Practical Logic;

or, the

ART OF THINKING.

A Text-Book for Schools and Colleges,

BY

D. S. GREGORY, D.D.,
PRESIDENT OF LAKE FOREST UNIVERSITY.

New York and Philadelphia
HINDS, NOBLE & ELDREDGE

BC 108
G 72

MOFFITT

PREFACE.

⟶•◦✸◦•⟵

NEXT to right and noble living, which is the highest thing
to which man may aspire, may be placed the right think-
ing which is essential to such living. Logic, as the Science
of the Laws of Thought, is very widely studied, in the higher
schools, as an aid to the pupil in thinking; yet it is the set-
tled conviction of many of the best educators that this Sci-
ence, as it is ordinarily presented, does very little toward
training to think or preparing for thinking. In short, there
seems to be a growing feeling that it rather serves, in case
of the average mind, to cram the memory and paralyze the
thinking powers. The author of this volume shares to some
extent in this conviction and feeling; hence the present
attempt to construct a Practical Logic, by the use of which
intelligent teachers may train inquiring minds to correct
thinking.

The only way to learn to think is by thinking; the only
way of training a pupil to think is by making him practise
thinking. Assuming the correctness of this principle, Logi-
cal Praxis is made the prominent and essential feature of
the work. Each principle of thought is turned into a Rule,

iii

and then made part of the mental property and power of the student by abundant exercises.

The best training in thinking must be intelligent and systematic. Accordingly the foundation for this is laid by a comprehensive and systematic presentation of the forms and laws of thinking. The processes of formation and unfolding, of involution and evolution, are presented in succession. Beginning with the simplest process of observation, the praxis is carried, by successive stages, up to the highest and most complex processes of constructive thinking, and the mind capable of such work trained intelligently and systematically to clear, distinct, connected, continuous, and constructive thought.

To the various writers on the subject of Logic, the author would acknowledge his indebtedness, and especially to Ueberweg, Hamilton, Thomson, Whately, Mill, Jevons, Atwater, McCosh, Davis, Bowen, and Day.

To teachers he would suggest that Part I. may be used in the earlier stages of training, and the remaining parts reserved for a later stage. In the use of the text-book the teacher will ordinarily do his best work for his pupil by drawing largely on his own resources for material for praxis. Each locality, school-room, branch of study, and experience will suggest innumerable topics of fresh and living interest, which may be profitably substituted for those given in the text-book.

D. S. GREGORY.

LAKE FOREST UNIVERSITY,
LAKE FOREST, ILL., August, 1881.

CONTENTS

INTRODUCTION.

PART I.

The Logic of Conception or the Term.

CHAPTER I.

PART II.

The Logic of Judgment or the Proposition.

CHAPTER I.

THE FORMATION OF JUDGMENTS OR PROP-OSITIONS 93

CHAPTER II.

THE UNFOLDING OF JUDGMENTS. . 121

———∞∘‡∘∞———

PART III.

The Logic of Reasoning or the Syllogism.

CHAPTER I.

CHAPTER II.

———◦◦ↂ◦◦———

PART IV.

The Logic of Construction or the System.

CHAPTER I.

THE FORMATION OF CONSTRUCTION OR SYSTEM 200

CHAPTER II.

THE UNFOLDING AND TESTING OF SYSTEMS. 207

———◦◦ↂ◦◦———

PRACTICAL LOGIC.

———oo;o;oo———

INTRODUCTION.

I. THE NATURE OF LOGIC.

What is Logic? — This question has been variously answered. Whately says it may be considered as "the Science and also as the Art of Reasoning." Hamilton defines it as "the Science of the formal and necessary Laws of Thought as Thought." Dr. Watts called his work, "The Art of Thinking." According to his view, "Logic is the art of directing the reason aright in acquiring the knowledge of things, for the instruction both of ourselves and others."

Without stopping to discuss these definitions, which would be unintelligible to the ordinary student at the outset, it is clear that these different authors must either be defining different things, or defining the same thing from different points of view, or defining different things from different points of view. "Reasoning" and "thought" are different things, the former being only one form of the latter. The points of view of science and art are different, the one being theoretical, the other practical.

Definition.— As treated in the present work, Logic is the

9

Practical Science of the Principles or Laws which govern the various forms of correct Thinking or Thought.

This definition will be best explained by considering the following Topics:

Topic I.—The Object-Matter of Logic.

Topic II.—The Practical Aim of Logic.

Topic III.—The Principles or Laws of Thinking or Thought.

Topic First.— The Object-Matter of Logic is found in the Forms of Thinking or Thought.

1. What is Thinking or Thought?

(1.) In a **loose sense,** any operation of the human soul is sometimes spoken of as thought. The man knows, feels and purposes or wills; any act of knowing, feeling, or willing may, in this loose sense, be called thinking. The term is evidently not used in this loose sense in the definition of Logic.

(2.) In a **stricter sense,** thinking or thought is confined to the operations of the intellect or power of knowing. In popular phrase, it is any act of the head as distinguished from the heart and will of the man. In this sense, "thinking" is synonymous with "knowing."

The main office of the human intellect is **to know,** *i. e.,* first, to apprehend objects in themselves and their phenomena or attributes; and secondly, to apprehend objects or their phenomena in their connections or relations. The first of these forms may be termed *simple apprehension,* or intuition, or simple knowledge; the second, *thought-knowledge,* or thought. Logic has to do with thought-knowledge, or thought.

To state the same thing in another and fuller form, the **cognitive power** or **intellect** of man performs its entire office of knowing in four different ways; in other words, it has four different faculties:

1st. The intellect **acquires** the fundamental facts of knowledge of things material and spiritual by the senses and consciousness; and has, therefore, a simple cognitive faculty. Its office is to gather the material for the use of the higher faculties of thought.

2d. The intellect **keeps** the acquired knowledge in such shape as to be able to reproduce it for use at any time when it may be needed by the higher faculties; and has, therefore, a conservative faculty or memory.

These two faculties furnish knowledge and keep it at command for use, and their operations are often spoken of, in a loose sense, as **thought**; but, using thought in the strict sense, it is often truly said of one who uses these two powers with great ease, "He never had a thought in his life. He is a mere man of memory."

3d. The intellect **compares** the knowledges acquired and conserved, and connects them into conceptions, judgments, and arguments; and has, therefore, a comparative, or elaborative, faculty.

4th. The intellect **groups in systems,** according to the law of the true, the beautiful, or the good, the knowledges acquired by the simple cognitive faculty, kept and reproduced by the conservative faculty, and connected in thought by the comparative faculty; and has, therefore, a system-making, or constructive, faculty. This is also discursive.

The term "thinking" or "thought" is often applied to all four forms of intellectual action. This is evidently not the sense in which it is used in the definition of Logic.

(3.) Thought or thinking, **strictly speaking,** is the operation or product of the operation of the third and fourth faculties only, *i. e.*, of the **comparative** and **constructive faculties** only. These faculties are the **thought faculties;** their operation is **thinking;** and the product of their operation is **thought.** Ordinarily, the word *thought* is used for any or all three: the faculty, its exercise, its product. These faculties are also called **discursive,** since they proceed from simple knowledges to new results founded upon them.

2. What are the Forms of Thinking or Thought?

The forms of thinking or thought are the forms in which the discursive or thought faculties act, or the products of that action.

(1.) As thinking is embodied in **language,** the most common forms of thought may be learned by an examination of thought expressed in language. Take the following example: Light is opposed to darkness; feathers are light; therefore, feathers are opposed to darkness.

This is in the form of a syllogism. A syllogism embodies an argu‑ment, or process of reasoning. In it two propositions are compared and a conclusion reached which is expressed in a third proposition. This is thought as **reasoning**.

These three propositions are verbal expressions of judgments, in which the mind compares and connects two terms. This is thought as **judgment.**

Each of the terms in these propositions is a thought, and must be understood, as is shown by the example given, if any correct think‑ing is to be done. This is thought as **conception.**

When a series of terms, propositions, arguments, etc., is grouped together to make a larger whole of thought, the result is thought as **system.**

(2.) Or looking at the subject from the **thought side,** instead of from the language side, the same result is reached.

The **comparative faculty** acts in three ways: a. By comparing the objects or knowledges, given by the simple cognitive faculty and re‑tained by the conservative faculty, and connecting them by resembling attributes or marks, thus forming notions or concepts, classes and general terms. This is thought as **conception.** b. By comparing concepts or general terms and connecting them by agreement or dis‑agreement, resulting in judgments and propositions. This is thought as **judgment.** c. By comparing judgments or propositions and connect‑ing them by the principle of reason and consequent, resulting in argu‑ments including syllogisms. This is thought as **reasoning.**

The **constructive faculty** also acts in three ways: a. Grouping by the Law of the True, or in the form of scientific construction, result‑ing in **scientific systems.** b. Grouping by the Law of the Beautiful, or in the form of artistic or æsthetic construction, resulting in **artistic,** or **æsthetic, systems,** including all art productions. c. Grouping by the Law of the Good, or in the form of practical construction, result‑ing in **practical systems,** including inventions, plans of conduct, etc.

It will at once be seen that the second of these forms of construc‑tion falls within the sphere of Æsthetics, leaving only the first and third in the sphere of Logic.

The distinct **Forms of Thought** with which Logic deals are, therefore, as follows:

Conception, embodied in the **general term;**

Judgment, embodied in the **proposition;**

Reasoning, embodied in the **argument ;**

Construction, embodied in **system, scientific** and **practical.**

The facts concerning the workings of the Human Soul may be tabulated so as to present their relations to the eye.

THE HUMAN SOUL, or Man, the Conscious Subject,	Knows, and, therefore, has a **Cognitive Power,** or Intellect ; Feels, and, therefore, has an **Emotive Power,** or Sensibility ; Wills, and, therefore, has a power of Endeavor, or **Conative Power,** or Will.	Thought in Widest sense.
Man, by the COGNITIVE POWER,	Acquires knowledges by the **Simple Cognitive Faculty ;** Keeps knowledges by the **Conservative Faculty,** or Memory ; Compares knowledges, or works out their Relations by the **Comparative Faculty ;** Constructs knowledges into Systems by the **Constructive Faculty.**	Thought in the Middle sense.

MAN, by the DISCURSIVE Faculties,	Compares, by the **Comparative Faculty,**	Simple knowledges, in **Conception ;** Conceptions, in **Judgment ;** Judgments, in **Reasoning.**	Thought in the Strict sense.
	Constructs, by the **Constructive Faculty,**	The True, in **Scientific System ;** The Beautiful, in **Æsthetic System ;** The Good, in **Practical System.**	

Topic Second.—The Practical Aim of Logic is to train to Correct Thinking or Thought.

Logic, as here treated, is a **practical science,** aiming to lead the thinker to a systematic knowledge of the laws of

2

thought, **in order** to find in these the rules by which **to train to skill in right thinking.**

1. What is Practical Science or Art?

A **science** is a complete and systematic presentation of the facts and principles in any sphere of knowledge, in accordance with truth. Hamilton draws from Aristotle the distinction between Philosophy "Theoretical" and "Practical." "**Theoretical,** called likewise speculative and contemplative, **philosophy** has for its highest end mere truth or knowledge. **Practical philosophy,** on the other hand, has truth or knowledge only as its proximate end,—this end being subordinate to the ulterior end of some practical action." Notwithstanding Hamilton's objections to the expressions, they are in common use, and will doubtless continue in use. "**Science**" and "**Art**" are also often used to express substantially the same distinction.

The sciences and arts are both systematic forms of human knowledge. The aim of a science is to give systematic knowledge of something; that of an art, to give skill in doing something. The one calls for the study of scientific principles; the other for the intelligent application to practice of rules based upon these principles. A science presents truths to be grasped; an art, exercises to be performed.

Practical Science, or Art, as it is sometimes called, is a form of science in which the systematic knowledge of the subject treated is subordinate to the training to skill in some activity.

2. How far is Logic Theoretical and how far Practical?

Logic is a **theoretical science,** or science proper, so far as it aims to give a systematic view of the laws of thought; it is a **practical science,** or art, so far as it subordinates this to its aim to train to skill in applying the laws of thought in avoiding error and arriving at truth. From the **scientific** side, Logic should present in systematic shape the laws which govern the various forms of thought, or the laws by which the mind must be governed when it thinks correctly, *i. e.,* when it conceives, judges, reasons, systematizes correctly. From the **practical** side, Logic should turn these

laws into rules and train the mind to think correctly and efficiently, by training it to use these rules of thought intelligently and skilfully. It should, if it is to be of the most service, train the thinker at once to accuracy of thought in reaching truth and avoiding error, and to power in using the thought-faculties,— in other words, it should train both to skill and power. In accordance with this view, Professor Bain remarks, that although " Logic, no doubt, has a certain theoretic aspect, . . . its chief aim must ever be practical. Had the subject not been wanted as an aid to the search of truth, it would never have been called into existence."

3. Logic aims at Correct Thinking, or at Truth.

Logic is defined, in the " Port Royal Logic," as " the science of the operations of the understanding in the pursuit of truth." Logic aims at **correct thinking.** Such thinking is, from one point of view, thinking that is done in accordance with the laws of thought which are treated in works on Logic. From another point of view it is thinking which, by conformity to the laws of thought, arrives at **truth.**

(1.) **Truth.**—In order to understand the meaning of these statements concerning truth, there is need of considering: the nature and criterion of truth; the modes of arriving at truth; the degrees of assurance in the grasp of truth.

a. The Nature and Criterion of Truth.

According to Hamilton, **truth** is " the correspondence or agreement of a cognition with its object." Or, including both thought and statement, truth is the agreement of a thought or statement with the reality which the thought or statement concerns. **Error** is the opposite, or the want of harmony between a thought or statement and its object.

The **criterion,** or test of truth, arises out of its nature as thus stated. Does it correspond with the reality ? " Man is mortal." " The sun shines." " Madagascar is inhabited." " The earth is spherical." In deciding whether these statements are true, the question to be asked of each is, *Does it agree with fact or reality ?*

b. Modes of Arriving at Truth.

The truth in any given case is arrived at in one or other of two ways:

First, by the use of one's own powers intuitive or discursive. These give knowledge in the narrower sense. The **intuitive powers** furnish immediate knowledge, or *a priori* knowledge: (*a.*) By external or sense perception, of matter and its phenomena; (*b.*) By internal perception or self-consciousness, of spirit and its operations; (*c.*) By intuition proper, of the self-evident and necessary notions and principles which underlie and condition all human experience. The **discursive powers** furnish mediate or *a posteriori* knowledge by the processes of thought, conception, judgment, reasoning, and construction.

Secondly, by acceptance of the statements of others. These give *belief* in the narrow and strict sense, or the acceptance of truth on the ground of testimony. Most of man's knowledge in the wide sense, and that the most valuable part of it, is derived from this source. The witnesses gain their knowledge either: (*a.*) By the use of their intuitive powers, which lays the foundation for **testimony proper**; or, (*b.*) By the use of their discursive or thought powers, which lays the foundation for **authority.**

c. Degrees in the Assurance of Truth.

The mind does not lay hold of all its knowledge with the same degree of certainty. Distinction is made between *belief, opinion, probable truth, certain truth.*

Certainty is the consciousness of the necessity of agreement between a thought and its object, in whichever of the above ways that thought may be reached. It absolutely excludes the admission of any opposite supposition. Where this is not the case, **doubt** and **uncertainty** arise.

Considered with reference to the degree of certainty, there appear, at the two extremes:

Knowledge, in the strictest sense, where the consciousness of necessity is absolute, or certainty perfect;

Opinion, or the admission of something where the evidence is not such as to necessitate a perfect certainty.

Probability appears in the approximation of the imperfect certainty of opinion to the perfect certainty of knowledge.

Belief is used in various loose senses, but the distinction given above will, it is thought, commend itself as the fundamental sense. Belief is the acceptance of truth on the ground of testimony, including testimony proper and authority.

(2.) Truth by Thought.—The aim of Logic is to arrive at truth through the powers of thinking or thought. The grasp of the truth reached will evidently depend upon the kind of truth and the accuracy of the thinking. Correct thinking will give a more or less certain grasp of the truth reached as the result of it. In mathematical and intuitive truth the result reached is absolutely certain. In other regions of thought the results of thought are more or less probable. These varying degrees of certainty may be illustrated by examples. It is *certain* that two and two cannot but make four ; that things which are equal to the same thing are equal to each other ; that every event must have a cause. It is *probable* that the first day of January, 1900, will be cold. It is extremely probable that the sun will rise to-morrow. It is probable that a young man of good capacity, character, and habits will succeed in business. It is the *opinion* of certain astronomers that the moon is uninhabited. It is the *belief* of most intelligent men that the earth is about 93,000,000 miles from the sun.

Topic Third.—The Principles or Laws of Thinking or Thought.

Logic deals with the principles or laws which govern thought.

Every rational human activity proceeds according to definite laws, known or unknown. The highest degree of intelligence and efficiency in any such activity requires that the laws be known and correctly made use of in directing the activity. This is true of the various forms of thought; they have their laws which govern their action. There are laws of conception, laws of judgment, laws of reasoning, laws of construction. Logic should enable the thinker to ascertain and apply these laws, and thus aid him in correct thinking and save him from incorrect thinking. It is likewise true that the highest degree of intelligence and efficiency in thinking requires a thorough knowledge of the laws of thought and a correct use of them in guiding the exercise of thought. Practical Logic should aim to give the thinker the most thorough knowledge of the laws and the greatest efficiency in using them in thinking.

Besides the **special laws** which govern the various forms of thought, there are certain **general laws,** certain axioms or fundamental laws and certain postulates with which Logic sets out. The special laws will be unfolded in con-

nection with the treatment of the various forms of thought; at the outset must be presented the fundamental laws and postulates.

1. The Fundamental Laws of Thought.

Logic, like other sciences, has certain fundamental principles upon which the more special laws rest. These are usually reduced to four:

The Law of Identity, or Affirmation;

The Law of Contradiction, or Negation;

The Law of Excluded Middle, or Exclusion;

The Law of Reason and Consequent, or Sufficient Reason.

(1.) The Law of Identity, or Affirmation.

The Law of Identity may be stated as follows: Everything is identical with itself, or is what it is, and we may affirm this of it. This has been formulated: A is A; or A = A. Whatever is, is.

The identity may be: *a. Absolute*, or that of total sameness of a thing or thought with all its parts; or, *b. Relative*, or that of partial sameness of a thing or thought with each or some of its parts. The logical concept or notion expressed by the general term, *man*, is made up of the following elements: being, material, organized, animated, rational, terrestrial. *Man* is, therefore, totally identical with all these elements; so that it may be correctly affirmed that *man* is material, organized, animated, rational, terrestrial being. *Man* is partially identical with any of these elements; so that it may be correctly affirmed that *man* is material; *man* is organized, etc.

The Law of Identity gives the logical right to affirm such total or partial identity in all cases where it exists. It is at the basis of all consistent **affirmative thinking,**— of all positive conceptions, logical definitions, affirmative judgments, and categorical arguments.

(2.) **The Law of Contradiction, Negation,** or, as Hamilton terms it, Non-contradiction, may be stated as fol-

lows: Everything is not what it is not, and we may affirm this of it. Or, conflicting attributes cannot co-exist in and may not be affirmed of the same object. This has been formulated: A isn't not -A. Nothing can both be and not be.

The logical concepts expressed in the following pairs of general terms are contradictories: black and not-black; round and not-round; good and wicked; finite and infinite. We are logically excluded from affirming the co-existence of these mutually contradictory thoughts or things. A thing cannot be *black* and *not-black* at the same time and in the same sense. A door cannot be *open* and *shut* (not-open) at the same time and in the same sense. Black-white, round-square, good-wickedness, finite-infinitude, combine mutual contradictories, and are, therefore, logically excluded from correct thought by this law.

The law of non-contradiction is the complement of that of identity. Its importance arises from the fact that it is at the basis of all logical **negation and distinction** in thought,— of all negative conceptions and judgments.

(3.) **The Law of Excluded Middle, or Exclusion,** may be stated as follows: Of two contradictories one must be true and the other false. If one is affirmed, the other is thereby denied. One excludes the other, and hence there can be no medium affirmation between the two. This axiom has been formulated: A either is or is not. A either is or is not B. Everything must either be or not be.

E. g., An intra-mercurial planet, Vulcan, exists or does not. The moon either is inhabited or it is not. Bacon either was Shakespeare or he was not. The two propositions, Vulcan exists, Vulcan does not exist, are first tested by the Laws of Identity and Contradiction. If by the Law of Identity it is true that Vulcan exists, then, by the Law of Exclusion, the proposition, **Vulcan does not exist,** must be false. If by the Law of Contradiction it be true that Vulcan does not exist, then, by the Law of Exclusion, the proposition, **Vulcan exists,** must be false.

The importance of the Principle of Exclusion arises from

its being the foundation of all **disjunctive judgments,** *i. e.*, " of judgments in which a plurality of judgments are contained, and which stand to each other in such a reciprocal relation that the affirmation of one is the denial of the other."

(4.) **The Law of Reason and Consequent, or Sufficient Reason.**— The Law is stated as follows: All continuous thought must be rationally connected. The Law has been formulated : Infer nothing without a ground or reason. The starting-point in continuous thinking is the affirmation of some knowledge by which the mind is necessitated to affirm or posit something else. This starting-point is called the *logical reason, ground,* or *antecedent,* or, as Hamilton suggests, *condition;* that something else which the mind is necessitated to affirm or posit is called the *logical consequent,* or the *conditioned;* the relation between the reason and consequent is called the *logical connection* or *consequence.*

Reason and consequent involve not only cause and effect, but every case where an antecedent compels the mind to affirm something else. as logically following it. It includes the relations of *whole to part, cause to effect, substance to attribute,* etc., with the reversed relations of *part to whole, effect to cause, attribute to substance,* etc.

The axiom, as presented by Hamilton, takes a positive and a negative form.

(a.) **Positive Form.**—" When a reason is explicitly or implicitly given, then there must exist a consequent; and, *vice versa,* when a consequent is given, there must also exist a reason." The presence of a tree as a *whole* always implies the presence of any or all of its *parts,* — roots, trunk, branches. The presence of any attribute, as *intelligence,* always implies the presence of the substance of which it is an attribute,— *mind.*

(b.) **Negative Form.**—Where there is no reason, there can be no consequent (either implicitly or explicitly). Where there is no consequent, there can be no reason. The absence of *mind* involves the absence of *memory* as an attribute of mind. The absence of *will* implies the absence of *moral accountability,* of which it is an attribute.

The logical significance and value of the Law of Reason and Consequent lies in this, "that, in virtue of it, **thought is constituted into a series of acts** all indissolubly connected; each necessarily inferring the other." Without it, continuous and connected thought or reasoning would be impossible.

2. The Postulates of Logic.

There are certain fundamental postulates, or practical propositions, assumed at the outset of the treatment of Logic. The two here emphasized respect the reality of truth, and the requirement of full, explicit statement.

(1.) **The First Postulate.**—There is such a thing as truth, which can be ascertained, and on which all minds, acting in accordance with the laws of thought, must agree.

Without this assumption there can be no starting-point for thought, and no goal for the activity of the thought-power. No two minds could otherwise have any common basis from which to start together or on which to come together in thinking or discussion.

(2.) **The Second Postulate.**—This, as stated by Hamilton, is, "to be allowed to state explicitly in language all that is implicitly contained in thought." Logic deals ultimately with **thought,** and it has to do with **language** only as expressing thought. It is, therefore, proper to ask, in connection with any term, proposition, or argument, "What is the thought in this?" or, in other words, "What is the full and exact meaning of this?" and to state in full this meaning. Abridged forms are to be completed, rhetorical forms to be translated into plain language, and expressions changed, if need be (provided the thought be preserved), until the thought is brought out naked and entire. Mill states this postulate as follows: "Logic postulates to be allowed to assert the same meaning in any words which will

express it; we require the liberty of exchanging a proposition for any other that is equipollent (that is, having equal power and reach) with it."

------∘∘✦∘∘------

II. THE DIVISIONS OF LOGIC.

WHAT are the Divisions under which Logic should be presented? This question has been variously answered. The answer should, in any case, depend upon the point of view and object of the work.

The most common division is, perhaps, into Pure Logic and Applied Logic. Hamilton divides it into Pure and Modified. Regarded as a Practical Science, it is, perhaps, better to base its divisions on the various Forms of Thought.

1. Distinction of Pure and Applied Logic.—The logical writers who follow the common division find it necessary to define and distinguish Pure and Applied Logic, or Theoretical and Practical Logic. As these terms will constantly be met with in the works on Logic, a brief explanation of them will here be given.

(1.) Pure Logic is the Science of the Necessary and Formal Laws of Thought as Thought. It treats of the necessary laws of thought, in the strict sense of discursive thought, as they are in themselves, whatever may be the object-matter to which they are applied. In this sense Logic is a science of abstractions, like pure mathematics or metaphysics. As furnishing the principles implied in and underlying the construction of all other Sciences, it has also been called "the science of sciences."

(2.) Applied Logic treats of the application of the principles, or laws of thought, unfolded in Pure Logic, to the investigation of truth. It assists in ascertaining and fol-

lowing right processes of thought and in avoiding wrong processes.

This division is the same as the distinction of the School-men, of Logica Docens and Logica Utens; of the Wolfian School in Germany into Theoretical and Practical; also, as General and Special, Abstract and Concrete.

The following **Outline** presents the common Divisions of Logic from this point of view:

LOGIC, the Science of the Laws of Discursive Thought, comprises —

I. THEORETICAL, or PURE, LOGIC, or the Science of these Laws in themselves, including —

1. **Laws of Conception.**
2. **Laws of Judgment.**
3. **Laws of Reasoning.**

II. PRACTICAL, or APPLIED, LOGIC, including —

1. **The Doctrine of Fallacies,** or the modes of avoiding incorrect thinking.
2. **Method,** or the right modes of ascertaining truth.

2. Distinction of Pure and Modified Logic.—Sir William Hamilton divides Logic into Pure and Modified: confining attention to Abstract or General Logic.

(1.) Pure Logic, in the Hamiltonian sense, "considers Thought Proper simply and in itself, and apart from the various circumstances by which it may be affected in its actual application. But human thought, it is evident, is not exerted except by men and individual men." It is, therefore, variously modified by individual peculiarities, original and acquired, and by the circumstances of the thinker. Hence arises —

(2.) Modified Logic, which considers " the conditions to which thought is subject, arising from the empirical circum-

stances, external and internal, under which exclusively it is the will of our Creator that man should manifest his faculty of thinking."

For Hamilton's Divisions, see Hamilton's "Logic," page 49.

3. Divisions based on the Forms of Thought.—In treating Logic as a Practical Science, it is more convenient and satisfactory, if not more logical, to base the divisions on the various Forms of Thought,— Conception, Judgment, Reasoning, and System.   It is **more convenient and satisfactory,** since by this method, first, the learning of the principles will go hand in hand with their use ; secondly, the scientific view will be kept in strict subordination to the practical end aimed at.   It is **more logical,** since in this way it is believed that, first, the subjects of Fallacies and of Method will fall into their natural places, in connection with the presentation of the laws of correct thinking ; secondly, the whole subject will take such shape as is best to train to skill and power in right thinking and in testing the products of thought.

According to this view, Logic will be treated under four Divisions :

PART FIRST.   Logic of Conception, or of the Term.

PART SECOND.   Logic of Judgment, or of the Proposition.

PART THIRD.   Logic of Reasoning, or of the Syllogism.

PART FOURTH. Logic of Construction, or of the System.

Part I.

THE LOGIC OF CONCEPTION OR THE TERM.

THE aim of the Logic of Conception is to train the mind to skill in dealing with the first and fundamental Form of Thought.

Definition.—Conception is that form of thought in which we compare various acquired knowledges and connect them by resembling marks or attributes, thus forming Concepts, Classes, and General Terms.

This definition suggests, as a **first** subject for treatment, **the formation of conceptions.** The every-day practical necessity for studying and logically testing the work of thinkers, as embodied in their scientific, philosophic, and literary productions, suggests, as a **second** subject, **the unfolding of conceptions.** The Logic of Conception will, therefore, be treated in two Chapters.

CHAPTER I.

THE FORMATION OF CONCEPTIONS.

WORKS on Logic are usually mainly confined to the work of unfolding thought; but as the process of forming conceptions furnishes the key to their unfolding, it will be first

considered. The definition of conception suggests the **four elements** of the process, to be treated in as many **Sections:**

First, the gathering of the materials for conception, *i. e.*, the knowledges or objects of thought. This is the work of **Observation.**

Second, the placing of these materials side by side, noting the resembling parts, marks, or attributes, and gathering these into thoughts, called **concepts.** This is **Conception proper.**

Third, the gathering of the objects, to which these concepts or bundles of common attributes apply, into **classes.** This is **Classification.**

Fourth, the embodying of both concepts and classes in **names,** or **general terms.** This is **Denomination.**

The skilful thinker will need to have command of the laws of these four elements of Conception: Observation, Conception proper, Classification, and Denomination. The last three will be seen in their **formation** to involve comparison as an essential element. In the treatment of the three in the First Chapter, both the *process* and *product* will be considered. The **unfolding** of the products of the three — the concept proper, the class, and the term — by Partition, Division, and Definition, will be the work of the Second Chapter.

Section I.— Observation.

Strictly speaking, Observation is a **condition rather than an element** of conception. It must always precede the proper work of conception, since, without careful examination of the objects or facts about which the work of thinking is done, no material would be furnished in fit shape for the use of thought in its first form.

Definition.—Observation is the mental process by which we gain a minute and comprehensive knowledge of objects and their make-up.

The Instruments of Observation are the Senses and Consciousness. In gaining a knowledge of the external or material world, the ob-

server must make use of his *five senses.* This is **observation in the narrow sense.** In gaining a knowledge of the facts of the inner world, or world of mind, he must make use of *consciousness,* or internal perception. This is sometimes known as **reflection,** or **introspection,** and, with observation by the senses, makes up **observation in the wide sense.**

Topic First. The Predicables, or Things Knowable or Nameable.—The first thing in order to observe well for the purpose of correct thinking is to know the kinds of things that may be known, or what the observer may expect to find. This will furnish him with the clew needed to make his observation exact and complete in gathering his material for thinking.

From another point of view, the kinds of things knowable or nameable are called **the categories** (from a Greek word meaning *to predicate*), or **the predicates** (from the Latin, meaning *to assert*), or **the predicables,** since they sum up what may be predicated or asserted of anything.

It should manifestly be the aim of every intelligent man to acquire the power to know as much as possible of what may be known and named.

1. The Predicables.—Starting with **being,** or **thing,** as the conception including all things in the universe, a simple classification may be made which will be of practical value to the observer. Being always appears as **substance** having **properties** or **attributes.** Properties may be divided into four kinds, reducible to three:

1st. Properties of quality, or those which constitute anything what it is.

2d. Properties of action, or those which manifest the active and passive powers of any being.

3d. Properties of condition, or those which express the connections of beings with space and time.

4th. Properties of relation, or those which express the connections of beings with other beings.

The properties or attributes of condition and **relation** are sometimes known together as *properties of relation in the wide sense,* and the scheme thus reduced to three kinds of properties.

Substance and **property** and the kindred terms need to be carefully distinguished.

1. **Substance** is used in two kindred meanings: **first,** as *being,* in contrast to and independent of its properties, as that which exists absolutely and of itself, *absolute being;* **second,** "as conjoined with the attributes" and furnishing their basis, that which stands under and supports the attributes, the thing back of all phenomena "which is and abides." In the latter and more common meaning, *substance* is divided into *matter* and *spirit,* or that which is extended and that which thinks. **Subject** is used in the more recent philosophy, especially German, to denote the spirit, "the basis of the various mental phenomena." *Conscious subject* means the thinker or the mind itself. **Object** is a term for that about which the knowing subject is conversant. **Subjective** is applied to that which belongs to or proceeds from the conscious subject; **objective,** to that which belongs to or proceeds from the object known.

2. Various **properties**—called, also, attributes, qualities, parts, marks, characteristics, phenomena, etc.—are the **materials** to be gathered, in connection with *substance,* for conception. These terms are often used in a loose sense as synonymes. The **first three**—property, attribute, quality—are the most important, from the point of view of logic, and need to be carefully distinguished; the others sufficiently explain themselves. **Property** may be regarded as the widest of the three terms, and as including *whatever belongs to* or pertains to any object of knowledge. **Quality,** etymologically, is that which makes anything what it is, and may, therefore, be properly regarded as including the essential properties, called, in the Scheme given, *properties of quality.* With Aristotle and Descartes, **attributes** are real properties, essential and inherent. They may be restricted to properties of quality, **or** extended so as to include properties of action.

Properties may be distinguished as intrinsic and extrinsic. The **intrinsic properties** of any object of knowledge are those which are inherent in the object itself. In the Categories the *properties of quality* and action may be regarded as intrinsic. Intrinsic properties may be looked upon as including what are sometimes called peculiar prop-

erties and inseparable accidents. The **extrinsic properties** of any object are those which arise from its connection with something external rather than from its own nature. They include the properties of condition and relation.

Properties are also distinguished as essential and non-essential. An **essential property** is one of those which make any object, class, or species what it is, as, in *man*, the faculties of sense and intelligence; in *body*, the dimensions of length, breadth, and thickness. An essential property might appropriately be called a *quality*, in the strict etymological sense. The essential properties of any object, or those which make it what it is, are known as its **essence** or (in the old Logic) its **definition.** **Non-essential properties** are those which do not belong to the essence of an object. Essential properties are substantially the same as intrinsic, and non-essential as extrinsic. The former may be looked upon as embracing properties of quality and of action; the latter, properties of condition and relation.

Note.—Logicians have, from the earliest times, made use of the distinctions of peculiar property (often called simply property) and of *accidental property*, or *accident*. A **peculiar property** has been defined to be one which is common to the whole of a class of objects, but is not necessary to mark off that class from other classes. "Capable of speaking correctly" is said to be a peculiar property of man, not embraced in the definition or essence of man, "rational animal." "It is, however," as Thomson has shown, "*a part* of the essence, for *rational* implies it. In like manner, all the properties seem to be implicitly contained in every perfect definition. No criterion can be given for distinguishing between the essence and the inseparable accompaniment of the essence; and a larger acquaintance with the nature of things makes it evident that, what one science regards as a property, another must consider as essential, and that there is no one paramount quality which is absolutely essential and can never be degraded to the rank of a property."

An **accidental property**, or **accident**, is one which may indifferently belong or not belong to the objects of any class without affecting their essence. The birthplace of a man and the clothes he wears are accidents which have no necessary effect upon his manhood. Accidents are separable or inseparable. A **separable accident** is one that can be changed, as the *clothes* of a man, his *position*, and many other circumstances. An **inseparable accident** is one that can never be changed, although it may have no necessary connection with essential properties, as the *birthplace* of a man, his *height*, etc. Thomson has, however, shown that it is often difficult, if not impossible, to distinguish accident from essential property. Writing in England, he says: "It is an accident to the people of this country that they were born in it; because we might conceive them to have been born elsewhere; but then it has modified their nature or essence, and we understand by Englishman not merely one who was born within the four seas, but a man of particular feelings, views, and privileges, which are parts of his very nature. Here accident and genus or property seem to become confused."

3 *

It is, therefore, proposed to abandon these distinctions as at least **unneces-sary for logical purposes.**

The Scheme of Predicables, therefore, becomes :

$$
\textbf{BEING,} \begin{cases} \textbf{Substance,} \\[1em] \begin{array}{l} \textbf{Properties, or} \\ \text{Modes of} \\ \text{Substance,} \end{array} \begin{cases} \begin{array}{l} \text{Quality,} \\ \text{Action,} \end{array} \Big\} \ \textbf{Intrinsic and Essential.} \\[1em] \begin{array}{l} \text{Condition,} \\[0.5em] \text{Relation,} \end{array} \Big\} \ \textbf{Extrinsic and Non-essential.} \end{cases} \end{cases}
$$

The old Aristotelian logicians looked upon all existing things as being divided by nature into ten classes or **categories.** These, according to Aristotle, are : substance, quantity, quality, relation, place, time, posture, possession, action, passion. A thing that can be known or named comes under one or other of these categories. As will be seen at a later stage in the study of Logic, the categories will not stand the test of the laws of accurate division.

It will readily be seen that these categories of Aristotle may all be placed under one or other of the categories of the simpler scheme previously given.

2. Use of the Predicables.—The accurate and intelligent observer must consciously or unconsciously make use of some such scheme in order to make his observations intelligent and complete ; otherwise he will never know when he has learned the most important facts in any given case, nor when he has learned all the main facts.

The scheme will decide the **general questions** to be asked when attention is called to any object of knowledge.

1st. What is it in its substance—spiritual or material ? This will bring out what is included under Aristotle's category of *substance.*

2d. What are its properties of quality? This will embrace Aristotle's category of *quality.*

3d. What are its properties of action? This will embrace Aristotle's categories of *action* and *passion.*

4th. What are its properties of condition? This will take in Aristotle's categories of *time, place, quantity,* and *posture.*

5th. What are its properties of relation? This will include Aristotle's categories of *relation* and *possession.*

Topic Second. Observation of Things Predicable.— The practical work of observation lies at the foundation of cor-

rect thinking, since such thinking must depend upon first ascertaining the exact facts about which it is to be done. The tendency is to careless and superficial observation. Perhaps more errors in science arise from want of proper observation of facts than from any other source. Hence the necessity of securing, in the earlier stages of training, the careful study and diligent practice of the processes and rules of exact observation.

I. Processes and Products of Observation in General.

Whately styles the operation of the mind, in contemplating any object, **simple apprehension.** It is often called **intuition,** or **immediate knowledge.** The result of this operation may be called the **simple notion.** This notion may take various forms, from that of the vaguest **percept** to that of the complete, concrete **thing.**

In beginning the work of observation we apprehend objects, whether material or mental, with various properties or parts. We perceive a tree with its trunk, branches, and leaves, with their forms, colors, qualities, etc. We thus gain what is called a **percept** of the tree. We may subsequently give special attention to any particular part or property of the tree, as its height, or color, or firmness of texture. This is called **abstraction,** or the drawing away of a part or property from the concrete whole. The result is an **abstract,** or **abstract notion,** of these parts or attributes, of height, color, etc. The most important element in accurate observation is **mental analysis,** in which the attention is voluntarily turned to particular parts or properties of any object of knowledge. This process of mental separation is continued until many constituent parts of the object are brought out. In examining material objects, these parts may evidently be regarded either as spacial parts or as attribute parts. The first point of view leads to what is called physical partition, the second to mental analysis proper.

Physical partition is the simplest form of mental analysis. The analysis of **tree** into roots, trunk, branches, leaves, brings out the spacial parts. Such partition is of special service in the earlier stages of mental training.

Praxis.—Name in an orderly manner the parts of the following objects: 1. A peach. 2. A piano. 3. A ship. 4. A book. 5. A house. 6. A landscape. 7. A mountain view. 8. A telephone. 9. A telescope. 10. A locomotive.

Mental analysis proper, the more important form of observation, deals with attributes rather than with spacial parts. It belongs to a more advanced stage of mental training. **Water** may thus be analyzed in thought into the separate properties named weight, liquidity, transparency, refracting power, solvent power. A **dime** may be analyzed into the attributes or parts, material substance, heavy, round, small, white, coin.

Note.—It will be obvious that **chemical analysis, involving intricate processes** of thought, belongs to a different range of mental activity. It would bring out of *water* its chemical components, oxygen and hydrogen, and out of a *gold dollar* its chemical components, gold and the alloy of silver and copper.

The result of the careful application of these processes of abstraction and mental analysis is the notion of the complete concrete object, or **thing,** which, according to Horn Tooke, is the same as **think,** a *thing* being what one thingeth or thinketh.

Praxis.—Analyze and describe in an orderly way the following objects: 1. A diamond. 2. A gold dollar. 3. A painting. 4. A piece of wood. 5. A flower. 6. A rose. 7. A forest. 8. A sunset at sea. 9. A church service. 10. An act of memory.

II. Exact or Scientific Observation and its Rules.

The general and superficial observation thus far considered, however well it may serve the purposes of common life, is insufficient for the purposes of accurate thinking. **Scientific observation** must be made accurate and **exact** by intelligent conformity to certain rules, and must be made **complete** by careful use of the scheme of things knowable.

1. The Rules of Observation, which need to be grasped and practised in order to reach the best results, are three.

They are substantially Hamilton's Laws of Integrity, Parcimony, and Harmony.

Rule 1st. Observe all the essential facts, parts, or properties in any given case.

Rule 2d. Admit no fact, part, or property that does not belong to the case in hand.

Rule 3d. Avoid all delusive mixtures of inference with the facts of observation.

Rule 1st is needed to guard against the common fault of incomplete observation. Through the careless use of the powers, or the holding of some false theory, or the blinding influence of prejudice, men are liable not to see *all* the facts. The honest observer should see to it that none of these things stand in the way of completeness or integrity of observation. Rule 2d is intended to guard against the danger of receiving as facts things that are not such, and of receiving as facts of the region under observation things which belong to some other sphere of facts. This danger arises in the same way as the preceding. Rule 3d is to guard against the introduction of unsound or irrelevant inferences among the facts of observation. The sources of this danger are the same as the preceding. Here is the fruitful source of much of the scientific and philosophic error in all ages.

2. Scientific Observation, in order to the best results, while conforming to these rules, **must make intelligent use of the categories.** The observer must make use of the questions, already given in connection with the scheme, in order to bring out the facts of all kinds.

The character of this observation will appear more fully in the later stages of the study of Logic. The mode of using the scheme may here be cursorily illustrated, and the main things in the process suggested, by the observation of a *white-oak tree* in the school-yard or campus. *Question first* will bring out the fact of material substance or constitution. *Question second* will give the facts of extension, of organization, of life, and of unity of structure and plan in the tree, the facts of cupule-bearing and half-covered fruit, and the other facts peculiar to the white-oak. *Question third* will furnish the facts of growth, of resisting violence, of counteracting pressure, etc. *Question fourth* will lead to the facts concerning the height, size, shape, habitat,

C

etc., of the tree, and those concerning its time of planting, length of life, periods of growth, etc. *Question fifth* will direct to the facts concerning the position of the tree with reference to the school-building, to other trees and objects on the grounds, to other trees belonging to the class, oak, to the industrial arts in which its wood is used, etc.

Praxis.—Observe systematically and describe carefully the following objects: 1. An inkstand upon the writing-desk. 2. A clock upon the mantel-piece. 3. A student's lamp upon the table. 4. A Worcester's Dictionary in the library-case. 5. A stove in the room. 6. A ship at sea. 7. Jupiter as the evening star. 8. The centre-table in the library. 9. The feeling of home-sickness in the student. 10. The contemplation of Church's Niagara.

Note.—The teacher will do well to use as an adjunct some such work as the little Manual, published by Eldredge & Brother, entitled "The Cultivation of the Senses." This will prepare the way for the application of the right principles to the more difficult work of introspection and analysis of mental objects.

Section II.—Conception Proper.

Conception proper is the **first essential element** in the first Form of Thought. The work of Observation makes ready the material for it; Conception proper begins the work of comparing that material, arriving at the thought-connections, and gathering up and combining the results in a thought.

Topic First.—The Process of Concept-Forming.

Definition.—Conception proper is the mental process of fixing upon resembling parts, marks, or properties of objects, and grasping them singly or together as **attribute thoughts** or **concepts.**

This element of conception always involves a **comparison** of two or more objects of knowledge, and has more or less direct reference to the process of **classification** by similar properties. The concept may, indeed, be said to be formed for the purpose of being used to classify objects, and this gives it its chief value. There is need, therefore, of considering two things: *first*, the gathering of similars by comparison; *second*, the grasping of similars in thought by conception.

I. The Gathering of Similars by Comparison.

Comparison in the formation of concepts proper begins with the work of fixing upon similar properties. Observing objects side by side, we note and affirm differences and resemblances, and then fix upon and abstract the **resemblances or properties common** to the objects.

1. The **simplest connecting act** in thought is in finding a single point of resemblance, and withdrawing, or abstracting, this from the points of difference.

E. g., *water* has materiality, weight, liquidity, refracting power, solvent power, transparency, etc. A *dime* has materiality, weight, whiteness, hardness, malleability, roundness, smallness, the stamp of a coin, etc. *Air* has materiality, fluidity, elasticity, invisibility, etc. Examining these three objects side by side, they are all found to have in common *materiality*. They resemble each other in this point, or, in other words, this is a characteristic *common* to them all.

2. A **more important connecting act** in this first stage of thought is that of finding and seizing upon several or all the points of similarity in the objects compared.

It will readily be seen that the same objects may be observed from different points of view. A *gold* or *silver coin* may be observed as a *substance* having essential attributes of its own, or as a *piece of money* used in the work of commercial exchange. The observer should first fix upon his point of view, and then seek the resemblances from that point of view.

Considered as a *substance*, a *sovereign* is material, of yellow color, extremely malleable, of circular shape, nineteen times heavier than water, etc. As a *piece of money* it is of the metal gold, of comparatively high value, being worth five dollars, of the kind which is the standard of values in most countries, a coin, etc. Considered as a *substance*, a *silver dollar* is material, of white color, moderately malleable, of circular shape, ten times heavier than water, etc. As a *piece of money* it is a coin, fashioned of the metal silver, of moderately high value, being worth one hundred cents, etc. Treating the gold and silver coins as *substances* merely, they resemble each other in

being material, having color, being malleable, having circular shape, being of high specific gravity, etc. These are the resembling or common properties or parts. Treating them as *pieces of money*, they resemble each other in being coins, composed of metal shaped into circular form and valuable for the purposes of exchange.

3. The **most important connecting act** in this stage of thought is that of finding and fixing upon the essential points of similarity in the objects compared. **Scientific thinking,** as will be seen further on, must fix mainly upon the essential points of resemblance, rather than upon the non-essential.

Praxis.—State, concerning the properties of the following objects, whether they are intrinsic or extrinsic; whether essential or non-essential; whether properties of quality, action, condition, or relation: 1. *Of George Washington,*—born in Virginia in 1732, studied mathematics under a private instructor, tall, wise, just, brave, president, led the armies of his country, the friend of Hamilton, the father of his country, died in 1799. 2. *Of Great Britain,*—populous, fertile, insular, powerful, manufacturing, agricultural, commercial, philanthropic, missionary, kingdom, colonizing, literary, modern, small, nation.

