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THE

PRACTICAL ARITHMETIC:

DESIGNED FOR SUCH

INSTITUTIONS

AS REQUIRE A GREATER NUMBER OF EXAMPLES THAN ARE GIVEN IN THE

ELEMENTARY ARITHMETIC.

BY

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PREFACE.

While many distinguished teachers unite in pronouncing my Elementary Arithmetic as the best work of the kind, well adapted to the purpose of teaching the science, as well as the art, of Arithmetic; others have often expressed to me their belief that its usefulness would be greatly increased by the addition of more examples of a practical kind; and in many cases I have been strongly urged to omit the answers.

I am convinced that for certain grades of institutions it would be well to have a greater variety of examples for practice, but I do not so readily see the advantage of omitting the answers. I have always been inclined to believe the omission of the answers gave an opportunity for the pupil, and in some cases for the teacher, to pass over many principles without thoroughly understanding them, since a result would frequently be obtained which might perhaps be quite erroneous; and having no answer with which to compare it, he dismisses the subject with the belief that he has conquered the difficulty, and that he understands clearly the principle especially designed to be brought out by the example.

With these facts in view, I have prepared the Practical

Arithmetic, which is designed not to supply the place of the Elementary Arithmetic, but is designed for the use of such institutions as require a greater number of examples than are given in that work.

In many cases the rules have been rewritten and condensed, so as to bring the same rule to apply to as large a class of operations as possible.

The arrangement of the subjects is in many respects different from that given in the Elementary Arithmetic. The whole is divided into chapters, and under each chapter the examples begin a new numbering, so that under the same chapter there are no two examples of the same number.

In forming these examples, great care has been taken to make them as practical as possible. The answers have been omitted in the body of the work, but they are given in the Appendix, where they are arranged in reference to the chapter and section.

The questions designed to test the pupil's knowledge of the principles and rules, are also given in the Appendix. In the construction of these test questions, care has been taken to draw out the actual knowledge of the pupil, as in many cases they cannot be correctly answered without a thorough knowledge of the subject. It is believed these questions will be found of valuable assistance to the teacher in reviews.

GEO. R. PERKINS.

ALBANY, JUNE, 1851.

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THE

PRACTICAL ARITHMETIC.

CHAPTER I.

ARITHMETIC.

§ 1. Every single thing is called a unit. A Number, then, must represent one or more units.

If the units represented by a number have no reference to particular things, the number is called an *abstract* number.

Thus, the number Eight is an abstract number, because it does not mean eight apples, or eight dollars, or eight any particular things, but, simply, eight.

If the units represented by a number have reference to particular things, the number is called a *concrete* or *denominate* number.

Thus, the number Eight, meaning eight apples, or eight dollars, or eight any particular things, is a denominate number.

Note.—Units or numbers are called denominate, because they denominate or *name* particular things.

§ 2. Arithmetic treats of numbers, whether abstract or denominate. As a science, it is a knowledge of the properties and relations of numbers; as an art, it is a practical facility in computing by numbers

Numbers may be expressed by words, by letters, or by figures. These two latter methods form different systems of notation.

CHAPTER II.

NOTATION.

§ 3. Notation is the expressing of numbers by letters or figures.

The use of letters for this purpose was adopted by several early nations, especially by the Romans; hence called the

ROMAN METHOD.

This method employed seven capital letters, viz.: I for *one;* V for *five;* X for *ten;* L for *fifty;* C for *a hundred;* D for *five hundred;* M for *a thousand.* By means of these letters, repeated or variously combined, any number may be expressed; thus:

I	stands	for	......One.	XV	stands	for	Fifteen.
II	"	"	......Two.	XVI	"	"	Sixteen.
III	"	"	......Three.	XVII	"	"	Seventeen.
IV	"	"	......Four.	XVIII	"	"	Eighteen.
V	"	"	......Five.	XIX	"	"	Nineteen.
VI	"	"	......Six.	XX	"	"	Twenty.
VII	"	"	......Seven.	XXI	"	"	Twenty-one.
VIII	"	"	......Eight.	L	"	"	Fifty.
IX	"	"	......Nine.	C	"	"	One hundred.
X	"	"	......Ten.	D	"	"	Five hundred.
XI	"	"	......Eleven.	DC	"	"	Six hundred.
XII	"	"	......Twelve.	M	"	"	One thousand.
XIII	"	"	......Thirteen.	MM	"	"	Two thousand.
XIV	"	"	......Fourteen.	MMM	"	"	Three thousand.

A letter placed *before* another letter of greater value, takes away its own value from the greater; as V, five; IV, one from five, or four.

A letter placed *after* another letter of greater value, adds its own value to the greater; as V, five; VI, five and one, or six.

The repeating of a letter repeats the value of the letter.

A horizontal line over a letter increases it a thousand-fold; thus, D, five hundred; $\overline{\text{D}}$, five hundred thousand.

1–5. Write, after the Roman method, seventeen; forty-two; twenty-six; ninety-eight; one hundred and three.

6–11. Write eighty-two; fifty-seven; seventy-nine; four hundred and thirty; six hundred and eighty; two thousand and seven.

12–14. Write three hundred thousand; nine hundred and sixty thousand; one million.

15. How many dollars are represented by bank bills, marked as follows: X; XX; V; I; II; III?

16. How many dollars are represented by bank bills, marked X; C; V; L; I?

17–28. Read the following numbers: XXVI; CXLIV; XCVIII; MCCCXII; MDCCCL; MDCCCCLXXII; $\overline{\text{D}}$; $\overline{\text{D}}$MMDCCLXX; $\overline{\text{M}}$; $\overline{\text{X}}$XCIV; MDCLXXXVIII; MDCCLXXV.

Roman numbers are now used chiefly to mark volumes, chapters, or lessons; to indicate the hours on clock or watch faces; for dates upon tombstones, tablets, &c.; and to designate the year of the Christian era.

Note.—Among the Romans each I represented a finger; the whole hand spread out, thus,  represented *five*. This character was afterwards written V. X, ten, is merely two fives written one above the other; thus, ($\genfrac{}{}{0pt}{}{\text{V}}{\Lambda}$.)

C, the initial of Centum, the Latin for one hundred, was often written ⊏; this character cut in two, will leave L, fifty, for its lower half.

M, the initial of Mille, the Latin for one thousand, was originally written CIƆ. The right-hand portion of this character is IƆ, which represents five hundred, and is now expressed by D.

ARABIC NOTATION.

§ 4. The method of notation in common use is the Arabic. This method employs ten characters, viz.:

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine,	Naught.

Each of these characters, except the *Naught*, is called a *digit*;* and the first nine taken together are called the nine digits.

The digits are also called *significant* figures.

§ 5. The significant figures have unchanging values; that is, they always represent *Units* or *Ones*; but the units which they represent differ in value.

If a significant figure stand disconnected from other figures, the value of its unit is called its *simple* value. Thus, 6 means six units of simple value.

But if a significant figure stand in connection with other figures, the value of its unit is called its *local* value, because this value depends upon the *place* which such figure occupies, in relation to the figures with which it is connected.

Thus, in the number 3456, which consists of four significant figures standing in connection with each other, *each figure expresses units*; but units of different values.

The 6 occupies the right-hand or *first place*; and its units are said to be of the *first order*: their value is their simple value; that is, they represent six *single ones.*

The 5 occupies the *second place*; and its units are said to be of the *second* order: their value is *ten* times greater than

* From the Latin, *digitus*, a finger; because the ancients used to do their reckoning on their fingers. Originally 10, ten, was also called a digit.

the value of the units of the first order; that is, they represent 5 *tens.*

The 4 occupies the *third* place; and its units are said to be of the *third* order: their value is *ten* times greater than the value of the units of the second order, and one hundred times greater than the value of the units of the first order; that is, they represent 4 *hundreds.*

The 3 occupies the *fourth* place; and its units are said to be of the *fourth* order: their value is *ten* times greater than the value of the units of the third order, and one thousand times greater than the value of the units of the first order; that is, they represent 3 *thousands.*

Each unit, then, of the 6, represents *a simple unit:* each unit of the 5 represents *a ten:* each unit of the 4 represents *a hundred:* each unit of the 3 represents *a thousand.*

So we might proceed with any number of figures thus connected together. Hence we discover this fundamental property:

Every figure in a number represents a value ten times greater than that of the figure next to it at its right hand.

§ 6. The *naught,* 0, represents the *absence* of number. It shows that in the place which it occupies, *no* value is to be expressed.

Thus, in the preceding number 3456, for the 4 and 5, substitute naughts, thus, 3006. These naughts show that there are no units of the second order, or *tens,* and no units of the third order, or *hundreds,* in the number.

Again, 7 standing alone, represents seven units of the first order, or of simple value: a naught written at the right of the 7, thus, 70, shows that there are no units of the first order in the number. The 7 now occupies the second place, and represents units of the second order, or 7 *tens.*

Two naughts written at the right of the 7, thus 700, show that there are no units either of the first or of the second order in the number. The 7 now occupies the third place, and represents units of the third order, or 7 *hundreds,* and so on.

Hence this property:

Every naught at the right of a significant figure increases the value which that figure represents ten-fold.

And conversely:

Every naught removed from the right of a significant figure, diminishes the value which that figure represents ten-fold.

NOTE.—It will be seen that the office of the *naught* is to keep the significant figures in their proper places; so that they shall correctly express the *order of units* which they are intended to represent. Annexing them to, or taking them from, the right of a digit, increases or diminishes its value, by causing that digit to occupy a place further to the left or further to the right.

§ 7. We have already seen that the first place of a number is occupied by *units,* the second place by *tens,* the third place by *hundreds,* and the fourth place by *thousands.*

In writing numbers, then, which do not contain more than four places, the pupil will probably find no difficulty.

Begin with the units of the highest order mentioned, and in whatever place *no* units are required, be careful to write 0.

Express in figures seven thousand nine hundred and four.

In this example the highest order of units is thousands, of which there are 7: the next order is hundreds, of which there are 9: the next order is tens, of which there are 0: the next order is simple units, of which there are 4. The whole number is 7904.

29–43. Express in figures:

Twenty. (Two tens and no units.)

Thirty-seven. (Three tens and seven units.)

Ninety-eight.

Three hundred and thirty-seven.

Four hundred and seven.

Two thousand four hundred and thirty-seven.

Six thousand four hundred and seven.

Eight thousand and seven.

Nine thousand and twenty-seven.

Four thousand and six.

Three thousand.

One thousand and one.

One thousand and one hundred.

One hundred and one.

One thousand one hundred and one.

§ 8. Suppose it be required to express in figures the number Sixty-one millions, nine hundred and thirty-four thousands, four hundred and sixty-five.

There are three denominations in this number: Millions, Thousands, and Units.

Write, first, the figures that express how many millions there are in the number: 61. Write, next, the figures that express how many thousands there are in the number: 934. Write, finally, the figures that express how many units there are in the number: 465. Connecting these groups in a line,

61,934,465,

we have the number that was to be expressed.

The following are the names of the first eight groups, or periods, counting from the right towards the left: *Units, Thousands, Millions, Billions, Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions.* These periods may be extended indefinitely.

Each period contains *three* places. In the first or right-hand place, the units of the period must be written: in the second place, the tens must be written: in the third place, the hundreds must be written.

In expressing large numbers, then, by figures, begin with the highest denomination, and write out its period as required, and so proceed with all the periods; *taking care that each period except the left-hand one, has its three places occupied either by digits or by naughts.*

Express in figures the number Seven trillions, six millions, and thirty-one.

This number involves *five* periods, (although but three are heard in reading,) Trillions, Billions, Millions, Thousands, and Units. In the highest or trillions' period, the unit's place only is occupied: 7. In the next, or billions' period, there are no hundreds, tens, or units; therefore write 000. In the millions' period, there are no hundreds, no tens, but simply 6 units; therefore write for this period 006. The thousands' period requires no digits; therefore write for that period 000. In the units' period there are no hundreds to be expressed; therefore write 0 in the hundreds' place: in the tens' place write 3: in the units' place write 1.

The whole number, then, is 7,000,006,000,031.

The following will exhibit the periods, and the places in each period, as high as Octillions.

PERIODS.	PLACES. Hundreds of	Tens of	Units of
OCTILLIONS, 10th.	3	2	1
SEPTILLIONS, 9th.	4	6	8
SEXTILLIONS, 8th.	9	0	7
QUINTILLIONS, 7th.	8	4	3
QUADRILLIONS, 6th.	0	0	1
TRILLIONS, 5th.	0	0	0
BILLIONS, 4th.	7	4	1
MILLIONS, 3d.	8	9	1
THOUSANDS, 2d.	4	5	3
UNITS, 1st.	2	2	1

3 2 1, 4 6 8, 9 0 7, 8 4 3, 0 0 1, 0 0 0, 7 4 1, 8 9 1, 4 5 3, 2 2 1

44–49. Express in figures the following numbers: (two periods.) Twenty-seven thousand three hundred; nine hundred and forty thousand and two hundred; thirty-six thousand four hundred and fifty-six; five hundred and one thousand; ninety-eight thousand; eleven thousand.

50–53. (Three periods.) Forty-six millions, nine hundred and thirty thousand, six hundred and fifty-nine; three hundred and seven millions, eight hundred and two thousand, five hundred and nine; nine hundred and eighty-one millions, seven hundred; ten millions, ten thousand and ten.

54–57. (Four periods.) Ninety-six billions, forty-eight millions, seventy three thousand and ninety-eight; eight hundred and seven billions and six; ninety billions and four thousand and ten; eight hundred billions, six millions and seven.

58–62. (Five periods.) Forty-eight trillions; six hundred and nine trillions; nine hundred and eighty trillions, four billions and seven; three trillions and two; nine trillions and two thousand.

63–68. Thirty-six sextillions and ninety-eight; four quadrillions, eight trillions, five thousand and ninety-four; thirty-five quadrillions, ninety-eight billions and sixty-three; nine hundred millions, seven hundred thousand, five hundred and ninety; eighty-six septillions and five billions; nine hundred billions and forty-six thousand.

Note.—The preceding method of Notation is the French, which is now in almost universal use. The English method counts six places in the period.

NUMERATION.

§ 9. Numeration is the reverse process of Notation. Notation is the method of expressing in figures numbers that are written in words; Numeration is the method of expressing in words, numbers that are written in figures.

Notation answers to writing, and Numeration answers to reading.

To read large numbers with facility, separate them, as before, into periods of three places each, counting from units. Then commencing at the left hand, read the figures in each period, adding the name of the period. The name of the unit's period need not be added.

Thus, for example, read the following: 37192854675. Separate the figures into periods as directed. This may be done in the imagination, or by the aid of the comma. The number will then be read,

37 billions, 192 millions, 854 thousands, 675.

69–75. Read the following numbers: 678210; 5493678; 456321980; 779146005; 42567000123901; 327980060; 3298765430000098.

76–80. 563428670009; 358920761; 987678932; 4560007980540068; 33492677005316896321.

Let the following sentences be read:

81. The distance of the earth from the nearest fixed star is supposed to be about 20000000000000 miles.

82. The distance of the moon from the earth is 236847 miles.

83. The planet Mercury is distant from the sun 36814721 miles.

84. Venus is distant from the sun 68791752 miles.

85. Mars is 144907630 miles distant from the sun.

86. Jupiter is 494499108 miles distant from the sun.

87. The diameter of the sun is 883246 miles.

88. The circumference is 2774799 miles.

89. The square miles on his surface is 2450830241208.

90. The number of cubic miles is 360781001204398299.

91. The amount of tea consumed in the United States for the years 1842–1846, was 73376290 pounds.

92. The amount of coffee consumed for the same time was 561707046 pounds.

NOTE.—To convey some idea of the number in a trillion, it may be stated that not a trillion seconds have elapsed since the birth of Christ.

CHAPTER III.

ADDITION.

§ 10. ADDITION is the process of uniting two or more numbers so as to form one number; thus, 8 and 6 are 14.

Numbers of the same kind or denomination only can be added; thus we may add 8 oranges to 6 oranges, or 8 dollars to 6 dollars, or, simply, 8 (units) to 6 (units); but we cannot add 8 oranges to 6 dollars, or 8 dollars to 6 units.

The single number, formed by uniting two or more numbers, is called the SUM.

The sign or symbol used for addition is $+$. Thus, $6 + 8$ means 6 *added to* 8. The symbol $=$, which is not confined to addition, means *equal to;* thus, $6 + 8 = 14$, is the same as if written 6 added to 8 is equal to 14. The expression may be read 6 and 8 are 14.*

* The symbol $+$ is often read *plus*, which is a Latin word, meaning *more*.

Let the pupil *commit to memory* the following

ADDITION TABLE.

2+ 2= 4	3+ 2= 5	4+ 2= 6	5+ 2= 7	6+ 2= 8
2+ 3= 5	3+ 3= 6	4+ 3= 7	5+ 3= 8	6+ 3= 9
2+ 4= 6	3+ 4= 7	4+ 4= 8	5+ 4= 9	6+ 4=10
2+ 5= 7	3+ 5= 8	4+ 5= 9	5+ 5=10	6+ 5=11
2+ 6= 8	3+ 6= 9	4+ 6=10	5+ 6=11	6+ 6=12
2+ 7= 9	3+ 7=10	4+ 7=11	5+ 7=12	6+ 7=13
2+ 8=10	3+ 8=11	4+ 8=12	5+ 8=13	6+ 8=14
2+ 9=11	3+ 9=12	4+ 9=13	5+ 9=14	6+ 9=15
2+10=12	3+10=13	4+10=14	5+10=15	6+10=16
2+11=13	3+11=14	4+11=15	5+11=16	6+11=17
2+12=14	3+12=15	4+12=16	5+12=17	6+12=18
7+ 2= 9	8+ 2=10	9+ 2=11	11+ 2=13	12+ 2=14
7+ 3=10	8+ 3=11	9+ 3=12	11+ 3=14	12+ 3=15
7+ 4=11	8+ 4=12	9+ 4=13	11+ 4=15	12+ 4=16
7+ 5=12	8+ 5=13	9+ 5=14	11+ 5=16	12+ 5=17
7+ 6=13	8+ 6=14	9+ 6=15	11+ 6=17	12+ 6=18
7+ 7=14	8+ 7=15	9+ 7=16	11+ 7=18	12+ 7=19
7+ 8=15	8+ 8=16	9+ 8=17	11+ 8=19	12+ 8=20
7+ 9=16	8+ 9=17	9+ 9=18	11+ 9=20	12+ 9=21
7+10=17	8+10=18	9+10=19	11+10=21	12+10=22
7+11=18	8+11=19	9+11=20	11+11=22	12+11=23
7+12=19	8+12=20	9+12=21	11+12=23	12+12=24

NOTE.—Let the foregoing table be *thoroughly* committed to memory. Time bestowed here will save time in all after arithmetical processes. The preceding combinations of digits lie at the foundation of the combinations of larger numbers. Thus, if the scholar see instantly that 7 and 6 are equal to 13, he will also instantly see that 17 and 6 are equal to 23; the right-hand figure being the same in all sums that differ by one or more tens.

It will be well for the teacher to *skip about* in examining the pupil, and to require prompt answers, preventing, if possible, the obtaining of the sum required by adding *one at a time.*

After having exercised the pupil in adding *two* numbers, let the question combine three, four, or more; thus, 7+8+6+9?

§ 11. Where the sums of the several columns are less than ten:—

Add together 2432, 3343, and 4122.

Write the numbers so that the figures of the same kind shall fall in the same column; that is, so that units may be under units, tens under tens, &c. Draw a line under the whole, as in the example.

Add, first, the column of units. Write the sum, 7, underneath. Next, add the tens; write their sum, 9, under the column of tens. Next, add the hundreds; write the sum, 8, under the hundreds. Lastly, add the thousands, and set the sum, 9, under the column of thousands. The whole sum is nine thousand eight hundred and ninety-seven.

Thousands.	Hundreds.	Tens.	Units.
2	4	3	2
3	3	4	3
4	1	2	2
9	8	9	7

1. Add 6264, 2532, and 1203.
2. Add 4132, 1001, and 1423.

Add the following:

(3.)	(4.)	(5.)	(6.)	(7.)
5153	3642	2001	87543	3026178
4245	6037	3658	12425	2513421

NOTE.—Let the pupil *prepare* himself in his seat to perform the addition in the class. At the recitation, let one be required to add, say, the sums in Ex. 3. The scholar called upon, will answer, at once, eight, nine, three, nine, which figures the class will write in their proper places.

§ 12. Where the sums of the several columns equal or exceed ten:—

Find the sum of the following numbers: 3758, 4903, 7006, 3713, 3721.

Place the numbers as directed in (§ 11) the preceding example. The sum of the numbers in the units' column is 21; that is, 2 tens and 1 unit. Set the 1 under the units' column, and carry the 2 to the next, or tens' column. The sum of the tens' column, thus increased, is 10 tens; that is, 1 hundred and no tens. Place a naught under the tens' column, and carry the 1 to the hundreds' column. The sum of the hundreds' column, so increased, is 31 hundreds; that is, 3 thousands and 1 hundred. Set the 1 under the hundreds' column, and carry the 3 to the thousands' column. The sum of this column, so increased, is 23 thousands, or 2 tens of thousands and 3 thousands. Set the 3 under the thousands' column, and carry the 2 to the tens of thousands' place; or, what is the same thing, set down the whole of the sum of the last column.

Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
	3	7	5	8
	4	9	0	3
	7	0	0	6
	3	7	1	3
	3	7	2	1
2	3	1	0	1

From what has thus far been explained, the pupil will be able to understand the following

RULE.

I. *Place the numbers to be added so that the figures of the same kind shall fall in the same column.*

II. *Commence at the right, and add each column successively: if the sum of any column be less than* 10, *place each sum under the column added; but if it equal or exceed* 10, *place the right-hand figure of the sum under the column added, and carry the left-hand figure or figures to the next column.*

III. *Write down the whole sum of the last column.*

PROOF OF ADDITION.

Begin at the top, and add the columns downward. This will vary the order in which the figures were added, and will be likely to rectify error.

EXAMPLES.

Prepare the following before recitation, and let them be performed aloud in the class; thus (Ex. 8), "twelve, twenty:" the class writes the proper figure in its place; "four, nine, eighteen, twenty-one," and so on.

(8.)	(9.)	(10.)	(11.)
56430	7921341	62849765	2809576832
12798	82345678	98765432	95300765
34457	79013265	23456789	298
21325	7890275	64698705	46543298764

(12.)	(13.)	(14.)
7920658437	49827088221	54952138017
5987693219	30765098762	79876320441
1439758436	25421765765	57632768719
2870129876	379866	28495762804
5432194562	2996	998799

(15.)	(16.)	(17.)
34567890	43345678	123423434
2357911	21123355	23785432
234567	27893	9876543
24897	54689	751002
64	734321	10200

18. Add 123405, 2354210, 794327, and 36547, together
19. Add 275602, 345607, 4567801, and 365, together.
20. Add 100375, 406780, 4673005, 4112, and 2478, together.

21. Add 1034001, 78954, 379205, 367001, and 45637, together.

22. What is the sum of the following numbers: Three thousand six hundred and fifty, seven thousand eight hundred and thirty-two, eleven thousand five hundred and sixty-seven, ten thousand and fifty-six, four hundred and seventy-two?

23. What is the sum of the numbers, four thousand three hundred and seventy-three, three thousand one hundred and fourteen, one thousand two hundred and twenty-three, six hundred and fifty-four?

24. Find the number of days in a year—the days of the respective months being as follows: January 31, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, December 31.

25. A man drew five loads of bricks: in the first load he had 1209, in the second load 1453, in the third load 1101, in the fourth load 1212, and in the fifth load 1303. How many bricks were there in all?

26. If there are shipped from the United States, 15624 barrels of flour to Sweden, 250 barrels to Holland, 205154 barrels to England, 6401 to Texas, 19602 to Mexico, what is the whole amount?

27. In 1837 the United States exported 100232 hogsheads of tobacco; in 1838 they exported 100592; in 1839 they exported 78995; in 1840 they exported 119484; in 1841 they exported 147828. How many hogsheads of tobacco were exported during these five years?

28. If the cotton crop of the United States is estimated at 1360532 bales for the year 1839, 2177835 bales for the year 1840, 1634945 bales for the year 1841, and 1683574

bales for the year 1842, how many bales will the four years' crops amount to?

29. In 1839 the Onondago Springs produced 2864718 bushels of salt; in 1840 they produced 2622305 bushels; in 1841 they produced 3340769 bushels; in 1842 they produced 2291903 bushels. What is the whole number of bushels during the above four years?

30. The United States exported in bullion and specie—in 1838, 3508046 dollars; in 1839, 8776743 dollars; in 1840, 8417014 dollars; in 1841, 10034332 dollars. How much was exported during these four years?

31. The amount of tea consumed in the United States, during 1842, was 1348265 pounds; in 1843, it was 12785748 pounds; in 1844, it was 13054327 pounds; in 1845, it was 17162550 pounds; and in 1846, it was 16891020 pounds. What was the whole number of pounds during these five years?

32. The amount of coffee consumed in the United States, during the year 1842, was 107383567 pounds; in 1843, it was 85916666 pounds; in 1844, it was 149711820 pounds; in 1845, it was 94358939 pounds; and in 1846, it was 124336054 pounds. What was the whole number of pounds during these five years?

33. The number of acres of public land sold by the United States government, in the year 1841, was 1164796 acres; in the year 1842, it was 1129217 acres; in 1843, it was 1605264 acres; in 1844, it was 1754763 acres; and in 1845, it was 1843527 acres. What was the whole number of acres sold during these five years?

34. The United States revenue for letter postage, under the new law, was as follows: for the year 1842, it was 3953315 dollars; for 1843, it was 3738307 dollars; for 1844, it was 3676162 dollars; and for 1845, it was 3660231

dollars. What was the whole number of dollars during these four years?

35. In 1843, the amount of gold coined at the United States mint and branches was as follows: at Philadelphia, 4062010 dollars; at the branch at New Orleans, 3177000 dollars; at Dahlonega, 582782 dollars; at Charlotte, 287005 dollars. How many dollars of gold were coined in all?

36. The population of Europe is estimated at two hundred and thirty-three millions, two hundred and forty thousand and forty-three; of Asia, at six hundred and eight millions, five hundred and sixteen thousand and nineteen; of Africa, at one hundred and one millions, four hundred and ninety-eight thousand, four hundred and eleven; of America, at forty-eight millions, seven thousand one hundred and fifty; of Oceanica, one million, eight hundred and thirty-four thousand, one hundred and ninety-four. What is the population of the globe?

37. The number of square miles in Europe is estimated at three millions, eight hundred and seven thousand, one hundred and ninety-five; in Asia, seventeen millions, eight hundred and five thousand, one hundred and forty-six; in Africa, eleven millions, six hundred and forty-seven thousand, four hundred and twenty-eight; in America, thirteen millions, five hundred and forty-two thousand, four hundred; in Oceanica, three millions, three hundred and forty-seven thousand, eight hundred and forty. What is the number of square miles of land upon the globe?

38. An army consists of 52714 infantry, 5110 horse, 6250 dragoons, 3927 light-horse, 928 gunners, 1410 pioneers, 250 sappers, and 406 miners. What is the whole number of men?

The quantity and value of teas and coffee consumed annually, from 1821 to 1846, in the United States, were as follow:

Years.	Teas consumed.		Coffee consumed.	
	Pounds.	Value.	Pounds.	Value.
1821	4586223	$1080264	11886063	$2402311
1822	5305588	1160579	18515271	3899042
1823	6474934	1547695	16437045	2835420
1824	7771619	2224203	20797069	2513950
1825	7173740	2346794	20678062	1995892
1826	8482483	3443587	25734784	2710536
1827	3070885	942439	28354197	1130607
1828	6289581	1771993	39156733	3695241
1829	5602795	1531460	33049695	3052020
1830	6873091	1532211	38362687	3180479
1831	4656681	1057528	75700757	5796139
1832	8627144	2081339	36471241	2516120
1833	12927043	4775081	75057906	7525610
1834	13193553	5422275	44346505	4473937
1835	12331638	3594293	91753002	9381689
1836	14484784	4472342	77647300	7667877
1837	14465722	5003401	76044071	7335506
1838	11978744	2559546	82872633	7138010
1839	7748028	1781824	99872517	9006685
1840	16860784	4059545	86297761	7615824
1841	10772087	3075332	109200247	9855273
1842	13482645	3567745	107383567	8447851
1843	12785748	3405627	85916666	5923927
1844	13054327	3152225	149711820	9054298
1845	17162550	4809611	94358939	5380532
1846	16891020	3983337	124336054	7802894
Totals.				

39–42. What number of pounds of tea was consumed for the 10 years commencing 1821? for the 10 years commencing 1831? from 1841 to 1846 inclusive? What number from 1821 to 1846 inclusive?

43–46. What was the value of teas consumed during the same periods of time?

47–50. What number of pounds of coffee was consumed for the 10 years commencing 1821? for the 10 years commencing 1831? from 1841 to 1846 inclusive? from 1821 to 1846 inclusive?

51–54. What was the value of the coffee consumed during the same periods of time?

NOTE.—The preceding table furnishes ample materials for all necessary discipline of the pupil in addition. It is recommended that exercises from the table be performed in the following manner. Let one pupil be called upon to add eight, ten, or twelve of the numbers; another pupil a different set of numbers; or one pupil one column of figures, another the next column, and so on; or let one pupil add a column upward, another add the same column downward. Let these exercises be performed aloud: the pupil omitting all intermediate words, and uttering only the sum as it is increased by the addition of the successive figures. The pupil must not be allowed to delay at each figure. If he cannot run the column rapidly, he should be drilled again in the Table, § 10.

55–58. Add the columns in the preceding table, *two figures at a time.*

Thus (commencing at the right hand), "six, twenty-one, twenty-five, thirty-four," &c.

CHAPTER IV.

SUBTRACTION.

§ 13. SUBTRACTION is the process of taking a less number from a greater, to find the *difference*; as, 5 from 14 leaves 9.

The greater number is called the *Minuend*; the smaller number is called the *Subtrahend.*

NOTE.—The termination *nd* means *to be.* Thus Minue*nd* means the number *to be diminished;* Subtrahe*nd* means the number *to be subtracted.*

The *difference* of two numbers is also called the *remainder.*

The symbol for subtraction is —. Thus 5 − 3 = 2, is the same as if written 5 diminished by 3 equals 2. The expression may be read, five *less** three is two.

1–12. How many are 3 from 5? 3 from 12? 3 from 7? 3 from 9? 3 from 13? 3 from 20? 3 from 8? 3 from 11? 3 from 10? 3 from 14? 3 from 19? 3 from 17?

13–25. How many are 4 from 6? from 8? 10? 20? 18? 17? 14? 16? 19? 11? 7? 9? 12?

26–39. How many are 5 from 7? from 10? 13? 16? 19? 8? 11? 14? 17? 20? 9? 12? 15? 18?

40–52. How many are 6 from 8? from 11? 14? 17? 20? 9? 12? 15? 18? 10? 13? 16? 19?

53–64. How many are 7 from 9? from 12? 15? 18? 10? 13? 16? 19? 11? 14? 17? 20?

65–75. How many are 8 from 10? from 13? 16? 19? 11? 14? 17? 20? 12? 15? 18?

76–85. How many are 9 from 11? 14? 17? 20? 12? 15? 18? 13? 10? 19?

NOTE.—Let the pupil be exercised on the preceding questions until he can answer with great promptness.

§ 14. In which no figure of the subtrahend is larger than the corresponding figure of the minuend.

From 796 subtract 375.

Place the subtrahend under the minuend, as in the example; units under units, &c.

Commence at the units' column and subtract: 5 from 6 leaves 1; place the 1 underneath, and so proceed with each succeeding column.

Hundreds.	Tens.	Units.	
7	9	6	Minuend.
3	7	5	Subtrahend.
4	2	1	Difference.

* The symbol — is often read *minus*, a Latin word, meaning *less.*

(86.)	(87.)	(88.)	(89.)	(90.)
687	7649	69985	879465	987654321
486	5438	59831	729355	821350011

NOTE.—Let the pupil *prepare* himself in his seat to perform the subtraction in the class. At the recitation, let one be required to subtract, as in Ex. 88. The scholar called upon will answer at once the results, without any intermediate words, thus: "four, five, one," &c., which figures the class will write in their proper places.

§ 15. In which figures of the subtrahend are larger than their corresponding figures of the minuend.

From 8053 subtract 4967.

8053
4967
3086

We cannot subtract 7 from 3; therefore we add 10 to the 3, and say, 7 from 13 leaves 6. Having thus increased the minuend figure 3 by 10 units, we balance that excess by adding 1 ten to the 6 of the subtrahend, making 7 tens. But the 7 cannot be subtracted from the 5 tens. Add, then, 10 tens to the 5, making 15 tens, and then say, 7 from 15 leaves 8. Having added 10 tens to the 5 of the minuend, we restore the balance by adding 1 hundred to the 9 of the subtrahend, making 10. But we cannot subtract 10 from 0. Then we add 10 hundred to the 0, and say 10 from 10 leaves 0. Before subtracting the thousands, we must add 1 to the 4 thousands, to compensate for the 10 hundred added to the 0. We then say, 5 from 8 leaves 3.*

* The following is another mode of performing this example:

We cannot subtract the 7 from the 3. We therefore take 1 ten from the tens' figure of the minuend, leaving that figure, 4 (which we place in brackets over the 5, marking out the 5), and counting the 1 ten as ten units, we add it to the 3 units, making 13 units, which sum we place in brackets over the 3, and mark out the 3. We can now subtract the 7 from the 13. We next seek to subtract the 6 from the 4, which we cannot do. We must then seek one from the hundreds' place to be added to the 4. But there are no hundreds there. We then go to the thousands' place. Taking one from the 8, we have 7 left. Place the 7 in brackets over the 8 and mark out the 8. The 1 thousand we carry to

Thousands.	Hundreds.	Tens.	Units.
:	[9]	[14]	:
[7]	[~~10~~]	[~~4~~]	[13]
~~8~~	~~0~~	~~5~~	~~3~~
4	9	6	7
3	0	8	6

(91.)	(92.)	(93.)	(94.)	(95.)
9034	8087	87315	64281	5987650
7941	4759	19848	38796	4898562

(96.)	(97.)	(98.)	(99.)
34678	789347	10345678937	9345678201
13787	120305	902134124	3279609167

NOTE.—See note, § 14. Let the teacher, also, write exercises upon the blackboard, to be performed by the pupil, or by the class in unison. Let *promptness*, as well as accuracy, be aimed at. While one pupil is calling out the results, another may be appointed to watch, and correct any error, while a third may write down the answers with the corrections under them, &c.

From what has thus far been explained in subtraction, the pupil will be able to understand the following

RULE.

I. *Place the less number under the greater, so that units may stand under units, tens under tens, &c.*

II. *Commence at the right, and subtract each figure of the subtrahend from the corresponding figure of the minuend.*

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the hundreds' place, where it counts 10 hundred; place the 10 over the naught, and mark out the 0. Then take 1 hundred from the 10 in the brackets, leaving 9, which place in second brackets above, and mark out the 10; then add the 1 hundred, counting it as 10 tens, to the 4, and you have 14 tens, which place within second brackets over the 4 and mark out the 4.

Now we proceed with the subtraction: 6 from 14 leaves 8; 9 from 9 leaves 0; 4 from 7 leaves 3.

It will be noticed that the minuend appears in three different forms; yet the sum is the same in all. Thus, in the minuend proper, the sum is 8 thousands, 0 hundreds, 5 tens, 3 units; in the minuend in the first brackets, the sum is 7 thousands, 10 hundreds, 4 tens, 13 units; in the second brackets, 7 thousands, 9 hundreds, 14 tens, 13 units; each form being equal to 8053.

These explanations are intended to show the reasons of the process. The pupil should perform similar operations without writing down the steps.

If a figure of the subtrahend be greater than the corresponding figure of the minuend, add 10 *to the minuend figure before subtracting, and then carry* 1 *to the next figure of the subtrahend.*

PROOF OF SUBTRACTION.

§ 16. If the work be correct, the difference added to the subtrahend will equal the minuend.

Perform and prove the first 8 of the following

EXAMPLES.

(100.)	(101.)	(102.)	(103.)
7654321	549876509	7950042689	8877665938
6578906	498763218	6592719897	1988329876

(104.)	(105.)	(106.)	(107.)
8947629	976807136	8765421987	4987670981
98421	7690234	400000901	768432108

108. From seven millions, three hundred and sixty-five thousand, two hundred and thirty-nine, take three hundred and forty-two thousand and thirteen.

109. From one million and eleven, subtract thirteen.

110. From three hundred and sixty-five thousand, take three hundred and sixty-five.

111. America was discovered in 1492. How many years from that time till the year 1844.

112. If a man receive 11345 dollars, and pay out of it 9203 dollars, how much will he have remaining?

113. In 1842 the Onondaga Salt Springs yielded 2291903 bushels of salt, and in 1826 they yielded 827505 bushels. How many more bushels were produced in 1842 than in 1826?

114. In 1842 the United States shipped to England 205154 barrels of flour, to Scotland 3830 barrels. How many more barrels were sent to England than to Scotland?

115. Two men start together from the same place, and travel in the same direction; one goes 63 miles each day, and the other goes 37 miles. How far apart will they be at the end of the first day?

116. George Washington was born in the year 1732; he died in the year 1799. To what age did he live?

117. At an election 12572 votes are taken, of which the successful candidate received 7391. How many votes did the other candidate receive?

118. And what was the first one's majority?

119. The coinage of the United States mint for 1843 was in value 11967830 dollars, and in 1846 it was 6633965 dollars. How much greater in value was the coinage in 1843 than in 1846?

120. The total number of pieces coined in 1843 was 114640582, and in 1844 it was 9051834. How many more pieces were coined in 1843 than in 1844.

121. In the year 1846, the value of the gold coin produced at the mint was 4034177 dollars; the value of the silver coin was 2558580 dollars; and the copper coin was 41208 dollars. How much greater was the value of the gold than the silver, and how much greater than the copper? Also, how much did the silver exceed the copper?

122. In 1835, the number of post-offices in the United States was 10770; extent of post-roads 112774 miles: in 1845, the number of offices was 14183; and extent of roads 143940 miles. How many offices were added during these 10 years, and how many additional miles of road were added?

123. In 1840 the population of New York was 2428921, and in 1830 it was 1913006. What was the increase during this 10 years?

QUESTIONS INVOLVING ADDITION AND SUBTRACTION.

124. A. lets B. have 60 bushels of wheat, worth 70 dollars, a fine horse worth 150 dollars, and 27 dollars' worth of butter. B. in turn gives A. his note for 110 dollars, and the rest in cash. What is the amount of cash?

125. A. borrows of B., at one time, 375 dollars; at a second time he borrows 95 dollars, and at a third time he borrows 413 dollars; he has paid him 319 dollars. How much does he still owe him?

126. A person left a fortune of 10573 dollars to be divided between two sons and one daughter; the first son received 4309 dollars, the other son had 4987 dollars. How much did the daughter receive?

127. Two persons are 375 miles apart; they travel towards each other; at the end of one day, one has travelled 93 miles and the other 57 miles. How far apart are they?

128. A farmer sold a span of horses for 150 dollars, a cow for 27 dollars, some cheese for 83 dollars, and 7 tons of hay for 56 dollars. He purchased 10 yards of broadcloth worth 45 dollars, a cook-stove for 23 dollars, and a pleasure carriage for 80 dollars. How much money will he have left?

129. In the year 1840, the coinage of the United States mint was as follows: 1675302 dollars of gold, 1726703 dollars of silver, and 24627 dollars of copper; in the year 1841 the gold coin amounted to 1091597, the silver to 1132750, and the copper to 15973. How much was the whole value for each year? How much greater was the

whole coinage in 1840 than in 1841? In each year, how much greater was the value of the silver than that of the gold and copper respectively?

130. A person dying, left 60000 dollars, to be divided among his widow, two sons, and two daughters, in the following manner: To each son he gave 10500 dollars, to each daughter 9250 dollars, and the residue to the widow. What was the widow's portion?

131. In 1836 the number of volumes in the Royal Library of Paris was 700000, and the number of manuscripts 80000. In the same year the Vienna Library consisted of 300000 volumes and 16000 manuscripts. How many volumes and how many manuscripts in both libraries? How many more volumes and how many more manuscripts in the Paris library than in the Vienna? The total number of volumes exceed the total number of manuscripts by how many?

132. At a certain election the number of votes polled for two opposing candidates was 544 and 431 respectively. What was the total number of votes polled, and what was the majority of the successful candidate?

CHAPTER V.

MULTIPLICATION.

§ 17. MULTIPLICATION is the process of repeating one of two numbers as many times as there are units in the other.

The number to be repeated is called the *multiplicand.*

The number denoting how many times the multiplicand is to be repeated is called the *multiplier.*

Both multiplicand and multiplier are called *factors*.* The result obtained is called the *product*.

The symbol for multiplication is ×. Thus 3 × 7 means 3 times 7, or 3 multiplied by 7.

MULTIPLICATION TABLE.

2× 1= 2	4× 1= 4	6× 1= 6	8× 1= 8	11× 1= 11
2× 2= 4	4× 2= 8	6× 2=12	8× 2= 16	11× 2= 22
2× 3= 6	4× 3=12	6× 3=18	8× 3= 24	11× 3= 33
2× 4= 8	4× 4=16	6× 4=24	8× 4= 32	11× 4= 44
2× 5=10	4× 5=20	6× 5=30	8× 5= 40	11× 5= 55
2× 6=12	4× 6=24	6× 6=36	8× 6= 48	11× 6= 66
2× 7=14	4× 7=28	6× 7=42	8× 7= 56	11× 7= 77
2× 8=16	4× 8=32	6× 8=48	8× 8= 64	11× 8= 88
2× 9=18	4× 9=36	6× 9=54	8× 9= 72	11× 9= 99
2×10=20	4×10=40	6×10=60	8×10= 80	11×10=110
2×11=22	4×11=44	6×11=66	8×11= 88	11×11=121
2×12=24	4×12=48	6×12=72	8×12= 96	11×12=132
3× 1= 3	5× 1= 5	7× 1= 7	9× 1= 9	12× 1= 12
3× 2= 6	5× 2=10	7× 2=14	9× 2= 18	12× 2= 24
3× 3= 9	5× 3=15	7× 3=21	9× 3= 27	12× 3= 36
3× 4=12	5× 4=20	7× 4=28	9× 4= 36	12× 4= 48
3× 5=15	5× 5=25	7× 5=35	9× 5= 45	12× 5= 60
3× 6=18	5× 6=30	7× 6=42	9× 6= 54	12× 6= 72
3× 7=21	5× 7=35	7× 7=49	9× 7= 63	12× 7= 84
3× 8=24	5× 8=40	7× 8=56	9× 8= 72	12× 8= 96
3× 9=27	5× 9=45	7× 9=63	9× 9= 81	12× 9=108
3×10=30	5×10=50	7×10=70	9×10= 90	12×10=120
3×11=33	5×11=55	7×11=77	9×11= 99	12×11=132
3×12=36	5×12=60	7×12=84	9×12=108	12×12=144

The product of a factor multiplied by itself is called *a square*. A factor, which, multiplied by itself, produces a given number, is called the *square root* of that number. Thus 16 is the *square* of 4, because it is the product of 4 by itself; 4 is the *square root* of 16, because 4 multiplied by itself produces 16.

EXAMPLES.

1–11. What is the square of 2? of 3? of 4? of 5? of 6? of 7? of 8? of 9? of 10? of 11? of 12?

12–22. What is the square root of 25? of 49? of 4?

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* From a Latin word, signifying *to make;* because multiplied together they *make* the product.

of 9? of 16? of 36? of 64? of 81? of 100? of 144? of 121?

23–27. Multiply 8 by the square root of 16; 9 by the square root of 4; 12 by the square root of 49; 8 by the square root of 64; 9 by the square root of 81.

28–36. How many are 3 times 4? 3 times 6? 8? 10? 12? 3? 9? 7? 5?

37–47. How many are 4 times 2? 4? 6? 8? 10? 12? 3? 5? 7? 9? 11?

48–58. Multiply by 5 the following numbers: 3; 6; 9; 12; 2; 5; 8; 11; 4; 7; 10.

59–69. Multiply by 6 the following numbers: 3; 6; 9; 12; 2; 5; 8; 11; 4; 7; 10.

70–80. Multiply by 7 the following numbers: 3; 6; 9; 12; 2; 5; 8; 11; 4; 7; 10.

81–135. Multiply the following numbers: 3; 6; 9; 12; 2; 5; 8; 11; 4; 7; 10—first by 8; then by 9; then by 10; then by 11; then by 12.

136–142. What numbers must be written on the right of the sign = in the following expressions? $6\times5=$; $6\times6=$; $9\times2-4=$; $(6-3)^*\times5=$; $(8+2)\times5=$; $(4+9-2)\times11=$; $(12+4-11+3)\times7=$.

NOTE.—The pupil will remember that the multiplier and the multiplicand may be interchanged without altering the product; thus, $8\times7=7\times8=56$.

§ 18. When the multiplier consists of one figure.

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* When a parenthesis is made to embrace two or more terms, the whole is to be considered as one quantity. In the above expression, the 6 — 3, which is 3, is to be multiplied by 5; so in the expression $(12+4-11+3)\times7$, we first find the value of $12+4-11+3$, which is 8, and then multiply 8 by 7, and find 56.

Multiply 697 by 3.

Place the multiplier under the unit figure of the multiplicand. First, multiplying the 7 units by the 3 units, we obtain 21 units, or two tens and 1 unit. We write the 1 unit under the units' column, and reserve the 2 tens for the tens' column. We next multiply the 9 tens by the 3—the product is 27 tens; adding the 2 reserved, we have 29 tens, or 2 hundreds and 9 tens. Write the 9 under the tens' columns, and reserve the 2 to carry to the hundreds Finally, we multiply the 6 hundreds by the 3, and find the product 18 hundreds, to which, adding the 2 reserved, we have 20 hundreds, or 2 thousands and 0 hundreds. Write the naught under the hundreds and the 2 in the thousands' place. Our product then is 2091 *

Thousands.	Hundreds.	Tens.	Units.	
......	6	9	7	Multiplicand.
			3	Multiplier.
2	0	9	1	Product.

(143.)	(144.)	(145.)	(146.)	(147.)
1234	234156	612378	897654	1003456
2	3	4	5	6

(148.)	(149.)	(150.)	(151.)
205670678	6531023456	891030756078	6289376
7	8	9	7

NOTE.—Let the pupil *prepare*, in his seat, to perform the preceding examples in his class. The recitation will consist in his answering the figures successively, as they are to be written in the product. Let the teacher dictate examples for the slate, and make free use of the blackboard until the operations can be performed very promptly and accurately.

* Annexed is a different illustration of the principles involved in the above operation. The product of the 7 by the 3 is 21 units, or 2 tens and 1 unit; the product of the 9 tens by the 3 is 27 tens; the product of the 6 hundreds by the 3 is 18 hundreds: which are written in their appropriate columns and added together. The whole product is as before.

697	Multiplicand.
3	Multiplier.
21	Units.
27	Tens.
18	Hundreds.
2091	Product.

152–160. Multiply 31486 by 2; 3; 4; 5; 6; 7; 8; 9; and add the products.

161–169. Multiply 8976201 by 9; 8; 7; 6; 5; 4; 3; 2; and add the products.

170–172. Multiply 652081 by 8, and by 3; then give the difference of the products.

173–180. What is the sum of the following products? 8×7654; 9×872150; 8624×6; 9021763×9; 5765×3; 1453217×7; 7×7123541.

§ 19. When the multiplier consists of more than one figure.

Multiply 367 by 84.

367	Multiplicand.
84	Multiplier.
1468	
2936	
30828	Product.

Place the multiplier under the multiplicand, units under units, and tens under tens.

Multiplying first by the 4 units, we find 1468 for the product. We are next to multiply by the 8 tens. Now, it is obvious that 1 unit, taken ten times, that is, multiplied by 1 ten, must produce 10 units or 1 ten. So 7 units (as in the example), multiplied by 8 tens, must produce 56 tens, or 5 hundreds and 6 tens. Therefore, set the first figure, 6, of this second product under the tens' column, and reserve the 5 to carry to the hundreds. The next step is the multiplication of tens by tens, which must produce hundreds, since 1 ten, taken 1 ten times, is equal to 1 hundred. Therefore 8 tens times 6 tens are 48 hundreds; to which add the 5 hundreds reserved, and we obtain 53 hundreds; equal to 5 thousands and 3 hundreds. Place the 3 under the hundreds' column, and carry the 5 to the next column, and so proceed throughout. The sum of these partial products will give the total product, 30828.

NOTE.—If the multiplier consists of three figures, its left-hand or hundreds' figure, multiplied into the units of the multiplicand, will give hundreds for the first figure of the product, which must of course be set down under the hundreds' column; hundreds and tens, multiplied together, will give thousands; hundreds and hundreds, multiplied together, will give ten thousands, &c.

If the multiplier consists of four figures, its left-hand or thousands figure multiplied into units, will give thousands for the first figure of the product, which must be set down under the thousands' column. Thousands multiplied into tens, gives tens of thousands; into hundreds, gives hundreds of thousands; and so on.

It would be necessary to annex ciphers to the figures in these several products, to show their true places, if these places were not determined by the position of the figures with relation to other figures, whose places are known.

§ 20. Take again the first example, viz. the multiplication of 697 by 3. Since 697 is to be repeated 3 times, this may be done by writing the 697 three times, and then performing the addition as in the example.

$$\begin{array}{r} 697 \\ 697 \\ \underline{697} \\ 2091 \end{array}$$

All questions in multiplication may be performed by addition. Hence, *multiplication is sometimes defined as being a concise way of performing several additions.*

§ 21. The pupil will now be able to apply understandingly the following

RULE.

I. *Place the multiplier under the multiplicand, units under units,* &c.

II. *If the multiplier consists of one figure, multiply by it each figure of the multiplicand successively. Place underneath, the right-hand figure of each product, and carry the left-hand figure or figures to the succeeding product.*

III. *If the multiplier consists of more than one figure, multiply by each figure successively. Place the right-hand figure of each partial product under the figure by which* *you multiply. The sum of these partial products will be the total product sought.*

PROOF OF MULTIPLICATION.

§ 22. Interchange multiplicand and multiplier, and see if the same result be obtained.

Or, carefully repeat the multiplication.

EXAMPLES.

(181.)	(182.)	(183.)	(184.)
36984176	809653217	97654321	87509313349
39	68	97	45

(185.)	(186.)	(187.)	(188.)
80265947	79829054	320008999	1357902684
356	289	794	936

NOTE.—See note under § 18.

189–195. Multiply 123456789 by 4287; by 98321; by 65943; by 876542; by 7689769; by 987654321; and add their products.

196. What is the difference of 20946781×87634 and 598421765×30896?

197–198. Write the proper number on the right of the sign $=$: $(9426176-4319832) \times 3426 =$
$69428 \times 876 - 98140 \times 34 =$

199–200. Multiply 69821 by the square of 22; by the square root of 144.

§ 23. We know from what has been said (§ 6), that the annexing of a naught to a number is the same as multiplying that number by 10; that the annexing of two naughts is the same as multiplying by 100, &c. Hence,

When the multiplier or multiplicand, or both, have one or more naughts at the right,

Multiply by the significant figures, as by § 21, and to the product annex as many naughts as there are at the right of both multiplier and multiplicand.

EXAMPLES.

201–204. Multiply 76429 by 10; by 100; by 1000; by 10000.

205–208. Multiply 98740 by 20; by 200; by 2000; by 20000.

209–212. Multiply 654200 by 30; by 300; by 3000; by 30000.

213–216. Multiply 800032000 by 40; by 400; by 4000; by 40000.

217, 218. Multiply 307210000 by 3780000; by 93600700.

§ 24. When the multiplier is a composite number.

A *composite number* is one which may be produced by multiplying two or more numbers together. Thus, 35 is a composite number, since it may be produced by multiplying together 5 and 7. The 5 and 7 are called the component parts or *factors* of 35.

Multiply 48 by 35. If we multiply 48 by 5, one of the factors of 35, we obtain 240 for a product. This product multiplied by 7, the other factor of 35, will give 1680; that is, 1680 is 7 times 5 times 48, or 35 times 48. Hence this

RULE.

Multiply the given number by one of the factors of the *multiplier, and the product thus obtained by another factor, and so on. The last product will be the one sought.*

EXAMPLES.

219. Multiply 365 by 28. The factors of 28 are 4 and 7, or 2 and 2 and 7.

220–224. Multiply 374 by 24; by 18; by 36; by 63; by 108.

225–230. Multiply 987623 by 84; by 35; by 81; by 77; by 64; by 40.

231–237. Multiply 829 by 144; by 21; by 99; by 30; by 15; by 24; by 12.

MISCELLANEOUS EXAMPLES: INVOLVING ADDITION, SUBTRACTION, AND MULTIPLICATION.

238. Suppose I buy 15 loads of bricks, each load containing 1250 bricks: how many bricks have I?

239. In an orchard there are 107 apple-trees, each producing 19 bushels of apples. How many bushels does the whole orchard yield?

240. If a person travel 17 days at the rate of 37 miles each day, how many miles will he travel in all?

241. If a person buy 175 barrels of salt, each weighing 304 pounds, how many pounds in all will he have?

242. Suppose I purchase the following bill of merchandise:

3 Firkins of butter, at 15 dollars each.
7 Hogsheads of molasses, at 23 dollars each.
12 Bags of coffee, at 11 dollars each.
5 Boxes of raisins, at 2 dollars each.
3 Boxes of lemons, at 5 dollars each.

How many dollars must I give for the whole?

243. How many dollars will the following bill of goods amount to?

52 Yards of black broadcloth, at 4 dollars per yard.
40 Yards of Brussels carpeting, at 2 dollars per yard
2 Sofas, at 56 dollars each.
9 Mahogany chairs, at 5 dollars each.
5 French bedsteads, at 7 dollars each.

244. If the railroad extending between Albany and Buffalo, a distance of 326 miles, cost 25649 dollars per mile, what was the entire cost?

245. How many bushels of potatoes may be produced from 13 acres of land, if each acre produces 212 bushels?

246. How much must be paid for constructing 18 miles of plank-road, at 4211 dollars per mile?

247. How much will 543 cords of wood cost, at 5 dollars per cord?

248. In one year there are 8766 hours. How many hours in 1848 years?

249. In one cubic foot there are 1728 cubic inches. How many cubic inches in 17 cords of wood, each cord containing 128 cubic feet?

250. What will 13 square miles of land cost, at 17 dollars per acre, there being 640 acres in one mile?

251. How many miles will a steam locomotive pass in 7 days of 24 hours each, if it move at the rate of 45 miles each hour?

252. If the earth move in its orbit 68000 miles per hour, how far will it move in 365 days of 24 hours each?

253. If one mile of railroad require 116 tons of iron, worth 53 dollars per ton, what will be the cost of sufficient iron to construct a road of 78 miles in length?

254. In an orchard of 105 apple-trees the average pro-

duce of each tree is 7 barrels of fruit, worth 3 dollars per barrel. What was the income of the orchard?

255. A ship, after sailing 23 hours with the velocity of 8 miles per hour, encounters a storm, by which she is driven directly back during 5 hours, with a velocity of 15 miles per hour. At the end of the 28 hours, how far is the ship from the place of departure?

256. A man owing 1213 dollars, gives in payment 3 horses, valued at 150 dollars each, 15 cows, at 27 dollars each, and 143 dollars in cash. How much remains unpaid?

257. A person purchased a farm of 126 acres, at 36 dollars per acre: at one time he sold 57 acres, for 43 dollars per acre; at another time he sold 37 acres, for 51 dollars per acre. How many acres has he left, and at what cost?

258. A person having a journey of 600 miles to perform, travels 47 miles daily for 5 days; during the next 4 days he travels 54 miles each day. How many more miles must he go to complete his journey?

259. If I give 3 horses, each worth 150 dollars, and 13 cows, each worth 29 dollars, for 50 acres of land, valued at 16 dollars per acre, do I gain or lose, and how much?

260. In one year there are 31557600 seconds. How much does one trillion exceed the number of seconds in 1850 years?

261. A person bought two farms: one of 87 acres, at 54 dollars per acre, the other of 101 acres, at 37 dollars per acre: he paid 8140 dollars. How much is still due?

262. How many cubic inches in a hogshead of wine of 63 gallons, there being 231 cubic inches in one gallon?

CHAPTER VI.

DIVISION.

§ 25. DIVISION is the process of finding how many times one number is contained in another.

The number to be divided is called the *dividend.*

The number by which we divide is called the *divisor.*

The number of times which the dividend contains the divisor is called the *quotient.*

The number which is left over after the division is performed, is called the *remainder.*

The symbol for division is $\div$. Thus $8 \div 2$, means 8 divided by 2. Division is also often represented by placing the divisor under the dividend with a line between them; thus $\frac{8}{2}$ denotes that 8 is to be divided by 2.

NOTE.—This symbol is often employed to express an accurate quotient where there is a remainder after division. Thus $13 \div 3 = 4\frac{1}{3}$; which means that the quotient of 13 divided by 3 is 4; to which the remainder 1, divided by 3, or a third part of 1, is to be added.

1–12. How many times is 2 contained in 2? in 6? in 8? in 12? in 4? in 10? in 14? in 18? in 16? in 20? in 24? in 22?

13–24. How many times is 3 contained in 3? in 9? in 15? in 21? in 27? in 33? in 36? in 6? in 12? in 18? in 24? in 30?

25–34. How many times is 4 contained in 8? in 16? 24? 32? 40? 48? 12? 20? 28? 36?

35–45. How many times is 5 contained in 10? 20? 30? 40? 50? 60? 15? 25? 35? 45? 55?

46–54. How many times is 6 contained in 12? 24? 36? 48? 60? 18? 30? 42? 54?

55–65. How many times is 7 contained in 14? 28? 42? 56? 70? 84? 21? 35? 49? 63? 77?

66–76. How many times is 8 contained in 16? 32? 48? 64? 80? 96? 24? 40? 56? 72? 88?

77–87. How many times is 9 contained in 18? 36? 54? 72? 27? 45? 63? 81? 99? 90? 108?

88–100. How many times is 10 contained in 20? 40? 60? 90? 80? 70? 30? 50? 100? 130? 120? 110? 140?

101–109. How many times is 11 contained in 22? in 44? in 66? in 33? in 77? in 110? in 88? in 121? in 132?

110–120. How many times is 12 contained in 24? 48? 60? 96? 84? 144? 120? 132? 36? 72? 108?

121–129. What is the quotient of $108 \div 12$? $36 \div 9$? $77 \div 11$? $63 \div 9$? $100 \div 5$? $\frac{84}{12}$? $\frac{84}{7}$? $\frac{45}{9}$? $\frac{27}{3}$?

130–134. What numbers are to be written on the right of the symbol = in the following expressions?* $(5+7)\div 4 =$? $(6+8-2)\div 3=$? $(12+12-6)\times 2\div 9=$? $(8-4+7)\times 4\div 11=$? $[(9-3)\times 6+4]\div 10=$?

§ 26. When the divisor consists of one figure.

Let us divide 973 by 7.

Divisor.	Dividend.
7)	973
	139 Quotient.

Arrange the numbers as in the example. First, see how many times the 7 is contained in the 9 hundreds: we find it contained 1 time, and 2 hundreds remainder. Write the 1 underneath. To the next figure, 7, which is tens, add the 2 hundreds, or 20 tens, remainder. We then have 27 tens, the same result as if we had prefixed the 2 to the 7. Next, we see how many times 7 is contained in 27

* See note at the bottom of page 29.

which is 3 times, and 6 tens remainder; we place the 3 for the next figure of the quotient, and conceive the 6 to be prefixed to the next figure of the dividend, making 63, which is the same as adding 6 tens or 60 units to the 3 units. Finally, we find 7 is contained in 63 9 times.

We find, then, 7 to be contained in 973, 139 times. Hence 7 repeated 139 times, or, what is the same thing, 139 repeated 7 times must equal 973.

NOTE.—The above example might have been performed another way. We know that 7 is contained in 973 at least once. Subtracting 7 from 973, we have 966 for a remainder. 7 must be contained in 966 at least once. Subtracting 7 from 966 we have 959 for a remainder. We might go on thus until we had performed 139 subtractions, which would exhaust the number, 973. Hence 7 would be contained in 973, 139 times. *Division may therefore be called a concise way of performing several subtractions.*

EXAMPLES.

(135.)	(136.)	(137.)	(138.)
3) 36903	4) 6728910	5) 8931069	6) 31894372

(139.)	(140.)	(141.)
7) 897654321	8) 921076865	9) 234567890

(142.)	(143.)	(144.)
9) 76421068	8) 765324816	7) 890123456

(145.)	(146.)
6) 7890123456	5) 6789012345

NOTE.—Let the pupil practice on the preceding and similar examples, performing the operation aloud in the class, by naming at once the successive quotients which go to make the entire quotient.

147–154. Divide 347891 by 2; by 3; 4; 5; 6; 7; 8; 9.

155–162. Divide 76541307 by 2; 3; 4; 5; 6; 7; 8; 9.

163–170. Divide 897643 by 2; 3; 4; 5; 6; 7; 8; 9.

171–178. Give the quotients of 7653908 ÷ 2; ÷ 3; ÷ 4; ÷ 5; ÷ 6; ÷ 7; ÷ 8; ÷ 9.

179–183. Divide the following numbers by 6: 543218; 39876541; 79654870002; 46895318; 9998887.

184–213. Divide the preceding numbers by 3; by 4; 7; 8; 9; 5.

The foregoing method is called Short Division. The pupil will now understand the following

RULE.

I. *Place the divisor at the left of the dividend, keeping them separate by a curved line, and draw a straight line underneath the dividend.*

II. *Seek how many times the divisor is contained in the left-hand figure or figures of the dividend, and place the result directly beneath, for the first figure of the quotient.*

III. *If there is no remainder, divide the next figure of the dividend for the next figure of the quotient. But when there is a remainder, conceive it to be prefixed to the next succeeding figure of the dividend before making the next division. If a figure of the dividend, which is required to be divided, is less than the divisor, write 0 in the quotient, and consider that figure as a remainder.*

NOTE.—The final remainder, if there be one, may be placed over the divisor with a line between them, and annexed to the quotient. (§ 25, note.) This will denote that the remainder is still to be divided by the divisor.

§ 27. When the divisor consists of more than one figure, or Long Division.

Divide 4703598 by 354.

	Quotient.
354) 4703598 (	13287
354	1st product.
1163	Thousands.
1062	2d product.
1015	Hundreds.
708	3d product.
3079	Tens.
2832	4th product.
2478	Units.
2478	5th product.

It requires 3 figures, 470, of the dividend, to contain the divisor. This is contained once in 470; we place the 1 at the right of the dividend for the first figure of the quotient. Multiplying the divisor by this quotient figure, and subtracting the product from 470, we have 116 for a remainder, to which we annex the next figure, 3, of the dividend, thus forming the number 1163. We now seek how many times the divisor is contained in 1163, which is 3 times. We place the 3 for a second figure of the quotient. Multiplying the divisor by this second figure, and subtracting the product from 1163, we find 101 for a second remainder; to which annexing 5, the next figure of the dividend, we have 1015. Thus we proceed till all the figures of the dividend have been brought down.

From the above work we readily deduce the following

RULE.

I. *Place the divisor at the left of the dividend, keeping them separate by a curved line.*

II. *Seek how many times the divisor is contained in the fewest figures of the dividend that will contain it; set the figure expressing the number of times at the right of the dividend for the first figure of the quotient, keeping dividend and quotient separate by means of a curved line.*

III. *Multiply the divisor by this quotient figure, and subtract the product from those figures of the dividend used, and to the remainder annex the next figure of the dividend;*

then find how many times the divisor is contained in this new number, and write the result in the quotient.

IV. *Continue the operation, as before, until all the figures of the dividend have been brought down.*

NOTE 1.—Having brought down a new figure, if the number thus formed be less than the divisor, it will contain it 0 times; we therefore write 0 in the quotient, and bring down another figure.

NOTE 2.—If in multiplying the divisor by any quotient figure we obtain a product which exceeds the number we sought to divide, we must make the quotient figure smaller.

NOTE 3.—If a remainder should be found larger than the divisor, the quotient figure must be taken larger.

PROOF.

§ 28. The dividend, when there is no remainder, is a composite number, of which the divisor and quotient are factors. The final remainder, if any, is evidently part of the dividend. Consequently, if the work be right, the product of the divisor and quotient, with the remainder, if any, added, will equal the dividend. Or, we may prove or test the work as we proceed, by adding together the successive products and the remainder, if any. If the work is right, this sum will equal the partial or entire dividend, as the case may be. Thus, in the previous example, 354, the first product, and 1062, the second product, with the remainder, 101, when added together, the sum is equal to 4703, the partial dividend, &c.

EXAMPLES.

214–218. Divide 826190 by 24; 48; 96; 112; 144; and annex the remainders, if any, to the quotients, according to § 26; "Note," under the rule.

219–224. Divide 9281746 by 27; 44; 98; 76; 236; 294; and annex, &c.

225–235. Dividend 829765149. Divisors 486; 928; 714; 386; 403; 907; 6172; 8316; 5793; 2405; 3006. Give the exact quotients.

236–243. Divide 7200651897 by 2498; 76389; 32174; 98263; 45208; 301987; 567891; 890123.

244–250. Divide 8976014236 by 298701; 4853684; 9130821; 1280319; 7600994; 3268753; 91465923.

251–257. Divide 123456789 by 789; 5763447 by 678509; 1521808704 by 6503456; 243166625648 by 3471032; 166168212890625 by 12890625; 11963109376 by 109376; 24892456 by 36546.

§ 29. When the divisor ends with one or more naughts. We have seen (§ 6) that a number is multiplied by 10, by annexing a naught; by 100, by annexing two naughts, &c. Conversely, a number is divided by 10, by cutting off one naught from the right; by 100, by cutting off two naughts from the right, &c.

So if, instead of naughts, significant figures are cut off from the right of a number, the number is still divided by 10, 100, &c., while the figures cut off, are remainders after the division.

Let us divide 2475 by 20.

```
2|0 ) 247|5
      ---
      123   15 remainder.
```

Having cut off the 5 from the right of the dividend, and the 0 from the right of the divisor, which is, in effect, dividing both dividend and divisor by 10, we divide 247 by 2. We obtain 123 for a quotient and 1 for a remainder. This remainder is 1 ten, since it is a part of the 7 of the dividend which occupies the tens' place; annexing the 5 units which was cut off to the 1 ten which remained, we have 1 ten and 5 units, or 15 for the true remainder.

NOTE.—This case may be comprised under that wherein the divisor is a composite number. (See forward, § 29.) Thus, taking the preceding example, the divisor $20 = 2 \times 10$. Dividing 2475 first by 10, which division is effected by cutting off the right-hand figure, 5, we have 247 for the first quotient, and 5 for the first remainder. Next, dividing 247 by 2, we find 123 for the quotient sought, and 1 for the second remainder.

Now, by the rule under the case referred to, we find the true remainder to be $1 \times 10 + 5 = 15$.

Hence the following

RULE.

Cut off from the right of the dividend as many figures as there are naughts at the right of the divisor; divide what remains by the divisor without the naughts at its right. To the final remainder annex the figures cut off from the dividend, for the true remainder.

EXAMPLES.

258–263. Divide by 20 the following numbers: 17284; 365920; 9873542; 345678901; 135794680; 379653219.

264–271. Divide 69543218937 by 240; by 300; by 480; by 700; by 690; by 4000; by 80000; by 900000.

272–273. Divide 7123545 by 421000; 1212121212 by 42000.

274–275. Divide 123456789 by 12300; 7296148731 2498000.

276–279. Divide 87369841 by 3000; 970000; 8103030; 6090300.

280–285. Divide 943821900 by the following numbers: 78910; 36800; 42700; 9865200; 437001000; 9843020.

286–289. Divide 376549281 by 370×480; by $630 \times 30 \div 10$; by $82 \times 500 \div 100$; by $8700 \times 300 \div 1000$.

When a divisor is a composite number, it may sometimes be convenient to divide by its factors.

Thus, if we wish to divide 944 by 105, we may resolve the 105 into its factors, $3\times5\times7$, and divide as in the example. The only difficulty lies in obtaining the true remainder; that is, the remainder which would have resulted from dividing by 105 at once.

```
3 ) 944
    ---
5 ) 314  2, 1st rem.
    ---
7 ) 62   4, 2d rem.
    ---
     8   6, 3d rem.
```

Since each unit of the 62 is 5 times as great as each unit of 314, it follows, that each unit of the 3d remainder, 6, which is a part of 62, is also 5 times as great as each unit of 314. Hence the remainder 6 is the same as 5 times 6, or 30 units of the same kind as those of 314; but the 2d remainder, 4, being a part of 314, and of the same order, should be added to 30, making 34, for the true remainder arising from dividing 314 by 35 or 5×7. Again, since each unit of 314 is 3 times as great as each unit of 944, it follows, that each unit of the 34 is also 3 times as great as each unit of 944. Hence the remainder 34 is the same as 3 times $34=102$ units of the same kind as 944; but the first remainder, 2, being a part of 944, is of the same order; so that $102+2=104$, is the true remainder required.

The operation may be arranged thus: $(6\times5+4)\times3+2=104$.

Hence the following

RULE.

Divide the given sum by one of the factors of the divisor, and the quotient thus obtained by another factor, and so on. The last quotient will be the quotient required.

Multiply the last remainder by the divisor preceding the last, and add in the preceding remainder; multiply this sum by the next preceding divisor, and add in the next preceding remainder; and so on.

EXAMPLES.

290–295. Divide 8217 by $35=5\times7$; 33678 by $15=5\times3$; 9591 by $72=9\times4\times2$; 10859 by $49=7\times7$; 926541 by $81=9\times9$; 987654 by $63=7\times3\times3$. Give the true remainders of the preceding.

QUESTIONS INVOLVING THE FOUR GROUND-RULES.

296. A person owes to one man 375 dollars, to another he owes 708 dollars, to a third man he owes 911 dollars. How much does he owe to the three men?

297. A farmer has sheep in five fields: in the first, he has 917; in the second, 249; in the third, 413; in the fourth, 1000; and in the fifth, he has 197. How many sheep has he in the five fields?

298. A person owes to one man 302 dollars, to another man he owes 707 dollars, and has owing to him 2000 dollars. How much will remain after paying his debts?

299. A farmer receives for his wheat 103 dollars, for his corn 60 dollars, for his butter 511 dollars, for his cheese 1212 dollars, for his pork 601 dollars. He pays towards a new farm 1000 dollars, for a new wagon 50 dollars, for hired help on his farm 290 dollars, for repairing house 173 dollars. How much money has he remaining?

300. A person wills 1200 dollars to his wife, 300 dollars for charitable purposes, and what remains is to be equally divided among 6 children. Allowing his property to amount to 8562 dollars, how much would each child have?

301. A man gave 13558 dollars for a farm: he then sold 73 acres, at 75 dollars per acre; the remainder stood him in at 59 dollars per acre. How many acres did he purchase?

302. Four boys divide 336 apples as follows: the first takes one-sixth of the whole; the second takes one-fourth of what was left; the third takes one-half of what was then left; the fourth has the remainder. What number of apples did each boy have?

303. Three men are to share equally in the sum of 1236 dollars. How many dollars will each have?

304. Divide 1245 acres of land equally between five brothers.

305. It is about 95000000 miles from here to the sun. Now, admitting that it requires 8 minutes for light to pass from the sun to the earth, how many miles does it pass in one minute?

306. Allowing 22 bricks to be sufficient to make one cubic foot of masonry, how many cubic feet are there in a work which requires 100000 bricks?

307. The circumference of the earth is about 25000 miles. How long would it require for a person to travel around it, if he could pass uninterruptedly at the rate of 200 miles per day?

308. In 1845 the extent of post-roads in the United States was 143940 miles, and the amount paid for the transportation of the mail during the same year was 2905504 dollars. How much was the average expense per mile?

309. The distance of Uranus from the sun is about 1860624000 miles. How many hours would it require to pass this distance at 18 miles per hour? Also, how many days, and how many years, counting 24 hours to the day, and 365 days to the year?

310. How many barrels of apples, at 3 dollars per barrel, can I buy for 2568 dollars? And if one tree produce 8 barrels, how many trees will yield the required amount?

311. An estate of 8100 dollars was divided among nine children in the following way: the first had 100 dollars and one-tenth of the remainder; after this the second had 200 dollars and one-tenth of the residue; again, the third had 300 dollars and one-tenth of the remainder, and so on: each succeeding child had 100 dollars more than the one immediately preceding, and then one-tenth of what still remained. What was the share of each?

312. A. and B. each owe C.: A. owes 1472 dollars, which is less than what B. owes him, and yet the difference between A.'s and B.'s debts is 719 dollars. How much does B. owe C.?

313. Admitting the earth to move 68000 miles per hour, how far will it move in one day; and how far in a year of 365 days?

314. If the President of the United States expends daily 60 dollars, how much will he be able to save at the end of the 365, out of his salary of 25000 dollars?

315. An army, consisting of 4525 men, have 103075 loaves of bread. At the end of 21 days, 500 men are killed in a battle. Now, if each man in each day eat one loaf of bread, how many days after the battle will the bread sustain the army?

316. Two locomotives start from the same place, and move in the same direction; the first goes 25 miles each hour, the second only 15 miles. After the first has passed a distance of 100 miles, it commences a backward motion, maintaining the same velocity until it meets the second locomotive. How many hours after starting will they meet? And at what distance will they meet from the starting-point?

317. One hundred miles of railroad track are to be laid with heavy rail, requiring 116 tons to the mile. After receiving iron, at 52 dollars per ton, to lay 58 miles, the price per ton was increased so as to make the whole cost of the entire road 612944 dollars. What was the latter price per ton of the iron?

318. A person bought two farms, one of 97 acres, at 51 dollars per acre, and the other of 111 acres, at 47 dollars per acre. He paid 9539 dollars cash, and for the balance he gave 5 horses. What were the horses valued at?

319. A person having a salary of 1700 dollars saves 970 dollars at the end of the year. How much on an average were his daily expenses, if we count 365 days to the year?

320. A man travelled 832 miles in 20 days: during the first 9 days he went 37 miles daily; during the next 5 days, he went 41 miles daily. How many miles each day did he travel during the last 6 days?

CHAPTER VII.

GENERAL PROBLEMS AND PRINCIPLES.

PROBLEMS FOUNDED UPON THE FOREGOING RULES.

§ 30. *a.* The sum of two numbers, and one of those numbers being given, to find the other number: *Subtract the given number from the sum.*

NOTE.—Let the *pupil* illustrate by an example this and each succeeding problem.

b. The difference between two numbers and the larger number being given, to find the smaller number: *Subtract the difference from the larger number.*

c. The difference between two numbers and the smaller number being given, to find the larger: *Add the smaller number and the difference together.*

d. The sum and the difference of two numbers being given, to find the two numbers: *The difference added to the sum will give twice the larger number.*

NOTE.—Let the pupil illustrate and explain; and show how the smaller number may be found.

e. The product of two numbers, and one of those numbers being given, to find the other number: *Divide the product by the given number.*

f. The dividend and quotient being given, to find the divisor: *Divide the dividend by the quotient.*

g. The quotient and divisor being given, to find the dividend: *Multiply the quotient and divisor together.*

NOTE.—Let the pupil be required to illustrate each of the above problems.

EXAMPLES.

1. The sum of two numbers is one hundred and forty-seven millions, two hundred and eight thousand, eight hundred and sixty-six; one of the numbers is twenty-three millions, seven hundred and eighty-five thousand, four hundred and thirty-two. What is the other number?

2. A gentleman dying bequeathed 387984 dollars to his two children: one obtained 44836 dollars. What was the share of the other?

3. The difference between two numbers is 10144; the minuend is 69975. What is the subtrahend?

4. The difference between two numbers is 150110; the subtrahend is 729355. What is the minuend?

5. The sum of two numbers is 1809004332; their difference is 166304310. What are the two numbers?

6. A product is 539902; one of its factors is 23. What is the other factor?

7. Divide the sum of the following numbers by 147: 31969217, 182681240, 456703100.

8. A person was desirous of knowing the amount of money bequeathed to each of two children. He ascertained that together they had received 48465 dollars; and that one had received 20891 dollars more than the other. How much did each receive?

9. The product of two numbers is 374671924092; one of the numbers is 302014. What is the other number?

10. A dividend is 4703598; the quotient is 13287. What is the divisor?

11. A divisor is 6503456; the quotient is 234. What is the dividend?

12. At a certain election the whole number of votes received by two opposing candidates was 8737; the successful candidate's majority was 375. How many votes did each receive?

13. If the distance between two planets is 239000000 miles when they are on opposite sides of the sun, and 49000000 miles when on the same side of the sun, how far is each from the sun, if we suppose their orbits perfectly circular?

14. In an orchard of 1813 trees, there are 37 rows. How many trees in each row?

15. A certain number of dollars is divided among 365 persons, each person receiving 97 dollars. How many dollars was divided?

16. The construction of 13 miles of plank road cost 74334 dollars. How much was that per mile?

17. In a field of maize there are 243 rows of 187 hills in each row. How many hills in all?

18. I have two casks of wine, which together contain 67 gallons, and one contains 17 gallons more than the other. How many gallons does each contain?

19. Two persons starting from the same place find that when they travel in opposite directions they are at the end of one hour 18 miles apart, but when they travel in the same direction they are at the end of one hour only 4 miles apart. How far does each go in one hour?

20. After taking 371 dollars from a certain sum of money,

I find 275 dollars remaining. How many dollars were there at first?

21. Two brothers being asked their ages, the elder said he was 59 years old, and that his brother was just 15 years younger. What was the age of the younger?

22. If I give 785 dollars for 157 barrels of apples, how much do I pay per barrel?

23. How many barrels of apples can I buy for 785 dollars, at 5 dollars per barrel?

24. In one cubic foot there are 1728 cubic inches. How many cubic feet are there in 174528 cubic inches?

25. In 101 cubic feet there are 174528 cubic inches. How many cubic inches in one foot?

PRINCIPLES EVOLVED FROM DIVISION.

§ 31. We are now prepared to understand certain important relations which divisor, dividend, and quotient bear to each other.

The product of the divisor and quotient is always equal to the dividend. Hence,

a. The divisor and the quotient may be interchanged; that is, if the dividend be divided by the quotient, the result will be the divisor; thus, 4) 36 (9 quot., 9) 36 (4 quot.

b. If the divisor remain the same, *multiplying* the dividend by a given number has the effect to multiply the quotient by the same number; thus, 36, the same dividend as above, multiplied by 2 and divided by the same divisor, 4, will give 18 for a quotient, or 2 times the quotient above.

c. If the dividend remain the same, multiplying the divisor by any number has the effect to divide the quotient by the same number. Thus, retaining the dividend, 36, if we multiply its divisor by 3, $3 \times 4 = 12$, the quotient is 3, or the same as $9 \div 3$.

d. If the divisor and dividend be multiplied by the same number, there will be no change in the quotient; thus, 36×3 divided by 4×3 will give 9 for the quotient, as before.

Still further:

e. If the divisor remain the same, *dividing* the dividend by a given number has the effect to divide the quotient by the same number; thus, $36 \div 3$, or 12 divided by 4, will give $9 \div 3$, or 3 for a quotient.

f. If the dividend remain the same, dividing the divisor by any number has the effect to multiply the quotient by the same number; thus, $4 \div 2$, or 2 is contained in 36, 9×2 or 18 times.

g. If dividend and divisor be divided by the same number, the quotient will remain the same; thus, $36 \div 2$, or 18 divided by $4 \div 2$, or 2 will give 9 for a quotient as before.

CHAPTER VIII.

PRIME AND COMPOSITE NUMBERS.

PRIME NUMBERS.

§ 32. A COMPOSITE NUMBER, § 24, is one which can be resolved into other factors besides itself and units; thus, 12 is a composite number, because it may be resolved into $2 \times 3 \times 2$.

A *prime number* is one that *cannot* be resolved into other factors besides itself and unity. Thus, 11 is a prime number, because no two or more numbers, less than 11, and greater than 1, multiplied together, will produce 11.

Two numbers are said to be prime to each other when they have no common factor. Thus, 8 and 9 are prime to each other, although neither is a prime number.

The following are some of the prime numbers:

1	29	71	113	173	229	281	349	409	463
2	31	73	127	179	233	283	353	419	467
3	37	79	131	181	239	293	359	421	479
5	41	83	137	191	241	307	367	431	487
7	43	89	139	193	251	311	373	433	491
11	47	97	149	197	257	313	379	439	499
13	53	101	151	199	263	317	383	443	503
17	59	103	157	211	269	331	389	449	509
19	61	107	163	223	271	337	397	457	521
23	67	109	167	227	277	347	401	461	523

§ 33. The pupil may be aided to determine whether a number is prime or not by remembering the following facts:

1. If any number terminate with 0 or an *even** digit, the whole will be divisible by 2.

2. If any number terminate with 0 or 5, the whole will be divisible by 5.

3. When the number expressed by the two right-hand figures is divisible by 4, the whole will be divisible by 4.

4. When the number expressed by the three right-hand figures is divisible by 8, the whole will be divisible by 8.

5. If the sum of the digits of any number be divisible by 3 or 9, the whole number will be divisible by 3 or 9.

NOTE.—We see that any number formed by a succession of zeros placed at the right of 1, as 10, 100, 1000, &c., will contain 9 a certain number of times and 1 over. If the numbers are formed by zeros placed at the right of 2, as 20, 200, 2000, &c., 9 will be contained a

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* An *even* digit is one that can be exactly divided by 2; as, 2, 4, 6, 8. An *odd* digit is one that cannot be so divided.

certain number of times and 2 over. In numbers of the form of 30, 300, 3000, &c., the remainders will be 3; and so on, for other similar kind of numbers.

