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Piggybacking on Insider Trades

*Asani Sarkar*

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Piggybacking on Insider Trades

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## ABSTRACT

The effect of piggybacking (copying of insider trades by other traders) on insider trades is considered in the context of an asymmetric information leader-follower model. When the piggybacker is able to observe the insider's trades perfectly, insider information is revealed exactly in the separating equilibrium. In order to signal poor information, the insider restricts trading. A surprising result is that these strategic effects may cause price informativeness and insider's trading volume to be negatively related to information quality for some parameter values. The main policy implication is that requiring trade intentions disclosure from insiders will reduce trading volumes and market efficiency for some parameter values.



Insider trading spawns a host of people who try to piggyback on these trades. Outsiders can monitor publicly available statistics such as the SEC's "Official Summary of Insider Trading." Others, such as bankers and brokers, may observe insider trades through their relationship with insiders. Rational insiders must condition their behavior upon the presence of piggybackers. The strategic interaction that results between insiders and piggybackers affects insiders' equilibrium behavior in ways which are important to our understanding of how financial asset markets operate.

The insider-piggybacker interaction is modelled as a leader-follower game between two traders who use trade size as the strategic variable. It is based upon the one-shot version of Kyle's [1985] trading model. In stage one noise traders and the insider (leader) trade simultaneously. The former trade a fixed amount while the leader's trading volume is based on his private piece of information about the random value of a single risky asset. In stage two, the piggybacker (follower) trades after observation of the leader's quantity choice in stage one. When all the orders are in, the market-maker sets the price of the asset to clear the market.

If the leader's trade is observed exactly (perfect piggybacking) then the leader has an incentive to restrict trades in order to signal poor information. In a separating equilibrium, however, the follower correctly infers the leader's information. In this setting the use of valuable information is costly to the leader because the follower successfully piggybacks on his trades and drives up the asset price. I show, in fact, that the leader may throw away valuable information

by reducing his trades in response to better information. This is consistent with empirical evidence that the size of insider transactions is unrelated to the value of information.<sup>1</sup> As a consequence, when information precision increases, prices may become less informative (unlike Grossman and Stiglitz [1981]) and markets more liquid (unlike Kyle).<sup>2</sup>

The perfect piggybacking model is comparable to the limit pricing paper of Milgrom and Roberts [1981], who solve a signalling model in which prices convey information about an incumbent firm's cost function. Gal-Or [1987] constructs a leader-follower model with asymmetric information in the context of a product market. The difference with this paper is that here the pricing rule is endogenously determined and non-linear decision rules are also worked out. The idea of trade size conveying information is present in Easley and O'hara [1987], Golsten and Milgrom [1985], Madhavan [1988], and numerous empirical papers (see footnote 1 for some references).

Under noisy piggybacking, the follower observes the sum of noise trade and the leader's trade. Therefore observation of the leader's trade only conveys statistical information to the follower. A striking result is that arbitrarily small amounts of noise trading is enough to reverse the comparative static effects of piggybacking. The leader's

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<sup>1</sup>See, for example, Scholes [1972], Jaffe [1974], Seyhun [1986], and Givoly and Palmon [1985].

<sup>2</sup>Markets are more liquid in the sense of "depth" not volume (see Kyle). In liquid markets prices are relatively insensitive to movements in trades.

trade is now positively related to both its information precision and the amount of noise trading and inversely related to the follower's information quality. These comparative static results are consistent with those in Mathews and Mirman [1983] who consider a limit pricing model where the entrant has a noisy observation of the incumbent's price.

Other extensions to the perfect piggybacking model show that (1) information sharing between the two insiders makes the leader strictly better off and (2) as the number of follows go to infinity, each follower's trades approach zero but total trades of all followers increase without bound.

An interesting application of the models is in analyzing pre-trade disclosure laws. The noisy piggybacking model is taken as the benchmark case of no-regulation. A policy requiring trade intentions (TI) disclosure can be interpreted as the leader having to reveal his trades exactly, which is the perfect piggybacking model. The resulting analysis suggests that the TI policy reduces trading volume and may lower market efficiency. I propose instead a policy of deregulating insider trading along with a lump-sum grant to outsider traders financed by a tax on insiders' profits. An additional advantage of this policy is that it maintains the benefit of insider trading, which is to increase the informativeness of the pricing system through the trading process.

The paper is organized as follows. Section 1 surveys the relevant literature. Sections 2 and 3 deal with the perfect piggybacking model. Section 4 describes the model with noisy piggybacking.

Section 5 considers other extensions to the perfect piggybacking model. Section 2 suggests policy implications and Section 7 concludes.

### Section 1. Review of Literature

Markets communicate information between rational market participants in different ways. In the context of securities markets, the focus has long been on the role of equilibrium prices as a reflector of information possessed by various traders. Grossman and Stiglitz, Diamond and Verrecchia [1981], Hellwig [1980], and others have used the concept of a competitive rational expectations model to study the information content of equilibrium prices. Information is incorporated into prices by the trades of insider traders who have private information about the uncertain value of the asset. These traders are rational in the sense that, although they do not know actual outcomes, they are aware of the underlying stochastic model. Traders therefore understand that prices convey information and use this knowledge to form expectations which are correct in equilibrium. The link between information and prices via trades provides an explicit mechanism for information transmission between informed and uninformed traders. Also Grossman and Stiglitz, in particular, show how changes in the precision of private information affect price informativeness and other variables of interest.

A fundamental problem with the competitive paradigm is that informed traders are assumed to be price-takers even though they influence prices through trades. This "schizophrenic" behavior, to

use Hellwig's term, leads to certain problems--chiefly, the paradox discussed by Grossman and Stiglitz. If traders are price-takers and prices are fully-revealing, no trader will want to be informed. But then, there will be no information for prices to reflect and information will have value.

Different approaches have been taken to get over this problem. One way is to add noise to the system, keeping the price-taking assumption intact. This can be done through noisy aggregate supply of the asset, adding noise traders, etc. Another is to work with a "large market," as in Hellwig, where each informed trader is made "small" relative to the market in a specified manner.

A last method involves dropping the price-taking assumption and, instead, allow insiders to act strategically. There are now several papers in this category. Kihlstrom and Postlewaite [1987] look at a well-informed dominant trader who sets prices in a futures market. Realizing that other traders may be able to infer his information from market prices, the monopolist uses a randomized pricing strategy to optimally determine how much of his information to use. In Grinblatt and Ross [1985], uninformed price-taking investors try to infer the monopolist insider's information from the market-clearing price of a risky asset. The monopolist insider is modelled as a Stackelberg leader. Aware of his information leakage, the insider does not use his information to the extent a price-taker would. Gould and Verrecchia [1985] look at a price-setting specialist who has private information about a risky asset. A trader, upon observing prices, can infer information as before. The specialist is assumed to

exogenously add noise to his pricing rule. Grinblatt and Ross allow the insider to add noise to demand and find that adding noise is not an optimal strategy.

Allen [1987] asks: what is the social value of asymmetric information? He shows that cheaper information is Pareto-inferior because (1) risk-sharing opportunities are reduced as prices become better signals and (2) uninformed agents have to trade with more informed traders. An important implication of the model is that insider trading may improve the incentives of current and future managers, as originally suggested by Manne [1966]. Dye [1984] has also demonstrated this result in the context of a principal-agent model.

Another strand of the literature looks at equilibrium trades as information signals. Empirical research provides considerable support to the contention that insider trades have informative content. Scholes [1972] finds that secondary offerings (many of which are issued by insiders) may act as a signal to other investors that the seller has adverse information about the firm. Jaffe [1974] studies the information content of the SEC's "Official Summary of Insider Trading" and finds that uninformed outsiders can get significant abnormal returns from replicating these trades. And, finally, Givoly and Palmon [1985] content that "significant abnormal returns are generated in the wake of these trades themselves . . . outside investors follow the footsteps of insiders."

The theoretical literature primarily focuses on market-makers and their ability to infer insiders' information by observing their



market-orders. The seminal article in this literature was by Kyle, where a monopolist insider behaves strategically by explicitly taking into account the effect of his trades on the price established. Uninformed liquidity traders camouflage the insider from competitive market-makers who infer information through observation of aggregate trades in the market. Easley and O'hara presents a model with two exogenously fixed order size levels--large and small. Since informed traders wish to trade larger amounts at any given price, trade size conveys information to the market-maker. In Glosten and Milgrom trade size is fixed but the act of trading (whether the trader buys or sells, for example) is informative to a specialist who sets bid-ask spreads. Madhavan distinguishes between continuous and periodic trading mechanisms. In the continuous dealer market, dealers learn from the sequence of traders' market-orders, while traders learn from dealers' bid-ask quotes. In the periodic batch system, traders submit price-dependent orders so that prices act as information signals.

## Section 2. The Model With Perfect Piggybacking

There are three kinds of traders who exchange among themselves a risky asset for a riskless asset: uninformed noise traders who trade randomly;<sup>3</sup> two insider traders with private informations  $\tilde{s}_1$  and  $\tilde{s}_2$

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<sup>3</sup>Noise-traders as a group lose money in equilibrium. Their presence is often justified by considering them as life-cycle or liquidity constrained traders. An alternative assumption is noisy total asset supply. Allen uses the assumption of a stochastic birth rate and fixed total supply to obtain random per capita asset supply.

about the liquidation value  $\tilde{v}$  of the risky asset; and market-makers who set prices efficiently conditional on the aggregate quantities traded in the market.

Initially, noise traders and the insider who receives information first (leader) trade simultaneously. Noise traders trade a fixed quantity  $\tilde{u}$  while the leader trades  $\tilde{x}_1$  based on his private information  $\tilde{s}_1$ . In the second stage, the other insider (follower) trades after observing  $\tilde{x}_1$  and his private information  $\tilde{s}_2$ . Finally, market-makers set a price and trade the quantity that clears the market.

Denoting  $I_1$  as the leader's information set and  $I_2$  as the follower's information set, we have:

$$(2.1) \quad I_1 = \{s_1\} \quad I_2 = \{s_2, x_1\}$$

Further, the insiders' signals are of the form:

$$(2.2) \quad \tilde{s}_k = \tilde{v} + \tilde{e}_k, \quad k = 1, 2$$

where  $\tilde{e}_k$  is a random noise term uncorrelated with  $\tilde{v}$  and independent of  $\tilde{e}_j$ ,  $j \neq k$ . All random variables are assumed to be normally distributed with zero mean and constant variance.

$$(2.3) \quad \tilde{v} \sim N(0, \Sigma_0)$$

$$\tilde{e}_k \sim N(0, \Sigma_{e_k}), \quad k = 1, 2$$

$$\tilde{u} \sim N(0, \Sigma_u).$$

These distributional assumptions, though restrictive, allow me to find a unique equilibrium under linear trading rules and derive

comparative static results. Also, I will only consider pure strategies. Non-linear trading rules are discussed in Section 5.

