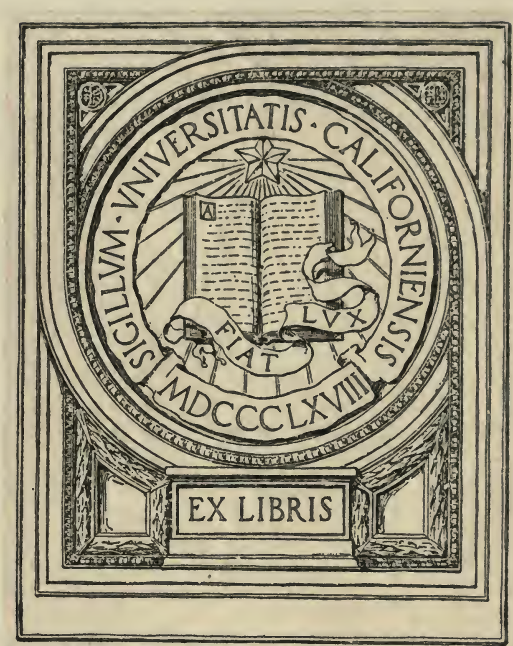
RACTICAL YROSTATIC BALANCING

HERBERT CHATLEY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PRACTICAL
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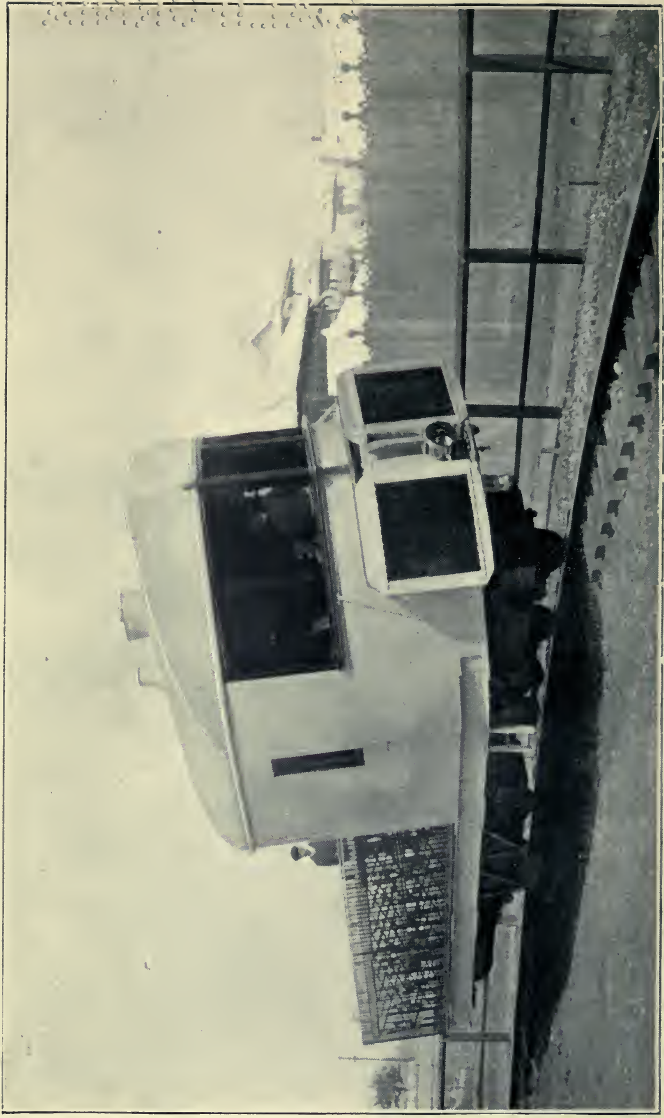
Practical Gyrostatic Balancing.



[*Photo by* A. E. WALSHAM.

The Brennan Mono-Rail.

[*Frontispiece.*

PRACTICAL
GYROSTATIC BALANCING

BY

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INTRODUCTION

THE subject of gyrostatic control is daily assuming more and more importance. The devices invented by Obry, Brennan, and Schlick have made it abundantly clear that by means of this piece of apparatus the automatic balancing of all kinds of locomotives, on land and in the sea or air, can be effected to an extent that would a few years since have seemed impossible. My attention has been particularly directed to the subject in connection with its applications to aerial engineering, and it has seemed to me that a small and inexpensive book on the general problem would prove useful to many inventors and engineers. May I hope that this little compilation will serve for this purpose?

HERBERT CHATLEY.

TANG SHAN, CHIH-LI,
NORTH CHINA, 1912.

PRACTICAL GYROSTATIC BALANCING

CHAPTER I

THE GYROSTATIC PARADOX—CONSTRUCTION OF GYROSTATS

THERE is a well-known saying to the effect that the commonplace is frequently the most marvellous, and the subject to which this little book is devoted forms no mean illustration of this aphorism. In childhood the top in one of its various forms, peg, humming or tee-totum, supplies an unending source of amusement. This amusement finds its cause probably in the eccentric behaviour of the top. It stands up when according to ordinary notions it should fall; it falls when it might be expected to stand up; and it performs the most weird movements without apparent cause. To those who have experimented with a Spinning Top (usually made in Germany) further marvels disclose themselves. The top walks along a thin string with the skill of a Blondin, turns somersaults, and generally performs in a most diabolical fashion. Not the least remarkable of the properties of this top is the effect of resistance which is felt by the hand when any attempt is made to alter its inclination or rotation. One could almost imagine that the toy possessed some form of life in its purposeful exercise of force against force.

Before proceeding to study the general behaviour of the spinning top we must, however, devote some attention to several important details of the phenomena which occur with the humbler varieties.

In the first place, there seems to be something very peculiar, when attention is given to the matter, in the mere fact that a top remains vertical when spinning, whereas it can in no way be persuaded to do so when still. This really is the essential property of all spinning mechanisms intended for balancing purposes. **A top is in unstable equilibrium when still, and stable equilibrium when spinning** at or above a certain rate. Apart from all questions of form or purpose, we have this first and rather paradoxical fact to grasp.

Now consider another point. A top is spun and strikes the ground obliquely. For a moment or two it persists in this oblique state, but at the same time performs a circular movement of large radius. Gradually (if on a smooth surface) it becomes less inclined, until finally the axis of spin is perpendicular. There is thus shown the further property of actually overcoming the force of gravity, and we have to note that under certain conditions a top **can do work against the force of gravity.**

Let us now study our top a little. I take as the most complete example in ordinary use, the peg top. It consists of a pear-shaped piece of wood, in the apex of which is inserted a steel peg, circular in section, about 2 inches long by a $\frac{1}{4}$-inch diameter. The wooden pear is usually about 4 inches long and

3 inches diameter, and the peg projects from 1 inch to nothing, according to the age of the top. The end of the peg is rounded, except in the case of small tops, which are occasionally pointed. For spinning, the pointed peg is preferable, but, of course, is somewhat dangerous. A number of shallow grooves about $\frac{1}{8}$-inch wide are turned in the conical surface. It is usual to employ a fine whipcord (such as is fitted to window-blinds), about 3 feet over all, a knot being formed at one end, the other is frayed out. The frayed end is carefully damped—good boys never under any circumstances wet this with the tongue! —plastered on to the base of the cone, and the cord is then wound from the peg tightly round the cone until the widest portion is reached. There are two methods of throwing a top, overhand and underhand (girls are only privileged to use the latter method), but there is the same essential principle involved. The top is thrown smartly away, the string being held. The friction between the string and the surface of the top causes the latter to spin rapidly, the increasing speed being allowed for by the taper of the top. In the whip or humming top the same result is produced by a rapidly moving string, which coils itself round the top by reason of its momentum, and is thrown off again by centrifugal acceleration. The surface speeds of the edge of a top often reach 50 feet per second—this corresponds to a note of about 60 frequency, a pitch of quite common occurrence. At this speed the top can easily lift itself from any inclination (provided, of course, there is no contact of the moving surface), so that there is an ability to overcome a torque of at least say $\frac{1}{12}$ of a lb.-foot. It will be found

that the larger and heavier the top, the more easily can it resist gravity. It is next necessary to understand in what manner this resistance is exercised, and whether any other process occurs at the same time. Since work is done some energy must be expended, and the manner in which this energy is absorbed and transformed is best seen in Foucault's gyrostat, gyroscope, or spinning top (*gyro*, to turn; *sto*, to stand; and *skopeo* to show or see). This is illustrated in fig. 1.

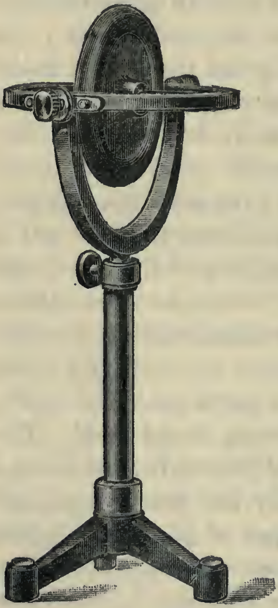

FIG. 1.—Laboratory Gyroscope.

The Gyroscope.—It must first be pointed out that in order to show all the possible variations of torque and angular motion which can occur with a spinning body, it should possess freedom of rotation in three directions, and the little unavoidable restraint which must occur should be as equally distributed as possible. In the case of a top, all the constraint is at one end of the axis of spin. In the gyrostat, there are three spindles, each supported in a pair of bearings. There are thus six bearings, which correspond to the six cardinal points of space. This arrangement is identical with that employed in supporting the mariner's compass, and known as "gimbal mounting." The gyroscope consists, then, of the following parts:—

(*a*) *The Fly-wheel and Spindle, or Top,* which is the reservoir of momentum (or, rather, "moment of momentum"; see later).—This is a heavy metal disc or thick-rimmed wheel, very nicely balanced on the spindle, and rotating in the central plane of the whole apparatus. The spindle is stiff and pointed at both ends, the conical ends resting in hard metal bearings in the first gimbal ring. The shank of the spindle usually bears a prepared drum on which string can be wound. A small electric motor is, however, far preferable as the source of power.

(*b*) *First Gimbal Ring.*—This is a circular hoop containing two adjustable bearings for the fly-wheel spindle; and at right angles to the line joining these bearings, spindle ends are rigidly fitted to the ring for transmitting the weight to the next ring.

(*c*) *Third Gimbal Ring.*—This is similar in arrangement to the second, but the two spindle ends are not always supplied. If they are fitted to a fixed fork or ring the instrument is much more useful, but frequently there is only one spindle supported in a fixed bearing.

It is very unfortunate that no definite system of nomenclature has been adopted for describing the various parts of the gyroscope, because in practice the parts change their relative positions so much, that, unless some definite system is adopted, there is some likelihood of error in observation.

To avoid any uncertainty we will number the parts of each section.

