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A PLANNING PROCEDURE FOR EFFICIENT PROJECT SELECTION AND DECENTRALIZATION OF DISTRIBUTIONAL JUDGEMENTS

by

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Abstract

This paper presents a solution to the following, frequently encountered, planning problem. A community wishes to choose a project from a variety of alternatives. The construction of the project is undertaken by a monopolistic firm. An intermediary (who cannot observe either technology or preferences) is charged with the following responsibilities: (i) evaluation of the most efficient size of each project, (ii) comparison of the surpluses generated by the various projects at their efficient levels and selection of a project that maximizes the surplus, (iii) evaluation of how much each consumer pays and how much should be paid to the firm, and (iv) decentralization in the making of distributional judgements regarding the surplus that remains after payments have been made to the firm, i.e. there is no social planner with an *a priori* welfare function; the decision originates from the community itself. The nature of the problem is such that it cannot be solved using existing methods of planning.

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1. Introduction

This paper is concerned with the following planning problem. A community wishes to choose a project from a variety of alternatives. In particular, the choice of a project means that a particular public good will be provided to the community. The construction of the project is undertaken by a monopolistic firm. An intermediary is found and is charged with the following responsibilities: (i) evaluation of the most efficient size of each project -- or "internal" efficiency, (ii) comparison of the surpluses generated by the various projects at their efficient levels and selection of a project that maximizes the surplus -- or "external" efficiency, (iii) evaluation of how much each consumer pays and how much should be paid to the firm, and (iv) decentralization in the making of distributional judgements, i.e. after the firm has been paid, the choice of a distribution scheme for the surplus that remains is made by the consumers themselves. This is a problem which arises quite frequently. Accurate information on both preferences and technology is generally unobservable to a third party, especially when the demand side has a number of consumers and the supply side is monopolistic. In addition, distributional or welfare judgements are often suggested endogenously by the consumers themselves and not imposed by a (usually fictitious) social planner. A planner with a specific welfare function may not exist. The objective of this paper is to propose a simple solution to this frequently encountered problem.

We shall follow in the tradition of planning models that are dynamic in nature, since they have generally achieved the greatest degree of success in resolving incentive-compatibility problems. In this genre, the classic is the MDP (after Malinvaud (1971, 1972) and Drèze and de la Vallée

Poussin (1971)) planning procedure. Typically, the focus of planning problems has been on the issue of internal efficiency and not on efficiency questions across projects. The intermediary is presumed to know the technology of the firm providing the public good However, it does not observe consumers' preferences. In addition, it makes its own judgements about how the surplus is to be distributed. Within this class of problems, the MDP procedure and its extensions have been rather successful. Under truthful revelation of preferences and costs, the MDP procedure converges in a monotonic and feasible manner to an individually rational and internally efficient allocation. Malinvaud conjectured that the convergence property would hold even if the consumers were to adopt Nash equilibrium strategies in the reporting game at each instant in time. This conjecture was later proved to be true by Roberts (1979) and Henry (1979) using the *local game* formulation, i.e. consumers are myopic and attempt to maximize their incremental utilities at each instant. Later results by Champsaur and Laroque (1982) and Truchon (1984) demonstrate that under certain modifications of the basic procedure, the Malinvaud conjecture about convergence under Nash-manipulation holds even in a *global game*, i.e. consumers maximize their final utilities.

These results, which show that the internal efficiency properties are not destroyed when the consumers play Nash equilibrium reporting strategies, are remarkable largely because the equilibrium reports are generally untruthful. To resolve the question of external efficiency, we need to measure the true social surplus generated by different projects operated at their respective internally efficient levels. To measure such true surpluses, *truthful* information is required.

Fugigaki and Sato (1981) have devised a generalization of the MDP algorithm, for which truthful reporting on the part of the consumers is a unique dominant strategy equilibrium in the local game. However, for this result to hold, the consumers must receive equal shares of the surplus. When distributional judgements are decentralized (which is the situation that we study), such an "equal-sharing" rule will not generally be demanded by the consumers as part of every Nash equilibrium of a game where consumers report their willingnesses to pay and their desired distribution schemes. An equal-sharing rule may be forced by restricting the consumers to reports of equal-sharing schemes. This is undesirable, however, since it would imply that an egalitarian distribution function is being imposed on the consumers by a social planner.

