RAY'S A... IC THIRD BOOK

RAY'S

PRACTICAL ARITHMETIC

... THOUSAND... EDITION

VAN ANTWERP, BRAGG & CO.,
CINCINNATI and NEW YORK.

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ECLECTIC EDUCATIONAL SERIES.

RAY'S ARITHMETIC, THIRD BOOK

PRACTICAL ARITHMETIC,

BY

INDUCTION AND ANALYSIS.

By JOSEPH RAY, M.D.
LATE PROFESSOR OF MATHEMATICS IN WOODWARD COLLEGE.

ONE THOUSANDTH EDITION—IMPROVED.

VAN ANTWERP, BRAGG & CO.,
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PREFACE.

Few, if any, works on Arithmetic, have received the approbation which has been bestowed upon "Ray's Arithmetic, Part Third;" and the constantly increasing demand for it having rendered *another* renewal of the stereotype plates necessary, it has been made the occasion for remodeling and greatly improving the volume in all its parts.

The Inductive and Analytic methods here adopted, lead the learner to an understanding of the principles from which the rules are derived, and teach him to regard rules as *results* rather than *reasons:* he will understand the "*why* and *wherefore*" of every operation performed, and gain a thorough knowledge of the principles of Arithmetic with its applications, while the faculties of the mind are strengthened and disciplined.

The leading features of the work are:

1st. Every principle is clearly explained by an *analysis* or *solution* of simple examples, from which a Rule is derived. This is followed by graduated exercises designed to render the pupil familiar with its application.

2d. The arrangement is strictly philosophical; no principle is anticipated: the pupil is never required to perform any operation until the principle on which it is founded has first been explained. For this reason, those processes of reduction that require the use of fractions, are introduced *after* fractions.

3d. The subject of Fractions, a thorough understanding of which is almost a knowledge of Arithmetic, has received that attention which its use and importance demand.

4th. The subject of Proportion is introduced immediately after Decimals; this enables the instructor to treat Percentage and its various applications, either by *proportion*, or by *analysis*, as he may prefer.

5th. Particular attention has been given to render the work *practical;* the weights and measures are referred to, and conform to the legal standards; while pounds, shillings, and pence, being no longer used in actual business, are only introduced under Exchange.

While Federal Money may be considered in connection with decimals, yet it is truly a species of compound numbers, and *is so regarded* in all the ordinary computations of business. Hence, the propriety of assigning it the place which it occupies in this work.

While cancellation is introduced, it is not made a *hobby*, or an arithmetical machine, by which results can be obtained merely in a mechanical manner.

The object throughout has been to combine *practical utility* with *scientific accuracy;* to present a work embracing the best methods, with all real improvements. How far this object has been secured, is submitted to those engaged in the laborious and responsible work of education.

☞ An APPENDIX has been added to this edition, presenting the *Metrical System of Weights and Measures.* This has been compiled chiefly from modern French works on this subject; and was submitted, in manuscript, to the critical examination of PROF. H. A. NEWTON, *of Yale College*, to whom we are under great obligations for such corrections, additions, and emendations as his intimate knowledge of the subject suggested.

CONTENTS.

OBSERVATIONS TO TEACHERS.

INTELLECTUAL ARITHMETIC should be thoroughly studied by all, and especially by the young, before commencing PRACTICAL. For this purpose, attention is called to "Ray's Arithmetic, Second Book," which has been carefully prepared, and is now published with important improvements.

When admissible, pupils studying Arithmetic should be taught in classes; the presence of the class being a stimulus to both *teacher* and *pupil.* This arrangement also economizes time, since the same *oral illustrations*, necessary for the instruction of a single pupil, serve for a class.

The time occupied at each recitation ought not to be less than thirty minutes, nor more than one hour. The class should not be too large; and, if possible, the attainments of its members equal.

Every school should have a blackboard, on which pupils can solve the questions and explain the method of solution.

A prime object in recitations is to secure *attention;* to do this, the exercises must be *interesting*, and all must be kept *employed.* Let as many be called out as can obtain positions at the blackboard, and let all solve the *same* question at once.

When the solutions are completed, let some one be called on to explain the process, giving the *reason* for each step of the operation. Exercises thus conducted animate the class; and by requiring the learner to explain every process, and assign a reason for every step, he learns to *rely on his own reasoning powers.*

In assisting pupils to overcome difficulties, it is preferable to do it indirectly, by making such suggestions, or asking such questions, as will enable the learner to accomplish the object.

Frequent Reviews will be found of great benefit.

The pupil should be rendered familiar with the answers to the questions in the REVIEW at the foot of the page. This review is intended to *aid* the teacher, but not to prevent his asking other questions, or presenting different illustrations.

ARITHMETIC.

1. A Unit, or one, is a single thing of any kind; as, one apple, one dollar, one pound.

2. Number is a term signifying one or more units; as, one, five, seven cents, nine men.

3. Numbers are expressed by ten characters, called Figures; as, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

4. Arithmetic treats of numbers, and is the art of computing by them. The fundamental rules are five; Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

I. NOTATION AND NUMERATION.

ART. 1.	A single thing is a *Unit*, or					*one;*	written,	1
One	unit	and	one	more,	are	*two;*	..	2
Two	units	and	one	more,	are	*three;*	..	3
Three	units	and	one	more,	are	*four;*	..	4
Four	units	and	one	more,	are	*five;*	..	5
Five	units	and	one	more,	are	*six;*	..	6
Six	units	and	one	more,	are	*seven;*	..	7
Seven	units	and	one	more,	are	*eight;*	..	8
Eight	units	and	one	more,	are	*nine;*	..	9

ART. 2. These nine characters are called *digits*, or *significant figures*, because they denote something.

The character, 0, called *cipher*, *naught*, or *zero*, is employed to denote *nothing:* thus, to show that there are no cents in a purse, write, *the number of cents is* 0.

REMARK.—The *cipher* is sometimes termed an auxiliary digit, because it *helps* the other digits in expressing numbers.

REVIEW.—ART. 1. What is a single thing? What are one unit and one more of the *same kind?* Two units and one more, &c.? 2. What are these nine characters called? Why? What does naught, or zero denote? REM. What is the cipher termed? Why?

ORDER OF TENS.

ART. **3**. Nine units and one more are called *Ten:* it forms a unit of a *second* or higher order called *Tens.*

Ten is represented by the same figure, 1, as a *single* thing, or unit of the first order; but,

To distinguish ten, the 1 is written in the *second* order, the second place from the right hand:

The first order, or right hand place, is filled with a cipher. The cipher, in the first place, denotes that there are *no* units of the first order.

The number *ten* is written thus; . . .		10
One ten and one unit, are .	*eleven;*	11
One ten and two units, . .	*twelve;*	12
One ten and three units, . .	*thirteen;*	13
One ten and four units, . .	*fourteen;*	14
One ten and five units, . .	*fifteen;*	15
One ten and six units, . .	*sixteen;*	16
One ten and seven units, . .	*seventeen;*	17
One ten and eight units, . .	*eighteen;*	18
One ten and nine units, . .	*nineteen;*	19
Two tens, ...	*twenty;*	20
Two tens and one unit, . . .	*twenty-one;*	21
Two tens and five units, . .	*twenty-five;*	25
Three tens, ...	*thirty;*	30
Four tens, ...	*forty;*	40
Five tens, ...	*fifty;*	50
Six tens, ...	*sixty;*	60
Seven tens, ...	*seventy;*	70
Eight tens, ...	*eighty;*	80
Nine tens, ...	*ninety;*	90

REVIEW.—3. What are nine units and one more called? Ten forms a unit of what order? How is it written? When two figures are written together, what is the place on the right called? *Ans.* The first place, or first order, or unit's place. What the place on the left? *Ans.* The second place, or second order, or tens' place.

3. In writing ten, why is a naught put in unit's place? What are one ten and one unit? How written? What are two tens? How written?

ART. **4**. The numbers between 20 and 30, 30 and 40, &c., may be expressed by considering the tens and units of which they are composed.

Thus, the number twenty-six is composed of two tens and 6 units; and is expressed by writing 2 in the place of tens, and 6 in the place of units, thus, ... 2 6

Ninety-seven has 9 tens and 7 units; written, 9 7

ORDER OF HUNDREDS.

ART. **5**. Ten *tens* are *One Hundred*, which forms a unit of the *third* order called *Hundreds*.

It is represented by the same figure, 1, as a *single* thing, or unit of the first order; but,

The 1 is written in the *third* order, while the orders of tens and units are each filled with a cipher, thus, 1 0 0.

The mode of writing *one* unit, *two* units; *one* ten, *two* tens, &c., has been explained.

In writing *one* hundred, *two* hundreds, &c., place the figure representing the hundreds in the third order, and fill the place of units and tens with ciphers, thus:

Two hundred	2 0 0	Six hundred	6 0 0
Three hundred . . .	3 0 0	Seven hundred . . .	7 0 0
Four hundred . . .	4 0 0	Eight hundred . . .	8 0 0
Five hundred	5 0 0	Nine hundred . . .	9 0 0

ART. **6**. With the three orders of UNITS, TENS, and HUNDREDS, all the numbers between one and one thousand may be expressed.

In the number three hundred and sixty-five, are three hundreds, six tens or sixty, and 5 units; written, 3 6 5.

The number four hundred and seven, contains 4 hundreds, 0 tens, and 7 units; written, 4 0 7.

In the number seven hundred, are 7 units of the third order, and no units of the first and second orders, 7 0 0.

REVIEW.—4. How are the numbers between 20 and 30, 30 and 40, written? 5. Of what order do ten *tens* form a unit? In writing one hundred, what orders are filled with ciphers?

ORDER OF THOUSANDS.

ART. **7**. Ten hundreds, or ten units of the order of hundreds, are called *One Thousand*, which forms a unit of the *fourth* order.

One thousand is expressed thus, ... 1000
Two thousand, thus, ... 2000
Three thousand, thus, ... 3000

ART. **8**. PRINCIPLES OF NOTATION AND NUMERATION.

1. Numbers are represented by the 9 digits and cipher.

2. The cipher or naught (0) has no value; it is used merely to fill vacant orders.

3. The number expressed by any figure depends upon the order or place which it occupies;

Thus, 2 in the first order is 2 units; in the second order, 2 tens or twenty; in the third order, 2 hundreds, &c.

4. The number expressed by a figure standing alone, is called its *simple* value; the number expressed when combined with other figures, its *local* value.

5. The local value of a figure increases from right to left *tenfold:* ten units of the first order make one unit of the second; ten units of the second, one of the third.

Invariably, *ten units of any order make one unit of the next higher order.*

NOTE.—When units are named, those of the first order are meant.

ART. **9**. The name of each of the first nine orders is learned from the *Table of Orders.*

REVIEW.—6. With what three orders may the numbers between one and one thousand be expressed? In writing the number *three hundred and five*, what order would be filled with a cipher? Why?

7. Ten hundreds make a unit of what order? How expressed? In writing one thousand, what orders are filled with ciphers? Why?

8. By what characters are numbers represented? What value has the cipher? For what is it used? Upon what does the number expressed by a figure depend? What is 2 in the first place? In the second? In the third? What is the *simple* value of a figure? The *local* value? How does it increase? What is meant by tenfold?

9. Where learn the names of the first nine orders? Repeat the table.

TABLE OF ORDERS.

9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.
Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units

ART. **10**. For convenience, the different orders are divided into *periods*, of three orders each.

The first three orders, that is, Units, Tens, Hundreds, constitute the first or UNIT PERIOD.

The second three orders, that is, Thousands, Tens of Thousands, Hundreds of Thousands, constitute the second or THOUSAND PERIOD.

The next three orders, that is, Millions, Tens of Millions, and Hundreds of Millions constitute the third or MILLION PERIOD.

ART. **11**. *Periods according to the Common Method:*

First	period,	Unit.	Seventh	period,	Quintillion.
Second	period,	Thousand.	Eighth	period,	Sextillion.
Third	period,	Million.	Ninth	period,	Septillion.
Fourth	period,	Billion.	Tenth	period,	Octillion.
Fifth	period,	Trillion.	Eleventh	period,	Nonillion.
Sixth	period,	Quadrillion.	Twelfth	period,	Decillion.

For other methods of Numeration, see "*Ray's Higher Arithmetic.*"

REVIEW.—10. Why are the orders divided into periods? How many orders constitute a period? What is the name of the first period, and of what orders is it composed? The name of the second period, and of what orders is it composed? The name of the third period?

ART. **12**. This Table shows the division into Periods.

5th Period. Trillions.			4th Period. Billions.			3d Period. Millions.			2d Period. Thousands.			1st Period. Units.		
Hundreds of Trillions	Tens of Trillions	Trillions	Hundreds of Billions	Tens of Billions	Billions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
6	5	4	3	2	1	9	8	7	6	5	4	3	2	1

ART. **13**. The art of reading numbers when they are written in figures, is called

NUMERATION.

Rule for Numeration.—1. *Begin at the right hand, and point off the numbers into periods of three figures each.*

Beginning at the left hand, read each period as if it stood alone; then add the name of the period.

10, *ten;* or *one* ten and *no* units.

12, *twelve;* or *one* ten and *two* units.

25, *twenty-five;* or *two* tens and *five* units.

60, *sixty;* or *six* tens and *no* units.

200, *two hundred*; or 2 hundreds, 0 tens, and 0 units.

305, *three hundred and five;* or 3 hundreds, 0 tens, and 5 units.

7405, *seven thousand four hundred and five;* or 7 thousands, 4 hundreds, 0 tens, and 5 units.

45068, *forty-five thousand and sixty-eight;* or 4 tens of thousands, 5 thousands, 0 hundreds, 6 tens, and 8 units.

REVIEW.—12. Name the periods. 13. What is Numeration? Rule?

EXAMPLE.

368271927

	three hundred and	sixty	eight millions,	two hundred and	seventy	one thousand,	nine hundred and	twenty	seven.
Read thus,	3	6	8	2	7	1	9	2	7

EXAMPLES IN NUMERATION,

To be copied, separated into periods, and read.

5	901	10000	200000	20020	300300
63	1000	12000	456000	106307	909090
90	1005	13200	682300	400001	8600050
100	1050	50004	704208	302404	2102102
104	1085	62001	800141	800010	4080400
147	1100	70400	900016	700010	1000011
208	1108	80090	601020	1030725	3036000
280	3003	97010	700400	4050607	9001003
403	4050	40305	800002	6601000	33007820
729	3045	76052	910103	7406035	61189602
710	9699	83991	700100	9725014	99099099

130670921	23004090901
6900702009	942029307029

ART. **14**. *Examples to be written in figures, then read.*

1. One unit of the third order.

2. Two units of the third order, and three units of the first order.

3. Three units of the fourth order, and two units of the second order.

4. Five units of the fifth order, two units of the third order, and four units of the first order.

5. Nine units of the seventh order, and five units of the third order.

6. Seven units of the fifth order, and five units of the second order.

7. Six units of the ninth order, and four units of the fourth order.

8. Two units of the eighth order, five units of the fifth, and two units of the the third.

9. What is 5 in the first order? 4 in the second? 3 in the sixth? 8 in the third? 7 in the fourth? 9 in the seventh? 4 in the fifth? 2 in the eighth? 8 in the sixth? 4 in the ninth? 5 in the tenth?

ART. **15**. The art of expressing numbers by figures or letters, is called

NOTATION.

Rule for Notation.—*Write each figure in the order to which it belongs, and fill the vacant orders with ciphers.*

EXAMPLE.—Write in figures, the number forty-five thousand and twenty-six.

Put 4 in ten thousands' place for forty thousand;
5 in thousands' place for five thousand;
0 in hundreds' place, as there are no hundreds;
2 in tens' place for twenty; and,
6 in units' place for six: written, 45026.

EXAMPLES IN NOTATION.

NOTE.—Learners should be taught to put a dot for each order, and a large dot for the commencement of each period.

1. Four units.
2. One ten and six units.
3. Eighteen.
4. Two tens, or twenty.
5. Two tens and four units.
6. Twenty-eight.
7. Three tens, or thirty.
8. Three tens and two units.
9. Thirty-seven.
10. Four tens and one unit.
11. Forty-six.
12. Five tens and nine units.
13. Sixty-five.
14. Nine tens and seven units.
15. Eighty-seven.
16. One hundred and four.
17. One hundred and two tens.
18. One hundred and thirty.
19. One hundred and seventy-five.
20. Two hundreds and three units.
21. Three hundreds and four tens.

22. Four hundreds, three tens, and five units.
23. Five hundred and two.
24. Six hundred and twenty-five.
25. Two hundreds, two tens, and two units.
26. Nine hundreds, nine tens, and nine units.
27. Eight hundred and seven.
28. Eight hundred and seventy.
29. Nine hundred and one.
30. Six hundreds and six units.
31. Three hundred and nine.
32. One hundred and ninety.
33. Two hundred and two.
34. One unit of the third order and one unit of the first order.
35. Two units of the second order and one unit of the first order.
36. Three units of the third order. What orders have ciphers in them?
37. Five units of the third order and three units of the first order.
38. Eight units of the third order, two units of the second order, and seven units of the first order.
39. One unit of the fourth order.
40. One thousand and twenty —or one thousand, no hundreds, two tens, and no units.
41. Twenty-five thousand and six—or two tens of thousands, five thousands, no hundreds, no tens, and six units.
42. Three hundred and forty-five—or three hundreds, four tens, and five units.
43. Seven hundred and sixty —or seven hundreds, six tens, and no units.
44. Three thousand four hundred and six—or three thousands, four hundreds, no tens, and six units.
45. Forty-two thousand and thirty—or four tens of thousands, two thousands, no hundreds, three tens, and no units.
46. Thirty thousand—or three tens of thousands, no hundreds, no tens, no units.
47. One hundred and sixty-three thousand—or one hundred thousand, six tens of thousands, three thousands, no hundreds, no tens, and no units.
48. Three hundred and forty-one thousand, five hundred and sixty-three—or three hundred thousands, four tens of thousands, one thousand, five hundreds, six tens, and three units.
49. Four tens of millions, five millions, no hundreds of thousands, eight tens of thousands, three thousands, no hundreds, two tens, and six units.
50. Eight hundreds of millions, seven tens of millions, no millions, seven hundreds of thousands, four tens of thousands, three thousands, five hundreds, seven tens, and nine units.

51. Two thousand eight hundred and four.
52. Four thousand and twenty-nine.
53. Six thousand and six.
54. Twenty-two thousand seven hundred and sixty-five.
55. Eighty thousand, two hundred and one.
56. Ninety thousand and one.
57. Thirty thousand and thirty.
58. Four hundred and ten thousand, two hundred and five.
59. Eight hundred thousand, six hundred and sixty-nine.
60. Nine hundred thousand and one.
61. Five hundred thousand and fifty.
62. One hundred thousand and ten.
63. Nine hundred and nine million and ninety thousand.
64. One hundred million, ten thousand and one.
65. Ninety-one million, seven thousand and sixty.
66. Seventy million and four.
67. Seven hundred millions, ten thousand and one.
68. One billion, one million and forty.
69. Forty billions, two hundred thousand and five.
70. Seven hundred and twenty-six billions, fifty millions, one thousand, two hundred and forty-three.

ROMAN NOTATION.

ART. **16**. The common method of representing numbers, by *figures*, is termed the *Arabic*. Another method, by means of *letters*, is termed the *Roman*.

The letter I stands for *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* and M, *one thousand.* Numbers are represented on the following principles:

1. Every time a letter is repeated, its value is repeated: thus, II denotes two, XX denotes twenty.

2. When a letter of *less* value is placed before one of *greater* value, the less is taken from the greater; if placed after it, the value of the greater is increased: thus, IV denotes four, while VI denotes six.

REVIEW.—15. What is Notation? What is the Rule? 16. What is the common method of Notation termed? What other method? What number is represented by the letter I? By V? By X? By L? By C? By D? By M? What is the effect of repeating a letter?

16. What the effect of placing a letter of less value *before* one of greater value? *After* one of greater value? What of placing a line over a letter?

3. A line over a letter increases its value a *thousand times.* Thus, $\overline{V}$ denotes 5 0 0 0, $\overline{M}$ denotes *one million.*

TABLE OF ROMAN NOTATION.

I	One.	XXI	Twenty-one.
II	Two.	XXX	Thirty.
III	Three.	XL	Forty.
IV	Four.	L	Fifty.
V	Five.	LX	Sixty.
VI	Six.	XC	Ninety.
IX	Nine.	C	One hundred.
X	Ten.	CCCC	Four hundred.
XI	Eleven.	D	Five hundred.
XIV	Fourteen.	DC	Six hundred.
XV	Fifteen.	DCC	Seven hundred.
XVI	Sixteen	DCCC	Eight hundred.
XVII	Seventeen.	DCCCC	Nine hundred.
XVIII	Eighteen.	M	One thousand.
XIX	Nineteen.	MM	Two thousand.
XX	Twenty.	MDCCCLVI	1856.

ART. **17**. The preceding illustrations show the three methods of expressing numbers:

1ST. By *words*, or ordinary language.

2D. By *figures*, termed the Arabic method.

3D. By *letters*, termed the Roman method.

II. ADDITION.

1. If you have 2 cents and find 3 cents, how many will you then have? *Ans.* 5 cents.

2. I spent 12 cents for a slate, and 5 cents for a copy-book: how many cents did I spend? *Ans.* 17 cents.

3. John gave 6 cents for an orange, 7 cents for pencils, and 9 cents for a ball: how many cents did they all cost? *Ans.* 22 cents.

ART. **18**. *The process of uniting two or more numbers into one number, is termed Addition.*

The number obtained by addition, is the *Sum* or *Amount.*

REMARK.—When the numbers to be added are of the *same* denomination, that is, all cents, or all yards, &c., the operation is called *Simple Addition.*

ART. 19. OF THE SIGNS.

The Sign +, called *plus*, means *more.* When between two numbers, it shows that they are to be added; thus, $4 + 2$ means that 4 and 2 are to be added together.

The sign of *equality*, =, denotes that the quantities between which it stands equal each other.

The expression $4+2=6$, means that the sum of 4 and 2 is 6; read, 4 *and* 2 *are* 6, or 4 *plus* 2 *equals* 6.

ADDITION TABLE.

2 + 0 = 2	3 + 0 = 3	4 + 0 = 4	5 + 0 = 5
2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6
2 + 2 = 4	3 + 2 = 5	4 + 2 = 6	5 + 2 = 7
2 + 3 = 5	3 + 3 = 6	4 + 3 = 7	5 + 3 = 8
2 + 4 = 6	3 + 4 = 7	4 + 4 = 8	5 + 4 = 9
2 + 5 = 7	3 + 5 = 8	4 + 5 = 9	5 + 5 = 10
2 + 6 = 8	3 + 6 = 9	4 + 6 = 10	5 + 6 = 11
2 + 7 = 9	3 + 7 = 10	4 + 7 = 11	5 + 7 = 12
2 + 8 = 10	3 + 8 = 11	4 + 8 = 12	5 + 8 = 13
2 + 9 = 11	3 + 9 = 12	4 + 9 = 13	5 + 9 = 14
6 + 0 = 6	7 + 0 = 7	8 + 0 = 8	9 + 0 = 9
6 + 1 = 7	7 + 1 = 8	8 + 1 = 9	9 + 1 = 10
6 + 2 = 8	7 + 2 = 9	8 + 2 = 10	9 + 2 = 11
6 + 3 = 9	7 + 3 = 10	8 + 3 = 11	9 + 3 = 12
6 + 4 = 10	7 + 4 = 11	8 + 4 = 12	9 + 4 = 13
6 + 5 = 11	7 + 5 = 12	8 + 5 = 13	9 + 5 = 14
6 + 6 = 12	7 + 6 = 13	8 + 6 = 14	9 + 6 = 15
6 + 7 = 13	7 + 7 = 14	8 + 7 = 15	9 + 7 = 16
6 + 8 = 14	7 + 8 = 15	8 + 8 = 16	9 + 8 = 17
6 + 9 = 15	7 + 9 = 16	8 + 9 = 17	9 + 9 = 18

REVIEW.—17. What are the three methods of expressing numbers? 18. What is Addition? What the sum or amount? REM. What is Simple Addition? 19. What does the sign plus mean? What does it show? What does the sign of equality denote? Give an example.

Art. 20. 1. James had 63 cents, and his father gave him 35 cents: how many cents had he then?

Place the *units* and the *tens* of one number under the *units* and the *tens* of the other, that figures of the same unit value may be more easily added.

Solution.—Write the numbers as in the margin; then say 5 units and 3 units are 8 units, which write in units' place; 3 tens and 6 tens are 9 tens, which write in tens' place. The sum is 9 tens and 8 units, or 98 cents.

	tens.	units.	
	6	3	cents.
	3	5	cents.
Ans.	9	8	cents.

In this example, *units* are added to *units*, and *tens* to *tens*, since only numbers of the same kind, that is, having the *same unit value*, can be added. Thus,

3 units and 2 tens make neither 5 units nor 5 tens; as, 3 apples and 2 plums are neither 5 apples nor 5 plums.

2. I own 3 tracts of land: the first contains 240 acres; the second, 132 acres; the third, 25 acres: how many acres in all?

Since units of *different* orders can not be added together, place *units* under *units*, *tens* under *tens*, &c., that figures to be added may be in the most *convenient position*.

Solution.—Begin at the right, and say 5 units and 2 units are 7 units, which write in units' place; 2 tens and 3 tens and 4 tens are 9 tens, which write in tens' place:

Lastly, 1 hundred and 2 hundreds are 3 hundreds, which write in hundreds' place, and the work is complete.

240	acres.
132	acres.
25	acres.
397	acres.

Questions to be solved and explained as above.

The sign $ stands for the word dollars, and when used is placed *before* the figures.

3. There are 43 sheep in one pasture, 21 in another, and 14 in another: how many sheep in all? *Ans.* 78.

Review.—20. Why are units placed under units, and tens under tens? Can numbers of different unit values be added? Why not? Give an example. What sign is used for the word dollars?

4. I owe one man $210, another $142, and another $35: what is the amount of the debts? *Ans.* $387.

5. Find the sum of 4321, 1254, 3120. *Ans.* 8695.

6. The sum of 50230, 3105, 423. *Ans.* 53758.

ART. **21**. Where the *sum* of the figures in a column does not exceed 9, it is written under the column added.

When the sum of the figures exceeds 9, *two* or *more* figures are required to express it. To explain the method,

TAKE THIS EXAMPLE.

1. James bought a reader for 74 cents, an atlas for 37 cents, a slate for 25 cents: how much did all cost?

	T. U.	
Reader	7 4	cents.
Atlas	3 7	cents.
Slate	2 5	cents.
	1 6	
	1 2	
Ans.	1 3 6	cents.

1ST SOLUTION.—By adding the figures in the first column, the sum is 16, which is 1 ten and 6 units. Write the 6 units in the order of units, and the 1 ten in the order of tens.

The sum of the figures in the tens' column is 12 tens, which is 1 hundred and 2 tens. Write the 2 tens in the order of tens, and the 1 hundred in the order of hundreds.

Lastly, unite the figures in the column of tens. The sum is 1 hundred, 3 tens, and 6 units, or 136 cents.

```
 74
 37
 25
---
136
```

2D SOLUTION.—The preceding example is usually performed thus: 5 units 7 units and 4 units are 16 units, which is 1 ten and 6 units. Write the 6 units in units' place, and carry the 1 ten to tens' place.

Then, 1 ten 2 tens 3 tens and 7 tens are 13 tens, which is 1 hundred and 3 tens; write the 3 tens in tens' place, the 1 hundred in hundreds' place, and the work is completed.

The 1 ten derived from the sum of the figures in the 1st column, and added to the 2d, is said to be *carried*.

REVIEW.—21. When the sum of a column does not exceed 9, where is it written? If greater than 9? What is understood by carrying the tens? In what does it consist? Why does the addition begin with the units' column? 22. What is the rule for addition? The *proof?*

When the sum of the figures in a column exceeds 9, reserving the tens, or left hand figure, and adding it to the figures in the next column, is called *carrying the tens.*

2. Add the numbers, 3415, 503, 1870, and 922.

Solution.—Write units of the same order under each other. Then say, 2 and 3 are 5, and 5 are 10 *units*, which is no (0) units, written in units' place, and 1 ten, carried to the tens; 1 and 2 are 3, and 7 are 10, and 1 are 11 *tens*, which is 1 ten, written in tens' place, and 1 hundred, carried to the hundreds; 1 and 9 are 10, and 8 are 18, and 5 are 23, and 4 are 27 *hundreds*, which is 7 hundreds, written in hundreds' place, and 2 thousands, carried to the thousands; 2 and 1 are 3, and 3 are 6 *thousands*, written in thousands' place.

$$\begin{array}{r} 3415 \\ 503 \\ 1870 \\ 922 \\ \hline 6710 \end{array}$$

Carrying the tens is simply adding *tens* to *tens*, *hundreds* to *hundreds*, &c., on the principle (Art. 20), that only numbers of the same unit value can be added.

The addition begins at the right hand column, with the unit of the lowest order, so that,

If the sum of the units in any column exceeds 9, the tens can be carried to the sum of the next higher order.

Art. 22. RULE

For Addition.—1. *Write the numbers to be added, so that figures of the same order may stand under each other, units under units, tens under tens, &c.*

2. *Begin at the right hand, and add each column separately. Place the units obtained by adding each column under it, and carry the tens to the next higher order. At the last column write the whole amount.*

Proof.—Separate the numbers into two or more divisions; find the sum of each; then add the amounts together. If their sum equal that found by the rule, the work is correct.

Remarks.—1. Another method of proof consists in adding the columns downward, taking the figures in a *different* order from that in which they were taken before.

2. For the method of proof by casting out the 9's, (too difficult for a work of this grade,) see "*Ray's Higher Arithmetic.*"

3. Add the numbers 853, 516, 29, 6, 34, 580.

SOLUTION.—Writing the numbers and adding them, their sum is 2018.

PROOF.—If the numbers be separated by a line between 29 and 6, the sum of the numbers in the first division will be 1398, and in the second, 620. Adding these numbers together, their sum is 2018, as before.

```
 853
 516
  29
   6—1398
  34   620
 580  ----
----  2018
2018
```

4. Find the sum of 3745, 2831, 5983, 7665.

In adding long columns of figures, it is necessary to retain the numbers carried. This may be done by placing them in small figures under their proper columns, as 3, 2, 1, in the margin.

```
 3745
 2831
 5983
 7665
 ----
20224
 321
```

EXAMPLES FOR PRACTICE.

(5)	(6)	(7)	(8)	(9)	(10)
184	204	103	495	384	1065
216	302	405	207	438	6317
135	401	764	185	348	5183
320	311	573	825	843	7102
413	109	127	403	483	3251
101	43	205	325	834	6044
1369	1370	2177	2440	3330	28962

(11)	(12)	(13)	(14)	(15)
3725	5943	82703	987462	6840325
5834	6427	102	478345	7314268
4261	8204	6005	610628	3751954
7203	7336	759	423158	6287539

16. 23+41+74+83+16= how many? *Ans.* 237.

17. 45+19+32+74+55= how many? *Ans.* 225.

18. 51+48+76+85+4 = how many? *Ans.* 264.

19. 263+104+321+155= how many? *Ans.* 843.

20. 94753+2847+93688+9386+258+3456 are how many? *Ans.* 204388.

21. January has 31 days, February 28, March 31, April 30, and May 31: how many days are there in these five months? *Ans.* 151.

22. June has 30 days, July 31, August 31, September 30, October 31: how many days in all? *Ans.* 153.

23. The first 5 months have 151 days, the next 5 have 153 days, November has 30, and December 31: how many days in the whole year? *Ans.* 365.

24. I bought 4 pieces of muslin: the first contained 50 yards, the second 65, the third 42, the fourth 89: how many yards in all? *Ans.* 246 yds.

25. I owe one man $245, another $325, a third $187, a fourth $96: how much do I owe? *Ans.* $853.

26. General Washington was born A. D. 1732, and lived 67 years: in what year did he die? *Ans.* 1799.

27. From the creation of the world to the flood was 1656 years; thence to the siege of Troy, 1164 years; thence to the building of Solomon's Temple, 180 years; thence to the birth of Christ, 1004 years: in what year of the world did the Christian era begin? *Ans.* 4004.

28. A has 4 flocks of sheep; in the 1st are 65 sheep and 43 lambs; in the 2d, 187 sheep and 105 lambs; in the 3d, 370 sheep and 243 lambs; in the 4th, 416 sheep and 95 lambs: how many sheep and lambs has he?

Ans. 1038 sheep, and 486 lambs.

29. A man bought 30 barrels of pork for $285, 18 barrels for $144, 23 barrels for $235, and 34 barrels for $408: how many barrels did he buy, and how many dollars did he pay? *Ans.* 105 barl., and $1072.

30. The first of four numbers is 287; the second, 596; the third, 841; and the fourth, as much as the first three: what is their sum? *Ans.* 3448.

31. The Pyramids of Egypt were built 337 years before the founding of Carthage; Carthage was founded 49 years before the destruction of Troy; Troy was destroyed 431 years before Rome was founded; Carthage was destroyed 607 years after the founding of Rome, and 146 before the Christian era. How many years before Christ were the Pyramids built? *Ans.* 1570.

32. Add three thousand and five; forty-two thousand, six hundred and twenty-seven; 105; three hundred and seven thousand and four; 800,791; three hundred and twenty thousand, six hundred. *Ans.* 1474132.

33. Add 275,432; four hundred and two thousand and thirty; three hundred thousand and five; 872,026; four million, two thousand, three hundred and forty-seven. *Ans.* 5851840.

34. Add eight hundred and eighty millions, eight hundred and eighty-nine; 2,002,002; seventy-seven million, four hundred and thirty-six thousand; two hundred and six million, five thousand, two hundred and seven; 49,003; nine hundred and ninety million, nineteen thousand, nine hundred and nineteen.

Ans. 2155513020.

III. SUBTRACTION.

ART. **23**. 1. If you have 9 apples, and give 4 away, how many are left? *Ans.* 5. Why? *Because* 4 *and* 5 *are* 9.

2. Frank had 15 cents; after spending 7, how many were left? *Ans.* 8. Why?

3. If you take 8 from 13, how many are left? *Ans.* 5.

The operation in the preceding examples is termed Subtraction. Hence, *Subtraction is the process of finding the difference between two numbers.*

The larger number is called the *Minuend;* the less, the *Subtrahend;* and the number left after subtraction, the *Difference* or *Remainder.*

REMARKS.—1. The word Minuend means, *to be diminished;* Subtrahend, *to be subtracted.*

2. In Addition (See Art. 20), numbers of the same kind are added; in Subtraction they are taken from each other; therefore, Subtraction is the reverse of Addition.

3. When the given numbers are of the same denomination, the operation is called *Simple Subtraction.*

ART. **24**. The Sign —, is called *minus*, meaning *less.* Placed between two numbers, it denotes that the one on the right is to be taken from that on the left.

Thus, $8 - 5 = 3$, shows that 5 is to be taken from 8; it is read, 8 minus 5 equals 3. Here, 8 is the minuend, 5 the subtrahend, and 3 the remainder.

SUBTRACTION TABLE.

2 — 2 = 0	3 — 3 = 0	4 — 4 = 0	5 — 5 = 0
3 — 2 = 1	4 — 3 = 1	5 — 4 = 1	6 — 5 = 1
4 — 2 = 2	5 — 3 = 2	6 — 4 = 2	7 — 5 = 2
5 — 2 = 3	6 — 3 = 3	7 — 4 = 3	8 — 5 = 3
6 — 2 = 4	7 — 3 = 4	8 — 4 = 4	9 — 5 = 4
7 — 2 = 5	8 — 3 = 5	9 — 4 = 5	10 — 5 = 5
8 — 2 = 6	9 — 3 = 6	10 — 4 = 6	11 — 5 = 6
9 — 2 = 7	10 — 3 = 7	11 — 4 = 7	12 — 5 = 7
10 — 2 = 8	11 — 3 = 8	12 — 4 = 8	13 — 5 = 8
11 — 2 = 9	12 — 3 = 9	13 — 4 = 9	14 — 5 = 9

6 — 6 = 0	7 — 7 = 0	8 — 8 = 0	9 — 9 = 0
7 — 6 = 1	8 — 7 = 1	9 — 8 = 1	10 — 9 = 1
8 — 6 = 2	9 — 7 = 2	10 — 8 = 2	11 — 9 = 2
9 — 6 = 3	10 — 7 = 3	11 — 8 = 3	12 — 9 = 3
10 — 6 = 4	11 — 7 = 4	12 — 8 = 4	13 — 9 = 4
11 — 6 = 5	12 — 7 = 5	13 — 8 = 5	14 — 9 = 5
12 — 6 = 6	13 — 7 = 6	14 — 8 = 6	15 — 9 = 6
13 — 6 = 7	14 — 7 = 7	15 — 8 = 7	16 — 9 = 7
14 — 6 = 8	15 — 7 = 8	16 — 8 = 8	17 — 9 = 8
15 — 6 = 9	16 — 7 = 9	17 — 8 = 9	18 — 9 = 9

ART. **25**. When numbers are small, the difference between them may be ascertained in the *mind;* when large, the operation is most easily performed by writing them.

EXAMPLES.

1. A man having $135, spent $112: what sum had he left?

Since only things of the same *unit value* can be added (Art 20), the difference between things of the same unit value only can be found; hence,

Place *units* under *units*, *tens* under *tens*, &c., that the figures between which the subtraction is to be made, may be in the most *convenient position.*

SOLUTION.—After arranging the numbers, say 2 (units) from 5 (units) leave 3 (units), which put in units' place; then 1 (ten) from 3 (tens) leaves 2 (tens), which put in tens' place; then 1 (hundred) from 1 (hundred) leaves 0, and there being no figures on the left of this, the place is vacant; hence the number of dollars left is 23; that is, 135—112=23.

hund.	tens.	units.	
1	3	5	minuend.
1	1	2	subtrahend.
	2	3	remainder.

2. A farmer having 245 sheep, sold 123: how many had he left? *Ans.* 122.

3. A man bought a farm for $751, and sold it for $875: how much did he gain? *Ans.* $124.

What is the difference

4. Between 734 and 531? . . . *Ans.* 203.

5. Between 8752 and 3421? . . . *Ans.* 5331.

6. Between 529 and 8? . . . *Ans.* 521.

7. Between 79484 and 25163? . . *Ans.* 54321.

REVIEW.—23. What is Subtraction? Give an example. What is the minuend? The subtrahend? Remainder? REM. What does minuend mean? What subtrahend?

23. Why is subtraction the reverse of addition? 24. What does the sign minus mean? When placed between two numbers what does it denote?

ART. **26**. When the lower figure in each order is *not greater* than the upper, the less is subtracted from the greater, and the difference written beneath; but,

When the lower figure in any order is *greater* than the upper, a difficulty arises, which we will now explain.

James had 13 cents; after spending 5, how many had he left?

5 can not be subtracted from 3, but can be from 13; 5 from 13 leaves 8; hence he had 8 left.

13
5
8

1. From 73 subtract 45.

SOLUTION.—Here, 5 units can not be taken from 3 units. Take 1 (ten) from the 7 (tens), and add this 1 (ten) or 10 units to the 3 units, which will make 13 units in the units' place; then, subtract the 5 units, and there will remain 8 units, to be put in units' place. Since 1 ten is taken from the 7 tens, there remain 6 tens in the tens' place. Subtract 4 tens from 6 tens and put the remainder in tens' place. The difference of the two numbers is thus found to be 2 tens and 8 units, or 28.

	T. U.
	73
	45
Dif.	28

tens.	units.
6	13
4	5
2	8

Instead of actually taking 1 ten from the 7 tens, and adding it to the 3 units, as is done in the margin, the operation is performed in the *mind:* thus,

5 from 13 leaves 8, and 4 from 6 leaves 2.

In such cases, the value of the upper number is not changed, since the 1 ten which is taken from the order of tens is added to the number in the order of units.

EXPLANATIONS.

Taking a unit of a higher order and adding it to the units of the next lower, so that the figure beneath may be subtracted from the sum, is called *borrowing ten.*

After increasing the units by 10, instead of considering the next figure of the upper number as *diminished* by 1,

---

REVIEW.—25. When the numbers are small, how is their difference easily found? When they are large? In writing numbers for subtraction, why place units under units, tens under tens?

the result will be the same, if the next figure of the lower number be *increased* by 1.

Thus, in the previous example, instead of diminishing the 7 tens by 1, add 1 to the 4 tens, which makes 5;

5 from 7 leaves 2, the same as 4 from 6.

Hence, when a figure in the lower number is greater than that above it, add 10 to the upper figure, then subtract the lower figure from the sum; and,

To compensate for the 10 added to the upper figure, increase the next lower figure by 1.

This process depends on the principle, that *if any two numbers be equally increased, their difference will remain the same.*

The 10, added to the upper number, is equal to 1 of the next higher order added to the lower number; ten units of any order being always equal to 1 of the order next higher.

2. Find the difference between 805 and 637.

	805
	637
Dif.	168

SOLUTION, (1ST METHOD.)—Writing the less number under the greater, with units of the same order under each other, it is required to subtract the 7 units from 5 units, which is impossible.

The five can not be increased by borrowing from the next figure, because it is 0; therefore, borrow 1 hundred from the 8 hundreds, which leaves 7 hundreds in hundreds' place; this 1 hundred makes 10 tens; then, borrowing 1 ten from the 10 tens, and adding it to the 5 units, 9 tens will be in the tens' place, and 15 units in the units' place.

Subtracting 7 units from 15 units, 8 units are left, to be written in units' place; next, subtracting 3 tens from 9 tens, there are left 6 tens, to be written in tens' place; lastly, subtracting 6

REVIEW.—26. When the lower figure is less than the upper, how is the subtraction performed? What do you understand by borrowing ten? Illustrate the process, by subtracting 45 from 73. After borrowing ten, what may be done in order to avoid diminishing the next upper figure by 1? On what principle does this process depend?

hundreds from 7 hundreds, there remains 1 hundred, to be written in hundreds' place. The difference is 168.

Or, (2D Method.)—If the 5 units be increased by 10, say 7 from 15 leaves 8; then, increasing the 3 by 1, say 4 from 0 can not be taken, but 4 from 10 leaves 6; then, increasing 6 by 1, say 7 from 8 leaves 1, and the whole remainder is 168, as before.

QUESTIONS TO BE SOLVED BY BOTH METHODS.

3. From 73 take 48. . . . . . *Ans.* 25.
4. From 340 take 150. . . . . . *Ans.* 190.
5. From 508 take 325. . . . . . *Ans.* 183.
6. From 4603 take 3612. . . . . . *Ans.* 991.
7. From 8765 take 7766. . . . . . *Ans.* 999.

REMARKS.—1. The SECOND method is generally used; it is more convenient, and less liable to error, especially when the upper number contains ciphers.

2. Begin at the *right* to subtract, so that if any lower figure is greater than the upper, 1 may be borrowed from a higher order.

When each figure in the lower number is less than the one above it, the subtraction may commence at the *left* hand.

ART. **27**. If 5 subtracted from 8 leaves 3, then 3 added to 5 must produce 8; that is,

If the difference of two numbers be added to the less, the sum will be equal to the greater.

RULE

For Subtraction.—1. *Write the less number under the greater, placing units under units, tens under tens, &c.*

2. *Beginning at the right hand, subtract each figure from the one directly over it, and write the remainder beneath.*

3. *If the lower figure exceeds the upper, add ten to the upper figure, subtract the lower from it, and carry one to the next lower figure, or take one from the next upper figure.*

PROOF.—Add the remainder to the subtrahend; if the sum is equal to the minuend, the work is correct.

For proof by casting out the 9's, see "*Ray's Higher Arithmetic.*"

	(8)	(9)	(10)	(11)
Minuends,	7640	860012	4500120	3860000
Subtrahends,	1234	430021	2910221	120901
Remainders,	6406	429991	1589899	3739099
Proof . . .	7640	860012	4500120	3860000

12. Take 1234567 from 4444444. *Ans.* 3209877.
13. Take 15161718 from 91516171. *Ans.* 76354453.
14. Take 34992884 from 63046571. *Ans.* 28053687.
15. 153425178 — 53845248 = *Ans.* 99579930.
16. 100000000 — 10001001 = *Ans.* 89998999.
17. Take 17 cents from 63 cents. *Ans.* 46 cents.

18. A carriage cost $137, and a horse $65: how much more than the horse did the carriage cost? *Ans.* $72.

19. A tree 75 feet high was broken; the part that fell was 37 feet long: how high was the stump? *Ans.* 38 ft.

20. America was discovered by Columbus in 1492: how many years had elapsed in 1837? *Ans.* 345.

21. I deposited in the bank $1840, and drew out $475: how many dollars had I left? *Ans.* $1365.

22. A man has property worth $10104, and owes debts to the amount of $7426: when his debts are paid, how much will be left? *Ans.* $2678.

23. A man having $100000, gave away $11: how many had he left? *Ans.* $99989.

24. Subtract 19019 from 20010. *Ans.* 991.

25. Required the excess of nine hundred and twelve thousand and ten, above 50082. *Ans.* 861928.

26. Take 4004 from four million. *Ans.* 3995996.

27. Subtract 1009006, from two million, twenty thousand, nine hundred and thirty. *Ans.* 1011924.

REVIEW.—26. REM. After borrowing ten, which of the two methods is generally used? Why? Why begin at the right hand to subtract? 27. Give the rule for subtraction. Method of proof.

IV. MULTIPLICATION.

Art. 28. 1. If 1 orange cost 2 cents, what will 3 cost?

Analysis.—*Three oranges will cost 3 times as much as one. That is, 2 cents taken 3 times:* $2+2+2=6$. *Ans.*

2. If 1 lemon cost 3 cents, what will 4 lemons cost?

$3+3+3+3=12$. *Ans.*

3. In an orchard there are 4 rows of trees, in each row 21 trees: how many trees in the orchard?

Solution.—1st. By writing 21 four times, as in the margin, and adding, the whole number of trees is 84.

1st row,	21	trees.
2d row,	21	trees.
3d row,	21	trees.
4th row,	21	trees.
	84	

2d. Instead of writing 21 four times, write it once, place the number 4 under, it being the number of times 21 is to be taken, and say, 4 times 1 (unit) are 4 (units), which put in units' place: then, 4 times 2 (tens) are 8 (tens), to be put in tens' place; the result is 84, the same as found by *addition.*

$$\begin{array}{r} 21 \\ 4 \\ \hline 84 \end{array}$$

The latter method is termed *Multiplication.*

DEFINITIONS.

Multiplication is a short method of Addition, when the numbers to be added are equal.

Multiplication is also taking one number as many times as there are units in another.

The number to be taken is the *multiplicand.*

The number denoting how many times the multiplicand is taken, is the *multiplier.* The result is the *product.*

Thus, 4 times 5 are 20; 5 is the *multiplicand,* 4 the *multiplier,* and 20 the *product.*

The multiplicand and multiplier are together called *factors,* because they make or produce the product.

Review.—28. What is multiplication? What is the multiplicand? What does the multiplier denote? What is the product? What are the multiplicand and multiplier together called? Why? Rem. When is multiplication termed simple?

REMARK.—When the multiplicand is of one denomination, the operation is called *Simple Multiplication.*

ART. **29.** The Sign ×, read *multiplied by*, or *times*, denotes that the numbers between which it is placed are to be multiplied together.

Thus, 4×3=12, shows that 4 multiplied by 3, or 4 times 3, are 12, or equal 12.

In the table, the sign ×, may be read *times:* thus,
2 times 2 are 4; 2 times 3 are 6; and so on.

MULTIPLICATION TABLE.

1 × 0 = 0	2 × 0 = 0	3 × 0 = 0	4 × 0 = 0
1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 0 = 0	6 × 0 = 0	7 × 0 = 0	8 × 0 = 0
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96

9×0= 0	10×0= 0	11×0= 0	12×0= 0
9×1= 9	10×1= 10	11×1= 11	12×1= 12
9×2= 18	10×2= 20	11×2= 22	12×2= 24
9×3= 27	10×3= 30	11×3= 33	12×3= 36
9×4= 36	10×4= 40	11×4= 44	12×4= 48
9×5= 45	10×5= 50	11×5= 55	12×5= 60
9×6= 54	10×6= 60	11×6= 66	12×6= 72
9×7= 63	10×7= 70	11×7= 77	12×7= 84
9×8= 72	10×8= 80	11×8= 88	12×8= 96
9×9= 81	10×9= 90	11×9= 99	12×9=108
9×10= 90	10×10=100	11×10=110	12×10=120
9×11= 99	10×11=110	11×11=121	12×11=132
9×12=108	10×12=120	11×12=132	12×12=144

ART. **30**. Here are several rows of stars. Counting upward, there are 5 rows of 4 stars each; 5 rows contain 5 times as many as 1 row; 5 times 4 stars are 20 stars.

* * * *
* * * *
* * * *
* * * *
* * * *

Counting across, there are 4 rows of 5 stars each; 4 rows contain 4 times as many as one row; 4 times 5 stars are 20 stars, the same as before.

Hence, *the product of two numbers is not altered by changing the order of the factors.*

What is the difference between 6×5 and 5×6? 9×8 and 6×12? 10×12 and 12×10?

REMARK 1.—The product is always of the same *kind* or *denomination* as the multiplicand.

PRINCIPLES AND EXAMPLES.

1. What is the product of 5 multiplied by 3; that is, what is the amount of 5 taken 3 times? *Ans.* 15.

3 times 5 are equal to 5+5+5=15.

Here, the multiplicand, 5, is an *Abstract* number; that is, it denotes no *particular thing*, and the product, 15, is also abstract.

REVIEW.—29. What does the sign (×) of multiplication denote? Give an example. 30. Does it alter the product to make either of the factors the multiplier? Illustrate this.

2. What will 3 yards of muslin cost, at 5 cents a yard?

ANALYSIS.—*Three yards will cost three times the price of one yard; that is,* 5 *cents taken* 3 *times, which is*

5 cts.+5 cts.+5 cts.=15 cts. *Ans.*

Here, the multiplicand is a *Concrete* number; that is, it denotes some *particular* thing, as *Cents;* the product, 15 cents, denoting the *same thing*, is also concrete.

Hence, in general, if the multiplicand is *money*, the product will be *money* of the same name; if it is *pounds*, the product will be *pounds*, &c.

REM. 2.—The multiplier shows the *number of times* the multiplicand is to be taken; hence, it must always be considered an *abstract* number.

In Example 2, 5 cents are not multiplied by 3 yards, but are taken *three times;* as three yards will cost *three times* the price of 1 yard.

To speak of multiplying 3 dollars by 2 yards, or 25 cents by 25 cents, is as absurd as to propose to multiply 3 apples by 2 potatoes.

ART. 31. *When the Multiplier does not exceed* 12.

EXAMPLES.

1. How many yards of cloth are there in 3 pieces, each containing 123 yards?

SOLUTION.—Having placed the multiplier under the multiplicand, as in the margin, say, 3 times 3 units are 9 units, which write in units' place; 3 times 2 tens are 6 tens, which write in tens' place; 3 times 1 hundred are 3 hundreds, which write in hundreds' place.

OPERATION.

123	multiplicand.
3	multiplier.
369	product.

2. What will 2 houses cost at $231 each? *Ans.* $462.
3. What will 3 horses cost at $132 each? *Ans.* $396.
4. What is the product of 201×4? *Ans.* 804.
5. What is the product of 2301×3? *Ans.* 6903.

REVIEW.—30. REM. 1. What is the denomination of the product? Show that when the multiplicand is an abstract number, the product is also abstract. Show that when the multiplicand is a concrete number, the product must be concrete.

REM. 2. What does the multiplier show? What must it be considered?

6. At $43 an acre, what will 5 acres of land cost?

SOLUTION.—Say, 5 times 3 (units) are 15 (units); write the 5 (units) in units' place, and carry the 1 (ten); then, 5 times 4 (tens) are 20 (tens), and 1 (ten) carried make 21 (tens), which write as in the margin.

OPERATION.

```
 $43
   5
----
$215
```

7. What is the product of 347×12?

The product of each of the figures by 12 is known from the Mul. Table; multiply then by 12, as by a single figure.

```
 347
  12
----
4164
```

PROOF.—Since 11 times 347 and 1 time 347 are equal to 12 times 347, therefore, take 1 from the multiplier, and use the remainder instead of the former multiplier; to the product, add the multiplicand; the result must be the same as the first product.

```
 347
  11
----
3817
 347
----
4164
```

RULE

For Multiplication.—1. *Write the multiplicand, with the multiplier under it, and draw a line beneath.*

2. *Begin with units; multiply each figure of the multiplicand by the multiplier, and carry as in addition.*

PROOF.—Take 1 from the multiplier, and use the remainder for a multiplier; to the product thus found, add the multiplicand; if the sum equals the first product, the work is correct

REM.—Begin at the right hand to multiply, so that the excess of tens in any lower order may be carried to the order next higher.

EXAMPLES FOR PRACTICE.

	(8)	(9)	(10)	(11)
Multiplicand,	5142	4184	3172	41834
Multiplier,	5	6	5	7
Product, . .	25710	25104	15860	292838

REVIEW.—31. How are the numbers written, Rule? Where do you begin to multiply? How do you multiply? How do you prove the operation? REM. Why begin at the right hand to multiply?

	EXAMPLES.	ANS.		EXAMPLES.	ANS.
12.	$49 \times 3 =$	147.	17.	$19645 \times 8 =$	157160.
13.	$57 \times 4 =$	228.	18.	$44386 \times 9 =$	399474.
14.	$128 \times 5 =$	640.	19.	$708324 \times 7 =$	4958268.
15.	$367 \times 6 =$	2202.	20.	$964578 \times 9 =$	8681202.
16.	$1427 \times 7 =$	9989.	21.	$96432 \times 10 =$	964320.

22. Multiply 46782 by 11. *Ans.* 514602.
23. Multiply 86458 by 12. *Ans.* 1037496.
24. Multiply 680323 by 11. *Ans.* 7483553.
25. Multiply 1236839 by 12. *Ans.* 14842068.

ART. **32**. *When the Multiplier exceeds* 12.

26. What is the product of 43×25?

ANALYSIS.—Since 25 is equal to 2 tens and 5 units, that is, 20+5, multiply by 5, and write the product, 215 (units), in units' place; then multiply by the 2 tens, and set the product, 86 (tens), in tens' place.

$$\begin{array}{rl} 43 & \\ 25 & \\ \hline 215 & = 5 \text{ times } 43 \\ 86 & = 20 \text{ times } 43 \\ \hline 1075 & = 25 \text{ times } 43 \end{array}$$

Multiplying by 5 units gives 5 times 43, and multiplying by 2 tens gives 20 times 43; add them, because 5 times 43 and 20 times 43=25×43.

Hence, multiply by the units' figure of the multiplier, and write the product in units' place, because it is *units;* multiply by the tens' figure, and write the product in tens' place.

Therefore, *in multiplying by a figure of any order, write the product in the same order as the multiplier.*

NOTE.—The products of the multiplicand by the separate figures of the multiplier, are *partial* products.

GENERAL RULE

For Multiplication.—1. *Write the multiplier under the multiplicand, placing figures of the same order under each other.*

REVIEW.—32. If the *partial* multiplier be in units' place, where is the product written? Why? Where should the right hand figure of each product always be written? NOTE. What are partial products? Give the general rule for multiplication. What is the method of proof?

2. *Multiply by each figure of the multiplier in succession, beginning with units, always setting the right hand figure of each product under that figure of the multiplier which produces it.*

3. *Add the several partial products together; their sum will be the product sought.*

PROOF.—Multiply the multiplier by the multiplicand: the product thus obtained must be the same as the first product.

For proof by casting out the 9's, see "*Ray's Higher Arithmetic.*"

27. Multiply 2345 by 123.

```
  2345 multiplicand.
   123 multiplier.
  ----
  7035   =     3 times 2345
 4690    =    20 times 2345
2345     =   100 times 2345
------
288435   =   123 times 2345
```

PROOF.

```
   123
  2345
  ----
   615
  492
 369
246
------
288435
```

28. Multiply 327 by 203.

When a cipher is in the multiplier, fill the corresponding order in the product with 0, or leave it vacant, and multiply by the other figures.

Remember *to place the right hand figure of each partial product under the multiplying figure.*

By observing the above principle, the multiplication may begin with any figure of the multiplier.

```
  327
  203
  ---
  981
654
-----
66381
```

	EXAMPLES.	ANS.		EXAMPLES.	ANS.
29.	235 × 13 =	3055.	38.	624 × 85 =	53040.
30.	346 × 19 =	6574.	39.	976 × 97 =	94672.
31.	425 × 29 =	12325.	40.	342 × 364 =	124488.
32.	518 × 34 =	17612.	41.	376 × 526 =	197776.
33.	279 × 37 =	10323.	42.	476 × 536 =	255136.
34.	869 × 49 =	42581.	43.	2187 × 215 =	470205.
35.	294 × 57 =	16758.	44.	3489 × 276 =	962964.
36.	429 × 62 =	26598.	45.	1646 × 365 =	600790
37.	485 × 76 =	36860.	46.	8432 × 635 =	5354320.

47. Multiply 6874 by 829. *Ans.* 5698546.
48. Multiply 2873 by 1823. *Ans.* 5237479.
49. Multiply 4786 by 3497. *Ans.* 16736642.
50. Multiply 87603 by 9865. *Ans.* 864203595.
51. Multiply 83457 by 6835. *Ans.* 570428595.
52. Multiply 31624 by 7138. *Ans.* 225732112.

53. What will 126 barrels of flour cost, at $6 a barrel? *Ans.* $756.

54. What will 823 barrels of pork cost, at $12 a barrel? *Ans.* $9876.

55. What will 675 pounds of cheese cost, at 13 cents a pound? *Ans.* 8775 cents.

56. What will 496 bushels of potatoes cost, at 24 cents a bushel? *Ans.* 11904 cents.

57. If a man travel 28 miles a day, how many miles will he travel in 152 days? *Ans.* 4256 miles.

58. There are 1760 yards in one mile; how many yards are there in 209 miles? *Ans.* 367840 yds.

59. There are 24 hours in a day, and 365 days in a year: if a ship sail 8 miles an hour, how far will she sail in a year? *Ans.* 70080 miles.

60. Sound moves 1130 feet in a second: how far will it move in 109 seconds? *Ans.* 123170 feet.

61. Multiply two thousand and twenty-nine by one thousand and seven. *Ans.* 2043203.

62. Multiply eighty thousand four hundred and one by sixty thousand and seven. *Ans.* 4824622807.

63. Multiply one hundred and one thousand and thirty-two by 20001. *Ans.* 2020741032.

CONTRACTIONS IN MULTIPLICATION.

CASE I.

ART. **33**. *When the Multiplier is a Composite Number.*

Any number produced by multiplying together two or more numbers, is termed a *composite* number; and,

The numbers which, multiplied together, produce a number, are its *component parts*, or factors.

Thus, 21 is a composite number: the factors are 7 and 3, because 7 multiplied by 3 produces 21. So, also, 12 is a composite number; $6\times2=12$; or, $3\times4=12$; or, $2\times2\times3=12$.

1. What will 15 oranges cost, at 8 cents each?

Since $5\times3=15$, it follows that 15 is a composite number, of which the factors are 5 and 3.

Cost of 1 orange,	8	cents.
	5	
Cost of 5 oranges,	40	cents.
	3	
Cost of 15 oranges,	120	cents.

ANALYSIS.—Since 15 are 3 times 5, 15 oranges will cost 3 times as much as 5 oranges. Therefore, instead of multiplying 8 by 15, first find the cost of 5 oranges, by multiplying 8 cents by 5; then take 3 times that product for the cost of 15 oranges.

PROOF.—$8\times15=120$.

Rule for Case I.—*Separate the multiplier into two or more factors.* 2. *Multiply the multiplicand by one of the factors, and this product by another factor, till every factor is used; the last product will be the one required.*

REM.—Do not confound the *factors* of a number with the *parts* into which it may be *separated.* Thus, the factors of 15 are 5 and 3, while the parts into which 15 may be separated, are any numbers whose *sum* equals 15; as, 14 and 1; or, 2, 9, and 4.

EXAMPLES FOR PRACTICE.

2. What will 24 acres of land cost, at $124 an acre? *Ans.* $2976.

3. How far will a ship sail in 56 weeks, at the rate of 2395 miles per week? *Ans.* 134120 miles.

4. How many pounds of iron are there in 54 loads, each weighing 2873 pounds? *Ans.* 155142 pounds.

5. How many gallons of wine in 63 vats, each containing 1673 gallons? *Ans.* 105399 galls.

6. Multiply 2874 by 72. *Ans.* 206928.

7. Multiply 8074 by 108. *Ans.* 871992.

CASE II.

ART. **34**. *When the Multiplier is* 1 *with ciphers annexed to it; as,* 10, 100, 1000, *&c.*

Placing *one* cipher on the right of a number, (Art. 8), changes the units into tens, the tens into hundreds, and so on, and, therefore, *multiplies the number by* 10.

Annexing *two* ciphers, changes units into hundreds, tens into thousands, &c., and *multiplies the number by* 100.

Annex one cipher to the right of 25, and it becomes 250, the product arising from multiplying it by 10.

Annex two ciphers to 25, and it becomes 2500, the product of 25×100. Hence the

Rule for Case II.—*Annex as many ciphers to the Multiplicand as there are ciphers in the multiplier, and the number thus formed will be the product required.*

1. Multiply 245 by	100.	*Ans.* 24500.
2. Multiply 138 by	1000.	*Ans.* 138000.
3. Multiply 428 by	10000.	*Ans.* 4280000.
4. Multiply 872 by	100000.	*Ans.* 87200000.

CASE III.

ART. **35**. *When there are ciphers at the right hand of one or both of the factors.*

Find the product of 2300×170.

```
 2300
 170
------
 161
 23
------
391000
```

ANALYSIS.—The number 2300 may be regarded as a composite number, of which the factors are 23 and 100; and 170 as a composite number, of which the factors are 17 and 10.

By Art. 33, the product of 2300 by 170 will be found by multiplying 23 by 17, and this product by 100, and the resulting product by 10; that is, by finding the product of 23 multiplied by 17, and then annexing to the product 3 ciphers, as there are 3 ciphers at the right of both factors. Hence the

Rule for Case III.—*Multiply without regarding the ciphers on the right of the factors; then annex to the product as many ciphers as are at the right of both factors.*

1. Multiply	2350	by	60.	*Ans.*	141000.
2. Multiply	80300	by	450.	*Ans.*	36135000.
3. Multiply	10240	by	3200.	*Ans.*	32768000.
4. Multiply	9600	by	2400.	*Ans.*	23040000.
5. Multiply	18001	by	26000.	*Ans.*	468026000.
6. Multiply	8602	by	1030.	*Ans.*	8860060.
7. Multiply	3007	by	9100.	*Ans.*	27363700.
8. Multiply	80600	by	7002.	*Ans.*	564361200.
9. Multiply	70302	by	80300.	*Ans.*	5645250600.
10. Multiply	904000	by	10200.	*Ans.*	9220800000.

☞ For additional problems, see Ray's Test Examples.

V. DIVISION.

ART. **36**. 1. If you divide 6 apples equally between 2 boys, how many will each have?

ANALYSIS.—*It will require two apples to give each boy* 1; *hence, each boy will have as many apples as there are times* 2 *apples in* 6 *apples, that is* 3.

How many times 2 in 6?—*Ans.* 3. Why? *Because* 3 *times* 2 *are* 6.

2. If you divide 8 peaches equally between 2 boys, how many will each have? *Ans.* 4. Why?

3. How many times 2 in 10? Why?

REVIEW.—33. What is a composite number? What are its component parts or factors? Give an example. How multiply by a composite number, Rule? Illustrate this method. REM. In what respect do the *factors* of a number differ from its *parts?* Give an example.

34. If one cipher is placed on the right of a number, how are the orders changed? If two ciphers? How multiply by 10, 100, 1000, &c.?

DEFINITIONS.—The process by which the preceding examples are solved, is called Division; hence,

Division is the process of finding how many times one number is contained in another.

The number by which to divide, is the *divisor.*

The number to be divided, is the *dividend.*

The number denoting *how many times* the divisor is contained in the dividend, is the *quotient.*

ART. **37**. How many times 3 in 12? *Ans.* 4 times.

Here, 3 is the divisor, 12 the dividend, and 4 the quotient.

Since 3 is contained in 12 four times, 4 times 3 are 12; that is, the divisor and quotient multiplied, produce the dividend.

Hence, since 3 and 4 are factors of the product 12, the divisor and quotient correspond to the factors in Multiplication; the dividend, to the product.

		Factors.		Product.
BY MULTIPLICATION, . . .		3×4	=	12
	Dividend.	Divisor.		Quotient.
BY DIVISION,	12	divided by 3	=	4
Or	12	divided by 4	=	3

Hence, Division is *the process of finding one of the factors of a product, when the other factor is known.*

ART. **38**. If 7 cents be divided equally among 3 boys, each boy would receive 2 cents, and there would be 1 cent left, or *remaining* undivided.

The number left after dividing, is called the *remainder.*

REMARKS.—1. Since the remainder is a part of the dividend, it must be of the same denomination. If the dividend be dollars, the remainder will be dollars: if pounds, the remainder will be pounds.

2. The remainder is always *less* than the divisor; for, if it were equal to, or greater than it, the divisor would be contained at least once more in the dividend.

3. If the dividend denotes things of one denomination only, the operation is called *Simple Division.*

REVIEW.—36. What is Division? What is the number by which to divide? What the number to be divided? What the number denoting how many times the divisor is contained in the dividend?

ART. **39**. A boy has 8 cents: how many lemons can he buy at 2 cents each?

ANALYSIS.—*He can buy* 4, *because* 4 *lemons at* 2 *cents each, will cost* 8 *cents. If he did not know that* 4 *times* 2 *are* 8, *the operation would be thus:*

The boy would give 2 cents for 1 lemon, and then have 6 cents left.

After giving 2 cents for the 2d lemon, he would have 4 cents left;

Then giving 2 cents for the third, he would have 2 cents left;

Lastly, after giving two cents for the fourth, he would have nothing left: having taken 2 cents 4 *times* from 8 cents, and each time received one lemon.

	8	cents.
1st lemon	2	cents.
Left, ...	6	cents.
2d lemon	2	cents.
Left, ...	4	cents.
3d lemon	2	cents.
Left, ...	2	cents.
4th lemon	2	cents.
Left, ...	0	cents.

The natural method of performing this operation is by Subtraction; but,

When it is known *how many times* 2 can be subtracted from 8, instead of subtracting 2 four times, say, 2 in 8 four times, and 4 times 2 are 8; which, subtracted from 8 *once*, nothing is left.

The last method is by Division, and it differs from the first in this: that the subtractions, instead of being performed separately, are all made at *once*.

Hence, Division may be termed *a short method of making many subtractions of the same number.*

The divisor is the number subtracted; the dividend

REVIEW.—37. Three in 12, 4 times; what is 3 called? 12? 4? To what is the product of the divisor and quotient equal? To what do they correspond? To what does the dividend correspond? What is division?

38. When there is a number left after dividing, what is it called? REM. Of what denomination is the remainder? Why? Why is the remainder always *less* than the divisor? When the dividend denotes things of one denomination only, what is the operation called?

39. To what natural method do the operations in division belong? Illustrate by an example. What may division be termed? What is the divisor? Dividend? Quotient?

the number from which the subtractions are made; the quotient shows how many subtractions *have been* made.

Art. 40. Division is denoted by three Signs:

1st. 3)12 means that 12 is to be divided by 3.

2d. $\frac{12}{3}$ means that 12 is to be divided by 3.

3d. 12÷3 means that 12 is to be divided by 3.

Use the 1st sign when the divisor does not exceed 12; draw a line under the dividend, and write the quotient beneath.

If the divisor exceeds 12, draw a curved line on the right of the dividend: place the quotient on the right of this.

The sign, ÷, in the Table, is read *divided by.*

EXAMPLES.

2)8	15)45(3	15	
4	45	$\frac{15}{5}=3$	21÷3=7.

DIVISION TABLE.

1÷1= 1	2÷2= 1	3÷3= 1	4÷4= 1
2÷1= 2	4÷2= 2	6÷3= 2	8÷4= 2
3÷1= 3	6÷2= 3	9÷3= 3	12÷4= 3
4÷1= 4	8÷2= 4	12÷3= 4	16÷4= 4
5÷1= 5	10÷2= 5	15÷3= 5	20÷4= 5
6÷1= 6	12÷2= 6	18÷3= 6	24÷4= 6
7÷1= 7	14÷2= 7	21÷3= 7	28÷4= 7
8÷1= 8	16÷2= 8	24÷3= 8	32÷4= 8
9÷1= 9	18÷2= 9	27÷3= 9	36÷4= 9
10÷1=10	20÷2=10	30÷3=10	40÷4=10
5÷5= 1	6÷6= 1	7÷7= 1	8÷8= 1
10÷5= 2	12÷6= 2	14÷7= 2	16÷8= 2
15÷5= 3	18÷6= 3	21÷7= 3	24÷8= 3
20÷5= 4	24÷6= 4	28÷7= 4	32÷8= 4
25÷5= 5	30÷6= 5	35÷7= 5	40÷8= 5
30÷5= 6	36÷6= 6	42÷7= 6	48÷8= 6
35÷5= 7	42÷6= 7	49÷7= 7	56÷8= 7
40÷5= 8	48÷6= 8	56÷7= 8	64÷8= 8
45÷5= 9	54÷6= 9	63÷7= 9	72÷8= 9
50÷5=10	60÷6=10	70÷7=10	80÷8=10

9÷9= 1	10÷10= 1	11÷11= 1	12÷12= 1
18÷9= 2	20÷10= 2	22÷11= 2	24÷12= 2
27÷9= 3	30÷10= 3	33÷11= 3	36÷12= 3
36÷9= 4	40÷10= 4	44÷11= 4	48÷12= 4
45÷9= 5	50÷10= 5	55÷11= 5	60÷12= 5
54÷9= 6	60÷10= 6	66÷11= 6	72÷12= 6
63÷9= 7	70÷10= 7	77÷11= 7	84÷12= 7
72÷9= 8	80÷10= 8	88÷11= 8	96÷12= 8
81÷9= 9	90÷10= 9	99÷11= 9	108÷12= 9
90÷9=10	100÷10=10	110÷11=10	120÷12=10

PRINCIPLES AND EXAMPLES.

ART **41**. 1. I wish to put 15 hats into boxes, each containing 3 hats: how many boxes do I need?

1ST SOLUTION.—I need as many boxes as 3 hats are contained *times* in 15 hats; that is, 5 boxes.

```
Hats. Hats.
 3) 15 (5, boxes.
    15
    --
```

2. Having 15 hats, I wish to separate them into 5 equal lots: how many hats will there be in each lot?

2D SOLUTION.—Putting *one* hat into each lot will require 5 hats; hence, there will be *as many* hats in each lot, as there are *times* 5 hats in 15.

```
Hats. Hats.
 5) 15 (3, hats in each.
    15
    --
```

The first solution shows, that by Division a number can be separated into parts containing a *certain number* of *units*, and the *number* of parts found.

The second solution shows, that by Division a given number can be separated into a certain number of *equal* parts, and the *number of units* in each part found.

REVIEW.—40. How many signs are used to denote division? What is the first? Second? Third? Illustrate their meaning.

41. What does the first solution show? What the second? REM. How does it appear that the divisor and dividend are both of the same denomination? Is the quotient an abstract or a concrete number? What does it show? What may it represent?

MENTAL EXERCISES.

Solve the examples on the *left* like the first, and those on the *right*, like the second of the preceding SOLUTIONS.

3. How many lemons at 3 cents each, can you buy for 12 cents?

4. If you pay 12 cents for 4 lemons, how much will each cost?

5. How many barrels of flour, at 4 dollars a barrel, can you purchase for $20?

6. If you pay $20 for 5 barrels of flour, how many dollars will a barrel cost?

7. At six dollars a yard, how many yards of cloth can you purchase for $30?

8. If you pay $30 for 5 yards of cloth, how many dollars will a yard cost?

9. How many lead pencils, at 5 cents apiece, can you buy for 35 cents?

10. If you pay 35 cents for 7 lead pencils, how much will that be for each?

11. A man has 63 pounds of butter, and wishes to put it into boxes, each containing 7 pounds: how many boxes will be required?

12. A man has 63 pounds of butter, to put into 9 boxes: how many pounds must he put into each box?

13. Into how many parts, of 3 each, can 24 be separated?

14. If 24 is separated into 8 equal parts, how many will there be in each part?

REM.—1. The divisor and dividend are both of the same denomination. This follows from that view of division, which shows it to be a short method of making several subtractions of the same number.

Since it is only numbers of the same denomination whose difference can be found, those only of the same denomination can be divided.

2. The quotient is an *abstract* number, and shows *how many times* the divisor is contained in the divided. But, it may represent the *number* of units in some concrete number, as in examples 1 and 2, Art. 41.

ART. 42. EXAMPLES.

1. Two in 6 how many times? *Ans.* 3 times.

2. How many times is 2 contained in 60?

ANALYSIS.—Two in 6 units of the *first* order, 3 times: in 6 units of the *second* order, (10 times as large,) it is contained 10 times as many times.

$$2\overline{)60}$$
$$\quad 3$$

3. How many times is 2 contained in 600?

ANALYSIS.—Two is contained 3 times in 6 units of the first order; but, in 6 units of the third order, which are 100 times as large, it is contained 100 times as often; that is, 300 times.

$$2\overline{)600}$$
$$300$$

4. Three feet make one yard: how many yards in 60 feet? *Ans.* 20.

5. Two pints make one quart: how many quarts in 400 pints? *Ans.* 200.

6. How many times 3 in 6000? *Ans.* 2000.

7. How many times 4 in 80000? *Ans.* 20000.

ART. **43**. 1. How many times is 2 contained in 468?

Here, the dividend is composed of 3 numbers; 4 hundreds, 6 tens, and 8 units; that is, of 400, 60, and 8.

	Divisor.		Dividend.		Quotient.	
Now,	2	in	400	is contained	200	times.
	2	in	60	.. ..	30	times.
	2	in	8	.. ..	4	times.
Hence,	2	in	468	is contained	234	times.

The same result can be obtained without actually separating the dividend into parts:

Thus, 2 in 4 (hundreds) 2 (hundred) times, which write in hundreds' place; then, 2 in 6 (tens), 3 (tens) times, which write in tens' place; then, 2 in 8 (units), 4 (units) times, which write in units' place.

Dividend.
Divisor 2)468
Quotient 234

2. How many times 3 in 693? *Ans.* 231.

3. How many times 4 in 848? *Ans.* 212.

4. How many times 2 in 4682? *Ans.* 2341.

5. How many times 4 in 8408? *Ans.* 2102.

6. How many times 3 in 36936? *Ans.* 12312.

7. How many times 2 in 88468? *Ans.* 44234.

REVIEW.—43. Explain how many times 2 is contained in 468.

Art. 44. SHORT DIVISION.

1. How many times is 3 contained in 129?

Solution.—Here, 3 is not contained in 1; but consider the 1 and 2 as forming 12 tens, then 3 is in 12 tens, 4 (tens) times, which write in tens' place; 3 in 9 (units), 3 times, which write in units' place.

$$\begin{array}{r} 3\,)\,129 \\ \hline 43 \end{array}$$

2. How many times is 3 contained in 735?

Solution.—Here, 3 is contained in 7 (hundreds), 2 (hundred) times, and 1 (hundred) over; the 1 hundred united with the 3 tens, makes 13 tens, in which 3 is contained 4 (tens) times and 1 (ten) left; this 1 ten united with the 5 units, makes 15 units, in which 3 is contained 5 times.

$$\begin{array}{r} 3\,)\,735 \\ \hline 245 \end{array}$$

OF REMAINDERS.

3. How many times is 3 contained in 743?

After dividing, there is 2 left, which ought to be divided by the divisor 3:

$$\begin{array}{l} 3\,)\,743 \\ \hline 247\ldots2 \text{ Rem.} \end{array}$$

But the method of doing this will not be explained until the pupil has studied Fractions.

The division is merely indicated by placing the divisor under the remainder, thus, $\frac{2}{3}$.

The quotient is written thus, $247\frac{2}{3}$; read, 247, and *two divided by three;* or, 247, with a *remainder, two.*

Instead of performing all the operation mentally, the work may be written, as in the following solution:

Solution.—In this operation, say 3 in 7 (hundreds), 2 (hundred) times; then multiply 3 by 2 (hundreds), and subtract the product, 6 (hundreds), from 7 (hundreds), which leaves a remainder, 1 (hundred); to this remainder unite the 4 (tens), making 14 (tens), which contains 3, 4 (tens) times, with a remainder 2 (tens).

To this rem. unite the 3 units, and 3 in 23 (units), 7 times, with a remainder 2.

OPERATION.

$$\begin{array}{l} 3)743(247\tfrac{2}{3}. \\ \;\;\;6 \\ \hline \;\;\;14 \text{ tens.} \\ \;\;\;12 \\ \hline \;\;\;\;\;23 \text{ units.} \\ \;\;\;\;\;21 \\ \hline \;\;\;\;\;\;\;2 \text{ Rem.} \end{array}$$

DEFINITIONS.—When the division is performed mentally, and merely the result written, it is termed *Short Division;* when the entire work is written, *Long Division.* Short Division is used when the divisor does not exceed 12.

4. How many times 3 in 462? . . . *Ans.* 154.
5. How many times 5 in 1170? . . . *Ans.* 234.
6. How many times 4 in 948? . . . *Ans.* 237.

ART. 45. RULE

For Short Division.—1. *Write the divisor at the left of the dividend, with a curved line between them. Begin at the left hand, divide successively each figure of the dividend by the divisor, and write the result in the same order in the quotient.*

2. *If there is a remainder after dividing any figure, prefix it to the figure in the next lower order, and divide as before.*

3. *If the number in any order does not contain the divisor, place a cipher in the same order in the quotient, prefix the number to the figure in the next lower order, and divide as before.*

4. *If there is a remainder after dividing the last figure, place the divisor under it, and annex it to the quotient.*

EXPLANATION.—To *Prefix,* means to place *before,* or at the *left* hand. To *Annex,* is to place *after,* or at the *right* hand.

PROOF.—Multiply the quotient by the divisor, add the remainder, if any, to the product: if the work is correct, the sum will be equal to the dividend.

REM.—This method of proof depends on the principle, Art. 37, that a dividend is a product, of which the divisor and quotient are factors.

If the remainder be subtracted from the dividend, and the result divided by the quotient, the quotient thus obtained will be the divisor.

REVIEW.—44. How many times 3 in 129? Explain it. How is a remainder to be written? When is the operation termed short division? When long division? When is short division used? When long? What is the difference between long and short division?

45. In dividing, how are the numbers written, Rule? Where do you begin to divide? Where is each quotient figure placed?

7. Divide 653 cents by 3.

	Dividend.	
Divisor . .	3)653	
Quotient. . .	217	Rem. 2.

PROOF.

217	
3	
651	= cents divided.
2	= remainder.
653	= the dividend.

	(8)	(9)	(10)	(11)
	6)454212	7)874293	9)2645402	8)3756031
Ans.	75702	124899	$293933\frac{5}{9}$	$469503\frac{7}{8}$
	6	7	9	8
Proof.	454212	874293	2645402	3756031

OF PARTS OF NUMBERS.

NOTE.—When any number is divided into two equal parts, one of the parts is called *one-half* of that number.

If divided into three equal parts, one of the parts is called *one-third.* If into four equal parts, *one-fourth.* If into five equal parts, *one-fifth;* and so on. Hence,

To find *one-half* of a number, divide by 2; to find *one-third,* divide by 3; *one-fourth,* divide by 4; *one-fifth,* by 5, &c.

12. Divide 8652 by 2. . . . *Ans.* 4326.
13. Divide 406235 by 3. . . . *Ans.* $135411\frac{2}{3}$.
14. Divide 675043 by 4. . . . *Ans.* $168760\frac{3}{4}$.
15. Divide 984275 by 5. . . . *Ans.* 196855.
16. Divide 258703 by 6. . . . *Ans.* $43117\frac{1}{6}$.
17. Divide 523408 by 6. . . . *Ans.* $87234\frac{4}{6}$.
18. Divide 8643275 by 7. . . . *Ans.* $1234753\frac{4}{7}$.

REVIEW.—45. How do you proceed when there is a remainder, after dividing any order? When the number in any order does not contain the divisor? When there is a remainder after dividing the last figure? What is the method of proof? REM. Upon what principle does the proof depend? What other method of proof?

45. NOTE. When you divide a number into two equal parts, what is one part called? If into three equal parts, what is one part called? If into four equal parts? If into five equal parts? How do you find one-half of a number? One-third? One-fourth? One-fifth? When a number is divided by 7, (example 18,) what part of the number is found?

19. Divide 6032520 by 8. *Ans.* 754065.
20. Divide 9032706 by 9. *Ans.* 1003634.
21. Divide 1830024 by 10. *Ans.* $183002\frac{4}{10}$.
22. Divide 603251 by 11. *Ans.* 54841.
23. Divide 41674008 by 12. *Ans.* 3472834.

24. If oranges cost 3 cents each, how many can be bought for 894 cents? *Ans.* 298 oranges.

25. If 4 bushels of apples cost 140 cents, how much is that a bushel? *Ans.* 35 cts.

26. If flour cost $4 a barrel, how many barrels can be bought for $812? *Ans.* 203 barl.

27. A carpenter receives $423 for 9 months' work: how much is that per month? *Ans.* $47.

28. There are 12 months in 1 year: how many years are there in 540 months? *Ans.* 45 yrs.

29. There are 4 quarts in 1 gallon: how many gallons are there in 321276 quarts? *Ans.* 80319 gals.

30. At $8 a barrel, how many barrels of flour can be bought for $1736? *Ans.* 217 barl.

31. There are 7 days in one week: how many weeks are there in 734566 days? *Ans.* 104938 wks.

32. A number has been multiplied by 11, and the product is 495: what is the number? *Ans.* 45.

33. The product of two numbers is 3582: one of the numbers is 9: what is the other? *Ans.* 398.

34. Find one-half of 56. . . *Ans.* 28.
35. Find one-half of 3725. . . *Ans.* $1862\frac{1}{2}$.
36. Find one-third of 147. . . *Ans.* 49.
37. Find one-fourth of 500. . . *Ans.* 125.
38. Find one-fifth of 1945. . . *Ans.* 389.
39. Find one-sixth of 4476. . . *Ans.* 746.
40. Find one-seventh of 2513. . . *Ans.* 359.
41. Find one-eighth of 5992. . . *Ans.* 749.
42. Find one-ninth of 8793. . . *Ans.* 977.
43. Find one-tenth of 1090. . . *Ans.* 109.

44. Find one-eleventh of 4125. . . . *Ans.* 375.

45. Find one-twelfth of 5556. . . . *Ans.* 463.

46. I divided 144 apples equally among 4 boys; the eldest boy gave one-third of his share to his sister: what number did the sister receive? *Ans.* 12.

47. James found 195 cents, and gave to Daniel one-fifth of them: Daniel gave one-third of his share to his sister: how many cents did she receive? *Ans.* 13.

ART. 46. LONG DIVISION.

1. Divide 3465 dollars equally among 15 men.

SOLUTION.—When the divisor exceeds 12, it becomes necessary to write the process, as in "OPERATION," Example 3, page 50.

Fifteen is not contained in 3 (thousands), therefore, there will be no thousands in the quotient. Take 34 (hundreds) as a *partial* dividend; 15 is contained in it 2 (hundred) times; that is, 15 men have each $200, which requires in all 15×2 (hundred) = 30 hundreds.

```
     thous. hund. tens. units.   hund. tens. units.
15)3465(231
   30 hund.
   ---
    46 tens.
    45
    ---
     15 units.
     15
     ---
```

Subtract 30 (hundreds) from 34 (hundreds) and 4 (hundreds) remain, to which bring down the 6 (tens), and you have 46 (tens) for a second partial dividend.

46 contains 15, 3 (tens) times, giving each man 3 ten (30) dollars more, requiring for all, 15×3 (tens) = 45 tens of dollars.

Subtract 45 and bring down the 5 (units), you have 15 (units) for a 3d partial dividend, in which the divisor, 15, is contained once, giving to each man 1 (unit) dollar.

Hence, each man receives 2 hundred dollars, 3 ten dollars, and 1 dollar; that is, 231 dollars.

By this process, the dividend is separated into parts, each containing the divisor a certain number of times.

Divisor.	Parts.	Quotients.
15	3000	200
	450	30
	15	1
	3465	231

The first part, 30 (hundreds), contains the divisor 2 (hundred) times; the second part, 45 (tens), contains it 3 (ten) times; the third part, 15 (units), contains it 1 (unit) time.

The several parts together, equal the given dividend: the several partial quotients make up the entire quotient.

2. In 147095 days, how many years, each 365 days?

SOLUTION.—Taking 147 (thousand) for the first partial dividend, we find it will not contain the divisor; hence use four figures.

```
365)147095(403 years.
    1460
    ----
      1095
      1095
      ----
```

Again, after multiplying and subtracting, as in the preceding example, and bringing down the 9 (tens), the partial dividend, 109 (tens), will not contain the divisor; hence, write a 0 (no tens) in the quotient, and bring down the 5 (units): the last partial dividend is 1095 (units). which contains the divisor 3 (units) times.

