PRACTICAL STRUCTURAL DESIGN

by

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PRACTICAL STRUCTURAL DESIGN

A TEXT AND REFERENCE WORK FOR ENGINEERS, ARCHITECTS, BUILDERS, DRAFTSMEN AND TECHNICAL SCHOOLS; ESPECIALLY ADAPTED TO THE NEEDS OF SELF-TUTORED MEN

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PREFACE

COMMENCING in the year 1914 a series of articles by the author of this book appeared in the pages of *Building Age*, with the title "Design of Beams, Girders and Trusses." The articles were completed early in 1916, and in reply to an insistent demand they were prepared for appearance in book form with practically an equal amount of new material. The present book is the result.

Before writing the articles the subject matter had been tried out on a number of classes of students, some of them in evening schools and some in private classes organized for the purpose of preparing the students to pass state examinations to obtain a license to practice architecture. The writer in the intervals of a busy professional life has managed to find time to teach in evening schools a deserving class of men who entered the offices of architects and contractors at too early an age.

Samuel Butler says: "There are plenty of things that most boys would give their ears to know; these and these only are the proper things for them to sharpen their wits upon." It happens that a great many boys have a taste for drafting and like to watch construction work. Under the guidance of woefully ignorant teachers, lacking practical experience outside the class room, drafting is thought to be an end; and equipped with a certain facility in elementary drafting these unfortunate boys go forth to seek employment. They find it, and after laboring a few years discover that draftsmen who are merely draftsmen are truly unfortunate beings. Never enough to go around in brisk times, they are a drug on the labor market in dull times. The boys were given what they were ready to give their ears to know, but their immature judgment was at fault and the judgment of their teachers was no better.

When a realization comes of the fact that a man must "learn more to earn more," Samuel Butler is there again with a wise remark as follows: "The rule should be never to learn a thing till one is pretty sure one wants it, or that one will want it before long so badly as not to be able to get on without it." No one knows the truth of this remark more than the men who made a false start and got into the low paid trade of drafting. The pity is that some one with sufficient intelligence had not made them understand at the age of sixteen that a draftsman should know something more than just enough to draw lines on paper, copying examples of the work of other men. If this sort of knowledge were pounded into them they would "give their ears" to know enough to fit them for advancement.

The realization comes to few before the age of twenty-five. Many are by that time married and to these men the first "lay-off" comes shortly after marriage, because periods of business depression are just far enough apart to allow this. Few, if any, have attended High School and few have graduated from High School. This is the sort of material that came to the author for many years in his evening class work. Not many of the men could find time outside of class hours to work problems. None of them were sure of themselves when it came to working problems. The men most illy prepared to understand a mathematical demonstration were most eager to know "why." Many hours were spent in trying out ways to demonstrate truths in structural mechanics and the mechanics of materials, so that men skilled only in arithmetic could understand them. The men generally resented a seeming attempt to cram them with formulas and rules without a "step by step" explanation of the work. When the author began to teach this class of pupils he was told by instructors who had tried it before that most of them were too thick-headed to do anything with. The author found it otherwise. Where men were dull it was generally because they were overworked or were struggling in deep financial sloughs.

The men wished to learn. That much was certain. If they did not wish to learn they could have had a better time elsewhere and been in pocket by the amount of the fees they paid for instruction. The author told them at the beginning of each term that if they failed to get their money's worth they could blame him, for he was there to teach them what they wished earnestly to learn. They were a great inspiration to him and those evenings in the class room in an atmosphere of dogged earnestness and intense hopefulness will ever remain fragrant in his memory. They paid him for many hours he put in when he was tired, trying to think of plain statements of elementary truths. He has heard from a number of the men since, who told him that the work he did was an inspiration to them. The work was not undertaken for the small amount the schools could afford to pay, but was done principally because he liked the service. The great regret of his life has been that he could never secure financial inducement to enable him to make teaching his life work.

The book is written to reach the men who cannot attend evening classes in mechanics of materials and structural design. It is written also to be used as a text book in such classes. The articles in *Building Age* were mimeographed by a number of teachers for use in manual training and high schools and it is hoped these teachers will use the book as a text. The author has done his best to make the subject matter plain. The book is peculiarly adapted for the use of self-tutored men and the author would like to hear from such readers, so that explanations and statements they fail to grasp may be ironed out in future editions.

There is no royal road to learning. In addition to listening to lectures or reading books the student must do a certain amount of thinking and reasoning. It is not enough to accept a statement as true. The reason why it is true must be understood. The burden on memory is lessened as reasons are grasped. For a man studying alone the mistake should never be made of putting an entire evening on the work. Study one hour each evening, rain or shine, and study hard. A rest of an hour is good and then another hour, or even half an hour, of study will be found to clear matters up wonderfully. Early in the game start teaching the office boy, for to teach is one of the best ways to learn. If the office boy cannot be interested then the studying is not being done right, for a man who is studying in the proper spirit becomes somewhat enthusiastic over his progress. The most commonplace facts are wonderful. That is why the newly found knowledge should be passed on to the office boy, for men who know will not care to have things they know placed before them as fresh discoveries.

One considerable difficulty in the path of the self-tutored man is eliminated when he finds a ready listener and pupil. The tiresomeness of studying alone is hard to describe and accounts for so many quitting early in the work. The writer usually organized a class of some sort when he had to bone in a hurry and found it of advantage. Even so, difficulties arise, but they are much like the troubles the old man spoke about on his death bed as having been very real when they came, but as a matter of fact most of them never happened. Walter Bagehot, in "Physics and Politics," says: "Everybody who has studied mathematics knows how many shadowy difficulties he seemed to have before he understood the problem, and how impossible it was, when once the demonstration had flashed upon him, ever to comprehend those indistinct difficulties again, or to call up the mental confusion that admitted them."

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PRACTICAL STRUCTURAL DESIGN

CHAPTER I

External Forces

A STRUCTURE is a combination of parts designed to hold in equilibrium definite forces and in this book the word "structure" is limited to buildings.

The intention being to give a modern treatment in the plainest possible manner, it is necessary to settle upon a definite terminology, — that is, upon a system of shorthand symbols to use — for it is best to present rules in a condensed manner in order that every step in an operation may be quickly apprehended. A rule thus written becomes a formula. Formulas are merely mathematical shorthand, and when a formula is seen it is not algebra. Algebra is useful in deriving a formula, but when the formula is presented it requires only the use of common arithmetic to solve it.

Let W = a uniformly distributed load.

Let w = a unit of a distributed load. On a panel w is the load per square foot. On a beam or girder w is the load per lineal foot. Then it follows that $W = w$ multiplied by the span.

P = a concentrated load. When several concentrated loads are used the different loads are designated by subscripts, as $P_{\prime}$ — $P_{\prime\prime}$ — $P_{\prime\,\prime\prime}$ — $P_{\prime\prime\,\prime\prime}$ etc.

S = clear span between supports.

L = length of span used for obtaining a bending moment. That is, a beam may extend from one support to another with a span, S, measured from face to face of support, and this will be used in obtaining W, the total load on the span. To find the bending effect of the load it is necessary to use a length measured from center to center of supports, or from points back of the face of the supports and this length is L, which is always greater than S.

l = any portion of L or S used to obtain a distance from the support to the center of gravity of a load. When distances are required to several loads, subscripts corresponding to those under the respective loads are used. Thus to load $P_{\prime}$ we use $l_{\prime}$; to load $P_{\prime\prime}$ we use $l_{\prime\prime}$ etc.

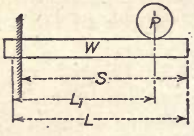


Fig. 1 — The Cantilever Beam

The use of the letters is illustrated in Fig. 1, in which a cantilever beam is shown; and in Fig. 2, in which is shown a beam on two supports. To illustrate a uniformly distributed load the weight of the beam is used.

Sometimes the letters a, b, c, etc., are used to designate portions of a span or lengths less than the whole, but these will be dealt with as they may arise. The letters x, y, z, etc., are used similarly and will be dealt with when required.

Dead Load. The weight of the structure and permanent loads.

Live Load. The load the structure is designed to carry in addition to the dead load. The live load consists of machinery, merchandise, people, etc.

Impact. The effect of a live load in motion. It is added to the live load and varies from twenty-five per cent for a slowly moving load on a rigid structure to one hundred per cent for a live load suddenly applied.

Wind. This force acts perpendicularly to the pressed surface, so is horizontal on the sides of a building and is a diagonal load when it acts on a sloping roof.

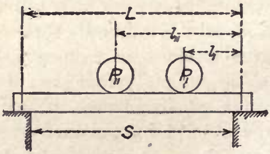


Fig. 2 — Beam Resting on Two Supports

Moments

A cantilever beam is supported at one end and may be loaded at any point with a concentrated load. The load tends to bend the beam down. The uniformly distributed weight of the beam tends to bend the beam also, so on all loaded beams there must be considered the actual weight of the beam plus added loads it may carry. The name "cantilever" gives a hint that the bending action is similar to that of a lever.

All formulas and rules for obtaining the effect of loads on beams are derived from the fundamental principle of the lever. It makes no difference whether the beam rests on one, two, or more supports, the principle of the lever, as represented by the cantilever beam in Fig. 1, gives the key for obtaining the desired information.

All formulas and rules for obtaining the strength of a beam to resist the bending effect are similarly based on the principle of the lever.

The bending effect is termed a "Bending Moment" and the resisting effect, dependent upon the form, size, and material, of the beam, is termed the "Resisting Moment." The bending moment is first found and then a beam having an equal resisting moment is used to carry the load.

In structural design the moment may be compared to the common denominator in problems involving fractions. There are two quantities which must be reduced to a common measure before operations involving both can be performed. This explains why engineers invariably equate (make equal) the bending moment and resisting moment instead of working by rules derived by other men. Each man who does much designing work derives rules for himself because only by so doing can he be certain of their correctness. When the underlying principle of moments is understood no man should have trouble in verifying rules which he may run across in his work or reading.

Tables are published of resisting moments of standard size beams from which a designer may readily obtain a beam to resist a bending moment, which is calculated for each case. Spans and also loads to be carried on the spans vary considerably, every building presenting a number of different combinations. All rules and formulas apply to beams which are secured against sidewise bending.

When the resisting moment is greater than the bending moment there is obtained a factor of safety.

Let M = moment. This may be either bending or resisting moment.

M_b = bending moment. The subscript is used only when both moments are used in the same expression, and there must be some distinguishing mark.

M_r = resisting moment, the subscript being used only when the subscript b is used for the bending moment.

The factor of safety $= \frac{M_r}{M_b}$

A Moment is the product of a force multiplied by the distance through which it acts.

In Fig. 1 the load $P_{\prime}$ acts through a distance $L_{\prime}$. The formula is

$$M = PL_{\prime}$$

Forces act through the center of gravity of bodies, and a load is a force, for it tends to bend the beam down. The length $L_{\prime}$ is measured from the center of the support to the center of gravity of the load.

For a uniformly distributed load the center of gravity is at the center, which for the beam will be one-half of L, so the formula for the bending moment due to the uniformly distributed load is

$$M = W \times \frac{L}{2} = \frac{WL}{2}$$

The total moment on the beam, when W is the weight of the beam, is

$$M \text{ or } M_b = PL_{\prime} + \frac{WL}{2}$$

In Fig. 6 a cantilever beam carries two concentrated loads. For this condition

$$M = PL_{\prime} + PL_{\prime\prime} + \frac{WL}{2}$$

For more than two loads the formula will be the same, it being only necessary to obtain the moment for each load and for the weight of the beam and add them together.

When the load is in pounds and the distance to the center of gravity is in feet the bending moment is in foot pounds. When the distance is in inches the bending moment is in inch pounds. A bending moment in foot pounds is converted into inch pounds by multiplying by 12. A bending moment in inch pounds is converted into foot pounds by dividing by 12.

The following examples will illustrate the foregoing formulas:

1. A cantilever beam projecting 10 ft. beyond a wall and weighing 50 lbs. per lineal foot carries a concentrated load of 400 lbs. at a point 7 ft. from the wall. Find the bending moment in foot pounds.

The total load is 500 lbs. (weight of beam) + 400 lbs. = 900 lbs. Assume the beam to be fastened in the wall 1 ft. and the center of bearing is then 6 in. from the face of the wall. The length used for the beam will be 10 ft. 6 in. (10.5 ft.) and the distance to the concentrated load will be 7 ft. 6 in. (7.5 ft.).

$$M_b = (400 \times 7.5) + \left(\frac{500 \times 10.5}{2}\right) = 5625 \text{ ft. lbs.}$$

Add a load of 200 lbs. 4 ft. from the wall and a load of 300 8 ft. from the wall.

$$M_b = (400 \times 7.5) + (200 \times 4.5) + (300 \times 8.5) + \left(\frac{500 \times 10.5}{2}\right) = 9074 \text{ ft. lbs.}$$

The reader will notice that the distance to the center of gravity has in all cases been measured back from the face of the support, but the length used in computing the weight of the beam was the clear length. In this way all the weights are those clear of the supports, for the portion of the beam resting on the support has no effect on the bending moment.

It is wrong to use only the distance from the face of the support for cantilever beams, or the clear span between supports for simply supported beams, when figuring a bending moment, as it throws all the bearing on the edge. By using the longer distance in computing bending moments a stiffer beam is secured. Commercial designers in competitive work invariably use the distance measured from the face of supports, as they thereby save a little material and are enabled to cut down cost. All designs should be prepared by men who have no other interest than that of securing for the owner a design which is not "skinned."

Continuous beams with uniform moment of inertia are not subject to all the limitations of simply supported beams. This will be discussed later.

Reactions

The loads acting downward exert an action on the beam, which is resisted by the strength of the bearing. Considered theoretically the bearing exerts an upward force pushing against the downward force and equal in amount. This is an example of the old saying, "Action and reaction are always equal and in opposite directions."

The reactions being equal to the load, it follows that the reaction of a cantilever beam is equal to the sum of the weight of the beam and all loads it may carry.

R = reaction.

For beams carried on two or more supports a method will be given later for computing the reactions on each support.

Shear

Shear is a downward cutting force exerted at the edge of each support. It is called shear because if the material is soft the edge of the support will cut it. A piece of butter resting on the edges of two upturned knife blades is as good an example as any, perhaps, of true shear.

V = shear. It is always equal to the reaction at the support, and at other points on the beam varies according to laws to be hereafter explained.

The use of the capital V to designate shear may be explained by its resemblance to a sharp cutting edge. Mathematicians may give another reason, but the writer is interested in fixing a fact in the mind of the student.

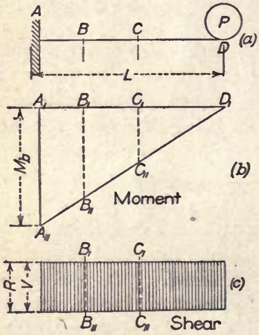


Fig. 3 — Concentrated Load at End of Beam

Graphical Methods for Moment, Shear, etc.

In Fig. 3 a concentrated load acts at the point D on the beam, AD. The beam is drawn to any scale and the load shown on it, as in the figure. Underneath is drawn the line $A_{,}D_{,}$ and the bending moment at the support is computed. Plot this to any scale (say 1000 lbs. = 1 in.) on the vertical line $A_{,}A_{,,}$. Connect $A_{,,}$ to $D_{,}$ by a straight line $A_{,,}D_{,}$ as shown at (*b*). To find the bending moment at any intermediate point on the beam drop a vertical line across the diagram and the length of the line intercepted between the upper and lower lines of the bending moment triangle gives the bending moment. For example the bending moment at B is given by the line $B_{,}B_{,,}$ and the bending moment at C is given by the line $C_{,}C_{,,}$. The vertical lines are forces and the horizontal lines are lengths, the closing line of the triangle being merely a closing line. There is no bending moment at *D*.

At (*c*) is drawn the shear diagram. The load being concentrated at the end, the shear of course is constant from the point of application of the load to the support. The shear may be found at any point by measuring a vertical line at the point across the shear diagram.

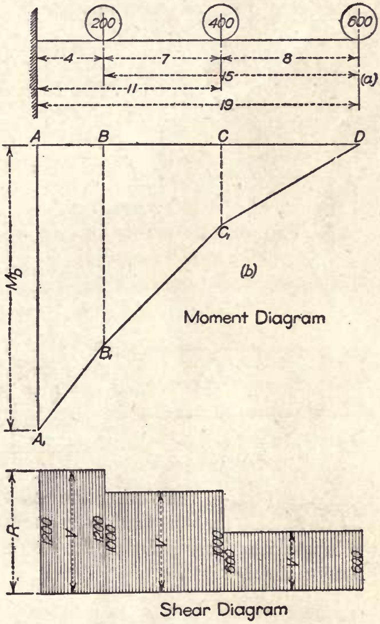


Fig. 4 — Several Loads on a Beam

Relation between Shear and Moment Diagram

Assume line $C_{,}C_{,,}$ to be dropped across the shear diagram. Then the area of the shear diagram to the right of the line, that is to the free end, gives the bending moment at *C*. Similarly, the area of the shear diagram to the right of line $B_{,,}B_{,}$ gives the bending moment at *B*. The area is in foot pounds because the vertical dimension is expressed in pounds and the horizontal dimension in feet. The relation is true in all cases and must not be forgotten.

In Fig. 4 is shown the case of several loads on a beam. The bending moment at $D = 0$. The bending moment at $C = 8 \times 600$, and this is plotted as line $CC_{,}$. The bending moment at $B = (15 \times 600) + (7 \times 400)$, and this is plotted as line $BB_{,}$. The bending moment at $A = (19 \times 600) + (11 \times 400) + (4 \times 200)$, and this is plotted as line $AA_{,}$.

The shear diagram, at (*c*), shows the shear at the right end to be constant and equal to the load at the end until it reaches the next load. It immediately changes to the sum of the two loads and continues as a constant amount to the next load, when it immediately becomes the sum of the three loads, continuing thus to the point of support.

In Fig. 5 are shown the diagrams for a uniformly distributed load.

The shear at the end = O and the moment at the end = O. The shear diagram is a triangle and the shear at intermediate sections varies as the vertical depth of the triangle at the respective sections.

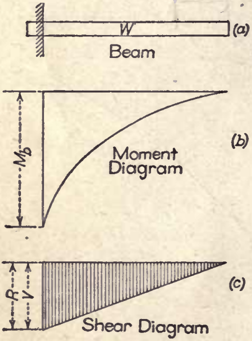


Fig. 5 — Diagrams for a Uniformly Distributed Load

The bending moment diagram is formed by a closing line which is a semi-parabola. This may be proven by dividing the uniform load into any number of loads equal in amount and finding the moment, as in Fig. 4, for each load and connecting the ends of the moments drawn to scale. The closing line is a broken line, which approaches a curve, depending upon the number of unit loads into which the uniform load is divided. By using a sufficient number the closing line becomes a parabolic curve.

If we assume the unit load to be one foot long and call it w, and the length from any point to the free end is l feet, the moment at any point is

$$M_b = \frac{wl^2}{2}.$$

By using this formula points may be plotted one foot apart and the ends connected by using a French curve.

In Fig. 6 is illustrated the effect of concentrated loads combined with a uniformly distributed load. When a beam carries concentrated loads the shear and moment diagrams are drawn as described for the uniform load. Then the moments and shears due to the concentrated loads are drawn *upward* from the top horizontal line of the diagrams. The total moment at any point is the distance, BB, or CC, from the top closing line to the bottom closing line, the intermediate horizontal line being disregarded.

In Fig. 7 is shown the actual shear when the loads have a definite width and when the beam rests on a support of a definite width. At the face of the support the shear is a maximum and it is zero at the end of the support. It is not usual to show this in shear diagrams, for it complicates the drafting work without enough benefit to pay for the trouble. The slight difference increases the factor of safety.

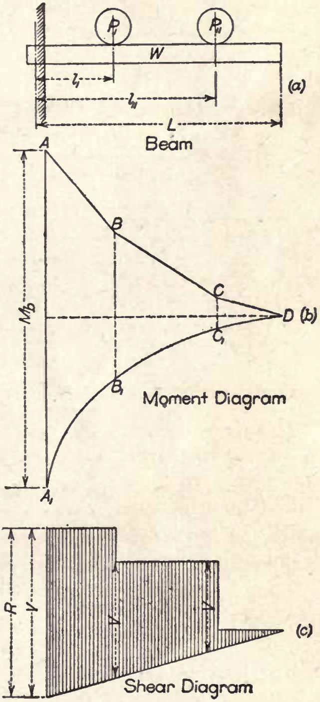


Fig. 6 — A Cantilever Beam Carrying Two Concentrated Loads

Beams Resting on Two Supports

To determine bending moments on beams on two or more supports, it is necessary to find first the amount of the reactions.

In Fig. 8 a concentrated load is carried on a beam resting freely on two supports. For convenience we adopt the conventional method of beginning at the left end as in reading. The word "conventional" has the same root form as the word "convenience" so may be easily remembered.

Common sense assures us that if the load is in the middle one-half will be carried by each support. Let us imagine the concentrated load to be stationary and the ends of the beam pushed up by the reactions. The reactions being equal to the load, there is no movement, but the forces actually exist.

Each reaction being one-half of P, then $R_1 = \frac{P}{2}$ and $R_2 = \frac{P}{2}$. This gives two cantilever beams with moments acting about (around) the load, with a lever arm $= \frac{L}{2}$. The bending moment under the load (middle of span) is

$$M = \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}.$$

The reader is not to forget that in computing reactions, which take into account only the weight, we use the clear span (S), but in computing the moment we use the length (L) from the center of bearing on the support.

In Fig. 9 a uniform load (represented by the weight of the beam) is carried on two supports on which it rests freely.

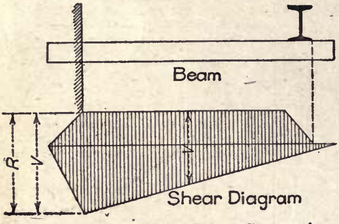


Fig. 7 — A Cantilever Beam Supporting an I-beam

Each reaction $= \frac{W}{2}$, in which $W = wS$.

This gives two cantilevers with moments acting about the center of gravity of the beam, which being in the middle of the span makes the length of the moment arms $= \frac{L}{4}$ measured from the middle of the span to the center of gravity of each half span. The bending moment is

$$M = \frac{W}{2} \times \frac{L}{4} = \frac{WL}{8}, \text{ or } \frac{wSL}{8}.$$

The load $\frac{W}{2}$ acts as a uniformly distributed load and not as a concentrated load at the end, although it is equal to the reaction.

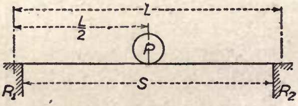


Fig. 8 — Concentrated Load at Mid-span with Beam on Two Supports

The reader will notice that the multiplication sign (×) is not used between letters, for when several letters are written together in formulas it means they are to be multiplied together. If the multiplication sign were used it might be mistaken for the letter x when written. The multiplication sign is always used between figures. Thus, WL stands for $W \times L$, but 67 means sixty-seven and does not mean $6 \times 7 = 42$.

In Fig. 10 a concentrated load is assumed to be at the middle of an opening spanned by a beam of uniform weight. The reactions are as follows: $R_1 = R_2 = \frac{P}{2} + \frac{W}{2} = \frac{P + W}{2}.$ The bending moment is

$$M = \frac{PL}{4} + \frac{WL}{8}.$$

In Fig. 11 several concentrated loads are shown on a span and the reactions are to be found.

Commencing at the left end take moments about R_1. Thus the bending moment at the left support is

$$M = Pa + P_, a_, + P_{,,} a_{,,}.$$

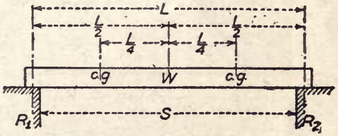


Fig. 9 — Uniform Load on Beam Resting on Two Supports

This moment will have a tendency to carry the far end of the beam, at R_2, downward unless a supporting force is exerted to hold it in position.

Here the principle of moments is again applied. The moment of a force is equal to the force multiplied by the distance, or arm, through which it acts, so it is necessary to have at R_2 an upward moment equal to the downward moment. This is obtained by dividing the downward moment by the span length.

$$R_2 = \frac{Pa + P_, a_, + P_{,,} a_{,,}}{S}$$

R_1 = (sum of the loads) — R_2.

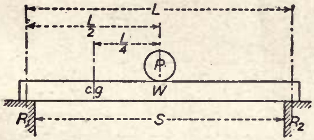


Fig. 10 — Concentrated Center Load and Uniform Load on Beam Resting on Two Supports

This is proved when we consider that the sum of the reactions is equal to the total load. The amount of each reaction may be checked by taking moments from the right end instead of the left and working as before.

Example: Let $a = 3$ feet and $P = 200$ lbs.
$a_, = 7$ " " $P_, = 300$ "
$a_{,,} = 11$ " " $P_{,,} = 250$ "
750 = total load.

Span = 15 feet.

$$R_2 = \frac{(3 \times 200) + (7 \times 300) + (11 \times 250)}{15} = 363.33 \text{ lbs.}$$

$$R_1 = 750 - 363.33 = 386.67 \text{ lbs.}$$

Checking by taking moments about the right end.

$$R_1 = \frac{(4 \times 250) + (8 \times 300) + (12 \times 200)}{15} = 386.67$$

$$R_2 = 750 = 386.67 = 363.33 \text{ lbs.}$$

The same results may be obtained by common proportion, but upon examination the method by moments is seen to be exactly the same thing, but with less work. If the reactions were found by the common school arithmetical process of proportion the work would be longer and mistakes more apt to occur. The method of moments is the shortest and neatest way of working.

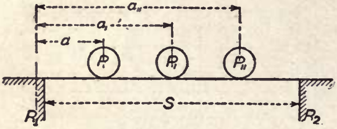


Fig. 11 — Several Concentrated Loads on Beam Having Two Supports

In Fig. 12 three concentrated loads are shown on a beam of which the weight is uniformly distributed. First find the reactions due to the concentrated loads. Then add to each reaction half the weight of the uniformly distributed load.

Example. — Assume a beam having a weight of 50 lbs. per lineal foot on a 15-ft. span, carrying the concentrated loads given in the last example. What are the reactions?

Answer. — Weight of beam = 15 × 50 = 750 lbs.

The reaction at each end = $\frac{750}{2}$ = 375 lbs.

$R_1 = 386.67 + 375 =$ 761.67 lbs.
$R_2 = 363.33 + 375 =$ 738.33 "

Total load 1500.00 "

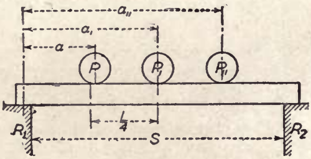


Fig. 12 — Uniform Load and Several Concentrated Loads on Beam Having Two Supports

When the bending moment is wanted at any section of a beam we assume the beam at the section to be supported at the section and moments are taken about it as if it were a cantilever beam. The reaction is an upward force creating an upward moment and the loads are downward forces creating a downward moment. The difference between the moments in opposite directions is the bending moment at the chosen section. When a beam is freely supported on two or more supports, this is always a positive (downward) moment, the beam being in tension on the lower side and in compression on the upper side between supports.

Bending Moment at any Point on a Beam on Two Supports

The loads are shown in position and amount in Fig. 13. First find the reactions.

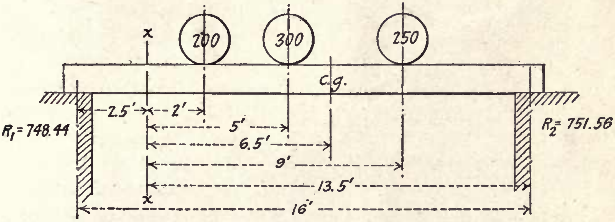


Fig. 13 — Example of Uniform Load and Several Concentrated Loads on Beam on Two Supports

$$R_2 = \frac{(4.5\times200) + (7.5\times300) + (11.5\times250) + (8\times750)}{16} = 751.56 \text{ lbs.}$$

$$R_1 = (200 + 300 + 250 + 750) - 751.56 = 748.44 \text{ lbs.}$$

Check for R_1:

$$R_1 = \frac{(4.5\times250) + (8.5\times300) + (11.5\times200) + (8\times750)}{16} = 748.44$$

In finding the reactions the weight of the beam = 15 × 50 = 750 lbs. This was multiplied by half the length of the span, which gave the quantity above, 8 × 750.

What is the bending moment at the section *xx*?

The section *xx* is 2 ft. from the face of the left support. The span face to face of supports = 15 ft., but the length center to center of bearings = 16 ft., assuming a bearing 1 ft. long on each support. Therefore the moment arm from the section to the reaction at the right end = 13.5 ft. The moment arms to the loads from the section are marked on the figure. There is one moment arm, however, of 6.5 ft. to be explained. It is the length from the section to the center of gravity of that portion of the beam lying between the section and the support at the right end. The total clear span = 15 ft. and the section is 2 ft. from the left support. The length of the beam therefore to the right support = 13 ft., and one-half = 6.5 ft. The weight = 13 × 50 = 650 lbs.

$$M = (13.5 \times 751.56) - [(2 \times 200) + (5 \times 300) + (6.5 \times 650) + (9 \times 250)] = 1771.06 \text{ ft. lbs.}$$

In Fig. 14 the section *xx* is taken 5 ft. from the left support. This leaves the load of 200 lbs. to the left of the section,

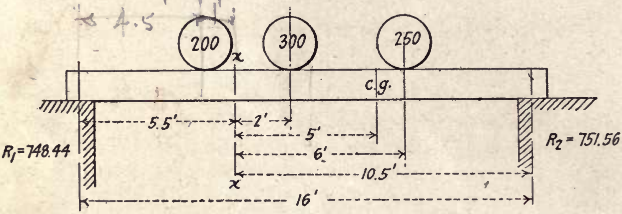


Fig. 14 — Another Example of Uniform Load and Several Concentrated Loads on Beam on Two Supports

so it is omitted from the calculations. The load was used in obtaining the reactions, but in this present example it will be noticed that the moment arm from the section is only 10.5 ft. to the reaction. The beam length = 15 − 5 = 10 ft. and the weight = 10 × 50 = 500 lbs., with a moment arm to the center of gravity = 5 ft.

$$M = (10.5 \times 751.56) - [(2 \times 300) + (5 \times 500) + (6 \times 250)] = 3291.38 \text{ ft. lbs.}$$

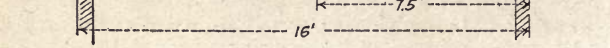


Fig. 15 — Still Another Example of Uniform Load and Several Concentrated Loads on Beam on Two Supports

In Fig. 15 the section *xx*, is taken 8 ft. from the left support. The beam length = 15 − 8 = 7 ft. and the weight = 7 × 50 = 350 lbs., with a moment arm of 3.75 ft. to the center of gravity.

$$M = (7.5 \times 751.56) - [(3 \times 250) + (3.75 \times 350)] = 3574.2 \text{ ft. lbs.}$$

The method given for obtaining the bending moment at any section is used for any number of loads, three loads being used in the examples for the sake of clearness. The computations being illustrative the reactions have been given to fractions of a pound and the moments have been given to fractions of foot pounds. In actual work, it is generally considered that the nearest unit is sufficiently exact.

Rule for Obtaining the Bending Moment; at any Section of a Beam

Multiply one end reaction by the length from it to the section. From the moment thus obtained subtract the sum of the moments of the loads lying between the section and the chosen reaction, using as moment arms the length in each case from the center of gravity of the load to the section. The portion of the beam included between the section and the reaction is to be counted as a load.

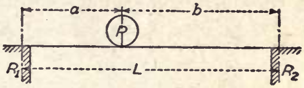


Fig. 16 — Concentrated Load at Any Point on Beam on Two Supports as Shown

A floor is merely a shallow beam, usually with a width of 12 inches.

A beam is a secondary girder and the load is usually uniformly distributed.

A girder is uniformly loaded when it carries the floor slab directly without the floor load going first to beams. When the floor rests on beams the reactions at the ends of the beams are concentrated loads going to the girders.

A girder is generally carried on walls or columns and beams are generally carried on girders. A rafter is a girder and purlins are beams, or joists.

For a load concentrated at any point, referring to Fig. 16,

$M = \frac{Pab}{L}$, for load only.

The derivation of the formula is as follows:

$$R_2 = \frac{Pa}{L}, \text{ and } R_1 = P - R_2,$$

$$\text{Then } M = R_2 b, \therefore = \frac{Pab}{L}.$$

The maximum bending moment for a single load is always under the load. If the weight of the beam is included the moment under the load is

$$M = R_2 b - \frac{wb^2}{2},$$

in which w = weight of beam per lineal foot.

Example. — A beam with a length of 15 ft. between centers of supports carries a load of 600 lbs. at a point 5 ft. from the left support. The weight of the beam per lineal foot is 9 lbs. What is the bending moment?