To summarize, the procedures that are available do not solve our problem. Moreover, they cannot be easily extended to do so either. This provides the motivation for the planning procedure presented here.

The basic algorithm employed for adjusting the level of a given public good is the MDP algorithm. The tax rules for the consumers and the compensation paid to the firm differ from the classical recommendations. The procedure operates in continuous-time and is given by a system of differential equations. It induces a local game at each point in its trajectory. We follow Roberts (1979) in modelling the agents as myopic players. Despite its obvious limitations, this case is an important one to study for several reasons: (i) since Nash equilibrium is generally a solution concept for games of complete information, it is less restrictive to assume that each player has knowledge of the marginal willingnesses to pay and marginal costs of others at the current moment in time and does not possess this information for the entire time-path -- in this sense myopia makes weaker informational assumptions than non-myopia; (ii) the difference in perspective between the intermediary and the agents is more realistic; (iii) in many developing countries the agents are not sure when the current plans (and, perhaps, the government) will end, so they optimize as if the current moment is the last. The case of non-myopic players is an open question and will be dealt with in future research.

Our results have two, rather interesting, aspects. The firm is induced to report truthfully. The consumers generally misrepresent their marginal benefits. The *aggregate* of their reports, however, is equal to the true aggregate marginal benefits. Moreover, the equilibrium distribution scheme recommends that each consumer's share of the surplus is equal to his/her share of the aggregate marginal benefit.

2. Preliminaries

We shall focus on a given project P, (i.e. we fix a public good) and apply the procedure to P. Let $\mathcal{P}$ be the class of projects. There is a group of agents endowed with a private good X which is available for use in any project. We assume that utility functions are additively separable in the private good. This provides a common numeraire good with which surpluses from alternative projects can be measured. Having measured the surplus generated by every P in $\mathcal{P}$, we have a choice criterion.

A *project P* comprises a private good space for each agent i, $X_i \subseteq \mathbb{R}$, a public good space $Y \subseteq \mathbb{R}_+$ and a set of agents, $N = \{1,\ldots,n\}$. $\{1,\ldots,n-1\}$ is a set of *consumers* and agent n is a *firm*. In the sequel, for any variable g_i defined only for all i in $K \subseteq N$, we shall write $(g_i)_{i\in K}$ as g and $(g_j)_{j\in K\setminus\{i\}}$ as g_{-i}. Each $i \in N$ is characterized by a pair (u_i, ω_i) where u_i: $X_i \times Y \to \mathbb{R}$ is i's utility function and $\omega_i \in \mathbb{R}_+$ is i's initial endowment of the private good with $\omega_n = 0$. We assume the existence of *willingness-to-pay functions* v_i: $Y \to X_i$ and a *cost function* c: $Y \to X_i$ such that for all $i \in N\backslash\{n\}$, for all $x_i \in X_i$, for all $x_n \in X_n$ and all $y \in Y$, $u_i(x_i, y) = x_i + v_i(y)$ and $u_n(x_n, y) = x_n - c(y)$, i.e. the utility functions are linear in the private good. For all $i \in N\backslash\{n\}$, v_i is strictly concave and C^2 and c is strictly convex and C^2. In addition, for all $i \in N\backslash\{n\}$, $v_i' \geq 0$, $v_i(0) > 0$ and $c' > 0$, $c(0) = 0$. Finally, we also assume that there exists $\bar{y} \in Y$ such that $\sum_{i \in N\backslash\{n\}} v_i(\bar{y}) < c(\bar{y})$. In the sequel, we use $\alpha_i(y)$ to denote $v_i'(y)$ and $\beta(y)$ to denote $c'(y)$ which are to be interpreted as the "true" marginal willingness to pay by i and the "true" marginal cost to n for a level y of the public good.

A *feasible allocation* of resources in P is a vector $z \in Z \equiv \{(x, y) \in \mathbb{R}^{n+1}: \sum_{i \in N}(\omega_i - x_i) = c(y)\}$. $z \in Z$ is *internally efficient in* P if there exists no $z' \in Z$ such that for all $i \in N$, $u_i(z_i') \geq u_i(z_i)$ with strict inequality for some i. $z \in Z$ is *individually rational* if for all $i \in N$, $u_i(z_i) \geq u_i(\omega_i, 0)$.

