PRACTICAL BUSINESS ARITHMETIC

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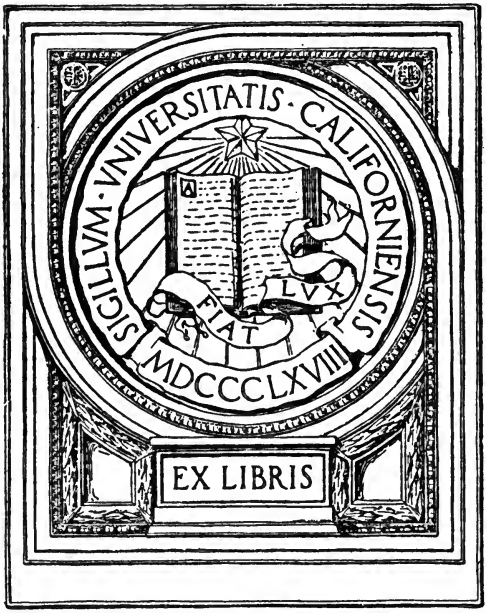

Florian Cajori.

PRACTICAL BUSINESS ARITHMETIC

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GINN & COMPANY

BOSTON · NEW YORK · CHICAGO · LONDON

66.8

The Athenæum Press
GINN & COMPANY · PROPRIETORS · BOSTON · U.S.A.

PREFACE

THIS work has been prepared with the belief that it will be of genuine service to all interested in business education. It is particularly planned for students pursuing a commercial course in business schools, high schools, and normal schools.

The constant aim of the authors has been to develop the subject in such a way as to make it possible for the student to realize both the utilitarian and the cultural value of arithmetic. The topics have been selected with great care, and a logical unfolding of the whole subject has been kept in view. An attempt has been made to give problems which appeal to the needs and interests of the business student, and so to grade and group these problems as to make the mind-furnishing and mind-developing qualities of the subject go hand in hand. Inherited puzzles and manufactured conditions which give a false notion of business have been studiously avoided. The subject as a whole has been modernized, and the exercises made simple, natural, and straightforward.

The most important part of the arithmetic, considered from a business standpoint, is that part devoted to the four fundamental processes and fractions. Particular attention has therefore been devoted to the chapters in this part of the book. The need for speed and accuracy is emphasized in many different ways. There are many speed exercises, and the student is taught to check his work at every step. To make the work more real, many self-checking problems, taken from actual business transactions, are given.

Many classes in high schools study business arithmetic before they have taken up the subject of bookkeeping. To bring all

the work of the text within the understanding of such classes, the principles of debit and credit and of simple account-keeping are developed in the chapter on subtraction.

The method of introducing all new topics is inductive rather than deductive. The student is led to discover as much as possible for himself. Useless lists of so-called "principles" and all worthless definitions have been omitted; but principles which portray business customs and definitions which are understandable and valuable have been carefully stated. No arbitrary rules are given. When a rule is thought necessary to promote facility and rapidity in numerical calculation, the student is induced to make it for himself.

Many new topics have been added, and many of the obsolete topics which have so long encumbered the arithmetics of the schools have been eliminated. The simple exercises on graphic methods of representing statistics, the exercises on plotting and on reading scales, and the exercises on calculation tables, tariffs, freight and express schedules, price lists, stock and bond quotations, etc., will, it is believed, be welcomed by progressive teachers. On the other hand, the elimination of cube root and its applications, compound proportion and compound partnership, unreal fractions of all kinds, all of the useless matter commonly given under denominate numbers, present worth and true discount, and various other obsolete topics, will add to the effectiveness of the course.

Many students who can solve the difficult problems of a textbook often fail in the solution of the ordinary problems of business. One reason for this is that the problems of business are never labeled according to the case or the principles involved in their solution. Recognizing this, the authors have avoided the usual division of the topics into cases. General principles are developed and applied through groups of related problems. These problems enable the student to view a question from all sides and to acquire a knowledge of current business methods as well as skill in numerical calculation.

To make the problems vivid and lifelike numerous photo-

graphs of actual business papers have been reproduced. These facsimiles serve two good and useful purposes, — one, to place the problems before the student just as they will come to him in real business; the other, to give him that familiarity with common business forms which of itself is an invaluable part of any training in business arithmetic. Pictures and diagrams have been freely used whenever they seemed likely to throw light on either principles or problems.

The abundance of oral work given in connection with every chapter will, it is thought, add to the value of the book. These exercises are used to illustrate new principles, to prepare the student for written work, to introduce and develop short processes, to cultivate rapidity and accuracy in calculation, and to teach close and accurate thinking. Such oral work as is given is an absolute business requirement and a tool for proper training in analysis and expression.

The authors wish to acknowledge their indebtedness to Dr. David Eugene Smith, Professor of Mathematics, Teachers College, Columbia University, New York, who read the complete manuscript and much of the proof, and kindly made numerous suggestions for the betterment of the book; to Mr. George M. Clough for the larger part of the material in the chapter on life insurance; to Mr. George Abbot of Brown Bros. & Co., Boston, and to Mr. H. T. Smith, Assistant Cashier of the Shawmut National Bank, Boston, for valuable assistance on the chapters on interest and banking.

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PRACTICAL BUSINESS ARITHMETIC

FUNDAMENTAL PROCESSES

CHAPTER I

INTRODUCTION

1. It is assumed at the outset that the student is familiar with the ordinary symbols of operation; that he can read and write numbers; that he can add, subtract, multiply, and divide integers; that he can do simple work in United States money and in common and decimal fractions; and that he knows many of the most common uses of arithmetic.

2. In this course in business arithmetic he may learn more about methods of working with numbers; the uses of arithmetic in the most important lines of business and in the ordinary affairs of everyday life; how to acquire skill in handling numbers; how to check results; and how to make problems and solve them. Besides all this, he may learn a great deal about system and economy in the home and in the office; current business practices and usages; business phraseology and literature; the quantitative side of commerce and industry; and many other useful and interesting items of information pertaining to his active participation in life.

3. The fundamental processes are the foundation of all arithmetic. The student should therefore be able to perform these essential processes with *speed*, *absolute accuracy*, and *intelligence* before he attempts to take up the more advanced work.

Where work in the fundamental processes is not thought to be advisable it may of course be omitted.

CHAPTER II

NOTATION AND NUMERATION

ORAL EXERCISE

1. How many different figures are used to express numbers?

2. What is the meaning of the syllable *teen* in the numbers from 13 to 19 inclusive?

3. What is the meaning of the syllable *ty* in such numbers as 20, 30, 40, 45, 75, 87, 96?

4. What name is given to 10 tens? to 10 hundreds? to 1000 thousands? to 1000 millions?

5. In 7, 70, 700, 7,000, and 70,000 how does the 7 change in value? In 7007 how do the values of the 7's compare?

6. What is the value of the cipher in any number? Why is it used? Explain the use of the ciphers in 900,905.

7. Upon what two things does the value of a figure depend? Illustrate your answer, using the number 121,000,121.

8. Mention five things that are counted in thousands; three things that are counted in millions; two things that are counted in billions. Can you think of any use for trillions?

9. Read aloud the following:

a. The coinage of the mints at Philadelphia, New Orleans, and San Francisco during a recent year amounted to 176,999,132 pieces, of a value of $136,340,781. Of this $99,065,715 was in gold coin, $24,298,850 in silver dollars, and $12,976,216 in fractional silver and minor coins.

b. In the United States Bureau of Engraving and Printing there are printed yearly about 20,000,000 sheets of United States notes, certificates of deposit, bonds, and national currency to the amount of about $500,000,000. In addition to this there are printed about 1,000,000,000 internal revenue stamps, and more than 3,000,000,000 postage stamps.

THE ARABIC SYSTEM

4. This is the common system of notation. It is generally called the **Arabic system** because the numerals which it employs were introduced into Europe by the Arabs.

The Arabic numerals 1, 2, 3, and so on to 9 originated in India about 2000 years ago. When only these numerals were used, the system proved to be cumbersome, and all mathematical operations involved great difficulty. About 1200 years ago the cipher 0 was added, thus making a system sufficiently ample and simple for ordinary purposes of analysis and investigation. The Arabs introduced the system into Europe in the twelfth century, but it was not until about 300 years later that it displaced the clumsy Roman system.

5. The distinctive feature of the Arabic system is the **place value** of the numerals employed. The value of an Arabic numeral depends as much upon its place in the number as upon its simple or digit value.

Thus, in the Roman system, VII = 5 + 1 + 1. In the Arabic system, 511 = 5 *hundreds* + 1 *ten* + 1. 5 has not only the unit value *five*, but also the place value *hundreds;* and the 1 following has not only the unit value *one*, but also the place value *ten*.

6. The successive places a figure may occupy in a number are called **orders of units.**

7. *Orders of units increase from right to left and decrease from left to right in a tenfold ratio.* Therefore,

8. The Arabic system of notation is properly called a **decimal system**, from the Latin *decem*, meaning ten.

9. A comma (**separatrix**) or a greater space than that between other figures may be used to separate a number into periods.

Thus, twenty-five thousand four hundred twenty-one may be written 25,421 or 25 421.

ORAL EXERCISE

Read aloud the following numbers:

1. 1,482.	**3.** 375,214.	**5.** 8 217 000 214.
2. 7,009.	**4.** 278,900.	**6.** 7 000 421 817.

10. For convenience in reading, the successive orders of units are divided into groups of three figures each, called **periods**. The first four periods are shown in the following numeration table. The number used for illustration is sixty-seven billion, four hundred twenty-one million, five thousand, two hundred sixteen, and seven hundred fifty-one thousandths.

NUMERATION TABLE

PERIODS:	Billions			Millions			Thousands			Units				Thousandths		
ORDERS:	Etc.	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
		6	7,	4	2	1,	0	0	5,	2	1	6	.	7	5	1

11. In reading integers do not use the word *and*. In decimal fractions *and* has an office to perform, and if it is used in reading integers, misunderstandings may occur.

Thus, 400.011 is read *four hundred and eleven thousandths;* but
.411 is read *four hundred eleven thousandths;* and
411. is read *four hundred eleven.*

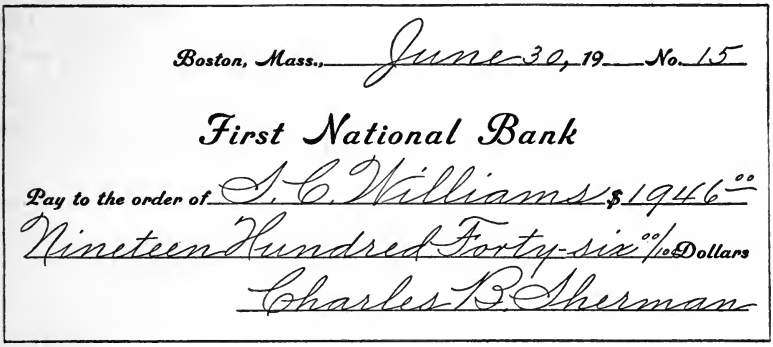
WRITTEN EXERCISE

Write in figures the following:

1. Six million, six thousand, five.
2. Seven hundred fifty-three billion.
3. Four million, one hundred twenty-five.
4. Three hundred twenty-one million, six.
5. Three million four dollars and five cents.
6. Ten billion, one thousand, one hundred three.
7. Twenty-seven and one hundred twenty-five thousandths.
8. Sixty-two thousand and four hundred twenty-five thousandths.
9. Three million four hundred twenty thousand one dollars and fifteen cents.

12. Integers should be read in the **shortest way possible.**

Thus, 1946 should be read *nineteen hundred forty-six*, not *one thousand nine hundred forty-six*. The space for writing the amount on a check,



note, or other business paper is generally limited to one line, and it is important that the amount be expressed in the fewest words possible.

ORAL EXERCISE

Read aloud the following:

1. In a recent year the railroad trackage of the world was about 550,400 mi., distributed as follows: North America, 237,600 mi.; Europe, 179,500 mi.; Asia, 75,400 mi.; South America and West Indies, 29,100 mi.; Australasia, 16,900 mi.; Africa, 11,900 mi.

2. The trackage in North America in the same year was distributed approximately as follows: United States, 208,000 mi.; British North America, 18,900 mi.; Mexico, 9,200 mi.; Central America, 900 mi.; Newfoundland, 600 mi.

3. In the same year the railways of the United States aggregated about one half the total mileage of the world, and over this enormous trackage about 44,500 locomotives and 1,562,900 coaches and cars carried about 696,950,900 passengers and 1,306,628,800 tons of freight.

4. In the same year the aggregate capital stock of these railways was about $6,500,000,000, the gross earnings about $1,908,800,000, and the net earnings $592,509,000.

THE ROMAN SYSTEM

ORAL EXERCISE

1. Make a list of the Roman numerals used in the headings marking the divisions of this book, and read the list so prepared.

2. What symbol ordinarily appears on a watch face for four?

13. This system of writing numbers is called **Roman notation** because it was first used by the Romans. It is now rarely used except for numbering books and their parts, for writing inscriptions on buildings, and for marking the hours on the dials of clocks and watches. It employs seven capital letters:

I	V	X	L	C	D	M
1	**5**	**10**	**50**	**100**	**500**	**1000**

14. Other numbers are expressed by a combination of these letters on the general principle that

A combination of letters arranged from left to right in the order of value is equal to the sum of the constituent letters.

15. But the use of the same letter four or more times is avoided by employing the sub-principle that

When one letter precedes another of greater value the value of the two is that of their difference.

Thus, II = 2; VIII = 8; and CCC = 300. But IV or IIII = 4; XL = 40; XC = 90; and CD = 400.

ORAL EXERCISE

1. Multiply twenty-seven by itself in Roman numerals.

2. Why is the Arabic system better than the Roman system?

3. Read the following inscription: MDCCCXLVIII — Charlestown High School — MCMVI.

Nineteen hundred was formerly written MDCCCC, but it is now often written MCM.

4. Read the following numbers of chapters in a book: XXIX, XXXVIII, LXIX, LII, LXVII, LXXVI, LXXIX, CLIII.

5. Read the following numbers of years: MDCCXCV, MCMVII, MDCCLXXVI, MCMIX, MDCCCXCVIII.

WRITTEN EXERCISE

1. Write in the Roman system: 19, 88, 99, 124, 1907, 1910.

2. Write the largest possible number using the six following numerals: 1, 0, 8, 0, 9, 5.

3. Write in Arabic numerals the following number: five billion, two hundred seventeen million, two hundred ten thousand, and fifteen thousandths.

4. Write in the Roman system the following historical years: the discovery of America; the landing of the Pilgrim Fathers at Plymouth; the declaration of independence.

5. Write in Arabic numerals the number in problem 3 increased by two hundred seventy-one and four hundred fifteen thousandths; diminished by two thousand, four hundred sixty, and eleven thousandths.

16. A **unit** is a standard quantity by which other quantities of the same kind are measured.

The simplest form of a unit is a *single entire thing* by which other similar things can be measured by integral enumeration. Thus, the unit of distance is an inch; a group of 12 in. taken in succession is a foot; 3 ft. is a yard; and so on.

17. Numbers that have units of the same kind are called **like numbers**.

Thus, \$12 and \$15, and 8 hr. and 3 hr., are like numbers.

ORAL EXERCISE

Name the unit in each of the following:

1. A barrel of sugar sold by the pound.
2. A car load of apples bought by the barrel.
3. A car load of lumber sold by the thousand feet.
4. Sixty-four thousand bricks sold by the thousand.
5. Forty and one-half yards of carpet sold by the yard.
6. Twenty-five hundred pounds of beef bought by the hundredweight.
7. When the value in a five-dollar gold piece is thought of, what is the unit?

CHAPTER III

UNITED STATES MONEY

ORAL EXERCISE

Read the following expressions, supplying the missing word or words:

1. The denominations of United States money used in business are dollars, ——, and ——.

2. —— —— mills or —— —— cents equal one dollar.

3. The —— is not a coin, but it is sometimes used in making calculations.

4. The first two figures at the right of dollars denote ——, and the third figure denotes ——.

5. The two figures denoting cents express —— of a dollar; the figure denoting mills expresses —— of a dollar.

6. One thousandth of a dollar is —— mill; seven mills are —— —— of a dollar.

7. Fifteen hundredths of a dollar are —— ——; nine tenths of a dollar are nine —— or —— cents.

8. \$25 = —— ¢; 3700 ¢ = \$ ——; \$17.85 = —— ¢; 4925 ¢ = \$ ——; \$79 = —— ¢.

9. State a short method of reducing dollars to cents; dollars and cents to cents; cents to dollars.

18. The following **kinds of currency** are in daily use in the United States at the present time: gold coins; silver dollars; subsidiary coins (small change); gold certificates; silver certificates; United States notes and treasury notes of 1890; national bank notes.

The coins now authorized by the United States government are as follows:

1. The *gold* double eagle, eagle, half eagle, and quarter eagle.
2. The *silver* dollar, half dollar, quarter dollar, and dime.
3. The *nickel* five-cent piece and the *bronze* one-cent piece.

19. Gold or silver in bars or ingots is called **bullion.**

The paper money of the United States is at present as follows:

1. *Gold certificates*, issued for gold deposited in the U. S. Treasury.
2. *Silver certificates*, issued for silver deposited in the U. S. Treasury.
3. *United States notes* (*greenbacks*), promises of the government to pay to the holder on demand a definite number of gold or silver dollars.
4. *National bank notes*, issued by national banks under the supervision of the National Government. These notes are secured by U. S. bonds and are redeemable on demand in lawful money.
5. *Treasury notes*, which were issued for silver bullion deposited in the U. S. Treasury. These notes are not now issued.

ORAL EXERCISE

1. What is meant by *money*, *currency*, *legal tender?*

In such exercises as the above the student should not try to repeat definitions, but should explain the terms in his own way.

2. Name the gold coins of the United States; the silver coins; the paper money; give the value of each of the gold coins.

3. Read in three ways: \$4.8665; \$25.87½; \$178.475.

4. Name the largest gold and silver coins that will exactly express each of the following amounts: \$27.95; \$28.24; \$75.82.

20. When it is desirable to express United States money in written words, the cents should be written in fractional form, as in the following note:

\$1050 45/100 New York, June 30, 19—

Thirty days after date I promise to pay to the order of Charles B. Kingsbury

Ten Hundred Fifty 45/100 Dollars

at First National Bank

Value received

No. 24 Due July 30, 19 F. M. Everett

CHAPTER IV

ADDITION

ORAL EXERCISE

1. Find the sum of 1, 2, 3, 4, 5, 6, 7, 8, and 9.

2. Read each of the numbers in problem 1 increased by 2; by 5; by 3; by 7; by 8; by 9; by 17; by 23.

3. Find the sum of 8, 7, 9, 5, 6, 11, and 12.

4. Read each of the numbers in problem 3 increased by 12; by 15; by 18; by 24; by 42; by 19; by 16.

5. Illustrate what is meant by like numbers.

21. *Only like numbers can be added.*

22. To secure speed and accuracy in addition name *results only* and express these in the fewest words possible.

Thus, in adding 2, 4, 7, 8, 3, 2, and 8 say *6, 13, 21, 4, 6, 34*; do *not* say *2 and 4 are 6 and 7 are 13 and 8 are 21 and 3 are 24 and 2 are 26 and 8 are 34.*

ORAL EXERCISE

Name the sum in each of the following problems:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
3	2	2	8	1	5	8	1	3	5	5	1	3	4	2
2	1	4	2	3	2	2	3	3	1	4	7	2	5	7
1	6	3	1	6	1	3	6	4	6	4	2	1	2	3
2	8	2	2	4	3	7	4	2	2	3	7	5	8	5
8	4	1	3	4	4	4	9	8	7	2	3	2	6	4
4	8	4	4	3	7	7	5	3	3	1	4	8	4	2
5	6	3	5	2	2	3	8	6	2	0	5	2	5	1
6	0	6	2	3	1	4	2	2	5	7	2	6	3	4
3	8	1	7	7	6	1	1	1	1	7	7	1	2	3
3	6	2	2	4	2	2	4	3	4	2	1	1	1	2
2	2	3	5	1	8	3	2	2	3	1	3	8	6	2
4	1	5	1	2	3	2	4	1	2	4	4	9	8	7

23. Addition is the basis of all mathematical processes. It constitutes a large part of all the computations of business life and concerns, to some extent, every citizen of to-day. Ability to add rapidly and accurately is therefore a valuable accomplishment.

24. Rapid addition depends mainly upon the ability to group; that is, to instantly combine two or more figures into a single number. In reading it is never necessary to stop to name the individual letters in the words. All the letters of a word are taken in at a glance; hence the whole word is known at sight. Words are then grouped in rapid succession and a whole line is practically read at a glance. This is just the principle upon which rapid addition depends. From two to four figures should be read at sight as a single number, and the group so formed should be rapidly combined with other groups until the result of any given column is determined. This can be done only by intelligent, persistent practice.

25. The following list contains **all possible groups of two figures each.**

ORAL EXERCISE

Pronounce at sight the sum of the following groups:

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
1.	1	1	2	2	4	1	3	3	4	3	1	4	2	4	7
	1	3	1	2	1	5	2	3	2	6	7	3	5	6	7
2.	8	9	8	5	6	4	5	5	7	1	5	6	6	8	9
	9	9	8	5	1.	4	3	4	2	8	6	6	9	6	1
3.	8	7	7	4	9	7	6	7	5	3	2	4	5	7	6
	2	3	5	8	3	8	7	9	9	8	9	9	8	4	2

The above exercise may be copied on the board and each student in turn required to name the results from left to right, from right to left, from top to bottom, and from bottom to top. The drill should be continued until the sums can be named at the rate of 150 per minute. This is the first and most important step in grouping.

ORAL EXERCISE

Name the sum in each of the following problems:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
6	7	3	5	6	7	9	9	9	1	2	5	8	2	8
3	1	4	2	4	7	9	8	8	4	7	2	3	7	2
8	7	7	5	8	2	5	9	4	5	8	3	5	4	1
7	9	6	9	3	9	8	4	7	1	1	7	9	5	9
3	8	5	3	8	1	6	4	9	2	6	5	7	3	4
9	4	7	7	2	9	8	5	1	3	5	7	6	5	5
5	6	6	8	2	4	4	3	6	3	6	8	7	4	6
5	6	5	5	7	5	4	2	1	3	6	4	9	4	8
2	3	2	1	1	2	3	1	1	2	5	3	8	1	9
4	3	3	1	4	2	1	5	6	4	5	9	7	6	6

Name the results only and make groups of two figures each. Thus, in problem 1, beginning at the bottom and adding up, say *6, 16, 28, 43, 52.*

16–45. Add the numbers in the exercise on page 10 by groups of two figures each.

26. It is practically as easy to add 54 and 9, 59 and 6, etc., as it is 4 and 9, 9 and 6, etc. 4 and 9 are always equal to 1 ten and 3 units, and 9 and 6 to 1 ten and 5 units. Hence in adding 54 and 9 think of the tens as increased by 1, call the units 3, and the result is 63; in adding 59 and 6 think of the tens as 6, the units as 5, and the result as 65.

ORAL EXERCISE

Pronounce at sight the sum of each of the following groups:

1.	27	48	59	77	58	52	59	75	95	84	39	59	84	76	91
	7	8	6	8	7	8	8	6	9	7	6	5	9	8	8
2.	75	59	77	88	74	23	24	44	89	78	67	37	56	58	68
	8	9	9	5	6	8	9	9	9	9	9	7	7	4	5
3.	37	49	38	37	45	95	98	87	54	72	63	42	73	97	88
	5	8	7	6	9	8	7	7	9	9	8	9	8	5	9

27. In combining numbers between 10 and 20 think of them as one ten and a certain number of units and not as a certain number of units and 1 ten.

Thus, in combining 17 and 18 think of 28 and 7, or 35; in combining 19 and 15 think of 29 and 5, or 34; and so on.

ORAL EXERCISE

Pronounce at sight the sum of each of the following groups:

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
1.	12	17	12	16	11	12	18	16	17	11	19	13	18	12	17
	15	17	12	13	14	11	18	12	18	19	15	13	12	14	19
2.	13	11	15	19	14	19	17	15	13	19	16	14	18	18	12
	18	16	16	14	15	16	16	13	11	18	14	14	11	15	19
3.	11	17	12	17	15	15	12	18	16	14	19	14	19	17	11
	11	14	13	13	17	15	17	16	16	13	19	18	13	11	15

The above exercise contains all combinations possible with the numbers from 11 to 19 inclusive. Drill on the exercise should be continued until results can be named at the rate of 120 per minute.

28. Numbers between 10 and 20 may be combined with numbers above 20 in practically the same manner as in § 27

Thus, in adding 62 and 12 think of 72 and 2, or 74; in adding 79 and 17 think of 89 and 7, or 96.

ORAL EXERCISE

Pronounce at sight the sum of the following numbers:

1.	25	48	59	87	91	75	86	75	48	78	57	89	37	56	75
	17	17	16	14	18	18	19	12	16	13	16	14	17	18	14
2.	29	47	83	92	36	54	59	78	67	92	77	86	53	78	85
	13	14	19	14	19	13	18	15	13	13	19	19	17	14	14
3.	31	32	45	69	74	95	98	92	96	87	86	34	43	64	38
	19	17	19	15	8	18	14	19	15	17	19	18	18	19	17

ORAL EXERCISE

1. Count by 7's from 1 to 85.

SOLUTION. 8, 15, 22, 9, 36, 43, 50, 7, 64, 71, 8, 85.

Count by :

2. 2's from 39 to 55.
3. 5's from 11 to 86.
4. 6's from 15 to 63.
5. 5's from 2 to 107.
6. 7's from 11 to 60.
7. 8's from 25 to 89.
8. 9's from 31 to 112.
9. 8's from 32 to 192.
10. 7's from 18 to 102.
11. 6's from 72 to 126.
12. 9's from 10 to 136.
13. 9's from 17 to 152.
14. 8's from 10 to 138.
15. 7's from 19 to 152.
16. 6's from 20 to 128.
17. 6's from 15 to 111.
18. 9's from 12 to 102.
19. 8's from 17 to 113.
20. 7's from 24 to 108.
21. 6's from 27 to 117.
22. 4's from 19 to 183.
23. 11's from 14 to 102.
24. 12's from 17 to 161.
25. 13's from 17 to 121.

26. Beginning at 1 count by 4's to 17; going on from 17 count by 7's to 52; from 52 count by 9's to 133; from 133 count by 5's to 158; from 158 count by 12's to 206; from 206 count by 13's to 271.

This exercise furnishes one of the best possible drills in addition, and it should be continued until the successive results can be named at the rate of 150 per minute.

29. If the student is accurate and rapid in making groups of two figures each, he is ready for practice in groups of three figures each. In the following exercise are **all the possible groups of three figures each.**

ORAL EXERCISE

Name at sight the sum of each of the following groups:

4, 2, and 3 should be thought of as 9 just as p-e-n is thought of as pen.

1.	4	1	9	8	1	1	3	1	8	1	4	5	1	7	8
	1	3	1	2	2	3	1	7	3	3	1	4	4	1	4
	3	3	2	1	7	5	6	3	1	9	4	1	6	4	1

2.	1	6	1	4	1	2	1	1	1	1	7	6	9	8	1
	4	1	2	1	2	2	9	1	1	6	6	6	5	5	5
	9	2	5	2	3	1	1	8	7	8	1	1	1	1	7
3.	6	5	2	5	2	3	9	2	2	2	2	6	1	1	2
	1	1	3	3	3	2	2	8	7	6	5	1	1	1	2
	5	5	6	2	4	3	2	2	2	2	2	1	5	4	4
4.	3	2	1	2	2	6	2	6	5	5	7	1	1	1	1
	2	2	1	7	6	8	6	2	2	2	2	1	1	6	9
	2	2	3	7	9	2	7	6	9	8	5	2	1	9	9
5.	9	8	9	8	7	3	4	5	6	6	5	4	3	3	4
	1	1	1	1	1	5	8	7	7	7	5	4	4	4	4
	8	8	7	7	7	5	4	5	9	8	6	7	9	8	6
6.	5	6	6	9	5	7	3	4	9	6	6	8	3	3	3
	5	7	6	4	4	3	4	4	4	8	7	4	9	4	4
	5	7	9	9	4	4	6	4	8	6	6	8	9	5	4
7.	3	4	6	9	8	5	4	3	3	2	3	3	4	5	8
	8	7	6	9	9	9	7	8	3	5	3	7	7	8	8
	9	9	6	9	9	9	8	8	9	6	8	9	7	9	8
8.	8	5	4	3	3	5	2	3	3	4	5	7	7	5	4
	8	8	9	8	7	2	4	3	7	6	7	9	8	7	6
	9	5	6	7	3	5	9	6	7	8	9	9	9	8	7
9.	3	3	2	2	3	3	4	5	7	9	9	9	7	3	6
	6	3	4	4	3	6	6	7	8	7	6	5	6	3	4
	9	5	8	7	4	8	6	7	8	7	5	4	3	3	2
10.	2	2	3	4	5	7	2	2	3	4	5	7	9	6	6
	4	9	6	5	6	7	4	8	5	5	6	7	9	6	5
	5	9	6	8	8	8	4	9	9	7	7	7	6	5	4
11.	8	8	9	2	2	3	4	5	6	8	8	9	6	8	7
	5	8	3	3	7	5	5	5	8	8	5	4	5	7	3
	3	2	2	8	9	7	5	9	9	6	5	4	3	2	2

This exercise should be drilled upon until the sums of the groups, in any order, can be named at the rate of 120 per minute.

ORAL EXERCISE

1–15. Turn to the exercise on page 10 and find the sum of the numbers given.

Name results only, and make groups of three figures each. Thus, in problem 1, say *9, 23, 37, 43.*

Add from the bottom up and check the work by adding from the top down.

Find the sum of the following problems:

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
1	3	1	4	2	2	2	4	4	5	1	2	9	5	4
1	1	3	3	3	1	6	3	9	5	7	4	0	7	3
1	1	4	1	5	2	2	4	5	0	2	4	1	2	1
2	1	3	1	3	1	4	1	8	8	9	2	8	0	1
2	4	1	4	6	4	5	8	3	2	0	3	0	0	6
2	2	3	8	1	1	2	1	7	1	1	5	2	5	8
2	4	2	2	2	2	2	3	8	3	5	7	2	6	1
5	2	1	4	5	3	7	6	2	7	3	7	2	6	6
1	2	9	4	3	2	3	1	8	2	2	1	6	0	7
5	1	8	3	4	2	1	2	9	9	6	7	2	3	3
3	5	2	3	3	6	9	3	3	1	2	8	2	6	3
1	3	1	3	3	1	0	5	6	3	7	0	4	1	1
3	2	7	2	4	3	0	2	8	8	4	7	2	5	9
5	4	2	5	2	4	8	5	1	2	3	3	2	3	2
2	4	1	4	4	3	2	2	0	4	3	0	5	2	1
1	2	1	2	6	6	4	4	6	6	3	6	2	5	8
8	6	2	3	3	3	5	2	4	4	3	3	2	8	2
1	2	6	5	1	1	1	3	0	5	6	1	6	2	1
4	4	1	3	7	2	9	3	7	9	1	5	7	5	7
3	5	2	2	2	6	2	2	3	1	7	3	3	7	2
4	2	5	6	1	3	1	3	0	3	2	2	1	3	1
4	2	1	2	1	2	2	7	7	7	1	1	9	2	2
9	7	2	2	3	8	3	1	2	3	9	1	2	5	2
1	3	4	4	4	1	7	7	1	0	0	8	4	8	4
2	1	3	7	3	2	5	7	6	5	5	2	4	4	3
1	6	2	1	5	5	3	2	3	2	8	1	3	6	3
2	3	1	1	2	1	1	2	1	2	1	5	7	1	1

30. It is always an advantage to find **groups of figures aggregating 10 and 20** in the body of a column.

These groups should be added immediately to the sum already obtained by simply combining the tens of the two numbers. It is not a good plan, however, to take the digits in irregular order in order to form groups of 10 and 20.

ORAL EXERCISE

Find the sum of the following problems, taking advantage of groups of 10 and 20 wherever possible:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
1	2	7	6	5	2	5	3	4	3	7	8	2	5	9
9	8	3	4	5	5	4	3	2	5	5	4	7	8	9
7	4	5	1	8	5	6	7	8	5	5	6	3	2	1
3	6	5	9	2	7	9	8	7	4	0	2	5	8	1
2	7	7	2	4	3	1	2	3	6	9	7	5	2	5

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
4	1	9	1	4	6	5	7	1	2	4	4	1	6	1
3	1	2	6	1	2	2	2	4	2	2	3	9	3	1
3	8	9	3	5	2	3	1	5	6	4	3	2	1	8
7	1	4	6	5	4	4	7	5	1	8	7	8	7	0
2	6	1	2	2	4	3	3	5	7	6	2	3	4	9
1	3	5	2	3	2	3	4	4	2	4	5	7	5	0
6	5	9	2	4	4	8	6	6	5	3	1	8	1	1

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.	43.	44.	45.
2	3	9	8	6	6	6	6	5	8	6	6	2	7	6
2	8	4	5	6	7	7	5	7	4	3	9	8	9	7
9	9	7	6	8	7	8	6	8	9	8	5	9	9	4
9	7	9	9	7	9	7	8	7	7	9	6	9	2	9

46.	47.	48.	49.	50.	51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
38	42	25	35	46	14	21	12	18	29	57	17	13	14	15
8	1	4	2	4	5	0	3	2	5	5	4	6	2	7
1	2	1	9	7	6	6	3	4	7	6	8	6	7	2
1	7	3	9	9	0	6	8	4	8	9	8	8	5	8
2	4	6	5	2	9	8	6	7	4	5	5	2	3	6

31. When three figures are in consecutive order the sum may be found by multiplying the middle figure by 3; when five figures are in consecutive order the sum may be found by multiplying the middle figure by 5; etc.; or the sum of any number of consecutive numbers may be found by taking one half the sum of the first and last numbers and multiplying it by the number of terms.

ORAL EXERCISE

By inspection find the sum of:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
7	10	13	16	19	22	25	28	31	34	37	40	43	46	49
8	11	14	17	20	23	26	29	32	35	38	41	44	47	50
9	12	15	18	21	24	27	30	33	36	39	42	45	48	51

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
11	16	21	26	31	36	41	46	51	56	61	66	71	76	81
12	17	22	27	32	37	42	47	52	57	62	67	72	77	82
13	18	23	28	33	38	43	48	53	58	63	68	73	78	83
14	19	24	29	34	39	44	49	54	59	64	69	74	79	84

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.	43.	44.	45.
7	10	13	16	19	22	25	28	31	34	37	40	43	46	49
8	11	14	17	20	23	26	29	32	35	38	41	44	47	50
9	12	15	18	21	24	27	30	33	36	39	42	45	48	51
10	13	16	19	22	25	28	31	34	37	40	43	46	49	52
11	14	17	20	23	26	29	32	35	38	41	44	47	50	53
12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
13	16	19	22	25	28	31	34	37	40	43	46	49	52	55
14	17	20	23	26	29	32	35	38	41	44	47	50	53	56
15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
16	19	22	25	28	31	34	37	40	43	46	49	52	55	58
17	20	23	26	29	32	35	38	41	44	47	50	53	56	59

32. When a figure is repeated several times the sum may be found by multiplication.

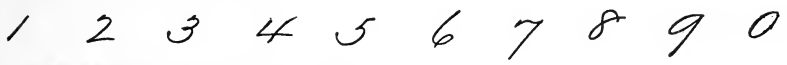
ORAL EXERCISE

By inspection find the sum of the following groups:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
4	3	4	5	3	7	8	8	15	6	7	8	15	13	9
9	7	4	5	3	7	5	7	15	6	8	7	14	13	8
9	8	4	5	9	7	5	9	15	12	7	8	15	13	8
9	8	9	5	9	8	6	9	8	12	7	7	14	7	9
9	8	9	9	8	8	6	9	8	12	7	8	15	7	8

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
3	7	4	2	7	5	12	2	4	6	8	9	8	5	16
3	7	4	2	7	5	5	2	4	6	8	9	8	5	16
3	7	4	2	4	5	5	2	4	6	8	9	8	5	16
2	2	7	8	4	4	5	3	5	4	3	5	8	5	16
2	2	7	8	2	4	5	3	5	4	3	5	8	5	20
2	2	7	8	2	4	5	3	5	4	3	5	9	8	1

33. In all written work make plain, legible figures of a uniform size, write them equal distances from each other, and be sure that the units of the same order stand in the same vertical column.



34. Many of the errors that occur in business are in simple addition. Errors in addition result from two main causes: irregularity in the placing of figures; poor figures.

35. In business it is important that figures be made rapidly; but rapidity should never be secured at the expense of legibility.

WRITTEN EXERCISE

Copy and find the sum of:

1.	2.	3.	4.	5.	6.
1745	1842	1249	4271	6229	1481
1862	1695	1810	8614	4813	1862
7529	4716	6241	9217	7142	4129
8721	8412	1728	8214	6212	2412

7.	8.	9.	10.	11.	12.
4216	2110	4142	1061	4113	4112
8912	8420	4347	1875	8217	1012
4729	1641	1012	6214	8614	1862
8624	1722	1816	1931	1692	1721
4829	1837	4112	1648	1591	1692
6212	4216	4210	1721	1686	1486
4110	4117	1618	1728	2172	4112
4210	1832	4060	1421	1754	1010

36. The simplest way to **check addition** is to add the columns in reverse order. If the results obtained by both processes agree, the work may be assumed to be correct.

37. In adding long columns of figures it is generally advisable to record the entire sum of each column separately; then if interruptions occur, it will not be necessary to re-add any portions already completed. After the total of each column has been found the entire total may be determined by combining the separate totals of the columns.

38. The best way to test the accuracy of columns added in this manner is to begin at the left and repeat the addition in reverse order. The entire total of each column should again be written and the complete total of the problem found by adding the separate totals of the several columns. If the results obtained by the two processes agree, the work may be assumed to be correct.

39. Example. Find the sum of 54669, 15218, 36425, 45325, and 68619. Check the result.

SOLUTION. Beginning at the bottom of the right-hand column, add each column in regular order and write the entire totals as shown in (*a*). Beginning at the top of the left-hand column again add each column and write the entire totals as shown in (*b*). Next add the totals obtained by the first and second additions and compare the results. Since the total shown by (*a*) is equal to the total shown by (*b*), the result, 220,256, is assumed to be correct. All work in addition should be carefully checked.

(*b*)	54669	(*a*)
19	15218	36
28	36425	12
21	45325	21
12	68619	28
36	220256	19
220256		220256

WRITTEN EXERCISE

See how many times the following numbers can be written in one minute. Write each number in form for vertical addition.

1. 426579.
2. 123987.
3. $7983.21.
4. $4080.91.
5. $70812.34.
6. $41182.50.

Thus, in repeating the number in problem 1 write it as follows:

4 2 6 5 7 9
4 2 6 5 7 9
4 2 6 5 7 9
4 2 6 5 7 9
Etc.

Be sure that the spacing between the lines and between the columns is uniform. Increase the speed gradually until from 150 to 200 figures can be written per minute.

40. Skill in writing figures from dictation should be cultivated. The dictation should be slow at first, but it should be gradually increased until the requisite speed is acquired.

41. In calling off numbers to another great care should be taken in order that no errors may be made. In reading United States money the word *dollars* should be called with each amount. The word *cents* may be omitted in all cases except where there are no dollars.

Thus, in calling $400.37 say *four hundred dollars, thirty-seven;* in calling $25.11 say *twenty-five dollars, eleven;* in calling $1573.86 say *fifteen hundred seventy-three dollars, eighty-six;* in calling $5.31 say *five dollars, thirty-one.*

WRITTEN EXERCISE

Write from dictation and find the sum of:

1. $75.18, $123.95, $147.25, $9.50, $181.45, $172.16, $84.98, $314.95, $49.10, $69.90, $312.60, $415.90.

2. $3140.19, $310.92, $3164.96, $3162.19, $18.62, $410.95, $690.18, $10.75, $3100.40, $300.40, $200.50, $100.90, $410.80, $100.85, $310.60, $80.90, $399.80, $412.60.

WRITTEN EXERCISE

Copy, find the sum, and check:

1.	2.	3.
$ 6925 64.21	$ 624 2654.92	$ 215 1621.54
1345 678.50	125 9216.05	126 2141.67
379 0269.17	217 9203.04	213 1472.03
672 1234.23	913 1426.04	146 2141.07
421 2612.45	125 9212.46	964 8968.03
124 9212.54	132 1652.17	124 2472.42
128 2964.16	214 1621.46	986 8638.68
217 9216.54	315 1259.06	214 1262.49
692 3469.48	211 4201.58	896 9849.61
246 8627.16	317 4257.69	126 2479.14
549 2165.18	426 9237.50	341 6218.49

4.	5.	6.
$ 126 724.45	$ 1275 246.48	$ 125 1624.64
437.26	751 2047.72	721 266.44
12.91	126 056.42	411 200.54
7.69	216 4414.79	1500 000.98
647 686.42	161 47.92	196 9214.54
541 64.95	721 46.54	240 1206.25
712.78	12.75	129 863.64
92.14	126.92	1264.92
569.86	2146.01	151 2692.07
11.46	326 45.91	169 2162.04
7269.42	4260.01	1721.90
6924.75	3001.98	416.75
12798.69	75.06	6721.90
124.60	1.92	14.95
1.59	4120.78	4261.74

7.	8.	9.
$1 2 6 4 1 2.80	$2 1 4 6 2 1.6 4	$1 6 2 0 1 7 7.2 5
9 8 5 3 6.4 1	9 2 6 8 4.7 5	2 5 1 6 2 1 4.2 9
6 8 1 2.0 0	3 5 7 8 9.4 4	1 2 6 1 4 2 6.4 1
2 3 2 7 1 4.1 8	1 6 9 2 9.7 5	1 0 1 9 0 2 7.6 0
9 8 2.7 5	1 8 9 2 7.9 5	9 9 1 8 3 3.4 0
1 2 5 6 8.3 0	3 7 4 2 6 1.4 2	7 2 1 1 6 4.7 4
2 2 9 9 4 2.7 8	1 0 2 6 4 0 6.4 7	3 8 9 9 4 6.3 6
2 6 0 5 4.9 0	6 2 1 4 5 7.9 2	1 9 4 1 7 2.4 5
2 2 6 4 9 1.3 7	7 4 2 4 6 7.4 8	9 1 6 9 3 8.5 5
8 3 6 2.4 0	6 9 2 4 1 6.2 4	1 2 0 1 2.4 0
4 2 6 8 3 4.7 5	5 2 1 2 0 7.8 4	6 4 1 2.7 5
8 2 9 6 4.3 5	3 7 2 9 2.2 0	1 6 1 6 9 1 2.6 2
3 5 9 4 8 2.2 0	1 2 0 4 0.0 0	7 5 1 8.9 5
7 9 6 0.0 0	7 2 4 5.6 9	1 6 2 1 9.7 5
8 6 5 2 1.4 6	3 6 0 4 7.4 6	1 4 1 7 5 2 1.6 4
7 5 4 3 7 6.5 0	6 9 4 1 1 6.5 2	1 1 2 1 4 9 0.7 5
1 0 1 6 8 2 3.2 5	7 2 1 2 0 5.1 1	6 9 8 7 6.6 2
6 2 9 4 3.1 2	6 2 1 4 7 6.0 1	7 2 4 1 3 8.7 5
4 8 3 6.9 0	7 4 1 6 2 7.0 3	1 2 6 4 2 1.2 6
9 2 7 8 5 9.4 6	6 2 1 2 6 4.7 5	2 1 7 2 2 1.6 4
2 6 2 5 8 2.7 4	6 9 1 2 8.6 0	6 9 2 1 4.7 5
1 2 6 1 2 9.4 5	1 0 1 2 0 1 6.5 9	2 1 1 1 6 9 4.4 1
9 6 8 5 2.1 7	9 9 2 0 1 7.2 9	9 9 9 9 4 0.6 9
1 4 9 1 2.1 8	1 2 1 2 0 2.6 0	7 2 1 2 6.7 3
2 5 1 9 2 6.8 6	1 0 1 4 2 7.6 5	2 5 1 3 2 7.1 4
4 6 5 3 2.4 5	2 4 1 6 5 9.6 4	7 5 9 7 8 3.9 6
2 2 6 1 4.9 2	6 9 3 9 8.9 5	1 6 1 2 6.4 7
4 6 2 2 8 4.2 0	7 0 1 2 0.9 2	7 2 1 2.4 5
9 7 0 5 2.0 4	6 9 5 4.8 7	2 7 7 2 6 9.1 8
6 8 0 5 2.6 5	1 2 6 7 0 2 6.5 4	1 8 9 2 4 7.9 0
1 2 6 1 2 5.6 5	3 2 1 6 1 4.7 0	6 9 2 4 6.7 8
3 7 5 8 0.5 4	1 6 2 3 6 7.9 0	2 4 1 2 6.2 5

42. Some accountants practice adding two columns at once when the columns are short. The method generally employed is similar to the method explained for combining groups in regular addition.

43. Example. Find the sum of 83, 72, 89.

SOLUTION. Beginning at the bottom and adding up, think of 89 and 72 as 159 and 2, or 161; of 161 and 83 as 241 and 3, or 244.
In adding name results only. Thus say *159*, *161*, *241*, *244*.

$$\begin{array}{r} 83 \\ 72 \\ 89 \\ \hline 244 \end{array}$$

ORAL EXERCISE

By inspection give the sum of each of the following groups:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
43	64	52	37	65	38	52	85	93	68	58	76	83	57	62
25	18	29	56	27	43	67	34	72	75	46	39	47	25	39

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
53	52	61	34	91	68	48	24	78	54	94	57	92	76	43
46	43	37	76	13	47	69	96	76	35	36	44	37	31	56

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.	43.	44.	45.
65	44	46	48	67	44	53	25	54	46	33	16	67	83	88
86	57	65	25	48	57	45	31	65	39	64	34	43	82	25
75	21	34	31	39	21	67	69	87	87	77	25	41	98	31

HORIZONTAL ADDITION

44. In some kinds of invoicing and in short-extending the items of an account numbers to be added are written in horizontal lines. Much time may be saved by adding these numbers as they stand. After careful practice it will be found possible to add numbers written in horizontal lines with as much facility as numbers written in vertical columns.

45. In adding numbers written horizontally care should be exercised to combine only units of the same order. It is generally best to add from left to right and to verify the work from right to left. Grouping may be employed to advantage in horizontal addition.

WRITTEN EXERCISE

Copy and add the following numbers horizontally. Verify the work.

Thus, in problem 1, beginning at the left, say *10*, *20*, *32*, *52*. In verifying the work from the right say *20*, *32*, *42*, *52*.

1. 8, 2, 1, 1, 7, 1, 4, 6, 2, 3, 8, 9.
2. 7, 9, 6, 5, 4, 8, 7, 4, 3, 7, 3, 1, 3.
3. 6, 2, 4, 8, 3, 1, 7, 6, 4, 2, 8, 9, 4, 2.
4. 15, 23, 46, 83, 29, 35, 42, 15, 21, 26.
5. 64, 48, 56, 35, 47, 87, 32, 45, 67, 91.
6. 52, 64, 86, 28, 76, 41, 15, 32, 12, 87.
7. 32, 48, 24, 62, 85, 14, 63, 54, 78, 94, 23, 45.
8. 42, 76, 49, 81, 17, 42, 17, 19, 21, 43, 64, 17.
9. 45, 48, 34, 46, 48, 53, 25, 42, 35, 56, 70, 10.
10. 291, 196, 855, 578, 210, 354, 102, 232, 241, 162.
11. 469, 388, 962, 764, 351, 899, 111, 232, 190, 175.
12. 15^{25}, 50^{25}, 16^{84}, 31^{42}, 86^{38}, 19^{10}, 23^{12}, 10^{13}, 64^{80}, 40^{10}.

It is frequently desirable to express dollars and cents without the dollar sign and the decimal point. This may be done by slightly raising the cents of the amount. Thus, \$ 17.17 may be written 17^{17}; \$ 2.08 may be written 2^{08}.

13. 15^{25}, 8^{93}, 4^{88}, 21^{84}, 16^{35}, 18^{46}, 29^{14}, 44^{60}, 62^{90}, 84^{60}, 40^{50}.
14. 76^{75}, 84^{97}, 67^{05}, 95^{74}, 68^{63}, 52^{21}, 13^{25}, 42^{18}, 60^{95}, 80^{13}, 90^{62}.

46. It is important that the student acquire the ability to carry a series of numbers in mind. The following exercises are suggestive of what may be done to cultivate ability in this direction.

The dictation suggested should not be slower than at the rate of one hundred twenty words per minute. Nothing should be written by the students until all of the numbers of a problem have been called by the teacher; then one student may be sent to the blackboard and required to write the numbers from memory. If the numbers are correctly written, the teacher may require another student to give the sum of them without using pen or pencil. The numbers may be written on the board in either vertical or horizontal order as the teacher may direct.

ORAL EXERCISE

From the teacher's dictation mentally find the sum of each of the following problems:

1. 6, 9, 8, 4, and 8 are how many?
2. 14, 17, 20, and 5 are how many?
3. 24, 17, 16, and 9 are how many?
4. 5, 6, 7, 1, and 3 are how many?
5. 6, 2, 8, 1, and 7 are how many?
6. 364, 436, and 657 are how many?
7. 438, 212, and 750 are how many?
8. 859, 441, and 769 are how many?
9. 2140, 3160, and 4000 are how many?
10. 200, 415, 600, and 920 are how many?
11. 857, 643, 237, and 500 are how many?
12. $4150, $4050, and $5000 are how many?
13. $5.15, $2.15, and $6.70 are how many?
14. $167.14, $232.86, and $150 are how many?

WRITTEN REVIEW EXERCISE

1. Find the sum of all the integers from 2165 to 2260 inclusive.

2. Find the sum of all the integers from 1137 to 1200 inclusive.

3. Complete the following sales sheet. Add by columns and by lines and check the work by adding the vertical and horizontal totals.

SUMMARY OF SALES FOR WEEK ENDING AUG. 25

	PINE	OAK	MAPLE	SPRUCE	WALNUT	CHERRY	TOTAL
Monday	1216 18	16161 47	649 58	860 40	315 64	186 50	
Tuesday	5160 40	3214 90	316 40	160 50	513 80	216 54	
Wednesday	6152 18	2150 18	163 59	430 17	968 52	756 14	
Thursday	1216 18	2160 50	130 98	115 67	413 60	314 75	
Friday	4160 80	1215 40	315 16	218 90	411 50	132 75	
Saturday	3165 80	2115 72	218 50	165 37	118 50	17 05	
Total							

4. Add the following by columns and by lines, and check the work by adding the vertical and horizontal totals :

21162	49	962	18	1245	76	54168	97	52	19		
176	19	1278	95	52698	13	7529	87	95162	87		
2164	89	7524	16	47612	87	6842	23	5948	23		
76	95	87	14	2150	49	172	93	1745	86		
51276	92	18187	95	75	19	162	14	5290	18		
9834	18	92923	15	25	91	162	18	14	95		
754	95	2167	92	2584	16	9176	92	3164	82		
1356	05	1314	93	7125	95	2167	18	2645	97		
756	92	142	18	167	42	926	44	3167	18		
75162	19	82195	78	72162	18	9165	97	168	44		
7162	95	4167	18	7156	95	172	18	1	56		
2	15	6843	82	3954	05	60	65	9	18		
8	85	9162	19	5144	65	8162	18	91684	57		
2416	45	1829	32	4217	64	1492	95	8647	64		
168	94	257	16	417	86	952	17	347	18		

5. Complete the following sales sheet. Add by columns and by lines and then check the work by adding the vertical and horizontal totals.

SUMMARY OF CLERKS' DAILY SALES

NAMES OF CLERKS	MONDAY		TUESDAY		WEDNESDAY		THURSDAY		FRIDAY		SATURDAY		TOTAL FOR WEEK	
J. E. Snow	167	18	194	67	98	46	241	80	175	66	314	90		
W. B. Moore	78	20	65	14	50	42	60	93	51	19	64	86		
T. B. Welch	112	40	118	64	192	40	146	18	110	50	140	12		
E. H. Ross	164	90	143	18	192	64	214	10	110	60	190	18		
Minnie Davis	165	19	214	78	120	42	167	18	164	27	140	51		
Ada Benton	68	49	90	81	64	75	120	14	142	16	60	90		
Elmer S. Frey	240	18	920	41	718	52	167	59	840	72	143	86		
Joseph White	22	49	72	86	51	47	62	14	91	26	72	15		
Margaret Dix	47	26	91	18	21	64	18	42	61	19	64	86		
F. O. Beck	127	16	95	27	114	82	162	15	102	15	112	61		
L. O. Avery	214	91	218	46	920	41	172	14	152	86	142	71		
B. W. Snyder	162	14	153	46	118	64	162	14	182	15	69	58		
Ella Harding	21	27	18	92	17	65	28	64	59	18	72	41		
Carrie Simpson	21	18	45	30	16	98	42	41	20	68	75	98		
W. F. Baldwin	162	10	114	80	115	90	116	84	117	41	200	60		
E. O. Burrill	84	90	90	10	116	80	114	30	65	20	300	75		
Total														

6. Without copying, find the total population of the United States at each census from 1860 to 1900 inclusive. Check.

POPULATION OF THE UNITED STATES AT EACH CENSUS FROM 1860 TO 1900

STATES AND TERRITORIES	1860	1870	1880	1890	1900
Alabama	964,201	996,992	1,262,595	1,513,017	1,828,697
Alaska			33,426	30,329	63,592
Arizona		9,658	40,440	59,620	122,931
Arkansas	435,450	484,471	802,525	1,128,179	1,311,564
California	379,994	560,247	864,694	1,208,130	1,485,053
Colorado	34,277	39,864	194,327	419,198	539,700
Connecticut	460,147	537,454	622,700	746,258	908,420
Dakota	4,837	14,181	135,177		
Delaware	112,216	125,015	146,608	168,493	184,735
District of Columbia	75,080	131,700	177,624	230,392	278,718
Florida	140,424	187,748	269,493	391,422	528,542
Georgia	1,057,286	1,184,109	1,542,180	1,837,353	2,216,331
Hawaii					154,001
Idaho		14,999	32,610	84,385	161,772
Illinois	1,711,951	2,539,891	3,077,871	3,826,351	4,821,550
Indiana	1,350,428	1,680,637	1,978,301	2,192,404	2,516,462
Indian Territory				179,321	392,060
Iowa	674,913	1,194,020	1,624,615	1,911,896	2,231,853
Kansas	107,206	364,399	996,096	1,427,096	1,470,495
Kentucky	1,155,684	1,321,011	1,648,690	1,858,635	2,147,174
Louisiana	708,002	726,915	939,946	1,118,587	1,381,625
Maine	628,279	626,915	648,936	661,086	694,466
Maryland	687,049	780,894	934,943	1,042,390	1,188,044
Massachusetts	1,231,066	1,457,351	1,783,085	2,238,943	2,805,346
Michigan	749,113	1,184,059	1,636,937	2,093,889	2,420,982
Minnesota	172,023	439,706	780,773	1,301,826	1,751,394
Mississippi	791,305	827,922	1,131,597	1,289,600	1,551,270
Missouri	1,182,012	1,721,295	2,168,380	2,679,184	3,106,665
Montana		20,595	39,159	132,159	243,329
Nebraska	28,841	122,993	452,402	1,058,910	1,066,300
Nevada	6,857	42,491	62,266	45,761	42,335
New Hampshire	326,073	318,300	346,991	376,530	411,588
New Jersey	672,035	906,096	1,131,116	1,444,933	1,883,669
New Mexico	93,516	91,874	119,565	153,593	195,310
New York	3,880,735	4,382,759	5,082,871	5,997,853	7,268,894
North Carolina	992,622	1,071,361	1,399,750	1,617,947	1,893,810
North Dakota				182,719	319,146
Ohio	2,339,511	2,665,260	3,198,062	3,672,316	4,157,545
Oklahoma				61,834	398,331
Oregon	52,465	90,923	174,768	313,767	413,536
Pennsylvania	2,906,215	3,521,951	4,282,891	5,258,014	6,302,115
Rhode Island	174,620	217,353	276,531	345,506	428,556
South Carolina	703,708	705,606	995,577	1,151,149	1,340,316
South Dakota				328,808	401,570
Tennessee	1,109,801	1,258,520	1,542,359	1,767,518	2,020,616
Texas	604,215	818,579	1,591,749	2,235,523	3,048,710
Utah	40,273	86,786	143,963	207,905	276,749
Vermont	315,098	330,551	332,286	332,422	343,641
Virginia	1,596,318	1,225,163	1,512,565	1,655,980	1,854,184
Washington	11,594	23,955	75,116	349,390	518,103
West Virginia		442,014	618,457	762,704	958,800
Wisconsin	775,881	1,054,670	1,315,497	1,686,880	2,069,042
Wyoming		9,118	20,789	60,705	92,531
Total					

7. Arrange the following data in tabular form, in six columns. Add by columns and by lines and check the work by finding the sum of the vertical and horizontal totals.

The attendance at a state fair for a week was as follows: Monday: officials, 384; other adults, 4162; children, 875; single carriages, 489; double carriages, 164. Tuesday: officials, 437; other adults, 5286; children, 374; single carriages, 315; double carriages, 100. Wednesday: officials, 311; other adults, 11,438; children, 986; single carriages, 721; double carriages, 209. Thursday: officials, 280; other adults, 21,865; children, 8219; single carriages, 914; double carriages, 286. Friday: officials, 118; other adults, 8211; children, 452; single carriages, 136; double carriages, 59. Saturday: officials, 118; other adults, 9164; children, 762; single carriages, 148; double carriages, 56.

8. Arrange in tabular form, in seven columns, with proper headings, the following data. Show (*a*) the total departmental sales, (*b*) the total monthly sales, and (*c*) the total yearly sales. Check the results.

The sales of E. H. Robinson & Co. for the year ending June 30, 1908, were as follows: July, 1907: books, $4162.18; shoes, $9162.17; millinery, $5218.19; dry goods, $27,162.50; gloves, $2816.49; furniture, $9267.50. August: books, $2160.59; shoes, $4162.87; millinery, $6714.92; dry goods, $28,146.92; gloves, $1624.80; furniture, $7247.95. September: books, $6216.45; shoes, $4167.95; millinery, $3142.89; dry goods, $24,167.46; gloves, $2140.17; furniture, $8175.96. October: books, $2786.90; shoes, $4562.18; millinery, $3147.98; dry goods, $22,162.49; gloves, $2478.67; furniture, $8692.14. November: books, $4675.82; shoes, $4864.19; millinery, $6416.90; dry goods, $24,160.92; gloves, $2841.16; furniture, $6418.46. December: books, $8746.90; shoes, $4621.19; millinery, $5162.19; dry goods, $27,127.46; gloves, $4846.19; furniture, $10,614.92. January, 1908: books, $4641.19; shoes, $2462.18; millinery, $4018.60; dry goods, $28,562.14; gloves, $2417.90; furniture, $8642.14. February: books, $2418.64; shoes, $4267.32; millinery, $3742.24; dry goods, $22,140.86;

gloves, $2019.30; furniture, $4867.32. March: books, $4416.95; shoes, $8618.94; millinery, $8437.46; dry goods, $24,162.18; gloves, $2814.92; furniture, $7596.54. April: books, $2486.14; shoes, $2876.90; millinery, $3249.84; dry goods, $22,172.14; gloves, $1865.92; furniture, $8714.95. May: books, $2834.16; shoes, $3547.24; millinery, $4214.90; dry goods, $28,137.56; gloves, $2272.18; furniture, $8416.59. June: books, $2816.32; shoes, $4756.19; millinery, $3952.84; dry goods, $24,167.49; gloves, $2467.14; furniture, $8619.42.

9. Arrange the following data in tabular form, in nine columns, with proper headings. Find the amount of milk delivered by each patron, the amount received at the creamery each day, and the amount received during the week. Check.

There was received at a creamery, during the first week of June, milk as follows: Sunday: from C. D. Allen, 415 lb.; L. B. Brown, 695 lb.; W. D. Carroll, 425 lb.; J. H. Dean, 165 lb.; F. A. Ellis, 726 lb.; J. L. Frey, 920 lb.; I. T. Good, 214 lb.; E. H. Lord, 170 lb. Monday: from C. D. Allen, 416 lb.; L. B. Brown, 702 lb.; W. D. Carroll, 426 lb.; J. H. Dean, 175 lb.; F. A. Ellis, 729 lb.; J. L. Frey, 964 lb.; I. T. Good, 216 lb.; E. H. Lord, 172 lb. Tuesday: from C. D. Allen, 420 lb.; L. B. Brown, 711 lb.; W. D. Carroll, 419 lb.; J. H. Dean, 186 lb.; F. A. Ellis, 728 lb.; J. L. Frey, 963 lb.; I. T. Good, 218 lb.; E. H. Lord, 174 lb. Wednesday: from C. D. Allen, 432 lb.; L. B. Brown, 709 lb.; W. D. Carroll, 430 lb.; J. H. Dean, 176 lb.; F. A. Ellis, 724 lb.; J. L. Frey, 962 lb.; I. T. Good, 217 lb.; E. H. Lord, 178 lb. Thursday: from C. D. Allen, 428 lb.; L. B. Brown, 709 lb.; W. D. Carroll, 427 lb.; J. H. Dean, 178 lb.; F. A. Ellis, 729 lb.; J. L. Frey, 966 lb.; I. T. Good, 217 lb.; E. H. Lord, 173 lb. Friday: from C. D. Allen, 432 lb.; L. B. Brown, 700 lb.; W. D. Carroll, 420 lb.; J. H. Dean, 170 lb.; F. A. Ellis, 746 lb.; J. L. Frey, 980 lb.; I. T. Good, 246 lb.; E. H. Lord, 170 lb. Saturday: from C. D. Allen, 450 lb.; L. B. Brown, 721 lb.; W. D. Carroll, 417 lb.; J. H. Dean, 178 lb.; F. A. Ellis, 740 lb.; J. L. Frey, 920 lb.; I. T. Good, 314 lb.; E. H. Lord, 180 lb.

CHAPTER V

SUBTRACTION

ORAL EXERCISE

State the number that, added to the smaller number, makes the larger one in each of the following:

1.	3	4	4	5	6	7	8	8	9	9	9	9	8	8	7
	1	2	1	3	2	3	3	2	3	1	6	4	4	1	2
2.	12	11	12	11	12	11	12	11	10	11	10	11	10	12	10
	9	2	3	9	8	3	4	8	4	7	6	4	7	5	3
3.	18	17	16	17	16	15	14	15	14	13	13	16	15	14	13
	9	8	7	9	8	6	9	7	8	4	7	9	8	5	9
4.	13	14	14	15	16	17	18	18	19	19	19	19	18	18	17
	11	12	11	13	12	13	13	12	13	11	16	14	14	11	12
5.	22	21	22	21	22	21	22	21	20	21	20	21	20	22	20
	19	12	13	19	18	13	14	18	14	17	16	14	17	15	13
6.	38	27	26	37	26	35	44	25	34	53	43	36	45	54	73
	29	18	17	29	18	26	39	17	28	44	37	29	38	45	69
7.	42	51	72	81	92	71	32	41	70	61	90	81	30	62	50
	39	42	63	79	88	63	24	38	64	57	86	74	27	55	47

47. A **parenthesis** ( ) signifies that the numbers included within it are to be considered together. A **vinculum** $\overline{\quad}$ has the same signification as a parenthesis.

Thus, $15 - (4 + 2)$, or $15 - \overline{4 + 2}$ signifies that the sum of 4 and 2 is to be subtracted from 15.

48. Examples. 1. Find the difference between 849 and 162.

SOLUTION. 2 from 9 leaves **7**. 6 cannot be subtracted from 4, but 6 from 14 leaves **8**. Since 1 of the 8 hundreds has been taken, there are but 7 hundreds remaining. 1 from 7 leaves **6**.

CHECK. $687 + 162 = 849$.

$$\begin{array}{r} 849 \\ 162 \\ \hline 687 \end{array}$$

The above is a common method of subtraction. For practical computation, however, the "making change" method is best. It is easily understood and is much more rapid when once learned. The "making change" method is illustrated in the following example and solution.

2. Find the difference between 7246 and 4824.

SOLUTION. Think "$4 + 2 = 6$," and write 2; "$2 + 2 = 4$," and write 2; "$8 + 4 = 12$," and write 4; "1 and $4 \times 2 = 7$," and write 2.

CHECK. $2422 + 4824 = 7246$.

$$\begin{array}{r} 7246 \\ 4824 \\ \hline 2422 \end{array}$$

ORAL EXERCISE

1. $16 + 23 + ? = 54?$
2. $27 + 14 + ? = 72?$
3. $17 + 36 + ? = 62?$
4. $19 + 17 + 12 + ? = 57?$
5. $25 + 14 + 11 + ? = 75?$
6. $18 + 17 + 16 + ? = 70?$
7. $16 + 18 + 16 = 25 + ?$
8. $72 + 17 + 11 = 37 + ?$
9. $14 + 18 + 38 = 42 + ?$
10. $12 + 16 + 12 + 14 + ? = 75?$
11. $16 + 15 + 19 + 15 + ? = 93?$
12. $18 + 17 + 15 + 29 + ? = 98?$

WRITTEN EXERCISE

1. Without copying the individual problems, find quickly the sum of the twenty differences in the following:

\$2140.50	\$4157.50	\$5000.24	\$9000.72	\$3145.62
714.23	1236.80	249.17	1246.18	2000.79
\$5500.89	\$1624.14	\$1985.72	\$1379.54	\$1742.18
2799.14	957.80	645.92	923.18	842.16
\$9275.17	\$2446.80	\$3169.14	\$3156.19	\$4756.83
842.99	1321.44	874.36	1400.72	2738.44
\$7514.85	\$7291.80	\$1756.92	\$8721.13	\$1872.14
721.92	1642.95	921.74	2049.79	742.12

2. Copy the following table and show (*a*) the total exports for each year given; (*b*) the excess of exports for each year given; (*c*) the total exports and imports for the eleven years; (*d*) the total excess of exports for the eleven years. Check.

IMPORTS AND EXPORTS IN THE UNITED STATES FOR TEN YEARS

YEAR ENDING JUNE 30	EXPORTS		TOTAL EXPORTS	IMPORTS	EXCESS OF EXPORTS
	Domestic	Foreign			
1895	$793,392,599	$14,145,566		$731,969,965	
1896	963,200,487	19,406,451		779,724,674	
1897	1,032,007,603	18,985,953		764,730,412	
1898	1,210,291,913	21,190,417		616,050,654	
1899	1,203,931,222	23,092,080		697,148,489	
1900	1,370,763,571	23,719,511		849,941,184	
1901	1,460,462,806	27,302,185		823,172,165	
1902	1,355,481,861	26,237,540		903,320,948	
1903	1,392,231,302	27,910,377		1,025,719,237	
1904	1,491,744,641	25,910,377		991,090,978	
1905	1,491,744,641	26,817,025		1,117,513,071	
Total					

49. The common method of **making change** is to add to the price of the goods purchased a sum that will equal the amount offered in payment.

Thus, if a person buys groceries amounting to 74¢ and tenders $1 in payment, the mental process of the clerk in making the change is as follows: "74¢ + 1¢ + 25¢ = $1"; the customer should receive as change a 1-cent, piece and a quarter of a dollar.

Obviously, the change may usually be made in a number of ways. In the above example two dimes and a 5-cent piece might be given instead of the quarter of a dollar. But, as the different bills and coins are usually sorted in the till, the experienced clerk generally makes change in the simplest way; that is, with the largest possible denominations. In the following exercise name the largest coins and bills that could be used.

ORAL EXERCISE

1. Name the coins and the amount of change to be given from $1 for each of the following purchases: 17¢; 24¢; 31¢; 38¢; 45¢; 52¢; 59¢; 66¢; 73¢; 80¢; 87¢; 18¢; 25¢; 32¢; 29¢; 46¢; 53¢; 60¢; 67¢; 74¢; 81¢; 88¢.

2. Name the coins and the amount of change to be given from $2 for each of the following purchases: $1.19; $1.26; $1.33; $1.40; $1.47; $1.54; $1.61; $1.68; $1.75; $1.82; $1.89; $1.20; $1.27; $1.34; $1.41; $1.48; $1.55; $1.62; $1.69; $1.76; $1.83; $1.90.

3. Name the bills and coins and the amount of change to be given from $5 for each of the following purchases: $1.21; $1.28; $1.35; $1.42; $2.22; $2.29; $2.36; $4.43; $3.49; $4.50; $3.51; $3.56; $4.57; $2.58; $1.63; $2.64; $1.65; $1.70; $2.71; $3.72; $2.77; $3.84; $1.91; $2.85; $2.92.

4. Name the bills and coins and the amount of change to be given from $10 for each of the following purchases: $4.93; $3.86; $7.70; $2.44; $8.37; $5.30; $3.23; $5.17; $4.24; $3.31; $8.38; $2.45; $6.52; $4.59; $3.66; $5.73; $4.80; $3.87; $2.88; $7.81; $9.74; $5.67; $3.60; $4.53; $2.46; $3.29; $8.32; $7.25; $2.18; $7.49; $9.42; $3.67; $1.93.

50. It is frequently necessary to find the difference between a minuend and several subtrahends. If the "making change" method of subtraction is employed, the operation is a simple one.

51. Example. From a farm of 578 A. I sold at one time 162 A., at another 98 A., and at another 121 A. How many acres remained unsold?

SOLUTION. Arrange the numbers as shown in the margin. Eleven $(1+8+2)$ and seven are 18; write 7. Three (1 carried $+2$), eighteen $(3+9+6)$ and nine are twenty-seven; write 9. Four (2 carried $+1+1$) and one are 5; write 1.

CHECK. $197+121+98+162=578$.

578 A.
162 A.
98
121
197 A.

WRITTEN EXERCISE

Find the amount each person has remaining on deposit:

1. A. Deposit, $900; checks, $210, $175, $198.
2. B. Deposit, $875; checks, $157, $218, $157.
3. C. Deposit, $750; checks, $120, $117, $121, $118.
4. D. Deposit, $960; checks, $128, $109, $118, $117.

5. E. Deposit, $967; checks, $192, $102, $117, $128, $146.

6. F. Deposit, $998; checks, $119, $117, $105, $123, $173.

Do not neglect to check all work. The bank clerk who makes an error a day in work like the above, and who fails to discover and correct this error, will not long retain his position.

7. Copy the following, supplying the missing terms and checking the results:

$$\begin{array}{rrrr}
\$148.90 + & \$149.75 + & \$421.77 = & \$???.?? \\
118.60 + & 172.12 + & ???.?? = & ???.?? \\
242.30 + & ???.?? + & 210.96 = & ???.?? \\
???.?? + & 168.72 + & 130.41 = & ???.?? \\
\hline
\$718.95 + & \$698.75 + & \$978.60 = & \$???.??
\end{array}$$

The following problem shows a portion of a bank discount register. In the first column are recorded the amounts of several notes that have been discounted; in the second, the discount charges; and in the third, the collection and exchange charges. The proceeds of any note is the difference between the amount (face) of the note and the total charges upon it.

8. Copy and complete the following bank record. Check the work. ($j + i + h$ should equal g.)

Face of Paper		Discount		Coll. & Exch.		Proceeds	
729	14	7	29		73	a	
862	29	4	31		86	b	
725	74	7	26		73	c	
832	16	12	48	1	26	d	
426	19	6	39		43	e	
378	36	3	78		38	f	
g		h		i		j	

52. The **complement** of a number is the difference between the number and a unit of the next higher order.

Thus, 2 is the complement of 8, 23 is the complement of 77, and 152 is the complement of 848. 3 and 7, 24 and 76, 250 and 750, are complementary numbers. Observe that *when two numbers of more than one figure each are complementary, the sum of the units' figure is 10 and the sum of the figures in each corresponding higher order is 9.*

53. Since numbers are read from left to right, in finding the complement of a number, begin at the left to subtract.

54. In beginning at the left to subtract take 1 from the highest order in the minuend and regard the other orders as 9's, except the last, which regard as 10.

55. Example. A man gave a 100-dollar bill in payment for an account of $77.52. How much change should he receive?

SOLUTIONS. (*a*) Begin at the left. 7 from 9 leaves **2**; 7 from 9 leaves **2**; 5 from 9 leaves **4**; 2 from 10 leaves **8**. Or

(*b*) 7 and **2** are 9; 7 and **2** are 9; 5 and **4** are 9; 2 and **8** are 10. $22.48.

$100.00
77.52
$22.48

This method of finding the *amount of change* is used by many clerks and cashiers. The work is in all cases proved by counting out to the customer the bills and coins necessary to make the amount of the purchase equal to the amount offered in payment.

ORAL EXERCISE

State the difference between the following amounts:

1.	$1.00	$1.00	$1.00	$1.00	$1.00	$1.00	$1.00	$1.00
	.22	.29	.36	.85	.78	.64	.57	.56
2.	$1.00	$2.00	$3.00	$4.00	$5.00	$6.00	$7.00	$8.00
	.54	1.36	2.02	2.17	2.23	5.01	5.23	7.21
3.	$10.00	$10.00	$10.00	$10.00	$10.00	$10.00	$10.00	$10.00
	8.75	5.63	4.68	5.35	2.38	2.89	1.51	8.35
4.	$50.00	$50.00	$50.00	$50.00	$50.00	$50.00	$50.00	$50.00
	28.14	17.49	11.52	16.84	14.89	12.52	19.64	21.87

5. If $100 is offered in payment for each of the following bills, what amount of change should be returned? $27.42; $89.17; $64.11; $53.41; $18.75; $23.14; $37.48; $87.37.

6. If $20 is offered in payment for each of the following bills, what amount of change should be returned? $4.72; $8.17; $19.21; $17.41; $2.46; $17.48; $11.42; $7.43; $12.64; $11.42; $4.96; $1.16; $7.25; $15.98; $16.87; $14.17; $13.56.

ORAL EXERCISE

State the amount of change in each of the following problems:

	Cost of Items Purchased	Amount Paid		Cost of Items Purchased	Amount Paid
1.	17¢, 13¢, 42¢	$2	14.	$1.25, $0.75, $2.18	$20
2.	27¢, 23¢, 14¢	$2	15.	$1.50, $2.70, $1.18	$20
3.	45¢, 55¢, 13¢	$5	16.	$4.60, $1.40, $2.13	$20
4.	64¢, 16¢, 87¢	$5	17.	$1.50, $1.20, $2.30	$10
5.	23¢, 14¢, 27¢	$2	18.	$3.17, $4.11, $4.98	$50
6.	63¢, 17¢, 59¢	$5	19.	$4.25, $0.75, $3.18	$20
7.	49¢, 84¢, 37¢	$5	20.	$1.29, $2.17, $1.50	$20
8.	78¢, 42¢, 67¢	$5	21.	$1.64, $1.66, $2.50	$20
9.	52¢, 69¢, 88¢	$5	22.	$1.59, $23.41, $118	$200
10.	75¢, 86¢, 54¢	$5	23.	$24.17, $20.83, $15	$100
11.	89¢, 46¢, 72¢	$5	24.	$11.48, $10.52, $50	$100
12.	76¢, 54¢, 29¢	$5	25.	$18.91, $12.09, $45	$100
13.	75¢, 25¢, 89¢	$10	26.	$21.27, $2.73, $50.50	$100

56. $19-7=9$ (the minuend minus 10) + 3 (the complement of the subtrahend); $191-17=91$ (the minuend minus 100) + 83 (the complement of the subtrahend); $1912-178=912$ (the minuend minus 1000) + 822 (the complement of the subtrahend), and so on.

57. This principle makes it a simple matter to find the difference between a subtrahend and several minuends.

58. Examples. The following examples illustrate the application of the principle:

	1.	2.	3.
	316	299	311
	+478	+488	+111
	−118	−111	−219
	=676	=676	=203

Solutions. **1.** 2 (the complement of 8), 10, 16; $16-10=6$. 9 (the complement of 1), 16, 17; $17-10=7$. 9, 13, 16; $16-10=6$.

2. 9, 17, 26; $26-10=16$; that is, **6** and 1 to add to the minuends. 9, 18 (9+8+1), 27; $27-10=17$; that is, **7** and 1 to add to the minuends. 9, 14, 16; $16-10=6$.

3. 1, 2, 3. $3-10$ is impossible, so subtract 1 ten from the minuend (or add 1 ten to the subtrahend). 9, 10. $10-10=0$. 8, 9, 12. $12-10=2$.

59. Example. The following problem shows a concrete application of the foregoing principle:

DEPOSITORS' LEDGER

DEPOSITOR	BALANCE	CHECKS	DEPOSITS	BALANCE
A	$ 74	$ 25	$ 86	$ 135
B	$ 86	$ 11	$ 99	$ 174
C	$ 92	$ 79	$ 81	$ 94

SOLUTION. Here is a depositors' ledger. The data in the first three columns being given, it is required to find the new balance.

The process is as follows: A. 6, 11, 15, **5**; 8, 16, 23, **13**; balance, $135. B. 9, 18, 24, **4** and 1 to add to the minuend. 10, 19, 27, **17**; balance, $174. C. 1, 2, **4** and 1 to take away from the minuend. 7, 10, 19, **9**; balance, $94.

WRITTEN EXERCISE

Find the new balances, the total old balance, the total checks, the total deposits, the total new balances, and check the work:

1.

DEPOSITOR	BAL.	CHECKS	DEPOSITS	BAL.
A	$ 758	$ 128	$ 421	*a*
B	921	154	175	*b*
C	934	214	122	*c*
D	862	162	218	*d*
E	478	187	126	*e*
F	921	215	124	*f*
G	756	157	137	*g*
H	864	128	142	*h*
I	926	214	121	*i*
J	752	221	124	*j*
K	878	162	218	*k*
	l	*m*	*n*	*o*

2.

DEPOSITOR	BAL.	CHECKS	DEPOSITS	BAL.
A	$ 428	$ 125	$ 718	*a*
B	726	128	296	*b*
C	832	279	318	*c*
D	456	154	421	*d*
E	298	275	568	*e*
F	728	178	188	*f*
G	762	218	215	*g*
H	837	316	176	*h*
I	493	121	219	*i*
J	862	128	188	*j*
K	925	125	211	*k*
	l	*m*	*n*	*o*

60. $48 - 29 = \overline{48 + 1}$ (30, the next higher order of units than 29, $-$ 29) $-$ 30, or 19; $128 - 59 = \overline{128 + 1} - 60$, or 69.

61. This principle may be applied to advantage in billing items in which the gross weights and the tares are recorded.

The **gross weight** is the weight of merchandise, together with bag, cask, or other covering; the **tare** is the weight of the bag, cask, or other covering

of merchandise; the **net weight** is the difference between the gross weight and the tare.

62. Example. The gross weights and tares, in pounds, of 3 bbl. of sugar are: 332 – 19, 337 – 18 335 – 18. Find the total net weight.

SOLUTION. The numbers would be written on the bill horizontally, as shown in the margin. Adding the units of the tare, the result is 25; 30 (the next higher order of units than 25) minus 25 equals 5; 5 added to the units of the gross weight equals 19; 19 – 30 is impossible, so write **9** and subtract 2 tens (the difference between the tens in 30 and 19) from the gross weight or add 2 tens to the tens of the tare. Adding 2 tens to the tens of the tare, the result is 5; 10 – 5 = 5; 5 added to the tens of the gross weight equals 14; 14 – 10 = **4**. Adding the hundreds in the gross weight, the result is **9**. Net weight is 949 lb.

332 – 19	337 – 18	335 – 18	949#

WRITTEN EXERCISE

Copy the following bills. Verify the net weights given and supply all missing terms.

1.

CHICAGO, ILL., June 12, 19

Messrs. McGraw, Sprague & Co.,
Tonawanda, N. Y.

Bought of PHILIP ARMOUR & CO.

Terms 60 days

	3	tubs Lard							
		70-14 69-14 71-15	167	10#	16	70			
	6	casks Shoulders							
		422-64 428-70 424-65							
		420-64 426-70 424-70	2141	11#	235	51			
	5	casks Hams							
		410-70 411-70							
		408-70 412-71 416-69	****	12#	204	84	****		

2.

Chicago, Ill., July 20, 19

Messrs. A. M. THOMPSON & CO.
Rochester, N.Y.

Bought of **Nelson, Morris & Co.**

Terms 30 days

6	tubs Lard 72-17 70-14 69-14 71-14 71-15 70-16	***	$0.11	36	63	
6	casks Shoulders 421-65 426-70 424-72 422-64 427-72 421-60	****	.12	256	56	
6	casks Hams 409-72 412-70 414-71 410-73 412-70 416-71	****	.12	245	52	*****

3. The gross weights and tares of 6 casks of shoulders are as follows: 428 – 68, 419 – 70, 423 – 65, 432 – 72, 436 – 69, 434 – 65 lb. Find the total net weight.

4. The gross weight and tares of 12 tubs of lard are as follows: 71 – 14, 70 – 15, 69 – 14, 71 – 15, 72 – 17, 73 – 17, 69 – 15, 71 – 16, 72 – 15, 73 – 16, 74 – 17, 75 – 17 lb. Find the total net weight.

5. The gross weights and tares of 10 bbl. of sugar are as follows: 319 – 18, 331 – 19, 329 – 17, 334 – 20, 338 – 21, 325 – 18, 326 – 16, 325 – 19, 327 – 19, 321 – 17 lb. Find the total net weight.

BUSINESS TERMS AND RECORDS

63. A **debit** is an expression of value received; a **credit** is an expression of value delivered.

A buys of B 100 bu. wheat for $100 cash; the value received (debit) by A is *100 bu. wheat* and the value parted with (credit), *$100.* A sells C 50 bu. wheat for $75, C agreeing to pay for the same in 10 da.; the value received by A is *C's express* or *implied promise* to pay for the wheat in 10 da. and the value parted with is *50 bu. wheat.*

64. An **account** is a collection of related debits and credits.

65. Some of the **common accounts** kept in business are the cash account; personal accounts; the merchandise account; the expense account; the proprietary account.

66. A **resource** is any property on hand or any amount owed to a person or concern; a **liability** is any amount owed by a person or concern. The excess of resources over liabilities is the **net capital** or **present worth**; the excess of liabilities over resources, the **net insolvency.**

67. A **gain** is any sum realized in excess of the cost of a business or of business transactions; a **loss** is any sum spent or incurred in excess of the returns of a business or of business transactions. The excess of gains over losses is the **net gain**; the excess of losses over gains, the **net loss.**

68. The **cash account** is kept for the purpose of showing the receipts and payments of cash and the amount of cash on hand.

Cash

19–					19–				
Jan.	1	L. O. White	1200	–	Jan.	6	Office expense	169	50
	5	B. E. Smith	175	–		12	Rent of store	75	–
	9	Mdse.	69	50		12	Balance	1200	–
			1444	50				1444	50
Jan.	12	Balance	1200	–					

The receipts of cash are entered on the left or debit side, and the payments, on the right or credit side, of the account. The excess of debits at any time is the amount of cash on hand.

69. Personal accounts are kept for the purpose of showing whether persons owe us or we owe them, and how much in either case.

J. E. King & Co.

19–						19–					
Feb.	3	Mdse.	721	60		Feb.	15	Discount	7 22		
								Cash	714 38	721	60
Mar.	2	Mdse.	114	90		Apr.	1	Cash		114	90
Apr.	15	Mdse.	610	80		Apr.	25	Discount	6 11		
								Cash	604 69	610	80
May	10	Mdse.	421	—							

On the left (debit) side of these accounts are placed the amounts which the persons owe us or which we pay them; on the right (credit) side, the amounts which we owe them or which they pay us. When the debits of an account are in excess of the credits, the account owes us for the amount of the excess; when the credits are in excess of the debits, we owe the account for the amount of the excess.

70. The **merchandise account** is kept for the purpose of showing the cost of goods purchased, the proceeds of goods sold, and the gain or loss resulting from such dealings.

Merchandise

19–						19–					
Mar.	1	Unsold goods	960	—		Mar.	5	Cash		430	60
	6	W. E. Haynes	160	70			7	F. C. Hill		529	40
	9	L. W. Lawton	720	05			14	L. W. King		360	15
	12	W. E. Haynes	160	85			16	R. O. Cook		360	—
	18	Gain	275	05			18	Cash		65	90
							18	Unsold goods		530	60
			2276	65						2276	65
Mar.	18	Unsold goods	530	60							

On the left (debit) side is entered the cost of goods purchased and on the right (credit) side the proceeds of goods sold. When the goods are all disposed of the excess of credits is a gain; the excess of debits, a loss. When it is desired to show the gain or loss on merchandise before the goods are all disposed of, it is necessary to first enter in the credit side of the account the present market value of the unsold goods.

71. The **expense account** is kept for the purpose of showing the cost of outlays incurred in carrying on the business.

Expense

19–					19–				
Apr.	1	Stationery		16 50	Apr.	9	Coal on hand		12 40
	6	Coal		19 40		9	Loss		55 30
	6	Clerk hire		20 90					
	9	Drayage		10 90					
				67 70					67 70
Apr.	9	Coal on hand		12 40					

Such outlays are entered on the left (debit) side of the account. Ordinarily there are no credit entries. When the expense items are all used the debit of the account is a loss. When it is desired to show the loss or gain on expense and there are unused expense items on hand, it is first necessary to enter in the credit side of the account the present value of such items.

72. The **proprietary account** is kept for the purpose of showing whether the proprietor owes the business or whether the business owes him, and how much in either case.

F. W. Simpson, Proprietor

19–					19–				
May	10	Cash		160 –	May	1	Cash		6000 –
	31	Present Worth		6009 90		31	Net Gain		169 90
				6169 90					6169 90
					June	1	Present Worth		6009 90

On the right (credit) side are entered all sums invested and the net gain, and on the left (debit) side all sums withdrawn and the net loss. The excess of credits is the present worth of the business.

ORAL EXERCISE

1. In the cash account on page 41 what are the total receipts? the total payments? the balance of cash on hand?

2. At the top of page 42 is your account with J. E. King & Co. On what dates did you sell the firm merchandise? When and how were payments made on account? What was the balance of the account May 10?

3. In the account with merchandise, page 42, what is the cost of the purchases? the proceeds of the sales? How would the value of the unsold goods be determined in business? Verify the amount of the gain. Is it correct?

4. Verify the amount of the loss in the expense account, page 43. Is it correct?

5. What are the total withdrawals in the account with F. W. Simpson, Proprietor, page 43? the total investment?

WRITTEN EXERCISE

1. Copy the cash account on page 41 and continue it with the following items: Jan. 12, receive cash of Jones & Co., $75; Jan. 14, pay cash for groceries, $165.62; Jan. 15, receive cash for groceries, $189.75; Jan. 18, pay cash to office help, $129.74; Jan. 20, pay cash for stationery, $11.75; Jan. 22, receive cash for groceries, $126.94; Jan. 24, receive cash of H. W. Conant, $200,67. Balance the account as shown in the model.

2. Copy the purchases and sales of the merchandise account, page 42. Assuming that the value of the unsold goods is $327.61, find the gain and close the account.

3. Copy the purchases and sales of the merchandise account, page 42. Assuming that the value of the unsold goods is $50, find the gain or loss and close the account. Assuming that all of the goods are sold, find the gain or loss and close the account.

4. Arrange the following data in the form of your account with Benj. F. Butler. June 1, buy of Benj. F. Butler on account (without making payment) dry goods amounting to $627.96; June 10, pay him for invoice of June 1 less $6.28 discount; June 28, buy of him dry goods amounting to $472.69 and pay cash to apply on the bill, $172.69; July 15, buy of him on account dry goods amounting to $369.71; July 31, pay him cash to apply on bill of July 15, $79.79; Aug. 2, sell him lace amounting to $14.60. Find the balance of the account and tell whether such balance is a resource or a liability.

5. Using the above data, write Benj. F. Butler's account of his dealings with you. Balance the account.

6. Copy the account with F. W. Simpson, Prop., page 43. Continue the account through June, using the following items: June 6, make an additional investment of $1000; June 25, withdraw for personal use $160; June 30, the net gain for the month, which is to remain as an additional investment, is $369.75. Find the present worth and close the account.

ORAL EXERCISE

Classify the following as resources, liabilities, losses, or gains:

1. A personal account showing a debit balance of $150.

2. A personal account in which the credit balance is $270.

3. A merchandise account in which there are no goods on hand and the purchases aggregate $7160 and the sales, $8249.50.

4. The total losses of a business are $480, and the net gain, $640.90. What are the total gains?

5. The total liabilities of a concern are $2400, and the present worth, $6280.50. What are the total resources?

WRITTEN EXERCISE

Copy the following statements, supplying the missing terms:

1

Statement of Losses and Gains, July 31, 19-

		Gains				
Merchandise		sales			3470	60
		purchases	4160.10			
		on hand	960.10			
		cost of sales			3200	—
		gain				??? ??
		Losses				
Expense		cost			???	??
		office safe on hand			400	—
		loss				??? ??
F. O. Ely's net gain						192 00

2.

Statement of Resources and Liabilities, July 31, 19–

	Resources				
Merchandise	goods on hand	960	10		
Real Estate	store and lot	4000	—		
Cash	balance on hand	1290	60		
H. E. Gale	owes the business	610	90		
J. C. Bush	" " "	200	40		
F. C. Miller	" " "	150	—	????	??
	Liabilities				
V. M. Cary	business owes him	614	20		
L. S. Martin	" " "	300	80		
C. H. Jones	" " "	???	??	1345	60
	F. O. Ely's present worth			????	??

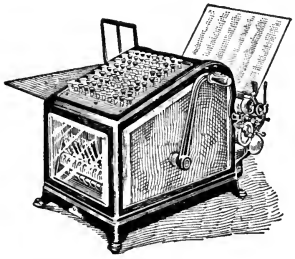
3. A merchant purchased a stock of hardware amounting to $45,112.18 and sold from this stock goods amounting to $31,136.85. He then took an account of stock and found that the value of the hardware on hand was $18,438.50. Find the amount of his gain.

4. C. E. Cyr's resources and liabilities at the close of a month were as follows: dry goods on hand, $1629.40; store and lot, $3000; cash in bank, $1400.60; C. O. Bond owes the business $400; L. E. Young, $390.10; and J. O. Snow, $209.90. The business owes Roe & Co. $750; and Doe & Co. $90.75. Make a statement of resources and liabilities.

5. At the close of the same month C. E. Cyr's business accounts show the following results: stock of dry goods on hand at the beginning of the month, $1270.40; purchases of dry goods for the month, $3229.60; sales of dry goods for the month, $3762.90; market value of the dry goods on hand at the close of the month, $1629.40; expense for the month, $413.95; value of expense items on hand, $250. Make a statement of losses and gains.

6. A real estate agent had property on hand Jan. 1 to the amount of $8155.60. During the year he bought property

costing $4150.60, added buildings at a cost of $6190.40, and paid taxes $250.90. April 15 a house valued at $1690 was destroyed by fire, and for this loss the insurance company paid him $1300. During the year he sold property for $9260.50 and received for rents $840.80. If the expenses of the sales aggregated $240.19 and the value of the property on hand Dec. 31 was $11,250.60, what was his net gain or loss for the year?



73. Banks and other business houses having a large amount of adding to do, frequently use an **adding machine.** Because it cannot be used to advantage for many kinds of addition, this machine has not done away with the necessity for the hand-and-mind method of addition; on the other hand, by its rapid and accurate work, it has put a premium on the hand-and-mind method. Business men will no longer tolerate a bookkeeper who is slow and inaccurate in his additions; but the person who can add with speed, accuracy, and intelligence is more than ever in demand. In the margin is a picture of an adding machine such as is commonly used. The operation of subtraction, or of combined addition and subtraction, may usually be performed on an adding machine.

ORAL REVIEW EXERCISE

1. Find the sum of 45, 45, 45, 45, 45, and 60.
2. Find the sum of 61, 62, 63, 64, 65, 66, and 67.
3. Find the sum of 102, 103, 104, 105, 106, 107, and 108.
4. Find the sum of all the integers from 6 to 12, inclusive.
5. How many days from Apr. 15 to June 2? from Mar. 15 to May 3? from July 30 to Sept. 5?
6. Count backwards rapidly by 5's from 96; by 7's from 97; by 13's from 100; by 12's from 135; by 14's from 99.
7. Subtract each of the following amounts from $50: $24.19, $21.76, $42.14, $13.98, $47.29, $19.32, $16.38, $11.43.

8. State the sum of each of the following groups:

82¢ 79¢ 74¢ 52¢ 92¢ 38¢ 73¢ 69¢ 86¢ 63¢ 42¢ 26¢ 81¢ 27¢
35¢ 18¢ 87¢ 31¢ 85¢ 57¢ 99¢ 34¢ 75¢ 28¢ 95¢ 19¢ 93¢ 41¢

98¢ 46¢ 89¢ 72¢ 59¢ 30¢ 91¢ 80¢ 73¢ 53¢ 66¢ 24¢ 76¢ 43¢
55¢ 15¢ 45¢ 14¢ 88¢ 77¢ 97¢ 54¢ 78¢ 47¢ 62¢ 49¢ 32¢ 11¢

39¢ 13¢ 90¢ 40¢ 96¢ 21¢ 84¢ 56¢ 58¢ 22¢ 48¢ 37¢ 50¢ 25¢
29¢ 12¢ 94¢ 17¢ 83¢ 61¢ 65¢ 33¢ 44¢ 16¢ 70¢ 36¢ 51¢ 23¢

9. State the difference between each of the above groups.

In subtracting 91 and 27 think of 71 and 7, or 64; in subtracting 52 and 29 think of 32 and 9, or 23; and so on.

10. State the difference between \$2 and the sum in each of the above groups; between \$5; between \$10.

11. What change should I receive from \$2 if I spent:

a. 26¢ and 43¢?
b. 17¢ and 59¢?
c. 28¢ and 52¢?
d. 17¢ and 58¢?
e. 25¢ and 37¢?
f. 42¢ and 39¢?
g. 19¢ and 37¢?
h. 16¢ and 29¢?
i. 15¢, 14¢, and 31¢?
j. 11¢, 43¢, and 37¢?
k. 19¢, 34¢, and 47¢?
l. 28¢, 11¢, and 47¢?

12. Add each of the following numbers to each of the numbers below: 2, 8, 7, 6, 5, 4, 9, 11, 12, 3, 14, 15, 16, 13, 18, 17, 19.

First add by lines and then by columns. Thus, to add 7 by lines say 7, 8, 11, 9, 12, 10, 13, 14, 17, 15, 18, 16, 19, 20, and so on; to add 7 by columns say 8, 20, 32, 44, 56, 68, 80, 92, 104, 116, 11, and so on.

	a	b	c	d	e	f	g	h	i	j	k	l
1.	1	4	2	5	3	6	7	10	8	11	9	12
2.	13	16	14	17	15	18	19	22	20	23	21	24
3.	25	28	26	29	27	30	31	34	32	35	33	36
4.	37	40	38	41	39	42	43	46	44	47	45	48
5.	49	52	50	53	51	54	55	58	56	59	57	60
6.	61	64	62	65	63	66	67	70	68	71	69	72
7.	73	76	74	77	75	78	79	82	80	83	81	84
8.	85	88	86	89	87	90	91	94	92	95	93	96
9.	97	100	98	101	99	102	103	106	104	107	105	108
10.	109	112	110	113	111	114	115	118	116	119	117	120

WRITTEN REVIEW EXERCISE

In all exercises of this kind a time limit should be set for the work. The work should also be checked before answers are submitted for examination. Accuracy is of paramount importance in business. One error that passes unnoticed by the student in ten problems of this character is a failure.

1. Without copying, find quickly the missing terms in the following statement of government receipts and expenditures for the fiscal year closing June 30 in a recent year. Check.

Receipts		
From customs	$262,068,483	
Internal revenue	232,435,695	
Miscellaneous	46,682,565	
Total		
Expenditures		
Civil and miscellaneous	$132,229,913	
War	115,337,786	
Navy	102,757,073	
Indians	10,437,196	
Pensions	142,558,335	
Interest	24,618,766	
Total		
Surplus		

2. Without copying, find the totals and grand totals of the following table. Check the results.

Coinage of the Mints of the United States

Calendar Years	Gold	Silver	Minor	Totals
1793 to 1894	$ 1,732,552,323 00	$ 681,909,719 10	$ 25,391,531 79	
1895	59,616,357 50	5,698,010 25	882,430 56	
1896	47,053,060 00	23,089,899 00	832,718 93	
1897	76,028,485 00	18,487,297 30	1,526,100 25	
1898	77,985,757 50	23,034,033 45	1,124,835 14	
1899	111,344,220 00	26,061,519 90	1,837,451 86	
1900	99,272,942 50	36,295,321 45	2,031,137 39	
1901	101,735,187 50	30,838,460 75	2,120,122 08	
1902	61,980,572 50	30,116,369 45	2,429,736 17	
1903	45,721,773 00	25,996,536 25	2,484,691 18	
1904	233,402,428 00	15,695,609 95	1,683,529 35	
Grand totals				

CHAPTER VI

MULTIPLICATION

ORAL EXERCISE

1. Which of the following numbers are **concrete**; that is, refer to some particular kind of object or measure? 12; $5\frac{1}{2}$; 12 ft.; 2.5 da.; 15 yd.; 18 men; 200; \$12; $172\frac{4}{5}$.

2. Which of the above numbers are **abstract**; that is, do not refer to any particular kind of object or measure?

3. $5 + 4 + 2 + 8 + 9 = ?$

4. $9 + 9 + 9 + 9 + 9 = ?$ 5 times $9 = ?$

5. Could the sum of the numbers in problem 3 be found by any shorter process?

6. What is the first process in problem 4 called? the second?

7. 9 times $27 = ?$ 9 times 29 bu. $= ?$

8. If 1 bu. of rye weighs 56 lb., what will 12 bu. weigh?

74. In problems 7 and 8 it is seen that *the multiplier is always an abstract number;* and *the multiplicand and product are like numbers.*

75. Three 5's are equal to five 3's; \$3 multiplied by 5 is equal to \$5 multiplied by 3; 4 trees multiplied by 125 is equal to 125 trees multiplied by 4.

76. It is therefore seen that *the product is not affected by changing the order of the factors regarded as abstract numbers.*

77. The multiplicand and multiplier together are called **factors** (makers) of the product; the product of two abstract integers is sometimes called a **multiple** of either of the factors.

78. Sometimes a number is used several times as a factor. Numbers so used are indicated by a small figure, called an **exponent**, written above and at the right of the factor.

Thus, 4 used *twice* as a factor is written 4^2, 5 used *four times* as a factor is written 5^4, and 6 used *five times* as a factor is written 6^5.

79. The product arising from using a number two or more times as a factor is called a **power** of that number.

Thus, 4 is the second power of 2; 64 is the third power of 4 and the sixth power of 2.

Too much attention should not be given to the definitions like the above. They are valuable only as they help to make clear the matter in the exercises. They are rarely heard in business and therefore should not be memorized.

ORAL EXERCISE

1. Multiply at sight each number below by 2; by 3; by 4; by 5; by 6; by 7; by 8; by 9.

Name the products by lines from left to right and from right to left; also by columns from left to right and from right to left. Name results only. Thus, to multiply lines by 4 say 20, 36, 8, 24, 40, 12, 28, 44, 16, 48, 32, 52, 68, 84, and so on up to 100; and backwards, 100, 80, 96, 64, and so on back to 20. To multiply columns by 4 say 20, 68, 36, 84, and so on to 52, 100; and backwards 100, 52, 80, 32, and so on to 68, 20. Continue the work until results can be named at the rate of 120 or more per minute.

5	9	2	6	10	3	7	11	4	12	8	13
17	21	14	18	22	15	19	23	16	24	20	25

2. Multiply as instructed in problem 1 and add 8 (carried) to each product. Also multiply as instructed and add 6, 4, 7, 2, 5, 3, and 9 to each product.

Name results only. Thus, to multiply by lines say 20, 28; 36, 44; 8, 16; and so on.

3. Multiply by 2: 27, 35, 81, 36, 28, 32, 47, 93, 56, 39, 54, 45, 52, 86, 75, 67, 59. Also by 4, 3, 5, 8, 6, 7, 9.

4. Find the cost of each of the following: 20 lb. crackers at 8¢; 9 lb. coffee at 34¢; 7 lb. tea at 57¢; 11 lb. beef at 17¢; 120 lb. sugar at 4¢; 134 lb. sugar at 5¢.

5. Find the cost of each of the following: 44 yd. at 9¢; 37 yd. at 8¢; 123 yd. at 6¢; 214 yd. at 4¢; 52 yd. at 12¢; 29 yd. at 8¢; 8 yd. at $1.03; 7 yd. at $1.01; 5 yd. at $1.35.

6. Beginning at 0 count by 9's to 81; by 10's to 150; by 11's to 154; by 12's to 108; by 13's to 117; by 14's to 126; by 15's to 135; by 16's to 144; by 17's to 153; by 18's to 162; by 19's to 171; by 20's to 180.

80. Examples. 1. Find the cost of 2150 lb. at 5¢.

SOLUTION. Since 1 lb. costs 5¢, 2150 lb. will cost 2150 times 5¢; but 2150 times 5¢ is equal to 5 times 2150¢. 5 times $21.50 (2150¢) equals $107.50, the required result.

$$\begin{array}{r} \$21.50 \\ 5 \\ \hline \$107.50 \end{array}$$

2. Multiply 224 by 46.

SOLUTION. In multiplying one number by another, there is no practical advantage in beginning with the lowest order of units of the multiplier; in fact, in some multiplications (see page 140) there is a decided advantage in beginning with the highest order. The arrangement of work for both methods is shown in the margin.

$$\begin{array}{r} 224 \\ 46 \\ \hline 1344 \\ 896 \\ \hline 10304 \end{array} \qquad \begin{array}{r} 224 \\ 46 \\ \hline 896 \\ 1344 \\ \hline 10304 \end{array}$$

CHECK. The work may be checked by multiplying first by one method and then by the other, or by interchanging the multiplier and multiplicand and re-multiplying. (See also pages 83 and 84.)

3. Multiply 2004 by 1275.

SOLUTION. When one of two numbers to be multiplied contains a number of zeros or ones, it is always easier to take that number as the multiplier. Since the product of any number multiplied by 0 is 0, the product of 1275 multiplied by the tens and hundreds of the multiplier need not be written.

$$\begin{array}{r} 1275 \\ 2004 \\ \hline 5100 \\ 2550 \\ \hline 2555100 \end{array} \qquad \begin{array}{r} 1275 \\ 2004 \\ \hline 2550 \\ 5100 \\ \hline 2555100 \end{array}$$

CHECK. The problem may be checked the same as problem 2.

When two numbers are to be multiplied, it is generally easier to take as the multiplier the number having the least number of places. Thus, to find the cost of 1647 A. of land at $27 per acre, take 27 as the multiplier.

If one of the two numbers to be multiplied has two or more digits alike, it is easier to take that number as the multiplier. Thus, to multiply together 6729 and 7777, it is easier to take 7777 as the multiplier.

ORAL EXERCISE

1. Find the value of 51 T. of hay at $17 per ton.
2. Find the cost of 175 lb. of sugar at 5¢ per pound.
3. How much will a boy earn in 87 hr. at 9¢ an hour?
4. What is the cost of a flock of 52 sheep at $7 per head?
5. At the rate of 47 mi. an hour, how far will a person travel in 12 hr.?
6. What is the cost of 12 pr. of shoes at $4.50 per pair, and 8 pr. of boots at $3.50 per pair?

7. What must be paid for handling 12 loads of freight at $2.25 per load?

8. In an orchard there are 13 rows of trees, each containing 21 trees. How many trees in the orchard?

9. If you buy 5 pencils at 9¢ each and 9 penholders at 5¢ each, and some stationery costing 25¢, how much change should you receive from a two-dollar bill? from a ten-dollar bill?

10. I bought 6 cd. of wood at $5.75 per cord. If a fifty-dollar bill is offered in payment, how much change should be received?

11. I bought 12 bu. of wheat at $1.05. If I gave in payment two ten-dollar bills, what change should I receive?

12. My average marketing expenses per day are $2.10. If I offer a twenty-dollar bill in payment for 7 days' expenses, what change should I receive?

13. I sold 16 chairs at $7 each, and 5 tables at $9 each. If two one-hundred-dollar bills are offered in payment, how much change should I return? If a one-hundred-dollar bill, a fifty-dollar bill, and a twenty-dollar bill are offered in payment, how much change should I return?

WRITTEN EXERCISE

In the following problems find the missing numbers by multiplying across and adding down. Check the results by comparing the sum of the line products with the sum of the multiplicands multiplied by one of the multipliers.

1.	2.	3.
$15 \times 211 = ?$	$9 \times 1475 = ?$	$12 \times \$16.50 = ?$
$15 \times 346 = ?$	$9 \times 2618 = ?$	$12 \times \$27.75 = ?$
$15 \times 318 = ?$	$9 \times 1575 = ?$	$12 \times \$14.95 = ?$
$15 \times 721 = ?$	$9 \times 1792 = ?$	$12 \times \$29.86 = ?$
$15 \times 936 = ?$	$9 \times 4936 = ?$	$12 \times \$49.88 = ?$
$15 \times 849 = ?$	$9 \times 7289 = ?$	$12 \times \$39.62 = ?$
$15 \times 218 = ?$	$9 \times 8728 = ?$	$12 \times \$86.99 = ?$
$15 \times ? = ?$	$9 \times ? = ?$	$12 \times ? = ?$

4.	5.	6.
$12 \times 192 = ?$	$98 \times 2178 = ?$	$16 \times \$18.10 = ?$
$12 \times 721 = ?$	$98 \times 1692 = ?$	$16 \times 17.20 = ?$
$12 \times 836 = ?$	$98 \times 2536 = ?$	$16 \times 21.40 = ?$
$12 \times 456 = ?$	$98 \times 2892 = ?$	$16 \times 25.85 = ?$
$12 \times ? = ?$	$98 \times ? = ?$	$16 \times ? = ?$

Problems such as the above are very helpful. They afford a variety of work and suggest a simple method by which the student may test the correctness of his results. The teacher should add as many more problems as circumstances require.

7. A produce dealer bought 2145 bu. of potatoes at 23¢ a bushel, and sold them at 47¢ a bushel. What did he gain?

8. A drover bought 125 head of cattle at $15.75 per head. He sold 65 head at $23.40, 15 head at $13.75, and 45 head at $17.75. Did he gain or lose, and how much?

9. A grocer bought 14 bu. of apples at 35¢ per bushel and 12 bu. of potatoes at 37¢ per bushel. He sold the apples at 30¢ a peck and the potatoes at 20¢ a peck. What did he gain?

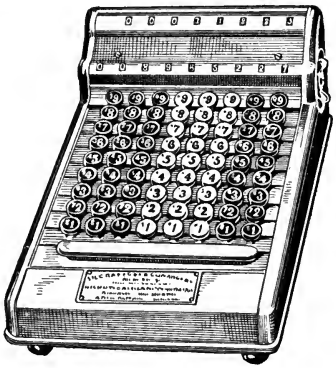
10. A speculator bought 1247 bbl. of apples at $1.35 per barrel. After holding them for three months he sold them at $3.75 per barrel. If he paid $74.82 for storage, and his loss by decay was equal to 37 bbl. of apples, what was his gain?

11. The gross weight in pounds, and tare in pounds, of 25 tubs of lard are as follows: 71 – 14, 70 – 15, 69 – 14, 72 – 16, 71 – 14, 72 – 15, 70 – 15, 69 – 14, 71 – 15, 70 – 15, 69 – 14, 71 – 16, 71 – 15, 71 – 14, 70 – 15, 68 – 14, 73 – 16, 73 – 15, 70 – 14, 70 – 14, 71 – 15, 73 – 16, 74 – 18, 71 – 13, 73 – 16. Find the cost at 11¢ per pound.

12. The gross weight in pounds, and the tare in pounds, of 25 casks of hams are as follows: 400 – 78, 420 – 68, 420 – 71, 403 – 71, 409 – 71, 418 – 68, 412 – 72, 407 – 67, 423 – 69, 419 – 67, 426 – 68, 403 – 70, 399 – 69, 400 – 69, 425 – 71, 413 – 72, 399 – 67, 412 – 72, 418 – 68, 409 – 71, 408 – 70, 412 – 68, 402 – 71, 421 – 71, 403 – 71. Find the cost at 14¢ per pound.

SHORT METHODS IN MULTIPLICATION

81. There are many short methods in multiplication, but of these only a few are practical, either because they generally apply to problems that in themselves are not practical or because they have been supplanted by the elaborate use of tables and mechanical devices. Many practical tables are in use for figuring pay rolls, interest, discount, and the like. (See pages 224 and 315.) Multiplying machines are also used in many offices. In the margin is a picture of a multiplying machine.



82. The short methods given herewith have a wide application. They are not dependent upon formal rules, and are suggestive of many other ways in which the student may exercise his own ingenuity to shorten his work in multiplication.

Multiplication by Powers and Multiples of Ten

ORAL EXERCISE

1. 40 is how many times 4? 60 is how many times 6? 100 is how many times 10? 150 is how many times 15?

2. Give a short method for multiplying an integer by 10.

3. 400 is how many times 4? 600 is how many times 6? 1000 is how many times 10? 1500 is how many times 15?

4. Give a short method for multiplying an integer by 100; by 1000; by 10000.

5. How does the product of 40×66 compare with the product of $4 \times 66 \times 10$? the product of 400×59 with the product of $4 \times 59 \times 100$?

6. Give a short method for multiplying an integer by any number of 10's, 100's, or 1000's.

7. Multiply 270 by 300.

SOLUTION. In the accompanying illustration it will be seen that $270 \times 300 = 27 \times 3 \times 1000$ or 81,000.

$$\begin{aligned} 270 &= 27 \times 10 \\ 300 &= 3 \times 100 \\ \hline 81000 &= 81 \times 1000 \end{aligned}$$

8. Formulate a rule for finding the product when there are zeros on the right of both factors.

9. \$7 is how many times \$0.70? \$90 is how many times \$0.90? \$500 is how many times \$0.50?

10. State a short method for multiplying United States money by 10; by 100; by 1000.

11. Read aloud the following, supplying the missing words:

(*a*) Annexing a cipher to an integer multiplies the integer by ——; annexing two ciphers to an integer —— the integer by ——.

(*b*) Removing the decimal point in United States money one place to the right —— the number by 10; removing the decimal point two places to the right —— the number by ——.

12. Multiply \$14.70 by 10; by 100; by 1000.

83. In the above exercise it is clear that

Annexing a cipher to an integer multiplies the integer by 10; and

Removing the decimal point one place to the right multiplies the number by 10.

ORAL EXERCISE

1. Read aloud the following numbers multiplied by 10; by 100; by 1000: 17; 285; 3712; \$413.45; \$1926.75; 4165.95.

2. Read each of the following numbers multiplied by 20; by 400; by 600; by 5000: 16; 19; 37; 49¢; 64¢; \$122; \$2.60.

3. By inspection find the cost of:

a. 750 lb. coffee at 30¢.
b. 500 lb. cocoa at 40¢.
c. 650 lb. chocolate at 30¢.
d. 300 bbl. lump salt at \$3.
e. 200 bbl. oatmeal at \$4.50.
f. 170 bx. wool soap at \$3.
g. 650 yd. silk at \$1.20.
h. 140 bu. beans at \$3.50.
i. 500 bu. beans at \$2.50.
j. 240 gro. jet buttons at \$3.
k. 500 doz. half hose at \$5.50.
l. 800 yd. taffeta silk at \$1.20.

84. When the multiplier is a number a little less than 10, 100, or 1000, the multiplication may be shortened as shown in the following examples.