First. — For load only.

$$M = \frac{Pab}{L} = \frac{600 \times 5 \times 10}{15} = 2000 \text{ ft. lbs.}$$

Second. — For load and beam.

$$R_2 = \frac{Pa}{L} + \frac{wL}{2} = \frac{600 \times 5}{15} + \frac{135}{2} = 267.5 \text{ lbs.}$$

$$M = R_2 b - \frac{wb^2}{2} = (267.5 \times 10) - \left(\frac{9 \times 10^2}{2}\right) = 2675 - 450$$
$$= 2225 \text{ ft. lbs.}$$

Referring to Fig. 17, the load P acts through the center of gravity, but the effect is lessened because the load is distributed over a portion of the beam instead of acting at a point as it would were the load round like a ball.

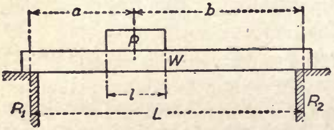


Fig. 17 — Partially Distributed Load on a Beam Resting on Two Supports as Shown

Consider the load of 600 lbs. in the last example to be distributed over a length of 3 ft. What is the total bending moment under the load?

$$M = R_2 b - \frac{wb^2}{2} - \frac{Pl}{8}$$

Writing this out it appears

$$M = (267.5 \times 10) - \frac{9 \times 10^2}{2} - \frac{600 \times 3}{8} = 2675 - 450 - 225$$
$$= 2000 \text{ ft. lbs.}$$

The formulas for the reactions are the same as though the load was concentrated at a point.

For two equidistant loads on a beam, as in Fig. 18, the formula for the reactions reads

$$R_1 = R_2 = P + \frac{wL}{2}.$$

The moment due to the loads only is a maximum under each load and is constant at all points on the beam between the loads.

$$M \text{ (for loads only)} = Pa.$$

This can be proved. The loads being placed on the beam at the same distance from the end, call each load P. Then the total load $= 2P$. One-half of the total load goes to each end, so the reaction must be $\frac{2P}{2} = P$. The moment is equal to the reaction multiplied by the arm through which it acts, therefore $M = Pa$.

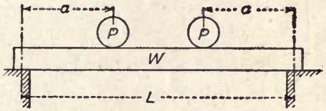


Fig. 18 — Two Equidistant Concentrated Loads on Beam on Two Supports as Shown

Adding the weight of the beam gives us under the load,

$$M = R_2 a - \frac{wa^2}{2},$$

in which w = weight per lineal foot of beam

a = length of beam between load and reaction.

To prove that the moment is the same under the middle of the beam as it is under each load we must multiply the reaction by half the span and subtract from it the load multiplied by half the span minus the length of the arm from the load to the reaction. The full written-out expression is as follows:

$$M = P \times \frac{L}{2} - P\left(\frac{L}{2}a\right) = \frac{PL}{2} - \frac{PL}{2} + Pa = Pa.$$

Note. — In the above expression algebra has been used for the first time. The product of two positive (+) quantities is positive. The product of two negative (−) quantities is positive. The product of a positive (+) and a negative (−) quantity is negative. In the above expression, where the subtraction is indicated, the load P is negative and the first quantity within the parenthesis is positive, for when no sign is written the positive (+) sign is understood. The second quantity is negative. By clearing away the parenthesis and multiplying each quantity within by the load we obtain $+\frac{PL}{2}$ and $-\frac{PL}{2}$, which of course cancel each other, leaving Pa, for P = reaction when weight of beam is neglected.

Example (Fig. 19). — A beam 15 ft. long, weighing 9 lbs. per lineal foot, carries two loads of 300 lbs. each at points distant 5 ft. from the ends. Find the bending moment.

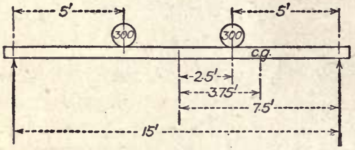


Fig. 19 — Diagram Illustrating the Example

First. — Under each load for the loads only.

$$M = Pa = 300 \times 5 = 1500 \text{ ft. lbs.}$$

This moment is constant for each point on the beam between the loads.

Second. — Under each load for the load and the weight of the beam.

$$R_1 = R_2 = P + \frac{W}{2} = 300 + \frac{9 \times 15}{2} = 367.5 \text{ lbs.}$$

$$M = (367.5 \times 5) - \left(\frac{9 \times 5^2}{2}\right) = 1725 \text{ ft. lbs.}$$

This moment is not constant, for the weight of the beam between the loads must be considered. The increase is slight and usually is negligible, except in the case of concrete beams, in which the dead load often equals or exceeds the live load. This example will be worked out in detail, there being four distinct steps.

First. — Total moment at middle of beam $= R_2 \times \frac{L}{2}$.

Second. — Moment due to weight of half the beam. The weight $= \frac{wL}{2}$. Multiply this by the distance from the middle of the span to the center of gravity of half the beam $= \frac{L}{4}$. This reduces to $\frac{wL}{2} \times \frac{L}{4} = \frac{wL^2}{8}$, to be subtracted from the total moment, which took into consideration the weight of the beam. This gives

$$M = \left(R_2 \times \frac{L}{2}\right) - \frac{wL^2}{8}.$$

Third. — The moment due to the concentrated load must be subtracted. The load is multiplied by the distance from the middle of the beam to the load $= \left(\frac{L}{2} - a\right)$; this giving $P \times \left(\frac{L}{2} - a\right) = \frac{PL}{2} + Pa$, and as this is to be subtracted the minus sign is placed before $\frac{PL}{2}$.

Fourth. — The whole expression now appears,

$$M = R_2 \times \frac{L}{2} + Pa - \frac{PL}{2} - \frac{wL^2}{8}.$$

When adding positive and negative quantities add each kind separately. Take the difference of the sums and prefix the sign of the greater sum. By inclosing the two negative quantities in a parenthesis the sign of the second is changed, so the full expression may be written

$$M = \left(Pa + \frac{R_2 L}{2}\right) - \left(\frac{PL}{2} + \frac{wL^2}{8}\right).$$

The arithmetical work is simple, as here shown

$$M = (300 \times 5 + 367.5 \times 7.5) - \left(300 \times 7.5 + \frac{9 \times 225}{8}\right) = 1753 \text{ ft. lbs.}$$

Graphical Methods

To divide a line into any number of equal parts use the geometrical principle of similar triangles.

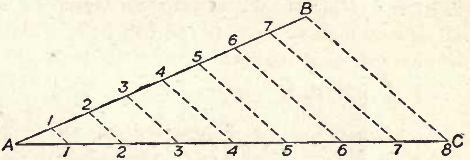


Fig. 20 — Division of Line into Equal Parts

In Fig. 20 let AC represent a line that is to be divided into any number of equal parts. The length is such that no regular scale can be used readily for the purpose. Set off from one end a line, AB, at any angle. This line is drawn to some scale and divided into as many equal parts as it is desired to divide the line AC. Connect B to C and through the divisions on the line AB draw lines parallel to BC, as shown.

A number of curves are used by engineers and scientists, the most useful of which, and the only one used by structural designers, is the parabola. This curve is formed by cutting a section

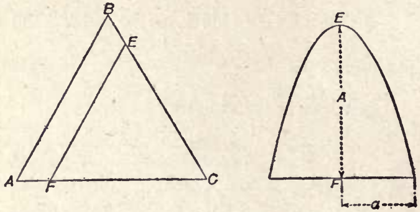


Fig. 21 — Section of Cone Developed into Parabola

through a cone parallel to the slope, as shown by the line *EF* in Fig. 21. The major axis in all curves is designated by the letter *A* and the minor axis, perpendicular to the major axis, by the small letter *a*. The axes of a parabola are infinite, so for this curve *A* designates height and *a* one-half the base.

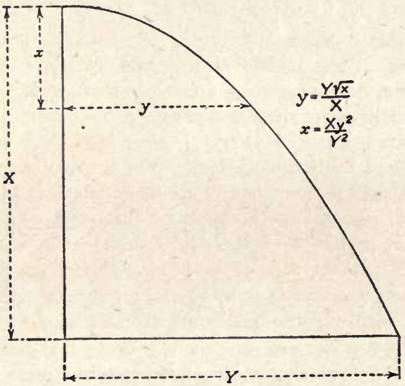


Fig. 22 — The Parabola and its Equations

In Fig. 22 let X and x = the abscissas and Y and y the ordinates of the parabola. The abscissas are parallel to and the ordinates are perpendicular to the major axis. Assuming X and Y each with a value of 1, and X divided into 10 equal parts, the following values are obtained, numbering the spaces from the bottom up.

Points on X	Ordinates (Y)
1	0.949
2	0.894
3	0.837
4	0.775
5	0.707
6	0.633
7	0.548
8	0.447
9	0.317

Assuming X and Y each with a value of 1, with X divided into 8 equal parts, the following values are obtained:

Points on X	Ordinates (Y)
1	0.936
2	0.866
3	0.791
4	0.707
5	0.612
6	0.50
7	0.353

To construct a parabola by using a table of ordinates erect a perpendicular at the middle of the span having a height equal to the bending moment, using any convenient scale. Divide the line into 8 or 10 equal parts and through the division points draw horizontal lines parallel with the beam. Multiply the half span by the value of the ordinate for any line and set off to scale the length of the ordinate. Connect the ends by means of a French curve and thus obtain the parabola. In this method the scale for all horizontal lines is the scale used in drawing the beam.

In Fig. 23 another method is shown. Divide the span of the beam into any number of equal parts, numbering from each end as shown. Multiply the maximum moment by the product of the two figures under any line and divide by the product of the two equal figures under the maximum moment line. The result is the length of the perpendicular at the two numbers. Connecting the upper ends of the perpendiculars by using a French curve, the parabola is drawn. The scale used in drawing the perpendicular lines is the scale used in setting off the value of the bending moment at the middle of the span.

Example. — Compute the moments at the points shown in Fig. 23.

$$\frac{1000 \times 6 \times 4}{5 \times 5} = 960 \qquad \frac{1000 \times 7 \times 3}{5 \times 5} = 840$$

$$\frac{1000 \times 8 \times 2}{5 \times 5} = 640 \qquad \frac{1000 \times 9 \times 1}{5 \times 5} = 360$$

The parabola in Fig. 23 was constructed by using perpendiculars computed as shown and the curve drawn with a French curve. The middle perpendicular line was divided into 10 equal parts and the ordinates to the major axis measured off on the horizontal lines to check the accuracy of the curve. This is recommended as an exercise for the student.

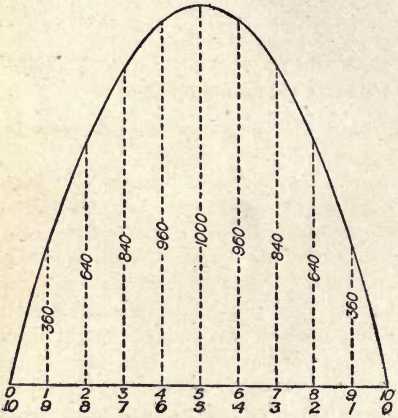


Fig. 23 — Ordinate Method for Constructing Parabola

A graphical method for constructing a parabola is shown in Fig. 24. The perpendicular representing the bending moment at the middle of the beam is set up and a rectangle drawn, with a height equal to the bending moment and a width equal to the span. The horizontal lines of the rectangle are divided into any number of equal parts and the vertical end lines into half this number. In the example the horizontal lines are divided into 8 parts and the vertical lines into 4 parts. From the apex radiating lines are drawn to the end divisions and vertical lines are drawn through the horizontal divisions. A curve is drawn through the intersections of the radiating and vertical lines.

All the common methods in use for drawing parabolas have been given in order that the students may have a choice of methods as well as to show that in even the most ordinary matters there are several ways of accomplishing a result. The man who knows only one way for doing anything is apt to be dogmatic and think his way alone is right. An ingenious man will discover for himself

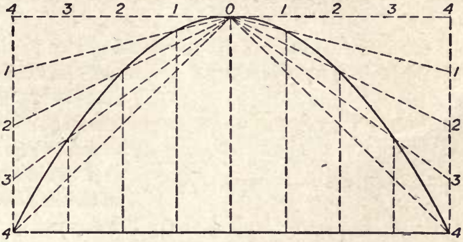


Fig. 24 — Graphical Method for Constructing Parabola

many ways to shorten his work when he knows he is not compelled to adhere to some method he has been shown.

When the height of the parabola is greater than the span more accurate results may be obtained than when the height and span are nearly equal, or when the height is less than the span. Use a scale that will give results with the accuracy required for the work. In some work the height of the parabola may be much

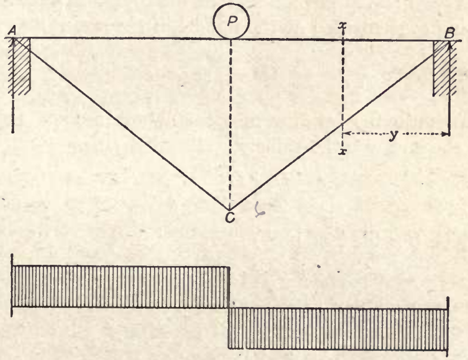


Fig. 25 — Graphical Solution for One Concentrated Load Resting on a Beam

less than the span and still give results sufficiently accurate. When the height of the parabola is not greater than one-tenth the span, a semicircle may be used. For cantilever beams the vertex of the parabola is at the support. For beams on two or more supports the vertex of the parabola between the supports is at the top.

For one concentrated load in the middle of a beam resting freely on two supports drop a perpendicular line under the load to represent the amount of the bending moment, as shown in Fig. 25. Connect the lower end, C, of the line by straight lines to A and B. Measuring vertically from any point on the center line of the beam to the bounding line, ACB, gives the bending moment at that point. The moment at any point may be computed also if the arithmetical method is preferred to the graphical method. The load being concentrated at the middle of the span,

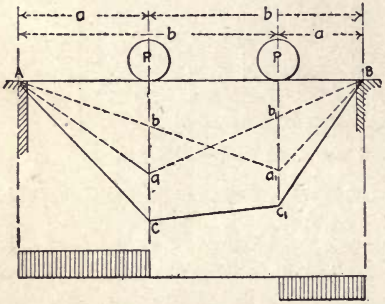


Fig. 26 — Graphical Method for Two Concentrated Loads Resting on a Beam

$$M = \frac{PL}{4}.$$

To compute the bending moment at any section, xx, distant y from the end, use the principle of similar triangles

$$\frac{L}{2} : \frac{PL}{4} :: y : M$$

from which (for the load only)

$$M = \frac{\frac{PL}{4}y}{\frac{L}{2}} = \frac{PLy}{4} \times \frac{2}{L} = \frac{Py}{2}.$$

The shear diagram is shown below. For a concentrated load the shear is constant from the load to each end and the shear is always equal to the reaction.

In Fig. 26 two concentrated loads are shown. Call the length from either end a and that from the other end b. Then under each load (for the load only)

$$M = \frac{Pab}{L}.$$

Through each load drop a perpendicular and set off the bending moment on each line for the load above that line. Connect the ends of the lines to the ends of the center line of the beam, thus making two triangles. Under each load is the moment due to the load, plus the moment due to the other load shown by the intercepts, Pb and P_1b_1. From a set off $ac = Pb$ and from a_1 set off $a_1c_1 = P_1b_1$. The total moment under $P = Pc$ and under $P_1 = P_1c_1$. Connect the points by the lines acc_1B, thus forming a bending moment diagram (for the loads only). The bending moment at any point on the beam is obtained by measuring from the center line (AB), of the beam to the bounding line of the bending moment curve.

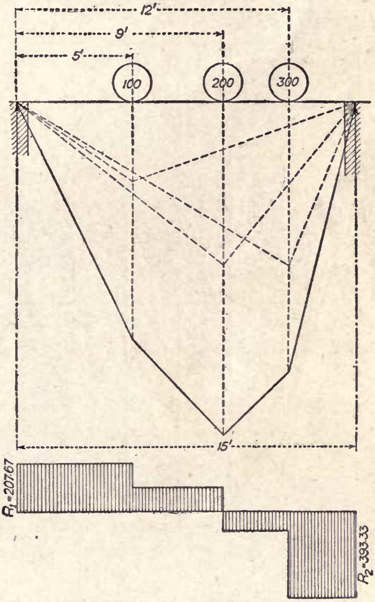


Fig. 27 — Graphical Method for Several Loads on a Beam

In Fig. 27 is illustrated the application of the method just described to three loads. Any number of loads may be similarly treated, no matter how unequal in weight nor how unevenly spaced.

Assume any number of equal loads equally spaced as in Fig. 28. This amounts to a uniform load, and, triangles being drawn as shown, for each load, a bending moment diagram is formed of which the boundary is a parabola. Knowing this to be true a parabola with a height represented by

$$M = \frac{wL^2}{8} = \frac{WL}{8}.$$

is drawn to represent the bending moment for uniformly loaded beams. The bending moment at any point is obtained by measuring vertically from the center line of the beam to the parabola. The bending moment at any point of a uniformly loaded beam is computed as follows, calling the distance from the end of the beam to the point in question x:

$$M_x = \frac{wlx}{2} - \frac{wx^2}{2}.$$

A moving load, for example a single wheel, occupies each point on the beam as it travels. If a triangle is drawn for each position of the load and the triangles are connected by a bounding line a parabola will be formed with a middle ordinate equal to

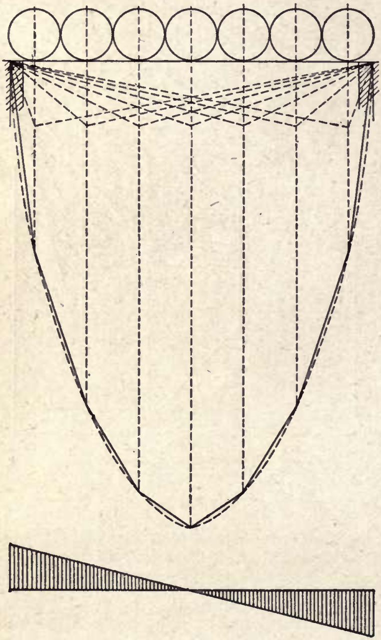


Fig. 28 — Graphical Method for Uniformly Distributed Load on a Beam

$$M = \frac{PL}{4},$$

for the continuous line across the span, of units each equal to the load on the wheel, compares with a uniform load. Consequently, to ascertain the bending moment at any point on a beam due to a single traveling load, construct a parabola with a middle ordinate as above and measure the ordinate to the curve at the point where the desired bending moment is to be found.

In Fig. 29 is shown the effect of concentrated loads plus the uniform load due to the weight of the beam. The moment due to the uniform weight of the beam is computed and a center line measured upward to represent this moment. Construct a parabola.

Below the center line of the beam construct a moment diagram representing the effects of the concentrated loads. The bending moment at any point is shown by the line intercepted by the upper and lower boundaries of the moment curves. For example, at a distance y from the left end of the beam the bending moment is shown by the length of the line xx_1.

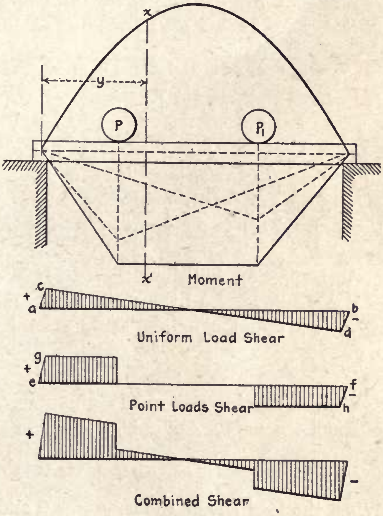


Fig. 29 — Graphical Method for Combination of Concentrated and Uniformly Distributed Loads on a Beam

In Fig. 29 the reactions are drawn to scale so that *ac* and *bd* each represent one-half the uniform load. Draw the horizontal line *ab* and connect *c* to *d*. The two triangles represent the shear to scale at all points on the beam. The horizontal measurements are lengths and the vertical measurements are loads. Since beams usually weigh much less than any load they carry it is commonly stated that the maximum moment and zero shear, or point where shear passes through zero, occur always under a concentrated load. The student can see that this statement must be qualified when the uniform load is considerable. The combined shear diagram in Fig. 29 is obtained by adding the end reactions due to the different load systems and making the sloped lines parallel to those in the diagram of uniform load shear.

General Method for Position of Maximum Bending Moment

First. — Find the reactions.

Second. — Starting from either end add the loads until a point is reached where the sum of the loads equals or exceeds the reaction at that end. This is the point of maximum bending moment.

General Method for Locating Point of Zero Shear

First. — Call the left reaction positive (+) and the right reaction negative (−).

Second. — Call each load negative (−) and successively subtract from the left reaction each load, prefixing the proper sign until the sign of the sum of the quantities changes from positive (+) to negative (−). This is the point of zero shear and maximum bending moment.

An inspection will show the two rules to be identical.

For a uniform load the shear at any point distant x from either support is found as follows:

Shear at $$x = \frac{wL}{2} - wx.$$

When a moving uniform load is passing over the beam, a train of small trucks, for example, the maximum shear at any point

$$x = \frac{w}{2L}(L - x)^2.$$

When the load covers the span $x = 0$ and the maximum shear at the ends $= \frac{wL}{2}$.

A crane travels on a girder with two wheels equally loaded and separated by a constant distance. The maximum bending moment is under one wheel when this wheel is as far from one support as the center of gravity of the total load is from the opposite support.

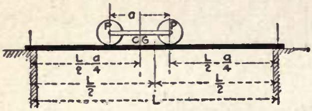


Fig. 30 — Two Equal Rolling Loads at Fixed Distance Apart

Referring to Fig. 30, if a is less than 0.586 L,

$$M = \frac{P(L - \frac{a}{2})^2}{2\,L}.$$

The maximum moment will occur twice on the span as the load moves along, the moments being equal and distant one-fourth a from the middle of the span. The maximum shear is at one end when one of the wheels is directly over the edge of the support.

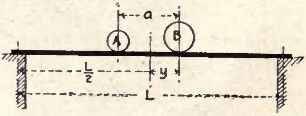


Fig. 31 — Two Unequal Rolling Loads at Fixed Distance Apart

In Fig. 31 is illustrated the case of two unequal loads at fixed distance apart, moving across a span. The maximum moment is under the heavier load. The maximum end shear is under the heavier load when it is over the edge of the support.

Let w = weight of lighter wheel load (A).

W = total load = $A + B$.

a = distance center to center of wheels.

y = distance from heavier wheel to mid-span.

M_{max} = maximum bending moment under heavier load.

$$y = \frac{wa}{2\,W}$$

$$M_{max} = \frac{W\left(\frac{L}{2} - y^2\right)}{L}$$

In Fig. 32 is shown a graphical method for ascertaining the bending moment when a load occupies a definite length on the beam. The triangle is first drawn as though the entire weight was concentrated at the center of gravity of the load. Drop vertical lines from the ends of the load to intersect the triangle. Connect the points of intersection by a straight line. Use this line as the base of a parabola, which is then constructed as shown. This method is also used if one end of the load rests on one abutment.

Overhanging Beams

When beams overhang one or two supports the methods for obtaining reactions, bending moments, and shears are no different from those used for cantilever beams and beams resting freely on two supports. The three examples following are from Greene's "Structural Mechanics" (3d ed.).

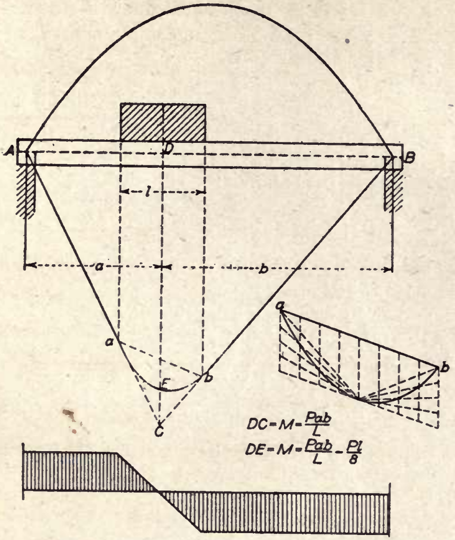


Fig. 32—Graphical Method for Concentrated Load with Wide Bearing

Studying Fig. 33 we see that the reactions are as follows:

$$R_1 = \frac{750 \times 25}{20} = 937.5 \text{ lbs.}$$

$$R_2 = 750 - 937.5 = -187.5 \text{ lbs.}$$

The weight of the beam has been neglected and the load at the extreme left tends to revolve the beam around R_1 so an additional

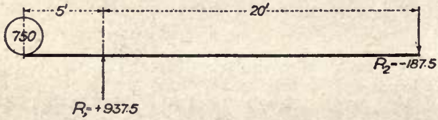


Fig. 33 — Concentrated Load on Short Overhanging Beam

load of 187.5 lbs. will be required at R_2 to hold down the right end. The negative sign indicates an upward pull.

The bending moment at R_1 is

$$M = -750 \times 5 = -3750 \text{ ft. lbs.},$$

indicating a negative bending moment, tending to make the beam convex at this support, showing tension to exist in the upper part and compression in the lower part of the beam. At section $x-x_1$

$$M = -R_2 \times 10 = -187.5 \times 10 = -1875 \text{ ft. lbs.}$$

also negative because R_2 is negative.

Check the moment at R_1 as follows, using R_2:

$$M = -187.5 \times 20 = -3750 \text{ ft. lbs.}$$

This beam has a negative bending moment at all points, which indicates tension above, and compression below, the neutral axis.

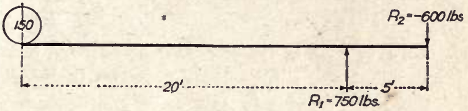


Fig. 34 — Concentrated Load on Long Overhanging Beam with Supports Close Together

A positive bending moment indicates tension below, and compression above, the neutral axis.

In Fig. 34 the reactions are

$$R_1 = \frac{150 \times 25}{5} = 750 \text{ lbs.}$$

$$R_2 = 150 - 750 = -600 \text{ lbs.}$$

Note that the divisor is, in all cases, the distance between supports. Both reactions are large compared with the load, showing the absurdity of considering a beam to be fastened at supports when it runs only a short distance into a wall. No beam should be considered as tied unless it runs into a wall a couple of feet at least and actually carries enough load to counteract the amount of negative reaction.

Several loads are shown in Fig. 35 on a beam resting on two supports and overhanging one of the supports.

$$R_1 = \frac{100\times18+200\times16+150\times13+300\times11+50\times8+80}{16} = 665.625 \text{ lbs.}$$

$$R_2 = 880 - 665.625 = 214.375 \text{ lbs.}$$

Fig. 36 illustrates an example given in Volume 1 of "Building Construction," edited by F. M. Simpson, in which the weight of the beam is considered. The total length of the beam is 28 ft. and the weight is 100 lbs. per lineal foot.

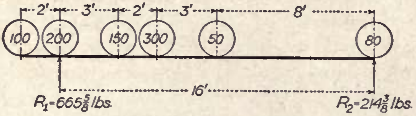


Fig. 35 — Beam Carrying Several Concentrated Loads with Short Overhang at One End

$$R_1 = \frac{23 \times 1000 + 14 \times 1200 + 9 \times 2800 - 5 \times 500}{20} = 3125 \text{ lbs.}$$

$$R_2 = 5500 - 3125 = 1875 \text{ lbs.}$$

For all the overhanging beam cases the amount and location of bending moment and shear at any point on the beam may be found by the rules previously given for cantilever beams and beams resting on two supports. Notice that the distance from either reaction to the center of gravity of the weight of the beam is equal to the distance from the other reaction to the center of gravity of the weight of the beam for uniformly distributed loads covering the whole beam. This applies as well to the beam alone, for the weight of a beam is uniformly distributed.

In Fig. 37 the beam weighs 20 lbs. per lineal foot $= 21 \times 20 = 420$ lbs. Half is carried on each support, for the overhang is equal at

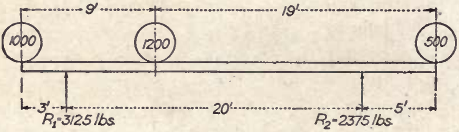


Fig. 36 — Beam Carrying Several Concentrated Loads, with Both Ends Overhanging

either end. Each support also carries the concentrated load nearest to it in this particular example.

$$R_1 = R_2 = 210 + 250 = 460 \text{ lbs.}$$

On the moment diagram the upper curved line (parabola with vertex at the support) under the overhanging end shows moment due to the cantilever end of the beam. The straight line under it shows moment due to the concentrated load on the extreme end. The lower slightly curved lines, AC and BD, represent the combined moments under the cantilever ends. This line at each point is the sum of the two moments, so is a mean between the parabola and straight line.

The bending moment at the middle of the span between the two supports is found as follows, there being a positive and a negative moment to consider:

$$-M = 250 \times 10.5 + (10.5 \times 20) \times 5.25 = -3827.5 \text{ ft. lbs.}$$

$$+M = 460 \times 7.5 = 3450 \text{ ft. lbs.}$$

Actual

$$M = +M - M = -3827.5 + 3450 = -377.5 \text{ ft. lbs.}$$

This negative moment is set off at the middle of the span measuring down from the line AB. The parabola CED is drawn. The bending moment at any point is found by scaling the length intercepted between the line AB and the bounding line $ACEDB$ of the bending moment diagram. All lengths measured horizontally are distances and all lengths measured vertically are forces.

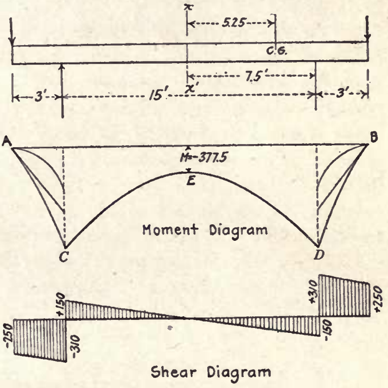


Fig. 37 — Graphical Method for Beam with Two Overhanging Ends

When the positive moment is greater than the negative moment the point E is set off above the line AB, so the parabola in such case is partly above and partly below this line. The curve above indicates positive and the curve below indicates negative moment. The maximum moment is where the shear changes sign. Where the moment curve crosses the line AB there is no moment, this point being termed the " point of reverse moment " or " point of contraflexure," or " point of inflection."

The shear diagram is evident. After drawing all the diagrams the student should check the moment at different sections by means of the shear diagram. Drop a vertical line through the shear diagram from any point on the beam. The area of the shear diagram between this section and the nearest support is equal to the bending moment at the section. When a beam rests on two supports and the moment is desired at any point the beam is assumed to be fixed there and to be pushed up by a force equal in amount to the reaction. Thus it is a cantilever beam, and by reference to Fig. 3 it will be found that for a cantilever beam the moment at any section is equal to the area of the shear diagram between that section and the free end.

The principle of the lever applies in all cases and moment effects are additive; therefore the effect of additional concentrated loads on the beam may be readily found. This is recommended as an exercise, arithmetical computations being checked by graphical methods.

In the figures an arrow point indicates the center of gravity of the bearing area. The clear span is S and the length of the beam is L_1. The moment span is L. To simplify all computations use

$$L = \frac{S + L_1}{2},$$

and instead of

$$M = \frac{w \times S \times L_1}{8}$$

use the formula

$$M = \frac{wL^2}{8},$$

the average length being used in all cases, as it is close enough for all practical purposes.

In examples involving loads concentrated at some point one side of the middle of a span the distance to the nearer support has been termed a and the distance to the farther support b. Then

$$M = \frac{Pab}{L}.$$

The custom in modern text books is to use the letter a for the shorter length and designate the longer length by describing it as the difference between L and a. Thus

$$M = \frac{Pa(L - a)}{L}.$$

The older method was given for the reason that it is so frequently met with in handbooks and trade papers, but the method of modern text books is preferable and should be used by the student in his work.

Equivalent Distributed Loads

A convenient method to use in figuring bending moments when one or more concentrated loads must be considered in addition to a uniformly distributed load, is to reduce the concentrated loads to equivalent distributed loads. Suppose we take the expression last given for the effect of a concentrated load at some point of the beam:

Let $$M = \frac{Pa(L - a)}{L} = \frac{WL}{8}.$$

The problem is to find the value of W, the uniformly distributed load.

Arrange it thus, $$M = \frac{WL}{8}.$$

Eliminating fractions, $$8M = WL$$

Dividing, $$W = \frac{8M}{L}.$$

The student can see that after obtaining the bending moment for the concentrated load the bending moment had only to be equated to that for a uniformly distributed load. If he does a little thinking he will see that if the concentrated load is off center very far the bending moment is greater than it is at the center of the span, yet the equation of the uniformly distributed load was made on the basis of the maximum moment being at the center of the span.

The method of equivalent uniformly distributed loads is in common use in many designing offices, but only because it saves a little time and because beams come in stock sizes. It always gives a trifle larger beam than is necessary, so it is a safe method to use. When the greatest possible economy is desired, or the beam size selected is on the border line between a heavy and a light beam, the exact method should be used to obtain the size, as thereby considerable saving may be effected. The exact method should be used also when a built-up girder is to be designed.