Mechanism for Driving.—The motor power is in small models supplied by a piece of string wound round the top spindle, but it is by far preferable to have a small, well-balanced motor, the current being led by wires and rings at the trunnions of the first gimbal ring. The field magnets will be fixed to the first gimbal ring, and the armature will be on top the spindle. It would be preferable to have the armature in two parts (with field magnets to match), so that the centre of gravity should remain at the centre of the fly-wheel. We shall now have in the central (first) gimbal ring the following parts:—

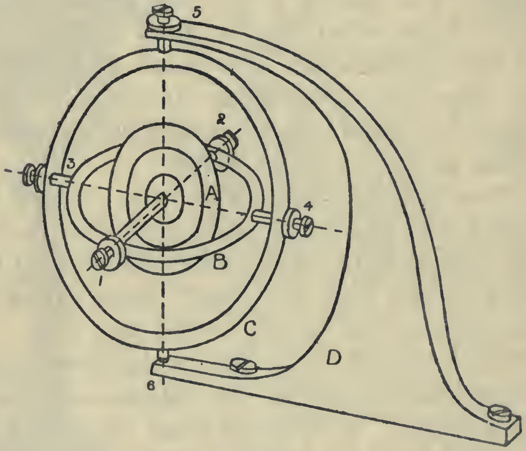

FIG. 2.—Diagram of Gyroscope.

(1) Spindle carrying armature, fly-wheel, and commutator.

(2) Field-magnets with brass framepieces fixed to first gimbal ring.

(3) Gimbal ring with ring insulation at trunnions.

The wiring will have to be carried to the trunnion of the second and third gimbal frames until the fixed frame is reached, when proper terminals will be fitted.

The fly-wheel can also be rotated by means of gearing through the trunnions, as shown in diagram; but the transmission losses are considerable, and the mechanism is intricate.

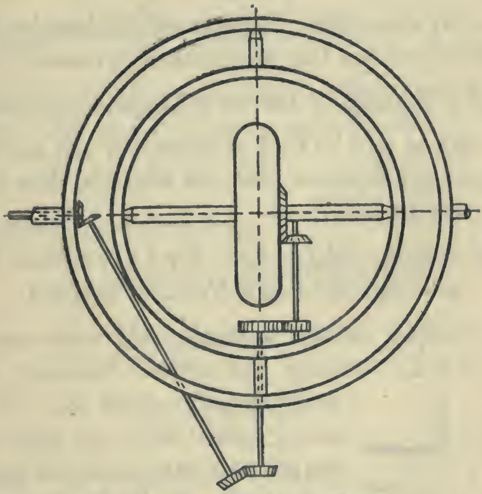

FIG. 3.—Gear-driven Gyroscope.

The fly-wheel in a large example can be driven by a petrol or other heat motor, self-contained entirely within the first ring, but electric motors are always preferred.

Lastly, the wheel may be operated by a jet, in turbine fashion. W. Schlick's apparatus for steadying ships operates in this manner, the steam being admitted through the trunnions. He also uses a hydraulic (turbine) type of brake to check the rotation.

Power required to Drive.—It will be as well at this point to consider the power absorbed in a rotating fly-wheel.

We have to consider the following points :—

(1) Inertia of fly-wheel.
(2) Friction at journals.
(3) Air resistance.

To arrive at a notion of the speed attained by the fly-wheel we must use the well-known dynamical rule :—

Torque = Moment of Inertia × Angular Acceleration.

The torque will be that produced by the motor, less the resisting torque, so that we may put the balance of moments as follows :—

Applied torque = (M.I. × Ang. Acc.) + (Friction Torque at Journals) + (Air Friction Torque).

The friction at the journals will not vary very much, and by keeping the actual diameter of the bearing very small (*i.e.* as small as consistent with the load on it), we get two very desirable features: (*a*) comparatively low speed, and therefore constant friction; (*b*) small consumption of energy. In any case, there is no great variation in the journal friction with the speed, but the work consumed is proportional to the diameter of the journal.

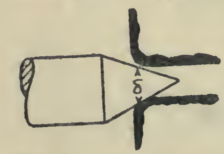

FIG. 4.—Cone Bearing.

The friction of the air, on the other hand, increases as the square of the velocity, and this item soon becomes of great importance.

The air friction varies as the square of the speed, the area, and the value of surface, $S = fav^2$.

Assuming a disc fly-wheel, on any element of width dr, on the faces, we have an area $2\pi r \, . \, dr$, a velocity ωr, and a coefficient of friction f.

$$fav^2 = f \, . \, 2\pi r \, . \, \omega^2 r^2 \, . \, dr.$$

Integrating between limits of radius we have, for one side face,

$$\int_0^R 2 \, . \, f\pi r^3 \, . \, \omega^2 \, . \, dr = 2\pi\omega^2 f \int_0^R r^3 \, . \, dr = \frac{f\omega^2\pi R^4}{2} \, .$$

For both sides we have $f\omega^2\pi R^4$, to which must also be added the friction on the drum (say breadth b),

$$fav^2 = f \, . \, 2\pi R b \, . \, \omega^2 R^2 = 2\pi f\omega^2 R^3 b.$$

To find resisting torque take

$$fav^2 r = f \, . \, 2\pi r^4 \omega^2 \, . \, dr$$

$$\text{torque (sides)} = 2\int_0^R 2\pi f r^4 \omega^2 \, . \, dr = 4\pi f\omega^2 \int_0^R r^4 \, . \, dr$$

$$= \frac{4}{5}\pi f\omega^2 R^5,$$

$$\text{(drum)} \qquad fav^2 r = 2\pi f\omega^2 R^4 b,$$

Using foot-lb.-seconds unit, $f = 0{\cdot}0000015$ lb. per square foot at 1 foot per second.*

If d is diameter of bearings, and μ the coefficient of friction, W the total weight of fly-wheel and spindle, the torque caused by friction is $\frac{\mu W d}{2}$ for the two bearings. $W =$ say $gm\pi R^2 b$ for a disc.

Taking the fly-wheel as a disc b thick, and m mass

---

* To avoid this air friction the gyro can be run in a vacuum. This is done in the Brennan stabiliser.

per unit volume, the moment of inertia $=\dfrac{bm\pi R^4}{2}$. If a is angular acceleration, and T applied torque

$$T=\frac{bm\pi R^4 a}{2}+\frac{\mu gm\pi R^2 bd}{2}+\left[\frac{4}{5}\pi f\omega^2 R^5+2\pi f\omega^2 R^4 b\right],$$

$$a=\frac{2T-2\left\{\dfrac{\mu gm\pi R^2 bd}{2}+\left[\dfrac{4}{5}\pi f\omega^2 R^5+2\pi f\omega^2 R^4 b\right]\right\}}{bm\pi R^4}.$$

If we consider this equation, we notice that there are the following quantities to be considered:—

T, the torque.
μ, the coefficient of journal friction.
f, the coefficient of air friction.

Value of f is about ·0000015 (author's result deduced from comparison of Zahn, Unwin, and Lanchester's figures).

μ, the coefficient of friction with cone bearings, should not exceed ·01.

Splitting up the expression for a, as given above, into separate terms, we have

$$a=\frac{2T}{bm\pi R^4}-\left[\frac{\mu gd}{R^2}+\frac{8}{5}\frac{f\omega^2 R}{bm}+\frac{4f\omega^2}{m}\right]$$

$$=\left(\begin{array}{c}\text{Acceleration}\\\text{produced by}\\\text{unresisted}\\\text{torque}\end{array}\right)-\left[\left(\begin{array}{c}\text{Retarda-}\\\text{tion due}\\\text{to axle}\\\text{friction}\end{array}\right)+\left(\begin{array}{c}\text{Retarda-}\\\text{tion due}\\\text{to face air}\\\text{friction}\end{array}\right)+\left(\begin{array}{c}\text{Retarda-}\\\text{tion due}\\\text{to rim air}\\\text{friction}\end{array}\right)\right].$$

By enclosing the mechanism in a vacuum, the third and fourth terms practically vanish, and we are only left with the second term.

If we take the expression

$$a=\frac{2T}{bm\pi R^4}-\frac{\mu gd}{R^2},$$

we know that if an angular velocity of $2\pi n$ radians per second is required, the angular acceleration being a, then $2\pi n = ta$, and $t = \frac{2\pi n}{a}$.

Therefore the time occupied in obtaining any required speed is, in seconds,

$$t = \frac{2\pi n}{\frac{2\mathrm{T}}{bm\pi \mathrm{R}^4} - \frac{\mu g d}{\mathrm{R}^2}}.$$

[Note throughout that these expressions refer to a disc fly-wheel.]

The torque applied, T, will of course need to be in excess of the resisting torque caused by friction under the most unfavourable conditions, and the balance will determine how long the gyrostats will take to attain a certain speed.* Once this speed has been reached, it is only necessary to provide a continuous torque slightly in excess of the frictional resistance. It would certainly seem desirable to have the prime mover of such a character that it could exercise a great torque for a short time, and a small torque for a long time, so as to economise power and time.

The work done will of course be the product of the torque and angular velocity

$$\frac{2\mathrm{T}\pi n}{550} = \frac{\mathrm{T}n}{87{\cdot}5},$$

where n is revolutions per second and T is f lbs.-feet of torque exerted by the motor. Some additional work will be done by the motor at the time external

---

* With an electric motor there is a counter-torque due to induced E.M.F., so that the motor torque must be considered as a function of the velocity.

torques are being overcome (*i.e.* when balancing occurs). This work will be absorbed in friction at the trunnions, and in causing the framework to rotate or "precess," and, later, reinforces the kinetic energy of rotation.

Having now a clear idea as to how much energy is required to spin the gyrostat, and its general construction: we can proceed to notice the phenomena which it manifests.

The fly-wheel is **spinning** at a high speed, its axis being horizontal, and the axis of the second gimbal ring vertical. The direction of spin is from left to right (*i.e.* clockwise) as the observer faces the fly-wheel.

The following peculiarities may be observed first:—

(*a*) If the frame C be rotated, the inner ring remains in its original position.

(*b*) If the whole apparatus be carried about the room, the axis 1–2 (fig. 2) (the revolving spindle) remains always in the same direction.

(*c*) If any attempt be made to alter the plane of the ring 1–4–2–3, considerable resistance is experienced.

(*d*) If a weight be hung at 1, or a downward pressure be applied at that point, one of two things will happen—

(1) If the weight is considerable, or the wheel is not going very rapidly, the point 1 will descend obliquely, moving towards the original position of 3, *i.e.* will rotate about the axis of the second ring 5–6 (fig. 2). If weight is taken off oscillation occurs.

(2) If the weight is small or the wheel is moving with great velocity, the point 1 will **not** descend but will rotate about the axis 5–6, as before, carrying with

it, of course, the weight. This rotation is termed "precession."

(*e*) If an upward force is applied at 1, the point precesses in the reverse direction (*i.e.* towards 4), rising only if the velocity of the wheel is small.

(*f*) If a downward force is applied at the point 2, precisely the same effect is produced as in experiment (*e*).

(*g*) If an upward force is applied at point 2, precisely the same effect is produced as with (*d*). All these experiments lead to one conclusion, viz.: **A force which ordinarily would turn the spindle in its own plane, when the fly-wheel rotates, produces precessional rotation at right angles to its plane (and also at right angles to the direction of spin).**

Now some further experiments—

(*h*) Apply a sideward push at 1 towards 3 (*i.e.* try to rotate the first gimbal ring). The point 1 rises, indicating a torque produced.

(*i*) Push in reverse direction and the point 1 sinks.

(*j*) Push the point 2 towards 4 and the same result as in (*h*) is produced.