85. Examples. **1.** Multiply 123 by 99.

SOLUTION. *99 is 100 diminished by 1; hence, multiply 123 by 100 and then by 1 and subtract the results.* The product is 12,177. Check by retracing the steps in the process.

$$\begin{array}{r} 12300 \\ 123 \\ \hline 12177 \end{array}$$

2. Multiply 145 by 96.

SOLUTION. *96 is 100 diminished by 4; hence, multiply 145 by 100 and then by 4 and subtract the results.* The product is 13,920. Check by retracing the steps in the process.

$$\begin{array}{r} 14500 \\ 580 \\ \hline 13920 \end{array}$$

WRITTEN EXERCISE

1. Find the total cost of:

5260 bu. rye at 99¢.
1521 bu. rye at 92¢.
1640 bu. wheat at 98¢.
2994 bu. millet at 97¢.
1112 bu. wheat at 97¢.
2160 bu. millet at 96¢.
834 bu. millet at 95¢.
246 bu. wheat at 92¢.
998 bu. millet at $1.04.
998 bbl. apples at $1.05.
893 bkt. peaches at $1.05.
993 bu. clover seed at $3.35.

MULTIPLICATION BY 11 AND MULTIPLES OF 11

86. Example. Multiply 237 by 11.

SOLUTION. To multiply by 11 is to multiply by 10 + 1. Hence, annex a cipher to 237 and add 237; or, better still, add the digits as follows: **7**; 3 + 7 = **10**; 3 + 2 + 1 (carried) = **6**; bring down **2**; therefore, the result is 2607.

ORAL EXERCISE

1. Multiply each of the following by 11:

14; 26; 45; 19; 16; 34; 36; 49; 64; 125; 112; 115; 128; 192; 175; 116; 142; $4.95; $9.62; $4.41; $6.82; $5.21; $3.65; $4.31; $21.12; $14.21; $18.32; $3.26.

2. Find the cost of 11 yd. at 27¢; at 91¢; at 86¢; at 95¢; at $1.49; at $1.23; at $2.17; at $2.31; at $2.40; at $2.50; at $2.75; at $4.35; at $3.15; at $3.10; at $8.13.

87. Examples. 1. Multiply 46 by 22.

SOLUTION. 22 is 11 times 2. Multiply 46 by 11 and by 2, as follows: $2 \times 6 = 12$; write 2 and carry 1. $4 + 6 = 10$; $2 \times 10 + 1$ (carried) $= 21$; write 1 and carry 2. $2 \times 4 + 2$ (carried) $= 10$; write 10. The result is 1012.

$$\begin{array}{r} 46 \\ 22 \\ \hline 1012 \end{array}$$

2. Find the cost of 122 bu. of potatoes at 66¢ per bu.

SOLUTION. $6 \times 2 = 12$; write 2 and carry 1. $2 + 2 = 4$; $6 \times 4 + 1$ (carried) $= 25$; write 5 and carry 2. $1 + 2 = 3$; $6 \times 3 + 2$ (carried) $= 20$; write 0 and carry 2. $6 \times 1 + 2$ (carried) $= 8$. Write 8. The result is $80.52.

$$\begin{array}{r} 122 \\ .66 \\ \hline 80.52 \end{array}$$

WRITTEN EXERCISE

In the following problems make all the extensions mentally.

1. Find the total cost of:

11 lb. coffee at 42¢.
14 doz. eggs at 11¢.
64 lb. cheese at 11¢.
33 bu. carrots at 56¢.
11 bu. potatoes at 65¢.
88 bu. wheat at $1.13.
115 bu. rye at 99¢.
215 bu. peas at 77¢.
344 bu. oats at 44¢.
300 bu. grain at 85¢.
115 bu. barley at 88¢.
400 bbl. apples at $1.65.

2. Find the total cost of:

77 bu. peaches at $1.85.
151 bu. corn at 66¢.
265 bu. onions at 22¢.
135 bu. apples at 33¢.
241 bu. turnips at 44¢.
112 bu. tomatoes at 55¢.
820 bu. rye at 88¢.
327 bu. oats at 33¢.
314 bu. peas at 66¢.
110 bu. pears at $1.66.
880 bu. barley at $1.17.
100 bu. quinces at $1.60.

A careful computer checks his work *at every step*. The student who forms the habit of doing this in all his computations will soon find himself in no need of printed answers to problems involving only numerical calculation.

Checks for multiplication have already been mentioned. To guard against large errors, it is also important to form a rough estimate of an answer before beginning the solution. Thus, in finding the cost of 211 yd. of lining at 32¢, at once see that the result will be a little more than $63.00 (210 times 30¢); this will do away with such absurd results as $6752, $675.20, or $6.75.

3. Copy and find the amount of the following bill:

Boston, Mass., July 21, 19

Mrs. GEORGE W. MUNSON
168 Huntington Ave., City

Bought of S. S. PIERCE COMPANY

Terms Cash

15	cs. Horse-radish	$0.66				
25	lb. Huyler's Cocoa	.44				
31	gal. N. O. Molasses	.33				
55	lb. Japan Tea	.38				
212	" Raisins	.11				
77	pkg. Yeast Cakes	.44				

Multiplication of Numbers from 11 to 19 Inclusive

88. Example. Multiply 18 by 17.

Solution. $7 \times 8 = 56$; write 6 and carry 5. $7 + 8$ (that is $\overline{7 \times 1 + 1 \times 8}$) $+ 5$ (carried) $= 20$; write 0 and carry 2. $1 \times 1 + 2$ (carried) $= 3$; write 3.

$$\begin{array}{r} 18 \\ 17 \\ \hline 306 \end{array}$$

The foregoing method may be summarized as follows:

Multiply the units of the multiplicand by the units of the multiplier and write the result as the first figure of the product. Add the units in the multiplicand and multiplier and write the result as the second figure of the product. Finally bring down the tens of the multiplicand. Carry as usual.

89. In a similar manner multiply together all numbers of two figures each whose *tens* are alike.

90. Example. 1. Multiply 92 by 97.

Solution. $7 \times 2 = 14$; write 4 and carry 1. $2 + 7 = 9$; $9 \times 9 + 1$ (carried) $= 82$; write 2 and carry 8. $9 \times 9 + 8$ (carried) $= 89$. The result is 8942.

$$\begin{array}{r} 92 \\ 97 \\ \hline 8924 \end{array}$$

91. The above method may be so modified as to cover all numbers of two figures each whose *units* are alike.

92. Example. Multiply 92 by 52.

SOLUTION. $2 \times 2 = 4$; write 4. $9 + 5 = 14$; $2 \times 14 = 28$; write 8 and carry 2. $5 \times 9 + 2$ (carried) $= 47$; write 47. The result is 4784.

$$\begin{array}{r} 92 \\ 52 \\ \hline 4784 \end{array}$$

ORAL EXERCISE

State the product of:

1. 16×15.	**5.** 14×16.	**9.** 19×18.	**13.** 27×23.
2. 17×18.	**6.** 18×13.	**10.** 24×25.	**14.** 31×38.
3. 19×13.	**7.** 18×14.	**11.** 23×21.	**15.** 37×32.
4. 15×19.	**8.** 15×14.	**12.** 24×26.	**16.** 34×32.

WRITTEN EXERCISE

In the following problems make all the extensions mentally.

1. Find the total cost of:

42 lb. cocoa at 48¢.
45 lb. cocoa at 43¢.
54 lb. coffee at 24¢.
15 lb. raisins at 13¢.
17 lb. biscuits at 12¢.
27 bx. salt at 57¢.
23 lb. coffee at 24¢.
19 lb. candy at 18¢.
32 lb. chocolate at 22¢.
85 lb. Oolong tea at 35¢.

2. Find the total cost of:

36 yd. wash silk at 26¢.
54 doz. whalebones at 94¢.
97 yd. Amazon cloth at 97¢.
17 gro. bone buttons at 19¢.
18 yd. gunner's duck at 17¢.
87 yd. flannel at 27¢.
19 yd. cottonade at 14¢.
17 yd. York denim at 15¢.
16 yd. cotton cheviot at 19¢.
17 yd. Hamilton stripe at 12¢.

MULTIPLICATION BY NUMBERS OF TWO FIGURES ENDING IN 1

93. Example. Multiply 412 by 31.

SOLUTION. Write 2 in the product. $\overline{3 \times 2} + 1$ (the tens' figure of the multiplicand) $= 7$; write 7 in the product. $\overline{3 \times 1} + 4$ (the hundreds' figure of the multiplicand) $= 7$; write 7 in the product. $3 \times 4 = 12$; write 12. The result is 12,772.

$$\begin{array}{r} 412 \\ 31 \\ \hline 12772 \end{array}$$

94. In a similar manner multiply by all such numbers as **301**, 101, and 901.

95. Example. Multiply 126 by 201.

SOLUTION. Write 26 in the product. $2 \times 6 + 1$ (the hundreds' figure of the multiplicand) $= 13$. Write 3 and carry 1. $2 \times 12 + 1$ (carried) $= 25$. The result is 25,326.

$$\begin{array}{r} 126 \\ 201 \\ \hline 25326 \end{array}$$

The two processes just explained are the best for making mental extensions on a bill and the like. For general work, however, many persons prefer the following methods:

First problem	Second problem
412 = once the number	126 = once the number
1236 = 30 times the number	252 = 200 times the number
12772 = 31 times the number	25326 = 201 times the number

WRITTEN EXERCISE

Find the product of:

1. 214×21.
2. 315×31.
3. 425×61.
4. 386×91.
5. 465×121.
6. 215×401.
7. 746×201.
8. 859×301.

MULTIPLICATION BY NUMBERS FROM 101 TO 109 INCLUSIVE

96. Examples. 1. Find the cost of 64 bu. of wheat at \$1.02.

SOLUTION. $2 \times 64 = 128$; write 28 and carry 1. $1 \times 64 + 1 = 65$; write 65. The result is \$65.28.

$$\begin{array}{r} 64 \\ 1.02 \\ \hline 65.28 \end{array}$$

Some persons may prefer to work this problem as follows: 64 bu. at \$1 = \$64; 64 bu. at $2\,¢$ = \$1.28; \$64 + \$1.28 = \$65.28.

2. Find the cost of 251 bu. of barley at \$1.04.

SOLUTION. $4 \times 51 = 204$; write 04 in the product and carry 2. $4 \times 2 + 2$ (carried) $+ 1$ (the right-hand figure of the multiplicand) $= 11$; write 1 and carry 1. $1 \times 25 + 1$ (carried) $= 26$; write 26. The result is \$261.04.

$$\begin{array}{r} 251 \\ 1.04 \\ \hline 261.04 \end{array}$$

97. Similarly multiply by such numbers as 201, 302, and 405.

98. Example. Find the cost of 124 bu. of beans at \$2.05.

SOLUTION. $5 \times 24 = 120$. Write 20 and carry 1. $5 \times 1 + 1$ (carried) $+ 2 \times 4$ (the right-hand figure of the multiplicand) $= 14$; write 4 and carry 1. $2 \times 12 + 1$ (carried) $= 25$; write 25. The result is \$254.20.

$$\begin{array}{r} 124 \\ 2.05 \\ \hline 254.20 \end{array}$$

Some persons may prefer the following solution: 124 bu. at \$2 = \$248; 124 bu. at $5\,¢$ = \$6.20; \$248 + \$6.20 = \$254.20. The student should try to exercise his own ingenuity in all this work.

WRITTEN EXERCISE

Find the value of:

1. 215 T. coal at $6.05.
2. 224 bu. rye at $1.02.
3. 215 bu. wheat at $1.02.
4. 318 bu. barley at $1.05.
5. 124 bbl. apples at $2.05.
6. 116 bbl. onions at $1.08.
7. 232 bbl. potatoes at $2.05.
8. 302 bu. peas at 74¢.
9. 104 bu. corn at 89¢.
10. 103 bu. beets at 85¢.
11. 205 bu. turnips at 54¢.
12. 215 bu. pears at $1.05.
13. 411 bu. plums at $1.08.
14. 206 bu. parsnips at 93¢.

Miscellaneous Short Methods

99. When one part of the multiplier is contained in another part a whole number of times, the multiplication may be shortened as shown in the following examples.

100. Examples. 1. Multiply 412 by 357.

SOLUTION. 35 is 5 times 7. $7 \times 412 = 2884$, which write as the first partial product. $5 \times 2884 = 14{,}420$, which write as the second partial product.

CHECK. Interchange the multiplier and multiplicand and remultiply. $4 \times 357 = 1428$; $3 \times 1428 = 4284$. Add. Since the results by both multiplications agree, the work is probably correct.

412	357
357	412
2884	1428
14420	4284
147084	147084

2. Multiply 214 by 756.

SOLUTION. 56 is 8 times 7. $7 \times 214 = 1498$, which write as the first partial product. $8 \times 1498 = 11{,}984$, which write as the second partial product. The sum of these partial products, 161,784, is the entire product.

Check as in problem 1. (See also pages 83 and 84.)

214
756
1498
11984
161784

WRITTEN EXERCISE

Find the product of:

1. 319×248.
2. 927×279.
3. 728×287.
4. 848×369.
5. 12816×10217.
6. 14416×12525.

101. In multiplying together *any* two numbers of two figures each, the work may be shortened as in the following example.

102. Example. Multiply 35 by 23.

SOLUTION. $3 \times 5 = 15$; write 5 and carry 1. $3 \times 3 + 1$ (carried) $+$ $2 \times 5 = 20$; write 0 and carry 2. $2 \times 3 + 2$ (carried) $= 8$; write 8. The result is 805.

$$\begin{array}{r} 35 \\ 23 \\ \hline 805 \end{array}$$

WRITTEN EXERCISE

Find the product of:

1. 23×25.
2. 72×21.
3. 56×35.
4. 34×52.
5. 67×51.
6. 86×42.
7. 75×24.
8. 66×82.

WRITTEN REVIEW EXERCISE

1. Multiply 45,216 by 14 412 in two lines of partial products.
2. Multiply 31,216 by 10,217 in two lines of partial products.
3. I bought 15 A. of land at \$275 per acre and laid it out in 100 city lots. After expending \$6750 for grading and taxes, \$257 for ornamental trees, and \$250 for advertising, I sold 15 lots at \$625 each, 35 lots at \$415 each, and exchanged the remainder for a farm of 120 A., which I immediately sold at \$195 per acre. Did I gain or lose, and how much?
4. Copy and find the amount of the following bill:

Rochester, N.Y., July 26, **19**

Mr. F. C. GORHAM
120 Spring Street, City

Bought of C. E. Ferguson & Son

Terms 30 days

37 bu.	Oats	\$0.40
50 "	Corn	.67
76 "	Wheat	1.02
75 "	Rye	1.04
95 "	Beans	4.00
16 "	Clover Seed	3.50
26 "	Millet	.99

CHAPTER VII

DIVISION

ORAL EXERCISE

1. What is the product of 12 times 15? How many times is 15 contained in 180? What is $\frac{1}{12}$ of 180?

2. How much is 11 times \$17? How many times is \$17 contained in \$187? What is $\frac{1}{11}$ of \$187?

3. What is the product of 9 times 12 ft.? How many times is 12 ft. contained in 216 ft.? What is $\frac{1}{15}$ of 225 ft.?

4. When one factor and the product are given, how is the other factor found?

103. The process of finding either factor when the product and the other factor are given is called **division.**

104. The known product is called the **dividend**; the known factor, the **divisor**; the unknown factor, when found, the **quotient.**

105. The part of the dividend remaining when the division is not exact is called the **remainder.**

While definitions such as the above should not be memorized, the ideas which they express should be thoroughly understood.

106. Since 6 times 7 ft. = 42 ft., 42 ft. ÷ 7 ft. = 6, and 42 ft. ÷ 6 = 7 ft. It is therefore clear that

1. *If the dividend and divisor are concrete numbers, the quotient is an abstract number;* and

2. *If the dividend is concrete and the divisor abstract, the quotient is a concrete number like the dividend.*

In §106 it will be seen that there are **two kinds of division**: 42 ft. ÷ 7 ft. = 6 is sometimes called *measuring*, because 42 ft. is measured by 7 ft.; 42 ft. ÷ 6 = 7 ft. is sometimes called *partition*, because 42 ft. is divided into 6 equal parts.

ORAL EXERCISE

1. Divide by 2: 18, 32, 78, 450, 642, 964, 893.
2. Divide by 3: 27, 57, 72, 423, 642, 963, 845.
3. Divide by 4: 64, 88, 92, 488, 192, 396, 728.
4. Divide by 5: 65, 85, 95, 135, 275, 495, 725.
5. Divide by 6: 84, 96, 54, 246, 546, 672, 846, 636.
6. Divide by 7: 63, 84, 91, 217, 497, 714, 791, 921.
7. Divide by 8: 72, 56, 88, 248, 640, 128, 144, 152.
8. Divide by 4: 56, 96, 77, 241, 168, 128, 920, 848.
9. Divide by 6: 78, 96, 56, 272, 848, 190, 725, 966.
10. Divide by 9: 98, 72, 49, 279, 819, 720, 189, 918.

ORAL EXERCISE

1. 16 ft. ÷ 2 = ? 24 ft. ÷ 8 ft. = ?
2. $25 ÷ 5 = ? $29.75 ÷ 5 = ? $129.78 ÷ 9 = ? 13.40 ÷ 4 = ?
3. 126 yd. ÷ 3 yd. = ? $125 ÷ 25 = ? $6.25 ÷ $1.25 = ?
4. If 9 T. of coal cost $49.50, what is the cost per ton?

SOLUTION. $49.50 ÷ 9 = $5; subtracting 9 times $5, the result is $4.50 undivided; $4.50 ÷ 9 = $0.50. Therefore the quotient is $5.50.

$$9\overline{)\$49.50}\quad \$5.50$$

5. At $1.75 a yard, how many yards can be bought for $35?

SOLUTION. The divisor contains cents and it is therefore better to first change both dividend and divisor to cents. It is found that $35 would buy 20 times as many yards as $1.75, or 20 yd.

$$175\overline{)3500}\quad 20$$

6. If 5 T. of coal cost $31.25, what is the cost per ton?
7. At $2.50 per yard how many yards can be bought for $550?

ORAL EXERCISE

1. How many weeks in 98 da.?
2. What is $\frac{1}{25}$ of 2250 bbl. of apples? $\frac{1}{15}$? $\frac{1}{5}$? $\frac{1}{45}$?
3. The quotient is 8 and the dividend 128. What is the divisor?
4. How many times can 18 be subtracted from 75, and what will remain?

5. At 15¢ per pound, how many pounds of beef can be bought for $6.30?

6. The quotient is 5, the divisor 23, and the remainder 2. What is the dividend?

7. If 5 men earn $17.50 a day, how much can 8 men earn in 2 da. at the same rate?

8. What is the nearest number to 150 that can be divided by 9 without a remainder?

9. If there are 960 sheets in 40 qr. of paper, how many sheets in 5 qr.? in 11 qr.?

10. If 6 bbl. of apples are worth $9, what are 24 bbl. worth at the same rate? 36 bbl.?

11. If 17 bbl. of flour cost $85, what will 25 bbl. cost at the same rate? 32 bbl.? 48 bbl.? 34 bbl.?

12. If 8 be added to a certain number, the sum will be 24 times the number. What is the number?

13. If 20 yd. of cloth cost $60, for how much per yard must it be sold to gain $25? to gain $15?

14. A grocer sold 250 oranges at 5¢ each and gained $5. How much did he pay a dozen for the oranges?

15. A grocer pays $3 for 20 doz. of eggs. At what price per dozen must he sell them in order to gain $1.50?

16. At $2.50 per yard, how many yards of cloth can be bought for $75? for $150? for $2500? for $750?

17. How many days' labor at $3.50 per day will pay for 2 T. of coal at $7 a ton and 5 lb. of tea at 70¢ per pound?

18. A clothier pays $96 for a dozen overcoats. At how much apiece must he retail them to gain $48 on the lot?

19. A man exchanged 1140 bu. of wheat at $1 per bushel for flour at $6 per barrel. How many barrels did he receive?

20. It was found that after 15 had been subtracted 5 times from a certain number the remainder was 4. What was the number?

21. A man contracts a debt of $175 which he promises to pay in weekly installments of $3.50 each. After paying $35, how many more payments has he to make?

107. Examples. 1. Divide 4285 by 126.

Complete Operation	Required Work
$34\frac{1}{126}$	$34\frac{1}{126}$
$126\overline{)4285}$	$126\overline{)4285}$
$\underline{378}$ = 3 times 126	$\underline{378}$
505 undivided	505
$\underline{504}$ = 4 times 126	$\underline{504}$
1 undivided	1

Check. $34 \times 126 + 1 = 4285$

The remainder cannot always be written as a part of the quotient. Thus in the problem, "At \$7 per head how many sheep can be bought for \$37," we cannot say, "$5\frac{2}{7}$ sheep," but "5 sheep and \$2 remaining."

2. A farmer received \$283.25 in payment for 275 bu. of wheat. How much was received per bushel for the wheat?

Solution. \$283.75 ÷ 275 = \$1 and \$8.25 undivided. \$8.25 ÷ 275 = \$0.03. \$1.03 per bushel was therefore received for the wheat.

Check. 275 times \$1.03 = \$283.25.

\$1.03
275)\$283.25
275
8 25
8 25

108. Work in division may be abridged by omitting the partial products and writing only the partial dividends.

109. Example. Divide \$614.80 by 232.

Solution. 2 times 2 plus **0** = 4; 2 times 3 plus **5** = 11. 2 times 2 + 1 = 5, and 5 plus **1** = 6. Bring down 8. 6 times 2 plus **6** = 18; 6 times 3 plus 1 = 19, and 19 + **1** = 20; 6 times 2 plus 2 = 14, and 14 plus **1** = 15. Bring down 0 and proceed as before.

\$2.65
232)\$614.80
150 8
11 60
0 00

WRITTEN EXERCISE

1. Find the value of 8800 lb. of oats at 45¢ per bushel of 32 lb.

2. How many automobiles, at \$650 each, can be purchased for \$4,225,000?

3. By what number must 8656 be multiplied to make the product 8,223,200?

4. If 120 bbl. of flour cost \$660, what will 829 bbl. cost at the same rate?

5. The product of two numbers is 1,928,205. If one of them is 621, what is the other?

6. If 380 T. of coal can be bought for \$3040, how many tons can be bought for \$3600?

7. How many cords of 128 cu. ft. in a pile of wood containing 235,820 cu. ft.? What is it worth at \$4.50 per cord?

8. A speculator sold a quantity of apples that cost \$2500 for \$4750. If his gain per barrel was \$1.12½, how many barrels did he buy?

9. A man received a legacy of \$11,375 which he invested in railroad stock. He paid a broker \$125 to buy stock at \$112.50 per share. How many shares were bought?

10. A dealer bought 250 T. of coal by the long ton of 2240 lb. at \$4.50 per ton. He retailed the same at \$6.75 per short ton of 2000 lb. What was the total gain?

11. In a recent year there were produced in the United States 550,935,925 bu. of wheat on 44,074,874 A. What was the yield per A.? What was the yield worth at 44.9¢ per bu.?

12. Copy and complete the following table of corn statistics. Check the work. (The total yield multiplied by the price per bushel should equal the total valuation.)

PRINCIPAL CORN-GROWING STATES IN A RECENT YEAR

STATE	YIELD IN BUSHELS	FARM PRICE PER BUSHEL	FARM VALUATION	
Illinois	334 133 680	44¢	147 018 819	20
Iowa		44¢	133 337 277	04
Nebraska		44¢	114 814 627	40
Missouri		44¢	66 669 962	92
Indiana	143 396 852	44¢		
Texas	136 702 699	44¢		
Total				

13–15. Make and solve three self-checking problems in division.

SHORT METHODS IN DIVISION

Powers and Multiples of 10

ORAL EXERCISE

1. How many times is 10 contained in 50? 100 in 800? 1000 in 9000?

2. Cutting off a cipher in 30 divides it by what number?

3. Cutting off two ciphers in 800 divides it by what number?

4. Cutting off three ciphers in 11,000 divides it by what number?

5. Read aloud, supplying the missing words:

a. The number of 10's in any number may be found by cutting off the units' figure; the number of 100's by cutting off the —— and —— figures; the number of —— by cutting off the hundreds' and tens' and units' figures.

b. In 4561 there are 456 tens and 1 unit, or $456\frac{1}{10}$ tens; 45 —— and 61 units, or $45\frac{61}{100}$ hundreds; and —— thousands and 561 units, or $4\frac{561}{1000}$ thousands.

6. How many times is \$0.10 contained in \$1? \$0.01 in \$1? \$0.001 in \$1?

7. What is $\frac{1}{10}$ of \$1? $\frac{1}{100}$ of \$1? $\frac{1}{1000}$ of \$1?

8. Read aloud, supplying the missing words: \$0.60 is —— of \$6; \$0.06 is —— of \$6; \$0.006 is —— of \$6.

9. Formulate a short method for dividing United States money by 10; by 100; by 1000.

10. By inspection find the quotient of:

a. $736 \div 10$.	*e.* \$271 ÷ 100.	*i.* 2140 lb. ÷ 100.
b. $1531 \div 100$.	*f.* \$519.50 ÷ 10.	*j.* 3145 lb. ÷ 100.
c. $16351 \div 1000$.	*g.* \$84.50 ÷ 100.	*k.* 3416 ft. ÷ 1000.
d. $311219 \div 10000$.	*h.* \$2150 ÷ 1000.	*l.* 1279 posts ÷ 100.

11. Read aloud, supplying the missing amounts:

a. $6400 \div 1600 =$ ——; $640 \div 10 =$ ——.

b. $27000 \div 9000 =$ ——; $2700 \div 900 =$ ——; $270 \div 90 =$ ——; $27 \div 9 =$ ——.

c. $18801 \div 90 =$ —— $\div 9$; $214200 \div 700 = 2142 \div$ ——.

12. How is the quotient affected by like changes in both the dividend and divisor?

13. Divide 1323 by 400.

SOLUTION. Cut off the two ciphers in the divisor and two digits in the right of the dividend, thus dividing both dividend and divisor by 100. 4 is contained in 13 three times with a remainder 1 *hundred*. Adding to this remainder the 23 units remaining in the dividend after dividing by 100, the true remainder is 123, which write in fractional form.

$$4|00\overline{)}13|23 \quad 3\tfrac{123}{400}$$
$$12$$
$$123$$

14. Read aloud, supplying the missing amounts: $1611 \div 400 = \text{——}$; $2847 \div 700 = \text{——}$; $1531 \div 300 = \text{——}$; $16139 \div 4000 = \text{——}$.

15. Formulate a rule for dividing a number by any multiple of ten.

16. State the quotient of:

a. $1231 \div 30$.	*f*. $96131 \div 400$.	*k*. $63571 \div 3000$.
b. $9647 \div 40$.	*g*. $84199 \div 700$.	*l*. $16657 \div 4000$.
c. $6551 \div 50$.	*h*. $64137 \div 800$.	*m*. $36119 \div 6000$.
d. $4273 \div 70$.	*i*. $45117 \div 900$.	*n*. $18177 \div 9000$.
e. $8197 \div 90$.	*j*. $25121 \div 500$.	*o*. $42113 \div 7000$.

ORAL REVIEW EXERCISE

The diagram on the opposite page is a portion of the New York Central time-table giving the distances between many of the stations from New York City to Suspension Bridge, and the time taken by two different trains to travel this route.

1. How many miles between New York City and Poughkeepsie? between Poughkeepsie and Utica? between Utica and Syracuse? between Syracuse and Rochester? between Rochester and Buffalo? between Buffalo and Niagara Falls?

2. What is the distance between New York City and Syracuse? between Poughkeepsie and Niagara Falls? between Rochester and Suspension Bridge?

3. How many miles between Ludlow and each station below it? between Poughkeepsie and each station below it? between Tarrytown and each station below it?

4. How many miles between Montrose and each station below it? between Oscawana and each station below it?

5. At 2¢ per mile, what is the fare from New York to Niagara Falls? from Poughkeepsie to Syracuse? from Buffalo to Utica? from Troy to Yonkers?

6. At 2¢ per mile, what is the fare from Rochester to Syracuse? from Rensselaer to Suspension Bridge? from Albany to Niagara Falls? from Syracuse to Buffalo? to Albany?

7. How long does it take train No. 93 to travel the first 30 mi. toward Poughkeepsie? the first 74 mi. toward Albany?

8. How long is train No. 93 in making the run from Fishkill Landing to Camelot? This is approximately how many miles an hour?

9. How long does it take train No. 73 to make the run from Utica to Syracuse? How long does it take train No. 73 to make the run from Fishkill Landing to Chelsea? This is approximately how many miles an hour?

10. Add each number in the column marked "Miles" to the one immediately below it.

Miles	NORTH AND WEST BOUND	Midnight Express	Poughkeepsie Local
		73	93
	New York		
0	**Grand Cent. Sta.** .....Lv.	12 10	6 01
4	**125th St. Sta.** ..... "	12 23	6 13
5	**138th St. Sta.** ..... "		6.15
7	High Bridge ..... "		6.21
8	Morris Heights..... "		6.25
10	Kings Bridge..... "		6.29
11	Spuyten Duyvil..... "		6.33
13	Riverdale ..... "		.....
14	Mt. St. Vincent..... "		.....
15	Ludlow ..... "		.....
16	Yonkers..... "	12.46	6.43
18	Glenwood ..... "	.....	6.46
20	Hastings-on-Hudson "	.....	6.52
21	Dobbs' Ferry..... "	.....	6.59
22	Ardsley-on-Hudson "	.....	7.01
23	Irvington..... "	.....	7.05
26	Tarrytown ..... "	1.09	7.12
30	Scarborough..... "	.....	7.19
31	Ossining..... "	1.25	7.25
35	Croton-on-Hudson .. "	.....	7.31
37	Oscawana..... "	.....	7.34
38	Crugers..... "	.....	7.37
39	Montrose ..... "	.....	7.41
42	Peekskill ..... "	1.47	7.49
47	Highlands..... "	.....	7.59
50	Garrison ..... "	x	8.06
53	Cold Spring..... "	x	8.12
56	Storm King ..... "	.....	8.16
58	Dutchess June ..... "	.....	8.21
59	Fishkill Landing..... "	2 24	8.27
63	Chelsea..... "	2.31	8.34
65	New Hamburg..... "	.....	8.40
69	Camelot..... "	.....	8.46
74	**Poughkeepsie**.....Ar.	2.53	8 55
74	**Poughkeepsie**.....Lv.	3.05	.....
80	Hyde Park..... "	.....	.....
84	Staatsburgh..... "	.....	.....
89	Rhinecliff (Rh'b'k).. "	.....	.....
95	Barrytown..... "	.....	.....
99	Tivoli..... "	.....	.....
105	Germantown ..... "	.....	.....
109	Linlithgo ..... "	.....	.....
111	Greendale ..... "	.....	.....
115	Hudson ..... "	4.47	.....
119	Stockport ..... "	.....	.....
122	Newton Hook..... "	.....	.....
125	Stuyvesant ..... "	.....	.....
131	Schodack Landing.. "	.....	.....
135	Castleton ..... "	.....	.....
142	Rensselaer ..... "	.....	.....
143	**Albany**.....Ar.	5.50	.....
149	**Troy** ..... "	6 50	.....
238	**Utica**.....Ar.	8 40	.....
291	**Syracuse**..... "	9.55	.....
371	**Rochester**..... "	11.38	.....
440	**Buffalo** ..... "	1 15	.....
463	**Niagara Falls**.....Ar.	2 13	.....
464	**Suspension Bridge**..... "	2 20	.....

See Note A

Thus, 9, 12, 16, 24, 34, 45, 58, etc. In adding 89 and 95 think of 179 and 5, or 184; in adding 143 and 149 think first of 243 and 49 and then of 283 and 9, or 292.

11. Multiply each number in the column marked "Miles" by 5; by 8; by 3; by 7; by 6; by 4; by 9.

The numbers in the portion of the time-table illustrated may be used for such other exercises as may seem necessary at this point. Students should be impressed with the importance of being able to add, subtract, multiply, and divide numbers in any relative position.

12. Five parts of 120 are 15, 18, 32, 10, and 20. Find the sixth part, and multiply it by 15.

13. From a flock of 170 sheep I sold at different times 12, 18, 32, and 9. How many sheep remained?

14. Multiply each of the following numbers by 11: 21, 32, 43, 54, 65, 76, 87, 98, 61, 28, 37, 14, 21, 62.

15. At 22¢ per yard, what will 18 yd. cost? 21 yd.? 36 yd.? 56 yd.? 29 yd.? 73 yd.? 94 yd.? 72 yd.?

16. Multiply each number in problem 15 by 33; by 44.

17. Multiply each number in problem 15 by 10; by 100; by 30; by 300; by 500.

18. What will 102 bu. of wheat cost at 68¢ per bushel? at 82¢ per bushel? at 91¢ per bushel? at 99¢ per bushel?

19. Find the cost of 32 bu. of apples at 45¢ per bushel; at 38¢ per bushel; at 42¢ per bushel; at 28¢ per bushel; at 15¢ per bushel; at 21¢ per bushel.

20. I have on hand at the opening of business Monday morning cash amounting to $800. I pay out $80, $40, and $30 and have on hand at the close of the day $860. How much cash did I receive during the day?

Postal information. All mailable matter for transmission by the United States mails within the United States or to Cuba, Mexico, Hawaii, Porto Rico, Canada, and the Philippine Islands is divided into four classes: first-class matter, second-class matter, third-class matter, and fourth-class matter.

First-class matter includes letters, postal cards, and anything sealed or otherwise closed against inspection. The rate for first-class matter is 2¢ per ounce or fraction thereof. The cost of an ordinary postal card is 1¢; of a reply postal card, 2¢.

Second-class matter includes newspapers and periodicals entirely in print. When sent by publishers or news agents, the rate is 1¢ per pound or fraction thereof; when sent by others, 1¢ for each 4 oz. or fraction thereof.

Third-class matter includes books, circulars, pamphlets, proof sheets and manuscript copy accompanying the same, and engravings. The rate is 1¢ for each 2 oz. or fraction thereof.

The limit of weight in third-class matter is 4 lb., except single books in separate packages, on which the weight is not limited.

Fourth-class matter includes all mailable matter not specified in the preceding classes, such as merchandise and samples of every description and kind and specie. The rate is 1¢ for each ounce or fraction thereof.

All kinds of postal matter may be *registered* at the rate of 8¢ for each package in addition to the regular rates of postage.

The rates on *special delivery letters* are 10¢ per letter in addition to the regular postage. Any matter on which a special delivery stamp is affixed is entitled to special delivery.

Foreign rates of postage are as follows: letters 5¢ per *half* ounce; postal cards, 2¢; newspapers and other printed matter, 1¢ per every 2 ounces.

21. What is the postage on a letter weighing $\frac{1}{2}$ oz.? $4\frac{1}{2}$ oz.? $1\frac{1}{2}$ oz.? $3\frac{1}{4}$ oz.? $2\frac{1}{4}$ oz.? $4\frac{1}{4}$ oz.?

22. Find the total cost of mailing the following to points in Canada: a book, weighing $32\frac{1}{2}$ oz., which you have registered; a package of jewelry, weighing 19 oz., which you have registered.

23. What will be the total cost of mailing the following articles at your post office to points within the United States: an ordinary letter, weighing $2\frac{1}{4}$ oz.; a registered letter, weighing $1\frac{1}{2}$ oz.; a book, weighing 3 lb. 8 oz.; and a bundle of papers, weighing 10 oz.?

24. Find the total cost of mailing the following to points within the United States: a special delivery letter, weighing $1\frac{1}{4}$ oz.; a registered letter, weighing $2\frac{1}{2}$ oz.; some printers' proofs, weighing 18 oz.; some separate manuscript for printer, weighing 12 oz.; a pamphlet weighing 6 oz.

25. Find the mailing price of each of the following articles:

	Article	List Price	Weight when Packed
a.	A pair of opera glasses	$12.50	2 lb. 8 oz.
b.	A pair of ladies' gloves	$ 2.50	6 oz.
c.	A copy of Star-Land	$ 1.20	1 lb. 8 oz.
d.	A copy of Whittier's Poems	$ 1.60	1 lb. 12 oz.
e.	A copy of Footprints of Travel	$ 1.25	1 lb. 8 oz.

26. A publishing house advertises books at the following prices. If the wrapping used in preparing the books for mailing weighs 4 oz. in each case, what is the weight of the book?

Book	List Price	Mailing Price
a. Wilderness Ways	45¢	50¢
b. Ways of Woodfolk	50¢	60¢
c. Friends and Helpers	60¢	70¢
d. Triumphs of Science	30¢	35¢
e. Industries of To-day	25¢	30¢

27. A publisher sends 20,000 copies of his magazine by mail. If each magazine and wrapper weighs $14\frac{1}{2}$ oz. and the total number is weighed at the post office in bulk, what will the publisher have to pay for postage?

28. A subscriber mails two issues of the above magazine to a friend. What will be the cost for postage?

29. 25,000 copies of a monthly magazine weighing $14\frac{1}{4}$ oz. were sent by mail. What is the cost to the publisher of mailing?

30. Find the total cost for mailing the following: printers' proof weighing $18\frac{1}{4}$ oz.; manuscript and printers' proof in one package, weighing $28\frac{1}{2}$ oz.; a book, weighing 22 oz.; a special delivery letter, weighing $\frac{3}{4}$ oz.; two ladies' pocketbooks, weighing 14 oz.

WRITTEN REVIEW EXERCISE

1. Find the total cost of the articles in problem 3 of the oral exercise, page 56. Find the total of the products in the oral exercise, page 60.

2. A mechanic earns \$125 per month and his monthly expenses average \$72. If he saves the remainder, how long will it take him to pay for a house costing \$4352?

3. I spent \$24,800 for apples at \$2.50 per barrel. The loss from decay was equal to 74 bbl. What was my gain, if the remainder of the apples sold for \$3.75 per barrel, and my expenses for storage were \$675.80?

4. Without copying find (*a*) the total number of railway employees in the United States in 1903 and (*b*) the total number per hundred miles of line in the same year.

RAILWAY EMPLOYEES IN THE UNITED STATES

CLASS	1904			1903		
	TOTAL NUMBER	NUMBER PER 100 MILES	AVERAGE DAILY WAGES	TOTAL NUMBER	NUMBER PER 100 MILES	AVERAGE DAILY WAGES
General officers	5,165	2	$11.61	4,842	2	$11.27
Other officers	5,375	3	6.07	5,201	3	5.76
General office clerks	46,037	22	2.22	42,218	21	2.21
Station agents	34,918	16	1.93	34,892	17	1.87
Other stationmen	120,002	57	1.69	120,724	59	1.64
Engineers	52,451	25	4.10	52,993	26	4.01
Firemen	55,004	26	2.35	56,041	27	2.28
Conductors	39,645	19	3.50	39,741	19	3.38
Other trainmen	106,734	50	2.27	104,885	51	2.17
Machinists	46,272	22	2.61	44,819	22	2.50
Carpenters	53,646	25	2.26	56,407	27	2.19
Other shopmen	159,472	75	1.91	154,635	75	1.86
Section foremen	37,609	18	1.78	37,101	18	1.78
Other trackmen	289,044	136	1.33	300,714	147	1.31
All other employees	244,747	115	1.98	257,324	125	1.93

5. Without copying find (*a*) the total number of railway employees in the United States in 1904 and (*b*) the total number per one hundred miles of line in the same year.

6. Find the total salaries paid to railway employees in 1903; in 1904.

7. Find the average daily wages paid to railway employees in 1903; in 1904.

8. During a certain week a contractor employed help as follows: 34 hands, 8 hr. per day, for 5 da., at 15¢ per hour; 16 hands, 9 hr. per day, for 6 da., at 25¢ per hour; 29 hands, 10 hr. per day, for 6 da., at 18¢ per hour. Find the amount due the employees.

9. In a recent year there were produced on 27,842,000 A. in the United States 863,102,000 bu. oats, valued on the farm at 31.3¢ per bushel. What was the average yield per acre? what was the value of the year's crop?

10. Complete the following schedule by finding the vertical and horizontal totals. Check the work by comparing the sum of the vertical totals with the sum of the horizontal totals.

Salary and Expense Schedule

Fish and Game Commission of Massachusetts

For the month ending June 30, 1906

No.	Names	Commissioners		Incidentals	Clerical	Deputies		Total
		Salary	Expenses			Salary	Expenses	
1	F. M. Ford	375 —	179 60					
2	W. D. Snow	375 —	184 90					
3	M. L. Covey			70				
4	E. E. Trine			14 96				
5	J. H. Kaplan				125 —			
6	W. S. Osborn				100 —			
7	L. C. Nixon					275 —	72 90	
8	J. F. Wilcox					275 —	124 60	
9	A. M. Cook					275 —	101 —	
10	C. M. Closs					275 —	112 80	
11	G. A. Mason					275 —	142 30	
12	A. D. Curtis					275 —	60 50	
13	D. E. Henry					275 —	60 20	
14	G. H. Ingalls					275 —	75 90	
15	J. M. Cox					275 —	125 60	
16								

11. Without copying, find quickly the total amount of the following manufacturer's time sheet. Check the work.

TIME SHEET FOR WEEK ENDING JULY 29

NAME	M.	T.	W.	T.	F.	S.	TOTAL TIME	RATE PER HOUR	AMOUNT	
Harry Ball	9	8	10	10	10	9		12¢		
John Cook	8	8	10	9	9	8		12¢		
James Easton . . .	9	9	9	10	8	8		15¢		
Frank King	7	6	8	9	9	10		20¢		
Paul Mason	8	8	8	8	8	8		25¢		

12. From the following data make a statement of losses and gains: Market value of groceries on hand May 1, $4469.40. Bought groceries during the month: for cash, $1279.60; on credit, $2150.40. Sold groceries during the month: for cash, $2160.40; on credit, $2640.10. Gross expenses at the close of the month, $590.50. Account against J. E. Brown & Co. which cannot be collected, $79.80. Market value of groceries on hand at the close of the month, $2842.60. Required, the net gain or net loss.

13. In the following table find (*a*) the total number of tickets sold each day, (*b*) the total number of each class sold during the week, and (*c*) the aggregate number of tickets sold during the week. Check the work.

TICKETS OF ADMISSION SOLD AT A STATE FAIR

CLASS	PRICE	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	TOTAL
Children	$0.35	1240	1242	4165	3169	3146	1240	
Adults	.75	6129	6129	12168	17246	12174	9167	
Single carriages	.75	68	126	329	278	278	74	
Double carriages	1.25	49	114	215	210	210	62	
Total								

14. In the above table find (*a*) the daily receipts from tickets and (*b*) the aggregate receipts for the week. Check the work.

Copy the following time sheets and find (*a*) the total number of hours worked on each order, (*b*) the total number of hours worked each day, (*c*) the amount earned on each order, and (*d*) the total amount earned during the week. Check the work.

15.

BOSTON ELEVATED RAILWAY COMPANY

Time worked by Fred H. Bowles

During the week ending Aug. 5, 19 06.

Rate per hour 35¢. Occupation Electrician

Order No.	Sat.	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Total hours	Amount	
518	3		6							
725					4	4	6			
692	5		1	10						
437					6	6	3			
865	1		2				1			

16.

BOSTON ELEVATED RAILWAY COMPANY

Time worked by John D. Smith

During the week ending July 29, 19

Rate per hour 25¢ Occupation Carpenter

Order No.	Sat.	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Total hours	Amount	
497	2		4							
832			2		10					
693	3			10						
472	1		3							
913			1			10	10			

CHAPTER VIII

AVERAGE

ORAL EXERCISE

1. A earns \$3, B earns \$4, and C earns \$5 per day. What do the three earn in 1 da.? If \$12 were paid to these men in equal parts, how much would each receive?

2. What sum is intermediate between 6, 7, and 8? between 6, 8, and 10? between 6, 12, and 18?

110. The process of finding a number that is intermediate between two or more other numbers is called **average.**

111. Example. What is the average weight of 3 bales of cotton weighing 460, 449, and 475 lb., respectively?

SOLUTION. The aggregate of the 3 bales of cotton is 1384 lb. 1384 lb. divided into three equal parts shows the mean or average weight to be $461\frac{1}{3}$ lb.

To find the average of consecutive numbers, add the highest number to the lowest, and divide by 2.

$$\begin{array}{r} 460 \\ 449 \\ 475 \\ \hline 3)\overline{1384} \\ \hline 461\frac{1}{3} \end{array}$$

WRITTEN EXERCISE

1. A tapering board is 14 in. wide on one end and 18 in. on the other. What is the average width of the board?

2. A manufacturing pay roll shows that 15 hands are employed at \$1.25 per day, 12 hands at \$1.75 per day, 16 hands at \$2.25 per day, 32 hands at \$2.50 per day, and 5 hands at \$6.50 per day. Find the average daily wages.

3. A merchant's sales for a year were as follows: January, \$12,156; February, \$14,175; March, \$16,152; April, \$12,175; May, \$12,465.95; June, \$12,476.05; July, \$15,145.40; August, \$12,431.46; September, \$17,245.90; October, \$18,256.45; November, \$19,250.65; December, \$19,654.20. What were his average sales per month?

4. In a certain school of 300 pupils, 85 are 14 yr. of age; 50, 15 yr. of age; 25, 16 yr. of age; 75, 17 yr. of age; 50, 18 yr. of age; 15, 19 yr. of age. What is the average age of the school?

5. The attendance for a certain school for a week was as follows: Monday, 727 pupils; Tuesday, 732 pupils; Wednesday, 756 pupils; Thursday, 761 pupils; Friday, 734 pupils. What was the average daily attendance for the week?

6. What should a ground feed made from 50 bu. of oats worth 28¢ per bushel, 30 bu. of barley worth 78¢, and 60 bu. of corn worth 59¢ sell for in order to make 10¢ per bushel on each ingredient used to make the mixture?

7. Find the aggregate weight and the average weight per box of 100 bx. of cheese weighing 65, 64, 62, 60, 61, 65, 62, 64, 61, 62, 61, 60, 60, 61, 62, 60, 68, 65, 66, 64, 62, 61, 65, 66, 62, 64, 67, 58, 62, 59, 59, 60, 62, 64, 66, 67, 58, 60, 65, 58, 62, 69, 62, 65, 68, 69, 61, 65, 62, 61, 65, 68, 59, 62, 64, 58, 62, 65, 71, 70, 58, 67, 58, 62, 64, 58, 62, 64, 65, 69, 65, 65, 62, 64, 60, 60, 65, 60, 65, 65, 62, 60, 62, 64, 60, 72, 64, 70, 61, 62, 60, 60, 59, 65, 60, 70, 58, 62, 61, 64 lb., respectively.

8. Counting 8 hr. to a day, find the total amount and the average daily wages in the following contractor's time sheet:

TIME SHEET FOR WEEK ENDING JUNE 30

NAME	M.	T.	W.	T.	F.	S.	HOURS	DAYS	DAILY WAGES	AMOUNT	
C. E. Ames	8	8	8	8	8	8			$1.75		
W. O. Bye	9	10	9	10	10	8			2.00		
M. E. Carey	10	9	9	10	8	10			2.00		
W. D. Frey	6	8	9	10	7	8			2.25		
G. W. Jones	10	10	10	8	10	8			2.25		
D. O. Munn	4	4	4	6	8	6			2.50		
E. H. Post	6	6	6	6	4	4			3.00		
L. C. Roe	10	10	10	10	4	4			3.25		
J. H. Small	6	8	8	10	12	12			3.25		
H. M. Young	8	8	8	8	8	8			3.50		
Total											

CHAPTER IX

CHECKING RESULTS

112. It has been seen in the preceding exercises on statistics, time sheets, etc., that various ruled forms provide for practical and convenient methods of checking results. While it is possible to give a great variety of these problems it is also necessary to give a great many problems that do not furnish such a check.

113. It is very important that all results be checked. The most common methods of checking addition, subtraction, and division have already been mentioned. Multiplication may be proved by dividing the product by either factor, or as explained on page 52.

114. The properties of 9 and 11 may also be applied to advantage in checking results, especially results in multiplication and division.

PROPERTIES OF 9 AND 11

Properties of 9

115. Any number of 10's is equal to the same number of 9's plus the same number of units; any number of 100's is equal to the same number of 99's plus the same number of units; any number of 1000's is equal to the same number of 999's plus the same number of units; and so on.

Thus, 10 = one 9 + 1; 40 = four 9's + 4; 100 = one 99 + 1; 300 = three 99's + 3; 500 = five 99's + 5.

116. Any number may be resolved into one less than as many multiples of 10 as it contains digits.

Thus, 946 = 900 + 40 + 6; 42175 = 40000 + 2000 + 100 + 70 + 5.

117. The excess of 9's in any multiple of a power of 10 multiplied by a single digit is the same as the significant figure in that number. Hence,

The excess of 9's in any number is equal to the excess of 9's in the sum of its digits.

Thus, the excess of 9's in $241 = 2 + 4 + 1$, or 7. The excess of 9's in $946 = 9 + 4 + 6$, or 19; but 19 contains 9, and the excess of 9's in $19 = 1 + 9$, or 10; but 10 contains 9, and the excess of 9's in $10 = 1 + 0$, or 1; the excess of 9's in 946 is therefore shown to be 1.

118. In finding the excess of 9's in any number, omit all 9's and all combinations of two or three digits which it is seen at a glance will make 9 or some multiple of 9.

Thus, in finding the excess of 9's in 9458, begin at the left, reject the first digit 9, the sum of the next two digits, 9, and the single 8 will be the excess of 9's in the entire number.

Properties of 11

119. Any number of 10's is equal to the same number of 11's minus the same number of units; any number of 100's is equal to the same number of 99's plus the same number of units; any number of 1000's is equal to the same number of 1001's minus the same number of units; and so on.

Thus, 40 = four 11's − 4; 500 = five 99's + 5; 7000 = seven 1001's − 7.

120. It is therefore clear that even powers of 10 are multiples of 11 plus 1 and odd powers of 10 are multiples of 11 minus 1.

Thus, 10^2 or 100 = nine 11's + 1; 10^3 or 1000 = ninety-one 11's − 1; 10^4 or 10,000 = nine hundred nine 11's + 1.

121. From the foregoing it is evident that:

The excess of 11's in any number is equal to the sum of the digits in the odd places (increased by 11 or a multiple of 11 if necessary) minus the sum of the digits in the even places.

Thus, the excess of 11's in 45 is 1 (5 − 4); the excess of 11's in 125 is 4 ($\overline{5-2} + \overline{1-0}$); the excess of 11's in 2473 is 9 [14 (11 + 3) − 7 + 2 (4 − 2) = 9].

Checking Addition and Subtraction

122. Examples. 1. By casting out the 9's, show that the sum of 935, 651, 782, and 465 is 2833.

$935 = 8$
$651 = 3$
$782 = 8$
$465 = 6$
$2833 = \overline{7}$

Solution. The sum of the digits in 935 is 17; but since 17 contains 9, find the sum of the digits in 17 and the result, 8, is the excess of 9's in the entire number. In like manner find the excess of 9's in 651, 782, and 465. Since 935 is a multiple of $9 + 8$, 651 a multiple of $9 + 3$, 782 a multiple of $9 + 8$, 465 a multiple of $9 + 6$, the sum of these numbers, 2833, should equal a multiple of $9 + (8 + 3 + 8 + 6)$, or $9 + 25$. 25 is a multiple of $9 + 7$, and 2833 is a multiple of $9 + 7$; hence, the addition is probably correct.

2. By casting out the 11's, show that the sum of 648, 217, 451, and 688 is 2004.

$648 = 10$
$217 = 8$
$451 = 0$
$688 = 6$
$2004 = \overline{2}$

Solution. $\overline{8-4} + \overline{6-0} = 10$, the excess of 11's in 648. $\overline{7-1} + \overline{2-0} = 8$, the excess of 11's in 217. $12\ (11+1) - 5 + \overline{4-0} = 11$; but 11 contains 11, hence, the excess of 11's in 451 is 0. $\overline{8-8} + \overline{6-0} = 6$, the excess of 11's in 688. Since 648 is a multiple of $11 + 10$, 217 a multiple of $11 + 8$, 451 a multiple of 11, and 688 a multiple of $11 + 6$, the sum of these numbers, 2004, should be a multiple of $11 + (10 + 8 + 6)$, or $11 + 24$. 24 is a multiple of $11 + 2$ and 2004 is a multiple of $11 + 2$; hence, the addition is probably correct.

123. Subtraction may be proved either by casting out the 9's or 11's in practically the same manner as addition.

The difference between the excess of 9's or 11's in the minuend and subtrahend should equal the excess of 9's or 11's in the remainder; or the sum of the excess of 9's or 11's in the subtrahend and remainder should equal the excess of 9's or 11's in the minuend.

These methods are but little used for checking addition and subtraction. Addition is generally checked as explained on page 20, and subtraction as explained on page 32. On the other hand, long multiplications and divisions are almost always checked by applying the properties of 9 or 11.

Checking Multiplication and Division

124. Examples. 1. By casting out the 9's show that the product of 64×95 is 6080.

$95 = 5$
$64 = 1$
$6080 = \overline{5}$

Solution. The excess of 9's in 95 is 5, and in 64, 1. Since 95 is a multiple of $9 + 5$ and 64 a multiple of $9 + 1$, the product of 64×95 should be a multiple of 9 plus (1×5). 1×5 or 5 equals the excess of 9's in 6080; hence, the work is probably correct.

2. By casting out the 11's show that the product of 46×95 is 4370.

SOLUTION. The excess of 11's in 95 is 7, and in 46, 2. Since 95 is a multiple of $1 + 17$ and 46 a multiple of $11 + 2$, the product of 46×95 should be a multiple of 11 plus (2×7) or 14; but 14 is a multiple of $11 + 3$. Since the product 4370 is a multiple of $11 + 3$, the work is probably correct.

$$\begin{array}{r} 95 = 7 \\ 46 = 2 \\ \hline 4370 = 3 \end{array}$$

125. Division may be proved either by casting out the 9's or 11's in practically the same manner as multiplication. The excess of 9's or 11's in the quotient multiplied by the excess of 9's or 11's in the divisor should equal the excess of 9's or 11's in the dividend, minus the excess of 9's or 11's in the remainder, if any.

Casting out the 9's will not show an error caused by a transposition of figures; but casting out the 11's will show such an error. The method of casting out the 11's is therefore considered the better proof.

WRITTEN EXERCISE

1. Determine without dividing whether \$2.64 is the quotient of $\$1375.44 \div 521$.

2. Determine without multiplying whether \$1807.50 is the product of 482 times \$3.75.

3. Determine without adding whether 4231 is the sum of 296, 348, 924, 862, 956, and 845.

4. Multiply 34,125 by 729 in two lines of partial products and verify the work by casting out the 9's.

5. Find the cost of 173,000 shingles at \$4.27 per thousand, in two lines of partial products, and verify the work by casting out the 11's.

6. Find the cost of 126,000 ft. of clear pine at \$24.60 per thousand, in two lines of partial products, and verify the work by casting out the 9's.

7. Find the cost of 2,191,000 ft. of flooring at \$32.08 per thousand, in two lines of partial products, and verify the work by casting out the 11's.

FRACTIONS

CHAPTER X

DECIMAL FRACTIONS

ORAL EXERCISE

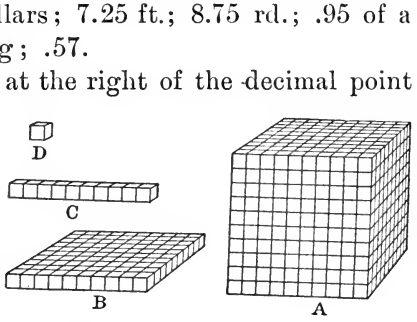
1. In the number \$7.62 what figure stands for the dollars? the tenths of a dollar? the hundredths of a dollar?

2. What name is given to the point which separates the whole number of dollars from the part of a dollar?

3. Read: 3.5 dollars; 3.5 ft.; 27.5 lb.; .7 of a dollar; .5 of a ton; 16.6; .9; 9.25 dollars; 7.25 ft.; 8.75 rd.; .95 of a dollar; .85 of a pound sterling; .57.

4. What is the first place at the right of the decimal point called? the second place?

5. In the accompanying diagram what part of A is B? What part of B is C? What part of C is D?



6. What part of A is C? What part of A is D?

7. If A is a cubic inch, what is B? C? D?

8. In a pile of 10,000 bricks one brick is what part of the whole pile? 10 bricks is what part of the whole pile? 100 bricks is what part of the whole pile? 1000 bricks is what part of the whole pile?

9. How may one tenth be written besides $\frac{1}{10}$? one hundredth besides $\frac{1}{100}$? one thousandth besides $\frac{1}{1000}$?

126. Units expressed by figures at the right of the decimal point are called **decimal units.**

127. A number containing one or more decimal units is called a **decimal fraction** or a **decimal.**

NOTATION AND NUMERATION

ORAL EXERCISE

1. Read: 0.7; 0.03; 0.25. How many places must be used to express completely any number of hundredths?

2. Read: 0.004; 0.025; 0.725. How many places must be used to express completely any number of thousandths?

3. Read: .0005; .00007; .000009; .0037; .00045; .000051; .0121; .00376; .000218; .1127; .01525; .004531; .16067.

4. How many places must be used to express completely any number of ten-thousandths? any number of hundred-thousandths? any number of millionths?

128. In **reading decimals** pronounce the word *and* at the decimal point and omit it in all other places.

Thus, in reading 0.605 or .605 say *six hundred five thousandths;* in reading 600.005 say *six hundred and five thousandths.*

129. The relation of integers and decimals with their increasing and decreasing orders to the left and to the right of the decimal point is shown in the following

NUMERATION TABLE

PERIODS:	Millions			Thousands			Units				Thousandths			Millionths		
ORDERS:	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
	9	8	7,	6	5	4,	3	2	1	.	2	3	4	5	6	7

130. Hundredths are frequently referred to as **per cent**, a phrase originally meaning *by the hundred.*

131. The symbol % stands for hundredths and is read *per cent.*

Thus 45% = .45; 48% of a number = .48 of it.

ORAL EXERCISE

Read:

1. 0.073.
2. 0.00073.
3. 3004.025.
4. 300.4025.
5. 532.002.
6. 60.0625.
7. 63.3125.
8. 126.8125.
9. 31.08%.
10. 126.75%.
11. 2150.1875.
12. 3165.00625.
13. 131.3125 T.
14. 240.0125 A.
15. A tax of 1.0625 mills.
16. A tax of 9.1875 mills.
17. Read the number in the foregoing numeration table.
18. Read the following, using the words "per cent": .17; 28; .85; .67; .425; .37½.
19. Read the following as decimals, not using the words "per cent": 25%; 75%; 87%; 62½%; 27.15%.
20. Read aloud the following:

a. The value of a pound sterling in United States money is $4.8665.

b. A meter (metric system of measures) is equal to 39.37079 in.; a kilometer, to 0.62137 mi.

c. 1 metric ton is equal to 1.1023 ordinary tons; 1.5 metric tons are equal to 1.65345 ordinary tons.

d. A flat steel bar 3 in. wide and 0.5 in. thick weighs 5.118 lb.

e. The circumference of a circle is 3.14159 times the length of its diameter.

WRITTEN EXERCISE

Write decimally:

1. Five tenths; fifty hundredths; five hundred thousandths.
2. Nine hundred and eleven ten-thousandths; nine hundred eleven ten-thousandths; five hundred and two thousandths.
3. One hundred seventy-four millionths; one hundred seventy-four million and seven millionths; seven million and one hundred seventy-four millionths.
4. Seven thousand and seventy-five ten-thousandths; two hundred fifty-seven ten-millionths; two hundred and forty-six millionths; two hundred forty-six millionths.

5. Four million ten thousand ninety-seven ten-millionths; four million ten thousand and ninety-seven ten-millionths; five hundred millionths; five hundred-millionths.

6. Six hundred six and five thousand one hundred-thousandths; six hundred six and fifty-one hundred-thousandths; fifty-six and one hundred twenty-eight ten-billionths.

7. Seventeen thousand and eighteen hundred seventy-six millionths; seventeen thousand and eighteen hundred seventy-six ten-thousandths; twenty-one hundred sixteen hundredths.

132. In the number 2.57 there are 2 integral units, 5 tenths of a unit, and 7 hundredths of a unit. In the number 2.5700 there are 2 integral units, 5 tenths of a unit, 7 hundredths of a unit, 0 thousandths of a unit, and 0 ten-thousandths of a unit. 2.5700 is therefore equal to 2.57. That is,

Decimal ciphers may be annexed to or omitted from the right of any number without changing its value.

ORAL EXERCISE

Read the following (a) as printed and (b) in their simplest decimal form:

1. 0.700.	3. 16.010.	5. 0.50.	7. 0.7000.
2. 5.2450.	4. 18.210.	6. 0.00950.	8. 12.9010.

ADDITION

ORAL EXERCISE

1. What is the sum of 0.4, 0.05, 0.0065?
2. What is the sum of 0.3, 0.021, 0.008?
3. Find the sum of seven tenths, forty-four hundredths, and two; of four tenths, twenty-one hundredths, and six thousandths.

133. Example. Find the sum of 12.021, 256.12, and 27.5.

SOLUTION. Write the numbers so that their decimal points stand in the same vertical column. Units then come under units, tenths under tenths, and so on. Add as in integral numbers and *place the decimal point in the sum directly under the decimal points in the several numbers added.*

$$\begin{array}{r} 12.021 \\ 256.12 \\ 27.5 \\ \hline 295.641 \end{array}$$

WRITTEN EXERCISE

Find the sum of:

1. 7.5, 165.83, 5.127, 6.0015, and 71.215.
2. 257.15, 27.132, 5163, 8.000125, and 4100.002.
3. 0.175, 5.0031, .00127, 70.2116001, and 21.00725.
4. 51.6275, 19.071, 0.000075, 21.00167, and 40,000.01.
5. 2.02157, 2.1785, 2500.00025, 157.2165, and 7.0021728.
6. Copy, find the totals as indicated, and check:

$1241.50	$9215.45	$1421.12	$1421.32	?
1.52	1275.92	1.46	1618.40	?
349.21	3725.41	2.18	1920.41	?
2975.47	7286.95	7.96	10.20	?
27.14	8276.92	14.21	41.64	?
9218.49	7271.44	1240.80	126.18	?
5.17	8926.95	7216.80	24.17	?
12627.85	8972.76	4.75	240.20	?
721.92	7214.25	8.16	960.80	?
11.41	8142.76	.47	1860.45	?
1.21	8436.14	.92	9270.54	?
.72	8435.96	9.26	75.86	?
14178.21	7926.14	1490.75	45.95	?
2172.14	9214.72	1860.54	75.86	?
726.95	1241.16	9265.80	72.18	?
85.21	4214.71	625.50	9260.14	?
75.92	8726.19	240.75	1.20	?
72604.25	2140.12	60.50	7.40	?
124.61	7146.14	120.41	8.32	?
2114.62	7214.86	4101.08	2860.14	?
?	?	?	?	?

7. Find the sum of twenty-one hundred sixty-five and one hundred sixty-five ten-thousandths, thirty-nine and twelve hundred sixty-five millionths, twenty-seven hundred thirty-six and one millionth, four and six tenths, six hundred and six thousandths, and six hundred sixty-five thousandths.

SUBTRACTION

ORAL EXERCISE

1. From the sum of 0.7 and 0.4 take 0.5.

2. From the sum of 0.07 and 0.21 take 0.006.

3. From seventy-four hundredths take six thousandths.

4. To the difference between .43 and .03 add the sum of .45 and .007.

5. Goods on hand at the beginning of a week, $24.50; goods purchased during the week, $35.50; goods sold during the week, $36; goods on hand at the close of the week, $36.50. What was the gain or loss for the week?

134. Example. From 14.27 take 5.123.

SOLUTION. Write the numbers so that the decimal points stand in the same vertical column. The minuend has not as many places as the subtrahend; hence suppose decimal orders to be annexed until the right-hand figure is of the same order, then subtract as in integers and *place the decimal point in the remainder directly under the decimal points in the numbers subtracted.*

$$\begin{array}{r} 14.27 \\ 5.123 \\ \hline 9.147 \end{array}$$

WRITTEN EXERCISE

Find the difference between:

1. 7.2154 and 2.8576.
2. 17.2157 and 1.0002.
3. 1.0005 and .889755.
4. $1265.45 and $87.99.
5. 9 and 5.2675.
6. 16 and 5.0000271.
7. .0002 and .000004.
8. 24.503 and 17.00021.

9. The sum of two numbers is 166.214. If one of the numbers is 40.21, what is the difference between the numbers?

10. The minuend is 127.006 and the remainder 15.494. What is the sum of the minuend, subtrahend, and remainder?

11. From the sum of ninety-nine ten-thousandths, one hundred fifty-one and five thousandths, two hundred fifty-two and twenty-five millionths, six tenths, and eighteen and one hundred seventy-five thousandths take the sum of twelve hundred fifteen millionths, and one hundred eighty-eight thousandths.

12. From the sum of two hundred fifty-seven thousandths and eight and one hundred twenty-six millionths take the sum of five hundred ten thousandths and two and one hundred twenty-four ten-thousandths.

13. A merchant had, at the beginning of a year, goods amounting to $8165.95. During the year his purchases amounted to $5265.90 and his sales to $9157.65. At the close of the year he took an account of stock and found that the goods on hand were worth $7216.56. What was his gain or loss for the year?

14. A provision dealer had on hand Jan. 1, goods worth $4127.60. His purchases for the year amounted to $4165.95 and his sales to $6256.48. Dec. 31 of the same year his inventory showed that the goods on hand were worth $3972.50. If the amount paid for freight on the goods bought amounted to $237.50, what was his gain or loss on provisions?

15. I had on hand Jan. 1, lumber amounting to $4210.60. During the year my purchases amounted to $3126.50, and my sales to $4165.85. I lost by fire lumber valued at $506.75, for which I received from an insurance company $500. Dec. 31, my inventory showed the lumber to be worth $5209.08. How much did I gain or lose on lumber during the year?

16. At the beginning of a year my resources were as follows: cash on hand, $1262.50; goods in stock, $1742.85; account against A. M. Eaton, $146.50. At the same time my liabilities were as follows: note outstanding, $156.85; account in favor of Robert Wilson, $521.22. During the year I made an additional investment of $1250.65, and withdrew for private use $275. I sold for cash during the year goods amounting to $1250.75, and bought for cash goods amounting to $530.90; I also paid Robert Wilson $320 to apply on account. At the close of the year my inventory showed goods in stock valued at $750.48. What was my gain or loss for the year and my present worth at the close of the year?

Do not fail to check all problems. No phase of arithmetic is more important.

MULTIPLICATION

ORAL EXERCISE

1. How many times .4 is 4? .77 is 7.7? .999 is 9.99?

2. 44 is how many times .44? 22 is how many times .022? 1 is how many times .001? .01 is how many times .0001?

3. Read aloud the following, supplying the missing terms: Removing the decimal point one place to the right multiplies the value of the decimal by —— ; two places, —— the value by —— ; three places, —— the value by ——.

4. Multiply 12.1252 by 1000; by 100; by 100,000.

5. Multiply $9.375 by 100; by 10,000; by 100,000.

6. Multiply 5.15 by 10; by 100; by 1000; by 10,000.

7. Multiply .000016 by 1000; by 100,000; by 1,000,000.

8. Multiply $67.50 by 10; by 100; by 1000; by 10,000.

9. Multiply .0037 by 10; by 100; by 1000; by 10,000,000.

10. What part of 1 is .1? of 7 is .7? of 29 is 2.9?

11. What part of 84 is .84? of 129 is 1.29? of 1275 is 12.75?

12. What part of .6 is .006? of .64 is .0064?

Read aloud the following, supplying the missing terms:

a. Each removal of the decimal point one place to the left —— the value of the decimal by 10.

b. To divide a decimal by —— is to find one tenth (.1) of it, or to —— it by .1.

13. Give a short method for multiplying a number by .1; by .01; by .001; by .0001.

14. Multiply .009 by .1; by .01; by .001.

15. Multiply 217.59 by .1; by .01; by .001.

16. Multiply 54.65 by .01; by .00001; by .000001.

17. Multiply 2.375 by .1; by .01; by .001; by .0001.

18. Multiply 25.215 by .1; by .01; by .001; by .0001.

19. Multiply 2111 by .01; by .001; by .0001; by .00001.

20. Compare $2400 \times \$0.06$ with $100 \times 24 \times \$0.06$ or with $24 \times \$6$.

21. Compare 3000×612.251 with $1000 \times 3 \times 612.251$, or with 3×612251.

22. Multiply 21.25 by 2400.

SOLUTION. 2400 is 24 times 100. Multiply by 100 by removing the decimal point two places to the right. The result is 2125. 24 times 2125 equals 51,000, the required product.

In multiplying begin with either the lowest or the highest digit in the multiplier as shown in the margin.

2125	2125
24	24
8500	4250
4250	8500
51000	51000

23. Formulate a brief rule for multiplying a decimal by any number of 10's, 100's, 1000's, etc.

24. Find the cost of :

a. 500 lb. at 18¢. *d.* 600 lb. at 29¢. *g.* 900 lb. at 34¢.
b. 150 lb. at 14¢. *e.* 300 lb. at 41¢. *h.* 700 lb. at 51¢.
c. 200 lb. at 26¢. *f.* 400 lb. at $12\frac{1}{2}$¢. *i.* 1400 lb. at 5¢.

135. Examples. 1. Multiply 41.127 by 4.

SOLUTION. 41.127 is equal to 41,127 thousandths. 41,127 thousandths multiplied by 4 equals 164,508 thousandths, or 164.508. That is, thousandths multiplied by a whole number must equal thousandths.

41.127
4
164.508

2. Multiply 41.127 by .04.

SOLUTION. The multiplier, .04, is equal to 4 times .01 ; therefore, multiply by 4 and by .01. Multiplying by 4, as in problem 1, the result is 164.508. Multiplying by .01, by simply moving the decimal point in the product two places to the left, the result is 1.64508.

41.127
.04
1.64508

It will be seen that *the number of decimal places in the product is equal to the decimal places in the multiplicand and multiplier.*

It should not be necessary to memorize the above rule. The student should know at a glance that the product of tenths and tenths is hundredths, of tenths and hundredths is thousandths, and so on.

ORAL EXERCISE

1. In multiplying 24.05 by 3.14 can you tell before multiplying how many integral places there will be in the product? how many decimal places? Explain.

2. How many integral places will there be in each of the following products : 2.5×4.015? 27.51×3.1416? 321.1×201.51? 1.421×42.267? $126.5 \times .01$? 1020×5.01? $.105 \times 6$? 2.41×10.05? How many decimal places will there be in each of the above products?

3. What are 400 bbl. of apples worth at $2.12 per barrel? at $1.27\frac{1}{2}$ per barrel?

4. I bought 60 lb. of sugar at $0.04\frac{1}{2}$ and gave in payment a five-dollar bill. How much change should I receive?

5. A and B are partners in a manufacturing business, A receiving 52% and B 48% of the yearly profits. The profits for a certain year are $5000. Of this sum how much should A and B, respectively, receive?

WRITTEN EXERCISES

Find the product of:

1. 3.121×152.
2. $3121 \times .152$.
3. 31.21×15.2.
4. 12.14×265.
5. $9.004 \times .021$.
6. $.3121 \times .0152$.
7. 2.531×31000.
8. $.1724 \times 18000$.
9. $.15539 \times 2002$.

10. A man owned 75% of a gold mine and sold 50% of his share. What is the remainder worth if the value of the whole mine is $425,000?

11. A man bought a farm of 240 A. at $137.50 per acre. He sold 75% of it at $150 per acre, and the remainder at $175 per acre. What was his gain?

12. Copy and complete the following table of statistics. Check the results. (The total yield multiplied by the price per bushel should equal the total valuation.)

LARGEST WHEAT-GROWING STATES IN A RECENT YEAR

STATE	YIELD IN BUSHELS	FARM PRICE PER BUSHEL	FARM VALUATION
Minnesota	68,344,256	92.4¢	
Kansas	65,019,471	92.4¢	
North Dakota	53,892,193	92.4¢	
South Dakota	31,556,784	92.4¢	
Total			

13–15. Make and solve three self-checking problems in multiplication of decimals.

DIVISION

ORAL EXERCISE

1. Divide by 8: 64 ft., .64, .064, 6.4.
2. Divide by 9: 63 in., .63, .063, 6.3.
3. Divide by 16: \$640, \$6.40, 6.4, .64, .064.
4. Divide by 15: \$15.75, \$7.50, \$0.75, 30.45, 3.045, .3045.
5. Divide 337.5 by 45.

$$
\begin{array}{r}
7.5 \\
45\overline{)337.5} \\
315 \quad = 45 \text{ times } 7 \\
\hline
22.5 \text{ undivided} \\
22.5 \quad = 45 \text{ times } .5
\end{array}
$$

CHECK. 45 times 7.5 = 337.5; hence, the work is probably correct.

136. In the above exercise it is clear that *when the divisor is an integer, each quotient figure is of the same order of units as the right-hand figure of the partial dividend used to obtain it.*

ORAL EXERCISE

1. 500 is how many times 50? \$75 is how many times \$7.50?
2. Divide 50 by 5; 500 by 50. How do the quotients compare?
3. Divide 7.50 by 15; \$75 by 150. How do the quotients compare?
4. 720 is how many times 72? 9 is how many times .9?
5. Divide 720 by 9; 72 by .9; 7.2 by .09; .72 by .009.

137. It has been seen that multiplying both dividend and divisor by the same number does not change the quotient.

138. Therefore, to divide decimals when the divisor is not an integer:

Multiply both dividend and divisor by the power of 10 that shall make the divisor an integer, and divide as in United States money.

139. Divide 0.3375 by 0.45.

$.3375 \div .45 = 33.75 \div 45$. $33.75 \div 45 = .7$, with a remainder of 2.25. $2.25 \div 45 = .05$. The quotient is therefore .75.

$$\begin{array}{r} .75 \\ .45\overline{)33.75} \\ 31\,5 \\ \hline 2\,25 \\ 2\,25 \\ \hline \end{array}$$

Observe that *the divisor may always be made an integer if the decimal point in the dividend is carried to the right as many places as there are decimal places in the divisor.*

Should there be a remainder after using all the decimal places in the dividend, annex decimal ciphers and continue the division as far as is desired.

ORAL EXERCISE

Divide :

1. 1 by 1.
2. 1 by .1.
3. 1 by 10.
4. .1 by .1.
5. 1 by .01.
6. 1 by 100.
7. 1 by .001.
8. .10 by .10.
9. .01 by .01.
10. 1 by 1000.
11. 1 by .0001.
12. 1 by 10,000.
13. 1 by .00001.
14. .001 by .001.
15. 1 by 100,000.
16. 1 by .000001.
17. .0001 by .0001.
18. .00001 by .00001.
19. 33 by .11.
20. 33 by 110.
21. .33 by .11.
22. 3.3 by 1.1.
23. .0001 by 1.
24. 33 by .011.
25. 33 by 1100.
26. .0001 by .1.
27. 3300 by .11.
28. 330 by .011.
29. 33 by .0011.
30. 33 by 11000.
31. .0001 by .01.
32. .033 by .011.
33. .0001 by .001.
34. .0033 by .0011.
35. .0001 by .0001.
36. .0001 by .00001.

WRITTEN EXERCISE

Divide :

1. 5842 by .046.
2. 2.592 by .108.
3. 1.750 by 8750.
4. .00338 by .013.
5. 1.728 by .0024.
6. 2200 by .44.
7. 231.6 by 579.
8. 950 by 19,000.
9. 81.972 by .00009.
10. 115.814 by .00079.
11. 16 by .0064.
12. 1.86 by 31,000.
13. 1600 by 64,000.
14. .0004 by 20,000.
15. 100 by .000001.

Find the sum of the quotients:

16.	17.	18.
$8.1 \div .9$.	$72 \div 8$.	$125 \div 250$.
$81 \div .09$.	$72 \div .8$.	$12.5 \div 2.5$.
$8.1 \div .09$.	$7.2 \div .8$.	$1.25 \div 2.5$.
$.81 \div 900$.	$72 \div .08$.	$12.5 \div 250$.
$.0081 \div 9$.	$.72 \div .08$.	$125 \div 2500$.
$8.1 \div 900$.	$72 \div .008$.	$.125 \div .025$.
$810 \div .009$.	$72 \div 8000$.	$12500 \div .25$.
$.0081 \div 9000$.	$72 \div .0008$.	$125 \div 25000$.
$81000 \div .009$.	$.072 \div .008$.	$12500 \div .025$.
$81 \div .000009$.	$72 \div .00008$.	$125 \div 250000$.
$8100 \div 90000$.	$.0072 \div .0008$.	$.125 \div .00025$.
$.00081 \div 90000$.	$.00072 \div .00008$.	$12500 \div .0025$.

19.	20.	21.
$8.8 \div 2.2$.	$17 \div 68$.	$36 \div .072$.
$.88 \div .22$.	$1.7 \div 6.8$.	$3.6 \div .072$.
$88 \div .0022$.	$.17 \div .68$.	$.36 \div .072$.
$8.8 \div 2200$.	$1.7 \div 680$.	$360 \div .072$.
$880 \div 2200$.	$170 \div 680$.	$.036 \div .072$.
$8.8 \div 2.200$.	$.017 \div .068$.	$3.6 \div 72000$.
$880 \div .2200$.	$1.7 \div 68000$.	$36 \div 720000$.
$8800 \div 2200$.	$1700 \div 6800$.	$360 \div .00072$.
$880 \div 22000$.	$1700 \div 68000$.	$3600 \div .0072$.
$880 \div .00022$.	$.0017 \div .0068$.	$.0036 \div .0072$.
$88000 \div .0022$.	$.00017 \div .00068$.	$3.6 \div .000072$.
$88000 \div .00022$.	$.000017 \div .000068$.	$.00036 \div .00072$.

22. The product of two numbers is 0.00025. If one of the numbers is 0.0025, what is the other?

23. A retailer bought 450 yd. of cloth for \$1237.50 and sold it at \$3.25 per yard. How much did he gain per yard?

24. A drover bought a flock of sheep at the rate of \$3.30 per head. He sold them at a profit of \$0.20 per head and received \$700. How many sheep were there in the flock and what was his gain?

25. Copy and complete the following table. Check the results.

LARGEST OAT-GROWING STATES IN A RECENT YEAR

STATE	YIELD IN BUSHELS	FARM PRICE PER BUSHEL	FARM VALUATION	
Illinois		31¢	36,376,005	12
Iowa		31¢	37,920,192	00
Wisconsin		31¢	26,887,699	65
Minnesota		31¢	26,405,335	93
Total				

26–28. Make and solve three self-checking problems in division of decimals.

DIVIDING BY POWERS AND MULTIPLES OF TEN

ORAL EXERCISE

1. 6.4 is what part of 64? \$0.17 is what part of \$1.70?

2. Compare (as in problem 1) \$240.60 with \$24,060; 17.75 ft. with 1775 ft.

3. Compare (as in problem 1) .1 with 1; .01 with 1; .001 with 1; .0001 with 1.

4. Read aloud the following, supplying the missing terms: Removing the decimal —— place to the —— divides the value of the decimal by 10; two places, —— the value of the decimal by ——; three places, —— the value of the decimal by ——.

5. Compare the quotient of $28 \div .7$ with the quotient of $\overline{28 \times 10} \div \overline{.7 \times 10}$; the quotient of $28 \div .7$ with the quotient of $280 \div 7$.

6. Compare the quotient of $16.4 \div 40$ with the quotient of $\overline{16.4 \div 10} \div \overline{40 \div 10}$; the quotient of $16.4 \div 40$ with the quotient of $1.64 \div 4$. What is the quotient of 56.77 divided by 7000?

SOLUTION. Removing the decimal point three places to the left and dropping the ciphers of the divisor is equivalent to dividing both dividend and divisor by 1000 and does not change the value of the quotient.

$$7\overline{).05677} \quad .00811$$

Buying and Selling by the Hundred

ORAL EXERCISE

1. Compare $460 \div 100 \times \$2$ with $4.60 \times \$2$.
2. Find the cost of 450 lb. of guano at \$4 per cwt.
3. Find the cost of 600 lb. of wire nails at 34¢ per cwt.
4. Find the cost of 4950 paving stones at \$8 per C.

Solution. C stands for 100. 4950 paving stones are 49.5 times 100 paving stones. Since 1 hundred paving stones cost \$8, 49.5 hundred paving stones will cost 49.5 times \$8, or \$396.

$$\begin{array}{r} 49.5 \\ 8 \\ \hline 396.0 \end{array}$$

WRITTEN EXERCISE

Find the cost:

	Quantity	Price per Hundredweight		Quantity	Price per Hundredweight
1.	450 lb.	55¢	5.	1600 lb.	71¢
2.	510 lb.	77¢	6.	2600 lb.	15¢
3.	640 lb.	60¢	7.	4900 lb.	70¢
4.	330 lb.	56¢	8.	3100 lb.	88¢

Buying and Selling by the Thousand

ORAL EXERCISE

1. Compare $\overline{3500 \div 1000} \times \$9$ with $3.500 \times \$9$.
2. Compare $\overline{12200 \div 1000} \times \$5$ with $12.2 \times \$5$.
3. Find the cost of 7150 feet of lumber at \$11 per M.

Solution. M stands for thousand. 7150 feet are 7.15 times 1000 feet. Since 1 thousand feet of lumber cost \$11, 7.15 thousand feet will cost 7.15 times 11, or \$78.65.

$$\begin{array}{r} 7.15 \\ 11 \\ \hline 78.65 \end{array}$$

Find the cost of:

4. 8500 tiles at \$8 per M; at \$9 per M.
5. 4500 bricks at \$6 per M; at \$7 per M.
6. 7500 shingles at \$12 per M; at \$14 per M.
7. 3200 ft. lumber at \$14 per M; at \$12 per M.
8. 15,000 ft. lumber at \$11 per M; at \$12 per M.
9. 12,000 ft. lumber at \$16 per M; at \$15 per M.

WRITTEN EXERCISE

1. Find the cost of 17,500 shingles at $4 per M.

2. What is the cost of 2700 envelopes at $2.25 per M ?

3. Find the cost of 27,560 feet of oak lumber at $21 per M.

4. Find the total cost of :

125 bolts at $2.75 per C.
750 bolts at $3.50 per C.
450 fence posts at $6 per C.
275 lb. nails at $3.50 per cwt.
750 lb. wire at $3.75 per cwt.
750 lb. guano at $4.75 per cwt.

5. Find the total cost of:

7600 shingles at $4 per M.
14,400 ft. plank at $9 per M.
24,560 bricks at $3.50 per M.
9000 tiles at $9.375 per M.
2320 ft. lumber at $23 per M.
1,270,500 bricks at $6.75 per M.

6. Find the total cost of :

760 lb. bran at $.60 per cwt.
5875 lb. bran at $.70 per cwt.
5970 lb. meal at $1.12 per cwt.
4275 lb. meal at $1.10 per cwt.
5600 lb. feed at $1.10 per cwt.
500 lb. oatmeal at $2.50 per cwt.

7. Find the total freight on:

8000 lb. oil at 70¢ per cwt.
1500 lb. fish at 58¢ per cwt.
5100 lb. salt at 73¢ per cwt.
4950 lb. ale at 52¢ per cwt.
9900 lb. beef at 72¢ per cwt.
4950 lb. pork at 57¢ per cwt.

8. Find the amount of the following bill:

Bangor, Me., July 27, 1906

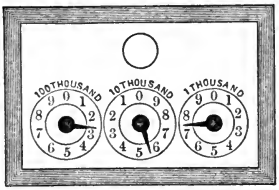
Messrs. Gray, Salisbury & Co.,
Springfield, Mass.

Bought of McGraw, Eldridge & Co.

Terms 5% off if paid in 10 days; 30 days net

		22000 ft. Clear Pine	23¢ per M				
		16000 Cedar Posts	6¢ per C				
		12500 ft. Pine Plank	20¢ per M				
		16500 Cedar R.R. Ties	12¢ per C				
		25000 Shingles	4¢ per M				
		92000 ft. Flooring	31¢ per M				

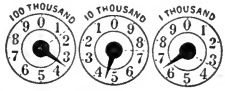
140. The accompanying illustration shows the three dials of a gas meter. Each division on the dial at the right denotes 100 cu. ft. of gas consumed; each division on the center dial 1000 cu. ft.; and each division on the dial at the left 10,000 cu. ft. The dials are read from left to right by simply taking the figures which the hands have just passed and adding two ciphers to them.



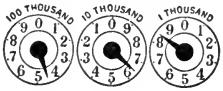
Thus, the above dial registers 20,000 cu. ft. + 5000 cu. ft. + 700 cu. ft. = 25,700 cu. ft.; but it is only necessary to write 257 (2, 5, 7) and add two ciphers to get this result.

WRITTEN EXERCISE

1. Read the accompanying meters and find the cost of the gas consumed during the period Jan. 1 to Feb. 1 at \$1.20 per 1000 cu. ft.



Jan. 1. 1906



Feb. 1, 1906

2. The following is the number of cubic feet of gas used in a residence for the six months ending July 1: January, 2900; February, 3200; March, 3700; April, 2900; May, 2700; June, 1200. Find the total gas bill for the six months at \$0.90 per 1000 cu. ft.

3. Assuming that gas is worth \$0.95 per 1000 cu. ft., find the amount of the following bill, less 10%.

GAS DEPARTMENT
87 West Street
Office Hours from 8 A.M. to 5 P.M.

Boston, Mass., Dec 1, 1906

Mrs Fred M. Allison,
34 Atkinson St.

To THE BOSTON GAS AND ELECTRIC LIGHT CO., Dr.

For Gas supplied by meter				
2210 0 cu. ft. as shown by Meter Dial	Nov. 29, 1906			
1860 0 cu. ft. as shown by Meter Dial	Oct. 30, 1906			
350 0 cu. ft. at \$1.00 per 1000 cu. ft.			350	
Less 10%			35	315
Discount of 10% allowed if paid on or before	Received payment for the Company			
Dec. 10, 1906	Dec. 4, 1906			
	C. E. Bannigan			

Buying and Selling by the Ton of 2000 Pounds

ORAL EXERCISE

1. Compare $\overline{8000 \div 2000} \times 8$ with $\overline{8000 \div 1000} \times 4$.
2. Compare $\overline{7000 \div 2000} \times 18$ with 7×9.
3. Find the cost of 4250 lb. coal at \$8 per ton.

Solution. 4250 lb. is 4.25 times 1000 lb. If the cost of 2 thousand pounds is \$8, the cost of 1 thousand pounds is \$4. Since 1 thousand pounds of coal cost \$4, 4.25 thousand pounds will cost 4.25 times \$4, or \$17.

4.25
4
17.00

WRITTEN EXERCISE

1. At \$9 per ton, find the cost of the hay in the following weigh ticket. Also find the cost at \$8.75 per ton.

SCALES OF E. H. ROBINSON & CO.

No. 250 Clyde, N.Y., July 26, 1906

Load of Hay

From C. A. Brown To W. D. Eaton

Gross weight 4810 lb.

Tare 1860 lb.

Net weight 2950 lb.

Geo. W. Ball
Weigher

2. At \$7.50 per ton find the cost of the coal in the following weigh ticket. Also find the cost at \$6.95 per ton.

WELLINGTON-WILD COAL CO.
126 Main Street, Rochester, N.Y.

No. 573 Nov. 16, 1906

Mrs. E. M. Robinson,
360 Monroe Ave., City

2930 lb. Stove Coal

Teamster Mann Received by E. M. Robinson

3. What will 8650 lb. of hay cost at $12 per ton?

4. Find the cost of 2150 lb. of coal at $6 per ton.

5. At $32 per ton, what is the cost of 26,480 lb. of phosphate?

6. Find the cost of 54,260 pounds of coal at $5.80 per ton.

7. Find the cost of 12 loads of coal weighing 4100, 3900, 4306, 4100, 4060, 4300, 3286, 3980, 3850, 4130, 3700, 3950 lb. net, at $5.20 per ton.

8. Find the total cost of: 5265 lb. hard coal at $8.40 per ton; 12,200 lb. soft coal at $3 per ton; 8275 lb. cannel coal at $11.75 per ton; 34,160 lb. egg coal at $6.20 per ton; 12,275 lb. nut coal at $5.75 per ton; 8753 lb. grate coal at $5.80 per ton; 24,160 lb. stove coal at $6.50 per ton.

9. During the month of January, in a recent year, there were consumed in a manufacturing plant 72 loads of coal weighing as follows: 6100, 6500, 6700, 6840, 7210, 6680, 7250, 8400, 6100, 6100, 6250, 6380, 6480, 6300, 6500, 6160, 6410, 6370, 6410, 6570, 6480, 6240, 6370, 6430, 6480, 6300, 7400, 7580, 7620, 7240, 7110, 7220, 7420, 7480, 6390, 6100, 6250, 6250, 6900, 6270, 6280, 6290, 6270, 6390, 6420, 6120, 6120, 6200, 6300, 6120, 6430, 6430, 8100, 6100, 6200, 6310, 6204, 6160, 6170, 6240, 6390, 6140, 6240, 7190, 7240, 7140, 7200, 6340, 8420, 6310, 7420, 6120 lb. net. Find the cost at 5.87\frac{1}{2}$ per ton.

WRITTEN REVIEW EXERCISE

1. Of what number is 25.56 both the divisor and quotient?

2. The sum of the divisor and quotient is 414.06. If the divisor is .6, what is the dividend?

3. In what time will 3 boys at $.75 per day earn as much as 2 men earn in 75 da. at $2.25 per day?

4. A merchant sold a quantity of flour for $370 and realized a gain of $34. If the selling price was $7.40 per barrel, what was the cost per barrel?

5. What number is that which is 165 times as great as 82.5?

6. If 450 bbl. of beef sold for $5872.50, what was the selling price per hundred barrels?

7. What will be the cost, at 15¢ per yard, of a paper border for a room 8 yd. wide and 12 yd. long?

8. If .25 be added to a certain number, 15 may be subtracted from it 75 times. What is the number?

9. Wood costing $3.50 per cord is sold for $4.10 per cord. How many cords must be handled to gain $240?

10. Find the cost of 8 bbl. of pork weighing 280, 281, 286, 290, 285, 277, 285, and 290 lb. net, at $8.50 per hundred pounds.

11. A flock of 200 sheep was bought for $700. 10 of the sheep died, and the remainder of the flock was sold at $3.95 per head. What was the gain or loss?

12. A hardware merchant had .5 of his capital invested in hardware stock, .25 of it invested in government bonds, and the remainder, $4896.25, on deposit in City National Bank. What was his entire capital?

13. A, B, and C bought a stock of goods for $7500, A contributing $2500, B $3000, and C the remainder. They sold the goods for $8400 and divided the profits equally. How much of the $8400 should A, B, and C, respectively, receive?

14. A, B, and C unite in forming a manufacturing establishment. A invests .4 of the entire money put into the business; B, .3; C, the remainder, $4500. What was the total amount invested, and what was A's and B's investment, respectively?

15. A fails in business. The excess of his liabilities over resources is $7500. It is found that he can pay his creditors but $.25 on the dollar. B is given $750 in payment for the amount owed him. What was the full amount of A's indebtedness, and how much did he owe B?

16. What is the total freight on 12,250 lb. of hardware at \$.65 per hundredweight and 15,670 lb. of hardware at \$.60 per hundredweight?

17. A merchant bought 250 yd. of cloth at \$3.50 per yard, and 150 yd. at \$4.25. At what average price per yard should the whole be sold to realize an average profit of \$1 per yard?

18. What is the cost of 25 bbl. of sugar containing 312, 304, 309, 317, 330, 325, 315, 318, 317, 305, 319, 320, 325, 330, 335, 330, 325, 315, 315, 320, 320, 330, 330, 315, 315 lb. net, at $5\frac{3}{8}¢$ per pound?

19. A received \$1088 from the sale of his barley crop. If he received \$0.85 per bushel for the barley and his farm produced an average of 32 bu. to the acre, how many acres did it take to produce the barley?

20. A shoe manufacturing pay roll shows that 40 hands are employed at \$1.45 per day, 50 hands at \$1.40 per day, 10 hands at \$3 per day, 40 hands at \$2.50 per day, and 5 hands at \$8 per day. Find the average daily wages.

21. A hardware merchant found that his stock of goods, Jan. 1, amounted to \$34,350.65. During the year he bought goods amounting to \$211,165.45, and sold goods amounting to \$220,540.45. Dec. 31, he took an account of stock and found that the goods on hand at cost prices were worth \$81,275.64. What was his gain or loss for the year?

22. Without copying the following figures, find (*a*) the sum of each line, and (*b*) the sum of each column. Prove the work by adding the line totals and comparing the sum with the sum of the column totals.

17.035	18.0135	186.02	126.42	6.009
8.005	5.07	142.004	.0634	3.14
32.972	18.0981	165.42	1.7538	9.314
126.83	4.931	.628	6.75	.048
95.16	6.815	.8467	8.41	.062
101.215	21.214	21.221	2.61	18.641

23. Copy and find the amount of the following bill:

Buffalo, N.Y., Dec. 15, 1906

Mr. John P. Kennedy,
369 State St., City.

Bought of F. M. Upton & Co.

Terms 30 da.

Nov. 1	3 loads Nut Coal		
	2000 2400 2600 7000#	4.50	
18	4 loads Nut Coal		
	2100 2400 2500 2400 ****	4.50	
Dec. 12	5 loads Egg Coal		
	2600 2000 2200 2100 2800 ****	4.25	

24. Find the cost, at $12.75 per ton, of the hay in the following weigh ticket. Also find the cost at $10.75 per ton.

SCALES OF E. H. ROBINSON & CO.

No. 278 Clyde, N.Y., Aug. 5, 19__

Load of Hay

From E. D. Snow To C. M. Sherman

Gross weight 4670 lb.

Tare 2170 lb.

Net weight 2500 lb.

J. M. Brennan
Weigher

25. Find the cost at $14.75 per ton of six loads of hay, the gross weights and tares of which were as follows: 4920 – 1848, 4810 – 1850, 5220 – 1960, 5820 – 2140, 4980 – 1920, 4910 – 1980 lb.

CHAPTER XI

FACTORS, DIVISORS, AND MULTIPLES

FACTORS

ORAL EXERCISE

1. Name two factors of 63; of 88; of 144; of 128.
2. What are the factors of 49? of 77? of 35? of 21?
3. Name three factors of 45; of 66; of 24; of 60; of 80.
4. Name a factor that is common to 35 and 77; 36, 63, and 81.
5. Name three factors that are common to 30, 60, and 210.
6. Which of the following numbers have no factors except itself and one? 11, 27, 15, 37, 49, 62, 73, 81, 23.

141. An **even number** is an integer of which two is a factor. An **odd number** is an integer of which two is not a factor. A **prime number** is a number that has no integral factor except itself and one. A **composite number** is a number that has one or more integral factors besides itself and one.

Numbers are **mutually prime** when they have no common factor greater than one.

WRITTEN EXERCISE

1. Make a list of all the odd numbers from 1 to 100 inclusive; of all the prime numbers; of all the even numbers; of all the composite numbers.

ORAL EXERCISE

1. Is 2 a factor of 28? of 125? of 42? of 49? By what means do you readily determine this?
2. Is 5 a factor of 125? of 170? of 224? of 1255? of 1056? By what means do you readily determine this?
3. When is a number divisible by 10? by 3? by 9?

Tests of Divisibility of Numbers

142. A number is divisible by:

1. *Two*, when it is even, or when it ends with 0, 2, 4, 6, or 8.
2. *Three*, when the sum of its digits is divisible by 3.
3. *Four*, when the number expressed by its two right-hand figures is divisible by 4.
4. *Five*, when it ends with 0 or 5.
5. *Six*, when it is even and the sum of its digits is divisible by 3.
6. *Eight*, when the number expressed by the last three right-hand figures is divisible by 8.
7. *Nine*, when the sum of its digits is divisible by 9.
8. *Ten*, when its right-hand figure is a cipher.

ORAL EXERCISE

Name one or more factors of each of the following numbers:

1. 184.	**5.** 6984.	**9.** 51625.	**13.** 14128.
2. 2781.	**6.** 2750.	**10.** 83870.	**14.** 66438.
3. 1449.	**7.** 8975.	**11.** 13599.	**15.** 31284.
4. 638172.	**8.** 71168.	**12.** 123125.	**16.** 17375.

Factoring

143. Factoring is the process of separating a number into its factors.

144. Example. Find the prime factors of 780.

Solution. Since the number ends in a cipher, divide it by the prime factor 5; since the resulting quotient is an even number, divide it by 2. Since 78 is an even number, divide it by 2; since the sum of the digits in the resulting quotient is divisible by 3, divide by 3. The prime factors are then found to be 5, 2, 2, 3, and 13.

5	780
2	156
2	78
3	39
	13

WRITTEN EXERCISE

Find the prime factors of:

1. 112.	**4.** 786.	**7.** 968.	**10.** 408.	**13.** 2718.	**16.** 6900.
2. 126.	**5.** 392.	**8.** 689.	**11.** 650.	**14.** 3240.	**17.** 2064.
3. 288.	**6.** 315.	**9.** 1098.	**12.** 762.	**15.** 3205.	**18.** 7400.

Cancellation

ORAL EXERCISE

1. $(4 \times 15) \div (4 \times 3) = 15 \div 3$. Why?

2. Divide $\overline{2 \times 5 \times 7}$ by $\overline{5 \times 2}$; $\overline{8 \times 7 \times 5}$ by $\overline{5 \times 2 \times 7}$.

3. $\dfrac{3 \times 7 \times 8}{7 \times 3} = ?$ $\dfrac{5 \times 2 \times 8 \times 3}{2 \times 8 \times 3} = ?$ $\dfrac{2 \times 9 \times 7 \times 5}{5 \times 7 \times 2 \times 3} = ?$

4. What effect on the quotient has rejecting equal factors in both dividend and divisor?

145. Cancellation is the process of shortening computations by rejecting or canceling equal factors from both dividend and divisor.

146. Example. Divide the product of 6, 8, 12, 32, and 84 by the product of 3, 4, 6, and 24.

$$\frac{\overset{2}{\cancel{6}} \times \overset{2}{\cancel{8}} \times \overset{2}{\cancel{12}} \times \overset{4}{\cancel{32}} \times \overset{28}{\cancel{84}}}{\underset{3}{\cancel{3}} \times \underset{1}{\cancel{4}} \times \underset{1}{\cancel{6}} \times \underset{1}{\cancel{24}}} = 2 \times 2 \times 2 \times 4 \times 28 = 896.$$

Solution. Do not form the products, but indicate the multiplication by the proper signs and write the divisor below the dividend as shown above. 3, 4, and 6 in the divisor are factors of 6, 8, and 12, respectively, in the dividend; hence, reject 3, 4, and 6 in the divisor and write 2, 2, and 2, respectively, in the dividend; then cancel the common factor 8 from 24 in the divisor and 32 in the dividend, retaining the factors 3 and 4, respectively; next cancel the common factor 3 in the divisor from 84 in the dividend and there remains the uncanceled factors 2, 2, 2, 4, and 28 in the dividend. Hence, the quotient is $2 \times 2 \times 2 \times 4 \times 28$, or 896.

WRITTEN EXERCISE

1. $\overline{14 \times 21 \times 48} \div \overline{7 \times 21 \times 6} = ?$

2. $\overline{128 \times 48 \times 88} \div \overline{64 \times 24 \times 4} = ?$

3. Divide $128 \times 18 \times 36$ by $64 \times 18 \times 12$.

4. $\dfrac{12 \times 16 \times 24 \times 8 \times 92 \times 28}{6 \times 8 \times 23 \times 7} = ?$

5. If 18 T. of hay cost $270, what will 25 T. cost at the same rate?

6. How many days' work at $2.75 will pay for 2 A. of land at $110 per acre?

7. If 75 bbl. of flour may be made from 375 bu. of wheat, how many bushels will be required to make 120 bbl. of flour?

8. If 45 men can complete a certain piece of work in 120 da., how many men can complete the same piece of work in 30 da.?

9. The freight on 350 lb. of evaporated apricots is $1.47. At that rate how much freight should be paid on 7350 lb. of evaporated apricots?

10. If 15 rm. of paper are required to print 400 copies of a book of 300 pp., how many reams will be required to print 32,000 copies of a book of 300 pp.?

DIVISORS AND MULTIPLES

Common Divisors

ORAL EXERCISE

1. Name a factor that is common to 35 and 49.
2. Name two factors that are common to both 48 and 64.
3. Name the greatest factor that is common to 75 and 100.

147. A **common divisor** is a factor that is common to two or more given numbers. The **greatest common divisor** (g. c. d.) is the greatest factor that is common to two or more given numbers.

148. Example. Find the g. c. d. of 24, 84, and 252.

Solutions. (*a*) Separate each of the numbers into its prime factors. The factor 2 occurs twice in all the numbers and the factor 3 once in all the numbers. None of the other factors occur in all the numbers; hence, $2 \times 2 \times 3$, or 12, is the greatest common divisor of 24, 84, and 252.

(*a*)

$$24 = 2 \times 2 \times 2 \times 3$$
$$84 = 2 \times 2 \times 3 \times 7$$
$$252 = 2 \times 2 \times 3 \times 3 \times 7$$

(*b*) The common prime factors of two or more given numbers may be found by dividing the numbers by their prime factors successively until the quotients contain no common factor, as shown in the margin.

(*b*)

$$\begin{array}{r} 2)24-84-252 \\ 2)12-42-126 \\ 3)6-21-63 \\ 2-7-21 \end{array}$$

Ever since decimal fractions came into quite general use the subject of greatest common divisor has been stripped of most of its practical value. When fractions like $\frac{1261}{1643}$ were quite generally used, it was necessary to reduce them to their lowest terms before they could be conveniently handled in an operation. For this purpose, the greatest common divisor (here 97) was found and canceled from each term, thus greatly simplifying the fraction (here $\frac{13}{17}$). Now, however, the greatest common divisor of the terms of the fractions used in business is easily found by inspection, and the need for finding the greatest common divisor is slight.

ORAL EXERCISE

1. What is the greatest common divisor of 65 and 75? of 12 and 32? of 75 and 125?

2. What is the greatest common divisor of 12, 30, and 96? of 8, 24, and 42? of 36, 90, and 96?

3. What divisor should be used in reducing $\frac{75}{125}$ to its lowest terms? $\frac{128}{640}$? $\frac{64}{320}$? $\frac{16}{160}$? $\frac{48}{240}$? $\frac{72}{240}$?

WRITTEN EXERCISE

Find the greatest common divisor of:

1. 48, 240. 2. 42, 28, 144. 3. 88, 144, 220.

4. A real estate dealer has four plots of land which he wishes to divide into the largest number of building lots of the same size. If the plots contain 168, 280, 182, and 252 square rods, respectively, how many square rods will there be in each building lot?

Common Multiples

ORAL EXERCISE

1. Name a multiple of 7; of 9; of 16; of 64.

2. Name two other multiples of each of the above numbers.

3. Name two multiples that are common to 3 and 4; to 5 and 9; to 8 and 12. Which of the multiples just named is the least common multiple?

149. A **common multiple** is any integral number of times two or more given numbers. The **least common multiple** (l. c. m.) of two or more numbers is the least number which is an integral number of times each of the given numbers.

150. Example. Find the l. c. m. of 28, 42, and 84.

SOLUTIONS. (*a*) Resolve each of the numbers into its prime factors. The factor 2 occurs twice in 28 and in 84, the factor 3 occurs once in 42 and 84, the factor 7 occurs once in each of the numbers. Therefore, the least common multiple is $2 \times 2 \times 3 \times 7$, or 84; or

(*a*)

$28 = 2 \times 2 \times 7$
$42 = 2 \times 3 \times 7$
$84 = 2 \times 2 \times 3 \times 7$

(*b*) Arrange the numbers in a horizontal line and divide by any prime factor that will exactly divide any two of them. Divide the numbers in the resulting quotient by any prime factor that will divide any two of them, and so continue the operation until quotients are found that are prime to each other. Find the product of the several divisors and the last quotients and the result is the l.c.m. $2 \times 2 \times 3 \times 7 = 84$, the l. c. m.

(*b*)

2)	28	42	84
2)	14	21	42
3)	7	21	21
7)	7	7	7
	1	1	1

All numbers that are factors of other given numbers may be disregarded in finding the l. c. m. Thus the common multiples of 4, 8, 16, 32, 64, and 80 are the same as the multiples of 64 and 80.

ORAL EXERCISE

State the least common multiple of:

1. 6, 5, and 3.
2. 6, 8, 12, and 24.
3. 4, 5, 15, and 30.
4. 2, 4, 7, 8, 48, 24.
5. 6, 42, 84, 168, 336.
6. 5, 15, 75, 150, 300.

WRITTEN EXERCISE

Find the least common multiple of:

1. 6, 7, 8, and 5.
2. 6, 18, 24, and 84.
3. 12, 24, 36, and 96.
4. 32, 46, 92, and 128.
5. 4, 20, 12, and 48.
6. 62, 78, 30, and 142.
7. 35, 105, 125, and 225.
8. 114, 240, 72, and 320.
9. What number is that of which 2, 3, 5, and 11 are the only prime factors?

CHAPTER XII

COMMON FRACTIONS

ORAL EXERCISE

1. When a quantity is divided into 3 equal parts, what is each part called? into 8 equal parts? into 12 equal parts?

2. The shaded part of A is what part of the whole hexagon? the shaded part of B? the shaded part of C?

3. In the shaded part of A how many sixths? in the shaded part of B?

4. One half of the hexagon is how many sixths of it? How many sixths in the whole hexagon?

5. In the unshaded part of B how many thirds? Two thirds are how many sixths?

6. In the unshaded part of C how many sixths?

7. Read the following fractions in the order of their size, the largest first: $\frac{1}{3}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{9}$.

8. Complete the following statement: Such parts of a unit as .5, .25, $\frac{1}{3}$, $\frac{5}{6}$, etc., are called ——.

151. Common fractions are expressed by two numbers, one written above and one below a short horizontal line.

152. The number written above the line is called the **numerator** of the fraction, and the number written below, the **denominator** of the fraction.

153. The *numerator* tells the *number of parts* expressed by the fraction; the *denominator names the parts* expressed by the fraction.

Thus, in the fraction $\frac{3}{4}$, 4 tells that a number has been divided into four equal parts and 3 shows that three of these parts have been taken.

154. It is clear that the *greater the number* of equal parts into which a unit is divided, the *less is the value* of each part; and the *less the number* of equal parts into which a unit is divided, the *greater the value* of each part. Hence,

Of two fractions having the same denominator, the one having the greater numerator expresses the greater value; and

Of two fractions having the same numerator, the one having the smaller denominator expresses the greater value.

155. The **terms of a fraction** are the numerator and denominator taken together.

156. A **unit fraction** is a fraction whose numerator is one.

Thus $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ are unit fractions. $\frac{1}{3}$ in. is read *one third of an inch.*

157. An **improper fraction** is a fraction whose numerator is equal to or greater than its denominator.

Thus, $\frac{6}{6}$, $\frac{5}{3}$, and $\frac{25}{5}$ are improper fractions. The value of an improper fraction is always equal to or greater than one.

158. A **mixed number** is the sum of a whole number and a fraction.

Thus, $2\frac{1}{7}$ and $4\frac{2}{5}$, read *two and one seventh* and *four and two fifths,* are mixed numbers.

ORAL EXERCISE

1. What takes the place of the denominator in .5? in .25?

2. Read aloud the following fractions in the order of their size, the largest first: $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{16}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{100}$.

3. Read aloud the following fractions in the order of their size, the smallest first: $\frac{2}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{2}{7}$, $\frac{1}{3}$, $\frac{2}{9}$, $\frac{1}{10}$, $\frac{1}{6}$, $\frac{2}{7}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{7}$.

4. Read aloud the following: $\frac{1}{2}$ mi.; $\frac{3}{4}$ T.; $27\frac{1}{2}$ yd.; $\frac{1}{1728}$ cu. ft.; $275\frac{3}{4}$ A.; $250\frac{5}{16}$ lb.; £ $18\frac{9}{20}$; £ $271\frac{1}{4}$; $\frac{1}{144}$ sq. ft.

5. Of the total cotton produced in the United States in a recent year the principal cotton-growing states contributed approximately as follows: North Carolina, $\frac{1}{25}$; South Carolina, $\frac{1}{12}$; Georgia, $\frac{1}{6}$; Florida, $\frac{1}{100}$; Alabama, $\frac{1}{10}$; Mississippi, $\frac{1}{7}$; Louisiana, $\frac{1}{14}$; Texas, $\frac{1}{4}$; Arkansas, $\frac{1}{11}$; Tennessee, $\frac{1}{50}$. Name the principal cotton-growing states, in the order of production, for this year.

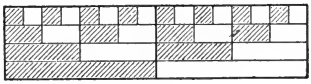
REDUCTION

To Higher Terms

ORAL EXERCISE

1. How many halves in 1? how many fourths? how many eighths? how many sixteenths?

2. How many fourths in $\frac{1}{2}$? how many eighths? how many sixteenths?



3. How many eighths in $\frac{1}{4}$? how many sixteenths?

4. How many fourths in $\frac{2}{16}$? how many eighths in $\frac{4}{16}$? how many halves in $\frac{8}{16}$?

5. What effect is produced upon the value of a fraction by multiplying or dividing both terms of a fraction by the same number?

6. Change 14 gal. to quarts. Compare the size of the units in 14 gal. with the size of the units in 56 qt.; the number of units; the value of the two numbers.

7. Change $\frac{1}{2}$ to twelfths; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$.

8. Name three fractions equal in value to $\frac{1}{8}$; to $\frac{2}{3}$; to $\frac{3}{4}$.

159. It has been seen that *multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.*

160. A fraction is reduced to **higher terms** when the given numerator and denominator are expressed in larger numbers.

ORAL EXERCISE

1. Reduce to twelfths: $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{6}$.

2. Reduce to sixteenths: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{5}{8}$.

3. Reduce to twentieths: $\frac{3}{4}$, $\frac{1}{5}$, $\frac{3}{10}$, $\frac{3}{5}$, $\frac{4}{5}$.

4. Reduce to twenty-fourths: $\frac{3}{8}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{3}{4}$, $\frac{7}{8}$.

5. Reduce to thirty-seconds: $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{3}{4}$, $\frac{3}{16}$, $\frac{1}{16}$, $\frac{9}{16}$, $\frac{7}{16}$.

6. Reduce to one-hundredths: $\frac{3}{4}$, $\frac{1}{2}$, $\frac{4}{5}$, $\frac{9}{10}$, $\frac{4}{25}$, $\frac{9}{20}$, $\frac{1}{4}$, $\frac{7}{25}$.

7. Reduce $\frac{3}{4}$ and $\frac{5}{8}$ to fractions having the denominator 24.

To Lowest Terms

ORAL EXERCISE

1. $\frac{8}{24}$ equals how many thirds? $\frac{12}{24}$ equals how many halves?

2. Name the largest possible unit fraction. Why is this the largest possible unit fraction?

3. Change $\frac{6}{12}$ to the largest possible unit fraction; $\frac{8}{16}$; $\frac{25}{100}$; $\frac{50}{200}$; $\frac{125}{1000}$. Express $\frac{14}{16}$ in its simplest form. Reduce $\frac{25}{200}$ to its lowest terms.

161. A fraction is reduced to its **lowest terms** when the numerator and denominator are changed to numbers that are mutually prime.

162. Example. Reduce $\frac{96}{108}$ to its lowest terms.

Solution. 6 is a common factor of 96 and 108; dividing both terms by 6, the result is $\frac{16}{18}$. 2 is a common factor of 16 and 18; dividing both terms by 2, the result is $\frac{8}{9}$.

$$\frac{96}{108} = \frac{16}{18} = \frac{8}{9}$$

ORAL EXERCISE

1. Reduce to fifteenths: $\frac{1}{3}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{4}{5}$.

2. Reduce to eighths: $\frac{8}{24}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{12}{32}$, $\frac{14}{16}$, $\frac{1}{4}$.

3. Reduce to fiftieths: $\frac{1}{2}$, $\frac{3}{5}$, $\frac{24}{100}$, $\frac{7}{10}$, $\frac{8}{25}$, $\frac{16}{200}$.