Note that the market described has the character of an auctions market because prices are determined only at the final stage. The insiders place market orders while the market-makers choose a pricing rule  $P$  as a function of total trade (insider trade plus noise trade). Insiders' market orders and noise trade, along with the pricing rule  $P$ , determines equilibrium trading price at the final stage. Figure 1 summarizes the sequence of moves.

Modelling the trading protocol in this manner allows the follower's piggybacking to affect the leader's trading strategy and the exploitation of his private information. So long as insiders are not able to exploit all their information instantaneously, piggybacking will have real effects on insiders' behavior. The model is able to capture these effects in a relatively simple and stylized setting.

The equilibrium concept followed is that of sequential equilibrium, introduced by Kreps and Wilson [1982].

## 2.1 Sequential Equilibrium

A sequential equilibrium is a strategy triple  $(X_1, X_2, P)$  and a set of beliefs on the follower's part such that:

- (1) The leader's trading strategy  $X_1$  and the follower's strategy  $X_2$  are best responses to each other.
- (2) For any  $x_1$ ,  $X_2(x_1)$  maximizes the follower's expected profits where these expectations are taken with respect to some beliefs over the leader's information.

(3) Given  $X_1$  and  $X_2$ ,  $P$  satisfies the following efficiency condition.

$$(2.4) \quad p = E(\tilde{v} | y = x_1 + x_2 + \mu) + \Gamma \tilde{y}.$$

Condition (2.4) is derived by assuming that market-makers earn zero expected profits conditional on  $y$ , the total trade in the market.<sup>4</sup> (It is required that market-makers take their expectations with respect to the same beliefs as the follower.) Thus equilibrium prices satisfy semi-strong efficiency. The linear relation between  $p$  and  $y$  follows from the assumption of normality, which implies that  $\Gamma$  is the regression coefficient of the linear project of  $\tilde{v}$  and  $\tilde{y}$  and is given by the normal equation:

$$(2.5) \quad \Gamma = \frac{\text{Cov}(v, y)}{\Sigma_y}$$

where  $\text{Cov}(v, y)$  is the covariance between  $v$  and  $y$  and  $\Sigma_y$  is the variance of  $y$ . Kyle has interpreted  $1/\Gamma$  as market liquidity or "depth."

## 2.2 Market Liquidity

Market liquidity is defined as the volume of trading required to change prices by one dollar and is measured as  $1/\Gamma$ .

Intuitively, liquid markets are those that allow investors to trade large volumes of stocks in a short period of time without changing prices by large amounts.

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<sup>4</sup>This assumption can be justified by interpreting (2.4) as the equilibrium outcome of a Bertrand game between at least two market-makers who only observe  $y$ , as noted by Kyle.

The sequential equilibrium concept defined in 2.1 requires insiders' and market-makers' strategies to be optimal with respect to given beliefs over  $\tilde{s}_1$ . Condition two requires that these beliefs be given by Bayes' rule along the equilibrium path but places no restriction on beliefs off-the-equilibrium path. In general, I will look for a separating equilibrium which will be defined rigorously below. In these equilibria, the leader's information is revealed perfectly to the follower and so the impact of piggybacking on the leader's trading strategy is maximized.

Let  $z = X_1(s_1)$  be the leader's trade and  $s^*$  the follower's beliefs about the leader's information  $s_1$  upon observing  $z$ . Then  $s^* = X_1^{-1}(z)$  if  $X_1$  is one-to-one. Define the follower's problem as:

$$(2.6) \quad \max_{x_2} E\{[(v - \Gamma y)x_2] | s^*, s_2\}.$$

The follower maximizes his expected profits conditional on his information set  $I_2$ . Condition (2.6) incorporates the fact that  $p = \Gamma y$  (see equation 2.4). The first-order condition for this problem yields  $x_2 = [E(v | s^*, s_2) - \Gamma z] / \Gamma$ . The second-order condition is satisfied by  $\Gamma > 0$ . It is shown in the appendix that:

$$(2.7) \quad E(v | s^*, s_2) = t_1(1-T)s^* + Ts_2$$

$$(2.8) \quad T = \frac{t_2(1-t_1)}{1-t_1t_2} \quad \text{where } t_i = \frac{\Sigma_0}{\Sigma_0 + \Sigma_{e_i}}, \quad i = 1, 2$$

$$= \frac{1/\Sigma_{e_2}}{1/\Sigma_{e_0} + 1/\Sigma_{e_1} + 1/\Sigma_{e_2}}$$

Note that  $t_i \in [0, 1]$  and is a measure of the unconditional precision of  $s_i$ . For example,  $t_i = 1$  implies that  $s_i$  is a perfect signal. Second, since  $1/\Sigma_{e_i}$  is the precision of  $s_i$  (conditional on  $v$ ),  $T$  is the proportion of total precision contributed by  $s_2$ . Similarly,  $t_1(1-T)$  is the proportion explained by  $s_1$ . These, then, are the weights placed by the follower on his signal  $s_2$  and his inference  $s^*$  in learning about  $r$ . Re-writing the follower's first-order condition:

$$(2.9) \quad x_2 = [t_1(1-T)s^* + Ts_2 - \Gamma z]/2\Gamma.$$

Denote  $V(s_1, s^*, z)$  as the leader's expected profits when his information is  $s_1$ , the follower's inference is  $s^*$  and the leader chooses  $z$ .<sup>5</sup>  $V(s_1, s^*, z) = E[\{(v - \Gamma z - \Gamma x_2 - \Gamma \mu)x_1\} | s_1]$ . Substituting (2.9) for  $x_2$  and using the facts that (i)  $E(\tilde{v} | s_1) = t_1 s_1$  and (ii)  $E(\tilde{s}_2 | s_1) = t_1 s_1$  yields the following form for  $V$ :

$$(2.10) \quad V(s_1, s^*, z) = (t_1 s_1 - \frac{t_1(1-T)s^*}{2} - \frac{Tt_1 s_1}{2} - \frac{\Gamma z}{2})z.$$

### 2.3 Separating Equilibrium Strategy

$X_1(s_1)$  is a separating equilibrium strategy if it is one-to-one and satisfies the following incentive compatibility (IC) condition:

$$(2.11) \quad X_1(s_1) = \underset{z}{\operatorname{argmax}} V(s_1, X_1^{-1}(z), z)$$

$X_1(s_1)$  is a linear separating equilibrium strategy if  $X_1(s_1)$  is linear in  $s_1$ .

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<sup>5</sup>This exposition follow Mailath [1987].

### Section 3. Effect of Piggybacking on Insider Trades

A unique separating equilibrium will be shown to exist when  $X_1(s_1)$  is linear in  $s_1$  leaving non-linear trading rules for Section 5. However, comparative static results will only be derived for the linear separating equilibrium.

Suppose  $X_1$  is linear and satisfies  $X_1 = A_1 s_1$ . Then  $X_1^{-1}(z) = z/A_1$  and the IC condition (2.11) requires

$$(3.1) \quad t_1(1-T)s_1/2 - t_1(1-T)z/A_1 - \Gamma z = 0.$$

The following proposition characterizes the equilibrium.

Proposition 3.1. If  $t_1 \in (0, 1)$ ,  $t_2 > 0$ , and  $\Sigma_\mu > 0$ , then there is a unique separating equilibrium under linear trading strategies for the model described in Section 2. The equilibrium  $X_1$ ,  $X_2$ , and  $P$  are given by:

$$(3.2) \quad X_1(s_1) = A_1 s_1, \quad X_2(x_1, s_2) = B_1 x_1 + B_2 s_2,$$

$$P(y) = \Gamma y, \quad y = x_1 + x_2 + \mu$$

where  $A_1$ ,  $B_1$ ,  $B_2$ , and  $\Gamma$  are defined as

$$(3.3) \quad A_1 = t_1 T/2\Gamma, \quad B_2 = T/2\Gamma, \quad B_1 = (1/T) - 1.5,$$

$$\Gamma = (Q\Sigma_0/\Sigma_\mu)^{1/2}/2, \quad Q = t_1(1-T) + T - t_1 T^2/4,$$

and  $t_1$ ,  $t_2$ ,  $T$  are defined in (2.8).

Proof: The solution to (3.1) gives the equilibrium  $A_1$ .  $B_1$  and  $B_2$  are obtained from (2.9) by substituting  $s^* = z/A_1$  and the equilibrium

value for  $A_1$ . Finally,  $\Gamma$  is derived from the pricing rule (details are given in the appendix).

Figure 2 illustrates the linear separating equilibrium described in Proposition 3.1. The example assumes  $(\Sigma_0/\Sigma_\mu)^{1/2} = 1$ ,  $t_1 = 0.75$ ,  $t_2 = 0.5$ .  $V_1$  and  $V_2$  are the leader's iso-profit curves for  $s_1 = 1.2$  and  $s_1 = 3$ , respectively.  $L$  is the leader's linear separating equilibrium strategy. Mailath shows that  $L$  must be tangent to  $V_1$  at  $[1.2, L(1.2)]$  and to  $V_2$  at  $[3, L(3)]$ . The figure illustrates that  $L$  fulfills this requirement.

To gain insight into the effect of piggybacking by the follower, consider two special cases.

Corollary 3.1. Suppose piggybacking has no value to the follower. In equilibrium,

- (i) The leader trades more than he does with piggybacking. Further, his trades are a strictly increasing function of his information precision.
- (ii) Market efficiency (represented by the informativeness of the price system) is a strictly increasing function of information precision.
- (iii) Market liquidity (defined in 2.2) is a strictly decreasing function of information precision.

Proof: The formal proof is given in the appendix.



Intuitively, the follower does not benefit from piggybacking if  $E(v|s^*, s_2) = E(v|s_2)$ . In other words, observation of the leader's trade  $z$  (and so  $s^*$ ) provides the follower no information about  $v$  that is not already contained in  $s_2$ . A sufficient (but not necessary) condition for this is  $s_1 = s_2 = s$  which means that both insiders have the same piece of information. It can be shown that the resulting outcome has the characteristics of a symmetric information Stackelberg equilibrium. The leader has a strict first-mover advantage, which he exploits by pre-committing to a large position in the asset. He trades more than the follower and obtains higher expected profits.