(*k*) Push the point 2 towards 3 and the same result as in (*i*) is produced.

We now have a further practical result—**If the precessional rotation be externally produced a torque appears in the frame exactly of the same kind as would cause such precession by the previous rule.**

CHAPTER II

MECHANICAL PRINCIPLES

The essential relations between the turning moment, velocity of precession, velocity of rotation, and the weight of the mechanism can be expressed without any elaborate mathematics, if we remember the principle of the triangle of rotations—

Angular displacements (rotations), velocities, accelerations or momenta, and torques can be combined by triangles just like linear quantities and forces if their axis be taken as the directions.

Thus a line 5 units long, with an arrow to the right, can indicate an angular rotation or a torque whose numerical value is 5 units, whose *axis* of rotation is horizontal and in the plane of this paper, and whose motion is clockwise looking at it from the left side.

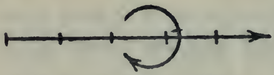

Fig. 5.—Rotation Vector.

Further, we must know the mechanical law—

$$\text{Torque } (\doteq \text{turning moment}) \atop \text{(in foot-lb.)} \quad = \frac{\text{Change of angular momentum}}{\text{Time of change}}.$$

Angular momentum is the sum of the linear momenta of all the parts of the rotating body, each momentum being multiplied by the radius of the part from the centre of rotation. Thus—

Linear momentum = mass × linear velocity.

Angular momentum = total of (mass × linear velocity × radius) [for all parts of the body].

Since also linear velocity = angular velocity × radius,

Angular momentum = total of (mass × radius × radius) × angular velocity [for all parts of the body],

Total of (mass × radius × radius) is called "Polar Moment of Inertia,"

so that

$$\text{Torque} = \frac{\text{Polar moment of inertia} \times \text{change of angular velocity of rotation}}{\text{Time of change}}.$$

If we now draw two lines representing the angular momentum before and after a change in the direction of the axis ("precession"), we see that the third side of the triangle is the "Change of Angular Momentum."

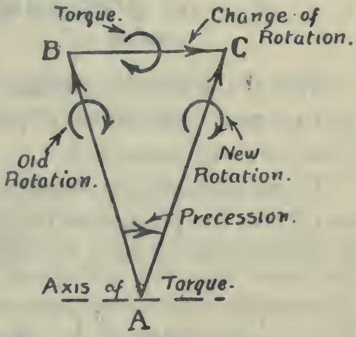

FIG. 6.—Combination of Rotations.

A B is the angular momentum about the old axis.

A C is the angular momentum about the new axis.

B C is the change of angular momentum.

[*Note.*—A B and B C have the arrows in the same direction round the triangle. A C, their resultant, has the arrow in the reverse direction.]

If A C is *numerically* the same as A B, for *small* values of the angle B A C we can say

$$BC = AB \times \text{angle } BAC \text{ (measured in radians).}$$

If now the axis is precessing with an angular velocity "ω," we have

Angle B A C $= \omega \times$ time of change, so that time of change

$$= \frac{\text{angle } BAC}{\omega};$$

so that we have

$$\text{Torque} = \frac{\text{Polar moment of inertia} \times \text{angular velocity of rotation} \times \text{angle } BAC}{\text{Time of change}}$$

$$= \text{Polar moment of inertia} \times \text{angular velocity of rotation} \times \text{angular velocity of precession } (\omega).$$

This is the most important rule in connection with gyroscopes, and it only remains to say how the rotations are connected.

If we look at the rotation of the fly-wheel from the front, the precession from on top, and the torque from the left, we have the following table:—

	Rotation.	Torque.	Precession.
1.	Clockwise.	Clockwise.	Clockwise.
2.	Clockwise.	Anti-clockwise.	Anti-clockwise.
3.	Anti-clockwise.	Clockwise.	Anti-clockwise.
4.	Anti-clockwise.	Anti-clockwise.	Clockwise.

A forced precession causes a torque *opposed* to that which would produce it.

As already mentioned, these mutual relations exist when the three planes of rotation or torque are perpendicular to one another, and the engineer without special mathematical practice is at a loss to deal with two practical cases which sooner or later are bound to occur—

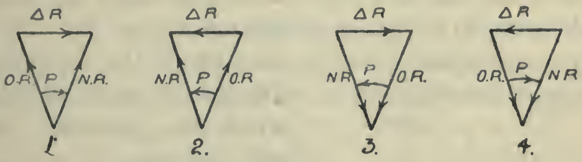

Fig. 7.—The Four Cases of Combination.

(*a*) The torque or precession is not at right angles to the plane of rotation;

(*b*) Appreciable friction appears at the bearings of one or more axes.

Some notions on these points follow, which do not involve any very abstruse calculations.

Gyrostatic Action with Oblique Torque.

If the torque which acts on a fly-wheel spindle is not in a plane perpendicular to the plane of rotation, it may be regarded as consisting of two rectangular components, one of which is in that plane. The other is in a plane parallel to the plane of rotation. The latter produces no precessional effect. Thus, if initially or by reason of the precession the spindle of the fly-wheel becomes inclined to the plane of the torque, the latter can be resolved into two components, parallel and perpendicular to the spindle. The component whose axis is parallel to the spindle has no effect in altering the momentum of the fly-wheel, and it is the other component alone which needs to be considered. If the spindle rotates until it is perpendicular to the plane of the torque, *i.e.* the axis of the torque coincides with the axis of the spindle, all precessional effect ceases. This is the principle upon which the gyrostatic compass depends.

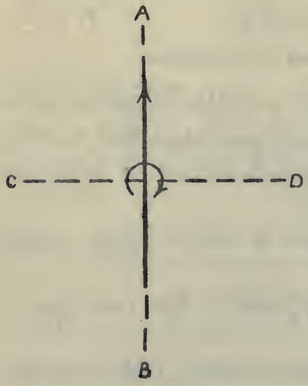

Fig. 8.—Rotation of Vectors.

As an example, if a fly-wheel be spinning about an axis A B, in the plane of a torque whose axis is C D, the precession will cause it to rotate until its spindle coincides with axis C D. The torque has then no effect on it, and it remains in that position.

[*Note.*—In an ordinary experimental gyroscope the weights producing torque are carried round with the spindle, so that this diminution of torque does not occur and the precession is continuous.]

A further question that arises is this: What is the greatest value of a torque that can be balanced by the precessional motion? Or, to put it in another way, What torque will force the spindle of the gyroscope to move in the direction which it would follow if not rotating? Experiment shows that this depends to a considerable extent on the frictional resistance to precession. If the axis of precession be of little friction, the spindle does not move so much in the direction of the torque as when the friction is great. In fact, if the friction be great enough there will be no precession, and the spindle will fall as if not rotating, a grinding noise being heard at the bearings, due to the precessional torque. The fact of the matter is simply this: the disturbing torque causes a precessional torque which causes precessional acceleration. The precessional velocity depends on the resistances (inertia and friction), and the disturbing torque will only be balanced to the extent corresponding to the instantaneous value of the precessional velocity. For this reason the spindle always does yield or "wobble" to some extent in the direction of the disturbing torque, and the latter cannot be wholly balanced until the precessional velocity reaches the value

$$a = \frac{T}{I\omega}.$$

The action can best be understood if we consider the spindle to move for a very short time in the direction of the torque. Thus the fly-wheel spindle A C is depressed through an angle A o B. This motion may be regarded as a precession, and causes a new "Precessional Torque" in the direction B D. The precession (in the direction B D) causes a further torque, which is opposed to the applied torque. By considering the matter in this

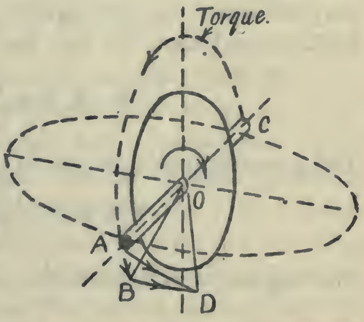

FIG. 9.—Diagram showing Precessional Evolution.

way we see that the torque first causes an angular acceleration in its own plane, as in ordinary plane motion; this acceleration causes angular velocity which, again, causes torque; this torque causes angular acceleration and velocity, which is the "precession"; and this, again, is accompanied by a counter-torque which acts against the original torque. We thus see that the fact that torque produces precession is explicable from the fact that precession produces torque, just as radial acceleration is accompanied by centripetal constraint.

This conception can be expressed analytically as follows:—The torque T causes an inertia acceleration $I_2\ddot{\theta}$ when I_2 is the moment of inertia of the fly-wheel and one gimbal ring about a diameter perpendicular to the spindle. At the end of an infinitesimal term δt, the velocity

$$\dot{\theta} = \ddot{\theta} \,.\, \delta t = \frac{T}{I_2} \,.\, \delta t \,.$$

The gyrostatic torque due to this angular velocity is

$$T_2 = I\omega\dot{\theta} = \frac{I\omega T}{I_2} \,.\, \delta t \,.$$

The angular acceleration due to this torque is

$$\ddot{\phi} = \frac{T_2}{I_2} \,,$$

and the angular velocity (real precession) is

$$\dot{\phi} = \ddot{\phi} \,.\, \delta t = \frac{I\omega T}{I_2^{\,2}} \,.\, \delta t^2 ,$$

which causes a torque

$$\Delta T . = I\omega\left(\frac{I\omega T}{I_2^{\,2}} \,.\, \delta t_1^{\,2}\right)$$

so that the original torque T is diminished by ΔT, and becomes

$$\mathrm{T}-\Delta\mathrm{T}=\mathrm{T}-\frac{\mathrm{I}^2\omega^2\mathrm{T}}{\mathrm{I}_2^{\,2}}\cdot\delta t_1^{\,2}.$$

(All these expressions are only applicable for the time δb.)

Hence we see that the rate of diminution or balancing of T depends on the square of the angular momentum of the fly-wheel, and varies inversely as the square of the moment of inertia of the fly-wheel and gimbal ring about a diametral axis perpendicular to the axis of rotation.

If now T_2 is opposed by friction, obviously $\ddot{\phi}$ will be less, and therefore ΔT also.

Effect of Friction

The difficulties which arise in connection with the axle friction are only completely soluble by an elaborate mathematical analysis, but a very good notion of the action of friction can be obtained without introducing any further complications. The friction at the bearings of the fly-wheel spindle has no effect on the gyrostatic torque, beyond absorbing some of the angular momentum. This loss is made up by the motor in an electrically-driven machine, but in a clockwork or string-impelled wheel the diminution of the momentum will, of course, be accompanied by a corresponding diminution of the gyrostatic torque.

With regard to the frictional moments at the bearings of the gimbals, we need only remember that every torque which is not in the plane of the rotation of the fly-wheel causes a precession, and every precession causes a torque. Thus, if the precessional rotation is in a horizontal plane about a vertical axis, and there is friction at the bearings, there will be a precession in a vertical plane about a horizontal axis to a value determined by the frictional moment. This frictional precession will usually appear as a diminution of an already existing motion, and not as a positive quantity. It is, however, quite apparent when a well-balanced gyroscope is running without any external load, and, as already mentioned, is the cause of the yield or wobbling of the gimbal frames. Obviously, it is desirable to minimise these frictional moments as much as possible, and roller or ball-bearings will be useful in this connection. It should also be noticed that the friction may be due to forces over and above those of the weight of the parts, for whenever a torque acts it has to be transmitted through the bearings, and will cause pressures (equal and opposite at the two corresponding bearings) thereon.