Compare the following objects, fixing upon some resembling property, and stating to what class of properties it belongs: 1. Snow, light, chalk, lime. 2. Book, parchment roll, Rosetta stone, paper manuscript. 3. Oak-tree, rose, elephant, man. 4. Memory, argument, fence, watch, world.

Compare the following objects, fixing upon the resembling properties, and stating to what class they belong: 1. Wood, coke, charcoal, bituminous coal. 2. Plumbago, charcoal, diamond. 3. Star, student's lamp, sun. 4. Tree, carriage, watch, poem. 5. Poem, painting, statue, anthem, temple. 6. Triangle, polygon, dodecahedron, globe.

Note.—The teacher may, with great profit to the student, devote much time to the processes of Observation and Comparison. They lie at the very basis of correct thinking, so that their importance cannot well be over-estimated.

II. The Grasping of Similars by Conception.

The work of observation and comparison up to the present point has only brought out **common attributes** without fixing them in a **thought** binding them together into logical

unity. The attributes of the *sovereign* as money are named each by itself. The work of conception brings together all these attributes into **one thought,** which, as being the product of conception, is called a **concept,** or **attribute-thought.** This concept, which is named *sovereign* (in accordance with the laws of naming to be considered under Section Fourth) embraces in itself the characteristics—coin, fashioned of the metal gold, of comparatively high value, being worth five dollars, of the kind which is the standard of values in most countries, etc.

The value of the product of thought reached by the process of grasping together properties will depend upon the method and principles followed.

1. It is obvious that **any of the kinds of properties** already considered may be fixed upon and embodied in the concept or attribute-thought, and that this may be done in various ways.

a. A **single property** of any kind may be fixed upon, in which case the result may be looked upon as a **simple concept,** although the mental act is one of simple *grasping,* and not of *grasping together.*

b. The **properties of any particular object** may be grasped together, without special attention to other objects, or to the principle of similarity. This may also be regarded as an **unapplied concept,** which may be applied later to similar objects in the work of classification.

c. The **similar properties of various objects** may be grasped together, keeping in view the principle of similarity. This is the **concept in the strictest sense.**

2. **The Rules which must govern** concept-forming, if the best results are to be reached, may be reduced to two.

Rule 1st.—In order to the best thought, essential properties should be grasped in preference to others.

The **loose thinking** of common life is characterized by its seizing upon non-essential properties. In observing an *individual man* the separable accident of wearing broadcloth may be observed, abstracted, and embodied in the concept *broadcloth-wearing.* Such a concept brings out nothing essential to man. **Scientific thinking,** on the other

4

hand, fixes chiefly upon essential properties, so that it embodies the very nature of the objects of thought. In observing a *man*, it fixes upon the animal and rational properties which make him what he is, and embodies these in the concept *man*, or *humanity*. The products of scientific thinking will be found of the utmost value in the work of classifying objects.

Rule 2d.—In order to the best, the only adequate, thought in this form, all the essential properties should, so far as possible, be grasped.

It is obvious that any number of abstractions may be drawn from any object. Strictly speaking, we can never be certain that all the possible properties have been abstracted. There may always remain innumerable unobserved or undetected properties. But ordinarily all the essential properties may be more or less clearly detected and grasped, and the perfection of the concept as a group of properties will depend upon the completeness with which it takes in the essence of the object of thought.

Observing carefully an **animal,** the properties of organized being, of life, of sentiency, and of voluntary motion, are fixed upon as essential properties. These are all embodied in the concept, *animal.* If but one of these sets of properties should be embodied, the concept would be of comparatively little value. Observing some **virtuous act,** as the Prophet Daniel's act of praying to the true God notwithstanding the prohibitory decree of the king, the characteristics, conformity to the law of right, and intelligent, intentional action, are fixed upon and embodied in the concept *virtue*, or *virtuousness.* If any one of these essential characteristics is omitted in our conception of *virtue*, the thought will be incomplete and of little value scientifically.

Praxis.—Gather up into concepts the similar properties of the following groups of objects, stating the kind of property in each case: 1. A piece of crayon, a chair, a lamp, a book, a tree, a stone. 2. A man, an eagle, a lion, a serpent. 3. A horse, a tiger, an elephant, a lap-dog. 4. A cat, a leopard, a hyena. 5. A vulture, a hawk, a falcon. 6. Love, patience, joy, gratitude. 7. Faith, hope, charity. 8. Cathedral of Milan, Madonna of Raphael, Paradise Lost of Milton. 9. Great Britain, United States, Germany.

Topic Second.—The Product of Concept-Forming.

The product of gathering up the abstracted properties of

objects in thought is a **thought property** or **group of properties.** It would be appropriately named a **notion** (from *notæ*, marks, characteristics), if that word were not used in such loose and varied senses. **Concept proper** is, perhaps, the best name. But whether spoken of as notion, concept proper, or attribute thought, the **essential thing** in it is always the grasping in thought of certain observed **properties** of objects.

The properties contained in any concept make up its **content.** The same thing has also been denoted by *internal quantity, intension, comprehension, depth, marks,* etc. The content of the concept *man* is made up of animal and rational properties. The content of *triangle* is plane figure, three-sided, rectilineal.

In connection with concepts proper, Logic gives prominence to their **reciprocal relations by content.** These relations may be considered from **two points of view:** first, that of identity, and, second, that of congruity.

1. Compared by content, concepts proper are distinguished as identical and different. They are —

1st. **Identical,** when they coincide in their marks, or comprise the same properties. Identity is either *absolute* or *relative.* **Absolute identity,** or sameness, does not strictly exist between concepts, but **relative identity,** or similarity, does exist. The terms of a complete definition approach most nearly to absolute identity, both comprising the same marks or properties, *e. g.,* "Body is extended substance."

2d. **Different,** when they do not comprise the same properties. Difference is again either **absolute** or **relative.**

2. Compared by content, concepts proper are also divided by logicians into congruent and conflictive.

1st. **Congruent notions** are such as may be connected in thought with the same object, as good, wise, powerful, etc.

2d. **Conflictive notions** are such as may not be connected in thought. Conflictive opposition is either contradictory or contrary. **Immediate or contradictory opposites** are "directly, immediately, and absolutely repugnant" to each other, as exemplified in yellow, not-yellow; walking, not-walking. Of these conflictives there can be two only, and one of them must be true. In **contrary opposition,** on the other hand,

more than two conflictive characters are possible, as yellow, blue, red, etc.; sitting, standing, lying, etc. If one of these be not predicated of any person, it does not follow that any one other must be. Thus, though I cannot at once sit and stand, yet I may be neither sitting nor standing,—I may lie; but I must either sit or not sit; I must either stand or not stand, etc.

These relations of concepts by content may be represented to the eye by **diagram.** *Squares* may be used to represent the sphere or content of concepts and also the objects of which they may or may not be predicated. The overlapping parts of the double squares and the dotted lines indicate the partial coincidence of the properties. The sign plus (+) may indicate congruence, the sign minus (—) confliction.

Praxis.—State and illustrate by diagram the relation by content of the following concepts: 1. Running, lying.　2. Blue, not-blue. 3. White, black.　4. Money, memory.　5. Learning, virtue.　6. Saint, sinner.　7. Grace, unmerited favor.　8. Yellow, blue, red.　9. Walking, standing, sitting, running.　10. Wealth, poverty.　11. Beauty, virtue.　12. Old, middle-aged, young.　13. Tall, short.

Give five examples of each of the following relations of concepts by content: 1. Identity absolute.　2. Identity relative.　3. Difference absolute.　4. Difference relative.　5. Congruence.　6. Contradictory opposition.　7. Contrary opposition.

Section III.—Classification.

The **second essential element** of conception, in the wide sense, may be defined as grasping in one thought, called a **class,** all the objects to which the attributes included in any concept or notion are common.　Hence the process is called **classification.**　From another point of view it may be defined as extending the application of the content of a concept or notion to all the objects to which it is applicable. Hence the process is also called **generalization.**

A **dime** has the property of roundness.　When we extend the application of this property to all bodies that possess it, and so connect them all with dime into one thought, the result is the class, round bodies.　A dime has the property of whiteness.　When we extend the application of this property in the same manner as before, the result is the class, white bodies.　Making use of both round and white, the result is the class, round, white bodies.　Classifying by the mark, stamped as coin, the result is the class of coins.

It is thus evident that the work of classification is simply the general application of one or more properties of a concept to objects.　The resemblance of properties or attributes furnishes the key to the work. If bodies had no *differences*, there would be but one great, monotonous mass of existing things; if they had no *resemblances*, no two could be brought together into a group, and there would be no possibility of thought.　Classification is possible because objects have both different and resembling properties.

Topic First.—Process of Classification.

Classes may either be considered singly, or in systems
4*

or combinations. Hence the ordinary distinctions and rules.

1. In forming **single classes,** it is obvious that the thinker may make use of accidental, peculiar, or essential properties. In order to reach the most valuable scientific results, classification should keep in view the most important properties.

Rule.—Classify by essential properties rather than by non-essential.

Gold might be classified, by the property of color, with yellow objects ; **silver** in the same way with white objects. Such a classification would, however, be of no scientific value. Taking the resembling essential properties of the two : (1), they are elements or simple substances ; (2), they possess metallic lustre ; (3), they are good conductors of heat and electricity,—they may be classified with other objects having like properties, as **metals.** Such classification is of scientific value.

Praxis.—Classify each of the following with like objects by various non-essential and essential properties : 1. Porter's " Human Intellect." 2. A comet. 3. The north star. 4. The Temple of Solomon. 5. The Parthenon. 6. The Washington Monument. 7. The Mississippi River. 8. The Mer de Glace. 9. Mount Vesuvius. 10. Victoria of England. 11. Ulysses S. Grant. 12. Jefferson Davis. 13. Moses. 14. Jesus.

2. Objects of knowledge are so related that they may be arranged in **systems of classes.** Such classification requires the application to classes of the process used in forming single classes, while keeping in view the wider relations of things. It is a successive classification of classes.

Rule.—Classify the lower classes under higher by fixing upon properties common to the lower.

Certain **figures** are classified, by the number and relation of their straight sides, as triangles, squares, parallelograms, polygons, etc. All these classes have the common characteristic, being bounded by straight lines, and may, therefore, be classed as *rectilinear figures.* Certain **other figures** are classified, by the various character of their curved boundary-lines, as circles, ellipses, parabolas, hyperbolas, etc. All these

classes have the common characteristic, being bounded by curved lines, and may, therefore, be classed as *curvilinear figures*. Rectilinear and curvilinear figures have, as a common characteristic, plane surface, and may, therefore, be classed as *plane figures*. Certain **other figures** are classed, by the character of their bounding surfaces, as tetrahedrons, cubes, parallelopipeds, etc. They are in common bounded by plane surfaces, and may, therefore, be classed as *plane solids*. Certain **other figures** are classed, by the character of their bounding surfaces, as spheres, cones, paraboloids, etc. They are in common bounded by curved surfaces, and may, therefore, be classed as *curved solids*. Both plane and curved solids have in common, *solidity*, and may, therefore, be classed as *solid figures*. Plane figures and solid figures have in common, extension, which is the subject-matter of Geometry, and may, therefore, all be classed as *geometrical figures*. The result is a **System of Classes:**
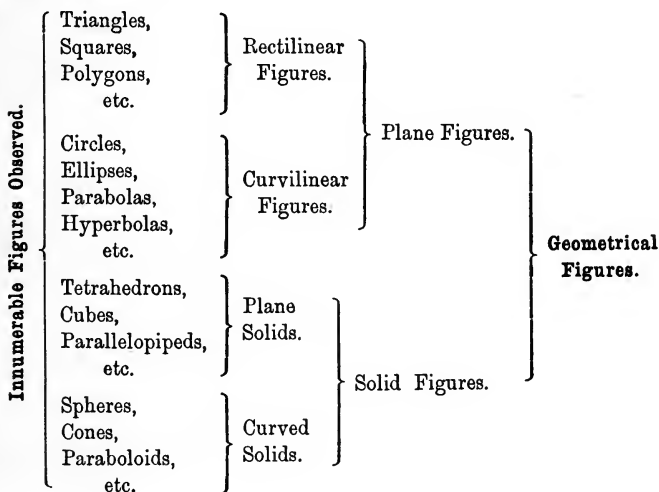
Such systems are found on the most extensive scale in the classification of animals and plants, in Zoology and Botany. Exercises in forming systems of classes may be drawn from these sciences.

Praxis.—Classify the following collections or masses of objects in single classes and in systems of classes: 1. The articles in a schoolroom. 2. The objects in a school or college campus. 3. The struc-

tures in New York city. 4. The objects comprised in a farm. 5. The objects embraced in a Pennsylvania landscape. 6. The objects in the heavens as revealed by a powerful telescope. 7. The operations of the human soul. 8. The things seen. 9. The things unseen.

Topic Second.—Results of Classification.

The product of the general application of the concept to all the objects to which it is common is a thought-group, or a thought-system, of objects, *i. e.*, a **class** or a **system of classes.**

In the general notion as class, the essential thing is always the grasping together of **individuals.** The individuals contained in any such general notion make up its **extent.** The extent has also been denoted by *external quantity, extension, breadth*, etc.

In connection with the class notion and extent, Logic gives prominence to two things : **first,** the relations of general notions as classes to one another by extent ; **second,** the reciprocal relations of extent and content, or of the class and the concept proper.

I. Relations of Classes to One Another.

1. Compared by extent, general notions as classes stand to each other in **five mutual relations :** exclusion, co-extension, subordination, co-ordination, and intersection.

1st. **Exclusion.**—One class excludes another when no part of the one coincides with any part of the other ; *e. g.*, horse and syllogism. No horse is ever a syllogism, and *vice versa.*

2d. **Co-extension.**—One class is co-extensive with another when each includes exactly the same species ; *e. g.*, living being and organized being. Using life as including plant life, every living being is an organized being, and *vice versa.*
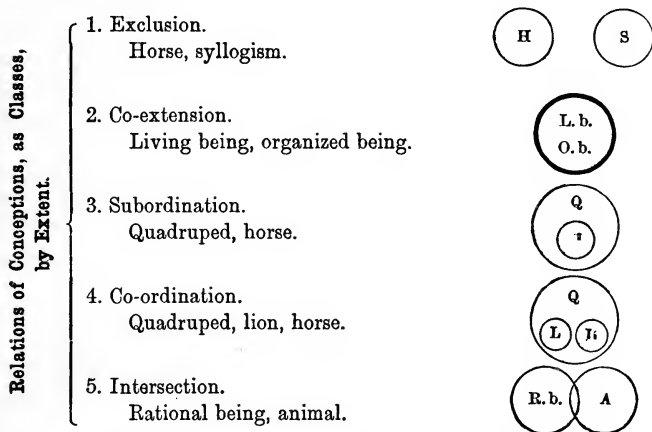
3d. **Subordination.**—One class is subordinate to another (which is called the *superordinate*) when the former is included in the latter as a part of it ; *e. g.*, dog, horse, under quadruped. Every dog is a quadruped, as is also every horse.

4th. **Co-ordination.**—Two or more classes are co-ordinate when they are co-exclusive, yet all immediately comprehended under the same

higher class; *e. g.*, dog, horse, while immediately subordinate to the higher class, quadruped, are co-exclusive and, therefore, co-ordinate.

5th. **Intersection.**—Two classes intersect each other when each is partially included in the other; *e. g.*, rational and animal. Some rational beings are animals and some are not, and *vice versa.*

These relations may be symbolized by Euler's **circular notation,** in which the extent of classes is represented by circles, and the relations of classes by the relative positions of the circles.

Relations of Conceptions, as Classes, by Extent.	
1. Exclusion. Horse, syllogism.	
2. Co-extension. Living being, organized being.	
3. Subordination. Quadruped, horse.	
4. Co-ordination. Quadruped, lion, horse.	
5. Intersection. Rational being, animal.	

Starting from **Inclusion,** other logicians divide the relations of classes into those of (1), Inclusion, embracing, (*a*), Co-extension, and (*b*), Subordination; (2), Intersection; (3), Exclusion, embracing, (*a*), Co-ordination, and (*b*), Non-co-ordination.

2. Special Relations arising from Classification.

Out of classes and systems of classes arise various **logical distinctions** which, as they occur constantly in science and philosophy, in the writings of the modern as well as ancient masters, should be understood by the student who expects to read and think for himself.

(1.) The **simpler forms of classification** give rise to the distinctions of genus, species, differentia, individual.

In any series of higher and lower classes, each higher class is a **genus** to those next below it. Those classes next below the genus are its **species.** Caucasian, Mongolian, Malaysian, Negro, and American Indian are species of the genus, man. Or, if European is considered as a genus, German, Frenchman, Englishman, etc., are the species. **Differentia,** or **specific difference,** is the characteristic or property, simple or complex, which distinguishes one species from others under the same genus. Red is the differentia of red rose, or that which distinguishes it from white, yellow, and other species of the genus, rose. An **individual** is one of the single objects of which a species or genus is always made up. It is only capable of physical or mechanical partition, and can never be a genus. Washington and Napoleon are individuals.

Note.—**Species,** in its peculiar use in Natural History, needs to be carefully distinguished from species in Logic. In Natural History, species means only "such a class of animals as has, or might have, descended from a single original pair, and the varieties of which may permanently interpropagate among themselves." The sub-species are named **varieties.** Greyhound, spaniel, terrier, bull-dog, etc., are varieties of the species, dog.

(2.) **Systems of classes** give rise to the logical distinctions of summum genus, infima species, subaltern genera and species, proximate genera and species, superordinates, subordinates, co-ordinates, and disparates.

The **highest class** in any system of classes is known as *summum genus;* the **lowest class,** which can never be a genus, as *infima species.* The **absolute highest genus** is *being,* which includes all the existences in the universe. In classifying any department of knowledge, it is usual, however, to assume and start from some **relative highest genus.** In Botany, this genus is *plant;* in Zoology, *animal.* **Subaltern genera,** or sub-classes, are those which are species of a higher genus. **Subaltern species,** or sub-species, are species of some higher species considered as a genus to those lower than itself. White oak, black oak, scarlet oak, yellow oak, etc., are subaltern species of oak. Oak is a species of the genus, mastwort, or cup-bearing trees, and constitutes, with chestnut, beech, hazel, and hornbeam, the subaltern genera, or sub-classes, of that class.

This may be illustrated by the following **tabular example:**

Designations.	Classes.
Summum Genus.	Being or Thing.
Species or Subaltern Genera.	Organic (Inorganic).
Intermediate or Sub-Species.	Animal (Plant).
Infima Species.	Man (Brute).
Individuals.	Washington (Other Men).

Genera and species, which are next to each other in order of ascent or descent in any system of classes, are known as **proximate genera** and **species,** or nearest classes and species; as animal and man, in the example just given. The higher genus in relation to a lower is called the **superordinate genus,** or next in rank above; the lower in relation to the higher, the **subordinate,** or next in rank below. The species under any genus are **co-ordinates,** or of equal rank. This may be illustrated by the following **example**:

Assumed Highest Genus,—Cup-bearing Trees.

Species.	Oak.	Chestnut.	Beech.	Hazel.	Hornbeam.
	Red,	American,	American,	American,	Ironwood,
	White,	Spanish,	Red,	Beaked,	Hornbeam,
	Black,	Dwarf,	etc.	etc.	etc.
	etc.	etc.			

(with **Co-ordinates.** bracketing the species rows)

Oak, chestnut, etc., are superordinates with reference to the co-ordinate species respectively embraced under them. The co-ordinate species, red, white, etc.; American, Spanish, etc., are subordinates to oak and chestnut respectively, and these last to the higher genus, cup-bearing trees, which embraces also beech, hazel, and hornbeam. Any one of these co-ordinates, considered in relation to a higher or lower part in the divisions of any of the other co-ordinates in the system of classes, is called **disparate.** Red oak as compared with chestnut is disparate.

II. Reciprocal Relations of Concepts and Classes.

The concept and class notions are both very closely connected with one another, and embodied in **one word.** From one point of view the word man means the rational and animal **properties** which make **man** what he is. It has, therefore, **content** or contained properties. From another point of view man means all the **individuals** that have these

common properties, or all mankind. It has, therefore, **extent** or comprehended objects. **The Rule expressing the relation** of content and extent is, that as the content increases the extent diminishes, and as the extent increases the content diminishes.

In other words, the greater the number of properties in a concept, the less the number of objects that have all these properties in common, and the greater the number of objects in a class, the less the number of properties common to them all. This may be illustrated by the following **diagram of concept and class in content and extent:**

CONCEPT and CLASS.	CONTENT, *i. e.*, the properties contained in the concept.	EXTENT, *i. e.*, the objects embraced in the class.
Body.	Extended substance.	Stone, Plant, Brute, Man, etc.
Living Body.	Body with life.	Plant, Brute, Man.
Animal.	Body with life and sensation.	Brute, Man.
Man.	{ Body with life, sensation, and reason. }	Man.
Washington.	{ Body with life, sensation, reason, Father of his country. }	Individual.

From this diagram it is apparent that the concept, body, which has, as its content, only extended substance, has the greatest extent, embracing stone, plant, brute, man, etc., while the lowest concept, man, which has, as its content, extended substance with life, sensation, and reason, has the least extent, embracing only mankind. Washington, with still broader content, has, as its extent, only an individual. Being, the concept of least possible content, containing simply existence, is the absolute highest class, and has the greatest possible extent, embracing all things material and spiritual.

Praxis.—Give five illustrations of each of the following relations of classes: 1. Exclusion. 2. Co-extension. 3. Subordination. 4. Co-ordination. 5. Intersection.

State and illustrate, by diagram and by circular notation, the relations of the classes: 1. Man, horse. 2. Dog, ox, alligator. 3. Book, manuscript. 4. Magazine, daily paper. 5. Planet, body moving round the sun. 6. Aryan, European, Frenchman. 7. Faith, hope, love. 8. Affection, desire. 9. Man, animal. 10. Plant, tree. 11. House, barn. 12. Botany, Geology. 13. Mathematics, Astronomy.

Illustrate by three examples each: 1. Genus, species, differentia,

individual. 2. Highest class, lowest species, sub-class, sub-species, superordinate, subordinate, co-ordinate, disparate. 3. The varying relation of content and extent.

Section IV.—Denomination.

When, by the processes of conception, concepts and classes have been formed, they need to be embodied in language in order that they may be fixed and made subject to recall for further use. This is **the third essential element** in Conception.

Topic First.—The Process of Naming.

Language is the expression of thoughts by means of words spoken or written. It is the medium of communication between men. It fixes thoughts which would otherwise be vague, or fleeting, or confined to some individual, and makes them the property of all. It thus greatly facilitates the progress of our thinking. In short, while it is true that some of our processes of thought may be carried on without any language, it is nevertheless true that without it thought would practically cease, while communication would become impossible.

I. Modes of Naming.

In giving names to our conceptions, the aim should be to embody them as perfectly as possible and bring them as fully as may be under the recall and control of ourselves and others. It is evident that this aim is not always kept in view. Things are named in various ways, and the names, judged by the mode in which they are given, are oftener non-logical than logical.

1. The name is sometimes **purely arbitrary.** This is often the case with the strictly proper name. "It denotes an individual, but does not indicate or imply any attribute of that individual. . . It is an unmeaning mark or sign which we connect in our minds with an object, so that when this sign meets our eyes or ears we may think of

that individual." The most profane of men may be named Christopher, *Christ-bearer.*

2. The name is sometimes given from some **accidental circumstance** or property. In proper names this is illustrated by such Bible names as Moses, *drawn out;* Isaac, *laughter.* In common or class names, the same process is illustrated by moon, *measurer;* planet, *wanderer;* vulture, *flyer;* lord, *loaf-keeper.*

3. The name sometimes embodies **some prominent essential property** or mark. This is illustrated by such words as sun, *shiner;* man, *thinker;* animal, *breather;* barometer, *weight-measurer.*

4. The perfect or strictly logical name aims to embody as completely as possible **the entire essence** of a conception. As such naming is difficult in the case of complex conceptions, it is usually necessary to fix upon some prominent essential property, in accordance with the principle already given. The essential marks in the conception, man, are rational and animal, but the Aryan people who named man seized upon the essential mark, *thought,* and so called him *man,* i. e., *thinker.*

5. Names, as languages are constituted, are often, in fact, **little more than mere hints,** which start the mind on its work of interpretation. This has been shown by Hamilton to be one of the necessities of language,—since, unless the vocabulary becomes almost infinite so as to express all our single notions, the same words must be used to express a multitude of thoughts, more or less differing from each other. See Hamilton's *Logic,* p. 437.

II. Rules for Naming.

The Rules for giving names to our conceptions naturally arise from the aim in naming.

Rule 1st.—Name a conception what it is.

The science of the human soul should be named, not mental philosophy, nor intellectual philosophy, nor metaphysics, nor philosophy, but *psychology.*

Rule 2d.—Make the name self-interpreting if possible.

A name is **notative** when it suggests its own marks (*notæ*), and thus becomes self-interpreting. It is **symbolical** when it serves as a symbol or label of properties or marks which it does not suggest. Names should be notative, if possible, in order to give the mind the best start in its work of interpretation. It is a fact to be noted, that many

names which were **originally notative** have lost their power of suggestion except to men who are educated. To one who would best understand thought as expressed in English, acquaintance with the languages from which the English has drawn its words becomes a necessity. To one acquainted with Latin, triangle, quadruped, biped, become notative and self-interpreting. To one having the mastery of Greek, democracy, oligarchy, oxygen, mythology, philosophy, become self-interpreting. To one understanding Anglo-Saxon, lord, wicked, battle, war, orchard, become self-interpreting. To one versed in Philology and History, heathen, villain, church, sincere, saunter, become self-interpreting.

Rule 3d.—Make the name as simple as possible.

As the genius of our language is Saxon, let the preference be given to Saxon, and, if possible, let the name be a single word. Pierce is better than penetrate; love is stronger than affection, and hate than animosity; working is more forceful than operation. Psychology, as being one word, is better than intellectual philosophy; arithmetic than the art of computation.

Rule 4th.—In naming a system of conceptions or classes, use a system of names.

In a system of names one may be made to suggest all the other names and thoughts. Such system is thus of immense advantage, especially in the various Sciences. In the *Natural History Sciences*, which deal largely with classes, a system of distinctions has been adopted by which the precise place of each logical genus and species in the great system of classes may be accurately fixed. In **Zoology,** the Animal Kingdom is separated by Agassiz into Branches, Classes, Orders, Families, Genera, Species, Varieties.

Praxis.—Test the following names by the rules for naming, stating whether they are notative or symbolical: 1. Intellectual Philosophy, for science of the human soul. 2. Paternal ancestor, for father. 3. Affection, for love. 4. Sierra Nevada Mountains. 5. Telegraph. 6. Geology. 7. Geography. 8. Academy. 9. School of herring. 10. Accident. 11. Blackboard. 12. Candlestick. 13. Ambition. 14. Navy. 15. Book. 16. Bible. 17. Volume. 18. Parchment. 19. Paper. 20. Pen.

Topic Second.—Products of Naming.

The products of naming concepts and classes are the various kinds of **terms** in which our notions are embodied. The **divisions** are based, (1), either upon something in the term itself; or (2), upon something in the relations of terms.

I. Kinds of Terms arising out of the Nature of the Term itself.

The term involves in itself three elements,— mark or property, object, name.

1. **Considered as made up of marks,** terms are divided, (1), by the presence or absence of such marks into positive and non-positive; (2), by the separation or connection of the attributes with objects, into abstract and concrete.

(1.) All terms are either positive or non-positive. A **positive term** is one that implies the presence of some real mark or property, as man, tree, good. A **non-positive term** is one that implies the absence of such mark or property, as not-man, uncertain, deaf. Non-positive terms are either *negative* or *privative.* A **negative term** is one that implies simply the absence of any real mark, as not-tree, not-good, uncertain. Terms **apparently negative** are often positive in reality, as *immortal,* the word meaning not only not subject to death, but living for ever. So terms **apparently positive** are often negative, as *idle,* which is equivalent to not working, or not disposed to work. **Privative terms** are equivalent to a positive and negative term taken together. They mark the absence of certain properties, and the presence of others, from which the presence also of the former might naturally have been expected. Such terms are, blind, unkind, unholy. *Blind* is not equivalent to not seeing, nor to not capable of seeing, but signifies deprivation of sight in some being which might have been expected to have it.

(2.) All terms are either abstract or concrete. **Abstract terms** are those which embody abstracts or marks or properties as apart from the objects to which they properly belong, as coldness, hardness. Of the innumerable abstracts formed, the mind suffers the greater number to pass without naming, but fixes some by names. Thus in observing some individual man, the abstracts, life, intelligence, feeling, self-

activity, etc., are seized upon and fixed singly by names; or several of them together, under one name, as intelligence, feeling, etc., under rationality; or all the marks together under humanity. **Concrete terms** present the marks or qualities in connection with the objects to which they belong, or (as indicated by the derivation of the word from the Latin *con* and *cresco*, or *con* and *cerno*) with which they are grown together or seen together, as the adjective terms, cold, hard, and the substance terms or substantives, ice, iron, man.

2. **Considered as embodying objects,** terms are divided, (1), by the number of objects embodied, into singular and universal; (2), by the connection of the objects with their marks, into connotative and non-connotative.

(1.) All terms are either singular or universal. **Singular terms** are those in which our percepts or simple apprehensions are embodied, or our general notions as connected with our perceptions; as, Shakespeare, the Great Eastern, this man. They begin with embodying simple no- tions, but gradually rise toward the expression of thought proper or general notions. They are of **three kinds:** proper names, individual- ized common names, and collective names. **Proper names** are singular concrete terms which denote an individual, but do not necessarily indicate or express any properties of that individual; as, George Washington, Alexander Hamilton. There is, however, a tendency in the progress of thought to connect with and designate by the proper term the peculiar qualities of the individual denoted by it. We say of a man he is a Washington or a Cæsar—meaning to bring out his patriotism and equanimity or his ambition and universal genius. An **individualized common term** is one which expresses the simple notion of an object as it is presented to us in the concrete with more or less of its properties; as, this table, this man, yonder mountain. It is usually formed by adding some individualizing or limiting word to a common or general term; as, this table, that man, an organ, my hat. The **collective term** is also properly a singular made up of many objects brought together into the unity of a mass, rather than that of a class; as, the House of Commons, the army, a regiment, a forest.

The **universal term** is that in which the general notion, embracing concept proper and class, is embodied. It is *universal*, as it embraces all the objects possessing the common marks or properties involved in it as an attribute term. It is *common*, or *general*, since it is applicable to any and every one of these objects, as living, or man, is applicable

to every individual of the human race. It differs from the *collective term*, which embraces a number of things joined together in one mass, as regiment, Congress, since the collective is not applicable to each and every object under it. Every being embraced under the general term, man, is a man; but not every soldier embraced under the collective term, army, is an army. When the concept proper, or complement of marks or properties in a general term, is made prominent, it is used as a **concept,** or **attribute, term;** when the class, or complement of objects embraced in it, is made prominent, it is considered as a **class term.** In the propositions, Jesus was man; Jesus was a man,—the meaning of the first is, that Jesus had the marks or properties of a man; of the second, that he belonged to the class, man. In the first proposition man is a concept or attribute term; in the second, a class term.

(2.) All terms are either connotative or non-connotative. A **connotative term** is one which denotes an object, and notes along with it a mark or property. A **non-connotative term** is one which signifies an object only or a property only. All proper names are non-connotative, since they denote objects, but connote no property; as, Washington, London. All abstracts of qualities, as whiteness, length, are non-connotative, as they denote only properties without connoting any objects. All adjectives, as white, just, and all concrete general names, as bird, fish, are connotative, since they denote objects and connote properties.

3. **Considered as words,** terms are divided, (1), by self-interpretation, into notative and non-notative or symbolical; and (2), by the number of words constituting the term, into simple and complex.

(1.) All terms are either **notative** or **symbolical.** This distinction has already been defined and illustrated under the Second Rule of Denomination.

(2.) All terms are either simple or complex. A **simple term** is one which consists of only one word. But some words cannot be used as terms, although they may form parts of terms. Hence arise **complex terms,** which are made up of combinations of words. With reference to their being used as terms, words are either **categorematic** (from a Greek word, to assert or predicate), *i. e.,* such that they can stand alone as complete terms in propositions; or **syncategorematic** (from the Greek, to assert or predicate along with), *i. e.,* such that they can only form parts of terms, since they must be used *with* other words to

make up complete terms. To the former belong the noun, adjective, and certain parts of the verb. There are, however, those who contend that in the last analysis *only nouns can form terms.* Such sentences, as " Dictionaries are useful," must be completed by adding *books* or *things;* thus, " Dictionaries are useful *books.*" Adverbs, prepositions, conjunctions, etc., are syncategorematic. We speak of " the conservation of energy," " the conflict of religion and science," thus uniting many conceptions in one, and embodying them in a phrase. In the statement, " This is a faithful saying, and worthy of all acceptation, that *Christ Jesus came into the world to save sinners,*" the part italicized is a term expressed in a sentence. **Complex terms** are formed by combining syncategorematic with categorematic words. Any of the objects and properties included under the Predicables may thus be combined in complex terms.

II. Kinds of Terms arising out of the Relations of Terms.

Terms are divided, 1, by their relation to one another, into relative and non-relative or absolute; and, 2, by their relation to the objects of which they are predicated, into compatible and incompatible.

1. All terms are either relative or non-relative. A **relative term** is one which implies some other of which we may predicate it as its *correlative,* as father, son; ruler, subject; cause, effect. **Non-relative** or **absolute terms** are such as do not imply any such relative object or correlative, as tree, stone.

2. All terms are either compatible or incompatible. **Compatible terms** are such as can be applied to the same object at the same time. **Contrary terms** are the most opposed that can be conceived as applicable to the same object at the same time, as wise and foolish, good and bad. They are not compatible, however, when used in a strict sense; since anything which is absolutely good cannot be in any sense bad. **Incompatible terms** are such as are entirely excluded from application to the same object in the same sense at the same time. All **contradictory terms** are incompatible, as wise and not-wise, black and not-black.

These various distinctions of terms, embodying important distinctions in thought, are to be met with more or less frequently in all the profounder discussions in science, philosophy, and theology. Most of them will be found to

be of value in the subsequent portions of Logic. They may readily be presented in outline form by the student.

Praxis.—Apply all the foregoing distinctions, as far as possible, to the following words : 1. Government. 2. Industry. 3. Art. 4. Agriculture. 5. Joy. 6. Jupiter. 7. This earth. 8. The consolations of philosophy. 9. Intemperance. 10. Foolish. 11. Sobriety. 12. Hopefulness. 13. Psychology. 14. Virtue. 15. Non-relative. 16. Absolute. 17. Immortal. 18. Deaf. 19. From. 20. Life.

Select, from the page preceding the praxis, the following kinds of terms or words : 1. Negative. 2. Privative. 3. Simple. 4. Complex. 5. Concrete. 6. Abstract. 7. Relative. 8. Absolute. 9. Singular. 10. Universal. 11. Syncategorematic. 12. Notative. 13. Symbolical. 14. Connotative. 15. Non-connotative. 16. Abstract. 17. Concrete. 18. Collective. 19. Attribute. 20. Class.

CHAPTER II.

THE UNFOLDING OF CONCEPTIONS.

CONCEPTION, in its three essential elements, conception proper, classification, and denomination, has been found to result in three products :

First, the **Concept Proper,** embracing content or contained properties ;

Second, the **Class,** embracing extent or included individual objects ;

Third, the **Term,** embodying both concept proper and class, and, therefore, to be regarded either as an attribute term or as a class term.

The processes of unfolding these products, or of ascertaining accurately and exhibiting systematically and completely what is contained in them, are the processes at the foundation of all right and full understanding of the materials of which discourse, whether spoken or written, is made up. It is evident at once that a man who does not understand what is involved in such conceptions as cause, force, expe-

rience, persistence, can neither think nor discourse intelligently concerning them, and can neither hear nor read intelligently anything that others may say or write, which involves these conceptions.

As the products of conception are three, the processes of unfolding are three :

First, the unfolding of the content of the concept proper. This has been named Metaphysical Analysis, but has also been called **Logical Partition.**

Second, the unfolding of the extent of the class.   This is known as **Logical Division.**

Third, the unfolding of the term.   This will be known as **Logical Definition.**

Logical Partition, Division and Definition will, therefore, furnish the subjects of the three Sections embraced under the Unfolding of Conceptions.

Section I.—Logical Partition.

Logical Partition is that form of analysis which takes a concept proper, as a complex of properties or attributes, and unfolds the component properties.   In other words, Logical Partition is the complete and orderly statement of the parts of the content of a concept, or the separation of a complex attribute into its component attributes.

The thought-whole analyzed in partition is the concept proper which is an attribute or intensive whole.

The mind contemplates the objects presented to it under three kinds of wholes :

1st. **Mathematical** or Quantitative Wholes, or Wholes of strict Intuition.   This includes two kinds :

a. The Numerical, based on Time.

b. The Geometrical, based on Space.

2d. **Essential** or Physical Wholes, or Wholes of Observation. This includes two kinds :

a. The Substance, as composed of substance and attributes.

b. The Causal, as composed of cause and effects.

3d. **Logical** Wholes, or Wholes of Discursion or Thought. This includes two kinds:

  a. The Attribute or Intensive Whole, or Whole of Content.

  b. The Class or Extensive Whole, or Whole of Extent.

A mathematical whole, called also a quantitative, an intuitive, an integrate, whole, is, according to Hamilton, one composed of integral, or, more properly, integrant parts. It is a whole every part of which lies out of every other part, while all the parts together make up the integer or complete whole. Thus in the integrate spacial whole of the human body, the head, body, and limbs, its integrant parts, are not contained in, but each lies out of, each other. When the parts of an integrate spacial whole are separate and accidentally thrown together, the result is a **mass whole,** as a *gallon* of water, a *pile* of wheat, a *block* of wood. When the parts of an integrate numerical whole are thus separate and accidentally thrown together, the result is a **collective whole,** as an army, a forest. These wholes are analyzed by *mechanical* or *physical partition.*

An essential whole, called also a physical whole and a whole of observation, is the kind of whole with which observation brings us in contact. It consists of substance and properties either of quality or of action. The parts do not lie out of each other, but substance and property permeate and modify each other. Thus in gold the material substance is inseparably connected and blended with the properties of quality and action, metallic and reflecting the yellow rays of light. These wholes are analyzed by the process of *mental analysis* already described.

A logical whole, called also a whole of thought, is the product of the power of conception, and is, therefore, a creation of thought. As a concept proper it is analyzed by logical partition; as a class whole, by logical division.

Logical analysis by partition and division, therefore, deals with the logical whole in its two forms, partition having particularly to do with the logical whole as an attribute whole. The aim of partition, to unfold the content of an attribute whole, will, in connection with the nature and make-up of this whole as already learned from the formation of the concept proper, suggest the forms and rules of the process.

Topic First.—The Forms of Logical Partition.

The purpose of the thinker in partition is to attain to completeness in the work of unfolding the marks or properties of the concept. Such completeness may be either relative or absolute. This gives the two forms of partition.

I. Relatively Complete Partition.

A partition is relatively complete when complete from the thinker's point of view or for his special purpose. It is obvious that it is not always the aim to bring out all the possible properties included in the four predicable classes. Thus the chemist may desire to bring out the properties of *gold* as an element or as a metal ; the banker, as a medium of exchange; the encyclopædist, in these and all other aspects. It is thus manifest that any one of many points of view may be made available, the choice being always governed by the object of the thinker.

The point of view may be some one of the four kinds of properties, and the aim to reach the component parts from this point of view. The concept *man* may be parted by qualitative properties into rationality and animality ; or by active properties, or as a causal agency, into self-acting, thinking, feeling, etc. ; or by properties of condition, into temporal, terrestrial, etc. ; or by relative properties, into dependent, responsible, sinful, etc.

Or the point of view may be a single aspect of some one of the four kinds of predicable properties. *E. g.*, taking active properties as the starting-point, *man* as a causal agency operates in many different spheres, and may, therefore, have the properties unfolded with reference to any one of these spheres. The thinker may be a physicist and so may regard man materially, as counterpoising more or less weight and excluding other objects from the same space. He may be a chemist and so may regard man chemically, as forming, by decomposition, nitrogen, carbon, and other chemical elements. He may be a physiologist and so may regard man organically, as breathing, digesting, etc. He may be a political economist and so may regard man industrially, as farming, manufacturing, trading, or as producing, transporting, consuming, etc. He may be a psychologist and so may

regard man spiritually, as thinking, feeling, willing, etc. He may be a theologian and so may regard man religiously, as recognizing, longing after and worshipping God, etc.

II. Absolutely Complete Partition.

A partition is absolutely complete when the aim is to give an exhaustive analysis of a concept, or to present all the kinds of properties.

In such partition the various characteristics of *man*, as given from the four points of view, would all be embraced. Or, to take another example, *gold* may be parted by qualitative properties, as material, solid, elementary substance, etc.; by active properties, as reflecting the yellow rays of light, conducting heat and electricity, counterpoising great weight, etc.; by relative attributes (including condition and relation proper), as being mainly confined to particular regions of the earth, being of great value as a precious metal, being the standard of values in exchange, etc.

Topic Second.—The Rules of Logical Partition.

The rules for logical partition are determined by its aim to unfold systematically, from some definite point of view, the properties or attributes contained in a given concept.

Rule 1st.—The thinker in partition should first fix upon a single complement of attributes, should then determine upon the proper point of view for the purpose he has in mind, and should finally adhere to this point of view throughout the entire partition.

This is the **law of unity.** The danger of violating it arises from the fact that language uses the same term or the same form of expression for very different concepts or bundles of properties. *Man*, from the point of view of the physiologist, has very different marks from man as considered in social science or in psychology or theology. Physiology considers *man* as a material, organized, living being; social science, as a member of society and having certain social wants and instincts; psychology, as a spirit embodied; theology, as a creature and subject of God. The law of unity requires that the proper point of view be fixed upon and prohibits the mixing up of properties belonging to man from these various points of view.

Rule 2d.—A partition should be complete from its point of view, or inclusive of the whole complement of properties divided.

This is a form of the general **law of completeness** or **adequacy** or **integrity.** So far as a partition is incomplete it omits something essential to the conception, and thus fails to give that distinct view which requires that all the parts be presented in their proper relation to each other. Moreover, incomplete partitions are necessarily partial or one-sided, and will inevitably lead to positive error. If, for example, in analyzing *faith* as a Christian virtue, we recognize only the marks, knowledge, assent or intellectual belief, and sentiment or response of the heart, leaving out all moral disposition or purpose, we make faith involuntary, and so take from it the essential element of all virtue. Such faith ceases to be a virtue. Mr. Mill falls into a like error in analyzing *cause*, as invariable antecedence, thereby omitting efficiency, the principal and essential property involved in causation.

Prof. Day, in writing of the general Law of Adequacy in analysis, says: " The practical importance of a careful observance of this Law of Logical Analysis is to be seen in the fact that by far the greatest part of erroneous opinion in all departments of knowledge arises from the incomplete apprehension of the objects of knowledge. Most dissensions in science and in belief would be ended by a complete survey of all the constituent elements of the matter in dispute. It is mainly because the parties look, one at one element, the other at another, and each to the exclusion from his view of some element or character important to a correct opinion, that any dissension arises." This holds with special force in partition, since this process deals with the properties, involved in the essential nature and make-up of things, upon which all scientific classification depends.

If, for example, *murder* is analyzed into the elements, taking of human life, deliberate purpose, then the act of the sheriff in hanging a murderer, or the killing of another in self-defence, would be murder. The essential element of malice is omitted in the analysis. Or, again, if *virtue* is analyzed as embracing intelligence and conformity to the law of right, omitting intention, then the act of every hypocritical Pharisee in giving alms might be termed virtuous. On the other hand, if right intention is embraced in the partition, and conformity to the law of right omitted, the acts of the fanatic and enthusiast might be termed virtuous. It is only by taking in all the elements that error is escaped. Or, once more, if the characters of the *rose* are

given, as a shrub, producing flowers, having thorns, the rose might be confounded with any thorn-bush. All such possibilities of error are eliminated when the characters are fully enumerated as they are in the scientific text-books of Botany.

Rule 3d.—A partition should be exclusive, *i. e.*, it should shut out all marks or characters not belonging to the subject.

This rule corresponds to the **Law of Parcimony** under observation. It is violated if *education* is made to embrace, drawing out of the powers, putting them to use by their proper exercise, in the study of the physical sciences, in a scientific school. The use, the kind of study, and the place are none of them essential to the process, and they should, therefore, be excluded. So *money* may be analyzed into the characters, stamped metal, means of exchange. This, however, would not apply to most of the money in use in civilized lands, as most of it is not metallic. Money embraces the characters, representative of value, means of exchange, passing current, so that metallic is not an essential characteristic.

Rule 4th.—A partition should be orderly in the arrangement of the component elements.

This requires that some principle of arrangement should be seized upon and made use of in the statement of the elements of the complex thought analyzed. It also requires that in any continued process of partition the elements obtained should be arranged so as to bring out the relations of co-ordination and subordination.

In analyzing *man*, in its intrinsic elements, by partition, we may begin with the visible and tangible and proceed to the higher invisible and intangible. The resulting partition will be, animal attributes or animality and rational attributes or rationality. Analyzing animality, we may again proceed from lower elements to higher. The result will be, attributes of matter or corporeity, of organization, of life, of sentiency, of voluntary motion. On the same principle of procedure, *rationality* will yield the properties of intelligence, emotion, and endeavor. The rule given requires such orderly procedure and arrangement in the work of partition. In the partition of *man*, it

would forbid the mingling of the two sets of attributes and the co-ordination of any of the set of attributes resulting from the second step in the partition, as *sentiency*, with animality or rationality.

In an exhaustive process of partition each of these elements should be still further divided into its component properties, until the ultimate elements are reached. For example, *corporeity* would give extension in length, breadth and thickness, weight, etc. *Organization, life*, etc., would each be found to yield component elements co-ordinate with those of corporeity.

Praxis.—Give exhaustive Partitions of the following Concepts, testing the work by the Rules: 1. Money. 2. Englishman. 3. The love of God. 4. Life. 5. Salvation. 6. Genius. 7. Despair. 8. Forgiveness. 9. Heaven. 10. Duty. 11. Manliness. 12. Wisdom. 13. Justice. 14. Beauty. 15. Prophet. 16. Foresight. 17. Value. 18. Fortitude. 19. Egotism. 20. Selfishness. 21. History. 22. Philosophy. 23. Benevolence. 24. Charity. 25. Eternity. 26. Omnipotence. 27. Politeness. 28. Explanation. 29. Confirmation. 30. Design.