3. Divide 4056 by 13. . . . . *Ans.* 312.

RULE

For Long Division.—1. *Place the divisor on the left of the dividend, draw a curved line between them, and another on the right of the dividend.*

2. *Find how many times the divisor is contained in the fewest left hand figures of the dividend that will contain the divisor, and place this number in the quotient at the right.*

3. *Multiply the divisor by this quotient figure; place the product under that part of the dividend from which it was obtained.*

4. *Subtract this product from the figures above it; to the remainder bring down the next figure of the dividend, and divide as before, until all the figures of the dividend are brought down.*

5. *If, at any time, after bringing down a figure, the number thus formed is too small to contain the divisor, place a cipher in the quotient, and bring down another figure, after which divide as before.*———PROOF. Same as in Short Division.

REVIEW.—46. Repeat the Rule for Long Division. Where is the divisor placed? Where the quotient? How commence dividing?

46. After obtaining the first quotient figure, how proceed? After bringing down a figure, if the partial dividend thus formed is too small to contain the divisor, what is to be done? What the method of proof?

NOTES.—1. The product must never be *greater* than the partial dividend from which it is to be subtracted. When so, the quotient figure is *too large*, and must be diminished.

2. After subtracting, the remainder must always be *less* than the divisor. When the remainder is not less than the divisor, the last quotient figure is *too small*, and must be increased.

3. Example 1 shows that *the order of each quotient figure is the same as the lowest order in the partial dividend from which it was obtained.* It will be a useful exercise to name the order of each quotient figure immediately after obtaining it.

☞ For casting out the 9's, see "*Ray's Higher Arithmetic.*"

4. Divide 78994 by 319.

```
                                 PROOF.  247 Quotient.
319)78994(247                            319 Divisor.
    638                                 ----
    ----                                2223
    1519                                247
    1276                               741
    -----                              -----
     2434                              78793
     2233                        Add     201 Remainder.
     -----                             -----
      201 Rem.                         78994 = the Dividend.
```

5. Divide	11577	by 14.	. . . *Ans.*	$826\frac{13}{14}$.
6. Divide	48690	by 15.	. . . *Ans.*	3246.
7. Divide	1110960	by 23.	. . . *Ans.*	$48302\frac{14}{23}$.
8. Divide	122878	by 67.	. . . *Ans.*	1834.
9. Divide	12412	by 53.	. . . *Ans.*	$234\frac{10}{53}$.
10. Divide	146304	by 72.	. . . *Ans.*	2032.
11. Divide	47100	by 54.	. . . *Ans.*	$872\frac{12}{54}$.
12. Divide	71104	by 88.	. . . *Ans.*	808.
13. Divide	43956	by 66.	. . . *Ans.*	666.
14. Divide	121900	by 99.	. . . *Ans.*	$1231\frac{31}{99}$.
15. Divide	25312	by 112.	. . . *Ans.*	226.
16. Divide	381600	by 123.	. . . *Ans.*	$3102\frac{54}{123}$.
17. Divide	105672	by 204.	. . . *Ans.*	518.
18. Divide	600000	by 1234.	. . . *Ans.*	$486\frac{276}{1234}$.

19. Divide 47263488 by 4674. *Ans.* 10112.
20. Divide 26497935 by 2034. *Ans.* $13027\frac{1017}{2034}$.
21. Divide 48905952 by 9876. *Ans.* 4952.
22. Divide 4049160 by 12345. *Ans.* 328.
23. Divide 552160000 by 973. *Ans.* $567482\frac{14}{973}$.

24. At $15 an acre, how many acres of land can be bought for $3465? *Ans.* 231 acres.

25. If a man travel 26 miles a day, in how many days will he travel 364 miles? *Ans.* 14 days.

26. If $1083 be divided equally among 19 men, how many dollars will each have? *Ans.* $57.

27. A man raised 9523 bushels of corn on 107 acres: how much was that on one acre? *Ans.* 89 bush.

28. In 1 hogshead there are 63 gallons: how many hogsheads in 14868 gallons? *Ans.* 236 hds.

29. A President receives $25000 a year (365 days): how much is that a day? *Ans.* $68 a day, and $180 over.

30. The yearly income from a railroad is $37960: how much is that per day? (365 days=1 year.) *Ans.* $104.

31. The product of two numbers is 6571435; one of the factors is 1235: what is the other? *Ans.* 5321.

32. Divide one million two hundred and forty-seven thousand and four hundred by 405. *Ans.* 3080.

33. Divide 10 million four hundred and one thousand by one thousand and six. *Ans.* $10338\frac{972}{1006}$.

CONTRACTIONS IN DIVISION.

CASE I.

ART. **47**. *When the Divisor is a Composite Number.*

1. A man paid $255 for 15 acres of land: how much was that per acre?

ANALYSIS.—By taking *one-third* of $255, you obtain the value of *one-third*, (5 acres,) of the land; dividing this quotient (85) by 5, gives the value of 1 acre.

Dollars.
3)255 = the value of 15 acres.
5)85 = the value of 5 acres.
17 = the value of 1 acre.

The preceding analysis shows that instead of dividing by the composite number 15, whose factors are 3 and 5, we may first divide by one factor, then divide the quotient thus obtained by the other factor.

2. Find the quotient of 37, divided by 14.

SOLUTION.—Dividing by 2, the quotient is 18 *twos* and 1 unit remaining. Dividing by 7, the quotient is 2, with a remainder of 4 *twos*; the whole remainder then, is 4 *twos* plus 1, or 9.

```
2)37
  ———
7)18 and 1 over.
  ———
   2 and 4 twos left.
```

Rule for Case I.—*Divide the dividend by one of the factors of the divisor; then divide the quotient thus obtained by the other factor.*

2. *Multiply the last remainder by the first divisor; to the product add the first remainder; the amount will be the true remainder.*

NOTE.—When the divisor can be resolved into more than two factors, you may divide by them successively. The true remainder will be found by multiplying each remainder by all the preceding divisors, except that which produced it. To their sum add the remainder from first divisor.

3. Divide 2583 by 63. $(63=7\times9)$ *Ans.* 41.
4. Divide 6976 by 32. $(32=4\times8)$ *Ans.* 218.
5. Divide 2744 by 28. $(28=7\times4)$ *Ans.* 98.
6. Divide 6145 by 42. $(42=6\times7)$ *Ans.* $146\frac{13}{42}$.
7. Divide 19008 by 132. . . . . *Ans.* 144.
8. Divide 7840 by 64. . . . . *Ans.* $122\frac{32}{64}$.
9. Divide 14771 by 72. . . . . *Ans.* $205\frac{11}{72}$.
10. Divide 10206 by 81. . . . . *Ans.* 126.

CASE II.

ART. **48**. *To divide by* 1 *with ciphers annexed; as* 10, 100, 1000, *&c.*

REVIEW.—46. NOTE. When any product is greater than the partial dividend from which it is to be subtracted, what must be done?

47. How may division be performed, when the divisor is a composite number? How is the true remainder found? NOTE. When the divisor can be resolved into more than two factors, how may the division be performed? How is the true remainder obtained?

To multiply 6 by 10, annex one cipher, thus, 60. On the principle that division is the reverse of multiplication, to divide 60 by 10, *cut off* a cipher.

Had the dividend been 65, the 5 might have been separated in like manner as the cipher; 6 being the quotient, 5 the remainder. The same will apply when the divisor is 100, 1000, &c.

Rule for Case II.—*Cut off as many figures from the right of the dividend as there are ciphers in the divisor; the figures cut off will be the remainder, the other figures, the quotient.*

1. Divide 34872 by 100.

OPERATION. 1|00)348|72

348 Quo. 72 Rem.

2. Divide 2682 by 10. . . . *Ans.* $268\frac{2}{10}$.
3. Divide 4700 by 100. . . . . *Ans.* 47.
4. Divide 37201 by 100. . . . *Ans.* $372\frac{1}{100}$.
5. Divide 46250 by 100. . . . *Ans.* $462\frac{50}{100}$.
6. Divide 62034 by 100. . . . *Ans.* $620\frac{34}{100}$.
7. Divide 18003 by 1000. . . . *Ans.* $18\frac{3}{1000}$.
8. Divide 375000 by 1000. . . . . *Ans.* 375.

CASE III.

ART. **49**. *To divide, when there are ciphers on the right of the divisor.*

1. Divide 4072 by 800.

OPERATION.

1|00)40|72

8)40

5 Quo 72 Rem.

SOLUTION.—Regard 800 as a composite number, the factors 100 and 8, and divide as in the margin.

In dividing by 800, separate the two right hand figures for the remainder, then divide by 8.

8|00)40|72

5 Quo. 72 Rem.

REVIEW.—48. How do you divide by 10, 100, 1000, &c.? On what principle does the rule for case II depend? 49. How do you divide when there are ciphers on the right of divisor, Rule for case III?

2. Divide 77939 by 2400.

SOLUTION.—Since 2400 equals 24×100, cut off the two right hand figures, the same as dividing by 100; then divide by 24.

```
24|00)779|39(32 1139/2400
       72
       --
        59
        48
        --
        11
```

Dividing by 100, the remainder is 39; dividing by 24, the remainder is 11. To find the true remainder, multiply 11 by 100, and add 39 to the product, (Art. 47, Rule); this is the same as annexing the figures cut off, to the last remainder. Hence, the

Rule for Case III.—1. *Cut off the ciphers at the right of the divisor, and as many figures from the right of the dividend.*

2. *Divide the remaining figures in the dividend by the remaining figures in the divisor.*

3. *Annex the figures cut off to the remainder, which gives the true remainder.*

3. Divide 73005 by 4000. . . . *Ans.* $18\frac{1005}{4000}$.
4. Divide 36001 by 9000. . . . *Ans.* $4\frac{1}{9000}$.
5. Divide 1078000 by 11000. . . . . *Ans.* 98.
6. Divide 40167 by 180. . . . *Ans.* $223\frac{27}{180}$.
7. Divide 907237 by 2100. . . . *Ans.* $432\frac{37}{2100}$.
8. Divide 364006 by 6400. . . . *Ans.* $56\frac{5606}{6400}$.
9. Divide 76546037 by 250000. . *Ans.* $306\frac{46037}{250000}$.
10. Divide 43563754 by 63400. . . *Ans.* $687\frac{7954}{63400}$.

Exercises in more difficult contractions are in "*Ray's Higher Arithmetic.*"

PROOF OF MULTIPLICATION BY DIVISION.

ART. **50**. Division (Art. 37), is a process for finding one of the factors of a product when the other factor is known: therefore,

If the product of two numbers be divided by the multiplier, the quotient will be the multiplicand: Or, if divided by the multiplicand, the quotient will be the multiplier.

REVIEW.—50. What is division? If the product of two factors be divided by either of them, what will be the quotient?

1. What number multiplied by 7895, will give 434225 for a product? *Ans.* 55.

2. If 327 be multiplied by itself, the product will be 106929. Give the proof.

3. The product is 10741125; the multiplier 375: what is the multiplicand? *Ans.* 28643.

4. The product is 63550656, and the multiplicand 60352: what is the multiplier? *Ans.* 1053.

☞ For additional problems, see Ray's Test Examples.

REVIEW OF PRINCIPLES.

ART. **51**. NOTATION and NUMERATION show how to express numbers by *words*, by *figures*, or by *letters*.

For other *scales of notation* than the *decimal* or *tens' scale*, an interesting subject for advanced students, see "*Ray's Higher Arithmetic.*"

ART. 52 BY ADDITION,

The *aggregate* or *sum* of two or more numbers is found, (Art. 18). Thus, when the separate cost of several things is given, the *entire* cost is found by addition.

EXAMPLE.—A bag of coffee cost $23, a chest of tea $38, a box of sugar $11: what did all cost? *Ans.* $72.

ART. 53. BY SUBTRACTION,

The *difference* between two numbers is found. Thus, *if the sum of two numbers be diminished by either of them, the remainder will be the other.*

Hence, by Addition, *if the difference of two numbers be added to the less, the* SUM *will be the greater.*

REVIEW.—51. What do Notation and Numeration show? 52. What is found by addition? Give an example.

53. What is found by subtraction? Having the sum of two numbers, and one of them, how is the other found? When the smaller of two numbers and the difference are given, how is the greater found? When the difference and greater are given, how is the less found?

EXAMPLE 1. The sum of two numbers is 85; the less number is 37: what is the greater?

2. The sum of two numbers is 85; the greater number is 48: what is the less?

3. The difference of two numbers is 48; the less number is 37: what is the greater?

4. The difference of two numbers is 48; the greater number is 85: what is the less?

ART. 54. BY MULTIPLICATION,

Is found the *amount* of a number taken as *many times* as there are units in another. Art. 28.

Hence, having the cost of a *single* thing, to find the cost of any number of things, multiply the *cost* of *one* by the *number* of things.

1. If 1 yard of tape cost 3 cents, what will 5 yards cost?

ANALYSIS.—*Five yards are* 5 TIMES 1 *yard; therefore* 5 *yards will cost* 5 TIMES *as much as* 1: *the entire cost is found by multiplying the price of* 1 *yard by the* NUMBER *of yards.*

The divisor and quotient being given, *the dividend is found by multiplying together the divisor and quotient.* Art. 37.

2. A divisor is 15; the quotient is 12: what is the dividend?

3. An estate was divided among 7 children; each child received $525: what sum was divided? *Ans.* $3675.

ART. 55. BY DIVISION,

Is found *how many times* one number is contained in another. Art. 36. This enables us,

1. *To divide any number into parts, each part containing a certain number of units.*

2. *To divide a number into any given number of equal parts.*

Thus, if the cost of a number of things and the price of *one* are given, the *number* of things is found by division.

1. James spent 35 cents for oranges, and paid 5 cents each: how many did he buy?

SOLUTION.—He got *one* orange for each *time* 5 cents are contained in 35 cents; 5 in 35, 7 *times;* therefore, he bought 7 oranges.

Knowing the cost of a given number of things, we obtain the price of *one*, by dividing the whole cost into as many equal parts as there are things.

2. If 4 oranges cost 20 cents, what does one cost?

SOLUTION.—If 20 cents be divided into 4 equal parts, each part will be the cost of 1 orange. Placing 1 cent to each part, will require *four* cents. Hence, there will be as many cents in each part, as 4 cents are contained *times* in 20 cents; 4 in 20, 5 times; hence, in each *part* there will be 5 cents, the cost of one orange.

If the product of two factors be divided by either of them, the quotient will be the other. Art. 37.

Hence, *if the dividend and quotient be given, find the divisor by dividing the dividend by the quotient.*

Therefore, having the product of *three* numbers, and *two* of them given, the *third* can be found by *dividing* the product of the *three* numbers by the product of the *two given numbers.*

3. A dividend is 2875; the quotient, 125: find the divisor. *Ans.* 23.

4. The product of three numbers is 3900: one number is 12, another 13: what is the third? *Ans.* 25.

ART. 56. PROMISCUOUS EXAMPLES.

1. In 4 bags are $500: in the first, 96; the 2d, 120; the 3d, 55: what sum in the 4th bag? *Ans.* $229.

2. Four men paid $1265 for land; the first paid $243; the 2d, $61 more than the first; the 3d, $79 less than the 2d: how much did the 4th man pay? *Ans.* $493.

3. I have 5 apple-trees; the first bears 157 apples; 2d, 264; 3d, 305; 4th, 97; 5th, 123: I sell 428, and 186 are stolen: how many apples are left? *Ans.* 332.

REVIEW.—54. What is found by multiplication? Give an example. When the divisor and quotient are given, how is the dividend found?

55. What is found by division? What does it enable us to do? Give examples. If the dividend and quotient are given, how is the divisor found? Having the product of three numbers, and two of them given, how is the other found?

4. In an army of 57068 men, 9503 are killed; 586 join the enemy; 4794 are prisoners; 1234 die of wounds; 850 are drowned: how many return? *Ans.* 40101.

5. On the first of the year a man is worth $123078; during the year he gains $8706; in January he spends $237, in February $301; in each of the remaining ten months, he spends as much as in the first two: how much had he at the end of the year? *Ans.* $125866.

6. In a building there are 72 rooms; in each room 4 windows, and in each window 24 lights: how many lights are there in the house? *Ans.* 6912.

7. A merchant has 9 pieces of cloth, of 73 yards each: and 12 pieces, of 88 yards each: how many yards in all? *Ans.* 1713 yards.

8. I spend 99 cents a day: how many cents will I spend in 49 years, of 365 days each? *Ans.* 1770615.

9. An Encyclopedia consists of 39 volumes; each volume has 774 pages of two columns each; each column 67 lines; each line 10 words; and every 10 words 47 letters: how many pages, lines, words, and letters, in the work?

Ans. { 30186 pages, 4044924 lines, 40449240 words, 190111428 letters.

10. The Bible has 31173 verses: in how many days can I read it, reading 86 verses a day? *Ans.* $362\frac{41}{86}$ days.

11. I bought 28 horses for $1400; 3 died: for how much each must I sell the rest, to incur no loss? *Ans.* $56.

12. How many times can I fill a 15 gallon cask, from 5 hogsheads of 63 gallons each? *Ans.* 21 times.

13. A certain dividend is 73900; the quotient 214; the remainder 70: what is the divisor? *Ans.* 345.

14. Multiply the sum of 148 and 56 by their difference; divide the product by 23. *Ans.* 816.

15. How much cloth, at $6 a yard, will pay for 8 horses at $60 each, and 14 cows at $15 each? *Ans.* 115 yards.

16. A cistern of 360 gallons, has 2 pipes; one will *fill* it in 15 hours, and the other *empty* it in 20 hours. If both pipes are left open, how many hours will the cistern be in filling? *Ans.* 60 hours.

17. Two men paid $6000 for a farm; one man took 70 acres at $30 an acre, the other the remainder, at $25 an acre: how many acres in all? *Ans.* 226 acres.

SUGGESTION.—In the four following examples, obtain the required number by reversing the operations.

18. What is the number, from which, if 125 be subtracted, the remainder will be 222? *Ans.* 347.

19. What is the number, to which, if 135 be added, the sum will be 500? *Ans.* 365.

20. Find a number, from which, if 65 be subtracted, and the remainder divided by 15, the quotient will be 45? *Ans.* 740.

21. What is the number, to which if 15 be added, the sum multiplied by 9, and 11 taken from the product, the remainder will be 340? *Ans.* 24.

22. If 98 be subtracted from the difference of two numbers, 27 will remain; 246 is the less number: what is the greater? *Ans.* 371.

GENERAL PRINCIPLES OF DIVISION.

ART. **57.** The value of the quotient depends on the relative values of divisor and dividend. These may be changed by Multiplication and Division, thus:

1st. *The Dividend may be multiplied, or the Divisor divided.*

2d. *The Dividend may be divided, or the Divisor multiplied.*

3d. *Both Dividend and Divisor may be multiplied, or both divided, at the same time.*

ILLUSTRATIONS.—Let 24 be a dividend, and 6 the divisor; the *quotient* is 4: $(24 \div 6 = 4)$.

If the dividend (24), be *multiplied* by 2, the quotient will be multiplied by 2: for, $24 \times 2 = 48$; and $48 \div 6 = 8$, which is the former quotient (4), *multiplied* by 2.

Now, if the divisor (6), be *divided* by 2, the quotient will be multiplied by 2; for, $6 \div 2 = 3$; and $24 \div 3 = 8$, which is the former quotient (4), *multiplied* by 2. Hence,

PRINCIPLE 1.—*If the dividend be multiplied, or the divisor be divided, the quotient will be multiplied.*

Art. 58. Take the same Example, $24 \div 6 = 4$.

If the dividend (24), be *divided* by 2, the quotient will be divided by 2: for, $24 \div 2 = 12$; and $12 \div 6 = 2$, which is the former quotient (4), *divided* by 2.

And, if the divisor (6), be *multiplied* by 2, the quotient will be divided by 2; for, $6 \times 2 = 12$; and $24 \div 12 = 2$, which is the former quotient (4), *divided* by 2. Hence,

Principle II.—*If the dividend be divided, or the divisor be multiplied, the quotient will be divided.*

Art. 59. Take the same Example, $24 \div 6 = 4$.

If the dividend (24), and divisor (6), be *multiplied* by 2, the quotient will not be changed; for, $24 \times 2 = 48$; and $6 \times 2 = 12$; $48 \div 12 = 4$; the former quotient (4), *unchanged.*

And if the dividend (24), and divisor (6), be *divided* by 2, the quotient will not be changed; for, $24 \div 2 = 12$; and $6 \div 2 = 3$; $12 \div 3 = 4$; the former quotient (4), *unchanged.* Hence,

Principle III.—*If both dividend and divisor be multiplied or divided by the same number, the quotient will not be changed.*

Art. 60. *If a number be multiplied, and the product divided by the same number, the quotient will be the original number.*

For, $24 \times 2 = 48$; and $48 \div 2 = 24$, the original number, on the principle, that if the product be *divided* by the multiplier, the quotient will be the multiplicand. Also,

If a number be divided, and the quotient multiplied by the same number, the product will be the original number.

For, $24 \div 2 = 12$; and $12 \times 2 = 24$, the original number, on the principle, that if the quotient be *multiplied* by the divisor, the product will be the dividend.

Hence, the operations of multiplication and division by the same number, destroy (*cancel*) each other.

Review.—57. On what does the value of the quotient depend? How may the divisor and dividend be changed? If the dividend be multiplied, what effect on the quotient? If the divisor be divided?

58. If the dividend be divided, what effect on the quotient? If the divisor be multiplied?

59. If both divisor and dividend be multiplied by the same number, what effect on the quotient? If both be divided by the same number?

VI. CANCELLATION.

Most TEACHERS defer this subject until after Factoring.

1. I bought 3 oranges at 10 cents each, and paid for them with pears at 3 cts. each: how many pears did I give?

SOLUTION.—Ten cents multiplied by 3, give 30 cents, the cost of the oranges.

Then it will take as many pears, as 3 cents are contained times in 30 cents; that is, $30 \div 3 = 10$, the number of pears.

Here, 10 is multiplied by 3 and the product divided by 3; *but a number is not changed by multiplying it, and then dividing the product by the same number*, (Art. 60); hence, multiplying by 3, and then dividing by 3, may be *omitted*, and 10 taken as the result; hence,

OPERATION.

$$\begin{array}{r} 10 \\ 3 \\ \hline 3)30 \\ \hline \end{array}$$

Ans. 10 pears.

ART. 61[a]. When a number is to be multiplied and then divided by the same number, *both operations* may be *omitted*, and a line drawn across the common multiplier and divisor, as in the margin.

OPERATION.

$$\frac{10 \times \not{3}}{\not{3}} = 10.$$

REM.—In the above example, 10 and 3 form the dividend, and 3 the divisor. In arranging the numbers, place the dividend above a horizontal line, and the divisor below it.

2. How many barrels of molasses at $13 a barrel, will pay for 13 barrels of flour at $4 a barrel? *Ans.* 4.

3. If I buy 41 cows at $11 each, and pay in horses at $41 each, how many horses are required? *Ans.* 11.

4. If I buy 10 lemons at 3 cents each, and pay in oranges at 5 cts. each, how many oranges will I give?

SOLUTION.—Ten times 3 cents are 30 cents, the cost of the lemons: 30 cts. divided by 5 cents, equal 6, the number of oranges.

But, as 10 is a composite number, whose factors are 5 and 2, ($5 \times 2 = 10$), indicate the operation as in the margin on the left.

OPERATION.

$$\begin{array}{r} 3 \\ 10 \\ \hline 5)30 \\ \hline \end{array}$$

Ans. 6.

$$\frac{\not{5} \times 2 \times 3}{\not{5}} = 6.$$

Since 5, 2, and 3 are to be multiplied together, and their product divided by 5, *omit* 5 both as multiplier and divisor (Art. 60), draw a line across it, and only multiply 2 by 3.

5. Multiply 17 by 18, and divide the product by 6.

OPERATION.

$$\text{Dividend, }\quad \frac{17\times\cancel{6}\times 3}{\cancel{6}}=51. \quad \text{Divisor,}$$

SOLUTION.—Instead of multiplying 17 by 18, and dividing the product by 6, separate 18 into the factors 6 and 3, ($6\times3=18$), then *cross* the factor 6, which is common to both multiplier and divisor: after which, multiply 17 by 3.

6. In 15 times 8, how many times 4? *Ans.* 30.
7. In 24 times 4, how many times 8? *Ans.* 12.
8. In 37 times 15, how many times 5? *Ans.* 111.
9. Multiply 36 by 40, and divide the product by 30 multiplied by 8.

OPERATION.

$$\begin{array}{l} \text{Dividend,} \\ \text{Divisor,} \end{array}\quad \frac{36\times40}{30\times\ 8}=\frac{\cancel{6}\times3\times2\times\cancel{5}\times\cancel{4}\times\cancel{2}}{\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{2}}=6.$$

SOLUTION.—Indicate the operation to be performed, then resolve (separate) the numbers into factors. Now the quotient will not be changed (Art. 59) by dividing both divisor and dividend by the same number, which is done by *erasing* the same factors in both.

OPERATION.

$$\begin{array}{l} \text{Dividend,} \\ \text{Divisor,} \end{array}\quad \frac{\overset{6}{\cancel{36}}\times\overset{\cancel{5}}{\cancel{40}}}{\underset{\cancel{5}}{\cancel{30}}\times\ \cancel{8}}=6.$$

SECOND SOL.—Indicate the operation as in the margin.

As 8 is a factor of 40, divide 40 by 8, and write the quotient 5 over 40; *cross out* (cancel) 40 and 8.

Then, as 36 and 30 have a common factor 6, divide each by 6, and write the quotients 6 and 5 as in the operation: cancel 36 and 30. Next, canceling the common factor 5 in both dividend and divisor, the result is 6, as before.

10. In 36 times 5, how many times 15? *Ans.* 12.
11. In 14 times 9, how many times 6? *Ans.* 21.

ART. **61**^{b}. The process of shortening the operations of arithmetic, by omitting equal factors from the dividend

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REVIEW.—60. If a number be multiplied, and the product divided by the same number, what will be the quotient? On what principle? If a number be divided and the quotient multiplied by the same number, what will the product be? On what principle?

and divisor, is termed *Cancellation.* It depends on the principle explained in Arts. 58 and 59.

NOTE.—To cancel is to *suppress* or *erase.* When the same factor is omitted in both dividend and divisor, it is said to be canceled.

RULE FOR CANCELLATION.

When there are common factors in a dividend and its divisor, shorten the operation by canceling all the factors common to both: proceed with the remaining factors as the question may require.

REM.—1. Canceling is merely dividing both dividend and divisor by the same number, which (Art. 59) does not alter the quotient.

2. The pupil should observe that *one* factor in the dividend will cancel only *one equal* factor in the divisor.

3. Some prefer to place the numbers forming the dividend on the *right* of a vertical line, and those forming the divisor on the *left.*

1. Multiply 42, 25, and 18, together, and divide the product by 21×15. *Ans.* 60.

2. I sold 23 sheep at $10 each, and was paid in hogs at $5 each: how many did I receive? *Ans.* 46.

3. How many yards of flannel at 35 cents a yard, will pay for 15 yards of calico at 14 cts.? *Ans.* 6 yards.

4. What is the quotient of $21\times11\times6\times26$, divided by $13\times3\times14\times2$? *Ans.* 33.

5. The factors of a dividend are 21, 15, 33, 8, 14, and 17; the divisors, 20, 34, 22, and 27: required the quotient. *Ans.* 49.

6. I bought 21 kegs of nails of 95 pounds each, at 6 cents a pound; paid for them with pieces of muslin of 35 yards each, at 9 cents a yard: how many pieces of muslin did I give? *Ans.* 38.

NOTE.—Other applications of Cancellation will be found in Fractions, Proportion, &c. The pupil will apply it more readily, when acquainted with Factoring.

REVIEW.—61*a*. REM. How are numbers arranged for cancellation? 61*b*. What is cancellation? Upon what principle does it depend? What does *cancel* mean? When is a factor canceled? What is the rule? REM. What is canceling?

VII. COMPOUND NUMBERS.

TO TEACHERS.—While placing Fractions immediately after Simple Whole Numbers is philosophical, and appropriate in a Higher Arithmetic for *Advanced* pupils, the experience of the author convinces him that, in a book for *Young* learners, Compound Numbers should be introduced here, instead of after Fractions, as is done by some authors. His reasons are,

1st. The operations of Addition, Subtraction, Multiplication, and Division of *Compound Numbers*, are analogous to the same operations in *Simple Numbers*, and serve to illustrate the principles of the fundamental rules. The principle of *Notation* is the same in each.

2d. The subject of Fractions is important and difficult. Before studying it, most pupils require more mental discipline than is demanded in the elementary rules. This is acquired by the study of Compound Numbers.

3d. The *general principles* involved in their study, do not require a knowledge of fractions. The Examples involving fractions are few, and are introduced, (as they should be,) with other exercises in that subject.

☞ TEACHERS who prefer it, can direct their pupils to defer Compound Numbers until they have studied Fractions to page 169.

DEFINITIONS.

ART. **62**. When two numbers have the *same unit*, they are of the *same kind* or denomination: thus, 3 dollars, 5 dollars, are of the same denomination; both *dollars*.

When they have *different* units, they are of *different* denominations: thus, 3 dollars, and 5 cents, are of different denominations; *dollars* and *cents*.

ART. **63**. A *simple number* denotes things of the *same unit value:* thus, 3 yards, 2 dollars, 5 pints, are each simple numbers. All abstract numbers are *simple*.

ART. **64**. A *compound number* is two or more numbers of *different unit values* used to express one quantity: thus, 3 dollars 5 cents, 2 feet 3 inches, are each *compound*.

REM.—1. In Compound Numbers, *denomination* or *order*, denotes the

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REVIEW.—62. When are two numbers of the same denomination? Give an example. When of different denominations? Give an example.

63. What does a Simple Number denote? Give an example. 64. What is a Compound Number? Give an example.

name of the unit considered. Thus, *dollar* and *cent* are denominations of money; *foot* and *inch*, of length; *pound* and *ounce*, of weight.

2. Compound Numbers are analogous to Simple Numbers in this particular; a certain number of units of each order is collected into a group, and forms a unit of a higher order or denomination. But,

They differ in this: that in compound numbers 10 units of one order *do not uniformly* make one of the next higher.

3. The simplest class of Compound Numbers is Federal money, because we pass from one denomination to another according to the *scale of tens*.

FEDERAL OR UNITED STATES MONEY,

ART. **65**, Is the currency of the United States, established by the Federal Congress, in 1786.

While U. S. money *may* be treated decimally, it is a species of Compound Numbers, being so *regarded in ordinary business transactions*.

Its denominations, or the names of its different orders, are mill, cent, dime, dollar, eagle.

Ten units of each denomination make one unit of the next higher denomination.

TABLE.

10 mills,	marked m., make	1 cent,	marked ct.
10 cents	....	1 dime,	..... d.
10 dimes	....	1 dollar,	..... $.
10 dollars	....	1 eagle,	..... E.

Also, 5 cents	.... make	one-half dime.
25 cents	....	one-quarter of a dollar.
50 cents	....	one-half of a dollar.
75 cents	....	three-quarters of a dollar.
100 cents	....	one dollar.

The coins of the United States are of bronze, nickel, silver, and gold. Their denominations are:

1st. Bronze: cent. 2d. Nickel: three-cent piece, five-cent piece.

3d. Silver: dollar, half dollar, quarter dollar, dime.

4th. Gold: double eagle, eagle, half eagle, three-dollar piece, quarter eagle, dollar.

REVIEW.—64. REM. 1. What does *denomination* or *order* denote? 2. In what are Simple and Compound Numbers analogous? In what do they differ? 3. What is the simplest class of Compound Numbers?

NOTATION AND NUMERATION.

Art. 66. Accounts are kept in dollars, cents, and mills; or in dollars, cents, and parts of a cent.

Eagles and dollars are called *dollars;* dimes and cents, *cents.*

NUMERATION TABLE.

Hundreds of dollars.	Tens of dollars, or eagles.	Dollars.	Tens of cents, or dimes.	Cents.	Mills.	
			.0	4	3	read 4 cents and 3 mills, or 43 mills.
			.2	1	4	read 21 cents and 4 mills, or 214 mills.
		3	.0	4	5	read 3 dollars 4 cents and 5 mills.
	7	6	.2	5	0	read 76 dollars and 25 cents.
6	8	1	.3	4	5	read 681 dollars 34 cents and 5 mills.

The second line may also be read, 2 dimes, 1 cent, 4 mills; the fourth line, 7 eagles, 6 dollars, 2 dimes, 5 cents. This method of reading is not customary.

The third line may also be read, 304 cents and 5 mills, or 3045 mills; the fourth line, 7625 cents, or 76250 mills; the lower line, 68134 cents and 5 mills, or 681345 mills.

A period (.), is used as a separating point, to separate the cents and dollars. Some use the comma.

Thus, 2 dollars, 2 dimes, and 2 cents, or 2 dollars and 22 cents, are written, $2.22.

Art. 67. The Table shows that cents occupy the first two places to the right of dollars, and mills the place to the right of cents, the third from dollars. Hence the

Rule for Numeration.—*Read the number to the left of the period as dollars, and the first two figures on the right of the period as cents; and if there be a third figure, as mills.*

Review.—65. What are the denominations of United States money? How many units of either denomination make a unit of the next higher? Repeat the table. How many cents in a half dime? In a quarter dollar? In a half dollar? In a dollar? Of what are the coins of the United States? Which are copper? Which silver? Which gold?

EXAMPLES TO BE COPIED AND THEN READ.

$18.62 5	$ 70.01 5	$6.12	$ 29.00
$20.32 4	$100.28 3	$3.06	$100.03
$79.05	$150.00 2	$4.31	$ 20.05
$46.00 3	$100.00 3	$5.43	$ 40.00 7

ART. 68. RULE FOR NOTATION.

Write the dollars as in whole numbers ; place a period on the right of dollars, next to this write the cents, then the mills.

If the cents are less than ten, place a cipher next to the dollars; if there are no cents, put two ciphers in the place of cents.

EXAMPLES TO BE WRITTEN.

1. Twelve dollars, seventeen cents, eight mills. $12.17 8
2. Six dollars, six cents, six mills. . . . . $ 6.06 6
3. Seven dollars, seven mills. . . . . $ 7.00 7
4. Forty dollars, fifty-three cents, four mills. . $40.53 4
5. Two dollars, three cents. . . . . $ 2.03
6. Twenty dollars, two cents, two mills. . . $20.02 2
7. One hundred dollars, ten cents. $100.10
8. Two hundred dollars, two cents. $200.02
9. Four hundred dollars, one cent, eight mills. $400.01 8

REDUCTION OF U. S. MONEY.

ART. 69. Reduction consists in changing the denominations or orders, without altering the value.

1st. *Reduction Descending* is changing numbers from a higher to a lower denomination; as, from dollars to cents.

2d. *Reduction Ascending* is changing numbers from a lower to a higher denomination; as, from cents to dollars.

REVIEW.—66. In what denominations are accounts kept? What are eagles and dollars together called? What dimes and cents? How are dollars and cents separated? 67. What places do cents occupy? What place mills? What is the Rule for Numeration?

68. How write any sum of U. S. money, Rule? 69. In what does Reduction consist? What is Reduction descending? What ascending?

ART. **70**. As there are 10 mills in 1 cent, in any number of cents there are 10 times as many mills as cents.

Therefore, to reduce *cents* to mills, multiply by 10; that is, annex one cipher.

Hence, conversely, to reduce *mills* to cents, divide by 10; that is, cut off one figure on the right.

As there are 100 cents in 1 dollar, in any number of dollars there are 100 times as many cents as dollars.

Therefore, to reduce *dollars* to cents, multiply by 100; that is, annex two ciphers.

Hence, conversely, to reduce *cents* to dollars, divide by 100; that is, cut off two figures on the right.

As there are 1000 mills in a dollar, in any number of dollars there are 1000 times as many mills as dollars.

Therefore, to reduce *dollars* to mills, multiply by 1000; that is, annex three ciphers.

Hence, conversely, to reduce *mills* to dollars, divide by 1000; that is, cut off three figures on the right.

ART. **71**. As the operations of reduction consist in multiplying or dividing by 10, 100, or 1000, the work can be shortened by simply moving the point.

ILLUSTRATIONS.—In *Multiplying*, move the point as many places to the *right*, as there are ciphers in the multiplier.

Thus, \$2.50 = 250 cents; \$2.50 5 = 2505 mills.

In *Dividing*, move the point as many places to the *left*, as there are ciphers in the divisor.

Thus, 275 cents = \$2.75; 4285 mills = \$4.28 5

1. Reduce 17 cts. to mills. *Ans.* 170 m.
2. Reduce 28 cts. to mills. *Ans.* 280 m.

REVIEW.—70. How are cents reduced to mills? Why? Mills to cents? Dollars to cents? Why? Cents to dollars? Dollars to mills? Why? Mills to dollars? 71. In what do the operations of reduction consist? How can the work be shortened? Illustrate.

3. Reduce 43 cts. and 6 m. to mills. *Ans.* 436 m.
4. Reduce 70 cts. and 6 m. to mills. *Ans.* 706 m.
5. Reduce 106 m. to cents. *Ans.* 10 cts. 6 m.
6. Reduce 490 mills to cents. *Ans.* 49 cts.
7. Reduce 9 dollars to cents. *Ans.* 900 cts.
8. Reduce 14 dollars to cents. *Ans.* 1400 cts.
9. Reduce 104 dollars to cents. *Ans.* 10400 cts.
10. Reduce $60 and 13 cts. to cents. *Ans.* 6013 cts.
11. Reduce $40 and 5 cts. to cents. *Ans.* 4005 cts.
12. Reduce 375 cts. to dollars. *Ans.* $3.75
13. Reduce 9004 cts. to dollars. *Ans.* $90.04
14. Reduce 4 dollars to mills. *Ans.* 4000 m.
15. Reduce $14 and 2 cts. to mills. *Ans.* 14020 m.
16. Reduce 2465 mills to dollars. *Ans.* $2.46 5
17. Reduce 3007 mills to dollars. *Ans.* $3.00 7
18. Reduce 3187 cents to dollars. *Ans.* $31.87

ADDITION OF U. S. MONEY.

Art. **72**. 1. Add together 4 dollars, 12 cents, 5 mills; 7 dollars, 6 cents, 2 mills; 20 dollars 43 cents; 10 dollars, 5 mills; and 16 dollars, 87 cents, 5 mills.

Rule—1. *Write the numbers to be added, units of the same denominations under each other; dollars under dollars, cents under cents, mills under mills, because only numbers of the same denomination can be added.*

2. *Add as in Addition of Simple Numbers, and place the separating point directly under the separating points above.*

OPERATION.

$	cts.	m.
4.	12	5
7.	06	2
20.	43	0
10.	00	5
16.	87	5
$58.	49	7

Proof.—The same as in Addition of Simple Numbers.

Review.—72. How write numbers in addition of U. S. money? Why? How is the addition performed? Why? Where place the point?

2. What is the sum of 17 dollars, 15 cents; 23 dollars, 43 cents; 7 dollars, 19 cents; 8 dollars, 37 cents; and 12 dollars, 31 cents? *Ans.* \$68.45

3. Add 18 dollars, 4 cents, 1 mill; 16 dollars, 31 cents, 7 mills; 100 dollars, 50 cents, 3 mills; and 87 dollars, 33 cents, 8 mills. *Ans.* \$222.19 9

4. William had the following bills for collection: \$43.75; \$29.18; \$17.63; \$268.95; and \$718.07: how much was to be collected? *Ans.* \$1077.58

5. Bought a gig for \$200; a watch for \$43.87 5; a suit of clothes for \$56.93 7; hat for \$8.50; and a whip for \$2.31 3: what was the amount? *Ans.* \$311.62 5

6. A person has due him, five hundred and four dollars, six cents, three mills; \$420, 19 cents, 7 mills; one hundred and five dollars, fifty cents; \$304 and 5 mills; \$888, forty-five cents, five mills: how much is due to him? *Ans.* \$2222.22

7. Add five dollars, seven cents; thirty dollars, twenty cents, three mills; one hundred dollars, five mills; sixty dollars, two cents; seven hundred dollars, one cent, one mill; \$1000.10; forty dollars, four mills; and \$64.58 7 *Ans.* \$2000.

SUBTRACTION OF U. S. MONEY.

Art. **73**. From one hundred dollars, five cents, three mills, take \$80, 20 cents, 7 mills.

OPERATION.

	\$	cts.	m.
	100 .	05	3
	80 .	20	7
Ans.	\$19 .	84	6

Rule.—*Place the less number under the greater, dollars under dollars, cents under cents, &c. Subtract as in Simple Numbers, placing the separating point under the points above.*

Proof.—As in Subtraction of Simple Numbers.

2. From \$29.34 2 take \$17.26 5 *Ans.* \$12.07 7
3. From \$46.28 take \$17.75 *Ans.* \$28.53
4. From \$20.05 take \$ 5.50 *Ans.* \$14.55

5. From $3, take 3 cts. . . . *Ans.* $2.97
6. From $10, take 1 mill. . . . *Ans.* $9.99 9
7. From $50, take 50 cts., 5 mills. *Ans.* $49.49 5
8. From one thousand dollars, take one dollar, one cent, and one mill. *Ans.* $998.98 9
9. B owes 1000 dollars, 43 cents, 5 mills; if he pay nine hundred dollars, sixty-eight cents, seven mills, how much will he still owe? *Ans.* $99.74 8

MULTIPLICATION OF U. S. MONEY.

ART. **74.** 1. What will 13 cows cost, at 17 dollars, 12 cents, 5 mills each?

SOLUTION.—Consider the cost, $17.12 5, as reduced to its lowest denomination, viz.: 17125 mills. Then, since 13 cows will cost 13 times as much as 1, multiply the cost of one, by 13, which gives for the cost of 13 cows, 222625 mills, the product being of the same denomination as the multiplicand, Art. 30. Finally, reduce the mills to dollars, Art. 70. Hence, the

OPERATION.

```
$17.12 5
      13
--------
  51375
 17125
--------
$222.62 5
```

Rule.—*Multiply as in Simple Numbers; the product will be the answer in the lowest denomination of the multiplicand, which may then be reduced to dollars by pointing.*

PROOF.—As in Multiplication of Simple Numbers.

2. Multiply $7, 83 cts. by 8. *Ans.* $62.64
3. Multiply $12, 9 cts., 3 m. by 9. *Ans.* $108.83 7
4. Multiply $23, 1 ct., 8 m. by 16. *Ans.* $368.28 8
5. Multiply $35, 14 cts. by 53. *Ans.* $1862.42
6. Multiply $125, 2 cts. by 62. *Ans.* $7751.24
7. Multiply $40, 4 cts. by 102. *Ans.* $4084.08
8. Multiply 12 cts., 5 m. by 17. *Ans.* $2.12 5
9. Multiply $3, 28 cts. by 38. *Ans.* $124.64

REVIEW.—73. How are numbers written in subtraction? Why? How is the subtraction performed? Where is the separating point placed? 74. How is multiplication performed? How is the product pointed?

10. What cost 338 barrels of cider, at 1 dollar, 6 cents a barrel? *Ans.* $358.28

11. Sold 38 cords of wood, at 5 dollars, 75 cts. a cord: to what did it amount? *Ans.* $218.50

12. At 7 cts. a pound, what cost 465 pounds of sugar? *Ans.* $32.55

NOTE.—Instead of multiplying 7 cents by 465, multiply 465 by 7, which gives the same product, Art. 30. But, in fixing the denomination of the product, remember that 7 cents is the true multiplicand.

13. What cost 89 yards of sheeting, at 34 cts. a yard? *Ans.* $30.26

14. What will 24 yards of cloth cost, at 5 dollars, 67 cents a yard? *Ans.* $136.08

15. I have 169 sheep, valued at $2.69 each: what is the value of the whole? *Ans.* $454.61

16. If I sell 691 bushels of wheat at 1 dollar, 25 cts. a bushel, what will it amount to? *Ans.* $863.75

17. I sold 73 hogsheads of molasses, of 63 gallons each, at 55 cts. a gallon: what is the sum? *Ans.* $2529.45

18. What cost 4 barrels of sugar, of 281 pounds each, at 6 cents, 5 mills a pound? *Ans.* $73.06

19. Bought 35 bolts of tape, of 10 yards each, at 1 cent a yard: what did it cost? *Ans.* $3.50

20. If I earn 13 cts. an hour, and work 11 hours a day, how much will I earn in 312 days? *Ans.* $446.16

21. I sold 18 bags of wheat, of 3 bushels each, at $1.25 a bushel: what is the amount? *Ans.* $67.50

22. What cost 150 acres of land, at 10 dollars, 1 mill per acre? *Ans.* $1500.15

23. What cost 17 bags of coffee, of 51 pounds each, at 14 cents, 7 mills per pound? *Ans.* $127.44 9

ART. 75. DIVISION OF U. S. MONEY.

The object in Division of United States money is,

1st. *To find how many times one sum of money is contained in another of the same order or denomination.* *Or*, 2d. *To divide a sum of money into a given number of equal parts*, Art. 41.

1. How much cloth at 7 cts. a yard, will $1.75 buy?

ANAL.—*As* 1 *yard costs* 7 *cts., there will be as many yards as* 7 *cts. are contained times in* 175 *cts.* 175÷7=25.

Here, the divisor and dividend are the same denomination, *cents;* and the quotient, *how many* yards, is an abstract number.

2. Divide 65 dollars equally among 8 persons.

OPERATION.

```
   $  cts. m.
8)65.00 0
  8.12 5
```

SOLUTION.—In this case it is required to divide $65 into 8 equal parts, that is, to find one-eighth of it. One-eighth of $65 is $8, with a remainder $1 = 100 cts. One-eighth of 100 cts. is 12 cts., with a remainder of 4 cts. which equals 40 mills. One-eighth of 40 mills is 5 mills; hence, one-eighth of $65 is $8.12 5

```
   Mills.
8)65000
   8125
 $8.12 5
```

The operation may be performed by reducing the dollars to mills, then dividing by 8, and after this, reducing the quotient to dollars.

3. A farmer received $29.61 cents, for 23 bushels of wheat: how much was that per bushel?

SOLUTION.—To divide $29.61 into 23 equal parts, annex a cipher, which reduces it to 29610 mills; then divide by 23 as in Simple Numbers.

The quotient, 1287, is mills, (Art. 41) which reduce to $1.28 7, (Art. 70).

But, 1287 is not the *exact* quotient, as there is a remainder of 9 mills. It is, however, less than a mill of being exact, which is sufficiently accurate for business purposes.

OPERATION.

```
23)29610(1287 mills.
   23          =$1.28 7+
   66
   46
   201
   184
    170
    161
      9 Rem.
```

The sign + is annexed to denote a remainder.

Rule for Division.—1. *To find how many times one sum of money is contained in another, reduce both sums to the same denomination, and divide as in Simple Numbers.*

2. *To divide a sum of money into any number of equal parts, reduce the sum to mills; divide as in Simple Numbers; the quotient will be mills, which reduce to dollars.*

Proof—As in Division of Simple Numbers.

4. How many yards of calico, at 8 cents a yard, can be bought for $2.80? *Ans.* 35 yards.

5. How many yards of ribbon, at 25 cents a yard, can be purchased for $3? *Ans.* 12 yards.

6. At $8.05 a barrel, how many barrels of flour will $161 purchase? *Ans.* 20 barrels.

7. At 7 cents 5 mills each, how many oranges can be bought for $1.20? *Ans.* 16 oranges.

8. At $1.12 5 per bushel, how many bushels of wheat can be purchased for $234? *Ans.* 208 bush.

9. If 4 acres of land cost $92.25, how much is that an acre? *Ans.* $23.06 2+

10. Make an equal division of $57 and 50 cents among 8 persons. *Ans.* $7.18 7+

11. A man received $25 and 76 cts. for 16 days' work: how much was that a day? *Ans.* $1.61

12. I bought 755 bushels of apples for $328, 42 cts., 5 mills: what did they cost a bushel? *Ans.* $0.43 5

13. My salary is $800 a year: how much is that a day, 313 working days in the year? *Ans* $2.55 5+

14. Divide ten thousand dollars equally among 133 men: what is each man's share? *Ans.* $75.18 7+

15. A man purchased a farm of 154 acres, for two thousand seven hundred and five dollars and one cent: what did it cost per acre? *Ans.* $17.56 5

16. I sold 15 kegs of butter, of 25 pounds each, for $60: how much was that a pound? *Ans.* 16 cts.

17. I bought 8 barrels of sugar, of 235 pounds each, for $122.20: what did 1 pound cost? *Ans.* $0.06 5

Review.—75. What is the object of division? How is the number of times one sum of money is contained in another found, Rule 1? How is a sum of money divided into equal parts, Rule 2?

ART. **76**.—PROMISCUOUS EXAMPLES.

1. I owe A \$47.50; B \$38.45; C \$15.47; D \$19.43: what sum do I owe? *Ans.* \$120.85

2. A owes \$35.25; B \$23.75; C as much as A and B, and \$1 more: what is the amount? *Ans.* \$119.

3. A paid me \$18.38; B \$81.62; C, twice as much as A and B: how much did I receive? *Ans.* \$300.

4. I went to market with \$5; I spent for butter 75 cts., for eggs 35 cts., for vegetables 50 cts., for flour \$1.50: how much money was left? *Ans.* \$1.90

5. A lady had \$20; she bought a dress for \$8.10, shoes for \$1.65, eight yards of calico at 75 cts. a yard, and a shawl for \$4: what sum was left? *Ans.* 25 cts.

6. I get \$50 a month, and spend \$30.50 of it: how much will I have left in 6 months? *Ans.* \$117.

7. A farmer sold his marketing for \$21.75: he paid for sugar \$3.85, for tea \$1.25, for coffee \$2.50, for spices \$1.50: how much had he left? *Ans.* \$12.65

8. I owe A \$37.06, B \$200.85, C \$400, D \$236.75, and E \$124.34; my property is worth \$889.25: how much do I owe more than I am worth? *Ans.* \$109.75

9. Bought 143 pounds of coffee at 13 cts. a pound: after paying \$12.60, what was due? *Ans.* \$5.99

10. A owed me \$400: he paid me 435 bushels of corn, at 45 cts. a bushel: what sum is due? *Ans.* \$204.25

11. If B spend 65 cts. a day, how much will he save in 365 days, his income being \$400? *Ans.* \$162.75

12. Bought 21 barrels of apples, of 3 bushels each, at 35 cts. a bushel: what did they cost? *Ans.* \$22.05

13. What cost four pieces of calico, each containing 19 yards, at 23 cts. a yard? *Ans.* \$17.48

14. If 25 men perform a piece of work, for \$2000, and spend, while doing it, \$163.75, what will be each man's share of the profits? *Ans.* \$73.45

15. If 16 men receive \$516 for 43 days' work, how much does each man earn a day? *Ans.* 75 cts.

16. C earned \$90 in 40 days, working 10 hours a day: how much did he earn an hour? *Ans.* 22 cts. 5 m.

17. A merchant, failing, has goods worth \$1000, and \$500 in cash, to be equally divided among 22 creditors: how much will each receive? *Ans.* \$68.18+

MERCHANTS' BILLS.

A Bill or Account, is a written statement of articles bought or sold, their prices, and entire cost.

18.

Bought 9 pounds Coffee,	at \$0.16 per lb.		\$
4 pounds Tea,	.. 1.25	do.	
45 pounds Sugar,	.. .09	do.	
17 pounds Cheese,	.. .13	do.	

What is the amount of my bill? *Ans.* \$12.70

19.

Bought 8 yards Silk,	at \$1.10 per yd.		\$
18 yards Muslin,	.. .25	do.	
25 yards Linen,	.. .15	do.	
12 yards Calico,	.. .35	do.	
6 yards Gingham,	.. .65	do.	

What is the whole amount? *Ans.* \$25.15

20. *Cincinnati*, Feb. 20th, 1860.

MR. WILLIAM RAY, *Bought of* W. B. SMITH & Co.

5 Eclectic Third Readers,	at \$ 0.35 each.	\$
12 dozen Olney, ...	.. 10.50 per doz.	
6000 Quills, ...	.. 1.60 per M.	
5 Quires Paper, ...	.. .25 per quire.	
3 Copies of Hutton, . . .	.. 4.50 each.	

Rec'd Payment, W. B. SMITH & Co. \$152.10

21. *Boston*, July 5th, 1860.

MR. JOHN JONES, *Bought of* C. BRADFORD.

27 Spelling Books, ...	at \$0.19 each. . .	\$
25 Eclectic Readers, ...	.. .27 do. . .	
8 Ainsworth's Dictionaries,	.. 4.50 do. . .	
27 Greek Readers, ...	.. 2.25 do. . .	
18 Bibles, ...	.. 1.50 do. . .	
75 Testaments, ...	.. .31 do. . .	

Rec'd Payment, C. BRADFORD. \$158.88

☞ For Fractional Exercises in U. S. Money, see page 169.

REDUCTION OF COMPOUND NUMBERS.

Art. 77. Reduction is the process of changing the denomination of a number, without altering its value.

Ex.—Since 3 feet make 1 yard, yards may be changed to feet by *multiplying* by 3; and, feet to yards, by *dividing* by 3:

5 yards $=5\times3=15$ feet: and 15 feet $=15\div3=5$ yards.

Hence, as shown in United States money,

Reduction Descending consists in changing a number from a *higher* to a *lower* denomination: *Reduction Ascending*, in changing a number from a *lower* to a *higher* denomination.

Rem.—The Tables teach the names of the different units, and the number of units of one order or denomination which make a unit of the next higher order: they are analogous to the Table of Orders in Simple Numbers.

Art. 78. DRY MEASURE

Is used in measuring grain, vegetables, fruit, coal, &c.

TABLE.

2 pints (pt.)	make	1 quart,	marked	qt.
8 quarts	. . . .	1 peck,	. . . . . .	pk.
4 pecks	. . . .	1 bushel,	. . . . . .	bu.

The *standard unit* of dry measure is the *Bushel;* a circular measure of $18\frac{1}{2}$ inches diameter, 8 inches deep, and contains $2150\frac{2}{5}$ cubic inches.

☞ Those who resort to Tables of Unit Values, to avoid each successive step of an operation, lose a valuable exercise.

Notes.—1. There are two other denominations of Dry Measure, the *quarter* and *chaldron.* The quarter contains 8 bushels, of 70 pounds each, used in England in selling wheat.

The chaldron, in England, and in some of the U. S., contains 36 bu.; in other States 32 bu., and is used for measuring coal.

2. When Grain and Seeds are bought and sold by *weight*, 60 pounds of Wheat, 60 of Clover seed, 56 of Rye, Corn, or Flax seed, 32 of Oats, 42 of Timothy seed, and 48 of Barley, make 1 bushel.

For information in detail in regard to foreign and domestic weights and measures, see "*Ray's Higher Arithmetic.*"

Review.—77. What is Reduction? Give an example. In what does reduction descending consist? In what does reduction ascending consist? Rem. What do the Tables teach?

TO TEACHERS.—Numerous questions should be asked on each Table, similar to the following:

1. How many pints in 1 quart? in 2? in 3? in 4? in 5? in 6? in 7? in 9? in 10?

2. How many quarts in 1 peck? in 2? in 3? in 4? in 5? in 6? in 7? in 8? in 9? in 10?

3. How many pecks in 1 bushel? in 2? in 3? in 4? in 5? in 6? in 7? in 8? in 9? in 10?

4. How many quarts in 2 pints? in 5? in 6? in 8? in 9? in 11? in 12? in 13? in 14?

5. How many pecks in 8 quarts? in 16? in 24? in 35? in 40? in 49? in 56? in 65?

6. How many bushels in 4 pecks? in 12? in 20? in 11? in 15? in 27? in 32? in 39?

ART. **79**. THE PRECEDING EXAMPLES SHOW, THAT

To reduce quarts to pints, multiply the number of quarts by the number of pints in a quart.

To reduce pecks to quarts, multiply the number of pecks by the number of quarts in a peck.

To reduce bushels to pecks, multiply the number of bushels by the number of pecks in a bushel. Hence,

REDUCTION DESCENDING is performed by *Multiplication: the multiplier being that number of the lower order or denomination, which makes a* UNIT *of the next higher.*

ART. **80**. To reduce pints to quarts, divide the pints by the number of pints in a quart.

To reduce quarts to pecks, divide the quarts by the number of quarts in a peck.

To reduce pecks to bushels, divide the pecks by the number of pecks in a bushel. Hence,

REDUCTION ASCENDING *is performed by Division: the divisor being that number of the lower order or denomination, which makes a* UNIT *of the next higher.*

REVIEW.—78. For what is Dry Measure used? Repeat the Table. What is the *standard unit* of Dry Measure?

7. Reduce 3 bushels to pints.

SOLUTION.—To reduce bushels to pecks, multiply by 4, because there are 4 pecks in a bushel. To reduce pecks to quarts, multiply by 8, because there are 8 quarts in a peck, or 8 times as many quarts as pecks. To reduce quarts to pints, multiply by 2, because there are 2 pints in a quart.

OPERATION.

$$\begin{array}{rl} 3 & \text{Bushels.} \\ 4 & \\ \hline 12 & = \text{pk.} \\ 8 & \\ \hline 96 & = \text{qt.} \\ 2 & \\ \hline 192 & = \text{pt.} \end{array}$$

8. Reduce 192 pints to bushels.

SOLUTION.—To reduce pints to quarts, divide by 2, because there are 2 pints in a quart. To reduce quarts to pecks, divide by 8, because there are 8 quarts in a peck. To reduce pecks to bushels, divide by 4, because there are 4 pecks in a bushel.

OPERATION.

$$\begin{array}{rl} 2)\underline{192} & \text{Pints.} \\ 8)\underline{96} & = \text{qt.} \\ 4)\underline{12} & = \text{pk.} \\ 3 & = \text{bu.} \end{array}$$

The two preceding examples show that *Reduction Descending* and *Ascending prove each other.*

9. Reduce 7 bushels, 3 pecks, 4 quarts, 1 pint, to pints.

SOLUTION.—In solving this example, multiply the bushels by 4, which make 28 pecks; to these add the 3 pecks. Then, multiply the pecks (31) by 8, and add the 4 quarts; multiply the quarts (252) by 2, and add the 1 pint.

OPERATION.

$$\begin{array}{rlll} \text{Bu.} & \text{pk.} & \text{qt.} & \text{pt.} \\ 7 & 3 & 4 & 1 \\ 4 & & & \\ \hline 31 & \multicolumn{3}{l}{= \text{pk. in 7 bu. 3 pk.}} \\ 8 & & & \\ \hline 252 & \multicolumn{3}{l}{= \text{qt. in 7 bu. 3 pk. 4 qt.}} \\ 2 & & & \\ \hline 505 & \multicolumn{3}{l}{= \text{pints in the whole.}} \end{array}$$

REVIEW.—78. NOTE 1. What other denominations of Dry Measure? What does the quarter contain? For what is it used? What does the chaldron contain? For what is it used? NOTE 2. How many pounds in a bushel of wheat? How many in other grains?

79. How are quarts reduced to pints? Pecks to quarts? Bushels to pecks? How is reduction descending performed? What the multiplier?

80. How are pints reduced to quarts? Quarts to pecks? Pecks to bushels? How is reduction ascending performed? What is the divisor?

10. Reduce 505 pints to bushels.

SOLUTION.—To reduce pints to quarts, divide by 2, and there is 1 left; as the dividend is pints, this remainder is 1 pint.

To reduce quarts to pecks, divide by 8, and 4 quarts are left.

To reduce pecks to bushels, divide by 4, and 3 pecks are left.

OPERATION.

Pt. in a qt.	2)505	
Qt. in a pk.	8)252	qt. 1 pt.
Pk. in a bu.	4)31	pk. 4 qt.
	7	bu. 3 pk.

Ans. 7 bu. 3 pk. 4 qt. 1 pt.

The remainder is always of the same denomination as the dividend, Art. 38. Hence,

ART. 81. GENERAL RULES.

TO REDUCE FROM A HIGHER TO A LOWER ORDER,

Rule.—*Multiply the highest denomination given, by that number of the next lower, which makes a unit of the higher; add to the product the number, if any, of the lower denomination.*

Proceed in like manner with the result thus obtained, till the whole is reduced to the required denomination.

TO REDUCE FROM A LOWER TO A HIGHER ORDER,

Rule.—*Divide the given quantity by that number of its own denomination which makes a unit of the next higher.*

Proceed in like manner with the quotient thus obtained, till the whole is reduced to the required denomination.

The last quotient, with the several remainders, if any, annexed, will be the answer.

PROOF.—Reverse the operation: that is, reduce the answer back to the denomination from which it was derived. If this result is the same as the quantity given, the work is correct.

11. Reduce 2 bu. to pints. . . . . . *Ans.* 128 pt.
12. 12 pk. to pints. . . . . . *Ans.* 192 pt.
13. 8 bu. to quarts. . . . . . *Ans.* 256 qt.
14. 1 bu. 1 pk. to pints. . . . *Ans.* 80 pt.
15. 2 bu. 2 qt. to pints. . . . *Ans.* 132 pt.

REVIEW.—81. What is the general rule for reducing from a higher to a lower order? From a lower to a higher? What the method of proof?

16. Reduce 4 bu. 2 pk. 1 qt. to pints. *Ans.* 290 pt.
17. 7 bu. 3 pk. 7 qt. 1 pt. to pt. *Ans.* 511 pt.
18. 3 bu. 1 pt. to pints. . . . *Ans.* 193 pt.
19. 384 pt. to bushels. . . . *Ans.* 6 bu.
20. 47 pt. to pecks. *Ans.* 2 pk. 7 qt. 1 pt.
21. 95 pt. to bu. *Ans.* 1 bu. 1 pk. 7 qt. 1 pt.
22. 508 pt. to bu. *Ans.* 7 bu. 3 pk. 6 qt.

ART. 82. TROY OR MINT WEIGHT

Is used in weighing gold, silver, jewels, liquors, &c.

TABLE.

24 grains (gr.)	make 1	pennyweight,	marked pwt.
20 pennyweights ...	1	ounce, ...	oz.
12 ounces	1	pound, ...	lb.

NOTE.—The *standard unit* of weight in the United States, is the *Troy pound,* containing 5760 grains.

For interesting historical and other information with respect to coins, see "*Ray's Higher Arithmetic.*"

TEACHERS should ask questions on each Table, as on Dry Measure.

1. Reduce 13 lb. 11 oz. 16 pwt. 14 gr. to grains.

SUGGESTION.—When the denominations to be added are small, add while multiplying; when large, beginners should add after multiplying.

OPERATION.

```
lb.  oz.  pwt.  gr.
13   11   16    14
12
----
156 oz.
 11 oz. to be added.
----
167 oz. (Carried up.)
```

```
  167 oz. (Brought up.)
   20
 ----
 3340 pwt.
   16 pwt. to be added.
 ----
 3356 pwt.
   24
-----
13424
6712
-----
80544 gr.
   14 gr. to be added.
-----
80558 gr.
```

REVIEW.—82. For what is Troy Weight used? Repeat the Table. NOTE. What is the standard unit of weight in the United States?

2. Reduce 4 lb. to grains. . . . *Ans.* 23040 gr.
3. 5 lb. 4 oz. to ounces. . . . *Ans.* 64 oz.
4. 9 lb. 3 oz. 5 pwt. to pwt. . . *Ans.* 2225 pwt.
5. 14 lb. 11 oz. 19 pwt. 23 gr. to gr. *Ans.* 86399 gr.
6. 8 lb. 9 oz. 13 pwt. 17 gr. to gr. *Ans.* 50729 gr.
7. 171 gr. to pennyweights. *Ans.* 7 pwt. 3 gr.
8. 505 gr. to ounces. *Ans.* 1 oz. 1 pwt. 1 gr.
9. 12530 gr. to pounds. *Ans.* 2 lb. 2 oz. 2 pwt. 2 gr.
10. 805 pwt. to pounds. *Ans.* 3 lb. 4 oz. 5 pwt.
11. 25591 gr. to pounds. *Ans.* 4 lb. 5 oz. 6 pwt. 7 gr.

Art. 83. APOTHECARIES WEIGHT

Is used by Apothecaries in compounding medicines.

TABLE.

20 grains (gr.)	make	1 scruple,	marked	℈.
3 scruples	. . . . .	1 dram,	. . . . . .	ʒ.
8 drams	. . . . .	1 ounce,	. . . . .	℥.
12 ounces	. . . . .	1 pound,	. . . . .	℔.

The grain, ounce, and pound, in Apothecaries and Troy weight, are the same: but the *ounce* is differently divided.

☞ Questions should be asked on the Table, as before.

1. Reduce 3 ℔ to grains. . . . *Ans.* 17280 gr.
2. 4 ℔ 5 ʒ 2 gr. to grains. . . . *Ans.* 23342 gr.
3. 7 ℔ 2 ℈ to scruples. . . . *Ans.* 2018 ℈.
4. 7 ℔ 2 ℥ 1 ℈ to grains. . . . *Ans.* 41300 gr.
5. 67 ℥ to pounds. . . . . . . *Ans.* 5 ℔ 7 ℥.
6. 431 ʒ to pounds. . . . . *Ans.* 4 ℔ 5 ℥ 7 ʒ.
7. 975 ℈ to pounds. . . . . *Ans.* 3 ℔ 4 ℥ 5 ʒ.
8. 6321 gr. to pounds. *Ans.* 1 ℔ 1 ℥ 1 ʒ 1 ℈ 1 gr.
9. 30941 gr. to pounds. *Ans.* 5 ℔ 4 ℥ 3 ʒ 2 ℈ 1 gr.
10. 29239 gr. to pounds. . . *Ans.* 5 ℔ 7 ʒ 19 gr.

Review.—83. For what is Apothecaries Weight used? Repeat the Table. What is said of the grain, ounce, and pound, of Apothecaries and Troy weight? What is differently divided?

ART. 84. AVOIRDUPOIS WEIGHT

Is used in weighing heavy articles; as, groceries, coarse metals and medicines at wholesale.

TABLE.

16 drams (dr.) . . .	make 1 ounce, . .	marked	oz.
16 ounces	1 pound, ...		lb.
25 pounds	1 quarter, ...		qr.
4 quarters or 100 lb. . .	1 hundred weight, .		cwt.
20 hundred weight . . .	1 tun, ...		T.

NOTES.—1. The *standard* Avoirdupois pound of the United States is determined from the Troy pound, and contains 7000 grains Troy.

2. Formerly, 28 pounds were allowed for a quarter, 112 pounds for a hundred weight, 2240 pounds for a tun; these, called the *long* hundred and *long* tun, are chiefly used at the Custom House.

1. Reduce 2 cwt. to pounds. . . . *Ans.* 200 lb.
2. 3 cwt. 3 qr. to pounds. . . . *Ans.* 375 lb.
3. 1 T. 2 cwt. to pounds. . . . *Ans.* 2200 lb.
4. 3 T. 3 qr. to pounds. . . . *Ans.* 6075 lb.
5. 4 cwt. 1 qr. 19 lb. to pounds. . *Ans.* 444 lb.
6. 5 T. 3 qr. 15 lb. to pounds. . . *Ans.* 10090 lb.
7. 2 cwt. 3 qr. 2 lb. 12 oz. to ounces. *Ans.* 4444 oz.
8. 2 cwt. 17 lb. 3 dr. to drams. . . *Ans.* 55555 dr.
9. 1 T. 6 cwt. 4 lb. 2 oz. 10 dr. to dr. *Ans.* 666666 dr.
10. 4803 lb. to cwt. . . . . . . *Ans.* 48 cwt. 3 lb.
11. 22400 lb. to tuns. . . . . . *Ans.* 11 T. 4 cwt.
12. 2048000 dr. to tuns. . . . *Ans.* 4 T.
13. 64546 dr. to cwt. *Ans.* 2 cwt. 2 qr. 2 lb. 2 oz. 2 dr.
14. 97203 oz. to tuns. *Ans.* 3 T. 3 qr. 3 oz.
15. 544272 dr. to T. *Ans.* 1 T. 1 cwt. 1 qr. 1 lb. 1 oz.
16. In 52 parcels of sugar, each containing 18 lb., how many hundred weight? *Ans.* 9 cwt. 1 qr. 11 lb.

REVIEW.—84. For what is Avoirdupois Weight used? Repeat the Table. NOTE 1. From what is the standard Avoirdupois pound determined? 2. What is the *long* hundred and *long* tun? Where used?

ART. 85. LONG OR LINEAR MEASURE:

Used in measuring lengths, breadths, thickness, &c.

TABLE.

12	inches (in.). . make	1 foot, . . marked	ft.
3	feet	1 yard, ...	yd.
$5\frac{1}{2}$	yards or $16\frac{1}{2}$ feet . .	1 rod, perch, or pole,	rd.
40	rods or 220 yards .	1 furlong, ...	fur.
8 1760	furlongs or yd. = 5280 ft. } . .	1 mile, ...	mi.

ALSO, 60 geographic, or $69\frac{1}{2}$ statute miles, make 1 degree.
360 degrees make a great circle, or circumference of the earth.
3 miles make 1 league, used in measuring distances at sea.
4 inches make 1 hand, used in measuring the hight of horses.
6 feet make 1 fathom, used in measuring the depth of water.

NOTE.—The *standard unit* of length is the *yard.*

1. Reduce 2 yd. 2 ft. 7 in. to inches. *Ans.* 103 in.
2. 7 yd. 11 in. to inches. . . . . *Ans.* 263 in.
3. 12 mi. to rods. . . . . *Ans.* 3840 rd.
4. 7 mi. 6 fur. to rods. . . . . *Ans.* 2480 rd.
5. 9 mi. 31 rd. to rods. . . . . *Ans.* 2911 rd.
6. 133 in. to yards. . . . *Ans.* 3 yd. 2 ft. 1 in.
7. 181 in. to yards. . . . . *Ans.* 5 yd. 1 in.
8. 2240 rd. to miles. . . . . . . *Ans.* 7 mi.
9. 2200 rd. to miles. . . . . *Ans.* 6 mi. 7 fur.

☞ For examples involving Fractions, see page 170.

ART. 86. LAND OR SQUARE MEASURE

Is used in measuring land, or any thing in which both length and breadth are considered.

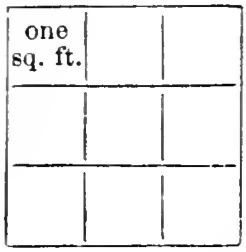
ART. 87. A figure having four equal sides, and four right angles (corners), is a *square.*

Hence, *A square inch* is a square, each side of which is a *linear* inch; that is, 1 inch in *length.*

REVIEW.—85. For what is Long Measure used? Repeat the Table. NOTE. What is the standard unit of length?

A *square foot* is a square, each side of which is a *linear* foot.

3 feet.



3 feet.

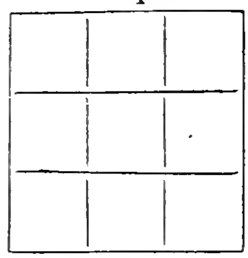
A *square yard* is a square, each side of which is a *linear* yard (3 feet).

The figure shows that 1 square yard, that is, 3 feet square, contains 9 square feet.

The number of small squares in any large square, is equal to the number of units in one side of the large square multiplied by itself. Thus,

In a square figure, each side of which is 8 inches, there are 64 square inches: in 1 foot square, or 12 inches on each side, there are $12 \times 12 = 144$ square in.

3 in. square.

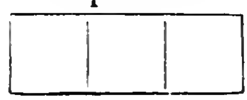


Art. 88. By 3 *inches square*, we mean a square figure, each side 3 inches: but,

3 *square inches* are 3 small squares, each an inch long, and an inch wide; and,

3 *inches square* contain 9 *square* inches.

3 square in.



The difference between 4 in. square, and 4 square in., is 12 square inches; between 6 miles square, and 6 square miles, is 30 sq. miles.

TABLE.

144	square inches	make	1	square foot,	marked sq. ft.
9	square feet	. . . .	1	square yard, . . .	sq. yd.
$30\frac{1}{4}$	square yards	. . .	1	sq. rod or perch, . .	P.
40	perches	. . . . . .	1	rood, ...	R.
4	roods	. . . .	1	acre, ...	A.
640	acres	. . . .	1	square mile, . . .	sq. mi.

ALSO, $7\frac{92}{100}$ inches	. . . .	1	link, ...	l.
4 rods or 66 feet	. .	1	chain, ...	ch.
80 chains	. . . .	1	mile, ...	mi.
1 square chain	. . .	16	perches, ...	P.
10 square chains	. . .	1	acre, ...	A.

Note.—Land is measured with a surveyor's or Gunter's chain; it is 4 rods or 66 feet in length, and is divided into 100 links.

Review.—86. For what is Land or Square Measure used? 87. What is a square? What is a square inch? A square foot? A square yard? To what is the number of small squares in a large square equal?

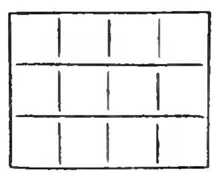
1. Reduce 8 sq. yd. to square inches. *Ans.* 10368 sq. in.
2. 4 A. to perches. . . . . *Ans.* 640 P.
3. 1 sq. mi. to perches. . . . . *Ans.* 102400 P.
4. 2 sq. yd. 3 sq. ft. to sq. in. . *Ans.* 3024 sq. in.
5. 5 A. 2 R. 20 P. to perches. . . . *Ans.* 900 P.
6. 960 P. to acres. . . . . *Ans.* 6 A.
7. 3888 sq. in. to square yards. . . *Ans.* 3 sq. yd.
8. 243 P. to acres. . . . *Ans.* 1 A. 2 R. 3 P.
9. 603 P. to acres. . . . *Ans.* 3 A. 3 R. 3 P.
10. 4176 sq. in. to sq. yd. . *Ans.* 3 sq. yd. 2 sq. ft.

ART. **89**. A *Rectangle* is a figure having four sides and four right angles. See the figure below.

The *Area* or Superficial content of a figure, is the number of times it contains its *unit of measure.*

The unit of measure for surfaces, is a *square* whose side is a linear unit, as a square inch, a square foot, &c.

1. How many square inches in a board 4 inches long and 3 inches wide?



SOL.—Dividing each of the longer sides into 4 equal parts, the shorter sides into 3 equal parts, and joining the opposite divisions by straight lines, the surface is divided into squares.

In each of the longer rows there are 4 squares, that is, as many as there are inches in the longer side; and there are as many such rows as there are inches in the shorter side. Hence,

The whole number of squares in the board is equal to the product obtained by multiplying together the numbers representing the length and breadth; that is, $4 \times 3 = 12$. Hence the

RULE FOR FINDING THE AREA OF A RECTANGLE.—*Multiply the length by the breadth: the product will be the superficial contents.*

REVIEW.—88. What is meant by 3 inches square? By 3 sq. inches? What is the difference between 4 inches square, and 4 square inches?

89. What is a Rectangle? What is the Area of a figure? What is the unit of measure for surfaces?

NOTE.—Both the length and breadth, if not in units of the same denomination, must be made so, before multiplying.

2. In a floor 16 feet long and 12 feet wide, how many square feet? *Ans.* 192 sq. ft.

3. How many square yards of carpeting will cover a room 5 yds. long and 4 yds. wide? *Ans.* 20 sq. yd.

4. How many square yards of carpeting will cover two rooms, one 18 feet long and 12 feet wide, the other 21 feet long and 15 feet wide? *Ans.* 59 sq. yd.

5. How many square yards in a ceiling 18 feet long and 14 feet wide? *Ans.* 28 sq. yd.

6. In a field 35 rods long and 32 rods wide, how many acres? *Ans.* 7 A.

ART. **90**. The Area of a Rectangle being equal to the product of the length by the breadth, and as the product of two numbers, divided by either of them, gives the other (Art. 37); therefore,

Rule.—*If the area of a rectangle be divided by either side, the quotient will be the other side.*

ILLUSTRATION.—In Example 1, Art. 89, if the area, 12, be divided by 4, the quotient, 3, is the width; or, divide 12 by 3, the quotient, 4, is the length.

NOTE.—Dividing the area of a rectangle by one of its sides, is really dividing the number of squares in the rectangle by the number of squares on one of its sides. Thus,

In dividing 12 by 4, the latter is not 4 linear inches, but the number of inches in a rectangle 4 in. long and 1 in. wide. See figure, Art. 89.

1. A floor containing 132 square feet, is 11 feet wide: what is its length? *Ans.* 12 ft.

2. A floor is 18 feet long, and contains 30 square yards: what is its width? *Ans.* 15 ft.

3. A field containing 9 acres, is 45 rods in length: what is its width? *Ans.* 32 rd.

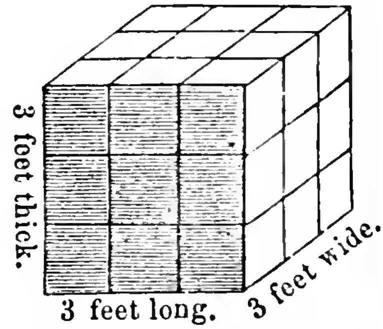
4. A field 35 rods wide, contains 21 acres: what is its length? *Ans.* 96 rd.

REVIEW.—89. What is the Rule for finding the Area of a Rectangle? **90.** The area and one side being given, how may the other side be found?

ART. 91. SOLID OR CUBIC MEASURE:

Used in measuring things having length, breadth, and thickness; as, timber, stone, earth.

ART. 92. A Cube is a SOLID, having 6 equal faces, which are squares.



If each side of a cube is 1 inch long, it is called a cubic inch; if each side is 3 feet (1 yard) long, as in the figure, it is a cubic or solid yard.

The base of a cube, being 1 square yard, contains $3 \times 3 = 9$ square feet; and 1 foot high on this base, contains 9 solid feet; 2 feet high contain $9 \times 2 = 18$ solid feet; 3 feet high contain $9 \times 3 = 27$ solid feet. Also, it may be shown that 1 solid or *cubic* foot contains $12 \times 12 \times 12 = 1728$ solid or *cubic* inches.

Hence, *the number of small cubes in any large cube, is equal to the length, breadth, and thickness, multiplied together.*

ART. 93. Any solid, whose corners resemble a cube, is a rectangular solid: boxes and cellars are of this form.

The solid contents of a rectangular solid are found, as in the cube, by multiplying together the length, breadth, and thickness.

TABLE.

1728 cubic inches (cu. in.)	make	1 cubic foot, marked	cu. ft.
27 cubic feet	. . . .	1 cubic yard, . . .	cu. yd.
40 feet of round or 50 feet of hewn timber	. . .	1 tun, ...	tn.
128 cubic feet $= 8 \times 4 \times 4 =$ 8 ft. long, 4 ft. wide, and 4 ft. high.		1 cord, ...	C.

NOTE.—A cord foot, is 1 foot in length of the pile which makes a cord. It is 4 feet wide, 4 feet high, and 1 foot long; hence, it contains 16 cubic feet, and 8 cord feet make 1 cord.

1. Reduce 2 cu. yd. to cubic inches. *Ans.* 93312 cu. in.
2. 28 cords of wood to cu. ft. *Ans.* 3584 cu. ft.

REVIEW.—91. For what is Cubic Measure used? 92. What is a cube? A cubic inch? Cubic yard? How many sq. ft. in one side of a cubic yard? How many cubic feet in a solid 3 ft. long, 3 ft. wide, and 1 ft. thick? If 2 ft. thick? If 3 ft. thick?

3. Reduce 34 cords of wood to C. ft. *Ans.* 272 C. ft.
4. 1 cord of wood to cu. in. *Ans.* 221184 cu. in.
5. 63936 cu. in. to cu. yd. *Ans.* 1 cu. yd. 10 cu. ft.
6. 492480 cu. in. to tn. round timber. *Ans.* 7 tn. 5 cu. ft.
7. How many cubic feet in a rectangular solid, 8 ft. long, 5 ft. wide, 4 ft. thick? See Art. 93. *Ans.* 160 cu. ft.
8. How many cubic yards of excavation in a cellar 8 yd. long, 5 yd. wide, 2 yd. deep? *Ans.* 80 cu. yd.
9. How many cubic yards in a cellar, 18 feet long, 15 feet wide, 7 feet deep? *Ans.* 70 cu. yd.
10. In a pile of wood 40 feet long, 12 feet wide, and 8 feet high, how many cords? *Ans.* 30 C.

Art. 94. CLOTH MEASURE

Is used in measuring cloth, muslin, ribbons, tape, &c.

TABLE.

$2\frac{1}{4}$ inches (in.)	make	1 nail,	marked na.
4 nails (or 9 in.)	. .	1 quarter of a yard,	qr.
4 quarters (36 in.)	.	1 yard,	yd.
3 quarters	. . . . .	1 Ell Flemish,	E. Fl.
5 quarters	. . . . .	1 Ell English,	E. En.
6 quarters	. . . . .	1 Ell French,	E. Fr.

This is a species of long measure, the yard being 36 inches in both.

1. Reduce 19 yd. to nails. . . . . . *Ans.* 304 na.
2. 14 E. Fl. to nails. . . . . *Ans.* 168 na.
3. 5 yd. 2 qr. 3 na. to nails. . . . *Ans.* 91 na.
4. 13 E. En. 1 qr. to nails. . . . *Ans.* 264 na.
5. 23 E. Fr. 3 qr. 2 na. to nails. . . *Ans.* 566 na.
6. 159 na. to yards. . . . *Ans.* 9 yd. 3 qr. 3 na.
7. 287 na. to E. Fr. . . *Ans.* 11 E. Fr. 5 qr. 3 na.
8. 6 yd. to Ells Flemish. . . . . . *Ans.* 8 E. Fl.

Reduce the yards to quarters; then the quarters to Ells Flem.

REVIEW.—92. To what is the number of small cubes in any large cube equal? 93. What is a rectangular solid? How is the solid contents found? Repeat the Table. 94. For what is Cloth Measure used? Repeat the Table. Of what is Cloth Measure a species?

9. Reduce 9 yd. 3 qr. to Ells Flemish. *Ans.* 13 E. Fl.
10. 12 yd. 1 qr. to E. Fl. . . . *Ans.* 16 E. Fl. 1 qr.
11. 37 E. Fl. to yards. . . . . . *Ans.* 27 yd. 3 qr.
12. 36 E. En. to Ells French. . . . *Ans.* 30 E. Fr.
13. 22 E. En. 4 qr. to Ells Fr. . . *Ans.* 19 E. Fr.
14. 47 E. En. to yards. . . . . *Ans.* 58 yd. 3 qr.

Art. 95. WINE OR LIQUID MEASURE:

For measuring all liquids except beer, milk, and ale.

TABLE.

	4	gills (gi.)	make	1 pint,	marked pt.
	2	pints	. . . . .	1 quart,	. . . . qt.
	4	quarts	. . . . .	1 gallon,	. . . . gal.
	63	gallons	. . . . .	1 hogshead,	. . . hhd.
	4	hogsheads	. . .	1 tun,	. . . . . T.
Also,	$31\frac{1}{2}$	gallons	. . . . .	1 barrel,	. . . . bl.
	42	gallons	. . . . .	1 tierce,	. . . . tr.
	84	gallons	. . . . .	1 puncheon,	. . pn.
	126	gallons	. . . . .	1 pipe,	. . . . . p.

The *Standard Unit* of Liquid Measure is a *gallon* of 231 cubic inches.

1. Reduce 17 gal. to pints. . . . *Ans.* 136 pt.
2. 13 gal. to gills. . . . . *Ans.* 416 gi.
3. 2 hhd. to pints. . . . . *Ans.* 1008 pt.
4. 5 T. to gills. . . . . . . *Ans.* 40320 gi.
5. 3 T. 3 hhd. to gallons. . . . . . *Ans.* 945 gal.
6. 1 hhd. 60 gal. 1 pt. to pints. . . *Ans.* 985 pt.
7. 2 hhd. 17 gal. 3 qt. to gills. . . *Ans.* 4600 gi.
8. 2 T. 62 gal. 1 pt. to gills. . . *Ans.* 18116 gi.
9. 96 gi. to gallons. . . . . *Ans.* 3 gal.
10. 6048 gi. to hogsheads. . . . . . . *Ans.* 3 hhd.
11. 32256 gi. to tuns. . . . . *Ans.* 4 T.
12. 4050 gi. to hogsheads. *Ans.* 2 hhd. 2 qt. 2 gi.

Review.—95. For what is Liquid Measure used? Repeat the Table. Note. What is the standard unit of Liquid Measure?

13. Reduce 30339 gi. to T. *Ans.* 3 T. 3 hhd. 3 gal. 3 gi.
14. 10125 gi. to T. *Ans.* 1 T. 1 hhd. 1 gal. 1 qt. 1 pt. 1 gi.
15. 3 puncheons to gills. . . . . . *Ans.* 8064 gi.
16. 5 pipes to quarts. . . . . . . *Ans.* 2520 qt.
17. 5712 pt. to tierces. . . . . *Ans.* 17 tr.

ART. 96. ALE OR BEER MEASURE

Is generally used in measuring ale, beer, and milk.

TABLE.

2 pints (pt.)	make	1 quart,	marked qt.
4 quarts	. . . . .	1 gallon, ...	gal.
36 gallons	. . . . .	1 barrel, ...	bl.
54 gallons	. . . . .	1 hogshead, . .	hhd.

NOTE.—The beer gallon contains 282 cubic inches. In most places milk is now sold by Wine Measure.

1. Reduce 4 hhd. to pints. . . . . . *Ans.* 1728 pt.
2. 7 hhd. 3 qt. to pints. . . . . . *Ans.* 3030 pt.
3. 1000 pt. to barrels. . . . . . *Ans.* 3 bl. 17 gal.
4. 443 pt. to hhd. . *Ans.* 1 hhd. 1 gal. 1 qt. 1 pt.

ART. 97. TIME MEASURE.

Time is a measured portion of duration. The parts into which time is divided are shown in this

TABLE.