4. Change to twentieths: $\frac{1}{2}$, $\frac{7}{10}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{9}{10}$, $\frac{3}{5}$.

5. Reduce to lowest terms: $\frac{6}{16}$, $\frac{8}{10}$, $\frac{8}{12}$, $\frac{14}{16}$, $\frac{9}{12}$, $\frac{6}{8}$.

WRITTEN EXERCISE

1. Reduce to sixteenths: $\frac{110}{160}$, $\frac{180}{320}$, $\frac{7}{8}$, $\frac{25}{80}$, $\frac{3}{4}$, $\frac{160}{640}$.

2. Reduce to lowest terms: $\frac{720}{1728}$ cu. ft., $\frac{48}{160}$ A., $\frac{448}{2240}$ T.

3. Reduce to lowest terms: $\frac{100}{320}$ mi., £$\frac{165}{240}$, $\frac{4800}{5760}$ lb., $\frac{28}{80}$ mi.

4. Reduce to three-hundred-twentieths: $\frac{1}{8}$ mi., $\frac{1}{4}$ mi., $\frac{1}{16}$ mi.

5. Reduce to their simplest common fractional form: $\frac{1840}{2240}$ T., $\frac{1260}{2400}$ T., $\frac{92}{160}$ A., $\frac{124}{160}$ A., $\frac{360}{640}$ sq. mi., $\frac{492}{640}$ sq. mi., $\frac{248}{320}$ mi.

Integers and Mixed Numbers to Improper Fractions

ORAL EXERCISE

1. How many quarts in 1 gal.? in 3 gal.?
2. How many sixths in 1? in 3? in 5? in 7?
3. How many fifths in 1? in $1\frac{1}{5}$? in $1\frac{3}{5}$? in $3\frac{1}{5}$?
4. Express as fourths: $6\frac{1}{4}$, $12\frac{3}{4}$, 13, 87, $61\frac{1}{4}$, $28\frac{1}{2}$.
5. Express as eighths: 15, 12, $10\frac{1}{2}$, $1\frac{1}{4}$, $2\frac{5}{8}$, $1\frac{3}{8}$, $9\frac{7}{8}$.
6. Express as halves: 27, 14, $30\frac{1}{2}$, $171\frac{1}{2}$, $182\frac{1}{2}$, 249.

WRITTEN EXERCISE

Reduce to improper fractions:

1. $83\frac{1}{3}$.
2. $166\frac{2}{3}$.
3. $333\frac{1}{3}$.
4. $666\frac{2}{3}$.
5. $180\frac{5}{32}$.
6. $212\frac{1}{64}$.
7. $265\frac{1}{48}$.
8. $319\frac{5}{16}$.
9. $146\frac{15}{32}$.
10. $3150\frac{5}{9}$.
11. $1625\frac{1}{6}$.
12. $2150\frac{5}{12}$.

Improper Fractions to Integers or Mixed Numbers

ORAL EXERCISE

1. How many quarters of a dollar in \$25? $\frac{100}{25} = ?$
2. Change to integers: $\frac{160}{4}$, $\frac{153}{9}$, $\frac{192}{16}$, $\frac{288}{12}$, $\frac{4860}{60}$, $\frac{1920}{32}$.
3. Express $28\frac{1}{4}$ as fourths; express $\frac{113}{4}$ as a mixed number.
4. Change to mixed numbers: $\frac{27}{4}$, $\frac{173}{3}$, $\frac{157}{8}$, $\frac{161}{4}$, $\frac{214}{3}$.
5. What is the value of: $\frac{288}{12}$ lb.? $\frac{128}{16}$ lb.? $\frac{168}{4}$ bu.? $\frac{244}{8}$ pk.? $\frac{366}{12}$ ft.? $\frac{4800}{160}$ A.? $\frac{960}{320}$ mi.? $\frac{250}{12}$ lb.? $\frac{720}{144}$ sq. ft.?

WRITTEN EXERCISE

Reduce to integers or mixed numbers:

1. $\frac{856}{320}$ mi.
2. $\frac{1930}{160}$ A.
3. $\frac{9655}{2240}$ T.
4. $\frac{1728}{160}$ A.
5. $\frac{82750}{2000}$ T.
6. $\frac{69230}{2000}$ T.
7. $\frac{24560}{16}$ lb.
8. $\frac{95040}{1728}$ cu. ft.
9. $\frac{92000}{640}$ sq. mi.

163. When expressing **final results** reduce all proper fractions to their lowest terms and all improper fractions to integers or mixed numbers.

To Least Common Denominator

ORAL EXERCISE

1. How many pounds in 1 T. 500 lb.? 5 T. + 1000 lb. = ? lb. 5 T. 1000 lb. = ? T.

2. How must numbers be expressed before they can be added or subtracted?

3. $\frac{1}{2} = \frac{?}{8}$; $\frac{1}{2} + \frac{3}{8} = ?$ $\frac{1}{4} = \frac{?}{16}$; $\frac{1}{4} - \frac{1}{16} = \frac{?}{16}$; $\frac{1}{3} = \frac{?}{6}$; $\frac{1}{3} - \frac{1}{6} = ?$

4. What kind of fractions can be added or subtracted?

5. Express $\frac{3}{4}$ as sixteenths. Add $\frac{3}{4}$ and $\frac{5}{16}$; $\frac{1}{4}$ and $\frac{9}{16}$; $\frac{3}{4}$ and $\frac{7}{8}$.

6. Express $\frac{1}{2}$ as eighths. Subtract $\frac{1}{2}$ and $\frac{3}{8}$; $\frac{1}{2}$ and $\frac{3}{16}$; $\frac{3}{4}$ and $\frac{1}{16}$.

164. Two or more fractions whose denominators are the same are said to have a **common denominator;** if this denominator is the smallest possible, the fractions are said to have a **least common denominator.** Two or more fractions having the same denominator are sometimes called **similar fractions.**

ORAL EXERCISE

Change to similar fractions:

1. $\frac{1}{2}, \frac{1}{4}$.
2. $\frac{1}{2}, \frac{1}{3}$.
3. $\frac{1}{3}, \frac{1}{5}$.
4. $\frac{2}{3}, \frac{1}{2}$.
5. $\frac{3}{5}, \frac{4}{10}$.
6. $\frac{1}{5}, \frac{1}{4}$.
7. $\frac{2}{3}, \frac{3}{4}$.
8. $\frac{1}{2}, \frac{4}{5}$.
9. $\frac{3}{5}, \frac{2}{3}$.
10. $\frac{2}{5}, \frac{7}{8}$.
11. $\frac{2}{3}, \frac{7}{9}$.
12. $\frac{1}{2}, \frac{3}{16}$.
13. $\frac{2}{5}, \frac{7}{10}$.
14. $\frac{3}{4}, \frac{9}{10}$.
15. $\frac{3}{8}, \frac{7}{10}$.
16. $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$.
17. $\frac{1}{3}, \frac{1}{9}, \frac{4}{5}$.
18. $\frac{1}{4}, \frac{1}{8}, \frac{5}{32}$.
19. $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$.
20. $\frac{1}{2}, \frac{3}{8}, \frac{5}{16}$.

WRITTEN EXERCISE

Change to fractions having the least common denominator:

1. $\frac{7}{8}, \frac{5}{32}, \frac{18}{64}$.
2. $\frac{3}{5}, \frac{9}{10}, \frac{1}{25}$.
3. $\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{9}$.
4. $\frac{2}{5}, \frac{7}{9}, \frac{3}{10}, \frac{2}{3}$.
5. $\frac{5}{6}, \frac{7}{8}, \frac{5}{12}, \frac{7}{32}$.
6. $\frac{2}{3}, \frac{3}{5}, \frac{7}{36}, \frac{5}{48}$.
7. $\frac{1}{4}, \frac{7}{16}, \frac{9}{32}, \frac{17}{64}$.
8. $\frac{9}{10}, \frac{5}{24}, \frac{7}{12}, \frac{1}{8}$.
9. $\frac{1}{12}, \frac{5}{9}, \frac{2}{3}, \frac{1}{48}$.
10. $\frac{11}{16}, \frac{5}{32}, \frac{1}{4}, \frac{19}{64}$.
11. $\frac{15}{320}, \frac{3}{4}, \frac{17}{160}, \frac{5}{8}$.
12. $\frac{10}{640}, \frac{6}{16}, \frac{1}{32}, \frac{5}{32}$.

Change the fractions to form for addition or subtraction:

13. $31\frac{5}{16}, 7\frac{1}{10}$.
14. $134\frac{1}{64}, 112\frac{5}{20}$.
15. $6126\frac{5}{12}, 178\frac{5}{16}$.

ADDITION

165. It has been seen that *only like numbers and parts of like units can be added.*

ORAL EXERCISE

State the sum of:

1. $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$.
2. $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{4}$.
3. $\frac{1}{7}$, $\frac{3}{7}$, $\frac{4}{7}$.
4. $\frac{2}{11}$, $\frac{5}{11}$, $\frac{6}{11}$.
5. $\frac{1}{5}$, $\frac{4}{5}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{7}{5}$.
6. $\frac{1}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$.
7. $2\frac{1}{2}$, $3\frac{1}{2}$, $12\frac{1}{2}$, $19\frac{1}{2}$.
8. $5\frac{1}{4}$, $12\frac{1}{4}$, $7\frac{1}{4}$, $10\frac{1}{4}$.
9. $7\frac{7}{8}$, $2\frac{5}{8}$, $8\frac{1}{8}$, $1\frac{1}{8}$, $2\frac{1}{8}$.
10. $2\frac{1}{3}$, $5\frac{2}{3}$, $8\frac{1}{3}$, $12\frac{1}{3}$, $10\frac{2}{3}$.
11. $1\frac{1}{6}$, $10\frac{5}{6}$, $15\frac{1}{6}$, $18\frac{1}{6}$, $12\frac{1}{6}$.
12. $5\frac{7}{16}$, $2\frac{9}{16}$, $1\frac{5}{16}$, $8\frac{3}{16}$, $3\frac{1}{16}$.

By horizontal addition find the sum of:

13. 2 pieces of gingham containing 41^1 and 43^2 yd.

In the dry-goods business fourths (quarters) are very common fractions. They are usually written without denominators by placing the numerators a little above the integers. Thus, 51^1 equals $51\frac{1}{4}$, 54^2 equals $54\frac{2}{4}$ ($54\frac{1}{2}$), and 52^3 equals $52\frac{3}{4}$.

14. 4 pc. stripe containing 42^1, 38^1, 40^2, and 49 yd.
15. 3 pc. fancy plaid containing 42^1, 40^2, and 41 yd.
16. 4 pc. duck containing 48^1, 47^3, 46^2, and 40^2 yd.
17. 2 pc. monument cotton containing 54^2 and 55^2 yd.
18. 4 pc. dress silk containing 32^1, 34^2, 35^3, and 32^2 yd.

166. Examples. **1.** Find the sum of $\frac{7}{8}$ and $\frac{2}{5}$.

SOLUTION. $\frac{7}{8}$ and $\frac{2}{5}$ are not similar fractions; hence, make them similar by reducing them to equivalent fractions having a least common denominator. $\frac{7}{8} = \frac{35}{40}$ and $\frac{2}{5} = \frac{16}{40}$. $\frac{35}{40} + \frac{16}{40} = \frac{51}{40} = 1\frac{11}{40}$.

l. c. m. of 8 and $5 = 40$

$\frac{7}{8} = \frac{35}{40}$; $\frac{2}{5} = \frac{16}{40}$

$\frac{35}{40} + \frac{16}{40} = \frac{51}{40} = 1\frac{11}{40}$

2. Find the sum of $56\frac{1}{3}$, $34\frac{1}{8}$, $52\frac{3}{4}$.

SOLUTION. By inspection determine the least common denominator of the given fractions; then make the fractions similar and add them, as shown in the margin. The result is $1\frac{5}{24}$, which added to the sum of the integers equals $143\frac{5}{24}$, the required result.

$56\frac{1}{3} = 8$

$34\frac{1}{8} = 3$

$52\frac{3}{4} = \frac{18}{29}$

$143\frac{5}{24}$ $\quad \frac{29}{24} = 1\frac{5}{24}$.

WRITTEN EXERCISE

Find the sum of:

1. $\frac{7}{16}$, $\frac{5}{8}$.
2. $\frac{9}{16}$, $\frac{17}{64}$.
3. $2\frac{1}{8}$, $17\frac{1}{4}$.
4. $12\frac{1}{2}$, $19\frac{5}{16}$.
5. $\frac{2}{3}$, $4\frac{1}{5}$, $19\frac{1}{4}$.
6. $2\frac{1}{2}$, $4\frac{3}{4}$, $25\frac{9}{16}$.
7. $12\frac{3}{5}$, $172\frac{7}{10}$.
8. $8\frac{1}{4}$, $\frac{3}{5}$, $\frac{19}{10}$, $27\frac{1}{8}$.
9. $52\frac{5}{8}$, $59\frac{1}{2}$, $57\frac{3}{4}$, $52\frac{1}{16}$.
10. $60\frac{5}{8}$, $18\frac{3}{4}$, $21\frac{5}{16}$, $142\frac{1}{16}$.
11. $20\frac{1}{4}$, $12\frac{1}{5}$, $18\frac{1}{3}$, $92\frac{1}{5}$, $75\frac{3}{7}$.
12. $140\frac{3}{4}$, $260\frac{1}{8}$, $145\frac{1}{2}$, $216\frac{7}{8}$, $390\frac{1}{3}$.

13. A carpet dealer sold at different times $125\frac{5}{8}$ yd., $272\frac{1}{4}$ yd., $169\frac{1}{2}$ yd., $186\frac{3}{4}$ yd., $241\frac{1}{4}$ yd., $265\frac{1}{8}$ yd., $296\frac{7}{8}$ yd., and $314\frac{5}{8}$ yd. of Axminster carpet, at \$2.65 per yard. If it cost him \$2.45 per yard, what was his gain?

14. A dry-goods merchant bought 50 pc. of dress silk at \$1 per yard. If the pieces contained 42^1, 43^2, 44^2, 47^3, 44^1, 45^2, 40^3, 46^2, 45^1, 42, 47^1, 48^2, 40^3, 40^1, 40^2, 40^3, 50^2, 40^3, 47^2, 48^3, 40^3, 45^1, 40^2, 45^2, 44^2, 47^3, 46^2, 41^1, 51^3, 42^3, 53^2, 57^2, 53^1, 51^1, 48^3, 47^2, 40^1, 45^2, 45^2, 40^3, 40^1, 45^3, 47^2, 48^1, 51^1, 52^2, 57^2, 61^3, 60^2, 50^1 yd., respectively, and he sold the entire purchase at \$1.25 per yard, what was his gain?

Short Methods in Addition

ORAL EXERCISE

1. $\frac{1}{7}+\frac{1}{9}=\frac{16}{63}$. Observe that the numerator of the sum is equal to the sum of the denominators in the given fractions.

2. $\frac{1}{5}+\frac{1}{8}=?$ Give a short method for adding any two simple fractions whose numerators are 1.

3. $\frac{2}{7}+\frac{2}{5}=\frac{24}{35}$. Observe that the numerator of the sum is equal to the sum of the denominators multiplied by the numerator of either of the given fractions.

4. $\frac{2}{3}+\frac{2}{7}=?$ Give a short method for adding any two fractions whose numerators are alike.

5. Find the sum of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

SOLUTION. $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$; $\frac{7}{12}+\frac{1}{5}=\frac{47}{60}$, the required result.

ORAL EXERCISE

State the sum of:

1. $\frac{1}{2}, \frac{1}{3}$.
2. $\frac{1}{4}, \frac{1}{5}$.
3. $\frac{1}{5}, \frac{1}{6}$.
4. $\frac{1}{7}, \frac{1}{8}$.
5. $\frac{1}{9}, \frac{1}{6}$.
6. $\frac{1}{3}, \frac{1}{5}$.
7. $\frac{1}{2}, \frac{1}{8}$.
8. $\frac{1}{4}, \frac{1}{6}$.
9. $\frac{1}{9}, \frac{1}{7}$.
10. $\frac{2}{7}, \frac{2}{9}$.
11. $\frac{2}{5}, \frac{2}{7}$.
12. $\frac{2}{5}, \frac{2}{3}$.
13. $\frac{1}{4}, \frac{1}{3}$.
14. $\frac{3}{8}, \frac{3}{7}$.
15. $\frac{4}{5}, \frac{4}{9}$.
16. $\frac{5}{8}, \frac{5}{9}$.
17. $\frac{3}{4}, \frac{3}{5}$.
18. $\frac{4}{7}, \frac{4}{5}$.
19. $\frac{7}{9}, \frac{7}{8}$.
20. $\frac{6}{7}, \frac{6}{10}$.
21. $\frac{1}{5}, \frac{1}{8}, \frac{1}{3}$.
22. $\frac{1}{4}, \frac{1}{6}, \frac{1}{2}$.
23. $\frac{1}{9}, \frac{1}{8}, \frac{1}{7}$.
24. $\frac{2}{3}, \frac{2}{5}, \frac{2}{9}$.

167. The most common business fractions are usually small and of such a nature that they may be added with equally as much ease as integers. The following exercise will be found helpful to the student in learning to add these fractions in practically the same manner that he adds integers.

168. Example. Find the sum of $\frac{5}{16}$, $\frac{1}{8}$, $\frac{3}{4}$, and $\frac{1}{2}$.

SOLUTION. By inspection determine that the least common denominator is 16. Then mentally reduce each fraction to 16ths and add as in whole numbers. Thus, 5, 7, 19, $\frac{27}{16}$, $1\frac{11}{16}$.

ORAL EXERCISE

Find the sum of:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{5}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{7}{9}$
$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{7}{9}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{2}{3}$
$\frac{1}{2}$	$\frac{1}{9}$	$\frac{3}{10}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{6}$

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$\frac{1}{3}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{7}{8}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{5}$	$\frac{1}{2}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{6}$	$\frac{3}{7}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$
$\frac{1}{5}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{7}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$
$\frac{4}{5}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{5}{4}$
$\frac{2}{3}$	$\frac{7}{10}$	$\frac{3}{8}$	$\frac{3}{7}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{12}$	$\frac{5}{7}$	$\frac{1}{6}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{1}{5}$	$\frac{7}{10}$
$\frac{1}{3}$	$\frac{9}{10}$	$\frac{5}{12}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{7}{8}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{3}{10}$
$\frac{4}{5}$	$\frac{7}{10}$	$\frac{7}{12}$	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{11}{16}$	$\frac{5}{12}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{10}$

Exercises similar to the foregoing should be continued until the student can name the successive results in the addition without hesitation.

169. The ordinary mixed numbers that come to an accountant should be arranged for addition practically the same as integers. In adding, the fractions should be combined first and then the integers.

170. Example. Find the sum of $2\frac{1}{2}$, $5\frac{1}{4}$, and $3\frac{7}{8}$.

SOLUTION. By inspection determine that the least common denominator of the fractions is 8. Mentally find the sum of the fractions and the result is $1\frac{5}{8}$. Add this result to the integers and the entire sum is $11\frac{5}{8}$.

$$\begin{array}{r} 2\frac{1}{2} \\ 5\frac{1}{4} \\ 3\frac{7}{8} \\ \hline 11\frac{5}{8} \end{array}$$

ORAL EXERCISE

State the sum of:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$2\frac{1}{2}$	$3\frac{1}{3}$	$3\frac{1}{4}$	$8\frac{1}{5}$	$14\frac{1}{2}$	$5\frac{3}{4}$	$4\frac{1}{5}$	$2\frac{7}{8}$	$3\frac{1}{3}$	$14\frac{1}{7}$
$3\frac{1}{4}$	$2\frac{1}{6}$	$5\frac{1}{3}$	$7\frac{1}{2}$	$17\frac{3}{4}$	$13\frac{1}{2}$	$7\frac{1}{3}$	$16\frac{2}{5}$	$17\frac{1}{8}$	$16\frac{2}{9}$

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$9\frac{3}{4}$	$5\frac{2}{3}$	$1\frac{1}{6}$	$1\frac{3}{7}$	$8\frac{1}{5}$	$4\frac{1}{4}$	$5\frac{1}{3}$	$4\frac{1}{9}$	$4\frac{1}{5}$	$4\frac{1}{2}$
$10\frac{1}{2}$	$4\frac{1}{4}$	$2\frac{5}{6}$	$6\frac{1}{2}$	$3\frac{3}{5}$	$2\frac{1}{3}$	$2\frac{2}{3}$	$5\frac{2}{3}$	$2\frac{2}{3}$	$1\frac{3}{8}$
$13\frac{1}{4}$	$11\frac{1}{2}$	$7\frac{7}{12}$	$5\frac{1}{2}$	$2\frac{1}{10}$	$4\frac{1}{5}$	$4\frac{1}{9}$	$6\frac{5}{6}$	$6\frac{4}{5}$	$7\frac{3}{4}$
$10\frac{1}{4}$	$12\frac{1}{12}$	$8\frac{5}{12}$	$13\frac{4}{7}$	$4\frac{9}{10}$	$3\frac{3}{4}$	$6\frac{7}{9}$	$2\frac{1}{3}$	$3\frac{3}{10}$	$12\frac{1}{4}$

21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
$4\frac{1}{2}$	$1\frac{1}{2}$	$5\frac{1}{6}$	$3\frac{1}{4}$	$8\frac{1}{4}$	$4\frac{1}{2}$	$5\frac{2}{3}$	$4\frac{7}{9}$	$4\frac{1}{2}$	$4\frac{1}{8}$
$5\frac{1}{4}$	$5\frac{2}{3}$	$2\frac{2}{3}$	$1\frac{3}{7}$	$6\frac{3}{4}$	$2\frac{1}{8}$	$1\frac{1}{6}$	$1\frac{1}{9}$	$2\frac{1}{5}$	$6\frac{1}{2}$
$3\frac{3}{4}$	$5\frac{3}{4}$	$2\frac{5}{6}$	$2\frac{5}{7}$	$7\frac{1}{5}$	$4\frac{1}{5}$	$9\frac{5}{6}$	$7\frac{5}{6}$	$6\frac{2}{3}$	$9\frac{3}{8}$
$6\frac{7}{8}$	$5\frac{2}{7}$	$7\frac{3}{4}$	$6\frac{1}{2}$	$2\frac{3}{5}$	$3\frac{1}{4}$	$4\frac{1}{9}$	$5\frac{1}{3}$	$6\frac{4}{5}$	$7\frac{5}{8}$
$3\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$7\frac{1}{2}$	$3\frac{1}{5}$	$3\frac{1}{2}$	$7\frac{5}{9}$	$2\frac{2}{3}$	$9\frac{3}{5}$	$8\frac{3}{4}$
$4\frac{1}{4}$	$1\frac{1}{3}$	$2\frac{5}{6}$	$2\frac{5}{7}$	$5\frac{1}{5}$	$3\frac{1}{8}$	$3\frac{7}{9}$	$7\frac{1}{6}$	$4\frac{1}{5}$	$2\frac{1}{2}$
$3\frac{1}{8}$	$4\frac{1}{3}$	$8\frac{1}{12}$	$3\frac{4}{7}$	$2\frac{1}{4}$	$3\frac{1}{5}$	$2\frac{2}{3}$	$9\frac{5}{6}$	$3\frac{3}{10}$	$6\frac{1}{4}$
$5\frac{1}{8}$	$2\frac{7}{12}$	$8\frac{5}{12}$	$13\frac{3}{7}$	$5\frac{7}{10}$	$3\frac{1}{4}$	$12\frac{1}{3}$	$15\frac{2}{3}$	$9\frac{7}{10}$	$12\frac{1}{2}$

Exercises similar to the above should be continued until the student can add with great facility. If the principles of grouping have not been well mastered, simple addition should be carefully reviewed.

WRITTEN EXERCISE

Copy or write from dictation and find the sum of:

1.	2.	3.	4.	5.	6.
$1649\frac{1}{3}$	$1672\frac{3}{4}$	$1436\frac{1}{2}$	$2110\frac{1}{3}$	$6214\frac{1}{8}$	$1214\frac{1}{3}$
$4372\frac{1}{6}$	$1485\frac{7}{8}$	$1390\frac{3}{4}$	$1640\frac{1}{9}$	$1745\frac{1}{4}$	$2167\frac{1}{4}$
$8431\frac{2}{3}$	$1635\frac{1}{4}$	$2415\frac{1}{5}$	$3680\frac{1}{2}$	$3146\frac{3}{4}$	$3159\frac{1}{6}$
$5132\frac{7}{9}$	$1264\frac{1}{2}$	$1867\frac{1}{2}$	$4590\frac{7}{9}$	$1864\frac{1}{3}$	$9275\frac{1}{4}$
$1654\frac{1}{3}$	$1269\frac{3}{4}$	$1639\frac{1}{8}$	$2169\frac{2}{9}$	$2839\frac{5}{8}$	$7215\frac{3}{4}$
$1831\frac{5}{6}$	$1748\frac{1}{8}$	$4136\frac{3}{5}$	$8432\frac{2}{3}$	$6241\frac{7}{8}$	$5261\frac{1}{2}$
$1831\frac{3}{4}$	$1936\frac{5}{8}$	$1652\frac{1}{4}$	$4041\frac{1}{2}$	$4036\frac{3}{4}$	$7215\frac{2}{3}$
$1462\frac{1}{2}$	$5413\frac{1}{8}$	$3116\frac{1}{2}$	$6542\frac{5}{6}$	$8130\frac{5}{16}$	$5144\frac{3}{4}$
$1851\frac{1}{4}$	$2114\frac{7}{16}$	$1439\frac{1}{10}$	$1862\frac{2}{3}$	$2148\frac{7}{16}$	$6257\frac{2}{3}$
$1114\frac{7}{12}$	$1116\frac{9}{16}$	$2243\frac{7}{10}$	$3246\frac{8}{9}$	$1439\frac{9}{16}$	$2186\frac{5}{12}$

7.	8.	9.	10.	11.	12.
$9124\frac{1}{4}$	$7249\frac{1}{5}$	$1649\frac{1}{6}$	$7529\frac{1}{5}$	$7365\frac{1}{5}$	$2814\frac{1}{2}$
$2716\frac{1}{2}$	$2724\frac{4}{5}$	$2724\frac{1}{2}$	$6214\frac{1}{2}$	$2614\frac{1}{9}$	$2910\frac{1}{4}$
$2514\frac{1}{3}$	$8692\frac{1}{2}$	$8695\frac{1}{3}$	$1825\frac{4}{5}$	$1583\frac{1}{3}$	$2817\frac{1}{8}$
$2967\frac{1}{2}$	$2476\frac{1}{4}$	$1565\frac{1}{9}$	$8614\frac{3}{5}$	$1695\frac{1}{5}$	$2714\frac{1}{8}$
$2964\frac{1}{4}$	$8695\frac{1}{2}$	$2724\frac{7}{9}$	$9215\frac{3}{5}$	$1762\frac{1}{9}$	$2913\frac{7}{8}$
$6875\frac{1}{3}$	$6214\frac{1}{4}$	$8619\frac{1}{3}$	$6719\frac{3}{4}$	$1875\frac{5}{9}$	$2874\frac{5}{8}$
$8875\frac{3}{4}$	$7241\frac{1}{2}$	$2924\frac{2}{3}$	$8516\frac{1}{4}$	$1629\frac{7}{9}$	$2619\frac{3}{8}$
$2658\frac{1}{2}$	$8614\frac{1}{2}$	$6529\frac{1}{9}$	$7528\frac{1}{8}$	$7214\frac{2}{9}$	$1472\frac{1}{8}$
$8425\frac{2}{3}$	$4725\frac{1}{4}$	$8592\frac{1}{2}$	$7216\frac{5}{8}$	$2510\frac{1}{9}$	$2615\frac{3}{4}$
$8273\frac{3}{4}$	$1649\frac{1}{4}$	$2725\frac{1}{9}$	$6729\frac{1}{8}$	$2625\frac{2}{3}$	$1813\frac{1}{4}$
$1782\frac{3}{4}$	$1286\frac{1}{2}$	$8647\frac{2}{9}$	$3514\frac{1}{4}$	$8614\frac{1}{3}$	$1962\frac{1}{2}$
$8695\frac{1}{4}$	$6248\frac{1}{4}$	$8725\frac{5}{9}$	$1686\frac{3}{4}$	$2729\frac{1}{5}$	$1862\frac{1}{2}$
$2472\frac{1}{2}$	$1286\frac{1}{2}$	$6219\frac{7}{9}$	$1725\frac{1}{8}$	$2816\frac{1}{3}$	$1759\frac{1}{2}$
$6273\frac{1}{6}$	$8537\frac{3}{4}$	$8413\frac{1}{3}$	$2538\frac{3}{8}$	$2814\frac{1}{3}$	$2864\frac{1}{4}$
$9685\frac{5}{6}$	$6982\frac{1}{4}$	$7226\frac{5}{6}$	$1758\frac{5}{8}$	$2716\frac{2}{3}$	$1624\frac{1}{4}$
$9685\frac{11}{12}$	$3685\frac{1}{5}$	$1825\frac{1}{6}$	$2752\frac{7}{8}$	$1762\frac{1}{9}$	$1729\frac{1}{2}$
$1925\frac{5}{12}$	$2614\frac{3}{5}$	$4725\frac{1}{3}$	$2114\frac{1}{5}$	$1875\frac{1}{9}$	$1805\frac{3}{4}$
$4212\frac{1}{12}$	$8796\frac{4}{5}$	$2816\frac{2}{3}$	$2216\frac{1}{4}$	$2614\frac{2}{3}$	$1721\frac{7}{8}$
$2729\frac{1}{12}$	$1592\frac{3}{4}$	$2519\frac{1}{3}$	$1872\frac{1}{5}$	$2075\frac{1}{3}$	$1465\frac{3}{8}$

SUBTRACTION

ORAL EXERCISE

1. 172 A. − 154 A. = ? $\frac{5}{7} - \frac{1}{7} = ?$ 1 bu. − 3 pk. = ?

2. Find the difference between $\frac{1}{4}$ and $\frac{1}{8}$; $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{3}{7}$ and $\frac{2}{5}$.

171. It is clear that *only like numbers and parts of like units can be subtracted.*

172. Examples. 1. Find the difference between $\frac{7}{8}$ and $\frac{5}{12}$.

SOLUTION. The given fractions must be reduced to equivalent fractions having a least common denominator. The least common denominator is 24. $\frac{7}{8} = \frac{21}{24}$ and $\frac{5}{12} = \frac{10}{24}$. $\frac{21}{24} - \frac{10}{24} = \frac{11}{24}$, the required result.

2. From $21\frac{1}{3}$ take $17\frac{1}{2}$.

SOLUTION. Change the given fractions to similar fractions as in example 1. $\frac{3}{6}$ cannot be subtracted from $\frac{2}{6}$, hence 1 is taken from 21 and mentally united to $\frac{2}{6}$, making $\frac{8}{6}$. $\frac{3}{6}$ from $\frac{8}{6}$ leaves $\frac{5}{6}$, and 17 from 20 leaves 3. The required result is therefore $3\frac{5}{6}$.

ORAL EXERCISE

Find the value of:

1. $2\frac{3}{4} - \frac{1}{3}$.
2. $2\frac{5}{7} - \frac{1}{4}$.
3. $3\frac{7}{9} - \frac{4}{5}$.
4. $7\frac{5}{8} - 1\frac{3}{8}$.
5. $4\frac{5}{7} - 1\frac{2}{3}$.
6. $6\frac{7}{8} - 4\frac{5}{16}$.
7. $7\frac{1}{8} - 3\frac{5}{16}$.
8. $12\frac{1}{4} - 6\frac{1}{8}$.
9. $30 - 11\frac{7}{8}$.
10. $45 - 16\frac{3}{4}$.
11. $11\frac{1}{2} - 6\frac{2}{3}$.
12. $70\frac{3}{4} - 20\frac{1}{2}$.

The following is a recent clipping from a daily paper. It shows the prices of wheat on the Chicago market. The first line of prices is for wheat to be delivered in July, and the second line for wheat to be delivered in September.

CHICAGO WHEAT QUOTATIONS

DELIVERY	PREVIOUS CLOSING	OPENING	HIGHEST	LOWEST	CLOSING
July	$91\frac{1}{4}¢$	$90\frac{7}{8}¢$	$92\frac{7}{8}¢$	$90\frac{3}{4}¢$	$92\frac{1}{4}¢$
September	$87\frac{5}{8}¢$	$87\frac{1}{4}¢$	$88\frac{1}{4}¢$	$87¢$	$87\frac{1}{2}¢$

13. What was the difference between the highest and the lowest price of July wheat? of September wheat?

14. What was the difference between the opening and the closing price of September wheat? of July wheat?

15. What was the difference between the opening price and the previous closing (yesterday's closing) price of July wheat? of September wheat?

16. A bought 1000 bu. July wheat at the lowest price and sold the same at the closing price. What was his gain?

SUGGESTION. $1\frac{1}{2}\not{c} = \$0.015$; 1000 times $\$0.015 = \$?$

17. B bought 1000 bu. September wheat at the opening price and sold it at the highest price. What was his gain? Had he bought at the lowest price and sold at the closing price, what would have been his gain?

18. C bought 25,000 bu. July wheat at the opening price and sold it at the highest price. What was his gain?

WRITTEN EXERCISE

Find the value of:

1. $39 - 11\frac{5}{6}$.
2. $85 - 21\frac{3}{4}$.
3. $168 - 45\frac{5}{6}$.
4. $264\frac{9}{16} - 131\frac{1}{3}$.
5. $165\frac{1}{2} - 41\frac{1}{32}$.
6. $245\frac{3}{4} - 17\frac{3}{16}$.
7. $177\frac{2}{3} - 17\frac{3}{16}$.
8. $2150 - 121\frac{15}{16}$.
9. $\frac{5}{8} - \frac{1}{4} - \frac{1}{3}$.
10. $\frac{7}{8} - \frac{9}{16} - \frac{1}{8}$.
11. $2\frac{1}{2} + 1\frac{5}{8} - 1\frac{7}{12}$.
12. $25\frac{1}{2} - 8\frac{3}{4} - 15\frac{1}{2}$.

173. When the numerators of any two fractions are alike, the subtraction may be performed as in the following examples.

174. Examples. **1.** From $\frac{1}{7}$ take $\frac{1}{9}$. **2.** From $\frac{3}{5}$ take $\frac{3}{8}$.

SOLUTIONS. **1.** $9 - 7 = 2$, the new numerator. $9 \times 7 = 63$, the new denominator. Therefore, the required result is $\frac{2}{63}$. **2.** $8 - 5 \times 3 = 9$, the new numerator. $8 \times 5 = 40$, the new denominator. Therefore, $\frac{9}{40}$ is the required result.

ORAL EXERCISE

State the value of:

1. $\frac{1}{2} - \frac{1}{3}$.
2. $\frac{1}{2} - \frac{1}{4}$.
3. $\frac{1}{2} - \frac{1}{6}$.
4. $\frac{1}{3} - \frac{1}{7}$.
5. $\frac{1}{3} - \frac{1}{8}$.
6. $\frac{1}{3} - \frac{1}{9}$.
7. $\frac{1}{5} - \frac{1}{7}$.
8. $\frac{1}{5} - \frac{1}{8}$.
9. $\frac{1}{5} - \frac{1}{9}$.
10. $\frac{1}{4} - \frac{1}{5}$.
11. $\frac{1}{4} - \frac{1}{6}$.
12. $\frac{1}{4} - \frac{1}{7}$.
13. $\frac{1}{4} - \frac{1}{8}$.
14. $\frac{1}{6} - \frac{1}{7}$.
15. $\frac{1}{6} - \frac{1}{8}$.
16. $\frac{2}{3} - \frac{2}{7}$.
17. $\frac{3}{4} - \frac{3}{5}$.
18. $\frac{3}{4} - \frac{3}{7}$.
19. $\frac{4}{5} - \frac{4}{7}$.
20. $\frac{5}{6} - \frac{5}{7}$.
21. $\frac{5}{6} - \frac{5}{9}$.
22. $\frac{5}{6} - \frac{5}{8}$.
23. $\frac{7}{8} - \frac{7}{9}$.
24. $\frac{2}{7} - \frac{2}{9}$.
25. $12\frac{1}{2} - 6\frac{1}{3}$.
26. $13\frac{1}{4} - 2\frac{1}{5}$.
27. $14\frac{1}{5} - 7\frac{1}{6}$.
28. $16\frac{3}{5} - 12\frac{3}{8}$.

MULTIPLICATION

ORAL EXERCISE

1. 12 times 2 A. are how many acres? 12 times 2 fifths ($\frac{2}{5}$) are how many fifths? $\frac{2}{5} \times 4 = ?$

2. 32 mi. divided by 4 equals how many miles? $\frac{1}{4}$ of 32 mi. equals how many miles? Multiplying by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, etc., is the same as dividing by what integer?

3. If 5 men can dig 125 bu. of potatoes in 1 da., how many bushels can 3 men dig in the same time? $\frac{3}{5}$ of 125 bu. equals how many bushels?

175. Example. Multiply $\frac{3}{8}$ by 248.

Solutions. (*a*) 248 times 3 eighths = 744 eighths $= \frac{744}{8} = 93$; but,

$$\frac{3}{8} \times 248 \overset{(a)}{=} \frac{744}{8} = 93$$

(*b*) If the multiplication is indicated as in the margin, the work may be shortened by cancellation.

$$\frac{\overset{31}{\cancel{248}} \text{ times } 3}{\cancel{8}} \overset{(b)}{=} 93$$

176. Therefore, to find the product of an integer and a fraction, *find the product of the integer and the numerator, and divide it by the denominator.*

Before actually multiplying, indicate the multiplication and cancel if possible.

ORAL EXERCISE

1. If 1 yd. of cloth costs $\$0.87\frac{1}{2}$ ($\$\frac{7}{8}$), what will 16 yd. cost? 48 yd.? 128 yd.? 72 yd.?

2. When oats cost $\$0.33\frac{1}{3}\,¢$ ($\$\frac{1}{3}$) a bushel, how much must be paid for 29 bu.? for 36 bu.? for 129 bu.?

3. A boy earns $\$0.75$ ($\$\frac{3}{4}$) a day. How much will he earn in 18 da.? in 40 da.? in 84 da.? in 128 da.? in 160 da.?

4. When property rents for $720 a year, what is the rent for $\frac{1}{2}$ yr.? for $\frac{1}{4}$ yr.? for $\frac{1}{3}$ yr.? for $\frac{1}{12}$ yr.? for $\frac{1}{6}$ yr.?

5. A ship is worth $48,000. What is $\frac{1}{8}$ of the ship worth? $\frac{7}{16}$ of the ship? $\frac{5}{6}$ of the ship? $\frac{7}{8}$ of the ship? $\frac{7}{12}$ of the ship?

WRITTEN EXERCISE

Find the product of:

1. $98 \times \frac{5}{8}$.
2. $80 \times \frac{2}{3}$.
3. $50 \times \frac{7}{24}$.
4. $97 \times \frac{7}{10}$.
5. $92 \times \frac{5}{16}$.
6. $188 \times \frac{1}{64}$.
7. $\frac{7}{8}$ of 95.
8. $\frac{5}{7}$ of 25.
9. $\frac{4}{5}$ of 88.
10. $\frac{9}{16}$ of 51.
11. $\frac{5}{24}$ of 99.
12. $\frac{7}{16}$ of 77.
13. $784 \times \frac{5}{7}$.
14. $459 \times \frac{1}{9}$.
15. $400 \times \frac{1}{16}$.
16. $510 \times \frac{7}{10}$.
17. $990 \times \frac{1}{64}$.
18. $800 \times \frac{15}{16}$.
19. $\frac{5}{6}$ of 2420.
20. $\frac{7}{8}$ of 2500.
21. $\frac{2}{3}$ of 3240.
22. $\frac{5}{8}$ of 5117.
23. $\frac{7}{16}$ of 7254.
24. $\frac{7}{16}$ of 1024.

177. Example. Multiply 25 by $4\frac{3}{8}$.

SOLUTION. $\frac{3}{8}$ of $25 = 7\frac{5}{8}$ or $9\frac{3}{8}$. Write $\frac{3}{8}$ as shown in the margin, and carry 9 to the product of the integers. $4 \times 25 + 9 = 109$. Therefore, 25 multiplied by $4\frac{3}{8} = 109\frac{3}{8}$.

$$\begin{array}{r} 25 \\ 4\frac{3}{8} \\ \hline 109\frac{3}{8} \end{array}$$

178. Therefore, to find the product of a mixed number and a whole number, *multiply the integer and the fraction separately and find the sum of the products.*

ORAL EXERCISE

Find the cost of:

1. $15\frac{3}{4}$ lb. of fish at 9¢.
2. $7\frac{3}{8}$ yd. of cloth at \$3.
3. 16 lb. of beef at $10\frac{1}{2}$¢.
4. $16\frac{1}{2}$ lb. of sugar at 5¢.
5. 12 yd. of cloth at $11\frac{1}{4}$¢.
6. $6\frac{3}{4}$ bu. turnips at 32¢.
7. $12\frac{1}{2}$ bu. of oats at 39¢.
8. $10\frac{1}{2}$ yd. of calico at 4¢.
9. $16\frac{1}{2}$ yd. of ribbon at 20¢.
10. $8\frac{1}{2}$ gal. of molasses at 25¢.

WRITTEN EXERCISE

1. Find the total cost of:

124 lb. beef at $9\frac{1}{2}$¢.
$112\frac{1}{2}$ lb. beef at 5¢.
136 lb. pork at $5\frac{1}{2}$¢.
$114\frac{5}{8}$ lb. fish at 7¢.
156 lb. pork at $7\frac{1}{2}$¢.
$131\frac{7}{16}$ lb. fish at 9¢.

2. Find the total cost of:

27^3 yd. crêpe at \$2.
28^2 yd. satin at \$2.
25^3 yd. dress silk at \$2.50.
18^1 yd. velvet ribbon at \$2.
12^3 yd. fancy stripe at \$0.50.
43^2 yd. English serge at \$1.75.
43^2 yd. English camel's hair at \$2.
8 pc. fancy black ribbon at $\$2.87\frac{1}{2}$.

3. A merchant bought 25 pc. of striped denim containing 41^1, 41^1, 42^2, 43^2, 42^1, 44^2, 43^1, 40^2, 42^1, 45^3, 42^1, 40^2, 41^2, 47^3, 45^1, 41^1, 43^2, 47^2, 44^3, 42^3, 43^2, 39^1, 42^1, 48^2, and 47 yd., at 11¢ per yard. If he sold the first 11 pc. at 15¢ per yard and the remainder at 17¢ per yard, what was his gain?

4. Copy and find the amount of the following bill:

Little Falls, N.Y., June 7, 19—

Messrs. Sibley, Lindsay & Curr,
Rochester, N. Y.

Terms 60 das. *Bought of* **The Eureka Mills**

	5	pc. Bombay Gingham 41′ 42³ 45′ 40 41 210 yd.	8¾¢
	5	pc. Pontiac Seersucker 42′ 41 39³ 40² 42 ???	7¼¢
	10	pc. York Denim 40 42² 41³ 40 41² 41 39 42 41 40 ???	15½¢

179. The expressions $\frac{1}{2}$ of $\frac{3}{5}$ and $\frac{1}{2} \times \frac{3}{5}$ have the same meaning; hence, the sign of multiplication may be read *of*, or *multiplied by*, when it immediately follows a fraction.

180. Examples. 1. Multiply $\frac{3}{5}$ by $\frac{2}{3}$.

SOLUTION. To multiply $\frac{3}{5}$ by $\frac{2}{3}$ is to find $\frac{2}{3}$ of $\frac{3}{5}$.

Let the line AF in the accompanying diagram represent a unit divided into 5 equal parts.

Then AD will represent $\frac{3}{5}$. Subdivide each of the five equal parts into 3 equal parts and the line AF will represent a unit divided into 15 equal parts, each of which is $\frac{1}{15}$ of the whole. It is then clear that $\frac{1}{3}$ of $\frac{1}{5}$ equals $\frac{1}{15}$. Since $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, $\frac{1}{3}$ of $\frac{3}{5}$ is $\frac{3}{15}$. But $\frac{2}{3}$ of $\frac{3}{5}$ is 2 times $\frac{1}{3}$ of $\frac{3}{5}$; therefore, $\frac{2}{3}$ of $\frac{3}{5}$ equals $\frac{6}{15}$.

2. Find the product of $2\frac{1}{2}$, $\frac{4}{5}$, and $\frac{7}{15}$.

SOLUTION. Reduce the mixed number $2\frac{1}{2}$ to an improper fraction and obtain $\frac{5}{2}$. Cancel, and there remains in the numerators 2 times 7, and in the denominators 15, from which obtain the fraction $\frac{14}{15}$.

$$\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{5}}} \times \frac{7}{\underset{3}{\cancel{15}}} = \frac{14}{15}$$

181. Hence, to multiply a fraction by a fraction:

Reduce the mixed numbers and integers to improper fractions and cancel all factors common to the numerators and denominators.

Find the product of the remaining numerators for the required numerator, and the product of the remaining denominators for the required denominator.

ORAL EXERCISE

1. How many yards in $\frac{2}{3}$ rd.? feet in $\frac{3}{5}$ rd.?

2. When barley is worth $25\frac{1}{2}¢$ per bushel, what is the value of $\frac{1}{2}$ bu.? of $\frac{3}{4}$ bu.?

3. A book, the retail price of which was \$5, was sold at wholesale for $\frac{4}{5}$ of the retail price, with $\frac{1}{10}$ off from that for cash. Find the selling price of 10 books.

WRITTEN EXERCISE

Reduce to their simplest form:

1. $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{3}{7}$.
2. $\frac{3}{4}$ of $\frac{8}{9}$ of $2\frac{1}{3}$.
3. $7\frac{1}{5} \times 25 \times \frac{5}{8}$.
4. $3\frac{5}{8} \times 4\frac{1}{2} \times 20$.
5. $50 \times \frac{7}{10} \times 7\frac{3}{5}$.
6. $1\frac{2}{3} \times 4\frac{1}{2} \times 8\frac{4}{5}$.

7. A saves \$9.75 per week and B $\frac{2}{3}$ as much. How much more will A have than B at the end of the year?

8. A merchant bought a piece of cloth containing $43\frac{1}{2}$ yd. at \$1.50 per yard. He sold $\frac{3}{4}$ of it at $\$1.62\frac{1}{2}$ a yard, and the remainder at $\$1.37\frac{1}{2}$ a yard. Did he gain or lose, and how much?

The following is a recent clipping from a daily paper. It shows the prices of corn on the New York market.

NEW YORK CORN QUOTATIONS

DELIVERY	PREVIOUS CLOSING	OPENING	HIGHEST	LOWEST	CLOSING
July	$56\frac{1}{2}¢$	$55\frac{7}{8}¢$	$56\frac{1}{2}¢$	$54\frac{7}{8}¢$	$55\frac{3}{8}¢$
September	$55\frac{1}{4}¢$	$54\frac{7}{8}¢$	$55\frac{1}{8}¢$	$54\frac{1}{4}¢$	$54\frac{5}{8}¢$

9. D bought 25,000 bu. September corn at the opening price and sold it at the highest price. What was his gain? Had he bought at the lowest price and sold at the highest price, what would he have gained?

10. E bought 12,500 bu. July corn at the lowest price and sold it at the closing price. What was his gain? Had he bought at the lowest price and sold at the highest price, what would he have gained?

11. A gold dollar weighs 25.8 Troy grains. For every 90 parts of pure gold there are ten parts of alloy. How many grains of each kind in a gold dollar? in a 5-dollar gold piece?

12. A 5-cent piece weighs 77.16 Troy grains. For every part of nickel there are three parts of copper. How many grains of each kind in a 5-cent piece?

13. The second general coinage act (1834) of the United States made one silver dollar weigh approximately as much as sixteen gold dollars, and this ratio of **sixteen to one** has been maintained up to the present time. What is the weight of a silver dollar? If silver coins are $\frac{9}{10}$ pure, how much pure silver in 10 silver dollars?

Short Methods in Multiplication

182. When mixed numbers are large, they may be multiplied as shown in the following example.

183. Example. Multiply $255\frac{1}{3}$ by $24\frac{2}{5}$.

Solution. Multiply the fractions together and obtain $\frac{2}{15}$, which write as shown in the margin. Multiply the integer in the multiplicand by the fraction in the multiplier and obtain 102. Multiply the fraction in the multiplicand by the integer in the multiplier and obtain 8. Multiply the integers together and add the partial products. The result is $6230\frac{2}{15}$.

$$
\begin{array}{r}
255\frac{1}{3} \\
24\frac{2}{5} \\
\hline
\frac{2}{15} = \frac{2}{5} \text{ of } \frac{1}{3} \\
102 = \frac{2}{5} \text{ of } 255 \\
8 = 24 \text{ times } \frac{1}{3} \\
\left.\begin{array}{r} 1020 \\ 510 \end{array}\right\} = 24 \text{ times } 255 \\
\hline
6230\frac{2}{15} = 24\frac{2}{5} \text{ times } 255\frac{1}{3}
\end{array}
$$

WRITTEN EXERCISE

Multiply:

1. $975\frac{1}{4}$ by $18\frac{1}{2}$.
2. $876\frac{3}{4}$ by $21\frac{7}{8}$.
3. $720\frac{7}{8}$ by $21\frac{3}{4}$.
4. $445\frac{1}{2}$ by $46\frac{3}{4}$.
5. $512\frac{7}{16}$ by $16\frac{1}{2}$.
6. $450\frac{9}{16}$ by $20\frac{3}{4}$.

SQUARING NUMBERS ENDING IN $\frac{1}{2}$ OR 5

184. Examples. 1. Multiply $9\frac{1}{2}$ by $9\frac{1}{2}$.

SOLUTION. $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$, which write as shown in the margin. $\frac{1}{2}$ of the integer in the multiplicand plus $\frac{1}{2}$ of the integer in the multiplier is equal to either the integer in the multiplicand or multiplier. Therefore, add 1 to the integer in the multiplicand and multiply by the multiplier. $9 \times 10 = 90$. Then, $9\frac{1}{2} \times 9\frac{1}{2} = 90\frac{1}{4}$.

$$\begin{array}{r} 9\frac{1}{2} \\ 9\frac{1}{2} \\ \hline 90\frac{1}{4} \end{array}$$

2. Find the cost of 8.5 T. of coal at \$8.50 per ton.

SOLUTION. The principles embodied in this example are practically the same as those in problem 1. $.5 \times .5 = .25$, $8 \times 9 = 72$. Therefore, 8.5 tons of coal at \$8.50 per ton will cost \$72.25.

$$\begin{array}{r} 8.5 \\ 8.5 \\ \hline 72.25 \end{array}$$

3. Find the cost of 75 A. of land at \$75 per acre.

SOLUTION. This problem is similar to example 2, the only difference being in the matter of the decimal point. Since the decimal point has no particular bearing upon the steps in the process of multiplying, proceed to find the product as in example 2. $5 \times 5 = 25$, which write as shown in the margin. $7 \times 8 = 56$, which write to complete the product. 75 acres of land at \$75 an acre will therefore cost \$5625.

$$\begin{array}{r} 75 \\ 75 \\ \hline 5625 \end{array}$$

ORAL EXERCISE

Multiply:

1. $1\frac{1}{2}$ by $1\frac{1}{2}$.	**6.** $6\frac{1}{2}$ by $6\frac{1}{2}$.	**11.** $13\frac{1}{2}$ by $13\frac{1}{2}$.	**16.** $16\frac{1}{2}$ by $16\frac{1}{2}$.
2. $2\frac{1}{2}$ by $2\frac{1}{2}$.	**7.** 7.5 by 7.5.	**12.** $14\frac{1}{2}$ by $14\frac{1}{2}$.	**17.** $17\frac{1}{2}$ by $17\frac{1}{2}$.
3. $3\frac{1}{2}$ by $3\frac{1}{2}$.	**8.** 8.5 by 8.5.	**13.** $15\frac{1}{2}$ by $15\frac{1}{2}$.	**18.** $18\frac{1}{2}$ by $18\frac{1}{2}$.
4. $4\frac{1}{2}$ by $4\frac{1}{2}$.	**9.** 9.5 by 9.5.	**14.** 11.5 by 11.5.	**19.** 195 by 195.
5. $5\frac{1}{2}$ by $5\frac{1}{2}$.	**10.** 10.5 by 10.5.	**15.** 12.5 by 12.5.	**20.** 205 by 205.

WRITTEN EXERCISE

In the following problems make all the extensions mentally.

1. Find the total cost of:

85 lb. of tea at 85¢.	55 lb. tea at 55¢.
75 gal. sirup at 75¢.	75 bbl. flour at \$7.50.
45 gal. sirup at 45¢.	650 bbl. oatmeal at \$6.50.
$2\frac{1}{2}$ bu. beans at \$2.50.	25 doz. cans olives at \$2.50.
35 gal. molasses at 35¢.	95 cs. salad dressing at 95¢.
65 cs. horseradish at 65¢.	750 lb. cream codfish at $7\frac{1}{2}$¢.
$4\frac{1}{2}$ cs. baking powder at \$4.50.	$3\frac{1}{2}$ cs. baking powder at \$3.50.

MULTIPLICATION OF ANY NUMBERS ENDING IN $\frac{1}{2}$ OR .5

185. Examples. 1. Multiply $7\frac{1}{2}$ by $6\frac{1}{2}$.

SOLUTION. $\frac{1}{2}$ of the integer in the multiplicand plus $\frac{1}{2}$ of the integer in the multiplier is equal to $\frac{1}{2}$ of $\overline{6+7}$, or $6\frac{1}{2}$, which added to $\frac{1}{2}$ of $\frac{1}{2}$ equals $6\frac{3}{4}$. Write $\frac{3}{4}$ as shown in the margin, and carry 6. $\overline{6\times 7}+6 = 48$. Therefore, $7\frac{1}{2}\times 6\frac{1}{2} = 48\frac{3}{4}$.

$$\begin{array}{r} 6\frac{1}{2} \\ 7\frac{1}{2} \\ \hline 48\frac{3}{4} \end{array}$$

2. Multiply $7\frac{1}{2}$ by $9\frac{1}{2}$.

SOLUTION. $\frac{1}{2}$ of $\overline{7+9} = 8$, with no remainder. $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$, which write as shown in the margin, and carry 8. $\overline{7\times 9}+8 = 71$. Therefore, $7\frac{1}{2}\times 9\frac{1}{2} = 71\frac{3}{4}$.

$$\begin{array}{r} 7\frac{1}{2} \\ 9\frac{1}{2} \\ \hline 71\frac{1}{4} \end{array}$$

Observe that: (1) in finding $\frac{1}{2}$ of any number (dividing a number by 2) there is either nothing remaining or 1 remaining; (2) in finding $\frac{1}{2}$ of an even number there can be no remainder, and in finding $\frac{1}{2}$ of an odd number there is always a remainder 1. Hence, to multiply numbers ending in $\frac{1}{2}$ or .5:

Mentally determine the sum of the integers in the multiplicand and multiplier. If it is an even number, write $\frac{1}{4}$ (.25 or 25) in the product. If it is an odd number, write $\frac{3}{4}$ (.75 or 75) in the product. Multiply the integers and to the product add $\frac{1}{2}$ of their sum.

ORAL EXERCISE

Multiply:

1. $3\frac{1}{2}$ by $7\frac{1}{2}$.
2. $4\frac{1}{4}$ by $5\frac{1}{2}$.
3. $16\frac{1}{2}$ by $4\frac{1}{2}$.
4. $17\frac{1}{2}$ by $2\frac{1}{2}$.
5. $14\frac{1}{2}$ by $6\frac{1}{2}$.
6. $21\frac{1}{2}$ by $9\frac{1}{2}$.
7. 3.5 by 8.5.
8. 7.5 by 6.5.
9. 5.5 by 8.5.

WRITTEN EXERCISE

Make the extensions in each of the following problems mentally.

1. Find the total cost of:

6.5 T. coal at \$8.50.
2.5 T. hay at \$17.50.
15.5 cd. wood at \$3.50.

8.5 T. coal at \$9.50.
16.5 T. hay at \$11.50.
14.5 cd. wood at \$5.50.

2. Find the total cost of:

45 bu. beans at \$2.50.
35 bbl. flour at \$6.50.
45 bbl. flour at \$8.50.

350 bu. wheat at \$1.05.
350 bu. beans at \$2.50.
85 bbl. oatmeal at \$7.50.

DIVISION

ORAL EXERCISE

1. 8 A. $\div 4 = ?$ 8 ninths $(\frac{8}{9}) \div 4$?

2. If 2 lb. of coffee costs \$0.66$\frac{2}{3}$ (\$$\frac{2}{3}$), what will 1 lb. cost? Divide $\frac{2}{3}$ by 2. What is the effect of dividing the numerator of a fraction?

3. $\frac{4}{5} \div 2 = ?$ $\frac{1}{2}$ of $\frac{4}{5} = ?$

4. Because $\frac{4}{5} \div 2 = \frac{1}{2}$ of $\frac{4}{5}$, therefore, $\frac{1}{4} \div 5 = \frac{1}{5}$ of $\frac{1}{4}$, or $\frac{1}{4} \times \frac{1}{5}$. $\frac{1}{4} \times \frac{1}{5} = ?$

5. What is the quotient of $\frac{1}{3} \div 5$? of $\frac{1}{2} \div 8$? of $\frac{1}{5} \div 2$?

6. Because $\frac{1}{3} \div 5 = \frac{1}{5}$ of $\frac{1}{3}$, therefore $\frac{2}{3} \div 5 = 2$ times $\frac{1}{3}$ of $\frac{1}{5}$. That is, $\frac{2}{3} \div 5 = \frac{1}{5}$ of $\frac{2}{3}$, or $\frac{2}{3} \times \frac{1}{5}$. $\frac{2}{3} \times \frac{1}{5} = ?$

7. How much is $\frac{3}{4} \div 5$? $\frac{5}{6} \div 3$? $7\frac{1}{2}$ $(\frac{15}{2}) \div 8$? $3\frac{1}{2} \div 6$?

8. What is the effect of multiplying the denominator of a fraction?

186. In the above exercise it is clear that

Dividing the numerator of a fraction by an integer divides the whole fraction; and,

Multiplying the denominator of a fraction by an integer divides the whole fraction.

ORAL EXERCISE

Find the quotient of:

1. $\frac{8}{9} \div 4$.	**4.** $\frac{3}{4} \div 12$.	**7.** $\frac{7}{12} \div 4$.	**10.** $\frac{3}{5} \div 9$.	**13.** $\frac{1}{5} \div 19$.
2. $\frac{8}{13} \div 2$.	**5.** $\frac{5}{8} \div 12$.	**8.** $\frac{7}{16} \div 9$.	**11.** $\frac{1}{5} \div 6$.	**14.** $\frac{3}{16} \div 5$.
3. $\frac{15}{9} \div 5$.	**6.** $\frac{9}{10} \div 3$.	**9.** $\frac{5}{16} \div 7$.	**12.** $\frac{1}{8} \div 5$.	**15.** $\frac{7}{16} \div 5$.

187. Examples. 1. Divide $28\frac{7}{8}$ by 7.

SOLUTION. First divide the integers and the result is 4; then divide the fraction by 7 and the result is $\frac{1}{8}$. Therefore, $28\frac{7}{8} \div 7 = 4\frac{1}{8}$.

$$7\overline{)28\frac{7}{8}} \quad 4\frac{1}{8}$$

2. Divide $26\frac{1}{2}$ by 8.

SOLUTION. Divide 26 by 8 and the result is 3 with a remainder 2. Join the remainder, 2, with the fraction, $\frac{1}{2}$, making $2\frac{1}{2}$. Reduce $2\frac{1}{2}$ to an improper fraction and the result is $\frac{5}{2}$. $\frac{5}{2} \div 8 = \frac{5}{16}$. Therefore, $26\frac{1}{2} \div 8 = 3\frac{5}{16}$.

$$8\overline{)26\frac{1}{2}} \quad 3\frac{5}{16}$$

ORAL EXERCISE

Divide:

1. $16\frac{1}{2}$ by 4.
2. $18\frac{1}{5}$ by 9.
3. $25\frac{3}{4}$ by 2.
4. $17\frac{1}{2}$ by 8.
5. $32\frac{3}{5}$ by 4.
6. $27\frac{1}{2}$ by 7.
7. $19\frac{7}{8}$ by 9.
8. $20\frac{2}{3}$ by 10.
9. $21\frac{1}{2}$ by 8.
10. $24\frac{6}{7}$ by 6.
11. $45\frac{5}{9}$ by 5.
12. $40\frac{2}{3}$ by 10.
13. $8\frac{1}{2}$ by 5.
14. $14\frac{3}{4}$ by 7.
15. $11\frac{1}{2}$ by 9.
16. $26\frac{1}{2}$ by 10.

ORAL EXERCISE

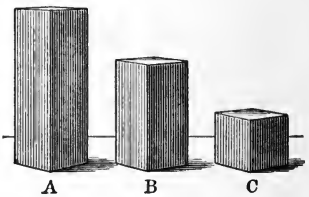
1. How many eighths in one? $1 \div \frac{1}{8} = ?$

2. What is the value of: $1 \div \frac{1}{10}$? $3 \div \frac{1}{5}$? $17 \div \frac{1}{8}$? $125 \div \frac{1}{12}$? $250 \div \frac{1}{4}$?

3. Read aloud the following, supplying the missing word: *To divide an integer by a unit fraction, multiply the integer by the —— of the fraction.*

4. What is the value of $25 \div \frac{1}{3}$? $2.5 \div \frac{1}{2}$? $7.5 \div \frac{1}{4}$? $25.5 \div \frac{1}{12}$? $54 \div \frac{1}{4}$? $48 \div \frac{1}{5}$? $29 \div \frac{1}{3}$? $2\frac{1}{2} \div \frac{1}{6}$?

5. If B, in the accompanying diagram, is 1, what is C? How many blocks like C in B? $1 \div \frac{1}{2} = ?$



6. If A is 1, what is B? A is how many times B? That is, $A \div B = ?$ $1 \div \frac{2}{3} = ?$

7. If $1 \div \frac{2}{3} = \frac{3}{2}$ $(1\frac{1}{2})$, then $2 \div \frac{2}{3} = ?$

8. What is the value of $4 \div \frac{2}{3}$? $5 \div \frac{3}{4}$? $12 \div \frac{2}{5}$? $15 \div \frac{2}{3}$?

9. Read aloud the following, supplying the missing words: If A is 1, B is ——, and C is ——. If B is contained in A $\frac{3}{2}$ $(1\frac{1}{2})$ times, it is contained in C $\frac{1}{3}$ of $\frac{3}{2}$ times or —— times. That is, $\frac{1}{3} \div \frac{2}{3} = \frac{1}{3} \times \frac{3}{2} =$ ——.

10. What is the value of $\frac{1}{2} \div \frac{1}{4}$? $\frac{2}{3} \div \frac{2}{5}$? $\frac{2}{7} \div \frac{2}{3}$? $\frac{3}{4} \div \frac{5}{6}$?

188. The **reciprocal of a fraction** is 1 divided by that fraction.

Thus, the reciprocal of $\frac{2}{3}$ is $1 \div \frac{2}{3}$, or $\frac{3}{2}$. That is, *the reciprocal of a fraction is the fraction inverted.*

189. Reciprocal numbers, as we use the terms in arithmetic, are numbers whose product is 1.

Thus, 4 and $\frac{1}{4}$, $\frac{4}{5}$ and $\frac{5}{4}$, $\frac{1}{6}$ and 6, $\frac{2}{3}$ and $\frac{3}{2}$, are reciprocal numbers, because their product is equal to 1.

190. It has been seen that the brief method for dividing a fraction or an integer by a fraction is to *multiply the dividend by the reciprocal of the divisor.*

The principles of cancellation should be used whenever possible. Integers and mixed numbers should be reduced to improper fractions before applying the rule.

WRITTEN EXERCISE

Divide:

1. $\frac{7}{8}$ by $\frac{2}{3}$.
2. $7\frac{1}{2}$ by $\frac{1}{8}$.
3. 95 by $\frac{5}{8}$.
4. 88 by $\frac{3}{4}$.
5. 16 by $\frac{3}{5}$.
6. $15\frac{1}{2}$ by $\frac{1}{5}$.
7. $\frac{3}{4}$ by $\frac{2}{3}$.
8. $4\frac{5}{7}$ by $\frac{2}{3}$.
9. $\frac{9}{10}$ by $\frac{7}{8}$.
10. $6\frac{2}{3}$ by $1\frac{1}{2}$.
11. 160 by $4\frac{1}{2}$.
12. 250 by $3\frac{2}{3}$.
13. $\frac{7}{8}$ by $\frac{4}{5}$.
14. $\frac{3}{5}$ by $\frac{2}{7}$.
15. 169 by $4\frac{7}{8}$.
16. 640 by $5\frac{3}{8}$.
17. 625 by $83\frac{1}{3}$.
18. $920\frac{3}{5}$ by 73.

191. Examples. 1. Divide 2190 by $48\frac{2}{3}$.

SOLUTION. Multiplying both dividend and divisor by the same number does not affect the quotient; hence, multiply the dividend and divisor by 3 and obtain for the new dividend and divisor 6570 and 146, respectively. Divide the same as in simple numbers and obtain the result 45. Or,

Reduce both the dividend and divisor to thirds, obtaining $\frac{6570}{3}$ and $\frac{146}{3}$. Reject the common denominators and divide as in whole numbers.

$$\begin{array}{r} 48\frac{2}{3})2190 \\ 3 \quad\quad 3 \\ \hline 146) \quad 6570 (45 \\ \quad\quad 584 \\ \hline \quad\quad 730 \\ \quad\quad 730 \\ \hline \end{array}$$

2. Divide $65\frac{1}{2}$ by $12\frac{1}{3}$.

SOLUTION. Multiply both dividend and divisor by 6, the least common denominator of the fractions, and divide as in simple numbers. The result is $5\frac{23}{74}$. Or,

Reduce both the dividend and divisor to sixths, obtaining as a result $\frac{74}{6}$ and $\frac{393}{6}$. Reject the common denominator and divide as in simple numbers.

$$\begin{array}{r} 12\frac{1}{3})65\frac{1}{2} \\ 6 \quad\quad 6 \\ \hline 74) \quad 393 (5\frac{23}{74} \\ \quad\quad 370 \\ \hline \quad\quad 23 \end{array}$$

WRITTEN EXERCISE

Divide:

1. $270\frac{1}{2}$ by $12\frac{1}{2}$.
2. $508\frac{1}{4}$ by $30\frac{2}{3}$.
3. $1431\frac{1}{5}$ by $20\frac{1}{2}$.
4. $962\frac{1}{2}$ by $31\frac{1}{4}$.
5. $650\frac{3}{4}$ by $26\frac{1}{2}$.
6. $1680\frac{1}{8}$ by $45\frac{1}{2}$.
7. 7552 by $78\frac{2}{3}$.
8. $470\frac{3}{4}$ by $17\frac{1}{8}$.
9. $1054\frac{1}{2}$ by $168\frac{1}{3}$.

FRACTIONAL RELATIONS

ORAL EXERCISE

1. If f in the accompanying diagram is 1, what is e? d? c? b? a?

2. What part of e is f? of d? of c? of b? of a? What part of 6 is 1? of 5? of 4? of 3? of 2?

3. What part of a is e? d? c? b? What part of 6 is 2? 3? 4? 5?

4. What part of d is f? What part of b is e? What part of $\frac{1}{2}$ ($\frac{3}{6}$) is $\frac{1}{6}$? What part of $\frac{5}{6}$ is $\frac{1}{3}$ ($\frac{2}{5}$)?

5. What part of 7 bu. is 1 bu.? What part of 7 eighths ($\frac{7}{8}$) is 1 eighth ($\frac{1}{8}$)?

6. What part of $\frac{3}{5}$ is $\frac{2}{5}$?

SOLUTION. $\frac{3}{5}$ and $\frac{2}{5}$ are similar fractions; hence they may be compared in the same manner as concrete integral numbers. 2 is $\frac{2}{3}$ of 3; therefore, $\frac{2}{5}$ is $\frac{2}{3}$ of $\frac{3}{5}$; or,

$$\frac{2}{5} \text{ is } \frac{\frac{2}{5}}{\frac{3}{5}} \text{ of } \frac{3}{5}. \quad \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{5} \times \frac{5}{3} = \frac{2}{3}.$$

7. $\frac{3}{4}$ is what part of $1\frac{3}{4}$ ($\frac{7}{4}$)? of $2\frac{3}{4}$? of $5\frac{1}{4}$?

8. $\frac{1}{8}$ is what part of $\frac{1}{2}$?

SOLUTION. $\frac{1}{2} = \frac{4}{8}$. $\frac{1}{8}$ is $\frac{1}{4}$ of $\frac{4}{8}$, therefore, $\frac{1}{8} = \frac{1}{4}$ of $\frac{1}{2}$; or,

$$\frac{1}{8} \text{ is } \frac{\frac{1}{8}}{\frac{1}{2}} \text{ of } \frac{1}{2}. \quad \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{8} \times \frac{2}{1} = \frac{1}{4}.$$

192. To find what fraction one number is of another, *take the number denoting a part for the numerator of the fraction, and the number denoting the whole for the denominator.*

ORAL EXERCISE

1. If a piece of work can be performed in 12 da., what part of it can be performed in 5 da.? in 7 da.?

2. If A can do a piece of work in 15 da., what part of it can he do in 1 da.? in 2 da.? in 5 da.? in $7\frac{1}{2}$ da.?

3. If B can do a piece of work in $7\frac{1}{2}$ da., what part of it can he do in 1 da.? in 2 da.? in 5 da.? in $5\frac{1}{2}$ da.? in $6\frac{1}{2}$ da.?

4. I bought a farm for $2000 and sold it for $3000. What part of the cost was realized? what part of the cost was gained?

5. A watch costing $75 was sold for $60. What part of the cost was realized? What part of the cost was lost?

6. A and B hired a pasture together. A pastured 5 cows, 7 wk., and B pastured 7 cows for the same length of time. What part of the price should each pay?

7. A can do a piece of work in 8 da. which B can do in 9 da. How many days will it take them if they join in the completion of the work?

WRITTEN EXERCISE

1. What part of 100 is $33\frac{1}{3}$? $12\frac{1}{2}$? $66\frac{2}{3}$? $8\frac{1}{3}$? 25? 75? 125? $16\frac{2}{3}$? $83\frac{1}{3}$? $62\frac{1}{2}$? $22\frac{2}{9}$? $9\frac{1}{11}$? $56\frac{1}{4}$? $6\frac{2}{3}$?

2. What part of $1 is $33\frac{1}{3}¢$? $66\frac{2}{3}¢$? $25¢$? $75¢$? $16\frac{2}{3}¢$? $8\frac{1}{3}¢$? $6\frac{2}{3}¢$? $3\frac{1}{3}¢$? $6\frac{1}{4}¢$? $62\frac{1}{2}¢$? $87\frac{1}{2}¢$? $37\frac{1}{2}¢$? $14\frac{2}{7}¢$?

3. What part of 1000 is 125? $166\frac{2}{3}$? $666\frac{2}{3}$? 625? $333\frac{1}{3}$?

4. What part of $10 is 3.33\frac{1}{3}$? $1.25? 1.66\frac{2}{3}$? 8.33\frac{1}{3}$? $2.50? $6.25? 6.66\frac{2}{3}$?

5. A, B, C, and D hired a pasture for $45. A pastured 4 cows for $4\frac{1}{2}$ mo.; B, 6 cows for $3\frac{1}{2}$ mo.; C, 4 cows for $1\frac{1}{2}$ mo.; D, 5 cows for 3 mo. How much should each pay?

ORAL EXERCISE

1. If a in the accompanying diagram is 10 in. high, how high is b? c? d? 10 is $\frac{1}{2}$ of what number? $\frac{1}{3}$ of what number? $\frac{1}{4}$ of what number?

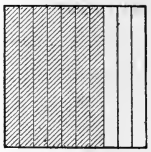
2. If 225 is $\frac{5}{8}$ of a certain number, what is $\frac{1}{8}$ of the number? $\frac{8}{8}$ of the number?

3. 192 is $\frac{3}{5}$ of what number? $\frac{3}{16}$ of what number?

d c b a

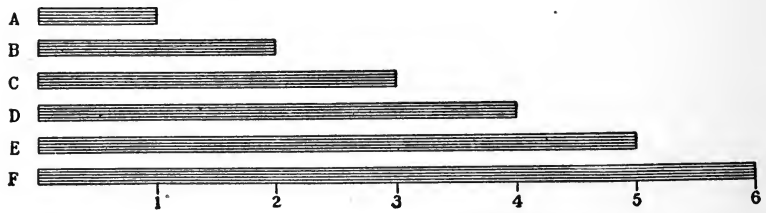
4. After making a payment of $3500 I find that I still owe for $\frac{2}{3}$ of the cost of my house. What was the cost of my house? How much still remains unpaid?

WRITTEN EXERCISE



1. The square in the margin represents the total population of the state of New York (state census of 1905), and the shaded area represents the urban (city) population. If the suburban (country) population is 2,420,001, what is the entire population of the state? the urban population?

2. In a recent year the population of Massachusetts was 3,002,000, and there were three persons living in the cities of the state to every one person living in the country. Represent this graphically as in problem 1, and find the city population and the country population for the state.



3. Suppose that *C* in the diagram represents the population of the United States in 1870, *A* the population in 1830, and *F* the population in 1900. If the population in 1870 was 38,400,000 (round numbers), what was the population (round numbers) in 1900? In 1830?

4. Suppose that *F* in the diagram represents the population of the United States in 1900, and *C* the proportion of this population living in cities in 1900. What proportion of the population lived in cities in 1900? Suppose that *F* represents the population in 1860 and *A* the proportion of this population living in cities. Assuming that the city population in 1860 was 5,240,554, find the total population for the same year.

5. The total population of New Jersey (state census of 1905) is 2,144,134, and the urban population, 1,286,480. Represent this graphically and find the country population.

CONVERSION OF FRACTIONS

ORAL EXERCISE

1. What is the denominator of the decimal .6? of .75?
2. What is the numerator of .4? of .04? of .004? of .0004?
3. Write as a common fraction .7; .23; .079; .0013; .00123.

193. A decimal may be written as a common fraction.

194. Examples. 1. Reduce .0625 to a common fraction.

SOLUTION. .0625 means $\frac{625}{10000}$; but $\frac{625}{10000}$ may be expressed in simpler form. Dividing both terms of the fraction by 625, the result is $\frac{1}{16}$.

$$\frac{625}{10000} = \frac{5}{80} = \frac{1}{16}$$

WRITTEN EXERCISE

Reduce to a common fraction or to a mixed number:

1. 0.375.	5. 0.9375.	9. 0.0335.	13. 260.675.
2. 0.0625.	6. $1.66\frac{2}{3}$.	10. $0.0056\frac{1}{4}$.	14. 126.1875.
3. 0.0016.	7. 0.4375.	11. 181.875.	15. 175.0625.
4. 0.5625.	8. 0.125.	12. 171.245.	16. 172.0075.

195. A common fraction may be written as a decimal.

196. Example. Reduce $\frac{3}{8}$ to a decimal.

SOLUTION. $\frac{3}{8}$ equals $\frac{1}{8}$ of 3 units. 3 units equals 3000 thousandths. $\frac{1}{8}$ of 3000 thousands equals 375 thousandths (.375).

$$8\overline{)3.000} \quad .375$$

ORAL EXERCISE

1. Reduce to equivalent decimals: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{16}$, $\frac{1}{12}$, $\frac{3}{16}$, $\frac{1}{9}$, $\frac{1}{11}$.
2. Reduce to common fractions: .5, .25, .50, .75, $.33\frac{1}{3}$, $.66\frac{2}{3}$, $.16\frac{2}{3}$, $.12\frac{1}{2}$, .6, .4, .60, .40, .2, $.83\frac{1}{3}$, .20, $.08\frac{1}{3}$, .375, .125, $.37\frac{1}{2}$, $.87\frac{1}{7}$, .875, .0625, $.11\frac{1}{9}$, $.09\frac{1}{11}$.

WRITTEN EXERCISE

Reduce to equivalent decimals:

1. $\frac{7}{8}$.	3. $\frac{9}{16}$.	5. $\frac{9}{320}$.	7. $\frac{17}{480}$.	9. $\frac{5}{6400}$.	11. $21\frac{5}{8}$.
2. $\frac{5}{16}$.	4. $\frac{7}{16}$.	6. $\frac{11}{16}$.	8. $\frac{7}{25}$.	10. $5\frac{7}{12}$.	12. $165\frac{17}{20}$.

APPROXIMATIONS

197. Since results beyond two or three decimal places are seldom required in business, **approximations in multiplication** are frequently desired. In problems involving dollars and cents, it is sufficient to carry the decimal places in the final results just far enough to obtain accurate cents. In order to make sure that a product is correct to the nearest cent, it is usually necessary to carry the partial products to three decimal places.

198. Example. If \$1 put at compound interest (see page 314) for 10 yr. at $4\frac{1}{2}$ % amounts to \$1.55297, what will \$4125.67 amount to in the same time at the same rate?

SOLUTION. It has been seen (page 52) that in multiplying there is no advantage in beginning with the lowest order of the multiplier. In this example it will be seen that there is a decided advantage in beginning with the highest order of the multiplier.

In beginning the multiplication note that 4000 times .00007 = .28 and write 8 in the hundredths' place. Complete and point off the first partial product as shown in the process at the left. The other partial products are then formed in natural order

FULL PROCESS	CONTRACTED PROCESS
1.552 97	1.55297
4125.67	4125.67
6211.88	6211.88
155.297	155.297
31.059 | 4	31.059
7.764 | 85	7.765
.931 | 782	.932
.108 | 7079	.109
6407.041 | 7399	6407.04

The work is given in full and in contracted form. Examine both processes. Note that in the full process all of the work on the right of the vertical line is wasted; also note how much better for practical purposes the contracted form is than the other. In this problem the first two steps are the same by either process. Multiplication by 20 would give a figure in the fourth place. Instead of writing down the product of 20 times .00007, add the nearest ten to the next column. $\overline{2 \times 9} + 1 = 19$; $\overline{2 \times 2} + 1 = 5$; etc. In multiplying by the next 5 it is not necessary to take the 7 in the multiplicand into account; in multiplying by .6 both 7 and 9 in the multiplicand may be rejected; in multiplying by .07, the 7, 9, and 2 in the multiplicand may be rejected. When any figure in the multiplicand is dropped, it may be marked off as follows: 1.55$\backslash$2$\backslash$9$\backslash$7. In finding the sum of the partial products do not set down the result for the third decimal place, but carry the nearest ten (3) to the second decimal place. The required result is finally found to be \$6407.04.

199. Approximations in division are also frequently desired.

200. Example. If 10.134 A. of land cost $889.26, what is the cost per acre?

$$
\begin{array}{r}
\$87.75 \\
10134\overline{)\$889260} \\
\underline{8107} \quad = \text{approximately.} \quad 8 \text{ times } 1013\ (4) \\
785 \\
\underline{709} \quad = \text{approximately} \quad 7 \text{ times } 101\ (34) \\
76 \\
\underline{71} \quad = \text{approximately} \quad 0.7 \text{ times } 10\ (134) \\
5 \\
\underline{5} \quad = \text{approximately } 0.05 \text{ times } 1\ (0134)
\end{array}
$$

SOLUTION. Since the decimal point appears in both dividend and divisor, it is better to first multiply each by such a power of ten as shall make the divisor integral. In such problems as this a result correct to the nearest cent is all that is required. Since 10's (an approximation for the last two figures in the dividend) divided by 10000's (an approximation for the divisor) is less than 0.01, the last two figures of the dividend will not affect the quotient, and they may therefore be rejected. Hence, also, the divisor may be considered 1013 and may be continually contracted; but in multiplying the divisor by each quotient figure, mentally multiply the figure cut off and carry the nearest ten. When a figure is rejected in the divisor, it may be marked off as explained in § 198. The work may be further abridged by omitting the partial products and writing down the remainders only as explained on page 67.

$$
\begin{array}{r}
\$87.75 \\
10134\overline{)\$889260} \\
785 \\
76 \\
5
\end{array}
$$

WRITTEN EXERCISES

1. Divide 20,000 by 3.1416 correct to .01.

2. Find the product of 10.48×3.14159 correct to two decimal places.

3. If $1 placed at simple interest for 1 yr. 7 mo. at $3\frac{1}{2}\%$ will amount to $1.05541, what will $1869.75 amount to in the same time at the same rate?

4. The estimated population of Continental United States for 1906 was 92,500,000 and the area was 3,602,990 sq. mi. What was the average population per square mile for this year, to the nearest unit?

THE SOLUTION OF PROBLEMS

201. The steps in the solution of a problem are: (1) reading the problem to find what is given and what is required; (2) determining from what is given how to find what is required; (3) outlining a process of computation and then performing it; (4) checking results.

202. A problem should be thoroughly understood before any attempt is made to solve it; and when the relation of what is given to what is required has been discovered, the process of computation should be briefly indicated and then performed as briefly and rapidly as possible.

203. To insure accuracy the work should always be checked in some manner. If the answer to the problem is estimated in advance, it will prove an excellent check against absurd results.

Thus, 42 doz. boys' hose at $48 a dozen is equal to approximately $40 \times \$50$; $9\frac{2}{3}\%$ of 1290 bu. is equal to approximately $\frac{1}{10}$ of 1290 bu.; etc.

204. Example. A tailor used 30 yd. of flannel in making 18 waistcoats; at that rate how many yards will he require in making 45 waistcoats?

SOLUTION

1. The quantity needed in making 18 waistcoats is given and the quantity needed in making 45 waistcoats is required.

2. One waistcoat requires $\frac{30}{18}$ yd.; 45 waistcoats will require 45 times $\frac{30}{18}$ yd.

3. $\frac{\overset{15}{\cancel{45}} \text{ times } \overset{5}{\cancel{30}}}{\underset{3}{\cancel{18}}} = 75$; that is 75 yd. of flannel are required in making 45 waistcoats.

4. $\frac{30}{18}$ yd. $= \frac{5}{3}$ yd.; $\frac{75}{45}$ yd. $= \frac{5}{3}$ yd.; therefore the work is probably correct.

205. If reasons for conclusions, processes, and results are given, they should be brief and accurate. It is also a mistake to try to use the language of the book or teacher. Such artificial work stifles thought and conceals the condition of the learner.

The subject of analysis should not be unduly emphasized. A correct solution may generally be accepted as evidence that the correct analysis has been made.

ORAL EXERCISE

In the following problems first find each result as required, and then give a brief, accurate explanation of the steps taken in the solution. Do not use pen or pencil.

1. If 2 T. cost $8, what will 5 T. cost?

SUGGESTION. $20; since 2 T. cost $8, 5 T., which are $2\frac{1}{2}$ times 2 T., will cost $2\frac{1}{2}$ times $8, or $20.

2. 24 is $\frac{6}{7}$ of what number? $\frac{3}{8}$ of what number? $\frac{2}{13}$ of what number?

3. 220 is $\frac{1}{2}$ less than what number? 450 is $\frac{1}{3}$ less than what number?

4. A, having spent $\frac{1}{5}$ of his money, finds he has $84 left. How much had he at first?

5. $124 is $\frac{1}{3}$ more than what sum of money? $300 is $\frac{1}{4}$ more than what sum of money?

6. A man sold $\frac{5}{16}$ of an acre of land for $35. At that rate what is his entire farm of 100 acres worth?

7. A man bought a stock of goods and sold it at $\frac{1}{4}$ above cost. If he received $275, what was the cost of the goods?

8. B bought a stock of goods which he sold at $\frac{1}{5}$ below cost. If he received for the sale of the goods $240, what was the cost and what was his loss?

9. $\frac{9}{16}$ of the students in a high school are girls and the remainder are boys. If the number of boys is 350, how many scholars in the school?

10. A bought a quantity of wheat which he sold at $\frac{1}{2}$ above cost. If he received $300 for the wheat, what did it cost him and what was his gain?

11. A bought a quantity of dry goods and sold them so as to realize $\frac{1}{3}$ more than the cost. If the selling price was $720, what was the cost and what was the gain?

12. D bought a stock of carpeting which he was obliged to sell at $\frac{1}{6}$ below cost. If he received $750 for the sale of the carpeting, what was the cost of same, and what was his loss?

WRITTEN EXERCISE

In the following problems give both analysis and computation.

1. If $\frac{1}{2}$ lb. of tea cost 21¢, what will $9\frac{1}{2}$ lb. cost?

COMPUTATION	ANALYSIS
$9\frac{1}{2} = \frac{19}{2}$ $19 \times 21¢ = \$3.99$	$9\frac{1}{2} = \frac{19}{2}$; $9\frac{1}{2}$ is therefore 19 times $\frac{1}{2}$. If $\frac{1}{2}$ lb. cost 21¢, $9\frac{1}{2}$ lb. will cost 19 times 21¢, or \$3.99.

2. If $\frac{1}{2}$ of a pound of tea cost 42¢, what will $35\frac{1}{2}$ lb. cost?

3. If a drain can be dug in 17 da. by 45 men, how many men will it take to dig $\frac{1}{5}$ of it in 3 da.?

4. In what time will 3 boys at $\$0.62\frac{1}{2}$ per day earn as much as 4 men at \$2.25 each per day will earn in $45\frac{3}{4}$ da.?

5. A spends \$72 per week or $\frac{3}{4}$ of his income; B saves \$48 per week or $\frac{3}{8}$ of his income. How long will it take A to save as much as B saves in five weeks?

6. If 115 bbl. of wheat are required to make 23 bbl. of flour, how many barrels will be required to make 50 bbl. of flour? 117 bbl. of flour? 259 bbl. of flour?

ORAL REVIEW EXERCISE

1. $.05 \times 6 \times 0 \times 2\frac{1}{2} = ?$

2. \$0.75 is what part of \$3?

3. What is the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{10}$?

4. Find the value of $.45 + (.25 \times 5) - .04$.

5. 60 is $\frac{3}{5}$ of what number? $\frac{3}{4}$? $\frac{3}{7}$? $\frac{4}{5}$? $\frac{5}{6}$?

6. At 25¢ a yard, what will $2\frac{1}{2}$ yd. of cloth cost?

7. $\frac{6}{7}$ is $\frac{1}{2}$ of what number? $\frac{3}{5}$ is $\frac{3}{4}$ of what number?

8. If $\frac{3}{4}$ of an acre of land costs \$75, what will 50 A. cost?

9. If $\frac{2}{5}$ of a number is 84, what is 5 times the same number?

10. The dividend is $4\frac{1}{2}$ and the quotient is $6\frac{3}{4}$; what is the divisor?

11. If 6 bu. of apples cost \$15, what will 80 bu. cost at the same rate?

12. At \$460 per half mile, what will be the cost of grading 6 mi. of road?

13. How much will 4 carpenters earn in 10 da. at the rate of $2.25 per day?

14. At $4.50 per cord, what will be the cost of $4\frac{1}{2}$ cd. of wood? of $6\frac{1}{2}$ cd.? of $12\frac{1}{2}$ cd.? of $7\frac{1}{2}$ cd.?

15. A bought a horse for $96 and sold it for $\frac{7}{8}$ of its cost. What part of the cost was the loss sustained?

16. A bought $4\frac{1}{4}$ yd. of velvet at $5.20 per yard and gave in payment a $50 bill. How much change should he receive?

17. I sold 5 A. of land for $375 and sustained a loss equal to $\frac{1}{6}$ of the original cost of the land. What did the land cost per acre?

18. D and E agree to mow a field for $36. If D can do as much in 2 da. as E can do in 3, how should the money be divided?

19. N sold a watch to O and received $\frac{1}{3}$ more than it cost him. If O paid $64 for the watch, what did it cost N? What per cent did N gain?

20. A earns $125 per month. Of this sum he spends $75 and saves the remainder. What part of his monthly earnings does he save? What per cent?

WRITTEN REVIEW EXERCISE

1. Find the cost of 1100 eggs at $23\frac{3}{4}¢$ per dozen.

2. Counting 2000 lb. to a ton, find the cost of $5\frac{7}{8}$ T. of steel at $1\frac{5}{16}¢$ per pound.

3. When flour is sold at $6.02 per barrel of 196 lb., what should be paid for $55\frac{1}{2}$ lb.?

4. I bought 300 bbl. of flour at $5.75 per barrel. At what price must I sell it per barrel in order to gain $150?

5. The cost of 200 bu. of wheat was $204.50 and the selling price $212.35. What was the gain per bushel?

6. A can do a piece of work in $5\frac{1}{2}$ da. and B in $7\frac{1}{2}$ da. If they join in the completion of the work, how long will it take them?

7. How much will 7 men earn in 6 da., working 10 hr. per day, at 25 ¢ per hour?

8. At \$2.50 per day of 8 hr., how much should a man receive for $11\frac{1}{2}$ hours' work?

9. A boy works $4\frac{1}{2}$ da. at the rate of \$5.75 per week of 6 da. How much does he earn?

10. W, in $\frac{1}{3}$ of a day, earns \$1.25, and Y, in $\frac{1}{2}$ of a day, earns $\$0.87\frac{1}{2}$. How much will the two together earn in $40\frac{1}{2}$ da.?

11. A and B together can do a piece of work in 10 da. If A can complete the work alone in 16 da., how long will it take B to do it?

12. Nov. 1, in a recent year, was on Tuesday. How much did B earn during November if he was employed every working day at the rate of \$3.75 per day?

13. A farm is divided into 6 fields containing, respectively, $25\frac{5}{8}$, $26\frac{7}{16}$, $32\frac{3}{4}$, $56\frac{7}{8}$, $35\frac{9}{16}$, and $52\frac{1}{16}$ A. How much is the farm worth at \$37.50 per acre?

14. Find the total cost of: 630 lb. sugar at $4\frac{3}{4}$ ¢; 375 lb. tea at $38\frac{1}{2}$ ¢; 240 lb. crackers at $5\frac{7}{8}$ ¢; 65 lb. rice at $7\frac{9}{16}$ ¢; $52\frac{1}{2}$ lb. raisins at $7\frac{1}{2}$ ¢; and 250 lb. coffee at $24\frac{3}{4}$ ¢.

15. A retailer bought 5 bbl. of flour at \$6.50 per barrel, 12 bu. potatoes at 75 ¢ per bushel, and gave in payment a fifty-dollar bill. How much change should he receive?

16. Five garden lots measuring $2\frac{7}{8}$, $10\frac{1}{2}$, $12\frac{5}{8}$, $6\frac{7}{16}$, and $8\frac{9}{16}$ A. respectively, were bought at $\$212.87\frac{1}{2}$ per acre and sold at \$250.50 per acre. Find the gain resulting from the transaction.

17. I bought 4120^2 yd. of silk at \$1.02 per yard and sold $\frac{3}{4}$ of it at \$1.50 per yard, and the remainder for \$1600. What was the average price received per yard, and how much did I gain?

18. A, B, C, and D hire a pasture for \$419.50. A put in 25 head of cattle for 4 wk.; B, 31 head for 5 wk.; C, 44 head for 6 wk.; and D, 40 head for 8 wk. How much should each be required to pay?

19. A grain dealer bought $6750\frac{1}{2}$ bu. of corn at $60\frac{1}{2}¢$ per bushel, and $2130\frac{1}{4}$ bu. of oats at $32\frac{3}{4}¢$ per bushel. He sold the corn at $69\frac{7}{8}¢$ per bushel, and the oats at $29\frac{5}{8}¢$ per bushel. Did he gain or lose, and how much?

20. A grocer bought 15 bbl. of molasses, each containing 50 gal., at $25\frac{1}{2}¢$ per gallon. He retailed $150\frac{1}{2}$ gal. of it at $30¢$ per gallon, $170\frac{1}{4}$ gal. at $28¢$ per gallon, and the remainder at $35¢$ per gallon. Did he gain or lose, and how much?

21. Find the cost of 25 bx. of cheese weighing: $67-4$, $62-4$, $61-3$, $72-4$, $81-5$, $64-4$, $66-3$, $65-5$, $61-4$, $62-3$, $64-4$, $66-3$, $65-5$, $61-4$, $62-3$, $64-4$, $67-3$, $65-5$, $60-3$, $62-4$, $67-4$, $65-4$, $60-4$, $68-3$, $65-4$ lb., respectively, at $11\frac{3}{4}¢$ per pound.

22. A dry-goods merchant bought 25 pc. of Scotch cheviot containing 42^1, 40^2, 45^3, 41^1, 40^1, 45^2, 42^1, 43^3, 38^1, 35^1, 36^2, 41^2, 44^1, 45^2, 39^1, 37^1, 42^2, 47, 41, 42^1, 43^3, 40^1, 47^1, 38, 31 yd., respectively, at $39\frac{1}{2}¢$ per yard. If he sold the entire purchase at $43\frac{3}{4}¢$ per yard, did he gain or lose, and how much?

23. C. W. Bender fails in business. He owes A \$712.25; B, \$1421.25; C, \$625.25; D, \$1460.75; his entire resources amount to \$2109.75. What fractional part of his indebtedness can he pay? what per cent? How many cents on \$1? If his creditors accept payment on this basis, how much will each receive?

24. A dry-goods merchant bought 12 pc. of striped denim containing 40^1, 45^1, 40^1, 48^2, 41^2, 40^3, 45^2, 41^1, 44^2, 39^2, 51^1, 38 yd., respectively, at $14\frac{3}{4}¢$ per yard; 15 pieces of cashmere containing 39^1, 41^2, 42^1, 45^2, 39, 52, 40, 45, 46, 51, 47^2, 42^1, 41^1, 47^1, 48 yd., respectively, at \$1.12 per yard; 10 pc. wash silk containing 35^1, 30, 31^2, 30, 30, 30, 32^3, 32, 31^1, 32 yd., respectively, at $31¢$ per yard. He gave in payment, cash, \$300, and a 60-da. note for the balance. What was the amount of the note?

25. Find the amount of the following bill:

Boston, Mass., Apr. 15, 19

MESSRS. CHARLES H. PALMER & CO.

Springfield, Mass.

Bought of EDGAR W. TOWNSEND & CO.

Terms: cash

250	lb. Rio Coffee	$0.24\frac{7}{8}$
450	" Mocha Coffee	$.20\frac{1}{8}$
172	doz. Eggs	$.14\frac{7}{8}$
990	lb. White Sugar	$.04\frac{3}{4}$
900	" Brown Sugar	$.03\frac{1}{8}$
975	" Granulated Sugar	$.06\frac{5}{8}$
172	" Butter	$.16\frac{1}{2}$
3021	" Ham	$.13\frac{1}{8}$
280	" Cream Codfish	$.07\frac{5}{8}$
11	pails Mackerel	$1.87\frac{1}{2}$
120	lb. Raisins	$.07\frac{5}{8}$
480	" Starch	$.03\frac{1}{8}$
225	" Japan Tea	$.26\frac{1}{2}$
210	" Young Hyson Tea	$.24\frac{7}{8}$
420	" Oolong Tea	$.27\frac{7}{8}$
157	" Pearl Tapioca	$.03\frac{1}{2}$
17	pkg. Yeast Cakes	$.37\frac{1}{2}$
375	lb. Java Coffee	$.23\frac{5}{8}$

26. C's salary is $17.50 per week of 48 hr. How much should he be paid for 11 da., working 9 hr. per day?

27. A man earning $2.75 per day of 10 hr. lost $7\frac{1}{2}$ hr. during one week of 6 da. How much should he receive for the week's work?

28. E begins work at 7:30 A.M. and quits work at 6:30 P.M. If he is paid at the rate of $3.75 per day of 8 hr. and he takes the noon hour off for lunch, how much should he receive for his day's labor?

29. A factory foreman is paid $3.75 per day of 8 hr. and $0.50 an hour for overtime. How much should he be paid for a week in which he begins work at 7 o'clock A.M., quits work at 7:30 o'clock P.M., and takes $1\frac{1}{2}$ hr. off each day for lunch?

30. Copy the following time sheet and find: (*a*) the total number of hours worked on each order; (*b*) the total number of hours worked each day; (*c*) the amount earned on each order; and (*d*) the total amount earned during the week.

BOSTON ELEVATED RAILWAY CO.

Time worked by *E. M. Doe*, during the week ending *Aug. 15.*

Rate per hour, *25 cents.* Occupation, *Lineman.*

Order No.	Sat.	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Total Hours	Amount	
420	$2\frac{1}{4}$		$4\frac{1}{2}$							
715			$2\frac{1}{4}$		$9\frac{1}{2}$					
960				$7\frac{1}{2}$						
318	$4\frac{3}{4}$		$1\frac{1}{4}$							
420				$2\frac{1}{4}$		$4\frac{1}{4}$	$2\frac{3}{4}$			
715						$4\frac{3}{4}$	$7\frac{1}{2}$			
Total hr.										

31. A foreman in a shoe factory receives \$5 per day and \$0.50 per hour for overtime. His time for two weeks is as follows: Monday, $10\frac{1}{2}$ hr.; Tuesday, 12 hr.; Wednesday, 8 hr.; Thursday, $8\frac{1}{2}$ hr.; Friday, $12\frac{1}{2}$ hr.; Saturday, 10 hr.; Monday, 11 hr.; Tuesday, $12\frac{1}{2}$ hr.; Wednesday, 10 hr.; Thursday, 8 hr.; Friday, $8\frac{1}{2}$ hr.; Saturday, $9\frac{1}{2}$ hr. How much should he be paid for the two weeks' work, assuming that a day's work is 8 hr.?

32. The following is a manufacturer's piece-labor ticket. Copy it and find the totals and amounts as indicated.

PIECE LABOR

Workman's No. 964 Name D. A. Fain

Week ending July 16, 1906. Examined by C. O. Love

Articles	M.	T.	W.	T.	F.	S.	Total	Price	Extensions		Amount	
Hours Labor	4	4	4		4			25¢				
Shelves	10		20		20	12		13½¢				
Hopper Caps		8						29½¢				
No. 4 Caps						10		18¾¢				

CHAPTER XIII

ALIQUOT PARTS

206. An **aliquot part** of a number is a part that will be contained in the number an integral number of times.

Thus, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10.

ORAL EXERCISE

1. How many cents in $\$\frac{1}{2}$? in $\$\frac{1}{4}$? in $\$\frac{1}{8}$? in $\$\frac{1}{6}$?

2. What aliquot part of \$1 is 25¢? 50¢? $6\frac{1}{4}$¢? $12\frac{1}{2}$¢?

3. Read aloud the following, supplying the missing terms: $16 \times 50¢ = 16 \times \$\frac{1}{2} = \frac{1}{2}$ of \$16; $16 \times 25¢ = 16 \times \$\frac{1}{4} = \frac{1}{4}$ of \$16; $16 \times 12\frac{1}{2}¢ = 16 \times \$$ —— = —— of \$16; $16 \times 6\frac{1}{4}¢ = 16 \times \$$ —— = —— of \$16.

4. Give a short method for finding the cost when the quantity is given and the price is 50¢; 25¢; $12\frac{1}{2}$¢; $6\frac{1}{4}$¢.

5. What is the cost of 160 yd. of dress goods at \$1? at 50¢? at 25¢? at $12\frac{1}{2}$¢? at $6\frac{1}{4}$¢?

6. How many cents in $\$\frac{1}{3}$? in $\$\frac{1}{6}$? in $\$\frac{1}{12}$? in $\$\frac{1}{15}$? in $\$\frac{1}{7}$? in $\$\frac{1}{5}$? in $\$\frac{1}{10}$?

7. What aliquot part of \$1 is $33\frac{1}{3}$¢? $16\frac{2}{3}$¢? $8\frac{1}{3}$¢? $6\frac{2}{3}$¢? $14\frac{2}{7}$¢? 20¢? 10¢?

8. Read aloud the following, supplying the missing terms: $140 \times 14\frac{2}{7}¢ = 140 \times \$\frac{1}{7} = \frac{1}{7}$ of \$140; $90 \times 6\frac{2}{3}¢ = 90 \times \$$ —— = —— of \$90; $90 \times 20¢ = 90 \times \$$ —— = —— of \$90.

9. Read aloud the following, supplying the missing terms: $240 \times 33\frac{1}{3}¢ = 240 \times \$$ —— $= \frac{1}{3}$ of \$240; $240 \times 16\frac{2}{3} = 240 \times \$\frac{1}{6} =$ —— of \$240; $240 \times 12\frac{1}{2}¢ = 240 \times \$$ —— = —— of \$240.

10. Give a short method for finding the cost when the quantity is given and the price is $33\frac{1}{3}$¢; $16\frac{2}{3}$¢; $8\frac{1}{3}$¢; $6\frac{2}{3}$¢; $14\frac{2}{7}$¢.

11. Find the cost of 960 yd. of cloth at $33\frac{1}{3}$¢; at $16\frac{2}{3}$¢; at $8\frac{1}{3}$¢.

ORAL EXERCISE

State the cost of:

1. 240 lb. tea at 50¢; at $33\frac{1}{3}$¢; at 25¢.
2. 360 lb. coffee at $33\frac{1}{3}$¢; at 25¢; at 20¢; at $12\frac{1}{2}$¢.
3. 720 gal. cider at $6\frac{1}{4}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
4. 2400 doz. eggs at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 20¢; at 25¢.
5. 2400 yd. prints at $8\frac{1}{3}$¢; at $6\frac{2}{3}$¢; at $6\frac{1}{4}$¢; at $12\frac{1}{2}$¢.
6. 960 yd. cotton at $6\frac{1}{4}$¢; at $8\frac{1}{3}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
7. 2040 yd. plaids at 50¢; at $33\frac{1}{3}$¢; at 25¢; at 20¢; at $16\frac{2}{3}$¢.
8. 480 lb. lard at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 10¢.
9. 3600 lb. raisins at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 20¢; at 25¢; at $33\frac{1}{3}$¢.
10. 480 yd. lining at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at 10¢; at $12\frac{1}{2}$¢; at $6\frac{2}{3}$¢.
11. 4200 yd. silesia at 10¢; at 20¢; at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at $14\frac{2}{7}$¢.
12. 1500 yd. plaids at $1; at 50¢; at $33\frac{1}{3}$¢; at 25¢; at 20¢.
13. 420 yd. stripe at 10¢; at $12\frac{1}{2}$¢; at $14\frac{2}{7}$¢; at $16\frac{2}{3}$¢; at 25¢.
14. 120 yd. gingham at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
15. 1240 yd. wash silk at 25¢; at 50¢; at $33\frac{1}{3}$¢; at 20¢.
16. At the rate of 3 for 50¢, what will 27 handkerchiefs cost?
17. At $33\frac{1}{3}$¢ per half dozen, what will 12 doz. handkerchiefs cost? 17 doz.? 25 doz.? $7\frac{1}{2}$ doz.? $4\frac{1}{2}$ doz.?
18. A merchant bought cloth at $33\frac{1}{3}$¢ per yard and sold it at 50¢ per yard. What was his gain on 1680 yd.?

ORAL EXERCISE

1. What is the cost of $12\frac{1}{2}$ yd. of silk at 96¢ per yard?

SUGGESTION. The cost of $12\frac{1}{2}$ yd. at 96¢ = the cost of 96 yd. at $12\frac{1}{2}$¢. *Interchanging the multiplicand and multiplier considered as abstract numbers does not affect the product.*

2. Find the cost of 25 yd. of silk at $1.72 per yard.

SUGGESTION. The cost of 25 yd. at $1.72 (172¢) = the cost of 172 yd. at 25¢.

3. Find the cost of:

a. 25 yd. at 16¢. *c.* $6\frac{1}{4}$ lb. at 32¢. *e.* 25 yd. at 84¢.

b. $12\frac{1}{2}$ yd. at 48¢. *d.* $12\frac{1}{2}$ lb. at 80¢. *f.* $12\frac{1}{2}$ yd. at $1.75.

TABLE OF ALIQUOT PARTS

Nos.	$\frac{1}{2}$'s	$\frac{1}{4}$'s	$\frac{1}{8}$'s	$\frac{1}{16}$'s	$\frac{1}{3}$'s	$\frac{1}{6}$'s	$\frac{1}{12}$'s	$\frac{1}{15}$'s	$\frac{1}{5}$'s	$\frac{1}{10}$'s
1	.50	.25	.12$\frac{1}{2}$	.06$\frac{1}{4}$	.33$\frac{1}{3}$	.16$\frac{2}{3}$	.08$\frac{1}{3}$	.06$\frac{2}{3}$	.20	.10
10	5.	2$\frac{1}{2}$	1$\frac{1}{4}$	.62$\frac{1}{2}$	3$\frac{1}{3}$	1$\frac{2}{3}$	.83$\frac{1}{3}$	.66$\frac{2}{3}$	2.	1.
100	50.	25.	12$\frac{1}{2}$	6$\frac{1}{4}$	33$\frac{1}{3}$	16$\frac{2}{3}$	8$\frac{1}{3}$	6$\frac{2}{3}$	20.	10.
1000	500.	250.	125.	62$\frac{1}{2}$	333$\frac{1}{3}$	166$\frac{2}{3}$	83$\frac{1}{3}$	66$\frac{2}{3}$	200.	100.

WRITTEN EXERCISE

In the three problems following make all the extensions mentally.

1. Without copying, find quickly the total cost of:

84 lb. tea at 50¢.
75 lb. tea at 33$\frac{1}{3}$¢.
72 lb. coffee at 25¢.
84 lb. coffee at 33$\frac{1}{3}$¢.
25 lb. coffee at 28¢.
88 lb. candy at 12$\frac{1}{2}$¢.
24 lb. tapioca at 6$\frac{1}{4}$¢.
6$\frac{1}{4}$ lb. tea at 64¢.
25 lb. cocoa at 52¢.
12$\frac{1}{2}$ lb. cocoa at 48¢.
360 lb. codfish at 6$\frac{2}{3}$¢.
66 lb. crackers at 8$\frac{1}{3}$¢.
25 lb. chocolate at 36¢.
25 cs. horseradish at 64¢.

2. Without copying, find quickly the total cost of:

25 yd. silk at 84¢.
12$\frac{1}{2}$ yd. silk at 96¢.
750 pc. lace at 6$\frac{2}{3}$¢.
112 yd. ticking at 6$\frac{1}{4}$¢.
210 yd. plaids at 33$\frac{1}{3}$¢.
128 gro. buttons at 12$\frac{1}{2}$¢.
68 yd. lansdowne at 50¢.
77 yd. duck at 14$\frac{2}{7}$¢.
6$\frac{1}{4}$ gro. buttons at 32¢.
155 yd. cheviot at 20¢.
96 yd. gingham at 8$\frac{1}{3}$¢.
84 yd. shirting at 12$\frac{1}{2}$¢.
25 doz. spools thread at 25¢.
168 yd. striped denim at 8$\frac{1}{3}$ ¢.

3. Without copying, find quickly the total cost of:

25 bu. corn at 64¢.
25 bu. corn at $0.72.
12$\frac{1}{2}$ bu. oats at $0.36.
25 bu. beans at $2.80.
12$\frac{1}{2}$ bu. wheat at $1.04.
12$\frac{1}{2}$ bu. millet at $1.24.
25 bu. clover seed at $3.60.
50 bu. clover seed at $3.75.
25 bu. corn at $0.84.
25 bu. corn at $0.44.
25 bu. oats at $0.35.
12$\frac{1}{2}$ bu. rye at $1.04.
6$\frac{1}{4}$ bu. wheat at $1.20.
6$\frac{1}{4}$ bu. wheat at $1.12.
25 bu. timothy seed at $2.40.
50 bu. timothy seed at $2.75.

ORAL EXERCISE

1. Multiply by 10: 4; 15; .07; 8¢; $1.12; $24.60; $12.125.

2. Multiply by 100: 3; 17; .09; 12¢; $1.64; $21.17.

3. Multiply by 1000: 7; 29; .19; 15¢; $1.75; $23.72.

4. What aliquot part of $10 is $2.50? Find the cost of 16 articles at $10 each; at $2.50 each.

5. Find the cost of 84 bu. of wheat at $1.25.

SOLUTION. $1.25 is $\frac{1}{8}$ of $10. 84 bu. at $10 = $840; $\frac{1}{8}$ of $840 = $105.

6. Formulate a short method for finding the cost when the quantity is given and the price is $1.25.

SOLUTION. $1.25 is $\frac{1}{8}$ of $10; hence, *multiply the quantity by 10 and take $\frac{1}{8}$ of the product.*

7. Formulate a short method for finding the cost when the quantity is given and the price is $2.50; 3.33\frac{1}{3}$; 1.66\frac{2}{3}$.

8. Find the cost of 168 yd. of cloth at $1.25; at $2.50; at 3.33\frac{1}{3}$; at 1.66\frac{2}{3}$.

9. What aliquot part of $100 is $25? $12.50? $6.25?

10. Find the cost of 72 chairs at $25 each.

SOLUTION. 72 chairs at $100 = $7200; but the price is $25, which is $\frac{1}{4}$ of $100; therefore, $\frac{1}{4}$ of $7200, or $1800, is the required cost.

11. Give a short method for multiplying any number by 25; by 12$\frac{1}{2}$; by 6$\frac{1}{4}$; by 33$\frac{1}{3}$; by 8$\frac{1}{3}$.

12. Find the cost of 25 T. coal at $7.20; of 6$\frac{1}{4}$ T.; of 12$\frac{1}{2}$ T.

13. What aliquot part of 1000 is 250? 500? 125? 62$\frac{1}{2}$? 333$\frac{1}{3}$? 166$\frac{2}{3}$? 200? 100? 83$\frac{1}{3}$? 66$\frac{2}{3}$?

14. Formulate a short method for multiplying a number by 250.

SOLUTION. Since $250 = \frac{1000}{4}$, *multiply by 1000 and take $\frac{1}{4}$ of the product.*

15. Give a short method for finding the cost when the quantity is given and the price is $125; 166\frac{2}{3}$.

16. Multiply 84 by 50; by 25; by 12$\frac{1}{2}$; by 16$\frac{2}{3}$; by 33$\frac{1}{3}$.

17. Multiply 160 by 2$\frac{1}{2}$; by 1$\frac{1}{4}$; by 12$\frac{1}{2}$; by 125; by 62$\frac{1}{2}$.

18. Multiply 240 by 3$\frac{1}{3}$; by 1$\frac{2}{3}$; by 33$\frac{1}{3}$; by 16$\frac{2}{3}$; by 333$\frac{1}{3}$.

19. Find the cost of 250 sofa beds at $32 each.

SOLUTION. The cost of 250 beds at $32 = the cost of 32 beds at $250. The cost of 32 beds at $1000 = $32,000; but the price is $250, which is $\frac{1}{4}$ of $1000; therefore, $\frac{1}{4}$ of $32,000, or $8000, is the required cost.

20. Find the cost of 720 couches at $12.50 each.

21. Find the cost of 440 lb. sugar at $2\frac{1}{2}¢$.

SOLUTION. $2\frac{1}{2}¢$ is $\frac{1}{4}$ of $10¢$. The cost of 440 lb. at $10¢ = $44; but the price is $2\frac{1}{2}¢$, therefore, $\frac{1}{4}$ of $44, or $11 = the required cost.

22. Formulate a short method for finding the cost when the quantity is given and the price is $1\frac{1}{4}¢$.

SOLUTION. $1\frac{1}{4}¢ = \frac{1}{8}$ of $10¢$; hence, *point off one place in the quantity and take $\frac{1}{8}$ of the result.*

23. Give a short method for finding the cost when the quantity is given and the price is $2\frac{1}{2}¢$; $3\frac{1}{3}¢$; $1\frac{2}{3}¢$.

24. Find the cost of 160 lb. at $2\frac{1}{2}¢$; at $1\frac{1}{4}¢$; at $3\frac{1}{3}¢$; at $1\frac{2}{3}¢$. Also of 240 lb. at each of these prices.

25. Find the cost of 2400 lb. at $2\frac{1}{2}¢$; at $1\frac{1}{4}¢$; at $3\frac{1}{3}¢$; at $1\frac{2}{3}¢$. Also of 360 lb. at each of these prices.