Given an allocation z which is internally efficient in P, let $S(P) \equiv \sum_{i \in N}(u_i(z_i) - u_i(\omega_i))$ denote the *surplus generated by* P. $P \in \mathcal{P}$ is *externally efficient* if there exists no $P' \in \mathcal{P}$ such that $S(P') > S(P)$.

3. The Planning Procedure

A *planning procedure* is a dynamic mechanism which accepts messages from the agents and recommends an adjustment in the allocation of resources at each instant in time. A procedure is to be interpreted as a survey that is performed before any construction actually takes place. Once a procedure is applied to all the P in $\mathcal{P}$ and the relevant information is collected, construction of the chosen project begins.

We shall restrict our attention here to a planning procedure which operates in continuous-time. Let Δ denote the $(n-2)$-dimensional simplex. At every instant $t \in [0, \infty)$, each consumer i sends a message $m_i(t) = (a_i(t), \delta_i(t)) \in \mathbb{R} \times \Delta$ and the firm n sends a message $m_n(t) = b(t) \in \mathbb{R}$. Consumer i's report consists of two values: a reported marginal willingness to pay, $a_i(t)$ and a surplus sharing scheme, $\delta_i(t)$. The firm reports a value for its marginal cost $b(t)$.

The following notation will be used. $\delta^i_j(t)$ denotes agent i's share of the surplus recommended by the surplus sharing scheme $\delta_j(t)$. Let $\#^m(\delta)$ denote the number of consumers who report a surplus sharing scheme δ, given that the profile of reports is $m(t)$. $W(m(t)) \equiv \{\delta \in \Delta: \forall\delta' \in \Delta, \#^m(\delta) \geq \#^m(\delta')\}$. An average of the elements of $W(m(t))$ is computed. $\delta_w(t) \equiv \frac{\sum_{\delta\in W(m(t))} \delta}{|W(m(t))|}$. Observe that $\delta_w(t) \in \Delta$.

The planning procedure introduced in this paper is a system of differential equations [1] - [3]. We shall ignore the case $y = 0$ and use an overdot to represent the time derivative.

$\forall t \in [0, \infty)$, given an initial position $(x(0), y(0))$:

$$\dot{y}(t) \equiv \textstyle\sum_{i\in N\setminus\{n\}} a_i(t) - b(t) \qquad [1]$$

$\forall i \in N\setminus\{n\}$,

$$\dot{x}_i(t) = -\delta^i_w(t)\textstyle\sum_{j\in N\setminus\{n\}} a_j(t)\dot{y}(t) + \frac{1}{2}\delta^i_w(t)[\dot{y}(t)]^2 \qquad [2]$$

$$\dot{x}_n(t) = b(t)\dot{y}(t) + \frac{1}{2}[\dot{y}(t)]^2 \qquad [3]$$

The level of the public good is adjusted using the classical MDP method. Consumers are taxed in the following way: at each instant in time the surplus sharing scheme that gets the maximum number of recommendations is chosen. If there is no unique "winner", then an average is computed from all the joint winners. This is $\delta_w(t)$. The i-th consumer is asked to pay the proportion $\delta_w^i(t)$ of the aggregate of the reported marginal willingnesses to pay. The rate of accumulation of the surplus is given by the expression $[\dot{y}(t)]^2$. The surplus is split in half. One half goes to the firm. The i-th consumer's share of the remaining half of the surplus is $\delta_w^i(t)$.

A stationary point of the system [1]-[3] is an allocation $(x(t), y(t))$ such that $\dot{y}(t) = 0$. The first result shows that under truthful revelation of the relevant information about the firm and about the consumers as a whole, the procedure given above converges to an individually rational and internally efficient allocation. Thus, it shares the convergence properties of other procedures discussed in the literature. We shall write $\alpha_i(y(t))$ and $\beta(y(t))$ as $\alpha_i(t)$ and $\beta(t)$ respectively.