The uniform load has been found. Divide it by the concentrated load and get a factor we will call m. Then divide the span by the distance from the concentrated load to the nearest support and call the result x.

$$m = \frac{W}{P} \quad ; \qquad x = \frac{L}{a}.$$

Example. — A beam weighing 200 lbs. on a span of 12 ft. carries a concentrated load of 750 lbs. 3 ft. from one end. What is the equivalent distributed load? What is the bending moment?

Ans. $$M = \frac{Pa(L-a)}{L} = \frac{750 \times 3 \times 9}{12} = 1687.5 \text{ ft. lbs.}$$

$$W = \frac{8M}{L} = \frac{8 \times 1687.5}{12} = 1125 \text{ lbs.}$$

$$m = \frac{W}{P} = \frac{1125}{750} = 1.5.$$

$$x = \frac{L}{a} = \frac{12}{3} = 4.$$

The equivalent distributed load producing a bending moment equivalent to the bending moment produced by the concentrated load = 1125 lbs. and to this must be added the weight of the beam, 200 lbs. The total bending moment is

$$M = \frac{WL}{8} = \frac{1325 \times 12}{8} = 1987.5 \text{ ft. lbs.}$$

The student is advised to compute a table, following the above example, with the concentrated load assumed to be placed at various points on the beam, the table giving values of m and x, to be used in shortening labor in future work.

Such tables are in use giving values of m and x for a dozen or more points on a beam. The following table gives these values for ten points:

When	$x = 2$	$m = 2$
	$x = 3$	$m = 1.78$
	$x = 4$	$m = 1.5$
	$x = 5$	$m = 1.28$
	$x = 6$	$m = 1.11$
	$x = 7$	$m = 0.98$
	$x = 8$	$m = 0.875$
	$x = 9$	$m = 0.79$
	$x = 10$	$m = 0.72$

In using the table first find the value of x by dividing the total span by the distance from the nearest support to the load. Then multiply the load by m in the table opposite the value found for x. Do this for each concentrated load in turn, add the weight of

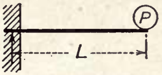
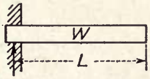
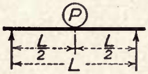
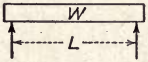
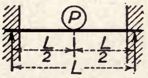
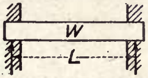
Loading	Maximum Bending Moment	Relative Strength	Relative Deflection in terms of stress
	$M = PL$	$\frac{1}{8}$	3.2
	$M = \frac{WL}{2}$	$\frac{1}{4}$	2.4
	$M = \frac{PL}{4}$	$\frac{1}{2}$	0.8
	$M = \frac{WL}{8}$	1	1
	$M = \frac{PL}{8}$	1	0.4
	$M = \frac{WL}{12}$	$\frac{3}{2}$	0.2

Fig. 38 — Reference Table Showing the Strength and Stiffness of Beams

the beam, and then find the bending moment for the total uniformly distributed load so obtained.

The method of equivalent uniformly distributed loads is applicable only to bending moments. When a load is uniformly distributed the end reactions are equal and the shear is always equal to the reaction, so if the reactions and shears are considered at each end as being one-half the equivalent distributed load the beam may be weak in shearing resistance and the supports may be improperly designed. It is necessary, therefore, to compute the reactions by the exact method and at the same time find the maximum shear. The beam will be designed for maximum shear and the supports will be properly taken care of.

The accompanying table of strength and stiffness of beam is valuable for daily reference in beam calculations. The subject of deflection will be taken up later. This table, Fig. 38, takes as a basis the uniformly distributed load on a beam resting freely on two supports. In the first column is shown the loading condition; in the second column the formulas for ascertaining the bending moments; in the third column the relative loads, and in the fourth column the relative deflection due to these loads. For example, the cantilever beam carrying a concentrated load at one end will support only one-eighth the load the same size beam with the same span will carry when freely supported at the two ends. The deflection under this load will be 3.2 times the deflection of the freely supported beam carrying 8 times the load. The uniformly distributed load on a beam securely fastened over supports is 1.5(3/2) times the load carried on the same beam on the same span when freely supported and the deflection is greatly lessened, being only 0.3 the deflection of the freely supported beam carrying two-thirds the load of the restrained beam.

Restrained Beams

In Fig. 39 is shown a beam tied into the supports. This is known as a restrained beam. A restrained beam carrying one centrally concentrated load will be first considered.

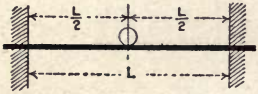


Fig. 39—Beam Resting on Supports

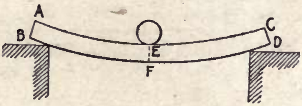


Fig. 40—Beam Deflected under Loads

When a beam is simply supported, that is rests on supports without being fastened in place, it deflects under load as shown in an exaggerated manner in Fig. 40, so the ends AB and CD slope and are no longer vertical. At E there is compression and at F there is tension, but no tensile or compressive stresses exist at A, B, C and D.

When a beam having a uniform moment of inertia (that is, a beam symmetrical in form with material uniform throughout) is restrained, the ends have no slope when the beam carries a load. The case is shown in Fig. 41, where the ends are extended to some distance, b, where a load is placed which has sufficient weight to hold the ends in the original positions. Tension under such conditions exists at A and C with compression at B and D. At the point where the beam ceases to be horizontal and bends down there is neither tension nor compression, the only existing force being shear. This point is termed the "point of contraflexure," the "point of reverse moment," or the "point of inflection."

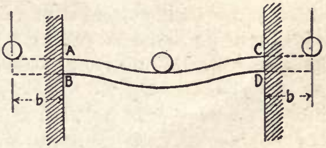


Fig. 41

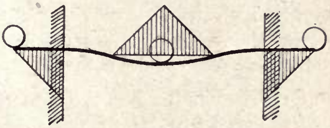


Fig. 42

In Fig. 42 the shaded triangles represent the moments of the actual center load and the two assumed end loads. These loads, as well as the length b, may actually exist, but the same effect will be obtained by riveting or otherwise fastening the ends of the beams to, or in, solid supports; therefore the loads and the moment areas beyond the point of support are said to be imaginary or assumed. The condition created is that of a simply supported beam, having a length measured between the points of contraflexure, carried on the ends of two short cantilevers. An expression must be found for the force creating such a condition and also for the lengths of the cantilevers, that is, the distance from the point of support to the point of contraflexure.

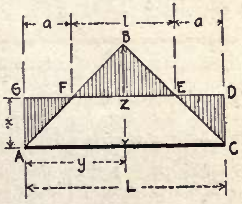


Fig. 43

In Fig. 43 let the triangle ABC represent the moment area due to a concentrated load at midspan of a freely supported beam, AC. The two end triangles AGF and CDE are the moment areas of the loads causing the restraint. An inspection will show that the combined areas of the two end triangles must equal the area of the triangle FBE and that the points of contraflexure lie vertically beneath E and F.

Assume a rectangle $AGDC$ placed on the span AC. The area of this rectangle times the distance to the center of gravity from either support must equal the area of the triangle ABC times the distance of its center of gravity from the same support. This is known as equating moment areas, or,

$$(AC \times AG) \times y = \frac{AC \times Z}{2} \times y.$$

Let $AG = x$. The span, AC, is common to both sides of the equation and the length, y, of the moment arm is one-half of AC, so may be eliminated, as it also is common to both sides of the equation, which is treated as follows;

Eliminating y, $AC \times x = \frac{AC \times Z}{2}$.

Eliminating the common factor AC, $x = \frac{Z}{2}$.

From the similarity of triangles, since $AG = DC = x = \frac{Z}{2}$, then $AF = FB$, and $CE = EB$, for F lies in the line AB and E lies in the line CB, and the line GD, parallel to AC, intersects the line AB in F and the line BC in E.

The length $EF = GF + ED$; therefore the area of the triangle FBE = area of triangle AGF + area of triangle CDE. The length $GF = ED = \frac{AC}{4}$; therefore the points of contraflexure are $\frac{1}{4}L$ from the supports.

Since $Z = \frac{PL}{4}$ and $x = \frac{Z}{2}$,

then $x = \text{negative moment} = -\frac{1}{2} \times \frac{PL}{4} = -\frac{PL}{8}$.

The positive moment

$$= \frac{Z}{2} = \frac{1}{2} \times \frac{PL}{4} = +\frac{PL}{8}.$$

Let $\frac{Pl}{4} = \frac{PL}{8}$; then $Pl = \frac{PL}{2}$ and $l = \frac{L}{2}$ (eliminating P).

Since $l = \frac{L}{2}$ and $l + 2a = L$,

$$a = \frac{L - l}{2} = \frac{L}{4}.$$

The uniformly loaded beam restrained at the ends is shown in Fig. 44. The reasoning follows that for the beam carrying one center concentrated load. The bending moment diagram, however, is a parabola with height, $Z = \frac{WL}{8}$, the area of which is equal to the product of the base times two-thirds the height; then

$$ACxy = \frac{2ACZy}{3}$$

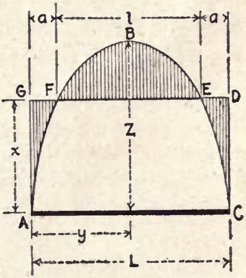


Fig. 44

Eliminate like quantities from the two sides of the equation and

$$x = \frac{2Z}{3}.$$

Then x = negative moment =

$$-\frac{2}{3} \times \frac{WL}{8} = -\frac{WL}{12}, \text{ at each end.}$$

The center positive moment $= \frac{1}{3} \times \frac{WL}{8} = + \frac{WL}{24}.$

Now take unit load per lineal foot = w.

Let $\frac{wl^2}{8} = \frac{wL^2}{24}$; then $wl^2 = \frac{wL^2}{3}$ and $l^2 = \frac{L^2}{3}$ (eliminating w).

$$l = \sqrt{\frac{L^2}{3}} = \frac{L}{\sqrt{3}} = \frac{L}{1.732}.$$

When $L = 1$, $l = \frac{1}{1.732} = 0.5773L.$

Since $l = 0.5773L$ and $l + 2a = L$,

$$a = \frac{L - l}{2} = \frac{1.0000 - 0.5773}{2} = 0.2113\ L.$$

Continuous Beams

Continuous beams, that is beams running over a number of supports, are designed by methods which are an extension of the principles used for overhanging beams and restrained beams. The only instance of continuous beams in wood construction appears in the placing of floors over joists or closely spaced beams. On account of the excessive deflection of wood this is not justifiable, for the theory underlying continuous and restrained beams requires that the supports be immovable. The slightest settlement causes increased stress and sometimes a reversal of stress. As it is merely a matter of properly designed connections, continuous girders and beams are sometimes used in steel structures. They are not common, however.

The maximum stresses are invariably over the supports, and lack of realization of this fact has caused distress to some designers. The principle of continuous beams finds application in reinforced concrete work. Owing to the monolithic character of reinforced concrete there is no other proper way for designing in this material. To assume that the bending moment on a span $= \frac{WL}{8}$ does not make it so, and designers who assumed that the greater stiffness thus secured would permit the use of a smaller moment over supports suffered in reputation thereby.

By methods involving the use of higher mathematics it can be proved that the sum of the moment in the middle of the span and the moment at one support $= \frac{WL}{8}$. In the study of restrained beams this has been shown, for, disregarding the signs,

$$\frac{WL}{12} + \frac{WL}{24} = \frac{3WL}{24} = \frac{WL}{8}.$$

The smaller moment, however, is in the span and the larger moment is over the support. To assume $M = \frac{WL}{12}$ does not make the moment over the support $= \frac{WL}{24}$ based on the following reasoning; $\frac{WL}{8} - \frac{WL}{12} = \frac{3WL}{24} - \frac{2WL}{24} = \frac{WL}{24}$. The moment over the support is $\frac{WL}{12}$, or nearly this amount, no matter what assumption may be made for the bending coefficient in the middle of the span.

When the moment of inertia of the beam, or slab, is constant, the tension over the support is that due to a moment $= -\frac{WL}{24}$, if the span is designed for $M = +\frac{WL}{12}$. However, if the amount of steel is reduced so the resisting moment will be barely sufficient to take care of this amount of bending moment, the moment of inertia is thereby altered. Changing the moment of inertia has the effect of greatly increasing the tension in the steel over the supports. The positive and negative moments should total $\frac{1}{6}WL$ and not $\frac{1}{8}WL$, this in effect making the positive and negative moments for interior spans and supports $=\frac{1}{12}WL$.

When one panel is loaded and an adjacent panel is not loaded there will be an uplift in the unloaded panel, for it opposes only the dead load to the combined dead and live load on the loaded panel. To make the positive and negative moment coefficients in each panel equal gives the necessary stiffness and increased weight.

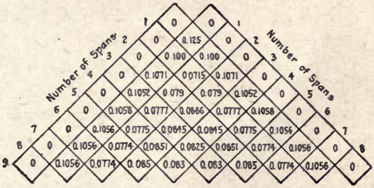


Fig. 45 — Coefficients for Maximum Negative Bending Moments for Uniform Loads over the Supports of Continuous Beams with Equal Spans. $M = Cwl^2$, in which C = coefficient.

Example: $M = \frac{wl^2}{8} = 0.125\ wl^2$.

Assuming spans equal in length and loaded uniformly, the negative bending moment coefficients to use over supports are shown in Fig. 45. Each square represents a support and the coefficients are given as decimal instead of common fractions. The moments are constant at the edges and across the tops.

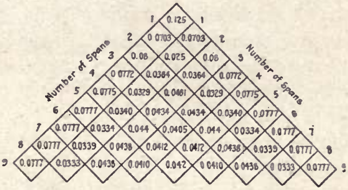


Fig. 46 — Coefficients for Maximum Positive Bending Moments for Uniform Loads on Equal Spans of Continuous Beams. $M = Cwl^2$, in which C = coefficient.

Examples: $M = \frac{wl^2}{8} = 0.125\ wl^2$.

Assuming spans equal in length and loaded uniformly, the positive bending moment coefficients to use for the spans are shown in Fig. 46. The coefficients, however, are the theoretical coefficients for beams with constant moment of inertia. They should not be used for the reasons given above, for it is best to have the positive and negative moments equal.

Assuming spans equal in length and loaded uniformly, the coefficients to use in figuring the reactions on the supports are given in Fig. 47. These coefficients give the total reaction due to the load from the middle of one span to the middle of the adjacent span, one full panel length. The moments on each side of the support differ, as shown in Fig. 46, so the shear at the edge of the supports is proportional to the moment coefficients in the spans. Usually, however, it is safe to use half the reaction for the shear on each side of a support. The three figures are all based on the assumption that the ends of the beams are freely supported.

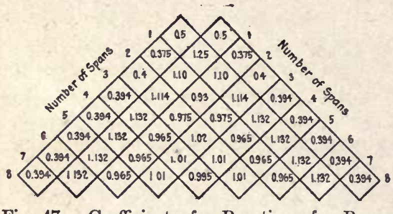


Fig. 47 — Coefficients for Reactions for Beams under Uniform Load over Several Equal Spans and Freely Supported at the Ends. $R = Cwl$.

Spans are not always equal in length for continuous beams and the beams are not always uniformly loaded. A complete discussion of such conditions is best treated graphically and will be taken up in another chapter.

When spans are unequal the reactions for continuous beams must be computed for each span separately. The total reaction on any intermediate support is equal to the sum of the reactions for the adjacent spans.

Let M_1 = moment at left end of span.

M_2 = moment at right end of span.

R_1 = reaction at left end.

R_2 = reaction at right end.

w = load per lineal foot.

l = span. When the moment is in foot pounds the span is in feet.

$$R_1 \times l - M_2 = -M_1 + \frac{wl^2}{2},$$

by taking moments about R_2 from which

$$R_1 = \frac{wl}{2} - \frac{M_1 - M_2}{l} \text{ and}$$

$$R_2 = \frac{wl}{2} - \frac{M_2 - M_1}{l}.$$

The reaction from a continuous slab is carried by the beam and constitutes the uniform load on the beam, plus the weight of the beam. This in turn is carried by the girder and, together with the weight of the girder, constitutes the uniform load on the girder. The girder, if continuous, transmits to the columns the effect of the unbalanced moments over supports, of slab, beam, and girder. Columns must be proportioned to carry this load made up of the direct weights plus the unbalanced moments in the system of framing.

Building ordinances require thât the effects of unbalanced moments must be considered in design. The author in his own practice designs in this manner. It is the practice of all reputable designers. The majority of reinforced concrete buildings in the past have been designed by firms engaged in the business of selling steel. A large part of the work of the author for several years consisted in checking these "free" (so-called) designs and he has not found one in which anything more than the direct load was considered. The practice being somewhat common and structural engineers in building departments having passed designs so prepared, he has had no other alternative, for it was what is termed "common practice." After the date on which the present book is placed on the market he will approve no more designs following the old "common practice" and will force designers to fully recognize the effect of unbalanced moments.

The "free design" is usually given in the following way. An architect is employed by an owner to prepare plans for a reinforced concrete structure. The architect not being an engineer specialist has a choice of either employing a reputable engineer to prepare the engineering design, for which he must pay a considerable part of his own fee if he has no engineer employed on salary, or he merely prepares general drawings and in his specifications states that competitive engineering plans will be accepted. Sometimes he fixes the stresses and other necessary designing specifications, but more commonly omits to do so. Many architects believe that reinforced concrete design is as standard as steel design, which is not the case.

Contractors who bid on the work apply to steel selling concerns for designs. These are prepared and the contractor is given a lump sum price for the steel, which includes the cost of making the design. Frequently the amount of concrete is guaranteed by the steel salesman. Sometimes the above procedure is varied by a draftsman being employed to make the building plans and the owner takes up the design of the structure directly with some steel salesman. In this case the plans are furnished without cost to the owner, provided the firm supplying the plans sells the steel. A price is fixed for the plans in case the owner purchases the steel elsewhere. The price for the design must then be added to bids for the steel, which usually results in the owner purchasing the steel from the firm giving him the " free " design. There is severe competition in the steel-selling game, the result being that few designs thus made are as good as they should be.

The designers employed by steel salesmen are first-class men in nearly every instance, but in order to hold their positions they work for their employers rather than for the owners, which is quite natural. Few of these men, when they finally go into business for themselves, design exactly as they were forced to design when they were required to assist in selling steel. If the owner employs a reputable engineer in private practice to do the engineering work and pays him a fee in addition to that paid to the architect, he will receive designs which conform to the best modern practice and all contractors will bid on these designs. The difference in cost between the certainty of good work and the uncertainty affecting the " skinning " of designs by steel salesmen will be very small. Frequently the competition between contractors will entirely wipe out the difference.

Owing to the large number of unsatisfactory designs received under the competitive method of having steel salesmen furnish the engineering work, and to the failures occurring during construction, the practice should be abandoned. The average owner, and many architects, consider the engineer as an expert juggler with figures and a sort of human attachment to a slide rule and table book. When a steel-selling concern boasts of the ability of the engineers employed by it the engineers are really only highly trained men doing clerical work, for computing the strength of parts is nothing more. The engineer really is, or should be so considered, a man whose only interest is the safeguarding of the interests of his employer. No man can serve two masters, and the designer who is trying to help his employer make the largest possible profit on the sale of steel cannot be a disinterested designer for the owner who does not employ him directly but merely obtains his technical services through a salesman as an intermediary.

Sometimes architects and owners obtain competitive designs on specifications prepared by a reputable engineer in private practice and insert in the specifications a clause that the successful bidder must deposit some definite amount to pay the cost of having the designs checked by the engineer, whose name is given. The author does a great deal of work of this sort and is glad to see that many architects are now insisting upon having competitive designs thus checked. When they all do it there will undoubtedly be a radical change in " common practice."

When an engineer is independently employed to furnish engineering service he should not accept the work if it stops with the furnishing of the plans and details. He should insist upon being retained as an adviser during the progress of the work. The work of the best designers may often be discredited when the lowest bidder is not possessed of enough experience, or honesty, to put into the fabrication of the structure the quality of work which the designer put into the design. That many poorly designed buildings stand to-day is due to the fact that the work was performed by honest, experienced contractors.

CHAPTER II

Internal Forces

THE term "bending moment" is a description of the breaking effect of external forces on a beam. The term "resisting moment" is a description of the effect of internal forces in a beam set up to resist breaking. Beams not stayed laterally will bend to one side, in which case the full value of the resisting moment is not obtained.

The action of the resisting forces may be illustrated by simple framework, for a frame is merely a light beam containing little or no superfluous material. The ideal frame contains no superfluous material, but if this is obtained by an increase in cost of fabrication the frame is not ideal from the standpoint of the user, regardless of the mathematically ideal condition.

The designer soon finds that mathematical analysis of stresses treats frames as lines through the center of gravity of pieces. It is with pieces that the designer has to deal. In a mathematical design of a frame all forces act at points, and when a line of infinitesimal thickness, dealt with by the mathematician, is replaced by a piece of wood or steel or concrete, certain stresses are set up around the points. Then joints are constructed and an analysis must be made of the forces around the joints, this involving the design of rivets and fastenings to keep the joints from moving.

A simple beam contains some superfluous material, but much of this material acts to transmit stresses in various directions and thus takes care of internal stresses which otherwise would interfere with the direct action assumed to take place along the lines connecting the points around which the forces act.

The strongest frame is a triangle, for a triangle with sides of fixed length cannot distort under direct stresses acting along the center lines of the pieces of which it is composed.

The capital letters *A*, *B*, and *C* are used to designate the three angles of a triangle, and the corresponding small letters are used to designate the sides opposite the angles. By placing the capital letters so small b will represent the base and small a will represent the altitude (height) of a triangle the student has a mnemonic idea of the relations of the sides and angles. In the following figures

$$a = BC \text{ (line from } B \text{ to } C)$$
$$b = AC \text{ (line from } A \text{ to } C)$$
$$c = AB \text{ (line from } A \text{ to } B)$$

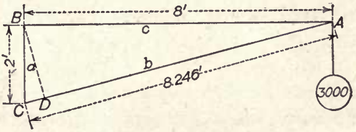


Fig. 48 — Frame with Inclined Strut

Then in Fig. 48 $\qquad b = \sqrt{a^2 + c^2} = 8.246,$

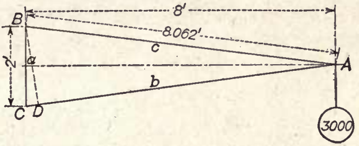


Fig. 49 — Frame with Inclined Strut and Inclined Tie

in Fig. 49 $\qquad b = c = \sqrt{(a/2)^2 + (\text{line } A\text{–}a)^2} = 8.062,$

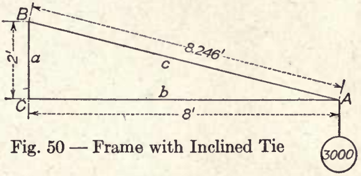


Fig. 50 — Frame with Inclined Tie

in Fig. 50 $\qquad c = \sqrt{a^2 + b^2} = 8.246.$

The rule is known as the "Rule of Pythagoras" and is given in every school arithmetic as follows:

In every right-angled triangle the square on the hypothenuse is equal to the sum of the squares on the other two sides.

In Fig. 48

$$cP = 8 \times 3000 = 24{,}000 \text{ ft. lbs. in member } C.$$

The stress in $c = \frac{cP}{a} = \frac{24{,}000}{2} = 12{,}000$ lbs. tension, for the stress *acts away from* the point B.

$$bP = 8.246 \times 3000 = 24{,}738 \text{ ft. lbs.}$$

for member b.

The stress in $b = \frac{bP}{a} = \frac{24{,}738}{2} = 12{,}369$ lbs. compression, for the stress *acts toward* the point C.

In Fig. 49 $cP = bP = 8.062 \times 3000 = 24{,}186$ ft. lbs.

Stress in c = stress in $b = \frac{24{,}186}{2} = 12{,}093$ lbs., the character of the stress in each member being determined by whether the member *pulls* from the point of fastening or *pushes* toward the support.

In Fig. 50 $cP = 8.246 \times 3000 = 24{,}738$ ft. lbs. Stress in $c = \frac{cP}{a} = \frac{24{,}738}{2} = 12{,}369$ lbs. tension.

$$bP = 8 \times 3000 = 24{,}000 \text{ ft. lbs.}$$

Stress in $b = \frac{bP}{a} = \frac{24{,}000}{2} = 12{,}000$ lbs. compression.

The examples show that *the stress at any point in a horizontal member of a frame is equal to the bending moment at the point divided by the depth of the frame at that point. The stress in an inclined member is equal to the stress in the corresponding horizontal member times the ratio of their respective lengths.*

A frame is so made that certain members are in tension and other members are in compression, the shear being carried by the vertical and inclined members. The lines of travel of the stresses are plainly exhibited. The same lines exist in a solid beam, so in a beam it is also true that the horizontal stress is equal to the bending moment divided by the depth of the beam. On one side of the neutral axis the stress is tension and on the other side the stress is compression. This will be fully explained later.

Assume that the frame is made of some material, wood for example, in which a fiber stress of 500 lbs. per square inch can be used in either tension or compression. Referring to Fig. 49, where the stress in each member is equal, each member will require an area $= \frac{12{,}093}{500} = 24.186$ sq. ins. Extracting the square root gives the dimensions of each piece as 5.85 ins. $\times$ 5.85 ins., which, of course, will be commercial 6 ins. × 6 ins. wood, after surfacing. The pieces therefore can be 6 ins. square. The lines of stress pass through the centers of the pieces, otherwise some twisting and bending strains will be set up. Twisting is called torsion in such cases.

The actual construction of such a bracket frame is shown in Fig. 51, the stresses for which are given in Fig. 50. The lower piece rests on an angle at the wall and a plate may be placed between the end and the wall if the pressure exerted is greater than the wall can stand. The area of the plate will be such that the pressure will be distributed to an extent calculated to keep the allowable compressive load on the wall within proper limits. The load P represents the reaction at the outer end of the frame, the reaction at the wall end being carried on the angle support. The diagonal is a rod which will be about 1 in. in diameter if of steel, for steel can be stressed to 16,000 lbs. per square inch in tension. To anchor the rod in the wall a bolt 1 in. in diameter extends into the far side and there a plate is fixed or it may be anchored in a concrete block in the wall.

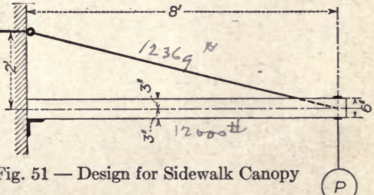


Fig. 51 — Design for Sidewalk Canopy

The circumference of a circle is 3.1416 × the diameter. The circumference of a 1-in. rod is 3.1416 ins., so for each inch in length there is an area of 3.1416 sq. ins. An adhesion of concrete to steel of 75 lb. per square inch is customary, so the adhesion per inch of length of the anchor rod in the concrete = 3.14 × 75 = 236 lbs. The total length of the rod $= \frac{12,369}{236} = 52.41$ ins. Instead of using one straight rod with a ring in the end it can be made U-shaped, each leg embedded in the concrete 27 ins. A much lower stress may be obtained by running the tie rod higher, an angle of about 45 degrees being good.

This example was worked to illustrate the simplicity of such computations and to show that all lines of stress must pass through points. In Fig. 51 the tie rod is shown to run quite a distance into the bottom member in order to have all the forces acting properly. In practical work the vertical bolt suspending the load will have an eye ring at the top to receive the tie. The eccentricity will not be large enough to make a great difference, but even this small amount can be greatly reduced by running the tie at a steeper angle.

Assume that the lines of stress are contained within a beam, Fig. 52, anchored in the wall in the usual way. The size of the beam is proportioned to take care of the tensile, compressive, and shearing stresses and the anchorage is the only item to be now considered. The wall reaction is equal to the sum of all the loads on the beam, so the weight of the wall resting on the beam must be equal to, or exceed, the reaction. Weight implies bulk and bulk implies bottom surface, and as the wall must be of a definite width to rest on the beam a moment is created.

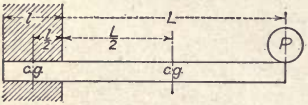


Fig. 52—Action of Moments in Anchorage of Cantilever Beam

In Fig. 52 the weight of the wall multiplied by the distance from the face to the center of gravity must be equal to the moment obtained by multiplying the loads on the beam by the respective distances from their centers of gravity to the face of the wall.

The illustration of how the stresses are obtained is true for beams or frames resting on two or more supports, the stress being equal to the moment at a point divided by the depth at that point. When the lower member is a square piece of timber the distance is measured from its center and when the upper member is a rod the distance is measured to the center of the rod. That is, all distances are measured between centers of gravity of the parts, or members, of a frame, the total over-all height from the top to the bottom being equal to the distance center to center plus half the thickness of each piece.

When a frame or truss is composed of angles or other rolled shapes the distance is always measured between centers of gravity of the top and bottom chords. Thus when depth is mentioned it is not the over-all depth. The stress obtained is the stress on the center line passing through the center of gravity, the stress being slightly larger at the outer edge of the rolled section and slightly smaller at the inner edge, when there is bending. In pieces acting as plain ties or plain struts, so .the stress is pure tension or pure compression without bending stress, the stress is equal over the entire area. In a frame pieces are generally so placed that the stresses are purely tension or compression, but a beam is a solidly filled frame and the stress is greatest at top and bottom, reducing to zero on the line where the compressive force changes to tension. It is necessary then to find the position of the center of gravity of the beam on each side of the neutral plane.

The question may be asked, "Why is the depth used as a divisor?" Referring to Fig. 48 and Fig. 49 a dotted line is shown from *B* to *D*. The moment is obtained by multiplying the load by the horizontal distance from the wall. To resist this moment, which means to prop up the member *AB*, there must be some force exerted at a distance *BD* from the point *B*. That is, the bending moment and the moment to resist it are taken about the point *B*. The bending moment at $B = 8 \times 3000 = 24{,}000$ ft. lbs. This is resisted by some force acting about the point *B* with an arm $= BD$. Thus the upward pushing force in the member *AC* must equal the downward moment divided by the length *BD*. This upward force is a reaction and is compressive.

To obtain a reaction multiply the loads by the distance through which they act and divide by the span length between supports. The obtaining of tensile and compressive stresses is the same thing. First a downward bending moment is obtained and then a reaction is found by dividing, not by the span of the beam, but by the span between supports. There is an upper support to which the tension member is fastened and a lower support against which the compression member abuts. The distance between them is the span between supports. This span is the distance measured on the shortest line between the lines representing the direction of the forces, so it is perpendicular to the direction of the inclined member. For all practical purposes the lower support is at *D* and not at *C*. The member is merely carried on to the point *C*.

It is correct to multiply the load by the arm *BA* and divide by the arm *BD* in all cases, but considerable work must be done to obtain the length of the arm *BD*. This requires a knowledge of geometry and trigonometry and the use of tables of functions of angles. To obtain the length of the inclined member and use this as a moment arm and then divide by the vertical distance between the centers of gravity of the top and bottom members, is the shortest method and commonly used, for the result is correct.

The experienced structural designer uses tables of squares to lighten the labor of making the computations involved in obtaining the hypothenuse of a triangle. Barlow's Tables contain the squares, cubes, square roots and cube roots of all numbers from 1 to 10,000. Smoley's Tables for the Use of Structural Designers contain the squares of all lengths up to 100 ft., varying by sixteenths of an inch.

Moment of Resistance

In Fig. 53 is illustrated a beam loaded so there is a tendency to bend, which tendency is resisted by compression in the upper fibers and tension in the lower fibers. The beam rests on two supports. If it were a cantilever beam the compression would be in the lower fibers and the tension in the upper fibers. The two triangles show graphically the forces, the horizontal shading representing the compressive force and the vertical shading representing the tensile force. The overlapping portions of the triangles neutralize each other, and the force triangles in Fig. 54 are the result. In the case considered the beam is uniform in cross section and the stresses in tension and compression are equal; therefore the triangles are equal in size and the neutral plane where the triangles meet is in the center of area of the cross section.

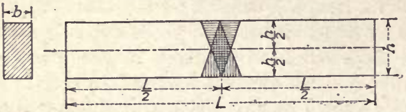


Fig. 53 — Compressive and Tensile Force Triangles in a Beam

The neutral plane is a plane in the center of gravity of the section, parallel with the upper and lower surfaces, where there is no tension or compression. The stresses above and below are opposite in nature; therefore the only stress along the neutral plane is horizontal shear. The word "neutral" implies that in this plane opposite forces are neutralized.

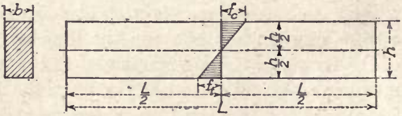


Fig. 54 — The Usual Method of Representing Action of Resisting Forces in a Beam

This definition of the neutral plane is important. In all text books the illustrations refer to beam sections of homogeneous material, with equal tensile and compressive unit stresses. Therefore the center of gravity of the beam section is the center of gravity of the area of the section. For this reason a great many students claim that the neutral plane is always in the center of gravity, which is correct, but they insist that this section of gravity is at mid-depth, which is not always correct.