Give the component elements of the following Concepts, stating the kind of whole and the point of view, and showing that the Partition is in each case made in conformity to the Rules given: 1. The violet. 2. The diamond. 3. Botany. 4. Habit. 5. Hope. 6. Affection. 7. Religion. 8. Art. 9. Fine Arts. 10. The orange. 11. Carbon. 12. Monsoon. 13. Partition.

Examine the following Partitions, stating the kind of whole and the point of view, showing whether they conform to the Rules, and, in case they do not, correcting or completing the Partition according to the Rules:

1. Government = Intelligent power, ordered by law, controlling action.

2. Duelling = Fighting of two persons, mutual agreement, intent to kill, deadly weapons.

3. Lie = Enunciation of what is false, intent to deceive, violation of some obligation to give to others the truth.

4. Novel = Fictitious story, central interest in love, artistic construction.

5. Contract = Two parties, mutual promise, mutual obligation.

6. Charity = Compassion and sympathy for the needy, kindly and affectionate provision for the need, wise administering of the relief.

7. Circle = A curved line, drawn round a given point.

8. Planet = A star wandering in the heavens.

9. Triangle = A plane figure, three sides, three angles equal to two right angles.

10. Parallelogram = A plane figure, four-sided, opposite sides equal and parallel, opposite angles equal.

11. Fluid = Material substance, yielding easily to pressure, parts readily changing relative position without separation, gaseous form.

12. Whale = A large fish, living in cold regions, useful, yielding oil.

13. Education = Instruction, moral discipline, training.

Section II.—Logical Division.

Logical Division is that form of logical analysis which takes a conception as a genus or class whole and unfolds its component species. In the words of Ueberweg: " Division is the complete and orderly statement of the parts of the extent of a notion, or the separation of a genus into its species."

Note.—The student needs to distinguish carefully between partition and division. The former takes a concept proper or attribute whole and separates it into its component properties; the latter takes a genus or class whole and separates it into its component species made up of individuals.

The grounds or principles of division are found in the concept proper, or the common properties by which the objects in the class were originally classified. These properties embodied in the concept proper, and making up its content, have been called the **base,** since they are at the foundation of both concept and class. **The possible principles of division** in any given case are, therefore, only limited by the number of properties and combinations of properties, intrinsic and extrinsic, contained in the base and unfolded by partition.

Thus the class **man** has a content or *base of two complex intrinsic properties*, animality and rationality. The class may be divided by any property embraced in these. It may be analyzed into *animal parts*,—by the material properties, of extension in length, of weight and of color, giving tall and short; heavy and light; white, tawny, and black: by the properties of organization, giving sanguine, nervous, and bilious; etc., etc. Or man may be divided into *rational*

parts,—by different properties of intelligence, giving cultivated and uncultivated; enlightened and barbarous; learned and unlearned; imitative and creative; etc.: by the comparative prominence of the intelligence, sensibility, and will, giving intellectual, sentimental, and practical. Or it may be divided by *both animal and rational parts combined,*—by language, giving Aryan, Semitic, and Turanian; by race constitution, giving Caucasian, Mongolian, etc.; and the like. **Man** has also a *base of many extrinsic properties*, or properties of condition and relation, which may also furnish innumerable other principles of division. It may thus be divided by *relation to place*, as European, Asiatic, etc.; islanders and dwellers on the continents; men of the torrid, temperate, and frigid zones; and the like: by *relation to time*, into ancient and modern; or ancient, mediæval, and modern; antediluvian and postdiluvian; old, middle-aged, and young; and the like: or by *relation proper*, into bond and free; rulers and ruled; and the like.

Topic First.—The Forms of Logical Division.

The principal forms of logical division are the artificial or dichotomous and the natural. Either of these may be single and unextended or complex and extended.

1. The simplest form of division is the artificial or *dichotomous*, or that which arrives at two members which are contradictories.

For example: animals are rational and irrational, or vertebrate and invertebrate; angles are right and not-right or oblique; oblique angles are acute and not-acute or obtuse; the ancients were Jews and Gentiles, or Greeks and barbarians, or bond and free.

Such division is said by the logicians to be **strictly logical,** considering merely the form of the thought and not requiring any knowledge of what the concepts mean in order to assure us that the division is correct and exhaustive. But, as Ueberweg has remarked, "it labors under the **defect** that the species classed under the negation are left indefinite. Through the unimportance of the principle of division, or by reason of the number of species included in the negative and contradictory notion, the division may become worthless."

Thus, the division of the universe into partridges and not-partridges is of no value, both because of the worthlessness of the ground of division and the indefiniteness of the negative notion.

The process of **dichotomous division** may be **extended** until the lowest species or individuals are reached. There are two forms of this extended dichotomous division, a loose form and a strict one.

In the **loose form** the principles of division are seized upon successively as the new occasions of division arise. This is illustrated by what is known, from its author, the Greek logician Porphyrius, as the **Tree of Porphyry,** which, starting with **substance** as the *highest genus*, closes with man as the *lowest species*, and with Socrates, Plato, etc., as the *individuals.*
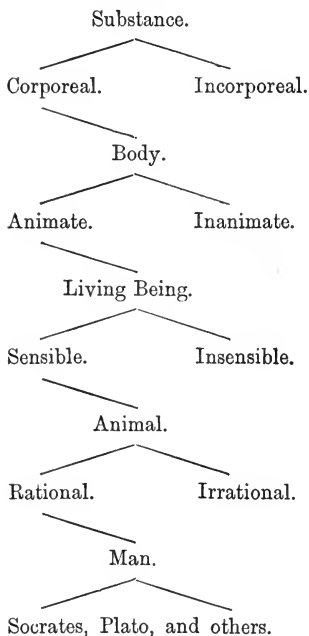
It will be observed that the successive principles of division are the qualities implied in corporeal, animate, sensible (or sentient), and rational. It is evident that the divisions on the negative side, incorporeal, insensate, etc., are also capable of like subdivision with those on the positive side.

In the **stricter form** of dichotomous division, one principle of division is carried through the entire series of subdivisions. In this case it is necessary to select at the outset some mark or attribute of the original class, as the principle on which the successive divisions shall be made. This may be illustrated by dividing **man** or **mankind** by *religion* as the principle of division.

Substance.

Corporeal.   Incorporeal.

Body.

Animate.   Inanimate.

Living Being.

Sensible.   Insensible.

Animal.

Rational.   Irrational.

Man.

Socrates, Plato, and others.

```
                            Mankind
                               |
              The|ists                Atheists
          Mon|otheists                Polytheists
      Chris|tians                     Non-Christians
    Pa|pists                          Anti-Papists
  Jes|uits                            Non-Jesuits
  _____
      |
Loyola and others
```
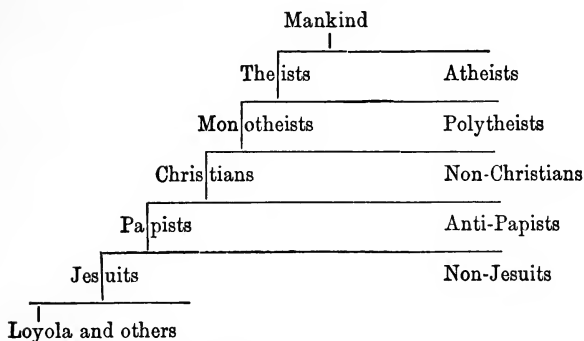
In this example, religion, in the various forms in which it appears among mankind, furnishes successive principles of division. The successive marks or characteristics used are, a personal God, the one God, God in Christ, the control of the Pope, Jesuitical principles.

2. **The most perfect form of division is natural division.** "It founds itself," as Ueberweg has said, "on the essential modifications of the essentially constitutive (or intrinsic) attributes. It depends on the essential parts of the notion or class to be divided. It is called **natural division** in the same sense as the system which results from a continuous series of such divisions is to be called a **natural system.**"

It is evident that divisions of this kind cannot be formed in any way according to an external uniform scheme. It is incorrect to look for an equal number of members of division in all cases in divisions of this kind. Thus the *animal kingdom*, divided by plan of structure, gives, by the four distinct kinds of structure, vertebrates, articulates, molluscs, and radiates. These four divisions are again taken up and subdivided in the Natural System of Zoology. Confining the natural subdivision to the vertebrates, we find at least five subdivisions recognized by zoologists,—mammals, birds, reptiles proper, amphibians, and fishes. Again, *human duties*, divided by the object toward which they are directed, naturally fall into the divisions, individual, social, and theistic. The student may also turn to the classification of the *cupule-bearing trees*, as already given, for another illustration of natural division. *The natural divisions are seldom dichotomous.*

3. From both dichotomous and natural division often arise the **trichotomy** or threefold division, and the **polytomy** or manifold division.

From the examples already presented, it is manifest that natural division is often found to be trichotomous or polytomous. It is likewise true that these forms may arise from a condensed statement of extended dichotomous division. *Angles* are divided, by the degrees of difference in the direction of the sides, into acute, right, and obtuse. This is a trichotomy condensed from an extended dichotomy, as follows: angles are right and not-right; angles not-right are acute and obtuse. The trichotomy is drawn from this. *Mankind* are Christians, Jews, Mohammedans, polytheists, and atheists, is a polytomy condensed from an extended dichotomous process, as follows:

Mankind

| The | ists | Atheists |

| Mon | otheists | Polytheists |

Christians   Non-Christians

Jews and Mohammedans
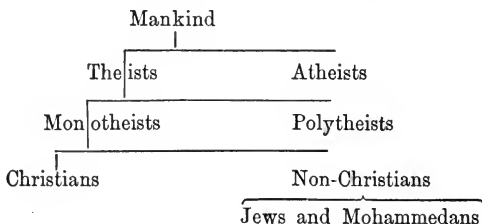
The trichotomy often arises because the parts of the class divided are not sharply marked off or separated from each other. Thus, men divided by color are white, tawny, and black. The present condition of a sentient being may be one of pleasure, of indifference, or of pain. Men divided by age are young, middle-aged, and old. Action considered morally is good, indifferent, or bad.

Topic Second.—The Rules of Logical Division.

The rules for division naturally arise out of its nature and aim. They spring either from the principle of division, from the various relations of the parts or species to the whole or class divided, or from the relations of the divisions and subdivisions to each other.

Rule 1st.—In a logical division the first requirement is to fix upon the one principle of division suited to the purpose in view, and the next to adhere to it throughout.

Several particulars need to be noted and emphasized in connection with this rule.

1. There must be **some principle of division** in every case as the reason or ground for the division. This is self-evident, for, as Hamilton has said, "otherwise there would be no division determined, no division carried into effect."

2. The principle of division is always to be sought in **some common mark** or property, intrinsic or extrinsic, of the class to be divided, and should be clearly and definitely grasped.

In general, it is manifest that the essential or intrinsic properties (those of quality and action) have most to do with determining the character of the class and its species. These properties must, therefore, furnish the most important principles, or those of natural division. In dividing man, rationality and animality furnish more characteristic divisions than the extrinsic properties (those of relation).

The particular end which the thinker has in view must, however, regulate the choice of the principle of division, so that in certain circumstances that principle is found in extrinsic or relative properties. Man is divided by intrinsic properties, mental and physical constitution, into Caucasian, Mongolian, etc. Geographically, man may need to be divided, by the relative property, place of abode, into European, Asiatic, etc.

In all cases the principle of division, whether intrinsic or extrinsic, should be clearly and definitely grasped. Failure in this inevitably leads to incoherent, uncertain, and unsatisfactory results. Thus when sentences are divided into indicative, interrogative, imperative, and exclamatory, no principle of division is apparent; we are left uncertain whether these are all the kinds of sentences and whether they should all enter into a proper division.

3. Every division should have **only one principle.**

The result of not complying with this requirement is what is called **cross-division.** This fault brings confusion and perplexity. The division of *governments* into monarchical, republican, despotic, aristocratic, and hereditary, violates this principle. The first, second, and fourth of these divisions have as their ground, the persons by whom the authority is exercised; the third has its ground in the extent of

the control; the fifth in the tenure of office. Monarchy and aristoc-racy may be despotic or hereditary, or both or neither. In short, the divisions cross each other in various ways and the whole is hopelessly confused. The same thing is illustrated by the division of *books* into poetry, history, Latin, French, German, morocco, and cloth. Three principles of division are made use of: the subject-matter, the lan-guage in which written, and the kind of binding. This results in many and perplexing cross-divisions.

4. The principle of division should always be **one of some importance and value.**

This excludes all useless and foolish divisions, but especially the counterfeit of dichotomy known as **division by infinitation.** To divide the universe of being into man and not-man; or the animal kingdom into parrots and not-parrots, may have a show of logic, but the result, as already seen, is absolutely worthless.

5. The principle of division should always be **suited to the purpose.** Very different divisions of the same class may be required for different ends.

For the purposes of Philology, a division of *conjunctions*, by the words from which they are derived, into verbal, adjective, substantive, phrase or prepositional, and composite, might possibly be of some service; but as a division for the purposes of grammar (which gives attention to the thought embodied rather than the origin of words) it has no relevancy and is of no value. For the purposes of Grammar, the principle of division should be, by the relations of the sentences or parts of sentences to each other, into co-ordinate and subordinate. These again should be subdivided, by the special forms of co-ordination and subordination, into copulative, adversative, etc., final, conditional, etc. For the purposes of Philology it would be as absurd to divide *man* into producers, transporters, and consumers, as it would to divide man, for the purposes of Political Economy, into Aryan, Semitic, and Turanian.

Rule 2d.—A division should be complete or inclusive of all the species of the class divided.

These species into which a class is divided are called the **members of the division.**

If these species or members of the division taken to-

gether do not exactly equal the class, then the division is evidently only partial and imperfect. This rule may be transgressed in various ways, as has been shown by writers on logic.

1. The rule is transgressed when **members** of a division are **left out.** For example, when we divide the *actions* of men into good and bad. To this we should add, *indifferent.*

2. The rule is transgressed when a **subdivision** is **co-ordinated with a division,** as when we divide *mathematical figures* into solids and plane surfaces. It should be solids or surfaces, since this is the fundamental division (by the number of dimensions), and plane and curved surfaces are subdivisions of surfaces by another principle.

3. We violate the rule when we bring in a **dividing member too much,** as when we divide *mathematical figures* into solids, surfaces, lines, and points. Here the last two elements, lines and points, must be excluded, since lines and points, though elements of mathematical figures, are not themselves figures.

Rule 3d.—The members of a division should be reciprocally exclusive.

This requires that each specific part brought out should be entirely different from every other such part.

1. This rule is violated by placing a **subdivision above or beside a division** under which it belongs, as when *human actions* are divided into necessary, free, and moral. Free actions are either moral or indifferent. In this case, therefore, a subdivision of free actions, which is included under it, is placed by the side of it. Or, again, when the sphere of Natural History is divided into the animal, vegetable, and mineral kingdoms, and the vertebrates; vertebrates is subordinate to animal, and as a subdivision of it should be excluded from enumeration with it.

2. The rule is also violated when **more principles of division than one** are used. For example, when we divide *human actions* into necessary, free, useful, and detrimental, two principles of division are used, necessity and utility, and the result is that the enumeration covers the whole class of human actions twice.

Rule 4th.—A division should proceed immediately from proximate genera to proximate species.

Divisions should, as far as possible, be continuous, that is, the notion must first be divided into its proximate, and then into its remoter parts, and this without overleaping any one part; or, in other words, each part must be immediately subordinated to its own whole. It is, therefore, improper to divide *animals* into elephants, birds, fishes, etc. According to Cuvier, as modified by Agassiz, the system of Zoology which gives the true division of animal is as follows:

Kingdom ....*Animal.*
Branch............*Vertebrates*, Articulates, Mollusks, and Radiates.
Class..............*Mammals*, Birds, Reptiles, Fishes, etc.

Elephants belong under mammals. In the division given, the intermediate classes, vertebrates and mammals, are overleaped.

Such an overleaping is, however, sometimes allowed for the sake of brevity; but this only when the omitted members can be readily supplied in thought. This is illustrated by the common mathematical division, already given, of triangles, into right, acute, or obtuse.

Rule 5th.—A division should be orderly in the arrangement of the specific parts into which the class is divided,— *i. e.*, the parts should be placed in proper co-ordination and subordination.

This is simply the requirement that in the statement of a system of division everything should be put in its own place. The rule may be illustrated by the Intellect or Power of Cognition, beginning with the Simple Cognitive Faculty.

COGNITIVE POWER OR INTELLECT (divided by progressive stages of *knowing*) :—

1. **Simple Cognitive Faculty** (by kind of knowledges *acquired*),—

    (1.) Internal Perception or Self-Consciousness, giving knowledge of self;

    (2.) External Perception or Sense, giving knowledge of external world;

    (3.) Intuitive Perception or Intuition Proper, giving knowledge of first truths.

2. **Conservative Faculty or Memory** (by psychological elements in *keeping* knowledges),—

(1.) *Retention,* keeping knowledges, out of consciousness;

(2.) *Reproduction* or Association of Ideas, bringing back knowledges by linking them together;

(3.) *Representation* or Imagination, vividly imaging the knowledges reproduced;

(4.) *Recognition,* connecting the present image with the past knowledge.

3. Comparative Faculty or Thought (by material compared),—

(1.) *Conception* or Comparison of Objects, forming concepts, classes, and terms;

(2.) *Judgment* or Comparison of Concepts, forming judgments and propositions;

(3.) *Reasoning* or Comparison of Judgments, forming arguments and conclusions.

4. Constructive or System-making Faculty (by law followed),—

(1.) *Scientific Construction* or Construction by the True, giving scientific system;

(2.) *Artistic Construction* or Construction by the Beautiful, giving æsthetic system;

(3.) *Practical Construction* or Construction by the Good, giving practical system.

It will be observed that neither of the four main divisions can change place with any other. Simple cognition, or the power of acquiring our simple and fundamental knowledges, must act before there can be anything for memory to conserve; conservation or memory must act before comparison can have any material to elaborate; comparison must do its work in order to furnish the materials for construction. The powers subordinate to these four must likewise take their proper places of subordination.

Praxis.—Give Divisions of the following Classes, stating clearly the Principles of Division and whether Artificial or Natural, and testing the work by the Rules: 1. The Races of Men. 2. The Nations of the Earth. 3. Languages. 4. Fruits. 5. Heavenly Bodies. 6. Commerce. 7. Art. 8. Industries. 9. Governments. 10. Churches. 11. Emotions. 12. Desires. 13. Ships. 14. Triangles. 15. Quadrilaterals. 16. Laws. 17. Life. 18. Dogs. 19. Metals. 20. The Carnivora. 21. Plants. 22. Roses. 23. Stars. 24. Processes of Rhetorical Invention. 25. Physical Forces. 26. Colors. 27. Divisions of time. 28. Flowering Shrubs.

29. The ruminants. 30. Insects. 31. Forms of religion. 32. Civiliza. tions. 33. Laws. 34. Societies. 35. Educational institutions. 36. Mechanic arts. 37. Wars. 38. International alliances. 39. Homicides. 40. Social conditions. 41. Human relationships. 42. The rocks. 43. Occupations. 44. Systems of unbelief. 45. Monotheistic systems. 46. Periods of human history. 47. Theistic systems. 48. Diversities of genius. 49. Poets. 50. Phases of religious character. 51. Temperaments. 52. Influences in formation of character. 53. Influences of the Crusades upon European civilization. 54. Aims in life. 55. Motives influencing human conduct. 56. Benefits of international commerce.

Give extended and complete Divisions, Dichotomous or Natural, of the following Classes, stating the Principle and testing by the Rules: 1. The vegetable kingdom. 2. Furniture. 3. Birds. 4. Cereals. 5. Fishes. 6. Creeds of Christendom. 7. The Sciences. 8. Foods for man. 9. Views of the origin of the universe. 10. Forces of civilization.

Examine each of the following Divisions, stating the Principle of Division, showing whether the Division is Natural or Artificial and whether it conforms to the Rules; and, in case it does not, showing wherein it fails and correcting and completing it:

1. Triangle = equilateral and equiangular.

2. Triangle = right-angled, isosceles, and scalene.

3. Literature = history, oratory, and poetry.

4. Literature = writings historical, religious, poetical, classical, and current.

5. Government = democracy, oligarchy, aristocracy, and monarchy.

6. Government = absolute, limited, constitutional, and free.

7. Government = empires, kingdoms, dukedoms, and republics.

8. The fine arts = the arts of free beauty, the arts of dependent beauty, and the arts of utility.

9. The arts of free beauty = music, sculpture, painting, and poetry.

10. The arts of dependent beauty = architecture, landscape-gardening, embroidery, and decorative painting.

11. Rectilinear figures = triangles, quadrangles, rectangles, parallelograms, and polygons.

12. Sentence = simple, compound, and complex.

13. Proposition = categorical, hypothetical, conditional, and disjunctive.

14. Proposition = affirmative, hypothetical, and negative.

15. Man = foot and horsemen.

16. Man = white, black, copper-colored, olive-colored, etc.

17. Thought = memory, conception, and reasoning.

18. Poetry = didactic, lyric, and epic.

19. Poetry = didactic, lyric, epic, and the ballad and sonnet.

20. Matter = solid, liquid, aeriform. and radiant.

21. Duties to self = self-conservation, self-culture, and self-conduct or direction.

22. Carnivora = cats, dogs, civets, weasels, bears, seals, whales, etc.

23. Mental faculties = sense-perception, memory, conception, abstraction, judgment, reasoning, and taste.

Section III.—Logical Definition.

Definition in general is the mental separation of an object of thought embodied in language from every other object of thought. **Logical definition** is the accurate unfolding of the signification of the terms which embody thought.

The various forms of definition and the indefiniteness of view on the whole subject of definition make it necessary to consider with greater care, the kinds of definition and the rules of logical definition.

Topic First.—The Kinds of Definition.

The word **definition** is used in both a wide and loose and in a narrow and strict sense. For definition in the former sense, Hamilton has suggested the name of **declaration,** signifying *throwing light upon, clearing up.* It may also be called **rhetorical definition,** in distinction from definition in the narrow and strict sense, which is called **logical definition.**

I. Rhetorical Definition.

The object of rhetorical definition or declaration is to give the meaning of a word loosely, or as it is popularly understood and for common use, rather than exactly and for scientific ends. It does not necessarily undertake to unfold essential properties, but freely uses those that **are**

accidental, relative, or extrinsic. It is called **description**
when it makes use of a number of concrete characteristics,
as when we say that the Caucasian is tall, white, graceful.

1. Various **popular modes** of defining words may be in-
cluded under rhetorical definition. These should be dis-
tinguished in order to guard against certain common errors
and fallacies.

(1.) **Etymological definition** traces the root of a word
back to its origin and defines accordingly. It sometimes
throws much light upon the meaning of a word and adds
great force to the word.

There are, as has been shown by Trench in his "Study of Words,"
most important lessons of history, romance, poetry, and morals wrapped
up in even our commonest words. In bringing out this meaning by
etymological definition it is necessary, however, to guard very care-
fully against **two errors** in particular,—that of fixing upon a wrong
etymology, and that of assuming that what the word meant at the
beginning it means now. Horne Tooke furnished an illustration of
the **first error** in confounding the root of *truth* with that of *trow*,
meaning think, and then concluding that "truth is what one troweth,"
or simply a matter of opinion. A better philology finds for truth a
root which would make it signify *reality*. The **second error** may be
illustrated by assuming that *villain* is still simply a villager, because
that was the original meaning, or that *knave* is still merely a boy,
because that is what the word once meant.

(2.) **Definition by word analysis,** or by unfolding the
various roots of which a word is made up, bringing out
and combining their significations,—is also of value; but,
since it involves, in most cases, a knowledge of the roots
of words, it is liable to lead to the same errors as etymo-
logical definition.

For example, the word *edify*, separated into its two component parts,
one meaning a temple and the other to make, would be defined etymo-
logically as the making or building of a temple. This may be strikingly
suggestive of the greater work of spiritual building signified by the
word as now used, but it can hardly be taken in the literal sense.

Note.—The subject of word analysis is treated of in such works as Webb's "Manual of Etymology," and Swinton's "Word Analysis." The student of Logic ought to be familiar with it.

2. Rhetorical Definition may also proceed **by the various thought wholes,** already considered. It may define words, in the looser way, as essential, as mathematical, or as logical wholes, by giving concrete characteristics, by using synonymes, or by casual substitution of phrases.

Such careless definition sometimes takes the form of **mere description,** or the naming of one or more concrete characteristics, as when we say, "Man is a risible animal," "Man is a two-legged animal without feathers," "The east is where the sun rises." It sometimes becomes only the enumeration of synonymes, as in much of the *definition of the Dictionaries*, as, "Law is a rule, decree, or statute," "Religion is piety." It sometimes becomes little more than a careless or **casual substitution of phrases,** narrative or descriptive, perhaps presenting some consequence or attendant circumstance, as "Wisdom leads to virtue and blessedness."

Some names are **not definable** except by rhetorical definition. It is obvious that an **individual** cannot be logically defined, since practically we cannot form a notion comprising all the essential marks which it has in common with any other notion or thing. *Description* is the process applicable to individuals. On the other hand, **simple notions,** or those containing a single or simple mark, cannot be logically defined, since they have only one mark and, therefore, no differential or distinguishing element. *Being*, for example, having only one mark, *existing*, cannot be unfolded, as there is no complex content to unfold. It can only be distinguished from *nothing* or *non-entity*, which is a mere negation, or defined by some synonyme, as *thing*, *existence*.

One office of Logic is to make plain the insufficiency of all such loose forms of definition, while giving command of the stricter forms of logical definition.

II. Logical Definition.

Logical definition separates a conception, as expressed by a word, from all other conceptions by fixing upon and presenting the essential and distinctive property or properties.

7 *

1. Strict or perfect logical definition has two forms.—The general term, as has been shown, may be considered either as embodying a class or a concept proper, in other words, either as a class term or as a concept term. Logical definition should, therefore, regard the general term from both these points of view. In other words, it is of two forms : one defining the general term as a class term and the other as a concept term ; the former dealing with extent or contained objects, the latter with content or contained properties.

Note.—The failure to recognize this twofold form has led to various differences of statement concerning the nature of logical definition. The *old logical definition* was confined to the conception as a genus or *class*. Professor Davis proposes to confine it to the conception as a concept proper. Logical definition thus becomes substantially synonymous with Partition as that subject has already been presented. Other logicians confine it to language or terms, and make it apply chiefly to class terms. The view here taken is that it applies to terms as embodying both classes and concepts. It is thus to be distinguished from Logical Division and Partition, which deal with thought directly rather than indirectly through language.

(1.) Definition of the Class Term.—If the term to be defined is regarded or used as a class term, the definer is required, by the principles of logical definition,—

First, to name the next higher *genus* to which the class, considered as a species, belongs ; and,

Secondly, to name the *difference* (*differentia*), or specific difference or that which distinguishes the class, considered as a species, from all the other co-ordinate species under that higher genus.

The genus and difference together make up the *essence* of the term, because they embrace the essential characteristics or marks of the class embodied in the term.

Thus, in the definition, Man is a rational animal, it is meant that *animal* is the next higher *genus* to which man belongs as a species, and that *rational* is the *difference* or that which distinguishes man from the other co-ordinate species, irrational animal or brute.

(2.) Definition of the Concept Term.—If the term to be

defined is regarded or used as an attribute or concept term, the definer is required, by the principles of logical definition,—

First, to state *the properties of the higher genus* to which the term, considered as a species, belongs; and

Secondly, to state *the properties which distinguish the term, considered as a species*, from other species under the higher genus.

Thus, in the definition, Man is rational animal, the meaning is that the concept term, man, includes animal properties or animality, and rational properties or rationality. The properties of the higher genus are included under animality, and those of the species under rationality.

2. **Certain imperfect forms of logical definition** are also distinguished by logicians. These are known as definition by division, by colligation, by resolution, and by composition. They approach the strict standard of definition more nearly than does rhetorical definition. They are in fact statements of the results of Division and Partition.

The **first two forms** are simply different statements of the results of Division as already treated. Definition by **division** unfolds a class term into its constituent species or individuals, as when we state that, "The animal kingdom consists of radiates, mollusks, articulates, and vertebrates." Definition by **colligation**, which is the reverse of definition by division, gathers up and unites the constituent species or individuals of a genus or species, as when we say that, "The Earth, Mars, Mercury, Venus, Jupiter, etc., are the planets." The **second two forms** are simply different statements of the results of Partition as already treated. **Resolution** brings out of a concept term its component properties, as when we say that, "Man is rational animal." **Composition,** the reverse of resolution, gathers up and unites the component properties, as when we say that, "Rational animal is man."

3. By an **extended process of logical definition** an ultimate and indefinable term is reached. In making such a complete explication of a term it is necessary to proceed by

defining successively the genus of each new definition until a simple notion is reached.

Professor Davis has illustrated this process in tabular form by an extended definition of *carnivore.*

" A carnivore is a flesh-eating (= *differentia*) mammal (= genus).
A mammal is a vertebrate (= g) suckling its young (= d).
A vertebrate is an animal (= g) having an internal skeleton (= d).
An animal is a sentient (= d) organism (= g).
An organism is a living (= d) being (= g)."

The process comes to a close when the simple notion, being, is reached. The result of the definition embraces all the properties connoted by the concept term, carnivore, and all that would be brought out by a Partition of that term. Stated as a *definition by resolution*, it becomes, "Carnivore includes flesh-eating, suck-giving, internal-skeletoned, sentient, living, existing."

III. Nominal, Real, and Genetic Definition.

Logicians, from another point of view, distinguish definition as nominal, real, or genetic. The first has to do with the mere name of the object of thought; the second with its reality or essential properties; the third with the cause which generates it.

Nominal or **verbal definitions,** or definitions of names or words, comprise the loose forms given under rhetorical definition or declaration, as when we say, "The word circle signifies a uniformly curved line." A **real definition** is a definition of the thought or reality embodied in a word. It unfolds essential marks, and is, therefore, strictly analytic. It comprises the forms of logical definition already given. Thus we define a circle as "a line returning upon itself, of which all the parts are equidistant from a given point." A **genetic** or **causal definition** is one which states the rise or production of a thing as the result of some working cause. It adds something to what is contained in the defined term, and hence is always synthetic. The genetic definition of a circle is, "A circle is formed when we draw around, and always at the same distance from, a fixed point, a movable point which leaves its trace, until the termination of the movement coincides with the commencement." Only such notions as relate to quantities repre-

sented in space and time, in other words only mathematical notions, can be genetically defined.

Topic Second.—The Rules of Logical Definition.

The rules for logical definition are determined by its nature and aim. They spring either from peculiarities in the origin and use of language, or from the nature of the thought embodied in the language.

Rule 1st.—In logical definition the first step is to study carefully the term to be defined.

The object of such study is to guard against the common errors in defining, which arise from the ambiguities of language. It is obvious, therefore, that logical definition requires in general a knowledge of language and the principles of interpretation. In particular it calls for a knowledge of the kinds and sources of ambiguity in the use of terms.

Professor Jevons has presented very forcibly the importance of a thorough acquaintance with the great imperfections of language. He says, "Comparatively few terms have one single clear meaning and one meaning only, and whenever two or more meanings are unconsciously confused together, we inevitably commit a logical fallacy. If, for instance, a person should argue that 'punishment is an evil,' and according to the principles of morality 'no evil is to be allowed even with the purpose of doing good,' we might not at the first moment see how to avoid the conclusion that 'no punishments should be allowed,' because they are evil. A little reflection will show that the word evil is here used in two totally different senses; in the first case it means physical evil or pain; in the second, moral evil; and because moral evil is never to be committed, it does not follow that physical evils are never to be inflicted, for they are often the very means of preventing moral evil."

In studying the subtle variations in the meaning of even our common words, it is necessary to distinguish between terms as univocal and equivocal. **Univocal** terms are those which can suggest to the mind no more than a single mean-

F

ing. **Equivocal** terms are such as have two or more different meanings.

1. **Strictly univocal terms** are not liable to mislead. The names of *individual objects, persons, or events* are usually fixed and certain in their meaning, as George Washington, Westminster Abbey, the Atlantic Ocean. The instances of univocal terms, outside of individual names, are found chiefly in *technical and scientific language.* Steam-engine, railway train, oxygen, hydrogen, sulphuric acid, etc., are examples of what may be found in connection with every well-defined science. It will be seen, however, on looking more closely, that **general terms are not strictly univocal.** The same word has been found to embody both the concept proper and the class. Hence the first inquiry, even in the case of words commonly called univocal, should be, *Is the term here used as a concept term or as a class term ?* The word *man* may be used in one case to express the attributes of humanity, and in another to express the species or individual human beings, and clear thinking requires that the thinker should know precisely which is meant in any given case.

2. **Equivocal terms** are exceedingly numerous. Equivocal terms are either properly ambiguous or homonymous.

(1.) A **properly ambiguous** (from Latin *ambigo*, to wander, hesitate, or be in doubt) term is one that has come to be used in different significations. Equivocation from ambiguity arises in two different ways:

1st. **Through association,** *i. e.*, from the transfer of the meaning from the thing originally denoted by the word to some other thing habitually and intimately associated with it. The word *church* originally denoted the building in which religious worshippers assemble. It has come to mean the particular body of worshippers accustomed to assemble in any one place ; or any body of persons holding the same opinions and connected in one organization, as the Church of England, the Roman Catholic Church ; or the church of Christendom ; or the clergy and religious authorities of any sect or country. The word differs entirely in meaning as used by a member of the Anglican, Greek, Roman Catholic, Congregational, Presbyterian, or any other existing church.

2d. **Through analogy,** *i. e.*, from the transfer of meaning to analogous objects. We speak of a sweet taste, a sweet flower, a sweet tune, a sweet face, a sweet poem, from the analogy or resemblance between the pleasure given by the flower, tune, etc., and that given by something sweet to the taste, as a lump of sugar.

The use of the same word in different significations renders it necessary in many cases to ask the question, *What is the signification in which the word is here used?* When the philosopher asserts that "*experience* proves the eternity of matter," the first question gives rise to such as follow: Whose experience? The philosopher's? All men's? All men's in all ages? All human experience plus human speculation?

There are some ambiguous words which should be carefully studied in order that an intelligent answer may be given to the question, *Precisely what does this word mean in the present instance?* The word *all* is an example of such ambiguity. In the proposition, "All these soldiers are individual persons," all is used distributively, or one by one. In the propositions, "Not all men are soldiers," "All men are not soldiers," all with the negative attached is not equal to *none*, but only to *not some*, so that the all in this case is only equal to *some.*

Words often change their meaning in the course of time, so that in studying and testing the works of past thinkers, there is need to ask the question, *What was the meaning of the term to be defined, in the day when this author wrote?* When the authors of King James's version of the Bible represent the Psalmist as praying, "Let thy tender mercies speedily *prevent* us," careful inquiry should be made into the use of the word *prevent*, about the opening of the seventeenth century. Such inquiry will reveal the fact that the word, which now means to go before one to hinder him, then meant to go before to anticipate or supply his wants.

(2.) **Homonyms** are terms which, though of different origins, have accidentally assumed the same form either in sound, or in spelling, or in both. Examples of the *first kind* are seen in such words as right, wright, write, rite, or rein, rain, reign, etc. Examples of the *second kind* are such words as lead, the metal, and lead, as in following the guidance of another. Examples of the *third kind* are such words as mass, a heap, and mass, a Roman Catholic religious service. An important instance of this kind of equivocation is found in grammar, "as between the numeral *one*, derived from an Aryan root, through the Latin *unus*, and the indeterminate pronoun, *one* (as in, '*one* ought to do *one's* duty'), which is really a corrupt form of the French word *homme* or man. The Germans to the present day use *man* in this sense, as in, *man sagt*, i. e., one says."

Too great care cannot well be given to the study of the terms to be defined. It is obvious, from the examples

given, that any failure to grasp the precise signification in which a single important word is used may utterly vitiate a whole system of thought.

Rule 2d.—A logical definition should bring out the essence of the term defined. This requires scientific accuracy.

The non-essential or accidental properties are not sufficiently characteristic for a definition. The worthlessness of the well-known Platonic definition, "Man is a two-legged animal without feathers," as containing only non-essential marks, was easily shown by Diogenes when he presented a plucked chicken as Plato's man.

Since general terms embrace both concept and class, use is to be made of both Partition and Division in framing logical definitions. In the case of a *class term* the definition should bring out *the next higher genus*, and the *differentia*, or characteristic of the term defined considered as a species under that genus. In the case of a *concept term* the definition should bring out *the properties of the next higher genus*, and the *differentia*, or characteristics of the term considered as the marks of a species under that genus.

Definition of the term as a class term is much the more common form. Such definition becomes easy when the student has once learned to *put the term defined under the next higher class, and then to bring out the distinguishing characteristics*. Rhetoric is defined by first putting it under the next higher class, *art*, or practical science, and then distinguishing it from all other co-ordinate species of art by stating its object, *discourse*,— "Rhetoric is the art of discourse." Patriotism is defined by first putting it under the next higher class, *love*, and then naming the special object, *one's country*, which distinguishes it from all other forms of love,—"Patriotism is love of one's country."

Rule 3d.—A logical definition should be adequate or precisely equal to the term defined. This forbids making the definition too wide or too narrow, deficient or redundant.

It follows that a good definition may be **tested by simple conversion,** or by letting the subject and predicate change places. If the common definition, "Man is a rational animal," be adequate, then

the converse will be true, "Every rational animal is man." Strictly speaking, we are not absolutely certain of the truth of this converse, for although it may be true of this earth, there may be in other worlds rational animals that are not men. The definition is, therefore, on this supposition, said to be **too wide,** embracing not only man, but all possible rational animals in other worlds. To make it perfectly adequate it is necessary to add the relative property expressed by *terrestrial* or some such term, as, "Man is a rational animal of this earth." The converse will then be strictly true, "All rational animals of this earth are men." On the other hand, if man be defined as a *praying animal,* the definition is said to be **too narrow.** It is not true in the strict sense that no animals that do not pray are men. The definition, in other words, embraces only a part of men. Definitions are **redundant** when they add to the essential characteristics derivative or unessential marks, as, "Man is a rational animal that laughs;" they are **deficient** when they omit some essential characteristic, as, "Man is an animal." To the latter belong definitions **by co-ordinate** and **subordinate notions,** as, "An odd number is that which is distinguished from an even by unity," "Man is an American."

Rule 4th.—A logical definition should be expressed in language as perfect as possible.

This forbids absurdity, ambiguity, verbosity, tautology, and obscurity of language, as well as circular, negative, and figurative definitions.

The language in a definition should be clear and significant and not vague, ambiguous, or senseless.

When Mr. Spencer defines the virtue of *patriotism* as *national egoism,* his definition is probably accepted by the mass of readers without thought. But egoism is selfishness, which of course is not a virtue at all, and patriotism is not a national but an individual sentiment. The definition is, therefore, absurd. The same objection holds against Mr. Spencer's definition of *evolution,* "Evolution is a change from an indefinite incoherent homogeneity to a definite coherent heterogeneity, through continuous differentiations and integrations." The definition is pronounced obscure both by common readers and by those who understand the strict meaning of the scientific and mathematical phraseology. A British critic has translated the definition into English, as follows: "Evolution is a change from a nohowish untalk-

8

aboutable-all-alikeness, to a somehowish and in-general-talkaboutable-not-all-alikeness, by continuous somethingelsifications and stickto-getherations."

The language in a definition should be precise and free from surplus words.

Dr. Johnson's definition of *oats*, "Oats is a grain *which in England is generally given to horses, but in Scotland supports the people*," violates this principle. The specific difference, expressed by the words italicized, is entirely unessential. Dr. James, in the "Anxious Inquirer," says, "It is a great principle that *subjective religion, or in other words*, religion *in us*, is produced and sustained by fixing the mind on *objective religion*, or the facts and doctrines of the word of God." Ruskin says of this, "Put entirely out the words I have put in italics, and the sentence has a meaning, but by its verbosities it is extended into pure nonsense; for 'facts' are neither 'objective' nor 'subjective' religion; they are not religion at all. The belief of them, attended with certain feelings, is religion; and it must always be religion '*in us*,' for in whom else should it be (unless in angels; which would not make it less subjective)."

The language in definition should not be tautological, *i. e.*, a definition should not contain the name of the thing defined, nor a derivative, synonymous, or correlative term, for this would be to define a thing by itself.

This is violated by such definitions as "Life is the vital force." It is also violated by what is called the *circle* or *dialellon*, as, "A board is a thin plank," "A plank is a thick board." John Stuart Mill's final definition of *cause* is a flagrant violation of this principle. It is as follows: "We may define, therefore, the cause of a phenomenon to be the antecedent, or concurrence of antecedents, on which it is consequent invariably, and subject only to the absence of preventing or counteracting causes." The essential idea of cause, *efficiency*, is left out; the last and perhaps the most emphatic word in the definition of *cause* is *causes;* and the affirmation that the *consequent* is *invariable* is followed immediately by the assertion of a *variable condition*.

The language in a definition should be perspicuous.

The aim of definition is to place the thought before the mind with more distinctness; hence, **terms more unintelligible than the one de-**

fined should be avoided. This is violated by Aristotle's definition, "The soul is the first entelechy or energy of a natural organized body possessing life potentially." Definition **by negative marks** is also forbidden by this principle, where definition by positive marks is possible. To define man as not a brute or not an angel gives no clear conception of what he is. **Figures of speech** are not ordinarily suitable for definition, *e. g.*, "Memory is the warder of the brain;" "The Divine nature is a circle whose centre is everywhere and the circumference nowhere." Such definitions make thought obscure rather than distinct.

Praxis.—Define the following terms Etymologically, by Analysis (where possible), Rhetorically and Logically, stating the kind of Whole in each case: 1. Proposition. 2. Development. 3. Sincere. 4. Lord. 5. Heathen. 6. Tawdry. 7. Saunter. 8. Slave. 9. Faculty. 10. Operation. 11. Education. 12. Vulture. 13. Instinct. 14. Virtue. 15. Patriotism. 16. Fanaticism. 17. Ox. 18. Gas. 19. Ice. 20. Oxygen. 21. Diamond. 22. Electricity. 23. Sun. 24. Moon. 25. Loadstone. 26. Gold. 27. Sophomore. 28. Voyage. 29. Battle. 30. War. 31. Sentence. 32. Grammar. 33. Rhetoric. 34. Logic. 35. Arithmetic. 36. Straight line. 37. Circle. 38. Point. 39. Sphere. 40. Vice. 41. Ghost. 42. Spirit. 43. Tribulation. 44. Passion. 45. Vexation. 46. Rage. 47. Love. 48. Desire. 49. Expectation. 50. Loafer.

Note.—See Trench "*On the Study of Words.*"

Define the words from number 36 to number 39 inclusive, nominally, really, and genetically.

Examine each of the following definitions, stating of what kind it is, showing whether it conforms to the Rules, and, in case it does not, showing wherein it fails, and correcting and completing it:

1. Grammar is the science of language.

2. Philology is the science of language.

3. A triangle is a rectilinear figure having three sides and three angles.

4. A square is a quadrilateral having all the angles right angles, all the sides equal, and the opposite sides parallel.

5. Malaria is that which induces fever.

6. A cone is a solid generated by the revolution of an angle about one of its sides.

7. Virtue is a voluntary act done in obedience to the law of God for the sake of everlasting happiness.

8. Logic is the art of reasoning.—WHATELY.
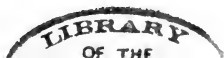
9. Logic is the light-house of the understanding.

10. Truth is the agreement of a cognition with its object.—HAM-ILTON.

11. Truth is accordance with the reality.

12. A whale is a large fish inhabiting the polar seas, and furnishing oil and whalebone as articles of commerce.

13. Happiness is the reflex of unimpeded energy.—HAMILTON.

14. Life is that condition of an organized being in which it is capable of performing its functions.—PORTER.

15. Life is definable as the continued adjustment of internal relations to external relations.—SPENCER.

16. Science is systematized knowledge.

17. Mind is the unextended.—BAIN.

18. Matter is the permanent possibility of sensation.—MILL.

19. Mind is a conscious string of sensations.

20. A sphere is a solid generated by a revolution of a semicircle about its diameter as an axis.

21. A sphere is a solid or volume bounded by a surface, every point of which is equally distant from a point within, called the centre.—WORCESTER.

22. Education is the training of the intellectual powers, principally by the study of the physical sciences.

23. Knowledge is power.

24. Net-work is anything reticulated or decussated at equal distances with interstices between the intersections.—DR. JOHNSON.

25. A saunterer is one who is going to the Holy Land.

26. Law is a lawful command.

27. Gratitude is a lively sense of future favors.

28. Gratitude is a virtue of acknowledgment.

29. A ruler is one who establishes laws.

30. A circle is a curved line returning upon itself, the parts of which are at an equal distance from the central point.

31. Logic is the electric light of the intellect, the cynosure of truth, the physic of the mind.

32. Man is an animal walking on two feet.

33. Man is a bimanous mammal.

34. Monarchy is a form of political government in which one man is sovereign.

35. Wealth is that which furthers the well-being of man.

36. The soul is the principle by which we live, feel, move, perceive, and understand.—ARISTOTLE.

37. Beauty is the feeling we experience in recognizing unity amidst variety.

38. A dragon is a serpent breathing flame.

39. Fine Art is the embodiment of thought in sensuous form.

40. Man is a rational being.

41. A cat is a domestic animal.

42. A dog is a digitigrade quadruped, having fixed claws, four toes, and a recurved tail.

43. Memory is that power of the human soul which recalls past knowledge.

44. Philosophy is the science of principles.—UEBERWEG.

45. Philosophy is the love of wisdom.

46. Dirt is matter in the wrong place.—LORD PALMERSTON.

47. A perception is an impression made on the mind.

48. Mathematics is the science of extension.

49. Snow is frozen mist.

50. A carnivore is flesh-eating, suck-giving, internal-skeletoned, sentient, living, existing.

51. A seal is a species of fish.

52. Honesty is a species of policy distinguished from other co-ordinate species by being the best.

53. Dancing is a refined and sublimated modification of circum-ambulatory locomotion.

54. Man is physically a living machine.

55. A conjunction is a word that connects words and sentences.

56. Matter is that in which is discerned the promise and potency of all terrestrial life.—TYNDALL.

57. God is the not-ourselves which makes for righteousness.—MATTHEW ARNOLD.

58. Religion is cosmic emotion.—CLIFFORD.

59. Evolution or development is essentially a combination of causes working toward a particular end.—McCOSH.

60. The conic section is that mathematical figure which divides into these four forms—circle, ellipse, parabola, hyperbola.