ORAL EXERCISE

By inspection find the cost of:

1. 25 lb. tea at 54¢.
2. 25 lb. tea at $33\frac{1}{3}¢$.
3. 125 lb. tea at 64¢.
4. $6\frac{1}{4}$ A. land at $112.
5. 25 T. coal at $8.40.
6. 25 T. coal at $5.20.
7. 18 T. coal at $6.25.
8. 164 A. land at $25.
9. 72 T. coal at $6.25.
10. 250 yd. silk at 88¢.
11. 250 yd. silk at 96¢.
12. 25 pc. lace at $6.60.
13. 250 yd. silk at $1.12.
14. 192 A. land at $12.50.
15. 165 gro. buttons at $33\frac{1}{3}¢$.
16. $1\frac{1}{4}$ yd. silk at 88¢.
17. 64 pc. lace at $1.25.
18. 125 yd. silk at $1.12.
19. 1250 bbl. beef at $24.
20. 78 yd. velvet at $2.50.
21. $2\frac{1}{2}$ bu. potatoes at 44¢.
22. 640 bu. apples at $12\frac{1}{2}¢$.
23. 840 yd. prints at $16\frac{2}{3}¢$.
24. $12\frac{1}{2}$ bu. potatoes at 64¢.
25. 84 bookcases at $12.50.
26. 810 bbl. pork at $12.50.
27. 125 yd. crepon at $3.60.
28. $12\frac{1}{2}$ yd. cheviot at $1.04.
29. 24 oak sideboards at $125.
30. $12\frac{1}{2}$ yd. gunner's duck at 48¢.

WRITTEN EXERCISE

In the following problems make all the extensions mentally. See how many of the problems can be done in 10 minutes.

1. Without copying, find the total cost of:

425 lb. at 10 ¢.	2500 lb. at 64 ¢.	24 lb. at $1\frac{1}{4}$ ¢.
310 lb. at 20 ¢.	1600 lb. at 25 ¢.	48 lb. at $2\frac{1}{2}$ ¢.
100 lb. at 14 ¢.	1893 lb. at $3\frac{1}{3}$ ¢.	$2\frac{1}{2}$ lb. at 96 ¢.
1000 lb. at 27 ¢.	2500 lb. at 14 ¢.	125 lb. at 24 ¢.
1000 lb. at $4\frac{1}{2}$ ¢.	1400 lb. at 25 ¢.	192 lb. at $3\frac{1}{3}$ ¢.
1250 lb. at 44 ¢.	1250 lb. at 88 ¢.	88 lb. at $12\frac{1}{2}$ ¢.

2. Without copying, find the total cost of:

88 yd. at $1\frac{1}{4}$ ¢.	174 yd. at 10 ¢.	24 yd. at 12 ¢.
72 yd. at $3\frac{1}{3}$ ¢.	123 yd. at 11 ¢.	78 yd. at $3\frac{1}{3}$ ¢.
104 yd. at $2\frac{1}{2}$ ¢.	127 yd. at 11 ¢.	165 yd. at 20 ¢.
480 yd. at $6\frac{1}{4}$ ¢.	246 yd. at 25 ¢.	114 yd. at $6\frac{2}{3}$ ¢.
360 yd. at $8\frac{1}{3}$ ¢.	171^2 yd. at 10 ¢.	1280 yd. at $6\frac{1}{4}$ ¢.
121 yd. at 11 ¢.	178^3 yd. at 10 ¢.	192 yd. at $33\frac{1}{3}$ ¢.

3. Copy and find the total cost of:

450 lb. at $1\frac{1}{2}$ ¢.	249 lb. at 25 ¢.	$6\frac{1}{4}$ lb. at 88 ¢.
820 lb. at $1\frac{1}{2}$ ¢.	240 lb. at $3\frac{1}{3}$ ¢.	92 lb. at $2\frac{1}{2}$ ¢.
1200 lb. at $4\frac{1}{4}$ ¢.	200 lb. at $3\frac{1}{2}$ ¢.	$12\frac{1}{2}$ lb. at 24 ¢.
1400 lb. at $6\frac{1}{2}$ ¢.	450 lb. at $6\frac{2}{3}$ ¢.	18 lb. at $4\frac{1}{2}$ ¢.
7961 lb. at 50 ¢.	$79\frac{1}{2}$ lb. at 40 ¢.	125 lb. at 18 ¢.
1293 lb. at 30 ¢.	$78\frac{1}{2}$ lb. at 50 ¢.	648 lb. at $6\frac{1}{4}$ ¢.
1480 lb. at 40 ¢.	750 lb. at $33\frac{1}{3}$ ¢.	1900 lb. at $4\frac{1}{4}$ ¢.

4. Copy and find the total cost of:

750 gal. at $8\frac{1}{3}$ ¢.	99 gal. at 30 ¢.	360 gal. at 5 ¢.
488 gal. at $6\frac{2}{3}$ ¢.	60 gal. at $6\frac{2}{3}$ ¢.	625 gal. at 64 ¢.
640 gal. at $6\frac{1}{4}$ ¢.	50 gal. at 76 ¢.	810 gal. at $1\frac{1}{2}$ ¢.
194 gal. at 50 ¢.	25 gal. at 74 ¢.	920 gal. at $2\frac{1}{2}$ ¢.
176 gal. at 25 ¢.	$12\frac{1}{2}$ gal. at 88 ¢.	165 gal. at $6\frac{2}{3}$ ¢.
280 gal. at $12\frac{1}{2}$ ¢.	79 gal. at $33\frac{1}{3}$ ¢.	240 gal. at $62\frac{1}{2}$ ¢.
720 gal. at $33\frac{1}{3}$ ¢.	20 gal. at $1.79.	666 gal. at $66\frac{2}{3}$ ¢.
366 gal. at $16\frac{2}{3}$ ¢.	$6\frac{1}{4}$ gal. at $1.96.	1680 gal. at $16\frac{2}{3}$ ¢.

ORAL EXERCISE

1. How much less than $1 is 75¢? what fractional part of $1 less?

2. Find the cost of 144 pc. of lace at 75¢ per piece.

SOLUTION. At $1 per piece the cost would be $144; but the cost is not $1 but $\frac{1}{4}$ less than $1. Deducting $\frac{1}{4}$ of $144, the result is $108, the required cost.

3. Find the cost of 124 bookcases at $7.50.

SOLUTION. $7.50 is $\frac{1}{4}$ less than $10. $1240 less $\frac{1}{4}$ of itself = $930, the required result.

4. Formulate a rule for multiplying a number by .75; by $7\frac{1}{2}$; by 75; by 750.

5. How much more than $1 is 1.12\frac{1}{2}$? What fractional part of $1 more?

6. Find the cost of 84 yd. of silk at 1.16\frac{2}{3}$ per yard.

SOLUTION. At $1 per yard, the cost would be $84; but 1.16\frac{2}{3}$ is $\frac{1}{6}$ more than $1. Adding $\frac{1}{6}$ of $84 to itself, the result is $98, the required cost.

7. Formulate a short method for finding the cost when the quantity is given and the price is 1.12\frac{1}{2}$; 1.16\frac{2}{3}$; 1.33\frac{1}{3}$; $11.25; $112.50.

8. How much less than $1 is 87$\frac{1}{2}$¢? what fractional part of $1 less? Formulate a short method for multiplying a number by 87$\frac{1}{2}$.

9. Formulate a short method for multiplying a number by .83$\frac{1}{3}$; by 1.25.

10. Compare the cost of 87$\frac{1}{2}$ yd. at 64¢ with the cost of 64 yd. at 87$\frac{1}{2}$¢.

ORAL EXERCISE

State the cost of:

1. 24 yd. at 75¢.
2. 75 yd. at 24¢.
3. 192 yd. at 87$\frac{1}{2}$¢.
4. 240 yd. at 83$\frac{1}{3}$¢.
5. 87$\frac{1}{2}$ yd. at $2.48.
6. 176 yd. at 1.12\frac{1}{2}$.
7. 87$\frac{1}{2}$ yd. at $2.88.
8. 25 yd. at 4¢.
9. 28 yd. at 75¢.
10. 27 yd. at 75¢.
11. 75 yd. at 84¢.
12. 75 yd. at 16¢.
13. 270 yd. at 11$\frac{1}{9}$¢.
14. 144 yd. at 11$\frac{1}{9}$¢.
15. 11$\frac{1}{9}$ yd. at 18¢.
16. 1125 yd. at 64¢.
17. 1125 yd. at 32¢.
18. 1125 yd. at 48¢.

WRITTEN REVIEW EXERCISE

1. Find the total of the costs called for in problems 1–15 in the oral exercise at the top of page 151.

2. Find the total cost of the items in the oral exercise at the bottom of page 154; of the items in the oral exercise at the bottom of page 156.

3. Find the total cost of:

84 yd. at 7¢.	98 yd. at 9¢.	72 yd. at 75¢.
112½ yd. at 5¢.	79 yd. at 11¢.	87½ yd. at 88¢.
112½ yd. at 6¢.	17 yd. at 16¢.	320 yd. at 11¢.

4. Find the total cost of:

71 yd. at 22¢.	85 yd. at 30¢.	30 yd. at 7½¢.
31 yd. at 44¢.	17 yd. at 25¢.	24 yd. at 8⅓¢.
82 yd. at 88¢.	12½ yd. at 39¢.	56 yd. at 83⅓¢.
71 yd. at 72¢.	250 yd. at 64¢.	124 yd. at $1.12½.

5. Find the total cost of:

192 lb. at 3⅓¢.	167 lb. at 12½¢.	1151¾ lb. at 10¢.
384 lb. at 6¼¢.	184 lb. at 37½¢.	1721½ lb. at 15¢.
378 lb. at 6¼¢.	2164 lb. at 2½¢.	2911½ lb. at 33⅓¢.
149 lb. at 6¼¢.	1369 lb. at 2½¢.	2706 lb. at 33⅓¢.

6. Copy and find the amount of the following bills, less 3%:

a.

Rochester, N.Y., Aug. 2, 19

Mr. C. G. Garlic
North Rose, N.Y.

To Smith, Perkins & Co., *Dr.*

Terms: cash, less 3%.

330 lb.	Granulated Sugar	6¼¢
32 "	Butter	22¢
64 "	Cheese	16⅔¢
75 "	Young Hyson Tea	24¢
155 "	Dried Apples	8¢
300 "	Brown Sugar	3½¢
60 "	Oolong Tea	51¢
125 "	Rio Coffee	28¢
250 "	Mocha Coffee	24¢

b.

Buffalo, N.Y., Aug. 5, 19

Mr. George A. Collier
Savannah, N.Y.

Bought of George H. Buell & Co.

Terms: cash, less 3%.

	72 pr. Boys' Hose	$12\frac{1}{2}¢$				
	18 doz. Linen Handkerchiefs	2.50				
	18 " Lace Handkerchiefs	$3.33\frac{1}{3}$				
	78 yd. Silk Velvet	$3.33\frac{1}{3}$				
	75 pc. Black Ribbon	$28¢$				
	347 yd. Pontiac Seersucker	$6\frac{1}{4}¢$				
	186 " Washington Cambric	$12\frac{1}{2}¢$				

ORAL EXERCISE

1. At $33\frac{1}{3}¢$ per pound, how many pounds of coffee can be bought for $12?

Solution. $.33\frac{1}{3} = \$\frac{1}{3}$; 3 pounds can be bought for $1; then, 12×3 lb. $= 36$ lb., the required result.

2. When the cost is given and the price is $25¢$, how may the quantity be found?

Solution. When the price is $25¢$, the quantity is 4 times the cost; hence, *multiply the cost by 4.*

3. Give a short method for finding the quantity when the cost is given and the price is $20¢$; $33\frac{1}{3}¢$; $12\frac{1}{2}¢$; $6\frac{1}{4}¢$; $6\frac{2}{3}¢$; $16\frac{2}{3}¢$.

4. Formulate a short method for dividing any number by 125.

Solution. 125 is $\frac{1}{8}$ of 1000; then the quotient by 125 will be 8 times the quotient by 1000. Therefore, *divide by 1000 and multiply the result by 8.* Or, $\frac{1}{125} = \frac{8}{1000}$. Therefore, *multiply by 8 and move the decimal point three places to the left.*

5. Give a short method for dividing by $6\frac{1}{4}$.

Solution. $6\frac{1}{4} = \frac{1}{16}$ of 100; then the quotient by $6\frac{1}{4}$ will be 16 times the quotient by 100. Therefore, *move the decimal point two places to the left and multiply the result by 16.* Or, $\frac{1}{6} = \frac{16}{100}$. Therefore, *multiply by 16 and move the decimal point two places to the left.*

6. Give a short method for dividing a number by $12\frac{1}{2}$; by $16\frac{2}{3}$; by $33\frac{1}{3}$; by $6\frac{1}{4}$; by $66\frac{2}{3}$; by $333\frac{1}{3}$; by $166\frac{2}{3}$.

7. Formulate a short method for dividing a number by .75.

SOLUTION. .75 increased by $\frac{1}{3}$ of itself = 1. When the divisor is 1 the quotient is the same as the dividend. Hence, to divide a number by .75 *increase the number by $\frac{1}{3}$ of itself.*

8. At 75¢ per bushel, how many bushels of wheat can be bought for \$144? for \$192? for \$240? for \$780? for \$1260? for \$360? for \$1350? for \$810?

9. At \$7.50 per dozen, how many dozen men's gloves can be bought for \$1440?

SOLUTION. \$7.50 + $\frac{1}{3}$ of itself = 10. To divide by 10 is to point off one place to the left. \$1440 + $\frac{1}{3}$ of itself = \$1920; \$1920 ÷ \$10 = 192, the number of pairs of gloves.

10. State a short method for dividing a number by $7\frac{1}{2}$; by 75; by 750.

ORAL EXERCISE

Find the quantity:

	Cost	Price per Yard		Cost	Price per Pound
1.	\$65	$33\frac{1}{3}$¢	**7.**	\$75	$6\frac{2}{3}$¢
2.	\$250	25¢	**8.**	\$12	$1\frac{2}{3}$¢
3.	\$120	$6\frac{1}{4}$¢	**9.**	\$25	$1\frac{1}{4}$¢
4.	\$215	$2\frac{1}{2}$¢	**10.**	\$38	$1\frac{1}{2}$¢
5.	\$126	$12\frac{1}{2}$¢	**11.**	\$125	\$1.25
6.	\$125	20¢	**12.**	\$420	$12\frac{1}{2}$¢

WRITTEN EXERCISE

Find the quantity:

	Cost	Price per Yard		Cost	Price per Bushel
1.	\$570.00	75¢	**6.**	\$1721.00	$33\frac{1}{3}$¢
2.	\$612.00	75¢	**7.**	\$1842.50	25¢
3.	\$274.50	$7\frac{1}{2}$¢	**8.**	\$1785.50	$87\frac{1}{2}$¢
4.	\$281.50	$12\frac{1}{2}$¢	**9.**	\$2142.00	$33\frac{1}{3}$¢
5.	\$864.50	$12\frac{1}{2}$¢	**10.**	\$2720.50	$16\frac{2}{3}$¢

CHAPTER XIV

BILLS AND ACCOUNTS

BILLS

207. A detailed statement of goods sold, or of goods bought to be sold, is called either a **bill** or an **invoice**. A detailed statement of goods bought to be used or consumed, such as office furniture, stationery, and fuel, or a statement of services rendered, or of a work performed, is called a **bill**.

Thus, a physician's statement of services rendered, or a transportation company's bill for work performed, and the charges for the same, is called a *bill;* but a statement of a quantity of silk bought or sold by a dry-goods merchant in the course of trade is called either a *bill* or an *invoice.*

208. The models following show a variety of current practices in billing. They will therefore be found helpful as studies.

1. GROCERIES

Boston, Mass., Oct. 15, **19**

Messrs. SMITH, PERKINS & CO.
Rochester, N.Y.

Bought of E. E. GRAY COMPANY

Terms 30 days **Telephone,** Main 167

3	bbl. Rolled Oats	$6.25	18	75		
10	" Gold Medal Flour	6.50	65	00		
5	bx. Wool Soap	3.10	15	50	99	25

This is one of the simplest bill forms; it is the form that is common in a great many lines of business.

2. Groceries

Boston, Mass., Nov. 12, 19

Messrs. E. O. Sherman & Co.
Charlestown, Mass.

Bought of **S. S. PIERCE COMPANY**

Terms 30 da.; 3% 10 da.

10 Red Label Hams	146	lb.	$0.23	$33.58
20 mats Java Coffee	1500	"	.25	375.00
12 6-lb. tins Mustard	72	"	.36	25.92
15 6-lb. tins Cocoa	90	"	.34	30.60
				$465.10

Goods bought by the mat, chest, case, etc., are frequently billed by the pound. The above bill shows the form in such cases.

3. Hardware

The following bill is sometimes used in the hardware business. The first number after the name of the article is the quantity; the number above the horizontal line following, the price; and the number below the line, the grade. Thus, the first item in the bill shows that 12 doz. porcelain knobs in all were sold, of which 6 doz. were No. 8 at $1.25 and 6 doz. No. 16 at $1.33⅓.

New York, Apr. 6, 19

Messrs. L. M. Shipman & Son,
Springfield, Mass.

Bought of *The Eureka Hardware Company*

Terms 3% 10 da.; net 30 da.

12	doz. Porcelain Knobs	6 $\frac{1.25}{\#8}$ 6 $\frac{1.33\frac{1}{3}}{\#16}$	15	50		
15	" Steelyards	3 $\frac{.50}{\#17}$ 12 $\frac{.25}{\#64}$	142	50		
25	" Jack Planes	5 $\frac{6.25}{\#10}$ 5 $\frac{6.00}{\#27}$ 15 $\frac{6.66\frac{2}{3}}{\#42}$	161	25	319	25

4. Wholesale Dry Goods

CHICAGO, Mar. 15, 19

Messrs. C. E. Boynton & Son
Buffalo, N. Y.

Bought of **MARSHALL FIELD & CO.**
Franklin Street and Fifth Avenue

TERMS 30 da.

124	12	pc. Humboldt Jeans 42 40 42½ 40 40⅜ 43½ 41 43 41½ 43½ 40½ 42½	500	33⅓%	166	67		
125	12	pc. Washington Cambric 40 37½ 40½ 46½ 37½ 43½ 39 32½ 40½ 41½ 39½ 40½	479	6⅔%	29	94		
129	12	pc. Cotton Flannel 40 39 42 40 42 40 42 41 42 41 43 43	495	8⅓%	41	25		
132	12	pc. Kellogg Opera Flannel 41 43 42 41 43 39 42 40 40 42 44 41	498	37½%	186	75		
127	12	pc. Monument Cotton 42 40 40 40 43 39½ 41 43 46 39 42 43½	499	6⅔%	33	27		
135	6	pc. Creton Plaids 37 41 39½ 41½ 37 42	238	11%	26	18		
141	6	pc. Whipcord 39 41½ 42½ 46 41 43	253	99%	250	47		
	25	doz. Whalebones		90%	22	50		
167	75	yd. Black Ribbon		75%	56	25		
	100	doz. spools Thompson Thread		37½%	37	50	850	78

In the wholesale dry-goods business the price is generally for a yard, and the number of yards to the piece varies in some kinds of cloth. The first item in the above bill is followed by a series of numbers, 41, 42, etc.; these represent the number of yards in each of the 12 pc. Immediately following these numbers is recorded the total number of yards in the 12 pc. The total number of yards should be found by horizontal addition.

5. Manufacturer's

The following is a bill for neckwear. The different styles are distinguished by the marks at the left of the quantity. This form is common among manufacturers, jobbers, and wholesalers. Bills on which trade discounts (see page 242) are allowed are arranged as shown in this bill.

New York, Oct. 10, 19

Messrs. J. E. Whiting & Co.
Boston, Mass.

Bought of Johnson Bros., Sons & Co.

Terms Net 30 days

721	$1\frac{1}{2}$	doz. Neckwear	$4.50	6	75		
1026	$\frac{1}{2}$		27.00	13	50		
1025	$1\frac{1}{2}$		27.50	41	25		
1020	$\frac{3}{4}$		9.00	6	75		
923	$2\frac{1}{2}$		18.00	45	00		
1015	$1\frac{3}{4}$		24.00	42	00		
				155	25		
		Less 2%		3	11	152	14

6. Furniture

In the following bill the goods were sold delivered on the cars (f. o. b.) Boston, but the shippers prepaid the freight to Bangor. The freight is a part of the selling price and is added to the amount of the bill, as shown in the model.

BOSTON, May 23, 19—

Messrs. J. D. Lyman & Bro.
Bangor, Me.

Bought of E. M. PRAY, SONS & CO.
Manufacturers of Fine Furniture

TERMS Net 30 da.

65	5	Turkish Rockers	24.25	123	75		
396	12	Mahogany Hall Stands	67.50	810	—		
626	10	Mahogany Office Chairs	17.25	172	50		
178	12	Roman Chairs	19.25	231	—		
				1337	25		
		Freight		12	10	1349	35

7. Wholesale Coal

F. H. OSBORN & CO.
SHIPPERS OF
Anthracite, Bituminous, and Gas Coal

Boston, Nov. 29, 19

Sold to C. W. Graham & Co.,
162 Causeway St., City

Terms Cash

		27800# Grate	4.25	59	08		
		26900# Egg	4.25	57	16		
		29600# Stove	4.50	66	60		
		29700# Lump	2.75	40	84	223	68
		Received payment, F. H. Osborn & Co. per O.					

The above is a form of bill that is generally used for wholesale transactions in coal. It shows that the coal has been paid for, and is called a **receipted bill.**

8. Retail Coal

Boston, Jan. 31, 19

Mr. Ira B. Chapin,
212 Huntington Ave., City

Terms Cash

Bought of **F. M. Everett & Co.**

Jan.	3	2 loads Grate Coal					
		6340-2140 6380-2180 8400#	5.00	21	—		
	18	2 loads Grate Coal					
		6380-2140 6400-2140 8500#	5.00	21	25	42	25
		Paid, F. M. Everett & Co. per W. E. G.					

On page 106 is a form of coal bill used by many retailers. The foregoing bill shows another form sometimes used by retailers. The numbers at the left of the hyphen are the gross weights, and the numbers at the right the tares of the different loads.

9. China and Glassware

Boston, Nov. 6, *19*

M THE WENTWORTH-STRATTON CO.
Rochester, N.Y.

Bought of Osgood, Traver & Son

Terms 60 da. net; 2% 10 da.

1	Dinner Set, 130 pieces; viz.:							
	1 doz. Plates, 8 in.			1 88				
	1 " " 7 "			1 63				
	1 " " 6 "			1 38				
	1 " " 7 " (deep)			1 63				
	1 " Fruit Saucers, 4 in.			75				
	1 " Individual Butters			50				
	1/12 doz. Covered Dishes, 8 in.	$12.00		1 00				
	1/12 " Casseroles, 8 in.	13.50		1 13				
	1/4 " Dishes, 8 in.	2.50		63				
	1/12 " " 10 "	4.50		38				
	1/12 " " 12 "	7.50		63				
	1/12 " " 14 "	10.50		88				
	1/6 " Bakers, 8 in.	4.50		75				
	1/12 " Sauce Boats	4.00		33				
	1/12 " Pickles	3.00		25				
	1/12 " Bowls	2.00		17				
	1/12 " Sugars	6.00		50				
	1/12 " Creams	2.79		23				
	1 " Handled Teas			2 00				
	1/2 " " Coffees	2.33		1 17				
	1/12 " Pitchers	6.00		50				
	1/12 " Covered Butters and Drainers	9.00		75		19	07	
25	more Dinner Sets as above	19.07				476	75	
						495	82	
	Crates					7	50	
	Carting					2	10	505 42

The above form is common in the china and glassware business. In this case a charge is made for the crates used in packing and the prices do not include delivery. The cost of the crate and the cost for carting are therefore made a part of the bill.

10. LUMBER

The H. M. Bickford Co.

Boston, Mass., Oct. 8, *19*

Sold to L. A. Hammond & Co.
Paterson, N.J.

Terms Fgt. net cash; bal. in 5 da. less $1\frac{1}{2}\%$

23,289 ft.	$\frac{1}{2} \times 2\frac{1}{2}$	#1	N. C.	Ceiling	$18.50	$430.85
3,520 "	"	2	"	"	17.00	59.84
10,307 "	$\frac{3}{8} \times 2\frac{1}{2}$	1	"	"	13.50	139.14
1,690 "	"	2	"	"	12.50	21.13
						$650.96
Less freight (45,200 lb. at 24¢)						108.48
						$542.48

Lumber is generally sold by the thousand feet. In the above bill the goods were sold free on board cars (f. o. b.) Paterson, N.J., but the shippers have not prepaid the freight. They find that these charges are $108.48 and deduct this amount from the total of the bill. In the wholesale lumber business the prices quoted usually include the cost of delivery, and when the freight charges are not known at the time of making the shipment, they are paid by the consignees and deducted from the amount of the bill on the arrival of the goods. The freight bill is then sent to the shippers for credit.

WRITTEN EXERCISE

1. Study the model bill, page 160. Increase the price of each article 25¢ and then copy and find the amount of the bill.

2. Study the first model bill, page 161, and then copy and find the amount of it at the following prices: hams, 27¢; coffee, 23¢; mustard, 31¢; cocoa, 39¢.

3. Study the second model bill, page 161, and then copy and find the amount of it at the following prices: porcelain knobs #8, $1.12\frac{1}{2}$; #16, $1.25; steelyards #64, $11; #17, $8.33\frac{1}{3}$; jack-planes #14, $6; #21, $6.25; #48, $6.75.

4. Apr. 15, you bought of S. S. Pierce Co., Boston, Mass., for cash: 25 gal. finest New Orleans molasses at 48¢; 15 gal. fancy sugar-house sirup at 49¢; 75 lb. raw mixed coffee at 29¢; 25 lb. raw Pan-American coffee at 19¢; 5 cartons Fowle's entire-wheat flour at 39½¢; ½ bbl. Franklin Mills flour at $6.75; ⅛ bbl. pastry flour at $5.25. Write the bill.

5. Mar. 19, Frank M. Richmond & Co., New York City, sold to Charles M. Thompson, Poughkeepsie, N.Y., 12 doz. porcelain knobs: 3 doz. #71 at $6.35, 9 doz. #74 at $6.75; 12 doz. shingle hatchets: 6 doz. #16 at $9.75, 6 doz. #34 at $12.50; 7 doz. steel squares: 3 doz. #91 at $35, 4 doz. #73 at $33. Terms: 30 da. Write the bill.

6. Study the model bill on page 162. Increase the prices of the articles marked 124 and 132 five cents each and the remainder of the articles one cent each; then copy and find the amount of the bill.

7. Nov. 15, J. B. Ford & Co., Albany, N.Y., bought of the Clinton Mills, Little Falls, N.Y., 10 pc. percale shirting containing 42, 48, 52[1], 58, 62, 38, 49, 51, 54, and 46[1] yd., at 7½¢; 10 pc. fine wool cheviot containing 58[1], 42, 49, 51, 44[2], 46, 48, 41[2], 39, and 42 yd., at $1.12½; 5 pc. cashmere containing 49[3], 40[1], 48[3], 49[1], and 49 yd. at $1.37½. Terms: 60 da., or 3% discount for cash within 10 da. Write the bill.

8. Study the first model bill on page 163. Increase the prices of styles 1026, 1025, 1020, and 923, 25¢ each and diminish the prices of the other styles 25¢ each; then copy and find the amount of the bill.

9. Sept. 24, Geo. W. Fairchild, Buffalo, N.Y., bought of E. M. Lawrence & Co., New York City, silk ribbon as follows: 12 pc. #1142 at $2.25; 5 pc. #1321 at $1.25; 25 pc. #171 at $4.37½; 8 pc. #1927 at $1.75; 36 pc. #2114 at $1.66⅔; 15 pc. #1371 at $1.33⅓; 15 pc. #624 at $4.37½; 12 pc. #909 at $1.87½; 25 pc. #1008 at $3.33⅓; 25 pc. #1246 at $4.75; 18 pc. #2119 at $1.12½. Terms: 30 da., or 2% discount for cash in 10 da. Write the bill.

10. Study the second model bill on page 163. Increase the price of the articles marked 65 and 396, 25¢ each, and diminish the price of the other articles $12\frac{1}{2}$¢ each; then copy and find the amount of the bill. Freight added, $14.70.

11. July 20, The Hayden Furniture Co., Rochester, N.Y., bought of John H. Pray & Son, Boston, Mass., 25 #31 card tables at $11; 24 #94 china closets at $27.50; 15 #16 dining sets at $85; 25 #3060 fancy rockers at $9.25; 15 #35 music cabinets at $2.75; 25 #26 mahogany office chairs at $12.50; 12 #89 oak sideboards at $125. Terms: 30 da. The prices are free on board Boston, and the shipper prepaid the freight, $34.50. Write the bill.

12. Study the first model bill on page 164. Increase the price of the stove coal 25¢ per ton and the price of each of the other kinds $12\frac{1}{2}$¢ per ton; then copy and find the amount of the bill. Receipt the bill for F. H. Osborn & Co.

13. May 19, C. E. Williams & Co., Cleveland, O., bought of Fairbanks & Co., Scranton, Pa.: 3 car loads of stove coal weighing 20,500, 26,400, and 25,600 lb., respectively, at $4.75 per ton (2000 lb.); 1 car load grate coal weighing 21,900 lb. at $4.25 per ton; 1 car load cannel coal weighing 22,500 lb. at $7.75 per ton. Terms: 30 da., or 3% discount for cash in 10 da. Write the bill.

14. Study the second model bill, page 164, then copy and find the amount of it at $6.25 per ton for each sale.

15. Copy the bill in problem 14 in accordance with the model shown on page 106. Make the price of the coal $6.66\frac{2}{3}$.

16. Study the model bill on page 165. Increase each price given five cents and then copy and find the amount of the bill. Cost of crates used in packing, $6.40; carting, $2.80.

17. July 15, Henry Nelson & Co., Portland, Me., bought of Jones, Stratton & Co., New York City, 5 doz. plates, 8 in., at $1.50; 35 doz. plates, 7 in., at $1.35; 15 doz. plates, 6 in., at $1.10; 10 doz. plates, 5 in., at 90¢; 65 doz. handled teas at $1.85 Terms: 30 da. Cost of crate used in packing, $2; cartage, 75¢. Write the bill.

18. June 25, F. E. Winter & Co., Batavia, N.Y., bought of E. M. Page & Co., Chicago, Ill., provisions as per problems 3, 4, and 5, page 40. Terms: note at 60 da. Write the bill, using current prices.

Find the net weight of each quantity as explained in §§ 60–62.

19. Jan. 1, John P. Alven, 100 Vine St., bought of E. E. Gray Co., Boston, Mass., 2 lb. *café des invalides* at 38¢; 2 gal. maple sirup at $1.35; 1 pkg. magic yeast at 5¢; 5 cartons Fowle's entire-wheat flour at 22¢; 3 cartons Franklin Mills flour at 23¢; 16 lb. pastry flour at $3\frac{1}{2}$¢; 5 gal. fancy sugar-house sirup at 56¢; 5 gal. dark molasses at 41¢; $6\frac{1}{2}$ lb. red frosting sugar at 12¢; $7\frac{1}{2}$ lb. rock-candy crystals at $9\frac{3}{4}$¢; 3 lb. C. & B. coffee extract at 25¢; 1 lb. postum cereal at 22¢; 2 lb. Chance's bread soda at 10¢; 3 lb. cream tartar at 40¢; $1\frac{1}{2}$ lb. Pyle's saleratus at 8¢; 50 lb. granulated sugar at $5\frac{1}{2}$¢; 10 lb. powdered sugar at $5\frac{3}{4}$¢; 5 lb. cut-loaf sugar at $6\frac{1}{2}$¢; 5 gal. finest P. R. molasses at 59¢; 5 gal. finest N. O. molasses at 61¢; $1\frac{1}{3}$ doz. bottles maple sirup at $3.75. Write the bill.

20. Study the model bill on page 166. Increase each price 50¢, make the freight charge 28¢ per hundred pounds, and then copy and find the net amount of the bill.

21. Nov. 1, J. B. Bickwell & Co., Worcester, Mass., bought of the Northern Lumber Co., St. Johnsbury, Vt., on 60 days' credit: 3 M extra spruce clapboards at $52.50; 25 M lath at $3.75; 1500 ft. 2″ choice cypress lumber at $65 per M; 1200 ft. 2″ spruce at $23 per M; 750 ft. rift hard pine at $65 per M; less freight, $42.50. Write the bill.

22. June 15, Helen M. Stone, Cambridge, Mass., sends Frank M. Spaulding a bill for tuition and supplies to date as follows: tuition, one term, $37.50; music lessons, $9.75; 1 Practical Elements of Elocution, $1.65; 1 Allen & Greenough's Cæsar, $1.35; 1 Allen & Greenough's Cicero, $1.55; 1 Myer's General History, $1.65. Write the bill and receipt it for Helen M. Stone.

STATEMENTS

FOLIO 72

Albany, N.Y. Feb. 1, 19__

Mr. C. B. McMenimen,
St. Louis, Mo.

In account with Baker, Taylor & Co.

Jan.	5	To mdse.	300	10		
	12	" "	599	90		
	25	" "	97	10	997	10
	16	By cash	300	10		
	30	" "	371	30	671	40
					325	70

209. A **statement** is an abstract of a customer's account, showing under proper dates the details and totals of debits and credits and the balance remaining unpaid.

FOLIO 72

Albany, N.Y. Mar. 1, 19__

Mr. C. B. McMenimen,
St. Louis, Mo.

In account with Baker, Taylor & Co.

Feb.	1	To account rendered	325	70		
	8	" mdse.	300	60		
	12	" "	460	20	1086	50
	9	By mdse. returned	12	40		
	15	" cash	300	—		
	27	" "	361	90	674	30
					412	20

The first model on the preceding page is a statement of C. B. McMenimen's account for January. It shows that the charges aggregate $997.10, the credits $671.40, and that the balance remaining unpaid is $325.70.

The second model on the preceding page is a statement of C. B. McMenimen's account for January and February. The items on the January statement are summarized in the record "To account rendered, $325.70." The first item on the March statement will be "To account rendered, $412.20."

WRITTEN EXERCISE

1. During March, F. E. Smith, Buffalo, N.Y., bought merchandise of The Hayden Furniture Co., Rochester, N.Y., as per bills rendered: namely, Mar. 3, $400.80; Mar. 15, $360.90; Mar. 20, $200.70; Mar. 26, $260.90; Mar. 28, $130.50. During the same time he made cash payments on account as follows: Mar. 15, $400.80; Mar. 23, $360.90. On Mar. 27 he also returned goods for credit amounting to $18.60. Render a statement of F. E. Smith's account.

2. During April the above account was charged for merchandise as follows: Apr. 15, $720.50; Apr. 27, $260.90. The account was also credited for cash as follows: Apr. 16, $200.70; Apr. 28, $100.00. Render the April statement.

3. Copy and find the balance of the following statement:

Boston, Mass., Feb. 1, 19

MRS. C. M. SHERMAN
931 BEACON ST., City

In account with SPENCER, MEAD & CO.

Jan.	1	Account rendered		13	64		
	3	2 pr. Gloves	2.50				
		3 yd. Velvet	3.75				
		12 " Black Silk	2.10				
	12	6 pr. Hose	35¢				
		2 Hats	9.00				
		30½ yd. Muslin	12½¢				
		Cr.					
	5	2 pr. Gloves	2.50				
	15	1 Hat	9.00				

PAY ROLLS

PAY ROLL For the week ending Sept. 18, 190

No.	NAME	Number of Hours' Work Each Day M	T	W	T	F	S	Total No. of Hours	Wages per Hour	Total Wages		REMARKS
1	John W. Alden	9	9	9	9	9	9	54	25¢	13	50	
2	C. E. Brown	8½	9½	8½	8½	8	8	51	25¢	12	75	
3	Fred W. Dunn	9½	10	9½	10	10	10	59	35¢	20	65	
4	N. F. Good	7¼	9¼	9½	9¼	10	9¼	54¼	33⅓¢	18	08	
5	Frank M. Innes	8	8	8	8	8	8	48	35¢	16	80	
6	J. P. Kaplan	9	9	9	9	9	9	54	37½¢	20	25	
7	D. N. Martin	7	8	9	9½	9½	9¼	52	35¢	18	20	
8	Fred Reed	8	9	7	8	7½	8½	48	27½	13	20	
9	A. L. Spaulding	10	8	9	9	9	9	54	18¾	9	84	
10	James Spaulding	10	7½	9½	7	9	9	52	21¢	10	92	
		9¼	8½	8½	8½	8¾	8¾	526¼		154	19	

This form is most common among manufacturing establishments, but it is also used by printers, contractors, and builders.

Checks are sometimes used in paying off employees, but most large concerns find the envelope system the most convenient and satisfactory. To pay off employees by the envelope system it is necessary for the bookkeeper to find first the amount of money required and then the bills and fractional currency that are necessary to pay each employee. The amount required is the total of the pay roll, and the bills and fractional currency desired may be found as shown in the following illustration. This illustration, called a **change memorandum**, shows the method of finding just the denominations wanted for the pay roll at the top of the page. A change memorandum may be proved correct as shown in the **pay-roll memorandum** at the top of page 173.

No.	BILLS					COINS				
	$20	$10	$5	$2	$1	50¢	25¢	10¢	5¢	1¢
1		1		1	1	1				
2		1		1		1	1			
3	1					1		1	1	
4		1	1	1	1				1	3
5		1	1		1	1	1		1	
6	1						1			
7		1	1	1	1			2		
8		1		1	1			2		
9			1	2		1	1		1	4
10		1				1	1	1	1	2
	2	7	4	7	5	6	5	6	5	9

When the amount of the pay roll and the necessary bills and fractional currency have been determined, a check payable to the order of *Pay Roll* is written. A pay-roll memorandum similar to the accompanying form is then attached to the check and both are sent to the bank. The pay-roll memorandum should foot the same as the pay-roll book, and is therefore a check upon the correctness of the change memorandum.

FIRST NATIONAL BANK
Westfield, Mass.
PAY-ROLL MEMORANDUM
NELSON & CO.
require the following:

Pennies	9		09
Nickels	5		25
Dimes	6		60
Quarters	5	1	25
Halves	6	3	00
Dollars	5	5	00
2's	7	14	00
5's	4	20	00
10's	7	70	00
20's	2	40	00
		154	19

In a large pay roll the adept bookkeeper frequently *estimates* the kind of change required. This is done by scanning the pay roll first to find the number of pennies required, then the number of nickels, etc. The experienced book-keeper can make a very accurate estimate.

PAY ROLL For the week ending Nov. 6, 190

No.	NAME	Time in hours M	T	W	T	F	S	Total time	Rate per hour	Amount advanced	Amount due	Bills and silver necessary $20	$10	$5	$2	$1	50c	25c	10c	5c	1c	REMARKS
1	Ralph Roe	9	9	9	9	9	9	54	44¢		24 —	1			2							
2	Emma Tyng	9	9	9	9	9	9	54	44¢		24 —	1			2							
3	John Walsh	8	9	8	9	9	9	52	35¢		18 20		1	1	1	1			2			
4	Edward Laine	8½	8½	9	9	9	9	53	35¢		18 55		1	1	1	1	1			1		
5	Dora Small	9½	9	8½	9	8½	9½	54	22½		12 —		1		1							
6	Ellen Dale	9	8½	8½	8½	9	9½	53	33⅓	5 —	12 67		1		1		1		1	1	2	
7	J. Cox	9	9	9	9	9	9	54	35¢		18 90		1	1	1	1	1	1	1	1		
8	Charles Frey	8	9	9	9	9	9	53	25¢		13 25		1		1	1		1				
9	Benj. Green	9	9	9	9	9	9	54	18⅔		9 84			1	2		1	1		1	4	
10	F. M. Sperry	8	7½	12	12	9	9	57½	22½		12 78		1		1		1	1			3	
11	Ada Neal	10	10	10	9	8	8	55	14¢		8 13			1	1	1			1		3	
12	Ora Snell	9	9	9	8	8	8	51	35¢		17 85		1	1	1		1	1	1			
13	E. H. Spring	9	9	8	9	9	9	53	30¢		15 90		1	1			1	1	1	1		
14	Sam'l Dobson	9	9	9	9½	9½	8	54	22½		12 —		1		1							
		124	124	127	128	124	124	751½		5 —	218 07	2	10	7	16	5	7	6	7	5	12	

40 100 35 32 5 35 10 7 5 7 5 2 = 218 07

WRITTEN EXERCISE

1. Study the model pay roll, page 172, and find the amount of it at the following wages per hour: #1, 18¢; #2, 21$\frac{2}{9}$¢; #3, 25¢; #4, 35¢; #5, 33$\frac{1}{3}$¢; #6, 35¢; #7, 37$\frac{1}{2}$¢; #8, 35¢; #9, 27$\frac{1}{2}$¢; #10, 18$\frac{2}{9}$¢. Make a change memorandum.

2. Study the model pay roll on page 173, and then find the amount of it at the following wages per hour: #1, 50¢; #2, 45¢; #3, $33\frac{1}{3}$¢; #4, 35¢; #5, $27\frac{1}{2}$¢; #6, $37\frac{1}{2}$¢; #7, 25¢; #8, $33\frac{1}{3}$¢; #9, $44\frac{4}{9}$¢; #10, $22\frac{2}{9}$¢; #11, $22\frac{2}{9}$¢; #12, $14\frac{2}{7}$¢; #13, $12\frac{1}{2}$¢; #14, 30¢.

3. Make a pay roll memorandum from problem 2.

WRITTEN REVIEW EXERCISE

1. Find the amount of each of the following bills:

New York, May 31, *19*

MESSRS. GRAY, SALISBURY & CO.

Rochester, N.Y.

Bought of J. E. PAGE, SONS & CO.

Terms: net, 60 da.; 2% 10 da.

CASE	PIECES	DESCRIPTION OF ARTICLES	YDS.	PRICE	ITEMS	AMOUNT
#364	10	Velveteen				
		42^1 40 40 46 38^1				
		40 42 42 41 39		25¢		
#359	12	Corduroy				
		36 38^1 39^2 42 41^2 39^2				
		37 37 41 45 41 40^1		$66\frac{2}{3}$¢		
#371	15	Gray Homespun				
		39 38 35 42 41				
		45 39 41 34 37				
		41 40 41 38 42^3		$83\frac{1}{3}$¢		
#360	6	Storm Serge				
		40 42^1 43 42 39 42^1		44¢		
#373	24	Fine English Serge				
		42 38 42 42 40^2 42^1				
		40 39 40 41 40^1 43				
		42 42 38^2 38 41 42				
		43 44 41 40 37^1 37		$1.37\frac{1}{2}$		
#381	24	Groveland Flannel				
		32 40 39 42 41 45				
		45 46 35 41 38 41				
		37 42 43 40 37 42				
		37 40 42 41 44 41		$33\frac{1}{3}$		

2. Make out a bill for the following order. Bill the English breakfast tea at 41¢; Finest oolong tea at 65¢; Young Hyson tea at 97½¢; Choice Japan tea at 59¢; Orinda kaughphy at $1.90; raw Java coffee at 30½¢; gluten flour at 30¢ a carton and $7.75 per barrel. Assume that half a chest of tea weighs 75 lb., and a mat of coffee 70 lb.

E. M. BARBER & SON

RETAIL GROCERS

Springfield, Mass., Aug. 13, 19

S. S. Pierce Company,
Boston, Mass.

Gentlemen:

Please ship us via B. & A. R.R., the following goods:

3 hf. cht. English Breakfast Tea
3 " " Finest Oolong Tea
5 " " Young Hyson Tea
25 lb. Choice Japan Tea
5 5-lb. cans Orinda Kaughphy
7 mats Raw Java Coffee
5 hf. bbl. Gluten Flour
25 5-lb. cartons Gluten Flour

Respectfully yours,

E. M. Barber & Son.

3. Boston, Mass., Apr. 16, E. O. Burrill, Philadelphia, Pa., bought of Jones, Talcott & Co., on account, 30 da., 25 Turkish rugs $4\frac{1}{2} \times 7$ at $10.25; 750 yd. matting at 55¢; 225 yd. linoleum at 27½¢; 25 Turkish rugs $8\frac{1}{2} \times 12$ at $21.75; 25 Persian rugs 6×9 at $12.25; 12 Persian rugs 7×11 at $16.25; 10 rolls, each containing 150 yards, Brussels carpeting at $2.25; 275 yd. Moquette carpeting at $1.75. Find the amount of the bill.

4. Fill the following order: English breakfast tea, 47¢; Formosa oolong tea, $62\frac{1}{2}$¢; Japan tea, $62\frac{1}{2}$¢; Ceylon Pekoe tea, $90\frac{5}{8}$¢. 70 lb. to each half-chest.

THE WESTERN TELEGRAPH COMPANY

INCORPORATED

21,000 OFFICES IN AMERICA CABLE SERVICE TO ALL THE WORLD

Receiver's No.	Time Filed	Check

SEND the following message subject to the terms of the Company, which are hereby agreed to

Newton, Mass, May 16, 190_.

To E. E. Gray Co.,
New York City.

Express	one	half-chest	each	English
Breakfast	Formosa	Oolong	Japan	and
Ceylon	Pekoe			

Hart, Taylor & Hart.

5. You sold Shepard, Farmer & Co., the following: 5 M extra cedar shingles at $3.50; 15 M clear cedar shingles at $3.00; 20 M extra spruce clapboards at $45.00; 15 M clear spruce clapboards at $43.00; 1230 ft. random hemlock boards at $13.00; 2760 ft. planed spruce boards at $19 00; 2090 ft. rough spruce boards at $16.50; 18 M spruce lath at $3.25; 6493 ft. $1 \times 4''$ rift flooring at $26.00. Write the bill.

6. Copy and complete the following time card:

Time worked by *C. E. Small*, for the week ending *Aug. 13.*
Rate per hour, $29\frac{1}{2}$¢. Occupation, *Painter.*

No.	Hours Worked							Total Hours for Each Account	Amount for Each Account	
	Sat.	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.			
501	$2\frac{3}{4}$		$4\frac{1}{4}$							
724			$2\frac{3}{4}$		$9\frac{1}{2}$					
1029	$4\frac{1}{2}$			$8\frac{1}{4}$						
476	$1\frac{1}{2}$			$2\frac{1}{4}$						
910						$10\frac{3}{4}$	$9\frac{1}{2}$			
735						$1\frac{1}{2}$	$\frac{1}{2}$			
Check										

TIME SLIP

Friday, 4/26, 1906

	IN	OUT	IN	OUT
1	651	1159	1256	459
2	645	1159	1257	459
3	644	1159	1232	459
4	700	1159	1257	459
5	700	1159	1259	459
6	640	1159	1259	459
7	756	1159	1259	459
8	759	1159	104	506
9	756	1159	1255	459

TIME SLIP

Saturday, 4/27, 1906

	IN	OUT	IN	OUT
1	753	1200	1258	459
2	703	1204	1256	504
3	753	1150	1256	504
4	655	1159	1259	459
5	655	1159	1259	459
6	701	1159	1255	459
7	654	1150	1259	459
8	654	1158	1259	459
9	654	1159	1254	503

The above slips show an actual record of time for 9 employees for 2 da. in a large printing establishment. These records are made by a large mechanical timekeeper and at convenient times are copied in the pay-roll book. Fractions are recorded to the nearest $\frac{1}{4}$ of an hour. In the above slips, the time each employee arrived in the morning is recorded in the first column, the time each went away at noon in the second, the time each returned at noon in the third, and the time each went away in the afternoon in the fourth. Thus, #1 arrived at 7:53, Saturday, went away at 12:00, returned at 12:58 and worked until 4:59; time, 8 hr.

7. Copy the following pay roll, enter the time for Friday and Saturday (from the above slips), find the amount of the pay roll as in previous exercises, and make a change memorandum and a pay-roll memorandum.

PAY ROLL For the Week Ending April 27, 1906

No.	Name	Number of Hours' Work Each Day						Total No. of Hours	Wages per Hour	Total Wages		Remarks
		M.	T.	W.	T.	F.	S.					
1	A. B. Comer	9	8	9	9				$55\frac{5}{9}¢$			
2	W. D. Ball	9	9	9	$9\frac{1}{4}$				$44\frac{4}{9}¢$			
3	A. M. Snow	9	8	8	$9\frac{1}{4}$				$44\frac{4}{9}¢$			
4	R. O. Mark	8	9	9	9				$33\frac{1}{3}¢$			
5	Miss Mary Cane	9	$8\frac{1}{2}$	9	9				$33\frac{1}{3}¢$			
6	Miss Ellen Kyle	8	$8\frac{1}{2}$	9	9				$35¢$			
7	D. M. Garson	9	$8\frac{1}{2}$	8	$9\frac{1}{4}$				$35¢$			
8	S. D. Lane	9	$8\frac{1}{2}$	$8\frac{1}{4}$	9				$25¢$			
9	Miss Cora Knapp	9	9	$8\frac{1}{4}$	8				$22\frac{2}{9}¢$			

EXPRESSAGE AND FREIGHTAGE

WRITTEN EXERCISE

1. I wish to express five separate packages from Boston, Mass., to Cincinnati, O. The rate per 100 lb. is quoted at $2.00. If the packages weigh 15 lb., 73 lb., 86 lb., 126 lb., and 29 lb., respectively, what will be the express charge?

Small packages are usually sent by express. The charge varies with the distance and is stated at so much per 100 lb. The following table shows the rate for smaller weights, when the rate per hundred pounds is $2.00, $2.50, $3.00, $3.50, $4.00, and $4.50:

CHARGES FOR PACKAGES WEIGHING LESS THAN 100 POUNDS

WHEN THE RATE IS:												
$2.00		$2.50		$3.00		$3.50		$4.00		$4.50	
1 lb.	$ .25	1 lb.	$ .25	1 lb.	$ .25	1 lb.	$ .25	1 lb.	$ .25	1 lb.	$ .30
2	.35	2	.35	2	.35	2	.35	2	.35	2	.35
3	.45	3	.45	3	.45	3	.45	3	.45	3	.45
4	.50	4	.55	4	.60	4	.60	4	.60	4	.60
5	.55	5	.60	5	.65	5	.70	5	.70	5	.75
7	.60	7	70	7	.75	7	.80	7	.85	7	.90
10	.70	10	.75	10	.80	10	.90	10	1.00	10	1.00
15	.75	15	.85	15	.90	15	1.00	15	1.10	15	1.15
20	.85	20	1.00	20	1.10	20	1.20	20	1.25	20	1.30
25	1.00	25	1.10	25	1.20	25	1.30	25	1.50	25	1.50
30	1.00	30	1.15	30	1.30	30	1.50	30	1.60	30	1.70
35	1.00	35	1.25	35	1.40	35	1.60	35	1.70	35	1.90
40	1.00	40	1.25	40	1.50	40	1.75	40	1.85	40	2.00
45	1.00	45	1.25	45	1.50	45	1.75	45	2.00	45	2.25
50	1.00	50	1.25	50	1.50	50	1.75	50	2.00	50	2.25

Pound rates (2¢, 2½¢, 3¢, etc.) are charged for everything over 50 lb. Weights between those named in the table are charged at the rate for the next higher weight.

2. The express charge from Boston to Chicago is quoted at $2.50 per hundred pounds. Find the express charges on four separate packages, weighing 47 lb., 16 lb., 12 lb., and 15 lb., respectively, sent from Boston to Chicago.

3. A publisher sent a package of books by express, C. O. D., from Boston to Detroit. The rate is quoted at $2.00 per 100 lb. If the books are worth $75 and weigh 56 lb., how much should the express company collect, expressage included?

4. The express rate from Lake View, Mich., to Boston is quoted at $3.00 per 100 lb. Find the amount of express to pay this distance on 10 pkg., weighing 12 lb., 10 lb., 9 lb., 21 lb., 27 lb., 34 lb., 86 lb., 121 lb., 127 lb., and 54 lb., respectively.

5. If the express rate from St. Joseph, Mo., to Boston, Mass., is quoted at $4.50 per 100 lb., which is the cheaper and how much, to send three separate 2-lb. packages from St. Joseph to Boston by mail or by express?

6. The express rate from Boston to St. Albans, Mo., is quoted at $3.50 per 100 lb. Find the express charges on 17 separate parcels of merchandise sent from Boston to St. Albans, when the weights are as follows: 15 lb., 17 lb., 25 lb., 14 lb., 18 lb., 35 lb., 72 lb., 37 lb., 42 lb., 64 lb., 92 lb., 121 lb., 146 lb., 5 lb., 15 lb., 31 lb., 41 lb.

7. Find the amount of the following freight bill:

Date of W. B., July 15 19 *W. B. No.* 4679 *Albany, N.Y.,* July 20 19

L. K. Palmer & Co.

To THE INTERSTATE TRANSPORTATION COMPANY, *Dr.*

For Transportation from Boston *to* Albany

	Weight	Rate	Amount	
200 bbl. Potatoes	32100#			
40 crates Onions	2500#			
	????#	25¢	??	??
Advance charges			2	15
			??	??

Received payment F. O. Bennett, Freight Agent

No. Car 37145

Bulky goods are generally sent by freight. The articles are divided into different classes, according to quantity and character, and are subject to different rates. All railroads follow some official classification. All official classifications divide freight into six different classes.

Such freight as organs and pianos in cases, furniture, statuary, etc., is generally designated as first-class matter. Baled hay, iron, etc., in car loads, is generally designated as fifth-class matter. Building blocks, brick, etc., in car-load lots, is generally designated as sixth-class matter. First-class rates are the highest and sixth-class rates are the lowest charged.

Between most points, shipments weighing less than 100 lb. are charged as 100 lb., irrespective of weight.

BOSTON & ALBANY RAILROAD

LOCAL FREIGHT TARIFF BETWEEN

BOSTON, MASS.

AND

MILES	STATIONS	RATE PER 100 LB. Classes						MILES	STATIONS	RATE PER 100 LB. Classes					
		1	2	3	4	5	6			1	2	3	4	5	6
21	So. Framingham	10¢	9¢	7¢	6¢	5¢	4¢	98	Springfield . .	21¢	18¢	15¢	13¢	11¢	11¢
32	Westboro . .	11¢	10¢	9¢	7¢	6¢	5¢	108	Westfield . .	22¢	20¢	16¢	14¢	13¢	11¢
44	Worcester . .	13¢	12¢	10¢	8¢	8¢	6¢	146	Athol	29¢	25¢	21¢	15¢	14¢	13¢
62	Webster . . .	17¢	15¢	13¢	11¢	10¢	9¢	150	Pittsfield . .	29¢	25¢	21¢	15¢	14¢	13¢
83	Palmer . . .	19¢	16¢	14¢	12¢	11¢	10¢	202	Albany . . .	30¢	27¢	22¢	15¢	14¢	13¢

8. Using the table, find the amount of freight to charge on 27,500 lb. sixth-class matter, from Boston to Pittsfield.

9. Using the above table, find the amount of freight to charge on 27,290 lb. sixth-class matter and 890 lb. first-class matter from Boston to Albany; to Westfield.

10. Using the above table, find the amount of freight to charge on 14,790 lb. fifth-class matter and 2170 lb. second-class matter from Boston to Palmer; to Worcester; to Pittsfield; to Springfield.

11. Using the above table, find the amount of freight to charge on 75 lb. first-class matter, 125 lb. second-class matter, 1250 lb. third-class matter, 7290 lb. fourth-class matter, 21,490 lb. fifth-class matter, and 64,640 lb. sixth-class matter from Boston to South Framingham; to Westboro; to Webster; to Springfield; to Athol; to Albany.

DENOMINATE NUMBERS

CHAPTER XV

DENOMINATE QUANTITIES

REVIEW OF THE COMMON TABLES[1]

ORAL EXERCISE

1. Which of the following numbers are concrete? which are abstract? which are denominate?

a. 16	*f.* 150	*k.* 36 min.
b. 24 yr.	*g.* 21 yd.	*l.* 5 yd. 2 ft.
c. 64 hr.	*h.* 65 A.	*m.* 3 yr. 4 mo.
d. 12 men	*i.* 17 books	*n.* 10 T. 75 lb.
e. 15 desks	*j.* 34 houses	*o.* 5 A. 61 sq. rd.

2. Define an abstract number; a concrete number; a denominate number; a simple number; a compound number.

3. Which of the numbers in question 1 are simple? which are compound?

ORAL EXERCISE

1. Repeat the table of avoirdupois weight.

2. Repeat the table of long measure; of surveyors' long measure; of square measure; of surveyors' square measure.

3. Repeat the table of cubic measure; of dry measure; of liquid measure; of time; of angular measure; of United States money; of English money.

4. Name a number expressing distance; two numbers expressing area; two expressing value; three expressing capacity.

5. How many statute miles in a degree of the earth's surface at the equator? how many geographical miles? How many feet in a statute mile? how many inches?

[1] Tables of weights and measures may be found in the Appendix.

REDUCTION

ORAL EXERCISE

1. Change 42 ft. to inches; to yards.
2. Express 15 yd. as feet; as inches.
3. Reduce 80 qt. to gallons; to pints.
4. Change 128 qt. to pecks; to bushels.
5. Express 120 pt. as quarts; as gallons.
6. What part of a yard is 2 ft.? $\frac{1}{2}$ ft.? $\frac{1}{4}$ ft.?
7. Reduce 5 bu. to pecks; to quarts; to pints.

Reduction Descending

210. Example. Reduce 4 T. 75 lb. to ounces.

Solution. Since 1 T. = 2000 lb., 4 T. = 4 times 2000 lb. = 8000 lb.; and with the 75 lb. added this = 8075 lb. Since 1 lb. = 16 oz., 8075 lb. = 8075 times 16 oz. = 129,200 oz., the required result.

8075 times 16 oz. = 16 times 8075 oz.; therefore 8075 times 16 oz. is found as shown in the margin.

2000
4
8075
16
129200, No. of oz.

WRITTEN EXERCISE

Reduce:

1. 115′ 6″ to inches.
2. 12 bu. 4 qt. to pecks.
3. £16 15*s.* to shillings.
4. 211 rd. 3 ft. to inches.
5. $3\frac{1}{2}$ rd. to feet.
6. $1\frac{1}{2}$ T. to ounces.
7. 12 A. to square feet.
8. 161 cd. to cubic feet.

ORAL EXERCISE

1. How many pecks in $\frac{1}{4}$ bu.? in $\frac{1}{2}$ bu.?
2. Change .25 A. to square rods; .375 A.; 75 A.
3. Reduce $\frac{1}{2}$ gal. to pints. Express $\frac{1}{2}$ rd. as inches; as yards.

WRITTEN EXERCISE

Reduce:

1. $\frac{2}{3}$ mi. to feet.
2. .75 cd. to cubic feet.
3. $\frac{119}{160}$ A. to square feet.
4. $\frac{7}{8}$ yd. to inches.
5. .375 mi. to feet.
6. $\frac{1}{12}$ hr. to seconds.

Reduction Ascending

211. Example. Express 176 qt. dry measure in higher denominations.

Solution. Since 8 qt. = 1 pk., divide by 8 and obtain as a result 22 pk. Since 4 pk. = 1 bu., divide by 4 and obtain as a result 5 bu. 2 pk.

8)176 qt.
4)22 pk.
5 bu. 2 pk.

WRITTEN EXERCISE

Reduce to higher denominations:

1. 3840 ft.
2. 1054 pt.
3. 14,400 sec.
4. 2000 sq. in.
5. 816 pk.
6. 106,590 ft.
7. 43,560 sq. in.
8. 27,900 lb. avoir.
9. 15,120″
10. 51,200 cu. ft.
11. 145,152 cu. in.
12. 27,900 oz. avoir.

ORAL EXERCISE

1. Reduce $\frac{1}{2}$ ft. to the fraction of a yard.
2. Change .16 cwt. to the decimal of a ton.
3. What part of a yard is 1 in.? 2 in.? $\frac{1}{2}$ in.?
4. What decimal part of an acre is 16 rd.? 40 rd.?
5. What part of 35 bu. is 7 bu.? of $1\frac{1}{2}$ bu. is $\frac{1}{2}$ bu.?

WRITTEN EXERCISE

1. Reduce $1\frac{7}{8}$ in. to the fraction of a foot; of a yard.
2. Reduce 10*s*. 9*d*. to the fraction of a pound sterling.

Solution. The successive divisors for reducing pence to pounds sterling are 12 and 20 respectively. Divide 9*d*. by 12 and the result is .75*s*. Put with this the 10*s*. in the problem and the result is 10.75*s*. Divide 10.75*s*. by 20 and the result is £.5375. Or

12) 9*d*.
20)10.75*s*.
£.5375

10*s*. 9*d*. = 129*d*. £1 = 240*d*. Therefore 10*s*. 9*d*. = $\frac{129}{240}$ = £.5375.

3. Reduce 4 yd. $1\frac{1}{2}$ ft. to the decimal of a rod.
4. Reduce 10*s*. 6*d*. 2 far. to the decimal of a pound sterling.
5. Reduce 5 T. 721 lb. to tons and decimal of a ton; 6 T. 1750 lb.; 12 T. 290 lb.; 29,240 lb.; 28,390 lb.
6. Find the cost of 1750 lb. of coal at $6.25 per ton; of 2170 lb.; of 690 lb.; of 1360 lb.; of 3240 lb.; of 32590 lb.

ADDITION AND SUBTRACTION

ORAL EXERCISE

State the sum of:

1.	2.	3.	4.
12 ft. 1 in.	5 lb. 8 oz.	15 rd. 5 ft.	10 mi. 8 rd.
6 3	6 3	17 2	8 40

5.	6.	7.	8.
5 rd. 2 ft.	11 ft. 2 in.	5 bu. 1 pk.	5 mi. 20 rd.
8 $2\frac{1}{2}$	8 1	8 0	17 13
7 $2\frac{1}{2}$	3 3	9 1	11 10

State the difference between:

1.	2.	3.	4.
90 mi. 300 rd.	75 rd. $12\frac{1}{2}$ ft.	30 yd. 2 ft.	44 bu. 3 pk.
75 120	26 $4\frac{1}{2}$	17 $1\frac{1}{2}$	29 1

5.	6.	7.	8.
11 mo. 12 da.	12 mo. 31 da.	11 mo. 15 da.	98 gal. 2 qt.
6 6	8 17	2 9	69 1

212. Examples. 1. Three jars of butter weighed 48 lb. 7 oz., 45 lb. 9 oz., and 53 lb. 11 oz. Find the total weight.

SOLUTION. Arrange the numbers as in simple addition, so that units of the same order stand in the same vertical column. Adding the first column at the right, the result is 27 oz. = 1 lb. 11 oz.; write 11 oz. and carry 1 lb. Adding the pounds, the sum is 147.

48 lb.	7 oz.
45	9
53	11
147 lb.	11 oz.

2. From a barrel containing 379 gal. 1 qt. of molasses, 17 gal. 3 qt. were sold. How much remained unsold?

SOLUTION. Arrange the numbers as in simple subtraction, so that units of the same order stand in the same vertical column. 3 qt. cannot be subtracted from 1 qt.; therefore mentally take 1 gal. (4 qt.) from 37 gal. and add it to 1 qt., making 5 qt. 5 qt. − 3 qt. = 2 qt. Inasmuch as 1 gal. was added to 1 qt., there are but 36 gal. remaining in the minuend; 36 gal. − 17 gal. = 19 gal.

37 gal.	1 qt.
17	3
19 gal.	2 qt.

WRITTEN EXERCISE

Find the sum of:

1.	2.	3.	4.
£140 6*s.*	£139 5*s.*	84 T. 75 lb.	279 T. 840 lb.
159 3	214 5	96 14	364 210
162 4	921 3	78 79	872 220
139 2	141 7	37 41	146 140
167 4	10 9	19 63	214 180
129 3	171 8	84 79	926 230
136 4	215 7	97 13	210 420
147 2	321 5	87 125	75 750

Find the difference between:

5.	6.	7.	8.
11 mo. 17 da.	11 mo. 1 da.	8 mo. 14 da.	9 mo. 17 da.
8 31	9 31	2 29	2 31

9. From a pile of wood containing $74\frac{1}{2}$ cd., $28\frac{1}{2}$ cd. and $15\frac{1}{4}$ cd. were sold. How much remained unsold?

10. I owned a farm of 340 A. when I bought an adjoining field of $74\frac{1}{4}$ A. I then sold $140\frac{3}{4}$ A. What is the remainder of the farm worth at $75 per acre?

11. An English merchant had on hand Jan. 1 goods valued at £5927 10*s.*; during the following six months he bought goods at a cost of £4920 10*s.* and sold goods to the amount of £7926 4*s.* If the value of the goods on hand July 1 of the same year was £4120 10*s.*, what has been the gain or loss in English money? in United States money?

Finding the Difference between Dates

213. In the foregoing problems in addition and subtraction only compound numbers of two denominations were used. These are practically the only compound numbers met with in business, if the case of finding the difference between two dates is excepted.

214. The difference between two dates may be found by compound subtraction, or by counting the actual number of days from the given to the required date.

In business transactions involving long periods of time, the difference is generally found by compound subtraction; but in transactions involving short periods of time, the difference is generally found by counting the exact number of days.

215. Examples. 1. A mortgage dated Oct. 15, 1901, was paid Apr. 6, 1907. How long had it run?

SOLUTION. Write the later date as the minuend and the earlier date as the subtrahend. April being the 4th and October the 10th month, write 4 and 10 respectively instead of the names of the months. Consider 30 da. a month and 12 mo. a year and subtract as usual.

1907 yr.	4 mo.	6 da.
1901	10	15
5 yr.	5 mo.	21 da.

2. Find the difference between Apr. 21 and July 27.

SOLUTION. Write the number of days remaining in April, the number in May and June, and finally the number in July up to and including July 27. The sum of these numbers is the required time expressed with exactness. Observe that *the total time excludes the first and includes the last day of the given dates.*

9 da.	in April
31 da.	in May
30 da.	in June
27 da.	in July
97 da.	from April 21 to July 27

ORAL EXERCISE

State the exact number of days between:

1. Mar. 12 and Apr. 16.
2. Apr. 27 and May 31.
3. May 31 and July 18.
4. June 7 and Aug. 16.
5. July 1 and Oct. 1.
6. June 30 and Sept. 1.
7. July 31 and Nov. 7.
8. Aug. 31 and Dec. 1.

WRITTEN EXERCISE

Find the exact number of days between:

1. Apr. 2 and Nov. 25.
2. Mar. 1 and Sept. 18.
3. Mar. 15 and Nov. 2.
4. Apr. 21 and Dec. 31.
5. Mar. 18 and Nov. 27.
6. Mar. 17 and July 28.
7. June 16 and Sept. 18.
8. June 19 and Nov. 29.

9. Find the difference between Jan. 3, 1907, and each of the following dates: May 15, 1904; Sept. 6, 1905; Apr. 8, 1901; Mar. 12, 1889. Find the difference by compound subtraction.

MULTIPLICATION AND DIVISION

ORAL EXERCISE

Multiply:	*Divide:*
1. 3 ft. by 6.	**7.** 27 yd. by 9.
2. $1\frac{1}{2}$ mi. by 8.	**8.** 225 ft. by $7\frac{1}{2}$ ft.
3. 9 lb. 4 oz. by 2.	**9.** 48 ft. 6 in. by 2.
4. 18 lb. 1 oz. by 9.	**10.** 540 yd. by 18 yd.
5. 17 yd. 2 in. by 9.	**11.** 164 lb. 12 oz by 4.
6. 19 gal. 1 qt. by 3.	**12.** 640 mi. 160 rd. by 20.

216. Examples. **1.** How much hay in 8 stacks each containing 5 T. 760 lb.?

SOLUTION. 8 times 760 lb. = 6080 lb. = 3 T. 80 lb.; write 80 in place of pounds and carry 3. 8 times 5 T. = 40 T.; 40 T. + 3 T. carried = 43 T. The required result is therefore 43 T. 80 lb.

$$\begin{array}{r} 5 \text{ T. } 760 \text{ lb.} \\ \times \quad 8 \\ \hline 43 \text{ T. } 80 \text{ lb.} \end{array}$$

2. An importer paid £ 87 10*s.* for 50 pc. of bric-a-brac. What was the cost per piece?

SOLUTION. Since 50 pc. cost £ 87 10*s.*, 1 pc. costs $\frac{1}{50}$ of £ 87 10*s.* $\frac{1}{50}$ of £ 87 = £ 1 with an undivided remainder of £ 37; write £ 1 in the quotient and add £ 37 to the next lower denomination; £ 37 10*s.* = 750*s.* $\frac{1}{50}$ of 750*s.* = 15*s.*

$$\begin{array}{r} \text{£ } 1 \quad 15s. \\ 50\overline{)\text{£ } 87 \quad 10s.} \end{array}$$

3. At 10*s.* 6*d.* per yard, how many yards can be bought for £ 15 15*s.*?

SOLUTION. The dividend and divisor are concrete numbers; therefore reduce them to the same denomination before dividing. £ 15 15*s.* = 3780*d.*, 10*s.* 6*d.* = 126*d.* 3780*d.* ÷ 126*d.* = 30; that is 30 yd. can be bought.

£ 15 15*s.* = 3780*d.*
10*s.* 6*d.* = 126*d.*
3780*d.* ÷ 126*d.* = 30, no. of yd.

ORAL EXERCISE

1. At 72 ¢ per gross what will 2 doz. buttons cost? 4 doz.? 7 doz.?

2. How many 3-oz. packages can be put up from 4 lb. of pepper?

3. Find the cost of 3 T. of bran at 30 ¢ per hundredweight; of 5 T. at 50 ¢ per hundredweight.

4. How many 1-lb. packages can be put up from 15 T. of breakfast food?

5. When coal is $6 per ton what will 7000 lb. cost? 6400 lb.? 3600 lb.?

6. Find the cost of 2400 lb. of flour at $2.25 per hundredweight; of 4400 lb.; of 3200 lb.

7. At $12\frac{1}{2}$¢ per quire what will 480 sheets of paper cost? 240 sheets? 2880 sheets? 720 sheets?

8. I buy 3 qt. of milk per day. If I pay 5¢ per quart, what is my bill for July and August?

9. I bought 3 gro. pens at 60¢ a gross and sold them at the rate of 2 for 1¢; what was my gain or loss?

10. I bought $3\frac{1}{2}$ bu. of apples at $1.00 per bu. and sold them at 50¢ a peck. What was my gain?

11. I sold $4\frac{1}{2}$ cd. of wood for $27 and thereby lost $9 on the cost. What was the cost per cord?

12. A dealer bought 5 rm. of paper at $1.25 per ream and retailed it at 20¢ a quire. What was his gain?

13. At $4.80 per ream what will 3 qr. of paper cost? At $3.60 per ream what will 1 qr. cost? 7 qr.?

14. If the gross weight of a load of straw is 3380 lb. and the tare 1580 lb., what is the straw worth at $4.00 per ton?

15. A dealer bought pens at 60¢ a gross and retailed them at the rate of 6 for 5¢. What did he gain on 1 gro.? on 6 gro.? on 8 gro.?

WRITTEN EXERCISE

1. Find the cost of 10 pwt. 7 gr. of old gold at $1.25 per pennyweight; of 12 pwt. 4 gr. at $1.10 per pennyweight.

2. I bought $3\frac{1}{8}$ A. of city land at $125 an acre and sold it at 50¢ per square foot. Did I gain or lose and how much?

3. Give the length of a double-track railroad that can be laid with 352,000 rails 30 ft. long.

4. I bought a barrel of cranberries containing $2\frac{1}{2}$ bu. at $4 per bushel and retailed them at 15¢ a quart. Did I gain or lose and how much?

5. From a farm of 375 A. I sold $25\frac{3}{4}$ A. What is the remainder worth at \$125 per acre?

6. Find the cost (*a*) in English money and (*b*) in United States money of 360 doz. cotton hose at 5*s*. 2*d*.

SOLUTION. (*a*) 5*s*. 2*d*. = $5\frac{1}{6}$*s*. 360 times $5\frac{1}{6}$*s*. = 1860*s*. = £ 93, the cost in English money.

(*b*) £ 1 = \$ 4.8665. 93 times \$ 4.8665 = \$ 452.58, the cost in United States money.

7. Copy and find the amount of the following invoice:

London, England, Apr. 3, 19—.

Messrs. W. P. Firth & Co.,
New York City.

Terms Net 30 ds. Bought of E. M. LLOYD & SON

#1	150 doz.	Fancy Cotton Hose	5/2	
#2	250 "	Black Cotton Hose	4/3	
	2 cases		12/-	

5/2, 4/3, and 12/– in the price column = 5*s*. 2*d*., 4*s*. 3*d*., and 12*s*., respectively.

8. The distance around a square garden is 48 rd. 12 ft. Find the length of one side of it.

9. Reduce \$2500 to English money.

SOLUTION. £ 1 = \$ 4.8665. \$2500 ÷ 4.8665 = 51.372. 51.372 × £ 1 = £ 51.372. .372 × 20*s*. = 7.44*s*. .44 × 12*d*. = 5.28*d*. .28 × 4 far. = 1.12 far. Hence \$ 2500 = £ 51.7*s*. 5*d*. 1 far.

10. Find the value in United States money of a post-office money order for £ 5 18*s*. 6*d*.; for £ 3 12*s*.

11. Change \$100 to English money; \$ 135; \$ 250; \$ 1250.

12. A coal dealer bought 448 T. of coal by the long ton at \$4 per ton and sold it by the short ton at \$5.25 per ton. Did he gain or lose and how much?

13. A druggist bought by avoirdupois weight 5 lb. of peppermint oil at $2.50 per pound and retailed it at 50¢ an ounce, apothecaries' weight. What was his gain?

217. Farm products which are handled in bulk are frequently bought and sold by the bushel. The statutory weights of the bushel for some of the common commodities are shown in the following table:

STATUTORY WEIGHTS OF THE BUSHEL

COMMODITIES	WEIGHT IN AVOIRDUPOIS POUNDS	EXCEPTIONS
Barley	48	Ala., Ga., Ky., and Penn., 47; Ariz., 45; Cal., 50.
Beans	60	N. H. and Vt., 62.
Clover Seed	60	
Corn, Shelled	56	Ariz., 54; Cal. 52.
Oats	32	Me., N.J., Va., 30; Md., 26.
Potatoes, Irish	60	Md., Penn., and Va., 56.
Rye	56	Cal., 54.
Wheat	60	

218. Example. What will 4260 lb. of wheat cost at 80¢ per bushel?

SOLUTION. In examples of this character the principles of cancellation may be applied to advantage.

$$\frac{\overset{71}{\cancel{4260}} \times 80¢}{\underset{1}{\cancel{60}}} = \$56.80$$

In problems 1–4 in the following exercise the price is per bushel in each case.

WRITTEN EXERCISE

1. Find the total value of:

6640 lb. wheat at 84¢.
4230 lb. wheat at 95¢.
1260 lb. wheat at 90¢.
6120 lb. wheat at $87\frac{1}{2}$¢.

2. Find the total value of:

3264 lb. oats at 25¢.
2400 lb. oats at 48¢.
2560 lb. oats at $37\frac{1}{2}$¢.
6951 lb. oats at 32¢.
1920 lb. oats at $33\frac{1}{3}$¢.
3840 lb. oats at $29\frac{1}{2}$¢.

3. Find the total value of:

3660 lb. clover seed at $4.50.
1200 lb. clover seed at $4.75.
2472 lb. clover seed at $4.20.
5040 lb. shelled corn at $47\frac{1}{2}$¢.
2800 lb. shelled corn at 56¢.
2240 lb. shelled corn at 73¢.

4. Find the total value of :

3793 lb. rye at \$1.12.
9240 lb. rye at \$1.25.
6720 lb. rye at \$1.12½.
6160 lb. rye at 90¢.
3080 lb. rye at 97½¢.
7924 lb. rye at \$1.12.

5. The gross weights and the tares of ten loads of wheat were 4260 – 1260, 4310 – 1260, 3890 – 1260, 4160 – 1260, 3860 – 1260, 4180 – 1260, 4370 – 1260, 4290 – 1260, 4370 – 1260, 4480 – 1260 lb., respectively. Find the value of the wheat at \$1.12½ per bushel.

ORAL REVIEW EXERCISE

1. Find the cost of 2500 lb. of hay at \$12 per ton.
2. What is a ton of wheat worth at 90¢ per bushel?
3. Change 4860 lb. to tons ; 3640 lb.; 4280 lb.; 6240 lb.
4. Change 2.5 T. to pounds; .75 T.; 2.03 T.; 11.004 T.
5. Change 6 mi. to rods; 50 rd. to feet; 330 ft. to rods.
6. How much more than 1 ton does 70 bu. of oats weigh?

WRITTEN REVIEW EXERCISE

1. Find the total cost of :

3260 lb. at \$5.25 per ton.
3840 lb. at \$7.50 per ton.
4560 lb. at \$6.87½ per ton.
4960 lb. at \$8.00 per ton.
5800 lb. at \$6.25 per ton.
5200 lb. at \$5.25 per ton.

2. Find the total cost of :

3500 lath at \$3 per M.
3600 lb. hay at \$9 per ton.
3150 lb. pork at \$4.50 per cwt.
1500 brick at \$8 per M.
4260 lb. coal at \$4 per ton.
60 lb. beef at \$4.75 per cwt.

3. Find the total value of :

Commodity	Gross weight	Tare	Price
A load of coal	6460 lb.	2140 lb.	\$6.25 per T.
A load of straw	3680 lb.	1680 lb.	\$3.25 per T.
A load of wheat	4160 lb.	1620 lb.	85½¢ per bu.
A load of oats	4760 lb.	1560 lb.	31½¢ per bu.
A load of coal	4230 lb.	1530 lb.	\$7.25 per T.
A load of paper rags	3260 lb.	1260 lb.	½¢ per lb.
A load of old iron	3480 lb.	1280 lb.	¼¢ per lb.
A load of corn meal	4160 lb.	1620 lb.	75¢ per cwt.

4. A church was lighted by kerosene lamps and the amount of oil consumed each evening was $1\frac{1}{2}$ qt. If the church was lighted 2 evenings each week for 1 yr., what was the cost of the oil at 14¢ per gallon?

5. An American lady shopping in Paris bought 10 yd. of lace at 20 francs per yard; 6 pr. of gloves at 10 francs per pair. What was the amount of the bill in United States money?

6. A local dealer bought 448 T. of coal, by the long ton, at $5.50 per ton and sold it by the short ton at $6. If the waste and loss amounted to 2 short tons, how much did he gain?

7. Without copying, find the amount of the following invoice:

Leith, Scotland, May 16, 19—

INVOICE OF HOSIERY

Shipped by The J. M. Roberts Co. *In the steamship* Winifredian

To Edward M. Davidson & Co.
Boston, Mass.

Marks	Number	Quantity		Article and Description	Price	Extension	Amount
D	1611	1	cs.	Mdse, as follows:			
	622	51	doz.	Black Cotton Hose	6/4		
	670	30	"	" " "	7/2		
	676	6½	"	" " "	6/6		
	650	15	"	Black Lace Hose	12/4		
	680	6	"	" " "	16/6		
	686	3	"	" " "	18/4		

8. Find, by compound subtraction, the difference between Sept. 14, 1908, and each of the following dates: Jan. 8, 1881; Feb. 7, 1883; Mar. 9, 1890; Apr. 27, 1895; May 20, 1897; June 17, 1899; July 25, 1900; Aug. 15, 1901; Sept. 24, 1903; Oct. 19, 1904; Nov. 18, 1905; Dec. 15, 1906.

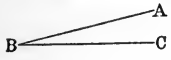
CHAPTER XVI

PRACTICAL MEASUREMENTS

DISTANCES AND SURFACES

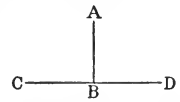
Distances

219. An **angle** is the divergence of two lines from a common point.



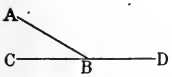
Thus the divergence of the lines BA and BC from the point B is the angle ABC.

220. A **right angle** is the angle formed when one straight line so meets another as to make the two adjacent angles equal. The lines forming the angles are **perpendicular** to each other.



Thus the two angles ABC and ABD are right angles, and the lines AB and CD are perpendicular to each other.

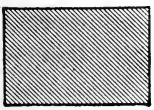
221. An **acute angle** is less than a right angle; an **obtuse angle** is greater than a right angle.



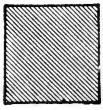
Thus the angle ABC is an acute angle, and the angle ABD is an obtuse angle.

222. A **surface** is that which has *length* and *width*, but not measurable thickness. A level surface, as the surface of still water, is called a **plane surface** or a **plane**.

223. A **rectangle** is a plane figure bounded by four straight lines and having four right angles.



A **square** is a rectangle whose sides are all equal.



224. A **triangle** is a plane figure bounded by three straight lines and having three angles.

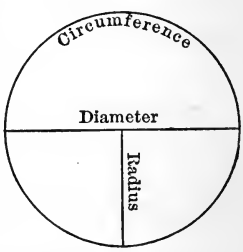
A triangle is called **equilateral** when all its sides are equal; **isosceles** when any two of its sides are equal; **scalene** when no two of its sides are equal.

225. A **right-angled triangle** is a triangle having a right angle.

A triangle containing an acute angle is sometimes called an **acute-angled triangle**; a triangle containing an obtuse angle, an **obtuse-angled triangle.**

226. The **perimeter** of a plane figure is the distance around it.

227. A **circle** is a plane figure bounded by a regularly curved line, every point of which is equally distant from a point within called the *center*. The **circumference** of a circle is the curved line which bounds it; the **diameter** is any straight line passing through the center and terminating in the circumference; the **radius** is one half the diameter. An **arc** is any part of the circumference of a circle.



ORAL EXERCISE

1. Measure very accurately the diameter and the circumference of each of several circular objects, such as an ink-well cover, a coin, a ring, a plate, or a wheel. Record the measurements in each case.

2. Divide each circumference by its diameter, carrying the result to four decimal places.

3. Find the average of the several quotients.

4. How many times the diameter of a circle is its circumference?

5. A piece of circular stove pipe 7 in. in diameter is approximately 22 in. in circumference; the circumference is approximately how many times its diameter? If the diameter of a circle is 21 in., what is its circumference?

228. It is proved in geometry that the *circumference of a circle is* 3.1416 *times the diameter.*

229. Therefore, to find the circumference of a circle when the diameter is given, *multiply the diameter by 3.1416.*

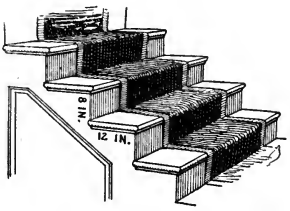
230. And, conversely, to find the diameter of a circle when the circumference is given, *divide the circumference by 3.1416.*

WRITTEN EXERCISE

1. Draw neat figures to represent each of the following: rectangle, triangle, square, circle, right-angled triangle, equilateral triangle, isosceles triangle, scalene triangle, radius of a circle, arc of a circle.

2. A parlor is 18 ft. 6 in. long and 12 ft. 3 in. wide. What will be the cost, at 28¢ per foot, of a molding extending around the room?

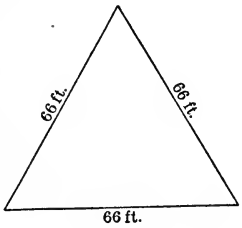
3. The circumference of a circle is 113.0976 ft. What is the length of the longest straight line that can be drawn across the circle? Find the circumference of a circle whose radius is 21 ft.

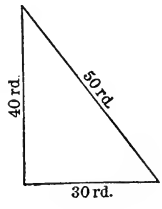


4. What will be the cost, at 75¢ per yard, of carpeting a stairway of 18 steps, the tread of each stair being 12 in. and the riser 8 in.?

5. How many telegraph poles, 10 rd. apart, will be required for 150 mi. of railroad?

6. Find the cost, at 75¢ per rod, of fencing the fields illustrated in the accompanying triangles:





7. A rectangular field is 100 rd. long and 60 rd. wide. How many posts set 1 rd. apart will be required to inclose the field and to divide it into four equal fields?

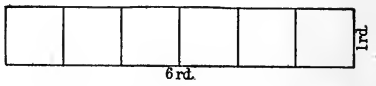
Areas

ORAL EXERCISE

1. What is the area of a square 1 rd. on each side?

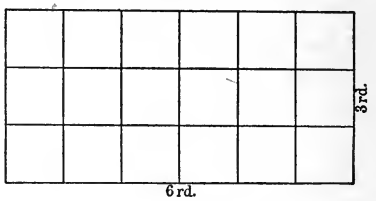
2. How many squares 1 rd. on each side in a rectangle 6 rd. long and 1 rd. wide?

3. How many rectangles, each 6 rd. by 1 rd., in a rectangle 6 rd. by 3 rd.?



4. How many square rods in the area of a rectangle 6 rd. long and 3 rd. wide?

5. How many square rods in the area of a rectangle 16 rd. long and 132 ft. wide?



SOLUTION. 132 ft. = 8 rd. A rectangle 1 rd. on a side contains 1 sq. rd. But the given rectangle is 16 times 1 rd. long and 8 times 1 rd. wide. Therefore the required area is $16 \times 8 \times 1$ sq. rd. or 128 sq. rd.

132 ft. = 8 rd.

8 × 16 sq. rd. = 128 sq. rd.

231. In the foregoing exercise it is clear that *the product of the length and width of a rectangle equals the area.*

ORAL EXERCISE

Find the areas of rectangles having the following dimensions. Make use of the short method explained in §§ 180–182.

1. $6\frac{1}{2}$ ft. by $6\frac{1}{2}$ ft.

2. $7\frac{1}{2}$ rd. by $7\frac{1}{2}$ rd.

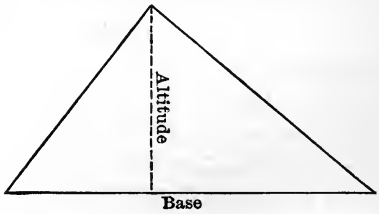
3. 6.5 rd. by 6.5 rd.

4. 9.5 rd. by 9.5 rd.

5. 12.5 ft. by 4.5 ft.

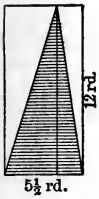
6. 14.5 rd. by 6.5 rd.

232. The dimensions of a triangle are called the base and the altitude. The **base** is the side on which the triangle appears to stand; the **altitude** is the perpendicular distance from the base to the highest point of the triangle.



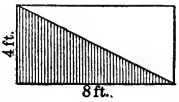
ORAL EXERCISE

1. How does the area of the triangle on the right compare with the area of a rectangle 8 ft. by 4 ft.?



2. Compare the area of the triangle on the left with the area of a rectangle 12 rd. by $5\frac{1}{2}$ rd.

3. What is the area of a triangle whose base is 8 ft. and whose altitude is $9\frac{1}{2}$ ft.?



4. The area of a triangle equals what part of the area of a rectangle having the same base and altitude?

233. In the above exercise it is clear that *one half the product of the base and altitude of a triangle equals the area.*

ORAL EXERCISE

State the areas of triangles whose bases and altitudes, respectively, are as follows:

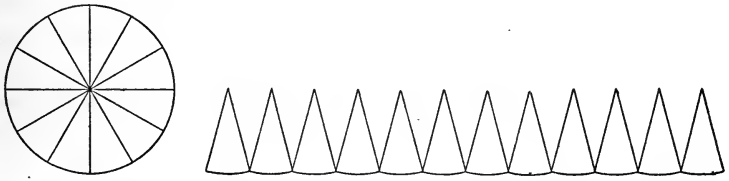
1. 20 ft., 18 ft.

2. 12 ft., 16 ft.

3. 12 ft., $4\frac{1}{2}$ ft.

4. $19\frac{1}{2}$ ft., 8 ft.

234. If a circle be divided as in the figure on the left and the parts rearranged as in the figure on the right, it will be clear



that the area of the circle equals the area of the twelve triangles. The altitude of each triangle is the radius of the circle, and the sum of the bases, the circumference.

235. It is therefore clear that *one half the product of the circumference and radius of a circle equals the area.*

When a circle is divided as in the above figure, the parts are not exact triangles; but it is proved in geometry that the area of a circle is the same as that of a triangle having a base equal to the circumference and an altitude equal to the radius.

ORAL EXERCISE

1. The base of a triangle is 8 in. and the height 11 in. What is the area?

2. A field contains 1280 sq. rd. If the width is 32 rd., what is the length?

3. A man sold a lot 10 rd. long and 8 rd. wide at the rate of $260 per acre. How much did he receive?

4. A porch is 20 ft. long and 6 ft. wide. How many square feet of oilcloth will be required to cover it?

5. A canvas on which a portrait is painted contains 1440 sq. in. If the width is 3 ft., what is the length?

WRITTEN EXERCISE

1. A circular pavilion has a radius of $56\frac{1}{2}$ ft. What is the area of the floor space?

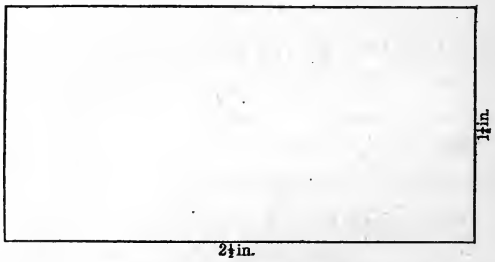
2. A city lot contains $\frac{1}{4}$ A. If it is 200 ft. long, what is its width, and what is its value at 50¢ per square foot?

3. The floor of a restaurant 50 ft. long and 40 ft. wide is covered with tiles 8 in. square. How many tiles will be required?

4. A small park, 50 rd. long and 40 rd. wide, has a walk inclosing it. If the walk is 1 yd. wide, how many square feet does it contain?

5. How many square feet of slate will be required to furnish blackboard surface for a schoolroom 30 ft. wide and 42 ft. long, if the slate is 1 yd. wide and extends across one end of the room and one third the length on each side?

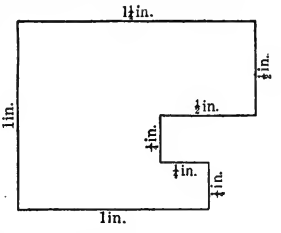
6. The accompanying diagram represents a field of wheat.



It is drawn on a scale of $\frac{1}{32}$ in. to the rod. How much will it cost, at 50¢ per rod, to build a fence around the field?

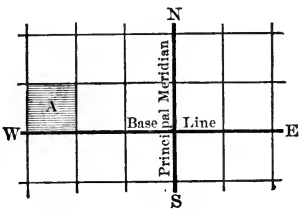
7. If the field in problem 6 yields an average of $16\frac{1}{8}$ bu. of wheat to the acre, for a certain season, what is the crop worth at \$0.95 per bushel?

8. The accompanying diagram represents a field of corn. It is drawn on a scale of $\frac{1}{64}$ in. to the rod. If the field yields an average of 28 bu. to the acre for a certain year, what is the crop worth at 55¢ per bushel?



Public Lands

236. In the more recently settled parts of the United States, public lands are surveyed by selecting a north and south line as a **principal meridian** and an east and west line intersecting this as a **base line**. Other lines are then run, at intervals of 6 miles, both east and west of the principal meridian, and north and south of the base line. These lines divide the land into tracts 6 mi. square, called **townships**. The lines of townships running north and south are called **ranges**.



Thus A in the above diagram may be described as Tp. 1 N., R. 3 W.; that is, the first township north of the base line, in the third range west of the principal meridian.

237. Each township is divided into 36 tracts, each 1 mile square, called **sections**. The numbering of sections in every township is as shown in the diagram at the left.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
B	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Township

Sections are divided into *halves* and *quarters;* quarter sections are subdivided into *halves* and *quarters.*

If diagram 3 is B of diagram 2, and diagram 2 is A of diagram 1, C of diagram 3 may be described as the S.E. ¼ of S.E. ¼, Sec. 19, Tp. 1 N., R. 3 W.

ORAL EXERCISE

1. How many chains in a mile? how many rods? how many feet? How many rods in a chain? how many feet?

2. How many acres in a field 50 ch. by 40 ch.? in a field 40 ch. square? in a field 80 ch. by 80 ch.?

3. A field has an area of 4 A. If it is 10 ch. long, how wide is it and what will it cost to fence it at 50¢ per rod? at 60¢?

WRITTEN EXERCISE

1. Make a diagram of a township and locate N. $\frac{1}{2}$, Sec. 20.

2. Draw a diagram illustrating principal meridian, base line, range line, and township lines, and mark Tp. 2 S., R. 2 E. and Tp. 1 N., R. 3 W.

3. Find the value, at $12.50 per acre, of Tp. 2 N., R. 3 W.

4. Find the cost at $25 per acre of N.E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 20, Tp. 1 N., R. 4 W.

Square Root and Its Applications

ORAL EXERCISE

1. What is meant by *factor?* by *exponent?* by *power of a number?*

2. State the second power of each of the following numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9. How much is 12^2, 13^2, 14^2, 15^2, 16^2?

3. Name *one* of the *two equal* factors of each of the following numbers: 2, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196.

238. The **square** of a number is the product arising from using the number *twice* as a factor. The **square root** of a number

wo equal factors of the number.

quare root of a number may be indicated by writ-
er under the *radical sign* $\sqrt{\ }$ or by placing the
ove and to the right of the number.

r $196^{\frac{1}{2}}$ indicates the square root of 196.

square root of a number is readily derived from the process by which the square is formed.

241. Example. What is the square of 42?

SOLUTION. Since $42 = 40 + 2$, the square of 42 may be found as follows:

$$\begin{array}{r} 40+2 \\ 40+2 \\ \hline (40\times 2)+2^2 \\ 40^2+(40\times 2) \\ \hline 40^2+2(40\times 2)+2^2= \end{array} \qquad \begin{array}{r} 40^2 = 1600 \\ 2(40\times 2) = \ 160 \\ 2 \ = \ \ 4 \\ \hline 1764 \end{array}$$

242. In the preceding process it is shown that *the square of a number is equal to the square of the tens plus twice the product of the tens by the units, plus the square of the units.*

243. $1^2 = 1$, $10^2 = 100$, $100^2 = 10000$, and so on; $9^2 = 81$, $99^2 = 9801$, $999^2 = 998001$, and so on. It is therefore evident that the square of an integral number contains twice as many figures or one less than twice as many figures as the number. Hence, if an integral number be separated into groups of two figures each, from right to left, there will be as many figures in the square root as there are groups of figures in the number.

244. Examples. 1. What is the square root of 529?

SOLUTION. Beginning at the right, separate the number into periods of two figures each. The greatest square in 5 is 4 and the square root of 4 is 2, the tens' figure of the root. Find the remainder, affix the second period, and the result is 129. This remainder is equal to twice the product of the tens by the units, plus the square of the units (§ 242). Twice 2 tens is 4 tens (40) and 4 tens (40) is contained in 129, 3 times; hence, 3 is the units' figure of the root. Twice the tens multiplied by the units plus the square of the units is the same as twice the tens plus the units multiplied by the units. Therefore, annex 3 units to the 4 tens and multiply by 3; the result is 129. The square root of 529 is thus shown to be 23.

$$\begin{array}{r} 5\ 29(23 \\ 4 \\ \hline 43)1\ 29 \\ \ \ 1\ 29 \\ \hline \end{array}$$

2. What is the square root of (*a*) 13.3225; (*b*) of .0961?

$$\begin{array}{r} 13\ .32\ 25(3.65 \\ 9 \\ \hline 6.6)4\ .32 \\ \ \ 3\ .96 \\ \hline 7.25)\ \ .36\ 25 \\ \ \ \ \ .36\ 25 \\ \hline \end{array} \qquad \begin{array}{r} .09\ 61(.31 \\ .09 \\ \hline .61).00\ 61 \\ \ \ .00\ 61 \\ \hline \end{array}$$

245. The process of finding the square root of a number may be summarized as follows:

Beginning at the units, separate the number into groups of two figures each.

Find the greatest square in the left-hand group and write its root for the first figure of the required root.

Subtract the square of the root figure from the left-hand period and annex the second period for a dividend.

Take twice the root figure already found, considered as tens, and divide the dividend by it.

Annex the quotient to both the root and the trial divisor and multiply by the units.

Continue in like manner until all the periods have been used. The result will be the square root.

If a number contains a decimal, begin at the decimal point and indicate groups to the left for the integral part of the root, and to the right for the decimal part of the root. If the last period on the right of the decimal point has but one figure, annex a decimal cipher, as each decimal period *must* contain two figures.

To find the square root of a common fraction, extract the square root of the numerator and denominator separately. If the terms of the fraction are not perfect squares, reduce the fraction to a decimal and then extract the square root.

WRITTEN EXERCISE

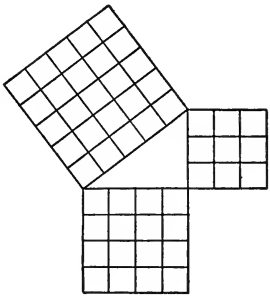
Find the square root of:

1. 324.	**5.** 576.	**9.** 9025.	**13.** $\frac{49}{64}$.
2. 484.	**6.** 1024.	**10.** 3364.	**14.** $\frac{169}{324}$.
3. 676.	**7.** 7225.	**11.** 70.56.	**15.** $\frac{6561}{8281}$.
4. 729.	**8.** 3969.	**12.** 150.0625.	**16.** $\frac{190969}{743044}$.

246. It has been seen that the area of a square is the product of its two equal sides. It therefore follows that *the square root of the area of a square equals one of its sides.*

247. The **hypotenuse** is the side opposite the right angle in a right triangle.

248. In the accompanying illustration it will be seen that the square on the hypotenuse is equal to the sum of the squares on the other sides. Hence,



249. To find the hypotenuse *take the square root of the sum of the squares of the base and altitude;* and

250. To find the base or the altitude *take the square root of the difference between the squares of the hypotenuse and the other side.*

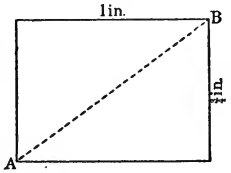
WRITTEN EXERCISE

1. A square field contains 5.625 A. What is the length of one of its sides?

2. Find the side of a square containing the same area as a field 160 rd. long by 90 rd. wide.

3. What is the hypotenuse of a right-angled triangle the base of which is 30 ft. and the altitude 40 ft.?

4. The accompanying diagram represents a piece of land. It is drawn on the scale of $\frac{1}{80}$ in. to the rod. The land is divided into two fields by the line AB. Find the cost, at 50¢ per rod, of fencing the two fields.



5. What will be the cost, at $1.75 per chain, of fencing a square field containing 1.6 A.?

Roofing

251. Roofing is usually measured by the *square* of 100 sq. ft.

252. The **size of slates** used for roofing varies from 6 in. by 12 in. to 16 in. by 24 in.

Contractors and builders generally use prepared tables for estimating the amount of slate to be used. The number of slates per square varies with the size of the slate. Thus, slates 16 in. by 24 in. require 86 per square; slates 6 in. by 12 in. require 533 per square; etc.

253. All shingles average 4 in. in width and are put up in **bundles** of 250. The shingles most commonly used are 16 in. or 18 in. long. 16-inch shingles are generally laid $4\frac{1}{2}$ in. and 18-inch shingles $5\frac{1}{2}$ in. to the weather.

254. A shingle 4 in. wide laid $4\frac{1}{2}$ in. to the weather will cover 18 sq. in. A square contains 14,400 sq. in. 14,000 sq. in. ÷ 18 sq. in. = 800. It is therefore clear that 800 16-*inch shingles will cover a square of roof.*

255. A shingle 4 in. wide laid $5\frac{1}{2}$ in. to the weather will cover 22 sq. in. 14,400 sq. in. ÷ 22 sq. in. = 655. It is therefore clear that 655 18-*inch shingles will cover a square of roof.*

In practice 655 per square is called 700 per square.

ORAL EXERCISE

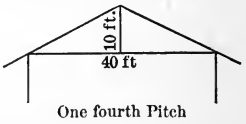
1. How many bundles in 1000 shingles? in 7500 shingles? in 26,000 shingles?

2. What will be the cost, at \$4 per square, of tinning a roof 20 ft. by 15 ft.?

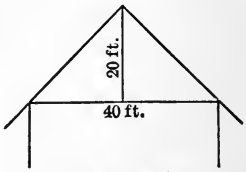
3. A certain roof requires 7610 shingles. How many bundles of shingles must be bought to cover it?

A dealer will not sell a fractional part of a bundle of shingles.

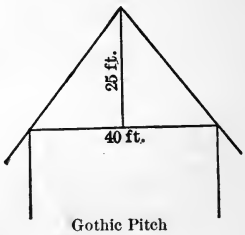
4. How many slates at 300 to the square will be required for a flat roof 30 ft. by 20 ft.?



One fourth Pitch



One half Pitch



Gothic Pitch

256. The rise in the rafters for each foot in the base of the gable is called the **pitch of the roof.**

257. When the rise of the roof is 6 in. per foot, the roof is said to have **one-fourth pitch.**

258. When the rise of the rafters is 12 in. per foot, the roof is said to have **one-half pitch.**

259. When the rise of the rafters is 15 in. per foot, the roof is said to have **five-eighths**, or **Gothic pitch.**

When the rise of the rafters is 6 in. per foot, the perpendicular height of the gable is $\frac{1}{4}$ of the width of the building; when the rise is 12 in. per foot, the height of the gable is $\frac{1}{2}$ the width of the building; when the rise is 15 in. per foot, the height of the gable is $\frac{5}{8}$ of the width, or $1\frac{1}{4}$ times $\frac{1}{2}$ the width of the building. Hence the names one-fourth pitch, one-half pitch, etc.

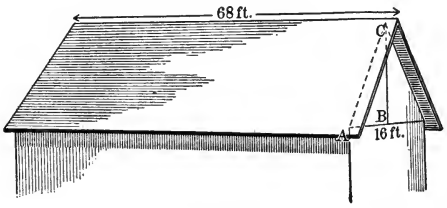
ORAL EXERCISE

Find the height of the gable:

	Width of Building	Pitch of Roof		Width of Building	Pitch of Roof
1.	30 ft.	$\frac{1}{2}$	**3.**	24 ft.	Gothic
2.	50 ft.	12 in. per ft.	**4.**	36 ft.	$\frac{1}{4}$

WRITTEN EXERCISE

1. The accompanying diagram represents the roof of a shed 16 ft. wide. If the ridge-pole is 68 ft., the pitch of the roof one half, and the projection of the rafters 18 in., how many shingles 16 in. long, laid $4\frac{1}{2}$ in. to the weather, will be required to cover the roof?



Solution

$\frac{1}{2}$ of 16 ft. = 8 ft. = the base of the triangle ABC.

The pitch of the roof is $\frac{1}{2}$; $\frac{1}{2}$ of 16 ft. = 8 ft. = the altitude of the triangle ABC.

$8^2 + 8^2 = 128$; $128^{\frac{1}{2}} = 11.31$, number of feet in the hypothenuse of ABC.

18 in. = 1.5 ft.; 11.31 ft. + 1.5 ft. = 12.81 ft. = the length of the rafters or the width of each side of the roof.

$2 \times 68 \times 12.81$ ft. = 1742.16 sq. ft. = the entire surface of the roof.

1742.16 sq. ft. = 17.4216 squares; 17.4216 × 800 shingles = 13937 shingles.

As bundles of shingles are not broken it will be necessary to buy 14000 shingles.

2. A building is 40 ft. wide. If the length of the ridge-pole is 80 ft. and the projection of the rafters 20 in., how many shingles 18 in. long and laid $5\frac{1}{2}$ in. to the weather will be required for the roof, the pitch being $\frac{1}{2}$?

3. A building is 30 ft. wide. If the length of the ridge-pole is 60 ft. and the projection of the rafters 15 in., how many shingles 16 in. long and laid $4\frac{1}{2}$ in. to the weather will be required for the roof, the pitch being $\frac{1}{4}$?

Plastering

260. Plastering is usually measured by the **square yard.**

261. There is no uniform rule with respect to the **allowance** to be made for doors, windows, and other openings.

What allowance, if any, shall be made for openings is usually stated in the contract covering the work. In some sections it is customary to make allowance for one half the area of the openings; in others, for the full area of the openings; in still others, for a stated number of square feet.

In giving the dimensions of a room carpenters, architects, and mechanics write the length first, then the width, and finally the height. They also usually write 5″ for 5 in., 5′ for 5 ft., and 5′ × 5′ for 5 ft. by 5 ft.

ORAL EXERCISE

1. What is the perimeter of a square room 20′ on a side?
2. What is the perimeter of a dining room 18′ × 12′ × 9′?
3. How many square feet in the four walls of the room in problem 2, not allowing for openings? in the ceiling? in the four walls and the ceiling?
4. How many square yards in the four walls of a room 24′ × 16′, not allowing for openings?
5. At 25¢ per square yard, what will it cost to plaster 945 sq. ft.? 1080 sq. ft.? 1440 sq. ft.?

WRITTEN EXERCISE

1. What will it cost, at 27 ¢ per square yard, to plaster the walls and ceiling of a hall 60′ × 40′ × 24′, making an allowance of 40 sq. yd. for openings?
2. Find the cost, at 26¢ per square yard, of plastering the walls and ceiling of a room 18′ × 16′ 6″ × 8′ 6″, making full allowance for 2 doors each 7′ 6″ × 4′, 3 windows 6′ × 4′.

3. What will be the cost of plastering, with hard finish, at 34¢ per square yard, the walls of the rooms in the following dwelling?

First Floor. Parlor, 14′ × 12′; sitting room, 12′ × 12′; dining room, 12′ × 10′; kitchen, 12′ × 10′; pantry, 8′ × 6′. All rooms on this floor are uniformly 8′ 6″ high.

Second floor. Front chamber, 14′ × 12′; back chamber, 12′ × 12′; middle chamber, 10′ × 9′; hall, 23′ × 4′. All rooms on this floor are uniformly 8′ high.

Allowance is made for 40 openings of 17 sq. ft. each.

Painting

262. Painting is usually measured by the **square yard.**

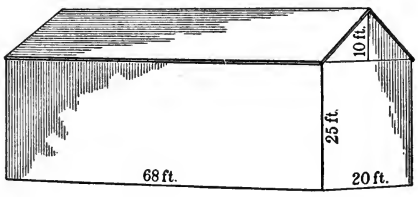
263. It is customary to make no allowance for windows, the painting of window sills and sashes being considered as expensive as the painting of the surface area of the entire window.

WRITTEN EXERCISE

1. What will it cost, at 25¢ per square yard, to paint the walls of a room 20′ × 16′ × 12′, no allowance being made for doors or windows?

2. At $6\frac{1}{4}$¢ per square yard, what will it cost to kalsomine the walls and ceiling of a room 24′ × 18′ × 12′, allowing for a door 9′ × 4′, 2 windows 7′ × 4′, and a wainscot 3′ high around the regular surface of the room?

3. Find the cost, at 24¢ per square yard, of painting, with two coats, the outside walls of a tobacco barn 68′ × 20′ × 25′ with gables extending 10′ above the ends of the walls.



4. What will be the cost, at 22¢ per square yard, of painting the outside walls of a barn 100′ × 40′ × 20′ with gables extending 10′ above the walls? with gables extending $12\frac{1}{2}$′ above the walls?

Flooring

264. Flooring is measured by the **square** or by the **thousand square feet.**

Professional floor layers charge by the square, the price being from 75¢ to $1.50 per square. Carpenters usually work by the day in laying floors.

Spruce flooring is 4″ or 5½″ in width; hardwood flooring is 2″ or 2½″ in width. In flooring there is considerable waste in forming the tongue and the groove of the boards. When flooring is 3″ or more in width, it requires about 1¼ sq. ft. of material for every square foot of surface to be covered; when flooring is less than 3″ in width, it requires 1⅓ sq. ft. for every square foot of surface to be covered.

265. Example. How many feet of spruce flooring will be required for a room 32′ × 24′?

Solution. $32 \times 24 = 768$, the number of square feet to be covered.
$1\frac{1}{4} \times 768$ sq. ft. = 960 sq. ft., the quantity of flooring required.

WRITTEN EXERCISE

1. Find the cost at $45 per thousand square feet of a hardwood floor for a room 20′ × 16′.

2. A pavilion is 70′ × 50′. If the flooring is of spruce, what will be the cost at $27 per thousand square feet?

3. In a two-story dwelling the floor area measures 35′6″ × 26′. The first floor is to be of hardwood and the second floor of spruce. Find the quantity of flooring needed.

4. What will be the cost of a hardwood floor in a room 30′ × 28′, if the labor and incidentals cost $25.50, the lumber $30.50 per M., and 60 sq. ft. are allowed for waste?

5. Find the cost of laying an oak floor 20′ × 15′, reckoning the labor and incidentals at $9.50, the floor boards at $83½ per thousand, and estimating that there is a waste of 40 sq. ft.

6. The floors in a three-story dwelling are each 55′ 4″ × 33′ 10″. The first floor is to be of hardwood worth $50 per thousand square feet and the other floors of spruce worth $27 per thousand square feet. If it costs $1.10 per square for labor, what will be the total cost of laying the three floors?

Carpeting

266. Carpet is sold by the **yard.** Such floor covering as oilcloth and linoleum are frequently sold by the **square yard.**

267. In determining the number of yards of carpeting required for a room it is necessary to know whether the strips are to run lengthwise or crosswise.

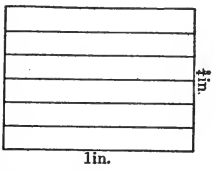
Carpets are generally laid lengthwise of a room; but when the matter of expense is an item, it is sometimes more economical to lay the strips crosswise.

When the length of the strips required is not an even number of yards, there is usually some waste in matching the pattern. Merchants will sell fractional lengths but not fractional widths of carpeting. It is therefore frequently necessary to cut off or turn under a part of a strip.

ORAL EXERCISE

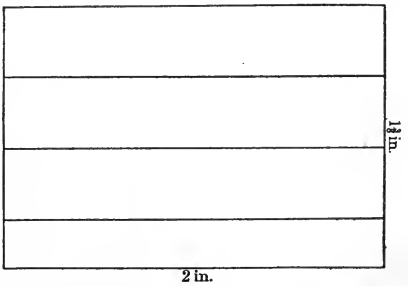
1. How many yards of carpet, 1 yd. wide, must be purchased for a room 5 yd. long by 4 yd. wide?

2. The accompanying diagram represents a room drawn on the scale of $\frac{1}{24}$ of an inch to the foot. Find the dimensions of the room.



3. How many strips of carpet, 1 yd. wide, laid lengthwise of the room, will be required for problem 2? How many feet in each strip? How many yards of carpet will be required for the room?

4. The accompanying diagram represents a room drawn on the scale of $\frac{1}{8}$ in. to the foot. How many strips of carpet, 1 yd. wide, laid lengthwise of the room, will be required to cover it? What part of a strip must be cut off or turned under in this case?



5. How many feet in each strip in problem 4? If there is no waste in matching the pattern, how many feet of carpet will be required? how many yards?

6. If the strips in problem 4 are run crosswise of the room, how many will be required? what will be the length of each strip? If the strips in problem 4 are laid crosswise of the room, it is found that there will be a waste of 6 in. per strip in matching. Under these conditions, how many yards will be required?