Now let us seek the remainder, when 2846 is divided by 9. The number 2846 is made up of 2000, 800, 40 and 6. 2000 will contain 9 a certain number of times and 2 over; 800 a certain number of times and 8 over; 40 a certain number of times and 4 over. If then the sum of these remainders, together with the 6 units, will exactly divide by 9, the whole number is divisible by 9; that is, a number, is divisible by 9 when the sum of its digits is divisible by 9.

The same will be true of 3, since 3 is a divisor of 9; that is, a number is divisible by 3 when the sum of its digits is divisible by 3.

Find the prime factors of 868.

We see at a glance that 868 is divisible by 2. Again, that the quotient 434 is divisible by 2. Dividing the second quotient 217 by 7,* we obtain the prime quotient 31. Therefore the prime factors of 868 are 2, 2, 7, 31; so that $2 \times 2 \times 7 \times 31 = 868$.

$$\begin{array}{r|l} 2 & 868 \\ \hline 2 & 434 \\ \hline 7 & 217 \\ \hline & 31 \end{array}$$

EXAMPLES.

1–8. What are the prime factors of 12? 14? 15? 16? 18? 20? 22? 24?

9–16. What are the prime factors of 25? 26? 27? 28? 30? 32? 33? 34?

17–24. What are the prime factors of 35? 36? 38? 39? 40? 42? 44? 45?

25–32. What are the prime factors of 46? 48? 49? 50? 51? 52? 54? 55?

33–41. Find the prime factors of 56; 57; 58; 60; 62; 63; 64; 65; 66.

* Having determined by inspection that a number is not divisible by 2, 3, 5, 8, 9, or 10, we must try the prime numbers in succession, beginning with 7.

42–50. Find the prime factors of 68; 69; 70; 72; 85; 87; 90; 96; 98.

51–57. Find the prime factors of 102; 111; 119; 125; 138; 146; 155.

58–63. Find the prime factors of 154; 166; 178; 209; 234; 259.

64–69. Find the prime factors of 309; 366; 375; 404; 473; 524.

70–76. What are the prime factors of 984? of 1040? of 1368? of 1224? of 6584? of 78903? of 62148?

77–83. What are the prime factors of 6918? of 76540? of 63142? of 78900? of 6432? of 97563? of 89706?

84–90. Resolve into their prime factors the following numbers: 7498; 56234; 49750; 3333; 99939; 48765; 92890.

Greatest Common Divisor.

§ 34. The numbers 12, 24, 48, 60, can each be divided by 2, 3, or 6; 2, 3, 6 are therefore common divisors of those numbers; but 12 is the *greatest* number that will divide them all; therefore 12 is their *greatest* common divisor.

To show the principle of this, let us resolve the above numbers into their prime factors:

$12 = 2 \times 2 \times 3$; $24 = 2 \times 2 \times 3 \times 2$; $48 = 2 \times 2 \times 3 \times 2 \times 2$; $60 = 2 \times 2 \times 3 \times 5$.

Now, evidently, each of these numbers may be divided by one of its factors, or by the product of two or more of them. Thus each may be divided by 2; but this will be their *least* common divisor above unity.* So each may be divided by 2×2.

* When numbers are prime to each other, § 32, they have no common divisor greater than unity.

But the *greatest* common divisor must be the product of *all* the factors common to all the numbers; thus, $2\times2\times3=12$, the greatest common divisor.

Take another example. What is the greatest common divisor of 492, 744, 906? $492=2\times2\times3\times41$; $744=2\times2\times3\times2\times31$; $906=3\times2\times151$. The factors common to all the numbers are 3×2; therefore $3\times2=6$ is their greatest common divisor.

EXAMPLES.

91–94. What is the greatest common divisor of 360 and 276? of 592 and 599? of 315 and 405? of 1825 and 2655?

95–99. Find the greatest common divisor of 506 and 308; of 404 and 364; of 1112 and 616; of 808 and 728; of 1518 and 924.

100–103. Find the greatest common divisor of 492, 744, and 906; of 246, 372, and 522; of 1476, 2232, and 2718; of 738, 1116, and 1566.

104–106. Find the greatest common divisor of 252, 380, 454, and 500; of 756, 1140, 1362, and 1500; of 1764, 2660, 3178, and 3500.

107–109. Find the greatest common divisor of 632, 706, 834, and 8834; of 1896, 2502, 2862, and 1640; of 3528, 4424, 1942, and 5164.

Hence we have the following

RULE.

Resolve each number into its prime factors. The product of all the factors common to all the numbers will be the greatest common divisor.

§ 35. There is another mode of determining the greatest common divisor.

a. Any number that will exactly divide the less of two numbers and their difference, will also divide the greater number. Suppose the two numbers to be 18 and 48. 6 will divide the less number 18, and the difference 30; consequently it will divide 48. For the number of times the divisor is contained in the less number plus the number of times it is contained in the difference, must be the number of times it is contained in the larger number.

b. The same may be said of any number that will exactly divide the less number and the difference between *any number of times the less number* and the greater number. Thus taking the number above: 18, the less number, $\times 2 = 36$; 36 subtracted from 48 leaves 12; 6 will divide the remainder 12, and the less number 18. Consequently it will divide the larger number 48.

What is the greatest common divisor of 276 and 360?

```
276 ) 360 ( 1
      276
      ---
       84 ) 276 ( 3
            252
            ---
             24 ) 84 ( 3
                  72
                  --
                  12 ) 24 ( 2
                       24
                       --
```

The greatest divisor cannot exceed the less number 276. But 276 will not divide the other number 360 without a remainder 84. Therefore 276 is not a common divisor.

Now the number that will exactly divide 84 and 276 (*a*) will also divide 360. We seek such a divisor. It cannot exceed 84. Trying 84, we find it will not divide 276 without a remainder, 24; 84 is therefore not the greatest common divisor.

Again, the number that will exactly divide 24 and 84 (*b*) will also divide 276. This divisor cannot exceed 24. But 24 will not divide 84 without a remainder 12; 24 is therefore not the greatest common divisor.

Lastly, the number that will exactly divide 12 and 24 will also divide 84 (*b*). Trying the less number 12, we find it exactly divides the other. Therefore 12 is the greatest common divisor of 276 and 360.

Hence the following

RULE.

Divide the greater number by the less, then the less number by the remainder: thus continue to divide the last divisor by the last remainder, until there is no remainder. The last divisor will be the greatest common divisor.

NOTE.—If the greatest common divisor of more than two numbers be required, find first the greatest common divisor of two of them, then of the divisor so found, and one of the remaining numbers, and so on.

EXAMPLES.

110–117. Find the greatest common divisor of 365 and 511; of 115 and 161; of 203 and 261; of 145 and 185; of 120 and 350; of 420 and 864; of 560 and 768; of 936 and 1170.

118–121. Find the greatest common divisor of 805 and 1127; of 1421 and 1827; of 1015 and 1295; of 888 and 999.

122–124. Find the greatest common divisor of 345, 483, and 609; of 783, 435, and 555; of 2842, 3654, and 2030.

125–127. Find the greatest common divisor of 1602, 1603, 1604; of 311, 400, 510; of 823, 800, 672.

128–130. Find the greatest common divisor of 185, 259, 407; of 86, 430, 473; of 505, 707, 4343.

131–132. What is the greatest common divisor of 2233, 2030, 1827, 3045, 4060? of 3885, 5550, 6105?

LEAST COMMON MULTIPLE.

§ 36. The *multiple* of a number is a product of which such number is a factor. Thus, 4, 6, 8, are multiples of 2.

A *common multiple* of two or more numbers is a product of which all such numbers are factors. Thus, 48 is a common multiple of 2, 3, 4, 6, 8.

The *least common multiple* of two or more numbers is the *least* product of which all such numbers are factors. Thus, while, as above, 48 is a common multiple of 2, 3, 4, 6, 8, because it can be exactly divided by each of those numbers, 24 is their *least* common multiple, because it is the least number that can be so divided.

Evidently a number or a series of numbers can have an infinite number of multiples.

The multiple of two numbers prime to each other, is their product.

Find the least common multiple of 8, 16, 24.

Let us resolve these numbers into their prime factors. $8=2\times2\times2$; $16=2\times2\times2\times2$; $24=2\times2\times2\times3$. The least common multiple of these numbers must contain *all* their prime factors *once*. We then note, first, all the factors of 8: $2\times2\times2$. Next, seeing that 16 has an additional factor, 2, and that 24 has an additional factor, 3, we take the product of all these different factors: $2\times2\times2\times2\times3=48$, which is the least common multiple of 8, 16, 24. We have then this rule, for finding the least common multiple of two or more numbers.

Resolve each number into its prime factors. Multiply these factors together, using such as are common to two or more numbers BUT ONCE. *The product so found will be the least common multiple.*

NOTE.—If a factor be used more than once, though the result would be a multiple of the given numbers, yet it would not be the least common multiple.

§ 37. It may be sometimes more convenient to adopt the following method.

Find the least common multiple of 10, 18, 21.

We divide first by the prime factor 2, since it will divide two of the numbers without a remainder, and place the quotients 5 and 9 together with the undivided number 21, in the line below. We then divide by another prime number 3, for the same reason, and set the quotients, with the undivided number 5, below. There can be no further division since no two of the remaining numbers have a common divisor. The divisors multiplied into the quotients in the last line, will give the least common multiple required.

$$\begin{array}{r|lll} 2 & 10, & 18, & 21 \\ \hline 3 & 5, & 9, & 21 \\ \hline & 5, & 3, & 7 \end{array}$$

$$2\times3\times5\times3\times7=630$$

NOTE.—The principle of this operation is the same as before. It consists in finding the prime factors of a series of numbers and taking their product. Thus, $10=2\times5$; $18=2\times3\times3$; $21=3\times7$. 2 being a common factor of 10 and 18, as in the above divisor, is used but once. One of the 3's being common to 9 and 21, is used but once as in the second division above. These multiplied into the 5, the remaining 3 and the 7, will produce the result sought. Hence the following

RULE.

Write the numbers in a horizontal line; divide them by any prime number which will divide two or more of them without a remainder; place the quotients with the undivided numbers, if any, for a second horizontal line; proceed with this second line as with the first; and so continue until there are no two numbers which can be exactly divided by the same divisor. The continued product of the divisors, and of the numbers in the last horizontal line, will give the least common multiple.

133–138. What is the least common multiple of 12, 16, and 24? of 12, 15, and 24? of 11, 77, and 88? of 37 and 41? of 24, 60, 45, 180? of 2, 4, 6, 8?

139–143. What is the least common multiple of 3, 5, 7, 9? of 2, 3, 4, 5, 6, 7, 8, 9? of 7, 14, 16, 18, 24? of 1, 2, 3, 4, 5, 6, 7, 8, 9, 11? of 12, 15, 16, 18, 20, 24?

144–147. What is the least common multiple of 36, 40, 45, 60, 72, 90? of 10, 20, 25, 50? of 5, 9, 15, 18, 36, 135, 162? of 115, 184, 230, 460?

148–151. What is the least common multiple of 140, 168, 210, 280, 420? of 3, 5, 7, 14, 35, 42? of 17, 19, 34, 38, 209? of 11, 13, 26, 99, 100?

152–156. What is the least common multiple of 8, 10, 12, 13, 375? of 34, 75, 88, 99? of 2, 3, 5, 7, 11, 13, 17? of 4, 6, 8, 10, 19? of 20, 21, 24, 48?

CANCELATION.

§ 38. Suppose we are required to divide 35 times 99 by 63. As these numbers are composite, we have $\frac{35\times99}{63}=\frac{5\times7\times9\times11}{7\times9}$. Now we know (§ 31, *g.*) that dividend and divisor may be divided by the same number without altering their relation to each other; in other words, without affecting the value of the quotient. We then divide the dividend and divisor of the preceding expression by 7 and by 9; it becomes $\frac{5\times1\times1\times11}{1\times1}=55$. We may perform this division by drawing a line through the common factors, thus, $\frac{5\times\not7\times\not9\times11}{\not7\times\not9}$, and operating upon the remaining ones; the expression then becomes $\frac{5\times11}{1}=55$.

This rejecting of common factors is called *cancelation.* It is a great saving of labor.

When *all the factors* in either dividend or divisor are canceled, write 1 in their place.

Divide the product 21 times 22 times 65 by 1001.

$$\frac{21\times22\times65}{1001}=\frac{3\times7\times2\times11\times5\times13}{7\times11\times13}=\frac{3\times\not{7}\times2\times\not{11}\times5\times\not{13}}{\not{7}\times\not{11}\times\not{13}}=\frac{30}{1}=30$$

or the process may be arranged thus:

$$\begin{array}{c} 3 \qquad 2 \qquad 5 \\ \not{21}\times\not{22}\times\not{65} \\ \hline \not{1001} \\ \not{143} \\ \not{13} \end{array}$$

First cancel the factor 7 in 1001 and in 21, writing the other factors 143, and 3 below and above the respective numbers. Next cancel the 11 in 143 of the divisor and 11 of the dividend, writing the other factors, 13 and 2, below and above the respective numbers as before. Next, cancel the 13 of the divisor and the 13 of the dividend, writing the other factor 5 over the 65. The expression will then stand $\frac{3\times2\times5}{1}=\frac{30}{1}=30.$

EXAMPLES.

157. Divide $2\times3\times8\times5\times7$ by $2\times4\times15$.

158. A man had 34 filberts in each of 49 different piles. He was to distribute these among 7 boys and 7 girls. How many did each boy and girl receive?

159–160. 8×12 is how many times 8? how many times 12?

161–162. 4×72 is how many times 6? how many times 12?

163–169. 72×48 is how many times 6? 8? 12? 16? 24? 32? 48?

170–172. What is the value of 36×100 divided by 10×18? divided by 2×20? divided by 9×10?

173–175. What is the value of $99\times360\times365$ divided by 11×73? divided by 33×18? divided by 44×5?

176–179. What is the value of 33×77 divided by 121? divided by 21? divided by 7? divided by 3?

180–183. What is the value of $36\times42\times52$ divided by $2\times3\times4$? divided by 32×13? divided by 9×21? divided by 21×13?

184–187. What is the value of $12 \times 11 \times 10$ divided by $2 \times 3 \times 4$? divided by 3×5? divided by 3×11? divided by 4×5?

188–190. What is the value of $9 \times 40 \times 100$ divided by $2 \times 3 \times 4 \times 5$? divided by $6 \times 8 \times 10$? divided by $3 \times 6 \times 25$?

CHAPTER IX.

FRACTIONS.

§ 39. A FRACTION is a *part** of a unit. If an apple be divided into 2 equal parts, each part will be *one-half* of the apple; that is, $1 \div 2$, or $\frac{1}{2}$. If the apple be divided into 3 equal parts, each part will be *one-third* of the apple; that is, $1 \div 3$, or $\frac{1}{3}$, &c.

Suppose 3 apples are to be divided among 5 boys. Cut each apple into 5 equal parts or *fifths*, and give one part or fifth of each apple to each boy. He will then have *one-fifth* of three apples, or, what is the same thing, *three-fifths* of an apple: in figures, $\frac{3}{5}$.

We see, then, that the number of parts into which a thing or a unit is divided, is expressed by the figure below the line, while the number of such parts as are taken or used is expressed by the figure above the line.

The expression $\frac{3}{5}$ may be read, one-fifth of three; or 3 divided by 5; or *three-fifths*. The latter is the usual mode.

Read the following expressions: $\frac{2}{3}$, $\frac{8}{10}$, $\frac{4}{16}$, $\frac{9}{11}$, $\frac{20}{35}$, $\frac{25}{86}$, $\frac{42}{120}$, $\frac{47}{238}$, $\frac{96}{102}$, $\frac{120}{485}$, $\frac{1230}{3444}$, $\frac{19876}{27548}$.

The number above the line is called the *numerator:* the

* The term fraction is from a Latin word signifying *to break*, meaning a broken part of a unit.

number below the line is called the *denominator*. These are also called the *terms* of the fraction.

If the numerator and the denominator of a fraction be equal, the value of the fraction is unity: $\frac{12}{12}=1$. If an apple be divided into 12 equal parts, the 12 parts or twelfths will make the whole apple.

If the numerator be less than the denominator, the fraction is called a *proper* fraction; as $\frac{3}{5}$, $\frac{9}{12}$, $\frac{10}{20}$.

If the numerator equal or exceed the denominator, the fraction is called an *improper* fraction; as $\frac{4}{4}$, $\frac{13}{12}$, $\frac{46}{14}$.

When a whole number* and a fraction are connected, the expression is called a *mixed number*. Thus, $4\frac{1}{2}$, $3\frac{1}{7}$, $48\frac{39}{177}$, are mixed numbers. The whole number is called the *integral* part of the expression, and the fraction is called the *fractional* part.

A fraction of a fraction is called a *compound fraction*. Thus, $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{9}$, $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{8}{5}$ of $\frac{11}{10}$, $\frac{3}{7}$ of $\frac{3}{8}$ of $\frac{3}{9}$ of $\frac{5}{6}$, &c., are compound fractions.

Any number may be made to assume the form of an improper fraction, by writing under it a unit for the denominator. Thus, 2, 3, 4, 5, 7, &c., are the same as $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$, $\frac{5}{1}$, $\frac{7}{1}$, &c.

Fractions sometimes occur, in which the numerator or denominator, or both, are themselves fractional; such expressions are called *complex fractions*.

Thus, $\frac{3\frac{1}{2}}{4}$, $\frac{4}{7\frac{1}{3}}$, $\frac{2\frac{1}{5}}{3\frac{1}{7}}$, $\frac{10\frac{1}{9}}{9\frac{1}{10}}$, &c., are complex fractions.

A fraction is said to be inverted when the numerator and denominator exchange places. Thus, the fractions $\frac{2}{3}$, $\frac{5}{4}$, $\frac{7}{8}$, $\frac{6}{10}$, $\frac{4}{9}$, $\frac{3}{7}$, when inverted, become $\frac{3}{2}$, $\frac{4}{5}$, $\frac{8}{7}$, $\frac{10}{6}$, $\frac{9}{4}$, $\frac{7}{3}$.

These fractions are called COMMON or VULGAR FRACTIONS,

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* A whole number is also called an *Integer*; thus, 5, 9, 24, 146, are integers.

as distinguished from another kind, to be hereafter treated, called Decimal Fractions.

§ 40. It will be seen that Common Fractions are founded upon *Division*. The numerator is the dividend, the denominator is the divisor. The *fraction itself expresses the quotient, or the value of the quotient resulting from the division.* Thus, divide 6 by 3; the quotient may be represented by $\frac{6}{3}=2$. Divide 3 by 6; the quotient $\frac{3}{6}$, *three-sixths.*

From the relations of divisor, dividend, and quotient, as seen in § 31, we may readily infer the following

PROPOSITIONS.

I. *That, multiplying the numerator by any number is the same as multiplying the fraction by the same number.*

II. *That, multiplying the denominator by any number is the same as dividing the fraction by the same number.*

III. *That, multiplying both numerator and denominator by any number does not alter the value of the fraction.*

IV. *That, dividing the numerator by any number is the same as dividing the fraction by the same number.*

V. *That, dividing the denominator by any number is the same as multiplying the fraction by the same number.*

VI. *That, dividing both numerator and denominator by the same number does not alter the value of the fraction.*

REDUCTION OF FRACTIONS.

§ 41. REDUCTION is the process of changing the *form* of an expression without altering its *value*. Thus, the integer 1 may be reduced to the fraction $\frac{5}{5}$; the fraction $\frac{12}{12}$ may be reduced to the integer 1.

Suppose an apple be cut into 12 equal parts: 6 of those parts are equal to one-half the apple; that is, $\frac{6}{12}=\frac{1}{2}$. It will be seen that $\frac{1}{2}$ is the result of the division of the numerator and denominator of $\frac{6}{12}$, by 6.

There is evidently the same relation between 1 divided into 2 parts, as there is between 6 divided into 12 parts. We may therefore find an equivalent value of a fraction, in *lower* terms, by dividing numerator and denominator by the same number. (§ 40, Prop. VI.)

Hence to reduce a fraction to its *lowest* terms, *Divide its numerator and denominator by their greatest common divisor.*

Reduce $\frac{492}{744}$ to its lowest terms. The greatest common divisor of 492 and 744 is 12. § 34, 35. Dividing by 12, we have $\frac{41}{62}$ for the answer.

NOTE.—We may frequently discover numbers, by inspection, which will divide both numerator and denominator without a remainder. When this is the case, we need not resort to the rule for obtaining the greatest common divisor, until we have divided by such numbers.

EXAMPLES.

1–5. Reduce to their lowest terms $\frac{4}{6}$; $\frac{3}{9}$; $\frac{3}{12}$; $\frac{7}{21}$; $\frac{24}{48}$.

6–11. Reduce to their lowest terms $\frac{32}{48}$; $\frac{60}{96}$; $\frac{72}{144}$; $\frac{12}{72}$; $\frac{81}{108}$; $\frac{144}{240}$.

12–16. Reduce to their lowest terms $\frac{49}{343}$; $\frac{1049}{6302}$; $\frac{316}{405}$; $\frac{172}{1118}$; $\frac{275}{440}$.

17–20. Reduce to their lowest terms $\frac{99}{9999}$; $\frac{4114}{4560}$; $\frac{33522}{41223}$; $\frac{58760}{66105}$.

21–25. Reduce to their lowest terms $\frac{744}{906}$; $\frac{276}{360}$; $\frac{592}{999}$; $\frac{315}{405}$; $\frac{1825}{2655}$.

26–30. Reduce to their lowest terms $\frac{308}{506}$; $\frac{364}{404}$; $\frac{616}{1112}$; $\frac{728}{808}$; $\frac{924}{1518}$.

31–40. Reduce to their lowest terms $\frac{756}{1140}$; $\frac{1362}{1500}$; $\frac{1764}{2660}$; $\frac{3178}{3500}$; $\frac{834}{8834}$; $\frac{1896}{2502}$; $\frac{1640}{2862}$; $\frac{3528}{4424}$; $\frac{1942}{5164}$; $\frac{5826}{15492}$.

§ 42. To reduce an improper fraction to a whole or mixed number.

Reduce $\frac{12}{4}$ to a whole number.

As there are four-fourths, $\frac{4}{4}$, in a whole thing or unit, in $\frac{12}{4}$ there will be as many whole things or units as 4, the denominator, is contained times in 12, the numerator, which is 3. Therefore $\frac{12}{4}=3$.

Reduce $\frac{95}{13}$ to a mixed number.

As there are $\frac{13}{13}$ in a whole one, there will be as many whole ones in $\frac{95}{13}$ as 13 is contained times in 95, which is 7, and 4 thirteenths over. Therefore $\frac{95}{13}=7\frac{4}{13}$.

It will thus be seen that the division expressed by an improper fraction may be actually *performed* as in division proper. Hence this

RULE.

Perform the division expressed by the fraction.

EXAMPLES.

41–50. Reduce to whole numbers the following fractions: $\frac{9}{3}$; $\frac{27}{9}$; $\frac{96}{8}$; $\frac{144}{12}$; $\frac{288}{6}$; $\frac{2208}{48}$; $\frac{104}{13}$; $\frac{192}{16}$; $\frac{42000}{42}$; $\frac{55550}{5}$.

51–64. Reduce to mixed numbers $\frac{60}{25}$; $\frac{80}{7}$; $\frac{90}{12}$; $\frac{56}{6}$; $\frac{10}{9}$; $\frac{131}{7}$; $\frac{460}{44}$; $\frac{384}{37}$; $\frac{4198}{7}$; $\frac{29817}{365}$; $\frac{3307}{97}$; $\frac{3313}{101}$; $\frac{3407}{86}$; $\frac{33045}{3203}$.

65–68. Reduce to a whole or mixed number $\frac{971}{89}$; $\frac{534}{89}$; $\frac{76818}{378}$; $\frac{76818}{25606}$.

§ 43. To reduce a whole or a mixed number to an improper fraction.

Reduce 5 to fourths. In a unit there are 4 fourths. In 5 there will be as many times 4 fourths as there are units; that is, $5=5\times4$ fourths $=\frac{20}{4}$.

Reduce $5\frac{3}{4}$ to fourths. $5=\frac{20}{4}$ as before; to which, if the 3 fourths be added, the sum will be 23 fourths; that is, $5\frac{3}{4}=\frac{5\times4+3}{4}=\frac{23}{4}$.

The denominator of the required fraction being given, find the numerator by the following

RULE.

Multiply the whole number by the denominator of the fraction to which it is to be reduced. To the product add the numerator of the fractional part, if any.

EXAMPLES.

69–75. Reduce 9 to 4ths; 5ths; 6ths; 7ths; 8ths; 9ths; 10ths.

76–86. Reduce to improper fractions $4\frac{1}{2}$; $3\frac{1}{3}$; $7\frac{2}{5}$; $8\frac{1}{3}$; $7\frac{1}{2}$; $9\frac{3}{8}$; $12\frac{4}{5}$; $16\frac{2}{3}$; $24\frac{7}{12}$; $36\frac{31}{32}$; $19\frac{273}{498}$.

87–96. Reduce to improper fractions $81\frac{252}{365}$; $37\frac{37}{41}$; $3\frac{321}{331}$; $7\frac{17}{19}$; $365\frac{81}{83}$; $1234\frac{29}{31}$; $77\frac{7}{11}$; $84\frac{960}{1231}$; $763\frac{81}{93}$; $41\frac{65}{928}$.

97–99. Reduce $484\frac{36}{408}$ to an improper fraction; $376\frac{47}{48}$; $296\frac{43}{44}$.

§ 44. To reduce compound fractions to simple ones.

Take the compound fractions $\frac{2}{3}$ of $\frac{5}{7}$. This expression is the same as $\frac{5}{7}$ taken $\frac{2}{3}$ of a time, or $\frac{5}{7}\times\frac{2}{3}$. $\frac{2}{3}$ is evidently $2\times\frac{1}{3}$: the expression then becomes $2\times\frac{1}{3}\times\frac{5}{7}$; $\frac{1}{3}$ of $\frac{5}{7}$ is the same as $\frac{5}{7}\div3=\frac{5}{7\times3}$ (§ 40, Prop. II.), 2 times $\frac{5}{7\times3}=$(Prop. I.) $\frac{2\times5}{3\times7}=\frac{10}{21}$. Thus, the numerators of the compound fraction have been multiplied together and the denominators together.

Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{9}{10}$ of $\frac{7}{12}$ to a simple fraction.

Canceling the factors common to numerator and denominator, the expression becomes

$$\frac{\not{2}}{\not{3}}\times\frac{\not{3}}{5}\times\frac{\overset{3}{\not{9}}}{\underset{5}{\not{10}}}\times\frac{7}{\underset{4}{\not{12}}}=\frac{3\times7}{5\times5\times4}=\frac{21}{100}.$$

Hence the following

RULE.

First, cancel the factors common to the numerators and denominators of the given fraction; then multiply the remaining numerators together for a new numerator, and the remaining denominators together for a new denominator.

NOTE.—When a fraction is to be multiplied by a whole number, the whole number may be changed to an improper fraction by writing 1 for its denominator; thus, $4=\frac{4}{1}$.

EXAMPLES.

100–101. Reduce $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{8}{15}$ of $\frac{5}{12}$ to simple fractions.

102–109. Reduce to their simplest forms the following fractions: $\frac{3}{7}$ of $\frac{14}{35}$; $\frac{7}{8}$ of $\frac{4}{9}$ of $\frac{5}{11}$; $\frac{3}{4}$ of $\frac{4}{5}$; $\frac{5}{7}$ of $\frac{7}{9}$ of $\frac{3}{5}$; $\frac{3}{8}$ of $\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{1}{7}$; $\frac{3}{7}$ of $\frac{14}{15}$; $\frac{10}{12}$ of $\frac{3}{7}$ of $\frac{14}{17}$; $\frac{1}{2}$ of $\frac{5}{2}$ of $\frac{10}{3}$ of $\frac{15}{36}$.

110–116. Simplify the following: $\frac{1}{2}$ of $\frac{2}{7}$ of $\frac{3}{8}$ of $\frac{7}{15}$; $\frac{1}{3}$ of $\frac{3}{8}$ of $\frac{5}{7}$ of $\frac{16}{21}$ of $\frac{49}{50}$; $\frac{1}{4}$ of $\frac{4}{9}$ of $\frac{18}{35}$; $\frac{7}{8}$ of $\frac{5}{6}$ of $4\frac{1}{2}$; $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$; $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$; $\frac{9}{10}$ of $\frac{13}{18}$ of $\frac{7}{9}$ of $\frac{36}{43}$.

117–121. Simplify $\frac{4}{7}$ of $\frac{9}{12}$ of $\frac{6}{7}$ of $\frac{13}{14}$; $\frac{26}{88}$ of $\frac{11}{32}$ of $\frac{15}{29}$; $\frac{4}{7}$ of $\frac{9}{13}$ of $\frac{26}{65}$ of 21; $\frac{8}{9}$ of $\frac{14}{32}$ of $\frac{25}{60}$ of 19; $\frac{4}{7}$ of $\frac{12}{15}$ of $\frac{18}{36}$ of 62.

§ 45. To reduce fractions to a common denominator.

We know (Prop. III.) that the value of a fraction is not changed by multiplying its numerator and denominator by the same number. If, then, we multiply the numerator and denominator of each of a series of fractions by the product of the denominators of all the other fractions, we shall retain the values of the respective fractions, and at the same time they will have a common denominator.

Reduce $\frac{1}{2}$, $\frac{5}{3}$, and $\frac{4}{7}$ to a common denominator.

The numerator of $\frac{1}{2}$	becomes	$1\times3\times7=21$
The denominator of $\frac{1}{2}$	"	$2\times3\times7=42$
The numerator of $\frac{5}{3}$	"	$5\times2\times7=70$
The denominator of $\frac{5}{3}$	"	$3\times2\times7=42$
The numerator of $\frac{4}{7}$	"	$4\times2\times3=24$
The denominator of $\frac{4}{7}$	"	$7\times3\times2=42$

The fractions, then, are $\frac{21}{42}$, $\frac{70}{42}$, $\frac{24}{42}$.

Hence the following

RULE.

Multiply each numerator by all the denominators except its own for a new numerator, and all the denominators together for a common denominator.

NOTE.—Mixed numbers must be reduced to improper fractions, compound fractions to their simplest form, and all the fractions to their lowest terms, before multiplying.

EXAMPLES.

122–129. Reduce to common denominators $\frac{1}{2}$ and $\frac{2}{3}$; $\frac{3}{4}$ and $\frac{4}{5}$; $\frac{5}{6}$ and $\frac{7}{8}$; $\frac{8}{9}$ and $\frac{9}{10}$; $\frac{10}{11}$ and $\frac{11}{12}$; $\frac{12}{13}$ and $\frac{13}{14}$; $\frac{14}{15}$ and $\frac{15}{16}$; $\frac{17}{18}$ and $\frac{19}{20}$.

130–136. Reduce to common denominators $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{7}$; $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$; $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$; $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{8}{9}$; $\frac{9}{10}$, $\frac{10}{11}$, and $\frac{11}{12}$; $\frac{12}{13}$, $\frac{13}{14}$, and $\frac{14}{15}$; $\frac{18}{20}$, $\frac{6}{7}$, and $\frac{4}{5}$.

137–139. Reduce to common denominators $\frac{1}{2}$ of $\frac{2}{3}$, $4\frac{1}{2}$, $5\frac{1}{2}$; $\frac{2}{3}$ of $\frac{3}{4}$, of 5, $7\frac{1}{3}$, $5\frac{1}{5}$; $\frac{3}{7}$ of $\frac{6}{7}$, $\frac{2}{3}$ of $4\frac{1}{2}$, $\frac{3}{8}$ of $7\frac{1}{6}$.

§ 46. When the *least* common denominator is required.

Find the least common denominator of $\frac{5}{12}$, $\frac{7}{16}$, $\frac{11}{24}$. The least common multiple of 12, 16, 24 (§ 36), is 48. It is evident that the denominator of each fraction is multiplied by a certain factor to produce this multiple: that is, 12 by 4; 16 by 3; 24 by 2. Now if the numerator of such fraction be multiplied by the same factor, each fraction will retain its value, and all will have a common denominator; thus, $\frac{20}{48}$, $\frac{21}{48}$, $\frac{22}{48}$. Hence, the following

RULE.

Find the least common multiple of the denominators for the least common denominator.

For each new numerator multiply the numerator of each fraction by that factor of the multiple, of which the denominator of such fraction is the other factor.

EXAMPLES.

140–146. Reduce to the least common denominator $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$; $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$; $\frac{5}{6}$ and $\frac{2}{3}$; $\frac{4}{7}$ and $\frac{5}{9}$; $\frac{3}{4}$, $\frac{8}{9}$, and $\frac{5}{7}$; $\frac{3}{4}$, $\frac{9}{10}$, and $\frac{12}{13}$; $\frac{5}{9}$, $\frac{4}{5}$, and $\frac{3}{4}$.

147–155. Reduce to the least common denominator $\frac{5}{12}$, $\frac{7}{15}$, $\frac{11}{24}$; $\frac{1}{2}$ of $\frac{3}{7}$ of $\frac{7}{12}$, $\frac{3}{20}$, and $\frac{7}{15}$; $3\frac{1}{2}$, $4\frac{1}{3}$, $\frac{6}{5}$; $\frac{8}{9}$, $\frac{7}{15}$, $\frac{13}{20}$; $\frac{4}{15}$, $\frac{5}{11}$, $6\frac{3}{22}$; $\frac{1}{2}$, $\frac{2}{3}$, $3\frac{1}{4}$, and $\frac{1}{5}$; $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{7}$, $\frac{4}{21}$; $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{20}$; $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{9}{10}$, $\frac{7}{120}$.

156. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$ to equivalent fractions having a common denominator.

Addition of Fractions.

§ 47. In addition of whole numbers, we have seen that units can only be added to units, tens to tens, &c. 3 units and 4 tens will make neither 7 units nor 7 tens; but 4 tens $=40$ units; and 40 units$+$3 units$=$43 units.

So in fractions, 3 *fourths* cannot be added to 4 *sixths*, for the result will be neither 7 *fourths* nor 7 *sixths*. But 3 fourths$=$9 *twelfths* and 4 sixths$=$8 *twelfths*; and 9 twelfths and 8 twelfths$=$17 twelfths, or 1 and 5 twelfths; that is, $\frac{3}{4}+\frac{4}{6}=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}=1\frac{5}{12}$. Hence, for the addition of fractions, the following

RULE.

Reduce the given fractions to a common denominator. Over this denominator place the sum of their numerators.

Note.—Seek the *least* common denominator of the fractions. If the result be an improper fraction, it must be reduced to a whole or mixed number.

EXAMPLES.

157–162. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$? of $\frac{1}{6}$ and $\frac{1}{8}$? $\frac{1}{5}$, $\frac{1}{10}$, $\frac{3}{20}$? of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$? of $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$? of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{11}{12}$?

163–168. What is the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$? of $\frac{7}{8}$, $\frac{11}{12}$, $\frac{13}{44}$? of $\frac{17}{18}$, $\frac{19}{20}$, $\frac{23}{24}$? of $\frac{27}{28}$, $\frac{29}{30}$, $\frac{3}{32}$? of $\frac{1}{18}$, $\frac{2}{9}$, $\frac{3}{15}$? of $\frac{6}{7}$, $\frac{3}{8}$, $\frac{7}{56}$?

169–174. What is the sum of $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$? of $4\frac{1}{3}$, $3\frac{1}{2}$, $\frac{1}{5}$? of $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{12}$? of $\frac{3}{4}$, $\frac{4}{5}$, $\frac{1}{7}$, $\frac{1}{8}$? of $\frac{1}{17}$, $\frac{1}{19}$, $\frac{1}{23}$? of $\frac{3}{14}$, $\frac{5}{18}$, $\frac{20}{27}$?

175–180. What is the sum of $10\frac{1}{3}$, $12\frac{1}{4}$, $15\frac{1}{2}$? of $\frac{3}{11}$, $\frac{4}{13}$, $\frac{1}{7}$? of $\frac{30}{40}$, $\frac{42}{44}$, $\frac{48}{50}$? of $\frac{60}{72}$, $\frac{48}{96}$, $\frac{1}{2}$? of $\frac{6}{8}$, $\frac{4}{6}$, $\frac{3}{5}$? of $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{40}$?

181–186. What is the sum of $2\frac{1}{3}$, $4\frac{1}{5}$, $6\frac{1}{7}$? of $2\frac{1}{2}$, $4\frac{1}{3}$, $6\frac{1}{4}$? of $3\frac{1}{2}$, $5\frac{1}{3}$, $6\frac{1}{4}$? of $5\frac{1}{5}$, $10\frac{1}{10}$, $15\frac{1}{15}$? of $3\frac{1}{3}$, $6\frac{1}{6}$, $9\frac{1}{9}$? of $2\frac{1}{2}$, $4\frac{1}{3}$, $5\frac{1}{9}$?

SUBTRACTION OF FRACTIONS.

§ 48. Subtract $\frac{1}{7}$ from $\frac{1}{5}$. This cannot be done, because the fractions have different denominators. If we reduce both to *thirty-fifths*, $\frac{1}{7}$ becomes $\frac{5}{35}$, and $\frac{1}{5}$ becomes $\frac{7}{35}$; and $\frac{7}{35}-\frac{5}{35}=\frac{2}{35}$. Hence, to subtract one fraction from another, the following

RULE.

Reduce the fractions to a common denominator: over this denominator place the difference of the numerators.

EXAMPLES.

187–195. Find the difference between $\frac{1}{8}$ and $\frac{1}{7}$; $\frac{2}{3}$ and $\frac{5}{6}$; $\frac{8}{9}$ and $\frac{7}{11}$; $\frac{4}{5}$ and $\frac{3}{7}$; $\frac{9}{13}$ and $\frac{12}{17}$; $\frac{3}{4}$ and $\frac{11}{21}$; $\frac{3}{5}$ and $\frac{8}{11}$; $\frac{4}{5}$ and $\frac{15}{17}$; $\frac{3}{9}$ and $\frac{8}{11}$.

196–203. Subtract $\frac{1}{5}$ from $\frac{1}{2}$; $\frac{1}{6}$ from $\frac{1}{2}$; $\frac{1}{8}$ from $\frac{1}{2}$; $\frac{1}{4}$ from $\frac{2}{7}$; $\frac{1}{6}$ from $\frac{11}{12}$; $\frac{1}{360}$ from $\frac{1}{18}$; $\frac{17}{35}$ from $\frac{109}{105}$; $\frac{51}{120}$ from $\frac{360}{480}$.

204–209. $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{5}-\frac{1}{10}=$; $\frac{6}{7}-\frac{5}{9}$ of $\frac{18}{25}=$; $\frac{1}{2}$ of $\frac{3}{7}-\frac{1}{7}$ of $\frac{3}{8}=$; $3\frac{1}{5}-2\frac{1}{9}=$; $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5}-\frac{1}{8}$ of $\frac{2}{7}=$; $\frac{3}{2}$ of $\frac{4}{3}$ of $\frac{5}{4}$ of $\frac{6}{5}-\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}=$.

210–213. $(\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\frac{6}{5})-(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5})=$; $(2\frac{1}{2}+3\frac{1}{3}+4\frac{1}{4})-(2\frac{1}{3}+3\frac{1}{4}+4\frac{1}{5})=$; $(6\frac{1}{8}+7\frac{1}{10})-(4\frac{1}{5}+3\frac{1}{12})=$; $(\frac{1}{20}+\frac{1}{40})-(\frac{1}{30}+\frac{1}{50})=$.

214–218. What is the value of $(\frac{1}{2}+\frac{1}{3})-(\frac{1}{4}+\frac{1}{5})$? of $(\frac{1}{6}+\frac{1}{7})-(\frac{1}{8}+\frac{1}{9})$? of $(\frac{1}{10}+\frac{1}{11})-(\frac{1}{12}+\frac{1}{13})$? of $(\frac{1}{14}+\frac{1}{15})-(\frac{1}{16}+\frac{1}{17})$? of $(\frac{1}{18}+\frac{1}{19})-(\frac{1}{20}+\frac{1}{21})$?

219–222. What is the value of $(2\frac{1}{2}+3\frac{1}{3})-(1\frac{1}{3}+2\frac{1}{4})$? of $(4\frac{1}{5}+5\frac{1}{6})-(3\frac{1}{6}+4\frac{1}{8})$? of $\frac{14}{16}$ of $\frac{13}{18}-\frac{1}{12}$ of $\frac{3}{20}$? of $(\frac{1}{10}+\frac{1}{20})-(\frac{1}{30}+\frac{1}{40})$?

Multiplication of Fractions.

§ 49. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

We know, § 44, that $\frac{2}{3}$ multiplied by $\frac{4}{5}$, or $\frac{2}{3}\times\frac{4}{5}$, is the same as $\frac{2}{3}$ of $\frac{4}{5}$. Hence, for multiplication of fractions, we must use the same rule as for reducing compound fractions to simple ones.

RULE.

First, cancel the factors common to the numerators and denominators of the given fraction; then multiply the remaining numerators together for a new numerator, and the remaining denominators together for a new denominator.

EXAMPLES.

223–233. Multiply $\frac{1}{2}$ by $\frac{1}{3}$; $\frac{1}{2}$ by $\frac{1}{4}$; $\frac{1}{3}$ by $\frac{2}{5}$; $\frac{1}{3}$ by $\frac{7}{8}$; $\frac{4}{5}$ by $\frac{10}{16}$; $\frac{3}{5}$ by $\frac{5}{7}$; $\frac{4}{9}$ by $\frac{18}{27}$; $\frac{8}{11}$ by $\frac{55}{56}$; $\frac{7}{9}$ by $\frac{63}{77}$; $\frac{13}{14}$ by $\frac{28}{120}$; $\frac{11}{17}$ by $\frac{68}{120}$.

234–240. Multiply together $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$; $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$; $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{9}$; $\frac{3}{7}$, $\frac{8}{9}$, and $\frac{7}{10}$; $\frac{5}{6}$, $\frac{4}{7}$, and $\frac{6}{11}$; $\frac{3}{4}$, $\frac{7}{9}$, and $\frac{13}{16}$; $\frac{5}{8}$, $\frac{9}{14}$, and $\frac{16}{70}$.

241–243. What is the product of $\frac{2}{3}$ of $\frac{4}{5}$ by $\frac{7}{10}$ of $\frac{3}{5}$? of $\frac{4}{7}$ of $\frac{11}{12}$ by $\frac{2}{3}$ of $\frac{12}{14}$? $\frac{2}{4}$ of $\frac{7}{8}$ by $\frac{3}{9}$ of $\frac{7}{10}$ of $\frac{11}{12}$?

244–246. $3\frac{1}{2}\times4\frac{1}{3}\times\frac{1}{14}=$; $4\frac{1}{2}\times3\frac{1}{3}=$; $3\frac{1}{5}\times4\frac{1}{6}\times\frac{1}{5}=$.

247–249. Multiply together the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$; $\frac{3}{7}$, $\frac{4}{5}$, $\frac{5}{9}$, $\frac{14}{3}$; $3\frac{1}{3}$, $4\frac{1}{4}$, $5\frac{1}{5}$.

250–254. Multiply together $\frac{3}{40}$, $\frac{7}{18}$, $\frac{5}{7}$; $\frac{3}{7}$ by 4; 7 by $\frac{3}{4}$; $7\frac{1}{2}$ by $3\frac{1}{2}$; $16\frac{1}{2}$ by 5.

255. Multiply the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, by the sum of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.

256. Multiply the sum of $\frac{1}{2}$ of $\frac{2}{4}$, $\frac{3}{4}$ of $\frac{3}{5}$ by the sum of $\frac{1}{3}$ of $\frac{1}{4}$, $\frac{1}{5}$ of $\frac{1}{4}$.

257. Multiply $\frac{3}{7}$ of $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{4}{11}$ by $\frac{11}{12}$ of $\frac{6}{7}$ of $\frac{8}{9}$.

258. Multiply the sum of 3, $3\frac{1}{2}$, $3\frac{1}{3}$, $3\frac{1}{4}$, by the sum of $2\frac{1}{3}$, $3\frac{1}{4}$, $4\frac{1}{5}$.

Division of Fractions.

§ 50. Divide $\frac{4}{7}$ by $\frac{5}{8}$. *First method.* If the fractions be reduced to a common denominator, their numerators may be operated upon as if they were whole numbers. Thus, $\frac{4}{7}=\frac{32}{56}$; $\frac{5}{8}=\frac{35}{56}$. And $\frac{32}{56}$ divided by $\frac{35}{56}$ is the same as 32 divided by 35, or $\frac{32}{35}$.

Second method. $\frac{4}{7}$ divided by 1 will give $\frac{4}{7}$ for a quotient; divided by $\frac{1}{8}$ (§ 31, *f*), will give 8 times as large a quotient as when divided by 1, or $\frac{4}{7}\times\frac{8}{1}=\frac{32}{7}$; divided by $\frac{5}{8}$ (§ 31, *c*), will give one-fifth as large a quotient as when divided by $\frac{1}{8}$, or $\frac{1}{5}\times\frac{32}{7}=\frac{32}{35}$, the same result as found by the first method, we see that in fact the dividend $\frac{4}{7}$ has been multiplied by $\frac{8}{5}$; that is, by the divisor with its terms inverted. Hence, for dividing one fraction by another, the following

RULE.

Invert the terms of the divisor, and proceed as in multiplication.

NOTE.—If either dividend or divisor be a whole number, make it an improper fraction by giving to it 1 for a denominator.

EXAMPLES

259–269. Divide $\frac{1}{2}$ by $\frac{1}{4}$; $\frac{1}{5}$ by $\frac{1}{6}$; $\frac{1}{7}$ by $\frac{1}{8}$; $\frac{1}{9}$ by $\frac{1}{10}$; $\frac{1}{11}$ by $\frac{1}{12}$; $\frac{1}{12}$ by $\frac{1}{13}$; $\frac{1}{14}$ by $\frac{1}{15}$; $\frac{2}{3}$ by $\frac{3}{4}$; $\frac{3}{5}$ by $\frac{4}{6}$; $\frac{5}{6}$ by $\frac{6}{7}$; $\frac{7}{8}$ by $\frac{9}{10}$.

270–274. Divide $\frac{3}{4}$ by $\frac{1}{3}$; $\frac{6}{7}$ by $\frac{6}{11}$; $\frac{8}{15}$ by $\frac{12}{20}$; $\frac{3}{11}$ by $\frac{8}{12}$; $\frac{9}{13}$ by $\frac{11}{16}$.

275–278. Divide $4\frac{1}{3}$ by $17\frac{1}{2}$; $\frac{15}{17}$ by 10; $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{1}{6}$ of $\frac{5}{8}$; $3\frac{1}{3}$ of $2\frac{1}{2}$ by $4\frac{1}{4}$.

279. Divide $\frac{1}{2}$ by $\frac{3}{8}$ of $\frac{4}{5}$.

280. Divide the sum of $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$, by the sum of 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

281. Divide the sum of $\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{8}{7}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{11}{10}$, $\frac{12}{11}$, $\frac{13}{12}$, $\frac{14}{13}$, by the sum of 1, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{13}$.

282. Divide $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{3}{8}$ by $\frac{2}{3}$ of $\frac{2}{5}$ of $\frac{1}{7}$ of $\frac{5}{8}$.

283–287. Divide the sum of 1, $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, by the sum of $1\frac{1}{3}$, $2\frac{1}{4}$, $3\frac{1}{5}$; the sum of $\frac{1}{3}$ of $\frac{4}{5}$, $\frac{1}{4}$ of $\frac{5}{6}$, by the sum of $\frac{1}{5}$ of $\frac{6}{7}$, $\frac{1}{6}$ of $\frac{7}{8}$; $\frac{3}{5}$ of $\frac{10}{11}$ of $\frac{22}{27}$ by $\frac{1}{9}$ of $\frac{3}{4}$ of $\frac{5}{8}$; $\frac{7}{8}$ of $\frac{16}{27}$ of $\frac{1}{2}$ by $\frac{5}{8}$ of $\frac{4}{5}$ of 12; $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ by $\frac{1}{3}$ of $\frac{1}{5}$ of 8.

Reciprocals.

§ 51. The reciprocal of any number is found by dividing 1 by the number. Thus, the reciprocals of 2, 3, 4, are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

The reciprocal of a fraction, for example, of $\frac{2}{3}$ is $1 \div \frac{2}{3} = 1 \times \frac{3}{2} = \frac{3}{2}$. Hence, the reciprocal of a fraction is the fraction inverted.

Operations in division may therefore be included under those of multiplication, by making the reciprocal of the divisor the multiplier.

EXAMPLES.

288–295. What are the reciprocals of 7, 8, 9, 11, 18, 24, 96, 108?

296–303. What are the reciprocals of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$?

304–309. What are the reciprocals of $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, $5\frac{1}{7}$, $9\frac{3}{6}$, $12\frac{9}{11}$?

310–313. What are the reciprocals of $\frac{3}{4}$ of $\frac{3}{7}$? $\frac{2}{3}$ of $\frac{7}{5}$? $\frac{8}{9}$ of $\frac{11}{12}$? $\frac{3}{7}$ of $\frac{2}{3}$?

314–318. Perform the following by using the reciprocals of the divisors: $\frac{3}{4} \div 4$; $\frac{9}{10} \div \frac{2}{3}$; $\frac{6}{7} \div 8$; $4\frac{1}{2} \div 7$; $3\frac{1}{6} \div 2\frac{7}{8}$.

MISCELLANEOUS EXAMPLES IN COMMON FRACTIONS.

319–324. Reduce to their lowest terms $\frac{588}{672}$; $\frac{3387}{4516}$; $\frac{7721}{7763}$; $\frac{15636}{16939}$; $\frac{505}{515}$; $\frac{18999}{21110}$.

325–329. Reduce to mixed numbers $\frac{515}{505}$; $\frac{37}{5}$; $\frac{59}{17}$; $\frac{2371}{907}$; $\frac{101}{97}$.

330–334. Reduce to improper fractions $3\frac{1}{2}$; $15\frac{11}{13}$; $3\frac{7}{17}$; $1\frac{1}{901}$; $100\frac{11}{17}$.

335–339. Reduce to their simplest forms $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{20}{27}$; $\frac{7}{8}$ of $\frac{1}{2}$ of $\frac{6}{14}$ of 3; $\frac{1}{10}$ of $\frac{3}{7}$ of $\frac{21}{25}$ of $3\frac{1}{3}$; $\frac{6}{7}$ of $\frac{14}{3}$ of $\frac{1}{2}$ of 100.

340–344. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, to equivalent fractions having a common denominator; so $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$; $3\frac{1}{2}$, $\frac{7}{5}$, $\frac{3}{4}$, $\frac{3}{10}$; $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{11}$; $\frac{3}{5}$, $\frac{5}{7}$, $\frac{7}{11}$, $\frac{11}{13}$.

345–346. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$? of $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$?

347. From a piece of cloth $\frac{1}{2}$ and $\frac{1}{3}$ of the whole was cut off. What part of the whole was thus taken away?

348–350. From $\frac{1}{2}$ subtract $\frac{1}{3}$; from $\frac{1}{10}$ subtract $\frac{1}{11}$; from $\frac{3}{4}$ subtract $\frac{4}{7}$.

351. A tree 150 feet high had $\frac{1}{5}$ broken off in a storm. What was the length broken off?

352. A. and B. together possess 1477 sheep, of which A. owns $\frac{4}{7}$ and B. $\frac{3}{7}$. How many belong to each man?

353. A. owns $\frac{3}{11}$ of a ship, valued at 15422 dollars: he sells to B. $\frac{2}{3}$ of his share. What is the value of what A. has left; also, what is the value of B.'s part?

354. A cotton-mill is sold for 30000 dollars, of which A. owns $\frac{1}{5}$ of the whole, B. and C. each own $\frac{1}{2}$ of $\frac{1}{3}$ of the whole. How many dollars does each one claim?

355. A. and B. have a melon, of which A. owns $\frac{3}{8}$ and B. $\frac{5}{8}$: C. offers them one shilling, to partake equally with them of the melon, which was agreed to. How must the shilling be divided between A. and B.?

356. A farmer had $\frac{1}{5}$ of his sheep in one field, $\frac{1}{6}$ in a second field, and the residue, which was 779, in a third field. How many sheep had he in all?

357. If I divide 616 dollars between A., B., C., and D., by giving A. $\frac{1}{4}$ of the whole, B. $\frac{5}{14}$ of the remainder, C. $\frac{8}{9}$ of what then remained, and D. the balance, how much will each receive?

358. In Fahrenheit's thermometer there are 180 degrees between the boiling and freezing points; in that of Reaumur there are 80. What fraction of a degree in the latter expresses a degree of the former?

359. The receipts of Jenny Lind's first concert in New York were 26000 dollars; the expenses were 4000 dollars. Jenny received 1000 dollars as her regular nightly stipend, and $\frac{1}{2}$ the net* proceeds in addition. How much did she receive?

360. Of the proceeds of her first concert Jenny Lind donated as follows: to the Fire Department Fund $\frac{6}{19}$; to the Musical Fund Society $\frac{4}{19}$; Home for the Friendless $\frac{1}{19}$; Society for Relief of Indigent Females $\frac{1}{19}$; Dramatic Fund Association $\frac{1}{19}$; Home for Colored Aged Persons $\frac{1}{19}$; Asylum for Destitute Females $\frac{1}{19}$; Orphan Asylum $\frac{1}{19}$; Roman Catholic Orphan Asylum $\frac{1}{19}$; Protestant do. $\frac{1}{19}$; Old Ladies' Asylum, the remainder, 500 dollars. How much

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* Net or neat means over and above expenses.

did the generous singer give away? and how much did each society receive?

361. In the year 1850 there were probably 800000 baskets of peaches brought into New York city. If the population were 510000, what fraction will express how many baskets that was to each person?

362–363. A journeyman's wages per week were 10 dollars: of that sum he spent $\frac{1}{7}$ for the 6 working days for meat. What fraction will express the amount he spent each day? If 10 dollars are 1000 cents, how many cents did he spend each day?

364. Paid 137 dollars for flour, at $6\frac{6}{7}$ dollars a barrel. How many barrels did I buy?

365. There were 37 bushels of potatoes in a cart: $\frac{1}{3}$ of them were divided among 3 families of 4 persons each; $\frac{1}{6}$ of them among 2 families of 7 persons each; $\frac{1}{9}$ of them between 3 persons; and the remainder among 18 persons. What part of a bushel had each person by the 1st division? What part had each by the 2d? what by the 3d? what by the 4th?

366. Two persons, A. and B., being 95 miles apart, travel towards each other, both starting at the same time. They meet at the end of 6 hours, when they discover that A. travelled $1\frac{1}{2}$ miles more than B. each hour. How many miles did each go?

367. A boy, after losing $\frac{1}{2}$ of his kite string, added 30 feet, and then found that it was just $\frac{4}{5}$ of the original length. What was the length at first?

368. A person commencing business with a certain capital, found at the end of the first year that he had increased it $\frac{1}{2}$, but at the end of the next year, having been unfortunate in business, his capital amounted to 3000 dollars, which

was $\frac{1}{2}$ of what he had at the end of the first year. What was the capital he commenced with?

369. A. owns $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{7}{8}$ of a ship; B. owns $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{8}{9}$ of the whole; C. owns $\frac{1}{4}$ of $\frac{5}{6}$ of $\frac{5}{7}$ of the whole; D. owns the remainder. How much does A.'s part exceed $\frac{1}{4}$ of the whole? How much does B.'s part fall short of $\frac{1}{4}$ of the whole? How much does C.'s part fall short of $\frac{1}{4}$ of the whole? How much does D.'s part exceed $\frac{1}{4}$ of the whole?

370. Divide 88 dollars as follows: to A. give 1 dollar more than $\frac{1}{4}$ of the whole; to B. give 10 dollars more than $\frac{1}{5}$ of the remainder; to C. give 14 dollars more than $\frac{1}{6}$ of the second remainder; and to D. the balance. What is each one's part?

CHAPTER X.

DECIMAL FRACTIONS.

§ 52. Suppose 1 to be divided into 10 equal parts; each one of these parts is 1 *tenth*, or $\frac{1}{10}$; two parts are 2 *tenths*, or $\frac{2}{10}$, &c. Now, if each *tenth* be divided into ten equal parts, each of these subdivisions will be a *hundredth*; that is, $\frac{1}{10} \div \frac{10}{1} = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$. So $\frac{1}{100} \div \frac{10}{1} = \frac{1}{100} \times \frac{1}{10} = \frac{1}{1000}$, &c.

Such fractions as the above, which decrease or increase *only* in a *tenfold* ratio, are called *Decimal* Fractions.* So that Decimal Fractions must always have denominators of the following form: 10, 100, 1000, 10000, &c.

In treating of whole numbers, we saw (§ 5) that the successive orders of units had a tenfold increase from right to

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* Decimal, from a Latin word signifying *ten*.

left, or decrease from left to right. This being true, also, of decimals, they may be written down and operated upon as if they were whole numbers; that is, their denominators may be omitted. The only care necessary is to *distinguish the decimal from the integer by a separatrix or point.* Thus, seven and 3 tenths is written 7·3; six and 9 hundredths is written 6·09.

The *first* place at the right of the decimal point is *tenths;* the *second* place is *hundredths;* the *third* place, *thousandths;* the *fourth* place, *ten-thousandths;* the *fifth* place, *hundred-thousandths:* the *sixth* place, *millionths,* &c., as in the following

TABLE.

&c. &c.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	Ten Millionths.	Hundred Millionths.	Billionths.	Ten Billionths.	&c. &c.

In notation, where a decimal place does not require a digit, 0 must be written. Thus, 3 hundredths is written 0·03, the naught showing that no tenths are to be expressed; 7 thousandths is written 0·007, the naughts showing that no tenths and no hundreds are to be expressed, &c. We also write a naught at the left of the decimal point when there are no *units.* The naught is thus necessary to keep the decimal digit in its proper place.

Every naught *prefixed* to a decimal carries it one place further to the right, and thus decreases its value 10 times. Thus, $0·1 = \frac{1}{10}$; $0·01 = \frac{1}{100}$; $0·001 = \frac{1}{1000}$, &c.

Naughts *annexed* to a decimal do not alter its value, since they multiply its numerator and denominator by the same number. Thus, $0{\cdot}1 = \frac{1}{10}$; $0{\cdot}10 = \frac{10}{100}$; $0{\cdot}100 = \frac{100}{1000}$.

§ 53. To express decimals in figures.

Write the decimal as a whole number. Prefix as many naughts as are necessary to make the decimal places equal to the number of naughts of the denominator. Be careful to place the POINT *at the left of the number.*

For example: Express in figures three hundred and fifty-seven *millionths.* I write first the 357. There are 6 naughts in 1000000*ths* ($\frac{1}{1000000}$). I then prefix to the 3 decimal places already written, 3 naughts, 0·000357.

EXAMPLES.

1–9. How many decimal places in 1 hundredth? in 1 thousandth? in 1 millionth? in 1 ten-thousandth? in 1 hundred-thousandth? in 1 billionth? in 1 ten-millionth? in 1 hundred-millionth? in 1 ten-billionth?

10–19. Write 37 thousandths; 3 hundredths; 48 millionths; 95 hundred-millionths; 490 hundred-thousandths; 1240 ten-millionths; 10000004 hundred-millionths; 96 billionths; 9301 hundred-millionths; 27101 millionths.

20–27. Write eight hundred and four thousand *ten-millionths;* seven million and four hundred *millionths;* seventy-four million and eighty-one *billionths;* eight hundred and ninety-six thousand *hundred-millionths;* four thousand and seven *hundred-thousandths*; eight hundred million and four thousand *ten-billionths;* sixty billions and seventy-four *trillionths;* eight hundred billions and ninety-nine *ten-billionths.*

NOTE.—The teacher will exercise the pupils in similar numbers, until they can write them with rapidity and accuracy.

28–37. Express decimally the following fractions: $\frac{84}{100}$; $\frac{96}{1000}$; $\frac{77}{10000}$; $\frac{104}{100000}$; $\frac{10007}{100000}$; $\frac{44}{1000000}$; $\frac{7}{10000000}$; $\frac{12}{10000000}$; $\frac{1365}{100000}$.

38–50. Express the following decimally: $\frac{84}{10}=8\frac{4}{10}=8{\cdot}4$: $\frac{94}{10}$; $\frac{813}{100}$; $\frac{418}{10}$; $\frac{4189}{10}$; $\frac{4674}{100}$; $\frac{8961}{1000}$; $\frac{7461}{10000}$; $\frac{54982}{1000}$; $\frac{478619}{100}$; $\frac{2826018}{1000000}$; $\frac{189765}{10000}$; $\frac{84108}{10000}$; $\frac{897654821}{100000}$.

NOTE.—Perform the division indicated by the fraction.

§ 54. To *read* decimals expressed in figures.

Read the figures as if they were whole numbers, and add the name of the right-hand decimal place.

Thus, 0·7 is read seven *tenths;* 0·06 i read six *hundredths;* 0·004 is read four *thousandths;* 0·1070004 is read one million, seventy thousand and four *ten-millionths.*

NOTE.—If the pupil numerate, beginning at the left, thus, "tenths," "hundredths," "thousandths," &c., till he reach the last figure, he will ascertain the name of the right-hand decimal place.

EXAMPLES.

51–78. Read the following expressions:

0·8	0·10876	3·0017	$27\frac{46812}{100000}$
0·90	0·0001007	4·90018	$9\frac{467}{10000}$
0·407	0·1000012	6·000001	$8\frac{42}{10000}$
0·001	0·6750912	49·100007	$21\frac{1}{10000000}$
0·6945	0·80700176	86·0010007	$36{\cdot}21\frac{1}{6}$
0·87601	0·80000001	44·62000016	$48{\cdot}4081\frac{1}{9}$
0·00076	0·901010101	0·1001000100	$9{\cdot}10000101\frac{69}{70}$

An expression made up of an integer and a decimal is called a *mixed number;* as, 26·41.

Addition of Decimals.

§ 55. Add 7·8, 9·04, 78·005, 801·7604.

We arrange the numbers so that *tenths* will stand under *tenths, hundredths* under *hundredths*, &c., as units stand under units, tens under tens, &c. We then add as in whole numbers. As many figures must evidently be pointed off on the right of the sum for decimals, as are equal to the greatest number of decimal places in any of the numbers added.

Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten thousandths.
		7	·	8			
		9	·	0	4		
	7	8	·	0	0	5	
8	0	1	·	7	6	0	4
8	9	6	·	6	0	5	4

Hence, for the Addition of Decimals, the following

RULE.

I. *Place the numbers to be added so that the figures occupying the same decimal place shall fall in the same column. Add as in whole numbers.*

II. *From the right of the sum, point off for decimals as many figures as equal the greatest number of decimal places in any one of the given numbers.*

Note.—If the numbers are properly set down, the decimal point in the sum will fall directly under those in the numbers added.

EXAMPLES.

79–81. Add 0·123, 0·012, 0·675, and 0·0045; 0·14145, 0·23235, 0·34345, and 0·45455; 0·617, 9128, 76435, and 476280.

82–87. What is the sum of 0·3456, 0·3465, 0·3546, 0·3564? of 0·3645, of 0·3654, 0·4356? of 0·4365, 0·4536, 0·4563? of 0·4635, 0·4653, 0·5346, 0·5364? of 0·6345, 0·6354, 0·6435, 0·6453? of 0·6534, 0·6543, 0·8765, 0·9876?

88–93. What is the sum of 1·234, 6·0045, 10·034? of

0·0036, 0·01701, 0·4005? of 37·38, 365·1, 63·36, 67·1? of 100·001, 19·001, 48·5, 3·47? of 12·2001, 21·012, 212·1, 122·11? of 401·104, 365, 390·91, 1000·1?

94–98. What is the sum of 256·7, 365·07, 17·071, 3·365? of 0·1924, 0·4501, 0·7512, 0·78301, 0·00019? of 884·12, 100·001, 303·044, 6·398, 48·485? of 971·914, 87·372, 547·006, 533·014, 384·009? of 203·145, 207·37, 0·017, 0·099, 0·083?

99–103. What is the sum of 12078·5, 60075·8, 6·085, 66·07, 301·38? of 44·369, 27·036, 64·027, 125·125? of 105·317, 206·004, 6·001, 0·009, 0·478? of 17·286, 3704, 1076, 1710·1, 0·03457? of 34689·14, 40057·82, 6078·65, 47083·9, 34·567?

§ 56. Subtraction of Decimals.

RULE.

Place the numbers as in addition of decimals, subtract as in whole numbers. Point off in the result as in addition of decimals.

Subtract 0·000001 from 0·1.

In examples of this kind naughts may be supposed to be annexed to the minuend, which (§ 40, Prop. III.) does not change the value of the decimal.

0·1
0·000001
0·099999

EXAMPLES.

104–110. From 898·7604 subtract 47·9631; 701·0001; 37·2896; 0·4972; 1·0001; 897·6795; 2·461.

111–116. From 92581·31 subtract 8461·1; 94·0009; 0·695816; 82000; 0·000036; 41·498.

117. From 3 millions and 1 millionth subtract 1 tenth.

118. From 96 billions, 2 thousand and 7, subtract 84 ten millionths.

119. From 82 millions 3 hundred, subtract 7 and 9 hundred-thousandths.

120–122. From 345·345 subtract 54·123; from 1245·3478 subtract 340·0122; from 3456·12347846 subtract 479·100345.

123–125. Subtract 99·9 from 1023·4; 0·13047 from 0·4785; 0·00675 from 0·11232.

126–129. Subtract 10·9807 from 219·307; 365·365 from 4017·37; 301·627 from 505·0005; 404·3737 from 900·1301.

Multiplication of Decimals.

§ 57. A tenth taken once must give 1 *tenth* for a product; if taken only one-tenth of a time, the product will be one-tenth of a tenth, or one-hundredth; that is, $\frac{1}{10}\times\frac{1}{10}=\frac{1}{100}$, or decimally expressed, $0\cdot1\times0\cdot1=0\cdot01$. This is evidently true, since if the tenth part of any thing be divided into 10 equal parts, each subdivision will be a hundredth part of the whole. So $\frac{1}{10}$ of $\frac{1}{100}=\frac{1}{1000}$, and so on.

Multiply 0·136 by 0·78. If we supply the denominators of these decimals, which denominators are always understood, we shall have $0\cdot136=\frac{136}{1000}$; $0\cdot78=\frac{78}{100}$.

Hence, multiplying $\frac{136}{1000}$ by $\frac{78}{100}$, we find

$$\frac{136}{1000}\times\frac{78}{100}=\frac{136\times78}{100000}=\frac{10608}{100000}=0\cdot10608.$$

From which we see that the number of decimal places in the product, always denoted by the number of naughts in the denominator which is understood, is equal to the number of decimal places in both factors. Hence we have this

RULE.

Multiply as in whole numbers. From the right of the product point off as many figures for decimals as there are decimal places in both the factors. If there be not enough figures in the product, prefix naughts.

Multiply 0·125 by 0·37.

In this example, the multiplicand has 3 decimal places, and the multiplier 2. Therefore the product must have five places. And since there are but 4 figures in the product, we prefix 1 naught before placing the decimal point.

```
 0·125
  0·37
 -----
   875
  375
 -------
 0·04625
```

EXAMPLES.

130–138. Multiply 943·078 by 8; by 12; by 14; by 28; by 39; by 121; by 696; by 1240; by 67932.

139–147. Multiply 270·4601 by 2·1; 7·09; 6·003; 92·804; 0·073; 0·2946; 0·94820; 0·765921; 1023·6921.

148–154. Multiply 0·49801 by 36·296; 492·12; 37·009; 6·219786; 0·000006; 0·0000009; 0·000000008.

155–163. Multiply 0·00074 by 0·19; 0·028; 0·0036; 0·00048; 0·000096; 0·0000084; 0·00000097; 3648·1; 8936·0004.

§ 58. A decimal number may be multiplied by 10, 100, 1000, &c., by removing the decimal point as many places to the right as there are naughts in the multiplier. If the number do not contain so many figures, annex naughts.

EXAMPLES.

164. Multiply 82·146 by 10.

165. Multiply 76·92 by 1000.

166–171. Multiply 4610·4 by 10; 100; 1000; 10000; 100000; 1000000.

172–179. Multiply 0·47692 by 10; 100; 1000; 10000; 100000; 1000000; 10000000; 100000000.

180–187. Multiply 3·7 by 10; 100; 1000; 10000; 100000; 1000000; 10000000; 100000000.

Division of Decimals.

§ 59. In multiplication of decimals, we know that the number of decimal places in the product is equal to the sum of those in both the factors. Now, since the product divided by one of the factors must produce the other factor or quotient, it follows that in division the decimal places of the dividend must be equal to the number of places in both divisor and quotient. Hence, the number of decimal places in the quotient must equal the excess of those in the dividend over those in the divisor.

Divide 5·81224 by 5·432.

Dividing 581224 by 5432, we find 107 for the quotient. Since 5 figures of the dividend, and only 3 figures of the divisor are decimals, it follows that two figures of the quotient 107 must be decimals, so that 1·07 is the quotient sought. Hence the following

RULE.

Divide as in whole numbers; point off as many decimal places in the quotient as those in the dividend exceed those in the divisor; if there are not as many, supply the deficiency by prefixing naughts.

Note.—Division of Decimals may be explained as follows:

Suppose dividend and divisor to be whole numbers, the quotient will be a whole number. If the dividend be divided by 10, that is, if it contain *one* decimal, the quotient (§ 31, *e*) will be divided by 10; that is, it will contain one decimal; and generally *as many times* as the dividend is divided by 10 will the quotient be so divided. But if the divisor also be divided by 10, the quotient just obtained will be multiplied by 10 (§ 31, *f*), and in general *as many times* as the divisor is divided by 10, so many times will the quotient be multiplied by 10; that is, for every decimal place in the divisor one decimal place in the quotient must be cancelled.

It is thus seen that in the effect upon the quotient, each decimal place in the divisor cancels a decimal place in the dividend, and that the excess of decimal places in the dividend over those in the divisor, that is, the number of uncancelled 10's by which it is divided, must be expressed by the same number of decimal places in the quotient.

Divide 0·123428 by 11·8.

In this example, the dividend contains 6 decimal places, and the divisor but 1; the quotient must, therefore, contain 5. As there are but 4 figures in the quotient, supply the deficiency by prefixing a naught before placing the decimal point.

```
11·8 ) 0·123428 ( 0·01046
         118
         ───
          542
          472
          ───
          708
          708
          ───
```

EXAMPLES.

188–192. Divide 7·11 by 3·1; 24·06 by 8·02; 67·2336 by 6·003; 96·97662 by 37·2987; 2146·078488 by 37·84.

193–198. Divide 3·810688 by 1·12; 0·109896 by 0·241; 1·12264556 by 1·0012; 0·01764144 by 0·0018; 0·07056545 by 0·0073; 0·1411309 by 0·00365.

§ 60. When there are not as many decimal places in the dividend as in the divisor, naughts may be annexed (§ 40, Prop. III.) to the dividend. When the number of decimal places is the same in dividend and divisor, the quotient will be a whole number. Thus, $\frac{6}{10} \div \frac{2}{10} = \frac{6}{2} = 3$; that is, $0·6 \div 0·2 = 3$.

EXAMPLES.

199–205. Divide 0·7 by 0·07; 0·25 by 0·0005; 0·25 by 0·00005; 0·125 by 0·000005; 122·418 by 3·4005; 244·431 by 1·2345; 365·2 by 9·13.

206–213. Divide 234·31 by 0·4967; by 0·28160; by 2·00076; by 7·892165; by 22·872003; by 41·9865432; by 221·762980; by 3·4076321.

214–225. Divide 827640·32167 by 8·2; by 9·03; by 11·416; by 327·0489; by 7260·19876; by 9831·00014; by 63·222219; by 92·4234767; by 38·91765890; by 21814·26; by 8·4; by 9·701.

NOTE.—The annexing of naughts to the dividend is obviously to reduce dividend and divisor to a common denominator.

Where the decimal places of the divisor are fewer than those of the dividend, naughts are always supposed to be annexed to the divisor; thus, 0·8215 ÷ 0·5 = 0·8215 ÷ 0·5000. Of course, then, if the dividend contain the divisor, the first figure of the quotient will be a whole number.

§ 61. When there is still a remainder, we may continue to annex naughts to it and to divide, until a sufficiently accurate result is obtained. The sign + annexed to the quotient shows that it is larger than is written.

NOTE.—The pupil will remember that every naught annexed to a remainder adds another decimal place to the dividend.

EXAMPLES.

226. Divide 0·8215 by 0·5.

227–231. Divide 4·1175 by 0·5; by 25; by 35; by 45; by 55.

232–238. Divide 20 by 0·003; 37·4 by 4·5; 7·85 by 3·43; 0·478 by 0·58; 0·9009 by 0·4051; 68·283 by 9·22; 845·6501; by 37·37.

§ 62. To divide a decimal by 10, 100, 1000, &c.

$\frac{1}{100} \div \frac{10}{1} = \frac{1}{100} \times \frac{1}{10} = \frac{1}{1000}$; that is, $0·01 \div 0·1 = 0·001$. Hence the following

RULE.

Remove the decimal point as many places to the left as there are naughts in the divisor: when there are not figures enough in the dividend prefix naughts.

EXAMPLES.

239–242. Divide 41497·6 by 10; by 100; by 1000; by 10000.

243–247. Divide 67·4 by 10; by 100; 1000; 10000; 100000.

248–253. Divide 0·341 by 10; 100; 1000; 10000; 100000; 1000000.

PROMISCUOUS EXAMPLES IN DECIMALS.

254. Bought 4 loads of wood: the first contained 0·97 cords, the second contained 1·03 cords, the third contained 0·945 cords, the fourth contained 1·005 cords. What did the four loads measure in decimals?

255. In the month of May the amount of rain was 3·15 inches, in June it was 4·05 inches, in July it was 2·97 inches, and in August it was 3·03 inches. How much rain fell during these four months?

256. During three successive days the mean* range of

* If the sum of a series of unequal quantities be divided by the number of quantities, the quotient is called the *mean* or *average* of these quantities, since it will, when repeated as many times as there are unequal quantities, just equal their sum. Thus, the average of 2, 4, 6, 8, and 10 (5 quantities), is $(2+4+6+8+10) \div 5 = 30 \div 5 = 6$.

the barometer was 29·04 inches, 29·51 inches, and 29·73 inches respectively. What is the sum of these heights?

257. In 1844, the whole number of school districts of New York was 10990, and the whole number of children in said districts, between the ages of 5 and 16 years, was 696548. What was the average* number for each district?

258. In New York, the total number of volumes in the 11018 school-district libraries was 1145250. What was the average number for each library?

259. In one mile there are 1760 yards, and in one rod there are $5\frac{1}{2}=5{\cdot}5$ yards. How many rods in one mile?

260. If light passes 191515 miles in a second, how many seconds will it require to pass from the sun to the earth, a distance of 95500000 miles?

261. If a cubic inch of pure water weigh 252·458 grains avoirdupois, of which 7000 make one pound, what is the weight of the Imperial or English gallon, which contains 277·274 cubic inches?

262. If one Imperial gallon contain 277·274 cubic inches, how many cubic inches in 8 gallons or one bushel, and how many cubic feet of 1728 inches each?

263. If one cubic inch of pure water weigh 252·458 grains avoirdupois, how many grains will 1728 cubic inches, or one cubic foot, weigh, and how many pounds of 7000 grains each?

264. If at each stroke of the piston-rod of a locomotive engine a distance of 13·25 feet is passed over, how many strokes must be made in passing a distance of 93 miles?

265. In one mile there are 5280 feet, and in one rod there are 16·5 feet. How many rods in one mile?

266. How many feet in circumference must a wheel be

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* See note on preceding page.

so as to roll over just 100 times in going a distance of one mile?

267–269. If the circumference of the forward wheel of a carriage is 15·25 feet, and the circumference of the hind wheel 17·75 feet, then in a journey of 10 miles, how many times will each revolve? and how many more times will the one revolve than the other?

270. If 37·03 acres of land cost 2000 dollars, how much was it per acre?

271–274. If I purchase 43·25 acres of land at 55·5 dollars per acre, and sell 31·25 acres for 2500 dollars, then how much did I give for the whole? How much did I receive per acre for what I sold? How much more did I receive for what I sold than the whole cost me? and how many acres remained unsold?

275. From a cistern containing 3000 gallons, 73·5 barrels, of 31·5 gallons each, are drawn off. How many gallons remain?

Reduction of Common Fractions to Decimals.

§ 63. Reduce $\frac{3}{8}$ to a decimal.

We cannot divide 3 by 8; but reducing the 3 to *tenths*, that is, multiplying it by 10, we have $3=30$ tenths, which divided by 8 gives 3 tenths for a quotient. But there are 6 tenths remainder. Reducing these to *hundredths*, we have 60 hundredths, which divided by 8 gives 7 hundredths for a quotient. But there are 4 hundredths remaining. Reducing these to *thousandths*, we have 40 thousandths, which divided by 8 gives 5 thousandths for a quotient.

Thus, $\frac{3}{8} = 3$ tenths $+ 7$ hundredths $+ 5$ thousandths

$=0{\cdot}375$. Hence, to reduce a common fraction to a decimal, we have this

RULE.

Perform the division expressed by the fraction, annexing as many naughts to the numerator as are necessary to produce a sufficiently exact quotient. In the quotient point off as many decimal places as there have been naughts annexed.

NOTE.—After having annexed one 0, if the dividend will not contain the divisor, write 0 in the quotient, and so on.

EXAMPLES.

276–297. Reduce to their equivalent decimal fractions the following common fractions: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{7}$; $\frac{1}{6}$; $\frac{1}{12}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{6}{7}$; $\frac{7}{8}$; $\frac{8}{9}$; $\frac{9}{11}$; $\frac{10}{11}$; $\frac{11}{12}$; $\frac{13}{14}$; $\frac{14}{15}$; $\frac{15}{16}$; $\frac{16}{17}$; $\frac{17}{18}$; $\frac{18}{19}$; $\frac{19}{20}$.

298–329. Reduce to decimals the following: $\frac{3}{5}$; $\frac{3}{7}$; $\frac{3}{8}$; $\frac{3}{10}$; $\frac{3}{13}$; $\frac{4}{7}$; $\frac{4}{9}$; $\frac{4}{11}$; $\frac{4}{13}$; $\frac{5}{7}$; $\frac{5}{8}$; $\frac{5}{9}$; $\frac{5}{11}$; $\frac{6}{11}$; $\frac{6}{13}$; $\frac{6}{17}$; $\frac{7}{9}$; $\frac{7}{11}$; $\frac{7}{13}$; $\frac{8}{11}$; $\frac{8}{13}$; $\frac{8}{15}$; $\frac{8}{17}$; $\frac{9}{13}$; $\frac{9}{17}$; $\frac{11}{14}$; $\frac{13}{22}$; $\frac{16}{28}$; $\frac{19}{33}$; $\frac{47}{51}$; $\frac{83}{113}$; $\frac{114}{702}$.

330–333. What is the decimal value of $\frac{4}{5}$ of $\frac{6}{7}$? of $\frac{3}{4}$ of $\frac{8}{11}$ of $\frac{7}{12}$? of $\frac{2}{3}$ of $\frac{7}{8}$ divided by $\frac{6}{7}$ of $\frac{4}{5}$? of $\frac{2}{3}$ of $\frac{6}{7}$ diminished by $\frac{2}{4}$ of $\frac{4}{9}$?

334–336. Find the decimal value of $\frac{6}{7}+\frac{9}{11}+\frac{3}{5}+\frac{5}{7}+\frac{4}{15}$; of $\frac{6}{7}\times\frac{9}{11}\times\frac{3}{5}\times\frac{5}{7}\times\frac{4}{15}$; of $(\frac{6}{7}+\frac{9}{11})\times\frac{3}{4}$.

337. A man received $7\frac{2}{15}$ of a dollar at one time, $3\frac{1}{4}$ dollars at another, and $5\frac{1}{3}$ of a dollar at another. How much did he receive in all?

It will be seen, as in some of the preceding examples, that the figures of the quotient are repeated: $\frac{1}{3}$ giving 0·333, &c., and $\frac{1}{7}$ giving 0·1428571428, &c.; $\frac{1}{6}$ giving 0·1666, &c.; $\frac{1}{12}$ giving 0·08333, &c.

These are called *repeating* decimals. The figures that are repeated are called the *repetend*, and are distinguished by a dot placed over the first and last; as 0·333, &c.$=0{\cdot}\dot{3}$; 0·1428571428, &c.$=0{\cdot}\dot{1}4285\dot{7}$; 0·08333, &c.$=0{\cdot}08\dot{3}$; &c.

When decimal figures precede the repetend they are called the *finite part* of the decimal. Thus, in 0·08333, &c., which is the decimal value of $\frac{1}{12}$, 0·08 is the finite part.

Reduction of Decimals to Common Fractions.

§ 64. Change 0·375 to a common fraction.

$0{\cdot}375=\frac{375}{1000}=\frac{3}{8}$. Hence, to reduce a decimal to a common fraction, the following

RULE.

Erase the decimal point; supply the decimal denominator, and reduce the fraction to its lowest terms.

EXAMPLES.

338–351. Reduce to their equivalent common fractions the following decimals: 0·5; 0·15; 0·25; 0·375; 0·225; 0·435; 0·575; 0·486; 0·656; 0·0025; 0·00375; 0·000225; 0·1001; 0·36984.

352–357. Reduce to decimals the following: 0·0982; 0·00764; 0·00025; 0·5005; 0·0125; 0·01250505.

§ 65. To reduce repeating decimals to common fractions.