61. The sensibility takes that to be good which warrants or promises pleasure, and affects us pleasantly ;—the desires rest on pleasant feelings.

62. The feeling of the pleasant is the immediate consciousness of the furtherance of life.—UEBERWEG.

63. Justice is a square number.

64. The idea of the good is the sun in the kingdom of ideas.—PLATO.

8 *

65. Nature (Heaven and earth and all that is therein) is the body of God.

66. The state is man writ large.

Define the principal Terms used in the following Sciences, testing the definitions by the Rules: 1. Arithmetic. 2. Geometry. 3. Botany. 4. Zoology. 5. Grammar. 6. Physical Geography. 7. Rhetoric. 8. Psychology. 9. Natural Philosophy. 10. Astronomy. 11. Geology. 12. Ethics. 13. Political Economy. 14. Science of Government.

———o◦°o°◦o———

SUMMARY OF RESULTS

THE aim of the Logic of Conception is to train to the best thinking and fullest appreciation of thought in the first form. The degree of perfection or imperfection with which the mind grasps its conceptions constitutes what is called the **logical quality of conception.** Our grasp of conceptions is **perfect** in proportion as it is clear, distinct, and adequate: **imperfect** in proportion as it is obscure, confused, and inadequate.

A conception is **clear** when it is simply distinguishable from others; **obscure** when it is not. This may be illustrated by experience in gazing upon a tree. When the light falls upon it we readily distinguish it from other trees and objects of the landscape, and the view is clear; but when the mist or the twilight settles around it we can no longer distinguish it from other objects, and the view becomes obscure. We have a clear conception of *man* when we distinguish it from inorganic matter, plant, animal, etc.; so long as we are unable to do this our conception of it is obscure.

A conception is **distinct** when we not only distinguish its object from all others, but also grasp the constituent marks or parts of that object. In every-day life we may know the hand-writing or features of a person from those of all others and yet not be able to give the characteristics of either. This is true in conception,—we may be able to discriminate *man* from mineral, plant, animal, etc., and yet not be able to give the characteristics of man. Our conception is **confused** or **indistinct.** Distinctness requires us not only to discriminate between

an object and all others, but also to know the distinctive marks or parts of that object. Our conception of man becomes distinct when we see that it includes animality, rationality, and terrestriality; until then it is confused.

A conception is **adequate** when we not only grasp the constituent marks, but also the marks of these marks; **inadequate** when we fail to do this. **Perfect adequacy** of conception is reached by carrying out the complex processes of Partition, Division, and Definition until the lowest component attributes, constituent species, and characteristic marks are reached. The extent to which these processes must be carried to reach a **practical adequacy** of conception in any given case will depend upon the exigencies of the thinking or the aims of the thinker. The conception of *man* is adequate when we not only know the three marks given above, but have also gone further and grasped the marks of *animality*, as corporeity, organization, life, sentiency, voluntary motion; of *rationality*, as intuition of first truths and the power of thinking and acting in the light of such truths; of *terrestriality*, as limitation to the earth with its conditions of time and space.

It will be readily seen that **clearness** is chiefly attained through Definition; **distinctness** through Partition and Division; **adequacy** through the extended processes of Partition, Division, and Definition.

A conception is **true** when it corresponds with the reality. **The aim of the Practical Logic of conception is fully attained** when the training results in the ability of the thinker to reach true conceptions which are clear, distinct, and adequate.

PART II.

THE LOGIC OF JUDGMENT OR THE PROPOSITION.

⸻⦾⦾⦾⸻

THE aim of the Logic of Judgment is to train the mind to skill in dealing with the second Form of Thought.

Definition.—Judgment is that form of thought in which we compare two notions and mentally affirm their union or disunion, on the ground of a like union or disunion apprehended in the objects or realities which the notions represent. The result of the operation of judging is a complex form of thought known as a **Judgment,** the verbal expression of which is called an **Assertion** or **Proposition.** The connection between judgment and proposition is so intimate that the two terms are used interchangeably.

Note.—The definitions of judgment have been various. Some have defined it to be, the affirmation of the agreement or disagreement, or of the congruence or confliction, of two notions. According to Thomson, it is "an expression that two notions can or cannot be reconciled—that the mark of the one may or may not henceforward be assigned to the other." Manifestly judgment as thought is much more than mere affirmation, whether mental or verbal, of the agreement or disagreement of two notions. The question whether the form of words, "Man is intelligent," or, "Man is patent elliptic," is a judgment or embodies a judgment, is not to be decided by affirmation of any kind. It depends upon the knowledge of connection existing or not existing between the realities or objects represented by the words and notions.

Ueberweg comes nearer the presentation of the *essence* of judgment, when he makes it the comparison of two notions, whose forms are different from but belong to each other, and the mental affirmation of their union or disjunction on the ground of like relation apprehended between the objective realities

which the notions represent. The all-important thing is "the consciousness, whether or not the analogous combination exists between the corresponding objective elements. As the individual conception corresponds to the individual existence, so the judgment in its various forms corresponds to and is the subjective copy of the various objective relations."

The desired skill in judgment can only be acquired by the knowledge and use of the principles which govern the forming and unfolding of judgments. The subject will, therefore, be considered under **two Chapters,** one treating of the formation of judgments, the other of their unfolding.

CHAPTER I.

THE FORMATION OF JUDGMENTS OR PROPOSITIONS.

THE formation of judgments is manifestly a most important work of thought. Processes of reasoning and systems of science and philosophy are made up of combinations of judgments, and if the judgments are not properly and thoroughly established, *i. e.,* if they are not true, then the arguments and systems cannot be expected to prove true. It is, therefore, necessary to inquire carefully into both the **process and products of judgment-forming.**

Section I.— The Process of Judgment-Forming.

The definition of judgment already given suggests for consideration the following Topics : **first,** ascertaining and combining the elements of judgment ; **second,** finding the reasons or grounds upon which the truth of judgments depends, or the verification of judgments.

Topic First.—The Elements of Judgment.

The elements of judgment are ascertained by analyzing judgment either as embodied in the proposition or as a form of thought. From the former point of view, it is

made up of two terms (so called because they are the *termini* or boundaries of the proposition) united by the verb *to be* as copula (*bond*); from the latter point of view, it is composed of two notions united by some connecting link of thought.

I. The Terms or Notions in Judgment.

The terms or notions are distinguished as the **Subject** or **Subject Notion,** or that about which the assertion is made, and the **Predicate** or **Predicate Notion,** or that whose union or non-union with the subject is affirmed. In logical formulæ the subject is usually expressed by **S** and the predicate by **P.**

The various notions, already considered, resulting from the processes of conception, constitute the material which may possibly form the terms of judgments. The following kinds have already been distinguished : (1.) The **simple notion,** called also simple apprehension, and percept. This is the result of immediate cognitions by the senses and consciousness. In observation this notion has as yet no name given, but may be known by the indefinite " it." An orange, as an object hitherto unseen and unknown, might be called " it." (2.) The **simple abstract notion,** or part abstracted from the object observed, but not yet combined with others into a concept. By observation we get, from the hitherto unseen and uninvestigated orange, the *abstracts*, yellow, round, sweet, juicy, etc. (3.) The **general notion,** as the **concept proper** or bundle of properties or marks expressed in the concept term. By conception proper the various abstracted properties are gathered up in thought in the concept, *orange.* (4.) The **general notion,** as the **class** or group of objects to which the bundle of attributes in the concept applies. By classification the concept *orange* is applied.

It will be seen that only part of these can enter into the strictly logical judgment.

II. The Connecting Links of Judgment.

The two terms of a proposition are always united by the **copula,** which, according to the view of most logicians, is always the present tense indicative of the verb **to be,** either with or without the negative particle. The real **quality** of

judgment, however, or that which makes it what it is, is the mental union or separation of two terms or notions, on the ground of a more or less clearly apprehended connection or absence of connection between them. The various **links** by which this union in judgment is affected are to be found in the **predicables** already given.

1. While the connecting link of judgment in language is always the verb *to be*, which to the logician signifies *connection* rather than *existence*, it is obvious that the copula does not always appear in this form in propositions as we find them. E. g.,

"Columbus discovered America;" "Napoleon was the emperor of France." Hence arises the necessity for the practical application of the Second Logical Postulate, in reducing judgments to the **normal form,** S *is* P, or S *is not* P. Under this any change of logical form is permissible, provided it brings out the thought more fully, without changing it. "I am," means "I *am* existing," or, "I *am* a being." "Columbus discovered America," means, "Columbus *is* the one who discovered America." "Napoleon was the emperor of France," means, "Napoleon *is* he who was the emperor of France." "Stars twinkle," means, "Stars *are* things that twinkle." The same postulate permits the restoration of all inversions and displacements of parts of sentences to the normal form, S is P, or S is not P. E. g., "Great is Diana of the Ephesians" becomes "Diana of the Ephesians *is* great."

2. A judgment, however, is not a mere form of words, *two terms* joined by the verb *to be*. "Man is intelligent." "Man is round-square horizontal." One of these is a judgment; the other is not. The difference is that in the one case there is a connection in thought, while in the other there is none. This connection has been variously presented.

(1.) It has been said that the affirmative judgment is always based upon the Axiom of Identity; the negative on the Axiom of Contradiction. In accordance with this view judgment has been defined to be the affirmation of agreement or disagreement.

This is true, but it is necessary to go below these generalities to the special features in which the agreement or disagreement is found. E. g., in the judgment, "Man is a terrestrial, rational animal," the copula represents equality or *identity.* This is true in all perfect definitions. Or again, in the judgment, "Man is intelligent," the copula expresses the relation of *substance and property,* or *genus and species,* and the judgment is interpreted as meaning, either that intelligence is an attribute of humanity, or that man is a species of the genus intelligent beings. Or again, in the judgment, "The life was the light of men," the copula may express the relation of *substance and active property* or *cause and effect.* The judgment is thus seen to involve certain special principles of connection which underlie the mere agreement or disagreement.

(2.) According to the Aristotelian logic, every judgment predicates of the subject either a genus, or a property, or a definition, or an accident.

These forms of predication have been illustrated by suitable judgments. "Envy is a passion." The relation is that of *genus to species.* "Man has the faculty of speech." The relation is that of *peculiar property* to substance or subject. "A state is a community governed by its own laws." The relation is that of identity of the essential properties, or *essence* of a thing,—by which the definition is constituted,—with the thing itself. "Life is sweet." The relation is that of an *accidental property* to its subject.

These predicable classes have been reduced by Thomson to *definition* and *attribute,* the latter including genus, property, and accident.

(3.) The **Predicables** as given, page 30, furnish the simplest statement of what may be predicated in any judgment. Of any subject may be predicated its substance and whatever belongs to it as its properties.

Thus of *man* may be predicated the substance of the thing itself, as "Man is man;" or some of the properties of quality, as "Man is rational," or all of them (the essence or definition), "Man is rational animal;" or active properties, as "Man is the moulder of nature;" or relative properties, as "Man is of few days," "Man is terrestrial," "Man is finite," etc.

When notions or terms are thought together by one or other of these various connections the product is a judgment.

III. The Elements Combined.

The various notions or terms are united either in judgments of observation or in strictly logical judgments. These are both included under logical judgments in the wider sense.

Note.—Hamilton gives the name of **primitive judgment** to the *judgment of existence* implied in all our cognitions. This is not, however, *judgment as thought*, and, therefore, is not to be treated in Logic.

1. The **judgment of observation** follows upon observation. In starting with an orange, assumed to be a thing never before known, the observer has no name for the object. The mental analysis by which the abstracts are formed may be looked upon as made up of a succession of judgments: "It is yellow;" "It is sweet;" "It is round;" etc. All the predicates of these judgments, when gathered up, give the concept, which is finally embodied in the word *orange*, and then used in classifying all like objects as *oranges*. The judgment of observation may be seen to be the mental union of simple apprehensions or percepts and abstracts.

2. The judgment of observation thus prepares the way for and gradually approaches the **strictly logical judgment,** which makes use of the concept and class, as, "The orange is yellow;" "Oranges are yellow." It will readily be seen that the strictly logical judgment will take different forms, as the subject and predicate are concept or class notions. The various relations of the notions in logical judgments as embodied in propositions may be brought out in the following form, using the notions *man* and *mortal:*

Subject.	Copula.	Predicate.
Concept proper,		Concept proper,
(*Man* = humanity)	is	(*mortal.*)
Class,	or	Class,
(*Man* = mankind)	is not	(*a mortal.*)

The strictly logical judgment is the form of judgment of which Logic mainly treats. In the logical proposition the two terms may both be concept terms, giving a **proposition of content,** as, "Man is mortal;" or both class terms, giving a **proposition of extent,** as, "Man is a mortal."

The subject term in the latter form may be either an individual, as,

"Garibaldi;" or an individualized general term, as "this man;" or a general term taken partially, as "some men;" or a general term taken universally, as "all men." This form of logical judgment may, therefore, be either, "Garibaldi is a mortal," or "This man is a mortal," or "Some men are mortals," or "All men are mortals."

Note.—The strictly **impersonal judgments**, expressed in the classical languages without subject (except as the subject in the third person singular is involved in the termination of the impersonal verb) and in the English with "it" as the subject, as, "it rains," "it thunders," properly come under the logical judgments. Says Ueberweg, "In the so-called judgments without subjects the sum total of the existence surrounding us, thought of indefinitely, or an indefinite part of it, takes the place of the subject."

Praxis.—Examine carefully the following judgments, stating them in the normal form (S is P, or S is not P), naming the subject and predicate, and bringing out the precise connecting link in each case: 1. Truth is stronger than error. 2. The human race was one in its origin. 3. A square is rectangular. 4. A square is an equilateral rectangle. 5. "Few and short were the prayers we said." 6. "Flashed all their sabres bare." 7. Man is risible. 8. Not all the ills of earth can mar my joy. 9. Not all men are virtuous. 10. A horse may be white. 11. He that destroys a usurper does right. 12. Great is the work of life. 13. There was no possibility of substantiating the allegations. 14. "In jewels and gold men cannot grow old." 15. "From peak to peak the rattling crags among leaps the live thunder." 16. It is wrong to put an innocent man to death. 17. There is no place like home. 18. "None but the brave deserve the fair." 19. "The most sublime act is to put another before thee." 20. Life every man holds dear.

Topic Second.—Verification or Proof of Judgments.

When a so-called judgment, expressed in a proposition, is brought before the mind, the question is naturally asked concerning it, **What reason is there for believing it to be true?** A so-called judgment is decided to be true, doubtful, or false, by the presence or absence of **proof,** *i. e.*, of something which makes the reality of the connection of the two notions more or less evident to the mind.

Practically, in all our intercourse with men and books, judgments of every form are constantly being presented to our minds for con-

sideration. "Geometry is the science of extension." "Things which are equal to the same thing are equal to each other." "Logic is the art of reasoning." "The weather is cold." "If the weather remains as at present, the streams will be frozen over." In short, every sentence read, heard, or uttered involves one or more judgments, and no such judgment is anything more to us than an empty assertion until we have grasped some proof that the expressed connection of its parts agrees with the corresponding reality. The verification or confirmation of judgments is, therefore, a most important part of this form of thought.

Judgments have been divided, by the sources from which the predicate is drawn, into analytic and synthetic. The predicate notion may either be brought out of the subject notion by analysis, or brought to it from without. **Proofs** are accordingly either **analytic or synthetic,** the former being drawn by analysis from the terms of the proposition itself; the latter being brought from outside the terms of the proposition.

An **analytic** or **explicative judgment** is one in which what is affirmed in the predicate is already contained in the definition of the concept or general term which forms the subject. "Man is rational," is an analytic judgment; since the predicate, rational, is involved in the notion, man, as brought out by partition or by the definition, "Man is a rational animal." Such judgments are also called *a priori*, or judgments not grounded on but prior to experience. The simple study of what is contained in the subject notion gives the predicate without resort to the testimony of experience. E. g., in the judgment, "Body is extended," the instant the thinker understands what is meant by the term "body," he knows that "extended" is comprehended in it. A **synthetic** or **ampliative judgment** is one in which the predicate adds something which is not contained in the conception or definition of the subject. E. g., "Man is a sinner," "Neptune is the most remote of the planets," are synthetic judgments. The predicate adds to the subject something which it brings from outside and which no analysis could have discerned in the subject.

In connection with the various forms of judgment analytic and synthetic the **nature** of the proof, and the **canons** or **rules** governing it, will be set forth.

I. Proof of Analytic Judgments.

Analytic judgments furnish within themselves the material for their own verification. This is to be brought out by analysis, *i. e.*, by partition or division of the subject or predicate or both.

The proposition, "**All trees are organic**," is proved by analyzing "organic." The proposition is regarded as one of extent, affirming that the genus, "organic beings" includes the species, "trees." Organic beings are divided, by the presence or absence of a nervous system and power of causation, into animals and plants. Plants are divided, by the size and duration of the stem or ascending axis, into herbs, shrubs, and trees. The result reached may be expressed in tabular form:

$$\text{Organic beings} = \begin{cases} \text{Animals,} \\ \text{Plants} \end{cases} = \begin{cases} \text{Herbs,} \\ \text{Shrubs,} \\ \textbf{Trees;} \end{cases}$$

from which it is apparent that the lower species "trees" is included under the higher genus "organic." The proposition, "**Duelling is murder**," is analytic. Regarded as a proposition of content, its proof is reached by partition of the terms. "Murder" includes the generic mark, taking of human life, and the differential or specific marks, deliberately, unlawfully, maliciously. "Duelling," where it results in death, is found to include the same marks, taking of human life, deliberately, unlawfully, maliciously. The two are thus found to agree. Duelling is, therefore, murder, *i. e.*, the relation affirmed to exist between the two, in the proposition to be proved, corresponds to the reality. The proof of the proposition, "**Labor is a blessing to man**," is to be found by an analysis of the terms. Regarded as a proposition of extent, it affirms that "labor" is one species of the genus or class "blessings to man." By partition "blessing to man" has the active properties or characteristics, meeting some fundamental and natural need of man, giving satisfaction or happiness. There are, therefore, as many "blessings to man" as he has fundamental and legitimate needs to be satisfied. Analyzing "blessings to man" by division, we, therefore, find that the genus includes the desires for habitual activity physical and rational, for knowledge, for power, for property, for help in dependence and helplessness, for deliverance from sin, etc. Anything which meets and satisfies any one of these desires is a blessing

to man. "Labor" analyzed by partition is found to include the marks, exertion of the powers, habitual, with rational aim, or, in other words, habitual rational activity. "Labor," as meeting the fundamental and legitimate need for habitual rational activity is a "blessing to man." Continuing the process of thought still further, we may conclude from the analysis, that "knowledge" is a blessing to man, "power" is a blessing to man, "wealth" is a blessing to man, "the sustaining power of divine providence" is a blessing to man, "salvation from sin" is a blessing to man, etc.

General Rule.—The analysis must be accurate and complete.

It is obvious that this method of proof must render certain the truth of the propositions which admit of its application. All analytic proof is, therefore, said to be **demonstrative** in its force.

II.—Proof of Synthetic Judgments.

Synthetic judgments require that their proofs be sought outside of the judgments themselves. No analysis of terms will furnish the proof that, "Duelling is a relic of barbarism," or that, "The Feudal System was beneficial." The proof must be brought from outside sources.

The precise source or place outside will depend upon the species of synthetic judgment to be proved. Synthetic judgments are divided, by the place outside the proposition from which the predicate is brought, into intuitive and empirical.

Intuitive, or *a priori*, judgments are those whose predicates are brought from within the mind itself, from some fundamental or thought necessity. In these the predicate could never be unfolded from the subject, as in the judgment, "Every event must have a cause." It is a law of our thinking that compels us to connect "must have a cause" with the subject, "every event." **Empirical,** or *a posteriori*, **judgments** are those of which the predicates are brought from outside the mind. They have their ground in experience. The judgment, "Body is extended substance," is analytic, since "extended substance" is seen to be comprehended in "body," or to be identical with it; but the

9*

judgment, " Body is heavy," is a synthetic judgment, since the mark " heavy " is not comprehended in " body." The latter is an empirical judgment, since only experience, examining bodies and measuring pressure by muscular effort, enables us to predicate " heavy " of " body."

1. Proof of Intuitive Judgments. — These draw their proofs from the mind itself. The proofs are intuitions or fundamental truths, accepted by all, and lying at the foundation of all human knowledge and activity.

For the proposition, **" Suicide is wrong,"** the proof is to be found in man's intuitive convictions of duty. Every one knows intuitively that man, as a creature under the moral government of God, is bound to make the most and the best of himself, and that to fail in this is wrong. Duty is intuitively seen to require that he should preserve himself, improve himself, and use his powers for the true end of life. " Suicide " is intuitively seen to break the first of these requirements, and, therefore, seen to be " wrong." The propositions, " I exist," " I am thinking," rest upon the intuitive belief in the veracity of our consciousness.

As so-called intuitions are often urged in proof of various false judgments, it becomes necessary to keep clearly in mind the **tests of intuition.** These may be given in the following rules :

Rule 1st.—Every intuition is self-evident. The mind, on the bare contemplation of it, must see its truth at once, without requiring any foreign evidence or outside proof.

Rule 2d.—Every intuition is necessary. The mind cannot help believing and acting upon its truth. That " Space exceeds my widest imagination of space," and that " Every event must have a cause," one cannot help believing.

Rule 3d.—Every intuition is catholic or universal. It must be entertained by all men intelligent and understanding what is meant by it. An intuition is sometimes described as being " What all men everywhere and always believe."

Rule 4th.—Every intuition is accepted by all men practically. Intuitive truths may not be consciously apprehended and stated by the majority of mankind, but they are assumed and acted upon by all men, even by those who deny their belief in them.

The notions of being, personal identity, time, space, causation, the axioms of Mathematics, Logic, Ethics, etc., are among these self-evident, necessary, and universal cognitions of men.

It is evident that all such proofs, properly tested, must render certain the truth of propositions based upon them. Intuitive proofs are, therefore, said to be, like analytic proofs, **demonstrative** in force.

2. Proofs of Empirical Judgments. — Empirical judgments, or those based upon something outside of the proposition and of the mind itself, rest for their proofs upon the experience of the thinker himself or of others. Knowledge in the form of **experience** has been seen (p. 16) to include the observation and thinking of the man himself, and the observation and experience of others given in testimony and authority. This suggests the kinds of empirical judgments to be established.

(1.) **Judgments from Observation.**—When the judgment to be verified is based upon our own observation of things external or internal, its truth is tested by careful application of the Rules of Observation already given (p. 33). Thus, "I see my uplifted hand in all its parts;" "I am conscious of exertion in lifting my hand," are judgments of observation. Their truth evidently depends upon the trustworthiness of the senses and consciousness, assumed in all observation, and upon strict compliance with the Rules of Observation.

(2.) **Judgment from Thought.** — Many empirical judgments are reached by the processes of Reasoning Inductive and Deductive. These must be tested by the Canons of Reasoning, which will be presented in Part III.

(3.) **Judgments from Testimony and Authority.**—**Testimony** is the statement of others concerning matters of fact which they have observed in their own consciousness or in the world around them. **Authority** is the statement of others concerning matters of opinion which they have reached by the processes of conception, judgment, and reasoning. The testimony or authority may be recorded on

monuments or in writings, books, etc., or given by word of mouth.

As almost all human knowledge is received on testimony or authority, the question, **What are the tests of testimony and authority?** becomes a most important one. The tests are to be found either, **first,** in the ability, character and number of witnesses or authorities, or, **secondly,** in the character of that which they present. Out of these arise the Rules to be observed in judging of the truth or falsity of judgments received from others.

Rule 1st.—A witness or authority must be competent, *i. e.*, must have the opportunity, the ability, and the disposition to know the facts testified to, or to think out the judgments presented on his authority.

a. **Want of opportunity** to observe destroys the value of any so-called testimony. The testimony of A concerning what B says that C did is mere **hearsay,** and of little evidential value. **Negative testimony** is of little value. The testimony of a thousand witnesses that they did not see A kill B is not sufficient to countervail the statement of one good witness that he did see A kill B. **Want of ability** to observe the facts in any given case may make the testimony worthless. A blind man's testimony to mere objects of sight is worthless. Certain spheres of observation require special skill, so that only the testimony of **experts,** or those trained for the purpose, may be of value in those spheres. A man acquainted with the phenomena of electricity will be able to detect important facts which would entirely escape the notice of the ordinary observer. Testimony regarding the distance, size, form, and appearance of any object requires a trained judgment to make the observation trustworthy. Thus the testimony of an expert,— *e. g.*, of a practical astronomer to the fall of a meteor,— may become of more value than that of hundreds of ordinary observers. **Want of disposition** to observe accurately vitiates testimony. This may result, through habitual carelessness, in imperfect observation, or, through prejudice, in warped views of things. There are men who, from the first cause, never see anything worth seeing, and others who, from the second cause, always see things double or quadruple or as they expect or wish to see them.

b. **Want of opportunity or ability or disposition** to think out the

conclusions for which one is quoted as an authority must, of course, destroy the weight of the authority. In order to be **an authority** in any department of thought a man must have had special opportunity of acquaintance with that department, must have shown himself pos· sessed of extraordinary ability to deal with it and of unusual mastery of it, and must be disposed to seek and discern the truth in it. The authorities in *Law* are the men who have shown themselves masters of legal science. The authorities in *Physical Science*, are, according to Professor Tait, "the advanced, best, ablest scientific thinkers." The "competent authorities" in Physics are not the men who simply observe and experiment, but the men of exact science, who, largely by the aid of mathematics, have advanced the bounds of the science. Professor Tait names as such authorities in Great Britain, "Brewster, Faraday, Forbes, Graham, Rowan Hamilton, Herschel, and Talbot," in the immediate past, and "Andrews, Joule, Clerk Maxwell, Balfour Stewart, Stokes, William Thomson, and such like," in the present. The authorities in *Theology*, *Philosophy*, etc., are the men who are masters in these departments.

The **utterance of a competent authority** in any department has great weight even when not accompanied with the reasons, because he is rightly supposed to know whereof he affirms. The word of the average man, even if he is admitted to be familiar with his subject, has just as much weight as the reasons by which he supports it, and it has weight at all only as he presents his reasons along with it. He is **not an authority.** Assertions made concerning Theology, Metaphysics, etc., by experimental physicists who have given absolutely no attention to those difficult departments, are worth just as much as the counter assertions made concerning Experimental Physics by theologians who know nothing of that department. In all such cases, however distinguished a man may be in his own department, his words concerning the unknown department should have only so much weight as is given by the reasons with which he accompanies them.

Rule 2d.—A witness or authority must be credible, *i. e.*, must be of such a character as to be worthy of belief.

a. Whatever the opportunities or natural ability of a witness, if he is shown to be careless in observing, credulous in receiving statements, addicted to falsehood, under the influence of prejudice, or swayed by motives that would warp his view of the facts, the value of his testimony is just so far impaired.

b. The value of authority is equally affected by the credibility of the one giving the opinion. If the judge who renders a certain decision can be shown to be corrupt, or to be in any way wanting in principle, his decision will come so far short of commanding assent as authority.

Rule 3d.—Concurrence in testimony or authority increases the probability of its truth.

a. The force of **concurrence in testimony** is broken when there is evidence of **collusion** or pre-arrangement. **Precise agreement** in stating the general facts and all the details of any occurrence is looked upon as proof of collusion; whereas **incidental variation** in non-essential particulars, along with general agreement, shows the absence of collusion and the truthfulness of the witnesses. Where there has been no opportunity for collusion, concurrent testimony may become absolutely conclusive even where all the witnesses are **noted liars.** In such cases we cannot account for the agreement except on the ground that what the witnesses independently state is true.

b. The force of **concurrence in authority** is subject to the same limitations as that in testimony. Too precise agreement in statement of matters of opinion indicates probable collusion. No weight is to be attached to the concurrence of many judges, if it can be shown that the successive decisions have all followed some one original and leading decision. If, however, there is evidence that each arrived at his decision by independent thought, the authority may become of the greatest weight, even when the word of each one separately could command little or no respect. The cumulative force of **convergent evidence or argument** is also to be considered. The convergence of several lines of proof is often sufficient to render certain what perhaps no one of these lines alone would fully establish. This is illustrated in the proof that there is a personal God. The consent of mankind, the principle of causation, the order of the universe, the intuition of the infinite, the voice of conscience, and the yearnings of the affections, all converge towards the common centre, a personal God, and the strength of the proof lies in this convergence, rather than in the separate arguments taken alone.

Rule 4th.—Things absurd or impossible are not to be believed on the ground of testimony or authority, although

things strange, wonderful, or even miraculous may be believed on such ground.

Whatever is **absurd** or **impossible,** *i. e.,* logically contradictory or beyond the reach of power to accomplish, cannot, of course, be believed. No testimony or authority could make one believe in a triangle with four sides, or in Mill's conceived world in which two and two make five. It must be observed, however, that what is merely **contrary to experience** is not necessarily absurd or impossible. The King of Siam had never in his experience known water to be transformed into a solid upon which men could walk; but every one sees that this was not sufficient reason for his pronouncing the missionary, who told him of such a thing, a liar and impostor, since human experience is very limited.

There is need to note especially **the natural inclination** of men to pronounce everything absurd and impossible which contradicts their settled convictions, their preconceptions or their prejudices, or which is repugnant to their feelings. It was once, by the majority of mankind, pronounced impossible for the earth to turn on its axis and move through space with incredible rapidity without our perceiving it. It was declared absolutely impossible that information should be transmitted thousands of miles in the fraction of a second, or that a man should converse with his friend hundreds of miles away. It must be borne in mind that **the impossible** is only that which is logically contradictory or beyond the reach of power; and that, therefore, before any particular thing can be pronounced impossible, the laws and limitations of thought and power must be comprehended and found to forbid its accomplishment. A thing may, therefore, be perfectly credible, though it be **strange, unaccountable,** or even **unintelligible.** "What is strange or unaccountable to one mind may be perfectly familiar and plain to another. For the most limited intellect or experience to make itself the standard of the possible, would be as absurd as a man's making his visible horizon the limit of space." Even testimony to **supernatural and miraculous events** may be entirely worthy of belief, if there be any Supernatural Power in the universe, and such events may and ought to be believed if the witnesses are competent and credible and concur in their statements. It is a remarkable fact that the greatest scientists and philosophers,—such men as Bacon, Locke, Descartes, Newton, Herschel, Brewster, and Faraday,—have unhesitatingly believed in **miracles** on the ground of such testimony, regarding them, not as events without any adequate cause, but as

events into whose production a higher, Unseen Cause entered. In all such cases, however, the witnesses to the supernatural events should be subjected to the most rigid scrutiny and cross-examination, according to the established rules of testimony.

It is evident that the proofs of empirical judgments never give the judgments the absolutely demonstrative force which belongs to the proofs of analytic and intuitive judgments, but simply render them **more or less probable.** As the entire practical ongoing of human life depends upon such judgments from experience, *i. e.*, from observation, thought, testimony, and authority, the meaning and truth of Butler's statement, that **"probability is the guide of life,"** becomes apparent.

Probability varies in different cases. It may in one case practically amount to certainty; in another the balance may be as a thousand, or a million, or vastly more, to one, against the truth of the judgment. The **rational conduct** of human affairs varies accordingly. Where the balance of probabilities is in favor of the truth of a judgment, men base their action upon it, in all the ordinary affairs of life, with a confidence increasing as the degree of probability rises. When the probabilities are as fifty-one to forty-nine that certain goods will greatly advance in price, the enterprising merchant hesitatingly invests in them; as the probabilities become as seventy-five to twenty-five, he invests more eagerly; as the probabilities approach certainty, he secures control of all that his capital will enable him to command. Where **great and permanent practical interests** are involved, even the lowest degree of probability should, in accordance with the dictates of common sense, be acted upon. The man wrecked in mid-ocean wisely clings to his solitary plank even when the probabilities that he will be saved are only as one to a thousand or even one to a million. **The balancing of probabilities and deciding the course in view of them is manifestly an essential part of man's rational and moral discipline in this world.**

Note.—Professor Jevons says of the **Theory of Probabilities:** "It is the very guide of life, and hardly can we take a step or make a decision of any kind without correctly or incorrectly making an estimate of probabilities. . . . The whole cogency of inductive reasoning rests upon probabilities. The truth or untruth of a natural law, when carefully investigated, resolves itself into a

high or low degree of probability, and this is the case whether or not we are capable of producing precise numerical data."—Jevons' *Principles of Science,* p. 217.

Praxis.—Examine critically the following judgments or propositions,— first, stating of each whether it is analytic or synthetic; secondly, if analytic, developing the proof from the judgment itself; thirdly, if synthetic, showing whence its proofs are to be derived and bringing the proofs of the judgments from observation, testimony, and authority from the proper sources:

1. Washington is the capital of the United States.
2. George Washington was a true patriot.
3. Columbus discovered America.
4. New Orleans is situated on the Mississippi.
5. England is across the Atlantic Ocean.
6. There is such a country as China.
7. Madagascar is inhabited.
8. Civilization has been progressive from the earliest ages.
9. The Aztecs reached a high degree of civilization.
10. Lying is never justifiable.
11. The Allegheny Mountains were formerly submerged.
12. The Himalayas are the highest mountains on the globe.
13. The feudal system was beneficial.
14. Honesty is the best policy.
15. Education cannot be effected by mere class-room instruction or lecturing.
16. The sum of the three angles of a triangle is equal to two right angles.
17. Two straight lines cannot inclose a space.
18. The earth is between 93,000,000 and 94,000,000 miles from the sun.
19. Wrong-doing blinds the conscience.
20. Falsehood is dangerous.
21. The story of Christ's life and death is true.
22. Joan of Arc was a religious enthusiast.
23. In a right-angled triangle the hypothenuse is the longest side.
24. Any two sides of a triangle are together greater than the third.
25. Christianity is of divine origin.
26. The study of the classics is necessary to the highest culture.
27. North America was once inhabited by a race of Indians of higher civilization than the existing tribes.

28. Christianity is the religion which meets the needs of man.

29. A triangle cannot have more than one angle as great as a right angle.

30. The moon revolves round the earth.

31. The best science recognizes a God.

32. Probability is the guide of life.

Section II.—The Products of Judgment.

The process of judging results in **judgments** which are embodied in **propositions.** These products need to be carefully classified and divided, since the unfolding of judgments depends upon a knowledge of their kinds and characteristics, and since judgments constitute the material of Reasoning, the third Form of Thought.

Judgments of content and **extent** and **analytic** and **synthetic judgments** have already been considered in treating the process of judgment (pp. 97–99). **For further logical purposes** the chief divisions of judgments are based on the various **ways of making the predication** or assertion, since the assertive element is the main one in judgment. This gives rise to the following divisions:

First, by the **quality** of the predication, whether affirmative or not, into affirmative and negative judgments. This division is treated under **Quality of Judgments.**

Second, by the **extent** of the predication, whether total or not, into universal or total and particular or partial judgments. This division is treated under **Quantity of Judgments.**

Third, by the **directness** of the predication, whether direct or indirect, into categorical and hypothetical. This division is treated under **Relation of Judgments.**

Fourth, by the **degree of certainty** of the predication, whether certain or not, into certain including demonstrative and assertory, and not-certain including probable and possible. This division, as it has reference to the results in the mind of the thinker himself, will be treated, in summing up the results of thinking in its second form, at the close of Part II., under **Modality of Judgments.**

Since the divisions of **scientific syntax** in Grammar depend upon the forms and combinations of logical judgments or logical propositions,

for grammatical purposes there is still another division of judgments, which needs to be considered:

Fifth, by **combination,** whether single or not, into simple, and multiple or combined including complex and compound. This is treated under **Grammatical Combination of Judgments.**

Topic First.—Quality of Judgments.

By the quality or character of the predication judgments are either affirmative, as, "Belgium is populous;" or negative, as, "The vicious are not wise." In the former there is indicated the union of the two notions by some link of connection, and they are, therefore, said to agree, by the principle of Identity; in the latter there is indicated by the negative the separation of the two notions, which are, therefore, said to disagree, by the principle of Contradiction.

It follows from the nature of negation that a **negative copula** always excludes everything in the predicate,—the whole, the species, the individuals,—entirely from the subject. E. g., "No men are angels" cuts off the entire class "angels" and all that is included in it from the class "men." "Some men are not artists" cuts off the entire class "artists" from these "some men." This is called the **distribution** of the predicate, or the taking of it in its entire signification.

It should be observed that the **negative particle** is not always connected with the copula, but may be placed in other parts of the proposition; yet in every judgment really negative it belongs only to the copula. By the second Logical Postulate it is always permissible to put the negative into its proper place, with the verb *to be,* in reducing any proposition to the normal form, S *is not* P. "No human knowledge is perfect" may thus be changed into, "All human knowledge *is not* perfect." In many **apparently negative propositions** the force of the negative particle does not fall on the copula, but upon one of the terms. E. g., "*Not* to submit *is* madness" is really an affirmative proposition, since the force of the "not" falls on the words "to submit." The meaning is, "Non-submission (or resistance) *is* madness." Again, "A person *not* vicious *is* virtuous" is equivalent to, "A non-vicious person *is* virtuous," and is, therefore, an affirmative proposition. In like manner **propositions apparently affirmative** may be really negative, the force of the negative particle being in some way involved in the thought, if not in the form of expression. E. g., "Only a few

men are wise;" "Few men are wise;" "But few men are wise," are all substantially negative propositions, since they are equivalent to, "Most men *are not* wise." On the other hand, "A few men are wise," is an affirmative proposition. Great care should manifestly be exercised in ascertaining the precise quality of all such propositions.

Topic Second.—Quantity of Judgments.

The quantity of judgments depends upon the extent of the predication. Certain logical distinctions, which arise from the combination of quantity with quality, may also be most conveniently treated under this Topic.

I. Kinds of Judgments by Quantity.

The predicate notion of a judgment may be affirmed or denied either of the whole of a subject or of a part of it only. Having once formed the notion, "orange," we may affirm that, "This orange is yellow," or, "Some oranges are yellow," or, "All oranges are yellow." Hence judgments by this division are universal or total and particular or partial.

1. **Universal** or **total judgments** include the strictly universal, or those in which the notion of the subject is taken in its entire extent; the judgments in which a definite part of the notion of the subject is taken; the judgments with individualized, singular, or collective subjects; and equivalent or substitutive judgments.

Universal judgments in the strict sense are those in which the predicate notion is affirmed or denied of the entire subject notion, *i. e.*, of all that is comprehended or contained under it, whether attributes or objects. The subject is, in this case, a logical whole taken in all its parts. "All men are mortal;" "Every man is mortal," are universal judgments, the subject embracing the total number of objects in the class "man." The subject in all universal judgments, whether affirmative or negative, is said to be **distributed,** because what is predicated is predicated of each and every object in the entire whole. Universal judgments include those in which **a definite part** of the subject is taken, as, "These men are Japanese." They also include judgments with **individualized subjects,** as, "This man is sober;" and judgments with

singular subjects, as, "Bucephalus is a horse;" "France is not an empire." This follows from the fact that the predicate notion is affirmed or denied of the whole subject. The same is true of judgments whose subjects are **collective wholes,** as army, forest; **mass wholes,** as, wheat, rice; **material wholes,** as gold, stone.

From the predication of the *definition*, or *essence*, of a notion, there arises a peculiar kind of universal judgment in which the subject and predicate are equal and identical. This is known as the **equivalent,** or **substitutive judgment,** in distinction from the simple **attributive judgment** or ordinary universal. For example, "Body is extended substance;" "Man is a rational animal." In all such judgments the notions or terms of both subject and predicate are taken in their entire meaning, or **distributed.**

The **signs** of universal judgments are *all, every, each, both, any, none, neither, always, never, whoever, wherever, whatever,* etc. Care must be taken, however, to guard against the ambiguous use of such signs, especially against such use of the word **all.** The word *all* in its proper logical sense means "each and every;" but it stands sometimes for "all taken together," as, "All these claims upon my time overpower me." Hence may arise an ambiguity, since instead of *all,* in its proper sense of "all taken together," we are liable, in our interpretation, to put *all* in its logical sense of "each and every." The example could not mean, "Every single claim upon my time overpowers me."

2. **Particular or partial judgments** embrace the ordinary form including the purely indefinite and the semi-definite judgments; and the more unusual forms called numerically definite and plurative judgments.

Particular or partial judgments are those in which the predicate notion is affirmed or denied of a number of objects less than the whole denoted by the subject notion, as, "Some men are poets," "Some rulers are not just." In particular judgments the naked subject must always be restricted either by implication or by some restrictive term. The **signs** of particular judgments, are, *some, not all, not every, a few, there are—that, a* or *an, one, two, three,* etc., *sometimes, somewhere,* etc.

The word **some,** as used in introducing particular judgments embodied in propositions, is, as Hamilton has shown, ambiguous. In some instances it introduces a **semi-definite judgment,** as, "Some men are poets," i. e., *some at most, not all.* In other instances it introduces a **strictly indefinite judgment,** as, "Some men reason," i. e., *some at*

least, perhaps all. The latter is the old logical meaning of *some*, and the judgment is wholly indefinite; the former meaning makes the judgment semi-definite, since it excludes *all.* In which sense the word is used in any given instance must be determined by examining the thought or, in connected discourse, the context. **Numerically definite judgments,** are those in which the predicate notion is affirmed or denied of a definite number or proportion of the objects included in the subject, as, "Ten men in a thousand are wise." Considering the "ten men" alone as the subject, the judgment would be regarded as universal, since the predicate is affirmed of all the ten. Of like nature are **plurative judgments** which embrace more than half but not all the subject. These may be *numerically definite*, as, "Forty men out of the fifty on the steamer perished;" or *indefinite*, as, "Most men are not poets." In the numerically definite form the sign is found in *numbers* expressing more than half of the whole embraced in the subject. In the indefinite plurative judgment the signs are found in such expressions as, *more than half, the majority, many,* etc.

When the predication approaches more nearly to covering the whole of the subject, as in **approximately universal judgments,** such terms are used as *most, almost every one, the large majority,* etc. On the other hand the following signs are **nearly total negatives:** *few, very few, hardly* or *scarcely any, little, small, slight, rare, seldom,* etc.

II. Logical Distinctions from Quantity and Quality Combined.

Two subjects—the normal forms of judgments as they appear in the syllogism, and the distribution of terms—are dependent upon both Quality and Quantity, and will be most naturally treated and best understood in immediate connection with these topics.

1. **Normal Forms of Judgment.**—Men in their thinking combine quality and quantity in judgments. To facilitate the use of judgments in the syllogism logicians have formed a complete scheme of judgments combining quality and quantity, and have affixed to each form a symbol by which both quality and quantity are briefly expressed. The possible combinations are four, two of which are subdivided as shown in the following form:

	Quantity.	Quality.	Symbol.	Formula.
Judgments by Quality and Quantity.	Universal Affirmative, Attributive,		A,	All S is (some) P.
	[Substitutive,		U,	All S is (all) P.]
	Universal Negative,		E,	No S is (any) P.
	Particular Affirmative, Attributive,		I,	Some S is (some) P.
	[Substitutive,		Y,	Some S is (all) P.]
	Particular Negative,		O,	Some S is not (any) P.

These may be illustrated by examples:

All men are (some) mortals	A.
[All men are (all) rational animals . . .	U.]
No men are (any) angels	E.
Some men are (some) mortals	I.
[Some men are (all) the poets	Y.]
Some men are not (any) artists . . .	O.

The judgments in most common use are A, E, I, and O, and the logical processes are usually confined mainly to these.

2. **Distribution of Terms.**—As already indicated, a term is said to be distributed when it is taken in its entire signification embracing each and every object included under it. From the principles already presented a general statement of the terms distributed, or taken in their full extent, in the various judgments and also of those undistributed, or not taken in their full extent, can readily be made. These may be embodied in Rules.

Rule 1st.—All universals,—A, U, and E,—and no particulars,—I, Y, and O,—distribute the subject.

Rule 2d.—All negatives,—E and O,—and all substitutive affirmatives,—U and Y,—but no attributive affirmatives,—A and I,—distribute the predicate.

From the nature of quantity and quality, as seen in the statements made and examples given, it appears that the six kinds of judgments have their terms distributed or undistributed, as follows:

A distributes the subject only.

U	"	both subject and predicate.
E	"	both subject and predicate.
I	"	neither subject nor predicate.
Y	"	the predicate only.
O	"	the predicate only.

Praxis.—State of each of the following judgments,—first, to which of the six forms it belongs, and whether its terms are distributed or undistributed and why, marking the judgment by its appropriate letter; secondly, if the judgment is particular, whether it is definite, semi-definite, numerically definite, plurative, etc., and if universal, whether singular, attributive, substitutive, etc.; and, thirdly, if ambiguous, wherein the ambiguity consists:

1. All oaks are trees.   2. Some men have genius.   3. Poets are men of genius.   4. Body is extended substance.   5. This inkstand is made of glass.   6. The senate has adjourned.   7. Birds breathe and fly.   8. "All Jerusalem went out to meet him."   9. Salt is chloride of sodium. 10. Some men reason.   11. Some men seek reputation.   12. A few were saved.   13. He that does not heed, stumbles.   14. Nine boys out of ten prefer play to study.   15. Forty of the fifty sailors perished.   16. Not every mistake is culpable.   17. Milton was blind.   18. All men are not liars.   19. God is good.   20. Gold is a heavy metal.   21. With rare exceptions men are selfish.

Topic Third.—Relation of Judgments.

The relation of judgments depends upon the manner of predication.   The predication may be made either simply and positively or may be made to depend upon something else.   The first gives rise to the categorical judgment; the second to the hypothetical.

Note.—The ordinary grammatical division of propositions as embodied in sentences is based upon the mental states embodied.   It embraces the following kinds of sentences:

Sentences

Expressing Cognition or Intellect, including,—
  Interrogative, showing search for ground of judgment,
  Hypothetical, showing certain grounds only as still in doubt,
  Categorical, showing the comparison and connection completed;
Expressing Emotion or Sensibility,—
  Exclamatory, embodying feeling;
Expressing Conation or Will, including,—
  Optative, indicating wish or choice,
  Imperative, indicating determination or volition.

Interrogative sentences may have the same terms as the other sentences expressing cognition, and are treated in Logic in the same manner as those sentences. The elements of *emotion* and *will* do not enter into the *thought* of the proposition, in the strict sense. In so far as the sentences based upon them express thought in the proper sense, they may be treated as propositions expressing cognitions, and so become either categorical or hypothetical. See Davis' *Logic,* p. 82.