7. If the carpet in problem 4 is laid the most economical way, what will it cost at $1.50 per yard?

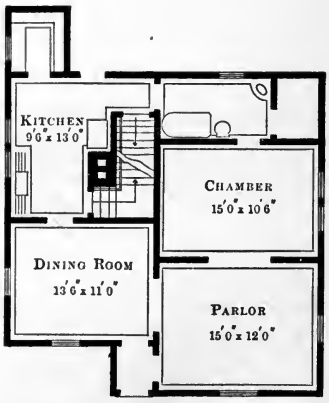
268. Example. How many yards of carpet $\frac{3}{4}$ yd. wide will be required for a parlor floor $20' \times 16'\ 6''$, if the strips run lengthwise and there is a waste of 6 in. on each strip for matching the pattern?

$$16'\ 6'' = \tfrac{3}{2}\tfrac{3}{}' = \tfrac{3}{6}\tfrac{3}{}\text{ yd.}$$
$$\tfrac{3}{6}\tfrac{3}{}\text{ yd.} \div \tfrac{3}{4}\text{ yd.} = 7\tfrac{1}{3}\text{ or 8 strips}$$
$$20' + 6'' = 20\tfrac{1}{2}'$$
$$8 \times 20\tfrac{1}{2}' = 164' = 54\tfrac{2}{3}\text{ yd.}$$

Solution. Since the strips run lengthwise of the room, the width of the room divided by the width of the carpet equals the number of strips required. $\frac{3}{6}\frac{3}{}$ yd. $\div \frac{3}{4} = 7\frac{1}{3}$, the no. of strips; but since an even number of strips must be purchased, $7\frac{1}{3}$ strips must be called 8 strips. The length of the room is 20′ and there is a waste of 6 in. per strip; hence $20\frac{1}{2}'$ of carpet must be purchased for each strip. 8 times $20\frac{1}{2}' = 164' = 54\frac{2}{3}$ yd., the required result.

WRITTEN EXERCISE

1. How many yards of carpeting 1 yd. wide will be required to cover the chamber in the accompanying floor plan, if the strips are to run lengthwise and there is no waste in matching the pattern?



2. Find the number of yards of carpet required to cover the room in problem 1 if the strips run across the room and there is a waste of 6 in. per strip in matching the pattern.

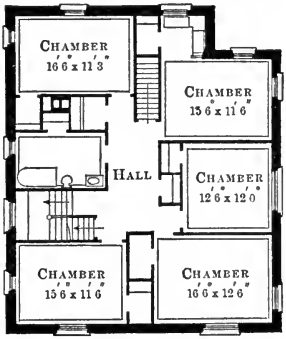
3. If the chamber is carpeted in the most economical way, what will be the cost at $1.25 per yard?

4. How many yards of carpet $\frac{3}{4}$ yd. wide will be required for the parlor in the foregoing floor plan? The strips are to run lengthwise and there is no waste in matching the pattern.

The cheaper grades of carpet are usually 1 yd. wide. The expensive grades, such as Brussels, Wilton, etc., are $\frac{3}{4}$ yd. wide.

5. How many yards of carpet $\frac{3}{4}$ yd. wide will be required for the dining room in the foregoing floor plan? The strips are to run lengthwise and there is a waste of 6 in. per strip in matching the pattern.

6. A rug $18' \times 24'$ is placed centrally on a floor $24' \times 30'$ and filling is used to cover the remainder of the room. If the rug cost \$29.50 and the filling $27\frac{1}{2}¢$ per yard, what is the cost of covering the floor?



7. The five chambers in the accompanying diagram are to be covered with carpet 1 yd. wide, that can be matched without waste. The strips in each room are to run in the direction requiring the smaller number of yards. At $85¢$ per yard, what will it cost to cover the five floors?

PAPERING

269. Wall paper is usually sold in double rolls 18 in. wide and 16 yd. long.

Single rolls 18 in. wide and 8 yd. long are sometimes used, but it is generally found more economical to use double rolls. These dimensions vary more or less.

Allowances for openings, such as doors and windows, are made in different ways by different paper hangers. Some make a uniform allowance for each opening, while others make allowance for the exact measurements of the openings.

Any whole number of rolls left over after papering may usually be returned to the dealer.

ORAL EXERCISE

1. What will the border for a room 15′ × 18′ cost at 33⅓¢ per yard?

2. 18 in. = $\frac{3}{2}$ ft. 30 ft ÷ $\frac{3}{2}$ ft. = 30 ft. × $\frac{2}{3}$ ft. = 20. Divide 21 ft. by 18 in.

3. A wall is 15 ft. long and 9 ft. high. If there are no openings, how many strips will be required to cover it? How many full strips can be cut from each double roll of paper? What part of a strip will run to waste? How many rolls will be required for the wall?

4. Suppose that in problem 2 there is a door 3′ × 8′. What is the length of the regular surface of the wall? Fractional strips must be counted as full strips. Why? How many strips of paper will be required to cover the regular surface of the wall? Will dealers sell a fractional part of a roll of paper? How many rolls, then, will be required for the regular surface of the walls?

5. There is a small surface over the door in problem 5 that has not been considered. What may be used to cover this surface?

270. Obviously, to estimate the quantity of paper required for a room:

From the perimeter of the room subtract the width of the openings. Find $\frac{2}{3}$ of this remainder and the result will be the number of strips required. Divide the number of strips required by the number of full strips that can be cut from each roll of paper and the result is the required number of rolls.

By this method the ends of the rolls are supposed to be utilized for the surface above the doors and above and below the windows and other irregular places.

The height of the room, in papering, will be understood to mean the distance from the baseboard to the frieze.

To estimate the paper required for a ceiling, take $\frac{3}{4}$ of the width of the room for the number of strips required. Divide the number of strips required by the number of full strips that can be cut from each roll and the result is the number of rolls of paper required.

271. Example. How many double rolls of paper will be required for the walls and ceiling of a room $21' \times 18' \times 8'$, allowing for 2 doors and 3 windows, each $3\frac{1}{2}$ ft. wide?

SOLUTION

$(21' + 18') \times 2 = 78'$, the perimeter of the room.
$5 \times 3\frac{1}{2}' = 17\frac{1}{2}'$, the total width of the openings.
$78' - 17\frac{1}{2}' = 60\frac{1}{2}'$, the perimeter of the regular surface of the walls.
$\frac{2}{3}$ of $60\frac{1}{2} = 40\frac{1}{3}$, the number of strips of paper necessary for the regular surface.
$48' \div 8' = 6$, the number of strips in each roll.
$40\frac{1}{3}$ strips $\div$ 6 strips $= 6\frac{1}{18}$, or practically 7 rolls of paper required for the walls.
$\frac{2}{3}$ of $18 = 12$, the number of strips required for the ceiling.
$48' \div 21' = 2\frac{2}{7}$, or practically 2, the number of strips in each roll.
12 strips $\div$ 2 strips $= 6$, the number of rolls required for the ceiling.
6 rolls + 7 rolls = 13 rolls required for the walls and ceiling.

WRITTEN EXERCISE

1. The rooms in the floor plan, page 210, are $9'$ high. What will it cost, at $95¢$ a roll, to paper the walls and ceiling of the parlor, making allowance for 2 double doors, each $6'$ wide, 1 single door $3\frac{1}{2}'$ wide, and 2 windows, each $3\frac{1}{2}'$ wide?

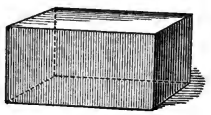
2. How many rolls of paper will be required for the walls and ceiling of the dining room in the floor plan, page 210, allowing for 1 double door $6'$ wide, 1 single door $3\frac{1}{2}'$ wide, and 2 windows each $3\frac{1}{2}'$ wide?

3. At $43¢$ per roll how much will it cost to paper the walls and ceiling of the chamber in the floor plan, page 210, allowing for 2 windows, each $3\frac{1}{2}'$ wide, 1 double door $6'$ wide, and 1 single door $3\frac{1}{2}'$ wide.

SOLIDS

Rectangular Solids

272. A **solid** is that which has *length*, *width*, and *thickness*.

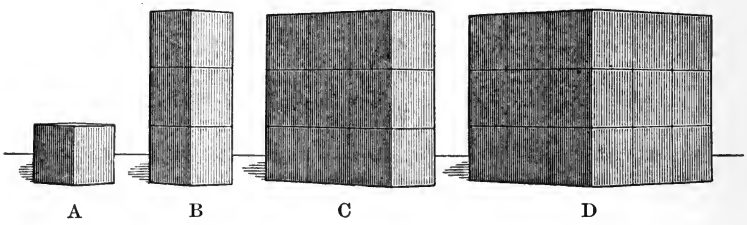


273. A **rectangular solid** is a solid bounded by six rectangular surfaces.

274. A **cube** is a rectangular solid having six square faces.

ORAL EXERCISE

1. If A in the accompanying series of diagrams is 1 cu. ft., how many cubic feet in B? in C? in D?



2. How many cubic feet in a block of granite 6 ft. long, 1 ft. wide, and 1 ft. high? in a block 6 ft. long, 3 ft. wide, and 1 ft. high? in a block 6 ft. long, 3 ft. wide, and 3 ft. high?

3. Find the volume of a rectangular solid 6 ft. by 4 ft. by 2 ft.; a rectangle 10 ft. by 9 ft. by 9 ft.

4. A cellar is 40 ft. square and 6 ft. deep. How many cubic yards of earth were removed in excavating it?

SOLUTION. A cube 1 ft. on the side contains 1 cu. ft. The given cube is 40×1 ft. long, 40×1 ft. wide, and 6×1 ft. high. Therefore, it contains $6 \times 40 \times 40 \times 1$ cu. ft., or 9600 cu. ft.; and 9600 cu. ft. $= 355\frac{5}{9}$ cu. yd., the required result.

$6 \times 40 \times 40 \times 1$ cu. ft. $= 9600$ cu. ft.

9600 cu. ft. $\div 27 = 352\frac{5}{9}$ cu. yd.

275. In the foregoing exercises it is clear that *the product of the three dimensions of a solid equals the volume or solid contents.*

WRITTEN EXERCISE

1. A box car is 50 ft. 6 in. long, 8 ft. 4 in. wide, and 3 yd. high. What is its volume?

2. A piece of timber is 60 ft. long and 18 in. square. How many cubic feet does it contain?

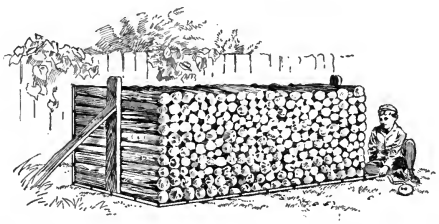
3. A village constructs a reservoir for a water supply. The length is 100 yd., the width 70 yd., and the depth 15 ft. What will be the cost, at 23¢ per cubic yard, of excavating the reservoir?

Wood

276. Wood is measured by the cord.

277. A **cord** of wood or stone is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. It contains 128 cu. ft.

The word "cord," as practically used in wood measure, generally means a pile 8 ft. long and 4 ft. high, the price depending on the length of the stick.



278. Example. How many cords of wood in a pile 32 ft. long, 8 ft. wide, and 4 ft. high?

SOLUTION. $\frac{4 \times 32 \times 8}{128} = 8$; that is, there are 8 cd. in the pile.

4

WRITTEN EXERCISE

1. How many cords in a pile of wood 60 ft. long, 4 ft. wide, and 6 ft. high?

2. A pile of wood contains 5 cd. If it is 4 ft. wide and 4 ft. high, how long is it?

3. A pile of tan bark contains 150 cd. If it is 4 ft. wide and 8 ft. high, how long is it?

4. A pile of wood contains 8 cd. It is 64 ft. long and as high as it is wide. What is the height of the pile?

Lumber

279. A **foot of lumber**, sometimes called a **board foot**, is a board 1 ft. long, 12 in. wide, and 1 in. thick, or its equivalent. An exception to this is made in the measurement of boards *less than* 1 *in.* in thickness. A square foot of the surface of such boards is regarded as a foot of lumber regardless of the thickness. Boards more than one inch in thickness, planks, joists, beams, scantling, and sawed timber are generally measured by the board foot.

Thus, a board 12 ft. long, 12 in. wide, and 1 in. thick contains 12 *sq. ft. of surface*, or 12 *board feet;* a board 12 ft. long, 12 in. wide, and $\frac{1}{2}$, $\frac{3}{4}$, or $\frac{7}{8}$ in. thick contains 12 *sq. ft. of surface*, or 12 *board feet;* but a board 12 ft. long, 12 in. wide, and $2\frac{1}{2}$ in. thick contains 30 *board feet.*

Scantling is timber $3\frac{1}{4}$ in. wide and from 2 in. to 4 in. thick; *joists* are narrow and deep sticks of lumber; *planks* are thick boards; lumber heavier than joists or scantling is usually called *timber.*

Except when sawed to order and in cherry, black walnut, etc., where the price is 15¢ a board foot and upward, the width of a board is reckoned only the **next smaller half inch.** Thus, a board $10\frac{1}{4}$ in. wide is reckoned as 10 in., and a board $10\frac{3}{8}$ in. wide is reckoned as $10\frac{1}{2}$ in.

The **average width** is used in measuring boards that taper uniformly. Thus, a tapering board 12 ft. long, 8 in. wide, at one end and 6 in. wide at the other and 1 in. thick averages 7 in. wide and contains 7 ft. of lumber.

ORAL EXERCISE

1. How many square feet in the surface of a board 12 ft. long, 8 in. wide, and 1 in. thick? How many board feet?

2. How many board feet in a board 12 ft. long, 4 in. wide, and $\frac{7}{8}$ in. thick?

3. How many feet, board measure, in a board 12 ft. long, 12 in. wide, and 2 in. thick?

4. How many feet of lumber in 65 boards each 12 ft. long, 6 in. wide, and 1 in. thick?

280. In **charging** or **billing lumber** the number of pieces is entered first; then the thickness and width in inches and the length in feet; and finally, the article.

Thus, in billing 12 pc. hemlock, 2 in. thick, 6 in. wide, 12 ft. long, the form would be: 12 pc. $2'' \times 6''$, $12'$, hemlock.

ORAL EXERCISE

1. How many board feet in 6 planks, $1\frac{1}{2}'' \times 12''$, $14'$?

SUGGESTION. By inspection eliminate 12 in the dividend.
Then, $1\frac{1}{2} \times 6 \times 14 = 126$, the required number of board feet.

2. How many feet, board measure, in 6 planks $2'' \times 8''$, $18'$?

SUGGESTION. By inspection cancel a 12 in the dividend (6×2).
Then, $8 \times 18 = 144$, the required number of feet, board measure.

3. How many feet of lumber in 6 pc. of scantling $4'' \times 4''$, $16'$?

SUGGESTION. Mentally picture the problem arranged in form for cancellation $\left(\frac{6 \times 4 \times 4 \times 16}{12}\right)$. Cancel a 12 in the dividend ($\frac{1}{12}$ of $\overline{6 \times 4}$). Then, $2 \times 4 \times 16$, or 128, equals the required number of feet of lumber.

4. How many feet of lumber in 5 sticks, $2'' \times 6''$, $16'$?

SUGGESTION. Mentally picture the problem in form for cancellation $\left(\frac{5 \times 2 \times 6 \times 16}{12}\right)$. Cancel a 12 in the dividend ($\frac{1}{12}$ of $\overline{2 \times 6}$). Then, 5×16, or 80, equals the required number of feet of lumber.

5. How many feet of lumber in a plank $3'' \times 12''$, $16'$? in 6 planks? in 10 planks? How many feet of lumber in a board $2'' \times 6''$, $12'$? in 5 boards? in 20 boards?

281. Obviously, the number of board feet in lumber 1 in. or less in thickness is $\frac{1}{12}$ *of the product of the length in feet by the width in inches;* and the number of board feet in lumber more than 1 in. in thickness is $\frac{1}{12}$ *of the product of the length in feet by the width and thickness in inches.* But the work may be materially shortened by mentally cancelling 12 from the dividend as illustrated in the foregoing exercise.

ORAL EXERCISE

State the number of feet, board measure, in the following hemlock:

1.	5 pc., $3'' \times 4''$, $14'$.	**13.**	12 pc., $2'' \times 8''$, $18'$.
2.	6 pc., $2'' \times 4''$, $20'$.	**14.**	6 pc., $8'' \times 10''$, $20'$.
3.	6 pc., $2'' \times 4''$, $20'$.	**15.**	30 pc., $2'' \times 6''$, $20'$.
4.	20 pc., $2'' \times 6''$, $14'$.	**16.**	6 pc., $8'' \times 10''$, $21'$.
5.	12 pc., $2'' \times 8''$, $14'$.	**17.**	25 pc., $3'' \times 8''$, $14'$.
6.	25 pc., $3'' \times 4''$, $12'$.	**18.**	10 pc., $2'' \times 6''$, $13'$.
7.	25 pc., $2'' \times 6''$, $20'$.	**19.**	15 pc., $2'' \times 6''$, $18'$.
8.	25 pc., $3'' \times 8''$, $16'$.	**20.**	15 pc., $2'' \times 6''$, $12'$.
9.	10 pc., $3'' \times 4''$, $14'$.	**21.**	16 pc., $2'' \times 6''$, $10'$.
10.	10 pc., $2'' \times 8''$, $18'$.	**22.**	10 pc., $8'' \times 10''$, $15'$.
11.	14 pc., $2'' \times 6''$, $20'$.	**23.**	15 pc., $8'' \times 10''$, $12'$.
12.	10 pc., $3'' \times 6''$, $20'$.	**24.**	200 pc., $2'' \times 6''$, $20'$.

WRITTEN EXERCISE

How many feet, board measure, in each of the following?

1. 100 joists, $4'' \times 4''$, $16'$.
2. 65 boards, $\frac{3}{4}'' \times 6''$, $12'$.
3. 12 timbers, $8'' \times 8''$, $40'$.
4. 70 joists, $2'' \times 10''$, $32'$.
5. 8 beams, $10'' \times 10''$, $24'$.
6. 10 beams, $12'' \times 12''$, $30'$.

7. At \$19 per M, find the total cost of:

6 joists, $2'' \times 8''$, $12'$.
12 joists, $2'' \times 8''$, $13'$.
30 joists, $2'' \times 8''$, $15'$.
5 joists, $2'' \times 8''$, $18'$.
17 joists, $2'' \times 6''$, $16'$.
30 joists, $2'' \times 8''$, $16'$.

8. At \$16 per M, find the total cost of:

7 beams, $9'' \times 9''$, $20'$.
24 joists, $2'' \times 10''$, $18'$.
150 boards, $\frac{7}{8}'' \times 5''$, $12'$.
27 planks, $1\frac{1}{2}'' \times 14''$, $14'$.
16 beams, $9'' \times 9''$, $18'$.
75 planks, $2\frac{1}{2}'' \times 8''$, $12'$.
576 boards, $1'' \times 9''$, $16'$.
40 scantlings, $2'' \times 4''$, $12'$.

9. Find the cost, at \$10 per M, of the lumber required to fence both sides of a railroad 10 mi. long. The boards used are $1'' \times 6''$, $16'$, and the fence is 5 boards high.

10. Copy and find the amount of the following bill:

Boston, Mass., Sept. 12, *19*

Mr. JOHN D. MOREY
Somerville, Mass.

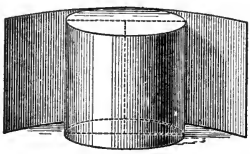
Bought of *E. M. LIVINGSTONE & SON*

Terms 30 days net

20	pc.	3" x 4", 14'	Hemlock	280'	$15.00
10	"	2" x 6", 16'	"		12.00
25	"	3" x 8", 16'	"		12.00
50	"	2" x 4", 20'	"		15.00
16	"	3" x 8", 14'	"		15.00
25	"	2" x 6", 20'	"		12.50
100	"	2" x 6", 18'	"		13.50

Cylinders

282. A **cylinder** is a solid bounded by a uniformly curved surface and two equal parallel circles.



Two circles are parallel when all the points of one are equally distant from all the points of the other. The curved surface of a cylinder is called its *lateral surface;* the parallel circles its *bases.*

283. If the lateral surface of a cylinder be exactly covered with paper, it will be found that the paper is in the form of a rectangle whose length and width are equal to the circumference and height, respectively, of the cylinder. Hence,

The product of the circumference and height of a cylinder equals the area of its lateral surface.

ORAL EXERCISE

1. If the accompanying diagram is a solid 4 ft. square and 12 ft. high, what is the area of its six sides?

2. Give a brief rule for finding the entire surface (lateral surface and bases) of a rectangular solid; of a cylinder.

3. How many cubic feet in a block 2 in. square and 1 in. high? in a block 2 in. square and 10 in. high?

4. The area of the base of a cylinder is 22 ft. If the cylinder is 1 ft. high, what is its volume? if it is 12 ft. high?

284. In the foregoing exercise it is clear that *the area of the base multiplied by the height of the cylinder equals the volume.*

WRITTEN EXERCISE

1. What will be the cost, at 40¢ per cubic yard, of excavating for a cistern 10 ft. in diameter and 23 ft. deep?

2. A man dug a well 6 ft. in diameter and 38 ft. deep. How much should he receive if he was paid $1 for each cubic yard of earth removed?

3. What will be the cost, at $12\frac{1}{2}$¢ per square foot, of a sheet-iron smokestack $2\frac{1}{2}$ ft. in diameter and 30 ft. high?

Stone Work

285. Stone work is usually measured by the **perch**, which is a mass of stone $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, containing $24\frac{3}{4}$ cu. ft.

In some localities the perch contains $16\frac{1}{2}$ cu. ft.

286. **Masonry** is measured by the *cubic yard* or the *perch*.

In measuring stone work, such as the walls of cellars and buildings, masons take the distance around the outside of the wall (the girt) for the length. In this way the corners are measured twice, but this is considered offset by the extra work required in building the corners.

The work around openings, such as doors and windows, is also more difficult than the straight work and on this account no allowance is usually made for openings, unless they are very large.

WRITTEN EXERCISE

1. How many perches of stone will be required for an 18-in. foundation $72' \times 40' \times 10'$?

2. How many perches of masonry in the 18-in. walls of a cellar $40' \times 30' \times 8'$?

3. How many cubic yards of masonry in the foundation walls of a house $42' \times 32'$ if the walls are $2\frac{1}{2}$ ft. wide and 8 ft. high? (Solve (*a*) by mason's and (*b*) by actual measure.)

Brick Work

287. **A common brick** is 8 in. long, 4 in. wide, and 2 in. thick.

Bricks vary in size, but the common brick may be taken as a unit for measuring brick work. Contractors and builders do not follow any uniform rule for estimating the number of bricks required for a wall. It is sufficiently accurate, however, to reckon 22 common bricks, laid in mortar, for each cubic foot of wall. **In estimating material** for a brick wall actual measurements are taken and an allowance made for doors and windows and other openings. **In estimating labor** girt measurements are taken and usually a stated allowance made for openings such as doors and windows. The allowance to be made for openings is generally covered by contract. In some localities a uniform number of cubic feet is deducted for each opening; in others one half the volume of all openings is deducted; in still others nothing whatever is deducted.

WRITTEN EXERCISE

1. How many common bricks will be required for a wall 84 ft. long, $16\frac{1}{2}$ ft. high, and $1\frac{1}{2}$ ft. thick?

2. Find the cost of the bricks required to build a wall 300 ft. long, 12 ft. high, and 18 in. thick, at \$6 per thousand.

3. How many bricks will be required for the four walls of a building $80' \times 50' \times 25'$ if the walls are 18 in. thick and 500 cu. ft. is allowed for openings? (Solve (*a*) by mason's measure, making allowance for the openings, and (*b*) by actual measure.)

CAPACITY

Bins

288. The **stricken bushel** is used in measuring grain. The **heaped bushel** is used in measuring such things as large fruits, vegetables, coal, and corn on the cob. A stricken bushel equals 2150.42 cu. in. A heaped bushel equals 2747.71 cu. in.

ORAL EXERCISE

1. How many bushels of wheat in 2,150,420 cu. in.?

2. State a rule for finding the exact number of stricken bushels in a bin. What part of a stricken bushel is 1 cu. ft.?

Solution. 2150.42 cu. in. = 1 bu., stricken measure. 1728 cu. in. = 1 cu. ft. Therefore, 1 cu. ft. = 172800 ÷ 215042, or approximately .8 of a bushel, stricken measure.

$$2150.42\overline{)1728.000}\ .8+$$
$$1720\ 336$$
$$7664$$

3. Find the approximate capacity, in stricken bushels, of a cubical bin the inside of which measures 10 ft. on a side; in cubic inches of 800 bu. of wheat.

4. State a brief rule for finding the approximate number of stricken bushels in a bin; the approximate number of cubic feet in any number of stricken bushels.

5. How many bushels of potatoes in a bin containing 2,747,710 cu. in.? State a rule for finding the exact number of heaped bushels in any number of cubic inches. Reduce a cubic foot to a decimal of a heaped bushel.

SOLUTION. 2747.71 cu. in. = 1 bu., heaped measure. Therefore, 1 cu. ft. = 172800 ÷ 274771, or approximately .63 of a bushel, heaped measure.

```
                .63 —
2747.71)1728.0000
        1648 626
          79 3740
          82 4313
```

6. Find the approximate capacity, in heaped bushels, of 1000 cu. ft.; in cubic feet, of 630 bu.

7. State a short method of reducing cubic feet to heaped bushels; heaped bushels to cubic feet.

8. Find (*a*) the approximate capacity and (*b*) the exact capacity, in stricken bushels, of a bin $10' \times 5' \times 4'$.

SOLUTIONS

$10' \times 5' \times 4' = 200$ cu. ft.
(*a*) 200 × 1728 cu. in. = 345600 cu. in.
345600 cu. in. ÷ 2150.42 = 165.31 + bu.

(*b*) $10' \times 5' \times 4' = 200$ cu. ft.
.8 of 200 cu. ft. = 160 bu.

9. Find (*a*) the approximate capacity and (*b*) the exact capacity, in heaped bushels, of the bin in problem 14.

SOLUTIONS

$10' \times 5' \times 4' = 200$ cu. ft.
(*a*) 200 × 1728 cu. in. = 345600 cu. in.
345600 cu. in. ÷ 2747.71 = 125.77 bu.

(*b*) $10' \times 5' \times 4' = 200$ cu. ft.
.63 of 200 cu. ft. = 126 bu.

ORAL EXERCISE

1. Find the approximate capacity in bushels of a wheat bin 10 ft. long, 8 ft. wide, and 5 ft. high.

2. A square bin 10 ft. high contains, by approximate measurements, 800 bu. What is its width?

3. Approximately, how many bushels of potatoes may be stored in a bin 10 ft. long, 5 ft. wide, and 4 ft. high?

WRITTEN EXERCISE

Find the approximate capacity in stricken bushels of:

1. A bin 12 ft. square and 4 ft. deep.

Inside dimensions are given in all the problems of this and similar exercises.

2. A box 6 ft. long, $2\frac{1}{2}$ ft. wide, and $3\frac{1}{2}$ ft. deep.

3. A wagon box 10 ft. 6 in. long, 4 ft. wide, and 2 ft. deep.

4. A farmer wishes to construct a square granary 15 ft. on each side that will hold 800 bu. of grain. How deep must the bin be made? (Approximate rule.)

5. A man wishes to construct a coal bin that will store 200 bu. of stove coal. If the bin is 20 ft. wide and 5 ft. deep, what must be the length? (Approximate rule.)

6–8. Find the exact capacity, in stricken bushels, of problems 1–3.

9–11. Find the approximate capacity, in heaped bushels, of problems 1–3.

Cisterns

289. A gallon equals 231 cu. in.

ORAL EXERCISE

1. How many gallons in 462 cu. in.? in 1386 cu. in.?

2. How many gallons of water in a vat 22 in. long, 7 in. high, and 3 in. wide?

3. Give a rule for finding the exact number of gallons in a vessel. How many gallons in a cubic foot?

Solution. 231 cu. in. = 1 gal. 1728 cu. in. = 1 cu. ft. Therefore, 1 cu. ft. $= \frac{1728}{231}$ gal. = 7.48 + gal., or approximately $7\frac{1}{2}$ gal.

4. Find the approximate capacity, in gallons, of a vat 5 ft. square and 4 ft. high.

Solution. 5 ft. × 5 ft. × 4 ft. = 100 cu. ft. 100 times $7\frac{1}{2}$ gal. = 750 gal.

5. State a rule for finding the approximate capacity, in gallons, of a vessel.

WRITTEN EXERCISE

Find the capacity (approximate and exact), in gallons, of:

1. A cistern 6 ft. square and 12 ft. deep.

2. A cistern 6 ft. in diameter and 10 ft. deep.

3. A tank 5 ft. long, 4 ft. wide, and 6 ft. deep.

4. A cistern 15 ft. in diameter and 20 ft. deep.

CALCULATION TABLES

290. Persons who have a great deal of computing to do frequently use machines (see pages 47 and 55) and calculation tables to aid them in their work. The table on page 225 will give a good idea of the arrangement of calculation tables that are used in making up and proving bills and invoices, computing wages, finding percentages, etc. The following examples will illustrate a few of the many uses of such tables.

291. Examples. **1.** Multiply 58 by 42.

SOLUTION. Under 58 and opposite 42 find 2436.

2. How many square yards in a floor $38' \times 46'$?

SOLUTION. Under 46 and opposite 38 find 1748; that is, 1748 sq. yd.

3. Find the cost of 495 yd. wash silk at 39¢.

SOLUTION. Under 495 and opposite 39 find 19,305; that is, $193.05.

4. Find the cost of 48,000 bricks at $4.95 per M.

SOLUTION. Under 495 and opposite 48 find 23,760. Since the zeros in 48,000 have been rejected, there are but two places to point off. Result $237.60.

5. Find the cost of 46 hr. of labor at $25\frac{3}{4}$¢ per hour.

SOLUTION. Under 46 and opposite 25 find 1150 ($11.50); under 46 and opposite $\frac{3}{4}$ find 34.50 (35¢). $11.50 + 35¢ = $11.85, the required result.

ORAL EXERCISE

By the aid of the table state the product of:

1. 27×26.	**5.** 39×27.	**9.** $87 \times 46¢$.	**13.** $35 \times 93¢$.
2. 27×58.	**6.** 45×58.	**10.** $93 \times 32¢$.	**14.** $93 \times 42¢$.
3. 45×46.	**7.** 37×46.	**11.** $48 \times 93¢$.	**15.** $46 \times 87¢$.
4. 47×39.	**8.** 49×58.	**12.** $47 \times 87¢$.	**16.** $38 \times 93¢$.

17. Find the cost of 49,500 lb. of old rags at $\frac{3}{4}$¢.

18. Find the cost of 93,000 bricks at $5.25 per M.

19. Find the cost of 37 days' labor at $1.35 per day; at $5.25.

20. Find the cost of 109 hours' labor at 27¢; at 39¢; at 46¢.

21. Find the cost of 49,500 lb. freight at 31¢ per hundredweight; of 46,000 lb. at 27¢ per hundredweight.

Calculation Table

Multiplier	27	39	46	58	Multiplier	87	93	109	128	Multiplier	135	147	495	525	Multiplier
1	27	39	46	58	1	87	93	109	128	1	135	147	495	525	1
2	54	78	92	116	2	174	186	218	256	2	270	294	990	1050	2
3	81	117	138	174	3	261	279	327	384	3	405	441	1485	1575	3
4	108	156	184	232	4	348	372	436	512	4	540	588	1980	2100	4
5	135	195	230	290	5	435	465	545	640	5	675	735	2475	2625	5
6	162	234	276	348	6	522	558	654	768	6	810	882	2970	3150	6
7	189	273	322	406	7	609	651	763	896	7	945	1029	3465	3675	7
8	216	312	368	464	8	696	744	872	1024	8	1080	1176	3960	4200	8
9	243	351	414	522	9	783	837	981	1152	9	1215	1323	4455	4725	9
10	270	390	460	580	10	870	930	1090	1280	10	1350	1470	4950	5250	10
11	297	429	506	638	11	957	1023	1199	1408	11	1485	1617	5445	5775	11
12	324	468	552	696	12	1044	1116	1308	1536	12	1620	1764	5940	6300	12
13	351	507	598	754	13	1131	1209	1417	1664	13	1755	1911	6435	6825	13
14	378	546	644	812	14	1218	1302	1526	1792	14	1890	2058	6930	7350	14
15	405	585	690	870	15	1305	1395	1635	1920	15	2025	2205	7425	7875	15
16	432	624	736	928	16	1392	1488	1744	2048	16	2160	2352	7920	8400	16
17	459	663	782	986	17	1479	1581	1853	2176	17	2295	2499	8415	8925	17
18	486	702	828	1044	18	1566	1674	1962	2304	18	2430	2646	8910	9450	18
19	513	741	874	1102	19	1653	1767	2071	2432	19	2565	2793	9405	9975	19
20	540	780	920	1160	20	1740	1860	2180	2560	20	2700	2940	9900	10500	20
21	567	819	966	1218	21	1827	1953	2289	2688	21	2835	3087	10395	11025	21
22	594	858	1012	1276	22	1914	2046	2398	2816	22	2970	3234	10890	11550	22
23	621	897	1058	1334	23	2001	2139	2507	2944	23	3105	3381	11385	12075	23
24	648	936	1104	1392	24	2088	2232	2616	3072	24	3240	3528	11880	12600	24
25	675	975	1150	1450	25	2175	2325	2725	3200	25	3375	3675	12375	13125	25
26	702	1014	1196	1508	26	2262	2418	2834	3328	26	3510	3822	12870	13650	26
27	729	1053	1242	1566	27	2349	2511	2943	3456	27	3645	3969	13365	14175	27
28	756	1092	1288	1624	28	2436	2604	3052	3584	28	3780	4116	13860	14700	28
29	783	1131	1334	1682	29	2523	2697	3161	3712	29	3915	4263	14355	15225	29
30	810	1170	1380	1740	30	2610	2790	3270	3840	30	4050	4410	14850	15750	30
31	837	1209	1426	1798	31	2697	2883	3379	3968	31	4185	4557	15345	16275	31
32	864	1248	1472	1856	32	2784	2976	3488	4096	32	4320	4704	15840	16800	32
33	891	1287	1518	1914	33	2871	3069	3597	4224	33	4455	4851	16335	17325	33
34	918	1326	1564	1972	34	2958	3162	3706	4352	34	4590	4998	16830	17850	34
35	945	1365	1610	2030	35	3045	3255	3815	4480	35	4725	5145	17325	18375	35
36	972	1404	1656	2088	36	3132	3348	3924	4608	36	4860	5292	17820	18900	36
37	999	1443	1702	2146	37	3219	3441	4033	4736	37	4995	5439	18315	19425	37
38	1026	1482	1748	2204	38	3306	3534	4142	4864	38	5130	5586	18810	19950	38
39	1053	1521	1794	2262	39	3393	3627	4251	4992	39	5265	5733	19305	20475	39
40	1080	1560	1840	2320	40	3480	3720	4360	5120	40	5400	5880	19800	21000	40
41	1107	1599	1886	2378	41	3567	3813	4469	5248	41	5535	6027	20295	21525	41
42	1134	1638	1932	2436	42	3654	3906	4578	5376	42	5670	6174	20790	22050	42
43	1161	1677	1978	2494	43	3741	3999	4687	5504	43	5805	6321	21285	22575	43
44	1188	1716	2024	2552	44	3828	4092	4796	5632	44	5940	6468	21780	23100	44
45	1215	1755	2070	2610	45	3915	4185	4905	5760	45	6075	6615	22275	23625	45
46	1242	1794	2116	2668	46	4002	4278	5014	5888	46	6210	6762	22770	24150	46
47	1269	1833	2162	2726	47	4089	4371	5123	6016	47	6345	6909	23265	24675	47
48	1296	1872	2208	2784	48	4176	4464	5232	6144	48	6480	7056	23760	25200	48
49	1323	1911	2254	2842	49	4263	4557	5341	6272	49	6615	7203	24255	25725	49
50	1350	1950	2300	2900	50	4350	4650	5450	6400	50	6750	7350	24750	26250	50

Multiplier	27	39	46	58	Multiplier	87	93	109	128	Multiplier	135	147	495	525	Multiplier
⅛	3 38	4 88	5 75	7 25	⅛	10 88	11 63	13 63	16 00	⅛	16 88	18 38	61 88	65 63	⅛
¼	6 75	9 75	11 50	14 50	¼	21 75	23 25	27 25	32 00	¼	33 75	36 75	123 75	131 25	¼
⅜	10 13	14 63	17 25	21 75	⅜	32 63	34 88	40 88	48 00	⅜	50 63	55 13	185 63	196 88	⅜
½	13 50	19 50	23 00	29 00	½	43 50	46 50	54 50	64 00	½	67 50	73 50	247 50	262 50	½
⅝	16 88	24 38	28 75	36 25	⅝	54 38	58 13	68 13	80 00	⅝	84 38	91 88	309 38	328 13	⅝
¾	20 25	29 25	34 50	43 50	¾	65 25	69 75	81 75	96 00	¾	101 25	110 25	371 25	393 75	¾
⅞	23 63	34 13	40 25	50 75	⅞	76 13	81 38	95 38	112 00	⅞	118 13	128 63	433 13	459 38	⅞

22. Find the cost of 48,000 ft. of lumber at \$16 per M; of 93,000 ft.; of 52,500 ft.; of 49,500 ft.; of 58,000 ft.

23. An agent sold 240 (10 × 24) excursion tickets at \$4.95. How much did he receive? $360 \times \$5.25 = ?$ $310 \times \$1.47 = ?$

24. Find the cost of 45 rm. of paper at \$1.35; at \$1.28; at \$1.09; at 93¢; at \$4.95. Also find the cost of 38 rm. at each of the above prices; of 29 rm.; of 37 rm.; of 46 rm.

25. Find the cost of 4600 lb. of coal at \$6.40 per ton (\$3.20 per thousand pounds); at \$8.40; at \$4.60; at \$6.80; at \$7.20; at \$7.40; at \$9.20; at \$5.60. Also find the cost of 2700 lb. at each of the above prices; of 3900 lb.; of 8700 lb.; of 9300 lb.; of 10,900 lb.; of 12,800 lb.; of 13,500 lb.; of 14,700 lb.; of 49,500 lb.; of 52,500 lb.

WRITTEN EXERCISE

1. By the aid of the table find the total cost of:

525 bolts at \$1.70 per C. 128 bolts at \$1.90 per C.
495 bolts at \$2.40 per C. 525 bolts at \$2.70 per C.
135 bolts at \$1.60 per C. 495 bolts at \$3.50 per C.

2. By the aid of the table find the total cost of:

1280 ft. lumber at \$28 per M. 5250 ft. lumber at \$27 per M.
1350 ft. lumber at \$29 per M. 3800 ft. lumber at \$27 per M.
4950 ft. lumber at \$19 per M. 4600 ft. lumber at \$18 per M.

3. By the aid of the table find the total amount of the following time sheet:

TIME SHEET FOR WEEK ENDING JULY 14

NAME	M.	T.	W.	T.	F.	S.	TOTAL TIME	RATE PER HOUR	AMOUNT	
A. M. Ball	$8\frac{1}{2}$	9	$7\frac{1}{2}$	8	8	8		27¢		
J. B. King	$8\frac{1}{4}$	$7\frac{3}{4}$	9	8	8	8		39¢		
C. E. Frey	$9\frac{1}{4}$	9	$8\frac{3}{4}$	8	7	5		46¢		
W. D. Hall	7	9	8	8	8	8		58¢		
M. F. Hill	$9\frac{3}{4}$	$8\frac{1}{4}$	7	8	8	8		87¢		
D. M. Muir	$8\frac{1}{4}$	$7\frac{1}{4}$	$5\frac{1}{4}$	$9\frac{1}{4}$	$8\frac{3}{4}$	$6\frac{3}{4}$		93¢		

PERCENTAGE AND ITS APPLICATIONS

CHAPTER XVII

PERCENTAGE

ORAL EXERCISE

1. .50 may be read *fifty hundredths*, *one half*, or *fifty per cent.* Read each of the following in three ways: .25, .30, $12\frac{1}{2}\%$.

2. Read each of the following in three ways: $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{25}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{4}$, 2 %, $2\frac{1}{2}\%$, 125 %, $6\frac{1}{4}\%$, $8\frac{1}{3}\%$, $66\frac{2}{3}\%$, 250 %, 375 %.

3. 50 % of a number is .50 or $\frac{1}{2}$ of the number. What is 50 % of \$600? 25 %? $12\frac{1}{2}\%$? 10 %? 40 %? 20 %? 75 %?

292. **Per cent** is a common name for hundredths.

293. The **symbol** % may be read *hundredths* or *per cent.*

294. **Percentage** is the process of computing by *hundredths* or *per cents.*

ORAL EXERCISE

Express as per cents:

1. .28.	**3.** $.00\frac{1}{2}$.	**5.** $.33\frac{1}{3}$.	**7.** $.62\frac{1}{2}$.	**9.** .5.
2. .37.	**4.** $.14\frac{2}{7}$.	**6.** $.28\frac{4}{7}$.	**8.** .0075.	**10.** .2.

Express as decimal fractions:

11. 20 %.	**13.** 72 %.	**15.** $\frac{1}{2}\%$.	**17.** 125 %.	**19.** $\frac{1}{50}\%$.
12. 45 %.	**14.** 18 %.	**16.** $\frac{1}{4}\%$.	**18.** 250 %.	**20.** 375 %.

Express as common fractions:

21. 1 %.	**23.** $2\frac{1}{2}\%$.	**25.** $133\frac{1}{3}\%$.	**27.** $87\frac{1}{2}\%$.	**29.** $\frac{1}{2}\%$.
22. 2 %.	**24.** $3\frac{1}{2}\%$.	**26.** $266\frac{2}{3}\%$.	**28.** $112\frac{1}{2}\%$.	**30.** 175 %.

Express as per cents:

31. $\frac{1}{5}$.	**33.** $\frac{1}{11}$.	**35.** $1\frac{1}{4}$.	**37.** $\frac{6}{5}$.	**39.** $\frac{5}{6}$.
32. $\frac{1}{9}$.	**34.** $\frac{9}{10}$.	**36.** $2\frac{3}{4}$.	**38.** $1\frac{7}{8}$.	**40.** $\frac{24}{6}$.

IMPORTANT PER CENTS AND THEIR FRACTIONAL EQUIVALENTS

PER CENT	FRACTIONAL VALUE	PER CENT	FRACTIONAL VALUE	PER CENT	FRACTIONAL VALUE	PER CENT	FRACTIONAL VALUE
$12\frac{1}{2}\%$	$\frac{1}{8}$	75 %	$\frac{3}{4}$	$83\frac{1}{3}\%$	$\frac{5}{6}$	$6\frac{1}{4}\%$	$\frac{1}{16}$
25 %	$\frac{1}{4}$	100 %	1	20 %	$\frac{1}{5}$	$6\frac{2}{3}\%$	$\frac{1}{15}$
$37\frac{1}{2}\%$	$\frac{3}{8}$	$16\frac{2}{3}\%$	$\frac{1}{6}$	40 %	$\frac{2}{5}$	$8\frac{1}{3}\%$	$\frac{1}{12}$
50 %	$\frac{1}{2}$	$33\frac{1}{3}\%$	$\frac{1}{3}$	60 %	$\frac{3}{5}$	$11\frac{1}{9}\%$	$\frac{1}{9}$
$62\frac{1}{2}\%$	$\frac{5}{8}$	$66\frac{2}{3}\%$	$\frac{2}{3}$	80 %	$\frac{4}{5}$	$14\frac{2}{7}\%$	$\frac{1}{7}$

295. The **terms** used in percentage are the base, the rate, and the percentage. The **base** is the number of which a per cent is taken; the **rate**, the number of hundredths of the base to be taken; the **percentage**, the result obtained by taking a certain per cent of the base.

In the expression "12 % of \$50 is \$6," \$50 is the base, 12 %, the rate, and \$6, the percentage.

296. The base plus the percentage is sometimes called the **amount**; the base minus the percentage, the **difference**.

FINDING THE PERCENTAGE

297. Example. What is 15 % of \$660?

SOLUTION. 15 % of a number equals .15 of it. .15 of \$660 = \$99, the required result.

\$660
.15
\$99.00

298. Obviously, *the product of the base and rate equals the percentage.*

The *base* may be either *concrete* or *abstract*. The *rate* is always *abstract*. The *percentage* is always of the *same name as the base.*

ORAL EXERCISE

1. What aliquot part of 1 is $.12\frac{1}{2}$? .25? .50? $.16\frac{2}{3}$? $.33\frac{1}{3}$? .20? $.06\frac{1}{4}$? $.06\frac{2}{3}$? $.08\frac{1}{3}$? $.11\frac{1}{9}$? $.14\frac{2}{7}$? $37\frac{1}{2}\%$? $62\frac{1}{2}\%$? $66\frac{2}{3}\%$?

2. Formulate a short method for finding $12\frac{1}{2}\%$ of a number.

SOLUTION. $12\frac{1}{2}\% = .12\frac{1}{2} = \frac{1}{8}$; hence, to find $12\frac{1}{2}\%$ of a number, *divide by 8.*

3. State a short method for finding 25 % of a number; 50 %; $16\frac{2}{3}\%$; $33\frac{1}{3}\%$; 20 %; $6\frac{1}{4}\%$; $6\frac{2}{3}\%$; $8\frac{1}{3}\%$; $11\frac{1}{9}\%$.

To guard against absurd answers in exercises of this character estimate the results in advance as explained on pages 58 and 142.

4. Find 50% of 960. Also 25%; $37\frac{1}{2}$%; $12\frac{1}{2}$%; $62\frac{1}{2}$%; 75%; $16\frac{2}{3}$%; $33\frac{1}{3}$%; $66\frac{2}{3}$%; $83\frac{1}{3}$%; 20%; 40%; 60%; $6\frac{1}{4}$%.

5. By inspection find:

a. 50% of \$792.
b. $37\frac{1}{2}$% of \$320.
c. $12\frac{1}{2}$% of \$880.
d. $16\frac{2}{3}$% of \$669.
e. 25% of \$1729.
f. $6\frac{2}{3}$% of \$6600.
g. $6\frac{1}{4}$% of 3296.
h. $83\frac{1}{3}$% of 4560.
i. $66\frac{2}{3}$% of 2460.
j. $33\frac{1}{3}$% of 2793.
k. $8\frac{1}{3}$% of 24,960.
l. 20% of 12,535.

ORAL EXERCISE

1. Find 10% of 720; of \$15.50; of 120 men; of \$127.50.

2. What aliquot part of 10% is 5%? $2\frac{1}{2}$%? $1\frac{1}{4}$%? $3\frac{1}{3}$%? $1\frac{2}{3}$%?

3. Formulate a short method for finding $1\frac{1}{4}$% of a number.

SOLUTION. $1\frac{1}{4}$% of a number is $\frac{1}{8}$ of 10% of the number; hence, to find $1\frac{1}{4}$% of a number, *point off one place to the left and divide by 8.*

4. State a short method for finding 5% of a number; $2\frac{1}{2}$%; $3\frac{1}{3}$%; $1\frac{2}{3}$%.

5. By inspection find:

a. 5% of 720.
b. $2\frac{1}{2}$% of 840.
c. $3\frac{1}{3}$% of 1560.
d. $1\frac{1}{4}$% of 1840.
e. $1\frac{2}{3}$% of \$366.
f. $2\frac{1}{2}$% of \$720.
g. $3\frac{1}{3}$% of \$3900.
h. $1\frac{2}{3}$% of 120 mi.
i. $1\frac{1}{4}$% of 1632 A.

ORAL EXERCISE

1. Compare 24% of \$25 with 25% of \$24; 24% of \$2500 with 25% of \$2400. What is 32% of \$25?

SOLUTION. 32% of \$25 = 25% of \$32 = $\frac{1}{4}$ of \$32 = \$8, the required result.

2. What is 125% of \$880?

SOLUTION. 125% = 1.25 = $\frac{1}{8}$ of 10; $\frac{1}{8}$ of \$8800 (10 times \$880) = \$1100.

3. Find 125% of 400; of 640; of 3200; of 160; of 1280.

4. Formulate a short method for finding $166\frac{2}{3}$% of a number; $333\frac{1}{3}$% of a number; 250% of a number.

5. Compare 88% of 12,500 bu. with 125% of 8800 bu.

6. Find 32% of \$125; of \$1250; of \$12,500; of \$125,000.

7. Find 250% of \$720; of \$3200; of \$28,800; of \$64,800.

ORAL EXERCISE

By inspection find:

1. 48 % of 250.
2. 32 % of 125.
3. 128 % of 250.
4. 16 % of 2500.
5. 180 % of 625.
6. 160 % of 875.
7. 240 % of 7500.
8. 125 % of $240.40.

WRITTEN EXERCISE

1. A farmer sold 640 bu. wheat, receiving $1.05 per bushel for $87\frac{1}{2}$ % of it and 85¢ per bushel for the remainder. What was the total amount received?

2. A grocer compromised with his creditors, paying 60 % of the amount of his debts. If he owed A $756, B $1250, and C $3750, how much did each receive?

3. A merchant sold 360 bbl. apples for $1200. If he received $3.50 per barrel for $66\frac{2}{3}$ % of the apples, what was the price received per barrel for the remainder?

4. A man bought a house for $12,864.75; he expended for improvements $33\frac{1}{3}$ % of the first cost of the property, and then sold it for $20,000. Did he gain or lose, and how much?

5. A commission merchant bought 1200 bbl. apples and after holding them for 3 mo. found that his loss from decay was 10 %. If he sold the remainder at $3.75 per barrel, how much did he receive?

6. A merchant prepaid the following bills and received the per cents of discount named: 4 % on bill of $875.50; 6 % on bill of $378.45; 2 % on bill of $940.50; $3\frac{1}{2}$ % on bill of $400. What was the net amount paid?

FINDING THE RATE

ORAL EXERCISE

1. 8 is what part of 40? what per cent of 40?
2. 90 is what per cent of 270? of 360? of 450?
3. 70 is what per cent of 560? of 630? of 700?
4. The base is 900 and the percentage 450; what is the rate?

299. Example. $35.50 is what per cent of $284?

SOLUTIONS. *a.* $35.50 is $\frac{3550}{2840}$ or $\frac{1}{8}$ of $284. $284 is 100% of itself; hence, $35.50, which is $\frac{1}{8}$ of $284, must be $\frac{1}{8}$ of 100%, or $12\frac{1}{2}$%. Or,

(*a*)

$$\frac{3550}{2840} = \frac{1}{8} = 12\tfrac{1}{2}\ \%$$

b. Since the product of the base and the rate is the percentage, the quotient obtained by dividing the percentage by the base is the rate.

(*b*)

$$284\overline{)35.50}\quad .125 = 12\tfrac{1}{2}\ \%$$

300. Obviously, *the percentage divided by the base equals the rate.*

ORAL EXERCISE

What per cent of:

1. 95 is 19?
2. 4.8 is 1.2?
3. $35 is $17\frac{1}{2}$?
4. 225 A. are 75 A.?
5. 34 bu. are 34 bu.?
6. 34 bu. are 68 bu.?
7. 1.6 is .008?
8. 1 yd. is 1 ft.?
9. 2 da. are 8 hr.?
10. 4 T. are 3000 lb.?
11. 1 yr. are 4 mo.?
12. 2 mi. are 80 rd.?

WRITTEN EXERCISE

1. A man bought a house for $7500 and sold it for $8700. What per cent did he gain?

2. In a certain city, school was in session 190 da. A lost 38 da. What per cent of the school year did he attend?

3. An agent sold a piece of property for $8462.50 and received $338.50 for his services. What per cent did he receive?

4. A commission agent sold 28,600 bu. of grain at 50¢ per bushel and received for his services $357.50. What per cent did he receive on the sales made?

5. Smith and Brown engaged in business, investing $18,000. Smith invested $10,440, and Brown the remainder. What per cent of the total capital did each invest?

6. An agent for a wholesale house earned $165.55 during the month of May. If the goods sold amounted to $1505, what per cent did he receive on the sales made?

FINDING THE BASE

ORAL EXERCISE

1. What is 5 % of 240 bu. ?

2. 12 bu. is 5 % of how many bushels ?

3. 160 is 8 % of what number ? 4 % ? 2 % ? 1 % ? $\frac{1}{2}$ % ? $\frac{1}{4}$ % ?

4. The multiplicand is 400 and the multiplier 10; what is the product ? The product is 2000 and the multiplicand 100; what is the multiplier ? The product is 4000 and the multiplier 20 ; what is the multiplicand ?

5. In percentage what name is given to the product ? to the multiplicand ? to the multiplier ? When the base and rate are given, how is the percentage found ? When the percentage and base are given, how is the rate found ? When the percentage and rate are given, how is the base found ?

ORAL EXERCISE

1. 25 is $\frac{1}{2}$ of what number ? 25 is 50 % of what number ?

2. 12 is $\frac{1}{15}$ of what number ? 24 is $6\frac{2}{3}$ % of what number ?

3. 25 is $\frac{1}{12}$ of what number ? 35 is $8\frac{1}{3}$ % of what number ?

4. 900 is $\frac{3}{4}$ of what number ? 600 is 75 % of what number ?

5. 130 is $\frac{1}{5}$ of what number ? 1300 is 20 % of what number ?

6. 444 is $\frac{4}{5}$ of what number ? 44.40 is 80 % of what number ?

7. 960 is $\frac{2}{3}$ of what number ? 96 is $66\frac{2}{3}$ % of what number ?

8. 65 is $\frac{5}{6}$ of what number ? 650 is $83\frac{1}{3}$ % of what number ?

9. 15 is $\frac{1}{16}$ of what number ? 150 is $6\frac{1}{4}$ % of what number ?

10. 100 is $\frac{1}{9}$ of what number ? 60 is $11\frac{1}{9}$ % of what number ?

11. 20 is $\frac{1}{7}$ of what number ? 200 is $14\frac{2}{7}$ % of what number ?

12. 375 is $\frac{3}{8}$ of what number ? 2700 is $37\frac{1}{2}$ % of what number ?

13. Anything is what per cent of itself ? of $\frac{1}{2}$ itself ? of twice itself ? of $\frac{3}{4}$ itself ? of $2\frac{1}{2}$ times itself ?

14. A farmer sold a horse for $66\frac{2}{3}$ % of its cost and received $80. How much did the horse cost ?

15. 20 % of the students of a high school are 18 yr. of age. If there are 170 such students, what is the aggregate attendance of the school ?

301. Example. 37.5 is 25 % of what number?

SOLUTION. 25 % or $\frac{1}{4}$ of the number = 37.5
$\therefore$ the number = $37.5 \div \frac{1}{4} = 150$.

302. Obviously, *the quotient of the percentage divided by the rate equals the base.*

WRITTEN EXERCISE

1. N invested 30% of the capital of a firm, H 35%, and W the remainder, $1400. What was the capital of the firm?

2. During the month of May the sales of a clothing merchant amounted to $4864.24, which was 8 % of the total sales for the year. What were the total sales for the year?

3. B sold his city property and took a mortgage for $4375, which was $17\frac{1}{2}$% of the value of the property. If the balance was paid in cash, what was the amount of cash received?

4. In compromising with his creditors, a man finds that his assets are $270,900, and that this sum is 43% of his entire indebtedness. What will be the aggregate loss to his creditors?

5. The aggregate attendance in the schools of a certain city for 1 da. was 43,225 students. If this number was 95% of the number of students belonging, how many students were absent?

6. The owner of city property received in rentals last year $1221.95. He paid for insurance $75, for repairs $353.75, and for taxes $175.20. If his net income was equal to 5% of the money invested, what was the value of the property?

7. A man bought a suit of clothes for $22.50, a pair of shoes for $5, a hat for $4, and a watch for $18.75, when he found he had expended $12\frac{1}{2}$% of his money. How much money had he at first? How much had he left after making these purchases?

8. In a recent year there were 5,737,372 farms in the United States having a total acreage of 831,591,744 A., of which 414,498,487 A. were improved and 424,093,287 A. were unimproved. What was the average number of acres to a farm? What per cent of farm land was improved? What per cent was unimproved? (Correct to three decimal places.)

PER CENTS OF INCREASE

ORAL EXERCISE

1. If $2\frac{1}{2}$ times a number is 50, what is the number?
2. If 2.5 times a number is 75, what is the number?
3. If 250% of a number is \$1250, what is the number?
4. If 250% of a number is 150, what is the number? If 250% is 125, what is the number?
5. If 300% of a number is \$5400, what is the number?

303. Examples. 1. A man sold a farm for \$3900 and thereby gained 30%. How much did the farm cost?

SOLUTION. 1.30 of the cost = \$3900.
∴ the cost = \$3900 ÷ 1.30 = \$3000.

2. What number increased by $33\frac{1}{3}$% of itself equals 180?

SOLUTION. $\frac{4}{3}$ of the number = 180
∴ the number = 180 ÷ $\frac{4}{3}$ = 135.

ORAL EXERCISE

What number increased by:

1. 10% of itself is 220?
2. 25% of itself is 125?
3. 50% of itself is 300?
4. 75% of itself is 700?
5. $6\frac{1}{4}$% of itself is 170?
6. $12\frac{1}{2}$% of itself is 180?
7. $66\frac{2}{3}$% of itself is 135?
8. $6\frac{2}{3}$% of itself is 480?
9. 125% of itself is 900?
10. $37\frac{1}{2}$% of itself is 440?
11. $11\frac{1}{9}$% of itself is 300?
12. $14\frac{2}{7}$% of itself is 328?
13. 200% of itself is 2700?
14. 300% of itself is 2800?

WRITTEN EXERCISE

1. I sold 375 bu. of wheat for \$427.50, thereby gaining 20%. How much did the wheat cost me per bushel?
2. A fruit dealer sold a quantity of oranges for \$6.75. If his gain was $12\frac{1}{2}$%, what did the oranges cost him?
3. My savings for March increased $33\frac{1}{3}$% over February. If my savings for March were \$84.36, what were my savings for February and March?

4. A merchant sold a quantity of cloth at \$1.50 per yard and thereby gained 20%. What per cent would he have gained had he sold the cloth at \$1.87½ per yard?

5. A merchant's total sales for this year were 12½% greater than his sales for last year. What were his sales for this year if the aggregate sales for the two years amounted to \$170,000?

6. A man paid \$42.50 for a second-hand wagon and after spending \$20.50 in repairs on it he found that it had cost him 5% more than a new wagon. What would have been the cost of a new wagon?

PER CENTS OF DECREASE

ORAL EXERCISE

1. What per cent of a number is left after taking away 33⅓% of it? What fractional part?

2. If ⅔ of a number is 600, what is the number? If 66⅔% of a number is 75, what is the number?

3. A man spent 40% of his money and had \$60 remaining. How much had he at first? How much did he spend?

304. Examples. **1.** A man sold a horse for \$332, thereby losing 17%. What was the cost?

SOLUTION. 0.83 of the cost = \$332.
∴ the cost = \$332 ÷ 0.83 = \$400.

2. What number decreased by 25% of itself equals 375?

SOLUTION. ¾ of the number = \$375.
∴ the number = \$375 ÷ ¾ = \$500.

ORAL EXERCISE

What number diminished by:

1. 6¼% of itself equals 75?
2. 8⅓% of itself equals 440?
3. 6⅔% of itself equals 280?
4. 10% of itself equals 270?
5. 33⅓% of itself equals 66?
6. 12½% of itself equals 210?
7. ¼ of itself equals 750?
8. ½% of itself equals 99.5?
9. 1% of itself equals 49.5?
10. 25% of itself equals 225?
11. 50% of itself equals 17¾?
12. 75% of itself equals 250?

WRITTEN EXERCISE

1. Of what number is 9581.88 77 %?

2. A merchant sold 1200 bu. of potatoes for $640, which was $16\frac{2}{3}$ % less than he paid for them. What was the cost per bushel?

3. In selling a carriage for $75 a merchant lost 25 % on the cost. What was the asking price if the carriage was marked to gain 25 %?

4. A newsboy sold 92 papers on Tuesday. If this number was $23\frac{1}{3}$ % less than the number sold on Monday, how many papers were sold on the two days?

5. A dealer sold a quantity of apples at $6 per barrel, and by so doing lost $16\frac{2}{3}$ %. If he paid $309.60 for the apples, how many barrels did he buy?

6. After paying $74.35 for mileage, $32.50 for hotel bills, and $13.15 for sundry items, a traveler finds that he has expended 40 % of his money. How much had he at first?

ORAL REVIEW EXERCISE

1. By inspection find $12\frac{1}{2}$ % of the following numbers:

a. $872.	*e.* $2464.	*i.* $1688.	*m.* $24.72.
b. 648 bu.	*f.* 2696 A.	*j.* 2072 A.	*n.* $168.48.
c. 1264 A.	*g.* 1624 ft.	*k.* 11,464 mi.	*o.* $176.24.
d. 960 mi.	*h.* 1832 mi.	*l.* 37,128 mi.	*p.* $2184.32.

2. By inspection find 10 % of each of the above numbers; 25 %; 125 %; 20 %.

3. State the missing term in each of the following:

No.	Base	Rate	Percentage	No.	Base	Rate	Percentage
a.	$600	$7\frac{1}{2}$ %	?	*f.*	966	$16\frac{2}{3}$ %	?
b.	$650	?	$39	*g.*	?	$8\frac{1}{3}$ %	15 bu.
c.	?	4 %	$18	*h.*	1275	$6\frac{2}{3}$ %	?
d.	900	?	720	*i.*	?	$6\frac{1}{4}$ %	21 mi.
e.	?	4 %	20	*j.*	400	?	600

4. By inspection find 10 % of each of the following :

a. $264.	*d.* $840.	*g.* $232.	*j.* $2448.
b. $920.	*e.* $750.	*h.* $144.	*k.* $1432.
c. $720.	*f.* $364.	*i.* $288.	*l.* $3624.

5. By inspection find $1\frac{1}{4}$ % of each of the above numbers; $1\frac{2}{3}$ % ; 1000 % ; 125 % ; $166\frac{2}{3}$ %.

6. By inspection find the numbers of which

a. $10\frac{1}{2}$ is $8\frac{1}{3}$ %.	*d.* 75 is 25 %.	*g.* 960 is 320 %.
b. 150 is $16\frac{2}{3}$ %.	*e.* 125 is 20 %.	*h.* 1920 is 32 %.
c. 170 is $33\frac{1}{3}$ %.	*f.* 750 is 250 %.	*i.* 240 is $33\frac{1}{3}$ %.

WRITTEN REVIEW EXERCISE

1. A collector charged 4 % on all amounts collected. If he remitted to his customers in one month $3720.48, how much did he receive for his services?

2. A father left to his son 60 % of his estate and to his daughter the remainder, $9390.88. What was the value of the estate and how much did the son receive?

3. A farmer planted 1 bu. 3 pk. of oats on an acre of ground and harvested 56 bu. What per cent of the yield was the planting? What per cent of the planting was the yield?

4. A merchant paid the following charges on a bill of goods: cartage $12.45, freight $65.32, insurance $41. If the charges represent 5 % of the face of the bill, what was the gross cost of the goods?

5. A merchant failed in business, his resources amounting to $12,840 and his liabilities to $24,000. What per cent of his indebtedness did he pay, and what was the aggregate loss to his creditors?

6. The density of population in Asia is approximately 125 per square mile, and in the United States, approximately 25 per square mile. What per cent greater is the density of population in Asia than that in the United States? What per cent less is the density in population in the United States than that in Asia?

7. A man had 6 A. of land; to one party he sold a piece 25 rd. by 20 rd., and to another party 140 sq. rd. What per cent of the field remained unsold?

8. In a recent year 176,774,300 lb. of fish were landed in Boston, and of this quantity Gloucester furnished 111,367,809 lb. What per cent was furnished by Gloucester? (Correct to the nearest .01.)

9. A owned property valued at $12,000 from which he received a yearly rental of $960. If he paid taxes amounting to $160, insurance $75.50, and made repairs amounting to $184.50, what per cent net income did he receive?

10. B owns a field 80 rd. square. During a certain year this field yielded on an average 25 bu. of wheat to an acre. The wheat when sold at $1 a bushel produced an amount equal to 25 % of the value of the field. What was the value of the field?

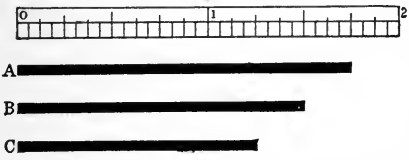
11. A landowner rented a field to a tenant and was to receive as rent $16\frac{2}{3}$ % of the grain raised. The owner of the field sold his share of the grain for 84¢ per bushel, receiving $298.20. If the tenant sold his share of the grain for the same price per bushel, how much did he receive?

12. Twenty years ago the value of knit goods produced in the United States was $39,271,900, of which New England produced 27 %; the value of the knit goods manufactured this year was $101,337,000, of which New England produced 18 %. What was New England's per cent of increase in 20 yr.? (Correct to the nearest .01.)

13. By a recent census report it was shown that the value of all personal property in the state of New York was approximately $500,000,000 and the value of all the real estate approximately $3,000,000,000. Draw parallel lines making a comparison of personal property and real estate. The real estate is what per cent greater than the personal property? The personal property is what per cent less than the real estate?

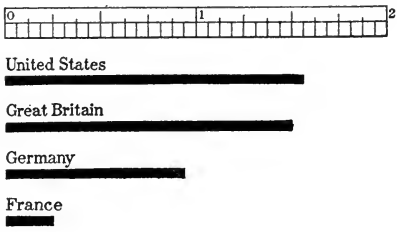
14. A young man entered a bank as cashier and at the end of the first year his salary was increased 25%; at the end of the second year he was given an increase of 20%; and at the end of the third year he was given an increase of 25%, which made his salary $4500. What salary did he receive at first?

15. A government statistician collected facts regarding wages and income from nearly two thousand private manufacturing concerns, and reported the following: the average wages of all employees, men, women, and children, per year was $263.06, and the average net profit for each employer was $2273. What per cent greater was the income of each employer than of each em-employee? (Correct to the nearest .01.)



16. The population of three cities during a certain year is illustrated by the accompanying lines, which are drawn on a scale of 12,500 inhabitants to each $\frac{1}{8}$ of an inch. What is the population of A, B, and C, respectively? The population of each city is what per cent of the population of the three cities?

17. The annual coal production in the United States, Great Britain, Germany, and France for a certain year is illustrated in the accompanying rectangles, drawn on the scale of 20,000,000 short tons to each $\frac{1}{8}$ of an inch. During that year, how many tons did the United States, Great Britain, Germany, and France, respectively, produce? The production of each country is what per cent of the production of the four countries? In the same year the rest of the world produced approximately 110,000,000 short tons. Illustrate graphically the world's coal production for this year. What was the world's approximate production this year?



18. The total value of the cotton crop to farmers in a recent year was $453,000,000 and the value of the cotton exported to England in the same year was $124,000,000. What per cent was exported to England? (Correct to the nearest .01.)

19. A saleswoman in a city store receives $9 per week. She pays $3.50 per week for board and room, 10¢ per day for car fare 6 da. in the week, 20¢ per day for 6 da. of each week for luncheon, and has incidental expenses amounting to $1.70. If she saves the remainder, what per cent of her weekly wages does she save? What per cent does she spend?

20. The production, in bushels, of grain in the United States in two recent years was approximately as follows:

Cereals	1903	1904
Corn	2,240,000,000	2,470,000,000
Wheat	640,000,000	550,000,000
Oats	780,000,000	900,000,000
Barley	131,000,000	130,000,000
Rye	30,000,000	27,000,000
Buckwheat	14,000,000	15,000,000

Find the per cent of increase or decrease of each cereal for 1904 as compared with the previous year. Then draw a series of parallel rectangles to compare the production of 1904 with the production of 1903. Also draw a series of rectangles to compare the production of 1904 with the production of a later year.

Suggestion. This may be represented by one series of rectangles. Each rectangle may be divided into two parts — one shaded and the other unshaded. The shaded part may be made to represent the yield for 1904 and the unshaded part the yield for 1903.

21. The silver produced by the leading sources in a recent year was approximately as follows:

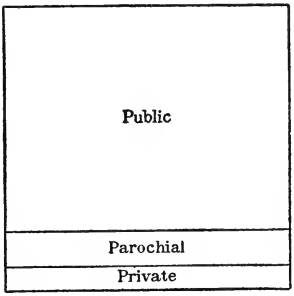
Mexico	60,000,000 oz.	Canada	4,500,000 oz.
United States	55,500,000 oz.	Peru	4,000,000 oz.
Bolivia	13,000,000 oz.	Spain	3,500,000 oz.
Australasia	8,000,000 oz.	Chili	3,500,000 oz.
Germany	6,000,000 oz.	Austria-Hungary	2,000,000 oz.

Draw a set of parallel rectangles to graphically represent the above numbers.

22. In the following table is shown the population in the United States in a certain year, at least ten years of age, engaged in gainful occupations, classified by sexes and kinds of occupations. Supply the missing terms. Check the work.

Kind of Occupation	Population Engaged in Gainful Occupations					
	Number			Per Cent of Total		
	Total	Male	Female	Total	Male	Female
Agricultural pursuits . . .	10,381,765	9,404,429	977,336	35.7	39.6	18.4
Professional services	1,258,739	828,163	430,576	—	—	—
Domestic and personal service	5,580,657	3,485,208	2,095,449	—	—	—
Trade and transportation . .	4,766,964	4,263,617	503,347	—	—	—
Manufacturing and mechanical pursuits	7,085,992	5,772,788	1,313,204	—	—	—
All occupations . . .	—	—	—	100.0	100.0	100.0

23. Suppose the accompanying diagram illustrates the distribution of school enrollment in the public, private, and parochial schools of the United States during a certain year. The private and parochial schools are what per cent of the public schools? of the entire school enrollment? The public schools are what per cent of the total enrollment? of the private and parochial schools combined?



24. The gold production, in ounces, in the eight principal gold-producing states in the United States in a recent year was approximately as follows: Colorado, 28,500,000; California, 17,000,000; Alaska, 8,500,000; Arizona, 4,000,000; Montana, 4,500,000; Nevada, 3,000,000; South Dakota, 7,000,000; Utah, 3,500,000. Compare these values by drawing a series of parallel rectangles.

CHAPTER XVIII

COMMERCIAL DISCOUNTS

ORAL EXERCISE

1. A set of Scott's works is marked \$12. If I buy it at this price, less $16\frac{2}{3}\%$, what does it cost me?

2. I buy \$90 worth of goods on 30 da. time, or 5% off for cash. What cash payment will settle the bill?

3. I owe B \$600, due in 30 da. He offers to allow me 5% discount if I pay cash to-day. I accept his offer and give him a check for the amount. What was the amount of the check?

305. A reduction from the catalogue (list) price of an article, from the amount of a bill of merchandise, or from the amount of a debt, is called a **commercial** or **trade discount.**

Business houses usually announce their terms upon their bill heads. The space allowed for recording the terms is usually limited, and bookkeepers find it necessary to use symbols and abbreviations to indicate them. Thus, if a bill is due in 30 da. without discount, the terms may be written $^{N}/_{30}$, or Net 30 da.; if the bill is due in 30 da. without discount, but an allowance of 2% is made for payment within 10 da., the terms may be written $^{2}/_{10}$, $^{N}/_{30}$, or 2% 10 da., net 30 da.

306. Manufacturers, jobbers, and wholesale dealers usually have printed price lists for their goods. To obviate the necessity of issuing a new catalogue every time the market changes, these lists are frequently printed higher than the actual selling price of the goods, and made subject to a trade discount.

307. The fluctuations of the market and the differences in the quantities purchased by different customers frequently give rise to two or more discounts called a **discount series.**

Large purchasers sometimes get better prices and terms than small purchasers. Thus, the average customer might be quoted the regular prices less a trade discount of 25%, while an especially large buyer might be quoted the regular prices less trade discounts of 25% and 10%.

308. When two or more discounts are quoted, one denotes a discount off the list price, another, a discount off the remainder, and so on.

The order in which the discounts of any series is considered is not material. Thus, a series of 25 %, 20 %, and 10 % is the same as one of 20 %, 10 %, and 25 %, or one of 10 %, 25 %, and 20 %.

309. Catalogue prices are generally estimated on the basis of credit sales, and a cash purchaser is given the usual trade discount and a special discount for early payment. This latter discount has the effect of encouraging prompt payments.

The list price is sometimes called the **gross price** and the price after the discount has been deducted the **net price.**

FINDING THE NET PRICE

310. Example. The list price of a dozen pairs of shoes is \$45. If this price is subject to a discount series of 20 % and 10 %, what is the net selling price?

SOLUTION. 20 % or $\frac{1}{5}$ of \$45 = \$9, the first discount.
\$45 − \$9 = \$36, the price after the first discount.
10 % or $\frac{1}{10}$ of \$36 = \$3.60, the second discount.
\$36 − \$3.60 = \$32.40, the net selling price.

ORAL EXERCISE

Find the net price:

	List Price	Trade Discount		List Price	Trade Discount		List Price	Trade Discounts
1.	\$4	25 %	8.	\$6	40 %	15.	\$4	25 % and $33\frac{1}{3}$ %
2.	\$15	20 %	9.	\$4	$12\frac{1}{2}$ %	16.	\$30	$33\frac{1}{3}$ % and 25 %
3.	\$90	$33\frac{1}{3}$ %	10.	\$24	$8\frac{1}{3}$ %	17.	\$35	20 % and 25 %
4.	\$20	10 %	11.	\$42	$16\frac{2}{3}$ %	18.	\$45	20 % and $16\frac{2}{3}$ %
5.	\$50	50 %	12.	\$35	20 %	19.	\$50	20 % and 25 %
6.	\$2.50	20 %	13.	\$100	25 %	20.	\$100	20 % and 10 %
7.	\$4.50	$16\frac{2}{3}$ %	14.	\$720	$33\frac{1}{3}$ %	21.	\$600	$16\frac{2}{3}$ % and 20 %

22. A piano listed at \$750 is sold less $33\frac{1}{3}$ %, 20 %, and 10 %. What is the net cost to the purchaser?

23. A dealer offers cloth at \$3.50 per yard subject to a discount of 20 %. How many yards can be bought for \$56?

WRITTEN EXERCISE

Find the net price:

	Gross Selling Price	Trade Discounts		Gross Selling Price	Trade Discounts
1.	$3360	25% and 10%	4.	$2500	20%, 10%, and 5%
2.	$3510	$33\frac{1}{3}$% and 20%	5.	$5400	25%, 20%, and 10%
3.	$4500	20% and $16\frac{2}{3}$%	6.	$3960	$33\frac{1}{3}$%, 20%, and $16\frac{2}{3}$%

7. The list price of cloth is $4.80 per yard, but this price is subject to discounts of 25% and 20%. How many yards can be bought for $288?

8. A hardware dealer sold 25 doz. 5-in. files at $2.50 and 25 doz. 12-in. files at $7.50, less 50% and 10%; 150 machine bolts at $21.50 per C, less 20% and 10%. What was the net amount of the bill?

9. Study the following model. Copy and find the net amount of the bill, using the discounts named in the bill, and the following prices: 5-in. pipe, $1.45; 1-in. pipe, 17¢; valves, $2.67.

Chicago, Ill., July 16, 19—

Mrs. E. M. Farnsworth,
Toledo, O.

Bought of GEORGE W. MUNSON & CO.

Terms 60 da. net

300	ft. 5 in. Iron Pipe	1.40	420			
200	" 1 in. " "	16¢	32			
			452			
	Less 50%		226			
			226			
	Less 10%		22 60	203	40	
20	1 in. Valves	2.40	48			
	Less 25%		12			
			36			
	Less $16\frac{2}{3}$%		6	30	233	40

10. One firm offers a piano for $400, subject to discounts of 20% and 20%; another offers the same piano for $400 less discounts of 25% and 15%. Which is the better offer? How much better?

11. A jobber bought a quantity of goods listed at $3600, subject to discounts of 25% and 20%. He sold the goods at the same list price, subject to discounts of 20% and 10%. Did he gain or lose, and how much?

12. Make out bills for the following, using the current date and the name and address of some dealer whom you know. Terms in each case, 60 da. net.

a. You bought 12 doz. hand saws, #27, at $18.50; $7\frac{1}{2}$ doz. mortise locks, #271, at $4.25; 25 doz. pocket knives, #27, at $7.50; and $1\frac{3}{4}$ doz. cheese knives at $8.25. Discount: 25%, 10%.

b. You bought $41\frac{1}{2}'$ of $2''$ extra strong iron pipe at $70¢$; $94\frac{1}{4}'$ of $1\frac{1}{4}''$ extra strong iron pipe at $31\frac{1}{2}¢$; $153\frac{1}{4}'$ of $\frac{1}{4}''$ iron pipe at $6\frac{1}{2}¢$; $88\frac{1}{2}'$ of $\frac{3}{8}''$ iron pipe at $7\frac{3}{4}¢$. Discount: 25%, 10%.

c. You bought 25 kitchen tables at $3.25; 25 dining-room tables at $8.75; 15 doz. dining-room chairs at $12.50; 12 antique rockers at $12.25; and 15 oak bedroom sets at $32.50. Discount: $16\frac{2}{3}$%, 5%.

FINDING A SINGLE RATE OF DISCOUNT EQUIVALENT TO A DISCOUNT SERIES

311. Example. What single rate of discount is equivalent to a discount series of 25%, $33\frac{1}{3}$%, and 10%?

Solution. Represent the list price by 100%. Then, 75% equals the price after the first discount, 50% the price after the second discount, and 45% equals the net selling price. 100%, the list price, minus 45%, the net selling price, equals 55%, the single rate of discount equivalent to the given discount series.

A single discount equivalent to a discount series may often be determined mentally (see §§ 312-313).

$$\begin{array}{l} 1.00 \\ \underline{.25} \ (25\% \text{ of } 100\%) \\ .75 \\ \underline{.25} \ (33\frac{1}{3}\% \text{ of } 75\%) \\ .50 \\ \underline{.05} \ (10\% \text{ of } 50\%) \\ .45 \end{array}$$

$100\% - 45\% = 55\%$

WRITTEN EXERCISE

1. Find a single rate of discount equivalent to a discount series of 50 %, 25 %, 20 %, and 10 %.

2. Which is the better and how much, a single discount of 65 % or a discount series of 25 %, 20 %, and 20 % ?

3. The net amount of a bill of goods was $450 and the discounts allowed were 25%, $33\frac{1}{3}$%, and 10 %. Find the total discount allowed.

4. I allowed a customer discounts of 50%, 10 %, and 10 % from a list price. What per cent better would a single discount of 65 % have been ?

5. Goods were sold subject to trade discounts of 25 %, 20 %, and 10 %. If the total discount allowed was $460, what was the net selling price of the goods ?

6. A quantity of goods was sold subject to trade discounts of 20 % and 20 %. The terms were 60 da. net or 5 % off for payment within 10 da. If a cash payment of $1026 was required 3 da. after the date of the bill, what was the list price of the goods sold ?

312. Since the first of a series of discounts is computed on 100 % of the list price, and the second on 100 % minus the first discount, it follows that *the sum of any two separate discounts exceeds the equivalent single discount by the product of the two rates per cent.*

Thus, in a discount series of 20 % and 20 % the apparent single discount is the sum of the two separate discounts or 40 %; but since the second discount is not computed on 100 %, but on 80%, 40% exceeds the true single discount by 20 % of 20 %, or 4 %; and the equivalent single discount is 40 % minus 4 %, or 36 %. Hence,

313. To find the single discount equivalent to a series of two discounts:

From the sum of the separate discounts subtract their product, and the remainder will be the equivalent single discount.

When two separate discounts cannot be reduced to a single discount *mentally*, proceed as in § 311; when they can, proceed as in § 313.

ORAL EXERCISE

State a single rate of discount equivalent to a discount series of:

1. 10% and 10%.
2. 20% and 20%.
3. 30% and 30%.
4. 40% and 40%.
5. 50% and 50%.
6. 20% and 10%.
7. 30% and 10%.
8. 40% and 10%.
9. 50% and 10%.
10. 60% and 10%.
11. 30% and 20%.
12. 40% and 20%.
13. 50% and 20%.
14. 60% and 20%.
15. 25% and 10%.
16. 35% and 10%.
17. 50% and 5%.
18. 10% and 5%.
19. 20% and 5%.
20. 40% and 5%.
21. 25% and 30%.
22. 25% and 40%.
23. 20% and 15%.
24. 40% and 15%.
25. 35% and 20%.
26. 45% and 20%.
27. 55% and 20%.
28. 60% and 25%.
29. 40% and 25%.
30. 60% and 20%.
31. 25% and $33\frac{1}{3}$%.
32. 45% and $33\frac{1}{3}$%.
33. 25% and 8%.
34. $8\frac{1}{3}$% and 24%.
35. $8\frac{1}{3}$% and 36%.
36. 35% and 10%.
37. 20% and $12\frac{1}{2}$%.
38. 40% and $12\frac{1}{2}$%.
39. 60% and $12\frac{1}{2}$%.
40. 12% and $12\frac{1}{2}$%.
41. 24% and $16\frac{2}{3}$%.
42. $16\frac{2}{3}$% and 20%.
43. $14\frac{2}{7}$% and 35%.
44. $16\frac{2}{3}$% and 25%.
45. $33\frac{1}{3}$% and 15%.
46. $66\frac{2}{3}$% and 15%.
47. $11\frac{1}{9}$% and 18%.
48. 36% and $11\frac{1}{9}$%.

314. When a discount series consists of three separate rates, the first two may be combined as in § 313 and then the result and the third may be combined in the same manner.

315. Example. Find a single rate of discount equivalent to a discount series of 25%, 20%, and 20%.

SOLUTION. — Combine the first two by thinking $25\% + 20\% - 5\% = 40\%$, the single discount equivalent to the series 25% and 20%. $20\% + 40\% - 8\% = 52\%$, or the single rate equivalent to the discount series 25%, 20%, and 20%.

ORAL EXERCISE

State a single rate of discount equivalent to a discount series of:

1. 20%, 25%, and 20%.
2. 20%, 15%, and 10%.
3. 20%, 20%, and 20%.
4. 10%, 10%, and 10%.
5. 20%, 20%, and 10%.
6. 25%, $33\frac{1}{3}$%, and 10%.
7. 20%, 10%, and 10%.
8. 40%, 10%, and 10%.
9. 50%, 10%, and 10%.
10. 30%, 10%, and 10%.
11. 20%, 25%, and 10%.
12. 20%, 20%, and 25%.

316. When it is not desirable to show the separate discounts, the net selling price may be found as shown in the following example.

317. Example. A mahogany sideboard listed at $175 is sold subject to trade discounts of 20% and 25%. Find the net cost to the purchaser.

SOLUTION. By inspection determine that a discount of 40% is equivalent to a series of 25% and 20%. Represent the gross cost by 100%.

$100\% - 40\% = 60\%$
60% of $\$175 = \$105$

Then $100\% - 40\% = 60\%$, the net cost to the purchaser; that is, the net cost of the sideboard is 60% of the list price. 60% of $\$175 = \$105$, the net cost to the purchaser.

ORAL EXERCISE

By inspection find the net cost of articles listed at:

1. $400, less 20% and 25%.
2. $300, less 20% and 20%.
3. $600, less 10% and 10%.
4. $200, less 30% and 30%.
5. $1000, less 50% and 50%.
6. $1000, less 30% and 10%.
7. $200, less 60% and 25%.
8. $400, less 20% and 40%.

WRITTEN EXERCISE

1. Find the net selling price of a piano listed at $450, less 20% and 20%.

2. Find the net selling price of an oak sideboard listed at $125, less 25%, $33\frac{1}{3}\%$, and 10%.

3. I bought 125 cultivators listed at $8.50, each subject to trade discounts of 20% and 25%. If I paid freight $30.50 and drayage $7.90, how much did the cultivators cost me?

4. The net cost of an article was increased $30 by freight, making the actual cost of it $630. What was the list price of the article, the rates of discount being 25% and $33\frac{1}{3}\%$?

5. You desire to buy 24,000 ft. choice cypress: one firm quotes you $60 per thousand feet, less trade discounts of 20% and 5%; another firm offers you the same lumber at $75 per thousand feet, less $33\frac{1}{3}\%$ and 8%. The terms offered by both firms are $^1/_{10}$, $^N/_{30}$. You accept the better offer and pay cash. How much does the lumber cost you?

WRITTEN REVIEW EXERCISE

1. Find the cost of 125 $1\frac{1}{2}''$ brass ells at \$1.25 each, less 25%, 20%, and 10%.

2. An agent bought 10 pianos listed at \$450 each, less $33\frac{1}{3}\%$ and 10%, and sold them for \$400 each, less 10% and 5%. Did he gain or lose and how much?

3. Apr. 15, E. L. Gano bought of W. L. Cunningham & Co. 5 phaetons listed at \$450 each, less 25% and 20%. Terms: $^{2}/_{30}$, $^{N}/_{60}$. How much ready money would settle the bill?

4. Study the following bill. Copy and find the net amount of it, using the discounts indicated in the bill, and the following prices: windmills, \$675; pumps, \$610; 1-in. iron pipe, $17\frac{1}{2}¢$; 4-in. iron pipe, 73¢; hose, 97¢; elbows, $21\frac{1}{2}¢$; valves, \$1.49.

Boston, Mass., Oct 24, 19__

Messrs. Cooper, Benedict & Co.,
Springfield, Mass.

Bought of E. M. McGREGOR & CO.

Terms 3/10, 1/30, N/60

3	Excelsior Windmills	650 ea	1950					
	Less 25% and 20%		780		1170			
5	Eureka Elevator Pumps	600 ea	3000					
	Less 33⅓% and 25%		1500		1500			
1800	ft. Iron Pipe, 1 in.	15¢	270					
200	" " " 4 in.	80¢	160					
			430					
	Less 20% and 25%		172		258			
300	ft. Engine Hose, 4 ply	1 ea	300					
	Less 50% and 10%		165		135			
100	Elbows	20¢	20					
50	Eureka Valves	1 50	75					
			95					
	Less 40% and 10%		43	70	51	30	3114	30

5. How much cash would settle the model bill (page 249) Oct. 30? Nov. 8? How much cash would settle the bill called for in problem 4, if it is paid for on the day it is written? If it is paid Nov. 15? Copy the model bill in the form that it would be written if cash accompanied the order; that is, copy it deducting the 3 % allowed for immediate payment.

6. Copy and find the net amount of the following bill:

Leith, Scotland, May 10, 19

Invoice of Wire Cloth

Shipped by the J. M. ROBERTS COMPANY

In the Steamship Winifredian To Edward M. Davidson & Co.
Philadelphia, Pa.

D	6 pc.,	each	34' x 5' 6"	1122 sq. ft.	1/3	70	2	6				
	6 "	"	40' x 6' 6"	****	1/4	***	*	*				
	6 "	"	42' x 7' 4"	****	1/5	***	**	*	—			
	3 "	"	48' x 7' 2"	****	1/5	**	**	*				
						***	**	*				
				Less 10%		**	**	*	***	**	*	

7. E. M. French & Co., Albany, N.Y., bought of Austin Bailey & Co., Boston, Mass., Apr. 12, 3 doz. pr. hinges, 8 in., at $4.20, and 3 doz. pr. hinges, 4 in., at $2.10, less 60 %, 10 %, and 10 %; 50 lb. brads, $\frac{5}{8}$ in., at 90¢, and 50 lb. brads, $\frac{3}{4}$ in., at 80¢, less 50 %, 10 %, and 5 %. Terms: $^{2}/_{10}$, $^{N}/_{30}$. Find the net amount of the bill Apr. 15.

8. D. M. DeLong, Portland, Me., sold S. H. Shapleigh & Co., Concord, N.H., on account 30 da., 2 % 10 da.: 35 cultivators listed at $7.50 each, less 20 % and 10 %; 15 doz. table knives listed at $9.75, less 10 %; 15 doz. hair curlers at 90¢, less 5 %; 15 doz. locks, No. 534, at $3.75, less 10 % and 5 %; $\frac{3}{4}$ doz. steel squares, No. 8, at $36, less 25 % and 10 %; $\frac{1}{3}$ gro. knives and forks, No. 760, at $12, less 20 % and 10 %; $\frac{3}{4}$ doz. cheese knives at $9.75, less $16\frac{2}{3}$ %. Find the net amount of the bill 5 da. after date.

9. Aug. 5, you buy of Gray, Salisbury & Son, New York City, 4000 lb. raisins at 16¢, less trade discounts of 25%, 20%, and 10%. Terms: $^{2}/_{10}$, $^{N}/_{30}$. You pay cash for freight $3.20. If you pay the bill Aug. 7, what will the raisins cost you?

10. Find the net amount of the following bill:

New York, Jan. 5, *19*

Mr. W. H. Meachum
Springfield, Mass.

To Leonard, Ross & Co., Dr.

Terms Net 60 da.

1/2 C	Machine Bolts	3/8 x 1 1/2"	$2.40
1/2 C	" "	3"	2.88
1/2 C	" "	6 1/2"	4.00
1/2 C	" "	1/2 x 3 1/2"	4.64
1/2 C	" "	5"	5.42
1/2 C	" "	6"	5.94
1/2 C	" "	9"	7.50
1/2 C	" "	10"	8.02
1/2 C	" "	5/8 x 4"	7.10
1/4 C	" "	4 1/2"	7.48
1/4 C	" "	3/4 x 5"	10.70
1/4 C	" "	10"	15.70
1/4 C	" "	16"	21.70
	Discounts: 50%, 10%, 5%		
5	doz. Files	5"	$2.50
5	" "	6"	3.10
2	" "	12"	7.50
3	" "	4"	3.00
2	" "	5"	3.20
1	" "	10"	7.40
1/2	" "	12"	10.20
	Discounts: 50%, 10%, 5%, 5%		

11. You desire to buy 200 lb. nutmeg. You find that S. S. Pierce Co., of your city, offer this article at 75¢ per lb., less a discount of 25%, and that Smith, Perkins & Co., New York City, offer it at 70¢ per lb., less discounts of 15% and 10%. The freight from New York to your city on a package of this kind is $1.50. The terms offered by both firms are: $^{1}/_{10}$, $^{N}/_{30}$. You accept the better offer and pay cash. How much does the nutmeg cost you?

CHAPTER XIX

GAIN AND LOSS

ORAL EXERCISE

1. What is $33\frac{1}{3}\%$ of \$660? How much is gained on goods bought for \$900 and sold at a profit of $33\frac{1}{3}\%$?

2. What per cent greater is \$75 than \$60? what per cent less is \$60 than \$75? Goods bought for \$100 are sold for \$150. What is the gain per cent?

3. What per cent less is \$80 than \$100? what per cent more is \$100 than \$80? Goods bought for \$100 are sold for \$90. What is the loss per cent?

4. If \$800 is increased by 25% of itself, what is the result? Goods bought for \$1400 are sold at a profit of 25%. What is the selling price?

5. If \$1500 is decreased by $33\frac{1}{3}\%$ of itself, what is the result? Goods bought for \$2700 are sold at a loss of $33\frac{1}{3}\%$. What is the selling price?

6. State a brief method for finding a gain of $6\frac{1}{4}\%$; a gain of $6\frac{2}{3}\%$; a gain of $8\frac{1}{3}\%$; a gain of 10%; a gain of $1\frac{1}{4}\%$; a gain of $1\frac{2}{3}\%$; a gain of $2\frac{1}{2}\%$; a gain of $3\frac{1}{3}\%$.

7. State a brief method for finding a loss of $11\frac{1}{9}\%$; a loss of $12\frac{1}{2}\%$; a loss of $14\frac{2}{7}\%$; a loss of $16\frac{2}{3}\%$; a loss of 20%; a loss of 25%; a loss of $9\frac{1}{11}\%$; a loss of $37\frac{1}{2}\%$.

8. State a brief method for finding a gain of $33\frac{1}{3}\%$; a gain of $22\frac{2}{9}\%$; a gain of 50%; a gain of $66\frac{2}{3}\%$; a gain of 75%.

318. The **gains** and **losses** resulting from business transactions are frequently estimated at some *rate per cent* of the cost, or of the money or capital invested.

Since no new principles are involved in this subject, illustrative examples are unnecessary.

FINDING THE GAIN OR LOSS

ORAL EXERCISE

By inspection find the gain or loss:

	Cost	Per Cent of Gain		Cost	Per Cent of Loss		Cost	Per Cent of Gain
1.	\$2900	50%	9.	\$1500	10%	17.	\$7500	20%
2.	\$1600	75%	10.	\$1600	$1\frac{1}{4}$%	18.	\$1400	25%
3.	\$5600	$28\frac{4}{7}$%	11.	\$3000	$1\frac{2}{3}$%	19.	\$2200	$9\frac{1}{11}$%
4.	\$2700	$33\frac{1}{3}$%	12.	\$4800	$2\frac{1}{2}$%	20.	\$8100	$11\frac{1}{9}$%
5.	\$2400	$37\frac{1}{2}$%	13.	\$3600	$3\frac{1}{3}$%	21.	\$6400	$12\frac{1}{2}$%
6.	\$1400	$42\frac{6}{7}$%	14.	\$3200	$6\frac{1}{4}$%	22.	\$2800	$14\frac{2}{7}$%
7.	\$3200	$62\frac{1}{2}$%	15.	\$4500	$6\frac{2}{3}$%	23.	\$9600	$16\frac{2}{3}$%
8.	\$2100	$66\frac{2}{3}$%	16.	\$8400	$8\frac{1}{3}$%	24.	\$3600	$22\frac{2}{9}$%

25–48. Find the selling price in each of the above problems.

WRITTEN EXERCISE

1. An importation of silks invoiced at £40 10*s.* was sold at a profit of 25%. Find the amount (in United States money) of the gain.

2. An importation of German toys invoiced at 43,750 marks was sold at a gain of $33\frac{1}{3}$%. Find the amount (in United States money) of the gain.

3. An article that cost \$1 was marked 10% above cost. In order to effect a sale, it was afterward sold for 10% below the marked price. Find the gain or loss on 250 of the articles.

4. A man bought a city lot for \$1150 and built a house on it costing \$2650. He then sold the house and lot at a gain of 5%. How much did he gain and what was his selling price?

5. A man bought a quantity of silk for \$450, a quantity of fancy plaids for \$120, and a quantity of velvet for \$90. He sold the silk at a gain of 25%, the plaids at a loss of 5%, and the velvet at a gain of $33\frac{1}{3}$%. What was his gain, and how much did he realize from the sale of the three kinds of material?

FINDING THE PER CENT OF GAIN OR LOSS

ORAL EXERCISE

By inspection find the per cent of gain or loss:

	Cost	Gain		Cost	Loss		Cost	Selling Price		Selling Price	Gain
1.	$100	$10	7.	$60	$15	13.	$80	$90	19.	$300	$60
2.	$150	$50	8.	$40	$10	14.	$90	$80	20.	$115	$23
3.	$140	$70	9.	$90	$45	15.	$60	$75	21.	$102	$17
4.	$140	$140	10.	$70	$14	16.	$75	$60	22.	$420	$60
5.	$200	$400	11.	$80	$16	17.	$10	$50	23.	$300	$200
6.	$300	$750	12.	$15	$10	18.	$50	$10	24.	$700	$100

WRITTEN EXERCISE

1. A milliner bought hats at $15 a dozen and retailed them at $3 each. What per cent was gained?

2. A stationer bought paper at $2 a ream and retailed the same at a cent a sheet. What was his per cent of gain?

3. A dry-goods merchant bought gloves at $7.50 a dozen pair and retailed them at $1.25 a pair. What was his per cent of gain?

4. A merchant imported 50 gro. of table knives at a cost of $1125. Two months later he found that the sales of table knives aggregated $920 and that the value of the stock unsold was $435. Did he gain or lose, and what per cent?

5. An importer bought a quantity of silk goods for £400 5*s*. After disposing of a part of the goods for $1200 he took an account of the stock remaining unsold and found that at cost prices it was worth $1047.82. Did he gain or lose, and what per cent?

6. Jan. 1, F. E. Smith & Co. had merchandise on hand valued at $2500. During the month they purchased goods costing $6000 and sold goods amounting to $7500. If the stock on hand at cost prices Feb. 5 was worth $2500, what was the per cent of gain on the sales?

FINDING THE COST

ORAL EXERCISE

By inspection find the cost:

	Loss	Rate of Loss
1.	$150	10 %
2.	$100	$1\frac{1}{4}$ %
3.	$200	$1\frac{2}{3}$ %
4.	$450	$2\frac{1}{2}$ %
5.	$220	$6\frac{1}{4}$ %
6.	$115	$8\frac{1}{3}$ %

	Gain	Rate of Gain
7.	$35	20 %
8.	$79	25 %
9.	$12	$11\frac{1}{9}$ %
10.	$19	$16\frac{2}{3}$ %
11.	$44	$22\frac{2}{9}$ %
12.	$15	$33\frac{1}{3}$ %

	Selling Price	Rate of Gain
13.	$1050	5 %
14.	$2040	2 %
15.	$3600	20 %
16.	$1400	$16\frac{2}{3}$ %
17.	$1800	$12\frac{1}{2}$ %
18.	$2400	$33\frac{1}{3}$ %

	Selling Price	Rate of Loss
19.	$950	5 %
20.	$900	50 %
21.	$150	$6\frac{1}{4}$ %
22.	$550	$16\frac{2}{3}$ %
23.	$240	$33\frac{1}{3}$ %
24.	$500	$22\frac{2}{9}$ %

25–36. Find the selling price in problems 1–12.

37–48. Find the gain or loss in problems 13–24.

49. B sold a farm for $2400, thereby losing 25 %. For how much should he have sold it to have gained 10 %?

50. By selling a piano at $400 a dealer realizes a gain of $33\frac{1}{3}$ %. What would be the selling price of the piano if sold at a gain of 25 %?

WRITTEN EXERCISE

1. A sleigh was sold for $64.80, which was 10 % below cost. What was the cost?

2. An office safe was sold at $102, which was 20 % above cost. What was the cost?

3. A merchant marks goods $16\frac{2}{3}$ % above cost. What is the cost of an article that he has marked $21.70?

4. An owner of real estate sold 2 city lots for $12,000 each. On one he gained 25% and on the other he lost 25%. What was his net gain or loss from the two transactions?

5. A merchant sold a quantity of goods to a customer at a gain of 25%, but owing to the failure of the customer he received in settlement but 88¢ on the dollar. If the merchant gained $645.15, what did the goods cost him?

6. A manufacturer sold an article to a jobber at a gain of 25%, the jobber sold it to a wholesaler at a gain of 20%, and the wholesaler sold it to a retailer at a gain of $33\frac{1}{3}$%. If the retailer paid $28 for the article, what was the cost to manufacture it?

7. A manufacturer sold an article to a wholesaler at a gain of 20%, the wholesaler sold the same article to a retailer at a gain of $33\frac{1}{3}$%, and the retailer to the consumer at a gain of 25%. If the average gain was $40, what was the cost to manufacture the article?

WRITTEN REVIEW EXERCISE

1. A merchant bought goods at 40% off from the list price and sold the same at 20% and 10% off the list price. What was his gain per cent?

2. I bought goods at 50% off from the list price and sold them at 25% and 25% off from the list price. Did I gain or lose, and what per cent?

3. Apr. 15 you bought of Baker, Taylor & Co., Rochester, N. Y., 4000 bbl. Roller Process flour listed at $4.50 a barrel, and 2000 bbl. of Searchlight pastry flour listed at $4.75 a barrel. Each list price was subject to trade discounts of 20% and 10%. You paid cash $16,000 and gave your note at 30 da. for the balance. What was the amount of the note?

4. May 18 you sold to F. H. Clark & Co., New York City, 2000 bbl. of the Roller Process flour, bought in problem 3, at $33\frac{1}{3}$% above cost. Terms: $^{2}/_{10}$, $^{N}/_{30}$. F. H. Clark & Co. paid cash. Find the cash payment.

5. May 30 you sold Smith, Perkins & Co., Albany, N.Y., the balance of the flour bought in problem 3, at an advance of $33\frac{1}{3}\%$ on the cost. Terms: $^{2}/_{10}$, $^{N}/_{30}$. The flour was paid for June 8. Find the cash payment.

6. What is the net gain on the transactions in problems 3, 4, and 5? the net gain per cent?

7. Dec. 15 you bought of E. B. Johnson & Co. 400 bbl. of apples at \$2.50 per barrel. Terms: $^{1}/_{10}$, $^{N}/_{30}$. You paid cash. Find the amount of your payment.

8. May 15 you sold F. E. Redmond the apples bought in problem 7, at \$4 a barrel. Terms: $^{1}/_{10}$, $^{N}/_{30}$. At the maturity of the bill Redmond refused payment and you placed the account in the hands of a lawyer who succeeded in collecting 75% of the amount due. If the lawyer's fee for collecting was 4%, what was your net gain or loss?

9. A tailor made 25 doz. overcoats with cloth worth \$2 a yard. 4 yd. were required for each coat and the cost of making was \$48 per dozen. He sold the overcoats so as to gain $33\frac{1}{3}\%$. How much did he receive for each?

10. Apr. 12 J. D. Farley & Son, Trenton, N.J., bought of Cobb, Bates & Co., Boston, Mass., a quantity of green Java coffee sufficient to yield 2400 lb. when roasted. If the loss of weight in roasting averages 4%, what will the green coffee cost at 30¢ a pound, less a trade discount of 10%? Arrange the problem in bill form.

11. If the coffee in problem 10 is retailed $33\frac{1}{3}\%$ above cost, and there is a loss of 1% from bad debts, what is the gain on the transactions in coffee? the gain per cent?

12. The Metropolitan Coal Co., of Boston, Mass., decides to bid on a contract for supplying 2240 T. of coal for the public schools of the city. It can buy the coal at \$4.50 per long ton delivered on board track, Boston. It costs on an average 75¢ per short ton to deliver the coal, and there is a waste of $\frac{1}{4}\%$ from handling. Name a bid covering a profit of 20%. Terms: cash.

13. Copy the following, supplying all missing terms:

Statement of Resources & Liabilities, Sept. 30, 19

	Resources						
Cash	on hand			1200	00		
Mdse.	per inventory			1205	00		
Accts. Rec	due the firm			705	00		
Real Estate	store property			4700	00		
Total resources						2500	00
	Liabilities						
Accts. Pay.	outstanding			600	00		
Mortgage Pay.	on store property			2000	00		
Total liabilities						800	00
Present Worth of firm						1700	00
Net investment of firm						1500	00
Net gain of firm						???	??
	Roof						
C. N. Dean's net investment		750	00				
C. N. Dean's ½ net gain		100	00				
C. N. Dean's present worth				850	00		
O. H. Bresee's net investment		750	00				
O. H. Bresee's ½ net gain		100	00				
O. H. Bresee's present worth				850	00		
Present Worth of firm						???	??

14. May 1 you began business investing $8000 in cash. 6 mo. later your resources and liabilities were as follows:

Resources		Liabilities	
Cash on hand,	$2500	Accounts Payable outstanding	$1387
Merchandise on hand,	1600	Notes Payable outstanding	3000
Real Estate per warranty deed,	5000		
Office Fixtures on hand	597		
Accounts Receivable unpaid	1950		

Make a statement showing your net gain or loss and your present worth Nov. 1. Find the per cent of gain or loss in problem 13; in problem 14.

15. Copy the following bill, supplying all missing terms:

New York, Apr. 15, 19

Messrs. D. K. Palmer & Co.,
Pittsfield, Mass.

To Page Manufacturing Co., Dr.

Terms Cash, less 10%

#24	20	Mahogany Sideboards 15%	3120		
#17	10	Parlor Tables 17½	170		
			3290		
		Less ?? %	329	????	
		Received payment.			
		Page Manufacturing Co.,			
		per D.			

16. If the sideboards in problem 15 retailed at $195 and the parlor tables at $21.25 and the delivery charges on sales amounted to $45.47, what was the per cent of gain or loss?

17. Copy the following bill, supplying all missing terms:

St. Louis, Mo., Feb. 18, 19

Messrs. C. K. Jones & Son,
Poughkeepsie, N. Y.

To F. M. Everett & Co., Dr.

Terms 2/10, 1/20, n/30

	25	doz. Pocket Knives 15 #20 7½ 10 #16 ??	172	50	
		Less ?? % and 10 %	48	30	
			124	20	
		Less ? %	2	48	121 72
		Paid Feb. 21, 19			
		F. M. Everett & Co.,			
		per C.			

18. How much must #16 pocket knives (problem 17) retail for apiece in order to gain $33\frac{1}{3}$ %? #20 pocket knives?

CHAPTER XX

MARKING GOODS

319. Merchants frequently use some private mark to denote the cost and the selling price of goods. The word, phrase, or series of arbitrary characters employed for private marks is called a **key**.

Many houses use two different keys in marking goods, one to represent the cost and the other the selling price. In this way the cost of an article may not be known to the salesmen, and the selling price may not be known to any except those in some way connected with the business. In large houses, when but one key is used, only the selling price is indicated on the article, it being deemed best to keep the actual cost of the article a secret with the buyers. In small houses, when but one key is used, both the cost and the selling price are frequently written on the article.

320. If letters are used to mark goods, any word or phrase containing *ten different letters* may be selected for a key. If arbitrary characters are used, any *ten different characters* may be selected for a key.

Some methods of marking are so complicated that it is necessary to always have a key of the system at hand for reference. Goods are so marked in order that important facts, such as the cost of goods, may be kept strictly private.

321. When a figure is repeated one or more times, one or two extra letters called **repeaters** are used to make the key word more secure as a private mark.

The following illustrates the method of marking goods by letters:

Cost Key	Selling-price Key
REPUBLICAN	PERTHAMBOY
1 2 3 4 5 6 7 8 9 0	1 2 3 4 5 6 7 8 9 0
Repeaters: S and Z	Repeaters: W and D

The cost is generally written above and the selling price below a horizontal line on a tag, or on a paster or box. Gloves No. 271, costing $5 a dozen and selling for $6.25 a dozen, might be marked as shown in the margin. Fractions may be designated by additional letters or characters. Thus, W may be made to represent $\frac{1}{2}$, K $\frac{1}{4}$, etc. in the above key. In marking goods for the retail trade, *all fractions* of a cent are called another whole cent.

WRITTEN EXERCISE

Using the keys given in § 321, write the cost and the selling price in each of the following problems:

	First Cost of Article	Freight	Gain	Loss		First Cost of Article	Freight	Gain	Loss
1.	$2.50	10%	20%		5.	$16.00	$2\frac{1}{2}$%	$37\frac{1}{2}$%	
2.	$1.00	10%	20%		6.	$40.00	5%	$16\frac{2}{3}$%	
3.	.50		$33\frac{1}{3}$%		7.	$ 3.60	$2\frac{1}{2}$%		
4.	$4.80	20%		25%	8.	$24.00			10%

Using the following key, write the cost and the selling price in each of the following problems:

Cost Key

Repeaters: □ ◺

Selling-price Key

Repeaters: × —

	First Cost of Article	Charges	Gain	Loss		First Cost of Article	Charges	Gain	Loss
9.	$10.00	5%	20%		12.	$15.00	$6\frac{2}{3}$%	25%	
10.	$20.00	10%	50%		13.	$18.00	10%	25%	
11.	$30.00	$6\frac{2}{3}$%		25%	14.	$12.00	5%	$33\frac{1}{3}$%	

322. Wholesalers and jobbers buy and sell a great many articles by the dozen. Retailers buy a great many articles by the dozen, but generally sell them by the piece. In marking goods, therefore, it is highly important that the student be able to divide by 12 with great rapidity.

To divide by 12 with rapidity, the decimal equivalents of the 12ths, from $\frac{1}{12}$ to $\frac{11}{12}$ inclusive, should be *memorized*.

TABLE OF TWELFTHS

TWELFTHS	SIMPLEST FORM	DECIMAL VALUE	TWELFTHS	SIMPLEST FORM	DECIMAL VALUE
$\frac{1}{12}$		$.08\frac{1}{3}$	$\frac{7}{12}$		$.58\frac{1}{3}$
$\frac{2}{12}$	$\frac{1}{6}$	$.16\frac{2}{3}$	$\frac{8}{12}$	$\frac{2}{3}$	$.66\frac{2}{3}$
$\frac{3}{12}$	$\frac{1}{4}$	.25	$\frac{9}{12}$	$\frac{3}{4}$	.75
$\frac{4}{12}$	$\frac{1}{3}$	$.33\frac{1}{3}$	$\frac{10}{12}$	$\frac{5}{6}$	$.83\frac{1}{3}$
$\frac{5}{12}$		$.41\frac{2}{3}$	$\frac{11}{12}$		$.91\frac{2}{3}$
$\frac{6}{12}$	$\frac{1}{2}$	.50	$\frac{12}{12}$	1	1.00

323. Example. What is the cost of one shirt when a dozen shirts cost $19?

SOLUTION. Divide by 12 the same as by any number of one digit and mentally reduce the twelfths in the remainder to their decimal equivalent. Thus, say or think $1\frac{7}{12}$, $1.58\frac{1}{3}$, practically $1.58.

ORAL EXERCISE

State the cost per article when the cost per dozen articles is:

1. $25.00.	7. $7.00.	13. $23.20.	19. $9.00.
2. $37.00.	8. $3.60.	14. $19.20.	20. $7.00.
3. $42.00.	9. $2.40.	15. $66.60.	21. $5.00.
4. $64.00.	10. $5.60.	16. $38.00.	22. $7.50.
5. $80.00.	11. $3.40.	17. $17.00.	23. $8.40.
6. $13.00.	12. $13.20.	18. $11.00.	24. $17.50.

ORAL EXERCISE

1. Hats costing $48 a dozen must be sold for what price each to gain 25%?

2. Rulers bought at $2 a dozen must be retailed at how much each to gain 50%?

3. Note books costing $1.60 per dozen must be retailed at what price each to gain $12\frac{1}{2}$%?

4. Erasers bought at $3.24 per gross must be retailed at how much each to gain $111\frac{1}{9}$%?

5. Matches costing $3.60 per gross boxes must be retailed at what price per box to gain 100%?

6. Envelopes bought at \$2 per M must be sold at what price per package of 25 to gain 100%?

7. Pickles bought at \$1.80 per dozen bottles must be sold at what price per bottle to gain $33\frac{1}{3}$%?

8. Mustard costing \$14.40 per gross packages must be retailed at what price per package to gain 20%? to gain 50%?

LISTING GOODS FOR CATALOGUES

324. In listing goods for catalogues dealers generally mark them so that they may allow a discount on the goods and still realize a profit.

325. Example. What should be the catalogue price of an article costing \$24 in order to insure a gain of 25% and allow the purchaser a discount of 20%?

SOLUTION. $\frac{1}{4}$ of \$24 = \$6, the gain.
\$30 = the selling price, which is 20% below the catalogue price.
.80 of the catalogue price = \$30.
∴ the catalogue price = \$30 ÷ .80 = \$37.50

WRITTEN EXERCISE

1. At what price must you mark an article costing \$400 to gain 25% and provide for a 20% loss through bad debts?

2. What should be the catalogue price of a library table costing \$25 in order to insure a gain of 20% and allow the purchaser a discount of 25%?

3. You list tea costing 30¢ a pound in such a way that you gain $33\frac{1}{3}$% after allowing the purchaser a trade discount of 20%. What is your list price?

4. You buy broadcloth at \$3.80 per yard. At what price must you mark it in order that you may allow your cash customers 5% discount and still realize a gain of 20%?

5. Having bought a quantity of oranges for \$3.00 per C you mark them so as to gain $33\frac{1}{3}$% and allow for a 20% loss through bad debts. What will be your asking price per dozen?

6. At what price must the articles in the following invoice be listed to gain 20 % and allow discounts of 25 % and 20 % ?

Boston, Mass., Nov. 24, *19*

Mr. Edgar C. Townsend
Rochester, N.Y.

Bought of WELLS, FOWLER & CO.

Terms Net 30 da.

#721	50	Oak Bookcases	$8.00	400			
#924	25	Gentlemen's Chiffoniers	12.00	300			
				700			
		Less 10%		70		630	

WRITTEN REVIEW EXERCISE

1. Using the word *importance*, with repeaters *s* and *w*, for the buying key, and the words *buy for cash*, with repeaters *t* and *m*, for the selling key, write the cost and selling price of the articles in the following bill. It is desired to gain 25 % on the pens and pencils, 20 % on the cards, and to provide for a loss of $12\frac{1}{2}$ % through bad debts.

Boston, Mass., Oct. 18, **19**

Messrs. WHITE & WYCKOFF
Holyoke, Mass.

Bought of C. E. Stevens & Co.

Terms Net 30 da.

100	gro.	Pens	$0.80	80			
25	"	Lead Pencils	3.20	80			
50	pkg.	Record Cards	.40	20			
				180			
		Less 12 1/2%		22	50	157	50

2. At what price must I mark the following shoes to gain 20 %?

Detroit, Mich., Sept. 7, 19

Messrs. M. L. Fisher
Buffalo, N. Y.