$\frac{1}{9}$=0·1111, &c.; $\frac{1}{99}$=0·01010101, &c.; $\frac{1}{999}$=0·001001001001, &c.; $\frac{3}{9}$=0·3333, &c.; $\frac{3}{99}$=0·03030303, &c.; $\frac{3}{999}$=0·003003003003, &c.; $\frac{6}{9}$=0·6666, &c. Hence we see that the numerator of the fraction is the repeating decimal (omitting the useless naught or naughts at the left), while the denominator of the fraction is as many 9's as there are figures in the repetend: 0·0909, &c.$=\frac{9}{99}$; $0{\cdot}\dot{1}4285\dot{7}=\frac{142857}{999999}$; 3·1666, &c.$=3{\cdot}1\dot{6}=3{\cdot}1\frac{6}{9}=3+\frac{1\frac{2}{3}}{10}=3\frac{1}{6}$. Hence this

RULE.

Make the repetend the numerator, and as many 9's as there are figures in the repetend the denominator, of the required fraction.

NOTE.—If there be a finite part to the decimal, write the whole decimal as a complex fraction (as above), and reduce it to a simple fraction.

EXAMPLES.

358–370. Reduce to common fractions the following repeating decimals: $0\cdot\dot{3}$; $0\cdot\dot{3}45\dot{6}$; $0\cdot\dot{4}35\dot{6}$; $0\cdot\dot{6}54\dot{3}$; $0\cdot\dot{1}$; $0\cdot\dot{1}\dot{2}$; $0\cdot08\dot{3}=\frac{8\frac{3}{9}}{100}$; $0\cdot0\dot{6}$; $0\cdot24\dot{3}$; $0\cdot\dot{0}14285\dot{7}$; $0\cdot\dot{9}$; $0\cdot\dot{0}1234567\dot{9}$; $0\cdot\dot{1}2332\dot{1}$.

371–385. Reduce to common fractions the following: $0\cdot123\dot{4}5\dot{6}$; $0\cdot0\dot{7}1428\dot{5}$; $0\cdot03\dot{5}7142\dot{8}$; $0\cdot0\dot{2}\dot{7}$; $0\cdot\dot{1}2\dot{3}$; $0\cdot\dot{3}2\dot{1}$; $0\cdot\dot{3}\dot{6}$; $0\cdot\dot{6}\dot{3}$; $0\cdot\dot{0}39\dot{8}$; $0\cdot\dot{6}34\dot{5}$; $0\cdot\dot{6}53\dot{4}$; $0\cdot\dot{5}64\dot{3}$; $0\cdot\dot{5}63\dot{4}$; $0\cdot\dot{7}\dot{2}$; $0\cdot\dot{5}\dot{4}$.

FEDERAL MONEY.

§ 66. Federal Money is the currency of the United States. This currency is expressed decimally. Its unit takes the name of *dollar;* its tenth is the *dime;* its hundredth is the *cent;* its thousandth is the *mill.* Thus, 2·345 expresses (§ 52) 2 units, 3 tenths, 4 hundredths, and 5 thousandths, or 2 and 345 thousandths. Now writing the symbol, $,* before the sum, thus, $2·345, it becomes at once 2 dollars,

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* This symbol probably represents a U placed upon an S to denote the currency of the U. S. (United States). It must *always* be written before numbers expressing dollars; and before numbers expressing parts of a dollar, unless these are denoted by the sign *cts.* placed after them.

3 dimes, 4 cents, 5 mills; or 2 dollars, 34 cents, 5 mills. The dime is always read as cents. There is another denomination of this currency, answering to the *ten*, called the *eagle*. This is read as dollars. Thus, $23 expresses 2 eagles and 3 dollars, but it is read 23 dollars. $84·6923 is read 84 dollars, 69 cents, 2 mills, and 3 tenths of a mill.

The following is the table of Federal Money:

10 mills* (marked *m.*)	make	one	cent,	marked	*ct.*
10 cents . .	"	"	dime,	"	*d.*
10 dimes . .	"	"	dollar,	"	$.
10 dollars . .	"	"	eagle,	"	E.

The coins of the United States are the double-eagle, eagle, half-eagle, quarter-eagle, dollar, made of gold; the dollar, half-dollar, quarter-dollar, dime, half-dime, and three cent piece, made of silver; the cent and half cent, made of copper.

The mill is not coined.

NOTE.—The gold for coinage is not pure, but consists of $\frac{22}{24}$ of pure gold, $\frac{1}{24}$ of silver, and $\frac{1}{24}$ of copper; or, as usually expressed, 22 *carats* of gold, 1 of silver, and 1 of copper. A carat is $\frac{1}{24}$ part of the whole.

The standard for silver is 1489 of pure silver to 179 of pure copper; which, in carats, is $21\frac{59}{139}$ of silver, and $2\frac{80}{139}$ of copper.

The copper coins are of pure copper. The three cent piece is $\frac{3}{4}$ silver and $\frac{1}{4}$ copper.

By an act of Congress, approved January 18, 1837, the gold and silver coin must consist of $\frac{900}{1000}=\frac{9}{10}$ pure metal, and $\frac{100}{1000}=\frac{1}{10}$ alloy. The alloy for silver must consist of pure copper, and the alloy for gold, of copper and silver, provided that the silver does not exceed one-half of the whole alloy.

The weight of the Eagle was fixed at 258 grains; the weight of the Dollar at 412½ grains; that of the Cent at 168 grains.

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* The word mill is from the Latin *mille*, meaning a thousand; cent from the Latin *centum*, meaning a hundred; dime is from the French word *disme*, meaning ten.

EXAMPLES.

386–392. Read the following: \$7·84; \$92·06; \$672·123; \$8961·006; \$4180·9673; \$901·001; \$3·03.

393–401. Read the following: \$6·82; \$7·448; \$9·02; \$3·01; \$4·07; \$6·93; \$48·761; \$217·001; \$36·987.

402–408. Express in figures thirty-seven cents; forty-four cents, three mills; six dollars, two cents; four dollars, eight mills; nine dollars, twenty cents, six mills; five thousand dollars, eight cents, nine mills; one million dollars, one and one-half cents.

NOTE.—Write the half-cent always as 5 mills.

409–412. Write $37\frac{1}{2}$ cts.; 2 dolls. $12\frac{1}{2}$ cts.; 4 dolls. $62\frac{1}{2}$ cts.; 5 dolls. $87\frac{1}{2}$ cts.

§ 67. From the table it is seen that a number expressing dollars will express cents by annexing two naughts to it, and will express mills by annexing three naughts; thus 3 dolls. become 300 cts. and 3000 mills.

So cents become mills by annexing one naught; thus, 7 cts. = 70 mills.

Reversely, mills become cents by cutting off a naught from the right; and become dollars by cutting off three naughts; thus, 8000 mills = 800 cts. = 8 dolls.

Cents become dollars by cutting off two naughts from the right; thus, 700 cts. = 7 dolls.

NOTE.—It is obvious that this reduction of dollars to cents or mills is simply the multiplication of the sum expressing dollars by 100 or 1000; and that the reduction of cents or mills to dollars is simply the division of the sum given by 100 or 1000.

EXAMPLES.

413–419. Reduce first to cents, then to mills, the following sums: \$8; \$894; \$620; \$34; \$936273; \$841904; \$123456.

420–426. Reduce the following sums to mills: 83 cts.; 91 cts.; 4 cts.; 378 cts.; 1234 cts.; 9100 cts.; 875618 cts.

427–441. Reduce to dollars the following sums: 841 cts.; 928 cts.; 4670 cts.; 12986 cts.; 4810 mills; 1234 m.; 4968 cts.; 321946 m.; 135792 cts.; 9800 m.; 9800 cts.; 3918762 m.; 4987621 cts.; 3076009 cts.; 4876543 m.

442–450. Reduce to cents the following sums: 8940 m.; 92801 m.; 1234567 m.; \$81·07; \$83·96; \$487·80; \$9654·21; \$13498·20; \$482·31.

451–456. Reduce to mills the following sums: \$0·83; \$98·436; \$2·076; \$281·296; \$4812·37; \$69874·983.

PROMISCUOUS EXAMPLES IN FEDERAL MONEY.

§ 68. *The rules that apply to operations in Decimals apply without change to operations in Federal Money.*

457. Bought a box of raisins for \$1·75, a bushel of apples for \$0·375, a cheese for \$3·175, a barrel of sugar for \$15·50. What did the whole amount to?

458. A farmer receives \$15·375 for a cow, \$75 for a horse, \$3·125 for some potatoes, \$5·55 for some poultry. How much did he receive in all?

459. A person bought some velvet for \$3·333, some broadcloth for \$18·75, some silk for \$12·50, some cotton cloth for \$5·405, a shawl for \$12·25, some carpeting for \$30·05. What did the whole amount to?

460. A person borrowed \$213·375, of which he has paid \$107·18. How much does he still owe?

461. Bought a cow for \$13·25, paid \$6·875. How much remains unpaid?

462. What will 185 pounds of coffee cost, at \$0·138 per pound?

463. Bought 8·375 cords of wood, at $2·50 per cord. What did it cost?

464. What will 121·5 gallons of molasses come to, at 41 cents per gallon?

465. The length of the Erie Canal is 364 miles, and it cost $7143790. What was the average expense per mile?

466. The Crooked Lake Canal is 8 miles long, and cost $156777. How much is this per mile?

467. In 1842, the whole number of children taught in the district schools of the State of New York was 598901; the whole amount disbursed for common schools was $1155419·90. How much was that per scholar?

468. The salary of the President of the United States is $25000. How much is that each day?

469. In one rod there are 16·5 feet. How many rods in 3573 feet?

470. Bought a farm of 137 acres for $5324. How much was that per acre?

471. If 35 miles of railroad cost $400000, how much was the average cost per mile?

472. A farmer sells his butter for $0·21 per pound, receiving $1613·22. How many pounds did he sell?

473. The butter made from the milk of 53 cows, during the summer, having been sold for $0·20 per pound, brought $1579·40. How many pounds were sold, and what was the average produce of each cow?

474. In a dairy of 46 cows, suppose each averages 2·5 gallons of milk daily, and that each gallon produces 1·1 pounds of cheese, how many pounds will be thus made in 5·7 months of 30 days each, and what will the whole bring at 15 cents per pound?

475. A farmer sold as follows:

15127	pounds of	cheese, at	6·75	cents per	pound.
400	" "	butter, "	15	" "	"
2400	" "	pork, "	5	" "	"
53	bushels "	wheat, "	125	" "	bushel.
73	" "	barley, "	50	" "	"
231	" "	corn, "	50	" "	"
262	" "	oats, "	30	" "	"

What did the whole amount to?

476. In 1845, the revenue or interest from the School Fund of the State of New York was $86828·96. During the same year there were employed 7147 teachers. If the above sum were equally divided among those teachers, what would each one receive?

477. A compositor worked nine months, and during that time set up at the rate of 7000 m's per day. How many thousand m's did he set up, reckoning 25 working days to the month? and how much did he receive at 15 cents per 1000 m's?

478. A man, in balancing his family accounts for one year, found his expenses as follows: for January, $98·41; for February, $81·33; for March, $102·28; for April, $125·26; for May, $74·38; for June, $73·47; for July, $65·98; for August, $87·21; for September, $70·34; for October, $122·08; for November, $79·68; for December, $52·77. His salary was $1050 per annum. What had he left at the end of the year?

479. A butcher, a shoemaker, and a tailor gave orders on each other in the way of their business, and at the end of a year settled accounts. The butcher's bill against the tailor was $61·84; against the shoemaker, $39·44. The shoemaker's bill against the butcher was $24·30; against

the tailor, $19·15. The tailor's bill against the butcher was $42·07; against the shoemaker, $39·39. Who received balances in cash?

480. Bought 116 feet of pine wood, at $4·50 per cord of 128 feet. How much did I pay for the load?

481. I bought 19 baskets of coal, at $12\frac{1}{2}$ cents per bushel;* $1\frac{3}{4}$ cords of wood, at $8 per cord; 3 tons hard coal, at $6·50 per ton; and paid 91 cts. for sawing and splitting the wood. How much did I pay for my fuel?

482. Mr. Holden's expenses for February were as follows: for the table $28·28; for sundries $45·83; for clothing $32·73; oil $0·68; rent $14·50; wages $6·50. How much in all?

483. A man takes 50 dollars to pay his grocer's bill, which is as follows: 38 doz. eggs, at $12\frac{1}{2}$ cts. a dozen; 34 pounds of white sugar, at 11 cts. a pound; 42 pounds of brown sugar, at 7 cts. a pound; 27 pounds codfish, at $3\frac{1}{2}$ cts. a pound; 3 brooms, at 18 cts. a piece; 7 gallons ale, at 25 cts. a gallon; 40 pounds butter, at 18 cts. a pound; 2 galls. molasses, at 40 cts. a gal.; $\frac{1}{2}$ gross matches, at $62\frac{1}{2}$ cts. a gross. How much change must the man receive?

484. The largest gold coin known is the dobraon of Portugal, of the value of $32·706. How many double-eagles are there in 75 dobraons?

485. A piece of silk is two-thirds as wide as a piece of mousseline-de-lain. It requires 10 yards of the latter for a dress. The mousseline is $87\frac{1}{2}$ cts. a yard, and the silk $62\frac{1}{2}$. What is the difference in price betweeen two dresses of equal fulness, one of the silk and the other of the mousseline?

486. The receipts from United States customs for the year 1847–8 were $31757071; and from lands, &c., $3679680.

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* Each basket contains 3 bushels.

The expenditures for the same time were for the army $27280163; navy, $9406737; civil and miscellaneous, $5585070. What was the excess of receipts over expenditures?

487. Receipt the following bill for its true amount.

HENRY PHELPS   To AARON MUNDIN,   Dr.

To 75 yds. Brussels carpeting, at $1·42 per yard;
" 63 skeins silk, at $2\frac{1}{2}$ cts. a skein;
" 1 piece cotton, 31 yds., at 11 cts. a yard;
" 1 Mousseline dress, 10 yds., at 92 cts. yd.;
" 1 box hooks and eyes, at $2·30;
" 1 Piano cover, $7·12$\frac{1}{2}$; Table do., $3·25.

488. Receipt the following bill for its true amount.

JOHN COX   To PHIL. BRADY,   Dr.

To 1 sup. broadcloth coat, $22;
" 1 vest, $5·37; 1 pair pants, $8·50;
" Overcoat, $26; 6 pairs gloves, at $0·33 per pair;
" Suspenders, $0·50; 4 pairs of drawers, $0·88 per pair;
" 1 doz. shirts, $1·87 each; 18 prs. socks, at $0·22 per pair.

489. What was the amount of my butcher's bill? The items were as follow:

10 pounds beef, at 14 cts. per pound; 6 pairs of fowls, average $2\frac{1}{2}$ pounds each, at 18 cts. per pound; kit mackerel, 25 pounds, at $5\frac{1}{2}$ cts. a pound; 38 pounds sausages, at 11 cts. a pound; fore-quarter lamb, 7 pounds, at 8 cts. a pound; 1 bushel of potatoes, 75 cts.; 30 pounds lard, at 10 cts. a pound.

490. How many volumes of good books, averaging 50 cts. a volume, could a man purchase with the sum he would spend for rum (two glasses a day, at 3 cts. a glass), during 30 years of his life, allowing 365 days to the year?

491. If a man spend 9 cts. a day for cigars, how much will he spend during a life of 70 years, in that worse than useless indulgence?

492. Bought $28\frac{1}{2}$ barrels of beef for \$285, and sold them at a profit of \$1·78 per barrel. How much did I seli them for?

§ 69. To find the value of articles estimated by the 100 or 1000.

What is the value of 9425 bricks, at \$3·25 per 1000?

Supposing the price to be \$3·25 for each brick, we multiply the price per brick by the number of bricks; that is, \$3·25 by 9425; or, what is the same thing, 9425 by 3·25, the number of dollars, since this is more convenient.

The product 30631·25 is evidently 1000 times too great. We therefore divide it by 1000 (§ 62), by removing the decimal 3 places to the left. The true result, then, is \$30·63125. Had the bricks been \$3·25 per 100, the decimal should have been removed two places to the left. Hence this

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    9425
    3·25
 -------
   47125
  18850
 28275
 -------
30631·25
```

RULE.

Multiply the number of articles by the number expressing the price per 100 *or* 1000. *From the right of the product point off two figures when the articles are estimated by the* 100, *or three figures when they are estimated by the* 1000.

NOTE.—The decimal figures pointed off by this rule are *in addition* to those which are pointed off by the usual rule for multiplication of decimals.

EXAMPLES.

493. What is the value of 1300 feet of hemlock boards, at \$5·50 per 1000?

494. What is the value of 675 feet of clear pine stuff, at \$25 per 1000?

495. What is the value of 11035 feet of timber, at \$2·25 per 100?

496. What is the value of 90422 bricks, at \$3·75 per 1000?

497. What must be paid for laying 875 bricks, at \$3·25 per 1000?

498. What cost 1689216 laths, at 8 cts. per 100?

499. A man carted 575 loads of bricks, each load containing 1800 bricks. What cost the whole, at \$8·25 per 1000?

500. What must be paid for planing 4976280 feet of boards, at 42 cts. per 1000 feet?

An abridged method for operations in Federal Money, *by the aid of aliquot parts.* (See § 116.)

§ 70. What cost 704 yards of cloth, at $12\frac{1}{2}$ cts. per yard?

The question may be answered in the usual way by multiplying 704 by 0·125, the cost of one yard in dollars. But there is a shorter method.

If the price of the cloth had been 1 dollar a yard, the 704 yards would have cost 704 dollars. But $12\frac{1}{2}$ cts. is $\frac{1}{8}$ of a dollar; consequently, the 704 yards must cost $\frac{704}{8}$ dollars=\$88.

At $12\frac{1}{2}$ cts. a yard, how many yards of cloth can be bought for \$88?

This question might be answered in the usual way by dividing 88 by 0·125. But as $12\frac{1}{2}$ cts. is $\frac{1}{8}$ of a dollar, 1 dollar would buy as many yards as $\frac{1}{8}$ is contained times in 1; that is, $1\div\frac{1}{8}=8$, and 88 dollars would buy 88 times as many yards; that is, $88\div\frac{1}{8}=704$, the number of yards. Hence this

RULE.

As the conditions of the question require division or multiplication, divide or multiply by the fractional part of the dollar which the price expresses.

TABLE

Of fractional parts of a dollar, called *aliquot* or exact parts.

cts.	$	cts.	$	cts.	$	cts.	$
5	$=\frac{1}{20}$.	$16\frac{2}{3}$	$=\frac{1}{6}$.	$33\frac{1}{3}$	$=\frac{1}{3}$.	$62\frac{1}{2}$	$=\frac{5}{8}$.
$6\frac{1}{4}$	$=\frac{1}{16}$.	$18\frac{3}{4}$	$=\frac{3}{16}$.	$37\frac{1}{2}$	$=\frac{3}{8}$.	$66\frac{2}{3}$	$=\frac{2}{3}$.
$8\frac{1}{3}$	$=\frac{1}{12}$.	20	$=\frac{1}{5}$.	50	$=\frac{1}{2}$.	75	$=\frac{3}{4}$.
10	$=\frac{1}{10}$.	25	$=\frac{1}{4}$.	$56\frac{1}{4}$	$=\frac{9}{16}$.	$83\frac{1}{3}$	$=\frac{5}{6}$.
$12\frac{1}{2}$	$=\frac{1}{8}$.	$31\frac{1}{4}$	$=\frac{5}{16}$.	$58\frac{1}{3}$	$=\frac{7}{12}$.	$87\frac{1}{2}$	$=\frac{7}{8}$.

NOTE.—$12\frac{1}{2}$ cts. would, by the table, be $\frac{1}{8}$ of a dollar; $1·$12\frac{1}{2}$ would be $\frac{9}{8}$; $2·$31\frac{1}{4}$ would be $\$\frac{37}{16}$; $4·75 would be $\$\frac{19}{4}$, &c.

EXAMPLES.

501–514. What would 678 baskets of peaches cost, at $12\frac{1}{2}$ cts. a basket? at $16\frac{2}{3}$ cts.? at $18\frac{3}{4}$ cts.? at 20 cts.? at 25 cts.? at $31\frac{1}{4}$ cts.? at $33\frac{1}{3}$ cts.? at $37\frac{1}{2}$ cts.? at 50 cts.? at $62\frac{1}{2}$ cts.? at $66\frac{2}{3}$ cts.? at 75 cts.? at $83\frac{1}{3}$ cts.? at $87\frac{1}{2}$ cts.?

515–521. What would 840 yards of cloth cost, at one shilling, New York currency ($12\frac{1}{2}$ cts.), per yard? at 3 shillings? at 4? at 5? at 6? at 7? at 2?

NOTE.—It would be more simple and more consistent that all accounts in the United States should be kept in Federal Money. Yet in most of the States the old colonial denominations of shillings and pence are more or less used. The unit of these denominations is not of the same value in all the States; thus, in New York, 1 shilling= $12\frac{1}{2}$ cts.; in New England=$16\frac{2}{3}$ cts. The reason of this is, that at the time of the adoption of Federal Money in 1786, the paper currency of the several colonies had depreciated in value. A colonial pound or shilling was not worth so much as a sterling (English) pound or shilling. But this depreciation being unequal in the several colonies, their shillings and pence varied in value. This variation continues to this day.

522–527. At one shilling, N. E. currency ($16\frac{2}{3}$ cts.), per pound, how many pounds of butter can be bought for $9?

for $12 ? for $54 ? for $926 ? for 8217 dolls. ? for 98127 dolls. ?

528–538. How many pecks of apples will $37 buy, at fourpence-ha'penny, N. E. currency ($6\frac{1}{4}$ cts.), a peck ? at ninepence ($12\frac{1}{2}$ cts.) a peck ? at one shilling a peck ? at sixpence ($8\frac{2}{3}$ cts.) a peck ? at 4 and 6 pence a peck ? at 2 and 6 pence ? at 3 and 6 pence ? at 5 and 6 pence ? at 3 shillings ? at 4 shillings ? at 5 shillings ?

539–549. How many brushes will $50 purchase at a sixpence, New York currency ($6\frac{1}{4}$ cts.), a piece ? at a shilling a piece ? at 18 pence ($18\frac{3}{4}$ cts.) a piece ? at 2 shillings ? at 2 and 6 pence ? at 3 shillings ? at 3 and 6 pence ? at 4 shillings ? at 5 ? at 6 ? at 7 ?

550–562. For 656 dollars, how many yards of carpeting can I buy, at $37\frac{1}{2}$ cts. per yard ? at 50 cts. ? at $56\frac{1}{4}$ cts. ? at $58\frac{1}{3}$ cts. ? at $62\frac{1}{2}$ cts. ? at $66\frac{2}{3}$ cts. ? at 75 cts. ? at $83\frac{1}{3}$ cts. ? at $\$1{\cdot}12\frac{1}{2}$? at $1·25 ? at $\$1{\cdot}33\frac{1}{3}$? at $1·50 ? at $\$1{\cdot}62\frac{1}{2}$?

563–575. What will 98 yards of carpeting cost, at $87\frac{1}{2}$ cts. per yard ? at $1·05 ? at $\$1{\cdot}06\frac{1}{4}$? at $\$1{\cdot}08\frac{1}{3}$? at $\$1{\cdot}12\frac{1}{2}$? at $\$1{\cdot}16\frac{2}{3}$? at $1·25 ? at $\$1{\cdot}33\frac{1}{3}$? at $\$1{\cdot}62\frac{1}{2}$? at $\$1{\cdot}37\frac{1}{2}$? at $\$1{\cdot}56\frac{1}{4}$? at $1·50 ? at $\$1{\cdot}87\frac{1}{2}$?

CHAPTER XI.

DENOMINATE NUMBERS.

§ 71. We have thus far treated of abstract numbers ; that is, of numbers as simple units, or as parts of a unit. We shall, in the present chapter, treat of *denominate* numbers, or numbers having reference to particular things. (§ 1.)

NOTE.—Federal Money, when considered as units, tenths, hundredths, &c., has an abstract character; when considered as dollars, cents, and mills, has a denominate character.

In multiplication, the multiplicand being repeated a certain number of times, or a certain fraction of a time when the multiplier is a fraction, it follows that the multiplier, considered as a multiplier, must always be regarded as an abstract number. And since the product is a repetition of the multiplicand, it must be like the multiplicand; that is, if the multiplicand is an abstract number, the product must be an abstract number; if the multiplicand is a denominate number, the product must be a denominate number of the same kind.

In division, when the quotient shows how many times the divisor is contained in the dividend, or what fraction of a time when the divisor is greater than the dividend, it follows that the quotient must be regarded as an abstract number, and that the divisor and dividend must be alike.

When, however, the process of division is rather the dividing of a dividend *into as many equal parts as are indicated by the divisor*, the quotient expressing the units in one of those parts is of the same kind as the dividend, while the divisor is to be regarded as an abstract number. See Example, § 91.

The following are the most important tables of weights and measures, &c., and must be thoroughly learned by the pupil.

§ 72. ENGLISH OR STERLING MONEY.

4	farthings (*qr.* or *far.*)	make	1 penny,	*d.*
12	pence	"	1 shilling,	*s.*
20	shillings	"	1 pound,	£

NOTE 1.—Farthings are often expressed as fractions of a penny Thus, 1 *far.*=$\frac{1}{4}$*d.*; 2 *far.*=$\frac{1}{2}$*d.*; 3 *far.*=$\frac{3}{4}$*d.*

NOTE 2.—The pound sterling was originally a bank note; but the note has fallen into disuse; and a gold coin, called a *sovereign*, of the value of $4·84, supplies its place.

NOTE 3.—The symbol £ is used because it is the first letter of the Latin word *libra*, which signifies a pound; *s.* stands for *solidus*, which signifies a shilling; *d.* for *denarius*, a penny; *q.* for *quadrans*, a quarter.

The stroke / often written between shillings and pence is a corruption of the long *ſ*.

§ 73. TROY WEIGHT.

24 grains (*gr.*)	make	1 pennyweight,	*pwt.*
20 pennyweights	"	1 ounce,	*oz.*
12 ounces	"	1 pound,	*lb.*

NOTE 1.—The original of all weights used in England was a *grain* of wheat, gathered out of the middle of the ear; 32 of these, well dried, were to make one pennyweight. But at a later period, it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, being the least weight now in common use.

Coins, precious metals, jewels, and liquors, are weighed by Troy weight.

NOTE 2.—This scale of weights is said to have been borrowed from *Troyes* in France—hence its name. Some, however, contend that the name has reference to the monkish title given to London of *Troy Novant.*

§ 74. APOTHECARIES' WEIGHT.

20 grains (*gr.*)	make	1 scruple,	℈
3 scruples	"	1 dram,	ʒ
8 drams	"	1 ounce,	℥
12 ounces	"	1 pound,	℔

NOTE.—This weight, as its name would imply, is used in weighing medicines in small quantities, as for prescriptions. But drugs and medicines in gross are bought and sold by Avoirdupois Weight. The pound and ounce Apothecaries' Weight are the same as in Troy Weight.

§ 75. AVOIRDUPOIS WEIGHT.

16	drams (*dr.*)	make	1 ounce,	*oz.*
16	ounces	"	1 pound,	*lb.*
25	pounds	"	1 quarter,	*qr.*
4	quarters	"	1 hundred weight,	*cwt.*
20	hundred weight	"	1 ton,	*T.*

NOTE 1.—By this weight are weighed all things of a coarse or drossy nature, as bread, butter, cheese, flesh, groceries, and some liquids; all metals, except gold and silver.

NOTE 2.—Formerly 28 pounds were estimated as 1 *qr.*, 112 pounds 1 *cwt.*, and 2240 *lbs.* 1 ton. These weights are still used for cheap and heavy articles, such as iron, coal, plaster, &c.

NOTE 3.—The pound Avoirdupois contains 7000 grains Troy, while the Troy pound contains only 5760 grains.

§ 76. LONG MEASURE.

12	inches (*in.*)	make	1 foot,	*ft.*
3	feet	"	1 yard,	*yd.*
$5\frac{1}{2}$	yards	"	1 rod, perch, or pole,	*rd.*
40	rods	"	1 furlong,	*fur.*
8	furlongs	"	1 mile,	*mi.*
3	miles	"	1 league,	*L.*
$69\frac{1}{6}$	miles, nearly,	"	1 degree,	*deg.* or °.

NOTE 1.—4 inches make 1 hand; 9 inches, 1 span; 18 inches, 1 cubit; 6 feet, 1 fathom; 3 feet, 1 pace.

NOTE 2.—The inch is subdivided sometimes into tenths; sometimes into halves, quarters, eighths, sixteenths; and sometimes into twelfths, called lines or primes.

NOTE 3.—A nautical or geographical mile is $\frac{1}{60}$ of a degree of the earth's circumference. And since one degree is $69\frac{1}{6}$ statute or legal miles, we have 1 nautical mile equal to $1\frac{11}{72}=1{\cdot}152\dot{7}$ statute miles $=6086\frac{2}{3}$ feet.

A *knot*, in nautical language, is a division of the log-line of $\frac{1}{120}$ of a nautical mile. A half-minute glass is used in connection with the log, by observing how many knots of the log-line are run, while the sand is running from the glass. As half a minute is $\frac{1}{120}$ of an hour, it follows that the number of knots thus run will be the number of miles the ship is making hourly. Hence, it is frequently said that a ship was running at the rate of a certain number of knots, by which is meant the number of nautical miles she was making hourly. In this sense, *knot* is used for a nautical mile.

NOTE 4.—The standard length of the yard in the United States, from which all other measures of length are derived, is the same as that of the Imperial yard of Great Britain. This yard is deduced from that of a pendulum which vibrates once in a second in vacuum at the level of the sea at London. Such a pendulum is found to be 39·13929 inches.

NOTE 5.—The French government derive their linear unit of measure from one quarter of the circumference of a great circle of the earth passing through the poles. Having determined by actual surveys the length of that portion of a quarter circle, which is comprised between the parallels of Dunkirk and Barcelona, they deduced the length of the entire quarter from the equator to the pole, and took one ten-millionth part of it for a *metre*. This method gave for the French metre 39·37079 English or United States inches, equal 3·2809 feet, nearly.

§ 77. CLOTH MEASURE.

$2\frac{1}{4}$	inches (*in.*)	make	1 nail,	*na.*
4	nails	"	1 quarter of a yard,	*qr.*
3	quarters	"	1 Ell Flemish,	*E. Fl.*
4	quarters	"	1 yard,	*yd.*
4	*qr.* $1\frac{1}{5}$ *in.*	"	1 Ell Scotch,	*E. S.*
5	quarters	"	1 Ell English,	*E. E.*
6	quarters	"	1 Ell French,	*E. Fr.*

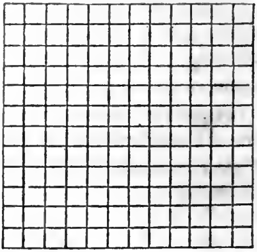
§ 78. Square Measure.

144	square inches (*sq. in.*)	make	1 square foot,	*sq. ft.*
9	square feet	"	1 square yard,	*sq. yd.*
$30\frac{1}{4}$	square yards	"	1 square rod or pole,	*P.*
40	square rods	"	1 rood,	*R.*
4	roods	"	1 acre,	*A.*
640	acres	"	1 square mile,	*M.*

Note 1.—This measure is used for measuring surfaces; such as boards, glass, pavements, plastering, flooring, painting, and any kind of materiel or work where *length* and *breadth* only are concerned. It is always employed for measuring land, and for this reason is sometimes called *Land Measure.*

1 foot=12 inches.

1 foot=12 inches.



A square is a figure having four equal sides, and all its angles *right angles;* that is, the sides are perpendicular to each other.

If the length of one of the sides is one inch, it is called a *square inch;* if the length of one of the sides is one foot, or 12 inches, it is called a *square foot,* which by the figure we see is composed of $12\times12=144$ *square inches.*

In a similar manner, if we had a square, each of whose sides was 3 feet, it would contain $3\times3=9$ *sq. feet,* or one *yard.*

Note 2.—The acre is always applied to surface or area. There is no such thing as an acre long. It is of such a magnitude as not to admit of being accurately given in the form of a square. The same is true of the rood.

Note 3.—In measuring land, Gunter's chain is used; its length is 4 rods, or 66 feet. It is divided into 100 links.

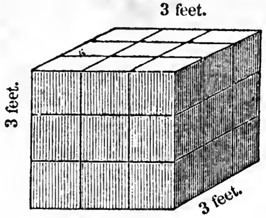
$7\frac{92}{100}$	inches	make	1 link,	*l.*
100	links, or 4 rods, or 66 feet,	"	1 chain,	*c.*
80	chains	"	1 mile,	*mi.*
10000	square links	"	1 square chain.	*sq. c.*
10	square chains	"	1 acre,	*A.*

§ 79. Solid or Cubic Measure.

1728	solid inches (*S. in.*)	make	1 solid foot,	*S. ft.*
27	solid feet	"	1 solid yard,	*S. yd.*
40	feet of round timber or } 50 feet of hewn timber	"	1 ton,	*Ton.*
128	solid feet	"	1 cord of wood,	*C.*

Note 1.—This measure is used in measuring solid bodies or spaces; that is, things having *length*, *breadth*, and *height* or *thickness*: such as earth, stone, timber, bales of goods, the capacity of rooms, &c.

Note 2.—A cube is a solid bounded by six equal squares, resembling a common tea-chest.



If the sides of a cube are each one inch long, it is called a *cubic inch*. If each side is one foot long, it is called a *cubic foot*, &c.

The figure represents a cube, each side of which is 3 feet or one yard in length; consequently, it represents one *solid* or *cubic* yard.

The top, which is equal to the base, contains $3\times3=9$ square feet; hence, if this was only one foot in height, it would contain 9 cubic feet; but as it is 3 feet in height, it must contain 3 times $9=27$ cubic feet. Hence, one cubic yard is equivalent to $3\times3\times3=27$ cubic feet.

In the same way one cubic foot is equivalent to $12\times12\times12=1728$ cubic inches.

Note 3.—A ton of round timber is such a quantity of timber in its natural state as, when hewed, will make 40 cubic feet.

Note 4.—A pile of wood 4 *ft.* wide, 4 *ft.* high, and 8 *ft.* long, makes a cord. One foot in length of such a pile is sometimes called a *cord foot.* It contains 16 solid feet: consequently, 8 cord feet make 1 cord.

§ 80. Wine Measure.

4	gills (*gi.*)	make	1 pint,	*pt.*
2	pints	"	1 quart,	*qt.*
4	quarts	"	1 gallon,	*gal.*
31½	gallons	"	1 barrel,	*bar*
63	gallons	"	1 hogshead,	*hhd.*
2	hogsheads	"	1 pipe,	*pi.*
2	pipes	"	1 tun,	*tun.*

Note.—The wine gallon contains 231 cubic inches. The Imperial gallon of Great Britain contains 277·274 cubic inches.

§ 81. Ale or Beer Measure.

2	pints (*pt.*)	make	1 quart,	*qt.*
4	quarts	"	1 gallon,	*gal.*
36	gallons	"	1 barrel,	*bar.*
1½	barrels	"	1 hogshead,	*hhd.*

Note.—The beer gallon contains 282 cubic inches. Milk is, or should be, measured by this measure.

§ 82. Dry Measure.

2	pints (*pt.*)	make	1 quart,	*qt.*
8	quarts	"	1 peck,	*pk.*
4	pecks	"	1 bushel,	*bu.*
32	bushels	"	1 chaldron,	*ch.*

Note 1.—By this are measured all dry wares; as grain, seeds, roots, fruits, salt, coal, sand, oysters, &c.

Note 2.—The standard unit of dry measure adopted by the United States is the Winchester bushel. This is made, by English statute, to contain $2150\frac{42}{100}$ cubic inches. It is a measure of cylindric form, 8 inches deep and 18½ inches in diameter.

§ 83. Time.

60	seconds (*sec.*)	make	1 minute,	*min.*
60	minutes	"	1 hour,	*hr.*
24	hours	"	1 day,	*da.*
7	days	"	1 week,	*wk.*
4	weeks	"	1 month,	*mo.*
13 / 365	*mo.*, 1 *da.*, 6 *hr.*, or / *da.*, 6 *hr.*	"	1 Julian year,	*yr.*

Note 1.—The true length of the solar year is 365·242217 days, or about 365 *da.* 5 *hr.* 48 *min.* 47½ *sec.*

Note 2.—As the year exceeds 365 days by very nearly 6 hours, at the end of every 4 years an additional day is given to the month of February. The years containing this extra day are called Bissextile or Leap Years. But since the excess of which we speak is not quite 6 hours, the addition of the extra day will in time give too many days to the calendar; therefore every closing year of a century (called a centennial year) that is not divisible by 400 is regarded as a common year.

Every year (except a centennial) that may be divided by 4 is a Leap year, and has 366 days. Thus, 1840, 1844, 1848, were leap years, as 1852, 1856, &c., will be; 1800 not being divisible by 400 was a common year, but the year 2000 will be a leap year.

Note 3.—In business transactions 30 days are considered a month, and 12 months a year.

The following lines will help the pupil to remember the number of days in each month:

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone:
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

Note 4.—It is very desirable to be able readily to determine the number of days from any particular date to any other date. For this purpose, we will give the following

TABLE,

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Ap'l	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
JANUARY	365	31	59	90	120	151	181	212	243	273	304	334
FEBRUARY...	334	365	28	59	89	120	150	181	212	242	273	303
MARCH......	306	337	365	31	61	92	122	153	184	214	245	275
APRIL.......	275	306	334	365	30	61	91	122	153	183	214	244
MAY	245	276	304	335	365	31	61	92	123	153	184	214
JUNE	214	245	273	304	334	365	30	61	92	122	153	183
JULY	184	215	243	274	304	335	365	31	62	92	123	153
AUGUST	153	184	212	243	273	304	334	365	31	61	92	122
SEPTEMBER..	122	153	181	212	242	273	303	334	365	30	61	91
OCTOBER....	92	123	151	182	212	243	273	304	335	365	31	61
NOVEMBER ..	61	92	120	151	181	212	242	273	304	334	365	30
DECEMBER ..	31	62	90	121	151	182	212	243	274	304	335	365

As an example, suppose we wish the number of days from November 6th to the 15th of next April. We find November in the left-hand vertical column, and April at the top line of the table, and at the intersection we find 151 days. So that from November 6th to April 6th is 151 days; consequently, adding 9, we find 160 for the number of days between November 6th and April 15th.

This table is constructed on the supposition of 28 days to February. When there are 29 days in February the proper allowance must be made.

§ 84. CIRCULAR MEASURE.

60 seconds ($''$)	make	1 minute,	$'$
60 minutes	"	1 degree,	$^\circ$
30 degrees	"	1 sign,	*s.*
12 signs or 360°	"	1 circle,	*cr.*

NOTE 1.—By this measure latitude and longitude, and the motions of the heavenly bodies, which appear to move in circles, are estimated.

NOTE 2.—Every circle, whether great or small, is supposed to be divided into 360 equal parts, called *degrees*.

NOTE 3.—The sun appears to pass completely around the earth in 24 hours; that is, it appears to move westward over an entire circle or 360° of longitude in 24 hours. Consequently, in one hour it will move over $\frac{1}{24}$ of 360°=15° of longitude. Hence, if the difference in the longitudes of two places is 15°, it will be noon at the more easterly place, just one hour before it is noon at the other place. And in all cases, the difference in time of any two places will be at the rate of one hour for every 15° of longitude between the two places. As an example, suppose the city of Washington to be 77° west of Greenwich: it is required to find what time it is at Washington, when it is noon at Greenwich.

Dividing 77° by 15°, we have $5\frac{2}{15}$ for the number of hours difference in time; that is, 5h. 8m. And as the apparent motion of the sun is westward, it must be earlier at Washington than at Greenwich. Therefore, when it is noon at Greenwich, it is 5h. 8m. before noon at Washington; that is, 6h. 52m. A. M.

§ 85. Measures, &c., not included in the foregoing tables.

12	individual things	make	1 dozen.
12	dozens or 144	"	1 gross.
12	gross	"	1 great gross.
20	individual things	"	1 score.
112	pounds	"	1 quintal of fish.
196	"	"	1 barrel of flour.
200	"	"	1 barrel of pork or beef.
24	sheets of paper	"	1 quire.
20	quires	"	1 ream.

A sheet folded in	2	leaves	makes	a folio.
" " " "	4	"	"	a quarto or 4to.
" " " "	8	"	"	an octavo or 8vo.
" " " "	12	"	"	a duodecimo or 12mo.
" " " "	18	"	"	an 18mo.

EXERCISES ON THE TABLES.

1–10. How many farthings in 2 pence? in 4? in 5? in 8? 10? 15? 20? 25? 50? 100?

11–20. How many farthings in 1 shilling? in 2 shillings? in 3? 5? 8? 15? 20? 25? 50? 100?

21–30. How many pence in 2 shillings? in 3? 5? 7? 9? 15? 20? 25? 50? 100?

31–40. How many shillings in 2 pounds? in 3? 5? 7? 9? 15? 21? 25? 50? 100?

41–42. How many pence in 3*qr.*+2*qr.*+6*qr.*+7*qr.*+8*qr.*? in £4+7*s.*+6*d.*+3*qr.*?

43. In a sovereign how many shillings? how many pence? how many farthings?

44. How many farthings in $^2/_6 + ^4/_9$ + 3*s.* + 6*qr.*?

45–60. How many grains in 2*pwt.*? in 5? in 7? 15? 25? 50? How many in 1*oz.*? 2*oz.*? in 5? 7? 15? 25? 50? in 1*lb.*? 2*lbs.*? 5*lbs.*?

61–74. How many pennyweights in 2*oz.*? in 5? 7? 9? 15? 20? 25? 50? in 1*lb.*? in 3*lbs.*? in 5? 9? 15? 25?

75–81. How many ounces in 2*lbs.*? in 4*lbs.*? in 9? in 15? in 25? in 50? in 100?

82–101. How many grains in 3℈? in 5? 7? 12? 18? in 1ʒ? in 5? 7? 12? 18? in 1℥? in 5? 7? 12? 18? in 1℔? in 5? 7? 12? 18?

102–110. How many scruples in 2ʒ? in 5? in 9? in 1℥? in 5? in 9? in 1℔? in 5? in 9?

111–118. How many drams in 2℥? in 5? in 7? in 12? in 6℔? in 9? in 15? in 16?

119–128. How many drams avoirdupois in 2*oz.*? in 7? in 9? in 1*lb.*? in 2? in 9? in 20? in 1*qr.*? in 1*cwt.*? in 1*T.*?

129–136. How many ounces avoirdupois in 2*lbs.*? in 7? in 9? in 15? in 25? in 100? in 1*cwt.*? in 1*T.*?

137–141. How many pounds in 1*T.*? in 3? n 20? in 50? in 100?

142–150. How many inches in 2*ft.*? in 7*ft.*? in 20? in 1*yd.*? in 5½*yds.*? in 1 rod? in 1 furlong? in 1 mile? in 1 degree?

151–153. How many feet in 100 *yds.*? in 1 mile? in 100?

154–158. How many inches in 18 hands? in 7 spans? in 20 cubits? in 1 fathom? in 40 paces?

159–164. How many inches in 16 nails? in 1 *qr.*? in 1 *E. Fl.*? 1 *E. S.*? 1 *E. E.*? 1 *E. Fr.*?

165–167. How many *sq. in.* in 42 *sq. ft.*? in 1 rood? in 1 acre?

168–169. How many *sq. yds.* in an acre? in 1 *sq. mi.*?

170–172. How many *sq. in.* in 1 *sq. yd.*? in 1 *T.* hewn timber? in 1 *C.*?

173–177. In 6 tuns, wine measure, how many *hhd.*? how many *bar.*? how many *gal.*? how many *qt.*? how many *pt.*?

178–180. How many *pts.* in 1 *hhd.*? how many *qts.*? how many *gi.*?

181–182. How many more pints in 1 *bar.*, beer measure, than in 1 *bar.*, wine measure? in 1 hogshead wine than in 1 *hhd.* beer?

183–186. How many quarts in 1 *bu.*? in 1 *ch.*? in 15 *ch.*? in 25?

187–189. How many pecks in 50 *bu.*? in 50 *ch.*? in 100 *ch.*?

190–197. How many seconds in 5 *min.*? in 15 *min.*? in 30 *min.*? in 45 *min.*? in 1 *hr.*? 12 *hr.*? 24 *hr.*? in 1 *wk.*?

198–200. How many hours in 1 *wk.*? 52 *wks.*? 365 days?

201–207. How many seconds, Circular Measure, in 2′? in 8′? in 15′? in 2°? in 8°? in 15°? in 1 cr.?

208–211. How many minutes in 2°? in 12°? in 25°? in 1 cr.?

212–214. How many units in 6 *doz.*? in 6 gross? in 6 great gross?

215–218. How many units in 5 score? in 25 score? in 100 score?

219–221. How many pounds in 5 *bar.* flour? in 25? in 100?

222–225. How many sheets paper in 5 quires? in 1 ream? in 7 reams? in 8 reams?

226–233. How many pence in 100 farthings? in 96*far.*? in 48*far.*? in 112*far.*? in 360*far.*? in 24*far.*? in 36*far.*? in 72*far.*?

234–239. How many shillings in 144*d.*? in 480*d.*? in 96*d.*? in 140*d.*+16*far.*? in 100*d.*+80*far.*? in 94*d.*+8*far.*?

240–246. How many pounds sterling in 360*s.*? in 240*s.*? in 60*s.*? in 400*s.*? in 640*s.*? in 58*s.*+144*d.*? in 85*s.*+180*d.*?

247–250. How many sovereigns in 100*s.*? in 750*s.*? in 372*s.*? in 200*s.*?

251–262. How many pennyweights of gold in 96*gr.*? in 300*gr.*? in 100*gr.*? in 1000*gr.*? in 1728*gr.*? How many ounces of gold in 100*pwt.*? in 1000*pwt.*? in 210*pwt.*? in 400*pwt.*? How many pounds of gold in 365*oz.*? in 720*oz.*? in 170*oz.*?

263–265. How many drams in 21℈? in 45℈? in 100℈?

266–268. How many ounces in 48ʒ? in 36ʒ? in 600ʒ?

269–272. How many pounds in 144℥? in 365℥? in 100℥? in 460℥?

273–276. How many tons in 400*cwt.*? in 1000*cwt.*? in 840*cwt.*? in 780*cwt.*?

277–281. How many pounds avoirdupois in 1600*oz.*? in 360*oz.*? in 500*oz.*? in 365*oz.*? in 711*oz.*?

282–286. How many yards in 132*ft.*? in 600*ft.*? in 927*ft.*? How many feet in 100*in.*? in 750*in.*?

287–289. In 600 quarters of cloth, how many yards? how many English Ells? how many French Ells?

290–292. In one square mile of land, how many acres? how many roods? how many *sq.* rods?

293–295. How many chains in 500 links? how many feet? In 660 feet how many chains?

296. How many cords of wood in 1280 cubic feet?

297. In a pile of wood 4 feet wide, 4 feet high, and 100 feet long, how many cords?

298. How many bushels in 100*pk.*?

299–300. How many hours in 300*min.*? in 1000*min.*?

REDUCTION OF DENOMINATE QUANTITIES.*

§ 86. When the quantity is to be reduced from a higher to a lower denomination, the process is called Reduction *Descending;* when from a lower to a higher, Reduction *Ascending.*

Reduction Descending.

CASE I.

Reduce £7 5*s.* 10*d.* 3*far.* to farthings.

£7 5*s.* 10*d.* 3*far.*
20
140*s.*
5*s.*
145*s.*
12
1740*d.*
10*d.*
1750*d.*
4
7000*far.*
3*far.*
7003*far.*

There are 20*s.* in a pound; therefore 7×20 will give the shillings in £7. But the 5*s.* of the given quantity are also to be reduced. Adding these to the shillings already found, we have 145*s.* in £7 5*s.* Multiplying 145, the number of shillings, by 12, because there are 12*d.* in a shilling, we obtain the number of pence in 145*s.*; adding the 10*d.*, we have 1750*d.* in £7 5*s.* 10*d.* Multiplying 1750 by 4,

* The term *quantity* is used here in preference to the term number, because it may include denominate fractions as well as denominate integers; and because numbers of different denominations often form but *one* quantity.

because there are 4 farthings in a penny, and adding the 3*far.* to the product, we have 7003*far.* in £7 5*s.* 10*d.* 3*far.*

CASE II.

Reduce £$\frac{1}{360}$ to its value in farthings, or in the fraction of a farthing.

We proceed as in Case I. Multiplying £$\frac{1}{360}$ by 20, we obtain for a product the value of £$\frac{1}{360}$ in the fraction of a shilling; that is, £$\frac{1}{360}=\frac{20}{360}s.=\frac{1}{18}s.$ Multiplying this by 12, we obtain its equivalent value in the fraction of a penny; that is, $\frac{1}{18}s.=\frac{12}{18}d.=\frac{2}{3}d.$ Multiplying this by 4, we obtain its equivalent value in the fraction of a farthing; that is, $\frac{2}{3}d.=\frac{8}{3}far.$, or $2\frac{2}{3}far.$

By cancelation the process becomes

$$£\tfrac{1}{360}=\frac{1}{\overset{}{\underset{\underset{3}{18}}{360}}}\times\frac{20}{1}\times\frac{\overset{2}{12}}{1}\times\frac{4}{1}=\frac{8}{3}=2\tfrac{2}{3}far.$$

CASE III.

Reduce $\frac{3}{8}yd.$, cloth measure, to its equivalent value in lower denominations.

We proceed as before, multiplying $\frac{3}{8}$, the number of yards, by 4, because there are 4 quarters in a yard, we obtain the number of quarters equivalent to $\frac{3}{8}$ of a yard; that is, $\frac{3}{8}yd.=\frac{12}{8}qr.=1\frac{1}{2}qr.$ Reducing the fractional part only of this result, we have $\frac{1}{2}qr.=\frac{1}{2}\times4na.=\frac{4}{2}na.=2na.$ Collecting the integers we have 1*qr.* 2*na.* for the answer.

CASE IV.

Reduce £0·778125 to its value in lower denominations.

We first multiply the given decimal by 20, as in cases 1 and 2, and obtain a product in shillings, and the decimal of a shilling. Reducing the fractional part of this product still further, we obtain its equivalent value in pence, and the decimal of a penny. Multiplying the decimal part of this second result by 4, to reduce it to farthings, we obtain 3*far.* for a final product. Collecting the integers, we have 15*s.* 6*d.* 3*far.* for the answer.

```
£0·778125
       20
15·562500s.
       12
 6·750000d.
        4
 3·000000far.
```

From the foregoing examples, we may deduce the following rule for the reduction of higher to lower denominations.

I. *Multiply the single quantity of a higher denomination, or the highest term of the compound quantity, by the number of the next lower denomination required to make one of that higher; the product will be in the lower denomination.*

II. *To this product add the term (if there be any) in the given quantity, which is of the same denomination as the product, and multiply as before, and so on.*

III. *The final product, or (if the single quantity be a* FRACTION) *the integers, if any, of the successive products, taken collectively, will give the result required.*

§ 87. Reduction Ascending.

CASE I.

Reduce 7003 farthings, to pounds, shillings, pence, and farthings.

Obviously the process of Reduction Ascending is the reverse of Reduction Descending.

In 7003*far.*, there can, of course, be but ¼ as many pence, since 4 farthings make 1 penny. Dividing, then, by 4, we obtain 1750*d.* for a quotient, and 3*far.* remainder. Next we divide the number of pence by 12, since there are 12 pence in a shilling, and obtain 145*s.* for a quotient, and 10*d.* re-

```
 4 ) 7003far.
12 ) 1750d.    3far. rem.
2|0 ) 14|5s.   10d. rem.
     £7 5s. 10d. 3far.
```

mainder. Lastly, dividing the number of shillings by 20, we obtain £7 for a quotient, and 5*s.* remainder. We have, then, for a total result, £7 5*s.* 10*d.* 3*far.*

CASE II.

Reduce $\frac{3}{5}$ *far.* to the fraction of a pound sterling.

We proceed as in Case I. Dividing $\frac{3}{5}$ *far.* by 4, we obtain for a quotient the value of $\frac{3}{5}$ *far.* in the fraction of a penny; that is, $\frac{3}{20}d.$ Dividing this by 12, we obtain for a quotient the value of $\frac{3}{20}d.$ in the fraction of a shilling; that is, $\frac{3}{240}s.$ Dividing this by 20, we obtain for a quotient the value of $\frac{3}{240}s.$ in the fraction of a £; that is, $\frac{3}{240}s.=£\frac{3}{4800}=£\frac{1}{1600}$.

By cancelation, the process becomes,

$$\tfrac{3}{5}\,far.=\frac{\cancel{3}}{5}\times\tfrac{1}{4}\times\frac{1}{\underset{4}{\cancel{12}}}\times\tfrac{1}{20}\text{ of a }£=£\tfrac{1}{1600}.$$

CASE III.

Reduce 2*da.* 10*hr.* 5*min.* to the fraction of a week.

As there are 60 minutes in 1 hour, to reduce any number of minutes to hours, we divide by 60. Then $5min.=\frac{5}{60}$, or $\frac{1}{12}hr.$ Adding to this quotient the 10*hr.* of the quantity to be reduced, we have $10\frac{1}{12}hr.$ Dividing this by 24, to reduce it to days, we have $10\frac{1}{12}ho.=\frac{121}{12}hr.$, and $\frac{121}{12}\div24=\frac{121}{288}$; that is, $10\frac{1}{12}hr.=\frac{121}{288}da.$ Again, dividing this quotient with the 2*da.* added in, by 7 to reduce it to weeks. $2\frac{121}{288}da.=\frac{697}{288}da.$, and $\frac{697}{288}\div7=\frac{697}{2016}$; that is, $2\frac{121}{288}da.=\frac{697}{2016}wk.$, the required fraction.

CASE IV.

Reduce 15*s.* 6*d.* 3*far.* to the decimal of a pound sterling.

This example is similar to the preceding. The only difference is, that the answer here is required in a decimal instead of a common fraction.

Dividing 3, the number of farthings, by 4, and expressing the quotient decimally, we have $\frac{3}{4}=0{\cdot}75$ that is, $3far.=0{\cdot}75d.$ Then 6*d.*

3*far.*=6·75*d.* Dividing by 12, we have $\frac{6·75}{12}$=0·5625; that is, 6·75*d.*=0·5625*s.* Adding to this quotient the 15*s.*, we have 15*s.* 6*d.* 3*far.* = 15·5625*s.* Dividing by 20, we find $\frac{15·5625}{20}$=0·778125; that is, 15·5625*s.*=£0·778125, for the required decimal.

The work may be conveniently arranged as follows:

4	3*far.*
12	6·75*d.*
2|0	15·5625*s.*
	0·778125 of a £.

We place the different denominations in a column, with the smallest denomination at the top; we then suppose naughts annexed to the 3 farthings, and divide by 4, and the quotient, which must be a decimal, we place at the right of the 6*d.*; we next divide 6·75*d.* with naughts annexed, by 12, and the quotient, which is also a decimal, we place at the right of the 15*s.*; finally, we divide the 15·5625*s.* by 20. In dividing by 20, we cut off the naught, and divide by 2, observing to remove the decimal point one place to the left.

From the foregoing examples we may deduce the following rule for the reduction of lower to higher denominations.

I. *Divide the single quantity of a lower denomination, or the lowest term of the compound quantity, by the number which is required of such denomination to make one of the next higher; the quotient will be in that higher denomination.*

II. *To this quotient add the term (if there be any) in the given quantity, which is of the same denomination as the quotient; and divide as before, and so on.*

III. *The final quotient, or (if the single quantity be a whole number) the final quotient with the intermediate remainders, will give the answer required.*

PROMISCUOUS EXERCISES IN REDUCTION OF DENOMINATE QUANTITIES, INCLUDING APPLICATIONS OF THE TABLES.

301. In £47 5*s.* 2*d.* 1*far.* how many farthings?

302. In 118567 farthings, how many pounds, shillings, and pence?

303. Reduce £75 to shillings.

304. Reduce 19*s.* 6*d.* to pence.

305. Reduce 25*s.* 3*d.* 2*far.* to farthings.

306. In 48926 grains, Troy Weight, how many pounds, ounces, pennyweights, and grains?

307. In 3605 pennyweights, how many pounds, ounces, and pennyweights?

308. In 1000 ounces, Troy Weight, how many pounds and ounces?

309. In 4*lb.* 6*oz.* 13*pwt.* 5*gr.* how many grains?

310. In 100*lb.* 1*gr.* how many grains?

311. In 4℔ 5℥ 1℈, how many drams?

312. In 1000 grains, Apothecaries' Weight, how many ounces, drams, scruples, and grains?

313. In 11521 grains, Apothecaries' Weight, how many pounds?

314. In 873450 drams, Avoirdupois Weight, how many tons?

315. Reduce 5*cwt.* 21*lb.* 4*oz.* to ounces.

316. Reduce 1*T.* 1*cwt.* 1*dr.* to drams.

317. Reduce 856702 drams to tons.

318. In 4355 inches, how many yards?

319. In 248 miles, how many inches?

320. How many inches in 360 degrees, of $69\frac{1}{6}$ miles to each degree, which is the circumference of the earth, nearly?

321. Reduce 12 Ells French to nails.

322. Reduce 11 Ells English, 3 quarters, to quarters.

323. Reduce 10 Ells Flemish, 3 quarters, 1 nail, to nails.

324. Reduce 4 yards to quarters.

325. In 1000 nails, how many yards?

326. How many inches in 6 yards, 3 quarters?

327. How many square inches in 10 square feet?

328. In 3 square miles, how many square rods or poles?

329. In 3 acres, 27 rods, how many square feet?

330. In 26025 square feet, how many square roods?

331. In 70000 square links, how many square chains?

332. How many square links in 5 acres?

333. In 17 cords of wood, how many cubic feet?

334. In 17 tons of round timber, how many cubic inches?

335. Reduce 17900345 cubic inches to tons of hewn timber.

336. In 1000 cord feet of wood, how many cords?

337. In 19 cubic feet, how many cubic inches?

338. In 16 hogsheads of wine, how many gills?

339. In 10000 gills of wine, how many barrels?

340. Reduce 2 pipes, 7 barrels, 3 quarts of wine, to pints.

341. Reduce 31752 gills of wine to barrels.

342. Reduce 201600 gills to tuns of wine.

343. Reduce 11 hogsheads of beer to pints.

344. In 100000 pints of beer, how many hogsheads?

345. In 10 hogsheads, 1 quart, 1 pint of beer, how many pints?

346. In 36 bushels, how many pints?

347. In 25 chaldrons, 29 bushels, how many quarts?

348. In 10000 pints, how many chaldrons?

349. In 1597 quarts, how many bushels?

350. In 30 days, how many seconds?

351. In 19 years, of $365\frac{1}{4}$ days each, how many hours?

352. In 25 years 6 days, how many seconds?

353. How many days from the birth of Christ to Christmas, 1843, allowing the years to consist of 365 days 6 hours?

354. A person was born May 3, 1795. How many days old was he May 3, 1821, paying particular attention to the order of leap year?

355. Suppose a person was born February 29, 1796;

how many birthdays will he have seen on February 29, 1844, not counting the day on which he was born?*

356. In 3 signs 18 degrees, how many seconds?

357. In 6 signs 9 degrees, how many degrees?

358. In 1000′ how many degrees?

359. In 10000″ how many degrees?

360. Reduce 45° 45′ 35″ to seconds.

361. In 1000 things, how many dozen?

362. How many buttons in $6\frac{1}{3}$ dozen?

363. In 80000 tacks, how many gross?

364. In three score and ten years, how many years?

365. In 15 quires of paper, how many sheets?

366. In a ream of paper, how many sheets?

367. Reduce $\frac{144}{360}$ of a yard to a fraction of a mile.

368. Reduce $\frac{27}{84}$ of a gill to the fraction of a gallon.

369. Reduce $\frac{330}{495}$ of a pound to the fraction of a ton.

370. Reduce $\frac{1}{3}$ of a mile to the fraction of a foot.

371. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{2}{7}$ of a yard to the fraction of a mile.

372. Reduce $\frac{1}{6}$ of $\frac{1}{8}$ of $\frac{24}{28}$ of a gallon to the fraction of a gill.

373. Reduce $\frac{2}{3}$ of $\frac{4}{9}$ of a hogshead of wine to the fraction of a gill.

374. Reduce $\frac{1}{2}$ of $\frac{3}{7}$ of $4\frac{1}{2}$ yards to the fraction of an inch.

375. Reduce $\frac{1}{7}$ of $\frac{3}{43}$ of a farthing to the fraction of a shilling.

376. Reduce $\frac{7}{59}$ of an ounce to the fraction of a pound avoirdupois.

377. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ of 1 rod to the fraction of an inch, of a foot, and of a yard.

* It must be recollected that the year 1800 was a common year, having no 29th of February.

378. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of 1 hour to the fraction of a month of 30 days, and to the fraction of a year of 365 days.

379. Reduce $\frac{3}{7}$ of 1 yard to lower denominations.

380. What is the value of $\frac{1}{2}$ of $\frac{3}{8}$ of 1 mile ?

381. Reduce $\frac{3}{7}$ of $\frac{5}{6}$ of 1 cwt. to lower denominations.

382. What is the value of $\frac{1}{6}$ of 14 miles, 6 furlongs ?

383. What is the value of $\frac{1}{3}$ of $\frac{3}{5}$ of 2 days of 24 hours each ?

384. What is the value of $\frac{1}{2}$ of $\frac{7}{8}$ of $\frac{3}{14}$ of an hour ?

385. Reduce $\frac{109}{450}$ of a solar day to lower denominations.

386. What is the value of $\frac{144}{175}$ of a pound avoirdupois ?

387. What is the value of $\frac{1}{10}$ of a bushel ?

388. What is the value of $\frac{6}{73}$ of a year of 365 days ?

389. What is the value of $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$ of an acre ?

390. Reduce £8 5*s.* 2*d.* 1*qr.* to the decimal of a £.

391. Reduce 3*qr.* 2*na.* to the decimal of a yard.

392. Reduce 1*ft.* 4*in.* to the decimal of a yard.

393. Reduce 3*lb.* 4*oz.* 8*pwt.* 1*gr.* Troy to the decimal of a pound.

394. Reduce 83*fur.* 7*rd.* 4*yd.* 2*ft.* to the decimal of a mile.

395. Reduce 3*h.* 30*min.* 10*sec.* to the decimal of a day.

396. Reduce £3 5*s.* 0*d.* 2*far.* to the value of a £.

397. Reduce 28 gallons of wine to the decimal of a hogshead.

398. Reduce 4*s.* 6$\frac{1}{2}$*d.* to the decimal of a £.

399. Reduce 18*s.* 3$\frac{3}{4}$*d.* to the decimal of a £.

400. Reduce 3 pecks, 5 quarts, and 1 pint to the decimal of a bushel.

401. Reduce 11*hr.* 16*m.* 15*sec.* to the decimal of a day.

402. Reduce 20 rods, 4 yards, 2 feet and 6 inches to the decimal of a furlong.

403. Reduce 42*min.* 36*sec.* to the decimal of an hour.

404. Reduce 30 days, 3 hours, 27 minutes, 30 seconds, to the decimal of a year, of 365·24224 days.

405. Reduce 5*hr.* 48*min.* 49·536*sec.* to the decimal of a day.

406. Reduce 0·9075*A.* to its value in lower denominations.

407. What is the value of £0·125 ?

408. What is the value of £0·66$\frac{2}{3}$?

409. Reduce 0·375 of a hogshead of wine to its value in lower denominations.

410. Reduce 0·121212 of a year of 365 days to its value in lower denominate numbers.

411. What is the value of 0·3355 of a pound avoirdupois ?

412. What is the value of 0·3322 of a ton ?

413. What is the value of 0·2525 of a mile ?

414. What is the value of 0·345 of a £ ?

415. What is the value of 0·121212 of a day ?

416. What is the value of 0·3456 of a £ ?

417. What is the value of 0·9875 of a £ ?

418. What is the value of 0·24224 of a solar day ?

419. What is the value of 0·375 of a great gross ?

420. What is the value of 0·75 of a score ?

421. What is the value of 0·485 of a quintal of fish ?

422. What is the value of 0·3434 of a barrel of flour ?

423. What is the value of 0·7575 of a barrel of pork ?

424. What is the value of 0·985 of a quire of paper ?

425. What is the value of 0·555 of a ream of paper ?

Addition of Denominate Numbers.

§ 88. If we wish to find the sum of £6 5*s.* 3*d.* 1*far.*, £7 1*s.* 10*d.* 2*far.*, £1 13*s.* 5*d.*, £4 18*s.* 0*d.* 2*far.*, we proceed as follows :

Placing the numbers of the same denomination in the same column, we add the column of farthings, which we find to be 5. But we know that 5 farthings are equivalent to 1 penny and 1 farthing; we therefore write down the 1 farthing under the column of farthings, and carry the penny into the next column, whose sum thus becomes 19 pence, which is the same as 1 shilling and 7 pence; we write down the 7 pence under the column of pence, and carry the shilling to the column of shillings, whose sum then becomes 38 shillings, which is the same as 1 pound and 18 shillings; we write down the 18 shillings under the column of shillings, and carry the pound to the column of pounds, whose sum then becomes 19 pounds; and since pounds is the highest denomination, we write down the whole.

£	s.	d.	far.
6	5	3	1
7	1	10	2
1	13	5	0
4	18	0	2
£19	18s.	7d.	1far.

Hence we deduce this general

RULE.

I. *Place the numbers so that those of the same denomination may stand beneath them in the same column.*

II. *Add the numbers in the lowest denomination; divide their sum by the number expressing how many of such denomination are required to make one of the next higher. Write the remainder under the column added, and carry the quotient to the next column; which add as before.*

III. *Proceed thus through all the denominations to the highest, whose sum must be set down entire.*

EXAMPLES.

(426.)

£	s.	d.
7	13	3
3	5	$10\frac{1}{2}$
6	18	7
0	2	$5\frac{3}{4}$
4	0	3
17	15	$4\frac{1}{2}$

(427.)

£	s.	d.
11	0	$5\frac{1}{2}$
2	4	4
0	5	$6\frac{1}{4}$
1	3	4
10	10	10

(428.)

£	s.	d.
5	5	5
8	1	$7\frac{3}{4}$
2	0	$1\frac{1}{2}$
13	0	$11\frac{3}{4}$
6	6	6

(429.)

lb.	oz.	pwt.	gr.
10	10	10	10
0	2	0	23
3	0	17	0
2	2	1	0
1	0	2	20

(430.)

lb.	*oz.*	*pwt.*	*gr.*
6	5	4	1
1	11	19	13
0	3	0	4
8	9	1	2
4	4	19	0

(431.)

lb.	*oz.*	*pwt.*	*gr.*
7	3	0	5
11	2	17	22
40	0	0	20
2	10	15	17
0	6	18	16

(432.)

℔	℥	ʒ	℈	*gr.*
8	10	7	2	19
10	0	6	0	10
0	1	2	1	15
5	1	2	1	15
8	0	5	1	13

(433.)

℔	℥	ʒ	℈
2	11	6	0
10	8	3	1
14	10	2	2
0	6	5	0
7	5	4	1

(434.)

ʒ	℈	*gr.*
1	0	18
2	1	15
3	2	13
4	0	0
6	1	7

(435.)

ton.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
10	18	2	23	15	1
1	15	0	0	14	15
12	0	1	3	0	10
0	13	0	24	1	11
2	2	2	0	7	8

(436.)

cwt.	*qr.*	*lb.*	*oz.*
4	3	20	5
5	0	12	3
1	2	0	8
0	3	24	13
1	2	20	10

(437.)

L.	*mi.*	*fur.*	*rd.*	*yd.*
1	2	6	37	4
6	0	0	30	5
0	1	4	0	3
2	0	1	1	0
3	2	0	25	1

(438.)

rd.	*yd.*	*ft.*	*in.*
10	4	2	8
1	3	0	5
8	2	1	6
1	1	0	4
0	2	1	9

(439.)

yd.	*qr.*	*na.*
15	1	2
13	0	3
20	2	2
0	3	0
8	1	1

(440.)

E. Fl.	*qr*	*na.*
3	2	3
15	1	2
9	2	0
8	0	1
10	0	0

(441.)

E. E.	*qr.*	*na.*
4	2	2
10	1	1
9	2	0
13	0	2
15	1	1

(442.)

Sq. yd.	*Sq. ft.*	*Sq. in.*
100	8	130
50	0	100
10	5	0
0	8	143
13	2	8

(443.)

M.	*A.*	*R.*	*P.*
0	100	1	30
10	600	2	10
8	40	1	12
0	0	3	2
4	4	0	20

(444.)

S. yd.	*S. ft.*	*S. in.*
4	26	1000
1	10	1541
0	20	80
10	17	11
8	25	59

(445.)

C.	*S. ft.*
10	120
8	100
2	80
0	119
12	6

(446.)

C.	c.ft.
3	7
10	4
12	1
8	6
15	3

(447.)

hhd.	gal.	qt.	pt.
4	30	3	1
10	25	0	1
25	0	2	0
0	60	0	1
13	45	3	0

(448.)

tun.	pi.	hhd.	gal.	qt.	pt.	gi.
1	1	1	37	3	1	3
10	0	0	50	0	1	2
11	0	1	13	1	0	1
4	1	0	25	2	0	0
8	0	1	18	0	1	3

(449.)

hhd.	gal.	qt.	pt.
2	50	3	1
10	30	1	0
11	25	0	1
25	1	1	0
6	52	3	1

(450.)

bar.	gal.	qt.
10	30	1
6	20	0
1	5	2
10	0	3
4	25	1

(451.)

ch.	bu.	pk.	qt.	pt.
1	30	3	7	1
0	30	2	3	0
10	19	1	0	1
5	10	2	4	0
4	4	0	5	1

(452.)

bu.	pk.	qt.	pt.
10	1	1	1
2	3	6	0
5	2	3	0
8	0	0	1
15	2	4	0

(453.)

da.	hr.	m.	sec.
15	18	50	49
1	13	59	59
4	23	0	2
10	11	1	4
0	2	10	15

(454.)

wk.	da.	hr.	m.	sec.
1	2	13	40	30
2	6	10	8	3
0	5	22	55	45
2	3	4	1	15
1	2	4	5	0

(455.)

cr.	s.	°	′	″
1	8	25	40	35
0	11	1	2	43
1	0	29	59	0
0	1	10	13	5
0	2	5	4	3

(456.)

s.	°	′
1	25	2
0	18	50
2	5	39
0	4	4
4	15	10

(457.)

°	′	″
13	10	19
1	40	35
2	48	39
0	30	40
10	45	45

Subtraction of Denominate Numbers.

§ 89. Subtract £15 13*s.* 10*d.* from £20 5*s.* 8*d.*

£	s.	d.
20	5	8
15	13	10
4	11	10

We place the numbers of the subtrahend directly under the numbers of the same denomination in the minuend. We cannot subtract 10*d.* from 8*d.*; we therefore increase the 8*d.* by 12*d.* making 20*d.*; then subtracting 10*d.* from the 20*d.* we have the difference 10*d.*, which we write under the column of pence. Having added 12*d.* to the minuend, we must equally increase the subtrahend, which we do by adding 1*s.* (the same as the 12*d.*) to the 13*s.*, making 14*s.* This cannot be subtracted from 5*s.*; we therefore increase the 5*s.* by 20*s.*, making 25*s.* Now, subtracting 14*s.* from 25*s.* we have 11*s.*, which we write under the column of shillings. Before subtracting the pounds, we add £1 to £15 to compensate for the 20*s.* added to the 5*s.*, and then say £16 from £20 leaves £4.

Hence, we have this general

RULE.

I. *Place the less number under the greater, so that the same denominations may stand in the same column.*

II. *Begin at the right, and subtract each number in the lower line from the one directly above it, and set the remainder below.*

III. *If any number in the lower line is greater than the one above it, add so many to the upper number as make one of the next higher denomination; then subtract the lower number from the upper one thus increased, and set down the remainder. Carry* 1, *expressing the increase of the upper line, to the next number in the lower line; after which subtract this number from the one above it, as before; and thus proceed till all the numbers are subtracted.*

PROOF.

If the work be right, the difference added to the subtrahend will equal the minuend, as in simple subtraction.

EXAMPLES.

(458.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
13	18	1	20	0	13
10	0	3	21	12	0

(459.)

A.	*R.*	*P.*
69	3	25
10	0	38

(460.)

℔	℥	ʒ	℈	*gr.*
24	7	2	1	16
16	10	3	2	17

(461.)

L.	*mi.*	*fur.*	*rd.*
16	2	7	39
5	0	7	8

(462.)

E. Fr.	*qr.*	*na.*
10	5	0
5	1	3

(463.)

ch.	*bu.*	*pk.*	*qt.*	*pt.*
30	10	1	1	0
10	8	3	6	1

(464.)

tun.	*pi.*	*hhd.*	*gal.*	*qt.*
10	1	1	50	1
1	0	0	60	3

(465.)

da.	*hr.*	*m.*	*sec.*
100	10	0	1
60	0	40	45

(466.)

yr.	*mo.*	*wk.*	*da.*
17	8	3	1
4	1	2	6

(467.)

m.	*fur.*	*rd.*
60	0	0
40	7	39

(468.)

C.	*S. ft.*
45	126
10	127

(469.)

C.	*Cord ft.*
100	6
80	7

(470.)

£	*s.*	*d.*
50	0	1
30	10	10

(471.)

yr.	*mo.*	*da.*
1838	9	6
1837	10	1

(472.)

yr.	*mo.*	*da.*
1839	2	4
1838	9	6

(473.)

yr.	*mo.*	*da.*
1840	9	15
1837	10	1

(474.)

yr.	*mo.*	*da.*
1840	3	5
1836	4	1

(475.)

yr.	*mo.*	*da.*
1850	9	3
1796	6	7

(476.)

yr.	*mo.*	*da.*
1850	11	2
1776	6	4

EXERCISES IN ADDITION AND SUBTRACTION.

477. Bought 20 yards of broadcloth for £18 5*s.* 3*d.*, 30 pounds of feathers for £8 2*s.* 4*d.*, 100 yards carpeting for £45 17*s.* 8*d.*, 10 pieces of cotton cloth for £8 18*s.* 1*d.*, 50 yards of calico for £2 0*s.* 10*d.* What was the cost of the whole?

478. Bought four hogsheads of sugar, weighing as follows: 1st weighed 8*cwt.* 1*qr.* 23*lb.* 10*oz.*; 2d weighed

9*cwt.* 2*qr.* 0*lb.* 3*oz.*; 3d weighed 10*cwt.* 0*qr.* 0*lb.* 8*oz.*; 4th weighed 8*cwt.* 3*qr.* 23*lb.* How much did the four weigh?

479. A man owns three farms: the first contains 69 acres, 3 roods, 10 rods; the second contains 100 acres, 5 rods; the third contains 150 acres, 2 roods. How many acres are there in all?

480. Suppose a note given August 3d, 1838, to be paid November 10th, 1843. How long was the note on interest, if we count 30 days to the month? and how long if the time is accurately computed?

481. A person buys 15*cwt.* 3*qr.* 20*lb.* of sugar, and sells 10*cwt.* 0*qr.* 11*lb.* How much remains unsold?

482. From a piece of cloth containing 37*yd.* 3*qr.* 2*na.* there has been taken at one time 6*yd.* 1*qr.*, at another time 10*yd.* 3*qr.* 3*na.* How much remains?

483. From a pile of wood containing 100 cords, I sold at one time 10*C.* 100*S. ft.*; at another time 18*C.* 59*S. ft.* How many cords remain unsold?

484. A farmer raises 100*bu.* 3*pk.* 2*qt.* of wheat from one field; 87*bu.* 1*pk.* 1*qt.* 1*pt.* from another field; he sells 53*bu.* to one person, and 37*bu.* 2*pk.* 1*qt.* to another person. How many bushels has he remaining?

485. Bought 5 loads of coal. The first weighed 2056 pounds; the second, 2250; the third, 2240; the fourth, 2310; the fifth, 2330. What was the entire weight? And how many tons of 2000 pounds each?

486. A person engages to build 100 rods and 10 feet of stone fence: at one time he builds 17 rods, 5 feet; at another time 37 rods, 15 feet. How much still remains to be built?

487. How much cloth in 3 pieces, measuring as follows: first piece 37 yards, 3 quarters, 1 nail; second piece 41 yards, 1½ Flemish Ells; third piece 43 yards, 1½ English Ells?

488. Bought 3 loads of wood: the first was 8 feet long, 4 feet wide, and 3 feet high; the second was 7 feet long, 4 feet wide, and 2 feet high; the third was 9 feet long, 3 feet wide, and 3 feet high. How many solid feet in the whole? How many cord feet, and how many cords?

489–491. George Washington was born Feb. 22, 1732; John Adams Oct. 19, 1735. How much earlier was the birth of the former than that of the latter? How long a time from the birth of each to January 1, 1851?

492. William Shakspeare was born April 23, 1564. How long since was that, estimating from January 1, 1851?

493. How long from the birth of Milton, Dec. 9, 1608, to the birth of George Washington?

494. The latitude of New Orleans is 29° 57′ 30″, that of Portland is 43° 36′. What is the difference in latitude of these two places?

495–496. The latitude of New York is 40° 42′ 40″. How far is it north of New Orleans? and how far south of Portland?

497–499. The latitude of Boston is 42° 21′ 23″. How far is it north of New York? how far north of New Orleans? how far south of Portland?

500. Henry Jenkins, of England, died Dec. 8, 1670, at the advanced age of 168 years, 9 months. When was he born?

501. America was discovered by Columbus Oct. 14, 1492. How long was that before the landing of the New England Pilgrims, Dec. 20, 1620?

Multiplication of Denominate Numbers.

§ 90. Multiply £13 5*s*. 10*d*. by 5.

First, we say 5 times 10*d*. is 50*d*., which equals 4*s*. and 2*d*.; we set down the 2*d*. and reserve the 4*s*. We then say 5 times 5*s*. equals

25*s.*, to which, adding the 4*s.* we have 29*s.*, which equals £1 9*s.*; we set down the 9*s.* and reserve the £1. Finally, we say 5 times £13 is £65, to which adding the £1, we have £66; this being the highest denomination, we set it down entire.

£	*s.*	*d.*
13	5	10
		5
66	9	2

Hence this general

RULE.

Commence at the right hand, and multiply the number in each denomination by the multiplier. Divide each product by the number expressing how many of the denomination of such product are required to make one of the next higher denomination. Write the remainder, if any, under the number multiplied, and carry the quotient to the next column.

The entire product of the highest denomination must be set down.

EXAMPLES.

502–507. Multiply £10 10*s.* 10*d.* by 3; by 5; by 6; by 7; by 8; by 9.

508–514. Multiply 8*cwt.* 0*qr.* 2*lb.* 4*oz.* 5*dr.* by 3; 4; 5; 6; 7; 8; 9.

515–523. Multiply 8*cwt.* 6*lb.* 7*dr.* by 3; 4; 5; 6; 7; 8; 9; 11; 12.

524–529. Multiply 8*gal.* 3*qt.* 1*pt.* 3*gi.* by 35; 42; 30; 54; 81; 49.

NOTE.—The multipliers in the preceding example may be regarded as composite numbers.

530. In 3 hogsheads of sugar, each containing 10*cwt.* 3*qr.* 5*lb.*, how many hundred weight?

531. How much cloth will it take for 7 suits of clothes, if each suit require 7*yd.* 3*qr.* 1*na.*?

532. How much wood can a horse draw in 13 loads, if he draw 1*C.* 19*S.ft.* at each load?

533. How long will it take a man to saw 6 cords of wood, if he employ 7*hr.* 30*min.* 45*sec.* to saw one cord, allowing 10 working hours for each day?

534. The circumference of a wheel is 15 feet 2 inches. What distance will this wheel measure on the ground, if it is rolled over 365 times?

535. Allowing the year to consist accurately of 365 days, 5 hours, 48 minutes, 49½ seconds, what will be the true length of 1843 years?

536. What will 35*cwt.* of cheese cost, at 15*s.* 6*d.* per hundred weight?

537. How much brandy in 84*pi.*, each containing 128*gal.* 2*qt.* 1*pt.* 3*gi.*?

538. In 21 loads of wood, each 1*C.* 1*c.ft.*, how many cords?

539. Suppose the piston-rod of a steam-engine to move 3*ft.* 4⅓*in.* at each stroke. Through what distance will it move in making 1000 strokes?

540. Bought as follows:

lb.			*s.*	*d.*		
18	of green tea,	at	12	3	per	pound.
12	of raisins,	"	1	2	"	"
27	of loaf sugar,	"	1	4	"	"
15	of English currants,	"	2	3	"	"
14	of citron,	"	3	6	"	"

What is the amount of the whole purchase?

541. What is the amount of the following bill of goods?

					£	*s.*	*d.*		
15	yards	of	broadcloth,	at	1	3	6	per	yard.
12	"	"	silk,	"		18	3	"	"
20	"	"	calico,	"		1	9	"	"
24	"	"	sheeting,	"		1	3	"	"
22	"	"	muslin,	"		3	4	"	"

DIVISION OF DENOMINATE NUMBERS.

§ 91. Divide £100 10*s.* 3*d.* equally among 17 men.

First, we say 17 in £100, is contained 5 times and £15 remaining; and since these £15, as well as the 10*s.*, are yet to be divided among the 17 men, we reduce the pounds to shillings, and add the 10*s.*, making 310*s.*; we find 17 to be contained 18 times in 310*s.* with 4*s.* remainder. We reduce the 4*s.* to pence, and add the 3*d.*, making 51*d.*, which divided among the 17 men, gives 3*d.* each.

