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PREFACE.

Geometry is one of the most attractive studies to artizans, on account of its practical utility. But attention has been heretofore confined principally to one branch of the subject—Plane Geometry. It is only of late years that Solid Geometry has been brought so extensively into notice in England.

The Science and Art Department has from year to year more strongly advocated the study of the latter branch of the subject, by rejecting greater numbers of those candidates, at their examinations of science classes, who have failed to come up to the gradually increasing standard of excellence in knowledge of its principles.

Foreign nations have been before us in this matter; and the recent Educational Exhibitions have shown that a knowledge of Solid Geometry is considered indispensable to the well-educated foreigner.

There are few English works upon this subject; and those which do exist are either too learned for the ordinary reader, or they are exceedingly expensive. This little treatise has been prepared expressly for those who are studying Geometry in classes in connection with the Science and Art Department. The aim of the writer, who has taught the subject to large classes of artizans for several years, has been to show, as far as possible, the principles upon which constructions are based, thereby helping the student to avoid the system of "cram," of which examiners so justly complain.

He should not rest satisfied until he can understand the "why" and "wherefore" of the point he is studying, and can reason out for himself any necessary deductions therefrom.

It is understood that, in future, candidates' attention must be confined almost entirely to Solid Geometry.

This book contains rather more than is necessary to correctly work the elementary papers set at the South Kensington Examinations.

H. A.

PREFACE TO THE NEW EDITION.

The subject of this little book is becoming year by year more popular, primarily on account of its vital importance as a study, comprising as it does the principles which form the basis of true and intelligent draughtsmanship, but also as a valuable educational agent.

The favour with which the former edition has been received amongst the teachers and students of our Institutes and classes has been very marked. Hence it was considered desirable to prepare a re-issue of the work, so revised, corrected, and added to, as to bring it up to date, and to still deserve the recognition and approval of those seeking a knowledge of the subject upon which it treats.

A chapter has been added upon Graphic Arithmetic; many methods of solution have been improved, and a large number of extra exercises inserted.

H. A.

Canonbury,

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SOLID GEOMETRY

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GEOMETRY.

CHAPTER I.

INTRODUCTION.

GEOMETRY is divided into two distinct branches—**Theoretical** and **Practical.** The former proves the principles of the science, whilst the latter applies those principles to construction. It is our duty to consider only the practical branch of the subject, although reference will be made to Euclid, and proofs of constructions given where advisable; so that the student may, if he choose, more clearly understand the solution of a problem by investigating the principles upon which it is based.

Practical Geometry is subdivided into two branches—**Plane** and **Solid.** The former describes the construction and properties of lines and figures, whilst the latter treats of the delineation of solid bodies upon plane surfaces.

It is necessary that the student should be provided with the following materials, to enable him to work out for himself the problems contained in this book :—

(1.) A DRAWING-BOARD. This should be quite square at its corners, and present a perfectly level surface. The size would, of course, depend upon the kind of work to be done; but a board 22 inches by 17 inches will be found very generally useful.

(2.) A T SQUARE. By means of this instrument, perpendicular and horizontal lines can be drawn parallel to the edges of the board; and if the head be so constructed as to turn upon the blade, lines at any angle with these perpendiculars, &c., can be obtained. The edge of the blade should be bevelled, as the instrument will not then throw a shadow where the line is to be drawn.

(3.) 2 SET-SQUARES (60° and 45°). These consist of two triangular pieces of wood or vulcanite. Those having angles of 60° and 45° are the most convenient. By means of these, lines can be drawn perpendicular to each other, or parallels in any direction can be determined. Thus, in figure 1, one set-square, *b*, rests against the edge of another, *a*. It will be seen that by sliding the former along the latter to other positions, as *c* and *d*, parallels or perpendiculars to the edge of the set-square *a*, can be drawn at any distance from each other.

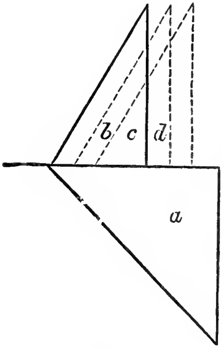


Fig. 1.

(4.) A SET OF MATHEMATICAL INSTRUMENTS, which should comprise, at least—a compass, with movable pen and pencil legs; a pair of dividers; bow pen and pencil compasses, to describe small circles and arcs; and a ruling pen. Indian ink should be used with the instruments, because it will not corrode them. After using, they should be wiped quite clean, to preserve them from rust.

(5.) A PROTRACTOR. This is an instrument used for setting out angles. It is made in several forms; but the most convenient for the student is the six-inch flat rule, with 180° marked round three of its edges. The method of using it is as follows :—Suppose an angle of 40° is to be made with a given line, A B, at some point, A, in it. The unmarked edge of the instrument should be so placed as to coincide with the line A B, the centre of that edge resting upon A. Then, if the required angle is to open from right to left, the numbers of the degrees upon the protractor must be read in that direction; and at the required 40°, a mark should be

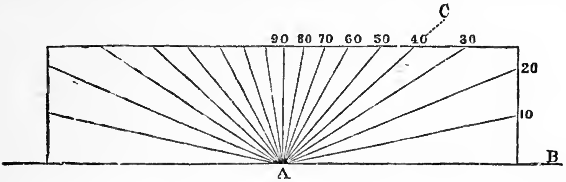


Fig. 2.

made upon the paper. Then, by removing the instrument, and joining the point found to A, an angle of 40° with the line A B will be determined.

(6.) PAPER AND PENCILS. *Cartridge Paper* is the cheapest and best which can be used for geometrical drawing purposes, as it is stout enough to prevent the points of the instruments from penetrating, if they are used carefully. The pencils should be those marked "H." and "H. B.;" the former, for what are termed construction lines, and the latter for completed figures, which should be drawn in firm dark lines.

(7.) DRAWING PINS. These are required to keep the paper in a fixed position upon the board. The best are those which have the pins soldered into the heads, but not penetrating quite through them. By using this kind, the annoyance of the pin coming through and pricking the finger, or unscrewing when taken out, is avoided.

CHAPTER II.

DEFINITIONS AND ELEMENTARY PROBLEMS.

A **point** has neither length, breadth, nor thickness. It merely denotes a position, and is shown in geometrical drawings thus, —⊙A.

A **straight line** has length, but not breadth nor thickness. It is the nearest distance between two given points.

An **angle** is the inclination to each other of two straight lines which meet in a point.

When one straight line meeting another straight line makes the angles on either side of it equal to one another, each of these angles is a **right angle**; and the lines are said to be mutually **perpendicular**. (Euclid, Bk. I., Def. 11.)

An angle is **acute**, when smaller than a right angle, and **obtuse** when greater.

The **complement** of an angle is that which it requires to complete a right angle.

The **supplement** of an angle is that which it requires to complete two right angles.

A **triangle** is a figure enclosed by three straight lines. When these lines are equal, the triangle is **equilateral**; when two of them only are equal, it is **isosceles**.

A **right-angled** triangle has one of its angles a right angle.

A **quadrilateral** figure is enclosed by four straight lines.

A **square** is a quadrilateral figure having all its sides equal and all its angles right angles.

A **rectangle** has two pairs of equal sides, and all its angles right angles.

A **parallelogram** is a figure having two pairs of parallel sides.

A **rhombus** is a quadrilateral figure having all its sides equal, but two of its angles acute.

The **diagonal** of a rectilineal (straight-lined) figure is the line which joins two opposite angular points.

A **polygon** is a figure having many sides. Polygons are **regular** or **irregular**, according as their sides are **equal** or **unequal**. Special names are given to polygons according to the number of their sides. Thus—

Pentagon, .	5 sides.	Nonagon, .	9 sides.
Hexagon, .	6 „	Decagon, .	10 „
Heptagon, .	7 „	Undecagon, .	11 „
Octagon, . .	8 „	Duodecagon, .	12 „

A **circle** is a space enclosed by a line which at all parts is equidistant from a fixed point, called the **centre.** The boundary line, or circumference, is also called a circle.

The straight line passing through the centre, and meeting the circumference in two points, is the **diameter.**

A **radius** is half a diameter.

Any part of the circumference of a circle is called an **arc.**

The straight line joining the extremities of an arc is called the **chord** of that arc.

A **semicircle** is half a circle.

The space enclosed by an arc and its chord is a **segment.**

The space enclosed by two radii and the intercepted arc is a **sector.**

A straight line touching a circle, but not cutting it, is a **tangent** to that circle.

A tangent is perpendicular to the radius which passes through the point of contact.

The mark (") means inches—thus 3·7" means 3·7 inches.

A line is named by letters placed at its extremities, as line A B.

Fig. 3.

An angle is named either by a single letter placed at the intersection of the two lines forming it, or by three letters, the middle one being that described above. Thus, angle B, or A B C.

Fig. 4.

A figure is named by letters placed at its angular points, as figure A B C.

Fig. 5.

A circle is named by a letter placed at its centre.

Parts of an inch are given as decimal fractions. Thus, 6·5" means six inches and five tenths of another inch; 3·25" means three inches and twenty-five hundredths of another inch. One-half is represented by ·5, one quarter by ·25 and three quarters by ·75.

Where the figures in this work do not agree with the dimensions given in the problems, the scale is $\frac{1}{2}$.

The problems of ***this chapter*** are to be worked with compass and ruler only. Perpendiculars and parallels are to be constructed by rule, and not drawn mechanically by aid of the T square or set-squares. Angles, too, are to be determined geometrically—that is, without the aid of the protractor.

Problem I.

To divide a finite straight line into two equal parts.

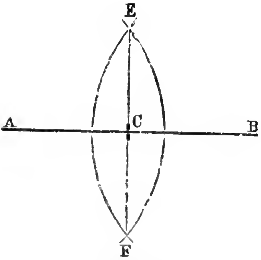


Fig. 6.

Let A B be the given straight line. With A and B, in turn, as centres, and with a radius obviously larger than half the line, describe arcs intersecting in E and F. Join E F. Then the point C, where E F meets A B, is the centre of the line. By extending this process and bisecting each half again, the line can be divided into four equal parts.

PROBLEM II.

At the given points A *and* B, *in the straight line* C D, *to erect perpendiculars.*

On either side of the point A mark off equal distances, as A E, A F. With E and F as centres—any radius, describe arcs intersecting in G. Join G A.

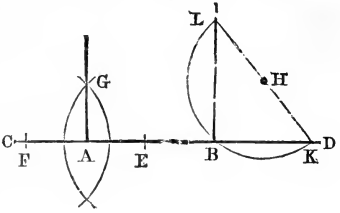


Fig. 7.

The above construction would not be convenient for determining a line perpendicular to C D, and to pass through B, when that point is near the extremity of the line. In this case, take *any* point, H, as centre, and describe an arc K B L, passing through the point B. Join K H, and produce it beyond H, until it meets the arc in L. Join LB.

PROBLEM III.

Through the given points A *and* B, *to draw lines perpendicular to* C D.

In this case the given points are *without* the given line. With A as centre, draw an arc, E L F, which will cut C D in two points E and F. Bisect E F (Prob. I.) in the point G. Then A G is the required perpendicular.

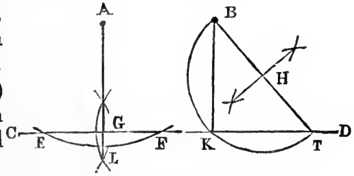


Fig. 8.

Another method is required when the given point, as B, is nearly over the extremity of the line. Draw any line, B T, intersecting C D in T. Bisect B T (Prob. I.), and, with H as centre—radius H B—describe the arc B K T, intersecting C D in K. Join B K.

Problem IV.

To bisect a given angle, B A C.

On A B and A C mark off equal distances, A E and A F.

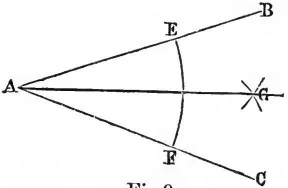


Fig 9.

With E and F as centres, and with a radius equal to more than half the distance E F, describe arcs intersecting in G. Then, a line, A G, will bisect the angle.

Problem V.

To draw a line, making an angle of 60°, *with the given line* A B, *at the point* A.

Before commencing this problem, the student will require a little instruction as to the conventional method of measuring angles. If he will take his compass, and keeping one leg stationary, will revolve the other about the hinge as a centre, he will notice that the opening between the legs will increase, until the two form one straight line. If this revolution could be continued far enough, a complete circle would be generated.

In England it is agreed, that the whole revolution shall be supposed to be divided into 360 equal steps, each step being a degree, written thus °. Consequently, when the movable leg has made one quarter of a revolution, it will have travelled through 90°. When half a revolution has been made, a straight line is formed, which theoretically is an angle of 180°; but in practical geometry no angle, greater than 179° is referred to. Any angle, therefore, is determined by the number of degrees which it contains. In the problem before us, we have to make an angle of 60°, which it is readily seen is one-sixth part of 360°.

With A as centre, draw any arc, C D, and as *the radius of a circle stepped round the circumference will divide it into six equal parts,* mark off C D, equal to the radius employed. Join A D, and D A B is the required angle. The student has now learned how to construct angles of 90° and 60°. (Prob. II. and III.) He can also bisect an angle. By the proper use of the constructions already described, many other angles can be determined. Thus, an angle of 30° is obtained by bisecting an angle of 60°; 45°, by bisecting 90°; 135°, by adding 90° to 45°; 120°, by doubling 60°; 15°, by bisecting 30°; and 75°, by adding 15° to 60°.

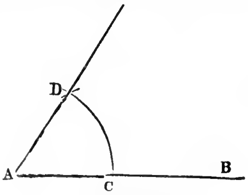


Fig. 10.

Problem VI.

To draw a line parallel to the given line, A B, *at a distance of* 1·2″ *from it.*

At any two points C and D, in the given line A B, construct two perpendiculars, 1·2″ in length (Prob. II). Join their extremities, E and F, and the required line will be determined.

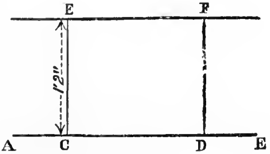


Fig. 11.

PROBLEM VII.

Through a given point, C, to draw a line parallel to a given line, A B.

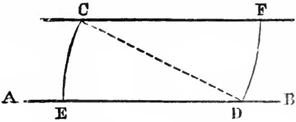


Fig. 12.

Take any point D, in A B, as centre, and with radius C D draw the arc C E. With C as centre, and radius C D, draw the arc D F. Make D F equal to C E, and join C F. Then C F is the required parallel.

Note.—The angles F C D, E D C, are equal, and are called alternate angles.

PROBLEM VIII.

Through a given point, C, *to draw a line meeting a given line* A B, *at an angle of* 60°.

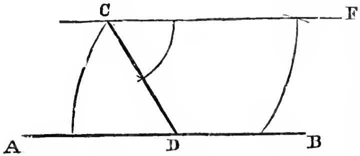


Fig. 13.

Through C, draw a line, C F, parallel to A B (Prob. VII.), and at the point C make C D so that the angle F C D shall be equal to the given one (60°). Then C D A will be an angle of 60°. (See note on Prob. VII.)

PROBLEM IX.

On a given straight line, A B, *to construct an Equilateral Triangle, a Square, and a Hexagon.*

With A and B as centres, radius A B, describe arcs intersecting in C. Join A C, B C. A B C is an equilateral triangle.

At B erect B E perpendicular to A B, making it equal in length to A B. With A and E as centres, radius A B, describe arcs intersecting in D. Join D E, A D, and A B E D is the required square.

With C as centre, radius A C, describe the circle

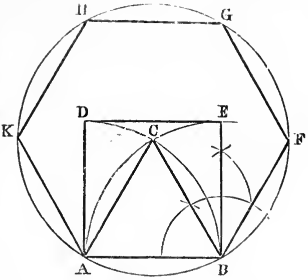


Fig. 14.

A B G. Then A B stepped round the circle will divide it into six equal parts in A B F G H and K. Join these points, as in the diagram, and the required hexagon will be determined.

Problem X.

To inscribe in a given circle a Square and an Octagon.

One figure is said to be inscribed in another when all the angular points of the former are in the boundary line of the latter.

Draw two diameters, A C and B D, perpendicular to each other. Join A B, B C, C D, and D A. Then A B C D is the required square. Bisect the arcs C D and A D (Prob. I.) in the points H and K. Draw lines from H and K to pass through the centre, E, meeting the circle in F and G. Join the points thus found, and the required octagon will be determined.

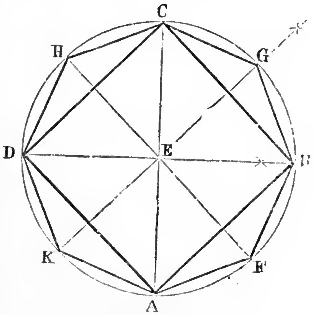


Fig. 15.

Problem XI.

In a given circle, to inscribe a Hexagon and an Equilateral Triangle.

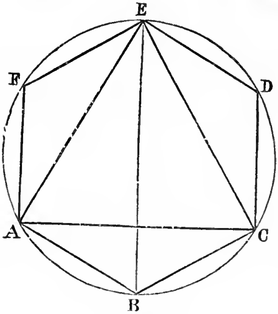


Fig. 16.

As the radius divides a circle into six equal parts, the hexagon is completed by joining the points of division. If the alternate points only be joined, an inscribed equilateral triangle will be determined.

Problem XII.

To construct a Rectangle, the diagonal of which shall be 2″ long, one side being ·8″ long.

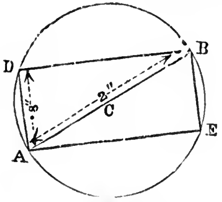


Fig. 17.

Draw a line, A B, 2″ long, and bisect it (Prob. I) in the point C. With C as centre, and A C as radius, describe a circle. Mark off A D and B E, each ·8″ long, and join A D, B D, A E, and B E. Then A B D E is the required rectangle. This solution is based upon the principle that all the angles in a semicircle are right angles.

Problem XIII.

To construct a Rhombus, having one of its angles 45°, its sides being 1·5″ long.

Draw A B 1·5″ long, and at A make an angle of 45°, by first constructing a right-angle, and then bisecting it. Make A D equal to A B, and with B and D as centres, radius A B, describe arcs intersecting in C. Then A B C D is the required rhombus.

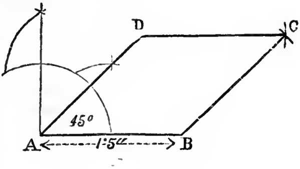


Fig. 18.

Problem XIV.

In a given straight line, A B, *to find a point,* F, *equidistant from two given points,* C *and* D.

Join the given points. Bisect C D in E (Prob. I.),

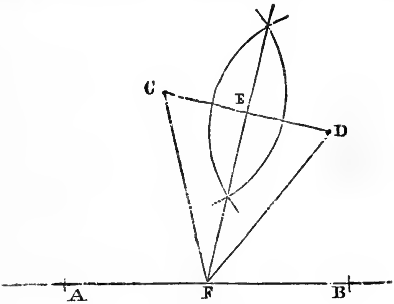


Fig. 19.

and draw E F perpendicular, meeting the line A B in F. Then the distances, F C, F D, will be equal.

Problem XV.

To describe a circle which shall pass through three given points, A, B *and* C.

Join A B. Bisect it by the perpendicular, D E. Join

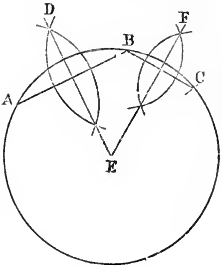


Fig 20.

B C, and bisect it by the perpendicular F E, intersecting D E in the point E. Then, with E as centre, draw a circle passing through either of the given points, and it will be found to contain the other two.

Note.—**The centre of a given circle can be determined by assuming any three points in its circumference, and proceeding as above.**

Problem XVI.

To draw two Tangents to the given circle, C, *each passing through one of the given points,* A *and* B.

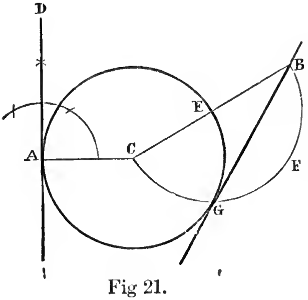


Fig 21.

(1.) Join A C, and at the point A draw A D perpendicular to it. This is the required tangent, as lines which touch circles are perpendicular to the radii at the points of contact.

(2.) Join B C, and bisect it in E. With E as centre, describe the semi-circle, B F C, meeting the circle in G. Then B G is the required tangent.

EXERCISES.

1. At a point A, in an indefinite line A B, construct the following angles, without using a protractor or scale of chords:—B A C = 15°, B A D = 37½°, B A E = 75°, B A F = 135°.
2. Draw a straight line 6" long, and divide into 32 equal parts, by continual bisection.

3. Make any triangle, and draw a line perpendicular to the base, and passing through the apex.

4. Draw a square, and by means of parallels to its sides—1" away—construct another one.

5. Draw an equilateral triangle, and on its three sides construct respectively, a square, a hexagon, and a rhombus with an angle of 60°.

6. Draw a line 3·5" long, and at one extremity erect a perpendicular 1·75" long. From the top of the perpendicular draw a line to make an angle of 30° with the given line.

7. Draw a circle of 1·75" radius; divide it into 6 equal parts. At each of the points of division draw a line tangent to the circle.

8. Draw a circle, and determine (as if unknown) its centre.

9. Draw a square on a 3″ line, and bisect the sides. Join the points of bisection. In the second square inscribe a circle.

10. Construct a square of 4¼″side, and place in it four equal circles, each touching one side and two diagonals.

11. Inscribe a circle in a rhombus of 2″ side and 2·25″ diagonal.

12. Draw a semicircle on a diameter of 3″ and in it inscribe a circle.

13. Find by construction the sixteenth part of 2·8″.

14. Draw any triangle whose sides are unequal (no side shorter than 2″), and bisect its three angles. The three bisecting lines will meet in one point.

15. Take any point inside an irregular triangle and find its perpendicular distances from the three sides.

16. Take any three points, not in one straight line, and find a fourth point equidistant from each of the three.

17. Draw a semicircle on a diameter of 3″, and then determine two tangents to it which shall meet at 90°.

18. Draw a semicircle on a diameter of 3″, and find a point in the curve 1″ from the diameter (measured perpendicularly).

CHAPTER III.

PROBLEMS TESTING NEATNESS AND ACCURACY OF DRAWING.

The problems of this chapter will not require a very extensive knowledge of geometry. They are intended to train the student to habits of neatness and exactness, without which his constructions can be of little value. A few hints are given, which have been found very useful by the writer.

If a line is intended to pass *through* a point, be careful to draw it neither a little *above* nor *below*, nor to the *right* nor to the *left* of that point.

Draw *from* a point, not *to* it.

Do not let the intersection of your arcs be too acute, as the exact point where the lines cut each other, in such a case, is not easily discerned.

Measure long distances in preference to short ones, where practicable. Thus, if an unequally divided line is to be copied, measure off upon an equal line the length of the greater segment in preference to that of the lesser.

In taking degrees from a protractor, be very careful to set the instrument exactly, and make the pencil mark in the same direction as that shown upon the protractor.

In drawing parallels with the T square and set squares, be sure that the fixed instrument is in its correct place, and that the moveable one has its edge close to that of the former.

So place your straight-edge that the part you rule by may not be in shadow.

Make your constructions as large as possible; and where dimensions are given, do not alter them.

A chisel-shaped point for the pencil is best, as it can be kept well up to the edges of your rulers.

Perpendiculars and parallels may, in all future problems, be mechanically determined by the aid of your T *square, &c.*

Problem XVII.

Make any four-sided figure, A B C D, and mark any point E, within it. Join E A, E B, E C, E D Divide E A into 3 equal parts, by trial with dividers, in the points 1, 2. By the aid of your set squares, draw lines through 1 and 2 parallel to A B. If the construction be accurate, the line B E will be also divided equally

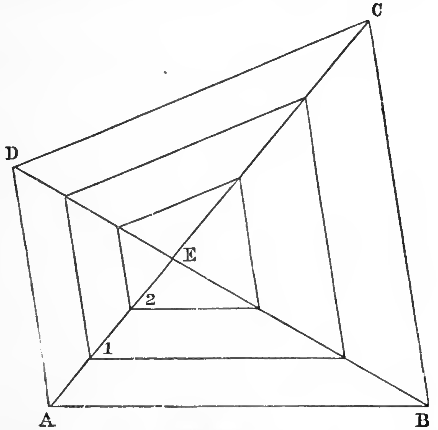


Fig. 22.

into 3 parts. Again, through the points of division of B E, draw parallels to B C, and so continue round the figure until they meet the first divided line, A E, in points 1 and 2. The exercise is rendered still more useful as a test of accurate drawing, by taking a greater number of sides for the first figure.

Problem XVIII

Draw three equal Circles of ·75" radius, each touching the other two.

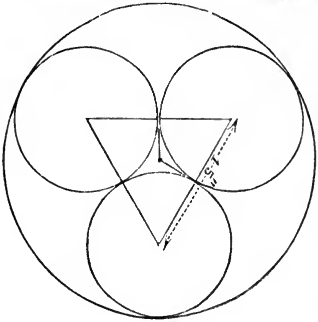


Fig. 23.

The centres of these circles will be the angular points of an equilateral triangle of 1·5" side. Construct this figure, and draw the circles. To test the accuracy of the drawing, bisect two sides of the equilateral triangle, and join their middle points to the opposite corners. The point where these lines intersect can be used as the centre of a circle circumscribing those first drawn.

Problem XIX.

To make a copy of a given geometrical pattern.

This figure is known as a running octagon and square pattern. The dotted lines show the method of construction. Notice that they are *all* either vertical, horizontal,

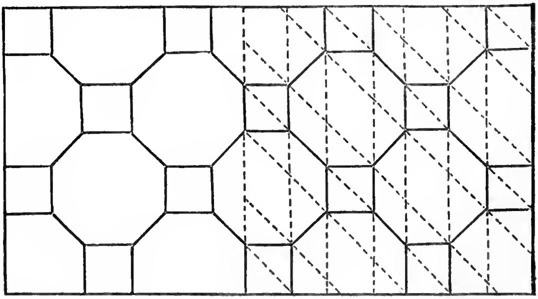


Fig. 24.

or at 45°; hence they can be drawn readily by the T square and 45° set square. The octagons and squares are to be equal-sided. Hence it will be noticed that the construction lines are not equally distant. A little consideration of the figure will tell the student how to space these. Any two contiguous edges of the octagon determine the relative distances apart. The pattern should be drawn much larger than the copy here given.

PROBLEM XX.

Construct a six-sided Polygon, A B C ... F, *from the following conditions:—*

Sides.	*Angles.*
$AB = 1{\cdot}5''$	$ABC = 100°$
$BC = 2''$	$BCD = 110°$
$CD = 2{\cdot}25''$	$CDE = 120°$
$DE = 2{\cdot}5''$	$DEF = 130°$.
$EF = 3''$	

Write down the length of the side A F, *and the magnitude of the angles* E F A, F A B, (*Science Exam.*)

The line A B (1·5″ long) must be drawn first: the angle A B C (100°) must then be laid off from the pro-

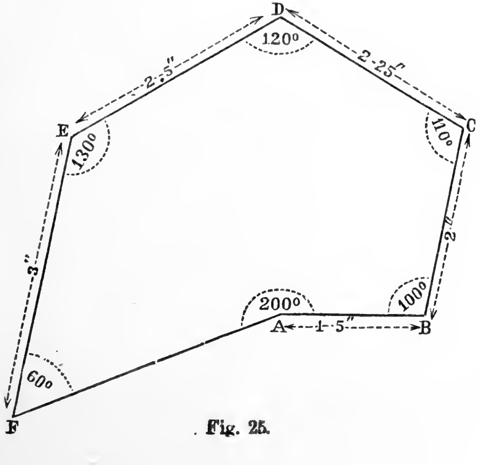


Fig. 25.

tractor. Make B C (2″) and the angle, B C D (110°) determines the direction of C D. By proceeding in this way, all the sides of the figure can be constructed, except A F, which should be 3″ long. The angles, E F A and F A B, will be found by the protractor to contain 60° and 200° (a re-entrant angle) respectively.

The sums of the angles should satisfy the following equation, n being the number of sides—

$$S=(2n-4)90^\circ. \quad (\textit{Euclid}, \text{ Bk. I., Prob. } 32.)$$

Problem XXI.

Draw a square of 2″ side. On each diagonal as a base, draw two equilateral triangles. In each of these four triangles inscribe a circle.

Make the square A B C D (Prob. IX.), and the diagonals A C and B D. On both sides of each of these construct

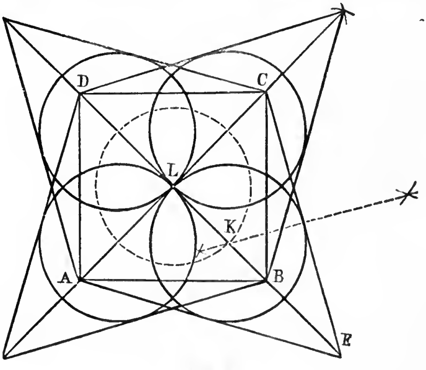


Fig. 26.

an equilateral triangle, (Prob. IX.) Produce the diagonals until they meet the vertices (highest points) of the triangles, and bisect one of the sides as E C. Then

K is the centre of the circle which is to be inscribed in E A C. Then, if another circle be drawn, with L as centre, and L K as radius, it will determine, by the points in which it intersects the diagonals of the square, the centres of the remaining circles.

EXERCISES.

1. Draw a square of 2·75" side, and inscribe another within it, having each of its corners in the sides of the first, and at 1" from its angular points. Draw the circles circumscribing these two squares.

2. Draw a square of 2·75" side. Inscribe in it four equal circles, each touching two others and two sides of the square.

3. Draw an equilateral triangle of 1·5" side; and the four circles, each touching one side of that triangle, and the other two, or those two produced; verify the construction by drawing the circle which would pass through the centres of the three exterior circles.

4. Draw a circle of 1·25" radius, with centre O. The corners of a polygon inscribed in this circle are so placed that the angles at the centre are as follows:—

A O B = 60°	D O E = 80°
B O C = 70°	E O F = 50°
C O D = 50°	

Write down the lengths of A B B C, and C D.

5. Construct an irregular six-sided figure ABCDEF, from the following data of sides and diagonals: AB = CD = 1·25"; BC = 2"; AF = EF = 1·5"; BD perpendicular to AB = DE = 2·5"; CE = 3·5". Write down lengths of CF and AE.

6. Fill a rectangular space 4" by 2·8" with a continuous hexagonal pattern, each hexagon to be of ·4" side.

7. Make any irregular four-sided figure and construct a similar one (same shape) whose sides shall each be double the length of those corresponding to it in the copy.

Note that to get this correct the angles *must be copied, or the figures must be made up of similar triangles.*

CHAPTER IV.

ON PROPORTION.

When two numbers or quantities are compared with each other, a ratio is formed. Thus, as 4 : 8 (read as four is to eight) is a ratio. It is readily seen in this instance that the latter number is *twice* the former. We should say, therefore, that 8 : 16 is the same ratio, because 16 is *twice* 8.

Sometimes a ratio is written in a fractional form. Thus $\frac{4}{8}$ is an equivalent expression to 4 : 8.

The first quantity in a ratio is called the *antecedent;* and the second, the *consequent.*

A proportion consists of a number of equal ratios. As 4 : 8 : : 10 : 20 (read as four *is to* eight, *so is* ten *to* twenty) is a proportion, because it consists of the two ratios 4 : 8 and 10 : 20, which are equal to each other. In this case 4 and 10 are the antecedents, and 8 and 20, the consequents.

If the proportion be true, the first antecedent, multiplied by the second consequent, equals the first consequent, multiplied by the second antecedent. This is called multiplying extremes and means.

In all the above instances, abstract numbers only have been used; but concrete quantities are compared in an exactly similar manner. Thus, 4″ : 8″ is the same ratio as 4 : 8, and 4″ : 8″ : : 10″ : 20″ is the same proportion as the one given above. If extremes and means be multiplied in this case, the result will be, that the rectangle made up with 4″ and 20″ as sides will equal in area another rectangle made up with 8″ and 10″ as sides.

When quantities are each in an equal ratio with those which follow them, they are said to be in continued proportion as 4 : 8 : 16 : 32, &c. Here 4 bears the same ratio to 8 that 8 does to 16, and 16 to 32. Then, any pair of alternate terms multiplied together will give the same result as that obtained by multiplying the term between them by itself. Thus, in the above continued proportion, $4 \times 16 = 8 \times 8$. This intermediate term is then called a mean proportional between the other two.

Problem XXII.

To divide a line A B 3″ *long in the point* C, *so that* A C : B C :: 3 : 5.

Draw the line A B, and at the point A draw a line A C, making *any* angle with A B. Take 3 equal distances of any length, from A to c', and 5 similar distances from c' to b'. Join b' B, and through c', draw a line c' c parallel to b B′. Then c is the required point. The solution of this problem depends upon the principle that in *similar triangles the similar sides are in the same proportion.* It will be readily seen that the triangles A b' B, and A c' c are similar, and that their sides A B and A c are in the same proportion to each other as A b' and A c'.

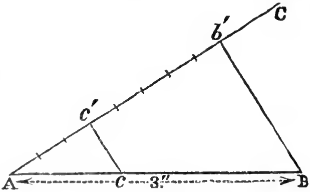


Fig. 27.

The line A B can be divided into any number of equal parts in the same manner, by taking the required number (n) of equal distances along A C, and by drawing a line through each point parallel to n B.

Problem XXIII.

To divide a line A B—3″ *long into* 3 *parts in the points* D *and* C, *so that* A B : A C : A D, *as* 8 : 6 : 5.

At the point A, in A B, draw as before a line, A C, at any angle. Mark off 8 equal distances. At the points 8, 6, 5, numbering from A, place the letters b' c' and d'. Then the line A b is divided in the proportion required. Join B b', and through c' and d' draw lines parallel to B b', and the line A B will be properly divided in C and D.

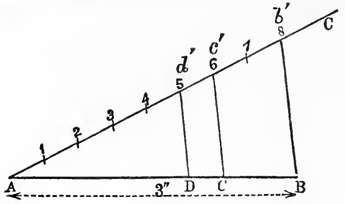


Fig. 28.

Many problems of a similar kind to the two preceding are solved in the same way. For instance, the required point C, *in* Problem XXII., *could be in* A B *produced, and must be so when the ratio of* A C : A B *is greater than unity. In that case, the line* A c' *must be divided as required, and* b' *joined to* B. *Then a parallel through* c' *will meet* A B *produced, and discover the point* C.

Problem XXIV.

To find a fourth proportional to three given lines, A, B, *and* C.

What is required in this problem is the consequent D in the proportion A : B : : C : D.

Take any indefinite line, X Y, and call it a *line of antecedents.* At one extremity make an angle, as X Y Z, and call the line Y Z a *line of consequents.* From the line X Y, cut off a distance, Y 1, equal to the antecedent A, and on Y Z, a distance, Y 2, equal

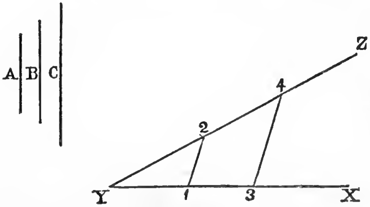


Fig. 29.

to the consequent B. Then, from Y measure the distance Y 3, equal to the given line C. Join 1 2, and through 3 draw 3 4, parallel to 1 2. Then Y 4 is the required line.

It will be clear that there is a fourth proportional smaller than either of these lines. In that case the proportion would read as C : B : : A : D, when C and A would be measured as before; but the point 3 would be joined to 2, and the parallel would be drawn through 1.

Problem XXV.

To find a third proportional to two given straight lines, A *and* B.

A line C is required such, that A : B : : B : C. These three lines will be in continued proportion.

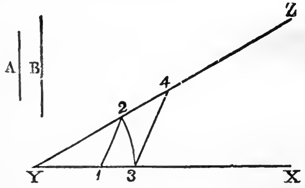


Fig. 30.

Proceed, as in the preceding problem, by constructing a line of antecedents, X Y, and a line of consequents, Y Z. Then measure on X Y the distance Y 1, equal to the antecedent A, and on Y Z, the distance Y 2, equal to the consequent, B. Describe an arc, 2 3, with centre, Y, and through the point 3 draw a line, 3 4, parallel to 2 1. Then the line Y 4 is the required third proportional.

Problem XXVI.

To find a mean proportional between two given lines A *and* C.

It will be noticed in the preceding problem that the second and third terms of the proportion are alike. If, therefore, extremes and means be multiplied, we shall find that A multiplied by C equals B multiplied by itself (B squared); or, in other words, the rectangle made up of A and C is equal in area to a square upon B. The term B of such a proportion is said to be a geometrical mean between the terms A and C.

In the present problem it is required to find this term B, when A and C are given.

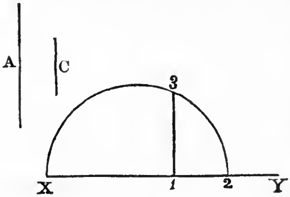


Fig. 31.

Draw a line, X Y, indefinite towards Y, and on it mark off the distances, X 1 and 1 2, equal to A and C respectively. On the line X 2, construct a semicircle, and raise a perpendicular at the point B, to meet the semicircle in 3. Then the distance 1 3 is the required mean proportional B.

Problem XXVII.

To divide a line, A B, *so that the rectangle on the whole line and the lesser segment may equal the square on the greater segment*—(*Euclid,* Bk. II., Prop. 11).

At one extremity, A, of the given line raise a perpendicular, A D, equal in length to half the line A B. Join D B. With D as centre, radius D A, describe the arc A E, cutting D B in E; and with B as centre, radius B E, describe arc E C, cutting A B in C. Then the line A B is divided in the point C, so that a rectangle made up of A B and A C will be equal to the square upon B C.

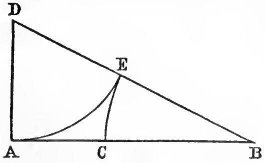


Fig. 32.

This is also called dividing a line in extreme and mean proportion, the terms reading thus, as A B : B C : : B C : A C. The greater segment is therefore a mean proportional between the whole line and the lesser.

EXERCISES.

1. A line A B is 2·7″ long. Divide it in the point C, so that A B : B C as 7·5 : 4.

2. Produce a line A B, 3″ long, to a point P, so that B P : A B : : 3 : 5.

3. Two lines, A B and C D, are 3″ and 4″ long respectively. Find a line, E F, so that A B. E F = C D^2.

4. A B is 3″ long, C D 2″. D E = 1·8″. Find a line, F G, such that C D : A B : : F G : D E.

5. A B is the mean proportional between two lines, 3″ and 1·8″ long. Find its length.

6. Divide a 4″ line in extreme and mean proportion, and prove by construction that the greater segment is a mean proportional between the whole line and the lesser segment.

7. A straight line AB is 2″ long. Determine by construction a second straight line CD, so that AB : CD = 7 : 11.

8. A straight line AB is 2·5″ long. Divide it into 11 equal parts.

9. Show by construction that the mean proportional between two straight lines 1″ and 2·25″ long is 1·5″ long.

10. Find by construction the $\frac{1}{17}$ of 2·55″.

11. Three straight lines are in the proportion of 3 : 5 : 7. The intermediate one is 2″ long. Find the lengths of the others.

12. Extend a straight line 2″ long, so that the produced portion may be $\frac{1}{5}$ of the whole line (when produced).

CHAPTER V.

ON THE CONSTRUCTION OF TRIANGLES, POLYGONS, &C.

A triangle is a figure having three sides and three angles; any two sides must be together greater than the third side (*Euclid*, Bk. I., Prob. 20).

The three angles of a triangle together make two right angles—(180°) (*Euclid*, Bk. I., Prob. 32).

The line on which the triangle stands is usually called the base; but any side may, for purposes of practical geometry, be considered as such.

The angle opposite the base is called the vertical angle, and the angular point is the vertex of the triangle.

Note.—The vertical angle of a polygon having an odd number of sides is that angle farthest from the base.

The altitude of a triangle is determined by a perpendicular to the base passing through the vertex.

Note.—When the vertex is not over the base, the latter must be produced.

The perimeter of a triangle is the sum of its sides.

Similar triangles are those having equal angles. A right-angled triangle has one right angle; the lines forming the right angle are called the base and perpendicular, the remaining side being the hypotenuse.

An isosceles triangle has two of its sides equal.

Note.—The angles at the base are also equal (*Euclid*, Bk. I., Def. 5).

PROBLEM XXVIII.

On a line, A B, *2″ long, to erect an Equilateral Triangle.*

Draw a line, A B, 2″ long. With a radius of 2″, and with A and B as centres, draw arcs intersecting in C. Join A C and C B. Then A B C is the required triangle.

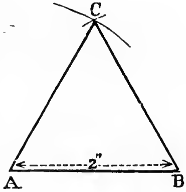


Fig. 33.

PROBLEM XXIX.

A Triangle, A B C, *has its sides* 2″, 1·6″ *and* 2·1″ *respectively. Construct the figure and determine the inscribed circle.*

Draw a line, A B, 2″ long. With A as centre, and with a radius of 1·6″, describe an arc. With B as centre, and with a radius of 2·1″, describe another arc, cutting the former in the point C. Join C A and C B. To determine the inscribed circle, two angles must be bisected, thus :—Mark two points, *e* and *f*, on A C and A B, at equal distances from A. Then, with *e* and *f* as centres, and with equal radii, draw two arcs intersecting in *g*. Join *g* A. Bisect the angle, C B A, in a similar manner. The point where the lines *g* A and *h* B meet is the centre of the inscribed circle.

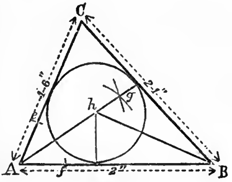


Fig. 34.

Note.—The exact length of the radius may be determined by dropping a perpendicular from the centre on one of the sides, (Prob. III.)

Problem XXX.

To construct an Isosceles Triangle—base 1″, the vertical angle to contain 40°.

This problem depends upon the principle that "all the angles in the same segment of a circle are equal." (*Euclid*, Bk. III., Prob. 21).

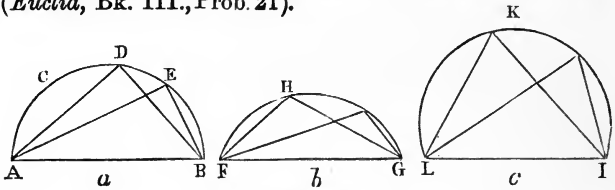


Fig. 35.

Thus the angles A D B, A E B, in the segment A C B (Fig. 35 *a*), are equal.

The same is seen in the segments F H G and L K I (Fig. 35 *b* and *c*).

The angles in a semicircle are right angles, as A E B.

The angles in a segment less than a semicircle are greater than a right angle, as F H G; and the angles in a segment greater than a semicircle are less than a right angle, as L K I.

It remains for us, then, to determine such a segment of a circle as shall contain angles of 40°, and whose chord or base line shall equal the base of the triangle.

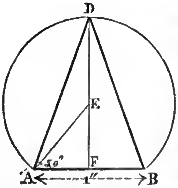


Fig. 36.

Draw the base, A B, 1″ long, and at the point A make A E at an angle of 50° with it. (This angle should always be 90° minus the given vertical angle.) Draw F E perpendicular to and bisecting A B. The intersection of these two lines is the centre of the required segment. As the triangle is to be isosceles, produce F E, to meet the circle in D. Then D A B is the required triangle.

Problem XXXI.

To construct a Triangle—base 1″, *altitude* 1·1″, *vertical angle* 40°.

This problem is similar to the preceding, in so far as finding the segment A D B. When that is obtained, mark off from the perpendicular, F D, the height (1·1″) required, and draw H C parallel to A B, cutting the circle in C and H. Join C A and C B, and A B C is the required triangle.

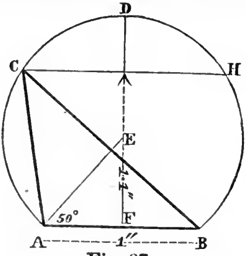


Fig. 37.

There are, therefore, two triangles satisfying the given conditions, one having its vertex in C, and the other in E.

If the vertical angle be greater than 90°, the line A E should be made below A B, at an angle equal to that required minus 90°. The remainder of the construction would be as before.

Problem XXXII.

The perimeter of a Triangle is 8″, *its sides are in the proportion of* 1·5, 2·1, *and* 2·8.

This problem is solved by dividing a line 8″ long into three parts, in the given proportion (Prob. XXII.), and then constructing a triangle, having its sides equal to those parts, (Prob. XXIX.)

Problem XXXIII.

To construct a Triangle whose base shall be 2", its perimeter 7", and one of the angles at the base 35°.

Draw the base, A B, 2″ long. At A make A C at an angle of 35° and 5″ long. These two lines will together

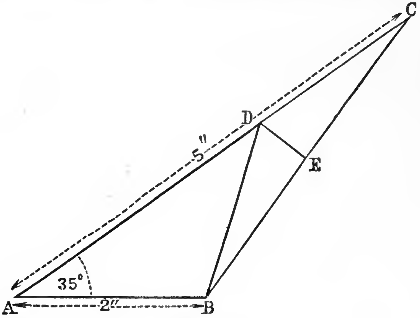


Fig. 38.

equal the perimeter. Join B C, and at the centre E, draw E D perpendicular to B C, cutting A C in D. Join D B, and A D B is the required triangle.

Note.—**The triangle B D C is isosceles, *i.e.* B D = C D.**

Problem XXXIV.

The angles of a Triangle being as 2 : 4 : 3, and the base 2″; to construct it.

Draw a line, A B, 2″ long. Produce it beyond A and, with A as centre, describe a semicircle (radius at pleasure). Divide the semicircle, with the dividers, into 9 (2 + 3 + 4) equal parts, and draw lines from the centre, A, to the points 2 and 6. The angles 2 A D, 2 A 6. and 6 A 9 will then be in the required proportion; and as they together make two right angles, they must be the angles of the required triangle. Through B, therefore, draw B C parallel to 2 A, till it meets A 6 produced in C; then A B C is the required triangle.

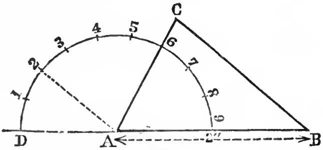


Fig. 39.

Problem XXXV.

The perimeter of a Triangle is 7", its angles are as 2 : 4 : 3. Construct it, and add the circumscribing circle.

The angles are determined by the construction explained in the preceding problem. A triangle, A B C, having angles equal to those found, must then be constructed. (The base of this figure may be assumed as of any length.)

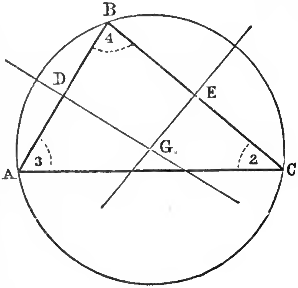


Fig. 40.

The length of the sides of the required triangle can be obtained by dividing a line 7″ long (the given perimeter) into three unequal parts, in the proportion of the three sides of the assumed triangle (Prob. XXII.) Construct then a triangle, with these three segments for sides, and the figure will be determined as required.

To circumscribe the triangle with a circle, find the centre of two of the sides, as D and E, and draw lines, D G and E G, perpendicular to them, intersecting in G. This is the centre of the required circle.

