the top part of the neck, causing a lowering in the meniscus reading of the pyknometer. The mensicus reading was made by means of a mirror held below the surface of the water, obviating the necessity of disturbing the heating coil about the neck of the bulb.

From the thermostat at 25° was led a stream of water through thick walled, jacketed tubing for the temperature regulation of the Pulfrich refractometer (new construction) which was used in the measurement of the indices of refraction. The room being at about 20° made necessary a small amount of heating to compensate the slight loss by radiation. This was accomplished by passing the tubing through the core of a small rheostat which could be warmed any desired amount. By this arrangement the refractometer could be kept at 25° C. without sensible deviation. The thermometer scale of the refractometer reads to .01° C.

## PREPARING VOLATILE MIXTURES OF KNOWN COMPOSITION.

The largest error, and the one most difficult of estimation, is to be found in the determination of the composition of the mixture. As this subject seems to have received very little attention it may be worth while to enter into it here, the object being to show that large systematic errors may be produced unless proper precautions are taken.

In the first place, all weighings must be reduced to vacuo, otherwise there will be a systematic error in the "per cent. composition," amounting in some cases to 0.1 per cent, due to the difference of density of the two components.

A quantity $a$ of liquid $A$ is first weighed out in the weighing bottle, then a quantity $b$ of the second liquid $B$ is poured into the bottle and the total amount, $a + b$, is determined. The proportion of $A$ in the mixture is then supposed to be $z = a/(a + b)$. If a series of observations be obtained, say of the deviation from the additive law of specific volume for the mixture $A - B$, as

$$\Delta v = v - v_C = v - [zv_A + (1 - z)v_B]$$

(where $v$ is the observed, and $v_C$ the calculated specific volume), $z$ being determined as described above, a smooth curve for $\Delta v$ and $z$ may be obtained. If, now, a second series of observations be made, always weighing out first the liquid $B$, a smooth curve is again obtained, but this curve will not coincide with the first. The discrepancy is caused by the evaporation of the first liquid when the cover of the weighing bottle is removed to add the second liquid. The numerator of the composition fraction for the liquid first weighed out will be too large.

The error due to this cause may be brought down to the other errors of the determination by subtracting from the weight of the first component a quantity determined by many trials as a suitable evaporation correction. This number is obtained by placing a quantity of the liquid $A$, for example, in the weighing bottle and making repeated weighings, between each weighing removing the stopper in the same manner and for the same length of time (five seconds) as was done when actually making up the mixture. The amount of evaporation so measured is surprisingly constant. Let it be $\Delta a$; then, instead of $z = a/(a + b)$, we have $z' = (a - \Delta a)/w$ where $w$ is the total weight. In these experiments $a$ had values from 6 in a very few cases to 60 gr., and $\Delta a$ varied from .0006 gr. in the case of acetic acid to .018 gr. in the case of carbon disulphide; the calculated fraction $z$ thus having a correction of from 1 in $10^5$ to 1 in 300, usually however about 1 in $10^4$. When the correction is applied the above mentioned discrepancy disappears.

One example is given here (Table I., Fig. 1) to show the manner of applying the evaporation correction.

TABLE I.[1]

*Evaporation Correction for Carbon Disulphide-Methylal.* 25° *C.*

$\Delta a$ ($CS_2$) = .018 ± .001 *gr.* $v_1$ ($CS_2$) = .79621.
$\Delta b$ (*Meth.*) = .017 ± .001 *gr.* $v_2$ (*Meth.*) = 1.17233.

| Wt. $CS_2$ | Wt. Meth. | Total. | % $CS_2$ | $\Delta v$ | % Corr. | Corr. to $\Delta v$. |
|---|---|---|---|---|---|---|
| | 11.318 | 12.978 | 12.793 | .00525 | +.128 | +.00049 |
| | 10.001 | 13.999 | 28.560 | 993 | +.122 | + 43 |
| | 8.963 | 14.525 | 38.290 | 1129 | +.118 | + 45 |
| 6.506 | | 14.885 | 43.708 | 1248 | −.120 | − 45 |
| | 8.029 | 14.736 | 45.513 | 1188 | +.117 | + 44 |
| | 7.187 | 15.300 | 53.026 | 1179 | +.113 | + 41 |
| | 6.224 | 15.531 | 59.92 | 1130 | +.110 | + 43 |
| | 6.024 | 15.892 | 61.12 | 1097 | +.109 | + 41 |
| 9.862 | | 15.781 | 62.493 | 1189 | −.113 | − 42 |
| 10.542 | | 16.126 | 65.371 | 1157 | −.109 | − 41 |
| 12.544 | | 16.670 | 75.251 | 966 | −.110 | − 42 |
| 15.087 | | 17.431 | 86.553 | 632 | −.104 | − 38 |

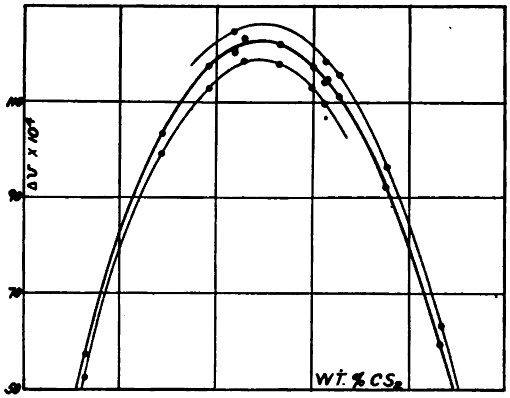


Fig. 1.

This case represents the greatest extreme, both in volatility and in the small quantity of liquid used, giving by far the largest corrections obtained in any of the results.

The sixth column of the table shows the corrections obtained for the percentage composition, while the last column gives the corresponding corrections to $\Delta v$. When the carbon disulphide is weighed

---

[1] For complete data see Table II. All the weights given in the table were made to 0.1 mmg. but as the evaporation correction could not be determined so accurately the weights are given only to the nearest milligram. All weights reduced to vacuo.

out first the percentage number is too great, and since the specific volume of $CS_2$ is smaller than that of methylal the calculated volume is too small, making the deviations of the specific volume from the additive law too large. The reverse is true when methylal is first weighed out; the deviations are too small. It will be seen that two curves are obtained which are brought together into one (the heavy curve of the figure) when the corrections are applied. The consideration of these systematic errors, due to errors in the percentage composition, applies, not only to densities, but to all other properties as well, and it is unfortunate that so little care has thus far been bestowed upon it.

## OPERATION.

When a given mixture has been made up it must be thoroughly shaken, and this must be done without wetting the space between the rim and the stopper of the weighing bottle, in order to insure all of each component being present. (Much time was lost in the beginning in locating errors finally found to be due to imperfect mixing, or to the loss of liquid in the rim space before mixing had become complete.) The stopper is then quickly removed, about 15 c.c. of the mixture is withdrawn by gentle suction from the bottom of the bottle into a slender stemmed pipette of just sufficient capacity. The pipette tube is then inserted into the pyknometer bulb to the bottom and the mixture is forced out gently until the bottle is full. Some of the liquid is allowed to run over; about 3 c.c. are left in the transferring tube. This procedure insures that very little of the liquid now in the pyknometer has been exposed to air, and until it was adopted it was found impossible to get concordant results. Some of the liquid is next removed from the graduated neck by inserting a capillary tube connected to a suction pump, and the exposed interior of the neck is dried. The stopper is inserted, and the bulb is placed in position in the thermostat at the lower temperature. When the meniscus reading has become constant the bulb and contents are weighed, more liquid is removed from the neck, as before, and the bulb is placed in the thermostat at the higher temperature, when similar observations are made. These observations give the specific volumes at the two temperatures.

The mixture is next brought to the refractometer temperature by placing the bulb in the first thermostat, after which three successive samples are taken from the bottom of the bulb for the determination of the indices of refraction with the sodium and three principal hydrogen lines. About 90 minutes are necessary for the completion of all the observations of a mixture of given composition.

## THE "ADDITIVE LAWS." NOTATION.

*Vapor Pressure.*—When the vapor pressure of pairs of liquids are plotted against the molecular proportion of one of the components, three types of curves are obtained. The simplest is a straight line, such as would be expected if the components exerted neither physical nor chemical influence on each other, but mixed in all proportions. Besides this curve are curves convex toward the axis, and curves concave toward the axis. The former are conceivably caused by a lessening of the number of evaporating molecules, on account of a union or by increased cohesion; the latter by an increased number of evaporating molecules, possibly caused by dissociation of polymerized molecules, or by a lessening of cohesion due to the mixing.[1] In any case the deviation from the straight line serves as a valuable criterion for the classification of binary mixtures, and, taken in connection with the deviation of other properties, may be expected to throw some light on the theory.

We shall consider the deviations from the simple additive law.

$$p_c = xp_1 + (1 - x)p_2,$$

where $x$ is the number of mols of $A$ in one mol of mixture, $p_1$ and $p_2$ being the respective vapor pressures of $A$ and $B$ in the pure state.

$$\Delta p = p - p_c, \tag{1}$$

where $p$ is the observed total pressure.

*Specific Volume.*—Let $v_1$ and $v_2$ be the respective specific volumes of the substances $A$ and $B$, and $v_c$ the calculated specific volume of the mixture, and let $w_1$ and $w_2$ be the weights of the two substances in the mixture. Then, assuming that there is no change

---

[1] See Zawidzki, loc. cit., p. 157 et seq.; also Ostwald, Allg. Chemie, II., p. 617; Dolezalek, Zeit. f. phys. Chem., 64, p. 727, 1908.

of volume on mixing, the total calculated volume is

$$V_c = w_1 v_1 + w_2 v_2 = (w_1 + w_2) v_c$$

whence

$$v_c = z v_1 + (1 - z) v_2$$

where $z$ is the weight of $A$ in one gram of the mixture. If $v$ is the observed specific volume of the mixture then

$$\Delta v = v - v_c \tag{2}$$

gives the deviation of the volume from the additive law.

*Indices of Refraction.*—The theory of dispersion for liquids is not as yet in a satisfactory state. The most common expression for the refractivity $R = N/d = (n^2 - 1)/(n^2 + 2)d$ (Lorenz-Lorentz), has strong theoretical support, and preserves a fair constancy even for change of state. Planck's theory gives the same expression with the assumption of a single absorption band for the substance, but does not lead to a simple expression for the refractivity when there is more than one absorption band.[1]

The well-known additive law

$$R_c = z R_1 + (1 - z) R_2, \tag{3}$$

where $z$ is the weight of $A$ per gram of the mixture, has been experimentally studied by a great number of observers, usually without much question as to whether the liquids investigated followed the dispersion formulæ. As, however, deviations from (3) tend to show the need, and perhaps indicate the direction, of further investigations both into the theory of dispersion and into the dispersion of particular liquids, such work has a distinct value.

The deviation of refractivity is

$$\Delta R = R - R_c. \tag{4}$$

If we multiply (3) by $d_c$, the calculated density, remembering that $R = N/d$, we have

$$N_c = y N_1 + (1 - y) N_2, \tag{5}$$

$y$ being the volume of $A$ in 1 c.c. of mixture.

---

[1] On the dispersion of liquids see also A. Trowbridge, Phys. Rev., XXVIII., p. 389, 1909.

Pulfrich gives the empirical equation

$$\frac{N - N_c}{N} = \alpha \frac{d - d_c}{d} = \alpha c = \alpha \frac{v_c - v}{v_c}, \tag{6}$$

or

$$\alpha = -\frac{\Delta N}{N} \Big/ \frac{\Delta v}{v_c}, \tag{7}$$

in which $\alpha$ is supposed to be constant, and $N = (n^2 - 1)/(n^2 + 2)$. Eq. (6) leads to the expression

$$R\frac{1 - \alpha c}{1 - c} = R_c = zR_1 + (1 - z)R_2,$$

which is Pulfrich's modification of the additive law.[1] We also have from (6)

$$\Delta R = (\alpha - 1)(v_c - v)N = -(\alpha - 1)N\Delta v. \tag{8}$$

In the following tables the values of $\alpha$ are calculated from (7), and $\Delta R$ from (8) except in two cases, where $\Delta v$ being too small, $\Delta R$ is calculated from (4).

## THE SUBSTANCES.

The substances benzol, ethyl acetate, ethyl iodide, carbon tetrachloride, acetic acid, chloroform, acetone, methylal and carbon disulphide were generously furnished by the Department of Chemistry of Clark University, and I am indebted to Mr. C. W. Bacon, a graduate student of that department, for carrying out their purification. It was obviously important to have had the substances in a state of high purity. The methods of purification, as carried out by Mr. Bacon, will be given in connection with each substance.

## RESULTS.

In the case of each mixture are given three tables. In the first table of each set, besides the proportions by weight of one of the substances, are given in order the values of $v_{25}$ and $v_{35.17}$, or $v_{50}$ as the case may be, and $n_C$, $n_D$, $n_F$ and $n_{G'}$. In the second table are given the deviations of specific volume $\Delta v_{25}$, $\Delta v_{35.17}$ or $\Delta v_{50}$, and the deviation of refractivity for the $D$ line, $\Delta R_D$, beside the constants

---

[1] Pulfrich, Zeit. f. phys. Chem., 4, 561, 1889; see also Winkelmann, Handbuch, 6, p. 650.

$\alpha$, of the Pulfrich equation, for each of the wave-lengths. In the last table of each set are given the molecular and weight proportions of the mixtures and the total vapor pressures as given by Zawidzki (*loc. cit.*), and the deviation $\Delta p$ of these total pressures from the simple additive law. Finally for each set of results has been drawn a figure showing the deviations of pressure, specific volume and refractivity plotted as functions of the weight composition of the mixture.