```
17 ) £100 10s. 3d. ( £5
      85
      15
       20
17 ) 310 ( 18s.
     17
     140
     136
       4
      12
17 )  51 ( 3d.
      51
```

Collecting, we have £5 18*s.* 3*d.*

NOTE.—We do not divide 100 pounds by 17 men, which is impossible; but we separate £100 into 17 equal parts. Each part is expressed by the quotient, and contains £5, § 71.

Or, adhering to the general definition of Division, we suppose a pound set apart for each man, and then find how many times £17, the number thus set apart, is contained in £100; the quotient will be an abstract number. The answer will, of course, be as many pounds to each man as there are parcels of £17 in £100; that is, as there are units in the quotient.

Had the divisor been a single digit, the work might have been performed by Short Division.

We therefore have this general

RULE.

Place the divisor on the left of the dividend, as in Simple Division. Divide first the number of the highest denomination by the divisor. Reduce the remainder, if any, to the next lower denomination (adding the number in the dividend of that denomination), and divide the sum by the divisor, and so on.

If there is a final remainder, express its division in a fractional form, and annex it to the quotient.

EXAMPLES.

542–548. Divide 25*yd.* 3*qr.* 1*na.* by 3; 4; 5; 6; 7; 8; 9.

549–562. Divide 27*cwt.* 3*qr.* 20*lb.* 13*oz.* 9*dr.* by 2; 3; 4; 5; 6; 7; 8; 9; by 27; 63; 81; 64; 42; 49.

NOTE.—The pupil will observe that these latter divisors are composite numbers.

563–567. Divide 10*lb.* 8*oz.* 16*pwt.* 3*gr.* by 13; 19; 23; 29; 31.

568. A man has 23 days allowed to walk 100*mi.* 4*fur.* 30*rd.* 1½*yd.* 2*ft.* How far must he walk each day?

569–572. Divide 10 *tuns* 2*hhd.* 17*gal.* 2*pt.* by 67; 51*A.* 1*R.* 11*P.* by 51; 4*gal.* 2*qt.* by 144; £113 13*s.* 4*d.* by 31.

573. Divide 673141*da.* 9*hr.* 58*m.* 24*sec.* by 1843.

574. Divide 1*mi.* 255*ft.* 10*in.* by 365.

575. Bought 15 sheep for £5 12*s.* 6*d.* How much did one sheep cost?

576. If 24*yds.* of cloth cost £18 6*s.*, how much is that per yard?

577. From a piece of cloth containing 128*yds.* 1*qr.*, a tailor made 18 coats, which took one-third of the whole piece. How many yards did each coat contain?

QUESTIONS INVOLVING THE FOUR PRECEDING RULES.

578. Twenty-four men agree to construct 7*mi.* 1*fur.* 24*rd.* of road: after completing $\frac{1}{6}$ of it, they employ 8 more men. What distance does each man construct before and after the 8 men were employed?

579. A silversmith has seven tea-pots, each weighing 1*lb.* 3*oz.* 13*pwt.* 11*gr.* What is the whole weight?

580. A farmer has 1000 bushels of apples, which he puts into 350 barrels. How many does each barrel hold?

581. If it require 1 sheet of paper to print 24 pages of a book, how many reams, allowing 18 quires to the ream, will it take to print 3000 copies, of 250 pages each?

582. An estate worth £2570 is to be divided as follows: the widow has one-third of the whole, the remainder is to be divided equally between seven children. How much does the widow receive, and how much does each child have?

583. Divide 100 acres, 3 roods, 8 rods of land, between four persons, A., B., C., and D., so that A. shall have one-sixth of the whole, B. one-fourth of the remainder, C. one-third of what then remains, and D. the rest. How much does each one have?

584. A., B., C. and D., having 13*cwt.* 1*qr.* 4*lb.* of sugar, they agree to divide it as follows: A. is to have one-half of the whole, B. is to have one-third of the remainder, C. is to have one-fourth of what then remains, and D. is to take what is left. What were their respective portions?

585. What is the weight of the following coins: 10 guineas, each weighing 5 *pwt.* $9\frac{1}{2}$ grains; 7 sovereigns, each weighing 1 *pwt.* $8\frac{1}{4}$ grains?

586. What is the weight of 13 crowns, each weighing 18 *pwt.* $4\frac{4}{11}$ grains; 14 shillings, each weighing 3 *pwt.* $15\frac{3}{11}$ *gr.*; 9 sixpences, each weighing 1 *pwt.* $19\frac{7}{11}$ *gr.*?

587. In one eagle there are $232\frac{2}{10}$ grains of pure gold, $12\frac{9}{10}$ grains of silver, and $12\frac{9}{10}$ grains of copper, and the same proportions of gold, silver, and copper for all other American gold coin. In 10 eagles, 7 half-eagles, 5 quarter-eagles, how many grains of gold, silver, and copper?

588. One pound of pure gold is sufficient for how many dollars of gold coin, if it require 23·22 grains for one dollar?

589. One pound of pure silver is sufficient for how many dollars of silver coin, if it require 371·25 grains for one dollar?

590. The aggregate ages of 35 individuals amount to 964*yr.* 11*mo.* 3*wk.* 6*da.* 13*hr.* 47*min.* 35*sec.* What would be the age of each of the 35, supposing their ages were all equal?

591. For 13 successive years a man peddled his wares from Nov. 7 to March 16 inclusive, in the Southern States; from March 21 to June 29, in the Western States; from July 14 to Oct. 2, in the Northern States; and 20 days of each year he passed in the Middle States. How long was he in each section of the country? and how long in all?

592. If one solar year is 365*da.* 5*hr.* 48*min.* 47·588*sec.*, what will be the length of 1000 years?

593. The moon occupies about 29·53 days, on an average, from change to change. How long will be required to make 235 changes or lunations?

594. The moon moves through an entire circumference of 360° in 27*da.* 7*hr.* 43*min.* 11·5*sec.* How far does it move each day?

595. If £1000 10*s.* 5*d.* 3*far.* be divided equally among 23 individuals, how much will each receive?

596. Bought 5 casks of sugar, each weighing 400*lb.* 13*oz.*; 4 casks of 375*lb.* 10*oz.* each. What was the weight of the 9 casks?

597. If a ship sail 3000 miles in 10*da.* 13*hr.*, what is the average hourly rate?

598. If we estimate one degree of the earth's equator at $69\frac{1}{6}$ miles, what will be the length of an arc of 14° 18′?

599. If the circumference of the earth at the equator is 24899 miles, through what distance is a point of the equator carried each hour by the earth's daily revolution?

600. If each individual of a city of 50000 inhabitants requires 4·75 gallons of water daily, how much will be annually required to supply the city?

Duodecimals.

§ 92. Duodecimals* are a kind of denominate numbers, whose denominations increase or decrease in a *twelve-fold* ratio. They are applied to the measurement of *surfaces* and *solids.*

The denominations of Duodecimals are the *foot* (*f.*), which is the unit; the inch or *prime* (′), which is $\frac{1}{12}$ of the *f.*; the *second* (″), which is $\frac{1}{12}$ of the prime; the *third* (‴), which is $\frac{1}{12}$ of the second, &c. The accents which are used to distinguish the denominations below feet are called *indices.*

The *Addition* and *Subtraction* of duodecimals are performed like addition and subtraction of other denominate numbers. It need only be remembered that 12 of any denomination make 1 of the next higher.

EXAMPLES.

(601.)	(602.)
17*f.* 7′ 8″	365*f.* 1′ 7″ 9‴
25*f.* 0′ 2″	521*f.* 10′ 10″ 11‴
30*f.* 10′ 11″	605*f.* 8′ 8″ 1‴
29*f.* 6′ 6″	731*f.* 3′ 0″ 8‴

603. What is the sum of 3*f.* 6′ 4″, 8*f.* 3′ 4″, 9*f.* 1′ 3″, and 10*f.* 10′ 10″?

604. What is the sum of 100*f.* 8′ 8″, 135*f.* 0′ 1″, 65*f.* 9′ 2″, 45*f.* 3′ 3″, and 200*f.* 6′ 6″?

* From a Latin word, *duodecim*, meaning *twelve.*

	(605.)	(606.)
From	87*f.* 3′ 4″	100*f.* 10′ 10″
Subtract	35*f.* 8′ 9″	90*f.* 6′ 3″

607. From 25*f.* 6′ 6″ subtract 18*f.* 9′ 10″.

608. From 100*f.* subtract 58*f.* 2′ 1″.

609–612. What is the sum, and what is the difference of 37*f.* 11′ 3″ and 13*f.* 1′ 11″? of 99*f.* 9′ 9″ and 31*f.* 10′ 10″?

613–615. From 100*f.* subtract 11*f.* 11′ 1″; from the remainder subtract 11*f.* 11′ 1″; and from this remainder subtract 11*f.* 11′ 1″. What are the three successive remainders?

616–619. What is the sum and difference of 47*f.* 1′ 1″ 1‴ and 13*f.* 11′ 11″ 11‴? of 101*f.* 10′ 10″ 10‴ and 19*f.* 3′ 3″ 1‴?

620. What is the sum of 6*f.* 3′ 4″, 4*f.* 4′ 0″ 4‴, and 13*f.* 6′ 6″ 2‴?

Multiplication of Duodecimals.

§ 93. Suppose we wish to multiply 14*f.* 7′ by 2*f.* 3′, we should proceed as follows:

	14*f.*	7′	
	2*f.*	3′	
	3*f.*	7′	9″
	29*f.*	2′	
Ans.	32*f.*	9′	9″

We begin on the right hand, and multiply the multiplicand through, first by the primes of the multiplier, then by the feet of the multiplier; thus, $3' \times 7' = \frac{3}{12} \times \frac{7}{12} = \frac{21}{144}$ of a foot, which is 21″=1′ 9″; we write down the 9″, and reserve the 1′ for the next product; again, $14f. \times 3' = 14 \times \frac{3}{12} = \frac{42}{12}$ of a foot, which is 42′; now adding in the 1′, which was reserved from the last product, we have $43'=3f.\ 7'$, which we write down, thus finishing the first line of products.

Again, we have $2f.\times 7'=2\times\frac{7}{12}=\frac{14}{12}$ of a foot, which is $14'=1f.\ 2'$; we write the $2'$ under the primes of the line above, and reserve the $1f.$ for the next product; $2f.\times 14f.=28f.$, to which, adding in the $1f.$ reserved from the last product, we have $29f.$, which we place underneath the feet of the line above. Taking the sum, we find $32f.\ 9'\ 9''$ for the answer, or $32f.+\frac{9}{12}f.+\frac{9}{144}f.$

It will thus be seen that, in the multiplication of Duodecimals, the sum of the indices of the factors is the number of the indices of the product, just as in decimals the sum of the decimal places in multiplier and multiplicand forms the number of decimal places in the product. Or each index (′) might be regarded as denoting the factor 12 in the denominator of a fraction, of which the number having the index is the numerator. Thus, $1'=\frac{1}{12}f.$; $2''=\frac{2}{12\times 12}f.=\frac{2}{144}f.$; $4'''=\frac{4}{12\times 12\times 12}f.=\frac{4}{1728}f.$, &c. $1'\times 1'=\frac{1}{12}\times\frac{1}{12}f.=\frac{1}{144}f.=1''$; $2''\times 5'''=\frac{2}{12\times 12}\times\frac{5}{12\times 12\times 12}f.=\frac{10}{248832}f.=10'''''$. The foot, being the unit, or integer, has no index.

Note.—It must be remembered that in Duodecimals, when not used to express *linear* measure, but surfaces or solids, the foot contains 144*sq. in.* or 1728*s. in.* Consequently, in the measurement of surfaces, $2'$ would be equal, not to 2*sq. in.*, but to $\frac{2}{12}$ of 144*sq. in.*, or 24*sq. in.* In the measurement of solids, $2'$ would be $\frac{2}{12}$ of 1728*s. in.*= 288*s. in.* The prime (square measure) is a strip of surface of 1 inch wide and 12 inches long; the prime (solid measure) is a slab 1 inch thick, 12 long, and 12 broad.

From what has been said, we infer the following

RULE.

Place the several terms of the multiplier under the corre-

sponding ones of the multiplicand. Beginning at the right hand, multiply the several terms of the multiplicand by the several terms of the multiplier successively, placing the right-hand term of each of the partial products under its multiplier. To each product-term annex as many indices as are in both its factors. The sum of the partial products will be the result required.

EXAMPLES.

621. What is the product of 3*f.* 7′ 2″ by 7*f.* 6′ 3″?

622–623. Multiply 7*f.* 8′ by 6*f.* 4′ 3″; 6*f.* 9′ 7″ by 4*f.* 2′.

624. What is the area of a marble slab whose length is 7*f.* 3′, and breadth 2*f.* 11′?

625. How many square feet are contained in the floor of a hall 37*f.* 3′ long, by 10*f.* 7′ wide?

626. How many square feet are contained in a garden 100*f.* 6′ in length, by 39*f.* 7′ in width?

627. How many yards of carpeting, one yard in width, will it require to cover a room 16*f.* 5′ by 13*f.* 7′?

628. How many yards of Brussels carpeting, 27 inches wide, will be required to cover a room 15*f.* 9′ by 16*f.* 7′?

629. How many square feet in 12 boards, averaging 12*f.* 8′ long by 1*f.* 9′ wide, each?

630. What will it cost to veneer a surface 7*f.* 6′ 3″ long, by 5*f.* 2′ 7″ wide, at $87\frac{1}{2}$ cts. per square foot?

631. How many cubic feet in a wall 80*f.* 9′ long, 1*f.* 8′ wide, and 3*f.* 4′ high?

632. How many solid feet in a pile of wood, 156 feet long, 4*f.* 8′ high, 6*f.* 4′ wide?

633. In one side of a house are 12 windows; in each window 12 lights: each light is 1*f.* 3′ by 11′. How many square feet of glass in the whole?

634. A room is 18*f.* long, 14*f.* 6′ wide, 9*f.* 8′ high. There are 4 windows in the room, each 5*f.* 6′ long by 3*f.* wide; and 2 doors, each 6*f.* 9′ high by 2*f.* 10′ wide. What will be the cost of plastering said room at $12\frac{1}{2}$ cents per square yard?

635. What will it cost to paint a house 42*f.* 6′ long, 28*f.* 6′ wide, 19*f.* 6′ high, at 13 cts. per square yard?

NOTE.—No deduction is made for windows. The painting of the sashes is considered equivalent to the painting of the surface over which the sashes stretch.

636. What is the square of 23*f.* 8′ 7″? What is the cube of the same?

637. How many bricks, each 8*in.* long, 4*in.* wide, and 2*in.* thick, are required to build a wall 180 feet long, 6*f.* 6′ high, and three bricks wide, no allowance being made for the mortar?

DIVISION OF DUODECIMALS.

§ 94. There are 27*f.* 0′ 7″ 9‴ 6⁗ in the surface of a piano-cloth. The breadth of the cloth is 3*f.* 7′ 2″. What is its length?

```
3f. 7′ 2″ ) 27f.  0′  7″  9‴  6⁗ ( 7f. 6′ 3″
            25f.  2′  2″
            ------------
             1f. 10′  5″  9‴
             1f.  9′  7″  0‴
             ----------------
                     10″  9‴  6⁗
                     10″  9‴  6⁗
                     -----------
```

Dividing the product 27*f.* 0′ 7″ 9‴ 6⁗ by 3*f.* 7′ 2″, one of its factors must give the other factor.

We therefore divide first the 27*f.* by 3*f.*, and find the quotient 7 feet. Multiplying the whole divisor by the quotient, we have 25*f.* 2′ 2″, which we subtract from the corresponding denominations of the dividend. To the remainder we annex another term of the dividend.

Dividing 1*f.* 10′ or 22′ by 3*f.*, we obtain 6′ for a quotient, by which we multiply the whole divisor. The product, 1*f.* 9′ 7″ 0‴, we subtract from the corresponding denominations of the dividend, and to the remainder annex the remaining term of the dividend. Dividing 10″ by 3*f.* we obtain 3″ for a quotient; multiplying the whole divisor by this, and subtracting, we find no remainder. The length of the cloth, then, is 7*f.* 6′ 3″.

There can be no difficulty as to how many indices we shall annex to any term of the quotient, if we remember that *the indices of the quotient added to those of the divisor must equal those of the dividend.* Thus, 9′′′′′ divided by $3'' = \frac{9}{12\times12\times12\times12\times12} \div \frac{3}{12\times12} = \frac{3}{12\times12\times12}$ or 3‴; so 36′′′′′′′ ÷ 6′′′′ = 6′′′′.

RULE.

Arrange the numbers as for denominate division. Divide the highest term of the dividend by the highest term of the divisor. Multiply the WHOLE *divisor by the quotient thus obtained, and subtract the product from the corresponding terms of the dividend. To the remainder annex the next denomination of the dividend. Divide the highest term of this partial dividend by the highest term of the divisor, as before, and so proceed, till the division is complete.*

NOTE 1.—If, on the multiplication of the *whole* divisor by the quotient figure, this is found too large, the quotient figure must be taken smaller.

2. If the highest term of a partial dividend will not contain the divisor, such term may be reduced to the next lower denomination, and the number in that denomination added, and the division then performed.

EXAMPLES.

638–640. Divide 32*f.* 9′ 9″ by 14*f.* 7′; by 29*f.* 2′; by 7*f.* 3′ 6″.

641. The area of a marble slab is 21*f.* 1′ 9″; its length is 7*f.* 3′. What is its breadth?

642. A carpenter bought 920 square feet of boards. He knew that their united length was 480 feet. What did he find their average breadth to be?

643. There were 4283*cu. ft.* 4′ of earth thrown out of a cellar. The cellar was 42*f.* 10′ long and 12*f.* 6′ wide. How deep was it?

644. I have a board fence containing 510*sq. ft.* 10′ 8″. Its height is 6*ft.* 4*in.* What is its length?

645. A block of marble for the Washington monument is 3*f.* 1′ wide, 2*f.* 3′ thick, and contains 37*S.f.* 6′ 11″ 3‴. What is its length?

Addition of Denominate Fractions.

§ 95. We have seen (§ 88) that whole numbers of different denominations cannot be added together; the same is true of fractions of different denominate values. Thus, $\frac{3}{4}$ of a peck and $\frac{3}{5}$ of a quart cannot be added together. But if *both quantities are made fractions of a peck, or both fractions of a quart, their sum may be found.*

$\frac{3}{4}pk.=\frac{6}{1}qt.$; and $\frac{6}{1}qt.+\frac{3}{5}qt.=6\frac{3}{5}qt.$ Again, $\frac{3}{5}qt.=\frac{3}{40}pk.$; and $\frac{3}{4}pk.+\frac{3}{40}pk.=\frac{30}{40}pk.+\frac{3}{40}pk.=\frac{33}{40}pk.=\frac{33}{5}qt.=6\frac{3}{5}qt.$, the same result as before.

EXAMPLES.

646–648. Add $\frac{1}{5}s.$ to $\frac{1}{4}d.$; $\frac{2}{3}qt.$ and $\frac{3}{4}pk.$; $\frac{4}{5}da.$ and $\frac{5}{7}hr.$

649–650. Add $\frac{1}{3}yd.$ $\frac{5}{6}ft.$ and $\frac{3}{8}in.$; £$\frac{3}{4}$ $\frac{4}{5}s.$ $\frac{5}{6}d.$ and $\frac{6}{7}qr.$

651–652. Add $\frac{1}{2}wk.$ $\frac{1}{6}da.$ and $\frac{1}{4}hr.$; $\frac{1}{5}yr.$ $\frac{2}{7}wk.$ $\frac{1}{12}da.$

653. What is the sum of $\frac{1}{2}$ of a *cwt.*, $\frac{1}{3}$ of a *qr.*, $\frac{1}{4}$ of a *lb.*?

654. What is the sum of $\frac{1}{10}$ of a bushel, $\frac{1}{6}$ of a peck, $\frac{1}{2}$ of a quart?

655. What is the sum of $\frac{1}{10}$ of a yard, and $\frac{1}{3}$ of a foot?

656. What is the sum of $\frac{2}{3}$ of a week, $\frac{1}{5}$ of a day, and $\frac{1}{3}$ of an hour?

657. What is the sum of $\frac{2}{3}$ of a bushel, $\frac{1}{3}$ of a peck, and $\frac{1}{7}$ of a quart?

658. An invalid laborer worked during the first week of harvesting, $\frac{5}{6}$*wk.*, counting 6 days to the week; during the 2d, $\frac{7}{10}$*da.*, counting 10 hours to the day; during the 3d, $\frac{17}{10}$*da.*; during the 4th, $6\frac{1}{2}$*hr.*; during the 5th, $\frac{3}{4}$*hr.* How much did he earn at the rate of $12\frac{1}{2}$ cts. per hour?

Subtraction of Denominate Fractions.

§ 96. As in addition, the fractions must be first reduced to the same denomination, afterwards to a common denominator. The operation is then the same as subtraction of common fractions.

From $\frac{1}{8}$ of a pound sterling subtract $\frac{1}{5}$ of a shilling.

$£\frac{1}{8}=\frac{5}{2}s.$; and $\frac{5}{2}s.-\frac{1}{5}s.=\frac{25}{10}s.-\frac{2}{10}s.=\frac{23}{10}s.=2\frac{3}{10}s.$ Or, $\frac{1}{5}s.=£\frac{1}{100}$; and $£\frac{1}{8}-£\frac{1}{100}=£\frac{25}{200}-£\frac{2}{200}=£\frac{23}{200}=\frac{23}{10}s.$ $=2\frac{3}{10}s.$, the same result as before.

EXAMPLES.

659–660. From $\frac{3}{8}$*da.* subtract $\frac{1}{5}$*min.*; from $\frac{9}{11}$*da.* subtract $\frac{2}{3}$*hr.*$+\frac{9}{13}$*min.*

661. From $\frac{1}{2}$ of $\frac{2}{3}$ of 15 yards of cloth subtract $\frac{1}{8}$ of $\frac{8}{15}$ of 1 quarter.

662. From $\frac{1}{7}$ of 5 acres of land, subtract $\frac{1}{4}$ of 3 roods.

663. From $\frac{2}{5}$ of an ounce, take $\frac{3}{8}$ of a pennyweight.

664. From $\frac{1}{9}$ of a hogshead, take $\frac{2}{3}$ of a quart.

665. A man had a field to plough, containing 3*A.* $\frac{1}{3}$*R.* $\frac{1}{2}$*P.*; $\frac{3}{5}$*A.* $\frac{2}{3}$*R.* $\frac{3}{4}$*P.* was ploughed the first half-day. How much remained to be ploughed?

666. A grocer lost from $\frac{3}{7}$ of a hogshead of molasses $\frac{2}{3}$*gal.* and $\frac{3}{4}$*qt.* How much of the hogshead, expressed decimally, leaked out?

667. Suppose a man consume $\frac{9}{24}$ of every day in sleep; $\frac{3}{24}$ of every day in eating; $\frac{2}{3}$*hr.* each day in amusement; $\frac{3}{4}$*hr.* each day in idleness: how many days, of 10 hours each, has he for work in the course of the year?

EXERCISES IN DENOMINATE FRACTIONS.

668. A person gave $\frac{1}{7}$ of a pound sterling for a hat, $\frac{1}{3}$ of a shilling for some thread, and $\frac{1}{2}$ of a penny for a needle. What did he pay for all?

669. What is the value of $\frac{1}{2}$ of a week, $\frac{1}{3}$ of a day, and $\frac{1}{4}$ of a minute?

670. What is the value of $\frac{1}{5}$ of a pound, $\frac{1}{4}$ of an ounce, and $\frac{1}{7}$ of a pennyweight, Troy?

671. If $4\frac{1}{3}$ pounds of sugar cost $43\frac{1}{3}$ cents, how much is it per pound?

672. If I pay $4·04 for $8\frac{3}{4}$ bushels of apples, how much do I give per bushel?

673. Four persons, A., B., C., and D., own a ship: of which A. owns $\frac{1}{5}$ of $\frac{2}{3}$ of the whole; B. owns $\frac{7}{8}$ of $\frac{2}{3}$ as much as A.; C. owns $\frac{3}{4}$ as much as B.; and D. owns the remainder. What are the respective parts owned by each?

674. From $\frac{1}{2}$ of $\frac{3}{4}$ of a day of 24 hours, take $\frac{1}{3}$ of $1\frac{1}{2}$ hour.

675. To $\frac{3}{4}$ of $4\frac{1}{2}$ days of 24 hours each, add $\frac{1}{3}$ of $\frac{1}{6}$ of $3\frac{1}{2}$ hours.

676. A certain sum of money is to be divided between 4 persons in such a manner that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth the remainder, which is $28. What is the sum?

677. A. received $\frac{1}{6}$ of a legacy, B. $\frac{1}{10}$, and C. the remainder. Now it is found that A. had $80 more than B. How much did each receive?

678. Eight detachments of artillery divided 4608 cannon-balls in the following manner: the first took 72 and $\frac{1}{9}$ of the remainder; the second took 144 and $\frac{1}{9}$ of the remainder; the third took 216 and $\frac{1}{9}$ of the remainder; the fourth took 288 and $\frac{1}{9}$ of the remainder. The balance was equally divided among the remaining four detachments. How many balls did each detachment receive?

679. Five persons divide 100 pounds of sugar as follows: the first takes $\frac{1}{7}$ of $\frac{3}{4}$ of the whole; the second takes $\frac{1}{6}$ of $\frac{3}{4}$ of the remainder; the third takes $\frac{1}{5}$ of $\frac{3}{4}$ of the remainder; the fourth takes $\frac{1}{4}$ of $\frac{3}{4}$ of the remainder; and the fifth had what was left. How much did each receive?

680–681. A person owning 100*A.* 3*R.* 4*P.* of land, bought $\frac{3}{4}$ of a farm of 97*A.* 1*R.* 30*P.*, and then divided $\frac{1}{2}$ of the whole equally among 3 sons. How much did each son have? and how much remained with the father?

682. What is $\frac{1}{8}$ of the sum of $\frac{3}{4}$ of $\frac{4}{7}$ of 13 weeks, and $\frac{1}{2}$ of $\frac{1}{5}$ of 30 days?

683. What must I pay for 38 eggs, at 2*s.* 2*d.* per dozen?

684. How much cheese, at 9*d.* 3*far.* per pound, ought I to receive for 13*lb.* 5*oz.* of veal, at $4\frac{1}{2}$*d.* per pound?

685. What is the value of $\frac{3}{4}$ of a year of $365\frac{1}{4}$ days + $\frac{1}{5}$ of 7 days of 10 hours each?

686. Add $\frac{1}{7}$ of 4 degrees of $69\frac{1}{6}$ miles each, and $\frac{1}{3}$ of 4 furlongs.

687. If I buy 113*lb.* 13*oz.* of butter, at $10\frac{1}{2}$*d.* per pound, and use 30*lb.*, how much per pound must I sell the remainder so as to receive as much as the whole cost?

688–691. A person gave $\frac{1}{2}$ of all his money for a dress coat, $\frac{1}{3}$ of the remainder for a pair of pantaloons, and $\frac{1}{4}$ of what then remained for a hat. He then found that he had remaining £3 14*s*. 6*d*. What was the cost of each article? and how much money had he at first?

692. From a piece of cloth containing 20*yd*. 2*qr*. 2*na*., 3 suits, each requiring $4\frac{1}{2}$*yd*., were taken; and $\frac{1}{3}$ of the remainder was sold for \$10·68$\frac{3}{4}$. How much was that per yard?

693. How many feet in $\frac{3}{4}$ of a statute mile + $\frac{1}{3}$ of a nautical mile?

694. How many inches in $\frac{1}{2}$ of a yard + $\frac{2}{3}$ of a metre?

695. How many feet in $\frac{3}{4}$ of a chain + $\frac{3}{10}$ of a furlong?

696. How many inches in $\frac{1}{2}$ of a hand + $\frac{1}{3}$ of a span + $\frac{1}{4}$ of a cubit?

697. How many inches in 1 Ell Scotch + $\frac{1}{2}$ of an Ell English + $\frac{1}{3}$ of an Ell French?

698. How many cubic inches in 3 gallons, 2 quarts, and 1 pint of wine?

699. How many cubic inches in $\frac{2}{3}$ of a gallon + $\frac{1}{2}$ of a quart of beer?

700. How many cubic inches in 1 bushel 3 pecks?

CHAPTER XII.

PERCENTAGE.

§ 97. The term *per cent.* is an abbreviation of the Latin words *per centum*, which mean by the hundred. Thus, 2 per cent. signifies 2 out of a hundred, or 2 *hundredths*; 3 per cent. 3 out of a hundred, or 3 *hundredths*, &c. Per cent. is applied to money, apples, beans, the pupils of a school, or to any thing else.

We have seen that hundredths may be expressed either as a common or as a decimal fraction; thus, 2 hundredths $=\frac{2}{100}=0{\cdot}02$; 3 hundredths $=\frac{3}{100}=0{\cdot}03$, &c. It is the decimal form that we use in all operations in percentage; 0·02, 0·03, &c., are called the *rates* per cent.

When the rate per cent. is more than 100, that is, more than $\frac{100}{100}$, it is expressed as an improper fraction; or, decimally, as a mixed number: thus, 106 per cent. $=\frac{106}{100}=1{\cdot}06$.

When the rate per cent. is less than 1, that is, less than $\frac{1}{100}$, it may be expressed by a decimal of three or four places. Thus $\frac{1}{2}$ of 1 per cent. $=0{\cdot}005$; which is read, not 5 thousandths per cent., but $\frac{5}{10}$ of 1 per cent.; $\frac{1}{4}$ of 1 per cent. $=0{\cdot}0025$; which is read $\frac{25}{100}$ of 1 per cent. Reduce the common fraction to a decimal, writing the first quotient figure as the *tenth of a hundredth*, or as a *thousandth*, &c.

We have seen, § 44, that to obtain a fractional part of any number, we must multiply the number by the fraction; thus, 0·06 of 25 is $0{\cdot}06\times 25$; 0·0025 of 50 is $0{\cdot}0025\times 50$. Hence, to compute the percentage of any number,

Multiply the given number by the rate per cent. expressed as a decimal. The product, pointed off by the rule for decimals, will be the percentage required.

EXAMPLES.

1–8. Express in figures the following: 2 per ct.; 8 per ct.; 12 per ct.; 50 per ct.; 106 per ct.; 140 per ct.; 260 per ct.: 1800 per ct.

9–17. Express in the required form $\frac{9}{100}$; $\frac{24}{100}$; $\frac{99}{100}$; $\frac{104}{100}$; $\frac{607}{100}$; $\frac{7281}{100}$; $\frac{5}{1000}$; $\frac{25}{1000}$; $\frac{350}{1000}$.

18–25. Express in decimals $\frac{1}{2}$ per ct.; $\frac{1}{4}$ per ct.; $\frac{2}{3}$ per ct.; $\frac{9}{10}$ per ct.; $\frac{3}{8}$ per ct.; $\frac{5}{6}$ per ct.; $\frac{1}{3}$ per ct.; $\frac{2}{5}$ per ct.

26–32. Express in decimal form $2\frac{1}{2}$ per ct.; $3\frac{1}{3}$ per ct.; $5\frac{4}{5}$ per ct.; $36\frac{1}{7}$ per ct.; $220\frac{1}{4}$ per ct.; $500\frac{3}{8}$ per ct.; $946\frac{3}{4}$ per ct.

33–37. What is 0·05 of \$1122? 0·06 of \$79468? 0·07 of \$8912·50? 0·08 of \$34567·623? 0·09 of \$3479021·05?

38–47. Find the percentage of \$987654·37 at each of the following rates: $\frac{1}{2}$ per ct.; $2\frac{1}{2}$ per ct.; $\frac{1}{4}$ per ct.; $3\frac{1}{4}$ per ct.; $\frac{3}{8}$ per ct.; $6\frac{3}{8}$ per ct.; 25 per ct.; 412 per ct.; 1900 per ct.; 43 per ct.

48–53. Find 38 per ct. of 4; 97 per ct. of 16; 500 per ct. of 7; 840 per ct. of $28\frac{1}{2}$; 365 per ct. of $\frac{1}{4}$; 92 per ct. of $\frac{3}{8}$.

54. What is 3 per cent. of 5789 pounds?

55. What is $4\frac{1}{2}$ per cent. of \$75·03?

56. What is 7 per cent. of 2345?

57. What is 30 per cent. of \$495?

58. A person laid out \$222 as follows: he gave 21 per cent. of his money for calicoes; 15 per cent. for thread; 45 per cent. for silks; and the remaining 19 per cent. for broadcloths. How many dollars did he expend for each?

59. A merchant having 500 barrels of flour, sold at one time 25 per cent. of it; at another time he sold 20 per cent. of the remainder. How many barrels did he sell at each time, and how many remained?

60. A farmer raising 1097 bushels of wheat, gives 10 per cent. of it for thrashing; 10 per cent. of the remainder for flouring. What per cent. of the whole will he have left?

61. A California miner having secured $15\frac{1}{2}$ pounds of gold dust, finds that it loses 5 per cent. in refining; he then gives 6 per cent. for coining. How much ought he to receive after it is coined?

62. Suppose at each stroke of the piston of an air-pump 10 per cent. of the air in the receiver is exhausted, what per cent. of the air will remain after the 1st, 2d, 3d, and 4th strokes, respectively?

63. A man laid out 25 per cent. of \$100 for clothing; at

the same time a sum of money was paid him equal to 25 per cent. of what he had after his purchases. How much less money did he bring home than he had when he left home?

64. A merchant invested \$6480 in trade, but lost 0·75 of it. How much did he save?

65. A. and B. invested \$100 each in a speculation. A. lost 100 per cent. of his investment, and B. made 100 per cent. on his. How much better off was B. than A.?

66. Out of a hogshead of molasses 36 per cent. leaked. How many gallons remained?

67. A coal dealer bought 17180 tons of coal, and sold 62 per cent. of it at \$4·50 per ton, and the remainder for \$4·87 per ton. How much did he sell the whole for?

Applications of Percentage.

§ 98. The principle of percentage has a very extensive application to mercantile transactions, and to the calculations of practical life. Commission and Brokerage, Rise and Fall of Stocks, Assessment of Taxes, Duties, Insurance, Profit and Loss, Interest, Discount, &c., involve chiefly the computation of percentage.

Commission, Brokerage, and Stocks.

§ 99. Commission is an allowance made to an agent for the purchase or sale, or care of property. This agent is called a factor, or correspondent, or commission merchant. Commissions are estimated at so much per cent. on the money employed.

Brokerage is merely the commission paid to a Broker, or dealer in stocks, money, or bills of exchange, for transacting business.

STOCKS are Government Funds; State Bonds; the *capital* of Banks; of Insurance, Railroad, and Manufacturing Companies, &c.

This capital, or *money paid in*, is divided into *shares*, which are owned by *stockholders*. The original cost of a share is its *par value*. If it sell in the market for more than its original cost, it is said to be *above par*, or at an advance; if it sell for less, it is *below par*, or at a discount.

The original cost of a share is usually $100, though it is sometimes $25, $50, $500, &c.

The rise or fall in stocks is a per cent. on the par value. Thus, a share, whose par value is $\frac{100}{100}$, at 16 per cent. advance, will bring $\frac{116}{100}$ of its original cost; at 16 per cent. discount, will bring but $\frac{84}{100}$ of its original cost.

The profits of these companies are, every year, or every half-year, divided among the stockholders. The amount so paid out is called a *dividend*.

EXAMPLES.

68. A lady had a bequest of $10500; she paid an agent $2\frac{1}{2}$ per cent. commission, per annum, to take care of the money for her. How much did the commission amount to?

69. I bought 27 shares of Providence Railroad stock at 13 per cent. discount, and sold them again at an advance of 2 per cent. How much did I gain by the operation? The par value was $100.

70. A gentleman paid a broker $\frac{3}{4}$ per cent. to invest $19278 in government funds. How much was the brokerage?

71. A man owns 46 shares of bank stock, par value $50, on which he received $4\frac{1}{2}$ per cent dividend. What was the amount of his dividend?

72. A factor sells 43 bales of cotton, at $375 per bale, and charges 2 per cent. commission. How much money must he pay to his principal?

73. A. sends to B., a broker, \$3605 to be invested in stock: B. is to receive 3 per cent. on the amount paid for the stock. What was the value of the stock purchased?

Since B. is to receive 3 per cent., it is plain that \$103 of A.'s money would purchase \$100 worth of stock. Hence, the amount expended for stock must be $\frac{100}{103}$ of \$3605.

In such cases as the above, when the given sum includes the factor's commission, and we desire to know what amount he must invest for his principal so that the balance may be his commission on the amount invested, we must divide the given sum by the percentage of the commission increased by a unit.

74. A factor receives \$60112, and is directed to purchase cotton at \$289 per bale: he is to receive 4 per cent. on the money paid for the cotton. How many bales did he purchase?

75. The par value of 125 shares of bank stock was \$100 per share. What is the present value, if the stock is worth 18 per cent. above par?

76. What is the value of 50 shares of bank stock, the par value of which was \$200 per share, on the supposition that it is 12 per cent. below par, or that it is worth only 88 per cent. of its par value?

77. A bank fails, and has in circulation \$108567. It can pay only 13 per cent. What amount of money has it on hand?

78–81. A person fails, who owes to A. \$3563·75, to B. \$4062·35, and to C. \$6723·33. He finds that he can pay 65 per cent. of his debts. How much ought A., B., and C. to receive respectively, and how much collectively?

82–84. A person in trade finds, during three successive years, that at the end of each year his money has increased 30 per cent. If he commenced with \$3000, how much will he have at the close of each successive year?

85. A tax of $7593·50 is to be collected. For collecting, 5 per cent. is given, which must be collected along with the tax. What was the whole sum collected?

86. Of $1000 worth of gold in California, 5 per cent. is paid for transportation to the United States, and 5 per cent. of the balance is paid for coining. What is the value of the coin received?

87. During a voyage at sea, the first week 20 per cent. of the provisions are consumed. During the second week 40 per cent. of the balance is used. What per cent. of the whole remains at the end of the second week?

88. How much is 30 per cent. of 30 per cent. of $999?

89. A person purchased 350 barrels of flour, and sold 20 per cent. of it at $6 per barrel. How much money did he receive?

90. Of 1000 fruit-trees 4 per cent. die the first year, and 5 per cent. of the remainder die the second year. How many died during the two years?

Assessment of Taxes.

§ 100. Taxes are moneys paid by the people for the support of government. They are assessed on the citizens in proportion to their property; except the *poll-tax*, which is so much for each individual, without regard to his property.

Before taxes can be assessed, an inventory of all the taxable property must be made.

Next the sum of the poll-taxes must be deducted from the whole sum to be raised, and then the remainder must be apportioned according to each individual's property.

To effect this apportionment, find what per cent. of the taxable property the sum to be raised is; then multiply each one's inventory by this per cent. expressed in decimals, and the product will be his tax.

Suppose a tax of $600 is to be raised in a town containing 60 polls. If the whole taxable property amounts to $37000, and each poll-tax is $0·75, what will be A.'s tax, whose property is inventoried at $653, and who pays one poll?

We find the sum of the polls to be $45·00, which we deduct from the $600, the amount to be raised. Dividing the remainder $555 by $37000, the value of the taxable property, we shall have the tax or per cent. on 1 dollar, expressed in decimals. Multiplying each individual's property by this per cent. will give each one's tax.

$$\begin{array}{rr} \$0·75 & \$600 \\ 60 & 45 \\ \hline \$45·00 & \$555 \end{array}$$

$$\frac{555}{37000} = \$0·015$$

Having determined this per cent., assessors facilitate their business by making out a table as follows:

$1	pays	$0·015	$10	pays	$0·15	$100	pays	$1·50
2	"	0·03	20	"	0·30	200	"	3·00
3	"	0·045	30	"	0·45	300	"	4·50
4	"	0·06	40	"	0·60	400	"	6·00
5	"	0·075	50	"	0·75	500	"	7·50
6	"	0·09	60	"	0·90	600	"	9·00
7	"	0·105	70	"	1·05	700	"	10·50
8	"	0·12	80	"	1·20	800	"	12·00
9	"	0·135	90	"	1·35	900	"	13·50
						1000	"	15·00

The pupil can easily understand the application of the table.

91. By the above table, what would be the tax on $425, there being no poll-tax?

92. By the same table, what must B. pay, who has 2 polls, and whose real and personal property is assessed at $762?

93. If C. pays 3 polls, and is assessed at $1250, how much ought he to pay?

94. What is the tax on $375, there being no polls?

95. How much is the tax on $1875?

96. How much is the tax on $1100?

97. How much is an 8 per cent. tax on an estate valued at $17000?

98. If a state tax of $\frac{1}{10}$ of 1 per cent. is levied on all the property of the state, how much must that county contribute whose property is valued at $9863473?

99–100. If all the taxable property of New York is estimated at $666089526, what would be the whole tax, if levied at 83 per cent. of 1 per cent.? What would the tax be at 31 per cent. of 1 per cent.?

101–103. If a town raise a tax of 2 mills on the dollar, how much must A. pay, whose property is estimated at $10500? How much must be paid on $37950? How much on $1500?

104. I own real* property to the amount of $45650, and personal* property to the amount of $4500. What will be the amount of my taxes if my state tax is $2\frac{1}{2}$ per cent.; my town tax 3 per cent., my city tax $4\frac{1}{8}$ per cent., and my school tax $\frac{3}{4}$ per cent.?

105. What is the difference between a tax of 37 per cent. on $9876, and a tax of 0·37 per cent.?

Custom-House Business.

§ 101. Duties are taxes levied by government upon goods imported into the country.

These duties, established by Congress, and collected by custom-house officers at the various ports of entry, constitute the *revenue* of a country.

Duties are either *specific* or *ad valorem.* A specific duty is a fixed sum imposed on a ton, hundred weight, hogshead, gallon, yard, &c., without regard to the value of the commodity.

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* *Real* estate or property consists of lands, houses, &c., which cannot be moved. *Personal* estate is property consisting of stocks, mortgages, money, furniture, goods, &c.

An ad valorem duty is a percentage computed on the cost of the article in the country from which it is imported.

Gross weight is the entire weight of merchandise, with the cask, box, bag, &c., containing it. *Net* weight is the weight of the merchandise after all deductions. Duties are computed on the net weight.

Draft is an allowance for waste. *Tare* is an allowance for the weight of the cask, box, &c., deducted after the draft. *Leakage* and *breakage* is an allowance of 2 per cent. for the waste of liquors in cask, paying duty by the gallon; of 10 per cent. on beer, ale, and porter in bottles; and of 5 per cent. on all other liquors in bottles.

The following is the allowance for draft:

	lb.	*lb.*		*lb.*	*lb.*
On	112	1	Between	336 and 1120	4
Between	112 and 224	2	"	1120 and 2016	7
"	224 and 336	3	Above	2016	9

NOTE.—Though not mentioned in a question, draft or leakage must be deducted before the other specific allowances are made.

In estimating ad valorem duties, no deductions of any kind are to be made.

EXAMPLES.

106. What is the duty on 150 bags of coffee, the gross weight of each bag being 158*lb.*, invoiced* at 7 cents per pound, the tare being 4 per cent. and duty 20 per cent. ?

107. At 40 per cent. ad valorem, what will be the duty on 346*lbs.* sewing-silk, bought at Florence at $2·50 per pound ?

108. What is the duty on 114 barrels of olive oil, at 9 cts. per gallon, allowing 2 per cent. for leakage ?

109. When there is a specific duty on tea of 12 cts. per pound, what must be paid on 175 chests, each weighing 112*lbs.*, tare 8 per cent. ?

110. What is the duty, at 3 per cent., on 47 bags of

* An invoice is a schedule of the articles imported, with the cost thereof.

pepper, each weighing 130 pounds gross, invoiced at 5 cts. per pound, the tare being 3 per cent.?

111. What is the specific duty on 20 chests of tea, at 10 cents per pound, the gross weight of the whole being 378 pounds, and tare on the whole being 56*lbs.*?

112. What is the duty, at 3 cents per pound, on 40 bags of Madeira nuts, each weighing 50 pounds, the tare being 3 per cent.?

113. What is the ad valorem duty, at 30 per cent., on an invoice of $3400 of broadcloths?

114. What is the ad valorem duty, at 55 per cent., on a case of silks invoiced at $8532?

115. What is the ad valorem duty, at 17 per cent., on 50 bags of coffee, each weighing 97 pounds, and which is invoiced at $9\frac{1}{2}$ cents per pound?

116. What is the ad valorem duty, at $37\frac{1}{2}$ per cent., on 2 gross of cutlery, invoiced at $352?

117. What is the ad valorem duty, at 40 per cent., on a case of silks, invoiced at $3192?

118. What is the ad valorem duty, at 50 per cent., on a case of Leghorn hats, invoiced at $1370?

119. What is the duty on 325 dozen bottles of porter, at 3 cents per bottle, allowance for breakage being made?

120. What is the duty on 400 dozen bottles of London Brown Stout, at 4 cents per bottle?

121. What is the duty on 1000 bottles of Madeira wine, at 8 cents per bottle?

122. What is the duty on 20 casks of wine, each containing 42 gallons, at 13 cents per gallon?

123. What is the duty on 5 barrels of Spanish tobacco, the gross weight of the whole being 637 pounds, tare 5 per cent., at $5\frac{1}{2}$ cents per pound?

124. What is the duty on 15 hogsheads of molasses, each

containing 63 gallons, at 8 cents per gallon, usual allowance of 2 per cent. for leakage being made?

125. What is the duty, at 3 per cent., on 7 hogsheads of sugar, the gross weight of the whole being 9430 pounds, tare being 15 per cent.?

126. What is the duty, at $37\frac{1}{2}$ per cent., on a bale of linen, invoiced at $2333?

127. What is the duty, at 15 per cent., on a package of indigo, invoiced at $200?

128. What is the duty, at 30 per cent., on 10 cases of French broadcloths, invoiced at $8575?

129. What is the duty, at 13 cents per gallon, on 35 casks of wine, each containing 56 gallons, usual leakage being deducted?

130. What is the duty, at 25 per cent., on a quantity of lace, invoiced at $9863?

Insurance.

§ 102. Insurance is a contract, by which an individual, or a company, bind themselves to make good any loss or damage of property by fire, or storms at sea, or other casualties.

Ships and their cargoes, houses, furniture, cattle, &c., are insured.

Life-insurance is a guaranty for the payment of a certain sum of money on the death of the insured. Health-insurance secures a weekly allowance during the sickness of the insured.

This insurance is effected in consideration of a sum of money, called the *premium*, which is paid beforehand, to the insurers or *underwriters*. The written agreement of indemnity is called a *policy*.

The premium is estimated at a certain rate per cent. on the amount insured.

EXAMPLES.

131. If A. gets his house insured for \$1800, at 41 cents on \$100, what will be the amount of the premium?

132. An insurance of \$12000 was effected on the ship Ocean, at a premium of 2 per cent. What did the premium amount to?

133. I effected an insurance of \$5230 on my dwelling-house and furniture for 1 year, at $\frac{3}{8}$ of 1 per cent. What did the premium amount to?

134. What is the amount of premium for insuring \$34567, at 60 cents on \$100?

135. What would be the premium for insuring a ship and cargo, valued at \$46370, from Boston to Liverpool, at $2\frac{1}{4}$ per cent.?

136–140. What is the insurance on a dwelling and furniture, valued at \$15000, at 1 per cent.? at $1\frac{1}{2}$ per cent.? at $2\frac{1}{8}$ per cent.? at $2\frac{1}{2}$ per cent.? at $2\frac{3}{4}$ per cent.?

141–142. A house, valued at \$2800, is insured at 45 cents on \$100. What was the premium? At 48 cents on \$100, what would have been the premium?

143. A person at the age of 38, effects an insurance on his life for a period of 7 years, for the sum of \$5000, at the rate of \$1·70 on \$100 per annum. What is the annual premium?

144. An insurance is taken for a person aged 21 years, for life for the sum of \$7500, at \$1·82 on \$100. What is the annual premium?

145. A person at the age of 18 years effects an insurance during life for the sum of \$10000, at the rate of \$1·69 on \$100. How much is the annual premium?

146. A life-insurance for the sum of \$8000 during life, is taken by a person 50 years old, at the rate of \$4·60 on \$100. What is the annual premium?

147. A person going to California, with the intention of returning at the end of 3 years, effects an insurance of \$5500 on his life for the benefit of his family, at $1\frac{1}{2}$ per cent. per annum. What was the annual premium?

148–150. What is the annual premium on a life insurance of \$18000, at $1\frac{1}{3}$ per cent.? what at 2 per cent.? what at $2\frac{1}{2}$ per cent.?

Profit and Loss.

§ 103. Profit and Loss signify the amount which the merchant gains or loses in his business transactions.

CASE I.

A. bought 20 yards of broadcloth, at \$1·50 per yard, and sold the same at \$2·50 per yard. *How much did he gain?* He gained \$2·50—\$1·50=\$1·00 per yd.; \$1·00 × 20, the number of yards, =\$20, the whole gain.

Suppose A. sold the cloth at \$1·25 per yard. *How much did he lose?* He lost \$1·50—\$1·25=\$0·25 per yard; \$0·25 × 20=\$5·00, the whole loss.

CASE II.

What *per cent.* did A. gain by the first operation, and what *per cent.* did he lose by the second operation?

Original cost per yard \$1·50: gain per yard \$1·00; per cent. gain $\frac{1 \cdot 00}{1 \cdot 50}=0 \cdot 66\frac{2}{3}$, or $66\frac{2}{3}$ per cent.

Original cost per yard \$1·50; loss per yard \$0·25; per cent. loss $\frac{25}{150}=0 \cdot 16\frac{2}{3}$, or $16\frac{2}{3}$ per cent.

CASE III.

How much per yard did A. sell his broadcloth for, at $66\frac{2}{3}$ per cent. profit? How much for, at $16\frac{2}{3}$ per cent. loss?

The gain per yard is $\$1{\cdot}50\times0{\cdot}66\frac{2}{3}$, which added to $\$1{\cdot}50$, the original cost, gives for the selling price $\$1{\cdot}50+\$1{\cdot}50\times0{\cdot}66\frac{2}{3}$, or, which is the same thing, $\$1{\cdot}50\times1{\cdot}66\frac{2}{3}=\$2{\cdot}50$.

Again, the loss per yard is $\$1{\cdot}50\times0{\cdot}16\frac{2}{3}$, which subtracted from $\$1{\cdot}50$, the original cost, gives for the selling price $\$1{\cdot}50-\$1{\cdot}50\times0{\cdot}16\frac{2}{3}$, or, which is the same thing, $\$1{\cdot}50\times0{\cdot}83\frac{1}{3}=\$1{\cdot}25$.

CASE IV.

What was the cost per yard of the broadcloth, if A. sold it at $\$2{\cdot}50$ per yard, gaining $66\frac{2}{3}$ per cent.?

What if he sold it at $\$1{\cdot}25$ per yard, losing $16\frac{2}{3}$ per cent.?

The gain is evidently $0{\cdot}66\frac{2}{3}$ of the original cost, and $\$2{\cdot}50$, the selling price, is equal to the original cost, $+\ 0{\cdot}66\frac{2}{3}$ of the original cost, or, what is the same thing, equal to $1{\cdot}66\frac{2}{3}$ of original cost; hence, the original cost was $\$2{\cdot}50\div1{\cdot}66\frac{2}{3}=\$1{\cdot}50$.

Again, the loss is $0{\cdot}16\frac{2}{3}$ of the original cost, and $\$1{\cdot}25$, the selling price, is equal to the original cost $-\ 0{\cdot}16\frac{2}{3}$ of the original cost, or, what is the same thing, equal to $0{\cdot}83\frac{1}{3}$ of original cost; hence the original cost was $\$1{\cdot}25\div0{\cdot}83\frac{1}{3}=\$1{\cdot}50$.

NOTE.—The preceding might be solved by the use of the *ratio*, § 114. In the case of gain the cost was $\frac{100}{166\frac{2}{3}}$ of the selling price; that is, $\$2{\cdot}50\times\frac{100}{166\frac{2}{3}}$; in the case of the loss, the cost was $\frac{100}{83\frac{1}{3}}$ of the selling price, or $\$1{\cdot}50\times\frac{100}{83\frac{1}{3}}$.

From the preceding demonstrations, we deduce the following

RULES.

I. *The total gain or loss is the difference between the first cost and the selling price.*

II. *The gain or loss upon a part, divided by the cost of that part, or the whole gain or loss, divided by the whole cost, will give the gain or loss per cent.*

III. *The first cost multiplied by* 1, *plus the gain per cent., or by* 1 *minus the loss per cent., expressed as a decimal, will give the selling price.*

IV. *The selling price divided by* 1 *plus the gain per cent., or by* 1 *minus the loss per cent., expressed as a decimal, will give the cost.*

MISCELLANEOUS EXAMPLES.

151. Bought 300 yards of broadcloth, at $2·25 per yard, and sold the same at $3·50 per yard. How much was gained?

152. A merchant bought 320 barrels of flour, at $5 per barrel, but finds that he must lose 10 per cent. in the sales. How much will he receive for the whole?

153. Suppose I buy 25 cords of maple wood, at $2·50 per cord, and sell it so as to make 25 per cent. What must I receive for the whole?

154. Bought a house and lot for $1400, and sold it for $1200. How much per cent. did I lose?

155. Bought 225 gallons of molasses for 26 cents per gallon, and sold the whole for $64·35. What did I gain per cent.?

156. Bought 75 pounds of coffee, at 10 cents per pound. At how much per pound must I sell it so as to gain $3 on the whole?

157. Bought 25 hogsheads of molasses, at $18 per hogshead, in Havana: paid duties, $16·30; freight, $25; cartage, $5·50; insurance, $25·25. What per cent. shall I gain, if I sell it at $28 per hogshead?

158. If I buy broadcloth for \$3·50 per yard, how much must I sell it at per yard so as to gain 25 per cent.?

159. If I buy cloth at \$3·50 per yard, how much must I sell it at per yard so as to lose 25 per cent.?

160. A person bought a city lot for \$800, and sold it so as to gain 40 per cent. How much did he sell it for?

161. A house which cost \$3000 was sold for \$2400. What per cent. was lost?

162. A house which cost \$2400 was sold for \$3000. What per cent. was gained?

163–164. If I buy an article for 2*s.* and 6*d.*, and sell it for 3*s.*, what per cent. do I gain? But if I sell it for 2*s.*, what per cent. do I lose?

165. If I buy eggs at 13 cents per dozen, and sell the same at 19 cents per dozen, what per cent. do I make?

166. If eggs which cost 19 cents per dozen, are sold at 13 cents per dozen, what is the loss per cent.?

167. I sold a house for \$4800, on which I gained 20 per cent. What did the house cost me?

168. I bought a railroad bond for \$1055, which was $5\frac{1}{2}$ per cent. above par. What was the par value?

169. Bought 50 shares of plank road stock at $3\frac{1}{2}$ per cent. below par, for \$4825. What was the par value of one share?

170. If I buy stock at $3\frac{1}{2}$ per cent. below par, and sell the same at $5\frac{1}{2}$ above, what per cent. do I gain?

171. If I buy a city lot for \$3150, at what price must I sell it so as to gain 40 per cent.?

172. I buy 500 barrels of flour, at \$5·37$\frac{1}{2}$, but am obliged to sell it at a loss of 15 per cent. What do I receive for the whole?

173. A. buys \$1000 worth of stock, which he sells to B. at a gain of 5 per cent. B. in turn sells the same to C. at a gain of 5 per cent. What did the stock cost C.?

174. A. buys $1000 worth of merchandise, which he sells to B. at a loss of 5 per cent.; B. in turn sells the same to C. at a loss of 5 per cent. How much did C. give for the merchandise?

175–176. A. buys an article for £2 3*s.* 6*d.*, and sells it to B. at a gain of 10 per cent.; B. in turn sells it to C. at a loss of 10 per cent. How much did C. pay for the same? What per cent. of original cost did he give?

177–178. If I buy 600 barrels of flour, at $5·25 per barrel, and sell $33\frac{1}{3}$ per cent. of the same at a profit of 10 per cent., and the balance at a profit of $12\frac{1}{2}$ per cent., how much shall I receive for the whole? And what per cent. shall I gain on the whole?

179. Sold a city lot for $1750, and find that I have lost $12\frac{1}{2}$ per cent. What did the lot cost?

180. Sold a city lot for $2000, and find that I have gained $15\frac{5}{33}$ per cent. What did the lot cost?

SIMPLE INTEREST.

§ 104. INTEREST is the sum paid for the use of money, by the borrower to the lender. It is estimated at a certain rate *per cent. per annum;* that is, a certain number of dollars for the use of $100, for one year. Thus, when $6 is paid for the use of $100, for one year, the interest is said to be at 6 *per cent.;* when $5 is paid for the use of $100 for one year, the interest is said to be at 5 *per cent.*, &c.

The rate *per cent.* is generally fixed by law. In the New England States it is 6 *per cent.*, while in the State of New York it is 7 *per cent.*

The sum of money borrowed, or upon which the interest is computed, is called the *principal.* The principal, with the interest added to it, is called the *amount.*

CASE I.

To find the interest on any given principal, for any whole number of years, at any given rate per cent.

What is the interest of \$365·50 for 3 years, at 7 per cent.?

The interest of \$365·50 for one year at 7 per cent. is \$365·50 × 0·07 = \$25·585; which multiplied by 3, the number of years, gives \$76·755 for the interest of \$365·50 for 3 years at 7 per cent. Hence the following

RULE.

Multiply the principal by the rate per cent., and the product so obtained by the number of years. Point off as usual.

EXAMPLES.

181–185. What is the interest of \$27 for 5 years, at 6 per cent.? of \$98? of \$279·50? of \$33120·01? of \$6958290·035?

186–190. What is the interest of \$68 for 13 years, at 7 per cent.? of \$142, for the same rate and time? of \$987·41? of \$654201·90? of \$9412860·007?

191–197. Find the interest on \$69582·57 for 2 years, at 5 per cent.; at 6 per cent.; at 7 per cent.; for 9 years, at 8 per cent.; at 10 per cent.; at 15 per cent.; at 24 per cent.

198–203. Find the interest of \$9812·17 for 48 years, at $1\frac{1}{2}$ per cent.; at $2\frac{1}{2}$ per cent.; at $3\frac{1}{3}$ per cent.; at $4\frac{2}{3}$ per cent.; at $5\frac{4}{5}$ per cent.; at $6\frac{7}{8}$ per cent.

204–210. What is the interest of \$375, at $5\frac{1}{2}$ per cent., for 11 years? for 13 years? for 15 years? for 17 years? for 19 years? for 23 years? for 27 years?

CASE II.

To find the interest on any given principal for any given time, *at* 6 *per cent.*

The interest on \$1 for one year, is \$0·06; and since 2 months is $\frac{2}{12}=\frac{1}{6}$ of a year, the interest on \$1 for 2 months is \$0·01; again, since 6 days is $\frac{6}{60}=\frac{1}{10}$ of 2 months when we reckon 30 days to each month, it follows that the interest on \$1 for 6 days is \$0·001. Hence, *if we call half the number of months* CENTS, *and one-sixth the number of days* MILLS, *we shall obtain the interest of* \$1 *for the given time, at* 6 *per cent.* Then the interest of \$1 being multiplied by the number of dollars in the given principal, will give the interest sought. As an example, suppose we wish the interest of \$125 for 1 year, 5 months, and 18 days, at 6 per cent.

\$0·085=int. of \$1 for 1 y. 5 m.=17 months.
3= " " " 18 days.

\$0·088=int. of \$1 for 1 y. 5 m. and 18 days.

If now we multiply \$0·088 by 125, the number of dollars in the principal, or, which is the same thing, if we multiply \$125 by 0·088, we shall find \$125×0·088=\$11, the interest sought.

EXAMPLES.

211. What is the interest of \$49·37 for 13 months and 15 days, at 6 per cent.?

212. What is the interest of \$608·62 for 1 year and 9 months, at 6 per cent.?

213. What is the interest of \$341·13 for 7 years and 9 days, at 6 per cent.?

214. What is the interest of \$100 for 16 years and 8 months, at 6 per cent.?

215. What is the interest of \$591·03 for 4 years, 3 months, and 7 days, at 6 per cent.?

216. What is the interest of \$0·134 for 4 months and 3 days, at 6 per cent.?

217. What is the interest of \$7·50 for 7 months, at 6 per cent.?

218. What is the interest of \$371·01 for 4 years and 15 days, at 6 per cent.?

219. What is the interest of \$57·92 for 3 years, 7 months, and 9 days, at 6 per cent.?

220. What is the interest of \$329 for 5 years and 13 days, at 6 per cent.?

221. What is the interest of \$47·39 for 1 year and 7 months, at 6 per cent.?

222–224. At 6 per cent. what will \$4650 amount to in 1 year and 3 months? in 2 years, 5 months, and 10 days? in 2 years, 1 month, and 6 days?

225–227. In 1 year, 6 months, and 6 days, at 6 per cent., how much will \$350 amount to? How much will \$490 amount to? How much will \$375 amount to?

228–230. At 6 per cent. what will \$1000 amount to in 3 years and 6 months? in 3 years and 6 days? in 4 years and 4 months?

CASE III.

To find the interest on any given principal for any given time, at any given rate per cent.

First method.

Find the interest of \$300 for 1 year, 3 months, and 12 days, at $4\frac{1}{2}$ per cent.

At 6 per cent. the interest would be \$23·10; at $4\frac{1}{2}$ per cent. it would be $4\frac{1}{2}$ *sixths* or $\frac{9}{12}=\frac{3}{4}$ of \$23·10=\$17·325; at 7 per cent. the interest would be 7 *sixths* of \$23·10=\$26·95, &c. Hence this

RULE.

Find the interest on the given principal for the given *time at* 6 *per cent.* as by Case II. *Then take as many* SIXTHS *of such interest as will equal the given per cent.*

EXAMPLES.

231. What is the interest of $19·41 for 1 year, 7 months, and 13 days, at 7 per cent.?

232. What is the interest of $530 for 3 years and 6 months, at 5 per cent.?

233. What is the interest of $5·37 for 4 years and 12 days, at 8 per cent.?

234. What is the interest of $4070 for 3 months, at 9 per cent.?

235. What is the interest of $3671 for 6 months, at 10 per cent.?

236. What is the interest of $4920·05 for 3 months, at 4 per cent.?

237. What is the interest of $40·17 for 3 months and 18 days, at 3 per cent.?

238. What is the interest of $37·13 for 5 months and 12 days, at 4½ per cent.?

239. What is the interest of $489 for 3 years and 4 months, at 5½ per cent.?

240. What is the interest of $700 for 1 year and 9 months, at 7 per cent.?

NOTE.—When the principal is given in English money, we must reduce the shillings, pence, and farthings to the decimal of a £; and then proceed as in Federal Money.

241. What is the interest of £75 13*s.* 6*d.* for 3 years and 5 months, at 6 per cent.?

242. What is the interest of £14 5*s.* 3½*d.* for 4 years, 6 months, and 14 days, at 7 per cent.?

243. What is the interest of £1 7*s.* 6*d.* for 2 years and 6 months, at 4½ per cent.?

244. What is the interest of £105 10*s.* 6*d.* for $9\frac{1}{2}$ months, at 5 per cent.?

245–247. What is the amount of $503·50 for 1 year and 8 months, at 5 per cent.? what at 6 per cent.? what at 7 per cent.?

248–250. What is the amount of $401·13 for 3 months, 13 days, at $5\frac{3}{4}$ per cent.? at $6\frac{1}{2}$ per cent.? at $6\frac{1}{3}$ per cent.?

Second method.

Find the interest of $126 for 3 years, 5 months, and 15 days, at 7 per cent.

TABLE OF ALIQUOT PARTS OF A YEAR OR MONTH.

mo.		*yr.*				
6	=	$\frac{1}{2}$	15*da.*	=	$\frac{1}{2}$	of 1*mo.*
4	=	$\frac{1}{3}$	10	=	$\frac{1}{3}$	"
3	=	$\frac{1}{4}$	6	=	$\frac{1}{5}$	"
2	=	$\frac{1}{6}$	5	=	$\frac{1}{6}$	"
1	=	$\frac{1}{12}$	3	=	$\frac{1}{10}$	"

	$126		
	0·07		
	$8·82	=	1 year's interest.
	3		
	$26·46	=	3 years' "
4 mos. = $\frac{1}{3}$ of a yr.	2·94	=	4 months' "
1 mo. = $\frac{1}{4}$ of 4 mos.	735	=	1 " "
15 dys. = $\frac{1}{2}$ of 1 mo.	3675	=	15 days' "

Ans. $30·5025 = 3 yrs. 5 mos. and 15 days' int.

Hence the following

RULE.

Multiply as by rule, Case I. *Then find the interest for months and days by means of aliquot parts.*

EXAMPLES.

251–254. What is the interest of \$39·42 for 1 year, 5 months, and 11 days, at 7 per cent.? of \$678·24? of \$9872·86? of \$27541·03?

255–257. What is the interest of \$47·13 for 7 months and 21 days, at 7 per cent.? at $9\frac{1}{2}$ per cent.? at $14\frac{1}{2}$ per cent.?

258–259. What is the interest of \$321·21 for 3 months and 15 days, at 6 per cent.? for 5 years, 9 months, and 21 days?

260–262. What is the interest of \$270 for 2 months and 8 days, at 7 per cent.? of \$57602·01? of \$4930016·02?

263–265. What is the interest of \$404·44 for 1 year, 5 months, and 4 days, at 7 per cent.? of \$808·88? of \$297654·03?

266. What is the interest of \$99·99 for 11 months and 29 days, at 5 per cent.?

267. What is the interest of \$37·50 for 6 months and 10 days, at $6\frac{1}{2}$ per cent.?

268. What is the interest of \$49·49 for 8 months and 8 days, at 7 per cent.?

269–271. What is the amount of \$4650, at 7 per cent., for 1 year and 10 days? for 2 years and 3 months? for 3 years, 4 months, and 12 days?

272–275. What is the amount of \$317·12 for 2 years, 5 months, 18 days, at $3\frac{1}{2}$ per cent.? at 4 per cent.? at 5 per cent.? at $5\frac{1}{2}$ per cent.?

Interest when the time is estimated in days.

§ 105. Thus far, we have considered the time, for which interest is to be computed, as estimated in months and days, counting a month as $\frac{1}{12}$ of a year, and a day as $\frac{1}{30}$ of a month, or $\frac{1}{360}$ of a year. But as some months have 31 days, while February has but 28 or 29, we, by the previous methods, obtain sometimes too much interest, and sometimes too little, though the error must always be small.

There is a more accurate method of computing interest by means of days.

Suppose we wish the interest of \$500 from May 15th to November 20th, at 7 per cent. We find $\$500\times0{\cdot}07=\$35$ for one year's interest of \$500, at 7 per cent. By Table under § 83, note 4, we find 189 days from May 15th to November 20th.

The interest for 189 days must be the same fractional part of one year's interest that 189 days is of 365 days. Hence, $\$35\times\frac{189}{365}=\frac{\$35\times189}{365}=\$18{\cdot}123+$ for the interest of \$500 from May 15th to November 20th, at 7 per cent.

Hence this

RULE.

Find the interest for one year. Multiply this by the time expressed in days, and divide the product by 365; *the quotient will be the interest sought.*

A note of \$37·37 was given May 3, 1848. How much was due on it Dec. 27, 1848, at 7 per cent.?

By the table under § 83, note 4, we find 238 days from May 3 to Dec. 27.