1. A **categorical judgment** is one in which the predicate is affirmed or denied of the subject simply and absolutely or without condition, as, "Captain Jack was a Modoc chief;" "Benedict Arnold was not a patriot." The affirmatives are based on the principle of Identity, the negatives on that of Contradiction.

2. A **hypothetical judgment** is one in which the predication is based upon some circumstance "which must be granted or supposed before the assertion becomes applicable." The supposition may be either a **condition** or an **alternative** or **both** these combined; and hypothetical judgments are, therefore, of three kinds, conditional, disjunctive, and dilemmatic.

(1.) A **conditional** or **conjunctive judgment** suspends the predication upon some **supposed circumstance** (called a **condition**), as, "If the sun shines the snow melts." This may be put into the form, "The snow is,—if the sun shines,—melting." "Melting" is predicated of "snow" upon the condition that "the sun shines." If it be true that "the sun shines," then it is true that "the snow melts." The supposed circumstance, "If the sun shines," is called the **antecedent;** the judgment suspended upon the condition is called the **consequent.** The relation between the two is that of reason and consequent, or cause and effect. The conditional judgment is, therefore, based upon the principle of Sufficient Reason. The signs of conditionals are, *if, when, in case of,* etc.

Conditional judgments may be **converted** into categorical form by changing the signs, *if, when, in case of,* etc., into such phrases as "the case of," "the circumstances in which," etc. Thus the conditional, "If the sun shines the snow melts," becomes "The case of the sun's shining is the case of the snow's melting."

(2.) A **disjunctive judgment** suspends the predication upon some alternative introduced by "*either — or.*" It involves two or more

judgments, all of which cannot be true, but one or more of which, by the principle of Excluded Middle, must be true. Thus in the disjunctive, " Either the Bible is false or holiness ought to be followed," there are two alternative judgments, " The Bible is false ; " and " Holiness ought to be followed." " Either London is in England or it is not," contains two alternative judgments, " London is in England ; " " London is not in England." One or other of them must be true; the other cannot be. The disjunctive needs to be carefully distinguished from the **partitive judgment,** which, under the form of a disjunctive, simply predicates of a genus its several species ; as, " All Africans are either bond or free." The genus, *Africans,* is in this case made up of the component species, *bond* and *free,* which are affirmed of it not alternatively nor disjunctively, but *concurrently.* The affirmation of the one is not a denial of the other.

Disjunctive judgments may be **converted** into categorical form by using all their members for one of the terms, and the phrase " possible cases," or, " the only alternative," or one like it, for the other term. The disjunctive, " This season is either Spring, Summer, Autumn, or Winter," becomes, " All the possible cases regarding this season are Spring, Summer, Autumn, and Winter." Disjunctives may also be converted into conditionals by taking the contradictory of one of the members for the antecedent and making the other members consequents. Thus, " If it is not Summer, it is either Autumn, Winter, or Spring."

(3.) A **dilemmatic judgment** is a hypothetical involving a combination of the conditional and the disjunctive. The disjunctive may fall either in the antecedent or in the consequent. Thus, " If a man falls into the sea, he will either sink or swim ; " " If man is either praiseworthy or blameworthy, he must be a free agent."

A dilemmatic judgment may be **converted** into categorical form by changing each of its elements, according to the principles laid down under hypotheticals and disjunctives.

Topic Fourth.—Grammatical Combination of Judgments or Propositions.

Judgments embodied in propositions are either single or combined. Combined judgments are combined by subordination or by co-ordination. Propositions are, therefore, simple, complex, or compound.

A simple proposition consists of only one subject and predicate. Both the subject and predicate may, however, be grammatically very complex, e. g., "A

legitimate and forcible argument may fail to win the assent of a prejudiced man." The kinds of judgments thus far treated are chiefly forms of simple judgments embodied in simple propositions.

A **complex proposition** consists of a principal judgment with one or more subordinate judgments, e. g., "Man who is born of a woman is of few days." The **subordinate elements** appear as *substantive, adjective,* or *adverbial* elements, so that in logic the complex sentence is treated as embodying a simple judgment. The office of the subordinate clauses is *explicative,* as, "Whoever is right, is safe;" or *restrictive,* as, "Men *who are avaricious* are discontented."

A **compound proposition** is made up of two or more co-ordinate judgments, as, "Art is long, and time is fleeting." For logical purposes the constituent judgments of a compound proposition require separate and independent statement. **Co-ordination** is either copulative, adversative, disjunctive, or causal. The co-ordination is **copulative** when two or more thoughts, which are considered independent, are so united together that the thought expressed in the co-ordinated judgment gives a greater extent to the thought of the preceding judgments, e. g., "Socrates and Plato were wise;" "Plato was a philosopher and Sophocles was a poet." The copulative connection may be either *annexive, enhansive, intensive,* or *ordinative.* The co-ordination is **adversative** when the judgments united in thought stand in opposition to one another, e. g., "Not the rich are happy, but the good." The opposition may be *contradictory, contrary,* or *restrictive.* The co-ordination is **disjunctive** when the judgments united in the one thought exclude one another, e. g. "Either he is here or he is not here." The disjunction is either *exclusive* as in the ordinary disjunctive judgments, or *separative* as in comparisons. The co-ordination is **causal** when the last of the co-ordinate judgments denotes the ground of the preceding judgment, or the conclusion from it, as, "Aristotle was an accurate thinker, for he formed conceptions and judgments well." The causal relation in the wide sense, may be either *reason,* or *cause proper,* or *conclusion* from reason, or *consequence* from cause.

Note.—For a full presentation of the principles of subordination and co-ordination, see Kühner's Latin and Greek Grammars, and Becker's German Grammar.

Praxis.—Examine and characterize the following judgments,—First, reducing them to the normal form; secondly, bringing out the connecting links; thirdly, indicating the sources of proof; fourthly, giving the quantity, quality, and relation, and showing the distribution of the terms; fifthly, stating whether simple or combined, and if combined showing whether complex or compound, and bringing out the particular relations of subordination or co-ordination:

1. No reptiles have feathers.
2. Grace is unmerited favor.
3. None are free who do not govern themselves.
4. He that ruleth his own spirit is greater than he that taketh a city.
5. George Eliot was the wife of George H. Lewes.

6. He that getteth silver is not satisfied with silver.

7. Thomas Jefferson prepared the first Anglo-Saxon Grammar produced in America.

8. There is no fireside, howsoe'er defended,
   But has one vacant chair.

9. Never morning wore to evening but some heart did break.

10. The rich are not necessarily happy, for happiness is not the result of external circumstances.

11. Those here present constitute the class in Logic.

12. All that glitters is not gold.

13. Man was originally a long-eared animal of arboreal habits.

14. A miracle is impossible.

15. No such thing as a miracle has ever been experienced.

16. Who steals my purse, steals trash.

17. Life every man holds dear.

18. If Christ rose from the dead, then Christianity is true.

19. Either Richard III. was a monster or Shakespeare was wrong.

20. If Socrates was innocent, Anytus was either deceived or perjured.

21. Wherever there is smoke, there is fire.

22. If Cæsar lives, he will rule or ruin.

23. He would have gone, but was prevented by sickness.

24. Goliath uttered his challenge and David accepted it.

25. First, the dawn; then, the rising sun; and last, the busy tide of life.

26. There are studies much vaunted, yet of little utility.

27. Some democracies are unstable.

28. Some honest men become bankrupt.

29. The world's no neuter; it will wound or save.

30. The country is generally flat or but slightly undulating.

31. Wealth may seek us; but wisdom must be sought.

32. He had the air of dignity, yet of deep feeling.

33. For man to tell how human life began is hard; for who himself beginning knew?

34. Thy father slew my father; therefore die.

35. We have no slaves at home—then why abroad?

36. He is very great in knowledge, and accordingly valiant.

37. I have the wish, but want the will to act.

38. The widow and her child returned to England, and lived almost hopeless in their old home.

CHAPTER II.

THE UNFOLDING OF JUDGMENTS.

THE best use of judgment in the practical work of think-ing requires that the thinker should be able to unfold what may be contained in any judgment, or implied in it, or im-mediately inferred from it. Hence the following Topics :

First, the development of contained judgments.
Second, the development of implied judgments.
Third, the development of inferred judgments.

Note.—Some logicians consider this subject as a part of **Reasoning.** Ac-cording to these, Reasoning is, either by inference from one judgment to an-other derived from it ; or from two judgments to a third, which could not be derived from either alone but is drawn from both combined. The latter is called **Mediate Inference;** the former **Immediate Inference.** The subjects of the present Chapter are, according to this view, treated under the head of Rea-soning. They are, however, properly to be treated under Judgment, for they all flow from the nature of conceptions as already presented and from the re-lations of these conceptions in judgments and propositions.

Section I.—Development of Contained Judgments.

That which is contained in any judgment may be brought out by analysis of the content or extent of its terms, the subject and predicate.

This form of analysis is of great service in careful thinking and especially in confirmation of judgments. It is applicable, of course, only to judgments in which at least one of the terms is complex or has component attributes or species. The process must conform to the laws of Partition and Division.

The proposition, " The highest civilization is dependent on Chris-tianity," may be analyzed, as a **proposition of content,** either by par-tition of the subject or of the predicate. The **subject,** "the highest civilization," includes as marks or attributes : the most righteous civil government; the completest development of the arts industrial and æsthetic ; the broadest and most liberal education ; the best manners and morals, or conduct in all relations ; the highest spirit of enter-prise and progress. The proposition may therefore be unfolded into the following : The most righteous civil government is dependent on

11

Christianity; The completest development of the arts industrial and æsthetic is dependent on Christianity; The broadest and most liberal education is dependent on Christianity; The best form of manners and morals, or conduct, in all relations is dependent on Christianity; The highest degree of enterprise and progress is dependent on Christianity. The **predicate** element, "Christianity," may be analyzed to meet the requirements of these propositions for proof. From this point of view, it includes the following marks: the perfect standard of justice; the true theory of activity and beauty; the grandest system of truth; the complete theory of responsibility and duty; the inspiring principles of progress. The **proposition** may, therefore, be unfolded into the following: The highest civilization is dependent upon Christianity as a perfect standard of justice; The highest civilization is dependent upon Christianity as the true theory of activity and beauty; The highest civilization is dependent on Christianity as embracing the grandest system of truth; The highest civilization is dependent on Christianity as the complete theory of responsibility and duty; The highest civilization is dependent on Christianity as containing the inspiring principles of progress.

Propositions of extent may be unfolded by the principles of division. Thus, "Free institutions are conducive to progress," may be unfolded through the **subject** as a genus, as including free governmental institutions, free educational institutions, free social institutions, free religious institutions, etc.; and through the **predicate,** as including political progress, educational progress, social progress, religious progress, etc.

To the development of contained judgments manifestly belongs also what Thomson names, "**Immediate Inference by the Sum of several Predicates.**" "Copper is a metal, red, malleable, ductile, etc.," is in no proper sense an immediate inference from the judgments, "Copper is a metal," "Copper is red, etc.," but a simple compounding of them. So these component judgments are simple constituents of the general judgment, and may be unfolded from it.

Praxis.—Develop the following propositions by Subject and Predicate, and suggest the sources of proof for the resulting propositions:

1. The studies of the High School Course are best fitted to prepare for the pursuits of business life.

2. The studies of the College Course are best fitted to prepare for the work of the professions.

3. The Fine Arts are favorable to a pure morality.

4. The study of the Ancient Classics is the best discipline for the mind.

5. Manly qualities are becoming to a student.

6. Proper protection of the various industries is essential to national prosperity.

7. The discipline of life is essential to man's development.

8. Division of labor is essential to national wealth.

Section II.—Development of Implied Judgments.

The implied judgment, according to Davis, " is one that actually exists together with the given judgment, either merely in thought or involved covertly in the expression." Several simpler and less important forms of implication need to be noted, but especially the more important form named obversion.

Topic First.—Simpler Forms of Implication.

These are chiefly forms of interpretation of the language or thought.

Such judgments may be **covertly implied in the language.** Thus, in the proposition, " Few men are wise," it is covertly implied by the language that " Most men are not wise." " Some men are rich," implies that " Some men are not rich." Such judgments are sometimes **implied in the thought.** Thomson places under immediate inference, what he names, **" Immediate Inferences of Interpretation."** It is not strictly *inference* but rather *implication*. Thus, in the judgment, " John loves Mary," it is implied that " John lives," that, " Mary lives," and that, " there is such a thing as love."

. The development of **active and passive forms** of judgments from each other may also be placed here. In the active form, " Napoleon conquered Europe," is implied the passive form, " Europe was conquered by Napoleon."

In any simple proposition many other propositions may be implied. Thus, " Yesterday I lifted one hundred pounds," implies judgments of the existence of yesterday, of the one hundred pounds, of myself, of the lifting, óf memory, of time, of personal identity, of will power, etc.

Topic Second.—Obversion.

Under implied judgments belongs also what Bain calls obversion. It is sometimes termed, **" Immediate Inference**

by Reciprocal Change of Positive and Privative Conceptions." In affirming one thing we impliedly deny the opposite. **Obversion** is the bringing out and denying of this opposite or reverse form.

Thus, "The road is level;" "The road is not inclined;" are not two facts, but the same fact from different sides. The second is not an inference from the first, but something implied in the first,—an obverse form of the first. "Whoever is wise is not foolish;" we must grant the obverse form if we grant the positive. In obversion the negative form may be taken either as **infinitated** or as simply **privative.** "Wise" implies the infinitated notion, "not-wise" or "non-wise," the two together making up the universe of being; and also the privative, "not-wise" or "unwise."

Each of the normal forms of judgment,—A, E, I, O,— has its obverse form. For developing these obverse implied judgments we have the following

Rule.—Obvert the predicate (*i. e.*, change it to the infinitated or privative form) and then change the quality of the judgment.

Note.—To avoid awkward compounds with **non** and **not,** in obverting and changing the quality of judgments, various prefixes and suffixes are often used, as, *in-, un-, dis-, less-,* etc.; and uncompounded negatives, as *unwise* and *foolish,* instead of not-wise. Great care needs, however, to be taken, as these terms are often not privatives, but only signify the existence of the quality in a low degree.

Taking **the four principal judgments** as embodied in propositions, in the order of the letters representing them, and applying the Rule given above, we get the obverse forms:

1. The normal form of the **universal affirmative,** A, is as follows: Every x is y; Every man is mortal.
Obverting the predicate, this becomes:

Every x is not-y; Every man is (not-mortal) immortal.

Changing the quality of the judgment from affirmative to negative, it becomes:

No x is not-y; No man is (not-mortal) immortal.

2. The normal form of the **universal negative,** E, is as follows: No x is y; No men are angels.

Obverting the predicate, this becomes:

No x is not-y; No men are not-angels.

Changing the quality of the judgment from negative to affirmative, it becomes:

Every x is not-y; All men are not-angels (excluded from angels).

3. The normal form of the **particular affirmative,** I, is as follows: Some x is y; Some men are wise.

Obverting the predicate, this becomes:

Some x is not-y; Some men are (not-wise) foolish.

Changing the quality of the judgment from affirmative to negative, it becomes:

Some x is not not-y; Some men are not foolish.

4. The normal form of the **particular negative,** O, is as follows: Some x is not y; Some men are not wise.

Obverting the predicate, this becomes:

Some x is not not-y; Some men are not (not-wise) unwise.

Changing the quality of the judgment from negative to affirmative, it becomes:

Some x is not-y; Some men are (not-wise) unwise.

Praxis.—State what is implied in the following propositions by the various forms of implication just explained:

1. Napoleon was an ambitious conqueror. 2. The diligent student will become wise. 3. Wellington was the soldier of duty. 4. John Howard was philanthropic. 5. Greece is a name of glory. 6. War is productive of evil. 7. The peacemakers are blessed. 8. Cold kills animals.

Section III.—Development of Inferred Judgments.

An inferred judgment, according to Davis, is "one that only virtually or potentially exists in the given judgment, and is derived from it." Its statement contains "something new, there is a step forward, a progress of thought. In the inferred judgment there is always either a different subject, or a different predicate, from that of the premise, and perhaps both."

The so-called inferred judgments may be reached from

11 *

other judgments either by Addition, Disjunction, Conversion, or Opposition. Of these forms the **last two** are the most important.

Topic First.—Inferred Judgments by Additions.

Determinants may be added to both terms of a judgment which is thereby rendered more definite, e. g., "A negro is a fellow-creature; therefore, a suffering negro is a suffering fellow-creature." The *original terms* of the judgment may themselves be made *determinants* or marks of new conceptions introduced into the judgment, e. g., "Oxygen is an element; therefore, the decomposition of oxygen is the decomposition of an element." On the same principles two judgments may be amalgamated; as, "Honesty deserves reward, and a negro is a fellow-creature; therefore, a negro who shows honesty is a fellow-creature deserving of reward." Care must be taken in all these forms of addition to **keep the distribution of the terms unchanged.**

Topic Second.—Inferred Judgments by Disjunction.

Since the members of a disjunctive judgment are *mutually exclusive*, we may infer from the disjunctive, "The teeth are either incisor, canine, bicuspid, or molar teeth," the judgment, "The molar teeth are neither incisor, canine, nor bicuspid." As the dividing members in a disjunctive judgment *exhaust the whole subject divided*, we may infer that the part of the whole not contained in one member must be in some other. Hence from the judgment just given come such inferred judgments as, "All teeth which are not molar are either canine, incisor, or bicuspid teeth."

Topic Third.—Inferred Judgments by Conversion.

Illative Conversion of judgments is such a transposition of the subject and predicate of a judgment that the **converse** or transposed form is a legitimate inference from the **convertend** or original judgment. Three general Rules must be observed in conversion :

Rule 1st. Before conversion reduce the proposition to the strict logical form, in which subject, copula, and predicate distinctly appear.

Rule 2d. No term not distributed in the convertend must be distributed in the converse. We may infer from *all* to

all, from *all* to *some*, and from *some* to *some*, but not from *some* to *all*.

Rule 3d. The transfer of the terms should be total. In other words, the whole **naked subject** (*i. e.*, the subject without its sign of quantity, every, all, some, etc.) must be transferred to the predicate, and the whole naked predicate must be transferred to the subject.

Confining attention mainly to the four attributive judgments, A, E, I, O,—since these are all the forms of which any special use is ordinarily made in compendiums of Logic,—it will be seen that there are three principal forms of conversion.

First, Simple Conversion when neither the quantity nor the quality is changed;

Second, Conversion by Limitation when the quantity is changed.

Third, Conversion by Negation or Contraposition, when the quality is changed.

1. **Simple Conversion** is where the terms can be transposed without change of either quantity or quality. This can, of course, occur only when both subject and predicate are distributed, as in **E,** and where both are undistributed, as in **I.**

(1.) Let **E,** "No one without a love of beauty can be an eminent artist," be given for conversion. The Rules should be applied in order. By Rule 1st, the proposition becomes, "Every one without a love of beauty *is not* any one who can be an eminent artist." By Rule 2d and Rule 3d, the converse becomes, "Any one who can be an eminent artist *is not* any one without a love of beauty." The converse is still E.

(2.) Let **I,** "Some good men are bad poets," be given. The proposition is already in strict logical form. By Rules 2d and 3d the converse becomes, "Some bad poets are good men." The converse is still I.

(3.) Substitutive and equivalent judgments, **U** and **Y,** are, of course, converted by simple transposition of the terms. "All bodies are extended substances" becomes, "All extended substances are bodies."

2. **Conversion by limitation** (*per accidens*) takes place

where it is necessary, in order to an illative transposition, that the quantity of the proposition should be changed from universal to particular, while the quality remains unchanged. This will, of course, occur where the subject is distributed and the predicate undistributed, *i. e.,* in **A.** As *some* may be inferred from *all,* **E** may also be converted by limitation.

(1.) Let **A,** " All poets are men," be given for conversion. It is already in strict logical form. In order to conform to Rule 2d, the predicate must be limited to " some men." By Rule 3d the converse becomes, " Some men are poets ; " or, " Some men are all the poets." The converse of A is I.

(2.) Let **E,** " No men are perfect," be given for conversion by limitation. Completing the form, limiting the quantity of the predicate, and then making a total transfer of the terms, the converse becomes, " Some perfect things are not men." The converse is O. By simple conversion it would be E.

3. **Conversion by Negation or Contraposition** takes place where it is necessary in order to illative transposition, that the quality of the judgment should be changed, while the quantity remains unchanged. This occurs in **O.**

Let **O,** " Some quadrupeds are not horses," be given for conversion. If converted simply, it would be, " Some horses are not quadrupeds," which is absurd. This result is avoided by *obverting,* or infinitating the proposition, *and then converting simply.* Infinitating the predicate, the proposition becomes, " Some quadrupeds are (things) not-horses ; " and by conversion, " Some things not- horses are quadrupeds." Thus the converse of O is I.

Topic Fourth.—Inferred Judgments by Opposition.

Opposition is the name given to the differences in quantity or quality, or both, between judgments having the same naked subject and predicate. Legitimate inferences follow from opposition.

Between the judgments, A, E, I, O, to which attention is here chiefly confined, there are **four kinds** of opposition,

which are exhibited by the following diagram, called the
Square of Opposition.

All men are true,   A ....Contrary.........E   No men are true.
  (Subalternans)   (Subalternans)

Some men are true,   Some men are not true.
  (Subalternate)   I ......Subcontrary........O   (Subalternate)
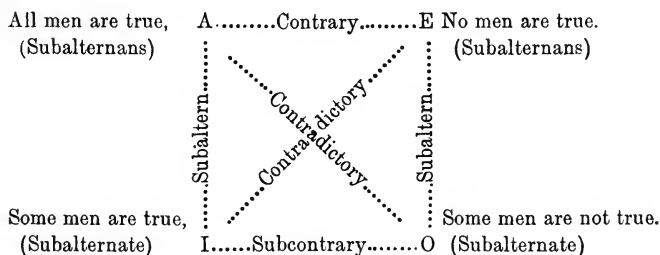
1. **Contradictory Opposition,** which is the only perfect
form, exists between the propositions **A and O, E and I,**
which differ both in quantity and quality. By the principles
of Contradiction and of Excluded Middle, of two contra-
dictory propositions both cannot be true and both cannot
be false.

Rule.—From the truth of either of two contradictories
the falsity of the opposite follows; and from the falsity of
either the truth of the opposite follows.

If A, "All men are true," be **sublated** (denied) then we can **posit**
(affirm) O, "Some men are not true." If it be not true that "All
men are true," then it is certain that, (at least) "Some men are
not true." If O, "Some men are not true," be denied, then A, "All
men are true," may be affirmed; but if the former be affirmed, the
latter may be denied.

Contradictory opposition is of special service in **indirect proof.**
Instead of showing an opponent's arguments false and his position,
therefore, unsustained, it is often better to prove the truth of the con-
tradictory and then infer the falsity of his position. E. g., if one
affirms that "All scientists are extreme evolutionists," which is A,
the best way to meet it is by establishing the contradictory O, "Some
scientists are not extreme evolutionists;" or, "Some one scientist, as
Prof. Tait, is not an extreme evolutionist." If this be established the
necessary inference is that A is false. The form of indirect proof
known as **reductio ad absurdum,** largely used in geometrical demon-
strations, instead of demonstrating a proposition directly, demonstrates

I

that its contradictory is absurd, and thence immediately infers the truth of the proposition.

2. Contrary Opposition is between the universal propositions **A** and **E**, which differ in quality only.

Rule.—From the truth of a judgment the falsity of its contrary opposite follows; from its falsity nothing follows.

Both A and E cannot be true. From the truth of A the falsity of E follows, and *vice versa.* E. g., if A, "All men have conscience," be true, then E, "No man has a conscience," is false; and if the latter be true then the former is false. From the falsity of one contrary *nothing follows* with regard to the other. If it be false that, "All men are poets," it does not follow that, "No men are poets." But *both may be false.* E. g., the propositions, A, "All men are poets," and E, "No men are poets," are both false, since the truth lies between the two and is expressed in I, "Some men are poets." In individual propositions, as, "Shakespeare was a poet," the opposition appears as the simple negative, "Shakespeare was not a poet."

3. Subcontrary Opposition is between the particular propositions **I** and **O,** which differ in quality only.

Rule.—If one sub-contrary be true, nothing follows in regard to the other; but if one be false, then the other must be true.

For example, if I, "Some wars are evil," be true, it does not follow from this that O, "Some wars are not evil," is true. But if I, "Some wars are evil," be false, then "Some wars are not evil," must be true.

4. Subalternate Opposition is between the propositions **A and I, E and O,** differing in quantity only.

Rule.—If the universal, A or E, be true, the particular I or O, must be true; and if the particular I or O, be false, then the universal, A or O, must be false.

If A, "All men are liars," is true, then I, "Some men are liars," is also manifestly true. If I, "Some men are perfect," is false, then A, "All men are perfect," is false.

The results may be summed up as follows:

		Contradictories.	Contraries.	Subalterns.
Universals.	If A is true,......O is false,....E false,..............I true.			
	If E " ,......I " ,....A " ,..............O " .			
	If A is false,....O is true,......E undetermined,...I undetermined.			
	If E " ,......I " ,......A " ,...O " .			

		Contradictories.	Subcontraries.	Subalterns.
Particulars.	If I is true,......E is false,....O undetermined,...A undetermined.			
	If O " ,......A " ,....I " ,...E " .			
	If I is false,......E is true,....O true, ...A false.			
	If O " ,......A " ',....I " , ...E " .			

Praxis.—Apply exhaustively the principles of implication and also the principles of immediate inference in its four kinds, to the following judgments, giving the quantity and quality of the judgments:

1. All the righteous are happy. 2. No human virtues are perfect.
3. Some possible cases are probable. 4. The just are (all) the holy.
5. Some men are all the poets. 6. All the insincere are dishonest.
7. No unjust act is unpunished. 8. Some unfair acts are unknown.
9. The unlawful is the (only) inexpedient. 10. No brutes are responsible. 11. Heaven from all creatures hides the book of fate. 12. Fair promises are not often to be trusted.

———o○;○;○o———

SUMMARY OF RESULTS.

THE aim of the Logic of Judgment is to train to the best thinking and fullest appreciation of thought in the second form. The perfection of thinking in judgment depends upon the certainty of the connection of the subject and predicate. This gives rise to what is called the **Modality of Judgments.**

1. By the degree of certainty of the predication to the mind of the thinker or others, all judgments have been divided into Demonstrative, Assertory, and Problematic.

(1.) A **demonstrative or apodictic judgment** is one that is certain to him who holds it, and that may be made certain to all sane minds suf-

ficiently intelligent to understand the signification of the judgment itself and its evidence. All analytic judgments are demonstrative, or are certain to him who holds them, and may be made certain to all other sane minds of sufficient intelligence to understand the significance of the terms. All intuitive judgments are also demonstrative, or have both subjective and objective certainty. These include the truths of Mathematics, the fundamental principles of Logic, the axioms of Ethics and Metaphysics. All judgments reached by immediate inference from these are also demonstratively certain.

(2.) An **assertory judgment** is one that announces what is known as actual. It is certain only to him who holds it, but not capable of being made certain to others of different moral disposition. "It commends itself to our moral nature, and in so far as other men are of the same disposition, they will accept it likewise." This holds especially of higher moral and religious truths. Moral and religious deterioration prevents their acceptance.

(3.) "A **problematic judgment** is one that is neither held with entire certainty by the thinking subject, nor can we show that it truly represents the object about which we judge. It is a mere *opinion*." Problematic judgments constitute one of the necessary stages in the progress towards truth. "Great discoveries are problems at first . . . Whenever we judge about variable things, as the future actions of men, the best course of conduct for ourselves under doubtful circumstances, historical facts about which there is conflicting testimony, we can but form a problematical judgment, and must admit the possibility of error at the moment of making our decision."

2. A simpler division of judgments, by the degree of certainty in the mind of the thinker, is into Certain, Probable, and Doubtful.

(1.) A **certain judgment** is one in which the knowledge that the connection between the subject and predicate corresponds to the reality is absolute and unquestionable. All analytic judgments, all intuitive judgments, all immediate inferences from certain judgments, all strict deductions from certain or necessary premises may become certain to the thinker.

(2.) A **probable judgment** is one in which the knowledge that the connection between the subject and predicate corresponds with the reality is not absolute and unquestionable. The boundary line between the **probable** and **doubtful** is not always clearly marked, since, in com-

mon language, the degrees of probability may reach all the way from the nearest approach to certainty that a judgment is true, down to the nearest approach to certainty that it is not true, *i. e.*, from the nearest possible to absolute certainty, to the nearest possible to absolute uncertainty; while the degrees of doubtfulness may have the same wide scope. It may be said, however, in general, that a strictly **probable judgment** is one which has the balance of proof in its favor, and that a **doubtful judgment** is one which has the balance of proof against it. As has already been seen, man receives most of the knowledge used in the conduct of life, in such a way that it is not certain, but at best only more or less probable. All the acquired perceptions of the senses and consciousness are mixed with inferences, and, therefore, only probable; while only the original or intuitive perceptions are certain. The conclusions of finite reason, especially by the inductive processes, are liable to error, and, therefore, cannot rise to certainty. The judgments based on testimony and authority can at best reach only a high degreee of probability. A judgment may be *possible* when it is not *probable*. "A thing is said to be *possible* when, though not actually in existence, all the conditions necessary for realizing its existence are given." It is possible, for example, that aerial transportation may some day take the place of transportation by steamer and railway, but not perhaps probable.

The aim of the Practical Logic of Judgment should be to train the thinker to skill in distinguishing clearly between the certain, the probable, and the doubtful; and in arriving at sound judgments, on the basis either of certainty or of probability, by which to govern the entire conduct of human life.

12

Part III.

THE LOGIC OF REASONING OR THE SYLLOGISM.

THE aim of the Logic of Reasoning is to train the mind to skill in dealing with the third Form of Thought.

Definition.—Reasoning is that form of thought in which we compare various judgments and, on the ground of some medium or cause, reach other judgments as inferences or conclusions from them. Reasoning may, therefore, be used as synonymous with **Mediate Inference.** The product of reasoning, as embodied in language, is usually known as the **Syllogism.**

Note.—Mediate inference is inference by a *medium,* or *middle notion* or *term.* It is thus distinguished from immediate inference which, as has been seen (p. 121), does not make use of any such third or middle term. The middle term is used where we cannot compare two things directly. We cannot compare two *lots* directly by placing one upon the other, but we can measure them both with a surveyor's chain, or other common measure, and thus ascertain their relative dimensions. So when two *notions* or *terms* cannot be directly compared and connected they may be indirectly by the use of a third notion or term. We may, e. g., wish to connect "John Baptist" and "priest" in the judgment, "John Baptist was a priest." Having no direct statement to that effect in the Bible, we must reach the conclusion by reasoning from the fact that the sons of priests were also priests. The process of thought is stated as follows :

<table>
<tr><td align="center">Analytic Form.</td><td align="center">Synthetic Form.</td></tr>
<tr><td>John Baptist was a priest ;
For he was the son of a priest ;
And the son of a priest was a priest.</td><td>The son of a priest was a priest ;
John Baptist was the son of a priest ;
∴ John Baptist was a priest.</td></tr>
</table>

Both terms are connected with a third term, "son of a priest," and thus connected with each other.

The most helpful logical presentation of Reasoning must treat of both the **formation** of reasonings or syllogisms and the **unfolding** of syllogisms. The present subject will, therefore, embrace two Chapters.

———∞∙ఠ∘∘———

CHAPTER I.

THE FORMATION OF REASONING OR MEDIATE INFERENCE.

THE formation of thought as reasoning must manifestly be placed at the foundation in all training to thought in its third form. It will be necessary to consider, in successive Sections, the **nature of reasoning** or mediate inference in general, and the **fundamental forms of reasoning,**—deduction and induction. The process and the products will be considered under each of the forms of reasoning.

Note.—Much of the modern depreciation of Logic, and especially of the Logic of the Syllogism, is doubtless due to the fact that the Science has been confined largely to the mechanical testing of barren forms. If this be all there is in the Logic of Reasoning, it would have to be admitted that it is not a very valuable means of knowledge; the old objection would hold, that "the premises, so far from being able to establish the truth of the conclusion, presuppose it." Take in illustration a syllogism commonly given: "All Cretans are liars; this man is a Cretan; therefore he is a liar." How do we know *all* before we know *each?* How do we know *all* before the character of this particular Cretan is decided? That is, until we are certain that this particular Cretan is a liar, we cannot be certain that *all* Cretans are liars.

The all-important thing in reasoning is **the finding of middle terms or connecting links** of argumentation; and even the testing of the various products of reasoning cannot proceed intelligently without some skill in finding these connecting links.

Section I.—The Process of Reasoning or Mediate Inference in General.

Topic First.—The Forms of Reasoning.

All reasoning necessarily proceeds from general principles to particulars or individuals, or from facts or particulars to general principles. Mediate Inference is, therefore,

divided into two chief kinds: Deduction, or Specialization, or Syllogism in the stricter sense; and Induction, or Generalization, or Syllogism in the looser sense.

Syllogism in the stricter sense in its chief forms is inference from the general to the particular or individual, and in all its forms inference proceeding from the general. **Induction** is inference proceeding from the individual or particular to the general. **Inference by analogy,** which proceeds from the individual or particular to a co-ordinate individual or particular, is a third form distinct from both, though able to be reduced to a combination of the other two. See Ueberweg's *Logic*, p. 333.

Deduction has also been called " the inference of subordination," or " inference by analysis of notions; " induction, " the inference of superordination; " analogy, " the inference of co-ordination."

The difference between deduction and induction may be illustrated by the methods of proving that the interior of the earth is in a molten condition. From the volcanic phenomena, *i. e.*, from the facts that the earth is in a molten condition under Mount Vesuvius, Mount Hecla, Mauna Loa, etc., it is inferred *inductively* that the whole interior is in such condition. From the process of the earth's formation by the condensation of intensely heated material (an origin probable on astronomical grounds), it is inferred *deductively* or *syllogistically* that the interior is in a molten condition. The one process starts from **facts;** the other from a **general principle.** They are usually thrown into syllogistic form, as follows:

Inductive Process.	Deductive Process.
The interior of the earth is molten;	The interior of the earth is molten;
For, it is molten under Vesuvius, etc.;	For the solar system was formed by condensation;
And Vesuvius, etc., fairly represents the whole.	And the earth is a part of the solar system.

The nature of analogy, as made up of induction and deduction, may be shown from the following example: " The Earth, a planet revolving in an orbit round our sun, turning on its axis, having an atmosphere, change of seasons, etc., supports organic life; Mars is a planet revolving in an orbit round our sun, turning on its own axis, having an atmosphere, change of seasons, etc.; hence Mars also will probably support organic life." It will be seen by examination that this consists

of an apparent induction and a deduction combined.   This may be exhibited, in full, as follows :

Ind.
⎡   The Earth supports organic life ;
⎢   The Earth is a planet revolving, etc., and fairly represents that
⎣   class of planets ;

∴ All planets revolving, etc., probably support organic life ;

Ded.
⎡   Mars is a planet so revolving, etc. ;
⎣ ∴ Mars probably supports organic life.

According to the common view both deduction and induction may be embodied in syllogistic form (as in the examples given).   The **elements of the reasoning,** as embodied in the syllogism, need, therefore, to be considered. As the validity of the reasoning depends, however, not upon the syllogistic forms, but upon the connecting link of thought embodied in the middle term, the subject of **finding middle terms** needs to be specially considered.

Note.—The question whether all reasoning can be reduced to the syllogism is one into which we have not space to enter.   Nor is there need to discuss it here, since it is freely admitted that *the validity of the reasoning depends upon the connecting links of thought and not upon the form;* and that the syllogism is of no special value in the formation of processes of reasoning, but only in formulating and testing them after they are formed.

Topic Second.—The Elements of Reasoning.

The elements of reasoning are ascertained by analyzing the process as embodied in the Syllogism.   The syllogism is composed of three terms and three propositions; and underlying the form, as the real basis of thought, is some mediating notion or cause.

I. The Terms and Propositions.

The terms or notions in the syllogism are distinguished as the major term, the minor term, and the middle term. The propositions in the usual form of statement are the major premise and the minor premise, constituting the antecedent or proof, and the conclusion or consequent.

The **conclusion** is the judgment to be proved.   In the formal syllogism it is placed last.   Its **subject,** represented by **S,** is the minor term;
    12*

its **predicate,** represented by **P,** is the major term.   The **middle term,** represented by **M,** is that with which the major and minor are compared in the premises.

The **major premise** is the judgment in which the major term or predicate of the conclusion is compared with the middle.

The **minor premise** is the judgment in which the minor term is compared with the middle.

This may be illustrated in concrete form and in formula, as follows:

$$\left\{\begin{array}{l} \text{All conquerors (M) are tyrants (P);} \\ \text{Napoleon (S) was a conqueror (M);} \\ \text{Napoleon (S) was a tyrant (P).} \end{array}\right. \quad \left\{\begin{array}{ll} \text{M is P;} & \text{Major Premise.} \\ \text{S is M;} & \text{Minor Premise.} \\ \therefore \text{S is P.} & \text{Conclusion.} \end{array}\right.$$

The conclusion is reached by comparison of both its terms with the third or middle term, " conqueror."

II. The Middle Term or Connecting Link.

The middle notion or term (originally called the *argument*) always represents the link of thought by which the two terms of the conclusion are brought together and the judgment proved.   It furnishes the **sufficient reason** for connecting the major and minor terms.   Reasoning is properly, therefore, **finding the sufficient reason**—in case of induction the **cause**—for the connection of the terms in the conclusion.

Various maxims have been formulated to express the connection embodied or implied in the middle term.   The principal are those of Aristotle and Kant, which apply respectively to propositions of extent and content, or to propositions made up of class terms and those made up of attribute terms.   The axiom of Sufficient Reason, or of Reason and Consequent, is, however, the best and most complete expression of this connection.

The so-called **dictum of Aristotle** places the relation of *genus* and *species* at the foundation of reasoning.   Whatever can be predicated affirmatively or negatively of any genus or class distributed can be predicated likewise of all or any of the species or individuals included under it.   If it can be affirmed of the genus *man,* that it is included in the higher genus *person,* then it can be affirmed of the species *slaves,* included under *man,* that it is included under *person.*   Or if it can be affirmed of the genus *man,* that it is excluded from the genus *brute,* then the same can be affirmed of the species *poets,* included under the genus, *man.*

The **formula of Kant** places the relation of a *complex property to its components* at the foundation of reasoning. Whatever is a component of a complex property of a thing is a property of the thing itself. The mark *brave*, which is a component of the complex mark *conqueror*, is also a mark of *Caesar*, the object to which the complex *conqueror* applies.

Others make the relation at the basis of reasoning that of *whole and part*. A part of a part is also a part of the whole.

The **real connecting principle** or basis in reasoning, *i. e.*, the real sufficient reason, is, perhaps, best expressed by the relation of **reason and consequent,** which, as has already been seen (p. 20), embraces *whole and part, cause and effect, substance and attribute, genus and species*, etc. Any form of reason and consequent may be at the *basis of deduction;* while the *basis of induction* is the *strictly causal relation* only.

Topic Third.—Finding and Verifying Arguments or Middle Terms.

From what has been thus far considered, it is obvious that reasoning essentially consists in finding and verifying arguments, or middle terms and causes, under the principle of Sufficient Reason or Reason and Consequent. This process differs in deduction and induction, inasmuch as these forms of reasoning differ.

Section II.— Deductive Reasoning.

Topic First. — Process of Finding and Verifying the Argument in Deduction.

Three things are essential in deduction: *first*, finding the proper middle term ; *second*, verifying the premises formed by the aid of it ; *third*, testing the conclusion.

I. Finding the Middle Term.

The first question is, *By what middle term can the two given terms be bound together or disjoined in the conclusion ?* The following Rules will guide the thinker in his quest :

Rule 1st.—Examine carefully, by the principles laid down in the Logic of Conception, the two terms to be connected or disjoined, in

order to ascertain which of the relations under reason and consequent is applicable to them.

Rule 2d.—Seek the proper mediating whole, concept proper, class, or cause, as the case may require.

Rule 3d.—Bring the middle term thus found into proper connection with the other terms and these with each other in syllogistic statement.

The application of these Rules may be illustrated by examples. Thus, in seeking a middle term to prove that "*The Persians worship a thing insensible,*" we find, by the first Rule, that this term must be an *individual* under the *genus*, "things insensible." By the second Rule, "the sun" furnishes such an individual. By the third Rule, these are brought together in syllogistic statement, in the order of proof, as follows:

> The Persians worship a thing insensible;   *Question.*
> For the Persians worship *the sun;* }
> And *the sun* is a thing insensible. }   *Proof.*

Again, in finding a middle term to prove that "*Judas was not a true apostle,*" we find, by the first Rule, that the major term, "true apostle," is a *genus* or *class term.* By the second Rule, "thief" furnishes a "genus" excluded from the genus, "true apostle." By the third Rule, this takes shape as follows:

> Judas was not a true apostle;   *Question.*
> For Judas was a thief; }
> And no thief was a true apostle. } *Proof.*

Once more, in proving that "*Plato* is mortal," we find, by the first Rule, that the major term, "mortal," is a *concept* or *attribute term.* By the second Rule, we find that the complex concept, "man," includes "mortal" as a component property; and, therefore, since the mark of a mark is a mark of the thing itself, "mortal" is a mark of "Plato." By the third Rule, this gives, stated in twofold syllogistic form:

> { Plato is mortal;   *Question.*   { Man is mortal; } *Proof.*
> { For Plato is man; } *Proof.*   { Plato is man; }
> { And man is mortal. }   { ∴ Plato is mortal.   *Conclusion.*

II. Verifying the Premises.

When the middle term has thus been found and connected with the major and minor terms, the question arises, *Are these premises true?* Hence the following Rule:

Rule 4th.—Test the premises by the principles already presented for the verification of judgments (p. 98), in order to be sure that only correct judgments have been grasped.

It is all important that correct judgments should be grasped and placed at the foundation as premises, since otherwise any inferences from them would be logically worthless. The *sources of the judgments* made use of in deduction are the following: intuition; thought proper inductive and deductive; and testimony and authority. The premises must be tested by the principles by which judgments from these various sources are proved.

III. Testing the Conclusion.

When the premises have been found to be true or probable, the question arises, *Does the conclusion follow from the premises?* Hence the following Rule:

Rule 5th.—Test the whole process by the principles of analysis presented in the Logic of Conception, and by the laws which govern the Syllogism as presented in the next Chapter under the Unfolding of Reasoning.

Partial understanding of the terms may lead to false conclusions. This may be prevented by a careful study and analysis of the concepts and terms involved, by means of Partition, Division, and Definition. **False conclusions** may also be drawn from correct premises. This may be prevented by the careful use of the formal rules of the Syllogism.

In all deductive reasoning, it should be remembered, that **the conclusion can never be any more certain than the premises.** Forgetfulness of this is the source of many and great errors in both Science and Philosophy.

Topic Second.—Products of Deductive Reasoning.

The product of deduction is the **Syllogism proper** in its various forms. Syllogisms are divided, by the form of the judgments embodied in them, into categorical and hypothetical. Categorical syllogisms are either simple or combined,—**simple** when they contain but one argument with its major and minor premises expressed or understood and its conclusion; **combined** when more than one process of

argument is involved.   The former may be called the **mono-syllogism**; the latter the **polysyllogism.**

I. Categorical Syllogisms.

A **categorical syllogism** is one in which the judgments are categorical (p. 117).

1. The **monosyllogism** may be in its statement either complete or incomplete.

The **complete form** is the ordinary form in which both the premises and the conclusion are expressed.   The **incomplete form** or the **enthymeme** (Greek, meaning *in the mind*) is that form in which one premise is unexpressed, or left to be supplied by the mind.   Thus:

> " Alexander the Great was brave;
>   For he was a conqueror."

In this case the *major premise*, "All conquerors are brave," is omitted. The *minor premise* may also be omitted.   Thus:

> " Conquerors are brave;
>   Therefore Alexander the Great was brave."

Note.—The enthymeme is the usual form in ordinary speech.   The premise left unexpressed is easily supplied in completing the syllogistic statement. It will also be seen that in common speech there are to be found many *abridged* and *disguised forms* of argument.   For example: "Hard study strengthens the mind, but wearies the flesh; so that what wearies, strengthens;" "Theft is a crime; yet some kinds were legal in Sparta."   In such cases the first step is to reduce the argument to the normal form.

2. The **polysyllogism** includes the various forms in which separate syllogisms are combined into wholes of connected reasoning.   Syllogisms may be attached, as **prosyllogisms,** to premises to prove them, or, as **episyllogisms,** to conclusions, making the conclusions premises for reaching further conclusions.   In the former case the prosyllogisms are subordinate to a principal syllogism, and the whole constituted is, therefore, a **complex syllogism,** which may be known as the **epichirema;** in the latter case the episyllogisms are co-ordinate with that to which they are attached, and the whole is, therefore, a **compound syllogism.**

(1.) The **Complex Syllogism,** or **Epichirema,** or reason-rendering

syllogism, is either **manifest** (*i. e.*, having all the parts fully expressed), or **occult** (*i. e.*, having some of the parts suppressed). Both the manifest and occult forms may be "either **single** or **double,** according as one or both of the premises are furnished with an auxiliary reason."

The **single epichirema,** in both its occult and manifest forms, may be illustrated by the following example:

Main Syllogism. Occult Prosyllogism.		Expanded Prosyllogism.
Vice is odious;	⎧	Whatever enslaves is a vice;
Avarice is a vice, for [it enslaves;]	⎨	Avarice enslaves;
∴Avarice is odious.	⎩	∴Avarice is a vice.

Omitting the expanded prosyllogism, we have the ordinary single epichirema in its occult form; omitting the occult prosyllogism, we have the same in its manifest form.

The **double epichirema,** in both its occult and manifest forms, may be illustrated by the following example:

Main Syllogism.	Occult Prosyllogisms.	Expanded Prosyllogisms.
Man has a spirit; for [he is rational; =]		Every rational being has a spirit;
		Man is a rational being;
		∴ Man has a spirit.
Man has a body; for [he fills space; =]		Whatever fills space has a body;
		Man fills space;
∴ Something that has a spirit has body.		∴ Man has a body.

Omitting the expanded prosyllogisms, we have the double epichirema in its occult form; omitting the occult prosyllogisms, we have the same in its manifest form.

