TR-100

TECHNICAL REPORT

POWER SPECTRUM ANALYSIS OF WAVE MOTION, SUBMARINE ROLL ANGLE, AND RELATIVE CROSS-FLOW VELOCITIES

CRUISE II
USS REDFIN (SS-272)

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ABSTRACT

This report discusses the general problem of recording and analyses of wave, submarine, and fluid motion data obtained from the USS REDFIN while hovering at keel depths (near 100 feet) at different relative headings. In particular, the problem of recording surface wave heights with the Sonic Surface Scanner is discussed. Power spectral estimates obtained by both analog and digital methods are presented. Composite graphs of power spectra of data are presented, and mean values of the motion are given in tabulated form.

FOREWORD

The advent of the nuclear submarine with its unlimited cruising range has increased the urgency of acquiring and analyzing data required to assess the ocean environment. This report presents one example of the type of analysis required to understand some of the characteristics of random-type data. Such data are exemplified by continuous recordings of submarine motions and relative flow across the deck of the hovering or slowly moving submarine, and digital recordings of the height of the sea surface above the submarine. The fact that so many interrelated variables can be sensed and recorded simultaneously is an indication of the submarine's potential as an oceanographic research platform. In addition, such simultaneous recording procedures make the data amenable to even more sophisticated analysis; namely, the estimation of cross-spectral densities connecting two continuously distributed random variables.

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Hydrographer

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I. INTRODUCTION

The submarine, USS REDFIN (SS-272), was utilized as a working platform by the Submarine Systems Section, U. S. Navy Hydrographic Office, to obtain digital recordings of surface wave motion and analog recordings of ship and fluid motions. This report is concerned with the spectral analysis of these recordings made while the REDFIN was hovering.

One of the objectives of the Submarine Systems Section is to collect and analyze data needed to study environmental effects on submarine motion, and the characteristics of the relative water motion in the proximity of the submarine. Such data, recorded while the submarine was hovering or underway at the lowest possible speed, have been analyzed by computing power spectra of the recordings.

Data presented in this report include composite graphs of spectra of surface wave motion (computed by digital estimation formulae), spectra of the submarine's roll angle, and spectra of the transverse and logitudinal relative flow velocity of water across the submarine's deck (obtained by an analog computer). Examples of spectra of in situ pressure fluctuations, estimated by both digital and analog methods, are presented. Differences in approach embodied in the digital versus analog method of power spectra estimation are discussed, and specific examples are presented to illustrate points of technique required for comparable resolution and statistical stability. Finally, spectra data presented in this report are summarized by presenting mean values of motion in tabular form.

The objective of this particular test was to study environmental effects on the motion of the submarine. During this test period, wave conditions were generally mild with the result that some data were recorded near the sensitivities of the instruments. Thus, energy spectra and related statistical parameters are representative of conditions encountered under lower sea states.

This report includes most of the data recorded on REDFIN Cruise II during the period 28 October to 5 November 1959. During this time, the REDFIN was hovering at a keel depth of 100 feet, and operating in an area centered about 180 miles east of Wilmington, North Carolina.

Hydrographic Office personnel participating in this test were Messrs. Q. H. Carlson (Senior Oceanographer), A. G. Alexiou, and D. E. Tidrick.

II. INSTRUMENTATION AND RECORDING PROCEDURES

A. Instrumentation

Wave, ship, and fluid motion observations discussed in this report were made from aboard the REDFIN. Instrumentation utilized and variables measured are included in the listing that follows. Locations of these instruments on the submarine are shown in Figure 1.

Pitch and Roll Angle	David Taylor Model Basin stable platform and potentiometer
Relative Flow Velocity	Litton Electromagnetic Flow Meter
In Situ Pressure Fluctuations	Wiancko Pressure Measuring System (U. S. Navy Mine Defense Laboratory)
Ship's Speed	Ship's Electromagnetic Log
Ship's Course	Ship's Gyrocompass
Depth (pressure)	Vibrotron and Bourns potentiometer in the pressure gauge line
Surface Wave Profile	Westinghouse Sonic Surface Scanner

Figure 2 illustrates a portion of a typical recording taken aboard the REDFIN. Most data were recorded on both magnetic and paper tape. Surface wave and in situ pressure fluctuation data, however, were recorded on magnetic tape only. A complete description of the instrumentation and measurement and recording procedures is given in the Hydrographic Office Technical Report No. 91 (Reference 10).

Nearly all pitch angle recordings, heave acceleration, and sway acceleration were practically flat and will not be discussed further in this report. Thus, the only submarine motion of interest is roll angle. Some cross-flow data with very low amplitudes were deleted.

Although attempts were made to provide analog recordings of in situ pressure fluctuations measured by the Wiancko pressure gauge, most of the collected data were deleted because the amplitude of the recording's envelope frequently exceeded the dynamic range of the pressure gauge. When this occurred, the relief valve in the gauge would open, resulting in intermittent jumps in the continuous recordings. Attempts to filter these pressure data before spectral analysis were not too successful, and most of the spectra of the Wiancko recordings were not considered valid.

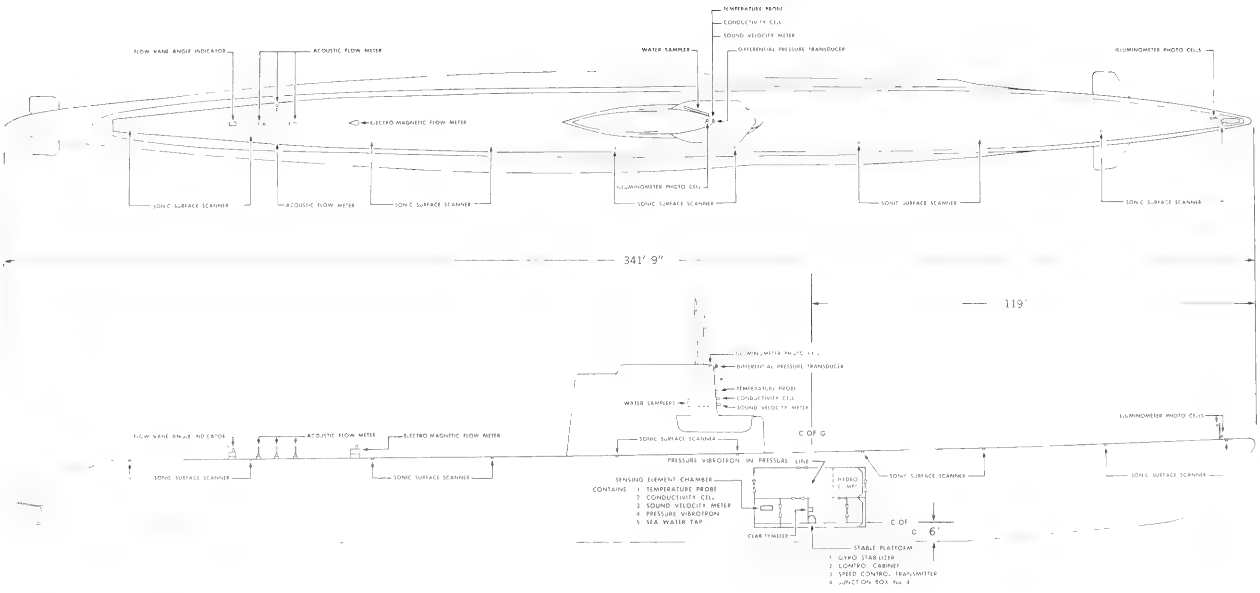

FIGURE 1. OCEANOGRAPHIC INSTRUMENT LOCATIONS– USS REDFIN (SS-272)

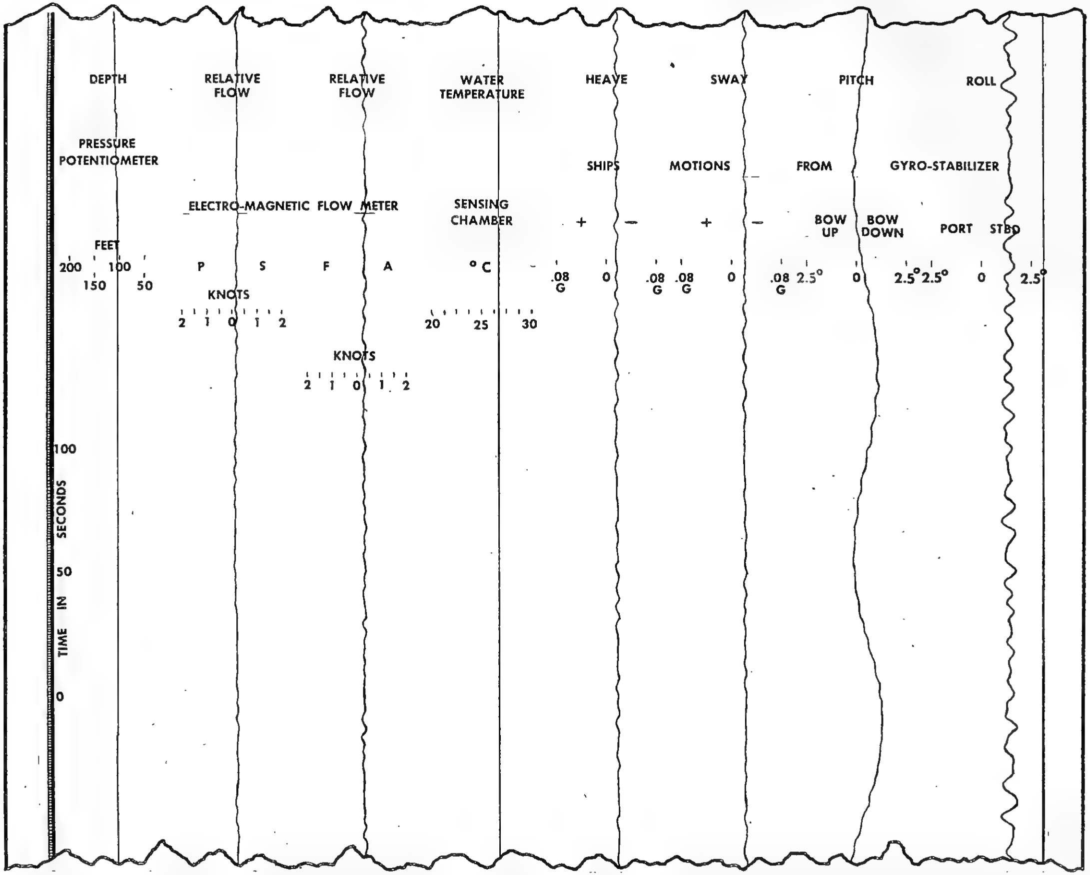

FIGURE 2. TYPICAL VARIABLES RECORDED ON THE OFFNER (DYNAGRAPH TYPE R) 8-CHANNEL RECORDER USS REDFIN, CRUISE II

Surface wave data were recorded by the Sonic Surface Scanner in the form of pulses on magnetic tape. Since this method may prove to be a useful way of obtaining wave data in the open ocean while submarines are on routine patrol, the theory and method of this technique are reviewed briefly in Appendix A.

B. Recording Procedure

The recording procedure was as follows: The REDFIN submerged to 100 feet (keel depth) and attempted to hold this depth under zero ground speed. Recordings of one-half hour duration were made under these conditions at various headings and wave conditions. Test conditions are summarized in Table 1. Exact headings were difficult to maintain; headings, relative to wave directions, shown in Table 1 are the average values during each run. Each recording was given two numbers for identification. For example, Run 3-1 is the first recording taken on Run 3. Except for surface wave data and in situ pressure fluctuations, each run was recorded on paper tape for visual analysis and monitoring of data recorded at sea, and, also, on magnetic tape for electronic analysis.

Most surface wave data in Table 1 were obtained by "hindcasting" from synoptic weather maps (Reference 9). There were a few visual observations, but these observations are considered unreliable, limited in many cases by darkness. The general difficulties of making visual wave observations at sea are discussed in Reference 11. Hindcast wave data provide an independent source of wave height, period, and direction. These are particularly useful as a check on the accuracy of visual observations made aboard a submarine, in the event no surface data are available from the Sonic Surface Scanner.

III. THEORY AND METHOD OF POWER SPECTRUM ANALYSIS

A. Spectrum Analysis

A preliminary analysis of most of the variables discussed in this report was presented in Reference 8. This report contains mean and maximum values of the peak-to-peak roll angles, and peak-to-peak cross-flow velocities. However, motion characteristics of such variables (including surface wave motion) can be more completely described by computing energy spectra of the recordings. Analog and digital recordings of motion data described in this report are assumed to be samples from quasi-stationary Gaussian processes whose mean values are zero. Such processes are completely described by their energy (power) spectra.

Suppose part of a given recording of the process occurs as shown in Figure 3. Although such functions are random, amplitudes of the process (determined from a line

TABLE 1. TEST CONDITIONS DURING CRUISE II, USS REDFIN, 28 OCTOBER TO 1 NOVEMBER 1959

Hovering Depth: 100 Feet (Keel Depth)

				Visual Observations		Surface Waves			
						Waves Hindcast From Synoptic Charts			
Run	Date	Time (Z)	Relative Heading (°T)	Height (Ft)	Direction (°T)	Height* (Ft)	Direction (°T)	Period* (Sec)	Period Band (Sec)
3-1	28 Oct	0835– 0916	165 105	–	120	6 9	315 225	5.1 7.5	2.5–10.0 Sea 3.0–12.0 Swell
3-2	28 Oct	0933– 1007	165 060	–	120	6 7	330 225	5.1 7.5	2.5–10.0 Sea 4.0–11.0 Swell
3-3	28 Oct	1110– 1140	085 030	–	120	6 5	340 225	5.1 8.0	2.5–10.0 Sea 6.0–10.0 Swell
3-4	28 Oct	1140– 1215	060 075	–	180	6 4	360 225	5.1 9.0	2.5–10.0 Sea 8.0–10.0 Swell
4-1	29 Oct	0442– 0510	068	–	–	3 –	022 –	4.0 –	1.5– 7.8 Sea –
4-2	29 Oct	0520 0550	113	–	–	3 –	022 –	4.0 –	1.5– 7.8 Sea –

Note: Two wave systems occurred during Run 3. The submarine's relative heading with respect to each wave system is shown in the sketch.

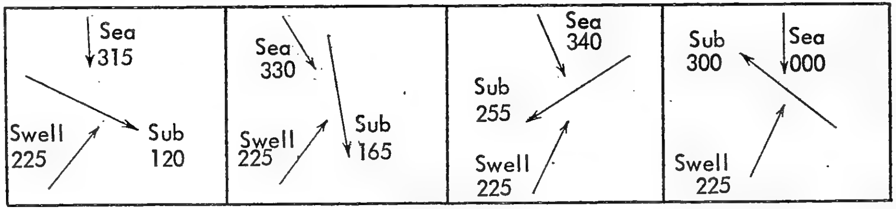

TABLE 1. TEST CONDITIONS DURING CRUISE II, USS REDFIN, 28 OCTOBER TO 1 NOVEMBER 1959 (Cont'd)

Run	Date	Time (Z)	Relative Heading (°T)	Visual Observations		Surface Waves			
						Waves Hindcast From Synoptic Charts			
				Height (Ft)	Direction (°T)	Height* (Ft)	Direction (°T)	Period* (Sec)	Period Band (Sec)
4-3	29 Oct	0600– 0640	158			3	022	4.0	1.5– 7.8 Sea
				–	–	–	–	–	–
4-4	29 Oct	0650– 0730	157			3	022	4.0	1.5– 7.8 Sea
				–	–	–	–	–	–
4-5	29 Oct	0745 0815	100			4	010	5.0	2.0– 8.3 Sea
				–	–	–	–	–	–
5-1	30 Oct	0445– 0530	015	4	075	5	060	5.5	2.5–10.0 Sea
						–	–	–	–
5-2	30 Oct	0537– 0636	063	4	075	6	090	5.5	2.5–10.0 Sea
						–	–	–	–
5-3	30 Oct	0647– 0720	128	4	075	7	090	6.5	3.0–11.0 Sea
						–	–	–	–
5-4	30 Oct	0734– 0822	180	4	075	7	090	6.5	3.0–11.0 Sea
						–	–	–	–
6-1	1 Nov	1235– 1320	102		210	5	225	4.6	2.0– 8.8 Sea
				–		–	–	–	–
6-2	1 Nov	1327– 1403	156		210	5	225	4.6	2.0– 8.8 Sea
				–		–	–	–	–
6-3	1 Nov	1407– 1433	160		210	5	225	4.6	2.0– 8.8 Sea
				–		–	–	–	–
6-4	1 Nov	1513– 1551	152		210	5	225	4.6	2.0– 8.8 Sea
						–	–	–	–

*HEIGHT: Average height of the one-third highest waves; PERIOD: Mean Period

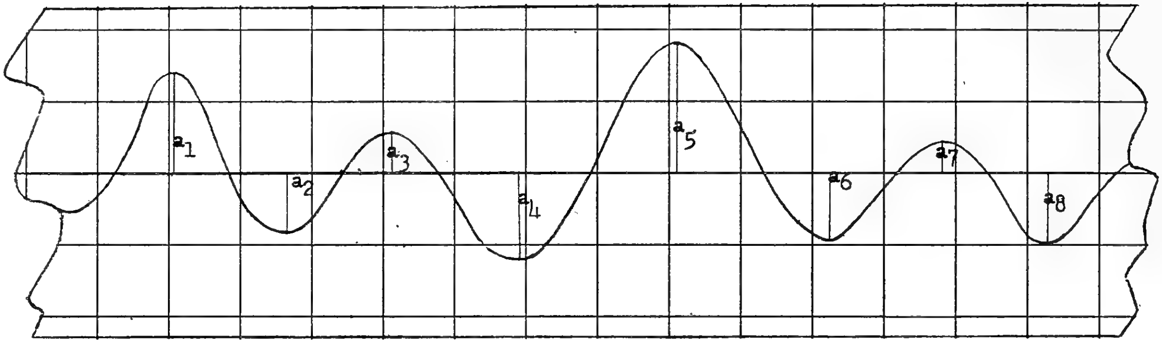

FIGURE 3. EXAMPLE OF A WAVE OR SHIP MOTION RECORD

connecting the zeros of the process) can be given statistically by a probability density function. If the dominant part of the random process occurs over a narrow interval of frequencies, amplitudes shown in Figure 3 can be given by the Rayleigh probability distribution:

$$f(x)\,dx = \frac{2x}{E}\, e^{-\frac{x^2}{E}}\, dx \tag{1}$$

Thus, the probability that a particular amplitude a_i will fall between x and x + dx is given by equation 1. The first moment about the origin of equation 1 gives the relationship between the mean wave amplitude $\bar{a}$ and the quantity E:

$$\bar{a} = \int_0^\infty \frac{2x^2}{E}\, e^{-\frac{x^2}{E}}\, dx = 0.866\sqrt{E} \tag{2}$$

The quantity E is given by the second moment about the origin of the Rayleigh distribution:

$$E = \int_0^\infty \frac{2x^3}{E}\, e^{-\frac{x^2}{E}}\, dx \tag{3}$$

$$\frac{E}{2} = \sigma^2 \tag{4}$$

where σ^2 is the variance of the process. Now E is related to the energy spectrum of the random process by:

$$E = \int_0^\infty [A(f)]^2\, dx \text{ and } \frac{dE}{df} = [A(f)]^2 \tag{5}$$

where $[A(f)]^2$ is called the energy density. From the energy (power) spectra and related statistical parameters, the process can be completely described for practical purposes. The term "power spectra" is freely used by aero- and hydro-dynamicists among others, although much of the work dealing with the energy spectra of random processes originated in the field of communications engineering (Reference 5).