Note.—The construction of the triangle is not shown in the figure.

PROBLEM XXXVI.

To construct a Right-angled Triangle whose base shall be 2" long and the hypotenuse 4".

Draw the base, A B, 2" long, and at the point A raise a perpendicular, indefinite in length. With B as centre, radius 4", cut this perpendicular in C. Join C B, and A B C is the required triangle. If the hypotenuse be bisected in D, and an arc be drawn passing through the points A and B, it will also pass through the point C, because, as we have already learned, the angle in a semicircle is a right angle.

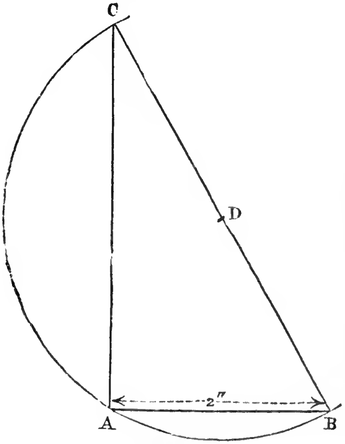


Fig. 41.

PROBLEM XXXVII.

To construct a Right-angled Triangle whose base, A B, *shall be 2" and its angle,* A C B, 38°.

As the three angles of every triangle together make two right-angles, and as in the required triangle one angle is to be a right angle, the remaining two must together make 90°. The angle, A C B, given in the problem is that opposite to the base; consequently, the other acute angle is equal in magnitude to 90° – 38° (52°). At the point B, therefore, set off this angle. The line B C will then meet an indefinite perpendicular through A, in the vertex, C, of the required triangle.

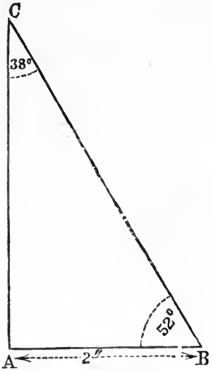


Fig. 42.

PROBLEM XXXVIII.

To construct a Right-angled Triangle whose hypotenuse shall be 3″ long and one of the acute angles 35°.

As a semicircle holds a right angle, bisect the given line B C, which is to be the hypotenuse, and describe a semicircle upon it. At one extremity of the diameter draw a line, making an angle of 35° with it. This line will intersect the semicircle in the point A, which is the third corner of the required triangle.

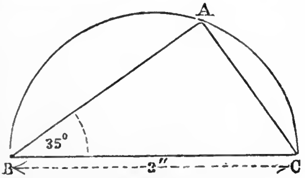


Fig. 43.

Problem XXXIX.

To construct a Right-angled Triangle, the base, A B, *and the perpendicular,* A C, *to be in the proportion of* 3 : 4, *and the hypotenuse,* B C, 3″ *long.*

Draw two lines, A 4, A 3, perpendicular to each other. From the angular point A mark off three equal distances (any length) along A 3, and four similar distances along A 4. Join 3 to 4, and produce the line 3 4, making 3 D, 3″ long. Through the point D draw a line, D C, parallel to the base, and meeting the perpendicular in C.

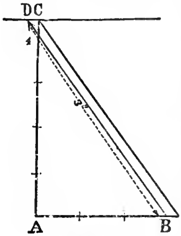


Fig. 44.

Through C draw B C parallel to 3 4, meeting the base line in B. Then A B C is the triangle required.

Problem XL.

To construct a Triangle whose perimeter shall be 6″, *altitude* 1·7″, *and one of the angles at the base* 42°.

As the altitude of the triangle is to be 1·7″, it is

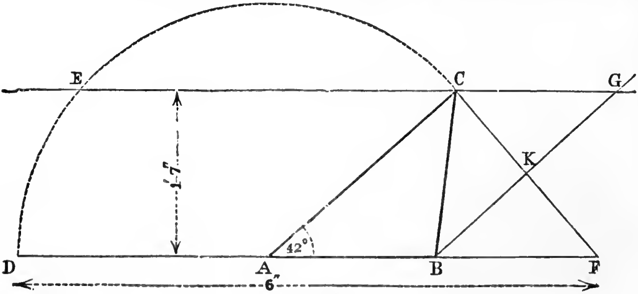


Fig. 45.

evident that the figure will stand between two parallels, D F and E C, that distance apart. Draw these, therefore, and at any point, A, in one of them make the angle F A C = 42°. One side, A C, of the figure is now determined. From A set off the distance A F, so that A C, together with A F, shall equal 6″. Join F C, and at its centre draw the perpendicular K B. Join B C, and the triangle will be completed.

Problem XLI.

To construct a Triangle whose vertical angle, A C B, *shall be* 36°, *the sides,* A C *and* B C, *in the proportion of* 3 : 4, *and the base,* A B, 1·5″ *long.*

At any point, X, make two lines, F X and D X, meeting at an angle equal to the given vertical angle. On F X, mark off three equal distances measuring from X (length at pleasure); and on D X, four of such distances. Join the points found, and cut off A B, equal to 1·5″. Through B draw B C parallel to E X, and the required triangle will be determined.

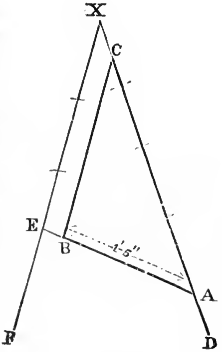


Fig. 46.

Polygons.

For definitions, &c., of Polygons, see Chapter II.

All the angles of a regular polygon are equal, and if they be added together their sum will equal twice as many right angles as the polygon has sides, less four. (*Euclid*, Bk. I., Prop. 32).

Thus, in an octagon, the sum of the interior angles will be (16 × 90°) — 360° = 1,080°. And as they are equal, each angle must be $\frac{1080°}{8} = 135°$.

Lines which bisect and are perpendicular to the sides of a regular polygon meet in one point—the centre.

Lines drawn from the angular points of a regular polygon to the centre divide the figure into a number of isosceles triangles.

A circle can be drawn to pass through all the angular points of a regular polygon. This is called the circumscribing circle.

A circle can also be drawn to which the sides of a polygon shall be tangent. This is the inscribed circle.

Both these circles have the same centre as the regular polygon.

Problem XLII.

A method of constructing any of the regular Polygons upon a given line, A B.

Let A B be the given line, and a pentagon the required polygon. Produce A B to F, making F A equal to A B. With centre, A, draw the semicircle, F E B, and divide it by trial into five equal parts. (The number of these divisions must correspond with the number of the sides of the polygon required.) Join A E—E being always the *second* division from the extremity of the semicircle. Then A B and A E will be two sides of the figure. Raise perpendiculars at the centres of these lines, meeting in O. With O as centre—radius O A—describe the circle, A B C D E, and mark off distances,

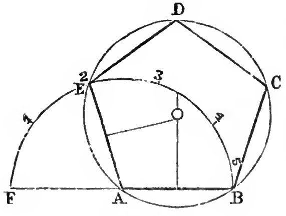


Fig. 47.

B C and C D, equal to A B. Join B C, C D, and D E, and the figure will be complete.

Problem XLIII.

An approximate method of inscribing a regular Polygon in a given circle.

Let A C B be the given circle, and let the required figure be a heptagon. Draw the diameter A B, and divide it into seven equal parts. (The number of parts is regulated by the required number of sides.) With A and B as centres—radius A B—describe two arcs intersecting in D. From the point D draw the line D2, passing through the second division of the diameter, and produce it, to meet the circle in E. The distance, A E, will divide the circle into seven equal parts; and if the points of division be joined, a heptagon will be inscribed in the circle.

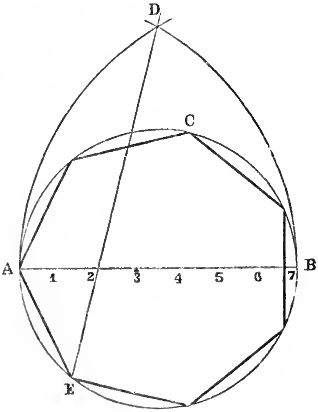


Fig. 48.

In the case of the hexagon, the radius of the circle will give the length of the side.

For an octagon, draw two diameters perpendicular to each other, and bisect the quadrants.

There are special methods for determining each of the regular polygons; but it is not necessary for the student to consider these if the two preceding problems be understood.

Problem XLIV.

To construct a Pentagon whose diagonal shall be 3" long

Draw a line, P Q, and mark any point, A, in

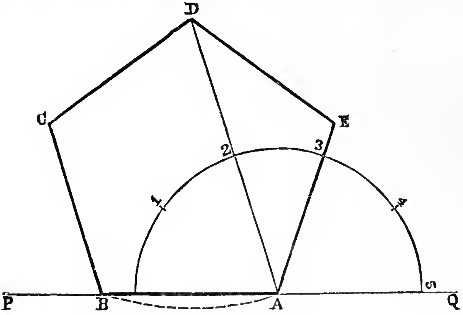


Fig. 49.

that line. With A as centre, describe a semicircle. Divide this semicircle into five equal parts, and draw the line A D, passing through the second division. Make A D 3" long, and with D as centre, radius 3", cut the line P Q in the point B. Then A B is the base of the pentagon. To complete the figure, draw arcs with A, D, and B as centres—radius, A B intersecting in E and C.

Join B C, C D, D E, and E A. Then A B C D E will be the required pentagon.

EXERCISES.

1. Construct a triangle, its sides 2" and 3", and the included angle 50°.
2. Draw an equilateral triangle, whose perimeter shall be equal to a square of 1" side.
3. Construct a triangle from either of the following conditions:—

Its sides as 3 : 4 : 5, and its perimeter 13″.
Its base 5″, altitude 2·5″, and its vertical angle 85°.

4. Construct a triangle whose perimeter shall be 7″; base, A B, 2″; and angle, B A C = 50°.

5. Construct a pentagon having its side 2″.

6. Place two equal lines, 1·5″ long, at an angle of 135°. Consider them as two sides of a polygon, and complete the figure.

7. Construct a triangle, two of whose sides are 2·5″ and 3·25″, the angle opposite the shorter one being 40°. Draw also the circumscribing circle.

8. Construct a triangle, A B C, having its angles 50°, 60°, 70° and circumscribing a circle of 1″ radius.

9. Construct a right-angled triangle whose base is half the length of its perpendicular, the hypotenuse being 4″.

10. Construct a right-angled triangle, base 1″, the acute angles being in the proportion of 2 : 1.

11. In a circle inscribe a heptagon.

12. Draw a pentagon, side 2″, divide it into five equa isosceles triangles, by drawing lines from its centre to the angular points, and inscribe a circle in each.

13. Draw a triangle whose sides are as 4 : 5 : 7, the longest side to be 3″ long.

14. Given a circle of 2·25″ diameter. Circumscribe this circle by a triangle whose sides are in the ratio of 3 : 4 : 6.

15. The line joining one corner of a square to the centre of the opposite side is 3″ long: draw the square.

16. Construct the triangle whose sides are 10′ 6″; 14′ 0″ and 16′ 3″ when ⅛ inch represents one foot.

17. Make an isosceles triangle, altitude 2″ and perimeter 6·5″.

18. Draw the regular polygon whose internal angles are of 140°; the perimeter of the figure to be 6·75″.

19. Draw an octagon whose longest diagonal is 3″.

20. Determine the regular heptagon whose inscribed circle is of 3·5″ diameter.

CHAPTER VI.

ON THE AREAS OF PLANE FIGURES.

The area of a plane figure is the amount of surface enclosed by its boundary line, called "the perimeter," and, although depending upon the position and length of the lines forming that perimeter, is not measured by the same standard. This is easily seen by cutting a piece of paper square, each side being 1″ long. The edges of it together measure 4″ in length; but the area is what is termed *one square inch.* Again, if we cut another piece of paper of the same shape, but each side 2″ in length, it is evident that it will be four times as large as the first piece, and contain therefore four square inches. The edges of the latter piece of paper will measure 8″; and thus we see that with only twice the perimeter we get four times the area.

The boundary line, therefore, of a figure does not alone determine the amount of its surface.

Shape plays a great part in settling the area of plane figures, and certain relations existing between them are demonstrated in works on Theoretical Geometry. A few of these are given, which the writer would advise all students to thoroughly comprehend before attempting the problems of this chapter.

a. The squares on the base and perpendicular of a right-angled triangle are together equal in area to the square on the hypotenuse (*Euclid,* Bk. I., Prob. 47).

b. Parallelograms (squares, rectangles, and rhomboid figures) or triangles upon the same or equal bases, and between the same parallels, are equal in area (*Euclid,* Bk. I., Probs. 35-38).

c. If a parallelogram and a triangle be upon the same base, and between the same parallels, the former will be double of the latter in area (*Euclid*, Bk. I., Prob. 41).

d. The area of a triangle is measured by a rectangle, having the altitude and half the base as sides.

Note.—The rectangle on the base and half the altitude gives the same area.

e. The areas of similar shaped figures are in the same proportion as the squares on the similar sides, (*Euclid*, Bk. VI.)

f. Circles are in the same proportion, as to area, as the squares on their diameters, (*Euclid*, Bk. VI.)

g. Perimeters being equal, the greatest space is enclosed by figures having equal sides, and the greater the number of sides the greater the area.

Note.—A circle, therefore, holds the greatest area with the shortest boundary line, called in this case the *periphery*.

PROBLEM XLV.

Given any irregular Triangle, A B C, *to construct an Isosceles Triangle upon the same base, equal to it in area.*

Referring to the commencement of the chapter, we find that triangles upon the same base and between the same parallels are equal.

It is clear, then, that if we draw a line, *ef*, through the apex C, of the given triangle, parallel to its base A B, that line will be the *locus** of the apices of all triangles on the same base, which are equal to the given one. If we bisect the base and raise a perpendicular *g* D, to meet

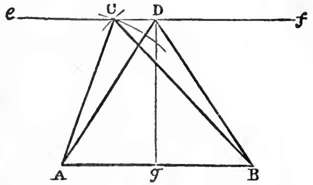


Fig. 50.

* If a line be determined in which, under certain conditions, a point must exist, that line is a *locus* of the point.

this parallel in D, we shall obtain the apex of the required figure, and can then complete it by joining this point to the extremities of the base.

Problem XLVI.

To make a Rectangle equal to a given Triangle, A B C.

If the two figures are to stand upon the same base, the rectangle must be half the height of the triangle; but if the height is to be the same, the rectangle must stand on half the base.

Problem XLVII.

To construct an Isosceles Triangle equal in area to a given Square.

The triangle will have its apex over the middle of the base of the square, and will be twice the height of it.

Problem XLVIII.

A Rectangle 2" by 1" being given, required a Square equal to it in area.

A B F E is the rectangle. Produce A B beyond B, making B G equal to B F. Find a mean proportional to A B and B G, (Prob. XXVI.) Then B C, the mean proportional found, is the side of the required square.

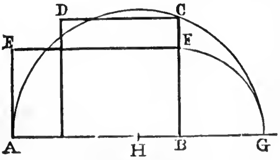


Fig. 51.

Problem XLIX.

Given one side of a Rectangle 2·5", to complete the figure, so that it may be equal in area to a square of 1·2" side.

This is the converse of the last problem. Set the given sides of the rectangle and square perpendicular to each other, as A B, B C. Join A C, and at its centre, D, set out the perpendicular, D E, cutting A B in E. With E as centre, describe an arc passing through C, and meeting A B, produced in the point F. Then B F is the second side of the rectangle, and the figure can be completed.

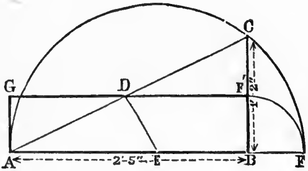


Fig. 52.

Problem L.

To construct a Triangle with sides 2", 1·7", and 1·2" respectively, and a Square equal to it in area.

Construct the triangle (Problem XXIX.), and make a rectangle equal to it in area, (Problem XLVI.) Convert this rectangle into a square, (Problem XLVIII.)

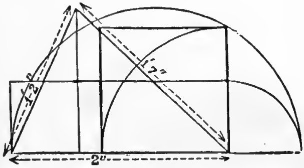


Fig. 53.

Problem LI.

To make a Square equal to two given Squares of ·8" and ·5" sides.

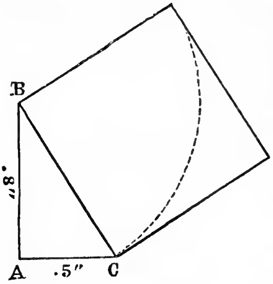


Fig. 54.

Upon two lines perpendicular to each other mark off the given lengths, as A B (·8") and A C (·5"). Join B C, and upon it construct the required square. This problem depends upon the principle *a* mentioned at the commencement of this chapter.

By similar means we are enabled to make a square of any given area, say 3 square inches. We place two lines, each 1" long, perpendicular to each other, as A B, B C (Fig. 55), and join A C. This line is the side of a square of 2 square inches area. We then make C D 1" long perpendicular to A C, and join A D. Then A D is the side of a square 3 square inches in area.

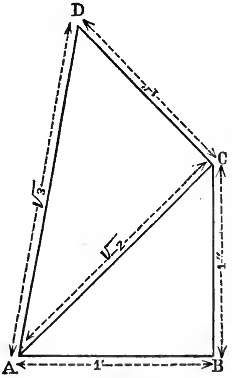


Fig. 55.

If lines 2" and 1" long be used as the base and perpendicular of a right-angled triangle, the hypotenuse will be the side of a square 5 square inches in area. It is usual to consider the lines as the square roots of the units of area enclosed in their respective squares, and to write them thus, $\sqrt{3}$, $\sqrt{5}$, &c., (read as the square root of 5, square root of 3, &c.)

If a semicircle be made upon a line 1" long, and the curve be bisected, the line joining the centre of the semicircle to either of its extremities is the side of a square, which will contain *half* a square inch, or, in other words, represents the root of $\frac{1}{2}$ (unit 1″).

The principle *a* is involved in this last construction, as the angle made by the lines joining the centre of the semicircle to the extremities of its diameter is a right-angle. The diameter plays the part of the hypotenuse.

Problem LII.

To determine a Circle equal in area to that of two given Circles added together.

The solution of this problem is similar to that of the preceding one, as the diameters of circles can be treated as regards area in the same manner as sides of squares. Make, therefore, a right-angled triangle whose base and perpendicular are respectively equal to the diameters of the given circles. Then half the hypotenuse is the radius of the required circle, equal in area to the sum of those given. (See Principle *f*.)

Problem LIII.

Having given two similar Polygons (regular or irregular): to construct a third Polygon, alike in shape but equal in area to that of the former two added together.

This is merely a modification of the two preceding problems, as we see, by Principle *e*, that the sides of similar polygons can be treated as regards area like those of squares or the diameters of circles. Place, therefore, a similar side of each of the two given figures perpendicular to each other, and assume the line which joins the opposite extremities of these, as the corresponding side in the required figure.

Note.—A square, circle, or polygon, can be found equal to more than two given squares, circles, or similar polygons. For, if the sum of the first two be added to the third, and the sum of these again to the fourth, &c., the result will be that a figure will be determined equal in area to the whole.

Problem LIV.

To determine the side of a Square of 3·5 square inches area.

This problem can be solved in two ways. We can by the method explained in Problem LI. find the root of 3 square inches, and at the extremity of this line raise a perpendicular equal in length to the root of one-half a square inch.* We can then make the hypotenuse upon these two lines, and it will be the side of a square of 3·5 square inches area.

The other method of working this problem is as follows:—Make a rectangle of 3·5 square inches area, one side of which it would be most convenient to assume as 1″ long, the other being 3·5″ long. Find a mean proportional between the sides of the rectangle, and it will be the line required.

It will be seen that, by this latter method, the sides of any square can be found containing fractional parts of a square inch; for if 1″ be used as one side of the rectangle, the other side must always be the same measure of inches in length as the required square contains in area.

Problem LV.

To make a Square equal in area to the difference between two given Squares.

Let A B and C D represent the sides of the given squares. At B raise a perpendicular, B E. With A as centre, radius equal to C D, describe an arc, intersecting the perpendicular in F. Then B F is the side of the required square.

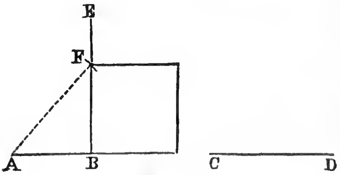


Fig. 56.

* It should be noted that, if the perpendicular be made one half a *linear* inch, only one quarter of a *square* inch is added.

This is easily proved; for as the squares upon the base and perpendicular of a right-angled triangle are together equal to the square upon the hypotenuse, therefore the square on the perpendicular must be equal to the difference of those upon the hypotenuse and base.

A circle equal in area to the difference between two given circles can be determined in an exactly similar manner by treating the respective diameters as sides of squares.

Problem LVI.

Given any Triangle, to make a similar one of double the area.

Let A B C be the given triangle. At one extremity of the base, as B, drop a perpendicular, B D, equal in length to A B. Join A D, and with A as centre, A D as radius, describe the arc, D E, meeting A B produced in E.

Then A E is the base of the required triangle, which can be completed by drawing through E the line E F parallel to B C, intersecting A C produced in F.

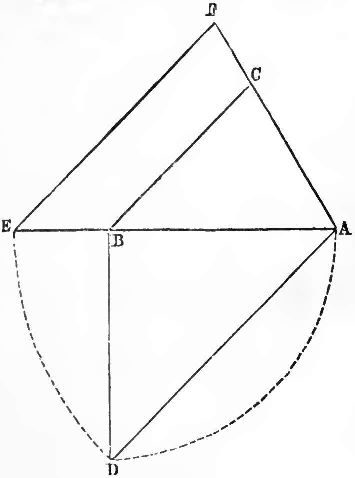


Fig. 57.

Here, again, the principle *a* is involved, but it is combined with the principle *e*.

Note.—The above construction will enable the student to make a triangle three or any number of times larger than the given one. For if he places a line, equal to A B, perpendicular to another line, equal to A E, the hypotenuse upon these two will be the base of a triangle three times the area of the given one.

Problems upon this principle could be multiplied indefinitely; but no intelligent student could feel any difficulty in solving them, if he understands the geometrical truths involved in the last few cases.

Problem LVII.

To reduce any irregular figure to a Triangle of equal area.

It is advisable, in working this problem for the first time, to assume a figure having no more than five

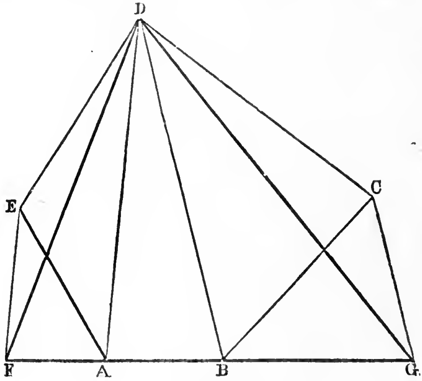


Fig. 58.

sides, and when the principle is understood, to apply it to more complicated cases.

Let A B C D E be the polygon. Produce the base, A B, in both directions. Join A D, and notice, in doing this we cut off a triangular piece, A E D, of the figure. Through E draw E F parallel to A D, and join D F.

Then, by principle *b*, it can be shown that the triangles, A D E and A D F, are equal; for they are both on the same base, A D, and have their vertices in the parallel, E F. But the triangle, A D F, has one of its sides in the line A B produced, and the figure F D C B is equal to the given polygon, but has only four sides. Hence their number has been reduced by one.

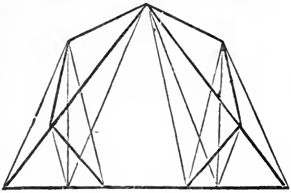


Fig. 59.

Joining B D, and proceeding as before, we lessen the number of sides again to three, and so obtain a triangle, F D G, equal in area to the original figure.

Note.—If the given polygon has many sides, proceed as before to reduce them, one at a time, till the required triangle is obtained, always commencing *from* the line to be used as base, on either side. The case of a heptagon reduced to an equal triangle is shown in Fig. 59, which the student will, by a little consideration, and by thoroughly comprehending the principles involved in the last case, be able to understand for himself without difficulty.

Problem LVIII.

To make an Isosceles Triangle with a vertical angle of 40 *the area of which shall be* 3·5" *square inches.*

Make an angle of 40°, as at C, and cut off equal distances, C A and C B, on each of the legs of the angle. Join A to B, and a triangle similar in shape to the one required in the problem will be obtained. A square equal in area to this figure can be deduced (Prob. L.), by making a rectangle first upon the same base, A B, and half the height of the triangle, and finding a mean proportional between the sides of the rectangle.

Produce the perpendicular, B F, and mark off B H equal to $\sqrt{3 \cdot 5}$, (Prob. LIV.) Then the two lengths, B E and B H, will represent the sides of two squares equal to the assumed and required triangles respectively. Join F C, and draw H K parallel to it, meeting B C produced in K.

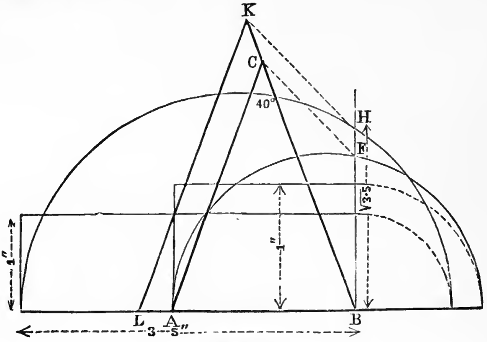


Fig. 60.

Draw K L parallel to A C, meeting A B produced in L, and B K L is the required triangle.

This problem depends upon another very useful principle—that of assuming a figure of wrong area, and deducing from it another of a required size.

Any figures can be compared in this way by finding the sides of squares equal to them in area, as similar figures are to one another as the squares upon their similar sides.

Problem LIX.

To construct a Rhombus, having an Angle of 60° *of* 6 *Square Inches area.*

Commence by drawing any rectangle holding the required area. In the figure, A B C D being 3″ by 2″, it contains 6 square inches. Convert this into a rhomboid A E F B having an angle of 60°, and on the same base

A B. Then find a mean proportional between the unequal sides A B and B F of the rhomboid. This is shown at B H.

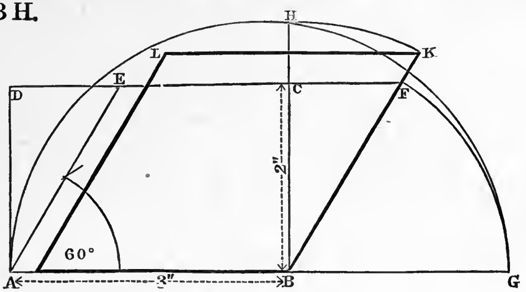


Fig 61 (½ *Scale*).

Make B K equal to B H, and complete the required figure. Note that the same principle which converts a rectangle into a square is used. (*Euclid VI.*)

Problem LX.

To make a Triangle equal to the given Triangle A B C, *and having* A D *for its base.*

Join D C, and through the point B draw B E parallel to C D, join E D, and the triangle A E D will be equal to the given one, A B C.

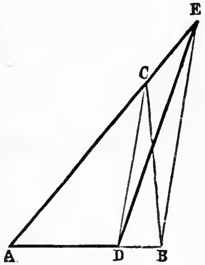


Fig. 62.

Problem LXI.

To make an Equilateral Triangle equal to any Irregular Triangle A B C.

Make an equilateral triangle on one of the sides as

A D C. Produce either of the sides D C, D A. Through B draw B E parallel to A C, and meeting D E in E. Then a mean proportional C F between D C and C E is the side of the required equilateral triangle C F G.

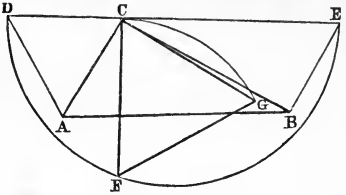


Fig. 63.

Problem LXII.

To make an Equilateral Triangle of 5 Square Inches area.

This problem can be solved by an exactly similar method to that adopted in Problem LVIII., by assuming an equilateral triangle on any base, and deducing therefrom one of the required area. It can also be solved by finding a square of 5 square inches area, and making an isosceles triangle equal to this square, afterwards deducing an equilateral triangle of the same area by the construction of the preceding problem.

Problem LXIII.

To make an Equilateral Triangle equal to the irregular Polygon (fig. 58).

This needs no detailed description. It is solved by Problems LVII and LXI.

Problem LXIV.

To divide a given Triangle A B C, *into any number* (4) *of equal parts by Lines parallel to the side* A B.

Divide one of the sides of the triangle, as B C, into the required number of parts. Upon B C describe a semicircle, and raise perpendiculars from the points 1′, 2′, and 3′, meeting the semicircle in 1, 2, and 3. With C as centre, draw arcs passing through 1, 2, and 3, intersecting B C in D, E and F.

Then parallels to the base drawn through these points will divide the triangle as required.

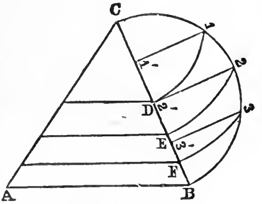


Fig. 64.

Problem LXV.

To bisect the Triangle A B C *by a Line perpendicular to the base* A B.

Bisect the base in the point D, and draw the line E C perpendicular to it. Find a mean proportional between the greater of the unequal segments A E, E B, and half the base.

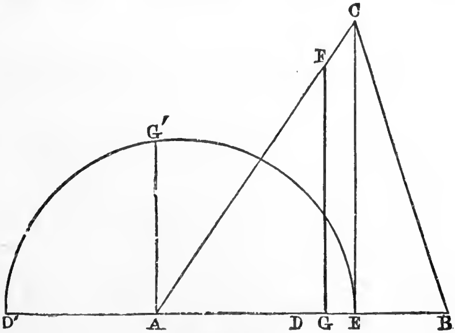


Fig. 65.

In the accompanying figure, A G′ is the mean proportional between A D and A E (A D′ = A D).

Measure the distance A G equal to A G′ from A along the base to G. At G raise the perpendicular G F. Then the triangle will be bisected as required.

NOTE.—The length of the mean proportional must be measured from the same extremity of the base, as the segment used for the mean proportional.

PROBLEM LXVI.

To divide the Triangle A B C *into any number of equal parts by Lines drawn through the apex.*

This is solved by dividing the base into the required number of parts and joining the points of division to the apex. The triangles thus formed are equal because they are on equal bases and of the same altitude.

PROBLEM LXVII.

To divide the Triangle A B C *into any number* (4) *of equal parts by Lines drawn through a point* P *in one of the sides.*

Divide one of the sides in which the given point does not occur into the given number of parts as 1, 2, 3.

Join 1 to P, and through C draw C D parallel to P 1, intersecting A B in D. Join P D. Then P C A D is one fourth of the whole triangle A B C.

Join 2 and 3 to P, and draw C E and C F parallel to 2 P and P 3 respectively, intersecting the base in E and F. Join P E and P F, and the problem will be solved

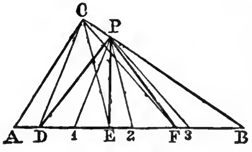


Fig. 66.

PROBLEM LXVIII.

To bisect any irregular figure by a Line drawn from one of its corners.

Let A B C D be the given figure and A the given corner. Draw the diagonals A C and B D. Bisect the

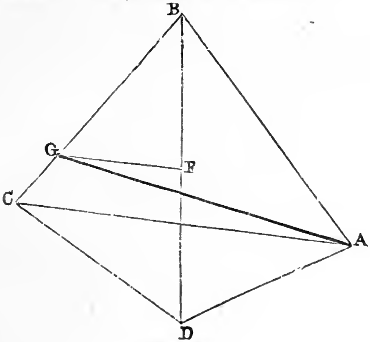


Fig. 67.

diagonal B D in F, and through the point F draw FG, parallel to A C, meeting B C in G. Join A G, and the figure is bisected.

PROBLEM LXIX.

To divide the Rectangle A B C *into any number* (3) *of equal parts, by Lines drawn through the given point* P.

Divide the side in which the given point is situated into three equal parts, and draw perpendiculars from each of the points of division. Bisect these perpendiculars and draw lines from the given point P, through each of the points of bisection.

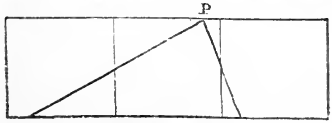


Fig. 68.

Note.—When the rectangle is to be divided into more than three parts, some of the dividing lines may not meet the base until it is produced. When such is the case the solution is correct for all those parts which have their bases in that of the rectangle, but incorrect for the others. Generally the remaining space has to be divided into two equal parts by the preceding problem. The student need not at this stage consider a more complex case.

Problem LXX

To divide a Square into any number (5) of equal parts, by Lines drawn through one of its corners.

Let A B C D be the required square. Divide the sides, B C and C D, each into five equal parts in the points 1, 2, 3, 4, &c. Join 2, 4, 5 and 7 to the corner A, and the figure will be divided as required.

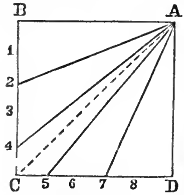


Fig. 69.

The triangles formed are of the same area, because their bases, and altitudes are equal. The quadrilateral figure, A 4 C 5, it will be seen, is made up of two smaller triangles, each equal to a half of either of the larger ones.

Problem LXXI.

To divide a Square into any number (5) of equal parts by Lines parallel to the diagonal.

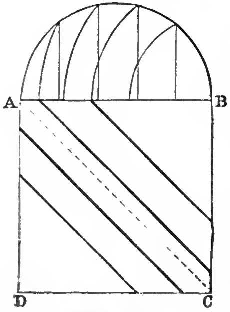


Fig. 70.

Let A B C D be the square, and A C its diagonal. Divide the triangles A B C and A D C each into five equal parts, by the construction explained in Problem LXIV. The square will then be divided into ten parts. By taking alternate divisions, as shown in fig. 70, and rubbing out the diagonal, the figure will be in five parts, as required by the problem.

Problem LXXII.

To bisect an irregular polygon by a straight line drawn through one of its angular points.

Reduce the figure to an equal triangle, having its apex in the given point and its base in the opposite side produced. Bisect the base of the triangle; and if the point falls within the original side of the figure, join to the apex. If it falls without, compensate for the external portion by the aid of principle *b.*

Problem LXXIII.

To construct a Rectangle of 3 *square inches area, its sides to be in the proportion of* 3 : 2.

Upon an indefinite line, A C, mark off two distances, A B, B C, which shall be in the proportion of 3 to 2.

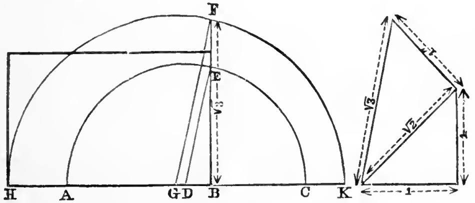


Fig. 71.

Find a mean proportional, B E, between them. Join E to D. Mark off B F upon B E produced, equal to $\sqrt{3}$ square inches, and draw F G parallel to E D. With G as centre, radius G F, describe the semicircle H F K, cutting the line A B in points H and K. Then a rectangle, with H B and K B—which are as 3 : 2—as sides, will contain 3 square inches.

PROBLEM LXXIV.

To construct a Square equal in area to any irregular quadrilateral figure.

Let A B C D be the given figure. Draw its diagonal, A C, and the lines B E and D F, perpendicular to A C. Bisect B E and F D in G and H. Find a mean proportional between the diagonal A C and the sum of the two lines, E G and F H. Then C K is the side of the required square.

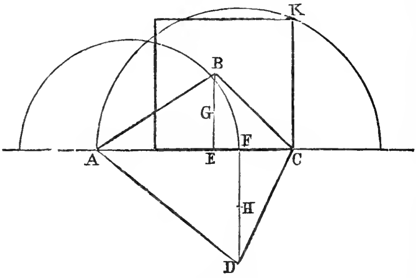


Fig. 72.

PROBLEM LXXV.

To construct an Isosceles Triangle equal to a Regular Pentagon (base 1").

If lines be drawn from each of the angular points of the polygon to its centre, the figure will be divided into five equal triangles. To solve the above problem we must either make an isosceles triangle, with a base equal to that of the pentagon, and with a height five times that of one of the small triangles, *or*, the height being the same, the base must be five times as long.

By means of this construction a square, equilateral triangle, &c., can be easily determined, equal to any given polygon.

EXERCISES.

1. Construct a triangle, perimeter 7″, area greater than that of any triangle of equal perimeter.

2. Determine an equilateral triangle, equal in area to the sum of 2 squares of 1 and 2 inches side.

3. Construct a square of 5 square inches area.

4. Construct a triangle—its sides as 7 : 8 : 9—area, 3 square inches.

5. Bisect a triangle, having its sides 3·5, 4, 4·5 inches by a line, either—

   a. Parallel to the shortest side;

   b. Or perpendicular to the longest;

   c. Or by a line drawn from a point at 1″ from either end of the longest side.

6. Draw an equilateral triangle of 1·5″ side, and a square equal to it in area.

7. Make any irregular figure of four or six sides, and construct an equilateral triangle equal in area.

8. Draw two circles equal to the sum and difference respectively of two other circles of 1·8″ and 3″ diameter.

9. Make a pentagon of 5 square inches area, and a second one twice as large as the first.

10. Construct a square, an equilateral triangle, and a hexagon. Determine by a square the area of the three figures added together.

11. Find by construction the value of the following expressions —the unit being 1 square inch.

$\sqrt{2}$ $\sqrt{5}$ $\sqrt{3}$ and $\sqrt{3\frac{3}{4}}$

12. Construct a pentagon having a diagonal 3″ long, and a square equal to it in area.

13. Draw a rectangle equal in area to a square of 1·75″ side, making the shorter sides 1·25″ long.

14. In a circle of 2·5″ diameter, inscribe a regular pentagon, and find its area as a square.

15. Draw a rhomboid which shall have an area of 4 square inches—two of its sides being 2·5″ long, and two of its angles 60°.

16. Draw any irregular four-sided figure, no side less than 1·5″. Make a second similar figure of three times its area.

17. Find an equilateral triangle, whose area is the same as a triangle, having *two* sides of 2″ and 3″ which include an angle of 60°.

18. Make an isosceles triangle on a base 2″ long equal in area to an equilateral triangle of 3″ side.

19. Bisect the five-sided irregular figure in Problem LVII. by a straight line drawn through point C.

20. Make an equilateral triangle equal in area to the difference between a square of 4″ side and a rectangle 5″ by 2″.

CHAPTER VII.

PROBLEMS ON THE LINE AND CIRCLE.

Facts to be remembered.—A tangent to a circle is perpendicular to the radius drawn through the point of contact. When two circles are tangential, the line joining their centres passes through the point of contact (*Euclid*, Bk. III., Prop. 12). Only two tangents can be drawn from one point to the same circle, and these are equal to each other.

PROBLEM LXXVI.

In the given Angle A, *to inscribe a circle of* 1·2″ *radius.*

Bisect the angle A by the line A D, and draw E F parallel to A B at a distance 1·2″ from it. This line will intersect A D in the point E. Describe the required circle, with E as centre, radius 1·2″. The points of contact, G and H, of the circle and legs of the angle can be determined by drawing the radii E G and E H perpendicular to A B and A C respectively.

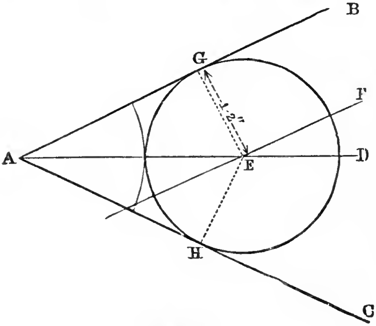


Fig. 73.

PROBLEM LXXVII.

To inscribe a succession of Circles in a given angle, each circle to be tangent to those which precede and follow it.

Bisect the angle, and with any point, E, as centre, describe a circle touching the lines A B, A C. Show the point of contact, H, as above, and at G, draw a line tangent to the circle (that is, perpendicular to A D) meeting A B in K. If we consider for a moment that K G and K B must both be tangent to the next circle,

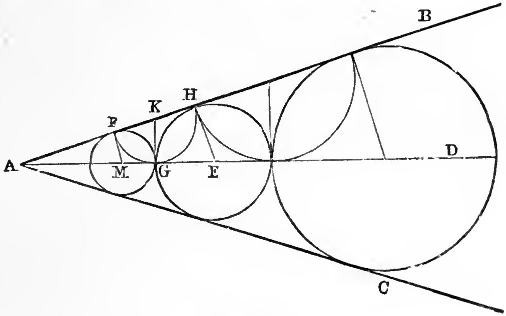


Fig. 74.

we shall see that by striking an arc, with K as centre and K G as radius, we shall obtain F, which is the point of contact of A B with that circle. F M must then be drawn perpendicular to A B, to determine the centre, M.

Describe the circle with M as centre and F M as radius. By similar construction, the centres of a series of circles can be found as required by the problem.

Problem LXXVIII.

To draw two Lines tangent to a given circle C, *meeting at any given angle* (57°).

From the centre, C, draw two radii, making an angle with each other equal to the supplement of the given one (180°–57°).

At the extremities of these radii, draw the required tangents, perpendicular to them.

PROBLEM LXXIX.

To inscribe a Circle in a given angle which shall pass through a given point C.

Bisect the given angle, and with E as centre describe a circle touching the lines A B, A D. Join C to the point A, intersecting the circle in G. Draw the radius G E, and a line C K parallel to G E will determine the centre, K, of the required circle, the radius being K C.

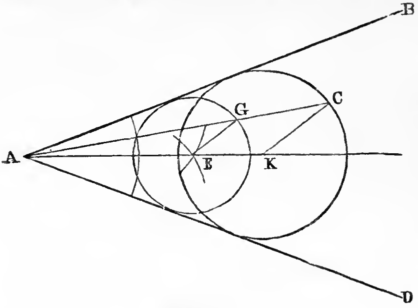


Fig. 75.

PROBLEM LXXX.

A Circle of ·75" *radius, has its centre* 1·5" *from a straight line* X Y: *required, a circle of* ·5" *radius to touch both the given line and circle.*

Through the centre, A, of the given circle draw a line, A B, perpendicular to the given line X Y. Make C F parallel to X Y at a distance from it equal to the given radius (·5"). With A as centre, radius ·5" greater than that of the given circle, describe the arc E F meeting the parallel, C F, in F. Then F is the centre of the required circle.

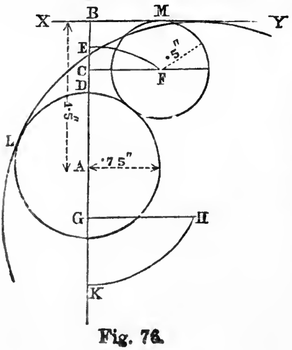


Fig. 76.

Note.—The line C F is the

locus of the centres of all circles with radius ·5″, to which X Y would be tangent.

Similarly, the circle E F is the locus of the centres of all circles, of ·5″ radius, which would be tangent to the given circle, A.

The large circle passing through L is another case, 2″ in radius and includes the given one, the construction being a modification of the above. Note that G H is 2″ from X Y, and D K is 2″ also.

Problem LXXXI.

Two Circles of 1″ and ·5″ radius respectively have their centres 2″ apart; required another circle of ·75″ radius to touch both the former externally.

Draw a straight line and mark off a distance, A B, of 2″. Describe the circles A and B, with their respective radii, as given in the question. To determine the centre

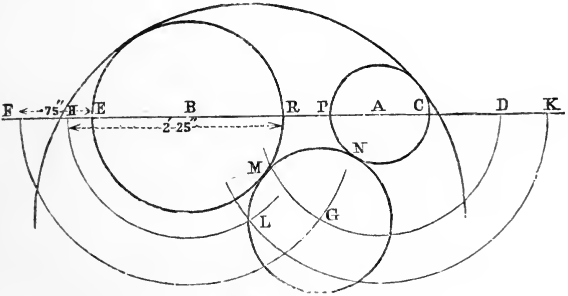


Fig. 77.

of the third circle, produce A B to D and F, making C D and E F each equal to the given radius, ·75″. With A and B as centres, describe the arcs D G and F G intersecting in G. Then G is the centre of the required circle.

The points of contact, M and N are determined by joining the centres of the circles.

Problem LXXXII.

Given the Circles as in Problem LXXXI.; to describe a Circle of 2·25″ radius which shall be tangent to both and include them.

The construction of this problem is shown in fig. 77. On the line A B, make R H and P K equal to the given radius (2·25″). With A as centre, describe the arc K L, and with B as centre describe the arc H L. These arcs will intersect in the point L, which will be the centre of the required circle.

In all cases where the required circle is to be external to the given one, the radius of the former should be added to that of the latter, but when the given circle is to be included, the radius of the required one should be measured across its diameter.

Problem LXXXIII.

A Line, A B, *is* 1·5″ *from the centre of a circle of* ·8″ *radius. A point,* P, *in the line is* 2″ *from the centre of the circle. Draw a second Circle to touch the line* A B *in the point* P, *and also the given circle* (1.) *externally,* (2.) *and to include it.*

(1.) Through the given point P draw an indefinite line, K L, perpendicular to A B. Mark off the distance P D equal to the radius of the given circle. Join C D, and bisect in the point E. Draw E F perpendicular to C D and intersecting K L in F. Then F is the centre. and F P the radius, of the required circle.

(2.) When the given circle is to be included, the length of the radius of the given circle must be measured off below the given point as P D′. Join C D′, and proceed as before. F′ is the centre of the required circle.

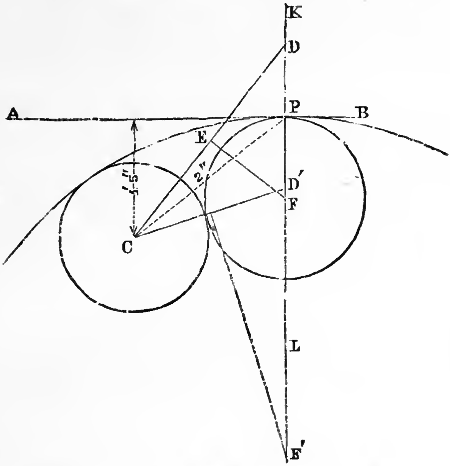


Fig. 78.

Problem LXXXIV.

The line and circle being given as in the preceding problem, required a second circle to be tangent to the line, and to touch the former at a given point P *in its circumference.*

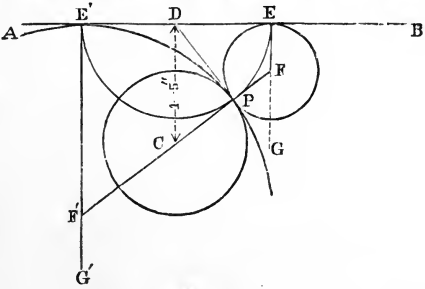


Fig. 79.

(1.) Join the centre C to the given point P, and produce it beyond P. Draw D P perpendicular to C P, meeting A B in D. With D as centre, radius D P, describe the arc E′ P E, meeting A B in E. Draw E G perpendicular to A B. The point F, where the lines E G and P C produced intersect, is the centre of the required circle.

(2.) When the given circle is to be included, the construction must be modified, in that the arc P E′ must be described in the opposite direction, and P C must be produced beyond C to intersect the perpendicular from E′. Then F′ is the centre of the required circle.

Problem LXXXV.

Two Points, A *and* B, *are* 2″ *apart.* A *is* 1·5″ *and* B ·8″ *from a straight line,* C D. *A Circle is required which shall pass through the points* A *and* B, *and touch the given line* C D.