```
$37·37=principal.       365)622·5842(1·705=int. sought.
  0·07=rate per cent.       365       37·37 =principal.
 ______                     ___      ______
$2·6159=one year's int.     2575    $39·075=am'nt. Ans.
    238=time in days.       2555
 ______                     ____
 209272                       2084
 78477                        1825
52318                         ____
______                         259
622·5842
```

EXAMPLES.

276. A note of $365 was given July 4, 1847. What will it amount to June 1, 1849, interest being 7 per cent.?

277. What is the interest on $100 from January 13th to November 15, it being leap-year, and interest being 6 per cent.?

278. What is the interest on $216 from March 10th to December 1st, interest being 5 per cent.?

279. What is the interest on $107 from April 12th to July 4th, interest being 7 per cent.?

280. What is the interest on $1000 from June 20th to August 13th, interest being 7 per cent.?

281. What is the interest on $730 from July 4th to December 25th, interest being 6 per cent.?

282. What is the interest on $63·37 from August 9th to December 31st, interest being 7 per cent.?

283–284. What is the amount of $210 at 5 per cent., from March 1st until the 25th of the following December? What is the amount of the same sum from July 4th until January 1st, at 7 per cent.?

285–287. What is the interest at $5\frac{1}{2}$ per cent. of $325 from April 1st until August 10th? from August 10 until Oct. 5th? from Oct. 5th until Dec. 8th?

288–290. From May 3d until August 8th, what is the interest on $75, at 5 per cent.? at 6 per cent.? at 7 per cent.?

Partial Payments.

§ 106. When notes, bonds, or obligations receive partial payments, or indorsements,* the rule adopted by the Supreme Court of the United States is as follows:

* From a Latin phrase, *in dorso*, meaning "upon the back;" because the payments are written across the back of the note.

RULE.

"*The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceed the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments taken together exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance, as aforesaid.*"

The above rule has been adopted by *New York, Massachusetts,* and by nearly all the other States of the Union.

CONNECTICUT RULE.

"*Compute the interest on the principal to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner, from one payment to another, till all the payments are absorbed; provided the time between one payment and another be one year or more. But if any payments be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation, for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above.*

"*If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.*"

$620. UTICA, NOV. 1, 1837.

For value received, I promise to pay Thomas Jones, or order, the sum of six hundred and twenty dollars, on demand, with interest. CHARLES BANK.

The following indorsements were made on this note: 1838, Oct. 6, received $61·07; March 4, 1839, $89·03; Dec. 11, 1839, $107·77; July 20, 1840, $200·50.

What was the balance due, Oct. 15, 1840, allowing 7 per cent. interest, according to the U. S. rule?

The pupil will find it convenient to arrange the work for finding the *multipliers* at 6 per cent. as follows:

	year.	*mo.*	*da.*	*mo.*	*da.*	*Multipliers at 6 per cent.*
Date of note............	1837	10	1			
1st indorsement.......	1838	9	6	11	5	$0·055\frac{5}{6}$.
2d indorsement	1839	2	4	4	28	$0·024\frac{2}{3}$.
3d indorsement	1839	11	11	9	7	$0·046\frac{1}{6}$.
4th indorsement	1840	6	20	7	9	0·0365.
Date of settlement ...	1840	9	15	2	25	$0·014\frac{1}{6}$
				35	14	$0·177\frac{1}{3}$.

The intervals of time are found by subtracting the earlier date from the one next below it, § 89, Ex. 471, &c.

To test the accuracy of the work, we may add the intervals together, making 35*mo.* 14*da.*; and the multipliers together, making $0·177\frac{1}{3}$ Now, subtracting the time when the note was given from the time of settlement, we also obtain 35 months and 14 days, which time gives $0·177\frac{1}{3}$ for multiplier.

It is well, in all cases where interest is to be cast on a note of many indorsements, to follow the above method, since by so doing, there is less chance for committing errors. In each particular computation of interest, when the value beyond the third place is one-half or more, add a unit to the decimal in the third place; when that value is less than one-half, neglect it.

Having found the *multipliers*, we continue the work as follows:

The amount of note, or principal, is	$620·000
Int. on the same to Oct. 6, 1838, at 7 per cent., is	40·386
Amount due on note, Oct. 6, 1838, is	660·386
The first indorsement is	61·070
	599·316
Interest from Oct. 6, 1838, to March 4, 1839, is	17·247
Amount due March 4, 1839, is	616·563
The second indorsement is	89·030
	527·533
Interest from March 4, 1839, to Dec. 11, 1839, is	28·414
	555·947
The third indorsement is	107·770
	448·177
Interest from Dec. 11, 1839, to July 20, 1840, is	19·085
	467·262
The fourth indorsement is	200·500
	266·762
Interest from July 20, 1840, to Oct. 15, 1840, is	4·409
Ans.	271·171

EXAMPLES.

$350. UTICA, May 1, 1836.

291. For value received, I promise to pay Isaac Clark, or order, three hundred and fifty dollars, with interest, at 6 per cent. N. BROWN.

Dec. 25, 1836, there was indorsed $50; June 30, 1837, $5; Aug. 22, 1838, $15; June 4, 1839, $100.

How much was due April 5, 1840, if interest is computed according to the U. S. rule?

292. How much was due according to the Connecticut rule?

NOTE.—We will here indicate the steps of the process under the Connecticut rule. First, find the amount of the principal sum for one year; that is, to May 1, 1837. Then find the amount of the first payment to the same date. Deduct the latter amount from the former. Next, find the amount of the new principal thus obtained for another year, that is, to May 1, 1838; then find the amount of the second payment to the same time, and deduct as before, and so on.

$\$108\frac{43}{100}$. UTICA, Dec. 9, 1835.

293. For value received, I promise to pay Peter Smith, or order, one hundred and eight dollars and forty-three cents, on demand, with interest, at 7 per cent.

JOHN SAVEALL.

March 3, 1836, there was indorsed \$50·04; Dec. 10, 1836, \$13·19; May 1, 1838, \$50·11.

How much remained due, according to the U. S. rule, Oct. 9, 1840?

294. How much according to the Connecticut rule?

NOTE.—After several steps, there will be a new principal, Dec. 9, 1838. The interest is to be computed upon this, not for one year, since there is no indorsement within the year, but up to the time of settlement.

$\$143\frac{50}{100}$. UTICA, Aug. 1, 1837.

295. For value received, I promise to pay D. Budlong, or bearer, one hundred and forty-three dollars and fifty cents, on demand, with interest. W. GOULD.

Dec. 17, 1837, there was indorsed \$37·40; July 1, 1838, \$7·09; Dec. 22, 1839, \$13·13; Sept. 9, 1840, \$50·50.

How much remains due, according to U. S. rule, Dec. 28, 1840, the interest being 7 per cent.?

296. How much according to Connecticut rule?

NOTE.—After a few steps we shall find a new principal, Aug. 1, 1838. We compute the interest on this up to Dec. 22, 1839, as there is no payment within a year. From the amount deduct the payment made Dec. 22, 1839. We have, then, another new principal, the interest on which is to be computed for one year, that is, to Dec. 22, 1840, and added; we find also the amount of the last payment to that date; deduct, and find amount of the balance, Dec. 28, 1840.

297. A note of $486 is dated Sept. 7, 1831.

March, 22, 1832, there was paid $125; Nov. 29, 1832, $150; May 13, 1833, $120.

What was the balance due, according to U. S. rule, April 19, 1834, the interest being 7 per cent.?

298. What was due according to Connecticut rule?

Problems in Interest.

§ 107. The principal, the rate per cent., the time, and the interest, are so related to each other, that any three of them being given, the remaining one can be found.

PROBLEM I.

Given the principal, the rate per cent., and the time, to find the *interest.*

RULE.

Multiply the interest of $1, *for the given time and given rate per cent. by the number of dollars in the principal.*

PROBLEM II.

Given the time, the rate per cent., and the interest, to find the *principal.*

It is obvious that the interest on a given sum is as many times greater than the interest on $1, as the given sum is times greater than $1. Hence the following

RULE.

Divide the given interest by the interest of $1 *for the given time at the given rate per cent.*

EXAMPLES.

299. The interest on a certain principal for 9 months and 10 days, at $4\frac{1}{2}$ per cent., is $1·01605. What was the principal?

300. What principal will, in 1 year, 7 months, and 15 days, at 6 per cent., give $9·75 interest?

301. What principal will, in 7 years and 9 days, at 6 per cent., give $16·86 interest?

302. What principal will, in 3 years and 6 months, at 5 per cent., give $92·75 interest?

303. What principal will, in 3 months and 9 days, at 8 per cent., give $90 interest?

304. If a man's property be invested at $5\frac{1}{2}$ per cent., how much is he worth, supposing his annual income to be $4372·50?

305. A widow is receiving $848 per annum. What is her property, supposing it invested at 6 per cent.?

306. The annual expenditures of an orphan asylum are $2753·00. What fund invested at 7 per cent. will produce that amount?

PROBLEM III.

Given the principal, the time, and the interest, to find the rate *per cent.*

If the rate per cent. be doubled, other things being the same, the interest will be doubled; if the rate per cent. is tripled, the interest will be tripled. And, in all cases, the interest at any particular rate per cent. is as many times greater than the interest at 1 per cent. as the given rate per cent. is times greater than 1 per cent. Hence we have this

RULE.

Divide the given interest by the interest of the given principal for the given time, at 1 *per cent.*

EXAMPLES.

307. The interest of \$100 for 9 months and 10 days is \$3·50. What is the rate per cent.?

In this example, we find the interest of \$100 for 9 months and 10 days, at 6 per cent., to be \$4·66$\frac{2}{3}$. The interest at 1 per cent. is \$0·77$\frac{7}{9}$; therefore, dividing \$3·50 by \$0·77$\frac{7}{9}$, we obtain 4$\frac{1}{2}$ for the rate per cent. required.

308. At what rate per cent. will \$530, in 3 years and 6 months, give \$92·75 interest?

309. At what rate per cent. will \$19·41, in 1 year, 7 months, and 13 days, give \$2·200339$\frac{1}{6}$ interest?

310. At what rate per cent. will \$5·37, in 4 years and 12 days, give \$1·73272 interest?

311. At what rate per cent. will \$4070, in 3 months, give \$91·575 interest?

PROBLEM IV.

Given the principal, the rate per cent., and the interest, to find the *time*.

If the time for which interest is computed be doubled, other things being the same, the interest will be doubled; if the time is tripled, the interest will be tripled. And in all cases, the interest for any particular time is as many times greater than the interest for one year, as the particular time is greater than 1 year. Hence, we have this

RULE.

Divide the given interest by the interest of the given principal, for 1 *year, at the given rate per cent.*

EXAMPLES.

312. In what time will \$37·13, at 4$\frac{1}{2}$ per cent., yield \$0·7518825 interest?

In this example, we find the interest of \$37·13 for 1 year, at $4\frac{1}{2}$ per cent., to be \$1·67085; therefore, dividing \$0·7518825 by \$1·67085, we get 0·45 years; this reduced to months and days gives 5 months and 12 days.

313. In what time will \$700, at 7 per cent., give \$85·75 interest?

314. In what time will \$100, at 6 per cent., give \$100 interest? That is, in what time will a given principal double itself at 6 per cent. interest?

315–329. In what time will a given principal double itself at 5 per cent. interest? at 6 per cent.? at 7? at 8? at 9? at 10? at 11? at 12? at $5\frac{1}{2}$ per cent.? at $6\frac{1}{2}$? at $7\frac{1}{2}$? at $8\frac{1}{2}$? at $9\frac{1}{2}$? at $10\frac{1}{2}$? at $11\frac{1}{2}$?

330–333. In what time will \$848 amount to \$965, at 4 per cent. interest? at 5 per cent.? at 6 per cent.? at 7 per cent.?

334–336. A note for \$636·50, at the time of its settlement, amounted to \$1748. How long was it on interest at 4 per cent.? at $5\frac{1}{2}$ per cent.? at $6\frac{1}{2}$ per cent.?

PROBLEM V.

Given the time, rate per cent., and *amount*, to find the *principal*.

This is the same as finding the *present worth* of a debt payable at some future time, without interest; that is, such a sum of money as will, if put at interest, for the given time, amount to the debt.

At 6 per cent. interest, the amount of \$1 for one year is \$1·06; therefore, the present worth of \$1·06 due one year hence is \$1. If the present worth of \$1·06 is \$1, the present worth of \$1 will be the same fractional part of \$1 that \$1 is of \$1·06; that is, $\frac{1}{1 \cdot 06}$ of \$1, or $\frac{\$1}{1 \cdot 06}$; so the present worth of \$2 is $\frac{\$2}{1 \cdot 06}$, &c.

Had the time been 6 months, the present worth of $1 would be $\frac{\$1}{1{\cdot}03}$; of $2, $\frac{\$2}{1{\cdot}03}$, &c.

At 7 per cent. interest, the present worth of $1 for one year would be $\frac{\$1}{1{\cdot}07}$; of $2, $\frac{\$2}{1{\cdot}07}$, &c. Hence the following

RULE.

Divide the sum, whose present worth is required, by the amount of $1 *for the given time at the given rate; the quotient will be the present worth.*

EXAMPLES.

337. What is the present worth of $622·75, due 3 years and 6 months hence, at 5 per cent.?

338. What is the present worth of $4161·575, due 3 months hence, at 9 per cent.?

339. What is the present worth of $7·10272, due 4 years and 12 days hence, at 8 per cent.?

340. Sold goods for $1500, to be paid one-half in 6 months, and the other half in 9 months. What is the present worth of the goods, interest being at 7 per cent.?

341. Sold goods for $1500, to be paid at the end of $7\frac{1}{2}$ months. What is the present worth of the goods, interest being at 7 per cent.?

342. What is the present worth of $50, payable at the end of 3 months, at $4\frac{1}{2}$ per cent.?

343. What is the present worth of $3471·20, due 3 years and 9 months hence, at 7 per cent.?

344. Bought a bill of goods for $1200, one-third payable in 3 months, one-third in 6 months, and the remaining one-third in 9 months. How much ready cash ought to pay for the goods, if we consider money worth 6 per cent.?

345. The amount of a note due 2 years, 7 months, and 13 days after date, is $71298·68. What is the principal, the rate being 6 per cent.?

Discount.

§ 108. Discount is an allowance made for the payment of money before it is due. It is found *by subtracting the present worth of the debt from the amount of the debt at the time when due.*

EXAMPLES.

346. What is the discount on $100, due 6 months hence, at 6 per cent.?

347. What is the discount on $750, due 9 months hence, at 7 per cent.?

348. What is the discount on $150, due 3 months and 18 days hence, at 6 per cent.?

349. What is the discount on $961·13, due 1 year and 5 months hence, at 7 per cent.?

350. What is the discount on $37·40, due at the end of 7 months, at 6 per cent.?

351–353. Bought a bill of goods, on 6 months' credit, amounting to $973·50. How much ought to be deducted if cash is paid at the time of receiving the goods, interest being considered at 6 per cent.? How much if interest is 7 per cent.? How much if interest is 8 per cent.?

354–356. A man purchases a farm of 97 acres, at $110 per acre, on a credit of 9 months. How much would he save by paying cash down for it, if interest is counted at 5 per cent.? How much if it is estimated at 6 per cent.? and how much if at $6\frac{1}{2}$ per cent.?

357–359. Bought a bill of goods of $1400, one-half on a credit of 6 months, and the other half on a credit of 9 months. If payment is made at the time of the purchase, how much ought to be deducted if 7 per cent. interest is considered? How much if 5 per cent. is reckoned? How much if $5\frac{1}{2}$ per cent.?

360–361. A person at the age of 18 years has a legacy of $500, which is to be paid to him when he is 21 years of age. How much ought to be discounted for ready cash, interest being 6 per cent.? How much, interest being 7 per cent.?

Compound Interest.

§ 109. In making contracts, it is often stipulated that the interest shall be paid annually or semi-annually, &c. If not paid at the specified time, the interest is added to the principal, forming a new principal, on which the next interest is to be computed. The final amount is called the *amount at compound interest.* If from this the original principal be subtracted, the remainder will be the *compound interest.*

What is the compound interest of $1000 for 3 years, at 7 per cent.?

Principal	$1000
Interest on $1000 for one year	70
First amount, or second principal	1070
Interest on $1070 for one year	74·90
Second amount, or third principal	1144·90
Interest on $1144·90 for one year	80·143
Third amount	1225·043
Original principal	1000
The compound interest required...... *Ans.*	$225·043

EXAMPLES.

362. What is the amount of $100 at 6 per cent. per annum, compound interest, for two years, the interest being payable semi-annually?

363. What is the compound interest of $630 for 4 years, at 5 per cent.?

364. What is the amount, at compound interest, of $50, for 3 years, at 5 per cent.?

365. What is the compound interest of $1000 for 4 years, at 6 per cent.?

366–368. What will $1700 amount to in 2 years, at 6 per cent. per annum, compound interest, the interest being payable semi-annually? How much, if interest is payable quarterly? How much, if payable annually?

369–370. What will be the compound interest of $333 for 2 years and 6 months, at 5 per cent. per annum, if interest is payable semi-annually? How much if the interest is payable quarterly?

Banking, and Bank Discount.

§ 110. A Bank is a corporation, chartered by law, for the purpose of receiving deposits, loaning money, dealing in exchange, and issuing bills or bank-notes, representing *specie*.

The money paid in to form the basis for the business of a bank is called the *capital stock*. This is divided into *shares*, and is owned by various individuals.

The affairs of a bank are managed by a *board of directors*, chosen annually by the stockholders. This board elect one of their number as *president*.

The *cashier*, appointed also by the directors, superintends the books, payments, and receipts of the bank. He and the president sign all the bills that are issued.

The *teller* is an officer who receives and pays money.

Money is borrowed from banks, on notes. These are promises to pay a certain sum at a specified time. The person who signs the note is called the *drawer* or *maker.* The person to whom the note is made payable, is called the *payee.* A note to be *negotiable,* that is, to pass from one to another and retain its value, must be *indorsed* by the payee. When so indorsed, a bank will *discount* it; that is, will deduct the interest from the amount for which the note is given (which amount is called the *face of the note*), and will lend the remainder.

This remainder is called the *present worth* or *proceeds.*

It is usual for the banks to take the interest for 3 days more than the time specified in the note; and the borrower is not obliged to make payment till these three days have expired, which are for this reason called *days of grace.*

The States establish the rate of interest by law. In New England it is 6 per cent.; in New York 7 per cent., though banks in this State are not permitted to take over 6 per cent., unless the notes discounted have more than 63 days to run.

§ 111. *Bank Discount* is the same as simple interest *paid in advance.** It is interest upon interest, and is, strictly speaking, usurious. Hence, to compute bank discount,

Cast the interest on the face of the note for 3 *days more than the specified time; the result will be the discount.*

The discount deducted from the face of the note will give the present worth or proceeds of the note.

EXAMPLES.

371. What is the bank discount on $1000 for 3 months, at 7 per cent.?

372–376. Find the bank discount on each of the following sums for the time and at the rate specified: $150 for 6 *mo.*,

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* This method of taking interest in advance, being usurious, has been discontinued by most banks, and instead thereof they deduct true discount, as found by § 108.

at 6 per cent.; $375 for 3 *mo.* 9 *da.*, at 7 per cent.; $400 for 9 *mo.*, at 7 per cent.; $29·30 for 7 *mo.*, at 5 per cent.; $472 for 10 *mo.*, at 7 per cent.

377. A note for $1800, payable in 60 days, was discounted at a bank at 6 per cent. What was received for the note?

NOTE.—Compute interest always for the *three days' grace.*

378. What is the present worth of a note for $6720 discounted at a bank, payable in 90 days, at 7 per cent.?

379. What will be the proceeds of a note for $857·50, payable in 30 days, discounted at 6 per cent. at a bank?

380–383. What will be the proceeds of the following notes, if discounted at a bank?

$650. ALBANY, Nov. 1, 1850.

For value received, I promise to pay John Norton, or order, six hundred and fifty dollars, in sixty days from date, the discount being made at 6 per cent.

HORACE ASHAM.

$848·25. ROCHESTER, Aug. 3, 1850.

Ninety days after date, I promise to pay Eli Stetson, or order, eight hundred and forty-eight dollars and $\frac{25}{100}$, for value received, the discount being made at 7 per cent.

ABRAM MOORE.

$69·28. LOCKPORT, Dec. 20, 1850.

Four months from date, I promise to pay Enoch Strasby, or order, sixty-nine dollars and twenty-eight cents, value received, the discount being made at 7 per cent.

ELLERY STRONG.

NOTE.—When the rate of interest is not specified, it is understood to be the *legal* rate of the State in which the transaction takes place.

$4280·75. BOSTON, Oct. 26, 1850.

Ninety days after date, we promise to pay Edwin Nicholson, or order, four thousand two hundred and eighty dollars and $\frac{75}{100}$, for value received. NAHUM & WALKUM.

$400. UTICA, Feb. 1, 1851.

Ninety days after date, I promise to pay at the Oneida Bank, to the order of Charles A. Mann, four hundred dollars, value received. JOHN JOHNSON.

384–385. What is the bank discount of the above note? And what the true discount as found by § 108?

$600. ALBANY, June 10, 1850.

Six months after date, I promise to pay at the Commercial Bank, to the order of Albertus Williams, six hundred dollars, value received. HIRON HARTER.

$300. ALBANY, June 10, 1850.

Three months after date, I promise to pay at the Commercial Bank, to the order of Albertus Williams, three hundred dollars, value received. HIRON HARTER.

386–389. What is the bank discount of each of the two foregoing notes? And what their true discounts?

390. What would be the bank discount of $900, the sum of both notes, for $4\frac{1}{2}$ months, the average time for which the above notes are made?

§ 112. When the present worth of a bankable note, the time for which it is to be discounted, and the rate per cent. are given, to find the amount or face of the note.

What must be the face of a bank-note which, when discounted for 4 months and 15 days, gives a present worth of $100, interest being 6 per cent.?

If we suppose the note to be $1, the bank discount for 4 months and 15 days will be $0·023; hence, $1 − $0·023 = $0·977, is the present worth.

If, then, $0·977 require $1 for the face of the note, $200 would require as many times $1 as $0·977 is contained times in $200. $200 ÷ $0·977 = $102·35 *Ans.* Hence this

RULE.

Divide the present worth, or the amount required to be raised, by the present worth of $1 *for the given time, and at the given rate of bank discount. The quotient will be the face of the note.*

EXAMPLES.

391. What must be the amount of a bankable note, so that when discounted for 3 months, at 6 per cent., it shall give a present worth of $600?

392. What must be the face of a bankable note, so that when discounted for 2 months, at 7 per cent., the borrower shall receive $50?

393. What must be the face of a bankable note, so that when discounted for 10 months, at 5 per cent., the present worth may be $1000?

394. What must be the face of a bankable note, so that when discounted for 7 months, at 7 per cent., the present worth may be $70·50?

395. What amount must I make my note, so that when discounted at the bank for 12 months, at 7 per cent., I may receive $100?

396. What must be the amount of a note, so that when discounted at the bank for 6 months, at 6 per cent., the borrower may receive $365?

397. A man bought a house for \$3287 cash. How large a note, payable in 90 days, must he take to a bank to realize that amount, at 6 per cent. discount?

398–401. For what sum must I draw my note, so that the bank proceeds, at 6 per cent. for 3 months, may be \$150? For what sum that the proceeds may be \$300? For what sum that the proceeds may be \$450? For what sum that the proceeds may be \$500?

402–403. I buy a bill of goods for \$675·50, without credit, and wish to make a bank-note for 60 days, which, discounted at 7 per cent., shall yield this sum. What must be the face of the note? What would be its face if discounted at 6 per cent. for the same time so as to give the same proceeds?

404. If a bank-note is discounted at 5 per cent. for 4 months, and yield \$101·75, what was its face?

405–407. I have three bank-notes, each discounted at 6 per cent.: the first for 3 months, the second for 4 months, and the third for 6 months; their proceeds were \$600, \$400, and \$300 respectively. What were their respective amounts?

408–410. I have three bank-notes, each discounted for 6 months: the first at 5 per cent., the second at 6 per cent., and the third at 7 per cent. They give equal proceeds, namely, \$1000. What was the face of each note?

CHAPTER XIII.

ANALYSIS AND RATIO.

ANALYSIS.

§ 113. If 4 men can do a piece of work in 9 days, in how many days will 6 men do the same work?

If 4 men can do the work in 9 days, 1 man will do it in $4 \times 9 = 36$ days. If 36 days be required for 1 man to do the work, 6 men will do it in $\frac{1}{6}$ of the time; that is, in 36 days $\div 6 = 6$ days.

This process of solution is termed ANALYSIS.

NOTE.—The word Analysis means the separating of any thing into its component parts or elements. It is applied here because the question is taken to pieces, and its various factors and their relations to each other are determined by the pupil's common sense rather than by any formal rule. It will be noted that the first thing to be ascertained in questions of this kind, is the value of 1 of each of the unknown quantities. This being found, the value required by the conditions of the question can be determined by multiplication.

EXAMPLES.

1–9. If 6 men can dig a field of potatoes in 12 days, in how many days will 2 men dig the same field? will 3 men? 4 men? 8 men? 9 men? 12 men? 18 men? 24 men? 36 men?

10–16. If 12 barrels of cider cost 18 dollars, what will 15 barrels cost? 18 barrels? 20 barrels? 25 barrels? 30 barrels? 50 barrels? 100 barrels?

17–22. It required 30 hands 8 days to load a vessel. How many days were required for 6 hands to load it? 8 hands? 10 hands? 12 hands? 20 hands? 60 hands?

23–28. It required 30 hands 8 days to load a vessel. How many hands were required to load it in 2 days? in 3 days? in 6 days? 10 days? 12 days? 16 days?

29. If a wheel revolve 12 times in going 10 rods, how many times will it revolve in going a mile?

30. A man paid 28 dollars for 16 gallons of wine. How much did he pay for 4 hogsheads?

31. If 20 *sq. ft.* of land cost $60, what would be the price of an acre at that rate?

32. A merchant invested in trade \$830, which was $\frac{7}{8}$ of all he possessed. What was his property?

NOTE.—\$830=$\frac{7}{8}$ of his property; $\frac{1}{8}$ of it is 830÷7.

33. A dying man bequeathed for public uses \$490000, which was $\frac{3}{16}$ of his property. How much did he possess?

34–40. If 10 yards of cloth cost \$12, what cost 15 yards? What cost 20 yards? What cost 25 yards? What cost 35 yards? What cost 45 yards? What cost 55 yards? What cost 65 yards?

41–44. If the interest of \$100 for one year is \$6, what will be the interest of \$50 for the same time? What will be the interest of \$100 for 6 months? what for 2 months? what for 4 months?

45–47. If 4 men mow a field in 12 hours, how many must be employed to mow it in 3 hours? How many to mow it in 4 hours? How many to mow it in 6 hours?

48–50. If a locomotive can run 40 miles in one hour, how far can it go in 10 minutes? How far in 12 minutes? How far in 15 minutes?

RATIO.

§ 114. Examples like the preceding may be performed by RATIO.

Ratio is the relation which one quantity bears to another. It is expressed as a quotient arising from the division of the first quantity by the second; thus, the relation of 7 to 8 is *seven-eighths*, or, expressed as a ratio, $\frac{7}{8}$, 7÷8, 7 : 8; the latter and most common form being the division symbol with the horizontal line between the dots omitted.

In any example, to obtain the required value by the use of ratio, it is simply necessary to multiply the number which is of the same denomination as the value sought, by the ratio between the other quantities, which ratio the conditions of the question will determine. Thus,

If 4 men consume 20 *lbs.* of meat in 3 weeks, how many pounds will 12 men consume in the same time? The denomination of the value sought is pounds; hence the number to be multiplied is 20. The ratio between the 12 and the 4 is $\frac{12}{4}$. Therefore 20 pounds $\times \frac{12}{4}$ = the value sought. By cancelation, $\overset{5}{\cancel{20}}$ pounds $\times \frac{12}{\cancel{4}} = 60$ pounds.

NOTE.—Every question must be carefully examined, to see whether the ratio is correctly expressed by the division of the greater number by the less, or of the less number by the greater.

If 4 men can do a piece of work in 9 days, in how many days will 6 men do the same work?

The denomination of the value sought is days; hence 9, the number of days, is to be multiplied.

The only difficulty now is to determine the ratio. The ratio cannot be $\frac{6}{4}$, for this would give a result showing that a longer time would be required for the 6 men to do the work than was required for the 4 men. The true ratio is $\frac{4}{6}$. This may be proved by analysis. If 4 men do the work in 9 days, it will take one man 4 times as long, which is 9×4 days; and 6 men can do it in one-sixth of this time, that is, in $\frac{9 \times 4}{6}$ days $= 9$ days $\times \frac{4}{6}$. In general, when it is not perfectly clear what the ratio is, ascertain it by analysis.

§ 115. There is no ratio between quantities of different denominations; as, for example, between 2 yards and 4 feet, between 7 dollars and 13 cents, or between £3 4*s.* and 3*s.* 4*d.* Before the ratio of two quantities, whether consisting of one or more denominations, can be determined, *they must be reduced to the same lowest denomination.* 2 yards = 6 feet;

\$7=700 cents ; £3 4*s*.=768*d*. ; 3*s*. 4*d*.=40*d*. Hence, the ratios of the preceding quantities are $\frac{6}{4}$; $\frac{700}{13}$; $\frac{768}{40}$.

EXAMPLES.

51–55. What is the ratio of 6 inches to 9 inches ? to 2 feet ? to 6 feet ? to 3 rods ? to 1 mile ?

56–60. What is the ratio of 7 pence to 8 shillings ? to 9*s*. 6*d*. ? to 18 shillings ? to £3 ? to £2 2*s*. 2*d*. 2*qr*. ?

61. What part of 50 men is 2 men ? that is, what is the ratio of 2 men to 50 men ?

62–65. What is the ratio of 7 per cent. to 8 per cent. ? to $4\frac{1}{2}$ per cent. ? to $5\frac{1}{2}$ per cent. ? to $7\frac{1}{2}$ per cent. ?

66. What part of 3 miles, 40 rods, is 27 feet, 9 inches ? that is, what is the ratio of 27*ft*. 9*in*. to 3*m*. 40*rd*. ?

67. What part of 1 day, 9*hr*. is 17 minutes, 4 seconds ?

68. What part of \$700 is \$5·30 ?

69. What part of 2 hogsheads is 3 pints ?

70. What part of \$3 is $2\frac{1}{2}$ cents ?

71. What part of 10 shillings, 8 pence, is 3 shillings, one penny ?

72. What part of 100 acres is 63 acres, 2 roods, 7 rods of land ?

73. In the Eagle there are $232\frac{1}{5}$ grains of pure gold, and $12\frac{9}{10}$ grains of silver, and the same quantity of copper. The silver and copper is each what part, by weight, of the gold ? And the silver and copper together is what part of the gold ?

74. In the United States standard silver coin of one dollar, there are $371\frac{1}{4}$ grains of pure silver, and $41\frac{1}{4}$ grains of copper. What fractional part is the copper of the silver ?

75. The silver in standard gold coin is what part of the silver in the same value of standard silver coin ?

76. The pound Troy contains 5760 grains, the pound

Avoirdupois contains 7000 grains. A pound Troy is what part of a pound Avoirdupois?

77. The Imperial gallon contains $277\frac{1}{4}$ cubic inches, nearly; the old wine gallon contains 231. What part of the Imperial gallon is the old wine gallon?

78. The solar year is 365 days, 5 hours, 48 minutes, 48 seconds. By what part of a day does this exceed 365 days?

PROMISCUOUS EXAMPLES.

79–83. If a ship's crew of 30 men consume $900 worth of provisions during a voyage of 60 days, how many dollars worth would they consume during a voyage of 117 days? of 30 days? of 45 days? of 72 days? of 99 days?

84–88. If a ship's crew of 30 men, during a voyage of 60 days, consume $900 worth of provisions, how many dollars worth would a crew of 7 men consume in the same time? how many would 15 men? 45 men? 72 men? 117 men?

89. If $\frac{2}{3}$ of a man's furniture be worth 860 dollars, what is the whole of it worth?

90. Five men earn $32 in a week. How much can 84 men earn in the same time?

91. Five men earn $32 in a week. How many weeks will it take 20 men to earn the same sum?

92–94. Five men earn $32 in a week. How many men will it take to earn the same sum in a day, or one-sixth part of a week? in an hour, allowing 10 working hours in a day? in a minute?

95. If a post 4 feet high cast a shadow of 12 feet, how long will that pole be that casts a shadow of 240 feet?

96. If a post 4 feet high cast a shadow of 12 feet, how long a shadow will a pole 90 feet high cast?

97. How high is yonder steeple? Its shadow is 14 feet long; and the shadow of this fence, $4\frac{1}{2}$ feet high, is 8 inches.

98. Suppose a poll-tax of $1344 be laid on a town containing 4370 inhabitants. What proportional tax must be laid on a town containing 721 inhabitants?

99. It took 140 laborers 9 days to dig a canal 14 rods long. How long would it take the same laborers to dig a canal 1*m.* 17*rd.* 2*yd.* 2*ft.* long?

100–106. If one gross of lead pencils is worth $1.50, what is 1 dozen worth? 2 dozen? $3\frac{1}{2}$ dozen? $4\frac{1}{3}$ dozen? $5\frac{2}{3}$ dozen? $7\frac{1}{2}$ dozen? 9 dozen?

107–112. If it require 84 bushels of apples to make 8 barrels of cider, how many barrels will 42 bushels make? How many will 100 bushels make? How many will 168 bushels make? How many bushels will be required to make 1 barrel? How many to make 6 barrels? How many to make 10 barrels?

113–121. If there are 165 feet in 10 rods, how many feet are there in 2 rods? in 4 rods? in 6 rods? in 15 rods? in 20 rods? in 25 rods? How many rods in 330 feet? How many rods in 66 feet? How many rods in 132 feet?

122–126. If 5 men can reap a field of grain in 3 days of 10 hours each, how many men would reap it in 10 hours? How many in 15 hours? How many in 25 hours? How many in 6 hours? How many in 30 hours?

127–137. If $100 gain $7 interest in 12 months, how much will it gain in 3 months? how much in 5 months? in 7 months? in 11 months? in 17 months? How much will $300 gain in 12 months? How much will $500 gain in 12 months? How much will $700 gain in 12 months? How much will be required to gain $10 in 12 months? How much to gain $18 in same time? How much to gain $24?

138–141. If $\frac{7}{8}$ of a ship is worth $49000, what is the whole worth? what is $\frac{1}{2}$ of it worth? what is $\frac{3}{8}$ worth? what is $\frac{5}{8}$ worth?

142–150. If a person can count 300 in one minute, how long will he require to count 45? how long to count 75? how long to count 225? How many can he count in 5 seconds? how many in 13 seconds? how many in 50 seconds? how many in 75 seconds? how many in 17 seconds? how many in 37 seconds?

Practice.

§ 116. Practice is the employment of the *ratio* of a multiplier or divisor to its unit of the same kind, instead of the employment of the given multiplier or divisor itself. Thus, if a bushel of apples be worth 50 cents, what will $18\frac{1}{2}$ bushels be worth? This answer may be obtained by multiplying 50 cents by $18\frac{1}{2}$ giving \$9·25. But the *ratio* of 50 cents to \$1.00 is $\frac{50}{100}=\frac{1}{2}$. Therefore, to find how many dollars $18\frac{1}{2}$ bushels are worth, it is only necessary to multiply $18\frac{1}{2}$, the number of bushels, by $\frac{1}{2}$; that is, to divide $18\frac{1}{2}$ by 2.

What is the interest of \$740 for a year and 6 months?

For a year the interest is $\$740\times0{\cdot}06=\$44{\cdot}40$. Now as 6 months is $\frac{1}{2}$ of a year, we find the interest for 6 months, by taking $\frac{1}{2}$ of \$44·40, which is \$22·20. Hence the interest for one year and 6 months is $\$44{\cdot}40+\$22{\cdot}20=\$66{\cdot}60$.

For tables of the *ratios* of particular parts of a dollar to their unit, see § 70, and of fractional parts of a year or month, to their units respectively, see § 104, Case III., *Second method.*

EXAMPLES.

151. What will 435 yards of cloth cost, at \$0·75 per yard?

152. If I receive 7 dollars for the use of \$100 for one

year, how much ought I to receive for the use of \$100 for 7 months and 18 days?

153. What cost $7\frac{1}{2}$ cords of wood, at \$2·75 per cord?

154. What is the value of $28\frac{3}{4}$ pounds of butter, at 11 cents per pound?

155. What is the value of $500\frac{1}{2}$ yards of tape, at $2\frac{1}{4}$ cents per yard?

156. What must I give for $13\frac{3}{4}$ bushels of oats, at $43\frac{3}{4}$ cents per bushel?

157. What cost $18\frac{3}{4}$ pounds of ham, at 8 cents per pound?

158. What cost $15\frac{3}{4}$ gallons of oil, at \$0·75 per gallon?

159. What cost 4000 quills, at \$2·25 per 1000?

160. What cost $27\frac{3}{4}$ yards of carpeting, at $87\frac{1}{2}$ cents per yard?

161. What is the value of 25 bushels of potatoes, at \$0·31$\frac{1}{4}$ per bushel?

162. What is the value of 54 spelling-books, at $12\frac{1}{2}$ cents per copy?

163. What is the value of $47\frac{1}{2}$ reams of paper, at \$3·25 per ream?

164. What is the value of $30\frac{1}{2}$ gross of almanacs, at \$2·25 per gross?

165. What cost $16\frac{3}{4}$ gallons of vinegar, at $16\frac{2}{3}$ cents per gallon?

166. What is the value of $5\frac{1}{3}$ bushels of walnuts, at \$1·62$\frac{1}{2}$ per bushel?

167. What cost $3\frac{1}{2}$ gross of matches, at \$1·125 per gross?

168. What cost 325 bushels of apples, at $37\frac{1}{2}$ cents per bushel?

169. What cost $16\frac{1}{2}$ yards of cloth, at \$$3\frac{3}{4}$ per yard?

170. If the interest on a certain sum of money is \$7·35 in one year, how much will it be for $5\frac{1}{2}$ months?

171. If the interest of \$100 for one year is \$6, how much is it for 10 months and 10 days?

172. If a steam locomotive pass 18 miles in 1 hour, how far will it move in $50\frac{1}{2}$ minutes?

173. If the interest of \$100 for 12 months is \$7, how much is it for $4\frac{1}{3}$ months?

174. What must I pay for $1\frac{1}{4}$ cords of wood, 128 feet in a cord, at $6\frac{1}{4}$ cents per foot?

175–180. For \$360 how many bushels of apples can I buy at 50 cents per bushel? how many at 25 cents per bushel? how many at $12\frac{1}{2}$ cents? how many at $33\frac{1}{3}$ cents? how many at 20 cents? how many at $16\frac{2}{3}$ cents?

181–185. Among how many beggars can \$12 be distributed by giving $6\frac{1}{4}$ cents to each? how many if each receive 10 cents? how many if each receive $12\frac{1}{2}$ cents? how many if each receive $16\frac{2}{3}$ cents? how many if each receive 20 cents?

186–190. If my income is \$600 per annum, how much will it be for 3 months? how much for 15 days? how much for 10 days? how much for 5 days? how much for 1 day?

191–195. How many yards of carpeting can be bought for \$300 at \$1·12½ per yard? how many at \$1·25 per yard? how many at \$1·87½ per yard? how many at \$2·06¼ per yard? how many at \$2·16⅔?

196–200. If the earth move 68000 miles per hour in its orbit, how far will it move in 35 minutes? how far in 45 minutes? how far in 55 minutes? how far in 1*hr.* 35*min.*? how far in 2*h.* 10*min.*?

Reduction of Currencies.

§ 117. *Currency* is money, whether specie, consisting of domestic and foreign coins, or bank-notes, redeemable in specie.

Foreign coins have, first, an *intrinsic* value, determined by their weight and purity; secondly, a *commercial* value, which is the price they will bring in the market; thirdly, a *legal* value, which is the value established by law.

Thus, the Pound Sterling (English) is represented by a gold coin called a sovereign. Its intrinsic value, as compared with our gold eagle of latest coinage, is \$4·861. Its commercial value depends upon the state of trade between this country and England. If the balance of trade be against us, requiring the transportation of coin to pay our debts, the sovereign will command a higher price than if we owe nothing abroad, and consequently require no specie for shipment. This mercantile value varies from \$4·83 to \$4·86.

The legal or custom-house value of the sovereign is \$4·84, as fixed by act of Congress in 1842.

§ 118. To reduce *Sterling* to *Federal* Money.

First method. £1=\$4·84; consequently, multiplying \$4·84 by the number of pounds, will give their value in Federal Money.

NOTE.—If there are shillings, pence, or farthings in the given quantity, they must be reduced to the decimal of a pound before multiplication.

Example. What is the value of £9 5*s.* in Federal Money?
£9 5*s.*=£9·25; \$4·84×9·25=\$44·77.

§ 119. To reduce Federal to Sterling Money.

\$4·84=£1; consequently, dividing the given number of dollars by 4·84, the number of dollars in a £, will give a quotient in pounds and the decimal of a pound. The decimal must be reduced to its equivalent value in shillings, pence, and farthings.

Example. Reduce \$44·77 to its value in Sterling Money. $44·77 \div 4·84 = 9·25$, the number of pounds sterling = £9 5*s.*

§ 120. *Method by ratio.* There is another mode of performing these reductions, which is a more accurate mercantile method.

The original value of the pound sterling, as fixed by act of Congress in 1799, was $\$4\frac{4}{9}$ or \$4·444+. This value is called the *par value* of £1; but it is now too small by a variable percentage of itself. Consequently this percentage, called the *premium* of exchange, must be added to the par value to give the current mercantile value of the pound.

Thus, suppose exchange on England is at 9 per cent. premium, $£1 = \$4\frac{4}{9} \times 1·09$ = par value of £1 plus the premium of exchange; if exchange be at 10 per cent. premium, $£1 = \$4\frac{4}{9} \times 1·10$, &c. So conversely, $\$1 = £\frac{9}{40} \div 1·09$, &c.

Example. Reduce £9 5*s.* to Federal Money, when the premium of exchange is 9 per cent.

£9 5*s.* = £9·25; $\$4\frac{4}{9} \times 9·25 = \$41·111+$. And $\$41·111 \times 1·09 = \$44·81099+$ *Ans.*

Reduce \$44·81099+ to British currency.

$44·81099 \div 4\frac{4}{9} = 44·81099 \times \frac{9}{40} = 10·08249+$, the number of pounds at par value. $£10·08249 \div 1·09 = £9·25$ nearly. So that £9 5*s.* is the answer.

Hence to change Sterling to Federal Money,

Reduce the pounds and decimal of a pound at their par value to dollars; then multiply the result by a percentage that will express the par value plus the premium of exchange.

To change Federal to Sterling Money.

Reduce the dollars and decimal of a dollar to pounds at their par value; then divide the result by a percentage that will express the par value plus the premium of exchange.

§ 121. Many of the States, at the present day, make use of the denominations of Sterling Money to some extent. But the value of the pound and its parts, as will be seen by the table, is not the same in all the States. (For the reason of this, see § 70, Note.)

TABLE.

\$1 in	South Carolina, Georgia,	$= 4s.\ 8d. = £\frac{7}{30}$, called Georgia currency
\$1 in	Canada, Nova Scotia,	$= 5s. = £\frac{1}{4}$, called Canada currency.
\$1 in	New England States, Virginia, Kentucky, Tennessee,	$= 6s. = £\frac{3}{10}$, called New England currency.
\$1 in	New Jersey, Pennsylvania, Delaware, Maryland,	$= 7s.\ 6d. = £\frac{3}{8}$, called Pennsylvania currency.
\$1 in	New York, Ohio, North Carolina,	$= 8s. = £\frac{2}{5}$, called New York currency.

We have, by the table, the value of \$1, expressed as the fraction of a pound in the various currencies. It is obvious that by inverting the ratio expressed by those fractions, we shall obtain the value of £1, of each of the above currencies, in the fraction of a dollar.

Hence, to reduce Federal Money to Canada or to any State currency,

Multiply the sum in Federal Money by the value of \$1 *expressed as the fraction of a pound of the currency to which the sum is to be reduced. If the product contain the decimal of a pound, reduce it to shillings and pence.*

To reduce Canada or any State currency to Federal Money,

Multiply the given sum, reduced to pounds and the decimal of a pound, by the value of £1 of the given currency, expressed as the fraction of a dollar.

§ 122. A table of some of the foreign coins at their custom-house value.

Pound Sterling or Sovereign	$4·84
Guinea, English	5·00
Crown, "	1·06
Shilling piece, English	·23
Louis-d'or, French	4·56
Franc, "	·186
Doubloon, Mexico	15·60
Silver Rouble of Russia	0·75
Florin or Guilder of the United Netherlands	0·40
Mark Banco of Hamburg	0·35
Real of Plate of Spain	0·10
Real of Vellon of do.	0·05
Milree of Portugal	1·12½
Tale of China	1·48
Pagoda of India	1·84
Rupee of Bengal	0·50
Specie dollar of Sweden and Norway	1·06
Specie dollar of Denmark	1·05
Thaler of Prussia and N. States of Germany	0·69
Florin of Austrian Empire and City of Augsburg,	0·48½
Lira Lombardo-Venetian Kingdom and of Tuscany,	0·16
Ducat of Naples	0·80
Ounce of Sicily	2·40
Pound of British Provinces, Nova Scotia, New Brunswick, Newfoundland, and Canada	4·00
Rix-dollar of Bremen	0·78¾
Thaler of Bremen	0·71
Mil-rees of Madeira	1·00
" of Azores	0·83⅓
Rupee of British India	0·41½
10 Thalers, German	7·80

Foreign coins may obviously be reduced to Federal Money, by *multiplying* the United States value of *one* coin by the number of coins. Federal Money may be reduced to its value in a required foreign coin, by *dividing* the given sum of money by the value of *one* such coin expressed in Federal Money.

PROMISCUOUS EXERCISES IN REDUCTION OF CURRENCIES.

201–210. Reduce the following sums, U. S. currency, to Sterling Money, at custom-house value: \$4·84; \$19·605; \$32·48; \$59·00; \$876·49; \$27·18; \$1264·36; \$22096·27; \$446987·84; \$2768912·76.

211–218. When the premium of exchange on England is 9 per cent., what is the value of the following sums in British currency? \$8·72; \$24·986; \$79·484; \$712·45; \$8694·36; \$79823·12½; \$89421·07; \$216549·48.

219–234. What is the value of the preceding sums, in British currency, when the premium of exchange is 10 per cent.? What, when it is 8½ per cent.?

235–240. Reduce the following sums, Sterling Money, at custom-house valuation, to U. S. currency: £9 5*s.*; £27 3*s.* 4*d.* 3*qr.*; £39 9*d.*; £270 14*s.* 9*d.* 2*qr.*; £4180 12*s.* 8*d.*; £69480 9*d.*

241–258. Find the value of each of the preceding sums, in U. S. currency, when the premium of exchange on England is 8 per cent.; is 9 per cent.; is 10 per cent.

259–263. Reduce \$100·20 to Canada and to the different State currencies.

264–268. Reduce \$37·37 to Canada and State currencies.

269–273. Reduce \$1000 to its equivalent value in Canada and State currencies.

274–278. Reduce £75 15*s.* 6*d.* of the respective currencies mentioned in the table to Federal Money.

279–283. Reduce £80 5*s.* 3*d.* of the different currencies to Federal Money.

284–288. Reduce £1000 of the different currencies to Federal Money.

289. How many sovereigns in $8496 ?

290. How many 5-franc pieces in $10765 ?

291–295. In $9284·47 how many Mexican doubloons ? how many 10-Thaler pieces ? how many Canada pounds ? how many rupees of Bengal ? how many ducats of Naples ?

296–301. Reduce to Federal Money 7498 rix-dollars of Bremen ; 25480 rupees of Bengal ; $4879\frac{1}{2}$ silver roubles of Russia ; 79682 sovereigns ; $729810\frac{3}{4}$ pagodas of India ; 1987629 francs.

302. Suppose I owe a Liverpool merchant £17496 8*s.*, what sum in Federal Money must I pay him, when exchange on England is 9 per cent. premium ?

303. I am indebted to a Liverpool house in the sum of $25000·75. How many pounds sterling must I pay to his order, when exchange on England is 10 per cent. premium ?

304. A New England merchant wished to pay £784 10*s.*, Georgia currency, to a merchant in Savannah. What sum in N. E. currency must he remit ?

305. How many 5-franc pieces must a Paris house remit to pay £9841 7*s.*, N. Y. currency ?

306. The rate of duty on imported dried plums, in 1842, was £1 8*s.* per *cwt.* How much is that per *lb.*, U S. currency ?

307. The duty on grain, not rated as corn or seeds, was 18*s.* per *cwt.* What is that per *cwt.*, U. S. currency ?

308. The duty on rose-wood was £6 per ton. What is that per *cwt.*, U. S. currency ?

309–314. In $1000 how many Ounces of Sicily ? how many ducats of Naples ? how many florins of Augsburg ?

how many rix-dollars of Bremen? how many Mexican doubloons? how many Louis d'ors?

315–325. In 1000 Mexican doubloons how many dollars? how many crowns? how many sovereigns? how many specie-dollars of Denmark? how many specie-dollars of Norway? how many pagodas of India? how many rupees of Bengal? how many milrees of Portugal? how many Mark bancos of Hamburg? how many English guineas? how many francs?

CHAPTER XIV.

PROPORTION.

§ 123. When the ratio of two quantities is the same as the ratio of two other quantities, the four quantities are in *proportion*. Thus, the ratio of 8 yards to 4 yards is the same as the ratio of 12 dollars to 6 dollars; therefore, there is a proportion between 8 yards, 4 yards, 12 dollars, and 6 dollars.

The usual method of denoting that four terms are in proportion, is by means of points or dots. Thus, the above proportion is written,

$$8 \text{ yards} : 4 \text{ yards} :: 12 \text{ dollars} : 6 \text{ dollars}.$$

Where two dots are placed between the first and second terms, and between the third and fourth; and four dots are placed between the second and third.

The two dots are equivalent to the sign of division, and the four dots correspond with the sign of equality. Thus, the above proportion may be written,

$$8 \text{ yards} \div 4 \text{ yards} = 12 \text{ dollars} \div 6 \text{ dollars}.$$

Either of the foregoing forms of this proportion may be read,

8 yards is to 4 yards as 12 dollars is to 6 dollars.

The first term of a ratio is called the *antecedent;* the second is called the *consequent.*

The first and fourth terms of a proportion are called the *extremes;* the second and third terms are called the *means.*

Since in a proportion the quotient of the first term divided by the second, is equal to the quotient of the third term divided by the fourth, we have, using the above proportion, $\frac{8}{4}=\frac{12}{6}$. If we reduce the fractions to a common denominator, they become $\frac{8\times6}{4\times6}=\frac{12\times4}{6\times4}$, or omitting the common denominator 4×6, which is, in effect, multiplying each fraction by 4×6, we have 8×6, or $48=12\times4$ or 48; that is, *the product of the extremes is equal to the product of the means.*

$$\text{Again, } \frac{8\times6=48}{12}=4, \text{ and } \frac{8\times6=48}{4}=12.$$

*Hence, if the product of the extremes be divided by either mean, the quotient will be the other mean.**

$$\text{Again, } \frac{12\times4}{8}=6, \text{ and } \frac{12\times4}{6}=8.$$

Hence, if the product of the means be divided by either extreme, the quotient will be the other extreme.

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* It is often required to find a *mean proportional* when the extremes are given; that is, one mean of a proportion in which the means are equal. Thus, 4 and 9 being the extremes, give a product of 36, which is equal to the product of the means. Hence the means may be 2 and 18, 3 and 12, or 6 and 6; of these 6 and 6 are the equal means; thus, 4 : 6 : : 6 : 9.

Therefore, to find a mean proportional when the extremes are given, the *square root* of their product must be found; that is, the number which being multiplied by itself will produce that product.

From the above properties, we see that if any *three* of the four terms which constitute a proportion are given, the remaining term can be found.

The method of finding the fourth term of a proportion, when *three* terms are given, constitutes the RULE OF THREE.

Let us now apply what has been explained.

If 8 yards of cloth are worth \$12, what are 24 yards worth?

The value sought must be as many times greater than \$12, as 24 yards is greater than 8 yards. Hence, there is the same ratio between \$12 and the *value sought*, as there is between 8 yards and 24 yards. Consequently, we have this proportion:

8 yards : 24 yards : : \$12 : *value sought*.

Taking the product of the means, we have $24 \times 12 = 288$. This, divided by the first term, which is one of the extremes, gives $\frac{288}{8} = 36$ for the other extreme or fourth term sought, which must be of the same kind as the third term; therefore \$36 is the value of 24 yards.

NOTE.—When we take the product of the means we do not multiply the 24 yards by 12 dollars, but simply multiply 24, the number denoting the yards, by 12, the number denoting the dollars. The product, 288, is neither yards nor dollars, but 288 units. When we divide this product by the first term of the proportion, we do not divide by 8 yards, but simply by 8, the number denoting the yards. The quotient, 36, gives the fourth term of the proportion; and since the fourth term is of the same denominate value as the third term, our fourth term, or answer, must be 36 dollars.

From the foregoing explanations, we deduce this first form of the

RULE FOR SIMPLE PROPORTION, OR SINGLE RULE OF THREE.

I. *Form a proportion by placing for the third term the quantity which is of the same denomination as the answer* *sought. Of the two remaining quantities, the larger must be taken for the second term, when the answer is to exceed the third term; but the smaller must be taken for the second term, when the answer is to be less than the third term.*

II. *Having written the three terms of the proportion, or, as usually expressed, having stated the question, then multiply the second and third terms together, and divide the product by the first term.*

NOTE.—Since there is a ratio between the first and second terms, they must be reduced to the same denominate value. Also, the third term must be reduced to its lowest denomination; then the quotient found by dividing the product of the means by the first term will be of the same denomination as the third term.

EXAMPLES.

1. If 25 *lbs.* of coffee cost $3·25, what will 312 *lbs.* cost?

2. What cost 6 cords of wood, at $7 for 2 cords?

3. What will 9 pairs of shoes cost, if 5 pairs cost £2 2*s.* 6*d.*?

4. If there are 9 weeks in 63 days, how many weeks in 365 days?

5. If a railroad car goes 17 miles in 45 minutes, how far will it go in 5 hours?

6. If $100 will gain $7 in one year, how long will it require to gain $100?

7. If 3 paces or common steps of a person are equal to 2 yards, how many yards will 480 paces make?

8. If 15 men can raise a wall of masonry, 12 feet, in one week, how many will be necessary to raise it 20 feet in the same time?

9. If 7 tons of coal, of 2000 pounds each, will last $3\frac{1}{2}$ months, of 30 days each, how much will be consumed in 3 weeks, or 21 days?

10. If $9\frac{1}{2}$ bushels of wheat make 2 barrels of flour, how many bushels will be required to make 13 barrels?

11. If a steamboat of 242 feet in length move 15 miles in one hour, how many seconds will it require to move its own length?

12. If a steamboat of 242 feet in length move 15 miles an hour, how many times its own length will it move in 11 hours?

13. A reservoir has a pipe capable of discharging 30 gallons in one minute. What time will be necessary to discharge 15 hogsheads?

14. If a man can mow 9 acres of grass in $3\frac{1}{2}$ days, of 10 hours each, how long will it require for him to mow 21 acres?

15. If 100 pounds of galena, or lead ore, yield 83 pounds of pure metal, how much pure metal will 7 tons of galena produce, if we reckon 2240 pounds to the ton?

If 12 barrels of flour are worth $54, what is the value of 42 barrels at the same rate?

In this example it is obvious that 2 times 12 barrels would be worth 2 times $54; 3 times 12 barrels would be worth 3 times $54; 4 times 12 barrels would be worth 4 times $54, and so on for other ratios. The ratio of 42 barrels to 12 barrels is $\frac{42}{12}$.

If we multiply $54 by this ratio, it will evidently give the value of 42 barrels.

$$\$54 \times \tfrac{42}{12}.$$

We may now employ the same rules for simplifying this expression as were used under § 114; that is to say, we may reject such factors as are common to both numerators and denominators. Thus, dividing the denominator 12, and the numerator 42, each by 6, it becomes

$$\$54 \times \frac{\overset{7}{\not{42}}}{\underset{2}{\not{12}}}, \text{ or } \$54 \times \tfrac{7}{2}.$$

Again, dividing the denominator 2 and \$54 of numerator each by 2, we have

$$\overset{27}{\cancel{\$54}} \times \frac{7}{\cancel{2}}, \text{ or } \$27 \times 7 = \$189. \quad \textit{Ans.}$$

If 200 sheep yield 650 pounds of wool, how many pounds will 825 sheep yield?

In this example, the answer is required to be in pounds; we therefore take 650 pounds for the third term. The ratio of 825 sheep to 200 sheep is $\frac{825}{200}$. Hence we have

$$650\textit{lb.} \times \frac{825}{200}.$$

Cancelling, we have

$$650\textit{lb.} \times \frac{\overset{33}{\cancel{825}}}{\underset{8}{\cancel{200}}}, \text{ or, } 650\textit{lb.} \times \frac{33}{8}.$$

Again, cancelling, we have

$$\overset{325}{\cancel{650}}\textit{lb.} \times \frac{33}{\underset{4}{\cancel{8}}} = \frac{325\textit{lb.} \times 33}{4} = 2681\tfrac{1}{4}\textit{lb.} \quad \textit{Ans.}$$

If $\frac{11}{13}$ of a pound of sugar cost $\frac{23}{26}$ of a shilling, how much will $\frac{9}{29}$ of a pound cost?

In this example, our third term is $\frac{23}{26}$ of a shilling. And since $\frac{9}{23}$ of a pound is less than $\frac{11}{13}$, we must obtain our ratio by dividing $\frac{9}{23}$ by $\frac{11}{13}$, which gives $\frac{9}{23} \times \frac{13}{11}$. Multiplying the third term by this ratio, we have $\frac{23}{26}$ of a shilling $\times \frac{9}{23} \times \frac{13}{11}$. To reduce this with the least labor, we must resort to the method of cancelling. Thus, cancelling the 23, which occurs in both numerator and denominator, also 13 of the numerator against a part of the 26 of the denominator, our expression will, by this means, become $\frac{1}{2}$ of a shilling $\times \frac{9}{1} \times \frac{1}{11} = \frac{9}{22}$ of a shilling.

NOTE.—This method of cancelling should be used when the nature of the question will admit, since it will always simplify the operation.

From the above explanation, we deduce this second form of the

RULE OF THREE.

Of the three quantities which are given, one will always be of the same kind as the answer sought; this quantity will be the third term. Then, if by the nature of the question, the answer is required to be greater than the third term, divide the greater of the two remaining quantities by the less, for a ratio; but if the answer is required to be less than the third term, then divide the less of the two remaining quantities by the greater, for a ratio. Having obtained the ratio, multiply the third term by it, and it will give the answer in the same denomination as is the third term.

NOTE.—Before obtaining the ratio, by means of the first two terms, we must reduce them to like denominations. See § 115.

EXAMPLES.

16. If a tree 38 feet 9 inches in height, give a shadow of 49 feet 2 inches, how high is that tree which, at the same time, casts a shadow of 71 feet 7 inches?

17. If $3\frac{1}{2}$ pounds of coffee cost $2\frac{1}{3}$ shillings, how much will $10\frac{1}{6}$ pounds cost?

18. If 6 men earn \$25 in 6 days, how much can they earn in 25 days?

19. If a locomotive move 95 miles in 4 hours, how far does it go each hour?

20. If it take 10 hours for 6 men to do a piece of work, how long will it take 15 men to do the same work?

21. Gave \$72 for 11 barrels of fish. How much will 88 barrels cost at the same rate?

22. If $43\frac{1}{3}$ pounds of cheese cost \$2·20, what will $216\frac{2}{3}$ pounds cost at the same rate?

23. If I pay \$3·90 for sawing 7 cords of wood, how much ought I to give for sawing $23\frac{1}{3}$ cords?

24. If $\frac{3}{10}$ of a ship is worth $2853, what is the whole worth?

25. If $\frac{4}{13}$ of my income is $533, what is my whole income?

26. A person failing in business, finds that he owes $7560, and that he only has $3100 to pay the debt with. How much can he pay to that creditor whose claim is $756?

27. If it require $5\frac{1}{2}$ bushels of wheat to make one barrel of flour, how many bushels will be required for 100 barrels of flour?

28. If 7 barrels of flour are sufficient for a family 6 months, how many barrels will they require for 11 months?

29. If it take 25 yards of carpeting, a yard wide, to cover a certain floor, how many yards of $\frac{3}{4}$ carpeting will be necessary to cover the same floor?

30. If a person travel 8 miles in 10 hours, how far will he travel in 5 days, by travelling 8 hours each day?

31. If 35 pounds of feathers cost $15, what will 100 pounds cost at the same rate?

32. If a man perform a certain piece of work in 18 days, when he works 8 hours per day, how many days will he require if he work 10 hours each day?

33. If a piece of board 12 inches wide and 12 inches long make one square foot, how many inches of length must be taken from a board 15 inches wide to make a square foot?

34. If 8 men can mow a field in 5 days, in how many days can 5 men mow it?

35. If $27\frac{1}{2}$ yards of cloth cost $60, how many yards can I buy for $100?

36. If $27\frac{1}{2}$ yards of cloth cost $60, what will $45\frac{5}{6}$ yards cost?

37. If $\frac{5}{8}$ of a ship is worth $9000, what is her whole value?

38. If $\frac{3}{16}$ of a city lot is sold for $500, what would $\frac{7}{16}$ of the same lot sell for at the same rate?

39. Admitting that the earth moves in its orbit about the sun, a distance of 597000000 miles, in 365 days 6 hours, how far on an average does it move each hour?

40. If the diurnal rotation of the earth move its equatorial portions about 24900 miles each day, how far is that in each hour?

41. If it require 10 years of $365\frac{1}{4}$ days, for light to pass from a fixed star to the earth, how many miles distant is it on the supposition that light moves 192000 miles in one second?

42. If by a leak of a ship, $\frac{3}{5}$ enough water run in, in 4 hours, to sink her, how long can she survive?

43. If I pay $25 for the masonry of 4000 bricks, how much ought I to pay for the work which requires 100000 bricks?

44. If a steamship require 14 days to sail a distance of 3000 miles, what time, at the same rate of sailing, would she require to sail 24900 miles?

45. Admitting the diameter of the earth to be 8000 miles and the loftiest mountain to be 5 miles in height, what elevation must be made on a globe of 16 inches diameter to represent accurately the height of such mountain?

46. If $100 in 12 months bring an interest of $7, how much will be the interest of $100 for 8 months?

47. If the interest of $100 for 12 months is $7, what will be the interest of $75 for the same time?

48. If in 12 months the interest of $100 is $7, how long must $100 be on interest to gain $10?

49. If a glacier of 60 miles in length move 50 inches per annum, in what time will it move its whole length?

50. If a staff of 10 feet in length give a shadow of 15 feet, how high is that tree whose shadow measures 90 feet?

51. Suppose sound to move 1100 feet in a second, how

many miles distant is a cloud, in which lightning is observed 16 seconds before the thunder is heard, no allowance being made for the motion of light?

52. If it require 30 yards of carpeting which is $\frac{3}{4}$ of a yard wide to cover a floor, how many yards of carpeting which is $1\frac{1}{4}$ yards wide will be necessary to cover the same floor?

53. If the earth move through 12 signs, or 360°, in $365\frac{1}{4}$ days, how far will it move in a lunar month of $29\frac{1}{2}$ days?

54. Suppose a steamboat capable of making 15 miles each hour, to move with a current whose velocity is $2\frac{1}{2}$ miles per hour, what will be the whole distance made during $13\frac{1}{2}$ hours? And what distance will the boat move in the same time against the same current?

55. If the magnetic influence move through the telegraphic wires at the rate of 200000 miles in one second of time, how many times could it pass around the world in one second, allowing the circumference of the earth to be 24899 miles?

56. If A. can do a piece of work in 7 days, and B. can do it in 8 days, what part of it can both do in $3\frac{1}{2}$ days?

57. A reservoir, whose capacity is 1000 hogsheads, has a supply-pipe, by means of which it receives 300 gallons each hour; it also has two discharging pipes, the first of which discharges $\frac{5}{6}$ of a gallon each minute, the second discharges $1\frac{1}{4}$ gallons per minute. The reservoir being empty, in what time will it be filled if the supply-pipe alone is opened? In what time, if the supply-pipe and the first discharging pipe are opened? In what time, if the supply-pipe and the second discharging pipe? And in what time if all three are opened?

COMPOUND PROPORTION.

§ 124. A *Compound Proportion* is an expression of the equality between the product of several ratios and a simple ratio.

If 6 men can mow 30 acres of grass in 5 days, by working 8 hours each day, how many acres can 4 men mow in 9 days of 10 hours each?

Before performing this example by the *Rule of Compound Proportion,* or Double Rule of Three, let us solve it, first by Analysis and then by Ratio.

ANALYSIS.—If 6 men can mow 30 acres in 5 days of 8 hours each, 1 man will mow $\frac{30}{6}=5$ acres in 5 days of the same length: 4 men will mow 4×5 acres or 20 acres, in 5 days.

If 4 men mow 20 acres in 5 days, in 1 day they will mow $\frac{20}{5}=4$ acres; in 9 days, $4\times9=36$ acres.

If 4 men mow 36 acres in 9 days of 8 hours each, they will mow $\frac{36}{8}$ or $4\frac{1}{2}$ acres, by working but 1 hour each day, and $4\frac{1}{2}\times10$ or 45 acres, by working 10 hours each day. The answer, then, is 45 acres.

NOTE.—In this Complex Analysis, as in Simple Analysis, we reason from the given quantity back to 1, then from 1 to the quantity required.

RATIO.—Had the number of days, as well as hours in each day been the same in both cases, the question would have been equivalent to the following:

If 6 men mow 30 acres of grass, how many acres will 4 men mow?

It is evident the number of acres sought would be the same fractional part of 30 acres that 4 men is of 6 men; that is, the quantity required is

$$\frac{4}{6} \text{ of } 30 \text{ acres.}$$

If, now, we take into account the number of days, still supposing the number of hours in each day to remain the same in both cases, our question would become:

If $\frac{4}{6}$ of 30 acres can be mowed in 5 days, how much can be mowed in 9 days?

The answer in this case is obviously

$$\tfrac{9}{5} \text{ of } \tfrac{4}{6} \text{ of 30 acres.}$$

Now, taking into account the number of hours in each day, our question will become as follows:

If $\frac{9}{5}$ of $\frac{4}{6}$ of 30 acres can be mowed in a certain time, when 8 hours are reckoned to each day, how much could be mowed when 10 hours are reckoned to each day?

This leads to the following final result:

$$\tfrac{10}{8} \text{ of } \tfrac{9}{5} \text{ of } \tfrac{4}{6} \text{ of 30 acres.}$$

By cancelling, we reduce this last expression to 45 acres.

As a second example, we will solve the following by ratio:

500 men, working 12 hours each day, have been employed 57 days to dig a canal of 1800 yards long, 7 yards wide, and 3 yards deep, how many days must 860 men, working 10 hours each day, be employed in digging another canal of 2900 yards long, 12 yards wide, and 5 yards deep, in a soil which is 3 times as difficult to excavate as the first?

In this example, the odd term is 57 days.

The different ratios will be as follows:

$\frac{500}{860}=\frac{25}{43}$ ratio of the men.

$\frac{12}{10}=\frac{6}{5}$ ratio of the hours.

$\frac{2900}{1800}=\frac{29}{18}$ ratio of lengths of the canals.

$\frac{12}{7}=$ ratio of widths of the canals.

$\frac{5}{3}=$ ratio of depths of the canals.

$\frac{3}{1}=$ ratio of the difficulty in excavation.

Multiplying successively these ratios and the odd term, we have

$$57 \text{ days} \times \tfrac{25}{43} \times \tfrac{6}{5} \times \tfrac{29}{18} \times \tfrac{12}{7} \times \tfrac{5}{3} \times \tfrac{3}{1}.$$

This becomes, after cancelling factors,

$$19 \text{ days} \times \tfrac{5}{43} \times \tfrac{6}{1} \times \tfrac{29}{1} \times \tfrac{2}{7} \times \tfrac{5}{1} \times \tfrac{1}{1} = 549\tfrac{51}{301} \text{ days.}$$

From the above work we see that questions of Compound Proportion may be solved by the following

RULE.

Among the terms given, there will be but one like the answer, which we will call the odd term. The other terms will appear in pairs or couplets. Form ratios out of each couplet in the same manner as in the Rule of Three; then multiply all the ratios and the odd term together, and it will give the answer in the same name and denomination as the odd term.

NOTE.—Before forming ratios from the couplets, they must be reduced to the same denominate value. See § 115.

EXAMPLES.

Solve the following problems, first by Analysis, and then by the Rule for Compound Proportion:

58. If a person travel 300 miles in 17 days, journeying 6 hours each day, how many miles will he travel in 15 days, journeying 10 hours a day?

59. If a marble slab 10 feet long, 3 feet wide, and 3 inches thick, weigh 400 pounds, what will be the weight of another slab of the same marble, whose length is 8 feet, width 4 feet, and thickness 5 inches?

60. If the expenses of a family of 10 persons amount to $250 in 23 weeks, how long will $600 support a family of 8 persons at the same rate?

61. 15 men, working 10 hours each day, have employed

18 days to build 450 yards of stone fence. How many men, working 12 hours each day, for 8 days, will be required to build 480 yards of similar fence?

62. If it require 1200 yards of cloth $\frac{5}{4}$ wide to clothe 500 men, how many yards which is $\frac{7}{8}$ wide will it take to clothe 960 men?

63. If 8 men will mow 36 acres of grass in 9 days, by working 9 hours each day, how many men will be required to mow 48 acres in 12 days, by working 12 hours each day?

64. If 11 men can cut 49 cords of wood in 7 days, when they work 14 hours per day, how many men will it take to cut 140 cords in 28 days, by working 10 hours each day?

65. If 12 ounces of wool make $2\frac{1}{2}$ yards of cloth that is 6 quarters wide, how many pounds of wool will it take for 150 yards of cloth, 4 quarters wide?

66. If the wages of 6 men for 14 days be 84 dollars, what will be the wages of 9 men for 16 days?

67. If 100 men in 40 days of 10 hours each, build a wall 30 feet long, 8 feet high, and 2 feet thick, how many men must be employed to build a wall 40 feet in length, 6 feet high, and 4 feet thick, in 20 days, by working 8 hours each day?

68. In how many days, working 9 hours a day, will 24 men dig a trench 420 yards long, 5 yards wide, and 3 yards deep, if 248 men, working 11 hours a day, in 5 days dig a trench 230 yards long, 3 yards wide, and 2 yards deep?

69. Suppose that 50 men, by working 5 hours each day, can dig, in 54 days, 24 cellars, which are each 36 feet long, 21 feet wide, and 10 feet deep, how many men would be required to dig, in 27 days, 18 cellars, which are each 48 feet long, 28 feet wide, and 9 feet deep, provided they work only 3 hours each day?

70. If 40 yards of cloth 3 quarters wide cost $45, what

will 36 yards of the same quality cost, which is 5 quarters wide?

71. If $400 will in 9 months gain $21, when the rate of interest is 7 per cent. per annum, how much will $360 gain in 8 months, if the rate per cent. is 6?

72. If $400 require 9 months to gain $21, when the rate per cent. is 7, how long a time will $360 require to gain $14·40, if the rate per cent. is 6?

73. If $400, to gain $21 in 9 months, require a rate of 7 per cent., what must be the rate per cent. for $360 to gain $14·40 in 8 months?

74. If it require $400 to gain $21 in 9 months, when the rate per cent. is 7, how much will be required to gain $14·40 in 8 months, when the rate per cent. is 6?

75. If the freight on 72 barrels of flour, for a distance of 95 miles, is $7·60, what would it be at the same rate on 120 barrels for a distance of 144 miles?

76. If from a dairy of 30 cows, each furnishing 16 *qts.* of milk daily, 24 cheeses of 55 pounds each are made, in 36 days, how many cows will be required, if each gives 4½ gallons of milk daily, to produce in 30 days 33 cheeses of 100 pounds each?

77. If I pay $45 for 40 yards of cloth which is 3 quarters wide, how many yards of the same quality of cloth which is 5 quarters wide ought $60 to buy?

78. If 6 persons eat 21 dollars' worth of bread in 4 months, when flour is sold at $7 per barrel, in how many months will 10 persons eat 50 dollars' worth, when flour is $5 per barrel?

79. If 21 dollars' worth of bread, when flour is $7 per barrel, will supply 6 persons 4 months, how many persons can be supplied for 8 months for 50 dollars, when flour is $5 per barrel?

80. If 21 dollars' worth of bread, when flour is \$7 per barrel, will supply 6 persons for 4 months, how many dollars' worth will be required to supply 10 persons during 8 months, if flour is \$5 per barrel?

Arbitration of Exchange.

§ 125. It sometimes happens that a merchant desires to pay a debt to a foreign creditor through three or four agents or brokers in different countries. There must, in such cases, be a *chain* of exchanges called *arbitration of exchange;* for example:

A merchant in Denmark owes a New York merchant \$280. How many specie-dollars of his own country must he remit through houses in Hamburgh, Amsterdam, and St. Petersburgh, if \$0·35 = 1 mark banco; 8 marks = 7 guilders; 15 guilders = 8 silver roubles; 21 roubles = 15 specie-dollars of Denmark?

$28000 \times \frac{1}{35} \times \frac{7}{8} \times \frac{8}{15} \times \frac{15}{21}$, which, after cancelling, becomes $266\frac{2}{3}$, the number of specie-dollars. Since 35 cts. = 1 mark, there will be $\frac{1}{35}$ as many marks as cents; as 8 marks = 7 guilders, there will be $\frac{7}{8}$ as many guilders as marks; in like manner there will be $\frac{8}{15}$ as many roubles as guilders, and $\frac{15}{21}$ as many specie-dollars as roubles.

It will be seen that examples of this kind require the multiplication of a given term by the product of a series of ratios; and that the process is the same as that of Compound Proportion, § 124. No independent rule is necessary.

EXAMPLES.

81–83. A New York merchant owes in London £375. How many dollars must he remit through houses in Naples and in Paris, if £20 = 121 ducats; 90 ducats = 414 francs;

500 francs = 91 dollars? How many dollars must he remit directly to London if the premium of exchange is 9 per cent. above the par value of $\$4\frac{4}{9}$ to the £? And how much was lost by the former circuitous method of remitting?

84–86. A merchant in New York orders £1000 due him in London to be forwarded to him by the following route: to Hamburgh, at 15 mark bancos per £; thence to Copenhagen, at 100 mark bancos for 43 rix-dollars; thence to Bourdeaux, at 4 rix-dollars for 18 francs; thence to New York, at 500 francs for 93 dollars. How many dollars did he receive? How many dollars would he have received had he ordered the £1000 direct to him, the premium of exchange being 7 per cent. above the par value of $\$4\frac{4}{9}$ to the £? And how much did he save by the above circuitous route?