Recordings of submarine motion (roll angle) and fluid motions (transverse and longitudinal relative flow velocity) illustrated in Figure 2, may be visualized by considering them frequency band-limited random signals continually oscillating above and below an equilibrium level of zero signal. Very often (especially in the case of wave and ship motions), the spectrum of such motions is characterized by a well-defined peak value which is skewed toward lower values of frequency. The area under the spectrum curve is related to mean amplitude values of the motion, and from the theoretical work of Longuett-Higgins (Reference 2) et al, the following relations connecting mean wave amplitude $\bar{a}$ to the means of the one-third highest ($\bar{a}_{\frac{1}{3}}$) and the one-tenth highest wave amplitudes ($\bar{a}_{\frac{1}{10}}$) were found:

$$\frac{\bar{a}_{\frac{1}{10}}}{\bar{a}} = 1.800 \qquad \frac{\bar{a}_{\frac{1}{3}}}{\bar{a}} = 1.416 \tag{6}$$

These amplitude relationships were verified on actual wave height data by J. K. A. Watters (Reference 12), who found the following mean wave height relationships $\bar{H}_i$ from a sample of wave records:

$$\frac{\bar{H}_{\frac{1}{10}}}{\bar{H}} = 1.94 \qquad \frac{\bar{H}_{\frac{1}{3}}}{\bar{H}} = 1.58 \tag{7}$$

From these we conclude that wave height values are given approximately by doubling the amplitude values. Finally, since: $\bar{a} = 0.866\sqrt{E}$ then

$$2\bar{a} = \bar{H} = 1.77\sqrt{E} \tag{8}$$

from which we get:

$$\bar{H}_{\frac{1}{3}} = 2.83\sqrt{E} \tag{9}$$

$$\bar{H}_{\frac{1}{10}} = 3.60\sqrt{E} \tag{10}$$

Thus, equations 8, 9 and 10 hold for mean values of roll angle and relative flow velocity, since recordings of roll angle and flow velocity are assumed to be samples from the same type stationary Gaussian random process.

B. Digital Estimation of Power Spectra

The practical problem of estimating energy spectra has received widespread attention and can be viewed from two distinct but practically equivalent points of view. One method of estimating the energy spectrum of a stationary Gaussian random process is to sample the continuous (analog) recording of the process at discrete intervals of time, obtain an estimate of the autocorrelation function, and then obtain raw or unsmoothed estimates of the energy spectrum by Fourier transformation of the autocorrelation function. The power spectrum is then obtained by suitably smoothing the raw or uncorrected spectral estimates. Since the study of such processes is the study of the sample variance of the process, it is essential that confidence intervals be given for each spectrum. The theory and procedures of digital estimation of power spectra are given in References 1, 3, and 6.

If the process can be "quantized" or digitized at the transducer or in the recording process, much of the manual data reduction can be eliminated. For example, the Sonic Surface Scanner measures instantaneous height of the sea surface above the transducers located on the hovering submarine's deck, and these data are recorded directly in digital form on magnetic tape. These values are played back, put on punched cards, and programmed through a high speed computer to obtain the estimated spectra of the surface wave heights. The method of recording and playback is outlined briefly in Appendix A.

As an example, consider Figure 4, which shows the power spectrum obtained from a one-half hour recording from transducer No. 1, located on the submarine's bow (Fig. 1). The dashed lines indicate the 90-percent confidence intervals (See Section C). For comparison, the spectrum of a simultaneous one-half hour recording from transducer No. 8 (located near the stern) is shown in Figure 5 with its 90-percent confidence intervals. Figure 6 shows the good agreement between the two measured spectra.

Amplitudes of submarine motion during this recording were low. The heave acceleration recording, for example, was practically flat. However, at higher sea states and lower wave frequencies, a submarine will move appreciably in response to surface wave motion. For example, heave displacement would be in phase with a pure swell, whose relative heading was 90-degrees, with the result that the fraction of the true recorded wave amplitude decreases with decreasing frequency (See Appendix A for a discussion of expected measurement errors). In general, surface wave spectra estimated from recordings of the Sonic Surface Scanner, will have to be corrected for the hovering submarine's motion. Furthermore, if the submarine is underway, an additional correction will be

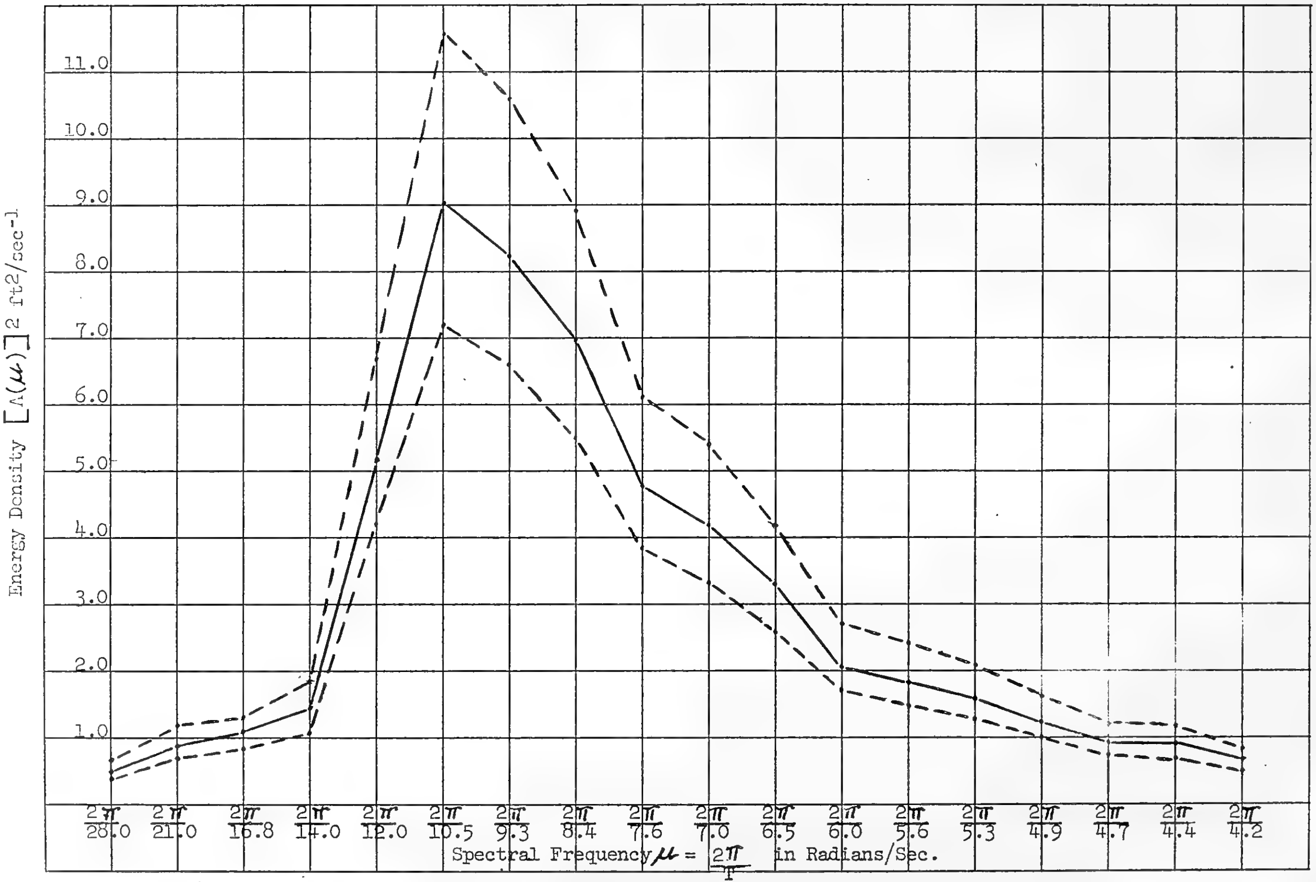

FIGURE 4. SURFACE WAVE SPECTRUM - TRANSDUCER NO. 1

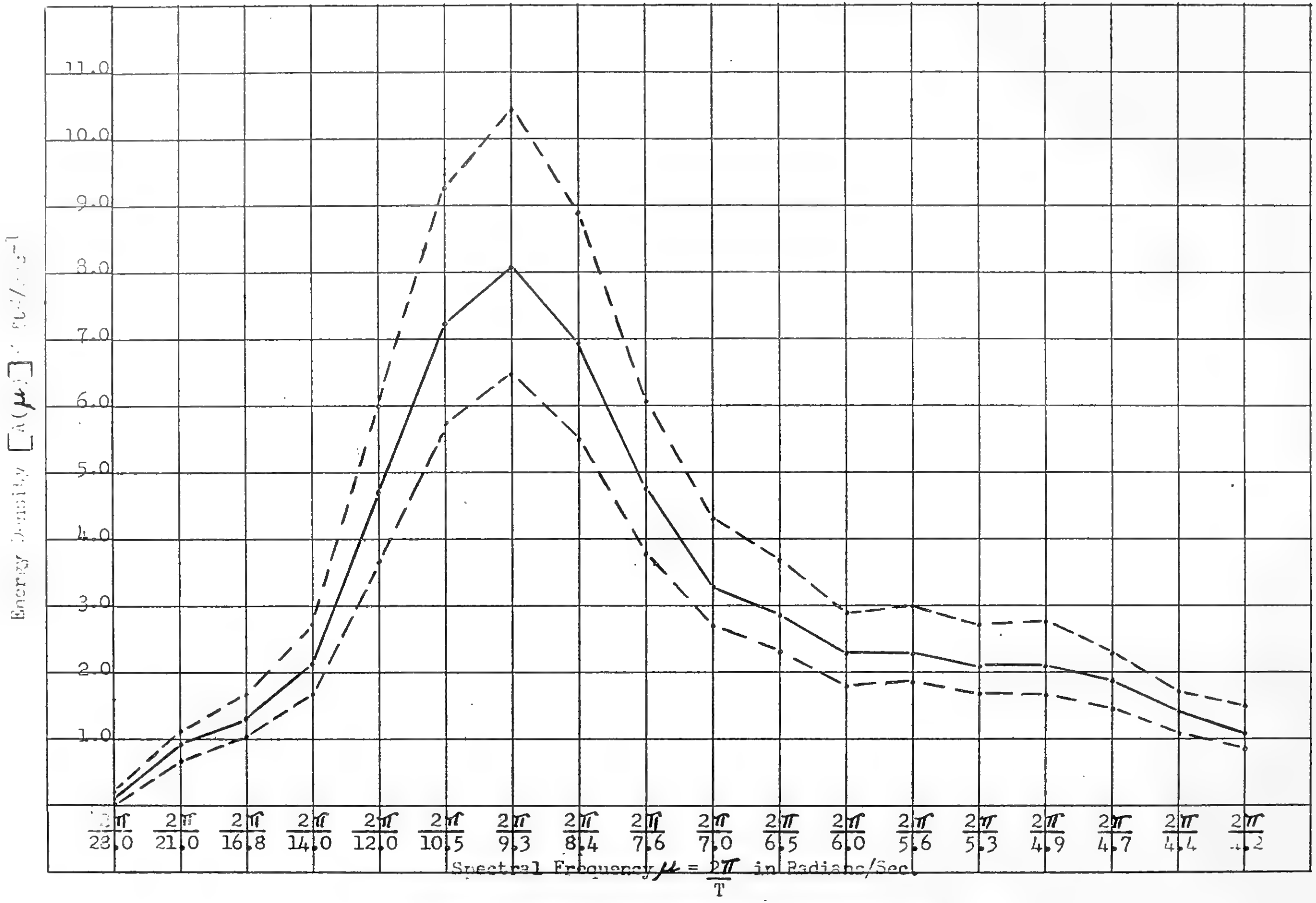

FIGURE 5. SURFACE WAVE SPECTRUM - TRANSDUCER NO. 8

necessary for its speed and relative heading.

Results of power spectra of surface wave motion obtained on the global circumnavigational cruise of the USS TRITON (SSRN-586)* indicate that such spectra can be measured successfully at speeds of 6 knots. At 6 knots, the TRITON's speed was equal to the phase speed of a 2-second wave. Thus, spectra given in terms of the frequency of encounter could be transformed in terms of the frequency relative to fixed coordinates.

C. Analog Estimation of Power Spectra

The digital method of estimating power spectra may not be practical if a large number of recordings are to be analyzed. This being the case, the recommended procedure would be to record analog data on a multichannel tape recorder and compute energy spectra by commercially available wave analyzers. This was the method used to analyze ship- and fluid-motion data presented in this report (References 4 and 7). Most of the analog spectra of submarine and fluid motions presented in this report were computed at the David Taylor Model Basin. The Hydrographic Office has recently acquired the electronic components required to estimate power spectra shown in the block diagram of Figure 7. An example of an analog spectrum of submarine roll angle estimated by the system at the Hydrographic Office is shown in Figure 8.

The analysis procedure is as follows: The random process to be analyzed is recorded aboard the submarine on magnetic tape at a tape speed of 1 7/8 inches per second. Each one-half hour run recorded at this speed is played on a 14-channel Ampex playback unit (Fig. 7) at 60 inches per second while being re-recorded on a 14-channel Precision recorder. The random signals are re-recorded also on an 8-channel Sanborn recorder so that the quality of the recordings can be inspected, and an indication of expected amplitudes of the random signals can be obtained. This results in a compression of 1/32 the original recorded tape length; the frequency components in the random signal are all increased by a factor of 32. The compressed recording is formed into a continuous loop and threaded into a special loop attachment. The Precision recorder is utilized again to drive the loop attachment at a speed of 30 inches per second for an additional frequency increase of 16 times and a total frequency increase of 32 x 16 = 512 times the original frequencies. Thus, a frequency of 0.10 cycles per second appears on the analyzer as a frequency of 51.2 cycles per second. The reason for analyzing in this frequency range is that beat frequency analyzers commonly used can easily accommodate frequencies at higher ranges, but could not handle gravity wave frequencies (e.g. 0.03 to 0.30 cycles per second).

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*U. S. HYDROGRAPHIC OFFICE. Oceanographic data report for the global circumnavigational cruise of the USS TRITON (SSRN-586). 16 February to 10 May 1960, by N. R. Mabry (in press).