The theory of gyrostatic action has received the attention of numerous physicists, and although it does not occupy a sufficiently important place in the ordinary mechanical text-books, fairly full explanations of the fundamental principles will be found in Prof. Perry's book of *Spinning Tops* (published by the S.P.C.K.), and Prof. Worthington's little book on the *Mechanics of Rotation* (Macmillan). In *Engineering*, numerous articles on the subject have appeared (vol. lxxxiii., 1907, p. 749; vol. lxxxiv., 1908, p. 385; vol. lxxxvi., 1910, p. 797). In the first named, there is a numerical error which is corrected in the later articles; but in that particular article a very clear idea of the reason for the relation between torque and precession is given. It is pointed out that if the fly-wheel be spinning and its spindle is precessing, any point on the fly-wheel has a linear velocity which is composed of two elements, one being the circumferential velocity of the point considered as part of the fly-wheel, and the other the precessional motion which varies according to the distance of the point from the axis of the precessional rotation. Owing to the spin of the wheel this distance varies periodically, and the precessional component of the point's velocity is alternately positive and negative as it descends below and rises above the level of the spindle. Hence there is a continuous alteration of its momentum, which can only be maintained by equal and opposite forces above and below the level of the spindle, *i.e.* a torque.

More elaborate treatment of the theory will be found in the works of Boys, Tollwer Preston, and a recent book by Harold Crabtree; but for full treatment of the subject, Kelvin and Tait's *Treatise on Natural Philosophy* (Cambridge Press), should be referred to.* In Lanchester's *Aerodonetics* (Constable), there is a long appendix dealing with the gyroscope and its applications, but there are no very precise indications of the manner of its application to the stability of flying machines, nor of the difficulties which arise in connection therewith.

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* See also Appendix B.

CHAPTER III

GYROSTATIC CONTROL

It is obvious that a device such as the gyroscope, which converts torque into precession, and can from precession produce torque, is applicable to the purpose of reinforcing the stability of bodies, and as a matter of fact it has been so employed for projectiles, cars, ships, torpedoes, and flying machines.

If the frame (second gimbal) is attached to the structure of any machine which becomes displaced by a disturbing torque which acts upon it, when the fly-wheel is spinning and the spindle is free to precess, there will be a reaction, depending on the angular momentum and precession, which will reduce the disturbance.

Such disturbing torques are, however, generally about a more or less fixed axis, so that as the precession proceeds the stabilising torque of the gyroscope will diminish. If the disturbing torque continuously acts in one direction (as in the case of an overturning vehicle), the precession will also be continuous, the righting torque will diminish, and the gyroscope will not save the body from being overturned, but will merely delay the fall. In order to successfully meet the disturbing torque, we must have two special features—

(1) A very slow precession (*i.e.* a high angular momentum), so that the counter-torque does not appreciably diminish during the time in which it is required.

(2) A counter-torque which rapidly develops to a value greater than the simultaneous value of the disturbing torque. This can only be arranged by forcing the precession, or employing the gyrostatic torque to produce a larger torque through a relay mechanism.

In the case of a disturbing torque which periodically varies (such as we have in ships and flying machines under some conditions), the more damping effect of the counter-torque will aid the natural diminution of the disturbing torque, and a simple, direct-acting gyroscope can be used to reduce the oscillations. A relay mechanism can, of course, be used in this connection, if desired.

In all cases the condition as to slow precession should be observed, for the reason that our fundamental formula equating the torque to the product of the polar moment of inertia, angular velocity, and angular precession refers to the instantaneous value of the angular precession. Since this precession must be attained from rest, if we allow a high value for the precession in computing the torque or angular momentum from each other, the torque will not be balanced until this value of the precession is reached. This will involve two disadvantages—

(1) Loss of time, there being an interval in which the precession is accelerating up to the required value.

(2) A large precessional displacement, which will reduce the component of the reacting torque in the plane of that which has to be balanced.

With regard to the magnitude of the reacting torque, unless the disturbing torque is itself subject to a natural diminution, the simple gyroscope (*i.e.* without relays or forced precession) cannot wholly restore the body controlled to its original position. We see this in the case of rifled projectiles, which, although they do maintain their course much better than smooth ones, gradually drift to one side or the other, according to the hand of the rifling, to an extent which has to be allowed for in the sighting.

This result is easily explained. The projectile accelerates in the gun, leaving the muzzle with a prescribed "muzzle velocity" (about 2000 feet per second in the case of military rifles), and during its passage through the barrel it has been rotated by the rifling through one complete revolution or thereabouts, so that it has an angular velocity of a thousand or more radians per second, and a corresponding angular momentum. The air resistance will sooner or later be slightly asymmetric, and there will then be a torque due to the combined effect of the weight and the asymmetric reaction. This torque will cause a precession; and although this precession will alternately act one way and then the other, the slight but unavoidable original bias, and the steady frictional reduction of the angular momentum, combined with the inability of the reacting torque to quite balance the disturbing one, will cause the drift.

If, however, we can force the precession, the value necessary to produce a counter-torque equal to the disturbing one may be passed, if *it is of a low value*, and then the counter-torque will exceed the disturbing one, and the body controlled will be righted. The commonest example of this is the bicycle. The angular momentum of the front wheel of the bicycle is altered if the steering handle be rotated, and it is a matter of common experience that if the handle be rapidly turned, or if the machine be travelling rapidly (so that the angular momentum of the wheel is great and the necessary precession is small), there is a righting torque.

The possibility of automatically producing this **forced** precession has exercised the minds of many inventors, but it is the peculiar merit of Louis Brennan to have discovered a most simple and effective means of doing this. This method is described later; but it is perhaps as well to say here that the essential idea is simply to attach a small roller to the fly-wheel spindle, so that when the spindle deflects in the plane of the disturbing torque by reason of the initial wobble, the roller touches a fixed surface, and the angular momentum of the fly-wheel itself forces the precession.

The indirect method is to arrange for the precession to operate the control of another machine, which will produce the righting torque. For land vehicles it would seem difficult to devise a relay mechanism which would be sufficiently rapid in its action, but marine and aerial vessels can easily be so fitted.

Most of the practical difficulties arise from the interaction of the gyrostatic control with other external forces, and also from the control coming into operation at times when it is not required.

CHAPTER IV

RELAYS

As we have seen, a simple gyroscope will not permanently resist a constant or increasing torque acting about a constant axis, so that overturning moments on mono-rail cars and aeroplanes about longitudinal axes cannot be met by an apparatus of this description. Even in the cases of periodic torque, such as occur in the longitudinal and lateral oscillations of ships, torpedoes, submarines, dirigibles, and sometimes of aeroplanes, the damping action of the precession of the simple gyroscope is not always sufficiently effective. Furthermore, the apparatus to be capable of producing a very large torque must have parts of considerable mass. Even if the highest speeds of electric motors or impulse steam turbines are employed, to obtain an appreciable torque with small precession the necessary moment of inertia involves fly-wheels of moderate size, and, in addition, the centrifugal stresses due to the high speed compel fairly massive construction.

Instead, therefore, of employing the direct action of the gyroscope to provide the counter-torques necessary, a common plan is to arrange it so that the precession will set in operation a further "relay" mechanism, which can produce counter-torques of the required value. The principal objection to this method lies in the delay which occurs before the counter-torque is developed. In the Whitehead torpedo the first gimbal ring carries a projecting pin which rests in a forked lever. This lever communicates with the admission valves to steering engines, the action being, of course, co-ordinated with that of the gimbal ring, so that the rudders are operated to correct the deviation recorded by the gyroscope. As a matter of fact, the gyroscope gimbal ring can be connected to the tillers direct, since the torque necessary to turn the rudders is much smaller than that required to right the vessel; but, as we shall see in studying the flying machine, this causes complications in connection with the trimming or rolling of the vessel, and to get proper control two-paired gyroscopes become necessary.

The relay mechanism may be of any one of numerous kinds, but electric or hydraulic appliances give the most rapid response. Thus the gimbal ring may be connected to a change-over switch, so that in the mean position there is no contact, but when the gimbal precesses either way it closes a circuit, and one of two motors is set in operation and moves a weight, turns a rudder, or performs some such operation which will restore the controlled body to its original position. Practical difficulties here occur in connection with the imperfect contact—sparking and armature heating. Again, the gimbal ring may control a three-way valve, such as is used for indicators in testing engines, and so admit to one or the other end of a cylinder the gas which will move the piston, which again will operate a torque-producing appliance.

A most ingenious relay mechanism is employed in the "Beauchamp Tower Steady Gun-platform" (see *The Engineer*, vol. lxvii., 1889, p. 324). Here the gyroscope is carried on a large gimbal mounting, on which the gun platform is arranged. Water under pressure passes through the gyroscope from one spindle bearing to the other. Some also escapes at the periphery of the wheels, and causes the angular momentum by the Barker's mill principle. The water issuing from the other bearing impinges on one of four parts, according to the position of the gyroscope relative to the gun-platform, and is then admitted to one of four hydraulic cylinders, which brings the gun-platform into a level position, or rather, since the changes are very rapid, maintains the platform in that position.

By introducing a second relay, the power of the gyroscope to regulate and indirectly produce large righting torques can be indefinitely extended, but the lag of the mechanism must always be allowed for, and in many cases may prove prohibitive.

CHAPTER V

GYROSTATIC CONTROL OF VEHICLES

ONLY one successful system for stabilising vehicles has yet been published, that of Louis Brennan. A German system exists, but Mr Brennan is understood to claim this to be a copy of his invention.

Vehicles on a proper road have minimum stability about an axis in direction of motion, and this stability is negative if the track is a mono-rail, the centre of gravity of the vehicle being above the rail.

Brennan's control consists of two fly-wheels mounted in gimbals, the axis of the fly-wheels being horizontal and ordinarily transverse to the track. The gimbal rings can rotate about vertical axes, and the two axes are geared together so as to rotate in opposite directions. The fly-wheel spindles carry the rotors of small electric motors, and the two revolve in opposite directions, so that the precessions due to alterations in the direction of motion of the vehicle are nullified by the equal and opposite rotations of the gimbals. The spindles of the fly-wheels project, and each carries two rollers, one larger than the other. The casing carries two running surfaces, which will be touched by these rollers if the fly-wheel spindles cant relatively to the frame. These rollers force the precession, and

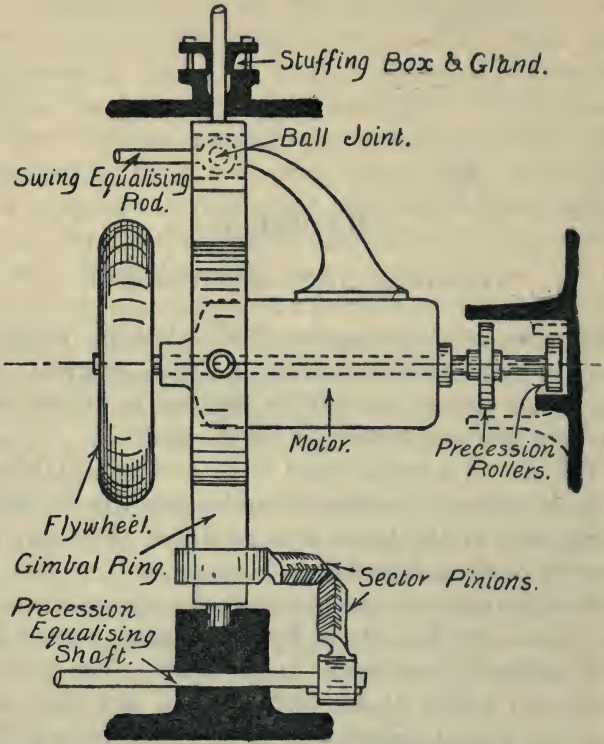

FIG. 10.—Brennan Gyroscope.