	60 seconds (sec.)	make	1 minute,	marked min.
	60 minutes	. . . . . .	1 hour, ...	hr.
	24 hours	. . . .	1 day, ...	da.
	365 days 6 hours, (365¼ da.)		1 solar year, . . .	yr.
	100 years	. . . .	1 century, ...	cen.
Also,	7 days	. . . .	1 week, ...	wk.
	4 weeks	. . . .	1 month, (nearly)	mon.
	12 calendar months	. . .	1 year, ...	yr.
	365 days	. . . .	1 common year, .	yr.
	366 days	. . . .	1 leap year, . . .	yr.

☞ See Notes on page 98.

REVIEW.—96. For what is Beer Measure used? Repeat the Table. 97. What is Time? Repeat the Table.

NOTE 1. The EXACT length of the SOLAR year is 365 days, 5 hours, 48 minutes, 48 seconds: but, is usually considered to be 365 days, 6 hours. Hence,

One year being regarded as 365 days, 6 hours, the odd 6 hours of each year, make, in 4 years, 24 hours, an additional day. This gives 366 days to every fourth year, called LEAP YEAR: and,

All leap years may be exactly divided by 4; thus, 1848, 1852, 1856, are leap years; while 1847, 1850, 1855, are not so: and,

The additional day in leap years is added to February, making this month 29 days, instead of 28, as in common years.

NOTE 2.—The difference between the SOLAR year and 365 days and 6 hours, is about three-fourths of a day in 100 years, or 3 days in 400 years. Hence,

For greater accuracy, it is agreed *that of the centennial* (hundredth) *years, only those which are exactly divisible by* 400 *shall be leap years:* thus, 1900 will be a common year, and 2000 a leap year.

Teachers and advanced students will find interesting and instructive historic information on this subject in "*Ray's Higher Arithmetic.*"

NOTE 3. The year, as used by civilized nations, is divided into 12 calendar months, and numbered in their order as follows:

January,	1st month,	31 days.	July,	7th month,	31 days.
February,	2d ..	28 ..	August,	8th ..	31 ..
March,	3d ..	31 ..	September,	9th ..	30 ..
April,	4th ..	30 ..	October,	10th ..	31 ..
May,	5th ..	31 ..	November,	11th ..	30 ..
June,	6th ..	30 ..	December,	12th ..	31 ..

Thirty days hath September, April, June, and November:
All the rest have thirty-one, save February, which alone
Hath twenty-eight; and one day more we add to it, one year in four.

1. Reduce 2 hr. to seconds. . . . *Ans.* 7200 sec.
2. 7 da. to minutes. . . . . . . *Ans.* 10080 min.
3. 1 da. 3 hr. 44 min. 3 sec. to sec. *Ans.* 99843 sec.
4. 9 wk. 6 da. 10 hr. 40 min. to min. *Ans.* 100000 min.

REVIEW.—97. NOTE 1. What is the exact length of the solar year? What are leap years? February has how many days in leap years? 2. Which of the centennial years are leap years? Why? 3. How is the year divided? Give the number of days in each month.

5. Reduce 1 mon. 3 da. 4 min. to min. *Ans.* 44644 min.

6. Reduce 1 yr. 20 da. 19 hr. 15 min. 33 sec. to seconds, allowing 365 days to a year. *Ans.* 33333333 sec.

7. How many seconds in an *exact* solar year?
(See Note 1, page 98.) *Ans.* 31556928 sec.

8. Reduce 10800 sec. to hours. . . . *Ans.* 3 hr.

9. 432000 sec. to days. . . . . *Ans.* 5 da.

10. 7322 sec. to hours. . *Ans.* 2 hr. 2 min. 2 sec.

11. 4323 min. to days. . . . *Ans.* 3 da. 3 min.

12. 20280 min. to weeks. . . *Ans.* 2 wk. 2 hr.

13. 41761 min. to mon. *Ans.* 1 mon. 1 da. 1 min.

ART. 98. CIRCULAR MEASURE

Is used in estimating latitude and longitude, and in measuring the motions of the heavenly bodies.

A circle is divided into 360 equal parts, called degrees; each degree into 60 parts, called minutes; each minute into 60 parts, called seconds.

TABLE.

60 seconds (″)	make 1 minute,	marked ′.
60 minutes	1 degree,	. . . . °.
30 degrees	1 sign,	. . . . . . s.
12 signs or 360° . . .	1 circle,	. . . . . . c.

1. Reduce 5° 3′ to minutes. . . . *Ans.* 303′.

2. 8° 41′ 45″ to seconds. . . . *Ans.* 31305″.

3. 3^s 25′ to minutes. . . . . *Ans.* 5425′.

4. 1^c to seconds. . . . . *Ans.* 1296000″.

5. 244″ to minutes. . . . . . . *Ans.* 4′ 4″.

6. 915′ to degrees. . . . . *Ans.* 15° 15′.

7. 1861′ to signs. . . . . *Ans.* 1^s 1° 1′.

ART. 99. MISCELLANEOUS TABLE.

24 sheets of paper	make 1 quire.
20 quires	1 ream.
2 reams	1 bundle.

REVIEW.—98. For what is Circular Measure used? How is every Circle divided? Repeat the Table.

OF BOOKS.

A sheet folded in 2	leaves is called	a folio.
A sheet folded in 4	. . . .	a quarto, or 4to.
A sheet folded in 8	. . . .	an octavo, or 8vo.
A sheet folded in 12	. . . .	a duodecimo, or 12mo.
A sheet folded in 18	. . . .	an 18mo.

OF THINGS.

12 things	. . . . make	1 dozen.
12 dozen	. . . .	1 gross.
12 gross, or 144 dozen	. . . .	1 great gross.
20 things	. . . .	1 score.
196 pounds of flour	. . . .	1 barrel.
200 pounds of beef or pork	. . . . .	1 barrel.
100 pounds of fish	. . . .	1 quintal.
18 inches	. . . .	1 cubit.
22 inches (nearly,)	. . . .	1 sacred cubit.

Art. 100. PROMISCUOUS EXAMPLES.

1. What cost 2 bu. plums, at 5 cts. a pt.? *Ans.* $6.40

2. 3 bu. 2 pk. peaches, at 50 cts. a pk.? *Ans.* $7.

3. 3 pk. 3 qt. barley, at 3 cts. a pt.? *Ans.* $1.62

4. At 15 cents a peck, how many bushels of apples can be bought for $3? *Ans.* 5 bu.

5. If salt cost 2 cents a pint, how much can be bought with $1.66? *Ans.* 1 bu. 1 pk. 1 qt. 1 pt.

6. I put 91 bushels of wheat into bags of 3 bu. 2 pk. each: how many bags were required? *Ans.* 26.

Reduce both quantities to pecks, and then divide.

7. What is the value of 1 lb. 3 pwt. of gold ore, at 3 cents a grain? *Ans.* $174.96

8. How many spoons, each weighing 2 oz. 5 pwt., can be made from 2 lb. 5 oz. 5 pwt. of silver? *Ans.* 13.

9. How many rings, weighing 5 pwt. 7 gr. each, can be made from 1 lb. 8 oz. 18 pwt. 1 gr. of gold? *Ans.* 79.

10. If 1 ℔ 4℥ of calomel be divided into doses of 15 grains each, and sold at 12 cts. 5 m. a dose, what will it amount to? *Ans.* $50.

What will be the cost of

11. 1℔ 1℥ 1ʒ opium, at 4cts. a ℈? *Ans.* $12.60
12. 6cwt. 1qr. raisins, at 3cts. a lb.? *Ans.* $18.75
13. 1T. 1cwt. rice, at $2.25 a qr.? *Ans.* $189.
14. 7lb. 8oz. copper, at 5cts. an oz.? *Ans.* $6.

15. A physician put up 316 doses of rhubarb, of 20gr. each: how much did he use? *Ans.* 1℔ 1℥ 1ʒ 1℈.

16. How many nails, weighing 4 drams each, are in a parcel weighing 15lb. 9oz. 12dr.? *Ans.* 999.

17. I bought 44cwt. 2qr. 2lb. of cheese; each weighed 9lb. 15oz.: how many cheese did I buy? *Ans.* 448.

18. How many kegs, of 84lb. each, can be filled from a hhd. of sugar of 14cwt. 1qr. 3lb.? *Ans.* 17.

19. How many boxes, containing 12lb. each, can be filled from 7cwt. 2qr. 6lb. of tobacco? *Ans.* 63.

20. If a family use 3lb. 13oz. of sugar in a week, how long will 6cwt. 10lb. last them? *Ans.* 160wk.

21. What will 2A. 3R. 5P. of land cost, at 20 cts. a perch? *Ans.* $89.

22. What cost 2sq. yd. 2sq. ft. of ground, at 5 cents a square inch? *Ans.* $144.

23. How many leaves, 3in. long and 2in. wide, can be cut from 1sq. yd. of paper? *Ans.* 216.

24. A farmer has a field of 16A. 1R. 13P., to divide into lots of 1A. 1R. 1P. each: how many lots will it make? *Ans.* 13.

25. How many cu. in. in a block of marble 2ft. long, 2ft. high, 2ft. wide? *Ans.* 13824cu. in., or 8cu. ft.

26. One cubic foot of water weighs 1000 oz. Avoirdupois: what do 5cu. ft. weigh? *Ans.* 312lb. 8oz.

27. What is the weight of a quantity of water occupying the space of 1 cord of wood, each cu. ft. of water weighing 1000 oz. Avoirdupois? *Ans.* 4T.

28. A cubic foot of oak weighs 950 oz. Avoirdupois: what do 2 cords of oak weigh? *Ans.* 7T. 12cwt.

29. How many pieces, of 13yd. 3qr. 2na. each, are there in 666 yards? *Ans.* 48.

30. How many suits of clothes, each containing 5yd. 1qr., can be made from 147 yards? *Ans.* 28.

31. How many coat patterns of 2 yd. 1 qr. 1 na. each, can be cut from 37 yd. of cloth? *Ans.* 16.

32. How many suits of clothes of 3 yd. 2 qr. each, can be cut from 70 Ells Flemish of cloth? *Ans.* 15.

33. Find the cost of 1 hogshead of wine, at 5 cents a gill. *Ans.* $100.80

34. Find the cost of 5 bl. of molasses, each containing 31 gal. 2 qt., at 10 cents a quart. *Ans.* $63.

35. At 5 cents a pint, what quantity of molasses can be bought for $2? *Ans.* 5 gal.

36. At 1 ct. 5 m. a gill, how much cider can be bought for $12? *Ans.* 25 gal.

37. How many dozen bottles, each bottle holding 3 qt. 1 pt., can be filled from 1 hhd. of cider? *Ans.* 6 doz.

38. How many kegs of 4 gal. 3 qt. 1 pt. each, can be filled from 1 wine hhd.? *Ans.* 12; and 4 gal. 2 qt. left.

39. How many bottles, each holding 1 gal. 1 qt. 1 pt., can be filled from 165 wine gal.? *Ans.* 120.

40. What will 1 hogshead of beer cost, at 3 cents a quart? *Ans.* $6.48

41. The human heart beats 70 times a minute: how many times will it beat in a day? *Ans.* 100800.

42. How many seconds in the month of February, 1840? (See Note 1, page 98.) *Ans.* 2505600 sec.

43. How many hours longer is January than February, A. D. 1839? (See Note 1, page 98.) *Ans.* 72 hr.

44. What are the leap years between 1837 and 1850? (See Note 1, page 98.) *Ans.* 1840, 1844, 1848.

45. The exact length of the *solar* year is 365 da. 5 hr. 48 min. 48 sec.: how many such years are contained in 1609403328 seconds? *Ans.* 51 years.

46. How much time will a man gain in 60 yr., of 365 da. each, by rising 30 min. earlier each day? *Ans.* 456 da. 6 hr.

47. If a ship sail 8 miles an hour, how many miles will she sail in 3 wk. 2 da. 3 hr.? *Ans.* 4440 mi.

48. If a planet move through 2° in a day, how long will it require to move through 2^s 4°? *Ans.* 32 da.

49. In what time will one of the heavenly bodies move through a quadrant, (90°), at the rate of 43′ 12″ per minute? *Ans.* 2 hr. 5 min.

WHAT WILL BE THE COST

50. Of 2 reams paper, at 20 cts. a quire? *Ans.* $8.
51. 3 quires paper, at 2 cts. a sheet? *Ans.* $1.44
52. 3 dozen apples, at 2 cts. each? *Ans.* 72 cts.
53. 4 doz. doz. oranges, at 3 cts. each? *Ans.* $17.28
54. 5 gross buttons, at 5 cts. a doz.? *Ans.* $3.
55. 1 bl. of flour, at 4 cts. a pound? *Ans.* $7.84
56. 1 bl. pork, at 12 cts. 5 m. a pound? *Ans.* $25.

57. A farmer started for market with 6 dozen dozen eggs; he broke half a doz. doz., and sold the remainder at 1 ct. each: what did they amount to? *Ans.* $7.92

☞ For additional problems, see Ray's Test Examples.

ADDITION OF COMPOUND NUMBERS.

ART. **101**. The process of uniting numbers of different denominations into one sum or amount, is termed *Addition of Compound Numbers.*

1. A farmer sold three lots of wheat: the first lot contained 25 bu. 3 pk. 4 qt.; the second, 14 bu. 5 qt. 1 pt.; the third, 32 bu. 1 pk. 1 pt.: find the amount.

SOLUTION.—In writing the numbers, place units of the *same* denomination under each other, since only numbers of the same name can be added. Art. 20.

Beginning with the column of the *lowest* denomination, and adding together the units in it, the sum is 2 pints, which is reduced to quarts by dividing by 2, the number of pints in a quart, and there being no pint left, write a 0 in the order of pints, and carry the 1 (quart) to the column of quarts.

OPERATION.

bu.	pk.	qt.	pt.
25	3	4	0
14	0	5	1
32	1	0	1
72	1	2	0

Then, adding the numbers in the column of quarts, the sum is 10 quarts, which, being reduced, make 1 peck and 2 quarts; write the 2 quarts in the column of quarts, and carry the 1 (peck) to the column of pecks. The number in the column of pecks added, make 5 pk., which reduces to 1 bushel and 1 pk.; write the 1 pk. in the column of pecks, and carry the 1 bu. to the column of bushels: the sum of the numbers in this column is 72, which is written in the same manner as the sum of the numbers in the left hand column in Simple Addition.

	bu.	pk.	qt.	pt.
(2.)	3	2	0	1
	4	0	6	1
	1	3	7	1
	9	2	6	1

	bu.	pk.	qt.	pt.
(3.)	7	3	7	1
	6	2	0	0
	9	2	4	1
	24	0	4	0

Rule for Addition.—1. *Write the numbers to be added, placing units of the same denomination under each other.*

2. *Begin with the lowest order, add the numbers, and divide their sum by the number of units of this denomination, which make a unit of the next higher. Write the remainder under the column added, and carry the quotient to the next column.*

3. *Proceed in the same manner with all the columns to the last, under which write its entire sum.*

PROOF.—The same as in Addition of Simple Numbers.

NOTE.—In writing Compound Numbers, if any intermediate denomination is wanting, supply its place with a cipher.

REM.—In adding Simple Numbers, we carry one for every ten, as ten units of a lower order make one unit of the next higher. But in Compound Numbers, we carry one for the *number* of units of each lower order, which make one unit of the next higher.

EXAMPLES IN ADDITION.

TROY WEIGHT.

	lb.	oz.	pwt.	gr.
(4.)	13	0	17	19
	42	11	19	23
	31	9	0	4

APOTHECARIES.

	℔	℥	ʒ	℈	gr.
(5.)	7	2	7	2	14
	4	6	5	0	17
	8	1	6	1	10

REVIEW.—101. What is Addition of Compound Numbers? Why place units of the same denomination under each other? Where do you begin to add? How is the addition performed? Repeat the Rule. REM. For what do you carry one?

AVOIRDUPOIS WEIGHT.

	T.	cwt.	qr.	lb.
(6.)	45	3	3	20
	14	14	1	14
	19	17	2	11

	cwt.	qr.	lb.	oz.	dr.
(7.)	31	3	17	14	14
	25	0	20	10	10
	37	2	16	11	13

LONG MEASURE.

	mi.	fur.	rd.
(8.)	14	7	24
	12	3	16
	11	2	19

	yd.	ft.	in.
(9.)	2	1	11
	5	2	9
	3	1	8

SQUARE MEASURE.

	A.	R.	P.
(10.)	41	1	11
	64	2	24
	193	3	35

	sq.yd.	sq.ft.	sq.in.
(11.)	15	8	115
	20	7	109
	14	5	137

CUBIC MEASURE.

	C.	cu. ft.	cu.in.
(12.)	13	28	390
	15	90	874
	20	67	983

	cu.yd.	cu.ft.	cu.in.
(13)	50	18	900
	45	17	828
	46	20	990

CLOTH MEASURE.

	yd.	qr.	na.
(14.)	14	3	2
	6	2	1
	5	1	3

	E. Fl.	qr.	na.
(15.)	11	1	2
	13	3	0
	12	1	1

	E.En.	qr.	na.
(16.)	18	0	3
	16	1	2
	11	3	1

WINE MEASURE.

	T.	hhd.	gal.	qt.
(17.)	9	1	40	1
	7	2	58	3
	8	3	61	2

	gal.	qt.	pt.	gi.
(18.)	40	3	1	3
	16	1	0	2
	71	2	1	2

TIME MEASURE.

	mon.	wk.	da.	hr.	min.	sec.
(19.)	2	3	7	23	51	40
	1	2	4	19	30	37
	3	1	5	13	27	18

CIRCULAR.

	°	′	″
(20.)	180	18	26
	101	35	42
	47	11	44

21. Five loads of wheat measured thus: 21 bu. 3 pk. 2 qt. 1 pt.; 14 bu. 5 qt.; 23 bu. 2 pk. 1 pt.; 18 bu. 1 pk. 1 pt.; 22 bu. 7 qt. 1 pt.: how much in all? *Ans.* 100 bu.

22. A farmer raised of oats 200 bu. 3 pk. 1 pt.; barley, 143 bu. 2 qt. 1 pt.; corn, 400 bu. 3 pk.; wheat, 255 bu. 1 pk. 5 qt.: how much in all? *Ans.* 1000 bu.

23. I bought a silver dish weighing 2 lb. 10 oz. 15 pwt. 21 gr.; a bowl weighing 1 lb. 1 oz. 16 pwt. 14 gr.; a tankard, 2 lb. 8 oz. 5 pwt. 12 gr.: what did all weigh?
Ans. 6 lb. 8 oz. 17 pwt. 23 gr.

24. A druggist mixed together four articles: the first weighed 3℥ 4ʒ 1℈; the second, 4℥ 3ʒ 2℈; the third, 4ʒ 18 gr.; and the fourth, 6℥ 5ʒ 2℈ 18 gr.: what was the weight of all? *Ans.* 1℔ 3℥ 2ʒ 16 gr.

25. A grocer sold 5 hhd. of sugar: the first weighed 8 cwt. 1 qr. 11 lb.; the 2d, 4 cwt. 2 qr. 14 lb.; the 3d, 5 cwt. 19 lb.; the 4th, 7 cwt. 3 qr.; the 5th, 7 cwt. 3 qr. 9 lb. what did all weigh? *Ans.* 33 cwt. 3 qr. 3 lb.

26. Add together, 13 lb. 11 oz. 15 dr.; 17 lb. 13 oz 11 dr.; 14 lb. 14 oz.; 16 lb. 10 dr.; 19 lb. 7 oz. 12 dr. and 17 lb. 9 oz. 9 dr. *Ans.* 99 lb. 9 oz. 9 dr.

27. Two men depart from the same place: one travels 104 mi. 1 fur. 10 rd. due east; the other, 95 mi. 6 fur. 30 rd. due west: how far are they apart? *Ans.* 200 mi.

28. A man has 3 farms: in the 1st are 186 A. 3 R. 14 P.; in the 2d, 286 A. 17 P.; in the 3d, 113 A. 2 R. 9 P.: how much in all? *Ans.* 586 A. 2 R.

29. Add together, 17 sq. yd. 3 sq. ft. 119 sq. in.; 18 sq. yd. 141 sq. in.; 23 sq. yd. 7 sq. ft.; 29 sq. yd. 5 sq. ft. 116 sq. in. *Ans.* 88 sq. yd. 8 sq. ft. 88 sq. in.

30. A has 4 piles of wood: in the first 7 C. 78 cu. ft.; the 2d, 16 C. 24 cu. ft.; the 3d, 35 C. 127 cu. ft.; the 4th, 29 C. 10 cu. ft.: how much in all? *Ans.* 88 C. 111 cu. ft.

31. Bought 5 pieces of cloth; the first contained 17 yd. 3 qr. 2 na.; the 2d, 13 yd. 2 qr. 1 na.; the 3d, 23 yd. 2 na.; the 4th, 27 yd. 1 qr. 2 na.; the 5th, 29 yd. 1 qr. 2 na.: find the amount. *Ans.* 111 yd. 1 qr. 1 na.

32. I sold to A, 73hhd. 43gal. 3qt. 1pt. of wine; to B, 27hhd. 3gal.; to C, 15hhd. 3qt. 1pt.; to D, 162hhd. 1qt.: how much in all? *Ans.* 277hhd. 48gal.

33. I sold beer as follows: 1bl. 28gal.; 1bl. 17gal.; 5bl. 2gal.; 1gal. 2qt. 1pt.; 7gal. 2qt. 1pt.; 18gal. 3qt.; and 33gal.: what is the quantity? *Ans.* 10bl.

34. What day of the year is the 1st of May in common years? *Ans.* 121st.

35. What the 4th of July in leap years? *Ans.* 186th.

36. Through how many degrees does a ship pass, in sailing from Cape Horn, latitude 55° 58′ 30″ south, to New York, lat. 40° 42′ 40″ north? *Ans.* 96° 41′ 10″.

SUBTRACTION OF COMPOUND NUMBERS.

ART. **102**. The process of finding the difference between two numbers of different denominations, is termed *Subtraction of Compound Numbers.*

1. I have 67bu. 1pk. 3qt. of wheat: how much will remain after selling 34bu. 2pk. 1qt.?

OPERATION.

bu.	pk.	qt.
67	1	3
34	2	1
32	3	2

SOLUTION.—In writing the numbers, the less is placed *under* the greater, for convenience.

Place units of the same denomination under each other, because a number can be subtracted only from one of the *same* name.

Then subtract 1 (quart) from 3 (quarts), which leaves 2 (quarts) to be placed in the column of quarts.

In the next column, 2 (pecks) can not be taken from 1 (peck), but we can take 1 bushel from 67 bushels, reduce it to pecks, and add them to the 1 peck; this gives 5 pecks, from which take 2 pecks, and 3 pecks are left, to be placed in the column of pecks.

Then take 34 bushels from 66 bushels, as in Simple Subtraction, and write the remainder, 32 bushels, in the column of bushels.

Instead of diminishing the upper number (67 bu.) by 1, the result will be the same to increase the lower number (34 bu.) by 1. Art. 26.

REVIEW.—102. What is Subtraction of Compound Numbers? In writing the numbers, why place the less under the greater?

	bu.	pk.	qt.	pt.		bu.	pk.	qt.	pt.
(2.) From	12	0	1	0	(3.)	5	0	0	0
Take	8	2	1	1		1	0	0	1
Ans.	3	1	7	1		3	3	7	1

Rule for Subtraction.—1. *Write the numbers, the less under the greater, placing units of the same denomination under each other.*

2. *Begin with the lowest order, and, if possible, take the lower number from the one above it, as in Simple Subtraction.*

3. *But, if the lower number of any order be greater than the upper, increase the upper number by as many units of that denomination as make one of the next higher; subtract as before, and carry one to the lower number of the next higher order. Proceed in the same manner with each denomination.*

PROOF.—As in Subtraction of Simple Numbers.

REM.—In Simple Subtraction, when any lower figure is greater than the upper, we borrow ten, *ten units of a lower order making a unit of the next higher.* In Compound Numbers, when the lower number of any order is greater than that above it, *borrow the number of units in that order which makes a unit of the next higher.*

TROY WEIGHT.

	lb.	oz.	pwt.	gr.
(4.) From	18	6	16	13
Take	9	6	18	6

AVOIRDUPOIS.

	T.	cwt.	qr.	lb.	oz.	dr.
(5.)	14	12	2	20	11	14
	10	9	3	14	12	11

LONG MEASURE.

	mi.	fur.	rd.
(6.) From	18	5	36
Take	11	4	38

	yd.	ft.	in.
(7.)	4	1	10
	2	1	11

SQUARE MEASURE.

	A.	R.	P.
(8.) From	327	3	28
Take	77	2	30

	sq.yd.	sq.ft.	sq.in.
(9.)	19	6	72
	16	6	112

TIME MEASURE.

	da.	hr.	min.	sec.
(10.) From	245	17	40	37
Take	190	11	44	42

CIRCULAR.

	°	′	″
(11.)	29	54	53
	19	54	57

12. If 2 bu. 1 pk. 1 qt. be taken from a bag of 4 bu., what quantity will remain? *Ans.* 1 bu. 2 pk. 7 qt.

13. From 100 bushels, take 24 bu. 1 pt.
Ans. 75 bu. 3 pk. 7 qt. 1 pt.

14. From 19 lb. 9 oz. of silver, take 9 lb. 9 oz. 10 pwt. 10 gr. *Ans.* 9 lb. 11 oz. 9 pwt. 14 gr.

15. A silver bar of 8 lb. 2 oz. 11 pwt., is divided into 2 parts; the less weighs 2 lb. 4 oz. 7 pwt. 16 gr.: find the weight of the greater. *Ans.* 5 lb. 10 oz. 3 pwt. 8 gr.

16. From 3 ℔ 3 ℥ 1 ʒ 1 ℈ 12 gr., take 1 ℔ 7 ℥ 2 ℈ 18 gr. *Ans.* 1 ℔ 8 ℥ 1 ℈ 14 gr.

17. I bought 46 lb. 9 oz. of rice: after selling 19 lb. 4 dr., how much remained? *Ans.* 27 lb. 8 oz. 12 dr.

18. A wagon loaded with hay weighs 32 cwt. 2 qr. 16 lb.; the wagon alone weighs 8 cwt. 2 qr. 17 lb.: what is the weight of the hay? *Ans.* 23 cwt. 3 qr. 24 lb.

19. It is about 25000 miles round the earth: after a man has traveled 100 mi. 1 fur. 1 rd., what distance will remain? *Ans.* 24899 mi. 6 fur. 39 rd.

20. I had 146 A. 2 R. of land. I gave my son 86 A. 2 R. 14 P.: how much was left? *Ans.* 59 A. 3 R. 26 P.

21. From 8 C. 50 cu. ft. of wood, there is taken 3 C. 75 cu. ft.: how much is left? *Ans.* 4 C. 103 cu. ft.

22. From 25 E. Fl., take 14 E. Fl. 1 qr. 3 na.
Ans. 10 E. Fl. 1 qr. 1 na.

23. From 11 yards of cloth, there is cut 3 yd. 2 qr. 2 na.: what remains? *Ans.* 7 yd. 1 qr. 2 na.

24. A hhd. of wine leaked; only 51 gal. 1 qt. 2 gi. remained: how much was lost? *Ans.* 11 gal. 2 qt. 1 pt. 2 gi.

25. From 5 da. 10 hr. 27 min. 15 sec., take 2 da. 4 hr. 13 min. 29 sec. *Ans.* 3 da. 6 hr. 13 min. 46 sec.

Review.—102. Why place units of the same denomination under each other? At which column begin to subtract? Why? How is the subtraction performed? Repeat the Rule. What is the proof?

102. Rem. In subtraction of Compound Numbers when the lower number of any order is greater than the upper, what is to be done?

ART. **103**. To find the Time between any Two Dates.

Subtract the first date from the last, numbering the months according to their order. See note 3, page 98.

NOTE.—In finding the time between two dates, and also in Interest, consider 30 days 1 month, and 12 months 1 year.

26. A note, dated April 14th, 1835, was paid February 12th, 1837 : find the time between these dates.

SOLUTION.—In writing the dates, *observe* that February is the 2d month, and April, the 4th. In subtracting, 14 days can not be taken from 12; therefore add 30 to the 12 days, subtract, and carry 1 to the 4 months. As 5 months can not be taken from 2 months, add 12 to the latter, subtract, and carry 1 to the years.

OPERATION.

yr.	mon.	da.
1837	2	12
1835	4	14
1	9	28

27. The Independence of the United States was declared July 4th, 1776 : what length of time had elapsed on the 5th of March, 1857? *Ans.* 80 yr. 8 mon. 1 da.

28. A certain man was born June 24th, 1822: what was his age Aug. 1st, 1848? *Ans.* 26 yr. 1 mon. 7 da.

29. A man was born Nov. 25th, 1807; his son was born June 28th, 1832: what is the difference of their ages? *Ans.* 24 yr. 7 mon. 3 da.

30. The latitude of the Cape of Good Hope is 33° 55′ 15″ south; that of Cape Horn, 55° 58′ 30″ south : find the difference of latitude. *Ans.* 22° 3′ 15″.

31. The latitude of Gibraltar is 36° 6′ 30″ north; that of North Cape, in Lapland, 71° 10′ north: what their difference of latitude? *Ans.* 35° 3′ 30″.

32. A ship departs from latitude 10° 25′ 48″ north, and sails north to latitude 50° : through how many degrees of latitude has she sailed? *Ans.* 39° 34′ 12″.

33. Take 5 quires 11 sheets from a ream of paper, and how much will remain? *Ans.* 14 quires 13 sheets.

34. A man started to market with 6 doz. doz. eggs : he broke half a doz. doz.: how many were left? *Ans.* 792.

MULTIPLICATION OF COMPOUND NUMBERS.

ART. **104**. The process of taking a number consisting of different denominations, a certain number of times, is termed *Multiplication of Compound Numbers.*

1. A farmer takes to mill 5 bags of wheat, each containing 2 bu. 3 pk. 4 qt.: how much in all?

SOLUTION.—Begin at the lowest denomination: 5 times 4 quarts are 20 quarts, which reduced, make 2 pecks and 4 quarts; write the 4 (quarts) in the column of quarts, and carry the 2 pecks.

OPERATION.

bu.	pk.	qt.	
2	3	4	amt. in 1 bag.
		5	number of bags.
14	1	4	amt. in 5 bags.

Next, multiply the 3 pk. by 5, making 15 pk., to which add the 2 pk. carried, making 17 pk., which, reduced, give 4 bu. and 1 pk.; write the 1 (pk.) in the column of pk., and carry the 4 bu.

Then, multiply the 2 bu. by 5, add to the product the 4 bu. carried, and we have 14 bu. to be written in the column of bu.

In Compound, as in Simple Multiplication, multiply the lowest denomination first, so as to carry from a *lower* to a *higher* order.

2. Multiply 2 bu. 5 qt. 1 pt. by 6. *Ans.* 13 bu. 1 qt.
3. Multiply 2 bu. 2 pk. 2 qt. by 9. *Ans.* 23 bu. 2 qt.

Rule for Multiplication.—1. *Write the multiplier under the lowest denomination of the multiplicand.*

2. *Multiply the lowest denomination first, and divide the product by the number of units of this denomination, which make a unit of the next higher; write the remainder under the denomination multiplied, and carry the quotient to the product of the next higher denomination.*

REVIEW.—103. How find the time between any two dates? 104. What is Multiplication of Compound Numbers? Why multiply the lowest denomination first? Repeat the Rule.

3. *Proceed in like manner with all the denominations, writing the entire product at the last.*

Proof.—The same as in Simple Multiplication.

Rem.—1. In Simple Multiplication, we carry one for every ten, because ten units of a lower order make one unit of the next higher.

In Compound Multiplication, we carry one for the number of units in each lower order which make a unit of the next higher.

2. The multiplier is always an *abstract* number, and shows *how many times* the multiplicand is to be taken.

4. If 4 bu. 3 pk. 3 qt. 1 pt. of wheat make 1 bl. of flour, how much will make 12 bl.? *Ans.* 58 bu. 1 pk. 2 qt.

5. What is the weight of 6 silver spoons, each weighing 2 oz. 11 pwt. 6 gr.? *Ans.* 1 lb. 3 oz. 7 pwt. 12 gr.

6. What is the weight of 10 bars of silver, each 10 oz. 10 pwt. 10 gr.? *Ans.* 8 lb. 9 oz. 4 pwt. 4 gr.

7. I put up 8 packages of medicine, of 4 ℥ 2 ℈ 15 gr. each: what did all weigh? *Ans.* 2 ℔ 8 ℥ 7 ʒ 1 ℈.

8. Find the weight of 9 hhd. of sugar, of 8 cwt. 2 qr. 12 lb. each. *Ans.* 3 T. 17 cwt. 2 qr. 8 lb.

9. How much hay in 7 loads, each weighing 10 cwt. 3 qr. 14 lb.? *Ans.* 3 T. 16 cwt. 23 lb.

10. If a ship sail 208 mi. 4 fur. 16 rd. a day, how far will she sail in 15 days? *Ans.* 3128 mi. 2 fur.

11. If a man travel 30 mi. 4 fur. 10 rd. a day, how far will he travel in 12 da.? *Ans.* 366 mi. 3 fur.

12. Multiply 130 A. 3 R. 30 P. by 4. *Ans.* 523 A. 3 R.

13. Multiply 23 cu. yd. 9 cu. ft. 228 cu. in. by 12. *Ans.* 280 cu. yd. 1 cu. ft. 1008 cu. in.

14. How many yards in 6 pieces of muslin, of 26 yd. 2 qr. 2 na. each? *Ans.* 159 yd. 3 qr.

15. Multiply 62 gal. 1 qt. 1 pt. by 8. *Ans.* 499 gal.

16. How many gallons in 5 casks, each containing 123 gal. 2 qt. 1 pt.? *Ans.* 618 gal. 1 pt.

Review.—104. Rem. 1. In Compound Multiplication, for what number do you carry one? 2. Is the multiplier concrete or abstract?

17. In a solar year are 365 da. 5 hr. 48 min. 48 sec.: how long has B lived, who is 12 years old?
Ans. 4382 da. 21 hr. 45 min. 36 sec.

18. Multiply 4° 11′ 15″ by 8. *Ans.* 1^s 3° 30′.

19. How many buttons in 3 great gross? *Ans.* 5184.

20. How much water in 48 casks, each containing 62 gal. 1 qt. 1 pt. 1 gi.? Here, 48 = 6 × 8. (See Art. 47.)

OPERATION.

gal.	qt.	pt.	gi.	
62	1	1	1	
			6	
374	1	1	2	= water in 6 casks.
			8	
2995	2	0	0	= water in 48 casks.

21. Multiply 2 bu. 3 pk. 5 qt. by 24. *Ans.* 69 bu. 3 pk.
22. 3 mi. 5 fur. 16 rd. × 60. *Ans.* 220 mi. 4 fur.
23. 6 A. 3 R. 30 P. × 56. *Ans.* 388 A. 2 R.
24. 8 cwt. 2 qr. 14 lb. 12 oz. 13 dr. × 22.
Ans. 9 T. 10 cwt. 1 qr. 9 oz. 14 dr.
25. 3 gal. 2 qt. 1 pt. 1 gi. × 112. (112 = 8 × 2 × 7.)
Ans. 6 hhd. 31 gal. 2 qt.

NOTE.—When the multiplier exceeds 12, and is not a composite number, it is most convenient to multiply each denomination, and reduce it separately, and write only the result.

26. Multiply 16 cwt. 2 qr. 24 lb. by 119.
Ans. 99 T. 12 cwt. 6 lb.
27. 37 yd. 3 qr. 2 na. × 89. *Ans.* 3370 yd. 3 qr. 2 na.
28. 47 gal. 3 qt. 1 pt. × 59.
Ans. 44 hhd. 52 gal. 2 qt. 1 pt.
29. A travels 27 mi. 3 fur. 35 rd. in 1 day: how far will he travel in 1 mon. of 31 days? *Ans.* 852 mi. 5 rd.

30. In 17 piles, each containing 7 C. 98 cu. ft.: what quantity of wood? *Ans.* 132 C. 2 cu. ft.

DIVISION OF COMPOUND NUMBERS.

ART. **105.** The process of dividing numbers consisting of different denominations, is termed *Division of Compound Numbers.*

The Divisor may be either a Simple, or Compound Number. This gives rise to two cases:

FIRST CASE.—To find how often one Compound Number is contained in another Compound Number.

SECOND CASE.—To divide a compound number into a given number of equal parts. (See example 1, below.)

NOTE.—Examples of the First Case are solved by reducing both divisor and dividend to the same denomination, and then dividing. They are treated of under Reduction. See examples 6, 8, page 100.

Examples of the Second Case, are usually considered under the head of Compound Division.

ART. **106**. 1. Divide 14 bu. 1 pk. 7 qt. 1 pt. of wheat equally among 3 persons.

OPERATION.

	bu.	pk.	qt.	pt.
3)	14	1	7	1
Ans.	4	3	2	1

SOLUTION.—Divide the highest denomination first, as in Simple Numbers, that if there be a remainder, it may be reduced to the next lower order, and added to it.

First, 3 in 14 is contained 4 times, and 2 bushels left; write the 4 in the order of bushels, and reduce the remaining 2 bushels to pecks, to which add the 1 in the order of pecks, and the sum is 9 pecks, which, divided by 3, gives a quotient of 3 pecks, to be written in the order of pecks.

Next, divide 7 quarts by 3, and the quotient is 2 quarts, with 1 quart remainder; write the 2 quarts in the order of quarts; reduce the 1 quart to pints, and add it to the 1 in the order of pints; the sum is 3 pints, and this, divided by 3, gives a quotient of 1 pint, which write in the order of pints.

(2.)

	bu.	pk.	qt.
7)	33	2	6
Ans.	4	3	2

(3.)

	da.	hr.	min.	sec.
5)	17	12	56	15
Ans.	3	12	11	15

Rule for Division.—1. *Write the quantity to be divided in the order of its denominations, beginning with the highest; write the divisor on the left.*

2. *Begin with the highest denomination, divide each number separately, and write the quotient beneath.*

3. *If a remainder occurs after any division, reduce it to the next lower denomination, and, before dividing, add to it the number of its denomination.*

PROOF.—The same as in Simple Division.

REM.—In Simple Numbers when a remainder occurs in dividing any order except the lowest, it is prefixed to the figure in the next lower order, which is equivalent to multiplying it by 10, and adding to the product the figure in the next lower order.

Hence, in both Simple and Compound Division, each remainder is multiplied by that number of units of the next lower order which make a unit of the same order as the remainder.

Each *partial* quotient is of the same denomination as that part of the dividend from which it is derived.

4. Divide 67 bu. 3 pk. 4 qt. 1 pt. by 5.
Ans. 13 bu. 2 pk. 2 qt. 1 pt.

5. Eight silver tankards of the same size, weigh 14 lb. 8 oz. 16 pwt. 16 gr.: what is the weight of each?
Ans. 1 lb. 10 oz. 2 pwt. 2 gr.

6. What will 1 dollar weigh; the weight of 10 dollars being 8 oz. 12 pwt. 12 gr.? *Ans.* 17 pwt. 6 gr.

7. Eleven bl. of sugar weigh 35 cwt. 1 qr. 17 lb. 3 oz. 7 dr.: find the weight of one. *Ans.* 3 cwt. 22 lb. 5 dr.

8. I traveled 39 mi. 7 fur. 8 rd. in 7 hours: at what rate per hour did I travel? *Ans.* 5 mi. 5 fur. 24 rd.

9. Divide 62 yd. 3 na. by 5. *Ans.* 12 yd. 1 qr. 3 na.

REVIEW.—105. What is Division of Compound Numbers? What may the divisor be? What is the first Case? What the second? NOTE. How are examples of the first Case solved?

106. In dividing a Compound Number, why divide the highest denomination first? When a remainder occurs, how proceed, Rule?

NOTE.—If the divisor exceeds 12, and is a composite number, we may divide by its factors in succession, as in Art. 47.

10. Divide 69 A. 1 R. 24 P. by 16.
(16 = 8 × 2, or 4 × 4.)

The true remainder is found in the same manner as in Division of Simple Numbers. (Art. 47).

OPERATION.

	A.	R.	P.
2)	69	1	24
8)	34	2	32
Ans.	4	1	14

11. 490 bu. 2 pk. 4 qt. ÷ 100. *Ans.* 4 bu. 3 pk. 5 qt.

12. 266 lb. 9 oz. 10 dr. ÷ 50. *Ans.* 5 lb. 5 oz. 5 dr.

13. 339 lb. 7 oz. 9 pwt. 18 gr. ÷ 42. *Ans.* 8 lb. 1 oz. 17 gr.

14. 114 da. 22 hr. 45 min. 35 sec. ÷ 54. *Ans.* 2 da. 3 hr. 5 min. 17 sec., and 17 sec. rem.

15. 45 T. 18 cwt. ÷ 17.

When the divisor exceeds 12, and is not a composite number, divide each denomination as in Long Division. Art. 46.

OPERATION.

```
     T.  cwt.
17)45  18(2T. 14cwt. Ans.
   34
   --
   11
   20
   --
  220 = cwt. in 11 tuns.
   18 = cwt. to be added.
  ---
17)238(14 cwt.
   17
   --
    68
    68
    --
```

16. 1027 lb. 1 oz. 8 dr. ÷ 23. *Ans.* 44 lb. 10 oz. 8 dr.

17. 17 lb. 7 oz. 6 pwt. 6 gr. ÷ 245. *Ans.* 17 pwt. 6 gr.

18. 309 bu. 2 pk. 2 qt. ÷ 78. *Ans.* 3 bu. 3 pk. 7 qt.

19. 788 mi. 4 fur. 9 rd. ÷ 319. *Ans.* 2 mi. 3 fur. 31 rd.

REVIEW.—106. Repeat the Rule for Division. REM. In both Simple and Compound Division, by what is each remainder multiplied? Of what denomination or order is each *partial* quotient figure?

Art. 107. PROMISCUOUS EXAMPLES.

1. If from 1 ℔ of Ipecac there be taken, at one time 4℥ 2ʒ 13gr., and at another 3℥ 1ʒ 2℈ 14gr., how much will be left? *Ans.* 4℥ 3ʒ 2℈ 13gr.

2. A silversmith has 3 pieces of silver, the first weighing 8oz. 10pwt. 12gr.; the 2d, 9oz. 3pwt. 5gr.; the 3d, 8oz. 9pwt. 7gr. If the loss in refining be 5pwt. 12gr., and the rest be made into 15 spoons of equal weight, what will each spoon weigh? *Ans.* 1oz. 14pwt. 12gr.

3. I have two farms, one 104A. 2R. 37P., the other, 87A. 1R. 38P.: I reserve for myself 40A. 1R., and divide the remainder equally among my 3 sons: required the share of each. *Ans.* 50A. 2R. 25P.

4. A boy residing 3fur. 25rd. from school, attends twice a day: how far does he travel in 30 days? *Ans.* 54mi. 3fur.

5. B loaned A money on the 27th of June, 1843, and A paid it February 3d, 1845; A then lent B a sum to be kept 5 times as long: how long is B to keep A's money? *Ans.* 8 yr.

6. A ship in latitude 35° 30′ north, sails 20° 35′ south; then 14° 20′ north; then 25° 4′ 30″ south; then 6° 19′ 20″ north: what is now her latitude? *Ans.* 10° 29′ 50″ north.

Art. 108. LONGITUDE AND TIME.

Difference of Longitude and Time between different places.

The circumference of the earth, like other circles, is divided into 360°, (equal parts), called *degrees of Longitude.*

The sun appears to pass entirely round the earth (360°) once in 24 hours, *one day;* and in 1 hour, over 15°. ($360 \div 24 = 15°$). And,

As 15° equal 900′ of a *degree*, and 1 hour equals 60 minutes of *time*, therefore, the sun passes in 1 min. of *time* over 15′ of a *degree.* ($900' \div 60 = 15'$). And, As 15′ equal 900″ of a *degree*, and 1 min. of *time* equals 60 sec. of *time*, therefore, in 1 sec. of *time* the sun passes over 15″ of a *degree*. (900″ ÷ 60 = 15″). Hence the

TABLE FOR COMPARING LONGITUDE AND TIME.

15° of longitude, = 1 hour of time.
15′ of longitude, = 1 min. of time.
15″ of longitude, = 1 sec. of time.

1. How many hr. min. and sec. of time correspond to 18° 25′ 30″ of longitude? *Ans.* 1 hr. 13 min. 42 sec.

ANALYSIS.—By inspection of the Table, it is evident that,

Degrees (°) of longitude divided by 15, give hours of *time*:
Minutes (′) of longitude divided by 15, give minutes of *time*:
Seconds (″) of longitude divided by 15, give seconds of *time*.

Hence, if 18° 25′ 30″ of lon. be divided by 15, the quotient will be the *time* in hr. min. and sec. corresponding to that longitude.

To find the time corresponding to any difference of longitude:

Rule.—*Divide the longitude by* 15, *according to the Rule for Division of Compound Numbers, and mark the quotient* hr. min. sec., *instead of* ° ′ ″.

Conversely: To find the longitude corresponding to any difference of time:

Rule.—*Multiply the time by* 15, *according to the Rule for the Multiplication of Compound Numbers, and mark the product* ° ′ ″ *instead of* hr. min. sec.

For short methods of operation, see "*Ray's Higher Arithmetic.*"

2. The difference of longitude between two places is 30°: what is their diff. of time? *Ans.* 2 hr.

3. The diff. of lon. between two places is 71° 4′: what the diff. of time? *Ans.* 4 hr. 44 min. 16 sec.

REVIEW.—108. At what rate does the sun appear to move in a day? In an hour? In a minute? In a second? What do degrees, minutes, and seconds of longitude divided by 15 give? Repeat the Rules.

4. The diff. of lon. between New York and Cincinnati is 10° 35′ : what the diff. of time? *Ans.* 42 min. 20 sec.

5. The diff. of time between Cincinnati and Philadelphia is 37 min. 20 sec. : what the diff. of lon.? *Ans.* 9° 20′.

6. The diff. of time between New York and St. Louis is 1 hr. 4 min. 56 sec. : what the diff. of lon.? *Ans.* 16° 14′.

7. The diff. of time between London and Washington is 5 hr. 8 min. 4 sec. : what the diff. of lon.? *Ans.* 77° 1′.

DIFFERENCE IN TIME.

ART. **109**. It is noon (12 o'clock), at any place when the sun is on the meridian of that place; and,

As the sun appears to travel from the east *toward* the west, when it is noon at any place, it is *after* noon *east* of that place, and *before* noon *west* of that place:

Hence, a place has *later* or *earlier time* than another, according as it is *east* or *west* of it. Therefore,

When the time at one place is given, the time at another, EAST *of this, is found by* ADDING *their difference of time: Or, if* WEST, *by* SUBTRACTING *their difference of time.*

8. When it is noon at Cincinnati, what is the time at Philadelphia? *Ans.* 37 min. 20 sec. past noon.

9. When it is 11 o'clock A. M. at New York, what is the time in lon. 30° east of New York? *Ans.* 1 P. M.

10. When 12 o'clock (noon) at Philadelphia, what is the time at Cincinnati? *Ans.* 11 hr. 22 min. 40 sec. A. M.

11. When it is 11 o'clock A. M. at New York, what is the time at St. Louis? *Ans.* 9 hr. 55 min. 4 sec. A. M.

12. Wheeling, Va., is in lon. 80° 42′ west: the mouth of the Columbia river in lon. 124° west: when it is 1 o'clock, P. M., at Wheeling, what is the time at the mouth of Columbia river? *Ans.* 10 hr. 6 min. 48 sec. A. M.

REVIEW.—109. When is it noon at any place? What the time east or west of that place? Why is the time later *east?* Why *earlier* west? Having the time at one place, how find the time at another?

VIII. FACTORING.

ART. **110**. DEFINITIONS.—1. An *integer* is a whole number; as, 1, 2, 3, &c.

DEF. 2. Whole numbers are divided into two classes; *prime* numbers, and *composite* numbers.

DEF. 3. A *prime* number can be exactly divided only by itself and unity, (1).

Thus, 1, 2, 3, 5, 7, 11, &c., are *prime*.

DEF. 4. A *composite* number (Art. 33) can be exactly divided by some other number besides itself and unity.

Thus, 4, 6, 8, 9, 10, &c., are *composite*.

DEF. 5. Two numbers are *prime to each other* when unity (1), is the only number that will exactly divide both. Thus, 4 and 5 are prime to each other.

REM.—Two prime numbers are always *necessarily* prime to each other. Also, two composite numbers are sometimes prime to each other: thus, 4 and 9 are prime to each other.

DEF. 6. An *even* number can be divided by 2 without a remainder. Thus, 2, 4, 6, 8, &c., are *even*.

DEF. 7. An *odd* number *can not* be divided by 2 without a remainder. Thus, 1, 3, 5, 7, &c., are *odd*.

REM.—All *even* numbers except 2, are composite numbers, while *odd* numbers are partly prime and partly composite.

DEF. 8. A *divisor* of a number will exactly divide it; that is, without a remainder: thus, 2 is a divisor of 4; 5 of 10, &c.

A divisor of a number is a measure of that number.

DEF. 9. One number is *divisible* by another, when the former contains the latter without a remainder. Thus, 6 is *divisible* by 2.

REVIEW.—110. What is an integer? How are the whole numbers divided? What is a prime number? Give examples. What a composite? Give examples. When are two numbers prime to each other? Give examples. What is an even number? An odd? Give examples.

DEF. 10. A *multiple* (dividend), of a number is the product arising from taking it a certain number of times: thus, 6 is a multiple of 2, because it is equal to 2 taken 3 times. Hence,

A multiple of a number can be divided by it without a remainder. Therefore, *every multiple is a composite number.*

DEF. 11. A *factor* of a number is a number that will exactly divide it: thus, 4 is a factor of 8, 12, 16, &c.

REM.—The terms, *factor, divisor* and *measure*, all mean the same thing. Every composite number being the product of two or more factors, each factor must exactly divide it, (Art. 37).

Hence, every *factor* of a number, is a *divisor* of that number.

DEF. 12. A *prime* factor of a number is a prime number that will exactly divide it: thus, 3 is a *prime* factor of 12; while 4 is a *factor* of 12, but not a *prime* factor.

Therefore, *all the prime factors of a number, are all the prime numbers that will exactly divide it:* thus, 1, 3, and 5, are all the prime factors of 15.

Every composite number is equal to the product of all its prime factors: thus, all the prime factors of 10 are 1, 2, and 5; $1\times2\times5=10$.

DEF. 13. An ALIQUOT part of a number, is a number that will exactly divide it: thus, 1, 2, 3, 4, and 6, are aliquot parts of 12.

RESOLVING NUMBERS INTO PRIME FACTORS.

ART. **111.** The smaller composite numbers may be resolved into their prime factors by *inspection;* thus,

$$6=2\times3;\ 8=2\times2\times2;\ 9=3\times3;\ 10=2\times5.$$

In the case of large numbers, their factors are found by trial; that is, by dividing by each of the prime num-

---

REVIEW.—110. REM. Are the even numbers prime or composite? Are the odd numbers? What is a divisor of a number? Give examples. When is one number divisible by another? Give examples.

110. What is a multiple? Give examples. A factor? Give examples. REM. What terms besides divisor are used in the same sense? Why is every factor a divisor? What is a prime factor? Give an example.

bers 2, 3, 5, 7, &c.; *the prime factors of any number, being all the prime numbers that will exactly divide it.*

In determining either the factors, or the prime factors, of a number, observe the following principles.

PRINCIPLE 1.—*A factor of a number is a factor of any multiple of that number.*

Thus, 3 is a factor of 6, and of any number of times 6; for, 6 is 2 *threes*, and any number of times 6 will be *twice* as many times 3.

PRINCIPLE 2. *A factor of any two numbers is also a factor of their sum.*

Since each number contains the factor a certain number of times, their sum must contain it as many times as both numbers.

Thus, 2 being a factor of 6 and 8, it is a factor of their sum; for, 6 is 3 *twos*, and 8 is 4 *twos*, and their sum is 3 *twos*+4 *twos*,=7 *twos*.

ART. **112**. From these two Principles, are derived

SIX PROPOSITIONS.

PROP. 1. Every number ending with 0, 2, 4, 6, or 8, is divisible by 2.

ILLUSTRATION.—Every number ending with 0, is either 10 or some number of tens; and, since 10 is divisible by 2, any number of tens will be divisible by 2. Prin. 1.

Again: any number ending with 2, 4, 6, or 8, may be considered a certain number of tens, plus the figure in units' place:

And, as each of the two parts of the number is divisible by 2, therefore, Prin. 2, the number itself is divisible by 2.

Conversely: *No number is divisible by 2, unless it ends with a 0, 2, 4, 6, or 8.*

PROP. II. Every number is divisible by 4, when the number denoted by its first two digits is divisible by 4.

REVIEW.—110. What is an aliquot part of a number? Give examples. 111. How may the smaller composite numbers be resolved into prime factors? What are the prime factors of 6? Of 8? Of 9? Of 10?

111. In determining the factors of a number, what two principles are used? Explain the first principle. The second.

Illustration.—Since 100 is divisible by 4, any number of hundreds is divisible by 4; and any number of more than two places of figures, may be regarded as a certain number of hundreds, plus the number denoted by the first two digits.

Then, since both parts of the number are divisible by 4, Prin. 2, the number itself is divisible by 4.

Conversely: *No number is divisible by* 4, *unless the number denoted by its first two digits is divisible by* 4.

Prop. III. Every number is divisible by 5, when its right hand digit is 0 or 5.

Illustration.—Ten being divisible by 5, and every number consisting of two or more places of figures, being composed of tens, plus the figure in the units' place:

Therefore, if this is 5, both parts of the number are divisible by 5; hence, Prin. 2, the number itself is divisible by 5.

Conversely: *No number is divisible by* 5, *unless its right hand digit is* 0 *or* 5.

Prop. IV. Every number whose first digits are 0, 00, &c., is divisible by 10, 100, &c.

Illustration.—If the first figure is a cipher, the number is either 10, or some multiple of 10; and,

If the first two figures are ciphers, the number is either 100, or some multiple of 100; hence, Prin. 1, the proposition is true.

Conversely: *No number is divisible by* 10, 100, *&c., unless it ends with* 0, 00, *&c.*

Prop. V. Every composite number is divisible by the product of any two or more of its prime factors.

Illustration.—Thus, the number 30 is equal to $2 \times 3 \times 5$; now, if 30 be divided by the product of either two of the factors, the quotient must be the other factor; if *not* so, the product of the three factors would not be 30: and,

The same may be shown of any other composite number.

Review.—112. When is a number divisible by 2? Why? When not divisible by 2? When is a number divisible by 4? Why? When not divisible by 4? When is a number divisible by 5? Why? When not divisible by 5? When is a number divisible by 10, 100, &c.? Why? When not divisible by 10, 100, &c.?

It follows, from Prop. 5, that *if any even number is divisible by 3, it is also divisible by 6.* For, if an even number, it is divisible by 2; and, being divisible by 2 and by 3, it is also divisible by their product, 2×3, or 6.

PROP. VI. Every prime number, except 2 and 5, ends with 1, 3, 7, or 9: a consequence of Prop. 1 and 3.

ART. **113**. 1. What are the prime factors of 30?

SOLUTION.—If 2 is exactly contained in 30, it will be a factor of 30. By trial, it is found to be a factor. Again,

OPERATION.

```
2)30
 ---
3)15
 ---
   5
```

If 3 exactly divides 30, it will be a factor of it; but, since 30 is 2 times 15, if 3 is a factor of 15, it will also be a factor of 30, (Art. 111, Prin. 1.)

To ascertain if 3 is a factor of 30, see if it is a factor of 15. Trial shows that 3 is a factor of 15; hence, it is a factor of 30.

For the same reason, whatever number is a factor of 5, is a factor of 15 and 30; but 5 is a prime number, having no factor except itself and unity; hence, the prime factors of 30, are 1, 2, 3, and 5.

2. Find the prime factors of 42. *Ans.* 1, 2, 3, 7.
3. Find the prime factors of 70. *Ans.* 1, 2, 5, 7.

RULE

FOR RESOLVING A COMPOSITE NUMBER INTO PRIME FACTORS.

Divide the given number by any prime number that will exactly divide it; divide the quotient in the same manner, and so continue to divide, until a quotient is obtained which is a prime number; the last quotient and the several divisors will constitute the prime factors of the given number.

REM.—1. It will generally be most convenient to divide, first by the smallest prime number that is a factor.

REVIEW.—112. By what is every composite number divisible? Why? When any even number is divisible by 3, by what is it also divisible? With what figures do all prime numbers, except 2 and 5, terminate?

113. Find the prime factors of 30, and explain the process. What is the rule for resolving a number into prime factors?

2. The *least* divisor of any number is a prime number; for, if it were composite, it might be separated into factors, which would be still *smaller* divisors of the given numbers. Art. 111, Prin. 1.

Hence, the prime factors of any number may be found, by first dividing it by the *least* number that will exactly divide it; then divide the quotient as before, and so on.

3. Since 1 is a factor of every number, either prime or composite, it is not usually specified in reckoning the factors of a number.

SEPARATE INTO PRIME FACTORS,

4.	8.	*Ans.* 2, 2, 2.	13.	26.	*Ans.* 2, 13.
5.	12.	*Ans.* 2, 2, 3.	14.	27.	*Ans.* 3, 3, 3.
6.	14.	*Ans.* 2, 7.	15.	28.	*Ans.* 2, 2, 7.
7.	15.	*Ans.* 3, 5.	16.	30.	*Ans.* 2, 3, 5.
8.	16.	*Ans.* 2, 2, 2, 2.	17.	32.	*Ans.* 2, 2, 2, 2, 2.
9.	18.	*Ans.* 2, 3, 3.	18.	34.	*Ans* 2, 17.
10.	20.	*Ans.* 2, 2, 5.	19.	35.	*Ans.* 5, 7.
11.	22.	*Ans.* 2, 11.	20.	66.	*Ans.* 2, 3, 11.
12.	24.	*Ans.* 2, 2, 2, 3.	21.	98.	*Ans.* 2, 7, 7.

22. What are the prime factors of 105? *Ans.* 3, 5, 7.
23. Of 168? . . . . *Ans.* 2, 2, 2, 3, 7.
24. 216? . . . . *Ans.* 2, 2, 2, 3, 3, 3.
25. 330? . . . . *Ans.* 2, 3, 5, 11.

To find the prime factors common to two numbers, resolve each into prime factors: then take the factors common to both.

WHAT PRIME FACTORS ARE COMMON

26. To 110 and 210? . . . . *Ans.* 2, 5.
27. 105 and 231? . . . . *Ans.* 3, 7.
28. 330 and 390? . . . . *Ans.* 2, 3, 5.
29. 231 and 330? . . . . *Ans.* 3, 11.

REVIEW.—113. REM. 1. What prime factor should be first taken as a divisor? 2. Why is the *least* divisor of any number a prime number?
REM. 3. Why is unity not reckoned among the prime factors of a number? How may the prime factors common to two numbers be found?

ART. **114**. Since any composite number is divisible not only by each of its prime factors, but also by the product of any two or more of them, (Art. 112, Prop. V.),

Therefore, to find the several divisors of a composite number, resolve it into its prime factors, and form from them as many different products as possible.

Thus, $30=2\times3\times5$, and its several divisors are 2, 3, 5, and 2×3, 2×5, and 3×5.

WHAT ARE THE SEVERAL DIVISORS

1. Of 42? . . . . . . *Ans.* 2, 3, 7, 6, 14, 21.
2. 105? . . . . . . *Ans.* 3, 5, 7, 15, 21, 35.
3. 20? . . . . *Ans.* 2, 5, 4, 10.
4. 24? . . . . . . *Ans.* 2, 3, 4, 6, 8, 12.

☞ For additional problems, see Ray's Test Examples.

IX. GREATEST COMMON DIVISOR.

ART. **115**. A *divisor* or *measure* of a number (Art. 110, Def. 8), is a number that will divide it without a remainder; thus, 2 is a divisor of 4; 3 of 6, &c.

A *common* divisor of two or more numbers, is a number that will divide each without a remainder; thus, 2 is a common divisor of 12 and 18.

The *greatest* common divisor of two or more numbers, is the greatest number that will divide each without a remainder; thus, 6 is the greatest common divisor of 12 and 18.

REM.—Two numbers may have several *common* divisors, but only one *greatest* common divisor.

G. C. D. should be read, *greatest common divisor.*

REVIEW.—114. How may the several divisors of a composite number be found? Why? 115. What is a divisor of a number? Give examples.

115. What is a *common* divisor of two or more numbers? Give example. What the *greatest* common divisor? Give example.

ART. **116**. *To find the greatest common divisor of two numbers.*

FIRST METHOD.

1. Find the greatest common divisor of 70 and 154.

SOLUTION.—Resolving the numbers into their prime factors, by inspection or by the rule (Art. 113), shows that $70 = 2 \times 5 \times 7$, and $154 = 2 \times 7 \times 11$.

OPERATION.

$$70 = 2 \times 5 \times 7$$
$$154 = 2 \times 7 \times 11$$

Ans. $2 \times 7 = 14$

Since 2 and 7 are factors of each of the numbers, both may be exactly divided by 2 or 7, or by their product: $2 \times 7 = 14$.

As 2 and 7 are the *only* prime factors common to the numbers, no number except 2, 7, and their product, $2 \times 7 = 14$, will exactly divide both of them: therefore, $2 \times 7 = 14$, is the G. C. D.

2. Find the G. C. D. of 6 and 10. . . . *Ans.* 2.
3. Of 30 and 42. . . . . . . *Ans.* $2 \times 3 = 6$.

Rule I.—*Resolve the given numbers into prime factors; the product of the factors which are common, will be the greatest common divisor.*

REM.—The *greatest* com. divisor of two numbers contains, as factors, all the other com. divisors of those numbers. Thus, 6, the *greatest* com. divisor of 30 and 42, contains, as factors, 2 and 3, the only remaining com. divisors of those numbers.

WHAT IS THE GREATEST COMMON DIVISOR

4. Of 42 and 54? . . . . . . *Ans.* $2 \times 3 = 6$.
5. 70 and 110? . . . . . . *Ans.* $2 \times 5 = 10$.
6. 105 and 165? . . . . . . *Ans.* $3 \times 5 = 15$.
7. 60 and 90? . . . . . *Ans.* $2 \times 3 \times 5 = 30$.
8. 140 and 210? . . . . . *Ans.* $2 \times 5 \times 7 = 70$.
9. 66 and 154? . . . . . . *Ans.* $2 \times 11 = 22$.
10. 154 and 280? . . . . . . *Ans.* $2 \times 7 = 14$.
11. 231 and 273? . . . . . . *Ans.* $3 \times 7 = 21$.

REVIEW.—115. REM. Can two numbers have more than one common divisor? 116. Find the G. C. D. of 30 and 42, by separating each into prime factors. What is Rule I?

116. REM. What factors does the G. C. D. of two numbers contain?

NOTE.—When there are more than two numbers, resolve each into *prime* factors; then take the product of the *common* factors.

WHAT IS THE GREATEST COMMON DIVISOR OF

12. 30, 42, and 66? . . . *Ans.* $2\times 3 = 6$.

13. 60, 90, and 150? . . *Ans.* $2\times 3\times 5 = 30$.

When the numbers are large, it is better to adopt

ART. 117. THE SECOND METHOD.

This method depends on the following principles:

1ST PRIN. *A divisor of a number, is a divisor of any multiple of that number;* Art. 111, Prin. 1.

2D PRIN. *A common divisor of* TWO *numbers, is a divisor of their* SUM; Art. 111, Prin. 2.

3D PRIN. *A common divisor of two numbers, is a divisor of their* DIFFERENCE.

Since each of the numbers contains the com. divisor a certain number of times, their difference must contain it as many times as the larger contains it more times than the smaller.

Thus, 2, being a divisor of 14 and 8, must be a divisor of their difference; for, 14 is 7 *twos*, and 8 is 4 *twos*, and their difference is 7 *twos* minus 4 *twos* = 3 *twos*.

Therefore, *if a number be separated into two parts, any number which will exactly divide the given number and one of its parts, will also exactly divide the other.*

In this case, either of the parts is the *difference* between the given number and the other part.

4TH PRIN. *The greatest common divisor of two numbers, is a divisor of their remainder after division.* See Solu.

1. Find the G. C. D. of 16 and 44.

SOLUTION.—As 16 is a divisor of itself, if it be a divisor of 44, it will be the G. C. D. required, since no number can have a divisor *greater* than itself. But 16 is contained twice in 44, with a remainder 12; hence, 16 is not the G. C. D. Since the G. C. D. is *a* divisor of 16 and 44, by first Prin., it will be a

OPERATION.

```
16)44(2
   32
   --
   12)16(1
      12
      --
       4)12(3
```

divisor of $16\times 2=32$; hence, by 3d Prin., it must be a divisor of $44-32=12$; that is, *the G. C. D. of two numbers is also a divisor of their remainder after division.*

Hence, the G. C. D. of 16 and 44 is also a com. divisor of 12 and 16, and it can not *exceed* 12. Since 12 is a divisor of itself, if it be a divisor of 16, it must be the G. C. D. sought.

By dividing 12 into 16, the remainder is 4; hence, 12 is not the G. C. D.; but, by Prin. 4th, the G. C. D. of 12 and 16 is a divisor of 4, their remainder after division; hence, the G. C. D. of 16 and 44 can not exceed 4, and must be a divisor of 4 and 12.

By dividing 12 by 4, there is no remainder; hence, 4 is a divisor of 12, and therefore, of $12\times 1+4=16$, Prin. 2d. And,

Since 4 is a divisor of 12 and 16, it must be a divisor of $16\times 2+12=44$; and since the G. C. D. can not exceed 4, and 4 is a divisor of 16 and 44, therefore, 4 is the G. C. D. sought.

2. What is the G. C. D. of 14 and 35? *Ans.* 7.
3. What is the G. C. D. of 9 and 24? *Ans.* 3.

Rule II.—*Divide the greater number by the less, and that divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remains; the last divisor will be the greatest common divisor.*

Note.—To find the G. C. D. of more than *two* numbers, first find the G. C. D. of two of them, then of that com. divisor and one of the remaining numbers, and so on for all the numbers; the last com. divisor will be the G. C. D. of all the numbers.

FIND THE GREATEST COMMON DIVISOR OF

4. 42 and 495.	*Ans.* 3.	8. 337 and 759.	*Ans.* 1.
5. 247 and 323.	*Ans.* 19.	9. 9873 and 69087.	*Ans.* 3.
6. 285 and 465.	*Ans.* 15.	10. 1814 and 1776.	*Ans.* 2.
7. 532 and 1274.	*Ans.* 14.	11. 693 and 1815.	*Ans.* 33.

Review.—117. On what principles does the second method of finding the G. C. D. depend? Explain the third principle. Find the G. C. D. of 9 and 24, and explain the fourth principle. What is Rule II?

117. Note. How is the G. C. D. of more than two numbers found?

12. Find the G. C. D. of 2145 and 3471. . . . *Ans.* 39.
13. Of 840 and 17017. . . . . . . *Ans.* 7.
14. Of 66284 and 153452. . . . . *Ans.* 908.
15. Of 40, 55, and 105. . . . . *Ans.* 5.
16. Of 70, 154, and 819. . . . . *Ans.* 7.
17. Of 120, 168, and 1768. . . . . . . *Ans.* 8.

☞ For additional problems, see Ray's Test Examples.

X. LEAST COMMON MULTIPLE.

Art. **118.** A *multiple* (dividend), of a number, is a number that can be divided by it without a remainder.

Thus, 12 is a multiple of 3, because 3 is contained in 12 an exact number of times, 4. Art. 110, Def. 10.

A *common* multiple (dividend), of two or more numbers, is a number that can be divided by *each*, without a remainder.

Thus, 24 is a common multiple of 3 and 4.

The *least* common multiple of two or more numbers, is the *least* number that can be divided by *each* without a remainder.

Thus, 12 is the *least* common multiple of 3 and 4.

Rem.—1. As the Com. Mul. of two or more numbers contains each of them as a factor, it is a composite number.

2. As the continued product of two or more numbers is divisible by each of them, a Com. Mul. of two or more given numbers may always be found by taking their continued product; and,

Since any multiple of this product will be divisible by each of the given numbers, (Art. 111, 1st Prin.), an unlimited number of Com. Multiples may be found for any given numbers.

Review.—118. What is a multiple of a number? Give an example. What is a *common* multiple of two or more numbers? What the *least* common multiple? Rem. 1. Is a common multiple of two or more numbers, a prime or composite number? Why? 2. How may a Com. Mul. always be found? How many Com. Multiples may numbers have? Why?

ART. **119**. *To find the least common multiple of two or more numbers.*

FIRST METHOD.

One number is divisible by another, when it contains all the prime factors of that number.

Thus, 30 is divisible by 6, because $30=2\times3\times5$, and $6=2\times3$; the prime factors of 6, which are 2 and 3, being also factors of 30.

One number is *not* divisible by another, unless it contains *all* the prime factors of that other.

Thus, 10 is not divisible by 6, because 3, one of the prime factors of 6, is not a factor of 10.

Hence, *a common multiple of two or more numbers must contain all the prime factors in those numbers;* and,

To be the *least* common multiple, (L. C. M.), it must *not* contain any prime factor not found in some one of the numbers.

☞ L. C. M. should be read, *least common multiple.*

1. What is the L. C. M. of 6 and 10?

OPERATION.

$$6=2\times3$$
$$10=\not{2}\times5$$
$$2\times3\times5=30 \text{ Ans.}$$

SOLUTION.—By factoring, $6=2\times3$, and $10=2\times5$. A number composed of the factors 2, 3, and 5, will contain all the factors in each of the numbers 6 and 10, and will contain no other factor; therefore, *cross out* (cancel) the factor 2, in one of the numbers; the product of the remaining factors, $2\times3\times5=30$, will be the L. C. M. of 6 and 10.

2. What is the L. C. M. of 6, 8, and 12?

OPERATION.

$$6=\not{2}\times3$$
$$8=2\times2\times2$$
$$12=\not{2}\times\not{2}\times\not{3}$$
$$3\times2\times2\times2=24 \text{ Ans.}$$

SOLUTION.—By factoring the numbers, the prime factor 2 occurs *once* in 6, *three* times in 8, and *twice* in 12; hence it must occur *three* times, and only three times, in the L. C. M.; therefore, after reserving it as a factor three times, *cancel* it in the other numbers.

The prime factor 3 occurs *once* in 6 and *once* in 12; hence, it *must* occur *once*, and only once, in the L. C. M. After reserving it once as a factor, cancel the other factor 3. The L. C. M. is then found by multiplying together the figures not canceled.

3. Find the L. C. M. of 12 and 30. . . *Ans.* 60.
4. Of 6, 10, and 18. . . . . *Ans.* 90.

Rule I.—*Separate the numbers into prime factors; then multiply together* ONLY *such of those factors, as are necessary to form a product that will contain all the prime factors in each number, using no factor oftener than it occurs in any one number.*

NOTE.—The solution of Ex. 2, shows that the same factor must be taken the *greatest* number of times it occurs in either number. After factoring, cancel (cross out) the needless factors.

FIND THE LEAST COMMON MULTIPLE OF

5. 6, 8, 9. *Ans.* 72.
6. 6, 15, 35. *Ans.* 210.
7. 10, 12, 15. *Ans.* 60.
8. 9, 15, 18, 24. *Ans.* 360.
9. 8, 15, 12, 30. *Ans.* 120.
10. 14, 21, 30, 35. *Ans.* 210.

SECOND METHOD.

ART. **120**. The L. C. M. of two or more numbers, contains all the prime factors of each of the numbers *once*, and no other factors.

For, if it did not contain all the prime factors of any number, it would not be divisible by that number; and, if it contained any prime factor not found in either of the numbers, it would not be the *least* common multiple.

Thus, the L. C. M. of 4 (2×2), and 6 (2×3), must contain the factors 2, 2, 3, and no others.

1. Find the L. C. M. of 6, 9, and 12.

OPERATION.

```
2)6  9  12
  ---------
3)3  9   6
  ---------
  1  3   2
```

$2\times3\times3\times2=36$ *Ans.*

SOLUTION.—Arranging the numbers as in the margin, we find that 2 is a prime factor common to two of them.

Hence, 2 must be a factor of the L. C. M.; therefore, place it on the left, and cancel it in the numbers of which it is a factor, by dividing by it.

Next, observe that 3 is a factor common to the quotients and the remaining number, and hence, (Art. 111,) is a factor of the given numbers, and must be a factor of the L. C. M.

therefore, place it on the left, and cancel it in each of the numbers in the 2d line, by dividing by it. As the numbers 3 and 2, in the 3d line, have no common factors to cancel, we do not divide them.

Thus we find, that 2, 3, 3, and 2, are all the prime factors in the given numbers; hence, their product, $2\times3\times3\times2=36$ is the L. C. M. of 6, 9, and 12.

In this operation, let the learner notice,

1st. The number 36 is a *common* multiple, because it contains all the prime factors in each of the numbers; it is the *least* C. M., because all the needless factors were canceled by dividing.

2d. To cancel needless factors, divide by a *prime* number.

By dividing by a *composite* number, in some cases, all the needless factors are not canceled; thus, in the preceding example, 6 will exactly divide two of the numbers; but,

In dividing by 6, a factor, 3, is left uncanceled in the multiple 9, and thus the *L. C. M.* is not obtained.

2. Find the L. C. M. of 6 and 10. . . . *Ans.* 30.
3. Of 15, 21, 35. . . . . *Ans.* 105.

Rule II.—1. *Place the numbers in a line, divide by any prime number that will divide two or more of them without a remainder, and place the quotients and undivided numbers in a line beneath.*

2. *Divide this line as before: continue to divide till no number greater than* 1 *will exactly divide two or more of the numbers.*

3. *Multiply together the divisors and the numbers in the lowest line, and their product will be the least common multiple.*

Rem.—If the given numbers contain no common factor, their product will be the L. C. M. Thus, the L. C. M. of 4, 5, and 9, is $4\times5\times9=180$.

Review.—119. When is one number divisible by another? Give an example. When not divisible? Give an example. What actors must the Com. Mul. contain?

119. What prime factors must the L. C. M. *not* contain? Find the L. C. M. of 6, 10, and 18, and explain the operation. What is Rule I?

119. Note. How often must the same factor be found in the L. C. M.?

FIND THE LEAST COMMON MULTIPLE OF

4.	9, 12.	*Ans.* 36.	8.	6, 10, 15, 18.	*Ans.* 90.
5.	14, 21.	*Ans.* 42.	9.	7, 11, 13, 3.	*Ans.* 3003.
6.	6, 9, 15.	*Ans.* 90.	10.	63, 12, 84, 7.	*Ans.* 252.
7.	4, 14, 35.	*Ans.* 140.	11.	54, 81, 63.	*Ans.* 1134.

12. Of 8, 12, 20, 24, 25. . . . . . . *Ans.* 600.
13. 9, 10, 24, 25, 32, 45. . . . *Ans.* 7200.
14. 98, 72, 64, 21, 18. . . . . *Ans.* 28224.
15. 2, 3, 4, 5, 6, 7, 8, 9. . . . *Ans.* 2520.

XI. COMMON FRACTIONS.

ART. **121**. A single thing, (Art. 1), is called a *unit*, or *one*, which may be divided into *equal* parts.

Thus, suppose 3 apples are to be equally divided between 2 boys: after giving one to each, there would remain one to be divided into *two equal* parts, to complete the division.

The equal parts into which a unit is divided are *fractions.*

ART. **122**. When a unit, or single thing, is divided into *two equal* parts, one of the parts is *one-half*.

If it is divided into *three equal* parts, one of the parts is *one-third;* two of the parts, *two-thirds*.

If divided into *four equal* parts, one of the parts is *one-fourth;* two of the parts, *two-fourths;* and three of the parts, *three-fourths*.

If divided into *five equal* parts, the parts are *fifths;* if into *six equal* parts, *sixths*, and so on. Hence,

When a unit is divided into equal parts, the parts are named from the number of parts into which the unit is divided.

REVIEW.—120. Ex. 1. Why divide by 2? By 3? Why multiply together the numbers 2, 3, 3, and 2? Why is 36 a Com. Mul. of 6, 9, and 12? Why the least? To cancel needless factors, why *not* divide by a *composite* number? What is Rule II?

120. REM. If the numbers contain no common factor, how is their L. C. M. found? 121. How do you divide 3 apples equally between 2 boys? When a unit is divided into equal parts, what are the parts called?

ART. **123**. The *value* of one of the parts depends on the *number* of parts into which the unit is divided.

Thus, if 3 apples of equal size be divided, one into 2, another into 3, and another into 4 *equal* parts, the *thirds* will be less than the *halves*, the *fourths* less than the *thirds*.

ART. **124**. Fractions are divided into two classes, *Common* and *Decimal*.

Common Fractions are expressed by two numbers, one above the other, with a horizontal line between them.

Thus, one-half is expressed by $\frac{1}{2}$; two-thirds by $\frac{2}{3}$.

The number *below* the line is the *denominator*: it *denominates*, or gives *name* to the fraction. It shows the *number* of parts into which the unit is divided.

The number *above* the line is the *numerator*: it *numbers* the parts, showing how many parts are taken.

Thus, in the fraction $\frac{3}{5}$, the denominator, 5, shows that the unit is divided into *five equal* parts, and the numerator, 3, shows that the fraction contains 3 of those parts.

The numerator and denominator together, are called the *terms* of the fraction. Thus, the terms of $\frac{3}{5}$, are 3 and 5.

ART. **125**. ANOTHER METHOD.

In the definition of numerator and denominator, reference is had to a *unit* only. This is the simplest method of considering a fraction; but, there is another mode:

ILLUSTRATION.—To divide 2 apples equally among 3 boys, divide each apple into *three equal* parts, making 6 parts in all; then give to each boy 2 of the parts, expressed by $\frac{2}{3}$.

REVIEW.—122. When a unit is divided into two equal parts, what is one part called? When divided into three equal parts, what is one part called? What two parts? When divided into four equal parts, what is one part called? Two parts? Three parts? When a unit is divided into equal parts, from what are the parts named?

123. On what does the value of one of the parts depend? Which is greater, 1-half or 1-third? 1-third or 1-fourth? 1-fourth or 1-fifth?

124. Into what two classes are fractions divided? How are common fractions expressed? What is the number below the line? Why? What the number above the line? Why? What are the terms?

In selecting one boy's share, take the 2 parts from *one* apple, or 1 part from each of the *two* apples; hence,

$\frac{2}{3}$ expresses either 2 thirds of 1 thing, or 1 third of 2 things.

Also, $\frac{3}{5}$ expresses 3 fifths of 1 thing, or 1 fifth of 3 things.

Therefore, the numerator of a fraction expresses *the number of units to be divided;* and the denominator the *divisor*, or *what part is taken* from each. Hence,

A fraction is expressed in the form of an *unexecuted* division,

In which, The DIVIDEND is the NUMERATOR;
The DIVISOR is the DENOMINATOR;
The QUOTIENT is the FRACTION itself.

$\frac{1}{3}$ is the quotient of 1 (numerator) $\div$ 3 (denom.);
$\frac{4}{5}$ is the quotient of 4 (numerator) $\div$ 5 (denom.);
$\frac{7}{6}$ is the quotient of 7 (numerator) $\div$ 6 (denom.).

ART. **126**. Since fractions arise from division, one of the *signs* of division (Art. 40,) is used in expressing them; the numerator being written above, and the denominator below, a horizontal line.

Thus, three-fifths is written $\frac{3}{5}$; two-sixths is written $\frac{2}{6}$.

TO READ COMMON FRACTIONS.

Read the number of parts taken, as expressed by the numerator; then the size of the parts, as expressed by the denominator.

TO WRITE COMMON FRACTIONS.

Write the numerator, place a horizontal line below it, under which write the denominator.

REM.—In reading, $\frac{2}{3}$ means *two-thirds of one.* There are two other methods, (Art. 125): thus, $\frac{2}{3}$ may be read, *one-third of two,* or *two divided by three;* but these methods are rarely used.

FRACTIONS TO BE READ.

$\frac{6}{7}$, $\frac{5}{8}$, $\frac{4}{9}$, $\frac{10}{11}$, $\frac{11}{17}$, $\frac{13}{17}$, $\frac{5}{21}$, $\frac{6}{29}$, $\frac{14}{45}$, $\frac{10}{65}$, $\frac{11}{90}$, $\frac{4}{100}$, $\frac{16}{240}$, $\frac{8}{1243}$.

REVIEW.—125. What do two-thirds express? What does the numerator of a fraction express? The denominator? In what form is a fraction expressed? What is the dividend? The divisor? The quotient? Give examples. 126. What sign is used in fractions? Why? How is a common fraction read? Give examples. How written? Give examples.

FRACTIONS TO BE WRITTEN.

Three-*sevenths* :	One-*twelfth* :	Fourteen-*twenty-ninths* :
Five-*ninths* :	Two-*thirteenths* :	Thirty-one-*ninety-thirds* :
Six-*tenths* :	Nine-*sixteenths* :	Twenty-three-*one-hundred-and-fourths.*
Seven-*elevenths* :	Eleven-*twentieths* :	

If an orange be cut into 8 equal parts, what fraction will express 3 of the parts?

If 3 apples be divided equally among 4 boys, what part of an apple will be given to each boy?

What part of 1 apple, is a third part of 2 apples?

What expresses the quotient of 5, divided by 8?

ART. **127.** A whole number may be expressed in the form of a fraction, by writing 1 *below* it for a denominator.

Thus, 2 may be written $\frac{2}{1}$ and is read two-*ones.*
3 may be written $\frac{3}{1}$ and is read three-*ones.*
4 may be written $\frac{4}{1}$ and is read four-*ones.*

But, 2 ones are 2; 3 ones, 3, &c.; hence, the value of the number is not changed.

ART. **128.** If 2 apples be divided, each into four equal parts, there will be 8 parts in all. Three of the parts (fourths) are expressed by $\frac{3}{4}$; 4 parts by $\frac{4}{4}$; 5 parts by $\frac{5}{4}$.

When the number of parts taken is *less* than the number into which the unit is divided, the value expressed is *less* than one, or the whole thing;

When the number of parts taken is *equal* to the number into which the unit is divided, the value, as $\frac{4}{4}$, is *equal* to 1;

When *greater* than the number into which the unit is divided, the value, $\frac{5}{4}$, is greater than 1. Hence,

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REVIEW.—126. Of the fractions to be read, which expresses parts of the largest size? Which the smallest? Which the least number? Which the greatest? Which the same? Which parts of the same size?

127. How may a whole number be expressed in fractional form? Does this change its value? Why not? 128. When is the value of a fraction less than 1? When equal? When greater? Illustrate by examples.

1st. *When the numerator is less than the denominator, the value of the fraction is less than* 1.

2d. *When the numerator is equal to the denominator, the value of the fraction is equal to* 1.

3d. *When the numerator is greater than the denominator, the value of the fraction is greater than* 1.

DEFINITIONS.

ART. **129.** 1. A *Fraction* is an expression of one or more of the equal parts of a unit, or one thing.

2. A *Proper Fraction* has a numerator less than the denominator; as, $\frac{1}{3}$, $\frac{3}{7}$, and $\frac{8}{9}$.

3. An *Improper Fraction* has a numerator equal to, or greater than the denominator; as, $\frac{3}{3}$, and $\frac{5}{4}$.

REM.—A *proper* fraction is so termed, because it expresses a value *less* than *one*. An *improper* fraction is *not properly* a fraction of a unit, the value expressed being *equal* to, or *greater* than *one*.

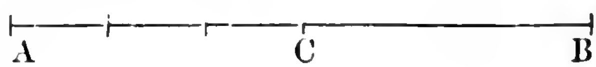
4. A *Simple Fraction* is a single fraction, either proper or improper; as, $\frac{1}{3}$, $\frac{3}{3}$, and $\frac{4}{3}$.

5. A *Compound Fraction* is a fraction of a fraction, or several fractions joined by *of;* as, $\frac{1}{2}$ of $\frac{1}{4}$, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.

6. A *Mixed Number* is a fraction joined to a whole number; as, $2\frac{1}{2}$, $3\frac{1}{4}$, and $5\frac{7}{8}$.

7 A *Complex Fraction* has a fraction in one or in both of its terms; as, $\dfrac{3\frac{1}{2}}{4}$, $\dfrac{1}{2\frac{1}{4}}$, $\dfrac{\frac{3}{4}}{\frac{2}{3}}$, $\dfrac{2\frac{1}{4}}{3\frac{1}{2}}$.

PARTS OF FRACTIONS.



ART. **130.** If a line, as A B, be divided into two equal parts, one of the parts, as A C, is termed one-half ($\frac{1}{2}$): that is, one-half of 1, or the whole thing.

REVIEW.—129. What is a fraction? A proper fraction? An improper fraction? REM. Why is a proper fraction so termed? An improper fraction? What is a simple fraction? A compound fraction? A mixed number? A complex fraction? Give examples of each. 130. What is the half of one-half? Why? The third of one-half? Why?

If a half be divided into 3 equal parts, as in the figure, one of the parts is one-third of one-half, expressed thus, $\frac{1}{3}$ of $\frac{1}{2}$; and this is one-sixth of the whole line: that is, $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$.

In the same manner, it may be shown, that $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$; that the $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$; that $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$; that $\frac{1}{2}$ of $\frac{1}{5}$ is $\frac{1}{10}$; &c.

If one apple be divided into three equal parts, and each of these parts into 3 other equal parts, into how many parts will the whole be divided?