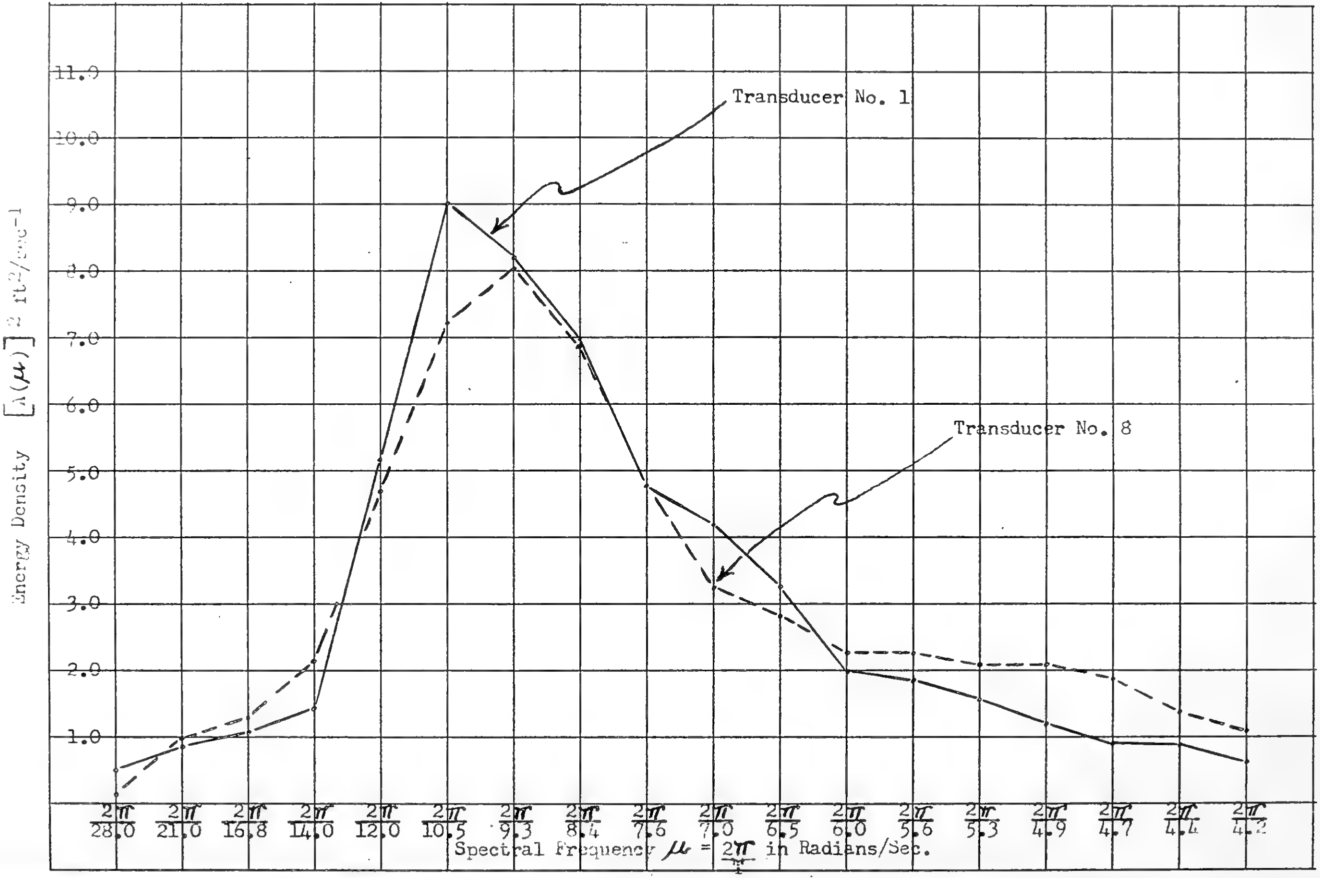

FIGURE 6. SURFACE WAVE SPECTRA - COMPARISON OF TRANSDUCER NO. 1 AND TRANSDUCER NO. 8

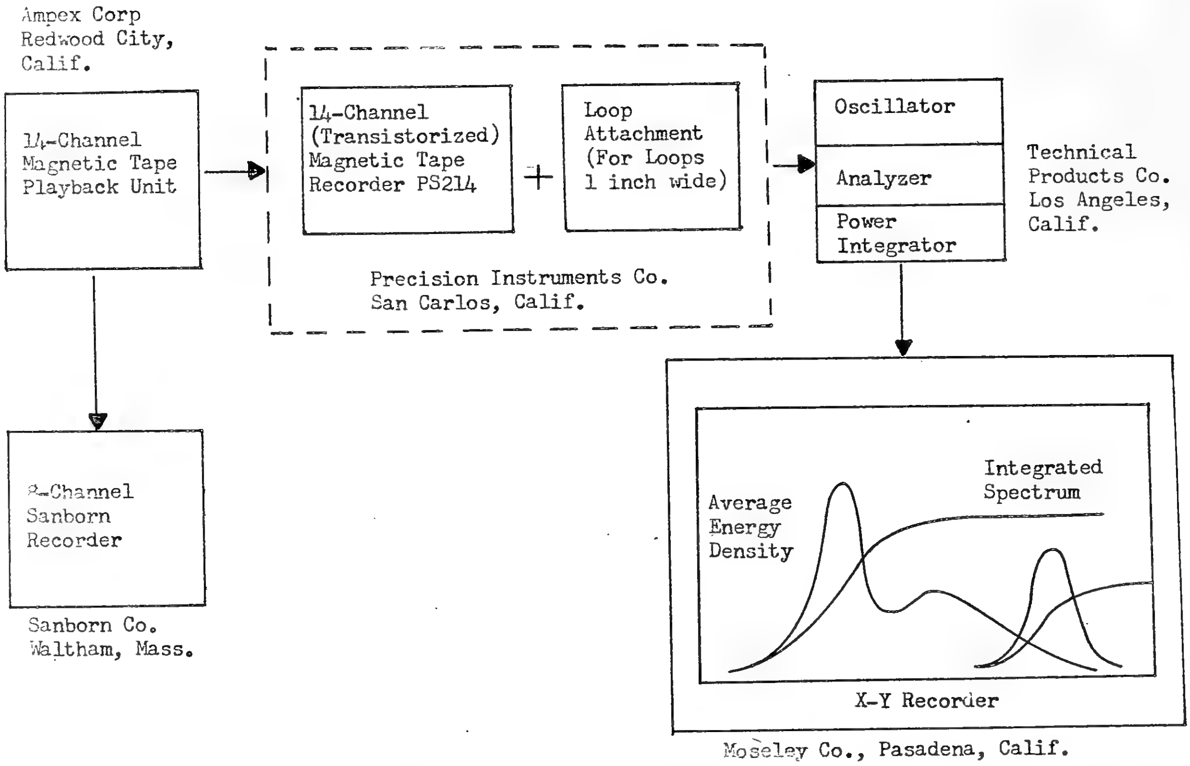

FIGURE 7. BLOCK DIAGRAM OF THE ANALOG WAVE ANALYSIS SYSTEM USED BY THE SUBMARINE SYSTEMS SECTION U. S. NAVY HYDROGRAPHIC OFFICE

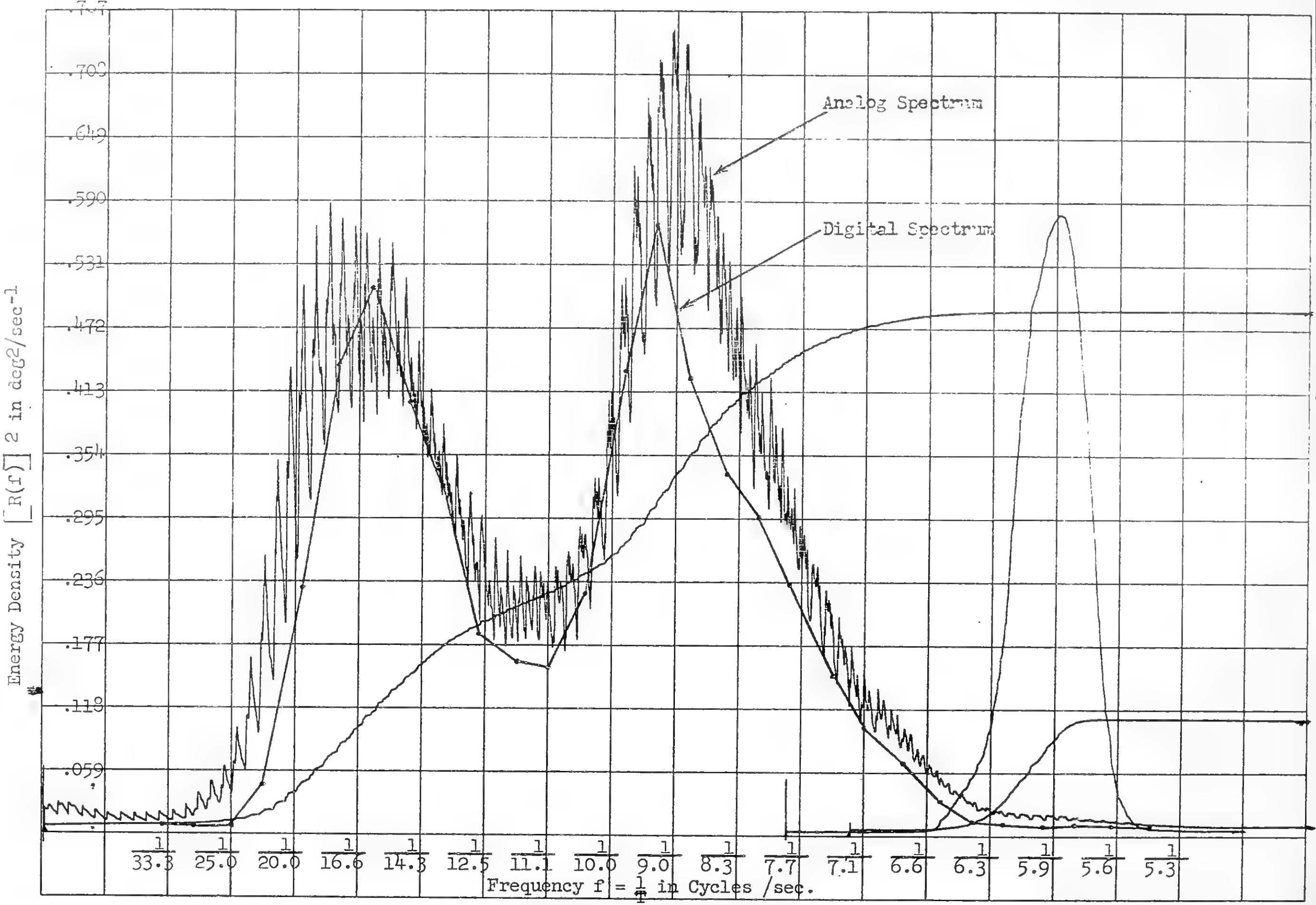

FIGURE 8. COMPARISON OF ENERGY DENSITY OF ROLL ANGLE ESTIMATED BY A WAVE ANALYZER AT THE HYDROGRAPHIC OFFICE AND DIGITAL SPECTRUM ESTIMATED BY DATATRON, WITH INTEGRATED SPECTRUM SUPERIMPOSED, RUN 4-1

The random signal output is fed into the analyzer system consisting of an oscillator, analyzer, and power integrator. These three units transform the random time signal into a continuous plot of the average energy density versus frequency, which is traced out on a Moseley X-Y Recorder. This system also provides for estimation of the integrated spectra. The block diagram of Figure 7 illustrates an integrated spectrum curve superimposed on the energy density curve. The small inset curves in the lower right hand corner of the X-Y Recorder represent the spectral outputs of the filter used to resolve the oridinate scales of the graphs of average energy density and the integrated spectrum. Details and circuit diagrams are given in "Instruction Booklets for TP-625 Wave Analyzer System," Technical Products Company, Los Angeles, California. A discussion of the problems associated with filter bandwidth, loop periods, time constants, and frequency scanning rates is given in Reference 4.

Figure 8 is an example of the power spectrum of a submarine roll angle recording and the power spectrum of the filter used in the analysis. This illustrates a typical power spectrum of the filter used to calibrate the ordinate scale of the energy density graphs. The broken line curve is the digital spectrum of the same recording which is superimposed for comparison. The digital spectrum superimposed is one of four spectra shown on Figure 9 (See Section D on comparison of spectral estimates).

This analog spectrum was estimated by the wave analyzer in the Hydrographic Office, and all the digital spectra contained in this report were estimated by a Burroughs 205 Electronic Computer, also located in the Hydrographic Office.

Such spectral estimates are distributed with a chi square distribution with 2N degrees of freedom, where N is the number of elemental bands covered by the filter. If the spectral density is not a fast changing function of frequency interval equal to the bandwidth of the filter, then N is approximately equal to the effective bandwidth of the filter divided by the elemental bandwidth. The elemental bandwidth is 1/T, where T is the loop period, and the effective bandwidth used was 6 cycles per second (Reference 4). Thus, for the analog spectra in this report:

$$\text{Number of degrees of freedom} = 2N = 2\ \frac{\text{effective bandwidth}}{\text{elemental bandwidth}} \qquad (11)$$

Confidence intervals for the estimated spectra can be determined from a table of the chi square distribution.

The 90-percent confidence intervals for the digital spectra are given by computing the number of degrees of freedom according to:

$$\text{Number of degrees of freedom} = \frac{2\ Tn}{m\ \Delta t} \qquad (12)$$

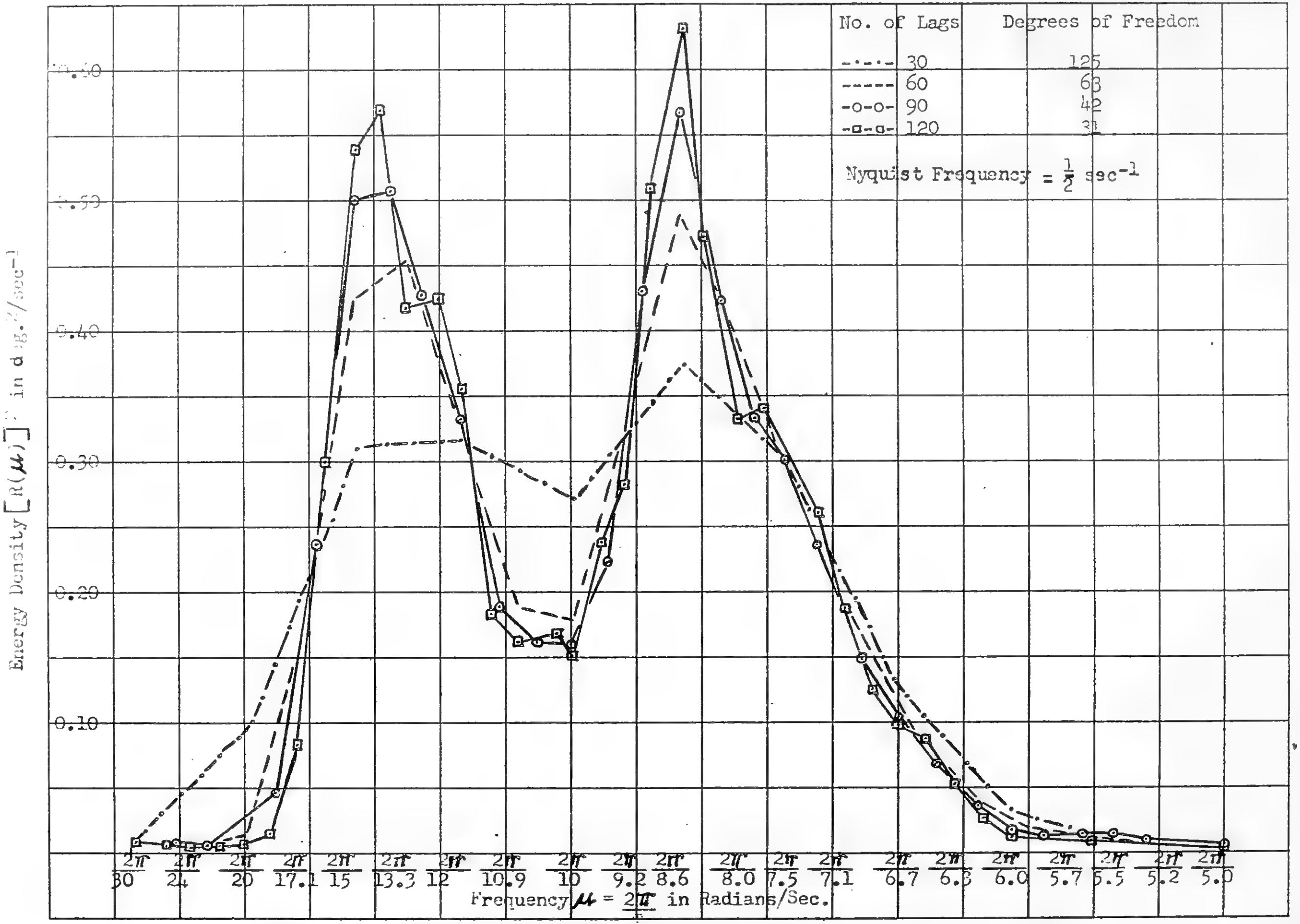

FIGURE 9. DIGITAL SPECTRA OF ROLL ANGLE FOR LAGS 30, 60, 90, and 120, RUN 4-1

where Tn is the length of the record in seconds, m is the number of lags, and Δt is the sampling interval. For example, the spectral estimates in Figures 4 and 5 are distributed with a chi square distribution with 70 degrees of freedom. The 90-percent confidence intervals can be determined from a table of chi square distribution. For 70 degrees of freedom, the true spectral density P_T (f) is bounded by the estimated spectral density P_E (f) according to:

$$\frac{P_E\ (f)}{1.30} < P_T\ (f) < \frac{P_E\ (f)}{0.76} \quad \text{or} \quad 0.77P_E(f)<P_T(f)<1.32P_E(f) \qquad (13)$$

Thus, if a single observed estimate with 70 degrees of freedom is observed to be 5 ft ^{2}/sec^{-1}, then we have 90-percent confidence that the true long-run value lies between 5/1.30 = 3.85 ft ^{2}/sec^{-1} and 5/0.76 = 6.58 ft ^{2}/sec^{-1}.

D. Some Considerations in Estimating Spectra

1. Resolution and Statistical Stability

In estimating power spectra, whether by digital or by analog methods, a choice must be made between a high resolution of the spectral estimates over a chosen frequency band and the statistical stability of the spectral estimate in that band. The two considerations are mutually opposed in that fulfillment of one implies some sacrifice of the other. For example:

a. Width of the "spectral window" determines the resolution of any peaks in the spectrum. Now, if the spectral window (i.e., the filter in the wave analyzer) is chosen too narrow, too much detail is obtained, and a truly significant peak may be overlooked in the resultant "blurring." However, if the spectral window is made too wide, the resulting spectral estimate may be too smooth, and here again some physically significant hump in the spectrum may be smoothed over.

b. The statistical stability of the estimate depends on the width of the spectral window and the length of the sample. When high resolution is desired, fewer Fourier components are averaged over the narrow spectral window, and the spectral estimate will have wider fluctuations over the totality of ensemble estimates. Conversely, less resolution with a wider filter gives a value which is expected to deviate less from the true but unknown spectrum.

2. Aliasing

a. If a signal g(t) contains no frequencies above f cycles per second, it is completely determined by giving its ordinate by a series of points spaced $\frac{1}{2f}$ seconds apart. For example, if waves with frequencies greater than 0.5 cycles per second are absent, the wave record can be sampled every second. However, if appreciable wave energy is present at frequencies above f cycles per second, this energy will contribute (by aliasing) to the spectrum of the lower frequency waves. Thus, the sampling interval must be determined by the high frequency cut-off.

b. The range of wave frequencies shown on the abscissas of Figures 4, 5, 10, and 11 represent the most important part of the spectral energy density associated with surface wave heights. Similarly, the range of frequencies in Figure 9 are associated with the dominant rolling motion of the submarine. Now, if a random process is characterized by oscillations about a slowly varying centerline, the spectral estimates at frequencies near zero may have appreciable values. However, since this spectral energy is not associated with frequencies in the gravity wave spectrum, which are the only frequencies of interest in this report, the graph is arbitrarily cut off at the lowest important frequency. Similarly, spectral densities at the high frequencies were not plotted since spectral densities decreased rapidly as the Nyquist frequency was approached.