### I. *Carbon Disulphide—Methylal.*

The carbon disulphide was washed with 5 per cent. sodium hydroxide solution and set away over quicklime for twenty-four hours being occasionally shaken. It was then poured into a bottle containing mercury and shaken frequently until the disagreeable odor had disappeared, decanted from mercury and distilled.

The fraction was collected boiling within 0°.2 C.

The methylal was dried with $CaCl_2$ and distilled over within a small fraction of a degree. The sample chosen came over within 0°.2 C.

The evaporation correction for $CS_2$ is .018 ± .001 gr. and for methylal .017 ± .001 gr. The total amount of mixture was about 15 gr.; thus the error in the weight composition is about $7 \times 10^{-5}$.

*Remarks on Tables II., III. and IV.*—This was the first set of observations completed, and as the temperature regulation of the refractometer had not been made perfect, readings for the same samples varied as much as 2′, which corresponds to about 2 units in the fourth decimal place of the index of refraction. As every determination in the table is a mean of at least three, the probable error is considerably less. The asterisk (*) after specific volume determinations indicates a slight amount of boiling in the pyknometer during the measurement at the higher temperature. The disturbance affected the meniscus to the extent of 1 division, introducing an uncertainty of 3 in $10^{-4}$ in the four results marked. The other volume errors are subject to the reading errors already mentioned of $2 \times 10^{-5}$. The composition error of $7 \times 10^{-5}$ produces uncertainty in the values of $v$ of $3 \times 10^{-5}$.

The value of $\alpha$ diminishes with the amount of $CS_2$ present, and with diminishing wave-length.

## TABLE II.

*Carbon Disulphide—Methylal.*

| No. | $x$(wt)$CS_2$ | $v_{25}$ | $v_{35.17}$ | $n_C$ | $n_D$ | $n_F$ | $n_{G'}$ |
|---|---|---|---|---|---|---|---|
| | 0.00000 | [1.17233] | 1.19043 | 1.34850 | 1.35018 | 1.35425 | 1.35759 |
| | 0.00000 | 1.17214 | 1.19044 | 1.34845 | 1.35018 | 1.35420 | 1.35759 |
| | 0.00000 | 1.17216 | 1.19043 | 1.34916 | 1.35078 | 1.35489 | 1.35832 |
| | Mean. | 1.17215 | 1.19043 | 1.34870 | 1.35038 | 1.35444 | 1.35783 |
| 1 | .12921 | 1.12932 | 1.14685 | 1.36939 | 1.37164 | 1.37730 | 1.38206 |
| 2 | .28682 | 1.07469 | 1.09108 | 1.39871 | 1.40178 | 1.40954 | 1.41639 |
| 3 | .38408 | 1.03949 | 1.05500* | 1.41928 | 1.42294 | 1.43220 | 1.44037 |
| 4 | .43588 | 1.02031 | 1.03538 | 1.43107 | 1.43502 | 1.44524 | 1.45440 |
| 5 | .45630 | 1.01293 | 1.02807 | 1.43595 | 1.44005 | 1.45054 | 1.45986 |
| 6 | .53139 | .98458 | .99905 | 1.45459 | 1.45922 | 1.47115 | 1.48179 |
| 7 | .60034 | .95819 | .97243* | 1.47286 | 1.47810 | 1.49146 | 1.50325 |
| 8 | .62226 | .94959 | .96319* | 1.47937 | 1.48471 | 1.49857 | 1.51091 |
| 9 | .62380 | .94911 | .96265 | 1.47964 | 1.48497 | 1.49889 | 1.51123 |
| 10 | .65262 | .93797 | .95114* | 1.48798 | 1.49357 | 1.50815 | 1.52061 |
| 11 | .75141 | .89891 | .91140 | 1.51859 | 1.52508 | 1.54205 | ——— |
| 12 | .86449 | .85309 | .86443 | 1.55901 | 1.56665 | 1.58690 | 1.60505 |
| | 1.00000 | .79620 | .80611 | | | | |
| | 1.00000 | .79623 | .80611 | | | | |
| | 1.00000 | .79621 | .80609 | | | | |
| | Mean. | .79621 | .80610 | 1.61440[1] | 1.62357 | 1.64830 | 1.67073 |

[1] It was impossible to get accurate readings for $CS_2$ in the refractometer. The values are reduced from Landolt & Börnstein's tables.

## TABLE III.

*Carbon Disulphide—Methylal.*

| No. | $\Delta v_{25}$ | $\Delta v_{35\ 17}$ | $\Delta R_{D25}$ | $a_C$ | $a_D$ | $a_F$ | $a_{G'}$ |
|---|---|---|---|---|---|---|---|
| 1 | .00574 | .00607 | .00030 | .773 | .767 | .734 | .742 |
| 2 | 1036 | 1088 | 91 | .650 | .638 | .630 | .566 |
| 3 | 1174 | 1219 | 126 | .595 | .579 | .574 | .561 |
| 4 | 1203 | 1248 | 136 | .576 | .565 | .545 | .503 |
| 5 | 1232 | 1301 | 151 | .548 | .534 | .529 | .515 |
| 6 | 1220 | 1285 | 166 | .519 | .503 | .492 | .472 |
| 7 | 1173 | 1273 | 170 | .518 | .488 | .472 | .472 |
| 8 | 1138 | 1189 | 161 | .463 | .444 | .425 | .410 |
| 9 | 1147 | 1196 | 175 | .483 | .467 | .449 | .432 |
| 10 | 1116 | 1152 | 179 | .469 | .448 | .424 | .475 |
| 11 | 924 | 976 | 163 | .452 | .425 | .400 | —— |
| 12 | 594 | 626 | 143 | .293 | .261 | .221 | .204 |

## TABLE IV.

*Carbon Disulphide—Methylal (Zawidzki).* 35°.17 C.

| $z$ and $x$ CS$_2$ | $p_{35.17}$ | $\Delta p$ | $z$ and $x$ CS$_2$ | $p_{35.17}$ | $\Delta p$ |
|---|---|---|---|---|---|
| 0.0000 | 587.7 | 0.0 | .6060 | 695.5 | 152.1 |
| .0496 | 613.3 | 29.2 | .6803 | 685.4 | 147.5 |
| .1044 | 638.9 | 58.9 | .7353 | 675.4 | 141.5 |
| .1651 | 660.1 | 84.5 | .7927 | 661.6 | 132.0 |
| .2719 | 686.0 | 118.2 | .8421 | 644.6 | 118.5 |
| .3480 | 696.7 | 134.5 | .8573 | 634.7 | 109.7 |
| .3904 | 700.4 | 141.2 | .9130 | 603.2 | 82.4 |
| .4542 | 703.2 | 148.7 | .9576 | 564.2 | 46.6 |
| .4942 | 701.4 | 149.8 | 1.0000 | 514.5 | 0.0 |
| .5377 | 700.3 | 152.0 | | | |

The values of $\Delta p_{35.17}$, $\Delta v_{25}$, $\Delta v_{35.17}$ and $\Delta R_{D25}$ are plotted in Fig. 2.

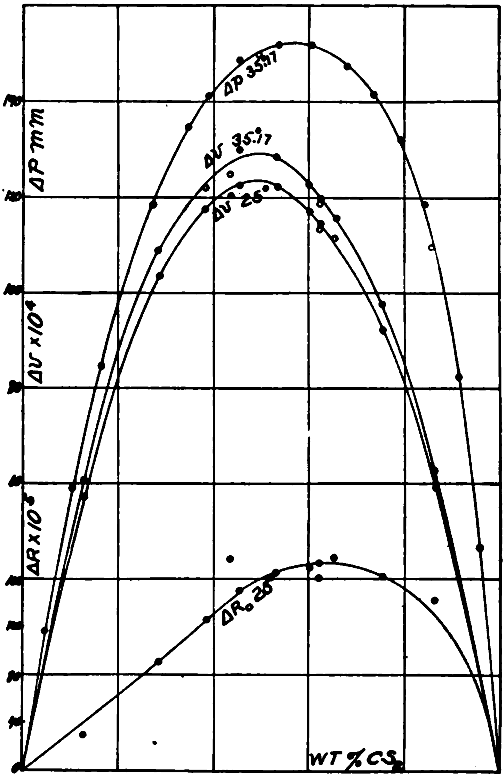


Fig. 2.

### II. *Carbon Disulphide—Acetone.*

The same $CS_2$ was used as in the last case. The acetone was mixed with 6 per cent. solid $KMnO_4$, boiled in a reflux condenser over a water bath for three hours, distilled off rapidly, dried over freshly ignited $K_2CO_3$ and distilled in the usual manner. Boiled at 56°.0 to 56°.3 C.

Evaporation correction for $CS_2$ = .018 ± .001 gr.; for acetone, .010 ± .001 gr. Corresponding probable error in the weight composition is $7 \times 10^{-5}$.

Table V.

*Carbon Disulphide—Acetone.*

| No. | $z$(wt)$CS_2$. | $v_{25}$ | $v_{35.17}$ | $n_C$ | $n_D$ | $n_F$ | $n_{G'}$ |
|---|---|---|---|---|---|---|---|
| | 0.00000 | [1.26913] | [1.28810] | 1.35450 | 1.35636 | 1.36112 | 1.36509 |
| | 0.00000 | 1.26906 | 1.28824 | 1.35450 | 1.35636 | 1.36112 | 1.36509 |
| | 0.00000 | 1:26903 | 1.28821 | 1.35475 | 1.35662 | 1.36140 | 1.36540 |
| | Mean. | 1.26905 | 1.28823 | 1.35458 | 1.35645 | 1.36121 | 1.36519 |
| 1 | .13245 | 1.21357 | 1.23204 | 1.37333 | 1.37576 | 1.38177 | 1.38701 |
| 2 | .29326 | 1.14319 | 1.16040 | 1.40047 | 1.40364 | 1.41171 | 1.41887 |
| 3 | .40329 | 1.09304 | 1.10938 | 1.42182 | 1.42567 | 1.43533 | 1.44390 |
| 4 | .51761 | 1.03962 | 1.05502 | 1.44776 | 1.45253 | 1.46387 | 1.47471 |
| 5 | .51799 | 1.03937 | 1.05478 | 1.44794 | 1.45253 | 1.46417 | 1.47522 |
| 6 | .51902 | 1.03847 | 1.05389 | 1.44819 | 1.45271 | 1.46439 | 1.47471 |
| 7 | .64733 | .97694 | .99106 | ——— | ——— | ——— | ——— |
| 8 | .71137 | .94547 | .95867 | 1.50093 | 1.50692 | 1.52263 | 1.53661 |
| 9 | .83283 | .88407 | .89605 | 1.54325 | 1.55043 | 1.56942 | 1.58644 |
| 10 | .86892 | .86551 | .87711 | 1.55702 | 1.56465 | 1.58477 | 1.60291 |
| | 1.00000 | .79620 | [.80590] | | | | |
| | 1.00000 | .79623 | .80609 | | | | |
| | 1.00000 | .79619 | .80611 | | | | |
| | Mean. | .79621 | .80610 | 1.61440 | 1.62357 | 1.64830 | 1.67073 |

*Remarks on Tables V., VI. and VII.*—Owing to the uncertainty of the evaporation correction to .001 gr. the compositions $z$ (V., VI.) are again subject to an error of $7 \times 10^{-5}$ which makes the calculated volumes and $\Delta v$ subject to an error of $3 \times 10^{-5}$. The observed volumes are measured to $2 \times 10^{-5}$. The indices are measured to the errors of the instrument (see "Operation"). The values of the deviations are plotted with the proportions by weight in Fig. 3.

### III. *Chloroform—Acetone.*

The acetone was purified as in the previous case. The chloroform was shaken with 5 per cent. NaOH solution, separated and shaken three times with distilled water. It was next dried over $CaCl_2$ for twenty-four hours, filtered and distilled. The distillate smelled

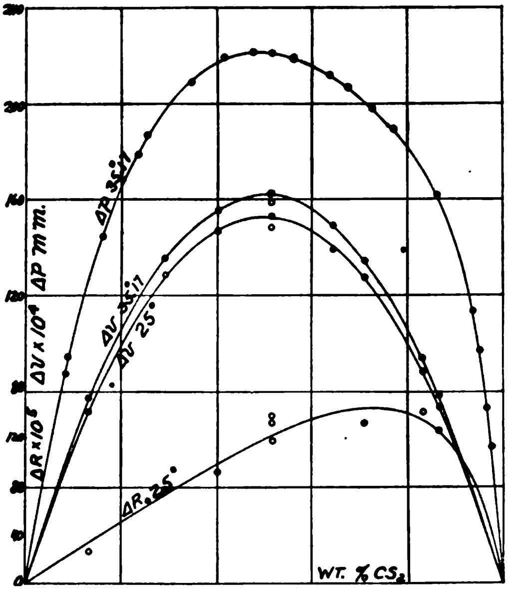


Fig. 3.