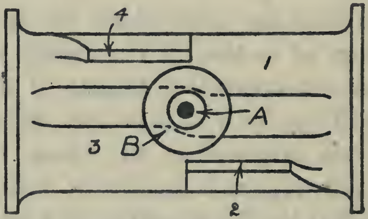

FIG. 11.—Rollers and Guides of Brennan Apparatus.

so produce a counter-torque more than equal to the disturbing one. This is the essential feature of the Brennan patent.

The two rollers are used, so that the oscillations due to the excess of the righting torque over the disturbing one may be rapidly extinguished.

The following example will illustrate the numerical values which occur in a practical case:—

Two fly-wheels (geared together as described), mass of each, 20 lb. $\div 32 = \frac{5}{8}$ units (mass-lb.).

$$\text{K} = \tfrac{3}{4} \text{ foot.}$$

$$\text{Moment of inertia} = \text{MK}^2 = \tfrac{5}{8} \times \tfrac{3}{4} \times \tfrac{3}{4} = \tfrac{45}{128}.$$

For the two, total polar moment of inertia $= \frac{45}{64}$, say ·70 units.

Speed 2400 revolutions per minute
= 40 revolutions per second
$= 2\pi \times 40$ radians per second, say 250.

[*Note.*—Roughly, revolutions per minute ÷ 10 = radians per second.]

Precession roller $\frac{1}{2}$ inch radius (smaller roller).

Peripheral speed of precession roller $= 250 \times \frac{1}{2} \times \frac{1}{12}$, say 10 feet per second.

Distance from axis of gimbal to precessional roller, say 1 foot.

Angular velocity of precession

$$= \frac{\text{Peripheral velocity of roller}}{\text{Distance from gimbal axis}}$$

$$= \frac{10}{1} = 10 \text{ radians per second.}$$

Torque $= \text{I}\omega a = {}·70 \times 250 \times 10$, say 1750 foot-lbs.

If the larger roller is 1 inch diameter and 10·8 inches from the gimbal axis, its peripheral speed is 20 feet per second, and the angular velocity of precession is

$$\frac{20}{0{\cdot}9}\text{, say 22 radians per second,}$$

if there is no slip. This roller being on a loose sleeve, the torque will be *upwards* of 3500 foot-lbs. Owing to the slip of the roller on the bearing, the actual torque will only be equal and opposite to the opposing torque. The loss of angular momentum due to the friction will be very small, being only due to the resistance to rolling.

The above example shows the precession which can be produced by the gyroscopes when the rollers are in contact with the fixed running surfaces. If there is $\frac{1}{10}$ of an inch clearance between the roller and the running surface when the car is horizontal, and the distance from the rollers to the gimbal axis is 1 foot (as is assumed in this example), the car can swing through an angle of $\frac{1}{10}$ inch ÷ 12 inches $= \frac{1}{120}$ radian, before the roller touches. If the height of the centre of gravity of the car is 5 feet, and its weight is 20 tons (say 40,000 lb.), then there is an overturning torque of

$$40{,}000 \times 5 \times \frac{1}{120} = \frac{20{,}000}{12} = 1700 \text{ foot-lbs. nearly.}$$

Thus, in case the car does begin to sway, the spindles will precess under a torque increasing from 0 to 1700 foot-lbs. As soon as the latter value is reached the rollers are forcibly precessed, and there is a counter-torque of 1750 foot-lbs., or a net torque of 50 foot-lbs., which returns the car to the vertical and beyond.

It will then swing to the other side, when again the forced precession will return it, and so the oscillation will go on until extinguished by the inertia and friction. In order to more rapidly stop the motion, the rollers and guides are so arranged that during the *reversal* of motion the larger roller comes in contact with the guide, so that the precession is more rapidly forced than during the first motion.

Order of Contact

Car sinking.—A touches 1; forced precession returns it.

Car rising.—B touches 2; forced precession (reverse hand) checks motion.

Car sinking on other side.—A touches 3; forced precession returns it.

Car rising.—B touches 4; forced precession checks motion.

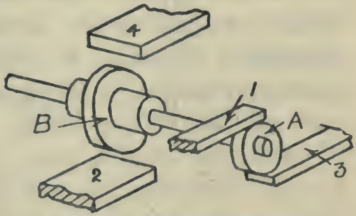

Fig. 12.—Rollers and Guides of Brennan Apparatus (Diagram).

The paired gyroscopes are enclosed in an air-tight casing, from which the air is pumped, so that the fly-wheels run in a fairly high vacuum. By this means the air friction is eliminated, and the fly-wheels will run for two hours or so after the current is cut off, so that accident on this score is rendered less likely. As is well known, Mr Brennan has made small and full-sized mono-rail cars, balanced on this principle, which have proved perfectly satisfactory.

It is quite possible that the relay may also be applied to vehicles, but the sensibility of the apparatus is hardly likely to be so perfect as that of Brennan. Thus the gimbals could be arranged to actuate a friction clutch, double acting, which would connect a continuously running motor (electric, petrol, or the like) with a traversing screw, lazy tongs, or system of levers, which would displace a jockey weight into a position in which it would produce a righting torque. Again, a hydraulic system might perhaps be devised by which the gyroscope would actuate a pump and transfer water so that its mass would cause a sufficient righting torque. The only real advantage of these methods lies in a possible smaller weight of the apparatus, but even this is doubtful. Brennan is understood to estimate the maximum weight of his gyroscopic control at 5 per cent. of that of the vehicle.

CHAPTER VI

GYROSTATIC CONTROL OF SHIPS

Two practical examples exist of the automatic balancing of ships. One is indirect in its action and serves to control the course of an unmanned vessel (the Whitehead torpedo). The second is to extinguish rolling, *i.e.* lateral oscillations. Both have proved very successful.

The torpedo control is very simple, consisting of a complete gyroscope of the experimental type, a lever, operating the throttle-valves of compressed air steering-engines, being moved by a pin attached to the first gimbal ring when the latter precesses. Motive power is obtained by a coiled spring, which is released when the torpedo is launched and transfers its energy to the fly-wheel spindle through two bevel pinions.

[An aerial torpedo could probably be controlled in a similar way.]

The mechanism for the control of the lateral rotations of a ship was invented by Dr Otto Schlick, and has been described by Sir W. White in the *Trans. Inst. Nav. Arch.*, 1907. A very large fly-wheel is used, the spindle being normally vertical, and suspended in a very massive gimbal frame supported in horizontal trunnions, the pedestals of which are attached to one of the transverse frames of the ship.

The fly-wheel is driven by a steam turbine, the steam being supplied and exhausted through the trunnions and spindle bearings. A hydraulic brake is fitted to the trunnions to prevent the precession when necessary. A lateral torque applied to the ship causes the gyroscope to precess, instead of causing the ship to roll.

It can be shown mathematically that the angle of precession varies as time integral of the torque.

[Torque = Time rate of change of angular momentum

$$= \frac{\text{Angular momentum} \times \text{precessional angle}}{\text{Time of precession}}$$

if a very short time is used.

Hence

$$\text{Precessional angle} = \text{integral}\left(\frac{\text{Torque}}{\text{Angular momentum}}\right) \times \text{Time of precession.}]$$

In the case of ships, the torque does not act continuously, but depends on the wave motion of the surface of the sea. In an uncontrolled ship the deck tends to set itself parallel to the instantaneous surface of the water, although this effect is complicated by the proper oscillations of the ship itself (such as would occur in still water). At any instant the torque is a function of the angle between the plane of the deck and the instantaneous tangent plane of the surface of the water. The gyroscope tends to maintain the plane of the deck in a horizontal position, so that as the ship passes over waves moving laterally, the torque will alternately increase and decrease, and reverse, causing a similarly changing precession. In order to avoid a complete rotation when precession occurs, the centre of gravity of the gimbal frame lies a little below the line of the trunnions. This leads to two other complications—

(1) Possible interaction of the natural vibrations of the gimbal considered as a pendulum, and the precessional vibrations.

(2) Rolling precession of the ship due to the restoring torque, which appears when the gimbal is displaced by precession.

In addition, it should be noticed that the gyroscope will affect the ship when the precession is prevented by a clamp. If the ship rolls under these circumstances (the spindle of the fly-wheel being perpendicular to the plane of the decks), the precession of the spindle with the ship will cause a torque tending to make the ship pitch, but owing to the enormous moment of inertia of the ship about a transverse axis the motion will be very small. Again, if the ship pitches, for analogous reasons there will be a torque producing a roll. This motion will, however, not be very great, because the angular velocity of pitching is generally small.

A complication also occurs if the gimbal is released when the ship is not vertical, as the gyroscope always tends to remain in its original position, so that there will be a "heel" in the direction which the ship instantaneously occupied when the gimbal was freed.

Numerical Example of Ship Control.—One fly-wheel. Weight 640 lb. Radius of gyration 3 feet.

$$\text{Moment of inertia} = \frac{640 \times 3 \times 3}{32} = 180 \text{ units.}$$

Speed 1800 revolutions per minute = 30 revolutions per second = $2\pi \times 30$ radians per second = say 190.

Rolling torque acting on ship at a certain instant, say 5000 foot-lbs. (This torque varies generally as the sine of the angle of heel.)

Precessional velocity $= \frac{5000}{180 \times 190} = \frac{50}{342}$, say $\frac{1}{7}$ radian per second.

Thus, if the periodic time of the wave motion is 5 seconds, and the value of the torque given is the mean, the gimbal will precess through $\frac{5}{7}$ radian or say 40°.

CHAPTER VII

CONSTRAINED STABILITY OF FLYING MACHINES

In numerous instances methods of controlling the stability of flying machines by gyroscopes or pendulums have been projected, and although I am not aware that in any case good results have been obtained, yet there can be little doubt as to the possibility of doing it. The principal objections are—

1. The weight of the apparatus.
2. Interference with steering.

Neither of these should be very serious. By using high speeds, the weight can be made very small, and by proper clutches, the gyroscopic action can be suspended during steering.

The gyroscope can be applied in two distinct ways—

(1) Directly, by rigidly connecting the gimbals to the frame of the machine; and

(2) Indirectly, by connecting the gimbals to the engine or directive organs, so as to actuate them in such a way as to produce a righting torque. (Maxim.)

The two figures show the method of arranging the gyroscopic control. Fig. 13 shows the indirect control, and fig. 14 the direct method, which is associated with the name of Brennan.