3. Comparison of Spectral Estimates

When the estimated analog spectrum of a process is to be compared to a digital estimate of the same spectrum, the equivalent spectral windows must be the same. Figure 9 illustrates 4 digital estimates of the analog spectrum shown in Figure 8. The analog spectrum in Figure 8 had 58 degrees of freedom, and the four digital spectra had 31, 42, 63, and 125 degrees of freedom. Therefore, the digital spectra with 63 degrees of freedom were superimposed on Figure 8 for comparison.

IV. POWER SPECTRA OF SURFACE WAVE, SUBMARINE, AND FLUID MOTIONS

A. General

Wave, ship, and fluid motion data analyzed in this report were taken while the REDFIN was executing a series of maneuvers which included hovering at different relative headings. During each hovering run, the REDFIN attempted to hold a keel depth of 100 feet at a fixed relative heading while making recordings of surface wave height (Sonic Scanner), roll angle, transverse relative flow velocity, longitudinal relative flow velocity, and in situ pressure fluctuations. Each recording was at least one-half hour in duration.

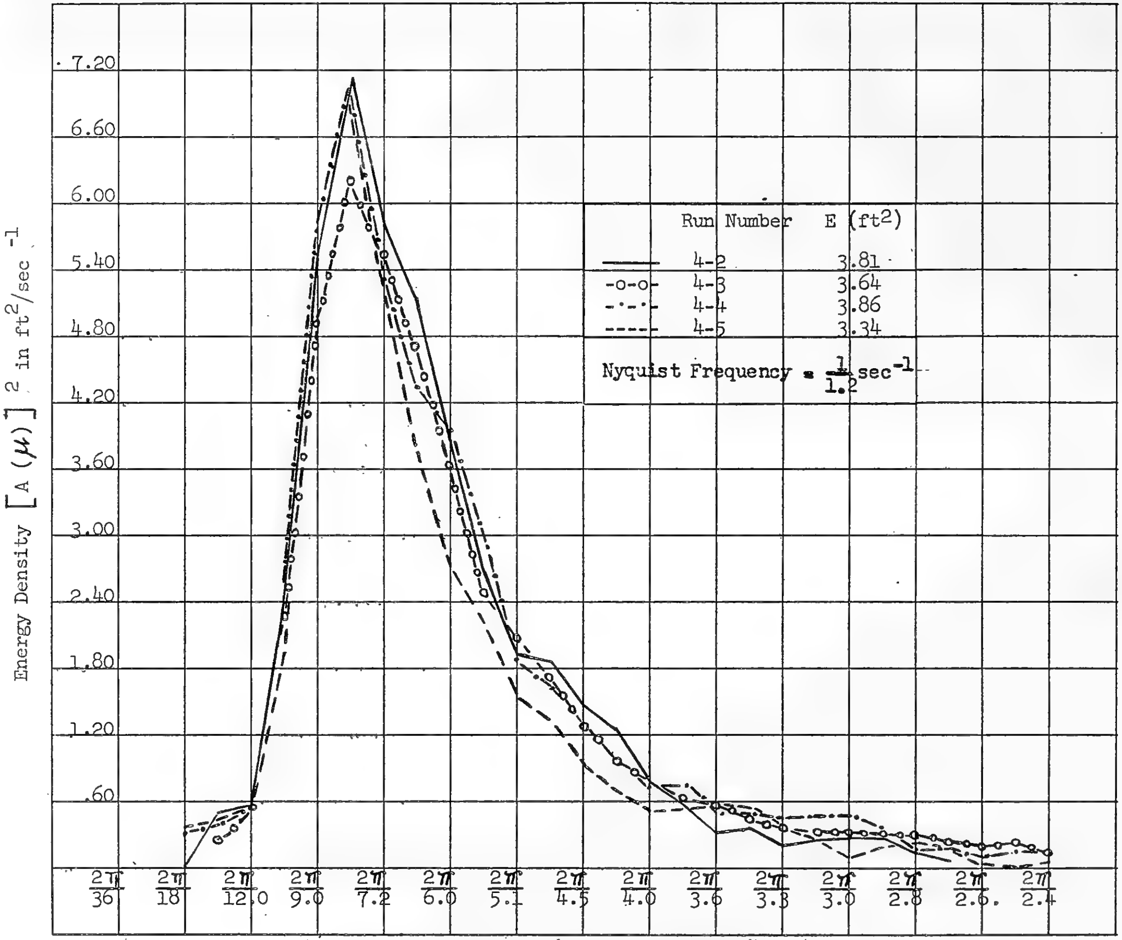

FIGURE 10. SPECTRA OF SURFACE WAVE HEIGHTS, RUN 4

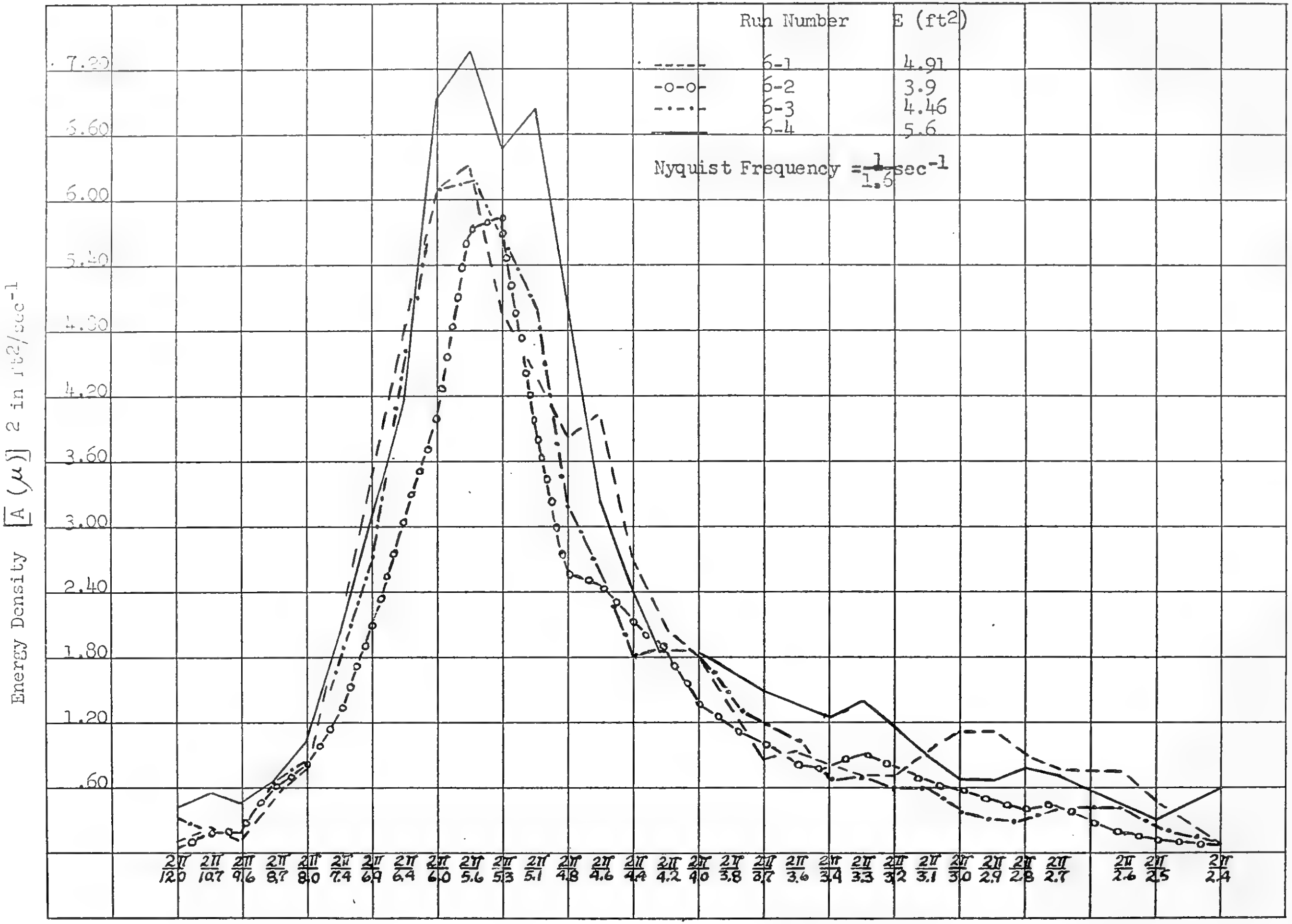

FIGURE 11. SPECTRA OF SURFACE WAVE HEIGHTS, RUN 6

Table 1 is a summary of test conditions for Runs 3, 4, 5, and 6 of Cruise II. The first recording began with Run 3. Wave conditions were generally mild except for the first four recordings of Run 3. On four of the runs (Runs 4-1, 5-1, 5-4, and 6-3), large variations in depth occurred because of temporary loss of buoyancy control. However, such large depth variations were of very short duration, and the submarine generally was able to return to its original hovering position in about 7 minutes. All runs where this occurred were continued for 30 minutes after depth was regained, and only this portion of the record was analyzed.

Submarine motions and relative flow velocity of water across the submarine's deck occur as a result of surface wave motion. Since motion characteristics of submarines will be required under various sea states, it is essential that some estimate of the surface wave spectrum be available. Such spectra can be obtained by hindcasting from the synoptic sea-level surface weather maps by methods described in H. O. Pub No. 603 (Reference 9). However, the exact functional form of the Pierson-Neumann spectrum and its dependence on such parameters as wind speed, wind duration, and fetch length are presently being improved. Hence, it is essential to obtain a series of surface wave spectra over the open ocean so that differences between measured and predicted spectra can be reconciled.

Although the Sonic Surface Scanner method is still in the experimental stage, it is a promising method of obtaining series of surface wave records in the open ocean. Subject to corrections for the motion of the submarine, it should be possible to obtain reliable spectral estimates of surface wave heights from such records.

B. Surface Wave Height Spectra

Surface wave height spectra for Runs 4 and 6 are shown in Figures 10 and 11, respectively. The sonic scanner was inoperative during Runs 3 and 5. Figures 10 and 11 indicate that wave conditions were stationary during each of these series of runs. No corrections have been applied to these surface wave spectra, since the amount of wave energy associated with high period waves is rather low. Table 2 presents the 90-percent confidence intervals for these spectra.

Each of the submarine and fluid motion spectra shown in the composite graphs (Figs. 12 to 21) was obtained in the same form as depicted in Figure 8. For ease of comparison and identification, however, a smooth curve was reconstructed from each analog spectrum.

C. Roll Angle Spectra

Figures 12 through 15 present analog energy spectra of roll angles for Runs 3 to 6. Each figure contains 4 spectra corresponding to a different relative heading. Each roll angle spectrum has a typical peak, occurring between 16 and 20 seconds, which is associated with the natural rolling period of the REDFIN. The natural rolling period of the REDFIN is about 18 seconds. Secondary peaks in Run 3, occurring at periods lower than 10 seconds, are associated with the lower period waves observed during this run (Table 1). Table 3 presents the 90-percent confidence intervals for all analog spectra discussed in this report.

TABLE 2. NINETY PERCENT CONFIDENCE INTERVALS FOR DIGITAL SPECTRA OF SURFACE WAVE HEIGHTS

Run No.	Degrees of Freedom	Lower Limit	Upper Limit
4-1	80	.78 $P_E(f)$	1.27 $P_E(f)$
4-2	80	.78	1.27
4-3	80	.78	1.27
4-4	80	.78	1.27
6-1	60	.74	1.39
6-2	60	.74	1.39
6-3	66	.76	1.37
6-4	68	.76	1.35

D. Transverse Relative Flow Velocity Spectra

Figures 16 and 17 present energy spectra of transverse relative flow velocity for Runs 4 and 6, respectively. Figure 16 presents 5 spectra while Figure 17 presents 2 spectra, each corresponding to different relative headings. Spectra for Runs 3 and 5 were deleted because the amount of energy associated with frequencies below the lower cut-off value (.03 cycles per second) of the wave analyzer made the determination of parameters extremely doubtful. In fact, the excessive energy produced some spectra which appeared as "white" noise (flat spectrum) from 0.03 to 0.20 cycles per second (5- to 33-second periods). This problem will require further study.

The two higher peaks in Figure 16, occurring in the first two recordings of Run 4, are associated with periods of 8.3 to 9.0 seconds, corresponding to dominant periods of surface wave motion. Peaks at higher period values are associated with the cross-flow energy induced by the rolling motion of the submarine, which tends to roll at its resonant period. Similar peaks occur in Figure 17, with the peaks centered at lower periods associated with the prevailing lower period wave motion.

TABLE 3. NINETY PERCENT CONFIDENCE INTERVALS FOR ANALOG SPECTRA

Run No.	Degrees of Freedom	Lower Limit	Upper Limit
3-1	46	.72 $P_E(f)$	1.47 $P_E(f)$
3-2	35	.68	1.61
3-3	31	.68	1.67
3-4	32	.67	1.59
4-1	58	.75	1.39
4-2	49	.72	1.45
4-3	41	.71	1.49
4-4	48	.72	1.45
4-5	38	.70	1.54
5-1	29	.68	1.64
5-2	59	.74	1.37
5-3	37	.70	1.52
5-4	25	.65	1.79
6-1	28	.68	1.67
6-2	40	.71	1.49
6-3	37	.70	1.52
6-4	24	.66	1.72

E. Longitudinal Relative Flow Velocity Spectra

Figures 18 through 21 present energy spectra of longitudinal relative flow velocity for Runs 3 to 6, respectively. Each figure, except Figure 20, contains four spectra corresponding to a different relative heading.

Relative headings for the first two recordings in Figure 18 are designated with respect to sea because these relative headings associated with sea waves had values that could result in larger longitudinal flow components, even though swell was greater in height and period bandwidth (Table 1). In the last two recordings, relative headings with respect to swell are considered most important, although the choice is arbitrary. The extreme peak in Run 3-1, however, probably was caused by the joint contribution of both sea and swell.

In Run 3-4, both sea and swell components were from abeam of the submarine, and this results in the lowest spectral values throughout the run because the longitudinal component is smallest.