Theorem 1: *The planning procedure specified by the system [1]-[3] converges to an allocation $z \in Z$ satisfying individual rationality and internal efficiency if for all $t \in [0, \infty)$, (i) $\sum_{i \in N \setminus (n)} a_i(t) = \sum_{i \in N \setminus (n)} \alpha_i(t)$, and (ii) $b(t) = \beta(t)$.*

Proof: By our assumptions, x and y and $V(t) \equiv \sum_{i \in N} u_i(x_i(t), y(t))$ are bounded above. By definition, $\dot{V}(t) \geq 0$. Also, $\dot{V}(t) = 0$ implies that $\dot{y}(t) = 0$ for all $t \in [0, \infty)$. Thus, the system [1]-[3] admits a Lyapunov function, V and, by Lyapunov's second method, converges to a stationary point. At a stationary point, the familiar Lindahl-Samuelson condition for Pareto-efficiency, i.e. internal efficiency, is met. Since utility of each agent is monotone and non-decreasing throughout the process, a stationary point is also individually rational. ■

The key assumptions in the theorem above are *(i)* and *(ii)*. Given that

$(\alpha_i)_{i\in N}$ and β are unobservable to the operator of the procedure, and given that the distributional parameters are determined in a decentralized manner, these assumptions are non-trivial. Hence, it is natural to treat the procedure as a non-cooperative game. The strategy of each agent i in N at t is $m_i(t)$. Thus, the strategy space is given by $\mathbb{R}^n \times \Delta^{n-1}$ and the payoffs at each $t \in [0, \infty)$ are the instantaneous increments in the agents' utilities and are given by:

$\forall m(t) = ((a(t), \delta(t)), b(t)),$

$\forall i \in N\setminus\{n\},$

$$\dot{u}_i(m(t)) = [\alpha_i(t) - \delta^1_w(t)\textstyle\sum_{j\in N\setminus\{n\}} a_j(t)]\dot{y}(t) + \frac{1}{2}\delta^1_w(t)[\dot{y}(t)]^2 \qquad [4a]$$

$$\dot{u}_n(m(t)) = [b(t) - \beta(t)]\dot{y}(t) + \frac{1}{2}[\dot{y}(t)]^2 \qquad [4b]$$

The payoff to each agent has two components -- the first one is the gain due to the simultaneous manipulation of private information and the second is the agent's share of the surplus. At every $t \in [0, \infty)$, the list $\{N, \mathbb{R}^n \times \Delta^{n-1}, \dot{u}\}$ defines a *local game* $\Gamma(t)$. For all $t \in [0, \infty)$, a *Nash equilibrium* of $\Gamma(t)$ is a list $m(t)$ such that for all $i \in N$, for all $(m_i'(t), m_{-i}(t)) \in \mathbb{R}^n \times \Delta^{n-1}$, $\dot{u}_i((m_i'(t), m_{-i}(t)) \le \dot{u}_i(m(t))$.

Armed with these definitions, we are ready to prove the main result of the paper. If all agents play their components of a Nash equilibrium strategy at every t, then the relevant information needed to meet the conditions of Theorem 1 is transmitted by the agents. The satisfaction of these conditions ensure that the true values $\sum_{i\in N\setminus\{n\}}\alpha_i(t)$ and $\beta(t)$ are made available to the intermediary for all $t \in [0, \infty)$. This provides a centralized availability of the accurate information about the project P

that is required for computing the true social surplus generated by P, $S(P)$. Once the procedure is applied to all P in $\mathcal{P}$, the intermediary can select one that generates the maximum surplus. Thus, external efficiency is ensured. To prove our result, we need at least three consumers.

Theorem 2: *Assume $n \geq 3$. At each $t \in [0, \infty)$, for any Nash equilibrium $((a(t), \delta(t)), b(t))$ of the local game induced by the planning procedure defined by [1]-[3], we have (i) $\sum_{i\in N\setminus\{n\}} a_i(t) = \sum_{i\in N\setminus\{n\}} \alpha_i(t)$, (ii) $b(t) = \beta(t)$. Moreover, the local game induced by the planning procedure defined by [1]-[3] has a non-empty set of Nash equilibria at each $t \in [0, \infty)$.*

Proof: Choose $t \in [0, \infty)$. Let $m^*(t) = ((a^*(t), \delta^*(t)), b^*(t))$ denote a Nash equilibrium for the the local game $\Gamma(t)$. We shall show that the properties *(i)* and *(ii)* are satisfied by $m^*(t)$ and then show that such an equilibrium indeed exists.