87. If a man receives \$27 for 16 barrels of cider, and he can buy 2 barrels of flour for \$11, and 3 tons of coal for 4 barrels of flour, and 45 pounds of tea for 2 tons of coal, how many pounds of tea ought he to receive for 7 barrels of cider?

88. If 25 pears can be bought for 10 lemons, and 28 lemons for 18 pomegranates, and 1 pomegranate for 48 almonds, and 50 almonds for 70 chestnuts, and 108 chestnuts for $2\frac{1}{4}$ cents, how many pears can I buy for \$1·35?

89. If 121 English guineas are equal to 125 pounds sterling, and £23 = 61 pagodas of India, and 5 pagodas = \$9, how many English guineas will be equal to \$100?

90. If 74 francs = 9 tales of China, 10 tales of China = 6 ounces of Sicily, and 5 ounces of Sicily = \$12, how many francs will be equal to \$172?

Partnership or Fellowship.

§ 126. Fellowship is the union of two or more individuals in trade, with an agreement to share the losses and profits in the ratio of the amount which each puts into the partnership. The money employed is called the *capital stock.* The loss or gain to be shared is called the *dividend.*

A., B., and C. entered into copartnership. A. put in $180, B. put in $240, and C. put in $480. They gained $300. What is each one's part of the gain?

$180 A.'s stock+$240 B.'s stock+$480 C.'s stock=$900 whole stock.

$\frac{180}{900}=\frac{1}{5}$ =A.'s part of the entire stock.

$\frac{240}{900}=\frac{4}{15}$=B.'s " " " " "

$\frac{480}{900}=\frac{8}{15}$=C.'s " " " " "

Hence, A. must have $\frac{1}{5}$ of $300= $60 for his gain.

B. " " $\frac{4}{15}$ of $300= $80 " " "

C. " " $\frac{8}{15}$ of $300=$160 " " "

$300 verification.

From the above we may deduce the following

RULE.

Make each partner's stock the numerator of a fraction, and the sum of their stock a common denominator; then multiply the whole gain or loss by each of these fractions, for each partner's share.

EXAMPLES.

91. A. and B. purchase a house for $2500, of which sum A. furnished $1200 and B. $1300. They receive $210 rent for the same. What part of this sum ought each to share?

92–94. A person failing in business finds that all his debts amount to $4500, and that he has only $2500 to meet these claims. How much ought A. to receive, whose claim is \$360? And how much B. whose claim is \$400? And how much is he able to pay on the dollar?

95. Two brothers, the one 18 years old and the other 21 years old, contribute in the ratio of their ages \$300 towards the support of an aged parent. What did each contribute?

96. Two persons, A. and B., hire a pasture for \$30, into which A. turned 3 cows and B. 5. What part of the \$30 ought each to pay?

97. Five persons, A., B., C., D., and E., are to share between them \$2400. A. is to have $\frac{1}{6}$; B. is to have $\frac{1}{4}$; C. is to have $\frac{3}{8}$; D. and E. are to divide the remainder in proportion to the numbers 5 and 7. How much does each one receive?

98. There are three horses belonging to three men, employed to draw a load of plaster a certain distance for \$26·45. It is estimated that A.'s and B.'s horses do $\frac{3}{4}$ of the labor; A.'s and C.'s horses $\frac{9}{10}$; B.'s and C.'s horses $\frac{13}{20}$. They are to be paid proportionally according to these estimates. What ought each man to receive?

99. A., B., and C. agree to contribute \$365 towards building a church, which is to be at the distance of 2 miles from A., $2\frac{7}{8}$ miles from B., and $3\frac{1}{2}$ from C. They agree that their shares shall be proportional to the reciprocals of their distances from the church. What ought each to contribute?

100. A person wills to his two sons and a daughter the following sums: to the elder son \$1200, to the younger son \$1000, and to his daughter \$600; but it is found that his whole estate amounts to only \$800. How much ought each child to receive?

101. Four persons, A., B., C., and D., together contribute \$500 towards the erection of a school-house, which is placed

at the distance of $\frac{1}{4}$ of a mile from A.'s residence, $\frac{1}{2}$ of a mile from B.'s, $\frac{3}{4}$ of a mile from C.'s, and 1 mile from D.'s. They contributed in the reciprocal ratio of their respective distances from the school-house. How much did each give?

Double Fellowship.

§ 127. When the stock of the several partners continues in trade for unequal periods of time, the profit or loss must be apportioned with reference both to the stock and time. In such cases the fellowship is called Double Fellowship.

Three partners, A., B., and C., put into trade money as follows: A. put in \$400 for 2 months; B. put in \$300 for 4 months; C. put in \$500 for 3 months. They gained \$350. How must they share this gain?

It is evident that \$400 for 2 months is the same as $\$400\times2=\$800$ for one month; \$300 for 4 months is the same as $\$300\times4=\$1200$ for one month; \$500 for 3 months is the same as $\$500\times3=\$1500$ for one month.

Hence \$800 A.'s money for one month, + \$1200 B.'s money for one month, + \$1500 C.'s money for one month = \$3500.

Therefore, by Single Fellowship,

A. must have $\frac{800}{3500}=\frac{8}{35}$ of \$350=\$80.
B. " " $\frac{1200}{3500}=\frac{12}{35}$ of 350=120.
C. " " $\frac{1500}{3500}=\frac{3}{7}$ of 350=150.

\$350 verification.

RULE.

Multiply each partner's stock by the time it was in trade; make each product the numerator of a fraction, and the sum of the products a common denominator; then multiply the whole gain or loss by each of these fractions, for each partner's share.

EXAMPLES.

102. Two persons, A. and B., enter into partnership, A. furnishing \$325 for 6 months, and B. \$200 for 8 months. What ought each to contribute to meet a loss of \$100?

103. In the construction of a piece of road, A. furnished 5 laborers, each of whom worked 9 days; B. furnished 7 laborers, each working 11 days: for the whole work they received \$150. What was each one's share of this sum?

104. To a certain school, A. sends 5 scholars during 35 days, and B. sends 4 during 38 days, and has to pay a rate bill of \$3·04. What was A.'s rate bill?

105. If I borrow \$300 for 7 months of A., \$400 for 8 months of B., and \$450 for 9 months of C.; for this accommodation I wish to divide \$100 among the three. What ought each to receive?

106. For the transportation of 100 barrels of flour a distance of 93 miles, I have to pay \$46·50 to 5 individuals, who performed the labor as follows: A. carried 50 barrels a distance of 70 miles, B. carried 10 barrels a distance of 93 miles, C. carried 40 barrels a distance of 53 miles, D. carried 50 barrels a distance of 23 miles, and E. carried 40 barrels a distance of 40 miles. How much ought I to pay to each?

107. Three farmers hired a pasture for \$55·50 for the season. A. put in 6 cows for 3 months, B. put in 8 cows for 2 months, C. put in 10 cows for 4 months. What rent ought each to pay?

108. On the first day of January, A. began business with \$650; on the first day of April following, he took B. into partnership with \$500; on the first day of next July, they took in C. with \$450: at the end of the year they found they had gained \$375. What share of the gain had each?

109. A., B., and C. have together performed a piece of work for which they receive $94. A. worked 12 days of 10 hours each; B. worked 15 days of 6 hours each; C. worked 9 days of 8 hours each. How ought the $94 to be divided between them?

110. A ship's company take a prize of $4440, which they agree to divide among them according to their pay and the time they have been on board. Now the officers and midshipmen have been on board 6 months, and the sailors 3 months: the officers have $12 per month, the midshipmen $8, and the sailors $6 per month; moreover, there are 4 officers, 12 midshipmen, and 100 sailors. What will each one's share be?

CHAPTER XV.

AVERAGE.

§ 128. If the sum of a series of *unequal* quantities be divided by the number of quantities, the quotient will be one of a series of *equal* quantities, whose sum will equal that of the former series. The value of this quotient is called the Average of the given quantities. Thus,

A laborer worked 5 hours on Monday, 6 on Tuesday, 3 on Wednesday, 9 on Thursday, 9 on Friday, and 10 on Saturday. How many hours work did he average each day? $5+6+3+9+9+10=42$; and 42 hours $\div 6=7$ hours' average work per day. Proof, $7\times 6=42$. Hence the following rule for determining the average:

Divide the sum of the given quantities by the number of quantities. The quotient will be the average.

EXAMPLES.

1–5. What is the average of 1, 2, 3? of 2, 3, 4, 7? of 5, 3, 4, 9, 4? of 4, 9, 8, 7, 2, 6? of 8, 4, 3, 9, 7, 12, 6?

6–10. Find the average of 4, 7, 6, 2, 12; of 12, 14, 19, 18, 21; of 6, 8, 13, 24, 30; of 36, 42, 96, 104; of 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

11. A gentleman expended in 1845, \$1250·75; in 1846, \$1196·38; in 1847, \$1341·67; in 1848, \$1275·96; in 1849, \$1060·07; in 1850, \$1196·27. What was his average yearly expenditure?

12. The following was the record of attendance for one week in a certain school: Monday *A. M.* 109, *P. M.* 94; Tuesday *A. M.* 109, *P. M.* 103; Wednesday *A. M.* 97, *P. M.* 91; Thursday *A. M.* 104, *P. M.* 100; Friday *A. M.* 88, *P. M.* 36. What was the average half-day attendance in that school?

13. Seven men weighed each as follows: 212*lbs.*, 135*lbs.*, $167\frac{1}{2}$*lbs.*, $196\frac{3}{4}$*lbs.*, $121\frac{1}{4}$*lbs.*, 102*lbs.*, 229*lbs.* What was their average weight? What was their aggregate weight?

14. What was the average cost of the following articles? The first cost £2 4*s.* 3*d.*; the 2d, £5 18*s.* 6*d.* 3*qr.*; the 3d, £14 3*s.* 2*d.*; the 4th, 19*s.* 2*qr.*

15. Your grandfather's age is 78 years 8 months; your father's is 54 years, 7 months, 22 days; your brother's 21 years, 3 months, 29 days; your sister's 16 years 4 months; your own 11 years, 6 months, 17 days. What is the average age of each?

16. At sunrise on 5 successive days the barometer was as follows: 29·38 inches; 29·41; 29·63; 29·87; 30·06. What was the average height of the mercury during this time?

17–18. The declination of the sun at noon on the first 7 days of January, 1851, was as follows: 23° 2′ 14″; 22°

57′ 10″; 22° 51′ 38″; 22° 45′ 39″; 22° 39′ 12″; 22° 32′ 19″; 22° 24′ 59″. What was the average declination of the sun during this time? At the same times respectively the sun was slow of clock 3*m.* 44*s.*; 4*m.* 12*s.*; 4*m.* 40*s.*; 5*m.* 8*s.*; 5*m.* 35*s.*; 6*m.* 2*s.*; 6*m.* 28*s.* What was the average time which the sun was slow of clock?

19. By observation the length of a pendulum vibrating once in a second of time, is found to be, at the equator, 39·01612 inches; at the Cape of Good Hope 39·07815 inches; at New York 39·10120 inches; at Paris 39·12929; at London 39·13929 inches. What is the average length for these five places?

20. The mean distances of the four satellites of Jupiter are as follows, the radius of Jupiter being taken as the unit: 6·04853; 9·62347; 15·35024; 26·99835. What is their average mean distance?

21–22. A locomotive made 7 successive trips over a track of 17 miles in the following times: 50*m.* 32*s.*; 49*m.* 3*s.*; 48*m.* 10*s.*; 40*m.* 30*s.*; 41*m.* 35*s.*; 45*m.* 45*s.*; 44*m.* 20*s.* What was the average time of one trip? What was the average time of running one mile?

23. A company of 6 California gold diggers find on a certain day gold as follows: the 1st, 7*oz.* 13*pwt.*; the 2d, 9*oz.* 14*pwt.*; the 3d, 6*oz.* 10*pwt.*; the 4th, 4*oz.* 4*pwt.*; the 5th, 10*oz.* 8*pwt.*; the 6th, 3*oz.* 2*pwt.* What was the average for each man?

24–25. On a certain day in January I noticed the thermometer to be as follows: at 6 A. M. 20°; at 7, 23°; at 8, 25°; at 9, 30°; at 10, 36°; at 11, 40°; at noon, 44°; at 1 P. M. 45°; at 2, 48°; at 3, 46°; at 4, 44°; at 5, 39°; at 6, 33°. What was the average during the forenoon, and what during the afternoon, if the observation at noon is not taken into the account?

EQUATION OF PAYMENTS.

§ 129. EQUATION OF PAYMENTS is a process by which we ascertain the *average time* for the payment of several sums due at different times.

Suppose I owe \$1000, of which \$100 is due in 2 months, \$250 in 4 months, \$350 in 6 months, and \$300 in 9 months. If I pay the whole sum at once, how many months' credit ought I to have?

A credit on \$100 for 2 months is the same as a credit on \$1 for 200 months.	$2mo. \times 100 = 200mo.$
A credit on \$250 for 4 months is the same as a credit on \$1 for 1000 months.	$4mo. \times 250 = 1000mo.$
A credit on \$350 for 6 months is the same as a credit on \$1 for \$2100 months.	$6mo. \times 350 = 2100mo.$
A credit on \$300 for 9 months is the same as a credit on \$1 for 2700 months.	$9mo. \times 300 = 2700mo.$
	\$1000 6000*mo.*

Hence, I ought to have the same as a credit on \$1 for 6000 months. But if I wish a credit on \$1000 instead of \$1, it ought evidently to be for only one-thousandth part of 6000 months, which is 6 months.

Hence this

RULE.

Multiply each sum by the time that must elapse before it becomes due; divide the amount of these products by the amount of the sums; the quotient will be the equated time.

EXAMPLES.

26. I purchased a bill of goods amounting to \$1500, of which I am to pay \$300 in 2 months, \$500 in 4 months, and the balance in 6 months. What would be the mean time for the payment of the whole?

27. A merchant owes \$500 to be paid in 6 months, \$600 to be paid in 8 months, and \$400 to be paid in 12 months. What is the average time of payment?

28. A. owes B. a certain sum: one-third is due in 6 months, one-fourth in 8 months, and the remainder in 12 months. What is the mean time of payment?

NOTE.—It makes no difference what the amount is which A. owes B., since it is certain fractional parts which becomes due at particular times. If we suppose the sum to be $1, then our work will be

	mo.		*mo.*
	$\frac{1}{3} \times$	6 =	2
	$\frac{1}{4} \times$	8 =	2
Remainder is $\frac{5}{12}$, and	$\frac{5}{12} \times$	12 =	5

29. A merchant has due him $300 to be paid in 2 months, $800 to be paid in 5 months, $400 to be paid in 10 months. What is the average time for the payment of the whole?

30. A merchant owes $1200, payable as follows: $200 in 2 months, $400 in 5 months, and the remainder in 8 months. He wishes to pay the whole at one time. What is the average time of such payment?

31. A merchant bought goods to the amount of $2400, for one-fourth of which he was to pay cash at the time of receiving the goods, one-third in 6 months, and the balance in 10 months. What was the equitable time for the payment of the whole?

32. Suppose I owe $100 payable on January 1st, $150 on February 5th, $300 on April 10th. If we count from January 1st, and allow 29 days to February, it being leap year, on what day ought the whole sum in equity to be paid?

NOTE.—Estimate the time in days. The 1st payment is $100 due in 0 days.

33. A merchant bought a bill of goods amounting to $1000. He agreed to pay $250 the first day of the next March, $250 on the 3d of the following May, $250 on the 4th of the following July, and the remaining $250 on the 15th of the

following September. What would be the equitable time for paying the whole?

NOTE.—As the sums are equal, it will simplify the operations tc consider each payment $1.

34. A person purchased a bill of goods amounting to $3450, and agreed to pay as follows: $1000 at the end of 3 months, $1000 at the end of 6 months, and the balance at the end of 9 months. What was the average time for which he received credit on the whole sum?

35. A person owes as follows: $300 due the 10th of March, $250 due the 28th of March, $450 due the 31st of March, and $100 due the 25th of the following April. At what time could the whole sum in equity be paid?

36. A person owes a certain sum of money, $\frac{1}{5}$ of which is due in $3\frac{1}{2}$ months, $\frac{1}{4}$ is due in $4\frac{1}{2}$ months, $\frac{1}{3}$ is due in 5 months, and the balance is due in 8 months. What is the mean time of payment?

37–38. A person purchases a farm for $7000, and agrees to pay as follows: $1000 at the end of 3 months; $1500 at the end of 4 months; $2000 at the end of 5 months; $2500 at the end of 6 months. At what time in equity ought he to pay the whole? Suppose he had agreed to pay $2500 at the end of 3 months, $2000 at the end of 4 months, $1500 at the end of 5 months, and $1000 at the end of 6 months; then, in equity, at what time ought the whole to be paid?

39. A sum of money is due as follows: $\frac{1}{2}$ on the 1st of July, $\frac{1}{4}$ on the 1st of August, $\frac{1}{8}$ on the 1st of September, $\frac{1}{16}$ on the 1st of October, and the balance on the 1st of November. At what time, estimating from the 1st of July, ought the whole in equity to be paid?

§ 130. Suppose $1000 to be due at the end of 6 months;

that 3 months before it is due $100 was paid, and that 1 month before the expiration of the 6 months $300 was paid. How long after the end of the 6 months may the balance of $600 remain unpaid?

NOTE.—The problem here is, when a debt due at some future period has received several partial payments before the time due, to find how long beyond this time the balance may in equity remain unpaid.

$3mo.\times 100=300mo.$; $1mo.\times 300=300mo.$; that is, $1 must have a credit of $300mo.+300mo.=600mo.$ The balance due is $600, which must have a credit equal to $600mo.\div 600=1mo.$ beyond the 6 months. Hence this

RULE.

Multiply each payment by the time it was paid before due; then divide the sum of the products thus obtained by the balance remaining unpaid; the quotient will be the equated time.

EXAMPLES.

40. Suppose $1496·41 to be due at the end of 90 days: that 84 days before it is due there is paid $500; 32 days before the 90 days expire there is paid $502·50. How long after the 90 days before the balance of $493·91 ought to be paid?

41. A. lent $200 to B. for 8 months; at another time he lent him $300 for 6 months. For how long a time ought B. to lend A. $800 to balance the favor?

42. A person owes $1000, due at the end of 12 months. At the end of 3 months he pays $100, one month after that he pays $100. How long beyond the 12 months may the balance of $800 remain unpaid?

43–44. A credit of 6 months on $500, and of 4 months on $1000, is the same as a credit of how many dollars for

8 months ? It is the same as a credit on $800 for how many months ?

§ 131. It is customary with many merchants to give a credit of from 3 to 6 months, on their bills of sale. In such cases, in settling up their accounts, which generally consist of various items of debit and credit at sundry times, it is very desirable to have some simple rule by which the cash balance can be found.

Suppose A. has the following account with B. :

1848.		Dr.	1848.		Cr.
Jan. 10.	To Merchandise . . .	$100	Feb. 8.	By Merchandise . . .	$50
March 26.	" " . .	400	April 23.	" " . . .	375

What is the *cash* balance, July 10, 1848, if interest is estimated at 7 per cent., and a credit of 30 days is allowed on all the different sums ?

If interest were not considered, the above account could be balanced as follows :

1848.		Dr.	1848.		Cr.
Jan. 10.	To Merchandise . .	$100	Feb. 8.	By Merchandise . . .	$50
March 26.	" " . .	400	April 23.	" " . . .	375
				" Balance	75
		$500			$500
	To Balance	$75			

Had no credit been given, the debits should be increased by the following items of interest. (§ 83, note 4, and § 105.)

On $100 for 182 days, at 7 per cent. $=100\times182\times\frac{0.07}{365}$.
" 400 " 106 " " " $=400\times106\times\frac{0.07}{365}$.

In like manner the credits should be increased by interest :

On $50 for 153 days, at 7 per cent. $=50\times153\times\frac{0.07}{365}$.
" 375 " 78 " " " $=375\times78\times\frac{0.07}{365}$.

But, since 30 days' credit is given on all sums, it follows that by the above, we should increase the debits by an excess of interest equal to the interest of the sum of debits, $500 for 30 days $=500\times30\times\frac{0.07}{365}$. In like manner we should increase the credits by an excess of interest equal to the interest of sum of credits, $425, for 30 days $=425\times30\times\frac{0.07}{365}$.

Now if, instead of diminishing the debit items of interest by $500 \times 30 \times \frac{0{\cdot}07}{365}$, and the credit items of interest by $425 \times 30 \times \frac{0{\cdot}07}{365}$, we merely diminish the debit items of interest by the interest on *merchandise* balance, \$75 for 30 days, which is $75 \times 30 \times \frac{0{\cdot}07}{365}$, the result will be the same. And since taking any sum from one side of a book account has the same effect as adding the same sum to the other side, it follows, that instead of diminishing the debit items of interest by $75 \times 30 \times \frac{0{\cdot}07}{365}$, we may increase the credit items of interest by this same quantity.

From which we see that the difference between $100 \times 182 \times \frac{0{\cdot}07}{365} + 400 \times 106 \times \frac{0{\cdot}07}{365}$ and $50 \times 153 \times \frac{0{\cdot}07}{365} + 375 \times 78 \times \frac{0{\cdot}07}{365} + 75 \times 30 \times \frac{0{\cdot}07}{365}$ is the *interest* balance.

The operations indicated in the foregoing work may be exhibited in a more condensed form, as follows:

DEBITS.	CREDITS.
\$ Days.	\$ Days.
$100 \times 182 = 18200$	$50 \times 153 = 7650$
$400 \times 106 = 42400$	$375 \times 78 = 29250$
	$75 \times 30 = 2250$
60600	
39150	39150

$\frac{0{\cdot}07}{365}$ of $21450 = \$4{\cdot}11 =$ *interest* balance.

Hence the foregoing account will become balanced as follows:

1848.		Dr.	1848.		Cr.
Jan. 10.	To Merchandise .	\$100·00	Feb. 8.	By Merchandise . .	\$50·00
March 26.	" " ·	400·00	April 23.	" " . .	375·00
July 10.	" balance of interest	4·11	July 10.	" balance	79·11
		\$504·11			\$504·11
July 10.	" Cash balance . .	\$79·11			

From the above, we deduce this

RULE.

Place such sum on the debtor or credit side as may be necessary to balance the account, which sum may be regarded as MERCHANDISE BALANCE. *Then multiply the number of dollars in each entry by the number of days from the time such entry was made, to the time of settlement; observing to* *multiply the merchandise balance by the number of days for which credit is given.*

Multiply the difference between the sum of the debit products, and the sum of the credit products, by the interest of $1 *for* 1 *day; the product will be the number of dollars in* INTEREST BALANCE, *which will be in favor of the debit side of account, when the sum of debit products exceeds the sum of credit products; but in favor of the credit side when the sum of credit products exceeds the sum of debit products. If then, the interest balance be added to, or subtracted from, the merchandise balance, as the case may require, it will give the cash balance.*

EXAMPLES.

45. Suppose A. has the following account with B.:

1848.		*Dr.*	1848.		*Cr.*
Jan. 1.	To Merchandise . . .	$200	Jan. 15.	By Merchandise . .	$300
March 3.	" " . . .	500	March 20.	" " . .	400
May 10.	" " . . .	100	May 3.	" " . .	200
June 6.	" " . . .	300	July 1.	" " . .	50
		1100			$950
		950			
	Merchandise balance	$150			

What is the cash balance of the above account on the 1st of July, 1848, provided each individual is allowed 90 days' time on his purchases, if interest is estimated at 7 per cent.?

NOTE.—The *interest* balance will be found in favor of the credit side; the *merchandise* balance is in favor of the debtor's side.

46–47. A. has the following account with B:

1850.		*Dr.*	1850.		*Cr.*
March 9.	To Merchandise . .	$18·38	March 28.	By Merchandise . .	$60·20
April 4.	" " . .	56·41	July 2.	" " . .	100·00
June 12.	" " . .	105·03	" 30.	" " . .	263·40
July 17.	" " . .	88·13	Aug. 20.	" " . .	75·75
		$267·95			499·35
					267·95
				Merchandise balance	$231·40

What was the cash balance of the above account, and in whose favor, on the 1st day of October, 1850, provided each individual is allowed 90 days' time on his purchases, interest being 6 per cent.?

What was the cash balance of the above, on the 1st day of January, 1851, the other conditions remaining the same?

48. Suppose A.'s account with B. to have been as follows:

1848.		*Dr.*	1848.		*Cr.*
Jan. 10.	To Merchandise .	$250·37	June 25.	By Merchandise . .	$37·51
Feb. 25.	" " . .	113·04	July 20.	" " . .	50·98
March 1.	" " . .	405·59	July 28.	" " . .	600·03
		769·00			$688·52
		688·52			
	Merchandise balance	$80·48			

What is the cash balance, and in whose favor, on the 1st of August, 1848, provided 6 months, or 180 days' time is given, interest being 6 per cent.?

NOTE.—In practice, when the cents in any of the entries, as in this example, are less than 50, we may, without sensible error, omit them; but when they are 50, or greater, we may consider them as an additional dollar.

49–50. A.'s account with B. is as follows:

1850.		*Dr.*	1850.		*Cr.*
September 2.	To Merchandise,	$212·14	September 13.	By Merchandise,	$300·00
" 25.	" "	405·21	" 30.	" "	212·12
October 24.	" "	303·60	October 28.	" "	404·80
Novemb'r 21.	" "	140·80			$916·92
Decemb'r 24.	" "	28·30			
		1090·05			
		916·92			
	Merchandise balance	$173·13			

What was the cash balance, and in whose favor, of the above account, on the 1st day of January, 1851, if each individual had a credit of 4 months or 120 days, interest being 7 per cent.?

What was the cash balance on the 21st day of June, 1851, the other conditions remaining the same?

Alligation Medial.

§ 132. Alligation Medial teaches the method of finding the *average* or *mean* value of a compound, when its several ingredients and their respective values are given.

A grocer mixes 140 pounds of tea, worth 8*s.* per pound; 200 pounds, worth 6*s.* per pound; and 160 pounds, worth 10*s.* per pound. What is a pound of the mixture worth?

140 pounds of tea, at 8*s.* per pound, are worth $140\times8=1120s.$, 200 pounds, at 6*s.*, are worth $200\times6=1200s.$; 160 pounds, at 10*s.*, are worth $160\times10=1600s.$ Therefore, the mixture, which is 500 pounds, is worth $1120+1200+1600=3920s.$; and one pound must be worth $\frac{3920}{500}=7\frac{21}{25}s.$

Hence, to find the mean value of a compound, composed of several ingredients of different values, we have this

RULE.

Divide the sum of the values of all the quantities by the sum of the quantities.

EXAMPLES.

51. A wine merchant mixed several sorts of wine, viz: 32 gallons, at 40 cents per gallon; 15 gallons, at 60 cents per gallon; 45 gallons, at 48 cents per gallon; and 8 gallons, at 85 cents per gallon. What is the value of a gallon of the mixture?

52. A farmer mixed together 7 bushels of rye, worth 72 cents per bushel; 15 bushels of corn, worth 60 cents per bushel; and 12 bushels of wheat, worth $1·20 per bushel. What is the value of a bushel of the mixture?

53. A goldsmith melts together 11 ounces of gold 23

carats fine, 8 ounces 21 carats fine, 10 ounces of pure gold, and 2 pounds of alloy. How many carats fine is the mixture?

NOTE.—A *carat* is a 24th part. Thus, 21 carats fine is the same as $\frac{21}{24}$ pure metal.

54. On a certain day, the mercury in the thermometer was observed to stand 2 hours at 62 degrees, 4 hours at 70 degrees, 5 hours at 72 degrees, 3 hours at 59 degrees, and 1 hour at 75 degrees. What was the mean temperature for the 15 hours?

55. Suppose a ship sail at the rate of 5 knots for 3 hours, at 7 knots for 5 hours, and 8 knots for 4 hours. What is her rate of sailing during the 12 hours?

56. A grocer mixes 30 pounds of sugar, worth 10 cents per pound; 40 pounds, worth $10\frac{1}{2}$ cents per pound; 24 pounds, worth 11 cents per pound; and 60 pounds, worth 13 cents per pound. What is a pound of the mixture worth?

57. A person bought 4 dozen of eggs, at $18\frac{3}{4}$ cents per dozen; 6 dozen, at 21 cents per dozen; $3\frac{1}{2}$ dozen, at 24 cts. per dozen; $5\frac{1}{2}$ dozen, at 25 cents per dozen. What was the average cost of one dozen?

58. A flour merchant bought 300 barrels of flour, at \$3·75 per barrel; 250 barrels, at \$3·87$\frac{1}{2}$ per barrel; 500 barrels, at \$3·93$\frac{3}{4}$ per barrel. What did the whole average per barrel?

59. A dairyman made during the first month, 26 cheeses, each weighing 85 pounds; during the second month, he made 25, each weighing 83 pounds; and during the third month he made 20, each weighing $80\frac{1}{2}$ pounds. What was the average weight of his cheese for the 3 months?

60. A dairyman having 30 cows, finds at a certain milking that 6 give 12 quarts of milk each; 8 give $10\frac{1}{2}$ quarts each; 10 give $9\frac{1}{2}$ quarts each; and the others give only 8 quarts each. What did each cow on an average give?

NOTE.—It will be seen that the *principle* of Equation of Payments and that of Alligation Medial are the same: in the one case, we operate upon *debts, and payments, and time;* in the other, upon *ingredients or quantities and values.*

ALLIGATION ALTERNATE.

§ 133. ALLIGATION ALTERNATE is the reverse of Alligation Medial; that is, it teaches the method of determining the proportional quantities of several ingredients, so that the compound shall have a given value.

Suppose we wish to mix teas, which are worth 4 and 6 shillings per pound, so that the mixture may be worth 5 shillings per pound; it is obvious that we must take equal quantities of each, since the price of the one is as much less than the average price, as the other is greater.

Again, suppose we wish to mix teas which are worth 4 and 7 shillings per pound, so that the mixture may be worth 5 shillings. In this case the 7 shilling tea is 2 shillings above the average price, whilst the 4 shilling tea is but 1 shilling below. It will be necessary to use twice as much of the 4 shilling tea as of the 7 shilling tea; and in all cases it is obvious that the *quantities to be used will be in the inverse ratio to the differences between their prices and the mean price.*

When there are more than two simples they may be compared together in couplets, one term of which must obviously exceed the average price, while the other must be less.

CASE I.

The rates of the several ingredients being given, to make a compound of a fixed rate.

RULE.

I. *Write the rates of the simples in a vertical column. Connect the rate of each ingredient which is less than the rate of the compound, with one or more rates greater than the rate of the compound; connect in the same way, each rate* *which is greater than the rate of the compound, with one or more rates which are less.*

II. *Write the difference between the rate of each one ingredient and the value of the compound, opposite the rate of each other ingredient with which the former is connected. If only one difference stands against any rate, it will be the required quantity of the ingredient of that rate; but if there be several, their sum will be the quantity required.*

How much sugar at 5, 6, and 10 cents per pound, must be mixed together, so that a pound of the mixture may be worth 8 cents?

8	5	2
	6	2
	10	3+2=5

Therefore, if we take 2 pounds at 5 cents, 2 pounds at 6 cents, and 5 pounds at 10 cents, we shall satisfy the conditions of the question. It is obvious, that *any other quantities of the several ingredients which are to each other as the numbers* 2, 2, *and* 5, will satisfy the question equally well; so that in Alligation Alternate the number of solutions are *indefinite;* all that we can do is to find the ratios of the quantities required.

In many cases the ingredients will admit of being connected in several ways, and then we shall obtain as many sets of ratios as there are methods of connecting them; for example:

How many pounds of raisins at 4, 6, 8, and 10 cents per pound, must be mixed, so that a pound of the compound may be worth 7 cents?

In this question the terms may be connected in seven distinct ways; therefore, we shall obtain seven sets of ratios, as follows:

7	4	3	1	1+3=4	3		
	6	1	3	1	1+3=4		
	8	1	3	3+1=4	1		
	10	3	1	3	3+1=4		

7	4	1	1+3=4	1+3=4
	6	1+3=4	3	1+3=4
	8	3+1=4	3	3+1=4
	10	1	3+1=4	3+1=4

How much tea at 5 shillings, 6 shillings, and 8 shillings per pound, must be mixed, so that the mixture may be worth 7 shillings per pound?

If we compound only the 5 and 8 shilling teas, we must take them in the ratio of 1 to 2, since 7 shillings is 1 shilling less than 8 shillings, and 2 shillings greater than 5 shillings. Hence, any one of the compounds in the following group (A) will be worth 7 shillings per pound.

	(1)	(2)	(3)	(4)	(5)	(6)	
5 shilling tea	1	2	3	4	5	6, &c.	(A).
8 shilling tea	2	4	6	8	10	12, &c.	
Sums,	3;	6;	9;	12;	15;	18, &c.	

If we now mix the 6 and 8 shilling teas, we see that it will be necessary to take equal quantities of each, since the average price is to be as much above 6 shillings as it is below 8 shillings. Hence, the following compound will also be worth 7 shillings per pound.

	(1)	(2)	(3)	(4)	(5)	(6)	
6 shilling tea	1	2	3	4	5	6, &c.	(B).
8 shilling tea	1	2	3	4	5	6, &c.	
Sums	2;	4;	6;	8;	10;	12, &c.	

Now, it is obvious, we may combine any one of these last results with any one of the former results. Thus, if we combine (1) of group (A) with (1) of (B), we have

	Pounds.
5 shilling tea....................	1
6 " "	1
8 " "	2+1=3

If we combine (1) of (A) with (2) of (B), we have

	Pounds.
5 shilling tea..	1
6 " "	2
8 " "	2+2=4

Combining (2) of (A) with (3) of (B), we have

	Pounds.
5 shilling tea	2
6 " "	3
8 " "	4+3=7

Combining (5) of (A) with (4) of (B), we have

	Pounds.
5 shilling tea..................	5
6 " "	4
8 " "	10+4=14

The number of combinations which could be made in this way is unlimited; hence the above class of questions in Alligation admit of an infinite number of answers.

EXAMPLES.

61–66. How much wine, at $1·12 per gallon and 48 cents per gallon, must be mixed together, that the composition may be worth 60 cents per gallon? 65 cts.? 72 cts.? 84 cts.? 91 cts.? $1·02?

67. How many gallons of wine and water must be mixed together, that the mixture may be worth 60 cents per gallon, the water being considered of no value, and the wine with which it is mixed being worth 90 cents per gallon?

68–71. Having gold of 12, 16, 17, and 22 carats fine, what proportion of each kind must I take, to make a compound of 18 carats fine? 19? 20? 21?

72–76. How much of each sort of grain, at 56, 62, and 75 cents per bushel, must be taken, so that the mixture may be worth 60 cents per bushel? 65 cts.? 68 cts.? 70 cts.? 72 cts.?

77–81. How much tea at 4 shillings, 5 shillings, 6 shillings, and 12 shillings per pound, must be mixed that the mixture may be worth 7 shillings per pound? 8*s.*? 9*s.*? 10*s.*? 11*s.*?

CASE II.

When one of the ingredients is limited to a certain quantity.

A person wishes to mix 10 bushels of wheat, worth $1 per bushel, with rye, worth 70 cents per bushel, and oats, worth 30 cents per bushel, so that the mixture may be worth 60 cents per bushel. How many bushels of rye and oats must he use?

Proceeding, according to Case I., we find the proportionate numbers to be 30, 30, and 50. Hence,

$$30 : 30 :: 10 : 10$$
$$30 : 50 :: 10 : 16\tfrac{2}{3}$$

So that he must make use of 10 bushels of rye and $16\frac{2}{3}$ bushels of oats. Hence, this

RULE.

Find the proportionate quantities of each ingredient, by Case I., as though there was no limitation; then, as the difference against the simple whose quantity is given, is to each of the other differences, so is the given quantity of that simple to the quantity required of each of the other simples.

EXAMPLES.

82–85. A grocer has 90 pounds of tea, worth 90 cents per pound, which he wishes to mix with three other qualities, valued at 80 cents, 70 cents, and 60 cents per pound. How much must he take of these three kinds, so as to be able to sell the mixture at 65 cents per pound? at 68 cts.? at 85 cts.? at $87\frac{1}{2}$ cts.?

86–91. A merchant has 90 pounds of spice worth 86 cents per pound, which he wishes to mix with three other sorts which are worth 30, 40, and 50 cents per pound, respectively. How many pounds must be used so that the compound may be worth 55 cents per pound? 60 cts.? 65 cts.? 70 cts.? 75 cts.? 80 cts.?

92–96. A merchant wishes to mix 100 pounds of sugar, worth 10 cents per pound, with three other kinds worth 9, 8, and 5 cents per pound, respectively. How many pounds must he use so that the compound may be worth $5\frac{1}{2}$ cts.? 6 cts.? $6\frac{1}{2}$ cts.? 7 cts.? $7\frac{1}{2}$ cts.?

CHAPTER XVI.

INVOLUTION AND EVOLUTION.

INVOLUTION.

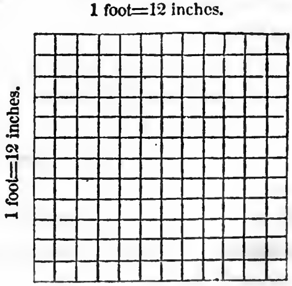
§ 134. The product arising from multiplying a number into itself is called the *second power*, or the *square* of that number. Thus, $3\times3=9$: 9 is the square of 3. If the square of a number be again multiplied by that number, the result is called the *third power*, or the *cube* of the number. Thus, $3\times3\times3=27$: the number 27 is the cube of 3.

The word *power* denotes the product arising from multiplying a number into itself a certain number of times; and the number thus multiplied is called the *root*. Thus, 9 is the second power of 3, and 3 is the *square root* of 9. In the same manner 27 is the third power of 3, and 3 is the *cube root* of 27.

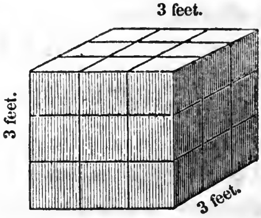
Involution is the process of raising a number to a given power.

To denote that a number is to be raised to a power, a small figure, called the *index* or *exponent*, is placed above and to the right of the number whose power is to be found, as 4^2. Here the exponent is 2, and denotes the 2d power of 4, or 4×4. So $3^3=3\times3$, &c.

The *second* power of a number is called the *square* of that number, because the surface of a geometrical square may be obtained by multiplying the number, expressing one side, by itself. Thus, if the side of the adjacent square is 12 *linear* units, or, as here, 12 inches long, its entire surface will be denoted by $12\times12=144$ *square* units, which in this case will be 144 square inches.



The *third* power of a number is called the *cube* of that number, because the solid contents of a geometrical cube, as in the adjacent figure, can be obtained by raising the number expressing one side, to the 3d power. Thus, $3 \times 3 \times 3 = 27$ cubic feet.



To raise a number to any power, *multiply the number by itself as many times as there are units less one in the exponent; the last product will be the power sought.*

EXAMPLES.

1–10. Find the square of each of the following numbers: 14; 19; 24; 36; 48; 57; 93; 111; 168; 233.

11–20. Cube each of the following numbers: 13; 18; 23; 35; 44; 56; 91; 148; 336; 221.

21–22. What is the 5th power of 47? the 9th power of 9?

23–26. What is the square of 0·75? of 1·14? of 34·09? of 4·781?

27–31. What is the cube of 0·61? of 0·13? of 0·202? of 0·65? of 3·021?

32–36. Find the the square of $2\frac{1}{4}$; of $3\frac{2}{3}$; of $4\frac{3}{5}$; of $7\frac{3}{8}$; of $9\frac{12}{13}$.

37–41. Find the cube of $\frac{1}{2}$; of $\frac{2}{3}$; $3\frac{4}{11}$; of $5\frac{4}{7}$; of $9\frac{2}{5}$.

42–44. What is the 5th power of $2\frac{3}{4}$? the 4th power of 0·25? of 0·375?

EVOLUTION.

§ 135. Evolution is the reverse of involution. *It is the process of finding the root of a given power.* Thus, 6 is the *square root* of 36, because 6 raised to the 2d power, that is, $6^2=36$, is the *square* of 6. So 4 is the *cube root* of 64, because 4 raised to the 3d power, that is, $4^3=64$, is the *cube* root of 4.

The symbol $\sqrt{}$, called the radical sign, is used to denote the square root of a number, as $\sqrt{9}=3$; $\sqrt{36}=6$. The 3d or cube root is denoted by the figure 3 written over the radical sign, $\sqrt[3]{8}=2$; $\sqrt[3]{64}=4$. In like manner $\sqrt[4]{}$ signifies the 4th root, &c.

§ 136. Before explaining the method of extracting the SQUARE ROOT, we will involve some numbers by considering them decomposed into *units, tens, hundreds,* &c.

What is the square of 25 ?

$25=20+5$ The 2 tens are written as 20 units.
$\quad 20+5$

$20^2+20\times5$=product by the units in the tens.
$\quad +20\times5+5^2$=product by the units.

$20^2+2\times20\times5+5^2$=square of $20+5$.

That is, *the square of a number consisting of tens and units equals the square of the tens (expressed in units), plus twice the product of the tens (expressed in units) into the units, plus the square of the units.*

What is the square of 252 ?

$252=200+50+2$ The 2 hundreds are written as 200 units, and
$\quad 200+50+2$ the 5 tens as 50 units.

$200^2+200\times50+200\times2$=product by hundreds (expressed in units).
$\quad 200\times50+50^2+50\times2$=product by tens (expressed in units).
$\quad +200\times2+50\times2+2^2$=product by units.

$200^2+2\times200\times50+50^2+2\times(200+50)\times2+2^2$

That is, *the square of a number consisting of hundreds, tens, and units, is equal to the square of the hundreds (expressed in units), plus twice the product of the hundreds into the tens (expressed in units), plus the square of the tens (expressed in units), plus twice the product* *of the sum of the hundreds and tens (expressed in units) into the units, plus the square of the units.*

Continuing in this way, we could show that *the square of the sum of any number of numbers is the square of the first number, plus twice the product of the first number into the second, plus the square of the second; plus twice the product of the sum of the first two into the third, plus the square of the third; plus twice the product of the sum of the first three into the fourth; plus the square of the fourth; and so on.*

§ 137. We will now extract the square root of 625. But first let us ascertain how many figures its root must have.

The smallest digit is 1; its square is 1. The largest digit is 9; its square is 81. The square of the *units*, then, must be either one or two figures; either units or units and tens.

The smallest number of 2 figures is 10; its square is 100. The largest number of 2 figures is 99; its square is 9801. The square of 2 figures or *tens* must, then, be 3 figures or hundreds, or 4 figures or thousands, &c.

Hence, if a number consist of 1 or 2 figures, its root must consist of 1 figure; if of 3 or 4 figures, its root must consist of 2 figures; if of 5 or 6 figures, its root must consist of 3 figures, and so on. That is, *the square will contain twice as many figures as the root, or twice as many, less one.*

Then, dividing any number into groups by placing a dot over the first or unit figure, and one over each second figure towards the left, we shall have as many figures in the root as there are dots. Thus the square root of $\dot{7}6\dot{2}0\dot{0}1\dot{6}$ will contain 4 figures; of $\dot{6}2\dot{5}$, 2 figures, units, and tens.

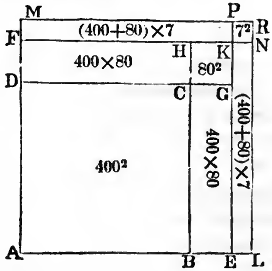
Then 625=square of the tens plus twice the tens into the units plus the square of the units. The square of the tens must be found in the hundreds of the number. The greatest number of tens whose square can be found in 6 (hundreds) is 2 tens,

Tens.		
2		$\dot{6}2\dot{5}$(25
Twice the tens.	Units.	
4 +	5	4
	=45	225
		225

which we write in the quotient. Subtracting the square 4 (hundreds) our remainder is 225=twice the tens into the units plus the square of the units, or, what is the same thing (twice the tens plus the units), ×the units To find the units' figure, by way of trial, we divide 225 by twice the tens; the quotient is 5. Adding, then, these units, 5, to twice the tens or 40, we have 45; which, multiplied by 5, will give a product that exactly equals 225.

Before giving a rule for the extraction of the square root, it may be well to illustrate, geometrically, the involution and evolution of a number.

$487^2=(400+80+7)^2=400^2+2\times400\times80\times80^2+2\times(400+80)\times7+7^2.$



The square ABCD may be enlarged to the square AEKF, by the addition of the two equal rectangles BG and DH, whose lengths are each equal to the side AB of the original square, and whose widths are equal to BE, the quantity by which the side of the square has been augmented, also a little square, CGKH, whose side is the same as the width of one of the equal rectangles.

Again, the square AEKF, having its side increased by EL, becomes augmented by the two rectangles EN, FP, and the little square KR. Thus we might continue to augment the square last found by the addition of two equal rectangles, and a little square; the length of each rectangle being equal to the side of the square which is to be augmented, and the width equal to the quantity by which the side of the square is increased; and the side of the little square being the same as the width of one of the rectangles. The diagram is adapted to the case of squaring $400+80+7=487$.

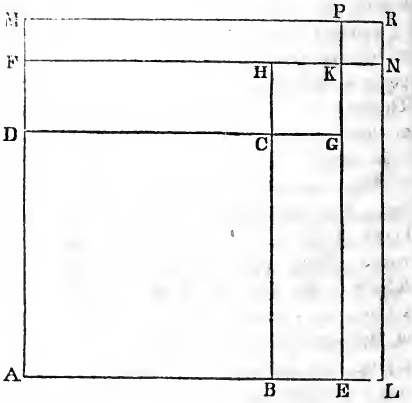
§ 138. We will now reverse the above process.

Let it be required to extract the square root of 527076; that is, we will seek the number of feet in the side of a square whose area shall contain 527076 square feet.

In this example there must be three figures in the root.

We know that the side of the square sought must exceed 700 *linear* feet, since the square of 700 is 490000, which is less than 527076; we also know that the side of this square must be less than 800 *linear* feet, since the square of 800 is 640000, which is greater than 527076. Hence the first, or hundreds' figure of the root, is 7; which is the greatest number whose square can be contained in 52, the first or left-hand period.

If we suppose each side of the square ABCD to be 700 *linear* feet, its surface will be 700×700=490000 *square* feet, which, subtracted from 527076 *square* feet, leaves 37076 *square* feet.



Hence it is necessary to increase the square ABCD, by 37076 *square* feet. We have seen that such increase is effected by the addition of two equal *rectangles*, and a little square. The surface of the two *rectangles* will evidently make by far the largest portion of the whole increase. The length of one of these rectangles is the same as the length of a side of the square ABCD, which has already been shown to be 700 *linear* feet. The length of the two rectangles taken together will be twice 700 *linear* feet, or, what would be the same thing, 700 *linear* feet added to 700 *linear* feet. If to BC, which is 700 *linear* feet, we add CD, which is also 700 *linear* feet, we shall have BC+CD, equal to 1400 *linear* feet, for the length of the two rectangles. Were we to multiply 1400 by the width of a rectangle, we should obtain the number of *square* feet in these rectangles, or nearly the 37076 *square* feet which require to be added. Consequently, if we divide 37076 by 1400, the quotient will give the approximate width of the *rectangles*. Using 1400 as a *trial divisor*, we find it to be contained between 20 and 30 times in 37076; hence the second or ten's figure of the root is 2. But besides the rectangles, there must be added the *little square* CGKH, each side of which is 20 *linear* feet; we will therefore add 20 to 1400, and thus obtain 1420 for the total length of the two *rectangles* and the side of the *little* square. Now, multiplying 1420 by 20, we obtain 28400 *square* feet for the total additions, which, subtracted from 37076, leaves

8676 *square* feet. The square AEKF thus completed is 720 feet on a side.

Again, a side of this square is to be further increased so that the added surface will amount to 8676 *square* feet. And, as before, the parts added will consist of two equal *rectangles* and a *little square*. The *trial* divisor, which is the sum of the length of the two new rectangles, is the same as the sum of two sides of the square AEKF.

If, now, to 1420 already found, we add 20, we shall have 1440, which is the sum of EK and KF, and which is our second *trial divisor*. We find this divisor contained between 6 and 7 times in 8676; hence our third or units' figure of the root is 6. Therefore 6 is the width of the second set of rectangles. The second *little square* KNRP, of the same width as the rectangles, must be 6 *linear* feet on a side; therefore, adding 6 to 1440, we find 1446 for the whole length of the new rectangles and a side of the second *little square*. Multiplying 1446 by 6, we obtain 8676 *square* feet as the sum of the series of additions to the square AEKF, thus forming the square ALRM, which is the square sought; each side being 726 feet.

The above work may be arranged as follows:

Linear feet.	NUMBER. *Square feet.*	ROOT. *Linear feet.*
700	527076(	700+20+6=726
1400=1st trial divisor	490000	
1420	37076	
1440=2d trial divisor	28400	
1446	8676	
	8676	
	0	

If we omit the ciphers on the right, the work will take the following condensed form:

Linear feet.	NUMBER. *Square feet.*	ROOT. *Linear feet.*
7	527076 (	726
14 =1st trial divisor	49	
142	370	
144=2d trial divisor	284	
1446	8676	
	8676	
	0	

CASE I.

From the above process, we deduce the following rule for the extraction of the square root of a whole number.

RULE.

I. *Separate the given number into periods of two figures each, counting from the right towards the left. When the number of figures is odd, it is evident that the left-hand, or first period, will consist of but one figure.*

II. *Find the greatest square in the first period, and place its root at the right of the number, in the form of a quotient in division, also place it at the left of the number. Subtract the square of this root from the first period, and to the remainder annex the second period; the result will be the* FIRST DIVIDEND.

III. *To the figure of the root, as placed at the left of the number, add the figure itself, and the sum will be the* FIRST TRIAL DIVISOR. *See how many times this trial divisor, with a naught annexed, is contained in the dividend; the quotient will be the next figure of the root; this must be annexed to the* TRIAL DIVISOR; *the result will be the* TRUE DIVISOR. *Multiply the true divisor by this last figure of the root, and subtract the product from the dividend, and to the remainder annex the next period, for a* NEW DIVIDEND.

IV. *To the last* TRUE DIVISOR *add the last figure of the root; the sum will be a new* TRIAL DIVISOR. *Continue to operate as before, until all the periods have been brought down.*

NOTE 1.—In case any dividend is not so great as its trial divisor, with a cipher annexed, write 0 as the next figure of the root; also place 0 at the right of the divisor, and form a new dividend by annexing a new period.

NOTE 2.—Approximate roots may be found by annexing decimal periods of two naughts each.

What is the square root of 11390625 ?

3	11390625(3375
63	9
667	239
6745	189
	5006
	4669
	33725
	33725
	0

EXAMPLES.

45. What is the square root of 11019960576 ?

46. What is the square root of 276793836544 ?

47–52. What is the square root of 12321 ? of 53824 ? of 30858025 ? of 16983563041 ? of 852891037441 ? of 61917364224 ?

CASE II.

To extract the square root of decimals.

Annex one cipher, if necessary, so that the number of decimals shall be even; then point them off into periods of two figures each, counting from the units' place towards the right. Extract the root, as in Case I.

NOTE.—If the given number has not an exact root, there will be a remainder after all the periods have been brought down, in which case the operation may be extended by forming new periods of ciphers.

EXAMPLES.

53. What is the square root of 3486·78401 ?

54–57. What is the square root of 6·5536? of 0·00390625? of 17? of 37·5?

58. What is the square root of 0·0000012321?

CASE III.

To extract the square root of a common fraction, or mixed number.

Reduce the vulgar fraction, or mixed number, to its simplest fractional form. Extract the square root of the numerator and denominator separately, if they have exact roots; if they have not, reduce the fraction to a decimal, and proceed as in Case II.

EXAMPLES.

59–60. What is the square root of $\frac{256}{324}$? of $\frac{10125}{12005}$?

61–62. What is the square root of $\frac{5}{9}$ of $\frac{4}{5}$ of $\frac{4}{7}$ of $\frac{7}{9}$? of $4\frac{21}{25}$?

63–65. What is the square root of $4\frac{1}{9}$? of $\frac{11}{17}$? of $\frac{13}{49}$?

EXAMPLES INVOLVING THE PRINCIPLES OF THE SQUARE ROOT.

§ 139. *A triangle* is a figure having three sides, and consequently three angles.

When one of the angles is right, like the corner of a square, the triangle is called a *right-angled triangle.* In this case the side opposite the right angle is called the *hypotenuse.*

It is an established proposition of geometry, that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Hence it follows that the square of the hypotenuse, diminished by the square of one of the sides, equals the square of the other side.

By means of these properties, it follows that two sides of a right-angled triangle being given, the third side can be found.

How long must a ladder be to reach to the top of a house 40 feet high, when the foot of it is 30 feet from the house?

In this example, it is obvious that the ladder forms the hypotenuse of a right-angled triangle, whose sides are 30 and 40 feet respectively. Therefore, the square of the length of the ladder must equal the sum of the squares of 30 and 40.

$$30^2 = 900$$
$$40^2 = 1600$$
$$\sqrt{2500} = 50, \text{ the length of the ladder.}$$

EXAMPLES.

66. Suppose a ladder 100 feet long, to be placed 60 feet from the roots of a tree; how far up the tree will the top of the ladder reach?

67. Two persons start from the same place, and go, the one due north, 50 miles, the other due west, 80 miles. How far apart are they?

68. What is the distance through the opposite corners of a square yard?

69. The distance between the lower ends of two equal rafters, in the different sides of a roof, is 32 feet, and the height of the ridge above the foot of the rafters is 12 feet. What is the length of a rafter?

70. What is the distance measured through the centre of a cube, from one corner to its opposite corner, the cube being 3 feet, or one yard, on a side?

We know, from the principles of geometry, that all similar surfaces, or areas, are to each other as the squares of their like dimensions.

71. Suppose we have two circular pieces of land, the one

100 feet in diameter, the other 20 feet in diameter. How much more land is there in the larger than in the smaller?

NOTE.—The circles will be to each other as the squares of their respective diameters.

72. Suppose, by observation, it is found that 4 gallons of water flow through a circular orifice of 1 inch in diameter in 1 minute. How many gallons would, under similar circumstances, be discharged through an orifice of 3 inches in diameter, in the same length of time?

73. What length of thread is required to wind spirally around a cylinder, 2 feet in circumference and 3 feet in length, so as to go but once around?

NOTE.—It is evident that if the cylinder be placed upon a plane, and be caused to roll once over, it will describe a rectangle, whose width is 2 feet, and length 3 feet; while the thread will form its diagonal, or line running from corner to corner.

74–77. A room is 16 feet long, 12 feet wide, and 10 feet high. What is the diagonal distance measured on the floor? What is the diagonal distance measured on the wall which forms the side of the room? What is the diagonal distance measured on the wall which forms the end of the room? And what the diagonal passing through the centre of the room?

78–80. There are three circular pieces of ground, the diameters of which are 697 feet, 185 feet, 153 feet. What is the diameter of a circular piece whose area is equal to the difference between the first and second? What the diameter of one whose area is equal to the difference between the first and third? What the diameter of one whose area is equal to the difference between the second and third?

§ 140. Before explaining the method of extracting the CUBE ROOT, we will involve the number 45, consisting of 4 *tens* and 5 *units* to the third power.

$$45=40+5$$
$$40+5$$
$$40^2+40\times5=\text{product by the units in the tens.}$$
$$40\times5+5^2=\text{product by the units.}$$
$$40^2+2\times40\times5+5^2=\text{square of } 40+5$$
$$40+5$$
$$40^3+2\times40^2\times5+40\times5^2$$
$$40^2\times5+\ 2\times40\times5^2+5^3$$
$$40^3+3\times40^2\times5+3\times40\times5^2+5^3=\text{cube of } 40+5.$$

That is, *the cube of a number consisting of tens and units, equals the cube of the tens (expressed in units), plus three times the square of the tens (expressed in units) into the units, plus three times the units in the tens into the square of the units, plus the cube of the units. And in general, the cube of the sum of any number of numbers is equal to the cube of the first number, plus three times the square of the first number into the second, plus three times the first into the square of the second, plus the cube of the second; plus three times the square of the* SUM OF THE FIRST TWO *into the third, plus three times the* SUM OF THE FIRST TWO *into the square of the third, plus the cube of the third, &c.*

EXAMPLES.

81–90. Express by symbols as above, 75^3; 89^3; 142^3; 365^3; 47^3; 96^3; 221^3; 496^3; 879^3; 999^3.

§ 141. We will now extract the cube root of 91125. It is first to be determined how many figures the root must have.

The smallest number of 2 figures is 10; its cube is 1000.

The largest number of 2 figures is 99; its cube is 970299.

A number, then, consisting of 4 to 6 figures must have a root of 2 figures. So, a number consisting of 7 to 9 figures, must have a root of 3 figures. If, then, we divide a number into groups, by placing a dot over the first or unit figure, and one over each 3d figure towards the left, we shall have as many figures in the root as there are dots. $9\dot{1}12\dot{5}$ has therefore 2 figures in its root.

The root must consist of tens and units; then 91125=cube of the tens plus 3 times the square of the tens into the units plus 3 times the tens into the square of the units plus the cube of the units. The cube of the tens will be found in the 91 (thousand). The greatest cube contained in 91 is 4 (tens), which we write in the quotient. Subtracting the cube 64 (thousands), our remainder is 27125=3 times the square of the tens into the units plus 3 times the tens into the square of the units plus the cube of the units. Or, what is the same thing (3×square of the tens plus 3×the tens×the units plus the square of the units), as one factor×the units as the other factor.

		91125 (45
		64
3×sq. of tens	=4800 trial div.	27125
3×tens×units	= 600	27125
sq. of units	= 25	
	5425 true divisor.	

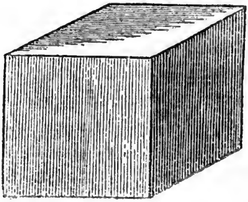
To find the units' figure of the root, by way of trial, we divide 27125 by 3×square of the tens; that is, by 4800. The quotient is 5.

Having now, as we suppose, the correct units' figure of the root as the one factor, let us ascertain the value of the other factor, or the true divisor. 3×sq. tens=3×1600=4800; 3×tens×the units=3×40×5=600; sq. of units=5^2=25. Then 4800+600+25=5425, the completed divisor. This, multiplied by 5, the quotient figure, gives 27125: a result showing that our quotient figure was neither too large nor too small. The cube root, then, of 91125 is 45.

Before giving a rule for the extraction of cube root, we will illustrate geometrically the involution and the evolution of a number.

Let it be required to cube 45, the number before employed, or suppose we are required to find the number of cubic inches in a cube whose side is 45 inches. Separating 45 into 40+5, we will suppose the cube (fig. 1) to be 40 inches on a side; then 40 × 40 × 40 will give the solid contents of this cube, represented by 40^3.

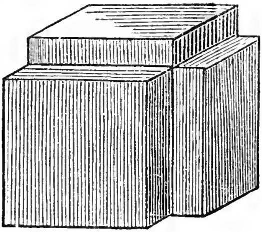
Fig. 1.



40^3=40×40×40
=64000

Let fig. 2 represent the cube increased by three equal *slabs;* then 3 (the number of slabs) times 40^2 (the surface of one of the slabs) multiplied by 5, the thickness of a slab, will give the solid contents of the slabs, represented by $3\times40^2\times5$.

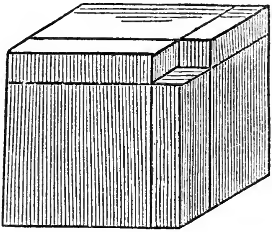
Fig. 2.



$40^2=40\times40$
$=1600$
$\times$ by 3
4800
$\times$ by 5
24000

Let fig. 3 represent the solid (as in fig. 2), further increased by *three equal corner* pieces; then 3 (the number of corner pieces) times 40 (the length of one corner piece) multiplied into 5^2, the surface of an end of a corner piece, will give the solid contents of the corner pieces represented by $3\times40\times5^2$.

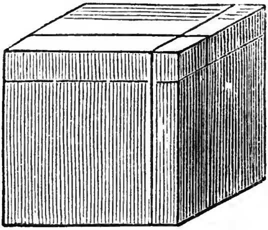
Fig. 3.



$3\times40=120$
$\times$ by $5^2=25$
600
249
3000

Let fig. 4 represent the solid (as in fig. 3), further increased by *a little corner cube,* each side of which is 5 inches; then $5\times5\times5$ will give the solid contents of this cube, represented by 5^3.

Fig. 4.



$5^3=5\times5\times5$
$=125$

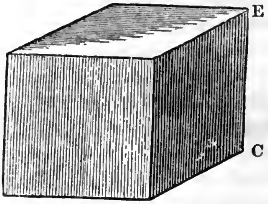
Then the *whole cube* thus increased will be represented by $45^3=40^3+3\times40^2\times5+3\times40\times5^2+5^3$

$$=64000+24000+3000+125=91125.$$

§ 142. Let it now be required to find the cube root of 382657176 We will suppose 382657176 to denote the number of cubic feet in a geometrical cube; we are required to find the number of *linear* feet in a side of this cube, that is, the length of one of its sides. 382657176 must give 3 figures for the root.

We know that the side of the cube sought must exceed 700 *linear* feet, since the cube of 700 is 343000000, which is less than 382657176; we also know that the side of this cube must be less than 800 *linear* feet, since the cube of 800 is 512000000, which is greater than 382657176. Hence the first figure of our root, or the figure in the hundreds' place, is 7; whose cube, 343, is the greatest cube contained in 382, the first or left-hand period. If we suppose each side of the cube, represented by figure 1, to be 700 *linear* feet, one of the equal faces, as the upper face DEFG, will be denoted by $700 \times 700 = 490000$ *square* feet. The solid contents of the cube will be represented by $700 \times 700 \times 700 = 700^2 \times 700 = 490000 \times 700 = 343000000$ *cubic* feet. Subtracting 343000000 *cubic* feet from 382657176 *cubic* feet, we find 39657176 *cubic* feet for a remainder.

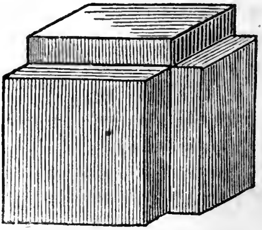
Fig. 1.



Hence it is necessary to increase the cube (figure 1), by 39657176 *cubic* feet. We have seen that such increase is effected by the addition of three equal *slabs*, three equal *corner pieces*, and an additional *cube;* and that the contents of the three *slabs* will make by far the largest portion of the whole increase.

The number of square feet in the face of one of these slabs will be the same as the number of square feet in the face of the cube (figure 1), which has already been shown to be 490000 *square* feet. The surface of the three slabs will be three times 490000 *square* feet; or, which would be the same thing, twice 490000 *square* feet, added to 490000

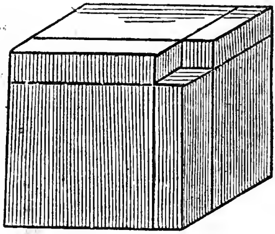
Fig. 2.



square feet.* If to AB (fig. 1), which is 700 *linear* feet, we add BC, which is also 700 *linear* feet, we shall have AB+BC equal to 1400 *linear* feet, which, multiplied by DB, equal to 700 *linear* feet, will give 980000 *square* feet, for the area ABDG+BCED, which, added to DEFG, which is 490000 *square* feet, will give 1470000 square feet, for the area of three faces of the cube (figure 1), which is the same as the area of the three slabs. Were we to multiply 1470000 by the thickness of the slabs, we should obtain the *cubic* feet in these slabs. And since the contents of the slabs make nearly the whole amount added, it follows that 1470000 multiplied by the thickness of slabs, will give nearly 39657176 *cubic* feet. Consequently, if we divide 39657176 by 1470000, the quotient will give the approximate thickness of the slabs. Using 1470000 as a *trial divisor*, we find it to be contained between 20 and 30 times in 39657176; hence the second or tens' figure of the root is 2.

We have already remarked that 1470000 multiplied by 20, the thickness of the *slabs*, will give their solid contents. But besides the *slabs*, there must be added three *corner pieces*, each of which is 700 feet long, and of the same thickness as the *slabs*, that is, 20 feet. Since each corner piece is the same length as a side of the cube, figure 1, it follows that adding 700 to 1400 or 700+700, the sum 2100 will represent the total length of the three *corner pieces*. Were we to multiply 2100 by 20, we should obtain the area of the three *corner pieces*, which might be added to 1470000, the area of the three *slabs*. But, since there is also to be added a *little cube*, each of whose sides is 20 *linear* feet, we will add 20 to 2100, and thus obtain 2120 for the total length of the three corner pieces, and of a

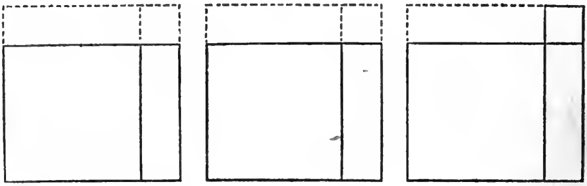
Fig. 3.



* It will be noted that the *peculiar steps* throughout this demonstration have reference to the mode of extracting the Cube Root which follows. The object of these processes is, to make use of what has been obtained in one stage of the work for the stage next succeeding; to obtain a new quantity by *adding to* one already in hand, instead of *multiplying* an original quantity; thereby saving much time and labor.

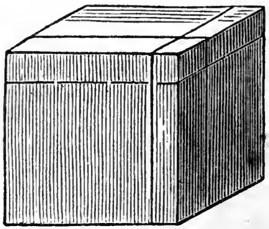
side of the little cube. Now, multiplying 2120 by 20, we obtain 42400 *square* feet for the surface of the three corner pieces and a face of the little cube; which, added to 1470000, the number of square feet in the faces of the three slabs, will give 1512400 *square* feet in all the additions. If we multiply 1512400 by 20, the thickness of these additions, we shall obtain 30248000 *cubic* feet for all the additions, which, subtracted from 39657176, leaves 9409176 *cubic* feet. The cube thus completed is 720 feet on a side, and is represented by figure 4.

Figure *a*.



The *surfaces* now obtained may be represented (figure *a*) by the parts included within the *heavy* lines. The three divisions of the figure, including the *dotted* lines, may be supposed to be three *entire* faces of the cube, figure 4.

Fig. 4.



But this cube is to be further increased by 9409176 *cubic* feet. And as before, the parts added will consist of three equal *slabs*, three equal *corner pieces*, and a *little cube*. The trial divisor, which is the area of the faces of the three slabs, is the same as three times the area of a face of the cube, figure 4, each of whose sides is 720 feet.

Now, to obtain this area, we have only to add to the surfaces already obtained, and represented within the heavy lines (figure *a*), three rectangles, each 700 feet by 20, and two little squares 20 feet by 20 feet.

If to 2120, a number which we already have, we add 20, we shall obtain 2140, the *linear* extent of the rectangles and squares desired, as in the dotted portions (figure *a*). And as these dotted portions have all the same width of 20 feet, if we multiply 2140 by 20, we shall obtain 42800 *square* feet for the area of the dotted portion (figure *a*), which, added to 1512400, the area of the parts included within the heavy lines, will give 1555200 *square* feet for the area of three slabs, each equal to one face of the cube (figure 4). This will be a second *trial divisor.* We find this divisor contained between 6 and 7 times in 9409176; hence our third figure of the root, or the figure in the units' place, is 6. Were we to multiply 1555200 by 6, it would give the *cubic* feet in the second set of *slabs.* But before multiplying, we will increase that sum by the surface of the second set of *corner pieces,* and of the second *little cube.* The length of each corner piece is the same as a side of the cube, figure 4, which is 720 feet; hence, adding 20 to 2140 already found, we obtain 2160, which, being 3 times 720, will be the linear extent of the three corner pieces. Were we to multiply 2160 by 6, we should find the surface of these three corner pieces, but as we wish also the area of one of the faces of the second *little cube,* we add 6 to 2160, and thus obtain 2166, which, multiplied by 6, will give 12996 for surface of second set of *corner pieces* and of second *little cube;* this added to 1555200, gives 1568196 for the surface of the whole second series of additions. Multiplying 1568196 by 6, we obtain 9409176 *cubic* feet, which have thus been added to the cube represented by figure 4; hence the cube whose side is 726 feet is the one sought. The above work may be arranged as follows:

1ST COLUMN.	2D COLUMN.	NUMBER.	ROOT.
Linear feet.	*Square feet.*	*Cubic feet.*	*Linear feet.*
700	490000	382657176(	700+20+6=726
1400	1470000=1st tr. divisor,	343000000	
2100	1512400	39657176	
2120	1555200=2d tr. divisor,	30248000	
2140	1568196	9409176	
2160		9409176	
2166		0	

If we omit the ciphers on the right, and omit unnecessary terms, the work will take the following condensed form:

1ST COLUMN.	2D COLUMN.	NUMBER.	ROOT.
Linear feet.	*Square feet.*	*Cubic feet.*	*Linear feet.*
7	49	382657176(	726
14	147=1st trial divisor,	346	
212	15124	39657	
214	15552=2d trial divisor,	30248	
2166	1568196	9409176	
		9409176	
		0	

NOTE.—In the extraction of the cube root, as just illustrated, it will be noticed that each divisor is a geometrical surface; that is to say, the product of two dimensions, width and breadth, for example; and of course the quotient must be the other dimension, that is, the thickness.

But it is important to remember that it is only squares and cubes, square roots and cube roots, that can have any relation to *geometrical* dimensions; any higher power of a number, as 4^5, or any other root as $\sqrt[5]{}$, cannot be illustrated by blocks. The *principle*, therefore, of *involution* and *evolution* is, strictly speaking, independent of surfaces and solids; it is purely *arithmetical.*

From the foregoing demonstration we may deduce the following

RULE.

I. *Separate the number whose root is to be found, into periods of three figures each, counting from the units' place towards the left. When the number of figures is not divisible by* 3, *the left-hand period will contain less than* 3 *figures.*

II. *Seek the greatest figure whose cube shall not exceed the first or left-hand period; write it after the manner of a quotient in division for the first figure of the root. Place this figure for the head of a first left-hand column, and its square for the head of a second left-hand column, and subtract its* *cube from the first period. To the remainder bring down a second period for the* FIRST DIVIDEND. *Add the figure in the root to the term of the* 1ST COLUMN *already found, for its next term, which multiply by the same figure, and add the product to the term already found in the* 2D COLUMN, *for its next term, which will be a* TRIAL DIVISOR.

III. *Find how many times the trial divisor, with two naughts annexed, is contained in the dividend; write the quotient for the next figure of the root. Annex this figure to the last term of the* 1ST COLUMN, *after having added to that term the preceding quotient figure; this will give the next term of the* 1ST COLUMN. *Multiply this term by the last found figure in the root, and add the product, after advancing it two places to the right, to the last term of the* 2D COLUMN, *for its next term. Multiply this term by the last found figure of the root, and subtract the product from the dividend, and to the remainder bring down the next period for a* NEW DIVIDEND.

Proceed as before until all the periods have been brought down.

NOTE 1.—When any dividend is not so great as the corresponding trial divisor with two ciphers annexed, write 0 for the next figure of the root, and to the dividend bring down the next period. Use the same trial divisor as before, but with *four* ciphers annexed.

NOTE 2.—The trial divisor, being less than the true divisor, will sometimes give too large a quotient figure; when the multiplication of the true divisor by this figure shows such to be the case, this figure must be made smaller.

NOTE 3.—By the above rule, which is different from the rule usually given by the aid of geometrical diagrams, we keep distinct all the geometrical magnitudes; thus our first column represents the numerical values of lines, the second column represents the numerical values of surfaces, and the third column corresponds to solids. And, as we are never required to multiply by any number greater than a digit, the labor of multiplying and adding results to the terms of the successive columns is far simpler than at first might be supposed.

By means of these auxiliary columns, the work bears a close analogy to Horner's method of solving numerical cubic equations. (See Treatise on Algebra.) The use of auxiliary columns becomes very apparent in the extraction of roots of the higher orders, as the fifth root, the seventh root, &c. (See Higher Arithmetic.)

What is the cube root of 913517247483640899?

1ST COLUMN.	2D COLUMN.	NUMBER.	ROOT
		913517247483640899(	970299
9	81	729	
18	243	184517	
277	26239	183673	
284	28227	844247483	
29102	282328204	564656408	
29104	282386412	279591075640	
291069	28241260821	254171347389	
291078	28243880523	25419728251899	
2910879	2824414250211	25419728251899	
		0	

EXAMPLES.

91. What is the cube root of 10077696?

92–95. What is the cube root of 2357947691? of 42875? of 117649? of 7256313856?

CASE II.

To extract the cube root of a decimal, *annex ciphers, if necessary, so that the decimals may be separated into equal periods of* 3 *figures each.*

Point off into periods of 3 *figures each, counting from units towards the right, and proceed as in whole numbers.*

NOTE.—If the given number has not an exact root, there will be a remainder after all the periods have been brought down. The process may be continued by annexing ciphers for new periods.

EXAMPLES.

96. What is the cube root of 0·469640998917 ?
97. What is the cube root of 18·609625 ?
98. What is the cube root of 1·25992105 ?
99. What is the cube root of 2 ?
100. What is the cube root of 9 ?
101. What is the cube root of 3 ?

CASE III.

To extract the cube root of a common fraction, or mixed number, *reduce the fraction, or mixed number, to its simplest fractional form. Extract the cube root of the numerator and denominator separately, if they have exact roots ; if they have not, reduce the fraction to a decimal, and extract the root by Case II.*

EXAMPLES.

102–104. What is the cube root of $\frac{2197}{4913}$? of $\frac{85169}{170723}$? of $17\frac{1}{8}$?

105–110. Find $\sqrt[3]{5\frac{1}{7}}$; $\sqrt[3]{\frac{81}{99}}$; $\sqrt[3]{\frac{2}{3}}$; $\sqrt[3]{\frac{7}{9}}$; $\sqrt[3]{4\frac{3}{5}}$; $\sqrt[3]{13\frac{9}{11}}$.

EXAMPLES INVOLVING THE PRINCIPLES OF THE CUBE ROOT.

§ 143. *It is an established theorem of geometry, that all similar solids are to each other as the cubes of their like dimensions.*

111. If a cannon-ball, 3 inches in diameter, weigh 8

pounds, what will a ball of the same metal weigh, whose diameter is 4 inches?

By the above theorem, we have

$$3^3 : 4^3 :: 8 \text{ pounds} : \text{the answer.}$$

112. The celebrated Stockton gun, which, in bursting, proved so fatal to many of our distinguished citizens, is said to have carried a ball 12 inches in diameter, which weighed 238 pounds. What ought to be the diameter of another ball of the same metal, which should weigh 32 pounds?

113. There are three balls, whose diameters are respectively 3, 4, and 5 inches. What is the diameter of a fourth ball, which is equal in weight to the three balls?

114–115. Which will weigh the most, three balls whose diameters are respectively 15, 22, and 41 inches, or three balls of the same metal whose diameters are respectively 20, 25, and 39 inches? What is the diameter of a ball whose weight is the average of the weights of the six balls?

116. A cooper having a cask 40 inches long and 32 inches at the bung diameter, wishes to make another cask of the same shape, which shall contain just twice as much. What will be the dimensions of the new cask?

117. What is the side of a cube, which will contain as much as a chest 8 feet 3 inches long, 3 feet wide, and 2 feet 7 inches deep?

118. How many cubic quarter inches can be made out of a cubic inch?

119. Required the dimensions of a rectangular box, which shall contain 20000 solid inches, the length, breadth, and depth being to each other as 4, 3, and 2.

CHAPTER XVII.

PROGRESSION.

ARITHMETICAL PROGRESSION.

§ 144. A SERIES of numbers, which succeed each other by a common difference, is said to be in *arithmetical progression.* When the terms are constantly increasing, the series is an *arithmetical progression ascending;* when constantly decreasing, the series is an *arithmetical progression descending.* Thus, 1, 3, 5, 7, 9, &c., is an ascending arithmetical progression; and 10, 8, 6, 4, 2, is a descending arithmetical progression.

The terms of an arithmetical progression may be fractional; as,

$\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, &c.;
$\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$, $2\frac{2}{3}$, 3, &c.

The first has a common difference of $\frac{1}{2}$; the second has a common difference of $\frac{1}{3}$.

In arithmetical progression, there are five things to be considered:

1. *The first term.* 2. *The last term.* 3. *The common difference.* 4. *The number of terms.* 5. *The sum of all the terms.*

These quantities are so related to each other, that any three of them being given, the remaining two can be found. We will demonstrate two of the most important cases.