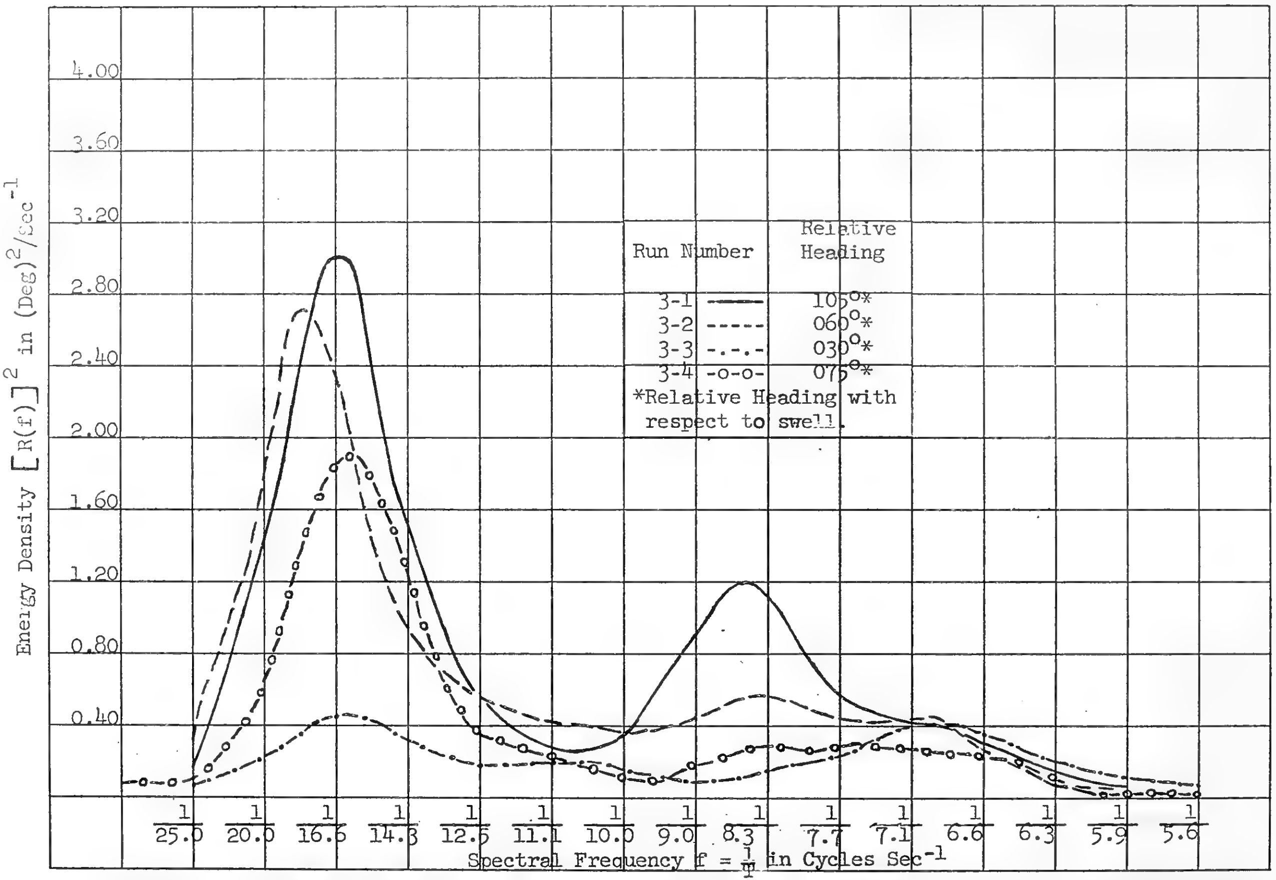

FIGURE 12. ROLL ANGLE SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 3

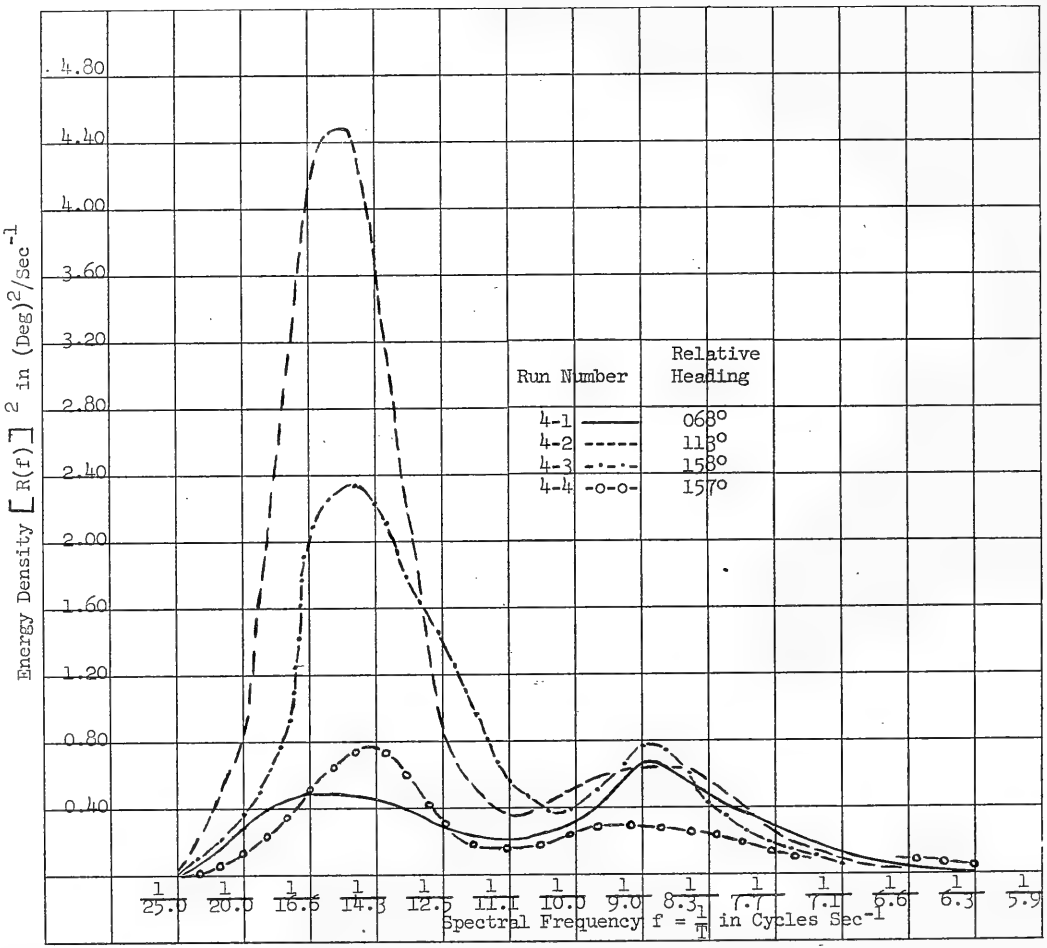

FIGURE 13. ROLL ANGLE SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 4

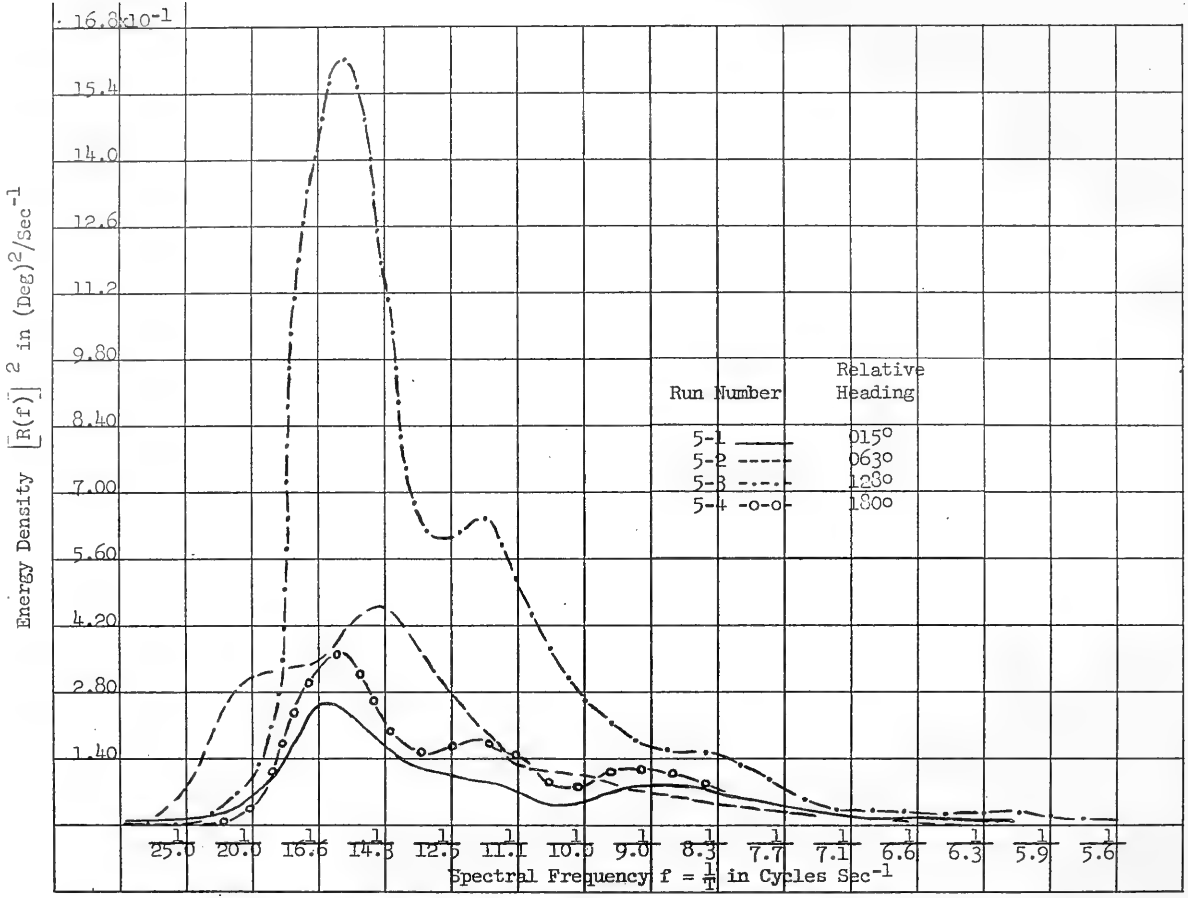

FIGURE 14. ROLL ANGLE SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 5

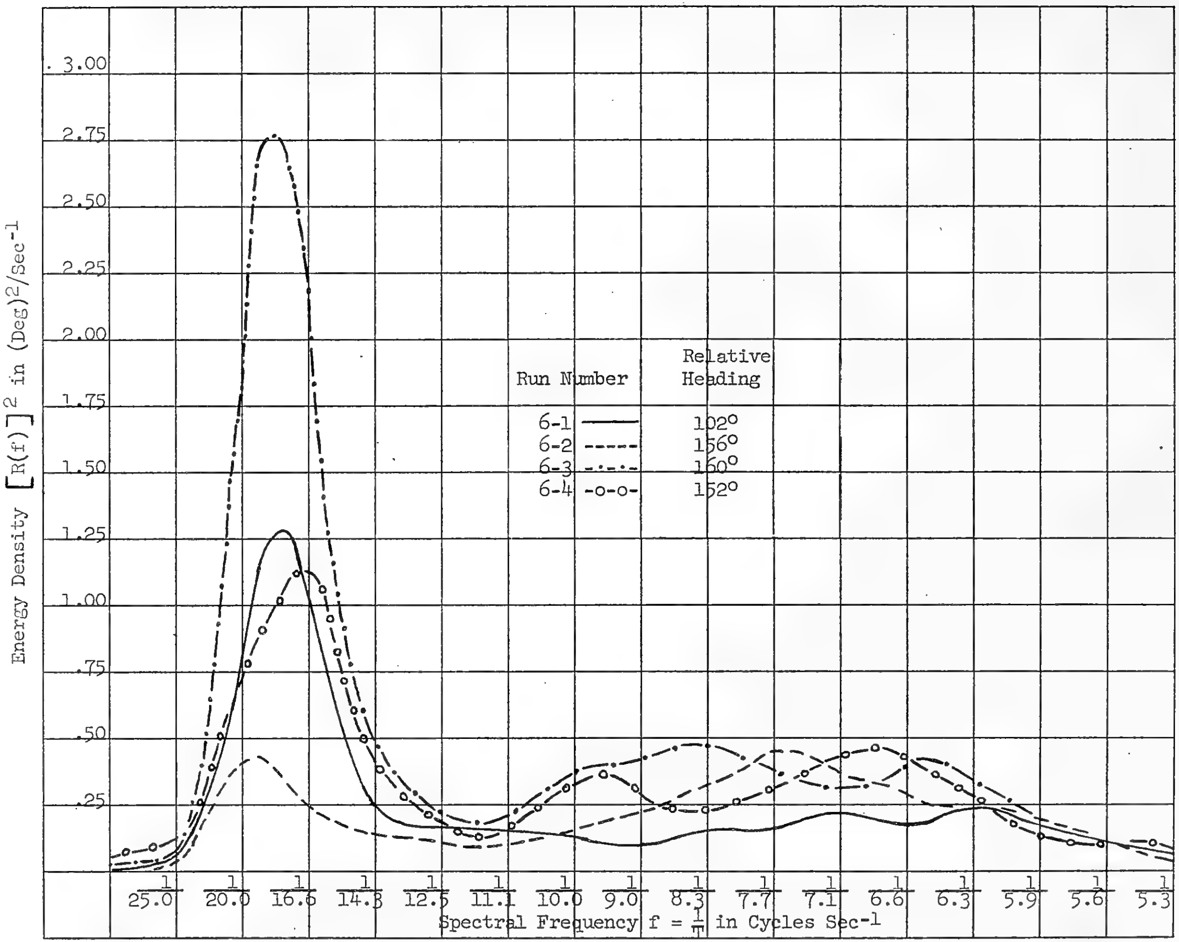

FIGURE 15. ROLL ANGLE SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 6

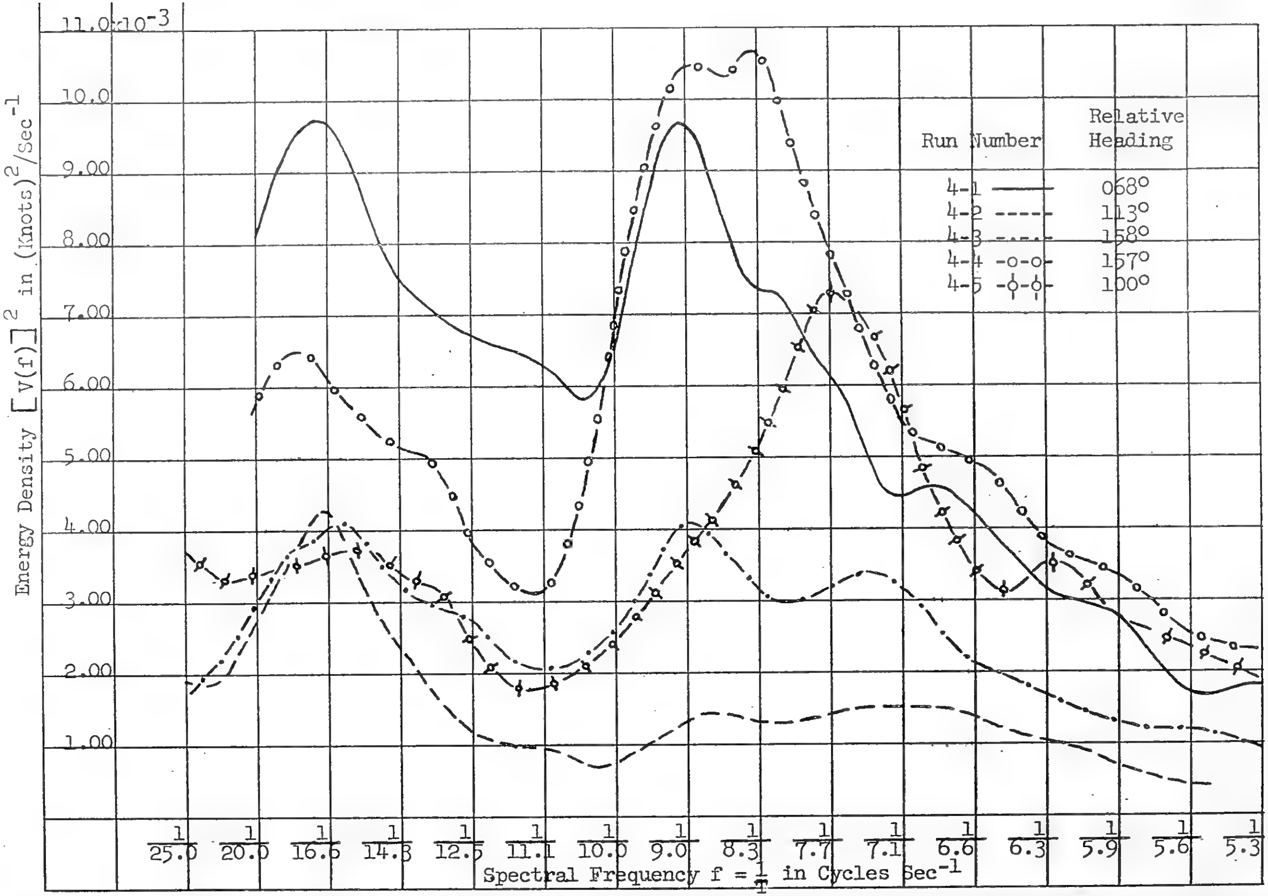

FIGURE 16. TRANSVERSE FLOW VELOCITY SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 4

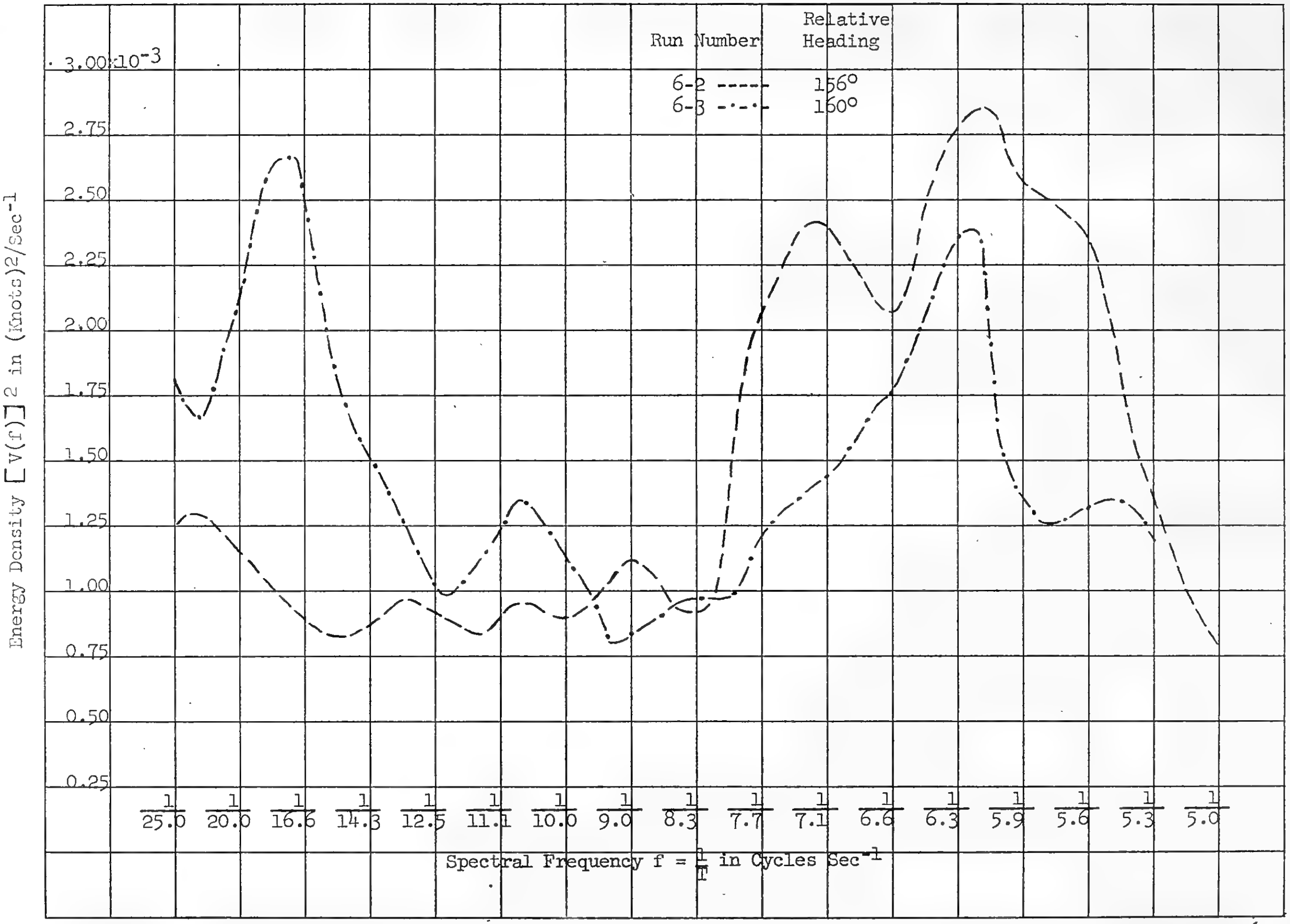

FIGURE 17. TRANSVERSE FLOW VELOCITY SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 6