## Table VI.

*Carbon Disulphide—Acetone.*

| No. | $\Delta v_{25}$ | $\Delta v_{35.17}$ | $\Delta R_{D25}$ | $a_C$ | $a_D$ | $a_F$ | $a_{G'}$ |
|---|---|---|---|---|---|---|---|
| 1 | .00715 | .00767 | .00027 | .855 | .836 | .895 | .899 |
| 2 | 1280 | 1356 | 77 | .762 | .755 | .766 | .747 |
| 3 | 1469 | 1559 | 92 | .779 | .755 | .764 | .755 |
| 4 | 1531 | 1633 | 140 | .710 | .662 | .698 | .626 |
| 5 | 1525 | 1628 | 133 | .700 | .676 | .675 | .579 |
| 6 | 1483 | 1589 | 118 | .720 | .705 | .700 | .696 |
| 7 | 1397 | 1493 | — | — | — | — | — |
| 8 | 1278 | 1341 | 133 | .666 | .650 | .640 | .637 |
| 9 | 881 | 936 | 141 | .516 | .497 | .485 | .479 |
| 10 | 732 | 781 | 124 | .508 | .477 | .458 | .438 |

### Table VII.

*Carbon Disulphide—Aceton (Zawidzki).* 35°.17 *C.*

| $x$CS$_2$ | $z$CS$_2$ | $p_{35.17}$ | $\Delta p$ | $x$CS$_2$ | $z$CS$_2$ | $p_{35.17}$ | $\Delta p$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 343.8 | 0.0 | .5716 | .6363 | 652.2 | 212.1 |
| 0.0624 | .0802 | 441.7 | 87.4 | .6144 | .6760 | 653.7 | 206.4 |
| .0690 | .0885 | 449.6 | 94.2 | .6713 | .7278 | 655.0 | 198.1 |
| .1271 | .1602 | 510.0 | 144.8 | .7208 | .7720 | 654.6 | 189.4 |
| .1857 | .2323 | 553.8 | 178.7 | .8280 | .8630 | 645.1 | 161.8 |
| .2038 | .2511 | 564.9 | 186.8 | .9216 | .9391 | 612.2 | 113.1 |
| .2815 | .3392 | 600.7 | 209.5 | .9378 | .9520 | 598.4 | 96.6 |
| .3526 | .4165 | 622.8 | 219.6 | .9584 | .9679 | 578.1 | 72.8 |
| .4100 | .4766 | 634.6 | 221.7 | .9692 | .9764 | 564.0 | 56.9 |
| .4502 | .5176 | 641.2 | 221.5 | 1.0000 | 1.0000 | 512.3 | 0.0 |
| .4953 | .5625 | 646.1 | 218.9 | | | | |

strongly of phosphine and also gave the test for chlorine. The whole material was then shaken up with NaOH, washed with water and dried as before. It gave no test for chlorine and after filtration was used without further distillation. Boiling point 61°.2 C. Evaporation corrections: Chloroform, .015 ± .001 gr.; acetone, .010 ± .001 gr.

### Table VIII.

*Chloroform—Acetone.*

| No. | $z$ Chl | $v_{25}$ | $v_{35.17}$ | $n_C$ | $n_D$ | $n_F$ | $n_{G'}$ |
|---|---|---|---|---|---|---|---|
| | 0.00000 | 1.27403 | 1.29334 | 1.35436 | 1.35620 | 1.36096 | 1.36495 |
| | 0.00000 | 1.27401 | 1.29330 | 1.35436 | 1.35621 | 1.36087 | 1.36483 |
| | 0.00000 | 1.27402 | 1.29325 | 1.35434 | 1.35621 | 1.36091 | 1.36474 |
| | Mean. | 1.27402 | 1.29330 | 1.35435 | 1.35621 | 1.36091 | 1.36484 |
| 1 | .16664 | 1.17445 | 1.19201 | 1.36261 | 1.36454 | 1.36941 | 1.37344 |
| 2 | .33039 | 1.07637 | 1.09208 | 1.37211 | 1.37417 | 1.37920 | 1.38334 |
| 3 | .45832 | .99950 | 1.01378 | 1.38122 | 1.38332 | 1.38848 | 1.39275 |
| 4 | .56298 | .93643 | .94957 | 1.38982 | 1.39196 | 1.39728 | 1.40171 |
| 5 | .64507 | .88682 | .89911 | 1.39721 | 1.39943 | 1.40494 | 1.40949 |
| 6 | .72276 | .83976 | .85122 | 1.40531 | 1.40757 | 1.41319 | 1.41790 |
| 7 | .79884 | .79411 | .80483 | 1.41385 | 1.41618 | 1.42196 | 1.42676 |
| 8 | .88409 | .74349 | .75348 | 1.42451 | 1.42692 | 1.43287 | 1.43784 |
| 9 | .94292 | .70889 | .71839 | 1.43237 | 1.43487 | 1.44097 | 1.44605 |
| | 1.00000 | .67573 | .68472 | 1.44042 | 1.44296 | 1.44919 | 1.45444 |
| | 1.00000 | .67570 | .68470 | 1.44063 | 1.44316 | 1.44939 | 1.45460 |
| | 1.00000 | .67570 | .68474 | 1.44064 | 1.44316 | 1.44939 | 1.45462 |
| | Mean. | .67571 | .68472 | 1.44057 | 1.44309 | 1.44932 | 1.45455 |

## TABLE IX.

*Chloroform—Acetone.*

| No. | $\Delta v_{25}$ | $\Delta v_{35.17}$ | $\Delta R_{D25}$ | $a_C$ | $a_D$ | $a_F$ | $a_{G'}$ |
|---|---|---|---|---|---|---|---|
| 1 | +.00013 | +.00013 | +.00017 | —— | —— | —— | —— |
| 2 | + 2 | − 15 | + 22 | —— | —— | —— | —— |
| 3 | − 30 | − 59 | + 34 | 5.4 | 5.8 | 5.8 | 5.4 |
| 4 | − 75 | − 111 | + 38 | 3.13 | 3.17 | 3.18 | 3.12 |
| 5 | − 125 | − 161 | + 28 | 1.89 | 1.94 | 2.04 | 2.02 |
| 6 | − 183 | − 222 | + 26 | 1.57 | 1.58 | 1.62 | 1.64 |
| 7 | − 195 | − 231 | + 20 | 1.39 | 1.42 | 1.45 | 1.44 |
| 8 | − 157 | − 178 | + 17 | 1.42 | 1.42 | 1.45 | 1.44 |
| 9 | − 97 | − 107 | + 9 | 1.29 | 1.36 | 1.38 | 1.36 |

## TABLE X.

*Chloroform—Acetone (Zawidzki).* 35°.17 *C.*

| $x$ Chl. | $z$ Chl. | $p_{35.17}$ | $\Delta p$ | $x$ Chl. | $z$ Chl. | $p_{35.17}$ | $\Delta p$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 344.5 | 0.0 | .4934 | .6672 | 255.5 | −63.6 |
| .0595 | .1152 | 332.2 | − 9.2 | .5083 | .6802 | 252.9 | −65.5 |
| .1217 | .2218 | 319.9 | −18.4 | .5135 | .6847 | 252.7 | −65.4 |
| .1835 | .3162 | 307.6 | −27.5 | .5523 | .7174 | 250.1 | −66.0 |
| .2630 | .4234 | 291.0 | −40.0 | .5812 | .7407 | 248.4 | −66.2 |
| .2897 | .4563 | 285.9 | −43.7 | .6622 | .8014 | 249.0 | −61.5 |
| .3613 | .5379 | 272.9 | −53.0 | .8022 | .8930 | 262.2 | −41.0 |
| .4008 | .5792 | 267.1 | −56.8 | .9177 | .9583 | 279.8 | −17.6 |
| .4240 | .6023 | 263.1 | −59.6 | 1.0000 | 1.0000 | 293.1 | 0.0 |

*Remarks on Tables VIII., IX. and X.*—A great deal of trouble was experienced in getting the results for this case. The acetone apparently absorbed moisture from the air every time the bottle was opened. The observations were repeated three times before satisfactory results were obtained. The results given are from the last set which alone are retained because they were made with acetone kept in several small bottles and also because great care was taken to thoroughly mix the liquids. These precautions seem to have entirely eliminated irregularities greater than ordinary errors of observation from the results. The error of .001 gr. in the evaporation correction implies an error of $3 \times 10^{-5}$ in $z$, since the amount of mixture made up in each case was never less than 30 gr. This implies an error well under the errors of observation for $\Delta v$ and $\Delta R$.

The errors in $n_D$ are about as before, but the agreement for differences of the indices for the different wave-lengths in any given case was excellent, seldom amounting to more than 3 or $4 \times 10^{-5}$. The deviations for this case are plotted in Fig. 4.

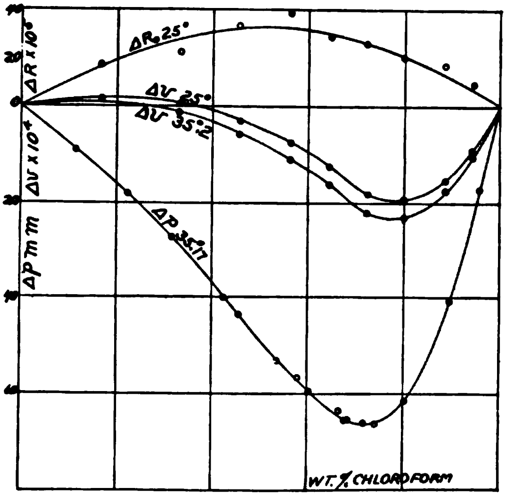


Fig. 4.

### IV. *Ethyl Iodide — Ethyl Acetate.*

The ethyl iodide was washed with water to remove alcohol, dried over $CaCl_2$ with molecular silver, filtered and distilled in the usual way. It was kept in a black bottle containing a little silver — boiling point 72°.2 C. The ethyl acetate was dried with freshly ignited $K_2CO_3$, filtered and distilled with a micro-burner in the usual manner. Boiling point 77°.2 C. The measurements were carried out at 25° and at 50° C., the latter being sufficiently near Zadwizki's temperature (49°.99) for the pressure measurements.

Evaporation corrections: Ethyl iodide, .0075 ± .0005 gr.; ethyl acetate, .0042 ± .0005 gr.

*Remarks on Tables XI.. XII. and XIII.*—The very great reluctance of ethyl iodide and ethyl acetate to mix is noteworthy. The smallest amount of any one mixture was 30 gr. The evaporation correction error of $5 \times 10^{-4}$ gr. implies an error in the composition of less than $2 \times 10^{-5}$, which is negligible in calculating $\Delta v$ and $\Delta R$. The remarks in the preceding case concerning indices hold

## TABLE XI.

*Ethyl Iodide—Ethyl Acetate.*

| No. | $x$ Iod. | $v_{25}$ | $v_{50}$ | $n_C$ | $n_D$ | $n_F$ | $n_{G'}$ |
|---|---|---|---|---|---|---|---|
| | .00000 | 1.11838 | 1.15874 | 1.36821 | 1.37004 | 1.37452 | 1.37822 |
| | .00000 | 1.11829 | 1.15866 | 1.36822 | 1.37006 | 1.37458 | ——— |
| | .00000 | 1.11821 | 1.15859 | 1.36822 | 1.37006 | 1.37456 | 1.37831 |
| | Mean. | 1.11829 | 1.15866 | 1.36822 | 1.37005 | 1.37455 | 1.37826 |
| 1 | .19082 | 1.00742 | 1.04358 | 1.37943 | 1.38150 | 1.38657 | 1.39077 |
| 2 | .35007 | .91416 | .94654 | 1.39162 | 1.39393 | 1.39964 | 1.40438 |
| 3 | .49517 | .82845 | .85737 | 1.40563 | 1.40822 | 1.41465 | 1.42004 |
| 4 | .59741 | .76738 | .79377 | 1.41808 | 1.42090 | 1.42800 | 1.43394 |
| 5 | .68529 | .71454 | .73880 | 1.43092 | 1.43402 | 1.44176 | 1.44833 |
| 6 | .74566 | .67763 | .70040 | 1.44119 | 1.44447 | 1.45274 | 1.45977 |
| 7 | .82792 | .62746 | .64814 | 1.45757 | 1.46121 | 1.47032 | 1.47811 |
| 8 | .89093 | .58845 | .60749 | 1.47253 | 1.47646 | 1.48639 | 1.49489 |
| 9 | .94970 | .55183 | .56927 | 1.48898 | 1.49318 | 1.50398 | 1.51323 |
| | 1.00000 | .52008 | .53619 | 1.50535 | 1.50992 | 1.52159 | 1.53163 |
| | 1.00000 | .52005 | .53623 | 1.50534 | 1.50992 | 1.52158 | 1.53160 |
| | 1.00000 | .52006 | .53623 | 1.50533 | 1.50992 | 1.52158 | 1.53157 |
| | Mean. | .52007 | .53622 | 1.50534 | 1.50992 | 1.52158 | 1.53160 |

## TABLE XII.

*Ethyl Iodide—Ethyl Acetate.*

| No. | $\Delta v_{25}$ | $\Delta v_{50}$ | $\Delta R_{D25}$ | $a_C$ | $a_D$ | $a_F$ | $a_{G'}$ |
|---|---|---|---|---|---|---|---|
| 1 | +.00328 | +.00369 | −.00020 | 1.266 | 1.263 | 1.294 | 1.330 |
| 2 | + 529 | + 577 | − 22 | 1.174 | 1.175 | 1.196 | 1.213 |
| 3 | + 638 | + 692 | − 25 | 1.152 | 1.156 | 1.177 | 1.197 |
| 4 | + 647 | + 697 | − 20 | 1.113 | 1.120 | 1.132 | 1.154 |
| 5 | + 620 | + 669 | − 10 | 1.065 | 1.065 | 1.081 | 1.091 |
| 6 | + 540 | + 587 | − 14 | 1.096 | 1.102 | 1.118 | 1.131 |
| 7 | + 444 | + 481 | − 5 | .993 | 1.041 | 1.060 | 1.070 |
| 8 | + 313 | + 337 | − 4 | 1.049 | 1.047 | 1.054 | 1.060 |
| 9 | + 167 | + 174 | − 0 | .970 | 1.000 | 1.007 | 1.010 |

here also. The deviations of specific volume, refractivity and vapor pressure form their respective additive laws are shown in Fig. 5.