What part of the whole will one piece be? What is $\frac{1}{3}$ of $\frac{1}{3}$?

If one orange be divided into 3 equal parts, and each part into 4 other equal parts, into how many equal parts will the whole be divided?

What part of the whole will 1 piece be? What is $\frac{1}{4}$ of $\frac{1}{3}$?

What is $\frac{1}{2}$ of $\frac{1}{3}$; $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{5}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{8}$?

Young Pupils may omit the Illustrations until they review the book.

GENERAL PRINCIPLES.

ART. **131**. If the numerator of the fraction $\frac{2}{7}$ be multiplied by 3, without changing the denominator, the result will be $\frac{6}{7}$.

$$\text{Thus,} \quad \frac{2}{7} \times 3 = \frac{6}{7}.$$

ILLUSTRATION.—Each of the fractions $\frac{2}{7}$ and $\frac{6}{7}$ having the same denominator, expresses parts of the same size: but, as the numerator of the second fraction ($\frac{6}{7}$), is 3 *times* that of the first, ($\frac{2}{7}$), it expresses 3 *times* as many of those equal parts as the first, and is 3 *times* as large. The same may be shown of any other fraction. Hence,

PROPOSITION 1.—*If the numerator be multiplied without changing the denominator, the value of the fraction will be multiplied as many times as there are units in the multiplier.*

Hence, *a fraction is multiplied, by multiplying its numerator.*

REVIEW.—131. What is the effect of multiplying the numerator of a fraction, without changing the denominator, Prop. I?

ART. **132**. If the numerator of the fraction $\frac{4}{5}$ be divided by 2, without changing the denominator, the result will be $\frac{2}{5}$.

$$\text{Thus,}\quad \frac{4\div 2}{5}=\frac{2}{5}.$$

ILLUSTRATION.—Each of the fractions $\frac{4}{5}$ and $\frac{2}{5}$ having the same denominator, expresses parts of the same size; but the numerator of the second fraction ($\frac{2}{5}$), is only *one-half* the numerator of the first ($\frac{4}{5}$); therefore, $\frac{2}{5}$ expresses one-half as many of those equal parts as the first ($\frac{4}{5}$), and is *one-half* the value. Hence,

PROP. II.—*If the numerator be divided without changing the denominator, the value of the fraction will be divided as many times as there are units in the divisor.*

Hence, *a fraction is divided by dividing its numerator.*

ART. **133**. If the denominator of the fraction $\frac{3}{4}$ be multiplied by 2, without changing the numerator, the result will be $\frac{3}{8}$.

$$\text{Thus,}\quad \frac{3}{4\times 2}=\frac{3}{8}.$$

ILLUSTRATION.—Each of the fractions $\frac{3}{4}$ and $\frac{3}{8}$ having the same numerator, denotes the same number of parts; but, in the second ($\frac{3}{8}$) the parts are *one-half* the size of those in the first ($\frac{3}{4}$): but, $\frac{1}{8}=\frac{1}{2}$ of $\frac{1}{4}$ (Art. 130); consequently, the *whole* value of the second fraction is *one-half* that of the first. Hence,

PROP. III. *If the denominator be multiplied without changing the numerator, the value of the fraction will be divided as many times as there are units in the multiplier.*

Hence, *a fraction is divided, by multiplying its denominator.*

ART. **134**. If the denominator of the fraction $\frac{2}{9}$ be divided by 3, without changing the numerator, the result will be $\frac{2}{3}$.

$$\text{Thus,}\quad \frac{2}{9\div 3}=\frac{2}{3}.$$

ILLUSTRATION.—Each of the fractions $\frac{2}{9}$ and $\frac{2}{3}$ having the same numerator, denotes the same number of parts; but, in the second fraction ($\frac{2}{3}$), the parts are 3 *times* the size of those in the first ($\frac{2}{9}$): but, $\frac{1}{9}=\frac{1}{3}$ of $\frac{1}{3}$ (Art. 130); consequently, the value of the second fraction is 3 *times* that of the first. Hence,

PROP. IV. *If the denominator be divided, without changing the numerator, the value of the fraction will be multiplied as many times as there are units in the divisor.*

Hence, *a fraction is multiplied, by dividing its denominator.*

ART. **135.** If the numerator of a fraction be multiplied by any number, its *value* (PROP. I,) will be *multiplied* by that number; if the denominator be multiplied, the *value* (PROP. III,) will be *divided* by that number.

Hence, if both terms are multiplied by the same number, the *increase* from multiplying the numerator, equals the *decrease* from multiplying the denominator: and the *value* is not changed.

Thus, $\frac{3\times 2}{5\times 2}=\frac{6}{10}$; and $\frac{3\times 3}{5\times 3}=\frac{9}{15}$.

ILLUSTRATION.—Multiplying both terms of the fraction $\frac{3}{5}$ by 2, gives $\frac{6}{10}$, in which the parts are *twice* as many, but only *one-half* the size. Multiplying both terms of $\frac{3}{5}$ by 3, gives $\frac{9}{15}$; *three times* as many parts, each part *one-third* the size. Hence,

PROP. V. *Multiplying both terms by the same number, changes its form, but does not alter its value.*

ART. **136.** If the numerator of a fraction be divided by any number, its *value* (PROP. II,) will be *divided* by that number; if the denominator be divided, the *value* (PROP. IV,) will be *multiplied* by that number.

Hence, if both terms are divided by the same number, the *decrease* from dividing the numerator, equals the *increase* from dividing the denominator: and the value is not changed.

Thus, $\frac{6\div 2}{12\div 2}=\frac{3}{6}$; and $\frac{6\div 3}{12\div 3}=\frac{2}{4}$.

REVIEW.—132. What is the effect of dividing the numerator of a fraction, without changing the denominator? 133. What of multiplying the denominator without changing the numerator?

134. What of dividing the denominator, without changing the numerator? 135. What of multiplying both terms by the same number?

Illustration.—Dividing both terms of the fraction $\frac{6}{12}$ by 2, it gives $\frac{3}{6}$; in which there are *one-half* as many parts, but each part is *twice* the size. Dividing both terms of $\frac{6}{12}$ by 3, gives $\frac{2}{4}$, *one-third* as many parts, each part *three times* the size. Hence,

Prop. VI. *Dividing both terms by the same number, changes its form, but does not alter its value.*

To Teachers.—By considering the numerator a dividend, the denominator a divisor, and the value of the fraction the quotient (Art. 125), the preceding propositions may be regarded as inferences from Art. 57, 58, 59. This short method is not best adapted to young pupils.

Art. 137. REDUCTION OF FRACTIONS

Is changing their form without altering their value.

CASE I.

Art. 138. *To reduce a fraction to its lowest terms.*

A fraction is in its lowest terms, when the numerator and denominator are prime to each other. Art. 110, Def. 5.

Thus, $\frac{3}{5}$ is in its lowest terms, while $\frac{24}{30}$ is not.

1. Reduce $\frac{24}{30}$ to its lowest terms.

FIRST OPERATION.

$$2)\frac{24}{30} = 3)\frac{12}{15} = \frac{4}{5} \text{ Ans.}$$

Solution.—Since the value of a fraction is not altered by dividing both terms by the same number, (Art. 136), and, as two is a common factor, divide both terms by it; the fraction then becomes $\frac{12}{15}$.

Again, since 3 is a factor of 12 and 15, divide them both by it; the result, $\frac{4}{5}$, can not be reduced lower.

SECOND OPERATION.

$$6)\frac{24}{30} = \frac{4}{5} \text{ Ans.}$$

Instead of dividing by 2, and then by 3, divide at once by 6, the greatest com. divisor of the two terms, and the result is the same.

Solve the two following Examples by *both* methods.

Note.—All subsequent Examples having a star, *, are intended to illustrate the principles on which the next succeeding rule is founded. The pupil should solve them and *explain* the operation, referring, at the *conclusion* of the exercise, to the rule which follows.

*2. Reduce $\frac{18}{30}$ to its lowest terms. . . . *Ans.* $\frac{3}{5}$.
*3. Reduce $\frac{60}{90}$ to its lowest terms. . . . *Ans.* $\frac{2}{3}$.

Rule for Case I.—*Divide the numerator and denominator by any common factor; divide the resulting fraction in the same manner, and so on till no number greater than* 1 *will exactly divide both terms.*

Or, *Divide the numerator and denominator by their greatest common divisor; the resulting fraction will be in its lowest terms.*

REM.—When the terms of a fraction are small, the first method is most convenient; when large, the second method.

REDUCE TO THEIR LOWEST TERMS,

4.	$\frac{12}{18}$.	*Ans.* $\frac{2}{3}$.	10.	$\frac{126}{198}$.	*Ans.* $\frac{7}{11}$.
5.	$\frac{30}{45}$.	*Ans.* $\frac{2}{3}$.	11.	$\frac{182}{196}$.	*Ans.* $\frac{13}{14}$.
6.	$\frac{60}{150}$.	*Ans.* $\frac{2}{5}$.	12.	$\frac{615}{915}$.	*Ans.* $\frac{41}{61}$.
7.	$\frac{42}{70}$.	*Ans.* $\frac{3}{5}$.	13.	$\frac{873}{1067}$.	*Ans.* $\frac{9}{11}$.
8.	$\frac{96}{112}$.	*Ans.* $\frac{6}{7}$.	14.	$\frac{777}{1998}$.	*Ans.* $\frac{7}{18}$.
9.	$\frac{60}{125}$.	*Ans.* $\frac{12}{25}$.	15.	$\frac{909}{2323}$.	*Ans.* $\frac{9}{23}$.

EXPRESS IN ITS SIMPLEST FORM,

16. The quotient of 391 divided by 667. *Ans.* $\frac{17}{29}$.
17. The quotient of 585 divided by 1287. *Ans.* $\frac{5}{11}$.
18. The quotient of 796 divided by 14129. *Ans.* $\frac{4}{71}$.

CASE II.

ART. **139**. *To reduce an improper fraction to a whole or mixed number.*

1. In 4 halves ($\frac{4}{2}$) of an apple, how many apples? in 6 thirds ($\frac{6}{3}$)? in 8 fourths ($\frac{8}{4}$)? in $\frac{9}{3}$? in $\frac{12}{4}$?

2. In 8 pecks, that is, in $\frac{8}{4}$ of a bushel, how many bushels? in $\frac{9}{4}$? in $\frac{10}{4}$? in $\frac{11}{4}$? in $\frac{13}{4}$?

REVIEW.—136. What is the effect of dividing both terms of a fraction by the same number? 137. What is Reduction of Fractions?
138. When is a fraction in its lowest terms? Give an example. How is a fraction reduced to its lowest terms, Rule?

3. In 9 fourths ($\frac{9}{4}$) of a dollar, how many dollars?

SOLUTION.—Since 4 *fourths* make one dollar, there are as many dollars as there are *times* 4 fourths in 9 fourths; that is, $2\frac{1}{4}$ dollars.

OPERATION.

$4)9$

Ans. $\$2\frac{1}{4}$.

4. Reduce $\frac{17}{5}$ to a mixed number.

SOLUTION.—Since 5 *fifths* make 1 (unit), there will be as many ones as there are times 5 in 17; that is, $3\frac{2}{5}$.

OPERATION.

$5)17$

Ans. $3\frac{2}{5}$.

*5. In $\frac{23}{10}$ of a dollar, how many dollars? *Ans.* $2\frac{3}{10}$.

*6. Reduce $\frac{25}{3}$ to a mixed number. *Ans.* $8\frac{1}{3}$.

Rule for Case II.—*Divide the numerator by the denominator: the quotient will be the whole or mixed number.*

7. In $\frac{53}{4}$ of a dollar, how many dollars? *Ans.* $13\frac{1}{4}$.
8. In $\frac{75}{4}$ of a yard, how many yards? *Ans.* $18\frac{3}{4}$.
9. In $\frac{125}{8}$ of a mile, how many miles? *Ans.* $15\frac{5}{8}$.
10. In $\frac{611}{24}$ of a day, how many days? *Ans.* $25\frac{11}{24}$.

REDUCE TO WHOLE OR MIXED NUMBERS,

11.	$\frac{19}{19}$.	*Ans.* 1.	15.	$\frac{6437}{298}$.	*Ans.* $21\frac{179}{298}$.
12.	$\frac{775}{25}$.	*Ans.* 31.	16.	$\frac{7536}{125}$.	*Ans.* $60\frac{36}{125}$.
13.	$\frac{171}{12}$.	*Ans.* $14\frac{3}{12}$.	17.	$\frac{3781}{19}$.	*Ans.* 199.
14.	$\frac{509}{11}$.	*Ans.* $46\frac{3}{11}$.	18.	$\frac{1325}{101}$.	*Ans.* $13\frac{12}{101}$.

CASE III.

ART. **140**. *To reduce a whole or mixed number to an improper fraction.*

1. In 2 apples, how many halves? In 3? In 4?
2. In 2 apples, how many thirds? In 3? In 4?
3. In 2 apples, how many fourths? In 3? In 4?
4. In $2\frac{1}{2}$ apples, how many halves? In $3\frac{1}{2}$? In $4\frac{1}{2}$?
5. In $2\frac{1}{3}$ apples, how many thirds? In $2\frac{2}{3}$? In $3\frac{1}{3}$?

REVIEW.—138. Why is the value of a fraction not altered by being reduced to its lowest terms? 139. How is an improper fraction reduced to a whole or mixed number, Rule?

6. In $5\frac{3}{4}$ dollars, how many fourths? Or, reduce $5\frac{3}{4}$ to an improper fraction.

SOLUTION.—Since there are 4 fourths in $1, in 5 dollars there are 5 *times* as many; 5 times 4 fourths are 20 fourths, and 20 fourths+3 fourths,= 23 fourths. *Ans.* $\frac{23}{4}$.

OPERATION.

$$\begin{array}{rl} 5\frac{3}{4} & \\ 4 & \\ \hline 20 = & \text{fourths in 5 dollars.} \\ 3 = & \text{fourths in fraction.} \\ \hline 23 = & \text{fourths in } 5\frac{3}{4}. \end{array}$$

*7. In $8\frac{3}{4}$ apples, how many fourths? *Ans.* $\frac{35}{4}$.

*8. Reduce $12\frac{3}{5}$ to an improper fraction. *Ans.* $\frac{63}{5}$.

Rule for Case III.—*Multiply the whole number by the denominator of the fraction; to the product add the numerator, and write the sum over the denominator.*

REM.—The analysis of question 6, shows that the whole number is really the multiplier, and the denominator the multiplicand; but the result will be the same (Art. 30), if the denominator be taken as the multiplier.

9. In $5\frac{3}{10}$ dollars, how many tenths? *Ans.* $\frac{53}{10}$.
10. In $15\frac{3}{6}$ yards, how many sixths? *Ans.* $\frac{93}{6}$.
11. In $26\frac{13}{24}$ days, how many 24ths? *Ans.* $\frac{637}{24}$.

REDUCE TO IMPROPER FRACTIONS,

12.	$8\frac{1}{2}$.	*Ans.* $\frac{17}{2}$.	16.	$21\frac{117}{583}$.	*Ans.* $\frac{12360}{583}$.
13.	$5\frac{1}{4}$.	*Ans.* $\frac{21}{4}$.	17.	$1\frac{999}{1000}$.	*Ans.* $\frac{1999}{1000}$.
14.	$3\frac{17}{55}$.	*Ans.* $\frac{182}{55}$.	18.	$14\frac{6}{71}$.	*Ans.* $\frac{1000}{71}$.
15.	$46\frac{5}{8}$.	*Ans.* $\frac{373}{8}$.	19.	$10\frac{1}{111}$.	*Ans.* $\frac{1111}{111}$.

ART. **141**. *To reduce a whole number to a fraction having a given denominator.*

1. Reduce 3 to a fraction whose denominator is 4.

SOLUTION.—Since there are 4 fourths in 1, in 3 there will be 3 times 4 fourths = 12 fourths; and hence, $3 = \frac{12}{4}$ *Ans.*

OPERATION.

$$\begin{array}{rl} 4 = & \text{fourths in 1.} \\ 3 & \\ \hline 12 = & \text{fourths in 3.} \end{array}$$

Rule.—*Multiply together the whole number and the denominator; beneath the product write the denominator.*

2. Reduce 4 to a Frac. whose denom'r is 7. *Ans.* $\frac{28}{7}$.
3. Reduce 8 to ninths. . . . . *Ans.* $\frac{72}{9}$.
4. 19 to nineteenths. . . . . *Ans.* $\frac{361}{19}$.
5. 37 to a fraction whose denom. is 23. *Ans.* $\frac{851}{23}$.

CASE IV.

Art. 142. *To reduce compound to simple fractions.*

1. Reduce $\frac{2}{3}$ of $\frac{4}{5}$ to a simple fraction.

OPERATION.

$$\frac{2}{3} \text{ of } \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}.$$

Analysis.—$\frac{2}{3}$ of $\frac{4}{5}$ is 2 times as much as $\frac{1}{3}$ of $\frac{4}{5}$, and $\frac{1}{3}$ of $\frac{4}{5}$ is 4 times as much as $\frac{1}{3}$ of $\frac{1}{5}$; but $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ (Art. 130); and hence, $\frac{1}{3}$ of $\frac{4}{5} = 4$ times $\frac{1}{15} = \frac{4}{15}$ (Art. 131), and $\frac{2}{3}$ of $\frac{4}{5} = 2$ times $\frac{4}{15} = \frac{8}{15}$.

In this operation, the numerators are multiplied together, as also are the denominators.

*2. Reduce $\frac{2}{3}$ of $\frac{5}{7}$ to a simple fraction. *Ans.* $\frac{10}{21}$.
*3. Reduce $\frac{3}{5}$ of $\frac{7}{8}$ to a simple fraction. *Ans.* $\frac{21}{40}$.

Rule for Case IV.—*Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

If mixed numbers occur, reduce them to improper fractions.

4. Reduce $\frac{1}{2}$ of $\frac{3}{5}$ of $2\frac{3}{4}$ to a simple fraction.

Sol.—$2\frac{3}{4} = \frac{11}{4}$, and $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{11}{4} = \frac{33}{40}$.

5. Reduce $\frac{7}{11}$ of $\frac{2}{3}$ to a simple fraction. *Ans.* $\frac{14}{33}$.
6. $\frac{3}{4}$ of $\frac{5}{8}$ to a simple fraction. *Ans.* $\frac{15}{32}$.
7. $\frac{2}{3}$ of $\frac{5}{7}$ of $1\frac{4}{9}$ to a simple fraction. *Ans.* $\frac{130}{189}$.

Review.—140. How is a mixed number reduced to an improper fraction, Rule? 141. How is a whole number reduced to a fraction having a given denominator, Rule?

8. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

OPERATION.

$$\frac{2}{\overset{}{\underset{1}{\not 3}}}\times\frac{\overset{1}{\not 3}}{\underset{1}{\not 4}}\times\frac{\overset{1}{\not 4}}{5}=\frac{2}{5}\ Ans.$$

SOLUTION.—After indicating the operation, the numerator of the result will be $2\times3\times4$; the denominator, $3\times4\times5$.

The value of a fraction not being altered by dividing both terms by the same number (Art. 136), *Cancel* the factors (3 and 4,) common to both terms.

As $3=3\times1$, and $4=4\times1$, the factors 1 and 1 will remain after canceling 3 and 4. Hence, the products of the remaining factors are $2\times1\times1$, and $1\times1\times5$, which give the terms of the required fraction in its simplest form.

* 9. Reduce $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ to a simple fraction. *Ans.* $\frac{5}{24}$.

*10. Reduce $\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{7}{8}$ to a simple fraction. *Ans.* $\frac{3}{8}$.

ART. **143**. Hence, to reduce compound to simple fractions by Cancellation,

Indicate the operation; cancel all the factors common to both terms, and multiply together the factors remaining in each.

REM.—As all the factors common to both terms are canceled by the operation, the result will be in its simplest form.

11. Reduce $\frac{3}{5}$ of $\frac{4}{9}$ of $\frac{7}{12}$ of $\frac{18}{35}$ to a simple fraction.

OPERATION.

$$\frac{\not 3}{5}\times\frac{\not 4}{\not 9}\times\frac{\not 7}{\not 12}\times\frac{\overset{2}{\not 18}}{\underset{5}{\not 35}}=\frac{2}{25}\ Ans.$$

SOLUTION.—First, cancel the factors 3 and 4 in the numerator, and 12 in the denominator, as $4\times3=12$.

Since 9 is a factor of 18, cancel the factor 9 in both terms, and write the remaining factor, 2, above 18; as 7 is a factor of 35, cancel the factor 7 in both terms, and write the remaining factor, 5, below 35. Then multiply the remaining factors as before.

REVIEW.—142. How are compound reduced to simple fractions, Rule? 143. How reduced by Cancellation? Why is the value of the fraction not altered? REM. Why is the result in its lowest terms?

REDUCE TO SIMPLE FRACTIONS,

12. $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{9}$. *Ans.* $\frac{1}{9}$.
13. $\frac{1}{9}$ of $\frac{3}{4}$ of $1\frac{1}{3}$. *Ans.* $\frac{1}{9}$.
14. $\frac{3}{5}$ of $\frac{6}{7}$ of $1\frac{17}{18}$. *Ans.* 1.
15. $\frac{3}{7}$ of $\frac{8}{3}$ of $1\frac{3}{4}$. *Ans.* 2.
16. $\frac{9}{13}$ of $\frac{7}{18}$ of $1\frac{6}{7}$. *Ans.* $\frac{1}{2}$.
17. $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{1}{8}$ of 5. *Ans.* $\frac{1}{4}$.
18. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{5}{9}$ of $\frac{9}{10}$. *Ans.* $\frac{1}{16}$.

For method of reducing complex to simple fractions, see page 167.

CASE V.

Art. 144. *To reduce fractions of different denominators to equivalent fractions having a common denominator.*

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator.

Solution.—The value of a fraction not being altered by multiplying both terms by the same number (Art. 135), multiply the numerator and denominator of each by the denominators of the other fractions; this will render the new denominator of each the *same;* since, in each case, it will consist of the product of the same numbers, that is, of all the denominators.

OPERATION.

$$\frac{1\times3\times4}{2\times3\times4}=\frac{12}{24}\ \text{new numer.}\ /\ \text{new denom.}$$

$$\frac{2\times2\times4}{3\times2\times4}=\frac{16}{24}\ \text{new numer.}\ /\ \text{new denom.}$$

$$\frac{3\times2\times3}{4\times2\times3}=\frac{18}{24}\ \text{new numer.}\ /\ \text{new denom.}$$

*2. Reduce $\frac{2}{5}$ and $\frac{3}{8}$ to a com. denom. *Ans.* $\frac{16}{40}$, $\frac{15}{40}$.

*3. $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{5}{6}$ to a com. denom. *Ans.* $\frac{30}{90}$, $\frac{36}{90}$, $\frac{75}{90}$.

Rule for Case V.—*Multiply both terms of each fraction by the product of all the denominators except its own.*

Note.—First reduce compound to simple fractions, and whole or mixed numbers to improper fractions.

4. Reduce $\frac{1}{2}$, $\frac{4}{3}$, and $\frac{7}{5}$ to a common denominator.

Suggestion.—Since the denominator of each new fraction consists of the product of the same numbers, (all the denominators of the given fractions,) we multiply them together but *once.*

OPERATION.

$1\times3\times5=15$ 1st num.
$4\times2\times5=40$ 2d num.
$7\times2\times3=42$ 3d num.

$2\times3\times5=30$ denom.

Observe, that, in each case, the result is obtained by multiplying the numerator and denominator by the same number.

REDUCE TO COM. DENOMINATORS,

5. $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{4}{5}$. *Ans.* $\frac{20}{40}$, $\frac{30}{40}$, $\frac{32}{40}$.
6. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. *Ans.* $\frac{12}{24}$, $\frac{8}{24}$, $\frac{6}{24}$.
7. $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{6}{7}$. *Ans.* $\frac{35}{70}$, $\frac{42}{70}$, $\frac{60}{70}$.
8. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. *Ans.* $\frac{60}{120}$, $\frac{80}{120}$, $\frac{90}{120}$, $\frac{96}{120}$.
9. $\frac{2}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, and $\frac{5}{8}$. *Ans.* $\frac{560}{840}$, $\frac{336}{840}$, $\frac{360}{840}$, $\frac{525}{840}$.
10. $\frac{6}{11}$, $\frac{4}{7}$, and $\frac{8}{13}$. *Ans.* $\frac{546}{1001}$, $\frac{572}{1001}$, $\frac{616}{1001}$.
11. $\frac{1}{2}$ of $\frac{3}{4}$, $2\frac{1}{3}$, and 3. *Ans.* $\frac{9}{24}$, $\frac{56}{24}$, $\frac{72}{24}$.
12. $\frac{3}{4}$ of $\frac{5}{6}$, and $\frac{1}{2}$ of $\frac{2}{5}$ of $\frac{3}{7}$ of $1\frac{2}{3}$. *Ans.* $\frac{35}{56}$, $\frac{8}{56}$.

ART. **145.** When the given fractions are expressed in small numbers, and the denominator of either fraction is a multiple of the denominators of the others, reduce them to a common denominator; thus,

Multiply both terms of each fraction by such a number as will render its denominator the same as the largest denominator; *obtain this number by dividing the largest denominator by the denominator of the fraction to be reduced.*

1. Reduce $\frac{1}{2}$ and $\frac{1}{6}$ to a com. denom.

OPERATION.

$$\frac{1\times3}{2\times3}=\frac{3}{6}.$$

$$\frac{1}{6}=\frac{1}{6}.$$

SOLUTION.—Since the largest denom., 6, is a multiple of 2, multiplying both terms of $\frac{1}{2}$ by $\frac{6}{2}=3$, reduces it to $\frac{3}{6}$. *Ans.* $\frac{3}{6}$ and $\frac{1}{6}$.

By similar process, Reduce to Com. Denominators,

	EXAMPLES.	ANSWERS.		EXAMPLES.	ANSWERS.
2.	$\frac{1}{2}$ and $\frac{3}{4}$ =	$\frac{2}{4}$, $\frac{3}{4}$.	6.	$\frac{1}{3}$, $\frac{5}{6}$, $\frac{7}{12}$ =	$\frac{4}{12}$, $\frac{10}{12}$, $\frac{7}{12}$.
3.	$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ =	$\frac{4}{8}$, $\frac{2}{8}$, $\frac{1}{8}$.	7.	$\frac{1}{2}$, $\frac{4}{7}$, $\frac{9}{14}$ =	$\frac{7}{14}$, $\frac{8}{14}$, $\frac{9}{14}$.
4.	$\frac{1}{3}$, $\frac{1}{2}$, $\frac{5}{6}$ =	$\frac{2}{6}$, $\frac{3}{6}$, $\frac{5}{6}$.	8.	$\frac{3}{4}$, $\frac{5}{8}$, $\frac{11}{16}$ =	$\frac{12}{16}$, $\frac{10}{16}$, $\frac{11}{16}$.
5.	$\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{10}$ =	$\frac{5}{10}$, $\frac{4}{10}$, $\frac{3}{10}$.	9.	$\frac{2}{3}$, $\frac{3}{4}$, $\frac{11}{12}$ =	$\frac{8}{12}$, $\frac{9}{12}$, $\frac{11}{12}$.

REVIEW.—144. How are two or more fractions reduced to a common denominator? Why is the value of each fraction not altered? Why does this operation render the new denominator of each the same?

CASE VI.

ART. **146**. *To reduce fractions of different denominators, to equivalent fractions having the least com. denominator.*

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to the least com. denominator.

OPERATION.

$$2\overline{)2\quad 3\quad 4}$$
$$\quad 1\quad 3\quad 2$$

$2\times3\times2=12$, least com. mul.

$$2\overline{)12} = 6 \qquad 3\overline{)12} = 4 \qquad 4\overline{)12} = 3$$

$$\left.\begin{array}{l} \frac{1\times6}{2\times6}=\frac{6}{12} \\ \frac{2\times4}{3\times4}=\frac{8}{12} \\ \frac{3\times3}{4\times3}=\frac{9}{12} \end{array}\right\} Ans.$$

SOLUTION.—Since multiplying both terms of a fraction by the same number, does not alter its value, a fraction may be reduced to another whose denominator is any *multiple* of the denominator of the given fraction.

Thus, $\frac{1}{2}$ may be reduced to a fraction, whose denominator is either 4, 6, 8, 10, 12, 14, 16, &c.

And, $\frac{2}{3}$ may be reduced to a fraction, whose denominator is either 6, 9, 12, 15, 18, 21, &c.

And, $\frac{3}{4}$ may be reduced to a fraction whose denominator is either 8, 12, 16, 20, 24, &c.

We find 12 to be the *least* denominator *common* to all of them. These denominators being *multiples* of the denominators of the given fractions, it follows, that 12, their *least common multiple*, is the least com. denominator to which the fractions can be reduced.

It now remains to reduce the fractions to TWELFTHS.

Thus, $\frac{1}{2}$ will be reduced to twelfths by multiplying both of its terms by 6, which is the quotient of the L. C. M., 12, divided by 2.

And, $\frac{2}{3}$ will be reduced to twelfths, by multiplying both of its terms by 4, the quotient of 12 divided by 3.

And, $\frac{3}{4}$ will be reduced to twelfths, by multiplying both of its terms by 3, the quotient of 12 divided by 4.

REVIEW.—145. When the denominator of one of the fractions is a multiple of the others, how reduce them to a com. denominator? How is the multiplier of each fraction obtained?

146. What are the denominators of the fractions to which one-half may be reduced? Two-thirds? Three-fourths?

With regard to this Operation, the pupil will notice,

1st. When a fraction is in its lowest terms, the denominator of any fraction to which it can be reduced must be a *multiple* of the denominator of the given fraction: hence,

Any denominator *common* to two or more fractions must be a *common multiple* of their denominators: therefore,

The *least common multiple* of their denominators, is the *least com.* denominator to which two or more fractions can be reduced.

2d. The values of the fractions are not altered, for both terms are multiplied by the same number (Art. 135.).

REDUCE TO THEIR LEAST COMMON DENOMINATORS,

*2. $\frac{1}{4}$ and $\frac{5}{6}$. . . . . *Ans.* $\frac{3}{12}$ and $\frac{10}{12}$.

*3. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{9}$. . . . . . . *Ans.* $\frac{9}{18}$, $\frac{6}{18}$, and $\frac{2}{18}$.

Rule for Case VI.—*Find the least common multiple of the denominators of the given fractions, and multiply both terms of each fraction by the quotient of the least common multiple, divided by the denominator of the fraction.*

NOTES.—1. Before commencing the operation, each fraction must be in its lowest terms.

2. Compound must be reduced to simple fractions, and whole or mixed numbers to improper fractions.

3. After the pupil is well acquainted with the nature of the operation, the multiplication of the denominators may be omitted, as the new denominators will be equal to the L. C. M.

4. The object of reducing fractions to a common denominator, is to prepare them for addition and subtraction, which can be performed only when the numbers are of the same *unit value.*

REVIEW.—146. What is the least denominator common to the fractions whose denominators are 2, 3, and 4? How reduce one-half to twelfths? Two-thirds? Three-fourths? Why is the L. C. M. of the denominators, the least common denominator? Why are the values of fractions not altered by the operation? What is the Rule for Case VI?

REDUCE TO THEIR LEAST COM. DENOMINATOR,

	EXAMPLES.		ANSWERS.		EXAMPLES.		ANSWERS.
4.	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	=	$\frac{6}{12}, \frac{4}{12}, \frac{3}{12}$.	7.	$\frac{3}{8}, \frac{4}{5}, \frac{9}{10}$	=	$\frac{15}{40}, \frac{32}{40}, \frac{36}{40}$.
5.	$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}$	=	$\frac{6}{18}, \frac{3}{18}, \frac{2}{18}$.	8.	$\frac{2}{3}, \frac{3}{4}, \frac{7}{8}$	=	$\frac{16}{24}, \frac{18}{24}, \frac{21}{24}$.
6.	$\frac{1}{2}, \frac{3}{4}, \frac{4}{5}$	=	$\frac{10}{20}, \frac{15}{20}, \frac{16}{20}$.	9.	$\frac{3}{4}, \frac{5}{8}, \frac{5}{9}$	=	$\frac{54}{72}, \frac{45}{72}, \frac{40}{72}$.

10. $\frac{2}{7}, \frac{5}{14}, \frac{9}{21}, \frac{11}{28}$. (See Note 1.) *Ans.* $\frac{8}{28}, \frac{10}{28}, \frac{12}{28}, \frac{11}{28}$.

11. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$. *Ans.* $\frac{30}{60}, \frac{40}{60}, \frac{45}{60}, \frac{48}{60}, \frac{50}{60}$.

12. $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{7}{4}$, $\frac{10}{12}$, $2\frac{2}{5}$. *Ans.* $\frac{90}{120}, \frac{105}{120}, \frac{100}{120}, \frac{288}{120}$.

☞ For additional problems, see Ray's Test Examples.

Art. 147. ADDITION OF FRACTIONS

Is the process of uniting two or more fractional numbers.

1. What is the sum of $\frac{1}{5}$ and $\frac{2}{5}$ and $\frac{3}{5}$?

Solution.—Since the denominators are the same, the numerators express parts of the same size: therefore, add

$$\left.\begin{array}{l} 1 \textit{ fifth}, \\ 2 \textit{ fifths}, \\ 3 \textit{ fifths}, \end{array}\right\} \text{ as you would add } \left\{\begin{array}{l} 1 \textit{ cent}, \\ 2 \textit{ cents}, \\ 3 \textit{ cents}, \end{array}\right.$$

The sum is 6 *fifths* ($\frac{6}{5}$) in one case; 6 *cents* in the other.

Hence, to add fractions having a Com. Denominator, *find the sum of the numerators; write the result over the denominator.*

EXAMPLES FOR MENTAL SOLUTION.

2. Add $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$.
3. $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$.
4. $\frac{1}{7}, \frac{2}{7}, \frac{5}{7}, \frac{6}{7}$.
5. Add $\frac{2}{8}, \frac{3}{8}, \frac{5}{8}, \frac{6}{8}$.
6. $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$.
7. $\frac{3}{12}, \frac{7}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}$.

Review.—146. Note 1. Before commencing the operation, what is required? 2. If there are compound fractions or mixed numbers?

Note 3. What may be omitted? Why? 4. What is the object in reducing fractions to a common denominator? 147. What is Addition of Fractions? How add fractions having a common denominator? Why?

ART. **148**. 1. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$?

SOLUTION.—Since the denominators are different, the numerators do not express things of the *same unit value*, and they can not be added together. The sum of 1 *half* and 1 *third*, is neither 2 *halves* nor 2 *thirds*. But, reduced to a common denominator, (Art. 144), 1 half = 3 sixths, and 1 third = 2 sixths; their sum is 5 sixths ($\frac{5}{6}$).

OPERATION.

$\frac{1}{2}=\frac{3}{6}$

$\frac{1}{3}=\frac{2}{6}$

$\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$ *Ans.*

*2. Add $\frac{1}{3}$ and $\frac{1}{4}$. . . . . *Ans.* $\frac{7}{12}$.

*3. Add $\frac{1}{2}$ and $\frac{3}{5}$. . . . . *Ans.* $\frac{11}{10}=1\frac{1}{10}$.

Rule for Addition.—*Reduce the fractions to a common denominator; add their numerators together, and place the sum over the common denominator.*

NOTES.—1. Reduce compound to simple fractions, and each fraction to its lowest terms, before commencing the operation.

2. Mixed numbers may be reduced to improper fractions, and then added; or the fractions and whole numbers may be added separately, then united.

3. After adding, reduce the result to its lowest terms. Art. 138.

4. What is the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$?

OPERATION.

$$\begin{array}{l|l}
2)\,3\;\;4\;\;6 & 12\div3=4,\text{ and }4\times2=8 \\
3)\,3\;\;2\;\;3 & 12\div4=3,\text{ and }3\times3=9 \\
\;\;\;\,1\;\;2\;\;1 & 12\div6=2,\text{ and }2\times5=10
\end{array}\Bigg\}\text{ New Num'rs.}$$

$2\times3\times2=12$, Least Com. Mult. *Ans.* $\frac{27}{12}=2\frac{3}{12}=2\frac{1}{4}$.

SUGGESTION.—In reducing the fractions to their least common denominator, omit writing the denominator beneath, until the sum of the numerators is obtained.

5. Add $\frac{4}{87}$, $\frac{17}{87}$, $\frac{29}{87}$, $\frac{48}{87}$, and $\frac{76}{87}$. *Ans.* 2.

6. Add $\frac{11}{144}$, $\frac{19}{144}$, $\frac{29}{144}$, $\frac{101}{144}$. . . . . . *Ans.* $1\frac{1}{9}$.

7. Add $\frac{999}{1000}$, $\frac{888}{1000}$, $\frac{777}{1000}$. . . . . *Ans.* $2\frac{83}{125}$.

REVIEW.—148. Why can not 1-half and 1-third be added, without reducing them to the same denominator? What the rule for addition?

8. Add $\frac{3}{4}$ and $\frac{5}{6}$. *Ans.* $1\frac{7}{12}$.
9. Add $\frac{5}{6}$, $\frac{1}{9}$. *Ans.* $\frac{17}{18}$.
10. Add $\frac{7}{8}$, $\frac{11}{12}$. *Ans.* $1\frac{19}{24}$.
11. Add $\frac{1}{4}$, $\frac{7}{8}$, $\frac{11}{12}$. *Ans.* $2\frac{1}{24}$.
12. Add $\frac{1}{8}$, $\frac{1}{9}$, $\frac{2}{11}$. *Ans.* $\frac{331}{792}$.
13. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$. *Ans.* $1\frac{9}{20}$.

14. Add $\frac{1}{12}$, $\frac{1}{13}$, $\frac{1}{14}$, and $\frac{1}{15}$. . . . . . *Ans.* $\frac{543}{1820}$.
15. Add $2\frac{1}{2}$ and $3\frac{1}{3}$. . . . . *Ans.* $5\frac{5}{6}$.
16. Add $\frac{4}{5}$, $7\frac{1}{2}$, and $8\frac{3}{4}$. . . . . . . *Ans.* $17\frac{1}{20}$.
17. Add $16\frac{2}{3}$, $12\frac{3}{4}$, $8\frac{3}{5}$, and $2\frac{1}{4}$. . . . *Ans.* $40\frac{4}{15}$.
18. Add $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{12}$. . . *Ans.* $\frac{2}{3}$.
19. Add $\frac{1}{5}$ of $1\frac{1}{2}$, $\frac{1}{9}$ of $2\frac{1}{2}$, $\frac{11}{6}$ of $\frac{1}{2}$, $\frac{7}{8}$. . *Ans.* $2\frac{133}{360}$.
20 Add $\frac{2}{5}$, $\frac{7}{16}$, $\frac{7}{50}$, $\frac{3}{140}$, and $\frac{3}{2800}$. . . *Ans.* 1.

☞ For additional problems, see Ray's Test Examples.

Art. 149. SUBTRACTION OF FRACTIONS

Is the process of finding the difference between two fractional numbers.

1. What is the difference between $\frac{2}{7}$ and $\frac{5}{7}$?

Solution.—Since the denominators are the same, the numerators express parts of the same size: therefore, subtract 2 *sevenths* from 5 *sevenths* as you would 2 *cents* from 5 *cents*.

Thus,	5 *sevenths*,	5 *cents*,
	2 *sevenths*,	2 *cents*,

Difference 3 *sevenths* ($\frac{3}{7}$) in one case; 3 *cents* in the other.

Hence, to find the difference between two fractions having a common denominator,

Find the difference between their numerators, and write the result over the common denominator.

QUESTIONS FOR MENTAL SOLUTION.

2. What is the difference between $\frac{1}{4}$ and $\frac{3}{4}$? $\frac{5}{8}$ and $\frac{7}{8}$? $\frac{5}{9}$ and $\frac{2}{9}$? $\frac{8}{10}$ and $\frac{3}{10}$? $\frac{11}{5}$ and $\frac{1}{5}$? $\frac{11}{8}$ and $\frac{3}{8}$?

Review.—148. Note. If there are Compound Fractions, what is required? What if each fraction is not in its lowest terms? How are mixed numbers added?

Art. 150. 1. Find the difference between $\frac{1}{2}$ and $\frac{2}{3}$.

	Operation.
Solution.—Since the denominators are different, the numerators do not express things of the same unit value: hence, one can not be subtracted from the other. Art. 25. Thus, the difference between 1 *half* and 2 *thirds* is neither 1 *half* nor 1 *third;* but, reduce them to a common denominator (Art. 144), and 2 thirds = 4 sixths, and 1 half = 3 sixths; their difference being 1 sixth ($\frac{1}{6}$).	$\frac{2}{3}=\frac{4}{6}$ $\frac{1}{2}=\frac{3}{6}$ $\frac{4}{6}-\frac{3}{6}=\frac{1}{6}$ *Ans.*

*2. The difference between $\frac{1}{3}$ and $\frac{1}{4}$. *Ans.* $\frac{1}{12}$.

*3. Between $\frac{1}{2}$ and $\frac{4}{5}$. . . . . *Ans.* $\frac{3}{10}$.

Rule for Subtraction.—*Reduce the fractions to a common denominator, find the difference of their numerators, and place it over the common denominator.*

Note.—Reduce compound to simple fractions, and each fraction to its lowest terms, before commencing the operation. After subtracting, reduce the result to its lowest terms.

4. What is the difference between $\frac{5}{6}$ and $\frac{3}{10}$?

OPERATION.

$6=\cancel{2}\times 3$	$30\div 6=5$, and $5\times 5=25$	New
$10=2\times 5$	$30\div 10=3$, and $3\times 3=\underline{\ \ 9}$	Nume'rs.

$3\times 2\times 5=30$, Least Com. Mult. *Ans.* $\frac{16}{30}=\frac{8}{15}$.

5.	$\frac{8}{9}-\frac{5}{9}=$ *Ans.* $\frac{1}{3}$.		10.	$\frac{1}{7}-\frac{1}{11}=$ *Ans.* $\frac{4}{77}$.
6.	$\frac{15}{16}-\frac{7}{16}=$ *Ans.* $\frac{1}{2}$.		11.	$\frac{5}{6}-\frac{3}{8}=$ *Ans.* $\frac{11}{24}$.
7.	$\frac{3}{4}-\frac{1}{3}=$ *Ans.* $\frac{5}{12}$.		12.	$\frac{5}{9}-\frac{1}{6}=$ *Ans.* $\frac{7}{18}$.
8.	$\frac{3}{5}-\frac{1}{4}=$ *Ans.* $\frac{7}{20}$.		13.	$\frac{4}{15}-\frac{1}{10}=$ *Ans.* $\frac{1}{6}$.
9.	$\frac{1}{4}-\frac{2}{9}=$ *Ans.* $\frac{1}{36}$.		14.	$\frac{8}{21}-\frac{3}{14}=$ *Ans.* $\frac{1}{6}$.

Rem.—In finding the difference between *mixed numbers,* either reduce them to improper fractions, and to a common denominator, and then make the subtraction; or, find the difference between the whole numbers and the fractions separately.

Review.—149. What is subtraction of fractions? How find the difference between two fractions having a com. denominator? Why?

15. From $3\frac{1}{2}$ subtract $1\frac{2}{3}$.

SUGGESTION.—As 4 sixths can not be taken from 3 sixths, borrow 1 from 3, reduce it to sixths, add them to the 3 sixths, making 9 sixths; then subtract.

OPERATION.

$$\begin{array}{ll} 3\frac{1}{2}=\frac{7}{2}=\frac{21}{6} & \text{Or, } 3\frac{1}{2}=3\frac{3}{6} \\ 1\frac{2}{3}=\frac{5}{3}=\underline{\frac{10}{6}} & \quad\;\; 1\frac{2}{3}=\underline{1\frac{4}{6}} \\ \qquad\quad\;\; \frac{11}{6}=1\frac{5}{6}. & \qquad\qquad\; 1\frac{5}{6}. \end{array}$$

16. From 5 subtract $\frac{2}{3}$.

SUGGESTION.—The subtraction may be made by reducing 5 to thirds; or, by borrowing 1 from 5, and reducing it to thirds, as in the above example.

OPERATION.

$$\begin{array}{ll} 5=\frac{15}{3} & \text{Or, } 5 \\ \quad\;\; \underline{\frac{2}{3}} & \qquad\;\; \underline{\frac{2}{3}} \\ \textit{Ans. } \frac{13}{3}=4\frac{1}{3}. & \qquad\; 4\frac{1}{3}. \end{array}$$

17.	$4\frac{3}{4}-2\frac{1}{4}=2\frac{1}{2}$.	23.	$11-4\frac{5}{9}=6\frac{4}{9}$.
18.	$8\frac{1}{3}-3\frac{2}{3}=4\frac{2}{3}$.	24.	$8-3\frac{3}{13}=4\frac{10}{13}$.
19.	$5\frac{2}{3}-4\frac{1}{2}=1\frac{1}{6}$.	25.	$\frac{3}{5}$ of $10-\frac{2}{3}$ of $6=2$.
20.	$7\frac{2}{3}-4\frac{3}{4}=2\frac{11}{12}$.	26.	$\frac{1}{2}$ of $\frac{3}{4}-\frac{1}{3}$ of $\frac{5}{6}=\frac{7}{72}$.
21.	$5-1\frac{1}{2}=3\frac{1}{2}$.	27.	$14\frac{1}{4}-\frac{2}{3}$ of $19=1\frac{7}{12}$.
22.	$8-\frac{3}{7}=7\frac{4}{7}$.	28.	$\frac{3}{8}$ of $72-\frac{3}{5}$ of $21=14\frac{2}{5}$.

☞ For additional problems, see Ray's Test Examples.

ART. 151. MULTIPLICATION OF FRACTIONS.

CASE I. *To multiply a fraction by a whole number.*

This operation consists in taking the fraction as many times as there are units in the multiplier.

1. If 1 apple cost $\frac{1}{5}$ of a cent, what cost 3 apples?

SOLUTION.—*Three* apples cost 3 *times* as much as *one;* that is, $\frac{1}{5}$ taken 3 times: $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{1}{5}\times3=\frac{3}{5}$. (Art. 131.)

2. If 1 lemon cost $\frac{3}{5}$ of a cent, what cost 4 lemons?
Ans. $\frac{3}{5}$ taken 4 times; $\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}=\frac{3}{5}\times4=\frac{12}{5}=2\frac{2}{5}$.

REVIEW.—150. Why can not one-half be taken from two-thirds without reducing to the same denominator? What the rule for subtraction?

150. NOTE. What is required if there are compound fractions? What if each fraction is not in its lowest terms? REM. How find the difference between two mixed numbers? Between a whole number and a fraction?

ANOTHER SOLUTION.—Since 3 fifths express three parts of the same size, we may multiply 3 *fifths*, as we would multiply 3 *cents*.

Thus,	3 *fifths*	And,	3 *cents*
multiplied by	4	multiplied by	4
the product is	12 *fifths* ($\frac{12}{5}$).	the product is	12 *cents*.

Hence, *To multiply a fraction by a whole number, multiply its numerator.*

EXAMPLES FOR MENTAL SOLUTION.

3. $\frac{1}{4} \times 3 =$ *Ans.* $\frac{3}{4}$.	7. $\frac{3}{7} \times 6 =$ *Ans.* $2\frac{4}{7}$.
4. $\frac{1}{3} \times 4 =$ *Ans.* $1\frac{1}{3}$.	8. $\frac{2}{3} \times 8 =$ *Ans.* $5\frac{1}{3}$.
5. $\frac{2}{3} \times 6 =$ *Ans.* 4.	9. $\frac{7}{9} \times 6 =$ *Ans.* $4\frac{2}{3}$.
6. $\frac{3}{5} \times 10 =$ *Ans.* 6.	10. $\frac{5}{8} \times 10 =$ *Ans.* $6\frac{1}{4}$.

11. What is the product of $\frac{7}{8}$ multiplied by 4?

SOLUTION.—Since a fraction is multiplied by multiplying its numerator, (Art. 131,) multiply 7 (eighths) by 4, and the product is 28 (eighths); which, reduced, equals $3\frac{1}{2}$.

OPERATION.

$\frac{7}{8} \times 4 = \frac{28}{8} = \frac{7}{2} = 3\frac{1}{2}$ *Ans.*

or,

$\frac{7}{8} \times 4 = \frac{7}{4)8} = \frac{7}{2} = 3\frac{1}{2}$.

Or, since a fraction is multiplied by dividing its denominator, (Art. 134,) if the denominator, 8, is divided by 4, the result is $\frac{7}{2} = 3\frac{1}{2}$, the same as by the first method.

*12. $\frac{5}{6} \times 3 =$ *Ans.* $2\frac{1}{2}$. | *13. $\frac{7}{12} \times 4 =$ *Ans.* $2\frac{1}{3}$.

Rule for Case I.—1ST. *Multiply the numerator of the fraction by the whole number, and under the product write the denominator.*

Or, 2D. *Divide the denominator by the whole number, when it can be done without a remainder; over the quotient write the numerator.*

14. $\frac{11}{17} \times 12 =$ *Ans.* $7\frac{13}{17}$. | 15. $\frac{2}{21} \times 7 =$ *Ans.* $\frac{2}{3}$.

NOTE.—When the denominator of the fraction, and the whole number contain common factors, employ Cancellation.

16. $\frac{5}{36} \times 9 =$ *Ans.* $1\frac{1}{4}$.
17. $\frac{25}{64} \times 8 =$ *Ans.* $3\frac{1}{8}$.
18. $\frac{9}{14} \times 21 =$ *Ans.* $13\frac{1}{2}$.
19. $\frac{11}{15} \times 10 =$ *Ans.* $7\frac{1}{3}$.
20. $\frac{5}{9} \times 9 =$ *Ans.* 5.
21. $3\frac{1}{3} \times 4 =$ (See sug.)

SUG.—In multiplying a mixed by a whole number, multiply the fractional part and the whole number separately, then unite their products; or, reduce the mixed number to an improper fraction, then multiply.

OPERATION. $3\frac{1}{3}$
4
Ans. $13\frac{1}{3}$.

Or, $3\frac{1}{3} = \frac{10}{3}$;
$\frac{10}{3} \times 4 = \frac{40}{3} = 13\frac{1}{3}$.

22. $18\frac{3}{4} \times 8 =$ *Ans.* 150.
23. $16\frac{2}{3} \times 3 =$ *Ans.* 50.
24. $10\frac{7}{9} \times 7 =$ *Ans.* $75\frac{4}{9}$.
25. $10\frac{5}{6} \times 9 =$ *Ans.* $97\frac{1}{2}$.

CASE II.

ART. **152**. *To multiply a whole number by a fraction.*

Multiplying by a whole number, is taking the multiplicand as many *times* as there are *units* in the multiplier.

Multiplying by a fraction, or *part* of a unit, is taking a part of the multiplicand. Therefore,

Multiplying by $\frac{1}{2}$, is taking 1 *half* the multiplicand.

Multiplying by $\frac{1}{3}$, is taking 1 *third* of the multiplicand.

Multiplying by $\frac{2}{3}$, is taking 2 *thirds* of the multiplicand, &c.

Hence, *To multiply by a fraction, is to take such a part of the multiplicand, as the multiplier is part of a unit.*

1. At 12 cts. a yard, what cost $\frac{1}{3}$ of a yard of ribbon?

SOLUTION.—If 1 yard cost 12 cents, 1 third of a yard will cost 1 third as much, that is, (Art. 54,) $12 \times \frac{1}{3} = 4$ cents. *Ans.*

2. At 12 cts. a yard, what cost $\frac{2}{3}$ of a yard of ribbon?

SOLUTION.—*Two*-thirds will cost *twice* as much as *one*-third; but, 1 third costs 4 cents, hence, 2 thirds cost $4 \times 2 = 8$ cents, that is, (Art. 54,) $12 \times \frac{2}{3} = 8$ cents. *Ans.*

REVIEW.—151. In what does the multiplication of a fraction by a whole number consist? What is the first method? The second method?
151. NOTE. When the denominator and whole number have common factors, how shorten the process? How multiply a mixed by a whole number?

3. What is the product of 20 multiplied by $\frac{3}{4}$?

SOLUTION.—Three-fourths are 3 times 1 fourth; 1 fourth of 20 is 5, and 3 times 5 are 15: *Ans.*

Or, $\frac{1}{4}$ of 20 is $\frac{20}{4}$; and 3 times $\frac{20}{4}$ are $\frac{20}{4} \times 3 = \frac{60}{4} = 15$.

EXAMPLES FOR MENTAL SOLUTION.

4.	$8 \times \frac{3}{4} =$	*Ans.* 6.	8.	$16 \times \frac{7}{8} =$	*Ans.* 14.
5.	$12 \times \frac{2}{3} =$	*Ans.* 8.	9.	$5 \times \frac{3}{4} =$	*Ans.* $3\frac{3}{4}$.
6.	$10 \times \frac{2}{5} =$	*Ans.* 4.	10.	$7 \times \frac{2}{3} =$	*Ans.* $4\frac{2}{3}$.
7.	$14 \times \frac{5}{7} =$	*Ans.* 10.	11.	$8 \times \frac{3}{5} =$	*Ans.* $4\frac{4}{5}$.

Rule for Case II.—1ST. *Divide the whole number by the denominator of the fraction: multiply the quotient by the numerator.*

Or, 2D. *Multiply the whole number by the numerator of the fraction, and divide the product by the denominator.*

REM.—1. The 2d rule is best, when the denominator of the fraction is not a factor of the whole number.

2. Since the product of two numbers is the same, whichever is the multiplier, (Art. 30,) the examples in this and the preceding case may be performed by the same rule.

3. When the multiplier is a whole number greater than 1, the multiplicand is *increased;* when it is a proper fraction, the multiplicand is *decreased*, the product being the same *part* of the multiplicand, that the multiplier is of unity.

4. Multiplying by a fraction always involves division. Thus, multiplying by one-third, is the same as dividing by three.

12.	$28 \times \frac{4}{7}$	$=$ *Ans.* 16.	15.	$31 \times \frac{2}{3}$	$=$ *Ans.* $20\frac{2}{3}$.
13.	$36 \times \frac{7}{9}$	$=$ *Ans.* 28.	16.	$29 \times \frac{3}{4}$	$=$ *Ans.* $21\frac{3}{4}$.
14.	$50 \times \frac{9}{10}$	$=$ *Ans.* 45.	17.	$37 \times \frac{4}{5}$	$=$ *Ans.* $29\frac{3}{5}$.

REVIEW.—152. In what does multiplication by a whole number consist? In what the multiplication by a fraction, or part of a unit? What is multiplying by one-half? By one-third? Two-thirds?

152. In multiplying by a fraction, what part of the multiplicand is taken? What is Rule for Case II, 1st? 2d Rule? REM. 1. Which is best?

18. Multiply 8 by $3\frac{2}{3}$.

In multiplying a whole by a mixed number, multiply by the integer and the fraction separately, then unite their products; or, reduce the mixed number to an improper fraction, then multiply.

OPERATION.

$$\begin{array}{l} 8 \\ \underline{3\frac{2}{3}} \\ 24 \\ \underline{5\frac{1}{3}} = 8 \times \frac{2}{3} \\ 29\frac{1}{3}\ \textit{Ans.} \end{array}$$

Or, $3\frac{2}{3} = \frac{11}{3}$

$8 \times \frac{11}{3} = \frac{88}{3} = 29\frac{1}{3}$.

19. $25 \times 8\frac{3}{5} =$ *Ans.* 215.
20. $45 \times 6\frac{1}{4} =$ *Ans.* $281\frac{1}{4}$.
21. $55 \times 9\frac{3}{7} =$ *Ans.* $518\frac{4}{7}$.
22. $64 \times 8\frac{7}{8} =$ *Ans.* 568.

CASE III.

Art. 153^a. *To multiply one fraction by another.*

1. What is the product of $\frac{4}{7}$ by $\frac{3}{5}$?

Solution.—To multiply $\frac{4}{7}$ by $\frac{3}{5}$, is to take $\frac{3}{5}$ of $\frac{4}{7}$ (Art. 152), and this is equal to $\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$ (Art. 131).

OPERATION.

$\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$. *Ans.*

Analysis.—Now, $\frac{3}{5}$ of $\frac{4}{7}$ is 3 times $\frac{1}{5}$ of $\frac{4}{7}$; and $\frac{1}{5}$ of $\frac{4}{7}$ is 4 times $\frac{1}{5}$ of $\frac{1}{7}$; but $\frac{1}{5}$ of $\frac{1}{7} = \frac{1}{35}$ (Art. 130); hence, $\frac{1}{5}$ of $\frac{4}{7}$ is 4 times $\frac{1}{35}$, and $\frac{3}{5}$ of $\frac{4}{7}$ is 3 times $\frac{4}{35} = \frac{12}{35}$. This is multiplying the numerators together, and the denominators together.

*2. What is the product of $\frac{3}{5}$ by $\frac{1}{4}$? . . . *Ans.* $\frac{3}{20}$.
*3. The product of $\frac{3}{4}$ by $\frac{5}{7}$? . . . . . *Ans.* $\frac{15}{28}$.

Rule for Case III.—*Multiply together the numerators for a new numerator, and the denominators for a new denominator.*

Rem.—Multiplying a fraction by a fraction, is the same as reducing a compound to a simple fraction.

4. $\frac{9}{13} \times \frac{3}{7} =$ *Ans.* $\frac{27}{91}$.
5. $\frac{11}{12} \times \frac{5}{9} =$ *Ans.* $\frac{55}{108}$.
6. $\frac{7}{15} \times \frac{8}{11} =$ *Ans.* $\frac{56}{165}$.
7. $\frac{8}{13} \times \frac{11}{16} =$ *Ans.* $\frac{11}{26}$.

Review.—152. Rem. 2. Why may the examples in this and the preceding case be solved by the same rule?

152. Rem. 3. When is the multiplicand *increased* by multiplying? When *decreased*? 4. What operation is involved in multiplying by a fraction?

NOTE 1.—Reduce mixed numbers to improper fractions.

8. Multiply $2\frac{1}{4}$ by $3\frac{1}{2}$.

SOL.—$2\frac{1}{4}=\frac{9}{4}$, and $3\frac{1}{2}=\frac{7}{2}$; then $\frac{9}{4}\times\frac{7}{2}=\frac{63}{8}=7\frac{7}{8}$ *Ans.*

9. $8\frac{3}{4}\times\frac{3}{7}$. *Ans.* $3\frac{3}{4}$.
10. $\frac{9}{10}\times17\frac{3}{11}$. *Ans.* $15\frac{6}{11}$.
11. $2\frac{1}{2}\times2\frac{1}{2}$. *Ans.* $6\frac{1}{4}$.
12. $16\frac{1}{2}\times16\frac{1}{2}$. *Ans.* $272\frac{1}{4}$.

NOTE 2.—After indicating the operation, employ CANCELLATION, when possible, as shown in Art. 61.

13. Multiply $\frac{1}{3}$ of 4 by $\frac{1}{6}$ of 5. . . *Ans.* $1\frac{1}{9}$.
14. Multiply $\frac{2}{7}$ of $4\frac{1}{3}$ by $\frac{1}{3}$ of $3\frac{1}{2}$. . . *Ans.* $1\frac{4}{9}$.
15. Multiply $\frac{3}{4}$ of $9\frac{1}{4}$ by $\frac{2}{3}$ of 17. . . *Ans.* $78\frac{5}{8}$.
16. Multiply $\frac{9}{10}$ of $7\frac{1}{3}$ by $\frac{11}{15}$ of $86\frac{3}{11}$. *Ans.* $417\frac{14}{25}$.
17. Multiply 3, $2\frac{1}{2}$, $\frac{5}{7}$, and $\frac{2}{3}$ of 5 together. *Ans.* $17\frac{6}{7}$.

18. $\frac{7}{8}\times\frac{3}{10}\times\frac{8}{9}\times\frac{5}{6}\times\frac{2}{3}\times\frac{6}{7}$ = *Ans.* $\frac{1}{9}$.
19. $\frac{1}{4}\times\frac{9}{7}\times\frac{4}{5}\times\frac{7}{9}\times\frac{5}{4}\times\frac{2}{3}$ of 6 = *Ans.* 1.
20. $\frac{6}{7}\times\frac{4}{9}\times1\frac{3}{4}\times\frac{1}{6}$ of $\frac{3}{4}\times\frac{5}{6}\times\frac{2}{5}$ of 20 = *Ans.* $\frac{5}{9}$.
21. $2\frac{1}{2}\times6\frac{2}{5}\times3\frac{1}{4}\times\frac{7}{13}$ of $2\times\frac{3}{7}$ = *Ans.* 24.

☞ For additional problems, see Ray's Test Examples.

ART. 153^b. MISCELLANEOUS EXAMPLES.

What will be the cost

1. Of $2\frac{1}{3}$ lb. of meat, at $1\frac{1}{5}$ cts. a lb.? *Ans.* $2\frac{4}{5}$ cts.
2. Of 3 yd. linen, at $\$\frac{2}{3}$ a yd.? of 5 yd.? of 7 yd.? of $6\frac{1}{2}$ yd.? $5\frac{3}{4}$ yd.? *Ans.* to last, $\$3\frac{5}{6}$.
3. Of $3\frac{1}{3}$ lb. of rice, at $4\frac{4}{5}$ cts. a lb.? *Ans.* 16 cts.
4. Of $3\frac{1}{5}$ tuns of iron, at $\$18\frac{3}{4}$ per T.? *Ans.* \$60.
5. Of $1\frac{2}{3}$ yd. of muslin, at $\$\frac{3}{20}$ per yd.? *Ans.* $\$\frac{1}{4}$.
6. Of $2\frac{1}{2}$ lb. of tea, at $\$\frac{4}{5}$ per lb.? *Ans.* \$2.
7. Of $5\frac{5}{9}$ cords of wood, at $\$1\frac{1}{5}$ per C.? *Ans.* $\$6\frac{2}{3}$.

REVIEW.—152. How may a whole be multiplied by a mixed number? 153^a. How multiply one fraction by another, Rule? NOTE 1. What is required when mixed numbers occur? 2. When employ Cancellation?

8. At the rate of $5\frac{1}{2}$ miles an hour, how far will a man travel in $7\frac{3}{4}$ hours? *Ans.* $42\frac{5}{8}$ mi.

9. I own $\frac{2}{3}$ of a steamboat, and sell $\frac{3}{5}$ of my share: what part of the boat do I sell? *Ans.* $\frac{2}{5}$.

10. At $\$6\frac{3}{4}$ per yard, what cost $\frac{2}{9}$ of a piece of cloth containing $5\frac{1}{2}$ yards? *Ans.* $\$8\frac{1}{4}$.

11. $\frac{3}{7}$ of $\frac{5}{9}$ of $16\frac{1}{2}$, $\times$ $\frac{2}{3}$ of $\frac{7}{8}$ of 15,=what? *Ans.* $34\frac{3}{8}$.

☞ For additional problems, see Ray's Test Examples.

Art. 154. DIVISION OF FRACTIONS.

Case I. *To divide a fraction by a whole number.*

The object in dividing a fraction by a whole number, is to separate the fraction 'nto a *given number* of equal parts, and find the *value* of one of the parts; or, to find *what part* a fraction is of a whole number.

1. If 3 yards of ribbon cost $\$\frac{6}{7}$, what cost 1 yard?

Analysis.—If 3 yards cost $\frac{6}{7}$ of a dollar, 1 yard, being $\frac{1}{3}$ of 3 yards, will cost $\frac{1}{3}$ of $\frac{6}{7}=\frac{2}{7}$ of a dollar.

Or, since a fraction is divided by multiplying its denominator (Art. 133), multiply 7 by 3, and the result, $\frac{6}{21}$, reduced, is $\frac{2}{7}$, as by the first method.

OPERATION.

$$\frac{6}{7}\div 3=\frac{3)6}{7}=\frac{2}{7}\ \textit{Ans.}$$

Or,

$$\frac{6}{7}\div 3=\frac{6}{7\times 3}=\frac{6}{21}=\frac{2}{7}.$$

Here, $\frac{6}{7}$ is divided into 3 *equal* parts, and the *value* of each part is $\frac{2}{7}$; thus, $\frac{6}{7}=\frac{2}{7}+\frac{2}{7}+\frac{2}{7}$; the *number* of parts corresponds to the *divisor*, and the *value* of each, to the *quotient.*

2. At 2 dollars a yard, what part of a yard of cloth can be bought for $\frac{1}{2}$ a dollar?

Analysis.—Had it been required to find how many yards, at \$2 a yard, could be bought for \$6, the \$6 should be divided by \$2; and, to find the *part* of a yard that $\$\frac{1}{2}$ will pay for, divide $\$\frac{1}{2}$ by \$2: to divide $\frac{1}{2}$ by 2, multiply the denominator (Art. 133); the quotient is one-fourth.

OPERATION.

$$\frac{1}{2}\div 2=\frac{1}{2\times 2}=\frac{1}{4}$$

Ans. $\frac{1}{4}$ yd.

*3. If 4 yards of muslin cost $\frac{8}{9}$ of a dollar, what will 1 yard cost? *Ans.* $\$\frac{2}{9}$.

*4. If 1 orange cost 3 cents, what part of an orange can be purchased for $\frac{1}{2}$ a cent? *Ans.* $\frac{1}{6}$.

Rule for Case I.—*Divide the numerator by the whole number, when it can be done without a remainder; write the quotient over the denominator.* Otherwise, *Multiply the denominator by the whole number, and write the product under the numerator.*

5. If 4 yards of cloth cost $\frac{12}{16}$ of a dollar, what will 1 yard cost? *Ans.* $\$\frac{3}{16}$.

6. If a man travel $\frac{9}{11}$ of a mile in 3 hours, how far does he travel in 1 hour? *Ans.* $\frac{3}{11}$ mi.

7. If 5 yards of tape cost $\frac{3}{20}$ of a dollar, what will 1 yard cost? *Ans.* $\frac{3}{100}$.

8. If 7 pounds of coffee cost $\frac{14}{25}$ of a dollar, what will 1 pound cost? *Ans.* $\$\frac{2}{25}$.

9. At 4 dollars a yard for cloth, what part of a yard will $\frac{16}{25}$ of a dollar buy? *Ans.* $\frac{4}{25}$ yd.

10. At 5 dollars a tun, what part of a tun of hay will $\frac{1}{2}$ a dollar purchase? *Ans.* $\frac{1}{10}$ T.

11. At 6 dollars a barrel, what part of a barrel of flour will $\$2\frac{2}{5}$ pay for?

OPERATION.

$2\frac{2}{5} = \frac{12}{5}$

$\frac{12}{5} \div 6 = \frac{2}{5}$ *Ans.*

Reduce a mixed number to an improper fraction, and it may be divided by a whole number, the same as a proper fraction.

12. If 5 bushels of wheat cost $3\frac{3}{4}$ dollars, what cost 1 bushel? *Ans.* $\frac{3}{4}$ of a dollar.

13. If 7 ounces of opium cost $8\frac{2}{5}$ dollars, what cost 1 ounce? *Ans.* $\frac{6}{5} = \$1\frac{1}{5}$.

14. If $\frac{18}{19}$ be divided into 9 equal parts, what will each part be? *Ans.* $\frac{2}{19}$.

REVIEW.—154. What is the object in dividing a fraction by a whole number? In Ex. 1, what does the divisor show? The quotient?

154. In Ex. 2, what does the quotient show? How divide a fraction by a whole number, Rule for Case I?

15. $4\frac{4}{5} \div 8 =$ *Ans.* $\frac{3}{5}$.	18. $3\frac{2}{3} \div 7 =$ *Ans.* $\frac{11}{21}$.
16. $\frac{3}{4} \div 5 =$ *Ans.* $\frac{3}{20}$.	19. $47\frac{2}{5} \div 15 =$ *Ans.* $3\frac{4}{25}$.
17. $12\frac{4}{7} \div 11 =$ *Ans.* $1\frac{1}{7}$.	20. $130\frac{2}{3} \div 18 =$ *Ans.* $7\frac{7}{27}$.

CASE II.

ART. **155.** *To divide a whole number by a fraction.*

Dividing a whole number by a fraction, is finding *how many times* the fraction is contained in the whole number.

1. At $\frac{2}{3}$ of a cent for 1 lemon, how many can be bought for 4 cents?

OPERATION.

$$\begin{array}{r} 4 \\ 3 \\ \hline \text{Thirds } 2)12 \text{ thirds.} \\ \hline \textit{Ans. } 6 \end{array}$$

SOLUTION.—In 4 cents there are 12 *thirds* of 1 cent (Art. 141). If 1 lemon costs 2 *thirds* of a cent, there will be as many lemons as 2 *thirds* are contained *times* in 12 *thirds;* that is, 6. *Ans.* 6 lemons.

Or thus: $\frac{1}{3}$ is contained in 4 as many times as there are thirds in 4, that is, 12 times; and 2 thirds in 4, *one half* as many times as 1 third; $12 \div 2 = 6$ times.

In this operation, the whole number is reduced to the same name—the same parts of a unit—as the divisor, that the divisor and dividend may be of the same denomination. Art. 41, Rem.

The whole number is multiplied by the denominator of the fraction, and the product divided by the numerator.

*2. At $\frac{1}{2}$ a cent each, how many apples can be bought for 3 cents? *Ans.* 6 apples.

*3. At $\frac{3}{4}$ of a dollar per yard, how many yards of cloth can you buy for 6 dollars? *Ans.* 8 yards.

Rule for Case II.—*Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.*

REVIEW.—155. What is dividing a whole number by a fraction? How many times is one-third contained in 1? In 2? In 3? In 4? How many times two-thirds in 1? In 2? In 3? In 4?

4. $4 \div \frac{2}{5} =$ *Ans.* 10.	7. $6 \div \frac{3}{7} =$ *Ans.* 14.
5. $16 \div \frac{3}{4} =$ *Ans.* $21\frac{1}{3}$.	8. $13 \div \frac{3}{5} =$ *Ans.* $21\frac{2}{3}$.
6. $8 \div \frac{2}{15} =$ *Ans.* 60.	9. $21 \div \frac{7}{11} =$ *Ans.* 33.

10. In one rod there are $5\frac{1}{2}$ yards: how many rods are there in 22 yards?

Reduce a mixed number to an improper fraction, and a whole number may be divided by it, as by a proper fraction.

OPERATION. $5\frac{1}{2} = \frac{11}{2}$

$$22 \times \frac{2}{11} = \frac{44}{11} = 4 \text{ rods. } Ans.$$

11. At $2\frac{2}{5}$ dollars for 1 yard of cloth, how many yards can be bought for $6? *Ans.* $2\frac{1}{2}$ yd.

12. At $3\frac{3}{4}$ cents a lb., how many pounds of rice can be bought for 30 cts.? *Ans.* 8 lb.

13. How many times $4\frac{3}{7}$ in 50? *Ans.* $11\frac{9}{31}$.

14. Divide 56 by $5\frac{4}{9}$. *Ans.* $10\frac{2}{7}$.

CASE III.

ART. **156**. *To divide a fraction by a fraction.*

The object in dividing a fraction by a fraction, is to find *how many times* the divisor is contained in the dividend; or, *what part* the divisor is of the dividend.

1. At $\frac{2}{10}$ of a dollar per yard, how many yards of muslin can be bought for $\$\frac{9}{10}$?

SUGGESTION.—Find how often 2 *tenths* are contained in 9 *tenths*, as you would find how often 2 *cents* are contained in 9 *cents;* that is, by dividing 9 by 2: $9 \div 2 = 4\frac{1}{2}$.

OPERATION.
Tenths 2)9 tenths.
Ans. $4\frac{1}{2}$.

Hence, *when two fractions have a common denominator, obtain their quotient by dividing the numerator of the dividend by the numerator of the divisor.*

2. How many times $\frac{2}{3}$ in $\frac{3}{4}$?

OPERATION.

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}; \quad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}:$$

$$\frac{9}{12} \div \frac{8}{12} = 9 \div 8 = 1\frac{1}{8} \ Ans.$$

We can not find how often 2 *inches* are contained in 3 *feet,* without expressing the divisor and dividend in the *same denomination, inches;* so, also, to find how often 2 *thirds* are contained in 3 *fourths*, reduce them to the *same denomination*, *twelfths*: $\frac{2}{3}$ are 8 *twelfths*, and $\frac{3}{4}$ are 9 *twelfths*; and 8 *twelfths* are contained in 9 *twelfths*, $9 \div 8 = 1\frac{1}{8}$.

No use is made of the common denominator, the *numerator* of the divisor being multiplied by the *denominator* of the dividend, and the *numerator* of the dividend by the *denominator* of the divisor.

This is easily performed by inverting the terms of the divisor, then proceeding as in Multiplication of Fractions, Art. 153.

$$\text{Thus, } \frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8} \text{ } Ans.$$

Another Solution.—If $\frac{3}{4}$ be divided by 2, the quotient, (Art. 133), is $\frac{3}{4 \times 2}$; but, since 2 is 3 times $\frac{2}{3}$, the divisor used is 3 times the given divisor; hence, multiply this quotient by 3 to obtain the true quotient: this gives $\frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$ *Ans.*

*3. At $\frac{2}{5}$ of a dollar each, how many knives can you buy for $\frac{4}{5}$ of a dollar? *Ans.* 2.

*4. At $\frac{1}{5}$ of a dollar per yard, how many yards of ribbon can be purchased for $\frac{3}{4}$ of a dollar? *Ans.* $3\frac{3}{4}$.

Rule for Case III.—*Invert the divisor; multiply the numerators together for a new numerator, and the denominators for a new denominator.*

Note.—Reduce compound to simple fractions, and mixed numbers to improper fractions, before commencing the operation.

FIND THE QUOTIENT OF

5. $\frac{3}{4} \div \frac{1}{4}$ = *Ans.* 3.
6. $\frac{1}{2} \div \frac{1}{4}$ = *Ans.* 2.
7. $\frac{1}{4} \div \frac{1}{2}$ = *Ans.* $\frac{1}{2}$.
8. $\frac{5}{6} \div \frac{2}{3}$ = *Ans.* $1\frac{1}{4}$.
9. $\frac{1}{2} \div \frac{1}{3}$ = *Ans.* $1\frac{1}{2}$.
10. $2\frac{1}{2} \div \frac{1}{16}$ = *Ans.* 40.
11. $4\frac{1}{2} \div 1\frac{1}{3}$ = *Ans.* $3\frac{3}{8}$.
12. $4\frac{3}{4} \div 5\frac{1}{8}$ = *Ans.* $\frac{38}{41}$.
13. $2\frac{1}{4} \div 7\frac{1}{2}$ = *Ans.* $\frac{3}{10}$.
14. $\frac{1}{2} \div \frac{1}{50}$ = *Ans.* 25.

15. Divide $\frac{3}{5}$ of $\frac{8}{9}$ by $\frac{6}{7}$ of $\frac{3}{4}$. . . . . . *Ans.* $\frac{112}{135}$.

16. Divide $\frac{1}{3}$ of $5\frac{1}{8}$ by $\frac{3}{4}$ of $17\frac{1}{2}$. . . . *Ans.* $\frac{41}{315}$.

Review.—155. How do you divide a whole number by a fraction? 156. What is the object in dividing a fraction by a fraction?

ART. **157**. The rules in the three preceding Articles may be embraced in this

General Rule for Division of Fractions.—*Express the divisor and dividend in the form of a fraction; invert the divisor; cancel all the factors common to both terms; then multiply together the numbers remaining in the numerator for a new numerator, those in the denominator for a new denominator.*

1. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{3}{5}$ of $\frac{4}{3}$.

OPERATION.

$$\frac{1\times \not{2}\times 5\times \not{3}}{\not{2}\times 3\times \not{3}\times 4}=\frac{5}{12}.$$

In this operation, cancel the factors 2 and 3 on both sides—above and below the line—then multiply together the factors remaining on each side.

2. $\frac{14}{15}\div 21 =$ *Ans.* $\frac{2}{45}$.
3. $8\frac{5}{9}\div 35 =$ *Ans.* $\frac{11}{45}$.
4. $8 \div \frac{16}{17} =$ *Ans.* $8\frac{1}{2}$.
5. $14 \div 8\frac{3}{4} =$ *Ans.* $1\frac{3}{5}$.
6. $\frac{21}{32}\div\frac{35}{48} =$ *Ans.* $\frac{9}{10}$.
7. $12\frac{5}{6}\div 4\frac{1}{8} =$ *Ans.* $3\frac{1}{9}$.
8. Divide $\frac{5}{18}$ of $\frac{2}{5}$ of $12\frac{3}{10}$ by $\frac{1}{5}$ of $8\frac{1}{5}$. . . *Ans.* $\frac{5}{6}$.

REDUCTION OF COMPLEX TO SIMPLE FRACTIONS.

ART. **158**^{a}. A complex fraction is the expression of an unexecuted division (Art. 125), in which the divisor or dividend, or both, are fractions.

Thus, $\frac{1\frac{1}{4}}{2\frac{1}{3}}$ indicates that $1\frac{1}{4}$ is to be divided by $2\frac{1}{3}$.

Hence, to reduce a complex to a simple fraction,

Regard the numerator as a dividend, the denominator as a divisor, and proceed as in Division of Fractions, Art. 157.

1. Reduce $\frac{1\frac{1}{4}}{2\frac{1}{3}}$ to a simple fraction.

OPERATION.

$$\left.\begin{array}{l}1\frac{1}{4}=\frac{5}{4}\\ 2\frac{1}{3}=\frac{7}{3}\end{array}\right\}\frac{5}{4}\div\frac{7}{3}=\frac{5}{4}\times\frac{3}{7}=\frac{15}{28}.\ \textit{Ans.}$$

In this operation, after reducing the mixed numbers to improper fractions, the numerator of the result is the product of the

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REVIEW.—156. When two fractions have a common denominator, how obtain their quotient? How find how often 2 inches are contained in 3 feet? How often two-thirds are contained in three-fourths? How divide a fraction by a fraction? 157. What is the General Rule?

extreme terms, 5 and 3; the denominator, the product of the *mean* terms, 4 and 7. Hence,

To Reduce a Complex to a Simple Fraction,

Reduce mixed numbers to improper fractions, then multiply together the extreme terms for a numerator, and the mean terms for a denominator.

Reduce these complex to simple fractions:

2. $\dfrac{\frac{6}{7}}{\frac{11}{5}}$ = *Ans.* $\frac{30}{77}$.

3. $\dfrac{\frac{2}{3}}{5}$ = *Ans.* $\frac{2}{15}$.

4. $\dfrac{2}{3\frac{2}{3}}$ = *Ans.* $\frac{6}{11}$.

5. $\dfrac{3\frac{1}{8}}{4\frac{5}{7}}$ = *Ans.* $\frac{175}{264}$.

6. $\dfrac{2\frac{1}{3}}{4\frac{1}{2}}$ = *Ans.* $\frac{14}{27}$.

7. $\dfrac{3\frac{3}{4}}{5\frac{5}{8}}$ = *Ans.* $\frac{2}{3}$.

Complex fractions may be multiplied together, or, one divided by another, by first reducing each to a simple fraction. Indicate the operation, and cancel.

8. Multiply $\dfrac{1\frac{1}{2}}{2\frac{1}{3}}$ by $\dfrac{\frac{3}{7}}{4\frac{1}{2}}$. . . . . *Ans.* $\frac{3}{49}$.

9. Multiply $\dfrac{7\frac{8}{15}}{9\frac{5}{12}}$ of $\dfrac{2\frac{1}{9}}{3\frac{2}{15}}$ by $\dfrac{1\frac{1}{4}}{\frac{5}{6}}$. . . . . . *Ans.* $\frac{38}{47}$.

10. Divide $\dfrac{1\frac{2}{3}}{2\frac{1}{2}}$ by $\dfrac{5\frac{1}{7}}{84\frac{6}{7}}$. . . . . *Ans.* 11.

Art. 158^b. MISCELLANEOUS EXAMPLES.

1. At $\frac{1}{2}$ a dollar per yd., how many yards of silk can be bought for \$3$\frac{1}{4}$? *Ans.* 6$\frac{1}{2}$ yd.

2. At $\frac{3}{5}$ of a dollar per pound, how many pounds of tea can be purchased for \$2$\frac{3}{10}$? *Ans.* 3$\frac{5}{6}$ lb.

3. Find the quotient of $\frac{1}{2}$ divided by 2; by $\frac{1}{2}$; by $\frac{1}{4}$; by $\frac{1}{6}$; by $\frac{1}{8}$; by $\frac{1}{1000}$. *Ans.* to last, 500.

4. At 3$\frac{3}{4}$ dollars per yard for cloth, how many yards can be purchased with \$42$\frac{1}{2}$? *Ans.* 11$\frac{1}{3}$ yd.

5. By what must $\frac{3}{8}$ be multiplied, that the product may be 10? *Ans.* 26$\frac{2}{3}$.

Review.—158^a. What is a complex fraction? How reduced to a simple fraction?

6. Divide $3\frac{3}{7}$ by $\frac{3}{7}$ of $1\frac{1}{2}$. . . . . *Ans.* $5\frac{1}{3}$.

7. Divide $\frac{4}{11}$ of $27\frac{1}{2}$ by $\frac{3}{10}$ of $21\frac{1}{4}$. . . *Ans.* $1\frac{29}{51}$.

8. Divide $\frac{1\frac{1}{2}}{\frac{2}{3}}$ by $\frac{2\frac{2}{5}}{2\frac{1}{6}}$. . . . . *Ans.* $2\frac{1}{32}$.

Art. 159. EXAMPLES IN U. S. MONEY.

1. Add $16.06¼; $9.12½; $5.43¾; $2.81¼
Ans. $33.43¾

2. I paid for books $9.12½; paper $4.43¾; a slate $0.37½; quills $1.62½: what did I pay? *Ans.* $15.56¼

3. Having $50.25, I paid a bill of $27.18¾: how much had I left? *Ans.* $23.06¼

4. From $32.31¼ take $15.12½ *Ans.* $17.18¾

5. From $5.81¼ take $1.18¾ *Ans.* $4.62½

Find the cost of

6. 9 yd. of muslin at 12½ cts. a yd. *Ans.* $1.12½

7. 21 lb. of sugar at 6¼ cts. a lb. *Ans.* $1.31¼

8. 15 yd. of cloth at $3.18¾ per yd. *Ans.* $47.81¼

9. 5½ yd. of linen at $0.62½ per yd. *Ans.* $3.43¾

10. 12½ yd. of ribbon at 18¾ cts. per yd. *Ans.* $2.34⅜

11. 13½ yd. of calico at 16⅔ cts. per yd. *Ans.* $2.25

12. 10¼ yd. of cloth at $3.37½ a yd. *Ans.* $34.59⅜

13. 17⅔ dozen books at $3.75 per doz. *Ans.* $66.25

14. At 18¾ cts. per yd., how many yards of muslin can be purchased for $2.25? *Ans.* 12 yd.

15. At 37½ cents per bu., how many bushels of barley can you buy for $5.81¼? *Ans.* 15½ bu.

16. If five yards of cloth cost $11.56¼, what cost one yard? *Ans.* $2.31¼

17. Seven men share $31.06¼ equally: what is the share of each man? *Ans.* $4.43¾

EXAMPLES IN LONG MEASURE.

18. Reduce 5 mi. to inches. . . *Ans.* 316800 in.

19. 2 mi. 2 rd. 2 ft. to feet. . . *Ans.* 10595 ft.

20. 20 yd. to rods.

OPERATION.

$5\frac{1}{2}$ yards = 11 halves, 20 yards = 40 halves.

11)40

3 rd. 7 half yd. left, = $3\frac{1}{2}$ yd. *Ans.* 3 rd. $3\frac{1}{2}$ yd.

Suggestion.—In reducing numbers from a lower to a higher denomination, when the divisor is a fractional number (Art. 155), reduce both divisor and dividend to like parts of a unit.

The remainder being of the same denomination as the dividend, (Art. 38), will be a fraction, which reduce to a whole or mixed number. Here, the remainder, 7 half yd., reduced, makes $3\frac{1}{2}$ yd.

21. Reduce 15875 ft. to miles. *Ans.* 3 mi. 2 rd. 2 ft.

22. 142634 in. to miles. *Ans.* 2 mi. 2 fur. 2 yd. 2 in.

23. How many steps, of 2 ft. 8 in. each, will a man take in walking 2 miles? *Ans.* 3960.

24. How many revolutions will a wheel, of 9 ft. 2 in. circumference, make, in running 65 mi.? *Ans.* 37440.

EXAMPLES IN SQUARE MEASURE.

25. Reduce 1 A. 3 R. 16 P. 25 sq. yd. to square yards. *Ans.* 8979 sq. yd.

26. 7506 sq. yd. to A. *Ans.* 1 A. 2 R. 8 P. 4 sq. yd.

27. 5 chains 15 links, to in. *Ans.* $4078\frac{4}{5}$ in.

28. How many acres in a field $40\frac{1}{2}$ rd. long, and 32 rd. wide? *Ans.* 8 A. 16 P.

EXAMPLES IN TIME MEASURE.

In these examples, the year is supposed to be $365\frac{1}{4}$ days.

29. Reduce 4 years to hours. . . . *Ans.* 35064 hr.

30. 914092 hr. to cen. *Ans.* 1 cen. 4 yr. 101 da. 4 hr.

31. In what time will a body move from the earth to the moon, at the rate of 31 miles per day, the distance being 238545 miles? *Ans.* 21 yr. $24\frac{3}{4}$ da.

REDUCTION OF FRACTIONAL COMPOUND NUMBERS.

CASE I.