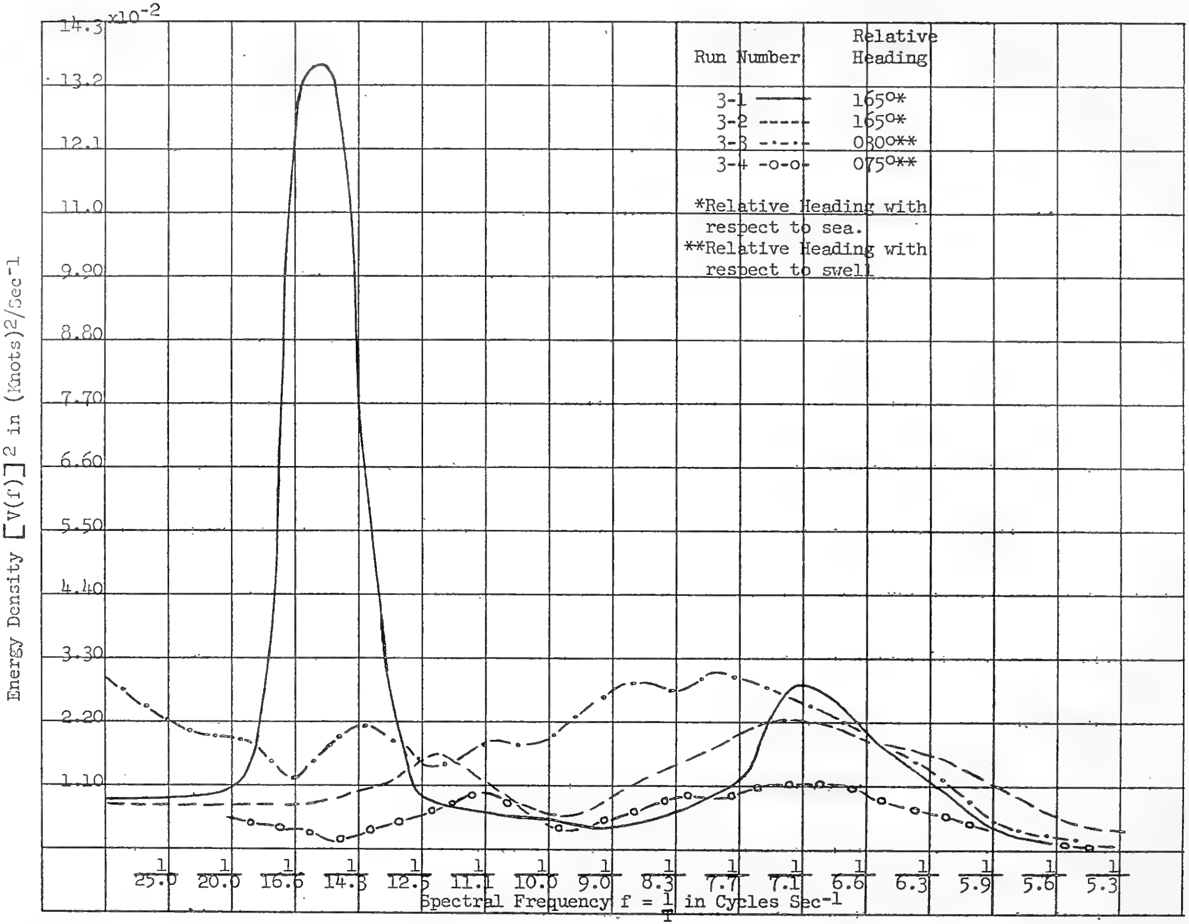

FIGURE 18. LONGITUDINAL FLOW VELOCITY SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 3

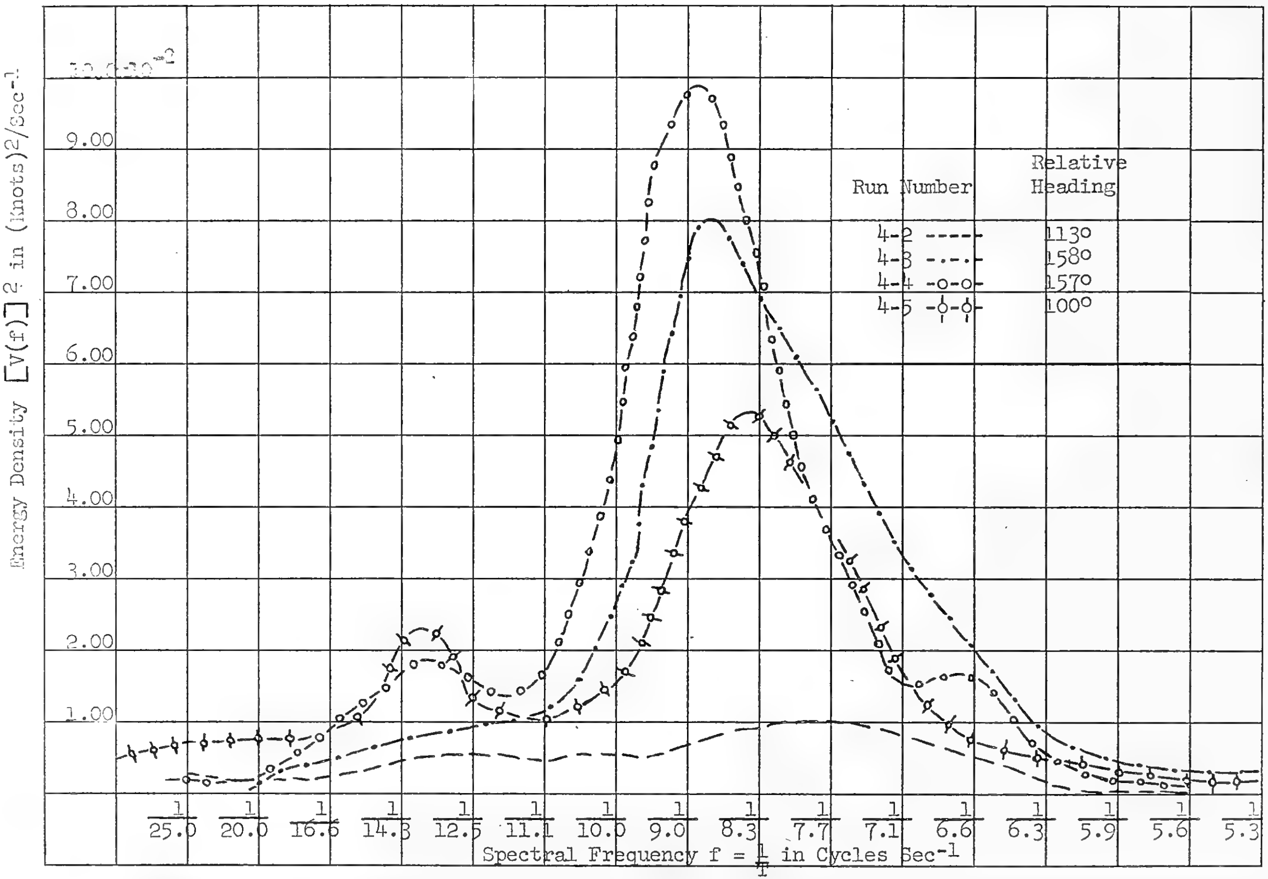

FIGURE 19. LONGITUDINAL FLOW VELOCITY SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 4

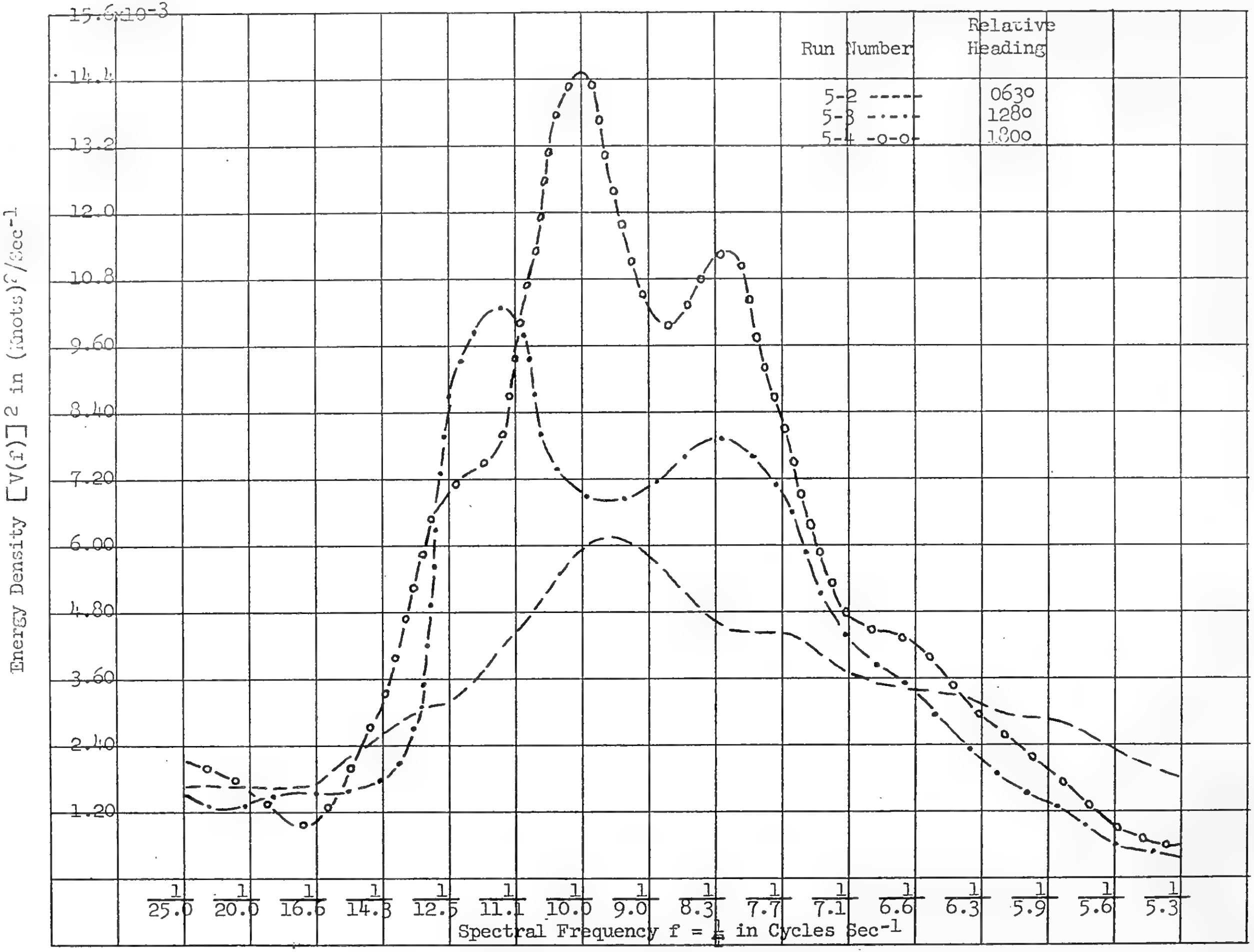

FIGURE 20. LONGITUDINAL FLOW VELOCITY SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 5

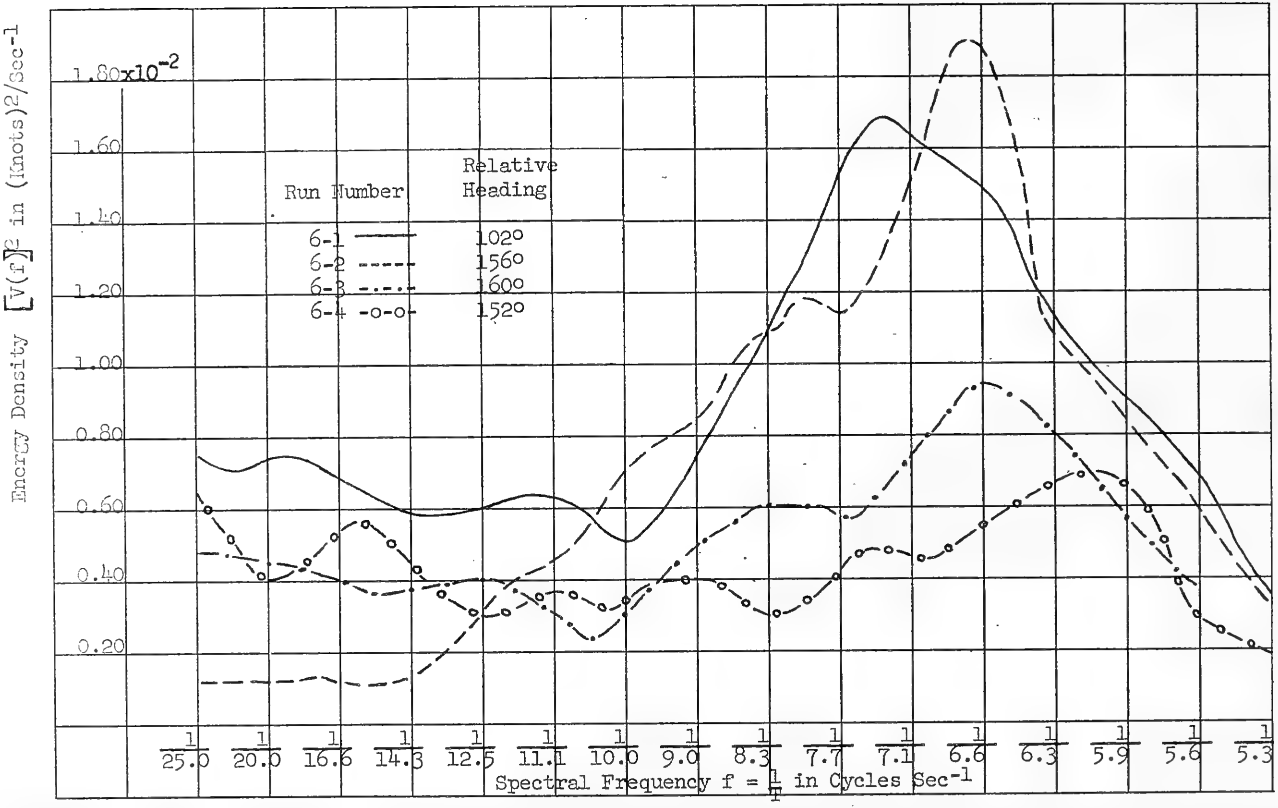

FIGURE 21. LONGITUDINAL FLOW VELOCITY SPECTRA FOR DIFFERENT RELATIVE HEADINGS UNDER HOVERING CONDITIONS, RUN 6

Similarly, in Runs 4 and 5 (Figs. 19 and 20), spectral energy values increased during those runs due to the favorable relative headings. During Run 6 (Fig. 21), peaks definitely shifted to lower period values, reflecting the low period wave values at the surface.

V. SUMMARY OF RESULTS

Table 4 compares values derived from the measured surface wave height spectrum to similar values obtained by the hindcasting methods outlined in H. O. Pub. No. 603. Significant wave heights obtained by hindcasting are lower than values obtained from the measured digital wave spectra. In addition, the highest periods from hindcasting are 7.8 to 8.8 seconds while the highest periods from the measured spectra are 12.0 to 14.4 seconds. Wind speeds are plotted on the Northern Hemisphere synoptic sea-level maps in increments of 5 nautical miles per hour. For fully-developed conditions, a significant wave height of 3.2 feet and a wave period band of 1.5 to 7.8 seconds occur when the wind speed is 14 knots; at 18 knots (fully developed) a significant wave height of 6.1 feet and wave period band of 2.5 to 10.0 seconds would occur. Thus, a 4-knot increase in wind speed (an increase of about 5 miles per hour) results in a significant wave height increase of 3 feet and an increase of 2.2 seconds at the high end of the wave-period band. In addition, high period swell of very low amplitude from many different sources, whose contribution could be detected in a measured wave spectrum, could never be hindcasted from the present oceanic weather maps. Thus, data presented in Table 4, from two different sources, appear to be fairly consistent.

TABLE 4. SUMMARY OF RESULTS DERIVED FROM SURFACE WAVE SPECTRA (DIGITAL) AND COMPARISON TO HINDCAST WAVE DATA

	Values from the Measured Wave Spectra		Values from Hindcast Wave Data	
Run No.	$\overline{H}_{1/3}$ (Ft)	T-Band (Sec)	$\overline{H}_{1/3}$ (Ft)	T-Band (Sec)
4-2	5.5	2.5-14.4	3	1.5-7.8
4-3	5.4	2.5-14.4	3	1.5-7.8
4-4	5.6	2.5-14.4	3	1.5-7.8
4-5	5.1	2.5-14.4	4	2.0-8.3
6-1	6.3	2.4-12.0	5	2.0-8.8
6-2	5.6	2.4-12.0	5	2.0-8.8
6-3	6.0	2.4-12.0	5	2.0-8.8
6-4	6.7	2.4-12.0	5	2.0-8.8

Table 5 presents a summary of some of the important parameters derived from the analysis of roll angle spectra. The first column gives the run number of each recording; the second, an indication of the success (or failure) of the submarine in attempting to hold to a fixed keel depth of 100 feet; and the next three, mean values of the peak-to-peak roll angles in accordance with equations 8, 9, and 10. The last three columns give mean values of peak-to-peak roll angles computed manually for comparison. The agreement in most cases is good.

Tables 6 and 7 summarize relative flow velocity data. Columns 3, 4, and 5 are the mean values derived from spectrum relationships in accordance with equations 8, 9, and 10. Columns 6, 7, and 8 are the mean values obtained manually. The agreement between the spectrum derived mean values and those computed manually is good in most cases.