(2.) The **compound syllogism,** made up of successive co-ordinate syllogisms, includes the **double syllogism,** in which the episyllogism is attached to the conclusion of a syllogism, making that conclusion a premise for reaching a new conclusion; and the **chain syllogism,** which is made up of successive co-ordinate syllogisms. In both these forms it may be either **manifest** or **occult.**

The **double syllogism** of the compound form does not need to be further subdivided. The **chain syllogism** in its occult form is usually known as the **sorites** (Greek, meaning *a heap*). The successive syllogisms in it are all equally abridged.

Both the manifest and occult forms may be illustrated by the following examples, in which the occult forms are contractions of the manifest forms:

Double Syllogism, Manifest Form.

1st, ⎧ Useful studies ought to be pursued;
⎨ Logic is a useful study;
⎩ ∴ Logic ought to be pursued.

2d. $\left\{\begin{array}{l}\text{A course which omits what ought to be studied is deficient;}\\ \text{A course which omits Logic omits what ought to be studied;}\\ \therefore \text{A course which omits Logic is deficient.}\end{array}\right.$

Double Syllogism.　　Occult Form.

$\left.\begin{array}{l}\text{Useful studies ought to be pursued;}\\ \text{Logic is a useful study;}\\ \therefore \text{Logic ought to be pursued;}\end{array}\right\}$ Main Syllogism.

$\left.\begin{array}{l}\text{Hence an educational course}\\ \text{which omits Logic is deficient.}\end{array}\right\}$ Episyllogism.

Chain Syllogism, Occult, Sorites.　　Chain Syllogism, Manifest.

$\left\{\begin{array}{l}\text{Bucephalus is a horse;}\\ \\ \\ \text{A horse is a quadruped;}\\ \\ \\ \text{A quadruped is an animal;}\\ \text{An animal is a substance;}\\ \therefore \text{Bucephalus is a substance.}\end{array}\right.$

I. $\left\{\begin{array}{l}\text{Bucephalus is a horse;}\\ \text{A horse is a quadruped;}\\ \therefore \text{Bucephalus is a quadruped.}\end{array}\right.$

II. $\left\{\begin{array}{l}\text{Bucephalus is a quadruped;}\\ \text{A quadruped is an animal;}\\ \therefore \text{Bucephalus is an animal.}\end{array}\right.$

III. $\left\{\begin{array}{l}\text{Bucephalus is an animal;}\\ \text{An animal is a substance;}\\ \therefore \text{Bucephalus is a substance.}\end{array}\right.$

The **sorites proper** is of two kinds,—the **progressive** or Aristotelian, in which the argument descends from whole to part; and the **regressive** or Goclenian, in which the argument ascends from part to whole, as in the following examples:

Regressive Sorites.

Bucephalus is a horse;
A horse is a quadruped;
A quadruped is an animal;
An animal is a substance;
∴ Bucephalus is a substance.

Progressive Sorites.

An animal is a substance;
A quadruped is an animal;
A horse is a quadruped;
Bucephalus is a horse;
∴ Bucephalus is a substance.

The sorites can thus readily be expanded into a manifest compound syllogism. It consists of "as many simple syllogisms as there are middle terms between the subject and predicate of the conclusion, *i. e.*, intermediate wholes and parts between the greatest whole and the smallest part, which the reasoning connects." In the example given,— taking the progressive form,—the greatest whole and smallest part are *substance* and *Bucephalus;* the middle terms are *horse, quadruped, animal.* This gives three simple syllogisms, by using successively these middle terms.

II. Hypothetical Syllogisms.

The **hypothetical syllogism** is that form of syllogism in which the reasoning turns upon some hypothetical judgment (p. 117) embodied in the major premise. Hypotheti-

cal syllogisms, whether monosyllogisms or polysyllogisms, are, therefore, primarily divided into conditional or conjunctive and disjunctive. These, as in the case of categoricals, may be either manifest or occult.

1. A **hypothetical monosyllogism** is one which contains but one argument, with its major and minor premises expressed or understood, and its conclusion. The suppressed or disguised premise gives the **hypothetical enthymeme** which is the most common form in ordinary speech. Both manifest and occult hypothetical arguments may be either conditional or disjunctive.

(1.) A **conditional,** or **conjunctive hypothetical syllogism** is one in which the reasoning turns upon a conditional or conjunctive judgment embodied in the major premise. This may be illustrated in both its manifest and occult forms by the following example:

Manifest Form.	Enthymeme.
If rains are plenty, the crops will be plenty;	If rains are plenty, crops will be plenty;
Rains are plenty;	. . . . .
∴ Crops will be plenty.	So crops will be plenty.

(2.) A **disjunctive hypothetical syllogism** is one in which the reasoning turns upon a disjunctive judgment embodied in the major premise. This may be illustrated by the following example:

Manifest Form.	Enthymeme.
Man is either an automaton or free;	Man is either an automaton or free;
He is a free being;	. . . . .
∴ He is not an automaton.	And so he is assuredly free.

2. The **hypothetical polysyllogism** includes the various forms in which hypothetical arguments may be brought together into wholes of connected reasoning. These wholes may arise from combining hypotheticals and disjunctives in the premises, or by combining entire arguments. The former gives rise to **dilemmatic syllogisms;** the latter to **compound hypothetical syllogisms,** including the **double form** and the **sorites.**

(1.) A **dilemmatic syllogism** is one having a dilemmatic judgment (p. 118) for its major premise, with a minor premise so affirming or

13 K

denying some member or members of the major as to lay the foundation for an inference. The forms depend upon the various combinations in the major premise. The combinations are as follows:

1st. **A single conditional antecedent with a disjunctive consequent,** as in the example:

> If the Senator aspires to a place, he will either rule or ruin:
> The Senator aspires to the place of President:
> ∴ He will either rule or ruin.
> Or, If A is B, either C is D or E is F;
> But A is B, ... ∴ either C is D or E is F.

2d. **A plurality of conditional antecedents all having one common consequent,** as in the example:

"If things are what we can help, we ought not to fret about them; and if they are what we cannot help, we ought not to fret about them;
But all things are either what we can or cannot help;,
∴ They are what we ought not to fret about."
Or, If A is B, X is Y, and if C is D, X is Y;
But either A is B or C is D; ... ∴ X is Y.

This form is what has been known as the *dilemma* in the strict sense, or the *horned syllogism.* It is so called because it confronts an opponent with two assumptions, on which it tosses him as on horns from one to the other, each being equally fatal to him.

3d. **A plurality of conditional antecedents each with its own consequent,** as in the example:

{ If men are virtuous they are wise,   Or, If A is B, C is D, }
{ And if they are vicious they are unwise; And if E is F, G is H; }
  But they are either virtuous or vicious;   But either A is B, or E is F;
∴ They are either wise or unwise.   ∴ Either C is D, or G is H.

(2.) The **compound hypothetical syllogism** includes the **double form,** in which the latter of two syllogisms is abridged, and appears as an episyllogism; and the **hypothetical sorites,** in which the successive syllogisms are all equally abridged. These may be illustrated by examples:

Double Form.
{ If the people are industrious, wealth increases;
{ They are industrious;   **Episyllogism.**
{ ∴ Wealth is increasing (and *hence the nation will become powerful*).

Hypothetical Sorites.
{ If Gladstone is virtuous, he is brave;   Or, If A is B, C is D;
{ If brave, he is magnanimous;   If C is D, E is F;
{ If magnanimous, he will relieve the Irish tenants; } If E is F, G is H;
{ But he is virtuous, and ∴ will relieve the Irish tenants. } But A is B, ∴ G is H.

Praxis.—Find middle terms for the following conclusions, according to the Rules given; verify the premises and test the conclusion; and mark by the appropriate vowels the quantity and quality of all the judgments: 1. Jupiter is a planet. 2. Education is valuable. 3. Religion is indispensable. 4. The crocodile is a reptile. 5. Few patriots have been disinterested. 6. No brutes are responsible. 7. Perseverance is a condition of success. 8. A sensualist is not truly free. 9. The elk is ruminant. 10. Good logicians are not true poets. 11. The immoral man is not happy. 12. The inactive man cannot be happy. 13. Astrology is not a science. 14. Astronomy is a science.

Give a complete outline of the kinds of Syllogisms, as presented in the preceding Section, and then construct one or more original syllogisms illustrating each of the kinds.

Section III.—Inductive Reasoning.

Topic First.—Process of Finding and Verifying the Cause in Induction.

Two things are essential in induction: **first,** fixing upon some assumed cause which works in the facts from which the inference is sought, and which furnishes the basis for a working hypothesis; **second,** testing and verifying this hypothesis.

Note.—Ueberweg has said truly: "Hypotheses are necessary in all sciences if the knowledge of causes is to be reached. Causes as such are not accessible to observation, and, therefore, at first can be thought only under the form of hypotheses, until, with the advance of the sciences, the previously problematic suppositions pass over into knowledge apodictically certain. . . Scientific hypotheses . . . are the results of regular reflection on experience, and, as premises in tentative deductions, form the necessary preliminaries to adequate knowledge."

I. Finding the Working Hypothesis.

In finding the cause in induction, the first question to be asked is, What working hypotheses, in themselves possible, can be formed, which agree with the facts of experience, so that the phenomena may all be taken into account and explained?

Induction derives its **data** from experience. **Experience** is the examination which is necessary to furnish us the facts from which to

make inferences. Such experience is obtained either by observation or by experiment.

Observation is the act of the mind in seizing upon facts as they are spontaneously presented in nature. Its nature and methods have already been unfolded (p. 26). **Experiment** is the process of voluntarily "putting in action causes and agents over which we have control, and purposely varying their combinations and noticing what effects take place." It vastly multiplies the possibilities of observation, and is thus of the greatest importance to science. The data drawn from experience for use in induction consist of **facts** or **phenomena**. A phenomenon means literally "that which appears to, or is known directly by, the senses," and then "that which is known as a fact to the mind." The word, therefore, includes all facts whether made known by the senses or consciousness. The word **fact** is substantially its equivalent in usage. It signifies literally "something done," and may be defined to be anything that exists or happens, whether in the world of matter or of mind.

Equally important with the data of induction is the **correct logical method of dealing with the facts.** This Bacon sought to furnish in his *Novum Organum* or New Instrument. Its aim is to direct the mind in seizing upon the facts in any given region, constructing hypotheses for their explanation, and, through the verification of these, reaching *perfected theories* or *general truths.*

"**The correct construction of hypotheses,**" says Ueberweg, "is a life and death question with Philosophy; for it is the science of the principles which underlie all the sciences, and requires more than any *other* to pass beyond mere experience, and to bring together by comparison very different departments of knowledge." Hence the importance of correct Rules carefully applied.

Rule 1st.—Observe, analyze, and classify the facts to be generalized and explained, in order to ascertain their reality and their various elements and relations.

This Rule guards against **two common sources of error** in induction. The **first** is that of assuming *what is not fact to be fact.* This is illustrated by the problem presented by Charles II. to the Royal Society: "Why does a live fish in water increase the weight while a dead fish does not?" The answer to the question, "Is it a fact?" would have saved the time spent in endeavoring to solve the imaginary problem. The **second** is the error from getting only a *partial view* of the facts or from failure to get them in their relations. This is illustrated in Stahl's method of accounting for combustion, by the extrication of a substance supposed to be contained in all combustible matter, called *phlogiston*, which went up in the flame. Combustion results in the visible residue of ashes and the invisible phlogiston which passes off. The error was in the non-observation of an important part of the actual residue,—the gaseous products of combustion. When these were at last taken into account, it was found that the gases with the ashes weighed much more than the substance burned, so that there was no room for *phlogiston.* See Mill's *Logic*, Book V., Ch. iv.

Rule 2d.—Correctly interpret the facts, i. e., **seek to find the appropriate cause for the facts and basis for the generalization.**

By **cause** in induction is meant "operating power," or, more strictly, "power which in operating originates new forms of being." It is anything which has **efficiency** and exerts it in producing change, and hence is often called *efficient cause.* It should be carefully distinguished from **law,** which has no efficiency, but is merely an expression of an established sequence of facts, or of the regular order in which a cause operates. A **condition** is "that which is prerequisite in order that something may be, and especially in order that a cause may operate." It is "prior to the production of an effect; but it does not produce it. It is fire that burns; but, before it burns, it is a *condition* that there be an approximation of the fire to the fuel, or the matter that is burned. . . The **cause** of burning is the element of fire, fuel is the **con-cause,** and the **condition** is the approximation of the one to the other."

The cause may be sought, first, in some known, or, secondly, in some unknown, force or forces. The search in the former case has to do with some *real cause* and is guided by the so-called Methods of Induction, and in the latter case must be reached by inductive assumption or assumption of *strictly hypothetical cause.* In the former case the results tend to take shape in contributions to exact science; in the latter they belong to the region of scientific question or metaphysical speculation. The quarrels of scientists and theologians very often arise from confounding the two.

(1.) Inductions of Real Cause.—The Canons of the Inductive Method used in the search for the real cause for any phenomenon, whether that cause is simple or complex, may have reference either to the **preliminary consideration** of *the happening or not happening of the event, the cause of which is sought;* or to the **more advanced problem** of *measuring the exact quantity of an effect,* if it be capable of being more or less, *and connecting it with the quantity of the cause.* To the first stage belong the methods of agreement and of difference; to the second, the methods of concomitant variation and of residues.

A. What can be learned of the real cause of an event from the happening or not happening of that event?

The Method of Agreement is applied in case of the uniform happening of an event. This gives rise to—

13 *

Canon First.—If in all observed cases of an effect or phenomenon one condition is uniformly present, that is probably the cause, or includes the cause, of the phenomenon or effect. In other words, " *the sole invariable antecedent of a phenomenon is probably its cause.*"

" To apply this method we must collect as many instances of the phenomenon as possible, and compare together their antecedents. Among these the causes will lie, but if we notice that certain antecedents are present or absent without appearing to affect the result, we conclude that they cannot be necessary antecedents."

The method of agreement is subject to a serious difficulty. **An antecedent may not be a cause.** Night or the cock-crowing or the rising of some diligent workman may uniformly precede the coming of the day without being the cause of it. Hence the necessity for tests by which to distinguish between simple antecedent and real cause.

The Method of Difference is applied in case of the uniform happening of an event in the case of the presence of some condition, and the uniform failure of it in case of the absence of that condition. This gives rise to

Canon Second.—If, in all instances in which a phenomenon does occur, one single condition is present, which is uniformly absent whenever such phenomenon does not occur, this constantly present or absent condition is presumed to be the cause of the phenomenon.

Thus we can clearly prove that *friction* is *one* cause of heat, because when two sticks are rubbed together they become heated; when not rubbed they do not become heated. Sir Humphrey Davy showed that even two pieces of ice when rubbed together in a vacuum produce heat, as shown by their melting, and thus completely demonstrated that the friction is the source and cause of the heat. We prove that *air* is the cause of sound being communicated to our ears, by striking a bell in the receiver of an air-pump, as Hawksbee first did in 1705, and then observing that when the receiver is full of air we hear the bell; when it contains little or no air we do not hear the bell.

B. What can be learned of the Real Cause of an event from the varying degree or quantity of an event?

" Every science and every question in science is," as Jevons has said, " **first, a matter of fact only, then a matter of quantity,** and by degrees becomes more and more precisely quantitative. Thirty years ago most of the phenomena of electricity and electro-magnetism were known merely as facts; now they can be for the most part exactly measured and calculated.

" There is in fact a **natural course of progress** through which we proceed in every such inquiry, as may be stated in the following series of questions.

1. Does the antecedent invariably produce an effect?
2. In what direction is that effect?
3. How much is that effect in proportion to the cause?
4. Is it uniformly in that proportion?
5. If not, according to what law does it vary?"

The Method of Concomitant Variations is applied, after phenomena begin to be measured, in cases where there is an increase or decrease of an event with a corresponding increase or decrease of the condition which, by the other methods, has been assumed to be the cause. This gives rise to

Canon Third.—Increase or diminution of the effect, accompanied by the increased or diminished intensity of the assumed cause, in cases which admit of increase and diminution, increases the assurance of the causal relation.

By the method of difference it may be shown that air is the cause of the transmission of sound, by striking a bell in the air and in a vacuum. Instead of this, the method of concomitant variations may be applied, by striking a bell in the receiver of an air-pump with a very little air, and then increasing and decreasing the density of the air. The sound, which is very faint with a little air, grows fainter and disappears as the air is exhausted, and becomes louder and fuller as air is added.

This method is made use of in seeking causes for events which go through **periodic changes**, alternately increasing and decreasing. It leads us to search for a cause which undergoes like periodic changes. The tides are thus proved to be due to the combined attraction of the moon and sun, since the periods of high and low, spring and neap, tides succeed each other in intervals corresponding to the *apparent* revolutions of those bodies round the earth.

But all these methods are subject to difficulty from the fact that causes are usually complex, or, in other words, that there is usually a **plurality of causes** co-operating in the production of any given effect. This gives rise to

The Method of Residues or of Residual Variations.—When there are several causes each producing a part of the effect, we desire to know how much is due to each cause. This leads to

Canon Fourth.—Subtract from any phenomenon such part as is known by previous inductions to be the effect of certain of the causes, and the residue of the phenomenon is the effect of the remaining causes.

This is illustrated by the method of ascertaining the exact weight of a load of hay or any other commodity in a cart, by weighing the cart and load together, and then subtracting the tare or weight of the cart alone, previously ascertained. Almost all the remarkable modern **predictions in astronomy** have been made by the use of the method of residues. Thus, after the effects of all known attractions were calculated in the case of Uranus, it was still

found that the planet was sometimes before and sometimes behind its calcu-
lated place. This residual effect pointed to the existence of some cause of
attraction not then known, and the exact place and size of the disturbing
body was calculated and the planet Neptune discovered.

" The **motions of several comets** have in this way been calculated, but it
is observed that they return each time a little later than they ought. This re-
tardation points to the existence of some obstructive power in the space
passed through, the nature of which is not yet understood."

When the same phenomenon may be the effect of any **one of various
causes,** there arises the necessity for excluding all the causes but that
which really operates in the given case. Ordinarily this is not a
difficult matter. It requires, however, that the attendant circum-
stances should be carefully noted and understood. A room may be
heated by the August sun, or by a fire in furnace or grate or stove, or
by any one of various other causes. Which is the operating cause
may be ascertained by the proper inspection, the real cause being
thus found and all others excluded.

(2.) Inductions of Hypothetical Cause.—When the cause
of any given phenomenon is unknown or beyond our reach,
the assumption of some hypothetical cause becomes a ne-
cessity of the human mind. Such cases are in the region
of tentative science or scientific speculation, rather than in
that of exact science. Rule 2d requires in such cases that
the cause or causes assumed should be appropriate and ade-
quate to account for all the facts.

**Rule 3d.—When the facts have been sufficiently investigated com-
bine them all under the cause, simple or complex, which seems best
suited to produce them, and which is at work in all similar facts.**
This gives **the working hypothesis,** which must be modified to suit the
further developments of investigation.

As the observation may be more or less complete, **various working
hypotheses** may be reached by the same thinker or by different
thinkers.

When the facts concerning the movements of bodies on the earth and in the
heavens have been to some extent observed, they may be referred to gravity
as the cause. When the investigation has been carried still further, the **work-
ing hypothesis of universal gravitation,** of Newton, may be stated : "*Any two
masses in the universe, whatever their material, attract each other by gravitation with
a force which varies directly as the mass and inversely as the square of the dis-
tance.*"

This is the work of the constructive imagination or of the power of scientific construction, and must always precede complete and established scientific theory.

II. Testing the Working Hypothesis.

Scientific thinking requires that to the most ingenious boldness in forming working hypotheses should be united the most cautious accuracy in testing them before their acceptance as truth. The **tests of hypotheses** are found in connection with the cause assumed, the facts to be explained, or the application of the deductive method.

"A riper inquiry," says Ueberweg, " recognizes that in all problems where we must proceed upon mere observation, and not with mathematical certainty, the scientific correctness of distinct hypotheses must be the first object of investigation. An essential advance in method in this sense was made in Astronomy, when in the Platonic school, and especially by Heraclides of Pontus, the question to be investigated was not stated in this way: What positions and motions of the heavenly bodies are to be necessarily accepted on empirical and speculative grounds? but in this: What *hypotheses* of regular motions, in themselves possible, can be formed which agree with the facts of observation, so that the phenomena may be 'preserved'?"

Rule 1st.—See that the hypothesis in each case embodies a cause or complex of causes which is appropriate, sufficient, and, if possible, known and true. This is the **cause test.**

All rival hypotheses should be considered and fairly tested according to this Rule. The direction of Ueberweg is as follows: " Let all the opposing fundamental opinions be brought under the view of different thoroughly testing hypotheses, and do not let the one opinion (as too often happens if it is the traditional one) be treated from outset as correct, necessary, sound, and rational, and those of opponents considered to be false, arbitrary, unsuitable, or foolish."

The Rule suggests **various particulars** to be noted in settling the claims of rival hypotheses.

1st. The hypothesis which is to be of service must embody **a cause.**

The hypothesis of evolution, as stated by Spencer, embodies no cause: " Evolution is a *change* from an indefinite, incoherent homogeneity to a definite, coherent heterogeneity through continuous differentiations and integration." " A change " is not a cause, but is rather the very thing to be explained. This is true of a vast region of so-called inductions, which are not inductions at all, because there is no cause at the foundation of the facts. For example, it might readily be concluded, from the fact that man and all the animals with which

we come in contact move the lower jaw in masticating food, that all animals do the same. The fact, however, is that the crocodile moves the upper jaw. This is mere *generalization*, and not induction in the proper sense.

2d. The hypothesis is to be preferred which embodies **an appropriate cause**.

The *universe* is found by scientific investigation to be a *thought-system*. Of various hypotheses concerning its production,—by chance, by self-origination through blind matter and force, by an Intelligent Author capable of planning and constructing such a thought-system, etc.,—the hypothesis of an Intelligent Author is the only scientific one, since such a cause is the only appropriate one for the effect.

3d. The hypothesis is to be preferred which embodies **a known cause**.

Induction assumes **the simplicity of nature**. That is, the Author of nature works as man would work, using the simplest means to attain the end in view, never introducing a new force where some already existing force will accomplish the object. On this principle Newton extended the familiar action of the known force of gravity on the earth's surface to the phenomena of the heavens.

4th. The hypothesis is to be preferred which embodies **a true cause**.

Newton's view of gravitation made use of a *true cause*, which "had been already known as an actual power in nature, in the power of weight upon the earth."

When no known agent can be found, it becomes necessary to assume some unknown, but appropriate and adequate, cause. Thus, the physicist in accounting for the phenomena of light, electricity, etc., assumes the existence of *ether*, an extremely tenuous substance, pervading all bodies and extending through the universe, which is the vibratory medium in the transmission of all these forms of energy. This is, of course, a strictly hypothetical cause. Some other hypothesis may, at some future time, take its place.

5th. The hypothesis is to be preferred which takes into account **the complex nature of causes** and makes the right ones prominent.

Almost universally in nature causes are manifold and complex, and none of the complex elements can be overlooked without falling into error. For example, about 1854, some excavators brought up some burnt brick and pottery from the depth of 60 and 72 feet, in the valley of the Nile. Assuming that they were found where they were made, and that the alluvium had been deposited upon them at the rate at which the Nile now makes its deposit, and that this was the only cause at work, it was calculated mathematically that the relics must be from 12,000 to 60,000 years old. One causal element omitted was the weight of the brick-bats in connection with the fact (also causal) that all the region is a vast quagmire during the inundation which covers it with water during a large part of the year. Sir Robert Stephenson afterwards found in the Delta near Damietta, at a far greater depth, a brick bearing the stamp of Mohammed Ali (1808). Some one said satirically that the main question in the first case should have been : How long will it take a brick to sink 72 feet in a quagmire? But although this might be the main question, all causes should be given their due weight in reaching the correct result.

Rule 2d.—See that the hypothesis in each case combines and explains all the facts. This is the **fact test.** This embraces **various particulars.**

1st. The hypothesis must **embrace the facts.**

This is the object in forming hypotheses, and forgetfulness of it is fatal to correct thinking. The question in inductive science should not be, **what must be?** but, **what is?** The old science, putting assumption and deduction in the place of induction from facts, taught that the orbits of the heavenly bodies *must be* circular, because "the circle is the perfect figure;" the true science teaches that the orbits of the heavenly bodies *are, in fact,* ellipses, because this alone agrees with the facts as explained by the laws of centrifugal and centripetal force in connection with gravitation and the motion of the bodies.

2d. The hypothesis must **explain all the facts.** A single fact clearly contradictory to any hypothesis calls for the modification or abandonment of the hypothesis.

It is manifest that even a **single fact** clearly contradictory to any hypothesis proves the hypothesis untenable, as that single fact, though there were no other such fact, would prove the principle embodied in the hypothesis not universal. The place occupied by **exceptional facts** is thus seen to be very important. As Jevons has said, "they are commonly the points from which we start to explore new regions of knowledge." As **all exceptions are not equally fatal** to the hypotheses to which they appear to be exceptional, Jevons (*Principles of Science*, pp. 644–672) has arranged them under **eight classes**:

(1.) "**Imaginary, or false, exceptions,** that is, facts, objects, or events which are not really what they are supposed to be.

(2.) "**Apparent but congruent exceptions,** which, though apparently in conflict with a law of nature, are really in agreement with it.

(3.) "**Singular exceptions,** which really agree with a law of nature, but exhibit remarkable and unique results of it.

(4.) "**Divergent exceptions,** which really proceed from the ordinary action of known processes of nature, but which are excessive in amount or monstrous in character.

(5.) "**Accidental exceptions,** arising from the interference of some entirely distinct but known law of nature. This is the largest class of exceptions.

(6.) "**Novel and unexplained exceptions,** which lead to the discovery of a new series of laws and phenomena, modifying or disguising the effects of previously known laws, without being inconsistent with them.

(7.) "**Limiting exceptions,** showing the falsity of a supposed law in some cases to which it has been extended, but not affecting its truth in other cases.

(8.) "**Contradictory or real exceptions,** which lead us to the conclusion that a supposed hypothesis or theory is in opposition to the phenomena of nature, and must therefore be abandoned." These exceptions are **the most important of all,** "since they lead to the entire rejection of a law or theory before accepted." No law of nature can fail; there are no such things as real exceptions to real laws. Where contradiction exists, it must be in the mind of the

experimentalist. Either the law is imaginary or the phenomena which conflict with it; if, then, by our senses we satisfy ourselves of the actual occurrence of the phenomena, the law must be rejected as illusory.

Rule 3d.—Apply the principles of deduction to the hypothesis, ascertaining what ought to happen in any given circumstances if the hypothesis be true, and test the predicted results by observation and experiment. This is the **prediction test.**

When any hypothesis embodies a real cause, it gives the thinker **the power of predicting** by deduction the particular phenomena which come under it. The verification of such predictions is one of the last and highest tests of an induction. "There is no more convincing proof of the soundness of knowledge than that it confers the gift of foresight." Astronomy furnished the earliest development of this power. Thales, the Father of Philosophy, predicted the eclipse which suddenly turned day into night during a battle between the Medes and Lydians. The recent discovery of Neptune is the most remarkable instance of this prevision. The method of prediction by deduction is equally applicable to all the physical and mental sciences.

"As we deduce more and more conclusions from any hypothesis and find them verified by trial, **the probability of the theory increases** in a rapid manner; but we never escape the risk of error altogether. **Absolute certainty** is beyond the powers of inductive investigation, and the most plausible supposition may ultimately be proved false.

"Such is the groundwork of similarity in nature, that two very different conditions may often give closely similar results. We sometimes find ourselves, therefore, in possession of **two or more hypotheses** which both agree with so many experimental facts as to have great appearance of truth. Under such circumstances we have need of some new experiment, which shall give results agreeing with one hypothesis but not with the other." This gives rise to what Bacon called an **Experimentum Crucis,** an "Experiment of the Finger Post." In Pascal's day his own hypothesis, that the mercury rose in the tube because of the pressure of the atmosphere, had as its rival the doctrine, that this phenomenon was due to nature's abhorrence of a vacuum. His experiment of causing a barometer to be carried to the top of the Puy-de-Dôme was the crucial experiment which established his own theory and negatived the rival hypothesis.

Rule 4th.—Avoid the common error of assuming unverified hypotheses, or such as are based upon other unverified hypotheses, as true.

The failure to conform to this general rule has been the bane of scientific investigation in both its physical and mental spheres in all ages. The spirit of speculation and the determination to believe one's own dreams to be the reality have overborne the spirit of the true philosopher. "The philosopher," says **Faraday**, "should be a man willing to listen to every suggestion, but determined to judge for himself. He should not be biased by appearances; have no favorite hypothesis; be of no school; and in doctrine have no master. He should not be a respecter of persons, but of things. Truth should be his primary

object. If to these qualities be added industry, he may indeed hope to walk within the veil of the temple of nature."

Topic Second.—Products of Inductive Reasoning.

The product of induction is a **generalization.** The process may be expressed in quasi-syllogistic form, as follows:

Mars, Jupiter, the Earth.........move in elliptical orbits round the sun;
These are (as good as—or fairly represent) all the planets;
∴ All the planets move in elliptical orbits round the sun.

Or letting M_1, M_2, etc., represent the different instances from which the inductions are made, we have the formula:

$$\left\{ \begin{array}{l} M_1, \text{ as well as } M_2, \ldots\ldots\ldots \text{ is P;} \\ M_1, \text{ as well as } M_2, \ldots\ldots\ldots \text{ is S;} \\ \therefore \text{ Every S is P.} \end{array} \right.$$

I. Varieties of Induction.

Inductions are divided by logicians into perfect and imperfect.

1. The so-called **perfect induction** takes place " when, by a perfect enumeration of all individuals or particulars, the whole sphere of the universal is exhausted." For example:

Mercury revolves on its axis; so do Venus, the Earth, Mars, Jupiter, and Saturn. But these are all the old planets. ∴ All the old planets revolve upon their axes.

This, however, is *enumeration* and *addition* rather than *inference.* It is ordinarily applicable, of course, only to spheres of objects so limited that all the individuals may be successively examined.

2. The so-called **imperfect induction** includes the cases in which the general is reached by inference, without the complete enumeration of objects. Sometimes only a very few objects out of an indefinite number are examined.

The conclusion in such cases may be made universal, **first,** by the *pure assumption of a real causal nexus* between the subject and predicate of the conclusion,—giving what may be called an **inductive guess,** often mistaken for induction; or, **secondly,** by the strictly inductive method of *finding some real, adequate and, if possible, true cause,* to connect the subject and predicate of the conclusion,—giving what may be called a **true induction.**

14

(1.) The **inductive guess** or **primary induction** may be illustrated by the following example:

Iron is heavier than water, so is silver, quicksilver, gold, etc.
∴ All the metals are heavier than water.

The primitive inductions thus formed are mostly false, as in this example, since some of the metals, as sodium and potassium, are lighter than water. A vast amount, not only of the thinking of common life but also of the so-called scientific induction, is of this nature, and, therefore, at the best only the **work of the imagination,** and at the worst mere **crude guess-work.**

(2.) The **true induction** is that in which a causal nexus, found in the nature or essential relation of the objects examined, is more or less completely established. The generalizations in such cases vary in degree of probability. The *highest degree* of probability is reached where some true and known cause is at work producing like effects in the various individual instances. The probability decreases as the cause recedes into the region of the unknown and hypothetical. The *true induction* may be illustrated by the following examples:

Mercury, Venus, Jupiter, etc., appear to be wanderers among the fixed stars;
These represent all the planets (since this apparent wandering is due to the motion of these stars and the earth);
∴ All the planets will probably appear to be wanderers among the fixed stars.

3. **Analogy** has already been shown (p. 136) to involve both induction and deduction, **the inductive being the principal element.** As analogy depends upon some assumed likeness, its **kinds** may be indicated by the kinds of properties (pp. 28–9) in which the likeness is found. That likeness may be in either essential or non-essential properties.

(1.) **Analogy based upon resemblance in essential properties** is the most valuable kind. The reasoning in this case rests upon the generic and essential nature of the objects coördinated in the analogy. This may be illustrated by the inference made by Franklin in November, 1749, which must be reckoned among inferences from analogy, since lightning and electrical phenomena were not yet known to be the *same* but only *similar:*

" The electric fluid, as it shows itself in experiments made by us, is attracted by projecting metallic points;

"This electric fluid and lightning agree in the properties, that they give light of the same color, have a quick motion, are conducted by metals, etc., etc.;

"Hence it is to be presumed that lightning will also be attracted by projecting metallic points."

(2.) **Analogy based upon resemblance in peculiar or accidental properties** is of comparatively little value, since these properties do not indicate any essential or causal principle lying back of them. This may be illustrated by the following examples:

"The American swan is white; therefore, the Australian swan is white."

"John Smith, a man with a red nose, is a drunkard; therefore, Timothy Jones, another man with a red nose, is a drunkard."

But the Australian swan, though in all essential respects the same as the American, differs in the non-essential property of color, being found to be black. In like manner the red nose may be the result of exposure to the sun, or of any other of many causes.

(3.) **Analogy based upon the resemblance of relations** is the most difficult to deal with of all the forms of analogy. **This is analogy in the strictest sense.** It is necessary in all inferences of this kind to consider with great care how far the analogy holds. In the direct form these characteristics of analogy may be illustrated by the relations of a *foot* to a *man* and a *mountain*. It is under the man as a support and under the mountain as a support, but its being that upon which man walks does not warrant the extension of this relation to the foot of a mountain. **Analogy from contradictories** is illustrated when, from the fact that *virtue* produces happiness, it is inferred by analogy that its contradictory moral quality, *vice*, will produce unhappiness.

II. Fallacies in Induction.

The most common fallacies in induction arise from failure, first, in dealing with the facts; or, secondly, in finding the cause.

The most common fallacy is that of **false generalization** (*fictae universalitatis*, or *unreal universality*). This makes a show, at least, of complete and conclusive induction.

(1.) This may result from *careless and incomplete observation of facts*, and may then be called the **fallacy of insufficient observation.** Thus, a French physician, it is said, once gave a Frenchman, who had typhoid fever, chicken soup; the patient recovered, and on the basis

of this one fact the doctor made the generalization,— "Chicken soup will cure a man who has typhoid fever." He afterwards used the same remedy in the case of an Englishman who had the same disease; the patient died, and the doctor reached and recorded the further generalization,—"Chicken soup cures a Frenchman, but kills an Englishman."

(2.) The false generalization may also result from the **hasty assumption of something as the cause which is not the cause** (*non causa pro causa*). That which is assumed as the cause in such cases may be either a *simple concomitant* or a *mere antecedent* (*post hoc ergo propter hoc*). The fallacy of assuming that a *simple concomitant* is a cause (*cum hoc ergo propter hoc*) is illustrated by the conclusion of the materialist, that since chemical action in the brain accompanies mental action, it is the cause of mental action; which is paralleled by the assumption, that because the small boy's boots always accompany the small boy, therefore, they are the small boy. The fallacy of assuming that a **mere antecedent** is a cause (*post hoc ergo propter hoc*) is illustrated by the inference, among the ancient Romans, that when a general engaged the enemy where the response of the augurs had been unfavorable, and suffered defeat, the cause of the disaster was the unfavorable character of the auspices.

Praxis.— Test the following conclusions reached by induction; state whether the induction is valid or not in each case; verify the induction when valid, and when not valid show what is the fallacy involved:

1. "The Jews are rogues,—The Carthaginians, faithless,—The Cretans, liars,—The French, braggadocios,—The Germans, mystics,—The rich, purse-proud,—The noble, haughty,—Women, frivolous,—The learned, pedants." 2. Matter is eternal. 3. Spirit is essentially immortal. 4. The Irish are malcontents. 5. All human languages had a common origin. 6. The great civilizations have all flourished in the North Temperate Zone. 7. Man is what circumstances make him. 8. "There's a divinity that shapes our ends, rough hew them how we will." 9. That which survives is fittest. 10. All the planets revolve on their axes. 11. Conceited men are always shallow. 12. Ignorant men are conceited. 13. Selfish men are not men of principle. 14. Man is born sinful. 15. The Christian nations are the progressive nations. 16. The Protestant nations are the foremost nations in the world. 17. The reach of gravitation is universal. 18. The best education is secured by means of the Classics. 19. The best education is secured by means of the Natural Sciences. 20. The best education is secured by means of the combined study of the Classics, Natural Sciences, and Mathematics. 21. The appearance of a comet is the harbinger of famine, pestilence, and war. 22. Friday is an unlucky day.

State, in the following cases, whether the facts are exceptional, and, if so, to what class each belongs; and show whether and how they

can be reconciled with the hypotheses to which they appear to be exceptional:

1. The rotation of the earth upon its axis gives to all the stars an apparent motion of rotation from east to west. The Pole Star seems not so to revolve. 2. According to the Newtonian view of gravitation all bodies are heavy. But flame, bubbles, clouds, etc., ascend, and were, therefore, regarded by the ancients as *essentially light*. 3. The Copernican theory teaches that the earth in revolving moves toward the east at the rate of a thousand miles or more an hour. It has been objected to it that, if this be so, then a stone dropped from the topmast of a ship at anchor ought to fall behind toward the west, just as a stone dropped from the mast-head of a moving ship would fall behind, owing to the motion of the ship. 4. The ancients held that the general tendency of bodies on the earth is downward. In the case of the loadstone held over iron, the iron had a tendency upward. This could not be explained by the hypothesis of *essential lightness*, since iron is one of the heaviest substances. 5. According to the theory of Torricelli and Pascal, the mercury ought to stand at a height of about 31 inches in the barometer. Boyle showed that in a perfectly cleansed tube it could be made to stand as high as 75 inches. 6. According to the hypothesis of the materialistic evolutionist, the development of the universe has been a continuous change and progress from the primordial atom, without break or interference of any other than material forces. Dr. McCosh, in *Christianity and Positivism* (Appendix, p. 344), enumerates eleven breaks in the continuity, among which are the following: " Chemical action cannot be produced by mechanical power." " Life, even its lowest forms, cannot be produced from unorganized matter." " Protoplasm can be produced only by living matter." "A living being can be produced only from a seed or germ." "An animal cannot be produced from a plant." " Sensation cannot be produced by insentient matter." The genesis of a new species of plant or animal has never come under the cognizance of man, either directly or indirectly. Consciousness cannot be produced out of mere matter or sensation. " We have no knowledge of man being generated out of the lower animals." "All human beings, even savages, are capable of forming certain high ideas, such as those of God and duty ;" the brute is not.

State and test the following hypotheses:

1. The Wolffian hypothesis of the origin of the Homeric Poems. 2. The hypotheses concerning the origin of the Four Gospels. 3. The hypotheses concerning the nature of Electricity. 4. The hypotheses concerning the nature of Heat. 5. The hypotheses concerning the composition of Comets. 6. The hypotheses concerning the origin of Life on our globe. 7. The hypotheses concerning the nature of Man. 8. The hypotheses concerning the nature of Beauty. 9. The hypotheses concerning the origin of the Universe.

Note.—For a complete and extended treatment of Induction, the teacher and student are referred to the following works: Jevon's *Principles of Science;* Mill's *System of Logic, Ratiocinative and Inductive.*

14 *   **L**

CHAPTER II.

THE UNFOLDING OF REASONING OR THE SYLLOGISM.

THE treatment of the formation of reasoning is naturally followed by the consideration of the unfolding and testing of its various kinds, as embodied in the Syllogism, and the presentation of the various forms of Fallacy or unsound reasoning.

Practical Logic should train the thinker to distinguish readily between a true syllogism and one that only seems to be a true one. This requires the treatment, in successive Sections, of the **Forms and Tests of Categorical and Hypothetical Syllogisms,** and the **kinds of Fallacies.**

Section I.—The Categorical Syllogism Unfolded.

In unfolding the categorical syllogism, the nature and kinds of which have already been presented (p. 141), the following Topics will be considered:

Topic I.—The Possible Forms of the Syllogism, or Figure and Mood.
Topic II.—The Testing of the Valid Forms.
Topic III.—Complex and Abnormal Forms.

Topic First.—The Possible Forms of the Simple Categorical Syllogism.

The possible forms of the single syllogism are determined by the various positions of the middle term, in the premises, with reference to the major and minor terms, and the possible combinations of the four normal judgments, A, E, I, O, in groups of three. The first gives rise to **Figure,** the second to **Mood.**

I. Figure of Syllogisms.

Syllogisms are divided into different Figures by the posi-

tion of the middle term. The **possible positions are four,** which give rise to as many Figures:

Figure I, middle term subj. of maj. prem. and pred. of minor.
Figure II, " " pred. of both maj. and min. premises.
Figure III, " " subj. " " " " "
Figure IV, " " pred. of maj. prem. and subj. of minor.

This may be expressed and illustrated as follows:

Fig. I. *sub prae.*	M	P	Every *virtue* is praiseworthy;	=A
	S	M	Eloquence is a *virtue;*	=A
	∴ S	P	∴ Eloquence is praiseworthy.	=A
Fig. II. *prae prae.*	P	M	No vice is *praiseworthy;*	=E
	S	M	Eloquence is *praiseworthy;*	=A
	∴ S	P	∴ Eloquence is not a vice.	=E
Fig. III. *sub sub.*	M	P	Every *virtue* is praiseworthy;	=A
	M	S	Every *virtue* is useful;	=A
	∴ S	P	∴ Something useful is praiseworthy.	= I
Fig. IV. *prae sub.*	P	M	Every virtue is *praiseworthy;*	=A
	M	S	Everything *praiseworthy* is useful;	= A
	∴ S	P	∴ Something useful is a virtue.	= I

In these examples the mnemonic *sub* and *prae* stand for *subject* and *predicate.* The wedge-shaped figure or line (▬▬▬) denotes a *judgment.* Its thick end turns toward the subject of extension, which is contained as a species under the predicate as a genus. The perpendicular stroke drawn through the line (▬┼▬) indicates *negation.* In the **Hamiltonian Notation,** of which this is a part, the heavy horizontal line (▬▬▬▬) used in the unfigured syllogism (p. 165), indicates *equality* between subject and predicate, or a substitutive judgment.

Note.—The syllogisms ordinarily used in the examples in Logic are made up of propositions of extent, and are, therefore, called **extensive syllogisms.** Hamilton introduces and insists upon the **intensive syllogism.** This is expressed by reversing the wedge-shaped figure, which in this case represents the copula as meaning "comprehends," instead of "is contained under," which is its meaning in the extensive syllogism. The two forms may be illustrated:

Ext. Syl.	The notion responsible is *contained under* the notion free-agent;	M	P
	The notion man is *contained under* the notion responsible agent;	S	M
	∴ The notion man is *contained under* the notion free-agent.	∴ S	P

<table>
<tr><td rowspan="3" style="writing-mode:vertical">Int. Syl.</td><td>The notion man comprehends the notion responsible;</td><td>S ———◄ M</td></tr>
<tr><td>The notion responsible comprehends the notion free;</td><td>M ———◄ P</td></tr>
<tr><td>∴ The notion man comprehends the notion free.</td><td>∴ S ———◄ P</td></tr>
</table>

In the first form the notions are *class notions;* in the second, *concepts proper.* In the second form *the premises* of the first form are *transposed.* With this slight change extensive and intensive syllogisms conform to the same rules, and are so nearly identical that the intensive form does not need separate treatment. In fact, both propositions of extent and of content are often used in the same syllogism. Thus:

All of the metals are positive;	Proposition of content.
Silver is one of the metals;	Proposition of extent.
∴ Silver is positive.	Proposition of content.

II. Mood of Syllogisms.

The **Mood** of a Syllogism is the arrangement of its propositions according to their respective quantity and quality. There are as many possible Moods as there are combinations of the four normal propositions, A, E, I, O, in syllogistic form.

It will be seen on examination that in the premises each of the four may be placed first, and then followed by each of the four successively, giving $4 \times 4 = 16$ combinations. Each of these 16 combinations may then be followed successively in the conclusion by each of the four judgments, A, E, I, O, giving $16 \times 4 = 64$ possible syllogistic combinations. These forms will be presented later, in gathering up the results of the application

The 16 Premise Forms.

A A	E A	I A	O A
A E	E E	I E	O E
A I	E I	I I	O I
A O	E O	I O	O O

of the Rules, and need not, therefore, be here given. The student, moreover, will be able readily to form the combinations for himself.

It will be found, when the proper tests are applied, that comparatively few of these combinations give valid syllogisms.

Topic Second. — The Testing of the Valid Forms.

Two methods have been employed in testing the validity of the various combinations:

First, By what Hamilton calls " the thorough-going quantification of the predicate."

Second, By comparing the spheres of the notions in the

various combinations and framing and using Rules based upon the results. This is the logical method.

1st. The Unfigured Syllogism.—By quantifying the predicate, Hamilton has sought to dispense with Figure altogether. By the explicit quantification of the terms the exact quantity of each is brought out. After the quantification the relation between the terms of the judgments may, according to Hamilton, be expressed by the sign of equality, and the subject and predicate may indifferently change places. The *figured* and *unfigured* form may be illustrated by example:

	Figured.—Fig. I.	Unfigured.
A	Men are rational;	All men = some rational;
A	Negroes are men;	All negroes = some men;
A	∴ Negroes are rational.	∴ All negroes = some rational.

If the object in introducing this new method is to simplify reasoning it is not attained, since while it apparently simplifies it really complicates it. The **fatal objection** to its general introduction is found in the fact, that the instant the predicate of a judgment is quantified, it ceases to be a *logical* or *qualitative whole* and becomes a simple *quantitative* or *mathematical whole.* The judgment is no longer a *logical,* but a simple *mathematical, judgment.* Davis enforces this position in "The Theory of Thought," p. 124:

"For, consider the meaning of 'all' in the predicate. It is not, it cannot be, the distributive, divisive, exemplar 'all,' but is always the total, indivisible, cumular 'all,' a mathematical whole. E. g., 'All men are bimana;' this is the distributive 'all,' meaning that all, each, and every man is in the class, or has the mark, *bimana.* But let us say 'All men are all bimana;' this does not mean 'Every man is all bimana,' nor 'All men are every bimana,' nor 'Every man is every bimana,' which is nonsense. It means 'All men (as a mathematical, total, collective whole) are all bimana' (as *ditto*). Thus 'all' in the predicate is never distributive, but cumular, and enforces the 'all' of the subject also to be cumular. So also the total predicate of a negative is a mathematical, not a distributed total; and 'some' in the predicate is a mathematical part. More generally, whenever the quantity of the predicate is designated, both terms are individuals, and the judgment is mathematical."