To determine the position of the points A and B, draw two lines parallel to C D at the respective distances of A and B from the given line.

Mark A upon the line which is 1·5″ from C D, and, with A as centre, radius 2″, intersect the other parallel in B.

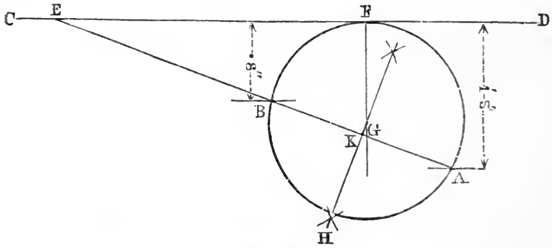


Fig. 80.

Join A B, and produce it beyond B to meet C D in E. A mean proportional between A E and B E will determine E F. Set the length of E F along the line C D from E to F. This latter point is that at which the required circle will touch the given line.

Bisect A B in K, and draw G H perpendicular to A B, intersecting a perpendicular to C D from F in the point G, which is the centre of the required circle.

PROBLEM LXXXVI.

A Point, P, *is 2″ from the centre of a Circle of ·75″ radius. Required a line drawn through* P, *and cutting the circle in* Q *and* R, *so that the intercepted segment,* Q R, *of the line shall be* 1″ *in length.*

Take a length equal to Q R (1″) in the compass, and mark off a chord, A B, equal to it. Join A B. From C draw C D perpendicular to A B. With C as centre, radius C D, describe a circle. Then from P draw P Q R tangential to this circle, and it will cut the given one in two points, Q and R, so that Q R will be 1″ long.

This is evident, as all chords in the same circle and equidistant from the centre are equal to one another (*Euclid*, Bk. III., Prop. 14).

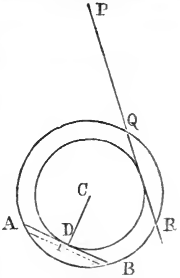


Fig. 81.

PROBLEM LXXXVII.

In a Quadrant (quarter of a circle) to inscribe a circle.

Make two lines, A B and A C, perpendicular to each other, and with A as centre, radius A B, describe the arc B C. Then A B C is a quadrant.

Bisect the angle, B A C, by the line A D. Draw E F through D, perpen-

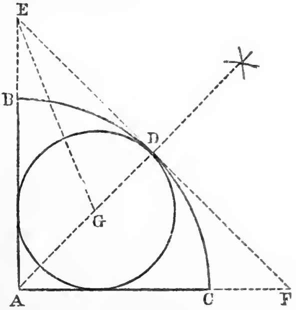


Fig. 82.

dicular to A D, and meeting A B and A C, produced in E and F. Bisect the angle, A E D, by the line E G, meeting A D in G. Then G is the centre of the required circle.

Problem LXXXVIII.

Two Points are 1·75" and 2·25" from the centre of a Circle of 1" radius, and 2" from each other. Draw the circle which, passing through these two points, shall touch the given circle.

To place the given points, &c., in position, draw the circle, C, of 1" radius, and with the same centre draw two arcs, radii 1·75" and 2·25" respectively. Take 2" in the compass, and, setting one leg upon any point in one of these arcs, draw a circle intersecting the other. These two points, A and B, will then be those given in the question.

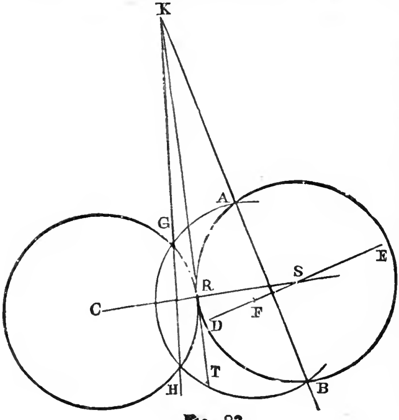


Fig 83.

Join them and draw D E through the middle point of A B, and perpendicular to it. With any point, F in E D, as centre, describe an arc, passing through A and B, and intersecting the given circle in G and H.

Join G H, and produce it beyond G, to meet A B produced beyond A in the point K.

From K draw K T tangent to the given circle. Then a line from C perpendicular to the tangent, K T, will, if produced, meet E D in S, which will be the centre of the required circle, and A S or B S will be the radius.

EXERCISES.

1. The centres of two circles are 2·5″ apart; their radii are ·75″ and ·5″. Draw any third circle to touch both, and include them.

2. Draw two lines crossing each other at an angle of 60°. Describe circles of ·5″ and ·75″ radii in the opposite acute angles, and circles of 1″ and 1·25″ radii in the opposite obtuse angles, all these circles to touch both lines.

3. A line is one inch from the circumference of a circle of 1″ radius; draw a circle to touch the given one and the line, from any one of the following conditions:—

   a. The circle to be 1″ radius.

   b. To touch the line in a point 2·75″ from the centre of the given circle.

   c. To touch the given circle in a point 1·75″ from the line.

4. Draw three circles of 1, 1·25, 1·75 inches radii, each circle touching the other two.

5. The distance between the centres (A) (B) of two circles is 2″; their radii are ·75″ and 1″ inch; draw a circle of 2″ radius to touch both the former, but to contain the smaller within it; the points of contact to be correctly determined

6. Draw two lines at an angle of 40°. Draw two circles, each touching the lines and one another, the radius of the smaller one to be 1″.

7. Two points, A and B, are 2″ and 3″ from the centre of a circle of 1″ radius, and 2·75″ from each other. Describe a circle to touch the given one, and to pass through A and B.

8. A straight line is 3·25″ from the centre of a circle of 1·75″ radius; draw any circle to touch the given circle and also the given line.

9. Draw a straight line, and mark a point P upon it. Assume another point Q, not in the line. Draw a circle to pass through Q and to touch the line in P.

10. Draw a circle of 2″ radius, and a chord in it 2″ long. Determine a circle of ·5″ radius to touch the line and circle interiorly.

11. Draw any irregular four-sided figure, no two sides to be equal. Determine a circle which shall pass through two corners of the figure and touch the opposite side.

12. Draw two lines A B and A C meeting at an angle of 60°. On A B mark off distances A D and A E equal to 1″ and 2·25″ respectively. Determine a circle to cut the line A B in D and E and to be tangential to A C.

CHAPTER VIII.

ON SCALES.

It is very seldom that drawings can be made equal in size to the objects themselves.

Sometimes, as in representations of parts of buildings, the drawings must be considerably smaller than the parts they depict. At other times, as in representations of the mechanism of a watch, they must be much larger to show with sufficient accuracy the minute details.

In both cases it is necessary that all parts of the drawing should bear the same relation to the size of the corresponding parts in the object.

To effect this, a scale is constructed which consists of a line, very accurately divided, in such a manner as to represent in a smaller space, the standards of length used in measuring the original object.

Thus a line 6″ long can be divided into 36 equal parts, and each part can be assumed to represent one inch. Then the whole line will represent one yard. Again, 12 of these parts will indicate 1 foot.

In this case, it is clear, the original yard is six times its length as shown upon the scale; or expressing the same fact in another form, the scale is $\frac{1}{6}$ of the original.

This fraction ($\frac{1}{6}$) is called the *representative fraction* of the scale.

If a line 2″ long be intended to show 1′ 6″, the representative fraction is $\frac{1}{9}$.

Scales are of no service, except they be very accurate. It is necessary, therefore, that the student should construct them with great care, and also that he should thoroughly test them before he uses them in making a drawing.

The same lengths should be taken upon different parts of the scale, so as to prove that the divisions are uniform.

A scale of merely equal parts is the simplest form. This is called a *plain scale.*

Diagonal scales are used for very minute divisions.

The following measures of length should be committed to memory:—

12 inches make 1 foot.
3 feet, or 36 inches make 1 yard.
$5\frac{1}{2}$ yards make 1 pole.
4 poles, or 22 yards make 1 chain.
10 chains, or 40 poles, or 220 yards make 1 furlong.
8 furlongs make 1 mile.
3 miles make 1 league.

Problem LXXXIX.

To construct a "Plain Scale" of $\frac{1}{60}$ to represent yards and feet. The scale to be long enough to measure 5 *yds.*

As the scale is $\frac{1}{60}$ part of the original, and it is to be long enough to show 5 yards, the whole of it will really be $\frac{5}{60}$ yard long. This will be $\frac{180}{60}''$, which is 3″. Draw a line therefore, 3″ long, and divide it into 5 equal parts. Each of these will then represent 1 yard. The first division on the left must be then divided into 3 equal parts to show feet. It is usual to number the divisions as shown in the figure. That is, having assumed that the division to the left, shows feet, the yards are numbered towards the right.

As an instance of the method of using such a scale, we will suppose that 13 feet is to be measured. As this is

Fig. 84.

4 yards 1 foot, we must place one leg of the dividers at the

point 4, and the other at 1′. The distance between these two figures will represent 13 feet.

PROBLEM XC.

Draw a Scale of 12·5 *yds. to* 1″. *It must be long enough to measure* 35 *yds.*

As 1″ is to represent $12\frac{1}{2}$ yards, 2″ will represent 25 yards. Take therefore a line 2″ long, and divide it into 5 equal parts. Each of these divisions will represent 5 yards. The first two must again be divided each into 5 equal parts. The result will be that 10 smaller divisions will be obtained, each showing on the scale 1 yard. The whole line will require to be made longer to

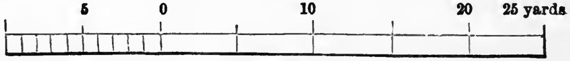


Fig. 85.

represent 35 yards. Add therefore to it a length equal to that which shows 10 yards, as it already represents 25.

PROBLEM XCI.

Draw a "Plain Scale," in which 3″ *shall represent a real length of* 1 *chain,* (22 yards.)

The first part of this problem is similar to the preceding. When the line 3″ long has been divided into 22 equal parts, each part will represent 1 yard. The

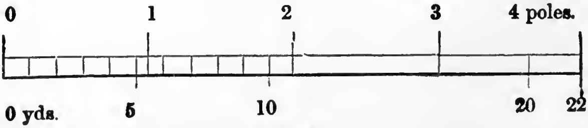


Fig. 86.

whole line must then again be divided into 4 equal parts, to show poles. The divisions will of course, if correct, be equal to 5½ yards.

PROBLEM XCII.

To show, by diagonal division, Tenths and Hundredths of 1″.

Divide a line 1 inch long into 10 parts, in points 1, 2, 3, &c. At each extremity of the line erect a perpendicular, and mark off upon one of these 10 equal distances, numbering them as in the diagram. Through each of these latter divisions draw horizontal lines, and join C to 9. Then parallels to C 9, through each of the divisions of the 1′-line, will complete the necessary construction.

To measure off ·47 of 1″, place one leg of the dividers upon the point E, which is on the horizontal line 7, and the other leg at the intersection of the diagonal line 4 with it. The student will see that the distance he measures is rather more than four-tenths, as shown upon the line A B, and less than five-tenths. The horizontal line 7, by its intersection with the diagonal through 4, gives the exact difference for the seven-hundredths.

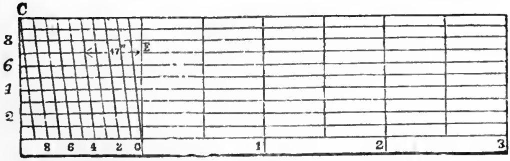


Fig. 87.

PROBLEM XCIII.

The Construction and Uses of the "Scale of Chords."

Construction.—Take any line A B, and on it describe a semicircle. Divide the semicircle into 18 parts, each division representing 10 degrees. With A as centre describe arcs passing through each of the points of division intersecting the line A B. Number these as in the figure, and complete the scale by drawing the double line and division marks.

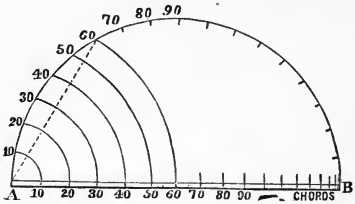


Fig. 87*a*.

Principle and Uses.—A line joining any two points in the circumference of a circle is called *a chord*, and the proportion the length of a chord bears to that of its radius is constant, so long as the angle subtended is the same. The chord of 60° is equal to the radius, and that is why the radius of a circle steps round the circumference 6 times as it is really the chord we measure at each step.

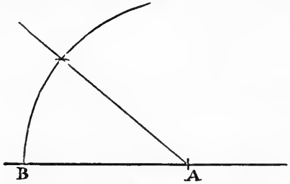


Fig. 87*b*.

By the aid of this scale, therefore, any given angle may be determined. Suppose we wish to set out an angle of 40° with the line A B (fig. 87*b*) at the point A. First we measure from our scale the chord of 60°, and with A as centre, we draw an arc.* Then we take the chord of 40° (from A to 40), and cut off its length along this arc. Then joining the point found to A, the angle required is determined.

* The arc drawn always has for its radius the chord of 60°.

EXERCISES.

1. Construct the following scales to show yards and feet :—

   a. $\frac{1}{36}$ to be long enough to measure 10 yards.
   b. $\frac{1}{80}$ „ „ „ 30 „

2. Construct the following scales to show feet and inches :—

   $\frac{1}{12}$ to be long enough to measure 8 feet.
   $\frac{1}{18}$ „ „ „ 10 „

3. A line 17·5 yards long is represented on a certain drawing by 3·5″. Construct the scale to show yards.

4. Draw a scale of feet of $\frac{1}{50}$; scale to show 30 feet.

5. Give the representative fractions of the following scales :—

   $\frac{1}{2}$″ to 1 ft. ; $\frac{3}{4}$″ to 10 yds. ; 5″ to the mile.
   22″ to the furlong ; $\frac{3}{8}$″ to the foot ; $1\frac{1}{4}$″ to $3\frac{3}{4}$ yds.

6. Give the representative fraction of a scale on which $3\frac{1}{2}$ inches represents 2247 feet.

7. Construct a scale of $\frac{1}{26}$ reading inches.

8. Determine $\frac{1}{96}$ of 2″ by diagonal division.

9. If 2 chains be represented by 4″. construct the same scale to show poles and yards.

CHAPTER IX.

GRAPHIC ARITHMETIC, ETC.

The results of ordinary arithmetical processes, such as Multiplication and Division of Numbers, Involution, and Evolution, etc., may be obtained *graphically*, by geometrical constructions, involving the use of ruler and compass only, independently of the ordinary figures or numerals. The principles upon which these solutions depend have been grouped under the general heading of *Graphic Arithmetic.*

USE OF A LINEAR UNIT.

Numbers ordinarily written in figures may be represented by straight lines. For instance, a line 4″ long being taken to represent 30, all other numbers can be indicated by lines to the same scale. Thus, 15, in this case, would be shown by a line 2″ long, and 10 by a line of $1\frac{1}{3}$″; and, it will be seen at once, that at the commencement of a problem in this subject, some linear dimension *must* be chosen to represent the number 1, and when the solution is finished, the result must be read as a *number* according to the scale pre-arranged. This selected length is called the LINEAR UNIT. When small numbers only are concerned, the unit may be of a moderate length, as ·5″, 1″, etc.; but when very large numbers have to be dealt with, the unit selected must obviously be small, to enable geometrical constructions to be made of a convenient size. Hence, units such as ·1″, ·05″, ·001″, etc., may at times be required.

PROBLEM XCIV.

To represent graphically the numbers 4, 11, 27 (unit = $\frac{1}{8}''$).

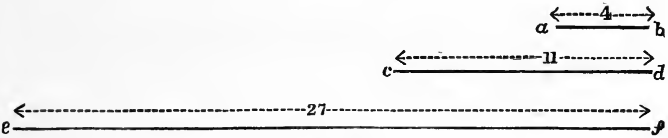


Fig. 88.

The line *ab* (fig. 88) being drawn $\frac{4}{8}''$ long, *cd* $\frac{11}{8}$, and *ef* $\frac{27}{8}$ from an ordinary inch scale, they will evidently indicate the numbers given, the unit or 1 being $\frac{1}{8}''$.

PROBLEM XCV.

The line ab (fig. 89) represents 21; what is the unit adopted?

Draw any line *ac*, at any angle with *ab*. On *ac* from any ordinary scale mark off 21 equal parts, *or* set out at *one* measurement a length *ac*, equal to 21 parts. Join

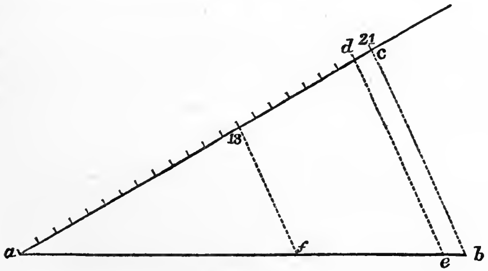


Fig. 89.

bc. Measure a length equal to 20 parts along *ac* to *d*. Draw *de* parallel to *bc*. Then *be* is $\frac{1}{21}$ of *ab*, and hence its length is the required unit.

Problem XCVI.

A line ab (*fig.* 89) *represents the number* 21, *what line will indicate* 13 *to the same unit?*

Proceed as in the previous problem to discover the unit. Then from No. 13 on the assumed line *ac* draw a parallel to *bc* to meet *ab* in *f*. Then *af* represents 13.

Problem XCVII.

To find a line to represent the following:

6 + 2 − 4 − 1 + 7 − 3 (unit ·25″).

Set out an indefinite straight line A X, and consider distances taken along it from left to right, as *positive* or + (plus), and those taken from right to left as *negative* or − (minus). The result will give the desired line.

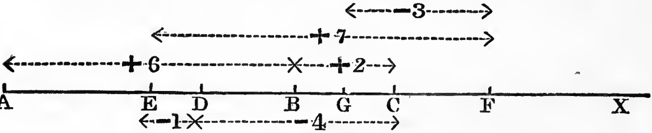


Fig. 90.

Thus, measure a length of 6 units from A to B; then 2 more from B to C. From C to D take 4, in the opposite direction, and from D to E, 1. From E to F measure 7 units, and from F to G, 3. Then A G (7 units) gives the result.

EXERCISES FOR PRACTICE.

1. Draw two straight lines to represent 36 and 27 (unit ·1″).
2. A line 5·1″ long represents 34; what is the unit used?
3. Represent graphically two forces acting at one point in directions which include 50°. Their intensities are to be 7 and 11 lbs. (unit $\frac{1}{4}''$ = 1 lb.).
4. Sum the following numbers graphically.
   a. (8 − 4 + 9 − 11 + 7) unit = ·2″.
   b. (− 6 + 7 − 2 + 19 − 9) unit = ·25″.
   c. (2·8 − 1·9 + 3·6 + 4·4 − 7·5) unit = 1″.

MULTIPLICATION.

Two *lines* can be multiplied together, and a third line can be obtained to represent their *product*. If the multiplier and multiplicand be taken to represent two numbers to some given scale, then the third line will indicate, *to the same scale*, the answer usually found as the product of the two numbers.

PROBLEM XCVIII.

To multiply a line A B *by a line* C D (*unit* = *a*).

Draw any straight line P Q, and mark off upon it P A′, equal to the given unit. At A′ raise a perpendicular to P Q, equal to one of the given lines, as A′ B′ = A B.

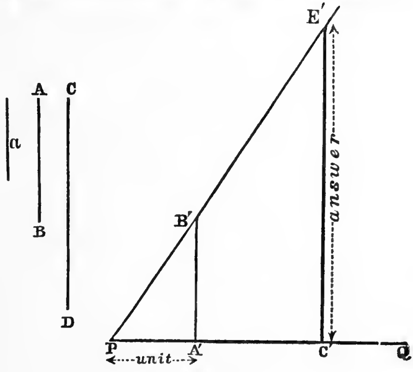


Fig. 91.

Join P B′. On P Q, mark off P C′ equal to the other line C D, and at C′ make C′ E′ perpendicular to P Q, to meet P B′ produced in E′. Then C′ E′ is the required product.

Proof.—The triangles P A′ B′ and P C′ E′ are similar, hence their corresponding sides are in the same proportion (Euclid vi.). Therefore, A′ B′ : C′ E′ : : P A′ : P C′, or $\frac{C' E'}{A' B'} = \frac{P C'}{P A'}$, or C′ E′ × P A′ = A′ B′ × P C′, *i.e.*, our answer multiplied by unity is equal to A B × C D, for A′ B′ and C′ P′ were made equal to them. The answer C′ E′ must be read to the scale of $a = 1$.

PROBLEM XCIX.

To find the continued product of three or more given lines as A B, C D, E F, etc. (fig. 92).

Proceed as in the last problem to multiply A B by C D. This gives C′ G as a first result. Then mark along A′ B′ produced, a distance A′ H equal to C′ G, and from P along P Q, a distance P E′ equal to the third multiplier E F. Join P H and produce to meet

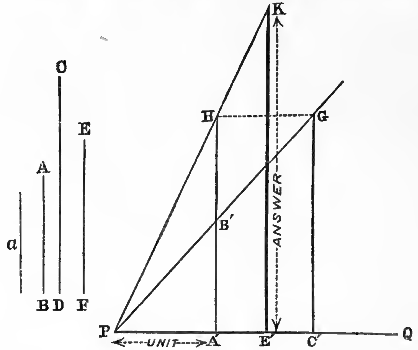


Fig. 92.

a perpendicular E′ K in K. Then E′ K is the required product, *i.e.*, it represents to the scale of *a* as unity, the result of multiplying the numbers indicated by A B, C D, and E F to the same scale. It should be noticed that if large numbers are to be multiplied, the unit adopted must be small. But that would lead at times to very large and inconvenient figures in the working. So to obviate this difficulty *a* may be considered as a line to represent 10 or 100 of the small units. But the student must not forget that his answer must be read correspondingly.

EXERCISES FOR PRACTICE.

1. A line A B 1·25″ long represents 13. Find the unit, and draw a line C D to represent 7. Then multiply A B by C D.
2. Find the continued product of 3 by 5 by $1\frac{1}{2}$ (unit ·2″).
3. Multiply 1·25 by 3·16 (unit 1″).
4. Multiply 173 by 112 (unit ·001″).

   Hint.—Take a line 1″ to represent $a=100$. Then read the answer to the scale when $a=10{,}000$, because each of the figures has been divided by 100.

DIVISION.

When we divide one number into another, we really complete a proportion, for the quotient is the fourth term of a proportional statement, made thus: As the *divisor* is to the *dividend*, so is *unity* to the *quotient*. To illustrate this, let 7 be divided into 21, answer 3. Then 7 : 21 :: 1 : 3. Therefore, in Graphic Arithmetic division is treated as finding the fourth proportional to three given lines (Prob. XXIV., page 32). A fraction is really a division. Thus, $\frac{4}{5}$ means 4 divided by 5. Hence, this fraction (in this subject) is represented by a line which is the result of the proportion. As 5 : 4 :: 1 : answer.

PROBLEM C.

To divide the line A B *by the line* C D (*unit* = *a*.)

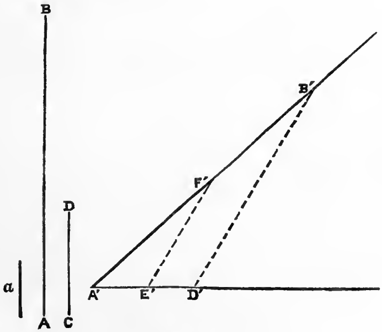


Fig. 93.

Draw two lines to meet at any angle. On one of them, mark off A′ D′ equal in length to the given divisor, and on the other one A′ B′ equal to the dividend. Join B′ D′. On the *same* line as A′ D′ make A′ E′ equal to the unit

a, and draw E′ F′ parallel to B′ D′ to meet A′ B′ in F′. Then A′ F′ is the answer to unit a. For A′ E′ F′ and A′ D′ B′ are similar triangles. Hence A′ E′ : A′ F′ : : A′ D′ : A′ B′, or $\frac{A'F'}{A'E'} = \frac{A'B'}{A'D'}$ *i.e.*, $\frac{A'F'}{1} = \frac{AB}{CD}$ or A′ F′ is the required quotient.

Problem CI.

To determine lines to represent the fractions $\frac{3}{7}$, $\frac{4}{5}$, $\frac{5}{8}$, etc. (unit ·75″).

Proceed as in the last problem to find lines to fulfil the part of fourth proportionals, to the statements made thus:—As the denominator : the numerator : : unity : the required answer.

EXERCISES FOR PRACTICE.

1. Divide 27 by 8 (unit $\frac{1}{4}$″).

2. A line A B, 2·8″ long, represents 25; find a line C D to represent 6, and obtain a third line to indicate the value of A B ÷ C D.

3. Determine graphically the value of $\frac{AB \times CD}{EF}$ where AB = 8 CD = $3\frac{1}{2}$, and EF = 5 (unit ·4″).

4. Find the value of x in the equation $x = \frac{27 \cdot 5}{11 \cdot}$ (unit ·1″).

5. Shew graphically the values of $\frac{7}{15}$, $\frac{2}{3}$, $\frac{5}{8}$ (unit $\frac{1}{2}$″).

INVOLUTION.

A line can be raised to any power in the same way as a number. Thus we may find a line C D to represent the square of a line A B; the meaning being that C D represents to the same scale as A B, the square of the number which A B indicates.

PROBLEM CII.

To determine straight lines to represent the square, cube, fourth, fifth, etc., powers of a given straight line (unit = a.)

Suppose A B, fig. 94, to be the given line, and a the given unit. To find lines to represent AB^2, AB^3, AB^4, etc. Draw two indefinite lines AE^1 and P F at right angles. Make P A equal to the unit a and A B on AE^1 equal to the given line. Join P B and draw B C perpendicular to it to meet P F in C. Then $AC = AB^2$.

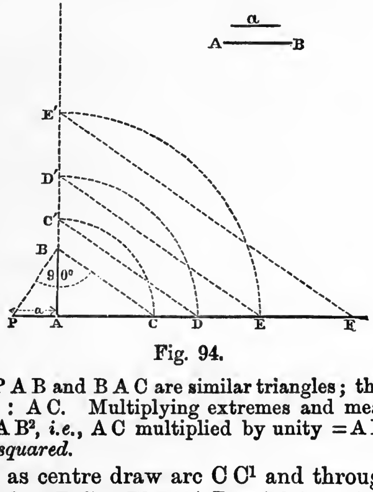


Fig. 94.

Proof.—P A B and B A C are similar triangles; therefore P A : A B :: A B : A C. Multiplying extremes and means, we have P A, $AC = AB^2$, *i.e.*, A C multiplied by unity = A B multiplied by itself or *squared.*

With A as centre draw arc CC^1 and through C^1 draw C^1D parallel to B C. Then $AD = AB^3$.

Proof.—A B C and A C′D are similar triangles; therefore $AB : AC :: AC^1 : AD$ or $AB \times AD = AC \times AC' = AC^2$. But $AC = AB^2$ hence $AB \cdot AD = AB^4$. Dividing by A B we have $AD = AB^3$.

To obtain higher powers, proceed in the same way to determine D′ and E′ and thence E and F. $AE = AB^4$; $AF = AB^5$.

Problem CIII.

To determine lines to represent graphically the powers of a fraction.

Let it be required to determine the square, cube, etc., of $\frac{11}{14}$ (unit 1″). Set out two lines PQ, PQ′ at any angle and describe arcs, R′ R and Q′Q within the angle, of radii, 11 and 14, taken from any scale. Along P Q set off P A equal to the given unit. Join R Q′ and R′ Q, and from A commence a series of lines alternately parallel to R′ Q and R Q′, meeting the two lines P Q and P Q′ in B, C, D, E, F, etc. Then P B represents the given fraction, and P C and P D represent its square and cube, and proceeding similarly, the other powers of the fraction could be obtained.

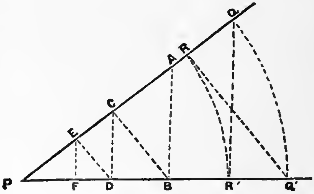


Fig. 95.

Proof.—$\frac{PB}{PA}=\frac{PR'}{PQ}=\frac{11}{14}$, but $\frac{PC}{PB}=\frac{PR}{PQ'}=\frac{11}{14}$; therefore, P C= $\frac{11}{14}$ of $\frac{11}{14}$ of P A = $\frac{11}{14}$ squared.

EXERCISES FOR PRACTICE.

1. Find a line to represent 3^3 (unit ·35″).
2. Find the fourth power of 1·3 graphically (unit 1″).
3. $a=1{\cdot}7$ $b=2{\cdot}3$; find the value of $(a+b)^2$; $(a\,b)^2$ and $(b-a)^2$ (unit ·5″).
4. What line will represent $(\frac{4}{5})^3$ if unity be 3″.

EVOLUTION.

Evolution is the process of obtaining the roots of numbers where their higher powers are given. This can be effected graphically, *i.e.*, a line can be found to represent the *n*th root of any given line.

Problem CIV.

To find the square root of any given line.

In Problem LI., page 54 (Plane Geometry), it was shown that the square roots of numbers could be found by taking advantage of Euclid's 47th Proposition, Book 1. Hence, adopting a line as a unit, the roots of 2, 3, 4, etc., could be found by that method, and in Exercise 11, page 69, that construction is expected to be used. But the *mean proportional* between any number and unity is its square root. Thus the mean proportional between 6 and 1 means that the new number (x) will satisfy the proportion, as $6 : x :: x : 1$. Therefore, multiplying extremes and means we have $6 = x^2$ or $x = \sqrt{6}$. It is best, therefore, in working the square root of a line to adopt this method, viz., to find the mean proportional between the *given line* and the linear unit which has been adopted. The result is the square root required.

Problem CV.

The given line A B (*fig.* 96) *represents* 10, *find a line to represent* $\sqrt{10}$.

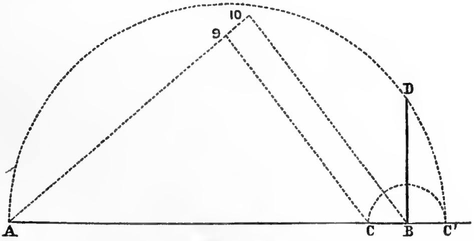


Fig 96.

First proceed to discover the unit (Problem XCV). Then find the mean proportional between the given line and the unit.

In the figure, A B is the given line, and B C is the unit. B C′ is made equal to B C and the mean proportional B D between A B and B C′ is evidently the square root of A B, for it fulfils the conditions A B : B D :: B D : B C′.

Problem CVI.

To find a line to represent the square root of a fraction, the unit being known.

Let it be required to find a line to represent $\sqrt{\frac{2}{3}}$ (unit 1″). Draw any two lines A P, A Q (fig. 97), and on them mark off 3 and 2 equal parts respectively. Join 3 to 2′. On A P mark off A B the unit (1″), and through B draw B C parallel to 3 2′. Then A C represents the fraction $\frac{2}{3}$ (unit 1″). Find then the mean proportional A D between A C and A B′ (unity); A B′ being equal to A B. Then A D represents $\sqrt{\frac{2}{3}}$ as required.

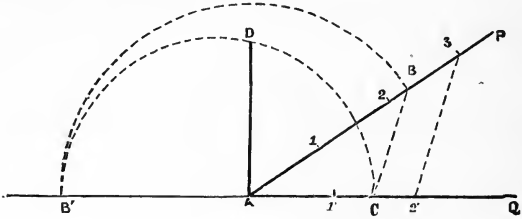


Fig. 97.

N.B.—In the Elementary Stage the other roots are not required, as the obtaining of the cube, fourth, fifth, etc., roots of a line involves the use of the logarithmic spiral, a subject included in the Advanced Stage; but the student who sees that the fourth root is the square root of the square root, will be able to find that and the eighth, sixteenth, etc., roots, from the above reasoning.

Problem CVII.

The line A B (fig. 98) *represents* $\sqrt{7}$; *what is the unit adopted?*

Make A′ B′ equal to A B, and first assume it to be equal to 7. Proceed to find the corresponding unit by taking 7 equal distances along B′ P (a line at any angle), and after joining 7 to A′, adding on one more (to point 8), and by a parallel to 7 A′, to meet A′ B′ produced, discovering

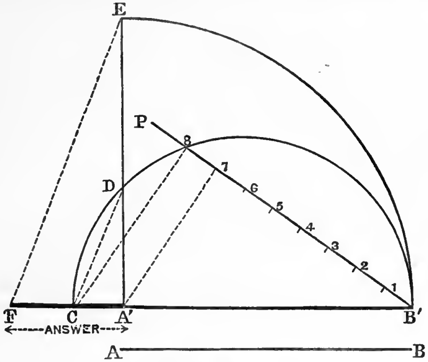


Fig. 98.

A′ C. Then find the mean proportional (A′ D) between A′ C and A′ B′. This line A′ D represents $\sqrt{7}$ when A′ C is the unit. Produce A′ D to E, making A′ E equal to A′ B′, and draw E F parallel to C D, to meet A′ B′ produced in F. Then A′ F is the unit corresponding to A′ E, or A B, as $\sqrt{7}$.

EXERCISES FOR PRACTICE.

1. Find the value of $3+\sqrt{3}$ (unit ·75″).

2. Find a line to represent the values of x in the following equations :—

$x=\sqrt{\frac{3}{2}}$ (unit ·5″).

$x=\ \frac{4}{5}+\sqrt{2}$ (unit 1″).

$x=(2+\sqrt{2})\ (2-\sqrt{2})$ unit = ·75″.

$x=\ \frac{4}{5}+2\sqrt{\frac{3}{2}}$ (unit = ·5″).

AREAS REPRESENTED BY STRAIGHT LINES.

In Problem LVII., page 58, a method is shown by which any irregular rectilinear figure may be reduced to an equivalent triangle. This may be further extended to obtain triangles of *equal altitudes* to represent the areas of given figures. Now, if triangles have equal altitudes, their areas must be proportional to their bases, as the area of each is obtained by multiplying the base by half the height, and the only quantities which vary are the bases.

Again, if the altitude be assumed to be 2″ (linear), then the number of linear inches in the bases will correspond with the number of superficial inches in the areas. Thus $\frac{b \times h}{2}$ = area. But $h = 2''$ therefore $\frac{b \times h}{2} = b$.

Consequently, if a series of figures be reduced to triangles by Problem LVII., and these be altered into other triangles of 2″ altitude, in each case, then the bases of the new figures obtained give in linear dimensions the true relative areas of the originals.

Problem CVIII.

Given any triangle A B C (*fig.* 99), *to determine another triangle equal to it in area, but having a given altitude* (2″).

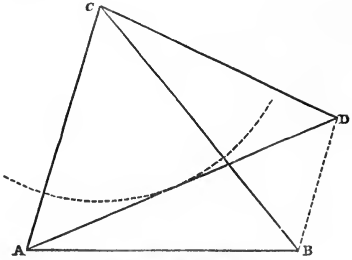


Fig 99 (½ *Scale*).

With the apex C as centre, describe an arc, with radius equal to the given altitude. From one extremity of the base as A, draw a tangent A D, and through the other extremity B draw B D parallel to A C, to meet A D in D. Join C D; A C D is the required triangle.

Note that A B C *and* A C D *are on one base* A C, *and between the same parallels* A C *and* B D. *The number of inches in* A D *is equal to the number of square inches in* A B C.

It should also be seen that if the bases were made equal, while the altitudes change, these latter would give the same results as above.

PROBLEM CIX.

A regular octagon (base ·75″), a square (2″ side), and an irregular five-sided figure (no side less than 1″) being given, to determine three straight lines to represent their areas.

Having drawn the figures, reduce each one to an equivalent triangle. Then by the previous problem convert the triangles into others of equal altitude, preferably 2″. The bases will give the three lines required.

EXERCISES FOR PRACTICE.

1. Make a triangle of 1″ altitude equal to a square of $1\frac{1}{4}$″ side.
2. Determine graphically, which has the greater area, a square 1·75″ side or a regular pentagon of 1·4″.
3. Show that if two triangles be of equal perimeter, and one of them be equilateral, *that* one will contain the *greater* area.
4. If a line 3·2″ long represents the area of a regular hexagon of 2″ side, what line will indicate to same unit the area of a regular octagon of 1·5″ side

SOLID GEOMETRY.

CHAPTER I.

INTRODUCTION, POINTS, LINES, ETC.

(This Chapter is to be studied simultaneously with Chapter II.)

SOLID or Descriptive Geometry enables us to represent the three dimensions of solids—length, breadth, and thickness—in two drawings. Thus, if the shape of the base of an ordinary rectangular instrument box be drawn upon the paper, it will indicate the length and breadth of the object, but not the height. To show the height, a drawing must be made of the front or end of the box.

Its base when in the ordinary position upon the table is horizontal, and the front is vertical; or, in other words, the base is part of a horizontal plane,* and the front, part of a vertical plane.

Thus we see that by representing the object as it appears, first upon a horizontal plane, and afterwards upon a vertical one, we can show its three dimensions.

* A plane is a perfectly level surface, like that of standing water.

Euclid defines a plane as "that upon which, any two points being taken, the straight line which joins those points lies wholly in that plane."

The student will understand that the views above described are those which would be seen by an observer placed at an infinite distance from the object. The rays of light by which the box is perceived, are supposed to be parallel, and not to converge towards the eye, as they actually do.

The drawing upon the H. P. (horizontal plane) is called the PLAN; that upon the V. P. (vertical plane), the ELEVATION.

Many elevations of one object can be made by assuming as many different positions for the vertical planes. By this means, end or profile views, front elevations, &c., can be obtained.

If we wish to show the arrangement of the internal parts of our box, we must suppose the object, cut by another plane in such a manner as to expose the required parts, and a drawing must be made of those parts upon that plane. Such a drawing is called a SECTION.

The horizontal and vertical planes are called co-ordinate planes, and their intersection is a straight line, which receives the name of ground-line, intersecting line, &c.

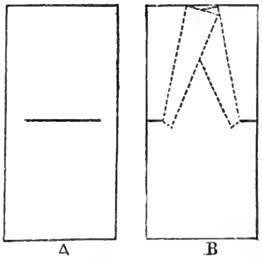


Fig. 99.

In this work, this line will be distinguished by the letters X Y.

A model of these planes can be made in a very simple manner:

Cut two pieces of card-board in the way indicated by the dark lines in figure 99, A and B. Then fold one of these pieces upon the lines shown, as dotted in figure 99B, and pass the folded piece of card-board half-way through the slit in the other piece. Unfold it, and a model of the co-ordinate planes with their intersection will be the result (fig. 100).

It will be seen that the two pieces of card-board will make four angles or corners. Such angles as these are called dihedral angles, to distinguish them from rectilineal angles.

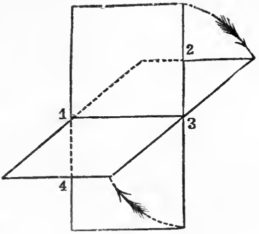


Fig. 100.

If the model be so held as to present to the holder a full view of the v. p., and the end of a pencil be imagined to represent a point, it will be noticed that there are several distinct positions which it can assume with regard to the co-ordinate planes. A point may be in *front* or *behind* the v. p., *above* or *below* the h. p., or *in* either or both of them.

The four dihedral angles are named as 1st, 2nd, 3rd, and 4th, according to the following table:—

A point in front of the v. p. and above the h. p. is in the first dihedral angle.

A point behind the v. p. and above the h. p. is in the second dihedral angle.

A point behind the v. p. and below the h. p. is in the third dihedral angle.

A point in front of the v. p. and below the h. p. is in the fourth dihedral angle.

If two lines be supposed to pass through the actual position of a point, one being perpendicular to the h. p., and the other to the v. p., the intersection of these lines with the respective planes will be the plan and elevation of the point. These imaginary lines are called *projectors*, and the plan and elevation, the *projections* of the point.

Thus a point in the first dihedral angle would have its plan upon the front part of the h. p., and its elevation upon the upper part of the v. p. A point in the third angle would have its plan upon the back part of the h. p., and its elevation upon the lower part of the v. p.

The student, to understand these facts, must use his model, and demonstrate for himself the positions of the various plans and elevations.

If a point be *in* * one of the co-ordinate planes, either its plan or elevation must be in X Y; and if it be *in both* planes, the plan and elevation will coincide with the actual point itself upon X Y.

In making drawings upon our paper, only one surface is used to represent both the co-ordinate planes; and for this purpose the v. p. is supposed to rotate upon X Y as a hinge, whilst the h. p. remains stationary. The model will best illustrate this. The piece of card-board which represents the v. p. should be rotated in the direction shown by the arrows in figure 100, until the upper part of the v. p. coincides with the back part of the h. p., and the lower part of the v. p. with the front part of the h. p.

All elevations which occur above X Y will be above X Y after this rotation.

When the co-ordinate planes are thus made to coincide, the representations of the projectors, which determine the plan and elevation of a point, will be found to form one straight line perpendicular to the ground line. Hence the rule that *plan and elevation of a point always fall in the same straight line, perpendicular to* X Y.

In the succeeding pages of this book the following nomenclature is (with a few exceptions) employed for indications of points and their projections :—

The plan of a point is indicated by an italic letter; thus, *a b*, &c.

The elevation by an italic letter with a dash; thus, *a′*, *b′*, &c.

A point itself, and any representation of it other than a projection, is indicated by a capital letter; as, A, B, &c.

[When a point coincides with its projection on either plane, it is indicated in the same manner as that projection.]

Problem I.

To determine the projections of the points A B C D E *and* F, *when in the following positions :—*

* When a point or line forms a part of a plane, that plane is said to contain the point or line.

A. 1·8″ *in front of the v. p. and* 1·6″ *above the h. p.*
B. 1·4″ „ „ 1·8″ *below* „
C. 1″ *behind* „ 1·7″ *below* „
D. 1·5″ „ „ ·5″ *above* „
E. 1″ „ „ *and in the h. p.*
F. *In both planes.*

Draw an indefinite projector, because the plan and elevation of point A must be upon the same line, perpendicular to X Y. The point being 1·8″ in front of the v. p., and 1·6″ above the h. p., the plan will fall upon the front portion of the h. p., 1·8″ from X Y, and the elevation upon the upper portion of the v. p., 1·6″ from X Y.

When the co-ordinate planes are made to coincide, the plan and elevation will be below and above X Y respectively.

Mark these distances upon the projector, and letter

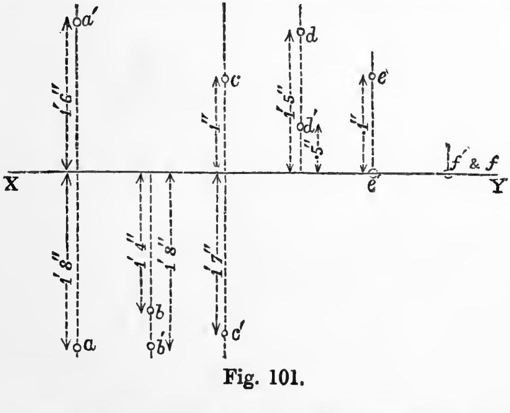


Fig. 101.

the points determined—*a* for the plan and *a′* for the elevation. This should be reasoned out by means of the card-board model.

The determination of the projections of points B C and D will present no particular difficulties, if the above be understood.

Point E is to be *on* the h. p., consequently its elevation will be *in* the ground-line. The plan is determined by its position in relation to the v. p., as before.

Point F being in *both* planes, its actual position must be upon the ground-line, and its projections must coincide with that position.

Problem II.

Given the projections of points A, B, C, *to determine their positions in relation to the Co-ordinate Planes.*

This problem is the converse of the preceding one.

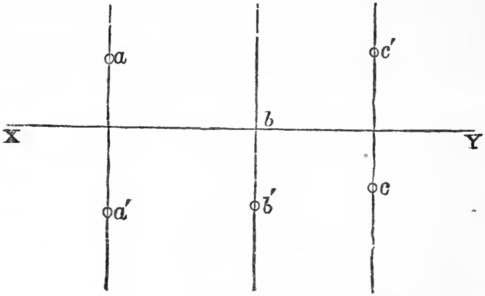


Fig. 102.

The model should be used to illustrate the method of discovering the actual position of point A. Its plan being above X Y, the point itself must be behind the v. p., and the distance of the plan from X Y is the actual distance of the point behind the v. p. Similarly, its elevation being below X Y, the point must be below the h. p., and the length of the projector which determines the elevation is the real distance of the point from the h. p.

Point B is below the h. p., but in the v. p., as the plan falls upon X Y.

Point C is above the h. p., and in front of the v. p.

Problem III.

Having given the projections of a point A, to determine its distance from X Y.

By distance from X Y is meant the length of the

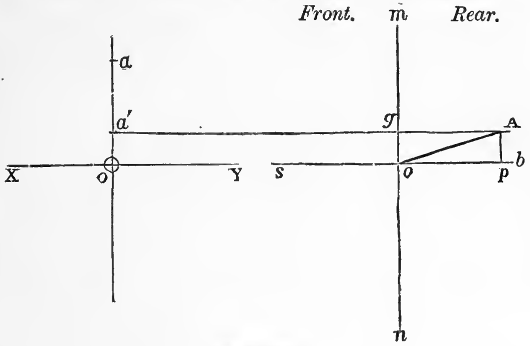


Fig. 103.

diagonal of a rectangle, made up of the two projectors of the point, and their representations upon the co-ordinate planes.

To determine this, a profile view must be taken of those planes. The projections of the point will show that its actual position is above the h. p. and behind the v. p.

Draw two lines, *s b* and *m n*, perpendicular to each other, to represent a profile view of the co-ordinate planes, and assume one side to be the front and the other the rear of the v. p., as in fig. 103. Measure the distance *o a* (fig. 103) along *o b* (fig. 103), to *p*, and the distance *o a′* along *o m*, to *g*. Draw *g* A and *p* A perpendicular to *o m* and *o b* respectively, and join *o* A. Then *p* A *g o* represents the rectangle mentioned above, and A *o* is its diagonal, or the real distance of the point A from X Y.

Problem IV.

A point P *is* 1" *in front of the v. p. and below the h. p., its distance from* X Y *is* 1.5"*; determine its plan and elevation.*

This is the converse of the preceding problem. A profile view must be made of the co-ordinate planes, and an indefinite line must be drawn parallel to the line which represents the v. p. 1″ in front of it. The point *o*, which indicates X Y, must be used as centre for an arc of 1·5″ radius, cutting the indefinite line in a point below that which represents the h. p. The distance of this intersection below that line is the real distance of the point *p* below the h. p.

Now, having the real position of the point as regards both planes, proceed to find its projections as before.

We have found that points may occupy positions in either of the four dihedral angles, and that their projections will determine those positions.

Lines also can occupy similar positions with regard to the co-ordinate planes, and their projections will follow the same laws as those of points.

But lines present another consideration to our minds. They may be horizontal, perpendicular, or oblique.

If a pencil be held in such a position that it is parallel to the h. p., its projection upon that plane will be equal in length to the pencil itself. Similarly, if the pencil be held parallel to the v. p., its projection upon that plane will be of exactly the same length.

But if the pencil be held in any other position, either its plan or its elevation, perhaps each of them will be represented by a shorter line.

When the pencil is held parallel to the v. p., but not parallel to the h. p., it is said to be *inclined* to the latter. Its elevation is, under these circumstances, equal in length to the line itself; but its plan is shorter, and the greater the inclination the shorter the plan, until at length, when the pencil stands vertically, its plan becomes a point.

If the elevation of a line be shorter than the line itself, whilst the plan remains of the same length, the line is inclined to the v. p., but parallel to the h. p.