The total tension must equal the total compression. When a beam is strong in compression and weak in tension, or vice versa, the neutral plane will not be in the center of area of the cross section if the beam is not proportioned for this condition, but it will be in the center of gravity. We will now explain this statement, which appears to be contradictory. Assume the stress triangles in Fig. 54 to be equal, although the tensile fiber stress may be lower than the compressive fiber stress. The fact that the two forces must balance makes the height of one triangle greater than the other, for "fiber stresses are directly proportional to their distance from the neutral axis." We therefore have two triangles of stress, one with a wide base (high fiber stress) and the other with a narrow base (low fiber stress). The heights obviously must vary in proportion in order that these "stress areas" will be equal.

The illustrations are of an imaginary beam section, a thin slice, so the position of the neutral plane is represented by a line called the "neutral axis" and the triangles represent the sides of wedges across the beam. Much of the difficulty met with by students in regard to the neutral axis arises from the fact that such illustrations are used. The material is not actually stretched and shortened as indicated, the triangles being imaginary, each with a base representing a force and not a length.

The neutral axis is in the center of gravity of a section, symmetrical or unsymmetrical, provided such position is the center of gravity of a couple in the section. If the stresses are different but the section is symmetrical the neutral axis will be nearer the side having the higher stress. If the section is unsymmetrical, to balance the difference in the fiber stresses, the neutral axis will be in the center of gravity of the section and also in the center of area. An example of this is the cast iron beam. Before taking it up a definition will be given of a couple. A couple consists of two equal opposite forces acting in parallel lines about a point. One illustration is a load on a beam and the reaction at the end of the beam. Another is the tensile force in a beam balanced by the compressive force. They are equal and opposite in amount, acting at the end of an arm measuring the distance between the centers of gravity of the two forces.

It is customary to use in cast iron a tensile fiber stress of 3000 lbs. per square inch and a compressive fiber stress of 10,000 lbs. per square inch.

A cast iron beam, therefore, is made in the form of an inverted *T*, the well-known cast iron window lintel being an example in point. First the beam was proportioned and the center of gravity of the cross section found. The tensile stress being the maximum at the lower surface, the average stress multiplied by the area of the section below the center of gravity gave the tensile force (total tensile stress). The area above the center of gravity times one-half the maximum compressive fiber stress gave the total compressive force. If they were not equal a new section would be chosen and after a few trials a section would be obtained in which the area below the neutral axis times the average tensile fiber stress equaled the area above the neutral axis times the average compressive fiber stress.

This particular example is interesting because it disproves a statement frequently met with in books of a certain class, namely that "the moments of the horizontal forces on the two sides of the neutral axis must be equal." Assuming this statement to be true, the distance from the center of gravity of the tensile area to the neutral axis must equal the distance from the center of gravity of the compressive area to the neutral axis. Having located the position of the neutral axis as above described, take a moment arm from the neutral axis to the center of area of each section and multiply the area by the average stress times the moment arm. One trial will show the falsity of the statement. The force (stress) areas on each side must be equal, and the moments do not balance about the neutral axis, except when the cross section is symmetrical and the tensile fiber stress is equal to the compressive fiber stress. The moment arm is measured from the center of gravity of the *stress triangle* on one side, not the center of area, to the center of gravity of the *stress triangle* on the other side. The moments of resistance will be equal, which is quite a different statement from that which makes the moments equal on the two sides of the neutral axis.

In Fig. 54 let the small f stand for the maximum fiber stress, that is the unit stress, which is generally expressed in pounds per square inch. Then f_c = unit compressive stress and f_t = unit tensile stress. When the stresses are equal the letter f is used without a subscript.

All forces must act through the center of gravity of bodies or areas in order to effect a movement of the whole without turning it, as about an axis. The center of gravity of a triangle is one-third the distance from the base. In Fig. 54 the triangle on one side of the neutral axis has a height equal to one-half the total height of the beam, therefore is represented by $\frac{h}{2}$. The width of the triangle at the base is represented by the unit stress at that place, f. The triangle is a force triangle and the area is equal to the total force exerted on it. This is found by the ordinary rule for areas of triangles—half the base multiplied by the height—or

$$A = \frac{f}{2} \times \frac{h}{2} = \frac{fh}{4}.$$

The length j is known as the moment arm. The moment arm is the distance between the centers of gravity of the tension and compression members. The lower triangle is the tension member, if we assume the beam to be a frame, and the upper triangle is the compression member, the neutral plane being the dividing line. Since the forces act at a point and this point is the center of gravity of the member, then the total force in compression, $\frac{fh}{4}$, acts to balance the total force in tension, $\frac{fh}{4}$, with a moment arm $= 2 \times \frac{2}{3} \times \frac{h}{2} = \frac{4h}{6} = \frac{2h}{3}$.

The moment arm times the compressive (or tensile) force gives the moment of resistance per unit of breadth,

$$M_r = \frac{2h}{3} \times \frac{fh}{4} = \frac{2fh^2}{12} = \frac{fh^2}{6}.$$

This reasoning has been based on a breadth equal to 1, or unity. To make practical use of the expression the breadth, designated by b, must be introduced. This gives the expression for the moment of resistance of a beam of homogeneous material and rectangular cross section commonly seen in text books,

$$M_r = \frac{fbh^2}{6}.$$

Some writers use d (depth) instead of h (height), but as d is used these days to indicate the depth from the top of a reinforced-concrete beam to the center of gravity of the steel reinforcement (the real depth of a reinforced-concrete beam) the letter h is preferred when the total over-all height or depth of a beam is meant. In a reinforced-concrete beam the concrete below the center of the steel is used solely for bond and protection and is not considered in computations to ascertain the strength of the beam.

The moment of resistance of a rectangular beam of homogeneous material is said to be " the section modulus times the fiber stress." This means that

$$S = \frac{bh^2}{6}$$

is an expression *denoting the effect of the shape of the beam* or the resistance it offers to destruction by loading. No matter what the material or what the stress used, the effect of the shape is the same and this is called the " section modulus," the word " modulus" meaning " measure." It is, therefore, the measure of the resistance of the shape.

Every shape has a section modulus designated by S in the steel manufacturers' handbooks. The calculation of the section modulus for a beam with rectangular cross section has been given, as it is the most simple section to handle without confusing the reader with a mass of figures.

To compute the section modulus for any shape first assume some axis passing through the center of gravity. Then divide the area into any number of layers desired by lines parallel to the axis. Find the area of each layer and the distance from the axis to the center of gravity of each layer. Multiply each layer by the square of the distance from the axis to the center of gravity of the layer and add the products. Thus is obtained the Moment of Inertia, an expression *denoting the disinclination of the body to move as a whole.* The Moment of Inertia divided by the distance from the axis to the highest stressed fiber gives the Section Modulus.

For a rectangular beam the moment of inertia is

$$I = \frac{bh^3}{12},$$

and this divided by the distance from the neutral axis to the most distant fiber gives the section modulus, as follows:

$$\frac{bh^3}{12} \div \frac{h}{2} = \frac{bh^3}{12} \times \frac{2}{h} = \frac{bh^2}{6} = S,$$

the distance from the neutral axis to the skin being $h \div 2$.

The section modulus is dependent entirely upon the shape and is independent of the weight of a beam and of the strength of the material in the beam. Tables giving the section modulus when once computed are good for all time. In steel manufacturers' handbooks tables are given of the section moduli for every shape rolled, so the proper beam may be selected when the bending moment is known and the fiber stress is known.

Let M = moment (bending moment = resisting moment) in inch pounds.

S = section modulus in inches.

f = allowable maximum fiber stress in pounds per square inch.

then $M = Sf$ and $S = \frac{M}{f}$.

In many books the expression $f = \frac{Mc}{I}$ is encountered. The moment divided by the section modulus gives the fiber stress. The section modulus $= \frac{I}{c}$, in which

I = moment of inertia.

c = distance from the neutral axis to the most stressed fiber. Sometimes y is used instead of c.

Therefore $$f = \frac{M}{I/c} = \frac{Mc}{I}.$$

One method for finding the Moment of Inertia and the Section Modulus for T-sections, *L*'s, etc., is to first assume a rectangular section having dimensions equal to the extreme outside dimensions of the shape. Find the properties (*i.e.*, I and S) for this rectangular section. Next take each hollow portion considered as a smaller rectangular section and find the properties. Adding the results for each of the pieces cut away and subtracting the sum from the properties for the entire section, the properties are found for the remainder.

Example. — What is the section modulus for a hollow rectangular section having an outside width of 8 ins. and an outside depth of 12 ins., the thickness of the shell being 1 in.? Axis horizontal.

$$S \text{ (for entire section)} = \frac{bh^2}{6} = \frac{8 \times 12 \times 12}{6} = 192$$

$$S \text{ (for interior section)} = \frac{bh^2}{6} = \frac{6 \times 10 \times 10}{6} = 100$$

$$S \text{ (for metal section of hollow shape)} = \overline{92} \text{ ins.}$$

Example. — What is the section modulus for a *T*-section 12 ins. deep over-all with an extreme width of 8 ins. and with stem and flanges each ½ in. thick? Axis horizontal.

$$S \text{ (for entire section)} = \frac{bh^2}{6} = \frac{8 \times 12 \times 12}{6} = 192 \text{ ins.}$$

$$S \text{ (for section on one side)} = \frac{3.75 \times 11 \times 11}{6} = 75.625 \text{ ins., and}$$

$2 \times 75.625 = 151.25$ ins.

The S for the section = 192 − 151.25 = 40.75 ins. The fiber stress is called by some writers the " skin stress," a very good term, for it is actually the stress in the outer skin, which is assumed to have no thickness, or has an infinitesimal thickness. The stress, within the elastic limit, varies uniformly as a straight line to zero at the neutral axis. Therefore on each layer between the skin and the neutral axis the stress is less than that assumed in the computations. The total stress is equal to the average stress multiplied by the distance from the neutral axis to the skin.

In Fig. 55 two beam sections are shown with the axes at right angles and the respective moments of inertia and section moduli are also given. The moment of resistance depends upon the square of the depth, so that for two rectangular beams of homogeneous material, having the same breadth, the beam having a depth twice as great as that of the other beam has a resisting moment four times as great. It will also be much stiffer, so there will be less deflection with a deep beam. The most economical beam, considering stiffness and strength, with a rectangular cross section, has a breadth between two-thirds and three-fourths the depth.

A beam of *I*-section is possible in steel and iron because of the strength of these materials. The material is so disposed that practically all the metal highly stressed is concentrated in the flanges, the web transmitting the stresses and taking care of shear. When a beam of *I*-section is required having a depth greater than can be properly rolled, one is made of a plate having angles riveted along the edges, this being known as a plate girder. When a still deeper girder is required a latticed girder is used, this being known as a truss.

Wooden beams are made only in solid form, rectangular or round, for wood is composed of distinct fibers, many of which would be completely detached from the main fibers in shaping the section to provide broad flanges. Steel and iron will transmit stresses equally well in all directions, so while in the filleted section connecting the flange to the web there is some concentration of stresses, this has been taken care of in designing the beam. In all solid and rolled shapes the maximum fiber stress is the skin stress. In built-up sections, such as plate or latticed girders, the maximum fiber stress is assumed to cover the flange member and the whole action is on the line passing through the center of gravity of the flange. The stress is transmitted from the web, or the web members, to the flange through rivets, which must be properly proportioned in size and properly spaced to take care of the shear.

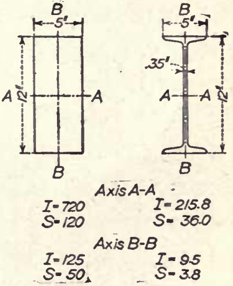


Fig. 55 — Comparison of Moments of Inertia and Section Moduli in a Rectangular Beam and in an I-beam

Elastic Limit

Within the limit of strength known as the "elastic limit," all materials may be stressed a number of times and recover their original dimensions. The elastic limit is a stress where the material is permanently deformed and stress in excess of the elastic limit causes rapid deformation. Up to the elastic limit the stress-strain curve is straight, but it curves after the elastic limit is passed. Fig. 56 is a stress-strain diagram of steel, iron, and wood and gives a good idea of the relative strengths and deformations. The illustration is copied from "Materials of

Construction," by the late Professor J. B. Johnson. His description of the apparent elastic limit is that when it is reached the deformation for each increment of stress is about double the deformation for the increment immediately preceding. The curve becomes a parabola, or is very nearly parabolic.

All the statements made about the moment of resistance of beams are true only within the elastic limit of the material, for they are based on "Hooke's Law" that "Stress is proportional to strain." When the elastic limit is reached the law is no longer true. A generation ago when steel was stressed to 16,000 lbs. per square inch it was said to have a factor of safety of 4, based on the ultimate strength of the steel, 64,000 lbs. per square inch. To-day the factor of safety is based on the elastic limit, and as this varies between 29,000 and 36,000 lbs. per square inch, averaging about 32,000 lbs., depending upon the hardness of the steel, the factor of safety is said to be 2, based on the elastic limit. There is a permanent set after the elastic limit is passed and a progressive weakening, even though the material may not fail until it is stressed up to four times the allowable working fiber stress.

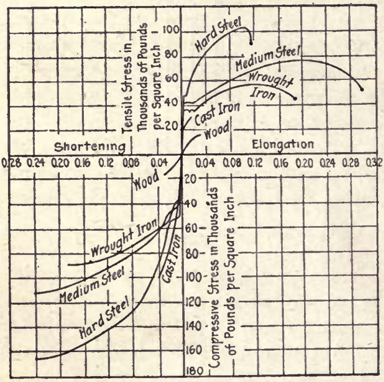


Fig. 56 — Stress-strain Diagram of Steel, Iron, and Wood

Modulus of Elasticity

The modulus of elasticity is a number obtained by dividing the stress by the deformation it causes. If it were a force it would be defined as a force which will stretch a unit piece of material to twice its length or compress it one-half. It is not a force, nor is it a pure number, for it is expressed in pounds.

A certain steel bar having an area of one square inch was stressed in tension 16,000 lbs. and the stretch carefully measured was found to be 0.000534 the length. What was the modulus of elasticity?

$$\frac{16,000}{0.000534} = 30,000,000 \text{ lbs. (in round numbers).}$$

In the example the bar was one inch square. It may make it more clear if the modulus of elasticity is defined as the ratio found by dividing the unit stress by the unit deformation, or unit strain. Stress is a force and strain is the deformation produced by a force.

The modulus of elasticity is used to compare the relative deformation of materials which must act together. Steel may be said to have a modulus of elasticity of 30,000,000. Concrete is made of so many different mixtures and the workmanship varies so greatly between specimens that an average value of the modulus of elasticity must be taken for each mixture. The average value of 1:2:4 concrete is 2,000,000 and the ratio between the moduli of elasticity of structural grade steel and 1:2:4 concrete is taken to be $\frac{30,000,000}{2,000,000} = 15$. The writer a few years ago called this the "ratio of deformation" instead of the "ratio between moduli of elasticity," and his term is very commonly used now. To give a clear explanation of the matter assume a piece of concrete with a unit cross-sectional area and beside it a piece of steel with the same area. An equal load is placed on each piece and the shortening measured. It will be discovered that the steel shortened one-fifteenth as much as the concrete, therefore the ratio of deformation = 15. Concrete can really have no modulus of elasticity, for it is a brittle material with an elastic limit very difficult to measure. When, however, concrete is tested it shows enough consistency in deformation to warrant the adoption of a value for the modulus of elasticity by means of which a workable ratio of deformation may be obtained.

Reinforced Concrete Beams

Whereas in beams of a uniform material there is a gradual and uniform increase in stress from the neutral axis to the top and bottom, in beams of reinforced concrete this is true only of the upper portion of the beam. Roughly, the tensile strength of concrete is about one-tenth the compressive strength. Since in a beam the tensile and compressive forces must be equal, the neutral axis in a beam of plain concrete under load will be very high.

The tensile stress is low and the compressive stress is high, so the height of the tensile force triangle will be greater than the height of the compressive force triangle. Steel is placed near the bottom of a reinforced concrete beam to give it increased tensile strength, and this lowers the neutral axis. The concrete is not relied upon to furnish any tensile strength, this being concentrated in the steel. The concrete below the neutral axis is therefore used only to furnish shearing strength and protect the steel from corrosion.

The total tensile stress is considered as being carried by the steel, and the compressive stress is carried by the concrete above the neutral axis, where the variation in fiber stress follows the

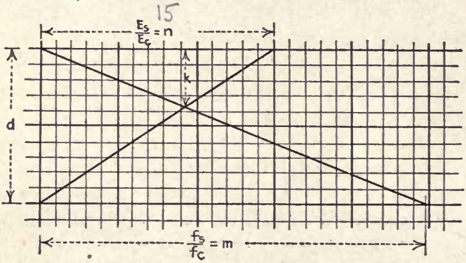


Fig. 57

straight line law already considered. Actually, the compressive stress varies as a parabola and not as a triangle, but to use the triangle is safe and the " straight-line method " alone is permitted in building ordinances and in all regulations issued by responsible officials in this and other countries.

To properly treat reinforced concrete design will require a book, and the author has one in preparation which will shortly follow the present book, and replace his " Reinforced Concrete, a Manual of Practice," written in 1907 and now out of print. The method he uses for determining the position of the neutral axis in a reinforced concrete will be given here, together with his method for determining the percentage of steel. The methods, and the resulting formulas, he believes to be original, as they have never appeared in any book to his knowledge, or to the knowledge of a large number of teachers and consulting engineers to whom he wrote about the matter.

We have in reinforced concrete a material (steel) stressed in tension which is from 20 to 30 times stronger than the compressive strength of the concrete. Furthermore, the weaker material has a deformation under load 15 times greater than the deformation of the stronger material.

Let E_s = modulus of elasticity of the steel.

E_c = modulus of elasticity of the concrete.

n = ration of deformation $= \frac{E_s}{E_c}$ (usually 15).

f_s = allowable fiber stress (working stress) in the steel.

f_c = allowable fiber stress (working stress) in the concrete.

m = stress ratio $= \frac{f_s}{f_c}$.

d = depth from top of concrete beam to center of gravity of the steel reinforcement.

k = depth from the top of the beam to the neutral axis (expressed as a percentage of d).

j = moment arm expressed as a per cent of d. When the value of d is given in inches then the moment arm is jd.

$$j = 1 - \frac{k}{3} \text{ and } jd = d - \frac{dk}{3}.$$

Referring to Fig. 57; on a piece of squared paper set off ten units vertically. Measuring to the right along the top lay off n. Measuring to the right along the bottom lay off m. Connect the ends as shown to form two triangles which cross at the neutral axis. The depth, k, to the neutral axis may then be measured on the paper. The two triangles show an obvious geometrical relation which enables us to find k by computation, thus:

$$k = \frac{n}{n + m}.$$

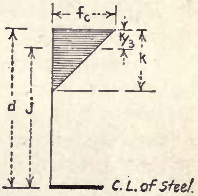


Fig. 58

Fig. 58 illustrates the triangle of compressive force. The area of the triangle $= \frac{f_c}{2} \times k$. The tensile force is equal to the area of the steel times the fiber stress, so the area of the steel will be obtained by dividing the compressive force by the steel fiber stress. Units are used throughout, so the steel area will be the ratio between the concrete and the steel. Then,

$$p \text{ (steel ratio)} = \frac{\frac{f_c k}{2}}{f_s} = \frac{f_c k}{2f_s} = \frac{k}{2} \times \frac{1}{\frac{f_s}{f_c}} = \frac{k}{2} \times \frac{1}{m} = \frac{k}{2m}.$$

Another expression for p is found as follows: The unit moment of resistance is expressed by a Resistance Factor, R. For the steel this is R_s and for the concrete this is R_c.

$$R_c = \frac{f_c}{2} kj \text{ and } R_s = pjf_s.$$

$$p = \frac{R_c}{jf_s}.$$

The moment of resistance of a concrete beam = Rbd^2, in which R is the resistance factor for the "balanced" beam, that is, a beam in which the tensile force exactly equals the compressive force. Fig. 59 is a chart for obtaining R and p for different stresses in steel and concrete. The stresses recommended by the Joint Committee on Concrete and Reinforced Concrete are 16,000 lbs. per square inch for the steel and 650 lbs. per square inch for 1:2:4 concrete.

For rectangular reinforced-concrete beams (and for slabs with width b, of 12 ins.) the following formulas are used, the moment being in inch pounds and all beam dimensions in inches.

$$M = Rbd^2$$

$$b = \frac{M}{Rd^2}$$

$$d = \sqrt{\frac{M}{Rb}}.$$

When the moment is in foot pounds, b will be in feet and d in inches, or b and d will be in inches and R will be divided by 12.

The Portland Cement Association, Chicago, Ill., is an organization supported by the cement manufacturers of the United States and Canada for the purpose of disseminating information about cement and concrete. Every man in the building business should have his name and address on the mailing list, for some of the Association bulletins deal with questions of design, while all the bulletins should be on file in the office of every one who has anything to do with construction work.

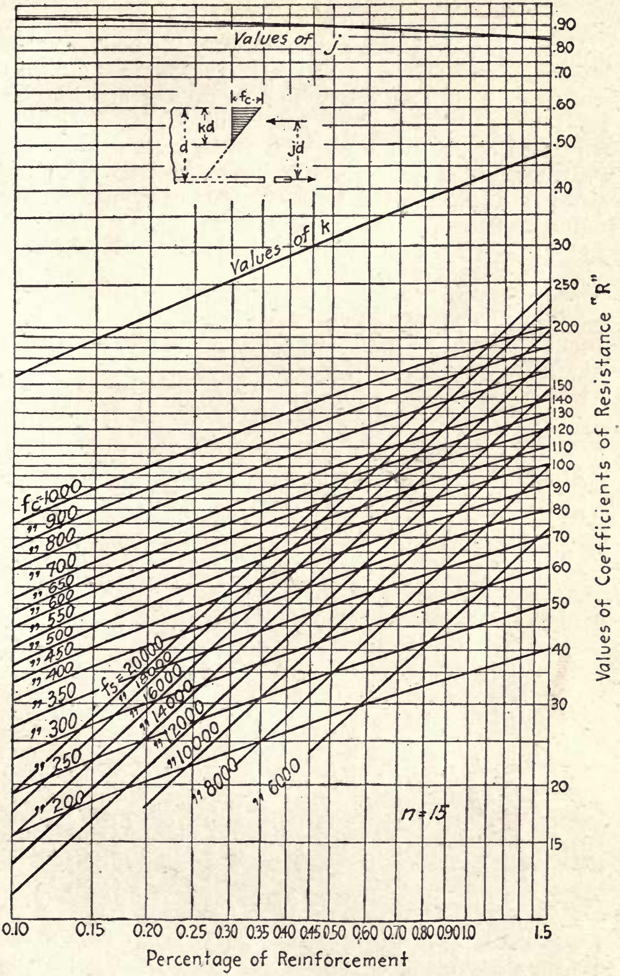


Fig. 59—Coefficients of Resistance of Reinforced Concrete Beams

Shearing Resistance

A beam may be strong enough to carry the load without a bending failure, which will crush the fibers at the top or pull them apart at the bottom, yet it may fail in shear. The direct shearing stress at any section on a beam is found by dividing the shear at the section by the area of the beam at the section. The direct shear, however, is seldom operative, this action being best represented by a punch making holes in a plate or by a large shear cutting a plate. The shearing stress which breaks a beam is diagonal tension resulting from the combined action of the horizontal and vertical shearing stresses.

The direct vertical end shear is equal to the maximum reaction. The horizontal shear is equal in amount and acts along the neutral plane where the fiber stress in bending changes from tension to compression, the stress being in reality a sliding of the fibers where they have no bending stress. The diagonal tension is the component of these two actions. (Fig. 60.) Referring again to the statement that the area of the shear diagram between any section and the nearest support, for a beam resting on two, or more, supports, equals the bending moment at the section, the unit shear at any section amounts to

$$s = \frac{V}{jd} = \frac{M_1 - M_2}{jd},$$

in which s = unit shear in pounds per square inch,
V = shear at the section in pounds,
M_1 = moment in inch pounds at one side of section,
M_2 = moment in inch pounds at the other side of section,
jd = moment arm in inches.

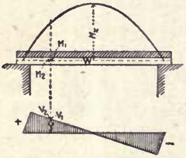


Fig. 60 — Relation between Moment and Shear

The allowable shearing stress in steel is 10,000 lbs. per square inch. After obtaining the size of beam to carry a certain load, divide the maximum reaction by the web thickness multiplied by the depth of the beam. This will give the shearing stress in pounds per square inch. If it exceeds 10,000 lbs. a larger beam is required. In the steel handbooks the total amount of shear for which a beam is safe is given in the tables of "Properties of Sections." For example, in taking out from the tables a beam of sufficient size to carry the load, the total amount of shear the beam is good for should be equal to or exceed the maximum reaction. Thin webs act like long slender columns and may fail by crippling. The crippling strength of the beams is also given in the tables and this should be equal to or exceed the maximum reaction.

Fig. 61 is a very old illustration used by many writers to explain shearing action in a beam. Let (*a*) represent a beam assumed to be composed of a number of planks not fastened together. When loaded the planks bend and slide on each other as shown. This sliding action is horizontal shear, which is zero at the top and bottom edges and a maximum along the neutral plane where the tensile stress changes to compressive stress.

Fig. 61 — Illustration of Horizontal Shear

Spike the planks together (*b*) and they will not separate when the beam bends under load. The sliding stress is pure shear on the spikes connecting the planks. The spikes act by bearing on the planks into which they are driven, and in this manner some tension is carried from outer to inner planks. The resultant force is called diagonal tension.

Imagine a beam of any material divided into a great number of horizontal layers. Along the imaginary joints shear exists which is resisted by the tensile strength of the material. In beams of steel or iron, in which materials the tensile strength is equal in all directions, the diagonal tension thus developed may be strong enough to tear the web along a diagonal line extending upward from the support. The web must be thick enough to resist the diagonal shear or, in the case of a plate girder, be strengthened by stiffeners.

Reinforced-concrete beams fail similarly in diagonal shear. This may be resisted by making the stem of the beam thick or, if it is desirable to use little concrete in the stem, stirrups are added to resist diagonal tension. There are other theories which endeavor to account for the diagonal cracks which appear sometimes in reinforced-concrete beams, but the generally accepted method for proportioning stirrups is based on shear expressed as diagonal tension, and since the desired result is accomplished and the computations are readily performed this method it is believed will persist.

Wood is composed of actual horizontal fibers, instead of the imaginary fibers, or horizontal planes, considered in analyzing shear in beams of homogeneous material. If wood were equally strong in all directions, a shearing failure in this material would also be indicated by the appearance of diagonal cracks.

Shear in Wooden Beams

In wooden beams the dangerous shear acts along the neutral plane and the beam may split, thus by shearing action being converted into two shallow beams, which will then break by bending, for the upper half must carry the whole load and the lower half carries the whole load when the upper half is destroyed. The strength in shear of wooden beams should be tested by the following formula. If the distributed load found by this formula is smaller than that found by the bending formula, increase the size of the beam.

$$W = \frac{4hbs}{3},$$

in which W = the load the beam will carry without failing in shear.
b = breadth in inches,
h = height in inches,
s = shearing stress per square inch, usually one-tenth the maximum fiber stress in bending.

The above formula is derived as follows:

$$V = \frac{W}{2} \text{ and } s = \frac{V}{jhb} = \frac{\frac{W}{2}}{\frac{2}{3}hb} = \frac{W}{2} \times \frac{3}{2hb} = \frac{3W}{4hb}$$

and, therefore

$$W = \frac{4hbs}{3}.$$

Modulus of Rupture

The modulus of rupture is a measure which represents a combination of all the forces that tend to break a beam; *i.e.*, the combined action of tension compression, shear, and crippling. It was formerly used in beam design to obtain the breaking load, which was divided by some factor of safety to determine the safe load. To-day it is used only for materials in which it is difficult to separate the different stresses, as, for example, clay, stone, and plain concrete. The moment of resistance, using an allowable safe fiber stress, is used in computations for beams of wood, steel, iron, and reinforced concrete.

The unit moment of resistance is a number which contains all the known quantities in an expression, leaving only the unknowns to be found. For example, the moment of resistance of a wooden beam in which we can use a maximum fiber stress of 1200 lbs. per square inch is

$$M_r = \frac{1200\, bh^2}{6},$$

and by dividing the fiber stress by 6 the unit moment of resistance equals 200, from which we get

$$M_r = 200\, bh^2 = Rbh^2.$$

Some men use R for wooden beams, but where the divisor is so small the only advantage is some slight simplification of the work, provided a table of values of R has been previously computed for the woods used. In reinforced-concrete work a number of factors enter into the formula for the resisting moment and the use of a table, or of a diagram which is really a graphical table, for all possible values of R is almost indispensable for the designer. Where a number of factors enter into a computation it is easy to forget to use some.

Deflection

The amount of deflection when a beam is loaded is measured on the bottom or top of the beam for convenience. The difference in elevation between the end of the beam and the middle is the deflection. The deflection actually used in computations is the deflection at the neutral axis, but the deflection measured on the bottom or top, which for obvious reasons is more readily obtained than the deflection of the neutral plane, is close enough for all practical purposes.

Deflection in beams and girders used in buildings is important only when the lower side carries a plastered ceiling. The deflection is limited to a maximum of one-three-hundred-and-sixtieth of the span to prevent cracks in the plaster. A greater deflection is not unsightly and is permissible when constant. Wood and steel beams straighten when the load is relieved and deflect when the load is increased. It is the movement that causes plaster to crack, so this must be limited. For beams and trusses under moving loads the deflection must be limited to an amount which will not set up dangerous vibrations, but with this the ordinary structural designer seldom has to deal, it being part of the work involved in the design of bridges. Deflection also affects appearance and a camber is given to trusses to hide deflection.

A beam may be amply strong so that it will not fail by bending, shearing, or crippling, and yet the deflection may be so great that it will not be suitable for use in the proposed location. The amount of deflection must then be found and if it exceeds the allowable deflection a deeper beam must be substituted. When using steel it is often possible to secure a deep beam which will weigh less than a beam of less depth of practically equal strength in bending and shear. For timber, experience indicates that the most economical beam, considering the two factors of strength and stiffness, has a breadth equal to two-thirds or three-fourths the depth.

Deflection Formulas

Deflection formulas as usually presented are formidable in appearance, so tables are given in the steel handbooks which enable the deflection in inches to be found by dividing a factor in the table by the depth of the rolled section in inches.

Similar information for wooden beams was not so readily obtainable until in 1913 the Yellow Pine Manufacturers' Association issued a book entitled "A Manual of Standard Wood Construction," following the lines laid down previously by the steel manufacturers in their handbooks. Copies of this book may be obtained from the secretary of the above association in St. Louis, Mo.

The "Structural Timber Handbook, for Pacific Coast Woods" is issued by the West Coast Lumbermen's Association, Seattle, Wash., and valuable books on the subject of wood design may be obtained from the National Lumber Manufacturers' Association, Chicago, Ill.

The complicated formulas for deflection are made to appear as follows, after certain substitutions and transformations of factors:

$$D = \frac{30 f L^2}{E h},$$

in which D = deflection in inches,
f = allowable maximum fiber stress in bending,
L = length in feet,
E = modulus of elasticity,
h = height of beam in inches.

Assuming common values:

For steel, f = 16,000 lbs. per square inch, $D = \frac{L^2}{60\,h}$.

For wood, f = 1300 lbs. per square inch, $D = \frac{L^2}{41\,h}$.

For wood, f = 1000 lbs. per square inch, $D = \frac{L^2}{44\,h}$.

For wood, f = 800 lbs. per square inch, $D = \frac{L^2}{46\,h}$.

See page 98.

Aids to Computation

In addition to the handbooks of the steel companies and the "Manual of Standard Wood Construction," designers use diagrams and slide rules to lighten their work on simple problems. The following are suggested in this connection:

The Wager timber scale for computing the strength of wooden beams, $1.

The Merritt beam scale for computing the strength of steel beams, $1.

Des Moines Bridge & Iron Company's calculator for steel beams, channels, angles, and tees, 25 cents.

The two first mentioned are made of heavy paper and the third is of celluloid. The writer has used them daily in his work for some years.

The most complete rule for this work is one designed by Benjamin Winslow. It enables one to design with any fiber stress, any span, any spacing, any system of loading, etc. The rule is made of German silver and costs $10. The size is 3½ ins. × 10½ ins. × 16 ins. Taking the place as it does of all pocket books, tables and diagrams, the writer feels that to omit recommending it to structural draftsmen and designers would be a neglect on his part of a plain duty. Mr. Winslow has also placed on the market a similar slide rule for reinforced-concrete design.

Slide rules of the Mannheim type are used to-day by all engineers, but a recent improvement is known as the Phillips slide rule. This rule enables one to multiply three factors at one setting and the arrangement of the graduations wonderfully increases the value of the slide rule for all purposes. This new rule sells for $5.

Example. — Determine the size of a wooden beam using a maximum fiber stress of 1000 lbs. per square inch to carry a uniformly distributed load of 6000 lbs. on a span of 14 ft.