The decision whether a given combination leads to a valid inference, and the proof of the validity or non-validity, must depend upon the comparison of the spheres of the notions as given in the premises of the apparent syllogism. The reciprocal relations of notions, already presented (pp. 40 and 45), comprehend all the relations essential in the comparison of notions in reasoning. These relations, as has been seen, may be made apparent to the senses by the use of geometrical figures.

2d. The proper method of testing the validity of the various combinations of judgments as premises is by comparing the spheres of the notions involved in these judgments. The valid forms are determined by **General Principles** arising out of the Logical Axioms; by **General Rules**

arising out of the relations of the terms and propositions of the Syllogism; and by **Special Canons** arising out of the nature of the particular Figures. Figure I. has always been considered the normal form of the syllogism, to which the other forms may be reduced. Hence, the principles which govern the **Reduction of Syllogisms** to this Figure need to be presented. For convenience of reference, a **Conspectus of Results** will also be given.

I. The General Principles.

At the foundation and applying equally to all the figures are three general principles embodying the axiom of Identity or Affirmation and of Contradiction or Negation.

First General Principle. Affirmative Conclusion.—If, when the major and minor terms are compared with the same middle term, they both agree with it, they may agree with each other. This is the basis of affirmative conclusions.

Second General Principle. Negative Conclusion.—If, on such comparison, one term agrees and the other disagrees with the same middle term, they disagree with each other. This is the basis of negative conclusions, which, therefore, result from the combination of one affirmative and one negative premise.

Third General Principle. No Conclusion.—If, on such comparison, both terms disagree with the same middle term, it is uncertain whether they agree or disagree with each other, and, therefore, no valid inference can be drawn in such cases. This is the case where both premises are negative.

II. The General Rules.

The general rules arising out of these general principles depend upon the relations of the terms and propositions of the syllogism. They may be reduced to seven, and are equally applicable to all the figures.

Rule 1st.—There must be three, and only three, terms in any valid **syllogism.** The major and minor terms would not otherwise be logically connected at all. This needs no illustration. It guards against the common **Fallacy of Four Terms,** which oftenest arises from the use

of equivocal terms (p. 82) or want of clear thought. In all cases the middle term needs to be carefully examined in order to make sure that it is used in precisely the same sense in both premises. Whenever it is not so used the case is one of substantially four terms. E. g. ·

"What we eat grows in the fields or is the flesh of animals;
Cooked food is what we eat;
∴ Cooked food grows in the fields or is the flesh of animals."

This is a case of **two middle terms.** In one premise, "what we eat" is used with reference to its mere *essence;* in the other, with reference to the *accident* or *property* of being cooked. This is the *Fallacy of Accident (Fallacia Accidentis).*

Rule 2d.—The middle term must be distributed at least once. The necessity for this arises from the fact that without it the major and minor terms might be compared with different parts of the sphere of the middle term, and so fail of being brought into logical connection. E. g.:

All poets (P) are men (M); = A
All orators (S) are men (M). = A

etc.

We cannot infer that "All poets are orators," or that "Some poets are orators," since the universal affirmative, A, does not distribute the predicate (p. 115), which is here the middle term. Such conclusions would result in the **Fallacy of Undistributed Middle.**

Rule 3d.—A term undistributed in the premises must not be distributed in the conclusion. Otherwise the conclusion would include more than is involved in the premises. The violation of this rule is called the **Fallacy of Illicit Process.** The fallacy may occur either with the major term or with the minor.

Illicit Process of the Major Term.

All birds (M) are winged (P); = A
A bat (S) is not a bird (M); = E
∴ A bat (S) is not winged (P). = E

The major term, "winged," is undistributed in the major premise (A), and distributed in the conclusion (E). Hence the inference is not valid, as may be seen from the above presentation of the relation of the spheres of the notions.

Illicit Process of the Minor Term.

Persons without imagination (M) are not true poets (P); = E
Good logicians often (S) are without imagination (M); = I
∴ Good logicians (S) are not true poets (P). = E

In this case the word "often" makes the judgment equivalent to, "Some good logicians are not true poets;" while the universal negative conclusion denies of "all good logicians" that they are "true poets."

Rule 4th.—The conclusion must always follow the weaker part. By this is meant that, if one premise is negative the conclusion must be negative, and, if one premise is particular the conclusion must be particular. This does not need illustration.

It follows that **universal conclusions** can be reached only from universal premises. It will appear subsequently that universal conclusions are not warranted in all cases by universal premises, since they would often involve the fallacies of undistributed middle or of illicit process.

Rule 5th.—No valid inference can be drawn where both premises are negative. This follows from the Third General Principle. The relation of the major and minor terms to each other is left wholly undetermined by the form of the judgment.

Three cases come under this Rule: where both premises are universal negative; where one is universal negative and the other particular negative; where both are particular negatives. The Rule, therefore, excludes, as invalid in all instances, four of the sixteen possible combinations,—E E, E O, O E, O O,—leaving only twelve possibly valid combinations.

A single illustration, coming under the **first case,** will suffice to assist the student in presenting for himself in diagram the various forms which the indeterminate relations of the major and minor terms may take.

$$\text{No poets (P) are angels (M)} ; \quad \text{No P is M} ; \; = \text{E}$$
$$\text{No men (S) are angels (M)} ; \quad \text{No S is M} ; \; = \text{E}$$
$$\therefore \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \qquad \therefore \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$$

By the terms of the judgments both "poets" and "men" are excluded from "angels," but they may stand to each other in any one of at least the five following relations (p. 40): 1st. They may be independent or coördinate. 2d. They may be coextensive. 3d. S may include P. 4th. P may include S. 5th. S and P may intersect each other.
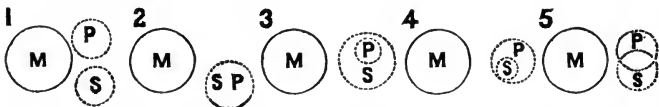
It will be found that, in the second and third cases of negative premises, the possible relations of the terms become even more complicated.

Rule 6th.—No valid inference can be drawn where both premises are particular. In such instances the precise connection of the spheres of the major and minor terms with that of the middle cannot be determined from the form of the judgments.

Three cases come under the Rule: where both premises are particular affirmative; where one is particular affirmative and the other particular negative; where both are particular negative.

The first case, I I, will furnish a sufficient illustration.

> Some poets (M) are intellectual (P); Some M is P; = I
> Some poets (M) are emotional (S); Some M are S; = I
> ∴ . . . . ∴ . . . .

In this case the "intellectual" and "emotional" poets might stand in either of the following relations: 1st. They might exactly coincide. 2d. They might wholly exclude each other. 3d. They might intersect each other, etc.

1 M (S P) **2** (P M S) **3** (M (P S)) etc.
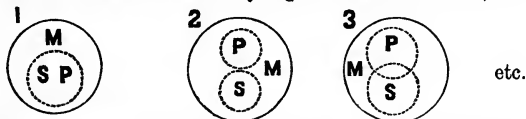
The same indeterminateness, in the relation of the major and minor terms to each other through the middle term, may be shown to exist in the other cases. The third case is likewise excluded from valid syllogistic forms by the Rule for negative premises.

The Rule, therefore, excludes three combinations not excluded by the previous Rule: I I, I O, O I; leaving but nine possibly valid combinations.

Apparent Exceptions.—Two apparent exceptions to this Rule need to be noted: first, syllogisms involving plurative judgments (p. 114); secondly, those in which one or both of the premises are substitutive judgments (p. 113). These are not, of course, strictly particular judgments.

Plurative Judgments, whether indefinite or numerically definite, give valid conclusions, as seen in the following examples:

Most men (M) are conceited (P); A ——Ignorant.———|————D
                                        | B   C
                                              Conceited.

Most men (M) are ignorant (S); E——Ignorant and conceited. |————G
                                                       F

∴ At least some conceited men (S) are ignorant (P).

It is obvious in this case that "most men" in the major premise may coincide more or less fully with "most men" in the minor, as illustrated in the diagrams. In the first case, the line A C represents the "ignorant," B D the "conceited," and A D "all men." The line B C represents the minimum of agreement, in the given case, when the "ignorant" and "conceited" differ to the utmost. In the second case, E F represents both the "ignorant" and the "conceited," and E G "all men." The line E F represents the maximum of agreement, when the "ignorant" and "conceited" agree to the utmost, i. e., coincide.

If this be given the **numerically definite** or proportional form, it may become·
80 out of every 100 men are conceited;
80 out of every 100 men are ignorant;
∴ At least 60 out of every 100 conceited men are ignorant.

15

In this instance the "80" of the major premise may agree more or less fully with the "80" of the minor, as illustrated by the diagram. The minimum of agreement, as shown in the following diagram, is D B, or 60 out of every 100; the maximum, E F, 80 out of every 100.
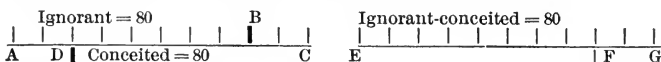
Substitutive Judgments, even when particular (Y), often result in making conclusions valid that would be invalid if the premises were mere particular attributives. Such judgments, whether universal or particular, always distribute the predicate (p. 115). They are not, however, strictly particular judgments. For example:

Some trees (P) are (all the) oaks (M);	= Y	Some P is all M ;
Some oaks (M) are white oaks (S);	= I or Y	Some M is (all) S ;
∴ Some white oaks (S) are trees (P).	= I	∴ All S is P.

Rule 7th.—No valid inference can be drawn from the combination of a particular major premise with a negative minor premise. This will appear from the comparison of the spheres of the notions in the four possible cases: I E; O E; I O; O O.

The Rule may be sufficiently illustrated by the first case, I E. The other combinations have also been already excluded from the valid combinations,— O E and O O by negative premises; I O and O O by particular premises.

Some iron ores (P) are magnetic (M); = I
No lead ores (S) are magnetic (M); = E
∴ . . . .

It is not determined whether the sphere of S is quite separated from the sphere of P, or intersects it, or falls wholly within it. If the attempt were made to draw the conclusion, "No lead ores are iron ores," the negative would distribute the predicate, "iron ores" (P), which is not distributed in the major premise, and would thus result in illicit process of the major term.

This Rule, therefore, excludes the combination I E, and leaves only eight out of sixteen, which can be valid in any case. These may be stated (numbered for convenient reference in treating the four Figures) as follows:

1. A A. 2. E A. 3. I A. 4. O A.   Only part of the remaining eight will
5. A E.   be found to hold true in any one of the
6. A I. 7. E I.   Figures.
8. A O.

III. Special Canons of the Figures.

Each of the four Figures has its special rules resulting from the relations of the terms, which may be embodied in a **Canon** for that Figure.

1. Figure I. is that which has the middle term as the sub-

ject of the major premise and the predicate of the minor. There follows, from the resulting relations of the terms,—

Canon 1st.—In Figure I. the requirements are :

{ **Major prem. universal to avoid fallacy of undistributed middle;**
{ **Minor prem. affirmative to avoid fallacy of illicit process of maj. term.**

Testing by this Canon the eight possible combinations left by the General Rules, only **six syllogistic forms** are found valid in Figure I.: AAA, AAI, EAE, EAO, AII, EIO. These are **reducible to four,** since AAI and EAO are but cases of particular or **weakened conclusions,** included in the universals, AAA and EAE.

Note.—In this Figure the process of testing by the General Rules and the Canon will be applied to the eight combinations successively, in order to prepare the student to apply the like process to the remaining three Figures.

No. 1, A A, by the successive addition of the four propositions, A, E, I, O, gives AAA, AAE, AAI, AAO. The second and fourth of these forms, AAE, AAO, drop out since the affirmative premises indicate agreement, while the negative conclusion would infer disagreement. The third, AAI, is included in AAA. Only **one valuable form,** AAA, remains. It conforms to the Canon, since its major premise is universal and its minor premise affirmative, and the syllogism thus guarded against fallacy. This **valid mood** is known among logicians by the mnemonic word, **Barbara,** the meaning of the consonants in which will be subsequently explained under Reduction. It is illustrated in the following example:

Barbara.	All that is composite is dissoluble;	= A	All M is P;	
	All material things are composite;	= A	All S is M;	
	∴ All material things are dissoluble.	= A	∴ All S is P.	

No. 2, E A, by the successive addition of the four propositions, A, E, I, O, gives EAA, EAE, EAI, EAO. The first and third of these forms, EAA and EAI, drop out, since the one negative premise always requires a negative conclusion by Rule 4th. The fourth, EAO, is included in EAE, drawing a particular conclusion when the universal is permissible. The **valid mood,** EAE, is known among logicians by the mnemonic word, **Celarent.** It stands the test of the Canon. It is illustrated in the following example:

Celarent.	No finite being is exempt from error;	= E	No M is P;
	All men are finite beings;	= A	All S is M;
	∴ No man is exempt from error.	= E	∴ No S is P.
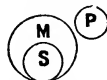
No. 3, I A, gives IAA, IAE, IAI, IAO, **none of which are valid,** since, besides the breach of the General Rules, the particular major premise, I, violates the Canon, and always results in undistributed middle.

No 4, OA, gives OAA, OAE, OAI, OAO, **none of which are valid** for the reasons given under No. 3.

No. 5, A E, gives AEA, AEE, AEI, AEO, **none of which are valid,** since the negative minor premise, E, violates the Canon, and results in illicit process of the major term.

No. 6, A I, gives AIA, AIE, AII, AIO. The first form, AIA, violates Rule 4th; the second and fourth, AIE and AIO, violate the General Principle of all affirmatives. The **fourth,** AII, is valid, standing the test of the Canon. The **valid mood** is known among logicians by the mnemonic word, **Darii.** It is illustrated as follows:

Darii.	All virtues are laudable ; = A	All M is P ;
	Some habits are virtues ; = I	Some S is M ;
	∴ Some habits are laudable. = I	∴ Some S is P.

No. 7, E I, gives EIA, EIE, EII, EIO. The first, second, and third forms, EIA, EIE, EII, violate Rule 4th. The **fourth,** EIO, is valid, standing the test of the Canon. The **valid mood** is known among logicians by the mnemonic word, **Ferio.** It is illustrated as follows:

Ferio.	No virtue is reprehensible ; = E	No M is P ;
	Some habits are virtues ; = I	Some S is M ;
	∴ Some habits are not reprehensible. = O	∴ Some S is not P.

No. 8, A O, gives AOA, AOE, AOI, AOO, **none of which are valid,** since the negative minor premise, O, violates the Canon.

The valid moods in Figure I. are **Barbara, Celarent, Darii, Ferio.** *The Figure is naturally and unconsciously used,* according to Lambert, *to prove qualities.* It follows from the "*Dictum de omni et nullo.*"

2. Figure II. is that which has the middle term as the predicate of both premises. There follows, from the resulting relations of the terms,—

Canon 2d.—In Figure II. the requirements are :
{ **Major prem. universal, to avoid illicit process of maj. term ;**
{ **One prem. negative, to avoid undistributed middle.**

Testing by this Canon the eight possible combinations left by the General Rules, only **six syllogistic forms** are found valid in Figure II.: EAE, EAO, AEE, AEO, EIO, AOO. These are **reducible to four,** since EAO and AEO are but cases of particular conclusions, included in the universals, EAE and AEE.

Leaving the student to test the various possible forms, it will be sufficient to illustrate the valid moods by examples. The moods are known among logicians as **Cesare, Camestres, Festino, Baroko.**

1. Cesare.	Nothing material has free will ; = E	No P is M ;
	All spirits have free will ; = A	All S is M ;
	∴ No spirit is material. = E	∴ No S is P.
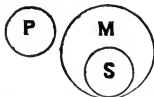
Cesare is a valid mood, as is seen by its conforming to the Canon, in its universal negative major premise, E.

Camestres.

All colors are visible;	= A
No sound is visible;	= E
∴ No sound is a color.	= E

All P is M;
No S is M;
No S is P.
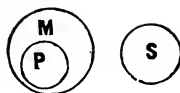
Camestres is a valid mood, since it conforms to the Canon in its universal major premise, A, and negative minor, E.

Festino.

No vice is praiseworthy;	= E
Some actions are praiseworthy;	= I
∴ Some actions are not vices.	= O

No P is M;
Some S is M;
∴ Some S is not P.
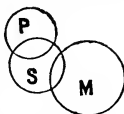
Festino is a valid mood, since it conforms to the Canon in its universal negative major premise, E.

Baroko.

All birds are oviparous;	= A
Some animals are not oviparous;	= O
∴ Some animals are not birds.	= O

All P is M;
Some S is not M.
∴ Some S is not P.

Baroko is a valid mood, since it conforms to the Canon in its universal major premise and negative minor.

Figure II. is naturally and unconsciously used, according to Lambert, *to prove differences.* It follows from a "*Dictum de diverso:*" "Things which are different do not belong to each other."

3. Figure III. is that which has the middle term as the subject of both premises. There follows, from the resulting relations of the terms,—

Canon 3d.—In Figure III. the requirements are:
{ Minor prem. affirmative to avoid illicit process of maj. term;
{ Conclusion particular to avoid illicit process of min. term.

Testing by this Canon the eight combinations of premises, **six** are found to be **valid** in this Figure: AAI, IAI, AII, EAO, OAO, EIO. These are known among logicians as **Darapti, Disamis, Datisi, Felapton, Bokardo (Dokamok), Ferison.**

1. Darapti.

All gilding is metallic;	= A
All gilding shines;	= A
∴ Some things that shine are metallic.	= I

All M is P;
All M is S;
∴ Some S is P.
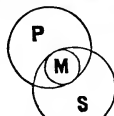
Darapti is a valid mood, since it conforms to the Canon, in its affirmative minor premise, A, and its particular conclusion, I.

2. Disamis.

Some acts of homicide are laudable;	= I
All acts of homicide are cruel;	= A
∴ Some cruel acts are laudable.	= I

Some M is P;
All M is S;
∴ Some S is P.

15 *
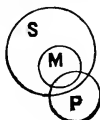
Disamis is a valid mood, since it conforms to the Canon, in its affirmative minor premise, A, and its particular conclusion, I.

<div style="float:right">3. Datisi.</div>

All acts of homicide are cruel;	= A	All M is P ;
Some acts of homicide are laudable;	= I	Some M is S ;
∴ Some laudable acts are cruel.	= I	∴ Some S is P.
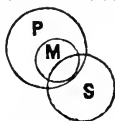
Datisi is a valid mood, since it conforms to the Canon, in its affirmative minor, I, and its particular conclusion, I.

<div>4. Felapton.</div>

No material substance is a moral subject ;	= E	No M is P ;
All material substance is extended ;	= A	All M is S ;
∴ Some extended thing is not a moral subject.	= O.	∴ Some S is not P.

Felapton is a valid mood, since it conforms to the Canon, in its affirmative minor premise, A, and its particular conclusion, O.

<div>5. Bokardo.</div>

Some syllogisms are not regular ;	= O	Some M is not P ;
All syllogisms are things important ;	= A	All M is S ;
∴ Some important things are not regular.	= O	∴ Some S is not P.
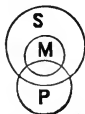
Bokardo is a valid mood, since it conforms to the Canon, in its affirmative minor premise, A, and its particular conclusion, O.

<div>6. Ferison.</div>

No truth is without result ;	= E	No M is P :
Some truths are misunderstood ;	= I	Some M is S ;
∴ Some things misunderstood are not without result.	= O	∴ Some S is not P.
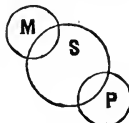
Ferison is a valid mood, since it conforms to the Canon, in its affirmative minor premise, I, and its particular conclusion, O.

Figure III. is naturally and unconsciously used, according to Lambert, *to prove examples and conceptions* (concepts proper). He founds it on a " *Dictum de exemplo :*" " When one finds things A which are B, then there are A which are B."

4. Figure IV. is that which has the middle term as the predicate of the major premise and the subject of the minor. There follows, from the resulting relations of the terms,—

Canon 4th.—In Figure IV. the requirements are :

{ If either prem. neg., maj. prem. universal to avoid illic. major;
{ If maj. prem. affirm., min. prem. universal to avoid undistrib. mid. ;
{ If min. prem. affirm., conclusion particular to avoid illicit minor.

Testing by this Canon the eight combinations of premises, **five** are found to be **valid** in this Figure: AAI, AEE, IAI, EAO, EIO. These are known among logicians as **Bramantip, Camenes, Dimaris, Fesapo, Fresison.**

1. Bramantip.

All greyhounds are dogs;   = A   All P is M;
All dogs are quadrupeds;   = A   All M is S;
∴ Some quadrupeds are greyhounds. = I ∴ Some S is P.
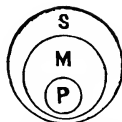
Bramantip is a valid mood, since it conforms to the Canon, in its universal minor premise, A, with particular conclusion, I.

2. Camenes.

All ruminating animals have four
    stomachs;   = A   All P is M;
No animal with four stomachs is
    carnivorous;   = E   No M is S;
∴ No carnivorous animal ruminates. = E ∴ No S is P.
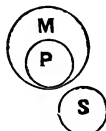
Camenes is a valid mood, since it conforms to the Canon, in its universal major premise, A, and its universal minor premise, E.

3. Dimaris.

Some practically virtuous men are
    necessarians;   = I   Some P is M;
All necessarians speculatively sub-
    vert the distinction of vice and
    virtue;   = A   All M is S;
∴ Some who speculatively subvert
    the distinction of vice and vir-
    tue are practically virtuous.   = I ∴ Some S is P.
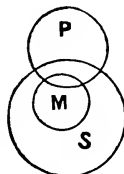
Dimaris is a valid mood, since it conforms to the Canon, in its universal minor premise, A, and its particular conclusion, I.

4. Fesapo.

No negro is a Hindoo;   = E   No P is M;
All Hindoos are blacks; = A   All M is S;
∴ Some blacks are not ne-
    groes.   = O ∴ Some S is not P.
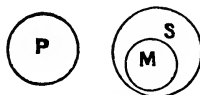
Fesapo is a valid mood, since it conforms to the Canon, in its universal major premise, E, its universal minor premise, A, and its particular conclusion, O.

5. Fresison.

No moral principle is an
    animal impulse;   = E   No P is M;
Some animal impulses are
    principles of action;   = I   Some M is S;
∴ Some principles of action
    are not moral principles. = O ∴ Some S is not P.
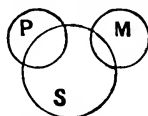
Fresison is a valid mood, since it conforms to the Canon, in its universal major premise, E, and in its particular conclusion, O.

Figure IV. is the reverse of Figure I. *It is naturally and uncon-
sciously used*, according to Lambert, *to prove reciprocities.* He founds
it on a " *Dictum de reciproco:*" " If no M is B, no B is this or that M;
if C is or is not this or that B, there are B which are or are not C."

IV. Collected Results.

For convenient reference the results of the testing of the various forms may be gathered up and tabulated.

1. Table of Moods, Valid and Invalid.

Maj. prem.	Min. prem.	Conclu.	Moods.	Tested.	Maj. prem.	Min. prem.	Conclu.	Moods.	Tested.
A.	A	A	**A A A**	V. C. 1.	E.	A	A	E A A	I. P. 2.
		E	A A E	I. C. 2. 3. 4. I. P. 1.			E	**E A E**	V. C. 1. 2. I. C. 3. 4.
		I	A A I	V. C. 1 (W). 3. 4. I. C. 2.			I	E A I	I. P. 2.
		O	A A O	I. P. 1.			O	**E A O**	V. C. 2 (W). 3. 4. I. C. 1.
	E	A	A E A	I. P. 2.		E	A	E E A	I. P. 3. R. 5.
		E	**A E E**	V. C. 2. 4. I. C. 1. 3.			E	E E E	I. P. 3. R. 5.
		I	A E I	I. P. 2.			I	E E I	I. P. 3. R. 5.
		O	**A E O**	V. C. 2 (W). 4. (W). I. C. 1. 3.			O	E E O	I. P. 3. R. 5.
	I	A	A I A	I. R. 4.		I	A	E I A	I. P. 2. R. 4.
		E	A I E	I. P. 1. R. 4.			E	E I E	I. R. 4.
		I	**A I I**	V. C. 1. 3. I. C. 2. 4.			I	E I I	I. P. 2.
		O	A I O	I. P. 1.			O	**E I O**	V. C. 1. 2. 3. 4.
	O	A	A O A	I. P. 2. R. 4.		O	A	E O A	I. P. 3. R. 4.
		E	A O E	I. R. 4.			E	E O E	I. P. 3. R. 5.
		I	A O I	I. P. 2.			I	E O I	I. P. 3. R. 5.
		O	**A O O**	V. C. 2. I. C. 1. 3. 4.			O	E O O	I. P. 3. R. 5.
I.	A	A	I A A	I. R. 4.	O.	A	A	O A A	I. P. 2. R. 4.
		E	I A E	I. P. 1. R. 4.			E	O A E	I. R. 4.
		I	**I A I**	V. C. 3. 4. I. C. 1. 2.			I	O A I	I. P. 2.
		O	I A O	I. P. 1.			O	**O A O**	V. C. 3. I. C. 1. 2. 4.
	E	A	I E A	I. P. 2. R. 4.		E	A	O E A	I. P. 3. R. 4. 5.
		E	I E E	I. R. 4.			E	O E E	I. P. 3. R. 4. 5.
		I	I E I	I. P. 2.			I	O E I	I. P. 3. R. 4. 5.
		O	I E O	I. R. 7.			O	O E O	I. P. 3. R. 4. 5.
	I	A	I I A	I. P. 3. R. 6.		I	A	O I A	I. P. 2. 3. R. 4. 6.
		E	I I E	I. P. 1. 3. R. 4. 6.			E	O I E	I. P. 3. R. 6.
		I	I I I	I. P. 3. R. 6.			I	O I I	I. P. 2. 3. R. 4. 6.
		O	I I O	I. P. 1. 3. R. 6.			O	O I O	I. P. 3. R. 6.
	O	A	I O A	I. P. 2. 3. R. 4. 5.		O	A	O O A	I. P. 3. R. 4. 5. 6.
		E	I O E	I. P. 3. R. 4. 6.			E	O O E	I. P. 3. R. 4. 5. 6.
		I	I O I	I. P. 2. 3. R. 4. 6.			I	O O I	I. P. 3. R. 4. 5. 6.
		O	I O O	I. P. 3. R. 6.			O	O O O	I. P. 3. R. 4. 5. 6.

Note.—In the column headed "Tested," **V** denotes valid; **I**, invalid; **P**, principle; **R**, rule; **C**, both canon and figure; **W**, weak (indicating a particular conclusion where a universal might be drawn). The student will find profitable exercise in applying the tests to all the forms and figures.

2. Things Proved by the Figures.

Fig. Proved.	Process.	Dictum.
I. Attribute.	Ascribes to the thing what we know of its attribute. It concludes from the genus to the species.	*Dictum de Omni et Nullo.* What is true of all A is true of every A.
II. Difference.	Leads to the discrimination of things, and relieves perplexity in our notions. Affords only negative conclusions.	*Dictum de Diverso.* Things which are different are not attributes of each other.
III. Example.	Affords examples and exceptions in propositions which appear general. Gives only particular conclusions.	*Dictum de Exemplo.* When we find things A which are B, in that case some A are B.
IV. Reciprocity.	Finds species in a genus in Bramantip and Dimaris; shows that the species does not exhaust the genus in Fesapo and Fresison; denies of the species what was denied of the genus in Camenes.	*Dictum de Reciproco.* If no M is B, no B is this or that M; if C is (or is not) this or that B, there are B which are (or are not) C.

3. Valid Moods in the Four Figures.

The valid moods in all the Figures have been embodied in five Latin hexameter lines:

Fig. 1. Barbara, Celarent, Darii, Ferio*que prioris;*

Fig. 2. Cesare, Camestres, Festino, Baroko (or Fakofo), *secundæ;*

Fig. 3. *Tertia* Darapti, Disamis, Datisi, Felapton,

Dokamok (Bokardo), Ferison *habet. Quarta insuper addit*

Fig. 4. Bramantip, Camenes, Dimaris, Fesapo, Fresison.

V. Reduction of Figures.

Figure I. has been looked upon by logicians as the normal Figure, to which all the others may be reduced. The object of **Logical Reduction** is to bring arguments of the last three Figures into the form of Figure I., and thus bring all alike to the test of Aristotle's *Dictum.* It is thus shown that this *Dictum,* which is clearly the regulating principle in Figure I., is also the regulating principle in all deductive rea-

soning, and that the process is, therefore, always substantially the same (p. 138).

Note.—Reduction is usually described as being of two kinds: Direct or Ostensive, and Indirect (*Reductio ad impossibile*). The latter method was the result of a mistaken notion of the logicians that Baroco and Bocardo could not be directly reduced, and is of no value theoretical or practical. Fakofo and Dokamok will be substituted for Baroco and Bokardo, and may be reduced by the direct method.

The mnemonic words in the last three Figures were designed to indicate not only the mood of syllogisms, but also **the principles by which they are to be reduced.** The valid forms in the four Figures must be kept in view in reduction.

The initial consonant, B, C, D or F, in the last three Figures indicates the mood in the first Figure to which the syllogism reduces. Thus, a syllogism in the mood Cesare, Fig. II., reduces to Celarent.

The inserted consonants, s, p, k, f, m, indicate the various processes in reduction. **S** indicates that the proposition symbolized by the vowel preceding it is to be **converted simply** (p. 127); **p,** by **limitation** or **per accidens** (p. 127); **k,** by **contraposition** (p. 128); **f,** by **infinitation** or **obversion** (p. 124). The letter **m** (*mutari*) indicates that the premises of the given syllogism are to be **transposed.** The **p** in **Bramantip** shows that, after converting simply, the premises warrant a universal conclusion.

The other consonants, b, d, l, n, r, t, are not significant, but are inserted for the sake of euphony, or of the metre in the mnemonic hexameters invented, to keep the moods and figures in mind, by Petrus Hispanus, who died in 1277 as Pope John XXII.

The **process** of reduction may be **illustrated** by the following examples:

Figure II.		Figure I.	
	No P is M;		No M is P;
Cesare =	All S is M;	**Celarent** =	All S is M;
	∴ No S. is P.		∴ No S is P.

Bashfulness is not something thoroughly good;	Nothing thoroughly good is bashfulness;
Modesty is something thoroughly good;	Modesty is something thoroughly good;
∴ Modesty is not bashfulness.	∴ Modesty is not bashfulness.

The **C** in Cesare indicates that the mood reduces to Celarent; the **s,** that the major premise is to be converted simply.

Figure III.		Figure I.	
	All M is P;		All M is P;
Darapti =	All M is S;	**Darii** =	Some S is M;
	∴ Some S is P.		∴ Some S is P.

All whales are mammalia;	All whales are mammalia;
All whales are water animals;	Some water animals are whales;
∴ Some mammalia are water animals.	∴ Some mammalia are water animals.

The **D** in Darapti indicates that the mood reduces to Darii; the **p**, that the preceding proposition, A, is to be converted by limitation.

Note.—The student can readily carry the work of reduction through all the figures and moods.

Topic Third.—Complex and Abnormal Forms.

In books and in conversation arguments usually appear in incomplete or irregular forms, and often combined as polysyllogisms manifest or occult. In dealing with these, the incomplete forms, except in the case of such regular forms as the Sorites, need to be completed and the irregular forms reduced to regularity. The general rules then become applicable.

The greater part of this work the student may be left to carry out for himself by aid of the principles already laid down. There is need, however, to present the principles which govern the **Sorites,** to consider briefly some **peculiar forms of argumentation,** and to exhibit especially the **calculation of probabilities.**

I. The Sorites Tested.

The Sorites, or chain of Enthymemes in Fig. I., has already been defined and illustrated (p. 143). There are **two ways of testing** the Sorites: by completing all the abridged syllogisms (p. 144), and then applying the usual tests; or by using a system of rules which may be immediately applied. The former method may be left to the student himself; only the latter needs to be illustrated.

From the nature of the Sorites the following principles result:

1. The first proposition furnishes the major premise of the first completed syllogism; the last proposition is the conclusion of the last syllogism and of the whole chain; the intermediate propositions are the minor premises of the successive syllogisms. The number of syllogisms must, therefore, equal the number of minor premises.

2. The major premise of each successive syllogism after the first is furnished by the conclusion of the preceding syllogism.

The reasoning must conform to the **Canon of Fig. I.**

Rule 1st.—Every major premise must be universal in order to avoid undistributed middle. It follows that only the last proposition in the progressive sorites and the first in the regressive may be particular, since any other particular premise would result in making the conclusion of its syllogism, or the next major premise, particular also.

Rule 2d.—Every minor premise must be affirmative in order to avoid illicit process of the major term. It follows that only one premise may be negative, the last proposition in the regressive sorites and the first in the progressive, since these only are not minor premises.

The **Sorites and its Rules** may be **illustrated** by the following examples, which are abridged to admit of compact parallel statement:

Regressive Sorites.	Progressive Sorites.
Some prosperous are avaricious;	No discontented are happy;
The avaricious are intent on gain;	All intent on gain are discontented;
The intent on gain are discontented;	All avaricious are intent on gain;
The discontented are not happy;	Some prosperous are avaricious;
∴ Some prosperous are not happy.	∴ Some prosperous are not happy.

It has been often asserted that Sorites cannot occur in any other than Fig. I. It has been shown, however, by Mill, that one step, and only one, step in a Sorites may be either in Figure II. or Figure III.

II. Peculiar Forms of Argumentation.

The usual form of direct proof of propositions is known among logicians as the **argumentum ad rem,** or proof of the thing itself. As variations from it or in contrast with it may be noted the *argumentum a fortiori,* the *argumentum ad judicium,* the *argumentum ad populum,* the *argumentum ad verecundiam,* the *argumentum ad ignorantiam,* the *argumentum ad hominem,* and the *reductio ad absurdum.*

The **argumentum a fortiori,** or, "by a stronger reason," is one involving comparative judgments. It is based upon the maxim, "What is greater than a greater is greater still than the thing." The argument is essentially mathematical or quantitative. Thus:

> Asia is larger than Africa;
> Africa is larger than Europe;
> ∴ By much more is Asia larger than Europe.

This may also be presented as follows:

> The Atlantic Ocean is as large as Lake Superior (and more);
> Lake Superior is as large as the Dead Sea (and more);
> ∴ The Atlantic Ocean is as large as the Dead Sea (and still more).

The argument *a fortiori* is also defined as "the proof of a conclusion deduced from that of a less probable supposition that depends upon it." For example, see Matthew vi. 30 and vii. 11.

The **argumentum ad judicium** is based upon the common judgments of mankind. Its maxim is, "What all men everywhere and always believe, is true," or the so-called principle of common sense on which the Scottish philosophy of Reid rests. The argument has great force when it is really based on the common judgment of mankind. The danger of appealing to this principle without sufficient grounds is, however, very great. Under the confident assertions, "Everybody says," "No one pretends to think," the greatest fallacies are often covered. The argument may be illustrated as follows:

The material world is a reality and our perception of it immediate, because all men, everywhere and always, have so believed.

The **argumentum ad populum** is based on an appeal to public opinion, or to passion or prejudice, rather than intelligence. It is often employed because no really good arguments are to be found, or because it is easier to appeal to the passions and prejudices of the masses than to their intelligence. It often puts forward as its major premise the false maxim, "*Vox populi Vox Dei*," "The voice of the people is the voice of God."

The **argumentum ad verecundiam** is an appeal to the feelings of reverence for certain persons or objects, instead of proceeding to prove the point in hand. The Scholastics used as a standing major premise the maxim, "It is foolish to affirm that Aristotle erred."

The **argumentum ad ignorantiam** is addressed to the ignorance of men. It sometimes consists in assuming that a position is correct because an adversary cannot show the contrary; sometimes, in taking advantage of men's ignorance to impose upon them by some shallow sophism, false statement, or confident assertion.

Under this may be included the *Fallacy of Interrogation*, in which a question is so put as to be equivalent to a confident assertion of some error. The **demand for an adequate conception** (p. 91) or description, often made by a browbeating lawyer upon a witness in court, is of the same character. It is only a few experts who can give anything more than a **clear notion** (p. 91) of the handwriting, features, or dress of the most intimate friend.

The **argumentum ad hominem** is an appeal to the practice, principles or professions of an opponent as confirming our own position or destructive to his. An opponent may thus be silenced, since the argument is *good against him*, even though it be *not good against the views* he advocates. As soon as he renounces such practice, principles or professions, the argument ceases to be of value as against him. Our Lord often used this method to silence the cavils of the Jews; for example, Matthew xxii. 41-45.

The **reductio ad absurdum** proves a proposition indirectly by proving the absurdity of its contradictory. It has already been considered (p. 129).

III. Calculation of Probabilities.

The **theory of probabilities,** or of chances, as it is sometimes designated, has in recent times received increased attention. In an elementary work there is only space for the simplest rules and cases.

Thomson has described **chance** as "*the amount of belief* with which we expect one or other, out of two or more uncertain events." **Uncertain,**

or **merely probable, events** are "those wherein no cause or law appears
to determine the occurrence of one rather than another." Jevons pro-
poses "to dispense altogether with this obscure word 'belief,' and to
say that the theory of probability deals with *quantity of knowledge*."
An event is merely "probable when our knowledge of it is diluted
with ignorance, and exact calculation is needed to discriminate how
much we do and do not know."

At the basis of the rules for the calculation of probabilities are the common-
sense principles which underlie all reasoning. "We must treat equals equally,
and what we know of one case may be affirmed of every case resembling it in
the necessary circumstances. The theory consists in putting similar cases on
a par, and distributing equally among them whatever knowledge we possess.
Throw a penny into the air, and consider what we know with regard to its
way of falling. We know that it will certainly fall upon a side, so that either
head or tail will be uppermost; but as to whether it will be head or tail, our
knowledge is equally divided. Whatever we know concerning head, we know
also concerning tail, so that we have no reason for expecting one more than
the other. The least predominance of belief to either side would be irra-
tional; it would consist in treating unequally things of which our knowledge
is equal."

The Rules concern either simple or combined probabilities.

**Rule 1st. — A single probability of any uncertain event is ascer-
tained by dividing the number of chances favorable to the event by
the total number of chances favorable and unfavorable.**

Thus the probability that the head will fall uppermost, when a penny is
thrown into the air, is expressed by $\frac{1}{2}$. The probability that a man blind-
folded will draw a white ball out of an urn containing 2 white balls and 8
black ones is expressed by $\frac{2}{10}$ or $\frac{1}{5}$. To take a different case, if the letters of the
word **Roma** are thrown down casually in a row, what is the probability that
they will form a significant Latin word? The possible combinations of the
four letters are $4 \times 3 \times 2 \times 1 = 24$. If all the combinations are examined, 7 will
be found to have a meaning, namely, *Roma, ramo, oram, mora, maro, armo*, and
amor. Hence the probability sought is $\frac{7}{24}$.

Rule 2d. — The probability of the independent recurrence of an
event is found **by multiplying together the fractions expressing the
single probabilities.**

Thus the probability of throwing head twice with a penny is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$;
the probability of throwing it three times is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$. This Rule will
be seen to rest on Rule 1st, since the denominator represents the possible com-
binations in the case, or the whole number of ways of the happening of the
compound event, and the numerator the number of ways favorable.

Rule 3d. — "In order to calculate the probability that an event al-
ready observed will be repeated any given number of times, **divide
the number of times the event has been observed, increased by one,**

by the same number increased by one and the number of times the event is to reour."

"Thus, if the tide had been observed 9 times, the chance that it would recur 10 times more would be $\frac{9}{9+10}\frac{+1}{+1}=\frac{10}{20}=\frac{1}{2}$. This is the same thing as if each reproduction of the observed event corresponded to putting a white ball in an urn where there were already, before commencing the trials, a white ball and as many black balls as it is supposed that the event observed should re-occur times."

Two or more probabilities if mutually dependent weaken each other, while if independent they strengthen each other.

Rule 4th. — In case of mutually dependent probabilities, or probabilities of probabilities, the total probability is reached **by multiplying together the several single probabilities.**

Thus, if the credibility (p. 105) of a witness be $\frac{1}{2}$, and his competency (p. 104), or ability to know the facts of which he testifies, be $\frac{2}{3}$, the total probability of his telling the truth is $\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}=\frac{1}{3}$. As certainty is represented by unity, the testimony will, in this case, be twice as likely to be false as it is to be true.

Rule 5th. — In case of independent probabilities the total probability is reached **by subtracting each separate probability from unity** (which gives the probability of the opposite event, in each case, or the probability of a probability), **multiplying the separate results together** (according to Rule 4th), **and subtracting this product from unity** (thus arriving at the probability of the original compound event).

Thus, the total probability that the Gospels are true may be made up from the probability arising from the character of the authors, represented by $\frac{2}{3}$; from the absence of any motive on the part of the authors to fabricate such accounts, represented by $\frac{3}{4}$; from the influence of the Gospels themselves upon the world, represented by $\frac{4}{5}$. Subtracting each of these from unity and multiplying the results together, we have, as the probability of imposture, $\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}=\frac{1}{60}$. This subtracted from unity gives $\frac{59}{60}$ as the probability of the truth of the Gospels.

Note. — See Thomson's *Laws of Thought;* Jevon's *Principles of Science;* Newcomb's *Algebra.*

Praxis. — In the following syllogisms show whether the premises are true. Name the middle, minor, and major terms. Name the mood and figure of each, showing whether valid or not. Reduce any mood in the other Figures to Fig. I. Bring out the relation of reason and consequent involved in connection with the middle term in each case, substituting the letters, S, P, M, for the terms in the general formula, and giving the relation of the notions by the circular notation.

1. No human weakness can belong to God; some attributes imputed

to the Deity by mythology are human weaknesses; hence (at least) some attributes imputed to the Deity by mythology cannot belong to Him.

2. Some who act in accordance with law do not do what is right with right intention; ∴ some who act legally are not morally disposed.

3. Every real, natural poem is naïve; those poems of Ossian which Macpherson pretended to discover are not naïve (but sentimental); hence they are not real, natural poems.

4. The sum total of the worlds belonging to our solar system must completely determine the orbit of Uranus; the known worlds of our solar system do not fully account for the orbit of Uranus; hence the whole of the worlds of our solar system are not known.

5. Passive mental states make men neither noble nor base, worthy of praise or of blame; the virtues do this; ∴ the virtues are not passive mental states.

6. All squares are rectilineal plane figures; some parallelograms are squares; ∴ some parallelograms are rectilineal plane figures.

7. No form of knowledge, which corresponds to a peculiar form of existence, is of merely didactic value; syllogism is a form of knowledge which corresponds to a peculiar form of existence (viz., to the real conformability to law); hence the syllogism is not of mere didactic worth.

8. All cetaceous animals are water animals; all cetaceous animals are mammalia; hence some mammalia are water animals.

9. Some persons accused of witchcraft have not believed themselves to be free from the guilt laid to their charge; all those accused of witchcraft were accused of a merely feigned crime; hence some who were accused of a merely feigned crime have not believed themselves free from the guilt laid to their charge.

10. *Jubeo* is not a verb *sentiendi vel declarandi; jubeo* takes the construction of the accusative and infinitive; hence at least one or some Latin verbs which take the construction of accusative and infinitive are not verbs *sentiendi vel declarandi.*

11. All squares are regular figures; some parallelograms are squares; ∴ some parallelograms are regular figures.

12. Some parallelograms are squares; all squares are regular figures; ∴ some regular figures are parallelograms.

13. All squares are parallelograms; no parallelogram has converging opposite sides; ∴ no square has converging opposite sides.

14. Good non-conductors of heat retain heat longer; woollen clothes are good non-conductors; ∴ woollen clothes retain heat longer.

15 Some things which retain heat longer are woollen clothes; things

which retain heat longer are good non-conductors; ∴ woollen clothes are good non-conductors.

Supply the conclusions to each of the following pairs of premises, and show whether the conclusion is valid, or why no conclusion can be drawn. Treat the syllogisms as required in the preceding examples.

1. All good reasoners are candid; some infidels are not candid; ∴ . . . . . .

2. The ox, deer, sheep, goat, etc., are ruminant; the ox, the deer, etc., are as good as all horned animals; ∴ . . . . . .

3. Oaks are vegetables; oysters are not oaks; ∴ . . . . . .

4. No good action results in evil; some alms-giving results in evil; ∴ . . . . . .

5. Animals are bodies having organization and sensation; frogs have organization and sensation; ∴ . . . . . .

6. Some of our tax laws are oppressive measures; all oppressive measures should be repealed; ∴ . . . . . .

7. Reptiles bring forth their young by eggs; the rat does not bring forth its young by eggs; ∴ . . . . . .

8. The connection of soul and body is to be believed; the connection of soul and body is incomprehensible; ∴ . . . . . .

9. True poets are men of genius; very unwise men have proved true poets; ∴ . . . . . .

10. All good men are sincere; Rousseau was sincere; ∴ . . . . . .

11. Political Economy is a profitable study; profitable study sharpens the intellect; ∴ . . . . . .

12. No truth is worthless; many truths are misapplied; ∴ . . . . . .

13. Most people are careless; most people are destitute of perfect health; ∴ . . . . . .

14. 90 out of every 100 men are imprudent; 90 out of every 100 are unsuccessful; ∴ . . . . . .

15. Elephants are stronger than horses; horses are stronger than men; ∴ . . . . . .

Section II.— Unfolding of the Hypothetical Syllogism.

Hypothetical syllogisms have already been defined and divided (pp. 144–146). They will now be considered in the order of the division given.

Topic First.— The Conditional or Conjunctive Syllogism.

The conditional syllogism may either be tested as it is,

16 *

or reduced to categorical form and then tested by the principles of categorical reasoning.

I. The Tests of Conditional Syllogisms.

The tests of conditional syllogisms arise out of their nature as directly embodying the principle of Reason and Consequent. From this it follows that, if the reason be present in any given case we may be sure of the presence of the consequent; and if the consequent be absent we may be sure of the absence of its reason. Hence the two forms of conditionals, the constructive and destructive, and the Rules applicable to these forms of reasoning.

Rule 1st. — Affirming the antecedent or reason affirms the consequent (*modus ponens*); **while denying the consequent denies the antecedent** (*modus tollens*).