The result on market efficiency is similar in spirit to the comparative static result in Grossman and Stiglitz. Efficiency is measured by  $\text{var}(v|p)$ --i.e., the variance of the asset value conditional on prices. This is a measure of the informativeness of equilibrium prices.<sup>6</sup> As in Grossman-Stiglitz, the leader's trades become more sensitive to changes in his information as  $t_1$  increases. So, movements in aggregate trade  $y$  become more informative about  $v$ . Since, prices are proportional to  $y$ , prices also become more informative.

Part (iii) re-affirms Kyle's result (formalizing Bagehot's [1971] intuition) that market-makers reduce liquidity to compensate themselves for bad trades with insiders.<sup>7</sup>

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<sup>6</sup>Strictly speaking, price informativeness is measured by  $\Sigma_0 - \text{var}(v|p)$ .

<sup>7</sup>Market-makers face an adverse selection problem due to the presence of informed traders. On average, they lose money to insiders which they make up by profiting with respect to noise traders.

Corollary 3.2. Suppose piggybacking is the follower's only source of information. Then the following is true.

(i) There is no separating equilibrium under linear trading strategies.

(ii) Semi-pooling equilibria and a unique non-linear separating equilibrium exist. In both kinds of equilibria the leader restricts his trade relative to the case where there is no value to piggybacking.

Proof: For the formal proof, the reader is referred to Propositions 2.1-2.3 in Sarkar [1988]. Intuitively, under the conditions of the corollary, the follower's information set is simply  $\{x_1\}$  and the leader's trading strategy  $X_1$  is linear in  $s_1$ ; and part (i) of the corollary says that there is no  $x_1 > 0$  which satisfies the (IC) condition 2.11. At the margin, the leader always benefits by reducing his trade because the gain from reducing the follower's piggybacking offsets the profits foregone from not exploiting his information. This result is similar to Corollary 2 in Gould and Verrecchia where a specialist with private information about a risky asset sets a price which is observed by a trader. They show that if the trader has no information other than his price observation, equilibrium would not exist unless the specialist pre-commits to adding noise to his pricing rule.

Since the follower's piggybacking has strong adverse effects on the leader's trades, the leader has an incentive to complicate his trading strategy, making it more difficult for the follower to infer

his information. Sarkar finds semi-pooling equilibria, where the leader trades only if his signal exceeds a critical value. In the unique non-linear equilibrium, his trades are equal to the first-best Stackelberg level for the best information but are strictly below that level elsewhere.<sup>8</sup>

The two corollaries consider two special cases of Proposition 3.1. When piggybacking has no value, the leader benefits from the advantage of trading first, akin to a Stackelberg leader. When the follower is solely dependent on piggybacking as a source of information about  $v$ , its adverse affect on the leader's trades is so strong that no separating equilibrium may exist. In general, piggybacking has less extreme but still significant effects on the leader's trades. In particular, the leader restricts his trading to mislead the follower into believing that his information is worse than it actually is. In a separating equilibrium, the leader does not succeed and his information is perfectly revealed. But the attempt to hide his information nevertheless constrains the leader to lower his equilibrium trading volume relative to a world where piggybacking has no value.

In fact, the next proposition shows that the strategic cost of using information for the leader can be so high that he may throw away valuable information.

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<sup>8</sup>For the semi-pooling equilibrium, the leader's signal distribution is truncated at both ends. For the non-linear equilibrium, the truncation is at the upper end only.

Proposition 3.2. Suppose the follower has less than perfect information. Then there exists a critical level of information precision  $t_1^*$  for the leader (which is a function of the follower's information precision  $t_2$ ) such that for  $t_1$  less than (greater to)  $t_1^*$ , the leader's trades and expected profits are a strictly increasing (decreasing) function of  $t_1$ .

Proof: See appendix.

Increases in the accuracy of the leader's information  $t_1$  also increases the value of piggybacking to the follower who reacts very strongly to the leader's trades. For  $t_1 > t_1^*$ , increases in the leader's trades prompt the follower to put so much volume on the market that market-makers set a very high price. Therefore, the leader finds it more profitable to reduce his trades when  $t_1 > t_1^*$  to restrain the follower's piggybacking. Figure 3 illustrates that the leader's trades are a single-peaked function of  $t_1$ .

Proposition 3.2 has an important implication for market efficiency, as described below. But, first, it is necessary to inquire about the behavior of liquidity. Figure 4 shows that market liquidity may increase with respect to both  $t_1$  and  $t_2$ . Due to the constraint on the leader's trades brought about by piggybacking, market-makers face a lower probability of bad trades with insiders. They are, therefore, less inclined to reduce liquidity to compensate for insider trades.

Proposition 3.3. When both insiders are well-informed ( $t_1$  and  $t_2$  close to 1), market efficiency is a decreasing function of the leader's information precision.

Proof: See appendix.

Market efficiency is positively related to both liquidity and the covariance between asset value  $v$  and total trades  $y$ . For low values of  $t_1$ , there is little piggybacking, markets are liquid and efficiency increases with  $t_1$  (Corollary 3.1). For high values of  $t_1$  and low values of  $t_2$ , piggybacking is effective so that trades are not informative (Proposition 3.2) but liquidity is increasing with  $t_1$  (see Figure 4) so efficiency also increases. When both  $t_1$  and  $t_2$  are high, further increases in  $t_1$  decreases both the covariance term and liquidity, so that prices become less information. Figure 5 illustrates.

The discussion above suggests that (i) piggybacking acts as a mechanism for reducing the volume of insider trading but (ii) the strategic behavior induced by piggybacking on insiders may cause a reduction in market efficiency.

#### Section 4. The Model With Noisy Piggybacking

Suppose that, in the model described in Section 2, the follower observes the sum of the leader's trade and the noise trade  $y_1 = (x_1 + \mu)$  instead of just  $x_1$ . Noise traders' activities, therefore, camouflage the leader's trades and the follower can no longer infer the leader's information perfectly. If, for example, the follower observes a high

$y_1$  he does not know whether the leader has good information and  $x_1$  is high or whether the leader has poor information but noise trading  $\mu$  is high. In this sense, the follower acts like Kyle's market-makers.

To solve the model assume that the leader's trading strategy is linear and satisfies  $X_1 = A_1 \tilde{s}_1$ . Given an observation of  $y_1$  and his private information  $s_2$ , the follower selects to trade an amount  $x_2$  that maximizes his expected profits.

$$\text{Max}_{x_2} E\{([v - \Gamma y_1 - \Gamma x_2]x_2) | y_1, s_2\}$$

The first-order condition for this problem yields:

$$(4.1) \quad x_2 = [E(v | y_1, s_2) - \Gamma y_1] / 2\Gamma \quad \text{where}$$

$$(4.2) \quad E(v | y_1, s_2) = a_2 s_2 + a_1 y_1 (1 - a_1), \quad a_1 = \frac{t_1 t_\mu A_1}{t_1 (1 - t_\mu) + t_\mu A_1^2},$$

$$a_2 = \frac{t_2 (1 - a_1 A_1)}{1 - t_2 a_1 A_1}, \quad t_\mu = \Sigma_0 / (\Sigma_0 + \Sigma_\mu), \quad t_i = \Sigma_0 / (\Sigma_0 + \Sigma_{e_i}),$$

$$i = 1, 2.$$

Details of the computation are given in the appendix.  $t_\mu$  can be interpreted as a measure of the noisiness of the follower's observation of  $y_1$ . (This interpretation of  $t_\mu$  is consistent with Mathews and Mirman.) For example, if  $\Sigma_\mu = 0$  then  $t_\mu = 1$  and this is the same as the follower observing  $x_1$ . Conversely,  $\Sigma_\mu$  infinity implies  $t_\mu = 0$  and  $y_1$  conveys no information to the follower.

Substituting  $x_2$  from (4.1) back into the leader's expected profit function and solving gives the leader's equilibrium trading strategy:

$$(4.3) \quad x_1 = A_1 s_1, \quad A_1 = \frac{t_1(1-a_2/2)}{\Gamma + a_1(1-a_2)}$$

Given  $\Gamma$ ,  $A_1$ ,  $a_1$  and  $a_2$  can be solved as a simultaneous equation system. The equilibrium is characterized below.

Proposition 4.1. For  $t_1 \in (0, 1)$ ,  $t_2 > 0$  and  $t_\mu \in (0, 1)$ , there is a unique equilibrium under linear strategies  $X_1$ ,  $X_2$ , and  $P$  where:

$$(4.4) \quad X_1(s_1) = A_1 s_1, \quad X_2(y_1, s_2) = B_1 y_1 + B_2 s_2, \quad P(y) = \Gamma y \text{ where}$$

$$A_1 = U + \frac{t_1 \Gamma}{6\Gamma}, \quad B_1 = \frac{a_1(1-a_2)}{2\Gamma} - 0.5, \quad B_2 = \frac{a_2}{2\Gamma}$$

$a_1$  and  $a_2$  are given in (4.2) and  $U$  is a constant term determined by  $t_1$ ,  $t_2$ , and  $t_\mu$ .  $U$  is described in equation (A10) of the appendix.  $\Gamma$  is defined in (A14) of the appendix.

Proof: See appendix.

As before, some intuition maybe obtained by looking at special cases of the result. First, when  $t_\mu = 1$  in equations (4.1)-(4.3)  $A_1$ ,  $B_1$ , and  $B_2$  have the same expressions as in the model where the follower observes  $x_1$  perfectly. In this case, of course,  $\Sigma_\mu = 0$  and so no equilibrium exists. Second, suppose  $t_\mu = 0$ . Then the follower's observation is perfectly noisy,  $B_1 = 0$  and there is no value to piggy-backing. In fact, a stronger result can be proved.

Corollary 4.1. Suppose the follower had no information other than his noisy observation  $y_1$ . Then in equilibrium,

- (i) The follower does not trade and makes zero profits.
- (ii) The leaders makes monopoly profits and trades the monopoly quantity.

Proof: Substitute  $t_2 = 0$  in equations (4.1)-(4.3). The proof that these are the monopoly trade and quantity levels is in the appendix.

The above discussion suggests that adding noise to the follower's observation mitigates the effect of piggybacking on the leader's trades. Without noise, Corollary 3.2 stated that there was no equilibrium with  $t_2 = 0$ . With some noise, the leader can effectively be a monopolist when  $t_2 = 0$ .

It may be expected that the leader can exploit his information more freely now that the threat of piggybacking is diluted. The next proposition shows that this is indeed true.