Answer. — Assume the beam to weigh 20 lbs. per linear foot $= 14 \times 20 = 280$ lbs. The total load $= 6280$ lbs.

$$M = \frac{6280 \times 14 \times 12}{8} = 1.5 \times 6280 \times 14 = 131{,}880 \text{ in. lbs.}$$

Assume a depth of 12 ins., which will give a beam 11.5 ins. the usual depth of a commercial size 12-in. beam.

$$M_r = \frac{fbh^2}{6} = 167\ bh^2 = 131{,}880 \text{ in. lbs.}$$

$$b = \frac{M_r}{167\ h^2} = \frac{131{,}880}{167 \times 11.5^2} = 5.9 \text{ ins.}$$

This calls for a commercial size beam 7 ins. $\times$ 12 ins., the actual size of which will probably be about 6.5 ins. $\times$ 11.5 ins. The weight per linear foot, assuming wood to weigh 35 lbs. per cubic foot, will be $\frac{6.5 \times 11.5 \times 35}{144} = 18.2$ lbs. This is so close to the weight assumed that we will let it stand.

Investigate for shear.

$$W = \frac{4bhs}{3} = \frac{4 \times 6.5 \times 11.5 \times 100}{3} = 9967 \text{ lbs.}$$

The load of 6280 lbs. is therefore safe.

The deflection is to be kept below $\frac{1}{360}$ of the span

$$= \frac{12 \times 14}{360} = 0.466 \text{ in.}$$

$$D = \frac{L^2}{44\ h} = \frac{14 \times 14}{44 \times 11.5} = 0.38 \text{ in.} = \frac{L}{443}.$$

Find the span on which the safe bending load is equal to the safe shearing load.

$$L = \frac{8\ M}{12\ W} = \frac{143{,}500}{1.5 \times 9967} = 9.67 \text{ ft.}$$

This beam cannot be safely loaded with more than 9967 lbs. on any span of less than 9 ft. 8 ins. no matter what the safe load in bending may be. (The moment used here is the actual resisting moment of the beam which had to be selected to carry the load, that is, $M = 167 \times 6.5 \times 11.5^2 = 143{,}500$ in. lbs., the actual bending moment as we have seen being 131,880 in. lbs. It is cheaper to use a commercial beam with a resisting moment larger than the bending moment than to trim the beam down to the theoretically exact size. This happens with rolled steel beams also. When a built-up plate girder or a latticed girder (truss) is used the difference between the bending moment and resisting moment can be cut to a smaller amount. Reinforced concrete is a material which permits of closer designing than rolled shapes, hence the differences in design shown by equally competent designers tackling the same problem when using reinforced concrete.

A formula to find the limiting span when bending and shear are considered is developed as follows for wood: M in inch pounds.

$$L = \frac{8M}{12W} = \frac{M}{\frac{1.5 \times 4bhs}{3}} = \frac{3\,M}{6bhs} = \frac{M}{2bhs}.$$

Find the deflection on the limiting span.

$$\text{The allowable deflection} = \frac{9.67 \times 12}{360} = 0.323 \text{ in.}$$

$$\text{The actual deflection} = \frac{L^2}{44h} = \frac{9.67 \times 9.67}{44 \times 11.5} = 0.184 \text{ in.}$$

Find the allowable safe uniformly distributed load the beam will carry on a span of 20 ft.

$$W = \frac{8\,M}{12L} = \frac{143{,}500}{1.5 \times 20} = 4785 \text{ lbs.}$$

$$\text{Allowable deflection} = \frac{12 \times 20}{360} = 0.667 \text{ in.}$$

$$\text{Actual deflection} = \frac{20 \times 20}{44 \times 11.5} = 0.79 \text{ in.}$$

The deflection is too great if the lower side of the beam is to be plastered, or the beam is to carry a plastered ceiling.

NOTE. — When a wooden beam has a depth in inches less than two-thirds the span in feet the deflection is apt to cause plaster to crack. Try a beam 14 ins. deep, the actual depth being 13.5 ins.

$$b = \frac{M_r}{167h^2} = \frac{143{,}500}{167 \times 13.5^2} = 4.71 \text{ ins.}$$

Try a commercial 6 ins. × 14 ins. = 5.5 ins. × 13.5 ins.

$$M_r = 167 \times 5.5 \times 13.5^2 = 167{,}500 \text{ in. lbs.}$$

This beam is seen to be excessively strong, but a beam 4.5 ins. × 13.5

ins. would have a resisting moment of only 137,000 ins. lbs. Allowable deflection = 0.667 ins. Actual deflection

$$= \frac{20 \times 20}{44 \times 13.5} = 0.6675 \text{ ins.}$$

The deflection in the formulas presented is dependent upon the stress, so the deflection found is that produced when the beam is fully stressed, that is, when the full resisting moment of 167,500 in. lbs. is developed. Under the load found for the 20-ft. span the moment is only 143,500 in. lbs., so the deflection will be less than that given.

This case may be dealt with as follows if it is desired to find the actual deflection. The divisor for the span squared is 41 for a fiber stress of 1300 lbs. per square inch, 44 for a fiber stress of 1000 lbs. per square inch, and 46 for a fiber stress of 800 lbs. per square inch. The divisor is seen to alter by 1 for each 100 lbs. change in unit fiber stress. Find the maximum fiber stress for the bending moment developed and then applying the proper divisor ascertain the actual deflection.

$$f = \frac{143{,}500}{\frac{bh^2}{6}} = \frac{6 \times 143{,}500}{5.5 \times 13.5^2} = 859.2 \text{ lbs. per square inch.}$$

The divisor for all practical purposes is 45.6.

$$D = \frac{L^2}{45.6h} = \frac{20 \times 20}{45.6 \times 13.5} = 0.65 \text{ ins.}$$

The load this beam can carry on a 20-ft. span with a deflection equal to 0.6675 in. is

$$W = \frac{8M}{12L} = \frac{167{,}500}{1.5 \times 20} = 5600 \text{ lbs.}$$

All the computations have been made with a slide rule, so in some cases the terminal figures in the results may differ slightly from those found by arithmetical computations, but when dealing with large quantities small differences in the units place make no material difference in results.

The calculations for deflection in wooden beams can never give exact results, for woods vary in texture throughout and the amount of moisture and seasoning also act to increase or decrease deflection.

CHAPTER III

Problems in Design of Beams

THE two standard steel handbooks are the "Carnegie Pocket Companion" and the "Cambria Steel Manual." The designer should have one or both of these books. The Bethlehem Steel Company issues a handbook which the designer should also possess, owing to the differences in shape and carrying capacity of the Bethlehem and standard beams.

The Cambria and Carnegie handbooks contain a great deal of text book matter and are very useful to students and to men who wish occasionally to refresh their memories on points of design. They contain the usual tables indispensable to structural designers. The handbook of the Lackawanna Steel Company and that issued by Jones & Laughlin contain the indispensable tables and some memory aiding text, but not as much as the first books mentioned.

For a uniformly distributed load the size of a beam is easily obtained. Tables give the uniformly distributed loads in pounds for all spans, varying by single feet which the different beams can carry. By reducing concentrated loads to their equivalents in uniformly distributed loads these tables may be used for any system of loading without first ascertaining the bending moment.

When concentrated loads are dealt with as such and the bending moments are found, the proper size beam may be found by looking up the bending moment in foot pounds, opposite which, on the same line, is found the size and weight of the beam. Beams must be secured (stayed) laterally to prevent side bending; otherwise the carrying capacity is less than that given in the tables.

The Carnegie book formerly gave a factor of strength, *C*, to use when the bending moment was used. It is designated as *C* in the Bethlehem book and as *F* in the Cambria book. In the 1913 edition of Carnegie this factor is not given, the bending moment in foot pounds being shown on the page containing the other properties of beams.

The factor of strength is obtained as follows:

The fiber stress is in pounds per square inch and the section modulus is in square inches, therefore the resisting moment is in inch pounds, or 12 times the bending moment in foot pounds. Two-thirds the moment in inch pounds is equal to 8 times the bending moment in foot pounds; which, in turn is equal to the total uniform load in pounds times the span in feet, for a freely supported beam.

Let S = section modulus in inches.

f = maximum fiber stress in lb. per sq. in.

Then $C = F = \frac{2}{3}fS$.

Let M = bending moment in foot pounds.

Then $C = F = 8M$.

Having computed the bending moment in foot pounds, multiply by 8 and in the table of properties of beams look for this value, or the nearest higher value, of F (or of C) in the Cambria or Bethlehem book. Following the line to the right, the beam is found which has this factor of strength. Each of the books mentioned contains a separate table of bending moments in foot pounds for each beam, so the designer has his choice of methods to use in obtaining a beam size when he has the bending moment instead of the uniformly distributed load.

Example. — A beam carrying several concentrated loads must resist a bending moment of 46,680 ft. lbs. What is the best size and weight of beam to use?

Carnegie (1913 edition): On page 184 it is shown that the resisting moment of a 12-in. I-beam weighing 31.5 lbs. per lin. ft. = 47,960 lbs., so this beam will be used.

Page 182 contains a description of all the factors shown on page 184, relating to the properties of beams. The student is now prepared to study pages 133, 140, 141, 164, 167 to 171 inclusive, 176 to 182 inclusive.

Cambria (1913 edition): On page 118 it is shown that a 12-in. I-beam weighing 31.5 lbs. per lin. ft. has a resisting moment of 48,000 ft. lbs.

The following pages should be studied by the student, 76, 77, 80 to 89 inclusive, 142 to 147 inclusive, 158 to 163 inclusive.

"Lackawanna Hand Book" (1915 edition): This book does not contain a table of bending moments for standard beams so the bending moment in foot pounds must be multiplied by 8 and the tables on pages 164 to 167 consulted. The size of beam is given in Col. 12 on page 167 and we find that a 12-in. I-beam, weighing 31.5 lbs. per lin. ft., will be required. On the pages mentioned is a column containing distances center to center of beams required to make the radii of gyration equal; a very useful table to use when designing columns.

In this book the student should read carefully pages 144 to 163 inclusive.

Jones & Laughlin, "Standard Steel Construction" (1916 edition: This book does not contain a table of bending moments for I-beams, neither does it contain a table of factors, C. or F. Our problem is solved as follows; Since the moment divided by the fiber stress equals the Section Modulus, divide the bending moment in foot pounds by the fiber stress, 16,000 lbs., and this gives the section modulus in feet. Multiply by 12 to obtain the section modulus in inches. Look up this value on pages 105–106. Proceeding in this fashion we get $(46{,}680 \div 16{,}000) \times 12 = 35$ in. $= S$. On page 106 the nearest value is 36, corresponding to a 12-in. I-beam weighing 31.5 lbs. per lineal foot.

The student should now become familiar with pages 95 to 175 inclusive, and with page 243 in this book.

Bethlehem (1911 edition): On page 38 a 9-in. girder-beam weighing 38 lbs. per lin. ft. has a resisting moment of 50,630 lbs. On page 39 a 12-in. Bethlehem I-beam weighing 28.5 lbs. per lin. ft. has a resisting moment of 48,050 ft. lbs.

To understand why the Bethlehem beams are stronger than standard I-beams of equal depth, read pages 3 to 9 inclusive. Then study pages 30, 31, 56, 66, 68, 99 to 103 inclusive.

In studying the pages mentioned the student should work examples in order to become familiar with the use of the tables. The tables of deflection factors should be thoroughly understood, which is not a difficult matter if the remarks on deflection in this chapter have been given proper attention.

After thoroughly mastering the subject matter on the pages enumerated the student should study pages 283 to 292 inclusive in Carnegie; 56 to 71 inclusive in Cambria; 104 to 107 in Bethlehem. The pages mentioned in each book cover the same subjects, so it is not necessary to use the three books, one giving all that is necessary. Should the student, however, possess the three books, it will be well to study the subjects thoroughly in one and then become familiar with the similar matter presented in the others.

Only rolled shapes have been considered so far. Compound shapes, *i.e.* plate girders and trusses, will be taken up later.

Practical Problems in Design

1. Find the resisting moment of flooring $\frac{5}{8}$ in. thick; $\frac{7}{8}$ in. thick; $1\frac{1}{8}$ in. thick; $1\frac{3}{4}$ ins. thick.

Answer.—$\frac{5}{8}$ in. = 0.625 in.; $\frac{7}{8}$ in. = 0.875 in.; $1\frac{1}{8}$ ins. = 1.125 ins.; $1\frac{3}{4}$ ins. = 1.75 ins. The width will be taken as 12 ins., as floor loads are generally given in pounds per square foot. The flooring is white pine having a fiber stress of 800 lbs. per square inch. [In all problems it is understood that by fiber stress is meant the maximum (skin) stress.] The unit moment of resistance = 800 ÷ 6 = 133.33.

$$M_r = 133.33 \times 12 \times 0.625^2 = 625 \text{ in. lbs.}$$
$$M_r = 133.33 \times 12 \times 0.875^2 = 1225 \text{ in. lbs.}$$
$$M_r = 133.33 \times 12 \times 1.125^2 = 2025 \text{ in. lbs.}$$
$$M_r = 133.33 \times 12 \times 1.75^2 = 4900 \text{ in. lbs.}$$

2. What is the greatest spacing permissible between joists if the deflection is to be limited the usual amount?

Flooring comes in long pieces and thus, extending over a number of supports, to each of which it is nailed, the thickness can be equal in inches to one-half the span in feet. This gives a maximum span for the $\frac{5}{8}$-in. of 2 × 0.625 = 1.25 ft. (15 ins.); $\frac{7}{8}$-in., 2 × 0.875 = 1.75 ft. (21 ins.); $1\frac{1}{8}$-ins., 2 × 1.125 = 2.25 ft. (27 ins.); $1\frac{3}{4}$-ins., 2 × 1.75 = 3.5 ft. (42 ins.).

Floors generally have greater stiffness than is here shown because of the tongue and groove along the edges, but this is frequently nullified by the fact that the loads brought on floors are more often concentrated than uniformly distributed. The above rule for deflection is arbitrary, and if the spans mentioned are actually used it will be well to check the deflection by a proper formula. Refer to the table of relative strength and stiffness of beams. The deflection formula gives deflection for uniform loads on beams resting freely on two end supports. First find the deflection by the formula and multiply it by the constant found in the column of relative deflections, opposite the condition of loading to which the case under consideration may apply.

3. Neglecting deflection, what is the greatest permissible spacing of joists for the following loads per square foot (including the weight of the flooring): 42 lbs.; 78 lbs.; 103 lbs.; 129 lbs.?

Flooring extends over several supports, so we may assume a condition of restraint and use the formula

$$M = \frac{wL^2}{12}, \text{ in foot pounds.}$$

The load is given in pounds per square foot, so the span should be in feet. The formula then becomes

$$M = \frac{wL^2 12}{12} \text{ in. lbs.}$$

which reduces to $M = wL^2$ in. lbs.

Similarly, for $M = \frac{wL^2 12}{8}$ in. lbs.

we obtain $M = 1.5wL^2$ in. lbs.

Another condition sometimes met with in wood and steel design and frequently used in reinforced concrete design is a partially restrained condition in which the beam rests freely on one end support and is fully restrained at the other support. For this condition the coefficient is 10 and

$$M = \frac{wL^2 12}{10} \text{ in. lbs., or } M = 1.2wL^2.$$

Using the expression $M = wL^2$, the spans for the various floor thicknesses are found as follows:

$$L^2 = \frac{M}{w}, \text{ or } L = \sqrt{\frac{M}{w}}.$$

Using the resisting moments in inch pounds obtained for each thickness,

$$\tfrac{5}{8}\text{-in. flooring: } L = \sqrt{\frac{625}{42}} = 3.85 \text{ ft.}$$

$$L = \sqrt{\frac{625}{78}} = 2.83 \text{ ft.}$$

The rest of the examples are left to the student as a useful exercise.

4. A floor is constructed of 2-in. (1.75-in.) planking laid over beams spaced 4 ft. 6 ins. center to center, the span of the beam from wall to girder being 18 ft. Find size of beam when the total load

per square foot, including weight of beam and floor, is 132 lbs. per square foot. Material yellow pine with an allowable fiber stress of 1300 lbs. per square inch. Deflection ignored.

Answer. — The total load on the panel is $132 \times 4.5 \times 18 = 10{,}700$ lbs.

$$M = 1.5 \times 10{,}700 \times 18 = 288{,}900 \text{ in. lbs.}$$

Try an 8-in. × 14-in. beam (7.5 ins. × 13.5 ins.)

$$M_r = \frac{1300 \times 7.5 \times 13.5^2}{6} = 296{,}156 \text{ in. lbs.}$$

Try for shear

$$W = \frac{4bhs}{3} = \frac{4 \times 7.5 \times 13.5 \times 130}{3} = 18{,}550 \text{ lbs.}$$

W. N. Twelvetrees, a British engineer, developed the following method for designing a beam in which the breadth is to be some definite proportion of the depth.

Let $n = \frac{h}{b}$, then $b = \frac{h}{n}$.

To design so $b = \frac{h}{2}$ $\qquad n = \frac{h}{\frac{1}{2}h} = \frac{1}{0.5} = 2$

$$b = \frac{2h}{3} \qquad n = \frac{h}{\frac{2}{3}h} = \frac{1}{0.67} = 1.5$$

$$b = \frac{3h}{4} \qquad n = \frac{h}{\frac{3}{4}h} = \frac{1}{0.75} = 1.33$$

$$M = Rbh^2 = R \times \frac{h}{n} \times h^2 = \frac{Rh^3}{n}.$$

Applying the method to the example under consideration:

Let $R = \frac{f}{6} = \frac{1300}{6} = 217.$

First. — Design so the breadth equals one-half the depth.

$$h = \sqrt[3]{\frac{nM}{R}} = \sqrt[3]{\frac{2 \times 288{,}900}{217}} = 13.84 \text{ ins.}$$

$$b = \frac{13.84}{2} = 6.92 \text{ ins.}$$

Use commercial size 7.5 ins. × 14.5 ins.

Second. — Design so the breadth equals two-thirds the depth.

Substituting in the formula $n = 1.5$, find $h = 12.6$ ins.

$$b = \frac{2 \times 12.6}{3} = 8.4 \text{ ins.}$$

Use a commercial size beam 9 ins. × 13 ins.

Third. — Design so the breadth equals three-quarters the depth. Substituting in the formula $n = 1.33$, find $h = 12.08$ ins.

$$b = \frac{3 \times 12.08}{4} = 9.06 \text{ ins.}$$

Use commercial size beam 9.5 ins. × 12.5 ins.

The student will have noticed that in all cases the exact size computed cannot be used and it is necessary to take a commercial size enough larger so the loss in dimensions through cutting will give a beam the size of the computed beam, or slightly larger. Small beams will run from $\frac{1}{4}$ in. to $\frac{3}{8}$ in. smaller than nominal size, but beams of the size here considered will seldom run less than $\frac{1}{2}$ in. smaller in each dimension than the nominal size, and if the superintendent of construction is not careful the loss will be even greater. The writer is acquainted with designers who use the nominal size always in their designs, assuming that the maximum fiber stress allowed is really less than the wood can stand. It is not good practice.

Assuming that the fiber stresses are based on the use of wood freely exposed to weather, then the following increases in fiber stress are allowable for long-leaf yellow pine:

Class A (moisture contents, 18 per cent). — Structures freely exposed to the weather, such as railway trestles, uncovered bridges, etc., let allowable stress equal $1 \times f$.

Class B (moisture contents, 15 per cent). — Structures under roof but without side shelter, freely exposed to outside air, but protected from rain, such as roof trusses of open shops and sheds, covered bridges over stream, etc., let allowable stress equal $1.15 \times f$.

Class C (moisture contents, 12 per cent). — Structures in buildings unheated, but more or less protected from outside air, such as roof trusses of barns, inclosed shops and sheds, etc., let allowable stress equal $1.4 \times f$.

Class D (moisture contents, 10 per cent). — Structures in buildings at all times protected from the outside air, heated in the winter, such as roof trusses in houses, halls, churches, etc., let allowable stress equal $1.55 \times f$.

For all woods other than long-leaf yellow pine the increases to be one-half those given. The shearing stress, however, cannot exceed one-tenth the fiber stress used for Class A structures.

Building ordinances in American cities do not recognize any difference in allowable stresses dependent on the moisture contents, so the fiber stresses permitted in cities apply to all structures. It would be better if the city ordinance requirements were based on Class D structures with proportionate decrease for structures in other classes.

The following table gives the allowable fiber stresses for wood in the city of Chicago (1916). Each designer should use the stresses permitted in the largest city nearest to the place where the building is to be erected.

Name of the Wood	Maximum Fiber Bending Stress and Tension with Grain	Compression with Grain	Compression across Grain	Shear with Grain
Douglas fir and long-leaf yellow pine	1300	1100	250	130
Oak..............................	1200	900	500	200
Short-leaf yellow pine..............	1000	800	250	120
Norway pine and white pine........	800	700	200	80
Hemlock..........................	600	500	150	60

The first column gives the name of the wood. The second column gives the maximum bending fiber stress and this is the maximum stress allowed if the wood is to be used as a tie in straight tension — something rarely possible because of the difficulty in making proper connections so the nails, screws or bolts will properly transmit the entire pull on the piece.

The third column gives the compressive stress per square inch on wood posts having a least breadth one-fifteenth the length. For lengths greater than fifteen times the least dimension, the compressive stress must be reduced, by a formula given in the ordinance, long slender pieces bending under load and causing additional strain on the concave side.

The fourth column gives the allowable bearing stress per square inch on the under side of a beam on the supports. The reaction is to be divided by the stress given in this column in order to obtain the number of square inches bearing surface. The student should pay attention to this column, for it explains the reason why steel and iron post caps are used instead of the old-fashioned wooden bolsters. If the load on a column is carried straight down on the ends of fibers, the full bearing capacity of the wood can be utilized. When a bolster is set between the foot of a post on one floor and the top of the post on the floor below, the compression across the grain of the wood in the bolster governs the carrying capacity of the post, or the bolster will crush.

The fifth column gives the allowable shearing stress with the grain, the use of this column having been explained in the examples when a test was made of the weight-carrying capacity of a beam so it would not fail in shear.

There is a shear parallel with the grain and if through some unavoidable circumstance it ever becomes necessary to design so a wide beam overhangs the sides of a support this shear will act. It should not exceed the safe shear with the grain.

There is a shear across the grain, that is, a tendency for the beam to be cut at the edge of the support. Provided the allowable compression across the grain is not exceeded, *i.e.*, sufficient bearing surface is provided, the effect of this shear is negligible.

The use of hangers and stirrups is common to-day. They save head room but increase the insurance rate, for the reason that metal is affected by intense heat. A large piece of timber will char on the surface and must be exposed to an intense flame for a long time before it begins to burn. The heat that will merely char a timber and do it little harm will heat wrought iron and steel to such an extent that the stirrup will be weakened and permit the suspended beam to drop. A study of a bending moment curve shows that at the bearing end of a beam there is practically no moment, so the area of a beam may be reduced nearly one-half at the supports without impairing the bearing capacity. If the strength of a stirrup is reduced one-half by fire the beam may drop.

Many types of stirrups are on the market, and before adopting anything other than a plain bent strap of steel or wrought iron the designer should require the manufacturers to furnish records of tests on the stirrups they propose to supply.

To design a stirrup: First obtain the area required for bearing, then the thickness to prevent straightening at the edge of the support, then check to see that the area of the vertical legs is sufficient in tensile strength to carry the load. This last item is generally taken care of when the other conditions are satisfied.

5. Design a strap hanger, or stirrup, for the 8-in. × 14-in. beam in the last example.

Answer. — The total load = 10,700 lbs. which gives a reaction = 5350 lbs. The allowable compression across the grain = 250 lbs. per square inch, so the bearing area in the stirrup $= \frac{5350}{250} = 21.4$ sq. ins. The width of the beam is 8 ins., therefore the width of the strap under the end of the beam $= \frac{21.4}{8} = 2.66$ ins. Make the strap 2.75 ins. wide, a stock width. Allowing a value of 10,000 lbs. per

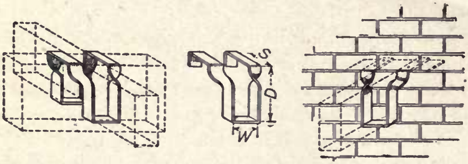


Fig. 62 — Various Styles of Stirrups

square inch tension for wrought iron the required area of the two legs $= \frac{5350}{10{,}000} = 0.535$ sq. ins. or 0.2675 sq. ins. for each leg. The thickness of metal required $= \frac{0.2675}{2.75} = 0.097$ in. (practically No. 10 gauge). Allowing a fiber stress of 14,000 lbs. per square inch for steel, the required area in the two legs $= \frac{5350}{14{,}000} = 0.382$ sq. in., or 0.191 sq. in. for each leg. The thickness of metal required $= \frac{0.191}{2.75} = 0.0695$ in. (practically No. 13 gauge). Each leg must rest on top of the girder with a length of not less than 4 ins.

This is thin metal and will surely straighten under the load, besides which it does not offer enough body to resist corrosion. Use a minimum thickness of $\frac{3}{8}$ in. The stirrup shown in Fig. 62 is double and the weight of the beam on either side tends to balance the weight of the beam on the opposite side of the girder. A stirrup $2\frac{3}{4}$ ins. wide of $\frac{1}{4}$-in. metal will therefore be all right and may be wrought iron or steel. A couple of holes drilled through the top for lag screws or spikes will take care of unequal loading on beams.

When a half stirrup is used it must be investigated for bending at the bearing edge. Assume a bearing length of 4 ins.

$$M = \frac{4 \times 5350}{2} = 10{,}700 \text{ in. lbs., half on each leg.}$$

This computation considers the legs as cantilever beams uniformly loaded.

The thickness of the metal is $\frac{3}{8}$ in. (0.375 in.) and the width is to be found. For wrought iron with a fiber stress of 10,000 lbs. per square inch, $R = \frac{10{,}000}{6} = 1667$. For steel with a fiber stress of 14,000 lbs. per square inch, $R = \frac{14{,}000}{6} = 2333$.

$$b = \frac{M}{Rh^2} = \frac{5350}{1667 \times 0.375^2} = 22.8 \text{ ins.,}$$

of which each leg will be one-half, or 11.4 ins. for wrought iron.

For steel, $b = \frac{5350}{2333 \times 0.375^2} = 16.3$ ins., of which each leg will be one-half, or 8.15 ins.

The reason for the low stresses used is due to the blacksmith work required to bend the metal to the required shape, the heating annealing the metal and restoring disturbed molecules to a normal condition. Cold working has a contrary effect — within limits — but may crack the metal, thus nullifying the effect of the strain which sets up internal stresses that apparently cause an increase in strength.

The effect of increasing the thickness of the metal is to make a considerable reduction in width on the supporting girder. Try a $\frac{1}{2}$-in. steel strap.

$$b = \frac{5350}{2333 \times 0.5^2} = 9.2 \text{ ins., of which each leg will be one-half,}$$

or 4.6 ins. By increasing the thickness $\frac{1}{8}$ in. the width of the strap has been reduced nearly one-half. The wide strap will weigh 10.4 lbs. per lineal foot. The narrow strap of thicker metal will weigh 7.8 lbs. per lineal foot, so will be the cheaper strap to use.

If a stirrup is not designed to be safe according to calculations such as those illustrated it should not be used. A lack of strength in bending is sometimes claimed to be taken care of by using a longer support and holding it down with lag screws or spikes.

The longer support increases the bending moment and the holding down strength of the fastenings must be investigated. The leg is sometimes run across the top of the girder and bent down on the other side and there fastened, which is sometimes good, but the increase in material added to the cost of fastenings and the cost of labor to drive them amounts to more than the cost of the additional thickness necessary to prevent straightening. A stirrup strong enough to carry a load without bending is more satisfactory than one confessedly weak with which fastenings must be used.

Tops of beams and girders should not be cut to make a seat for stirrups. This weakens the timber, so the under side of the floor planking should be cut to make pockets for the stirrups. A cheaper method is to lay a strip of wood to carry the flooring on top of the beam between stirrups. When the floor is double the under layer may be cut away at the stirrups, the upper layer being amply strong to carry over the small hole. Fig. 62 is reproduced from "Ryerson's Ready Reference." The stirrups illustrated are made of wrought iron and the recommendation is made in the book that the following sizes mentioned in the table should in general be used for the size of joist supported, the stirrups, unless otherwise specified, being furnished $\frac{1}{4}$ in. smaller than nominal size of timber or joist. Wall hangers rest on plates as shown.

TABLE OF STIRRUP SIZES AND CAPACITIES

Size of Joist or Timber to be Supported	Section of Stirrup	Capacity of Stirrup
2 × 8 to 3 × 10	$\frac{1}{4} \times 2\frac{1}{2}$	7,500 lbs.
4 × 10 to 4 × 12	$\frac{3}{8} \times 2\frac{1}{2}$	11,250 lbs.
6 × 12 to 3 × 14	$\frac{3}{8} \times 3$	13,500 lbs.
8 × 12 to 4 × 14	$\frac{1}{2} \times 3\frac{1}{2}$	21,000 lbs.
6 × 14	$\frac{1}{2} \times 4$	24,000 lbs.
8 × 14 to 10 × 14	$\frac{5}{8} \times 4$	30,000 lbs.

Another method for carrying the ends of joists on a girder when head room is to be saved and the joists cannot rest on top of girders is shown in Fig. 63. This depends upon shearing resistance of the spikes. First find the width of bearing required for each joist by dividing the reaction by the bearing strength across the grain. Use nails having a length practically three times the thickness of the bearing strip as a minimum, so they will go into the girder a depth about twice the thickness of the bearing strip. The number of nails to use depends on the reaction and the thickness of the nail. Divide the reaction in pounds by 100 to get the number of 20d. nails; by 150 for 30d. nails; by 175 for 40d. nails; by 200 for 50d. nails; by 225 for 60d. nails. There is considerable difference in weight between nails and spikes having the same designation and the above figures refer to nails. The nails should be spaced at least 3 ins. apart horizontally and this can be accomplished by putting half near the bottom of the strip and half near the top, thus staggering them. The size of nail to use will therefore be determined by the spacing when the reaction is considerable.

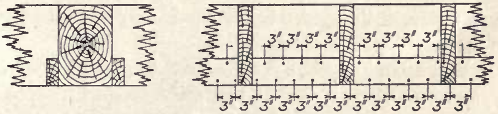


Fig. 63 — Wood End Bearings for Joists

The above described bearing strip support for joists is a cheap method. Formerly it was customary to use girders considerably larger than were necessary and seats were cut into them for the joists. This increased the labor cost and when water settled into the joints they rotted. The introduction of slow burning construction also acted to throw the gaining of joists into girders into disrepute because of the increased fire risk in the joints. When a nailed bearing strip is used it should be carefully computed. The bearing should be at least half an inch wider than the computed bearing. The ends of the joists should be carefully fitted. It is advisable with thick joists to top nail them to the girder to prevent twisting or winding. With thin joists a solid bridging should be inserted at the ends, nailed to the girder. When this is done many of the objections to the joiners pocket are introduced. Therefore metal hangers are better when for any reason it is not advisable to have the joists rest on top of a girder. In slow burning construction neither hangers or bearing strips are proper. The thickness of joists in such construction should be not less than half the depth and the minimum cross-sectional area should be 72 inches. All joists should rest on top of girders.

The maximum shear on a wooden beam is along the neutral axis and season checks are apt to occur here, so nailing strips should be as far from the neutral axis as possible, which indicates the bottom of the girder as a proper location. When a seating is cut into the bottom of a joist it is apt to cause the joist to split, and if season checks open at this point the joist will be greatly weakened. Half the depth of a joist may be cut out at the very end without weakening it for carrying purposes, for the bending moment at the end is zero, but if a joist splits for any considerable distance from the end it is greatly weakened. The principal objection, therefore, to the nailed bearing strip is the danger of splitting the joists at the upper edge of the seat. If the joists rest without cutting on the bearing strip and the strip is properly designed, it cannot be condemned. There will, however, in this case, be a projection of the girder below the ceiling equal to the width of the strip.

6. Design a laminated floor to carry a total load of 48 lbs. per square foot on a span of 16 ft. Use white pine with a fiber stress of 800 lbs. per square foot. Ignore deflection.

A laminated floor is a solid floor consisting of 2-in. planks spiked side by side. The width to use in designing is 12 ins., the load being in pounds per square foot.

$$M = 1.5 \times 48 \times 16^2 = 18{,}432 \text{ in. lbs.}$$

$$f = 800 \;\therefore\; R = 800 \div 6 = 133.$$

$$d = \sqrt{\frac{18{,}432}{133 \times 12}} = 3.4 \text{ in.}$$

Use 2 ins. $\times$ 4 ins., which will give an actual depth of $3\frac{5}{8}$ ins.