ART. **160.** *To reduce a fraction of a higher denomination, to a fraction of a lower.*

1. Reduce $\frac{1}{24}$ of a peck to the fraction of a pint.

SOLUTION.—To reduce pecks to pints, we multiply by 8 to reduce them to quarts, then by 2 to reduce them to pints.

In like manner, multiply the *fraction* of a peck by 8, to reduce it to the *fraction* of a quart, then by 2, to reduce it to the fraction of a pint.

OPERATION.

pk. qt. qt. pt.

$\frac{1}{24}\times8=\frac{1}{3}$; $\frac{1}{3}\times2=\frac{2}{3}$:

or,

$\frac{1}{24}\times8\times2=\frac{2}{3}$ pt. *Ans.*

Hence, fractional numbers may be reduced from a *higher* to a *lower* denomination, (Art. 81), by *multiplying* by that number of the next lower order which makes a unit of the higher.

*2. Reduce $\frac{1}{40}$ of a bu. to the fraction of a qt. *Ans.* $\frac{4}{5}$.

Rule for Case I.—*Multiply as in Reduction of Whole Numbers,* Art. 81, *according to the rules for the multiplication of fractions.*

REM.—The work in Cases I and II may often be shortened by Cancellation.

3. Reduce $\frac{1}{28}$ lb. Av. to the fraction of an oz. *Ans.* $\frac{4}{7}$.
4. $\frac{1}{16}$ of a lb. Troy, to the fraction of an oz. *Ans.* $\frac{3}{4}$.
5. $\frac{1}{20}$ of a yd. to the fraction of a na. *Ans.* $\frac{4}{5}$.
6. $\frac{7}{1280}$ of an A. to the fraction of a P. *Ans.* $\frac{7}{8}$.
7. $\frac{3}{350}$ of a dollar to the fraction of a ct. *Ans.* $\frac{6}{7}$.
8. $\frac{1}{1584}$ of a da. to the fraction of a min. *Ans.* $\frac{10}{11}$.
9. $\frac{3}{320}$ of a bu. to the fraction of a pt. *Ans.* $\frac{3}{5}$.

REVIEW.—160. How reduce pecks to pints? How the fraction of a peck to the fraction of a pint? How are fractional numbers reduced from a higher to a lower denomination?

CASE II.

Art. 161. *To reduce a fraction of a lower denomination, to a fraction of a higher.*

1. Reduce $\frac{2}{3}$ of a pint to the fraction of a peck.

Solution.—To reduce pints to pecks, we first divide by 2 to reduce them to quarts, and then by 8 to reduce them to pecks.

In like manner, divide the fraction by 2, to reduce it to the *fraction* of a quart, and then by 8, to reduce it to the *fraction* of a peck.

OPERATION.

pt. qt. qt. pk.

$$\frac{2}{3} \div 2 = \frac{1}{3};\quad \frac{1}{3} \div 8 = \frac{1}{24}:$$

or,

$$\frac{2}{3} \times \frac{1}{2} \times \frac{1}{8} = \frac{1}{24} \text{ pk. } Ans.$$

Hence, fractional numbers may be reduced from a *lower* to a *higher* denomination (Art. 81), by *dividing* the given fraction by that number of its own denomination which makes a unit of the next higher.

*2. Reduce $\frac{4}{5}$ of a qt. to the fraction of a bu. *Ans.* $\frac{1}{40}$.

Rule for Case II.—*Divide, as in Reduction of Whole Numbers,* Art. 81, *according to the rules for the division of fractions.*

3. Reduce $\frac{4}{5}$ of a na. to the fraction of a yd. *Ans.* $\frac{1}{20}$.
4. $\frac{3}{5}$ of a gr. T. to the fraction of a lb. *Ans.* $\frac{1}{9600}$.
5. $\frac{4}{9}$ of a ℈ to the fraction of a ℔. *Ans.* $\frac{1}{648}$.
6. $\frac{3}{5}$ of a pt. to the fraction of a bu. *Ans.* $\frac{3}{320}$.
7. $\frac{4}{7}$ of an oz. to the fraction of a cwt. *Ans.* $\frac{1}{2800}$.
8. $\frac{3}{5}$ of an in. to the fraction of an E. En. *Ans.* $\frac{1}{75}$.
9. $\frac{8}{9}$ of a min. to the fraction of a da. *Ans.* $\frac{1}{1620}$.
10. $\frac{5}{7}$ of a dr. to the fraction of a qr. *Ans.* $\frac{1}{8960}$.

Review.—160. What is Case 1? What is the rule for Case 1? 161. How reduce pints to pecks? How the fraction of a pint to the fraction of a peck? How are fractional numbers reduced from a lower to a higher denomination? What is Case 2? The Rule for Case 2?

CASE III.

ART. **162**. *To find the value of a fraction in integers* (whole numbers) *of a lower denomination.*

1. Find the value of $\frac{2}{5}$ of a day in integers.

SOLUTION.—$\frac{2}{5}$ of a da., are $\frac{2}{5}$ of 24 hr.; and $\frac{2}{5}$ of 24 hr. are found by multiplying by 2 and dividing by 5 (Art. 152). This gives $9\frac{3}{5}$ hr.

Again, $\frac{3}{5}$ of an hour are the same as $\frac{3}{5}$ of 60 minutes, which are 36 min. Hence, $\frac{3}{5}$ of a day = 9 hr. 36 min.

OPERATION.

$$\begin{array}{rr} 24 \text{ hr.} & 60 \text{ min.} \\ 2 & 3 \\ \hline 5)48 & 5)180 \\ \hline 9\frac{3}{5} \text{ hr.} & 36 \text{ min.} \end{array}$$

Ans. 9 hr. 36 min.

*2. Value of $\frac{4}{5}$ of a mi. in integers. *Ans.* 6 fur. 16 rd.

Rule for Case III.—*Take the number of units of the next lower denomination which forms a unit of the denomination of the fraction; multiply it by the numerator, and divide the product by the denominator.*

If this division produce a fraction, find its value in the same manner, and so on: the several quotients will be the answer.

3. What is the value of $\frac{3}{5}$ of a dollar? *Ans.* 60 cts.
4. Of $\frac{2}{5}$ of a mile? . . . . . . *Ans.* 3 fur. 8 rd.
5. Of $\frac{4}{5}$ of a lb. Troy? . . . . . *Ans.* 9 oz. 12 pwt.
6. Of $\frac{4}{7}$ of a lb. Av.? . . . . . *Ans.* 9 oz. $2\frac{2}{7}$ dr.
7. Of $\frac{5}{8}$ of an acre? . . . . . *Ans.* 2 R. 20 P.
8. Of $\frac{7}{8}$ of a T. of wine? . *Ans.* 3 hhd. 31 gal. 2 qt.

CASE IV.

ART. **163**. *To reduce a quantity having one or more denominations, to the fraction of another quantity composed of one or more denominations.*

1. Reduce 2 feet 3 inches to the fraction of a yard: that is, 2 ft. 3 in. is what part of 1 yard?

SOLUTION.—2 ft. 3 in. = 27 inches: 1 yd. = 36 inches; and since 1 inch is $\frac{1}{36}$ of 36 inches, 27 inches are $\frac{27}{36} = \frac{3}{4}$. *Ans.* $\frac{3}{4}$ yd.

2. Find what part 2 ft. 6 in. is of 6 ft. 8 in.

SOLUTION.—Reducing both quantities to the same denomination, the first is 30, and the second 80 in. Since 1 in. is $\frac{1}{80}$ of 80 in., 30 in. will be $\frac{30}{80}=\frac{3}{8}$.

OPERATION.

2 ft. 6 in. = 30 in.
6 ft. 8 in. = 80 in.

Ans. $\frac{30}{80}=\frac{3}{8}$.

*3. Reduce 2 pk. 4 qt. to the fraction of a bu. *Ans.* $\frac{5}{8}$.

*4. What part is 2 yd. 1 qr. of 8 yd. 3 qr.? *Ans.* $\frac{9}{35}$.

Rule for Case IV.—*Reduce both quantities to the lowest denomination in either; the less will be the numerator, and the greater the denominator of the required fraction, which reduce to its lowest terms.*

5. Reduce 13 hr. 30 min. to the frac. of a da. *Ans.* $\frac{9}{16}$.
6. 3 fur. 25 rd. to the fraction of a mile. *Ans.* $\frac{29}{64}$.
7. 2 ft. 8 in. to the fraction of a yard. *Ans.* $\frac{8}{9}$.
8. What part is 96 pages of 432 pages? *Ans.* $\frac{2}{9}$.
9. 15 mi. 3 fur. 3 rd. of 35 mi. 7 fur. 7 rd.? *Ans.* $\frac{3}{7}$.
10. A man has a farm of 168 A. 28 P.: if he sell 37 A. 2 R. 14 P., what part will he sell? *Ans.* $\frac{97}{434}$.

NOTE.—If one or both quantities contain a fraction, reduce them to a common denominator, and compare their numerators.

11. What part is 7 oz. $1\frac{7}{9}$ dr. of 1 lb. Av.? *Ans.* $\frac{4}{9}$.
12. 2 qt. $1\frac{1}{3}$ pt. of 1 bu. 1 qt. $1\frac{2}{3}$ pt.? *Ans.* $\frac{16}{203}$.
13. 1 yd. 1 ft. $1\frac{9}{11}$ in. of 3 yd. 2 ft. $8\frac{6}{7}$ in.? *Ans.* $\frac{1918}{5423}$.

ADDITION AND SUBTRACTION

OF FRACTIONAL COMPOUND NUMBERS.

ART. **164**. 1. Add $\frac{3}{4}$ of a yard, to $\frac{5}{6}$ of a foot.

SOLUTION.—Find the value of each of the quantities in integers (Art. 162), and then obtain their sum by the rule for addition in Art. 101. If their difference be required, subtract according to the Rule in Art. 102.

OPERATION.

	ft.	in.
$\frac{3}{4}$ yd. =	2	3
$\frac{5}{6}$ ft. =		10
Ans.	3	1

Rule for Addition or Subtraction.—*First, find the value of each fraction in integers* (Art. 162):

Then add or subtract, as may be required, according to the rules for the addition and subtraction of compound numbers.

NOTE.—If fractions occur in the lowest denomination, add or subtract by the rules for addition and subtraction of fractions.

2. Add $\frac{2}{3}$ of a da. to $\frac{3}{4}$ of an hr. *Ans.* 16 hr. 45 min.
3. Add together $\frac{1}{4}$ wk. $\frac{1}{4}$ da. $\frac{1}{4}$ hr. *Ans.* 2 da. 15 min.
4. $\frac{2}{3}$ wk. $\frac{5}{9}$ da. $\frac{2}{3}$ hr. $\frac{2}{3}$ min. *Ans.* 5 da. 6 hr. 40 sec.
5. $\frac{11}{12}$ gal. of wine, $\frac{1}{12}$ hhd. *Ans.* 6 gal. 1 pt. $1\frac{1}{3}$ gi.
6. How much land in $2\frac{6}{7}$ R., $\frac{2}{3}$ A., and 3 R. $28\frac{4}{7}$ P.? *Ans.* 2 A. 1 R. $9\frac{11}{21}$ P.
7. $\frac{7}{9}$ of a da., less $\frac{1}{18}$ of an hr., equals what? *Ans.* 18 hr. 36 min. 40 sec.
8. $\frac{5}{8}$ of \$1, less $\frac{3}{4}$ of a dime? *Ans.* 55 cts.
9. $\frac{3}{5}$ of an oz., less $\frac{7}{8}$ of a pwt.? *Ans.* 11 pwt. 3 gr.
10. $\frac{2}{9}$ mi., less $\frac{7}{11}$ fur.? *Ans.* 1 fur. 5 rd. 10 ft. 10 in.
11. $\frac{6}{7}$ da., less $\frac{2}{3}$ hr.? *Ans.* 19 hr. 54 min. $17\frac{1}{7}$ sec.
12. $\frac{7}{8}$ E. En., less $\frac{2}{7}$ yard? *Ans.* 3 qr. $\frac{13}{14}$ na.

ART. 165. PROMISCUOUS EXAMPLES.

1. Reduce $\frac{32989}{56981}$ to its lowest terms. . . *Ans.* $\frac{11}{19}$.

2. Find the sum of $\frac{3}{4}$ and $\frac{8}{9}$; of $\frac{5}{8}$ and $\frac{2}{7}$; of $\frac{3}{10}$ and $\frac{11}{15}$; of $2\frac{1}{2}$, $3\frac{2}{3}$, $\frac{5}{14}$, and $\frac{8}{21}$. *Ans.* to last, $6\frac{19}{21}$.

3. What is the difference between $\frac{3}{4}$ and $\frac{2}{3}$? $\frac{1}{2}$ and $\frac{4}{11}$? $3\frac{4}{7}$ and $1\frac{4}{5}$? $3\frac{5}{8}$ and $\frac{1}{3}$ of $3\frac{1}{2}$? *Ans.* to last, $2\frac{11}{24}$.

4. Add $\frac{5}{9}$ of $\frac{7}{10}$ to $\frac{2}{5}$ of $\frac{7}{12}$. . . . . . . *Ans.* $\frac{28}{45}$.

REVIEW.—162. Two-fifths of a day, are two-fifths of what? How find two-fifths of 24 hours? What is Case 3? What is the Rule?

163. What is Case 4? What the Rule? NOTE. If one or both quantities contain a fraction, how proceed? 164. What is the rule for the addition or subtraction of fractional compound numbers?

5. To the quotient of $1\frac{3}{4} \div 2\frac{1}{2}$, add the quotient of $5\frac{1}{2} \div 3\frac{1}{8}$. *Ans.* $2\frac{23}{50}$.

6. What number divided by $\frac{3}{5}$ will give 10 for a quotient? *Ans.* 6.

7. What number multiplied by $\frac{3}{5}$ will give 10 for a product? *Ans.* $16\frac{2}{3}$.

8. What number is that, from which if you take $\frac{3}{7}$ of itself, the remainder will be 16? *Ans.* 28.

9. What number is that, to which if you add $\frac{3}{7}$ of itself, the sum will be 20? *Ans.* 14.

10. A boat is worth \$900; a merchant owns $\frac{5}{8}$ of it, and sells $\frac{1}{3}$ of his share: what part has he left, and what is it worth? *Ans.* $\frac{5}{12}$ left, worth \$375.

11. I own $\frac{7}{12}$ of a ship, and sell $\frac{1}{3}$ of my share for \$$1944\frac{4}{9}$: what is the whole ship worth? *Ans.* \$10000.

12. What part of 3 cents, is $\frac{2}{3}$ of 2 cents? *Ans.* $\frac{4}{9}$.

13. What part of 368, is 176? *Ans.* $\frac{11}{23}$.

14. What number, $+\frac{1}{8}+\frac{1}{18}+\frac{13}{111}, =\frac{25}{37}$? *Ans.* $\frac{1007}{2664}$.

15. What number, $+\frac{3}{10}$ of $\frac{7}{12}$ of $4\frac{9}{14}, =1$? *Ans.* $\frac{3}{16}$.

16. From the quotient of $\frac{2}{3} \div \frac{5}{7}$, subtract the quotient of $\frac{5}{8} \div \frac{10}{11}$. *Ans.* $\frac{59}{240}$.

17. If I walk 2044 rods in $\frac{7}{15}$ of an hour, at that rate how far will I walk in $1\frac{14}{15}$ hr.? *Ans.* 8468 rd.

18. What part of $1\frac{1}{4}$ feet, is $3\frac{1}{3}$ inches? *Ans.* $\frac{2}{9}$.

19. Two men bought a bl. of flour; one paid \$$3\frac{1}{5}$, and the other \$$3\frac{2}{3}$: what part of it should each have?

Ans. One $\frac{48}{103}$, the other $\frac{55}{103}$.

20. A has \$2400; $\frac{5}{8}$ of his money, +\$500, is $\frac{5}{4}$ of B's: what sum has B? *Ans.* \$1600.

21. John Jones divided his estate among 2 sons and 3 daughters, the latter sharing equally with each other. The younger son received \$2200, which was $\frac{5}{12}$ of the share of the elder, his share being $\frac{16}{35}$ of the whole estate: find the share of each daughter. *Ans.* \$$1356\frac{2}{3}$.

XII. DECIMAL FRACTIONS.

ART. **166**. A *common* fraction (Art. 124) is one whose denominator may be *any* number; as, 2, 3, 4, &c.

A *decimal* fraction is one whose denominator is 10, or a number of 10's multiplied together; as, 100, 1000, &c.

EXPLANATION.—A *common* fraction arises from dividing a unit into *any* number of equal parts. A *decimal* fraction arises from dividing a unit by 10, or some multiple of 10 by itself; and its denominator is not *usually* expressed in figures.

ART. **167**. If a unit be divided into 10 equal parts, each part will be 1 *tenth;* thus, $\frac{1}{10}$ of $1 = \frac{1}{10}$.

If each tenth be divided into 10 equal parts, the unit will be divided into 100 equal parts, each part being 1 *hundredth;* thus, $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$.

If each hundredth be divided into 10 equal parts, the unit will be divided into 1000 parts, each part being 1 *thousandth* of the whole; thus, $\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000}$.

A comparison of the fractions $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, &c., shows that the value of each is *one-tenth* of that which precedes it; that is, they *decrease* in value *tenfold.*

ART. **168**. The orders of WHOLE numbers *decrease* from *left* to *right tenfold;* thus, 1 hundred is *one-tenth* of 1 thousand; 1 ten is *one-tenth* of 1 hundred; and 1 unit is *one-tenth* of 1 ten: in like manner,

The orders may be continued from the place of units toward the *right*, by the same law of decrease; and,

The first order on the right of units will then express tenths of units, that is *tenths;* the next order, tenths of tenths, or *hundredths;* the next, tenths of hundredths, or *thousandths;* and so on:

Hence, if a point (.) be placed to separate units and tenths, $\frac{1}{10}$ may be written in the order of *tenths;* thus, .1, as 1 (unit) is written in the order of units: and,

$\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, &c., may be written, .2, .3, .4, &c.

REVIEW.—166. What is a Com. Fraction? A Decimal? 167. What is 1 tenth? 1 hundredth? 1 thousandth? What the value of each?

Also, $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, &c., may be written in the order of *hundredths*, thus: .01, .02, .03, &c.: here,

There being no tenths, place a cipher in the order of tenths.

As the orders decrease from left to right, in the same manner as in whole numbers, Decimal Fractions may be expressed in figures, *without writing their denominators.*

NOTE.—$\frac{3}{10}$, $\frac{24}{100}$, $\frac{205}{1000}$, and .3, .24, .205 are *alike* decimal fractions.

ART. **169**. A figure in the *first* decimal place or order, expresses a fraction whose denominator is a unit with *one* cipher annexed; thus, $.2 = \frac{2}{10}$: and,

A figure in the *second* decimal place, expresses a fraction whose denominator is a unit with *two* ciphers annexed;

Thus, $.02 = \frac{2}{100}$: also,

A figure in the *third* place expresses a fraction whose denominator is a unit with *three* ciphers annexed;

Thus, $.002 = \frac{2}{1000}$; and so on: hence,

The denominator of a Decimal Fraction is 1, *with as many ciphers annexed as there are decimal places in the numerator.*

ART. **170**. TABLE OF DECIMAL PLACES OR ORDERS.

	Tens.	Units.	*(Decimal Point)*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hund.-thousandths.	Millionths.	Ten-millionths.	Hund.-millionths.	
			.	6								is read 6 tenths.
			.	2	5							25 hundredths.
			.	0	1	3						13 thousandths.
			.	0	3	0	5					305 ten-thousandths.
			.	0	0	1	0	8				108 hundred-thousandths.
			.	3	7	8	6	5	9			378659 millionths.
		5	.	0	0	0	0	1	2	4		5 and 124 ten-millionths.
	2	6	.	0	0	0	0	0	0	0	5	26 and 5 hundred-millionths.
PLACES,	2d	1st		1st	2d	3d	4th	5th	6th	7th	8th	

REM.—The orders called ten-thousandths, hundred-thousandths, &c., are, really, *tenths* of thousandths, *hundredths* of thousandths, &c.

ART. **171**. The table shows, that the *value* expressed by any figure in *decimals*, as well as in *whole numbers*, depends on the *place* from units which it occupies:

Thus, .2 expresses 2 tenths, $\frac{2}{10}$.
.02 .. 2 hundredths, $\frac{2}{100}$.
.002 .. 2 thousandths, $\frac{2}{1000}$.

But, $\frac{1}{10}$ of $\frac{2}{10}$ is $\frac{2}{100}$; $\frac{1}{10}$ of $\frac{2}{100}$ is $\frac{2}{1000}$, and so on.

Hence, *as the figures decrease from left to right tenfold*, such fractions are called *decimals*, from the Latin *decem*, meaning *ten*.

ANNEXING AND PREFIXING CIPHERS.

ART. **172**. Since the orders *decrease*, from left to right, in a tenfold ratio, and each cipher prefixed to a decimal removes it one place toward the *right*, therefore,

Each cipher prefixed to a Decimal, diminishes its value ten times.

Thus, prefix one cipher to the decimal .5, it becomes .05; prefix two, it becomes .005; three, .0005: each fraction expressing *one-tenth* of the preceding.

As the *value* expressed by a Decimal depends on the *place* from units which it occupies, and as its place is not changed by annexing ciphers, therefore,

Annexing ciphers to a Decimal does not alter its value.

Thus, $.2 = .20 = .200$; that is, $\frac{2}{10} = \frac{20}{100} = \frac{200}{1000}$.

REM.—1. Annexing ciphers to a decimal, is equivalent to multiplying both terms of a fraction by the same number, which does not alter its value. Art. 135.

2. The effect of annexing and prefixing ciphers to decimals, is the reverse of that in whole numbers.

REVIEW.—168. How do the orders of whole numbers decrease from left to right? Give an example. How may the orders be continued from the place of units toward the right?

168. What will the order next to units express? The next order? The next? How may $\frac{1}{10}$ be written without the denominator? $\frac{2}{10}$? $\frac{3}{10}$? $\frac{1}{100}$? $\frac{2}{100}$? $\frac{3}{100}$? In writing one-hundredth, why place a cipher in the order of tenths?

169. What is the denominator of the fraction, expressed by a figure in the first decimal place? In the second? In the third? What is the denominator of a decimal?

ART. **173**. As *ten* hundredths make *one* tenth, and *ten* tenths *one* unit, the decimals *increasing* from right to left as in whole numbers, Art. 8; therefore,

Whole numbers and decimals may be written in the same line, by placing the separating point between them; thus,

2 units and 2 hundredths are written 2.02

A whole number and a decimal, written together, is a *mixed decimal number.*

The decimal point (.)—called a *separatrix*—separates whole numbers and decimals.

ART. **174**. If, in UNITED STATES MONEY, the dollar be taken as the unit, dimes will be *tenths*, cents *hundredths*, and mills *thousandths* of a dollar; hence,

Any sum in U. S. Money may be expressed in dollars and decimal fractions of a dollar. Thus,

$2 and 15 cts., are $2, and 15 *hundredths;* written, $2.15

$1 and 8 mills, are $1, and 8 *thousandths;* written, $1.008

DECIMAL NUMERATION AND NOTATION.

ART. **175**. TO READ DECIMALS; *First, read the decimal as a whole number, then add the name of the right hand order.*

Thus,	.6	is read	six *tenths.*
	.06	..	six *hundredths.*
	.204	..	two hundred and four *thousandths.*

REM.—1. In reading *abstract mixed decimal numbers*, for example, 400.035, the word *units* should be added after the whole number, to distinguish it from the decimal .435 Thus, 400.035 is

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REVIEW.—170. Begin with tenths, and name the places of decimals in order. REM. What is the meaning of ten-thousandths?

171. On what does the value expressed by any figure depend? How do decimals decrease from left to right?

172. How does each 0 prefixed affect the value? Why? Give examples. How does annexing ciphers affect the value? Why? Give examples.

172. REM. 1. Annexing ciphers to a decimal is equivalent to what? 2. How does annexing and prefixing ciphers, compare with the same operation in whole numbers?

read, four hundred *units* and thirty-five *thousandths;* while .435 is read, four hundred and thirty-five *thousandths.* When the mixed number is *concrete,* (*yards,*) instead of saying 400 units, &c., say, 400 *yards* and 35 *thousandths.*

2. Another method: Read the whole number, add the word *decimal,* then read the fraction. Thus, read, 25.025, twenty-five, *decimal,* twenty-five thousandths; or, twenty-five, *decimal,* nought-two-five.

3. Before reading the decimal, let the pupil ascertain the name of the right hand order. Begin at the separatrix, point to each order successively, and pronounce its name; thus, tenths, hundredths, thousandths, ten-thousandths, &c.

EXAMPLES TO BE COPIED AND READ.

(1.)	(2.)	(3.)	(4.)	(5.)
.5	.0003	.00004	.000007	.0000008
.06	.0625	.00137	.000133	.00000009
.003	.2374	.02376	.001768	.1010101
.028	.2006	.31456	.040035	.00100304
.341	.0104	.01007	.360004	.00040005

(6.)	(7.)	(8.)
6.5	.6000000	40504.01037
60.04	.0080000	54000000.000054
184.173	.1020000	30701000.1037025

ART. **176**. TO WRITE DECIMALS; *First, write the number of parts expressed by the decimal, as a whole number; then prefix ciphers till the right hand figure stands in the required order.*

Thus, in writing twenty-five ten-thousandths, first write 25, then prefix two ciphers, so that, beginning at tenths to name the orders, 5 may stand in the order of *ten-thousandths;* .0025

REVIEW.—173. How are whole numbers and decimals written in the same line? What are they called? What the decimal point called?

174. If the dollar be taken as the unit, what are dimes? Cents? Mills? How express any sum in U. S. Money? Give examples.

175. What the rule for reading decimals? REM. 1. How read abstract mixed numbers? Why? 2. What other method? 176. What the rule for writing decimals?

NOTE.—To change a Com. Fraction whose denominator is 1, with ciphers annexed, as, $\frac{25}{10000}$, to a decimal form, *prefix* so many ciphers to the numerator, that the number of places in the decimal shall equal the ciphers in the denominator, Art. 169.

Express these Fractions and Mixed numbers in Decimals.

1. $\frac{5}{10}$, $\frac{5}{100}$, $\frac{135}{10000}$, $\frac{203}{1000000}$, $\frac{4006}{100000000}$.

2. $13\frac{7}{10}$, $24\frac{8}{100}$, $30\frac{25}{1000}$, $6\frac{4}{10000}$, $8\frac{105}{1000000}$.

EXERCISES IN WRITING DECIMALS.

3. Four *tenths.*

4. Two *tenths* and six *hundreaths.*

5. Thirty-five *hundredths.*

6. Eight *hundredths.*

7. Five *thousandths.*

8. Three hundred and four *thousandths.*

9. Four thousand one hundred and twenty-five *ten-thousandths.*

10. Two hundred and five *ten-thousandths.*

11. Eight *ten-thousandths.*

12. 20 thousand three hundred and four *hundred-thousandths.*

13. Six hundred and five *hundred-thousandths.*

14. Nine *hundred-thousandths.*

15. Three hundred thousand and four *millionths.*

16. 203 *millionths.*

17. Seven *millionths.*

18. Twenty-four *ten-millionths.*

19. Eighty-thousand and six *ten-millionths.*

20. Two *hundred-millionths.*

21. Nine hundred and seven *hundred-millionths.*

22. 20 million 20 thousand and three *hundred-millionths.*

23. One million ten thousand and one *hundred-millionths.*

24. Twenty units and twenty-five *hundredths.*

25. One hundred and six units and thirty-seven *thousandths.*

26. One thousand units and one *thousandth.*

27. Two hundred units and twenty-five *thousandths.*

28. 29 units and 29 *millionths.*

29. One million and five *billionths.*

30. Two hundred units and two *ten-billionths.*

31. Sixty-five units and six thousand and five *millionths.*

ART. **177.** Mixed decimal numbers may be read as though they were entirely decimals.

Thus, $6.5 = 6\frac{5}{10} = \frac{65}{10}$: it may be read, 65 *tenths.*

Hence, *when a mixed decimal number is expressed in decimals,* *it may be written as a whole number, and the point so placed, that the right hand figure shall stand in the required order.*

32. One hundred and forty-three *tenths.*

33. Three thousand two hundred and four *hundredths.*

34. Ten thousand five hundred and two *ten-thousandths.*

ADDITION OF DECIMALS.

ART. **178.** 1. Add 4 and 4 ten-thousandths; 28 and 35 thousandths; 8 and 7 hundredths; and 9404 hundred-thousandths.

OPERATION.

```
    4.0004
   28.035
    8.07
     .09404
Ans. 40.19944
```

SOLUTION.—Since only numbers of the same unit value can be added, write figures of the same order under each other.

Since ten units of any order (Art. 173) make one unit of the order next higher, add the figures in each order, and carry one for every ten, as in Addition of Simple Numbers, placing the separatrix under the points above.

*2. Find the sum of 3 units and 25 hundredths; 6 units and 4 tenths; and 35 hundredths. *Ans.* 10.

Rule for Addition.— *Write numbers of the same order under each other, tenths under tenths, hundredths under hundredths, &c.*

Then add, as in Addition of Simple Numbers, and point off in the amount as many places for decimals as are equal to the greatest number of decimal places in either of the given numbers.

PROOF.—The same as in Addition of Simple Numbers.

3. Add 21.611; 6888.32; 3.4167 *Ans.* 6913.3477

4. Add 6.61; 636.1; 6516.14; 67.1234; and 5.1233 *Ans.* 7231.0967

REVIEW.—177. How may mixed decimals be read? Give an example. When a mixed decimal is expressed in words, how written?

178. In writing decimals to be added, why place figures of the same order under each other? Why carry one for every ten? Where place the separating point? How prove Addition of decimals?

5. Add 4 and 8 tenths; 43 and 31 hundredths; 74 and 19 thousandths; 11 and 204 thousandths. *Ans.* 133.333

6. Add 45 and 19 thousandths; 7 and 71 hundred-thousandths; 93 and 4327 ten-thousandths; 6 and 401 ten-thousandths. *Ans.* 151.49251

7. Add 432 and 432 thousandths; 61 and 793 ten-thousandths; 100 and 7794 hundred-thousandths; 6.009; 1000 and 1001 ten-thousandths. *Ans.* 1599.69834

8. Add 16 and 41 thousandths; 9 and 94 millionths; 33 and 27 hundredths; 8 and 969 thousandths; 32 and 719906 millionths. *Ans.* 100.

9. Add 204 and 9 ten-thousandths; 103 and 9 hundred-millionths; 42 and 9099 millionths; 430 and 99 hundredths; 220.0000009 *Ans.* 999.99999999

10. Add 35 ten-thousandths; .00035; 35 millionths and 35 ten-millionths. *Ans.* .0038885

☞ For additional problems, see Ray's Test Examples.

SUBTRACTION OF DECIMALS.

ART. **179**. 1. From 20 and 14 thousandths, subtract 7 and 21 ten-thousandths.

SOLUTION.—Write numbers of the same order under each other, because the difference only between numbers of the same unit value can be found. Since annexing ciphers to a decimal does not alter its value, Art. 172, regard the place of ten-thousandths in the minuend, as occupied by a cipher. Then Subtract as in Simple Numbers.

OPERATION.

```
     20.014
      7.0021
     -------
Ans. 13.0119
```

*2. From 5 and .03 take 2 and .115 *Ans.* 2.915

Rule for Subtraction.—*Write the less number under the greater, tenths under tenths, hundredths under hundredths, &c. Subtract, as in Simple Numbers, and point off the decimal places as in Addition of Decimals.*

PROOF.—The same as in Subtraction of Simple Numbers.

REVIEW.—179. In Subtraction of decimals, why are tenths written under tenths, hundredths under hundredths, &c.?

3. From 24.0042 take 13.7013 *Ans.* 10.3029
4. 170.0035 take 68.00181 *Ans.* 102.00169
5. .0142 take .005 *Ans.* .0092
6. .05 take .0024 *Ans.* .0476
7. 13.5 take 8.037 *Ans.* 5.463
8. 3 take .00003 *Ans.* 2.99997
9. 29.0029 take 19.003 *Ans.* 9.9999
10. 5 take 125 thousandths. *Ans.* 4.875
11. 10000 take 1 ten-thousandth. *Ans.* 9999.9999
12. 1 take 1 millionth. *Ans.* .999999
13. 25 thousandths take 25 millionths. *Ans.* .024975

MULTIPLICATION OF DECIMALS.

Art. 180. Multiplication of decimals embraces two cases.

1. *To multiply together a decimal and a whole number.*
2. *To multiply together two decimals.*

Art. 181. 1. Multiply 125 thousandths by 9.

Solution.—Regard 125 as the numerator of a fraction, the denominator being 1000. Then, since a fraction is multiplied by multiplying its numerator, $125 \times 9 = 1125$ *thousandths* $= 1.125$, by proper pointing (Art. 177).

OPERATION.

$$\begin{array}{r} .125 \\ 9 \\ \hline Ans.\ 1.125 \end{array}$$

In this case, the number of decimal places in the product must equal the number in the multiplicand. If 9 be multiplied by .125, the product will also be 1.125 (Art 30).

*2. Multiply 35 hundredths by 7. *Ans.* 2.45

*3. Multiply 2 tenths by 8 tenths.

OPERATION. $.2 \times .8 = \frac{2}{10} \times \frac{8}{10} = \frac{16}{100} = .16$ *Ans.*

4. Multiply 2 hundredths by 4 tenths.

OPERATION. $.02 \times .4 = \frac{2}{100} \times \frac{4}{10} = \frac{8}{1000} = .008$ *Ans.*

Here, the decimals are converted into common fractions, for

Review.—179. How is the subtraction performed? Why? Give the Rule. 180. What two cases are embraced in Multiplication of decimals? 181. How multiply a decimal fraction by a whole number?

the purpose of explaining the principle on which the product is pointed.

The denominator of the product of two decimals will be 1, with as many ciphers annexed as there are ciphers in both the denominators. But the number of ciphers in each denominator is the same, Art. 169, as the number of places in the decimal.

Hence, the number of decimal places in the product, *must equal the number of decimal places in both factors.*

*5. Multiply 15 hundredths by 7 tenths. *Ans.* .105

General Rule for Multiplication.—*Multiply as in Simple Numbers; point off from the right of the product as many figures for decimals as there are decimal places in both multiplicand and multiplier; if there be not so many places in the product, supply the deficiency by prefixing ciphers.*

PROOF.—The same as in Multiplication of Simple Numbers.

6. Multiply	125.015	7. Multiply	.135
by	.001	by	.005
Ans.	.125015	*Ans.*	.000675

8.	Multiply 1.035 by 17.	*Ans.* 17.595
9.	19 by .125	*Ans.* 2.375
10.	4.5 by 4.	*Ans.* 18.
11.	.625 by 64.	*Ans.* 40.
12.	61.76 by .0071	*Ans.* .438496
13.	1.325 by .0716	*Ans.* .09487
14.	79000 by .079	*Ans.* 6241.
15.	1 tenth by 1 hundredth.	*Ans.* .001
16.	1 by 1 ten-thousandth.	*Ans.* .0001
17	43 thousandths by .0021	*Ans.* .0000903
18.	40000 by 1 millionth.	*Ans.* .04
19.	.09375 by 1 & 64 millionths.	*Ans.* .093756

ART. **182**. The operations of multiplying by 10, 100,

REVIEW.—181. To what is the denominator of the product of two decimals equal? To what the number of ciphers in each denominator?

1000, &c., may be shortened by removing the decimal point as many places to the *right*, as there are ciphers in the multiplier: and,

If there be not so many figures on the right of the point, *annex ciphers* to supply the deficiency.

Thus, $2.07 \times 10 = 20.7$

☞ For additional problems, see Ray's Test Examples.

DIVISION OF DECIMALS.

ART. **183**. Decimals may be divided when the divisor, or dividend, or both, are decimals.

Since the dividend is equal to the product of the divisor and quotient, it must contain as many decimal places as there are decimals in both divisor and quotient. Art. 181: hence,

There must be as many decimals in the quotient as the decimal places in the dividend *exceed* those in the divisor.

1. Divide 2.125 by 5 tenths.

SOLUTION.—Divide as in Simple Numbers; then, since there are *three* decimal places in the dividend, and *one* in the divisor, point off *two* decimals in the quotient.

OPERATION.
.5)2.1 2 5
Ans. 4.25

2. Divide 21 units by .5

SOLUTION.—In dividing a whole number by a fraction, the whole number is reduced to the same parts of a unit as the fraction, that the divisor and dividend may be of the same denomination.

OPERATION.
.5)21.0
Ans. 42.

So, in dividing a whole number by a decimal; reduce the dividend to the same denomination as the divisor, by annexing to it as many ciphers as there are decimal places in the divisor, and the quotient is a whole number.

3. Divide 83.1 by 4.

SOLUTION.—Divide the figures of the dividend by the divisor, as in whole numbers, and a remainder occurs. Then, to continue

---

REVIEW.—181. To what is the number of decimal places in the product equal? What is the General Rule for multiplication?

182. How multiply a decimal by 10, 100, &c.? 183. When may decimals be divided? How many decimal places must the quotient contain? Why?

the division, annex ciphers to the dividend, which does not alter its value, (Art. 172), and divide as before. Continue the division until there is no remainder, or until the quotient is sufficiently exact.

OPERATION.
4)83.100
Ans. 20.775

As there are *three* places of decimals in the dividend, and *none* in the divisor, there must be *three* in the quotient.

4. Divide 2.11 by .3

In this example, the division will not terminate. In such cases, it is to be carried to sufficient exactness: the sign + is annexed to denote that the division is not complete.

OPERATION.
.3)2.11000
Ans. 7.0333+

*5. Divide 1.125 by .03 — *Ans.* 37.5
*6. Divide 2 by .008 — *Ans.* 250.
*7. Divide 37.2 by 5. — *Ans.* 7.44

General Rule for Division.—*Divide as in Simple Numbers, and point off from the right hand of the quotient as many places for decimals as the decimal places in the dividend exceed those in the divisor; if there be not so many places, supply the deficiency by prefixing ciphers.*

PROOF.—The same as in Division of Simple Numbers.

NOTES.—1. When the divisor has *more* decimals than the dividend, annex ciphers to the dividend until its decimal places *equal* those of the divisor; the quotient will be a whole number.

2. After dividing all the figures of the dividend, if there be a remainder, annex ciphers to it, and continue the division till there is no remainder, or until the quotient is sufficiently exact. In pointing the quotient, regard the ciphers annexed as decimal places.

8. Divide 86.075 by 27.5 — *Ans.* 3.13
9. 24.73704 by 3.44 — *Ans.* 7.191
10. 206.166492 by 4.123 — *Ans.* 50.004

REVIEW.—183. What is the General Rule for Division? NOTE 1. When the number of places in the divisor exceeds those in the dividend, what is required? Why? What will be the quotient?

11. Divide 100.8788 by 454. *Ans.* .2222
12. .000343 by 3.43 *Ans.* .0001
13. 9811.0047 by .108649 *Ans.* 90300.
14. .21318 by .19 *Ans.* 1.122
15. 102048 by .3189 *Ans.* 320000.
16. .102048 by 3189. *Ans.* .000032
17. 9.9 by .0225 *Ans.* 440.
18. How often is 10 contained in .1? *Ans.* .01
19. 1 tenth contained in 1? *Ans.* 10.
20. 1 hundredth in 10? *Ans.* 1000.
21. 64 in 1 and 7 tenths? *Ans.* .0265625
22. 80 in 8 hundredths? *Ans.* .001
23. 1000 in 1 thousandth? *Ans.* .000001
24. Divide 1 thousandth by 1 thousandth. *Ans.* 1.
25. 1 ten-thousandth ÷ 1 ten-millionth. *Ans.* 1000.
26. 1 hundredth ÷ 4 millionths. *Ans.* 2500.
27. 1.5 ÷ .7 *Ans.* 2.142857+

ART. **184**. To divide a Decimal by 10, 100, 1000, &c., remove the decimal point as many places to the left as there are ciphers in the divisor:

And, if there are not so many figures on the left of the point, supply the deficiency by *prefixing* ciphers.

Thus, 18.3 divided by 10 = 1.83
18.3 divided by 100 = .183
18.3 divided by 1000 = .0183

ART. **185**. REDUCTION OF DECIMALS.

CASE I.—*To reduce a common fraction to a decimal.*

1. Reduce the fraction $\frac{3}{4}$ to a decimal.

SOL.—The numerator, 3, will not be changed by writing ciphers in the place of tenths, hundredths, &c. Since a fraction is expressed in the form of an unexecuted division (Art. 125), regard the operation as division of decimals, and perform it according to the rule, (Art. 183). Since the divisor has *no* decimal places, the quotient must have as many places as there are ciphers annexed.

OPERATION.

$4)3.00$

Ans. .75

*2. Reduce $\frac{1}{8}$ to a decimal. *Ans.* .125

Rule for Case I.—*Annex ciphers to the numerator, divide by the denominator, and point off in the quotient as many places for decimals as there are ciphers annexed to the numerator.*

NOTE.—When common fractions can not be *exactly* expressed in decimals, continue to divide till the quotient is sufficiently exact.

REDUCE THESE COMMON FRACTIONS TO DECIMALS:

3.	$\frac{4}{5}$.	*Ans.* .8	8.	$\frac{9}{400}$.	*Ans.* .0225
4.	$\frac{4}{25}$.	*Ans.* .16	9.	$\frac{1}{256}$.	*Ans.* .00390625
5.	$\frac{3}{40}$.	*Ans.* .075	10.	$\frac{5}{6}$.	*Ans.* .8333+
6.	$\frac{15}{16}$.	*Ans.* .9375	11.	$\frac{4}{33}$.	*Ans.* .121212+
7.	$\frac{1}{1250}$.	*Ans.* .0008	12.	$\frac{1}{11}$.	*Ans.* .090909+

CASE II.

ART. **186**. *To reduce a decimal to a common fraction, and to its lowest terms.*

1. Reduce .75 to a common fraction in its lowest terms.

SOL.—By *writing* the denominator, .75 becomes $\frac{75}{100}$, and, reduced to its lowest terms by the rule (Art. 138), it becomes $\frac{3}{4}$.

OPERATION.

$.75=\frac{75}{100}=\frac{3}{4}$ *Ans.*

*2. Reduce .6 to a common fraction, in its lowest terms. *Ans.* $\frac{3}{5}$.

Rule for Case II.—*Write the denominator under the decimal; it will then be a common fraction, which reduce to its lowest terms*, (Art. 138).

REDUCE TO COMMON FRACTIONS, IN THEIR LOWEST TERMS,

3.	.25	*Ans.* $\frac{1}{4}$.	7.	.033	*Ans.* $\frac{33}{1000}$.
4.	.375	*Ans.* $\frac{3}{8}$.	8.	.5625	*Ans.* $\frac{9}{16}$.
5.	4.02	*Ans.* $4\frac{1}{50}$.	9.	.34375	*Ans.* $\frac{11}{32}$.
6.	8.415	*Ans.* $8\frac{83}{200}$.	10.	.1484375	*Ans.* $\frac{19}{128}$.

NOTE.—When a decimal contains several places of figures, their value can be found *nearly* by inspection. To do this, take the *first* or the *first two* figures, as the numerator, of which 10 or 100 is the denominator, then reduce it to its lowest terms:

Thus .332125 is nearly 1 third; .258321 is nearly 1 fourth.

CASE III.

ART. **187**. *To reduce a decimal of one denomination to an equivalent decimal of another denomination.*

1. Reduce .25 qt. to the fraction of a pt.

OPERATION.

$$\begin{array}{r} \text{qt.} \\ .25 \\ 2 \\ \hline .50 \end{array}$$

Ans. .5 pt.

SOLUTION.—To reduce quarts to pints, we *multiply* by 2, the number of pints in a quart.

Therefore, multiply the *fraction* of a quart by 2, to reduce it to the *fraction* of a pint. Art. 81.

2. Reduce .5 pt to the fraction of a qt.

OPERATION.

$$\begin{array}{r} \text{pt.} \\ 2).5 \\ \hline \end{array}$$

Ans. .25 qt.

SOLUTION.—To reduce pints to quarts, we *divide* by 2, the number of pints in a quart.

Therefore, divide the *fraction* of a pint by 2, to reduce it to the *fraction* of a quart. Art. 81.

*3. Reduce .125 bu. to the fraction of a pk. *Ans.* .5

*4. Reduce .7 pk. to the fraction of a bu. *Ans.* .175

Rule for Case III.—*Multiply or divide as in Reduction of Whole Numbers*, Art. 81, *according to the rules for the Multiplication and Division of Decimals.* Arts. 181 and 183.

5. Reduce .0625 lb. Troy to the frac. of an oz. *Ans.* .75
6. .05 of a yd. to the fraction of a na. *Ans.* .8
7. .00546875 A. to the fraction of a P. *Ans.* .875

REVIEW.—185. What is Case 1? Rule for Case 1? NOTE. When com. fractions can not be exactly expressed in decimals, what is to be done?

186. What is Case 2? Rule for Case 2? NOTE. How find the value *nearly* of a decimal? 187. How reduce a decimal from a higher to a lower denomination? From a lower to a higher?

8. Reduce .0004375 mi. to the frac. of a rd. *Ans.* .14
9. .25 pt. to the fraction of a gal. *Ans.* .03125
10. .6 pt. to the fraction of a bu. *Ans.* .009375
11. .3 min. to the fraction of a da. *Ans.* .0002083+
12. .7 rd. to the fraction of a mi. *Ans.* .0021875

CASE IV.

ART. **188**. *To find the value of a decimal in integers of a lower denomination.*

1. Find the value of .3125 of a bu.

OPERATION.

```
     bu.
    .3125
        4
   ------
pk. 1.2500
        8
   ------
qt. 2.00
```

SOLUTION.—First multiply by 4, as in reducing bushels to pecks. Art 81. This gives 1 peck and .25 of a peck; then find the value of .25 pk., by multiplying it by 8, the number of quarts in a peck. This gives 2 quarts; hence, .3125 of a bushel equals 1 pk. 2 qt. *Ans.*

*2. Find the value of .875 of a yd. *Ans.* 3 qr. 2 na.

Rule for Case IV.—*Multiply the given decimal by that number which will reduce it to the next lower denomination* (Art. 81), *and point the product, as in Multiplication* (Art. 181). *Reduce this decimal in like manner, and so on; the several integers on the left will be the required answer.*

FIND THE VALUE IN INTEGERS,

3. Of .7 of a lb. Troy. . . . *Ans.* 8 oz. 8 pwt.
4. .8125 of a bu. . . . . . . *Ans.* 3 pk. 2 qt.
5. .3375 of an A. . . . . . . *Ans.* 1 R. 14 P.
6. .04318 of a mi. *Ans.* 13 rd. 4 yd. 1 ft. 5.8848 in.
7. .33625 of a cwt. . . . *Ans.* 1 qr. 8 lb. 10 oz.

CASE V.

ART. **189**. *To reduce a quantity composed of one or more denominations, to the decimal of another quantity of one or more denominations.*

1. Reduce 2 ft. 3 in. to the decimal of a yard.

SOLUTION.—By Art. 163, the result expressed as a common fraction, is $\frac{3}{4}$. This reduced to a decimal, becomes .75, the required answer.

OPERATION.

2 ft. 3 in. = 27 in.
1 yd. = 36 in.
$\frac{27}{36} = \frac{3}{4} = .75$ *Ans.*

*2. Reduce 2 pk. 4 qt. to the dec. of a bu. *Ans.* .625

Rule for Case V.—*Reduce the first quantity and that of which it is to be made a part, both to the same denomination; the less will be the numerator, and the greater the denominator of a common fraction, which reduce to a decimal.*

3. Red. 13 hr. 30 min. to the dec. of a da. *Ans.* .5625
4. 9 dr. to the dec. of a lb. Av. *Ans.* .03515625
5. .028 of a P. to the dec. of an A. *Ans.* .000175
6. 7 min. to the dec. of a da. *Ans.* .0048611+
7. 4 gal. 1 qt. 1.28 pt. to the dec. of a hhd. *Ans.* .07
8. What part is 3 pk. 7 qt. 1 pt. of 2 bu. 2 pk. 4 qt., expressed decimally? *Ans.* .375
9. What part is 99 pages, of a book of 512 pages, expressed decimally. *Ans.* .193359375
10. What decimal will express the part that 55 A. 2 R. 17 P. is of 229 A. 2 R. 16 P.? *Ans.* .2421875

ART. 190. PROMISCUOUS EXAMPLES.

1. What cost 9 yards of muslin, at \$0.4 per yd., and 12 yards at \$0.1875 per yd.? *Ans.* \$5.85
2. What cost 2.3 yards of ribbon, at \$0.45 per yd., and 1.5 yards at \$0.375 per yd.? *Ans.* \$1.5975
3. At \$2.6875 per yd., what cost 16 yd. of cloth? What $16\frac{1}{4}$ yd.? *Ans.* to last, \$43.671875
4. At \$0.75 per bushel, how much wheat can be bought for \$35.25? *Ans.* 47 bu.

REVIEW.—188. What is Case 4? What the Rule for Case 4? 189. What is Case 5? What the Rule for Case 5?

5. At \$2.5625 per yard, how much cloth can you buy for \$98.4? *Ans.* 38.4 yd.

6. What cost 6 cwt. 2 qr. of hops, at \$3.25 per cwt.?

Solu.—Reduce 2 qr. to the decimal of a cwt.; then, 6 cwt. 2 qr. $=$ 6.5 cwt.; \$3.25 $\times$ 6.5 $=$ *Ans.* \$21.125

7. What will be the cost of 7 hhd. 23 gal. of wine, at \$49 per hhd.? *Ans.* \$360.8888+

8. Of 343 yd. 3 qr. linen, at \$.16 per yd.? *Ans.* \$55.

9. What cost 14 bu. 3 pk. 4 qt. of corn, at \$0.625 per bushel? *Ans.* \$9.296875

10. What will 13 A. 2 R. 35 P. of land cost, at \$17.28 per acre? *Ans.* \$237.06

11. At \$1.24 per yard, how much cloth can be bought for \$19.065? *Ans.* 15.375 yd. $=$ 15 yd. 1 qr. 2 na.

12. At \$0.3125 per bu., how much corn can be bought for \$9.296875? *Ans.* 29.75 bu. $=$ 29 bu. 3 pk.

13. At \$4.32 per A., how much land can you buy for \$59.265? *Ans.* 13.71875 A. $=$ 13 A. 2 R. 35 P.

14. Add .34 yd. .325 qr. .4 na. *Ans.* 1 qr. 3.14 na.

To add or subtract decimals of different denominations, first reduce them to the same denomination. In this example, reduce the dec. of a yd. to qr., then add the dec. of a qr.; next reduce this result to na., and add the dec. of a na.

15. From 1.53 yards take 1.32 qr. *Ans.* 1 yd 3.2 na.

16. .05 of a year, (365.25 days,) take .5 of an hour. *Ans.* 18 da. 5 hr. 48 min.

17. .41 of a da. take .16 of an hr. *Ans.* 9 hr. 40 min. 48 sec.

18. In .4 T .3 hhd .8 gal. how many pt.? *Ans.* 964 pt.

19. Find the value of .3 of a year, (365.25 days,) in integers. *Ans.* 109 da. 13 hr. 48 min.

20. In .005 of a year how many sec.? *Ans.* 157788.

21. What decimal of a C. is 1 cu. in? *Ans.* .000004+

22. At \$690.35 per mile, what cost a road 17 mi. 3 fur. 15 rd. long? *Ans.* \$12027.19140625

In practice, only 3 or 4 places of decimals are generally used.

☞ For additional problems, see Ray's Test Examples.

XIII. RATIO.

ART. **191**. Ratio is the *quotient* arising from dividing one quantity by another of the *same denomination;*

Thus, the ratio of 2 to 6 is 3; as, 6 ÷ 2 gives the quotient 3.

The ratio of 2 to 8 is 4; of 2 yd. to 10 yds., 5.

ILLUSTRATIONS.—1. Two quantities to be compared, or to have a ratio to each other, must be of the same kind, and in the *same denomination*, that the one may be some part of the other.

Thus, 2 *yards* has a ratio to 6 *yards*. But, 2 *yards* has no ratio to 6 *dollars*, the one being no *part* of the other.

2. Since ratio is the relation of two numbers expressed by their *quotient;* and since the quotient of 2 and 6 may be 6 divided by 2, or 2 divided by 6, either may be used to express their ratio.

Thus, in comparing two lines, one of which is 2, the other 6 inches long, if the first is taken as the *unit* (1) or standard of comparison, the second is *three*, that is, it is 3 times the first. If 6 is taken as the unit of comparison, 2 is *one-third*.

In finding the ratio between two numbers, the French take the *first* as the *divisor*, the English the *last*. The French method being regarded the most simple, is now generally used.

3. Finding the ratio of two numbers, is finding what *part*, or what *multiple* one is of the other.

The following are equivalent expressions: What is the ratio of 2 to 6? What part of 2 is 6? What multiple of 2 is 6?

4. The ratio of \$2 to \$10 is 5; of \$2000 to \$10000 is also 5: hence, ratio shows only the *relative* magnitude of two quantities.

REVIEW.—190. What is ratio? Give examples. ILLUS. 1. Can quantities, not of the same kind, have a ratio? Why not? 2. What is the ratio of 2 to 6? 3. What are equivalent expressions?

ILLUS. 4. What does ratio show? In finding the ratio between two quantities of the same kind, but of different denominations, what is required?

1. What is the ratio of 3 to 6? of 3 to 9?
2. Of 2 to 12? of 3 to 15? of 7 to 21?
3. Of 6 to 18? of 6 to 30? of 5 to 30?
4. Of 2 to 3? *Ans.* $\frac{3}{2}=1\frac{1}{2}$. of 2 to 5?
5. Of 3 to 4? of 5 to 8? of 4 to 6?
6. Of 12 inches to 36 inches? of 2 feet to 9 feet?
7. Of 3 inches to 1 foot 9 inches? . . *Ans.* 7.

When the quantities are of the same kind, but of different denominations, reduce them to the same denomination

8. What is the ratio of 3 in. to 2 ft.? of 4 in. to 3 yd.?
9. Of 15 to 25? *Ans.* $1\frac{2}{3}$. of 25 to 15? *Ans.* $\frac{3}{5}$.
10. Of 4 to 10? of 10 to 4? of 6 to 16? of 16 to 6?

ART. **192**. A ratio is formed by two numbers, each of which is called a *term*, and both together, a *couplet.*

Thus, 2 and 6 together form a couplet of which 2 is the first term, and 6 the second.

The *first* term of a ratio is called the *antecedent;* the second, the *consequent.*

ART. **193**. RATIO IS EXPRESSED IN TWO WAYS

1st. In the form of a fraction, of which the *antecedent* is the *denominator*, and the *consequent* the *numerator.*

The ratio of 2 to 6 is expressed by $\frac{6}{2}$; of 3 to 12, by $\frac{12}{3}$.

2d. By a colon (:) between the terms of the ratio.

Thus, the ratio of 2 to 6 is written 2:6; of 3 to 8, 3:8.

ART. **194**. Since the ratio of two numbers is expressed by a *fraction*, of which the antecedent is the denominator, and the consequent the numerator, whatever is true of a fraction, is true of a ratio. Hence,

---

REVIEW.—192. By what is a ratio formed? What is each number called? What both together? What is the first term called? The 2D?
193. In how many ways is ratio expressed? What is the first method? The second? Give examples of each.

1st. *To multiply the consequent, or divide the antecedent, multiplies the ratio.* Arts. 131 and 133. Thus,

The ratio of 4 to 12 is 3; of 4 to 12×5, is 3×5; and
The ratio of $4\div2$ to 12, is 6, which is equal to 3×2.

2d. *To divide the consequent or multiply the antecedent, divides the ratio.* Arts. 132 and 134. Thus,

The ratio of 3 to 24 is 8; of 3 to $24\div2$, is $4, =8\div2$; and
The ratio of 3×2 to 24, is 4, which is equal to $8\div2$.

3d. *To multiply or divide both consequent and antecedent by the same number, does not alter the ratio.* Arts. 134, 135. Thus,

The ratio of 6 to 18, is 3; of 6×2 to 18×2, is 3; and
The ratio of $6\div2$ to $18\div2$, is 3.

ART. **195**. A single ratio, as 2 to 6, is a *simple ratio.*

A *compound ratio* is the product of two or more simple ratios. Thus,

The ratio $\frac{10}{3}$ multiplied by the ratio $\frac{6}{5}$, is $\frac{10}{3}\times\frac{6}{5}=\frac{60}{15}=4$.

In this case, 3 multiplied by 5, is said to have to 10×6, the ratio *compounded* of the ratios of 3 to 10 and 5 to 6.

ART. **196**. Ratios may be compared with each other, by reducing to a common denominator the fractions by which they are expressed: thus,

To find the greater of the two ratios, 2 to 5, and 3 to 8, we have $\frac{5}{2}$ and $\frac{8}{3}$, which, reduced to a common denominator, are $\frac{15}{6}$ and $\frac{16}{6}$; and, as $\frac{15}{6}$ is less than $\frac{16}{6}$, the ratio of 2 to 5, is less than the ratio of 3 to 8.

XIV. PROPORTION.

ART. **197**. Proportion is an equality of ratios. Four numbers are proportional, when the first has the same ratio to the second that the third has to the fourth.

REVIEW.—194. How is a ratio affected by multiplying the consequent, or dividing the antecedent? By dividing the consequent, or multiplying the antecedent? By multiplying or dividing both consequent and antecedent by the same number? Why? Illustrate each.

Thus, the two ratios, 2 : 4 and 3 : 6, form a proportion, since $\frac{4}{2} = \frac{6}{3}$, each being equal to 2

ART. **198.** PROPORTION IS WRITTEN IN TWO WAYS:

1st. By placing a double colon between the ratios.

Thus, 2 : 4 : : 3 : 6.

2d. By placing the sign of equality between them.

Thus, 2 : 4 = 3 : 6.

The first is read, 2 is to 4 as 3 is to 6; or, 2 has the same ratio to 4, that 3 has to 6. The second is read, the ratio of 2 to 4 *equals* the ratio of 3 to 6.

REM.—1. The least number of terms that can form a proportion is *four*, since there are *two* ratios each containing *two* terms.

But, one of the terms in each ratio may be the same; thus, 2:4::4:8. The number repeated is called a MEAN proportional between the other two terms.

2. The terms *ratio* and *proportion* are often confounded with each other. Two quantities having the same ratio as 3 to 4, are *improperly* said to be in the proportion of 3 to 4. A ratio subsists between *two* quantities; a proportion only between *four*.

ART. **199**. The first and last terms of a proportion are called the *extremes;* the second and third, the *means.*

Thus, in the proportion 2 : 3 : : 4 : 6, 2 and 6 are the extremes, and 3 and 4 the means.

ART. **200**. *In every proportion, the product of the means is equal to the product of the extremes.*

ILLUSTRATIONS.—If we have 3 : 4 : : 6 : 8,

the ratios of each couplet being equal (Art. 206), we must have . . . . . . $\frac{4}{3} = \frac{8}{6}$

Reducing these fractions to a common denominator (Art. 155), gives . $\frac{4 \times 6}{18} = \frac{3 \times 8}{18}$.

REVIEW.—195. What is a simple ratio? A compound ratio? Give examples of each. 196. How compare ratios with each other? 197. What is proportion? When are four numbers proportional? Give examples.

198. How is proportion written? How is the first read? The second? REM. 1. What the least number of terms that can form a proportion? 2. What of the terms ratio and proportion?

The denominators being the same, and the values of the fractions being equal, the numerators must be equal; that is, 4×6, the product of the *means*, must$=3\times8$, the product of the *extremes*.

REM.—The preceding shows that four numbers are *not* in proportion, when the product of the extremes is *unequal* to the product of the means. Thus, 2, 3, 5, and 8, are not in proportion, for 3×5 is not equal to 2×8.

Proportions have numerous properties, the full discussion of which belongs to Algebra. See "Ray's Algebra, First Book."

PROPORTION is divided into *Simple* and *Compound*.

ART. 201. SIMPLE PROPORTION.

Simple Proportion contains only simple ratios, Art. 195. It is sometimes called the RULE OF THREE, as three terms are given to find a fourth.

REM.—Some authors divide Proportion into *direct* and *inverse;* a distinction of no utility, and always embarrassing to the learner.

ART. **202**. Since the product of the means equals the product of the extremes, and the product of two factors divided by either of them, gives the other, Art. 37,

Therefore, *If the product of the means be divided by one of the extremes, the quotient will be the other extreme.* Or,

If the product of the extremes be divided by one of the means, the quotient will be the other mean.

Thus, in the proportion $2 : 3 :: 4 : 6$,

$3\times4\div2=6$, one of the extremes.
$3\times4\div6=2$, the other extreme.
$2\times6\div3=4$, one of the means.
$2\times6\div4=3$, the other mean.

REVIEW.—199. What are the first and last terms called? The second and third? 200. If four numbers are proportional, to what is the product of the means equal? REM. When are four numbers *not* in proportion? How is proportion divided?

201. What does Simple Proportion contain? What is it called? Why? 202. When 3 terms of a proportion are given, how find the fourth?

Hence, *If any three terms of a proportion are given, the fourth may be found by multiplying together the terms of the same name, and dividing their product by the other given term.*

1. The first three terms of a proportion are 2, 8, and 6: what is the fourth term?

SOLUTION.—The preceding shows that the 4th term will be found by taking the product of the 2d and 3d terms, and dividing by the 1st.

Or, by the nature of proportion, Art. 197, the ratio of the 3d term to the 4th = the ratio of the 1st to the 2d.

OPERATION.

$2 : 8 :: 6$

$\frac{8 \times 6}{2} = 24$. *Ans.*

Or,

$\frac{8}{2} = 4$, and $6 \times 4 = 24$.

Hence, *If the third term be multiplied by the ratio of the first to the second, the product will be the fourth term.*

EXAMPLES TO BE SOLVED BY EITHER METHOD.

2. The first three terms of a proportion are 5, 7, and 10: what is the fourth term? *Ans.* 14.

3. The last three terms are 8, 6, and 16: what is the first? *Ans.* 3.

4. The first, third, and fourth terms, are 5, 6, and 12: what is the second? *Ans.* 10.

5. The first, second, and fourth terms, are 3, 7, and 14: what is the third? *Ans.* 6.

6. Seven is to 14, as 9 is to what number? *Ans.* 18.

ART. **203**. 1. If 2 lb. of tea cost $4, at the same rate, what will be the cost of 6 lb.?

SOLUTION.—Two pounds have the same ratio to 6 lb., that the cost of 2 lb. ($4), has to the cost of 6 lb. Therefore, the first three terms are given, to find the fourth (Art. 202).

To find the result, multiply the 3d term by the 2d, and divide by the 1st; or, multiply the 3d term by the ratio (3) of the first to the 2d.

OPERATION.

```
lb.  lb.    $
 2 : 6  ::  4
            6
        2)24
```

Ans. $12.

In stating this question, (arranging the terms,) we

REVIEW.—202. If the third term of a proportion be multiplied by the ratio of the first to the second, what will be the product?

may, with equal propriety, place the $4 as the first term.

Thus, $4 : 2d term :: 2 lb. : 6 lb.

The operation of finding the 2d term in this arrangement, is like that of finding the 4th in the preceding.

It is more convenient to arrange the terms so that the *required* term shall be the *fourth;* then,

Since a ratio can subsist only between quantities of the same denomination, Art. 191, Illus. 1,

The third term of a proportion must be of the same denomination as that in which the answer is required.

2. If 3 men can dig a cellar in 10 days, in how many days will 5 men dig it?

SOLUTION.—Since the answer is to be *days*, write 10 *days* for the 3D term. The result will be found by multiplying the 3D term by the 2D, and dividing by the 1ST (Art. 201); and,

Since it will require 5 men a *less* number of days than 3 men, place the *less* number of men for the 2D term, and the *greater* for the 1ST.

OPERATION.

m.	m.	da.
5 :	3 ::	10
		3
	5)	30
		Ans. 6.

3. If 3 men can dig a cellar in 10 days, how many men will dig it in 6 days?

SOLUTION.—Since the answer is to be *men*, write 3 *men* for the third term. Since it will require a *greater* number of men in 6 days than in 10 days, place the *greater* number of days for the second term, and the *less* for the first.

STATEMENT.

da.	da.	m.
6 :	10 ::	3

Ans. 5 men.

*4. If 3 yd. cloth cost $8, what cost 6 yd.? *Ans.* $16.

*5. If 5 bl. flour cost $30, what cost 3 bl.? *Ans.* $18.

GENERAL RULE FOR SIMPLE PROPORTION.

1. *Write for the third term that number which is of the denomination required in the answer.*

2. *If, from the nature of the question, the answer should be greater than the third term, place the greater of the other two numbers for the second term, and the less for the first; but,*

If the answer should be less than the third term, place the less number for the second term, and the greater for the first.

3. *Multiply the third term by the second, and divide the product by the first; the quotient will be the answer in the denomination of the third term.*

NOTES.—1. If the first and second terms contain different denominations, reduce them to the same; if the third term consists of more than one denomination, reduce it to the lowest given.

2. Multiplying the 2d and 3d terms together, and dividing by the 1st, is really multiplying the 3d by the ratio of the 1st to the 2d.

It is most convenient, especially when the ratio is a whole number, to express it in its simplest form before multiplying.

3. After dividing, if there be a remainder, reduce it to the next lower denomination, and divide again, and so on.

6. If 3lb. 12oz. tea cost $3.50, what cost 11lb. 4oz.?

SOLUTION.—State the question; and, to express the 1st and 2d terms in the same denomination, reduce both to oz.

Then multiply the third term ($3.50) by 180, and divide the product by 60.

But it is shorter to multiply at once by 3, the ratio of 60 to 180.

OPERATION.

lb. oz.		lb. oz.		$
3 12	:	11 4	::	3.50
oz.		oz.		$
$\not{6}\not{0}$	:	$\not{1}8\not{0}$	::	3.50
1	:	3		3
				Ans. $10.50

7. If 2lb. 8oz. of tea cost $2, what quantity can you buy for $5? *Ans.* 6lb. 4oz.

Reduce the 3d term to ounces; the answer will then be in ounces.

8. If 4 hats cost $14, what cost 10 hats? *Ans.* $35.

9. If 3 caps cost 69 cents, what cost 11? *Ans.* $2.53

10. If 4yd. cloth cost $7, what cost 9? *Ans.* $15.75

11. If 8yd. cloth cost $32, what cost 12? *Ans.* $48.

REVIEW.—203. In stating a question in simple proportion, what number is put for the 3d term, Rule? How are the other two numbers arranged? How is the answer obtained? Of what denomination is the answer?

203. NOTE 1. If the first and second terms contain different denominations, what is required? What if the third term contains more than one denomination? What other explanations in Notes 2 and 3?

12. If 12 yd. cloth cost $48, what cost 8 yd.? *Ans.* $32.

13. If $32 purchase 8 yards of cloth, how many yards will $48 buy? *Ans.* 12 yd.

14. If $48 purchase 12 yd. of cloth, how many yards can be bought for $32? *Ans.* 8 yd.

15. A man receives $152 for 19 months' work: how much should he have for 4 mon. work?

SUGGESTION.—Since the product of the second and third terms is to be divided by the first, therefore, (Art. 71,) when the first term contains one or more factors common to either of the other terms, shorten the operation by *canceling* the common factors.

OPERATION.

mon. mon. $
19 : 4 :: 152

$$\frac{\overset{8}{\cancel{152}}\times 4}{\cancel{19}} = \$32 \text{ } Ans.$$

16. If 8 men perform a piece of work in 24 days, in what time can 12 men perform it? *Ans.* 16 days.

17. If 60 men perform a piece of work in 8 days, how many men will perform it in 2 days? *Ans.* 240.

18. If 15 oz. of pepper cost 25 cts., what cost 6 lb.?

SUGGESTION.—Instead of multiplying the 6 lb. by 16, to reduce it to ounces, *indicate* the multiplication, and cancel the factors common to the divisor and dividend. In this manner, operations may be shortened.

OPERATION.

oz. lb. cts.
15 : 6 :: 25
15 : 6 × 16 :: 25

$$\frac{\overset{2}{\cancel{6}}\times 16\times \overset{5}{\cancel{25}}}{\underset{\cancel{5}}{\cancel{15}}} = 160 \text{ cts. } Ans.$$

19. If 6 gal. of molasses cost 65 cts., what cost 2 hhd.? *Ans.* $13.65

20. If 5 cwt. 3 qr. 10 lb. of sugar cost $21.06, what will 35 cwt. 1 qr. cost? *Ans.* $126.90

21. If 1 yd. 2 qr. of cloth cost $2.50, what will be the cost of 1 qr. 2 na.? *Ans.* $0.62 5

22. If 90 bu. of oats supply 40 horses 6 days, how long will 450 bu. supply them? *Ans.* 30 da.

23. If 6 men build a wall in 15 days, how many men can build it in 5 days? *Ans.* 18 men.

24. If 15 bu. of corn pay for 60 bu. of potatoes, how much corn can be had for 140 bu. potatoes? *Ans.* 35 bu.

25. If 3 cwt. 1 qr. of sugar cost $22.60, what will be the cost of 16 cwt. 1 qr.? *Ans.* $113.

26. If a perpendicular staff, 3 ft. long, cast a shadow 4 ft. 6 in., what is the height of a steeple, whose shadow measures 180 ft.? *Ans.* 120 ft.

27. If a man perform a journey in 60 days, traveling 9 hours each day, in how many days can he perform it by traveling 12 hours a day? *Ans.* 45 da.

28. A merchant failing, paid 60 cts. on each dollar of his debts. He owed A $2200, and B $1800: what did each receive? *Ans.* A $1320. B $1080.

29. A merchant having failed, owes A $800.30; B $250; C $375.10; D $500; F $115. His property, worth $612.12, goes to his creditors: how much will this pay on the dollar? *Ans.* 30 cts.

30. If the 4 cent loaf weigh 9 oz. when flour is $8 a bl., what will it weigh when flour is $6 a bl.? *Ans.* 12 oz.

31. I borrowed $250 for 6 mon.: how long should I lend $300, to compensate the favor? *Ans.* 5 mon.

32. A starts on a journey, and travels 27 miles a day; 7 days after, B starts, and travels the same road, 36 mi. a day: in how many da. will B overtake A? *Ans.* 21 da.

33. If William's services are worth $\$15\frac{2}{3}$ a mon., when he labors 9 hr. a da., what ought he to receive for $4\frac{2}{5}$ mon., when he labors 12 hr. a da.? *Ans.* $\$91.91\frac{1}{9}$

34. If 5 lb. of butter cost $\$\frac{5}{8}$, what cost $\frac{3}{4}$ lb.? *Ans.* $\$\frac{3}{32}$.

35. If 6 yd. cloth cost $\$5\frac{3}{5}$, what cost $7\frac{3}{8}$ yd.? *Ans.* $\$6\frac{53}{60}$.

36. If $\frac{1}{3}$ bu. wheat cost $\$\frac{3}{8}$, what cost $\frac{1}{2}$ bu.? *Ans.* $\$\frac{9}{16}$.

37. If $1\frac{3}{4}$ yd. cloth cost $\$\frac{7}{24}$, what cost 2 yd.? *Ans.* $\$\frac{1}{3}$.

38. If $\$29\frac{3}{4}$ buy $59\frac{1}{2}$ yards of cloth, how much will $\$31\frac{1}{4}$ buy? *Ans.* $62\frac{1}{2}$ yd.

39. If .85 of a gallon of wine cost $1.36, what will be the cost of .25 of a gallon? *Ans.* $0.40

40. If 61.3 lb. of tea cost $44.9942, what will be the cost of 1.07 lb.? *Ans.* $0.78538

41. If $\frac{5}{7}$ of a yard of cloth cost $\$\frac{3}{5}$, what will $\frac{9}{11}$ of an Ell English cost? *Ans.* $\$\frac{189}{220}$.

42. If $\frac{3}{7}$ of a yd. of velvet cost $\$4\frac{2}{5}$, what cost $17\frac{3}{8}$ yd.? *Ans.* 178.38\frac{1}{3}$

43. A wheel has 35 cogs; a smaller wheel working in it, 26 cogs: in how many revolutions of the larger wheel, will the smaller gain 10 revolutions? *Ans.* $28\frac{8}{9}$.

44. If a grocer, instead of a true gallon, use a measure deficient by 1 gill, what will be the *true* measure of 100 of these *false* gallons? *Ans.* $96\frac{7}{8}$ gal.

45. If the velocity of sound be 1142 feet per sec., and the number of pulsations in a person 70 per min., what the distance of a cloud, if 20 pulsations are counted between the time of seeing a flash of lightning and hearing the thunder? *Ans.* 3 mi. 5 fur. 145 yd. $2\frac{1}{7}$ ft.

46. The length of a wall, by a measuring line, was 643 ft. 8 in., but, the line was found to be 25 ft. 5.25 in. long, instead of 25 feet, its supposed length: what the true length of the wall? *Ans.* 654 ft. 11.17 in.

NOTE.—A bushel of wheat is 60 lb.; of rye, 56 lb.; corn, 56 lb.; oats, 32 lb.; barley, 48 lb.; timothy seed, 42 lb.; clover seed, 60 lb.; flax seed, 56 lb. See Dry Measure, Note 2, page 83.

The simplest method for the following examples, is by Simple Proportion. The *first* term will be the number of pounds to the bushel; *second*, the weight; *third*, the price.

What will be the cost of

47. 2136 lb. wheat, at 75 cts. per bu.? *Ans.* $26.70

48. 1225 lb. wheat, at 81 cts. per bu.? *Ans.* $16.53+

49. 1000 lb. rye, at 63 cts. per bu.? *Ans.* $11.25

50. 3000 lb. oats, at 24 cts. per bu.? *Ans.* $22.50

ART. 204. OF CAUSE AND EFFECT.

The rule for Simple Proportion may be presented under another form, based on the obvious truth, that,

When a cause produces a certain Effect, if the Cause be increased or diminished, the Effect will be increased or diminished in the same ratio.

Thus, if the cause be doubled, the effect will be doubled, if trebled, the effect will be trebled. Hence this

PRINCIPLE.—*Any* CAUSE *is to another similar* CAUSE, *as the* EFFECT *of the first* CAUSE *is to the* EFFECT *of the second* CAUSE.

ILLUSTRATION.—If 3 men make 6 rods of road in a day, how many rods can 4 men make?

3 men. :	4 men. : :	6 rods. :	8 rods. *Ans.*
1st cause.	2d cause.	1st effect.	2d effect.

The effect of the 2d cause, being the 4th term of a proportion, is found by multiplying the 3d term by the 2d, and dividing the product by the 1st. Art. 202.

NOTE.—In reviewing, the questions may be stated by this principle; or it may be used at first instead of the Rule, Art. 203.

ART. 205. COMPOUND PROPORTION.

A *Compound Proportion* contains one or more compound ratios. Art. 195.

Thus, $\left.\begin{matrix} 2 : 6 \\ 4 : 8 \end{matrix}\right\} :: 7 : 42$, is a compound proportion.

Or, $2\times4 : 6\times8 :: 7 : 42$.

This, sometimes called the DOUBLE RULE OF THREE, is applied to the solution of questions requiring more than one statement in Simple Proportion.

1. If 2 men earn \$20 in 5 days, what sum can 6 men earn in 10 days?

Here, the sum earned depends on two things: the *number of men*, and the *number of days;* that is, on the ratio of 2 men to 6 men, and on the ratio of 5 days to 10 days.

First find the sum earned by 6 men in the *same time* with 2 men.

men.	men.	\$	\$	
2 :	6 : :	20 :	60	= sum earned by 6 men in 5 days.

Knowing the sum earned in 5 days, the sum earned in 10 days can be found thus:

days.	days.	\$	\$	
5 :	10 : :	60 :	120	= sum earned by 6 men in 10 days.

REVIEW.—204. On what is Cause and Effect based? What is the Principle? How find the effect of the 2d cause?

By examining these proportions, it is seen that the *ratio* of the third term to the fourth, depends on two ratios—

1st, the ratio of 2 men to 6 men; 2d, the ratio of 5 da. to 10 da.

The first ratio is 3, and the second 2; their product, $3 \times 2 = 6$, is the ratio of the third term to the required term. Hence,

The ratio, which the 3d term has to the 4th, is compounded of the ratios of 2 to 6 and of 5 to 10: write the two simple proportions

$$\text{Thus,}\quad \left.\begin{array}{l} 2 \text{ men} : 6 \text{ men} \\ 5 \text{ days} : 10 \text{ days} \end{array}\right\} :: \overset{\$}{20} : \text{fourth term.}$$

$$\text{Or, } 2 \times 5 : 6 \times 10 :: 20 : \text{fourth term.}$$

$20 \times 6 \times 10 = 1200$; $2 \times 5 = 10$; and $1200 \div 10 = \$120$. *Ans.*

*2. If a man travel 24 mi. in 2 da., by walking 4 hr. a day; at the same rate, how far will he travel in 10 da., walking 8 hr. a day? *Ans.* 240 mi.

Rule for Compound Proportion.—1. *Write for the 3d term, that number which is of the denomination required in the answer.*

2. *Take any two numbers of the same kind, and arrange them as in Simple Proportion.* Art. 203, Rule 2, page 201.

3. *Arrange any other two numbers of the same kind, in like manner, till all are used.*

4. *Multiply the third term by the continued product of the second terms; divide the result by the continued product of the first terms: the quotient will be the fourth term, or answer.*

NOTES.—1. If the terms in any couplet are of different denominations, reduce them to the same. If the third term consists of more than one denomination, reduce it to the lowest named.

2. The examples may be solved by two or more statements in Simple Proportion, or by Analysis, Art. 263. Also, by Cause and Effect, Art. 204, see statement, page 208.

REVIEW.—205. What does Compound Proportion contain? To what is it applied? In stating a question, what number is put for the third term, Rule? How are the other numbers arranged?

205. How is the fourth term found? NOTE 1. If the terms in any couplet are of different denominations, what is required? What if the third term contains more than one denomination?

3. If 6 men, in 10 days, build a wall 20 ft. long, 3 ft. high, and 2 ft. thick: in how many days could 15 men build a wall 80 ft. long, 4 ft. high, and 3 ft. thick?

STATEMENT BY CAUSE AND EFFECT.

1st cause.		2d cause.		1st effect.		2d effect.
6	:	15	: :	20	:	80
10		x		3		4
				2		3

The terms of the 2d cause and 1st effect form the divisors; those of the 1st cause and 2d effect, the dividends. The x shows the place of the required term.

EXPLANATION.—When the terms forming the divisor and the dividend, contain one or more common factors, shorten the operation by Cancellation.

In such cases, arrange the divisors on the left of a vertical line, and the multipliers (*dividends*), on the right.

OPERATION BY CANCELING.

		Divisors	Dividends	
Men	5	~~15~~	~~6~~	2
Length .		~~20~~	~~80~~	4
Hight .		~~3~~	4	
Thickness		~~2~~	~~3~~	
Days . .		x	~~10~~	2

$4 \times 4 \times 2 = 32$ da. *Ans.*

4. If 16 men build 18 rods of fence in 12 days, how many men can build 72 rd. in 8 da.? *Ans.* 96 men.

5. If 6 men spend \$150 in 8 mon., how much will 15 men spend in 20 mon.? *Ans.* \$937.50

6. I travel 217 mi. in 7 days of 6 hr. each, how far can I travel in 9 da. of 11 hr. each? *Ans.* $511\frac{1}{2}$ mi.

7. If \$100 gain \$6 in 12 months, what sum will \$75 gain in 9 mon.? *Ans.* \$3.375

8. If 100 lb. be carried 20 mi. for 20 cts., how far may 10100 lb. be carried for \$60.60? *Ans.* 60 mi.

9. To carry 12 cwt. 3 qr. 400 mi., costs \$57.12: what will it cost to carry 10 tuns 75 mi.? *Ans.* \$168.

10. If 18 men, in 15 da., build a wall 40 rd. long, 5 ft. high, 4 ft. thick; in what time could 20 men build a wall 87 rd. long, 8 ft. high, and 5 ft. thick? *Ans.* $58\frac{29}{40}$ da.

11. If 180 men, in 6 da. of 10 hr. each, dig a trench 200 yd. long, 3 yd. wide, 2 yd. deep; in how many days can 100 men, working 8 hr. a day, dig a trench 180 yd. long, 4 yd. wide, and 3 yd. deep? *Ans.* 24.3 da.

☞ For additional problems, see Ray's Test Examples.

XV. ALIQUOTS, OR PRACTICE.

ART. **206**. One number is an *aliquot* part of another, when it will exactly divide it (Art. 110). Thus, 5 cents, 10 cts., 20 cts., &c., are aliquot parts of $1.

TABLE OF ALIQUOT PARTS OF A DOLLAR.

100	cents = . . a dollar.	**20**	cents = $\frac{1}{5}$ of a dollar.
50	cents = $\frac{1}{2}$ of a dollar.	**10**	cents = $\frac{1}{10}$ of a dollar.
25	cents = $\frac{1}{4}$ of a dollar.	**5**	cents = $\frac{1}{20}$ of a dollar.
12$\frac{1}{2}$	cents = $\frac{1}{8}$ of a dollar.	**33$\frac{1}{3}$**	cents = $\frac{1}{3}$ of a dollar.
6$\frac{1}{4}$	cents = $\frac{1}{16}$ of a dollar.	**16$\frac{2}{3}$**	cents = $\frac{1}{6}$ of a dollar.

CASE I.

ART. **207**. *To find the cost of articles, when the price is an aliquot part, or aliquot parts of a dollar.*

1. What cost 24 yards of muslin, at 25 cts. a yd.?

SOLUTION.—If the price was $1 a yard, the cost would be $24; and, since 25 cents is $\frac{1}{4}$ of a dollar, the cost at 25 cts. a yard will be $\frac{1}{4}$ the cost at $1; and $\frac{1}{4}$ of $24=$6. *Ans.*

2. What cost 16 yards of calico, at 37$\frac{1}{2}$ cts. a yard?

SOLUTION.—If the price was $1 a yard, the cost would be $16. At 25 cts. a yard, the cost would be $\frac{1}{4}$ the cost at $1; $\frac{1}{4}$ of $16=$4.

Again; since 12$\frac{1}{2}$ cts. is $\frac{1}{2}$ of 25 cts., the cost at 12$\frac{1}{2}$ cts. a yard, will be $\frac{1}{2}$ the cost at 25 cts.; $\frac{1}{2}$ of $4=$2.

But, the cost at 37$\frac{1}{2}$ cts. a yard, is equal to the sum of the costs at 25 cts. and at 12$\frac{1}{2}$ cts.; $4+$2=$6. *Ans.*

REVIEW.—206. When is one number an aliquot part? Give examples.

*3. What cost 24 yd. silk at $62\frac{1}{2}$ cts. a yd.? *Ans.* \$15.

Rule for Case I.—*Take such aliquot parts of the cost at \$1, as may be necessary to find the cost at the given price.*

4. What cost 48 yd. of linen, at $68\frac{3}{4}$ cts. per yd.?

OPERATION.	\$48 =	cost of 48 yards,	at \$1.
50 cts. = $\frac{1}{2}$	24 =	" " "	at 50 cts.
$12\frac{1}{2}$ cts. = $\frac{1}{4}$	6 =	" " "	at $12\frac{1}{2}$ cts.
$6\frac{1}{4}$ cts. = $\frac{1}{2}$	3 =	" " "	at $6\frac{1}{4}$ cts.
Ans.	\$33 =	" " "	at $68\frac{3}{4}$ cts.

What will be the cost

5.	Of 173 bu. corn, at 25 cts. a bu.?	*Ans.* \$43.25
6.	45 lb. cheese, at $31\frac{1}{4}$ cts. a lb.?	*Ans.* $\$14.06\frac{1}{4}$
7.	54 yd. calico, at $43\frac{3}{4}$ cts. a yd.?	*Ans.* $\$23.62\frac{1}{2}$
8.	32 bu. rye, at $93\frac{3}{4}$ cts. a bu.?	*Ans.* \$30.00

9. What cost 20 yd. cloth, at $\$3.12\frac{1}{2}$ per yd.?

The cost at \$1 a yard is multiplied by the number of dollars (3); and for the $12\frac{1}{2}$ cts., an aliquot part, ($\frac{1}{8}$) of the cost at \$1 is taken; the results are added.

OPERATION.	\$20	= cost of 20 yd. at \$1.
	3	
$12\frac{1}{2}$ cts. = $\frac{1}{8}$	60	= cost at \$3.00
	2.50	= cost at $.12\frac{1}{2}$
Ans.	\$62.50	= cost at $\$3.12\frac{1}{2}$

Find the cost of

		ANSWERS.
10.	80 gal. of wine, at $\$2.37\frac{1}{2}$ a gal.	\$190.00
11.	36 bl. of flour, at $\$8.87\frac{1}{2}$ a bl.	\$319.50
12.	77 gal. of wine, at $\$1.62\frac{1}{2}$ a gal.	$\$125.12\frac{1}{2}$
13.	175 A. of land, at $\$14.37\frac{1}{2}$ an A.	$\$2515.62\frac{1}{2}$
14.	224 bl. of flour, at $\$3.43\frac{3}{4}$ a bl.	\$770.00
15.	462 yd. of cloth, at $\$1.06\frac{1}{4}$ a yd.	$\$490.87\frac{1}{2}$
16.	185 yd. of cloth, at $\$1.33\frac{1}{3}$ a yd.	$\$246.66\frac{2}{3}$
17.	150 yd. of satin, at $\$3.66\frac{2}{3}$ a yd.	\$550.00
18.	24 yd. of silk, at $\$1.16\frac{2}{3}$ a yd.	\$28.00

ART. **208**. CASE II.—*To find the cost of a quantity consisting of several denominations.*

1. What will be the cost of 4 A. 1 R. 20 P. of land, at $16.40 per A.?

OPERATION.		
	$16.40	= cost of 1 A.
	4	
	65.60	= cost of 4 A.
1 R. = $\frac{1}{4}$ A.	4.10	= cost of 1 R.
20 P. = $\frac{1}{2}$ R.	2.05	= cost of 20 P.
Ans.	$71.75	= cost of 4 A. 1 R. 20 P.