Proposition 4.2. If  $t_\mu > 0$ , then

- (i) The leader's trades are a strictly increasing function of his information precision.
- (ii) For a given quantity and quality of information, the leader's trades are a strictly increasing function of noise.

Since the complete solution (i.e., including  $\Gamma$  and the pricing function) to the model with noisy piggybacking cannot be obtained in closed-form, Proposition 4.2 cannot be proved analytically. However,



$A_1$  in (4.4) and  $\Gamma$  can be solved numerically as a system of two non-linear equations in  $A_1$  and  $\Gamma$ . Figure 6 shows how  $A_1$  behaves when  $t_1$  varies over the unit interval, given  $t_2$  and  $t_\mu$ . What's striking is the fact that arbitrarily small amounts of noise is sufficient for the comparative static effects of piggybacking to be reversed. Part (ii) of the proposition is intuitive since noise hides the leader's information from the follower. Figure 6 also illustrates that, given  $t_1$  and  $t_\mu$ , the leader's trades are a decreasing function of  $t_2$ . When the follower's information is more precise, he is less dependent on the noisy observation and so noise is a less effective masking device.

Proposition 4.2 implies that market efficiency is also an increasing function of the leader's information precision. The behavior of liquidity depends upon the level of  $t_2$ . For low values of  $t_2$ , the follower's trade is not sensitive to his own information. Therefore, market-makers view changes in total trades as due to either changes in the leader's information precision or noise trades. Kyle's analysis applies and liquidity declines with  $t_1$ . For high values of  $t_2$ , the follower has little need to piggyback and behaves more like a Stackelberg follower with a downward sloping reaction function. Changes in the leader's trades  $x_1$  causes the follower's trader to change by some proportion of  $(x_1 + \mu)$  in an offsetting manner. Total insider activity may decline and cause liquidity to increase with  $t_1$ .

#### Section 5. Other Extensions to the Model

In this section, I consider some further extensions to the model with perfect piggybacking. These extensions consider different

situations under which the effect of piggybacking on the leader's trades are diminished.

(a) Information Sharing by Leader

If the leader could credibly convey his information  $s_1$  to the follower, the latter would have no further need to piggyback on the leader's trade.<sup>9</sup> The follower's information set is now  $I_2 = \{s_1, s_2\}$ . Suppose the leader trades  $z$ . Solving the follower's problem in the usual way:

$$(5.1) \quad x_2 = [E(v|s_1, s_2) - \Gamma z] / 2\Gamma$$

$$\text{where } E(v|s_1, s_2) = t_1(1-T)s_1 + Ts_2.$$

The equilibrium trade for the leader is simple to obtain. Remember that the leader's expected profit function  $V$  in (2.10) was defined as a function of his information  $s_1$ , the follower's inference  $s^*$  and his trade  $z$ . Under direct revelation  $s^*$  is replaced by  $s_1$ . So his equilibrium trading strategy is simply the solution to:

$$(5.2) \quad V_z(s_1, s_1, z) = 0.$$

$V_z$  is the partial derivative of  $V$  with respect to  $z$ . The following proposition describes the direct revelation equilibrium and shows that the leader always does better from revealing his information directly as compared to having it inferred indirectly through his trades.

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<sup>9</sup>In other words, moral hazard problems associated with sales and purchases of information are ignored.

Proposition 5.1. (i) The linear strategy equilibrium of the direct revelation model is given by

$$X_1(s_1) = t_1 s_1 / 2\Gamma, \quad X_2(s_1, s_2) = [s_1(0.5-T) + Ts_2] / 2\Gamma,$$

$$\Gamma = (\Sigma_0^M / \Sigma_\mu)^{1/2} / 2$$

where  $\gamma = \frac{3t_1}{4} + \frac{T(1-t_1)}{2} + \frac{T^2(t_1 t_2 - 1)}{t_2}$  and  $T$  is defined in (2.8).

(ii) The leader's equilibrium trades and expected profits are always greater than their levels in the model with perfect piggybacking.

Proof: Part (i) follows directly from solving (5.1) to get the equilibrium  $X_1(s_1)$  and then substituting back into (5.1) to get the equilibrium  $X_2$ .  $\Gamma$  is obtained using (2.4). Part (ii) is proved in the appendix.

#### (b) Many Followers

Here I consider the effect of having several followers piggyback on the leader's trade. For expositional simplicity, I consider the case where  $n$  followers observe the leader's trade  $x_0$  but have no information of their own. Assume that all trading strategies are linear and take the following form.

$$(5.3) \quad x_0 = A_1 s_1$$

$$x_i = Cx_0, \quad i = 1, \dots, n.$$

Given a trade  $z$  by the leader and trades  $x_j$  by followers  $j \neq i$ , follower  $i$  chooses  $x_i$  to maximize his expected profits conditional on his inference about  $s_1$ . In other words, after observing  $z$ , the  $n$  followers play a Cournot game when choosing their trades. Then,  $x_i$  solves:

$$\text{Max}_{x_i} [E(v|s^*) - \Gamma x_0 - \Gamma x_i - \Gamma \sum_{j \neq i} x_j] x_i.$$

Assume that the  $n$  followers are symmetric so that  $x_i = x$  for each  $i = 1, 2, \dots, n$ . Solving for the followers' and leader's trades in the usual way,

$$(5.4) \quad x = \frac{ts_1(1-n)}{2\Gamma}$$

$$(5.5) \quad x_0 = x/(1-n).$$

Finally,  $\Gamma$  is solved using the efficiency condition (2.4). A case of particular interest is the equilibrium outcome when the number of followers increases without bound.

Proposition 5.2. As the number of followers go to infinity,

- (i) Each follower's trades go to zero.
- (ii) Total trades of all followers increase without bound.

Proof: Follows directly from (5.5).

### (c) Non-Linear Trading Strategies

So far, I have only worked with trading strategies which are linear in signals because of their tractability when working out

comparative static results. However, Theorem 1 in Mailath [1987] provides a general method of solving for separating equilibria when strategies are differentiable.

Consider again the  $V$  function defined in (2.10).

$$V(s_1, X_1^{-1}(z), z) = (t_1 s_1 - \frac{t_1(1-T)X_1^{-1}(z)}{2} - \frac{t_1 T s_1}{2} - \frac{\Gamma z}{2})z.$$

Then  $z(s_1)$  is a separating equilibrium strategy if it satisfies:

$$(5.6) \quad \frac{dz}{ds_1} = \frac{-V_2(s_1, s_1, z)}{V_3(s_1, s_1, z)}$$

Assume that  $s_1 \in (-\infty, \tilde{s}_1]$  and  $z(\tilde{s}_1) = t_1 \tilde{s}_1 / 2\Gamma$ . Solving the differential equation (5.6) yields the following unique non-linear separating strategy equilibrium.

$$(5.7) \quad z(s_1) = [CK^{1/T} + t_1 T s_1] / 2\Gamma$$

$$\text{where } K = 2\Gamma - (t_1 T s_1 / z),$$

$$C = [t_1 \tilde{s}_1 (1-T)] / [2\Gamma(1-T)]^{1/T}$$

Details of the solutions are given in the appendix. Note, first, that the linear solution  $z(s_1) = t_1 T s_1 / 2\Gamma$  is also a solution to (5.7) and, second, that the leader plays his Stackelberg strategy when  $\tilde{s}_1 = \bar{s}_1$ . For  $s_1 < \bar{s}_1$ , trades are strictly below the Stackelberg level. In general, therefore, the properties of this equilibrium is consistent with those described for the linear separating equilibrium. For all but the "best" information, piggybacking causes a reduction in the leader's trades.

(d) Randomization by the Leader

Since the leader's information is perfectly revealed in a separating equilibrium, incentives to mask information exist. In the semi-pooling equilibria described in Corollary 3.2, information is revealed imperfectly. Other papers in the literature allow the first-mover to pre-commit to a randomization mechanism. Examples are Gould and Verrecchia and also Grinblatt and Ross. In both these papers the informed agent's pricing rule includes a noise term which prevents perfect information revelation.

Gould and Verrecchia justify the pre-commitment assumption by referring to a noisy telephone connection between the informed specialist and the trader. In this paper, the leader has not been allowed to pre-commit to noise because it is hard to tell a plausible story that could justify pre-commitment in the present context. The noisy piggybacking model can be viewed as an alternative means of introducing these considerations whereby life-cycle traders mas the information of insiders.

Section 6. Policy Implications for Regulating  
Insider Trading

How does trading by an insider affect uninformed outsiders? What are the implications for a policy of regulating insider trading? The model described previously can suggest answers to these questions if it's assumed that the leader is not able to extract the full value of his information within a short time span. Otherwise he would not care about piggybacking. Also, since all traders are risk-neutral,

issues of risk-sharing addressed by Allen are ignored here. With risk-averse traders, information-release may be sub-optimal.

### 6.1 Pre-Trade Disclosures

Observation of public statistics (such as the SEC's "Official Summary of Insider Trading") is a noisy signal of insider information to uninformed outsiders. I use the noisy piggybacking model of Section 4 and put  $t_2 = 0$  (the follower becomes an outsider). This will be the benchmark case of no regulation. Following Grinblatt [12], I consider two regulatory regimes--pre-trade information disclosure and disclosure of trading intentions.

Proposition 6.1. With risk-neutral traders, a policy of mandating information disclosure (ID) is superior to trade intentions (TI) disclosure.

Proof: When  $t_2 = 0$ , the ID policy leads to the Stackelberg outcome described in Corollary 3.1 which is the first-best outcome in a symmetric information world. The TI policy can be interpreted as mandating the leader to reveal  $x_1$ . If traders follow linear trading strategies, Corollary 3.2 says that markets break down. Under non-linear trading strategies, Proposition 2.1 in Sarkar shows that trades are below the Stackelberg volume almost everywhere.

If the ID policy cannot be credibly implemented due to moral hazard problems, then a policy of not regulating insiders is second-best because it avoids the piggybacking induced by the TI policy. Without regulation, Corollary 4.1 states that the insider obtains

expected monopoly profits. To ensure participation of the outsider he can be given a lump-sum grant equal to the expected Stackelberg profit of the follower, financed by a tax on the leader's monopoly profits. This still leaves the leader with profits higher than his Stackelberg amount, so he has an incentive to remain the leader. The scheme is feasible because expected monopoly profits exceed total Stackelberg profits of the leader and follower combined.