If the deflection is not to exceed $\frac{1}{360}$ of the span, the deflection will be $\frac{16 \times 12}{360} = 0.534$ in.

$$\text{Actual deflection} = \frac{L^2}{46h} = \frac{16^2}{46 \times 3.625} = 1.535 \text{ ins.}$$

The formulas previously given for deflection, page 80, are based on the fiber stress and to avoid several trial computations to ascertain the depth with the reduced stress use the rule that:

The deflection in beams varies as the cube of the length in feet divided by the breadth in inches multiplied by the cube of the depth in inches.

Expressed as a formula it appears:

Depth varies as $\frac{L^3}{bh^3}$.

Let L = span in feet,
b = breadth in inches,
h = depth in inches (required for strength),
x = depth in inches (required for deflection),
D = deflection found in inches,
d = allowable deflection in inches.

Then $$x = \sqrt[3]{\frac{Dh^3}{d}} = \sqrt[3]{\frac{1.535 \times 3.625^3}{0.534}} = 5.15 \text{ ins.}$$

Use nominal 2-in. × 6-in. planks.

The formula is developed as follows:

$$\frac{dL^3}{bh^3} = \frac{DL^3}{bx^3},$$

which becomes $$\frac{dL^3}{bh^3} \times \frac{bx^3}{DL^3}.$$

Cancelling common factors, we get $\frac{dx^3}{Dh^3}$ and $x^3 = \frac{Dh^3}{d}$, the formula used above.

7. Assuming a floor with same load and fiber stress is to be carried on joists find the size required for joists 12 ins. center to center and 16 ins. center to center. Deflection to govern.

The allowable deflection $= \frac{16 \times 12}{360} = 0.534$ in.

$$h = \frac{L^2}{46D} = \frac{16^2}{46 \times 0.534} = 10.4 \text{ ins.}$$

The depth in this example needed to avoid undue deflection is based on the fiber stress used in design, for the breadth is governed by the depth. In the case of the laminated floor a constant breadth of 12 ins. was used and the deflection was fixed by a lower fiber stress than that used for strength only.

The bending moment for a width of 1 ft. = 18,432 in. lbs. (from the last example). The thickness of the joist will be

$$b = \frac{18{,}432}{133 \times 10.625^2} = 1.23 \text{ ins.}$$

Use nominal 1.5-in × 11-in. joists, 12 ins. center to center.

With joists spaced 16 ins. center to center the bending moment is increased one-third, 18,432 × 1.333 = 24,600 in. lbs.

$$b = \frac{24{,}600}{133 \times 10.625^2} = 1.64 \text{ ins.}$$

Use nominal 2-in. × 11-in. joists, 16 ins. center to center.

The increase in amount of lumber is about one-fourth, while the increased carrying capacity is one-third, so it will be better to use the 16-in. spacing than to use the 12-in. spacing. Thinner joists than 2-in. are not advisable when it is possible to avoid them, so this is another reason for using the wider spacing. To make the floors stiff and to avoid bending under load, or warping of the joists from any cause, lines of cross bridging should be used at intervals approximately twenty-four times the thickness of the joist. This for 2-in. joists will be 48-in. (4-ft.) centers.

Beams on a Slope

Let S = length of beam on slope,
L = horizontal span,
W = total load uniformly distributed on the slope,
W' = total load beam can carry,

then $W' = \frac{WS}{L}$.

The foregoing applies in the case of stringers supporting stairs and inclined rafters carrying a load on the upper surface. There is a horizontal and a vertical force acting when a beam is inclined and the resultant thrust increases the compression and decreases the tension in the fibers. It is usually unimportant and may be neglected when the slope is less than thirty degrees, but should be investigated in any case. When an inclined member of a truss carries a load on the upper surface in addition to the direct thrust, the member must be designed to take these loadings into account.

Let M = bending moment due to the load.
f = fiber stress.
A = area of member in cross section
W = direct load along axis of the member.
h = depth of the member.
I = moment of inertia of member, which is assumed here to be symmetrical.

then $f = + \frac{W}{A} \pm \frac{Mh}{2I}$.

In the above expression the first part gives the average fiber stress due to the direct load acting along the length of the member, that is, the push. The second half is the familiar expression for the fiber stress in a beam bending under load. Use it twice, once with a positive sign and once with a negative sign. The expression sometimes appears,

$$f = + \frac{W}{A} \pm \frac{M}{S},$$

in which S = section modulus.

Buckling of Beams

The author has been careful in calling attention to the fact that beam formulas and tables of carrying capacity of beams assume the beams to be stayed for lateral stiffness. If a beam is too long the upper half acts as a slender column having a least dimension equal to the breadth. When a floor is fastened to the upper surface along the length it is usually a sufficient stay. It is best, however, to have a stay as well for the lower edge of the beam. A familiar illustration is the cross bridging between wood floor joists placed at intervals of about 24 times the breadth of the joist. In steel beams the lateral stays should be spaced at intervals not exceeding 40 times the width of the flange. All stays prevent a sidewise buckling, and the stay at the lower edge prevents a blow from pushing the beam to one side, which would cause the loading to become eccentric and thereby increase the stresses. The effect of lateral deflection and eccentric loading is to set up the simultaneous action of a direct thrust plus bending.

Stiffness of Wood Beams

The following formula was evolved by Thomas Tredgold, a noted British authority on carpentry in the last century. A beam designed according to this formula will deflect less than $\frac{1}{360}$ the span.

b = breadth of beam in inches,
h = depth of beam in inches,
L = span in feet,
P = concentrated load in middle of span,
W = uniformly distributed load = $0.625P$,

$$h = \sqrt[3]{\frac{L^2PC}{b}} = \sqrt[3]{\frac{L^2WC}{b}},$$

$$b = \frac{L^2PC}{h^3} = \frac{L^2WC}{h^3},$$

C = a constant = 0.010 for fir and yellow pine.
= 0.013 for oak and white pine.

CHAPTER IV

Girders and Trusses

A METHOD frequently used by carpenters to strengthen joists and beams is shown in Fig. 64. Two pieces are nailed as indicated, the presumption being that they exert an arching action because the ends abut at the middle of the span and the nails hold the pieces in place when thrust is exerted.

Fig. 64 — A Poor Method for Reinforcing Joists

Wood shrinks when it dries, so the close contact is lost, and then considerable deflection must take place before the ends again meet, the bending being sufficient to cause failure in many instances. Provided the hoped-for arch action does occur there will be such a pushing against the nails that the wood is bound to split. However, assuming the arch action does take place and the nails do not split the pieces the reinforcement is not effective. For effective arch action there must be substantial abutments provided. If there are no substantial abutments a tie rod is necessary to tie the ends together and take the thrust. There being no tie rod, it is evident that the lower part of the joist will have to act as a tie. We know that when a beam is loaded the lower fibers are stressed in tension and the upper fibers are stressed in compression. To increase the tension in the bottom by adding to it the amount required to take care of the thrust in the diagonal reinforcing strips is not helpful. This old-time method is, therefore, based on a fallacy and should be abandoned.

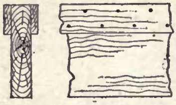


Fig. 65 — T-beam of Wood

In Fig. 65 is shown another method, the reinforcing being spiked along the top edge to make a beam of T-section. This raises the neutral surface so the increased area in compression is supposed to be offset by an increased area in tension.

Assume a joist 3 ins. × 14 ins. of wood in which a fiber stress of

1200 lbs. per square inch is used. A strip 1 in. × 4 ins. is spiked on each side along the top. How much is the strength increased?

The original strength, $M_r = \frac{fbh^2}{6} = \frac{1200 \times 3 \times 14^2}{6} = 117{,}600$ in. lbs., the neutral plane being in the middle of the joist. To find the position of the neutral plane in the T-section use the method of moments, taking moments about the lower edge:

The original piece,	$3 \times 14 \times 7 =$	294
One added piece	$1 \times 4 \times 12 =$	48
Second added piece	$1 \times 4 \times 12 =$	48
		390

$$\text{Area} = (3 \times 14) + (2 \times 4) = \frac{390}{50} = 7.08 \text{ ins.}$$

First the area of the beam was multiplied by the distance of the center of gravity above the bottom, the result being 294. Then the area of each added piece was multiplied by the distance of its center of gravity above the bottom of the beam. This was 12 ins., being half the depth of the piece added to the difference in depth of the beam and the piece. The products were added together, the sum being 390. Dividing by the total area, 50 sq. ins., the distance from the bottom to the center of gravity (center of area in this case) was found to be 7.08 ins.

The original moment arm $= \frac{2}{3} \times 14 = 9.333$ ins., that is, 9.333 ins. Before the pieces were added at the top the moment of resistance was equal to the area on one side of the neutral axis multiplied by the average fiber stress times the moment arm, that is:

$$\frac{1200}{2} \times 3 \times 7 \times 9.333 = 117{,}600 \text{ in. lbs.}$$

The strength of the beam is fixed by the fiber stress and the smaller stressed area. In this T-section the smaller area is the portion in tension below the neutral axis and the resisting moment $= \frac{1200}{2} \times 3 \times 7.08 \times 9.333 = 118{,}944$ in. lbs.

The increase in strength is very small, so the area added above the neutral axis was excessive. Better results would have been obtained by nailing one strip along the bottom and one along the top, thus increasing the area equally in tension and compression, without altering the position of the neutral axis. The proper method to follow is to increase the thickness by adding boards on one or both sides for the full depth. An example will be worked out:

In a mill-constructed building 7 ins. × 14 ins. white pine beams spaced 5 ft. on centers are used with a span of 20 ft. The allowable maximum fiber stress is 800 lbs. per square inch and the beams are to be strengthened so the total floor load can be increased to 100 lbs. per square foot, inclusive of floor, beams, and live load.

Testing first the strength of the beams against failure by longitudinal shear on the neutral axis, the unit shear being one-tenth the allowable fiber stress,

$$W = \frac{4 \times 7 \times 14 \times 80}{3} = 10{,}450 \text{ lbs.}$$

The total panel load will be 5 × 20 × 100 = 10,000 lbs., so the beam will carry the additional load without failing in shear.

$$M_b = 1.5 \times 10{,}000 \times 20 = 300{,}000 \text{ in. lbs.}$$

$$M_r = \frac{800 \times 7 \times 14^2}{6} = 182{,}933 \text{ in. lbs.}$$

Then the difference between the bending moment and the resisting moment is 300,000 − 182,933 = 117,067 in. lbs., which difference must be cared for by reinforcement. To secure equal deflection the reinforcement should be the same wood, white pine, but the difference will not be appreciable in this case, and to use yellow pine will give a smaller piece for reinforcement because of the higher allowable fiber stress. The beam is in an old building and quite likely the maximum deflection in the white pine has been reached, and there is a decided permanent set. The reinforcement should be added when the floor is unloaded in order to enable the old beam and the new pieces to deflect together when the live load is added, the difference in deflection between the two kinds of wood being cared for by the deflection due to dead load in the wood having the greatest deflection.

Assuming, therefore, yellow pine with a fiber stress of 1300 lbs. per square inch and a depth of 14 ins. the thickness is to be computed. Let

$$R = 1300 \div 6 = 217$$

and

$$b = \frac{M}{Rh^2} = \frac{117{,}067}{217 \times 14^2} = 2.76 \text{ ins.}$$

Use two $1\frac{3}{4}$-in. planks, one on each side. When surfaced the thickness will be practically $2\frac{3}{4}$ ins.

The load is uniformly distributed; the original beam is large enough to carry the required load without a shearing failure;

the diagram for bending moment due to a uniformly distributed load is a parabola; therefore the reinforcing planks need not extend the full length of the beam. They would have to extend the full length if there were danger of a longitudinal shearing failure, and the thickness of the reinforcement would also be governed by the requirement for shear.

Dividing; 182,933 ÷ 300,000 = 0.61, which shows that the resisting moment of the beam is 61 per cent of the bending moment created by the load. The ends of the reinforcing planks must extend each side of the middle of the span to the point where the bending moment is 61 per cent of the bending moment at the middle of the span. This may be obtained graphically by constructing a parabola with a base equal to 20 and a height about equal to this, the height divided decimally to any scale. At a height equal to 61 on the middle ordinate draw a horizontal line to intersect the parabola. From the point of intersection drop a perpendicular to the base. This defines the point where, theoretically, the reinforcement may end. Practically it should extend a little further.

The lengths of the reinforcing planks may be calculated by men who can solve a quadratic equation. The bending moment on a uniformly loaded beam at any point distant x from one end is as follows:

$$M_x = \frac{wLx}{2} - \frac{wx^2}{2}.$$

Substitute the values for M_x, w, and L and solve for x.

$W = \frac{10,450}{20} = 523$ lbs. per lineal foot (in even numbers).

$M_x = \frac{182,933}{12} = 15,244$ ft. lbs. (in even numbers). This moment is the resisting moment of the beam without reinforcement.

Then $15,244 = \frac{523 \times 20x}{2} - \frac{523x^2}{2}.$

Clearing of fractions,

$$2 \times 15,244 = 30,488 = 523 \times 20x - 523x^2.$$

Dividing by the coefficient of x^2,

$$58.3 = 20x - x^2.$$

Transposing, $x^2 - 20x = -58.3.$

Extracting the square root,

$$x = +\frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 - 58.3} = 3.55 \text{ ft.}$$

The reinforcing planks (theoretically) should have a length of $20 - (2 \times 3.55) = 13.90$ ft. Practically it will be best to make them 15 ft. long, which leaves 2 ft. 6 ins. without reinforcement at each end of the beam.

The planks must be attached to the beam by screws or nails, the latter being the cheaper. To get the best results the length must be not less than three times the thickness of the plank, in order that the nail may be embedded in the beam a depth at least twice the thickness of the plank. From a table of sizes of standard steel wire nails and spikes (in the steel manufacturers' handbooks) we find a 30d. nail is 4.5 ins. long, the length required. There must be enough nails used so the beam and planks will act together and the force to be resisted is shear, for if the beam bends and the planks do not bend there will be a sliding movement between them.

When a nail resists a shearing force three actions are set up: 1. A bending caused by the pull of one piece against the nail embedded in the other piece. 2. A shear in the nail which is caused if the nail is so stiff that it will not bend. 3. Bearing against the wood in which the nail is embedded. The size of the nail must be proportioned to care for the action most likely to cause a failure. When the material to be held is wood the bearing action of the nail against the wood is the only one to be considered, for if the nail furnishes area enough to transmit the shear it will be thick enough to resist bending and also thick enough not to shear across. Rivets in metal have to be similarly proportioned, but bending is seldom feared while failure by shear of the rivet or by insufficient bearing against the metal is practically, and usually, of equal importance. Both must be figured, whereas in the case of wood only the bearing value is considered. The bearing value is computed as follows:

The 30d. nail (not spike) is made from No. 5 wire, the diameter being 0.207 in. The cross-sectional area through a $1\frac{1}{2}$-in. plank is $0.207 \times 1.5 = 0.311$ sq. ins. The compressive value of the softer wood must be used, which is 700 lbs. per square inch with the grain, assuming the nail to bear on the end of the wood where it enters. The bearing value for one nail is found by multiplying the bearing area by the allowable fiber stress in compression with the grain, $0.311 \times 700 = 218$ lbs. This is the method to be used when no data is at hand giving the actual safe bearing values.

The actual safe bearing value for any nail is about two-thirds of the value as above computed. One reason is that there is no common gauge used by nail makers, so that while tables may show that nails are made of certain wire, not all tables give the diameter of the wire in decimals of an inch, and there being a number of wire and metal gauges in use we do not know the exact sizes of the nails used in the published experiments. The experiments referred to may have been made with nails not quite so thick as the nails used in computing the bearing value. A second reason for the actual bearing value being so small is that the nails push the fibers of the wood aside and start a splitting action, which is increased when the shearing action is set up. This second reason is no doubt much more important than the first. The method of figuring bearing value just illustrated is correct for bolts for which holes must be bored, but gives a value about 50 per cent too large for driven wire nails and for screws. The designer must not forget this. Having settled on the size of nail and the bearing value of each nail, the number and spacing must be determined.

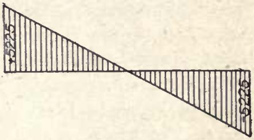


Fig. 66—Shear Diagram for Original Beam

Fig. 66 is the shear diagram for the uniformly loaded beam. At each end the shear = reaction = 10,450 ÷ 2 = 5225 lbs. The nails should be closer together near the ends, where the shear is a maximum, so theoretically the spacing should vary from nail to nail. Practically the spacing can be maintained at uniform intervals for each foot, which makes the diagram resemble Fig. 67, the reinforcement ending 2.5 ft. from each end.

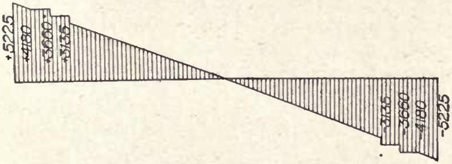


Fig. 67 — Shear Diagram for Reinforced Beam

The method to be described follows the common method for spacing rivets in the flanges of plate girders. There is another method which will later be illustrated, because it shows exactly how the stresses in the top and bottom flanges of plate girders affect the rivets used to connect the flanges to the web. In the present example the original beam may be said to represent the web, the reinforcing pieces the flange members, and the nails the rivets used in plate girders. The spacing of stirrups in a reinforced-concrete beam is another illustration of the transmission of stresses from one part of a beam to another by designing to resist shear. In a truss the sizes of the members are varied, for the panel lengths are equal. When the nails in a reinforced wooden beam are of the same size, the rivets in a plate girder are the same size and the stirrups in a reinforced-concrete beam are the same size, the variation in shear is taken care of by varying the spacing.

Two feet from the end of the beam the shear is $\frac{5225 \times 8}{10} = 4180$ lbs. The width of the original beam is 7 ins., and the two planks increase the width to 9.75 ins., or $\frac{4180 \times 7}{9.75} = 3000$ lbs., which will be carried by the original beam, leaving $\frac{4180 - 3000}{2} = 590$ lbs. to be carried by each plank. The nails must transfer this from the beam to the plank and they should be driven 1 in. from the edge, both top and bottom. Let

V = total vertical shear at the point considered,
r = resistance of one nail (bearing value),
d = distance in inches between lines of nails (in the present example $d = 12$ ins. vertically),
p = pitch of nails in inches (the horizontal distance center to center between nails);

then
$$p = \frac{rd}{V} = \frac{145 \times 12}{590} = 2.95 \text{ ins.}$$

Space the nails $2\frac{3}{4}$ ins. center to center along the upper and lower edge for at least 6 ins. at each end, the first nail being driven 1 in. from the end of the plank.

Shear 3 ft. from end $= \frac{5225 \times 7}{10} = 3660$ lbs.

Shear carried by each plank $= \frac{3660 - 3000}{2} = 330$ lbs.

$p = \frac{145 \times 12}{330} = 3.92$ ins. Space the nails $3\frac{3}{4}$ ins. center to center along the upper and lower edge of the plank for 1 ft.

Shear 4 ft. from end $= \frac{5225 \times 6}{10} = 3135$ lbs.

Shear carried by each plank $= \frac{3135 - 3000}{2} = 67.5$ lbs.

$$p = \frac{145 \times 12}{67.5} = 25.8 \text{ in.}$$

Nails should be driven not more than 12 ins., on centers, so, beginning at the end of the fourth foot from the end of the beam drive nails on 12-in. centers top and bottom. Along the neutral axis drive nails on 18-in. centers. The completed work is shown in Fig. 68.

No reduction in area was figured, as nails merely push wood fibers aside, but when bolts are used the effective depth of the beam is reduced by the thickness of each line of bolts. If bolts

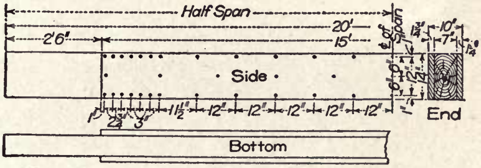


Fig. 68 — Beam Reinforced by Planks on the Sides

$\frac{3}{4}$ in. in diameter are used in two lines, and the hole for each bolt is $\frac{7}{8}$ in., the effective depth is reduced by $2 \times \frac{7}{8} = 1\frac{3}{4}$ ins. This is serious, for the strength of beams varies with the squares of the respective depths.

Sometimes beams are reinforced by nailing a plank or strip of steel along the bottom. Assume the same conditions as in the last example, and use a thin white pine plank on the bottom. Maintaining the breadth the problem is to obtain a new depth.

The fiber stress for white pine is 800 lbs., so $R = 800/6 = 167$ and $h = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{300{,}000}{167 \times 7}} = 16$ ins. The original depth is 14 ins., so a plank 2 ins. thick by 7 ins. wide must be spiked or bolted to the bottom. A thick plank like this must be fastened with bolts, and the holes will reduce the area, which will make necessary an increase in thickness. Methods for finding the length of the reinforcing plank and the pitch of the bolts have been given, the depth used being the full depth of the original beam plus half the thickness of the reinforcing plank.

To reinforce with a steel plate on the bottom use a moment arm = $\frac{2}{3} \times 14 = 9.333$ ins. The fiber stress in the steel will be 16,000 lbs. per square inch. Then area of plate = $\frac{117{,}067}{9.333 \times 16{,}000}$ = 0.783 sq. ins., and $0.783 \div 7 = 0.118$ ins., the thickness of the plate. Use lag screws to fasten the plate to the beam, the proper pitch being determined as in the last example, using the full depth. The objection to the use of the steel plate is that the compression in the upper half of the beam is increased, although the effect of adding the plate is to lower the neutral plane. The proper method for reinforcing a beam, or girder, in place is to add planks on one side or on both sides, but when fixtures or wires are in the way it may be best to use a steel plate on the bottom.

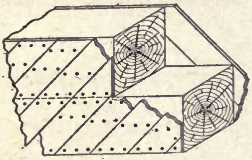


Fig. 69 — Compound Beam

Compound beams have been made consisting of two shallow beams superimposed (Fig. 69). If not carefully fastened together they act singly, because the line between them is in the position occupied by the neutral axis of a solid beam, having a depth equal to the combined depth of the two pieces. Several methods have been used to cause the two pieces to act together, one of which is shown in Fig. 69, the other in Fig. 70.

No matter how thoroughly the pieces are fastened together the strength of such a compound beam is only about 70 to 75 per cent of the strength of a beam of equal dimensions made from one piece of timber. The deflection of such a beam under load is much greater than the deflection of a beam of equal dimensions made from one piece of timber.

The diagonal side pieces shown in Fig. 69 should be preferably of a harder wood than the beam, and each should be not less than one-eighth the thickness of the beam, thus making a beam 25 per cent wider than the width of the pieces of which it is composed. The pieces should be diagonal and slope in opposite directions on the sides of the beam. Plenty of nails must be used.

In Fig. 70 the pins should be of hard wood or of metal. It is best to use two pieces in each hole, wedge-shaped, so they may be driven tight and have a bearing against the wood the full width of the beam. The shear being greatest along the neutral axis, it is here the pieces should join and the pins be driven. Between the pins should be vertical bolts with larger washers to hold the pieces together. The spacing of the pins will be determined in like manner as the pitch of nails is determined when reinforcing planks are used on the side. First determine the bearing value of the wood and the shearing value with the grain. Divide the shear where a pin is placed by the allowable bearing times the breadth to obtain the depth of the hole, half of which will be cut in each half of the beam. The shear divided by the breadth times the allowable unit shear with the grain gives the minimum distance allowable between pins. When the computations are completed it will be discovered that the pins get farther apart as the middle of the span is approached.

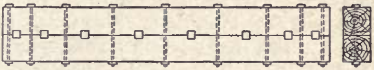


Fig. 70 — Compound Beam

Flitch plate girders, Fig. 71, are seldom used to-day, although very popular at one time. The only reason for referring to this type of compound girder here is to show wherein it fails. A flitched girder consists of a plate of steel, or wrought iron, between two planks, the whole construction being firmly bolted together. The writer, in wrecking old buildings, found a number of such beams evidently put together on a basis of relative fiber stresses, with no thought for relative deflections. He worked once in the office of an architect who tried to get him to design such a beam in this way and the man was greatly surprised when the proper method was shown to him. The method used is as follows: Assuming a maximum bending fiber stress of 1300 lbs. per square inch for wood and 16,000 lbs. per square inch for steel, the relative areas of wood and steel will be 16,000 ÷ 1300 = 12.5, or a $\frac{1}{4}$-in steel plate between two $\frac{7}{8}$-in. planks makes a girder having the strength of four $\frac{7}{8}$-in. planks.

Fig. 71 — Flitch Plate Girder

Referring to the deflection formulas it is seen that for a fiber stress of 16,000 lbs. in steel the deflection in inches on any span $= \frac{L^2}{60h}$, while for a fiber stress of 1300 lbs. in wood the deflection $= \frac{L^2}{41h}$. Therefore yellow pine deflects 60 ÷ 41 = 1.46 times as much as steel, under the respective fiber stresses given.

This question of deflection does not take into consideration the thickness of the material, for deflection is governed by the span and the depth.

The statement about relative deflections means that if the thicknesses are proportioned by the relative stresses, then the wood planks must be 1.46 times as deep as the steel plate between them. This will not do in practice, so it is necessary to obtain the relation between the stresses when the plate has a depth equal to the depth of the inclosing planks. The fiber stress in the wood divided by the fiber stress in the steel must equal the modulus of elasticity of the wood divided by the modulus of elasticity of the steel; that is,

$$\frac{f_w}{f_s} = \frac{E_w}{E_s}.$$

Then $f_w = \frac{f_s E_w}{E_s} = \frac{16{,}000 \times 1{,}500{,}000}{30{,}000{,}000} = 800$ lbs. per square inch.

This is a low stress for yellow pine, so a softer wood can be used. Assume a wood having a modulus of elasticity of 1,000,000, then $f_w = \frac{16{,}000 \times 1{,}000{,}000}{30{,}000{,}000} = 535$ lbs. per square inch.

If a steel fiber stress of 18,000 lbs. per square inch is assumed, the fiber stress in the wood = 600 lbs. per square inch. The computations show that for a flitch plate beam a soft, cheap wood is the kind to use. It is wasteful to use a wood in which a high fiber stress may be permitted.

To design a flitched girder the fiber stresses are first found. Then assume the depth and thickness of the steel plate. Find how much it will carry as a thin deep beam and deduct this load from the total load to be carried. The difference is to be carried by the two wood planks of which we know the depth and the fiber stress, so it is easy to find the thickness. The bolts are figured to transmit the shear. It is an interesting exercise to design a flitch girder, but a rolled steel beam or a trussed wooden girder will usually be cheaper.

Plate Girders

Plate girders are compound girders made of wrought iron or steel, the latter material being generally used to-day, for it may be used with a higher fiber stress, thereby reducing the weight. When rolled beams are not obtainable in a large enough size a plate girder is used, provided a rolled beam cannot be made to serve, by attaching plates to the flanges. Tables of plate girders are given in the steel handbooks, so the architect or builder finds it as easy to select a plate girder for much of his work as it is to select a rolled I-beam or channel for light loads on shorter spans. When the load, or the span, either or both, make a plate girder too heavy, a trussed girder is used.

Fig. 72 shows a plate girder. The thin vertical plate is known as the web and is made thick enough to carry the shear. It acts also as a long slender column, so must be safe against crippling. When proportioned to carry the shear and the thickness is greater than $\frac{1}{60}$ the depth between the rivets in the upper and lower flanges,

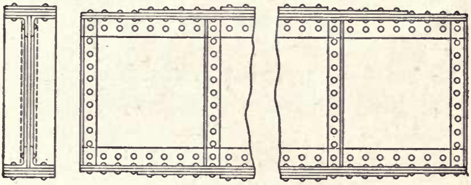


Fig. 72 — Plate Girder with Cover Plates and Stiffeners

the plate is safe against crippling. When designed to carry the shear and a thickness less than $\frac{1}{60}$ the depth is obtained, it is necessary to use stiffeners spaced regularly at intervals equal to the depth of the girder. Additional stiffeners are placed under concentrated loads and at the ends. Intermediate stiffeners are sometimes crimped over the flange angles, but it is as common to have fillers placed under them, the thickness of the fillers being equal to the thickness of the flange angle, so the stiffeners will be straight and have the ends resting on the outstanding leg of the flange angles. No scientific rules seem to be commonly accepted for designing intermediate stiffeners, the usual empirical method being to have the outstanding leg equal in width to $\frac{1}{30}$ the depth plus 2 in.

End stiffeners act as columns to carry the end shear, which is delivered to them by the web plate and carried to the bearing plate. Stiffeners under concentrated loads are designed as columns. The ends of all stiffeners designed as columns should be milled or ground to fit perfectly against the bottom flange angles.

Divide the load by a fiber stress of 12,500 lbs. per square inch for the area of the stiffeners and use fillers to keep the stiffeners straight.

The flange may be made solely of angles extending the whole length of the web plate, or of angles with plates riveted to them, the latter type being adopted when angles alone will not be sufficiently strong. The plates seldom extend the full length and if more than one flange plate is used the outer plates are very short, the lengths increasing progressively as they get closer to the angles. These plates are known as cover plates, and when different thicknesses are used the thinner plates are on the outside.

The resisting moment is determined as follows: one-eighth the area of the web is considered as forming part of the flange. This is the usual custom, but some engineers use only $\frac{1}{10}$ and some $\frac{1}{12}$.

$$M_r \text{ of web} = \frac{Rbd^2}{8}.$$

$$R = \frac{f}{6}.$$

b = thickness of web plate.

d = total depth of plate.

f = unit fiber stress (usually 16,000 lbs. per sq. in).

M_r of angles = Adf.

A = area in sq. ins. of the two angles on one edge of plate.

d = distance center to center of gravity of the angles on upper and lower edges of web plate.

f = unit fiber stress.

M_r of cover plates = Adf.

A = area in sq. in. of plates on one edge of web plate at middle of span.

d = distance center to center of gravity of cover plates.

f = unit fiber stress.

The total moment of resistance of the plate girder is the sum of the moments of resistance of the web, the angles, and the cover plates.

The rivets used to connect the flange angles to the web and to connect the cover plates to the angles must be spaced to take care of the shear, this being accomplished by using the following formula:

$$p = \frac{rd}{V},$$

in which V = total vertical shear at the section considered,

r = the resistance of one rivet,

d = distance in inches between the center of the upper row of rivets and the lower row of rivets,

p = pitch, center to center of rivets, in the flange.

The bending moment due to uniform load varies as a parabola, and as plate girders are generally designed for a uniform load the cover plates are varied in length to provide enough area for tension or compression. They extend a short distance past the point where they are no longer required, to allow for proper connections. By having the plates stop when no longer required some weight is saved and the design is the most economical possible. When concentrated loads must be cared for in addition to a uniform load the process is altered.

Let A = total area of angles and cover plates in one flange at mid-span,

a_1 = area of shortest cover plate,

a_2 = area of second cover plate,

a_x = area of longest plate, the plates being numbered progressively from 1 to the end, x being used to signify any general terminating number.

Similarly $l_1, l_2 \ldots l_n$, etc. = lengths of the plates, the letter n being used to designate the general number applying to the last plate, thus l_n = length of the last plate considered.

then,
$$l_n = \frac{L}{\sqrt{A}} \times \sqrt{a_1 + a_2 + \ldots a_x}.$$

The angles always extend the full length of the web plate.

In spacing rivets in the flanges of plate girders it is common practice to have the rivet spacing uniform between stiffeners, the amount of vertical shear considered as being taken at each stiffener. The minimum distance between centers of rivets is three diameters of the rivet, but not less than 3 ins. for $\frac{7}{8}$-in. rivets and $2\frac{1}{2}$ ins. for $\frac{3}{4}$-in. rivets. In the flanges the maximum pitch should not exceed six times the diameter of the rivets. The maximum pitch at ends of cover plates should not exceed four diameters of the rivets for a length equal to twice the depth of the girder. The maximum pitch in stiffeners is determined by the loading, if any, but should never exceed a maximum of 4½ ins. The load on an intermediate stiffener, and the load plus the end shear transmitted to an end stiffener, is divided by the bearing, or shearing, value of the rivet on the plate to obtain the number of rivets. These are equally spaced, but the maximum spacing is, as stated above, 4½ ins.

When it is necessary to splice the web of a girder the splice plates on each side of the web must be proportioned so the rivets will not be unduly stressed. The moment causing vertical bending in the girder makes the stress in the rivets greater as the bottom or top is approached. Sometimes an additional "moment splice" must be added on the sides of the web close to the flange angles.

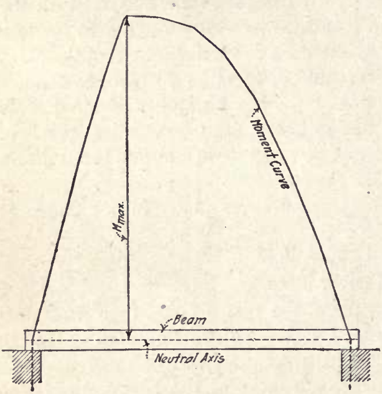


Fig. 73

The lengths of flange cover plates and the spacing of rivets may be done graphically. In fact the graphical method for obtaining the length of cover plates is that commonly used by designers. Fig. 29 shows a moment diagram on a beam carrying a uniform load and several distributed loads. Fig. 73 shows the combined curve for this condition, so that it will not be necessary to measure up and down from the neutral axis and add the lengths.