VI. CONCLUSIONS

This report is concerned with the measurement and analysis of certain variables applicable to oceanographic problems; the chief analysis tool is the energy spectrum. Specific conclusions are:

1. The Sonic Surface Scanner method appears to be a reliable method of obtaining surface wave records in the open ocean.

2. Parameters derived from the estimated spectrum obtained by the Sonic Surface Scanner compare reasonably well to similar parameters obtained by hindcasting.

3. As expected, the largest amplitudes of roll angle occur near the REDFIN's resonant frequency. A secondary maximum, associated with frequencies of maximum spectral energy of the surface wave motion, is observed in almost every roll angle spectrum.

4. Spectra of transverse relative flow velocities tend to exhibit two distinct peaks: one at frequencies near the resonant frequency of the REDFIN, and a secondary peak is well correlated with the surface wave motion.

5. Spectra of longitudinal relative flow velocities exhibit maxima whose frequencies are well-correlated with the frequency of maximum spectral energy of the surface wave motion.

TABLE 5. SUMMARY AND COMPARISON OF RESULTS DERIVED FROM ROLL ANGLE SPECTRA AND MANUAL COMPUTATIONS

Run No.	Actual Hovering Depths	Parameters Derived From Spectrum Analysis			Parameters Computed Manually		
		$\overline{R}$ (deg.)	$\overline{R}$ 1/3 (deg.)	$\overline{R}$ 1/10 (deg.)	$\overline{R}$ (deg.)	$\overline{R}$ 1/3 (deg.)	$\overline{R}$ 1/10 (deg.)
3-1	105-110	1.34	2.14	2.72	1.02	1.42	2.02
3-2	100-108	1.25	2.00	2.54	1.17	1.91	2.21
3-3	102-112	0.80	1.28	1.63	0.58	0.88	1.14
3-4	101-113	1.04	1.67	2.12	1.01	1.53	1.88
4-1	105-145*	0.80	1.28	1.63	0.77	1.12	1.38
4-2	112-125	1.39	2.22	2.82	1.31	2.19	2.78
4-3	102-112	1.19	1.90	2.42	0.91	1.50	1.82
4-4	102-113	0.74	1.18	1.50	0.68	1.11	1.47
4-5	94-113	-	No Data	-	1.49	2.18	2.71
5-1	101-170*	0.43	0.69	0.88	0.32	0.54	0.71
5-2	98-110	0.58	0.93	1.18	0.52	0.76	1.00
5-3	99-114	0.86	1.37	1.75	0.35	0.57	0.72
5-4	98-180*	0.34	0.55	0.69	0.42	0.62	0.75
6-1	96-103	0.82	1.32	1.67	0.74	1.25	1.60
6-2	97-102	0.79	1.27	1.62	0.73	1.09	1.33
6-3	98-172*	1.10	1.76	2.24	0.98	1.52	1.85
6-4	92-107	0.95	1.53	1.94	0.65	1.05	1.94

* Large variations in depth occurred when submarine was unable to control buoyancy. Hovering depth was usually regained after 6 or 7 minutes, and any data recorded during this time were not used to compute mean values, or used in spectrum analysis.

TABLE 6. SUMMARY AND COMPARISON OF RESULTS DERIVED FROM TRANSVERSE RELATIVE FLOW VELOCITY SPECTRA AND MANUAL COMPUTATIONS

Run No.	Actual Hovering Depths	Parameters Derived From Spectrum Analysis			Parameters Computed Manually		
		$\overline{V}$ (kn.)	$\overline{V}$ 1/3 (kn.)	$\overline{V}$ 1/10 (kn.)	$\overline{V}$ (kn.)	$\overline{V}$ 1/3 (kn.)	$\overline{V}$ 1/10 (kn.)
3-1	105-110	-	-	-	0.10	0.12	0.16
3-2	100-108	-	-	-	0.21	0.32	0.42
3-3	102-112	-	-	-	0.13	0.21	0.32
3-4	101-113	-	-	-	0.13	0.20	0.26
4-1	105-145*	0.11	0.19	0.24	0.13	0.21	0.31
4-2	112-125	0.06	0.10	0.12	0.12	0.20	0.27
4-3	102-112	0.08	0.13	0.16	0.10	0.14	0.19
4-4	102-113	0.08	0.12	0.16	0.09	0.13	0.16
4-5	94-113	0.09	0.15	0.19	0.11	0.17	0.23
5-1	101-170*	-	-	-	0.18	0.30	0.48
5-2	98-110	-	-	-	0.07	0.11	0.15
5-3	99-114	-	-	-	0.06	0.09	0.10
5-4	98-180*	-	-	-	0.08	0.13	0.19
6-1	96-103	No Data					
6-2	97-102	0.07	0.11	0.13	0.07	0.11	0.12
6-3	98-172*	0.06	0.10	0.13	0.07	0.11	0.12
6-4	92-107	No Data					

* Large variations in depth occurred when the submarine was unable to control buoyancy. Hovering depth usually was regained after 6 to 7 minutes, and any data recorded during this time were not used to compute mean values, or used in spectrum analysis.

These runs deleted because the excessive energy associated with low frequency components (less than 0.03 cycles per second) made results doubtful.

TABLE 7. SUMMARY AND COMPARISON OF RESULTS DERIVED FROM LONGITUDINAL RELATIVE FLOW VELOCITY SPECTRA AND MANUAL COMPUTATIONS

Run No.	Actual Hovering Depths	Parameters Derived From Spectrum Analysis			Parameters Computed Manually		
		$\overline{V}$ (kn.)	$\overline{V}$ 1/3 (kn.)	$\overline{V}$ 1/10 (kn.)	$\overline{V}$ (kn.)	$\overline{V}$ 1/3 (kn.)	$\overline{V}$ 1/10 (kn.)
3-1	105-110	0.30	0.48	0.62	0.21	0.43	0.70
3-2	100-108	0.19	0.31	0.39	0.12	0.18	0.23
3-3	102-112	0.22	0.35	0.45	0.19	0.32	0.43
3-4	101-113	0.13	0.21	0.27	0.37	0.57	0.71
4-1	105-145*	-	-	-	0.14	0.24	0.30
4-2	112-125	0.11	0.17	0.22	0.08	0.12	0.16
4-3	102-112	0.24	0.39	0.50	0.24	0.42	0.47
4-4	102-113	0.24	0.37	0.48	0.24	0.38	0.48
4-5	94-113	0.22	0.35	0.45	0.23	0.35	0.49
5-1	101-170*	-	-	-	0.13	0.24	0.37
5-2	98-110	0.10	0.16	0.20	0.09	0.13	0.19
5-3	99-114	0.11	0.18	0.22	0.12	0.18	0.24
5-4	98-180*	0.12	0.19	0.24	0.13	0.21	0.29
6-1	96-103	0.16	0.25	0.32	0.13	0.23	0.32
6-2	97-102	0.14	0.22	0.28	0.12	0.19	0.28
6-3	98-172*	0.12	0.18	0.23	0.12	0.18	0.22
6-4	92-107	0.10	0.16	0.21		No Data	

* Large variations in depth occurred when submarine was unable to control buoyancy. Hovering depth usually was regained after 6 to 7 minutes, and any data recorded during this time were not used to compute mean values, or used in spectrum analysis.

These runs deleted because the excessive energy associated with low frequency components (less than 0.03 cycles per second) made results doubtful.

6. Although spectra of roll angle and cross-flow velocities were presented with relative heading as parameter, no consistent relationship is indicated by these spectra data. This is attributed largely to the low sea state values which persisted throughout the test except on Run 3.

VII. FUTURE TEST AND ANALYSIS PLANS

1. Data discussed in this report are representative of lower sea states. To correctly assess the influence of environmental conditions on submarine motions, additional data are required during State 4 or 5 sea conditions. Such data should be taken under conditions of head seas (0-degrees relative heading), abeam seas (90-degrees), and following seas (180-degrees), in order to study the characteristics of motion at several different relative headings.

2. This report presents some examples of "auto" spectra, or the resulting transformation when the wave or ship motion record is autocorrelated (correlated with itself). The extension of the concept leads to cross-spectra, or the transformation resulting when simultaneous recordings of two different random processes are cross-correlated (correlated with each other). A random function of time representing a stationary Gaussian process (for example, sea surface wave motion) could be considered the input or forcing function. The output or response function would be submarine motion. Now if frequencies exist in the input such that an excitation response analogous to "resonance" is possible in the output, then such a frequency interdependence is clearly displayed by cross-spectrum analysis. Thus, the use of cross-spectrum analysis provides a useful tool in the study of cause and effect relationships between two random processes.

3. Recent communications between leading oceanographers indicate that the array of nine sonic scanners may be used to obtain the directional spectrum of the natural seaway under a variety of sea conditions. Such spectra would be estimated by cross-spectrum analysis. Sea surface wave motion is a two-dimensional phenomenon, and energy of wind-generated surface waves has been studied as a function of frequency alone (the notable exception being Stereo Wave Observation Project).

VIII. ACKNOWLEDGEMENTS

Most of the analog spectra were computed on the David Taylor Model Basin's wave analyzer; digital spectral estimates were provided by the Division of Computation at the Hydrographic Office. The many problems associated with the Sonic Surface Scanner

measurements and playback of the scanner data were solved at the Hydrographic Office by members of the Submarine Systems Section and the Electronics Branch. The assistance provided by members of the Processing Section in preparing text and related material for publication is greatly appreciated. Especially helpful in the analysis and presentation of these data were the comments, suggestions, and assistance of Mrs. Mary J. Middleton, whose contribution to this manuscript was invaluable.

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APPENDIX A

SURFACE WAVE HEIGHT DATA OBTAINED BY THE SONIC SURFACE SCANNER

APPENDIX A - SURFACE WAVE HEIGHT DATA OBTAINED BY THE SONIC SURFACE SCANNER

A. Digital Recording and Playback of Sonic Surface Scanner Data

An example of a sea surface profile obtained by Sonic Surface Scanner is presented in Reference 8. The Sonic Surface Scanner consists of ten transducers mounted on the deck. These are spaced 33 feet apart (Fig. 1). The transducers are mounted face upward and are essentially inverted echo sounders. Each transducer in turn measures distance between itself and the sea surface directly above in a 3° cone. By sequencing from one transducer to the next aft along the deck of the submarine, a wave profile, comparable to the length of the submarine, is obtained. The complete sequence takes approximately 0.7 seconds. The Sonic Scanner was designed to provide an accuracy to the nearest foot.

The output of all ten transducers are available, but the primary interest in this report is the power spectra of the output of one transducer. A recording from one transducer is, therefore, the instantaneous height of the sea surface above the submarine as a function of time. This is given in terms of the two-way travel time required for a sound wave front to travel from the transducer to the sea surface and return.

Assume the velocity of sound in ocean waters to be 5000 ft/sec (the resulting error in this assumption is very small). Travel time per foot can be expressed:

$$t = \frac{1}{5000} \text{ sec./ft.} = 0.2 \times 10^{-3} \text{ sec./ft.} = 0.2 \text{ millisec./ft.}$$

Since we are concerned with a two-way travel time (the time required for the sound beam to go from the submarine transducer to the sea surface and return), the effect is the same as if the travel time were doubled. Thus, the effective travel time per foot is:

$$t = 0.4 \text{ millisec./ft.}$$

For convenience in counting, t is expressed as:

$$t = 10 \text{ millisec./25 ft.}$$

In this way, changes in the height of sea surface above the submarine are measured in terms of the two-way travel time. These changing values of sea surface height above the submarine are recorded as pulses on an AM channel of a tape recorder every 0.7 seconds.

Assume the sequence of values recorded at Transducer No. 1 is desired. Appropriate pulse signals are monitored on an oscilloscope (Fig. A1), and the time interval between outgoing pulse and return echo is counted on the time interval counter. Time values are then printed out on a digital recorder. As a check on the quality of the digital printout, an Esterline-Angus Recording milliameter is used to record the analog output from the digital recorder, which appears as a staircase function and is smoothed out by the slow response time of the Esterline-Angus Recording milliameter.

Normally, the counter displays time in milliseconds, and this is proportional to the sea height above the submarine. For convenience in computation, it is desirable to have the print-out in feet directly. This is accomplished by introducing a 25 kc signal to the external time base of the time interval counter. The output of the printer is then given directly in feet and tenths of feet. These values are coded subsequently on punched cards for high speed computation of power spectra on the Datatron.

B. Effect of Submarine Motion on Surface Wave Measurements

Sonic Surface Scanner wave measurements at a keel depth of 100 feet and made under lower sea state conditions will not be affected seriously by the submarine moving in response to the subsurface wave motion. However, as the sea surface wave height grows under the action of the wind, or if low frequency swell propagates over the hovering submarine, the low frequency (high period) components will increase the amplitudes of motion of the submarine. In effect, wave measurements will be made with respect to a moving reference system.

Now, if water depth is great, the attenuation factor of a simple harmonic wave in the neighborhood of the hovering submarine is given by:

$$K_Z = e^{-\frac{2\pi Z}{L}} \qquad \text{for} \qquad L = \frac{gT^2}{2\pi}$$

where:

L = wave length
T = wave period
Z = depth to the submarine

Depth in the operating area off Wilmington, North Carolina, was 250 fathoms, and we will consider K_Z for $Z = 75$ feet and 100 feet. These two depths represent average depth to the Sonic Surface Scanner transducers and the average keel depth for Sonic Scanner spectra data shown in Figures 10 and 11. Expressing K_Z as a function of frequency f for fixed depth Z:

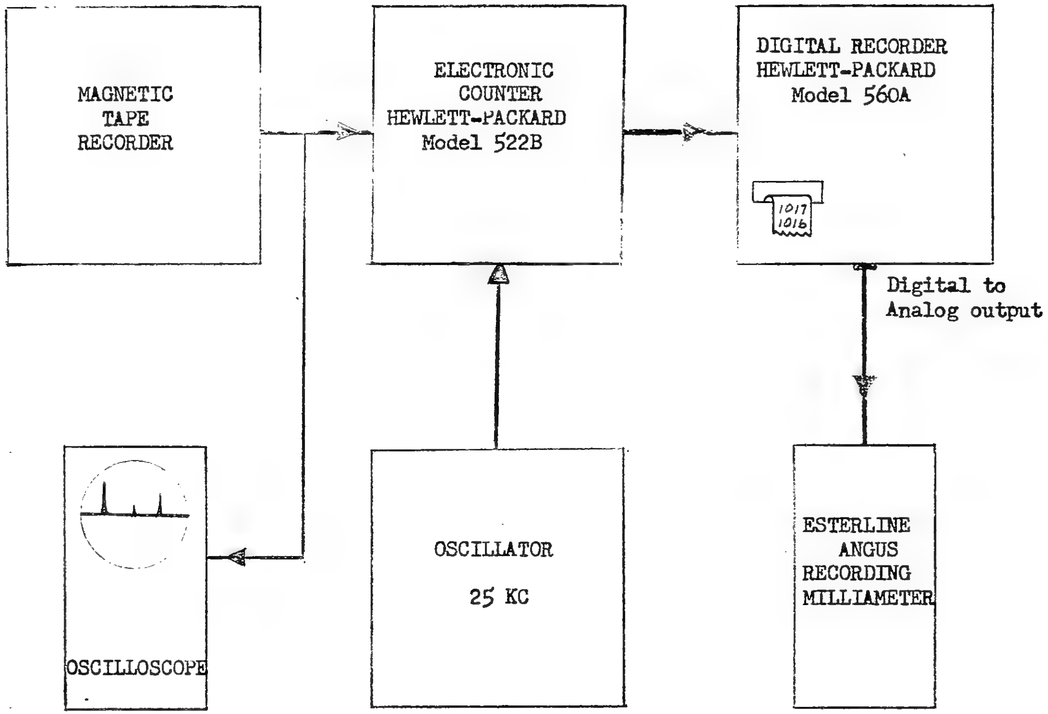

FIGURE A1. BLOCK DIAGRAM OF THE ANALOG TO DIGITAL RECORDING SYSTEM USED BY THE SUBMARINE SYSTEMS SECTION

$$K_Z = e^{-\frac{4\pi^2 Z}{g} f^2}$$

Figure A2 presents the wave height attenuation as a function of frequency alone for depths of 75 and 100 feet.

Thus, for a low frequency swell of .05 cycles per second (20-second period) only 20 percent of the surface wave height will be attenuated at 75 feet, and only 26 percent will be attenuated at 100 feet. If the submarine behaves as a particle, the heave amplitude of the neutrally buoyant, hovering submarine should be something like a simple harmonic motion in phase with the simple harmonic wave at the surface (neglecting for the moment, all other components of submarine motion). In any event, a correction must be applied to either the original data or possibly to the spectrum.

In general, the submarine will execute motion in all six degrees of freedom. The effect of pitching and rolling will tend to increase the path length over which the sound beam has to travel; this tends to make the recorded wave motion appear higher than the actual wave motion, which overcomes some of the difficulty mentioned in the preceding paragraph.

Both these difficulties can be ameliorated if submarine data are recorded at a depth of 200 or even 300 feet. However, with increased depth, there will be some "skipping" of data since, occasionally, a return echo will be too weak to gate the time interval counter.