When a line is inclined to both planes each of its projections is shorter than the line itself.

The inclination which a line makes with either of the co-ordinate planes is measured by the angle which the line makes with its projection upon that plane. Thus, the angle which a line makes with its plan is its inclination to the h. p., and the angle which it makes with its elevation, its inclination to the v. p.

It is usual to indicate the inclination of a line to the h. p. by the Greek letter θ (theta), and that to the v. p. by φ (phi), when the actual number of degrees is not required.

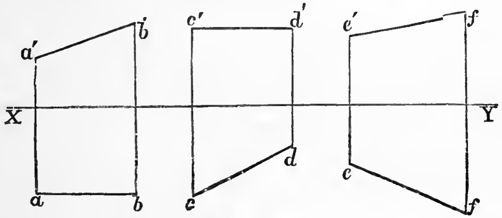


Fig. 104.

In fig. 104 the projections of three lines are shown.

The line A B is inclined to the h. p., but is parallel to the v. p.

The line C D is inclined to the v. p., but is parallel to the h. p.

The line E F is inclined to both planes.

Problem V.

To determine the projections of a Line, A B, 3" long, which is parallel to the v. p., and 1" in front of it; its extremities being ·5" and 1·2" above the h. p. respectively.

As the line is parallel to the v. p., its full length and inclination will be shown in the elevation. Draw two

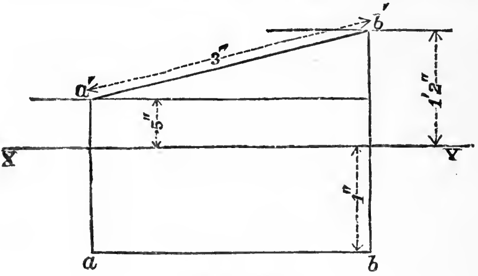


Fig. 105.

parallel lines ·5" and 1·2" above X Y, and assume any point upon one of them as the elevation of the extremity, A. With *a′* as centre, and radius equal to the length of the line, describe an arc, intersecting the remaining parallel in *b′*, which will be the elevation of the other extremity. Join *a′ b′* to complete the elevation.

The plan will be a line parallel to X Y, and 1" below it; the points *a* and *b* being determined by projectors from *a′* and *b′*.

Problem VI.

A Line, A B, 3" long, is inclined to the h. p. at 36°, and its plan makes an angle of 20° with X Y. Show the elevation.

In such problems as these, it is convenient to imagine the line as lying upon the surface of a cone.* If it have one extremity in the apex of that cone, the surface of the solid will be the *locus* of all lines having the same inclination as the side of the cone. Commence this problem by drawing a line, B a', upon the v. p., at an angle of 36°, with X Y, and 3″ long. A projector from a' will determine upon X Y the point a, which would be the centre of the base of a cone upon which the line must lie. Describe the arc B b c, as in the figure. This arc will represent part of the plan of the base of the solid; and the radii of this curve would be the plans of all the lines which lie upon the surface of the cone, having their extremities in the apex and the base of the solid. Draw $a\,b$ at an angle of 20° with X Y, and it will be the plan required. A projector from b will determine the point, b', and a' b' will be the required elevation.

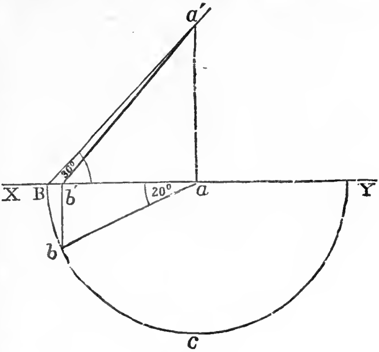


Fig. 106.

Problem VII.

Given, the projections of a Line, A B, *to determine* (1) *its traces;* (2) *its actual length; and* (3) *its inclinations to both planes of projection.*

If a line makes an angle with a plane, it either penetrates that plane, or would do so if it were produced far

* For description of cone, see Chapter II. (Solid Geometry.)

enough. The point where the penetration takes place is called the *trace* of the line upon that plane. The horizontal trace (*h. t.*) of a line, therefore, is the intersection of that line with the h. p.; and the vertical trace (*v. t.*) of a line is its intersection with the v. p.

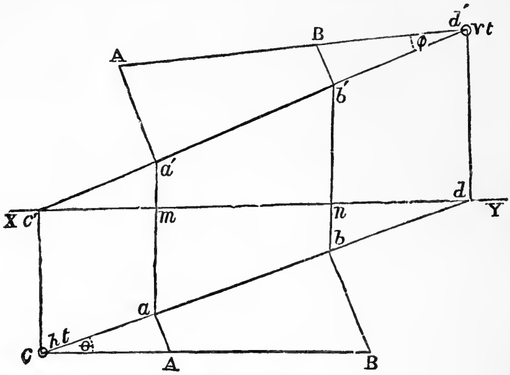


Fig. 107.

If a line be parallel to either plane, it will of course have no trace upon it.

Let *a′ b′*, *a b* be the projections of a line, A B. It is inclined to both the co-ordinate planes, and will therefore have a trace upon each of them, if produced. To discover the position of the h. t., produce the elevation until it intersects X Y in the point *c′*. Then the line *b′ c′* is the elevation of the line which penetrates the h. p. in the point C. If a projector be drawn from *c′* until it meets the plan, *a b*, produced in *c*, that point will be the h. t. required.

The v. t. of the line is determined by a similar construction. The plan must be produced beyond *b*, until it meets X Y in *d*, and a projector from that point, meeting the elevation produced in *d′*, will give the required v. t.

As both the projections of the line make angles with

X Y, neither of them is as long as the line itself. To determine the true length of A B, the following construction is necessary:—Conceive the real line as supported in its position above the h. p. by its vertical projectors, and the whole arrangement to revolve upon the plan as an axis, until it coincides with the h. p. This is called "*constructing*" the line into the h. p. The points A and B will then be upon the lines, A *a*, B *b*, perpendicular to the plan, *a b*, at distances equal to the heights of those points, as shown in the elevation. Thus the length *a* A is equal to *m a′*, and *b* B to *n b′*.

Then A B is the required real length of the line.

If A B be produced beyond A, it will pass through the h. t., and the angle which it makes with the plan will be the inclination of the line to the h. p.

In the figure the line is shown also "constructed" into the v. p. by similar means. It is produced beyond B′, and passes through the v. t. of the line, the angle shown being the inclination of A B to the v. p.

Note.—When the inclinations are small, it sometimes happens that the traces fall without the paper; but in that case the true length can be determined as above, and a parallel to the projection used, intersecting the true length found, will give the angle of inclination.

It is important that this problem should be well understood, as it is frequently necessary to determine the inclinations of edges of solids in the midst of complex drawings. The construction explained above offers a very ready means of doing this, although other methods are sometimes preferable.

Problem VIII.

A Triangle, A B C, *is represented in plan by an equilateral triangle of* 1″ *side. The points* A, B, *and* C *are* ·5″, 1·2″, *and* ·8″ *above the paper, respectively. What is the true shape of the figure, and the inclination of the side,* A B?

The real length of the sides of the triangle can be determined by the construction explained in the preceding problem from their projections. The triangle made up of the lengths thus found will be the true shape of the figure. The elevation need not necessarily be drawn, as the heights of the points are given in the question. Set

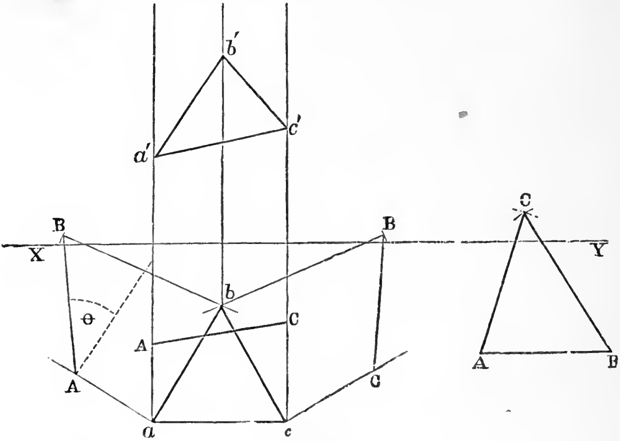


Fig. 108.

out perpendiculars from the extremities of each of the three lines in plan. Mark off along these perpendiculars distances equal to the heights of the points. Join as shown in diagram. Then A B, B C, C A are the real lengths of the sides of the triangle. The figure, A B C, is the one required, whose sides are equal to these three lines. The inclination of the line A B is shown by the angle θ which A B makes with its plan.

Problem IX.

To determine the plan and elevation of a Line (length at pleasure) inclined at 60° *to the h. p., and* 20° *to the v. p.*

If a number of lines lie upon the surface of a cone which stands with its base upon the paper, each having one of its extremities in the apex, these lines will all be equally inclined to the horizontal, and moreover make with the paper the same angle which the sides of the cone make with its base.

In fact, the surface of a cone whose base angle is 60° is the locus of all straight lines which pass through the apex, and are inclined at that angle to the h. p.

At any point A, therefore, in X Y, draw a line, A *b′*, making an angle of 60° with it. Draw *b′ b* perpendicular, and consider A *b′ b* as half elevation of a cone. Then an arc, having *b* for its centre, and *b* A for radius, will represent part of its plan.

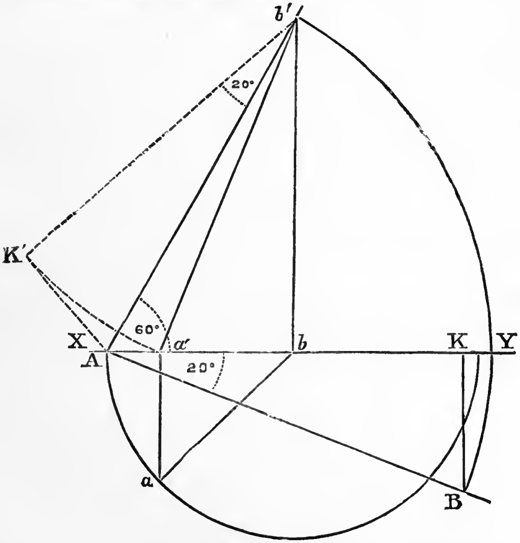


Fig. 109.

Now, if lines through B be conceived to lie upon the surface of the solid, only two of them—viz., those on the extreme right and left—will be shown in their full length in elevation. As the line travels round the solid, its elevation alters in length. When it is in such a position as this, it makes an angle with the v. p.

Our problem, therefore, is to determine the exact position upon the cone, when the line is inclined 20° to the v. p. There are, of course, four solutions—two when the line is in front of the cone, and two when behind.

At the point A, set out a line A B, as long as the side of the cone, and making an angle of 20° with X Y. Draw B K perpendicular to the ground line, and the length, A K, is that of the elevation of the line when it makes an angle of 20° with the v. p. With *b′* as centre, radius equal to A K, describe an arc intersecting X Y in *a′*. Join *a′ b′*. This is the required elevation. A projector from *a′* meeting the arc first drawn in *a* gives the plan of one extremity, A, of the line. Join *a b*, and the problem will be solved.

Another method of obtaining the same result (see fig. 109) is as follows:—Set out *b′* K′ at 20° to *b′* A. Draw A K′ perpendicular to *b′* K′. With *b′* as centre describe the arc K′ *a′*, and complete as before.*

PROBLEM X.

A line A B, 3″ *long, has its extremity,* B, *in the h. p., and* ·75″ *in front of* X Y; *its extremity,* A, *is in the v. p. The inclination of the line is* 40°. *Draw plan and elevation.*

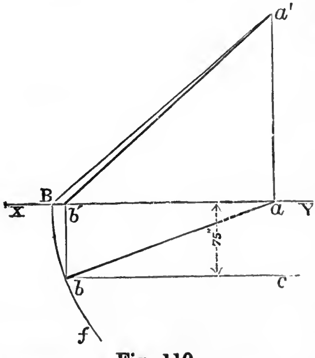


Fig. 110.

At any point, B, in X Y, make the line B *a′* at an angle of 40°. B *a′* must be 3″ long. Draw *a′ a* perpendicular to X Y, and describe the arc B *f*, with *a* as centre. Make *b c* parallel to X Y, at a distance ·75″ from it, intersecting the arc B *f* in *b*. Then *b a* is the plan of the required line, and its elevation, *b′ a′*, is determined by

* This is only one of the solutions, but the other three are not very difficult to determine, if the above be understood

drawing a projector through b to meet X Y in b', and joining b' a'. In this construction a cone has been assumed, having a base angle of 40°, the apex a' of the solid being in the v. p., and its sloping side 3″ long. The line in the question was then supposed to lie upon the surface of this cone, having one extremity in the apex and the other in the edge of the base, at a point ·75″ from the ground line. By this means it was ensured that the ends of the line should be in the co-ordinate planes, as required.

Problem XI.

Consider a', b', and c (fig. 111) *as three points, two of which* (a' and b') *are upon the v. p., and the other one upon the h. p. Consider these points joined by lines thus: a' to c and b' to c, what would be their lengths. Determine also a portion of each of these lines, 1″ long, measuring from c.*

First note that c is immediately under a', so that the line joining these stands ladder-wise towards the v. p., while the line joining c to b' is askew to each plane. Determine a' c', c' c, b' c', and b c, the projections of the lines. With c' as centre, revolve the plan c' c into the v. p. Join a' C. This gives the full length of a' C. Cut off 1″ from C

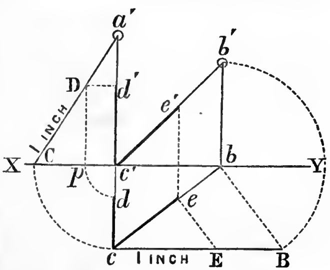


Fig. 111 (Scale ½).

to D, and project d' from D, and d from p. Next find true length of B C, as in Problem VII., and mark off 1″ on the "constructed" plan c E. Draw E e perpendicular to c b, and from e obtain e'. The darkened portions show the answers.

Problem XII.

Given the projections of a point A (fig. 112), *to determine those of a second point* B, 2″ *from* A, *and twice as far from each of the co-ordinate planes as* A *is.*

Commence by drawing *a′* P and *a* R parallel to X Y. Then draw two other parallels relatively twice as far from X Y. These are lettered Q and S. It is evident that the projections of B must occur on these two lines. From *a′*, with a radius of 2″, cut Q in B′. Join *a′* B′. This is the elevation of a line 2″ long measured from A, and *a* B is its plan. Note, however, that the line under these circumstances is parallel to the v. p. Now revolve the plan *a* B about *a*, until it crosses the level S in *b*. Join *a b*. Then *a b* is the plan of a 2″ line having one extremity in A; and if we consider the upper extremity to have maintained its former height above the h. p., then the new elevation of *b* will be at *b′*, and *b′ b* will be the projections required.

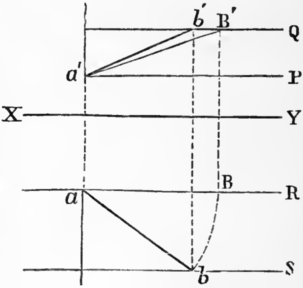


Fig. 112 (Scale ½).

Problem XIII.

Given a′ b′, a b the projections of a line, and c′ c those of a point; through C *to draw an equal line to* A B *and parallel to it. Then to discover the real form of the parallelogram made by joining their extremities towards the same parts.*

Let *a′ b*, *a b* (fig. 113) and *c′ c* be the given data. Parallel lines *always* have parallel projections and, *equal* parallel lines have *equal* parallel projections.

Hence, draw c' d' and c d equal and parallel to a' b', a b respectively. Then join a' to c', b' to d; a to c, and b to d. To find the true form of this figure it is absolutely necessary to divide it into two triangles by drawing the projections of *one* of its diagonals as a' d', a d; and it should be *noticed* that to discover the true form of any figure having more than three sides, it must first be divided into triangles, and the result obtained by building them

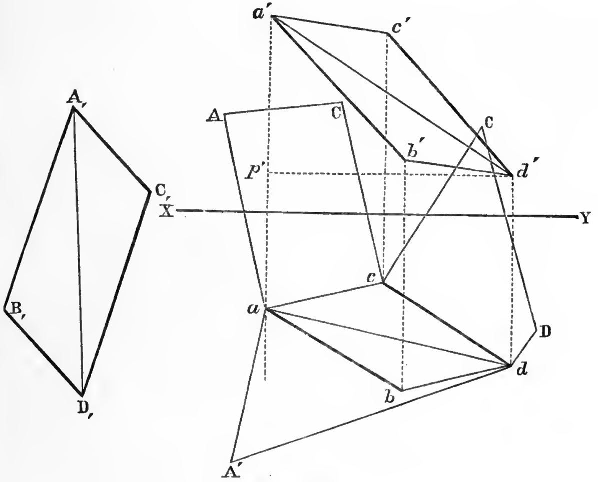


Fig. 113.

together in their proper relative positions afterwards. The true lengths of A C and C D are found as in Problem VIII., but a slight modification is adopted in the case of the diagonal. Instead of taking the full heights of points A and D, the latter is supposed to be on the h. p., and A′ is found by making a A′ equal to the DIFFERENCE only in the levels of A and D. Thus a A′ $= p'$ a' in the elevation. This often saves room, and facilitates a convenient construction. Then the figure A B C D is made up by first constructing the triangle A C D with the newly discovered lengths as sides, and the other half is finished by parallels.

PROBLEM XIV.

Given the projections of two straight lines, to determine whether they meet.

If they do meet, the intersection point in plan must be the plan of the point common to both lines; hence it must be perpendicularly under the crossing point of the elevations.

EXERCISES.

1. Show the plans and elevations of the following points, using the same ground-line for all. The distance between the lines joining the projections in each case may be 1″:—

A, 3″ above	the horizontal	plane,	1·8″ before	the vertical	plane.
B, 2″ above	,,	,,	3·5″ behind	,,	,,
C, 2·5″ below	,,	,,	·4″ before	,,	,,
D, in	,,	,,	2·5″ before	,,	,,

2. *a b*, two inches apart, are the *plans* of two points, of which A is 1·7, B 3· inches above the paper. What is the length, and inclination to the paper, of the line A B?

3. Draw the *plan* of a line three inches long when inclined at 40° and an elevation of it on any vertical plane not parallel to the line.

4. A line 3·5″ long is to be represented by its projections, according to each of the following conditions in turn—

   a. When inclined to the paper at 60°.

   b. When its ends are 1″ and 2·5″ above the paper.

5. The plan of a line is 2″ long and its elevation is 3″. The *projectors* of its extremities are 1″ apart, measured along X Y. What is its true length and inclination?

6. Draw the plan and elevation of a line (length at pleasure) which is inclined at 30° to the horizontal, and at 40° to the vertical plane.

7. Draw the plan and elevation of a point A, which is situated above the horizontal plane, 2″ behind the vertical plane, and is 3″ distant from X Y.

8. A line, A B, 3″ long, is inclined 30° to the vertical plane. Draw its projections when its plan makes an angle of 50° with X Y.

9. Draw an X Y, and mark a point *c* below it, and 1″ from it. Consider *c* as a point on the h. p. Find a point *a′* on the v. p., 3″ from *c*, and such that a line joining *a′* to *c* is inclined 25° to the h. p.

10. What are the traces of a line? When will a line have only one trace, and when none? A line has two traces 3″ apart, it is inclined 40°, and its h. t. is 1″ below X Y. Find its v. t.

11. Three lines, *a p*, *b p*, *c p*, meet in a point *p*, at angles of 110°, 120°, 130°. *a p*=1″, *b p*=1·5″, *c p*=2″. They are the plans of 3 sticks meeting in a point 3″ over *p*. Find the real length of each of the 3 sticks.

12. Draw the projections of a line 3″ long, inclined 57° to the h. p., and 33° to v. p.

Hint.—The student must note that when the inclinations θ and ϕ together make 90°, that the line takes a position to the two co-ordinate planes similar to the position of a ladder, *i.e.*, its projections must each be perpendicular to X Y, and also that the sum can never exceed 90°. (See Problem IX.)

13. Draw a line *a′ b′* above X Y (the lower end to be ·5″ from X Y), and at an angle of 30° with it. This is the elevation of a line inclined 40° to the v. p. The plan of *a′* is 1″ from X Y and below it. Finish the plan of the line.

14. Draw the projections of a line 3″ long, passing through the ground-line, and making 45° with each plane, then work the same problem if 30° be substituted for 45°.

15. Draw the projections of any line inclined to each co-ordinate plane. Assume the projections of any point not in this line. Take any point on the h. p. and join it to the plan of the first point. Assuming this new line to be the plan of a line intersecting the first line, find its elevation.

CHAPTER II.

ELEMENTARY PROBLEMS ON SOLIDS.

N.B.—(The problems of this chapter are to be studied simultaneously with those of Chapter I.)

The following are the solids commonly used to illustrate the principles of Solid Geometry :—

The **cube, prism,** and **pyramid**; the **sphere, cone,** and **cylinder.**

A **cube** is a solid having six equal faces, all squares.

A **right prism** is a solid having two equal and similar bases, with rectangular faces perpendicular to them.

If the faces be not perpendicular to the bases, the prism is oblique.

A **right pyramid** has one base and a number of triangular faces, meeting in a point over the centre of that base. This point is called the apex.

If the apex be not over the centre of the base, the pyramid is oblique.

Prisms and pyramids are named from the shapes of their bases, as square prism, hexagonal pyramid, &c.

A **sphere** is a solid whose surface is at all parts equidistant from a point within it, called the centre. If a semicircle revolve upon its diameter, it generates the surface of a sphere. All plane sections of a sphere are circles.

A **cone** may be defined as a pyramid with an infinite number of faces. It has a circular base, and its surface is generated by the revolution of a right-angled triangle upon its perpendicular, as an axis.

A **cylinder** bears a similar relation to a prism that a cone does to a pyramid. It is a prism with an infinite number of faces. Its bases are circles, and its surface is generated by the revolution of a rectangle upon one of its sides.

The **axis** of a prism or cylinder is a line joining the centres of the bases.

The **axis** of a cone or pyramid is a line joining the centre of its base to the apex.

A **tetrahedron** is a solid having four faces, each being an equilateral triangle.

An **octahedron** has eight faces, all equilateral triangles. It consists of two square pyramids, placed base to base (the height of each pyramid being equal to half the diagonal of the square).

Models of these solids should be seen by the student; and no teacher should be without them. Those made of wire, painted white, are the best, as they are so readily seen at a distance, when placed before a black board.

In figures 114 and 115 plans and elevations of these solids are shown, which it is most advisable the student should work out for himself.

Fig. A is the plan and elevation of a cube when it stands with its base upon the h. p. Its plan, *a b c d*, is a square which should be drawn first. In this figure it is assumed that neither of the faces of the solid is to be parallel to the v. p. Each corner of the square is the plan of one of the perpendicular edges of the cube, and *a b c d* is that of the upper surface also.

Projectors from *a b c d* will determine upon X Y the elevations of the four corners of the lower square. The perpendiculars, $a'\ e'$, $b'\ f'$, $c'\ g'$, and $d'\ h'$ must next be drawn, equal in length to the edge of the cube, and a line passing through $e'\ f'\ g'\ h'$ will complete the elevation.

The line $b'\ f'$ must be a dotted one, as it represents the perpendicular edge, which would be hidden from an observer placed in front of the object.

Fig. B is the plan and elevation of a hexagonal prism standing with its base upon the h. p. The construction of the projections is similar to that of the cube. It should be noticed that the angle which the line *a b*

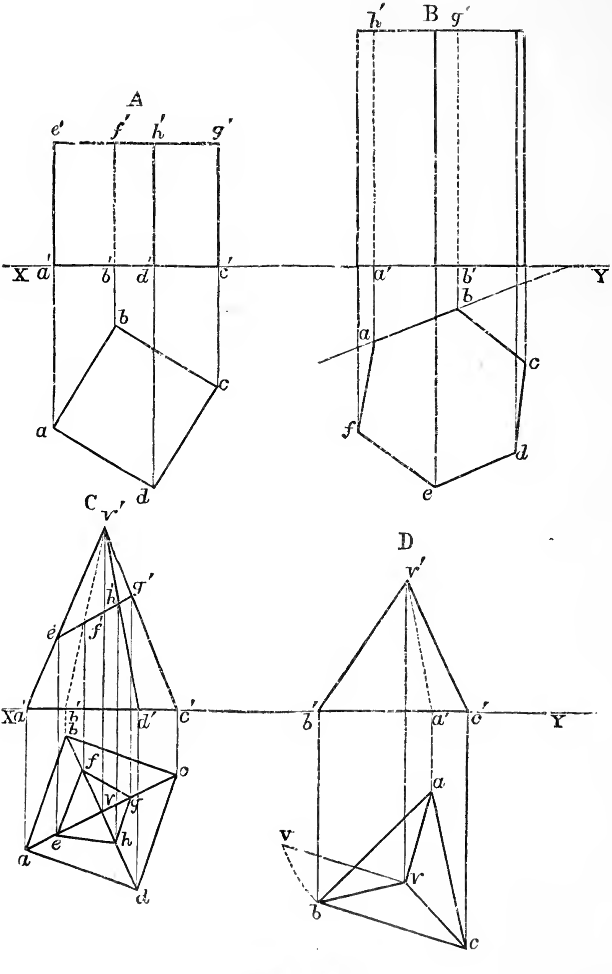


Fig. 114.

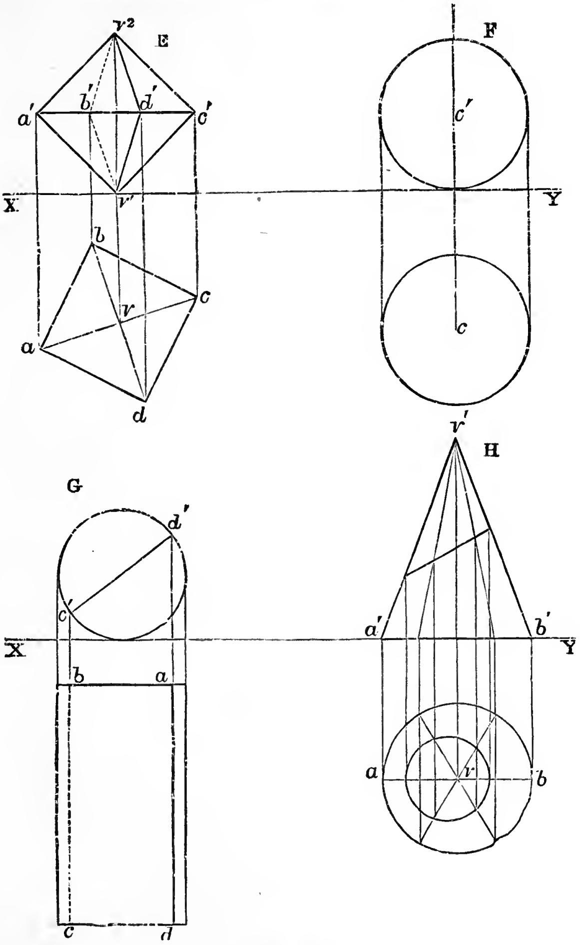


Fig. 115.

makes with X Y is the angle of inclination of the face, A B G H, to the v. p.

Fig. C is the plan and elevation of a pyramid with its base upon the h. p. Commence by drawing the square *a b c d*, and join the points *a c* and *b d*. This completes the plan, as the square is the plan of the base of the solid, and the diagonals that of the sloping edges. The point *v*, where the diagonals intersect, is the plan of the apex of the pyramid. Projectors from the four corners of the square will determine the elevation of the base upon X Y. That through *v* will contain the elevation of the vertex, and its exact position is obtained by measuring the height of the pyramid above X Y. The line *v′ b′* must be dotted.

A section line *e′ g′* is shown in the figure. This line is the elevation of a section made by a cutting plane. It is clear that the points *e′ f′ g′ h′* in that line are the elevations of the points where this cutting plane intersects the sloping edges of the pyramid, and as plan and elevation are always in the same straight line perpendicular to X Y, the plans of these points can be found by drawing projectors through *e′ f′ g′ h′* until they intersect the plans of the sloping edges in *e f g* and *h*. Join these and the plan of the section will be determined. This is not the real shape of the section. A method for determining this will be given in the next chapter

Fig. D is the plan and elevation of a tetrahedron resting with one of its faces upon the h. p. Its plan is an equilateral triangle, having lines joining each of its angular points to the centre of the figure. The point *v* is the plan of the axis of the solid. The elevation of the base is determined as before, and a projector *v′v* will contain the elevation of the apex; but as the height of the figure is not given, depending as it does upon the length of the edge, a special construction is necessary for determining it.

The plan of a sloping edge, that edge itself and the axis of the solid together form a right-angled triangle. The plan is the base, the axis is the perpendicular, and the sloping edge, the hypotenuse.

Now, if this triangle be imagined to revolve upon its base, until it is horizontal, it is clear that the height of the figure will be shown in the perpendicular of the triangle.

Set out, therefore, from *v*, a line *v* V, perpendicular to *a v*, and with *a* as centre, radius *a b*, describe an arc *b* V, intersecting the perpendicular in V. Then *v* V is the height of the tetrahedron. The completion of the elevation presents no further difficulty.

Fig. E is the plan and elevation of an octahedron having its axis vertical. The plan of it is the square *a b c d*, with its diagonals. The octahedron, as can be seen by a model, has three equal diagonals or lines upon which the solid can revolve. Two of these diagonals are shown in the plan, and the third is the vertical axis. As these are all equal, it only remains for us to measure either of the diagonals *a c* or *b d*, and use the length found as the height of the elevation. The elevation of the square will be a straight line a' b' c' d' mid-way between the apices v^1 and v^2.

Fig. F is the plan and elevation of a sphere. Both are circles; further explanation is unnecessary.

Fig. G is the plan and elevation of a cylinder when it rests with its side upon the h. p. and with its base, parallel to the v. p. In this case it is necessary to commence with the elevation which is a circle tangent to the ground line. The plan is a rectangle. A section is shown in elevation by the line c' d' and its plan, *a b c d* is determined from the elevation by projectors as before.

Fig. H is the plan and elevation of a cone, resting with its base upon the h. p. A section of this solid is given and the plan shown, which is determined by supposing the figure to be a pyramid. If the circle in plan be divided into any convenient number of parts and the points of division be joined to the centre, these lines will represent the plans of as many sloping lines upon the cone. Their elevations will be cut by the line of section and the plans of the points can be determined by projectors as in fig. H.

A curve (generally an ellipse) drawn through the plans of the points gives that of the entire section.

Problem XV.

A square block 4" edge by 1" thick rests upon the h. p., *another square block of 2" edge, 1" thick, stands upon the former over its centre; the similar edges of each solid being parallel. The upper surface of the second block is the base of a right pyramid 3" high. Draw a Plan of the whole and an Elevation upon a v. p., which makes an angle of 20° with any horizontal edge in the group.*

Commence with the plan, which is a square of 4" side having another square of 2" edge within it; the centres of both being coincident, and their sides parallel to each other. Draw the diagonals of the smaller square, and the plan of the group will be complete. Assume a ground line, making 20° with either of the sides of the square, and determine the elevation of the lower plinth, remembering that the height of the perpendicular edges is 1". Above this draw the elevation of the second plinth and the axis of the pyramid.

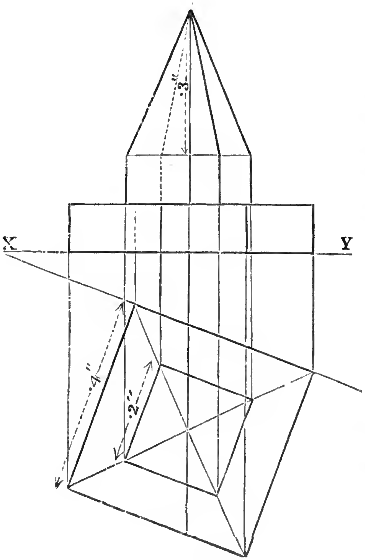


Fig. 116.

Mark off 3" upon the latter, measuring from the highest surface of the second block, and the elevation of the apex of the pyramid will be obtained. Join this point to the four corners of the top surface of the smaller plinth, and the elevation will be complete.

EXERCISES.

1. Draw plan and elevation of a cube of 2″ edge, when its base is horizontal and ·5″ above the paper; its horizontal edges making angles of 30° with the vertical plane.

2. Draw plan and elevation of a square prism (size at pleasure), when its long edges are horizontal, and one of its faces makes an angle of 27° with the paper.

3. Draw a plan and elevation of a cube, with a square prism standing upon it, the axes of both solids forming one straight line, the horizontal edges of the prism to make angles of 45° with those of the cube. The elevation is to be drawn upon a vertical plane making an angle of 30° with one face of the cube.

4. Draw plan and elevation of a tetrahedron of 1·5″ edge when its axis is vertical.

5. Show, by its projections, an octahedron when its axis is horizontal and perpendicular to the vertical plane.

6. A cone, base 1″ radius, 3″ high, is cut by a plane at 70° with the axis; the centre of the section being 2″ above the base. Show the plan of the cut.

7. A rectangular box, 4″ by 3″ and 1″ high, supports a cube of 2″ edge upon it, the sides of the latter being parallel to those of the former, and the axes of both being vertical and in the same straight line. A cone stands upon the cube over its centre, base 1″ radius, 3″ high. Draw a plan of the whole and an elevation upon a vertical plane, making an angle of 30° with a face of the box.

8. A circular slab, 2″ radius, 1″ thick, supports upon its upper surface and over its centre a cylinder, base 1″ radius, and 3″ high. Draw plan and elevation, and show upon the plan the projection of a section indicated upon the elevation, by a straight line joining the top left-hand point of the cylinder to the bottom right-hand point of the slab.

CHAPTER III.

ON ALTERATION OF THE GROUND LINE.

It is frequently necessary that different views of an object shall be given, so as to perfectly depict it. Thus we have *end* views, *side* views, etc., and in architectural drawings, *North* Elevation, *East* Elevation, &c. To understand the principles upon which such views are deduced and how they are connected, the student must assume that new vertical planes may be taken for *auxiliary* elevations, or new horizontal planes for plans, and the object can then be projected upon these new planes, thus giving the different views described above. Consideration of the problems which follow will make the subject clear.

DIVISION I.—NEW ELEVATIONS.

PROBLEM XVI.

Given the projections of three points, A, B, C (fig. 117); *to determine a New Elevation on a v. p. represented by* X_1 Y_1.

Let a' a, b' b, and c' c be the given projections. Note that the v. p. upon which the elevations are made is supposed to meet the h. p. in X Y, and that the observer is understood to be opposite to this v. p. Now, assume that a new v. p. is raised cutting the h. p. in X_1 Y_1,

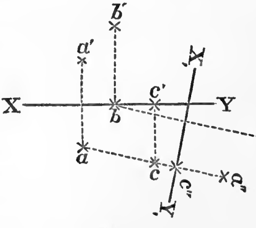


Fig. 117.

and that the observer takes a new position opposite to this plane. Then projectors through *a*, *b*, and *c* will give lines upon which the required new elevations must occur; the heights measured along these, above $X_1\ Y_1$, being equal to the heights of the points as indicated in the first elevation. Thus *b*, *b'* gives the height of B above the paper; hence *b″* is taken at that height above $X_{\prime\prime}Y_{\prime\prime}$ Note that in the case given B is on the first v. p., and C is on the h. p.; also, that C and A are on one projector in the new elevation.

Problem XVII.

Given the projections of 2 *lines* A B, B C (fig. 118), *to make New Elevations of them on other vertical planes.*

Having drawn *a b c* and *a' b' c'*, assume a new $X_{\prime\prime}\ Y_{\prime\prime}$ and as in the last case, proceed to project the three

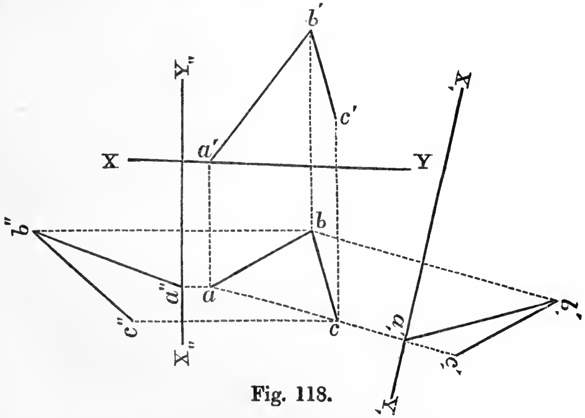


Fig. 118.

points, taking the heights from the first elevation. This gives the view *a″ b″ c″*. Then for practice take a third ground line X, Y, so that it is perpendicular to the line joining A to C, in the plan. This will show the student how several elevations can be deduced from one plan.

Problem XVIII.

A pentagonal prism lies with one face on the h. p., and with its ends parallel to the v. p. To determine a New Elevation on a given X, Y.

The elevation or end view should be drawn first, then the plan can be deduced therefrom, as in fig. 119. Let

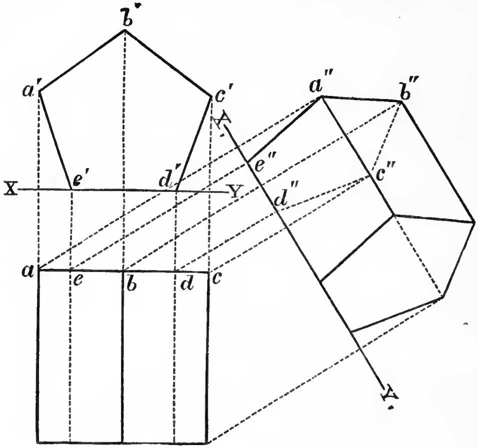


Fig. 119.

X Y be the new vertical plane. Project each corner of the pentagons perpendicularly to the new ground line, and take heights above it, equal to those in the first elevation. Note that the dotted edges are settled by determining which pentagon can be seen under the new disposition. It is clear that the lettered one must be hidden. Hence the dotting.

In fig. 120 a similar case to the above is shown, where the subject is a cylinder. The end view, a circle on X Y, is divided into a convenient number of equal parts. The plan is then striped by lines projected from the points of division. Then a second elevation is made upon X_1 Y_1, by treating the points found upon the plans of the base, as the corners of the prism were

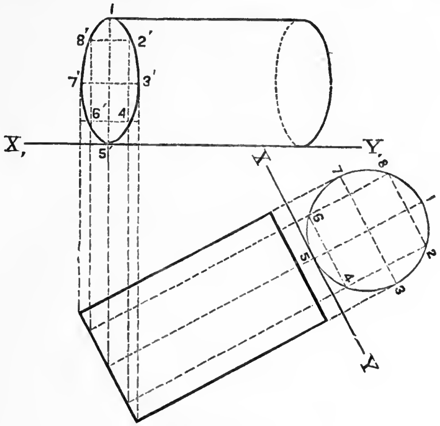


Fig. 120.

treated in the last figure. The ends, as they are oblique circles, will appear as ellipses, and the student must draw these curves as neatly as he can through the points he finds.

Problem XIX.

An octahedron is to be drawn, with one of its faces on the h. p., and two elevations are to be made, one upon a v. p. parallel to two edges of the solid, and the other upon a v. p. not parallel to any edge.

The octahedron is one of the *five regular solids,—i.e.,* the edges are all the same length, its faces the same shape and size, and its angular points are all equidistant from its centre. Before commencing this problem, it is necessary to call the student's attention to the fact that the opposite faces are parallel, and that their corners alternate. Now, as there are only 6 points or corners, it

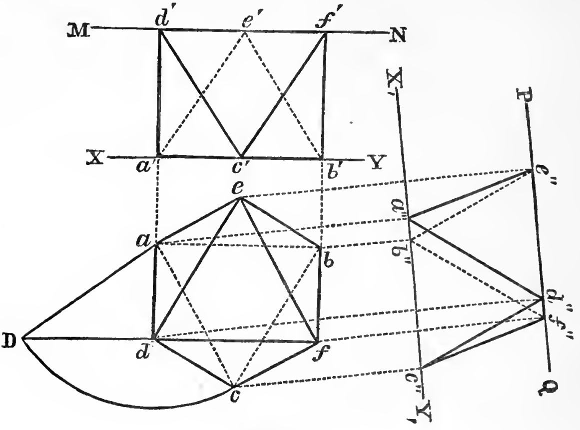


Fig. 121.

will be easily seen that when the solid has one triangular face on the h. p., whose plan therefore is an equilateral triangle, that the opposite face being parallel, it must be shown in plan as a concentric equilateral triangle with points reversed. Therefore, commence by drawing *a b c* (fig. 121) dotted, and *d e f* full (two equilateral triangles) and join the points as shown. This completes the plan. Next, draw an X Y parallel to *a b*, and project the lower triangle on X Y as *a′ b′ c′*. At this point it is necessary to find the distances between the levels of the two parallel faces. A special construction is needed, thus: Taking *a d* in plan, and noting that A is on the lower level while D is on the upper one, and that the line A D is in real length equal to *a c*, draw *d* D perpendicular to *a d* and with *a* as centre, *a c* as radius, describe the arc *c* D to intersect *d* D in D. Then *d* D is the thickness of the solid between parallel faces.

At a distance above X Y, equal to *d* D, draw M N parallel to it. Project points *d′ e′ f′* from their plans upon M N. Note which points are connected in the plan, and join the same relative points in elevation. The fact that D C F is in front to the observer, as discovered by the plan, settles the dotted edges. Next, take a new v. p., not parallel to any edge, as $X_{\prime}$ $Y_{\prime}$ and by using the level P Q obtain a second elevation as required.

In fig. 122 an instructive example of auxiliary elevations is given for practice which should be drawn. Note that the roof overhangs on three sides, but not at the

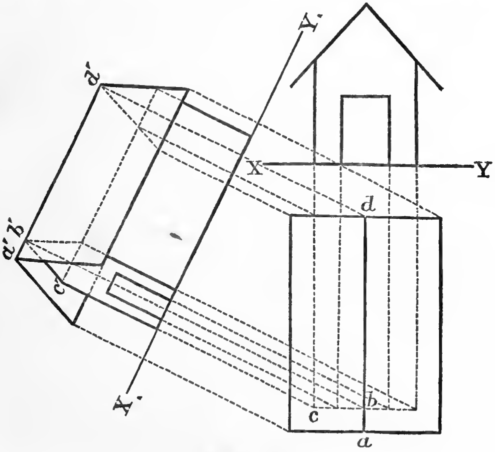


Fig. 122.

back. See also that the line *c′ b′* in the auxiliary elevation is obtained from *c* and *b* in the plan.

Division II.—NEW PLANS AND SECTIONAL VIEWS.

In commencing this chapter we noticed that several elevations could be projected from one plan. Thus, an end view of the instrument-box could be obtained by assuming a vertical plane parallel to that end, or an angular view by arranging our ground line accordingly.

It is now our intention to carry this principle further, and to determine several *plans* from one *elevation*, and *vice versa.*

Let us revert to our illustration of the box upon the table. If it be slightly inclined, it is easily seen that its plan will alter in shape. And if, instead of moving the box, we could suppose the table to be so tilted that its surface should make, with the base of the former, an angle equal to that which existed under the first conditions, the projection of the box upon the plane of the table would then be exactly the same shape as before.

Now, the removal of either of the co-ordinate planes is effected very easily by an alteration of the ground line. And, when it is desired that an object should have one of its surfaces inclined to either of the co-ordinate planes, the most ready means of effecting this is to project the solid first upon planes parallel to the surface in question, and afterwards, by the removal of X Y to assume a fresh plane, upon which the desired drawing can be made.

The consideration of the problems which follow will make this principle clear.

PROBLEM XX.

A Hexagonal Prism has its axis inclined 40° *to the paper, and one face parallel to the* v. p. *Draw plan and elevation.*

Draw the hexagon with one side parallel to X Y, which is the plan of the solid when standing with its base upon the paper. And by arranging the figure in this way the student will see that one face of the object will be parallel to the v. p.

The elevation must be deduced from this plan, as described in the preceding chapter.

Upon the elevation draw K′ K to represent the axis of the solid and produce it.

If a fresh X Y be now assumed, making an angle of 40° with the line K′ K, the elevation will then be that of the solid, with its axis inclined as regards the new h. p. The best way for the student to grasp this principle is to fold his paper upon the new X Y, so as to show a h. p. and a v. p. He will then see his first elevation under quite a different aspect. It will be that of a solid tilted over.

To determine the plan, projectors must be drawn through every point of the elevation, perpendicular to the assumed X Y, and lengths must be measured along each of these projectors, equal to the distances of the points in the first plan, from the first ground line (fig. 123).

Thus, taking the point A for an example, a projector a' 1 passes through a' perpendicular to the new X′ Y′. The distance a' a is transferred along this projector to the point a_1 beyond the ground line; that is, a' a is equal to 1a_1.

All the other points of the base are projected in the same way, and a new plan of it is thus obtained.

The plan of the other end of the solid is similarly determined by projectors through its points in elevation.

And as the first plan is that of both ends, the distances to be measured along the projectors first drawn will be exactly the same as before.

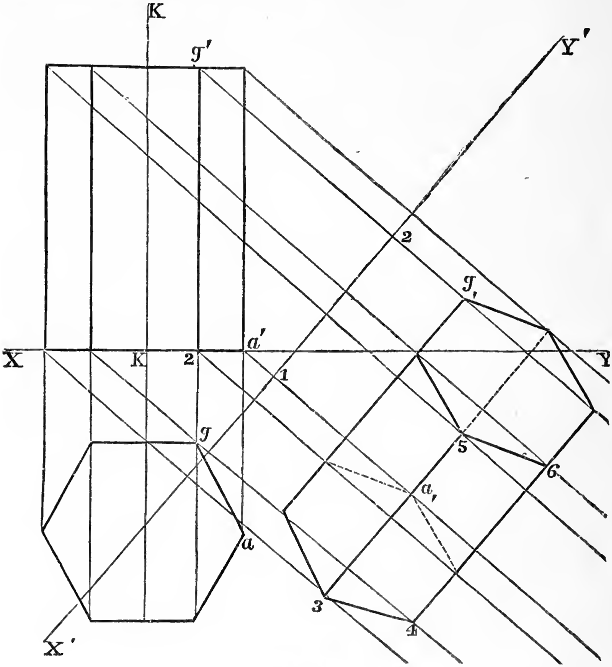


Fig. 123.

To illustrate this, take point g'. The distance measured upon the projector beyond X′ Y′ is equal to 2 g.

The whole plan is completed by joining the similar points in each base, as shown in the diagram.

It should be noticed that lines which are parallel in the solid are still parallel, however they may be projected, thus 3·4 is parallel to 5·6.

A little consideration of the position of the solid will show that that part of the base in which *a* is situated is hidden, and that the opposite end is wholly seen in plan. The edges dotted will indicate this.

PROBLEM XXI.

To draw the projections of a Hexagonal Pyramid when one of its triangular faces is, (1st), horizontal, and (2d), vertical.