CASE. I.

By our definition of an ascending arithmetical progression, it follows that the second term is equal to the first, increased by the common difference; the third is equal to the first, increased by twice the common difference; the fourth is equal to the first, increased by three times the common difference; and so on, for the succeeding terms.

Hence, to find the last term, when the first term, the common difference, and the number of terms are given, *to the first term add the product of the common difference into the number of terms, less one.*

EXAMPLES.

1. What is the 100th term of an arithmetical progression, whose first term is 2, and common difference 3?

2. What is the 50th term of the arithmetical progression, whose first term is 1, the common difference $\frac{1}{2}$?

3. A man buys 10 sheep, giving $1 for the first, $3 for the second, $5 for the third, and so on, increasing in arithmetical progression. What did the last sheep cost at that rate?

4. The first term of an arithmetical progression is $\frac{3}{4}$, the common difference $\frac{1}{8}$, and the number of terms 26. What is the last term?

5. A tapering board, 6 inches wide at the narrow end, and 12 feet long, is found to increase $\frac{1}{2}$ an inch for every foot in length. What is the width of the wide end?

6. A field of maize, consisting of 50 rows, has 20 hills in the first row, 23 in the second row, and so on, each row having three hills more than the preceding row. How many hills were in the last row?

7. A person makes 12 monthly deposits in a savings bank; the first deposit consisted of $25, the second of $30, the third of $35, and so on in arithmetical progression. How much did he deposit the 12th month?

CASE II.

From the nature of an arithmetical progression, we see that the second term added to the next to the last term is equal to the first added to the last; since the second term is as much greater than the first as the next to the last is less than the last. So we infer that the sum of any two terms equidistant from the extremes, is equal to the sum of the extremes.

Hence, it follows that the terms will average just half the sum of the extremes.

Therefore, to find the sum of all the terms, when the first term, the last term, and the number of terms are given, *multiply half the sum of the extremes by the number of terms.*

EXAMPLES.

8. The first term of an arithmetical progression is 2, the last term is 50, and the number of terms is 17. What is the sum of all the terms?

9. The first term of an arithmetical progression is 13, the last term is 1003, the number of terms is 100. What is the sum of the progression?

10. A person travels 25 days, going 11 miles the first day, 135 the last day; the miles which he travelled in the successive days form an arithmetical progression. How far did he go in the 25 days?

11. Bought 7 books, the prices of which are in arithmetical progression. The price of the first was 8 shillings, and the price of the last was 28 shillings. What did they all come to?

12. What is the sum of 1000 terms of an arithmetical progression, whose first term is 7 and last term 1113?

13. The first term of an arithmetical progression is $\frac{3}{7}$, and the last term $365\frac{4}{7}$, and the number of terms 799. What is the sum of all the terms?

14–15. How many strokes does the common clock make in 12 hours? How many strokes would be made by a clock which continues to strike through all the hours from 1 to 24?

16. A falling body moves $16\frac{1}{12}$ feet during the first second, and $144\frac{3}{4}$ feet during the fifth second. How far did it fall during the 5 seconds?

17. A person makes 12 monthly deposits, which are in arithmetical progression, in a savings bank. The first deposit is $20, the last $75. What is the whole sum thus deposited?

Geometrical Progression.

§ 145. A series of numbers which succeed each other by a constant multiplier, is called a *geometrical progression.*

This constant factor, by which the successive terms are multiplied, is called the *ratio.* When the ratio is greater than a unit, the series is called an *ascending geometrical progression;* when less than a unit, the series is called a *descending geometrical progression.* Thus, 1, 3, 9, 27, 81, &c., is an ascending geometrical progression, whose ratio is 3. And 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, &c., is a descending geometrical progression, whose ratio is $\frac{1}{4}$.

In geometrical, as in arithmetical progression, there are five things to be considered:

1. *The first term.* 2. *The last term.* 3. *The common ratio.* 4. *The number of terms.* 5. *The sum of all the terms.*

These quantities are so related to each other, that any three being given, the remaining two can be found.

We will demonstrate two of the most important cases.

CASE I.

By the definition of a geometrical progression, it follows that the second term is equal to the first term multiplied by the ratio; the third term is equal to the first term multiplied by the second power of the ratio; the fourth term is equal to the first term multiplied by the third power of the ratio; and so on, for the succeeding terms.

Hence, to find the last term, when the first term, the ratio, and the number of terms are given, *multiply the first term by the power of the ratio, whose exponent is one less than the number of terms.*

EXAMPLES.

18. The first term of a geometrical progression is 1, the ratio is 2, and the number of terms is 7. What is the last term?

19. The first term of a geometrical progression is 5, the ratio is 4, and the number of terms 9. What is the last term?

20. A person travelling, goes 5 miles the first day, 10 miles the second day, 20 miles the third day, and so on, increasing in geometrical progression. If he continue to travel in this way for 7 days, how far will he go the last day?

21–23. A person in business found that he was able to double his capital once in 3 years: if, then, he commence business with $1000, what will his capital amount to at the end of the 12th year? How much will it amount to at the end of the 18th year? How much at the end of the 24th year?

CASE II.

Let it be required to find the sum of all the terms of the geometrical progression, 2, 6, 18, 54, 162, 486.

A second progression is, 6, 18, 54, 162, 486, 1458; which is the result of the multiplication of each term of the first, by the ratio 3. The *sum* of the terms of this 2d progression is evidently three times as great as the *sum* of the terms of the first progression; while the *difference* between the sums of the terms of these two progressions must be $(3-1)=2$ *times* the sum of the terms of the first progression.

If we omit the first term of the first progression, it will agree with the second progression, after omitting its last term. Hence, the difference between the sums of the terms of these two progressions may be found by subtracting 2, the first term of the first progression, from 1458, the last term of the second progression, leaving 1456. And as this difference $=2$ times the sum of the terms of the first progression, dividing the difference by $2=3-1$, will give the sum of the terms required.

Therefore, to find the SUM OF THE TERMS of a geometrical progression, when the first term, the last term, and the ratio are given, *divide the difference between the first term and the last term multiplied by the ratio, by the difference between the ratio and* 1.

EXAMPLES.

24. The first term of a geometrical progression is 4, the last term is 78732, and the ratio is 3. What is the sum of all the terms?

25. The first term of a geometrical progression is 5, the

last term is 327680, and the ratio is 4. What is the sum of all the terms?

26. A person sowed a peck of wheat, and used the whole crop for seed the following year; the produce of the second year again for seed the third year, and so on. If in the last year his crop is 1048576 pecks, how many pecks did he raise in all, allowing the increase to have been in a fourfold ratio?

NOTE 1.—When the ratio of a geometrical progression is less than a unit, the first term will be the largest, and the last term the least; the progression will, in this case, be descending; but we may consider the series of terms in a reverse order; that is, we may call the last term the first, and the first the last, and treat the progression as ascending.

NOTE 2.—If a descending series be continued *to infinity*, the last term may be considered 0.

What is the sum of all the terms of the infinite series 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c.?

In this example, the difference between the ratio and 1, is $1-\frac{1}{2}=\frac{1}{2}$, and the first term, 1, divided by $\frac{1}{2}$, gives 2, for the sum of all the terms.

EXAMPLES.

27. What is the sum of the infinite series 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c.?

28. What is the sum of the infinite series $\frac{1}{10}$, $\frac{3}{100}$, $\frac{9}{1000}$, $\frac{27}{10000}$, &c.?

29. What is the sum of the infinite series $\frac{2}{10}$, $\frac{4}{100}$, $\frac{8}{1000}$, $\frac{16}{10000}$, &c.?

30–31. A ball falling from the height of 10 feet, by its elasticity bounds 5 feet; and again falling, bounds $2\frac{1}{2}$ feet, and so on, continuing to bound $\frac{1}{2}$ as high as it fell. What will be the whole distance made in the successive falls before coming to a state of rest? And what the whole distance made by its successive bounds?

32. Find the sum of the infinite series 1, $\frac{8}{9}$, $\frac{8^2}{9^2}$, $\frac{8^3}{9^3}$, &c.

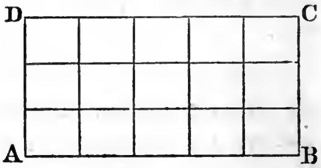
33. Find the sum of the infinite series 1, $\frac{4}{5}$, $\frac{4^2}{5^2}$, $\frac{4^3}{5^3}$, &c.

CHAPTER XVIII.

MENSURATION.

§ 146. For the reason of many of the rules which we shall give for measuring surfaces and solids, we shall refer to the principles of geometry. The reference being in all cases to the "Elements of Geometry."

Problem I.—*To find the area of a rectangle.*



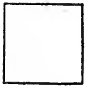
Suppose ABCD to be a rectangle whose length is 5 feet, and width 3 feet.

If we divide this rectangle into portions of one square foot each, by means of lines drawn parallel to the sides of the rectangle, we shall obtain 15 such squares; that is, the rectangle will contain 15 square feet. In this example there are 3 strips of 5 square feet in each, or 5 strips of 3 square feet each.

Hence, to find the area of a rectangle,

Multiply the length by the width, and the product will denote the number of squares of the same kind as the measure used in estimating the sides of the rectangle. If the measure be feet, the product will be square feet; if inches, square inches, &c. (B. IV., Prop. II., Scholium.)

NOTE.—When the width of the rectangle is the same as its length, it becomes a square; in which case we multiply the side of the square into itself.



EXAMPLES.

1. How many square feet in a floor which is 16 feet wide and $23\frac{1}{2}$ feet long? And how many yards of carpeting, one yard wide, will cover the floor?

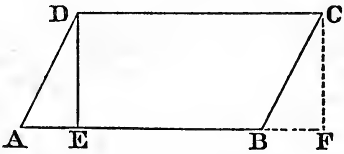
2. In a table 5 feet 3 inches long, and 3 feet 2 inches wide, how many square inches? And how many square feet?

3. In a rectangular field which is 13 rods long, and 7 rods wide, how many square rods? And what part is it of an acre?

4. How many square inches in a square board $10\frac{1}{2}$ inches on a side?

5. Which is the greater, a square board of 9 inches on a side, or a rectangular one 12 inches long and $7\frac{1}{2}$ wide?

PROBLEM II.—*To find the area of a parallelogram.*



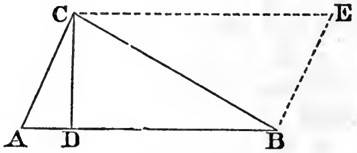
Let ABCD be a parallelogram having AB for its base and DE its altitude. If from C we draw CF perpendicular to the base AB, meeting it produced, at the point F, the figure EFCD will be a rectangle equivalent to the parallelogram, since the triangle AED is obviously equal to the triangle BFC. The base EF of the rectangle is equal to AB, the base of the parallelogram. The area of the rectangle is found (PROB. I.) by multiplying its base by its altitude; and since the parallelogram is equal to the rectangle, and since its base and altitude are respectively equal to the base and altitude of the rectangle, it follows that the area of the parallelogram may be found by multiplying its base by its altitude.

Hence, to find the area of a parallelogram,

Multiply the base by the altitude.

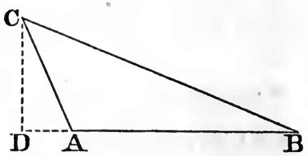
PROBLEM III.—*To find the area of a triangle.*

Let ABC be a triangle, having AB for its base and CD its altitude. By drawing CE parallel to the base AB, and BE parallel to the side AC, we shall form a parallelogram ABEC, evidently double the triangle ABC. The area of the parallelogram is found (PROB. II.) by multiplying the base AB into the altitude CD. And as the triangle is one-half the parallelogram, to find its area,



Multiply half the base by the altitude.

NOTE 1.—Either side of the triangle may be regarded as the base, and the altitude will be the perpendicular drawn from the opposite angle to the base, or to the base produced. In the annexed diagram, the perpendicular meets the base produced. The above rule applies equally well in this case, the area being found by multiplying half the base AB into CD.



When the three sides of a triangle are known, the area may be found as follows: *From the half sum of the three sides, subtract separately each side, take the square root of the continued product of the three remainders and half sum, and it will give the area.* (Geometry, B. II., Prop. IX.)

EXAMPLES.

6. What is the area of a triangle whose base is 12 feet, and altitude 3 yards?

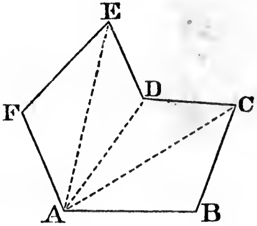
7. What is the area of a triangle whose sides are respectively 7, 11, and 12 feet?

8. What is the area of a triangle whose base is 14 rods, and whose altitude is 12 rods?

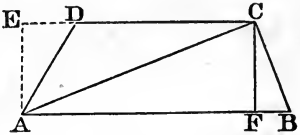
9. What is the area of a triangle whose sides are respectively 13, 14, and 15 yards?

10. In a triangular field, whose sides are 18, 80, and 82 feet, how many square yards?

NOTE 2.—The area of any figure which is limited by any number of right lines, as the field ABCDEF, may be found by dividing it into triangles, and then computing each triangle separately, and taking their sum.

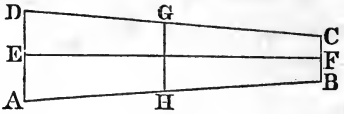


PROBLEM IV.—*To find the area of a trapezoid.*



Let ABCD be a trapezoid, having AB and CD for the parallel sides, CF for its altitude. If we draw AC, it will divide the trapezoid into two triangles ABC, CDA. The area of the triangle ABC may be found (PROP. III.) by multiplying half the base AB into the altitude CF; and the area of the triangle CDA is found by multiplying half the base CD into the altitude AE, or into its equal CF. Hence, to find the area of the trapezoid, which is the sum of the two triangles,

Multiply half the sum of the two parallel sides by the altitude.



NOTE.—This rule has a fine application in measuring a tapering board, as ABCD. In this case half the sum of the parallel sides, AD and BC, is found by measuring the width GH at the middle of the board. This average width GH being multiplied by the length EF, will give its area.

EXAMPLES.

11. If the parallel sides of a trapezoidal garden are respectively 4 and 6 rods; and the perpendicular distance between these sides is 8 rods, how many square rods in the garden?

12. How many square feet in a tapering board 16 feet long, measuring 15 inches wide at one end, and 10 inches at the other?

PROBLEM V.—*The diameter of a circle being given, to find its circumference.*

If the diameter of a circle is taken as a unit, the circumference will be 3·1415926, nearly. The exact value of the ratio of the circumference to the diameter has never been found. Its approximate value has been extended to more than 200 places of decimals. (Geometry, B. V., Prop. XIII., Scholium.)

Hence, when the diameter of a circle is known, to find its circumference,

Multiply the diameter by 3·1416.

EXAMPLES.

13. What is the circumference of the earth, on the supposition that it is 8000 miles in diameter?

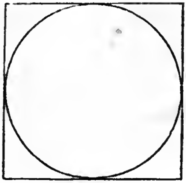
14. Suppose a cart wheel be 4*ft.* 9*in.* in diameter, over what distance would it pass in making 8 revolutions?

15. The hoop you drive is $3ft.$ $10in.$ in diameter. How many times will it revolve in being trundled to school, half a mile distant?

Problem VI.—*To find the area of a circle, when its diameter is known.*

RULE.

Multiply the circumference by one-fourth of the diameter. Or, what is equivalent, multiply the square of the diameter by 0·7854 $=\frac{1}{4}$ *of* 3·1416. (Geometry, B. V., Prop. XI.)



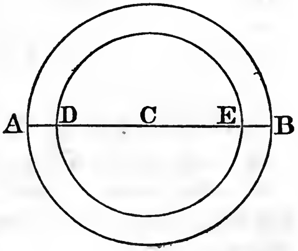
Note.—If a circle be inscribed in a square, its area will be to the area of the square as 0·7854 is to 1.

EXAMPLES.

16. How many acres in a circle one mile in diameter?

Note.—In a square mile there are 640 acres.

17. Which is the greater area, a circle 5 feet in diameter, or the sum of the areas of two other circles, the one being 4 feet in diameter and the other 3 feet?



From the preceding rule we may deduce a simple method of finding the area comprised between the circumferences of two concentric circles, which area is the difference between two circles.

The area of the circle whose diameter is AB, is found by multiplying its square by 0·7854. And the circle whose diameter is DE, is found by multiplying the square of this diameter by 0·7854. Hence, the difference of these areas is equal to the difference of the squares of the diameters multiplied by 0·7854.

PROBLEM VII.—*To find the volume of a prism, or of a cylinder.*

RULE.

Multiply the area of the base by the altitude. (Geometry, B. VII., Prop. XI.)

EXAMPLES.

18. How many cubic feet in a rectangular stick of timber 10 inches by 12 inches, and 36 feet long?

19. In a cylindrical log 14 feet long, and 14 inches in diameter, how many cubic feet?

20. How many cubic inches in a round bar of iron 20 feet long, and $\frac{3}{4}$ of an inch in diameter?

PROBLEM VIII.—*To find the volume of a pyramid, or of a cone.*

RULE.

Multiply the area of the base by one-third the altitude. (Geometry, B. VII., Prop. XVII.; and B. VIII., Prop. V.)

EXAMPLES.

21. The Egyptian pyramid Cheops covers a square of $763\frac{2}{5}$ feet on a side, and is 480 feet perpendicular height. How many cubic feet does it contain?

22. Suppose the mast of a ship to be a regular cone 87 feet long, and 2 feet in diameter at its base, how many cubic feet will it contain?

PROBLEM IX.—*To find the surface of a sphere, when its diameter is given.*

RULE.

Multiply the square of the diameter by 3·1416. (Geometry, B. VIII., Prop. XIII., Schol.)

EXAMPLES.

23. How many square miles on the surface of the earth, on the supposition that it is an exact sphere of 8000 miles in diameter?

NOTE.—In order to obtain a value true to a unit, we must use, for our multiplier, 3·14159265, instead of 3·1416.

24. How many superficial inches has a ball 6 inches in diameter?

PROBLEM X.—*To find the volume of a sphere, when its diameter is given.*

RULE.

Multiply the cube of the diameter by 0·5236, *which is* $\frac{1}{6}$ *of* 3·1416. (Geometry, B. VIII., Prop. XIII., Schol.)

EXAMPLES.

25. How many cubic inches in a ball 6 inches in diameter?

NOTE.—Compare the number of superficial inches and of cubic inches in a sphere 6 inches in diameter.

26. How many cubic inches in a ball of the celebrated Stockton gun, the diameter of which is 12 inches?

The following table of multipliers will be found very convenient for solving nearly all problems which can arise in mensuration of circles and spheres.

TABLE OF MULTIPLIERS.

1. Radius of a circle × 6·28318531 = Circumference.
2. Square of the radius of a circle × 3·14159265 = Area.
3. Diameter of a circle × 3·14159265 = Circumference.
4. Square of the diameter of a circle × 0·78539816 = Area.
5. Circumference of a circle × 0·15915494 = Radius.
6. Circumference of a circle × 0·31830989 = Diameter.
7. Square root of area of a circle × 0·56418958 = Radius.
8. Square root of area of a circle × 1·12837917 = Diameter
9. Radius of circle × 1·73205081 = Side of inscribed equilateral triangle.
10. Side of inscribed equilateral triangle × 0·57735027 = Radius of circle.
11. Radius of a circle × 1·41421356 = Side of inscribed square.
12. Side of inscribed square × 0·70710678 = Radius.
13. Square of radius of a sphere × 12·56637061 = Surface.
14. Cube of radius of a sphere × 4·18879020 = Volume.
15. Square of diameter of a sphere × 3·14159265 = Surface.
16. Cube of diameter of a sphere × 0·52359878 = Volume.
17. Square of circumference of a sphere × 0·31830989 = Surface.
18. Cube of circumference of a sphere × 0·01688686 = Volume.
19. Square root of surface of a sphere × 0·28209479 = Radius.
20. Square root of surface of a sphere × 0·56418958 = Diameter.
21. Square root of surface of a sphere × 1·77245385 = Circumference.
22. Cube root of volume of a sphere × 0·62035049 = Radius.
23. Cube root of volume of a sphere × 1·24070098 = Diameter.
24. Cube root of volume of a sphere × 3·89777707 = Circumference.
25. Radius of a sphere × 1·15470054 = Side of inscribed cube.
26. Side of inscribed cube × 0·86602540 = Radius.

PROBLEM XI.—*To find the volume of a frustum of a pyramid, or of a cone.*

RULE.

Find a mean proportional between the area of the two bases, to which add the sum of the bases, and multiply the result by one-third the altitude of the frustum.

Suppose a cistern in the form of a frustum of a cone to be 9 feet deep, having for diameters 10 feet and 6 feet. How many cubic feet will it contain?

$10^2 \times 0{\cdot}7854 = 100 \times 0{\cdot}7854 =$ area of one base.
$6^2 \times 0{\cdot}7854 = 36 \times 0{\cdot}7854 =$ " other "
$60 \times 0{\cdot}7854 =$ mean proportion between bases.
$196 \times 0{\cdot}7854 =$ sum.

And $196 \times 0{\cdot}7854 \times \frac{1}{3}$ of $9 = 461{\cdot}8152$ cubic feet for its volume.

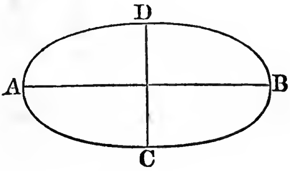
EXAMPLES.

27. Suppose a measure to be in the form of a frustum of a regular cone. If its top diameter is 6 inches, and the bottom diameter 9 inches, and it is 12 inches deep, how many cubic inches will it contain? and how many beer gallons of 282 cubic inches each?

28. There is a stick of timber in the form of the frustum of a regular pyramid, which is 30 feet long, and 30 inches square at one end and 13 inches square at the other. How many cubic feet does it contain?

PROBLEM XII.—*To find the area of an ellipse.*

NOTE.—A line drawn through the *centre* of an ellipse is called its diameter. The longest diameter is called the *transverse* diameter; the shortest is called the *conjugate* diameter. Thus AB is the transverse diameter, and CD is the conjugate diameter.



The area of an ellipse may be found by this

RULE.

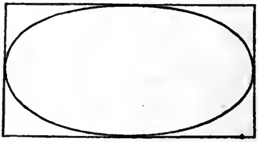
Multiply the product of the transverse and conjugate diameters by 0·7854.

EXAMPLES.

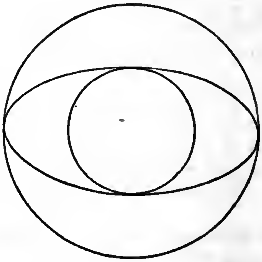
29. How many square feet in the surface of an elliptical pond, whose transverse diameter is 100 feet, and conjugate diameter 60 feet?

30. How many square inches in an elliptical table whose transverse diameter is 5 feet 3 inches, and conjugate diameter 3 feet 6 inches? And how many square feet?

NOTE 1.—If an ellipse be inscribed in a rectangle, its area will be to the area of the rectangle as 0·7854 is to 1.



NOTE 2.—We also infer that, if a circle be inscribed in an ellipse, and another circle be circumscribed about the same ellipse, the ellipse is a mean proportional between the areas of the two circles; that is, we shall have, area of inscribed circle is to the area of ellipse, as area of ellipse is to the area of circumscribed circle.



PROMISCUOUS QUESTIONS.

§ 147. 31. Suppose I purchase $1200 worth of goods, $\frac{1}{3}$ of which is on a credit of 3 months, $\frac{1}{3}$ on a credit of 6 months, and the remaining $\frac{1}{3}$ on a credit of 9 months. How much ready money ought to pay the purchase, interest being 7 per cent.?

32. In the above example, by the principles of equation

of payments, how much credit ought I to have on the whole sum of $1200 ?

33. Now, what is the present worth of $1200 due at the end of 6 months, interest being 7 per cent. ?

34. I employed A. and B. to ditch my meadow. A. was to receive $87\frac{1}{2}$ cents per rod, and B. was to have $112\frac{1}{2}$ cents per rod; each wrought until his wages amounted to $50. What was the amount of ditch dug by both ?

35. Three merchants, A., B., and C., enter into partnership. A. advances $1200, B. $800, and C. $600. A. leaves his money 8 months, B. 10 months, and C. 14 months in the business. They gain $500. What is the share of each ?

36. A. and B. have the same income: A. saves $\frac{1}{5}$ of his, but B., by spending $120 per annum more than A., at the end of 10 years finds himself $200 in debt. What was the income ?

37. Suppose a book to contain 365 pages, averaging 40 lines of 10 words each on each page. How many words would the book contain ?

38. There are 31173 verses in the Bible. How many days will it require to read it through, if 30 verses are read daily ?

39. After expending $\frac{1}{4}$ of my money, and $\frac{1}{4}$ of the remainder, I had remaining $72. How much had I at first ?

40. If I sell cloth at $1·50 per yard, and gain 25 per cent., how ought I to have sold it so as to lose 20 per cent. ?

41. Sold cloth at $1·50 per yard, and gained 25 per cent. What should I have lost per cent., if I had sold it at $0·96 per yard ?

42. If I buy cloth at $1·20 per yard, how must I sell it so as to gain 25 per cent. ?

43. A merchant has to make the following payments

at three different periods: $2832 in 3 months, $2560 in 9 months, and $1450 in 16 months. The creditor wishes to receive the whole sum of $6842 at once. When ought the payment to be made?

44. A father gives to his five sons $1000, which they are to divide according to their ages, so that each elder son shall receive $20 more than his next younger brother. What is the share of the youngest?

45. A company of 90 persons consists of men, women, and children. The men are 4 in number more than the women, the children 10 more than the adults. How many men, women, and children are there in the company?

46. The common-school fund for the State of New York was $1975093·15 in 1843, and during the same year there were in the State 677995 children between the ages of 5 and 16 years. How much would the above fund amount to per child?

47. Two persons, A. and B., being on opposite sides of a fish-pond, which is 536 feet in circumference, begin to walk around it at the same time, both in the same way: A. goes at the rate of 31 yards per minute, and B. at the rate of 34 yards per minute. In what time will B. overtake A.? And how far will A. have walked?

48. How much money which is 23 per cent. below par will pay a debt of $450?

49. A., B., and C. commence trade with $3053·25, and gain $610·65. A.'s stock, together with B.'s, is to the sum of A.'s and C.'s stock as 5 to 7; and C.'s stock, diminished by B.'s, is to C.'s increased by B.'s as 1 to 7. What was each one's part of the gain?

50. A., on preparing for a voyage to California, purchased of B. specie-dollars, at a premium of 3 per cent., to be paid in 18 months, with interest at 5 per cent. per annum, which

was to be added into the note. The amount of the note was $22145. How many specie-dollars did he receive?

51. Sold goods to the amount of $3000, one half to be paid in 3 months, the other half in 6 months. How much ought to be discounted for ready money, when money is worth 7 per cent. per annum?

52. The Falls of Niagara have receded nearly 50 yards within the last 40 years. How long, at this rate, has it taken them to recede from Queenstown, 7 miles below their present site?

53. It is found that the diameter of every circle is to its circumference very nearly in the ratio of 113 to 355. What, then, is the earth's circumference, its diameter being 7912 miles?

54. How many men must be employed to perform in 26 days what 60 men could do in 39 days?

55. If 72 sheep can graze in a field 36 days, how long might 144 sheep graze equally well?

56. If a locomotive pass from Albany to Schenectady, a distance of 17 miles, in 45 minutes, what time will it require, at the same rate, to go from Schenectady to Utica, a distance of 78 miles?

57. If A. and B., with C. working half time, can build a wall in 21 days; B. and C., with D. working half time, in 24 days; C. and D., with A. working half time, in 28 days; D. and A., with B. working half time, in 32 days; in what time would it be built by all together, and by each alone?

58. One-third of a quantity of flour being sold to gain a certain rate per cent., one-fourth to gain twice as much per cent., and the remainder to gain three times as much per cent.; it is required to determine the gain per cent. on each part, the gain upon the whole being 20 per cent.

59. A servant draws off one gallon each day, for 5

days, from a cask containing 10 gallons of wine, each time supplying the deficiency by the addition of a gallon of water; and then, to escape detection, he again draws off 5 gallons, each time supplying the deficiency by a gallon of wine. It is required to find how much water still remains in the cask.

60. Find the four smallest numbers, such that when each is divided successively by 2, 3, 4, 5, 6, 7, 8, and 9, the remainders shall in each case be 1.

61. Find the four smallest numbers, such that when each is divided by 2 the remainders shall be 1; when divided by 3, the remainders shall be 2; when divided by 4, the remainders shall be 3; and so on, until divided by 9, when the remainders shall be 8. In each case the remainder being 1 less than the divisor.

62. If 750 men require 22500 rations of bread for a month, how many rations will a garrison of 1200 men require for the same time?

63. How many yards of paper that is 30 inches wide will hang a room that is 20 yards in circuit, and 9 feet high?

64. There is a ladder with a hundred steps: on the first step is seated 1 pigeon; on the second 2; on the third 3; and so on, increasing by one for each step. How many pigeons were seated on the ladder?

65. If 9 porters drink in 8 days 12 casks of wine, how many casks will serve 24 porters for 30 days?

66. If 3 pounds of tea be worth 4 pounds of coffee, and 6 pounds of coffee be worth 20 pounds of sugar, how many pounds of sugar may be had for 9 pounds of tea?

67. If 48 feet of Cremona equal 56 English feet, 39·371 English inches equal one metre of France, how many Cremona feet is the French metre?

68. If a certain number of men can throw up an intrench-

ment in 10 days, when they work 6 hours per day, in what time would they do it if they work 8 hours per day?

69. If 12 men reap a field of wheat in 3 days, in what time can the same work be done by 25 men?

70. A ship's crew of 300 men were so supplied with provisions for 12 months, that each man was allowed 30 ounces per day; but after sailing 6 months, they find that it will take 9 months more to finish their voyage, and 50 of the crew have been lost. Required the daily allowance of each man for the last 9 months.

71. A., B., and C. are to share $1000 in the ratio of the numbers 3, 4, and 5; but C. dying, it is required to divide the whole sum equitably between A. and B.

72. The expense of repairing a school-house to the amount of $600 is paid by three individuals, A., B., and C., in the ratio of their nearness to it. What did each pay, if we suppose A. lived 1 mile distant, B. 2 miles, and C. 3 miles?

73. A merchant bought a piece of cloth for 240 dollars, and sold a portion, exceeding three-fourths of the whole by 2 yards, at a profit of 25 per cent. He afterwards sold the remainder at such a price as to clear 60 per cent. by the whole transaction; and had he sold the whole quantity at the latter price he would have gained 175 per cent. How many yards were there in the piece?

74. The whole number of volumes in the common-school libraries of New York, in 1843, was 874865. What would be their value at $37\frac{1}{2}$ cents per volume?

75. The whole number of children taught in New York during the year 1843, was 657782, and the whole number of schools was 10860. How many scholars on an average would each school consist of?

76. Suppose the Erie Canal to be 60 feet wide, and 6

feet deep, how many miles in length will it require to make one cubic mile of water?

77. A person owning $\frac{3}{5}$ of a copper-mine, sells $\frac{3}{4}$ of his interest in it for $1800. What, at this rate, is the value of the whole?

78. Suppose I buy a certain lot of oranges at 3 cents a piece, and as many more at 5 cents a piece, and sell them at 4 cents a piece; do I gain or lose by the operation?

79. Suppose I buy a certain number of oranges at 3 for one cent, and as many more at 5 for one cent, and sell them at 4 for one cent; do I gain or lose by the operation?

80. Suppose I expend a certain sum of money for oranges at $\frac{1}{3}$ of a cent a piece, and another equal sum for another lot of oranges at $\frac{1}{5}$ of a cent a piece, and sell them at $\frac{1}{4}$ of a cent a piece, do I gain or lose by the operation?

81. Suppose I expend a certain sum of money for oranges at 3 cents a piece, and another equal sum for another lot at 5 cents a piece; how much do I gain on each cent expended, if I sell them at 4 cents a piece?

82. If A. can do a piece of work in 3 days, B. in 4 days, and C. in 5 days, how many times longer will it take B. to do it alone, than it will take A. and C. together to do it?

83. If A. can accomplish a piece of work in $\frac{1}{3}$ of a day, B. in $\frac{1}{4}$ of a day, and C. in $\frac{1}{5}$ of a day, how many times longer will it take B. to do it alone, than it will take A. and C. together to do it?

84. What is the shortest piece of cloth which shall be at the same time an even number of yards, an even number of Ells Flemish, an even number of Ells English, and an even number of Ells French?

85. A man died, leaving $1000, to be divided between his two sons, one 14 and the other 18 years of age, in such a proportion, that the share of each being put to in-

terest at 6 per cent., should amount to the same sum when they should arrive at the age of 21. What did each one receive?

86. Divide $100 between A., B., and C., so that B. may have $3 more than A., and C. $4 more than B. How much must each one have?

87. A. can do a piece of work in 4 days, and B. can do the same in 3 days. How long would it take both together to do it?

88. A person wishes to dispose of his horse by lottery. If he sells the tickets at $2 each, he will lose $30 on his horse; but if he sells them at $3 each, he will receive $30 more than his horse cost him. What is the value of the horse, and the number of tickets?

89. Thomas sold 150 pine-apples at $33\frac{1}{3}$ cents a piece, and received the same amount of money that Henry did for watermelons at 25 cents a piece. How much money did each receive, and how many melons did Henry sell?

90. A man bought apples at 5 cents a dozen, half of which he exchanged for pears, at the rate of 8 apples for 5 pears; he then sold all his apples and pears at a cent a piece, and thus gained 19 cents. How many apples did he buy, and how much did they cost?

91. A person expended $23·40 for eggs. With one-half of his money he purchased a lot at 13 cents per dozen; with the other half of his money he purchased another lot at 9 cents per dozen. He afterwards sold them all together at 11 cents per dozen. Did he gain or lose by the operation?

92. Divide $1200 between A. and B., so that A.'s share may be to B.'s as 2 to 7.

93. A gentleman spends $\frac{2}{3}$ of his yearly income for board and lodging, $\frac{2}{3}$ of the remainder for clothes, and $\frac{2}{3}$ of

the residue he bestows for charitable purposes, and saves $100 yearly. What is his income?

94. If I buy an article for $4, and sell it for $5, how much per cent. do I gain?

95. If I give $5 for an article, and sell it for $4, how much per cent. do I lose?

96. What is the interest of $175 for 3 months, at 6 per cent.?

97. How many yards of Brussels carpeting, which is $\frac{3}{4}$ of a yard wide, will it require to cover a floor 18 feet by 20 feet?

98. Admitting the velocity of a cannon-ball to be 1600 feet per second, what time, at this velocity, would it require to move 95 millions of miles, which is the distance from the earth to the sun, counting $365\frac{1}{4}$ days to the year.

99. The Winchester bushel measure is of a cylindric form, 8 inches deep, and $18\frac{1}{2}$ inches in diameter, containing $2150\frac{2}{5}$ cubic inches. What must be the size of a cubical box which shall contain the same quantity?

100. The clocks of Italy go on to 24 hours; then how many strokes do they strike in one revolution of the index?

101. There is an island 20 miles in circumference, and three men, A., B., and C., start from the same point, and travel the same way around it; A. goes 3 miles per hour, B. goes 7 miles per hour, and C. goes 11 miles per hour. In what time will they all be together?

102. What is the discount of $175 for 3 months, at 6 per cent.?

103. If a ship and its cargo are worth $30000, and the cargo is worth 5 times as much as the ship, what is the value of the cargo?

104. What is the difference between six and one half times 7, and seven and one half times 6?

105. Three persons, A., B., and C., form a partnership: A. furnishes $1000, B. $600, and C. $450; at the end of 6 months, C. withdraws his capital, but no dividend is made until the end of the year, when it is found that the firm has gained $244·16. How is this gain to be divided between the partners?

106. Three persons, A., B., and C., engage to build a certain piece of wall for $244·16. While A. can build 10 rods, B. can build but 6, and C. but $4\frac{1}{2}$. When the wall is half completed, C. ceases to labor upon it, and A. and B. finish it. What part of the $244·16 ought each to receive?

107. A. and B. together can build a wall in 4 days, A. and C. can together build it in 5 days, B. and C. can together build it in 6 days. What time would it require for all together to accomplish it?

108. A note of $10000 given January 1st, 1840, has received the following indorsements: January 1st, 1841, indorsed $2952·28; January 1st, 1842, indorsed $2952·28; January 1st, 1843, indorsed $2952·28. How much remained due January 1st, 1844, interest being computed at 7 per cent.?

109. Two hunters, A. and B., kill a deer, whose weight they are desirous of knowing. For this purpose they rest a stick across the limb of a tree; then suspending the deer at the shorter extremity, they find that its weight is just counterpoised by the weight of A., who suspends himself by his hands at the other extremity. Without changing the point of support of the stick, they take the deer from the shorter extremity, and suspend it at the longer extremity of the stick, when it was found to be exactly balanced by B.'s weight, when suspended to the shorter extremity of the stick. Now, supposing A. to weigh 147 pounds, and

B. to weigh 192 pounds, what must have been the weight of the deer?

NOTE.—By the principle of the lever, when different weights at its extremities balance each other, they are to each other inversely as the lengths of the arms to which they are attached. Hence, in the first experiment, we know that the weight of A. is to the deer's weight, as the shorter arm is to the longer arm. In the second experiment, the deer's weight is to B.'s weight, as the shorter arm is to the longer arm. Consequently, A.'s weight is to the deer's weight, as the deer's weight is to B.'s weight.

APPENDIX.

CHAPTER I.

§ **1.** What is a unit? What is a number? Give an instance of a number. What is an abstract number? If a number be not abstract, what is it? What is the difference between an abstract and a denominate number? Give examples of each kind. What does denominate mean? If I say there are 365 days in a year, what kind of a number do I make use of? Why? How will you use the number 365 to make it an abstract number?

§ **2.** Of what does Arithmetic treat? What is it as a science? As an art? How many methods of expressing numbers are there? What are the different methods?

CHAPTER II.

§ **3.** What is Notation? What is Roman Notation? What letters stand for 3? for 8? for 10? for 16? for 52? for 70? &c., &c. What effect has a letter of less value when placed before a letter of greater value? when placed after? What effect has the repeating of a letter? What effect the horizontal line over a letter? For what are Roman letters used? What is the origin of the character I? V? X? L? C? D? Show this upon your slates. Write the present year of the Christian era on your slates.

§ **4.** Wherein does Arabic differ from Roman notation? Write on the board in backward order the Arabic characters. Write 3 digits and 2 naughts. What do you mean by a digit? What does the word *significant* mean?

§ **5.** How many values have figures? What do they *always* represent? What connection have their units with their values? What is meant by a *simple* value? Explain the *local* value of a figure. Write upon the board a number of 4 figures, and illustrate what is meant by first place, second place, &c.; first order, second order, &c. Does "place" apply to figures, or to the units which they represent? To which does "order" apply? What does the first order of units represent? the fourth? the third? the second? What *property* pertaining to figures in a number is inferred from the illustration given? What property pertains to figures with respect to the figure at its *left* hand?

§ **6.** Illustrate the value of the 0 in the following number, three thousand and three. Illustrate the value of the 0 at the right of a number; also, the effect of cutting off a 0 from the right of a number; two 00, three 000. What does the 0 represent, and what is its office?

§§ **7. 8.** How do you write a number that contains but 4 places? How will you write a number containing three denominations of figures? What is the difference between "places" and "denominations?" Suggest a number containing 5 denominations. Express it in figures. What is meant by "periods?" What is the difference between "periods," "denominations," and "places?" In expressing numbers by figures, what particular mistake will you be likely to make, against which the text cautions you? Must the left-hand period *always* be full? Why? What is meant by a period's being full? What is the service of the 0 in notation? Recite the first 10 periods, beginning with units. Which system of notation have you been using? How does it differ from the other system? and what is the name of that other?

§ **9.** Wherein do Notation and Numeration differ? What is necessary in order to read large numbers with facility? Write a number of 27 figures on the board and read it, and explain its divisions or groups. Give a rule in your own language for the reading of numbers.

ANSWERS.

§ **3.** **(1–5.)** XVII; XLII; XXVI; XCVIII; CIII. **(6–11.)** LXXXII; LVII; LXXIX; CCCCXXX; DCLXXX; MMVII. **(12–14.)** $\overline{\text{CCC}}$; $\overline{\text{DCCCCLX}}$; $\overline{\text{M}}$. **(15.)** XLI. **(16.)** CLXVI. **(17–28.)** Twenty-six; one hundred and forty-four; ninety-eight; one thousand three hundred and twelve; one thousand eight hundred and fifty; one thousand nine hundred and seventy-two; five hundred thousand; five hundred and two thousand seven hundred and seventy; one million; one million and ninety-four; one thousand six hundred and eighty-eight; one thousand seven hundred and seventy-five.

§ **7.** **(29–43.)** 20; 37; 98; 337; 407; 2437; 6407; 8007; 9027; 4006; 3000; 1001; 1100; 101; 1101.

§ **8.** **(44–49.)** 27300; 940200; 36456; 501000; 98000; 11000. **(50–53.)** 46930659; 307802509; 981000700; 10010010. **(54–57.)** 96048073098; 807000000006; 90000004010; 800006000007. **(58–62.)** 48000000000000; 609000000000000; 980004000000007; 3000000000002; 9000000002000. **(63–68.)** 36000000000000000000098; 40080000000005094; 35000098000000063; 900700590; 86000000000000005000000-000; 900000046000.

§ **9.** **(69–75.)** Six hundred and seventy-eight thousand two hundred and ten; five millions, four hundred and ninety-three thousand, six hundred and seventy-eight; four hundred and fifty-six millions, three hundred and twenty-one thousand, nine hundred and eighty; seven hundred and seventy-nine millions, one hundred and forty-six thousand and five; forty-two trillions, five hundred and sixty-seven billions, one hundred and twenty-three thousand, nine hundred and one; three hundred and twenty-seven millions, nine hundred and eighty thousand and sixty; thirty-two quadrillions, nine hundred and eighty-seven trillions, six hundred and fifty-four billions, three hundred millions, and ninety-eight. **(76–80.)** Five hundred and sixty-three billions, four hundred and twenty-eight millions, six hundred and seventy thousand and nine; three hundred and fifty-eight millions, nine hundred and twenty-thousand, seven hundred and sixty-one; nine hundred and eighty-seven millions, six hundred and seventy-eight thousand, nine hundred and thirty-two; four quadrillions, five hundred and sixty trillions, seven billions, nine hundred and eighty millions, five hundred and forty thousand and sixty-eight; thirty-three quintillions, four hundred and ninety-two quadrillions, six hundred and seventy-seven trillions, five billions, three hundred and sixteen millions, eight hundred and ninety-six thousand, three hundred and twenty-one. **(81.)** Twenty trillions. **(82.)** Two hundred and thirty-six thousand, eight hundred and forty-seven. **(83.)** Thirty-six millions, eight hundred and fourteen thousand, seven hundred and twenty-one. **(84.)** Sixty-eight millions, seven hundred and ninety-one thousand, seven hundred and fifty-two. **(85.)** One hundred and forty-four millions, nine hundred and seven thousand, six hundred and thirty. **(86.)** Four hundred and ninety-four millions, four hundred and ninety-nine thousand, one hundred and eight. **(87.)** Eight hundred and eighty-three thousand, two hundred and forty-six. **(88.)** Two millions, seven hundred and seventy-four thousand, seven hundred and ninety-nine. **(89.)** Two trillions, four hundred and fifty billions, eight hundred and thirty millions, two hundred and forty-one thousand, two hundred and eight. **(90.)** Three hundred and sixty quadrillions, seven hundred and eighty-one trillions, one billion, two hundred and four millions, three hundred and ninety-eight thousand, two hundred and ninety-nine. **(91.)** Seventy-three millions, three hundred and seventy-six thousand, two hundred and ninety. **(92.)** Five hundred and sixty-one millions, seven hundred and seven thousand and forty-six.

CHAPTER III.

§ **10.** What do you mean by Addition? Add 4 books to 3 slates. What is the one resulting number? Reasons? What is this number called? Write an expression on the board illustrating the sign plus, and the sign of equality. What is meant by numbers being of the same kind or denomination?

§ **11.** In setting down figures for addition, why must those of the same kind fall in the same column? Give an example of figures of different kinds. Write out an example of 3 numbers of 4 figures each, where the sum of each column shall be less than 10; add and explain.

§ **12.** Set down 6 numbers of 7 figures each; add and explain. Give the rule for addition. Why must you commence at the right hand to add? On what principle must you set down the right-hand figure of the sum under the column added? Why carry the left-hand figure to the next column? What directions does the note give you as to the mode of adding? What is the proof of addition?

ANSWERS.

§ **11.** (**1.**) 9999. (**2.**) 6556. (**3.**) 9398. (**4.**) 9679. (**5.**) 5659. (**6.**) 99968. (**7.**) 5539599.

§ **12.** (**8.**) 125010. (**9.**) 177170559. (**10.**) 249770691. (**11.**) 49448176659. (**12.**) 23650434530. (**13.**) 106014335610. (**14.**) 220957988780. (**15.**) 37185329. (**16.**) 65285936. (**17.**) 157846611. (**18.**) 3308489. (**19.**) 5189375. (**20.**) 5186750. (**21.**) 1904798. (**22.**) 33577. (**23.**) 9364. (**24.**) 365 days. (**25.**) 6278 bricks. (**26.**) 247031 barrels. (**27.**) 547131 hogsheads. (**28.**) 6856886 bales. (**29.**) 11119695 bushels. (**30.**) 30736135 dollars. (**31.**) 73376290 pounds. (**32.**) 561707046 pounds. (**33.**) 7497567 acres. (**34.**) 15028015 dollars. (**35.**) 8108797 dollars. (**36.**) 993095817 inhabitants. (**37.**) 50150009 square miles. (**38.**) 70995 individuals. (**39–42.**) 61630939 pounds; 117274121 pounds; 84148377 pounds; 263053437 pounds. (**43–46.**) 17581225 dollars; 34807174 dollars; 21993877 dollars; 74382276 dollars. (**47–50.**) 252971606 pounds; 746063693 pounds; 670907293 pounds; 1669942592 pounds. (**51–54.**) 27415498 dollars;

68457397 dollars; 46464775 dollars; 142337670 dollars. **(55–58.)** 142337670 dollars; 1669942592 pounds; 74382276 dollars; 263053437 pounds.

CHAPTER IV.

§§ **13, 14, 15, 16.** What is meant by Subtraction? of the numbers 6 and 8, one requiring to be subtracted from the other; which is the minuend? Why? What is the other called? Why? What is there peculiar in the termination of these words? After subtracting, what is the result called? Why? Write an expression on the board illustrating the symbols plus, minus, and equality. Write an example, wherein each figure of the subtrahend shall be less than its corresponding figure of the minuend; subtract and explain. Write an example, wherein figures of the subtrahend are larger than corresponding figures of the minuend; subtract and explain. Give the rule and the reason for every statement in it. How do you prove your work in subtraction?

ANSWERS.

§ **13.** **(1–12.)** 2; 9; 4; 6; 10; 17; 5; 8; 7; 11; 16; 14. **(13–25.)** 2; 4; 6; 16; 14; 13; 10; 12; 15; 7; 3; 5; 8. **(26–39.)** 2; 5; 8; 11; 14; 3; 6; 9; 12; 15; 4; 7; 10; 13. **(40–52.)** 2; 5; 8; 11; 14; 3; 6; 9; 12; 4; 7; 10; 13. **(53–64.)** 2; 5; 8; 11; 3; 6; 9; 12; 4; 7; 10; 13. **(65–75.)** 2; 5; 8; 11; 3; 6; 9; 12; 4; 7; 10. **(76–85.)** 2; 5; 8; 11; 3; 6; 9; 4; 1; 10.

§ **14.** **(86.)** 201. **(87.)** 2211. **(88.)** 10154. **(89.)** 150110. **(90.)** 166304310.

§ **15.** **(91.)** 1093. **(92.)** 3328. **(93.)** 67467. **(94.)** 25485. **(95.)** 1089088. **(96.)** 20891. **(97.)** 669042. **(98.)** 9443544813. **(99.)** 6066069034. **(100.)** 1075415. **(101.)** 51113291. **(102.)** 1357322792. **(103.)** 6889336062. **(104.)** 8849208. **(105.)** 969116902. **(106.)** 8365421086. **(107.)** 4219238873. **(108.)** 7023226. **(109.)** 999998. **(110.)** 364635.

(**111.**) 352 years. (**112.**) 2142 dollars. (**113.**) 1464398 bushels. (**114.**) 201324 barrels. (**115.**) 26 miles. (**116.**) 67 years. (**117.**) 5181 votes. (**118.**) 2210 votes. (**119.**) 5333865 dollars. (**120.**) 105588748 pieces. (**121.**) Gold exceeded silver by 1475597 dollars; gold exceeded copper by 3992969 dollars; silver exceeded copper by 2517372 dollars. (**122.**) 3413 post-offices; 31166 miles of road. (**123.**) 515915 inhabitants. (**124.**) 147 dollars. (**125.**) 564 dollars. (**126.**) 1277 dollars. (**127.**) 225 miles. (**128.**) 168 dollars. (**129.**) In 1840 total value was 3426632 dollars; in 1841, 2240320 dollars; 1840 exceeded 1841 by 1186312 dollars; in 1840, silver exceeded gold by 51401 dollars, silver exceeded copper by 1702076 dollars; in 1841, silver exceeded gold by 41153 dollars, silver exceeded copper by 1116777 dollars. (**130.**) 20500 dollars. (**131.**) 1000000 total volumes; 96000 total manuscripts; 400000 excess of volumes in Paris library above those in Vienna library; 64000 excess of manuscripts; 904000 total excess of volumes above manuscripts. (**132.**) 975 total number of votes; 113 number of votes in majority.

CHAPTER V.

§ **17.** What is Multiplication? What is the difference between the multiplier and the multiplicand? What are they called? Why? What is the difference between a factor and a product? Write an example by means of symbols, and show which is multiplier, which is multiplicand, which are the factors, which is the product. Suppose multiplier and multiplicand change places, what is the result? What is a square? What is a square root? Illustrate.

§§ **18, 19.** Perform an example, with a multiplier of one figure, and explain the process. Perform an example with a multiplier of 3 figures. Suppose there is a 0 in the multiplier, how do you proceed? Of what denomination is the product, if you multiply together units and units? units and hundreds? tens and hundreds? tens and tens? hundreds and thousands? tens and ten-thousands? How does the denomination of a product guide you as to the place which the first figure of any partial product should occupy?

§§ **20, 21.** What is another definition of multiplication? Show how this is true. What is necessary with regard to the numbers added that an exercise in addition might be turned into an exercise in multiplication? What is the rule for multiplication?

§ **22.** What is the method of proof?

§ **23.** Sometimes one or both factors will have 0's at the right; what must be done in such case? Why?

§ **24.** What do you understand by a composite number? Give an instance of a number that is composite, and of one that is not. If a multiplier be composite, how may you proceed? Give an example. Give the rule in your own words.

ANSWERS.

§ **17.** (**1–11.**) 4; 9; 16; 25; 36; 49; 64; 81; 100; 121; 144. (**12–22.**) 5; 7; 2; 3; 4; 6; 8; 9; 10; 12; 11. (**23–27.**) 32; 18; 84; 64; 81. (**28–36.**) 12; 18; 24; 30; 36; 9; 27; 21; 15. (**37–47.**) 8; 16; 24; 32; 40; 48; 12; 20; 28; 36; 44. (**48–58.**) 15; 30; 45; 60; 10; 25; 40; 55; 20; 35; 50. (**59–69.**) 18; 36; 54; 72; 12; 30; 48; 66; 24; 42; 60. (**70–80.**) 21; 42; 63; 84; 14; 35; 56; 77; 28; 49; 70. (**81–135.**) 24; 48; 72; 96; 16; 40; 64; 88; 32; 56; 80; 27; 54; 81; 108; 18; 45; 72; 99; 36; 63; 90; 30; 60; 90; 120; 20; 50; 80; 110; 40; 70; 50; 33; 66; 99; 132; 22; 55; 88; 121; 44; 77; 110; 36; 72; 108; 144; 24; 60; 96; 132; 48; 84; 120. (**136–142.**) 30; 36; 14; 15; 50; 121; 56.

§ **18.** (**143.**) 2468. (**144.**) 468312. (**145.**) 2449512. (**146.**) 4488270. (**147.**) 6020736. (**148.**) 1439694746. (**149.**) 52248187648. (**150.**) 8019276804702. (**151.**) 44025632. (**152–160.**) 62972; 94458; 125944; 157430; 188916; 220402; 251888; 283374; 1385384. (**161–169.**)

80785809; 71809608; 62833407; 53857206; 44881005; 35904804; 26928603; 17952402; 394952844. (**170–172.**) 5216648; 1956243; 3260405. (**173–180.**) 61232; 7849350; 51744; 81195867; 17295; 10172519; 49864787; 149212794.

§ **22.** (**181.**) 1542382864. (**182.**) 55056418756. (**183.**) 9472469137. (**184.**) 3937919100705. (**185.**) 28574677132. (**186.**) 23070596606. (**187.**) 254087145206. (**188.**) 1270996912224. (**189–195.**) 529259254443; 12138394951269; 8141111037027; 108215060743638; 949354188891741; 121932631112635269; 123011009127513387. (**196.**) 1665-3188645286. (**197–198.**) 17494334544; 57482188. (**199–200.**) 33793364; 837852.

§ **23.** (**201–204.**) 764290; 7642900; 76429000; 764290000. (**205–208.**) 1974800; 19748000; 197480000; 1974800000. (**209–212.**) 19626000; 196260000; 1962600000; 19626000000. (**213–216.**) 32001280000; 320012800000; 3200128000000; 32001280000000. (**217–218.**) 1161253800000000; 287550710470000000.

§ **24.** (**219.**) 10220. (**220–224.**) 8976; 6732; 13464; 23562; 40392. (**225–230.**) 82960332; 34566805; 79997463; 76046971; 63207872; 39504920. (**231–237.**) 119376; 17409; 82071; 24870; 12435; 19896; 9948. (**238.**) 18750 bricks. (**239.**) 2033 bushels. (**240.**) 629 miles. (**241.**) 53200 pounds. (**242.**) 363 dollars. (**243.**) 480 dollars. (**244.**) 8361574 dollars. (**245.**) 2756 bushels. (**246.**) 75798 dollars. (**247.**) 2715 dollars. (**248.**) 16199568 hours. (**249.**) 3760128 cubic inches. (**250.**) 141440 dollars. (**251.**) 7560 miles. (**252.**) 59568000 miles. (**253.**) 479544 dollars. (**254.**) 2205 dollars. (**255.**) 109 miles. (**256.**) 215 dollars. (**257.**) 32 acres at 198 dollars. (**258.**) 149 miles. (**259.**) Lose 27 dollars. (**260.**) 941618440000. (**261.**) 295 dollars. (**262.**) 14553 cubic inches.

CHAPTER VI.

§§ **25, 26.** What is Division? In the example 6÷2, what is the 6 called? What the 2? Why? What is the result called? What is the remainder? Write an example containing the symbol of division, and the symbol of equality. How is an accurate quotient sometimes to be expressed? Under what circumstances must it be so expressed? Give an example of your own upon the board, of the division of a number by a single digit. Explain as you go along. Show, by an example, the reason for the statement that division is a concise way of performing several subtractions.

§ **27.** What is the difference between short division and long division? What is the rule for short division? What for long division? Perform an example in each, and apply the rule step by step as you proceed. Illustrate the notes after rule for long division.

§§ **28, 29.** How do you prove your work in division? How do you proceed when your divisor ends with one or more naughts? Suppose a digit be cut off from the right of a number, what effect has it? What does the digit so cut off represent? How is the true remainder found after dividing by a divisor with naughts cut off? If there be a remainder after such division, is it of the same denomination as the digit or digits cut off? Illustrate by an example on the blackboard the division of a number by a composite divisor. Show how you find the true remainder, and give the reason for each step.

ANSWERS.

§ **25.** **(1–12.)** 1; 3; 4; 6; 2; 5; 7; 9; 8; 10; 12; 11. **(13–24.)** 1; 3; 5; 7; 9; 11; 12; 2; 4; 6; 8; 10. **(25–34.)** 2; 4; 6; 8; 10; 12; 3; 5; 7; 9. **(35–45.)** 2; 4; 6; 8; 10; 12; 3; 5; 7; 9; 11. **(46–54.)** 2; 4; 6; 8; 10; 3; 5; 7; 9. **(55–65.)** 2; 4; 6; 8; 10; 12; 3; 5; 7; 9; 11. **(66–76.)** 2; 4; 6; 8; 10; 12; 3; 5; 7; 9; 11.

(77–87.) 2; 4; 6; 8; 3; 5; 7; 9; 11; 10; 12. **(88–100.)** 2; 4; 6; 9; 8; 7; 3; 5; 10; 13; 12; 11; 14. **(101–109.)** 2; 4; 6; 3; 7; 10; 8; 11; 12. **(110–120.)** 2; 4; 5; 8; 7; 12; 10; 11; 3; 6; 9. **(121–129.)** 9; 4; 7; 7; 20; 7; 12; 5; 9. **(130–134.)** 3; 4; 4; 4; 4.

§ **26.** **(135.)** 12301. **(136.)** $1682227\frac{2}{4}$. **(137.)** $1786213\frac{4}{5}$. **(138.)** $5315728\frac{4}{6}$. **(139.)** $128236331\frac{4}{7}$. **(140.)** $115134608\frac{1}{8}$. **(141.)** $26063098\frac{8}{9}$. **(142.)** $8491229\frac{7}{9}$. **(143.)** 95665602. **(144.)** $127160495\frac{1}{7}$. **(145.)** 1315020576. **(146.)** 1357802469. **(147–154.)** $173945\frac{1}{2}$; $115963\frac{2}{3}$; $86972\frac{3}{4}$; $69578\frac{1}{5}$; $57981\frac{5}{6}$; $49698\frac{5}{7}$; $43486\frac{3}{8}$; $38654\frac{5}{9}$. **(155–162.)** $38270653\frac{1}{2}$; 25513769; $19135326\frac{3}{4}$; $15308261\frac{2}{5}$; $12756884\frac{3}{6}$; $10934472\frac{3}{7}$; $9567663\frac{3}{8}$; $8504589\frac{6}{9}$. **(163–170.)** $448821\frac{1}{2}$; $299214\frac{1}{3}$; $224410\frac{3}{4}$; $179528\frac{3}{5}$; $149607\frac{1}{6}$; $128234\frac{5}{7}$; $112205\frac{3}{8}$; $99738\frac{1}{9}$. **(171–178.)** 3826954: $2551302\frac{2}{3}$; 1913477; $1530781\frac{3}{5}$; $1275651\frac{2}{6}$; $1093415\frac{3}{7}$; $956738\frac{4}{8}$; $850434\frac{2}{9}$. **(179–183.)** $90536\frac{2}{6}$; $6646090\frac{1}{6}$; 1327581667; $7815886\frac{2}{6}$; $1666481\frac{1}{6}$. **(184–213.)** $181072\frac{2}{3}$; $13292180\frac{1}{3}$; 26551623334; $15631772\frac{2}{3}$; $3332962\frac{1}{3}$; $135804\frac{2}{4}$; $9969135\frac{1}{4}$; $19913717500\frac{2}{4}$; $11723829\frac{2}{4}$; $2499721\frac{3}{4}$; $77602\frac{4}{7}$; $5696648\frac{5}{7}$; $11379267143\frac{1}{7}$; $6699331\frac{1}{7}$; $1428412\frac{3}{7}$; $67902\frac{2}{8}$; $4984567\frac{5}{8}$; $9956858750\frac{2}{8}$; $5861914\frac{6}{8}$; $1249860\frac{7}{8}$; $60357\frac{5}{9}$; $4430726\frac{7}{9}$; $8850541111\frac{3}{9}$; $5210590\frac{8}{9}$; $1110987\frac{4}{9}$; $108643\frac{3}{5}$; $7975308\frac{1}{5}$; $15930974000\frac{2}{5}$; $9379063\frac{3}{5}$; $1999777\frac{2}{5}$.

§ **28.** **(214–218.)** $34424\frac{14}{24}$; $17212\frac{14}{48}$; $8606\frac{14}{96}$; $7376\frac{78}{112}$; $5737\frac{62}{144}$. **(219–224.)** $343768\frac{10}{27}$; $210948\frac{34}{44}$; $94711\frac{68}{98}$; $122128\frac{18}{76}$; $39329\frac{102}{236}$; $31570\frac{166}{294}$. **(225–235.)** $1707335\frac{339}{486}$; $894143\frac{445}{928}$; $1162136\frac{45}{714}$; $2149650\frac{249}{386}$; $2058970\frac{239}{403}$; $914845\frac{734}{907}$; $134440\frac{1469}{6172}$; $99779\frac{2985}{8316}$; $143235\frac{4794}{5793}$; $345016\frac{1669}{2405}$; $276036\frac{933}{3006}$. **(236–243.)** $2882566\frac{2029}{2498}$; $94262\frac{71979}{76389}$; $2238034\frac{5981}{32174}$; $73279\frac{37520}{98263}$; $159278\frac{12073}{45208}$; $23844\frac{73869}{301987}$; $12679\frac{361908}{567801}$; $8089\frac{446950}{890123}$. **(244–250.)** $30050\frac{49186}{298701}$; $1849\frac{1552520}{4853684}$; $983\frac{417193}{9130821}$; $7010\frac{978046}{12803319}$; $1180\frac{6841316}{76000994}$; $2746\frac{18498}{32688753}$; $98\frac{12353782}{914659923}$. **(251–257.)** $156472\frac{381}{789}$; $8\frac{335375}{678500}$; $23470056\frac{7856}{34771032}$; 12890625; 109376; $681\frac{4630}{36546}$.

§ **29.** **(258–263.)** $864\frac{4}{20}$; 18296; $493827\frac{2}{20}$; $17283945\frac{1}{20}$; 6789734; $18982660\frac{19}{20}$. **(264–271.)** $289763412\frac{57}{240}$; $231810729\frac{237}{300}$; $144881706\frac{57}{480}$; $99347455\frac{437}{700}$; $100787273\frac{567}{690}$; $17385804\frac{2937}{4000}$; $869290\frac{18937}{80000}$; $77270\frac{218937}{900000}$. **(272–273.)** $16\frac{387545}{421000}$; $28860\frac{1212}{42000}$. **(274–275.)** $10037\frac{1689}{12300}$; $2920\frac{1988731}{2498000}$. **(276–279.)** $29123\frac{841}{3000}$; $90\frac{69841}{97000}$; $10\frac{6339541}{8103030}$; $14\frac{3105641}{6090300}$. **(280–285.)** $11960\frac{58300}{78910}$; $25647\frac{12300}{36800}$; $22103\frac{23800}{42700}$; $95\frac{6627900}{9865200}$; $2\frac{69819900}{437001000}$; $95\frac{8735000}{9843020}$. **(286–289.)** $2120\frac{37281}{177600}$; $199232\frac{801}{1890}$; $918412\frac{361}{410}$; $144271\frac{1971}{2610}$. **(290–295.)** 234, r. 27; 2245, r. 3; 133, r. 15; 221, r. 30; 11438, r. 7; 15677, r. 3. **(296.)** 1994 dollars. **(297.)** 2776 sheep. **(298.)** 991 dollars. **(299.)** 974 dollars. **(300.)** 1177 dollars. **(301.)** 210 acres. **(302.)** 1st 56, 2d 70, 3d 105, 4th 105. **(303.)** 412 dollars. **(304.)** 249 acres. **(305.)** 11875000 miles. **(306.)** $4545\frac{10}{22}$ cubic feet. **(307.)** 125 days. **(308.)** $20\frac{26704}{143940}$ dollars. **(309.)** 103368000 hours, 4307000 days, 11800 years. **(310.)** 856 barrels, 107 trees. **(311.)** Each had 900 dollars. **(312.)** 2191 dollars. **(313.)** 1632000 miles in one day, 595680000 miles in one year. **(314.)** 3100 dollars. **(315.)** 2 days. **(316.)** They will meet in 5 hours, at a distance of 75 miles. **(317.)** 54 dollars. **(318.)** 125 dollars. **(319.)** 2 dollars. **(320.)** 49 miles.

CHAPTER VII.

§ **30.** What do you understand by a *problem*? by a *principle*? Show how problem *a* is founded upon the preceding rules. Illustrate problem *b*. Illustrate problem *c*. Illustrate problem *d*. Illustrate problem *e*; problem *f*; problem *g*. Can you give a practical example (not taken from the book) of the use of any one of the preceding problems?

§ **31.** What are the names of the quantities used in division? What effect has the multiplication of a divisor upon the result in division? What the division of a divisor? What the multiplication or division of a dividend? How is the *remainder* affected by such operations upon divisor or dividend? What relation has the *quotient* to the divisor? to the dividend? to the remainder? If the remainder be as large as the divisor, what is to be done? Can the remainder ever be as large as the quotient? Can it ever be exactly equal to the quotient? Give examples. Illustrate each principle in the section; in *a*, *b*, *c*, *d*, *e*, *f*, *g*.

ANSWERS.

§ **30.** (**1.**) 123423434. (**2.**) 343148. (**3.**) 59831. (**4.**) 879465. (**5.**) 1037654321, 771350011. (**6.**) 23474. (**7.**) 4567031. (**8.**) 34678 dollars, 13787 dollars. (**9.**) 1240578. (**10.**) 354. (**11.**) 1521808704. (**12.**) 4556 votes, 4181 votes. (**13.**) 144000000 miles, 95000000 miles. (**14.**) 49 trees. (**15.**) 35405 dollars. (**16.**) 5718 dollars. (**17.**) 45441 hills. (**18.**) 42 gallons, 23 gallons. (**19.**) 11 miles, 7 miles. (**20.**) 646 dollars. (**21.**) 44 years old. (**22.**) 5 dollars. (**23.**) 157 barrels. (**24.**) 101 cubic feet. (**25.**) 1728 cubic inches.

CHAPTER VIII.

§§ **32, 33.** What is the difference between a prime and a composite number? Give examples. To what extent can you determine upon inspection whether a number is prime or not? What is an *even* number? an odd? Show how and why it is that a number, the sum of whose digits is equal to 9, is itself divisible by 9. Show how and why the same thing is true of 3.

§ **34.** What is a divisor? a common divisor? the greatest common divisor? Show this, by analyzing numbers. Can prime numbers have common divisors? Give reason. What is the common divisor of two numbers that are prime to each other?

§ **35.** How is the greatest common divisor found by the process of long division? Explain this process, giving the reasons for each step as they are explained in (*a*) or (*b*). How would you proceed to find the greatest common divisor of three or four numbers?

§ **36.** What is a multiple? a common multiple? the least common multiple? What is the difference between the least common multiple and the greatest common divisor? Is the greatest common divisor of two or more numbers a factor of their least common multiple? If so, show how. What would be the other factor of such multiple? How many multiples may any number have? Show how to find the least common multiple by decomposing into primes.

§ **37.** Show the same by the process under present section. Explain how this process agrees with the former.

§ **38.** What is cancelation? Is it employed in subtraction or in addition? Give an example of cancelation. In what way is it useful?

ANSWERS.

§ **33.** **(1–8.)** 2×2×3; 2×7; 3×5; 2×2×2×2; 2×3×3; 2×2×5; 2×11; 2×2×2×3. **(9–16.)** 5×5; 2×13; 3×3×3; 2×2×7; 2×3×5; 2×2×2×2×2; 3×11; 2×17. **(17–24.)** 5×7; 2×2×3×3; 2×19; 3×13; 2×2×2×5; 2×3×7; 2×2×11; 3×3×5. **(25–32.)** 2×23; 2×2×2×2×3; 7×7; 2×5×5; 3×17; 2×2×13; 2×3×3×3; 5×11. **(33–41.)** 2×2×2×7; 3×19; 2×29; 2×2×3×5; 2×31; 3×3×7; 2×2×2×2×2×2; 5×13; 2×3×11. **(42–50.)** 2×2×17; 3×23; 2×5×7; 2×2×2×3×3; 5×17; 3×29; 2×3×3×5; 2×2×2×2×2×3; 2×7×7. **(51–57.)** 2×3×17; 3×37; 7×17; 5×5×5; 2×3×23; 2×73; 5×31. **(58–63.)** 2×7×11; 2×83; 2×89; 11×19; 2×3×3×13; 7×37. **(64–69.)** 3×103; 2×3×61; 3×5×5×5; 2×2×101; 11×43; 2×2×131. **(70–76.)** 2×2×2×11×13; 2×2×2×2×5×13; 2×2×2×3×3×19; 2×2×2×3×3×17; 2×2×2×823; 3×3×11×797; 2×2×3×5179. **(77–83.)** 2×3×7×13×19; 2×2×5×43×89; 2×131×241; 2×2×3×5×5×263; 2×2×2×2×2×3×67; 3×17×1913; 2×3×14951. **(84–90.)** 2×23×163; 2×31×907; 2×5×5×5×199; 3×1111; 3×7×4759; 3×5×3251; 2×5×7×1327.

§ **34.** **(91–94.)** 12; they have none; 45; 5. **(95–99.)** 22; 4; 8; 8; 66. **(100–103.)** 6; 6; 18; 18. **(104–106.)** 2; 6; 14. **(107–109.)** 2; 2; 2.

§ **35.** **(110–117.)** They have none; they have none; they have none; 5; 10; 12; 16; 234. **(118–121.)** 161; 203; 35; 111. **(122–124.)** 3; 3; 406. **(125–127.)** They have none; they have none; they have none. **(128–130.)** They have none; they have none; 101. **(131–132.)** 203; 555.

§ **37.** **(133–138.)** 48; 120; 616; 1517; 360; 24. **(139–143.)** 315; 2520; 1008; 27720; 720. **(144–147.)** 360; 100; 1620; 920. **(148–151.)** 840; 210; 7106; 128700. **(152–156.)** 39000; 336600; 510510; 4560; 3360.

§ **38.** **(157.)** 14. **(158.)** 119. **(159–160.)** 12; 8. **(161–162.)** 48; 24. **(163–169.)** 576; 432; 288; 216; 144; 108; 72. **(170–172.)** 20; 90; 40. **(173–175.)** 16200; 21900; 59130. **(176–179.)** 21; 121; 363; 847. **(180–183.)** 3276; 378; 416; 288. **(184–187.)** 55; 88; 40; 66. **(188–190.)** 300; 75; 80.

CHAPTER IX.

§ **39.** What is a fraction? What does the word mean? In how many ways may a given fraction be represented? In how many ways may a fraction in the common form be read? What is the name of the term above the line? and what does it denote? What the term below the line? Express by a fraction the value of unity. Show the difference between a proper and an improper fraction. Write a mixed number. What is meant by an integer? Write a compound fraction. What is the difference between a compound and a complex fraction? When is a fraction inverted? Write the nine digits as improper fractions. What two kinds of fractions are spoken of?

§ **40.** Upon what are common fractions founded? Illustrate. *What, then, does a fraction express?* What connection has § 31 with the subject of fractions? What propositions are deduced from that section? Illustrate each proposition by reference to the principle in division on which it is founded.

§ **41.** Define reduction. Illustrate it. What is meant by *lower* terms? by *lowest* terms? What have these expressions to do with reduction? What has the greatest common divisor to do with reduction to lowest terms?

§ **42.** Define an improper fraction, a whole number, a mixed number. How do you reduce the first to the second or the third? Give the rule.

§ **43.** Illustrate the reduction of a whole or mixed number to an improper fraction. Give rule.

§ **44.** Illustrate the reduction of compound fractions to simple ones. Give rule. In multiplying a fraction by a whole number, what form may the whole number take?

§ **45.** Define the term common denominator. How is this found? What is the change produced upon fractions by this process? Give the rule. Give an example, on the board, of the reduction of a mixed number and a compound fraction to a common denominator.

§ **46.** How does the *least* common denominator differ from the common denominator? Of what service is the least common multiple in this connection? Give the rule.

§§ **47–50.** What is necessary before fractions can be added? Rule. What is necessary before fractions can be subtracted? Rule. Give the rule for the multiplication of fractions. Explain and illustrate each step. Illustrate the two methods of division of fractions. What is the principle involved in the first method? What principles are involved in the second method? Give rule. Is this inversion of the terms of the divisor a mechanical help, or does it involve a principle?

§ **51.** What is a reciprocal? of an integer? of a fraction? How may an operation in division be included under that of multiplication? Illustrate.

ANSWERS.

§ **41.** **(1–5.)** $\frac{2}{3}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{3}$; $\frac{1}{2}$. **(6–11.)** $\frac{2}{3}$; $\frac{5}{8}$; $\frac{1}{2}$; $\frac{1}{6}$; $\frac{3}{4}$; $\frac{3}{5}$; **(12–16.)** $\frac{1}{7}$; $\frac{1049}{6392}$; $\frac{316}{405}$; $\frac{81}{559}$; $\frac{5}{8}$. **(17–20.)** $\frac{1}{101}$; $\frac{2057}{2280}$; $\frac{11174}{13741}$; $\frac{8}{9}$. **(21–25.)** $\frac{124}{151}$; $\frac{23}{30}$; $\frac{16}{27}$; $\frac{7}{9}$; $\frac{365}{531}$. **(26–30.)** $\frac{14}{23}$; $\frac{91}{101}$; $\frac{77}{139}$; $\frac{91}{101}$; $\frac{14}{23}$. **(31–40.)** $\frac{63}{95}$; $\frac{227}{250}$; $\frac{63}{95}$; $\frac{227}{250}$; $\frac{417}{4417}$; $\frac{316}{417}$; $\frac{820}{1431}$; $\frac{441}{553}$; $\frac{971}{2852}$; $\frac{971}{2582}$.

§ **42.** **(41–50.)** 3; 3; 12; 12; 48; 46; 8; 12; 1000; 11110. **(51–64.)** $2\frac{2}{5}$; $11\frac{3}{7}$; $7\frac{1}{2}$; $9\frac{1}{3}$; $1\frac{1}{9}$; $18\frac{5}{7}$; $10\frac{5}{11}$; $10\frac{14}{37}$; $599\frac{5}{7}$; $81\frac{252}{365}$; $34\frac{9}{97}$; $32\frac{81}{101}$; $39\frac{53}{86}$; $10\frac{1015}{3203}$. **(65–68.)** $10\frac{81}{91}$; 6; $203\frac{2}{9}$; 3.

§ **43.** **(69–75.)** $\frac{36}{4}$; $\frac{45}{5}$; $\frac{54}{6}$; $\frac{63}{7}$; $\frac{72}{8}$; $\frac{81}{9}$; $\frac{90}{10}$. **(76–86.)** $\frac{9}{2}$; $\frac{10}{3}$; $\frac{37}{5}$; $\frac{25}{3}$; $\frac{15}{2}$; $\frac{75}{8}$; $\frac{64}{5}$; $\frac{50}{3}$; $\frac{295}{12}$; $\frac{1183}{32}$; $\frac{3245}{166}$. **(87–96.)** $\frac{29817}{365}$; $\frac{1554}{41}$; $\frac{1314}{331}$; $\frac{150}{19}$; $\frac{30376}{83}$; $\frac{38283}{31}$; $\frac{854}{11}$; $\frac{104364}{1231}$; $\frac{71040}{93}$; $\frac{38113}{928}$. **(97–99.)** $\frac{16459}{34}$; $\frac{18095}{48}$; $\frac{13067}{44}$.

§ **44.** **(100–101.)** $\frac{3}{8}$; $\frac{2}{9}$. **(102–109.)** $\frac{6}{35}$; $\frac{35}{198}$; $\frac{3}{5}$; $\frac{1}{3}$; $\frac{1}{28}$; $\frac{2}{5}$; $\frac{5}{17}$; $\frac{125}{72}=1\frac{53}{72}$. **(110–116.)** $\frac{1}{40}$; $\frac{1}{15}$; $\frac{2}{35}$; $\frac{105}{32}=3\frac{9}{32}$; $\frac{1}{5}$; $\frac{5}{9}$; $\frac{91}{215}$. **(117–121.)** $\frac{117}{343}$; $\frac{195}{3712}$; $\frac{1512}{455}=3\frac{147}{455}$; $\frac{665}{216}=1\frac{17}{216}$; $\frac{496}{35}=14\frac{6}{35}$.

§ **45.** **(122–129.)** $\frac{3}{6}$, $\frac{4}{6}$; $\frac{15}{20}$, $\frac{16}{20}$; $\frac{40}{24}$, $\frac{42}{24}$; $\frac{80}{90}$, $\frac{81}{90}$; $\frac{120}{132}$, $\frac{121}{132}$; $\frac{180}{195}$, $\frac{182}{195}$; $\frac{224}{240}$, $\frac{225}{240}$; $\frac{340}{360}$, $\frac{342}{360}$. **(130–136.)** $\frac{35}{105}$, $\frac{21}{105}$, $\frac{15}{105}$; $\frac{12}{24}$, $\frac{8}{24}$, $\frac{6}{24}$; $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$; $\frac{360}{432}$, $\frac{378}{432}$, $\frac{384}{432}$; $\frac{1188}{1320}$, $\frac{1200}{1320}$, $\frac{1210}{1320}$; $\frac{2520}{2730}$, $\frac{2535}{2730}$, $\frac{2548}{2730}$; $\frac{630}{700}$, $\frac{600}{700}$, $\frac{560}{700}$. **(137–139.)** $\frac{4}{12}$, $\frac{54}{12}$, $\frac{66}{12}$; $\frac{75}{30}$, $\frac{220}{30}$, $\frac{156}{30}$; $\frac{288}{784}$, $\frac{2352}{784}$, $\frac{2107}{784}$.

§ **46.** **(140–146.)** $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$; $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$; $\frac{5}{6}$, $\frac{4}{6}$; $\frac{36}{63}$, $\frac{35}{63}$; $\frac{56}{63}$, $\frac{45}{63}$; $\frac{195}{260}$, $\frac{234}{260}$, $\frac{240}{260}$; $\frac{100}{180}$, $\frac{144}{180}$, $\frac{135}{180}$. **(147–155.)** $\frac{50}{120}$, $\frac{56}{120}$, $\frac{55}{120}$; $\frac{15}{120}$, $\frac{18}{120}$, $\frac{56}{120}$; $\frac{105}{30}$, $\frac{130}{30}$, $\frac{36}{30}$; $\frac{160}{180}$, $\frac{84}{180}$, $\frac{117}{180}$; $\frac{88}{330}$, $\frac{150}{330}$, $\frac{2025}{330}$;

$\frac{30}{60}$; $\frac{40}{60}$, $\frac{195}{60}$, $\frac{12}{60}$; $\frac{21}{210}$, $\frac{70}{210}$, $\frac{30}{210}$, $\frac{40}{210}$; $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$, $\frac{27}{60}$; $\frac{80}{120}$, $\frac{30}{120}$, $\frac{72}{120}$, $\frac{100}{120}$, $\frac{108}{120}$, $\frac{7}{120}$. **(156.)** $\frac{1260}{2520}$, $\frac{840}{2520}$, $\frac{630}{2520}$, $\frac{504}{2520}$, $\frac{420}{2520}$, $\frac{360}{2520}$, $\frac{315}{2520}$, $\frac{280}{2520}$.

§ **47.** **(157–162.)** $\frac{5}{4}=1\frac{1}{4}$; $\frac{7}{24}$; $\frac{9}{20}$; $\frac{133}{60}=2\frac{13}{60}$; $\frac{71}{120}$; $\frac{28}{12}=\frac{7}{3}=2\frac{1}{3}$. **(163–168.)** $2\frac{1}{3}$; $2\frac{23}{264}$; $2\frac{307}{360}$; $2\frac{83}{3360}$; $\frac{47}{90}$; $1\frac{5}{14}$. **(169–174.)** $7\frac{1}{12}$; $8\frac{1}{30}$; $\frac{31}{36}$; $1\frac{229}{280}$; $\frac{1151}{7429}$; $1\frac{44}{189}$. **(175–180.)** $38\frac{1}{12}$; $\frac{724}{1001}$; $2\frac{731}{1100}$; $1\frac{5}{6}$; $2\frac{1}{60}$; $\frac{13}{120}$. **(181–186.)** $12\frac{71}{105}$; $13\frac{1}{12}$; $15\frac{1}{12}$; $30\frac{11}{30}$; $18\frac{11}{18}$; $11\frac{17}{18}$.

§ **48.** **(187–195.)** $\frac{1}{56}$; $\frac{1}{6}$; $\frac{25}{99}$; $\frac{13}{35}$; $\frac{3}{221}$; $\frac{19}{84}$; $\frac{7}{55}$; $\frac{7}{85}$; $\frac{13}{33}$. **(196–203.)** $\frac{3}{10}$; $\frac{1}{6}$; $\frac{3}{8}$; $\frac{1}{28}$; $\frac{3}{4}$; $\frac{19}{360}$; $\frac{58}{105}$; $\frac{13}{40}$. **(204–209.)** $\frac{1}{2}$; $\frac{16}{35}$; $\frac{9}{56}$; $1\frac{4}{45}$; $\frac{37}{140}$; $2\frac{2}{3}$. **(210–213.)** 4; $\frac{3}{10}$; $6\frac{1}{40}$; $\frac{13}{600}$. **(214–218.)** $\frac{23}{60}$; $\frac{37}{504}$; $\frac{263}{8580}$; $\frac{479}{28560}$; $\frac{253}{23940}$. **(219–222.)** $2\frac{1}{4}$; $2\frac{3}{40}$; $\frac{223}{360}$; $\frac{11}{120}$.