Bought of ATWOOD & RANDALL

Terms 5/10, n/60

300	prs. Children's Lace Shoes 2ᵈᵒ	600		
	Less 5%	30	570	
	Paid Sept. 7, 19			
	Atwood & Randall,			
	per A.			
	Freight and other charges		17	50
			587	50

3. You list tea bought for 30¢ at an advance of $33\frac{1}{3}$ % on the cost. Finding small sale for the article you determine to sell so as to gain but $16\frac{2}{3}$ %. What trade discount should you allow?

4. What price per pound must be obtained for the following invoice of coffee to gain 25 % and allow 10 % for loss in roasting and $16\frac{2}{3}$ % for loss through bad debts?

Boston, Mass., Nov. 25, *19*

Messrs. Merchant & Co.
120 Main St., City

Bought of Cobb, Bates & Co.

Terms 30 days

2000 lb. Green Java Coffee	24¢	480	00	
Cartage		2	50	482 50

CHAPTER XXI

COMMISSION AND BROKERAGE

ORAL EXERCISE

1. A collected a bill of $350 and received 6 % for his services. How much did he make?

2. B bought $80 worth of eggs for a dealer who paid him $7\frac{1}{2}$ % for his services. How much did B make?

3. C receives $12 a week, and 5 % of his weekly sales. If he sold $350 worth of goods in a week, what was his income for the week?

326. An **agent** is a person who undertakes to transact business for another called the **principal.**

327. A great deal of the produce of the country and a large variety of manufactured articles are bought and sold through agents called commission merchants and brokers.

328. A **commission merchant** (sometimes called a factor) is an agent who has actual possession and control of the goods of his principal; a **broker** is an agent who arranges for purchases or sales of goods without having actual possession of them.

329. The sum charged by an agent for transacting business for his principal is called **commission** or **brokerage.**

Commission and brokerage are frequently computed at a certain per cent of the amount of property bought or sold, or of the amount of business transacted. Brokerage is also often a fixed rate per bushel, barrel, tierce, or other standard measure.

330. Agents frequently charge an additional commission, called **guaranty**, for assuming any risk or guaranteeing the quality of goods bought or sold.

The person who ships goods is sometimes called the **consignor**; the person to whom the goods are shipped, the **consignee.**

A quantity of goods sent away to be sold on commission is called a **shipment**; a quantity of goods received to be sold on commission, a **consignment.**

331. An **account sales** is an itemized statement rendered by a commission merchant to his principal. It shows in detail the sales of the goods, the charges thereon, and the net proceeds remitted or credited.

Buffalo, N.Y., June 18, *19*

Sale of Merchandise for Account of

E. H. Barker & Co., Poughkeepsie, N.Y.

By Hogg, Taylor & Hogg

June	5	200 bbl. Roller Process Flour	$6.00		1200	00
	12	300 " Old Grist Mill Flour	6.10		1830	00
		Charges				
June	2	Freight and Drayage		40 75		
	12	Commission 5%		151 50		
	18	Net proceeds remitted		2837 75		
				3030 00	3030	00

332. An **account purchase** is a detailed statement rendered by a purchasing agent to his principal. It shows in detail the quantity, grade, and price of goods purchased, the expenses incurred, and the gross (total) cost of the transaction.

CHICAGO, ILL., Mar. 15, 19—

Purchase of Merchandise for Account of F. H. Carpenter, Cincinnati, O.

By GRAY, DUNKLE & CO.

200	bbl. Greening Apples	3 00	600 —		
200	" Russet Apples	3 00	600 —		
200	" Baldwin Apples	3 25	650 —	1850	—
	Charges				
	Drayage		10 50		
	Commission, 2%		37 —	47	50
	Amount charged to your acct.			1897	50

ORAL EXERCISE

1. I sold 100 A. of land at $50 per acre on a commission of 3%. What was my commission?

2. A lawyer collected an account of $1000 and received for his services $40. What was his rate of commission?

3. A book agent received 25% on all books sold. In one week, after paying his expenses, $25, he netted $75. What was the gross amount of the week's sales?

4. I bought through a broker 1000 bu. of wheat quoted at $89\frac{7}{8}¢$ per bushel. If the broker charged $\frac{1}{8}¢$ per bushel for buying the wheat, what was his brokerage? How much did the wheat cost me?

SELLING ON COMMISSION

WRITTEN EXERCISE

1. Copy and complete the following letter:

JOHNSON & CO.
Produce Merchants

Boston, Mass., May 10, 19__

_____
(STUDENT'S NAME)

_____
(STUDENT'S ADDRESS)

Dear _____:

We have today shipped to your address, to be sold on our account and risk the following:

200 crates Eggs
100 boxes Cheese
200 tubs Creamery Butter

Please dispose of these goods to the best advantage.

Yours truly,
Johnson & Co.

2. May 15 you sell F. E. Spencer, Brattleboro, from Johnson & Co.'s consignment: 200 tubs, 10,000 lb., creamery butter at 23¢, and 100 crates, 3000 doz., eggs at 20¢, f.o.b. cars Brattleboro. You pay freight $16 and drayage $2.50. The terms are $^2/_{10}$, $^N/_{30}$. F. E. Spencer pays cash. Make a receipted bill for the transaction.

3. May 23 you sell Comstock & Co., Montpelier, from Johnson & Co.'s consignment: 100 crates, 3000 doz., eggs at 20¢, and 100 boxes, 6000 lb., cheese at 12¢, f.o.b. Montpelier. You pay freight $25 and drayage $4.50. Terms: $^2/_{10}$, $^N/_{30}$. Comstock & Co. pay cash. Make a receipted bill for the transaction.

4. Render Johnson & Co. an account sales for the goods shipped May 10. (See form, page 267.) The net proceeds are remitted by New York draft. Commission, 5%.

5. Find for Johnson & Co., the net gain on the shipment in problem 1. The eggs were bought at 12¢, the creamery butter at 15¢, and the cheese at 8¢. Johnson & Co. prepaid freight on shipment to you, $38.50.

6. Pay freight $20.50 on the merchandise enumerated in the following shipping invoice. This sum is 5% of the cost of the goods. Find the gross cost of the goods.

New York, Dec. 8, 19—

Invoice of Merchandise shipped to _____
(STUDENT'S NAME)

_____
(STUDENT'S ADDRESS)

To be sold for account of C. L. BROWN & CO.

	60	bx. Lemons				
	50	" Oranges				

7. Dec. 15 you sell Morgan & Co., Albany, N.Y., 60 bx. lemons at $4. Terms: $^2/_{10}$, $^N/_{30}$. Morgan & Co. pay cash. What is the amount of the cash payment?

8. Dec. 18 you sell Meachum & Co., Troy, N.Y., 50 bx. oranges at $4.50. Terms: $^2/_{10}$, $^n/_{30}$. Meachum & Co. pay for the goods Jan. 12. What is the amount of their payment?

9. Render C. L. Brown & Co. an account sales for the goods received Dec. 8, commission, 5%. Assume that on Dec. 5 you advanced them $50 on the consignment. Find C. L. Brown & Co's net gain or loss on the shipment in problem 6.

10. Prepare an account sales, under the current date, for the following, sold by you, for the account of Lewis, Grayson & Co., Rochester, N.Y.: 60 bbl. Pillsbury's flour at $6.25; 75 bbl. XXXX flour at $5.75; 45 bbl. star brand flour at $5; 100 bbl. XXX flour at $4.90; 50 bbl. peerless flour at $5.15. Charges: freight, $38.95; cartage, $12.60; cooperage, $6.25; commission, $3\frac{1}{2}$%; guaranty, $\frac{1}{2}$%.

BUYING ON COMMISSION

WRITTEN EXERCISE

1. B, a broker, bought for C, a speculator, 3000 bu. wheat at $90\frac{7}{8}$¢, on a commission of $\frac{1}{8}$¢ per bushel. What was the broker's commission, and what did the wheat cost C?

2. I bought through a broker 5000 bags coffee, each containing 130 lb., at $12\frac{1}{2}$¢. If the broker charged $10 for each 250 bags, how much did he earn on the transaction, and what did the coffee cost me?

3. I bought through a broker 20,000 bu. of wheat at $87\frac{7}{16}$¢, and three weeks later sold it through the same broker at $92\frac{7}{8}$¢. If the broker charged me $\frac{1}{8}$¢ per bu. for buying and the same for selling, what was my gain?

4. A firm of produce dealers bought through a broker 1500 bbl. pork at $12.50, and immediately sold it through another broker at $12.72\frac{1}{2}$. If each broker charged a commission of $2\frac{1}{2}$¢ per barrel, what was gained by the produce dealers?

5. You buy for your principal 1500 bbl. flour at $4.50, on a commission of 3%, and pay drayage $18.50. What is the cost of the purchase to your principal?

6. By your principal's instructions you put the flour (problem 5) in storage and later sold it at $5.25 a barrel, on a commission of 3%. The storage charges were 5¢ per barrel. What amount should you remit to your principal?

7. A broker bought cotton for a manufacturer as follows: 750 bales, 375,000 lb. at $10\frac{1}{2}$¢; 1500 bales, 750,000 lb. at $10\frac{3}{4}$¢; and 1000 bales, 500,000 lb. at $10\frac{3}{5}$¢. The broker's charges were $5 for each 100 bales. How much did he earn on the transaction, and what did the cotton cost the manufacturers?

8. Find the amount to be charged to Roe & Co.:

NEW YORK, N.Y., Mar. 15, 19

Purchased by ARAULT & Co.

For the account and risk of ROE & Co.

TELEPHONE, 690 MAIN — Poughkeepsie, N.Y.

20	hf. ch. Japan Tea	1200#	30¢			
20	hf. ch. Oolong Tea	1000#	45¢			
	Charges					
	Drayage			4	50	
	Commission, 2%, $; guaranty, ½%, $					
	Amount charged to your account					

9. Find the rate of commission and the amount due Brown Bros. Co. in the following account purchase.

ROCHESTER, N.Y., Apr. 20, 19

Purchased by BROWN BROS. Co.

For the account and risk of W. D. SNOW,

Telephone, 1291 Main — Springfield, Mass.

	600 bbl. Pillsbury's Best Flour	6.00			
	100 bbl. xxxx Flour	5.50			
	200 bbl. Peerless Flour	5.25			
	Charges				
	Cartage		15	00	
	Commission ? %		104	00	
	Amount due us				

WRITTEN REVIEW EXERCISE

1. An agent bought for me a consignment of flour. He charged 3% and received as his commission $38.40. I sold the flour at a gain of 20%. What was my gain?

2. A commission merchant sold 5000 bu. grain and charged $1\frac{1}{2}¢$ per bushel for selling. If the grain was sold at 49¢ per bushel, what sum did he remit to his principal?

3. I paid a grain merchant $22.26 for selling a quantity of grain. If he charged 2% commission and sold the grain at $1.06 per bushel, how many bushels did he sell?

4. The net proceeds of a consignment were $593.75. The following were the different charges: commission, $26; freight, $8.55; drayage, $3.40; storage, $9.20; advertising, $3; insurance, $6.10. What was the rate of commission?

5. During the months of July and August a college student traveled for the Lester Manufacturing Co., receiving a commission of 10% on all sales. After paying his expenses, $140.60, he had left as his net earnings $159.40. What was the value of the goods sold?

6. A commission merchant charged $3\frac{1}{2}\%$ commission and $1\frac{1}{2}\%$ guaranty for buying a stock of provisions. If the commission merchant received $22, what sum should the principal remit to cover cost of the provisions, commission, and guaranty?

7. B was given a difficult account for collection, with the assurance that he should receive 25% of all he might collect. He collected the account and remitted to the holder $198.42. What was the amount collected?

8. A firm of contractors employed an agent to collect their overdue accounts. As a special inducement for closing the accounts, they were to give him 6% on all collections made the first month, and $3\frac{1}{2}\%$ on all collections made the second month. The first month he returned to the firm $4013.80; the second month he returned $2798.50. The returns were made after taking out his commission. What was the agent's commission?

CHAPTER XXII

PROPERTY INSURANCE

FIRE INSURANCE

ORAL EXERCISE

1. One hundred persons have property valued at $500,000. They pay into a common fund $\frac{3}{5}\%$ of this sum; what is the amount of the fund?

2. These one hundred persons live in widely separated parts of the country. Is it likely that many of them will suffer losses by fire in the same year?

3. Suppose the losses to this property by fire for a year amount to $2500. What portion of the common fund remains on hand as a surplus? (No interest.)

4. If this surplus is divided among the 100 persons at the end of the year, how much should A, who paid in $30, receive?

5. What are the companies organized to receive and control the fund in problem 1 called?

333. Insurance is a contract whereby for a stipulated consideration one person agrees to indemnify another for loss on a specified subject by specified perils. The **main heads** of property insurance are fire insurance and marine insurance.

There are also companies which insure against steam-boiler explosions, failure of crops, death of cattle, burglary, interruption to business by strikes among employees, and numerous other hazards.

334. Fire insurance is insurance against loss of property or damage to it by fire.

Fire insurance contracts frequently cover loss caused by lightning, cyclones, and tornadoes. Fire insurance companies are liable for loss resulting from the use of water applied for the purpose of extinguishing flames; also for the destruction of buildings to prevent fire from spreading.

335. The **insurer**, sometimes called the **underwriter**, is the one who agrees to indemnify. The **insured** is the one to whom the promise of indemnity is made. The **premium** is the consideration agreed upon. The **policy** is the written contract.

336. Insurance companies are usually either stock companies or mutual companies. A **stock insurance company** is one in which the capital is subscribed, paid for, and owned by persons called stockholders, who share all the gains and are liable for all the losses. A **mutual insurance company** is one in which the policy holders share the gains and bear the losses.

In a mutual insurance company there are no stockholders, and the capital stock consists of the reserve earnings and investments of the company.

337. Policies of insurance are of various kinds. It is necessary to distinguish between the valued and the open policy. A **valued policy** is one that states the amount to be paid in case of loss. An **open policy** is one in which the amount to be paid in case of loss, not exceeding a certain sum, is left to be determined by evidence after the loss occurs.

Valued policies are very generally used in the insurance of ships, but not in the insurance of cargoes. Open policies are generally used in fire insurance.

338. The standard form of fire insurance policy states the maximum amount for which the company is liable, and this amount is used as a basis for computing premiums.

If a loss either total or partial occurs under such a policy, the company is bound to pay only so much of the sum stated in the policy as will indemnify the insured; *e.g.* if a building insured for $3000 is damaged by fire $400, only the actual loss, $400, can be recovered; but if the same building were damaged by fire $3500, the company could not be held for more than the sum stated in the policy, $3000.

339. Many fire insurance policies contain a **co-insurance** clause to the effect that the person insured shall keep his property insured for a certain per cent of its value, and that if he fails to do this, the company will pay him only that proportion of the loss which the per cent insured bears to the per cent named in the policy.

Thus, the value of a piece of property is $10,000, and the insured agrees to keep it insured for 80% of its value, or $8000, but fails to do so and carries only $6000 insurance. Should a loss occur, the company will pay only three fourths ($\frac{6000}{8000}$) of the amount of such loss.

340. The rate of premium is determined by the character of the risk and the length of time for which the policy is issued. It is sometimes stated as a per cent of the amount insured and sometimes as a certain rate on $100.

In some localities insurance agents sometimes charge a small fee for surveying the premises and making out a policy, but the practice is not common.

Insurance is usually effected for one or more years. **Short rates** are charges made for a term less than one year; they are proportionately higher than yearly rates.

ORAL EXERCISE

1. What is the cost of $6500 insurance at $\frac{4}{5}$ %?

2. What is the premium on a $4000 policy at $1\frac{1}{2}$ %?

3. What is the cost of $6000 worth of insurance at 75¢ per $100?

4. B insures a $6000 barn for $\frac{2}{3}$ value at $\frac{1}{2}$%. What quarterly premium should he pay?

5. A insures a $6000 house for $\frac{4}{5}$ value, at 50¢ per $100. What is the semiannual premium?

6. Goods worth $3000 are insured for $\frac{2}{3}$ value. If the annual premium is $30, what is the rate?

7. I insure $2400 worth of merchandise for $\frac{5}{6}$ of its value at 60¢ per $100. What premium must I pay?

8. I insure a stock of goods worth $8000 for $6000 at 2%. The goods became damaged by fire to the extent of $3000. Under a standard fire insurance policy how much can I recover? What will be my net loss, premium included?

9. A brick schoolhouse is insured at 50¢ per $100, the annual premium is $50, and the face of the policy $\frac{5}{8}$ of the value of the building. What is the value of the building?

10. A house valued at $20,000 is insured in one company for $8000, and in another for $7000. A fire occurs by which the house is damaged $6000. How much should each company pay?

ORAL EXERCISE

State the premium in each of the following problems:

	Face of Policy	Rate		Face of Policy	Rate
1.	$1600	$1\frac{1}{2}\%$	3.	$3500	$1.10 per $100
2.	$1000	$1\frac{1}{4}\%$	4.	$5000	$1.20 per $100

State the face of the policy in each of the following problems:

	Premium	Rate		Premium	Rate
5.	$9	2%	7.	$13.50	$1.35 per $100
6.	$15	$1\frac{1}{2}\%$	8.	$24.00	$1.60 per $100

State the rate of insurance in each of the following problems:

	Face of Policy	Premium		Face of Policy	Premium
9.	$1700	$25.50	11.	$3200	$130.00
10.	$1850	$37.00	12.	$6500	$40.00

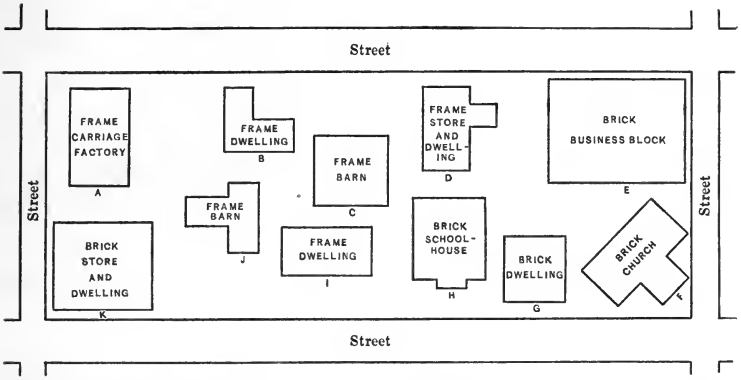
Table of Illustrative Rates

Risk	Annual Rate per $100	Risk	Annual Rate per $100
Frame carriage factory and contents	$1.75	Frame store and dwelling, and contents	$0.40
Frame dwelling and contents	.25	Brick store and dwelling, and contents	.25
Brick business block and contents	.50	Brick church and contents	.50
Frame barn and contents	1.00	Brick schoolhouse and contents	.50
Brick dwelling and contents	.17		

WRITTEN EXERCISE

1. A in the following diagram is a frame carriage factory worth $7000. Its contents are worth $8000. Both are insured at $\frac{4}{5}$ value. What is the amount of the annual premium?

2. B is worth $3400; its contents, $1200. C is worth $1500; its contents, $1100. All of this property is insured for 1 yr. at $\frac{5}{6}$ valuation. What is the annual premium? Two annual premiums in advance will pay for three years' insurance. At this rate what will it cost to insure the property for 3 yr.?



3. D is worth $4800. The contents of the store are worth $2400; of the dwelling, $800. What will it cost to insure all of this property at full value for 1 yr.? If three annual premiums in advance will pay for five years' insurance, what will it cost to insure the property for 5 yr.?

4. E is worth $20,000; its first-floor contents, $4500; its second- and third-floor contents, $7500. All are insured for 1 yr. at 75% valuation. What is the amount of the premium? A fire occurs, and the building and contents are damaged to the extent of $4500. If the contract of insurance (policy) contained an 80% co-insurance clause, how much will the company have to pay?

5. Suppose that E was insured in Company A for $18,000 at the rate in the table, and its contents in Company B for $10,000 at 75¢ per $100; that both policies contained an 80% co-insurance clause; and that the building was damaged to the extent of $3000 and the contents to the extent of $2500. How much would each company have to pay? What would be the net loss to the owner of the building? to the owner of the contents? (Premiums included in each case, but no interest.)

6. F is worth $10,000 and its contents, $3500. The property is insured for one year for $8100. If the policy contains an 80% co-insurance clause, what is the net loss to the insurance company, premium included, if the property is destroyed by fire?

7. A, the owner of G, has paid, annually for 5 yr., insurance on the dwelling and contents. The face of the policy is $6000. If the rate for five annual premiums in advance is the same as three separate annual premiums, how much would he have gained had he insured first for 5 yr.? (No interest.)

8. H is worth $15,000 and its contents $7500. Find the cost of insuring 80% of its value for 5 yr., three annual premiums in advance paying for five years' insurance.

9. For insuring I and J and contents at $\frac{3}{4}$ value, the owner paid an annual premium of $22.50. What is the value of the property, the value of J being $\frac{2}{7}$ of the value of I?

10. K, worth $15,000, is insured in three companies for $\frac{4}{5}$ value. Company A takes $\frac{1}{3}$ of the risk at the price in the table; Company B, $\frac{2}{5}$ of the risk at 50¢ per $100; Company C, the remainder at $\frac{2}{3}$ %. What was the total premium? The block becomes damaged by fire to the amount of $6000. How much will each company be obliged to pay?

11. I insured my block of buildings with the Ætna Insurance Co. for $75,000, at 75¢ per $100. The Ætna Insurance Co. later reinsured $15,000 with the Continental Insurance Co. at $\frac{3}{4}$% and $20,000 with the German-American Insurance Co. at $\frac{3}{4}$%. The block became damaged by fire $20,000. What was the net loss of the Ætna Insurance Co.? What was the net loss of the Continental Insurance Co.? of the German-American Insurance Co.?

MARINE INSURANCE

341. Marine insurance is insurance against loss to ships and cargoes by perils of navigation.

342. In marine insurance, the policies usually contain a clause to the effect that if a vessel or cargo, or both, are valued at more than the amount insured, the insurers will pay only such part of the loss, either partial or total, as the amount insured bears to the full valuation. This clause is called an **average clause.**

Thus, should a vessel valued at $20,000, and insured for $15,000, become damaged by fire to the extent of $8000, under an average clause policy the company will pay three fourths ($\frac{15000}{20000}$) of the loss, or $6000. Should the same vessel and cargo be wholly destroyed, the company will pay the full $15,000, which is three fourths of the entire valuation. In order to be fully protected in a marine risk, the insured must insure his property for full value. Some fire insurance policies contain a clause similar to the average clause of marine insurance policies.

WRITTEN EXERCISE

1. A vessel valued at $50,000 is insured (average clause policy) for $18,000 in Company A, and for $17,000 in Company B. A fire occurs by which the vessel is damaged $15,000. What is the amount to be paid by each company?

2. I paid $25.40 for insuring a shipment of goods by steamer from Boston to Manila. If the rate was $1\frac{1}{4}\%$, less 20%, what was the face of the policy? If the face of the policy was equal to the value of the goods, what would it cost to make the shipment by sailing vessel at $1\frac{7}{8}\%$, less 20%?

3. You take out a $7500 average clause policy on your stock of merchandise worth $9000. The premium is 75¢ per $100, which you pay in advance. A fire occurs by which the stock is damaged $3000. Estimate your total loss and the net loss to the company. (Premium included in each case.)

4. A of Boston instructed B of Sidney, Australia, to purchase $25,000 worth of hides. B made the investment as instructed and charged $1\frac{7}{8}\%$ commission. The hides were then shipped by steamer and insured at $1\frac{1}{2}\%$ for enough to cover the value of the hides and all charges. What was the amount of the policy and what was the premium?

5. A of New York ordered B of Duluth to buy on commission 6000 bu. of wheat and 6000 bu. of corn. B bought the wheat at 92¢ and the corn at 57¢ per bushel, and charged $1\frac{1}{2}$¢ per bushel commission. Before shipping the grain to A by boat, B took out a policy of insurance at $1\frac{1}{2}\%$ to cover the cost of the goods and all charges. What was the agent's commission? the insurance premium? What did the grain cost A?

CHAPTER XXIII

STATE AND LOCAL TAXES

ORAL EXERCISE

1. How are the expenses of towns, cities, counties, and states met?

2. A has property worth $5000 and B property worth $10,000. How should the taxes of these two men compare?

3. Mention several purposes for which taxes are raised in your city or town.

343. A **tax** is a sum levied for the support of government, or for other public purposes. Taxes are of two kinds: **direct taxes**, which are taxes levied on a person, his property, or his business; **indirect taxes**, which are taxes levied on imported goods, and on tobacco, liquors, etc., produced and consumed in the United States.

The expenses of town, county, city, and state governments are met by *capitation* or *poll taxes*, *property taxes*, and *license fees*. The expenses of the National Government are met chiefly by *import duties*, or *customs*, and *excise duties*.

344. A **capitation**, or **poll tax**, is a tax sometimes levied on each male inhabitant who has attained his majority. A **property tax** is a tax levied on real estate or on personal property. A **license fee** is a tax paid for permission to engage in certain kinds of business.

Real estate and personal property belonging to religious or charitable organizations are frequently exempt from taxation.

345. Property taxes are imposed in nearly all the states by practically the same method, namely:

1. Officers called assessors are elected in every city and town, whose business it is to set a valuation upon all property subject to taxation.

2. In most of the states a County Board of Equalization reviews the original assessments, and the judgment of this body is subsequently passed upon by the State Board of Equalization.

3. All the taxes for state purposes are then equitably apportioned among the different counties, cities, and towns. Each county, city, town, and school district also levies taxes for its own local expenses.

Real estate is usually assessed at from 25% to $33\frac{1}{3}$% less than its market value.

346. The **tax rate** is expressed as so many mills on the dollar or so many dollars on a hundred or a thousand dollars.

The methods of collecting taxes vary somewhat in the different states. A common method, which, on the whole, seems satisfactory, is for one collector in each city or town to collect all the taxes — state, county, city or town — at one time. If taxes are not paid, the property taxed may be sold. The purchaser of property sold for taxes is given only a tax title to it; but this title becomes complete after a certain period allowed the original owner for redemption. In some states if the poll tax is not paid, the person taxed may be committed to jail. The compensation of a collector is either a fixed salary or a commission on all taxes collected.

ORAL EXERCISE

1. If the rate of taxation is 12 mills on a dollar, how much tax must I pay on property assessed at $5000?

2. The tax rate is 13 mills on a dollar. B has property valued at $8000 and assessed at $\frac{3}{4}$ value. What is his tax?

3. C pays $1\frac{1}{4}$% tax on a city lot 100 ft. by 150 ft., valued at $1 per square foot, and assessed at $\frac{2}{3}$ value. What is the amount of his tax?

4. What tax must I pay on $80,000, at 5 mills on $1, the collector's commission being 1%?

SOLUTION. .005 of $80,000 = $400, the tax.
1% of the tax = 4, the collector's commission.
$404, my property tax.

5. What tax must I pay on $10,000 at $4\frac{1}{2}$ mills on $1, the collector's commission being 1%?

6. If the state tax is 2 mills, the county tax 3 mills, and the district school tax $\frac{1}{2}\%$, what should you pay on a farm assessed at $3000?

7. My total tax this year was $61.25. If I have property valued at $10,000, and my poll tax amounts to $1.25, what is the rate of taxation?

8. A collector turns over to the county treasurer $8000. If his commission was $1\frac{1}{2}\%$ what amount did he collect? If the property taxed was worth $800,000, what was the rate of taxation? Express this rate in three ways.

9. The assessed valuation of real and personal property in a certain city is $400,000,000. The city has a bonded indebtedness of $2,000,000, on which it pays 4% interest. Find the tax rate necessary to pay the interest.

WRITTEN EXERCISE

Find the total tax:

1. Valuation, $3600; rate, $0.016; 3 polls at $2.

2. Valuation, $4550; rate, $9\frac{1}{2}$ mills; 1 poll at $1.50.

3. Valuation, $2875; rate, $0.0175; 1 poll at $1.75.

4. Valuation, $5600; rate, $\$1.12\frac{1}{2}$ per $100; 1 poll at $2.

5. Valuation, $6000; rate, $13.40 per $1000; 2 polls at $1.00.

Find the valuation:

6. Total tax, $3800; rate, $0.015; 100 polls at $2.00.

7. Total tax, $11,295; rate $1.40 per $100; 250 polls at $1.50.

8. Total tax, $8850; rate, $15.00 per $1000; 225 polls at $1.00.

9. In a town 1040 persons were subject to a poll tax; the assessed valuation of real estate was $3,209,400, and of personal property $265,100. The polls were taxed $1.25 each. The tax levy was $42,994. What was the tax rate? What was the total tax of Charles B. Lester, who owned real estate valued at $6450, and personal property valued at $1250, and who paid for 2 polls?

10. In a town taxes were levied as follows: state tax, $4287; county tax, $9312.50; town tax, $93,156.20. There were 1850 polls assessed at $2 each. If the total property valuation was $6,245,800, what was the tax rate per thousand?

11. A town made provision by taxation for the following expenses: public schools $18,180; interest on borrowed money $2106; public highways $4720; officials' salaries $4620; general expenses $11,746; sinking fund $8000. The value of real and personal property was $2,450,600, and 2120 polls were assessed $1.50 each; $4531.80 was collected from license fees. What was the tax rate?

12. A died leaving property valued at $47,950 to B, his son, and property valued at $17,500 to C, a friend. The statutes of the state in which these three live provide that B, a lineal heir, and C, a collateral heir, shall pay to the state an **inheritance tax**. The rate for lineal heirs is 1%, and for collateral heirs 5%. What inheritance tax must B and C, respectively, pay when they come into possession of the property?

13. A city made the following appropriation for its public schools: teaching and supervision, $36,000; care and cleaning, $3360; fuel, $3000; repairs, $2000; text-books, $1700; supplies, $1700; printing, $300; contingent fund, $775; truant officer, $500; evening schools, $1305; transportation of pupils, $600; kindergarten, $1100; manual training, $700. The assessed value of real estate was $6,709,998 and of personal property $2,130,002. What was the tax rate for school purposes?

14. The market value of a certain street railway amounts to $20,881,000. This amount, less the company's real estate, machinery, etc., is subject to a state **corporation tax** of $17.25 per $1000. If the value of the real estate, machinery, etc., is $4,570,700, what is the corporation tax? This corporation tax is distributed according to trackage among the cities and towns in which the railway operates. If 80% of the trackage of the road lies within the city of B, how much of the state corporation tax will that city receive?

347. In order to facilitate clerical work a table may be used for computing taxes. The following table was made from the published tax lists of a city in Massachusetts:

TAX TABLE. RATE $18.60 PER $1000

	0	1	2	3	4	5	6	7	8	9
0	.0000	.0186	.0372	.0558	.0744	.0930	.1116	.1302	.1488	.1674
1	.1860	.2046	.2232	.2418	.2604	.2790	.2976	.3162	.3348	.3534
2	.3720	.3906	.4092	.4278	.4464	.4650	.4836	.5022	.5208	.5394
3	.5580	.5766	.5952	.6138	.6324	.6510	.6696	.6882	.7068	.7254
4	.7440	.7626	.7812	.7998	.8184	.8370	.8556	.8742	.8928	.9114
5	.9300	.9486	.9672	.9858	1.0044	1.0230	1.0416	1.0602	1.0788	1.0974
6	1.1160	1.1346	1.1532	1.1718	1.1904	1.2090	1.2276	1.2462	1.2648	1.2834
7	1.3020	1.3206	1.3392	1.3578	1.3764	1.3950	1.4136	1.4322	1.4508	1.4694
8	1.4880	1.5066	1.5252	1.5438	1.5624	1.5810	1.5996	1.6182	1.6368	1.6554
9	1.6740	1.6926	1.7112	1.7298	1.7484	1.7670	1.7856	1.8042	1.8228	1.8414

In the table the rate on each $1000 was made up as follows: state tax $.0807; county tax, $.5643; state highways, $.003; city tax, $17.952. The first figure of the number of dollars assessed is given at the left, and the second one at the top.

348. Example. What is the tax on a valuation of $16,400?

SOLUTION. Tax on $16,000 = $297.60 (1000 times .2976)
Tax on 400 = 7.44 (100 times .0744)
Tax on $16,400 = $305.04

WRITTEN EXERCISE

Using the table, find the tax on the following valuations:

1. $2485.	**5.** $8,478.	**9.** $34,500.	**13.** $20,000.
2. $1200.	**6.** $13,200.	**10.** $82,500.	**14.** $27,800.
3. $1050.	**7.** $14,700.	**11.** $98,250.	**15.** $71,690.
4. $4630.	**8.** $18,400.	**12.** $21,850.	**16.** $89,800.

Find the tax on the following valuations when the collector's commission is 1 %:

17. $5500.	**21.** $9500.	**25.** $19,000.	**29.** $21,000.
18. $7500.	**22.** $8700.	**26.** $26,000.	**30.** $89,000.
19. $2900.	**23.** $6500.	**27.** $85,000.	**31.** $10,000.
20. $4700.	**24.** $7250.	**28.** $78,000.	**32.** $21,000.

CHAPTER XXIV

CUSTOMS DUTIES

ORAL EXERCISE

1. The expenses of the National Government average about $1,500,000 per day. What is this per year?

SUGGESTION. To multiply by 15, multiply by 10 and add ½ of the result.

2. Name five sources of income to the National Government.

3. Name ten expense items of the National Government.

349. **Duties**, or **customs**, are taxes levied by the National Government on imported goods. They are imposed in two forms: ad valorem and specific. An **ad valorem duty** is a certain *per cent* levied on the net cost of the importation. A **specific duty** is a *fixed sum* levied on each article, or on each pound, ton, yard, or other standard measure, without regard to the cost.

Ad valorem duties are not computed on fractions of a dollar. If the cents of the net cost are less than fifty, they are rejected; if fifty or more than fifty, one dollar is added before computing the duty.

Some articles are subjected to both ad valorem and specific duties. Before specific duties are estimated allowance is usually made for tare and breakage. Specific duties are not computed on fractions of a unit. Fractions less than ½ of a unit are rejected; fractions ½ or more are counted a whole unit. The **long ton** of 2240 lb. is used in computing specific duties.

350. A **tariff** is a schedule exhibiting the different rates of duties imposed by Congress on imported articles. A **free list** is a schedule of imported articles exempt from duty.

351. A **customhouse** is an office established by the National Government for the collection of duties and the entry and clearance of vessels. A port at which a customhouse is established is called a **port of entry**; ports of entry and other ports are called **ports of delivery.**

The United States is divided into *collection districts*, in each of which there is a port of entry and one or more ports of delivery. All entries of goods and the payment of duties thereon must be made at the port of entry, after which the goods may be discharged at any port of delivery.

352. In the most important ports of the United States the customhouse business is distributed among three departments:

1. The **collector's office**, which takes charge of the entries and papers, issues the permits, and collects the duties.

2. The **surveyor's office**, which takes charge of the vessel and cargo, receives the permits, ascertains the quantities, and delivers the merchandise to the importer.

3. The **appraiser's office**, which examines imported merchandise and determines the dutiable value and the rate of duty on same.

One package of every invoice and one package, at least, out of every ten similar packages is sent to the appraiser's store for examination. Merchandise in bulk and all heavy and bulky packages uniform in size and quantity of contents are generally examined on the wharf.

353. A **manifest** is a memorandum, signed by the master of the vessel, showing the name of the vessel, its cargo, and the names and addresses of the consignors and consignees. An **invoice** is a detailed statement showing the particulars of the goods imported.

All invoices must be made out in the weights and measures of the country in which the goods are purchased; and if the goods are subject to an ad valorem duty they must be invoiced in the currency of the country into which they are imported. Invoices over $100 must be certified before a United States consul, who causes two copies of the invoice to be made. One of these is sent to the collector of the port at which the goods are to be entered and the other is kept on file in the consul's office.

When the merchandise is loaded on board the vessel the shippers are given **a bill of lading** which acknowledges the receipt of the several packages and agrees to deliver the same at destination. The vessel's commander keeps a copy of the bill of lading and from the several that have been issued makes out his manifest of cargo. The shippers mail the invoice and bill of lading to the purchaser, who fills out an entry therefrom and presents it and the invoice at the customhouse where the duties imposed by law on the several classes of merchandise are collected and a permit issued for the landing and delivery of the merchandise, subject to examination.

354. The **values of foreign coins** are periodically proclaimed by the Secretary of the Treasury, and these values must be taken in estimating duties unless a depreciation of the value of the foreign currency expressed in an invoice shall be shown by the consular certificate thereto attached. The following estimate of the values of foreign coins was recently proclaimed.

VALUES OF FOREIGN COINS

COUNTRY	STANDARD	MONETARY UNIT	VALUE IN U. S. GOLD
Brazil	Gold	Milreis	$ .546
Denmark, Norway, Sweden	Gold	Crown	.268
France, Belgium, Switzerland	Gold	Franc	.193
German Empire	Gold	Mark	.238
Great Britain	Gold	Pound sterling	4.866½
Japan	Gold	Yen	.498
Mexico	Silver	Dollar	.498
Netherlands	Gold	Florin	.402
Philippine Islands	Gold	Peso	.500
Russia	Gold	Ruble	.515

The *lira* of Italy, and the *peseta* of Spain, are of the same value as the franc. The *dollar*, of the same value as our own, is the standard of the British possessions of North America, except Newfoundland.

355. Depositing goods in a government or bonded warehouse is called **warehousing.**

Many importers buy foreign goods in large quantities, withdraw a part of them, and store the remainder in the government warehouse. The goods so deposited may be taken out at any time in quantities not less than an entire package, or in bulk, if not less than one ton, by the payment of duties, storage, and labor charges. Foreign goods are sometimes bought three or four months earlier than they can be placed on the market and are stored in the government warehouse until they are seasonable. In this way importers are able to make better selections and they also get better terms and prices.

356. A **bonded warehouse** is a building provided for the storage of goods on which duties have not been paid.

The importer must give bond for the payment of duties on all goods stored in a bonded warehouse. Goods remaining in bond are charged 10%

additional duty after 1 yr. Goods left in the government warehouse beyond 3 yr. unclaimed are forfeited to the government and sold under the direction of the Secretary of the Treasury. Goods may be withdrawn from a bonded warehouse for export without the payment of duty.

357. The two common forms of entry under which duties are collected are known as **inward foreign entry** and **warehouse entry.** The former is used for merchandise entered for consumption; the latter for merchandise that is placed in a bonded warehouse under charge of the government storekeeper.

358. Excise duties are taxes levied on certain goods produced and consumed in the United States. If goods, on which either excise or import duties have been paid, are exported, the amount so paid is refunded. The amount refunded is called a **drawback.**

TABLE OF DUTIES ON CERTAIN IMPORTS

ARTICLE AND DESCRIPTION	DUTY	
	Specific	Ad Valorem
Axminster rugs	10¢ per sq. yd.	40%
Barley, 48 lb. to the bushel	30¢ per bu.	
Barley malt, 34 lb. to the bushel	45¢ per bu.	
Beans, 60 lb. to the bushel	45¢ per bu.	
Brussels carpets	44¢ per sq. yd.	40%
Books		25%
Castile soap	$1\frac{1}{4}$¢ per lb.	
Cheese	6¢ per lb.	
China, porcelain, and crockery ware		60%
Clocks and watches		40%
Corn, 56 lb. to the bushel	15¢ per bu.	
Cotton tablecloths		50%
Hay	$4 per T.	
Ingrain carpets	22¢ per sq. yd.	40%
Knit woolens	44¢ per lb.	50%
Leather and leather goods		20%
Marble	65¢ per cu. ft.	25%
Plate glass	8¢ per sq. ft.	
Pocket knives, value not more than 50¢ per doz.	1¢ apiece	40%
Potatoes, 60 lb. to the bushel	25¢ per bu.	
Silk dress goods	11¢ per sq. yd.	50%
Sugar	$\frac{95}{100}$¢ per lb.	
Toilet soap, all descriptions	15¢ per lb.	
Wheat	25¢ per bu.	
Window glass	$1\frac{3}{8}$¢ per lb.	

FINDING A SPECIFIC DUTY

ORAL EXERCISE

Using the table on page 288, find the duty on:

1. 67,200 lb. of hay.
2. 48,000 lb. of barley.
3. 100 pc. plate glass 24″ x 30″.
4. 2400 lb. of window glass 10″ x 15″.
5. A quantity of cheese weighing 1000 lb.
6. A shipment of wheat weighing 240,000 lb.
7. A quantity of castile soap weighing 2100 lb.; tare 100 lb.

WRITTEN EXERCISE

1. *Using the table on page 288, find the total duty on:*

2500 bu. potatoes. 95,000 lb. barley. 44,800 lb. corn.
1275 lb. toilet soap. 24,000 lb. beans. 10,000 lb. cheese.
6500 lb. castile soap. 136,000 lb. barley malt. 30,000 bu. potatoes.

2. What is the duty on 175 bx. castile soap, each weighing 110 lb., if 5% is allowed for tare?

3. Calculate the duty on 10 hogsheads of sugar weighing 1060–105, 1040–105, 1160–112, 1240–120, 1180–116, 1100–102, 1090–101, 1100–100, 1005–100, 1210–118 lb., respectively.

4. Richard Roe & Co. imported from Canada 3750 bu. of potatoes invoiced at 20¢ per bushel. If the transportation and other charges amounted to $187.50, how much must be received per bushel for the potatoes in order to gain 25%?

FINDING AN AD VALOREM DUTY

ORAL EXERCISE

Find the total duty:

1. On 40 clocks invoiced at $4.50 each.
2. On 12 books invoiced at $1.50 each.
3. On 25 doz. pocket knives invoiced at 50¢ per doz.
4. On 100 sq. yd. ingrain carpet invoiced at $1 per yard.

WRITTEN EXERCISE

Find the duty on:

1. An Axminster rug, $12' \times 18'$, invoiced at £10.

For the values of foreign coins, see page 287.

2. A 200 lb. box of knit woolen goods invoiced at £100.

3. An importation of cotton table cloths invoiced at £100.

4. An importation of cotton table cloths invoiced at £255.

5. 300 bx. plate glass, each containing 25 plates $16'' \times 24''$.

6. 20 Axminster rugs, each $12' \times 18'$, invoiced at £8 6*s*. per rug.

7. An importation of china and crockery ware invoiced at 100 francs.

8. An invoice of knit woolens weighing 600 lb. and valued at £315 12*s*.

9. 200 blocks of marble, each $10' \times 4' \times 2'$, invoiced at 328,000 lira.

10. An importation of leather from Sweden invoiced at 6750 crowns.

11. 400 yd. of Brussels carpeting, $\frac{3}{4}$ yd. wide, invoiced at $2 per yard.

12. 4000 meters of Brussels carpeting, $\frac{3}{4}$ yd. wide, invoiced at 5 francs per meter.

A meter equals approximately 1.1 yd.

13. 4800 meters of silk dress goods, $\frac{3}{4}$ yd. wide, invoiced at 3.75 marks per meter.

14. A case of silk dress goods containing 200 yd., 1 yd. wide, invoiced at 1000 marks.

15. An invoice of leather goods from the Netherlands invoiced at 12,520 florins.

16. 5 cs. of silk dress goods, each containing 200 yd., $\frac{3}{4}$ yd. wide, invoiced at 20 marks per yard.

17. I bought an invoice of Swiss watches, paying 10750 fr. for them in Geneva. What was the total cost of the watches, including the duty?

INVOICES AND ENTRIES

WRITTEN EXERCISE

1. At what price per pair must the lace curtains in the following invoice be sold in order to realize a gain of $33\frac{1}{3}\%$?

No. 427 *Manchester, England,* Dec. 15, *19*

Invoice of Lace

Shipped by WILLIAM P. FIRTH & CO.

In the Steamer Catalonia *To* R. H. White Company
Boston, Mass.

Marks	No.	Quantity	Articles and Description	Price	Extension	Amount
W	317	50 doz. pr.	Lace Curtains	3/2/6	***-**-**	
			Less 2%		*-**-*	
					***-**-**	
			Insurance and Freight		4-10-6	
			Packing and Carting		16-6	
					***-**-**	
					$***.**	
			50% ad valorem duty		***.**	$***.**

2. Find the total cost of the following invoice:

Antwerp, Belgium, Apr. 2, *19*

Messrs. A. T. Stewart & Co.
New York City

Bought of SCHMIDT & WESTERFELDT

Terms 30 da.

S W	6	pc. Black Silk					
		39.00, 40.50, 39.00,					
		40.00, 41.00, 40.50	240	Rm. 4	Rm. ***		
		Insurance and freight			86		
		Cartage			6		Rm. ***
							$***.**
		50% ad valorem duty					**.**
		11¢ per yd. specific duty					**.**
							$***.**

39.00, 40.50, etc., above, equal the number of meters in each piece.

3. Copy the following invoice, supplying the missing terms:

Bradford, England, Dec. 5, 19

Invoice of Woolen Goods

Shipped by RADCLIFFE & SON

In the Steamship Winifredian — To R. H. Stearns & Co.

Terms 30 da. — Boston, Mass.

R/317	25	pc. Black Wool Crepon 68 69 69 68 69 60 55 60 56 54 60 60 60 68 68 60 45 65 65 55 60 65 65 60 60 1544 1/9	***	*	*
		Consul's fee		14 10	*** * *

4. If the foregoing invoice of goods were entered for immediate consumption, the following is the entry that would be made out. Complete the computation in the entry.

Manifest No. 650 Invoiced at Bradford, England, Dec. 15, 19

INWARD FOREIGN ENTRY OF MERCHANDISE

Imported by R. H. Stearns & Co. In the Steamer Winifredian

Fred M. Garson Master From Liverpool Arrived Jan. 5, 19

Mark	No.	Packages and Contents	Quantity	Free List	44c. per lb. + 60% ad valorem	Duty	Total
913	318	2 cs. Woolen Cloth £135.1 @ $4.8665 60% of $??? 256 lb. @ 44¢	1544 yds 256 lb.		135-2-0 $???.??	?????? ??????	??? ??

5. How much will R. H. Stearns & Co. have to receive per yard for the foregoing goods in order to realize a gain of 25%?

6. In a recent year the receipts from customs duties were $280,000,000, and from excise duties, $275,000,000. The customs duties for this year were what per cent greater than the excise duties? the excise duties were what per cent less than the customs duties?

7. Find the dutiable value and compute the duty on the following entries of merchandise:

a.

Manifest No. 370 Invoiced at Paris, France, Dec. 12, 19__

INWARD FOREIGN ENTRY OF MERCHANDISE

Imported by John Doe & Son In the Steamer Columbia
F. E. Langdon Master From Havre Arrived Jan. 3, 19__

Mark	No.	Packages and Contents	Quantity	Free List	50% ad valorem	$1\frac{3}{10}$c. per lb.	Duty	Total
◇	76	1 cs. Feathers			275.00 fr.			
◇	18	1 " Silk Ribbon			375.25 fr.			
◇	49	1 " Sewing Needles		25.00 fr.				
◇	73	1 " Flat Steel Wire	440 kilos 880 lb.	$?.??	650.25 fr.	23.25 fr. $?.??		
		50% of $???.??					???.??	
		88 lb. at 1.3¢					?.??	???.??

1 kilogram equals about $2\frac{1}{5}$ avoirdupois pounds. There is no duty charged on the value of the steel wire, nor on the quantity or value of the sewing needles; but the values of both of these quantities is reduced to United States money by the customhouse officials for statistical purposes.

b.

Manifest No. 715 Invoiced at Limoges, France, Dec. 11, 19__

INWARD FOREIGN ENTRY OF MERCHANDISE

Imported by Richard Roe & Co. In the Steamer Silesia
John P. Kennedy Master From Havre Arrived Jan. 3, 19__

Mark	No.	Packages and Contents	Quantity	Free List	60% ad valorem	65c. per lb. + 25% ad valorem	Duty	Total
W.H.K. C.O.D.	1/192 376/192	2 cs. Decorated China			845.00 fr.			
		1 " Celluloid Combs	4 kilos ? lb.		$???.??	133.50 fr. $??.??		
		60% of $???.					???.??	
		25% of $??.					??.??	
		? lb. @ 65¢					?.??	???.??

INTEREST AND BANKING

CHAPTER XXV

INTEREST

ORAL EXERCISE

1. A borrows $100 of B for 1 yr. At the end of the year what will A probably pay B besides the face of the loan?

2. C puts $100 in a savings bank and leaves it for 1 yr. What can he draw out at the end of the year besides the money deposited?

3. If you wished to borrow money of a bank in your town, what rate of interest would you have to pay?

4. If you loaned a man $500 for 1 yr., what would you require him to give you as evidence of the loan and security for its payment?

359. The compensation paid for the use of money is called **interest.** Interest is computed at a certain per cent of the sum borrowed. This per cent of interest is called the **rate,** and the sum upon which it is computed, the **principal.**

The rate of interest allowed by law is called the **legal rate.** Persons may agree to pay less than this rate, but not more, unless a higher rate by special agreement is permitted by statute. When an obligation is interest-bearing and no rate is mentioned, the legal rate will be understood. An agreement for interest greater than that allowed by law is called **usury.**

360. In the **commercial world,** 12 mo. of 30 da. each, or 360 da., are reckoned as 1 yr.

This method is not exact, but it is the most common because the most convenient. It has been legalized by statute in some states and is generally used in all the states.

SIMPLE INTEREST

The Day Method

ORAL EXERCISE

1. How many days in a commercial year?

2. What part of a year is 60 da.? 6 da.? What is the interest on \$1 for 1 yr. at 6%? for 60 da.? for 6 da.?

3. How do you find .01 of a number? .001 of a number? What is the interest on \$120 for 60 da. at 6%? for 6 da.?

4. State a short method for finding the interest on any principal for 60 da. at 6%; for 6 da.

5. 1 da. is what part of 6 da.? What is $\frac{1}{6}$ of .001? What is the interest on \$1200 for 1 da. at 6%? on \$180? on \$1500?

6. State a short method for finding the interest on any principal for 1 da. at 6%.

361. In the foregoing exercise it is clear that *0.001 of any principal is equal to the interest for 6 da. at 6%;* or *0.001 of any principal is equal to 6 times the interest for 1 da. at 6%.*

ORAL EXERCISE

1. Find the interest on each of the following for 6 da. at 6%.

a. \$250.	*e.* \$560.	*i.* \$678.	*m.* \$290.	*q.* \$890.
b. \$870.	*f.* \$435.	*j.* \$320.	*n.* \$150.	*r.* \$750.
c. \$358.	*g.* \$430.	*k.* \$100.	*o.* \$325.	*s.* \$580.
d. \$350.	*h.* \$470.	*l.* \$185.	*p.* \$990.	*t.* \$625.

2. Find the interest on each of the above amounts for 12 da. at 6%; for 18 da.; for 24 da.

3. Find the interest on each of the following for 1 da. at 6%.

a. \$360.	*e.* \$660.	*i.* \$600.	*m.* \$480.	*q.* \$840.
b. \$450.	*f.* \$900.	*j.* \$180.	*n.* \$780.	*r.* \$200.
c. \$300.	*g.* \$540.	*k.* \$720.	*o.* \$400.	*s.* \$330.
d. \$420.	*h.* \$240.	*l.* \$500.	*p.* \$120.	*t.* \$960.

4. Find the interest on each of the above amounts for 3 da. at 6%; for 2 da.

362. Example. Find the interest on \$450 for 54 da. at 6 %.

SOLUTION. Pointing off three places to the left gives \$0.45, or 6 times the interest for 1 da. Multiplying this result by 54 gives \$24.30, or 6 times the interest for 54 da. Dividing this result by 6 gives \$4.05, the required interest.

$54 \times \$0.45 = \$24.30$

$\$24.30 \div 6 = \$4.05$

By arranging the numbers as shown in the margin and canceling the work is greatly shortened.

$$\frac{\overset{9}{\cancel{54}} \times \$0.45}{\underset{}{\cancel{6}}} = \$4.05$$

WRITTEN EXERCISE

At 6% find the interest on each of the following problems. Reduce the time expressed in months and days to days.

	PRINCIPAL	TIME		PRINCIPAL	TIME		PRINCIPAL	TIME
1.	\$620	54 da.	7.	\$900.00	29 da.	13.	\$375.80	2 mo. 15 da.
2.	\$175	84 da.	8.	\$865.45	93 da.	14.	\$300.00	3 mo. 19 da.
3.	\$645	42 da.	9.	\$700.00	96 da.	15.	\$171.15	1 mo. 14 da.
4.	\$300	84 da.	10.	\$974.30	62 da.	16.	\$120.00	4 mo. 14 da.
5.	\$600	72 da.	11.	\$178.45	40 da.	17.	\$211.16	6 mo. 16 da.
6.	\$502	66 da.	12.	\$438.55	50 da.	18.	\$665.65	1 mo. 10 da.

ORAL EXERCISE

1. What is the interest on \$800 for 6 da. at 3 % ?

SOLUTION. 80¢ is the interest for 6 da. at 6 %. 3% is $\frac{1}{2}$ of 6%; therefore, $\frac{1}{2}$ of 80¢, or 40¢, is the interest for 6 da. at 3%.

2. If the interest at 6% is \$45, what is the interest for the same time at 3% ? at 12% ? at 2% ? at 1% ? at $1\frac{1}{2}$% ?

3. Formulate a short method for changing 6% interest to 8% interest.

SOLUTION. 8% is $\frac{1}{3}$ more than 6%; hence, *the interest at 6% increased by $\frac{1}{3}$ of itself equals the interest at 8%.*

4. State a short method for changing 6% interest to 7% interest; to 5% ; to 9% ; to $7\frac{1}{2}$% ; to $4\frac{1}{2}$%.

5. If the interest at 6% is \$120, what is the interest at 7% ? at 5% ? at 8% ? at 4% ? at $7\frac{1}{2}$% ? at $4\frac{1}{2}$% ?

363. In the foregoing exercise it is clear that *6% interest increased by $\frac{1}{2}$ of itself equals 9% interest; by $\frac{1}{3}$ of itself, 8% interest; by $\frac{1}{4}$ of itself, $7\frac{1}{2}$% interest; by $\frac{1}{6}$ of itself, 7% interest;* also that

6% interest decreased by $\frac{1}{3}$ of itself equals 4% interest; by $\frac{1}{4}$ of itself, $4\frac{1}{2}$% interest; by $\frac{1}{6}$ of itself, 5% interest; also that

6% interest divided by 2 equals 3% interest; by 3, 2% interest; by 6, 1% interest; by 4, $1\frac{1}{2}$% interest.

6% interest multiplied by 2 equals 12% interest.

6% interest is changed to 10% interest by dividing by 6 and removing the decimal point one place to the right; to any other rate by dividing by 6 and multiplying by the given rate.

WRITTEN EXERCISE

Using the exact number of days, find the interest on:

1. \$2500 from Sept. 18, 1906, to Feb. 6, 1907, at 9%; at $3\frac{1}{2}$%; at 4%; at 3%.

2. \$1700 from Nov. 20, 1906, to Jan. 16, 1907, at 8%; at $2\frac{1}{2}$%; at $5\frac{1}{2}$%; at $3\frac{1}{2}$%; at 4%.

3. \$2750 from Dec. 16, 1906, to Jan. 17, 1907, at 7%; at 2%; at 4%; at 5%; at 1%; at 10%.

4. \$6250 from Dec. 18, 1906, to Feb. 6, 1907, at $7\frac{1}{2}$%; at 10%; at $1\frac{1}{2}$%; at $4\frac{1}{2}$%; at 9%; at 8%; at 7%; at 3%.

The Banker's Sixty-Day Method

ORAL EXERCISE

1. 60 da. (2 mo.) is what part of a commercial year?

2. What is the interest on \$1 for 2 mo. at 6%? for 60 da.?

3. How can you find 0.01 of a number? What is the interest on \$50 for 60 da. at 6%? on \$370? on \$590? on \$214.55?

4. What fractional part of 60 da. is 30 da.? 20 da.? 15 da.? 10 da.? What is the interest on \$1680 for 60 da.? for 30 da.? for 20 da.? for 15 da.? for 10 da.?

5. State a simple way to find the interest on any principal for 60 da. at 6%; for 30 da.; for 20 da.; for 15 da.; for 10 da.

6. Read aloud the following, supplying the missing words:

a. 60 da. minus $\frac{1}{12}$ of itself equals 55 da.; 60 da. minus —— of itself equals 50 da.; 60 da. minus —— of itself equals 40 da.; 60 da. minus —— of itself equals 45 da.

b. 60 da. plus $\frac{1}{12}$ of itself equals 65 da.; 60 da. plus —— of itself equals 70 da.; 60 da. plus —— of itself equals 75 da.; 60 da. plus —— of itself equals 80 da.; 60 da. plus —— of itself equals 90 da.

7. What is the interest on \$600 for 60 da. at 6%? for 55 da.? for 50 da.? for 40 da.? for 45 da.?

8. What is the interest on \$1200 for 60 da.? for 65 da.? for 70 da.? for 75 da.? for 80 da.? for 90 da.?

9. State a short way to find the interest at 6% for 80 da.; for 90 da.; for 50 da.; for 65 da.; for 55 da.; for 75 da.; for 70 da.; for 40 da.; for 45 da.

364. In the above exercise it is clear that *removing the decimal point two places to the left in the principal gives the interest for 60 da. at 6%.*

365. Examples. 1. Find the interest on \$1950 for 20 da. at 6%.

SOLUTION. Removing the decimal point two places to the left gives the interest for 60 da. 20 da. is $\frac{1}{3}$ of 60 da. $\frac{1}{3}$ of \$19.50 = \$6.50.

\$19.50
\$6.50

2. What is the interest on \$8400.68 for 75 days?

SOLUTION. Removing the decimal point two places to the left gives the interest for 60 da. 75 da. is 60 da. increased by $\frac{1}{4}$ of itself; therefore, \$84.0068 increased by $\frac{1}{4}$ of itself or \$105.01 is the required interest. In the following exercise determine the separate interest mentally whenever it is possible to do so.

\$84.0068
21.0017
\$105.0085, or \$105.01

WRITTEN EXERCISE

1. Find the total amount of interest at 6% on:

\$8400 for 60 da.	\$8400 for 12 da.	\$7900 for 20 da.
\$8400 for 30 da.	\$8400 for 10 da.	\$7900 for 15 da.
\$8400 for 20 da.	\$7900 for 60 da.	\$7900 for 12 da.
\$8400 for 15 da.	\$7900 for 30 da.	\$7900 for 10 da.

2. Find the total amount of interest at 6% on:

$1600 for 60 da.	$1600 for 40 da.	$2800 for 75 da.
$1600 for 55 da.	$2800 for 60 da.	$2800 for 80 da.
$1600 for 50 da.	$2800 for 65 da.	$2800 for 90 da.
$1600 for 45 da.	$2800 for 70 da.	$7200 for 55 da.

3. Find the total amount of interest at 6% on:

$1500.60 for 30 da.	$832.60 for 90 da.	$8575.65 for 70 da.
$1800.72 for 20 da.	$720.18 for 10 da.	$6282.40 for 15 da.
$1200.64 for 15 da.	$440.70 for 40 da.	$1460.84 for 65 da.
$8400.60 for 10 da.	$479.64 for 50 da.	$1385.62 for 55 da.

4. Find the total amount of interest at 6% on:

$1800.40 for 90 da.	$7500.00 for 55 da.	$216.90 for 20 da.
$9200.50 for 80 da.	$8200.00 for 75 da.	$432.65 for 15 da.
$3240.64 for 70 da.	$6400.00 for 45 da.	$832.30 for 10 da.
$4125.18 for 45 da.	$1200.45 for 30 da.	$926.17 for 20 da.

ORAL EXERCISE

1. What is the interest on $215 for 6 da. at 6%? on $345? on $415? on $827.50? on $425.90? on $4520.60? State a simple way to find the interest on any principal for 6 da. at 6%.

2. What part of 6 da. is 3 da.? is 2 da.? is 1 da.? What is the interest on $720 for 6 da.? for 3 da.? for 2 da.? for 1 da.? State a brief method of finding the interest on any principal for 3 da. at 6%; for 2 da.; for 1 da.

3. Read aloud the following, supplying the missing words:

a. 6 da. minus $\frac{1}{6}$ of itself equals 5 da.; 6 da. minus —— of itself equals 4 da.

b. 6 da. plus $\frac{1}{6}$ of itself equals 7 da.; 6 da. plus —— of itself equals 8 da.; 6 da. plus —— of itself equals 9 da.

c. State a short method of finding the interest at 6% for 4 da.; for 5 da.; for 7 da.; for 8 da.; for 9 da.

366. In the above exercise it is clear that *removing the decimal point in the principal three places to the left gives the interest for 6 da. at 6%.*

367. Example. What is the interest on \$420 for 8 da. at 6%?

SOLUTION. Removing the decimal point three places to the left gives the interest for 6 da., or \$0.42. Since 8 da. is 6 da. plus $\frac{1}{3}$ of itself, \$0.42 increased by $\frac{1}{3}$ of itself, or \$0.56 is the required interest. In the following exercises determine the separate interests mentally whenever it is possible to do so.

\$.420
.140
\$.56

WRITTEN EXERCISE

1. Find the total amount of interest at 6% on:

\$800 for 6 da.	\$720 for 6 da.	\$1500 for 6 da.
\$800 for 3 da.	\$720 for 7 da.	\$1500 for 5 da.
\$800 for 2 da.	\$720 for 8 da.	\$1500 for 4 da.
\$800 for 1 da.	\$720 for 9 da.	\$1500 for 9 da.

2. Find the total amount of interest at 6% on:

\$1168 for 6 da.	\$1600 for 6 da.	\$2400 for 6 da.
\$1168 for 3 da.	\$1600 for 7 da.	\$2400 for 5 da.
\$1168 for 2 da.	\$1600 for 8 da.	\$2400 for 4 da.
\$1168 for 1 da.	\$1600 for 9 da.	\$2400 for 8 da.

3. Find the total amount of interest at 6% on:

\$640.50 for 8 da.	\$800.10 for 7 da.	\$213.80 for 50 da.
\$920.10 for 20 da.	\$240.80 for 90 da.	\$310.40 for 40 da.
\$280.40 for 15 da.	\$960.70 for 70 da.	\$135.90 for 10 da.
\$390.60 for 50 da.	\$845.60 for 90 da.	\$736.18 for 10 da.

ORAL EXERCISE

1. 600 da. is how many times 60 da.? If the interest on \$1 for 60 da. at 6% is \$0.01, what is the interest for 600 da.?

2. Give a rapid method for finding 0.1 of a number. What is the interest on \$500 for 600 da. at 6%? on \$350? on \$214.60? on \$359.80? on \$4500? on \$9243.80? on \$750? on \$2150?

3. What part of 600 da. is 300 da.? 200 da.? 150 da.? 75 da.? 120 da.? 100 da.? 50 da.?

4. What is the interest on \$1400 for 600 da.? for 300 da.? for 200 da.? for 150 da.? for 75 da.? for 120 da.? for 100 da.? for 50 da.?

5. State a brief method of finding the interest for 600 da. at 6%; for 300 da.; for 200 da.; for 75 da.; for 50 da.; for 150 da.; for 200 da.

6. If the interest on \$1 for 600 da. is \$0.10, what is the interest for 6000 da.? In how many days will any principal double itself at 6% interest?

7. What is the interest on \$1 for 6000 da. at 6%? on \$55? on \$75.60? on \$18.90? on \$350? on \$725? on \$9125.70.

8. What is the interest on each of the amounts in problem 7 for 3000 da.? for 2000 da.? for 1000 da? for 1500 da.?

9. What is the interest on \$2500 for 6000 da.? on \$2150? on \$7500? on \$790? on \$155.60?

10. What is the interest on each of the amounts in problem 9 for 6 da.? for 60 da.? for 600 da?

368. In the above exercise it is clear that *removing the decimal point in the principal one place to the left gives the interest for 600 da. at 6%; also that any sum of money will double itself in 6000 da.*

WRITTEN EXERCISE

Find the interest at 6% on:

1. \$240 for 3000 da. **5.** \$7420.50 for 600 da. **9.** \$1640 for 150 da.

2. \$318 for 6000 da. **6.** \$67218.90 for 30 da. **10.** \$1260.60 for 1 da.

3. \$912 for 2000 da. **7.** \$8400.50 for 400 da. **11.** \$17890 for 10 da.

4. \$316 for 1500 da. **8.** \$7500.79 for 1500 da. **12.** \$1696 for 100 da.

ORAL EXERCISE

1. How many times is 6 da. contained in 18 da.? in 24 da.? in 36 da.? in 42 da.? in 54 da.? in 48 da.?

2. What is the interest on \$150 for 6 da.? for 18 da.? for 48 da.? for 54 da.? for 36 da.? for 42 da.? for 12 da.?

3. What is the interest on \$350 for 60 da.? for 180 da.? for 240 da.? for 360 da.? for 420 da.? for 480 da.?

369. Example. Find the interest on \$375 for 48 da. at 6%.

SOLUTION. $37\frac{1}{2}¢$ equals the interest for 6 da. 48 da. is 8 times 6 da. Therefore, the interest for 48 da. is 8 times $37\frac{1}{2}¢$, or \$3.

\$0.375
\$3.000

WRITTEN EXERCISE

1. Find the total amount of interest at 6 % on:

$750 for 6 da.	$750 for 36 da.	$750 for 60 da.
$750 for 12 da.	$750 for 42 da.	$750 for 180 da.
$750 for 18 da.	$750 for 48 da.	$750 for 240 da.

2. Find the total amount of interest at 6 % on:

$725 for 18 da.	$690 for 6 da.	$450 for 540 da.
$824 for 36 da.	$129 for 60 da.	$727 for 180 da.
$729 for 42 da.	$475 for 600 da.	$286 for 240 da.
$850 for 54 da.	$8600 for 54 da.	$429 for 420 da.

3. Find the total amount of interest at 6 % on:

$317.40 for 240 da.	$217.18 for 18 da.	$360.40 for 24 da.
$218.60 for 180 da.	$420.50 for 24 da.	$860.50 for 48 da.
$419.80 for 420 da.	$240.70 for 540 da.	$900.60 for 66 da.
$425.60 for 120 da.	$290.60 for 180 da.	$400.80 for 84 da.

370. In some cases it is advisable to find the interest on the principal for 1 da. and then multiply by the number of days.

ORAL EXERCISE

1. What is the interest on $600 for 17 da. at 6 %?

SOLUTION. The interest for one day is .000$\frac{1}{6}$ of the principal, or 10¢. The interest for 17 da. is 17 times 10¢, or $1.70.

2. What is the interest on $6000 for 49 da. at 6%? on $300? on $240? on $3000? on $1800? on $840? on $600?

3. State the interest at 6% on:

a.	$600 for 19 da.	*e.*	$6000 for 37 da.	*i.*	$ 900 for 17 da.
b.	$300 for 37 da.	*f.*	$3000 for 43 da.	*j.*	$1500 for 40 da.
c.	$240 for 43 da.	*g.*	$2400 for 67 da.	*k.*	$ 600 for 139 da.
d.	$180 for 27 da.	*h.*	$1800 for 89 da.	*l.*	$ 300 for 179 da.

371. Frequently it is well to mentally divide the days into convenient parts of 6 or 60.

Thus, 97 da. = 60 da. + 30 da. + 6 da. + 1 da.; 71 da. = 60 da. + 10 da. + 1 da.; 49 da. = 8 times 6 da. + 1 da.

ORAL EXERCISE

Separate the days in the following exercise into 6 da. or 60 da., or into convenient parts of 6 da. or 60 da.

1. 8 da.	**7.** 7 da.	**13.** 86 da.	**19.** 17 da.
2. 67 da.	**8.** 22 da.	**14.** 55 da.	**20.** 25 da.
3. 27 da.	**9.** 11 da.	**15.** 84 da.	**21.** 85 da.
4. 13 da.	**10.** 63 da.	**16.** 14 da.	**22.** 89 da.
5. 72 da.	**11.** 37 da.	**17.** 97 da.	**23.** 19 da.
6. 43 da.	**12.** 23 da.	**18.** 99 da.	**24.** 29 da.

372. Examples. 1. Find the interest on \$840 for 31 da. at 6%.

SOLUTION. 31 da. = 30 da. + 1 da. The interest for 60 da. is \$8.40 and for 30 da. $\frac{1}{2}$ of this sum or \$4.20. The interest for 6 da. is \$0.84 and for 1 da. $\frac{1}{6}$ of this sum or \$0.14. Adding \$4.20 and \$0.14 the result is the required interest, or \$4.34.

\$8.40
\$4.20
.14
\$4.34

2. What is the interest on \$2500 for 121 da. at 6%?

SOLUTION. 121 da. = $\overline{2 \times 60 \text{ da.}}$ + 1 da. The interest for 60 da. is \$25 and for 120 da. twice this sum, or \$50. The interest for 6 da. is \$2.50 and for 1 da. $\frac{1}{6}$ of this sum, or \$0.42. Adding \$50 and \$0.42 the result is \$50.42, the required interest.

\$25.00
\$50.00
.42
\$50.42

WRITTEN EXERCISE

Find the interest:

	PRINCIPAL	TIME	RATE		PRINCIPAL	TIME	RATE
1.	\$420	3 mo.	6%	**11.**	\$450	4 mo.	$4\frac{1}{2}$%
2.	\$650	4 mo.	5%	**12.**	\$600	2 mo.	5%
3.	\$360	92 da.	4%	**13.**	\$720	8 mo.	3%
4.	\$250	30 da.	3%	**14.**	\$840	2 mo.	$1\frac{1}{2}$%
5.	\$380	24 da.	7%	**15.**	\$120	7 mo.	6%
6.	\$900	55 da.	6%	**16.**	\$280	9 mo.	$3\frac{1}{2}$%
7.	\$550	47 da.	3%	**17.**	\$885.90	20 da.	3%
8.	\$800	29 da.	5%	**18.**	\$240.00	21 da.	6%
9.	\$400	90 da.	4%	**19.**	\$420.18	25 da.	$2\frac{1}{2}$%
10.	\$270	11 da.	1%	**20.**	\$560.17	27 da.	6%

373. It has been observed that 6 times \$800 = 800 times \$6; that 0.01 of \$715 = 715 times \$0.01; etc. Hence,

374. *The principal in dollars and the time in days may be interchanged without affecting the amount of interest.*

375. Example. Find the interest on \$600 for 179 da. at 6 %.

SOLUTION. \$600 for 179 da. = \$179 for 600 da.; $\frac{1}{10}$ of the principal equals the interest for 600 da.; $\frac{1}{10}$ of \$179 = \$17.90, the required interest.

ORAL EXERCISE

State the interest at 6 % on:

1. \$60 for 27 da.
2. \$30 for 13 da.
3. \$20 for 171 da.
4. \$10 for 186 da.
5. \$15 for 145 da.
6. \$12 for 179 da.
7. \$10 for 131 da.
8. \$100 for 120 da.
9. \$200 for 189 da.
10. \$150 for 192 da.
11. \$360 for 91 da.
12. \$420 for 87 da.
13. \$540 for 21 da.
14. \$660 for 37 da.
15. \$750 for 56 da.
16. \$3600 for 218 da.
17. \$2000 for 183 da.
18. \$1200 for 155 da.
19. \$1800 for 181 da.
20. \$2400 for 218 da.

376. \$1500 on interest for 24 da. at 8 % = \$2000 (\$1500 + $\frac{1}{3}$ of itself) on interest for 24 da. at 6 %, or \$1500 on interest for 32 da. (24 da. + $\frac{1}{3}$ of itself) at 6 %. Hence,

377. *If either the principal or the time is increased or decreased by any fraction of itself, the interest is increased or decreased by the same fraction.*

378. Examples. 1. Find the interest on \$480 for 279 da. at $7\frac{1}{2}$ %.

SOLUTION. $7\frac{1}{2}$ % is $\frac{1}{4}$ more than 6 %. Increase the principal by $\frac{1}{4}$ of itself, and the result is \$600. Interchanging dollars and days, the problem is "Find the interest on \$279 for 600 da." Pointing off one place in the new principal, the result is \$27.90, the required interest.

2. Find the interest on \$2795.84 for 80 da. at $4\frac{1}{2}$ %.

SOLUTION. $4\frac{1}{2}$ % is $\frac{1}{4}$ less than 6 % interest. 80 da. decreased by $\frac{1}{4}$ of itself equals 60 da. The interest on \$2795.84 for 60 da. = \$27.96, the required result.

ORAL EXERCISE

State the interest on :

1. \$279.86 for 45 da. at 4 %.
2. \$478.65 for 45 da. at 4 %.
3. \$769.64 for 48 da. at $7\frac{1}{2}$ %.
4. \$217.49 for 80 da. at $4\frac{1}{2}$ %.
5. \$767.53 for 80 da. at $4\frac{1}{2}$ %.
6. \$2400 for 39 da. at 5 %.
7. \$2700 for 37 da. at 4 %.
8. \$2400 for 87 da. at $4\frac{1}{2}$ %.
9. \$1600 for 95 da. at $4\frac{1}{2}$ %.
10. \$3200 for 59 da. at $4\frac{1}{2}$ %.

The Six Per Cent Method

379. This method is best adapted to finding the interest when the time is *one year*, or *more than one year.*

ORAL EXERCISE

1. If the interest on \$1 for 1 yr. at 6 % is \$0.06, what is the interest on \$1 for 2 yr. ? for 3 yr. ? for 4 yr. ? for 6 yr. ? for 8 yr. ? for 10 yr. ?

2. If the interest on \$1 for 1 yr. at 6 % is \$0.06, what is the interest on \$1 for 1 mo. ? for 2 mo. ? for 3 mo. ? for 6 mo. ? for 10 mo. ? for 7 mo. ? for 8 mo. ?

3. What is the interest on \$1 for 1 yr. 6 mo. at 6 % ? for 2 yr. 6 mo. ? for 3 yr. 4 mo. ? for 3 yr. 6 mo. ? for 4 yr. 8 mo. ? for 1 yr. 10 mo. ? for 5 yr. 6 mo. ? for 2 yr. 9 mo. ?

4. What is the interest on \$50 for 1 yr. at 6 % ? for 1 yr. 6 mo. ? for 2 yr. ? for 3 yr. 6 mo. ? for 2 yr. 8 mo. ? for 1 yr. 10 mo. ? for 2 yr. 6 mo. ? for 4 yr. 6 mo. ? for 1 yr. 9 mo. ?

5. If the interest on \$1 for 1 mo. at 6 % is \$0.005 (5 mills), what is the interest for 1 da. ? for 2 da. ? for 3 da. ? for 4 da. ? for 6 da. ? for 12 da. ? for 18 da. ? for 28 da. ? for 24 da. ?

6. What is the interest on \$1 for 1 yr. 1 mo. 1 da. at 6 % ? for 2 yr. 3 mo. 3 da. ? for 1 yr. 10 mo. 6 da. ? for 4 yr. 4 mo. 24 da. ? for 1 yr. 5 mo. 12 da. ? for 2 yr. 1 mo. 1 da. ?

380. In the above exercise it is clear that:

\$0.06 = *interest on* \$1 *for* 1 *yr. at* 6 %.
\$0.005 = *interest on* \$1 *for* 1 *mo. at* 6 %.
\$0.000$\frac{1}{6}$ = *interest on* \$1 *for* 1 *da. at* 6 %.

ORAL EXERCISE

Find the interest on $1 at 6% for:

1. 1 yr. 4 mo. 12 da.
2. 1 yr. 8 mo. 18 da.
3. 1 yr. 7 mo. 24 da.
4. 1 yr. 9 mo. 27 da.
5. 2 yr. 6 mo. 6 da.
6. 3 yr. 4 mo. 9 da.
7. 5 yr. 3 mo. 3 da.
8. 4 yr. 8 mo. 4 da.

Find the interest at 6% on:

9. $250 for 2 yr.
10. $400 for 5 yr.
11. $700 for 4 yr.
12. $300 for 3 yr. 4 mo.
13. $500 for 4 yr. 2 mo.
14. $350 for 3 yr.
15. $450 for 2 yr. 3 mo.
16. $150 for 1 yr. 6 mo.
17. $50 for 1 yr. 2 mo. 6 da.
18. $10 for 2 yr. 6 mo. 6 da.

381. Example. What is the interest on $600 for 2 yr. 8 mo. 15 da. at 6%?

SOLUTION. Find the interest on $1 for 2 yr.; on $1 for 8 mo.; on $1 for 15 da. The sum of these interest items equals $0.1625, the interest on $1 for the given time at 6%. Multiplying this interest by the given number of dollars, 600, the product is the required interest, $97.50. Change to any other rate as in § 362.

$0.12 = int. on $1 for 2 yr.
.04 = int. on $1 for 8 mo.
.0025 = int. on $1 for 15 da.
$0.1625 = int. on $1 for the given time.
600 × $0.1625 = $97.50, int. on $600 for 2 yr. 8 mo. 15 da. at 6%.

Sometimes it is shorter to find the interest on $1 for the given time at any given rate, and multiply by the number of dollars in the principal. Thus to find the interest on $400 for 2 yr. 6 mo. at 8%, take 400 times 20¢ ($2\frac{1}{2} \times 8¢$); on $500 for 5 yr. 3 mo. at 4%, take 500 times 21¢ ($5\frac{1}{4} \times 8¢$); on $600 for 1 yr. 9 mo. at 4% take 600 times 7¢; etc.

ORAL EXERCISE

Find the interest:

	Principal	Time	Rate		Principal	Time	Rate
1.	$400	1 yr. 2 mo.	6%	7.	$840	1 yr. 6 mo.	6%
2.	$500	2 yr. 4 mo.	6%	8.	$100	3 yr. 6 mo.	5%
3.	$300	4 yr. 6 mo.	6%	9.	$960	4 yr. 2 mo.	6%
4.	$250	1 yr. 8 mo.	6%	10.	$300	3 yr. 4 mo.	3%
5.	$200	2 yr. 10 mo.	3%	11.	$240	2 yr. 6 mo.	4%
6.	$300	1 yr. 11 mo.	6%	12.	$180	1 yr. 8 mo.	6%

382. This method employs a series of tables in which interest computations are already worked out, and by the use of which the interest may be found on any sum, at given rates, for any time.

This method is used in banks, insurance offices, and kindred institutions, and it greatly lessens the work of computing interest. Many different systems are published, but the section of an interest table given on page 308 will illustrate the general plan followed.

ORAL EXERCISE

1. What is the interest (use the table, page 308) on \$8 for 5 da.? on \$80? (10 × \$8); on \$800? on \$8000?

2. What is the interest on \$10 for 7 da.? on \$100? on \$1000? on \$10,000? on \$70 for 5 da.? on \$700? on \$7000?

3. What is the interest on \$4 for 11 mo.? on \$40 for the same time? on \$400? on \$4000? on \$50,000 for 7 mo.?

383. Example. Find the interest on \$9980 for 7 da. at 6%.

SOLUTION: By the table, \$10.50 = interest on \$9000.
1.05 = interest on \$900.
.09 = interest on \$80.
\$11.64 = interest on \$9980.

WRITTEN EXERCISE

Using the table, find the interest on:

1. $8800 for 4 da.
2. $9600 for 5 da.
3. $7500 for 7 mo.
4. $8500 for 11 mo.
5. $17,000 for 1 da.
6. $29,000 for 1 da.
7. $71,000 for 7 da.
8. $87,000 for 11 da.

PROMISSORY NOTES

384. A written promise to pay a certain sum of money on demand, or at a specified time, is called a **promissory note.**

$ 243.50 New York, Jan. 6, 19

Two months after date I promise to pay to

the order of William B. Harris

Two Hundred Forty-three 50/100 Dollars

at First National Bank with int. at 5%

Value received

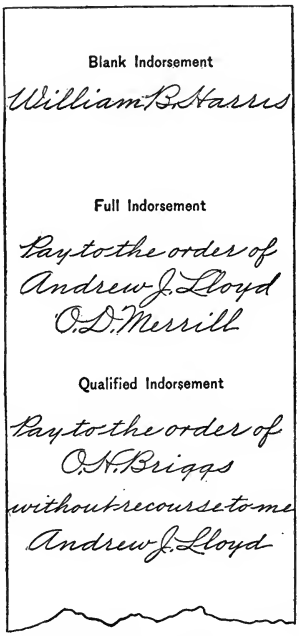
No. 64 Due 3/6/ Ellis B. Pitkin

385. In the foregoing note Ellis B. Pitkin is the **maker**; William B. Harris, the **payee**; and $243.50, the **face**. The note is **negotiable**; that is, it may be transferred by the payee to any other person by indorsement.

If the note were drawn payable to William B. Harris, *or bearer*, it would be transferable by delivery and would be negotiable. If the words *to the order of* were omitted, the note would not be transferable either by indorsement or by delivery; it would be payable to William B. Harris only, and would be called a **non-negotiable** note.

386. If the payee should sell the foregoing note, he would have to indorse it; that is, make it payable to the buyer by a writing on the back of the instrument. This **indorsement** may be made in either of the three ways shown in the margin.

William B. Harris sold the note to O. D. Merrill and effected the transfer by a **blank indorsement**. This is simply William B. Harris's signature. It makes the note payable to bearer. O. D. Merrill sold the note to Andrew J. Lloyd and effected the transfer by a **full indorsement**, an indorsement which specifies the one to whose order the note is made payable. By indorsing the note both William B. Harris and O. D. Merrill make themselves responsible for its payment in case the maker does not pay it. O. H. Briggs was willing to buy the note without Andrew J. Loyd's guarantee to pay it. The transfer was effected by a **qualified indorsement**. By this indorsement Andrew J. Lloyd avoids the responsibility of an ordinary indorser.



The note just considered is a **time note**; if the words *On demand* were substituted for the words *Two months after date* the form would be called a **demand note**. The note is **interest-bearing** because it contains a clause to that effect; it would draw interest after it became due without any interest clause. A demand note, in which there is no interest clause, draws interest after payment has been demanded.

387. A note in which two or more persons *jointly and severally* promise to pay is called a **joint and several note**; a note in which two or more persons *jointly* promise to pay, a **joint note.**

$300.00 Rochester, N.Y., Aug. 5, 19

Sixty days after date we jointly and severally promise to pay to the order of J. M. Cox & Son

Three Hundred Ten Dollars

Payable at Flour City National Bank

Value received

Arthur M. Black

Charles H. Palmer

David B. Small

No. 3 Due Oct. 4, 19

In a joint and several note, the holder may sue and collect of any one signer without proceeding against the others, or he may sue all of them together. In a joint note the signers must be sued jointly. The distinction between a joint and a joint and several note has been abolished by law in many of the states. The above form is a joint and several note. If the words *and severally* were omitted it would be a joint note.

The words *value received* in a note are equivalent to an acknowledgment that there has been a consideration. Their insertion is usual and advisable, but not legally required in all the states.

WRITTEN EXERCISE

Write interest-bearing notes as follows:

1. A demand note; amount, $1283.97; current date; payee, C. H. Good; maker (your name); interest at $5\frac{1}{2}\%$.

2. A time note; amount, $728.79; current date; time, 90 da.; payee, Snow & Co.; maker (your name); interest at $3\frac{1}{2}\%$.

3. A joint note; amount, $1795.73; current date; time, 6 mo.; payee, Ellis & Co.; maker (your name), and Richard Roe; interest at $4\frac{1}{2}\%$. Write a joint note under the same conditions.

4. Find the amount (face plus interest) due 87 da. after date in note No. 1; at the end of the time in note No. 2; at the end of the time in note No. 3.

EXACT INTEREST

388. Exact interest is simple interest for the exact number of days on the basis of 365 da. in a common year, or 366 da. in a leap year.

The United States Government takes exact interest, and its use is growing among business men. In strict justice it is the only correct method of computing interest.

389. The difference between the common year of 365 da. and the commercial year of 360 da. is 5 da., or $\frac{1}{73}$ of the common year.

If any sum were divided into 360 parts, each part would be larger than it would be if the sum were divided into 365 parts. Thus, $\frac{17}{360}$ and $\frac{93}{360}$ are greater than $\frac{17}{365}$ and $\frac{93}{365}$. It is therefore clear that exact interest is less than ordinary interest.

390. To find the exact interest, *compute interest in the usual way for the commercial year, and from the interest thus obtained subtract* $\frac{1}{73}$ *of itself.*

In many cases the work may be shortened by cancellation.

391. Example. Find the exact interest on \$3285 for 35 da. at 5%.

SOLUTION. $\frac{.05 \times 35 \times \$\overset{9}{\cancel{3285}}}{\underset{73}{\cancel{365}}} = .05 \times 35 \times \$9 = \$15.75.$

WRITTEN EXERCISE

Find the exact interest:

1. \$734.50 for 124 da. at 6%.
2. \$420.60 for 99 da. at $4\frac{1}{2}$%.
3. \$965.50 for 82 da. at $3\frac{1}{2}$%.
4. \$356.40 for 236 da. at 4%.
5. \$672.50 for 53 da. at $5\frac{1}{2}$%.
6. \$546.24 for 38 da. at $4\frac{1}{2}$%.
7. \$1240.35 for 50 da. at 6%.
8. \$1630.25 for 67 da. at 4%.
9. \$150,000 for 28 da. at 6%.
10. \$100,000 for 135 da. at 5%.
11. \$4653.28 for 182 da. at 4%.
12. \$45,000 for 42 da. at $2\frac{1}{2}$%.
13. \$3500 from July 17, 1907, to Nov. 26, 1907, at 3%; at $4\frac{1}{2}$%.
14. \$2315.89 from Mar. 11, 1907, to Sept. 1, 1907, at 6%; at 2%.
15. \$872.54 from Oct. 18, 1906, to Jan. 16, 1907, at 5%; at $7\frac{1}{2}$%.
16. £1006 6*s.* from Apr. 1, 1907, to Feb. 19, 1908, at 3%; at 2%.

PROBLEMS IN INTEREST

ORAL EXERCISE

1. If the principal is $100, the interest $12, and the time 2 yr., what is the rate?

2. If the principal is $150, the interest $18, and the time 3 yr., what is the rate?

3. If the principal is $200, the interest $24, and the rate 3%, what is the time?

4. If the principal is $160, the interest $12, and the rate 5%, what is the time?

5. If the interest is $108, the rate 6%, and the time 3 yr., what is the principal?

6. If the interest is $42, the rate 3%, and the time 3 yr. 6 mo., what is the principal?

7. If the amount is $60, the rate 4%, and the time 5 yr., what is the principal?

8. When the cash price of an article is $25, what should the sixty-day credit price be?

9. When the sixty-day credit price of an article is $50.50, what should the cash price be?

10. When money is worth 5%, what cash offer will be equivalent to a ninety-day credit of $101.25?

11. Which is the better and how much, a thirty-day credit offer of $100.50 or a cash offer of $98, money being worth 6%?

12. Which is the better and how much, a 60-da. credit offer of $404 or a cash offer of $402, money being worth 6%?

13. You offer a customer an article for $10 cash, or $10.40 on 4 mo. credit. If you consider the offers equal, how much is money worth to you at the present time?

14. One contractor offers to do a certain work for $1050 cash; another offers to do the same work for $1075, payable in 1 yr. If money is worth $7\frac{1}{2}\%$, which is the better offer? how much better?

WRITTEN EXERCISE

1. Which is the better for a tailor, to sell a suit for $65 cash, or for $73.15 on 9 mo. time, money being worth 6% ?

2. Which is the better, to sell carpet at $1.50 per yard cash, or at $1.68 per yard on 1 yr. time, money being worth 5% ?

3. Which is the more advantageous, to buy an article for $58.50 cash or for $61.80 on 6 mo. time, money being worth 6% ?

4. A merchant paid $160 cash for 4 sewing machines. After keeping them in stock 1 yr. 6 mo. he sold them for $190.80, on one year's time without interest. If money is worth 6% what was his gain or loss?

5. An invoice of merchandise listed at $2500, on which trade discounts of 20% and 10% were allowed, was purchased at 90 da. What was the actual cash value of the debt on the day of the purchase, money being worth 5% ?

6. A merchant bought 600 bbl. of flour at $7.50 per barrel. Terms: one half on account, 3 mo.; one half on account, 6 mo. At the end of 1 mo. he paid the cash value of the entire bill. How much did he gain, money being worth 6% ?

7. Sept. 8 you purchased of Edward Sprague & Son, at trade discounts of 20% and 25%, an invoice of coffee listed at $2006. Terms: 30 da. Sept. 20 you sent Edward Sprague & Son a check for the actual cash value of the bill. What was the amount of the check, money being worth 6%?

PERIODIC INTEREST

392. Periodic interest is simple interest on the principal increased by the simple interest on each installment of interest that was not paid when due.

As periodic interest can be legally enforced in only a few states, special contracts should be made if it is to be collected. Where technically illegal, periodic interest is often collected; as, when a series of notes is given for the interest on a note secured by a real-estate mortgage, such notes to draw interest if not paid when due.

393. Example. If payments of interest are due semiannually, what is the interest on $1000 for 3 yr. at 6%?

SOLUTION

$180 = interest on $1000 for 3 yr. at 6%.

$30 is the interest on $1000 for one semiannual period, 6 mo.

1st installment of interest, $30, was unpaid for	2 yr. 6 mo.
2d installment of interest, $30, was unpaid for	2 yr.
3d installment of interest, $30, was unpaid for	1 yr. 6 mo.
4th installment of interest, $30, was unpaid for	1 yr.
5th installment of interest, $30, was unpaid for	6 mo.
The sum of the periods for which interest was unpaid is	7 yr. 6 mo.

The interest on each $30 for the period it was unpaid is the same as the interest on $30 for the sum of the periods.

13.50 = interest on $30 for 7 yr. 6 mo., at 6%.

$193.50 = the total interest due.

WRITTEN EXERCISE

1. If payments of interest are due annually, what is the interest on $850 for 5 yr., at 8%?

2. If payments of interest are due quarterly, what is the interest on $1380 for 2 yr. 6 mo., at 4%?

3. What is the difference between the simple interest and periodic interest (payable annually) on $1800 for 6 yr. at 4%?

4. If payments of interest are due semiannually, what amount should be paid in settlement of a debt of $1450 which has run 5 yr. at 6%?

5. If payments of interest are due annually, what amount will settle a debt of $1500 for 5 yr., at 6%, if the first installment of interest was paid when due?

COMPOUND INTEREST

394. Compound interest is interest computed, at certain intervals, on the sum of the principal and unpaid interest.

Interest may be compounded annually, semiannually, quarterly, or even monthly. In most states the law does not sanction the collection of compound interest, but if it is agreed upon by the parties, the taking of it does not constitute usury. It is a general custom of savings banks to allow compound interest. Compound interest is also used by life insurance companies.

395. Example. What is the compound interest on $6000 for 4 yr., if the interest is compounded annually at 5%?

SOLUTION.

$6000	= 1st principal.
300	= interest 1st year.
6300	= amount, or the principal the 2d year.
315	= interest 2d year.
6615	= amount, or the principal the 3d year.
330.75	= interest 3d year.
6945.75	= amount, or the principal the 4th year.
347.29	= interest 4th year.
7293.04	= amount due at the end of the 4th year.
$7293.04	− $6000 = $1293.04, compound interest for 4 yr.

WRITTEN EXERCISE

1. If interest is compounded annually, what will be the amount of $600 for 5 yr. at 6%?

2. If interest is compounded semiannually, what will be the compound interest on $1500 for 2 yr. 6 mo. at 4%?

3. A man deposited $750 in a savings bank Jan. 1, 1905, and interest was added thereto every 6 mo. at the rate of 4%. No withdrawals having been made, what was the balance due Jan. 1, 1907?

11	1.24337	1.31209	1.38423	1.45997	1.53945	1.62285	1.71034	11
12	1.26824	1.34489	1.42576	1.51107	1.60103	1.69588	1.79586	12
13	1.29361	1.37851	1.46853	1.56396	1.66507	1.77220	1.88565	13
14	1.31948	1.41297	1.51259	1.61870	1.73168	1.85194	1.97993	14
15	1.34587	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893	15
16	1.37279	1.48451	1.60471	1.73399	1.87298	2.02237	2.18287	16
17	1.40024	1.52162	1.65285	1.79468	1.94790	2.11338	2.29202	17
18	1.42825	1.55966	1.70243	1.85749	2.02582	2.20848	2.40662	18
19	1.45681	1.59865	1.75351	1.92250	2.10685	2.30786	2.52695	19
20	1.48595	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330	20

ORAL EXERCISE

Refer to the table, page 315, *and give rapid answers to the following:*

1. What is the amount of \$1 for 12 yr. at 4%? at 3%? at 5%? at $4\frac{1}{2}$%? at $2\frac{1}{2}$%?

2. What is the amount of \$1 for 18 yr. at $4\frac{1}{2}$%? at $3\frac{1}{2}$%? at 2%? at 3%? at $2\frac{1}{2}$%?

3. What is the amount of \$1 for 9 yr. at 5%? at $4\frac{1}{2}$%? at $2\frac{1}{2}$%? at $3\frac{1}{2}$%? at 3%? at 4%?

4. What is the amount of \$1 for 20 yr. at 2%? at 5%? at $4\frac{1}{2}$%? at $3\frac{1}{2}$%? at $2\frac{1}{2}$%? at 3%?

5. What is the amount of \$10 for 10 yr. at 4%? for 20 yr. at 2%? for 5 yr. at 5%?

6. What is the amount of \$100 for 5 yr. at 2%? for 11 yr. at $3\frac{1}{2}$%? for 19 yr. at 5%?

396. Example. What is the compound interest on \$8000 for 10 yr., if interest is compounded annually at 5%?

SOLUTION. \$1.62889 = amount of \$1 for 10 yr. at 5%.
8000 × \$1.62889 = \$13031.12, amount due in 10 yr. at 5%.
\$13031.12 − \$8000 = \$5031.12, the compound interest.

1.	\$7500	4%	5 yr.	Annually
2.	\$2500	2%	12 yr.	Annually
3.	\$5600	$3\frac{1}{2}$%	20 yr.	Annually
4.	\$3350	5%	10 yr.	Semiannually
5.	\$2875	3%	17 yr.	Annually
6.	\$4600	4%	15 yr.	Semiannually

Sinking Funds

397. A **sinking fund** is a sum of money set aside at regular intervals for the payment of an existing or anticipated indebtedness.

The payment of a corporation or a public loan is sometimes facilitated by regularly investing a certain sum in some form of security. The interest from these investments from year to year constitutes a *sinking fund* which it is planned shall accumulate to an amount sufficient to redeem the debt when it falls due.

ORAL EXERCISE

1. In what time will any sum of money double itself at 4% simple interest? at 3%? at 6%? at $4\frac{1}{2}$%?

2. How long (approximately) will it take \$1 to double itself at $3\frac{1}{2}$%? compound interest, compounded annually? (See table, page 315.)

3. How long (approximately) will it take any sum to double itself at $4\frac{1}{2}$% compound interest, compounded annually? at 5% compound interest, compounded annually?

4. If you put \$1 at compound interest to-day, \$1 one year from to-day, and so on for 20 yr., how much would you have at the end of the twentieth year, interest being compounded annually at $4\frac{1}{2}$%? (See table below.)

398. In the following table is shown the amount at the close of a series of years of \$1 invested at different rates of compound interest *at the beginning* of each year.

Compound Interest Table

Yr.	2%	4%	$4\frac{1}{2}$%	Yr.	2%	4%	$4\frac{1}{2}$%
1	1.020000	1.040000	1.045000	11	12.412089	14.025805	14.464031
2	2.060400	2.121600	2.137025	12	13.680331	15.626837	16.159913
3	3.121608	3.246464	3.278191	13	14.973938	17.291911	17.932109
4	4.204040	4.416322	4.470709	14	16.293416	19.023587	19.784054
5	5.308120	5.632975	5.716891	15	17.639285	20.824531	21.719336
6	6.434283	6.898294	7.019151	16	19.012070	22.697512	23.741706
7	7.582969	8.214226	8.380013	17	20.412312	24.645412	25.855083
8	8.754628	9.582795	9.802114	18	21.840558	26.671229	28.063562
9	9.949721	11.006107	11.288209	19	23.297369	28.778078	30.371432
10	11.168715	12.486351	12.841178	20	24.783317	30.969201	32.783136

WRITTEN EXERCISE

1. At the beginning of each year for 10 yr. a certain railroad company put aside out of the profits of the previous year \$50,000 as a sinking fund. If this sum was invested at 4% compound interest, compounded annually, what did it amount to at the end of the tenth year?

2. Jan. 1, 1907, a certain city borrowed \$500,000 and agreed to pay the principal and compound interest, compounded annually, at $4\frac{1}{2}\%$, on Jan. 1, 1917. What sum must be invested in securities, paying $4\frac{1}{2}\%$ compound interest, compounded annually, on Jan. 1, 1907, and annually for 10 yr., in order to pay the loan when it becomes due?

3. On Dec. 31, 1907, a certain town borrowed \$40,000 with which to build a new high school. It was agreed that this amount together with compound interest, compounded annually, at $4\frac{1}{2}\%$, should be paid on Dec. 31, 1912. What sum must the town set aside and invest at $4\frac{1}{2}\%$ compound interest, compounded annually, on Jan. 1, 1905, and each year thereafter for 5 yr., in order to pay the debt when it becomes due?

WRITTEN REVIEW EXERCISE

1. What amount of interest (in United States money) at 6% will accrue on a debt of £84 12*s*. in 5 mo. 24 da.?

2. The yearly taxes on a house and lot which cost \$12,500 are \$162. How much should the house rent for per month to clear 6% on the investment?

3. A Chicago speculator bought 16,000 bu. of wheat at 85¢, and paid for it in 10 da. 46 da. from the date of purchase he sold the wheat for 92¢ per bushel, cash. If money was worth 4%, what did he gain?

4. A savings bank account was opened July 1, 1901, with a deposit of \$800. Interest was credited every 6 mo. at 4%. No withdrawals or subsequent deposits having been made, what was the balance of the account Jan. 1, 1907?

5. The note on page 308 was not paid until May 27. How much was due the holder of the note on that date?

6. Jan. 1, 1905, B invested $24,000 in a manufacturing business. July 1, 1907, he withdrew $33,000, which sum included the original investment and the net gains. What average yearly per cent of simple interest did the investment yield?

7. Derby & Co. offer B the following terms: $^{2}/_{10}$, $^{N}/_{30}$. Jan. 1, B bought a bill of goods amounting to $4000 which he paid Jan. 31. What rate of interest did he practically pay on the net amount of the bill by not taking advantage of the cash offer?

8. In a certain town the taxes are due Sept. 15 of each year, and all taxes unpaid by Oct. 15 are subject to interest from the date they are due, at 6%. The following taxes were paid on the dates named: Oct. 18, $68.40; Oct. 21, $22.50; Oct. 25, $132.75; Oct. 31, $98; Nov. 11, $176.80; Nov. 23, $326.30; Dec. 2, $45; Dec. 16, $13.25; Dec. 29, $21. How much interest was paid, the time being the exact number of days?

9. Jan. 1, 1902, F bought a piece of city property for $20,000, paid cash $4000, and gave a note and mortgage for 5 yr. without interest, to secure the balance. To cover the interest, which it was agreed should be met quarterly, he gave twenty notes for $240 each, one maturing every three months. The first five installments of interest were paid when due, and the balance of the mortgage and the interest were paid Jan. 1, 1907. Find the final payment.

10. Lester B. Ford keeps his deposit with the Second National Bank, and has left with the bank railroad stock valued at $1000 as collateral security for overdrafts, the bank charging 5% on all overdrafts that were not settled within 3 da. May 6 there was an overdraft of $280 that was settled May 13; May 28, $312.50, that was settled June 1; June 26, $156.75, that was settled July 8; Aug. 1, $456.20, that was settled Aug. 11. How much interest did Mr. Ford have to pay?

CHAPTER XXVI

BANK DISCOUNT

ORAL EXERCISE

1. What is meant by a promissory note? by the face of a note? by the time? by the maker? by the payee?

2. How would you word a promissory note for $600, dated at your place to-day, payable in 60 da. at a bank in your place, with interest at 5%, to C. B. Powell, signed by yourself?

3. What is meant by negotiable? by indorsing a note? Illustrate a blank indorsement; an indorsement in full; a qualified indorsement.

399. A **commercial bank** is an institution chartered by law to receive and loan money, to facilitate the transmission of money and the collection of negotiable paper, and, in some cases, to furnish a circulating medium.

400. If the **holder** (owner) of a promissory note wishes to use the money promised before it becomes due, a commercial bank will usually buy the note, provided the holder can show that it will be paid at **maturity**, that is, when it becomes due. This is called **discounting** the note.

$ 460.00 New York, May 10, 19

At this sight days Pay to

the order of Myself

Four Hundred Sixty 00/100 Dollars

Value received and charge the same to account of

To Frank G. Hall, Albany, N.Y.

No. 16

Geo. H. Catchpole

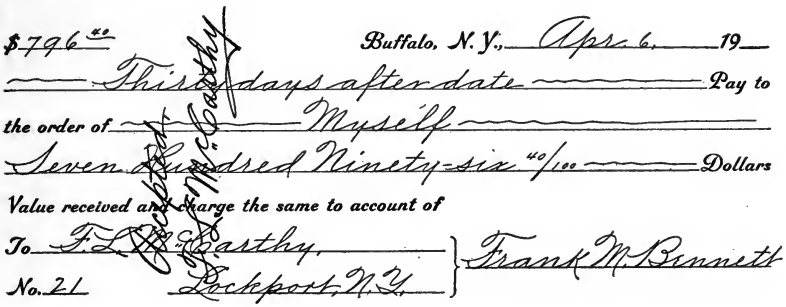
Accepted May 12 19

401. A **commercial draft** is now frequently used, instead of the promissory note, as security for the payment of goods sold on credit. Such a draft may be defined as a written order in which one person directs another to pay a specified sum of money to the order of himself or to a third person.

The circumstances under which the foregoing draft was drawn are as follows: Geo. H. Catchpole sold Frank G. Hill goods amounting to $460.80. Terms: 30-da. draft. The draft and an invoice were made out and sent to Frank G. Hill by mail. Frank G. Hill accepted the draft, that is, signified his intention to pay it by writing the word *accepted*, the date, and his name across the face. The draft was then returned to Geo. H. Catchpole, who may discount it the same as he would an ordinary promissory note.

The parties to a draft are the **drawer**, the **drawee**, and the **payee**. In the foregoing draft, George H. Catchpole is both the drawer and the payee, and Frank G. Hill is the drawee.

A draft payable after sight begins to mature from the date on which it is accepted. An acceptance *must*, therefore, be dated in a draft payable after sight, but it may or may not be dated in a draft payable after date.


796^{40} Buffalo, N. Y., Apr 6, 19—

Thirty days after date Pay to

the order of Myself

Seven Hundred Ninety-six 40/100 Dollars

Value received and charge the same to account of

To F. L. Casthy, Lockport, N. Y.

No. 21

Geo. H. McCatchpole

Frank M. Bennett

Some states allow three **days of grace** for the payment of notes and other negotiable paper. Days of grace are obsolete in so many of the states that they are not considered in the exercises in this book. Some states provide that when paper matures on Sunday or a legal holiday it must be paid the day preceding such Sunday or legal holiday; others provide that it must be paid on the day following. To hold all interested parties, the laws of any given state should always be observed. When the time of negotiable paper is expressed in months, **calendar months** are used to determine the date of maturity; but when the time is expressed in days, the **exact number of days** is used. Thus, a note payable 2 mo. after July 15 is due Sept. 15; but a note payable 60 da. after July 15 is due Sept. 13. Paper payable 1 mo. from May 31, Aug. 31, etc., is due Jan. 30, Sept. 30, etc.

402. The time from the date of discount to the maturity of paper is called the **term of discount**; the whole sum specified to be paid at maturity, the **value**, or **amount**, of the paper.

The term of discount is usually the *exact number of days* from the date of discount to the date of maturity. Some banks, however, find the term of discount by compound subtraction, and then reduce the time to days; *e.g.* the term of discount on a note due May 6 and discounted Mar. 1 is counted as 2 mo. 5 da., or 65 da. *In this text the term of discount is the exact number of days from the date of discount to the maturity of the paper.*

403. The reduction made by a bank for advancing money on negotiable paper not due is called **bank discount**. The value of negotiable paper at maturity, minus the bank discount, is called the **proceeds**.

Bank discount is always the simple interest for the term of discount on the whole sum specified to be paid at maturity.

404. The accompanying **maturity table** is sometimes used by bankers in finding the maturity of notes and drafts. The following examples illustrate its use.

MATURITY TABLE

1	Jan. − 1	13
2	Feb. + 2	14
3	Mar. − 1	15
4	Apr.	16
5	May − 1	17
6	June	18
7	July − 1	19
8	Aug. − 1	20
9	Sept.	21
10	Oct. − 1	22
11	Nov.	23
12	Dec. − 1	24

405. Examples. 1. Find the maturity of a note payable (*a*) 6 mo. from Apr. 27, 1906; (*b*) 6 mo. from Sept. 25, 1906.

SOLUTIONS. (*a*) Referring to the table, observe that April is the 4th month; adding 4 and 6, the result is 10, and the 10th month (see number on left) is October. The note is therefore due Oct. 27, 1906.

(*b*) September is the 9th month. $9 + 6 = 15$, and the 15th month (see number on right) is March of the next year. The note is therefore due Mar. 25, 1907.

2. Find the maturity of a note payable 90 da. from Jan. 18, 1907.

SOLUTION. $1 + 3 = 4$, and the 4th month is April. If the note were payable in 3 mo., it would be due Apr. 18. Referring to the table, note that 2 da. (1 da. + 1 da.) must be subtracted for January and March, and 2 da. added for February. The note is therefore due Apr. 18.

After the student has become familiar with the principles of the table it will not be found necessary to consult it.

ORAL EXERCISE

Find the maturity of each of the following notes:

	Date	Time		Date	Time
1.	Apr. 6, 1906	30 da.	6.	Jan. 30, 1907	30 da.
2.	Oct. 6, 1907	3 mo.	7.	Jan. 31, 1906	30 da.
3.	Nov. 9, 1906	60 da.	8.	May 10, 1907	90 da.
4.	Jan. 31, 1907	1 mo.	9.	June 19, 1907	60 da.
5.	Sept. 18, 1906	90 da.	10.	Nov. 15, 1907	30 da

Find the maturity of each of the following acceptances:

	Date	Time after Date		Date	Time after Date
11.	Apr. 3	30 da.	14.	Dec. 31	2 mo.
12.	May 5	60 da.	15.	Jan. 12	1 mo.
13.	Jan. 29	1 mo.	16.	Feb. 18	3 mo.

Find the maturity of each of the following acceptances:

	Date Accepted	Time after Sight		Date Accepted	Time after Sight
17.	Aug. 12	3 mo.	20.	Apr. 25	60 da.
18.	Sept. 18	2 mo.	21.	May 17	3 mo.
19.	Oct. 30	4 mo.	22.	June 18	30 da.

WRITTEN EXERCISE

Find the maturity and the term of discount:

	Date	Time	Discounted
1.	Jan. 16, 1907	3 mo.	Mar. 1
2.	Jan. 31, 1907	1 mo.	Feb. 3
3.	Feb. 12, 1907	90 da.	Mar. 2
4.	Feb. 24, 1907	60 da.	Apr. 1
5.	Mar. 31, 1907	90 da.	May 13

	Date of Draft	Time after Date	Date Accepted	Date Discounted
6.	Feb. 7	60 da.	Feb. 8	Feb. 9
7.	Mar. 12	30 da.	Mar. 12	Mar. 15

	Date of Draft	Time after Sight	Date Accepted	Date Discounted
8.	May 31	60 da.	May 31	June 3
9.	Mar. 17	90 da.	Mar. 20	Mar. 21

406. The following **time table** is frequently used by bankers in finding the exact number of days between any two dates:

TABLE OF TIME

FROM ANY DAY OF	TO THE SAME DAY OF THE NEXT											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
JANUARY	365	31	59	90	120	151	181	212	243	273	304	334
FEBRUARY	334	365	28	59	89	120	150	181	212	242	273	303
MARCH	306	337	365	31	61	92	122	153	184	214	245	275
APRIL	275	306	334	365	30	61	91	122	153	183	214	244
MAY	245	276	304	335	365	31	61	92	123	153	184	214
JUNE	214	245	273	304	334	365	30	61	92	122	153	183
JULY	184	215	243	274	304	335	365	31	62	92	123	153
AUGUST	153	184	212	243	273	304	334	365	31	61	92	122
SEPTEMBER	122	153	181	212	242	273	303	334	365	30	62	91
OCTOBER	92	123	151	182	212	243	273	304	335	365	31	61
NOVEMBER	61	92	120	151	181	212	242	273	304	334	365	30
DECEMBER	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of any month to the corresponding day of any other month, within a year, is found in the column of the last month directly opposite the line of the first month. Thus, from June 6 to Sept. 6 is 92 da.; from Apr. 1 to Oct. 1 is 183 da.; from Aug. 26 to Dec. 26 is 122 da. The exact number of days between *any two dates* is found as in the following illustrations:

407. Examples. **1.** How many days from Mar. 1 to May 11?

SOLUTION. From Mar. 1 to May 1 is 61 da. From May 1 to May 11 is 10 da. 61 da. + 10 da. = 71 da., the required result.

2. How many days from July 26 to Oct. 6?

SOLUTION. From July 26 to Oct. 26 is 92 da. From Oct. 26 back to Oct. 6 is 20 da. 92 da. − 20 da. = 72 da., the required result.

ORAL EXERCISE

By the table find the exact number of days from:

1. July 8 to Sept. 8.
2. Jan. 6 to Mar. 6.
3. Jan. 23 to June 23.
4. Feb. 13 to July 13.
5. Mar. 11 to Sept. 11.
6. Mar. 21 to Aug. 21.
7. May 31 to Aug. 1.
8. Feb. 23 to Sept. 23.
9. Mar. 24 to July 12.
10. May 11 to Aug. 31.
11. Aug. 15 to Dec. 10.
12. Nov. 25 to Mar. 25.

408. Examples. 1. Find the proceeds of a note for $3000, payable in 78 da., discounted at 6%.

SOLUTION. $0.013 = the rate for the term of discount.
$3000 \times $0.013 = $39, the bank discount.
$3000 - $39 = $2961, the proceeds.

2. A note for $6000 payable in 60 da. from May 10, 1907, with interest at 6%, is discounted May 25, at 6%. Find the maturity, the term of discount, the bank discount, and the proceeds.

SOLUTION. July 9, 1907 = the maturity.
45 da. = the term of discount.
$60 = the interest on the note for 60 da.
$6060 = the value of the note at maturity.
$45.45 = the bank discount.
$6014.55 = the proceeds.

409. The accompanying diagram illustrates a convenient outline for learning the proper method of computing bank discount. It will be observed that the first problem is an interest-bearing note, and the second problem a non-interest-bearing note. The items in black ink are taken from the problem, and the items in red ink are found as previously explained.

Memoranda	1	2	3
Date of Paper	Jan. 1	Mar. 1	
Face of Paper	$300	$600	
Time	2 mo.	3 mo.	
Rate of Interest	6%		
Rate of Discount	6%	6%	
Date of Discount	Jan. 30	May 1	
Maturity	Mar. 1	June 1	
Term of Discount	30 da.	31 da.	
Interest	$3	—	
Value	$303	$600	
Discount	$1.52	$3.10	
Proceeds	$301.48	$596.90	

WRITTEN EXERCISE

1. Assuming that the model note, page 9, was discounted July 2, at 6%, find the bank discount and the proceeds.

2. Assuming that the model note, page 308, was discounted Jan. 20, at 6%, find the bank discount and the proceeds.

3. Assuming that the model note, page 310, was discounted Aug. 26, at 6%, find the bank discount and the proceeds.

4. Assuming that the model draft, page 320, was discounted May 15, at 6%, find the bank discount and the proceeds.

5. Assuming that the model draft, page 321, was discounted April 12, at 6%, find the bank discount and the proceeds.

6. Find the proceeds of the following **joint note:**

$895.40 BALTIMORE, MD., May 25, 1907.