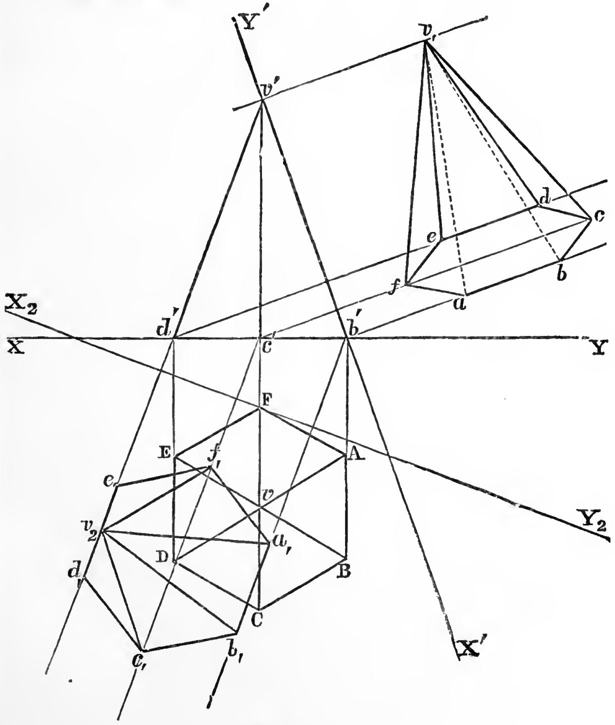


Fig. 124.

Commence with a plan and elevation of the solid, when standing with its base upon the paper, one side of the plan being perpendicular to X Y. It will be seen then, that two of the faces will be represented in the elevation by the lines b' v' and d' v'. If an X Y be assumed containing one of these lines as b' v', the elevation, with regard to the new h. p., will be that of the solid, when its triangular face, A B V, is horizontal, in fact when it is in that plane. A fresh plan of the base can then be determined by projectors perpendicular to the new X′ Y′; the distances of the points in plan from the first v. p. being transferred to the new plan. It should be noticed that the projector, b' a b, has two points upon it at distances from X′ Y′ equal to b' A and b' B respectively.

The apex of the pyramid must be projected in an exactly similar manner and v' v must be made equal to c' v.

In the diagram a plan of the solid is shown when the opposite triangular face is vertical. A new X_2 Y_2 is in that case drawn perpendicular to the line d' v'. Under those circumstances the plan of the vertex falls in the centre of the plan of one edge of the base.

Problem XXII.

To draw the plan and elevation of a Hexagonal Pyramid when one of the edges meeting in the vertex is (1*st*), *horizontal, or* (2*d*), *vertical.*

The solid, as before, must be drawn when resting with its base upon the paper, but it must have one of its diameters or sides parallel to X Y. The effect of putting the plan in this position will be that two opposite edges of the solid will be parallel to the v. p., and consequently their elevations will be as long as the edges themselves. And whenever a line in a solid is to be inclined and fresh plans or elevations of it determined, it is necessary, in the first place, that the object be so drawn that the line in question and one of its projections shall be equal in length.

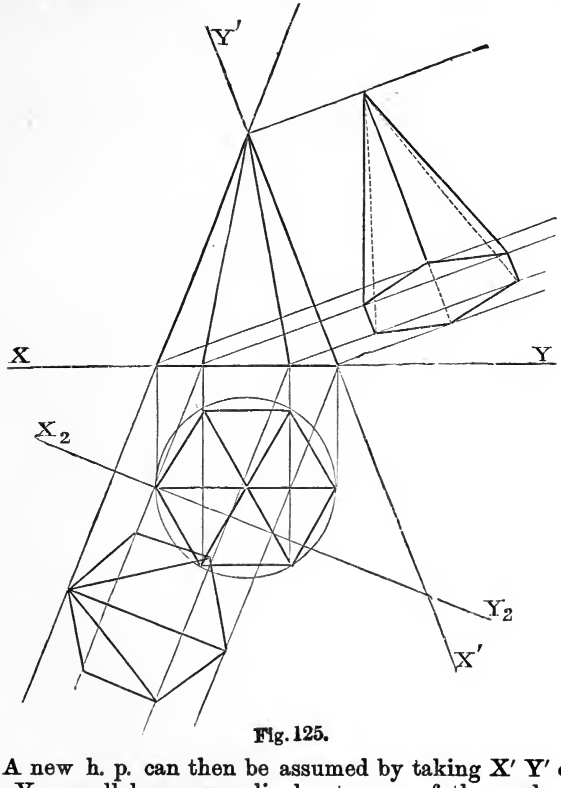


Fig. 125.

A new h. p. can then be assumed by taking X′ Y′ or X_2 Y_2, parallel or perpendicular to one of these edges, according as that line is to be horizontal or vertical.

The determination of the respective plans upon these planes is obtained by the same construction as in the preceding problem.

Problem XXIII.

To draw a plan and elevation of a Square Prism, when the diagonal of the solid is (1*st*), *horizontal, or* (2*d*), *vertical.*

It was shown in the preceding problem that when a line in a solid is to be inclined, and its projections

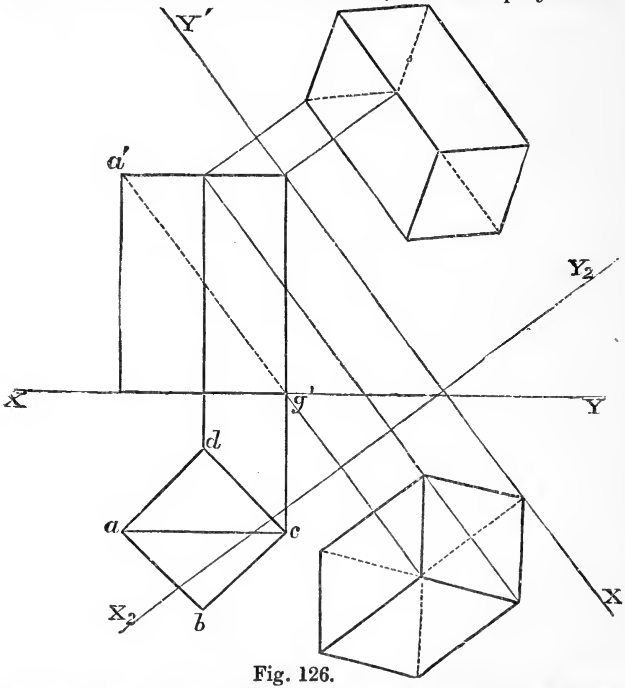


Fig. 126.

determined upon different planes shown by altered ground lines, that the solid must be so arranged at commencement that the line in question may be shown its full length in either plan or elevation, and that this is an invariable rule.

In the problem before us it is necessary so to place the prism that its diagonal may be parallel to the v. p.

The square, *a b c d*, which is first drawn, must therefore have its diagonal parallel to X Y. When the elevation is complete, the line *a′ g′* will represent that diagonal which is to be horizontal or vertical.

A parallel, X′ Y′, to it must be assumed as a ground line, and the plan of the solid determined as before. This will satisfy the first condition in the question.

The second part of the problem is solved by assuming the h. p. perpendicular to the diagonal; that is, by taking X_2 Y_2 at a right angle with *a′ g′*, and proceeding as before. It should be noticed that in the latter case two points coincide in plan, in fact that one point is the projection upon a horizontal plane of a vertical line.

PROBLEM XXIV.

To determine the plan and elevation of an Octahedron when it rests with one of its triangular faces upon the paper.

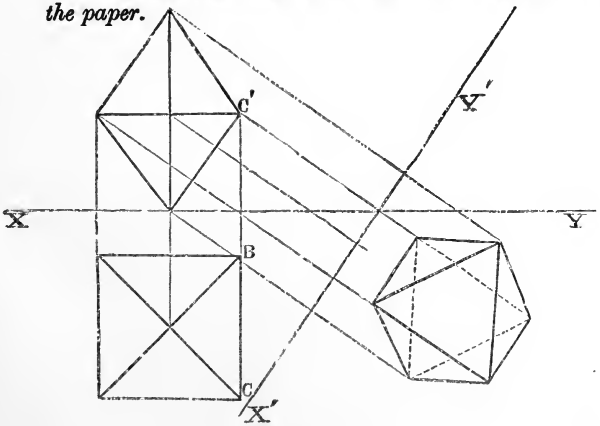


Fig. 127.

Commence with a plan of the solid when its axis is vertical and one of its edges perpendicular to X Y. We thus ensure that the elevations of the triangular faces which have B C for their common base are straight lines.

Assume an X′ Y′ parallel to either of these, as in fig. 127, and determine the plan as before.

The boundary line of an octahedron in this position is a hexagon.

Problem XXV.

Given the projections of any Solid; to determine other projections from them.

Let the figure $a\ c_1$ be the plan of a square prism, of which A′ B′ C′ D′ is the end elevation. And let a new

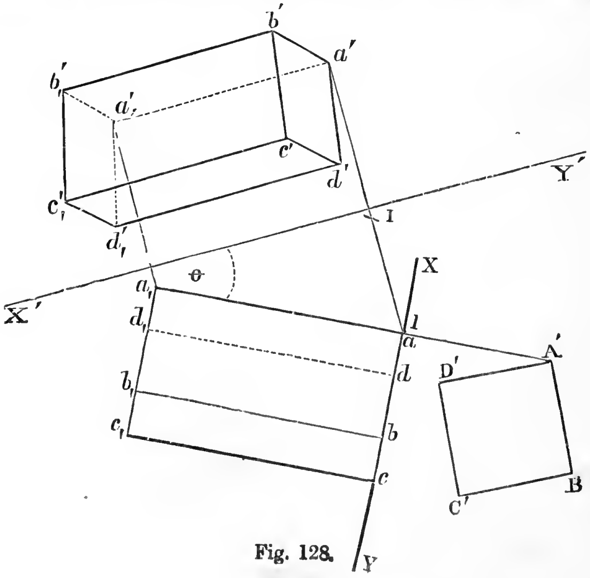


Fig. 128.

elevation be required upon a v. p., making an angle, θ, with the long edges of the solid.

Assume X′ Y′, making the required angle with either of the plans of the sides, as $a\ a_1$.

Projectors through *a b c* and *d* in the plan, perpendicular to X′ Y′, will contain the required elevations of the points A B C D. Now, as the heights of these points above the h. p. are shown in the given end elevation, it is only necessary to transfer them from one elevation to the other. The distance, 1*a*′, in both cases is the same.

The elevation of the end, A, B, C, D, is obtained in a similar manner; and as the solid is lying horizontally upon one of its edges, the heights of the corners are the same as those of *a b c d*.

Problem XXVI.

An Irregular Pyramid has for its base a triangle, A B C. A B = 3″; B C = 3·5″; A C = 4″; *the plan, d, of the fourth corner projected upon the plane of the base, is* 2″ *from* A *and* 1·5″ *from* B. *The true length of the remaining edge,* C D, *is* 3·7″. *Draw the plan of the Pyramid when standing on its base, and an elevation on a plane parallel to the edge* A D. *Determine the length of the edge* B D, *and the inclination of the face* A B D.

The whole of this question turns upon the proper arrangement of the ground line.

Draw a triangle, A B C, having its sides equal to those given in the question. With A as centre, radius 2″, describe an arc, and with B as centre, radius 1·5″, describe another arc, intersecting the former in *d*, which is the plan of the apex, D, of the pyramid.

Join *d* A, *d* B, and *d* C to complete the whole plan. The height of the apex, D, can be obtained by a similar construction to that employed for determining the height of a tetrahedron (Chap. II). At *d* in C *d*,

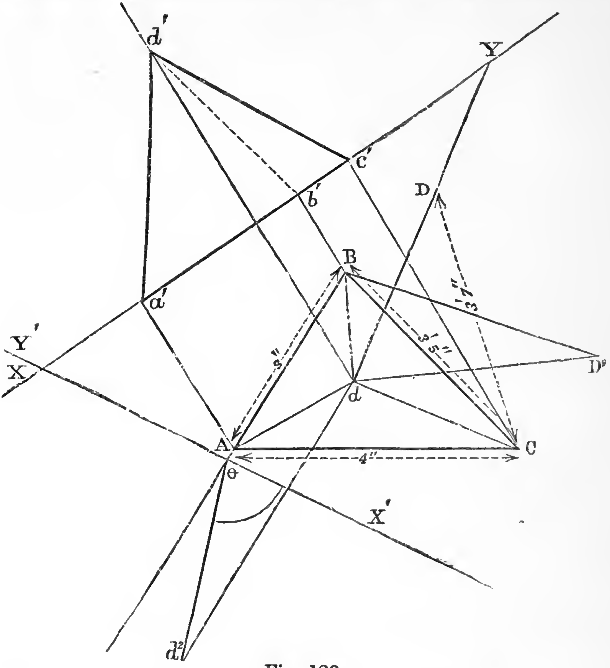


Fig. 129.

draw *d* D perpendicular to it, and with C as centre, radius 3·7″ (the real length of C D), describe an arc, intersecting *d* D in D. Then *d* D is the height of the pyramid.

The elevation upon a plane, parallel to the edge A D will present no difficulty.

To determine the length of the edge B D, set out a perpendicular to B *d* at the point *d*, making *d* D^2 equal to the height of the pyramid, as obtained above. Then B D^2 is the true length required, and D^2 B *d* is its inclination.

If a ground line be assumed perpendicular to the edge A B, and an elevation be made upon the new v. p., the whole face, A B D, will be shown as one line, and the angle it makes with X Y is the inclination of that face to the paper.

PROBLEM XXVII.

Given, the line 1′ 3′ *as the elevation of a section of a Square Prism (fig.* 130); *to determine its true shape.*

Assume the line 1′ 3′ to be the ground line of a plane upon which the plan of the figure 1 2 3 4 is to be projected. Set out projectors from each of the points 1′ 2′ 3′ 4′ perpendicular to the line 1′ 3′ and measure lengths equal to the distances of *a b c* and *d* from X Y. Then, by joining the points found, the true shape of the section will be determined.

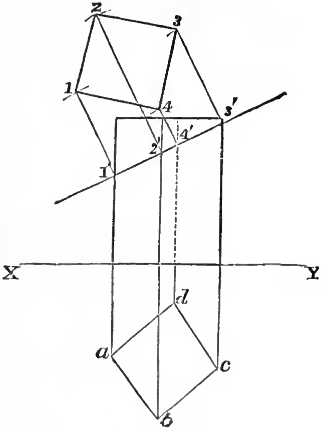


Fig. 130.

PROBLEM XXVIII.

Draw the plan of a Right Pyramid, whose base is a hexagon of 1·25″ *side, and its axis* 3·25″, *when it stands upright upon a horizontal plane. Give the section by a vertical plane which cuts off half of one edge and a quarter of the next, measuring downwards from the vertex.*

When the projections of the pyramid, under the conditions given in the question, have been determined, draw a line of section upon the plan, bisecting *v c* in *g*, and passing through *h*, *v h* being $\frac{1}{4}$ of *v b*.

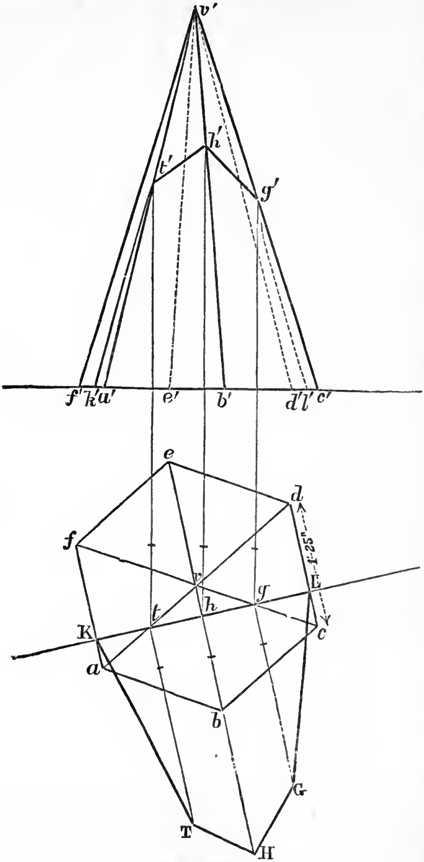


Fig. 131.

The line of section is drawn upon the plan, because the cutting plane is a vertical one. A moment's consideration will tell us that, if a cutting plane be perpendicular to either of the co-ordinate planes, the projection of the section will be a straight line upon that plane. Thus a horizontal section would be shown by a horizontal line upon an elevation.

K L drawn upon the plan, as explained above, shows that two edges, A F and C D, of the base would be cut. Determine the elevation of the section by projectors through the points in plan meeting the elevations of the lines upon which they occur. Two of these, *k'* and *l'*, will be upon X Y. The other three, *h' g'* and *t'*, are found in the usual manner.

The figure *k' t' h' g' l'* will be the elevation of the section.

The true shape is determined when the five points of the section are constructed into the h. p. upon the line K L as an axis.

Those two will remain stationary, and the others, G H and T, will fall in perpendiculars to K L, passing through *g h* and *t*. The distances *h* H, *g* G, and *t* T must be made equal to the heights of H, G, and T above the h. p., as shown in the elevation. Then the figure, K T H G L, will be the required true shape of the section.

Problem XXIX.

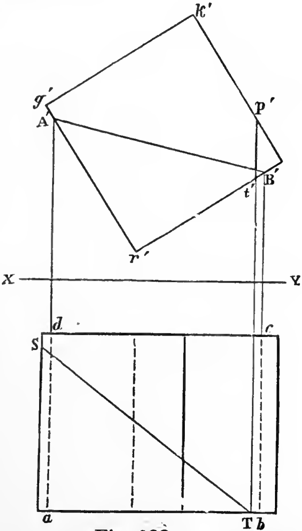


Fig. 132.

Figs. 132, 132*a*, 132*b*, 132*c*, are designed to help the student in determining the plans of sections from elevations, and elevations from plans.

Fig. 132 is the plan and elevation of a cube, with one face inclined 30°. In the elevation, a line A'B' represents a section of the solid perpendicular to the vertical plane. The projectors are shown by which the plan is determined, and *a b c d* is that plan. The student is recommended to try by the aid of a model of the cube (easily cut out of soap) to appreciate the section and its projections. On the plan of the cube, the line S T

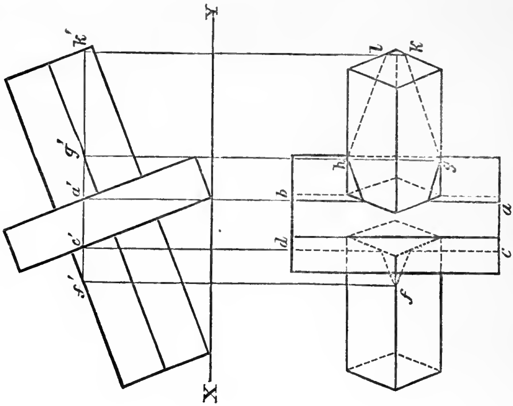


Fig. 132b.

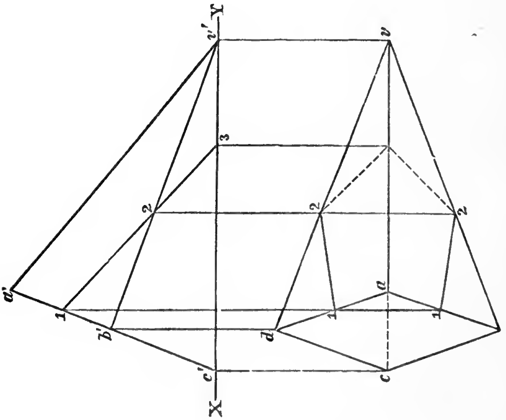


Fig. 132a.

represents a vertical section. The only line which is to be shown in the elevation is $p' t'$; the whole elevation of the section being $p' k' q' r' t'$.

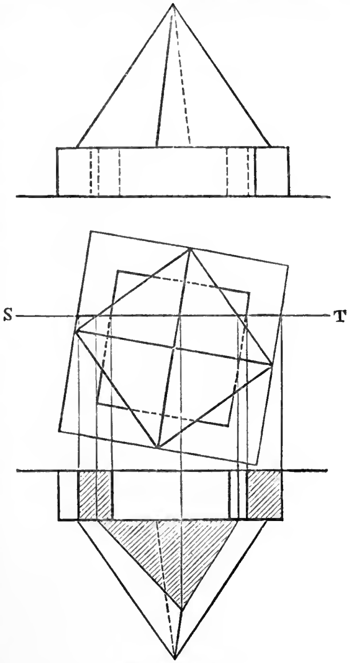
Fig. 132*a* is the plan and elevation of a square pyramid with one edge on the ground. The line 1, 3 in the elevation shows a section, and the plan is determined by the projectors as figured in the diagram.

The drawing is lettered, and the student should follow a line from elevation to plan thus: point 2′ is the elevation of a cut upon B V and D V; refer therefore by a projector to these lines in plan and settle points 2, 2, and so on.

Fig. 132*b* is the plan of a square prism passing through a square block. The line $f k'$ in the elevation represents a horizontal cut. This is a more complicated case, but the five projectors shown are sufficient to determine the plan.

Notice that in this case the plan of the section is really its true shape as it is horizontal, and thoroughly grasp the fact that the projection of a section always gives the true shape when the plane of projection is parallel to that of the cut.

Fig. 132*c* gives plan and elevation of a hollow square plinth, with a square pyramid above it. The line S T is given to represent a plane of section, and a



Figs. 132c.

new $X_1 Y_1$ being taken parallel to the cut, an elevation is obtained of one part *only* of the solid (the part nearest the assumed X Y). This is called a sectional elevation, and the parts in section are shaded by diagonal lines. The drawing speaks for itself as to the projectors required, and the student should carefully notice the lines which should be dotted as being unseen edges.

Note.—It has been assumed in preparing these figures that the student would not require a detailed description of how they are projected, previous lessons being considered sufficient for that purpose.

EXERCISES.

1. Draw plan and elevation of a square pyramid—base 1″ side, height 3″, when one of its long edges is inclined 20° to the paper.

2. A pyramid 3″ high has a pentagon A B C D E of 2″ side for its base. Show this solid by *one* elevation and *two* plans.
   When the two edges B V, C V are *horizontal*.
   When the edge E V is *vertical*.

3. A cylinder 4″ long, its base being a circle of 1·5″ radius, to be drawn in plan and elevation in one of the following positions:—
   a. Its axis inclined to the paper at 35°.
   b. When a vertical plane touches the two ends in opposite points of parallel diameters.

4. A hexagonal prism, base 1″ edge, and 3″ long, has its axis horizontal, one of its faces being inclined 12° to the paper. Draw plan and elevation, and a second elevation upon a vertical plane, making an angle of 40° with the plan of the axis.

5. A cube, 2″ edge, rests upon a circular slab, radius 3″, 1″ thick. The centre of the cube is over the centre of the slab. Draw plan and elevation of the whole, when a corner of the cube and an edge of the slab rest upon the paper.

6. Draw plan and elevation of a tetrahedron, 2″ edge.
   (1.) When one of its faces is vertical.
   (2.) When one edge is vertical.
   (3.) When that edge is horizontal.

7. A pyramid having for its *base* a square 2·5″ side, and its axis 3·25″ long, rests with one *face* on the horizontal plane. Draw its plan, and a sectional elevation on a vertical plane represented by a line bisecting the plan of the axis, and making an angle of 60° with it.

8. An hexagonal pyramid (side of base 1·5″, axis 4″) has one edge of its base horizontal, and the plane of that base inclined at 60°. Draw the plan, and show the real form of the section made by a horizontal plane bisecting the axis when the solid is so inclined.

CHAPTER IV.

ON OBLIQUE PLANES.

If the surface of a solid is not parallel to either of the co-ordinate planes, it is said to be *inclined* to those planes.

Thus, the lid of a mathematical instrument box, when it is slightly open, presents a surface inclined to the horizontal; whilst a shutter hung upon hinges, when partly open, is an instance of a surface inclined to the vertical plane of the window. If the surface of the lid could be produced until it met the ground, it would make by its intersection therewith a line. In the same way, the line upon which the shutter revolves, would represent the intersection of its inclined surface with the vertical plane of the window.

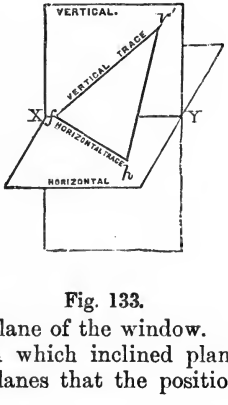


Fig. 133.

It is by means of the lines in which inclined planes would intersect the co-ordinate planes that the positions of such surfaces are indicated.

If a piece of cardboard be cut in the shape of a triangle and it be fitted into one of the angles of the model mentioned in Chap. I., *Solid Geometry*, as shown in fig. 133, its edges will mark two lines, $v' f$ and $f h$, upon the vertical and horizontal planes, meeting in a point f in X Y. These lines would be called the *traces* of the oblique plane. The trace $v' f$ upon the v. p. would be called the vertical trace (v. t.), and $f h$, that upon the horizontal plane, the horizontal trace (h. t.). When the co-ordinate planes are made to coincide by the revolution of the v. p. upon X Y, these traces will be represented as making a larger angle with each other than they actually do upon the oblique surface.

In all cases but *one*, the traces of oblique planes meet upon X Y.

In fig. 134, traces of planes in different positions are shown.

The first plane (fig. 134 *a*) is inclined to the h.p., but is perpendicular to the v. p.

The second (fig. 134 *b*) is the reverse of the above, as it is inclined to the v. p., but perpendicular to the h. p.

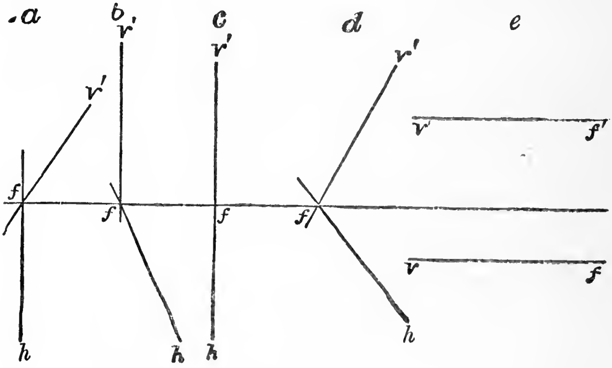


Fig. 134.

The third (fig. 134 *c*) is perpendicular to both the co-ordinate planes.

The fourth (fig. 134 *d*) is inclined to each of the co-ordinate planes.

The fifth plane (fig. 134 *e*) is inclined to both the co-ordinate planes, but is parallel to X Y.

As planes are infinite, so their traces are infinite, and must not be supposed to terminate upon X Y, in fact, it is advisable in all cases to draw these lines a short distance beyond the ground line.

The number of different positions in which the traces may occur is infinite, as any two lines meeting in X Y may represent an oblique plane.

Referring again to figure 134 *a*, the angle which the line *v' f* makes with X Y is the inclination of that plane to the horizontal.

Similarly, the angle which *f h*, in fig. **134*b*** makes with X Y shows the inclination of that plane to the vertical.

Neither of the lines *v' f'*, *f h*, in fig. **134*d*** will tell us the inclination of that plane. To determine this a special construction is necessary, which will be explained hereafter.

All lines which lie entirely upon a plane are said to be contained by it. If a pencil be laid upon an oblique surface in such a manner as to be parallel to its h. t., it will be seen that the pencil is horizontal. Again, if the pencil be placed perpendicular to the h. t., and lying in the plane, it will be inclined to the horizontal fully as much as the plane itself. Therefore, in any other position upon this plane, the pencil would be less inclined than the oblique surface.

If two oblique planes be parallel to each other their traces will be parallel.

An oblique plane is said to be "constructed" into either of the co-ordinate planes when its whole surface, with the points and figures upon it, is made to revolve upon one of its traces until it coincides with that plane.

When the inclination of a plane is given it must be understood to mean its inclination with the h. p., without it be otherwise stated.

PROBLEM XXX.

Given, an Oblique Plane, by its traces, to determine its inclinations to each plane of projection.

The angle of inclination of an oblique surface to the

h. p. is measured by the base angle of any vertical cone to which the plane is tangential. This is easily illustrated by taking a model of a cone and placing a piece of paper against its side. Then whatever the slope of the conical surface may be, the inclination of the sheet of paper will be the same. Let the paper be arranged so that one of its edges is upon the ground, and let the cone be vertical; then the former will represent the h. t. of a plane, and the latter the tangential cone. Note that the paper touches the cone in a *line* passing through the apex, and which is perpendicular to the h. t.

To find the inclination to the h. p., proceed as follows: With any point a in X Y, as centre, describe a semicircle, $c' \, b \, d'$, to touch the h. t. Complete the elevation of a semi-cone, whose plan is $c' \, b \, d'$, and having its apex in the v. t.

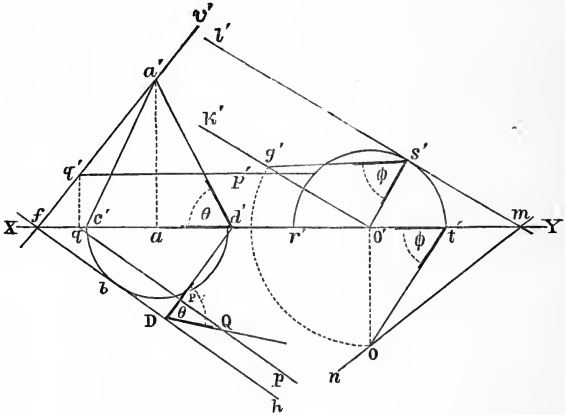


Fig. 135.

as $c' \, a' \, d'$. Then the angle at d', marked θ, is the required inclination to the h. p. Observe that this semi-cone fits under the oblique plane, and its back coincides with the v. p. Note also that the angle found is the bevel of the plane along *f. h*

2ND METHOD—Same figure. Draw the plan and elevation of *any horizontal* line in the plane.

Horizontal lines in oblique planes have their plans parallel to the horizontal trace.—The truth of this should be tried by an improvised model. Draw any line $p\ q$ parallel to $f\ h$. Consider it as the plan of a level line in the oblique plane. Then $q\ q'$ represents its height above the h. p., and $p'\ q'$ is therefore its elevation. Conceive a vertical section to be made of the given oblique plane at right angles to its h. t. Let D d' represent the trace of this section. This section will cut our horizontal line at P. Make P Q equal to the height of the horizontal line ($q'\ q'$) and join D Q. Then P D Q represents the section of the bevel of the plane turned over or rebatted on D P into the h. p. Hence the angle θ is the inclination required.

On the same figure the problem is worked whereby the inclination ϕ to the v. p. is discovered. It is simply a reversal of the work just described, assuming, *pro. tem.*, that the v. p. is the h. p., and *vice versa*. The thoughtful student will find no difficulty in following it out. He should note that the line O′ K′ plays the part of the horizontal P Q in the last problem; also that its plan is omitted, as O′ o gives the distance from the co-ordinate plane.

CASE II.—In fig. 136, a second example is shown where the traces are in one straight line. It is not advisable in such a case to adopt the method of the semi-cone. It is better to use the second solution, *i.e.*, to put a horizontal line on the plane, and to obtain the inclination by a vertical perpendicular section. This is shown in fig. 136, and the angle ϕ to the v. p. could be detected in a similar manner.

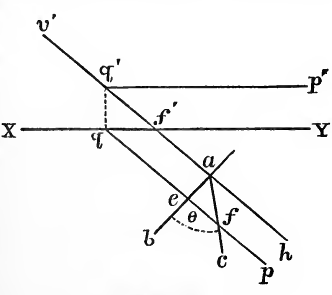


Fig. 136.

Problem XXXI.

To determine the inclination to both planes of projection of an Oblique Plane, having its traces parallel to X Y.

A right-angled triangle can be conceived as standing perpendicular to the v. p., and so fitting under the oblique one, that the hypotenuse of the triangle may be contained by it. The acute angles of such a triangle would then present the required inclinations.

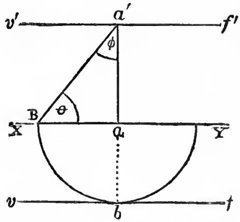


Fig. 137.

Take any point *a'* in *v' f'*, and draw *a' b* perpendicular to X Y, intersecting that line and h. t. in points *a* and *b*. With *a* as centre, and *a b* as radius, describe the arc *b* B, and join *a'* B. Then the angle θ is the inclination to the h. p., and ϕ is that to the v. p.

Note.—This is really only a repetition of the construction in the preceding problem, as a cone is generated by the revolution of a right-angled triangle upon its perpendicular. But the writer has found this case better understood by beginners as treated above, than by adopting the idea of a tangent cone, as in Problem XXX. In this case the sum of $\theta + \phi$ is a maximum, and is 90°.

Problem XXXII.

Given, either of the traces of an Oblique Plane and its inclination to the h. p., *to determine the remaining trace.*

Referring to fig. 135, let *f h* be the given h. t., and θ the given inclination.

Take any point *a* in X Y as centre, and describe an arc *b c'* tangent to *f h*, meeting X Y in *c'*. At *a* erect an indefinite perpendicular *a' a*, and at *c* make *c' a'*, making an angle with X Y equal to the given inclination, and meeting the perpendicular in *a'*. Join *a' f*, and *v' f* will be the given trace required.

If *v' f'* be given instead of *f h*, and it is required to find the latter, assume a point *c'* in X Y, and make *c' a'* at an angle with it equal to the given inclination, meeting the v. t. in *a'*. Draw *a' a* perpendicular to X Y, and with *a* as centre, radius *a c'*, describe the arc *a c' b*. Draw *f h* tangent to this arc, and it will be the h. t. required.

Problem XXXIII.

To determine the traces of a Plane inclined 60° *to the* h. p., *and* 42° *to the* v. p.

This is considered an *advanced* Problem. It is inserted here to make the series of questions upon traces of oblique planes complete. The student may, without any inconvenience, defer its study to a later part of the course.

To solve the above, it is necessary to determine two cones, whose generatrices* shall make the given angles of inclination with the co-ordinate planes. These two cones must have their axes *in* those planes, meeting in one point upon X Y. They must also envelop a common sphere, having its centre in X Y, at the point where the axes of the cones meet. Then the plane which touches both these cones is that required in the question.

Draw a line 1·2 perpendicular to X Y, and at any point, *c*, make an angle of 60°. The line *c a'* meets the perpendicular 1·2 in *a'*. Then *a' c a* will be the elevation of half a vertical cone. Describe the arc *c b* to represent part of the base of that cone in plan. Then, with *a* as centre, describe a circle *e g n*, which shall

* The hypotenuse of the right angled triangle which generates a conical surface is the generatrix of the sloping surface thereof.

be tangent to the line a' c. This is the elevation of the sphere enveloped by the cone.

Make the line a_2 k tangent to the circle e g n, and meeting X Y at an angle of 40°. Then the triangle a_2 k a will be the plan of the horizontal cone, also enveloping the sphere.

With a as centre, a k as radius, describe the arc k s t, to represent the elevation of part of this cone. Then v' f, drawn through the point a' and touching the

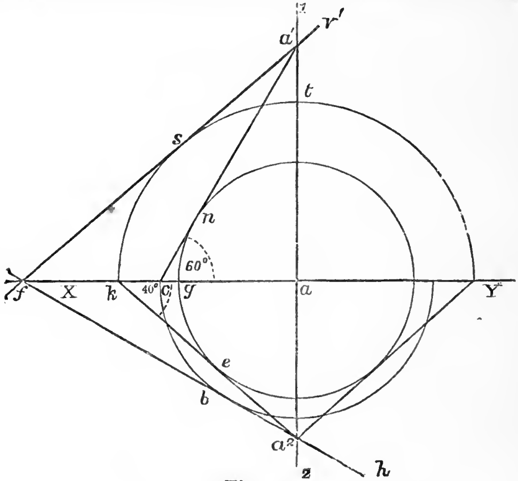


Fig. 138.

arc, k s t, in the point s, will be the required v. t., and f h passing through f and a_2, will be the required h. t.

Note.—The sum of the inclinations of an oblique plane must be between 90° and 180°.

PROBLEM XXXIV.

To determine the distance between two parallel Planes given by their traces.

Let v' f h and l' m n be the traces of the given planes. Proceed as if to find the inclination of the plane l' m n. Produce a a' beyond a' to meet v' f in a_2, and with a as centre, describe an arc tangent to f h, meeting X Y in b. Join b a_2, and the perpendicular distance between the lines b a_2 and c a' is that between the two given planes.

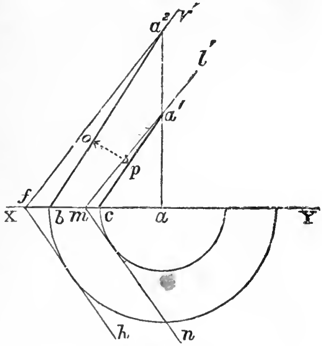


Fig. 139.

Problem XXXV.

To determine by its traces a Plane parallel to a given plane, and at a given distance from it.

This is the converse of the preceding problem; and, referring to fig. 139, let the given plane be v' f h, and o p the distance. Proceed as if to find the inclination of v' f h, and at a perpendicular distance, equal to o p, draw c a' parallel to b a_2. Then a' is one point in the vertical trace of the required plane; and as parallel planes have parallel traces, l' m and m n, drawn parallel to v' f and f h respectively, will be those of the plane required in the question.

Problem XXXVI.

To determine the Angle which exists between the traces of an oblique plane, when the co-ordinate planes are in their proper position.

If the inclined plane be "constructed" into the h. p., the v. t. will, when drawn upon the paper in its new position, make the angle required with the h. t. Imagine, once more, the cone fitting under the plane, and conceive the position of the line of contact after the "construction" into the h. p. has taken place. It will be perpendicular to the h. t. Draw, therefore, any line, *a b* (fig. 140), perpendicular to *f h*. Determine *a′ c*, as in Problem XXX. Produce *a b* beyond *b*, making *b* A equal to *c a′*. Join *f* A, and *h f* A is the angle between the traces. The method of proof of this construction would be, to cut a piece of paper so that it may have one angle equal to *h f* A, and to try if it would fit the traces given, when the horizontal and vertical planes are mutually perpendicular.

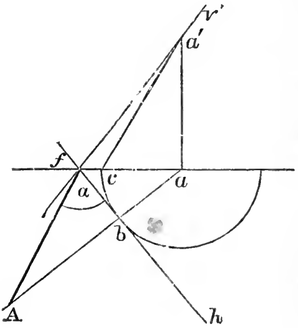


Fig. 140.

Problem XXXVII.

Given, the plan a *of a point,* A, *which is contained by the oblique plane, v′ f h, to determine its elevation.*

Four examples of this problem are shown in fig. 141. In the first case, as the plane is perpendicular to the v. p., the elevation of the point will fall upon the v. t., and its position, *a′*, is determined by a projector from *a*. In the second case, a line, *a b*, must be drawn parallel to the h. t. This line contains the plans of all the points which lie upon the plane, *v′ f h*, and are at the same height above the h. p. as A. At the point *b*, raise the perpendicular *b b′*, and the length of this line is the height of the point A above the paper. Through *b′* draw *a′ b′*

parallel to X Y, and the point a', where this parallel intersects a projector from a, is the elevation required. The third case is only a modification of the second, and

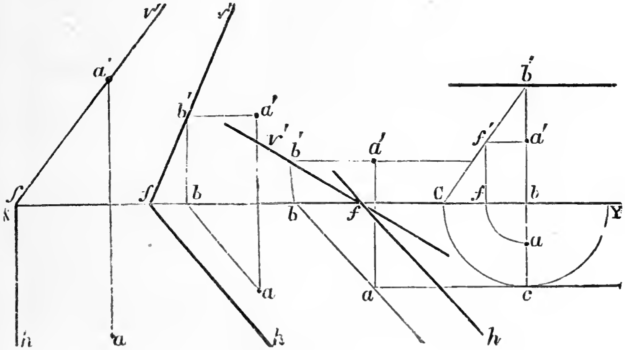


Fig. 141.

reference to the figure will be sufficient to explain its solution.

The fourth case requires a special construction. Draw the line b' c perpendicular to X Y, and passing through the point a. With b as centre, describe the arcs c C and a f. Join C b', and at the point f, draw f f' perpendicular to X Y, meeting C b' in f'. Through f', draw f' a' parallel to X Y, and the point a', where this line intersects b' c, is the elevation of A.

A little consideration, with the use of a model of the plane in the given position, will show the principle of the above construction.

b' C b is a right-angled triangle, which, in its original position, was perpendicular to the v. p., and contained the point A upon its hypotenuse. It is here shown constructed into the v. p., and the perpendicular $f f'$ indicates its height above the h. p.

Note.—If the elevation of the point be given, and the plan required, the construction would be the converse of the above.

Problem XXXVIII.

To determine the traces of a Plane, parallel to a given one, and passing through a given point.

Let $v' f h$ be the given plane, and $a' a$ the projections of a point, A. Through a draw $a b$ parallel to $f h$, and raise $b b'$ perpendicular to X Y. The point b', where a parallel ($a' b'$) to X Y meets this perpendicular, is in the vertical trace of the required plane. Its traces being parallel to those of $v' f h$, draw $l' m$ and $m' n$, as shown in the diagram.

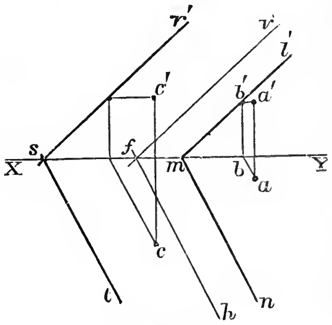


Fig. 142.

A second plane, $r' s t$, is shown in the figure passing through the given point C. The construction is similar, and needs no detailed explanation.

Problem XXXIX.

To determine a Line perpendicular to a given plane (which is inclined to h. p., *but at a right angle to the* v. p.), *to pass through a given point,* P.

The plan and elevation of a line which is perpendicular to an oblique plane, are perpendicular to the traces of that plane—the plan to the h. t., and the elevation to the v. t. This is easily proved by holding a pencil perpendicularly to any inclined surface, and viewing it in two directions, at a right angle with each other. Through the given projections of the point, draw *p′ r′* and *p r* perpendicular to the traces of the plane. The plan of the extremity, R, is determined by a projector through *r′*.

The given plane being inclined only to the h. p., the elevation of lines perpendicular to it will be fully as long as those lines themselves.

Perpendiculars to planes, which are inclined to both the horizontal and vertical planes, will be treated in a future chapter.

Problem XL.

To determine by its traces the Plane containing three given points.

Let *a′ b′ c′*, *a b c* be the projections of the given points, it is required to determine a plane which shall contain them.

If two points are contained by a plane, it is clear that the line joining those two points must also be contained by that plane. Also, if a line be contained by a plane, the traces of that line are in the traces of the plane. These two principles are sufficient to solve this problem; for the plane required must contain each of the three lines, A B, B C, A C, the traces of which will be points in the traces of the plane.

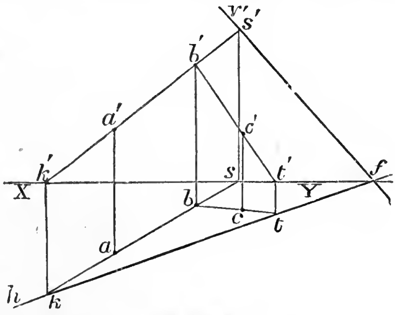


Fig. 143.

Join *a′ b′*, *a b*, *b′ c′*, *b c*, and produce *a′ b′* beyond *b′* to meet

X Y in k'. Then a perpendicular to X Y through k, intersecting the plan of A B produced in k, gives one point in the required h. t., and t which is the h. t. of the line, B C is a second point in that trace. The line $f\,h$, drawn through these points, is the h. t. of the required plane. Find s, the v. t. of the line A B, and draw $v'\,f$, passing through s, to meet $f\,h$ in f. Then $v'\,f\,h$ is the plane containing the three given points.

EXERCISES.

1. Draw any two lines meeting in X Y. Consider them as the traces of an oblique plane. Determine its inclination to the horizontal plane, and show the plan of a horizontal line lying in it 1″ above the horizontal plane.

2. The horizontal and vertical traces of a certain oblique plane, make angles of 30° and 60° respectively with X Y. Assume any point above the ground line as the elevation of a point contained by this plane, and determine its plan.

3. Draw a line parallel to X Y, at a distance of 1″ from it. Consider this as the horizontal trace of a certain plane inclined 40° to the horizontal plane, and determine the vertical trace.

4. The horizontal trace of a plane makes an angle of 30° with X Y; the vertical trace one of 50°. Determine the inclination of the plane and the true angle between the traces.

5. Draw two parallel planes, inclined 60° to the horizontal plane and 1″ apart; their horizontal traces to make angles of 40° with X Y.

6. Assume any three points, A, B, C, by their projections, and determine the oblique plane containing them.

7. Draw two parallel lines at angles of 40° with X Y, and above it; draw also two parallel lines from the points where the former intersect the ground line at angles of 30° with it. Consider these lines as the traces of two parallel planes, and determine the distance between them.

8. A triangle 2″ by 1·5″ by 1·8″ is the plan of a figure, the heights of whose corners are 1″, 1·5″, and ·7″ above the paper. Determine the plane containing the triangle.

CHAPTER V.

ON THE PROJECTION OF OBLIQUE SURFACES.

When a figure rests upon or is parallel to either of the co-ordinate planes, its projection upon that plane is the same in shape as the figure itself. But if its surface be inclined to those planes, its projections will differ from it in shape.

This is well illustrated by cutting a square and a circular hole in a piece of cardboard. When it lies upon the paper, the plans of the holes are a square and a circle; but if it be slightly rotated upon one of its edges until the surface is inclined, the plans of the holes upon the original h. p. will be altered in appearance.

The plan of the square may assume the shape of a rectangle, and that of the circle will be an ellipse. In both cases, the projections of the holes are narrower than their originals, and the more the surface of the cardboard is inclined, the greater the loss of width, until, when it stands vertical, the plans of both holes are straight lines.

An alteration, precisely analogous to this, would take place in elevation if the cardboard were inclined to the v. p.

The holes may be considered as forming parts of an oblique plane, when they are situated as above, the h. t. of which would be the edge upon which the cardboard rotated.

When the plane is inclined only to the h. p., the elevations of all lines contained by it are represented by straight lines coinciding with the v. t. The h. t. in such a case is perpendicular to X Y. And it is most convenient to arrange the co-ordinate planes in this position in relation to *any* oblique surface, for, if other elevations are required, they can be made after one is determined by proper alteration of the ground line.

Each corner of the square hole in the cardboard, when it revolves upon the edge, describes the arc of a circle in its journey, for it is at all times equi-distant from that edge.

And when the v. p. is assumed perpendicular to the oblique one, these arcs can be shown as parts of circles in elevation.

After working a few of the following problems, the student will appreciate the advantage gained by arranging the co-ordinate planes in the manner described above.

Note.—It is necessary to remind the student of a principle demonstrated in Chapter IV. If a line be contained by an oblique plane, and is parallel to its horizontal trace, that line is still horizontal.

Problem XLI.

To draw the plan of a Square when its surface is inclined 42°, and one of its sides is horizontal.

As the surface of the square is to be inclined 42°, commence by assuming the traces of a plane inclined at that angle, and rotate the figure from a horizontal position into this plane. The h. t.—*fh* is perpendicular to X Y, and the v. t. makes an angle of 42° with it.

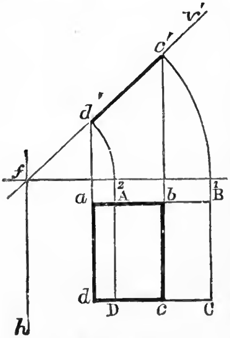


Fig. 144.

The square A B C D must be drawn with one of its sides parallel to *fh*. Then through the corners C and D, projectors must be determined meeting X Y in 1 and 2. With *f* as centre, describe the arcs 1 *c'* and 2 d', intersecting v. t. in d' and *c'*.