Rule for Case II.—*Multiply the price by the number of the denomination at which the price is rated, and find the cost of the lower denominations, by taking aliquot parts. Add the different costs; the sum will be the cost of the whole.*

Find the cost of

		ANSWERS.
2.	14 A. 2 R., at $10.00 per A.	$145.00
3.	28 A. 3 R., at $12.50 per A.	359.37\frac{1}{2}$
4.	5 A. 1 R. 10 P., at $12 per A.	$63.75
5.	12 A. 1 R. 10 P., at $18 per A.	221.62\frac{1}{2}$
6.	14 A. 3 R. 25 P., at $12.50 per A.	186.32\frac{13}{16}$
7.	3 yd. 2 qr., at $1.75 per yd.	6.12\frac{1}{2}$
8.	4 yd. 3 qr., at $1.50 per yd.	7.12\frac{1}{2}$
9.	56 yd. 3 qr., at $17.25 per yd.	978.93\frac{3}{4}$
10.	83 bu. 3 pk. 2 qt., at $6 a bu.	502.87\frac{1}{2}$
11.	24 bu. 3 pk. 7 qt., at $4 a bu.	99.87\frac{1}{2}$
12.	40 bu. 3 pk. 7 qt. 1 pt., at $3.20 a bu.	$131.15
13.	17 bu. 1 pk. 1 qt. 1 pt., at $2.56 a bu.	$44.28
14.	5 lb. 11 oz. butter, at 24 cts. per lb.	1.36\frac{1}{2}$
15.	3 lb. 13 oz. 15 dr. spice, at $2.56 a lb.	$9.91
16.	17 A. 3 R. 39 P., at $3.20 per A.	$57.58

☞ For additional problems, see Ray's Test Examples.

REVIEW.—207. What is Case 1? What the Rule for Case 1? 208. What is Case 2? What the Rule for Case 2?

XVI. PERCENTAGE.

ART. **209**. *Percentage* embraces calculations in which reference is made to a hundred as the *unit;* and,

The *per cent.* or *percentage* of any number or quantity, is so many *hundredths* of it. Thus,

1 per cent. is $\frac{1}{100}$; 2 per cent. $\frac{2}{100}$; 3 per cent. $\frac{3}{100}$.

Since $\frac{1}{100} = .01$, $\frac{2}{100} = .02$, and so on, (Art. 176), therefore, *per cent.* may be expressed decimally; thus,

4 per cent. is	.04	10 per cent. is	.10
5 per cent. is	.05	25 per cent. is	.25
9 per cent. is	.09	75 per cent. is	.75

100 per cent. ($\frac{100}{100}$, or the whole,) is 1.00

125 per cent. ($\frac{125}{100}$, or $\frac{25}{100}$ more than the whole,) is 1.25

$\frac{1}{2}$ per cent. is a half of 1 per cent.,$=\frac{1}{2}$ of $\frac{1}{100}$, is .005

$\frac{1}{8}$ per cent. is $\frac{1}{8}$ of 1 per cent.,$=\frac{1}{8}$ of $\frac{1}{100}$, is .00125

$3\frac{1}{2}$ per cent. is $.03\frac{1}{2}$,$=.035$ is 035

EXAMPLE.—Write 7 per cent. decimally. $4\frac{1}{2}$ per ct. $5\frac{1}{4}$ per ct. $10\frac{1}{2}$ per ct. $37\frac{1}{2}$ per ct. 120 per ct.

☞ THE SIGN %, is used in business, instead of the words "per cent.;" thus, 5 per cent. is written 5 %.

ART. **210**. CASE I.—*To find any per cent. of a number.*

1. What is 2 % (2 per cent.) of $125?

SOL.—To take 2 per cent. is to take 2 hundredths. Hence, multiply by 2 and divide by 100; or multiply by $\frac{2}{100}$ expressed decimally.

OPERATION.
```
     $125
      .02
 --------
Ans. 2.50
```

*2. What is 7 % of 175 dollars? *Ans.* $12.25

REVIEW.—209. What does Percentage embrace? What is the per cent. of any number? Give examples.

Rule for Percentage.—*Multiply by the rate per cent. expressed decimally; the product will be the per cent. required.*

Or, by PROPORTION (Art. 203). *As 100 is to the rate per cent. so is the given number or quantity to the required per cent.*

3. What is $5\frac{1}{3}$ % of $150?

SUGGESTION.—When the rate per cent. is a common fraction, or mixed number, multiply as explained in Art. 152; and in pointing the product, count *only two* decimals in the multiplier.

OPERATION.

$$\begin{array}{rl} \$150 & \\ .05\frac{1}{3} & \\ \hline 7.50 = 5 & \text{per cent.} \\ .50 = \frac{1}{3} & \text{per cent.} \\ \hline \textit{Ans. } \$8.00 = 5\frac{1}{3} & \text{per cent.} \end{array}$$

	ANSWERS.
4. What is 6 % of $250?	$15.00
5. What is 7 % of $162?	$11.34
6. What is 5 % of $118?	$5.90
7. What is 8 % of $11?	$0.88
8. What is 1 % of $278?	$2.78
9. What is $2\frac{1}{4}$ % of $68?	$1.53
10. What is $4\frac{1}{2}$ % of $220.50?	$9.922+
11. What is $7\frac{1}{2}$ % of $115.42?	$8.656+
12. What is $5\frac{3}{4}$ % of $243.16?	$13.981+
13. What is $3\frac{1}{4}$ % of $1250?	$40.625
14. What is 25 % of $25?	$6.25
15. What is $101\frac{1}{2}$ % of $2002?	$2032.03
16. What is 208 % of $650?	$1352.00
17. What is 1000 % of $24.75?	$247.50
18. What is $\frac{1}{10}$ % of $400?	$0.40
19. What is $\frac{3}{8}$ % of $464?	$1.74
20. What is $\frac{1}{12}$ % of $1950?	1.62\frac{1}{2}$

REVIEW.—209. How may per cent. be expressed? Give examples. What is 100 per cent.? What is 125 per cent.? $\frac{1}{2}$ per cent.? $3\frac{1}{2}$ per cent.? What sign is used for *per cent.*? Give examples.

21. If $8\frac{1}{2}$ % of \$72 be taken from it, how much will remain? *Ans.* \$65.88

22. I had \$800 in bank, and drew out 36 % of it: how much had I left? *Ans.* \$512.

23. What is the difference between $13\frac{1}{8}$ % of \$56, and $14\frac{2}{3}$ % of \$51? *Ans.* 13 cts.

24. A merchant in expending \$1764, paid 23 % for cloth; 31 % for calicoes; 9 % for linens; $3\frac{1}{4}$ % for silks; the remainder for muslin: how much did he pay for each? *Ans.* to last, \$595.35

25. A grocer bought 4 bags of coffee, of 75 lb. each: $12\frac{1}{3}$ % was lost by waste: what was the rest worth, at 14 cts. per lb.? *Ans.* \$36.82

26. A flock of 160 sheep increased 35 % in 1 year: how large was it then? *Ans.* 216.

27. I had 320 sheep; 5 % were killed: after selling 25 % of the rest, how many were left? *Ans.* 228.

28. A's salary is \$800 a year: he spends 18 % for rent; 15 % for clothing; 23 % for provisions; 12 % for sundries: how much is left? *Ans.* \$256.

Art. 211. CASE II.

To find what per cent. one number is of another.

1. What % of 8 dollars is \$2?

OPERATION.

$.08)2.00$

Ans. 25 per cent.

or,

$\frac{2}{8}=\frac{1}{4}=.25=25$ per cent.

Solution.—One per cent. of \$8 is $\$8\times.01=.08$, (Art. 210); and since \$2 contain this 25 times, it must be 25 per cent. of \$8. Or thus: the question is the same as to find how many *hundredths* \$2 are of \$8. Now 2 is $\frac{2}{8}$ of 8, and $\frac{2}{8}=\frac{1}{4}=.25=$ 25 per cent.

*2. What % of \$15, is \$3? *Ans.* 20.

Review.—210. How is the percentage of a number found, Rule? How, when the rate per cent. is a mixed number or a fraction? 211. What is Case 2? What the Rule for Case 2?

Rule for Case II.—*Divide the number which is considered the percentage, by 1 per cent. of the other number; the quotient will be the required rate per cent.*

Or, by Proportion.—*As the given number is to the percentage, so is* 100 *to the required rate per cent.*

3. What % of \$50, is \$6? *Ans.* 12.
4. What % of \$75, is \$4.50? *Ans.* 6.
5. What % of \$9, is \$3? *Ans.* $33\frac{1}{3}$.
6. What % of \$25, is \$0.25? *Ans.* 1.
7. What % of \$142.60, is \$7.13? *Ans.* 5.
8. What % of \$9, is \$9? *Ans.* 100.
9. What % of \$9, is \$13.50? *Ans.* 150.
10. What % of \$243, is \$8.505? *Ans.* $3.5=3\frac{1}{2}$.
11. What % of \$2, is 2 mills? *Ans.* $\frac{1}{10}$.
12. What % of \$3532, is \$13.245? *Ans.* $\frac{3}{8}$.
13. A man had \$300, and spent \$25: what % was that of the whole? *Ans.* $8\frac{1}{3}$.
14. A man owed \$500, and paid \$75 of it: what % of the whole debt did he pay? *Ans.* 15.
15. What % of any number is $\frac{3}{5}$ of it? *Ans.* 60.
16. A miller takes for toll, 6 qt. from every 5 bu. of grain ground: what % does he get? *Ans.* $3\frac{3}{4}$.

☞ For additional problems, see Ray's Test Examples.

APPLICATIONS OF PERCENTAGE.

Art. **212**. The principal applications of percentage are in Commission, Insurance, Stocks, Brokerage, Interest, Discount, Profit and Loss, Duties, and Taxes.

Art. 213. COMMISSION

Is the sum allowed to Agents for buying, selling, or transacting other business.

Those who transact business for others, are *Agents*, *Commission Merchants*, *Factors*, or *Correspondents*.

Commission is estimated at a certain rate *per cent.* on the amount employed in the transaction; it is calculated by the Rule, Art. 210.

1. At 5%, what does an agent receive for selling goods amounting to $240? *Ans.* $12.00

2. At $2\frac{1}{2}$%, what is the commission for selling goods amounting to $460? *Ans.* $11.50

3. What the commission on sales of $180 at 4%, and on sales of $119 at 3%? *Ans.* $10.77

4. What the commission on $240 sales at 3%, and on $225 sales at 5%? *Ans.* $18.45

5. What the commission on sales of $480 at $2\frac{1}{4}$%; $275 at $3\frac{1}{5}$%; and $216 at $2\frac{2}{3}$%? *Ans.* $25.36

6. What on sales of $275 at 3%; $341 at 15%; $964 at 25%; and $217 at $2\frac{1}{4}$%? *Ans.* $305.2825

7. An agent sells 25 bl. of molasses, at $13 each, at $2\frac{1}{2}$% commission: what will the agent and owner each receive? *Ans.* Agent, 8.12\frac{1}{2}$; Owner, 316.87\frac{1}{2}$

8. A factor sells 15 hhd. sugar, of 1114 lb. each, at 8 cts. a lb., charging $3\frac{1}{4}$% commission: what sum will he pay the owner? *Ans.* $1293.354

9. A factor sells 250 bl. pork, at $15 each; 175 bl. flour, at $7 each; and 1456 lb. feathers, at 25 cts. a lb.; his commission is 3%: what sum will he pay the owner? *Ans.* $5178.83

ART. **214.** When an agent invests MONEY, his commission is computed on the amount he expends.

10. An agent receives $210 to buy goods; after deducting his commission of 5%, what sum must he expend?

SOLUTION.—For each $1 expended, he receives 5 cts. Hence, for each $1.05, received, he must expend $1; therefore, he must expend as many dollars as $1.05 are contained times in $210:

That is,—$210 ÷ $1.05 = 200. *Ans.* $200.

PROOF.—$200 × .05 = $10; and $200 + $10 = $210.

Hence, *divide the given sum by $1, increased by the commission on $1; the quotient will be the sum to be expended; the difference between this and the given sum is the commission.*

11. A received $312 to buy goods: after deducting his commissions at 4%, what must he expend? *Ans.* $300.

12. B receives $1323.54 to buy goods, at 8% commission: what was his commission? *Ans.* $98.04

Art. 215. INSURANCE

Is security against loss by fire, navigation, &c. It is effected with a company, or an individual, who, for a certain per cent., agrees to make good the amount insured, for a certain period.

The Policy is the written contract between the parties.

The Premium is the sum paid for insurance.

The Insurer is called an Underwriter.

Insurance on ships, boats, &c., is *Marine Insurance.*

The Rate of insuring is a certain per cent. of the amount insured, and is calculated by Rule, Art. 210.

1. What is paid for insuring $2250 on a house, at $1\frac{1}{2}$% premium, the policy costing $1? *Ans.* $34.75

2. A steamboat is valued at $12600, the cargo at $14400; $\frac{2}{3}$ of their value is insured at $4\frac{1}{2}$%, the policy costing $1: what the cost of insurance? *Ans.* $811.

3. What is paid for insuring $\frac{3}{4}$ of a house worth $5000, at $\frac{1}{2}$%, the policy costing $1.50? *Ans.* $20.25

4. A has a ship valued at $21000, and a house at $1200: what will be the cost of insuring $\frac{4}{7}$ of the ship at $12\frac{1}{2}$%, and the full value of the house, at $1\frac{1}{2}$%, the two policies costing $1 each? *Ans.* $1520.

Review.—212. Where are the principal applications of percentage? 213. What is commission? How estimated?

215. What is insurance? How effected? What is the policy? The premium? An underwriter? Marine insurance? Rate? How calculated?

ART. **216**. Some persons wish to insure so as to cover the property and premium paid, in case of a loss.

5. A's shop is valued at \$190: at 5 %, what sum must be insured, so that he may recover the value of the property and the premium in case of loss?

SOLUTION.—Since the rate of insurance is 5 per cent., 5 cents of every dollar insured is premium, and the remaining 95 cents property; hence,

As often as the property to be insured contains 95 cts., so many dollars must be insured to cover that property and its premium.

But, $\$190 \div .95 = \$200$. *Ans.*

PROOF.—$\$200 \times .05 = \$10$; and $\$190 + \$10 = \$200$.

In such cases, *divide the value of the property insured, by the remainder left after subtracting the rate of insurance from* 1.

6. At 1 %, what sum must be insured on \$2475, to secure both property and premium? *Ans.* \$2500.

7. At $12\frac{1}{2}$ %, what sum must be insured on \$13125, to cover property and premium? *Ans.* \$15000.

8. At $1\frac{3}{4}$ % what sum must be insured on \$2358, to cover the premium and property? *Ans.* \$2400.

9. At $\frac{3}{8}$ % what must be paid for insuring \$2287.39, to cover both property and premium? *Ans.* \$8.61

ART. **217**. STOCKS.

STOCK is money or property invested in Manufactories, Railroads, Insurance Companies, Banks, Bonds, &c.

Stock is divided into shares, usually of \$50 or \$100 each. The owners are Stockholders.

The original cost of a share is its *par* or *nominal value.*

When \$100 of stock sells for \$100, the stock is said to be at *par;* when it sells for *more* than \$100, *above par;* and when for *less* than \$100, *below par.*

REVIEW.—216. How find the sum to be insured, to cover both property and premium?

217. What is stock? How divided? What is meant by par value?

When stocks are *above par* they are at an *advance;* and when *below par*, at a *discount.*

The *per cent.* is calculated on the par value. Art. 210.

1. What is the value of $1400 of stock, at 104 %; that is, 4 % above par? *Ans.* $1456.

2. What the value of $1400 of bank stock, at 96 %; or 4 % below par? *Ans.* $1344.

3. What the value of 11 shares railroad stock ($50 each) at 5 % advance? *Ans.* $577.50

4. What the value of 15 shares canal stock ($75 each), at 10 % below par? *Ans.* $1012.50

5. Bought $1500 of bank stock, at $104\frac{1}{2}$ %, and sold it at $108\frac{1}{3}$ %: find the gain. *Ans.* $57.50

6. Bought $1300 of stock, at 3 % below par, and sold it at $5\frac{1}{2}$ % above par: find the gain. *Ans.* $110.50

7. Bought $860 bank stock, at 4 % advance; sold at a discount of $2\frac{1}{2}$ %: find the loss. *Ans.* $55.90

ART. 218. BROKERAGE.

Dealers in money, stocks, &c., are BROKERS. The operation of finding the percentage is termed *Brokerage.*

The percentage is calculated, Art. 210, on the par value of the funds received or paid.

When bonds, notes, or coin, bring *more* than their *nominal* value, they are at a *premium;* when *less*, at a *discount.*

1. Find the premium, at $1\frac{1}{2}$ %, on $600. *Ans.* $9.

2. What sum is lost in exchanging $289 in bank notes, at a discount of $1\frac{1}{4}$ %? *Ans.* $$3.61\frac{1}{4}$

3. I sold $360, in bank notes, at $\frac{3}{8}$ % discount: how much did I receive? *Ans.* $358.65

REVIEW.—217. When is stock at par? Above par? Below par? At an advance? At a discount? On what is the per cent. calculated?

4. What sum must a broker pay for $134, in bank notes, at $2\frac{1}{2}$ % discount? *Ans.* $130.65

5. What sum must a broker pay for $200 in gold, at $\frac{5}{8}$ % premium? $201.25

6. A buys $500, in notes, at $\frac{1}{2}$ % discount, and sells at $\frac{1}{4}$ % premium: find his gain. *Ans.* $3.75

ART. 219. INTEREST.

INTEREST is an allowance made by the borrower to the lender, for the use of money.

PRINCIPAL is the sum loaned, for which interest is paid.

AMOUNT is the sum of the *principal* and *interest.*

PER ANNUM signifies *for one year.*

RATE PER CENT. *per annum*, is the number of dollars paid for the use of $100, or the number of cents paid for the use of 100 cents, for one year.

ILLUSTRATION.—B borrowed of A $200 for 1 year, and agreed to pay him $12 for the use of it, that is, $6 for the use of $100 for one year. Here, $200 is the *principal*, $12 the *interest*, $6 the *rate per cent.*, and $212 the *amount.*

REM.—1. The distinction between Interest and other computations in Percentage, as Commission, Brokerage, &c., is this: in the latter, the rate per cent. has *no regard to time*, whereas,

In Interest, *time* is always considered: and,

For periods of time greater or less than one year, the interest is proportionally greater or less than the interest for one year.

2. For brevity, the term *per annum* is seldom used in connection with *rate per cent.*, but is always understood. Hence,

In Interest, *rate* per cent. always means *rate per cent. per annum.*

LEGAL INTEREST is the *rate* established by law. Any rate *higher* than the legal rate is *Usury.*

REVIEW.—218. What are brokers? What is brokerage? On what is the percentage calculated? When are notes at premium? At discount?

219. What is Interest? Principal? Amount? What does per annum signify? What the rate per cent. per annum?

When no rate per cent. is named, the *legal* rate where the business is transacted, is understood.

NOTE.—The legal rate of Int. in Louisiana is 5 per cent.;
In N. York, S. Carolina, Michigan, Wisconsin, and Iowa, 7 per cent.;
In Georgia, Alabama, Mississippi, Florida, and Texas, 8 per cent.;
In the rest of the States, and in the U. S. Courts, 6 per cent.

CASE I.

ART. **220.** *To find the interest on any principal, at any rate per cent. for one or more years.*

1. Find the interest of $25 for 1 yr., at 6 %.

SOLUTION.—Since 6 per cent. is $\frac{6}{100}=.06$, it is only necessary, as in Percentage, Art. 210, to multiply the principal by the rate expressed decimally, and the product will be the Int. for 1 yr.

OPERATION.
$25
.06
Ans. $1.50

Or, thus: Six per cent. is $6 on $100, or 6 cents on $1. Hence, if the interest on $1 is 6 cts.=$.06, the interest on $25, for the same time, will be 25 times as much; 25 × $.06 = $1.50 *Ans.*

2. Find the interest of $50 for 3 yr., at 7 %.

SOLUTION.—The Int. of any sum for 3 yr. is, 3 times as much as its Int. for 1 yr.

Find the Int. of $50 for 1 yr., which is $3.50; multiply this by 3 to obtain the Int. for 3 yr.

OPERATION.
$50
.07
$3.50 = Int. for 1 yr.
3
Ans. $10.50 = Int. for 3 yr.

*3. Interest of $65 for 4 yr., at 5 %. *Ans.* $13.

Rule for Case I.—1. *Multiply the principal by the rate per cent. expressed decimally; the product will be the interest for one year.*

REVIEW.—219. REM. 1. What is the distinction between interest and commission? 2. What does rate per cent. in interest always mean?
219. What is legal interest? What is usury? When no rate is named, what is understood? NOTE. In what State is the legal rate of interest 5 per cent.? In what 7 per cent.? In what 8 per cent.? In what 6 per cent.?

2. *Multiply the interest for one year by the given number of years; the product will be the Int. required.*

3. *To find the amount, add the principal to the interest.*

Required the Interest	ANSWERS.
4. Of \$200.00 for 1 year, at 8 %.	\$ 16.00
5. \$150.00 for 1 year, at 5 %.	\$ 7.50
6. \$300.00 for 2 years, at 6 %.	\$ 36.00
7. \$275.00 for 3 years, at 6 %.	\$ 49.50
8. \$187.50 for 4 years, at 5 %.	\$ 37.50
9. \$233.80 for 10 years, at 6 %.	\$140.28

10. Find the amount of \$225.18 for 3 yr., at $4\frac{1}{2}$ %.

OPERATION.

```
   225.18
     .04½
  ---------
   90072=Int. at 4 %.
   11259=Int. at ½ %.
  ---------
 10.1331=Int. for 1 yr.
       3
  ---------
 30.3993=Int. for 3 yr.
225.18   =principal.
  ---------
$255.5793=amount.
```

SUGGESTION.—When the rate contains a fraction, as in this example, write it as $.04\frac{1}{2}$ or .045

When the fraction can not be exactly expressed in decimals, express it as a common fraction.

In adding the principal to the Int., place figures of the same order under each other.

What is the Amount of	ANSWERS.
11. \$215.00 for 1 year, at 6 %?	\$227.90
12. \$ 45.00 for 2 years, at 8 %?	\$ 52.20
13. \$ 80.00 for 4 years, at 7 %?	\$102.40
14. \$420.00 for 1 year, at $5\frac{1}{3}$ %?	\$442.40
15. \$237.16 for 2 years, at $3\frac{3}{4}$ %?	\$254.947
16. \$ 74.75 for 5 years, at 4 %?	\$ 89.70
17. \$ 85.45 for 4 years, at 6 %?	\$105.958

REVIEW.—220. What is Case 1? How find the interest on any principal, at any rate, for one or more years, Rule for Case 1?

18. \$325.00 for 3 years, at $5\frac{2}{5}$ %? *Ans.* \$377.65
19. \$129.36 for 4 years, at $4\frac{3}{8}$ %? *Ans.* \$151.998

CASE II.

ART. **221.** *To find the interest of any principal, at any rate per cent., for any number of months, or days.*

1. Find the interest of \$300 for 1 mon., at 6 %.

SOLUTION.—Since 1 mon. is $\frac{1}{12}$ of a yr., the Int. for 1 mon. is $\frac{1}{12}$ of the Int. for a yr. But the Int. of \$300 for 1 yr. is \$18.00, and $\frac{1}{12}$ of this is \$1.50 *Ans.*

OPERATION.

$$\begin{array}{r} \$300 \\ .06 \\ \hline 12)18.00 \\ \hline \$1.50 \end{array} \begin{array}{l} \\ \\ =\text{Int. for 1 yr.} \\ =\text{Int. for 1 mon.} \end{array}$$

*2. The Int. of \$240 for 2 mon., at 8 %. *Ans.* \$3.20

On this principle, the Int. for 3 mon. is $\frac{3}{12}=\frac{1}{4}$ the Int. for a yr.; for 4 mon. it is $\frac{4}{12}=\frac{1}{3}$; for 5 mon. $\frac{5}{12}$, &c.

3. Find the interest of \$288 for 1 day, at 5 %.

SOLUTION.—As 1 da. is $\frac{1}{30}$ of a mon., the Int. for 1 da. is $\frac{1}{30}$ of the Int. for 1 mon. The Int. for 1 mon. is \$1.20, and $\frac{1}{30}$ of this is \$.04 *Ans.*

OPERATION.

$$\begin{array}{r} \$288 \\ .05 \\ \hline 12)14.40 \\ \hline 30)1.20 \\ \hline \$.04 \end{array} \begin{array}{l} \\ \\ =\text{Int. for 1 year.} \\ =\text{Int. for 1 mon.} \\ =\text{Int. for 1 day.} \end{array}$$

*4. The Int. of \$360 for 2 days, at 6 %. *Ans.* \$0.12

On the same principle, the interest for 3 da. is $\frac{3}{30}=\frac{1}{10}$ of the Int. for 1 mon.; for 4 da. it is $\frac{4}{30}=\frac{2}{15}$; for 5 da., $\frac{5}{30}=\frac{1}{6}$; for 6 da.; it is $\frac{6}{30}=\frac{1}{5}$; for 7 da., $\frac{7}{30}$.

Rule for Case II.—*Find the interest of the given sum for one year,* (Art. 210).

To find the Int. for any number of months, take such a part of this, as the given number of months is part of a year.

To find the Int. for any number of days, take such a part of the Int. for 1 *mon., as the days are part of a month.*

5. Find the interest of \$50 for 5 mon., at 6 %.

SUGGESTION.—After finding the interest for 1 year, find the Int. for months, by taking aliquot parts (Art. 208); or,

Multiply by the number of mon., and divide by 12, (Art. 152).

OPERATION.

	\$50	
	.06	
	3.00	=Int. 1 yr.
4 mon.= $\frac{1}{3}$	1.00	=Int. for 4 mon.
1 mon.= $\frac{1}{4}$	.25	=Int. for 1 mon.
	\$1.25	=Int. for 5 mon.

To find the Int. for da., take aliquot parts of the Int. for 1 mon.; or, multiply by the number of da., and divide by 30.

Find the Interest of		ANSWERS.
6.	\$86.00 for 3 mon., at 6 %. . . .	\$1.29
7.	\$50.00 for 4 mon., at 8 %. . . .	\$1.33$\frac{1}{3}$
8.	\$150.25 for 6 mon., at 8 %. . . .	\$6.01
9.	\$360.00 for 7 mon., at 5 %. . . .	\$10.50
10.	\$204.00 for 11 mon., at 7 %. . . .	\$13.09
11.	\$726.00 for 10 days, at 6 %. . . .	\$1.21
12.	\$1200.00 for 15 days, at 6 %. . . .	\$3.00
13.	\$180.00 for 19 days, at 8 %. . . .	\$0.76
14.	\$240.00 for 27 days, at 7 %. . . .	\$1.26
15.	\$100.80 for 28 days, at 5 %. . . .	\$0.392

Find the Amount of		ANSWERS.
16.	\$228.00 for 9 mon., at 6 %. . .	\$238.26
17.	\$137.50 for 8 mon., at 6 %. . .	\$143.00
18.	\$150.00 for 18 days, at 5 %. . .	\$150.375
19.	\$360.00 for 11 days, at 6 %. . .	\$360.66
20.	\$264.00 for 9 days, at 6 %. . .	\$264.396

☞ For additional problems, see Ray's Test Examples.

REVIEW.—221. How find the interest of any sum for any number of months, Rule for Case II? For any number of days?

ART. 222. GENERAL RULE FOR INTEREST.

1. For one year. *Multiply the Principal by the rate per cent., expressed decimally.*

2. For more years than one. *Multiply the Interest for* 1 *year by the number of years.*

3. For months. *Take such part of the Interest for* 1 *year, as the number of months is part of one year.*

4. For days. *Take such part of the Int. for* 1 *month, as the number of days is part of one month.*

5. For years, months, and days, or for any two of these periods. *Find the Int. for each period, and add the results.*

6. To find the amount. *Add the Principal to the Int.*

Or, by PROPORTION. *As* 100 *is to the rate per cent., so is the Principal to the Int. for* 1 *yr.* Then, *As* 1 *yr. is to the given time, so is the Int. for* 1 *yr. to the Int. for the given time.*

NOTE.—In computing Int., regard 30 days 1 month, and 12 months, 1 year. Custom has made this lawful.

1. Find the interest of \$360 for 2 yr., 7 mon. 25 da., at 8 %, per annum.

OPERATION.

	\$360	
	.08	
	28.80	= Int. 1 yr.
	2	
	57.60	= Int. 2 yr.
6 mon. = $\frac{1}{2}$	14.40	= Int. 6 mon.
1 mon. = $\frac{1}{6}$	2.40	= Int. 1 mon.
15 da. = $\frac{1}{2}$	1.20	= Int. 15 da.
10 da. = $\frac{1}{3}$	.80	= Int. 10 da.
	\$76.40	= Int. for 2 yr. 7 mon. 25 da.

OPERATION BY PROPORTION.

2 yr. 7 mon. 25 da. = 955 da.

100 : 8 : : \$360

$$\begin{array}{r|l} \not{1}\not{0}\not{0} & \not{8}\ 4 \\ 5 & 36\not{0} \end{array} \quad \frac{36\times 4}{5} = \$28.80$$

360 : 955 : : \$28.80

$$\begin{array}{r|l} \not{3}\not{6}\not{0} & \not{9}\not{5}\not{5}\quad 191 \\ \not{7}\not{2} & \not{2}\not{8}.\not{8}\not{0}\quad .40 \end{array}$$

191 × .40 = \$76.40 *Ans.*

The operation may be further shortened by writing the proportions together, and canceling.

What is the Interest of

	ANSWERS.
2. \$350.00 for 7 yr. 3 mon., at 4 %?	\$101.50
3. \$150.00 for 4 yr. 2 mon., at 6 %?	\$ 37.50

Find the Interest of	ANSWERS.
4. \$375.40 for 1 yr. 8 mon., at 6 %.	\$ 37.54
5. \$ 92.75 for 3 yr. 5 mon., at 6 %.	\$ 19.01+
6. \$500.00 for 1 yr. 1 mon. 18 da., at 6 %.	\$ 34.00
7. \$560.00 for 2 yr. 4 mon. 15 da., at 8 %.	\$106.40
8. \$750.00 for 4 yr. 3 mon. 6 da., at 6 %.	\$192.00
9. \$456.00 for 3 yr. 5 mon. 18 da., at 5 %.	\$ 79.04
10. \$216.00 for 5 yr. 7 mon. 27 da., at 10 %.	\$122.22
11. \$380.00 for 3 yr. 9 mon. 9 da., at 15 %.	\$215.175

Find the Amount of	ANSWERS.
12. \$300.00 for 3 yr. 8 mon., at 6 %.	\$366.00
13. \$250.00 for 1 yr. 7 mon., at 6 %.	\$273.75
14. \$205.25 for 2 yr. 8 mon. 15 da., at 6 %.	\$238.603+
15. \$150.62 for 3 yr. 5 mon. 12 da., at 5 %.	\$176.601+
16. \$210.25 for 2 yr. 7 mon. 20 da., at 7 %.	\$249.087+
17. \$ 57.85 for 2 yr. 3 mon. 23 da., at 5 %.	\$ 64.542+

18. Find the interest of \$150, from January 9, 1847, to April 19, 1849, at 6 %. *Ans.* \$20.50

NOTE.—To find the Time between two dates, see Art. 103.

19. The interest of \$240, from February 15, 1848, to April 27, 1849, at 8 %. *Ans.* \$23.04

20. The interest of \$180, from May 14, 1843, to August 28, 1845, at 7 %. *Ans.* \$28.84

21. The interest of \$137.50, from July 3, to November 27, at 9 %. *Ans.* \$4.95

22. The amount of \$125.40, from March 1, to August 28, at $8\frac{1}{2}$ %. *Ans.* \$130.64+

23. The amount of \$234.60, from August 2, 1847, to March 9, 1848, at $5\frac{1}{4}$ %. *Ans.* \$242.024+

24. The amount of \$153.80, from Oct. 25, 1846, to July 24, 1847, at 5 %. *Ans.* \$159.546+

REVIEW.—222. How find the interest of any sum at any rate for 1 year, Rule? For two or more years? For months? For days? For years, months, and days? How find the interest by proportion?

ANOTHER METHOD FOR INTEREST.

Art. 223. *To find the interest of* $1, *at* 6 %, *for any time.*

At 6 %, the Int. of $1 for 1 year, (12 mon.) is 6 cts.	$ .06
For 1 mon. it will be $\frac{6}{12}=\frac{1}{2}$ ct., or 5 mills. . . .	.005
For 2 mon., $\frac{1}{2}\times 2=1$ cent.	.01
For 3 mon., $\frac{1}{2}\times 3=1\frac{1}{2}$ cents.	.015
For 4 mon., $\frac{1}{2}\times 4=2$ cents. (And so on.) . .	.02
Again: since 30 days = 1 mon.	
For 1 da. the Int. is $\frac{1}{30}$ of $\frac{1}{2}$ ct. $=\frac{1}{60}$ ct. $=\frac{1}{6}$ m. . . .	$ .000$\frac{1}{6}$
For 6 da., as $\frac{6}{30}=\frac{1}{5}$, it is $\frac{1}{5}$ of $\frac{1}{2}$ ct. $=\frac{1}{10}$ ct. = 1 m.	.001

Hence, to find the Interest of $1, at 6 %, for any time,

Take as many cents as equal half the even number of months; take 5 mills for each odd month, 1 mill for each six days, and one sixth of a mill for each remaining day.

1. Find the Int. of $1, for 9 mon. 12 da. at 6 %.

Solution.—The preceding shows that the Int. for 8 months is 4 cents; for 1 mon., 5 mills; for 12 da., 2 mills; the sum of these is $ .047, the required Int.

$.04
.005
.002
Ans. $.047

2. Find the Int. of $1 for 17 mon. 23 da., at 6 %.

Solution.—For 16 mon. the Int. is 8 cts.; for 1 mon., 5 mills; for 18 da., 3 mills; for 5 da., $\frac{5}{6}$ of a mill; their sum is $ .088$\frac{5}{6}$, the required Int.

$.08
.005
.003$\frac{5}{6}$
Ans. $.088$\frac{5}{6}$

3. Find the Int. of $1 at 6 %, for

16 mon.	*Ans.* $.08	11 mon.	*Ans.* $.055
13 mon.	$.065	2 mon. 1 da.	$.010$\frac{1}{6}$
4 mon. 18 da.	$.023	9 mon. 3 da.	$.045$\frac{1}{2}$
7 mon. 12 da.	$.037	14 mon. 4 da.	$.070$\frac{2}{3}$
10 mon. 13 da.	$.052$\frac{1}{6}$	17 mon. 27 da.	$.089$\frac{1}{2}$
5 mon. 17 da.	$.027$\frac{5}{6}$	33 mon. 20 da.	$.168$\frac{1}{3}$

Review.—223. How find the Int. of $1 at 6 per cent. for any time? Why?

ART. **224**. *To find the interest of any sum, at 6 %, for any given time.*

The interest of \$2 is twice as much as the Int. of \$1 for the same time; of \$3, three times as much, and so on. Hence, the

Rule for Finding the Interest at 6 %.—*Find the interest of* \$1 *at* 6 *per cent. for the given time* (Art. 223), *then multiply this by the given sum; the product will be the required Int.*

1. Find the interest of \$69, for 6 mon. 8 da., at 6 %.

SOLUTION.—The interest of \$1 for 6 mon. 8 da., at 6 per cent. (Art. 222), is $\$.031\frac{1}{3}$

As the Int. of \$69 will be 69 times that of \$1, for the same time, multiply the Int. of \$1 by 69, to obtain the result.

$$\begin{array}{r} \$.031\frac{1}{3} \\ 69 \\ \hline 279 \\ 186 \\ 23 \\ \hline \textit{Ans. } \$2.162 \end{array}$$

Find the Interest, at 6 %, of		ANSWERS.
2. \$65.00 for	8 mon.	\$2.60
3. \$36.00 for	11 mon.	\$1.98
4. \$28.00 for	1 yr. 4 mon. (16 mon.)	\$2.24
5. \$500.00 for	2 yr. 1 mon.	\$62.50
6. \$75.00 for	4 mon. 12 da.	\$1.65
7. \$186.00 for	9 mon. 23 da.	\$9.083
8. \$125.00 for	1 yr. 2 mon. 12 da.	\$9.00
9. \$210.25 for	3 yr. 4 mon. 24 da.	\$42.891
10. \$134.45 for	1 yr. 5 mon. 15 da.	\$11.764+
11. \$144.24 for	2 yr. 3 mon. 19 da.	\$19.929+

ART. **225**. From the illustrations in Art. 223, two rules are derived: the 2d is often used.

Rule I.—Since the Int. of \$1 at 6 %, for 1 mon. is half a cent, to find the Int. of any sum at 6 % when the time is mon.,

Multiply the sum considered as dollars, by half the number of mon., the product will be the Int. in cents.

REV.—224. How find the Int. of any sum at 6 per cent. for any time?

Rule II.—Since the Int. of $1, at 6 %, for 1 da. is $\frac{1}{60}$ of a ct., to find the Int. of any sum at 6 %, when the time is da.,

Multiply the sum considered as dollars, by the number of da., and divide the product by 60; *the quotient will be the Int. in cts.*

Rem.—1. In applying Rule 1, when the time is years and months, reduce to months. When the time is days, and either months or years, or both, find the Int. for the days separately, or, reduce the whole to days, and then find the Int.

2. Reduce the result obtained to dollars, by removing the decimal point two places toward the left. Art. 184.

3. The Int. of any sum at 5 per cent. is $\frac{5}{6}$ of the Int. at 6 pr. ct.; at 7 pr. ct., $\frac{7}{6}$ of 6 pr. ct., and so on. Hence, to find the Int. of any sum, at any rate, first find the Int. at 6 per cent., multiply this by the given rate, and divide the product by 6. Art. 152.

PARTIAL PAYMENTS ON BONDS, NOTES, &c.

Art. **226**. When partial payments have been made, the following rate for computing the Int., adopted by the Supreme Court of the U. States, is regarded

THE LEGAL RULE FOR PARTIAL PAYMENTS.

"*When partial payments have been made, apply the payment, in the first place, to the discharge of the interest then due.*

"*If the payment exceeds the Int., the surplus goes toward discharging the principal, and the subsequent Int. is to be computed on the balance of principal remaining due.*

"*If the payment be less than the Int., the surplus of Int. must not be taken to augment the principal, but Int. continues on the former principal, until the period when the payments, taken together, exceed the Int. due, and then the surplus is to be applied toward discharging the principal; and Int. is to be computed on the balance, as aforesaid.*"—Kent. C. J.

Rem.—A Partial Payment, is payment of part of a note, or other obligation, which payment should be indorsed upon it.

The rule is founded on the principle, *that neither interest nor payment shall draw interest.*

1. $350. *Boston*, July 1st, 1845.

For value received, I promise to pay to Edward Sargent, or order, on demand, Three hundred and fifty dollars, with interest at 6 %. LOWELL MASON.

On this note the following payments were indorsed:

March 1st, 1846, rec'd $44. Jan. 1st, 1847, rec'd $26.
Oct. 1st, 1846, rec'd $10. Dec. 1st, 1847, rec'd $15.

What was due, on settlement, March 16th, 1848?

OPERATION.

Principal, ...		$350.00
Interest to 1st payment (8 mon.), ...		14.00
Amount due March 1st, 1846, ...		364.00
1st payment (greater than Int.) deducted, ...		44.00
Balance due March 1st, 1846, ...		$320.00
Int. on bal. to Oct. 1st, 1846, (7 mon.) . .	$11.20	
2d payment, (less than Int. due,)	10.00	
Surplus Int. unpaid, Oct. 1st, 1846, . . .	1.20	
Int. continued on bal. from Oct. 1st, 1846, to Jan. 1st, 1847, (3 mon.)	4.80	6.00
Amount due Jan. 1st, 1847, ...		326.00
Deduct 3d payment, (greater than Int. due,) . . .		26.00
Balance due Jan. 1st, 1847, ...		$300.00
Int. on bal. to Dec. 1st, 1847, (11 mon.) . .	$16.50	
4th payment, (less than Int. due,)	15.00	
Surplus Int. unpaid Dec. 1st, 1847, . . .	1.50	
Int. continued on bal. from Dec. 1st, 1847, to March 16th, 1848, (3 mon. 15 da.) . . .	5.25	6.75
Balance due, on settlement, March 16th, 1848, . .		$306.75

SUGGESTION.—When any payment is less than the Interest on the principal or balance up to the time of payment, shorten the operation by deferring the computation of Int. until the payments, taken together, exceed the Int. due. Thus, in the above operation, instead of computing the Int. Oct. 1st, 1846, it might have been

REVIEW.—225. When the rate is 6 per cent., how find the interest when the time is months? When days? REM. How may the interest at any rate be found by Rules 1 and 2? Why?

226. What is the legal rule for partial payments?

deferred till Jan. 1st, 1847. The whole Int. then, would have been \$16, which, added to \$320, the preceding balance, =\$336.

2. A note of \$200 is dated Jan. 1, 1845, on which there is paid, Jan. 1, 1846, \$70: reckoning interest at 6 %, what was due Jan. 1, 1847? *Ans.* \$150.52

3. A note of \$300 is dated July 1st, 1843. Int. 6 %.
Indorsed, Jan. 1, 1844, \$109. July 1, 1844, \$100.
What was due Jan. 1st, 1845? *Ans.* \$109.18

4. A note of \$150 is dated May 10th, 1850. Int. 6 %.
Indorsed, Sept. 10, 1851, \$32. Sept. 10, 1852, \$6.80
What was due Nov. 10th, 1852? *Ans.* \$132.30

5. A note of \$200 is dated Mar. 5th, 1841. Int. 10 %.
Indorsed, June 5, 1842, \$20. Dec. 5, 1842, \$50.50
What was due June 5th, 1844? *Ans.* \$189.175

6. A note of \$250 is dated Jan. 1st, 1845. Int. 7 %.
Indorsed, June 1, 1845, \$6. Jan. 1, 1846, \$21.50
What was due July 1st, 1846? *Ans.* \$248.40

7. A note of \$180 is dated Aug. 1st, 1844. Int. 6 %.
Indorsed, Feb. 1, 1845, \$25.40 Aug. 1, 1845, \$4.30
Jan. 1, 1846, \$30.
What was due July 1st, 1846? *Ans.* \$138.535

8. A note of \$400 is dated March 1st, 1845. Int. 6 %.
Indorsed, Sept. 1, 1845, \$10. Jan. 1, 1846, \$30.
July 1, 1846, \$11. Sept. 1, 1846, \$80.
What was due March 1st, 1847? *Ans.* \$313.326

9. A note of \$450 is dated April 16th, 1846. Int. 8 %.
Indorsed, Jan. 1, 1847, \$20. April 1, 1847, \$14.
July 16, 1847, \$31. Dec. 25, 1847, \$10.
July 4, 1848, \$18.
What was due June 1st, 1849? *Ans.* \$466.50

10. A note of \$1000 is dated Jan. 1st, 1840. Int. 6 %.
Indorsed, May 1, 1840, \$ 18. Sept. 4, 1840, \$ 20.
Dec. 16, 1840, \$ 15. April 10, 1841, \$ 21.
July 13, 1841, \$118. Dec. 23, 1841, \$324.
What was due, Oct. 1st, 1843? *Ans.* \$663.80

ART. **227**. In Computing Interest, when PARTIAL PAYMENTS have been made, the following rule is applied where settlement is made *in a year*, or in *less than a year*, from the commencement of Int.

Find the amount of the principal for the whole time; then find the amount of each payment from the time it was made to the time of settlement. Add together the amounts of the several payments, and subtract their sum from the amount of the principal; the remainder will be the sum due on settlement.

1. A note of $320 is dated Jan. 1st, 1846. Int. 6 %.
Indorsed, May 1, 1846, $50. Nov. 16, 1846, $100.
What was due Jan. 1st, 1847? *Ans.* $186.45

2. A note of $540 is dated March 1st, 1847. Int. 8 %.
Indorsed, May 1, 1847, $90. July 1, 1847, $100.
Aug. 1, 1847, $150. Oct. 11, 1847, $180.
What was due Jan. 1st, 1848? *Ans.* $39.

ART. **228**. CONNECTICUT RULE.

"Compute the interest to the time of the first payment; if that be 1 year or more from the time the Int. commenced, add it to the principal, and deduct the payment from the sum total.

"If there be after payments made, compute the Int. on the balance due to the next payment, and deduct the payment as above; and, in like manner, from one payment to another, till all the payments are absorbed, provided the time between one payment and another be 1 year or more. But,

"If any payments be made before 1 yr.'s Int. has accrued, then compute the Int. on the principal sum due on the obligation, for 1 yr., add it to the Prin., and compute the Int. on the sum paid, from the time it was paid to the end of the yr., add it to the sum paid, and deduct that sum from the Prin. and Int. added as above.*

"If any payments be made of a less sum than the Int. arisen at

---

* If a year does not extend beyond the time of payment; but if it does, then find the amount of the principal, remaining unpaid, up to the time of settlement, likewise the amount of the payment or payments from the time they were paid to the time of settlement, and deduct the sum of these several amounts from the amount of the principal.

the time of such payment, no Int. is to be computed, but only on the principal sum for any period."—KIRBY'S REPORTS.

1. A note of $875 is dated Jan. 10th, 1831. Int. 6 %.
Indorsed, Aug. 10, 1834, $260. Dec. 16, 1835, $300.
Mar. 1, 1836, $ 50. July 1, 1837, $150.
What was due Sept. 1st, 1838? *Ans.* $446.983+

PROBLEMS IN INTEREST.

ART. **229**. There are *four* parts or quantities connected with each operation in Interest: these are, the

Principal: Rate per cent.: Time: Interest or Amount.

If any *three* of them are given, the *other* may be found.

PROBLEM I.—To find the INTEREST or AMOUNT, the Principal, Rate per cent. and Time being given.

This, the most important problem in Interest, is illustrated in Art. 220 to 223.

ART. **230**. PROBLEM II.—To find the TIME, the Principal, Rate per cent. and Interest being given.

1. I loaned $200 at 6 %, and received $36 for interest: how long was the money loaned?

SOLUTION.—The interest of $200 (Art. 220), at 6 per cent., for 1 year, is $12; then, since the given principal in 1 year produces $12 Int., it will require the same principal as many times 1 year to produce $36 Int., as $12 are contained times in $36; that is, 3. *Ans.* 3 yr.

OPERATION.
$200
·06
12.00
$36 ÷ 12 = 3.

*2. In what time will $60, at 5 %, gain $12 interest? *Ans.* 4 yr.

RULE FOR PROBLEM II.

Divide the given interest by the Int. of the principal at the given rate per cent. for one year; the quotient will be the time.

Or, BY PROPORTION.—*As the Int. of the principal for one year is to the given Int., so is one year to the required time.*

REVIEW.—229. What quantities are connected with each operation in interest? How many must be given, that the remaining may be found? What is the most important problem in interest?

230. What is Problem II? What the rule? How, by proportion?

NOTE.—When the quotient contains a common fraction, or a decimal, find its value in months and days. Art. 162 and 188.

3. A man loaned $375, at 8%, and received $90 interest: how long was it loaned? *Ans.* 3 yr.

4. How long will it take $225, at 4%, to gain $66 interest? *Ans.* $7\frac{1}{3}$ yr. = 7 yr. 4 mon.

5. How long will it take $250, at 6%, to gain $34.50 interest? *Ans.* 2.3 yr. = 2 yr. 3 mon. 18 da.

6. How long will it take $60, at 6%, to gain $13.77 interest? *Ans.* 3.825 yr. = 3 yr. 9 mon. 27 da.

7. $500, at 10%, to amount to $800? *Ans.* 6 yr.

SUGGESTION.—By subtracting the principal, $500, from the amount, $800, the remainder, $300, is the interest of the principal for the required time: then proceed as before.

8. In what time will $600, at 9%, amount to $798? *Ans.* 3 yr. 8 mon.

9. In what time will $200, at 6%, amount to $400; or, in what time will any sum of money double itself, at 6%? *Ans.* 16 yr. 8 mon.

SUGGESTION.—To find in what time any principal will double itself at any given rate per cent.,

Take any sum, as $100, for a principal, and find the time in which, at the given rate per cent., the Int. produced would equal the principal.

IN WHAT TIME WILL ANY PRINCIPAL DOUBLE ITSELF,

10. At 4% per annum? *Ans.* 25 yr.

11. At 5%? *Ans.* 20 yr.

12. At 7%? *Ans.* $14\frac{2}{7}$ yr.

13. At 8%? *Ans.* $12\frac{1}{2}$ yr.

14. At 10%? *Ans.* 10 yr.

15. At 12%? *Ans.* $8\frac{1}{3}$ yr.

16. At 16%? *Ans.* $6\frac{1}{4}$ yr.

17. In what time, at 15%, will any principal *treble* itself? *Ans.* $13\frac{1}{3}$ yr. Quadruple itself? *Ans.* 20 yr.

ART. **231.** PROBLEM III. To find the RATE PER CENT., the Principal, Interest, and Time being given.

REVIEW.—230. How find the time in which any principal will double itself, at any given rate per cent.?

231. What is problem III? What the Rule? How, by proportion?

1. I borrowed \$600 for 2 years, and paid \$48 interest: what rate % did I pay?

OPERATION.

$\$600 \times .01 = \$6.$

$\$6 \times 2 = \$12 =$ Int. for 2 yr. at 1 %.

$\$48 \div 12 = 4$ % *Ans.*

SOLUTION.—The Int. of \$600 for 1 year, at 1 per cent. (Art. 220), is \$6, and for 2 years, $\$6 \times 2 = \$12$.

As the given Int. (\$48), contains this Int. (\$12), exactly 4 times, the rate must have been 4 times as large; that is 4 per cent.

*2. A man paid \$28 for the use of \$80 for 5 years: what was the rate %? *Ans.* 7.

RULE FOR PROBLEM III.

Divide the given interest by the Int. of the principal at 1 *per cent. for the given time; the quotient will be the rate per cent.*

Or, by PROPORTION. *As the Int. of the principal at* 1 *per cent. for the given time, is to the given Int., so is* 1 *per cent. to the rate per cent.*

3. A merchant paid \$30 for the use of \$300 for 1 yr. 8 mon.: find the rate %. *Ans.* 6.

4. A broker paid \$200 for the use of \$1000 for 2 yr. 6 mon.: find the rate %. *Ans.* 8.

5. \$23.40 was paid for the use of \$260 for 2 years: what the rate %? *Ans.* $4\frac{1}{2}$.

6. \$110.40 was paid for the use of \$640 for 6 years: find the rate %. *Ans.* $2.875 = 2\frac{7}{8}$.

ART. **232**. PROBLEM IV. To find the PRINCIPAL, the Time, Rate per cent., and Int. being given.

1. What sum in 2 yr., at 6 %, will gain \$27 Int.?

SOLUTION.—The interest of \$1 for 2 years, at 6 per cent., is 12 cents, =\$.12 Since 12 cts. is the Int. of \$1 for the given time and rate, \$27 must be the Int. of as many times \$1, as 12 cts. are contained times in \$27. $\$27 \div \$.12 = 225$. *Ans.* \$225.

*2. What sum of money in 3 yr., at 5 %, will it take to gain \$8.25 Int.? *Ans.* \$55.

RULE FOR PROBLEM IV.

Divide the given interest by the Int. of $1 for the given time at the given rate per cent.; the quotient will be the principal.

Or, by PROPORTION. *As the Int. of $1 for the given time, at the given rate per cent., is to the given Int., so is $1 to the required principal.*

What Principal,

3. At 5 %, will gain $341.25 in 3 yr.? *Ans.* $2275.
4. At 6 %, will gain $2.26 in 16 mon.? *Ans.* $28.25
5. At 5 %, will produce a yearly interest of $1023.75? *Ans.* $20475.
6. At 9 %, will gain $525.398 in 12 yr. 3 mon. 20 da.? *Ans.* $474.40

PROBLEM V. To find the PRINCIPAL and INTEREST, the Time, Rate per cent., and Amount being given.

This is treated of under Discount. See Art. 238.

ART. 233. COMPOUND INTEREST

Is interest on the principal, and also on the interest itself, after the latter becomes due.

(Simple Interest is interest on the principal only.)

1. Find the compound Int. of $300 for 3 yr. at 6 %.

	$300.	Principal 1st year.
$300 × .06 =	18.	Interest 1st year.
	$318.	Principal 2d year.
$318 × .06 =	19.08	Interest 2d year.
	$337.08	Principal 3d year.
$337.08 × .06 =	20.2248	Interest 3d year.
	$357.3048	Amount for 3 yr.
	300.	Given principal.
	$ 57.3048	Compound Int. for 3 yr.

*2. Find the compound Interest of $200 for 2 years, at 8 %. *Ans.* $33.28

REVIEW.—232. What is Problem IV? What the Rule for Prob. IV?

RULE FOR COMPOUND INTEREST.

Find the amount of the given principal (Art. 222) *for the* 1st *yr., and make it the principal for the* 2d *yr.*

Find the amount of this principal for the 2d *yr., make it the principal for the* 3d *yr., and so on, for the given number of yr.*

From the last amount subtract the given principal; the remainder will be the compound interest.

NOTES.—1. When the interest is payable half-yearly, or quarter-yearly, find the Int. for a half, or a quarter year, and proceed in other respects as when the Int. is payable yearly.

2. When the time is years, months, and days, find the amount for the years, then compute the Int. on this for the months and days, and add it to the last amount.

Find the Amount, at 6 %, Compound Interest,

3. Of $500 for 3 years. . . . . *Ans.* $595.508

4. $800 for 4 years. . . . . *Ans.* $1009.98+

Find the Compound Interest

5. Of $250 for 3 yr., at 6 %. . . . *Ans.* $47.754

6. $300 for 4 yr., at 5 %. . . . *Ans.* $64.65+

7. $200 for 2 yr., at 6 %, payable semi-annually. *Ans.* $25.101+

8. Find the amount of $500 for 2 yrs., at 20 % compound interest, payable quarterly. *Ans.* $738.727+

9. What is the compound interest of $300 for 2 yr. 6 mon., at 6 %? *Ans.* $47.192+

10. What the compound interest of $1000 for 2 yr. 8 mon. 15 da., at 6 %? *Ans.* $171.353

ART. **234**. Since the amount of $2, for any given time, will be *twice* the amount of $1; the amount of $3, *three* times as much, &c.; Therefore,

If a table be formed containing the amounts of $1 for 1, 2, 3, &c., years, *any of these amounts multiplied by a given Principal will give its amount at Compound Interest for the same time and rate.* See Table, page 238.

TABLE

Showing the amount of $1, at 3, 4, 5, 6, 7 and 8 per cent., Compound Interest, for any number of years, from 1 to 25.

Yr.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1	1.03	1.04	1.05	1.06	1.07	1.08
2	1.060	1.0816	1.1025	1.1236	1.1449	1.1664
3	1.092727	1.124864	1.157625	1.191016	1.225043	1.259712
4	1.125509	1.169859	1.215506	1.262477	1.310796	1.360488
5	1.159274	1.216653	1.276282	1.338226	1.402551	1.469328
6	1.194052	1.265319	1.340096	1.418519	1.500730	1.586874
7	1.229874	1.315932	1.407100	1.503630	1.605781	1.713824
8	1.266770	1.368569	1.477455	1.593848	1.718186	1.850930
9	1.304773	1.423312	1.551328	1.689479	1.838459	1.999004
10	1.343916	1.480244	1.628895	1.790848	1.967151	2.158924
11	1.384234	1.539454	1.710339	1.898299	2.104851	2.331638
12	1.425761	1.601032	1.795856	2.012196	2.252191	2.518170
13	1.468534	1.665074	1.885649	2.132928	2.409845	2.719623
14	1.512590	1.731676	1.979932	2.260904	2.578534	2.937193
15	1.557967	1.800944	2.078928	2.396558	2.759031	3.172169
16	1.604706	1.872981	2.182875	2.540352	2.952163	3.425942
17	1.652848	1.947900	2.292018	2.692773	3.158815	3.700018
18	1.702433	2.025817	2.406619	2.854339	3.379932	3.996019
19	1.753506	2.106849	2.526950	3.025600	3.616527	4.315701
20	1.806111	2.191123	2.653298	3.207135	3.869684	4.660957
21	1.860295	2.278768	2.785963	3.399564	4.140562	5.033833
22	1.916103	2.369919	2.925261	3.603537	4.430401	5.436540
23	1.973587	2.464716	3.071524	3.819750	4.740529	5.871463
24	2.032794	2.563304	3.225100	4.048935	5.072366	6.341180
25	2.093778	2.665836	3.386355	4.291871	5.427432	6.848475

11. What, by the Table, will be the Amount of $70 for 9 yr., at 5 % compound interest?

SOLUTION.—By the Table, the amount of $1 for 9 yrs., at 5 pr. ct., is $1.551328; and $1.551328×70=$108.59296

12. Of $345 for 10 yr., at 6 %? *Ans.* $617.84+

13. $200 for 41 yr., at 6 %?

EXPLANATION.—Take any two periods whose sum is 41 years, thus, 20 yr.+21 yr.=41 yr., and find the amount for the 1st period: then regard this a new principal, and find its amount for the 2d period: the last amount will be the *Ans.*

The Tabular number for 20 yr. is 3.207135; this, multiplied by 200, gives $641.427, amount for 20 yr.: Tabular number for 21 yr. is 3.399564, which×641.427=$2180.572+ *Ans.*

What, by the Table, will be the Interest

14. Of $890 for 30 yr., at 6 % ? *Ans.* $4221.70+

15. $200 for 70 yr., at 5 % ? *Ans.* $5885.28+

ART. 235. DISCOUNT

Is a *deduction* made for the payment of money before it is due. For example,

If a debt of $106, due *one year* hence without interest, be paid at the *present time*, the *sum* paid, with *one year's* interest added, should make $106. And,

If the rate per cent. is 6, this sum would be $100; for, the *amount* of $100 at interest for 1 year, at 6 %, is $106. Art. 220.

The PRESENT WORTH of a debt payable at a future time without interest, is that sum which, at a specified rate % for the same time, would amount to the debt.

The DISCOUNT is the sum deducted for *present payment.*

ART. **236**. 1. Find the Present Worth of $224, due 2 yr. hence, without interest, money being worth 6 % per annum.

SOLUTION.—The amount of $1 for 2 years, at 6 per cent., is $1.12; hence, the present worth of each $1.12 of the given sum, is $1. And, the present worth of $224, will be as many times $1, as $1.12 is contained times in $224.

$224. ÷ $1.12 = 200. *Ans.* $200.

PROOF.—The amount of $200 for 2 years, at 6 per cent., is $224. Art. 220.

*2. Find the present worth of $81, due 2 yr. hence, no Int., money worth 4 % per annum. *Ans.* $75.

REVIEW.—234. What does the Table show? By means of it, how find the amount of any sum at compound interest?

235. What is discount? What the present worth of a debt, payable at a future time without interest? The discount?

OF PRESENT WORTH.

To find the present worth of a sum payable at a future time without interest:

Rule.—*Divide the given* DEBT *by the amount of one dollar for the given time, at the given rate per cent.; the quotient will be the* PRESENT WORTH.

To find the DISCOUNT, *subtract the* PRESENT WORTH *from the* DEBT.

Or, by PROPORTION. *As the amount of any principal,* (as $1, or $100,) *for the given time, at the given rate per cent., is to the principal, so is the given debt to the present worth.*

NOTE.—In the following Examples, the rate per cent. is 6, unless some other is given; and when the present worth of any sum is required, it is supposed not to be at interest.

What is the Present Worth of

		ANSWERS.
3.	$1300.00 due 5 years hence?	$1000.
4.	$4720.00 due 3 years hence?	$4000.
5.	$257.50 due 1 year hence? . . .	$242.924+
6.	$199.80 due 1 yr. 10 mon. hence?	$180.
7.	$675.00 due 5 yr. 10 mon. hence?	$500.
8.	$307.50 due 5 mon. hence?	$300.
9.	$493.20 due in 7 yr. 9 mon. 20 da.?	$335.89+

What is the Discount of

10.	$496.00 due 4 years hence?	$96.00
11.	$276.64 due 2 years hence?	$29.64
12.	$330.00 due 3 yr. 4 mon. hence? . . .	$55.00

13. Bought $260 worth of goods, on 8 mon. credit: what sum will pay the debt now? *Ans.* $250.

14. If money is worth 12 %, what is the present worth of $235.20, due 1 year hence? *Ans.* $210.

15. What is the discount, at 7 %, of $401.25, due 1 year hence? *Ans.* $26.25

16. What is the difference between the simple Int. and discount of \$1080 for 10 yr., at 6 %? *Ans.* \$243.

17. A man was offered \$1122 for a house, in cash, or \$1221, payable in 10 mon. without Int. He chose the latter: how much did he lose, supposing the note discounted at the rate of 12 % per annum? *Ans.* \$12.

ART. **237**. PAYMENTS AT DIFFERENT TIMES.

When payments without interest, are to be made at different times, to find the present value of the whole,

Find the present worth of each payment, and take their sum.

18. Find the present value of a debt of \$956.34, one-third to be paid in 1 yr., one-third in 2 yr., and one-third in 3 yr.; money being worth 5 %. *Ans.* \$870.60

19. Of a debt of \$1440, of which one-half is payable in 3 mon., one-third in 6 mon., and the remainder in 9 mon.; Int. at 6 % per annum. *Ans.* \$1405.044+

20. Of a debt of \$700, of which \$60 are to be paid in 6 mon., \$180 in 1 yr., \$260 in 18 mon., the remainder in 2 yr.; Int. 6 % per annum. *Ans.* \$645.167+

DISCOUNT AND INTEREST COMPARED.

ART. **238**. A comparison of Discount with Interest (Art. 219), shows that the *Present Worth* corresponds to the *principal*, the *debt* to the *amount*, and the *discount* to the *interest* of the principal for the given time, at the given rate per cent.; hence,

When the *time*, *rate per cent.*, and *amount* are given, *the principal is found* (Art. 236) *by dividing the amount by the amount of* \$1 *for the given time, at the given rate per cent.*

To find the interest, subtract the principal from the amount.

1. The amount is \$650; time, 5 yr.; rate, 6 %: what is the principal? *Ans.* \$500.

REVIEW.—236. How find the present worth, Rule? The discount? How, by proportion? 237. When payments without interest are to be made at different times, how find the present value of the whole?

2. What principal, at interest for 9 yr., at 5 %, will amount to $725? *Ans.* $500.

3. The amount is $571.20; time, 4 yr.; rate, 5 %: what is the interest? *Ans.* $95.20

4. A note, at interest for 2 yr. 6 mon., at 6 %, amounts to $690: find the interest. *Ans.* $90.

ART. 239. BANK DISCOUNT.

A PROMISSORY NOTE is a written promise by one or more persons to pay to another, a named sum of money, after a specified time has elapsed from its date.

A note is discounted when a bank receives it, and pays to the holder what remains after deducting from its *face* the *interest* on it, till it becomes due.

BANK DISCOUNT is the interest deducted from the face of the note.

The *bank discount*, therefore, is the *Simple Interest of the* FACE *of the note paid in advance.*

The time to elapse from any given date till a note becomes due, is termed *days to run.* By usage, a note or draft is not really due till *three* days after the time specified for payment.

These three days are called DAYS OF GRACE, but banks charge interest for them.

Hence, in calculating the interest, *three* days must be added to the time specified in the note.

The FACE of a note is the sum promised to be paid; the PROCEEDS, the sum realized when the note is discounted.

REM.—1. Before a bank discounts a note, it is required that one or more persons shall *indorse* it; that is, write their names upon the *back*, by which they become responsible for its payment.

REVIEW.—238. Comparing discount with interest, to what does the present worth correspond? The debt? Discount? When the time, rate per cent., and amount are given, how find the principal? The interest?

239. What is a promissory note? What is bank discount? What are *days to run?* When is a note *really* due? What are days of grace? What is the face of a note? What the proceeds?

2. Bank discount is different from true discount. Art. 235. The bank discount of $106 for 1 year, is $6.36, while the true discount, Art. 236, is $6. The difference, 36 cents, is *the interest of the true discount for the same time.*

For a fuller discussion of this subject, see "*Ray's Higher Arithmetic.*"

ART. **240**. In bank discount, as interest is always to be computed for three days more than the specified time of payment, the calculations involve the finding of Int. for days; see Rule 2, Art. 225.

The rate per cent. is always 6, unless some other is given.

1. What is the bank discount and proceeds of a note of $100, payable 60 days after date?

OPERATION.

Face of the note	$100.00	
Int. of $100 for 63 da., (Art. 225)	1.05	bank discount.
Ans.	$98.95	proceeds.

Find the Bank discount of a note of

		ANSWERS.
2.	$137, payable 90 days after date.	$2.12+
3.	$1780, payable 90 days after date.	$27.59
4.	$375, payable 30 days after date.	$2.06¼
5.	$165, payable 60 days after date.	$1.73¼
6.	$140, payable 4 mon. after date.	$2.87
7.	$80, payable 6 mon. after date.	$2.44

Find the Proceeds or Avails of a note of

8.	$180, due 30 days after date.	$179.01
9.	$960, due 30 days after date.	$954.72
10.	$875, due 90 days after date.	$861.43¾
11.	$3900, due 60 days after date.	$3859.05

REVIEW.—239. REM. 1. What is meant by indorsing a note? 2. Are bank discount and true discount the same? Show their difference by an example. 240. By what rule is bank discount generally calculated?

12. Find the proceeds of a note of $2580, due 100 days after date, discount 5%. *Ans.* $2543.09+

13. I bought 225 barrels flour, at $3.50 per bl.; sold it at $4 per bl., taking a note payable 6 mon. after date. If this note be discounted at 6 % per annum, what the gain by the transaction? *Ans.* $85.05

14. What the difference between true discount, and bank discount, of $535, for 1 year, at 7 %, not reckoning days of grace? *Ans.* $2.45

15. Omitting the 3 days of grace, find the difference between the true and bank discount of $1209, for 4 yr., at 6 % per annum. *Ans.* $56.16

ART. **241.** *To make a note, the proceeds of which, when discounted, shall be a given sum.*

1. For what sum, due 90 days hence, must I give a note, that when discounted at 6 % per annum, the proceeds will be $177.21?

SOL.—The bank discount of $1 for 93 da., is $.0155 (Art. 223); which, deducted, from $1, leaves $.9845, the proceeds of a note of $1, discounted for the same time. Therefore, for each $.9845 of proceeds, the note must contain $1; hence, the note must contain as many dol's as $.9845 is contained times in the proceeds.

$177.21 ÷ $.9845 = 180. *Ans.* $180.

PROOF.—Interest of $180 for 93 da.=$2.79;
and $180 — $2.79 =$177.21

Hence, *divide the proceeds of the note by the proceeds of* $1 *on the same conditions; the quotient will be the face of the note.*

2. For what sum must a note be made, at 3 mon., so that, when discounted at a bank at 6 %, the amount received will be $393.80? *Ans.* $400.

3. I wish to obtain $500 for 60 days: for what sum must the note be given? *Ans.* $505.305+

REVIEW.—241. How find the face of a note, which, when discounted, the proceeds shall be a given sum?

ART. 242. PROFIT AND LOSS

Are terms used to express the gain or loss in business.

In Profit and Loss, *four* quantities are considered:

1st, the *cost* price; 2d, the *selling price;* 3d, the *amount* of gain or loss; 4th, the *rate per cent.* of gain or loss.

ART. 243. CASE I.

To find the AMOUNT of profit or loss, when the cost price and rate per cent. of gain or loss are given.

1. A merchant bought a piece of cloth for $40, and sold it at 10 % profit: how much did he gain?

SOLUTION.—10 pr. ct. of $40 is $4, the required gain.

2. A merchant bought a bale of cotton for $80, which he sold at 8 % loss: what did he lose?

SOLUTION.—8 pr. ct. of $80 is $6.40, the loss.

Rule for Case I.—*Find the given per cent. of the cost price, and the result will be the gain or loss.* Art. 210.

NOTE.—The rate per cent. of gain or loss always refers to the *purchase* or *cost* price, and *not* to the selling price.

3. A merchant sold goods, that cost $150, and gained 10 %: what was his gain? *Ans.* $15.

4. A peddler sold goods, that cost $874, at a gain of 25 %: required his gain. *Ans.* $218.50

5. I bought goods for $500, and sold them at 12 % profit: what sum did they bring? *Ans.* $560.

6. Sold goods that cost $382.50, at a loss of 4 %: what sum did they bring? *Ans.* $367.20

ART. 244. CASE II.

To find the SELLING PRICE, when the cost price is known, so that a given rate % may be gained or lost.

REVIEW.—242. What are profit and loss? What four quantities are considered? 243. What is Case 1? What the Rule? NOTE. To what does the rate per cent. refer? 244. What is Case 2?

1. Tea costs 60 cents per lb.: at what rate must it be sold to gain 25 %?

SOLUTION.—25 % of 60 cts. is $60 \times .25 = 15$ cts., and 60 cts. + 15 cts. = 75 cts. per lb. *Ans.*

Or, thus: 25 per cent. $= \frac{25}{100} = \frac{1}{4}$, and $\frac{1}{4}$ of 60 cts. = 15 cts., which, added to 60 cts., gives 75 cts. for the selling price per lb.

If it were required to *lose* so much per cent., the percentage of the cost price must be *subtracted* from it.

Rule for Case II.—*Find the percentage of the cost price,* Art. 210, *and add it to, or subtract it from, the cost, as may be required; the result will be the selling price.*

2. Silk costs 90 cts. per yard: at what price per yd. must it be sold, to gain 25 %? *Ans.* $1.12\frac{1}{2}$
At what price, to lose 10 %? *Ans.* $0.81

3. If cloth cost $4.37\frac{1}{2}$ per yard, at what price per yd. must it be sold, to gain $33\frac{1}{3}$ %? *Ans.* $5.83+
At what price, to lose 20 %? *Ans.* $3.50

4. Cheese cost $8.50 per cwt.: at how much per cwt. must it be sold, to gain 20 %? *Ans.* $10.20
At how much, to lose 20 %? *Ans.* $6.80

5. Bought 40 yards of cloth for $300: at how much a yard must it be sold, to gain 20 %? *Ans.* $9.
At how much a yd., to lose 20 %? *Ans.* $6.

ART. 245. OF MARKING GOODS.

The rate per cent. of profit at which merchants mark the *selling* price of goods, is generally some aliquot part of 100. The following are those most used:

5 per cent. $= \frac{1}{20}$,	$12\frac{1}{2}$ per cent. $= \frac{1}{8}$,	25 per cent. $= \frac{1}{4}$,
$8\frac{1}{3}$ . . . . . . $= \frac{1}{12}$,	$16\frac{2}{3}$ . . . . . . $= \frac{1}{6}$,	$33\frac{1}{3}$ . . . . . $= \frac{1}{3}$,
10 . . . . . . $= \frac{1}{10}$,	20 . . . . . . $= \frac{1}{5}$,	50 . . . . . $= \frac{1}{2}$.

REVIEW.—244. What is the Rule for Case 2? 245. What part of any thing is 5 per cent.? 8 and 1 third? 10? 12 and a half? 16 and 2 thirds? 20? 25? 33 and 1 third? 50 per cent?

To mark goods for different rates of profit, *Add to the cost price such part of itself as the rate per cent. is part of* 100.

6. To make 10 % profit, what must calico be marked, that cost 10 cts. per yard? 15 cts.? 20 cts.? 30 cts.? 40 cts.? 50 cts.? 60 cts.? *Ans.* to last, 66 cts.

7. To make $12\frac{1}{2}$ % profit, how mark muslin that cost 8 cts. per yard.? 12 cts.? 16 cts.? 20 cts.? *Ans.* to last, $22\frac{1}{2}$ cts.

8. To make 20 % profit, how mark ribbons that cost 10 cts. per yd.? 15 cts.? 25 cts.? *Ans.* to last, 30 cts.

9. To make 25 % profit, how mark cloth that cost \$1 per yard? \$1.20? \$1.50? \$2? \$3? \$4? \$6? *Ans.* to last, \$7.50

10. To make $33\frac{1}{3}$ % profit, how mark ginghams that cost 25 cts. per yd.? 50 cts.? *Ans.* to last, $66\frac{2}{3}$ cts.

11. To make 50 % profit, how mark shawls that cost \$2? \$3? \$4? \$5? \$7? *Ans.* to last, \$10.50

ART. 246. CASE III.

To find the RATE PER CENT. of profit or loss, when the cost and selling price are given.

1. I sold cloth at \$5 a yard, that cost \$4 a yard: what was the gain %?

SOLUTION.—Take the difference between the cost and selling price, the gain is \$1 on what cost \$4; then find what per cent. \$1 is of \$4; this, (Art. 211), is $\frac{1}{4} = .25 = 25$ per cent. *Ans.*

Rule for Case III.—*Take the difference between the cost price and selling price, and find what per cent. this is of the cost price.*

2. A man paid \$75 for a horse, and sold him for \$105: what % did he gain? *Ans.* 40 %.

3. I bought a piece of cloth for \$30, and sold it for \$40: what was the % profit? *Ans.* $33\frac{1}{3}$ %.

REVIEW.—245. How mark goods to sell at different rates of profit? 246. What is Case 3? What the Rule?

4. Cloth cost 25 cts. a yard, and sold for 30 cts. a yd.: what the gain %? *Ans.* 20 %.

5. Muslin that cost 20 cts. a yard, is sold at 21 cts. a yard: what the % profit? *Ans.* 5 %.

6. Bought cloth at $8 a yard, and sold it at $9 a yard: what % profit did I make? *Ans.* $12\frac{1}{2}$ %.

7. Muslin that cost 30 cts. a yard, is sold at 24 cts. a yard: what % is lost? *Ans.* 20 %.

8. A bought 40 bales of cotton, at $40 each, and sold it at a profit of $704: what did he make? *Ans.* 44 %.

ART. 247. CASE IV.

To find the COST PRICE, when the selling price and rate per cent. of profit and loss are given.

1. Cloth sold at $5 a yard, pays 25 % profit: required the cost price per yard.

SOLUTION.—When the cost is $1, and 25 per cent. is gained, the selling price is $1.25 (Art. 244); hence, as many times as $1.25 is contained in the selling price, so many times is $1 contained in the cost: the cost price is as many dollars as $1.25 is contained times in $5. $5 ÷ $1.25 = 4. *Ans.* $4.

PROOF.—25 % of $4 is $1; and $4 + $1 = $5.

2. If, by selling cloth at $9 a yard, 10 % is lost, what was the cost price?

SOLUTION.—When the cost is $1, and 10 per cent. is lost, the selling price is $.90 (Art. 244); hence, as often as $.90 is contained in the selling price, so many times is $1 contained in the purchase price: the cost is as many dollars as $.90 is contained times in $9. $9 ÷ $.90 = 10. *Ans.* $10.

PROOF.—10 % of $10 is $1; and $10 — $1 = $9, the selling price.

*3. Cloth sold at $6 a yard, pays 20 % profit: what was the cost price per yd.? *Ans.* $5.

REVIEW.—247. What is Case 4? Rule? Rule by proportion?

*4. By selling cloth at $3 a yard, 25 % was lost: what was the cost price per yd.? *Ans.* $4.

Rule for Case IV.—*Add to, or subtract from,* $1, *as may be required, the per cent. of gain or loss on* $1; *divide the selling price by the result; the quotient will be the cost.*

Or, by PROPORTION. *As* 100 *increased by the per cent. of gain, or diminished by the per cent. of loss, is to* 100, *so is the selling price to the cost.*

Observe, that the per cent. of profit or loss is always calculated on the *cost*, and not on the selling price.

5. A jockey sold a horse for $75, and gained 25 %: what did the horse cost him? *Ans.* $60.

6. A jockey sold a horse for $75, and lost 25 %: what did the horse cost him? *Ans.* $100.

7. A grocer, by selling coffee at 22 cts. a lb., gains 10 %: find the purchase price per lb. *Ans.* 20 cts.

8. By selling cloth at $8.10 per yard, I gain $12\frac{1}{2}$ %: what the purchase price per yd.? *Ans.* $7.20

9. By selling tea at $1.19 per pound, I lost 15 %: what the cost price per lb.? *Ans.* $1.40

ART. 248. PROMISCUOUS EXAMPLES.

1. A buys 30 yd. muslin, at 6 cts. a yd., and sells it at 25 % profit: what does he gain? *Ans.* 45 cts.

2. B bought 40 yd. of cloth for $250: at what price per yd. must he sell to gain 20 %? *Ans.* $7.50

3. A bought 1 hhd. of wine for $75, and sold it for 40 cts. a qt.: what the % profit? *Ans.* $34\frac{2}{5}$ %.

4. The population of New York, in 1830, was 1918604; in 1840 it was 2428921: find the gain per cent. in 10 years. *Ans.* 26.5 % +

5. The population of Ohio, in 1830, was 937903; in 1840 it was 1519467: what was the gain per cent. in 10 years? *Ans.* 62 % +

6. If calico is sold at 42 cts. a yd., at a gain of 10 %, what was the cost price? what the gain %, if sold at 51 cts. a yard? *Ans.* Cost, $38\frac{2}{11}$ cts. a yd. Gain, $33\frac{4}{7}$ %.

7. Sold a bushel of rye for $1, and gained 25 %; purchased a bu. of wheat with the $1, and sold it at a loss of 25 %: what did I lose? *Ans.* 5 cts.

8. A merchant bought 14 pieces of cloth at $9.60 each; sold 5 pieces at $14.40 each, and 4 pieces at $12 each: at how much a piece must he sell the remainder, to gain 20 % on the whole? *Ans.* $8.256

9. Sold wine at $1.29 a gal., and *lost* 14 %: at what price per gal. must it sell, to *gain* 14 %? *Ans.* $1.71

10. Sold cloth at $1.36 a yd., and lost 15 %: what % would I gain by selling at $1.856 a yd.? *Ans.* 16 %.

11. Sold silk at $1.96 a yd., and gained 12 %: at what price per yd. should I sell, to lose 16 %? *Ans.* $1.47

12. Sold satin at $1.682 a yd., and gained 16 %: what % will I lose, by selling at $1.247 a yd.? *Ans.* 14 %.

13. How much cloth, at $5 a yd., must I buy, to clear $100 by selling it at 25 % profit? *Ans.* 80 yd.

14. A grocer buys 200 casks of raisins at $2.50 per cask; by selling at 5 cts. per pound, he gains 20 %: what was the weight of each cask? *Ans.* 60 lb.

15. Sold a quantity of corn, at $1 per bu., and gained 25 %; sold of the same to the amount of $59.40, and gained 35 %: at what rate did I sell: how many bu. in the last lot?
Ans. $1.08 per bu., and 55 bu.

☞ For additional problems, see Ray's Test Examples.

ASSESSMENT OF TAXES.

ART. **249.** A TAX, is a sum assessed on the citizens of a town, county, state, or district, for public purposes.

Taxes are of two kinds—*a property tax* and a *poll-tax.*

A PROPERTY TAX is a certain per cent. assessed on the *taxable* property held by each person.

A POLL-TAX is a *specific sum* assessed on male citizens over 21 years of age. Each person so taxed is called a poll.

REM.—In some States the whole tax is raised on *property;* in other States, partly on *property* and partly on *polls.*

ART. **250**. When a tax is to be assessed, first obtain a list or inventory of the amount of *taxable* property, from which the tax is to be collected.

If there be a poll-tax, make a list of the polls.

TO ASSESS A TAX, OBSERVE THIS

Rule.—1. *If there be a poll-tax, find its amount, by multiplying the tax on each poll by the number of polls. Subtract this from the whole amount of tax to be raised; the remainder will be the sum to be raised on property.*

2. *Divide the tax to be raised on property by the whole amount of property; the quotient will be the per cent. of tax on* $1.

3. *Multiply the per cent. of tax on* $1 *by the amount of each person's property, the product will be his property tax.*

4. *Add the poll-tax, if any, of each person to his property tax; the sum will be his whole tax.*

1. A tax of $500 is assessed in a district, to build a school-house; the property is valued at $125000: what the % of tax? *Ans.* .004, or 4 m. on $1.

What the tax on $1650? *Ans.* $6.60

2. A tax of $9057.60 is assessed in a county whose taxable property is valued at $534650; also, a list of 1258 polls, each taxed $1.25: what the % of tax on property? *Ans.* .014, or 1 c. 4 m. on $1.

NOTE.—In preparing tax lists, after finding the per cent. of tax, assessors make a table embracing the tax on dollars from 1 to 10; then on 10, 20, &c., to $100; then on 100, 200, &c., to $1000.

The following table, computed for the preceding example, is calculated by multiplying the tax on $1 by the number of dollars on which the tax is required.

REVIEW.—249. What is a tax? What a property tax? A poll-tax? 250. When a tax is to be assessed, what is to be first obtained?

250. How assess a tax, Rule? NOTE. How calculate a tax-table?

TAX TABLE.—RATE, 14 MILLS ON $1.

$1 pays $.014	$20 pays $.280	$300 pays $4.200
2 ...... .028	30 ...... .420	400 ...... 5.600
3 ...... .042	40 ...... .560	500 ...... 7.000
4 ...... .056	50 ...... .700	600 ...... 8.400
5 ...... .070	60 ...... .840	700 ...... 9.800
6 ...... .084	70 ...... .980	800 ...... 11.200
7 ...... .098	80 ...... 1.120	900 ...... 12.600
8 ...... .112	90 ...... 1.260	1000 ...... 14.000
9 ...... .126	100 ...... 1.400	2000 ...... 28.000
10 ...... .140	200 ...... 2.800	3000 ...... 42.000

3. What, by the Table, was A's tax; his property valued at $756, he paying for 2 polls?

OPERATION.

$700	is taxed	$9.800
50	....	.700
6	....	.084
$756	....	$10.584
2	polls pay	2.500

Ans. whole tax, $13.084

This operation is the the same as multiplication, Art. 32, and consists in taking the product of $.014 by 700, by 50, and by 6, and then finding their sum.

4. What, by the Table, was B's tax; his property valued at $1243, he paying for 3 polls? *Ans.* $21.152

5. C's property is valued at $3589, and he pays for 4 polls: what is his tax? *Ans.* $55.246

AMERICAN DUTIES.

ART. **251**. DUTY is a tax, levied by the Government on goods imported from a foreign country.

NOTE.—Duties are of two kinds, *Specific* and *Ad valorem*.

A SPECIFIC duty is a fixed *sum* per tun, gallon, yard, &c., without regard to value.

An AD VALOREM duty, (*according to value*), is a certain per cent. of the cost of the goods in the country from which they were imported.

In reckoning duties, deductions are made from the gross weight or measure. These are termed, *draft*, *tare*, and *leakage*.

REVIEW.—251. What is duty? Specific duty? Ad valorem?

DRAFT is made, that the quantity may hold out when retailed.

On each parcel weighing		112 lb., or less,	the draft is 1 lb.
From	112 lb. to	224 lb.,	the draft is 2 lb.
From	224 lb. to	336 lb.,	the draft is 3 lb.
From	336 lb. to	1120 lb.,	the draft is 4 lb.
From	1120 lb. to	2016 lb.,	the draft is 7 lb.
Over	2016 lb.		the draft is 9 lb.

TARE is an allowance (after deducting the draft), for the weight of the box, cask, &c., containing the goods.

GROSS weight is the weight before deducting draft and tare.

NET weight is the weight after deducting the draft and tare.

LEAKAGE is an allowance of 2 per cent. on all liquors in casks, paying duty by the gallon.

Duties are computed on what remains *after deducting all allowances.* The calculations are an application of percentage.

1. Find the duty on 3 boxes of Sugar, of 100 lb., 182 lb., and 264 lb., at 2 cts. a lb., allowing for draft, and deducting 15 % for tare. *Ans.* $9.18

First deduct the draft (6 lb.), then 15 per cent. of the remainder for tare, and compute the duty on the last remainder.

2. What the duty, at 20 % ad valorem, on 40 bales of wool, of 400 lb. each, cost, in Spain, 25 cts. a lb., the tare 5 %? *Ans.* $752.40

3. A merchant imports 75 cases of indigo, gross weight 196 lb. each: allowing 15 % for tare, what the duty at 5 cts. per lb.? *Ans.* $618.375

XVII. PARTNERSHIP.

ART. **252**. PARTNERSHIP is an association of persons for the transaction of business: such, is called a *firm* or *house;* and each member, a *partner*.

REVIEW.—251. What are the allowances for draft? What is tare? Gross weight? Net weight? Leakage? On what are duties computed?

The CAPITAL, or STOCK, is the amount of money or property contributed by the firm.

The DIVIDEND is the gain or loss shared among the partners.

1. A and B engaged in trade: A's capital was \$200; B's, \$300; they gained \$100: find each partner's share.

SOLUTION.—The whole capital is \$200+\$300=\$500. Of this A owns $\frac{200}{500}=\frac{2}{5}$, and, therefore, he should have $\frac{2}{5}$ of the gain: B owns $\frac{300}{500}=\frac{3}{5}$ of the capital, and should have $\frac{3}{5}$ of the gain.