Six months after date, for value received, we promise to pay to the order of Ralph D. Gibson Eight Hundred Ninety-five $\frac{40}{100}$ Dollars, at Exchange National Bank.

SETH M. BULLARD.

Discounted July 2, 1907, at 5%. ISAAC C. WATKINS.

7. Find the proceeds of the following **joint and several note:**

$1000.00 COLUMBUS, O., May 1, 1907.

Three months after date we jointly and severally promise to pay to the order of Wilson N. Burton One Thousand Dollars, at Second National Bank, Columbus, O., with interest at 6%.

Value received. JOHN M. SELLERS.

Discounted June 2, 1907, at 6%. DANIEL W. SHELDON.

8. Find the proceeds of the following **firm note:**

$1250.00 ST. LOUIS, MO., Aug. 20, 1907.

Ninety days after date we promise to pay to the order of C. M. Courtwright Twelve Hundred Fifty Dollars, at the National Bank of Redemption, with interest at 5%.

Value received. J. M. COX & SON.

Discounted Sept. 1, 1907, at 6%.

9. Sept. 26 you sold R. M. Stein, Portland, Me., a bill of hardware amounting to $2480, less 20%, 25%, and 10%. Terms: $\frac{1}{2}$ by 60-da. note with interest at 6%; $\frac{1}{2}$ on account 60 da. What was the amount of the note which was this day received?

10. Oct. 12 you discounted at Union Bank, at 6%, R. M. Stein's note received Sept. 26, the bank giving you credit for the proceeds. If the bank charges $\frac{1}{10}$% for collecting out-of-town paper, what was the amount of the proceeds credited?

A small fee called *collection and exchange* is sometimes charged on discounted paper payable out of town. The charge is by no means uniform, being controlled largely by the size of the depositor's account and the general custom of the banks in any given locality.

11. The following is a part of a page from a bank's discount register. Copy it, supplying all missing terms. The notes were all discounted June 17.

No.	Date of Paper	Time	When Due	Term of Discount	Rate of Discount	Value of Paper		Disc.		Coll. & Exch.		Proceeds Credited	
20	Apr. 25	3 mo.			6 %	2000	00						
21	May 1	3 mo.			6 %	3500	00			3	50		
22	Apr. 1	90 da.			6 %	1500	00						
23	Apr. 15	90 da.			6 %	900	60						
24	June 15	30 da.			6 %	378	90				38		

12. Sept. 15 the First National Bank notifies you that your bank account is overdrawn $1725.90. You immediately offer for discount, at 6 %, the following notes, the proceeds of which are to be placed to your credit: E. M. Robinson's 30-day note dated Sept. 1, for $300; C. E. Reardon's note payable 3 mo. from July 25, with interest at 6 %, for $427.65; C. W. Allen's 60-day note dated Aug. 1, for $321.17; F. H. Clark's 60-day note dated July 30, for $1500. What is your credit at the bank after discounting the notes?

13. Apr. 6, 1907, Peter W. Berger has on deposit in the First National Bank $523.87. He draws a check for $1176.45, and then discounts the following notes at the bank, at 6 %, receiving credit for the proceeds. What was the balance of his account after the notes were discounted and credited?

a.

$346.50 HARTFORD, CONN., Mar. 1, 1907.

Ninety days after date I promise to pay Peter W. Berger, or order, Three Hundred Forty-six $\frac{50}{100}$ Dollars, at First National Bank, Hartford, Conn.

Value received. HENRY S. LANE.

b.

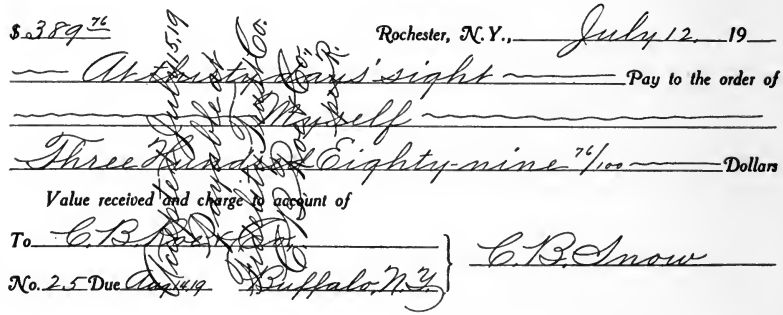
$575.00 HARTFORD, CONN., Feb. 1, 1907.

Aug. 1, 1907, I promise to pay Peter W. Berger, or order, Five Hundred Seventy-five Dollars, at Second National Bank, Hartford, Conn.

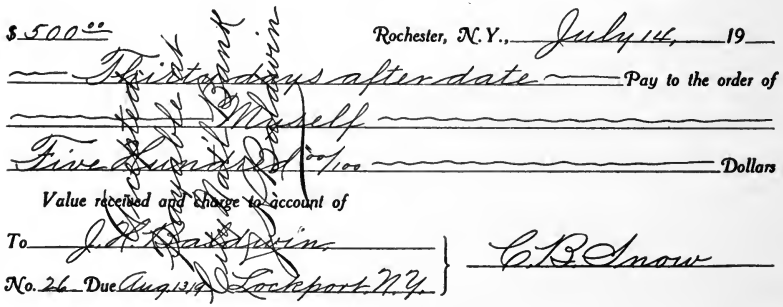
Value received. SAMUEL D. SKIFF.

14. July 18, C. B. Snow's bank balance is $312.90. He discounts at 6% the following drafts, and then issues a check in payment for 5 sewing machines at $75, less 20% and 25%. What is the amount of his balance after issuing the check?

a.



b.



BANK LOANS

410. The foregoing exercises have reference to paper bought or discounted by a bank. Money is frequently loaned upon the notes of the borrower, indorsed by some one of known financial ability, or secured by the deposit of stocks, bonds, warehouse receipts, or other collaterals. These notes, if drawn on time, are not interest-bearing, but the bank discounts them by deducting from their face the interest for the full time.

411. Loans are sometimes made on **call** or **demand notes**; that is, on notes that can be called or demanded at any time after they are made. These notes are interest-bearing and are drawn for the exact sum loaned.

Call or *demand loans* generally bear a lower rate of interest than loans on time. They are made principally to brokers and speculators, who use them to pay for stocks; but they are also made to merchants and others to some extent. Business men, however, generally prefer to borrow on time, for they do not wish to be embarrassed by having the loans called in at an unexpected time. *Time loans* are usually drawn for thirty, sixty, or ninety days. If the borrower requires money for a longer period, the bank will usually allow him to renew the note when it falls due.

WRITTEN EXERCISE

1. Jan. 7, 1907, E. L. Jennings & Co. desire to extend their business, and for this purpose borrow money at 6 % of the First National Bank of New York, on the following note. How much will the bank place to the credit of E. L. Jennings & Co.?

$5000.00 New York, Jan. 7, 19—

Ninety days after date we promise to pay to the order of First National Bank

Five Thousand 00/100 Dollars

at their banking house

Value received

No. 15 Due Apr. 7, 19 E. L. Jennings & Co.

2. You gave the Union National Bank, of your city, your note, for $1200, at 60 da., indorsed by Williams & Rogers. How much cash will the bank advance you, if discount is deducted at the rate of 6% ?

3. Howe & Rogers, Buffalo, N.Y., borrowed $12,000 of Merchants National Bank on their demand note secured by 300 shares of Missouri Pacific Railway stock, at $50. If the rate of interest was $2\frac{1}{2}$ %, how much was required for settlement 39 da. after the loan was made?

4. Jan. 1, 1906, C. W. Allen & Co., brokers, borrowed of First National Bank, Boston, Mass., $15,000 on the following **collateral note.** How much was required for full settlement of the loan 57 da. after it was made?

$15000.00 Boston, Mass., Jan. 2, 19

On demand for value received, we promise to pay to the order of First National Bank of Boston at their banking house Fifteen Thousand 00/100 Dollars with interest at 3½% per annum

As collateral security for the payment of the note and all other liabilities to said bank, either absolute or contingent, now existing or to be hereafter incurred, we have deposited with it:

100 shares Lake Shore & Michigan Southern Railway Stock

Should the market value of the same decline, we promise to furnish satisfactory additional collateral on demand, which may be made by a notice in writing, sent by mail or otherwise, to our residence or place of business. On the nonperformance of either of the above promises we authorize the holder or holders hereof to sell said collateral and any collaterals added to or substituted for the same, without notice, at public or private sale, and at or before the maturity hereof, he or they giving us credit for any balance of the net proceeds of such sale remaining after paying all sums absolutely or contingently due and then or thereafter payable from us to said holder or holders. And we authorize said holder or holders, or any person in his or their behalf, to purchase at any such sale.

C. W. Allen & Co.

FINDING THE FACE

412. Example. I wish to borrow $1980 of a bank. For what sum must I issue a 60-da. note to obtain the amount, discount being at the rate of 6%?

SOLUTION.

Let the face of the note = $1
Then the bank discount = $0.01
And the proceeds = $0.99
But the proceeds = $1980
$1980 ÷ $0.99 = 2000

∴ the face of the note is 2000 × $1, or $2000.

WRITTEN EXERCISE

1. What must be the face of a 30-da. note in order that when discounted at 6% the proceeds will be $1990? Of a 60-da. note, same conditions?

2. You wish to borrow $3940 cash. What must be the face of a 90-da. note in order that when discounted at 6% the proceeds will be the required sum?

3. Oct. 15, J. M. King bought of you goods amounting to $3500, less 20% and 10%. Terms: cash. Not having the money, he gave you his 60-da. note, dated Oct. 15, for an amount equivalent to the cash value of the goods. What was the face of the note, money being worth 6%?

4. You purchased through W. D. Allen, an agent, 3000 lb. coffee at $33\frac{1}{3}¢$. Commission 3%; guaranty 2%. You gave Mr. Allen a 30-da. note, which when discounted at 6% for its full term just covered the amount due. If the note bore interest at 5%, what was its face?

WRITTEN REVIEW EXERCISE

1. Find the proceeds of the following note, discounted Feb. 2 at 5%; collection charges $\frac{1}{8}$%.

$2700.00 LOS ANGELES, CAL., Dec. 27, 1906.

Mar. 27, 1907, we promise to pay to the order of F. M. Dunbar & Son Twenty-seven Hundred Dollars, at the Union Bank of Los Angeles, with interest at 4%.

Value received. GRAY & SALISBURY.

2. Copy the following discount memorandum, supplying all missing terms:

FIRST NATIONAL BANK

Discounted for

F. M. Dorman & Son,

Boston, Mass., June 20, 19

MAKER	AMOUNT	FROM	TO	TIME	RATE OF DISC.	RATE OF COLL. AND EXCH.	CHARGES	PROCEEDS CREDITED
Roe & Co.	1200 —	June 20	Sept. 12	84 da.	6%	1/8%	18 30	
Ellis Bros.	900 —	20	19	??	6%		?? ??	
Jameson & Co.	75 91	20	Aug. 14	??	6%	1/8%	??	
Rogers & Son	214 10	20	July 15	??	6%		? ??	
	2390 01						?? ??	
	?? ??							???? ??

CHAPTER XXVII

PARTIAL PAYMENTS

THE UNITED STATES METHOD

ORAL EXERCISE

1. A note for $500 bears interest at 6%. What amount would pay the note and interest at the end of 1 yr.?

2. Suppose that a payment of $130 was made at the end of 1 yr. After the accrued interest has been paid, how much is there left to apply to the face of the note?

3. After the $100 has been applied to the face of the note, what amount does the maker continue to keep? On what sum, therefore, should he pay interest after the first year?

4. The maker kept the remaining $400 another year. How much interest was then due? What was the total amount due?

5. If a payment of $224 was made at this time, what amount still remained unpaid? If the balance of the note was paid three years after it was issued, what was the amount of the payment?

413. Partial payments are payments in part of a note or bond.

Such payments may be made either before or after maturity. They should be acknowledged by indorsement on the back of a note or bond. Current forms for indorsing partial payments on notes are illustrated on page 336.

414. The **United States method** of partial payments (as suggested in problems 1–5 above) has been adopted by the Supreme Court of the United States, and made the *legal method* in nearly all the states.

This is the method ordinarily used by individuals when the time between the date of the note and its payment is *more than one year.*

415. Example. A note for $1200, dated Jan. 1, 1906, bearing interest at 6%, had payments indorsed upon it as follows: Mar. 1, 1906, $212; July 1, 1906, $15; Sept. 1, 1906, $515; Nov. 1, 1906, $175. How much was due upon the note at final settlement, Apr. 1, 1907?

SOLUTION

Face of note		$1200.
Interest from Jan. 1, 1906, to Mar. 1, 1906 (2 mo.)		12.
Amount due Mar. 1, 1906		1212.
Payment Mar. 1, 1906		212.
New principal, or amount to draw interest after Mar. 1, 1906		1000.
Interest from Mar. 1, 1906, to July 1, 1906 (4 mo.)	$20.	
Interest exceeds the payment and the principal remains unaltered.		
Interest from July 1, 1906, to Sept. 1, 1906 (2 mo.)	$10.	
Total interest due Sept. 1, 1906		30.
Amount due Sept. 1, 1906		1030.
Sum of the payments since July 1 ($15 + $515)		530.
New principal, or amount to draw interest after Sept. 1, 1906		500.
Interest from Sept. 1, 1906, to Nov. 1, 1906 (2 mo.)		5.
Amount due Nov. 1, 1906		505.
Payment Nov. 1, 1906		175.
New principal, or amount to draw interest after Nov. 1, 1906		330.
Interest from Nov. 1, 1906, to Apr. 1, 1907 (5 mo.)		8.25
Amount due at settlement, Apr. 1, 1907		$338.25

It will be observed in the foregoing example that the United States method provides: (1) that *the payment must first be applied to discharge the accrued interest;* (2) that *the surplus, if any, after paying the interest may be used to diminish the principal;* and (3) that *if any payment is less than the accrued interest, the principal remains unaltered until some payment is made with which the preceding neglected payment or payments is more than sufficient to discharge the accrued interest.*

CONDENSED FORM FOR WRITTEN WORK

DATES			INTEREST PERIODS			PER CENTS OF INTEREST	PRINCIPALS	INTERESTS ON PRINCIPALS	AMOUNTS OF PRINCIPALS	PAYMENTS
Yr.	Mo.	Da.	Yr.	Mo.	Da.					
1906	1	1								
1906	3	1		2	0	$.01	$1200.00	$12.00	$1212.00	$212.00
1906	7	1		4	0	.02	1000.00	20.00		15.00
1906	9	1		2	0	.01	1000.00	10.00	1030.00	515.00
1906	11	1		2	0	.01	500.00	5.00	505.00	175.00
1907	4	1		5	0	.025	330.00	8.25	338.25	
1	3	0	1	3	0	$.075	$338.25, balance due Apr. 1, 1907			

When there are many payments, the work may be simplified as shown in the foregoing outline. First write the date and the face of the note and then the dates and the amounts of the payments. Next find the interest periods and the per cents of interest. Test the accuracy of the work to this point (1) by finding the difference between the date of the note and the date of settlement and comparing it with the sum of the interest periods; and (2) by comparing the sum of the per cents of interest with the interest on $1 for the full time as shown by the sum of the interest periods. Complete the remainder of the work as suggested by the outline.

WRITTEN EXERCISE

1. Jan. 2, 1907, J. E. King & Co. borrowed of E. B. Peterson & Bro. $1000 and gave in payment a note payable in 6 mo., with interest at 5%. July 2, J. E. King & Co. made a payment of $500 and issued a new note at 90 da., with interest at 6% for the balance due. What was the face of the new note?

2. Jan. 30, 1906, you sold Irwin & Co. 5 Eureka Elevator Pumps at $475, less a trade discount of $16\frac{2}{3}\%$. Terms: note at 6 mo. with interest at 6%. What was the amount of the note? At the maturity of the note Irwin & Co. paid you cash $1000 and gave you a new note at 6 mo., with interest at 6% for the balance due. What was the face of the new note? Sept. 1, 1906, Irwin & Co. paid you $200, and Dec. 1, $300, on their note of July 30. What was due on the note Feb. 9, 1907?

3. On the note below indorsements were made as follows: May 1, 1906, $75; Sept. 2, 1906, $90; Oct. 2, 1906, $165; Jan. 2, 1907, $125.

$825.40 OMAHA, NEB., Jan. 2, 1906.

Apr. 2, 1907, I promise to pay Wilson & Allen, or order, Eight Hundred Twenty-five $\frac{40}{100}$ Dollars, at their office, with interest at 6%.

Value received. JOHN D. AVERILL.

What was due at the maturity of the note?

4. Find the amount due on each of the following notes July 1, 1907:

a.

1200^{00} Rochester, N. Y., Jan. 15, 1906.

Fifteen months after date I promise to pay to the order of Robert Hebblethwaite & Son Twelve Hundred ^{00}/_{100} Dollars at Alliance Bank with interest at 5%.

Value received

No. 12 Due Apr. 15, 1907. W. B. Martin

b.

2000^{00} Philadelphia, Pa., July 1, 1906.

On demand we promise to pay to the order of Richard Roe & Co. Two Thousand ^{00}/_{100} Dollars at their office, with interest at 6%.

Value received

No. 16 Due — Taylor Bros. & Co.

c.

Secured by Mortgage of Real Estate in Boston to be recorded in Suffolk Registry of Deeds.

12000^{00} Boston, Mass., Jan. 1, 1906.

Eighteen months after date, for value received I promise to pay to the order of Charles W. Lawson & Co. Twelve Thousand ^{00}/_{100} Dollars at their office, with interest at the rate of six per centum per annum during the said term, and for such further time as the said principal sum or any part thereof shall remain unpaid.

Signed in the presence of O. K. Bresee Chas. B. Sherman

Mar. 15, 1906, Paid $400 on
June 15, 1906, " $500 on
Dec. 15, 1906, " $50 on
Apr. 1, 1907, " $50 on

Received on the
within note:
Oct. 1, 1906.
Five Hundred Dollars (500)
Dec. 15, 1906.
Five Hundred Dollars (500)
Mar. 6, 1907.
Five Hundred Dollars (500)

Received 7/1/06, $2000 on
" 1/1/07, $5000 on
" 2/1/07, $1000 on
" 4/1/07, $1000 on

THE MERCHANTS' METHOD

ORAL EXERCISE

1. A note for $500 is dated July 1, 1906, payable in 1 yr. with interest at 6%. If no payments have been made, what is due on the note July 1, 1907?

2. A payment of $300 was indorsed on the note Jan. 1, 1907. What was the amount of this payment at the time the note became due?

3. If the value of the note at maturity is $530 and the value of the payment $309, what is the balance due?

4. By the United States method what is the balance due at maturity on the note described in problems 1 and 3? How does this balance compare with the balance in problem 3?

416. The **merchants' method** is based on custom rather than on legal authority. It is used by most banks and business men on *short-time* notes and other obligations.

The principles of the merchants' method are suggested in problems 1–3. This method provides that: (1) *the face of the note shall draw interest to the date of settlement;* (2) *interest shall be allowed on each payment from the time it is made to the date of settlement.*

417. Example. On a note for $600, dated May 13, 1907, payable on demand, with interest at 6%, payments were made as follows: June 28, 1907, $100; Aug. 28, 1907, $200. What was due at settlement, Sept. 28, 1907?

SOLUTION

Face of note May 13, 1907		$600.00
Interest from May 13, 1907, to Sept. 28, 1907 (4 mo. 15 da.)		13.50
Value of note Sept. 28, 1907, the date of settlement		$613.50
Payment June 28, 1907	$100.00	
Interest on this payment from Aug. 28, 1907, to Sept. 28, 1907 (3 mo.)	1.50	
Payment Aug. 28, 1907	200.00	
Interest on this payment from Aug. 28, 1907, to Sept. 28, 1907 (1 mo.)	1.00	
Value of the payments Sept. 28, 1907, the date of settlement		$302.50
Balance due Sept. 28, 1907, the date of settlement		$311.00

Some houses find the time by compound subtraction and some use the exact number of days. In the following exercise find the difference in time by compound subtraction in problems 1–2, and use the exact number of days in problems 3–7.

WRITTEN EXERCISE

1. Solve problem *a*, page 335, by the merchants' method for partial payments. Compare the results by the two methods.

2. On a note for $1200, dated Apr. 16, 1906, payable on demand, with interest at $4\frac{1}{2}$ %, payments were made as follows: June 15, 1907, $500; July 18, 1907, $200. What was due at settlement, Sept. 16, 1907?

3. June 15 you borrowed $25,000 at Traders' National Bank on your demand note secured by a deposit of 300 shares of New York, New Haven, and Hartford Railroad Stock at $170. June 27 you paid $5000, July 2, $10,000, and July 30, $5000. Aug. 2 you paid the remainder of the note and interest, and withdrew the collaterals. What was the amount of the last payment, money being loaned at $4\frac{1}{2}$ %?

4. The following is a partial page of the demand and loan register of a large bank. Copy it, supplying the amount of interest due Nov. 15, money being loaned at $4\frac{1}{2}$ %.

CHARLES W. SHERMAN

No.	Date Loaned	Amount Loaned	Date of Payment.	Part of Loan Paid	Balance of Loan	Interest	Collateral	Value of Collateral
347	Apr. 1	20,000 00	May 15	5,000 00	15,000 00	? ? ? ??	250 shares	
			July 1	5,000 00	10,000 00	? ? ? ??	Penn. R.R.	
			Sept. 1	6,000 00	4,000 00	? ? ? ??	Stock . .	31,250 00
			Nov. 15	4,000 00		? ? ? ??		
						? ? ? ??		

The balance due by the merchants' method may be found in the manner suggested by the above account. The interest is found on the face of the note to the date of the first payment. The payment is deducted and the interest found on the balance to the date of the second payment, and so on. The results obtained by this process are exactly the same as the results obtained by § 416.

5. Solve problem 4 by the United States method and compare the result with the merchants' method.

6. Assuming that the collateral note, page 330, has the following payments indorsed on its back, find the amount due at final settlement, Feb. 28, 1907. Indorsements: Jan. 15, 1907, $3000; Jan. 31, 1907, $5000; Feb. 5, 1907, $1000.

7. A collateral note dated at Philadelphia, Pa., July 10, 1907, for $20,000 payable at the Quaker City National Bank is indorsed as follows: Aug. 8, 1907, $3500; Sept. 12, 1907, $7500; Nov. 19, 1907, $4000; Dec. 31, 1907, $5000. What was due on the note Dec. 31, 1907, interest being at the rate of 4%?

To solve the problem copy and complete the following interest statement:

Philadelphia, Dec 29, 19

M. R. Roswell Post,
721 Chestnut St, City

To THE QUAKER CITY NATIONAL BANK, Dr.

To interest on demand loans, as follows:

$20000.00	from 7/10	to 8/8	29	days, $	??.??
$16500.00	from 8/8	to 9/12	35	days, $	??.??
$9000.00	from 9/12	to 11/19	??	days, $	??.??
$5000.00	from ??/??	to ??/??	??	days, $	??.??
					???.??

Please send us the above interest on or before the 31st inst.

Respectfully,
W. D. Brelsford
CASHIER

8. Make an interest statement, similar to the above, for problem 6.

9. Make an interest statement, similar to the above, for problem 3.

10. Bring to the class a canceled note on which partial payments are recorded. Find, by the United States method and by the merchants' method, the amount required to cancel the note. Which method is the better for the debtor? for the creditor?

CHAPTER XXVIII

BANKERS' DAILY BALANCES

418. Some **commercial banks** and **trust companies** pay interest on the daily balances of their depositors.

Whether interest shall be allowed on a depositor's account is usually determined by the size of his daily balances. As a rule, no interest is allowed on small balances subject to check. All balances not subject to check usually draw interest. In an active account, that is, an account in which the balance changes frequently, interest is seldom allowed except on an even number of hundred dollars, and all parts of a hundred dollars are rejected.

The form of the book in which accounts with depositors are recorded varies in different sections. What is known as the Boston individual ledger (see form, page 38) is extensively used. Another form of depositors' ledger is that shown in the example below.

419. Example. Verify the balance due on the following account Mar. 1, 1907, interest settlements being made monthly at 3%.

M. W. FARNHAM

EXPLANATION	DATE		F.	DEBIT		BALANCE		CREDIT		F.	DATE		EXPLANATION
	1907										1907		
						1056	25				Jan.	1	
						1656	25	600	00	15		7	Currency
						2556	25	900	00	15		11	N. Y. draft
Check	Jan.	15	14	510	00	2046	25						
						3746	25	1700	00	17	Jan.	22	N. Y. draft
Note	Jan.	25	16	210	00	3536	25						
Check		28	16	500	00	3036	25						
						3042	08	5	83	17	Jan.	31	Interest
						4042	08	1000	00	21	Feb.	8	N. Y. draft
Check	Feb.	15	20	500	00	3542	08						
Check		22	22	1340	00	2202	08						
						2209	49	7	41	23	Feb.	28	Interest

SOLUTION. The credit slip on page 341 shows a form used for recording the daily balances. Only two money columns are used, one for hundreds and the other for thousands. No interest is computed except on an even number of hundred dollars, and all parts of a hundred dollars are rejected.

Beginning with Jan. 1 the daily balance of M. W. Farnham's account for 6 da. was $1056.25; record $1000 on the credit slip as shown in the margin. A deposit of $600 was made Jan. 7, making the balance $1656.25 for the next 4 da.; record $1600 on the credit slip as shown in the margin. A deposit of $900 on Jan. 11 made the balance $2556.25 for the next 4 da.; record $2500 on the credit slip as shown in the margin. A withdrawal of $510 on Jan. 15 left a balance of $2046.25 for the next 7 da.; record $2000 on the credit slip as shown in the margin. A deposit of $1700 on Jan. 22 made the balance $3746.25 for the next 3 da.; record $3700 on the credit slip as shown in the margin. A withdrawal of $210 on Jan. 25 left a balance of $3536.25 for the next 3 da.; record $3500 on the credit slip. A withdrawal of $500 on Jan. 28 left a balance of $3036.25 for the next 4 da. This records the balance for each day in January. Adding these balances the result is $70,000, and the interest on this sum for 1 da. at 3% is $5.83. Adding $5.83 to $3036.25 gives the balance to the credit of the depositor Feb. 1 as $3042.08.

DAILY CREDIT BALANCES

M. W. Farnham

1907	Jan.		Feb.	
1	1		3	
2	1			
3	1			
4	1			
5	1			
6	1			
7	1	6		
8	1	6	4	
9	1	6		
10	1	6		
11	2	5		
12	2	5		
13	2	5		
14	2	5		
15	2		3	5
16	2			
17	2			
18	2			
19	2			
20	2			
21	2			
22	3	7	2	2
23	3	7		
24	3	7		
25	3	5		
26	3	5		
27	3	5		
28	3			
29	3			
30	3			
31	3			
Total	70	0	88	9
Interest	5	83	7	41

Enter the daily balances for February as shown in the margin. The result is found to be $88,900, and the interest on this sum for 1 da. at 3% is $7.41. $7.41 added to the balance of the depositor's account Feb. 28 gives $2209.41 as the balance to his credit beginning Mar. 1.

In practice the daily balances are usually written as shown in the February column of the accompanying credit slip. The total is then found by multiplication and addition. Thus, the total of the February column is $7 \times \$3000 + 7 \times \$4000 + 7 \times \$3500 + 7 \times \$2200$, or $88,900.

Some accountants also use the pure interest method in finding the amount due. Thus, the interest on $3000 for 7 da., plus the interest on $4000 for 7 da., plus the interest on $3500 for 7 da., plus the interest on $2200 for 7 da. equals $7.41, the same as by the first method.

In the examples which follow the student may use either of the three methods suggested.

WRITTEN EXERCISE

1. The Rochester Trust and Safe Deposit Co. allows interest to its depositors on daily balances at 3 % per annum, payable quarterly. Find the cash balance of the following account with Chas. M. Sherman, Apr. 1, 1907. Jan. 1, 1907, deposited $1200; Jan. 12 drew out $400; Jan. 30 deposited $800; Jan. 31 drew out $400; Feb. 10 deposited $800; Feb. 25 drew out $100; Mar. 10 deposited $800; Mar. 20 drew out $900; Mar. 25 deposited $300.

2. Mar. 1, 1907, Harvey & Smith's balance with the Fidelity Trust Co. was $2246. During the month they made the following deposits: Mar. 3, $2500; Mar. 9, $1750; Mar. 24, $2645.75; Mar. 28, $1310.50; Mar. 30, $500. They also drew out by check as follows: Mar. 4, $1050; Mar. 6, $2000; Mar. 8, $720; Mar. 12, $840.50; Mar. 16, $450; Mar. 19, $430; Mar. 23, $1000; Mar. 26, $150; Mar. 29, $267. How much interest should be credited at the end of the month, the rate being 3 % per annum? What was the balance of the account after the interest was credited?

3. Find the cash balance of the following account May 31, 1907, assuming that interest is allowed on daily balances at 3 % and added to the account monthly.

A. S. OSBORN

EXPLANATION	DATE		F.	DEBIT		BALANCE		CREDIT		F.	DATE		EXPLANATION
	1907										1907		
						1200	00	1200	00		Mar.	1	N. Y. draft
Check	Mar.	12		100	00	1500	00	400	00			12	Currency
						2000	00	500	00			25	Currency
Check		31		100	00	2400		500	00			31	N. Y. draft
						****	**	*	**			31	Interest
						****	**	700	00		Apr.	15	N. Y. draft
Note	Apr.	20		50	00	****	**	200	00			20	N. Y. draft
Check		30		1200	00	****	**	*	**			30	Interest
						****	**	250	00		May	10	Currency
Check	May	31		500	00	****	**	*	**			31	Interest

CHAPTER XXIX

SAVINGS-BANK ACCOUNTS

420. A **savings bank** is an institution, chartered by the state, in which savings or earnings are deposited and put to interest.

The deposits in a savings bank are practically payable on demand. Most banks reserve the right to require notice of withdrawal from 30 to 60 da. in advance; but this right is seldom exercised.

The period of time which must elapse before dividends of interest are declared is called the **interest term.** Dividends of interest are usually declared semiannually; but in some sections they are declared quarterly. The stated days on which balances begin to draw interest are called **interest days.** In some savings banks deposits begin to draw interest from the first of each quarter; in others, from the first of each month.

In nearly all savings banks, only such sums as have been on deposit for the *full time between the interest days* draw interest. Thus, if the interest days begin on the first of each quarter, only those sums that have been on deposit for the full quarter draw interest.

421. Interest is computed on an even number of dollars, and all fractions of a dollar are rejected. When interest is not withdrawn it is placed to the credit of the depositor and draws interest the same as any regular deposit. Savings banks therefore allow **compound interest.**

422. Examples. 1. In the Wildey Institution for Savings the interest term is 6 mo. and the interest days are Jan. 1, Apr. 1, July 1, and Oct. 1. Verify the balance due on the following account Jan. 1, 1907, at 4%.

SOLUTION. The account was opened July 1, 1906, by a deposit of $500. July 10 this sum was increased by a deposit of $10, making the balance $510; Aug. 14 this sum was diminished by a withdrawal of $20, making the balance $490; Oct. 4 this sum was diminished by a withdrawal of $200, making the balance $290. The account was similarly increased and diminished until Dec. 31, when there was a balance of $300.75 due the depositor.

The Wildey Institution for Savings

in account with

Mrs. Robert M. Sanborn

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1906 July	1	500	—					500	—
	10	10	—					510	—
Aug.	14					20	—	490	—
Oct.	4					200	—	290	—
	10	70	75					360	75
Dec.	10	40	—					400	75
	31					100		300	75
1907 Jan.	1			7	80			308	55

The smallest balance for the first interest period, July 1 to Oct. 1, was $490. The interest on $490 for 3 mo. at 4% is $4.90. The smallest balance for the second interest period, Oct. 1 to Jan. 1, was $290. The interest on $290 for 3 mo. at 4% is $2.90. $4.90 plus $2.90 equals $7.80, the dividend of interest due the depositor Jan. 1. Since this sum is not withdrawn, it is placed to the credit of the depositor, making his balance $308.55.

2. In the Warren Institution for Savings interest dividends are declared semiannually and the interest days are Jan. 1, Apr. 1, July 1, and Oct. 1. Verify the balance due on the following account Jan. 1, 1907, at 4%.

The Warren Institution for Savings

in account with

Miss Mabel C. Courtright

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1906 Jan.	1	500	—					500	—
Mar.	1	300	—					800	—
May	10	100	—					900	—
July	1			13	—			913	—
1907 Jan.	1			18	26			931	26

SOLUTION. The smallest balance for the first interest period was $500; the interest on $500 for 3 mo. at 4% is $5. The smallest balance for the second interest period was $800; the interest on $800 for 3 mo. at 4% is $8. $5 + $8 = $13, the total interest due the depositor July 1. $900 + $13 = $913. This balance remained unchanged for the next 6 mo. The interest on $913 for 6 mo. at 4% is $18.26. $913 + $18.26 = $931.26, the amount due the depositor Jan. 1, 1907.

WRITTEN EXERCISE

1. Solve example 1 above, assuming that the interest days are the first day of each month; also example 2.

2. Copy the following account, supplying the missing amounts. Interest at $4\frac{1}{2}$ %; interest days, Jan. 1, Apr. 1, July 1, and Oct. 1.

MANHATTAN SAVINGS BANK

IN ACCOUNT WITH *Mr. Chas. B. Sherman*

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1906									
Jan.	1	600	00					* * *	* *
Jan.	31					100	00	* * *	* *
Mar.	1	250	00					* * *	* *
May	6					50	00	* * *	* *
May	31	100	00					* * *	* *
July	1			* *	* *			* * *	* *

3. Copy and complete the following account. Interest at 4%; interest days, Jan. 1, Apr. 1, July 1, and Oct. 1.

FIDELITY SAVINGS BANK

IN ACCOUNT WITH *Mr. Frank M. Ellery*

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1906									
Jan.	1	300	00					* * *	* *
Mar.	6	200	00					* * *	* *
Mar.	30					125	00	* * *	* *
Apr.	17	165	50					* * *	* *
July	1	100	00	* *	* *			* * *	* *
Aug.	15					75	00	* * *	* *
Aug.	31	58	40					* * *	* *
Oct.	1	250	00					* * *	* *
Dec.	1					110	50	* * *	* *
1907									
Jan.	1			* *	* *			* * *	* *

CHAPTER XXX

EXCHANGE

DOMESTIC EXCHANGE

ORAL EXERCISE

1. Mention some objections to sending actual money by express.

2. If $50 sent by mail in a registered letter is lost, to what extent are the postal authorities liable?

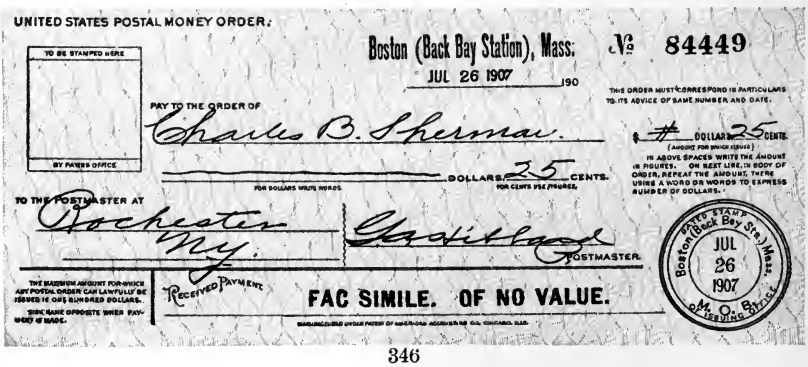
3. In what ways may you pay a debt at any distant point without actually sending the money?

423. The process of settling accounts at distant points without actually sending the money is called **exchange.**

Money Orders

424. Money orders, as issued by post offices, express companies, and banks are frequently used in making payments at a distance.

425. A **postal money order** is a government order for the payment of money, issued at one office and payable at another.

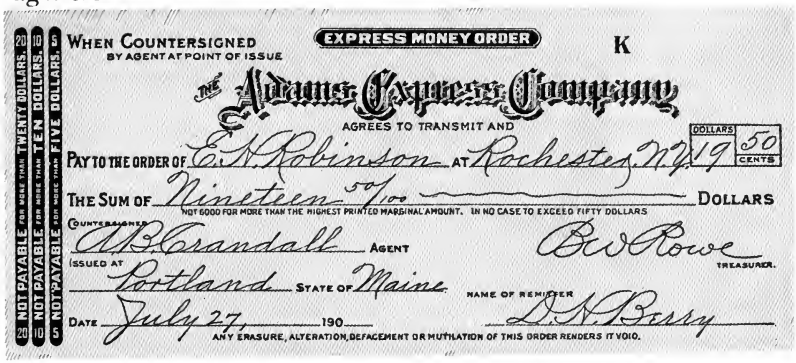


The fees (**rate of exchange**) charged for postal money orders are:
For orders for sums not exceeding

	$2.50	3¢	Over	$30.00 to $ 40.00	15¢
Over	2.50 to $ 5.00	5¢	Over	40.00 to 50.00	18¢
Over	5.00 to 10.00	8¢	Over	50.00 to 60.00	20¢
Over	10.00 to 20.00	10¢	Over	60.00 to 75.00	25¢
Over	20.00 to 30.00	12¢	Over	75.00 to 100.00	30¢

The maximum amount for which a single postal money order may be issued is $100. When a larger sum is to be sent, additional orders must be obtained. When an order is issued, the money is not sent from one post office to another. The transfer is merely a matter of bookkeeping, the money being received by the government at one office and paid out at another. If a postal money order is lost, a duplicate may be obtained from the Post Office Department at Washington.

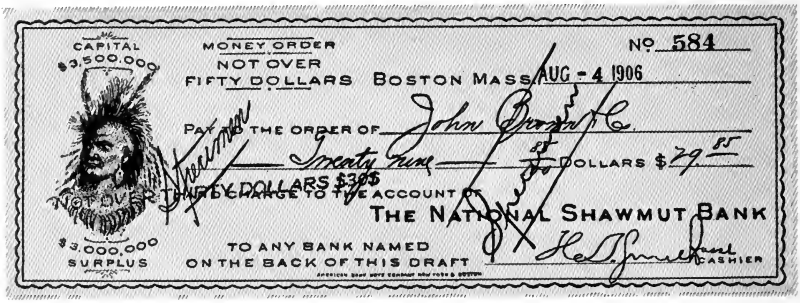
426. An **express money order** is an order for the payment of money, issued by an express company and payable at any of its agencies.



The fees charged for express money orders are the same as those for postal money orders. The maximum amount for which a single express money order may be issued is $50. A postal money order must not bear more than one indorsement; but an express money order may bear any number of indorsements.

427. A **bank money order** (see form, page 348) is an order for the payment of money issued by a bank and payable at certain other banks in different parts of the country.

The charge for a bank money order is usually the same as that for a postal money order.



428. A **telegraphic money order** is a telegram of an express or telegraph company, at any given place, ordering the payment of money at another designated place.

THE UNION TELEGRAPH CO.

INCORPORATED

23,000 OFFICES IN AMERICA CABLE SERVICE TO ALL THE WORLD

ROBERT C. CLOWRY, President and General Manager

SEND the following message subject to the terms on back hereof, which are hereby agreed to. Boston, Mass., July 27, 19

To The Union Telegraph Co.

Rochester, N.Y.

		Findable	Charles	Osgood
ten	East	Avenue	Fichant	Findelkind
	The	Union	Telegraph	Co.

These telegrams are usually in cipher; that is, in a language not understood by those who are unfamiliar with the system of abbreviations (code) used. The sender and the receiver must each have a code. The following code will illustrate the principle of telegraphing in cipher:

Code Word	Meaning
Fichant	One hundred dollars
Ficheron	One thousand dollars
Findable	Please pay —— of —— your city $——.
Findelkind	On production by him of positive evidence of his personal identity.

The principle of a telegraphic money order is the same as that of a postal money order; no money is transferred from one place to another. The charge for an order is usually 1% of the amount to be transmitted plus twice the rate for a single ten-word message.

The following are the rates for a ten-word message from Boston to the places named:

New York	$0.25	Chicago	$0.50	Galveston	$0.75
Philadelphia	$0.25	San Francisco	$1.00	Rochester	$0.35

ORAL EXERCISE

1. What was the total cost to the sender of the postal money order, page 346? the express money order, page 347? the telegraphic money order, page 348? the bank money order, page 348?

2. What will be the total cost of a postal money order for 27¢? $2.19? $5.28? $10.40? $18.90? $45.10? $35.89? $125 ($100 + $25)? $75.29? $49.82? $127.16?

3. What will be the total cost of an express money order for $6.20? $28.80? $19.50? $27.95? $48.90? $65 ($50 + $15)? $111? $37.59? $41.72? $65.59? $114?

4. What will be the total cost of a telegraphic money order from Boston to New York for $50? $75? $100? $125? $150? $200? $300? $400? $450? $500? from Boston to Philadelphia? from Boston to San Francisco? from Boston to Chicago?

5. Translate the following telegraphic money order: *Findable F. J. Reed*, 20 *Park St. ficheron findelkind.* How much will it cost for such an order from Boston to Galveston? from Boston to Chicago? from Rochester to Boston?

WRITTEN EXERCISE

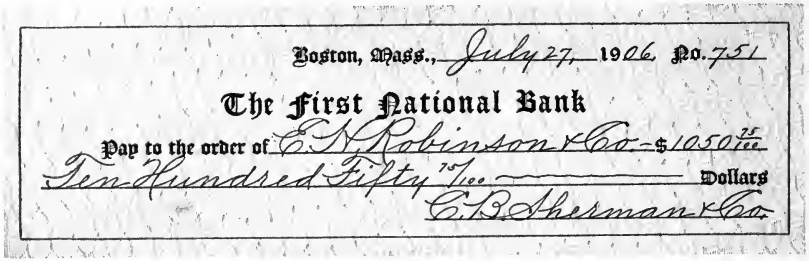
1. Find the total cost of 5 postal money orders for the following amounts: $3.10; $8.19; $25.06; $18.50; $20.

2. Find the total cost of six express money orders for the following amounts: $1.25; $10; $6.80; $16.25; $80; $19.50.

3. Find the total cost of the following telegraphic money orders: one from Boston to New York for $50; one from Boston to Philadelphia for $500; one from Boston to San Francisco for $175; one from Boston to Galveston for $300; one from Boston to Rochester for $250.

Checks and Bank Drafts

429. Business men usually keep their money on deposit with a commercial bank or trust company and make most payments, at home and at a distance, by **check**; that is, an order on a bank from one of its depositors for the payment of money.



Boston, Mass., July 27, 1906. No. 751

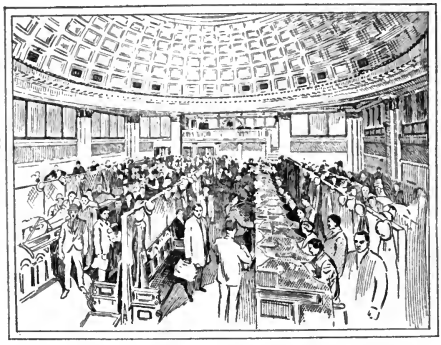
The First National Bank

Pay to the order of E. H. Robinson & Co. $1050 25/100

Ten Hundred Fifty 75/100 Dollars

C. B. Sherman & Co.

A check may be drawn for any amount so long as it does not exceed the balance on deposit to the credit of the drawer. It may be drawn payable to: (1) the order of a designated payee, in which case the payee must indorse it before the money will be paid over; (2) the payee, or bearer, in which case any one can collect it; (3) "Cash," in which case any one can collect it.



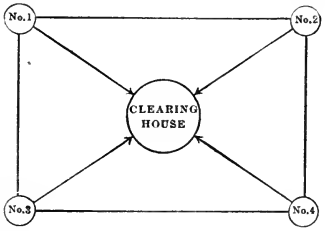
Interior View of a Clearing House.

C. B. Sherman & Co. and E. H. Robinson & Co. in the foregoing check both reside in Boston. On receiving the check, E. H. Robinson & Co. indorse it and deposit it for credit with their bank, say the National Shawmut Bank. The First National Bank and the National Shawmut Bank, as well as each of the other banks in the city, has many depositors who draw checks upon it which are deposited by the payees in various other city banks, and it also receives daily for credit from its own depositors checks drawn upon various other city banks.

Each bank therefore has a daily balance to settle or to be settled with each of the other banks. To some it has payments to make and from others it has payments to receive. If these balances were adjusted in money, each bank would have to send a messenger to each of the debtor

banks to present accounts and receive balances. This would be a risky and laborious task. To facilitate the daily exchanges of items and settlements of balances resulting from such exchanges there has been established in every large city a central agency, called a **clearing house**. This agency is an association of banks which pay the expense of conducting it in proportion to the average amount of their clearings.

Suppose, for example, that the banks constituting a clearing house are Nos. 1, 2, 3, and 4. No. 1 presents at the clearing house items against Nos. 2, 3, and 4, and Nos. 2, 3, and 4 present items against No. 1. So, likewise, with No. 2 and each of the other banks. In the clearing house there are usually two longitudinal columns containing as many desks as there are banks in the association. At a given time a settling clerk from each bank takes his place at his desk inside of one of the columns and a delivery clerk from each bank takes his place outside the column. Each delivery clerk advances, one desk at a time, and hands over to each settling clerk his exchange items against that bank. After the circuit of the desks has been completed each delivery clerk is at the point from which he started, and each settling clerk has on his desk the claims of all of the other banks against his bank. Each settling clerk then compares his claims against other banks with those of other banks against him and strikes a balance. This balance may be in favor of or against the clearing house. If No. 1 brought claims against Nos. 2, 3, and 4 aggregating $211,000 and Nos. 2, 3, and 4 brought claims against No. 1 aggregating $200,000, there is $11,000 due No. 1 from the clearing house. But if No. 1 brought to the clearing house exchange items aggregating $200,000 and took away exchange items aggregating $211,000, there is $11,000 due the clearing house from No. 1. So, likewise, with No. 2 and each of the other banks. When all of the exchanges have been completed, the clearing house will have paid out the same amount that it has received.

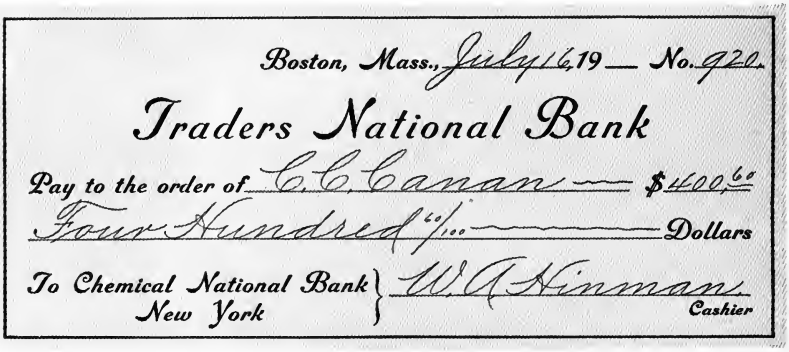


But all checks received by banks are not payable in the city. Suppose that A. W. Palmer, of Chicago, Ill., owes C. B. Andrews, of Westfield, Mass., $500 and that the amount is paid by a check on the City National Bank of Chicago. C. B. Andrews receives the check and offers it for credit at the Farmers and Traders Bank of Westfield, Mass. The Westfield Bank has no account with any Chicago bank, but it has with the First National Bank of Boston, and the check is sent to that bank for credit. The First National Bank wishes to increase its New York balance and the check is forwarded to Chemical National Bank of New York for credit. Chemical National Bank next mails the check to Commercial National Bank of Chicago, the

bank with which it has regular dealings in that city. Commercial National Bank sends the check to the clearing house and it is carried to the City National Bank by a messenger from that bank. Thus, all of a depositor's checks will in time be presented to the bank on which they are drawn. When presented, they will be charged to the depositor, cancelled, and later returned to him to be filed as receipts.

Banks frequently charge their depositors a small fee (**rate of exchange**) for collecting out-of-town checks. This fee is rarely over $\frac{1}{10}\%$, but there is no uniformity in the matter. Sometimes when a customer keeps a large bank account, no charge whatever is made for the collection.

430. It often happens that a person will find it necessary to make payment to one who does not care to take the risk of a private check or to one who should not be called upon to pay the cost of cashing a check. In such cases some other form of instrument of transfer must be used. A very common and convenient method of making a remittance is by means of a check of one banking institution upon another called a **bank draft.**


Boston, Mass., July 16, 19__ No. 920.

Traders National Bank

Pay to the order of C. C. Canan — $400.60

Four Hundred 60/100 — Dollars

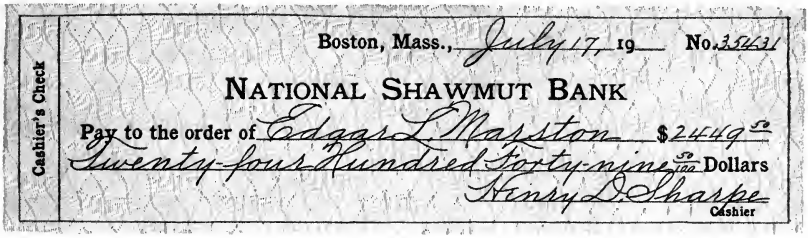
To Chemical National Bank, New York

W. A. Hinman, Cashier

Banks in the different cities frequently keep running accounts with each other and make periodical settlements. At the time of drawing the above draft Traders National Bank of Boston very likely has checks and drafts drawn upon New York banks which it has received from its depositors. These it sends to Chemical National Bank to cover the amount of the draft. Corresponding transactions may also take place in New York. Chemical National Bank may sell its draft on Traders National Bank and, to cover the amount, remit checks and drafts on Boston banks which it has received from its depositors. What is occurring between these two places is also occurring between all manner of places; but drafts upon New York banks and other financial centers are the most used in making remittances.

A bank draft is sometimes drawn payable to the one to whom it is to be sent. It is better, however, to have it drawn payable to the purchaser who may indorse it over to the person to whom it is to be sent. In this way the name of the sender appears on the draft, and when canceled, the draft will serve the purpose of a receipt. Banks usually sell drafts at a slight premium on the face. This premium is called **exchange**. It varies somewhat (see page 358), but is seldom more than $\frac{1}{10}\%$.

431. There are still other methods of transmitting funds through the instrumentality of a bank. A depositor may exchange his own check for that of a **cashier's check**. The latter, being a check of the cashier on his own bank, would pass among strangers better than a depositor's check.


Cashier's Check

Boston, Mass., July 17, 19__ No. 35431

NATIONAL SHAWMUT BANK

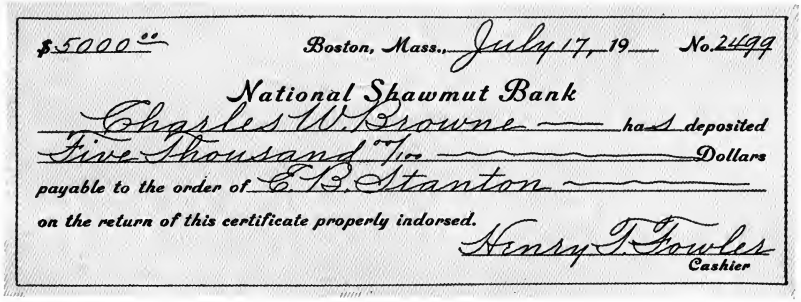
Pay to the order of Edgar L. Marston $2449 50/100

Twenty-four Hundred Forty-nine 50/100 Dollars

Henry D. Sharpe
Cashier

In New York City these checks are occasionally used instead of the New York draft. As New York exchange is in demand in all parts of the country, the expediency of the course is apparent.

432. By depositing a sum of money in a bank a person may receive a certificate, called a **certificate of deposit**. This will direct the payment of the sum deposited to any person whom the depositor may name.


$5000 00

Boston, Mass., July 17, 19__ No. 2499

National Shawmut Bank

Charles W. Browne has deposited

Five Thousand 00/100 Dollars

payable to the order of C. B. Stanton

on the return of this certificate properly indorsed.

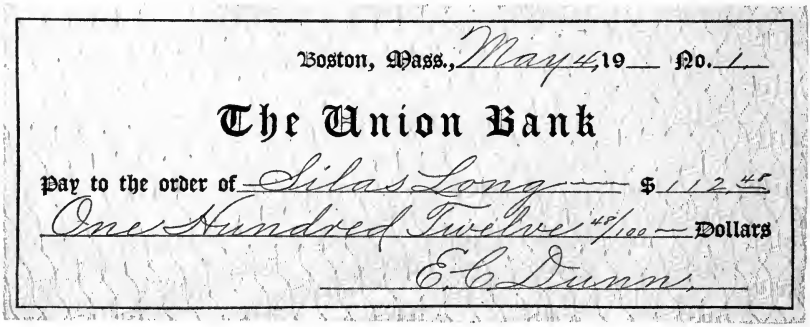
Henry T. Fowler
Cashier

The payee in a certificate of deposit will have no difficulty in getting the certificate cashed or the amount credited to him by his bank.

ORAL EXERCISE

1. Assuming that the bank which cashed the check on page 5 charged $\frac{1}{8}\%$ collection, what was the amount credited to the depositor?

2. Silas Long of New York deposited the following check. The bank deducted $\frac{1}{10}\%$ for collection. How much was placed to Silas Long's credit?



Boston, Mass., May 4, 19__ No. 1

The Union Bank

Pay to the order of Silas Long $112.45

One Hundred Twelve 45/100 Dollars

E. C. Dunns

3. B deposited three out-of-town checks in his bank as follows: \$300; \$700; \$750. If the bank charged $\frac{1}{10}\%$ collection, what amount was placed to B's credit?

4. Bring to the class a number of canceled checks and take several of them and trace them from the time they were issued until they were filed as receipts by the drawer. Show why a canceled check is the best kind of a receipt for the payment of money?

5. How much did the bank draft on page 352 cost the purchaser if the exchange was at $\frac{1}{10}\%$ premium?

WRITTEN EXERCISE

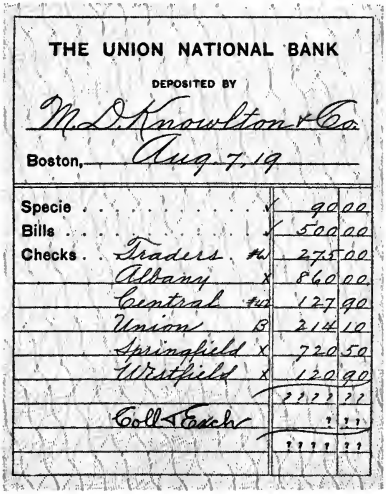
1. Find the cost of a bank draft for \$3958.75 at $\frac{1}{10}\%$ premium; of a bank draft for \$679.80 at $\frac{1}{20}\%$ premium; of a bank draft for \$768.54 at 50¢ per \$1000 premium.

2. To cover the cost of a bank draft bought at $\frac{1}{10}\%$ premium, I gave my bank a check for \$250.25. What was the face of the draft? What was the rate of premium per \$1000?

3. How large a bank draft can be bought for $850.85, exchange being at $\frac{1}{10}\%$ premium?

4. Find the proceeds of the accompanying deposit, $\frac{1}{10}\%$ collection and exchange being charged on the out-of-town checks.

When the receiving teller takes a deposit from a customer, he classifies the items on the deposit ticket, as shown in the accompanying illustration. If the coin and bills passed in count right, these items are checked (√) on the deposit slip; if a check on a clearing house bank is received, it is marked with the number of that bank in the clearing house; if a check on the teller's bank is received, it is marked "B"; if a check on an out-of-town bank is received, it is marked "X."



THE UNION NATIONAL BANK

DEPOSITED BY

M. D. Knowlton & Co.

Boston, Aug. 7, 19

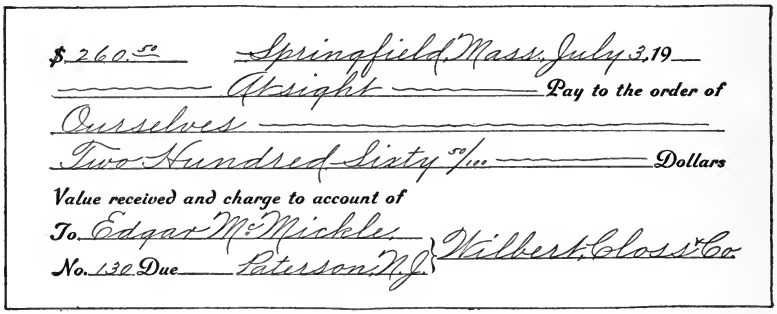
Specie		√	90 00
Bills		√	500 00
Checks	Traders	#6	275 00
	Albany	X	860 00
	Central	#2	127 90
	Union	B	214 10
	Springfield	X	720 50
	Whitfield	X	120 90
			2??? ??
	Coll & Exch		? ??
			???? ??

5. Write a bank draft using the following data: your address and the current date; drawer, Central National Bank; drawee, Chemical National Bank, New York; amount, $711.94; payee, C. E. Denison; cashier, your name. How large a check will pay for the draft at $\frac{1}{10}\%$ premium? Write the draft.

6. Suppose that the members of the class whose surnames begin with the letters from A to G inclusive have a deposit with Traders National Bank; that the members whose surnames begin with the letters from H to N inclusive have a deposit with City National Bank; that the members whose surnames begin with O to S inclusive have a deposit with First National Bank; and that the members whose surnames begin with T to Z inclusive have a deposit with Central Bank. Let each student write a check on his bank in favor of one of his classmates, and let this classmate indorse the check and deposit it with his bank. Then form a clearing house, strike a balance between the different banks, and have these balances adjusted by the payment of school money.

Commercial Drafts

433. Business men frequently employ the **commercial draft** as an aid in the collection of accounts that are past due.


$260.50 Springfield, Mass., July 3, 19—

At sight Pay to the order of

Ourselves

Two Hundred Sixty 50/100 Dollars

Value received and charge to account of

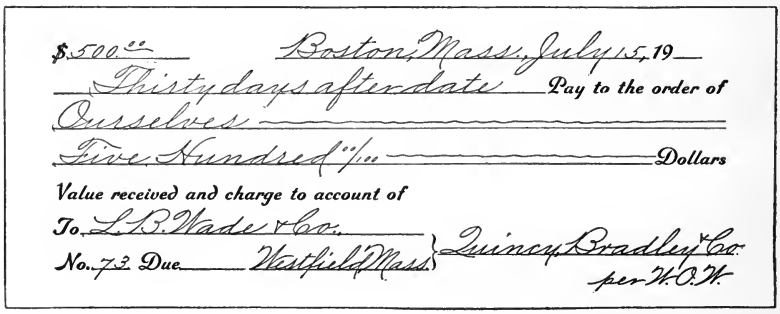
To Edgar McMickle

No. 130 Due Paterson, N.J.

Wilbert, Closs & Co.

The above is a common form of draft used for collection purposes. Edgar McMickle owes Wilbert, Closs & Co. $260.50. The amount is due, and Wilbert, Closs & Co. draw a draft on Edgar McMickle and leave it with their Springfield bank for collection. The Springfield bank forwards it to its correspondent in Paterson and this bank sends it by messenger to Edgar McMickle. When he pays the draft, the Paterson bank notifies the Springfield bank, and that bank deducts a small fee (**collection and exchange**) for collecting the draft, and credits Wilbert, Closs & Co. for the proceeds.

434. It has been seen (page 321) that the **time draft** is frequently used in connection with sales of merchandise.


$500.00 Boston, Mass., July 15, 19—

Thirty days after date Pay to the order of

Ourselves

Five Hundred 00/100 Dollars

Value received and charge to account of

To L. B. Wade & Co.

No. 73 Due Westfield, Mass.

Quincy, Bradley & Co.

per W. O. W.

Suppose Quincy, Bradley & Co. sell L. B. Wade & Co. a bill of merchandise amounting to $500. Terms: 30-da. draft for the amount of the bill. The draft, as above, and the bill in regular form would be drawn up and

sent to L. B. Wade & Co. for acceptance. The object of drawing a time draft in connection with sales of merchandise is twofold: (1) when accepted, the draft serves as a written contract; (2) since an acceptance is negotiable, it may be discounted and cash realized upon it before maturity. Such a draft is frequently left with a bank for collection instead of being remitted with the bill. The bank will then first present the draft for acceptance and later for payment.

ORAL EXERCISE

1. If you exchange your check for a cashier's check, is there any charge for the accommodation?

2. If the sight draft on page 356 was collected by a bank which charged $\frac{1}{8}\%$ collection, how much was placed to the credit of Wilbert, Closs & Co.?

3. You deposited in Shawmut National Bank $5000, received the certificate of deposit shown on page 353, and remitted it to E. B. Stanton on account. Would there be any exchange?

WRITTEN EXERCISE

1. The draft on page 356 was accepted July 17, and discounted July 25. If the bank charged $\frac{1}{10}\%$ collection and 6 % interest, how much was placed to the credit of the drawers?

2. Mar. 27 Wilson Bros., Chicago, Ill., drew a 30-da. draft on E. W. King, Toledo, O., in favor of themselves, payable 30 da. after date, for $3500, and mailed it for acceptance. Apr. 1 the draft was received accepted; Apr. 2 it was discounted at City Bank. If the charges were $\frac{1}{20}\%$ collection and 6 % interest, what amount was credited to Wilson Bros.?

3. Apr. 17 O. H. Brooks, Buffalo, N.Y., drew a sight draft on Slocum & Co., Hartford, Conn., in favor of himself, for $391, and left it with his bank (First National) for collection. First National Bank sent the draft to its Hartford correspondent (Commercial National), and 5 da. later informed O. H. Brooks that the draft had been collected, and the amount, less $\frac{1}{8}\%$ collection, placed to his credit. If O. H. Brooks's bank balance was $758.62 before the draft was drawn, what was it after the draft was credited? Write the draft and show the indorsements.

4. Aug. 9 you sold C. D. Mead & Co., San Francisco, Cal., 39 mahogany sideboards at $162.50, delivered the goods to the Interstate Transportation Co., and received a through **bill of lading** (receipt for the goods and an agreement to transport and deliver them to the consignee or to his order). You then drew a sight draft on C. D. Mead & Co. in favor of your bank, attached the draft to the bill of lading, and left it with your bank for collection. Your bank indorsed the draft and the bill of lading and sent them to First National Bank of San Francisco for collection and credit. Aug. 23 you received advice that the draft had been collected, and the amount, less $\frac{1}{4}\%$, placed to your credit. What was the amount credited?

When First National Bank of San Francisco received the draft, it notified C. D. Mead & Co. They paid the draft, and the bank gave them the bill of lading. When goods are shipped in this manner, the transportation company will not deliver the goods until the consignee presents the bill of lading.

Fluctuation of Rates of Exchange

435. It has been seen that money orders always sell for more than their face value, and that bank drafts frequently cost a little more than their face value. When exchange costs its face value, it is said to be **at par**; when it costs more than its face value, it is said to be **at a premium**; when it costs less than its face value, it is said to be **at a discount.**

On bank drafts for small sums, say $500 or less, exchange is usually at a uniform premium. This premium is to pay the banks for their trouble and the expense of shipping money to the centers on which the drafts are drawn, when balances at these points become low. But exchange on the trade centers of the country may be at par at one time, at a premium at another, and at a discount at still another. For example, during the late fall months, when the grain crops begin to be sent East, New York is sending a great many checks and drafts to the section of which Chicago is the trade center. Exchange on New York is then very plentiful in Chicago, and if a man in Chicago wished to buy a draft on New York for a large amount, say $10,000 or more, the Chicago banks will sell it to him at a discount. But if a man in New York at that time wished to buy a draft on Chicago for $10,000, he would have to pay a premium, because the New York banks would be anxious not to decrease their Chicago balances.

Early in the spring, when New York importers and jobbers are sending foreign and domestic manufactured goods for distribution in the West, a great many checks and drafts are being sent from the West to New York, and exchange is at a discount in New York and at a premium in Chicago. This principle applies at any trade centers between which exchange operations go on. Smaller places make their settlements in or through larger places, and the main exchange transactions go on between the few leading cities, with converging lines on New York.

The rate of exchange between two cities will never exceed the cost of shipping actual money from one of the cities to the other, except in time of panic or a financial unrest. Thus when the cost of sending money by express from New York to Chicago is $5 per $10,000, the discount in New York or the premium in Chicago will not greatly exceed $\frac{1}{20}\%$ ($5 per $10,000). To prevent the rates from going any higher the banks will arrange for the shipment of actual money from New York to Chicago.

As a rule no charge is made for cashing bank drafts on the trade centers of the country, like New York, Chicago, and Philadelphia.

436. It has been seen that banks frequently charge a small fee for collecting paper payable out of town.

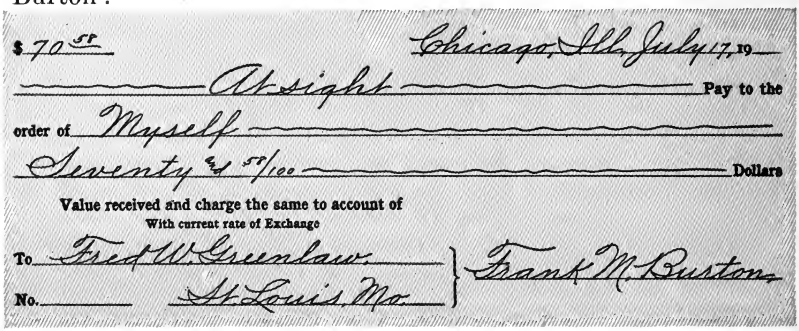
In some cases the rates of collection are more or less arbitrary; in others they are governed by trade movements, the same as rates of exchange. In still others the clearing house association fixes the rate.

ORAL EXERCISE

Find the cost of the following bank drafts:

1. $18,500 at $\frac{1}{20}\%$ discount; at 40¢ per $1000 premium.
2. $516.90 at $\frac{1}{10}\%$ premium; at 50¢ per $1000 discount.
3. $1600.80 at 75¢ per $1000 premium; at $\frac{1}{10}\%$ discount.
4. A draft for $4000 was bought for $3998. Was exchange at a premium or at a discount, and what rate?
5. J. E. Smith & Co. drew at sight on E. M. Barrows for $250 and made collection through their bank. If the bank charged $\frac{1}{10}\%$ for collection, for what amount did J. E. Smith & Co. receive credit?
6. During the late fall many checks and drafts are being sent to the southern cities in payment for shipments of cotton. At such times is exchange on New York likely to be at a discount or at a premium in New Orleans? in New York?

7. Frank M. Burton wishes to collect an account of $70.58 and for this purpose draws the following draft and leaves it with the National Express Co. for collection. If the express company charges 25¢ for collection, how much will it collect of Fred W. Greenlaw? how much will it pay Frank W. Burton?


$70.58 Chicago, Ill., July 17, 19

At sight Pay to the

order of Myself

Seventy & 58/100 Dollars

Value received and charge the same to account of

With current rate of Exchange

To Fred W. Greenlaw

No. St. Louis, Mo.

Frank M. Burton

Note that the draft contains the clause "With current rate of Exchange." This means that the drawee is requested to pay the face of the draft plus the cost of exchange. Nearly all express companies have arrangements by which they undertake the collection of notes and accounts. The process of collecting is simple. The note or draft covering the amount of the account is placed in a collection envelope furnished by the express company, and sent to its destination. If collection cannot be made, notice is given with reasons for refusal; if collection is made, the money is sent back in the collection envelope, and the amount, less collection charges, paid to the one for whom the collection was undertaken. The charge varies with the distance.

WRITTEN EXERCISE

1. A bank draft for $15,000 was bought for $14,992.50. Was exchange at a premium or at a discount, and what rate? At this rate find the cost of a draft for $17,121.98; a draft for $12,929.75; a draft for $127,162.89.

2. I gave the American Express Co. an account of $178.50 for collection. If the collection charges were $2.50 per $1000, how much did I receive from the company? At this rate what should be the proceeds from the collection of three drafts with amounts as follows: $125.60; $218.90; and $134.50?

3. An agent sold for me 1000 T. hay at \$17.50 per ton. He paid \$125 for cartage, \$75 for storage, charged $2\frac{1}{2}\%$ commission, and remitted the proceeds by a bank draft bought at $\frac{1}{8}\%$ premium. What was the face of the draft?

4. A Boston commission merchant sold for his principal in Chicago 27,518 lb. leather at $25\frac{7}{8}¢$ per pound. If he charged a commission of $4\frac{1}{2}\%$, how large a bank draft, bought at \$1.50 per \$1000 premium, should he remit to his principal?

5. Mar. 8 Edward Whitman & Co. drew a draft payable 30 da. after date on Elwood & Spears for \$375.98 and had it discounted at City Bank. If the rate of collection was $\frac{1}{8}\%$ and interest 5%, what were the proceeds of the draft?

6. Copy and complete the following letter of advice, assuming that the rate of collection is $\frac{1}{8}\%$ on Nos. 720 and 716, and $\frac{1}{10}\%$ on Nos. 692 and 710. Check the results.

NATIONAL EXCHANGE BANK

ALBANY, N.Y., *Feb. 12,* 19

MR. *Chas. W. Hamilton,* Cashier
Merchants National Bank
Boston, Mass.

DEAR SIR, — We credit your account this day for the proceeds of collections as stated below.

Respectfully yours,
L. H. PIERSON, Cashier

YOUR NO.	PAYER	AMOUNT		CHARGES		PROCEEDS	
720	*E. C. Mills*	*800*	*00*	*1*	*00*	*799*	*00*
716	*W. E. Frey*	*900*	*75*	*	**	***	**
692	*C. C. Bond & Co.*	*3750*	*50*	*	**	****	**
710	*W. D. Long & Son*	*37500*	*00*	**	**	*****	**
		42951	*25*	**	**	*****	**

FOREIGN EXCHANGE

Foreign Money

ORAL EXERCISE

1. Repeat the table for English money. (See Appendix page 441); for French money; for German money.

2. What is the value of a pound sterling in United States money? of a franc? of a mark?

3. Express $4866.50 in English money; £100 in United States money. Express $1930 in French money; 1000 fr. in United States money. Express $238 in German money; 10000 M. in United States money.

A pound sterling is commonly thought of as about $5; a shilling or a mark as about 25¢; a penny as about 2¢; a franc or lira as about 20¢; a guilder as about 40¢. In problems 4–6 use these approximations.

4. Express $100 as English money; as German money; as French money; 1500 guilders in United States money.

5. Express as United States money: £15; £8 5*s.*; £25 10*s.*; 100 M.; 1500 M.; 1750 M.; 75 fr.; 350 fr.; 200 fr.

6. A and B while abroad spent 3 wk. in Naples, Italy. If their expenses here averaged 25 lire apiece per day, how much was this in United States money for the 3 wk.?

WRITTEN EXERCISE

1. Express as pounds and decimals of a pound: £25 6*s.*; £150 15*s.*; £200 10*s.* 6*d.*; £300 12*s.* 9*d.*

2. Reduce to United States money: £25 10*s.*; £120 9*s.*

3. Reduce to United States money: 275 M.; 1500 M. 75 pf.; 315 fr.; 725 fr.; £115 10*s.* 6*d.* Reduce $1250 to English money; to French money; to German money.

4. In a recent year the funded debt of the German Empire amounted to 2,733,500,000 M., of which 1,240,000,000 M. bore interest at $3\frac{1}{2}\%$ and 1,493,500,000 M. at 3%. Express in United States money the interest on the funded debt for 1 yr.

The Metric System

437. The **metric system** is a system of measures having a decimal scale of relation. It was invented by France, and is now used in practical business in a large part of the civilized world. It has been authorized by law in Great Britain and the United States, but is not generally used in these countries except in foreign trade and in scientific investigations.

The **principal units** of the system are the **meter** for length, the **liter** for capacity, and the **gram** for weight. Submultiples and multiples of these units are easily learned when the meaning of the prefixes is known. The Latin prefixes, **deci, centi,** and **milli** mean respectively 0.1, 0.01, and 0.001 of the unit. The Greek prefixes **deca, hekto, kilo,** and **myria** mean respectively, 10, 100, 1000, and 10,000 times the unit.

Table of Length

10 **millimeters (mm.)**	= 1 **centimeter (cm.)**	=	.01 meter.
10 **centimeters**	= 1 decimeter (dm.)	=	.1 meter.
10 decimeters	= 1 **meter (m.)**	=	1. meter.
10 **meters**	= 1 dekameter (Dm.)	=	10. meters.
10 dekameters	= 1 hektometer (Hm.)	=	100. meters.
10 hektometers	= 1 **kilometer (Km.)**	=	1000. meters.
10 **kilometers**	= 1 myriameter (Mm.)	=	10,000. meters.

The units in common use are indicated by black-faced type.

Table of Square Measure

100 sq. millimeters	= 1 **sq. centimeter (sq. cm.)**	=	.001 sq. meter.
100 **sq. centimeters**	= 1 sq. decimeter (sq. dm.)	=	.01 sq. meter.
100 sq. decimeters	= 1 **sq. meter (sq. m.)**	=	1. sq. meter = 1 centare.
100 **sq. meters**	= 1 sq. dekameter (sq. Dm.)	=	100. sq. meters = 1 are.
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.)	=	10,000. sq. meters = 1 hectare.
100 sq. hektometers	= 1 **sq. kilometer (sq. Km.)**	=	1,000,000. sq. meters.
100 **sq. kilometers**	= 1 sq. myriameter (sq. Mm.)	=	100,000,000. sq. meters.

The **centare, are** (a.), and **hektare** are common terms in land measurements.

Table of Cubic Measure

1000 cu. millimeters	= **cu. centimeter (cu. cm.)**	=	.000001 cu. m.
1000 **cu. centimeters**	= 1 cu. decimeter (cu. dm.)	=	.001 cu. m.
1000 cu. decimeters	= 1 **cu. meter (cu. m.)**	=	1. cu. m.
1000 **cu. meters**	= 1 cu. dekameter (cu. Dm.)	=	1000. cu. m.
1000 cu. dekameters	= 1 cu. hektometer (cu. Hm.)	=	1,000,000. cu. m.
1000 cu. hektometers	= 1 **cu. kilometer (cu. Km.)**	=	1,000,000,000. cu. m.
1000 **cu. kilometers**	= 1 cu. myriameter (cu. Mm.)	=	1,000,000,000,000. cu. m.

The cubic meter is also called a **stere,** a unit used in measuring wood.

TABLE OF CAPACITY

10 milliliters (ml.)	= 1 **centiliter (cl.)**	=	.01 liter.
10 **centiliters**	= 1 deciliter (dl.)	=	.1 liter.
10 deciliters	= 1 **liter (l.)**	=	1. liter.
10 **liters**	= 1 dekaliter (Dl.)	=	10. liters.
10 dekaliters	= 1 **hektoliter (Hl.)**	=	100. liters.
10 **hektoliters**	= 1 kiloliter (Kl.)	=	1000. liters.

A liter is the same as a cubic decimeter.

TABLE OF WEIGHT

10 **milligrams (mg.)**	= 1 **centigram (cg.)**	=	.01 gram.
10 **centigrams**	= 1 decigram (dg.)	=	.1 gram.
10 decigrams	= 1 **gram (g.)**	=	1. gram.
10 **grams**	= 1 dekagram (Dg.)	=	10. grams.
10 dekagrams	= 1 hektogram (Hg.)	=	100. grams.
10 hektograms	= 1 **kilogram (Kg.)**	=	1000. grams.
10 **kilograms**	= 1 myriagram (Mg.)	=	10,000. grams.
10 myriagrams	= 1 quintal (Q.)	=	100,000. grams.
10 quintals	= 1 **tonneau (T.)**	=	1,000,000. grams.

The tonneau is usually called a **metric ton.**

TABLE OF APPROXIMATE VALUES

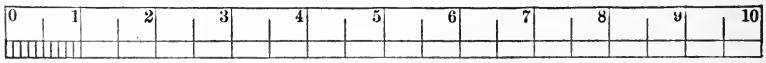
A meter	= $3\frac{1}{4}$ ft. or 1.1 yd.	A stere	= $\frac{3}{11}$ cd.
A kilometer	= $\frac{5}{8}$ mi.	A gram	= $15\frac{1}{2}$ gr.
A square meter	= $1\frac{1}{5}$ sq. rd.	A kilogram	= $2\frac{1}{5}$ lb. av.
An are	= 4 sq. rd.	A liter	= 1 qt.
An hectare	= $2\frac{1}{2}$ A.	An hektoliter	= $2\frac{4}{5}$ bu.
A cubic meter	= 1.3 cu. yd.	A metric ton	= 2200 lb.

ORAL EXERCISE

1. Name the prefix which means 10,000; 0.001; 100; 0.01; 10; 0.1; 1000.

2. Read the following: 2.5 m.; 72 mm.; 95.5 cm.; 302.05 km. Express 475.125 m. in millimeters; in hektometers.

3. Which of the divisions of the following scale are millimeters? centimeters?



1 decimeter

4. A certain tower is 200 m. high; this is approximately how many feet?

5. How many square meters in 1 a.? how many ares in 5 Ha.? in 25 Ha.?

6. How many liters in 1 cu. m.? in 5 cu. m.? Find the cost of 5 Kl. of milk at 5¢ a liter; at 4¢ a liter.

7. Find the length of your schoolroom in meters; the weight of any familiar object in kilograms.

8. Bought 1000 m. of cloth. How many yards was this?

9. An importer bought 1000 l. of liquors at 80¢ a liter. If he sold it at $3.50 per gallon, did he gain or lose, and how much?

10. The distance from Paris to Cologne is 510 Km.; from Cologne to Mainz 150 Km. Express these distances in miles.

WRITTEN EXERCISE

1. At $75 an acre find the cost of 75 Ha. of land.

2. Find the cost of 175.75 m. of lace at 65¢ a meter.

3. How many steres of wood in a pile 12 m. long, 1.5 m. wide, and 3 m. high? How many cords?

4. A merchant bought cloth at $1.14 per meter, including duties. For how much must he sell it per yard to gain $33\frac{1}{3}\%$?

5. I imported 1000 m. of silk (see duties, page 288) at 10 fr. per meter and sold it at $3 per yard. Did I gain or lose and how much, the silk being 1 yd. wide?

6. The distance between two places on a map is 15.5 cm.; this is $\frac{1}{10{,}000}$ of the actual distance. What is the actual distance in miles?

7. C bought cloth at $2 per meter, including duties, and sold it by the yard at a gain of 25%. What was the selling price per yard?

8. The speed rate of a certain express train is 64 Km. an hour; of a certain mail train, 48 Km. an hour. In a journey of 384 Km. what time will be saved by taking the express instead of the mail train.

Foreign Money Orders

438. Small sums are frequently sent from one country to another by means of foreign money orders.

The **international postal money order** and the **foreign express money order** or check are both extensively used for this purpose. These orders are usually drawn payable in the money of the country on which they are issued. They are similar in form to domestic money orders, but are issued on practically the same principle as the ordinary bank draft.

ORAL EXERCISE

1. D in Chicago wishes to send E in Havre, France, 780 fr. At 19.5¢ to the franc, how large an express money order (in francs) can he buy?

2. B in New York wishes to send $120 to C in Leipzig, Germany. At 24¢ to the mark, how large an express money order (in marks) can he buy?

3. At 1% premium find the cost of an international money order, payable in Great Britain, for each of the following amounts: $40; $50; $75; $100; $150; $200.

4. A in Boston bought an international money order for $20 and sent it to a friend in Liverpool, England. At 1% premium, what did the order cost? For how many pounds sterling (approximately) was it issued?

WRITTEN EXERCISE

1. I wish to send $100 to G in Holland. At $40\frac{1}{2}$¢ to the guilder, how large an express money order can I buy?

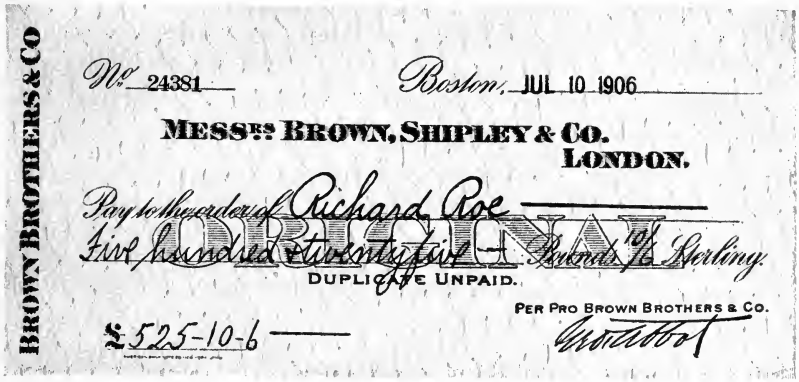
2. I wish to send $50 to a friend in Scotland. At $4.87 to the pound, how large an express money order can I buy?

3. C in Chicago sent D in Geneva an express money order for 256.41 fr. At 19.5¢ to the franc, how much did the order cost C?

4. E in Philadelphia sent F in Naples an international postal money order for 128.21 lira. At 19.5¢ to the lira, how much did the order cost E?

Bills of Exchange

439. Drafts of a person or a bank in one country on a person or a bank in another country are usually called **bills of exchange.**


BROWN BROTHERS & Co

No. 24381 Boston, JUL 10 1906

MESSRS BROWN, SHIPLEY & CO.
LONDON.

Pay to the order of Richard Roe
Five hundred twenty five 1/4 Pounds 10/6 Sterling.

ORIGINAL
DUPLICATE UNPAID.

£525-10-6

PER PRO BROWN BROTHERS & CO.

440. Bills of exchange may be divided into three classes: (1) **bankers' bills,** which are drawn by one banker upon another; (2) **commercial bills,** which are drawn by one merchant upon another; (3) **documentary bills,** which are drawn by one merchant upon another and secured by the assignment and transfer of a bill of lading and policy of insurance covering merchandise on its way to the market.

The foregoing form is a bankers' demand draft or check.

Bankers' bills of exchange are frequently issued in duplicate; that is, in sets of two of like tenor and amount. These bills are sometimes sent by different mails; but more frequently the original is sent and the duplicate is placed on file to be sent in case of necessity. Duplicate bills are so conditioned that the payment of one of them cancels the other. The bankers' sole bill of exchange is also used. This is preferred by many, inasmuch as it may be more easily negotiated by the payee when he resides in a city other than the one drawn upon. Commercial and documentary bills of exchange are usually issued in duplicate.

441. The **mint par of exchange** is the actual value of the pure metal in the monetary unit of one country expressed in terms of another.

The mint par of exchange is determined by dividing the weight of pure gold in the monetary unit of one country by the weight of pure gold in the monetary unit of another. Thus, the United States gold dollar contains 23.22 troy grains of pure gold and the English pound sterling, 113.0016 troy grains. $113.0016 \div 23.22 = 4.8665$. Since there is 4.8665 times as much pure gold in the pound sterling as in the gold dollar, the pound sterling is worth 4.8665 times \$1, or \$4.8665. The mint par of exchange is used mainly in determining the values on which to compute customs duties.

442. The **rate of exchange** is the market value in one country of the bills of exchange on another.

The price paid for bills of exchange fluctuates. When the United States owes Great Britain exactly the same amount that Great Britain owes the United States, the debts between these countries can be paid without the transmission of money, and exchange is at par. But when Great Britain owes the United States a greater amount than the United States owes Great Britain, exchange in the United States is at a discount and in Great Britain at a premium, and *vice versa*. The rates of premium or discount are limited by the cost of shipping gold bullion from one country to another. The cost of shipping gold from New York to London is about $\frac{5}{8}\%$. Therefore, when A in New York owes B in London, and A cannot buy a bill of exchange on London for less than \$4.88$\frac{1}{2}$ to \$4.89, it is cheaper for him to export gold. On the other hand, if D in London owes C in New York and C cannot sell a draft on D for more than \$4.83$\frac{3}{4}$ to \$4.84, it is cheaper for him to import gold. The greater part of exchange is drawn on Great Britain, France, Germany, Holland, Belgium, and Switzerland. Because London is the financial center of the world, probably more foreign exchange is drawn on Great Britain than on all the other countries combined.

443. **Exchange on Great Britain** is usually quoted at the number of dollars to the pound sterling; **exchange on France, Belgium,** and **Switzerland,** at the number of francs to the dollar; **exchange on Germany,** at the number of cents to each four marks; **exchange on Holland,** at the number of cents to each guilder.

The accompanying foreign exchange rates were quoted recently.

	60 Days	Demand
Sterling	4.85$\frac{1}{2}$	4.87$\frac{1}{2}$
Germany, reichsmarks	94$\frac{7}{8}$	95$\frac{3}{8}$
France, francs	5.16$\frac{7}{8}$	5.15
Belgium	5.18$\frac{3}{4}$	5.15$\frac{5}{8}$
Switzerland, francs	5.18$\frac{3}{4}$	5.15$\frac{5}{8}$
Holland, guilders	40	40$\frac{3}{8}$

In Great Britain 3 da. of grace are allowed on all bills drawn payable after sight, but drafts on Great Britain payable at sight or on demand have no grace. There are no days of grace allowed on any drafts drawn on Germany, and nearly all Europe, excepting Holland, where 1 da. of grace is allowed.

WRITTEN EXERCISE

1. Using the foregoing table of quotations, or current quotations clipped from any daily newspaper, find the cost of demand drafts for each of the following amounts:

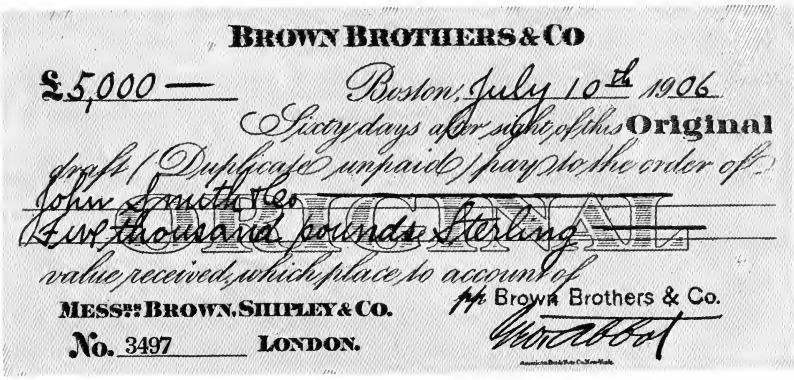
a.	£ 100.	*d.*	160 guilders.	*g.*	200 M.	*j.*	6000 M.
b.	£ 1200.	*e.*	240 guilders.	*h.*	160 M.	*k.*	4000 M.
c.	£ 1800.	*f.*	1200 guilders.	*i.*	2000 M.	*l.*	12000 M.

2. Find the cost of a 60-da. draft for each of the amounts in problem 1.

WRITTEN EXERCISE

1. F. M. Cole & Co., importers, Boston, owe Richard Roe, London, £ 525 10*s*., 6*d*., buy by check the draft illustrated on page 367, and remit it in full of account. If exchange on London is $4.87\frac{1}{2}$, what was the amount of the check?

2. Jordan, Marsh & Co. wish to import a quantity of woolen goods from Bradford, England. They make up an order and inclose in payment the following draft which they buy by check, at $4.85\frac{1}{2}$. What was the amount of the check?


BROWN BROTHERS & CO

£5,000 — Boston, July 10th 1906

Sixty days after sight of this Original

draft (Duplicate unpaid) pay to the order of

John Smith & Co

Five thousand pounds Sterling

value received, which place to account of

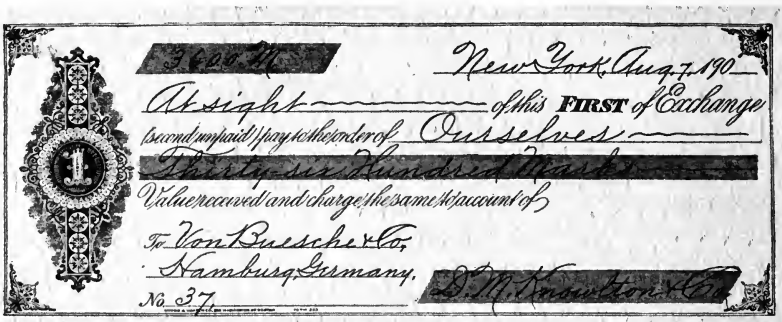
MESSRS. BROWN, SHIPLEY & CO.

No. 3497 LONDON.

Brown Brothers & Co.

3. 45 da. before the draft was due (problem 2) John Smith & Co. sold it to Baring Bros. at 2% discount. How much (in English money) did they receive? Write the indorsements which would appear on the back of the draft.

4. D. M. Knowlton & Co. drew the following **commercial bill of exchange** and sold it to Kidder, Peabody & Co. at 96⅜. How much was received for it?


3600 M.

New York, Aug. 7, 190—

At sight ——— of this FIRST of Exchange

(Second unpaid) pay to the order of Ourselves

Thirty-six Hundred Marks

Value received and charge the same to account of

To Von Buesche & Co.

Hamburg, Germany.

No. 37

D. M. Knowlton & Co.

Commercial bills of exchange are usually drawn by exporters against funds abroad which have accumulated to their credit from sales previously made. The exporter generally waits until the rates of exchange are high and then draws the draft as above.

5. Aug. 1 T. H. Reed & Co., exporters, Minneapolis, Minn., bought through their broker, 24,000 bu. No. 1 wheat at 84¢ per bushel and paid for same by check. What was the amount of the check, the broker's commission being ⅛¢ per bushel?

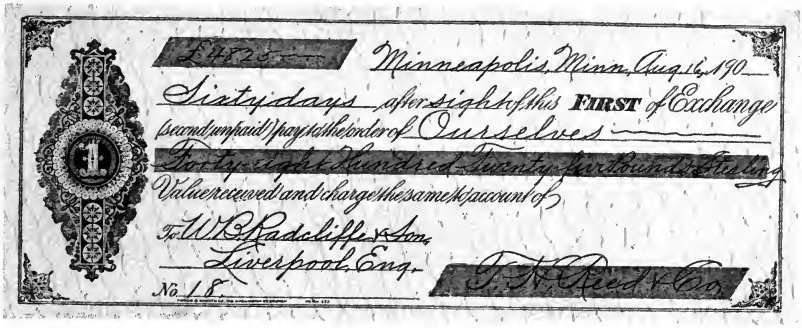
6. Aug. 2 the wheat was delivered and placed with City Elevator for storage. The storage rates were ¾¢ per bushel for the first 10 da. or fraction thereof, and $\frac{1}{40}$¢ per bushel for each additional day thereafter. On Aug. 15 the wheat was withdrawn from the City Elevator and delivered to the Soo Freight Line for shipment to W. B. Radcliffe & Son, Liverpool. What was the amount of the storage bill?

7. The wheat was sold to W. B. Radcliffe & Son at £1 12*s.* 2*d.* per quarter (8 bu. or 480 lb.). Make out the bill under date of Aug. 15.

8. On Aug. 15 a through bill of lading in duplicate was received from the Soo Freight Line. If the through freight rate from Minneapolis to Liverpool was 2*d.* per hundredweight, what was the amount of the freight bill?

9. Aug. 16, upon presentation of the bill of lading to the Western Assurance Co., the goods were insured for 10% more than their billed value and a certificate of insurance issued. What was the amount of the premium, the rate being $1\frac{1}{4}\%$?

10. T. H. Reed & Co., drew the following draft on W. B. Radcliffe & Son and attached it to the bill of lading and certificate of insurance. These documents, which constitute what is called a **documentary bill of exchange**, were then offered for sale and later sold to Kidder, Peabody & Co., at the rate of $\$4.84\frac{1}{8}$ per pound. How much was received for the bill?


£4825 — Minneapolis, Minn. Aug 16, 190—
Sixty days after sight of this FIRST of Exchange
(Second unpaid) pay to the order of Ourselves
Forty-eight Hundred Twenty-five Pounds Sterling
Value received and charge the same to account of
To W. B. Radcliffe & Son
Liverpool, Eng.
No 18.
T. H. Reed & Co.

11. Aug. 17 Kidder, Peabody & Co. sold the draft to American Express Co. at $\$4.84\frac{1}{4}$. If the American Express Co. paid by check, what was the amount of the check?

12. American Express Co. forwarded the bill to Provincial Bank, Liverpool, for collection, and this bank presented the draft to W. B. Radcliffe & Son for acceptance. Sept. 1 the wheat arrived by steamer and as the draft was stamped "Surrender documents only upon payment of draft" W. B. Radcliffe & Son had to pay the draft before they could get the documents or the goods. As the draft has 46 da. yet to run, the bank allowed W. B. Radcliffe & Son 1% discount. What was the amount paid by W. B. Radcliffe & Son?

Such drafts are frequently stamped "Surrender documents upon acceptance of the draft." In such cases the documents would be delivered to the consignee upon the acceptance of the draft, and he could then obtain possession of the goods.

13. What was T. H. Reed & Co.'s net gain or loss on the transactions in problems 5–10?

Letters of Credit and Traveler's Checks

444. A **traveler's letter of credit** is an instrument issued by a banker instructing his correspondents in specified places to pay the holder funds in any amount not exceeding a specified sum.

Brown Brothers & Co.

CIRCULAR LETTER OF CREDIT.

No B/B 13,683. New York, August 21st 1905

Gentlemen:

We beg to introduce to you Mr. Robert Thompson, or Mrs. Katharine Thompson — to whom you will please furnish such funds as either may require up to the aggregate amount of £1,200— Twelve hundred — pounds sterling against demand drafts on MESSRS. BROWN, SHIPLEY & Co. 123 PALL MALL, LONDON, each draft to be plainly marked as drawn under Brown Brothers & Co.'s Letter of Credit No B/B 13,683

We engage that such drafts shall meet with due honor in London, if negotiated on or before June 30th 1906 and request you to buy them at the rate at which you purchase demand drafts on London.

The amount of each draft must be inscribed on the back of this letter, and to this we wish to call your special attention. This letter itself should be cancelled and attached to the final draft drawn.

Please see to it that the drafts be signed in your presence, and carefully compare the signature with the one below. We are Gentlemen,

Your obedient Servants

Brown Brothers.

£1,200—

The signature of
Robert Thompson
Katharine Thompson

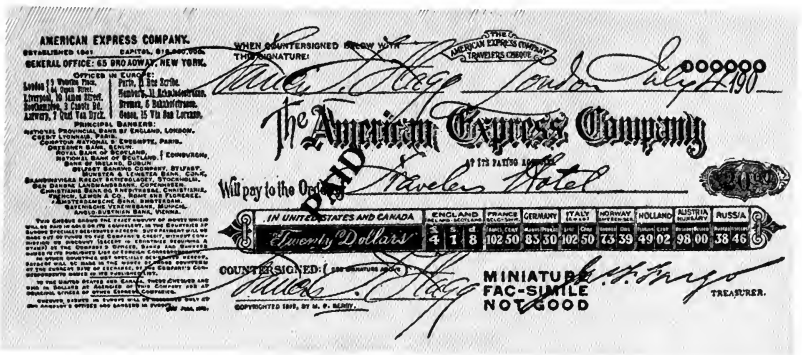
To Messieurs
The Bankers mentioned on the third page of this Letter of Credit.

THIS LETTER TO BE SURRENDERED WITH THE LAST DRAFT HEREUNDER

NOT EXCEEDING POUNDS.

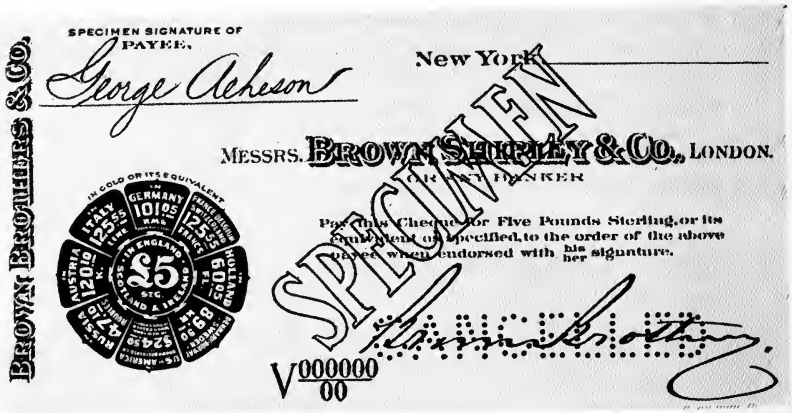
The purchaser of a letter of credit is required to subscribe his name upon the document as a means of identification later on. Other copies of the signature are left and forwarded to the leading foreign banks drawn upon. When the traveler desires funds, he presents his letter to the proper bank at the place in which he is stopping. The letter itself always specifies the banks that will honor the draft. When the letter is presented to a foreign banker for payment, he draws a sight draft on the London banker, which draft the traveler is required to sign. If the signatures on the letter and on the draft are identical, the amount desired is promptly paid and indorsed on the back of the letter. The indorsements on the back of a letter show at all times the balance available for the traveler. The bank making the last payment retains the letter to send to the drawee in London. Letters of credit are usually drawn payable in pounds sterling, but they are paid in the current money of the country in which they are negotiated. Banks usually charge 1% commission for issuing a letter of credit.

445. Another instrument frequently used by travelers is what is called a **traveler's check.**



When a check is purchased, the buyer signs his name in the upper left-hand corner. When he wishes funds, he presents his check to the correspondent of the express company or bank and signs his name either in the upper left-hand corner or on the back of the check. On the form above, he would sign his name in the lower left-hand corner; but on the form on page 374 he would sign his name on the back. The latter form is considered better because it is more difficult to forge another's signature when there is no signature near at hand from which to copy.

The terms of issue are cash for the face amount plus ½% commission.



ORAL EXERCISE

1. At \$4.85 to the pound sterling plus 1% commission, what did the letter of credit on page 372 cost?

2. At the same rate, find the cost of a letter of credit for £500; £1000.

3. At $\frac{1}{2}$% commission, what will be the total cost of 10 checks like the sample on page 373? of 20 checks? of 25 checks?

4. At \$4.85 to the pound plus $\frac{1}{2}$% commission, what was the cost of a traveler's check on page 374? of a book of 10 checks like the sample on page 374?

WRITTEN EXERCISE

1. On the letter of credit, page 372, the following payments are recorded on the back: Aug. 31, £200; Sept. 9, £400; Oct. 15, £250; Nov. 1, £100; Nov. 12, £200. The holder returns to New York on Nov. 20 and presents the letter to Brown Brothers & Co. for the refund. At \$4.85 to the pound, how much will Brown Brothers & Co. pay on the letter?

In this problem it is assumed that Brown Bros. & Co. refund 1% commission on the unused portion of the letter.

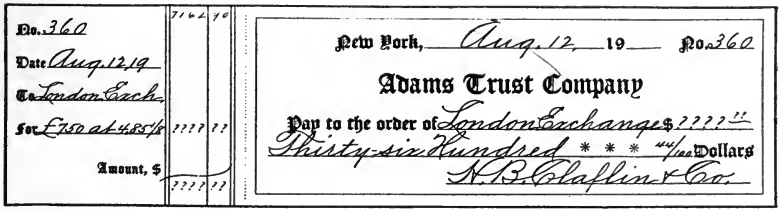
2. At 25¢ per word and 1% of the amount, find the cost of a twenty-one word cable money order from Boston to Paris for 25,000 fr. when exchange is quoted at $5.15\frac{5}{8}$.

Money may be cabled from one country to another on the same principle that it is telegraphed from one part of any country to another part. In a cable message a charge is made for each word in the address of the one to whom it is sent.

WRITTEN REVIEW EXERCISE

1. A broker sold for me a bill on Manchester, England, at $\$4.84\frac{1}{2}$ and charged $\frac{1}{8}\%$ brokerage. What was the face of the bill, if the proceeds were $\$5218.50$?

2. How much remains in the bank to the credit of H. B. Claflin & Co. after the following check was issued?



3. My agent in Brussels, Belgium, purchased for me carpet amounting to 35,000 fr., and his commission was 5%. I remitted him a draft to cover the cost of the carpet and the commission for buying. If exchange was $5.15\frac{5}{8}$, and I paid for the draft by check, what was the amount of the check?

4. My agent in Rotterdam sold for me 525 kegs of tobacco, each containing 50 lb., at $\frac{1}{2}$ guilder per pound, and charged me a commission of $4\frac{1}{2}\%$. I drew on him for the proceeds and sold the draft to a broker at $40\frac{3}{8}$. If the broker charged $\frac{1}{8}\%$ for his services, what did I receive as proceeds of the draft?

EQUATIONS AND CASH BALANCE

CHAPTER XXXI

EQUATION OF ACCOUNTS

ORAL EXERCISE

1. How long will it take \$5 to produce the same interest as \$10 for 10 da.? The use of \$100 for 1 mo. is equivalent to what sum for 2 mo.?

2. If I have the use of \$50 of A's money for 30 da., how much of my money should he have the use of for 15 da. in return for the accommodation?

3. The interest on \$40 for 2 mo. plus the interest on \$40 for 4 mo. is equal to the interest on \$80 for how many months?

4. D owes E \$100; \$50 is due in 2 mo. and the balance in 4 mo. In how many months may the whole be paid without loss to either party?

5. On Apr. 1 I bought a bill of goods amounting to \$200, payable as follows: \$100 in 3 mo. and the balance in 5 mo. In how many months may the whole sum be equitably paid?

6. A owes B \$400 and pays \$200 30 da. before the account is due. How long after the account is due may B have in which to pay the balance?

446. The process of finding the date on which the settlement of an account may be made without loss of interest to either party is called **equation of accounts.**

Sometimes one or more of the items in a personal account are not paid at maturity and the holder of the account suffers a loss; sometimes one or more of the items are paid before maturity and the holder of the account realizes a gain. To equitably adjust these items of loss and gain, accounts are equated. Retail accounts are not often equated; but wholesale and commission accounts are frequently equated, particularly foreign ones.

447. The time that must elapse before several debts, due at different times, may be equitably paid in one sum is called the **average term of credit**; the date on which payment may be equitably made, the **average date of payment**, the **equated date**, or the **due date.**

448. Any assumed date of settlement with which the several dates in the account are compared for the purpose of determining the actual due date is sometimes called the **focal date.**

The **face value** of each item should always be used in equating accounts. Items not subject to a term of credit and interest-bearing notes are worth their face value on the day they are dated. Items subject to a term of credit and non-interest-bearing notes are not worth their face value until maturity.

SIMPLE ACCOUNTS

ORAL EXERCISE

1. If I owe $200 due Jan. 1 and $400 due Jan. 31, when may both accounts be equitably paid in one sum?

SOLUTION. On Jan. 31, there is legally due $600 + $1 (the interest on $200 for 30 da.). Since more than the face of the account is due, the equitable date of settlement is *before* Jan. 31. It will take $600 one third as long as $200 to produce $1 interest. $\frac{1}{3}$ of 30 da. = 10 da. The whole account may therefore be paid 10 da. before Jan. 31, or Jan. 21, without loss to either party.

2. You sold Baker, Taylor & Co. goods as follows: Apr. 20, $600; Apr. 30, $600. How much is legally due on the account Apr. 30? On what day may the whole account, $1200, be paid without interest?

3. When is the following account due by equation?

A. B. COMER

1907										
Sept.	1	To mdse.	300	00						
	21	To mdse.	300	00						

4. Rowland & Hill bought goods of you as follows: Oct. 16, $400; Oct. 31, $800. How much was legally due on the account Oct. 31? On what date can the whole of the account, $1200, be paid without interest?

449. Example. On what date may the total of the following account be paid without interest?

F. M. PRATT & CO.

1907											
Jan.	1	To mdse.	20 da.	30	00						
	9	To mdse.	10 da.	120							
	15	To mdse.	15 da.	150							
	21	To mdse.	10 da.	300							
	26	To mdse.	10 da.	60							

SOLUTION. Take the latest date, Jan. 26, as the focal date. If settlement was made on Jan. 26, the holder of the account might charge interest on each item as shown in the accompanying statement.

The holder loses \$ 0.11 per day as long as the account remains unsettled. If settlement was made Jan. 26, the loss would be \$ 0.99, or 9 days' interest; therefore if the account were settled 9 da. before Jan. 26, the holder would lose nothing.

DATE		AMOUNT	DAYS	INTEREST
Jan.	1	\$ 30	25	\$ .125
	9	120	17	.34
	15	150	11	.275
	21	300	5	.25
	26	60	0	
		\$ 660		\$ .99

The amount of the account = \$ 660.
The interest on \$ 660 for 1 da. = \$ 0.11.
\$ 0.99 ÷ \$ 0.11 = 9, or the number of days.
Jan. 26 − 9 da. = Jan. 17, the equated date.

PROOF. The proof of the problem must show that the interest on the items *dated before Jan. 17*, the equated date, is offset by the discount on the items *dated after Jan. 17*. The following items are dated before Jan. 17 :

DATE	INTEREST PERIOD	ITEM	INTEREST
Jan. 1 to 17	16 da.	\$ 30	\$ .08
9 to 17	8	120	.16
15 to 17	2	150	.05
		Total interest,	\$ .29

The following items are dated after Jan. 17 :

DATE	DISCOUNT PERIOD	ITEM	DISCOUNT
Jan. 17 to 21	4 da.	\$ 300	\$ .20
17 to 26	9	60	.09
		Total discount,	\$ .29

The proof shows that the equated date, Jan. 17, is correct.

Any rate of interest may be used in equating an account. As a matter of convenience, always use 6 %. If items are subject to terms of credit, add the time to the date of the items before beginning to equate.

WRITTEN EXERCISE

In each of the following problems find the equated date and prove the work. Assume that all the dates are in 1907.

1. F. M. Drake, Dr.

Mar. 2, To mdse.	$120.
8, To mdse.	180.
11, To mdse.	60.
17, To mdse.	240.
23, To mdse.	150.

2. Louis M. Allen, Dr.

Apr. 3, To mdse.	$160.
9, To mdse.	250.
13, To mdse.	100.
19, To mdse.	280.
23, To mdse.	420.

3. Geo. M. Barton, Dr.

Aug. 3, To mdse., 60 da.	$360.
6, To mdse., 30 da.	240.
11, To mdse., 30 da.	300.
19, To mdse., 30 da.	60.
24, To mdse., 30 da.	180.

4. Leon H. Hazelton, Dr.

June 6, To mdse.	$200.
9, To mdse.	300.
14, To mdse.	400.
24, To mdse.	600.
27, To mdse.	330.

5. Carter & Co., Dr.

May 5, To mdse.	$180.
12, To mdse.	300.
16, To mdse.	230.
20, To mdse.	270.
23, To mdse.	360.

6. Lamson & Roe Co., Dr.

Dec. 1, To mdse., 3 mo.	$294.20.
10, To mdse., 3 mo.	698.40.
20, To mdse., 60 da.	136.60.
24, To mdse., 60 da.	740.60.
28, To mdse., 60 da.	700.40.

7. Brigham & Co., Dr.

Sept. 4, To mdse., 60 da.	$600.
9, To mdse., 60 da.	450.
12, To mdse., 60 da.	350.
17, To mdse., 60 da.	400.
22, To mdse., 30 da.	500.
30, To mdse., net,	150.

8. D. H. Beckwith & Co. Dr.

Nov. 3, To mdse., 2 mo.	$750.50.
8, To mdse., 2 mo.	432.25.
17, To mdse., net,	275.50.
22, To mdse., 2 mo.	210.50.
25, To mdse., 1 mo.	168.30.
28, To mdse., 1 mo.	240.50.

9. Brown, Kerr & Co., Dr.

Oct. 1, To mdse., 3 mo.	$210.
5, To mdse., 60 da.	840.
13, To mdse., 60 da.	720.
21, To mdse., 60 da.	660.
24, To mdse., 60 da.	540.
31, To mdse., net,	300.

10. D. M. Smith & Co., Dr.

July 3, To mdse.	$420.30.
8, To mdse.	325.70.
11, To mdse.	417.25.
16, To mdse.	186.24.
25, To mdse.	240.60.
29, To mdse.	126.84.

COMPOUND ACCOUNTS

ORAL EXERCISE

1. The following is your account with John D. Foster.

Had no payment been made, when would the account have been due? Since no payment was made until after maturity, you have *lost* the use of $400 for how many days? To offset this loss what should be the date of an interest-bearing note given to cover the balance of the account? Jan. 16 − 30 da. = Dec. ?, the date of an interest-bearing note given to cover the balance of the account.

2. The following is your account with Walter H. Wood.

WALTER H. WOOD

1907					1907				
Apr.	1	To mdse., 30 da.	600	00	Apr.	16	By Cash	300	00

Had no payment been made, when would the account have matured? By the payment recorded you have *gained* the use of $300 for how many days? To offset this gain, you should allow Walter H. Wood to keep the balance of the account how many days after maturity? May 1 + 15 da. = May ?, the date on which the balance is equitably due.

3. May 1 B sold C goods amounting to $500. Terms: 30 da. May 11 C made a payment of $250 on account. On what date is the balance of the account due?

4. Find the date of an interest-bearing note given for the balance of each of the following accounts, assuming that the terms in each case are 30 da.; assuming that the terms are cash.

	NAME	DR.	CR.
a.	H. H. Howard	Jan. 1, $400	Jan. 16, $300
b.	W. H. Lyman & Co.	Jan. 1, $400	Jan. 16, $100
c.	R. H. Delaney & Son	Jan. 1, $400	Jan. 16, $200

450. Examples. 1. Find the equated date for the following :

Irving E. Burdick

1907					1907			
Feb.	1	To mdse.	360 —		Feb.	18	By cash	180 —
	14	" "	240 —			24	" "	180 —

SOLUTION. Take as focal date the *latest* date in the account, Feb. 24.

DEBITS

DATE	ITEMS	INTEREST PERIODS	INTEREST
Feb. 1	$360	23 da.	$1.38
14	240	10	.40
	$600		$1.78

CREDITS

DATE	ITEMS	INTEREST PERIODS	INTEREST
Feb. 18	$180	6 da.	$.18
24	180	0	.00
	$360		$.18

$600 − $360 = $240, the balance of the account. $1.78 − $.18 = $1.60, the interest due the holder of the account on Feb. 24. The interest on $240 for 1 da. = $0.04. $1.60 ÷ $0.04 = 40, the number of days. If the account were settled Feb. 24 there would be *interest for 40 da. due* the holder of it. Therefore the balance of the account is due *40 da. before Feb. 24.* Feb. 24 − 40 da. = Jan. 15, the equated date.

PROOF. To prove the correctness of the above work it is necessary to show that a payment of $240 on Feb. 24 will result in no loss of discount to either party. This may be done by equating the account, using Jan. 15 as the focal date.

DEBITS

DATE	DISCOUNT PERIODS	ITEMS	DISCOUNT
Jan. 15 to Feb. 1	17 da.	$360	$1.02
15 to 14	30	240	1.20
		$600	$2.22

CREDITS

DATE	DISCOUNT PERIODS	ITEMS	DISCOUNT
Jan. 15 to Feb. 18	34 da.	$180	$1.02
15 to 24	40	180	1.20
		$360	$2.22

As there is no difference between the debit discount and the credit discount, the account is proved to be due by equation on Jan. 15, 1907.

2. Find the equated date for the following account:

Watson & Moore

1907					1907			
Apr.	1	To mdse. 60 da.	660 —		May	2	By cash	330 —
	24	" " 30 "	360 —			20	" 30 da. note	300 —
	30	" " 10 "	280 —				with int.	

SOLUTION. Assume May 31 to be the date of settlement.

DEBITS

DATE	TERM OF CREDIT	MATURITY	ITEM	INTEREST PERIOD	INTEREST
Apr. 1	60 da.	May 31	$660	0 da.	$.00
24	30	24	360	7	.42
30	10	10	280	21	.98
			$1300		$1.40

CREDITS

DATE	ITEM	INTEREST PERIOD	INTEREST
May 2	$330	29 da.	$1.595
20	300	11	.55
	$630		$2.145

$1300 − $630 = $670, the balance of the account. $2.145 − $1.40 = $0.745, the interest due Watson & Moore on May 31. The interest on $670 for 1 da. = $0.11⅙. $0.745 ÷ $0.11⅙ = 6.6 or 7, the number of days. If the account were settled May 31, Watson & Moore might deduct $0.75 from the balance of the account; therefore the balance of the account is not due until 7 da. after May 31, or June 7, 1907.

PROOF. The maturity of each item is used in the proof.