These arcs will represent the journey of the points C and D whilst being rotated into the plane *v′ f h*. Then *c′ d′* is the elevation of the whole square, because the sides A D and B C of the square being horizontal and perpendicular to the v. p., their elevations are points. The intersections of projectors through *d′* and *c′*, with lines parallel to X Y through A B C and D, are the plans of the four corners of the square.

Problem XLII.

To draw the plan of a Square when its plane is inclined 30°, the diagonal being horizontal.

This problem is only a modification of that preceding it. The plane having been determined by its traces, the square must be drawn upon the paper with its diagonal parallel to the h. t., as that line is to be horizontal. Proceed then, as before, to revolve each of the points, A B C and D into the oblique plane, and thus determine the elevation *a′ b′ c′ d′*. It should be noticed that *c′* is the elevation of both the points A and C, as the former is directly behind the latter. Projectors drawn through these points, to meet parallels, to X Y, through A B C D, will give the required plan, *a b c d*. The second diagonal of the square, B D, will be equally inclined with the plane, as it is perpendicular to the h. t.

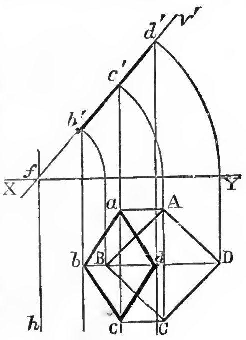


Fig. 145.

Problem XLIII.

A circle has the plane of its surface inclined 50° to the paper; draw plan and elevation.

In projecting curved lines, it is necessary to assume a number of points in those lines, and to determine, individually, the plans and elevations of each of them. When this has been done, a curve passing through the projections determined will give those of the original line, accurate so far as regards the assumed points. The correctness, therefore, of the drawing is augmented, if their number be increased. In the case before us it is most convenient to divide the circle into eight equal parts, so arranged that the diameter A E, which passes through two of them, shall be parallel to X Y. By this means we shall, after that line has been rotated into the oblique plane and its plan determined, have the minor (smaller) axis of the ellipse, which is the plan of the circle. Again, the points N B, G C, and F D, will be respectively opposite to each other, consequently fewer projectors will be required. The remainder of the construction is obvious.

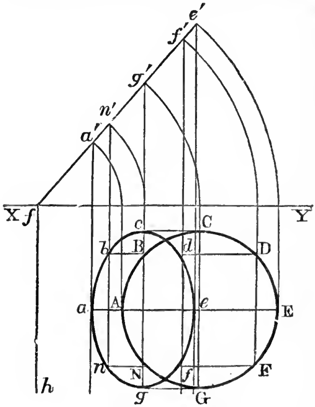


Fig. 146.

The ellipse should be very carefully drawn in by hand.

The elevation of the circle is the straight line *a′ e′* upon the v. t.

Problem XLIV.

A Pentagonal Surface is inclined 40°, the line joining two alternate corners of the figure is horizontal; draw plan and elevation.

Commence by drawing the pentagon **A B C D E**, and join B D. Consider this line as the h. t. of a plane inclined 40°. Assume an X Y perpendicular to it, and a v. t. making 40° with X Y. The figure must then be revolved upon B D into this oblique plane. The points B and D will not alter in position, and A and E can be treated in the usual way. But the point C, when the figure is thus revolved, will fall below the h. p., as when the portion D E A B is raised, D C B is depressed.

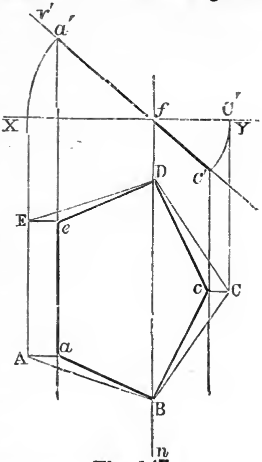


Fig. 147.

To determine the plan of C, a projector must be drawn through it, and the arc C′ c′ by its intersection with the v. t. will give the elevation of it. The plan is found as before, by a projector through c′ meeting a parallel to X Y through C.

Problem XLV.

The plane of a Hexagon of 1″ *side is inclined* 25°, *neither of its sides being horizontal; draw plan and elevation.*

After the plane is determined by its traces, the hexagon must be drawn upon the paper in such a position that neither of its sides is parallel to the h. t. The figure must then be rotated into the plane, and the plan determined as before.

Problem XLVI.

A Cube has one of its surfaces inclined 40°, *and neither of its edges horizontal; draw its plan and elevation.*

The projections of the square surface inclined 40°, and no edge horizontal, must be first determined by the construction explained in preceding problems. The cube can then be built upon it.

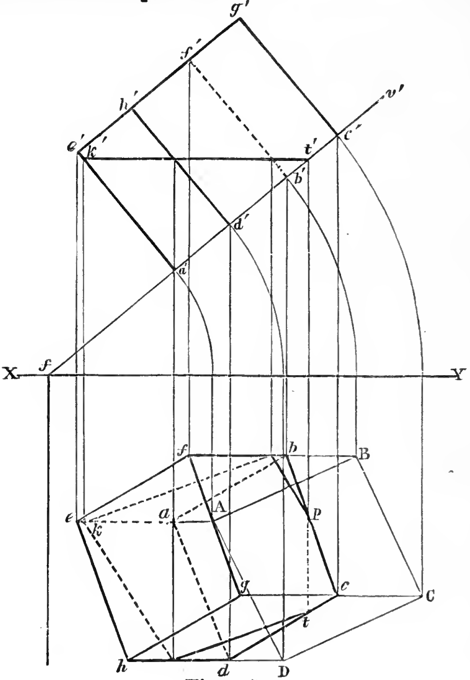


Fig. 147*a*.

A B C D is the figure first drawn; all the sides making angles with the h. t. Its plan is *a b c d*, and elevation *a' b' d' c'*.

At the four corners of the square, perpendiculars to its surface must be raised.

It was shown in Problem XXXIX., that if a line be perpendicular to a plane, its projections are perpendicular to the traces of that plane. The elevation of these perpendiculars, therefore, will be lines drawn through $a' b' c' d'$ at right angles to $v' f'$. The lengths of these elevations will be equal to the edges of the cube, as those edges are parallel to the v. p. Make $a' e'$ and $c' g'$ equal to A B and join $e' g'$. Then the elevation will be complete ($b' f'$ being a dotted line). To complete the plan, draw $b f$, $c g$, $a e$ and $d h$ parallel to X Y, and determine the points, $e f g h$ by projectors through $e' f' g' h'$. A glance at the position of the solid as represented by the elevation will show the student that the point A will be hidden in the plan. The edges $a e$, $a b$, and $a d$, which radiate from this point, must therefore be dotted. A horizontal section of the cube is shown in the elevation by the line $k' t'$. The plans of the several points of this section are determined by projectors through the elevations of those points, intersecting the plans of the edges upon which they occur. Notice that the point t' is the elevation of two points of section—one, upon the edge C D, and the other upon B C. These will be shown separately in plan in p and t.

As the section is a horizontal one, its plan is also its true shape.

Problem XLVII

An Equilateral Triangle, A B C, *is inclined* 40°; *the edge,* A B, *of the figure is horizontal and* ·5″ *above the paper; draw plan and elevation.*

Commence this problem by finding a line upon a plane inclined 40°, ·5″ above the paper, and horizontal. The line $k a'$, parallel to X Y at a distance ·5″ from it and intersecting the v. t., gives a', the elevation of such a line.

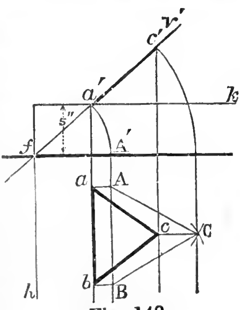


Fig. 143.

Construct this point into the h. p. by the arc a' A′, and draw A B parallel to $f h$. Upon it construct the equilateral triangle, and proceed to find its plan when rotated into the plane $v' f h$, as before.

PROBLEM XLVIII.

A Prism 2" long, its base being an equilateral triangle of ·5" side, has one of its rectangular faces inclined 30°, the edges of that face being each inclined to h. p.; draw plan and elevation.

The rectangle, A B C D, must be drawn with neither of its sides parallel to the h. t.

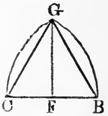


Fig.149.

The projections *a b c d*, *a′ b′ c′ d′*, are determined as before. To complete the plan and elevation of the solid, we have to project the points G and K, which are the apices of the triangles forming its bases. G is perpendicularly over the centre of C B, and K over A D. Bisect, therefore, *b′ c′* and *a′ d′* in *e′* and *f′*. Draw *f′ g′* and *e′ k′* perpendicular to the v. t. The height of these points above

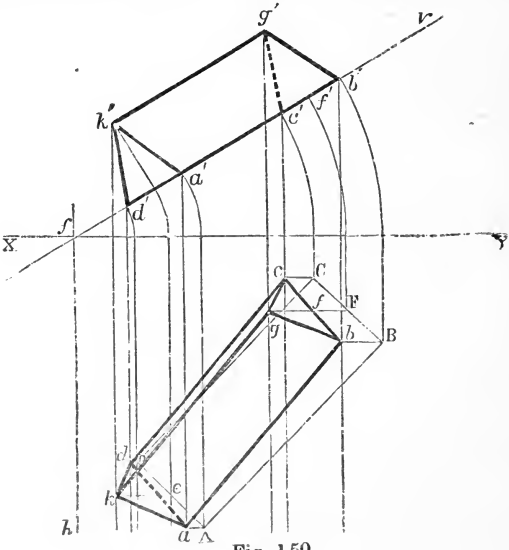


Fig. 150.

the base is the altitude of the equilateral triangles. This must be deduced from a supplementary drawing as shown in fig. 149. The length, F G, measured along $f' g'$ and $e' k'$, gives g' and k', the elevations of the points G and K. Join these to the proper points in the base, and the elevation of the solid will be complete. The plans g and k will be found upon lines perpendicular to the h. t. drawn through f and e, at the points where projectors through g' and k' meet them.

Problem XLIX.

A Tetrahedron has one of its faces inclined 24° *to the h. p. and a line bisecting that face is horizontal; draw plan and elevation.*

Draw an equilateral triangle, A B C, and show upon it a line B D bisecting it. Assume the h. t. of the oblique plane to be parallel to this line, and X Y perpendicular to it. The v. t. will therefore be drawn through f at an angle of 24° with the ground line. Rotate the figure into this plane and obtain its plan. The solid is then completed by projecting its axis. This is effected by finding the centre of the triangle,

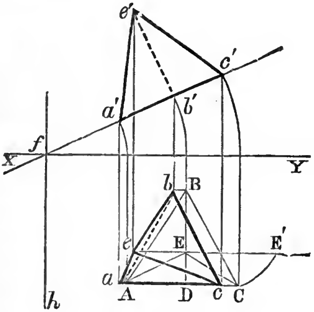


Fig. 151.

A B C, and the height of the tetrahedron, the construction for which is described in Chap. II. The point, E, can then be rotated into the plane $v' f h$, and the elevation of the axis $b' e'$ determined

The plan of the apex falls in a projector through e' at the point where it is intersected by a parallel to X Y through E.

If two lines, making a certain angle with each other, are contained by an oblique plane, the plans of those lines will make a larger angle. This is easily illustrated by rotating a 45° set square upon its longest edge. The right angle will be projected in plan as a constantly increasing angle during the rotation, until the set square becomes vertical, when the plan is a straight line. The relation between the magnitude of the real angle and that of its projections is sufficient to determine the inclination of the plane which contains the lines, and also that of the lines themselves.

If a figure rotate upon one of its edges, we have already found that the plan of any point in that figure will move in a line perpendicular to the axis of rotation.

We proceed, now, to discuss a few problems dependent for their construction upon the above principle.

PROBLEM L.

The plan of a 3″ line is 2″ long. What is its inclination?

Draw a line 2″ long, and at one extremity, *b*, erect a perpendicular. With the other extremity, A, as centre, radius 3″, describe an arc, intersecting the perpendicular in *b′*. Then, A B will represent the line, and A *b* its plan. The angle between these two measures is the required inclination.

PROBLEM LI.

Two lines, each of equal length, meet at an angle of 60°, *and are equally inclined. The plans of these lines contain a right angle. Determine the inclination of the plane containing them, and that of the lines themselves.*

Draw two lines, A B and B C, meeting at an angle of 60°. As they are to be so placed as to be equally inclined, the axis of rotation must be equidistant upon A B and B C from B. Join A C, and assume that the triangle revolves upon it. Produce it in either direction and consider it as the h. t. of the required oblique plane. The ground line must then be taken perpendicular to *h f*

The point B, when the triangle revolves upon A C, will travel in an arc, whose plan is a perpendicular to *h f*. Draw B E, and determine point *b* in it, such that the angle A B C shall be a right angle. This is effected by describing a semicircle upon A C. Join A *b*, *b* C. Then the plan of the two lines, when B is raised into the required position, is shown.

The elevation, *b′*, of B will be in a projector through *b′*, where it is intersected by the arc B′ *b′*.

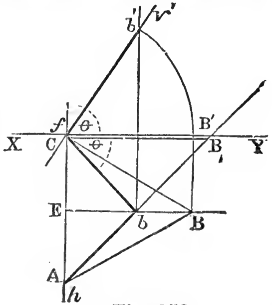


Fig. 152.

Furthermore, *b′* is a point in the v. t. of the required plane containing the two lines.

Join *b′ f*, and the angle which it makes with X Y is the inclination of that plane.

To determine the inclination of either of the lines, set *b* B_1 perpendicular to one of the plans, and mark off from it a distance equal to the height of the point B_1 above the paper, as shown in the elevation. Then the angle *b c* B_1 is the inclination of the lines.

By this construction the line B C is brought into the h. p.

Problem LII.

A *rectangle* A B C D, *revolves upon one of its diagonals until the plan of one of the opposite right angles contains an angle of* 120°. *Determine the inclination of the figure and that of the other diagonal.*

The construction of this problem is based upon the same principles as the preceding, but is rather more complicated.

The rectangle, A B C D, being drawn, the diagonal B D is assumed as the h. t., or axis of rotation. The plan of C will be in C *e*, drawn perpendicular to this diagonal, in such a position that the angle D C B, shall contain 120°. To discover this point, a segment of a circle containing that angle, and having B D for its chord, must be determined. (The method of construction is shown in fig. 36, *Plane Geometry*).

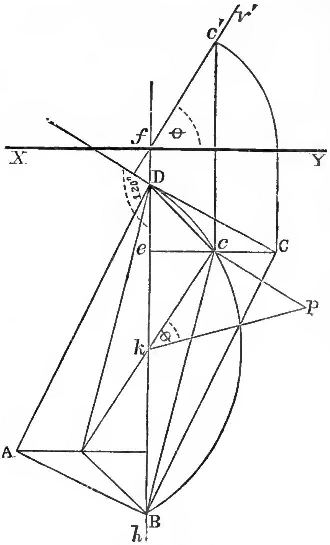


Fig. 153.

The elevation of C gives one point in the v. t., and the angle *v' f y* is the inclination of the plane of the rectangle.

If C be joined to *k*, *c k* will be the plan of half the remaining diagonal. The inclination of this line is shown by the angle C *k* P—and this is necessarily that of the whole diagonal.

EXERCISES.

1. A square has its surface inclined 40°, neither of its sides being horizontal. Draw plan and elevation.

2. A rectangle, 2″ long by 1″ broad, is inclined 50° to the paper, one of its diagonals being horizontal. Draw plan and elevation.

3. A circle of 3″ diameter lies in a plane inclined at 55° to the paper. Draw a plan and a second elevation of it, the ground line making an angle of 60° with the horizontal of the plane.

Note.—The second elevation is drawn upon a vertical plane having its X Y making an angle of 60° with the horizontal trace of the oblique plane.

4. A square prism, base 1·5″ by 4″ long, has one of its rectangular faces inclined 40°, the diagonal of that face being horizontal. Draw plan and elevation.

5. An isosceles triangle, whose vertical angle is 30° and base 1·5″, revolves upon that base so that the plans of its sides are at right angles. Determine the inclination of the plane of the triangle and of the sides.

6. A regular pentagon of 1·5″ side lies in a plane inclined at 70°, one *side* being horizontal. Draw its plan and elevation, the ground line being parallel to one of the sloping diagonals.

7. A pyramid having a square of 3″ side for its base, and its height half the diagonal of the square, is to be shown by its plan and elevation when one side of the bace is horizontal, and the plane of that base is inclined 50° to the paper.

8. An equilateral triangle of 3″ edge revolves upon one of its sides until the plan of the opposite angle is 80°. What is the inclination of the figure and of the other two sides?

9. A cylinder, base 1″ radius, 3″ long, has its base inclined 50°. Draw plan and elevation, and determine the true shape of a vertical section of the solid passing through the middle of its axis.

10. A hexagon is inclined 40°, neither of its diameters is horizontal. Draw plan and elevation.

11. A pentagon revolves upon one of its diagonals until the angle opposite is shown in plan as 120°. What is the inclination of the figure and that of the other diagonals?

12. The axis of a square pyramid, base 1″ side, 2″ long, is inclined 60°, one edge of the base being horizontal. Show true shape of a horizontal section bisecting the axis.

13. An octahedron, 2″ edge, has one edge horizontal and an axis inclined 30°. Draw plan and elevation.

CHAPTER VI.

ON THE INTERSECTION OF PLANES AND LINES.

If two planes meet one another, their intersection is a straight line, but if a line intersects a plane it does so in a point.

The angle between two planes which intersect is their *dihedral* angle, and is measured by the angle between the two lines, which would be made by a third plane cutting them perpendicular to their intersection.

PROBLEM LIII.

To determine the Plans and Elevations of the intersections of the given planes. Cases 1, 2, *and* 3.

In the first case, the planes slope in opposite directions.

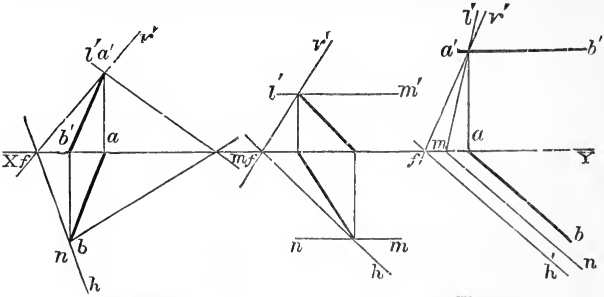


Fig. 154. Fig. 155.

The points a' and b, where the horizontal and vertical traces meet are in the intersection.

A is in the v. p. and B in the h. p.

The plan of A and the elevation of B are therefore upon X Y. Through *a'* and *b* draw projectors *a' a* and *b b'*. Join *a' b'* and *a b*. These, then, are the projections of the intersection of the given planes.

The inclination of the intersection can be determined by "constructing" it into the v. p. To do this, take *a* as centre, radius *a b*, and describe the arc *b* B, meeting X Y in B. Join *a'* B and the angle which it makes with X Y is the inclination of the intersection.

In the second case, the plane, *v' f h*, having its traces meeting in X Y, is intersected by another plane *l' m*, *m n* having its traces parallel to X Y.

The construction is similar to that of the first case, and needs no special explanation.

In the third case, the planes which intersect have their horizontal traces parallel to each other. In such a case the intersection is a horizontal line passing through the point where the vertical traces meet, and parallel to the horizontal traces.

Through *a'* draw a projector to determine *a*, its plan. Then, the elevation of the intersection is *a' b'* drawn through *a'* parallel to X Y, and the plan is *a b* drawn through *a*, parallel to the horizontal traces.

In fig. 156 the method is shown for determining the intersection of the planes *v' t'*, *v t*, and *l' m'*, *m n*, both of them being parallel to X Y. This forms a fourth case, requiring a special construction.

The student will see that, if the planes be themselves parallel, there can be no intersection, but if not, the intersection is a line parallel to both co-ordinate planes.

Conceive a right-angled triangle perpendicular to the v. p. to fit under each of the inclined planes; the bases of these triangles to be in the h. p., the perpendiculars in the v. p., and the hypotenuses in the oblique planes. These latter will cross each other in a point which is in the intersection of the planes.

Again, conceive these triangles to be constructed into each of the co-ordinate planes in turn, the axes of rotation being respectively their perpendiculars and bases. Then the points where one hypotenuse intersects the other, as shown upon these planes, will be the plan and elevation of one point in the intersection required. Draw K p perpendicular to X Y, meeting it in O. With O as centre, and radii O p and O f, describe the arcs p P and f F, intersecting X Y in P and F, Join P q and F K. Then the line $a' b'$ through a', and parallel to X Y, is the elevation of the intersection.

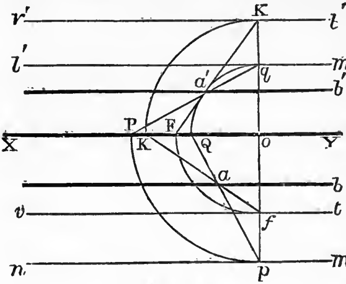


Fig. 156.

The converse construction as shown in the diagram determines b, a point in the plan, and $a\,b$, drawn through a, parallel to X Y, completes it.

Problem LIV.

To determine the projections of the intersection of a Line A B, *and a Plane*, $v' f h$.

In the first instance, we will suppose the line A B to be horizontal, as in fig. 157. It is clear that A B, being parallel to the h. p., will intersect the given plane in a point which is the same height above X Y as the line itself.

Produce the elevation $a' b'$ beyond a' until it meets $v' f$ in c'. Determine c, the plan of this point, and through it draw $c\,e$ parallel to $f\,h$.

This line contains the plans of all points contained by the oblique plane, which are at the same height above the h. p. as the line A B.

The point d, where $c\,e$ meets $a\,b$, is the plan of the intersection of the line and plane. Its elevation is d'.

When, as in the second case, the line is inclined to both the co-ordinate planes, a different construction is necessary. A vertical plane must be assumed con-

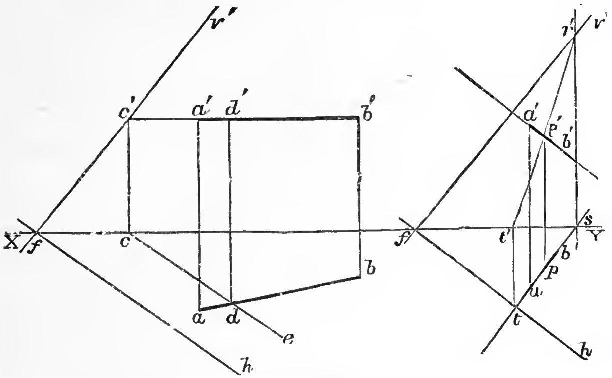


Fig. 157.

taining the given line, and the intersection of this plane with the given one must be determined. The point where this intersection crosses the given line is the intersection of the line with the plane. Or, the *plan* of the given line may be assumed to be the plan also of another line contained by the given plane. By this assumption two elevations would result, whose intersection would determine the projection of the desired point on the v. p.

Produce *a b* in both directions until it meets X Y and *f h* in *s* and *t*. Assume *t s* to be the h. t. of a vertical plane; *s r'* perpendicular to X Y is its v. t.

Determine the elevation *r' t'* of the intersection of these two planes by the construction described in a preceding problem. The point *p'* where *r' t'* meets *a' b'* is the elevation of the intersection of the line A B with the plane *v' f h*; *p* is its plan.

Problem LV.

To determine a Line perpendicular to the plane v' f h,

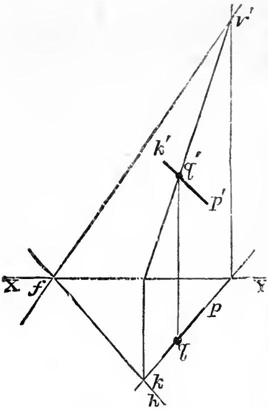


Fig. 158.

passing through a given point p' p, and to show its intersection with the plane.

Lines perpendicular to a plane have their projections perpendicular to the traces of that plane. Through the points p' and p, draw $p'\,k'$ and $p\,k$ perpendicular to $v'f$ and $f\,h$.

Determine the intersection Q, of the line and plane as in the preceding problem. Then the line, P Q will be perpendicular to the plane, and have one of its extremities contained by it.

PROBLEM LVI.

The point p, fig. 159, *is the plan of a Point contained by the oblique plane v'f h. It is required at the point* P, *to raise a perpendicular to the given plane* 1·5″ *long.*

Determine first the elevation of the point P, by the construction described in Prob. XXXVII. Through p' and p draw $p'\,r'$ and $p\,r$ at right-angles to $v'f$ and $f\,h$. This line P R must then be constructed into the h. p.

Assume therefore any second point, R, in it, and construct the points P and R into the h. p., as shown in the diagram. From P in P R mark off P Q – 1·5″ long

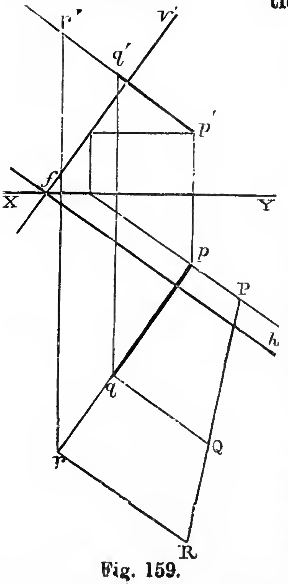


Fig. 159.

(the length required). Then if the point Q be projected back on to the h. p. as q, and the elevation q' be determined; the lines $p' q'$ and $p q$ will be the projections of a line P Q 1·5″ long perpendicular to the plane $v f h$, and passing through the point P.

Problem LVII.

To determine the Angle between the two given planes, $v' f h$ and $l' m n$.

The plan and elevation $a b$, and $a' b'$ of their intersection must be first determined. A plane, perpendicular to A B, must then be assumed, and the lines which this plane will make by its intersection with those given, will meet at the angle required.

Further, the intersection of the assumed plane with the two given ones and the h. p. will form a triangle, and if this figure be constructed into the h. p., its vertical angle will measure the dihedral angle.

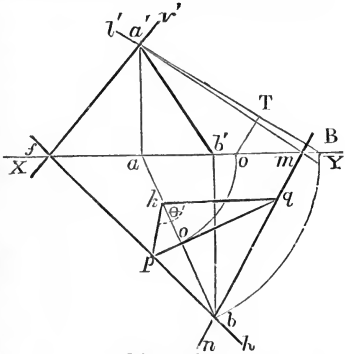


Fig. 160.

Draw $p q$, perpendicular to $a b$, and meeting the horizontal traces of the given planes in p and q, and intersecting $a b$ in o. This line, $p q$, is the base of the triangle mentioned above. With a as centre, radii $a o$, and $a b$, describe the arcs b B and o O, and join a' B.

By this means we construct the intersection of the two planes into the v. p. The altitude of the triangle is measured by a line, O T, passing through O, perpendicular to a' B From the point O, in $a b$, mark off a distance, *o k*, equal to O T, and join *k p*, *k q*. Then the angle, *p k q*, will be that between the given planes.

There are certain conditions, according to the relative position of the planes, under which this problem would require a modification of the construction for its solution.

For instance, the line *p q*, perpendicular to the intersection, may be parallel to one of the horizontal traces. In such a case, one point, as *q*, of the triangle would be infinitely distant. Then the line, *k q*, must be drawn parallel to *a b*, and the angle, *p k q*, would determine the dihedral angle as before.

Problem LVIII

To determine a Plane perpendicular to the given line, A B, *and passing through the point,* C, *in that line.*

The traces of the required plane will be perpendicular to the projections of the given line, and as the plane is to contain the point, C, it will also contain the horizontal, passing through C, which is perpendicular to A B.

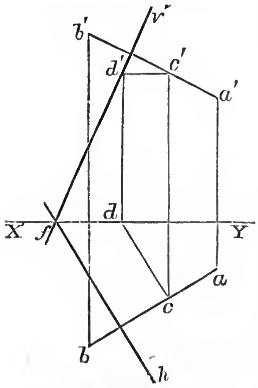


Fig. 161.

Through *c* draw *c d*, perpendicular to *a b*, meeting X Y in *d*. Determine *d′*, the elevation of *d*, by a projector through *d* meeting a parallel to X Y, through *c′*. Then *c′ d′*, *c d* are the projections of a horizontal line contained by the required plane, and *d′* is a point in the v. t. Draw therefore *v′ f* through *d′* perpendicular to *a′ b′*, and *f h* through *f* perpendicular to *a b*.

Problem LIX.

To determine the angle contained by two Straight Lines, A B *and* B C, *given by their projections.*

If the horizontal traces of the lines be joined, a third line will be formed, which, with those two given, will complete a triangle, the vertical angle of which is that required in the problem. If the triangle be constructed into the h. p., its base being the axis of rotation, its true shape will be determined, and consequently the required angle between the lines, A B, B C.

Determine *d* and *e*, the horizontal traces of A B and B C. Join *d e*. Then *d* B *e* is the triangle mentioned above. In constructing it into the h. p., *d* and *e* will be stationary, and the point, B, will travel in a vertical plane, perpendicular to *d e*.

Through *b* draw *b f*, perpendicular to *d e*, and produce it beyond *b*. The actual distance of point B from *f* is the length of the hypothenuse of a right-angled triangle, of which *b f* is the base, and *o b′* the perpendicular. Set off from *o*, along X Y *o f′*, equal to *b f*. Join *b′ f′*, and make *f* B in the plan equal to *b′ f′* Join B *d* and B *e*, and the angle, *d* B *e*, is that between the two given lines.

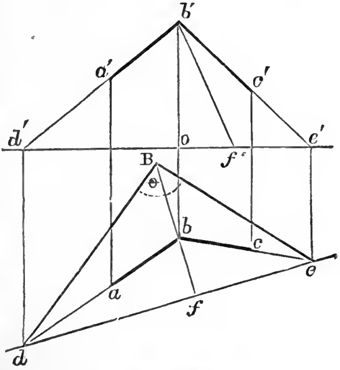


Fig. 162

CHAPTER VII.

ON THE PROJECTIONS OF FIGURES, HAVING GIVEN THE INCLINATIONS OF THEIR SURFACES, AND THAT OF A LINE BELONGING TO THEM.

It will be advisable at this point to recapitulate a few of the facts, mentioned in chapters IV. and V., upon lines lying in oblique planes.

It was there shown that if a line be contained by an oblique plane, it could only be horizontal when it was parallel to the h. t. of that plane. If such a line be perpendicular to the h. t., it will be inclined fully as much as the plane itself, and should it occupy any position upon the plane between these two, it will be inclined less than the plane. Thus we see, that the inclination of a line cannot exceed that of the plane which contains it. In this chapter, it will be our duty to treat of the projection of figures which lie upon oblique planes, their exact position upon them being fixed by the inclination of some line belonging to them.

Thus, if a square has its surface inclined at 40°, and one of its sides 20°, its position with regard to the co-ordinate planes is defined.

Problem LX.

To determine the projections of a line inclined 20°, *contained by a plane inclined* 40°.

It is most convenient, in drawing the traces of the

oblique plane, to assume the h. t. perpendicular to X Y. Let *v′ f h* be the traces of the plane inclined 40°.

Assume a point *a′* in the v. t., and set out *a′* B, making an angle of 20° with X Y. Draw *a′ a* perpendicular to X Y. With *a* as centre, radius *a* B, describe the arc B *b*, intersecting the h. t. in the point *b*. Join *a b*. Then *a b* is the plan of the required line.

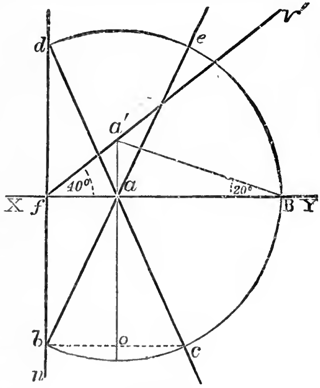


Fig. 163.

The first line *a′* B is in the v. p., and when the arc B *b* is described, one extremity, B, is brought round until it rests upon the h. t. of the oblique plane, the other extremity, *a′*, being fixed. We are sure, therefore, that *a b* is the plan of a line contained by the plane *v f′ h*, and it is inclined 20°, as its plan-length bears the same relation to the line itself as *a* B to *a′* B.

The elevation *a′ b′* is shown upon the v. t.

A second line, A C, is indicated in the figure, which has the plan, *c*, of one of its extremities on the other side of the horizontal line, *a′ o*, and at the same distance from it as *b* in *a b*. This line has the same inclination as the former one, and passes through the same point A.

We learn, therefore, that two lines can be contained by an oblique plane, equally inclined to the h. p. and passing through the same point.

If these two lines be produced towards *d* and *e*, the portions A D and A E will be in the second dihedral angle, the point *d* being equally distant with *b* from *f*. The lines A D and A B slope in opposite directions; in fact, the point *d* can be determined by continuing the arc B b until it meets the back portion of the h. t.

Problem LXI.

Given the plans of Lines, lying in an oblique plane, to determine their true lengths and inclinations.

Let a b, a c, be the plans of two lines, A B, A C, contained by the oblique plane v' f h. The elevation will in both cases be a' f. In Prob. VII. a method was given of determining the true length and inclination of a line when its projections are known, and the construction there adopted will solve the above problem, but a second method, which is preferable in this case, is shown in the diagram.

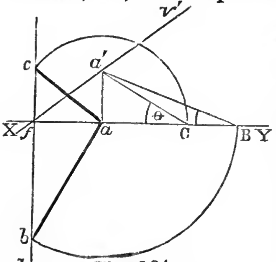


Fig. 164.

With a as centre, radii a c and a b, describe the arcs c C and b B, intersecting X Y in B and C.

Determine a', the elevation of A, and join a' C and a' B. Then the angles which these lines make with X Y are the respective inclinations of A B and A C. The lines a' B and a' C also represent their true lengths.

Problem LXII.

Draw the plan and elevation of Two Lines (any length) lying in a plane inclined 50°, and meeting in a point. The one is inclined at 25°, the other at 35°; determine the *true angle between them. The lines are to slope in opposite directions.*

Commence by drawing *v′ f h*, the traces of a plane inclined 50°. Assume *a′* as the point where the two lines meet, and set out *a′* B and *a′* C, making respectively angles of 25° and 35° with X Y.

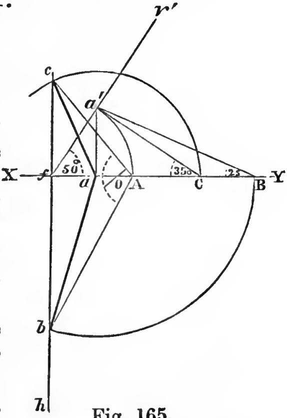


Fig. 165.

Determine *a*, the plan of *a′*, and as the two lines are to slope in opposite directions, with *a* as centre, radius *a* B, describe the arc B *b*, meeting the h. t. in *b*, and with the same centre, radius *a* C, describe the arc *c* C meeting the h. t. in *c*.

Join *a b a c*, and these lines will be the plans of those described in the question.

To determine the angle between them, construct *c* A *b* into the h. p. upon the h. t. The point A will travel in the v. p., and will, therefore after rotation, be in X Y. With *f* as centre, radius *f a′*, describe the arc *a′* A, intersecting X Y in A.

Join A *b*, A *c*, and *b* A *c* will be the required angle between the lines.

Problem LXIII.

A Square, C D E F, *has its surface inclined* 50°, *the side,* C D, *being inclined* 24°; *draw plan and elevation.*

In the first place, a line inclined 24°, and contained by a plane inclined 50° must be determined. This has been described in the preceding problems. *a b* is the plan of such a line.

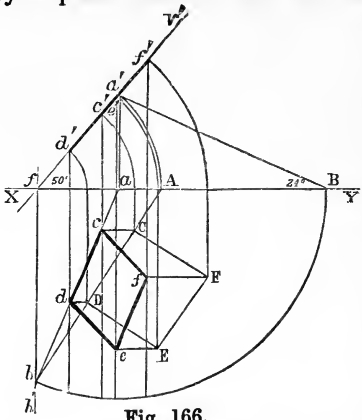


Fig. 166.

When this has been done, the line must be constructed into the h. p., and the square built upon it so that one side shall coincide with the constructed line. The square will then be in such a position that when it is revolved back into the oblique plane, the side which coincides with A B will be inclined 24°. With *f* as centre, radius *f a'*, describe the arc *a'* A. The point *b* will remain stationary. Join A *b*. If the problem be worked correctly, it will be found that the lines *a'* B and A *b* will be equal in length, and, upon consideration, we shall see that it should be so, as the former is the line, A B, constructed into the v. p., and A *b*, that same line, into the h. p.

Upon A *b* (at any part of it) construct the square, C D E F; rotate it back into the oblique plane, and determine its plan. This latter operation is fully described in Chap. V., and will present no difficulty.

The plan, *c d*, of the side, C D, should fall upon *a b*, proving that it is inclined as required by the problem.

Problem LXIV.

To draw the plan and elevation of a Hexagon whose surface is inclined 50°, *one of its diagonals being inclined* 20°.

This problem is introduced to show that the line, the inclination of which is given, need not be a side of a figure. It may be any line connected with it. Proceed as before to determine the plan and elevation of a line inclined 20°, contained by a plane inclined 50°. *a b* is the plan of such a line. Construct it into the h p., and A *b* will represent its true length.

Arrange a hexagon about this line so that its diameter may be upon A *b*. To do this, take any point, *o*, in A *b* as

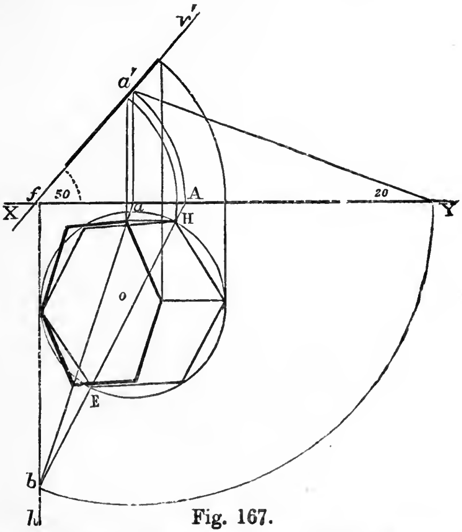


Fig. 167.

centre, and with a radius equal to the side of the required hexagon describe a circle. Then E and H, where this circle intersects A *b*, will be two points in the figure. Finish the hexagon and rotate it into the oblique plane, determining its plan as before.

Problem LXV.

The plane of a Pentagonal Figure of 1″ *side is inclined* 50°

The plan of one of its sides is ·75″ long; draw its plan and elevation.

This problem presents a slight modification of those preceding it. Instead of telling the student the inclination of a line in the figure, the relation between its true length and its projection upon the h. p. is given. The first thing, therefore, to be done is to determine at what angle a line 1″ long must be inclined, so that its plan may be ·75″ long. This is shown in a supplementary drawing in fig. 168.

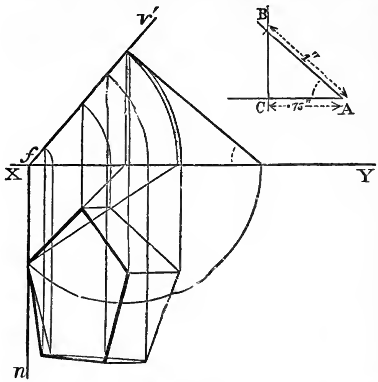


Fig. 168.

Take any line, A C, ·75″ long, and at one extremity, as C, erect an indefinite perpendicular, C B. With the other extremity as centre, and 1″ as radius, describe an arc meeting A B in B. Then the angle which A B makes with A C is the inclination of the line.

The problem now resolves itself into that of projecting a pentagon whose surface is inclined 50°, and one of its sides at an angle equal to θ. The construction of this needs no further explanation.

Problem LXVI.

A Cube of 1·5″ *edge is to be shown in plan and elevation when the plane of one face is inclined* 50°, *and one edge of that face is inclined* 30°.

Commence by determining the plan of the face, the position of which is given. This will present no difficulty. Upon the elevation show the edges of the solid perpendicular to that face, and also the straight line which represents the opposite face. The plan is

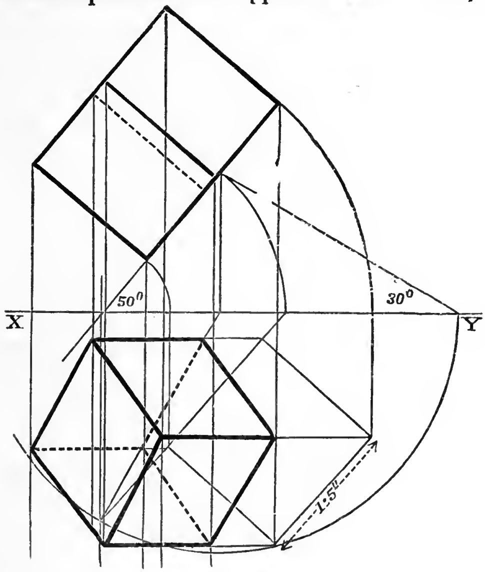


Fig. 169.

finished by determining those of the four points of the upper face and joining them to the proper points, which are already obtained.

Problem LXVII.

Given a b, the plan of a Line, A B, *contained by a given plane neither of whose traces is perpendicular to* X Y; *required the elevation.*

When a line is contained by a plane, the traces of the line are in the traces of the plane. If, then, the plan *a b* be produced beyond *a* until it meets *f h* in *d*, that point will be the h. t. of the line A B, and if it be produced beyond B until it meets X Y in *e*, and a perpendicular through *e* be drawn, intersecting the v. t. in *e'*, that point will be the v. t. of the line A B.

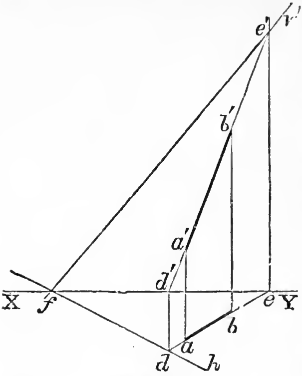


Fig. 170.

The point *d'* is the elevation of *d*, and *d'* joined to *e'* will give *e' d'*, which is the elevation of the line A B produced in each direction.

Projectors through *a* and *b* will determine *a' b'*, the part of *d' e'* which is the elevation of A B.

Problem LXVIII.

To determine the projections of a Line inclined 20° contained by the plane v' f h.

The line cannot be inclined more than the plane which contains it.

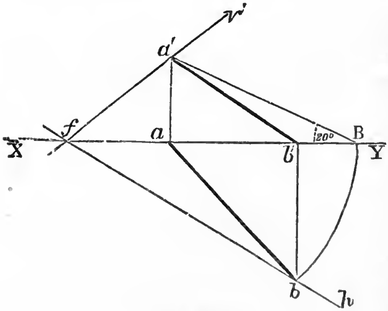


Fig. 171.

Assume a point *a'* in the v. t. of the given plane, and draw *a'* B, meeting X Y at an angle of 20°. This line is the elevation of one inclined 20°, contained by the v. p. Determine *a*, the plan of *a'*, and with *a* as centre, radius *a* B, describe the arc B *b*, intersecting the h. t. in the point *b*. Find *b′*, the elevation of *b*, and join *a* to *b* and *a′* to *b′*. Thus the line A B will be shown by its projections *a′ b′* and *a b*.

EXERCISES.

1. Draw the plan and elevation of a line inclined 40°, contained by a plane inclined 60°.

2. Draw the plan of a square of 3″ side when its plane is inclined at 63° and one side at 25°; add an elevation, the ground line being parallel to the shortest diagonal in plan.

3. A hexagon, 1″ side, has its surface inclined 40°, one side being inclined 20°. Draw plan and elevation.

4. A pentagon has its surface inclined 50°, one diagonal being inclined 30°. Show plan and elevation, and state the inclination of each of the other sides.

5. A right-angled triangle, 1·25″, 3″, and 3·25″, lies in a plane inclined 40°. Its hypotenuse is inclined 20°. Draw plan and elevation, and state the inclination of the base and perpendicular.

6. A square pyramid, base 1″ edge, 3″ long, is to be drawn when its base is inclined at 47°, and one edge of that base at 27°.

7. A square prism, base 1″ side, 3″ long, has its axis inclined 60°, one edge of the base is inclined at 20°. Draw plan and elevation, and show the true shape of a section made by a vertical plane bisecting the axis at an angle of 40°.

8. A triangle, A B C, A B, 3″, B C, 3·5″, and A C, 4″, lies in a plane inclined 70°, the line joining the middle point of A B to C is inclined 20°. Draw plan and elevation.

9. A tetrahedron, 2″ edge, has one face inclined 50°, whilst the line bisecting this face is inclined 40°. Draw plan and elevation.

10. An octahedron, 1·5″ edge, is so situate that the plane containing two of its diagonals is inclined 70°, one of these being inclined 45°. Draw plan and elevation of the solid.

11. A square, A B C D, 2″ side, is inclined 40°, the line joining A to the middle point of C D is inclined 15°. Draw plan and elevation, and determine the inclinations of A B and B C.

12. A cube, 3″ edge, has one of its faces inclined 60°, the plan of one of the edges of that face is 2·5″ long. Draw plan and elevation.

13. An equilateral triangle of 1″ side is the base of a prism 4″ long. One of its faces is inclined 45°, and an edge of the base belonging to that face is inclined 21°. Draw plan and elevation of the whole, and add a sectional elevation upon a vertical plane, bisecting the plan of the axis at an angle of 70°.

CHAPTER VIII.

ON THE PROJECTION OF OBLIQUE SURFACES, WHEN THE INCLINATION AND THE ANGLE BETWEEN TWO LINES IN THEM ARE GIVEN.

We have already seen that, if the inclinations of a figure and a line belonging to that figure are given, the position is fixed as regards the co-ordinate planes. This is also the case if the inclination of two lines upon the figure, meeting at a given angle, are known.

Thus, if an equilateral triangle has two of its sides inclined, at angles of 20° and 30° respectively, its projections upon the co-ordinate planes will be the same wherever that triangle may be placed.

But the oblique plane which contains them cannot be assumed at once. It must be determined by construction.

It is well that the student should understand this at once, as in all problems upon inclined surfaces which have previously claimed his attention, he has been enabled to assume the oblique plane at the commencement. We proceed, therefore, to explain the determination of a plane containing two lines, having given their inclinations and the angle between them.

Problem LXIX.

Two Lines, A B, B C, *indefinite in length, are mutually perpendicular; the former is inclined* 40°, *the latter* 20°; *required to determine by its traces a plane containing them.*

In the first place, the writer would recommend the student to cut out a piece of card-board of exactly the same shape as that shown in fig. 172; that is, he must prick through upon some paper the points A, D′, B, D, C. Let him then cut through the lines A D′, D′ B, B D, D C, and C A. After he has done this, let him fold the model upon the lines B A and B C, until the points D′ and D coincide.

If he then lets the model rest upon the lines A D′, D C, and A C, he will have an inclined surface represented by the triangle, A B C; the edges, A B, B C, being also inclined to the h. p. The longer edge, A B, will be less inclined, whilst the shorter edge, B C, will be more inclined. The line A C will represent the h. t. of the oblique plane, which contains A B C, and consequently the lines A B, B C. Now, if the v. p. be assumed perpendicular to A C, the elevation of the triangle upon that plane will be a straight line. This straight line will be the v. t. upon the assumed v. p. of the inclined plane containing the triangle A B C. Now, referring to the problem before us, the following construction is necessary; the reason for drawing every line being perfectly clear, if it be investigated, by aid of the model described above:—

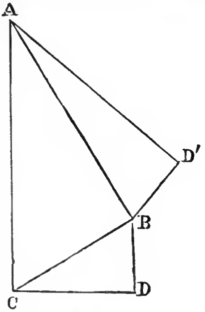


Fig. 172.