## 6.2 Regulatory Impact on Efficiency, Liquidity, and Price Volatility

Sequential trading with symmetrically informed leader and follower (the Stackelberg outcome) leads to liquid markets, both in terms of depth (prices are relatively insensitive to volume fluctuations) and volume. Better information improves efficiency but reduces depth due to the adverse selection effect on market-makers. Price volatility is inversely related to depth and positively related to the informativeness of trades. So volatility also increases in response to better information.

With the leader as insider and follower as uninformed outsider, markets are less liquid (lower volume and depth) and less efficient but with more stable prices. These effects arise due to strategic behavior by the insider. Regulation of insider trading has ambiguous effects on market parameters, however. The TI policy, for example, induces piggybacking which serves to restrict trading volumes and lower market efficiency but may increase liquidity since market-makers feel less threatened by insider trading. Further, as Grossman [1986] has argued in the context of futures trading, if information is



acquired at a cost piggybacking allows traders to free-ride on information collected and processed by other traders.

### 6.3 Are Piggybackers Insiders?

There is some controversy as to whether piggybackers should fall under current insider trading regulations since they may not have a fiduciary duty to shareholders. In this model both the piggybacker and the corporate insider profit at the expense of the liquidity traders. More importantly, in the non-linear equilibrium, the piggybacker makes profits without having any independent information. So the piggybacker also profits from the leader's information. The act of piggybacking makes him an insider.

## Section 7. Conclusion

Does the intense scrutiny of insider trades act as a mechanism for regulating insider trading? The question is explored in the context of an asymmetric information Stackelberg model with the insider as the leader and the piggybacker as the follower. Noise traders mask the insiders' information and provide liquidity to the market. Market-makers observe total trades and set prices which clear the market.

When piggybacking is perfect, the insider's trade is observed perfectly. The insider tries to mislead the follower and signal poor information by trading low. In a separating equilibrium, however the leader does not succeed and his information is perfectly revealed. But the attempt to do so restricts his equilibrium trading volume. So perfect piggybacking does regulate insider trades but at a cost. Because of the strategic cost of using information, the insider may

throw away valuable information. An increase in the precision of information may cause insiders to trade less, decreasing the informativeness of prices.

Under noisy piggybacking, the follower observes the sum of noise trades and the leader's trade. So piggybacking conveys only statistical information. I find that a small amount of noise is sufficient to drown the piggybacking effect. The insider's trades increase with respect to both his information precision and the amount of noise trading.

Although intuitively appealing, I suggest why randomizing by the leader is not plausible in this model. Also, if there is a credible way for the leader to reveal his information to the follower, he could avoid piggybacking and be better off.

The results of the models are used to analyze pre-trade disclosure laws. It is suggested that a policy of requiring trade intentions disclosure from insiders will have adverse market effects. Instead, a policy of deregulating insider trading along with a tax on insiders' profits used to subsidize outsider traders is proposed.

# APPENDIX

## Sections 2 and 3

First, (2.7) and (2.8) are derived. Since  $v$ ,  $s_1$ , and  $s_2$  are normally distributed  $E(\tilde{v}|s^*, s_2)$  will be linear in  $s^*$  and  $s_2$ . Suppose that  $E(\tilde{v}|s^*, s_2) = c_1 s^* + c_2 s_2$ , where  $c_1$  and  $c_2$  are given by the normal equation  $c = \Sigma^{-1} \text{cov}(v, s)$ .  $c, s$  are  $2 \times 1$  vectors  $[c_1 \ c_2]$  and  $[s^* \ s_2]$  and  $\Sigma$  is the variance-covariance matrix of  $s$ . The result is easily obtained using  $E(v|s_i) = t_i s_i$ ,  $i = 1, 2$ .

Second,  $\Gamma$  in (3.3) is derived.  $\Gamma$  is the regression coefficient of the linear projection of  $v$  on  $y$ , where  $y = x_1 + x_2 + u$ .

$$\Gamma = \frac{[A_1(1+B_1)+B_2]\Sigma_0}{[A_1(1+B_1)+B_2]^2\Sigma_{s_1} + B_2^2\Sigma_{s_2} + \Sigma_u + 2A_1(1+B_1)B_2\Sigma_0}$$

Substituting the equilibrium values for  $A_1$ ,  $B_1$ , and  $B_2$  from (3.3) in the text gives the following quadratic equation in  $\Gamma$

$$(A1) \quad 2\Gamma^2\Sigma_u + \Sigma_0[-t_1 - \Gamma^2(1-5t_1t_2/4)/t_2]/2 = 0$$

Since the second-order condition requires  $\Gamma > 0$ ,  $\Gamma$  is obtained as the positive square root of  $\Gamma^2$  that solves (A1).

## Proof of Corollary 3.1

$s_1 = s_2 = s$  and define  $t = \Sigma_0/(\Sigma_0 + \Sigma_s)$ . It is immediate that  $E(v|s^*, s) = E(\tilde{v}|s) = ts$ . The follower's maximization problem yields the first-order condition (given some trade  $z$  by the leader)

$$x_2 = [ts - \Gamma_s z]/2\Gamma_s.$$

Solving the leader's maximization problem yields the following equilibrium solution.

$$(A2) \quad z = ts/2\Gamma_s, \quad x_2 = ts/4\Gamma_s, \quad \Gamma_s = \frac{(3t\Sigma_0/\Sigma_\mu)^{1/2}}{4}$$

Finally, solve for the informativeness of the price system.  
 $\text{Var}(v|p) = E[v - E(v|p)]^2 = E(v-p)^2 = \Sigma_0(1-11t/16)$ . Part (iii) follows because  $\Gamma_s$  is increasing in  $t$ .  $\text{Var}(v|p)$  is decreasing in  $t$  which proves part (ii).  $z$  is increasing in  $t$ , as stated. To show that  $z$  is higher than its value in (3.3) note that (1)  $t \leq 1$  and (2)  $\Gamma > \Gamma_s$  because  $Q = \frac{3t_1}{4} + \frac{t_1(1-T^2)}{4} + T(1-t_1) > 3t_1/4$ .

### Proof of Proposition 3.2

I show that (1)  $\delta A_1/\delta t_1 > 0$  at  $t_1 = 0$  (2)  $\delta A_1/\delta t_1 < 0$  at  $t_1 = 1$  and (3)  $A_1$  is concave.

$$A_1 = t_1 T Q^{-1/2} \text{ ignoring the terms not involving } \Sigma_{e_1} \text{ and } \Sigma_{e_2}.$$

Differentiating with respect to  $t_1$ :

$$\frac{1}{A_1} \cdot \frac{\delta A_1}{\delta t_1} = \frac{1}{2t_1} \cdot [(T^2/t_2 Q) + 1] + \frac{\delta T}{\delta t_1} \cdot \frac{t_1}{TQ}$$

At  $t_1 = 1$ ,  $\delta A_1/\delta t_1 = -t_2 < 0$  because  $T = 0$ ,  $Q = 1$ , and  $\delta T/\delta t_1 = -t_2$ .

At  $t_1 = 0$ ,  $\delta A_1/\delta t_1 = t_2^{1/2} > 0$  because  $T = Q = t_2$  and  $\delta T/\delta t_1 = 0$ .

To show concavity, calculate the second derivative of  $A_1$ :

$$(A3) \quad \frac{\delta}{\delta t_1} \left( \frac{1}{A_1} \cdot \frac{\delta A_1}{\delta t_1} \right) = -1/2 t_1^{-2} + (T \cdot \delta T/\delta t_1)/t_1 t_2 Q - \\ (T^2 \cdot \delta Q/\delta t_1)/2 t_1 t_2 Q^2 + (\delta T/\delta t_1)/TQ - [t_1 (\delta T/\delta t_1)^2]/T^2 Q - \\ [t_1 (\delta Q/\delta t_1) \cdot (\delta T/\delta t_1)]/TQ^2$$

where  $(\delta Q/\delta t_1)/Q = 1/t_1 + (2T \cdot \delta T/\delta t_1)/Qt_2 - (5t_1 T \cdot \delta T/\delta t_1)/2Q$ .

Substitute the above expression into the RHS of (A3) and rewrite the RHS of A3:

$$(A4) \quad -1/2t_1^2 + T^4/(2t_1^2t_2^2Q^2) - T^2/(2t_1^2t_2Q) \\ + \delta T/\delta t_1 [T/t_1t_2Q - T^3/t_1t_2^2Q + (5T^3)/4t_1t_2Q^2] \\ + (\delta T/\delta t_1)^2 [5t_1^2/2 - 2t_1/t_2 - t_1/T^2]/Q^2$$

Consider the first three terms of (A4):

$$(A5) \quad -\frac{1}{2t_1^2} \cdot [1 + \frac{T^2}{t_2Q} \cdot \frac{(1-T^2)}{t_2Q}] < 0$$

Proof:  $Q \geq 0$  implies  $t_2Q - T^2 \geq t_1t_2(1 - \frac{5T^2}{4})$ .

If  $T^2 < 4/5$ , then we are done.

Suppose not and assume  $T = 1$ , the maximum possible value of  $T$ .

$$\text{Then } 1 + \frac{T^2}{t_2Q} \cdot \frac{(1-T^2)}{t_2Q} = 1 - \frac{t_1}{4(1-t_1/4)_2} > 0.$$

Next, consider the terms involving  $\delta T/\delta t_1$  in (A4).

$$(A6) \quad T \cdot \delta T/\delta t_1 \cdot [(t_2Q - T^2 + \frac{5}{4}t_1t_2T^2)/t_1t_2^2Q^2] \\ = T \cdot \delta T/\delta t_1 \cdot t_1t_2/t_1t_2^2Q^2 < 0 \text{ as } \delta T/\delta t_1 < 0.$$

Finally, arrange the last three terms of (A4);

$$(A7) \quad (\delta T/\delta t_1)^2 \cdot [(5t_1^2t_2T^2 - 4t_1T^2 - 2t_1t_2)/2t_2T^2]/Q^2$$

The numerator factors to  $4t_1T^2(t_1t_2-1) + t_2t_1(t_1T^2-2) < 0$  because  $t_1 < 1$ ,  $t_2 < 1$  and  $T < 1$ .

(A5)-(A7) together proves  $\delta^2 A_1/\epsilon t_1^2 < 0$ .