### V. *Acetic Acid—Benzol.*

The acetic acid was Kahlbaum's best, and was further purified by fractional freezing. The benzol was boiled for three hours with

10–15 per cent anhydrous aluminum chloride under a reflux condenser. Distilled off and washed with NaOH to remove HCl, then washed with water, and, after drying over $CaCl_2$, was distilled. The fraction was collected boiling at 80°.5 C.

Evaporation corrections: Acetic acid, .0006 ± .0001 gr.; benzol, .0050 ± .0005 gr.

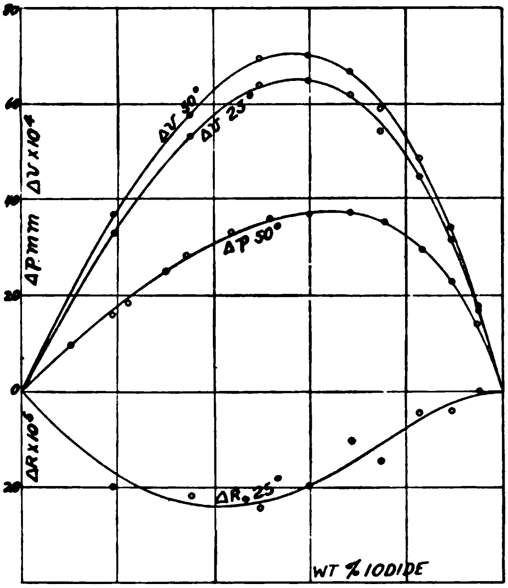


Fig. 5.

### TABLE XIII.

*Ethyl Iodide—Ethyl Acetate (Zawidzki).* 49°.99 C.

| $x$-Iod. | $z$-Iod. | $p_{49.99}$ | $\Delta p$ | $x$-Iod. | | $p_{49.99}$ | $\Delta p$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 280.4 | 0.0 | .4588 | .6001 | 350.7 | 36.8 |
| .0590 | .0999 | 294.3 | + 9.6 | .5486 | .6827 | 357.4 | 37.0 |
| .1148 | .1867 | 304.7 | 15.9 | .6340 | .7541 | 361.7 | 35.0 |
| .1376 | .2202 | 308.7 | 18.3 | .7388 | .8335 | 363.5 | 29.2 |
| .1946 | .2996 | 319.5 | 24.9 | .8253 | .8932 | 363.5 | 22.9 |
| .2288 | .3443 | 325.2 | 28.1 | .9098 | .9470 | 360.7 | 13.9 |
| .3057 | .4380 | 335.4 | 32.7 | 1.0000 | 1.0000 | 353.4 | 0.0 |
| .3745 | .5158 | 343.4 | 35.7 | | | | |

*Remarks in Tables XIV., XV. and XVI.*—The specific volumes and indices for the mixture acetic acid—benzol were easiest of all

## TABLE XIV.

*Acetic Acid—Benzol.*

| No. | $x$-Acetic. | $v_{25}$ | $v_{50}$ | $n_C$ | $n_D$ | $n_F$ | $n_{G'}$ |
|---|---|---|---|---|---|---|---|
| | .00000 | 1.14459 | 1.18091 | 1.49331 | 1.49794 | 1.50985 | 1.52019 |
| | .00000 | 1.14458 | 1.18093 | 1.49328 | 1.49794 | 1.50985 | 1.52012 |
| | Mean. | 1.14458 | 1.18092 | 1.49329 | 1.49794 | 1.50985 | 1.52015 |
| 1 | .11634 | 1.12768 | 1.16362 | 1.47782 | 1.48218 | 1.49323 | 1.50276 |
| 2 | .23614 | 1.10923 | 1.14433 | 1.46225 | 1.46628 | 1.47644 | 1.48517 |
| 3 | .34008 | 1.09194 | 1.12633 | 1.44890 | 1.45265 | 1.46208 | 1.47020 |
| 4 | .44857 | 1.07299 | 1.10638 | 1.43516 | 1.43861 | 1.44724 | 1.45460 |
| 5 | .54459 | 1.05549 | 1.08773 | 1.42300 | 1.42618 | 1.43414 | 1.44086 |
| 6 | .63968 | 1.03692 | 1.06822 | 1.41131 | 1.41418 | 1.42150 | 1.42765 |
| 7 | .73819 | 1.01687 | 1.04693 | 1.39907 | 1.40172 | 1.40827 | 1.41375 |
| 8 | .82321 | .99858 | 1.02776 | 1.38875 | 1.39119 | 1.39712 | 1.40206 |
| 9 | .91338 | .97830 | 1.00627 | 1.37803 | 1.38023 | 1.38552 | 1.38983 |
| | 1.00000 | .95797 | .98468 | 1.36779 | 1.36977 | 1.37440 | 1.37819 |
| | 1.00000 | .95799 | .98460 | 1.36774 | 1.36974 | 1.37443 | 1.37818 |
| | 1.00000 | .95789 | .98461 | 1.36783 | 1.36977 | 1.37444 | 1.37816 |
| | Mean. | .95795 | .98463 | 1.36779 | 1.36976 | 1.37442 | 1.37818 |

## TABLE XV.

*Acetic Acid—Benzol.*

| No. | $\Delta v_{25}$ | $\Delta v_{50}$ | $\Delta R_{D25}$ | $a_C$ | $a_D$ | $a_F$ | $a_{G'}$ |
|---|---|---|---|---|---|---|---|
| 1 | .00481 | .00554 | −.000041 | 1.043 | 1.030 | 1.030 | 1.030 |
| 2 | 872 | 976 | − 12 | 1.010 | 1.005 | 1.011 | 1.014 |
| 3 | 1082 | 1217 | − 82 | 1.032 | 1.028 | 1.029 | 1.019 |
| 4 | 1212 | 1351 | − 115 | 1.037 | 1.036 | 1.041 | 1.042 |
| 5 | 1255 | 1370 | − 157 | 1.048 | 1.049 | 1.050 | 1.053 |
| 6 | 1172 | 1286 | − 184 | 1.051 | 1,063 | 1.058 | 1.054 |
| 7 | 1006 | 1091 | − 229 | 1.088 | 1.093 | 1.100 | 1.102 |
| 8 | 764 | 843 | − 216 | 1.117 | 1.119 | 1.130 | 1.127 |
| 9 | 419 | 464 | − 118 | 1.114 | 1.121 | 1.126 | 1.195 |

to measure because of the low vapor pressure of acetic acid. The readings for the indices were especially sharp and constant, successive readings for a mixture agreeing within the error of the instrument, viz., 0′.5 or $5 \times 10^{-5}$, and the probable error of the measurements which are given in the tables is still smaller as they are all means of at least three; the sixth place is therefore retained for $\Delta R_{D25}$. The relative values of the indices for a given composition

TABLE XVI.

*Acetic Acid—Benzol (Zawidski).* 49°.99 *C.*

| $x$-Acetic. | $z$-Acetic. | $p_{49.99}$ | $\Delta p$ | $x$-Ac. | $s$-Ac. | $p_{49.99}$ | $\Delta p$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 267.1 | 0.0 | .3493 | .4136 | 228.0 | 34.8 |
| .0160 | .0170 | 265.9 | +2.2 | .3696 | .4365 | 224.3 | 35.4 |
| .0364 | .0413 | 265.2 | 5.8 | .5834 | .6530 | 189.5 | 45.8 |
| .0439 | .0504 | 264.4 | 6.6 | .6125 | .6801 | 184.0 | 46.5 |
| .0835 | .0996 | 261.1 | 11.7 | .6604 | .7242 | 175.3 | 48.0 |
| .1138 | .1377 | 259.0 | 16.0 | .8435 | .8809 | 126.0 | 37.4 |
| .1714 | .2088 | 250.2 | 19.4 | .9797 | .9851 | 68.0 | 8.3 |
| .2089 | .2535 | 245.2 | 22.3 | .9931 | .9949 | 59.2 | 2.4 |
| .2979 | .3566 | 236.0 | 32.0 | 1.0000 | 1.0000 | 55.4 | 0.0 |

are subject, as in the preceding case, to an error of about $2 \times 10^{-5}$, and the specific volumes are subject to a like error the error of the evaporation correction being negligible. From 30 to 60 gr. of mixture were used each time. For deviations, see Fig. 6.

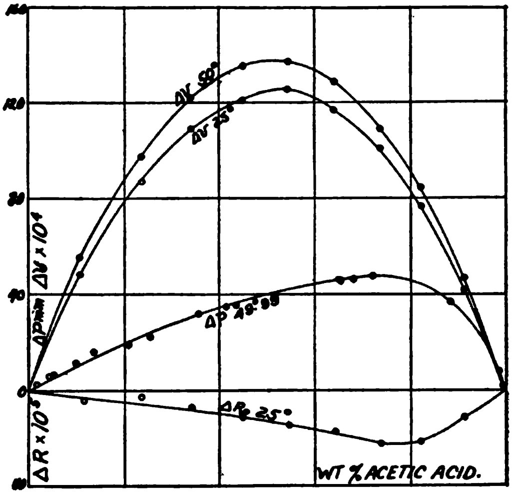


Fig. 6.

## VI. *Carbon Tetrachloride—Benzol.*

The benzol used was the same as in the preceding case.

The carbon tetrachloride was dried with $CaCl_2$ and distilled. It was allowed to stand over lime to remove traces of sulphur.

Evaporation correction: $CCl_4$, .0090 ± .0005 gr.; $C_6H_6$, .0050 ± .0005 gr.

## TABLE XVII.

*Carbon Tetrachloride—Benzol.*

| No. | $x\cdot CCl_4$ | $v_{25}$ | $v_{50}$ | $n_C$ | $n_D$ | $n_F$ | $n_{G'}$ |
|---|---|---|---|---|---|---|---|
| | 0.00000 | 1.14451 | 1.18089 | 1.49332 | 1.49794 | 1.50985 | 1.52016 |
| | 0.00000 | 1.14455 | 1.18090 | 1.49327 | 1.49794 | 1.50981 | 1.52011 |
| | 0.00000 | 1.14454 | 1.18093 | 1.49330 | 1.49794 | 1.50985 | 1.52014 |
| | Mean. | 1.14453 | 1.18091 | 1.49330 | 1.49794 | 1.50984 | 1.52014 |
| 1 | .16873 | 1.05794 | 1.09171 | 1.48973 | 1.49420 | 1.50562 | 1.51547 |
| 2 | .31954 | .98046 | 1.01198 | 1.48602 | 1.49030 | 1.50120 | 1.51058 |
| 3 | .43419 | .92179 | .95150 | 1.48261 | 1.48674 | 1.49718 | 1.50613 |
| 4 | .55495 | .85969 | .88749 | 1.47843 | 1.48232 | 1.49223 | 1.50070 |
| 5 | .64704 | .81251 | .83881 | 1.47476 | 1.47849 | 1.48788 | 1.49592 |
| 6 | .71042 | .77966 | .80492 | 1.47197 | 1.47555 | 1.48461 | 1.49231 |
| 7 | .79671 | .73562 | .75943 | 1.46775 | 1.47109 | 1.47954 | 1.48668 |
| 8 | .87639 | .69470 | .71713 | 1.46320 | 1.46629 | 1.47409 | 1.48069 |
| 9 | .93504 | .66449 | .68589 | 1.45941 | 1.46230 | 1.46961 | 1.47576 |
| | 1.00000 | .63110 | .65131 | 1.45465 | 1.45733 | 1.46401 | 1.46953 |
| | 1.00000 | .63109 | .65133 | 1.45460 | 1.45731 | 1.46397 | 1.46952 |
| | 1.00000 | .63107 | .65133 | 1.45467 | 1.45733 | 1.46403 | 1.46958 |
| | Mean. | .63109 | .65132 | 1.45464 | 1.45732 | 1.46400 | 1.46954 |

## TABLE XVIII.

*Carbon Tetrachloride—Benzol.*

| No. | $\Delta v_{25}$ | $\Delta v_{50}$ | $\Delta R_{D25}$ |
|---|---|---|---|
| 1 | +.00004 | +.00016 | +.000225 |
| 2 | − 1 | + 30 | + 398 |
| 3 | + 19 | + 54 | + 523 |
| 4 | + 10 | + 47 | + 497 |
| 5 | + 20 | + 56 | + 518 |
| 6 | − 10 | + 23 | + 491 |
| 7 | + 16 | + 45 | + 438 |
| 8 | + 13 | + 34 | + 318 |
| 9 | + 4 | + 17 | + 179 |

*Remarks on Tables XVII., XVIII. and XIX.*—Owing to the very small values of $\Delta v$ for this case the values of $\alpha$ have not been calculated. The magnitude of errors is in all respects as in the preceding case. For the plots of the deviations see Fig. 7.