Draw any two lines, A B, B C (fig. 173), perpendicular to each other.* Assume a point, A in A B, and draw A D′ making an angle of 20° (the angle at which A B is to be inclined) with A B. At the point B, make B D′ perpendicular to A D′. Then the distance, B D′, is the height to which the point B must be raised, in order that A B may meet the h. p. in A, at the given angle of 20°. Now, as the second line, B C, is to start from the same

* They are directed to be drawn perpendicular, because A B and B C are to be mutually at a right angle with each other. Should they have been set as meeting at any other angle, they should make that angle upon the drawing.

point, B, it is clear that the line B D must be made equal to B D′. But its direction is not yet known. To ensure the former condition, describe an arc, D′ D, with B as centre, and B D′ as radius. It follows then that, whatever be the position of B D, it must be a radius of the arc just drawn.

Furthermore, the line D C is to meet B C, at an angle of 40° (the angle at which B C is to be inclined).

Now, a line perpendicular to the radius of a circle is a tangent to it, therefore, D C must be drawn touching the arc D′ D and meeting B C in C at an angle of 40°.

The best method of doing this is to assume any line as E F (at 40° with B C) and, by means of the set square, to draw a parallel to E F tangent to the arc. It is unnecessary to draw the line B D.

We shall now have found two points, A and C, in the surface containing the two given lines which we can assume as being upon the h. p.

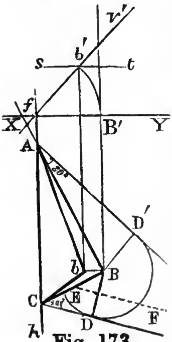


Fig. 173.

We will therefore join them, and consider the line A C as the h. t. of the oblique plane required. Our X Y must be taken perpendicular to A C. To determine the v. t., the triangle A B C must be rotated upon the side A C until the point B reaches a height equal to B D′. Then, the elevation of that point will be in the v. t. required. Through B draw a projector B B′, and, with *f* as centre, *f* B′ as radius, describe the arc B′ *b*′. Draw a line *s t* parallel to X Y, at a distance above it equal to B D′. This line will intersect the arc B′ *b*′ in *b*′, which is the elevation of B when in a position such that A B and B C make the given angles with the h. p.

If *b*′ be joined to *f*, the line *b*′ *f* will be the elevation of the whole triangle A B C, and its direction is that of the v. t. of the plane to be determined.

The plan of B will fall in a line perpendicular to the

h. t., its exact position being found by a projector through b'. Join b C, b A, and the lines thus formed will be the plans of A B, B C, when inclined as described in the problem.

Note.—The sum of the inclinations of the lines, together with the angle included between them, must not exceed 180°. When it equals that angle, the plane containing them is perpendicular to the h. p. The student will readily understand this when he remembers that the three angles of a triangle together make 180°, and that the lines will have their greatest inclination when the plane containing them is vertical. The angle between them is in that case the vertical angle of the triangle.

PROBLEM LXX.

Two sides of an equilateral triangle are inclined at angles of 34° *and* 40° *respectively; determine the plan and elevation of the figure.*

The plane must be determined containing two lines meeting at an angle of 60° (the angle of an equilateral triangle), and inclined as directed in the question by the construction explained in the preceding problem. Commence by drawing the equilateral triangle and produce two of its sides in opposite directions *away* from the point where they meet, and treat them in the same way as A B and A C, shown in Prob. LXIX. It is most convenient to assume the point

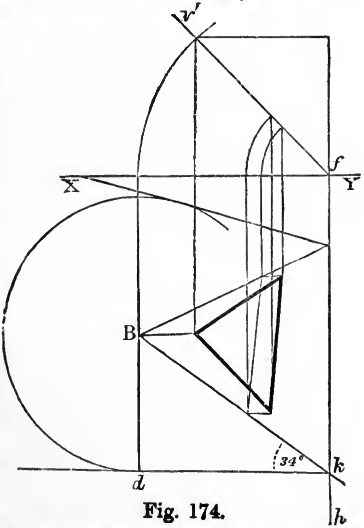


Fig. 174.

k in one of them, produced at some distance from a, as by doing so the traces of the plane, when determined, will be clear of the figure. At the point *k*, make *k d* at an angle of 34° with B*k*, and proceed as before.

When the traces, *v′ f* and *f h* of the oblique plane containing the triangle have been found, fold the figure into that plane, and thence determine the plan.

Problem LXXI.

Two Lines, A B *and* B C, *contain an angle of* 40°; *they are each* 2″ *in length; determine the plane containing them when they are so inclined, that the plan of* A B *is* 1·4″ *long, and that of* B C 1·6″.

This question is set in a similar manner to Prob. LXV. That is, instead of giving the inclination of two lines, the relation betwen their true length and that of their plans is stated. Take the length of the plan of A B, and at one extremity raise a perpendicular, with the other extremity as centre, radius 2″ (the length of A B), describe an arc intersecting the perpendicular. The angle contained by the base and hypotenuse of the triangle thus formed will be the inclination of the line A B.

Similarly, find the inclination of B C.

The problem then resolves itself into drawing the plan of two lines of given inclination, and containing a given angle between them.

Problem LXXII.

A Prism 2″ *long, whose base is an equilateral triangle of* 1″ *side, has two of the edges of that base inclined at angles of* 20° *and* 50° *respectively; draw its projections.*

Draw an equilateral triangle, and proceed, as in Prob. LXX., to determine the plane containing that triangle when two of its edges are inclined as described.

Determine the plan of the figure, and at the three points in elevation, which fall upon the v. t. of the plane, raise perpendiculars, each 2″ long.

Finish the elevation, and obtain the rest of the plan by projectors, through the points of the base which is highest, meeting the plans of the perpendicular lines.

Notice that the plan of the upper triangle will be firm lines, and any dotted edges which occur in the plan can be reasoned out upon this assumption.

Problem LXXIII.

One diagonal of an Octahedron 1·5″ *edge, is inclined* 25 ; *a second diagonal is inclined* 30°; *draw a plan and elevation of the solid.*

We have already learned that an octahedron has three diagonals or axes. Any two of these will also be the diagonals of one of the square sections of the solid.

Draw, therefore, a square of 1·5″ side, and show its diagonals. Consider that these latter are to be inclined as directed in the problem.

The plane containing the diagonals, and, consequently, the square, can be determined as in previous problems. The plan of the square, too, is readily deduced. There are two other points in the solid at the extremities of the third diagonal, which is perpendicular to the plane containing the square A B C D.

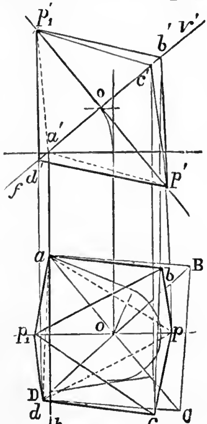


Fig. 175.

The projections of this line will be perpendicular to the traces of the plane $v' f h$. Draw, therefore, $p_1 p'$ perpendicular to $v'f$ and p, p—to $f h$ through the projections of the centre of the square.

The elevation p'_1 p' will be equal in length to the diagonal itself. Mark off therefore o p' and o p'_1 equal to OA. Join each of the four points a' b' c' d' to p'_1 and p', dotting the hidden lines a' p'_1, a' p', and the elevation will be complete.

The plans of the points P and P′ will be determined upon a line through O, parallel to X Y, by projectors through $p'_{,}$ and p'.

The edges, which should be dotted in the plan, are best determined by noticing from the elevation which vertex of the solid is uppermost, and deciding accordingly.

Problem LXXIV.

The three corners, A, B, *and* C, *of an Equilateral of* 1″ *side are* ·5″, ·75″, *and* ·3″ *above the paper respectively; determine the plane containing the figure and its plan.*

This is a modification of the cases explained in the previous part of this chapter. Instead of the inclination of two lines being given, we are told their lengths, and the heights of three points in them above the h. p. Such problems can be solved by one of two constructions—the more preferable of which, because the simpler, we proceed to explain.

Draw the equilateral triangle, A B C, and produce A B and B C, beyond A and C.*

With the points A B and C as centres, and with radii ·5″, ·75″, and ·3″, respectively, describe arcs as in the figure. Then a line, *p q*, tangent to the arcs which have A and B as their centres, will, when produced, meet the line B *n* in *n*. Similarly, a tangent, *r t*, to the arcs which have B and C for their centres, will meet B C produced in *k*. Then the angles, *p n* B, and *r k* B, are the angles of inclination of the lines A B and B C, when the points A B C are raised to the heights given in the problem.

The points *n* and *k* would be in the h. p., and the line, *n* K, joining them, is the h. t. of the plane containing the figure.

Take X Y, perpendicular to *k n*. To obtain the point *b′* in the v. t, draw a projector through B to meet X Y

* The line should be produced in directions *away* from the highest point.

in B′. Then with f as centre, radius fB′, describe the arc B′ b', and discover a point, b', in that arc which is ·75″ above X Y. Join $b' f$. Then $v' f h$ is the plane containing the figure when in the position required. Its plan and elevation can be determined in the usual manner.

The proof of the construction is made by noticing that the elevation of the three points, A B and C, are at their given heights above X Y.

When the *given heights* of any three points are considerable, so making the arcs, to be described in the construction, impracticable, the difficulty is remedied by subtracting an equal height from each of the given ones, and solving the problem with the new data thus obtained. Thus had A B C, in the present question, been 3·5″, 3·75″, and 3·3″ above the paper, the 3″ could have been subtracted from each, and the problem worked with the data ·5″, ·75″, and ·3″. The plans and elevations would have been the same as if the original numbers had been retained.

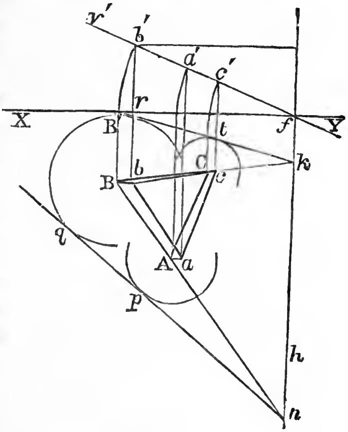


Fig. 176.

If the given height of the lowest point were subtracted from each, that point would be brought upon the h. p., as shown by the elevation.

Problem LXXV.

A Square, A B C D, *2″ sides, has its corners,* A B *and* C, *·8, 1·2″, and ·3″ above the paper respectively; draw its projections when in this position.*

This is a similar problem to the preceding, and can be solved by the same construction. It is, however, our intention here to describe the second method of solution, previously mentioned.

The principle involved is that of finding some line upon the figure which shall be horizontal, when the corners are in the positions assigned to them in the problem. Upon this line, when determined, the figure is supposed to rotate until the points attain their specified heights. In fact, it is used as the h. t. of the plane of the inclined square. Commence by drawing the square, A B C D. The line B C will join the points of the figure which are to be the highest and lowest of those whose heights are given. It will be an inclined line, and there will exist upon it some point whose height above the paper will be ·8″ (that of the remaining given point). Seeing that the difference in height of the extremities of B C is ·9″ (1·2″ minus ·3″) and that A is ·5″ (·8″ minus ·3″) above C, we can infer that a point $\frac{5}{9}$ of the distance from C to B will be that unknown spot in B C whose height will be ·8″, and equal to that of A. Join A 5, and consider it as the horizontal line upon which the figure A B C D is to revolve, in fact, the h. t. of its oblique surface when in the required position. Assume X Y perpendicular to A 5, and through B draw the projector B B′, and with a' as centre, radius a' B′, describe the arc B′ b'. The point b' will be upon this arc at the height above X Y, which B is above A, that is, ·4″ (1 2″ minus ·8″). Join $b'a'$ and consider it as the v. t. of

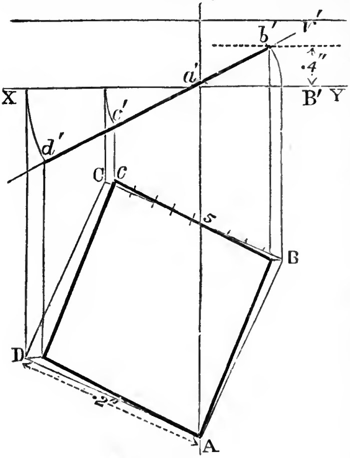


Fig. 177.

the plane, into which the square A B C D is to be revolved. The remainder of the construction will be understood by reference to the diagram.

Note.—By assuming A a' as the h. t., the point C falls below X Y. This is immaterial, as the projections of the figure are the same as if the whole of it were above the h. p.

EXERCISES.

1. Two lines meet at an angle of 90°,—one is inclined 25°, the other at 40°. Draw plan and elevation.

2. A square, A B C D, of 3″ side is to be represented in plan and elevation, when the lines joining A B and the middle points of C D, A D are inclined at 30°.

3. A pentagon has two of its sides inclined 12 and 20° ; draw plan and elevation. State the inclinations of the other sides and the diagonals.

4. Draw the plan of a cube of 3″ edge, when two edges are inclined at 25° and 50° to the paper, and add an elevation on a ground line parallel to that diagonal of the cube which has the shortest plan.

5. Draw the plan and two elevations of an equilateral triangle A B C of 3″ side when the sides A B, B C are inclined at 35° and 55° to the paper (one elevation to be on a plane parallel to the line A C).

6. Two lines, each 3″ long, are at right angles ; draw the plan of them when one is inclined to the paper at 25°, the other at 50° ; add an elevation on a plane (*i.e.* the ground line) parallel to the line joining the extremities of the given lines.

7. A hexagon, 1″ edge, has its diagonal inclined 40°, and one side of the base, which meets this diagonal, 30°. Draw plan and elevation.

8. A tetrahedron, 2″ edge, has two of its sides making angles of 40° and 25° with the paper. Draw plan and elevation.

9. The plans of two adjacent sides of a square of 3″ edge are 2·5″ and 3″ in length ; draw a plan and elevation of the figure.

10. An equilateral triangle of 3″ side has one edge inclined 40°, the plan of an adjacent side being 2·5″ long. Draw plan and elevation.

11. The three corners of an equilateral triangle of 3·25″ side are raised above the paper 1″, 1·75″, and 3″ ; draw its plan in this position ; add an elevation on a ground line parallel to the shortest side of the plan.

12. A triangular pyramid, its four faces being equal equilateral triangles of 3″ edge, is to be shown in plan and elevation when three of its corner are 1″, 1·5″, 2·5″ above the paper.

APPENDIX.

ADDITIONAL PROBLEMS AND EXAMINATION PAPERS.

N.B.—A few special solutions are here added, to show the reader that many roads may lead to the same end, and to induce him to suggest for himself methods independent of his course of instruction.

PROBLEM LXXVI.

Given both projections of three points, A, B, *and* C; *given also one projection of another point, D, to discover the other projection, assuming that the four points are contained by one plane.*

Let a' a, b' b, c' c, and d (fig. 178) be the given data; to find d'.

This could be solved by first obtaining the traces of the plane of A, B, and C (Prob. XL.), and afterwards by the aid of Prob. XXXVII., discovering the elevation d' of the point on the plane whose plan is d. But if the four points are contained by a plane, the lines joining them taken two and two must be in the plane, and hence *their intersection* also. Join, therefore, a to b and c to d. Consider a b and c d as plans of two lines. Join a' b' and project upon it, the elevation e' of the intersection of a line A B with a line joining C to D. Though c' and e' draw c' e' to meet a projector from d in d'. This is the required elevation.

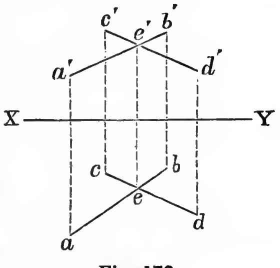


Fig. 178.

Problem LXXVII.

To determine the angle between the traces of an oblique plane when the co-ordinate planes are in their proper positions.

This Problem is identical with Problem XXXVI., but it is solved here by a different method, and two examples are given.

First, let *v′ f h* (fig 179) be the given plane; If *any* point in the v. t. (not on X Y) be joined to a point on the h.t., a triangle will be formed consisting of this line and the two intercepted portions of the traces. The angle of this triangle which occurs at X Y is the one required.

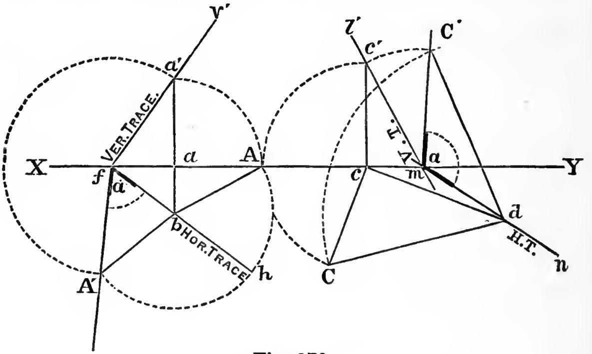


Fig. 179.

Construction:—Take *a′* and *b* on the given traces (preferably, vertically opposite as in the fig.) Join *a′ b.* With *a* as centre, describe an arc through *a′* to meet X Y in A. Then *b* A is the full length of a line joining *a′* to *b*. Make the triangle A′ *b f*, with sides equal to *a′ f*, *b* A, and *f b*. The required angle *a* is shown at *f*.

In the second case, the plane *l′ m n* is chosen with traces making an obtuse angle. Proceed as before to select points *a′* and *b* on the traces. Find the true length of a line joining *a′* to *b*, and then construct the triangle C′ *m d.*

PROBLEM LXXVIII.

Given the traces of a plane upon two co-ordinate planes to determine its trace upon a second vertical plane.

Let *v′ f h* (fig. 180) be the given traces to a given X Y. Let also X_1 Y_1 be the given new ground line. On the original plane, draw a horizontal line at any height. Its plan is *p q*, and *p p′* determines its height. At *q* in the new X_1 Y_1 erect *q′ q′* perpendicular to it and equal to *p′ p*. Then draw the new trace *h k* through *q′*.

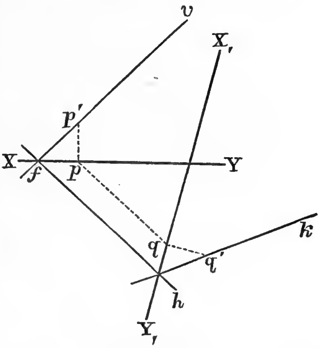


Fig. 180.

PROBLEM LXXIX.

Given the projections of a line A B, *to determine the traces of a plane inclined θ, to contain the given line.*

Let *a′ b′*, *a b* (fig. 181) be the given line. Assume any point in it as *c′ c′*, for the apex of a cone whose axis shall be vertical, and whose base angle is equal to *θ*. Find the h. t. of the given line (*t*). Then the plane or planes (as there are two to satisfy the conditions) will have the h.t.'s passing

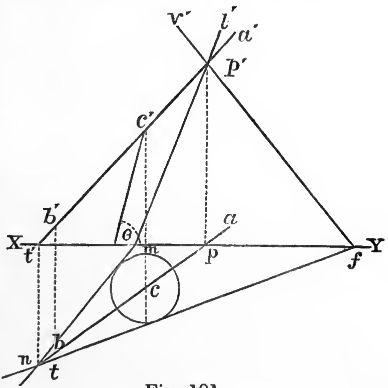


Fig. 181.

through *t*, and tangential to the plan circle of the cone. The planes *v′ f h* and *l′ m n* satisfy the conditions. Note that the v, t.'s of the plane pass through *p′*, the v. t. of A B.

Problem LXXX.

To determine the plan of the intersection of two given planes; when the meeting points of their traces are inaccessible.

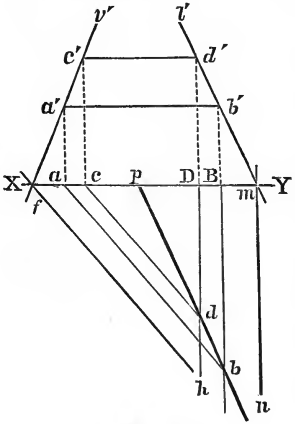


Fig. 182.

Let it be assumed that the points where the traces of the planes *v′ f h* and *l′ m n* (fig. 182) meet may not be used. On each plane draw two horizontal or level lines, at any heights, as A B and C D. Note where their plans cross, as at *b* and *d*. Then the plan of the intersection passes through these points.

Problem LXXXI.

To draw a plane parallel to X Y, *and* $\frac{3}{4}''$ *from it. It is to be inclined* 50° *to h. p. Then to discover the point in which a given line a′ b′, a b intersects the plane.*

Having drawn X Y, take any point O upon it as centre, and describe a circle of $\frac{3}{4}''$ radius. Make A B′ a line tangent to this circle at 50° with X Y. Draw A

B perpendicular to X Y through O. Consider the triangle A o B′ as a profile view of the co-ordinate planes and the required oblique plane. Then deduce the v. t. as a line v' f' parallel to X Y through A, and by the aid of the arc B′B, obtain the h. t. hf.

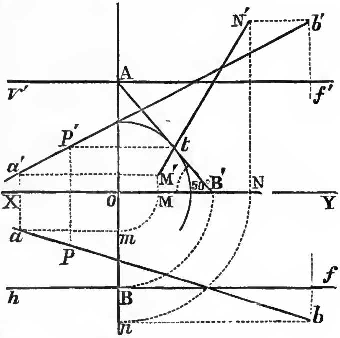


Fig. 183 (½ *Scale*).

Assume a' b', a b the projections of a line intersecting this plane, and by means of arcs and parallels, get the elevation of it upon the profile view. Thus, a, in plan, is projected to m, and rabatted by the arc m, M, and the projector M M′ to M′; N′ is obtained similarly. Then t, the crossing point of M′ N′ and A B′ gives the view (on the profile elevation) of the required intersection point. Transfer t to p', and hence obtain p ; p' p is the required intersection.

SPECIMEN EXAMINATION PAPER, 1884.

FIRST STAGE OR ELEMENTARY EXAMINATION.

Only eight questions are to be attempted.

Their relative values are shewn in brackets.

*Those questions marked * refer to diagrams on opposite page.*

Plane Geometry.

1. Construct a triangle whose sides are 10′·6″, 14′·0″, 16′·3″. Scale, $\frac{1}{8}=1'\cdot0''$. (6.)

2. Draw two lines intersecting at an angle of 52°, and between them place a line of 2·25″ long, making 58° with one of them. (8.)

*3. Describe a circle passing through the given points *a*, *b*, and touching the given line *cd*. (8.)

Solid Geometry.

4. Show by their traces :—

a. A line inclined at 37° to the vertical plane, and 53° to the horizontal plane.

b. A line inclined at 33° to both planes of projection. (8.)

*5. Represent by its traces a plane inclined at 65° to the horizontal plane, and containing the given point *aa′* (8.)

*6. *ab* is the plan of a line lying in the given plane. Determine the inclination of this line to the horizontal plane. 10.

7. The plans of two parallel horizontal lines are 1·75″ apart. The heights of the lines are 0·5″ and 1·75″ respectively above the horizontal plane. Determine the inclination of the plane containing the two lines. (10.)

*8. Standing at *a* with your eye 5′·0″ above the ground, you can just see the top of a vertical post at *c* over the head of a man 6′·0″ high standing at *b*. Determine and write down the height of the post. Scale, $\frac{1}{4}''=1'\cdot0''$. (9.)

*9. The given rectangle represents the elevation of a square. Determine the inclination of the plane of this square to the vertical plane. (10.)

*10. The figure represents the plans of two ordinary bricks placed one on the top of the other, the lower brick resting on the horizontal plane. The height of each brick is $2\frac{3}{4}''$. Make an elevation of the bricks on the line *ab*. Scale, $\frac{1}{4}$ full size. (12.)

*11. Draw the plan of one of the bricks (Question 10) when one of its shortest edges is in the horizontal plane, and the plane of its end is inclined at 52° to that plane. (14.)

*12. The plan and end elevation of a trestle are given. Draw an elevation of the trestle on AB. (16.)

*13. Make a sectional elevation of the trestle (Question 12) on CD. (18.)

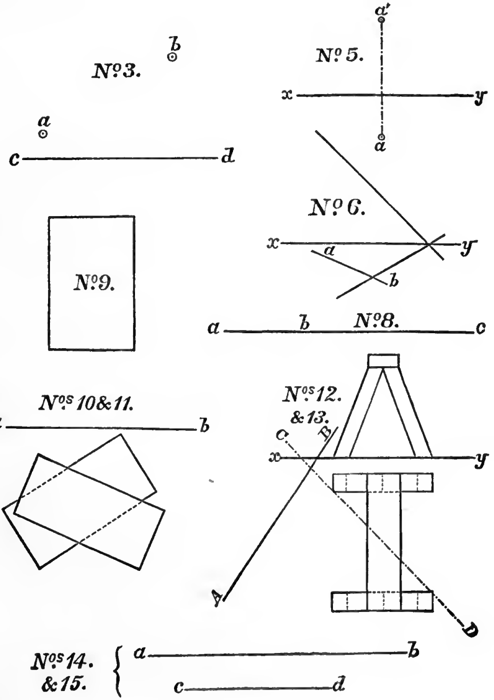
Graphic Arithmetic.

*14. If *ab* represents 26, what does *cd* represent, and what is the unit? (8.)

*15. *ab* (Question 14) represents 9. Obtain by construction a line which will represent the square root of *ab*. (10).

ELEMENTARY STAGE.

The Diagrams are to be accurately copied to three times the Scale.



SPECIMEN EXAMINATION PAPER, 1885.
First Stage or Elementary Examination.

Only eight questions are to be attempted.

Their relative values are shewn in brackets.

The diagrams on the next page give the necessary data.

Plane Geometry.

1. 13′·4″ are represented by 1 inch. Draw the corresponding scale, divided generally into ten feet lengths, with one such length sub-divided to show single feet. (8.)

2. Draw an equilateral triangle of $2\frac{1}{4}''$ side, and in it draw three equal circles, each touching 2 sides of the triangle and the remaining circles. (8.)

*3. Construct a rectangle equal in area to the given triangle, and one side of which is 1″ long. (8.)

Solid Geometry.

4. Show by their projections :—

a. A line parallel to the vertical plane, and inclined at an angle of 48° to the horizontal plane.

b. A line inclined at 35° to the vertical plane, and the plan of which makes an angle of 50° with the ground line. (8.)

*5. $l\,m$ is the horizontal trace of a plane inclined at 55° to the horizontal. Draw its vertical trace, and from the point whose projections are p and p' draw a horizontal line meeting the plane in a point 2″ from pp'. (10.)

*6. Draw a plane parallel to the ground line, 1″ distant from it, and making equal angles with the planes of projection; and find the projections of the point in which the given line AB meets this plane. (10.)

*7. Draw a plane to contain the given line aa', bb', and the horizontal trace of which makes an angle of 45° with the ground line. (10.)

*8. From the given point pp' draw a perpendicular to the given plane, and determine the real length between pp' and the point in which it intersects the plane. (10.)

*9. The isosceles triangle ABC is the plan of an equilateral triangle. Determine the inclination of the plane of the triangle to the horizontal plane. (12.)

*10. The elevation of a letter X is given. Supposing it cut out of wood of thickness equal to the breadth of the bars, draw its plan when standing $1\frac{1}{2}$ inches in front of the vertical plane. (12).

*11. Draw an elevation of the letter (Question 10) on the ground line x_2y_2. (14.)

*12. The front and side elevation of a hencoop are given. Draw its plan and an elevation on the ground line x_1y_1. (16.)

*13. Draw the section of the hencoop (Question 12) made by a vertical plane passing through the centre of the plan and making an angle of 55° with the sides. (16.)

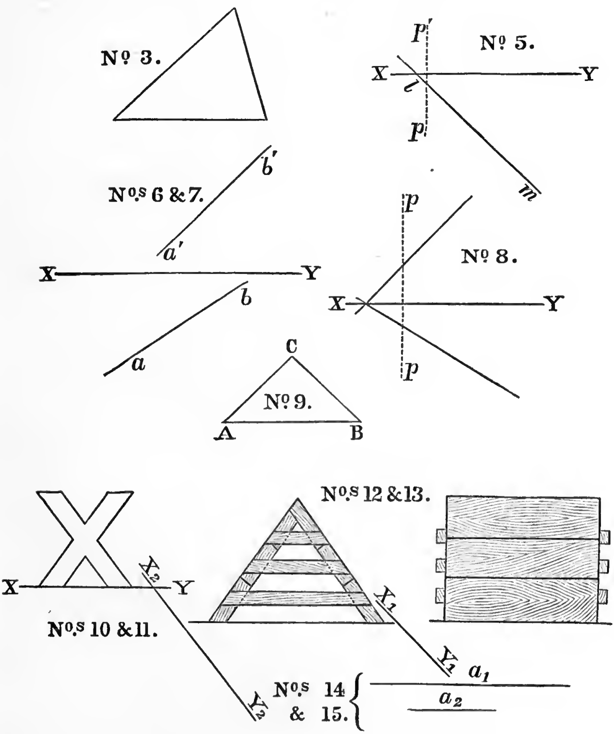
Graphic Arithmetic.

*14. Determine the product of the lines a_1, a_2, the unit being $\frac{1}{2}''$, and write down the number of units it contains. (10.)

*15. If a_1 (Question 14) represents $\sqrt{23}$ what is the unit? (10.)

ELEMENTARY STAGE.

The Diagrams are to be accurately copied to three times the Scale.



SPECIMEN EXAMINATION PAPER, 1886.

First Stage or Elementary Examination.

Only **eight** questions are to be attempted.

Their relative values are shewn in brackets.

*Those questions marked * refer to diagrams on the opposite page.*

Plane Geometry.

*1. The line *ab* represents 3′.9″. Construct a scale reading inches and showing 10′.0″. The scale to be correctly figured. (8.)

2. Construct a square $2\frac{1}{2}''$ side, and through an angle draw a line cutting off one-third of its area. (8.)

*3. Describe a circle touching the given line *ab* in *c*, and passing through the given point *p*. (8.)

Solid Geometry.

4. Represent by their traces :—

a. Two planes at right angles to each other and to the vertical plane of projection, and one of them inclined at 40° to the horizontal plane.

b. Two parallel planes not at right angles to either plane of projection. (8.)

5. Draw the trace of a plane parallel to and $2\frac{1}{4}''$ above the horizontal plane, and determine the projections of a point in this plane $3\frac{1}{8}''$ from the ground line. (10.)

*6. Obtain the projections of two points P and Q on the given line *ab*, *a′b′*; such that P is 1·4″ from the vertical plane, and Q 2″ from the horizontal plane. Determine and write down the real length of the line PQ. (10.)

*7. *ab* is the plan of one side of an equilateral triangle lying in the given plane. Complete the plan and draw the elevation of this triangle. (12.)

*8. The plan *abcv* of a triangular pyramid standing on the horizontal plane is given. The vertex of the pyramid is in the given plane *mon*. Draw the elevation of the pyramid and also a section on the line x_1y_1. (12.)

9. Draw the plan of a hexagon of $1\frac{1}{8}''$ side in any position, such that its plane is neither horizontal nor vertical. (12.)

10. The T square, supposed to have no thickness, rests with the angles *a* and *b* on the horizontal plane and its own plane is vertical. Draw an elevation on a plane making 35° with the plane of the T square. (12.)

*11. *ab*, *a′b′* are the projections of a given line; *cd* is the plan, and *p′* a point on the elevation of a second line intersecting the first. Determine the traces of the plane containing the two lines. (12.)

*12. The plan and elevation of a simple solid are given. Draw an elevation on a line parallel to x_1y_1. (14).

*13. Make a sectional elevation of the solid (Question 12) on the line AB. (16.)

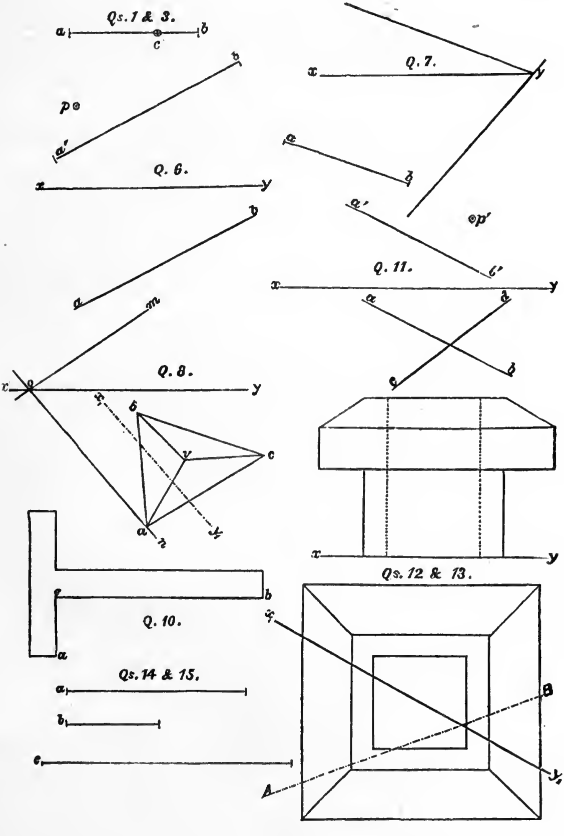
Graphic Arithmetic.

*14. The given line *a* represents the product of the given lines *b* and *c*. Determine and write down the length of the unit. (10.)

15. Taking one-third of *a* (Question 14) as unit, determine a line representing $\frac{\sqrt{c}}{b}$. (10.)

ELEMENTARY STAGE, 1886.

The Diagrams are to be accurately copied to three times the Scale.



SPECIMEN EXAMINATION PAPER, 1887.

First Stage or Elementary Examination.

Only **eight** questions are to be attempted.

The relative values are shewn in brackets.

*The questions marked * refer to diagrams on the opposite page.*

Plane Geometry.

*1. Draw the given figure *full size*, adhering strictly to the figured dimensions. (8.)

*2. Reduce the given figure to a square of equal area. (8.)

3. Construct a triangle whose sides *ab*, *bc*, *ca* are $3\frac{1}{4}''$, $2\frac{3}{8}''$, and $2''$ respectively. On *ac* construct a second triangle, *adc*, whose vertical angle *adc* is equal to the angle *abc* and the side *ad* $1\frac{5}{8}''$. N.B. The angles upon the same base and in the same segment of a circle are equal. (8.)

Solid Geometry.

4. *a*. Show by their traces two vertical planes making 60° with each other, and with the vertical plane of projection.

b. Show by their projections any two equal parallel lines not parallel to either plane of projection.

*5. State precisely what is represented by each of the four given figures (A), (B), (C), (D). (10.)

6. A is a point in the vertical plane $1\frac{1}{4}''$ above the horizontal plane; B is a point in the horizontal plane $1\frac{3}{4}''$ from the vertical plane. The real distance from A to B is $3''$. Draw the plan and elevation of the line joining A and B. (12.)

*7. a' is the elevation of a point in the given plane *lom*; *b* the plan of a point in the given plane no_1p. Determine the real length of the line joining these points. (12.)

*8. A vertical plane, of which *ht* is the horizontal trace, cuts the two given planes. Draw the elevations of the intersections, and determine the real angle they contain. (14.)

*9. *ab*, a line in the horizontal plane, is the base of a triangle, the real lengths of the other sides of which are $2\frac{1}{4}''$ and $1\frac{3}{4}''$. The height of the vertex is $1\frac{1}{2}''$ above the horizontal plane. Complete the plan and draw the elevation of the triangle. (14.)

10. Draw the plans of two spheres, $1''$ and $2\frac{1}{2}''$ in diameter, touching each other, and both planes of projection. (14.)

11. Draw the plan of a cube of $2\frac{3}{4}''$ side with one axis vertical (14.)

*12. Four equal bricks are arranged as shewn, their side faces being all in one plane. The dimensions of each brick are $9'' \times 4\frac{1}{2}'' \times 3''$ Draw the plan of the pile, and an elevation on a plane inclined at 35° to the long edges of the three lower bricks. *Scale* $\frac{1}{8}$ *full size.* N.B. The figure is *not* to be merely reproduced. (14.)

*13. Make a sectional elevation of the pile (Q. 12) by a vertical plane containing the diagonals of the horizontal faces of brick A. (14.)

Graphic Arithmetic.

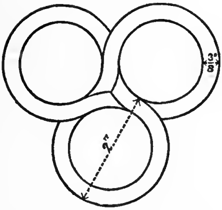
*14. Determine by construction a line representing the sum of the given lines A, B, divided by C. *Unit* $1\frac{1}{8}''$. (10.)

15. Supposing that the area of a circle is $\frac{22}{7}r^2$, where r is the radius; obtain by construction and write down the length of a line representing the area of a circle $3''$ in diameter. *Unit* $1''$. (14.)

ELEMENTARY STAGE, 1887.

The Diagrams are to be accurately copied to three times the Scale.

No 1.



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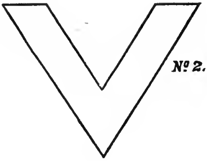

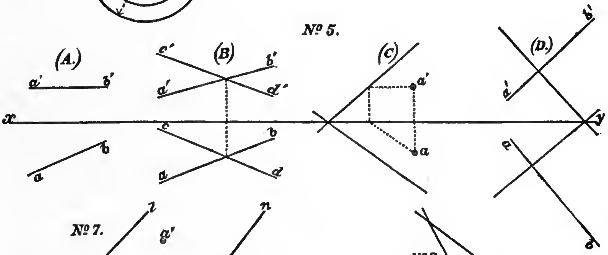

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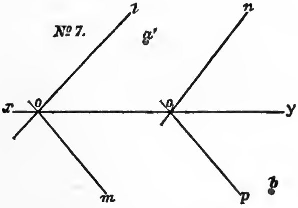

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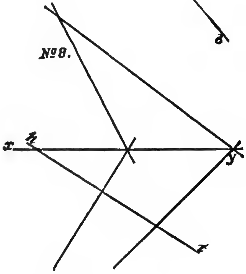

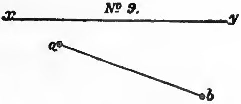

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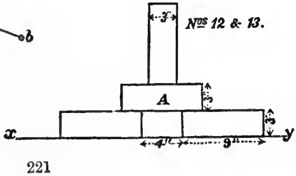

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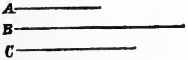


SPECIMEN EXAMINATION PAPER, 1888.

First Stage or Elementary Examination.

Only **eight** questions are to be attempted.

Their relative values are shown in brackets.

*Those questions marked * refer to diagrams on the opposite page.*

Plane Geometry.

*1. Given a scale of yards. Deduce from it a scale of feet to read to 1′·0″, and show 70′·0″. (8.)

*2. *aa*, *bb*, are given parallel lines; *p* is a given point. Through *p* draw a line cutting the given lines in points $1\frac{7}{8}''$ apart. (8.)

*3. The given figure represents a Maltese cross. Two dimensions and an angle are given. Draw the cross to a scale of $\frac{1}{8}'' = 1'\cdot0''$. (10.)

Solid Geometry.

4. A point 1·5″ from both planes of projection is distant 3·25″ from another point 2·25″ from both planes of projection. Obtain the projections of the two points. (10.)

*5. The given isosceles triangle, *abc*, is the plan of an equilateral triangle, whose base *ab* is *in* the horizontal plane. Determine the height of the vertex C above the horizontal plane. (10.)

*6. *vv*, *hh*, are the traces of a plane parallel to the ground line; *a* is a point *on* the ground line. Determine the length of a perpendicular drawn from *a* to the given plane. (10.)

*7. Draw the projections of a horizontal line lying in the given plane and 1·25″ above the horizontal plane of projection. (10.)

*8. O*a* is the horizontal trace of a plane inclined at 65° to the horizontal plane. O*i* is the plan of the intersection of this plane with a second plane whose horizontal trace is O*b*. Determine and write down the inclination of the second plane to the horizontal plane. (12.)

*9. *abcd* is a four-sided figure; draw its plan when *bd* is horizontal, and *ac* inclined at 62° to the horizontal plane. (12.)

10. A hexagonal right pyramid, side of base $1\frac{1}{4}''$, height $3\frac{3}{4}''$, stands on the horizontal plane. Draw the plan and make a section by a vertical plane, the horizontal trace of which is a line through one angle of the base passing $\frac{3}{8}''$ from the plan of the vertex. (14.)

11*. *abc* represents the plan of a wedge-shaped slice cut out of a cylindrical cheese, whose height is equal to half the radius *ab*. Draw a plan of the slice resting with one of its rectangular faces on the horizontal plane, and construct an elevation on a line making 25° with the short sides of that face. (14.)

*12. The letter A as given is cut out of material $\frac{3}{4}''$ thick, and stands on the horizontal plane. Draw the plan and make an elevation on a plane parallel to a diagonal of the rectangle at the top. (14.)

*13. Make a sectional elevation of the letter A (Question 12) on a line passing through the plan of the point *b′*, and making 35° with the plan of the line *b′c′*. (14.)

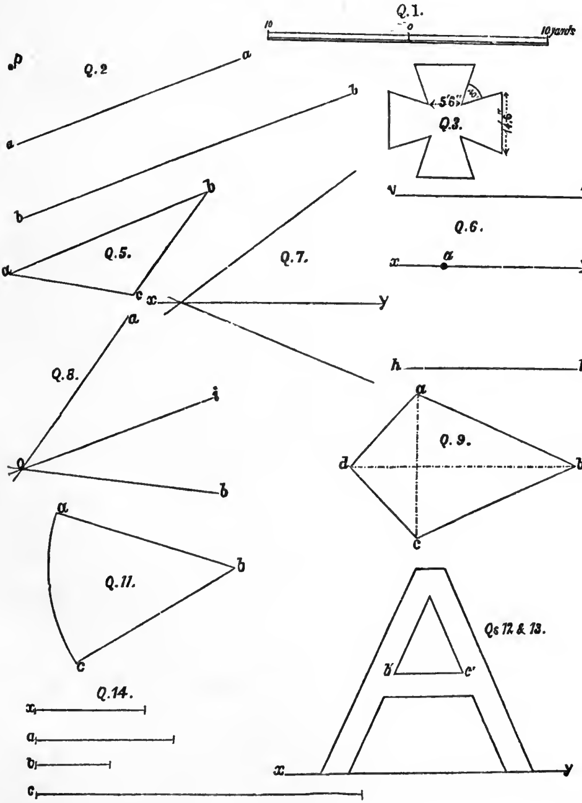
Graphic Arithmetic.

*14. The given line *x* represents the sum of the two lines *a* and *b* divided by *c* $\left(\text{in other words } x = \frac{a+b}{c}\right)$. Determine and write down the length of the unit. (10.)

15. Draw a curve which will give the squares of all quantities from 0 to 6, taking $\frac{1}{4}''$ as the unit. (12.)

ELEMENTARY STAGE, 1888.

The Diagrams are to be accurately copied to three times the Scale.



SPECIMEN EXAMINATION PAPER, 1889.

SECTION I. OR GEOMETRICAL DRAWING

AND

FIRST STAGE OR ELEMENTARY EXAMINATION.

Only eight questions are to be attempted.

The relative values are shown in brackets.

*Those questions marked * refer to diagrams on the opposite page.*

Candidates for examination in Geometrical Drawing only, may confine themselves to Section I.

Candidates for examination in the Elementary Stage of Practical, Plane, and Solid Geometry, must not attempt more than two questions in Plane Geometry.

SECTION I.—GEOMETRICAL DRAWING.

Plane Geometry.

*1. Convert the given scale into a diagonal scale reading inches. State the representative fraction of this scale. (8.)

2. Construct a scale of chords on a radius of 4·25″ to read to 5°. By means of this scale, plot an angle of 75°. (8.)

*3. Prick off the figure as given in Q. 7, and construct a similar figure, whose sides are to those of the given figure as 7 : 3. (8.)

*4. $a\ b\ c$ is a given triangle. Construct an isosceles triangle of equal area standing on $d\ e$ as base. (10.)

*5. Describe a circle of 1·25″ diameter touching each pair of adjacent lines $o\ a$, $o\ b$, $o\ c$, $o\ d$, produced if necessary. Describe two circles touching the three circles. (10.)

*6. The given figure is made up of circular arcs, all of $\frac{3}{4}$″ radius. Draw it full size. (10.)

*7. Construct a six-sided polygon $a\ b\ c\ d\ e\ f$, such that $e\ f = \frac{a\ e}{2} = \frac{3}{2} \cdot a\ f$; $d\ e = a\ d$ and $a\ b = \frac{2}{3} \cdot b\ c$. The rest of the data are given on the figure.
Scale $\frac{1}{2}$″ = 10′·0″. (10.)

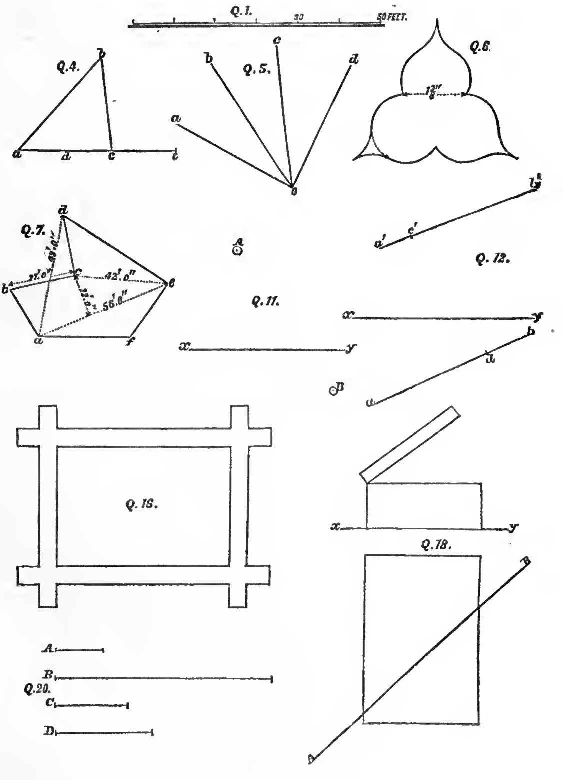
8. Construct a rhombus, side $2\frac{1}{2}$″, diagonal $4\frac{1}{4}$″. In this rhombus inscribe an ellipse.

Solid Geometry.

9. A pentagonal right prism, side of base 1·5″, height 1″, rests on a horizontal plane. On it is placed a right cone, whose base circle touches the sides of the top of the prism. The height of the cone is 2″. Draw the plan and an elevation on a plane, making 40° with one side of the base of the prism. (14.)

ELEMENTARY STAGE, 1889.

The Diagrams are to be accurately copied to three times the Scale.



N.B.—*The questions marked * refer to the diagrams on the preceding page.*

SECTION II.

10. A point is 2″ from the vertical and 1·75″ from the horizontal plane. Determine a point on the ground line 3·25″ from this point. (10.

*11. A is a point in the vertical plane; B a point in the horizontal plane. Draw the plan and elevation of the line AB. (10.)

*12. c' is the elevation of a point on the given line $a' b'$, $a b$; d is the plan of another point on the same line. Determine the real length of the line CD. (12.)