Proof of Proposition 3.2

$$\text{Var}(v|p) = E(v-p)^2 = \Sigma_0(1-T/2)(1-t_1/2)$$

Define price informativeness PI as  $PI = \Sigma_0[1-(1-T/2)(1-t_1/2)]$

$$(A8) \quad \delta PI / \delta t_1 = [1-T/2 + \delta T / \delta t_1 \cdot (1-t_1/2)] \cdot \Sigma_0 / 2$$

$$\text{where } \delta T / \delta t_1 = [t_2(t_2-1)] / (1-t_1 t_2)^2$$

Fix  $t_2$  and let  $t_1$  approach 1. In the limit, the RHS of (A8) tends to  $[1 - \frac{t_2}{2(1-t_2)}] \cdot \frac{\Sigma_0}{2}$  which is negative for  $t_2$  sufficiently close to 1.

Section 4

(4.2) is derived using the same technique for deriving (2.8) in the Section 2 appendix.

Proof of Proposition 4.1

Substituting  $a_1$  and  $a_2$  from (4.2) into (4.3) yields the following cubic equation in  $A$ :

$$(A9) \quad A_1^3 + pA_1^2 + qA_1 + \pi = 0$$

$$\text{where } p = -t_1 T / 2\Gamma, \quad q = t_1(1-t_\mu) / t_\mu(1-t_1 t_2)$$

$$\pi = -t_1 q(1-t_2) / 2\Gamma$$

The standard cubic solution (using reduced form) is applied to (A9) to give  $A_1 = + t_1 T / 6\Gamma$  where

$$(A10) \quad = L(1-m/3), \quad L = [-n/2 + (n^2/4 + m^3/27)^{1/2}]^{1/3},$$

$$m = (3q - p^2)/3, \quad n = (2p^3 - 9pq + 27\pi)/27$$

Given  $A_1$ , the variables  $a_1$ ,  $a_2$ ,  $B_1$ , and  $B_2$  are also defined.

#### Proof of Corollary 4.1

For a monopolist insider, the problem is

$$\max_x E[(v - \Gamma x - \Gamma \mu)x | s]$$

which gives as the first-order condition  $x = t_1 s / 2\Gamma$ . The monopoly equilibrium is then:

$$(A11) \quad X(s) = A_1 s \quad \text{where} \quad A_1 = t_1 / 2\Gamma, \quad \Gamma(t_1 \Sigma_0 / \Sigma_\mu)^{1/2} / 2$$

$\Gamma$  is derived using the condition  $\Gamma = \text{cov} \cdot (v, y) / \Sigma_y$ . The (unconditional) expected profits of the insider is:

$$(A12) \quad E\Pi_1 = E[(v - \Gamma A_1 s - \Gamma \mu)A_1 s] = (t_1 \Sigma_\mu \Sigma_0)^{1/2} / 2$$

Now substitute  $t_2 = 0$  in equations (4.1)-(4.4) of the text. This gives:

$$(A13) \quad a_2 = 0, \quad A_1 = t_1(\Gamma + a_1), \quad B_2 = 0, \quad \text{and} \quad B_1 + a_1 / 2\Gamma$$

$$(A14) \quad \Gamma = \frac{\Sigma_0 [A_1(1+B_1) + B_2]}{\Sigma_{y_1}(1+B_1)^2 + B_2^2 \Sigma_{s_2} + 2B_2(1+B_1)A_1 \Sigma_0}$$

$$\text{Using (A13), } \Gamma = \frac{\Sigma_0 \cdot A_1}{\Sigma_{y_1}(1+B_1)} \text{ which can be rewritten as } \Gamma = a_1(1+B_1)$$

using the definition  $a_1 = A_1 \Sigma_0 / \Sigma_{y_1}$ . Solve (A13) and (A14) to get

$B_1 = 0$ ,  $\Gamma = a_1$ ,  $A_1 = t_1 s / 2\Gamma$ ,  $B_2 = 0$ . Finally, solve the equation  $\Gamma = a_1$  using the definition of  $a_1$  and  $t_\mu$  in (4.2).

$$(A15) \quad \Gamma = (t_1 t_\mu)^{1/2} / s(1-t_\mu)^{1/2} = (t_1 \Sigma_0 / \Sigma_0)^{1/2} / 2$$

Comparing with (A11), both  $\Gamma$  and  $A_1$  are at their monopoly levels.  $B_1 = B_2 = 0$  so the follower does not trade. It is easily checked that the leader's expected profits are also at their monopoly levels.

#### Proof of Proposition 5.1

To prove part (ii), remember that  $A_1^P = t_1 T / 2\Gamma_P$  in the model with perfect piggybacking with  $A_i^I = t_i / 2\Gamma_I$  in the information-sharing model. The result follows because (1)  $T \leq 1$  and (2)  $\Gamma_P > \Gamma_I$ .

Claim 1:  $T \leq 1$

Proof:  $T = (t_2 - t_1 t_2) / (1 - t_1 t_2) \leq 1$  as  $t_2 \leq 1$

Claim 2:  $\Gamma_P \geq \Gamma_I$

Proof: From the definition of  $\Gamma_P$  in (3.3) and  $\Gamma_I$  in Proposition 5.1, need only show  $M \leq Q$ .

$$\begin{aligned} M &= 3t_1/4 + T(1-t_1)/2 + T^2(t_1 t_2 - 1)/t_2 \\ &\leq 3t_1/4 + T(1-t_1)/2 \quad \text{as } t_i \leq 1, i = 1, 2 \end{aligned}$$

$$\begin{aligned} Q &= 3t_1/4 + T(1-t_1) + t_1(1-T^2)/4 \\ &\geq 3t_1/4 + T(1-t_1) \quad \text{as } T \leq 1 \end{aligned}$$

$$> M$$



Proof of Non-Linear Equilibrium

The equilibrium is obtained by solving the following differential equation:

$$(A19) \quad dz/ds_1 = t_1(1-T) \cdot z / (t_1 s_1 - 2\Gamma z)$$

(A19) is of the form  $dz/ds_1 = f(z/s_1)$ . Make the substitution  $g = z/s_1$  and solve.

$$(A20) \quad s_1 = ce^G, \quad G = \int dg / [f(g) - g] \text{ where } f(g) \text{ is the RHS of (A19).}$$

$$\begin{aligned} G &= \int dg (t_1 - 2\Gamma g) / [g(2\Gamma g - t_1 T)] \\ &= \frac{1}{T} \log[(2\Gamma g - t_1 T)/g] - \log[2\Gamma g - t_1 T] \end{aligned}$$

Lastly, C is solved using the boundary condition  $z = t_1 \bar{s}_1 / 2\Gamma$  at  $s_1 = \bar{s}_1$ .

REFERENCES

- Allen, B., 1981, "Generic Existence of Completely Revealing Equilibria for Economies with Uncertainty when Prices Convey Information," Econometrica, 1173-1199
- Allen, Franklin, 1987, "Risk-Sharing, Taxing Information and Insider Trading," Working Paper, Department of Finance, University of Pennsylvania.
- Bagehot, W., pseud. 1971, "The Only Game in Town," Financial Analyst Journal, 2, 12-14.
- Diamond, Douglas W. and Robert E. Verrecchia, 1981, "Information Aggregation in a Noisy Rational Expectations Economy," Journal of Financial Economics, 9, 221-235.
- Dye, Ronald A., 1984, "Insider Trading and Incentives," Journal of Business, Vol. 57, No. 3.
- Easley, David and Maureen O'hara, 1987, "Price, Trade Size, and Information in Securities Markets," Journal of Financial Economics, 19, 69-90.
- Finnerty, J. E., 1976, "Insiders and Market Efficiency," Journal of Finance, 31, 1141-1148.
- Gal-Or, E., 1987, "First Mover Disadvantages with Private Information," Review of Economic Studies, LIV, 279-292.
- Givoly, Dan and Palmon, Dan, 1985, "Insider Trading and the Exploitation of Inside Information--Some Empirical Evidence," Journal of Business, Vol. 58, No. 1.

- Glosten, L. L. and P. R. Milgrom, 1985, "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneous Informed Traders," Journal of Financial Economics, 14, 71-100.
- Gould, John P. and Robert E. Verrecchia, 1985, "The Information Content of Specialist Pricing," Journal of Political Economy, Vol. 93, No. 1, 66-81.
- Grinblatt, Mark S., 1987, "On the Regulation of Insider Trading," Working Paper, Graduate School of Management, UCLA.
- Grinblatt, Mark S. and Stephen A. Ross, 1985, "Market Power in a Securities Market with Endogenous Information," Quarterly Journal of Economics, November.
- Grossman, Stanford J., 1986, "An Analysis of the Role of 'Insider Trading' on Futures Markets," Journal of Business, 59, No. 2, S129-S146.
- Grossman, S. J. and J. E. Stiglitz, 1981, "On the Impossibility of Informationally Efficient Markets," American Economic Review, 70, 393-408.
- Hellwig, M. F., 1980, "On the Aggregation of Information in Competitive Markets," Journal of Economic Theory, 22, 477-498.
- Jaffe, Jeffrey, 1974, "Special Information and Insider Trading," Journal of Business, 48, 410-428.
- Kihlstrom, Richard and Andrew Postlewaite, 1987, "Equilibrium in a Securities Market with a Dominant Trader Possessing Inside Information," Working Paper, Department of Economics, University of Pennsylvania.

- Kreps, David A. and Robert Wilson, 1982, "Sequential Equilibria," Econometrica, 50, 863-894.
- Kyle, A. S., 1985, "Continuous Auctions and Insider Trading," Econometrica, 53, 1315-1335.
- Madhavan, Anath, 1988, "Trading Mechanisms in Securities Markets," Working Paper, The Wharton School, University of Pennsylvania.
- Mailath, George, 1987, "Incentive Compatibility in Signalling Games with a Continuum of Types," Econometrica, 55, 1349-1365.
- Manne, H. G., 1966, Insider Trades and the Stock Market, Free Press, New York.
- Mathews, Steven A. and Leonard J. Mirman, 1983, "Equilibrium Limit Pricing: The Effects of Private Information and Stochastic Demand," Econometrica, 51, No. 4, 981-996.
- Milgrom, Paul and John Roberts, 1981, "Limit Pricing and Entry Under Incomplete Information," Econometrica, 50, 443-459.
- Sarkar, Asani, 1988, "Piggybacking on Insider Trades II: Some Additional Results," Working Paper, Department of Economics, University of Pennsylvania.
- Scholes, M. S., 1972, "The Market for Securities: Substitution versus Price Pressure and the Effects of Information on Share Prices," Journal of Business, 45, 179-211.
- Seyhun, Nejat H., 1986, "Insiders' Profits, Costs of Trading and Market Efficiency," Journal of Financial Economics, 16, 189-212.