The first part of the Rule gives the **constructive conditional,** which affirms the reason or antecedent, and then on the ground of this affirms the consequent. The second part gives the **destructive conditional,** which denies the consequent, and on the ground of this denies the reason. The two forms may be illustrated as follows:

	Antecedent.	Consequent.	
Constructive.	If General Grant has a fever,	he is sick;	Major premise.
	He has a fever;	(*Modus ponens*).	Minor premise.
	∴ He is sick.		Conclusion.

	Antecedent.	Consequent.	
Destructive.	If General Grant has a fever,	he is sick;	Major premise.
	He is not sick;	(*Modus tollens.*)	Minor premise.
	∴ He has not a fever.		Conclusion.

The absence of the particular reason or antecedent mentioned in any given case does not render certain the absence of the consequent, since antecedents or reasons are manifold and the consequent may follow from other antecedents. So the presence of the consequent does not argue the presence of a particular antecedent or reason, since it may be the consequent or effect of some other antecedent. Hence

Rule 2d. — Denying the antecedent does not deny the consequent; and affirming the consequent does not affirm the antecedent.

Antecedent.	Consequent.	
If there is fire in the stove;	the room will be warm;	Major premise.
There is no fire in the stove;	(Deny Ant.)	Minor premise.
∴		No conclusion.
If there is fire in the stove;	the room will be warm;	Major premise.
The room is warm.	(Affirm Conseq.)	Minor premise.
∴		No conclusion.

In the case of the **denial of the antecedent** the conclusion that the room will not be warm does not follow, since it may be warmed by a grate, or a furnace, or steam apparatus, or a warm sun in summer, or the presence of a large audience, or by being on fire, etc. In the case of the affirmation of the consequent, the particular antecedent does not follow, since the same thing may result from any one of the antecedents enumerated.

The whole may be illustrated by diagram:
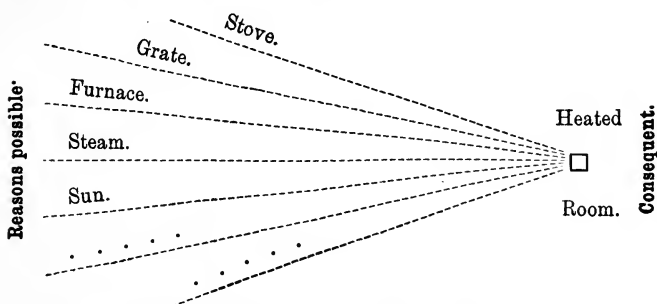
The **dotted lines** may represent the possible lines of reason or causation; the heated room, the consequent or effect. If the *stove* is present, then *the heated room* will be present, because that is a sufficient reason. If *the stove* is not present, the room *may still be heated*, since the *grate, furnace*, etc., may furnish the sufficient reason. If there is not *the heated room*, then all the antecedents must be absent,—*stove*, etc. If there be *the heated room*, no definite *a priori* conclusion concerning the agency of the *stove* is possible, since the consequent may result from any other of the antecedents.

II. Reduction of the Conjunctive Syllogism.

The conjunctive or conditional syllogism may readily be reduced to the categorical form, as already shown (p. 117),

and then tested by the Rules which apply to the various Figures.

Applying the principles of reduction to the syllogism just given, it becomes :

1st. The case of the presence of the heated stove is the case of a warm room;
 The present is the case of a heated stove;
 ∴ The present is the case of a warm room.

Or A.:   Every room in which a stove is heated is warm;
 A.   This is a room in which a stove is heated;
 A.   ∴ This room is a warm room.

2d. A.   Every room in which the stove is heated is warm;
 E.   This room is not warm;
 E.   ∴ The stove is not heated in this room.

3d. A.   Every room in which the stove is heated is warm;
 E.   This is a room in which the stove is not heated;
 E.   ∴ This room is not warm.   (Not valid.)

This form corresponding to the denial of the antecedent, under Rule 2d, involves illicit process of the major term.

4th. A.   Every room in which the stove is heated is a warm room;
 A.   This room is a warm room;
 A.   ∴ This room is one in which the stove is heated.   (Not valid.)

This form corresponding to the affirming of the consequent, under Rule 2d, involves undistributed middle, or substantially four terms.

Topic Second.—The Disjunctive Syllogism.

The tests of the disjunctive syllogism arise out of the nature of the disjunctive judgment, as embodying the principle of Excluded Middle, in connection with Reason and Consequent the principle of all reasoning.

A perfect disjunctive judgment embodies a **complete division of some genus** or class, and the alternatives presented are the species under that class, and are reciprocally exclusive (p. 71).

The major premise presents these species as alternatives.

The minor premise makes a categorical predication concerning one or other of the species or alternatives.

The conclusion draws an inference concerning the other species.

Rule 1st.—See that the disjunction exhausts the division, and that the disjunctives are reciprocally exclusive.

Rule 2d.—Affirming a part of the disjunctives, wholly or disjunctively, in the minor premise, denies all the others in the conclusion.

Rule 3d.—Denying a part of the disjunctives, in the minor premise, affirms the rest, in the conclusion, wholly or disjunctively, according as one or more may remain.

These may be illustrated by the following examples:

The Apostles must have been fanatics, or impostors, or true men;
They were neither fanatics nor impostors;
∴ They were true men.
The season of the battle of Lexington must have been spring, or summer, or autumn, or winter;
It was neither summer nor winter;
∴ It was either autumn or spring.

In the **first example** the character of the Apostles is analyzed into three possible exclusive phases, and we affirm that if they do not belong to one or other of the first two they must belong to the third. In the **second example** the seasons are analyzed into the four, and we affirm that since it was neither the second nor fourth it was one of the other two.

Topic Third.—The Dilemmatic Syllogism.

The dilemmatic, or conjunctivo-disjunctive, syllogism is subject to the Rules of conditionals and disjunctives. The most common forms are the following:

I. One Antecedent in the Major with Disjunctive Consequent.

This takes the form: If A is B, either C is D or E is F. By the rules of conditionals and disjunctives we have the **following** possible **cases** and results:

Affirm Antecedent.   A is B; ∴ either C is D or E is F.
Deny Cons. wholly.   Neither C is D nor E F; ∴ A is not B.
Deny Cons. disjunctively.   Either C is not D or E is not F.   No conclusion.
Denial of Antecedent.   A is not B.   No conclusion.
Affirmation of Consequent.   C is D or E is F   No conclusion.

II. Plurality of Antecedents in the Major with Common Consequent.

This takes the form: If A is B, X is Y, and if C is D, X is Y. This gives the **following cases** and results:

Affirm Antec. wholly.   A is B and C is D; ∴ X is Y.
Affirm Antec. disjunct.   A is B or C is D; ∴ X is Y.
Deny Consequent.   X is not Y; ∴ neither A is B nor C is D.
Deny Antecedents.   A is not B nor is C D.   No conclusion.
Affirm Consequent.   X is Y.   No conclusion.

III. Plurality of Antecedents in Major, each with its own Consequent.

This takes the form: If A is B, C is D, and if E is F, G is H. This gives the **following cases** and results:

Affirm Ant. wholly.   A is B and E is F; ∴ C is D and G is H.
Affirm Ant. disjunct.   A is B or E is F; ∴ C is D or G is H.
Deny Cons. wholly.   C is not D and C is not H; ∴ A is not B and E is not F.
Deny Cons. disjunct.   C is not D and G is not H; ∴ A is not B or E is not F.
Deny Antecedent.   A is not B and E is not F.   No conclusion.
Affirm Consequent.   C is D and G is H.   No conclusion.

Note.—All the forms enumerated are called dilemmatic syllogisms, but, as already stated (p. 146), the **dilemma**, in the strict sense, is only that form which has **a plurality of antecedents in the major, and a disjunctive minor.** This dilemma is sometimes rebutted by another with an opposite conclusion. Aristotle illustrates the process of rebuttal thus: "An Athenian mother said to her son, 'Do not engage in public affairs; for if you do what is just men will hate you, and if you do what is unjust the gods will hate you.' This the son rebutted by the following retort: 'I ought to enter into public affairs; for if I do what is unjust men will love me, and if I do what is just the gods will love me.'"

Praxis.—In the following examples, complete the syllogisms if incomplete. Name the kind in each case and formulate with letters and illustrate by diagram. Test each example by the Rules.

1. If men are virtuous they are wise, and if they are vicious they are unwise;

But they are either virtuous or vicious;

∴ They are either wise or unwise.

2. If the classics teach how to produce wealth they ought to be studied;

They do not so teach;

∴ They ought not to be studied.

3. Mahomet was either an enthusiast or an impostor;

He was an enthusiast;

∴ He was not an impostor.

4. If there be no future life, then either virtue receives its due reward in the present world, or there is no perfect government administered over men; neither of which is admissible.

5. The fact that I defended him is a proof that I hold him innocent.

6. If pain is severe, it will be brief; and if it last long, it will be slight; hence it should be borne patiently.

7. If this man were wise, he would not speak irreverently of Scripture in jest, and if he were good, he would not do so in earnest;

But he does it either in jest or in earnest;

∴ . . . .

Section III.—Conspectus of Fallacies.

A **fallacy** is any unsound or delusive mode of reasoning. **The principal fallacies in induction and deduction** need to be particularized and distinguished.

In order to acquire a complete command of the principles of reasoning and to guard against error, the thinker must make himself familiar with the most common kinds of fallacy. In the previous Sections, as Jevons has said, " we have considered, as it were, how to find the right road; it is our task here to ascertain the turnings at which we are most liable to take the wrong road."

Note.—With respect to the knowledge or intention of the reasoner, fallacies have been divided into paralogisms and sophisms. A **paralogism** is a fallacy which is unknown to the reasoner himself; a **sophism** is a false argument, understood to be so by the reasoner himself and intentionally used to deceive. This is not, however, a logical distinction, since it is not based upon the thought, but upon the mental and moral condition of the reasoner, and is, therefore, of no logical value.

Topic First.—Fallacies in Induction.

In induction we deal with matters of fact. The requirements of induction are summed up in two things:

1st. Exact Observation of the facts.
2d. Correct Interpretation of the facts.

All fallacies in induction arise from failure to conform to these requirements.

I. Fallacies from Failures in Exact Observation.

1. Neglect of observation, or ignoring of all facts (pp. 26–34, 148, 155).

2. Partial observation, giving incomplete view of the facts (pp. 33, 148, 155).

3. Neglect of exceptional, and especially contradictory, facts (p. 155).

4. Assuming what is not fact to be fact (pp. 33, 148).

5. Mixing illegitimate inferences with the facts (p. 33).

II. Fallacies from Failures in Correct Interpretation.

1. Neglect of all cause, or confounding induction with mere generalization (pp. 147, 153), including groundless universal conclusion from few unimportant facts (*fictæ universalitatis*) (p. 159).

2. Partial explanation of the facts, by assuming an improper or insufficient cause, including:

(1.) Assuming inappropriate cause (p. 154).

(2.) Assuming inadequate cause (p. 154).

(3.) Assuming a single cause where there is a complex of causes (p. 154).

3. Neglect of real cause for hypothetical cause (p. 154).

4. Fallacy of unreal reason, or assuming what is not a cause to be a cause (*non causa pro causa*) (p. 160), including:

(1.) Confounding antecedent and cause (*post hoc ergo propter hoc*) (p. 149).

(2.) Confounding concomitant, condition or occasion and cause (*cum hoc ergo propter hoc*) (p. 149).

(3.) Confounding law and cause (p. 149).

Note.—The most noted forms of the fallacy of **unreal reason** are the **lazy reason** (*ignava ratio*), **the reaper** (*ratio metens*), and the **controlling reason** (*ratio dominans*). These are all of the same character, and may be illustrated by an example of the first, which gave it its name:

Sumption.—" If I ought to exert myself to effect a certain event, this event either must take place or it must not;

Sub-sumption.—" If it must take place, my exertion is superfluous; if it must not take place, my exertion is of no avail;

Conclusion.—" Therefore, on either alternative, my exertion is useless."

In regard to the vice of this sophism, Krug, as quoted by Hamilton, says: " It is manifest that it lies in the sumption, in which the disjunct members are imperfectly enounced. It ought to have been thus conceived: If I ought to exert myself to effect a certain event, which I cannot, however, of myself effect, this event must either take place from other causes, or it must not take place at all. It is only under such a condition that my exertion can, on either alternative, be useless, and not if the event depend wholly or in part for its accomplishment on my exertion itself, as the *conditio sine qua non.*"

This shows that this so-called syllogism formally violates Rule 1st under disjunctives (p. 188), as applied to the dilemma.

5. Assuming unverified hypotheses as truth (pp. 156, 160).

Topic Second.—Fallacies in Deduction or Syllogism.

Deductive reasoning deals with truths or general principles. Its requirements are, therefore, summed up in two things:

1st. Correct Matter or Thought, or the grasping of true premises.

2d. Correct Form in Reasoning, or the proper unfolding of what is contained in the premises.

All fallacies in deduction result, therefore, from failure to comply with one or both these requirements. Those which result from some failure in the matter or thought are known as **Material Fallacies**; those resulting from some failure in the form of reasoning are known

as **Logical** or **Formal Fallacies**; those resulting from failure in both matter and form are known as **Semi-Logical Fallacies.**

I. Material Fallacies.

Material fallacies are those which arise outside of the mere form of thought, or verbal statement (*extra dictionem*), in the subject-matter or thought itself. They may take the form of unwarranted assumption of premises, or of irrelevant conclusion from the proper premises.

1. Unwarranted Assumption of Premises.

(1.) **Begging the question** (*petitio principii*), or virtual assumption of the thing to be proved or of that by which it is to be proved. This includes:

a. *Petitio principii* proper, where the assumption is openly made without show of proof.

b. Arguing in a circle, where the conclusion is virtually used to prove the premise.

E. g., John Knox and John Witherspoon are excellent men because they belonged to an excellent church, the Presbyterian Church; and the Presbyterian Church is an excellent one because it has contained such good men.

c. Assuming a resemblance without proving it, or where there is no such resemblance (*non tale pro tali*).

E. g., "All other religions are delusions; therefore, Christianity is a delusion."

(2.) Failure in Estimating Probabilities.

a. Over-estimation of dependent probabilities (p. 182).

b. Under-estimation of independent probabilities (pp. 108, 183).

2. Irrelevant Conclusion from Proper Premises.

(1.) **Fallacy from arguing to the wrong point.** This is also called *ignoratio elenchi*, or "ignoring the refutation," which refutation involves the establishment of the contradictory (p. 129). This includes:

a. Perverted argument from common consent (*argumentum ad judicium*) (p. 181).

b. *Argumentum ad populum* (p. 181).

c. *Argumentum ad verecundiam* (p. 181).

d. *Argumentum ad ignorantiam* (p. 181).

e. *Argumentum ad hominem* (p. 181).

(2.) Fallacy from simple *Confusion of Thought.* This includes:

a. Fallacy of accident (*fallacia accidentis*) and *the converse* (p. 167). This includes:

(a.) Arguing from a general rule to a special case, where some accidental circumstance renders the rule inapplicable.

(b.) Arguing from a special case to a general one. This is described by the

17   N

Latin phrase, " *a dicto secundum quid ad dictum simpliciter*," meaning " from a statement under a condition to a statement *simply* or without that condition."

(c.) Arguing from one special case to another special case.

b. Fallacy of the consequent, or *non sequitur*, where the reasoning is so loose and inconsequent that no one can discover any force in it.

c. Fallacy of many questions (*plures interrogationum*), which results from so combining two or more questions that no true answer can be given to them.

II. Logical or Formal Fallacies.

Logical fallacies are those which occur in the mere form of the statement (*in dictione*). They may ordinarily be discovered by the aid of the rules of deduction or the syllogism, without any knowledge of the subject-matter of the argument. They are violations of the Rules of Reasoning categorical and hypothetical.

1. Fallacies in Categorical Reasoning.

(1.) Violation of the Rules for Terms.

a. Four Terms (*quaternio terminorum*). Breach of Rule 1st (p. 166).

b. Undistributed Middle. Breach of Rule 2d (p. 167).

c. Illicit Process of Major or Minor. Breach of Rule 3d (p. 167).

(2.) Violation of the Rules for Premises.

a. Failure of conclusion to follow weaker part. Breach of Rule 4th (p. 168).

b. Conclusion from two negative premises. Breach of Rule 5th (p. 168).

c. Conclusion from particular premises. Breach of Rule 6th (p. 168).

d. Conclusion from particular major with negative minor. Breach of Rule 7th (p. 170).

2. Fallacies in Hypothetical Reasoning.

(1.) Violation of Rules for Conditionals.

Conclusion from denying antecedent or from affirming consequent. Breach of Rule 2d (p. 186).

(2.) Violation of Rules for Disjunctives.

a. Confounding partitive and disjunctive judgments (p. 118).

b. Disjunctive elements not exclusive and inclusive. Breach of Rule 1st (p. 188).

c. Conclusion not in accordance with the affirmation or denial of disjunction. Breach of Rules 2d and 3d (pp. 188, 189).

III. Semi-Logical Fallacies.

Semi-logical fallacies are fallacies partly material and partly formal.

These fallacies arise largely from the ambiguous use of terms. In such cases the term used in two senses is substantially equivalent to

two terms. The ambiguity must first be detected by examining into the meaning of the terms. The fallacy is so far material. When the ambiguity is fairly detected, the fallacy is at once transformed into the formal or logical fallacy of four terms. It includes:

1. Fallacy of Equivocation, consisting in the use of the same word in two distinct senses.

(1.) **Fallacy of ambiguous middle** (p. 82).

(2.) **Fallacy of homonymous terms** (p. 83).

2. Fallacy of Amphibology, consisting in ambiguous grammatical structure of a sentence.

E. g., "The Duke yet lives that Henry shall depose."

3. Fallacy of Composition and Division, arising from using a term distributively (pp. 112, 115) in one premise, and collectively (pp. 54, 113) in the other.

This is especially common in the use of "all" (p. 113), "not all" (p. 111), etc.

4. Fallacy of Etymology. This includes:

(1.) Fixing upon a wrong root (p. 76.)

(2.) Assuming that the original meaning of the root of a word decides the present meaning of the word (p. 76).

Note.—For enumerations of the sources of human error, see Bacon's *Novum Organum*, Lib. i.; Mill's *Logic*, Book V., ch. ii.; Hamilton's *Logic*, Lect. xxiii.

Praxis.—Examine the following arguments, completing them if incomplete, and reducing to regular form if irregular. Examine and define the important conceptions or terms. Name the kind of argument in each case, formulating with letters and illustrating by diagram. Present the proof of the premises. Test each example by the Rules, naming and explaining the fallacy, material, logical or semi-logical, wherever such fallacy exists. If categorical, reduce to Fig. 1.

1. A science which furnishes the mind with a multitude of useful facts deserves cultivation; but Logic is not such a science; ∴ Logic does not deserve cultivation.

2. Nuisances are punishable by law; to keep a noisy dog is a nuisance; ∴ to keep a noisy dog is punishable by law.

3. Twice two and three are seven; twice two and three are ten; ∴ seven is equal to ten.

4. If motion is possible, a body must move either in the place where it is, or in a place where it is not; but a body cannot move in a place

where it is, and of course it cannot move where it is not; ∴ motion is impossible.

5. What you bought yesterday you eat to-day; you bought raw meat yesterday; ∴ you eat raw meat to-day.

6. The Jews are avaricious; ∴ the prophet Daniel was avaricious.

7. All bodies that move themselves are animated; the stars move themselves; ∴ the stars are animated.

8. Mouse is a syllable; but a mouse eats cheese; ∴ a syllable eats cheese.

9. If it be fated that you recover from your present disease, whether you call in a doctor or not you will recover; again, if it be fated that you do not recover from your present disease, whether you call in a doctor or not you will not recover; But one or other of the contradictories is fatal; ∴ To call in a doctor is of no consequence.

10. Episcopacy is of Scripture origin; the Church of England is the only episcopal church in England; ∴ the Church established is the Church that should be supported.

11. Carbon is combustible; diamonds are composed of carbon; ∴ diamonds are combustible.

12. Rain has fallen, if the ground is wet; but the ground is not wet; ∴ rain has not fallen.

13. None but mortals are men; monarchs are men; ∴ monarchs are mortals.

14. Logic as it was cultivated by the Schoolmen proved a fruitless study; ∴ Logic as it is cultivated at the present day must be a fruitless study.

15. Men can live without animal food, and they can live without vegetable food, as has been often demonstrated; but all food is either animal or vegetable; ∴ men can live without food.

16. All birds are animals; no reptiles are birds; ∴ no reptiles are animals.

17. He who is most hungry eats most; he who eats least is most hungry; ∴ he who eats least eats most.

18. If rain has fallen, the ground is wet; but rain has not fallen; ∴ the ground is not wet.

19. Night must be the cause of day, for it invariably precedes it.

20. If Brandreth's pills are of any value, those who take them will improve in health; my friend who has been taking them has improved in health; ∴ they are of value.

21. He that can swim needs not despair to fly; for to swim is to fly in a grosser fluid, and to fly is to swim in a subtler.

22. The ground is wet, if rain has fallen; the ground is wet; ∴ rain has fallen.

23. All stars are self-luminous; all planets are not self-luminous; ∴ no planets are stars.

24. Some flowers are tulips; all flowers are beautiful; ∴ all the tulips are beautiful.

25. The probability of the existence of a God, derived from the existence of the universe, may be stated as $\frac{2}{3}$; from order in the universe, $\frac{4}{5}$; from conscience, $\frac{4}{5}$; from common belief of mankind, $\frac{2}{3}$, etc. These all fall far below unity or full proof; ∴ the proofs of the existence of a God are insufficient to warrant belief.

26. If the books in the Alexandrine Library be in conformity with the doctrines of the Koran, there is no need of them; if adverse, then also they should be burned.

27. If the ground is wet, rain has fallen; but rain has fallen; ∴ the ground is wet.

28. The hope of immortality is either a rational expectation or an illusion; but that belief cannot be an illusion which all the most enlightened peoples have adopted.

29. Personal deformity is an affliction of nature; disgrace is not an affliction of nature; ∴ personal deformity is not disgrace.

30. No idle person can be a successful writer of history; ∴ Hume, Macaulay, Hallam, and Grote must have been industrious.

31. Bacon was a great lawyer and statesman; and as he was also a philosopher, we may infer that any philosopher may be a great lawyer and statesman.

32. Nothing is better than wisdom; dry bread is better than nothing; ∴ *a fortiori* is dry bread better than wisdom.

33. If classical education is worth the cost, either it must be pre-eminently fitted to develop the mental powers, or it must furnish exceedingly valuable information; but neither alternative can be maintained, and so classical education is not worth the cost.

34. Men love to be humbugged; the President of the Bible Society is a man; ∴ he loves to be humbugged.

35. All power proceeds from will as its antecedent; a steam-engine has no will; ∴ it has no power.

36. What produces intoxication should be prohibited; the use of spirituous liquors produces intoxication; ∴ the use of spirituous liquors should be prohibited.

37. All the trees in the park make a thick shade; this tree is one of them; ∴ this tree makes a thick shade.

17 *

38. The object of war is durable peace; ∴ soldiers are the best peacemakers.

39. Improbable events happen almost every day; but what happens almost every day is a very probable event; ∴ improbable events are very probable events.

————◦◦⟐◦◦————

SUMMARY OF RESULTS.

The aim of the Logic of Reasoning is, in general, to train to the best thinking and fullest appreciation of thought in its third form. **The perfection of thinking as reasoning depends upon the degree of certainty that the right cause or middle term has been fixed upon.** As the finished result of Conception is clear and distinct thinking, and that of Judgment connected thinking, so that of Reasoning is **continuous thinking.**

The conclusions from induction are **probable truths** (judgments, p. 132), varying in probability all the way from mere hypotheses to perfected theories. The conclusions from deduction are always **certain truths** (judgments, p. 132) when the premises are certain and the reasonings correct, and **probable truths** when the premises are only probable.

The special aim of the Practical Logic of Reasoning should be to train the thinker to the highest degree of skill and certainty in using the various processes of induction and deduction in his own thinking, and to the greatest readiness and accuracy in grasping and testing these processes and their products as they are presented in the thinking of others.
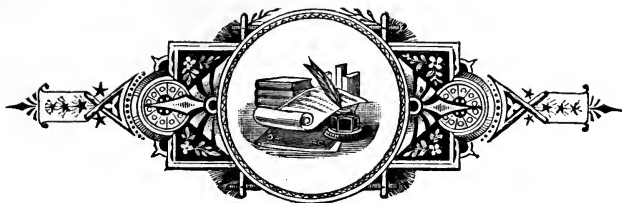
Part IV.

THE LOGIC OF CONSTRUCTION OR THE SYSTEM.

THE aim of the Logic of Construction should be to train the student to skill in dealing with the Fourth Form of Thought.

Definition. — Construction is that higher form of thought in which we combine mutually related products of the lower forms of thought, according to some rational principle, into one relatively complete whole (pp. 11–13). The product of construction is known as the **System.**

Ueberweg defines **system** as "the orderly combination of mutually-related knowledge into one relatively complete whole." System is either mechanical or rational. **Rational system** is that in which the combination is a result of the application of some rational principle; **mechanical system,** that in which such rational principle is wanting.

The alphabet, as arranged in the order, a, b, c, etc., is a mechanical system; as arranged in classes, — as vowels, semi-vowels, and consonants; or tonics, subtonics, and atonics, — it is a rational system. There are **three forms of rational system**: scientific system; artistic system; practical system. These all imply orderly arrangement, but they differ in the law by which that arrangement is effected; that of scientific system being according to the law of the true; that of artistic system according to the law of the beautiful; that of practical system according to the law of the good.

In **scientific system** the aim is to combine the related thoughts in

such a way that the totality will exactly express the truth and the whole truth. It is, therefore, said to be governed by the Law of the True. In **artistic** or **æsthetic system** the aim is to combine the related truths in such a way as to produce a totality which will express diversity in unity, or beauty. It is, therefore, said to be governed by the Law of the Beautiful. In **practical system** the aim is to combine forces and agencies as means so as to secure a whole by which some desired end or good may be secured. It is, therefore, said to be governed by the Law of the Good.

Artistic or æsthetic system belongs to Æsthetics; the other forms may be regarded as properly belonging to Logic (p. 12), and will be briefly considered.

Constructive thinking is manifestly the highest act of the human intellect, and should, therefore, be made prominent in the later stages of higher education. The **formation** and **unfolding** of systems will be briefly treated in successive Chapters.

———oo°o°oo———

CHAPTER I.

THE FORMATION OF CONSTRUCTION OR SYSTEM.

An understanding of the combination of mutually related thoughts into systems, by the constructive faculty, must prepare the way for unfolding and testing such systems. The process and the products will be considered briefly under each of the forms of construction.

Section I.—Scientific Construction.

Scientific construction is construction according to the law of the true. Its product is **Scientific System.**

Topic First. — Process of Forming and Verifying Scientific System.

Three things are essential in thinking in the form of scientific system: **First,** fixing upon some one sphere of

mutually related thoughts, or thoughts constituting a whole; **secondly,** maintaining logical consistency in the joining of all the parts under this whole; **thirdly,** verifying the agreement of the resulting combination, in its parts and as a whole, with the entire reality of the sphere which is being systematized.

These give the Laws of Scientific Construction; the Laws of Logical Unity or Logical Totality; of Logical Consistency; and of Logical Truthfulness.

I. The Law of Logical Unity or Totality.

The unity and totality of a system are determined by this, that all the individual thoughts contained in it depend on a common principle. A **principle,** in this sense, has been defined as " an absolutely or relatively original element on which a series of other elements depends." It is the unifying thought which binds together the otherwise disconnected and unorganized mass of thoughts. Hence arises

Rule 1st.—Seek a principle which will bring the thoughts to be systematized into unity under one sphere or whole.

The Law of Totality may also be presented as the *Law of Numerical Completeness*, which requires that a scientific view of any region of fact or truth shall present all the essential facts and truths, none added and none omitted. Any so-called science, *e.g.*, astronomy, may be rendered so far false by an addition to the facts or truths or subtraction from them.

Various principles, or points of view, may be made use of in systematizing any region of truth. The sphere may thus be enlarged or diminished. For example, the astronomer may aim to present the astronomy of the solar system or of the universe; he may give his work a mathematical or a descriptive form; he may present the solar system and universe as they are, or treat them historically, giving the stages in their development.

II. The Law of Logical Consistency or Correlation.

The logical consistency of a system requires the proper joining or correlation of all the parts under the whole or

totality. All the truths combined should be in their proper relation to each other.

The main relations to be kept in mind in such work are those of substance and properties, as brought out under Observation (p. 29); those of content of concepts (p. 40) and extent of concepts (p. 45); those of reason and consequent, as involved in induction and deduction (pp. 138–9). There should be perfect accuracy of thought in all the parts and relations of the system. Hence arises

Rule 2d. — See that all the parts are properly joined or articulated under the one whole.

Any science, *e. g.*, zoology, may be rendered false by any departure from the facts or laws of succession ; or from the relations of co-ordination, subordination, etc., brought out by logical and scientific division ; or from the relations of reason and consequent, as involved in induction and deduction.

III. The Law of Logical Truthfulness.

The logical truthfulness of a system requires that the entire system so constructed shall, in all its parts and as a whole, be in accordance with the reality, or the entire sphere or whole which is being systematized. This conformity with the reality is the crucial test of a system. From it arises

Rule 3d. — Test the system of thought constructed by the reality which it represents.

In any scientific system any want of conformity to the sphere of reality renders the system so far false. **Imaginary schemes,** such as the scheme of organized being as unfolded by Haeckel in his *History of Natural Evolution*, have no claim to the name of scientific system.

Topic Second. — Products of Scientific Construction.

Systems, as the products of scientific construction, are either absolutely or relatively complete.

Scientific System has sometimes been confounded with *systems of classes* (p. 44), but it is manifest that these merely form one of the elements used in constructing scientific systems. **Science** is used in various senses : " exact knowledge ;" " classified knowledge ;" etc. Ueberweg defines it as a " whole of knowledge in the form of the sys-

tem," in which sense it is substantially equivalent to *scientific system.* According to this view, "scientific knowledge finds its perfection in the combination of thoughts, one with the other, into a whole, which in its content and form represents the objective reality." "Science as such has its true existence only in the systematic form."

I. Relatively Complete System.

The Sciences, as we find them, usually deal with some relative whole and not with the entire universe of truth. They are inductive, deductive, or mixed, according to the method of thought employed.

1. The inductive sciences result from the employment of the inductive method of thought.

The **Inductive Method** involves three elements:

First, The scientific investigator starts with matters of fact.

Secondly, In reaching the materials for the science, he makes use of the principles of inductive reasoning chiefly.

Thirdly, These materials are given their proper systematic form by the principles of scientific construction (p. 201). Its **steps** are, as has been seen (pp. 148–9, 200–3): exact observation, correct interpretation, and scientific construction. The product is a system of thought wrought out from the facts.

2. The deductive sciences result from the employment of the deductive method of thought.

The **Deductive Method** involves three elements:

First, The scientific investigator starts with ideas or relations of ideas.

Secondly, In gathering the materials for the science he makes use of the principles of deduction chiefly.

Thirdly, These materials are given their proper systematic form by the same general principles of scientific construction made use of in the inductive method.

Its **steps** are, therefore: proper grasp of truth, or right judgments or general principles; correct unfolding of truth; and scientific construction of the results.

In it *induction* may be used in subordination whenever matters of fact are incidentally taken into account.

The product is a system of thought unfolded from fundamental thoughts or truths.

3. The mixed sciences arise from the joint employment of the inductive and deductive methods. This results from the presence of both facts and truths, both of which need to be wrought into the system. Astronomy furnishes an illustration of mixed scientific method.

II. Absolutely Complete System.

The absolutely complete science deals with all things, or the universe. It aims to construct the universal scientific system and the universal philosophy, both of which are included under complete scientific system in the wide sense.

Herbert Spencer distinguishes between knowledge, science, and philosophy as follows: " **Knowledge** of the lowest kind is *ununified* knowledge; **science** is *partially unified* knowledge; **philosophy** is *completely unified* knowledge." The **distinction usually made** between science and philosophy is as follows: **science** deals with facts and their order, or with the "*what;*" **philosophy** deals with general principles and reasons, or the "*why.*" It is impossible, however, to have any science so completely empirical as not to involve more or less of the principles or reasons of things, and equally impossible to have any philosophy so entirely transcendental as not to embrace a solid basis of fact or reality. In the highest sense *science,* as *scientific system,* embraces both facts and their reasons, both the "what" and "the why," or both **science proper** and **philosophy.**

1. The Complete Science.—Great thinkers have sought to construct the one universal scientific system, and with various success. The system of Comte may be presented as one of the best.

Comte starts with the suggestion of Descartes, that "*sound knowledge should advance from the simpler to the more complex phenomena.*" In this suggestion " lay the germ of a sound arrangement of the sciences, which scarcely, however, seems to have begun to bear fruit before the time of Ampère and Comte." Thomson, in his *Laws of Thought,* pp. 316–319, has presented the system drawn from Comte in brief form.

" **Mathematics,** or the science of quantity, is at once the most simple in its elements and the most general in its application, entering more or less into all the sciences of nature, and constituting almost the whole of that which comes next it in the order of dependence. **Astronomy,** or the science of the heavenly bodies, is the application of mathematical truths to the laws of matter and motion, matter and the motions of material bodies being the new concep-

tion which belongs to this science. **Physics,** being the science, or rather group of sciences, which is conversant with the general laws of the world so far as they relate to beings without life or organization, would come next; and it imports, in addition to the conceptions of Astronomy, those of light, of heat, of sound, of electricity, of magnetism, and many others. **Chemistry** would rank next, which is the science of the decomposition and combinations of the various substances that compose and surround the earth. Next in order of complexity would rank **Physiology,** founded on the additional conception of vegetable and animal life. To this would succeed **Anthropology,** or the science of man's nature; and to this **Social Science,** which ascertains the laws that govern men when combined in cities and nations.

Each of these departments may be divided into many branches, as Physics into Acoustics, Optics, Electricity, and the like; or Social Science into Morals, Politics, Political Economy, Law, and the like. . . . . There is a general correspondence between this classification and the order in which the various objects of science came into being. The heavenly bodies were first appointed their paths in the celestial spaces; then the surface of our earth was prepared for living creatures; then they were created after their kind, and man the last. The social life of man grew up last of all, when his race was multiplied on the globe; and ever as new elements appear, the conditions of society are being modified even to the present time."

We are now in a position to sketch the table of the Sciences.

" CLASSIFICATION OF THE SCIENCES.

Group.	Mode of Treatment.		
	Theoretical.	Historical.	Applied.
I. MATHEMATICS	Theoretical.	"	"
II. ASTRONOMY	"	"	"
III. PHYSICS	"	"	"
IV. CHEMISTRY	"	"	"
V. PHYSIOLOGY	"	"	"
VI. ANTHROPOLOGY	"	"	"
VII. SOCIAL SCIENCE	"	"	"

RELIGIOUS PHILOSOPHY."

2. The Complete Philosophy.— Thinkers have also aimed, in dealing with the question " Why?" to construct the universal philosophic system, and with equally various success. The common-sense philosophy may be accepted as the best.

The philosopher must seek to give a rational explanation of the ultimate facts to which all scientific investigation of phenomena leads. These ultimate facts are three: consciousness; the cosmos of matter and spirit; the being back of all on which all depends. A complete

philosophy must, therefore, have its psychological theory, its cosmological theory, and its ontological theory. The three are embraced in

THE COMMON-SENSE PHILOSOPHY.

I. PSYCHOLOGICAL THEORY....**Consciousness** is made up of two elements of knowledge: experience and intuition.

II. COSMOLOGICAL THEORY......**The Cosmos** is made up of two elements: spirit and matter.

III. ONTOLOGICAL THEORY.......**The Ultimate Being,** or **First Cause** of the Cosmos, is the infinite, personal Spirit, God.

Section II.—Practical Construction.

Practical construction is construction according to the law of the good. Its product is **Practical System.**

Topic First.—Process of Forming and Verifying Practical System.

Three things are essential to thinking in the form of practical construction: **First,** the intelligent fixing upon some one complex plan or aim; **secondly,** the careful preparation or gathering of ideas and forces which will serve as means to this end; **thirdly,** the best arrangement and adjustment of the means to secure the end in view. These give the laws of practical aim, practical adaptation, and practical unity.

I. The Law of Practical Aim.

Practical aim in constructive thinking requires that the view be fixed upon some beneficent, useful, rational, or moral end to be attained. Hence arises

Rule 1st.— Fix upon and define clearly in the mind the end to be attained.

II. The Law of Practical Adaptation.

Practical adaptation in constructive thinking requires that all the material made use of be such, and only such, as is suited to secure the proposed end. Hence arises

Rule 2d.— See that the suitable means are provided for attaining the proposed end.

III. The Law of Practical Unity.

Practical unity in constructive thinking requires that all the means be combined, arranged, and adjusted in such system as best to secure the end proposed. Hence arises

Rule 3d.—See that the means are properly correlated so as to secure the proposed end.

Topic Second.— Products of Practical Construction.

The Laws of Practical Construction govern in the production of all inventions, ideals, plans of life, etc. Success in life depends largely upon the possession of this power in proper development.

One of the highest forms of practical construction is found in *oratory*, in which the aim is to arrange thought in such a system as shall induce a change of view, of judgment, of feeling, or of purpose in an audience.

Illustrations will suggest themselves to the teacher and student. For the purpose of directing in the work, a few examples will suffice.

Praxis. —Study as practical systems: 1. A steam-engine. 2. A telephone. 3. A plough. 4. The speech of Daniel Webster, in the trial of John Francis Knapp, for the murder of Joseph White. 5. The oration of Demosthenes on the Crown.

———

CHAPTER II.

THE UNFOLDING AND TESTING OF SYSTEMS.

The best use of the power of construction in the work of thinking requires that the thinker should be able to grasp and unfold what may be contained in any system, and to test such system by the principles of construction, scientific and practical.

For the purposes of the brief discussion here proposed, the two forms of logical system need not be separated. Two things are of prime importance: **first,** the ascertaining of the elements of systems, and **secondly,** the testing of systems.

Section I.—Ascertaining the Elements.

The elements of any system may be learned from the Laws of Construction. In unfolding scientific constructions (to which attention will be confined) three things are embraced : **First,** the grasping of the totality involved in the system ; **secondly,** the study of the relations of the parts or the articulation of the system ; **thirdly,** the comparison of the system with the objective reality. The careful study of these elements is requisite to prepare for the testing of systems.

Topic First.—The Whole and its Principle.

In studying any system it is necessary first to seize upon it as a whole by ascertaining the principle which unites its elements.

A system is " an organized body of truth, or truths arranged under one and the same idea, which idea is as the life or soul which assimilates all those truths." In studying and unfolding any system, it is, therefore, necessary to inquire first for this organic idea or principle, which is the soul of the system. This holds in all three forms of system, scientific, æsthetic, and practical.

Trendelenburg distinguishes **" systems of arrangement,"** corresponding to systems of classes (p. 47); and **" systems of development,"** corresponding to the products of scientific construction (p. 202). The former arise under Conception, by Classification or Division; the latter, under Reasoning, by Induction and Deduction. The former take the form of the descriptive, classificatory, or natural history sciences, — as Botany, Zoology, etc.; the latter, of explanatory natural and mental sciences, — as Physics, Chemistry, Psychology, etc.

The principle or organic idea in systems of classes, is simply the *principle of classification* (p. 47) or *division* (p. 68), which has already been considered. *E. g.*, in Zoology the system of the animal kingdom is a system of classes and sub-classes, based on *plan of organic structure.*

The principle or organic idea in the higher form of system, or system in the stricter sense, is the *central truth* to which the inductive method leads, and with which the deductive method starts out.

Accordingly, Ueberweg has said : " The principles of knowledge are of two kinds, according as the *individual* or particular, or the *universal*, serves as the

starting-point of knowledge. The former do not correspond to the real principles, but form the natural foundations of propædeutic knowledge; the latter distinctly correspond to real principles and, accordingly, form the foundations of strictly scientific knowledge.

"The propædeutic or method of investigation proceeds *regressively* or **analytically** to the knowledge of real principles; the purely scientific or constructive method proceeds *progressively* or **synthetically** from principles to particulars or individuals. But it is by no means always desirable, in an exposition of the sciences, to thoroughly separate the analytic from the synthetic elements. Both are often to be combined with each other in the treatment of individual problems."

The construction and value of a system will, therefore, manifestly depend, in any given case, first of all, upon the correctness and completeness of the *principle* which unites its parts into a whole. Hence, in examining systems, arises

Rule 1st. — Ascertain the principle or organic idea of the system.

In a system of Ethics the idea of right or virtue is the principle. In the Moral System of the universe the idea of right as embodied in the control of the Moral Governor is the principle.

Topic Second.—The Articulation or Relation of the Parts.

In studying any system it is necessary, in the second place, to seize upon the relations of the parts to each other.

Every truth has relation to some other. In a system the various connections of related truths are brought out. Bishop Butler says, in his Sermons: "A *System, Economy*, or *Constitution*, is a one or a whole, made up of several parts, but yet the several parts even considered as a whole do not complete the idea, unless in the notion of a whole you include the relations and respects which these parts have to each other."

The relations of the thoughts to each other, in any system, may include any or all the possible relations of conception, judgment and reasoning. The aim in all systematic knowledge is "to unite the facts of knowledge so as to see them in their several bearings." Hence arises

Rule 2d.—See that the parts of the system are logically connected throughout.

Topic Third.—The Relation to the Objective Reality.

In studying any system, it is necessary, in the third place, to compare the thought-system with the reality which it represents.

18 *   O

"System applies not only to our knowledge, but to the objects of our knowledge. Thus we speak of the planetary *system*, the muscular *system*, the nervous *system*. *We believe that the order to which we would reduce our ideas has a foundation in the nature of things.* And it is this belief that encourages us to reduce our knowledge of things into systematic order."

The **final test** of the correctness of any system must be found, therefore, in its exact truthfulness. Hence arises

Rule 3d. — See that the system agrees exactly with the reality.

Section II.—Testing of Systems.

As the highest process in the formation of thought is the construction of systems, so the highest process in the unfolding of thought is the testing of systems.

The possibilities and dangers of error have been seen to be very great in Conception, Judgment and Reasoning, but they must evidently be as much greater in Systematizing, as this form of thought is higher and more difficult than the others. Mohammedanism and Buddhism in religion, Epicureanism and Utilitarianism in morals, and numberless other systems in all departments of thought, maintain their hold upon mankind simply because of the inability of the masses of mankind to ascertain their elements and put the systems themselves to the test.

Some **examples** of the testing of systems will best illustrate the kind of work to be done in order to avoid error. In a text-book of the scope of the present, it is impossible to find space for presenting such examples in detail. The work must, therefore, be confined to giving directions for testing systems, and referring the teacher and student to examples of such testing to be found elsewhere.

Topic First.—Directions for Testing.

The **first inquiry,** resulting from the carrying out of Rule 1st, is, What is the organic thought or principle which holds together the parts of the system?

The **second inquiry,** resulting from Rule 2d, is, Are the parts logically connected?

The **third inquiry,** resulting from Rule 3d, is, Does the system of thought agree with the facts or the reality?

Archbishop Whately has clearly marked out the course to be pursued in testing a *system of argument*. We quote his directions, which are as follows:

"First, then, of whatever length the reasoning may be, whether treatise, chapter, or paragraph, begin with the concluding assertion,—not necessarily the last sentence expressed, but the last point established,—and this, whether it be formally enunciated or left to be understood. Then, tracing the reason backwards, observe on what ground that assertion is made. The assertion will be your Conclusion; the ground on which it rests your Premises. The whole Syllogism thus obtained may be tried by the rules of Logic.

"If no incorrectness appear in this syllogism, proceed to take the premises separately, and pursue with each the same plan as with the conclusion you first stated. A premise must have been used as such, either because it required no proof, or because it had been proved. If it have not been proved, consider whether it be so self-evident as to have needed no proof. If it have been proved, you must regard it as a conclusion derived from other assertions which are premises to it, so that the process with which you set out will be repeated, viz., to observe on what grounds the assertion rests, to state these as premises, and to apply the proper rules to the syllogism thus obtained. Having satisfied yourself of the correctness of this, proceed, as before, to state its premises, if needful, as conclusions derived from other assertions. And thus the analysis will go on (if the whole chain of argument be correct) till you arrive at the premises with which the whole commences, which of course should be assertions requiring no proof; or, if the chain be anywhere faulty, the analysis will proceed till you come to some proposition, either assumed as self-evident though requiring proof, or incorrectly deduced from other assertions." See Whately's *Logic*, pp. 418, 419.

Topic Second.—Examples Illustrative.

The teacher of Logic will be able to furnish illustrations of this subject in every department of thought.

I. Familiar Subjects.

The tests should be applied first to familiar subjects. These are found in the text-books of Arithmetic, Geography, Physical Geography, Grammar, Rhetoric, Psychology, Ethics, etc., used in the study of these various departments.

One of the most important and useful of all mental processes is that of studying and grasping a science in its entirety as a system. It trains all the mental faculties,—simple cognition, memory, comparison and construction. Until a science is so grasped, it is not in any proper sense *mastered*, since the main thing in a science is not its separate facts and truths, but its whole of related facts and truths.

The best preparation for grasping and testing large and complex systems of thought is secured, by constantly training the student to analyze, outline, and test the parts and chapters of the text-books used.

II. More Difficult Subjects.

The logical training of the young is not, however, complete until this process of testing has been extended to more difficult and abstruse subjects. The following illustrations of such testing, found in various works,—some of which at least will be within the reach of every teacher of Logic,— may be of service. The illustrations may be extended at pleasure by the teacher.

1. Analysis of Part First of Paley's *Evidences of Christianity.* See Whately's *Logic,* Appendix III., pp. 421–427.

2. Mill's Criticism of the Theistic Argument for a First Cause, in *Three Essays on Religion.* Criticised in *Princeton Review,* September, 1878, Article "John Stuart Mill and the Destruction of Theism."

3. Herbert Spencer's *First Principles.* Criticised in *The Philosophy of Herbert Spencer,* by Professor Borden P. Bowne; and in *Mr. Spencer's Formula of Evolution,* by Malcolm Guthrie.

GENERAL SUMMARY.

THE last aim of all training in thinking should be to prepare for and lead to constructive thinking. The safe conduct of life, in the largest and best sense, will depend upon the thinker's power to know in system,—that is, to distinguish between true systems and false systems, as presented by others, and to construct true systems scientific and practical for himself. To help to prepare man intellectually for such conduct of life should be the aim of the Practical Logic of Construction.

INDEX.