First, we shall show that $b^*(t) = \beta(t)$. The payoff to agent n when all agents in $N\setminus\{n\}$ play their Nash equilibrium strategy is

$\dot{u}_n(m^*_{-n}(t), m_n(t)) =$

$$[b(t) - \beta(t)][\sum_{i\in N\setminus\{n\}} a^*_i(t) - b(t)] + \tfrac{1}{2}[\sum_{i\in N\setminus\{n\}} a^*_i(t) - b(t)]^2 \qquad [5]$$

Maximizing [5] with respect to $b(t)$ yields

$$\sum_{i\in N\setminus\{n\}} a^*_i(t) - 2b(t) + \beta(t) - \sum_{i\in N\setminus\{n\}} a^*_i(t) + b(t) = 0.$$

which reduces to

$$b(t) = \beta(t)$$

By definition of Nash equilibrium,

$$b^*(t) = \beta(t). \qquad [6]$$

The next step in the proof involves the agents in $N\setminus\{n\}$. Given that the other agents are playing their Nash equilibrium strategies, the payoff to each $i \in N\setminus\{n\}$ is

$\dot{u}_i((m_i(t), m^*_{-i}(t)) =$

$$[\alpha_i(t) - \delta_w^{i*}(t)\sum_{j\in N\setminus\{n,i\}} a_j^*(t) - \delta_w^{i*}(t)a_i(t)][\sum_{j\in N\setminus\{n,i\}} a_j^*(t) + a_i(t) - b^*(t)] + \tfrac{1}{2}\delta_w^{i*}(t)[\sum_{j\in N\setminus\{n,i\}} a_j^*(t) + a_i(t) - b^*(t)]^2 \quad [7]$$

Maximizing [7] with respect to $a_i(t)$, yields

$$\alpha_i(t) - \delta_w^{i*}(t)\sum_{j\in N\setminus\{n,i\}} a_j^*(t) - \delta_w^{i*}(t)a_i(t) = 0. \quad [8]$$

By definition of a Nash equilibrium, $a_i^*(t) = a_i(t)$ in [8]. Thus, we get

$$\alpha_i(t) = \delta_w^{i*}(t)\sum_{k\in N\setminus\{n\}} a_k^*(t) \quad [9]$$

For each $k \in N\setminus\{n\}$, we can derive the equation [9]. Summing the corresponding equations [9] over all $k \in N\setminus\{n\}$, we get

$$\sum_{i\in N\setminus\{n\}} \alpha_i(t) = \sum_{k\in N\setminus\{n\}} a_k^*(t) \quad [10]$$

By [6] and [10], the conditions for Theorem 1 are fulfilled in a Nash equilibrium. To check that a Nash equilibrium exists at every $t \in [0, \infty)$ choose $m(t) = ((a(t), \delta(t)), b(t))$ such that $a_i(t) = \alpha_i(t)$ for all $i \in N\setminus\{n\}$ and for all i, $k \in N\setminus\{n\}$, $\delta_k^i(t) = \alpha_i(t)/\sum_{j\in N\setminus\{n\}}\alpha_j(t)$ and $b(t) = \beta(t)$. By construction, no consumer can unilaterally affect $\delta_w^i(t)$ since $n \geq 3$. Given $\delta_w^i(t) \geq 0$ and the assumptions on u_k for all $k \in N$, $(a(t), \delta(t), b(t))$ is a Nash equilibrium since the conditions [6], [9] and [10] obtained above are met. ■

4. Conclusion

The objective of this paper was to address a specific problem in planning for which no resolution had been suggested by the existing literature on the subject. A particular mechanism was devised. This mechanism exploits the fact that efficiency in allocation problems with public goods requires accurate information about marginal willingnesses to pay at the aggregate level. The set of equilibria of the mechanism contains a continuum of equilibria.

In addition, the distribution scheme that emerges in an equilibrium has a specific form given by the conjunction of equations [9] and [10]. Thus, endogenizing the decisions regarding distributive justice has affected the neutrality of the mechanism that we use. A notion of distributive justice that offers "refunds" to consumers in proportion to their relative marginal willingnesses to pay for a project (this is the implication of [9] and [10]) may be theoretically justifiable by appealing to the sort of mechanism presented here.

We conclude with the observation that a broader question that this paper raises is: how are the results on mechanism design affected when notions of social welfare emanate from the agents themselves? It is typical to assume that social welfare correspondences and performance standards are arbitrarily pre-determined. Realism demands that the choice of such be part of every agent's strategy space. Can any given notion of social welfare be justified as an equilibrium outcome of some mechanism that takes account of this endogeneity? In a rather limited context, this paper has attempted to address this question.

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