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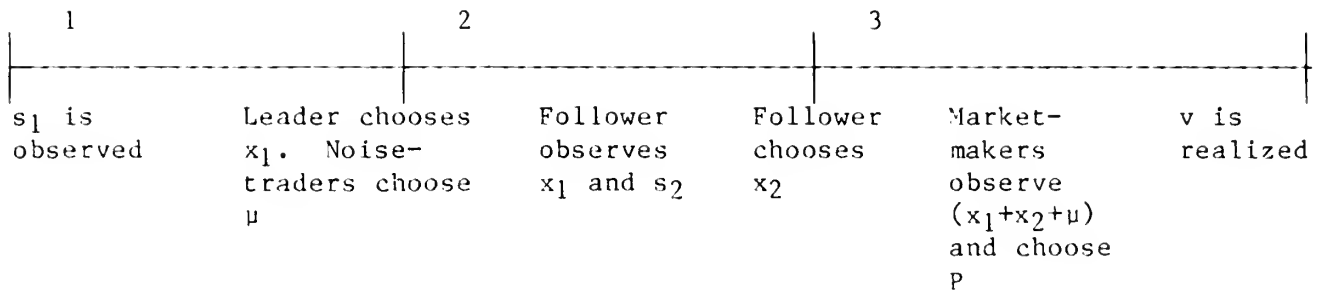
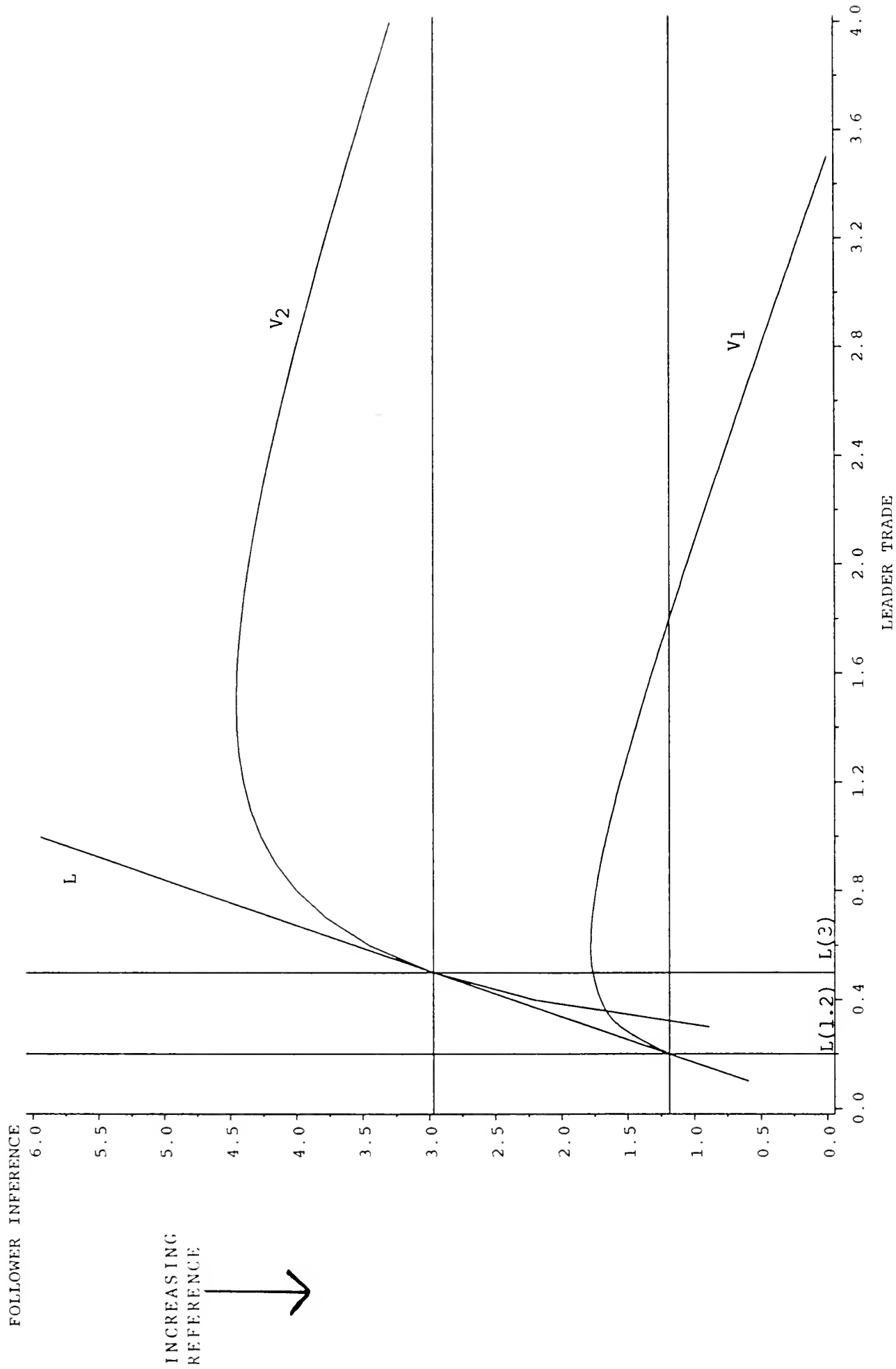


Figure 1. SEQUENCE OF MOVES

FIG 2: LINEAR SEPARATING EQUILIBRIUM



V1: LEADER ISO-PROFIT CURVE FOR  $S=1.2$   
V2: LEADER ISO-PROFIT CURVE FOR  $S=3$   
L : SEPARATING EQUILIBRIUM STRATEGY