## Table XIX.

*Carbon Tetrachloride—Benzol (Zawidzki).* 49°.99 *C.*

| $x \cdot CCl_4$ | $z \cdot CCl_4$ | $p_{19.99}$ | $\Delta p$ | $x \cdot CCl_4$ | $z \cdot CCl_4$ | $p_{49.99}$ | $\Delta p$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 268.0 | 0.0 | .3953 | .5630 | 294.5 | 10.7 |
| .0507 | .0952 | 271.8 | +1.8 | .5587 | .7138 | 301.0 | 10.7 |
| .1170 | .2070 | 277.6 | 4.9 | .6755 | .8041 | 305.2 | 10.2 |
| .1758 | .2959 | 281.5 | 6.5 | .7652 | .8653 | 306.8 | 8.2 |
| .2515 | .3983 | 285.4 | 7.3 | 1.0000 | 1.0000 | 308.0 | 0.0 |
| .2947 | .4515 | 288.3 | 8.5 | | | | |

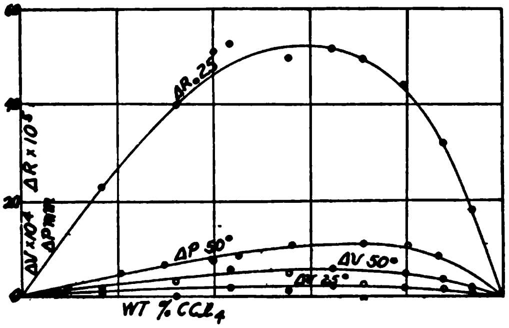


Fig. 7.

## General Discussion. Summary.

From an inspection of the tables and figures it is seen that, for the cases studied and in the given temperature interval:

1. The deviations of the specific volumes from additivity increase in absolute value with the temperature.

A possible exception to this conclusion is found in the case of chloroform—acetone, which for a small portion of the curve seems to show a deviation diminishing with rise of temperature. This is the only mixture of the present series which shows a systematic negative deviation and the only one in which the temperature coefficient of deviation is negative. It is interesting to note in connection with this case the great heat developed on mixing which is also exceptional among the mixtures here considered. Carbon tetrachloride—benzol, according to Brown[1] shows a small negative deviation at 20° C. In the measurements here given the deviation is slightly

---

[1] F. D. Brown, Trans. Chem. Soc., 39, p. 202, 1881.

positive at 25° and more so at 50° C. The results for these two mixtures indicate that further study is desirable to show more completely the nature of the temperature coefficient of specific volume deviation. It seems very probable that there are mixtures in which the deviations are negative at all temperatures, others in which the deviations are always positive, other in which the deviations grow from positive to negative, still others which show the reverse, and, possibly a few which show a mixture of effects. These possibilities must be settled before binary mixtures are understood

In this connection should be noticed the interesting analogous behavior of the partial and total vapor pressures; *i. e.*, that their deviations from additivity also increase in absolute value with the temperature. This was brought to my attention by Professor Rosanoff who has kindly furnished the following proof. Let $P_1$ and $P_2$ be the respective vapor pressures of the pure liquids $A$ and $B$, and let $p_1$ and $p_2$ be the respective partial vapor pressures of $A$ and $B$ in the mixture $A$-$B$. Also let $p_1'$ and $p_2'$ be the respective partial pressures calculated on the assumption of simple additivity, thus, $p_1' = P_1x$ and $p_2' = P_2(1 - x)$. We have

$$p_1 = P_1f(x),$$

hence

$$p_1 - p_1' = P_1\{f(x) - x\} = P_1\varphi(x).$$

Rosanoff and Easley (*loc. cit.*) have shown that $f(x)$ and, accordingly, $\varphi(x)$, in those cases in which the heat of dilution is moderate, is practically independent of the temperature. Hence, where this is true, for any given value of $x$, the deviation of the partial vapor pressure $p_1$ from additivity is

$$\Delta p_1 = p_1 - p_1' = kP_1,$$

where $k$ is constant. Since $P_1$ increases with the temperature, therefore must $\Delta p_1$ increase in absolute value with the temperature. Similarly for $\Delta p_2 = p_2 - p_2'$; and hence, also, *the deviation of total vapor pressure*, $\Delta p = \Delta p_1 + \Delta p_2$ *must increase in absolute value with the temperature.* The analogous behavior of specific volume deviations in these mixtures is striking and suggestive.

2. The algebraic sign of the deviation of specific volume is the same as that of the vapor pressure at the same temperature.

This conclusion, together with (1), suggests the possibility of the vapor pressure deviation changing sign as the temperature is varied, and also, judging from the vapor pressure and volume curves of chloroform—acetone, that there may exist curves of the mixed type, part positive and part negative. These curves have already been predicted but have not as yet been found.[1] It is possible that chloroform—acetone may yield such a curve at a low temperature.

3. The deviations of refractivity are of the same sign as the deviation of vapor pressure and specific volume in the cases of $CS_2$—methylal, $CS_2$—acetone and $CCl_4-C_6H_6$, and of opposite sign in the others. The deviations show an increase in absolute value with diminishing wave-length. The Pulfrich constant $\alpha$ is seen, in some cases to vary within wide limits. The Pulfrich equation is empirical and the constant $\alpha$ has no theoretical significance. The expression for the refractivity has not as yet taken a final form (see above), and until we know more about the dispersion of pure liquids it would seem to be very difficult to correlate deviations of refractivity $\Delta R$ with the deviations of density and vapor pressure. The values of $\Delta R$ are accordingly given here for but one wave-length, that of sodium. On account of the excellent uniformity shown by the differences of the indices for different wave-lengths the indices for all the wave-lengths are given in the hope that they may be useful for further calculation. Greater knowledge of the dispersion of binary mixtures may be expected from a study of their absorpton bands in the ultra-violet and in the infra-red.

It is the intention of the author to add results for other mixtures, making available for further study the binary systems *A-B*, *B-C*, *A-C*, to settle the question whether the deviations are additive from mixture to mixture or whether they depend upon the specific action of two given liquids upon each other.

The present results have been considered from the standpoint of Dolezalek's theory (*loc. cit.*), which attempts to explain all deviations by assuming dissociation of polymerized molecules, or combination of the two components, *i. e.*, putting the matter upon a

---

[1] See Zawidzki, Zs. für phys. Chem., LXIX., p. 630, 1909.

chemical basis. The results of these calculations will be published in a subsequent paper.

The measurements presented in this paper were made in the summer and fall of 1908.

CLARK COLLEGE, WORCESTER, MASS.,
March, 1910.

# ON ENTROPY.

BY W. S. FRANKLIN.

IT has perhaps occurred to every student of thermodynamics that there might be a simple and direct method of establishing the quantitative idea of entropy, considering that the idea is so widely applicable and that it is independent of the properties of particular substances and independent of particular processes. The difficulty is that the definite forward movement in nature which has been formulated as the law of increase of entropy is mixed up inextricably with changes of state of physical substances. This difficulty is met in the argument of Clausius by considering a cyclic process where no change of state is left outstanding, but it is apparently an unnecessary complication to base the discussion of a definitely forward moving quantity like entropy upon a cyclic process. Clausius's integral, moreover, is strictly applicable to reversible processes only, and in Clausius's argument, when limited in this way, the law of increase of entropy makes its appearance as follows: Consider a state *B* which is reached from state *A* by an irreversible process, then Clausius's integral extended over a reversible process leading from state *A* to state *B* gives a positive result. For example, a gas issues from an orifice in a high pressure tank, and if the gas is brought from its initial condition to its final condition by a reversible process, Clausius's integral extended over this reversible process gives the increment of entropy which is associated with the original irreversible process. To base the discussion of the law of increase of entropy upon reversible processes only (and this is substantially what Clausius's original argument does, considering that the application of Clausius's integral to irreversible processes is not legitimate) would seem to be to ignore a most important physical element, because the quantitative idea of entropy gains its significance chiefly in its application to irreversible processes.

The object of this paper is to call attention to a class of irreversible processes which are permanent or steady and therefore susceptible to quantitative treatment, and which lead to no change of state of any kind so that the entropy changes which are involved are associated solely with energy transformations. By a careful consideration of an irreversible process of this type the quantitative idea of entropy may be established in the most direct possible way. The quantitative idea of entropy when established in this way is quite primitive and highly generalized and it refers to the entropy increment involved in the conversion of work into heat or to the entropy increment involved in the flow of heat from a high-temperature region to a low-temperature region, and a further development of the idea of entropy is of course necessary in order to establish a measure of the entropy differences which are associated with changes of state. This extension of the primitive idea of entropy can be established only by a consideration of a cyclic process.

A further object of the paper is to discuss an apparent close relationship between entropy and time.

The simplest kind of irreversible process is that which takes place in an isolated system while the system is settling to thermal equilibrium. Such a process has a certain impetuous quality as, for example, in a conflagration, when a house is settling to thermal equilibrium with the surrounding air, and perhaps the physical nature of an irreversible process is most clearly and strongly suggested by speaking of such a process as a *sweeping process* or as a *sweep*. A careful consideration of the various sweeping processes which take place in nature leads to the recognition of three distinct types as follows: (*a*) Sweeping processes which take place in a closed system, (*b*) sweeping processes which take place in a system which is subjected to rapidly varying external action, and (*c*) sweeping processes which are perfectly steady and which involve no changes of state of any kind in the system under consideration. The first type is exemplified by the explosion of a mixture of hydrogen and oxygen in a closed vessel. The second type is exemplified in the departure from thermal equilibrium of a gas which is rapidly compressed under a piston or by the departure from thermal equilibrium of a vessel of water which is heated on a stove. In both cases the system never catches up, as it were, with the changing conditions but trails along behind them. The third type is exemplified by the flow of electric current through a wire from which heat is abstracted by a steady stream of air or water, and by the steady flow of heat from a region of high temperature to a region of low temperature. A number of years ago I suggested the terms "simple sweep," "trailing sweep" and "steady sweep," respectively, to designate these three types of irreversible processes. The entropy change which accompanies a simple sweep is involved in or associated with the change of state of the substance. The entropy change which accompanies a trailing sweep is associated in part with the conversion of work into heat, in part with the flow of heat from a region of high temperature to a region of low temperature, and in part with the changes of state of the substance. The entropy change which takes place in a steady sweep is due solely to the conversion of work into heat or to the transfer of heat from a region of high temperature to a region of low temperature, or both. Therefore a careful consideration of a steady sweep is the simplest basis for the discussion of the idea of entropy.

In establishing the idea of entropy it seems to be necessary to start out with the assumption that the entropy function exists, and then to justify this assumption by logical deductions and experimental verifications. Clausius, indeed, starts with the assumption that a non-compensated transfer of heat from a cold region to a hot region is impossible. This form of assumption may seem to obviate initial reference to the entropy function but it is not accompanied by a clear and complete definition of compensation in the thermodynamic sense, and indeed such a definition cannot be stated without the introduction of the idea of entropy. It is evident, therefore, that the idea of the entropy function is really involved at the very beginning of Clausius's classical argument, and a logical development of the second law of thermodynamics might perhaps be made less indefinite and more intelligible by frankly introducing the idea of entropy at the start in explicit terms. Indeed this is the procedure which is adopted by Professor Planck. It is difficult, however, to give a preliminary definition of entropy which is correct as far as it goes and which carries an appeal to one's primitive sense of physical things. Such an appeal is greatly to be desired and it is certainly possible because there is perhaps a more widespread intuitive sense touching the second law of thermodynamics than in the case of any other of the generalizations of physics.

Fire is the most familiar example of a sweeping process and its most striking characteristic is that its progress is not dependent upon any external driving cause; when once started it goes forward of itself and with a rush. Everyone perhaps will admit that the impetuous character of fire suggests a certain havoc, a certain degeneration or waste in the system in which the fire takes place, and the same is true of every sweeping process. Consider, for example, a charge of gunpowder which has been exploded in a large empty vessel; everything is there after the explosion, all of the energy is there and all of the material substance is there, and yet it cannot be exploded a second time! But the man on the street has heard so much during recent years of the conservation of energy and of the conservation of mass that the old proverb that "You can't eat your cake and have it" presents to his mind a very simple and inevitable fact or condition which he is at times tempted to ignore when he turns his attention to an unfamiliar matter like the steam engine; he tries in vain to rationalize steam engine theory in terms of the principles of conservation alone. Nearly all of the intuitive sense of the man on the street concerning such matters (and he has a great deal) is involved in the second law of thermodynamics which is not a law of conservation at all; it is a law of waste.

It may be assumed that every sweeping process brings about a definite amount of degeneration, an amount that can be expressed numerically. Thus a certain amount of degeneration may be assumed to be brought about when a compressed gas escapes through an orifice, when heat flows from a region of high temperature to a region of low temperature, when work is converted into heat by friction or by the flow of an electric current through a wire, and so on. In a simple sweep the degeneration lies wholly in the relation between the initial and final states of the substance. In a trailing sweep the degeneration may lie partly in the relation between the initial and final states of the substance which undergoes the sweep, partly in the conversion of work into heat, and partly in the flow of heat from a high temperature region to a low temperature region. In a steady sweep, however, the degeneration lies wholly in the conversion of work into heat, in the transfer of heat from a region of high temperature to a region of low temperature, or in both. Therefore the idea of thermodynamic degeneration as a measurable quantity can be reached in the simplest possible manner by a careful scrutiny of a steady sweep.

*Proposition* (*a*).—The thermodynamic degeneration which is represented by the direct conversion of work into heat at a given temperature is proportional to the quantity of work so converted. Consider, for example, a steady flow of electric current through a wire from which the heat is abstracted continuously so that the temperature of the wire remains constant. This is a steady process, that is to say, it remains unchanged during successive intervals of time, and therefore any result of the process must be proportional to the time. Thus the amount of degeneration occurring in a given interval of time is proportional to the time, but the amount of work which is degenerated into heat is also proportional to the time. Therefore the amount of degeneration is proportional to the amount of work which is converted into work at the given temperature.