§ **49.** **(223–233.)** $\frac{1}{6}$; $\frac{1}{8}$; $\frac{2}{15}$; $\frac{7}{24}$; $\frac{1}{2}$; $\frac{3}{7}$; $\frac{8}{27}$; $\frac{5}{7}$; $\frac{7}{11}$; $\frac{13}{60}$; $\frac{11}{30}$. **(234–240.)** $\frac{1}{24}$; $\frac{1}{120}$; $\frac{2}{9}$; $\frac{12}{45}$; $\frac{20}{77}$; $\frac{91}{192}$; $\frac{9}{98}$. **(241–243.)** $\frac{28}{125}$; $\frac{44}{147}$; $\frac{539}{5760}$. **(244–246.)** $1\frac{1}{12}$; 15; $2\frac{2}{3}$. **(247–249.)** $\frac{1}{5}$; $\frac{8}{9}$; $73\frac{2}{3}$. **(250–254.)** $\frac{1}{48}$; $1\frac{5}{7}$; $5\frac{1}{4}$; $26\frac{1}{4}$; $82\frac{1}{2}$. **(255.)** $1\frac{263}{1200}$. **(256.)** $\frac{7}{75}$. **(257.)** $\frac{5}{63}$. **(258.)** $127\frac{719}{720}$.

§ **50.** **(259–269.)** 2; $1\frac{1}{5}$; $1\frac{1}{7}$; $1\frac{1}{9}$; $1\frac{1}{11}$; $1\frac{1}{12}$; $1\frac{1}{14}$; $\frac{8}{9}$; $\frac{9}{10}$; $\frac{35}{36}$; $\frac{35}{36}$. **(270–274.)** $2\frac{1}{4}$; $1\frac{4}{7}$; $\frac{8}{9}$; $\frac{36}{88}$; $1\frac{1}{143}$. **(275–278.)** $\frac{26}{105}$; $\frac{3}{34}$; $3\frac{3}{5}$; $1\frac{49}{51}$. **(279.)** $1\frac{2}{3}$. **(280.)** $\frac{317}{137}$. **(281.)** $4\frac{706161}{845693}$. **(282.)** $3\frac{3}{5}$. **(283–287.)** $1\frac{78}{407}$; $1\frac{265}{533}$; $8\frac{8}{15}$; $\frac{7}{162}$; $\frac{625}{1024}$.

§ **51.** **(288–295.)** $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$; $\frac{1}{11}$; $\frac{1}{18}$; $\frac{1}{24}$; $\frac{1}{96}$; $\frac{1}{108}$. **(296–303.)** $\frac{3}{2}$; $\frac{4}{3}$; $\frac{5}{4}$; $\frac{6}{5}$; $\frac{7}{6}$; $\frac{8}{7}$; $\frac{9}{8}$; $\frac{10}{9}$. **(304–309.)** $\frac{2}{3}$; $\frac{3}{7}$; $\frac{4}{13}$; $\frac{7}{36}$; $\frac{2}{9}$; $\frac{11}{141}$. **(310–313.)** $3\frac{1}{9}$; $1\frac{1}{14}$; $1\frac{5}{22}$; $3\frac{1}{2}$. **(314–318.)** $\frac{3}{16}$; $\frac{27}{20}$; $\frac{3}{28}$; $\frac{9}{14}$; $\frac{76}{69}$. **(319–324.)** $\frac{7}{8}$; $\frac{3}{4}$; $\frac{1103}{1109}$; $\frac{12}{13}$; $\frac{101}{103}$; $\frac{9}{10}$. **(325–329.)** $1\frac{2}{101}$; $7\frac{2}{5}$; $3\frac{8}{17}$; $2\frac{557}{907}$; $1\frac{4}{97}$. **(330–334.)** $\frac{7}{2}$; $\frac{206}{13}$; $\frac{58}{17}$; $\frac{110}{101}$; $\frac{1711}{17}$. **(335–339.)** $\frac{1}{4}$; $\frac{1}{3}$; $\frac{9}{16}$; $\frac{3}{25}$; 200.

(340–344.) $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$; $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, $\frac{10}{60}$; $\frac{70}{20}$, $\frac{28}{20}$, $\frac{15}{20}$, $\frac{6}{20}$; $\frac{385}{1155}$, $\frac{231}{1155}$, $\frac{165}{1155}$, $\frac{105}{1155}$; $\frac{3003}{5005}$, $\frac{3575}{5005}$, $\frac{3185}{5005}$, $\frac{4235}{5005}$. **(345–346.)** $1\frac{1}{12}$; $2\frac{23}{60}$. **(347.)** $\frac{5}{6}$. **(348–350.)** $\frac{1}{6}$; $\frac{1}{110}$; $\frac{5}{28}$. **(351.)** 30 feet. **(352.)** A.'s 844, B.'s 633. **(353.)** A.'s 1402 dollars, B.'s 2804 dollars. **(354.)** A.'s claim 6000 dollars, B.'s 5000 dollars, C.'s 5000 dollars. **(355.)** A. must have $\frac{1}{8}$, B. $\frac{7}{8}$. **(356.)** 1230 sheep. **(357.)** A. had 154 dollars, B. 165, C. 264, and D. 33. **(358.)** $\frac{4}{9}$. **(359.)** 12000 dollars. **(360.)** She gave away in all 9500 dollars. To the Fire Department Fund 3000 dollars; to the Musical Fund Society 2000 dollars; and to each of the other societies 500 dollars. **(361.)** $1\frac{29}{57}$ baskets. **(362–363.)** $\frac{5}{21}$ dollars; $23\frac{17}{21}$ cents. **(364.)** $19\frac{47}{48}$ barrels. **(365.)** $1\frac{1}{36}$ bushels, $\frac{37}{84}$ of a bushel, $1\frac{10}{27}$ bushels, $\frac{37}{126}$ of a bushel. **(366.)** A. went 52 miles, B. 43 miles. **(367.)** 100 feet. **(368.)** 4000 dollars. **(369.)** $\frac{5}{64}$, $\frac{7}{540}$, $\frac{17}{168}$, $\frac{2179}{60480}$. **(370.)** A.'s 23 dollars, B.'s 23, C.'s 21, and D.'s 21.

CHAPTER X.

§ **52.** What is the difference between a common and a Decimal fraction? What gives the name decimal to the fraction? Which is the most, 3 units or 4 hundredths? Wherein does a decimal require a different treatment from a whole number? What is the use of the point? What are the names of the first six decimal places, beginning at the left hand? What is the difference in value between 1 ten and 1 tenth? between 1 hundred and 1 hundredth? What is the use of the 0 in decimal notation? of the naught prefixed? annexed?

§§ **53, 54.** How do you express decimals in figures? Give an example. How do you read decimals expressed in figures? Give an example.

§ **55.** Is the rule for the addition of decimals in any wise similar in principle to the rule for the addition of common fractions and of whole numbers? Why must as many figures in the product be pointed off as are equal to the greatest number of decimal places in any of the numbers added?

§ **56.** Is the principle in subtraction of decimals like that in subtraction of common fractions and of whole numbers? Show this. How must the *difference* be pointed off?

§ **57.** Give an example of the right mode of pointing off, in the product of one decimal by another, with reasons. Show the service of prefixing naughts to the product in certain cases.

§ **58.** How may a decimal number be multiplied by 10, 100, 1000, &c.?

§§ **59, 60, 61, 62.** Give the rule for pointing off the quotient in division of decimals. Give the reason for the rule. When is the quotient a whole number? When there is a remainder, what is to be done? What does the sign + mean annexed to a quotient? How do you divide a decimal by 10, 100, &c.? Give rule.

§ **63.** What is the difference between a common fraction and a decimal? How do you reduce a common fraction to a decimal? How do tenths become hundredths? Explain the reason for the rule. Are a repetend and a repeating decimal the same thing? What is the difference between them? What is the finite part?

§ **64.** Give the reasons for the rule.

§ **65.** How do you reduce repeating decimals to common fractions? Give the reason for the rule. What is to be done if there be a finite part to the decimal?

§ **66.** What is Federal Money? What is its symbol? What is the difference between 7·682 and $7·682? Are the denominations of Federal Money to be explained by decimals? If so, how? Is the gold or the silver, of which the United States coins are made, pure? Why is it not pure? What is the alloy?

§ **67.** Illustrate how the annexing or the cutting off of 0's changes the denominations of Federal Money.

§ **69.** Explain the method and the principle of dividing decimals by 10, 100, or 1000, &c.

§ **70.** What are aliquot parts? Explain the method of obtaining results by aliquot parts. Explain the rule. What is the origin of our denominations of shillings and pence? Why are the shillings of the different States of unequal value? In what year was Federal Money adopted?

ANSWERS.

§ **53.** (**1–9.**) 2; 3; 6; 4; 5; 9; 7; 8; 10. (**10–19.**) 0·037; 0·03; 0·000048; 0·00000095; 0·00490; 0·0001240; 0·10000004; 0·000000096; 0·00009301; 0·027101. (**20–27.**) 0·0804000; 7·000400; 0·074000081; 0·00896000; 0·04007; 0·0800004000; 0·060000000074; 80·0000000099. (**28–37.**) 0·84; 0·096; 0·0077; 0·00104; 0·10007; 0·000044; 0·0000007; 0·0000012; 0·01365. (**38–50.**) 9·4; 8·13; 41·8; 418·9; 46·74; 8·961; 0·7461; 54·982; 4786·19; 2·826018; 18·9765; 8·4108; 8976·54821.

§ **54.** (**51–78.**) Eight *tenths;* ninety *hundredths;* four hundred and seven *thousandths;* one *thousandth;* six thousand nine hundred and forty-five *ten thousandths;* eighty-seven thousand six hundred and one *hundred thousandths;* seventy-six *hundred thousandths;* ten thousand, eight hundred and seventy-six *hundred thousandths;* one thousand and seven *ten millionths;* one million and twelve *ten millionths;* six million, seven hundred fifty thousand, nine hundred and twelve *ten millionths;* eighty million, seven hundred thousand, one hundred and seventy-six *hundred millionths;* eighty million and one *hundred millionths;* nine hundred and one million, ten thousand, one hundred and one *billionths;* three, and seventeen *ten thousandths;* four, and ninety thousand and eighteen *hundred thousandths;* six, and one *millionths;* forty-nine, and one hundred thousand and seven *millionths;* eighty-six, and ten thousand and seven *ten millionths;* forty-four, and sixty-two million and sixteen *hundred millionths;* one billion, one million and one hundred *ten billionths;* twenty-seven, and forty-six thousand, eight hundred and twelve *hundred* *thousandths;* nine, and four hundred and sixty-seven *ten thousandths;* eight and forty-two *ten thousandths;* twenty-one, and one *ten millionths;* thirty-six, and twenty-one and one-sixth *hundredths;* forty-eight, and four thousand eighty-one and one-ninth *ten thousandths;* nine, and ten million, one hundred one and sixty-nine seventieths of *hundred millionths.*

§ **55.** **(79–81.)** 0·8145; 1·1718; 561843·617. **(82–87.)** 1·4031; 1·1655; 1·3464; 1·9998; 2·5587; 3·1718. **(88–93.)** 17·2725; 0·42111; 532·94; 170·972; 367·4221; 2157·114. **(94–98.)** 642·206; 2·1769; 1342·048; 2523·315; 410·714. **(99–103.)** 72527·835; 260·557; 317·809; 6507·42057; 127944·077.

§ **56.** **(104–110.)** 850·7973; 197·7603; 861·4708; 898·2632; 897·7603; 1·0809; 896·2994. **(111–116.)** 84120·21; 92487·3091; 92580·614184; 10580·31; 92581·309964; 92539·812. **(117.)** 2999999·900001. **(118.)** 96000002006·9999916. **(119.)** 82000302·99991. **(120–122.)** 291·222; 905·3356; 2977·02313346. **(123–125.)** 923·5; 0·34803; 0·10557. **(126–129.)** 208·3263; 3652·005; 203·3735; 495·7564.

§ **57.** **(130–138.)** 7544·624; 11316·936; 13203·092; 26406·184; 36780·042; 114112·438; 656382·288; 1169416·720; 63065174·696. **(139–147.)** 567·96621; 1917·562109; 1623·5719803; 25099·7791204; 19·7435873; 79·67754546; 256·450266820; 207·1510702521; 276867·86773521. **(148–154.)** 17·07577096; 245·0806812; 18·43085209; 3·09751462586; 0·00000298806; 0·000000448209; 0·00000000398408. **(155–163.)** 0·0001406; 0·00002072; 0·000002664; 0·0000003552; 0·00000007104; 0·000000006216; 0·0000000007178; 2·699594; 6·612640296.

§ **58.** **(164.)** 821·46. **(165.)** 76920. **(166–171.)** 46104; 461040; 4610400; 46104000; 461040000; 4610400000. **(172–179.)** 4·7692; 47·692; 476·92; 4769·2; 47692; 476920; 476900; 47692000. **(180–187.)** 37; 370; 3700; 37000; 370000; 3700000; 37000000; 370000000.

§ **59.** **(188–192.)** 2·3; 3; 11·2; 2·6; 56·71457. **(193–198.)** 3·4024; 0·456; 1·1213; 9·8008; 9·6665; 38·666.

§ **60.** **(199–205.)** 10; 500; 5000; 25000; 36; 198; 40. **(206–213.)** 471·73+; 832·07+; 117·11+; 29·68+; 10·24+; 5·58+; 1·05+; 68·46. **(214–225.)** 100931·7465+; 91654·520+; 725003·53+; 2530·6+; 114+; 84+; 13090+; 8954+; 20126+; 37·940+; 98528·6097+; 85314·94+.

§ **61.** **(226–231.)** 1·643; 8·235; 0·1647; 0·1176+; 0·0915; 0·0748+. **(232–238.)** 6666·666+; 8·3111+; 2·2886+; 0·824+; 2·223+; 7·405+; 22·629.

§ **62.** **(239–242.)** 4149·76; 414·976; 41·4976; 4·14976. **(243–247.)** 6·74; 0·674; 0·0674; 0·00674; 0·000674. **(248–253.)** 0·0341; 0·00341; 0·000341; 0·0000341; 0·00000341; 0·000000341. **(254.)** 3·95 cords. **(255.)** 13·2 inches. **(256.)** 88·28 inches. **(257.)** 63·38 nearly. **(258.)** 103·94+ volumes. **(259.)** 320 rods. **(260.)** 498·65+ seconds. **(261.)** 70000·039492 grains = 10·000005 pounds, nearly. **(262.)** 2218·192 cubic inches = 1·283 cubic feet, nearly. **(263.)** 436247·424 grains=62·32106 pounds, nearly. **(264.)** 37059·62+ times. **(265.)** 320 rods. **(266.)** 52·8 feet. **(267–269.)** 3462·29 times, nearly; 2974·65 times, nearly; 487·64 times, nearly. **(270.)** 54·01+ dollars. **(271–274.)** 2400·375 dollars; 80 dollars; 99·625 dollars; 12 acres. **(275.)** 584·75 gallons.

§ **63.** **(276–297.)** 0·5; 0·333+; 0·14285714+; 0·1666+; 0·08333+; 0·666+; 0·75; 0·8; 0·8333+; 0·85714285+; 0·875; 0·888+; 0·8181+; 0·90909+; 0·91666+; 0·92857+; 0·9333+; 0·9375; 0·94117+; 0·9444+; 0·94736+; 0·95. **(298–329.)** 0·6; 0·42857+; 0·375; 0·3; 0·230769+; 0·571428+; 0·4444+; 0·3636+; 0·307692+; 0·71428+; 0·625; 0·5555+; 0·4545+; 0·5454+; 0·461538+; 0·352941+; 0·7777+; 0·6363+; 0·53846+; 0·7272+; 0·61538+; 0·5333+; 0·470588+; 0·692308+; 0·52941+; 0·785714+; 0·590909+; 0·571428+; 0·575757+; 0·921568+; 0·734513+; 0·160112+

(330–333.) 0·685714+; 0·31818; 0·85069+; 0·349205+. **(334–336.)** 3·256277+; 0·080148+; 1·256493+. **(337.)** 15·716666+ dollars.

§ **64.** **(338–351.)** $\frac{5}{10}=\frac{1}{2}$; $\frac{15}{100}=\frac{3}{20}$; $\frac{25}{100}=\frac{1}{4}$; $\frac{375}{1000}=\frac{3}{8}$; $\frac{225}{1000}=\frac{9}{40}$; $\frac{435}{1000}=\frac{87}{200}$; $\frac{575}{1000}=\frac{23}{40}$; $\frac{486}{1000}=\frac{243}{500}$; $\frac{656}{1000}=\frac{82}{125}$; $\frac{25}{10000}=\frac{1}{400}$; $\frac{375}{100000}=\frac{3}{800}$; $\frac{225}{1000000}=\frac{9}{40000}$; $\frac{1001}{10000}$; $\frac{36984}{100000}=\frac{4623}{12500}$. **(352–357.)** $\frac{982}{10000}=\frac{491}{5000}$; $\frac{764}{100000}=\frac{191}{25000}$; $\frac{25}{100000}=\frac{1}{4000}$; $\frac{5005}{10000}=\frac{1001}{2000}$; $\frac{125}{10000}=\frac{1}{80}$; $\frac{1250505}{1000000000}=\frac{250101}{200000000}$.

§ **65.** **(358–370.)** $\frac{1}{3}$; $\frac{384}{1111}$; $\frac{44}{101}$; $\frac{727}{1111}$; $\frac{1}{9}$; $\frac{4}{33}$; $\frac{1}{12}$; $\frac{2}{30}$; $\frac{73}{300}$; $\frac{1}{70}$; 1; $\frac{1}{81}$; $\frac{101}{819}$. **(371–385.)** $\frac{41111}{333000}$; $\frac{1}{14}$; $\frac{1}{28}$; $\frac{1}{37}$; $\frac{41}{333}$; $\frac{107}{333}$; $\frac{4}{11}$; $\frac{7}{11}$; $\frac{398}{9999}$; $\frac{705}{1111}$; $\frac{66}{101}$; $\frac{627}{1111}$; $\frac{626}{1111}$; $\frac{8}{11}$; $\frac{6}{11}$.

§ **66.** **(386–392.)** Seven dollars and eighty-four cents; ninety-two dollars and six cents; six hundred and seventy-two dollars twelve cents and three mills; eight thousand nine hundred and sixty-one dollars and six mills; four thousand one hundred and eighty dollars ninety-six cents and seven and three-tenths mills; nine hundred and one dollars and one mill; three dollars and three cents. **(393–401.)** Six dollars and eighty-two cents; seven dollars forty-four cents and eight mills; nine dollars and two cents; three dollars and one cent; four dollars and seven cents; six dollars and ninety-three cents; forty-eight dollars seventy-six cents and one mill; two hundred and seventeen dollars and one mill; thirty-six dollars ninety-eight cents and seven mills. **(402–408.)** \$0·37; \$0·443; \$6·02; \$4·008; \$9·206; \$5000·089; \$1000000·015. **(409–412.)** \$0·375; \$2·125; \$4·625; \$5·875.

§ **67.** **(413–419.)** 800*ct.*=8000 mills; 89400*ct.*=894000*m.*; 62000*ct.* = 620000*m.*; 3400*ct.* = 34000*m.*; 93627300*ct.* = 936273000*m.*; 84190400*ct.* = 841904000*m.*; 12345600*ct.* = 123456000*m.* **(420–426.)** 830*m.*; 910*m.*; 40*m.*; 3780*m.*; 12340*m.*; 91000*m.*; 8756180*m.* **(427–441.)** \$8·41; \$9·28; \$46·70; \$129·86; \$4·81; \$1·234; \$49·68; \$321·946; \$1357·92; \$9·80; \$98; 3918·762; \$49876·21; \$30760·09; \$4876·543.

(442–450.) 894*ct.*; 9280·1*ct.*; 123456·7*ct.*; 8107*ct.*; 8396*ct.*; 48780*ct.*; 965421*ct.*; 1349820*ct.*; 48231*ct.* **(451–456.)** 830*m.*; 98436*m.*; 2076*m.*; 281296*m.*; 4812370*m.*; 69874983*m.*

§ **68.** **(457.)** \$20·80. **(458.)** \$99·05. **(459.)** \$82·288. **(460.)** \$106·195. **(461.)** \$6·375. **(462.)** \$25·53. **(463.)** \$20·9375. **(464.)** \$49·815. **(465.)** 19625·796+. **(466.)** \$19597·125. **(467.)** \$1·929+. **(468.)** \$68·493+. **(469.)** 216·5454+ rods. **(470.)** \$38·861+. **(471.)** \$11428·57$\frac{1}{7}$. **(472.)** 7682 pounds. **(473.)** 7897 pounds sold; average for each cow, 149 pounds. **(474.)** 21631·5 pounds, \$3244·725. **(475.)** \$1497·9225. **(476.)** \$12·149+. **(477.)** 1575 thousand *m's*; he received \$236·25. **(478.)** \$16·81. **(479.)** Butcher receives from tailor \$19·77, receives from shoemaker \$15·14, and the tailor receives from shoemaker \$20·24. **(480.)** \$4·078125. **(481.)** \$41·535. **(482.)** \$128·52. **(483.)** \$27·0225. **(484.)** \$122·6475. **(485.)** \$0·625. **(486.)** \$2164781. **(487.)** \$133·36. **(488.)** \$94·27. **(489.)** \$16·665. **(490.)** 1314 volumes. **(491.)** \$2299·50. **(492.)** \$335·73.

§ **69.** **(493.)** \$7·15. **(494.)** \$16·875. **(495.)** \$248·2875, **(496.)** \$339·0825. **(497.)** \$2·84375. **(498.)** \$1351·3728. **(499.)** \$8538·75. **(500.)** \$2090·0376.

§ **70.** **(501–514.)** \$84·75; \$113; \$127·125; \$135·60; \$169·50; \$211·875; \$226; \$254·25; \$339; \$423·75; \$452; \$508·50; \$550·875; \$593·25. **(515–521.)** \$105; \$315; \$420; \$525; \$630; \$735; \$210. **(522–527.)** 54 pounds; 72 pounds; 324 pounds; 5556 pounds; 49302 pounds; 588762 pounds. **(528–538.)** 592 pecks; 296 pecks; 222 pecks; 444 pecks; 49$\frac{1}{3}$ pecks; 88$\frac{4}{5}$ pecks; 63$\frac{3}{7}$ pecks; 40$\frac{4}{11}$ pecks; 74 pecks; 55$\frac{1}{2}$ pecks; 44$\frac{2}{5}$. **(539–549.)** 800 brushes; 400; 266$\frac{2}{3}$; 200; 160; 133$\frac{1}{3}$: 114$\frac{2}{7}$; 100; 80; 66$\frac{2}{3}$; 57$\frac{1}{7}$. **(550–562.)** 1749$\frac{1}{3}$ yards; 1312; 1166$\frac{2}{9}$; 1124$\frac{4}{7}$; 1049$\frac{3}{5}$; 984; 874$\frac{2}{3}$; 787$\frac{1}{5}$; 583$\frac{1}{9}$; 524$\frac{4}{5}$; 492; 437$\frac{1}{3}$; 403$\frac{9}{13}$. **(563–575.)** \$85·75; \$102·90; \$104·125; \$106·66$\frac{2}{3}$; \$110·25; \$114·33$\frac{1}{3}$; \$122·50; \$130·66$\frac{2}{3}$; \$159·25; \$134·75; \$153·125; \$147; \$183·75.

CHAPTER XI.

§ **71.** Explain the difference between an abstract and a denominate number. Which of the two is Federal Money to be considered? Which, any multiplier? Which, the product of two numbers? When is a quotient to be considered an abstract number, and when a denominate number? Give an example.

§ **72.** What is meant by Sterling Money? Is the pound in circulation? What do the symbols £, *s.*, *f.*, *d.*, *qr.*, express?

§ **73.** What was the original of all weights? Whence the term *Troy?* What are weighed by this weight?

§ **74.** Draw upon the board the symbols of the grain, the dram, the scruple, the ounce, the pound, Apothecaries' Weight? For what is this weight used?

§ **75.** Wherein does Avoirdupois Weight differ from Troy? What is said concerning the *qr.*?

§ **76.** Whence was our standard yard obtained? What is the French standard or unit of measure? How was it obtained? How is the inch often divided? How on the carpenters' rules, which you commonly see? What is the difference between a *knot* and a *nautical mile?*

§ **77.** Repeat the table of Cloth Measure.

§ **78.** Explain what is meant by *Square* Measure. Is an acre square? Why not? How long is Gunter's chain?

§ **79.** What is the difference between square and solid measure? between a square foot and a solid foot? Give examples. What is meant by round timber? What is a cord foot?

§ **80.** Repeat the table of Wine Measure.

§ **81.** Which is the larger, the wine or the beer gallon? the wine or the beer quart? How much larger? In which should milk be measured?

§ **82.** Repeat the table of Dry Measure. What articles *not* mentioned in the note are measured by this measure? What is the U. S. standard of Dry Measure?

§ **83.** What is leap year? What is the centennial year? What is the difference between a lunar, a calendar, and a business month?

§ **84.** Divide a circle in halves by a straight line. How many degrees in each half? How many in a quarter-circle? Explain latitude—longitude. How many miles in a degree? How many miles does the sun pass over in an hour?

§ **85.** Repeat the table.

§ **86.** Explain the difference between Reduction Ascending and Reduction Descending. Repeat the rule, and apply it to each of the four cases of reduction descending, viz.: (1st.) of a compound quantity to its lowest denomination; (2d.) of the fraction of a higher to the fraction of a lower denomination; (3d.) of the fraction of a higher to its value in lower denominations; (4th.) of the decimal of a higher to its value in lower denominations.

§ **87.** Give the rule for Reduction Ascending. Specify the four cases to which it may be applied, and apply and illustrate it.

§ **88.** Wherein does the principle involved in addition of denominate numbers differ from that of simple addition, and of addition of fractions? Give rule and explain it.

§ **89.** Why must a number of the subtrahend be placed under a number of the same denomination in the minuend? What distinct principles are embodied in the rule?

§ **90.** Give the rule and explain its principles.

§ **91.** Show in what way a divisor may be an abstract or a denominate number. Show how a quotient may be an abstract or a denominate number.

§ **92.** What is the meaning of Duodecimals? What are their denominations? To what are they generally applied? What is there peculiar in the addition or subtraction of duodecimals?

§ **93.** Illustrate the multiplication of duodecimals by the multiplication of decimals. What may the index (′) be considered? What is the rule for the annexing of indices to the product? Explain this. What is the strict value of 1′ in the measurement of surfaces? What in the measurement of solids? What as a *linear* measure, for which it is sometimes used?

§ **94.** Give an instance of the practical use of division of duodecimals. Go through with an operation, applying the rule and explaining each step. How can you illustrate the indices of the quotient by the decimal places in decimal division?

§§ **95. 96.** In the addition or suotraction of denominate fractions, what principle is involved different from that in addition and subtraction of common fractions?

ANSWERS.

§ **85.** **(1–10.)** 8; 16; 20; 32; 40; 60; 80; 100; 200; 400. **(11–20.)** 48; 96; 144; 240; 384; 720; 960; 1200; 2400; 4800. **(21–30.)** 24; 36; 60; 84; 108; 180; 240; 300; 600; 1200. **(31–40.)** 40; 60; 100; 140; 180; 300; 420; 500; 1000; 2000. **(41–42.)** $6\frac{1}{2}$; $1050\frac{3}{4}$. **(43.)** 1 sovereign=20*s.*=240*d.*=960*far.* **(44.)** 498. **(45–60.)** 48; 120; 168; 360; 600; 1200; 480; 960; 2400; 3360; 7200; 12000; 24000; 5760; 11520; 28800. **(61–74.)** 40; 100; 140; 180; 300; 400; 500; 1000; 240; 720; 1200; 2160; 3600; 6000. **(75–81.)** 24; 48; 108; 180; 300; 600; 1200. **(82–101.)** 60; 100; 140; 240; 360; 60; 300; 420; 720; 1080; 480; 2400; 3360; 5760; 8640: 5760; 28800; 40320; 69120; 103680. **(102–110.)** 6; 15; 27; 24; 120; 216; 288; 1440; 2592. **(111–118.)** 16; 40; 56; 96; 576; 864; 1440; 1536. **(119–128.)** 32; 112; 144; 256; 512; 2304; 5120; 6400; 25600; 512000. **(129–136.)** 32; 112; 144; 240; 400;

1600; 1600; 32000. (**137–141.**) 2000; 6000; 40000; 100000; 200000. (**142–150.**) 24; 84; 240; 36; 198; 198; 7920; 63360; 4382400. (**151–153.**) 300; 5280; 528000. (**154–158.**) 72; 63; 360; 72; 1440. (**159–164.**) 36; 9; 27; $37\frac{1}{5}$; 45; 54. (**165–167.**) 6048; 1568160; 6272640. (**168–169.**) 4840; 3097600. (**170–172.**) 46656; 86400; 221184. (**173–177.**) 24; 48; 1512; 6048; 12096. (**178–180.**) 504; 252; 2016. (**181–182.**) 36; 72. (**183–186.**) 32; 1024; 15360; 25600. (**187–189.**) 200; 6400; 12800. (**190–197.**) 300; 900; 1800; 2700; 3600; 43200; 86400; 604800. (**198–200.**) 168; 8736; 8760. (**201–207.**) 120; 480; 900; 7200; 28800; 54000; 1296000. (**208–211.**) 120; 720; 1500; 21600. (**212–214.**) 72; 864; 10368. (**215–218.**) 100; 500; 2000; 4000. (**219–221.**) 980; 4900; 19600. (**222–225.**) 120; 480; 3360; 3840. (**226–233.**) 25; 24; 12; 28; 90; 6; 9; 18. (**234–239.**) 12; 40; 8; 12; 10; 8. (**240–246.**) 18; 12; 3; 20; 32; $3\frac{1}{2}$; 5. (**247–250.**) 5; $37\frac{1}{2}$; $18\frac{3}{5}$; 10. (**251–262.**) 4; $12\frac{1}{2}$; $4\frac{1}{6}$; $41\frac{2}{3}$; 72; 5; 50; $10\frac{1}{2}$; 20; $30\frac{5}{12}$; 60; $14\frac{1}{6}$. (**263–265.**) 7; 15; $33\frac{1}{3}$. (**266–268.**) 6; $4\frac{1}{2}$; 75. (**269–272.**) 12; $30\frac{5}{12}$; $8\frac{1}{3}$; $38\frac{1}{3}$. (**273–276.**) 20; 50; 41; 39. (**277–281.**) 100; $22\frac{1}{2}$; $31\frac{1}{4}$; $22\frac{13}{16}$; $44\frac{7}{16}$. (**282–286.**) 44; 200; 309; $8\frac{1}{3}$; $62\frac{1}{2}$. (**287–289.**) 150; 120; 100. (**290–292.**) 640; 2560; 102400. (**293–295.**) 4; 330; 10. (**296.**) 10. (**297.**) $12\frac{1}{2}$. (**298.**) 25. (**299–300.**) 5; $16\frac{2}{3}$.

§ **87.** (**301.**) 45369. (**302.**) £123 10*s.* 1*d.* 3*far.* (**303.**) 1500. (**304.**) 234. (**305.**) 1214. (**306.**) 8*lb.* 5*oz.* 18*pwt.* 14*gr.* (**307.**) 15*lb.* 0*oz.* 5*pwt.* (**308.**) 83*lb.* 4*oz.* (**309.**) 26237. (**310.**) 576001. (**311.**) 425. (**312.**) 2℥ 0ʒ 2℈. (**313.**) 2℔ 0℥ 0ʒ 0℈ 1*gr.* (**314.**) 1*T.* 14*cwt.* 0*qr.* 11*lb.* 14*oz.* 10*dr.* (**315.**) 8340. (**316.**) 512257. (**317.**) 1*T.* 13*cwt.* 1*qr.* 21*lb.* 7*oz.* 14*dr.* (**318.**) 120*yd.* 2*ft.* 11*in.* (**319.**) 15713280. (**320.**) 1577664000. (**321.**) 288. (**322.**) 58. (**323.**) 133. (**324.**) 16. (**325.**) 62*yd.* 2*qr.* (**326.**) 243. (**327.**) 1440. (**328.**) 307200*P.* (**329.**) $138030\frac{3}{4}$. (**330.**) 2*R.* 15*P.* $161\frac{1}{4}$ *sq. ft.* (**331.**) 7. (**332.**) 500000. (**333.**) 2176. (**334.**)

1175040. (**335.**) 207 tons, 8 cubic feet, 1721 cubic inches (**336.**) 125. (**337.**) 32832. (**338.**) 32256. (**339.**) 9 barrels, 29 gallons. (**340.**) 3786. (**341.**) 31 barrels, 15 gallons, 3 quarts. (**342.**) 25. (**343.**) 4752. (**344.**) 231*hhd.* 26*gal.* (**345.**) 4323. (**346.**) 2304. (**347.**) 26528. (**348.**) 4*ch.* 24*bu.* 1*pk.* (**349.**) 49*bu.* 3*pk.* 5*qt.* (**350.**) 2592000. (**351.**) 166554. (**352.**) 789458400. (**353.**) 673155*da.* 18*hr.* (**354.**) 9496*da.* (**355.**) 11 birthdays. (**356.**) 388800″. (**357.**) 189°. (**358.**) 16° 40′. (**359.**) 2° 46′ 40″. (**360.**) 164735″. (**361.**) $87\frac{1}{3}$*doz.* (**362.**) 76 buttons. (**363.**) 555 gross, $6\frac{2}{3}$*doz.* (**364.**) 70 years. (**365.**) 360 sheets. (**366.**) 480 sheets. (**367.**) $\frac{1}{4400}$. (**368.**) $\frac{9}{896}$. (**369.**) $\frac{1}{3000}$. (**370.**) 1760 feet. (**371.**) $\frac{1}{36960}$. (**372.**) $\frac{4}{7}$. (**373.**) $597\frac{1}{3}$ gills. (**374.**) $34\frac{5}{7}$ inches. (**375.**) $\frac{1}{4816}$. (**376.**) $\frac{7}{944}$. (**377.**) $129\frac{15}{16}$ inches$=10\frac{53}{64}$ feet$=3\frac{39}{64}$ yards. (**378.**) $\frac{1}{2160}$ of a month$=\frac{1}{26280}$ of a year. (**379.**) 1*qr.* $2\frac{6}{7}$*na.* (**380.**) 1*fur.* 20*rd.* (**381.**) 1*qr.* $42\frac{6}{7}$*lb.* (**382.**) 2*mi.* 3*fur.* 26*rd.* 11*ft.* (**383.**) 9*hr.* 36*min.* (**384.**) 5*min.* $37\frac{1}{2}$*sec.* (**385.**) 5*hr.* 48*min.* 48*sec.* (**386.**) 13*oz.* $2\frac{144}{175}$*dr.* (**387.**) $3\frac{1}{5}$*qt.* (**388.**) 30*da.* (**389.**) 25*P.* (**390.**) £8·259375. (**391.**) 0·875 of a yard. (**392.**) 0·4444+ of a yard. (**393.**) 3·36684027777+ pounds. (**394.**) 10·3995265151+ miles. (**395.**) 0·145949074074+ of a day. (**396.**) £3·2520833+. (**397.**) 0·4444+ of a hogshead. (**398.**) 0·2270833+. (**399.**) £0·915625. (**400.**) 0·921875 of a bushel. (**401.**) 0·469618055+ of a day. (**402.**) 0·5219696+ of a furlong. (**403.**) 0·71 of an hour. (**404.**) 0·0827617+ of a year. (**405.**) 0·24224 of a day. (**406.**) 3*R.* 25·2*P.* (**407.**) 2*s.* 6*d.* (**408.**) 13*s.* 4*d.* (**409.**) 23*gal.* 2*qt.* 1*pt.* (**410.**) 44*da.* 5*hr.* 49*min.* 1·632*sec.* (**411.**) 5*oz.* 5·888*dr.* (**412.**) 6*cwt.* 2*qr.* 14*lb.* 4·8*oz.* (**413.**) 2*fur.* 0*rd.* 4*yd.* 1*ft.* 2·4*in.* (**414.**) 6*s.* 10*d.* 3·2*far.* (**415.**) 2*hr.* 54*min.* 32·7168*sec.* (**416.**) 6*s.* 10*d.* 3·776*far.* (**417.**) 19*s.* 9*d.* (**418.**) 5*hr.* 48*min.* 49·536*sec.* (**419.**) 648. (**420.**) 15. (**421.**) 54·32 pounds. (**422.**) 67·3064 pounds. (**423.**) 151*lb.* 8*oz.* (**424.**) 23·64 sheets. (**425.**) 11 quires 2·4 sheets.

§ **88.** (**426.**) £39 15*s.* $9\frac{3}{4}$*d.* (**427.**) £25 4*s.* $5\frac{3}{4}$*d.* (**428.**) £34 14*s.* 8*d.* (**429.**) 17*lb.* 3*oz.* 12*pwt.* 5*gr.* (**430.**) 21*lb.* 10*oz.*

3*pwt.* 20*gr.* **(431.)** 61*lb.* 11*oz.* 13*pwt.* 8*gr.* **(432.)** 32℔ 3℥ 0ʒ 2℈ 12*gr.* **(433.)** 36℔ 6℥ 5ʒ 1℈. **(434.)** 18ʒ 0℈ 13*gr.* **(435.)** 27*T.* 9*cwt.* 2*qr.* 21*lb.* 7*oz.* 13*dr.* **(436.)** 14*cwt.* 0*qr.* 22*lb.* 7*oz.* **(437.)** 14*L.* 0*mi.* 5*fur.* 15*rd.* 2*yd.* **(438.)** 22*rd.* 3*yd.* 0*ft.* 8*in.* **(439.)** 58*yd.* 1*qr.* **(440.)** 47*E. Fl.* 0*qr.* 2*na.* **(441.)** 52*E. E.* 2*qr.* 2*na.* **(442.)** 175*sq. yd.* 7*sq. ft.* 93*sq. in.* **(443.)** 23*M.* 106*A.* 0*R.* 34*P.* **(444.)** 26*s. yd.* 18*s. ft.* 963*s. in.* **(445.)** 35 cords 41*s. ft.* **(446.)** 50 cords 5*c. ft.* **(447.)** 54*hhd.* 36*gal.* 1*qt.* 1*pt.* **(448.)** 36*tuns* 0*pi.* 1*hhd.* 19*gal.* 0*qt.* 1*pt.* 1*gi.* **(449.)** 56*hhd.* 52*gal.* 1*qt.* 1*pt.* **(450.)** 33*bar.* 9*gal.* 3*qt.* **(451.)** 22*ch.* 23*bu.* 2*pk.* 4*qt.* 1*pt.* **(452.)** 42*bu.* 1*pk.* 7*qt.* **(453.)** 32*da.* 21*hr.* 2*min.* 9*sec.* **(454.)** 8*wk.* 6*da.* 6*hr.* 50*min.* 33*sec.* **(455.)** 4*cr.* 0*s.* 11° 59′ 26″. **(456.)** 9*s.* 8° 45′. **(457.)** 28° 55′ 58″.

§ **89.** **(458.)** 3*T.* 17*cwt.* 1*qr.* 24*lb.* 4*oz.* 13*dr.* **(459.)** 59*A.* 2*R.* 27*P.* **(460.)** 7℔ 8℥ 6ʒ 1℈ 19*gr.* **(461.)** 11*L.* 2*mi.* 0*fur.* 31*rd.* **(462.)** 5*E. Fr.* 3*qr.* 1*na.* **(463.)** 20*ch.* 1*bu.* 1*pk.* 2*qt.* 1*pt.* **(464.)** 9*tuns* 1*pi.* 0*hhd.* 52*gal.* 2*qt.* **(465.)** 40*da.* 9*hr.* 19*min.* 16*sec.* **(466.)** 13*yr.* 7*mo.* 0*wk.* 2*da.* **(467.)** 19*mi.* 0*fur.* 1*rd.* **(468.)** 34*C.* 127*s. ft.* **(469.)** 19*C.* 7*cord ft.* **(470.)** £19 9*s.* 3*d.* **(471.)** 11*mo.* 5*da.* **(472.)** 4*mo.* 28*da.* **(473.)** 2*yr.* 11*mo.* 14*da.* **(474.)** 3*yr.* 10*mo.* 4*da.* **(475.)** 54*yr.* 1*mo.* 26*da.* **(476.)** 74*yr.* 4*mo.* 28*da.* **(477.)** £83 4*s.* 2*d.* **(478.)** 36*cwt.* 3*qr.* 19*lb.* 5*oz.* **(479.)** 320*A.* 1*R.* 15*P.* **(480.)** 5*yr.* 3*mo.* 7*da.*; 1925*da.* **(481.)** 5*cwt.* 3*qr.* 9*lb.* **(482.)** 20*yd.* 2*qr.* 3*na.* **(483.)** 70*C.* 97*s. ft.* **(484.)** 97*bu.* 2*pk.* 2*qt.* 1*pt.* **(485.)** 11186*lb.*=5·593*T.* **(486.)** 45*rd.* $6\frac{1}{3}$*ft.* **(487.)** 124*yd.* 3*qr.* 1*na.* **(488.)** 233 cubic feet=14*c. ft.*, 9 cubic *ft.*=1*C.* 6*c. ft.* 9*cubic ft.* **(489–491.)** 3*yr.* 7*mo.* 27*da.*; 118*yr.* 10*mo.* 9*da.*; 115*yr.* 2*mo.* 12*da.* **(492.)** 286*yr.* 8*mo.* 8*da.* **(493.)** 123*yr.* 2*mo.* 13*da.* **(494.)** 13° 38′ 30″. **(495–496.)** 10° 45′ 10″; 2° 53′ 20″. **(497–499.)** 1° 38′ 43″; 12° 23′ 53″; 1° 14′ 37″. **(500.)** March 8th, 1502. **(501.)** 128*yr.* 2*mo.* 6*da.*

§ **90.** **(502–507.)** £31 12*s.* 6*d.*; £52 14*s.* 2*d.*; £63 5*s.*;

£73 15*s.* 10*d.*; £84 6*s.* 8*d.*; £94 17*s.* 6*d.* **(508–514.)** 24*cwt.* 0*qr.* 6*lb.* 12*oz.* 15*dr.*; 32*cwt.* 0*qr.* 9*lb.* 1*oz.* 4*dr.*; 40*cwt.* 0*qr.* 11*lb.* 5*oz.* 9*dr.*; 48*cwt.* 0*qr.* 13*lb.* 9*oz.* 14*dr.*; 56*cwt.* 0*qr.* 17*lb.* 14*oz.* 3*dr.*; 64*cwt.* 0*qr.* 20*lb.* 2*oz.* 8*dr.*; 72*cwt.* 0*qr.* 22*lb.* 6*oz.* 13*dr.* **(515–523.)** 24*cwt.* 0*qr.* 18*lb.* 1*oz.* 5*dr.*; 32*cwt.* 0*qr.* 24*lb.* 1*oz.* 12*dr.*; 40*cwt.* 1*qr.* 5*lb.* 2*oz.* 3*dr.*; 48*cwt.* 1*qr.* 11*lb.* 2*oz.* 10*dr.*; 56*cwt.* 1*qr.* 17*lb.* 3*oz.* 1*dr.*; 64*cwt.* 1*qr.* 23*lb.* 3*oz.* 8*dr.*; 72*cwt.* 2*qr.* 4*lb.* 3*oz.* 15*dr.*; 88*cwt.* 2*qr.* 16*lb.* 4*oz.* 13*dr.*; 96*cwt.* 2*qr.* 22*lb.* 5*oz.* 4*dr.* **(524–529.)** 296*gal.* 7*qt.* 1*pt.* 1*gi.*; 356 *gal.* 2*qt.* 1*pt.* 2*gi.*; 254*gal.* 4*qt.* 0*pt.* 2*gi.*; 458*gal.* 1*qt.* 0*pt.* 2*gi.*; 687*gal.* 1*qt.* 1*pt.* 3*gi.*; 415*gal.* 5*qt.* 1*pt.* 3*gi.* **(530.)** 32*cwt.* 1*qr.* 15*lb.* **(531.)** 54*yd.* 2*qr.* 3*na.* **(532.)** 14*C.* 119*s. ft.* **(533.)** 4*da.* 5*hr.* 4*min.* 30*sec.* **(534.)** 1*mi.* 255*ft.* 10*in.* **(535.)** 673141*da.* 10*hr.* 44*min.* $28\frac{1}{2}$*sec.* **(536.)** £27 2*s.* 6*d.* **(537.)** 10812*gal.* 1*qt.* 1*pt.* **(538.)** 23*C.* 5*c. ft.* **(539.)** 3361*ft.* $1\frac{1}{3}$*in.* **(540.)** £17 13*s.* 3*d.* **(541.)** £35 9*s.* 10*d.*

§ **91.** **(542–548.)** 8*yd.* 2*qr.* $1\frac{2}{3}$*na.*; 6*yd.* 1*qr.* $3\frac{1}{4}$*na.*; 5*yd.* 0*qr.* $2\frac{3}{5}$*na.*; 4*yd.* 1*qr.* $\frac{5}{6}$*na.*; 3*yd.* 2*qr.* 3*na.*; 3*yd.* 0*qr.* $3\frac{5}{8}$*na.*; 2*yd.* 3*qr.* $1\frac{8}{9}$*na.* **(549–562.)** 13*cwt.* 3*qr.* 22*lb.* 14*oz.* $12\frac{1}{2}$*dr.*; 9*cwt.* 1*qr.* 6*lb.* 15*oz.* 3*dr.*; 6*cwt.* 3*qr.* 23*lb.* 15*oz.* $6\frac{1}{4}$*dr.*; 5*cwt.* 2*qr.* 9*lb.* 2*oz.* $11\frac{2}{5}$*dr.*; 4*cwt.* 2*qr.* 15*lb.* 15*oz.* $9\frac{1}{2}$*dr.*; 3*cwt.* 3*qr.* 24*lb.* 6*oz.* $8\frac{1}{7}$*dr.*; 3*cwt.* 1*qr.* 24*lb.* 7*oz.* $11\frac{1}{8}$*dr.*; 3*cwt.* 0*qr.* 10*lb.* 10*oz.* $6\frac{1}{3}$*dr.*; 1*cwt.* 0*qr.* 3*lb.* 8*oz.* $12\frac{7}{9}$*dr.*; 1*qr.* 19*lb.* 8*oz.* $5\frac{10}{21}$*dr.*; 1*qr.* 9*lb.* 8*oz.* $4\frac{7}{27}$*dr.*; 1*qr.* 18*lb.* 10*oz.* $15\frac{25}{64}$*dr.*; 2*qr.* 16*lb.* 9*oz.* $1\frac{5}{14}$*dr.*; 2*qr.* 7*lb.* 0*oz.* $14\frac{43}{49}$*dr.* **(563–567.)** 9*oz.* 18*pwt.* $3\frac{12}{13}$*gr.*; 6*oz.* 15*pwt.* $14\frac{1}{19}$*gr.*; 5*oz.* 12*pwt.* $\frac{3}{23}$*gr.*; 4*oz.* 8*pwt.* $19\frac{28}{29}$*gr.*; 4*oz.* 3*pwt.* $2\frac{13}{31}$*gr.* **(568.)** 4*mi.* 2*fur.* 39*rd.* 3*yd.* 0*ft.* $7\frac{7}{23}$*in.* **(569–572.)** 39*gal.* 6*pt.*; 1*A.* 0*R.* 1*P.*; 1*gi.*; £3 13*s.* 4*d.* **(573.)** 365*da.* 5*hr.* 48*min.* 48*sec.* **(574.)** 15*ft.* 2*in.* **(575.)** 7*s.* 6*d.* **(576.)** 15*s.* 3*d.* **(577.)** 2*yd.* 1*qr.* 2*na.* **(578.)** 16*rd.* before, and 1*fur.* 20*rd.* after. **(579.)** 9*lb.* 1*oz.* 14*pwt.* 5*gr.* **(580.)** 2*bu.* 3*pk.* $3\frac{3}{7}$*qt.* **(581.)** 72 reams, 6 quires, and 2 sheets. **(582.)** The widow had £856 13*s.* 4*d.*, and each child had £244 15*s.* 2*d.* $3\frac{3}{7}$*far.* **(583.)** 16*A.* 3*R.* 8*P.*, 21*A.*, 21*A.*, 42*A.* **(584.)** 6*cwt.* 2*qr.* 16*lb.*, 2*cwt.* 0*qr.* 24*lb.*, 1*cwt.* 0*qr.* 12*lb.*, 3*cwt.* 1*qr.* 8*lb.* **(585.)** 3*oz.* 3*pwt.* $8\frac{3}{4}$*gr.* **(586.)** 1*lb.* 3*oz.* 3*pwt.* $15\frac{3}{11}$*gr.* **(587.)** 3424·95 *gr.* of gold, 190·275*gr.* of silver, 190·275*gr.* of copper. **(588.)** \$248·062. **(589.)** \$15·515. **(590.)** 27*yr.* 6*mo.* 3*wk.* 2*da.* 23*hr.* 42*min.* 30$\frac{1}{7}$*sec.* **(591.)** 4*yr.* 7*mo.* 17*da.* in Southern States; 3*yr.* 6*mo.* 14*da.* in Western States; 2*yr.* 7*mo.* 24*da.* in Northern States; 8*mo.* 20*da.* in Middle States: 11*yr.* 6*mo.* 15*da.* in all. **(592.)** 365242*da.* 5*hr.* 13*min.* 8*sec.* **(593.)** 6939·55*da.* **(594.)** 13° 10′ 34″·8+. **(595.)** £43 10*s.* 0*d.* 1*qr.* **(596.)** 3506*lb.* 9*oz.* **(597.)** 11·857+ miles. **(598.)** 989·0833+ miles. **(599.)** 1037$\frac{11}{24}$ miles. **(600.)** 86687500*gal.*

§ **92.** **(601.)** 103*f.* 1′ 3″. **(602.)** 2224*f.* 0′ 3″ 5‴. **(603.)** 31*f.* 9′ 9″. **(604.)** 547*f.* 3′ 8″. **(605.)** 51*f.* 6′ 7″. **(606.)** 10*f.* 4′ 7″. **(607.)** 6*f.* 8′ 8″. **(608.)** 41*f.* 9′ 11″. **(609–612.)** 51*f.* 1′ 2″; 24*f.* 9′ 4″; 131*f.* 8′ 7″; 67*f.* 10′ 11″. **(613–615.)** 88*f.* 0′ 11″; 76*f.* 1′ 10″; 64*f.* 2′ 9″. **(616–619.)** 61*f.* 1′ 1″; 33*f.* 1′ 1″ 2‴; 121*f.* 2′ 1″ 11‴; 82*f.* 7′ 7″ 9‴. **(620.)** 24*f.* 1′ 10″ 6‴.

§ **93.** **(621.)** 27*f.* 0′ 7″ 9‴ 6⁗. **(622–623.)** 48*f.* 8′ 7″; 28*f.* 3′ 11″ 2‴. **(624.)** 21*f.* 1′ 9″. **(625.)** 394*f.* 2′ 9″. **(626.)** 3978*f.* 1′ 6″. **(627.)** 24*yd.* 6*f.* 11′ 11″. **(628.)** 38$\frac{25}{36}$*yds.* **(629.)** 266*ft.* **(630.)** \$34·32+. **(631.)** 448 *cubic feet,* 7′ 4″. **(632.)** 4610$\frac{2}{3}$ *cubic ft.* **(633.)** 165*sq. ft.* **(634.)** \$10·903+. **(635.)** \$19·998+. **(636.)** 562*f.* 4′ 11″ 8‴ 1⁗; 13337*f.* 9′ 9″ 2‴ 1⁗ 4′′′′′ 7′′′′′′. **(637.)** 31590 bricks.

§ **94.** **(638–640.)** 2*f.* 3′; 1*f.* 1′ 6″; 4*f.* 6′. **(641.)** 2*f.* 11′. **(642.)** 1*f.* 11′. **(643.)** 8*f.* **(644.)** 80*f.* 8′. **(645.)** 5*f.* 5′.

§ **95.** **(646–648.)** 2*d.* 2$\frac{3}{5}$*far*; 6*qt.* 1$\frac{1}{3}$*pt.*; 19*hr.* 5*min.* 51$\frac{3}{7}$*sec.* **(649–650.)** 1*ft.* 10$\frac{3}{8}$*in.*; 15*s.* 10*d.* 2$\frac{62}{105}$*qr.* **(651–652.)** 3*da.* 16*hr.* 15*min.*; 75*da.* 2*hr.* **(653.)** 2*qr.* 8*lb.* 9*oz.* 5$\frac{1}{3}$*dr.* **(654.)** 5$\frac{1}{30}$*qt.* **(655.)** 7$\frac{3}{5}$*in.* **(656.)** 4*da.* 21*hr.* 8*min.* **(657.)** 3*pk.* 0*qt.* $\frac{2}{7}$*pt.* **(658.)** \$10·15$\frac{5}{8}$.

§ **96.** **(659–660.)** 8*hr.* 59*min.* 48*sec.*; 18*hr.* 57*min.* 29$\frac{53}{143}$*sec.* **(661.)** 4*yd.* 3*qr.* 3$\frac{11}{15}$*na.* **(662.)** 2*R.* 4$\frac{2}{7}$*P.* **(663.)** 7*pwt.* 15*gr.*

(664.) 6*gal.* 3*qt.* $\frac{2}{3}$*pt.* **(665.)** 2*A.* 1*R.* 10$\frac{5}{12}$*P.* **(666.)** 26*gal.* 1$\frac{1}{6}$*pt.* **(667.)** 387·29$\frac{1}{6}$ days of 10*hr.*, counting 365 days to the year. **(668.)** 3*s.* 2*d.* 3$\frac{1}{7}$*far.* **(669.)** 3*da.* 20*hr.* 15*sec.* **(670.)** 2*oz.* 13*pwt.* 3$\frac{3}{7}$*gr.* **(671.)** 10*cts.* **(672.)** 46$\frac{6}{35}$*cts.* **(673.)** $\frac{48}{360}$, $\frac{28}{360}$, $\frac{21}{360}$, $\frac{263}{360}$. **(674.)** 8*hr.* 30*min.* **(675.)** 3*da.* 9*hr.* 11*min.* 40*sec.* **(676.)** $112. **(677.)** $200, $120, $880. **(678.)** Each received 576 balls. **(679.)** 10*lb.* 11*oz.* 6$\frac{6}{7}$*dr.*, 11*lb.* 2*oz.* 9$\frac{1}{7}$*dr.*, 11*lb.* 11*oz.* 8*dr.*, 12*lb.* 7*oz.* 3$\frac{1}{2}$*dr.*, 53*lb.* 15*oz.* 4$\frac{1}{2}$*dr.* **(680–681.)** Each son received 28*A.* 3*R.* 36$\frac{1}{2}$*P.* There remained with the father 86*A.* 3*R.* 28$\frac{1}{4}$*P.* **(682.)** 5*da.* 6*hr.* **(683.)** 6*s.*10$\frac{1}{3}$*d.* **(684.)** 6*lb.* 2$\frac{4}{13}$*oz.* **(685.)** 274*da.* 12*hr.* 30*min.* **(686.)** 39*mi.* 5$\frac{11}{21}$*fur.* **(687.)** 1*s.* 2$\frac{77}{298}$*d.* **(688–691.)** Coat £7 9*s.*; pantaloons £2 9*s.* 8*d.*; hat £1 4*s.* 10*d.* At first he had £14 18*s.* **(692.)** $1·50. **(693.)** 5988$\frac{8}{9}$ feet. **(694.)** 44·24719+ inches. **(695.)** 12*rd.* 44*ft.* 6*in.* **(696.)** 9$\frac{1}{2}$*in.* **(697.)** 2*yd.* 2*na.* 1$\frac{1}{5}$*in.* **(698.)** 837$\frac{3}{8}$ cubic inches. **(699.)** 223$\frac{1}{4}$ cubic inches. **(700.)** 3763·235 cubic inches.

CHAPTER XII.

§ **97.** What is the principle of percentage? Does percentage apply to money alone? Are common fractions used in computations of percentage? Why not? What is the *rate*? How do you write a rate that is less than 1 per cent.? How, a rate greater than 100 per cent.? What rate does $\frac{100}{100}$ express? How is the fractional part of a number to be obtained? Give the general rule for obtaining the percentage.

§§ **98, 99.** What do you understand by applications of percentage? What is commission? Give an example. What is brokerage? What are stocks? What is the par value? Give an example of stock below par; above par. When will you prefer to buy? when to sell? What are dividends? Who are meant by stockholders? What is the difference between above par and *at a discount*?

§ **100.** If you were an assessor, state how you would proceed in laying taxes.

§ **101.** What do you understand by custom-house business? What officer of the United States government has the control of this department of public affairs? How are the expenses of the administration of government paid? What is revenue? What are duties? What is the difference between specific and ad valorem duties? Wherein does net weight differ from gross weight? Before computing duties, what allowances are made? What is draft? tare? leakage? breakage? What is the legal allowance for draft? What is an invoice?

§ **102.** State what you know concerning insurance. What is the premium? What is meant by indemnity? What is the policy? Are ships only insured?

§ **103.** What is meant by Profit and Loss? How will you ascertain the *amount* of gain or loss in a given transaction? How will you ascertain the *per cent.* gain or loss? How will you ascertain the *amount per yard*, per bale, per pound, per piece, &c., gain or loss? How will you determine the *cost* per yard, &c., knowing the sum for which it sold, and the gain or loss per cent.?

§ **104.** What is Interest? Why called simple? Wherein does interest differ from percentage? What has law to do with fixing the rate per cent. in any transaction? How do you find the interest, if principal, time, and rate are given? How if the rate be 6 per cent.? How if the rate be *any* per cent.? If your principal consist of pounds, shillings, pence, &c., what must be done? Explain aliquot parts. Give an example.

§ **105.** Show a more accurate method of finding the interest when the time is given in days.

§ **106.** What are partial payments? What is the U. S. rule for computing interest?

§ **107.** I give you the principal, the rate per cent., and the time; how will you find the *interest?* I give you the time, the rate, and the interest; how will you find the *principal?* I give you the principal, the time, and the interest; how will you find the *rate?* I give you the principal, the rate, and the interest; how will you find the *time?* I give you the time, rate, and amount; how will you find the *principal?*

§ **108.** Wherein does discount differ from the present worth?

§ **109.** What is compound interest? Must interest always be added annually to be compounded?

§ **110.** What is a Bank? How is it established? Relate all you know about it. What is specie? What are the officers of a bank called, and what are their duties? Tell all you can about notes of hand; why they are given; by whom, &c. What is the meaning of negotiable? Under what circumstances are notes of hand negotiable? What is the amount called for which the note is given? What do you understand by the proceeds of a note?

§ **111.** What is the precise difference between bank discount and discount? Which is the more equitable? Why? When the rate per cent. is not specified in a note, what rate is understood?

§ **112.** If I give you the present worth of a bankable note, the time and rate, how will you find the face of the note? Must interest on a bankable note be computed for the exact time specified in the note?

ANSWERS.

§ **97.** **(1–8.)** 0·02; 0·08; 0·12; 0·5; 1·06; 1·4; 2·6; 18. **(9–17.)** 0·09; 0·24; 0·99; 1·04; 6·07; 72·81; 0·005; 0·025; 0·35. **(18–25.)** 0·005; 0·0025; 0·0066$\frac{2}{3}$; 0·009; 0·00375; 0·00833$\frac{1}{3}$; 0·0033$\frac{1}{3}$; 0·004. **(26–32.)** 0·025; 0·0333$\frac{1}{3}$; 0·058; 0·36142857+; 2·2025; 5·00375; 9·4675. **(33–37.)** \$56·10; \$4768·08; \$623·875; \$2765·40984; \$313111·8945. **(38–47.)**

$4938·17185; $24690·85925; $2469·085925; $32098·117025; $3703·6288875; $62961·6910875; $246908·5925; $4069053·6044; $18765053·03; $424682·7791. **(48–53.)** 1·52; 15·52; 35; 239·4; 0·9125; 0·345. **(54.)** 173·67 pounds. **(55.)** $3·37635. **(56.)** 164·15. **(57.)** $148·50. **(58.)** $46·62 for calicoes; $33·30 for thread; $99·90 for silks; $42·18 for broadcloths. **(59.)** 125 barrels the first time, 75 the second time, and 300 remaining. **(60.)** 81 per cent. **(61.)** 13·8415*lb.*=13*lb.* 10*oz.* 1*pwt.* $23\frac{1}{25}$*gr.* **(62.)** After the 1st stroke 90 per cent.; after the 2d, 81 per cent.; after the 3d, $72\frac{9}{10}$ per cent.; after the 4th, $65\frac{61}{100}$ per cent. **(63.)** $6·50. **(64.)** $1620. **(65.)** $200. **(66.)** 40·32*gal.* **(67.)** $79725·508.

§ **99.** **(68.)** $262·50. **(69.)** $405. **(70.)** $144·585. **(71.)** $103·50. **(72.)** $15802·50. **(73.)** $3500. **(74.)** 200 bales. **(75.)** $14750. **(76.)** $8800. **(77.)** $14113·71. **(78–81.)** $2316·4375; $2640·5275; $4370·1645; $9327·1293. **(82–84.)** $3900; $5070; $6591. **(85.)** $7993·1578+. **(86.)** $902·50. **(87.)** 48 per cent. **(88.)** $89·91. **(89.)** $420. **(90.)** 88 trees.

§ **100.** **(91.)** $6·375. **(92.)** $12·93. **(93.)** $21. **(94.)** $5·625. **(95.)** $28·125. **(96.)** $ 6·50. **(97.)** $1360. **(98.)** $9863·473. **(99–100.)** $5528543·0658; $2064877·5306. **(101–103.)** $21; $75·90; $3. **(104.)** $5203·0625. **(105.)** $3617·5788.

§ **101.** **(106.)** $318·40704. **(107.)** $346. **(108.)** $316·7262. **(109.)** $2162·8464. **(110.)** $8·876955. **(111.)** $31·80. **(112.)** $57·9963. **(113.)** $1020. **(114.)** $4692·60. **(115.)** $78·3275. **(116.)** $132. **(117.)** $1276·80. **(118.)** $685. **(119.)** $114·66. **(120.)** $188·16. **(121.)** $78·40. **(122.)** $107·016. **(123.)** $33·07425. **(124.)** $74·088. **(125.)** $240·2355. **(126.)** $874·875. **(127.)** $30. **(128.)** $2572·50. **(129.)** $249·704. **(130.)** 2465·75.

§ **102.** **(131.)** $7·38. **(132.)** $240. **(133.)** $19·6125. **(134.)** $207·402. **(135.)** $1043·325. **(136–140.)** $150;

$225; $318·75; $375; $412·50. (**141–142.**) $12·60; $13·44. (**143.**) $85. (**144.**) $136·50. (**145.**) $169. (**146.**) $368. (**147.**) $82·50. (**148–150.**) $240; $360; $450.

§ **103.** (**151.**) $375. (**152.**) $1440. (**153.**) $78·125. (**154.**) $14\frac{2}{7}$ per cent. (**155.**) 10 per cent. (**156.**) $0·14. (**157.**) About 34 per cent. (**158.**) $4·37$\frac{1}{2}$. (**159.**) $2·625. (**160.**) $1120. (**161.**) 20 per cent. (**162.**) 25 per cent. (**163–164.**) 20 per cent.; 25 per cent. (**165.**) $46\frac{2}{13}$ per cent. (**166.**) $31\frac{11}{19}$ per cent. (**167.**) $4000. (**168.**) $1000. (**169.**) $100. (**170.**) $9\frac{63}{193}$ per cent. (**171.**) $4410. (**172.**) $2284·375. (**173.**) $1102·50. (**174.**) $902·50. (**175–176.**) £2 3*s.* 0*d.* $3\frac{3}{25}$*far.*; 99 per cent. (**177–178.**) $3517·50; $11\frac{2}{3}$ per cent. (**179.**) $2000. (**180.**) $1736·84$\frac{4}{19}$.

§ **104.** (**181–185.**) $8·10; $29·40; $83·85; $9936·003; $2087487·0105. (**186–190.**) $61·88; $129·22; $898·5431; $595323·729; $8565702·60637. (**191–197.**) $6958·257; $8349·9084; $9741·5598; $50099·4504; $62624·313; $93936·4695; $150298·3512. (**198–203.**) $7064·7624; $11774·604; $15699·472; $21979·2608; $27317·08128; $32380·161. (**204–210.**) $226·875; $268·125; $309·375; $350·625; $391·875; $474·375; $556·875. (**211.**) $3·332475. (**212.**) $6390·51. (**213.**) $143·786295. (**214.**) $100. (**215.**) $151·402185. (**216.**) $0·002747. (**217.**) $0·2625. (**218.**) $89·969925. (**219.**) $12·53968. (**220.**) $99·412$\frac{5}{8}$. (**221.**) $4·50205. (**222–224.**) $4998·75; $5332; $5235·90. (**225–227.**) $381·85; $534·59; $409·125. (**228–230.**) $210; $181; $260. (**231.**) $2·200339$\frac{1}{6}$. (**232.**) $92·75. (**233.**) $1·73272. (**234.**) $91·575. (**235.**) $183·55. (**236.**) $49·2005. (**237.**) $0·36153. (**238.**) $0·7518825. (**239.**) $89·65. (**240.**) $85·75. (**241.**) £15 10*s.* $3\frac{21}{100}$*d.* (**242.**) £4 10*s.* $7\frac{72}{100}$*d.*, nearly. (**243.**) £0 3*s.* $1\frac{1}{8}$*d.* (**244.**) £4 3*s.* 6*d.* 1·95*far.* (**245–247.**) $507·695$\frac{5}{6}$; $508·535; $509·374$\frac{1}{6}$. (**248–250.**) $407·729145625; $408·58990375; $408·3986241$\frac{2}{3}$. (**251–254.**) $3·993465; $68·70948; $1000·1755+; $2790·0593+. (**255–257.**) $2·1169+; $2·8729+; $4·38505+. (**258–259.**) $5·621175;

$111·941685. **(260–262.)** $3·57; $761·626576$\frac{2}{3}$; $65185·767+. **(263–265.)** $40·421+; $80·842+; $29748·866+. **(266.)** $4·94395. **(267.)** $4·9856125. **(268.)** $1·28646+. **(269–271.)** $4984·541$\frac{2}{3}$; $5382·375; $5745·85. **(272–275.)** $344·4979+; $348·409+; $356·2313+; $360·1423+.

§ **105.** **(276.)** $413·79. **(277.)** $5·047. **(278.)** $7·871. **(279.)** $1·703. **(280.)** $10·356. **(281.)** $20·88. **(282.)** $1·75. **(283–284.)** $8·601+; $7·289+. **(285–287.)** $6·415+; $2·742+; $3·134+. **(288–290.)** $0·996+; $1·195+; $1·395+.

§ **106.** **(291.)** $251·62. **(292.)** $252·12. **(293.)** $5·844. **(294.)** $5·80. **(295.)** $60·866. **(296.)** $60·72. **(297.)** $144·404. **(298.)** $143·55.

§ **107.** **(299.)** $29·03. **(300.)** $100. **(301.)** $40. **(302.)** $530. **(303.)** $4090·909. **(304.)** $79500. **(305.)** $14133·33$\frac{1}{3}$. **(306.)** $39328·57$\frac{1}{7}$. **(307.)** 4$\frac{1}{2}$ per cent. **(308.)** 5 per cent. **(309.)** 7 per cent. **(310.)** 8 per cent. **(311.)** 9 per cent. **(312.)** 5*mo.* 12*da.* **(313.)** 1*yr.* 9*mo.* **(314.)** 16*yr.* 8*mo.* **(315–329.)** 20*yr.*; 16$\frac{2}{3}$*yr.*; 14$\frac{2}{7}$*yr.*; 12$\frac{1}{2}$*yr.*; 11$\frac{1}{9}$*yr.*; 10*yr.*; 9$\frac{1}{11}$*yr.*; 8$\frac{1}{3}$*yr.*; 18$\frac{2}{11}$*yr.*; 15$\frac{5}{13}$*yr.*; 13$\frac{1}{3}$*yr.*; 11$\frac{13}{17}$*yr.*; 10$\frac{10}{19}$*yr.*; 9$\frac{11}{21}$*yr.*; 8$\frac{16}{23}$*yr.* **(330–333.)** 3*yr.* 5*mo.* 12*da.* nearly; 2*yr.* 9*mo.* 3*da.*+; 2*yr.* 3*mo.* 18*da.* nearly; 1*yr.* 11*mo.* 20*da.* nearly. **(334–336.)** 43*yr.* 7*mo.* 26*da.*+; 31*yr.* 2*mo.* 4*da.*+; 26*yr.* 10*mo.* 12*da.* nearly. **(337.)** $530. **(338.)** $4070. **(339.)** $5·37. **(340.)** $1437·227. **(341.)** $1437·126. **(342.)** $49·14. **(343.)** $2970·011. **(344.)** $1165·21+. **(345.)** $10269·146+.

§ **108.** **(346.)** $2·913. **(347.)** $37·411. **(348.)** $2·652. **(349.)** $86·713. **(350.)** $1·265. **(351–353.)** $28·35+; $32·92+; $37·44. **(354–356.)** $385·66+; $459·47+; $495·98+. **(357–359.)** $58·58+; $42·38+; $46·46+. **(360–361.)** $76·27+; $86·77+.

§ **109.** **(362.)** $112·55+. **(363.)** $135·769. **(364.)** $57·881.

(365.) \$262·477. **(366–368.)** \$1913·36+; \$1915·04+; \$1910·12. **(369–370.)** \$43·75+; \$44·04+.

§ **111.** **(371.)** \$18·085. **(372–376.)** \$4·575; \$7·438; \$21·23$\frac{1}{3}$; \$0·867; \$27·809. **(377.)** \$1781·10. **(378.)** \$6598·48. **(379.)** \$832·89+. **(380–383.)** \$643·175; \$832·81+; \$67·62+; \$4214·398+. **(384–385.)** \$7·23$\frac{1}{3}$; \$6·879+. **(386–389.)** \$21·35; \$5·425; \$20·29 nearly; \$5·16 nearly. **(390.)** \$24·15.

§ **112.** **(391.)** \$609·446. **(392.)** \$50·62. **(393.)** \$1043·932. **(394.)** \$73·546. **(395.)** \$107·594. **(396.)** \$376·483. **(397.)** \$3338·75+. **(398–401.)** \$152·36+; \$304·72+; \$457·08+; \$507·87+. **(402–403.)** \$683·877+; \$682·667+. **(404.)** \$103·518+. **(405–407.)** \$609·446+; \$408·371+; \$309·438+. **(408–410.)** \$1026·079+; \$1031·46+; \$1036·896+.

CHAPTER XIII.

§ **113.** What do you understand by Analysis? Give an example. What is to be first determined in a question of Analysis?

§ **114.** What is Ratio? How may a question in analysis be answered by ratio? What does this ratio show with respect to one of the given quantities? Is it always a multiplier? What care is specially necessary in determining the ratio?

§ **115.** Is there a ratio between quantities of different denominations? How may such quantities be reduced so as to have a ratio?

§ **116.** What is Practice? Has the ratio employed in Practice a relation to any one of the given quantities, or to something else?

§ **117.** What do you understand by Currency? What is the currency of the State in which you live? What values have coins? Explain. How can the state of trade affect the value of coin? What is the legal value of the sovereign?

§§ **118, 119.** What is one method of reducing Sterling to Federal Money? Federal to Sterling Money?

§ **120.** What is the method of reduction of currencies by ratio? What is the ratio in this case? Is it fixed or variable? Why?

§ **121.** Give the ratios of the dollar in the several States to the pound sterling; of the pound sterling to the dollar. How is Federal Money to be converted into State or Canada currency? How these latter currencies into Federal Money?

§ **122.** What are the custom-house values of some of the chief current foreign coins? How may foreign coins be reduced to Federal Money? Federal Money to foreign coins?

ANSWERS.

§ **113.** (**1–9.**) 36 days; 24 days; 18 days; 9 days; 8 days; 6 days; 4 days; 3 days; 2 days. (**10–16.**) \$$22\frac{1}{2}$; \$27; \$30; \$$37\frac{1}{2}$; \$45; \$75; \$150. (**17–22.**) 40 days; 30 days; 24 days; 20 days; 12 days; 4 days. (**23–28.**) 120 hands; 80 hands; 40 hands; 24 hands; 20 hands; 15 hands. (**29.**) 384 times. (**30.**) \$441. (**31.**) \$130680. (**32.**) \$$948\frac{4}{7}$. (**33.**) \$$2613333\frac{1}{3}$. (**34–40.**) \$18; \$24; \$30; \$42; \$54; \$66; \$78. (**41–44.**) \$3; \$3; \$1; \$2. (**45–47.**) 16 men; 12 men; 8 men. (**48–50.**) $6\frac{2}{3}$ miles; 8 miles; 10 miles.

§ **115.** (**51–55.**) $\frac{2}{3}$; $\frac{1}{4}$; $\frac{1}{12}$; $\frac{1}{33}$; $\frac{1}{10560}$. (**56–60.**) $\frac{7}{96}$; $\frac{7}{114}$; $\frac{7}{216}$; $\frac{7}{720}$; $\frac{14}{1013}$. (**61.**) $\frac{1}{25}$. (**62–65.**) $\frac{7}{8}$; $\frac{14}{9}$; $\frac{14}{11}$; $\frac{14}{15}$. (**66.**) $\frac{37}{22000}$. (**67.**) $\frac{64}{7425}$. (**68.**) $\frac{53}{7000}$. (**69.**) $\frac{1}{330}$. (**70.**) $\frac{1}{120}$. (**71.**) $\frac{37}{128}$. (**72.**) $\frac{10167}{16000}$. (**73.**) Silver and copper each $\frac{1}{18}$ of the gold; silver and copper together $\frac{1}{9}$ of the gold. (**74.**) $\frac{1}{9}$. (**75.**) $\frac{43}{12375}$. (**76.**) $\frac{144}{175}$. (**77.**) $\frac{924}{1109}$. (**78.**) $\frac{109}{450}$. (**79–83.**) \$1755; \$450; \$675; \$1080; \$1485. (**84–88.**) \$210; \$450; \$1350; \$2160; \$3510. (**89.**) \$1290. (**90.**) \$$537\frac{3}{5}$. (**91.**) $\frac{1}{4}$ of a week. (**92–94.**) 30 men; 300 men;

18000 men. **(95.)** 80 feet. **(96.)** 270 feet. **(97.)** $94\frac{1}{2}$ feet. **(98.)** $\$221\frac{1627}{2185}$. **(99.)** $\frac{594}{1591}$ of a day. **(100–106.)** \$0·125; \$0·25; \$0·4375; $\$0·541\frac{2}{3}$; $\$0·708\frac{1}{3}$; \$0·9375; \$1·125. **(107–112.)** 4 barrels; $9\frac{11}{21}$ barrels; 16 barrels; $10\frac{1}{2}$ bushels; 63 bushels; 105 bushels. **(113–121.)** 33 feet; 66 feet; 99 feet; $247\frac{1}{2}$ feet; 330 feet; $412\frac{1}{2}$ feet; 20 rods; 4 rods; 8 rods. **(122–126.)** 15 men; 10 men; 6 men; 25 men; 5 men. **(127–137.)** \$1·75; $\$2·91\frac{2}{3}$; $\$4·08\frac{1}{3}$; $\$6·41\frac{2}{3}$; $\$9·91\frac{2}{3}$; \$21; \$35; \$49; $\$142·85\frac{5}{7}$; $\$257·14\frac{2}{7}$; $\$342·85\frac{5}{7}$. **(138–141.)** \$56000; \$28000; \$21000; \$35000. **(142–150.)** 9*sec.*; 15*sec.*; 45*sec.*; 25; 65; 250; 375; 85; 185.

§ **116.** **(151.)** \$326·25. **(152.)** $\$4·43\frac{1}{3}$. **(153.)** \$20·625. **(154.)** \$3·1625. **(155.)** \$11·26125. **(156.)** $\$6·015\frac{5}{8}$. **(157.)** \$1·50. **(158.)** $\$11·81\frac{1}{4}$. **(159.)** \$9. **(160.)** $\$24·28\frac{1}{8}$. **(161.)** $\$7·81\frac{1}{4}$. **(162.)** \$6·75. **(163.)** $\$154·37\frac{1}{2}$. **(164.)** $\$68·62\frac{1}{2}$. **(165.)** $\$2·79\frac{1}{6}$. **(166.)** $\$8·66\frac{2}{3}$. **(167.)** $\$3·93\frac{3}{4}$. **(168.)** $\$121·87\frac{1}{2}$. **(169.)** $\$61·87\frac{1}{2}$. **(170.)** $\$3·36\frac{7}{8}$. **(171.)** $\$5·16\frac{2}{3}$. **(172.)** $15\frac{3}{20}$ miles. **(173.)** $\$2·52\frac{7}{9}$. **(174.)** \$10. **(175–180.)** 720 bushels; 1440 bushels; 2880 bushels; 1080 bushels; 1800 bushels; 2160 bushels. **(181–185.)** 192 beggars; 120 beggars; 96 beggars; 72 beggars; 60 beggars. **(186–190.)** \$150; \$25; $\$16·66\frac{2}{3}$; $\$8·33\frac{1}{3}$; $\$1·66\frac{2}{3}$. **(191–195.)** $266\frac{2}{3}$*yd.*; 240*yd.*; 160*yd.*; $145\frac{5}{11}$*yd.*; $138\frac{6}{13}$*yd.* **(196–200.)** $39666\frac{2}{3}$ miles; 51000 miles; $62333\frac{1}{3}$ miles; $107666\frac{2}{3}$ miles; $147333\frac{1}{3}$ miles.

§ **122.** **(201–210.)** £1; £4 1*s.*+; £6 14*s.* 2*d.* $2\frac{1}{2}$*far.*+; £12 3*s.* 9*d.* $2\frac{1}{2}$*far.* nearly; £181 1*s.* 10*d.* $1\frac{1}{4}$*far.*+; £5 12*s.* 3*d.* 3*far.*+; £261 4*s.* 7*d.* $1\frac{3}{4}$*far.*+; £4565 6*s.* 10*d.* 3*far.*+; £92352 17*s.* 2*d.* 1*far.*+; £572089 8*s.* 3*d.* 0*far.*+. **(211–218.)** £1 16*s.*; £5 3*s.* 1*d.* 3*far.*+; £16 8*s.* 1*d.* 2*far.*+; £147 1*s.* 3*d.* 2*far.*+; £1794 14*s.* 1*d.* 3*far.*+; £16477 5*s.*+; £18458 9*s.* 6*d.* 2*far.*+; £44700 11*s.* 7*d.* 1*far.*+. **(219–234.)** £1 15*s.* 8*d.*+; £5 2*s.* 2*d.* 2*far.*+; £16 5*s.* 2*d.*+; £145 14*s.* 6*d.* 3*far.*+; £1778 7*s.* 10*d.*+; £16327 9*s.* 1*d.* 3*far.*+; £18290 13*s.* 5*d.* 2*far.*+; £44294 4*s.* 2*d.* 3*far.*+; £1 16*s.* 1*d.* 3*far.*+; £5 3*s.* 7*d.* 2*far.*+;

£16 9*s.* 7*d.* 3*far.*+; £147 14*s.* 10*d.* 1*far.*+; £1802 19*s.* 6*d.* 2*far.*+; £16553 3*s.* 7*d.* 3*far.*+; £18543 10*s.* 9*d.* 1*far.*+; £44906 11*s.* 5*d.* 3*far.*+. **(235–240.)** $44·77; $131·50+; $188·94+; $1310·379+; $20234·265+; $336283·38+. **(241–258.)** $44·40; $130·41+; $187·38; $1299·55 nearly; $20067·039+; $333504·144; $44·811+; $131·622+; $189·115; $1311·582+; $20252·845+; $336592·188+; $46·22$\frac{2}{9}$; $132·829+; $190·85+; $1323·615+; $20438·651+; $339680·17$\frac{1}{3}$. **(259–263.)** £25 1*s.* Canada; £23 7*s.* 7$\frac{1}{5}$*d.* Georgia; £30 1*s.* 2$\frac{2}{5}$*d.* New England; £37 11*s.* 6*d.* Pennsylvania; £40 1*s.* 7$\frac{1}{5}$*d.* New York. **(264–268.)** £9 6*s.* 10·2*d.* Canada; £8 14*s.* 4·72*d.* Georgia; £11 4*s.* 2·64*d.* New England; £14 0*s.* 3·3*d.* Pennsylvania; £14 18*s.* 11·52*d.* New York. **(269–273.)** £250 Canada; £233 6*s.* 8*d.* Georgia; £300 New England; £375 Pennsylvania; £400 New York. **(274–278.)** $303·10 Canada; $324·75 Georgia; $252·58$\frac{1}{3}$ New England; $202·06$\frac{2}{3}$ Pennsylvania; $189·43$\frac{3}{4}$ New York. **(279–283.)** $321·05 Canada; $343·982$\frac{1}{7}$ Georgia; $267·541$\frac{2}{3}$ New England; $214·03$\frac{1}{3}$ Pennsylvania; $200·656$\frac{1}{4}$ New York. **(284–288.)** $4000 Canada; $4285·714$\frac{2}{7}$ Georgia; $3333·33$\frac{1}{3}$ New England; $2666·66$\frac{2}{3}$ Pennsylvania; $2500 New York. **(289.)** 1755$\frac{45}{121}$ sovereigns. **(290.)** 11575$\frac{25}{93}$ five francs. **(291–295.)** 595$\frac{19}{120}$ Mexican doubloons; 1190$\frac{19}{60}$ ten thalers; £2321 2*s.* 4$\frac{1}{5}$*d.* Canada; 18568·94 rupees of Bengal; 11605·5875 ducats of Naples. **(296–301.)** $5904·675; $12740; $3659·625; $385660·88; $1342851·78; $369698·994. **(302.)** $84760·33$\frac{7}{9}$. **(303.)** £5113 15*s.* 9*d.* 2*far.*+. **(304.)** £1008 12*s.* 10*d.* 1*far.*+ New England. **(305.)** 26455$\frac{15}{62}$ five francs. **(306.)** $0·003388. **(307.)** $4·356. **(308.)** $29·04. **(309–314.)** 416$\frac{2}{3}$ ounces of Sicily; 1250 ducats of Naples; 2061$\frac{83}{97}$ florins of Augsburg; 1269$\frac{53}{63}$ rix dollars of Bremen; 64$\frac{4}{39}$ Mexican doubloons; 219$\frac{17}{57}$ Louis-d'ors. **(315–325.)** $15600; 14716$\frac{52}{53}$ crowns; 3223$\frac{17}{121}$ sovereigns; 14857$\frac{1}{7}$ specie-dollars of Denmark; 14716$\frac{52}{53}$ specie-dollars of Norway; 8472$\frac{38}{46}$ pagodas of India; 31200 rupees of Bengal; 13866$\frac{2}{3}$ milrees of Portugal; 44571$\frac{3}{7}$ mark bancos of Hamburg; 3120 English guineas; 83870$\frac{30}{31}$ francs.

CHAPTER XIV.

§ **123.** Explain a proportion. What names are given to the terms of a ratio? What of a proportion? Wherein does a ratio differ from a proportion? Whence did the Rule of Three derive its name? What is meant by known quantities? What by the unknown? What is a mean proportional? How is it found? What propositions are true with respect to the various terms of a proportion? Repeat the first form of the Rule of Three. Explain its principles. Repeat the second form of this rule. Explain the difference between the two rules. Show the connection between the second form and the method by analysis.

§ **124.** Explain a compound proportion. By how many processes may a question involving complex conditions be answered? Illustrate by an example.

§ **125.** What is Arbitration of Exchange? What principle does it involve?

§ **126.** What is Partnership? What principle lies at the foundation of operations under this head? How will you ascertain each partner's gain or loss?

§ **127** Wherein does Double Fellowship differ from partnership?

ANSWERS.

§ **123.** (**1.**) \$40·56. (**2.**) \$21. (**3.**) £3 16*s.* 6*d.* (**4.**) $52\frac{1}{7}$ weeks. (**5.**) $113\frac{1}{3}$ miles. (**6.**) $14\frac{2}{7}$ years. (**7.**) 320 yards. (**8.**) 25 men. (**9.**) 2000 pounds. (**10.**) $61\frac{3}{4}$ bushels. (**11.**) 11 seconds. (**12.**) 3600 times. (**13.**) $31\frac{1}{2}$ minutes. (**14.**) $8\frac{1}{6}$ days. (**15.**) $13014\frac{2}{5}$ pounds. (**16.**) $56\frac{1}{4}\frac{9}{7}\frac{7}{2}$ feet. (**17.**) $6\frac{7}{9}$*s.* (**18.**) \$104·$16\frac{2}{3}$. (**19.**) $23\frac{3}{4}$ miles. (**20.**) 4 hours. (**21.**) \$576. (**22.**) \$11. (**23.**) \$13. (**24.**) \$9510. (**25.**) \$1732·25. (**26.**) \$310. (**27.**) 550 bushels. (**28.**) $12\frac{5}{6}$ barrels. **(29.)** $33\frac{1}{3}$*yd.* **(30.)** 32 miles. **(31.)** \$$42·85\frac{5}{7}$. **(32.)** 14*da.* 4*hr.* **(33.)** $9\frac{3}{5}$ inches. **(34.)** 8 days. **(35.)** $45\frac{5}{8}$ yards. **(36.)** \$100. **(37.)** \$14400. **(38.)** \$$1166\frac{2}{3}$. **(39.)** $68104\frac{56}{1461}$ miles. **(40.)** $1037\frac{1}{2}$ miles. **(41.)** 60590592000000 miles. **(42.)** 6*hr.* 40*min.* **(43.)** \$625. **(44.)** 116*da.* $4\frac{4}{5}$*hr.* **(45.)** $\frac{1}{100}$ of an inch. **(46.)** \$$4·66\frac{2}{3}$. **(47.)** \$5·25. **(48.)** $17\frac{1}{7}$*mo.* **(49.)** 76032 years. **(50.)** 60 feet. **(51.)** $3\frac{1}{3}$ miles. **(52.)** 18*yd.* **(53.)** $29\frac{37}{487}$ degrees. **(54.)** $236\frac{1}{4}$ miles with current, and $168\frac{3}{4}$ miles against it. **(55.)** $\frac{808}{24899}$ times. **(56.)** $\frac{15}{16}$. **(57.)** Supply-pipe alone $8\frac{3}{4}$*da.*; supply-pipe and 1st discharging pipe $10\frac{1}{2}$*da.*; supply-pipe and 2d discharging pipe $11\frac{2}{3}$*da.*; all together 15*da.*

§ **124.** **(58.)** $441\frac{3}{17}$ miles. **(59.)** $711\frac{1}{9}$ pounds. **(60.)** 69 weeks. **(61.)** 30 men. **(62.)** $3291\frac{3}{7}$*yd.* **(63.)** 6 men. **(64.)** 11 men. **(65.)** 30 pounds. **(66.)** \$144. **(67.)** 500 men. **(68.)** $288\frac{59}{207}$ days. **(69.)** 200 men. **(70.)** \$67·50. **(71.)** \$14·40. **(72.)** 8 months. **(73.)** 6 per cent. **(74.)** \$360. **(75.)** \$19·20. **(76.)** 80 cows. **(77.)** 32*yd.* **(78.)** 8*mo.* **(79.)** 10 persons. **(80.)** \$50 dollars' worth.

§ **125.** **(81–83.)** \$$1899·39\frac{11}{12}$; \$$1816·66\frac{2}{3}$; \$$83·73\frac{1}{4}$. **(84–86.)** \$5398·65; \$$4755·55\frac{5}{9}$; \$$643·09\frac{4}{9}$. **(87.)** $36\frac{171}{704}$ pounds. **(88.)** 375 pears. **(89.)** $20\frac{152}{549}$ English guineas. **(90.)** $982\frac{8}{81}$ francs.

§ **126.** **(91.)** A.'s share \$100·80, B.'s \$109·20. **(92–94.)** A. \$200; B. \$$222·22\frac{2}{9}$; and \$$0·55\frac{5}{9}$ on the dollar. **(95.)** \$$138·46\frac{2}{13}$, \$$161·53\frac{11}{13}$. **(96.)** \$11·25, \$18·75. **(97.)** \$400, \$600, \$900, \$$208·33\frac{1}{3}$, \$$291·66\frac{2}{3}$. **(98.)** \$11·50, \$5·75, \$9·20. **(99.)** \$161, \$112, \$92. **(100.)** \$$342·85\frac{5}{7}$, \$$285·71\frac{3}{7}$, \$$171·42\frac{6}{7}$. **(101.)** \$240, \$120, \$80, \$60.

§ **127.** **(102.)** \$$54·92\frac{68}{71}$, \$$45·07\frac{3}{71}$. **(103.)** \$$55·32\frac{48}{61}$, \$$94·67\frac{13}{61}$. **(104.)** \$3·50. **(105.)** \$$39·62\frac{14}{53}$, \$$60·37\frac{39}{53}$. **(106.)** A. \$17·50, B. \$4·65, C. \$10·60, D. \$5·75, E. \$8·00. **(107.)** A. \$13·50, B. \$12·00, C. \$30·00. **(108.)** A. \$195, B. \$112·50, C.

\$67·50 **(109.)** A. \$40, B. \$30, C. \$24. **(110.)** Each officer \$120, each midshipman \$80, each sailor \$30.

CHAPTER XV.

§ **128.** How do you find the average of a series of numbers? Define average. To what topics is the principle of average applied under this chapter?

§§ **129, 130.** What is Equation of Payments? Give the rule, and illustrate it.

§ **131.** What is the rule for finding the cash balance? Explain the principle involved in the rule. If the cents of an entry exceed 50, what may be done? if less than 50, what?

§§ **132, 133.** Define Alligation Medial. In what does it resemble equation of payments? What is Alligation Alternate? Show in what way Alligation may be of use to the dealer in groceries.

ANSWERS.

§ **128.** **(1–5.)** 2; 4; 5; 6; 7. **(6–10.)** $6\frac{1}{5}$; $16\frac{4}{5}$; $16\frac{1}{5}$; $69\frac{1}{2}$; $7\frac{1}{2}$. **(11.)** \$1220·$18\frac{1}{3}$. **(12.)** $93\frac{1}{10}$. **(13.)** Average $166\frac{3}{14}$*lb.*; aggregate $1163\frac{1}{2}$*lb.* **(14.)** £5 16*s.* 3*d.* $\frac{1}{4}$*qr.* **(15.)** 36*yr.* 6*mo.* $1\frac{3}{5}$*da.* **(16.)** 29·67 inches. **(17–18.)** $22° 44' 44\frac{3}{7}''$; 5*m.* 7*s.* **(19.)** 39·09281 inches. **(20.)** 14·5051475. **(21–22.)** 45*m.* 42$\frac{1}{[illegible]}$*s.*; 2*m.* $41\frac{36}{119}$*s.* **(23.)** 6*oz.* $18\frac{1}{2}$*pwt.* **(24–25.)** 29°; $42\frac{1}{2}°$.

§ **129.** **(26.)** $4\frac{8}{15}$*mo.*=4*mo.* 16*da.* **(27.)** $8\frac{2}{5}$*mo.*=8*mo.* 12*da.* **(28.)** 9*mo.* **(29.)** $5\frac{11}{15}$*mo.*=5*mo.* 22*da.* **(30.)** 6*mo.* **(31.)** $6\frac{1}{2}$*mo.* **(32.)** $64\frac{1}{11}$ days after Jan. 1st, or March 5th. **(33.)** $96\frac{1}{2}$ after the 1st of March, or, calling it 97 days, we have June 6th. **(34.)** $6\frac{9}{23}$*mo.*=6*mo.* 12*da.* nearly. **(35.)** $16\frac{19}{22}$ days after the 10th of March, or the 27th of March. **(36.)** $5\frac{9}{40}$ months. **(37–

38.) $4\frac{6}{7}$*mo.*; $4\frac{1}{7}$*mo.* **(39.)** $28\frac{15}{16}$ days after July 1st; that is, on the 30th of July.

§ **130.** **(40.)** $117\frac{1}{2}$ days. **(41.)** $4\frac{1}{4}$ months. **(42.)** $2\frac{1}{8}$ months. **(43–44.)** $875; $8\frac{3}{4}$*mo.*

§ **131.** **(45.)** $148·51 in favor of A. **(46–47.)** Cash balance of $229·26 in favor of B.; cash balance of $232·72 in favor of B. **(48.)** Cash balance of $98·92 in favor of A. **(49–50.)** Cash balance of $174·56 in favor of A.; cash balance of $180·28 in favor of A.

§ **132.** **(51.)** $0·502. **(52.)** $0·83$\frac{11}{17}$. **(53.)** $12\frac{25}{53}$ carats. **(54.)** $67\frac{11}{15}$ degrees. **(55.)** $6\frac{5}{6}$ knots. **(56.)** $11\frac{5}{11}$ cents. **(57.)** $22\frac{9}{38}$*ct.* **58.)** $3·86$\frac{19}{21}$. **(59.)** $83\frac{2}{71}$*lb.* **(60.)** $9\frac{29}{30}$*qt.*

§ **133.** **(61–66.)** 12, 52; 17, 47; 24, 40; 36, 28; 43, 21; 54, 10. **(67.)** Twice as much wine as water. **(68–71.)** 4, 4, 4, 9; 3, 3, 3, 12; 2, 2, 2, 15; 1, 1, 1, 18. **(72–76.)** 17, 4, 4; 10, 10, 12; 7, 7, 18; 5, 5, 22; 3, 3, 26. **(77–81.)** 5, 5, 5, 6; 4, 4, 4, 9; 3, 3, 3, 12; 2, 2, 2, 15; 1, 1, 1, 18. **(82–85.)** 90, 90, 810; 90, 90, 405; 10, 10, 10; $4\frac{2}{7}$, $4\frac{2}{7}$, $4\frac{2}{7}$. **(86–91.)** 62 pounds of each; 39 pounds of each; $25\frac{1}{5}$ pounds of each; 16 pounds of each; $9\frac{3}{7}$ pounds of each; $4\frac{1}{2}$ pounds of each. **(92–96.)** 100, 100, 2100; 100, 100, 900; 100, 100, 500; 100, 100, 300; 100, 100, 180.

CHAPTER XVI.

§ **134.** What is Involution? What is a square? a cube? What is the difference between a root and a product? between the power of a number and the factor of a number?

§ **135.** What is meant by Evolution? What is the difference between the 4th power of a number and the 4th root of a number?

§ **136.** To what is the square of two numbers (say tens and units) equal? the square of three numbers?

§ **137.** Explain how the number of figures in a square root may be determined. Illustrate by a diagram the involution of two or more numbers to the 2d power.

§ **138.** Illustrate the extraction of the square root by a geometrical diagram. What is the rule for the extraction of square root? Why must a naught be annexed to the trial divisor? If there are decimals, how must the number be pointed off? How is the square root of a common fraction found?

§ **139.** What is a triangle? What is a right-angled triangle? What is an hypotenuse? Give the established proposition in geometry concerning the square of the hypotenuse. What relation do similar surfaces or areas bear to each other?

§ **140.** The cube of two numbers equals what? of the sum of any number of numbers?

§§ **141, 142.** On what principle may the number of figures in the root of a cube be determined? Illustrate by a diagram the involution of two numbers to the 3d power. Illustrate by diagram the extraction of the cube root. Give the rule. Why must two naughts be annexed to the trial divisor? To what extent may the extraction of a root be illustrated geometrically? How is a decimal to be pointed off? How is the cube root of a decimal to be found? of a common fraction?

§ **143.** Similar solids are to each other as what? Illustrate. Can you give an example not found in the book of the use of a knowledge of square or cube roots?

ANSWERS.

§ **134.** (**1–10.**) 196; 361; 576; 1296; 2304; 3249; 8649; 12321; 28224; 54289. (**11–20.**) 2197; 5832; 12167; 42875; 85184; 175616; 753571; 3241792; 37933056; 10793861. (**21–22.**) 229345007; 387420489. (**23–26.**) 0·5625; 1·2996; 1162·1481; 22·857961. (**27–31.**) 0·216; 0·002197; 0·008242408; 0·274625; 27·570978261. (**32–36.**) $5\frac{1}{16}$; $13\frac{4}{9}$; $21\frac{4}{25}$;

$54\frac{25}{64}$; $98\frac{79}{169}$. **(37–41.)** $\frac{1}{8}$; $\frac{8}{27}$; $38\frac{75}{1331}$; $172\frac{323}{343}$; $830\frac{73}{125}$. **(42–44.)** $157\frac{283}{1024}$; 0·00390625; 0·019775390625.

§ **138. 45.)** 104976. **(46.)** 526112. **(47–52.)** 111; 232; 5555; 130321; 923521; 248832. **(53.)** 59·049. **(54–57.)** 2·56; 0·0625; 4·123 nearly; 6·123 nearly. **(58.)** 0·00111. **(59–60.)** $\frac{8}{9}$; $\frac{45}{49}$. **(61–62.)** $\frac{4}{9}$; $2\frac{1}{5}$. **(63–65.)** 2·027 nearly; 0·8044 nearly; 0·515+.

§ **139. (66.)** 80 feet. **(67.)** 94·34 miles nearly. **(68.)** 4·24264 feet nearly. **(69.)** 20 feet. **(70.)** 5·196 feet nearly. **(71.)** 25 times. **(72.)** 36 gallons. **(73.)** 3·60555 feet nearly. **(74–77.)** 20 feet; 18·86796+ feet; 15·54723+ feet; 22·36067+ feet. **(78–80.)** 672 feet; 680 feet; 104 feet.

§ **140. (81–90.)** $70^3+3\times70^2\times5+3\times70\times5^2+5^3$; $80^3+3\times80^2\times9+3\times80\times9^2+9^3$; $100^3+3\times100^2\times40+3\times100\times40^2+40^3+3\times(100+40)^2\times2+3\times(100+40)\times2^2+2^3$; $300^3+3\times300^2\times60+3\times300\times60^2+60^3+3\times(300+60)^2\times5+3\times(300+60)\times5^2+5^3$; $40^3+3\times40^2\times7+3\times40\times7^2+7^3$; $90^3+3\times90^2\times6+3\times90\times6^2+6^3$; $200^3+3\times200^2\times20+3\times200\times20^2+20^3+3\times(200+20)^2\times1+3\times(200+20)\times1^2+1^3$; $400^3+3\times400^2\times90+3\times400\times90^2+90^3+3\times(400+90)^2\times6+3\times(400+90)\times6^2+6^3$; $800^3+3\times800^2\times70+3\times800\times70^2+70^3+3\times(800+70)^2\times9+3\times(800+70)\times9^2+9^3$; $900^3+3\times900^2\times90+3\times900\times90^2+90^3+3\times(900+90)^2\times9+3\times(900+90)\times9^2+9^3$.

§ **142. (91.)** 216. **(92–95.)** 1331; 35; 49; 1936. **(96.)** 0·7773. **(97.)** 2·65. **(98.)** 1·08005+. **(99.)** 1·2599+. **(100.)** 2·08008+. **(101.)** 1·4422+. **(102–104.)** $\frac{13}{17}$; $\frac{23}{29}$; 2·577 nearly. **(105–110.)** 1·726 nearly; 0·9353 nearly; 0·8736 nearly; 0·9196 nearly; 1·6631 nearly; 2·3996 nearly.

§ **143. (111.)** $18\frac{26}{27}$ pounds. **(112.)** 6·1476 inches nearly. **(113.)** 6 inches. **(114–115.)** The weight of the first three is equal to the weight of the last three; the diameter of a ball of average weight is 30·2381 inches nearly. **(116.)** 50·3968 inches nearly in length, and 40·3175 inches nearly for diameter. (**117.**) 47·984 inches nearly. (**118.**) 64. (**119.**) 37·6412 inches nearly in length, 28·2309 inches nearly in breadth, and 18·8206 inches nearly in depth.

CHAPTER XVII.

§ **144.** What is an Arithmetical Progression? What is the difference between an ascending and a descending series? What five quantities are to be considered in arithmetical progression? How may the last term of a series be found, if the first term, the common difference, and the number of terms are given? Explain the principle. How may the sum of all the terms be found, when the first term, the last term, and the number of terms are given?

§ **145.** Will you explain the difference between an arithmetical and a geometrical progression? between the ascending and the descending series? What five quantities are to be considered? Given the first term, the ratio, and the number of terms, how will you find the last term? How will you find the sum of the terms, if first term, last term, and ratio are given? What is the value of the last term of an infinite series?

ANSWERS.

§ **144.** (**1.**) 299. (**2.**) $25\frac{1}{2}$. (**3.**) $19. (**4.**) $3\frac{7}{8}$. (**5.**) 12 inches. (**6.**) 167 hills. (**7.**) $80. (**8.**) 442. (**9.**) 50800. (**10.**) 1825 miles. (**11.**) £6 6*s*. (**12.**) 560000. (**13.**) 146217. (**14–15.**) 78; 300. (**16.**) $402\frac{1}{12}$ feet. (**17.**) $570.

§ **145.** (**18.**) 64. (**19.**) 327680. (**20.**) 320 miles. (**21–23.**) $16000; $64000; $256000. (**24.**) 118096. (**25.**) 436905. (**26.**) 1398101 pecks. (**27.**) $1\frac{1}{2}$. (**28.**) $\frac{1}{7}$. (**29.**) $\frac{1}{4}$. (**30–31.**) 20 feet; 10 feet. (**32.**) 9. (**33.**) 5.

CHAPTER XVIII.

§ **146.** What is a rectangle? How is the area of a rectangle found? of a parallelogram? of a triangle? of a trapezoid? the circumference of a circle, its diameter being given? the area of a circle, its diameter being known? How is the volume of a prism or of a cylinder found? How is the volume of a pyramid or of a cone found? the volume of a sphere, its diameter being given? the volume of the frustum of a pyramid or of a cone? How is the area of an ellipse found?

ANSWERS.

§ **146.** (**1.**) 376*sq. ft.*=$41\frac{7}{9}$*sq. yd.* (**2.**) 2394*sq. in.*=$16\frac{5}{8}$*sq. ft.* (**3.**) 91*sq. rods*=$\frac{91}{160}$ of an acre. (**4.**) $110\frac{1}{4}$*sq. in.* (**5.**) The rectangular piece contains 9*sq. in.* more than the square one. (**6.**) 54*sq. ft.* (**7.**) 37·95*sq. ft.* (**8.**) 84*sq. rods.* (**9.**) 84*sq. yd.* (**10.**) 80*sq. yd.* (**11.**) 40*sq.* rods=$\frac{1}{4}$ of an acre. (**12.**) $16\frac{2}{3}$*sq. ft.* (**13.**) 25132·8 miles nearly. (**14.**) 119·38 feet. (**15.**) 219·21 times. (**16.**) 502·656 acres. (**17.**) The first is equal to the sum of the other two. (**18.**) 30 cubic feet. (**19.**) 14·966 cubic feet. (**20.**) 106·029 cubic inches. (**21.**) $93244729\frac{1}{5}$ cubic feet. (**22.**) 91·1064 cubic feet. (**23.**) 201061930*sq. miles.* (**24.**) 113·0976*sq. in.* (**25.**) 113·0976 cubic inches. (**26.**) 904·7808 cubic inches. (**27.**) 537·2136 cubic inches= 1·909 beer gallons. (**28.**) $101\frac{23}{72}$ cubic feet. (**29.**) 4712·4*sq. ft.* (**30.**) 2078·1684*sq. in.*=14·4317*sq. ft.*

§ **147.** (**31.**) $1159·64 nearly. (**32.**) 6 months. (**33.**) 1159·42 nearly. (**34.**) $101\frac{37}{63}$ rods. (**35.**) A.'s $$184\frac{8}{13}$, B.'s $$153\frac{11}{13}$, C.'s $$161\frac{7}{13}$. (**36.**) $500. (**37.**) 146000 words. (**38.**) $1039\frac{1}{10}$ days. (**39.**) $128. (**40.**) $0·96. (**41.**) 20 per cent. (**42.**) $1·50. (**43.**) In 8 months. (**44.**) $160. (**45.**) 22 men, 18 women, 50 children. (**46.**) $2·91 nearly. (**47.**) $29\frac{7}{9}$*min.*, $923\frac{1}{9}$*yd.* (**48.**) $584·41+. (**49.**) A.'s $135·70, B.'s $203·55, C.'s $271·40. (**50.**) 2000 specie-dollars. (**51.**) $76·52+.

(52.) 9856 years. **(53.)** 24856·28+ miles. **(54.)** 90 men. **(55.)** 18 days. **(56.)** 3*hr.* $26\frac{8}{17}$*min.* **(57.)** All could do it in 16 days; A. in $52\frac{1}{2}$ days; B. in $57\frac{27}{29}$; C. in $44\frac{4}{19}$; D. in 280 days. **(58.)** $9\frac{3}{5}$ per cent.; $19\frac{1}{5}$ per cent.; $28\frac{4}{5}$ per cent. **(59.)** 2·418115599 gallons. **(60.)** 2521; 5041; 7561; 10081. **(61.)** 2519; 5039; 7559; 10079. **(62.)** 36000 rations. **(63.)** 72 yards. **(64.)** 5050 pigeons. **(65.)** 120 casks. **(66.)** 40 pounds of sugar. **(67.)** $\frac{39371}{24000} = 1\frac{15371}{24000}$ Cremona feet. **(68.)** $7\frac{1}{2}$ days. **(69.)** $1\frac{11}{25}$ days. **(70.)** 24 ounces per day. **(71.)** A.'s $\$428\frac{4}{7}$, B.'s $\$571\frac{3}{7}$. **(72.)** A. paid $\$327\frac{3}{11}$, B. paid $\$163\frac{7}{11}$, C. paid $\$109\frac{1}{11}$. **(73.)** 120 yards. **(74.)** $\$328074·37\frac{1}{2}$. **(75.)** Between 60 and 61. **(76.)** 77440 miles. **(77.)** \$4000. **(78.)** I neither gain nor lose. **(79.)** I lose $\frac{1}{60}$ of a cent on each orange. **(80.)** I neither gain nor lose. **(81.)** I gain $\frac{1}{15}$ of a cent for each cent employed. **(82.)** $2\frac{2}{15}$ times. **(83.)** 2 times. **(84.)** 15 yards. **(85.)** The younger had \$453·846 nearly; the elder had \$546·154 nearly. **(86.)** A. has \$30, B. \$33, C. \$37. **(87.)** $1\frac{5}{7}$ days. **(88.)** Value of horse \$150, number of tickets 60. **(89.)** Each received \$50, and Henry sold 200 melons. **(90.)** 48 apples for 20 cents. **(91.)** He gained 80 cents. **(92.)** A. has $\$266\frac{2}{3}$, B. has $\$933\frac{1}{3}$. **(93.)** \$2700. **(94.)** 25 per cent. **(95.)** 20 per cent. **(96.)** \$2·625. **(97.)** $53\frac{1}{3}$ yards. **(98.)** $9\frac{12284}{13149}$ years. **(99.)** 12·907 inches, nearly. **(100.)** 300 strokes. **(101.)** At the end of every 5 hours. **(102.)** \$2·586. **(103.)** \$25000. **(104.)** $\frac{1}{2}$ of a unit. **(105.)** A. has \$133·79−, B. has \$80·27+, C. has \$30·10+. **(106.)** A. ought to have \$135·85, B. \$81·51, C. \$26·80. **(107.)** $3\frac{9}{37}$ days. **(108.)** There was due \$2952·28. **(109.)** The deer weighed 168 pounds.

The total number of examples under the different chapters is as follows:

Chap. I., 0. Chap. II., 92. Chap. III., 58. Chap. IV., 132. Chap. V., 262. Chap. VI., 320. Chap. VII., 25. Chap. VIII., 190. Chap. IX., 370. Chap. X., 575. Chap. XI., 700. Chap. XII., 410. Chap. XIII., 325. Chap. XIV., 110. Chap. XV., 96. Chap. XVI., 119. Chap. XVII, 33. Chap. XVIII., 109.—Total, 3926.

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