Hence, A's gain will be $\frac{2}{5}$ of \$100=\$40. }
B's gain will be $\frac{3}{5}$ of \$100=\$60. } *Ans.*

*2. A and B form a partnership, with a capital of \$800: A's part is \$300; B's, \$500; they gain \$232: what the share of each? *Ans.* A's, \$87; B's, \$145.

TO FIND EACH PARTNER'S SHARE

Of the gain or loss, when each one's capital is used the same time.

Rule.—*Take such part of the whole gain or loss, as each partner's stock is part of the whole stock.*

Or, by PROPORTION. *As the whole stock is to each partner's stock, so is the whole gain or loss to each partner's gain or loss.*

PROOF.—Add together the several shares; if the work is correct, the sum will equal the whole gain or loss.

REM.—1. This rule is applicable, when required to divide a sum into parts having a given ratio to each other; as in Bankruptcy, General Average, &c.

2. Partnerships are generally governed by special agreements, which specify the method of dividing gains or losses.

3. A's stock was \$70; B's, \$150; C's, \$80; they gained \$120: what was each man's share of it?

Ans. A's, \$28; B's, \$60; C's, \$32.

4. A, B, and C traded together: A put in \$200; B, \$400; C, \$600: they gained \$427.26: find each man's share. *Ans.* A's, \$71.21; B's, \$142.42; C's, \$213.63

What was the gain %? *Ans.* 35.605 %.

5. Divide $90 among 3 persons, so that the parts shall be to each other as 1, 3, and 5. *Ans.* $10, $30, and $50.

6. Divide $735.93 among 4 men, in the ratio of 2, 3, 5, and 7. *Ans.* $86.58; $129.87; $216.45; $303.03

7. A person left an estate of $22361, to be divided among 6 children, in the ratio of their ages, which are 3, 6, 9, 11, 13, and 17 yr.: what are the shares?
Ans. $1137; $2274; $3411; $4169; $4927; $6443.

8. Divide $692.23 into 3 parts, that shall be to each other as $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{7}{8}$. *Ans.* $127.60; $229.68; $334.95

ART. 253. OF BANKRUPTCY.

A BANKRUPT is one who fails in business.

9. A man, failing, owes A $175; B, $500; C, $600; D, $210; E, $42.50; F, $20; G, $10; his property is worth $934.50: what will be each creditor's share?
Ans. A's, $105; C's, $360; E's, $25.50;
B's, $300; D's, $126; F's, $12.00; G's, $6.

NOTE.—Such questions may also be solved by finding what can be paid on $1, and multiplying this by each creditor's claim.

10. A man owes A $234; B, $175; C, $326: his property is worth $492.45: what can he pay on $1; and what will each creditor get? *Ans.* 67 cts. on $1;
A, $156.78; B, $117.25; C, $218.42

ART. 254. GENERAL AVERAGE

Is the method of apportioning among the owners of a ship and cargo, losses occasioned by casualties at sea.

11. A, B, and C freighted a ship with 108 tuns of wine. A owned 48, B 36, and C 24 tuns; they were obliged to cast 45 tuns overboard: how much of the loss must each sustain? *Ans.* A, 20; B, 15; C, 10 tuns.

REVIEW.—252. What is partnership? A firm? A partner? The Capital? Dividend? How find each partner's share of the gain or loss, Rule? How, by proportion? REM.—To what is this rule applicable?

12. From a ship valued at $10000, with a cargo valued at $15000, there was thrown overboard goods valued at $1125: what % was the general average, and what was the loss of A, whose goods were valued at $2150?

Ans. General average, $4\frac{1}{2}$ %; A's loss, $96.75

What the captain's loss, he owning $\frac{3}{8}$ of the ship?

Ans. $168.75

ART. 255. PARTNERSHIP WITH TIME.

1. A and B built a wall for $82; A had 4 men at work 5 days, and B, 3 men 7 days: how should they divide the money?

SOLU.—The work of 4 men 5 da. equals the work of 4×5, or 20 men 1 da.; and the work of 3 men 7 da., equals the work of 3×7, or 21 men 1 da.: it is then required to divide $82 into two parts having the same ratio to each other as 20 to 21.

Hence, A's part is $\frac{20}{41}$ of $82=$40.
B's part is $\frac{21}{41}$ of $82=$42. } *Ans.*

2. A put in trade $50 for 4 mon.; B, $60 for 5 mon.; they gained $24: what is each man's share?

SOLUTION.—$50 for 4 mon. equals $50×4=$200 for 1 mon.; and $60 for 5 mon. equals $60×5=$300 for 1 mon. Hence, divide $24 into two parts having the same ratio as 200 to 300.

This, (Art. 252), gives A $\frac{200}{500}=\frac{2}{5}$ of $24=$ 9.60
and B $\frac{300}{500}=\frac{3}{5}$ of $24=$14.40 } *Ans.*

Hence, to find each partner's share of the gain or loss, when *time* is regarded,

Multiply each partner's stock by the time it was employed; then take such part of the gain or loss as each partner's product is part of the sum of all the products.

OR, by PROPORTION. *Multiply each partner's stock by the time employed; then, as the sum of the products is to each partner's product, so is the whole gain or loss to each partner's share.*

REVIEW.—253. What is a bankrupt? NOTE. How may questions in bankruptcy be solved? 254. What is general average?
255. When time is regarded in partnership, how find each partner's share?

3. A and B hire a pasture for $54: A pastures 23 horses 27 da.; B, 21 horses 39 da.: what will each pay?

Ans. A, 23.28\frac{3}{4}$; B, 30.71\frac{1}{4}$

4. A put in $300 for 5 mon.; B, $400 for 8 mon.; C, $500 for 3 mon.: they lost $100: find each one's loss.

Ans. A's, 24.19\frac{11}{31}$; B's, 51.61\frac{9}{31}$; C's, 24.19\frac{11}{31}$

5. A, B, and C hire a pasture for $18.12: A pastures 6 cows 30 da.; B, 5 cows 40 da.; C, 8 cows 28 da.: what shall each pay? *Ans.* A, $5.40; B, $6; C, $6.72

6. Three men formed a partnership for 16 mon.: A put in at first $300, and at the end of 8 mon., $100 more; B put in at first $600, but, at the end of 10 mon., drew out $300; C put in at first $500, and, at the end of 12 mon., $400 more; they gained $759: find each man's share.

Ans. A's, $184.80; B's, $257.40; C's, $316.80

7. A and B are partners: A put in $800 for 12 mon., and B, $500. What sum must B put in at the end of 7 mon., to entitle him to half the yr.'s profits? *Ans.* $720.

☞ For additional problems, see Ray's Test Examples.

XVIII. EQUATION OF PAYMENTS.

ART. **256**. *Equation* or equality *of Payments* is the method of finding the *mean* or *average* time of making two or more payments, due at different times.

The rule for finding the mean or equated time, is based on the principle, that

The interest of any sum for any given period, is equal to the Int. of *half* the sum for *twice* the period; of *one-third* of the sum for *three* times the period, and so on. Thus,

The Int. of $2 for 1 mon. = Int. of $1 for 2 mon.
Int. of $4 for 5 mon. = Int. of $1 for 20 mon.

REVIEW.—256. What is Equation of Payments? On what principle is the rule for finding equated time based? Give examples.

EXAMPLE.—The Int. of \$4 for 5 mon., at 6 per cent., is 10 cents (Art. 224): the Int. of \$1 for 20 mon., is also 10 cents (Art. 223).

ART. **257.** 1. A owes B \$2 due in 3 mon., and \$4 due in 6 mon.: at what period can both sums be paid, neither party being the loser?

From Art. 255, it follows, that,

Int. of \$2 for 3 mon. = int. of \$1 for $2 \times 3 =$ 6 mon.
Int. of \$4 for 6 mon. = int. of \$1 for $4 \times 6 =$ 24 mon.
\$6 for - mon. = int. of \$1 for 30 mon.

Now find in what time \$6 will produce the same Int. as \$1 in 30 mon. At \$6 is 6 times \$1, it will produce the same Int. in $\frac{1}{6}$ of the time (Art. 256); that is, in 30 mon. $\div 6 = 5$ mon. *Ans.*

PROOF.—Int. of \$2 for 3 mon., at 6 %, $= 2 \times 1\frac{1}{2} =$ 3 cts.
Int. of \$4 for 6 mon., . . . $= 4 \times 3 =$ 12 cts.
Int. of \$6 for 5 mon., . . . $= 6 \times 2\frac{1}{2} =$ 15 cts.

*2. A owes B \$2 due in 4 mon., and \$6 due in 8 mon.: find the average time of paying both sums. *Ans.* 7 mon.

COMMON RULE FOR EQUATION OF PAYMENTS.

Multiply each payment by the time to elapse till it becomes due; divide the sum of the products by the sum of the payments; the quotient will be the equated time.

When one of the payments is due on the day from which the equated time is reckoned, its product is 0; but, in finding the sum of the payments, this must be added with the others. See Ex. 6.

3. A owes B \$8, due in 5 mon., and \$4 due in 8 mon.: find the mean time of payment. *Ans.* 6 mon.

4. A buys \$1500 worth of goods; \$250 are to be paid in 2 mon.; \$500 in 5 mon.; \$750 in 8 mon.: find the mean time of payment. *Ans.* 6 mon.

5. A owes B \$300; 1 third due in 6 mon.; 1 fourth

REVIEW.—257. What is the common rule for Equation of Payments? When one of the sums is to be paid down, how proceed?

in 8 mon.; the remainder in 12 mon.: what the average time of payment? *Ans.* 9 mon.

6. I buy $200 worth of goods; 1 fifth to be paid now; 2 fifths in 5 mon.; the rest in 10 mon.: what the average time of paying all? *Ans.* 6 mon.

ART. **258**. In finding the *Average* or *Mean* time for the payment of several sums due at different times, any date may be taken from which to reckon the time.

7. A merchant buys goods as follows, on 60 days credit: May 1st, 1848, $100; June 15th, $200: what the average time of payment? *Ans.* July 30th.

Counting from May 1st, it is 60 days to the first payment, and 105 days to the second.

$$\begin{array}{lr} \$100 \times \ \ 60 = & 6000 \\ \$200 \times 105 = & 21000 \\ \hline \$300 & 27000 \end{array}$$

$27000 \div 300 = 90.$

Ans. 90 days from May 1st, that is, July 30th.

Assuming April 1st as the day from which to count, the period is 120 days, which makes the same day of payment.

8. I bought goods on 90 days credit, as follows: April 2d, 1853, $200; June 1st, $300: what the average time of payment? *Ans.* Aug. 6th.

ART. **259**. The preceding rule, generally used, supposes *discount* and *interest paid in advance* to be equal; but this (Art. 239, Rem. 2) is not correct.

The following, based on *true* discount (Art. 235), is the

TRUE RULE FOR THE EQUATION OF PAYMENTS.

Find the present worth of each debt (Art. 237), *then find the* TIME (Art. 230), *at which the sum of the present worths will amount to the sum of the debts: this gives the true equated time.*

9. A owes $103 due in 6 mon., and $106 due in 12 mon.: find the true mean time of payment. *Ans.* 9 mon.

REVIEW.—258. To find the mean time, from what date do you reckon? 259. Is the common rule for Equation of Payments strictly accurate? What is the true rule for Equation of Payments?

XIX. ALLIGATION MEDIAL.

Art. **260**. Alligation medial is the method of finding the *mean* or *average* price of a mixture, when the ingredients composing it, and their prices, are known.

1. I mix 4 pounds of tea, worth 40 cts. a lb., with 6 lb. worth 50 cts. a lb.: what is 1 lb. of the mixture worth?

Solution.—4 lb. at 40 cts. per lb.=$1.60, and 6 lb. at 50 cts.=$3.00; making the total value of the 10 lb.= $4.60: hence, 1 lb. cost $\frac{1}{10}$ of $4.60, or 46 cts.

OPERATION.

4 lb. at 40 cts. cost	$1.60
6 lb. at 50 cts. cost	3.00
10 lb.	$4.60

$4.60 ÷ 10 = 46 cts., cost of 1 lb.

*2. Mix 6 lb. sugar, at 3 cts. a lb., with 4 lb. at 8 cts. a lb., what will 1 lb. of the mixture be worth? *Ans.* 5 cts.

Rule.—*Divide the whole cost by the whole number of ingredients; the quotient will be the average or mean price.*

Note.—The principles of this rule may be applied to the solution of many examples not embraced in the definition.

3. Mix 25 lb. sugar at 12 cts. a lb., 25 lb. at 18 cts., and 40 lb. at 25 cts.: what is 1 lb. of the mixture worth? *Ans.* $19\frac{4}{9}$ cts.

4. A mixes 3 gal. water, with 12 gal. wine, at 50 cts. a gal.: what is 1 gal. of the mixture worth? *Ans.* 40 cts.

5. I have 30 sheep: 10 are worth $3 each; 12, $4 each; the rest, $9 each: find the average value. *Ans.* $5.

6. On a certain day the mercury in the thermometer stood as follows: from 6 till 10 A. M. at 63°; from 10 A. M. till 1 P. M., 70°; from 1 till 3 P. M., 75°; from 3 till 7 P. M., 73°; from 7 P. M. till 6 A. M. of the next day, 55°. What was the mean temperature of the day, from sunrise to sunrise? *Ans.* $62\frac{7}{8}$.°

Multiply the number of hours by their mean temperature; divide the sum of the products by 24, the sum of the hours.

Review.—260. What is Alligation Medial? What the Rule?

XX. ANALYSIS.

ART. **261**. ANALYSIS is the separation of things into their elements or parts. In *Arithmetic*, it is the method of solution by reasoning according to the *nature* of the question, without reference to *special rules*.

EXAMPLES FOR MENTAL SOLUTION.

1. If 5 oranges cost 15 cts., what cost 4 oranges?

ANALYSIS.—1 orange is $\frac{1}{5}$ of 5 oranges, and will cost $\frac{1}{5}$ as much; $\frac{1}{5}$ of 15 cents is 3 cents, the cost of 1 orange; 4 oranges will cost 4 times as much as 1 orange; 4 times 3 cts.=12 cts. *Ans.*

Here, we first find the price of *one*, as it is easier to compute from the *value* of one, than from that of any other number.

2. If 5 sheep cost $20, what will 9 sheep cost?

3. If 5 bl. flour cost $40, what will 3 bl. cost?

4. If 3 lemons cost 12 cts., how many will 28 cts. buy?

ANALYSIS.—1 lemon is $\frac{1}{3}$ of 3 lemons, and will cost $\frac{1}{3}$ as much; but $\frac{1}{3}$ of 12 cents is 4 cents, the cost of 1 lemon. If 4 cents buy 1 lemon, 28 cents will buy as many lemons as 4 cents are contained times in 28 cents; that is, 7. *Ans.* 7 lemons.

5. If 5 barrels of flour cost $15, how many barrels of flour can be purchased for $21?

6. If 6 lb. of sugar cost 30 cts., how many pounds of sugar can be bought for 50 cts.?

7. If 7 yards of cloth cost $28, how many yards of cloth will $40 buy?

8. James had 28 cts., and spent $\frac{3}{7}$ of them for oranges at 2 cts. each: how many oranges did he buy?

ANALYSIS.—$\frac{1}{7}$ of 28 is 4, and $\frac{3}{7}$ are 3 times 4=12. If 2 cents buy 1 orange, 12 cents will buy as many oranges as 2 cents are contained times in 12 cents, that is 6. *Ans.* 6 oranges.

9. A man having $40, spent $\frac{3}{8}$ of it for cloth at $2 a yard: how many yards did he purchase?

10. $\frac{5}{6}$ of 48 are how many times 10?

REVIEW.—261. What is Analysis? What is it in Arithmetic?

11. $\frac{7}{9}$ of 45 are how many times 8?

12. I sold a watch for \$18, which was $\frac{3}{5}$ of its cost: how much did it cost, and what did I lose?

ANAL.—Since \$18 is 3 fifths of the cost, $\frac{1}{3}$ of 18 will be 1 fifth; and $\frac{1}{3}$ of 18 is 6. If 6 is 1 fifth, 5 fifths will be $5\times6=30$.

Hence, the cost is \$30, and \$30—\$18=\$12, the loss.

Here, we reason from *several* parts to *one* part; then from *one* part to the *required number*.

REM.—To avoid difficulty in analyzing examples of this kind, observe that the numerator of the fraction shows the *number* of parts taken. Art. 124. Thus, in the preceding example, 18 is *three* parts (fifths); and 1 part (fifth) is 1 third of 3 parts (fifths).

13. 8 is $\frac{2}{3}$ of what number? 12 is $\frac{3}{5}$ of what?

14. 16 is $\frac{4}{7}$ of what number? 15 is $\frac{5}{9}$ of what?

15. A farmer sold a wagon for \$45, which was $\frac{5}{7}$ of its cost; he paid for it with sheep at \$3 a head: how many sheep were required? *Ans.* 21.

16. 25 is $\frac{5}{8}$ of how many times 4?

17. 21 is $\frac{7}{11}$ of how many times 4?

18. $\frac{3}{4}$ of 40 is $\frac{5}{7}$ of what number?

ANALYSIS.—$\frac{1}{4}$ of 40 is 10, and $\frac{3}{4}$ is 3 times $10=30$. If 30 is $\frac{5}{7}$, $\frac{1}{5}$ of $30=6$, is $\frac{1}{7}$; and $\frac{7}{7}$, or the required number, are $7\times6=42$.

19. $\frac{2}{3}$ of 12 is $\frac{4}{9}$ of what number?

20. $\frac{8}{9}$ of 27 is $\frac{6}{7}$ of what number?

21. If $\frac{1}{3}$ bushel barley cost 20 cts., what cost $\frac{1}{4}$ bu.?

ANALYSIS.—If $\frac{1}{3}$ cost 20 cts., 1 bu. will cost 3 times as much, or 60 cts.; and $\frac{1}{4}$ bu. will cost $\frac{1}{4}$ of this, which is 15 cts. *Ans.*

22. If $\frac{2}{3}$ yd. muslin cost 24 cts., what cost $\frac{3}{4}$ yd.?

ANALYSIS.—$\frac{1}{3}$ yd. will cost $\frac{1}{2}$ as much as $\frac{2}{3}$; $\frac{1}{2}$ of 24 cts. is 12 cts., the cost of $\frac{1}{3}$ of a yd.; hence, 1 yd. will cost 3 times 12 cts., or 36 cts. If 1 yd. costs 36 cts., $\frac{1}{4}$ will cost 9 cts., and $\frac{3}{4}$ three times 9 cts., which are 27 cts. *Ans.*

Here, we reason from *several* parts to *one* part; then, from *one* part to the *whole;* then, from the *whole* to *a* part; lastly, from *one* part to *several* parts.

23. If $\frac{1}{4}$ yd. silk cost 20 cts., what cost $\frac{1}{5}$ yd.?
24. If $\frac{1}{5}$ bu. wheat cost 14 cts., what cost $\frac{9}{10}$ of a bu.?
25. If $\frac{3}{8}$ yd. linen cost 21 cts., what cost $\frac{6}{7}$ of a yd.?

ART. **262**. The solution of a question by analysis, is an *analytic solution.* Every operation in analysis depends directly on *elementary* or *self-evident* principles.

To solve questions *analytically*, determine the process to be pursued, by an examination of the *conditions* of the question, and the *relations* which the several quantities bear to each other.

In general, reason from a *given* number to *unity* (1), or *one part*, and then to the *required number.*

ART. **263**. PROMISCUOUS EXAMPLES.

1. If 15 bl. of flour cost \$40, what cost 24 bl.?

SOL.—If 15 bl. cost \$40, 1 bl. will cost $\frac{1}{15}$ of $\$40=\$2\frac{2}{3}$, and 24 bl. will cost 24 times as much as 1 bl.; $24\times\$2\frac{2}{3}=\$64$. *Ans.*

Or thus: Since 1 bl. is $\frac{1}{15}$ of 15 bl., 24 bl. are $\frac{24}{15}$ of 15 bl., and will cost $\frac{24}{15}$ as much. $\$40\times\frac{24}{15}=\$64$. *Ans.*

2. If 24 lb. of beef cost \$2.16, what will be the cost of 23 lb.? *Ans.* \$2.07

3. If 13 yd. of cloth cost \$32.50, what will be the cost of 14 yd.? *Ans.* \$35.

4. If \$8 will purchase 4 yd. of cloth, how many yards will \$24 buy? *Ans.* 12 yd.

5. If 159 yd. of muslin cost \$20.67, how many yards can be bought for \$34.71? *Ans.* 267 yd.

6. If 34 yd. cost \$147, what cost 9 yd.? *Ans* $\$38\frac{31}{34}$.

7. If 5 men perform a piece of work in 12 days, how many days will it take 3 men?

SOLUTION.—It will require 1 man 5 times as long as 5 men; that is, 5×12 days $=60$ da. Again, it will require 3 men $\frac{1}{3}$ as long as 1 man; $\frac{1}{3}$ of 60 days $=20$ days. *Ans.*

REVIEW.—261. What is Analysis? 262. On what does every operation in it directly depend? How ascertain the process to be pursued?

8. If 17 men can do a job of work in 25 days, in what time will 10 men do it? *Ans.* $42\frac{1}{2}$ da.

9. If 6 men can build a wall in 10 days, how many men can build it in 15 days? *Ans.* 4 men.

10. If 5 men consume a barrel of flour in 12 days, how long would it last 4 men? *Ans.* 15 da.

11. A man performs a piece of work in 7 days, working 10 hours a day: if he labors 12 hr. a day, how many days will be required? *Ans.* $5\frac{5}{6}$ da.

12. If 18 men can reap 72 acres of wheat in 7 days, how many days will 8 men require? *Ans.* $15\frac{3}{4}$ da.

13. If 5 hogs are worth 9 sheep, how many hogs will pay for 54 sheep? *Ans.* 30.

14. If 3 gal. wine are worth 7 gal. cider, how many gal. of cider are worth 42 gal. wine? *Ans.* 98 gal.

15. If a 3 cent loaf weigh 8 oz. when flour is $4 a bl., what should it weigh when flour is $5 a bl.? *Ans.* $6\frac{2}{5}$ oz.

16. If 3 stacks of hay will feed 12 horses 5 mon., how long will they feed 20 horses? *Ans.* 3 mon.

17. If $\frac{2}{5}$ of a yard of gingham cost 40 cents, find the cost of $\frac{3}{5}$ of a yard. *Ans.* 60 cts.

18. If $\frac{1}{3}$ of a yard of cloth cost $2, what will be the cost of $\frac{3}{4}$ of a yard? *Ans.* $4.50

19. If $\frac{5}{7}$ of a tun of hay cost $4.25, what will $\frac{11}{17}$ of a tun cost? *Ans.* $3.85

20. If $\frac{9}{11}$ C. of wood cost $2.52, what will $\frac{13}{14}$ of a C. cost? *Ans.* $2.86

21. If $\frac{3}{7}$ lb. of tea cost $\$\frac{1}{4}$, what cost $\frac{8}{9}$ lb.? *Ans.* $\$\frac{14}{27}$.

Examples in Art. 263 are often placed under Sim. Proportion. Solve, now, by *Analysis*, all the questions in Art. 203.

Art. **264**. 22. If 7 men eat 56 lb. of bread in 16 days, how many lb. will 21 men eat in 6 days?

Solution.—If 7 men eat 56 lb., 1 man will eat $\frac{1}{7}$ as much, which is 8 lb.; then, if 1 man in 16 days eat 8 lb., in 1 day he will eat $\frac{1}{16}$ of 8 lb. $=\frac{8}{16}=\frac{1}{2}$ lb. And, 21 men will eat $\frac{1}{2}\times21=10\frac{1}{2}$ lb. in 1 day, and in $10\frac{1}{2}\times6=63$ lb. *Ans.*

23. If 2 men earn \$16 in 4 days, how much will 7 men earn in 3 days? *Ans.* \$42.

24. If 2 men build 12 rods of a wall in 9 days, how many rods can 7 men build in 6 days? *Ans.* 28 rd.

25. If 15 oxen plow 11 A. in 5 da., how many oxen will plow 33 A. in 9 da.? *Ans.* 25 oxen.

Examples in Art. 264 are usually placed under Comp. Prop. Those in Art. 205 should now be solved analytically.

ART. **265**. 26. A cistern, of 250 gal., has two pipes; the first *fills* 41 gal. an hour, and the second *empties* 6 gal. an hour: if both pipes are open, how long will the cistern be in filling? *Ans.* 7 hr. 8 min. $34\frac{2}{7}$ sec.

27. A cistern of 600 gal. can be filled by pipe A, in 8 hours; and by pipe B, in 12 hours: in what time can it be filled by both pipes? *Ans.* $4\frac{4}{5}$ hr.

28. A cistern, of 900 gal., can be filled by pipe A, in 10 hr., and emptied by pipe B, in 12 hr.: in what time will it be filled, if both pipes are open? *Ans.* 60 hr.

29. A can do a job of work in 2 days, and B in 3 days: in what time can both do it, working together?

SOLUTION.—If A does it in 2 days, he does $\frac{1}{2}$ of it in 1 day: if B does it in 3 days, he does $\frac{1}{3}$ of it in 1 day; therefore, both do $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$ in one day, and the whole in $1\div\frac{5}{6}=1\frac{1}{5}$ da. *Ans.*

Or, Suppose the work to consist of any number of equal parts, say 6; then, if A does it in 2 days, he will do 3 parts in 1 da.; and if B does it in 3 da., he will do 2 parts in 1 da.; hence, both do $3+2=5$ parts in 1 da., and the whole in $6\div5=1\frac{1}{5}$ da.

30. A can perform a piece of work in 20 da., B, in 15 da., and C, in 12 da.: in what time can the three do it, working together? *Ans.* 5 da.

31. I hired 4 men to build a wall: A alone can do it in 12 da.; B, in 15 da.; C, in 18 da.; D, in 24 da.: how long will it take to do it, all working? *Ans.* $4\frac{4}{89}$ da.

32. A mows $\frac{1}{8}$ of a field in a day, and B $\frac{1}{9}$: in what time can both do it? *Ans.* $4\frac{4}{17}$ da.

33. A and B can mow a field in 12 da., and A alone in 20 da.: in what time can B mow it? *Ans.* 30 da.

34. A man and his wife ate a bu. of meal in 6 da.; when the man was absent, it lasted the woman 15 da.: how long would it last the man? *Ans.* 10 da.

35. A cistern has 3 pipes; the 1st will empty it in 4 min., the 2d in 8 min., the 3d in 15 min.: in what time will *all* empty it? *Ans.* 2 min. $15\frac{45}{53}$ sec.

36. A can mow $\frac{3}{5}$ of a meadow in 6 days, and B, $\frac{5}{7}$ in 10 da.: in what time can both mow it? *Ans.* $5\frac{5}{6}$ da.

37. Divide 35 cents between two boys, giving to one 9 more than the other. *Ans.* 13 and 22.

SUG.—Subtract 9 from 35, divide the remainder equally; then 9, added to one of the equal parts, will give the greater share.

38. Divide $3000 into two parts, one being $500 more than the other. *Ans.* $1250 and $1750.

39. A man left $3500 to his wife, son, and daughter; to the wife, $800 more than to the son; to the son, $300 more than to the daughter: find the share of each.

Ans. wife, $1800; son, $1000; daughter, $700.

40. The hour and minute hands of a watch are together at 12 o'clock: at what time are they next together?

SOL.—The min. hand moves over 60 min. while the hour hand moves over 5 min.; therefore, the min. hand moves over 12 min. while the hr. hand moves over 1 min. Hence, in moving over 12 min., the min. hand *gains* 11 min. on the hr. hand.

Now, 12 is $\frac{12}{11}$ of 11, that is, the *distance moved* over by the min. hand, is $\frac{12}{11}$ of the *distance gained.* When the min. hand is at 12, and the hr. hand at 1, the former must *gain* 5 min. to overtake the latter: $\frac{12}{11}$ of 5 min. $=\frac{60}{11}=5\frac{5}{11}$. *Ans.* $5\frac{5}{11}$ min. past 1.

41. At what time between 5 and 6 o'clock are the hr. and min. hands together? *Ans.* $27\frac{3}{11}$ min. after 5.

42. At what time between 8 and 9 o'clock, are they opposite to each other? *Ans.* $10\frac{10}{11}$ min. after 8.

43. A, B, and C are the partners: A put in $3276; the $\frac{5}{12}$ of what A put in was equal to $\frac{5}{9}$ of what B put in; and the difference between $\frac{11}{27}$ of what B put in, and the whole of what A put in, equaled $\frac{5}{4}$ of what C put in. They gained $7000; A received for his share a sum,

the $\frac{6}{5}$ of which equaled what he put in; C, a sum equal to $\frac{6}{5}$ of what he put in; and B the remainder. Find the amount B and C put in, and each one's share of the gain.

Ans. B put in $2457; C, $1820.
A's gain $2730; B's, $2086; C's, $2184.

Art. **266**. 44. A mixes sugar at 2 cts. per lb., with sugar at 5 cts. per lb., so that the mixture is worth 3 cts. per lb.: how much of each does he take?

Solution.—By taking 1 lb., at 2 cts., he *gains* 1 ct., and by taking 1 lb. at 5 cts., he *loses* 2 cts.; hence, that the gains and losses may be equal, he takes 1 lb. at 2 cts., and $\frac{1}{2}$ lb. at 5 cts., and in the same ratio for any quantity of the mixture; thus, 2 lb. at 2 cts., and 1 lb. at 5 cts.; 4 lb. at 2 cts., and 2 lb. at 5 cts., and so on, will make a mixture worth 3 cts. a lb. Art. 259.

45. In what ratio must I mix sugar at 4 cts. a lb., with sugar at 8 cts. a lb.; the mixture to be worth 5 cts. a lb.?

Ans. 3 lb. at 4 cts. to 1 lb. at 8 cts.

46. In what ratio must I mix sugar at 3 cts. a lb., with sugar at 8 cts. a lb.; the mixture to be worth 6 cts. a lb.?

Ans. 2 lb. at 3 cts. to 3 lb. at 8 cts.

47. How many pounds of tea, at 25 cts. per lb., must be mixed with 15 lb. at 30 cts. per lb.; the mixture to be worth 28 cts. per lb.?

Solution.—The ratio of the ingredients necessary for a mixture worth 28 cts. a lb., shows that for each $1\frac{1}{2}$ lb. at 30 cts., we must take 1 lb. at 25 cts. But 15 lb. $\div$ $1\frac{1}{2}$ lb. $=$ 10; hence, it will take 10 lb. at 25 cts.

48. How many pounds of sugar, at 8 cts. a lb., must I mix with 10 lb. at 11 cts. a lb., to make a mixture worth 10 cts. a lb.? *Ans.* 5 lb.

49. Mix two kinds of tea at 20 and 25 cts. a lb., to make a mixture of 25 lb. worth 24 cts. a lb.

Solution.—First find that 1 lb. at 20 cts. and 4 lb. at 25 cts. a lb., make a mixture worth 24 cts. a lb. Then (Art. 252), dividing 25 into two parts having the ratio of 1 to 4, will give 5 lb. at 20 cts. per lb., and 20 lb. at 25 cts. per lb.

The above examples are usually placed under a rule called *Alligation Alternate.* They properly belong to Algebra.

ART. **267**. 50. The sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of a certain number is 26: what is that number?

SOL.—The sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ is $\frac{13}{12}$; hence, $\frac{13}{12}$ of the number is 26, and $\frac{1}{12}$ is $\frac{1}{13}$ of $26=2$; and the number is $12\times2=24$. *Ans.*

51. One-third of a number exceeds $\frac{1}{4}$ of it by 8: find the number. *Ans.* 96.

52. Seven-tenths of a number exceeds $\frac{3}{5}$ of it by 7: what is that number? *Ans.* 70.

53. I spent $\frac{1}{3}$ and $\frac{1}{5}$ of my money, and had $35 left: what had I at first? *Ans.* $75.

54. What number, increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{5}$ of itself, gives the sum 73? *Ans.* 30.

55. A boy lost $\frac{4}{9}$ of his money, spent 20 cts., and had 15 cts. left: how much had he at first? *Ans.* 63 cts.

56. I spent $\frac{2}{3}$ of my money for books; $\frac{3}{5}$ of the rest for paper; I had 10 cts. left: what had I at first? *Ans.* 75 cts.

57. A received $2 for each day he worked, and lost $1 for each day idle; he worked 3 times as many days as he was idle; at the end of the time he received $25: how many days did he work? *Ans.* 15 da.

58. In an orchard, $\frac{1}{2}$ the trees bear apples; $\frac{1}{4}$, peaches; and $\frac{1}{5}$, cherries; the remaining 4, pears: how many trees in the orchard? *Ans.* 80.

59. A teacher, when asked the number of his pupils, replied, that if he had as many more, half as many more, and 1 fourth as many more as he now had, he would have 110: what the number of pupils? *Ans.* 40.

60. A traveler spent the 1st day, $\frac{1}{4}$ of his money; the 2d day, $\frac{1}{4}$ of the remainder, and so on, the 3d and 4th days, when he had $1.62 left: what sum had he at first? *Ans.* $5.12

61. A merchant increased his capital the first year by $\frac{1}{4}$ of itself; the 2d, he increased this sum by $\frac{3}{5}$ of itself; the 3d, he lost $\frac{1}{4}$ of all he had, which left him $3375. How much had he at first? *Ans.* $1875.

NOTE.—The examples in this article are sometimes placed under the rule of Position, now but little used, as Algebra is generally studied by the higher classes in common schools.

XXI. EXCHANGE OF CURRENCIES.

ART. **268**. EXCHANGE OR REDUCTION OF CURRENCIES, is the process of changing the currency of one country to that of another, without altering its value.

The *currency* of a country is its *money* or *circulating medium.*

ART. **269**. To change one currency to another, the different denominations in each must be known; also, the unit value of a denomination in one currency, in a denomination of the other.

TABLE OF ENGLISH, OR STERLING MONEY.

4 farthings (far.)	make	1 penny,	marked	d.
12 pence	..	1 shilling,	..	s.
20 shillings	..	1 pound, or sovereign	..	£.
21 shillings	..	1 guinea,	..	g.

NOTES.—1. Farthings are generally written as fractions of a penny. Thus, 1 far. is written $\frac{1}{4}$d., 2 far. $\frac{1}{2}$d., and 3 far. $\frac{3}{4}$d.

2. The present *legal* value of the pound sterling, according to act of Congress of 1842, is $4.84

The operations of Reduction, Addition, &c., of sterling money, are performed like those of other Compound Numbers.

1. Reduce £5 3s. $2\frac{1}{4}$d. to far. . . . *Ans.* 4953 far.
2. Reduce 8675 far. to £. . . . . . *Ans.* £9 $8\frac{3}{4}$d.
3. Add £3 $6\frac{1}{4}$d., £5 10s. $4\frac{1}{2}$d., £2 15s. $1\frac{1}{4}$d. *Ans.* £11 6s.
4. £17 6s. 5d.,—£8 5s. $11\frac{1}{2}$d. . . . *Ans.* £9 $5\frac{1}{2}$d.
5. Multiply £3 12s. $2\frac{1}{2}$d. by 8. *Ans.* £28 17s. 8d.
6. £25 $10\frac{1}{2}$d. ÷ 6, = what? *Ans.* £4 3s. $5\frac{3}{4}$d.
7. Find the value of £.625 (Art. 188.) *Ans.* 12s. 6d.

REVIEW.—268. What is Exchange of currencies? What is the currency of a country? 269. What must be known to change one currency to another? Repeat the table.

269. NOTE. How are farthings written? What is the legal value of the pound? How are the operations of reduction, addition, &c., of sterling money performed?

8. The value of .796875 of a £. *Ans.* 15s. $11\frac{1}{4}$d.

9. Reduce 7s. 6d. to the decimal of a £. *Ans.* .375

10. 8s. 9d. to the decimal of a £. (Art. 189.) *Ans.* .4375

ART. **270**. To compute Int. in pounds, shillings, &c.

Reduce the given shillings, pence, and farthings, to the decimal of a pound (Art. 189); *find the interest as in dollars and cents* (Art. 222); *reduce the resulting decimal figures to shillings, pence, and farthings* (Art. 188).

11. What is the interest of £75 10s. for 2 yr. 6 mon., at 4 %? *Ans.* £7 11s.

12. Of £85 12s. 6d. for 1 yr. 9 mon., at 6 %? *Ans.* £8 19s. $9\frac{3}{4}$d.

ART. **271**. 1. Reduce £12 sterling to U. S. Money.

SOLUTION.—Since £1 is worth \$4.84, £12 are worth 12 times as much; and \$4.84 × 12 = \$58.08 *Ans.*

2. Reduce £5 6s. 3d. to U. S. Money.

SUGGESTION.—Either reduce the shillings, pence, and farthings, to the decimal of a pound, and multiply \$4.84 by the result; or, multiply \$4.84 by the pounds, and find the value of the lower denominations, by taking aliquot parts (Art. 208).

OPERATION.

£5 6s. 3d. = £5.3125
£1 = \$4.84

Ans. \$25.7125

3. Reduce \$40.535 to sterling money.

SOLUTION.—Since £1 is \$4.84, there will be as many pounds in \$40.535 as \$4.84 is contained times in \$40.535

\$40.535 ÷ \$4.84 = 8.375, and £8.375 = £8 7s. 6d. *Ans.*

RULES.

1. TO REDUCE STERLING TO U. S. MONEY.—*Express sterling money in pounds and decimals of a pound: multiply this by the value of* £1, (\$4.84), *the product will be the value in dollars.*

Or, by PROPORTION. *As* £1 *is to the given sum, so is* \$4.84 *to the value of the given sum in dollars.*

2. TO REDUCE U. S. TO STERLING MONEY—*Divide the given*

sum by the value of £1, ($4.84), *the quotient will be the value in pounds and decimals of a pound.*

4. Reduce £25 to U. S. money. *Ans.* $121.
5. £15 8s. to U. S. money. *Ans.* $74.536
6. £36 15s. 9d. to U. S. money. *Ans.* $178.05+
7. $179.08 to sterling money. *Ans.* £37.
8. $124.388 to sterling money. *Ans.* £25 14s.
9. In $1000, how many £? *Ans.* £206 12s. $2\frac{3}{4}$d.+

NOTE.—The law of Congress, of July 31st, 1789, fixed the value of the pound sterling at $\$4\frac{4}{9}$, or $\$4.44\frac{4}{9}$, and $1. at 4s. 6d.

As the then *legal* or *nominal* value of the pound was below its real value, such a per cent. was afterward added to the nominal as was necessary to make it express the real value.

As it requires nearly 9 per cent. of $\$4.44\frac{4}{9}$ to be added to it, to make $4.84, when sterling funds or bills are estimated at $\$4\frac{4}{9}$ to a pound, and are 9 per cent. premium, they are really only at par.

ART. **272**. In buying or selling exchange on England, it is still customary to regard the pound as $\$4\frac{4}{9}$, and then to add the % premium.

1. What must be paid for a bill of exchange on London of £200, at 9 % premium?

SOLUTION.—$£200 \times 4\frac{4}{9} = \$888.88\frac{8}{9}$; $\$888.88\frac{8}{9} \times .09 = \$80$ premium; $\$888.88\frac{8}{9} + \$80 = \$968.88\frac{8}{9}$ *Ans.*

2. What for a bill of exchange on Liverpool of £150, at 8 % premium? *Ans.* $720.

3. What for a bill of exchange on London of £80 10s., at $9\frac{1}{2}$ % premium? *Ans.* $391.76+

ART. **273**. Previous to the adoption of Federal or U. S. Money, in 1786, accounts were kept in pounds, shillings, pence, and farthings.

Owing to the fact that the Colonies had issued *bills of credit,*

REVIEW.—270. How compute interest on sterling money? 271. How reduce sterling to U. S. Money? How reduce U. S. to sterling money?

271. NOTE. What was the legal value of the pound sterling in 1789? How was the real value found from this?

which *depreciated* more or less, the value of a *colonial* £ was less than that of a £ *sterling*.

The depreciation being greater in some colonies than in others, gave rise to the different State currencies. Thus,

In New England, Va., Ky., and Tenn.,	6s.	or £$\frac{3}{10}$=\$1.
In New York, Ohio, and N. Carolina,	8s.	or £$\frac{2}{5}$=\$1.
In New Jersey, Pa., Del., and Md.,	7s. 6d.	or £$\frac{3}{8}$=\$1.
In South Carolina and Georgia,	4s. 8d.	or £$\frac{7}{30}$=\$1.
In Canada and Nova Scotia,	5s.	or £$\frac{1}{4}$=\$1.

The process of changing any sum of U. S. Money to either of these currencies, or the reverse, involves the same principles as the exchanging of sterling money.

Hence, to reduce U. S. Money to the currency of a State,

Multiply the given sum, expressed in dollars, by the value of \$1 *expressed in the fraction of a pound; the product will be the value in pounds and decimals of a pound.*

To reduce a State currency to U. S. Money,

Express the given sum in pounds and decimals of a pound, then divide this by the value of \$1 *expressed in the fraction of a pound; the quotient will be the value in dollars.*

	ANSWERS.
1. Reduce \$120.50 to N. Eng. currency.	£36 3s.
2. \$75.25 to N. York currency. . . .	£30 2s.
3. \$98 to Penn. currency.	£36 15s.
4. £30 15s. N. E. currency to dollars. .	\$102.50
5. £25 17s. N. Y. currency to dollars. .	\$64.625
6. £29 8s. Georgia currency to dollars.	\$126.00

NOTE.—Any sum, in one currency, may be changed to that of another, by Sim. Proportion (Art. 203); or by short methods.

Thus, since 6 shillings New England currency are equal to 8 shillings New York currency, to change the former to the latter, ADD *one-third of the sum*; to change the latter to the former, SUBTRACT *one-fourth of the sum.*

REVIEW.—272. In buying or selling bills of exchange, how make the calculations? 273. In what were accounts kept previous to 1786?

Art. **274**. Any currency may be reduced to U. S. Money, or U. S. Money to any currency, by *multiplication* or *division*, as in Art. 271, when the value of a unit of the foreign currency expressed in U. S. Money is known.

The unit of French money is the franc, its value being $0.186 Bills of exchange on France are bought and sold at a certain number of francs to the dollar.

EXAMPLE.—At $1 for 5.30 francs, what will be the cost of a Bill on Paris for 1166 francs? *Ans.* $220.

In "*Ray's Higher Arithmetic*" may be found a complete table of all foreign coins, with their value in U. S. Money; also, valuable information respecting exchange with all civilized nations.

XXII. DUODECIMALS.

Art. **275**. Duodecimals are a peculiar order of fractions, which *increase* and *decrease* in a *twelve-fold ratio.*

Their name, from the Latin *duodecim*, signifies *twelve.*

The unit, 1 foot, is divided into 12 equal parts, called inches, or *primes*, marked thus, (′).

Each inch, or prime, is divided into 12 equal parts, called *seconds*, marked (″).

Each second, into 12 equal parts, called *thirds*, (‴).

Each third, into 12 equal parts, called *fourths*, (⁗).

Hence, 1′ inch, or prime, . . . $= \frac{1}{12}$ of a foot.

1″ second is $\frac{1}{12}$ of $\frac{1}{12}$ . . $= \frac{1}{144}$ of a foot.

1‴ third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$. $= \frac{1}{1728}$ of a foot.

TABLE.

12 fourths (⁗)	make	1 third, ...	marked ‴.
12 thirds	..	1 second, ...	.. ″.
12 seconds	..	1 inch or prime,	.. ′.
12 inches or primes .	..	1 foot, ...	.. ft.

The marks ′, ″, ‴, ⁗, called *indices*, show the different parts.

Duodecimals are added and subtracted like Compound Numbers; 12 units of each order making a unit of the next higher.

ART. 276. MULTIPLICATION.

Duodecimals are used in the measurement of *surfaces* and *solids*, as boards, solid walls, &c.

1. Find the superficial contents of a board 7 ft. 5 in. long, and 4 ft. 3 in. wide.

Length, multiplied by breadth, gives the superficial contents.

OPERATION.

	ft.	′	
	7	5	
	4	3	
	29	8	
	1	10	3″
Ans.	31	6	3

SOLUTION.—$5' = \frac{5}{12}$ of a foot; therefore, $5' \times 4$ ft. $= \frac{5}{12} \times 4 = \frac{20}{12} = 20$ inches, which is 1 ft. 8 in.; write the inches or primes in the order of primes, and carry the 1 ft.

Next, 7 ft. $\times$ 4 ft. $= 28$ ft., to which add the 1 ft. carried, the sum is 29 ft.; which write in the order of feet.

Again, $5' = \frac{5}{12}$, and $3' = \frac{3}{12}$; therefore, $5' \times 3' = \frac{5}{12} \times \frac{3}{12} = \frac{15}{144} = 15''$, which is $1'\ 3''$; write the 3 sec. in the order of seconds, and carry the 1 in.

Next, 7 ft. $\times$ 3 in. $= 7 \times \frac{3}{12} = \frac{21}{12} = 21'$, and $1'$ carried, make $22' = 1$ ft. $10'$. Writing these in their orders, and adding the two products, the entire product is 31 ft. $6'\ 3''$.

The product of any two denominations, is of that denomination denoted by the sum of their indices; thus, $3' \times 5' = 15''$, $3' \times 7 = 21'$, $7'' \times 4' = \frac{7}{144} \times \frac{4}{12} = \frac{28}{1728} = 28'''$. Hence,

Feet	multiplied by feet,	give (square) feet.
Feet	multiplied by inches,	give inches.
Inches	multiplied by inches,	give seconds.
Inches	multiplied by seconds,	give thirds.
Seconds	multiplied by seconds,	give fourths, and so on.

REVIEW.—273. Why was the value of a colonial pound less than that of a pound sterling? How reduce U. S. Money to a State currency?

273. How reduce a State currency to U. S. Money? 275. What are duodecimals? Whence their name? What are Primes? Seconds? Thirds? Fourths? Repeat the table. What are indices? What part of a foot is $1'$? $1''$? $1'''$? $1''''$? How are duodecimals added and subtracted?

Rule for Multiplication.—1. *Write the multiplier under the multiplicand, placing units of the same order under each other.*

2. *Multiply, first by the feet, next by the inches, and so on, recollecting that the product will be of that denomination denoted by the sum of their indices.*

3. *Add the several partial products together, and their sum will be the required product.*

The primes of the product of *two* duodecimal factors, are neither *linear* nor *square* in., but *twelfths* of a sq. ft. The primes of the product of *three* duodecimal factors, are *twelfths* of a cu. ft.

2. How many square feet in a board 5 ft. 3 in. long, and 1 ft. 5 in. wide? *Ans.* 7 sq. ft. 5′ 3″.

3. Multiply 5 ft. 7 in. by 1 ft. 10 in. *Ans.* 10 sq. ft. 2′ 10″.

4. 8 ft. 6 in. 9″ × 7 ft. 3 in. *Ans.* 62 sq. ft. 11″ 3‴.

5. 8 ft. 4 in. 6″ × 2 ft. 7 in. 4″. *Ans.* 21 sq. ft. 10′ 5″.

6. 4 ft. 5′ 6″ × 2 ft. 3′ 5″. *Ans.* 10 sq. ft. 2′ 2″ 9‴ 6⁗.

Another method of solution found in "*Ray's Higher Arithmetic.*"

XXIII. INVOLUTION.

ART. **277.** INVOLUTION is the multiplication of a number by *itself* one or more times.

A POWER is the product obtained by involution.

The ROOT, or *first power*, is the number multiplied.

If the number be taken *twice* as a factor, the *product* is the *second power*; $3 \times 3 = 9$, is the 2d power of 3.

If the number be taken 3 *times* as a factor, the product is the 3d *power*; $2 \times 2 \times 2 = 8$, is the 3d power of 2.

REVIEW.—276. For what are duodecimals used? Of what denomination is the product of any two denominations? What is the product of feet by feet? Feet by inches? Inches by inches? Inches by seconds? Seconds by seconds? Rule for multiplication? What do the primes of the product of two duodecimal factors represent? Of three?

And, if taken 4 times as a factor, the product is the 4th power; if 5 times, the product is the 5th power, and so on.

Hence, *the different powers derive their name from the number of times the root is taken as a factor.*

REM.—The given number is called the *root*, the different powers of the number being *derived* from it.

ART. **278.** The *second* power of a number is called the *square;* the *third* power the *cube.* These terms are derived thus:

ILLUSTRATIONS. 1. Take a line, say 3 feet long, its *first* power is the line itself.

2. If 3 feet be multiplied by itself, the product (Art. 87) will be $3\times3=9$ *square* feet. (See diagram of 3 feet square, page 91.) But $3\times3=9$ is the *second* power of 3; hence, the 2d power is called the *square.*

3. If each side of a *cube* is 3 feet, the cube (Art. 92) contains $3\times3\times3=27$ *cubic* ft. (See diagram, p. 94.) But $3\times3\times3=27$, is the *third* power of 3; hence, the 3d power is called the *cube.*

ART. **279.** The *number* denoting the power to which the root is to be raised, is the *index* or *exponent* of the power. It is placed on the right, a little higher than the root. Thus,

$2^1=2$, the 1st power of 2.
$2^2=2\times2=4$, the 2d power or square of 2.
$2^3=2\times2\times2=8$, the 3d power, or cube of 2.
$2^4=2\times2\times2\times2=16$, the 4th power of 2, &c.

To find the *second* power of 2, use it as a factor *twice;* thus, $2\times2=4$. To find the *third* power of 2, use it *three* times; thus, $2\times2\times2=8$, and so on.

REVIEW.—277. What is involution? What is a power? What is the root, or 1st power? The 2d power? The 3d? The 4th?

277. From what do the different powers derive their name? REM. Why is the given number called the root? 278. What is the second power of a number called? What the 3d? How are these terms derived? 279. What is the index of a power? How find the 2d power of 2? The 3d? The 4th?

Rule for Involution.—*Multiply the number by itself, till it is used as a factor as many times as there are units in the index of the power.*

EXAMPLES FOR MENTAL SOLUTION.

1. What is the square of 1? of 2? of 3? of 4? of 5? of 6? of 7? of 8? of 9? of 10? of 11?
2. What is the cube of 1? of 2? of 3? of 4?
3. What is the square of $\frac{1}{2}$? of $\frac{1}{3}$? of $\frac{2}{3}$? of $\frac{1}{4}$?
4. What is the cube of $\frac{1}{2}$? of $\frac{1}{3}$? of $\frac{2}{3}$? of $\frac{3}{4}$?
5. What is the fourth power of 2? fifth power of 2? fourth power of 3?

SLATE EXERCISES.

6. What is the 2d power or sq. of 65? *Ans.* 4225.
7. The third power or cube of 25? *Ans.* 15625.
8. The square of $16\frac{1}{2}$? . . . . . . *Ans.* $272\frac{1}{4}$.
9. The cube of $12\frac{1}{2}$? . . . . . . *Ans.* $1953\frac{1}{8}$.
10. The fourth power of 13? . . . *Ans.* 28561.
11. The fifth power of $\frac{2}{3}$? . . . . *Ans.* $\frac{32}{243}$.
12. The sixth power of 9? . . . . *Ans.* 531441.
13. The 4th power of .025? *Ans.* .000000390625
14. The value of 14^3? . . . . . . *Ans.* 2744.
15. The value of 19^4? . . . . . . *Ans.* 130321.
16. The value of $2\frac{1}{3}^5$? . . . . . . *Ans.* $69\frac{40}{243}$.
17. The value of $.09^6$? . . *Ans.* .000000531441

XXIV. EVOLUTION.

ART. **280**. EVOLUTION, or the *extraction of roots*, is the process of resolving numbers into *equal* factors.

When a number is resolved into *equal* factors, each factor is a ROOT of the number.

Hence, a *root* is a *factor* which, multiplied by itself *a certain number of times*, will produce the given number.

One of the *two* equal factors of a number, is the *second* root, or *square* root of that number. Thus,

$9=3\times3$; three being the *square* root of 9.

One of the *three* equal factors of a number, is the *third*, or *cube* root of that number. Thus,

$8=2\times2\times2$; two being the *cube* root of 8.

Also, one of the *four*, *five*, &c., equal factors of a number is the *fourth*, *fifth*, &c., root of that number.

Hence, the *name* of the root shows the *number of equal factors* into which the given number is resolved. Thus.

The *square* root of 25 is 5, as $5\times5=25$.
The *cube* root of 27 is 3, as $3\times3\times3=27$.
The *fourth* root of 16 is 2, as $2\times2\times2\times2=16$, &c.

Evolution is the *reverse* of Involution. In Involution, the *root* is given to find the *power;* in Evolution, the *power*, to find the *root.*

When one number is a *power* of another, the latter is a *root* of the former: thus, 8 is the *cube* of 2, and 2 is the *cube root* of 8.

Art. 281. Roots are Denoted in Two Ways:

1st. By $\sqrt{}$ called the *radical sign*, placed before the number.
2d. By a *fractional index* placed on the right of the number.

Thus, $\sqrt{4}$, or $4^{\frac{1}{2}}$, denotes the *square* root of 4.

And, $\sqrt[3]{27}$, or $27^{\frac{1}{3}}$, the *cube* root of 27.

Note.—The figure over the radical sign, denotes the *name* of the root. When the sign has no figure over it, 2 is understood; thus, $\sqrt[2]{25}$ and $\sqrt{25}$, each denotes the square root of 25. The *denominator* of the fractional index denotes the name of the root.

Art. 282. Any number whose *exact* root can be obtained, is a perfect power: as, 4, 9, 16, &c.

Review.—279. What is the rule for involution? 280. What is evolution? What is a root of a number? What the second or square root? What the third, or cube root? What does the name of the root show? Give examples. Rem. Why is evolution the reverse of involution?

281. How are roots denoted? Give examples. Note. What does the figure over the radical sign denote? What the denominator of the fractional index? 282. What is a perfect power? Give examples.

ART. **283**. Every root, and every power of 1, is 1.

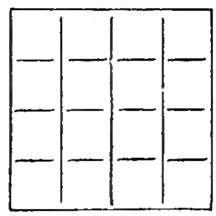
Thus, $\sqrt{1}$, $\sqrt[3]{1}$, $\sqrt[4]{1}$, and $(1)^2$, $(1)^3$, $(1)^4$, each $=1$.

EXTRACTION OF THE SQUARE ROOT.

ART. **284**. To extract the square root of a number, is to resolve it into TWO equal factors (Art. 280); or, to find a number, which, multiplied by itself, will produce the given number.

The extraction of the square root (Art. 278) is also the method of finding the number of units in the side of a square, when its superficial contents are known; or, knowing the superficial contents, it shows how to arrange them so as to form the largest square possible.

4 in. long.



4 in. wide.

$4 \times 4 = 16.$

EXAMPLE.—What is the side of a square board which contains 16 square inches?

SOLUTION.—Since $16 = 4 \times 4$, each side is 4 inches.

ART. **285**. THE FIRST TEN NUMBERS AND THEIR SQUARES ARE

Numbers.	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Squares.	1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

The numbers in the 1st line are the square roots of those in the 2d.

Since the sq. root of 1 is 1, and of 100 is 10, the sq. root of any number *less* than 100 consists of *one* figure.

That is, the square root of a number of fewer than *three* figures, must consist of only *one* figure.

Again, take the numbers 10, 20, 30, 40, &c., to 100.

Their squares are 100, 400, 900, 1600, &c., to 10000.

Since the square root of 100 is 10, and of 10000 is 100, the square root of any number greater than 100, and less than 10000, will consist of *two* figures.

REV.—283. What are the different roots and powers of 1? 284. What is it to extract the square root of a number? What else may it be considered? What is the side of a square containing 9 sq. in.? 25 sq. in.? 285. What the rule for pointing? Why?

The square root of a number of more than *two* figures and fewer than *five*, must consist of *two* figures.

Also, the square root of a number of more than *four* figures and less than *seven*, must consist of *three* figures.

Hence, if a dot (.) be placed over every alternate figure, beginning with units, the number of dots will be the number of figures in the root. This is the RULE FOR POINTING.

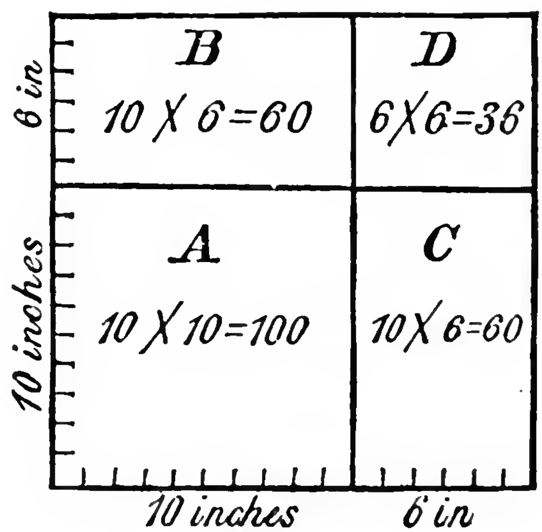
ART. **286.** 1. Extract the square root of 256; or, what is the same, arrange 256 sq. in. in the form of a square.

OPERATION.

```
       2̇56̇(10+6
       100    =16
    10|156       Ans.
     2|156
    20|
     6|
    26|
```

SOL.—To ascertain the number of figures in the root, begin at the unit's place, and place a dot over each alternate figure. This shows that the root consists of *two* figures.

Next find that the largest square in 2 (hundred) is 1 (hundred), the sq. root of which is 1 (ten), which put on the right, as in writing the quotient in division. Subtract the 100 from the given number, and 156 remain. While solving this example by figures, attend to arranging the squares.



After finding that the sq. root of the given number will contain two places of figures, (tens and units) and that the figure in tens' place is 1 (ten); form a square figure, A, 10 in. on each side, which contains (Art. 87) 100 sq. in.; taking this sum from the whole number of squares, 156 sq. in. remain, which correspond to the number, 156, left after subtracting above.

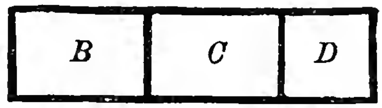
Without the diagram, it would be difficult to tell what operation to perform on the 156 to obtain the other figure of the root.

By examining the figure A, it is obvious, that to increase it, and at the same time preserve it a square, both length and breadth must be *increased equally*; and since each side is 10 in. long, it will take *twice* 10, that is, 20in. to encompass two sides of the square A. For this reason, 10 is doubled in the numerical operation.

Now determine the breadth of the addition to be made to each side of the square A.

By examining the figure, we see that after increasing each side equally, it will require a small square, D, of the *same breadth* as each of the figures B and C, to complete the entire square.

Hence, the superficial contents of B, C, and D, must be equal to the remainder (156); now the contents of B, C, and D, are obtained by multiplying their whole length by the breadth of D; this is made clearer by examining the figure in the margin.



It is now obvious that the figure (6) in the units' place, that is, the breadth of B and C, must be found by *trial*, and that it will be somewhat less than the number of times the length of B and C (20) is contained in the remainder (156). 20 is contained in 156 more than 7 times; let us try 7:—7 added to 20 makes 27 for the whole length of B, C, and D, and this, multiplied by 7, (Art. 88), gives 189 for the superficial contents of B, C, and D; which being more than 156, the breadth (7) was taken too great.

Next, take 6 for the length and breadth of D, and adding this to 20, gives 26 for the length of B, C, and D; multiplying this by the breadth (6) gives 156 for the superficial contents of B, C, and D.

Hence, the square root of 256 is 16; or, when 256 sq. in. are arranged in the form of a square, each side is 16 inches.

In performing the operation numerically as in the margin, it is not necessary to place a cipher on the right of the first figure of the root, either before or after doubling it, since that place is to be filled by the next figure of the root.

OPERATION.

$$\begin{array}{r l} & \dot{2}5\dot{6}(16 = \text{root.} \\ & 1 \\ \hline 26) & 156 \\ & 156 \end{array}$$

A	contains	10×10=100	sq. in.
B	..	10× 6= 60	..
C	..	10× 6= 60	..
D	..	6× 6= 36	..
		16×16=256	..

The superficial contents of the several parts of the figure added together, are equal to those of the whole found by squaring one side.

Another explanation of this subject is in "*Ray's Higher Arithmetic.*"

*2. Find the square root of 529. *Ans.* 23.

3. Find the square root of 56644.

OPERATION.

```
5̇66̇44̇(238.
4
43)166
   129
468)3744
    3744
```

Sol.—By the *Rule for Pointing*, (Art. 285), there are *three* periods; hence, the root will consist of *three* places of figures.

In performing the operation, 1st find the sq. root of 566 as in example 2d. Next consider 23 as so many *tens*, and find the last figure (8) as the figure 3 was found.

4. The sq. root of 915.0625

OPERATION.

```
9̇15̇.0̇62̇5(30.25
9
602)1506
    1204
6045)30225
     30225
```

Sol.—The product of two decimals will contain as many decimal places as there are decimals in both factors (Art. 187); if the root has *one* decimal place, its square will have *two*; if the root has *two* decimal places, its square will have *four*.

Hence, *in pointing a decimal number to extract its square root, put a dot over each alternate place, beginning with tenths.*

In solving this example, the 6 (tens) are not contained in 15; therefore place a cipher in the root and bring down another period. Hence, the square root of decimals is extracted in the same manner as that of whole numbers.

5. What is the square root of $\frac{4}{9}$?

Solution.—Since $\frac{4}{9}=\frac{2}{3}\times\frac{2}{3}$, the square root of $\frac{4}{9}$ is $\frac{2}{3}$.

Hence, *the square root of a common fraction is equal to the square root of its numerator divided by the square root of its denominator.*

Art. 287. TO EXTRACT THE SQUARE ROOT,

Rule.—1. *Separate the given number into periods of* TWO *places each, by placing a dot over the alternate figures, beginning with units.* (The left period will often have but one figure.)

Review.—286. How find the first figure of the root? Why double the first figure for a trial divisor? How find the second figure of the root?

286. Why add this to the trial divisor to get the complete divisor? How point a decimal number to extract its square root? Why?

2. *Find the greatest square in the left period, and place its root on the right, like a quotient in division. Subtract the square of this root from the left period, and to the remainder bring down the next period for a dividend.*

3. *Double the root found, and place it on the left for a trial divisor. Find how many times the divisor is contained in the dividend, exclusive of the right hand figure, and place the quotient in the root and also on the right of the divisor.*

4. *Multiply the divisor thus increased, by the last figure of the root; subtract the product from the dividend; to the remainder annex the next period for a new dividend.*

5. *Double the whole root found for a* NEW *trial divisor, and continue the operation as before, until all the periods are used.*

NOTES.—1. If any trial divisor is not contained in its dividend, place a cipher in the quotient, and bring down another period.

2. After bringing down the last period, and finding the figure of the root corresponding to it, if there is a remainder, *periods of ciphers* may be annexed, and the operation continued at pleasure.

3. To extract the sq. root of a common fraction reduce it to its lowest terms (Art. 138), then extract the sq. root of the numerator and denominator. If both terms are not perfect squares, reduce it to a decimal, then extract the sq. root.

To extract the sq. root of a mixed number, *first* reduce it to an improper fraction; or, reduce the fractional part to a decimal.

What is the Square Root of,

6.	625?	*Ans.* 25.	16.	.0196?	*Ans.* .14
7.	6561?	81.	17.	1.008016?	1.004
8.	390625?	625.	18.	.00822649?	.0907
9.	1679616?	1296.	19.	$\frac{25}{729}$?	$\frac{5}{27}$.
10.	5764801?	2401.	20.	$\frac{847}{1183}$?	$\frac{11}{13}$.
11.	43046721?	6561.	21.	$30\frac{1}{4}$?	$5\frac{1}{2}$.
12.	987656329?	31427.	22.	10?	3.162277+
13.	289442169?	17013.	23.	2?	1.41421+
14.	234.09?	15.3	24.	$\frac{2}{3}$?	.81649+
15.	145.2025?	12.05	25.	$6\frac{2}{5}$?	2.5298+

26. How many rods on each side of a square field of 6241 sq. rd.? *Ans.* 79 rds.

THE SQUARE ROOT BY FACTORING.

ART. **288.** Since any square is the product of two equal factors, if a *perfect* square be separated into its prime factors (Art. 113), its square root will be composed of half the equal factors. Thus,

$$441=3\times3\times7\times7;\ \text{hence}\ \sqrt{441}=3\times7=21.$$

Therefore, to obtain the square root of any perfect square, resolve the number into its prime factors, and take the product of one of each two equal factors.

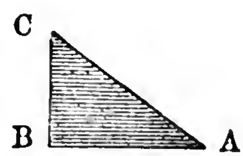
By Factoring, find the Square Root

1.	Of 16.	*Ans.* 4.	5.	Of 16×25.	*Ans.* 20.
2.	36.	*Ans.* 6.	6.	36×49.	*Ans.* 42.
3.	100.	*Ans.* 10.	7.	64×81.	*Ans.* 72.
4.	225.	*Ans.* 15.	8.	121×25.	*Ans.* 55.

ART. 289. APPLICATIONS OF THE SQUARE ROOT.

DEFINITIONS.—*A triangle* is a figure bounded by three straight lines. When one side is perpendicular to another, the angle between them is a *right angle*, and the triangle is a *right-angled triangle.*

The side *opposite* the right angle is called the *hypotenuse;* the other two sides, the *base* and *perpendicular.*



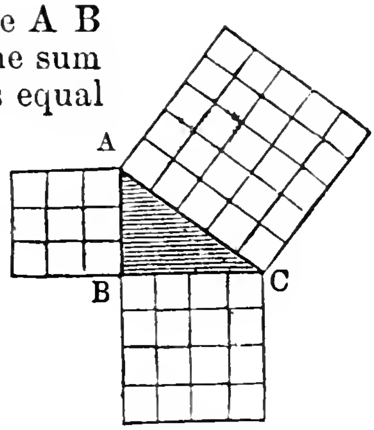
Thus, A B C is a right-angled triangle; A B being the base, B C the perpendicular, and A C the hypotenuse.

ART. **290.** It is a known principle, that *the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

REVIEW.—287. What is the rule for square root? NOTES. How proceed when any trial divisor is not contained in the corresponding dividend?

287. When there is a remainder after bringing down the last period, how proceed? How extract the square root of a common fraction? How, if both terms are not perfect squares? How, if a mixed number?

Thus, in the triangle A B C, the side A B is 3 feet; B C, 4 feet; and A C, 5 feet; the sum of the squares of A B and B C, 9+16, is equal to 25, the square of the side A C.



RULE FOR FINDING THE HYPOTENUSE WHEN THE BASE AND PERPENDICULAR ARE KNOWN.

To the square of the base, add the square of the perpendicular; the square root of the sum will give the hypotenuse.

RULE FOR FINDING EITHER SIDE WHEN THE HYPOTENUSE AND THE OTHER SIDE ARE KNOWN.

From the square of the hypotenuse, subtract the square of the other given side; the square root of the remainder will give the required side.

1. The base and perpendicular of a right-angled triangle are 30 and 40: what the hypotenuse? *Ans.* 50.

2. The base and perpendicular of a right-angled triangle are 81 and 108: what the hypotenuse? *Ans.* 135.

3. The hypotenuse of a right-angled triangle is 100; the base, 60: what the perpendicular? *Ans.* 80.

4. A castle 45 yd. high, is surrounded by a ditch 60 yd. wide: what length of rope will reach from the outside of the ditch to the top of the castle? *Ans.* 75 yd.

5. It is 36 yd. from the top of a fort, standing by the edge of the water, to the opposite side of a stream 24 yd. wide: how high is the fort? *Ans.* 26.83+ yd.

6. A ladder 60 ft. long, reaches a window 37 ft. from the ground on one side of the street, and without moving it at the foot, will reach one 23 ft. high on the other side: find the width of the street. *Ans.* 102.64+ ft.

REVIEW.—288. How extract the square root of a perfect square by factoring? 289. What is a triangle? A right angle? A right-angled triangle? What the hypotenuse? What the other two sides?

290. What is true of every right-angled triangle? Give an example. How find the hypotenuse when the base and perpendicular are known? How find either side when the hypotenuse and the other side are known?

7. A tree, 140 ft. high, is in the center of a circular island 100 ft. in diameter; a line 600 feet long, reaches from the top of the tree to the further shore: what is the breadth of the stream, the land on each side being of the same level? *Ans.* 533.43+ ft.

8. A room is 20 ft. long, 16 ft. wide, and 12 ft. high: what is the distance from one of the lower corners to the opposite upper corner? *Ans.* 28.28+ft.

ART. **291**. Since the area or superficial contents of a square equals the square of one of its sides, (Art. 87), hence, the

RULE FOR FINDING THE SIDE OF A SQUARE EQUAL IN AREA TO ANY GIVEN SURFACE.

Extract the square root of the given area; the root will be the side of the required square.

1. The superficial contents of a circle are 4096: what the side of a square of equal area? *Ans.* 64.

2. A square field measures 4 rd. on each side: what the length of one side of a square field having 9 times as many sq. rd.? *Ans.* 12 rd.

3. There are 43560 sq. ft. in 1 A.: what is each side of a square, containing 1 A, $\frac{1}{2}$ A, $\frac{1}{4}$ A?
Ans. 208.71+ft.; 147.58+ft.; and 104.35+ft.

4. A man has 2 fields; 10 A. and $12\frac{1}{2}$ A.: find the side of a sq. field equal in area to both. *Ans.* 60 rd.

EXTRACTION OF THE CUBE ROOT.

ART. **292**. To extract the cube root of a number, is to resolve it into THREE equal factors; or, to find a number which, when multiplied by itself *twice*, will produce the given number.

Thus, 4 is the cube root of 64, because $4 \times 4 \times 4 = 64$.

Roots.	1,	2,	3,	4,	5,	6,	7,	8,	9.
Cubes.	1,	8,	27,	64,	125,	216,	343,	512,	729.

REVIEW.—291. How find the side of a square equal in area to any given surface? 292. What is it to extract the cube root of a number?

ART. **293**. From Art. 278, it follows that the cube root of a number expresses the side of a cube whose solid contents are equal to the given number.

Hence, extracting the cube root, is finding the side of a cube when its solid contents are known; or, arranging a given number of cubes, so as to form the largest cube possible.

ART. **294**. From Art. 280, it follows that,

The cube root of 1 is 1;
The cube root of 1000 is 10;
The cube root of 1000000 is 100; and so on: hence,

The cube root of a number between 1 and 1000, consists of *one* figure; between 1000 and 1000000, of *two* figures; between 1000000 and 1000000000, of *three*, &c.: hence,

RULE FOR POINTING.—If a dot (.) be placed over every 3d figure of any given number, beginning with *units*, the number of dots will denote the number of figures in the cube root.

ART. **295**. 1. Extract the cube root of 13824; or, suppose 13824 cubic blocks, each 1 in. long, 1 in. wide, and 1 in. thick, are to be arranged in the form of a cube.

OPERATION.

$$
\begin{array}{r|l}
 & 1\dot{3}82\dot{4}(20+4 \\
 & 8000 \quad =24 \ \textit{Root.} \\
20\times20\times3=1200 & 5824 \\
20\times\ 4\times3=\ \ 240 & \\
4\times\ 4\ \ =\ \ \ 16 & \\
\hline
1456 & 5824
\end{array}
$$

SOLU.—First separate the given number into periods, by placing dots over the 4 and 3.

The root will consist of *two* figures.

We next find that the largest cube contained in 13 (thousand) is 8 (thousand), the cube root of which is 2 (tens), which place on the right, as in extracting the square root.

Subtract the cube of 2 (tens), which is 8 (thousand), from the given number, and 5824 remain.

While solving this example by figures, attend to arranging the cubic blocks. After finding that the cube root of the given number

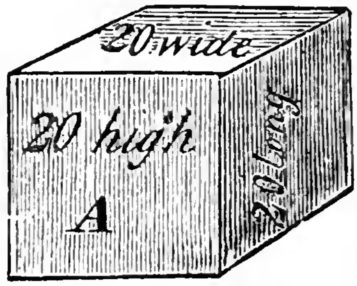
---

REVIEW.—292. Of what numbers are the nine digits the cube roots? 293. What does the cube root of a number express?

294. What the cube root of a number between 1 and 1000? Why? Of a number between 1000 and 1000000? Why? What the rule for pointing?

will contain two places of figures, (tens and units,) and that the figure in the tens' place is 2, form a cube, A, 20 (2 tens) inches long, 20 in. wide, and 20 in. high; this cube will contain (Art. 92) $20 \times 20 \times 20 = 8000$ cu. in.; take this sum from the whole number of cubes, and 5824 cu. in. are left, which correspond to the number 5824 in the numerical operation.

FIG. 1.



It is obvious that to increase the figure A, and at the same time preserve it a cube, the length, breadth, and hight, must each receive an equal addition.

Then, since each side is 20 in. long, square 20, which gives $20 \times 20 = 400$, for the number of sq. in. in each face of the cube; and since an addition is to be made to three sides, multiply the 400 by 3, which gives 1200 for the number of square inches in the 3 sides.

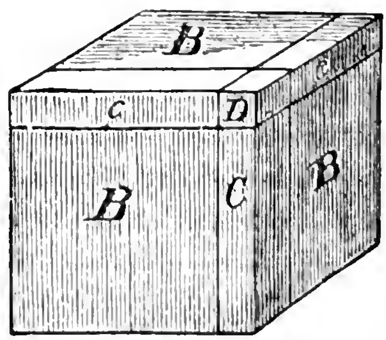
This 1200 is called the TRIAL DIVISOR; because, by means of it, the *thickness* of the additions may be determined.

By examining Fig. 2 (or the blocks, see Note 6), it will be seen, that after increasing each of the three sides equally, there will be required 3 oblong solids, C, c, c, of the same length as each of the sides, and whose thickness and hight are each the same as the additional thickness; and also a cube, D, whose length, breadth, and hight, are each the same as the additional thickness.

Hence, the solid contents of the first three rectangular solids, the three oblong solids, and the small cube, must together be equal to the remaining cubes (5824).

Now find the thickness of the additions. It will always be something less than the number of times the *trial divisor* (1200) is contained in the dividend (5824).

FIG. 2.



By trial we find 1200 is contained 4 times in 5824. Place the 4 in the quotient, and proceed to find the contents of the different solids: these added together, make the number to be subtracted, called the *subtrahend.*

The solid contents of the first three additions, B, B, B, are found, (Art. 93) by multiplying the number of sq. in. in the face by the thickness;

now there are 400 sq. in. in the face of each, and 400 × 3 = 1200 sq. in. in one face of the three; then multiplying by 4, (the thickness,) gives 4800 cu. in. for their contents.

The solid contents of the three oblong solids, C, c, c, are found (Art. 93) by multiplying the number of sq. in. in the face by the thickness; now there are 20 × 4 = 80 sq. in. in one face of each, and 80 × 3 = 240 sq. in. in one face of the three; then multiplying by 4, (the thickness,) gives 960 cu. in. for their contents.

Lastly, find the contents of the small cube, D, by multiplying its length (4) by its breadth (4), and that product by the thickness (4); this gives 4 × 4 × 4 = 64 cu. in.

If the solid contents of the several additions be added together, their sum, 5824 cu. in., will be the number of small cubes remaining after forming the first cube, A.

ADDITIONS.

B, B, B, =	4800	cu. in.	
C, c, c, =	960	,,	,,
D, =	64	,,	,,
Sum	5824		

Hence, when 13824 cu. in. are arranged in the form of a cube, each side is 24 in.; that is, the cube root of 13824 is 24.

In finding the solid contents of the additions, in each case the last multiplier is the thickness.

To produce the same result more conveniently, find the area of one face of each of the additional solids, then the sum of the areas, (as in the numerical operation,) and multiply it by the common thickness.

The *sum* of the areas of one face of each of the additional solids, is termed the COMPLETE DIVISOR. Thus,

In the preceding operation, 1456 is the *complete divisor.*

NOTE.—As the 1st figure of the root is always in the tens' place with regard to the 2d, annex to it a cipher before it is squared; or, omit the cipher, and multiply the square by 300 instead of 3.

REVIEW.—295. How obtain the 1st figure of the root? Why square it? Why multiply by 3? What is the product called? Why?

295. How obtain the 2d figure of the root? Why multiply the 1st figure by the 2d? Why multiply their product by 3? Why square the 2d figure of the root? How find the subtrahend? What is the complete divisor?

295. NOTE. Why is a cipher annexed to the 1st figure of the root before squaring? If the cipher is omitted, what must be done? What if the cipher is omitted in multiplying the 1st figure of the root by the 2d?

For the same reason, annex a cipher to the figure first obtained, before multiplying it by the 2d (the thickness):

Or, omit the cipher, and multiply by 30 instead of 3.

$$\text{Thus,}\quad \begin{array}{lr} 20\times 20\times 3= & 1200\\ 20\times 4\times 3= & 240\\ 4\times 4 \quad= & 16\\ \hline & 1456. \end{array} \qquad \text{Or,}\quad \begin{array}{lr} 2\times 2\times 300= & 1200\\ 2\times 4\times 30= & 240\\ 4\times 4 \quad= & 16\\ \hline & 1456. \end{array}$$

*2. What is the cube root of 1728? *Ans.* 12.

3. Find the cube root of 413493625.

OPERATION. $41\dot{3}49\dot{3}62\dot{5}\,(745$ *Ans.*

$$\begin{array}{lr|l} & & 343\\ \hline 7\times 7\times 300= & 14700 & 70493=\text{dividend.}\\ 7\times 4\times 30= & 840 & \\ 4\times 4 \quad= & 16 & \\ \hline & 15556 & 62224=\text{subtrahend.}\\ \hline 74\times 74\times 300= & 1642800 & 8269625=\text{dividend.}\\ 74\times 5\times 30= & 11100 & \\ 5\times 5 \quad= & 25 & \\ \hline & 1653925 & 8269625=\text{subtrahend.} \end{array}$$

EXPLANATION.—By the rule for pointing (Art. 294), the root will contain 3 figures. Find the 1st and 2d figures of the root as in the preceding examples. Then consider 74 as so many tens, and find the 3d figure in the same manner as the 2d was obtained.

4. Find the cube root of 515.849608

OPERATION. $51\dot{5}.\dot{8}49\dot{6}08\,(8.02$ *Ans.*

$$\begin{array}{lr|l} & & 512\\ \hline 8\times 8\times 300= & 19200 & 3849\\ \hline 80\times 80\times 300= & 1920000 & 3849608\\ 80\times 2\times 30= & 4800 & \\ 2\times 2 \quad= & 4 & \\ \hline & 1924804 & 3849608 \end{array}$$

EXP.—After obtaining the 1st figure, and bringing down the 2d period, we find the trial divisor is not contained in the dividend;

therefore, place a 0 in the root, and bring down another period. Hence, the cube root of a decimal is found in the same manner as that of a whole number, the periods being reckoned both ways from the decimal point.

ART. 296. TO EXTRACT THE CUBE ROOT,

Rule.—1. *Separate the given number into periods of 3 places each, by placing a dot over the units, a dot over the thousands, and so on.* (The left period often has only one or two figures.)

2. *Find the greatest cube in the left period, and place its root on the right, as in division. Subtract the cube of the root from the left period, and to the remainder bring down the next period for a dividend.*

3. *Square the root found, and multiply it by 300 for a trial divisor. Find how many times this divisor is contained in the dividend, and write the result in the root. Multiply the last figure of the root by the rest, and by 30; square the last figure of the root, and add these two products to the trial divisor; the sum will be the* COMPLETE *divisor.*

4. *Multiply the complete divisor by the last figure of the root, and subtract the product from the dividend; to the remainder bring down the next period for a new dividend, and so proceed until all the periods are brought down.*

NOTES.—1. When the product of the complete divisor by the last figure of the root is larger than the dividend, the figure of the root must be diminished.

2. After bringing down all the periods, if there be a remainder, the operation may be continued by annexing *periods of ciphers.*

3. If the divisor is not contained in the dividend, write a cipher in the root, and bring down another period for a *new* dividend.

4. When there are decimals in the given number, separate them into periods by placing dots over the *tenths*, *ten-thousandths*, and so on. The reasons for this are similar to those in Art. 286, Ex. 4.

5. To extract the cube root of a common fraction, reduce it to its lowest terms; then, if both terms are perfect cubes, extract the cube root of each; but, if either term is an imperfect cube, reduce the fraction to a decimal, and then extract the root.

☞ EVERY TEACHER should have a set of cubical blocks.

To TEACHERS.—Instead of finding the subtrahend by the rule, it may be obtained by finding separately the contents of each solid, then adding the whole together. This method, in connection with the blocks, is best adapted to give a clear idea of the nature of the operation.

What is the Cube Root of,

		ANSWERS.			ANSWERS.
5.	91125?	45.	15.	53.157376?	3.76
6.	195112?	58.	16.	.199176704?	.584
7.	912673?	97.	17.	$\frac{216}{343}$?	$\frac{6}{7}$.
8.	1225043?	107.	18.	$\frac{2744}{6859}$?	$\frac{14}{19}$.
9.	13312053?	237.	19.	$\frac{48778}{118638}$?	$\frac{29}{39}$.
10.	102503232?	468.	20.	$5\frac{104}{125}$?	$1\frac{4}{5}$.
11.	529475129?	809.	21.	2?	1.25992+
12.	958585256?	986.	22.	9?	2.08008+
13.	14760213677?	2453.	23.	200?	5.84803+
14.	128100283921?	5041.	24.	$9\frac{1}{6}$?	2.092+

25. The contents of a cubical cellar are 1953.125 cu. ft.: find the length of one side. *Ans.* 12.5 ft.

26. In 1 cu. ft., how many 3 in cubes? *Ans.* 64.

27. How many cubical blocks, each side of which is one-quarter of an inch, will fill a cubical box, each side of which is 2 inches? *Ans.* 512.

28. Find the difference between half a solid foot, and a solid half foot. *Ans.* 648 cu. in.

29. Find the side of a cubical mound equal to one 288 ft. long, 216 ft. broad, 48 ft. high. *Ans.* 144 ft.

30. The side of a cubical vessel is 1 foot: find the side of another cubical vessel that shall contain 3 times as much. *Ans.* 17.306+in.

REVIEW.—296. What the rule for extracting the cube root? NOTES. When the subtrahend is larger than the dividend, what is required?

296. When there is a remainder, how continue the operation? How proceed when there are decimals in the given number? How extract the cube root of a common fraction?

ART. **297**. It is a known principle, that spheres are to each other as the cubes of their diameters; and that

All *similar* solids are to each other as the cubes of their *corresponding* sides.

Hence, *the solid contents, or weight of two similar solids, have to each other the same ratio as the cubes of their like parts.*

31. A metal ball 6 in. in diameter weighs 32 lb.: what is the weight of one of the same metal, whose diameter is 3 in.? *Ans.* 4 lb.

32. If the diameter of Jupiter is 11 times that of the earth, how many times larger is it? *Ans.* 1331.

ART. 298. THE CUBE ROOT BY FACTORING.

The cube root of any *perfect* cube may be extracted *by resolving the given number into its prime factors, and multiplying together one of each three equal factors.*

1. Find the cube root of 216. . . *Ans.* $3\times2=6$.
2. Find the cube root of 27×64. . . . *Ans.* 12.
3. Find the cube root of 125×343. . . *Ans.* 35.

XXV. ARITHMETICAL PROGRESSION.

ART. **299**. An *Arithmetical Progression*, or *Series*, is a series of numbers which *increase* or *decrease*, by a *common difference*. If the series increase, it is called an *increasing* series; if it decrease, a *decreasing* series.

Thus, 1, 3, 5, 7, 9, 11, &c., is an *increasing* series.
20, 17, 14, 11, 8, 5, &c., is a *decreasing* series.

The numbers forming the series are called *terms;* the first and last terms are the *extremes;* the other terms, the *means.*

REVIEW.—297. What ratio have the solid contents of two similar bodies? 298. How extract the cube root of a perfect cube by factoring?

299. What is an arithmetical progression? When is the series increasing? Decreasing? Give examples. What are the extremes? The means?

ART. **300.** In every arithmetical series, five things are considered:

1st, the *first* term; 2d, the *last* term; 3d, the *common difference;* 4th, the *number* of terms; 5th, the *sum* of all the terms.

CASE I.

ART. **301.** *To find the* LAST TERM, *when the first term, the common difference, and the number of terms are given.*

1. I bought 10 yd. of muslin, at 3 cts. for the 1st yd., 7 cts. for the 2d, 11 cts. for the 3d, and so on, with a com. difference of 4 cts.: what did the last yd. cost?

SOLUTION.—To find the cost of the *second* yard, add 4 cts. *once* to the cost of the first; to find the cost of the *third*, add 4 cts. *twice* to the cost of the first; to find the cost of the *fourth*, add 4 cts. *three times* to the cost of the first, and so on. Hence,

To find the cost of the *tenth* yard, *add* 4 cts. *nine times* to the cost of the first; but 9 times 4 cts. are 36 cts., and 3 cts.+36 cts.=39 cts., the cost of the last yard, or *last term* of the progression.

2. The first term of a decreasing series is 39; the com. diff. 4; the number of terms 10: find the last term.

SOLUTION.—In this case, 4 must be *subtracted* 9 times from 39, which will give three for the last term. Hence, the

Rule for Case I.—*Multiply the common difference by the number of terms less one; if an increasing series, add the product to the* 1*st term: if a decreasing series, subtract the product from the* 1*st term: the result will be the required term.*

3. Find the last term of an increasing progression: the first term 2; the common difference 3; and the number of terms 50. *Ans.* 149.