In the case of the indirect control a plunging motion (rotation in a vertical plane about a transverse axis) will cause the gyroscope to precess (rotate in a horizontal plane about a vertical axis), and this precession is transmitted by gearing to cant a horizontal rudder. This rudder, when oblique to the direction of motion, is subject to fluid pressure, and so a turning moment is produced on the machine which rights it.

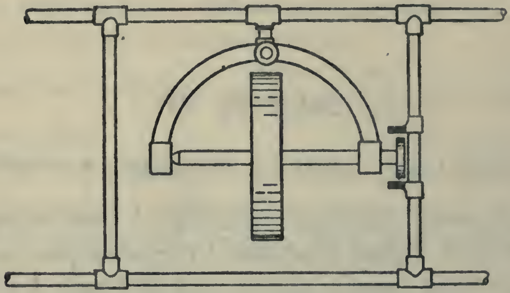

Fig. 13.—Diagram of Direct Control.

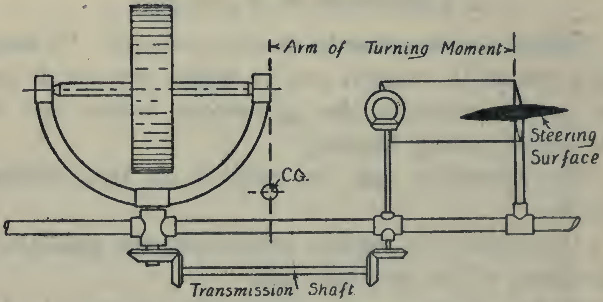

Fig. 14.—Diagram of Indirect Control.

Unfortunately, this precession will also occur if the machine deviates laterally (rotates in a horizontal plane about a vertical axis), because the fly-wheel spindle will remain in the original direction. In order to avoid this, one of two things can be done, either—

(1) The gyroscope frame must be braked so that it cannot rotate when the machine is to be laterally turned; or

(2) *Two* gyroscopes must be used, *running in opposite directions*, a further gear-wheel being inserted in the train of one of them, so that if they rotate about the vertical axis in *the same direction* (as they will do when the machine swings out of its direct path in a lateral direction), the gears neutralise each other and do not turn the rudder, *i.e.* the gyroscope frames will lock each other so that they can only rotate in opposite directions. The same result will be obtained if the two frames be directly geared together by spur-wheels. This coupling of two gyroscopes is also largely the invention of Brennan.

The direct method is not open to the objection mentioned, as the frame is free to rotate in a horizontal plane about a vertical axis. If the frame plunges, the spindle of the fly-wheel remains in the same position (the frame rocking on the suspension), and the roller on the projecting end of the spindle comes in contact with a fixed surface. On this surface it grips by friction, forcibly precessing the spindle in a horizontal plane, and so producing a righting torque.

Both these mechanisms have the further objectionable feature of hindering the vertical steering ("trimming") of the machine, so that the trimming rudders must be of such a size and position that the automatic torques can be overcome.

As suggested in the writer's articles on "The Stability of Flying Machines" (*Engineering*, March to October, 1910), if lateral stability is required, as well as longitudinal stability, an independent gyrostatic control would be the best arrangement. Stability, of course (rotation about a vertical axis), is easily assured by the use of fins and rudders, so that no third dynamic control is necessary.

If a lateral control is used with indirect mechanism, the precessional motion must be transmitted to the ailerons, so that when the machine rolls to one side, the ailerons must be rotated (to reverse hands on each side of the machine) so as to produce a righting torque just as is now done by the aviator, there being a compensation for "rotation of course" if the ailerons are integral with the main surfaces (as in the Wright system).

With the Brennan system there is no difficulty of this kind, but independent controls of lateral and longitudinal stability will be necessary if a complete system is required.

It is important, however, to notice that if the longitudinal direction of the machine is automatically maintained constant, the angle of attack and "attitude" of the lifting surfaces will continually vary as the relative wind alters, so that the lift and resistance will also vary. In all probability the machine will have a periodic rise and fall or "heaving" motion, instead of the periodic angular oscillations of the uncontrolled type.

A similar complication in the lateral stability will occur when turning. At present the machines cant so as to receive a horizontal thrust equal and opposite to the centrifugal force. If there is a lateral control so that the cant cannot occur, this centripetal thrust must be wholly supplied by the pressure on the rudders. The increased resistance will, of course, cause a slight descent in turning as at present, but there will be no diminution of lift to increase this effect. Compensating surfaces must be arranged to allow for the difference in the speeds of the two sides of the machine during turning.

CHAPTER VIII

THE GYROSTATIC ACTION OF ROTARY ENGINES, FLY-WHEELS, AND PROPELLERS ON AEROPLANES

As is well known, many successful aeroplanes have been fitted with seven-cylindered "Gnome" engines, which act as their own fly-wheels, the cylinder and crank-case revolving about a fixed crank. Many engineers and physicists have pointed out that there would be an appreciable gyrostatic effect, and, although it has been stated by some aviators that it is not apparent, it will probably be worth while to briefly discuss the values likely to occur.

The polar moment of inertia of a "Gnome" engine weighing 100 lb. (say a 50-h.p. engine) is 2 or 3 mass-lb.-foot^5 units. The angular velocity varies from 500 to 1500 revolutions per minute, say 900 revolutions per minute = about 100 radians per second. Hence the angular momentum is about 250 units. If the machine plunges or swings laterally, a gyrostatic torque will be exerted on the crank and through the crank to the frame of the machine, of an amount which depends on the rapidity of plunge or swing. Steering round a curve 600 feet radius at 60 feet per second, the angular velocity of the axis of the machine is $\frac{1}{10}$ radian per second, and the gyrostatic torque will be some 25 foot-lbs. This torque will cause a rotation of the machine about a transverse axis (plunging or rearing) of very small amount, on account of the great moment of inertia of the machine about that axis.

Thus, this moment of inertia of the machine about a transverse axis may easily amount to 300 units, so that the angular acceleration is

$$\frac{\text{Torque}}{\text{Moment of inertia}}=\frac{25}{300}=\frac{1}{12}\text{ radian per second.}$$

If a complete circle be turned, the axis of the machine will plunge or rear so as to completely overturn (or rather would do so if the conditions do not alter). [*Proof.*—The circle is about 3600 feet circumference, so that the machine will go round in one minute = 60 seconds. Angular displacement = angular acceleration × time$^2 = \frac{1}{12} \times 3600 = 300$ radians.] This looks very formidable but, as a matter of fact, the aviator is continually having to adjust the directive organs so as to balance gust and other torques of more than 25 foot-lb. value, so that it is quite likely that the gyrostatic torque has not been noticed.

If the rotation of the motor is clockwise, looking from the back of the machine, and the steering is along a left-handed curve (anti-clockwise when seen from above), the gyrostatic torque causes plunging (clockwise motion seen from the right of the machine). This will cause the angle of attack to decrease, so that the advance of the clockwise-plunge might automatically balance the action. If the motor or curve is of different hand the gyrostatic torque causes rearing.

The propellers of aeroplanes and other flying machines will have a similar action. They are, however, generally light, so that their moments of inertia are small, the lightness being, however, to some extent neutralised by the larger radius of gyration. Their effects can be investigated in the same manner as those of the rotary engine referred to above. The pairing of the propellers, as in the Wright aeroplane, neutralises the gyrostatic effect.

The same problem occurs in connection with ships, if the rotating parts have a moment of inertia which bears an appreciable ratio to the minimum moment of inertia of the ship itself. The heavy rotors of steam turbines or electric motors will have a slight gyrostatic effect, but, generally speaking, it is almost negligible because of the great mass of the ship itself and the low values of the imposed precessions. It should, however, be noticed that the moments of inertia of turbine rotors are comparable with, or even more than, those of the Schlick gyroscopes, so that their effects would be appreciable, if it were not that their axes are parallel to the longitudinal axis of the ship, so that their torques (due to the precession of steering or pitching) tend to produce deviation of trim or course, and not rolling. The rolling motion of a ship is, of course, the most important, because it is about the axis which has the smallest moment of inertia related to it.

APPENDICES

APPENDIX A

DESIGN OF A GYROSCOPE

The following notes illustrate the design of a fairly simple gyroscope, such as may be employed for experimental purposes or, in conjunction with some of the previous described devices, for controlling stability.

The value of the moment of inertia is first computed, using the usual fly-wheel approximation.

A discussion of the necessary strength of the wheel and spindle follows, succeeded by consideration of the spindle and air friction.

Following this comes the question of the power necessary to keep the fly-wheel running, and the time during which it will continue after the current is cut off.

Specification.—6-inch fly-wheel of maximum polar inertia possible. Surface to be of minimum frictional resistance, and wheel balanced as far as practically possible.

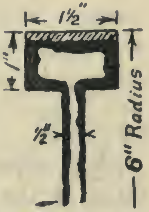

Fig. 15.—Rim of Fly-wheel.

Material: cast-iron, strengthened with a steel hoop. Section of rim about 1½ square inches.

Preliminary determination of polar inertia—

$$I = \Sigma(mr^2) = Mk^2.$$

Assume $R = k$.

Mass of Wheel.—$\pi \times$ radius $\times$ sectional area $\times$ unit mass

$$2 \times 3{\cdot}1416 \times 5\tfrac{1}{2} \text{ inches} \times 1\tfrac{1}{2} \text{ square inches} \times {\cdot}26 \div 32{\cdot}2.$$

$$\begin{array}{rl}
\log \pi = & 0{\cdot}4971 \\
\log 2 = & 0{\cdot}3010 \\
\log 5{\cdot}5 = & 0{\cdot}7404 \\
\log 1{\cdot}5 = & 0{\cdot}1761 \\
\log {\cdot}26 = & \bar{1}{\cdot}4150 \\
\cline{2-2}
 & 1{\cdot}1296 \\
\log 32{\cdot}2 = & 1{\cdot}5079 \\
\cline{2-2}
\log {\cdot}4185 \quad & \bar{1}{\cdot}6217 \\
\hline
\end{array}$$

say ·42. Mass is then rather less than ½ lb. (MASS).

Polar Moment of Inertia of Wheel.—

$$\begin{aligned}
\text{Moment of inertia} &= Mk^2 \\
&= {\cdot}42 \times (5{\cdot}5 \div 12)^2 \\
&= {\cdot}42 \times 5{\cdot}5 \times 5{\cdot}5 \div 144
\end{aligned}$$

$$\begin{array}{rl}
\log {\cdot}42 = & \bar{1}{\cdot}6217 \\
\log 5{\cdot}5 = & 0{\cdot}7404 \\
\log 5{\cdot}5 = & 0{\cdot}7404 \\
\cline{2-2}
 & 1{\cdot}1025 \\
\log 144 = & 2{\cdot}1584 \\
\cline{2-2}
\log {\cdot}08792 = & \bar{2}{\cdot}9441 \\
\hline
\end{array}$$

say ·09.

[*Note.*—The difference between this and the accurate value of the moment of inertia is inappreciable, as the assumption that $K = R$ is balanced by the additional inertia of the web, boss, and motor rotor.]

This wheel should then have a polar moment of inertia of about $\frac{1}{10}$ in feet and lb. of mass units.