The moment curve for a plate girder is similarly drawn. For a uniform load only, the curve is a parabola, so it is necessary to show but one-half the span. All horizontal measurements are made to the scale of the drawing, and all vertical measurements are made to the scale used for the bending moment. Make the drawing no larger than is necessary to obtain accurate data.

To set off the lengths of cover plates draw a vertical line through the point of maximum bending moment, Fig. 74. Begin at the bottom and set off first the amount of moment carried by the web ($\frac{1}{8}$, $\frac{1}{10}$ or $\frac{1}{12}$, according to specifications) and above this set off the amount carried by the flange angles. The number of cover plates having been determined, set off in succession the amount of moment carried by each. Through the points fixing the amount of moment carried by the web and the angles draw horizontal lines to the ends of the span. Through the points fixing the amount of moment carried by each cover plate draw horizontal lines beyond the moment curve 2 or 3 feet. This projection allows length in which to place a few rivets so the plates begin to be effective when needed.

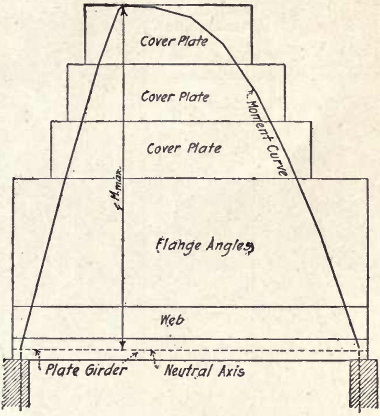


Fig. 74

The lengths of the cover plates are scaled from the diagram. When the plate girder carries a moving load on top it is usual to have the top cover plate next the angles extend the full length of the girder. This protects the angles against the entry of moisture in the joints and stiffens them near the ends against the effects of deflection of the frame carrying the load.

A similar diagram may be used for spacing rivets. The vertical scale represents the total maximum tensile (or compressive) stress in the girder, instead of the maximum moment. The number of rivets necessary to resist this stress is determined by dividing the stress by the safe allowable stress on each rivet. The vertical line is divided into as many parts as there are rivets required. Through the division points draw horizontal lines, Fig. 75, to an intersection with the boundary curve. From the points of intersection with the boundary curve drop vertical lines to the base. A rivet will be placed at each intersection thus determined. A similar method may be used for spacing stirrups in beams of reinforced concrete. The moment diagram for obtaining the lengths of cover plates may be used without change for determining rivet spacing by adopting a scale for the vertical lines proportioned to the ratio the total stress bears to the moment. By total stress is meant the product of the area of the flange angles plus the area of the cover plates multiplied by the unit stress.

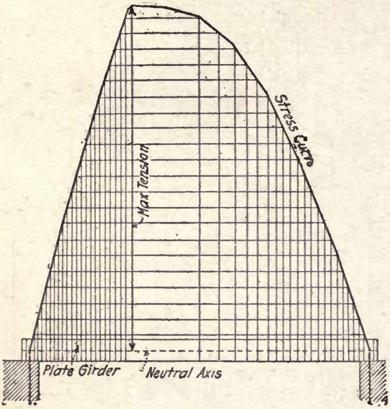


Fig. 75

Beams with Uniform Stress

When the shape of a beam resembles the shape of the bending moment diagram the stress is the same along the length. When the top and bottom of a beam are parallel the stress diminishes toward the ends. Cast-iron beams, therefore, are generally made with a "belly," for the material can be distributed at will since the beam is cast in a mold. This effects some saving in the cost of patterns and castings. Plate girders are sometimes made in this form, a familiar example being the main girders along the under side of railway cars. The stress equals the moment at any point divided by the depth of the beam at that point.

Trussed Beams

The trussed beam shown in Fig. 76 is the most simple form of truss. One-half of a uniformly distributed load is assumed to be concentrated over the strut in the middle, so, letting

$$P = \frac{W}{2}.$$

$$M = \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}.$$

Referring to Figs. 48, 49, 50, wherein the stress is shown to be equal to the moment divided by the depth, the compressive stress in the long horizontal member is,

$$C = \frac{PL}{4} \div d = \frac{PL}{4d}.$$

The length of the diagonal portion of the tie is found by the formula

$$t = \frac{\sqrt{L^2 + (2d)^2}}{2}$$

and the tensile stress in the diagonal is

$$T = \frac{Pt}{2d}.$$

The compressive stress in the vertical strut depends upon the construction of the horizontal member. If it is in two pieces joined over the strut, one-half the load, P, is carried by the strut. If it is in one piece, or composed of several planks so joined that they act as one piece, the strut carries $\frac{5}{8}P$ when P is a single concentrated load or $\frac{5}{8}W$ when W is a uniformly distributed load. This value must be used for P in the above formulas.

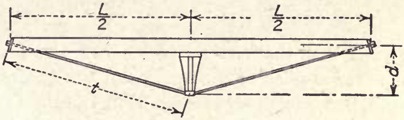


Fig. 76. — Single Strut Belly Rod Beam

When the single strutted beam carries a single concentrated load over the strut the latter carries the whole load, plus half the weight of the uniformly distributed load of the beam. The tensile and compressive stresses in the horizontal and diagonal members are found as explained above.

In Fig. 77 is shown a beam with struts at the third points.

The bending moment for a beam carrying two equal loads, P, at a distance $= L \div 3$ from each end, is

$$M = P \times \frac{L}{3} = \frac{PL}{3}.$$

The horizontal stress $C = \dfrac{PL}{3d}.$

The diagonal stress, $T = \dfrac{Pt}{d}.$

The compressive stress in each strut $= P.$

When the load is uniformly distributed $\frac{1}{3}$ is carried by each strut, so in the above expressions substitute $\frac{W}{3}$ for P.

When the horizontal member is in one piece and uniformly loaded, each strut carries $\frac{11}{30}$ of W.

The above formulas are true for any depth of truss of the single or double strut kind. The truss may be reversed so that the sloping members are above and the long horizontal member is below. Then the lower member carries tension and the upper members are in compression. The methods of computation are not altered

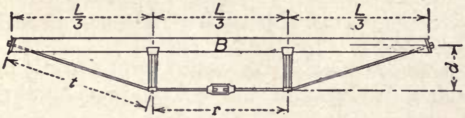


Fig. 77. — Double Strut Belly Rod Beam

except that the vertical pieces are ties instead of struts. With the load applied at the upper end the ties carry no stress from the load but are used merely to maintain the horizontality of the lower chord. With the load applied at the lower end each vertical carries the amount it would carry if the beam were inverted. With a double-tied beam the tie serves to hold the frame together in case of a rolling load, or a load applied other than vertically, in which case it does carry stress. Diagonal counters set between the ties will take care of such stresses and the ties merely serve to hold the frame together. It is advisable to have ties many times in trusses when an analysis shows they are not stressed, in order to carry the weight of the lower chord. If the lower chord must carry all of its own weight, or any load between supports, bending and shearing stresses will be set up in the lower chord in addition to the direct tensile stress. This is one reason for making all tension members of metal when possible.

Dimensions are on center lines. The tension rods should go through the ends of the compression member at the neutral axis. The plates at the ends should be normal to the direction of the tie. The area of each plate is obtained by dividing the tension in the rod by the allowable safe unit compressive stress on the end of the wood.

The area of each strut is obtained by dividing the compression in the strut by the safe unit compressive stress in the material of which it is made. The area of the end of the strut against the wood is found by dividing the compression in the struts by the allowable safe unit compressive stress across the grain of the wood.

The size of the long compression member is obtained by designing it as a column, plus the effect of bending caused by whatever load it may carry as a beam, with spans figured between end supports and vertical struts, or ties. The unit compressive stress on the end of wood may be used when the length of the member between supports does not exceed 15 times the least thickness. For longer pieces the unit compressive stress must be reduced by an appropriate column formula.

The sizes of metal tension members are fixed by dividing the total tension by the allowable safe unit tensile stress in the metal used. If threads are cut in a rod, this size must be at the root of the threads. If the rod has upset threads the full area of the rod may be used. The minimum size rod to use in any tie is $\frac{5}{8}$ in. diameter.

The size of a tension member made of wood is obtained by dividing the total tension by the allowable safe tensile stress in the wood and adding thereto an area equal to that caused by bolt holes and seating of other truss members.

Trusses

A truss is a system of framework forming a skeleton beam. The top chord is in compression; the bottom chord is in tension; the web members (interior braces and ties) carry the shear. The parts must be in equilibrium, that is each push must be balanced by a pull or a push from the opposite direction. This indicates the triangle as the perfect truss, for it cannot be changed in shape without breaking at the joints.

Trusses are of two kinds, — those with parallel chords and those with nonparallel chords. The parallel chord truss will first be considered. Fig. 78 shows the development of the Pratt truss from two panels to six panels. Assume a load = 1 at the middle vertical. Half the load goes to each support so the coefficient for the two middle diagonal ties = $\frac{1}{2}$. If the load is applied at the top, 1 is the coefficient for the middle vertical, but if the load is suspended at d the coefficient = 0.

In Fig. 78 (*b*) two panels have been added and it is assumed the panels are loaded equally, the loads being concentrated at the joints. The end panels not only carry the reactions from the original middle triangle but their own loads in addition. The diagonal ties *ef* are more heavily stressed on this account than are the ties *cd*, the stress being tension, for the whole load is suspended at the end points *f*. This increased tension throws on the

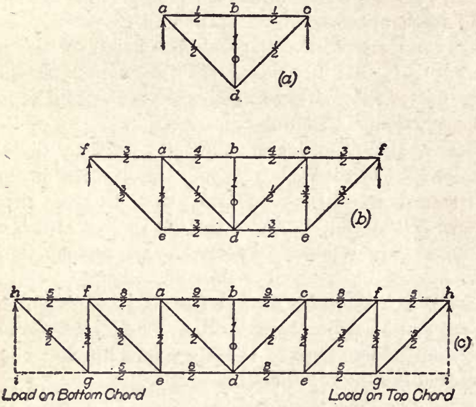


Fig. 78

middle of the upper chord the added load of the end panels in addition to the load carried by the middle triangle, so the middle panels of both upper and lower chords are more heavily stressed than the end panels.

The coefficient for *ad* and *cd* = $\frac{1}{2}$. When the unit load, 1, at the next joint is at *c* the coefficient for *ce* = $\frac{1}{2} + 1 = \frac{3}{2}$, but if the load is at *e* the coefficient for *ce* = $\frac{1}{2}$. The coefficient for *ef* is the sum of the coefficient for *dc* and the unit load on the line *ce*. The above explanations are based on the load being on one chord only. When there are loads on both chords there should be two sets of coefficients written down and their sum used. Using the positive sign (+) to indicate compression and the negative sign (−) to indicate tension, the algebraic sum is meant when the stresses are opposite in kind. To check what has been stated, note that there are three panel joints carrying equal loads, so $\frac{2}{3}$ of the total load goes to each support. If the load is uniformly distributed one-half a panel load will be concentrated at f, but this is carried directly on the supports and has no effect on the stresses in the framework.

In Fig. 78 (c) two more panels have been added. If the load is on the top chord the coefficients are as follows:

$$bd = 1 \qquad ce = \tfrac{3}{2} \qquad fg = \tfrac{5}{2}$$
$$cd = \tfrac{1}{2} \qquad ef = \tfrac{3}{2} \qquad gh = \tfrac{5}{2}$$

If the load is on the bottom chord:

$$bd = 0 \qquad ae = \tfrac{1}{2} \qquad fg = \tfrac{3}{2}$$
$$ad = \tfrac{1}{2} \qquad ef = \tfrac{3}{2} \qquad gh = \tfrac{5}{2}$$

Merely for illustration the end panels have been completed by dotted lines. The coefficient for $hi = \frac{6}{2}$ (that is, it carries the reaction). The difference, $\frac{6}{2} - \frac{5}{2} = \frac{1}{2}$ at h, acts vertically and creates no stress in the truss. The member gi carries no load when the weights all act vertically, but in case of wind or rolling loads causing horizontal or diagonal action on the frame there will be compression on the member gi at the end where the load is applied and tension in the same member at the opposite end. When the vertical post hi is omitted, the end h rests on the abutment and the truss is said to be suspended.

Fig. 79 shows the development of the Howe truss, which is merely the Pratt truss inverted. The verticals are in tension and the diagonals are in compression. The Pratt truss is usually the more economical and may be built of metal, or of metal and wood. The Howe truss is usually a combination of metal for tension members and wood for compression members. For maximum economy in metal trusses the compression members should be as short as possible, so the Howe truss is not well adapted for all metal construction.

Coefficients for the Howe truss are written as explained for the Pratt truss, with the stresses reversed in kind. The middle vertical, however, is opposite in character as affected by the load. That is, when the load is on the lower chord the coefficient = 1, but when it is on the upper chord the coefficient = 0. Practically, however, in the latter case the vertical does carry a portion of the weight of the lower chord in the middle panel. This is very small and the smallest sized rod used will more than take care of it.

A coefficient represents the proportion of panel load carried by the member on which it is written. Coefficients are used only when all the panels carry equal loads, the truss then being symmetrically loaded. Instead of starting from the middle panel and working to the ends, the coefficients may be obtained as follows: Count each panel load = 1. One-half the number of panel loads will be the reaction (expressed as fractional coefficients). From one end reaction subtract 1 successively at each panel joint and thus obtain the coefficient for each member in the

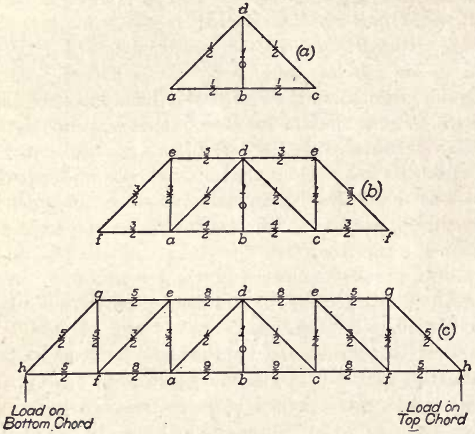


Fig. 79

following panel. The coefficient for the chord in any panel is the sum of the coefficients of the diagonals between that panel and the end support.

The weight (W) carried by each truss member is equal to the coefficient of the member multiplied by the unit panel load, P.

Let l = length of panel, center to center of verticals.

d = depth of truss, center to center of chords.

t = length of diagonal, center to center of chords.

then $t = \sqrt{l^2 + d^2}$.

Stress in diagonals $= \frac{Wt}{d}$.

Stress in chords $= \frac{Wl}{d}$.

Stress in verticals $= W$

The theory of coefficients is as follows: Assuming the truss to be symmetrically loaded with uniform loading on each panel, half the load on the middle panel alternately pulls and pushes (or pushes and pulls) on all the web members until the end of the truss is reached. Each panel load, as its point of application is reached, is added and the end web member carries half the entire load on the truss. Thus the load on the web members increases from the center to the ends and the load on a chord increases from the ends to the center. The end half panel load is carried by the abutment. It creates no stress in the truss.

For irregular and unsymmetrical loading find the reactions as for a simple beam similarly loaded with concentrated loads, and the panel joint where the maximum moment occurs is the point of zero shear. From this point the loads run up and down the web members to the ends, instead of from the center panel, as in the case of uniform and symmetrical loading. For unsymmetrically loaded trusses the weight per panel is used instead of the proportion of weight (coefficient).

A truss being merely a skeleton beam, a study of the manner in which the loads go to the abutments shows that the weight on each panel is really the shear on the panel. It is thus feasible and practical to consider the truss as a beam and from the reaction at either end subtract in succession the loads on the panel joints until the point of zero shear is reached. In Fig. 80 is shown a truss with the shear diagram. The shear on gh = 25,000 lbs.; on ef = 15,000 lbs.; on cd = 5000 lbs., the panel load being 10,000 lbs. concentrated at the joints. The skeleton truss lies on the center lines of the members, the panel length being 10 ft. and the height 10 ft. The length of a diagonal = 14.14 ft., so the compression in $gh = \frac{14.14 \times 25{,}000}{10} = 35{,}950$ lbs. The compression in $ef = \frac{14.14 \times 15{,}000}{10} = 21{,}750$ lbs. The compression in $dc = \frac{14.14 \times 5000}{10} = 7190$ lbs.

With the load considered as applied on the upper chord the tension in gf = 15,000 lbs.; the tension in ec = 5000 lbs.; and the tension in bd = 0. With the load considered as applied on the lower chord the tension in gf = 25,000 lbs.; the tension in ec = 15,000 lbs.; and the tension in bd = 5000 lbs.

The compression and tension per panel in the chords $= \frac{Wl}{d}$, therefore compression in eg = tension in fh = 25,000 lbs., for the ratio $\frac{l}{d} = \frac{10}{10} = 1$. The compression in de = tension in cf = 25,000 + 15,000 = 40,000 lbs. The tension in bc = 25,000 + 15,000 + 5000 = 45,000 lbs.

The object of the computations being to obtain the stresses so

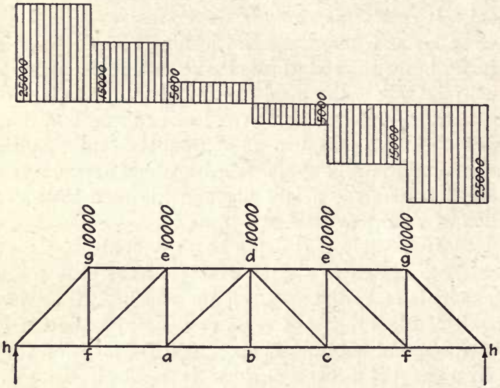


Fig. 80.

the members may be proportioned, the method above given of following the loads from joint to joint and obtaining the coefficients for uniformly and symmetrically loaded beams, or of obtaining the shear at panel joints for unsymmetrically loaded beams, is adequate and simple.

It can be proven that a truss is merely a skeleton beam by finding the shear and bending moment at each joint and then dividing the bending moment by the depth obtain the stresses in the chords, the web members carrying the shear. In Fig. 80 the end reactions each equal 25,000 lbs. Then

M, at f = 10 × 25,000 = 250,000 ft. lbs.
M, at g = 0, for the top chord rests on gh.
M, at c = (20 × 25,000) – (10 × 10,000) = 40,000 ft. lbs.
M, at e = 10 × 25,000 = 250,000 ft. lbs.

M, at b = (30×25,000) − (10×10,000 + 20×10,000) = 450,000 ft. lbs.
M, at d = (20 × 25,000) − (10 × 10,000) = 400,000 ft. lbs.

Dividing the moments by the depth,

$$\text{tension in } fh = \text{compression in } eg = \frac{250{,}000}{10} = 25{,}000 \text{ lbs.},$$

$$\text{tension in } cf = \text{compression in } de = \frac{400{,}000}{10} = 40{,}000 \text{ lbs.},$$

$$\text{tension in } bc = \frac{450{,}000}{10} = 45{,}000 \text{ lbs.},$$

Fig. 81 shows a truss having an odd number of panels. There is no stress in the dotted cross diagonals in the middle panel except

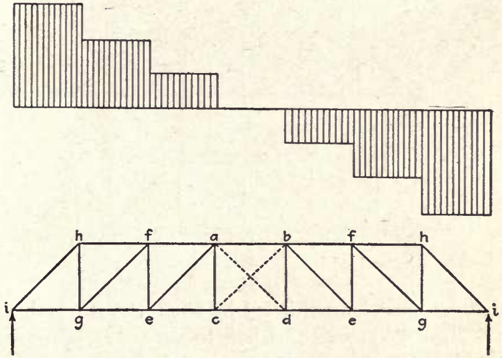


Fig. 81.

in case of wind or rolling loads, or otherwise unbalanced loading. Coefficients may readily be written for uniform and symmetrical loadings for this case, or the loads may be followed from the point of zero shear in cases of unsymmetrical loading, or the shear method may be followed.

In Fig. 82 (*c*) is shown a truss with a subvertical and subdiagonal at each end. Such an arrangement involves the consideration of an additional triangle in which half the weight is added to the load at *b* and is then carried to *a*, the other half being added to the load at *c*. This arrangement offers no difficulty when figured by the shear method, but sometimes causes trouble and confusion when an attempt is made to trace out the loads from the middle panel, or point of zero shear.

Some trusses have nonparallel chords. The shapes vary from those higher at one end, as in Fig. 82 (*a*), to those approaching an arch form as at (*b*). Part of the shear is carried by the sloping chord. When the chord stress is found by one of the preceding methods it is the horizontal stress. For a sloping chord the horizontal stress must be multiplied by the inclined length and the product divided by the panel length, the result being the axial (longitudinal) stress in the inclined member.

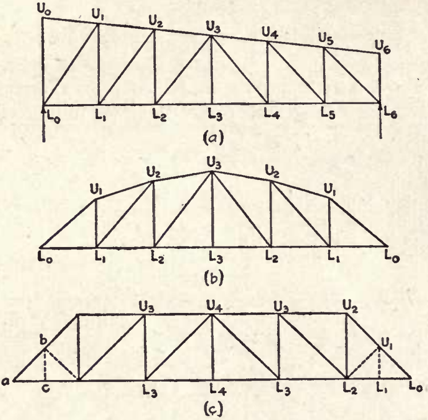


Fig. 82.

In the Warren truss (Fig. 83) the stresses in the web members are alternately tension and compression, the light lines indicating tension and the heavy lines compression. Each panel is an equilateral triangle and in the figure the truss is a single system. By using another set of triangles and placing the trusses side by side so one triangle overlaps another by half the width, we obtain a double system. Similarly, we may use a triple-system or a four-system truss. When two or more systems are used the result is a Latticed Truss, (Fig. 84).

Let W = total load on the truss, uniformly distributed,

P = load on each triangle,

n = number of triangles in the primary single system.

Then, in a single system truss, $P = \frac{W}{n}$.

In a double-system truss, $P = \frac{W}{(2n) - 1}$.

In a triple-system truss, $P = \frac{W}{(3n) - 1}$.

In a four-system truss, $P = \frac{W}{(4n) - 1}$.

Having found the panel load, P, each system is figured as a frame, and the combined strength of the systems determines the

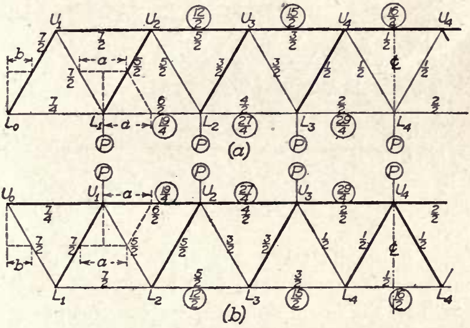


Fig. 83 — Warren Truss.

strength of the completed truss. The systems are connected together at every joint where the members meet or cross. The lower apices are the panel joints when the load is on the lower chord and the upper apices are the panel joints when the load is on the upper chord.

In Fig. 83 (*a*) the load is on the lower chord and in (*b*) the load is on the upper chord. The truss is here assumed to be uniformly and symmetrically loaded. Coefficients may be written by starting from the center line of the span. On the upper and lower chords are placed the summation of the coefficients for the chords in the respective panels. In Fig. 83 (*a*) the coefficient for L_0L_1 is one-half that for U_1U_2 and in (*b*) the coefficient for U_0U_1 is one-half that for L_1L_2. The dotted parallelograms at L_0 and L_1, and U_0 and U_1, represent to scale the panel load set off vertically with the parallelogram of forces completed by drawing horizontal lines to intersect the triangles. Then a = stress on U_1U_2 = stress on L_1L_2. It is twice b which represents the stress on U_0U_1 and L_0L_1. The thrust of the brace U_1L_0 = the pull of the tie U_0L_1. It is resolved at the point of support on the abutment into a horizontal component along the chord, and a downward vertical component, which latter is resisted by the upward reaction of the abutment.

A usual ratio of depth to span in trusses is one-tenth, but circumstances may alter this. It may be used in the absence of computations to ascertain the economic depth and economic

Fig. 84. — Multiple System Warren Truss. (Lattice Truss.)

ratio of depth to span. For Howe trusses the best angle for the diagonals is 45 degrees. When any different angle which indicates a panel length greater or less than the depth is adopted, the Pratt truss is better. For trusses of the Warren type the angle should be 60 degrees.

Deflection is usually taken care of by making the horizontal panel length at the upper end $\frac{1}{8}$ inch longer than the horizontal panel length at the lower end, in every ten feet of span. This does not alter the lengths of the verticals but does alter the lengths of diagonals and when the truss is in place the bottom chord will be cambered upward. Were it perfectly straight it would appear to the eye to sag. The amount of camber in inches is found as follows:

d = depth of truss in inches.

s = span of chord in inches.

c = camber in inches.

$$c = \frac{8d}{s}.$$

In some of the figures of trusses the spaces are lettered. This is the system introduced by Mr. Bow for the graphical analysis of frames. The member is indicated by the letters between which it lies. In addition to this system of lettering the spaces the joints are sometimes numbered. The spaces are lettered to identify the member in the graphical analysis and the joints are numbered only when the detail drawing of the joint is to be referred to.

Some of the trusses shown have the joints lettered with a capital U on the upper chord and a capital L on the lower chord. The subscript figure represents the number of the joint from the left end, the joint, or joints, at the abutments being O. In the drawings a joint is referred to by the U or L and the subscript indicating the number of the joint. A member is identified by giving the letter and subscript number of the joint at each end of the member. This method of identifying joints and members is common.

Architects and designers of buildings have to deal with the simpler forms of trusses, but when it is desirable to introduce the maximum economy into a design, that truss is most economical in which the stresses in the chords are constant from end to end. This points to a truss having the general outline of a bowstring girder. The top chord should be straight and not curved between joints. To obtain a curved outline for a roof it is easy to use fillers or vary the depths of the purlins resting on the trusses. For an exposed chord where the polygonal form would be unsightly the expedient is sometimes adopted of curving the segments, thereby introducing bent beams with arching action. This should never be done. It is better to use a false curved chord in segments to hide the short straight pieces.

The stresses in the top and bottom chord of a bowstring truss are found with sufficient accuracy by assuming the truss to be uniformly loaded. The moment divided by the depth gives the maximum stress at the center of the top chord and throughout the lower chord, the formula being

$$T = C = \frac{wl^2}{8d}.$$

in which T = total tension,
C = total compression,
l = length in feet,
w = uniform load per lineal foot,
d = depth in feet at center of span.

The chord assumes some of the functions of braces as the ends are approached, where the inclination of the chord increases, and the compression is nearly uniform throughout the length. The compression at any point distant y feet from the center is given by the following formula,

$$C = \sqrt{\left(\frac{wl^2}{8d}\right) + (wy)^2}.$$

The stress in the braces increases from the ends to the center, as in the case of the Queen truss, and may be figured the same way. The vertical rods at the joints are in tension and the braces are in compression. The center panel is usually as wide as the height, which decreases the angles at which the braces are set as they approach the ends.

Steeply pitched roofs of the Howe truss type may be figured by the method of coefficients when the loads are uniform and symmetrically placed. They may be figured by the cumulative load method or by the shear method when unsymmetrically or

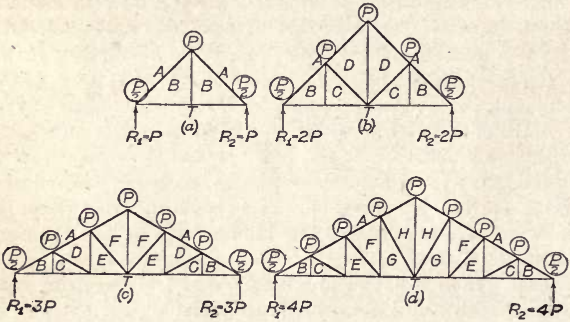


Fig. 85A. — (*a*) King Truss: (*b*) (*c*) (*d*) Queen Trusses.

irregularly loaded. In Fig. 85A the truss with one vertical is a King truss. When the vertical is a post it is a King Post truss and when the vertical is a tie it is a King Rod truss. At (*b*) and (*c*) are shown Queen trusses, these being known by the number of panels into which they are divided and, like the King truss, being Queen Post or Queen Rod trusses, as the verticals may be posts or rods.

In a system of roof framing all longitudinal members are called purlins and all members extending from the eaves to the ridge are rafters. The top chord of a truss is composed of rafters. Main purlins extend from truss to truss, resting on the joints at the upper ends of verticals. Intermediate rafters rest on the main purlins when the spacing between trusses is considerable and across the rafters sheathing is placed to carry the roof covering. By so doing all loads are concentrated at the upper ends of the verticals, so the truss rafters (upper chord) are in compression. Sometimes no intermediate rafters are used, the roofing being carried by purlins resting on the joints.

To obtain proper results the sloping rafter of the truss is divided into equal spaces and verticals are dropped to the bottom chord (or tie). Braces extend from the foot of one vertical to the top of another. If, through any error or because it is considered best, intermediate purlins rest on the truss rafters, or the roof is carried directly on these rafters, it will be necessary to design them to carry the bending stress in addition to the direct compression.

In the King truss, Fig. 85A (*a*), the load, P, when applied at the upper vertex causes no stress in the rod BB. When applied at the lower end the stress is tension and equal to the load. The stress AB = half the load, so the coefficient = $\frac{1}{2}$. The stress in the horizontal tie rod = half the load $\times \dfrac{\text{half the span}}{\text{height}}$, that is

$$T = \frac{P}{2} \times \frac{L}{2d} = \frac{PL}{4d}$$

In the Queen truss the action resembles an arch in that the compressive and tensile stresses increase towards the supports in the rafters and tie, and the stresses in the verticals and diagonals decrease toward the supports, for the inclined rafters carry part of the shear. Half of the load on each end panel is carried by the abutments and creates no stress in the truss.

In Fig. 85A (*b*) a load is assumed to be applied at the upper end of BC. If the load is at the lower end the rod BC carries this load to the rafter at the vertex of the triangle. If the load is applied directly to the rafter at the vertex there is no stress in BC. This will not again be referred to, as it applies to the rod in the end triangle in all trusses. Half the load is carried on AB and half goes down CD to the tie. The load on the top of the truss is increased by the load coming to the tie by the braces CD on both sides of the center; therefore it is $2P$, if the load on a joint is called P. The vertical rod, DD, however, carries only the load at the lower end. This load $2P$ from the top of the truss is carried halfway down the truss to the joint and there it has added to it the load $\frac{P}{2}$ of the brace AB, this brace therefore carrying a load $= \frac{3P}{2}$. The tie rod is in tension by an amount

$$= \frac{3P}{2} \times \frac{\text{half the span}}{\text{height}} \quad \text{or} \quad T = \frac{3P}{2} \times \frac{L}{2d} = \frac{3PL}{4d}.$$

In Fig. 85A (*c*) the rafter is divided into three equal parts. Each joint carries a load, P. The load on $DC = \frac{P}{2}$; on $DE = \frac{3P}{2}$; on $EF = \frac{5P}{2}\cdot$ The vertical FF carries $3P$. The rafter AF carries $\frac{3P}{2}$; AD carries $\frac{5P}{2}$; AB carries $3P$. The tie rod carries at the end a stress $= \frac{3PL}{2d}\cdot$ The stress in the tie rod on the section $TE = \frac{5PL}{4d}\cdot$ In actual practice the tie rod is uniform in size throughout the span.

With the examples given the student should have no trouble tracing the loads on the members of the truss shown at (*d*).

Each vertical is in tension by an amount equal to the load it carries. Each diagonal member is in compression by an amount $= \frac{xL}{d}$ in which x = amount of load on the member,

L = length of member,

d = the vertical height from the bottom to the top of the member.

All measurements are on center lines. The slope of the rafter is constant so the ratio is obtained once by dividing the slant length of the rafter by the height of the truss. The slopes change at each panel for the interior braces, so a ratio must be found for each separately.

Coefficients for Fink trusses, Fan trusses, and Pratt trusses with inclined rafters have been calculated for different degrees of slope and for varying numbers of panels, based on uniform symmetrical loads. Tables of these coefficients are given on pages 309–311, of the 1913 edition of the "Carnegie Pocket Companion" for trusses to be made of steel or wrought iron. Steel is commonly used except when corrosion is a grave danger, in which case wrought iron is preferred. All metal trusses are made of rolled shapes with riveted connections. The trusses illustrated may be a combination of steel rods for tension members and wood for compression members. Fink trusses are very generally used because most of the members are in tension and the struts are short. Partial loading can never cause maximum stresses in the parts of Fink trusses as they may in other forms of trusses.

Roof Loads

For information as to proper roof loads and the effect of wind the student is referred to pages 305–307, 1913 edition "Carnegie Pocket Companion." This will also be dealt with in the chapter on "Graphic Statics." Usually city ordinances specify that a roof shall be capable of carrying 40 lbs. per square foot of horizontal surface, in addition to its own weight, this allowing for wind, snow, live load, and roofing. Some cities require only 25 lbs. and others 30 lbs. For a steeply pitched roof 25 lbs. is proper, but for a very flat roof the designing load should not be less than 50 lbs. per square foot. Each joint in a frame carries a load, P, equal to the truss spacing times the panel length multiplied by the load per square foot.