4. I bought 100 yd. muslin, at 9 cts. for the 1st yard., 14 cts. for the 2d, and so on, increasing by the com. difference 5 cts.: find the cost of the last yd. *Ans.* $5.04

5. What is the 54th term of a decreasing series, the 1st term 140, and com. diff. 2? *Ans.* 34.

6. A lends $200 at simple interest, at 8 % per annum:

REVIEW.—300. What five things are considered in every series?

at the end of the 1st year $216 will be due; at the end of the 2d year, $232, and so on: what sum will be due at the end of 20 years? *Ans.* $520.

7. What is the 99th term of a decreasing series, the 1st term 329, and com diff. $\frac{7}{8}$? *Ans.* $243\frac{1}{4}$.

CASE II.

ART. **302**. *To find the* COMMON DIFFERENCE, *when the extremes and the number of terms are given.*

1. The first term of a series is 2, the last 20, and the number of terms 7: what the com. diff.?

SOLUTION.—The difference of the first and last terms is always equal to the com. diff. *multiplied* by the number of terms less one (Art. 301); therefore,

If the difference of the extremes be *divided* by the number of terms less one, the quotient will be the com. diff. Hence, the

Rule for Case II.—*Divide the difference of the extremes by the number of terms less one; the quotient will be the com. diff.*

2. The extremes are 3 and 300; the number of terms 10: find the com. diff. *Ans.* 33.

3. A travels from Boston to Bangor in 10 da.; he goes 5 mi. the first day, and increases the distance traveled each day by the same number of miles; and on the last day he goes 50 mi.: find the daily increase. *Ans.* 5 mi.

ART. **303** It is obvious that if the difference of the extremes be divided by the com. diff., the quotient, increased by unity (1), will be the number of terms.

1. The extremes are 5 and 49; the com. diff. 4: find number of terms. *Ans.* 12.

CASE III.

ART. **304**. *To find the* SUM *of all the terms of the series, when the extremes and number of terms are given.*

REVIEW.—301. What is Case 1? What is the Rule for Case 1? 302. What is Case 2? What is the Rule for Case 2?

1. Find the sum of 6 terms of the series whose first term is 1, and last term 11.

SOLUTION.—The series is . .	1,	3,	5,	7,	9,	11,
The order inverted is	11,	9,	7,	5,	3,	1,
The sum is	12,	12,	12,	12,	12,	12.

Since the two series are the same, their sum is *twice* the first series. But their sum is obviously as many times 12, (the sum of the extremes), as there are terms. Hence, the

Rule for Case III.—*Multiply the sum of the extremes by the number of terms; half the product will be the sum of the series.*

2. The extremes are 2 and 50; the number of terms, 24: find the sum of the series. *Ans.* 624.

3. How many strokes does the hammer of a clock strike in 12 hours? *Ans.* 78.

4. Find the sum of the first ten thousand numbers in the series, 1, 2, 3, 4, 5, &c. *Ans.* 50005000.

5. Place 100 apples in a right line, 3 yd. from each other, the first, 3 yd. from a basket: what distance will a boy travel who gathers them singly and places them in the basket? *Ans.* 17 mi. 380 yd.

6. A traveled one day 30 mi., and each succeeding day a quarter of a mile less than on the preceding day: how far did he travel in 30 days? *Ans.* $791\frac{1}{4}$ mi.

7. A body falling by its own weight, if not resisted by the air, would descend in the 1st second a space of 16 ft. 1 in.; the next second, 3 times that space; the 3d, 5 times that space; the 4th, 7 times, &c.: at that rate, through what space would it fall in 1 min.? *Ans.* 57900 ft.

XXVI. GEOMETRICAL PROGRESSION.

ART. **305.** A *Geometrical Progression*, or *Series*, is a series of numbers *increasing* by a common *multiplier*, or *decreasing* by a common *divisor*. Thus,

1, 3, 9, 27, 81, is an increasing geometric series.
48, 24, 12, 6, 3, is a decreasing geometric series.

REVIEW.—303. How find the number of terms, when the extremes and common difference are given?

The common multiplier or com. divisor, is called the *ratio.* In the 1st of the above series, the ratio is 3; in the 2d, 2.

The numbers forming the series are the *terms;* the first and last terms are *extremes;* the others, *means.*

ART. **306.** In every geometric series, 5 things are considered: 1st, the *first* term; 2d, the *last* term; 3d, the *number* of terms; 4th, the *ratio;* 5th, the *sum* of all the terms.

CASE I.

ART. **307.** *To find the* LAST TERM, *when the first term, the ratio, and the number of terms are given.*

1. The first term of an increasing geometric series is 2; the ratio 3; what is the 5th term?

SOLUTION.—The first term is 2; the second, 2×3; the third, $2\times3\times3$; the fourth, $2\times3\times3\times3$; and

The fifth, $2\times3\times3\times3\times3=2\times3^4=2\times81=162$. *Ans.*

Observe that each term after the first, consists of the first term multiplied by the ratio taken as a factor as many times *less one,* as is denoted by the number of the term. Thus, the *fifth* term consists of 2 multiplied by 3 taken *four* times as a factor. But 3, taken 4 times as a factor, is (Art. 277) the 4th *power* of 3. Hence,

The *fifth* term is equal to 2, multiplied by the 4th power of 3.

2. The first term of a decreasing geometric series is 192; the ratio 2; what is the *fourth* term?

SOLUTION.—The 2d term is $192\div2$; the 3d is 192 divided by 2×2; the 4th is 192 divided by $2\times2\times2$; that is, $192\div2^3=192\div8=24$, *Ans.* The required term is found by *dividing* the first term by the ratio raised to a power whose exponent is 1 less than the number of the term. Hence, the

Rule for Case I.—*Raise the ratio to a power* (Art. 279) *whose exponent is one less than the number of terms.*

If the series be increasing, MULTIPLY *the* 1*st term by this power, and the product will be the last term; if decreasing,* DIVIDE *the* 1*st term by the power, and the quotient will be the last term.*

REVIEW.—304. What is Case 3? What is the Rule for Case 3? 305. What is a geometrical series? Give examples. What the ratio? What the extremes? The means? 306. What five things are considered?

NOTE.—In finding high powers of the ratio, the operation may often be shortened by observing that the *product* of any two powers of a number, will give that power of the number which is denoted by the *sum* of their exponents. Thus,

The *third* power multiplied by the *fourth* power, will produce the *seventh* power.

$$2^3 \times 2^4 = 8 \times 16 = 128 = 2^7.$$

3. The first term of an increasing series is 2; the ratio, 2; the number of terms, 13: find the last term. *Ans.* 8192.

4. The first term of a decreasing series is 262144; the ratio, 4; number of terms, 9: find the last term. *Ans.* 4.

5. The first term of an increasing series is 10; the ratio, 3: what the tenth term? *Ans.* 196830.

6. What the 35th term of an increasing series, whose first term is 1, and ratio, 2? *Ans.* 17179869184.

7. Find the 35th term of an increasing series, the 1st term, 1; ratio, 3. *Ans.* 16677181699666569.

CASE II.

ART. **308**. *To find the* SUM *of all the terms of a geometric series.*

1. To obtain a General Rule, let us find the sum of 5 terms of the geometric series, whose 1st term is 4, and ratio 3.

SOLUTION.—Write the terms of the series as below; then multiply each term by the ratio, and remove the product one term toward the right: thus,

$$\begin{array}{ll} 4+12+36+108+324 & = \text{sum of the series.} \\ \quad\ 12+36+108+324+972 & = \text{sum} \times 3. \end{array}$$

Since the upper line is *once* the sum of the series, and the lower *three times* the sum, their difference is *twice* the sum; hence,

If the upper line be subtracted from the lower, and the remainder divided by 2, the quotient will be the sum of the series.

Performing this operation, we have 972—4=968; which, divided by 2, the quotient is 484, the required sum.

In this process, 972 is the product of the greatest term of the given series by the ratio; 4 is the least term, and the divisor 2, is equal to the ratio less one.

REVIEW.—307. What is Case 1? What the Rule? NOTE. How are high powers of the ratio most easily found?

Rule for Case II.—*Multiply the greatest term by the ratio; from the product subtract the least term, and divide the remainder by the ratio less* 1; *the quotient will be the sum of the series.*

Note.—When a series is decreasing, and the number of terms infinite, the last term is *naught.* In finding the sum by the rule, observe that the ratio is *greater* than 1.

2. The first term is 10; the ratio, 3; the number of terms, 7: what is the sum of the series? *Ans.* 10930.

3. A gave to his daughter on New Year's day $1; he doubled it the first day of every month for a year: what sum did she receive? *Ans.* $4095.

4. I sold 1 lb. of gold at 1 ct. for the 1st oz., 4 for the 2d, 16 for the 3d, &c.: what the sum? *Ans.* $55924.05

5. A sold a house having 40 doors, at 10 cts. for the 1st door, 20 for the 2d, 40 for the 3d, and so on: how much did he receive? *Ans.* $109951162777.50

6. B bought a horse at 1 ct. for the 1st nail in his shoes, 3 for the 2d, 9 for the 3d, &c.: what was the price, there being 32 nails? *Ans.* $9265100944259.20

7 Find the sum of an infinite series, the greatest term .3; the ratio, 10; that is, of $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}$, &c. *Ans.* $\frac{1}{3}$.

8. Find the sum of an infinite series, greatest term 100; ratio 1.04 *Ans.* 2600.

9. The sum of the infinite series $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c. *Ans.* $\frac{1}{2}$.

10. The sum of the infinite series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c. *Ans.* 1.

XXVII. PERMUTATION.

Art. **309**. *Permutation* teaches the method of finding in how many different positions any given number of things may be placed.

Thus, the two letters *a* and *b* can be placed in *two* positions, *ab* and *ba;* but if we add a third letter *c*, *three* positions can be made with each of the two preceding: thus,

Cab, *acb*, *abc*, and *cba*, *bca*, *bac*, making 2×3=6 positions.

By taking a *fourth* letter *d*, *four* positions can be made out of each of the six positions, making 6×4=24 in all.

Rule for Permutation.—*Multiply together the numbers, 1, 2, 3, &c., from 1 to the given number; the last product will be the required result.*

1. In how many different ways may the digits 1, 2, 3, 4, and 5 be placed? *Ans.* 120.

2. What number of changes may be rung on 12 bells? *Ans.* 479001600.

3. What time will 8 persons require to seat themselves differently every day at dinner, allowing 365 days to the year? *Ans.* 110 yr. 170 da.

4. Of how many variations do the 26 letters of the alphabet admit? *Ans.* 403291461126605635584000000.

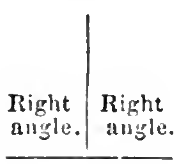
XXVIII. MENSURATION.

To Teachers.—As this short article on Mensuration is intended for pupils who may not have an opportunity of studying a more extensive course, only the more useful parts are presented.

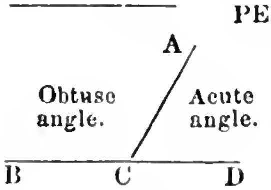
The definitions and illustrations are given in plain and familiar terms, not with a view to mathematical precision.

Art. 310. DEFINITIONS.

1. An Angle is the inclination of two straight lines meeting in a point, which is called the Vertex. It is the *degree* of the opening of the lines.



2. When one straight line stands on another so that it makes with it two equal angles, each of these angles is a Right Angle; and the straight line which stands on the other is said to be Perpendicular to it, or at Right Angles to it.



3. An Obtuse Angle is *greater* than a right angle: and an Acute Angle is *less* than a right angle.

Note.—An angle is named by 3 letters, the middle one being placed at the vertex, and the other two on the lines which form

---

Review.—308. What is Case 2? What the Rule? Note. What is the last term of a decreasing series of which the number of terms is infinite? 309. What is Permutation? What the Rule?

the angle. In the diagram the obtuse angle is called the angle A C B, and the acute angle, the angle A C D.

4. Parallel straight lines are everywhere equally distant from each other.

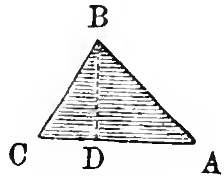
5. A Surface has length and breadth, without thickness.

6. A Plane is a surface, in which, if any two points be taken, the straight line joining them will be wholly in the plane.

7. A Figure is a portion of surface inclosed by one or more boundaries.

8. If a figure has equal *sides*, it is E-qui-lat′-er-al; if it has equal *angles*, E-qui-an′-gu-lar.

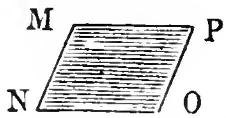
9. A Triangle is a figure bounded by 3 straight lines. The side on which the triangle stands, is the Base. The *perpendicular hight* is the shortest distance from the base to the opposite angle. Thus, A B C is a triangle; A C is the base, and B D the perpendicular hight.



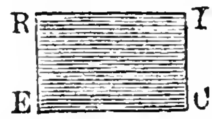
10. A Quad-ri-lat′-er-al is bounded by 4 straight lines.

11. A Polygon is bounded by more than 4 straight lines.

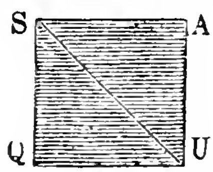
12. A Par-al-lel′-o-gram is a quadrilateral whose opposite sides are parallel.
Thus, M N O P is a *parallelogram.*



13. A Rect′-an-gle is a quadrilateral whose opposite sides are parallel; its angles right angles. Thus, R E C T is a *rectangle.*

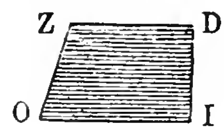


14. A Square is a quadrilateral whose sides are equal to each other; its angles right angles. Thus, S Q U A is a *square.*

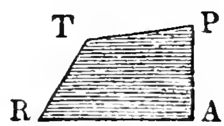


15. A Rhom′-bus is a quadrilateral whose sides are equal to each other; its angles, *not* right angles. See Fig. Def. 12.

16. A Trap′-e-zoid is a quadrilateral having only two sides parallel. Thus, Z O I D is a trapezoid; the sides Z D and O I being parallel.

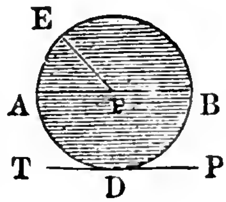


17. A Tra-pe′-zium is a quadrilateral having *no* two sides parallel to each other.
Thus, T R A P is a *trapezium.*



18. A DI-AG′-O-NAL is a line joining two angles of a figure not next to each other. Thus, S U (Fig. Def. 14) is a diagonal of the square.

19. A CIRCLE is a figure bounded by a curve line, called the *circumference*, every part of which is equally distant from a point within, called the *center*. A DIAMETER is a straight line passing through the center and terminated both ways by the circumference. A RADIUS is a straight line drawn from the center to the circumference; it is half the diameter.



Thus, A D B E is the circumference; A B the diameter; A F or B F the radius.

20. A TANGENT is a straight line which touches the circumference only in one point, called the point of contact.
Thus, T P is a tangent.

21. An ARC of a circle is any part of the circumference, as A F. A CHORD is a straight line joining the extremities of an arc.

22. MENSURATION is the art of finding the surface, and also the solid contents of bodies.

23. The AREA of a figure is the surface which it contains. The quantity of this surface is denoted by the number of times it contains a given surface called the *measuring unit.*

The MEASURING UNIT for surfaces is a square surface, whose side is some one of the common measures of length, such as a *square inch*, a *square foot*, &c. See Arts. 87, 88, and 89.

MEASUREMENT OF SURFACES.

ART. **311**. To find the superficial contents or AREA of a Parallelogram, Rectangle, Square, or Rhombus,

Rule.—*Multiply the length by the perpendicular breadth, the product will be the area.*

NOTE.—The learner must recollect (Art. 276) that feet in length multiplied by feet in breadth, produce *square* feet; and the same of the other denominations of lineal measure.

1. How many square feet in a floor 17 feet long and 15 feet wide? *Ans.* 255 sq. ft.

2. Find the sq. ft. in a board 2 ft. 3 in. wide and 12 ft. 6 in. long. *Ans.* 28.125 = $28\frac{1}{8}$ sq. ft.

3. The sq. ft. in a board 15 in. wide and 16 ft. long. *Ans.* 20 sq. ft.

4. How many sq. ft. in a board 1 ft. 2 in. in mean breadth, 12 ft. 6 in. long? *Ans.* 14 sq. ft. 84 sq. in., or $14\frac{7}{12}$ sq. ft.

5. At $1.50 per sq. ft., what cost a marble slab; the length 5 ft. 7 in.; breadth 1 ft. 10 in.? *Ans.* $15.354+

6. How many acres of land in a parallelogram; the length, 120 rd.; the breadth, 84 rd.? *Ans.* 63 A.

7. How many acres in a square field, each side of which is 65 rd.? *Ans.* 26 A. 1 R. 25 P.

8. How many acres in a field in the form of a rhombus; each side measures 35 rd; the perpendicular distance across it, 16 rd.? *Ans.* 3 A. 2 R.

9. Each side of the base of a pyramid is 693 ft. long: how many acres does it cover? *Ans.* 11 A. 4 P.

10. Find the difference between a floor 30 ft. sq., and two others each 15 ft. sq. *Ans.* 450 sq. ft.

11. If a room is 10 ft. long, how wide must it be to contain 80 sq. ft.? (See Art. 90.) *Ans.* 8 ft.

12. A board is 10 inches wide: what must be its length to contain 10 sq. ft.? *Ans.* 12 ft.

13. How many yd. of carpet $1\frac{1}{2}$ yd. wide, will cover a floor 6 yd. long, 5 yd. wide? *Ans.* 20 yd.

14. How many yd. of carpet $1\frac{1}{4}$ yd. wide, will cover a floor 21 ft. 3 in. long, 13 ft. 6 in. wide? *Ans.* $25\frac{1}{2}$ yd.

15. How many yd. of flannel $\frac{3}{4}$ yd. wide, will line 3 yd. of cloth, $1\frac{1}{2}$ yd. wide? *Ans.* 6 yd.

Plasterers', Pavers', Painters', and Carpenters' Work.

ART. **312**. Several kinds of artificers' work are measured by the preceding rule.

Plasterers', Pavers', and Painters' work, is computed in sq. yards: Glaziers' work, by the sq. ft., or by the pane:

Carpenters' and Joiners' work, some parts by the sq. yard; other parts by the SQUARE, which contains 100 sq. ft.

1. How many square yards in a ceiling 25 ft. 9 in long, and 21 ft. 3 in. wide? *Ans.* 60 sq. yd. 7 sq. ft.+

2. At 20 cts. a sq. yd., what will it cost to plaster a ceiling 22 ft. 7 in. long, 13 ft. 11 in. wide? *Ans.* $6.984+

3. A room is 20 ft. 6 in. long, 16 ft. 3 in. broad, 10 ft. 4 in. high: how many yd. of plastering in it, deducting a fireplace 6 ft. 3 in. by 4 ft 2 in.; a door 7 ft. by 4 ft. 2 in., and two windows, each 6 ft. by 3 ft. 3 in.? *Ans.* 110 sq yd. $8\frac{5}{12}$ sq. ft.

4. A room is 20 ft. long, 14 ft. 6 in. broad, and 10 ft. 4 in. high: what will the coloring of the walls cost, at 27 cts. per sq. yd., deducting a fireplace 4 ft. by 4 ft. 4 in., and two windows, each 6 ft. by 3 ft. 2 in.? *Ans.* $19.73

5. At 18 cts. per sq. yd., find the cost of paving a walk 35 ft. 4 in. long, 8 ft. 3 in. broad. *Ans.* $5.83

6. What will it cost to pave a rectangular yard, 21 yd. long, and 15 yd. broad, in which a footpath, 5 ft. 3 in. wide, runs the whole length of the yard; the path paved with flags, at 36 cts. per sq. yd., and the rest with bricks, at 24 cts. per sq. yd.? *Ans.* $80.01

7. At 10 cts. a sq. yd., what the cost to paint the walls of a room 75 ft. 6 in. in compass, 12 ft. 6 in. high? *Ans.* $10.486+

8. A house has 3 tiers of windows, 7 in a tier: the hight of the first tier is 6 ft. 11 in.; of the 2d, 5 ft. 4 in.; the 3d, 4 ft. 3 in.; each window is 3 ft. 6 in. wide: what cost the glazing, at 16 cts. per sq. ft.? *Ans.* $64.68

9. A floor is 36 ft. 3 in. long, 16 ft. 6 in. wide: what will it cost to lay it, at $3 a square? *Ans.* $17.943+

10. A room is 35 ft. long, and 30 ft. wide: what will the flooring cost, at $5 per square, deducting a fireplace 6 ft. by 4 ft. 6 in., and a stairway, 8 ft. by 10 ft. 6 in.? *Ans.* $46.95

11. At $3.50 per square, what cost a roof 40 ft. long, the rafters on each side 18 ft. 6 in. long? *Ans.* $51.80

Art. 313. To find the Area of a Triangle.

Rule.—*Multiply the base by the perpendicular hight, and take half the product for the area.*

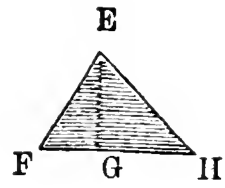
Or, when the sides are given, the following Rule:

1st. *Add the three sides together, and take half the sum.*

2d. *From the half sum take the 3 sides severally.*

3d. *Multiply the half sum and the 3 remainders together, and extract the square root of the product, which gives the area.*

1. Find the area of the triangle, E F G H, the base, F H, is 15 feet; the perpendicular hight, G E, 12 feet. *Ans.* 90 sq. ft.



2. The contents of a triangular space, the base 44 rd., perpendicular hight 18 rd. *Ans.* 2 A. 1 R. 36 P.

3. How many acres in a triangular field; the base 80 rd.; perpendicular hight, 67 rd.? *Ans.* 16 A. 3 R.

NOTE.—The area of any field or piece of land may be found by dividing it into triangles, and measuring the base and perpendicular hight of each triangle thus formed.

4. What cost the glazing of a triangular skylight, at 12 cts. per sq. ft., the base, 12 ft. 6 in., the perpendicular hight, 16 ft. 9 in.? *Ans.* 12.56\frac{1}{4}$

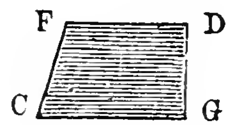
5. Find the area of a triangle, the sides being 13, 14, and 15 ft. *Ans.* 84 sq. ft.

6. The area of a triangle, the sides 2, 3, and 4 feet respectively. *Ans.* 2.9047375+sq. ft.

ART. 314. TO FIND THE AREA OF A TRAPEZOID.

Rule.—*Multiply the sum of the parallel sides by the perpendicular breadth; take half the product.*

1. The parallel sides of a trapezoid, F C G D, are 35 and 26 inches; its breadth 11 in.; required the area. *Ans.* 335$\frac{1}{2}$ sq. in.

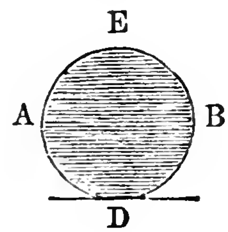


2. A field is the form of a trapezoid; one of the parallel sides is 25 rd., the other 19 rd.; the width 32 rd.: how many acres in it? *Ans.* 4 A. 1 R. 24 P.

ART. 315. TO FIND THE CIRCUMFERENCE OF A CIRCLE. WHEN THE DIAMETER IS GIVEN.

Rule.—*Multiply the diameter by* 3.1416, *the product will be the circumference.*

1. The diameter A B of the circle A D B E is 48 feet: what is the circumference? *Ans.* 150.7968 ft.



2. The diameter of a wheel is 4 feet: find the circumference. *Ans.* 12 ft. 6.7968 in.

3. What is the circumference of the earth, the mean diameter being 7912.4 mi.? *Ans.* 24857.59584 mi.

ART. 316. TO FIND THE DIAMETER OF A CIRCLE, WHEN THE CIRCUMFERENCE IS GIVEN.

Rule.—*Divide the circumference by* 3.1416, *the quotient will be the diameter.*

1. The circumference of a circle is 15 feet: what is the diameter? *Ans.* 4 ft. 9.295+in.

2. If the girt of a tree is 12 feet 5 inches, what its thickness or diameter? *Ans.* 3 ft. 11.428+in.

ART. 317. TO FIND THE AREA OF A CIRCLE.

Rule.—*Multiply the diameter by the circumference, and take one-fourth of the product.* Or, *Multiply the square of the diameter by* .7854; *or, for greater accuracy, by* .785398 Or, *Multiply the square of the radius by* 3.1416

1. Find the area of a circle, the diameter being 42 feet. *Ans.* 1385.4456 sq. ft.

2. Find the area of a space on which a horse may graze, when confined by a cord $7\frac{1}{2}$ rods long, one of its ends being fixed at a certain point. *Ans.* 1 A. 16.715 P.

ART. 318. TO FIND THE DIAMETER OF A CIRCLE, WHEN THE AREA IS GIVEN.

Rule.—*Divide the area by* .7854; *the square root of the quotient will be the diameter.*

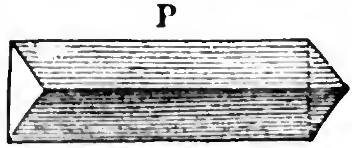
1. The area of a circle is 962.115: what its diameter and circumference? *Ans.* diam. 35: circum. 109.956

2. What length of halter will fasten a horse to a post in the center of an acre of grass, so that he can graze upon the 1 A. and no more? *Ans.* 7.1364+rd., or 117 ft. 9+in.

ART. 319. MEASUREMENT OF BODIES OR SOLIDS.

DEFINITIONS.—1. A BODY or SOLID, has length, breadth, and thickness or depth.

P



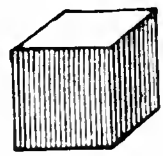
2. A PRISM is a solid whose ends, or bases, are parallel; its sides, parallelograms. Such a body is termed a RIGHT

PRISM when each of its bases is perpendicular to its other sides; and it is TRIANGULAR, QUADRANGULAR, &c., according as its base is a triangle, quadrangle, &c. Thus, P is a triangular prism.

B

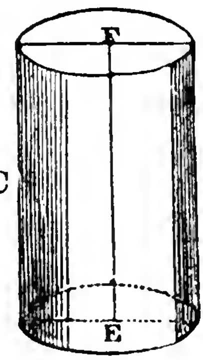
3. A PAR-AL-LEL-O-PI′-PED is a prism whose bases and also its other sides are parallelograms. Thus, B is a parallelopiped.

A



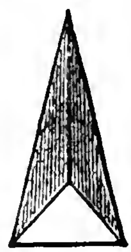
4. A parallelopiped is RIGHT when each of its faces is a rectangle. A common chest, a bar of iron, brick, &c., are instances of right parallelopipeds. When each face of a right parallelopiped, as A, is a square, it is termed a *cube*. A cube has 6 equal square faces.

C



5. A CYLINDER is a round prism, having circles for its ends. Thus, C is a cylinder, of which the line E F passing through the centers of both ends, is called the *axis*.

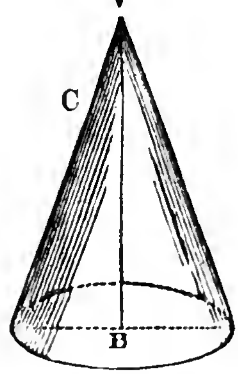
A



6. A PYRAMID is a solid having any plane figure for a base, and its sides triangles, whose vertices meet in a point at the top, called the VERTEX of the pyramid. A pyramid is TRIANGULAR, QUADRANGULAR, &c., according as its base is a triangle, quadrangle, &c. Thus, A is a triangular pyramid.

V

C



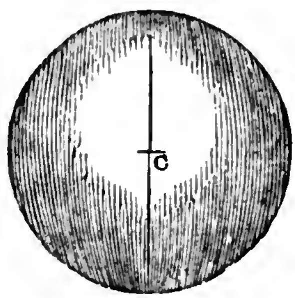
7. A body which has a circular base, and tapers uniformly to a point named the VERTEX, is called a CONE. The *axis* of a cone is a line passing through the vertex and the center of the base. Thus, C is a cone of which B V is the axis.

8. A FRUSTUM of any body, as a pyramid or cone, is what remains when the top is cut off by a plane parallel to the base.

9. A GLOBE, or SPHERE, is a body of such a figure, that all points of the surface are equally distant from a point within, called the *center*.

The *diameter* of a sphere, is a line passing through the center, and terminated both ways by the surface. The *radius* of a sphere is a line drawn from the center to the surface.

Thus, A B is a diameter: C A a radius; C being the center of the sphere.



10. The HIGHT or altitude of a solid, is a line drawn from its vertex or top, perpendicular to its base.

11. The CONTENTS or SOLIDITY of a body, is the space within it. The magnitude of this space is expressed by the number of times it contains a given space called the *measuring unit.*

12. The MEASURING UNIT for solids, is a cube whose base is the measuring unit for surfaces; as a cu. in., cu. ft., &c.

ART. **320**. To find the SOLID CONTENTS of a PARALLELOPIPED.

Rule.—*Multiply the length, breadth, and depth together: the product will be the solid contents.*

1. Find the solid contents of a parallelopiped: the length, 12 ft.; breadth, 3 ft. 3 in.; depth, 4 ft. 4 in. *Ans.* 169 cu. ft.

2. The solid contents of a rectangular stone: the length, 6 ft.; breadth, 2 ft. 6 in.; depth, 1 ft. 9 in. *Ans.* $26\frac{1}{4}$ cu. ft.

3. A block of marble, in the form of a parallelopiped, is in length, 3 ft. 2 in., breadth, 2 ft. 8 in.; depth, 2 ft. 6 in.: what its cost, at 81 cts. per cu. ft.? *Ans.* $17.10

4. How many solid feet in a box 4 ft. 10 in. long, 2 ft. 11 in. broad, and 2 ft. 2 in. deep? *Ans.* 30 cu. ft. 6′ 6″ 4‴.

ART. **321**. The principles of the preceding rule are applied to the measurement of

MASONS' AND BRICKLAYERS' WORK.

Masons' work is measured by the solid foot, or by the perch, which is $16\frac{1}{2}$ ft. long, 18 in. broad, 1 ft. deep;

And multiplying these numbers together, shows that a perch contains $24\frac{3}{4}$, or 24.75 cu. ft.

To find the number of PERCHES in any WALL, or solid body.

Rule.—*Find the contents in cubic feet, by multiplying together the length, breadth, and depth; then divide by* 24.75 *to obtain the contents in perches.*

1. How many perches in a wall 97 ft. 5 in. long, 18 ft. 3 in. high, 2 ft. 3 in. thick? *Ans.* 161.6+ P.

2. In a wall 53 ft. 6 in. long, 12 ft. 3 in. high, 2 ft. thick, how many perches? *Ans.* 52.95+ P.

3. What cost a wall 53 ft. 6 in. long, 12 ft. 6 in. high, 2 ft. thick, at $2.25 a perch? *Ans.* $121.59+

4. How many bricks in a wall 48 ft. 4 in. long, 16 ft. 6 in. high, 1 ft. 6 in. thick, 20 bricks to the cu. ft.? *Ans.* 23925.

5. At $5.875 per thousand bricks, allowing 20 bricks to wall the solid foot, what will it cost to build a wall 320 ft. long, 6 ft. high, 15 in. thick? *Ans.* $282.

6. How many bricks each 8 in. long, 4 in. wide, 2.25 in. thick, will be required for a wall 120 ft. long, 8 ft. high, 1 ft. 6 in. thick? *Ans.* 34560.

7. What the cost of building a wall 240 ft. long, 6 ft. high, 3 ft. thick, at $3.25 per 1000 bricks, each brick 9 in. long, 4 in. wide, 2 in. thick? *Ans.* $336.96

ART. 322. TO FIND THE SOLID CONTENTS OF A PRISM, OR OF A CYLINDER.

Rule.—*Find the area of the base, and multiply it by the perpendicular hight, the product will be the solid contents.*

1. Each side of the base of a triangular prism is 2 in.; its length 14 in.: find the contents. *A.* $\sqrt{588}$=24.2487+cu. in.

2. Find the contents of a cylinder 12 ft. long, the diameter of each end, 4 ft. *Ans.* 150.7968 cu. ft.

3. The cu. in. in a bu., each end $18\frac{1}{2}$ in. in diameter, depth 8 in. *Ans.* 2150.4252 cu. in.

4. If the bu. contain 2150.4 cu. in., what are the solid contents of a cylindrical tub 6 ft. in diameter and 8 ft. deep? *Ans.* 181.764 bu.

5. How many bu. in a box 15 ft. long, 5 ft. wide, 4 ft. deep? *Ans.* 241+ bu.

6. How many bu. in a box 12 ft. long, 3 ft. wide, 5 ft. deep? *Ans.* 144.6+ bu.

ART. 323. TO FIND THE SOLID CONTENTS OF A PYRAMID OR OF A CONE.

Rule.—*Multiply the area of the base by the perpendicular hight, and take one-third of the product.*

1. Find the solid contents of a square pyramid, the base, 5 ft. each side; perpendicular hight, 21 ft. *Ans.* 175 cu. ft.

2. The solid contents of a cone, base 10 ft. in diameter; perpendicular hight, 15 ft. *Ans.* 392.7 cu. ft.

3. The diameter of the base of a conical glass-house is 37 ft. 8 in.; the altitude, 79 ft. 9 in.: what space is inclosed? *Ans.* 29622.0227+cu. ft.

4. A sq. pyramid is 477 ft. high; each side of its base 720 ft.: find the contents in cu. yd. *Ans.* 3052800 cu. yd.

5. How often can a conical cup 9 in. deep, $1\frac{1}{2}$ in. diameter, be filled from a gal. or 231 cu. in.? *Ans.* 43.57+times.

ART. 324. TO FIND THE SOLID CONTENTS OF THE FRUSTUM OF A PYRAMID, OR OF THE FRUSTUM OF A CONE.

Rule.—1st. *Find the area or surface of each end by the preceding rules.* 2d. *Find the area of the mean base by multiplying the areas of the upper and lower bases together, extracting the square root of the product.* 3d. *Add together the areas of the upper, the lower, and the mean base; multiply their sum by one-third of the altitude, the product will be the solid contents.*

1. Find the solid contents of a block with square ends, each side of the lower base 3 ft.; and of the upper base 2 ft.: the altitude, 12 ft. *Ans.* 76 cu. ft.

2. The length of the frustum of a sq. pyramid is 18 ft. 8 in.; the side of its greater base 27 in.; that of its lesser base, 16 in.: what the contents? *Ans.* 61.2283950+cu. ft.

3. Find the contents of a glass in the form of the frustum of a cone; the diameter at the mouth $2\frac{1}{2}$ in.; at the bottom, 1 in.; the depth 5 in. *Ans.* 12.76275 cu. in.

ART. 325. TO FIND THE SOLID CONTENTS OF A GLOBE.

Rule.—*Multiply the cube of the diameter by* .5236

1. What are the solid contents of 3 globes, their diameters being 13, 15, and 30 inches, respectively?
Ans. 1150.3492; 1767.15; and 14137.2 cu. in.

Art. 326. To find the Area of the Surface of a body bounded by plane surfaces.

Rule.—*Find the area of the surfaces separately; then add.*

To find the Area of the Curved Surface of a Right Cone.

Rule.—*Multiply the circumference of the base by the slant hight, and take half the product.*

To find the Surface of a Globe.

Rule.—*Multiply the square of the diameter by* 3.1416

1. Each side of the base of a triangular pyramid is 5 ft. 4 in.; its slant hight, from the vertex to center of each side of the base 7 ft. 6 in.: find the area of its surface. *Ans.* 60 sq. ft.

2. What is the convex surface of a cone, whose side is 25 ft., and diameter of the base $8\frac{1}{2}$ ft.? *Ans.* 333.795 sq. ft.

3. Find the area of the curved surface and base of a right cone, the slant hight 4 ft. 7 in.; the diameter of its base 2 ft. 11 in. *Ans.* 27.679895+sq. ft.

4. If the earth be a perfect sphere 7912 mi. in diameter, what its superficial contents? *Ans.* 196663355.7504 sq. mi.

Art. 327. GAUGING

Is the method of finding the contents of any regular vessel, in gallons, bushels, barrels, &c.

When the vessel is in the form of a cube or parallelopiped, apply this

Rule.—*Take the dimensions in inches and multiply the length, breadth, and depth together;*

This product divided by 231, *will give the contents in wine gal.; or, divided by* 2150.4, *will give the contents in bu.*

1. How many wine gallons in a trough, 10 ft. long, 5 ft. wide, and 4 ft. deep? *Ans.* 1496+gal.

2. How many bushels in a box, 12 ft. long, 6 ft. wide, and 10 ft. deep? *Ans.* 578.57+bu.

Art. 328. To find the Contents of a Cistern, both ends Circular, the Upper and Lower Diameters equal.

Rule.—*Take all the dimensions in inches; then square the diameter, multiply this by the hight, and this product by* .7854; *this will give the contents in cubic inches, and this divided* *by* 231 *will give the contents in wine gallons, which may be reduced to barrels by dividing by* 31.5.

NOTE.—Since .7854÷231=.0034, therefore, when required to multiply by .7854 and divide the product by 231, shorten the operation (Art. 61), by multiplying at once by .0034.

1. How many barrels in a cistern, the diameter being 4 ft., the depth 6 ft.? *Ans.* 17.9+ bl.

2. How many barrels in a cistern, the diameter 6 ft., and depth 9 ft.? *Ans.* 60.43+ bl.

ART. **329**. TO FIND THE CONTENTS OF A CISTERN, BOTH ENDS CIRCULAR, AND DIAMETERS UNEQUAL.

Rule.—*Having the dimensions in inches, multiply together the diameters of the two ends; to the product add one-third of the square of the difference between these diameters:*

Then multiply this sum by the hight, and the product by .7854: *the result will be cu. in., which reduce as in* Art. 328.

1. What the contents, in wine gallons, of a cistern, the upper diameter 40 in.; lower diameter 30 in.; depth 50 in.? *Ans.* 209.66+ gal.

2. The contents in bl., of a cistern, the upper diameter 7 ft. 6 in.; lower diameter 10 ft.; depth, 12 ft. 6 in.? *Ans.* $179\frac{5}{7}$ bl.

ART. **330**[a]. TO FIND THE CONTENTS OF A CASK OR BARREL.

When the staves are straight from the bung to each end, consider the cask as the frustums of two equal cones and find the contents by the rule, Art. 324.

When the staves are curved, find the *mean* diameter of the cask, by adding to the head diameter, two-thirds of the difference between the bung and head diameters; or,

If the staves are but little curved, add six-tenths of the difference; and,

Having the mean diameter, find the solidity in the same manner as that of a cylinder, Art. 322.

Since multiplying the square of half the mean diameter by 3.1416 is the same as to multiply its square by .7854; and multiplying by .7854 and dividing by 231, (Art. 328, Note,) is the same as to multiply by .0034;

Therefore, to find the contents of a cask, when its dimensions are in inches, apply the following:

Rule.—*Multiply the square of the mean diameter by the length, and that product by* .0034; *the result will be wine gal.*

1. How many gallons in a cask, the staves much curved, the bung diameter 40 in., head diameter 31 in., length 50 in.?
Ans. 232.73 gal.

2. Find the contents of a cask, the staves nearly straight, bung diameter 32 in., head diameter 30 in., length, 40 in.
Ans. 132.38+ gal.

Art. 330^{b}. MISCELLANEOUS EXAMPLES.

1. A rectangular field is 15 rods long: what must be its breadth to contain one acre? *Ans.* $10\frac{2}{3}$ rd.

2. How many cubic feet in a room 24 ft. long, 18 ft. 6 in. wide, 10 ft. 7 in. high? *Ans.* 4699 cu. ft.

3. The area of a circle is 1 sq. ft.: what its diameter?
Ans. 13.5405+ in.

4. The solid contents of a globe are 1 cu. ft.: what the diameter? *Ans.* 14.8884+ in.

5. The sides of a triangle are 30, 40, and 50 feet: required the area. *Ans.* $66\frac{2}{3}$ sq. yd.

6. How many sq. ft. in a plank 12 ft. 6 in. long; one end 15 in. broad, the other 11 in.? *Ans.* $13\frac{13}{24}$ sq. ft.

7. Two circles, 10 and 16 ft. in diameter, have the same center: what their difference of area? *Ans.* 122.5224 sq. ft.

8. What will it cost to line a rectangular cistern, 6 ft. long, 2 ft. broad, 2 ft. 6 in. deep, with sheet lead, at 4 cts. a lb.; allowing 8 lb. of lead to each sq. ft. of surface? *Ans.* $16.64

9. At 25 cts. a bushel, what will oats cost to fill a bin 5 ft. long, 4 ft. wide, 4 ft. deep? *Ans.* $16.07+

10. What is the area of a circle, of which the circumference is 1448 ft.? *Ans.* 3 A. 3 R. 12 P. 25 sq. yd. 8+ sq. ft.

11. Required the surface of a cube, each side being 37 in.
Ans. 8214 sq. in.

Art. **331**. MECHANICAL POWERS.—This subject properly belongs to a more advanced work, and will be found appropriately treated in "*Ray's Higher Arithmetic.*"

Art. 332.—100 Promiscuous Questions.

1. The sum of three equal numbers is 1236: what is one of the numbers? *Ans.* 412.

2. The sum of two equal numbers, less 225, is 675: find one of the numbers. *Ans.* 450.

3. There are four equal numbers, whose sum divided by 3, is 292: find one of them. *Ans.* 219.

4. What cost 5 lb. 15 oz. of tea, at \$1.20 per lb.? *Ans.* \$7.12$\frac{1}{2}$

5. What cost 13 bu. 3 pk. potatoes, at \$1.45 per bu.? *Ans.* \$19.93$\frac{3}{4}$

6. Two men, A and B, purchased a farm of 320 acres; A paid \$1000, and B paid \$600: how many acres should each receive? *Ans.* A, 200; B, 120 acres.

7. In what time will a man, walking at the rate of 3$\frac{3}{4}$ miles an hour, travel 42$\frac{1}{2}$ miles? *Ans.* 11 hr. 20 min.

8. What number multiplied by 1$\frac{3}{8}$ will = 14$\frac{3}{4}$? *A.* 10$\frac{8}{11}$.

9. I have a number in my mind, which $\times$ 3, = 81 less than when $\times$ 6: what is the number? *Ans.* 27.

10. A man bought 4 yd. of cloth at \$$\frac{3}{5}$ per yd., and 10 yd. at \$$\frac{7}{8}$ per yd.: he paid with muslin at \$$\frac{1}{10}$ per yd.: how many yards were required? *Ans.* 111$\frac{1}{2}$ yd.

11. After spending $\frac{2}{3}$ of my money and $\frac{1}{4}$ of what was left, I had \$125 remaining: what sum had I at first? *Ans.* \$500.

12. Multiply the sum of 2$\frac{7}{16}$ and 1$\frac{5}{8}$ by their difference, expressing the product decimally. *Ans.* 3.30078125

13. I was married at the age of 21: if I live 19 yr. longer, I will have been married 60 yr.: what is my age? *Ans.* 62 yr.

14. Find the least Com. Mult. of 8, 12, 21, 36, and 48, and divide it by the greatest Com. Div. of 65 and 143. *Ans.* 77$\frac{7}{13}$.

15. How many French meters, each 39.371 English inches, are there in 3 mi. 5 fur. 110 yd.? *Ans.* 5934.317+

16. In what time can you count 800000000, at the rate of 250 a min., counting 10 hr. a da., 365 da. to the yr.? *Ans.* 14 yr. 223 da. 3 hr. 20 min.

17. Divide 12.625 by 16$\frac{2}{3}$. *Ans.* .7575

18. What does the rent of a house amount to from May 20, 1854, to May 10, 1855, at $250 per year? *Ans.* $\$243\frac{1}{18}$.

19. I bought an equal quantity of flour, butter, and sugar, for $47; the sugar was 12 cts., the butter 30 cts., and the flour 5 cts., a pound: how much of each did I buy? *Ans.* 100 lb.

20. A cistern is $\frac{2}{3}$ full of water; after 35 gal. are taken out, it is $\frac{3}{8}$ full: how many gal. will it contain? *Ans.* 120 gal.

21. I bought 60 barrels of flour at $5 a barrel; sold 23 barrels at $4 a bl.: at how much per bl. must I sell the rest, to gain $51 on the whole? *Ans.* $7.

22. How many boxes of 3 qr. 13 lb. each, can be filled from a hhd. of sugar containing 12 cwt. 1 qr. 7 lb.? *Ans.* 14.

23. What will it cost to gild a globe 10 inches in diameter, at 5 cents per square inch? *Ans.* $15.708

24. If 1 ox is worth 8 sheep, and 3 oxen are worth 2 horses, what is the value of each horse, the sheep being valued at $2.50 each? *Ans.* $30.

25. If $\frac{2}{3}$ of $1 buy $\frac{1}{5}$ of a sheep, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of an ox, what will 10 oxen cost? *Ans.* $200.

26. What number has to 54 the same ratio that 19 has to 9? *Ans.* 114.

27. Two-thirds of the ratio of $\frac{1}{2}$ to $\frac{3}{4}$, is three times the ratio of 3 to what? *Ans.* 1.

28. By working 13 hr. a day, a man can perform a piece of work in $5\frac{1}{4}$ days: in what time can he perform it by working 9 hr. a day? *Ans.* $7\frac{7}{12}$ da.

29. I bought 50 lb. of tea for $40, and sold it so as to clear $15: had I purchased $100 worth of tea, and sold it at the same rate, what sum would I have made? *Ans.* $37.50

30. A clock gains $7\frac{1}{2}$ min. in 24 hr. It is set right at noon on Monday: what will be the time by it at 6 o'clock on the following Thursday evening? *Ans.* 6 hr. $24\frac{3}{8}$ min.

31. If 7 men can mow 35 acres of grain in 4 days, how many acres will 10 men mow in $3\frac{1}{2}$ days? *Ans.* $43\frac{3}{4}$ A.

32. Bought $35\frac{7}{8}$ yd. linen at $\$\frac{3}{4}$ per yd., and sold $16\frac{1}{4}$ yd. at $\$1\frac{1}{4}$ per yd., and the rest at $\$\frac{5}{8}$ per yd.: what the gain by the transaction? *Ans.* $5.67+

33. If a man can build 10 cu. ft. of wall in an hr., what length of wall, 5 ft. high, and 2 ft. thick, can he build in 6 da., working 11 hr. a da.? *Ans.* 66 ft.

34. If 4 men *or* 6 women do a piece of work in 20 da., how soon can 3 men *and* 5 women do it? *Ans.* $12\frac{12}{19}$ da.

35. I bought 40 yd. of cloth at the rate of 5 yd. for $6, and 60 yd. more at the rate of 6 yd. for $9: I sold the whole at the rate of 5 yd. for $7: what did I gain? *Ans.* $2.

36. From a vessel containing 50 gal. wine, 10 gal. were drawn off and the vessel filled with water: if 10 gal. more be drawn, how many gal. *pure* wine will remain? *Ans.* 32 gal.

37. If 27 men do a piece of work in 14 days, working 10 hr. a day, how many hours a day must 24 boys work, to perform it in 45 da., 2 boys being equal to 1 man? *Ans.* 7 hr.

38. A ship starts at noon and sails west, 9 hours, going 16′ 40″ an hour: what time will it be at the place reached? *Ans.* 10 min. before 9.

39. If 10 men in 10.2 days of 9 hours each, dig a trench 20.4 ft. long, 3.3 ft. wide, 1.5 ft. deep, in how many days of 10 hours each, can 20 men dig one 40 ft. long, 4.5 ft. wide, 1.1 ft. deep? *Ans.* 9 da.

40. What was the cost of an article, which, when sold for $14, paid a profit of 20 %? *Ans.* $$11\frac{2}{3}$.

41. Two men hired a pasture for $45; A put in 4 horses, and B, 9 cows. If 3 cows equal 2 horses, what sum must each pay? *Ans.* A, $18; B, $27.

42. Two men, having started at the same time to travel toward each other met in $2\frac{1}{3}$ hr., one traveled 5 mi. an hr. faster than the other, and both together traveled 35 mi.: at what rate per hr. did each travel? *Ans.* $4\frac{1}{2}$; and $9\frac{1}{2}$ mi.

43. I sold corn for $14.85, and lost $17\frac{1}{2}$ %: for what should I have sold it to have gained $12\frac{1}{2}$ %? *Ans.* $20.25

44. How much grain must I take to mill, so that I shall have 2 bu. left for grinding, after paying toll at the rate of 4 qt. to the bushel? *Ans.* 2 bu. 1 pk. $1\frac{1}{7}$ qt.

45. If 32 men have food for 5 mon., how many must leave, for the food to last the rest 8 mon.? *Ans.* 12.

46. Received $1009.29 for a note having 60 da. to run, discounted in bank at 6 %: how much should I have received for it, discounted by true discount, at 12 %? *Ans.* $1000.

47. I bought 20 yd. cloth at 5 % less than first cost, and sold it at 10 % more than first cost; I gained $12: what was the first cost per yard? *Ans.* $4.

48. Bought a metallic plate 9 in. square, and $\frac{1}{2}$ in. thick, for \$3.24; what should I pay for a plate of the same metal 1 ft. 2 in. long, 11 in. wide, and $\frac{5}{8}$ in. thick? *Ans.* \$7.70

49. A vessel at sea has 120 persons on board, and provisions sufficient to last 3 mon.; they take from a wreck 60 persons more, how long will their provisions last? *Ans.* 2 mon.

50. A fox is 47 rd. before a dog; the fox runs 20 rd. and the dog 25 rd. in a minute: how many rods must the dog run to catch the fox? *Ans.* 235 rd.

51. For what sum must I give my note at the Bank of Boston, payable in 4 months, at 6 % discount, to obtain \$300?
Ans. \$306.27+

52. I have 40 gallons of wine worth \$1.50 per gal., which I wish to reduce to \$1.20 per gal.: how much water must I add? *Ans.* 10 gal.

53. A bank charged \$1.26 for discounting my note at 60 days: what was the amount of the note, the rate per cent. being 6? *Ans.* \$120.

54. A hare having 45 rd. the start of a dog, can run 25 rd. while the dog runs 28 rd.: how many rods must the dog run to catch the hare? *Ans.* 420 rd.

55. Wheat sold for \$1.50 per bu., pays a profit of $\frac{1}{5}$ of the cost: if sold for \$2 a bu. what % would it pay? *Ans.* 60 %.

56. I spent \$260 for apples at \$1.30 per bu.; after retaining a part for my own use, I sold the rest at 40 % profit, and cleared \$13 on the cost of the whole: how many bushels did I retain? *Ans.* 50 bu.

57. Two men start from the same place, and travel the same road: A at the rate of 8 miles an hour, and B at the rate of 10 miles an hour; A starts 5 hours before B: in what time will B overtake A? *Ans.* 20 hr.

58. A can mow 2 A. in 3 da.; B, 5 A. in 6 da.: in how many da. can they both together mow 9 A.? *Ans.* 6 da.

59. A and B together do a piece of work in 20 days: in what time can each do it separately, if A does three times as much as B? *Ans.* A, $26\frac{2}{3}$ da.; B, 80 da.

60. I bought two equal lots of oranges: for the first, I gave 5 cts. for 2 oranges; for the second, 8 cts. for 3: I sold them at the rate of 5 oranges for 14 cts., and gained 52 cts.: what number did I buy? *Ans.* 240 oranges.

61. A can do a piece of work in $2\frac{1}{2}$ da., B in $3\frac{1}{5}$ da.: in what time can both do it working together? *Ans.* $1\frac{23}{57}$ da.

62. A and B, at opposite points of a field 135 rd. in compass, start to go round it the same way, at the same time: A at the rate of 11 rd. in 2 min., and B, 17 rd. in 3 min.: how many rounds will each make, before the one will overtake the other? *Ans.* A, $16\frac{1}{2}$; B, 17.

63. A can mow a field in 3 days, B in 5 days, and B and C in 4 days: in what time can all three together mow it? *Ans.* $1\frac{5}{7}$ da.

64. A and B together can do a piece of work in 15 days; A and C in 12 days; B and C in 10 days: how long will it take all together to do it? *Ans.* 8 da.

65. I mixed 30 lb. of sugar at 10 cts. per pound; 25 lb. at 12 cts. per pound; 4 lb. at 15 cts. per pound; and 50 lb. at 20 cts.: what is 1 lb. of the mixture worth? *Ans.* $15\frac{25}{109}$ cts.

66. If 10 men or 18 boys can dig 1 acre in 11 da.; find the number of boys whose assistance will enable 5 men to dig 6 acres in 6 days. *Ans.* 189 boys.

67. How many yd. of paper, 24 in. wide, will cover the walls of a room 15 ft. long, 12 ft. wide, 10 ft. high? *Ans.* 90 yd.

68. A can do a piece of work in $\frac{1}{3}$ of a day; B in $\frac{1}{5}$ of a day; and C in a day: in what time can all three do it working together? *Ans.* $\frac{1}{9}$ da.

69. If a regiment of soldiers be arranged in the form of a square, there will be 28 men on each side, and 50 men over: what the whole number of men? *Ans.* 834 men.

70. What is the diagonal of a square, each of the sides of which is 40 ft.? *Ans.* 56.56+ft.

71. A man after doing $\frac{3}{5}$ of a piece of work in 30 days, calls an assistant; both together complete it in 6 days: in what time could the assistant do it alone? *Ans.* $21\frac{3}{7}$ da.

72. A received $\frac{11}{15}$ of an estate, and B the remainder; A had \$7420 more than B: what was the value of the whole estate? *Ans.* \$15900.

73. A room is 24 ft. long, 18 ft. wide, and 12 ft. high: what is the distance, measured diagonally, from one of the lower corners to the opposite upper corner? *Ans.* 32.3+ft.

74. If 1 lb. of tea be worth $2\frac{1}{2}$ lb. of coffee, and 1 lb. of coffee be worth $3\frac{1}{2}$ lb. of sugar, what will be the value of 56 lb. of tea, sugar being worth 10 cts. per lb.? *Ans.* \$49.

75. A, B, and C, can together perform a piece of work in 12 days; A alone can do it in 24 days; B alone in 34 days: in what time can C do it alone? *Ans.* $81\frac{3}{5}$ da.

76. A, B, and C, built a wall of 200 feet; A built as many feet as C and $\frac{1}{3}$ more; B built $\frac{3}{4}$ as much as A: how many feet did each build? *Ans.* A, 80 ft.; B, 60 ft.; C, 60 ft.

77. How many acres in a square field, the diagonal being 42.43 rd.? *Ans.* 5.62+A.

78. A man cleared $19 in 25 days, by earning $1.25 each day he worked, and spending 50 cts. each day he was idle: how many days did he work? *Ans.* 18 da.

79. A can do a job in 40 da., B in 60 da.; after both work 3 da., A leaves: when must he return, that the work may occupy but 30 da.? *Ans.* At the end of the 13th da.

80. A gentlemen left $\frac{5}{14}$ of his estate to A: $\frac{2}{3}$ of the remainder to B; and the rest to C; A's share was $1300 more than C's: what was the share of each?

Ans. A, $3250; B, $3900; C, $1950.

81. A cistern of 500 gal. has two flow pipes: one can fill it in 3 hours, and the other in 5 hours; a third pipe being added, the cistern was filled in 1 hour: in what time would the third pipe fill it alone? *Ans.* $2\frac{1}{7}$ hr.

82. I bought 60 gal. of wine at $1.20 per gal.: I sold 20 gal. at $1.50 per gal., and retained 5 gal. for my own use: at how much per gal. must I sell the remainder to gain 10 % on the whole cost? *Ans.* 1.40\frac{4}{7}$

83. I bought apples at the rate of 3 for 4 cts., and sold them at the rate of 2 for 3 cts.; I cleared 24 cts. on the whole number purchased: how many did I buy? *Ans.* 144 apples.

84. Purchased a house for $4500: it rented for $500 a yr.; paid for insurance $25; taxes $1\frac{8}{10}$ %, and repairs $134: what net % did it pay on the investment? *Ans.* $5\frac{7}{9}$ %.

85. How many square feet of flooring in a house of 4 floors, 60 ft. by 30 ft. within the walls, deducting from each floor a stairway 12 ft. 4 in. by 8 ft. 6 in.? *Ans.* 6780 sq. ft. 8′.

86. A, B, and C are partners: A put in $700; B, $600; C, $400; C's share of the gain was $260: what was the whole gain? *Ans.* $1105.

87. Ten per cent. of 120 is 8 less than 5 % of what number? *Ans.* 400.

88. A and B have the same annual income: A saves $\frac{1}{3}$ of his, while B, who spends annually $350 more than A, at the end of four years, is in debt $120: what is the annual income of each? *Ans.* $960.

89. Two Globes, each 5 in. in diameter, and 2 cubes, each 5 in. in length, were melted into one cube: how long was the side of this cube? *Ans.* 7.24+in.

90. Sold a cow for \$25, losing $16\frac{2}{3}$ %; bought another and sold it at a gain of 16 %; I neither gained nor lost on the two: what the cost of each? *Ans.* 1st, \$30: 2d, \$31.25

91. After a battle, in which an army of 24000 men were engaged, it was found that the number of slain was $\frac{1}{7}$ of those who survived; the number wounded was equal to $\frac{1}{2}$ of the slain: find the number wounded. *Ans.* 1500.

92. I cut 95 yd of cloth into 3 pieces, so that the first was three times the second, and the third one-fourth the first: what was the length of each? *Ans.* 1st, 60; 2d, 20; 3d, 15 yd.

93. A merchant purchased 60 yd. of cloth, at \$4 a yard, on a credit of 6 mon.; he sold it immediately for \$250: what did he gain, money being worth 6 %? *Ans.* \$16.99+

94. A drover expended \$1500 in horses, cows and sheep: the horses cost \$120, the cows \$30, and the sheep \$5 each: there were six times as many sheep as cows, and $\frac{1}{3}$ as many horses as sheep: find the number of each.

Ans. 10 horses; 5 cows; 30 sheep.

95. The tax in a certain town was 1 ct. 6 m. on the dollar, and polls \$1 each; A, who paid for 4 polls, was charged \$328 tax: what the value of his estate? *Ans.* \$20250.

96. A, B, and C do a piece of work in a certain time: A and B together do $\frac{5}{9}$ of it; B and C together do $\frac{2}{3}$: what part can B do alone? *Ans.* $\frac{2}{9}$.

97. A and B together received \$1000: if B had received \$100 less, his share would then have been one-half of A's: how much did each receive? *Ans.* A, \$600; B, \$400.

98. A and B have just \$500: $\frac{3}{4}$ of A's money is \$50 less than $\frac{2}{3}$ of B's: what sum has each? *Ans.* A, \$200; B, \$300.

99. If $\frac{2}{3}$ of the time past noon is equal to $\frac{2}{9}$ of the time to midnight, what is the hour? *Ans.* 3 P. M.

100. A lady spent in one store $\frac{1}{2}$ of all her money and \$1 more; in another, $\frac{1}{2}$ of the remainder and \$$\frac{1}{2}$ more; in another, $\frac{1}{2}$ of the remainder and \$1 more; and in another, $\frac{1}{2}$ of the remainder and \$$\frac{1}{2}$ more; she then had nothing left: what sum had she at first? *Ans.* \$20.

APPENDIX.

THE METRICAL SYSTEM

OF WEIGHTS AND MEASURES.

333. The **Metrical System** is so called because the **Meter** is the base, and the principal and invariable unit, upon which the system is founded.

This system was first employed by the French; and, hence, is frequently called the *French System of Weights and Measures.* Great confusion formerly existed in the weights and measures of France. Each province had its particular measures, which caused great embarrassment in commerce.

Government vainly endeavored to establish a uniformity, and to regulate all measures by those used in Paris.

In 1790, the French Assembly proclaimed the necessity of a complete reform, and invited other governments to join them in establishing a simple system, to be common to all nations.

The coöperation of other nations could not at the time be secured, and a commission, nominated by the Academy of Sciences, and composed of eminent scholars, was instructed to prepare a general system of measures.

The new system was adopted, and declared obligatory after Nov. 2, 1801. But its introduction was gradual. It had to struggle against the local customs, and, for a time, only increased the confusion by adding the new measures to the old.

In 1837, the Assembly enacted a law, rendering the *exclusive* use of the new system obligatory after Jan. 1, 1841; and imposed penalties against the further use of the old system.

It has since been adopted by Spain, Belgium, and Portugal, to the exclusion of all other weights and measures; and is in general or partial use in nearly all the states of Europe and America, and by scientific men throughout the world.

In 1864, the British Parliament passed an act allowing the metrical system to be used throughout the Empire; and in

1866, Congress authorized its use in the United States, and provided for its introduction into post-offices for the weighing of letters and papers.

The metrical system, like Federal Money, is founded on the *decimal* system of notation.

After the principal unit of each denomination is determined and named, the names of the *higher* denominations are formed by prefixing the *Greek* numerals, *deka* (10), *hecto* (100), *kilo* (1000), and *myria* (10000), to the name of the unit.

The names of the *lower* denominations are formed, in like manner, by prefixing the *Latin* numerals, *deci* (.1), *centi* (.01), and *milli* (.001).

MEASURES OF LENGTH.

334. The **Meter** is the unit for measure of *lengths*, and is very nearly one ten-millionth ($\frac{1}{10000000}$) part of the quadrant extending through Paris from the equator to the pole.

NOTE.—The meter is equal to 39.37 inches, nearly, or a little less than 1.1 yards. It is also nearly 3 feet, 3 inches, and 3 eighths of an inch, which may be remembered as *the rule of the three threes.* Five meters are about one rod.

TABLE.

10 Meters, marked M.	=1 Dekameter,	marked	D. M.
10 Dekameters . .	=1 Hectometer,	"	H. M.
10 Hectometers . .	=1 Kilometer,	"	K. M.
10 Kilometers . .	=1 Myriameter,	"	M. M.

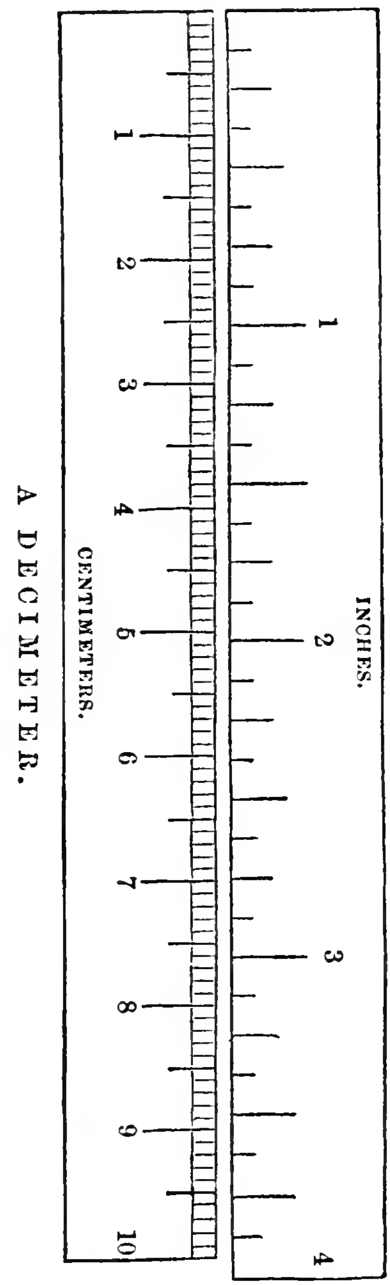
ALSO,

$\frac{1}{10}$ of a Meter . .	=1 Decimeter,	marked	d. m.
$\frac{1}{10}$ of a Decimeter .	=1 Centimeter,	"	c. m.
$\frac{1}{10}$ of a Centimeter .	=1 Millimeter,	"	m. m.

The figure in the margin represents the *exact* length of a decimeter, divided into 10 equal parts or centimeters, and these subdivided into 10 equal parts, or millimeters.

The adjoining figure represents a scale of four inches.

This illustration will aid the pupil in gaining a clearer idea of the relative length of the different *units of measure* of the two systems.



Meters are usually separated from the lower denominations by the decimal point. Thus, 7 meters, 3 decimeters, and 5 centimeters, are written 7.35 M.; read, 7 meters and 35 centimeters.

Great distances are reckoned by kilometers; thus, the length of a canal may be given as 48 kilometers, 7 hectometers; written, 48.7 K. M. In like manner, if the distance between two cities is 32 myriameters, 5 kilometers, it is written, 325. K. M.

Very small distances are reckoned by millimeters; for example, if the thickness of a piece of glass is 8 millimeters and 5 tenths, it is written, 8.5 m. m.

Millimeters, centimeters, meters, and kilometers, are the denominations most used; decimeters, dekameters, hectometers, etc., are not used in ordinary transactions; as in Federal money, the denominations of eagles, dimes, and mills are unused.

EXAMPLE.—How many millimeters in 4.27 M.?

SOLUTION.—Since 1 M. is equal to 1000 m. m., 4.27 M. will be equal to 4.27×1000 m. m., which is 4270 m. m.

OPERATION.

$$\begin{array}{r} 4.27 \\ 1000 \\ \hline 4270.00 \text{ m.m.} \end{array}$$

CONCLUSION.—Therefore, 4.27 M. equals 4270 millimeters. Hence, the

Rule for Reduction Descending.—*Multiply the given quantity by that number of the lower denomination which makes a unit of the required denomination.*

NOTE.—Since the multiplier is always 10, 100, 1000, etc., the operation may be performed by removing the point as many places to the right as there are ciphers in the multiplier.

EXAMPLE.—How many kilometers in 36429 M.?

SOLUTION.—Since 1000 meters = 1 kilometer, 36429 M. = as many kilometers, as 1000 is contained times in 36429; 36429 ÷ 1000 = 36.429 K. M.

OPERATION.

$$\begin{array}{r} 1000\,)\,36429 \\ \hline 36.429 \text{ K.M.} \end{array}$$

CONCLUSION.—Therefore, 36429 M. equal 36.429 K. M. Hence, the

Rule for Reduction Ascending.—*Divide the given quantity by that number of its own denomination which makes a unit of the required denomination.*

NOTE.—Since the divisor is always 10, 100, 1000, etc., the operation may be performed by removing the decimal point as many places to the left as there are ciphers in the divisor.

EXAMPLES FOR PRACTICE.

1. Reduce 30.75 M. to centimeters. *Ans.* 3075.
2. Reduce 4.5 K. M. to meters. *Ans.* 4500.
3. Reduce 75 m. m. to meters. *Ans.* .075.
4. Reduce 25 D. M. to decimeters. *Ans.* 2500.
5. Reduce 35 M. M. to hectometers. *Ans.* 3500.
6. Reduce 36.5 M. to dekameters. *Ans.* 3.65.
7. Reduce 36.5 M. to decimeters. *Ans.* 365.

MEASURES OF SURFACE.

335. The **Square Meter** is the unit of Square Measure. It is a square of which each side is a meter.

The square meter is equal to 100 square decimeters; for, if a square is constructed, each side of which is 1 meter, or 10 decimeters long, the area will be $10 \times 10 = 100$ *square* decimeters; hence, one square decimeter equals $\frac{1}{100}$ of a square meter.

TABLE.

100	Sq. Meters, ($M.^2$)	=1	Sq.	Dekameter,	marked	$D. M.^2$
100	" Dekameters,	=1	"	Hectometer,	"	$H. M.^2$
100	" Hectometers,	=1	"	Kilometer,	"	$K. M.^2$
100	" Kilometers,	=1	"	Myriameters,	"	$M. M.^2$

Also,

$\frac{1}{100}$	Sq. Meter . . .	=1	Sq.	Decimeter,	marked	$d. m.^2$
$\frac{1}{100}$	" Decimeter .	=1	"	Centimeter,	"	$c. m.^2$
$\frac{1}{100}$	" Centimeter .	=1	"	Millimeter	"	$m. m.^2$

Note.—Since 100 units of each order make one of the next higher, each order should occupy *two* places. Thus, 534.260780 $M.^2$ signifies 5 square decimeters, 34 square meters, 26 square decimeters, 7 square centimeters, and 80 square millimeters;

and may be read, five hundred and thirty-four square meters, two hundred and sixty thousand, seven hundred and eighty square millimeters.

EXAMPLES FOR PRACTICE.

1. Reduce 26304 d. m.^{2} to M^2. *Ans.* 263.04.
2. Reduce 1460 m. m.^{2} to M^2. *Ans.* .00146.
3. Reduce 14 D. M.^{2} to d. m^2. *Ans.* 140000.
4. Reduce 20 M. M.^{2} to D. M^2. *Ans.* 20000000.
5. Reduce 3294 M.^{2} to H. M^2. *Ans.* .3294.
6. Reduce 14.1 H. M.^{2} to M^2. *Ans.* 141000.

336. In measuring *land*, only three of the denominations of square measure are used, and these receive shorter names.

The **Are,** or square dekameter, is the unit of land measure.

NOTE.—The Are is equal to 119.6 square yards. The Are equals 100 square meters; the Centare, 1 square meter; and the Hectare, 10000.

TABLE.

100 Ares, (A.) . . =1 Hectare, marked H. A.

ALSO,

$\frac{1}{100}$ Are, ... =1 Centare, marked c. a.

EXAMPLES FOR PRACTICE.

1. Reduce 26.25 A. to centares. *Ans.* 2625.
2. Reduce 21 H. A. to ares. *Ans.* 2100.
3. Reduce 3.5 H. A. to M^2. *Ans.* 35000.
4. Reduce 14209 M.^{2} to ares. *Ans.* 142.09.
5. Reduce 3.8 A. to square meters. *Ans.* 380.
6. Reduce 27 d. m.^{2} to centares. *Ans.* .27.

CUBIC MEASURE.

337. The **Stere**, or *cubic meter*, is the unit of cubic or solid measure.

The cubic meter equals 1000 cubic decimeters; for, if a cube be taken, each side of which is one meter or ten decimeters, the cubic contents will equal $10\times10\times10=1000$ cubic decimeters; therefore, one cubic decimeter equals $\frac{1}{1000}$ of a cubic meter.

NOTE.—A cubic meter equals 1.308 cubic yards.

TABLE.

$\frac{1}{1000}$	Cubic M., (M.^{3})	=1	Cubic	d. m ,	marked	d. m.^{3}
$\frac{1}{1000}$	" d. m. . .	=1	"	c. m.,	"	c. m.^{3}
$\frac{1}{1000}$	" c. m . .	=1	"	m. m.,	"	m. m.^{3}

All solid bodies, except wood, are measured by cubic meters, and the lower denominations.

For the measurement of fire-wood and building timber, use the following

TABLE.

10 Steres, (S.) . . =1 Dekastere, marked D. S.

ALSO,

$\frac{1}{10}$ Stere . . . . . =1 Decistere, marked d. s.

EXAMPLES FOR PRACTICE.

1. Reduce 9 S. to decisteres. *Ans.* 90.
2. Reduce 4.19 d. s. to dekasteres. *Ans.* .0419.
3. Reduce 29 M.^{3} to c. m.^{3} *Ans.* 29000000.
4. Reduce 25 D. S. to decisteres. *Ans.* 2500.
5. Reduce 16.32 M.^{3} to dekasteres. *Ans.* 1.632.
6. Reduce 14000 c. m.^{3} to cubic meters. *Ans.* .014.

WEIGHTS.

338. The **Gram** is the unit of weight. It is used in weighing *all* substances.

NOTE.—The gram is equal to 15.432 Troy grains, which is the weight, in vacuum, of a cubic centimeter of water, at 39.2° Fahrenheit. At this temperature water attains its greatest density.

TABLE.

10 Grams, (G.)	=1	Dekagram,	marked	D. G.
10 Dekagrams	=1	Hectogram,	"	H. G.
10 Hectograms	=1	Kilogram,	"	K. G.

ALSO,

$\frac{1}{10}$ Gram . .	=1	Decigram,	marked	d. g.
$\frac{1}{10}$ Decigram .	=1	Centigram,	"	c. g.
$\frac{1}{10}$ Centigram .	=1	Milligram,	"	m. g.

The gram and its subdivisions are used in mixing and compounding medicines, and in all cases where great exactness is required.

The new five-cent coin, made of nickel and copper, and dated 1866, is 20 millimeters in diameter, and weighs 5 grams.

The kilogram is the principal and ordinary unit of weight, and is a little more than $2\frac{1}{5}$ pounds avoirdupois.

There are two other denominations employed in weighing heavy articles, the *quintal*, which equals 100 kilograms; and the *tonneau*, which equals 1000 kilograms.

The tonneau is greater than the short ton of 2000 pounds, and less than the long ton of 2240 pounds. Its exact weight is 2204.6 pounds Avoirdupois.

EXAMPLES FOR PRACTICE.

1. Reduce 1428.06 G. to kilograms. *Ans.* 1.42086.
2. Reduce 2.8 K. G. to grams. *Ans.* 2800.
3. Reduce 119 H. G. to decigrams. *Ans.* 119000.
4. Reduce 171300 G. to quintals. *Ans.* 1.713.
5. Reduce 3600 G. to kilograms. *Ans.* 3.6.
6. Reduce 19200 m. g. to grams. *Ans.* 19.2.
7. Reduce 4 quintals to grams. *Ans.* 400000.
8. Reduce 29 G. to centigrams. *Ans.* 2900.
9. Reduce 492 D. G. to quintals. *Ans.* .0492.

MEASURES OF CAPACITY.

339. The **Liter** is the unit of capacity. It is equal to a cubic decimeter.

NOTE.—The liter equals 1.0567 quarts, U. S. Wine measure. The denominations most used in this table are liter and hectoliter.

TABLE.

10 Liters, (L)	=1	Dekaliter,	marked	D. L.
10 Dekaliters	=1	Hectoliter,	"	H. L.
10 Hectoliters	=1	Kiloliter,	"	K. L.

ALSO,

$\frac{1}{10}$ Liter . .	=1	Deciliter,	marked	d. l.
$\frac{1}{10}$ Deciliter .	=1	Centiliter,	"	c. l.
$\frac{1}{10}$ Centiliter .	=1	Milliliter,	"	m. l.

The liter is equal to the cubic decimeter; the milliliter being the one-thousandth part of a liter, equals a cubic centimeter, and the kiloliter equals a cubic meter. The kiloliter or cubic meter of water weighs one tonneau; the liter of water weighs a kilogram; and the milliliter of water weighs a gram.

EXAMPLES FOR PRACTICE.

1. Reduce 24 L. to centiliters. *Ans.* 2400.
2. Reduce 183 H. L. to deciliters. *Ans.* 183000.
3. Reduce .02345 K. L. to liters. *Ans.* 23.45.
4. Reduce 1.4 L. to milliliters. *Ans.* 1400.
5. Reduce 3 M^3 to liters. *Ans.* 3000.
6. Reduce 40360 L. to hectoliters. *Ans.* 403.6.
7. Reduce 1400 L. to cubic meters *Ans.* 1.4.

340. In order that the pupil may become thoroughly acquainted with the value of the different denominations of the Metrical System, those in ordinary use are presented in the following TABLES:

MEASURES OF LENGTH.

NAMES.	Value in Meters.	Value in ordinary denominations.
Myriameter.	10000.	6.2137 miles.
Kilometer.	1000.	.62137 mile.
Hectometer.	100.	$328\frac{1}{12}$ feet.
Dekameter.	10.	393.7 inches.
Meter.	1.	39.37 inches.
Decimeter.	.1	3.937 inches.
Centimeter.	.01	.3937 inch.
Millimeter	.001	.0394 inch.

MEASURES OF SURFACE.

NAMES.	Value in M^2	Value in ordinary denominations.
Hectare.	10000.	2.471 acres.
Are.	100.	119.6 sq. yds.
Centare.	1.	1550 sq. in

MEASURES OF CAPACITY.

NAMES.	Value in Liters.	in Dry Mea're.	in Wine Mea're.
Kiloliter, or Stere.	1000.	1.308 cu. yds.	264.17 gal.
Hectoliter.	100.	2.8375 bush.	26.417 gal.
Dekaliter.	10.	9.08 qt.	2.6417 gal.
Liter.	1.	.908 qt.	1.0567 qt.
Deciliter.	.1	6.1022 cu. in	.845 gill.
Centiliter.	.01	.6102 cu. in.	.338 fluid oz.
Milliliter.	.001	.061 cu. in.	.27 fluid dra'm

WEIGHTS.

NAMES.	Value in Grams.	Quan'y of Water.	Ordinary weights.	
Tonneau.	1000000.	1 M^3, or 1 K.L.	2204.6 lb.	Av'pois.
Quintal.	100000.	1 Hectoliter.	220.46 lb.	"
Myriagram.	10000.	10 Liters.	22.046 lb.	"
Kilogram.	1000.	1 Liter.	2.2046 lb.	"
Hectogram.	100.	1 Deciliter.	3.5274 oz.	"
Dekagram.	10.	10 c. $m.^3$	.3527 oz.	"
Gram.	1.	1 c. $m.^3$	15.432 gr.	Troy.
Decigram.	.1	$\frac{1}{10}$ c. $m.^3$	1.5432 gr.	"
Centigram.	.01	10 m. $m.^3$	.1543 gr.	"
Milligram.	.001	1 m. $m.^3$	.0154 gr.	"

341. MISCELLANEOUS EXAMPLES.

1. The new 5-cent piece weighs 5 grams: how much will 100 such pieces weigh? *Ans.* 500 G.

2. Ten liters of a certain liquid weigh 92 K.G.: what is the weight of a deciliter? *Ans.* 920 G.

3. One hectogram of goods costs $5.35: what costs one kilogram? *Ans.* $53.50.

4. One decistere of wood is worth 28 cents: what is a stere worth? *Ans.* $2.80.

5. A hectoliter of wheat costs $6.25: what is the price of a dekaliter? *Ans.* $.625.

6. A hectoliter of wine costs $25.10: what is the price of a liter? *Ans.* $.251.

7. A kilogram of wool costs $1.875: what will be the cost of 100 kilograms, or 1 quintal? *Ans.* $187.50.

8. A liter of wine weighs 880 G.: what is the weight of a hectoliter? *Ans.* 88 K. G.

9. A wine merchant sold 1270 liters, 487 liters, 1563 liters, and 2345 liters: how many hectoliters did he sell? *Ans.* 56.65 H. L.

10. A vase weighing 24.67 K. G., contains 18.79 K. G. of liquid: what is the weight of the empty vase? *Ans.* 5.88 K. G.

11. A courier traveled 135 K. M. in one day: how far can he travel in 45 days? *Ans.* 6075 K. M.

12. How much will 135.60 M. of cloth cost at $1.16 a meter? *Ans.* $157.296.

13. Bought 25 hogsheads of wine, of 225 liters each, at the rate of $.156 a liter: how much did it cost? *Ans.* $877.50.

14. What is the cost of 21 pieces of cloth of 42 M. each, at $5.69 a meter? *Ans.* $5018.58.

15. A man bought 3.80 M. of velvet at $2.16 a meter: how much did it cost him? *Ans.* $8.208.

16. What will be the price of 5 deciliters of wine at 28 cents a liter? *Ans.* $.14.

17. What will be the cost of 45 H. A. of land at $3.32 an are? *Ans.* $14940.

18. What is the price of 15 L. of wine, if it costs $14.06 a hectoliter? *Ans.* $2.109.

19. A merchant paid $457.92 for cloth at $3 a meter: how many meters did he buy? *Ans.* 152.64 M.

20. A man traveled 665 K. M. in 7 days: how far did he go each day? *Ans.* 95 K. M.

21. A manufacturer bought 380 steres of wood for $454.10: how much was that a stere? *Ans.* $1.195.

22. If 235 L. of wine cost $34.545, what is the cost of one liter? *Ans.* $.147.

23. A tower is 142.695 M. high; it has 453 steps: what the height of each? *Ans.* .315 M.

24. What is the cost of 248.35 M. of gold thread, at the rate of $1.58 a meter? *Ans.* $392.393.

25. If 1 L. of a liquid weighs 1.25 K. G., how much will 25.8 L. weigh? *Ans.* 32.25 K. G.

26. I bought 346.75 K. G. of coffee at 56 cents a kilogram: what did it cost me? *Ans.* $194.18.

27. A traveler has to go 548 K. M. in 14 days: how far ought he to go in one day? *Ans.* 39.14+ K. M.

28. What must be paid for 367 S. of wood, at $3.175 a stere? *Ans.* $1165.225.

29. How many liters in 9684 flasks, each containing .25 L.? *Ans.* 2421 L.

30. How many hectoliters of oats in 4685 sacks, if each contains 1 H. L. 6 D. L.? *Ans.* 7496 H. L.

31. If 4957 loaves of sugar weigh 15684.08 K. G., what is the weight of one loaf? *Ans.* 3.164 + K. G.

32. What will be the price of a hectogram of sugar, if a loaf weighing 8 K. G., 6 D. G., and 6 G., costs $32.95? *Ans.* $.408 +.

33. How much cloth 1.85 M. wide will it take to line a cloak, made of 6.5 M. of cloth 1.25 M. wide? *Ans.* 4.39 + M.

34. A silver half-dollar weighs 12.441 G.: what is the weight of $11.50 in half dollars? *Ans.* 286.143 G.

35. If a meter of cloth costs $2.50, how much can be purchased for $72.10? *Ans.* 28.84 M.

36. What is the cost of a field of 12 H. A., if an are costs $6.35? *Ans.* $7620.

37. What is the cost of a field containing 3 A., at the rate of $.625 a square meter? *Ans.* $187.50.

38. If 3 H. L. of wine cost $$41\frac{1}{3}$, what is the price of 1 liter? *Ans.* $.137 +.

39. What quantity of wheat is contained in 792 sacks, each containing 1 H. L. and 2 D. L.?

Ans. 950.4 H. L.

40. A has a garden of 3.6 H. A.; he sold 30 M^2 to a neighbor: how much had he left? *Ans.* 3.597 H.A.

41. A block of marble .72 M. long, .48 M. wide, and .5 M. thick, costs $.864: what is the cost of one cubic meter? *Ans.* $5.

342. REDUCTION.

EXAMPLE.—Reduce 1 mile to meters.

OPERATION.

```
39.37)63360(1609.347+ M.
      3937
      -----
      23990
      23622
      ------
        36800
        35433
        -----
         13670
         11811
         -----
          18590
          15748
          -----
           28420
           27559
           -----
```

SOLUTION.—First, reduce the 1 mile to inches. It equals 63360 inches; and since 39.37 inches make one meter, 63360 inches will make as many meters as 39.37 is contained times in 63360 = 1609.347 + meters.

CONCLUSION.—Therefore, 1 mile equals 1609.347 + meters. Hence, the

Rule for reducing the ordinary denominations to the metrical system,—*Divide by the number of the given denomination making one of the required metrical denomination.*

EXAMPLE.—Reduce 2000 M. to yards.

SOLUTION.—Since 1 meter equals 39.37 inches, 2000 meters equal 2000× 39.37 inches = 78740 inches which, reduced to yards, equal 2187 yd. 8 in.

CONCLUSION.— Therefore, 2000 M. equal 2187 yd. 8 in. Hence, the

OPERATION.

```
     39.37
      2000
12)78740.00 in.
  3)6561 ft. 8 in.
    2187 yd.
```

Rule for reducing metrical denominations to the ordinary system.—*Multiply by the number expressing the equivalent of the unit of the given metrical denomination.*

EXAMPLES FOR PRACTICE.

1. Reduce 8 inches to centimeters. *Ans.* 20.32 + c. m.
2. Reduce 25 feet to meters. *Ans.* 7.62 + M.
3. How many K. M. from Cincinnati to Dayton, the distance being 60 miles? *Ans.* 96.56 K. M.
4. Mt. Everest, the highest mountain in the world, is 29100 ft. high; what is its height in meters? *Ans.* 8869.69. M.
5. Reduce 20 K. M. to miles. *Ans.* 12.427 + mi.
6. Reduce 49 M. to ordinary denominations. *Ans.* 9 rd. 4 yd. 3.13 in.
7. A merchant in Holland bought 360 M. of linen: how many yards did he buy? *Ans.* 393.7 yd.
8. How many hectares in a quarter section (160 acres) of land? *Ans.* 64.74 + H. A.

9. How many square yards in a roll of paper, 9 M. long and .5 M. wide? *Ans.* 5.38 + sq. yd.

10. Reduce $2\frac{1}{2}$ bushels to liters. *Ans.* 88.1 + L.

11. Reduce 32 L. to gallons. *Ans.* 8.45 + gal.

12. How many hectoliters in a barrel containing 42 gallons of vinegar? *Ans.* 1.5899 + H. L.

13. A grocer sold 4 lb. of tea: how much would it have weighed in metrical denominations?

Ans. 1.814 + K. G.

14. A load of hay weighing .92 tonneau, was sold at 1 cent a pound, Avoirdupois: for how much did it sell? *Ans.* $20.28 +

15. A has Manila cordage which he sells at 26 ct. a pound: how shall he mark it to sell by the kilogram? *Ans.* 57 cts.

16. B has sugars marked at 14, 16, and 20 ct. a pound: what is the price of each *by the kilogram?*

Ans. 31, 35, and 44 cts.

17. Bought 36 lb. of tobacco for $28.80: at how much per K. G. shall I sell it to gain $1.44?

Ans. $1.85 +

18. A *single* letter is allowed by law, to weigh 15 grams: how many such letters in a mail-bag, weighing 41 lb. 5 oz., if the bag alone weighs 8 lb. 3 oz. 15.2 dr.? *Ans.* 1000.

19. A man bought 25 lb. of tea at $1.80 a pound; he exchanged it for five times its weight in coffee, which he sold at $.86 a kilogram: did he gain or lose by the bargain, and how much?

Ans. Gained, $3.76 +.

THE END.