*Proposition* (*b*).—The thermodynamic degeneration which is represented by the transfer of heat from a given high temperature $T_1$ to a given low temperature $T_2$ is proportional to the quantity of heat transferred. Consider a steady flow of heat from temperature $T_1$ to temperature $T_2$ constituting a steady sweep, a sweep which remains unchanged in character in successive intervals of time. Any result of this sweep must be proportional to the time and therefore the degeneration which takes place in a given interval of time is proportional to the time; but the quantity of heat transferred is also proportional to the time. Therefore the amount of degeneration is proportional to the quantity of heat transferred from temperature $T_1$ to temperature $T_2$.

According to proposition (*a*), above, the thermodynamic degeneration which is involved in the conversion of work into heat at a given temperature is proportional to the amount of work so converted and the proportionality factor depends upon the temperature only. Therefore we may write

$$\varphi' = m_1 W, \tag{1}$$

$$\varphi'' = m_2 W, \tag{2}$$

in which $\varphi'$ is the degeneration involved in the conversion of an amount of work $W$ into heat at temperature $T_1$, and $\varphi''$ is the degeneration involved in the conversion of an amount of work $W$ into heat at temperature $T_2$, and $m_1$ and $m_2$ are factors which depend only upon $T_1$ and $T_2$, respectively. An amount of work $W$ having been converted into heat at temperature $T_1$, imagine the heat to flow to a lower temperature $T_2$, thus involving an additional amount of degeneration according to proposition (*b*) above. The conversion of work $W$ into heat at temperature $T_1$ and the subsequent flow of this heat to a lower temperature $T_2$ gives the same result as would be produced by the conversion of the work into heat at the lower temperature directly. Therefore the lower the temperature at which work is converted into heat the greater the amount of degeneration involved. That is to say, the factor $m_2$ in equation (2) is larger in value than the factor $m_1$ in equation (1), temperature $T_1$ being higher[1] than temperature $T_2$. Therefore, since $m_1$ and $m_2$ depend only upon $T_1$ and $T_2$, respectively, it is permissible to adopt the equation

$$T_1/T_2 = m_2/m_1, \tag{3}$$

as the definition of the ratio $T_1/T_2$.

Another way to express the definition which is involved in equation (3) is as follows: Considering that the factor $m_1$ is the smaller the higher the temperature $T_1$, we may adopt $k/m_1$ as the measure of the temperature $T_1$, and $k/m_2$ as the measure of the temperature $T_2$, giving

$$m_1 = k/T_1, \tag{4}$$

$$m_2 = k/T_2, \tag{5}$$

where $k$ is an indeterminate constant. Therefore equations (1) and (2) may be written in the general form

$$\varphi = kW/T, \tag{6}$$

---

[1] The idea of higher and lower temperature is not dependent upon any method of measuring temperature.

where $\varphi$ is the thermodynamic degeneration involved in the conversion of an amount of work $W$ into heat at temperature $T$, and $k$ is an indeterminate constant.

Since the factor $k$ in equation (6) is indeterminate, we may adopt as the unit of thermodynamic degeneration the amount which is involved in the conversion of one unit of work into heat as a temperature of one degree on the absolute scale; then the value of $k$ is unity and equation (6) becomes

$$\varphi = W/T, \tag{7}$$

in which $W$ is expressed in joules, $T$ in degrees centigrade and $\varphi$ in joules per degree. Thus one joule per degree is the degeneration involved in the conversion of one joule of work into heat at 1° C. on the absolute scale.

To convert an amount of work $W$ into heat at temperature $T_1$ involves $W/T_1$ units of degeneration. To convert the same amount of work into heat at temperature $T_2$ involves $W/T_2$ units of degeneration. Therefore to transfer an amount of heat equal to $W$ from temperature $T_1$ to temperature $T_2$ must involve an amount of degeneration equal to the excess of $W/T_1$ over $W/T_1$, or an amount equal to $W(1/T_2 - 1/T_1)$, or $H(1/T_2 - 1/T_1)$, where $H$ is the amount of heat transferred.

The word degeneration as used in the above discussion means the same thing as entropy increment, and the word degeneration is used because it carries with it a suggestion of irreparable waste. Furthermore, it is desirable to reserve the word entropy until with the help of Clausius's integral we are able to assign a definite entropy-difference to a given difference of state of a substance so that, choosing a zero state of the substance arbitrarily, we may speak of the entropy-of-the-substance in any other given state.

Starting with the above ideas of thermodynamic degeneration and remembering that such degeneration occurs only in sweeping processes, it is easy to establish the important proposition that the efficiency of a reversible engine depends only upon the boiler and condenser temperatures, and it is easy also to transform the above definition of temperature ratio as given in equation (3) to the form in which it was originally given by Lord Kelvin.[1]

## THE RELATIONSHIP OF ENTROPY AND TIME.

The above propositions concerning the entropy increment which is associated with a steady sweep suggest a relationship between entropy and time, and it is desirable to carry the inquiry further to determine whether this relationship is a fundamental one or not. It seems, indeed, at first sight, as though there could be no fundamental relationship between entropy and time because entropy increases with the utmost irregularity in different parts of the universe whereas time is thought of as a universal steady flux.

Imagine an isolated mechanical system involving no irreversible processes. After a sufficient length of time, using that term in the commonly accepted sense, the system returns to identically its initial state, and when the system has returned to its initial state it is unjustifiable to project into the system the idea that there has been a lapse of time. It seems, of course, absurd to make this statement because everyone realizes in looking at the ideal mechanical system that time has elapsed between its initial and final state, but this view of the matter involves the inclusion in our ideal system of one's own physical body and the projection of one's own consciousness into the aggregate of phenomena under consideration. Let us, therefore, consider our ideal system apart from its relation to any outside condition whatever. Then, to say that time has elapsed when the system has returned to its initial state is to introduce an arbitrary and meaningless difference between the initial and final state.

Consider a system in which no changes whatever are taking place. What we call the lapse of time finds no basis for its application to such a system because no progressive change of any kind is taking place in the system.

Consider a purely mechanical system (one in which no irreversible action takes place). Any change in state of this system after the lapse of what we call time can be completely specified in terms of

---

[1] These arguments are outlined in a very simple way in an article by W. S. Franklin in Popular Science Monthly, March, 1910.

the positions and velocities of the component parts of the system, and it must be remembered that in specifying the velocities of the component parts of the system no reference need be made to the time which has elapsed since the beginning. That is to say the complete specification of the change of state of the system may be made without reference to the lapse of time, and to say that time has elapsed is to introduce an arbitrary and meaningless difference between the initial and final states of the system in addition to the differences already completely specified in mechanical terms.

Consider a system in which irreversible processes take place. Such a system departs further and further from its initial condition without possibility of return and in this case a complete specification of change in the system can be made in terms of mechanical specifications and entropy specifications, and it would seem to be meaningless to add the further specification of lapse of time to what is otherwise complete.

What we call time, when reduced to its simplest terms, is a phenomenon of consciousness. And our sense of the inevitable forward movement of time is dependent upon the existence of irreversible processes everywhere about us, and especially inside of us. That is to say, our sense of the forward movement of time and the law of increase of entropy are based upon or grow out of the same fundamental conditions in nature.

The above argument would seem to indicate that the universal steady flux of time is an idea, and not a physical fact; although, taking the universe as a whole, local irregularities become individually negligible and the entire process of nature may be thought of as one vast steady sweep in which increase-of-entropy and passage-of-time, much as they differ in our local methods of measuring them, refer fundamentally to the same thing. The extent to which the idea of the steady flux of time pervades our habitual modes of thought seems to be an outgrowth of the methods that we have come to use in the expression of our rather complicated experiences relating to what we call coincidences in time. Indeed some of the most pervasive things of the human mind are the ideations which have been developed as bases for our system of language, and to make due allowance for them in a physical discussion is sometimes very difficult indeed, partly because we fail to recognize the ideations as such and partly because we are left without any adequate forms of expression if we set them aside. We always make time specifications with respect to a recurrent phenomenon like the rising of the sun, with finer gradations based upon the oscillations of a pendulum; our time specifications are simple counts of these recurrent phenomena; and the requirement of simplicity and directness of speech has led to the development of the idea of the steady flux of time and to the habitual projection of this idea into every objective condition we encounter in nature.

Two phases of two systems are simultaneous or coincident in time when they involve the loss of energy by one of the systems and the gain of energy by the other system. Thus it seems that the idea of simultaneity rests fundamentally upon the principle of the conservation of energy. If energy could be taken from one system, temporarily annihilated, and later delivered to another system, our idea of simultaneity would have to be modified; or if energy is taken from one system and if some time must elapse before its delivery to another system, as in the case of the transmission of energy by a beam of light, then again a modified conception of simultaneity would have to be developed. Indeed, this modified idea of simultaneity is already accomplished in terms of the idea that light has a definite velocity, and it seems as if the modern principle of relativity may lead to another solution of the same problem. The idea that it is everywhere *now* cannot be justified in the physical nature of things although an assumption to that effect is extremely convenient in speech. Also the idea that time is a universal steady flux cannot be justified in the physical nature of things although an assumption to that effect is extremely convenient in speech. Indeed, throughout the above discussion modes of expression are used which involve the common idea of time as a steady flux and to avoid the use of these common modes of expression would make the discussion extremely difficult to follow.

LEHIGH UNIVERSITY,
February 22, 1910.

# THE EFFECT OF PRESSURE ON THE ALUMINUM RECTIFIER.[1]

BY A. P. CARMAN AND G. J. BALZER.

THE peculiar action of the electrolytic cell with aluminum anode has been the subject of much investigation since it was first observed by Buff in 1857,[2] but the interest has been heightened within recent years, because of the increasing demand for a rectifier for alternating currents, and because the newer theories of solution and of electrons have suggested new explanations of the cell. Within a half dozen years, the important part played by the gas formed on the anode has been urged by several writers on the action of the cell.[3]

It suggested itself to the writers of the present paper, that light would be thrown on the problem by determining the exact effect of pressure on the action of the cell when traversed by an alternating current. For this purpose small cells were made, and placed under a cast-iron bell jar, so that the pressure could be raised from nearly zero to several atmospheres. The wave forms of the alternating current were photographed with an oscillograph, and at the same time, readings were made of the current and the electromotive force with Weston ammeter and voltmeter. All the cells had aluminum anodes and carbon cathodes. Fig. 1 gives all the description needed of the mechanical features of the cell. The evaporating dishes used were 10.5 cm. in diameter, and as indicated in the figure, were placed inside of larger dishes for packing the cell in ice. In most of the experiments, the temperature of the cell was kept about 2° C. The resistance of the cell was varied by varying the size of the anode; in some cells an aluminum wire of about 1 mm. diameter

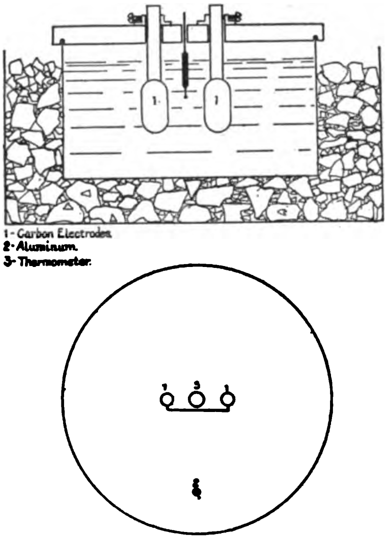
---

[1] Paper read before the American Physical Society, Boston, December, 1909.

[2] Buff, Liebig's Ann., 1857, p. 296.

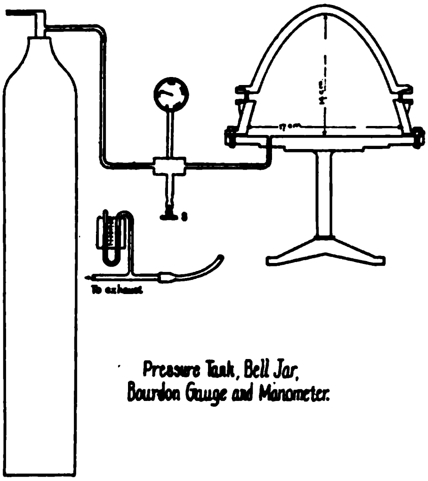
[3] Guthe, PHYS. REV., Vol. XV., p. 327, 1903. Schulze, Ann. d. Physik, 21, p. 955; 22, p. 543; 26, p. 372; 28, p. 787.

was used, and in others a vertical plate. Electrolytes used in the experiments were solutions of alum, of sodium borate, of bichromate of potassium and of potassium phosphate. When such a cell is placed in an alternating current circuit, the current becomes apparently unidirectional. The oscillograph curves show that the resulting current is practically a series of unidirectional pulses. That is, provided the impressed E.M.F. is not above a certain critical value, the current wave in one direction is practically cut out, the current passing freely from carbon to aluminum, but not from aluminum to carbon. To get this rectifying action quickly we employed the usual device of first forming the cell with a direct current. The alternating current was taken from the lighting and power circuits of the university system. The loads on the line were frequently changing, and so the wave forms changed. This hampered the work, and our curves often had to be taken late at night when the load on the lines was constant. In future work, a motor generator set, driven by a current from a storage battery, is to be used, but this was not available for the present experiments.



The Cell and Ice Jacket

Fig. 1.



Pressure Tank, Bell Jar, Bourdon Gauge and Manometer.

Fig. 2.