[*Note.*—Kinetic energy which can be stored at 500 revolutions per minute

$$= \tfrac{1}{2} I\omega^2 = \tfrac{1}{2} \times {\cdot}09 \times 4\pi^2 \times 500^2 \div 60^2$$

$$\begin{array}{rr}
\log \cdot 5 = & \bar{1}\cdot 6990 \\
\log \cdot 09 = & \bar{2}\cdot 9441 \\
\log 4 = & 0\cdot 6021 \\
\log \pi^2 = & 0\cdot 9943 \\
\log 500^2 = & 5\cdot 3980 \\
\cline{2-2}
 & 5\cdot 6375 \\
\log 3600 = & 3\cdot 5563 \\
\cline{2-2}
120\cdot 6 & 2\cdot 0812 \\
\cline{2-2}
\end{array}$$

The wheel should store about 120 foot-lbs. of energy at 500 revolutions per minute, and 12,000 foot-lbs. at 5000 revolutions per minute.]

Gimbal Mounting.—The wheel is to be accurately mounted on a steel spindle with conical ends. Concentric and attached to the spindle there is to be an armature in two sections, wound separately, the wires passing through the web of the wheel from one side to the commutator. Commutator to be fixed at end of spindle.

The spindle is to run in conical bearings in a gimbal ring, which will also carry the field-magnets and brush rocker for the motor. The field-magnets are to be in two sets corresponding to the armature. The gimbal ring is to carry, at right angles to the axes of the spindle, cone points to bear in cone-bearings on a second gimbal ring. This ring, again, must have attached a vertical rod, which is supported by a footstep-bearing with balls in the pedestal of the apparatus.

The terminals for the motor may be fixed to the inner gimbal ring, and connected by loosely coiled wire to the accumulators or other source of power, the connections being quite free to move, so that precession is not hindered. It will be preferable, however, to arrange for the cone-bearing of this ring to be ring-insulated, so that current may be passed in without any hindrance due to wires.

Strength of Wheel.—Required to know tension in rim at speed of 5000 revolutions per minute.

Centrifugal force due to unit length of circumference mass $m = m\omega^2 r$.

Do. for diameter $= 2r \times m\omega^2 r = 2m\omega^2 r^2$.

Double-rim tension $= 2m\omega^2 r^2$.

Rim tension $= m\omega^2 r^2$.

Mass per unit length = unit mass per unit volume × volume of inch length.

(Divide both sides by cross-section of rim.)

Tension per square inch = unit mass × $\omega^2 r^2$

$$= \frac{0{\cdot}26}{32 \times 12} \times 4\pi^2 \times 5000^2 \times 5{\cdot}5^2 \div 3600$$

$$\begin{array}{rl} \log 32{\cdot}2 = & 1{\cdot}5079 \\ \log 12 = & 1{\cdot}0792 \\ \hline & 2{\cdot}5871 \\ \hline \end{array}$$

$$\begin{array}{rl} \log {\cdot}26 = & \bar{1}{\cdot}4150 \\ \log 4 = & 0{\cdot}6021 \\ \log \pi^2 = & 0{\cdot}9943 \\ \log 5000^2 = & 7{\cdot}8980 \\ \log 5{\cdot}5^2 = & 1.4808 \\ \hline & 9{\cdot}8902 \\ \log 32{\cdot}2 \times 12 & 2{\cdot}5871 \\ \hline & 7{\cdot}3031 \\ \hline \end{array}$$

$$\begin{array}{rl} \log 7 = & 7{\cdot}3031 \\ \log 3600 = & 3{\cdot}5563 \\ \hline 5582 \text{ lb.} & 3{\cdot}7468 \\ \hline \end{array}$$

about 2 tons in the inch.

Problem: maximum speed attainable without exceeding strength of material—

$$T = m\omega^2 r^2 = 4m\pi^2 n^2 r^2$$

$$n^2 = \frac{T}{4m\pi^2 r^2}$$

[*Note.*—Breaking stress of cast-iron is about 20,000 lb.; of steel, say 70,000 lb.]

[Steel band shrunk on should not break under 50,000 lb. Allow factor of safety of 5. T = 10,000 lb.]

$$n^2 = \frac{10{,}000}{4 \times \dfrac{0{\cdot}26}{32{\cdot}2 \times 12} \times \pi^2 \times 5{\cdot}5^2}$$

$$= \frac{32{\cdot}2 \times 12 \times 10{,}000}{4 \times 0{\cdot}26 \times 9{\cdot}87 \times 5{\cdot}5^2}$$

$$\begin{array}{rr} \log 32{\cdot}2 = & 1{\cdot}5079 \\ \log 12 = & 1{\cdot}0792 \\ \log 10{,}000 = & 4{\cdot}000 \\ \hline & 6{\cdot}5861 \end{array}$$

$$\begin{array}{rr} \log 4 = & 0{\cdot}6021 \\ \log 0{\cdot}26 = & \bar{1}{\cdot}4150 \\ \log \pi^2 = & 0{\cdot}9943 \\ \log 5{\cdot}5^2 = & 1{\cdot}4808 \\ \hline \end{array}$$

$$\begin{array}{rr} & 2{\cdot}4922 \\ & 2\overline{)4{\cdot}0939} \\ \log 111{\cdot}4 & \overline{2{\cdot}0469} \end{array}$$

say 110 revolutions per second = 6600 revolutions per minute.

Fracture will probably occur when there is a stress of 50,000.

$$\begin{array}{rr} & 4{\cdot}0939 \\ & 0{\cdot}6990 \\ & 2\overline{)4{\cdot}7929} \\ 249{\cdot}1 & \overline{2{\cdot}3964} \end{array}$$

say 250 revolutions per second = 15,000 revolutions per minute. This speed would be very dangerous.

To further refine the question of strength of the wheel, we will consider the case when it is considered as a disc, following Dr Chree's solution. We have two formulæ (see Perry's *Applied Mechanics*, or Ewing's *Strength of Materials*)—

$$p^1 = \frac{\omega^2\rho}{4}\left\{(3+\mu)r_1^2 + (1-\mu)r_0^2\right\} \text{ (with hole)} \quad . \quad (1)$$

$$p^1 = \frac{\omega^2\rho}{8} r_1^2(3+\mu) \text{ (solid)} \quad . \quad . \quad . \quad . \quad (2)$$

where ω is angular velocity,
ρ is density $\left(\frac{\omega}{g}\right)$,
r_1 is radius of disc,
r_0 is radius of hole in disc,
μ is reciprocal of Poisson's ratio.

If T is not to exceed 10,000 lb. we have

$$10{,}000 = \frac{\omega^2 \times 0{\cdot}26}{12 \times 32{\cdot}2 \times 4}\left\{(3+\tfrac{1}{3})6^2 + (1-\tfrac{1}{3}){\cdot}25^2\right\}$$

$$\omega^2 = \frac{10{,}000 \times 12 \times 32{\cdot}2 \times 4}{0{\cdot}26(3\tfrac{1}{3} \times 36 + \tfrac{2}{3} \times {\cdot}25^2)}$$

$$= \frac{120{,}000 \times 128{\cdot}8}{0{\cdot}26 \times 120{\cdot}0416}$$

$$\begin{array}{rr} \log 120{,}000 = & 5{\cdot}0792 \\ \log 128{\cdot}8 = & 2{\cdot}1099 \\ \hline & 7{\cdot}1891 \end{array}$$

$$\begin{array}{lr} \log.\ 0{\cdot}26 = \bar{1}{\cdot}4150 & \\ \log 120{\cdot}04 = 2{\cdot}0793 & 1{\cdot}4943 \\ \log \omega^2 & 2)\overline{5{\cdot}7948} \\ \log \omega & \underline{2{\cdot}8974} \end{array}$$

$$\begin{array}{r} {\cdot}25 \\ {\cdot}25 \\ \hline 125 \\ 50 \\ \hline 3)\overline{{\cdot}0625} \\ {\cdot}0208 \\ 2 \\ \hline {\cdot}0416 \end{array} \qquad \begin{array}{r} 36 \\ 3\tfrac{1}{3} \\ \hline 108 \\ 12 \\ \hline 120 \\ 0{\cdot}0416 \\ \hline 120{\cdot}0416 \end{array}$$

$$\omega = 2\pi n \qquad n = \frac{\omega}{2\pi}$$

$$\begin{array}{rr} \log \omega = & 2{\cdot}8974 \\ \log 2\pi = & 0{\cdot}7982 \\ \hline & 2{\cdot}0992 \end{array}$$

$$\begin{array}{r} 125{\cdot}7 \\ 60 \\ \hline 7542{\cdot}0 \end{array}$$

Thus, about 7500 revolutions per minute are permissible when the strength of the web is considered. Flaws in the casting would, of course, greatly reduce the maximum safe speed.

Strength and Stiffness of Spindle.—Referring back, we see that the weight of the rim is 13·48 lb. (log is 1·1296; see calculation for mass). If we add 25 per cent. for mass of web and boss, we have say 17 lb. It is almost unnecessary to here calculate the size of the spindle, as the strength will be sufficiently assured by adopting a size convenient for properly attaching the armature and wheel; but assuming a length of 6 inches for the spindle, and taking the load as on a beam merely supported at the ends, we have

$$\Delta = \frac{1}{48}\,\frac{WL^3}{EI}$$

(take inch-lb. units)

$$= \frac{1 \times 17 \times 6 \times 6 \times 6}{48 \times 30{,}000{,}000 \times I}.$$

For a circular section $I = \frac{\pi R^4}{4}$. Take $R = {\cdot}25$ inch for practical construction, then we have

$$\Delta = \frac{17 \times 6 \times 6 \times 6 \times 4}{48 \times 30{,}000{,}000 \times 3{\cdot}14 \times {\cdot}25 \times {\cdot}25 \times {\cdot}25 \times {\cdot}25}.$$

$$\begin{array}{rr}
 & \log 17 = 1{\cdot}2304 \\
 & \log 6^3 = 2{\cdot}3346 \\
 & \log 4 = 0{\cdot}6021 \\ \cline{2-2}
 & 4{\cdot}1671 \\
\log 48 = 1{\cdot}6812 & \\
\log 30 \text{ millions} = 7{\cdot}4771 & \\
\log \pi = 0{\cdot}4971 & \\
\log {\cdot}25^4 = \bar{3}{\cdot}5916 & 7{\cdot}2470 \\ \hline
\log {\cdot}000832 = & \underline{\bar{4}{\cdot}9201}
\end{array}$$

say $\frac{1}{1000}$ inch, which is quite inappreciable.

Friction at Bearings.—With regard to the bearings, we have to consider that, according as the spindle is vertical or horizontal, so the pressure will come upon them rather differently. We may however assume, without great error, that when the spindle is horizontal, the lower half of each conical bearing is in much the same condition of pressure as the whole lower bearing when the spindle is vertical.

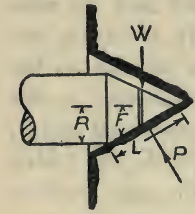

Fig. 16.—Cone Bearing.