DEBITS

DATE	INTEREST PERIOD	ITEM	INTEREST
May 31 to June 7	7 da.	$660	$ .77
24 to 7	14	360	.84
10 to 7	28	280	1.307
		$1300	$2.917

CREDITS

DATE	INTEREST PERIOD	ITEM	INTEREST
May 2 to June 7	36 da.	$330	$1.98
20 to 7	18	300	.90
		$630	$2.88

$2.917 − $2.88 = $0.037; as this is less than the interest on the balance of the account for ½ da. the solution is probably correct.

WRITTEN EXERCISE

Find the equated date and prove the work:

1. FRED L. UPSON

1907					1907				
Jan.	10	To mdse.	360		Jan.	25	By cash	180	
	30	To mdse.	240		Feb.	12	By cash	120	

2. VINTON L. BROWN & CO.

1907					1907				
Mar.	11	To mdse.	420		Mar.	27	By cash	540	
	23	To mdse.	300			31	By cash	180	
Apr.	6	To mdse.	300		Apr.	24	By cash	300	
	20	To mdse.	120						

3. ANSON L. JAMES

1907					1907				
Mar.	8	To mdse., 10 da.	240	60	Mar.	18	By cash	240	60
	12	To mdse., 10 da.	180	30		24	By 30-da. note with interest	300	
	19	To mdse., 10 da.	246						
	29	To mdse., 10 da.	381	24		31	By cash	257	54

The charge under Mar. 8 was paid when due, Mar. 18. Such items may be omitted in equating the account.

4. MACGREGOR & CO.

1907					1907				
Apr.	7	To mdse., 10 da.	127	54	Apr.	17	By cash	127	54
	25	To mdse.	218	99		30	By cash	100	
May	6	To mdse., 10 da.	87	43	May	16	By cash	206	42
	18	To mdse.	150			24	By mdse.	35	20
	27	To mdse., 10 da.	86	45					

5. DAVID J. UPHAM

1907					1907				
June	7	To mdse.	128	50	June	14	By cash	332	50
	10	To mdse.	432	75		25	By mdse.	67	40
	15	To mdse.	78	55		30	By cash	248	60
	21	To mdse.	246	80	July	15	By cash	500	
	29	To mdse.	312	30		28	By mdse.	88	54
July	3	To mdse.	186	40					
	14	To mdse.	66	36					

ACCOUNT SALES

451. The method of averaging an **account sales** is practically the same as the method of averaging an ordinary ledger account. The charges for freight, commission, guaranty, etc., constitute the **debits** and the sales the **credits** of the account.

Commission and guaranty are sometimes considered due on the date of the last sale, and sometimes on the average date of the sales. When goods are sold promptly, commission and guaranty are generally considered due on the date of the last sale; when the sales are large and there are long intervals between them, commission and guaranty are generally considered due on the average due date of the sales. When goods are sold for cash, the account sales is seldom averaged.

WRITTEN EXERCISE

1. Equate the account sales on page 267, assuming that both sales were made on 30 days' time, and that the commission is due on the date of the last sale.

2. Copy and complete the following account sales. Consider the commission as due on the date of the last sale.

Buffalo, N.Y., July 3, 19

Sale for the account of Wentworth, Stratton & Co.
Indianapolis, Ind.

By C. M. Ettenheimer & Sons
Commission Merchants

		Sales					
June	8	295 bbl.	Roller Process Flour, 60 da.	$5.75			**** **
	12	315 "	Old Grist Mill Flour, Cash	5.45			**** **
July	1	305 "	Roller Process Flour, 60 da.	5.67 1/2			**** **
	3	285 "	Old Grist Mill Flour, 30 da.	5.75			**** **
		Charges					
June	12	Freight and cartage			112	50	
	9	Insurance			60		
July	3	Storage			30		
	3	Commission, 5% of sales			***	**	
	*	Net proceeds due by equation			****	**	
					****	**	**** **

CHAPTER XXXII

CASH BALANCE

ORAL EXERCISE

1. When is the balance of the following account due?

JAMES B. SWEENEY

1907					1907				
Jan.	1	To mdse., 30 da.	600	00	Jan.	31	By cash	300	00

2. If no interest is charged on overdue balances, how much will settle the account Feb. 28?

3. If interest at 6% is charged on all amounts not paid at maturity, what is the cash balance of the above account Feb. 28?

4. Assuming that interest is charged on amounts not paid at maturity, find the cash balance of the above account March 30, at 6%.

452. The amount due upon an account at any given time is called the **cash balance** of an account.

When interest is not charged and discount is not allowed, the cash balance is the difference between the sides of an account. When interest is charged and discount is allowed, the cash balance is the difference between the sides of an account after interest has been added to overdue items and discount deducted from items not yet due.

Whether or not interest or discount is charged or allowed on ledger accounts is determined by custom or agreement. It is customary for wholesalers to charge interest on all overdue accounts. As a rule, retailers do not charge interest on the items of an overdue account, but they frequently close personal accounts at the end of the year and charge interest on the balances brought down from the date of closing to the date of settlement.

453. Example. What is the cash balance of the following account Aug. 1, 1907, interest being charged on overdue amounts at the rate of 6%?

George L. Stevens

1907						1907				
June	1	To mdse.	30	900	—	June	30	By cash		600 —
	9	" "	10	450	—	July	10	" "		300 —
	20	" "	10	300	—		18	" "		150 —

SOLUTION.

DEBITS

DATE	TERM OF CREDIT	MATURITY	ITEM	INTEREST PERIOD	INTEREST
June 1	30 da.	July 1	$900	31 da.	$4.65
9	10	June 19	450	43	3.23
20	10	30	300	32	1.60
			$1650		$9.48

CREDITS

DATE	ITEM	INTEREST PERIOD	INTEREST
June 30	$600	32 da.	$3.20
July 10	300	22	1.10
18	150	14	.35
	$1050		$4.65

The debit footing and interest: $1650 + $9.48 = $1659.48
The credit footing and interest: $1050 + $4.65 = $1054.65
The balance due Aug. 1, 1907 = $ 604.83

WRITTEN EXERCISE

1. Find the cash balance due June 1, 1907, on problem 4, page 383, money being worth 5 %.

2. Equate the following account and find the cash balance due Aug. 1, 1907, money being worth $4\frac{1}{2}$ %.

FREDERICK T. LAWRENCE

1907					1907			
May	4	To mdse., 60 da.	1360	May	14	By cash	360	
	17	To mdse., 30 da.	720	June	10	By cash	300	
	26	To mdse., 60 da.	1080		21	By cash	420	

To find the cash balance of an equated account: *Equate the account. Compute the interest on the balance of the account from the equated date to the date of settlement. Add the interest to the balance of the account and the result is the cash balance due.*

3–6. The following is a page from a sales ledger. Find the cash balance due on each account Aug. 1, money being worth 6 %.

O. N. Brown & Co.

1907					1907				
May	4	To mdse. 10 da.	$12	360 —	May	14	By cash	C 65	360 —
	17	" " " "	$16	720 —		19	" "	C 89	300 —
	26	" " " "	$29	1080 —		21	" "	C 103	420 —

F. M. Breton

1907					1907				
June	10	To mdse. 30 da.	$39	800 —	July	10	By cash	C 117	900 —
	30	" " 10 "	$54	750 —		15	" "	C 131	650 —
July	2	" " net	$58	1295 —					

D. N. Trenholm & Son

1907					1907				
May	23	To mdse. net	$31	180 —	May	25	By cash	C 111	180 —
	30	" " 30 da.	$42	900 —	June	29	" "	C 115	450 —
June	6	" " " "	$36	1000 —	July	5	" "	C 117	450 —
	22	" " " "	$45	540 —		6	" 30 da. note with int.	J 16	1000 —
	30	" " " "	$54	200 —					

Morey & Son

1907					1907				
May	11	To mdse. 2 mo.	$12	1200 —	June	25	By cash	C 114	780 —
	25	" " 30 da.	$27	780 —	July	11	" "	C 120	1200 —
June	5	" " " "	$32	1066 —		20	" 30 da. note with int.	J 20	1066 —
	19	" " " "	$42	630 —					
	30	" " " "	$52	420 —					

DIVIDENDS AND INVESTMENTS

CHAPTER XXXIII

STOCKS AND BONDS

STOCKS

454. A **corporation** or **stock company** is an artificial person created by law or under the authority of law for an association of individuals.

Being a mere creature of law a corporation possesses only those properties which its **charter** (the instrument which defines its rights and duties) confers upon it. These are such as are best calculated to effect the object for which it was created. Among the most important are legal immortality and power to act as a single person.

455. The **capital stock** of a corporation is the amount contributed by the stockholders to carry on the business. A **share** is one of the equal parts into which the capital stock is divided.

Shares of $100 are the rule in most companies, although there are some exceptions. Reading Railroad stock, for instance, is divided into shares of $50 each. Mining companies rather more often use other amounts than $100.

456. A **stock certificate** is an instrument signed usually by the president and treasurer of the company specifying that the holder is the owner of a certain number of shares of stock in the corporation. A **stockholder** is a person who owns one or more shares of stock.

Stockholders elect a few of their number to have general control of the company. These constitute a **board of directors,** which is in turn controlled by an executive committee. This executive committee is again controlled by a capitalist, who holds more of the stock than any other person. The average stockholder carries his stock merely for dividends and leaves the burden of the management to the directors.

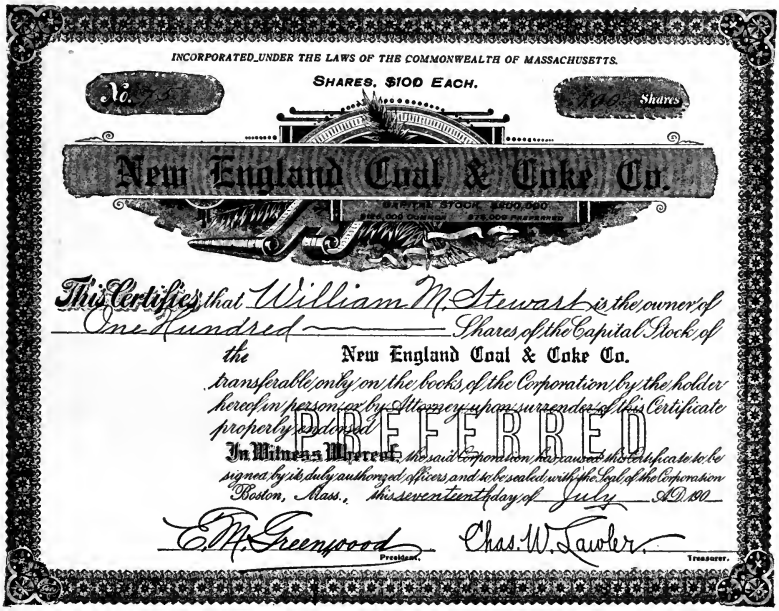
457. A **dividend** is a sum paid to the stockholders out of the net earnings of the company. An **assessment** is a sum levied upon stockholders to make up losses or deficiencies.

The board of directors decide upon the rate of dividend, which is frequently an even per cent on the *face value of the stock* of the corporation. If fractions are used in these rates, they are usually halves or fourths. Any portion of the profits remaining on hand after dividends have been declared is usually credited to **undivided profits**, an account which is opened to receive amounts set aside to be used in an emergency or in any manner which may be determined by the directors. Some corporations, notably national banks, carry a portion of the net profits to a **surplus fund** before declaring dividends. This fund, with certain restrictions, may be used in practically the same manner as the undivided profits account.

Shares of stock may be, and frequently are, non-assessable.

458. The two **leading kinds of stock** are preferred and common.

459. **Preferred stock** is stock which entitles the holder to a fixed rate of dividend which must be paid before anything can be divided among the stockholders.


INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS.

SHARES, $100 EACH.

No. Shares

New England Coal & Coke Co.

CAPITAL STOCK, $400,000

This Certifies that William M. Stewart is the owner of One Hundred Shares of the Capital Stock of the New England Coal & Coke Co.

transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

PREFERRED

In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation Boston, Mass., this seventeenth day of July A.D. 190

E. M. Greenwood President. Chas. W. Lawler Treasurer.

460. Common stock is stock which entitles the owner to an equal proportionate share of the net earnings of the company after the dividends on the preferred stock have been paid.

Preferred stock is usually bought for investment and common stock for speculation. But many companies have no preferred stock, and their common stock is so steadily a dividend payer, and thus so valuable, that it is not considered a speculative commodity. Preferred stock is usually given to secure some obligation of the company or to meet some special demand for capital when common stock may not be disposed of to advantage.


INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS.

SHARES, $100 EACH.

No. 73 — 53 Shares

New England Coal & Coke Co.

CAPITAL STOCK, $500,000
$300,000 COMMON — $200,000 PREFERRED

This Certifies that George W. Putnam is the owner of Fifty Shares of the Capital Stock of the New England Coal & Coke Co.

transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

COMMON

In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation.

Boston, Mass., this Tenth day of July A.D. 1902

E. M. Greenwood, President. Chas. W. Lawler, Treasurer.

461. The **par value** is the face value of stocks; the **market value** is the sum for which the stocks can be sold in the market.

462. If a company is prosperous and pays a higher rate of dividend than the money could earn in other ways, a share may sell for more than its face value. The stock is then said to be **above par**, or **at a premium**. If the company is not prosperous and pays a lower rate of dividend than could be earned on the money in other ways, a share may sell for less than its face value. The stock is then said to be **below par**, or **at a discount**.

463. A **stock broker** is a person who negotiates sales of stock. **Brokerage** is a commission charged by a stock broker for buying and selling securities.

Stocks are usually bought and sold through stock brokers. Brokerage is usually $\frac{1}{8}\%$ of the par value of the stock; a charge is also made both for buying and for selling.

464. When the price of stock is quoted at 97, $118\frac{3}{4}$, $160\frac{1}{2}$, it means that a share whose par value is \$100 can be bought for \$97, \$118.75, \$160.50. If a person buys stock through a broker at $160\frac{1}{2}$, it will cost him $\$160.50 + \$0.12\frac{1}{2}$ brokerage, or $\$160.62\frac{1}{2}$; if he sells stock through a broker for $160\frac{1}{2}$, he will receive as proceeds $\$160.50 - \$0.12\frac{1}{2}$, or $\$160.37\frac{1}{2}$.

Fractions in stock quotations are always halves, fourths, or eighths, and fractions of a share cannot be purchased. The bulk of the transactions in the stock exchange are in 100-share lots, although smaller lots are often purchased for investment.

ORAL EXERCISE

1. Examine the certificate of stock, page 389. What is the name of the company? From whom did the company get its right to carry on business as a corporation?

2. What is the entire capital stock of the company? Into how many shares is this divided? What per cent of the entire stock of the company does the holder of the certificate own?

3. What kind of stock is represented by the certificate? What is the difference between common and preferred stock?

4. What is the par value of each share? If the market value of each share is \$160, what is the certificate worth?

5. What sum must be laid aside to provide for the dividends on the preferred stock of the company, the rate being 6%? How much of this sum will the holder of the certificate receive?

6. Examine the stock certificate, page 390. What part of the stock of the company is common stock?

7. A 5% dividend on the common stock would require how much money from the treasury of the company? Of this sum how much would George W. Putnam receive?

Dividends and Assessments

WRITTEN EXERCISE

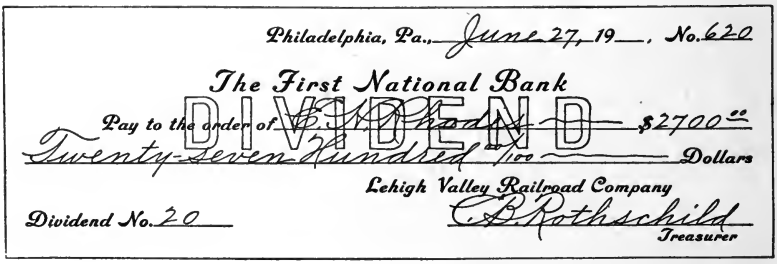
Unless otherwise specified the par value of a share will be understood to be $100.

1. A company with $3,500,000 capital declares an 8 % dividend. What does the holder of 250 shares receive?

2. B holds 450 shares of Pennsylvania Railroad stock. When the company declares a dividend $7\frac{1}{2}$ %, how much will he receive?

3. What annual income is derived from investing $48,000 in Union Pacific Railroad stock at 120, if $2\frac{3}{4}$ % semiannual dividends are declared?

4. E. H. Rhodes holds 600 shares of Lehigh Valley Railroad stock. If he received the following check as his annual dividend, what was the rate?


Philadelphia, Pa., June 27, 19—, No. 620

The First National Bank

Pay to the order of E. H. Rhodes $2700.00

Twenty-seven Hundred 00/100 Dollars

DIVIDEND

Lehigh Valley Railroad Company

Dividend No. 20

A. Rothschild
Treasurer

5. A company with $1,000,000 capital declares quarterly dividends of $1\frac{1}{4}$ %. What are the annual dividends? What is the amount received annually by D, who owns 475 shares?

6. A corporation with a capital of $125,000 loses $2500. What per cent of his stock must each stockholder be assessed to meet this loss? How much will it cost A, who owns 150 shares?

7. A company with a capital of $750,000 declares a semiannual dividend of $3\frac{1}{4}$ %. How much money does it distribute among its stockholders annually? What is the annual income of a man who owns 200 shares?

8. If the Reading Railroad declares a semiannual dividend of $2\frac{1}{2}\%$ on a capital stock of $500,000,000, what amount is annually distributed among the stockholders? What is the annual income to J. P. Morgan from this stock if he owns 7,500,000 shares having a par value of $50 each?

9. During a certain year a manufacturing concern with a capital of $750,000 earns $75,500 above all expenses. It decides to save $15,500 of this for emergencies and to divide the remainder in dividends. What is the rate? What would be the amount of A's dividend check if he owns 125 shares?

10. The capital stock of the Gramercy Finance Company is $1,500,000. The gross earnings of the company for a year are $375,000 and the expenses $215,000. What even per cent of dividend may be declared and what would be the amount of undivided profits if 10% of the net earnings are first set aside as a surplus fund?

11. A railway company has a capital of $3,500,000 and declares dividends semiannually. During the period from Jan. 1 to July 1 of a certain year the net earnings of the company were $191,000. Of this amount 10% is carried to surplus fund. What even rate per cent of dividend may be declared on the balance and how much will be carried to undivided profits?

12. A company with a capital stock of $500,000 gains during a certain year $38,750. It decides to carry $5000 of the profits to surplus fund and to declare an even per cent of dividends on the remainder. What sum was divided among the stockholders, and what sum was carried to undivided profits account? What was the annual income to F from this stock if he owned 500 shares?

13. During a certain year the gross earnings of a railroad having a capital stock of $100,000,000 were $65,150,000, and the operating expenses $45,150,000. If the company declared a semiannual dividend of $3\frac{1}{4}\%$ and carried the balance of the net earnings to undivided profits account, how much was divided among the stockholders? How much was the working capital of the company increased?

14. The capital stock of the National Shawmut Bank is \$3,000,000, and dividends are declared semiannually. The profits of the bank for a certain six months are \$185,750. 10% of this sum is carried to a surplus fund. The directors then vote to declare a dividend of $3\frac{1}{2}$% and carry the balance of the profits to undivided profits account. What amount was carried to surplus fund account? to dividend account? to undivided profits account?

Buying and Selling Stock

465. The following is an abbreviated form of the stock quotations for a certain day on the New York Stock Exchange:

Table of Sales and Range of Prices

Sales	Stocks	Open.	High.	Low.	Clos.	Net Changes
100	Adams Express	243	243	243	243	+ $\frac{1}{2}$
123,500	Amalgamated Copper	81	81$\frac{3}{4}$	79$\frac{5}{8}$	79$\frac{3}{4}$	−1 $\frac{1}{8}$
49,500	Am. Sugar Ref.	151	152	149$\frac{7}{8}$	150	+1
100	Am. Sugar Ref. pfd.	141	141	141	141	+ 1
9,300	Baltimore & Ohio	97$\frac{5}{8}$	98$\frac{1}{4}$	97$\frac{1}{8}$	97$\frac{1}{4}$	− $\frac{1}{4}$
12,900	Canadian Pacific	135$\frac{1}{2}$	135$\frac{5}{8}$	134$\frac{1}{4}$	134$\frac{1}{4}$	− $\frac{1}{2}$
1,600	Delaware & Hudson	188	188	186	186	−1 $\frac{1}{8}$
12,900	Del. Lak. & Western	388$\frac{1}{2}$	395	385	395	+10
1,200	General Electric	181$\frac{1}{4}$	182	181$\frac{1}{2}$	181$\frac{7}{8}$	+ $\frac{3}{8}$
500	Illinois Central	150$\frac{1}{4}$	150$\frac{1}{2}$	149$\frac{1}{8}$	149$\frac{7}{8}$	−1 $\frac{1}{8}$
7,900	Manhattan Elevated	169$\frac{1}{2}$	169$\frac{1}{2}$	167$\frac{3}{4}$	167$\frac{7}{8}$	+1
2,600	New York Central	136$\frac{1}{2}$	136$\frac{3}{4}$	135$\frac{3}{4}$	136	− $\frac{1}{4}$
500	N.Y. N.H. & H.	201	202	201$\frac{3}{4}$	202	+2
63,700	Pennsylvania	137$\frac{1}{4}$	137$\frac{1}{2}$	136$\frac{1}{2}$	136$\frac{5}{8}$	− $\frac{3}{8}$
4,700	Peoples Gas	109$\frac{1}{8}$	109$\frac{3}{8}$	108$\frac{3}{4}$	108$\frac{3}{4}$	+ $\frac{1}{4}$
85,700	Reading	75$\frac{7}{8}$	77	75$\frac{1}{4}$	75$\frac{3}{8}$	− $\frac{1}{8}$
100	Reading pfd.	88$\frac{1}{2}$	88	88$\frac{5}{8}$	88	− $\frac{1}{4}$
33,800	Southern Pacific	68$\frac{3}{8}$	68$\frac{3}{8}$	66$\frac{3}{4}$	66$\frac{3}{4}$	−1 $\frac{1}{2}$
303,700	Union Pacific	129$\frac{1}{8}$	130$\frac{7}{8}$	127$\frac{7}{8}$	128	− $\frac{7}{8}$
	Union Pacific pfd.	97	97$\frac{1}{2}$	97	97$\frac{1}{4}$	+ $\frac{3}{4}$
43,100	United States Steel	27$\frac{7}{8}$	28	27	27	− $\frac{3}{4}$
72,800	United States Steel pfd.	88	88$\frac{5}{8}$	87$\frac{1}{4}$	87$\frac{1}{4}$	− $\frac{3}{8}$
100	Wells Fargo Express	235	235	235	235	+4
400	Western Union	92	92$\frac{3}{8}$	92	92	− $\frac{1}{2}$

In the first column is shown the number of shares of stock sold; in the second, the name of the stock; in the third, fourth, fifth, and sixth respectively, the opening, highest, lowest, and closing prices of the day; in the last, the net charges between the closing price of yesterday and to-day. The plus sign signifies an advance; the minus sign a decline. Thus, on the day given 123,500 shares of Amalgamated Copper stock were sold. The opening price was \$81 per share; the highest price for the day, \$81.75; the lowest, \$79.62$\frac{1}{2}$; the closing, \$79.75, which shows a decline of \$1.12$\frac{1}{2}$ from the closing price of the preceding day.

ORAL EXERCISE

1. Find in the table (page 394) three cases where a quotation both for common stock and for preferred (pfd. stands for preferred) stock is given. Which is worth the more in each case? Under what circumstances may common stock sell for more than preferred stock?

2. What would 100 shares of American Sugar Refinery (common) cost if bought through a broker at the lowest price for the day, brokerage being $\frac{1}{8}\%$?

3. What would the seller of the stock realize on the sale?

SUGGESTION. The seller would receive the price for which it was sold minus the brokerage, $\frac{1}{8}\%$.

4. State the cost, at the opening price in the table, of 100 shares of each of the following stocks, assuming that the transactions take place through a broker who charges $\frac{1}{8}\%$ commission: Baltimore & Ohio; Canadian Pacific; General Electric; Manhattan Elevated; New York Central; Peoples Gas; Wells, Fargo Express; New York, New Haven and Hartford; Illinois Central.

5. At the highest price in the table, state the amount received from the sale of 100 shares of each of the following stocks, assuming that they are sold through a broker who charges $\frac{1}{8}\%$ commission: Southern Pacific; United States Steel (preferred); Western Union Telegraph; Reading (preferred); American Sugar Refinery (common); Pennsylvania; Amalgamated Copper; Union Pacific (preferred); Adams Express; Delaware, Lackawanna and Western; New York, New Haven, and Hartford.

WRITTEN EXERCISE

Find the cost, at the closing price in the table, of 2500 shares of the following stocks, including brokerage:

1. Canadian Pacific.
2. Amalgamated Copper.
3. American Sugar Refinery.
4. Pennsylvania.
5. Manhattan Elevated.
6. United States Steel (pref.).

At the closing price for the day find the amount received from the sale of 3500 shares of the following stocks sold through a broker:

7. Illinois Central.
8. Western Union.
9. Southern Pacific.
10. Delaware and Hudson.
11. Reading.
12. General Electric.
13. Canadian Pacific.
14. Amalgamated Copper.

466. Example. I bought 1000 shares Pennsylvania Railroad stock, at the lowest price in the table, and sold the same at $140\frac{1}{2}$. Allowing for brokerage both for buying and for selling, did I gain or lose, and how much?

SOLUTION. Since I bought through a broker, each share cost me $\$136.50 + \$0.12\frac{1}{2}$, or $\$136.62\frac{1}{2}$; and since I sold through a broker the proceeds of each share sold was $\$140.50 - \$0.12\frac{1}{2}$, or $\$140.37\frac{1}{2}$. $\$140.37\frac{1}{2} - \$136.62\frac{1}{2} = \$3.75$, gained on each share. Since \$3.75 is gained on 1 share, 1000 times \$3.75, or \$3750, is gained on 1000 shares.

$$\begin{array}{r} \$140.37\frac{1}{2} \\ 136.62\frac{1}{2} \\ \hline \$\ 3.75 \\ 1000 \\ \hline \$3750. \end{array}$$

In the following exercise it is understood that all sales and purchases are made through a broker who charges a commission of $\frac{1}{8}\%$ both for buying and for selling.

WRITTEN EXERCISE

Find the gain or loss on 500 shares of each of the following stocks bought at the opening price and sold at the price here given:

1. Pennsylvania, $141\frac{5}{8}$.
2. Western Union, 95.
3. Illinois Central, 157.
4. General Electric, 195.
5. Canadian Pacific, 131.
6. Southern Pacific, $69\frac{1}{8}$.
7. Peoples Gas, $97\frac{1}{8}$.
8. New York Central, 132.
9. Baltimore and Ohio, $98\frac{5}{8}$.
10. Manhattan Elevated, 170.
11. Amalgamated Copper, $84\frac{3}{4}$.
12. United States Steel (pfd.), $90\frac{1}{4}$.

13–24. Find the gain or loss on 1000 shares of each of the above stocks bought at the lowest price and sold at the highest price in the table.

25. F bought 500 shares of Peoples Gas at the opening price in the table and sold it so as to gain \$750. What was the quoted price when he sold it?

26. I bought some Western Union Telegraph stock at the opening price in the table and sold it for $94\frac{1}{2}$. If by so doing I gained $4500, how many shares did I buy?

27. I bought 2500 shares of General Electric at the lowest price in the table, held it a year, received 5% in dividends, and then sold it at $183\frac{3}{4}$. Did I gain or lose, and how much, money being worth $4\frac{1}{2}$%?

28. I gave my broker orders to buy 1500 shares Amalgamated Copper and to sell 2500 shares Canadian Pacific. If he buys at the lowest price in the table and sells at the highest price, what balance will he put to my credit?

29. At the closing price in the table, find the total cost of 500 shares American Sugar Refinery (preferred), 1500 shares General Electric, 1000 shares Manhattan Elevated, 100 shares Peoples Gas, 300 shares Delaware & Hudson, and 500 shares Illinois Central.

BONDS

467. A negotiable **bond** is a very formal promissory note issued by a government, railway, or industrial corporation for borrowed money.

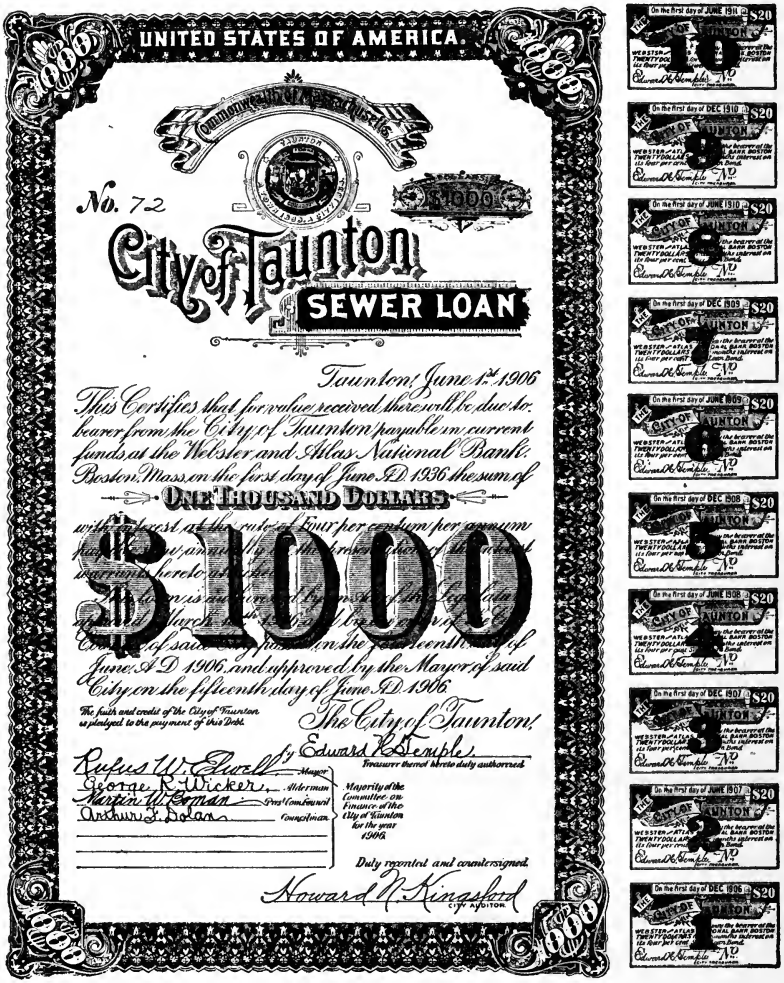
Bonds of corporation are generally issued in a series of like tenor and amount, and bear interest payable annually, semiannually, or quarterly. A bond is usually, though not invariably, issued for each $1000 borrowed.

The bonds of a business corporation are generally secured by a **mortgage** upon its property (an agreement by which the owners of the bonds may sell the property if the bonds and interest are not paid); but the bonds of a government have no security other than the honor of the people.

The bonds of a business corporation with reference to their security are of various kinds; the **first-mortgage bonds** usually stand highest, in that they have a first lien on the property covered by the mortgage. **Second-** and **third-mortgage bonds** take rank after the first. **Debenture bonds** are unsecured promises to pay; they are similar in principle to the unsecured paper of a merchant offered for discount.

468. With reference to the **form of contract** for the payment of principal and interest there are two kinds of bonds: coupon and registered.

469. A **coupon bond** is a bond to which are attached interest notes, or coupons, representing the interest due on the bond at stated periods of payment.



The interest notes may be cut off from the bonds at maturity and the amount of interest which they represent collected through a bank. If these notes are not paid when due, they bear interest at the legal rate.

470. A **registered bond** is a bond which has no separate contract for the payment of the interest. Such a bond must be recorded on the books of the corporation in the name of the holder to whom the interest is sent.

Coupon bonds are usually drawn payable to bearer and may be transferred by delivery or indorsement, or both. Registered bonds are always drawn payable to some designated person and can be transferred only by assignment and registry on the books of the corporation.

471. Bonds issued by the United States are called **government bonds** or **government securities**; bonds issued by a state, **state bonds** or **state securities**; bonds issued by a city, **municipal bonds** or **municipal securities.**

The names of the different government bonds are usually derived from the interest they bear and the time when they mature. Thus, "U. S. 2s, 1930" are United States bonds bearing interest at 2% and maturing in 1930; "U. S. 3s, 1908" are United States bonds bearing 3% interest and maturing in 1908; "U. S. 4s, 1925" are United States bonds bearing 4% interest and maturing in 1925.

472. Bonds, like preferred stock, pay a fixed income.

From the gross earnings of a company the operating expenses are first deducted; from the net earnings are deducted all fixed charges, such as interest on bonds; then the dividends on preferred stock are paid; and finally out of the remainder dividends on the common stock are paid.

ORAL EXERCISE

1. Examine the bond on page 398. With reference to the form of contract, what kind of a bond is it?

2. How many interest notes (coupons) would be attached to the full bond?

3. When was the bond issued? What date (of maturity) should be written on each interest note?

4. What is the face of the bond? What rate of interest does it bear? What sum should be written on each interest note?

5. How may coupon bonds be transferred? registered bonds?

6. If the bond on page 398 was quoted at $105\frac{7}{8}$ when it was purchased, how much did it cost, including $\frac{1}{8}\%$ brokerage? How much did the seller realize on it?

7. Has the city or town in which you live any bonded indebtedness (indebtedness secured by bonds)? If so, what are these bonds called and what rate of interest do they pay?

Buying and Selling Bonds

473. Bonds, like stocks, are usually bought and sold through brokers.

The broker's commission for buying and selling bonds is the same as for buying and selling stocks.

474. The following table is an abbreviated form of the sales, and opening, highest, lowest, and closing prices of bonds traded in on the New York Exchange on a recent date.

Table of Sales and Range of Prices

Sales	Bonds	Open.	High.	Low.	Clos.
$ 8000	Am. Hide & Leather 6s	$97\frac{1}{2}$	$97\frac{1}{2}$	$97\frac{1}{2}$	$97\frac{1}{2}$
241,000	Brooklyn Rapid Transit 4s	$89\frac{1}{4}$	$89\frac{1}{2}$	89	$89\frac{1}{4}$
1,000	Chesapeake & Ohio 6s, 1911	110	110	110	110
571,000	Chicago, Burlington & Quincy 4s	$101\frac{1}{2}$	$101\frac{7}{8}$	$101\frac{3}{8}$	$101\frac{7}{8}$
10,000	Denver & Rio Grande 4s	$100\frac{1}{2}$	$100\frac{7}{8}$	$99\frac{1}{2}$	$99\frac{7}{8}$
71,000	Erie 4s	108	$108\frac{1}{8}$	$107\frac{3}{4}$	$107\frac{3}{4}$
2,000	Illinois Central 4s, 1952	108	108	$107\frac{1}{2}$	108
12,000	Lackawanna Steel 5s	$106\frac{3}{8}$	$106\frac{3}{4}$	$106\frac{3}{8}$	$106\frac{3}{4}$
19,000	Missouri Pacific 4s	95	96	95	95
1,000	National Starch 6s	85	85	85	85
16,000	Northern Pacific 1st mtg. 4s	$105\frac{3}{4}$	$106\frac{1}{8}$	$105\frac{3}{4}$	$105\frac{3}{4}$
5,000	Pennsylvania $4\frac{1}{2}$s	$108\frac{3}{4}$	109	$108\frac{1}{2}$	109
11,000	Seaboard Air Line 4s	$89\frac{7}{8}$	90	$89\frac{1}{8}$	$89\frac{3}{8}$
17,000	Seaboard Air Line 5s	$103\frac{7}{8}$	$104\frac{1}{2}$	104	104
87,000	Union Pacific 1st mtg. 4s	$105\frac{3}{4}$	$105\frac{7}{8}$	$105\frac{5}{8}$	$105\frac{7}{8}$
1,000	United States reg. 4s., 1907	$104\frac{1}{2}$	$104\frac{1}{2}$	$104\frac{1}{2}$	$104\frac{1}{2}$
5,000	United States coupon 4s	$104\frac{1}{4}$	$104\frac{1}{4}$	104	$104\frac{1}{4}$

In the first column is shown the par value of the bonds sold; in the second, the name of the bonds and the interest they bear; in the third, fourth, fifth, and sixth, respectively, the opening, highest, lowest, and closing prices of the day. These prices are quoted at a rate per $100 of par value (amount of the bond). Thus, on the day given $241,000 worth of Brooklyn Rapid Transit bonds bearing 4% interest were sold. The opening price was $89.25 per $100 of par value, the highest price, $89.50, the lowest price, $89, and the closing price, $89.25 per $100 of par value.

475. Example. What is the cost of $50,000 (par value) Chicago, Burlington & Quincy 4% bonds at the highest price quoted in the table (page 400)?

SOLUTION. $100 of par value cost $101\frac{7}{8}$ + $0.12\frac{1}{2}$ brokerage, or $102.
∴ $50,000 of par value will cost 500 times ($50,000 ÷ $100) $102, or $51,000.

WRITTEN EXERCISE

1. What is the cost of $25,000 American Hide and Leather bonds at the opening price in the table?

2. I gave my broker orders to sell $10,000 Chesapeake and Ohio 6% bonds and buy $10,000 National Starch 6% bonds. If he sold at the highest price in the table and bought at the lowest price, what balance should he place to my credit?

3. Find the proceeds from the following sales: $1000 United States 4% registered bonds at the opening price in the table; $5000 United States 4% coupon bonds at the opening price in the table; $75,000 Chicago, Burlington & Quincy 4% bonds at the closing price in the table; $10,000 Erie 4% bonds at the lowest price in the table.

4. June 1, 1907, a certain city borrowed $250,000 with which to build a new high school, and issued $4\frac{1}{2}$% 10-yr. coupon bonds as security. If these bonds sold (through a broker) at $101\frac{5}{8}$, how much was received by the city? If A bought five $1000 bonds, how much did they cost him? If interest is payable semiannually, what date (of maturity) should the last interest note of each bond bear? What will be the amount of each interest note?

5. Find the total cost of the following purchases: $20,000 Erie 4% bonds at the closing price in the table; $2000 Illinois Central 4% bonds at the lowest price in the table; $5000 Lackawanna Steel 5% bonds at the lowest price in the table; $15,000 Missouri Pacific 4% bonds at the opening price in the table; $10,000 Northern Pacific first-mortgage 4% bonds at the lowest price in the table; $3000 Pennsylvania $4\frac{1}{2}$% bonds at the opening price in the table.

INCOMES AND INVESTMENTS

ORAL EXERCISE

1. A bought a 4% United States bond at $119\frac{7}{8}$. Not considering the question of the maturity of the bond, what rate of income did he receive on his investment?

SUGGESTION. $4 is what per cent of $120?

2. B bought 4% bonds having a market value of $79\frac{7}{8}$. What rate per cent of interest did he receive on his investment?

3. C bought $10,000 worth of 6% bonds quoted at $149\frac{7}{8}$, and $10,000 $4\frac{1}{2}$% bonds quoted at $112\frac{3}{8}$. What rate of income did he receive from both investments?

4. D bought a Seaboard Air Line 4% bond at the opening price in the table, also a Seaboard Air Line 5% bond at the opening price in the table. Interest being payable annually in each case, which will yield the larger income?

The **rates of interest** paid on bonds of high class security are very much lower at the present time than they were a generation ago. For example, in 1865 the National Government paid over 7% interest on 30% of its debt, 6% on 10% of its debt, 5% on 55% of its debt, and 4% on 5% of its debt. At the present time about one half of the United States bonds pay only 2% interest; and the average rate of interest paid on railroad bonds is about 4%.

476. As a general rule, a bond of undoubted security which bears a high rate of interest commands so large a premium as to reduce the actual return on the investment to the prevailing rates on other investments of as good security. (See problem 4 in the foregoing exercise.)

477. At the maturity of a bond only its face value and the interest accrued thereon are paid to the holder. In order to command a high price, therefore, a bond must pay a good rate of interest, be perfectly safe, and have a long period to run.

Thus, a 6% third-mortgage bond having 10 yr. to run, or a 6% first-mortgage bond having only 2 yr. to run, might not command as high a price as a 3% bond having a high class security and 30 yr. to run.

WRITTEN EXERCISE

1. A bought a 5% bond quoted at $149\frac{7}{8}$. What rate of interest did he receive on the money invested?

In the above and all similar problems the question of the maturity of the bond is not considered, and it is assumed that the transaction was effected through a broker who charged a commission of $\frac{1}{8}$%.

2. F invested $42,600 in Lackawanna Steel 5% bonds at the opening price in the table (page 400). What was his annual income?

3. Which gives the better income and how much, a 5% bond bought at $79\frac{7}{8}$ or a 6% bond bought at $119\frac{7}{8}$? 6% stock bought at $149\frac{7}{8}$ or $4\frac{1}{2}$% stock bought at $112\frac{3}{8}$?

4. G invested $24,312.50 in Adams Express Company stock at the closing price in the table on page 394. What was his annual income from a $3\frac{3}{4}$% quarterly dividend?

5. H invested $79,025 in Delaware, Lackawanna & Western Railroad stock at the closing price in the table, page 394. What will be his annual income when the dividends are $4\frac{1}{4}$% quarterly?

6. Which would be the more profitable as an investment, to buy Missouri Pacific first-mortgage 4% bonds, due in 1925, at $95\frac{7}{8}$, or Edison Electric Co. first-mortgage 5% bonds, due in 1925, at $104\frac{7}{8}$%?

7. When the current rate of interest is $4\frac{1}{2}$%, what price can I afford to pay for Chesapeake and Ohio 6% first-mortgage bonds? (Give the nearest $\frac{1}{8}$ in your answer.)

8. What sum must be invested in Illinois Central 4% bonds, at the opening price in the table, page 400, to realize an annual income of $2000?

SOLUTION. $4 = the income on $100 of the par value of the bonds.
$2000 ÷ $4 = 500.
∴ bonds having a par value of 500 × $100 must be purchased.
But the cost is $108 + $0.12½ or $108.12½.
∴ 500 × $108.12½ or $54062.50 must be invested.

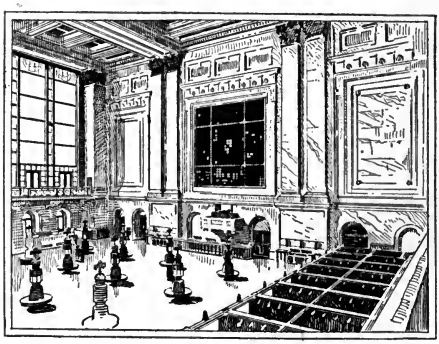
9. What sum must be invested in Missouri Pacific 4% bonds at the closing price in the table, page 400, to realize an annual income of $1500?

10. Using the closing price (with brokerage) in the tables on pages 394 and 400, find which gives the better income and how much: Illinois Central Railroad stock paying 6% dividends or Denver & Rio Grande 4% bonds; General Electric stock paying 8% dividends or Lackawanna Steel 5% bonds; New York, New Haven & Hartford Railroad stock paying 8% dividends or United States 4% coupon bonds; Manhattan Elevated Railroad stock paying 6½% dividends or Erie 4% bonds.

STOCK EXCHANGES

478. Stock exchanges are associations organized for the purpose of creating a regulated market for the buying and selling of stocks and bonds. The **principal stock market** of the United States is the New York Stock Exchange, an unincorporated association of 1100 members.

There are stock exchanges in Chicago, Philadelphia, Boston, and other large cities, but these are local institutions and their dealings are confined to local stocks. The New York Stock Exchange is a national institution which deals with the securities of the whole nation.



INTERIOR OF A STOCK EXCHANGE.

A membership in a stock exchange is called a "**seat.**" The price of a seat varies from $10,000 to $20,000 on local stock exchanges, to from $30,000 to $75,000 on the New York Stock Exchange. A stock exchange always maintains a **uniform rate of commission.** This, as has been seen, is usually ⅛%, or $12.50 per 100 shares; but as every purchase by a broker is usually followed by a sale, the commission on one transaction both ways amounts to ¼%, or $25 per 100 shares.

479. The principal ways in which **stocks** are **bought** and **sold** are as follows: "*cash*," that is, deliverable on the day of sale; "*regular*," that is, deliverable on the day following the sale; "*at three days*," that is, deliverable on the third day of the sale; "*buyer's option*," that is, deliverable at the option of the buyer at any time within the option period (from 4 to 60 days); "*seller's option*," that is, deliverable at the option of the seller any time within the option period.

By far the largest part of the sales are "regular." On "cash," "regular," and "at three days" sales no interest is paid; but on options over three days, interest at the legal rate on the selling price of the stock is paid by the buyer to the seller. To terminate an option of over three days, one day's notice is required.

480. A **margin** is a sum of money deposited with a broker to cover losses which he may sustain on behalf of his principal.

Stocks and bonds are frequently bought and sold on a margin. The process may be illustrated in the following:

June 8, A. M. Greyson deposited with Richard Roe & Co., his brokers, $4160, and instructed them to buy 400 shares of Atchison, Topeka and Santa Fé Railroad stock whenever they could do so at 104. On the same day the stock was bought in accordance with instructions. On June 14, pursuant to instructions, Richard Roe & Co. sold the stock at 107¼ and sent A. M. Greyson the following statement and a check for $5322.56.

New York, June 14, 19—

Mr. A. M. Greyson,
160 Wall Street,

In account current with **RICHARD ROE & CO.**

DATE		AMOUNT	DAYS	INTEREST	DATE		AMOUNT	DAYS	INTEREST
June 8	To 400 shares A.T. & S.F.R.R. at 104	41600 00	6	41 60	June 8	By cash, margin	4160 00	6	4 16
					14	By 400 shares A.T. & S.F.R.R. at 107¼	42900 00		
8	To brokerage	50 00			14	To interest			37 44
14	To brokerage	50 00							
14	To interest	37 44							
14	To check herewith	5322 56							
		47060 00		41 60			47060 00		41 60

By the above transactions A. M. Greyson has gained $1162.56.

The amount of margins demanded by a broker depends upon the character of the stocks traded in. On stocks that have a good market 10% of the market value is usually demanded; on stocks that have little or no market

20% of the market value or more is often required. The broker, of course, pays for the stock in full. In order to do this he is frequently obliged to borrow money from a bank. This he may usually do by depositing (**hypothecating**) stock as security (see page 328).

The speculators on the stock exchange may be divided into two classes: bulls and bears. A **bull** is a speculator who buys stocks in the expectation of selling them at a higher price. A **bear** is a speculator who sells stocks which he does not own, in the expectation that he can buy them at a lower price before the date on which they must be delivered. A bull who has bought is said to be "*long*" of stock; a bear who has sold is said to have sold *short*," or to be "*short*" of stock. A bull works for advancing prices; a bear for declining prices. A bull, when he sells at higher prices, is said to have "*realized*" his profits; when at lower prices, to have "*liquidated*." A bear, when he buys stock, is said to have "*covered*," no matter whether he bought at a gain or at a loss.

WRITTEN EXERCISE

1. On June 25 I purchased through a broker 300 shares of Amalgamated Copper at $87\frac{1}{2}$ b. 3 (buyer's option any time within 3 da.). On June 28 the stock was delivered and, pursuant to my instructions, sold for $89\frac{3}{4}$ cash. Did I gain or lose, and how much?

2. On Apr. 15 my broker purchased for me 500 shares Delaware & Hudson at $172\frac{1}{4}$ regular. On April 16 he sold the same at $174\frac{1}{8}$ cash. What was my gain?

3. On Sept. 15 I bought, through a broker, 250 shares Reading pfd. at $68\frac{3}{8}$ b. 30. On Sept. 25 my broker demanded the stock and, in accordance with my instructions, sold it for $70\frac{1}{2}$ regular. Did I gain or lose, and how much?

4. On Dec. 1 D bought of me through C, his broker, 2000 shares of Missouri Pacific at $99\frac{1}{4}$ s. 60 (seller's option any time within 60 da.). Dec. 17 C, pursuant to my instructions, delivered the stock which he had purchased for me on the previous day at 96 regular. Did I gain or lose, and how much?

5. On June 27 I ordered my broker to sell "short" for me 500 shares Baltimore & Ohio at $105\frac{1}{4}$ s. 30. July 7 the stock declined to $100\frac{1}{8}$. I ordered my broker, at this price, to "cover my short." Did I gain or lose, and how much?

6. Jan. 15 I deposited $4080 with my broker and instructed him to buy 400 shares of Baltimore & Ohio whenever he could do so at 102 regular. On the same day he bought the stock as directed. On Feb. 27 I ordered him to sell, and he did so at $105\frac{3}{4}$ cash. What was my net gain?

7. May 25 a speculator sent his broker a margin of $2000 with which to buy 100 shares Metropolitan Street Railway at 165 regular. The broker invested as directed. On May 27 the stock rose to $170\frac{1}{8}$ and the broker was authorized to sell. If he sold regular at this price, what was the speculator's gain? the broker's commission?

8. What is the balance due on the following account current:

New York, May 25, 19—

Mr. Charles M. Sanborn,
640 Wall St, City

In account current with **RICHARD ROE & CO.**

DATE		AMOUNT	DAYS	INTEREST	DATE		AMOUNT	DAYS	INTEREST
May 10	To 500 sh. Mich. Central at 150	75000 00	??	??? ??	May 10	By cash, margin	7500 00	??	?? ??
10	To brokerage ¼%	??? ??			25	By 500 sh. Mich. Central at 151½	????? ??		
25	To brokerage ¼%	??? ??							

PRODUCE EXCHANGES

481. Just as there are stock exchanges in many of the large cities to supply a regular market for the purchase and sale of securities, so there are **produce exchanges** (also called **boards of trade, chambers of commerce,** etc.) to supply a regulated market for the purchase and sale of farm crops.

Produce exchanges are important accessories of commerce. They promote just and equitable principles of trade; establish and maintain a uniformity in commercial usages; and acquire, preserve, and disseminate valuable business information. The more important produce exchanges, by inspecting and grading all of the important food products, protect the public against fraud and adulterations. The cereals, for example, are

inspected and graded according to their quality. There are usually four grades of wheat and corn, five of barley, and three of oats and rye; No. 1 wheat is the best quality; No. 4, the poorest; etc.

The **principal produce exchange** in the United States is the Chicago Board of Trade. On the floors of this exchange are bought and sold a large part of the cereals and the meat products of the Mississippi Valley and the West. The association thus practically determines the price of these commodities, not only for the United States, but for the world.

Commodities are bought and sold on the exchanges for present or for future delivery. Contracts for present delivery are called "**cash**" contracts; contracts for future delivery, "**futures.**" Speculative trading in grain and cotton is usually in "futures."

The established **brokers' commissions** for transactions on the Chicago Board of Trade are as follows: for grain, $\frac{1}{8}¢$ per bushel; for pork, $2\frac{1}{2}¢$ per barrel; for lard and ribs, $2\frac{1}{2}¢$ per 100 lb.

The **lowest margins** received are: on grain, $20 per 1000 bu.; on pork, $125 per 250 bbl.; on lard, $175 per 250 tierces; on ribs, $125 per 50,000 lb. Of course the margins demanded are sometimes considerably higher than the above figures.

In the accompanying table is shown the opening, highest, lowest, and closing prices of provisions for a certain day on the Chicago Board of Trade.

		Open.	High.	Low.	Close.
Wheat —	July	87	89½	87	88⅝
	Sept.	87	89	87	87⅝
	Dec.	89	89	87⅛	87¾
Corn —	July	57	59	55½	57⅜
	Sept.	54⅝	54⅞	54⅜	54⅝
	Dec.	47¾	47⅞	47½	47⅝
Oats —	July	31⅝	31⅝	31¼	31⅝
	Sept.	29½	29¾	29⅛	29⅝
Pork —	Sept.	12 90	13 05	12 90	13 02
	Oct.	12 95	13 07	12 95	13 05
Lard —	Sept.	7 17	7 20	7 15	7 20
	Oct.	7 20	7 27	7 20	7 27
Ribs —	Sept.	7 75	7 87	7 75	7 87
	Oct.	7 82	7 92	7 82	7 92

"Wheat—July" signifies wheat to be delivered in July; "Wheat—Sept." wheat to be delivered in September; etc. The usual time for future delivery is during the months of May, July, September, and December.

In the following exercise it is assumed that all transactions are effected through a broker who charges the usual commission.

WRITTEN EXERCISE

1. What will it cost me to buy 5000 bu. September wheat at the opening price in the table?

2. C bought 6000 bu. July oats at 27¢ per bushel and sold the same at the closing price in the table. What was his net gain?

3. B bought 15,000 bu. July corn at the lowest price and sold the same at the highest price in the table. Did he gain or lose, and how much? What per cent?

4. G bought 2250 tierces (765,000 lb.) of October lard at $\$7.26\frac{7}{8}$ and sold the same at the closing price in the table. Did he gain or lose, and how much?

5. F bought 1500 bbl. of September pork at the opening price and sold the same at the closing price in the table. Did he gain or lose, and how much?

6. D ordered his broker to sell 5000 bu. September corn and buy 5000 bu. December corn. If the broker sold at the highest price and bought at the lowest price in the table, what amount should he remit D?

7. A broker bought on his own account 10,000 bu. of each, September wheat, December corn, and July oats, at the opening price, and sold the same at the closing price in the table. Did he gain or lose, and how much?

8. H sold "short" 10,000 bu. September wheat at the highest price in the table. September wheat subsequently declined to $85\frac{1}{4}$ and he bought at this price to "cover his short." Did he gain or lose, and how much?

9. June 27 I deposited with my broker a margin of $\$200$ for the purchase of 5000 bu. of September wheat at the lowest price in the table. On July 25 I ordered him to sell. He did so, receiving $89\frac{3}{8}¢$ per bushel. How much should he pay me in settlement?

10. Aug. 5 I deposited with my broker $\$2500$ as a margin for the purchase of 5000 bbl. of October pork at the closing price in the table. On Sept. 2 I ordered him to sell at $\$13.07\frac{1}{2}$. He did so and remitted me a check for the amount due. What was the amount of the check?

CHAPTER XXXIV

LIFE INSURANCE

482. Life insurance companies, like fire insurance companies (page 274), are usually either *stock companies* or *mutual companies*.

There are also **assessment companies** and **fraternal beneficiary associations**. These usually depend upon monthly assessments or "calls" to pay death claims. They are required by law to hold but comparatively little, if anything, as a fund from which to pay losses.

483. Insurance rates are always a certain price per $1000 of insurance. They are payable annually, semiannually, or quarterly in advance.

484. The four leading **kinds of policies** are: ordinary life, limited life, endowment, and term.

485. An **ordinary life policy**, in consideration of premiums to be paid during the life of the insured, guarantees to pay at his death a stated sum of money.

486. A **limited life policy**, in consideration of premiums to be paid for a fixed number of years, guarantees to pay a stated sum of money at the death of the insured.

It will be observed that in an ordinary life policy the premiums are payable during the life of the insured, while in a limited life policy they are payable for a fixed number of years, when the policy becomes *paid up* (no more premiums due). The premium is higher on the latter form of policy.

487. An **endowment policy**, in consideration of premiums paid for a fixed number of years, guarantees to pay a stated sum of money to the insured at a certain time or to the *beneficiary* (one in whose favor the insurance is effected) in case of prior death.

488. A **term policy**, in consideration of premiums paid for a fixed time, guarantees to pay a stated sum of money if the insured dies within the term of insurance.

Thus, a person may insure his life for a limited number of years only. Since the company may never be called upon to pay the insurance, the premiums on these policies are low.

489. The **reserve** is that part of the premiums of a policy, with interest thereon, required by law to be set aside as a fund to be used in payment of the policy when it falls due.

The legal rate of interest on reserve funds varies slightly in different states. The higher the rate of interest, the smaller the reserve required.

490. The **surplus** of an insurance company is the excess of its assets (resources) over its liabilities.

This fund, with certain restrictions, may be used for such purposes as the company deems best. After retaining a surplus large enough to provide for contingencies, companies which issue policies on the **mutual** or **participating plan** divide the remainder of the surplus among such of its policyholders as are entitled to share in it. This is practically a return of an overcharge, but it is usually called the payment of a **dividend**.

491. Dividends may be used: (1) to reduce the next year's premium; (2) to purchase additional insurance, payable when the policy matures; (3) to shorten the time to run.

Dividends may also be left with the company, with the distinct understanding that there shall be no division of the same until the end of a certain period. As the policyholder receives no benefit unless he survives the selected period, it will be seen that the return should be somewhat larger. This plan is called *semi-tontine*, *distribution period*, *accumulated surplus*, *deferred dividend*, etc.

492. If a **policy is discontinued**, the insured may secure an equitable return for the reserve accumulated.

The insured usually has several options in this matter: (1) he may take the **cash value**, or practically all of the reserve value of the policy; (2) he may take a **paid-up policy** for such an amount as its reserve value will purchase; (3) he may take **extended insurance** for the face of the policy for as many years and days as its reserve value will purchase.

ANNUAL PREMIUM RATES FOR AN INSURANCE OF $1000

AGE	ORDINARY LIFE	20-PAYMENT LIFE	15-YEAR ENDOWMENT	20-YEAR ENDOWMENT
25	20.93	30.90	66.57	48.93
30	23.75	33.76	67.27	49.72
35	27.39	37.25	68.26	50.88
40	32.16	41.60	69.76	52.70
50	47.23	54.65	76.20	60.59

ORAL EXERCISE

1. What kind of a policy is that on page 413? Who is the beneficiary? the insured? What is the annual premium?

2. Should the beneficiary die in 1912, to whom would the policy be payable at the death of the insured in 1920?

3. Should the insured die after having paid one annual premium, how much would his heirs receive?

4. If the surplus earnings (dividends) on the policy amount to $1200, at the end of 10 yr., how much cash (see page 414) would the insured receive should he surrender the policy?

5. Should the insured decide to discontinue paying premiums after making five annual payments, how much paid-up insurance, exclusive of the surplus, might he receive?

6. How large a sum may the insured borrow on the policy after ten premiums have been paid?

7. If the company secures interest in advance by deducting it from the amount of the loan, and the insured should borrow $4000 for one year at 5%, what would be the amount of the check which he would receive from the company?

8. Had the insured taken out the policy when he was twenty-five years of age, what would be the annual saving, exclusive of interest, in the cost? How much would he have saved in 15 yr.? in 20 yr.?

9. If the insured should discontinue paying premiums after 5 yr. and take extended insurance, how much would the beneficiary receive should the insured die in 1914? in 1919?

10. If the insured had taken a life policy (see rates, page 411) for the same amount, instead of an endowment policy, and died after having paid ten full premiums, how much less would his insurance have cost, exclusive of dividends and interest?

11. If the insured should pay four full premiums on the policy, take extended insurance, and die 5 yr. later, how much would his beneficiary receive?

12. If the insured discontinues making payments after seven annual premiums had been paid, how much would he get in cash at the end of 20 yr. from date of issue, if living?

THE PENN MUTUAL

Number 1,000,000. Amount $10,000

LIFE INSURANCE COMPANY
OF PHILADELPHIA.

AGE 35
SUM INSURED $10,000
YEARLY PREMIUM $508.50

In Consideration of the Application for this Policy, hereby made a part of this contract,
The Penn Mutual Life Insurance Company of Philadelphia
insures the life of Edward H. Catchpole
of Rochester in the County of Monroe State of New York,
in the sum of Ten Thousand 00/100 Dollars, and promises
to pay at its Home Office, in the City of Philadelphia, unto
Edward H. Catchpole
his executors, administrators, or assigns, the said sum insured on the
tenth day of May in the year nineteen hundred and twenty-seven
or if the said insured should die before that time then to make said payment to
Marjorie P. Catchpole, if she survive him; otherwise
to his
executors, administrators, or assigns, upon receipt of satisfactory proof of the death of the insured, during the continuance in force of this Policy, upon the following conditions, namely:

The payment in advance to the Company, at its Home Office, of the sum of
Five Hundred Eight and 50/100 Dollars, at the date hereof, and of the
annual premium of Five Hundred Eight 50/100 Dollars,
at or before three o'clock P.M., on the tenth day of May
in every year during the continuance of this contract, or until
twenty full years' premiums shall have been paid:

This Policy shall participate annually in the surplus earnings of the Company in accordance with the regulations adopted by the Board of Trustees.

The extended insurance, paid-up insurance, and loan or cash surrender value privileges, benefits, and conditions stated on the second page hereof form a part of this contract as fully as if recited at length over the signatures hereto affixed.

ENDOWMENT IN 20 YEARS Regular

In Witness Whereof, The Penn Mutual Life Insurance Company of Philadelphia has caused this Policy to be signed by its President, Secretary, and Actuary, attested by its Registrar, at its Home Office, in Philadelphia, Pennsylvania, the tenth day of May 1907.

EXAMINED BY
Endt'r No. 3
A.D. Ed. 5, 1907

W. N. Kingsley Secretary. Harry J. Mock President.

Attest: A. J. Greene Registrar. Geo. G. Barnes Actuary.

No. 1367

Table of Extension, Paid-up, and Loan or Cash Values, provided for by the Policy, if no indebtedness exists against it

AT END OF YEAR	TERM OF EXTENSION FOR THIS POLICY	These Values are for $1000 Insurance. For this Policy multiply by 10			
		PURE ENDOWMENT AT END OF EXTENSION	PAID-UP INSURANCE ON SURRENDER	LOAN OR CASH SURRENDER VALUES	
3d	10 Years 279 Days	$	$ 169	$ 85	21
4th	14 " 89 "		224	126	48
5th	15 " "	49	278	169	21
6th	14 " "	131	332	213	46
7th	13 " "	210	384	259	37
8th	12 " "	286	436	306	94
9th	11 " "	359	487	356	27
10th	10 " "	429	538	407	45
11th	9 " "	497	587	456	84
12th	8 " "	562	636	508	08
13th	7 " "	624	684	561	28
14th	6 " "	684	731	616	55
15th	5 " "	742	777	674	00
16th	4 " "	798	823	733	77
17th	3 " "	851	868	796	05
18th	2 " "	902	913	861	01
19th	1 " "	952	956	928	91
20th	" "	Policy Matures		1000	00

Should any indebtedness exist it shall be deducted from the Cash Value of the Policy, and the other values shall be diminished proportionately

WRITTEN EXERCISES

1. If the insured in the foregoing policy should die just before the twelfth payment was due, how much would the estate receive above his total payments?

2. Suppose that the insured in the foregoing policy survives the endowment period, and the surplus earnings of the policy amounted to $3500. What would be the difference between the amount received and the amount paid, not reckoning interest?

3. The insured in the foregoing policy took out a $10,000 20-payment life policy at the same time he procured his endowment policy. The guaranteed cash value on the former was $2557.80 at the end of 10 yr., and the dividends for this term amounted to $83.22 per thousand. If the dividends on the endowment policy for this period amounted to $127.83 per thousand, which would have been the better investment, interest not being considered, and how much?

4. Assuming that the insured in the foregoing policy survived the endowment period and that the dividends which amounted to $350 per thousand were used to add to the value of the policy, how much less would he receive from the company than he would from investing the amount of the premiums in a savings-bank annually for 20 yr. at 4% interest?

5. What will be the first annual premium on a $15,000 ordinary life policy for a man 50 yr. old?

6. On his 25th birthday A took out a 20-yr. endowment policy for $5000; on his 35th birthday, a 15-yr. endowment policy for $6000; on his 40th birthday, a 20-payment life policy for $10,000. He died aged 43 yr., 6 mo. How much more did his heirs receive (dividends excepted) than he had paid the company?

7. B at the age of 25 yr. took out a 20-payment life policy for $5000. He died just before his twentieth payment became due. The company allowed $87.50 per thousand in dividends during this period, and these were used to reduce the annual premium. How much more did his heirs receive than was paid in premiums?

PARTITIVE PROPORTION, PARTNERSHIP, AND STORAGE

CHAPTER XXXV

PARTITIVE PROPORTION AND PARTNERSHIP

PARTITIVE PROPORTION

ORAL EXERCISE

1. A fails in business owing D $500, E $1500, and F $2500. If his resources are $1800, how much can he pay each of his creditors?

2. Two brothers, A and B, are engravers. A can earn $10 per day and B $5 per day. How much can they both earn in a day? What part of this amount can B earn? A?

3. They formed a partnership for one year and agreed to divide the net profits in proportion to the earning capacity of each. If the net profits for the year were $3600, what was the share of each?

4. C invests $3000, B $6000, and A $9000 in a manufacturing plant. The net profits for one year are $3600, and this sum is shared in proportion to the amount of capital invested. What amount does each receive as his share of the net profits?

5. A certain street was paved at a cost of $3000. The property owners on the street were A, who owned 200 ft. frontage, B, who owned 400 ft. frontage, and C, who owned 600 ft. frontage. If the cost of the paving was assessed on the property owners in proportion to the frontage owned, how much did each pay?

493. The process of dividing a number into parts proportional to several given numbers is called **partitive proportion.**

WRITTEN EXERCISE

1. Divide $42,770 among G, H, and I in proportion to $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8}$, respectively.

SUGGESTION. $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8} = \frac{2}{8}$, $\frac{4}{8}$, and $\frac{1}{8}$, respectively. Therefore, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8}$ stand in the same relation to each other as $\frac{2}{8}$, $\frac{4}{8}$, and $\frac{1}{8}$, or as 2, 4, and 1.

2. Divide the simple interest on $72,000 for 1 yr. 7 mo. at $3\frac{1}{2}\%$ among D, E, and F so that D's part will be twice E's part and one half of F's part.

3. An inheritance of $75,000 was divided among 3 sons and 4 daughters, so that each daughter received $\frac{1}{3}$ more than each son. How much did each son receive? each daughter?

4. A, B, and C were partners in a business. A put in $10,000, B $6000, and C $9000. Their net gain for a year was $17,500, shared in proportion to the amount of capital invested. What was each partner's share of the net gain?

PARTNERSHIP

ORAL EXERCISE

1. I invested $500 in a business and during the first year gained $1100. No withdrawals or subsequent investments having been made, what was my present worth at the close of the year?

2. Jan. 1 M invested $7500 in a factory. July 1 he found that his net loss was $1125. What was his present worth July 1, no withdrawals or subsequent investments having been made?

3. Answer problem 1 assuming that there was a withdrawal of $800 made during the year; problem 2 assuming that there was a subsequent investment of $1200 made on Mar. 1.

4. Apr. 1 B commenced business with a cash investment of $1500; Jan. 1 of the next year his present worth was $1875. What was his net gain or loss, no withdrawals or subsequent investments having been made?

5. July 1 D began business investing $25,000; Jan. 1 of the next year his net capital was $23,150. If no withdrawals or subsequent investments were made, did he gain or lose, and how much?

6. Answer problem 4 assuming that there were withdrawals amounting to $1000; problem 5 assuming that there was a subsequent investment of $5000.

7. June 1 F began business with a capital of $1750. During the 6 mo. following he lost $2750. What was the condition of his business Dec. 1?

8. Z began business on July 1 with a capital of $2500. 6 mo. later his net insolvency was found to be $1250. What was his net gain or loss?

9. A's business was insolvent $1250 on Jan. 1. From Jan. 1 to July 1 he gained $1750. What was the condition of his business July 1?

10. G gained $3750 during a certain year. He then found that his net capital was $1250. What was the condition of his business at the beginning of the year?

11. June 30, 1906, C's resources were $7500 and his liabilities $5000. June 30, 1907, his resources were $5000 and his liabilities $7500. What was his net gain or loss during this period?

12. Were the conditions in problem 11 reversed for the year stated, what would be the net gain or loss?

13. What is meant by *resources? liabilities? gain? loss?*

14. What is meant by *net gain? net loss? present worth? net capital? net insolvency?*

15. Read aloud the following, supplying the missing words: The condition of the business at the beginning + the —— —— or − the —— —— = the condition of the business at the close; and conversely, the condition of the business at the close + the —— —— or − the —— —— = the condition of the business at the beginning.

494. A **partnership** is an association of two or more persons who have agreed to combine their labor, property, and skill, or some of them, for the purpose of carrying on a common business and sharing its gains and losses.

Partnerships may be formed by either an oral or a written agreement, and in some cases by implication; but all important partnerships should be entered upon by an agreement in writing which definitely states all of the conditions relating to the business.

495. The members of a partnership are called **partners.**

Partners may be divided into four classes: (1) **Real,** or **ostensible,** those who are known to the world as partners and who in reality are such; (2) **nominal,** those who are known to the world as partners but who have no investment and receive no share of the gain; (3) **dormant,** or **silent,** those who are not known to the world but who nevertheless partake of the benefits of the business and thereby become partners; (4) **limited,** or **special,** those whose liability is limited.

Nominal partners, like real, or ostensible, partners, are liable to third parties for the debts of a business. Dormant partners are liable for the debts of the business as soon as their partnership connections become known to the world.

Ordinarily each partner is liable for all of the debts of the firm, but a special partner's liability is limited usually to the amount which he contributes to the firm's capital.

The method of forming a limited partnership is prescribed by statute. This differs somewhat in the different states. Such a partnership must usually have at least one member whose liability is not limited and who is the manager of the business.

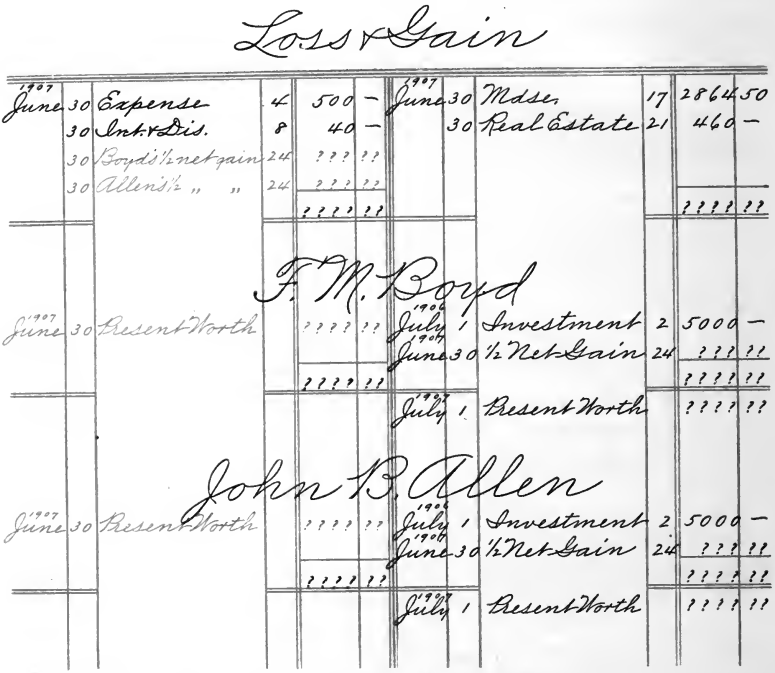
496. The **capital** of a partnership constitutes all the moneys and other properties contributed by the different partners to carry on the business.

Gains and Losses Divided Equally

497. The gains and losses of a business are divided among the partners in accordance with the agreement or contract entered into when the partnership was formed. If the partners invest equal sums and contribute equally in work, the gains are usually divided equally.

WRITTEN EXERCISE

1. Copy and complete the following ledger page:



In solving problems 2–4 use ledger paper as above.

If the student is not familiar with simple accounts, pages 41–47 should be reviewed.

2. Jan. 1, 1907, C. B. Johnson and B. H. Briggs engaged in a partnership business, each investing $3750. July 1, 1907, each partner withdrew $250. Jan. 1, 1908, their losses and gains were as follows:

Losses		Gains	
Expense	$104.75	Merchandise	$628.45
Merchandise Discounts	24.20	Interest and Discount	133.50
Real Estate	250.60	Stocks and Bonds	190.50

What was the present worth of each partner Jan. 1, 1908?

3. A, B, and C were partners for a year. Each invested $9500 and during the continuance of the partnership each withdrew $1000. The losses and gains at closing were as follows:

Losses		Gains	
Merchandise Discounts	$18.90	Merchandise	$4375.80
Expense	650.00	Interest and Discount	90.14

What was the net capital of each at closing?

4. O, P, and Q are partners sharing the gains and losses in equal proportions. O invested $8500, P $8200, and Q $8450. During their first year the gains were as follows: merchandise, $6457.10; real estate, $680.50; interest and discount, $29.90. If the cost of conducting the business was $1920.50, what was the present worth of each partner at the end of the year?

Gains and Losses Irregularly Divided

498. Sometimes the gains are divided according to certain arbitrary fractions which are not in proportion to the amount invested. In such cases the skill of a partner is frequently considered as being equal to a certain amount of capital. In some cases a certain amount is paid the heavier investor before other division of the gains or losses is made. In still other cases, a stated salary is paid to each partner before the gains or losses of the business are divided. This salary varies according to the ability of the several partners or according to the time each devotes to the business.

WRITTEN EXERCISE

1. A and B entered into partnership, each investing $7500. Because of the greater experience of A he was to be credited with $1200 before any other division of the gains or losses. The gains or losses were then to be divided equally. During the first year the gains were as follows: merchandise, $4111.10; real estate, $510. If the losses were $622.80, what was the present worth of each at the end of the year?

2. A and B entered into partnership, A investing $8000 and B $10,000. B doing no work, it was agreed that A should take $2000 from the gains before dividing, and that the net gain or loss should then be shared equally. The gains last year were $8900 and the losses $1400. What was the net gain of each?

3. C, D, and E entered into partnership Jan. 1, each investing $8500. The articles of agreement provided (1) that C should devote all his time to the business and D and E only a portion of their time; (2) that if losses occurred, they should be borne equally; (3) that if gains were realized, C should receive $\frac{1}{2}$ and D and E each $\frac{1}{4}$. During the year the gains were as follows: Merchandise, $8217.10; Stocks and Bonds, $612.50; Interest, $492.92. If the expenses were $2217.80, what was the present worth of each partner at the close of the year?

4. F and G entered into partnership, F investing $5000 and G $7500. Because of the greater skill of F it was agreed that he should be credited with $1500 a year before other division of the gains or losses. Then if losses occurred, F was to bear $\frac{2}{5}$ of them and G $\frac{3}{5}$; but if gains were realized, they were to be divided equally. During the first year the gains of the firm were as follows: Merchandise, $3129.50; Real Estate, $250; Stocks and Bonds, $575; Interest, $130.50. If the cost of conducting the business was $938.48 (exclusive of F's salary), what was each partner's net capital at the close of the year?

5. J, K, and L entered into partnership, J investing $20,000, K $10,000, and L nothing. The articles of agreement provided (1) that the gains or losses should be shared as follows: J, $\frac{3}{5}$, K, $\frac{1}{4}$, L, $\frac{3}{20}$; (2) that the capital should be kept intact; (3) that before any division of the profits was made, J should be credited with an annual salary of $1500. At the end of a year the resources were found to be $65,250 and the liabilities (not including J's salary), $16,750. What was each partner's share of the net gain? After the net gain was credited, what was the net capital of each partner?

Gains and Losses Divided According to Investment

499. Sometimes the gains and losses are divided in proportion to the amount invested; that is, according to the principles of partitive proportion.

500. Example. A and B engaged in business, agreeing to share the gains or bear the losses in proportion to the amount of capital invested. A invested \$2500 and B \$3500. They gained \$1800. What was the share of each?

Solution. \$2500 + \$3500 = \$6000, the total capital. Since the total capital is \$6000 and A put in \$2500, A's share is $\frac{2500}{6000}$, or $\frac{5}{12}$, and B's share is $\frac{3500}{6000}$, or $\frac{7}{12}$. Therefore, A should receive $\frac{5}{12}$ of \$1800, or \$750, and B should receive $\frac{7}{12}$ of \$1800, or \$1050.

ORAL EXERCISE

Find each man's gain or loss in each of the following problems:

	Investment	Gain		Investment	Loss
1.	A, \$3000; B, \$2000	\$500	6.	K, \$2000; L, \$4000	\$120
2.	C, \$1000; D, \$2000	\$150	7.	M, \$1500; N, \$2000	\$700
3.	E, \$1200; F, \$4800	\$1200	8.	O, \$1000; P, \$5000	\$600
4.	G, \$1500; H, \$4500	\$1800	9.	Q, \$1500; R, \$6000	\$750
5.	I, \$1500; J, \$7500	\$1500	10.	S, \$1750; T, \$3500	\$600

WRITTEN EXERCISE

1. A, B, and C invested \$2000, \$3000, and \$5000, respectively, in a wholesale dry goods business. During the first year the net profits were \$4155.80. What was the share of each?

2. D, E, and F invested \$2500, \$3250, and \$3500, respectively, in a manufacturing business. At the close of the first year their profits were found to be \$3774.37. What was the share of each?

3. G, H, and I formed a copartnership, G investing \$3000, H, \$2000, and I, \$1500. During the first six months their net gain was \$1829.10. How much was each man worth after his share of the net gain had been carried to his account?

4. Copy and complete the following statement:

Statement of Conditions, June 30, 1907.

		Losses and Gains					
	Mdse.	total cost	$5932 50				
	Mdse.	unsold	$1658 00				
	Mdse.	cost of sales			4274	50	
	Mdse.	total sales					5582 50
	Gain on Mdse	carried down			????	??	
					5582	50	5582 50
	Gain on Mdse.	brought down					???? ??
	Expense	total outlays			108	—	
	J. M. Robinson's	⅔ of net gain			????	??	
	O. H. Bressee's	⅓ of net gain			????	??	
					1308	—	1308 —
		Resources and Liabilities					
	Mdse.	unsold			1658	—	
	Cash	on hand			2067	—	
	Geo. M. Long	owes on account			369	—	
	Nathan Berry	" " "			665	—	
	F. M. Queen	" " "			466	50	
	Isaac Mann & Co.	owed on account					425 50
	Present Worth	of the firm					???? ??
					5223	50	5223 50
		Verification					
	J. M. Robinson's	net investment	$2400 00				
	" " "	⅔ of net gain	800 00				
	" " "	present worth			????	??	
	O. H. Bressee's	net investment	$1200 00				
	" " "	⅓ of net gain	400 00				
	" " "	present worth			????	??	
	Present Worth	of the firm					???? ??
					4800	—	4800 —

Interest Allowed and Charged

501. The inequalities in investments and withdrawals are frequently adjusted by allowing and charging interest upon same. When interest is allowed and charged on investments and withdrawals, the gains and losses are usually divided equally.

502. Example. June 1, 1907, C. H. Dean and E. D. Snow formed a partnership, C. H. Dean investing $5000 and E. D. Snow $4000. They agreed that the gains and losses should be divided equally, but that, owing to the unequal investments, each partner should be allowed interest at 6 % on all sums invested and charged interest at the same rate on all sums withdrawn, said interest to be adjusted at the time of closing the books. The profits for the first six months were $1050. What was the net capital of each partner after the interest was adjusted and the net gain carried to his account?

C. H. Dean

1906					1906				
Dec.	1	Net Capital	5540	00	June	1	Investment	5000	00
					Dec.	1	Interest	15	00
						1	½ Net Gain	525	00
			5540	00				5540	00
					Dec.	1	Net Capital	5540	00

E. D. Snow

1907					1907				
Dec.	1	Interest	15	00	June	1	Investment	4000	00
	1	Net Capital	4510	00		1	½ Net Gain	525	00
			4525	00				4525	00
					Dec.	1	Net Capital	4510	00

Solution. $5000 in 6 mo. will earn $150 interest. $4000 in 6 mo. will earn $120 interest. $\$150 + \$120 \div 2 = \$135$, the average interest earned. $\$150 - \$135 = \$15$; that is, C. H. Dean's interest is $15 above the average. $\$135 - \$120 = \$15$; that is, E. D. Snow's interest is $15 below the average. Therefore to adjust the interest on the investments, credit C. H. Dean's account $15 and charge E. D. Snow's account $15. ½ of $1050 = $525, the net gain of each. Credit each account with the net gain; then C. H. Dean's net capital is $5540 and E. D. Snow's net capital $4510.

WRITTEN EXERCISE

1. Copy and complete the following statement of conditions:

Statement of Conditions, Dec. 31, 1907.

	Resources			
Cash	on hand	1243 50		
Mdse.	unsold	4365 60		
Expense	items on hand	366 20		
Real Estate	estimated value	6500 —		
Accts. Rec.	total owed by customers	5387 25		
Notes Rec.	notes on hand	2833 25	20695 80	
	Liabilities			
Accts. Pay	total owed to creditors	2436 30		
Notes Pay.	outstanding	1050 —	?????	
	Present Worth of the firm		?????	
	Proprietor's Accounts.			
W. H. Brown's	investment	$10000 00		
" " "	int. due him	150 00		
" " "	net investment	?????		
Willis M. Harts	investment	$5000 00		
" " "	int. due the business	150 00		
" " "	net investment	4850 —		
	Net investment of the firm		?????	
	Net gain of the firm		2209 50	
	Verification			
W. H. Brown's	net investment	$10150 00		
" " "	½ of net gain	1104 75		
" " "	present worth	?????		
Willis M. Hart's	net investment	$4850 00		
" " "	½ of net gain	?????		
" " "	present worth	?????		
	Present Worth of the firm		17209 50	

2. W. H. Burgess and Otis Clapp began business July 1, 1906, the former investing $12,000 and the latter $10,000. They agreed that the gains and losses should be divided equally, but that, because of the inequality in the investments, interest at 6 % should be allowed on investments and charged on withdrawals. July 1, 1907, the firm's resources and liabilities (partners' accounts excluded) were as follows:

Resources		Liabilities	
Cash	$4150.00	Accounts Pay.	$7500.
Accounts Rec.	8150.60	Notes Pay.	4900.
Mdse.	18210.50		
Notes Rec., on hand	4250.00		
Street Railway Stock	3000.00		
Store and Lot	5200.00		
Office Fixtures	500.00		

Make a statement, as in problem 1, showing the present condition of the business.

3. Aug. 1, 1906, F. E. Greene and W. B. Linden formed a partnership for the purpose of carrying on a manufacturing business. F. E. Greene invested $8500 and W. B. Linden, $10,750. It was agreed that interest at 6% should be allowed and charged on investments and withdrawals and that the gains and losses should be divided equally. At the close of the first year the resources and liabilities (partners' accounts excluded) were as follows:

Resources		Liabilities	
Cash	$2355.20	Notes Pay.	$1158.25
Mdse.	5284.85	Accounts owed by the business	2100.00
Notes Rec.	2840.00		
Accounts owing the business	4170.50		
Office Fixtures	450.00		

Feb. 1, 1907, F. E. Greene withdrew $750 and W. B. Linden $600. Make a statement showing the condition of the business at the close of the year.

4. James B. Westfall and John L. Manning began a common business on Sept. 1, 1906, the former investing $14,500 and the latter $13,935. They agreed that interest at 6% should be

allowed and charged on investments and withdrawals, respectively, and that the gains and losses should be divided equally. Sept. 1, 1907, a trial balance of their general ledger was as follows :

	Debits	Credits
James B. Westfall		$14500.00
John L. Manning		13935.00
Cash	$13368.64	
Merchandise	31664.00	20000.00
Office fixtures	510.50	
Horse and wagon	405.00	
Real estate	7000.00	
Expense	445.80	
Collection and exchange	12.20	
Mdse. discounts	58.50	
Accounts receivable	6852.84	
Accounts payable		8864.75
Bills payable		3000.00
Interest and discount		17.73
	$60317.48	$60317.48

The merchandise unsold was found to be worth $13,827.35 ; the real estate, $7500 ; the office fixtures, $500 ; the horses and wagons, $400; and the expense items on hand, $102.50. There was due on the merchandise account for freight, $138.50, and on the expense account for telephone service, $25. Make a statement showing the condition of the business Sept. 1, 1907. (See model, page 431.)

Gains and Losses Divided According to the Average Investment

503. That sum which, invested for a certain period, is equivalent to two or more sums invested for different periods, is called an **average investment.** The gains and losses of a business are sometimes divided in proportion to the average investment.

504. Example. April 1, 1906, A and B formed a partnership and agreed to share the gains or losses according to average net investment. A furnished $10,000 of the capital and

B $7500. July 1 A withdrew $1500 and B $500. Apr. 1, 1907, their net gain was found to be $12,800. What was the net gain of each partner?

SOLUTION

A had in $10,000 for 3 mo., when he withdrew $1500, leaving $8500 for the remaining 9 mo.

B had in $7500 for 3 mo., when he withdrew $500, leaving $7000 for the remaining 9 mo.

A's $10000 for 3 mo. = $30000 for 1 mo.
A's $8500 for 9 mo. = $76500 for 1 mo.
A's average net investment = $106500 for 1 mo.
B's $7500 for 3 mo. = $22500 for 1 mo.
B's $7000 for 9 mo. = $63000 for 1 mo.
B's average net investment = $85500 for 1 mo.

$106500 + $85500 = $192000, the firm's average net investment for 1 mo.

A's share is $\frac{106500}{192000}$, or $\frac{71}{128}$.
B's share is $\frac{85500}{192000}$, or $\frac{57}{128}$.
Therefore, A should receive $\frac{71}{128}$ of $12800, or $7100.
And B should receive $\frac{57}{128}$ of $12800, or $5700.

WRITTEN EXERCISE

1. Apr. 1 R and C formed a partnership for 1 yr., the former investing $4500 and the latter $6000. They agreed to share the gains and losses in proportion to the average net investment. Aug. 1 R invested $1500, and C withdrew $1000. On closing the books at the end of the year the net loss was found to be $1290. What was each partner's present worth after his account was charged with his share of the net loss?

2. June 1, 1906, E and F formed a copartnership for the purpose of carrying on a real estate business. E invested $25,000 and F $15,000. They agreed to share the gains and losses in proportion to the average net investment. Sept. 1, 1906, E withdrew $1000 and F $500. Dec. 1, 1906, each withdrew $1000. Mar. 1, 1907, F invested $5000. June 1, 1907, the partnership was dissolved. After all resources were converted into cash and all liabilities to outside parties paid, the amount of cash in bank was $50,890. What amount was due each partner?

WRITTEN REVIEW EXERCISE

1. Apr. 1, 1907, W. L. Cutter and O. M. Woodward formed a copartnership for the purpose of carrying on a dry goods business. W. L. Cutter invested $20,500 and O. M. Woodward $18,500. They agreed to allow interest at 6% on investments, charge interest at the same rate on withdrawals, and divide the gains and losses equally. July 1, 1907, W. L. Cutter withdrew $500. Oct. 1 O. M. Woodward withdrew $1000 and W. L. Cutter $750. At the close of the year the resources and liabilities, exclusive of partners' accounts, were as follows:

Resources		Liabilities	
Cash in bank	$2130.60	Accounts owed by the business	$7260.00
Stocks and bonds on hand	6450.00	Notes payable unredeemed	1200.00
Goods in stock	16095.00		
Notes receivable on hand	6150.00		
Office fixtures on hand	500.00		
Accounts owing the business	12260.52		

Make a statement showing the condition of the business Apr. 1. 1908.

2. July 1, 1906, A. B. Curtis and B. H. Barton formed a partnership and invested $7500, of which A. B. Curtis furnished $\frac{3}{5}$ and B. H. Barton, $\frac{2}{5}$. Jan. 30, 1907, their resources were as follows: merchandise, unsold, $2172.70; cash on hand, $2823.96; real estate on hand, $3100; account against James Noble, $840.10; account against A. H. Cook & Co., $1156.84. On the same date their liabilities were as follows: account in favor of D. M. Frost & Co., $218.60; account in favor of J. B. Neal & Co., $385. During the year the merchandise bought cost $6807.50 and the sales aggregated $7154.90. The cost of carrying on the business was $530.10. Make a statement (see page 424) showing the present condition of the business. Divide the net gain in proportion to the investments.

3. Copy and complete the following statement of conditions:

Statement of Conditions, July 31, 1907.

	Losses & Gains					
Mdse.	total cost	$29078.12				
Mdse.	unsold	$18078.12				
Mdse.	cost of sales		????.??			
Mdse.	total sales				20150.50	
Gain on Mdse.	carried down		????.??			
			20150.50		20150.50	
Gain on Mdse.	brought down				????.??	
Wamsutta Mills Stock	estimated value	$18725.-				
Wamsutta Mills Stock	cost	$15000.-			3725.-	
Expense	cost		41.50			
C. N. Palmer's	⅓ net gain		????.??			
D. O. Mills's	⅓ " "		????.??			
M. W. Newbury's	⅓ " "		????.??			
			12874.62		12874.62	
	Resources & Liabilities					
Mdse.	unsold		18078.10			
Cash	on hand		12271.52			
Accts. Rec.	owing the business		24800.-			
Wamsutta Mills Stock	estimated value		18725.-			
Notes Rec.	on hand		5000.-			
Accts. Pay.	owed by the business				10150.-	
Present Worth	of the firm				????.??	
			78874.62		78874.62	
	Verification					
C. N. Palmer's	investment	$25000.-				
" " "	interest due him	300.-				
" " "	⅓ of net gain	2908.21	28208.21			
D. O. Mills's	investment	$20000.-				
" " "	⅓ of net gain	2908.21	22908.21			
M. W. Newbury's	investment	$15000.-				
" " "	int. due the business	300.-				
" " "	net investment	14700.-				
" " "	⅓ of net gain	2908.21	17608.20			
Present Worth	of the firm				????.??	
			68724.62		68724.62	

4. Jan. 1, 1906, C. H. Smith and W. W. Osgoodby formed a copartnership for the purpose of carrying on a real estate business. C. H. Smith invested $15,000 and W. W. Osgoodby $10,000. They agreed to share the gains and losses in proportion to the average net investment. July 1, 1906, C. H. Smith withdrew $1000 and W. W. Osgoodby $750. On closing the books at the end of the year the net gain was found to be $8685. What was each partner's present worth after his account was credited with his share of the net gain?

5. Frank M. Congdon, E. H. Robinson, and O. B. Moulton are partners in a dry goods house under the firm name of E. H. Robinson & Co. On commencing business Aug. 1, 1907, Frank M. Congdon invested $17,500, E. H. Robinson $20,000, and O. B. Moulton $12,000. The articles of agreement provided: (1) that each partner should be allowed interest at 6% on investments and charged interest at the same rate on withdrawals; (2) that because of special skill and experience Frank M. Congdon should be credited $1500 before any other division of the gains and losses; (3) that then the gains should be divided equally. Aug. 1, 1908, the results of the year's business were as follows: cost of merchandise purchased, $81,240; value of merchandise on hand, $14,280.95; sales of merchandise, $78,756; cost of real estate, $18,000; cost of permanent improvements on real estate, $1200; present estimated value of real estate, $25,000; notes in favor of the firm, $11,500; interest accrued on these notes, $112; cost and present value of horses and wagons, $1250; general expenses for the year (exclusive of the amount due Congdon), $1800; traveling expenses for the year, $1200; accounts owing the firm, $20,160.90; cash on hand, $19,033.10; mortgage on the firm's real estate, $12,000; interest accrued on the mortgage, $480; notes outstanding, $3500; accounts owed by the firm, $11,260. Show in proper statements the financial condition of the partners.

CHAPTER XXXVI

STORAGE

SIMPLE STORAGE

ORAL EXERCISE

1. I stored my piano in a warehouse from June 16 to October 1 at $1 per month or fraction thereof. What sum must I pay in settlement?

2. I rented a room in a storage warehouse from Sept. 1 to Dec. 18 at $6.50 per month or fraction thereof. What amount did I have to pay?

3. What must I pay for the storage of 5000 bu. of wheat stored from Dec. 3 to Apr. 15 at 4¢ per bushel per month or fraction thereof? for the storage of 10,000 bu. of corn stored from Dec. 1 to Mar. 1 at $3\frac{1}{2}$¢ per bushel per month?

505. **Storage** is a charge made for storing goods in a warehouse.

506. The **term of storage** is the period of time for which a certain rate is charged.

The term of storage is usually, though not invariably, 30 da.; and in estimating charges, a part of a term is counted the same as a full term.

507. The **rates of storage** are sometimes fixed by an agreement between the contracting parties, sometimes by boards of trade, chambers of commerce, or associations of warehousemen, and sometimes by legislative enactment.

508. **Simple storage** is storage estimated at the time of the withdrawal of the goods from the warehouse.

ORAL EXERCISE

1. Verify the following storage bill:

Boston, Feb. 1, 19—

Mr. D. H. Sutherland.
215 Causeway St.

To Quincy Market Cold Storage and Warehouse Co., Dr.
Main Office, 133 Commercial Street

FOR STORAGE

DATE RECEIVED	QUANTITY	MERCHANDISE	STORAGE LOT NO.	DATE DELIVERED	QUANTITY	MO.	RATE	AMOUNT
May 14	490 cs.	Eggs	4546	Sept. 4	140 cs.	4	40¢	56 —
				Nov. 26	200 "	7	70¢	140 —
				Jan. 3	150 "	8	80¢	120 — 316 —

2. When were the eggs received for storage? If there are 30 doz. in a case, how many dozen were received?

3. Suppose the rate in the bill were 10¢ per case per month or fraction thereof for the first 3 mo., and 5¢ per case per month after the first 3 mo. What would this rate be for 4 mo.? for 7 mo.? for 9 mo.? for 10 mo.? for 11 mo.?

4. Using the rate in the bill, find the storage on 150 cs. eggs stored from July 1 to Jan. 14; on 500 cs. eggs stored from July 3 to June 14; on 350 cs. eggs stored from June 14 to Mar. 4; on 12,000 doz. eggs stored from June 14 to Nov. 18.

5. The storage rate on poultry is $\frac{1}{4}$¢ per pound per month. Find the storage on 1000 lb. from Jan. 10 to Feb. 6; on 800 lb. from Jan. 10 to Feb. 18; on 1200 lb. from Jan. 10 to May 27; on 1600 lb. from Jan. 10 to July 3.

6. In a certain warehouse the rate of storage on cheese is 8¢ per 100 lb., for each month or fraction thereof. At that rate find the storage on 1000 lb. cheese from May 3 to July 15; on 20,000 lb. from May 3 to Aug. 26; on 7500 lb. from May 3 to Sept. 12; on 10,000 lb. from May 3 to Oct. 6; on 5 T. from June 15 to Oct. 28; on 10 T. from June 15 to Nov. 17.

509. Example. The following memorandum of flour stored for you by the Central Storage Co.: received Nov. 1, 2000 bbl., and Nov. 16, 3000 bbl.; delivered Nov. 8, 1000 bbl., and Dec. 5, 4000 bbl. If the rate of storage was 5¢ per barrel per month or fraction thereof, what was the bill to render?

SOLUTION

RECEIPTS AND DELIVERIES	TERM	RATE	STORAGE
Nov. 1, received 2000 bbl.			
Nov. 8, delivered 1000 bbl., which were in storage	7 da.	5¢	$ 50
1000 bbl., balance in storage			
Nov. 16, received 3000 bbl.			
4000 bbl., balance in storage			
Dec. 5, delivered 4000 bbl., 1000 of which were in storage	34 da.	10¢	100
3000 of which were in storage	19 da.	5¢	150
Total storage,			$ 300

WRITTEN EXERCISE

1. In a certain warehouse the storage charges on flour are 3¢ per barrel per month or fraction thereof. Nov. 1, I stored 500 bbl.; Dec. 1, I withdrew 100 bbl.; Jan. 1, I stored 600 bbl.; Mar. 1, I withdrew 1000 bbl. What was the storage on the first withdrawal? 400 bbl. of the second withdrawal was in storage for how many months? What was the total storage due Mar. 1?

2. How much is due on the following account?

Boston, Mass., Jan. 10, 19

Received from C. W. Washburn & Co.,
210 Atlantic Ave.

Article Poultry — Lot No. 4888

bkt. 27 bx. lb. — Weight 5400 lb.

Room 6 — Section 4 — Rate ¼¢ per lb. per mo.

DELIVERIES AND CHARGES									CREDITS			
DATE		QUANTITY			MONTHS	RATE	AMOUNT		AMOUNT		DATE	REMARKS
Feb.	6	5	cs.	1000#	1	¼¢	?	??	11	50	Feb. 18	Cash
	18	9	"	1800#	2	½¢	?	??				
May	27	6	"	1200#	?	?¢	??	??				
July	5	7	"	1400#	?	?¢	?	??				

3. The following is a memorandum of apples stored by you for T. B. Welch & Co. : received Nov. 28, 5000 bbl., Dec. 15, 1000 bbl., and Dec. 18, 3000 bbl.; delivered Dec. 28, 2000 bbl., Feb. 1, 1000 bbl., and Feb. 10, 6000 bbl. Render a bill for the storage, charges being 5¢ per barrel per month or fraction thereof.

4. Copy and complete the following bill :

Boston, Jan. 30, 19—

Messrs. John Smith & Co.,
1286 Boylston St.

To EASTERN COLD STORAGE CO., Dr.
28 to 44 North Street

FOR STORAGE

LOT	DATE IN	DATE OUT	NO. MONTH	ARTICLE	WEIGHT	RATE PER 100 LB.	EXTENSION	AMOUNT
7592	19 Jan. 10	Feb. 6	1	200 lb. Butter	10000#	18¢		
		Apr. 9	3	100 " "	5000#	54¢		
		May 19	5	350 " "	17500#	90¢		
		June 3	5	150 " "	7500#	90¢		

AVERAGE STORAGE

510. When there are frequent receipts and deliveries of goods, it is customary for some warehouses to average the time and charge a certain rate per month of thirty days. The process is called **average storage.**

511. Example. The following is a memorandum of the receipts and deliveries of flour stored by the Eastern Storage Co. for A. M. Briggs & Co.: received Apr. 10, 2000 bbl., and Apr. 30, 3000 bbl.; delivered May 8, 1000 bbl., and June 9, 4000 bbl. The storage charge being $4\frac{1}{2}$¢ per barrel per term of 30 da. average storage, what was the amount of the bill to render?

SOLUTION. The solution of this problem is clearly shown in the following statement of account:

ACCOUNT OF FLOUR RECEIVED AND DELIVERED BY
EASTERN STORAGE CO.,
For A. M. BRIGGS & CO.

DATE		RECEIPTS	DELIVERIES	BALANCE	TIME IN STORAGE	QUANTITY IN STORAGE FOR 1 DA.
1907						
Apr.	10	2000 bbl.		2000 bbl.	20 da.	40000 bbl.
	30	3000 bbl.		5000 bbl.	8 da.	40000 bbl.
May	8		1000 bbl.	4000 bbl.	32 da.	128000 bbl.
June	9		4000 bbl.	0000 bbl.	00 da.	00000 bbl.
		5000 bbl.	5000 bbl.			30)208000 bbl.

Average storage for 1 mo. = $6933\frac{1}{3}$ bbl.

$6933\frac{1}{3}$ bbl. at $4\frac{1}{2}¢ = \$312$, the amount of the bill to render.

WRITTEN EXERCISE

1. The Quincy Storage and Warehouse Co. received and delivered on account of Boynton Travers & Co. sundry barrels of apples as follows: received Dec. 1, 1906, 1000 bbl., Dec. 26, 2000 bbl.; delivered Feb. 1, 500 bbl., Mar. 1, 1000 bbl., Mar. 15, 1100 bbl., Mar. 31, 400 bbl. If the charges were 6¢ per barrel per term of 30 da. average storage, what was the amount of the bill to render?

2. The Central Storage Warehouse Co. received and delivered on account of A. S. Osborn & Co. sundry bushels of wheat as follows: received Oct. 1, 17,600 bu., Nov. 15, 3600 bu., Dec. 18, 4200 bu., Dec. 27, 4320 bu.; delivered Oct. 31, 10,000 bu., Dec. 4, 10,720 bu., Dec. 19, 4000 bu., Dec. 28, 5000 bu. If the charges were $1\frac{3}{4}¢$ per bushel per term of 30 da. average storage, what was the amount of the bill to render?

3. Metropolitan Storage Co. received and delivered on account of Chas. B. Sherman sundry barrels of flour as follows: received Nov. 15, 1906, 1800 bbl., Nov. 30, 1000 bbl., Dec. 18, 600 bbl., Jan. 30, 3000 bbl.; delivered Dec. 1, 1000 bbl., Dec. 31, 1900 bbl., Jan. 31, 600 bbl., Feb. 5, 600 bbl., Apr. 30, 2300 bbl. If the charges were $5\frac{1}{2}¢$ per barrel per term of 30 da. average storage, what was the amount of the bill to render?

WRITTEN REVIEW EXERCISES

1. I bought wheat at $0.80 per bushel. Allowing 6% for waste and incidentals and 2% for storage, how much must I receive per bushel for the wheat to realize a gain of $0.12 per bushel?

2. A produce dealer bought 150 T. cabbage at $5.50 per ton. He paid 90¢ per ton for storage and then sold the cabbage at a clear profit of 25%. How much did he receive per ton and what was his gain?

3. Nov. 1 a speculator bought 5000 bbl. apples at $2.25 per barrel and put them in storage. Feb. 1 he withdrew them from the storage warehouse. He had them sorted and repacked, when he found that he had only 4600 bbl. of sound apples. These he sold at $3.50 per barrel. If the storage charges were 5¢ per barrel per month or fraction thereof, and the charges for repacking were $500, did he gain or lose, and how much? what per cent?

4. Dec. 15, 1906, A. L. Farley bought 1000 bbl. flour at $4 and placed it with the Union Warehouse Co. for storage. Jan. 15 he bought 3000 bbl. flour at $4.15 and placed it with the same warehouse company for storage. On Feb. 15 he withdrew 2000 bbl. from storage and sold it at $5.85, on Mar. 25 he withdrew 1000 bbl. and sold it at $5.62½, on Apr. 1 he withdrew 1000 bbl. and sold it at $5.87½. If the storage charges were 5¢ per barrel per month or fraction thereof, and cartage and incidentals cost $100, did he gain or lose, and how much?

APPENDIX

TABLES OF MEASURES

MEASURES OF CAPACITY

Liquid Measure

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
= 231 cubic inches

Dry Measure

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel
= 2150.42 cubic inches

Barrels and **hogsheads** vary in size; but in estimating the capacity of tanks and cisterns 31.5 gal. are considered a barrel, and 2 bbl., or 63 gal., a hogshead.

A **heaped bushel**, used for measuring apples, corn in the ear, etc., equals 2747.71 cu. in. A **dry quart** equals 67.2 cu. in., and a **liquid quart** 57.75 cu. in.

MEASURES OF WEIGHT

Avoirdupois Weight

16 ounces = 1 pound
100 pounds = 1 hundredweight
2000 pounds = 1 ton

Troy Weight

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound

Apothecaries' Weight

20 grains = 1 scruple
3 scruples = 1 dram
8 drams = 1 ounce
12 ounces = 1 pound

Comparative Weights

1 lb. troy or apothecaries' = 5760 gr.
1 oz. troy or apothecaries' = 480 gr.
1 lb. avoirdupois = 7000 gr.
1 oz. avoirdupois = 437½ gr.

The ton of 2000 lb. is sometimes called a **short ton**. There is a ton of 2240 lb., called a **long ton**, used in all customhouse business and in some wholesale transactions in mining products.

In weighing diamonds, pearls, and other jewels, the unit generally employed is the **carat**, equal to 3.2 troy grains. The term "carat" is also used to express the number of parts in 24 that are pure gold. Thus, gold that is 14 carats fine is $\frac{14}{24}$ pure gold and $\frac{10}{24}$ alloy.

Miscellaneous Weights

1 keg of nails = 100 pounds
1 cental of grain = 100 pounds
1 quintal of fish = 100 pounds
1 barrel of salt = 280 pounds
1 barrel of flour = 196 pounds
1 barrel of pork or beef = 200 pounds

A cubic foot of water contains 6¼ gal. and weighs 62½ lb., avoirdupois.

MEASURES OF EXTENSION

Long Measure

12 inches	= 1 foot
3 feet	= 1 yard
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod
320 rods, or 5280 feet	= 1 mile

Surveyors' Long Measure

7.92 inches	= 1 link
25 links	= 1 rod
4 rods, or 100 links	= 1 chain
80 chains	= 1 mile

City lots are usually measured by feet and decimal fractions of a foot; farms, by rods or chains.

Miscellaneous Long Measures

4 inches	= 1 hand
6 feet	= 1 fathom
120 fathoms	= 1 cable length
1.15 miles, nearly,	= 1 knot, or
1 nautical or geographical mile	

Square Measure

144 square inches	= 1 square foot
9 square feet	= 1 square yard
$30\frac{1}{4}$ square yards	= 1 square rod
160 square rods	= 1 acre
640 acres	= 1 mile

The **hand** is used in measuring the height of horses at the shoulder. The **fathom** and **cable length** are used by sailors for measuring depths at sea. The **knot** is used by sailors in measuring distances at sea. Three knots are frequently called a **league.**

Surveyors' Square Measure

625 square links	= 1 square rod
10 square rods	= 1 square chain
10 square chains	= 1 acre
640 acres	= 1 square mile
36 square miles	= 1 township

Cubic Measure

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard
128 cubic feet	= 1 cord
1 cubic yard	= 1 load (of earth, etc.)
$24\frac{3}{4}$ cubic feet	= 1 perch

The square rod is sometimes called a **perch.** The word **rood** is sometimes used to mean 40 sq. rd. or $\frac{1}{4}$ A. In the government surveys, 1 sq. mi. is called a **section.**

The perch of stone or masonry varies in different parts of the country; but it is usually considered as 1 rd. long, 1 ft. high, and $1\frac{1}{2}$ ft. thick, or $24\frac{3}{4}$ cu. ft.

Angular Measure

60 seconds	= 1 minute
60 minutes	= 1 degree
90 degrees	= 1 right angle
360 degrees	= 1 circumference

Angular (also called circular) measure is used principally in surveying, navigation, and geography for measuring arcs of angles, for reckoning latitude and longitude, for determining locations of places and vessels, and for computing difference of time.

A minute of the earth's circumference is equal to a geographical mile. A degree of the earth's circumference at the equator is therefore equal to about 69 statute miles.

MEASURES OF TIME

60 seconds	= 1 minute	12 months	= 1 year
60 minutes	= 1 hour	360 days	= 1 commercial year
24 hours	= 1 day	365 days	= 1 common year
7 days	= 1 week	366 days	= 1 leap year
30 days	= 1 commercial month	100 years	= 1 century

September, April, June, and November have 30 da. each; all of the other months have 31 da. each, except February, which has 28 da. in a common year and 29 da. in a leap year.

Centennial years that are divisible by 400 and other years that are divisible by 4 are leap years.

In running trains across such a broad stretch of country as the United States, it is highly important to have a uniform time over considerable territory. Recognizing this, in 1883, the railroad companies of the United States and Canada adopted for their own convenience a system of **standard time.** This system divides the United States into **four time belts,** each covering approximately 15° of longitude, $7\frac{1}{2}°$ of which are east and $7\frac{1}{2}°$ west of the governing meridian. The region of **eastern time** lies approximately $7\frac{1}{2}°$ each side of the 75th meridian, and the time throughout this belt is the same as the local time of the 75th meridian. Similarly, the regions of **central, mountain,** and **Pacific time** lie approximately $7\frac{1}{2}°$ each side of the 90th, 105th, and 120th meridians, respectively, and the time throughout each belt is determined by the local time of the governing meridian of that belt. There is just one hour's difference between adjacent time belts. Thus, when it is 11 o'clock A.M. by eastern time, it is 10 o'clock A.M. by central time, 9 o'clock A.M. by mountain time, and 8 o'clock A.M. by Pacific time. Since railroad companies change the time at important stations and termini, regardless of the longitude of such stations and termini, the boundaries of the time belts are quite irregular.

MEASURES OF VALUE

United States Money	*English Money*
10 mills = 1 cent	4 farthings = 1 penny
10 cents = 1 dime	12 pence = 1 shilling
10 dimes = 1 dollar	20 shillings = 1 pound sterling
10 dollars = 1 eagle	= $4.8665

The term "eagle" is seldom used in business. The mill is not a coin, but the name is frequently used in some calculations. In Canada the units of money are the same as in the United States. 1 far. = $\frac{5}{9}$¢; 1*d.* = $2\frac{2}{9}$¢; 1*s.* = $24\frac{1}{3}$¢.

French Money	*German Money*
100 centimes = 1 franc = $0.193	100 pfennigs = 1 mark = $0.238

MISCELLANEOUS MEASURES

Counting by 12	*Counting Sheets of Paper*
12 things = 1 dozen	24 sheets = 1 quire
12 dozen = 1 gross	20 quires = 1 ream
12 gross = 1 great gross	= 480 sheets

BUSINESS ABBREVIATIONS

A . . . acre
Apr. . . April
Aug. . . August
bbl. . . barrel; barrels
bdl. . . bundle; bundles
bg. . . bag; bags
bkt. . . basket; baskets
bl. . . bale; bales
bu. . . bushel; bushels
bx. . . box; boxes
cd. . . cord; cords
ch. . . chain; chains
c.i.f. . . carriage and insurance free
Co. . . company; county
c.o.d. . . collect on delivery
coll. . . collection
Cr. . . creditor; credit
cs. . . case; cases
ct. . . cent; cents; centime
cu. ft. . cubic foot; cubic feet
cu. in. . cubic inch; cubic inches
cu. yd. . cubic yard; cubic yards
cwt. . . hundredweight
d. . . . pence
da. . . day; days
Dec. . . December
doz. . . dozen; dozens
Dr. . . debtor; debit; doctor
E. . . . east
ea. . . each
e.g. . . *exempli gratia*, for example
etc. . . *et cætera*, and so forth
far. . . farthing; farthings
Feb. . . February
f.o.b. . . free on board
fr. . . franc; francs
ft. . . foot; feet
gal. . . gallon; gallons
gi. . . gill; gills
gr. . . grain; grains
gro. . . gross
hhd. . . hogshead; hogsheads
hf. cht. . half chest; half chests
hr. . . hour; hours
i.e. . . *id est*, that is
in. . . inch; inches
Jan. . . January
kg. . . keg; kegs
l. . . . link; links
lb. . . pound; pounds
Mar. . . . March
mdse. . . merchandise
Messrs. . . *Messieurs*, Gentlemen; Sirs
mi. . . . mile; miles
min. . . . minute; minutes
mo. . . . month; months
Mr. . . . Mister
Mrs. . . . Mistress
N. . . . north
No. . . . number
Nov. . . . November
Oct. . . . October
oz. . . . ounce; ounces
p. . . . . page
pc. . . . piece; pieces
per. . . . by the; by
per cent. . *per centum*, by the hundred
pk. . . . peck; pecks
pkg. . . . package; packages
pp. . . . pages
pr. . . . pair; pairs
pt. . . . pint; pints
pwt. . . . pennyweight; pennyweights
qr. . . . quire; quires
qt. . . . quart; quarts
rd. . . . rod; rods
rm. . . . ream; reams
Rm.(or M.) *Reichsmark*, Mark
s. . . . . shilling; shillings
S. . . . . South
sec. . . . second; seconds
sq. ch. . . square chain; square chains
sq. ft. . . square foot; square feet
sq. mi. . . square mile; square miles
sq. rd. . . square rod; square rods
sq. yd. . . square yard; square yards
T. . . . . ton
tb. . . . tub; tubs
Tp. . . . township; townships
viz. . . . *videlicet*, namely; to wit
via . . . by way of
wk. . . . week; weeks
wt. . . . weight; weigh
yd. . . . yard; yards
yr. . . . year; years

BUSINESS SYMBOLS

$^{a}/_{c}$	account	=	equal; equals	1^{3}	one and three fourths
$^{a}/_{s}$	account sales	′	foot; feet; minutes	℔	per; by
+	addition	C	hundred	%	per cent; hundredth; hundredths
( ), $\overline{\ }$	aggregation	″	inch; inches; seconds	£	pounds sterling
&	and	×	multiplication	∵	since
.....	and so on	#	number, if written before a figure; pounds, if written after a figure	−	subtraction
@	at; to			∴	therefore
$^{c}/_{o}$	care of	1^{1}	one and one fourth	M	thousand
¢	cent; cents	1^{2}	one and two fourths; one and one half	$^{5}/_{6}$	5 shillings 6 pence; five sixths
√	check mark				
°	degree				
÷	division				
$	dollar; dollars				

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