To vary the pressure, the cell was placed under a cast-iron bell jar, the arrangement being shown in Fig. 2. Small windows of heavy glass were placed so that the cell would be watched, and the temperatures read. To obtain uniform pressure, compressed air was taken from a cast-iron cylinder which had been charged to a pressure of about 1,000 lbs. per sq. in. from the compressor of the liquid air plant. The Duddell oscillograph was fitted with a home-made vertical slide, and the photographs were taken on 4 × 5 inch glass plates, the amplitude of the waves on the plate being generally about 4 cm. Fig. 4 shows the effect on the alternating current of the increase of pressure on a cell with a solution of sodium borate, the temperature being kept constant at 2.1° C. The pressures here increase successively from 8 mm. of mercury to 21 atmospheres. Fig. 5 shows the successive curves for the same cell as the pressure increases. These curves show that the rectifying action decreases with increasing pressure, and that the cell practically recovers its action when the pressure is removed. Similar curves were taken for cells with other electrolytes, showing that pressure decreased the rectifying action, though not always as much as in this case. In Fig. 6 we have curves for increasing pressures with a potassium bichromate cell, and in Fig. 7 we have similar curves for an alum cell. Readings of the current and of the electromotive forces were also made, in most cases, the instruments being kept in only during the times of reading. The readings of the voltmeter placed across the cell show only relative values, because the resistance of the low reading Weston voltmeter was not large compared with the resistance of the cell. A low reading electrostatic voltmeter was not available. Table I. shows the course of these readings for the alum cell with increasing and decreasing pressures.

These readings show the apparent decrease of resistance with increasing pressure, but the oscillograph curves are needed in any interpretation of the readings.

The experiments described above were all made at practically the same temperature, about 2° C. It suggested itself to us to try the effect of changes of temperature. Fig. 8 gives a series of oscillograph curves made for an alum cell, with temperatures increasing from 2.1° C. to 50° C. It is seen that the increase of temperature

PHYSICAL REVIEW
CLXX

PLATE I
To face page 778

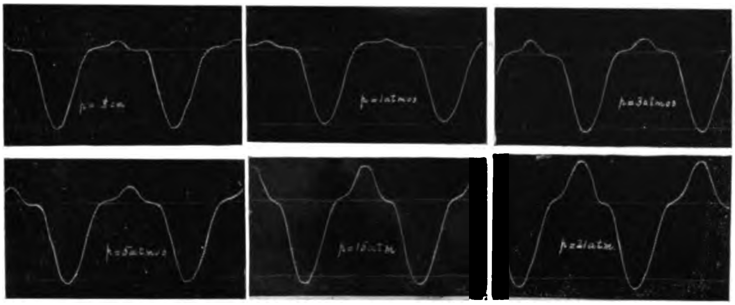


Fig. 3.

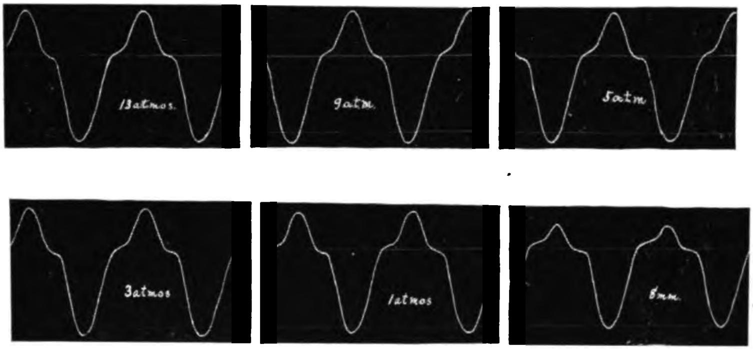


Fig. 4.

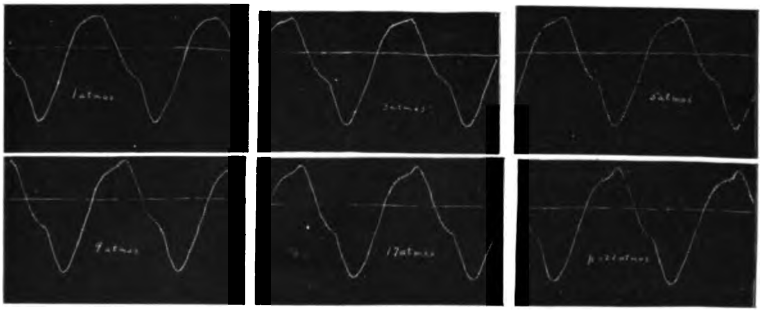


Fig. 5.

CARMAN AND BALZER

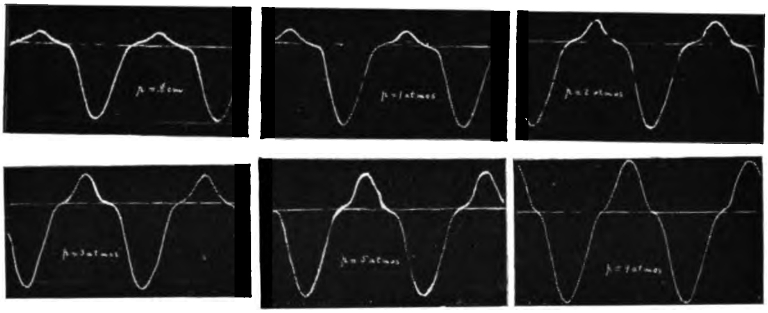


Fig. 6.

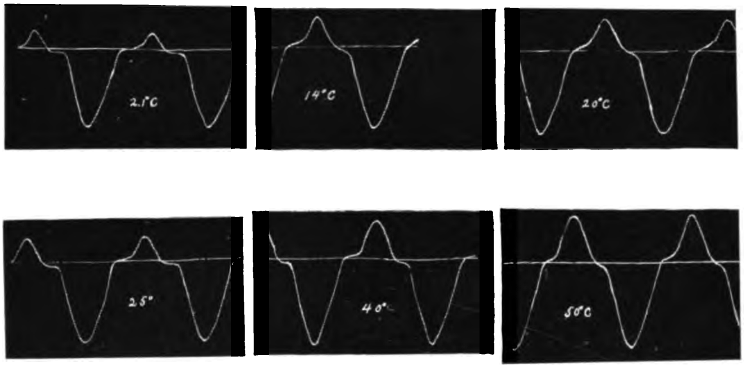


Fig. 7.

CARMAN AND BALZER

### Table I.

*Cell with Aluminum Anode and with Alum Solution.*

| Pressure. | E.M.F. Across Cell. | E.M.F. Impressed. | Current (Amp.). |
|---|---|---|---|
| .8 cm. | 24.5 | 32.2 | .21 |
| 1 atmosph. | 23.5 | 32.3 | .22 |
| 2 | 22.1 | 32.2 | .24 |
| 3 | 21.5 | 32.1 | .25 |
| 5 | 19.7 | 32.2 | .26 |
| 7 | 19.0 | 32.3 | .27 |
| 9 | 18.5 | 32.4 | .28 |
| 11 | 17.8 | 32.5 | .29 |
| 13 | 17.4 | 32.3 | .29 |
| 15 | 17.0 | 32.2 | .29 |
| 17 | 16.5 | 32.3 | .30 |
| 21 | 16.3 | 32.2 | .31 |
| 17 | 16.4 | 32.3 | .33 |
| 15 | 16.8 | 32.3 | .33 |
| 13 | 16.9 | 32.2 | .33 |
| 11 | 17.0 | 32.2 | .32 |
| 9 | 17.0 | 32.1 | .32 |
| 7 | 17.5 | 32.2 | .30 |
| 5 | 18.2 | 32.2 | .29 |
| 3 | 18.6 | 32.2 | .28 |
| 2 | 19.3 | 32.3 | .27 |
| 1 | 20.5 | | .26 |
| .8 cm. | 25.6 | | .17 |

decreases the rectifying action for this particular solution. The cell did not recover the rectifying action when the temperature was lowered again—a fact which has perplexed us. Of course the temperature fall was slow, and during this fall it was difficult to keep other conditions perfectly steady, but there was certainly a permanent change following the rise in temperature. This temperature effect probably depends also upon the temperature at which the cell is originally formed.

Numbers of investigators of the action of the aluminum cell have studied the character of the solid oxide film which is formed on the aluminum anode. The differences in the explanations of the cell are largely differences in the part ascribed to this solid film. Some writers have thought that the cell's action depended upon active selective properties of the film, while Schulze, who has published five or six investigations on this within the last few years, makes the action of the film wholly or almost wholly mechanical. According to him, it supports the gas film, which forms between the solid oxide deposit and the aluminum plate.

He says:[1] "The electrolytic valve action is not due to the rigid layer, which forms on the valve metals, but to a much thinner gas film which exists in the pores of the rigid layer and separates the electrolyte from the metal. If the valve metal is the kathode, then the free electrons contained in it pass through this gas layer with a comparatively small fall of potential. If the valve metal is the anode, a high difference of potential is necessary because there are then no free electrons in the electrolyte. I conclude, that the negative electrolytic ions then take the place of the electrons, and the ions, on account of their greater mass, experience a much higher resistance in the gas layer than that which the free electrons encounter in the opposite direction. This explains the so-called valve action."

The effect of pressure on the rectifying action, or "valve action" (Ventilwirkung), as Schulze calls it, is readily explained in accordance with Schulze's conceptions. The gas layer is reduced in thickness by the pressure and hence the resistance of the layer is decreased. We also observed a phenomenon which puzzled us at the time, but which is readily explained on the supposition of gas layers imprisoned in the body of the rigid film or behind it. The phenomenon was that at higher pressures a white fluffy deposit resembling in appearance the white of a boiled egg began to form at the aluminum anode and this increased as the pressure increased. This fluffy deposit would sometimes leave the electrode and rise to the top, or sink to the bottom. As the pressure was released the deposit decreased, and at low pressure dropped to the bottom almost entirely. Evidently the solid film was broken by the pressure of the gas, and the fluffy deposit was the disintegrated film.

The effect due to rising temperature which we observe is not explained so evidently. It would seem that the solid oxide film was rendered more porous by the temperature increase, and hence

---

[1] Schulze, Ann. d. Physik, 28, p. 787.

the gas layers were dissipated, that is, the formation on the plate was destroyed. This would accordingly not recover immediately. The temperature effects however require more investigation.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
December, 1909.

# PROCEEDINGS
# OF THE
# AMERICAN PHYSICAL SOCIETY.

## THE RELATION BETWEEN ZERO SHIFT AND SIZE OF WIRE IN THE MOVING COIL GALVANOMETER.[1]

BY WALTER P. WHITE.

THE displacement of zero usually produced by the deflection of a moving coil galvanometer is at present one of the most serious sources of inconvenience or error with that instrument and is not so well understood as most other features of the design. It increases, of course, with the magnetic impurity of the materials in the coil, and is diminished by the use of a radial field. For both of these further study seems desirable. It also depends on the coil constants, and in the following way: Let $F$ be the proportion of "dead" material (frame, horizontal wire, mirror, etc.) in the coil, let $X$ be the proportion of magnetic impurity in the coil material lying in the field, $K$ the moment of inertia of the coil, $T$ the periodic time, $R$ the critical or total circuit resistance, $V$ the voltage sensitiveness, $d$ the diameter of the wire in the coil. The torque due to magnetic impurity will be taken as proportional to the square of the field strength and to the total amount of magnetic impurity lying in the field — which is true to a very close approximation. Then, from equations previously given[2] it is easy to show that the zero displacement for a given deflection is proportional to

$$\frac{X}{1-F}\,\frac{RT}{K}\,d^4, \quad \text{or to} \quad \frac{X}{1-F}\,\frac{V^2R^2}{T^2}\,d^4.$$

That is, for a coil of given performance, zero shift increases with the size of wire used and with the dead material, and when so expressed is independent of the field strength, shape of coil, etc., except as the shape

---

[1] Abstract of a paper presented at the Washington Meeting of the Physical Society, April 22–3, 1910.

[2] Everyday Problems of the Moving Coil Galvanometer, Walter P. White, PHYS. REV., XXIII., 382, 1906.

affects the amount of dead material. It increases with the excellence of the galvanometer as measured by the smallness of $K$, that is, by the delicacy of the coil; for a given coil it is less for shorter periods. If, however, the shorter period is added to a given sensitiveness, which involves a further increase in the galvanometer performance, the shift increases. If the shift is diminished by diminishing the size of wire in the coil, this may increase the coil resistance. Hence an *unnecessarily* small coil resistance may prove a detriment in moving coil galvanometers. If finer wire is used, the resistance and moment of inertia of the coil can be kept constant by increasing the width. This change will require a stronger field, hence in such a case the use of a stronger magnet actually contributes toward reducing the error from magnetic impurity in the coil. Of course, a decrease in the size of wire increases the relative weight of the insulation, and therefore increases the constant, $F$. An exact estimate of this effect is impossible, as long as the constants of the particular wire and the distribution of the magnetic impurity between metal and insulation are unknown, but a rough calculation indicated in one case (for silk insulation) that the magnetic difficulty is not likely to be diminished by reducing the diameter of the wire below 2 mils (.05 mm.).

For ballistic galvanometers, a long period is desired, and it is clear from the first expression above that as far as zero shift is concerned the increase can best be obtained by increasing $K$, the moment of inertia of the coil. In that case, since $T$ only increases as the square root of $K$, the zero shift will actually diminish. The ballistic sensitiveness, however, is proportional to the square root of $T/K$, hence this change reduces the sensitiveness and may sometimes be undesirable. Since, however, the second expression above becomes, for the ballistic galvanometer,

$$\frac{X}{1 - F} V^2 R^2 d^4$$

the increase of period will not increase the zero shift unless the sensitiveness is actually increased at the same time. As to the factor $F$, the practice of weighting the coil increases it and hence is undesirable as far as magnetic difficulties are concerned. It is better to increase either the number of turns of wire, or, if this would give too high resistance, the width of the coil. Of course, an extreme increase in coil width also increases the factor $F$ unduly. The best arrangement in any given case may depend upon the balance of advantage among a number of different factors, which is too complicated for formulation.