13. Construct an equilateral triangle of $2\frac{1}{8}$″ side. Consider the triangle to be the plan of a triangle ABC, such that the heights of the angles A, B, C, above the horizontal plane, are $\frac{7}{8}$″, $1\frac{5}{8}$″, and $1\frac{1}{4}$″ respectively. Draw the plan of a horizontal line lying in the plane of the triangle and passing through C. (12.)

14. Draw the traces of two planes not at right angles to either plane of projection, and determine the angle which the intersection of these two planes makes with the vertical plane of projection. (12.)

15. The horizontal trace of a vertical plane makes 42° with the ground line. Determine the elevation of a line lying in this plane, inclined at 30°, and passing through the point where the given plane cuts the ground line. (12.)

*16. The given figure represents a picture frame. The plane of the picture is inclined at 55° and the long edges of the frame are horizontal. Draw the plan, neglecting the thickness. (14.)

17. A hexagonal right pyramid, side of base $1\frac{1}{2}$″, height $3\frac{1}{2}$″, stands on the horizontal plane. It is truncated by a plane parallel to and $1\frac{5}{8}$″ from its base. Suppose this truncated pyramid to be tipped about an edge of the base till the plane of one of the six side faces is horizontal. Draw the plan. (16.)

*18. The given figure represents the end view and part of the plan of an open box made of wood $\frac{3}{16}$″ thick. Complete the plan and make a sectional elevation on the line AB. (18.)

Graphic Arithmetic.

19. A line 2·6″ long represents $4\frac{1}{2}$ units. Obtain lines representing 13; $\frac{10}{3}$, and $2\sqrt{6}$. (10.)

*20. If the given line A represents the unit, what numbers do the lines B, C, and D severally represent? Determine also a line representing $\frac{B}{C \times D}$. (12.)

SPECIMEN EXAMINATION PAPER, 1890.

SECTION I. OR GEOMETRICAL DRAWING

AND

FIRST STAGE OR ELEMENTARY EXAMINATION.

Only **eight** questions are to be attempted.

Their relative values are shown in brackets.

*Those questions marked * refer to diagrams on the following page* (228).

SECTION I.—GEOMETRICAL DRAWING.

Plane Geometry.

*1. The given line A B represents a length of 45 feet. Construct and figure correctly the scale to which A B is drawn. (10.)

2. Construct a square of 1⅝″ side. Through each angle draw a line parallel to a diagonal of the square, thus obtaining a second larger square. Repeat the process with the second square, obtaining a third square. (8.)

*3. Draw a triangle whose sides are respectively *twice* the length of those of the given triangle *a b c*. Inscribe a circle in this triangle, and circumscribe a circle about it. (12.)

4. Three lines A, B, C, are parallel; A is ⅝″ from B, and 2″ from C, B lying between A and C. Draw a line D cutting A, B, and C at an angle of 50°. Describe a circle touching D, B, and C. (12.)

5. Construct a regular pentagon *a b c d e*, on a side of 1·5″. Join the centre points of the sides *a b*, *d e*, and circumscribe the four-sided figure thus obtained by a circle. What is this figure called? (12.)

6. Construct a right-angled triangle *a b c*, in which *a c*, the hypothenuse, is 3·25″, and *a b* is 1·75″. (12.)

7. Triangles standing on the same base and having the same altitude are equal.

On a line A B, 2″ long, as base, construct a triangle A C B whose altitude is 2¼″ and angle A B C 105°. On A B as base describe a second triangle A D B equal to the triangle A C B, and having the side B D parallel to A C. (10.)

8. In a circle of 2⅝″ diameter, place seven equi-distant radii. From the outer extremity of any radius, draw a line to the outer extremity of the next radius but one, and continue the process round the circle, always missing out a radius. (12.)

Solid Geometry.

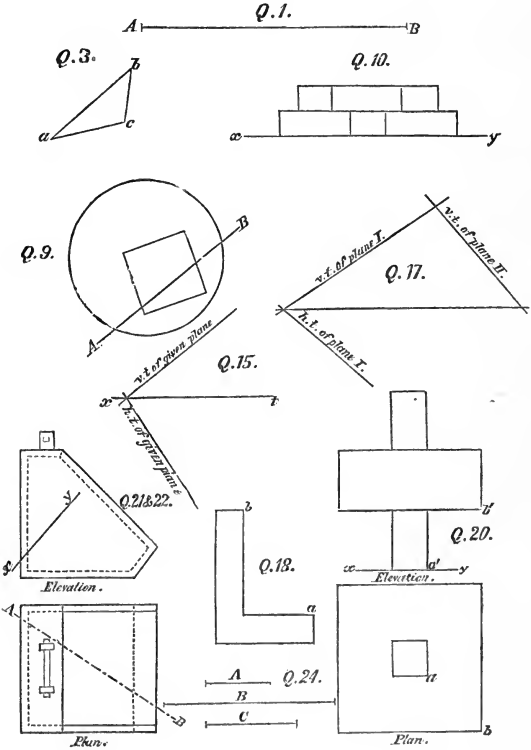
*9. The given figure represents the plan of a cube resting on a right cylinder 1·75″ high. Make a section of the solids on the line A B. (14.)

*10. The figure represents a portion of a wall made up of three whole and three half bricks. Draw the plan, and also an elevation on a line making 60° with the line of the wall.

N.B.—The thickness of the wall is half a brick. (16.)

ELEMENTARY STAGE, 1890.

The Diagrams are to be accurately copied to three times the Scale.



N.B.—*The questions marked * refer to diagrams on the opposite page* (228).

First Stage or Elementary Examination.

Plane Geometry.

11. Draw a triangle the sides of which are respectively twice the length of those of the given triangle $a\,b\,c$ (fig. to Question 3), and produce the sides indefinitely. Draw four circles each touching the three lines. (10.)

12. Construct a regular pentagon $a\,b\,c\,d\,e$ on a side of 1·5″. Join the centre points of the sides $a\,b$ and $d\,e$, and circumscribe the four-sided figure thus obtained by a circle. What is this figure called? (8.)

13. In a circle of $2\frac{3}{8}''$ diameter place seven equi-distant radii. From the outer extremity of any radius, draw a line to the outer extremity of the next radius but one, and continue the process, always missing out a radius. (8.)

Solid Geometry.

14. A point P is $\frac{3}{4}''$ in front of the vertical and 1″ above the horizontal plane of projection. Show it by its plan and elevation, and through it draw a plane inclined at 50° to the horizontal plane and perpendicular to the vertical plane. (10.)

*15. The traces of a plane are given. Determine the plan and elevation of a point which is in the given plane $\frac{1}{2}''$ in front of the vertical plane, and the plan of which is $1\frac{1}{4}''$ from the horizontal trace of the given plane. (10.)

16. The horizontal and vertical traces of a plane make respectively angles of 55° and 30° with the ground line. Determine the plane which bisects the obtuse dihedral angle between the horizontal plane and the given plane. (10.)

*17. The traces of a plane are given and the vertical trace of a second plane. This second plane is inclined at 65° to the horizontal plane. Determine the inclination of the first plane to the horizontal plane, and the plan and elevation of the line in which the planes intersect. (12.)

*18. The letter L, as given, rests on a plane inclined at 35°, with the points a and b on the horizontal trace of the plane. Draw its plan. (10.)

19. An equilateral triangle $a'\,b'\,c'$ of $1\frac{1}{2}''$ side is the end elevation of a prism 2″ long. Draw its plan when the long edge through A is in the horizontal plane, and the face containing the edge A B is inclined at an angle of 40° to the horizontal plane. (12.)

*20. The plan and elevation of a solid, made up of two square prisms, are given. Draw its plan when, instead of standing vertically as in the figure, the edges through the points marked A and B are both in the horizontal plane. (16.)

*21. The plan and side elevation of a wooden coal box are given. Draw the elevation on the given ground line $x\,y$. (14.)

*22. Draw the section of the coal box made by the vertical plane, the horizontal trace of which is the line A B. (14.)

Graphic Arithmetic.

23. Taking $\frac{1}{4}''$ as unit, obtain lines representing—

$$15,\ \sqrt{15},\ \text{and}\ \frac{5}{\sqrt{15}}. \qquad (10.)$$

*24. Taking the given line A as unit of length, determine and write down the number of units of area in the rectangle contained by the given lines B and C. (12.)

SPECIMEN EXAMINATION PAPER, 1891.

SECTION I. OR GEOMETRICAL DRAWING
AND
FIRST STAGE OR ELEMENTARY EXAMINATION.

Only **eight** questions are to be attempted.
The relative values are shown in brackets.
*The questions marked * refer to diagrams on the opposite page.*

SECTION I.—GEOMETRICAL DRAWING.

Plane Geometry.

1. Draw a scale, the representative fraction of which is $\frac{1}{360}$ reading yards. (10.)

*2. Draw the figure from the given dimensions. (10.)
N.B.—It is not to be merely copied as given.

3. Draw two parallel lines AB and CD $1\frac{3}{8}''$ apart; draw a line AD making the angle BAD = 35° = the angle CDA, and through D and A draw two parallel lines DB and AC, so that the figure ABDC is a rhombus. (12.)

*4 Construct a similar figure to the given one *abcd*, in which AB shall correspond to *ab*. (12.)

5. What is meant by the mean proportional between two lines? Find the mean proportional between two lines respectively $1\frac{1}{4}''$ and $2''$ long. (10.)

6. Construct a triangle ABC, AB = $2\frac{1}{4}''$, BC = $1\frac{3}{4}''$, CA = $2\frac{1}{2}''$. Take a point D on the side AB $1\frac{5}{8}''$ from A, and through D draw a line dividing the triangle into two equal parts. (14.)

7. Describe a circle $2''$ in diameter, and in it describe three equal circles each touching the other two. (10.)

8. Draw an ellipse, the distance between the foci being $2\frac{1}{4}''$, and the major axis $3''$ long. (12.)

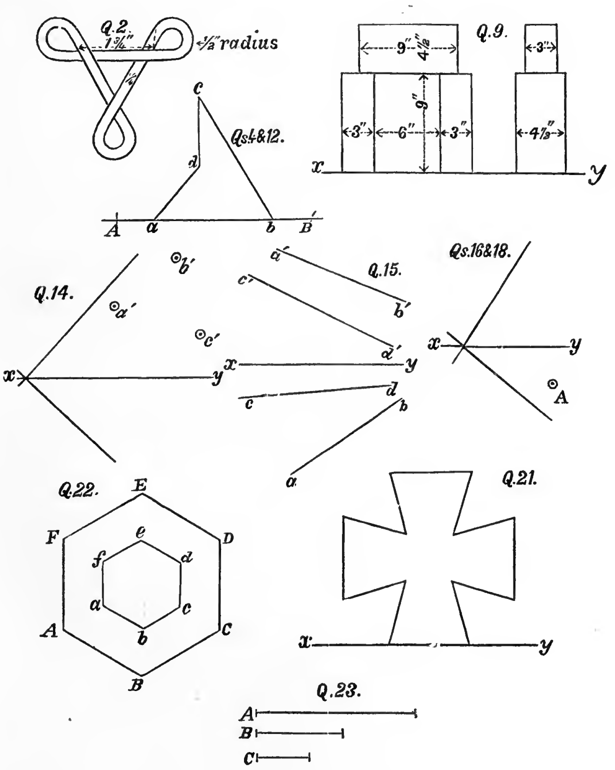
Solid Geometry.

*9. Three bricks each $9'' \times 4\frac{1}{2}'' \times 3''$ are arranged as shown by the front and end elevations. Draw the plan and construct the section by a vertical plane containing a diagonal of the top brick. Scale $\frac{1}{6}$. (16.)

10. A cube of $2''$ side stands on the horizontal plane, and a right cone (radius of base $\frac{3}{4}''$, height $1\frac{3}{4}''$) stands on the top of the cube, so that two of its edges are tangents to the base of the cone. Draw the plan and an elevation of the cube and cone. (14.)

ELEMENTARY STAGE, 1891.

The Diagrams are to be accurately copied to three times the Scale.



N.B.—*The questions marked * refer to diagrams on the preceding page* (231).

FIRST STAGE OR ELEMENTARY EXAMINATION.

Plane Geometry.

11. Construct a scale of feet for a drawing on which 7 feet 6 inches is represented by 1·5 inches. The scale is to be correctly divided and figured. (8.)

*12. Reduce the figure *abcd* to a triangle with AB as base, and vertex on *bc*. (10.)

13. In a circle $2\frac{3}{8}''$ in diameter inscribe a triangle having angles of 62° and 44°. (8.)

Solid Geometry.

*14. a', b', c' are the elevations of three points in the given plane. Determine the *true form* of the triangle ABC. (14.)

*15. A horizontal line 1·25″ above the horizontal plane is terminated by the two given lines ab, $a'b'$; cd, $c'd'$. Determine the real length of this line. (10.)

*16. Determine the intersection with the given plane of a vertical line passing through the given point A. (10.)

17. The horizontal trace of a vertical plane makes 35° with *xy*. Obtain the elevation of a line lying in this plane, inclined at 45°, and passing through *xy*. (10.)

*18. A line $2\frac{1}{2}''$ long, inclined at 40°, lies in the given plane and is terminated by the traces of this plane. Determine the projections of the line. (14.)

19. A regular hexagon of 1·25″ side has one side in the horizontal plane. The plane of the hexagon is vertical and inclined at 43° to the vertical plane of projection. Draw the elevation of the hexagon. (10.)

20. Draw the plan of a square of $2\frac{1}{2}''$ side in such a position that two adjacent sides are inclined at 50° and 35° respectively. (12.)

*21. The given figure represents a Maltese cross cut out of material $\frac{3}{4}''$ thick. Draw the plan and an elevation on a plane making 55° with *xy*. (14.)

*22. The two concentric, similarly-situated hexagons ABCDEF, *abcdef*, represent respectively the base and top of a truncated right hexagonal pyramid standing on the horizontal plane. The height of the truncated portion is 1·5″. Determine the *true form* of the section of the solid by a plane containing CD and *af*. (14.)

Graphic Arithmetic.

*23. If A represents the product of B and C, what length of line will represent the product of A and C? (10.)

24. If the area of an equilateral triangle of $2\frac{1}{4}''$ side is represented by $3\frac{1}{4}''$, what is the unit? (12.)

SPECIMEN EXAMINATION PAPER, 1892.

SECTION I. OR GEOMETRICAL DRAWING

AND

FIRST STAGE OR ELEMENTARY EXAMINATION.

Only **eight** questions are to be attempted.

Their relative values are shown in brackets.

*Those questions marked * refer to diagrams on the following page* (234).

SECTION I.—GEOMETRICAL DRAWING.

Plane Geometry.

*1. The given line AB represents a length of 35 feet. Construct and figure correctly the scale to which AB is drawn. (10.)

2. Draw a line AB $2\frac{1}{4}''$ long. Through A draw AC perpendicular to AB and $1''$ long. On CB describe a triangle CBD, having a right angle at D, and equal sides, CD, BD; D and A to be taken on opposite sides of CB.

Describe a circle passing through A, B, and C. (10.)

3. Describe an equilateral triangle of $1\frac{1}{4}''$ side. Through each vertex draw lines parallel to the opposite sides, thus forming a second triangle.

In the larger triangle inscribe three equal circles, each touching two sides and each other. (12.)

4. Construct a regular octagon of $1\frac{1}{8}''$ side, and reduce it to a triangle of equal area. (12.)

5. Construct a triangle with base AB $2\frac{3}{4}''$ long, angle BAC $=50^\circ$, and side BC $2\frac{1}{4}''$ long.

How many triangles fulfilling these conditions can be drawn? (12.)

*6. Draw the figure from the given dimensions.

N.B.—*No marks will be awarded for a reproduction of the figure as given.* (12.)

7. The sides of a triangle are respectively $3''$, $3\frac{1}{4}''$, and $2\frac{1}{2}''$ long. Construct a triangle of equal area, and the base of which is $4\frac{1}{2}''$ long. (14.)

8. The axes of an ellipse are $4''$ and $3''$ long respectively. Draw the curve, and determine the foci. (10.)

Solid Geometry.

9. A sphere of $1\frac{1}{2}''$ radius is divided by a plane $\frac{3}{4}''$ distant from centre of sphere. Draw the plan and elevation of the larger portion of the sphere when the cut surface rests on the horizontal plane. (14.)

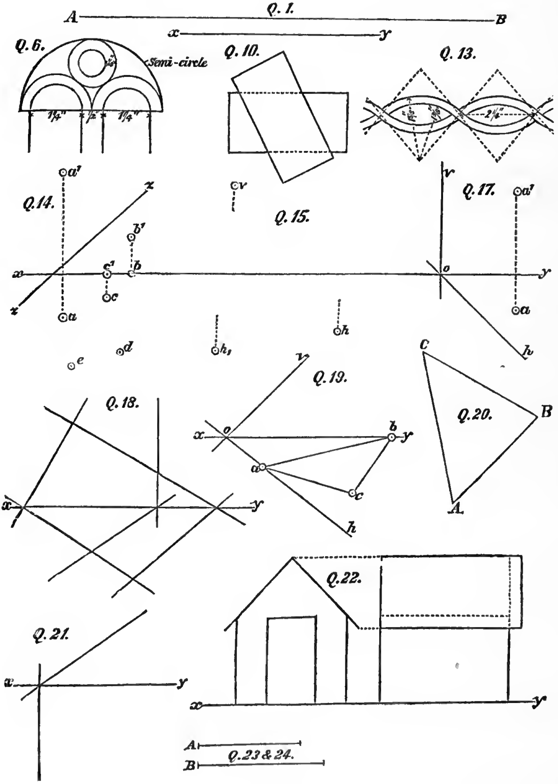
*10. The plan of two bricks, one on the horizontal plane, and the other resting on the first, is given.

The thickness of each brick is $\frac{1}{3}$rd of its length.

Draw the elevation on the ground line, *xy*. (14.)

ELEMENTARY STAGE, 1892.

The Diagrams are to be accurately copied to three times the Scale.



N.B.—*The questions marked thus * refer to diagrams on the opposite page* (234).

First Stage or Elementary Examination.

Plane Geometry.

11. Construct a triangle, with sides $2\frac{1}{4}''$, $1\frac{7}{8}''$, $1\frac{1}{8}''$. Supposing that you have four such triangles, how many different parallelograms can you make up of them, using all four? Draw these parallelograms. (10.)

12. Describe a circle A, of $1\frac{1}{4}''$ diameter, touching internally a circle B, of $3\frac{1}{2}''$ diameter. Describe a circle of $2''$ diameter touching both circles A and B, the latter internally. (10.)

*13. The figure shows a double riband, the curves being all circular arcs of two radii. The method of construction is indicated. Draw the riband in strict accordance with the figured dimensions.

N.B.—*No marks will be given for a reproduction of the figure.* (12.)

Solid Geometry.

*14. The heights of the points D and E are respectively $1\frac{3}{4}''$ and $1\frac{1}{4}''$ above the horizontal plane. Obtain the elevations of all five points, A to E, on the line *z z*. (12.)

*15. *v* and *h* are the vertical and horizontal traces of a line; h_1 is the horizontal trace of a second line, which bisects the first. Draw the projections of the second line. (12.)

*16. The traces of a plane both make 40° with *x y*. Draw the projections of two lines lying in this plane, one parallel to the horizontal, and the other parallel to the vertical plane of projection. (12.)

*17. Determine a plane parallel to the given plane *v o h*, and passing through the given point *a a′*. Determine the traces of a second plane parallel to and $1\frac{1}{4}''$ from the given plane *v o h*. (12.)

*18. Determine the intersection of the three given planes.

N.B.—Obtain first the intersection of the given oblique planes and then the point in which this intersection cuts the given vertical plane. (12.)

*19. *a b c* is the plan of a triangle lying in the given plane *v o h*. Determine the *elevation* of this triangle. (12.)

*20. The triangle ABC lying in the horizontal plane is rotated about the side AB till its plane is inclined at 50°. Draw the *plan* of the triangle. (12.)

*21. Draw the *plan* of an octagon of $1\frac{1}{8}''$ side lying in the given plane, and having one side in each plane of projection. (12.)

*22. Two elevations of a dog-kennel are given. Draw the *plan* and an *elevation* on a line parallel to a diagonal of the base, and showing the doorway. (16.)

Graphic Arithmetic.

*23. The line A represents the square root of the line B. Determine and write down the length of the unit. (12.)

*24. If $1\frac{1}{2}''$ is the unit, determine and write down the length of the line which will represent the product of A and B. (12.)

SPECIMEN EXAMINATION PAPER, 1893.

SECTION I. OR GEOMETRICAL DRAWING
AND
FIRST STAGE OR ELEMENTARY EXAMINATION.

Only eight questions are to be attempted.
The relative values are shown in brackets.
*The questions marked * refer to diagrams on the opposite page.*

SECTION I.—GEOMETRICAL DRAWING.

Plane Geometry.

1. Construct and figure correctly a scale of 0° 8″ to 10′ 0″ capable of giving dimensions up to 90′ 0″. (10.)

*2. Draw a regular trefoil as shown in the given sketch, adhering strictly to the figured dimensions. (12.)

3. Construct a triangle with sides 3″, $2\frac{1}{4}$″, and $1\frac{1}{4}$″. Construct a similar triangle with a perimeter of $8\frac{1}{2}$″. (12.)

N.B.—In similar triangles the sides are respectively proportional.

4. Draw two parallel lines $\frac{3}{4}$″ apart, and cut at an angle of 75° by two other parallel lines also $\frac{3}{4}$″ apart. Describe all the circles of $\frac{5}{8}$″ radius which touch any two of these lines and cut none. (12.)

N.B.—The lines are supposed to be produced indefinitely.

*5. Draw two parallel lines $1\frac{1}{4}$″ apart, and describe a circle cutting these lines in chords respectively 1″ and $1\frac{5}{8}$″ long. (*See* sketch.) (8.)

6. From a point *o* equal lines *oa*, *ob* ($2\frac{1}{2}$″ long) are drawn, including an angle of 50°. Describe the circle to which these lines are tangent at the points *a* and *b*. On *oa* produced determine a point such that tangents drawn to the same circle shall include an angle of 70°. (12.)

*7 Two equal figures of X are given, the equal lines forming which cross at an angle of 60°. Draw these figures in such a position that the points *a*, *c*, *a′*, *c′*, and the intersections of the lines *cd*, *c′d′*, and *ab*, *a′b′*, produced fall at the angles of a regular hexagon. (12.)

8. Construct a square equal to the *difference* of the areas of two equilateral triangles whose sides are 3″ and $1\frac{3}{4}$″ respectively. (12.)

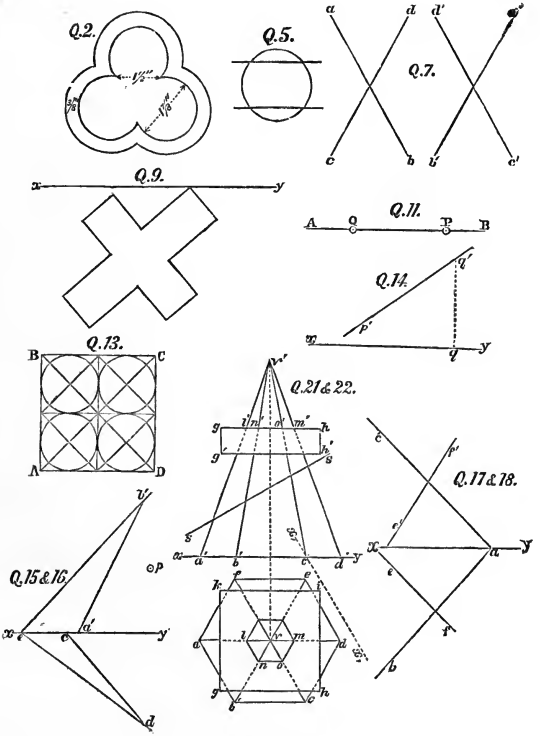
Solid Geometry.

*9. A cross composed of material $\frac{3}{8}$″ thick lies on the horizontal plane as shown. Make an elevation of the cross on the line *xy*. (14.)

10. Draw two concentric circles of $2\frac{1}{2}$″ and $1\frac{5}{8}$″ diameter. The larger circle represents the plan of a right cylinder (height $\frac{7}{8}$″) resting on the horizontal plane. The smaller circle represents the plan of a sphere resting on the cylinder. Make a section of these solids by a vertical plane passing $\frac{1}{2}$″ from the centre of the sphere. (14.)

ELEMENTARY STAGE, 1893.

The Diagrams are to be accurately copied to three times the Scale.



N.B.—*The questions marked * refer to diagrams on the preceding page* (237).

Plane Geometry.

* 11. Draw a circle of 1″ radius, tangent to the line AB at Q, and draw another circle touching the first circle, and tangent to AB at P. (10.)

12. Construct a triangle ABC having AB = 2′, AC = 1·75″, angle CAB = 40°; and draw a square equal in area to the triangle ABC. (10.

*13. Draw the figure, *not of the same size as the diagram*, but taking the length of the sides of the square ABCD at 2′. (12.)

Solid Geometry.

*14. q, q' are the projections of a point on a line whose elevation is p' q'. Find the plan (pq), the line being inclined at 30° to the horizontal plane of projection. (10.)

*15. $a'b'$ is the elevation of one line, cd the plan of another line, both in the given plane ($b'e$, ed). Find the other projections of the two lines, and the projections of their point of intersection. (12.)

*16. p' is the elevation of a point in the given plane ($b'e$, ed). Find the plan p of the point; and through the point pp' draw the projections of a line, in the given plane, and inclined at 30°. (12.)

*17. Find the intersections of the line ef, $e'f'$ with the given plane ($c'a$, ab). (12.)

*18. Determine the angles which the given plane ($c'a$,ab) makes with the vertical and horizontal planes of projection. (14.)

19. Draw the plan of an equilateral triangle of 1·5″ side, its plane inclined at 60°, one side inclined at 50°, and one vertex in the horizontal plane. (12.)

20. A circle of 3″ diameter is tilted up so that its *plan* is an ellipse whose minor axis is 2′. Draw the ellipse, and give the angle of inclination of the plane of the circle when in that position. (12.)

*21. Draw the elevation of the pyramid, and of the block (gi, $g'h'$) through which it passes, on a ground line (such as x_1,x) parallel to the sides ab, de of the base. (14.)

*22. Find the *plan* and the *true form* of the section of the pyramid by a plane perpendicular to the vertical plane of projection, and represented in elevation by SS, its vertical trace. (12.)

Graphic Arithmetic.

23. Find the line which represents the cube of a line 1½ inches long to a unit of 1 inch. (12.)

24. Find the line which represents the fraction ⁵⁄₇ to a unit of ½ inch. (10.)

SPECIMEN EXAMINATION PAPER, 1894.

SECTION I. OR GEOMETRICAL DRAWING

AND

FIRST STAGE OR ELEMENTARY EXAMINATION.

Only **eight** questions are to be attempted.

The relative values are shown in brackets.

*Questions marked * refer to diagrams on the following page* (240).

SECTION I.—GEOMETRICAL DRAWING.

Plane Geometry.

1. Construct a scale of 3·5″ to 30′, to show feet, and correctly figured. Draw to the scale a triangle, its sides, respectively, 20′, 15′, and 12′ long. (10.)

*2. Divide the line AB into three consecutive parts, to each other in the proportion of $1 : \frac{3}{4} : \frac{1}{2}$. (8.)

*3. The figure represents a continuous outline composed of portions of three circles and of their common tangents. Draw the figure *to the indicated dimensions.* (10.)

4. Draw a quadrilateral figure ABCD, with the following dimensions:

AB = 2″, BC = 1·5″, AD = 1·5″. The diagonal BD = 2″, the diagonal AC = 2·5″.

Find the length of the side of a square equal in area to the quadrilateral. (12.)

5. Draw a circle of 1″ radius; and (*a*) draw a triangle circumscribing the circle, its angles being, respectively, 75°, 70°, and 35°; (*b*) join the points where these lines are tangent to the circle, and write down the values of the angles of the triangle thus formed. (12.)

[NOTE.—The angle subtended by a chord at the centre of a circle is double of that subtended by the same chord at the circumference.]

*6. Draw the figure from the indicated dimensions. (14.)

7. Inscribe a pentagon in a circle of 1·5″ radius. Draw the diagonals, and the circle circumscribing the pentagon formed by their intersection. (12.)

8. The minor axis of an ellipse is $2\frac{1}{4}$″ long, and the distance between the foci is 2″. Find the major axis and draw the curve. (10.)

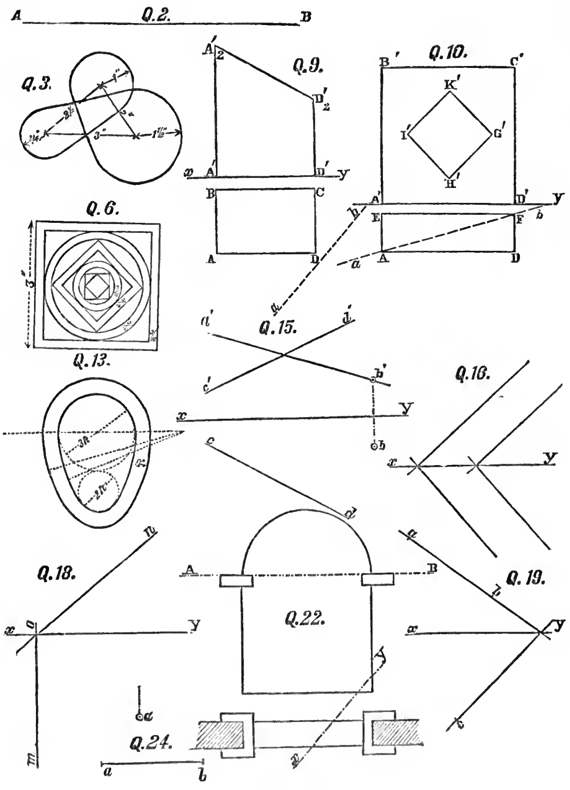
Solid Geometry.

*9. The figure represents the plan (ABCD) and elevation ($A'A'_2D'_2D'$) of a truncated prism with a rectangular base. Draw an elevation of the solid in a direction perpendicular to the line *a b*. (14.)

*10. The figure represents the plan (AEFD) and elevation (A′B′C′D′) of a vertical block with a square hole (in elevation I′K′G′H′) through it. Draw the section of the block made by a vertical plane represented in plan by the line *ab*, and shade the portions of the block cut by the plane. (16.)

ELEMENTARY STAGE, 1894.

The Diagrams are to be accurately copied to three times the Scale.



N.B.—*Questions marked * refer to diagrams on the opposite page* (240).

FIRST STAGE OR ELEMENTARY EXAMINATION.

Plane Geometry.

11. In a circle of $1\frac{1}{2}''$ diameter, place a chord $1\frac{3}{8}''$ long. At each extremity of this chord draw a tangent to the circle. Describe a second *larger* circle, touching the first and also the tangents. (10.)

12. Construct a square *abcd* of $2\frac{1}{2}''$ side. Bisect this square by a line drawn from a point on the side *ab* distant $1''$ from *a*. Reduce this square to a parallelogram of which the bisecting line is a diagonal. (10.)

*13. The figure represents the section of an oval sewer, the outline of which is made up of circular arcs. The construction is indicated. Draw the section in strict accordance with the figured dimensions. Scale $\frac{3}{8}'' = 1$ foot. (12.)

Solid Geometry.

14. Show in *plan* and *elevation* :—

a. A point $2''$ from the ground line and $1\frac{1}{4}''$ from the vertical plane of projection.

b. A line parallel to and $1\frac{1}{4}''$ from the vertical plane of projection, and inclined at 40° to the horizontal plane. (10.)

*15. A line of which *a′b′* is the elevation passes through the given point *bb′*, and intersects the given line *cd, c′d′*. Draw the plan of this line and determine its inclination to the horizontal plane. (12.)

*16. Determine the distance apart of the given parallel planes. (10.)

17. An isosceles triangle (base $2''$, sides $2\frac{3}{4}''$) rests with its base on the horizontal plane, and is rotated about the base till the height of the vertex is $\frac{3}{4}''$ above the horizontal plane. It is then further rotated till the height of the vertex is $2''$. Determine the angle contained by the planes of the triangle in these two positions. (12.)

*18. The plan *a* of a point lying in the plane *mon* is given. From this point draw, in plan and elevation, a line lying *in* the plane and inclined at 35° to the horizontal plane. From the same point draw a second line also in the given plane and making 45° with the first line. (12.)

*19. *a*, *b* and *c* are points on the respective traces of a given plane as shown. Determine the true form of the triangle ABC. (12.)

20. A pentagon *abcde* (side $1\frac{1}{4}''$) is the plan of a right prism (height $2\frac{1}{2}''$) standing on the horizontal plane. This prism is cut by an oblique plane which passes through the *lower* edge *cd* and the *upper* point *a*. Draw an elevation of the truncated prism on a line parallel to *ab*. (14.)

21. Draw the plan of a right hexagonal pyramid (side of base, $1\frac{1}{8}''$, height $2\frac{3}{4}''$) lying with one triangular face on the horizontal plane. Draw also an elevation on a plane parallel to any side of the base which is not horizontal. (12.)

*22. The elevation of a semi-circular headed window is given, and also a plan at AB. Draw an elevation of the window on a ground line parallel to *xy*. (16.)

Graphic Arithmetic.

23. Determine a line whose length shall represent $\sqrt{\frac{7}{2}}$, taking $\frac{3}{4}''$ as the unit. (12.)

*24. If the given line *ab* represents the fraction $\frac{7}{4}$, determine the unit. (12.)

SPECIMEN EXAMINATION PAPER, 1895.

SECTION I. OR GEOMETRICAL DRAWING
AND
FIRST STAGE OR ELEMENTARY EXAMINATION.

Only **eight** questions are to be attempted.

The relative values are shown in brackets.
*Questions marked * refer to diagrams on the opposite page* (243.)

SECTION I.—GEOMETRICAL DRAWING.

Plane Geometry.

*1. The given line AB represents a length of 15 yards. Construct a scale of yards by which single yards may be measured up to 40 yards. The scale must be correctly figured. (10.)

2. Draw an isosceles right-angled triangle having its equal sides $2\frac{1}{2}''$ long. Within the triangle inscribe *two* equal circles, each touching the other and two sides of the triangle. (10.)

*3. Draw the pattern shown (including the dotted lines). The arcs are all to be of $\frac{1}{2}''$ radius. (10.)

4. Construct a regular heptagon of $1\frac{5}{8}''$ side, and within it inscribe a circle. (12.)

5. The diagonals of a parallelogram are $2\frac{1}{2}''$ and $2''$ long respectively and are perpendicular to one another. Construct the figure, and a similar parallelogram of $2''$ side, having its sides parallel to and equi-distant from those of the first. (10.)

6. O is a point within a quadrilateral figure ABCD. Construct the figure from the following dimensions:—

Angles—$AOB = 115°$, $BOC = 65°$, $AOD = 85°$
Lengths—$OA = 1\frac{1}{4}''$, $OB = 1\frac{5}{8}''$, $BC = 1\frac{3}{4}''$, $BD = 3''$

Reduce the quadrilateral to an isosceles triangle of equal area, having AB for base. (14.)

*7. Draw the figure from the given radii. (12.)

8. Draw a line AB $3''$ long, and describe two circles, each of $\frac{3}{4}''$ radius, *touching* AB at A and B respectively. Then describe a third circle, of $1''$ radius, touching both the other circles. (10.)

9. Find, and mark clearly, the third proportionals, greater and less, to two lines $1\frac{1}{2}''$ and $2\frac{1}{4}''$ long. (8.)

10. Construct a triangle having its sides in the ratio 5 : 4 : 2, the longest side being $2\frac{3}{4}''$ long. (8.)

Solid Geometry.

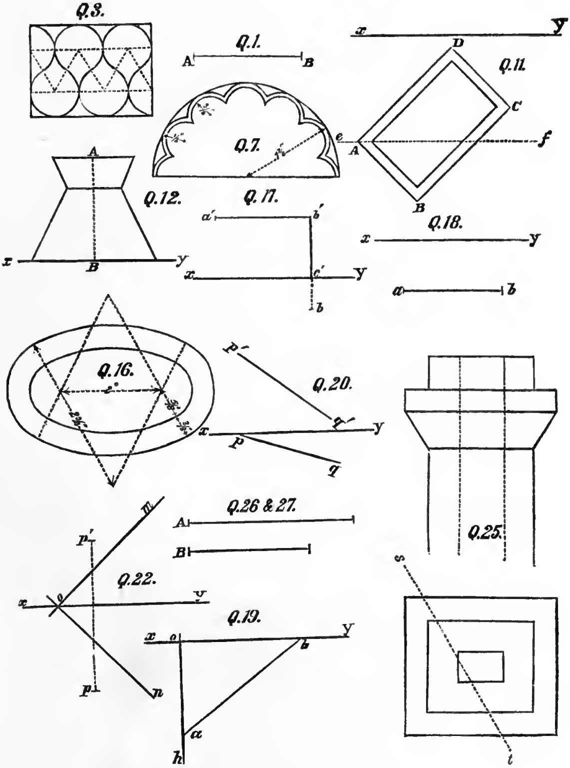
*11. ABCD is the plan of a box without a lid, $1\frac{1}{4}''$ high, made of material of uniform thickness. Draw an elevation on the line *xy*, showing the section of the box made by the vertical plane represented in plan by the line *ef*. (16.)

*12. The figure is the elevation of a solid composed of two truncated cones. Draw its plan, and show upon the given elevation the section made by a vertical plane passing through the axis AB of the solid, and inclined at 45° to the vertical plane of projection. (12.)

13. A right prism has for base a regular hexagon of $1\frac{1}{2}''$ side, and its axis is $1''$ long. Draw an elevation of the prism standing with one rectangular face on the ground, the planes of the bases making angles of 45° with the vertical plane of projection. (14.)

ELEMENTARY STAGE, 1895.

The Diagrams are to be accurately copied to three times the Scale.



N.B.—*Questions marked * refer to diagrams on the preceding page* (243).

Plane Geometry.

14. On a chord 2″ long describe the segment of a circle containing an angle of 57°. Then describe a circle of ·75″ radius touching the arc of the segment internally and also touching the chord. (10.)

15. Construct a rectangle, one side 2″ long, having its area half that of an equilateral triangle of $2\frac{1}{2}$″ side. (10.)

*16. Draw the figure from the given dimensions. (12.)

Solid Geometry.

*17. *a′b′* and *b′c′* are the elevations of two lines, each 2″ long. Find the plans of the lines, the plan of point *b′* being at *b*. (10.)

*18. *ab* is the plan of a horizontal line $1\frac{1}{4}$″ above the horizontal plane. Find its elevation, and determine the plan and elevation of an isosceles triangle having the given line for base, and its vertex in *xy*. (10.)

*19. *oh* is the horizontal trace of a plane, and *ab* is the plan of a line in that plane inclined at 25° to the horizontal plane. Determine the vertical trace of the plane, and the plan of a line in the plane, passing through *a* and at right angles to the line of which *ab* is the plan. (12.)

*20. Determine the traces of a plane containing the given line *pq*, *p′q′*, and inclined at 45° to the horizontal plane of projection. (12.)

21. A rectangle, sides $1\frac{1}{4}$″ and $2\frac{1}{2}$″, rests with a short side in the horizontal plane, and is rotated about that side until its plan is a square of $1\frac{1}{4}$″ side. Determine its inclination to the horizontal plane, and draw an elevation on a vertical plane parallel to the horizontal sides of the rectangle. (12.)

*22. Determine the perpendicular distance of the point *pp′* from the given plane *mon*. (12.)

23. A right equilateral triangular prism, axis 2″, edge of base 1·75″, lies with one rectangular face in the horizontal plane, and one triangular base in the vertical plane of projection. Draw its plan, and also the plan of a sphere of 1″ radius which touches the horizontal plane, the vertical plane, and the prism. (12.)

24. A hemisphere of 1·25″ radius is placed with its plane surface vertical, and inclined at 48° to the vertical plane of projection. Draw its elevation. (12.)

*25. Plan and elevation are given of the upper part of a chimney. Draw a sectional elevation on the line *st*. The part in section may be lightly shaded. (16.

Graphic Arithmetic.

*26. Two lines A and B are given. Find the line representing the value of $\frac{A}{2B}$, the unit being $1\frac{1}{4}$″. (12.)

*27. The line B represents $\sqrt{6}$. Determine the unit. (12.)

SPECIMEN EXAMINATION PAPER, 1896.

SECTION I. OR GEOMETRICAL DRAWING AND
FIRST STAGE OR ELEMENTARY EXAMINATION.

Only **eight** questions are to be attempted.
The relative values are shown in brackets.
*Questions marked * refer to diagrams on following page* (246.)

SECTION I.—GEOMETRICAL DRAWING.

Plane Geometry.

1. A line $4\frac{3}{4}''$ long represents a distance of 4′. Construct a scale by which feet and inches may be measured up to 4 feet. The scale must be neatly finished and correctly figured. (12.)

2. Within a circle of $1\frac{1}{4}''$ radius inscribe a regular pentagon. About the same circle describe another regular pentagon, having its sides parallel to those of the inscribed pentagon. (10.)

*3. Draw figure shown, making sides of square $2\frac{1}{2}''$ long. (10.)

4. Construct a rectangle having one of its sides $1\frac{5}{8}''$, and its diagonals 2″ long. Make a *similar* rectangle having its *shorter* sides $1\frac{1}{2}''$ long. (8.)

5. Draw two lines AB, AC, making an angle of 25° at A. Describe a circle of $\frac{3}{4}''$ radius touching AB and having its centre on AC. From A draw a second tangent to the circle, marking clearly the point of contact. (8.)

6. The foci of an ellipse are $2\frac{1}{2}''$ apart and its major axis is $3\frac{1}{2}''$ long. Describe half the curve. (10.)

*7. Draw the pattern according to the given dimensions. (12.)

*8. Draw the "cyma recta" moulding shown, adhering to the given dimensions. The curve is composed of two quarter-circles of equal radii, tangential to one another and to the lines AB and CD respectively. (10)

9. Construct an irregular pentagon, ABCDE, from the following data:—
Sides—AB = 2″, BC = $\frac{3}{4}''$, AE = $\frac{3}{4}''$.
Angles—ABC and BAE each = 135°. BCD and AED each = 95°.
Reduce the figure to a triangle of equal area, having AB (produced if necessary) for base and D for vertex. (12.)

*10. Draw the curve every point of which is at equal distances from the line PQ and the point F. The curve, which is a parabola, need not be shown below the line ST. (12.)

Solid Geometry.

*11. The plan is given of a *cube*, having a cylindrical hole pierced through its centre. A vertical plane, represented by the line *lm*, cuts off a portion of the solid. Draw an elevation on the line *xy*, supposing the part of the solid in front of *lm* removed. The part in section should be clearly indicated by lightly shading it. (14.)

*12. Plan and elevation are given of a solid composed of a half-cylinder placed upon a prism. Draw a new elevation, when the horizontal edges of the prism make angles of 45° with the vertical plane of projection. (16.)

13. Draw the plan of a right pyramid $2\frac{1}{2}''$ high, base an equilateral triangle of 2″ side. The pyramid stands on its base, and the upper part is cut off by a horizontal plane 1″ above the base. Indicate the section clearly by light shading. (12.)

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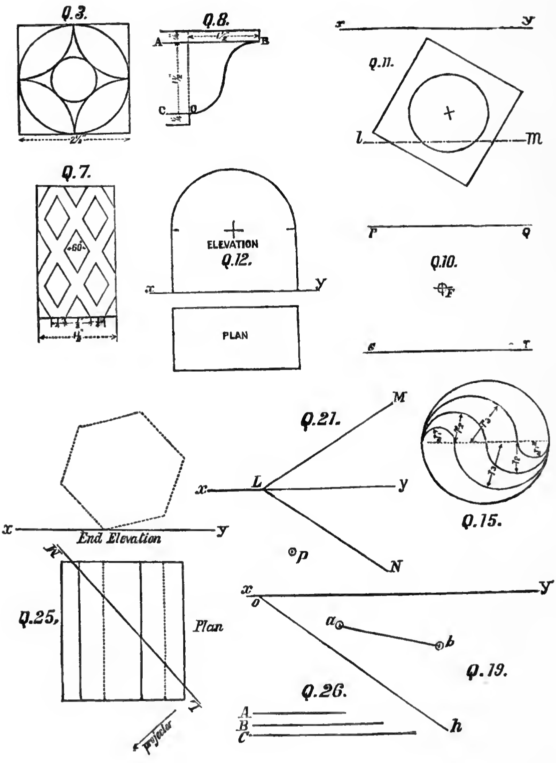
The Diagrams are to be accurately copied to three times the Scale.



N.B.—*Questions marked * refer to diagrams on the preceding page* (246).

Plane Geometry.

14. ABC is an equilateral triangle of $2\frac{1}{2}''$ side. Describe a circle of $\frac{3}{4}''$ radius round C, and draw another circle which will touch the former one, and will *touch* the line AB at A. (10.)

*15. Divide a circle of $1\frac{1}{2}''$ radius, similarly to the given diagram, the radii of the smaller circles (r_1, r_2, and r_3) being $\frac{3}{8}''$, $\frac{3}{4}''$, and $1\frac{1}{8}''$ respectively. (10.)

16. ABCD is a rectangle, AB = $1\frac{1}{2}''$, BC = $2\frac{1}{2}''$. Find two points (P and Q) on the line BC, such that the angles, APD, AQD, shall be each 70°. (10.)

Solid Geometry.

17. A line, 2″ long, is the plan of a square of $1\frac{3}{4}''$ side. Show the position in plan of the angles of the square. (10.)

18. The vertical trace of a plane makes an angle of 48° with the *xy* line. The plane is inclined at 60° to the horizontal plane. Draw the horizontal trace. (12.)

*19. *oh* is the horizontal trace of a plane, and *ab* the plan of a line lying in the plane. The real length of AB being 2″, draw the vertical trace of the plane, and the elevation of the line. (12.)

20. *abc* is an equilateral triangle of $2\frac{1}{2}''$ side, and is the plan of a triangle of which the angles A, B, and C are respectively 1″, 2″, and 3″ above the horizontal plane of projection. Determine the real angle ABC. (12.)

*21. Through *p*, a point in the horizontal plane, draw the plan of a line inclined at 40° to the horizontal plane, in the given plane MLN. (12.)

22. Draw the plan of a regular pentagon of $1\frac{3}{4}''$ side, when two adjacent sides are inclined at 25° to the horizontal plane. (12.)

23. Draw the projections of two spheres (of 1″ and $\frac{1}{2}''$ radius respectively), touching one another and the horizontal plane. (10.)

24. The base of a right pyramid is an equilateral triangle of 2″ side. The height of the pyramid is $2\frac{1}{2}''$. Draw the plan of the pyramid when it lies with one triangular face (not the base) on the ground. (12.)

*25. The plan and end elevation are given of a hexagonal right prism lying with one long edge on the ground. Draw a section and elevation on the line LM. The parts in section may be lightly shaded. (16.)

Graphic Arithmetic.

*26. If A represents the unit, determine a line $= \frac{B \times C}{2}$. (12.)

27. If a line $2\frac{1}{2}''$ long represent $\sqrt{2}$, draw a line $= \frac{\sqrt{3}}{2}$. (10.)