FIG 3 : TRADING STRATEGY OF LEADER VERSUS T1

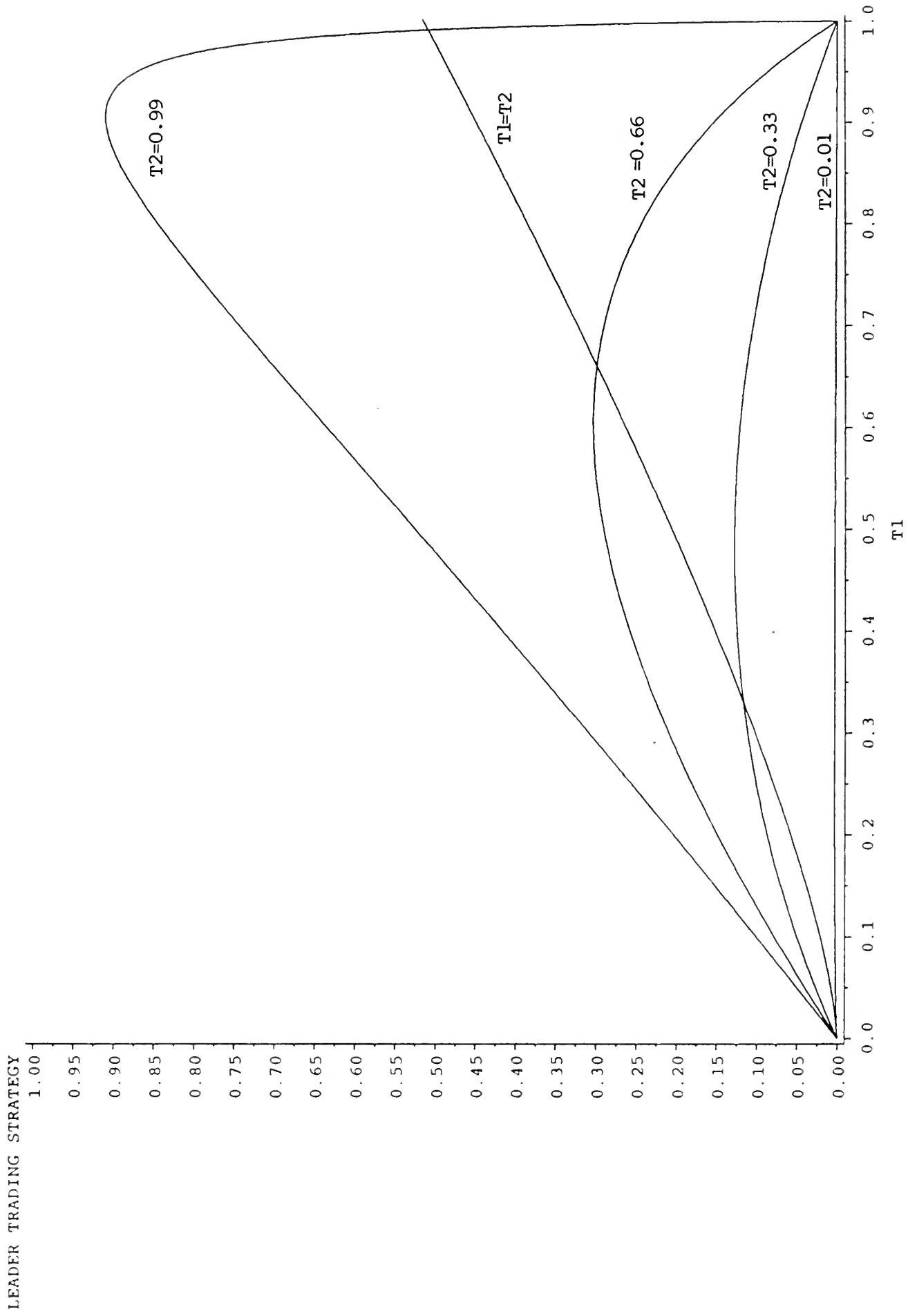




FIG 4 : MARKET LIQUIDITY VERSUS T1

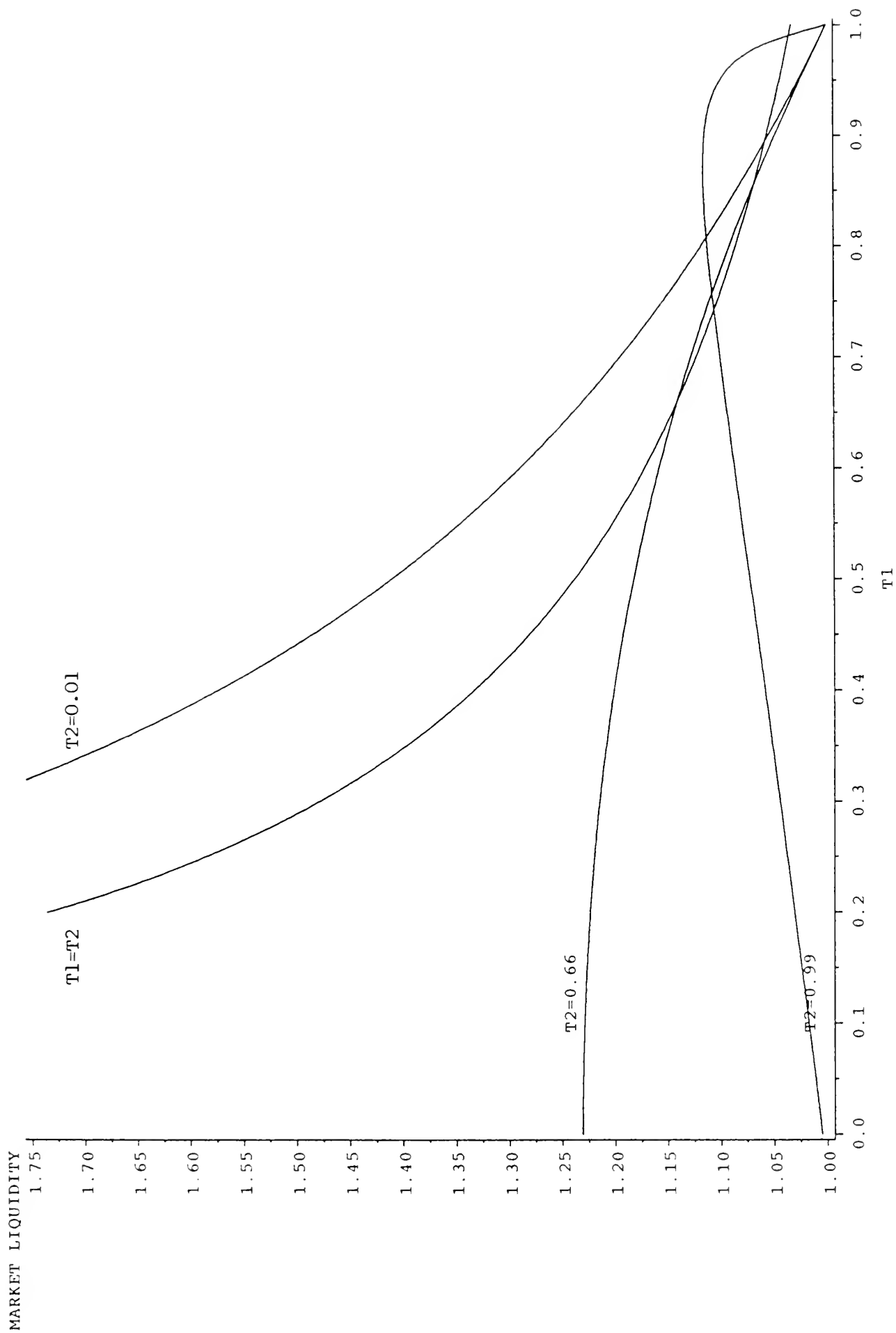
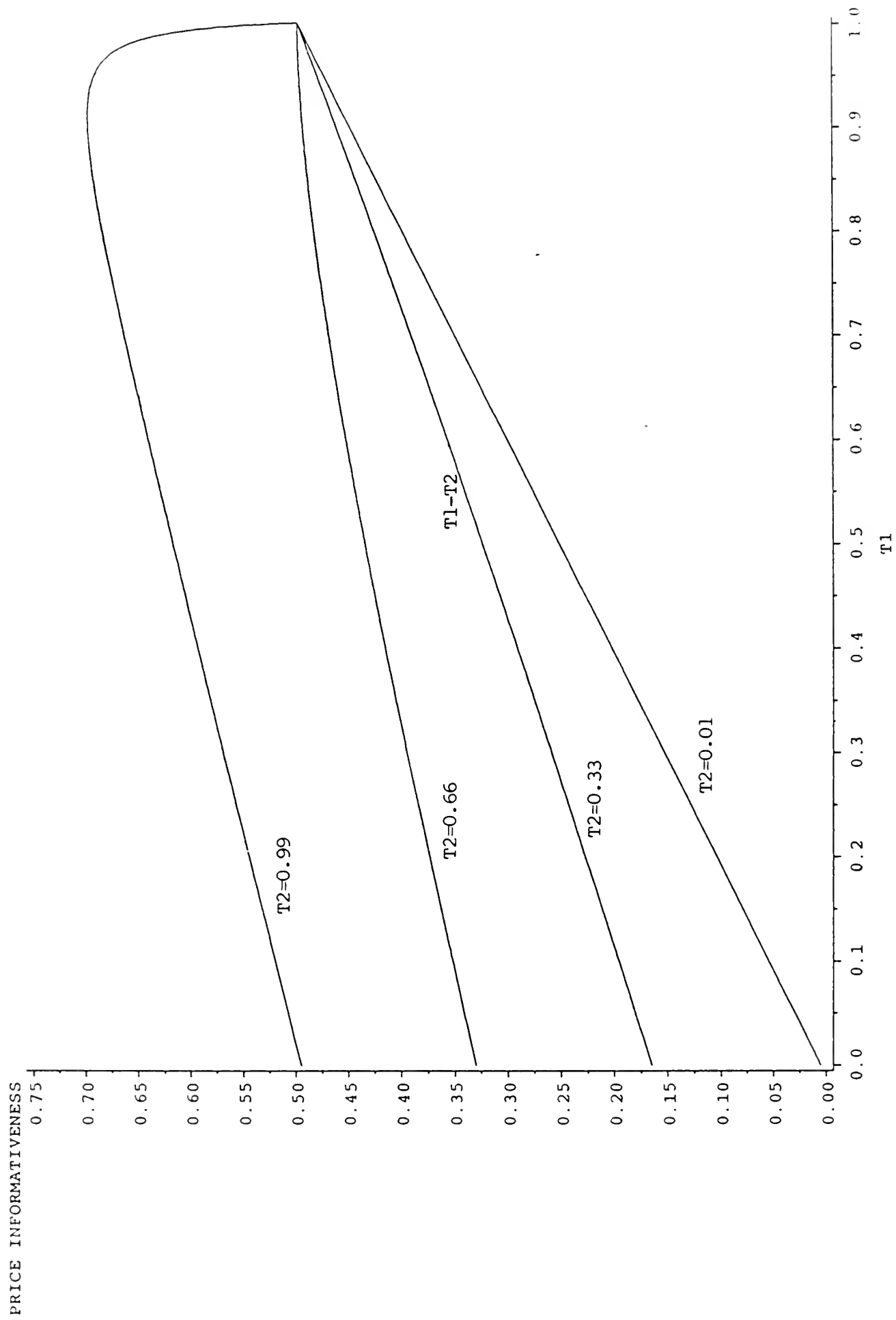
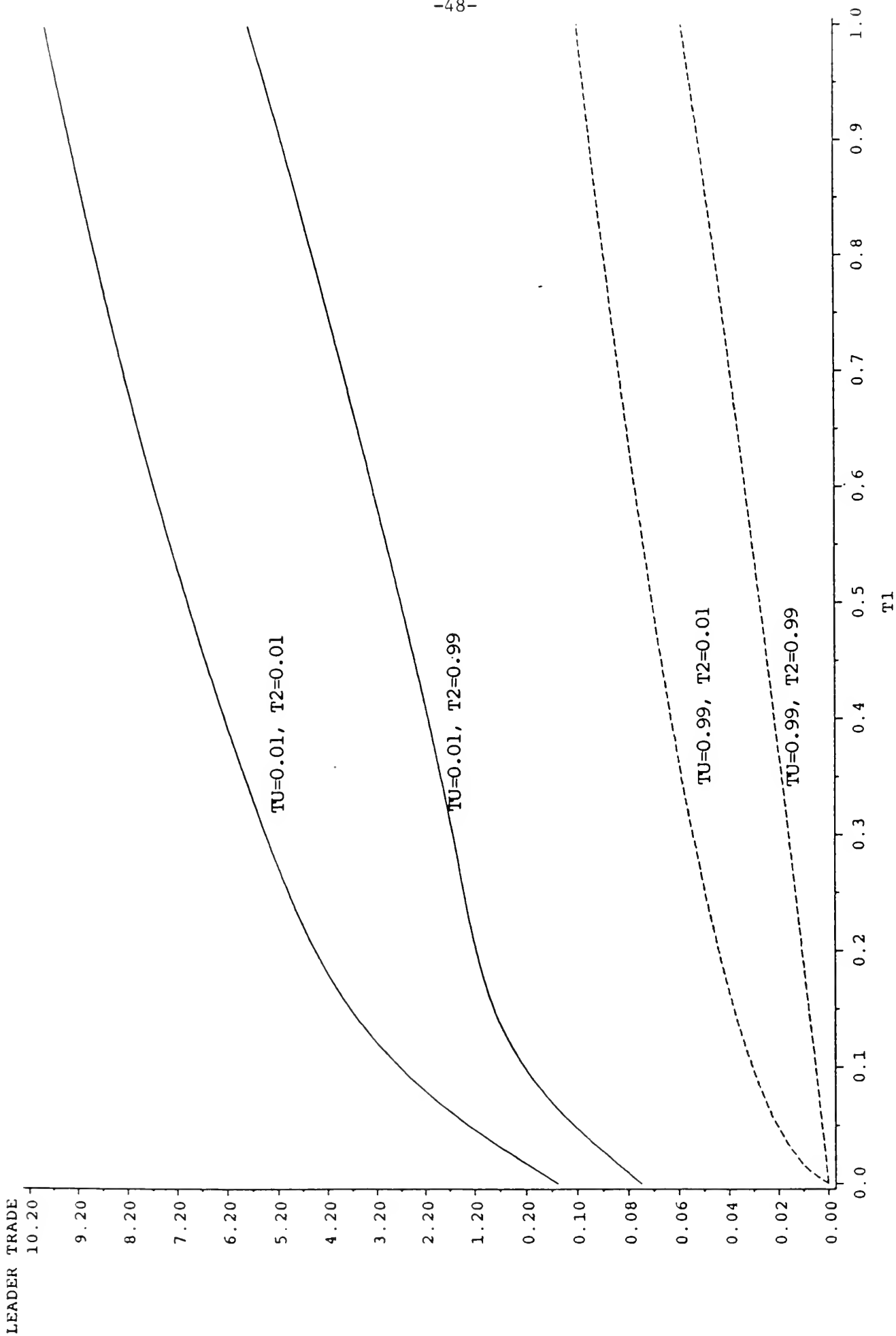


FIG 5 : PRICE INFORMATIVENESS VERSUS T1





T1: INFORMATION PRECISION OF LEADER  
T2: INFORMATION PRECISION OF FOLLOWER  
TU: MEASURE OF NOISE TRADE














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