## Neutral Contacts and Switches.[1]

By Walter P. White.

THE potentiometer is a more recent instrument than the wheatstone's bridge. In many cases, apparatus and methods for electrical measurements are now adapted to resistance measurements, where low-resistance contacts are imperative and thermal electromotive forces must take care of themselves. With the newer methods of measuring electrical quantities by means of electromotive force rather than by resistance, this necessity for small contact resistance largely disappears, and it becomes possible to gain a decided advantage by using neutral contacts, that is, contacts which are free from electromotive force. Such a contact is also needed at one point in a potentiometer system.

One requirement for a neutral contact is of course obvious, namely, that both metals be alike thermoelectrically. Another less familiar requirement is that there shall be no temperature gradient across the contact surface, since in that case the surface layer of the metal, always different from the inside, will form with it a thermoelement. The necessary elimination of temperature gradient is most easily secured by making the contact between two thin strips of metal (from .1 to .3 mm. thick), one of them backed by cork or other thermal insulator. The easiest way to get homogeneous metal is also to cut strips from the same sheet, hence practically the essential of a neutral contact is the use of two thin strips of the same sample of metal. When one side of such a combination was placed in direct contact with a plate heated 45° above room temperature, the resulting electromotive force in the contact was too slight to measure, that is, less than .1 microvolt. Under ordinary temperature conditions, therefore, such contacts are practically quite neutral. The metal need not be copper, but if it is not, junctions where the switch metal joins copper must enter as pairs oppositely directed in the circuit, and these must be small, close together, and well shielded, so that the temperatures of the two junctions shall always be the same.

Three forms of neutral contact have been found useful: (1) A sliding contact, much like an ordinary switch, can be arranged with thin strips of metal. (2) Two tongues of metal can be pressed together directly without sliding. Even with copper such an arrangement requires surprisingly little cleaning, and it produces a switch of very simple construction. One rather complicated switchboard was constructed in this way without a single soldered joint. The resistance of such a contact with about 1 kg. pressure is about the same as that of the best sliding

---

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22–3, 1910.

switches. (3) A tongue of copper pulled through an ordinary wooden clothespin can be clamped upon a similar strip, forming a remarkably cheap and very effective neutral contact for semi-permanent connections, or even, in some cases, for use as a switch contact.

The last form of contact has decided advantages aside from its neutrality. Compared with a binding post, it is quicker to manipulate and yet much less likely to become loose, is much more adaptable and convenient, and is inferior only in its slightly greater bulk. A cable with such contacts at its ends, ready for instant connection to almost anything, is a very convenient laboratory appliance. By the aid of a couple of strips of celluloid, equally convenient two-pole contacts of this type can also be made.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
WASHINGTON, D. C.,
May 5, 1910.

## A SPECIAL FORM OF SURFACE BOLOMETER FOR TOTAL RADIATION EXPERIMENTS.

BY E. F. NORTHRUP.

THE apparatus consists, essentially, of two flat spirals of very fine insulated wire which form two arms of a wheatstone bridge. The flat spirals used are each 4½ cm. in diameter. They are wound with 2 mil silk insulated nickle wire and cemented to very thin sheets of mica. These flat disk spirals are mounted with their faces opposed in a box which replaces a Lummer-Brodhun screen on a photometer bench. When equal radiation from two incandescent lamps falls on the two disks, the wheatstone bridge is balanced. If one lamp gives more total radiation than the other the balance of the bridge is restored by moving the bolometer along the photometer bench to obtain the balance in precisely the same way as a Lummer-Brodhun is moved to obtain a balance for equal luminosity. Thus the device enables the total radiation of two sources to be compared with great sensitiveness in the same manner as a photometer screen allows the luminous radiation of two sources to be compared.

The surface bolometer has the advantages over the thermopile in being more sensitive, in being able to have the sensitiveness adjusted by varying the current through the bridge, in permitting a balance to be obtained, (giving a null method) by varying the resistance along a slide wire, and in having an exceedingly small thermal capacity and time lag as compared with a thermopile. The instrument may be modified to read radiation

---

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22–3, 1910.

in absolute measure and hence become a pyrheliometer. It may be adapted for use as a sensitive and accurate sunshine recorder, and gives promise of being adapted for use as a radiation pyrometer. Slides will be shown fully explaining its construction and uses.

## Some Phosphorescent Salts of Sodium with Cadmium.[1]

By C. W. Waggoner.

In an earlier work[2] the writer studied the phosphorescent decay of some simple compounds, among which was a compound of cadmium sulphate and manganese sulphate, made by simply evaporating to dryness a water solution of the two salts. On taking up this compound for further work a year later it was found that it had absorbed enough water from the atmosphere to destroy its phosphorescent properties, and it was necessary to drive off this water to cause it to regain its original properties of phosphorescence. On testing the original cadmium sulphate, in the anhydrous condition, from which the compounds had been made, it was found to give a yellow phosphorescence when excited by an ultra-violet source. A micro-chemical test showed the presence of a very small trace of sodium and zinc. Some phosphorescent-free cadmium sulphate was procured and a number of phosphorescent compounds were made by adding different salts of sodium to the cadmium sulphate.

From sixteen soluble salts of sodium which were tried, eleven produced phosphorescence when added to the cadmium sulphate. The color of the phosphorescence when excited by the ultra-violet from an iron spark, varied from blue to yellow depending upon the salt used.

With a new form of phosphoroscope decay curves were taken where the intensity was sufficient to measure with a spectrophotometer and the curves all show the same general characteristics as those determined for the $MnSo_4 - CdSO_4$ compounds in the paper referred to above.

The distribution of the energy in the phosphorescent spectrum was determined for $CdSO_4 + NaMnO_4$, $CdSO_4 + MnCl_2$, and $CdSO_4 + MnSO_4$. The curves show a decided similarity, giving the maximum value in all three at about $.566\mu$.

West Va. University.

## The Effect of Temperature on the Ionization of a Gas.[1]

By J. H. Clo.

This paper is an account of an attempt to measure the ionization of a gas at higher temperatures than have hitherto been investigated, and at the same time to obtain sufficient accuracy to detect any small

---

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22–3, 1910.

[2] Phys. Rev., Vol. XXVII., p. 209, 1908.

variation due to change in temperature. The ionization was measured by the usual electrometer method, radium being used as the ionizing agent. It was found necessary to keep the density of the gas constant, in order to eliminate errors due to certain disturbances caused by change of density. As soon as the temperature at which the metallic electrodes began to give off electrons, was reached, it became necessary to eliminate this effect, partly by prolonged heating and partly by balancing the heat leak by means of an auxiliary chamber. Experiments were made on hydrogen and on air. No effect was noticeable. For hydrogen the ionization was constant to about 0.2 per cent. up to temperatures above 400° C., and for air the same uniformity was found to hold up to 500° C., while readings of almost the same constancy were taken in air up to 600° C.

UNIVERSITY OF CHICAGO.

## THE INDUCTANCE OF A METAL TUBE BENT INTO THE FORM OF A CIRCULAR RING.[1]

BY F. W. GROVER.

AN approximate formula for the inductance of a circular ring with circular cross-section has been derived by applying Maxwell's principle of the geometric mean distance to the cross-section of the ring. The formula derived rests upon the assumption that the radius of cross-section is small in comparison with the mean radius of the ring. To obtain the correction due to the departure from this condition in any given case, Max Wien[2] derived a more exact expression by integrating over the cross-section of the ring Maxwell's series formula for the mutual inductance of two parallel, coaxial circles. Wien's expression corrected by Terezawa[3] for an error in the coefficient of $\rho^2/R^2$ is

$$L = 4\pi R\left[\left(1 + \frac{\rho^2}{8R^2}\right)\log\frac{8R}{\rho} - 1.75 + \frac{\rho^2}{24R^2}\right] \qquad (1)$$

where $R =$ mean radius of the ring.

$\rho =$ radius of the cross-section.

This expression was first given by Rayleigh[4] without proof. In this expression, all terms of the order of $\rho^2/R^2$ are included, which gives an accuracy ample for all practical cases. Formula (1) differs from Max-

---

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22–3, 1910.

[2] Wied. Ann., 53, pp. 932–935, 1894.

[3] Tokyo Math. Phys. Soc. (2), 5, p. 84, 1909.

[4] Collected Papers, Vol. II., p. 15.

well's expression principally in the last term, the error being, however, negligible in most cases.

The author of this paper has employed Wien's method to obtain

1. The inductance of a circular ring whose cross-section is a circular line; that is, the practical case of a ring of circular cross-section traversed by a current of very high frequency. The inductance in this case is to the same order of accuracy as the preceding

$$L = 4\pi R\left[\left(1 + \frac{\rho^2}{4R^2}\right)\log\frac{8R}{\rho} - 2\right]. \tag{2}$$

2. The general case of a circular ring whose cross-section is bounded by two concentric circles of radii $r$ and $\rho$, that is, of a tube whose walls have a thickness $(\rho - r)$. For this case

$$L = 4\pi R\left[\left(1 + \frac{r^2+\rho^2}{8R^2}\right)\log\frac{8R}{\rho} - 1.75 - \frac{r^2}{2(\rho^2 - r^2)} + \frac{2\rho^2 + r^2}{32R^2} + \frac{r^4}{(\rho^2 - r^2)^2}\left(1 + \frac{r^2}{8R^2}\right)\log\frac{\rho}{r} - \frac{\rho^4 + r^2\rho^2 + r^4}{48R^2(\rho^2 - r^2)}\right]. \tag{3}$$

For $\beta^2 = \rho^2 - r^2$ small in comparison with $\rho$ we find on expanding (3)

$$L = 4\pi R\left[\left\{1 + \frac{\rho^2}{4R^2}\left(1 - \frac{1}{2}\frac{\beta^2}{\rho^2}\right)\right\}\log\frac{8R}{r} - 2 + \frac{1}{6}\frac{\beta^2}{\rho^2}\left(1 + \frac{3}{8}\frac{\rho^2}{R^2}\right) + \frac{\beta^4}{24\rho^4} + \cdots\right]$$

which for the limiting case of a tube with infinitely thin walls $\beta = 0$ is the same as (2) the expression already found for this case.

If $a = r/\rho$ is small, that is, if the cross-section is nearly solid, we find on expansion

$$L = 4\pi R\left[\left\{1 + \frac{\rho^2}{8R^2}(1 + a^2)\right\}\log\frac{8R}{\rho} - 1.75 + \frac{\rho^2}{24R^2} - \frac{a^2}{2}\left(1 + \frac{\rho^2}{48R^2}\right) - \frac{a^4}{2}\left(1 + \frac{\rho^2}{8R^2}\right) - a^4\log a - \cdots\right]$$

which when $a = 0$ (the case of a solid ring of circular cross-section) goes into (1) the equation of Rayleigh.

For a ring for which $R = 20$ cm., $\rho = 0.5$ cm. The inductance with varying values of the radius of the inner wall of the tube is

| $r$ cm. | $L$ cm. | |
|---|---|---|
| 0 | 1010.032 | solid cross-section |
| 0.125 | 1003.210 | |
| 0.250 | 987.528 | |
| 0.375 | 968.045 | |
| 0.500 | 947.308 | infinitely thin walls |

## A Test Object for Photometry by Visual Acuity.[1]

By Herbert E. Ives.

Acuity of vision, or the ability to distinguish detail, is sometimes used to measure illumination. Acuity is determined by means of test objects, of which there are two classes. With objects of the first class the illumination of the object is changed until certain detail is distinguished. With objects of the second class the illumination is taken as it occurs and the details observed are of different sizes, of continually varying size, or are changed in size by change of distance from the eye. Objects of both classes are open to objection, because memory, visual accommodation, retinal induction and psychological factors of recognition complicate the results. An ideal test object should possess details whose fineness could be changed uniformly, accompanied by no change in illumination, flux of light entering the eye, or distance.

These requirements are admirably met by a test object composed of two coarse transmission diffraction gratings laid one over the other. On rotation of one about an axis normal to their surfaces, broad dark bands appear whose separation changes uniformly and without the observer being cognizant of change until the bands reach the size necessary for visibility.

The distance of the bands from each other varies inversely as the chord of the angle. As the angle need vary only about 20 degrees, the chord and arc are practically the same, so that the visual acuity is directly as the angle between the grating lines.

## A Graphic Recorder for Cooling Curves.[1]

By C. B. Thwing.

The method consists in comparing the rate of cooling of the test piece with the rate of cooling of an adjacent neutral body. The instrument consists of two galvanometers recording on a single chart carried by one clock. The needles are depressed on the chart eight times per minute. The chart has two speeds, one of three fourths of an hour, one of one and one half hours. One galvanometer is attached to a thermocouple imbedded in close contact with the metal of the neutral body; the other measures the E.M.F. of a series of short thermocouples, one junction of which is imbedded in the neutral body, the other junction in the test piece. The size of the test piece need not exceed 25 mm. in length, and the same in diameter, having a cavity 8 mm. in diameter and 12 mm. deep.

The temperature range of the differential pyrometer is 50°, more or less as desired.

---

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22-3, 1910.

# INDEX TO VOLUME XXX.

## ERRATA.

Page 342. The equation a little above the middle of the page, which now reads

$$L_1 = \frac{2ma}{d_1^{\,2}}\left[2cf + \frac{a(\overline{c - a}^2 + 2c^2)}{3}f''\right]$$

should read

$$L_1 = \frac{2ma}{d_1^{\,2}}\left[2cf + \frac{c(a^2 + c^2)}{3}f''\right]$$

the expression being the same as that found when $a > c$.