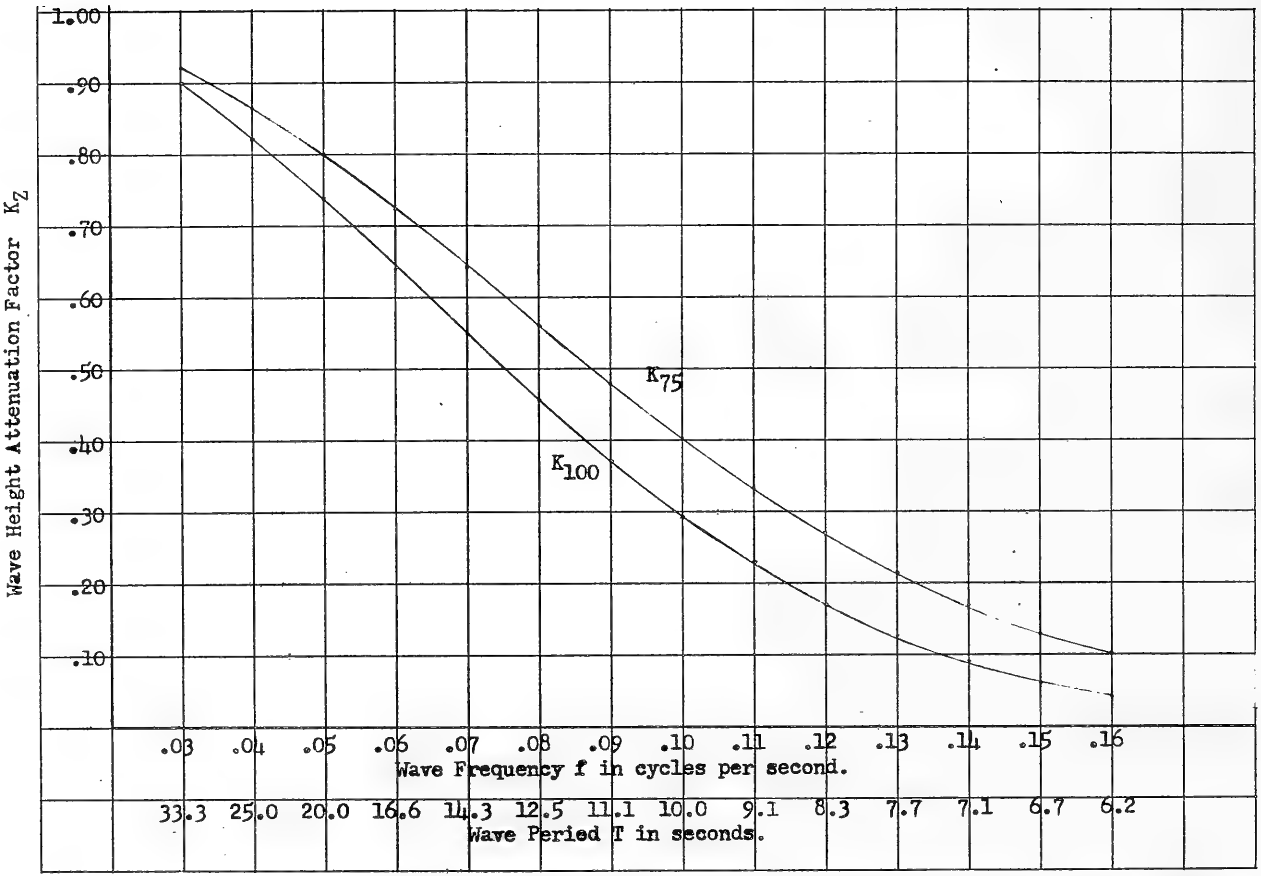

FIGURE A2. FRACTION OF A UNIT SIMPLE HARMONIC WAVE HEIGHT REMAINING AT DEPTHS OF 75 AND 100 FEET

APPENDIX B

POWER SPECTRAL ESTIMATES
OF
IN SITU PRESSURE FLUCTUATIONS

APPENDIX B - POWER SPECTRAL ESTIMATES OF IN SITU PRESSURE FLUCTUATIONS

A. General

One of the variables the Submarine Systems Section attempted to record was pressure fluctuation, as measured by a Wiancko differential pressure gauge. This gauge was mounted on the sail with the diaphragm parallel to the deck of the submarine (Fig. 1). Thus, when the submarine hovered at a keel depth of 100 feet, the gauge's rubber diaphragm was actually 60 feet below the surface.

Total pressure change occurring at the diaphragm of the differential pressure gauge of the hovering submarine will be composed of, (1) pressure change due to surface wave motion, (2) linear motions of the submarine (heave, sway, and surge), and (3) angular motions (roll, pitch, and yaw).

Pressure changes at the gauge due to surface wave motion alone are independent of the submarine's relative heading. Total motion of the submarine, hence total pressure change associated with the submarine's motion, is definitely a function of relative heading. Even under stationary wave conditions, power spectra of the Wiancko recordings will, in general, be different for different relative headings.

B. Spectral Estimates of Filtered Signals

The problem of recording pressure fluctuations on the hovering REDFIN was complicated by the limited dynamic range of the Wiancko differential pressure gauge. To prevent damage to the gauge, a relief valve eliminates differential pressure in the interior chamber of the gauge when the dynamic range of the gauge is exceeded. A typical in situ pressure recording is shown in Figure B1. The small oscillations are pressure changes due to surface wave motion and linear and angular components of the submarine motion. The very long "period" oscillations are associated with the slowly changing dive angle of the hovering submarine. The amplitude ratio of the large to small oscillations is about 20 to 1. This highly compressed recording represents approximately 8 minutes of recording. Many of the Wiancko pressure recordings were characterized by breaks or "jumps" which occurred when the amplitude of the long-period oscillations became too large. When this happened, the relief valve would relieve the differential pressure momentarily, and the recording would begin anew at the center of the chart.

Power spectra of the recording shown in Figure B1 were estimated by the wave analyzer system at the Hydrographic Office and are shown in Figure B2. In Figure B2, three different estimates are superimposed to show the effect of filtering. Curve A is the spectrum of the unfiltered signal while Curves B and C are the resultant spectra of first passing the signal through a Krohn-Hite Band-Pass Filter, Model 320A. Cut-off limits were arbitrarily set at 0.08 to 0.5 cycles per second (periods of 2 to 12.5 seconds) for Curve B and 0.09 to 0.5 cycles per second (periods of 2 to 11.1 seconds) for Curve C. Filter response curves for the frequency band used to obtain Curve B are shown in Figure B3. It should be noted that the shape of the filter response curve allows a fraction of the energy associated with frequencies less than 0.08 cycles per second to pass, but blocks off the very low frequencies.

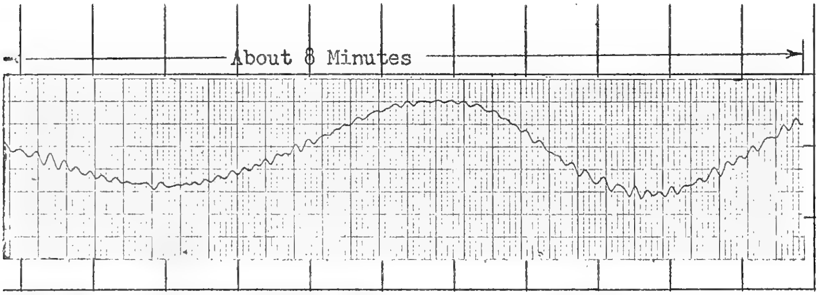

FIGURE B1. COMPRESSED RE-RECORDING OF IN SITU PRESSURE FLUCTUATIONS

A digital spectral estimate of this same run was obtained by re-recording the filtered run with pass-band limits set at 0.05 to 0.5 cycles per second. In Figure B4, the dashed curve is the power spectrum of this filtered recording; the solid curve, surface power spectrum; and the dashed-dot curve, the residual power spectrum at 60 feet obtained by attenuating the surface power spectrum according to:

$$\left[A(\mu)\right]^2_{60\text{ feet}} = \left(e^{-\frac{2\pi Z}{L}}\right)^2 \left[A(\mu)\right]^2_{\text{Surface}}$$

where Z and L are given in Appendix A.

The difference between the in situ power spectrum and the residual spectrum at 60 feet due to surface wave motion alone is attributed to pressure fluctuations associated with the submarine motion; i.e. roll angle and cross-flow.

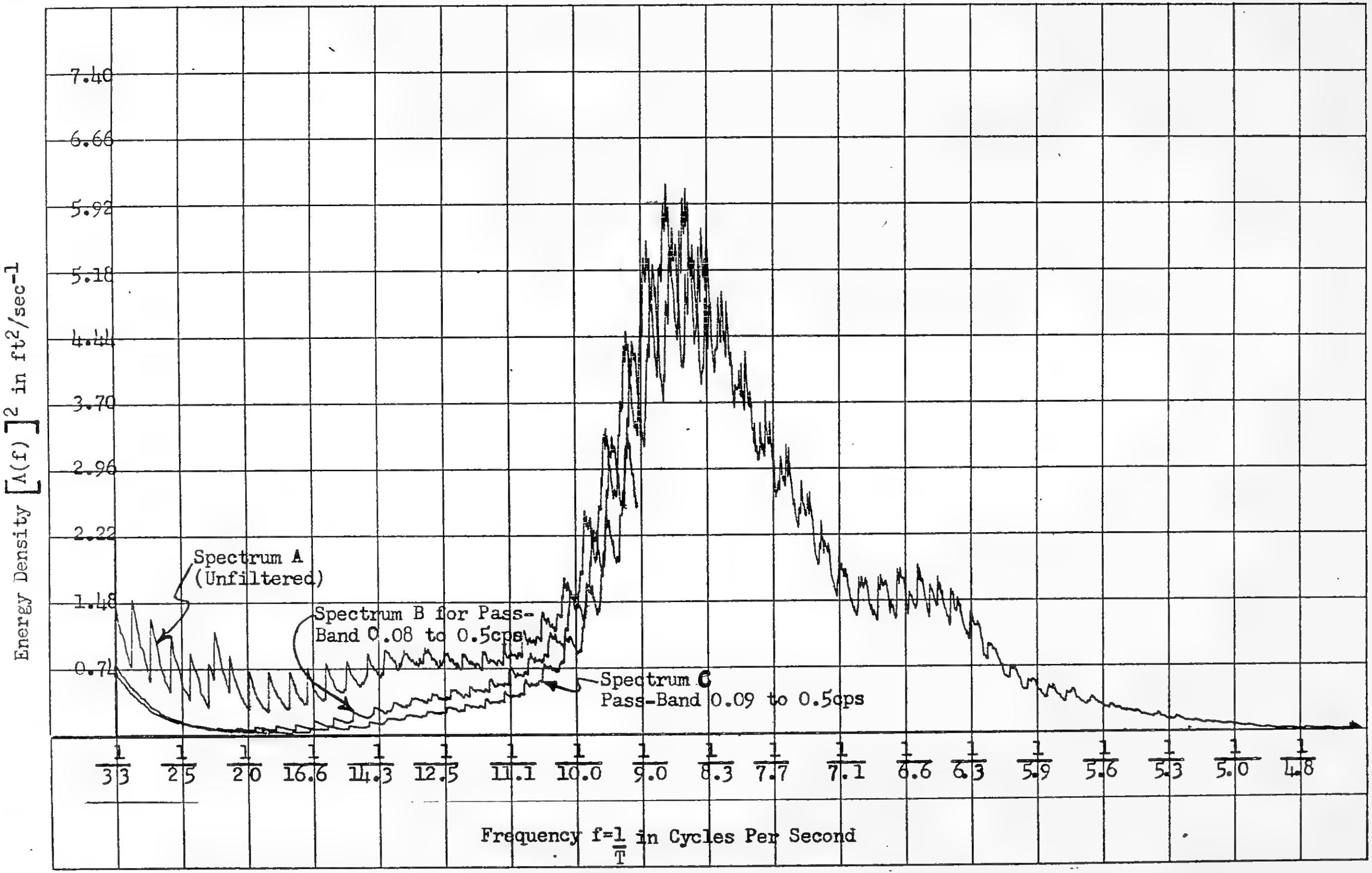

FIGURE B2. COMPARISON OF POWER SPECTRAL ESTIMATES OF FILTERED AND UNFILTERED SIGNALS, RUN 4-4

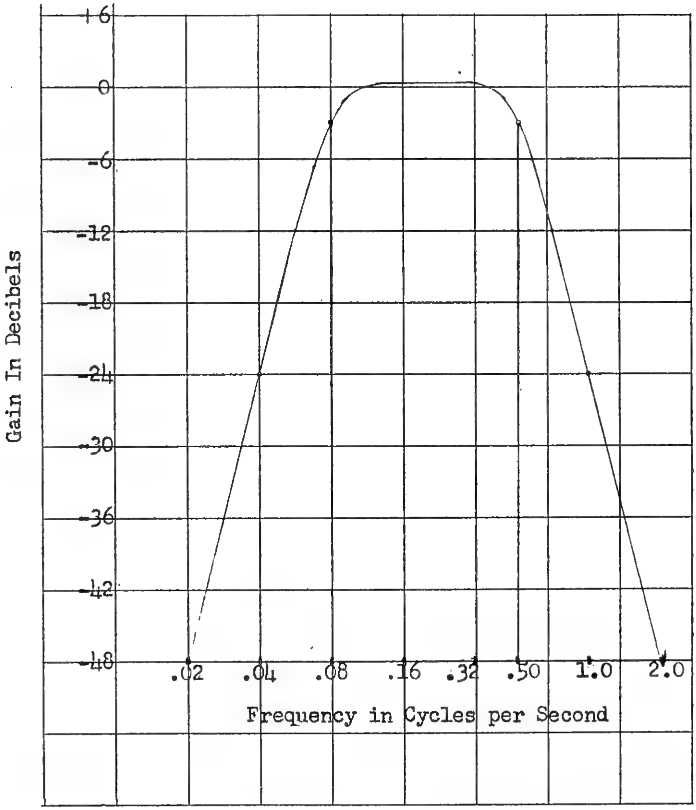

FIGURE B3. BAND PASS RESPONSE OF THE MODEL 330-A KROHN-HITE ULTRA-LOW FREQUENCY BAND-PASS FILTER

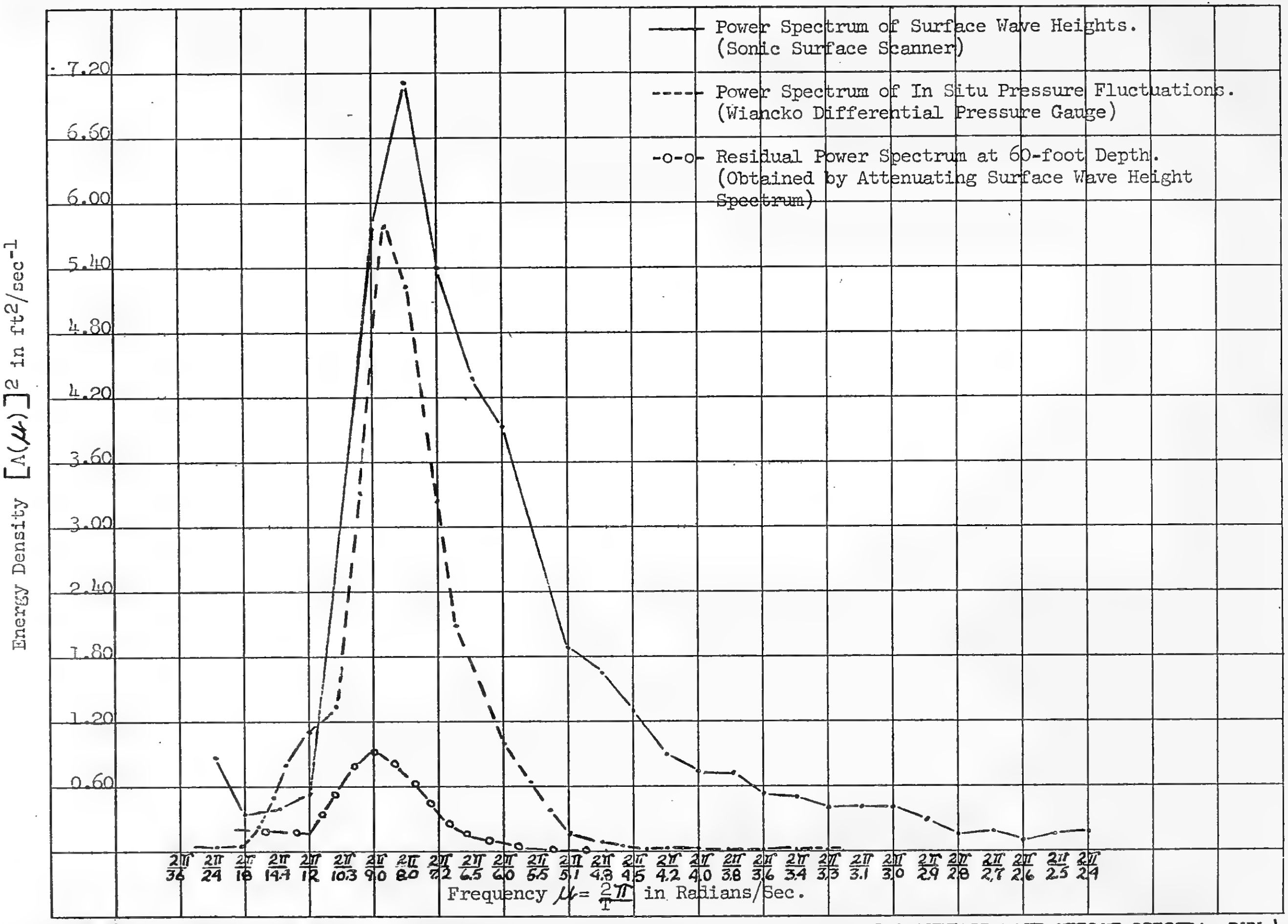

FIGURE B4. POWER SPECTRA OF IN SITU PRESSURE FLUCTUATIONS COMPARED TO ATTENUATED SURFACE WAVE HEIGHT SPECTRA, RUN 4-4

U. S. Navy Hydrographic Office
POWER SPECTRUM ANALYSIS OF WAVE MOTION, SUBMARINE ROLL ANGLE, AND RELATIVE CROSS-FLOW VELOCITIES, CRUISE II, USS REDFIN (SS-272), by P. S. DeLeonibus and M. J. Middleton, February 1961. 61 p., 27 figs., 7 tables, 2 app. (H. O. TR-100)

This publication presents a discussion of digital and analog recordings of surface wave motion and ship and fluid motions, respectively. These recordings were made while a submarine was hovering at different relative headings. Results of power spectral analysis of resultant data are given.

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2. Oceanography - Submarine
3. Wave Motion
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