The normal pressure on this one half is $\frac{\omega}{2\cos\theta}$, and the area of half the conical surface being $\frac{\pi R_1 L}{2}$ (for area of cone $= \pi R_1 L$), and since

$$L = \frac{R_1}{\sin\theta},$$

we have

$$\text{area} = \frac{\pi R^2}{2\sin\theta}.$$

$$\text{Pressure in unit area} = \frac{\text{Total pressure}}{\text{Total area}}$$

$$= \frac{\omega}{2\cos\theta} \div \frac{\pi R^2}{2\sin\theta} = \frac{\omega\tan\theta}{\pi R^2} = \rho.$$

Total pressure on elemental ring dl wide $= 2\pi rp \,.\, dl$,

and since $$dl = \frac{dr}{\sin\theta},$$

we have friction moment on each strip is

$$\frac{2\pi r^2 \mu p \,.\, dr}{\sin\theta}.$$

Integrate

$$\int^{R_1} \frac{2\pi r^2 \mu p \,.\, dr}{\sin\theta} = \frac{2\pi\mu p}{\sin\theta}\int_0^{R_1} r^2 \,.\, dr = \frac{2\pi\mu p R^3}{3\sin\theta}.$$

Substituting again for p, we have

$$\frac{2\pi\mu W R_1^3 \tan\theta}{3\pi R^2 \sin\theta} = \frac{2\mu W R_1 \tan\theta}{3 \sin\theta} = \frac{2\mu W R}{3 \cos\theta}.$$

In this case it is proposed to make $R = 0{\cdot}25$ inch, μ with good lubrication need not exceed ·01, and θ say 30°, so that we have

$$\text{Moment of friction} = \frac{2 \times {\cdot}01 \times 17 \times {\cdot}25}{3 \times {\cdot}866}.$$

$$\begin{array}{rcr}
 & & \log 2 = 0{\cdot}3010 \\
 & & \log {\cdot}01 = \bar{2}{\cdot}0 \\
 & & \log 17 = 1{\cdot}2304 \\
 & & \log {\cdot}25 = \underline{\bar{1}{\cdot}3979} \\
\log 3 = 0{\cdot}4771 & & \bar{2}{\cdot}9293 \\
\log {\cdot}5 = \underline{\bar{1}{\cdot}9375} & & \underline{0{\cdot}4146} \\
 & \log {\cdot}033 & \underline{\underline{\bar{2}{\cdot}5147}}
\end{array}$$

About ·033 lb.-inch moment of friction; or, if we suppose μ may rise to 0·02, friction moment is ·07.

Variation of Friction with Speed.—Although the friction at the cones is very small, it may be worth while to consider the manner in which it may vary at different speeds when lubricated.

At low pressures the frictional resistance varies directly as the speed. Since the load in lb. per square inch of the bearing will be quite small in this mechanism, we may therefore assume that the frictional resistance will vary in this way.

From Mr Beauchamp Tower's experiments it would appear that the coefficient of friction at 200 feet per minute is about twice the initial (dynamic) friction, at 400 three times, at 600 four times.

The speed attained by the periphery of the journal cannot exceed $2\pi rn$ where r is outside radius of the bearing (0·25 inch).

$$\begin{array}{rr} & \log 2 = 0{\cdot}3010 \\ & \log \pi = 0{\cdot}4972 \\ & \log 0{\cdot}25 = \bar{1}{\cdot}3979 \\ & \log 5000 = 3{\cdot}6990 \\ \hline \text{inches } 7554{\cdot}0 & 3{\cdot}8951 \\ & \log 12 = 1{\cdot}0792 \\ \hline \text{feet per minute } 653{\cdot}1 & \underline{2{\cdot}8159} \end{array}$$

Taking the previously calculated moment of friction as the initial of dynamic friction (it is probably in excess of this), we can assume that there will be about the following values:—

Speed (revolutions per minute).	Moment of Friction.
0	·07 lb.-inches.
2500	·17 ,,
5000	·28 ,,

Air Friction.—Next must be considered the air friction against the fly-wheel.

We have in our disc four surfaces to surface (neglecting the central boss)—

(1 and 2) The sides.
(3) The rim surface.
(4) Inner rim surface.

Air friction follows the law $F = fav^2$ very nearly, so that on an element dr wide and r radius rotating with a speed ω we have

$$fav^2 = f \,.\, 2\pi r \,.\, dr \,.\, \omega^2 r^2$$

and

$$\text{torque} = fav^2 r.$$

For the sides we have, integrating

$$F_1 = \int^R 2f\pi r \,.\, dr \,.\, \omega^2 r^2 = 2\pi f \omega^2 \int_0^R r^3 \,.\, dr = \frac{f\omega^2 \pi R^4}{2},$$

or for both sides

$$F_{1+2} = f\omega^2\pi R^4$$

or

$$\text{torque} = \frac{4f\omega^2\pi R^5}{5}.$$

The rim surfaces have a constant radius. The outer one gives us

$$F_3 = fav^2 = f \,.\, 2\pi Rb \,.\, \omega^2 R^2 = 2\pi f\omega^2 R^3 b$$

and

$$\text{torque} = 2\pi f\omega^2 R^4 b,$$

and the inner surface gives similarly

$$F_4 = fav^2 = f \,.\, 2\pi R_0 c \,.\, \omega^2 R_0{}^2 = 2\pi f\omega^2 R_0{}^3 c$$

and

$$\text{torque} = 2\pi f\omega^2 R_0{}^4 c.$$

Altogether we have

$$f\omega^2\pi R^4 + 2\pi f\omega^2 R^3 b + 2\pi f\omega^2 R_0{}^3 c = f\omega^2\pi(R^4 + 2R^3 b + 2R_0{}^3 c)$$

and

$$\text{torque} = f\omega^2\pi\left(\frac{4}{5}R^5 + 2R^4 b + 2R_0{}^4 c\right)$$

f seems to be as nearly as can be ascertained at present ·000002 lb.-foot-units,* so that we can now find what air-friction moment must be overcome by the motor at any speed.

We will take the maximum speed desired at say 5000 revolutions per minute (it will then store about 12,000 foot-lbs. of kinetic energy).

The web may be assumed as ½ inch thick, so that $b = 1\frac{1}{2}$ inches and $c = 1$ inch.

---

* This quantity is in great dispute, and a somewhat lower value is given in the text, but for design it will be well to err on the side of excess.

Total air-friction resisting moment

$$=f\left(\frac{4}{5}\mathrm{R}^5+2\mathrm{R}^4b+2\mathrm{R}_0{}^4c\right)\pi\times\omega^2,$$

or, since $\omega=2\pi n$ (where n is revolutions per second) $=\dfrac{\pi\mathrm{N}}{30}$ (where N is revolutions per minute), for this wheel the moment is

$$\frac{2}{1{,}000{,}000}\left[\frac{\left(\frac{4}{5}\times6^5\right)+(2\times6^4\times1\tfrac{1}{2})+(2\times5^4\times1)}{12^5}\right]\frac{\pi^3\mathrm{N}^2}{900}$$

$$=\frac{1}{500{,}000}\times\frac{(6220+3888+1250)}{248{,}832}\times\frac{31\times\mathrm{N}^2}{900}.$$

This can be written sufficiently nearly as

$$\frac{11{,}500}{500{,}000\times250{,}000\times30}\times\mathrm{N}^2.$$

$$\begin{array}{rr}
 & \log 11{,}500=4{\cdot}0607\\
\log 500{,}000=5{\cdot}6990 & \\
\log 250{,}000=5{\cdot}3979 & \\
\log 30=1{\cdot}4771 & \\
\hline
12{\cdot}5740 & 12{\cdot}5740\\
\cline{2-2}
 & \bar{9}{\cdot}4867
\end{array}$$

so that the torque is

·000000003 N^2 *foot*-lb.,

or ·00000004 N^2 *inch*-lb.

At 5000 revolutions per minute the torque is 1 inch-lb., which is more than the friction moment at the bearings, so that the total torque at 5000 revolutions per minute is upwards of 2 inch-lbs., including air friction on armature (supposed to be smoothly formed), and eddy resistance about the field magnets, etc.

Power required.—Using the rule

$$\frac{\text{torque} \times \text{angular velocity}}{550} = \text{horse-power},$$

we have

$$\frac{\frac{2}{12} \times \frac{2\pi \times 5000}{60}}{550} = \text{about } \tfrac{1}{6} \text{ horse-power.}$$

The acceleration of the fly-wheel will depend to some extent on the design of the electric motor. At zero speed the acceleration will be as indicated in the first chapter, but as the induction and velocity increases, the resisting torque, due to the distortion and rupture of the magnetic field, will steadily increase until the final speed is reached at which the motor torque is balanced by the frictional torque, and also that due to the back E.M.F.

When the current is cut off, there is only the frictional torque resisting the motion, plus a slight magnetic torque due to the residuous magnetism of the field magnets. If we say there is a mean resisting torque of 3 lb.-inches, then the angular retardation in radians per second per second is

$$a = \frac{\frac{3}{12}}{\cdot 09} = \frac{3}{12} \times \frac{100}{9} = \frac{100}{36} \text{ say } 3.$$

If the running velocity is, as assumed, 5000 revolutions per minute, or 500 radians per second, the wheel will run for $\frac{500}{3} = 166$ seconds, or say 3 minutes at least. As a matter of fact, it will run much longer than this, because the resisting torque decreases as the speed decreases. If the wheel runs in a vacuum, and is at a lower speed (say 300 revolutions per minute or so), the resisting torque is almost inappreciable (say $\frac{1}{10}$ of an inch-lb.), and the retardation falls to

$$a_1 = \frac{\frac{1}{10} \times \frac{1}{12}}{\cdot 09} = \frac{1}{120} \times \frac{100}{9} =$$

say $\frac{1}{10}$ of a radian per second per second, which, if it does not alter, will need 500 seconds to stop the motion, or say 10 minutes. If the time be worked out by the calculus, it will be found to total to an hour or even more in most such cases.

APPENDIX B

MATHEMATICAL THEORY OF THE GYROSCOPE

The difficulties in connection with the analysis of gyroscopic motion arise from the complicated geometry; and it is necessary to understand the use of spherical co-ordinates and the transformation of axes before the principles of the conservation of energy and momentum can be applied to give a general solution. If the gyroscope be initially in a position in which the axis of rotation (the spindle), the axes of precession (bearings of the second gimbal), and the axis of the torque (bearings of the first gimbal), are all at right angles to one another, we can express in a fairly simple way the velocities and displacements of the various parts with reference to these three axes; but as the motions proceed the axes cease to be rectangular to one another, and it is necessary to conceive of two sets of axes, one rigidly fixed in space and another which moves with the gyroscope. The motions of the various parts are still easily expressible in terms of the moving axes, but it is necessary to geometrically relate these motions to the fixed axes if the kinetic energy and momentum are to be in an useful form.

The famous "Equations of Lagrange" are next employed, one for each degree of rotational freedom (*i.e.* three), and from these differential equations can be constructed which relate together the moments of inertia about the three axes, the displacements, and the angular velocities.

An alternative method is to enter the quantities relative to the two systems of axes in "Euler's equations," which show the relations between the moments of inertia, torques, angular accelerations, and angular velocities, and from these can be written expressions for the total energy and total angular momentum. These quantities, except for frictional losses, being constant, mutual relations can be deduced which will enable any problem of three-dimensional rotation to be solved. The various cases will be found exhaustively discussed in Kelvin and Tait's *Natural Philosophy.*

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