The Signs Used for Stresses

The author mentioned that the positive (+) sign indicates compression and the negative (−) sign indicates tension. This is the way he was taught, and thirty years ago this use of the signs was common with American and British writers. There was a certain mnemonic aid in using the signs thus, for compression thickens a body and tension makes it thinner, so the "minus sign" expressed the idea of thinness. In drawings the pieces in compression were indicated by heavy lines and the pieces in tension by light lines.

Continental European writers used the signs in a directly opposite sense, for strict mathematical analysis in which careful attention must be paid to the signs of quantities resulted in bringing compression out at the end with a negative sign and tension with a positive sign. The well-trained mathematician needs no aid from mnemonics. The result of late years has been to unsettle American and British authors, and a reader of modern books must be careful to ascertain just how the signs are used by the author. It is to be hoped that at some not distant day all writers will agree upon a definite use of the signs, but for the purposes of the present work the author believes the mnemonic value, as given above, is too great to be neglected.

CHAPTER V

Joints and Connections

FIG. 85B was copied from a sheet of drawings forming part of a set made for the building of a public school in a middle western state. The writer knows from experience in checking designs that this is a common type of roof. Amateur architects and many young draftsmen have a fondness for constructing wooden trusses with light tension members of wood. The student who read Chapter IV carefully knows how simple a matter it is to find the stresses in the members of a simple truss. There are many hand-books on the market which contain diagrams and formulas for the design of trusses, so it is not a difficult thing to proportion the members of any form of truss.

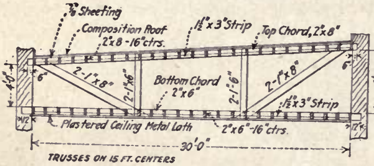


Fig. 85B.

The student is advised to check the design of the truss shown in Fig. 85B. It will be a very useful exercise. The joists are on 16-in. centers. The joists under the roof are 2 in. × 8 in. and carrying sheathing on which is placed the composition roofing. The ceiling joists are 2 in. × 6 in. In the design assume a fiber stress in tension and compression of 800 lbs. per sq. in., the loads per square foot of horizontal surface to be as follows: composition roofing, 5 lbs.; joists, 5 lbs.; sheathing, 4 lbs.; plastering, 5 lbs.; roof load 30 lbs.; truss, 4 lbs.

In the specifications appeared the following clause: " The truss is to be well and securely nailed together, but the number of nails at each joint shall not be less than shown on the drawings." The draftsman had placed eight dots at each joint, probably meaning each dot to represent a nail. The size of nail to use was not given. On the advice of the author the roof was changed so it appeared as if inverted, putting all the diagonal members in compression and substituting vertical wrought iron rods for the thin boards used in the original design. Before accepting the change, however, the architect attempted to retain his truss by substituting in each joint four $\frac{5}{8}$-in. bolts, saying that he had been architect for more than twenty schoolhouses, in which he had used the form of truss shown in his drawings.

When a member is in compression the joint is not so hard to construct, or design, as a connection at the end of a tension member. It is an elemental fact that when a piece in compression rests on another piece there must be a bearing area of the proper size to prevent crushing. It is only in joints for pieces in tension that the average draftsman seems to forget elementary principles. He laps one piece over another, specifies that the contractor shall "nail it securely," and passes on to something else. It is either because of laziness or downright ignorance that such things happen. Experienced contractors are often life savers for incompetent draftsmen, for we can hardly call them designers.

When there is a pull in a board used to transmit tension in a truss the joint must be strong enough to resist the pull. It is necessary to decide on the size of nail to use and then divide the total pull by the resisting power of one nail to obtain the number of nails to use. To avoid splitting the wood the nails should be separated by a space at least twenty times the thickness of the nail. Even this rule must be modified by the sort of wood used, as some wood is brittle and cannot stand many nails in a small space. Wire nails do not cut through the fibers, but spread them apart so it is not necessary to make up the area occupied by the nails, but this must be considered in using cut nails. The proper size of nail to use is governed largely by the length. It must be not less than three times the length of the thinnest outside piece. If it cannot go two-thirds its length into wood, because the wood is not thick enough, the end must be firmly clinched.

A rule often blindly given in pocket books for the lateral resisting value in pounds for nails is as follows:

$P = Cd$, in which

P = total number of pounds transverse load per nail,

C = a coefficient varying from 4.5 to 12, depending on the nail, whether wire or cut, and on the wood,

d = the size of nail in pennyweights.

A more logical rule given by Mr. H. D. Dewell, in *Western Engineering*, Vol. 7, page 291, is as follows for Douglas Fir,

$$P = 4000\ d^2,$$

in which d = diameter of nail in inches.

For other woods multiply the result by the following coefficients:

Long-leaf yellow pine	1.05
White pine	.78
Norway pine	.65
White oak	.78

For common wood screws use the constant 4375 instead of the constant 4000 used for nails.

Nails or common wood screws are generally thought of first for fastening timber because they are cheap and the labor cost of driving them is low. Their usefulness, however, is limited to thin pieces carrying little stress. When the load is too great to be transmitted properly by nails or common screws, or the pieces are too thick, lag screws may be used on account of the low labor cost as compared with that required for bolts, for which holes must first be bored. Mr. H. D. Dewell, in *Engineering News*, Vol. 76, page 797, gives the following recommended working values for lag screws.

Description	Pounds per screw
Metal plate lagged to timber, ¾ × 4½-in. screw	1030
Metal plate lagged to timber, ⅞ × 5-in. screw	1200
Timber planking lagged to timber, ¾ × 4½-in. screw	900
Timber planking lagged to timber, ⅞ × 5-in. screw	1050

Generally speaking the resistance of lag screws varies with the ratio of their diameters, so the values above given may be used as a basis for other sizes.

The strength of nails, lag screws, and bolts in wood cannot be computed the same as rivets in metal, for the rivets may shear, but this is impossible with joints in wood. Nails, screws, or bolts will bend, for the wood will crush long before the shearing strength of the metal is reached. It is necessary, therefore, to use bearing values obtained by experiments. Mr. H. D. Dewell, in *Engineering News*, Vol. 76, page 115, described the result of tests made with bolts. Two thin pieces of timber were fastened to a thick piece, by bolts passing through with washers on the ends. The two side pieces were placed on the table of a testing machine and the center piece was pressed down until failure resulted.

The following table is recommended as giving proper safe loads corresponding to a slip not exceeding $\frac{1}{32}$ in. They are for end bearing with bolts having a driving fit and the thickness of each side piece equal to, or greater than, one-half the thickness of the main timber. For side bearing (across the grain) the values can be taken at six-tenths the values given for end bearing in the table.

The values in the table are for double shear, that is, for three pieces of timber having two shearing planes. For cases of single shear, two timbers bolted together, use one-half the values given in the table.

The working values given are for Douglas fir. For other timbers the values are to be multiplied by the factors following:

Long-leaf yellow pine	1.05
White pine	.78
Norway pine	.65
White oak	.78

TABLE OF WORKING STRENGTH OF ONE BOLT IN TIMBER JOINT IN DRY TIMBER, AS FOR USE IN INTERIOR OF BUILDING

(Bolt in double shear bearing against end of grain.)

Size of Bolt Ins. diam.	Thickness of One Side Piece $\overline{\overline{<}}$ One-half Thickness of Center Timber =			
	2 Ins.	3 Ins.	4 Ins.	6 Ins.
⅝	1057	1275	1460	1460
¾	1450	1665	1980	2100
⅞	1900	2130	2450	2850
1	2460	2664	2970	3705

For single shear take one-half the above values. For bearing against the side of grain take six-tenths the above values.

Joints in timber are sometimes made with a large pin on which a timber rests. The pieces are joined at an angle less than 90 degrees and the bearing per square inch on the fibers will be something less than the allowable safe bearing with the grain and considerably greater than the allowable safe bearing across the grain for broad surfaces.

Let n = the bearing per sq. in. on the diametrical area of the pin, having a driving fit.

p = the allowable bearing per sq. in. on the ends of the fibers,

q = the allowable bearing per sq. in. perpendicular to the direction of the fibers.

$n = \frac{1}{3}p + \frac{2}{3}q.$ (Dewell formula)

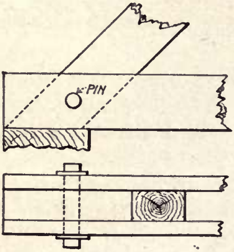


Fig. 86.

Example. — A brace rests against a 2-inch pipe driven through the bottom chord of a truss to act as a pin joint. The allowable bearing on the ends of the fibers is 1200 lbs. per square inch and the allowable cross bearing is 300 lbs. per square inch.

What bearing can be used on the diametral area of the pin?

Answer. — $(\frac{1}{3} \times 1200) + (\frac{2}{3} \times 300) =$ 600 lbs. per sq. in.

Mr. H. D. Dewell was Chief Structural Engineer, Panama-Pacific International Exposition, San Francisco, 1915. The possibilities of timber construction were probably never better treated than in the work under his charge, and a great many experiments and detailed studies were made in order to design properly. The results were given by him in a series of articles in the June to December issues (inclusive), 1916, of *Western Engineering*, San Francisco, Cal. The author has made free use of the articles by Mr. Dewell and discarded much material prepared for this chapter, based on older writings, some of which were speculative and some of which were founded on experimental work conducted by men not so skilled as are the modern experimenters.

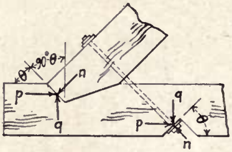


Fig. 87.

Standards are well enough in their places, but men should not blindly use standards without knowing all reasons and the authority. Standard washers should not be used, merely on the advice of an advertisement writer, to carry certain loads until their sufficiency has been checked by computations for the wood with which they are to be used. If a washer is too small it will be drawn into, and crush the fibers of, the wood.

Bolts placed through the joints of trusses will have washers as shown in Fig. 87. One washer will bear on the side of the fibers and the other washer will bear at an angle to the fibers. Using the proper table giving the safe bearing strength of the wood perpendicular to the fibers proportion the side bearing washer accordingly. For the safe pressure to allow under the washer set at an angle to the fibers, and also to determine the safe bearing at the foot of the sloping member, several formulas have been proposed.

The Jacoby formula is as follows:

$$n = p \sin^2\theta + q \cos^2\theta,$$

as given in his book "Structural Details." In *Engineering News*, Vol. 68, Professor Malverd A. Howe published the result of some tests made to determine the allowable bearing pressure on inclined surfaces for various timbers, and recommended the formula

$$n = q + (p - q) \left(\frac{\theta^\circ}{90^\circ}\right)^{\frac{5}{2}}$$

in which n = the allowable unit stress on a surface which makes an angle θ with the direction of the fibers,

p = the allowable unit stress against the ends of the fibers,

q = the allowable unit stress on the sides of the fibers.

The author in attempting to simplify the formula of Professor Jacoby and make it fit closer to some experiments on timber other than yellow pine, which the Jacoby formula closely fitted at low angles, developed the following straight-line formula.

$$n = \frac{P\theta}{100}, \text{ minimum value equal to } q. \quad \text{Straight line } 80^\circ \text{ to } 90^\circ.$$

Later, learning of the Howe formula, he attempted to simplify it and developed the following formula:

$$n = \frac{\theta}{0.81} \left(\frac{\theta}{100}\right)^2, \text{ minimum value equal to } q.$$

In the two formulas of the author the angle θ is expressed in figures, as 10, 20, 30, etc.

In Fig. 88 the four formulas are platted. The diagram appeared in the July, 1916, *Western Engineering*, with the Jacoby and Howe formulas, the author adding here the curves produced by his own formulas.

The student should now be able to determine the size and number of nails, screws, lag screws, or bolts for all the joints of the truss shown in Fig. 85B. It being assumed that wire nails and common screws merely push the fibers aside, no additional area is required on account of the holes. Lag screws and bolts, however, cut the fibers, so it is necessary to add to the members a width equal to the diameters of the holes. By the time the student has worked through the problem of detailing all the joints he will probably learn that there is not area enough for fastenings. The problem will serve to show why timber should not be used for members in tension unless the load from other members can be transferred by end thrust. It is best when using wood to have all joints in compression. Use wrought iron or steel for tension members.

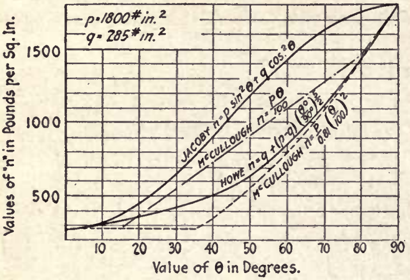


Fig. 88.

Fig. 89 illustrates several types of joints and fastenings used in framing timber. Bolts in carpentry should be used, when possible, only to hold abutting portions of timber together. It is not always possible to so use them, but the hint is enough to set a man to studying seriously every joint he makes in order to hold to the rule if possible. Straps are more expensive than bolts and are not so good. A detail that is too common and which should never be used is shown at (*a*); what happens when the wood shrinks is shown at (*b*), for it is impossible to tighten the joint. Water getting into the toe of a joint often causes the fibers to decay and this added to shrinkage ruins the truss. A preferable method is shown at (*c*). The bolt may be tightened from time to time. This form of joint, however, should never be designed with part of the load carried by the abutting end and part by the inclined bolt. Many experiments have demonstrated that under test the two do not act together. The weaker system will act first and give way before the other gets into action. It is best to design for the entire load to be taken by the dap and use the inclined bolt only for the purpose of holding the pieces in contact. All bolts will be screwed tight so that washers should be designed with area enough to develop the safe working stress in the bolts without exceeding the safe bearing on the wood. If the safe bearing is exceeded, which will be the case if the washers are too small, the fibers will be cut around the edges of the washers and decay will set in.

At (*d*) is shown a method sometimes adopted for upper chord joints and at (*e*) is shown what happens when the wood shrinks. A proper joint is shown at (*f*) with the center lines of all pieces meeting at a common point. This idea of all the center lines meeting at a common point is also illustrated at (*g*) and (*h*). This is necessary to avoid rotation which would cause a bending moment. When part of a piece of timber is cut out for a dap the stress is concentrated in the rest of the piece, which alters the position of the center lines. This must be considered in the design and will be discussed when we take up the design of joints.

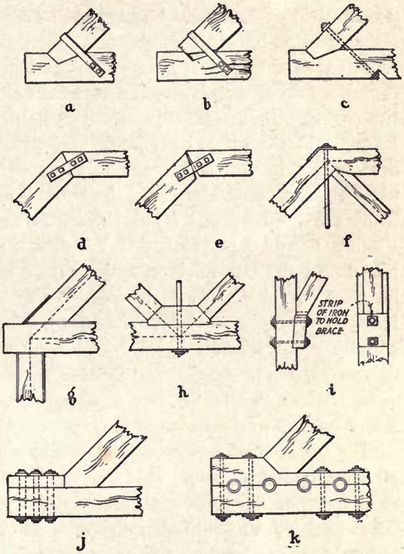


Fig. 89

At (*i*) is shown a brace abutting on a post. If a waling is spiked along a line of posts, or blocks with the fibers in a horizontal position are bolted to the posts, and the braces rest on the sides of the fibers, there will be settlement when shrinkage occurs. When blocks, or cleats, are used as shown they should have the fibers vertical and the dap should be deep enough to transmit all the load to the post. The bolts shown are used only to hold the cleats in place. To design the bolts, however, it is well to have them strong enough to carry half the load, which will maintain the integrity of the joint in case the cleats through shrinkage, or decay of dap, lose one-half their strength. To prevent the braces from being pushed off to one side they may be nailed to the cleats or a thin strip of iron may be inserted in a saw cut, one-half in the cleat and one-half in the foot of the brace.

At (j) is shown a method often used in making a joint at the foot of a raking member. The vertical bolts are assumed to resist the thrust. It is a poor joint. The habit of many draftsmen is to guess at the number of bolts, and they seldom add additional area to supply that lost in the bolt holes. A better joint is shown at (*k*). The black circles are the ends of pipes used as shear pins. They are designed to take all the thrust and the joint is made by spiking the two pieces together, after which the bolt holes are bored and the bolts driven and tightened. Enough bolts should be used to take half the tension in the lower chord; as direct tension, not cross bearing. After the bolts are tightened the spike can be removed and holes bored for the pins which are then driven through.

In making joints in woodwork study the problem carefully. *Avoid pockets where moisture may collect.* Cracks seriously weaken timber framework, and as cracks usually start from interior angles all complicated framing joints should be avoided. Frame all pieces so the center lines *through stressed areas* will meet at a common point and try to make all jointing lines straight. Jogs in joints not only are apt to start cracks, but there is a danger of shrinkage, causing unequal bearing, which will set up bending moments about the joint, so it is best to have straight joints which admit of adjustment. If through any mischance the two faces do not meet perfectly, insert a thin sheet of lead, which, in course of time, will equalize the bearing. Engineering principles were seldom used for timber joints in former generations, much of the work being done by carpenters who had no engineering training and by draftsmen who blindly copied old examples. Design the joint in accordance with engineering principles, not forgetting that wood shrinks and that when moist it rots. The joint is bound to be all right if it is simple and the computations show it to be adequate for the work it must do.

In studying joints of members they are considered as single sticks. Methods for making single sticks out of a number of pieces will be shown, but, while it makes very little difference for tension pieces, it is bad practice to use several pieces to form compression members when single sticks can be obtained.

In Fig. 90 is shown the truss illustrated in Fig. 80, the computations for which were made as an exercise. We will now proceed to design some of the joints. Assume the wood to be Yellow

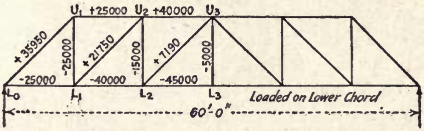


Fig. 90.

Pine, Grade 1 (Underwriter's Code), in which the allowable safe stresses are as follows:

Tension	1600 lbs. per sq. in.
Compression (end bearing)	1200 lbs. per sq. in.
Compression (side bearing)	350 lbs. per sq. in.
Shear (with the grain)	120 lbs. per sq. in.

The stresses in the steel are as follows:

Tension	16,000 lbs. per sq. in.
Shear	10,000 lbs. per sq. in.
Bearing	20,000 lbs. per sq. in.

For the strength of bolts and lag screws in combined shear, bearing and cross bending, see pages 138 and 139.

The compression in L_0U_1 is 35,950 lbs. and the tension in L_0L_1 25,000 lbs. The lower chord is always dimensioned to take care of the maximum tension, which in this case is 45,000 lbs. This leaves considerable excess material in the end panels, which is available for cutting to form connections. Similarly, the upper chord is dimensioned for the maximum compression and the size is uniform throughout.

Before proceeding to design the connection joints at the ends of a truss and at the ends of panels it is necessary to know the sizes of the members. If material has to be cut out for the formation of joints the required area can be added to the member provided it has not enough excess material to provide the necessary area for the details. Proceeding in this manner we will study all the usual methods for making joints in tension and compression members and then design the joints.

It is not always possible to get a single stick to use as a tension or compression chord in a truss, and it is necessary to build them up in nearly every instance. If a single stick is used the area is increased to allow for the hole through which the largest vertical rod passes. In the truss now being considered the sizes of the rods are found as follows:

The rod at U_3 will be 1 in. diameter if the threaded ends are upset, and if the ends are not upset the diameter will be $1\frac{3}{8}$ ins. This rod may be smaller as the stress is very low, but it is not advisable to use a rod of smaller diameter for such a length. The stress in rod at U_2L_2 will be the same as the rod at U_3, for the stress is 15,000 lbs. and the allowable stress in the steel is 16,000 lbs. per square inch. The net area of the rod at $U_1L_1 = \frac{45{,}000}{16{,}000} = 1.56$ sq. ins., and we will use a rod $1\frac{1}{2}$ ins. diameter if the threaded ends are upset. If upset ends are not used the loss due to cutting of the threads will require the use of a rod $1\frac{3}{4}$ ins. diameter. The following table gives the dimensions of rods with upset screw ends. To obtain the diameter at the root of the thread cut in a bolt use the figures in the second column for the outside diameter and the diameter at the root of the thread will be found in the third column. The screw threads in the table are Franklin Institute standards. In the 1913 edition of the "Carnegie Pocket Companion" the tables for upset screw threads are American Bridge Company standards.

REMARKS. — As upsetting reduces the strength of iron, bars having the same diameter at root of thread as that of the bar invariably break in the screw end, when tested to destruction, without developing the full strength of the bar. It is therefore necessary to make up for this loss in strength by an excess of metal in the upset screw ends over that in the bar.

To make one upset end for 5-inch length of thread, allow 6-inch length of rod additional.

UPSET SCREW ENDS FOR ROUND AND SQUARE BARS

Diameter of Round or Side of Square Bar, Inches	Round Bars				Square Bars			
	Diameter of Upset Screw End, Inches.	Diameter of Screw at Root of Thread, Inches.	Threads per Inch. No.	Excess of Effective Area of Screw End over Bar. Per cent.	Diameter of Upset Screw End, Inches.	Diameter of Screw at Root of Thread, Inches.	Thread per Inch. No.	Excess of Effective Area of Screw End over Bar. Per cent.
1/2	3/4	.620	10	54	3/4	.620	10	21
9/16	3/4	.620	10	21	7/8	.731	9	33
5/8	7/8	.731	9	37	1	.837	8	41
11/16	1	.837	8	48	1	.837	8	17
3/4	1	.837	8	25	1 1/8	.940	7	23
13/16	1 1/8	.940	7	34	1 1/4	1.065	7	35
7/8	1 1/4	1.065	7	48	1 3/8	1.160	6	38
15/16	1 1/4	1.065	7	29	1 3/8	1.160	6	20
1	1 3/8	1.160	6	35	1 1/2	1.284	6	29
1 1/16	1 3/8	1.160	6	19	1 5/8	1.389	5 1/2	34
1 1/8	1 1/2	1.284	6	30	1 5/8	1.389	5 1/2	20
1 3/16	1 1/2	1.284	6	17	1 3/4	1.490	5	24
1 1/4	1 5/8	1.389	5 1/2	23	1 7/8	1.615	5	31
1 5/16	1 3/4	1.490	5	29	1 7/8	1.615	5	19
1 3/8	1 3/4	1.490	5	18	2	1.712	4 1/2	22
1 7/16	1 7/8	1.615	5	26	2 1/8	1.837	4 1/2	28
1 1/2	2	1.712	4 1/2	30	2 1/8	1.837	4 1/2	18
1 9/16	2	1.712	4 1/2	20	2 1/4	1.962	4 1/2	24
1 5/8	2 1/8	1.837	4 1/2	28	2 3/8	2.087	4 1/2	30
1 11/16	2 1/8	1.837	4 1/2	18	2 3/8	2.087	4 1/2	20
1 3/4	2 1/4	1.962	4 1/2	26	2 1/2	2.175	4	21
1 13/16	2 1/4	1.962	4 1/2	17	2 5/8	2.300	4	26
1 7/8	2 3/8	2.087	4 1/2	24	2 5/8	2.300	4	18
1 15/16	2 1/2	2.175	4 1/2	26	2 3/4	2.425	4	23
2	2 1/2	2.175	4	18	2 7/8	2.550	4	28
2 1/16	2 5/8	2.300	4	24	2 7/8	2.550	4	20
2 1/8	2 5/8	2.300	4	17	3	2.629	3 1/2	20
2 3/16	2 3/4	2.425	4	23	3 1/8	2.754	3 1/2	24
2 1/4	2 7/8	2.550	4 1/2	28	3 1/8	2.754	3 1/2	18
2 5/16	2 7/8	2.550	4 1/2	22	3 1/4	2.879	3 1/2	22
2 3/8	3	2.629	3 1/2	23	3 3/8	3.004	3 1/2	26
2 7/16	3 1/8	2.754	3 1/2	28	3 3/8	3.004	3 1/2	19
2 1/2	3 1/8	2.754	3 1/2	21	3 1/2	3.100	3 1/4	21
2 9/16	3 1/4	2.879	3 1/2	26	3 5/8	3.225	3 1/4	24
2 5/8	3 1/4	2.879	3 1/2	20	3 5/8	3.225	3 1/4	19
2 11/16	3 3/8	3.004	3 1/2	25	3 3/4	3.317	3	20
2 3/4	3 3/8	3.004	3 1/2	19	3 7/8	3.442	3	23
2 13/16	3 1/2	3.100	3 1/4	22	3 7/8	3.442	3	18
2 7/8	3 5/8	3.225	3 1/4	26	4	3.567	3	21
2 15/16	3 5/8	3.225	3 1/4	21	4 1/8	3.692	3	24
3	3 3/4	3.317	3	22	4 1/8	3.692	3	19
3 1/8	3 7/8	3.442	3	21	4 3/8	3.923	2 7/8	24
3 1/4	4	3.567	3	20	4 1/2	4.028	2 3/4	21
3 3/8	4 1/8	3.692	3	20	4 5/8	4.153	2 3/4	19
3 1/2	4 1/4	3.798	2 7/8	18				

The following table gives weights and areas for square and round bars and rods.

WEIGHTS AND AREAS OF SQUARE AND ROUND BARS AND CIRCUMFERENCES OF ROUND BARS

(One cubic foot of steel weighing 489.6 lbs.)

Thickness of Diameter in inches	Weight of □ Bar One Foot Long	Weight of ○ Bar One Foot Long	Area of □ Bar in Sq. Inches	Area of ○ Bar in Sq. Inches	Circumference of ○ Bar in Inches
0					
1/16	.013	.010	.0039	.0031	.1963
1/8	.053	.042	.0156	.0123	.3927
3/16	.119	.094	.0352	.0276	.5890
1/4	.212	.167	.0625	.0491	.7854
5/16	.333	.261	.0977	.0767	.9817
3/8	.478	.375	.1406	.1104	1.1781
7/16	.651	.511	.1914	.1503	1.3744
1/2	.850	.667	.2500	.1963	1.5708
9/16	1.076	.845	.3164	.2485	1.7671
5/8	1.328	1.043	.3906	.3068	1.9635
11/16	1.608	1.262	.4727	.3712	2.1598
3/4	1.913	1.502	.5625	.4418	2.3562
13/16	2.245	1.763	.6602	.5185	2.5525
7/8	2.603	2.044	.7656	.6013	2.7489
15/16	2.989	2.347	.8789	.6903	2.9452
1	3.400	2.670	1.0000	.7854	3.1416
1/16	3.838	3.014	1.1289	.8866	3.3379
1/8	4.303	3.379	1.2656	.9940	3.5343
3/16	4.795	3.766	1.4102	1.1075	3.7306
1/4	5.312	4.173	1.5625	1.2272	3.9270
5/16	5.857	4.600	1.7227	1.3530	4.1233
3/8	6.428	5.049	1.8906	1.4849	4.3197
7/16	7.026	5.518	2.0664	1.6230	4.5160
1/2	7.650	6.008	2.2500	1.7671	4.7124
9/16	8.301	6.520	2.4414	1.9175	4.9087
5/8	8.978	7.051	2.6406	2.0739	5.1051
11/16	9.682	7.604	2.8477	2.2365	5.3014
3/4	10.41	8.178	3.0625	2.4053	5.4978
13/16	11.17	8.773	3.2852	2.5802	5.6941
7/8	11.95	9.388	3.5156	2.7612	5.8905
15/16	12.76	10.02	3.7539	2.9483	6.0868
2	13.60	10.68	4.0000	3.1416	6.2832
1/16	14.46	11.36	4.2539	3.3410	6.4795
1/8	15.35	12.06	4.5156	3.5466	6.6759
3/16	16.27	12.78	4.7852	3.7583	6.8722
1/4	17.22	13.52	5.0625	3.9761	7.0686
5/16	18.19	14.28	5.3477	4.2000	7.2649
3/8	19.18	15.07	5.6406	4.4301	7.4613
7/16	20.20	15.86	5.9414	4.6664	7.6576
1/2	21.25	16.69	6.2500	4.9087	7.8540

SQUARE AND ROUND BARS — *Continued*

Thickness of Diameter in Inches	Weight of □ Bar One Foot Long	Weight of ○ Bar One Foot Long	Area of □ Bar in Sq. Inches	Area of ○ Bar in Sq. Inches	Circumference of ○ Bar in Inches
9/16	22.33	17.53	6.5664	5.1572	8.0503
5/8	23.43	18.40	6.8906	5.4119	8.2467
11/16	24.56	19.29	7.2227	5.6727	8.4430
3/4	25.71	20.20	7.5625	5.9396	8.6394
13/16	26.90	21.12	7.9102	6.2126	8.8357
7/8	28.10	22.07	8.2656	6.4918	9.0321
15/16	29.34	23.04	8.6289	6.7771	9.2284
3	30.60	24.03	9.0000	7.0686	9.4248
1/16	31.89	25.04	9.3789	7.3662	9.6211
1/8	33.20	26.08	9.7656	7.6699	9.8175
3/16	34.55	27.13	10.160	7.9798	10.014
1/4	35.92	28.20	10.563	8.2958	10.210
5/16	37.31	29.30	10.973	8.6179	10.407
3/8	38.73	30.42	11.391	8.9462	10.603
7/16	40.18	31.56	11.816	9.2806	10.799
1/2	41.65	32.71	12.250	9.6211	10.996
9/16	43.14	33.90	12.691	9.9678	11.192
5/8	44.68	35.09	13.141	10.321	11.388
11/16	46.24	36.31	13.598	10.680	11.585
3/4	47.82	37.56	14.063	11.045	11.781
13/16	49.42	38.81	14.535	11.416	11.977
7/8	51.05	40.10	15.016	11.793	12.174
15/16	52.71	41.40	15.504	12.177	12.370

For designing plate washers the following table will be useful.

TANK IRON AND STEEL, WEIGHT OF SUPERFICIAL FOOT

Thickness in Inches	Weight in Lbs.		Thickness in Inches	Weight in Lbs.	
	Iron	Steel		Iron	Steel
1/32 = .03125	1.27	1.30	5/16 = .3125	12.63	12.88
1/16 = .0625	2.52	2.57	3/8 = .375	15.16	15.46
3/32 = .09375	3.79	3.87	7/16 = .4375	17.68	18.03
1/8 = .125	5.05	5.15	1/2 = .5	20.21	20.61
5/32 = .15625	6.32	6.45	9/16 = .5625	22.73	23.19
3/16 = .1875	7.58	7.73	5/8 = .625	25.26	25.77
7/32 = .21875	8.84	9.02	3/4 = .75	30.31	30.92
1/4 = .25	10.10	10.30	7/8 = .875	35.37	36.08
9/32 = .28123	11.38	11.61	1 = 1.	40.42	41.23

The low temperature (as compared with iron) at which steel plates have to be finished causes a slight springing of the rolls, leaving the plate thicker in the center. This, combined with greater density, causes steel plates, if kept up to full thickness on the edges, to weigh more than iron. Both iron and steel over 72 inches wide are liable to run even heavier than the weights given above.

Tables of sizes and weights of plates and bars for all the gauges in use are given in all the steel handbooks.

The areas of washers will be as follows, the cross bearing strength of the wood being 350 lbs. per square inch; U_3 and $L_3 = \frac{5000}{350} = 14.3$ sq. ins. The washer should extend across the chord, which we will assume is 8 ins. wide, so the width will be, $\frac{14.3}{8} = 1.78$ ins. Make it 2 ins. wide. The thickness $= \frac{4 \times (2 \times 350)}{2} = 1400$ in. lbs. bending moment considering it to be two cantilever beams, one on either side of the bolt extending to the edge of the chord.

The thickness $t = \sqrt{\frac{6M}{fb}} = \sqrt{\frac{14{,}000 \times 6}{16{,}000 \times 2}} = 0.51$ in. (make it $\frac{1}{2}$ in.). Tables of standard square and round washers may be used, if available, but the area must be sufficient to keep the stress on the side of the wood down to the allowable limit. The washer may be round or square, but it is best usually to have to go across the width of the chord. If square the above washer will be 3.81 ins. $\times$ 3.81 ins. If round the diameter will be $4\frac{3}{8}$ in.

The washers at U_2 and L_2 will have the following area: $\frac{15{,}000}{350} = 42.9$ sq. ins. If extended across the chord the dimensions will be $5\frac{1}{2}$ ins. $\times$ 8 ins. If square the dimensions will be 6.6 ins. $\times$ 6.6 ins. If round, the diameter will be $7\frac{1}{2}$ ins. Assuming a washer of steel with dimensions $5\frac{1}{2}$ ins. $\times$ 8 ins. the thickness will be obtained by using the following formula:

$$t = 1.73\sqrt{\frac{WLb}{f(L^2 + b^2)}}$$

in which t = thickness in inches,
W = total load centrally applied,
L = length of plate (width of chord) — bolt hole,
b = width of plate = bolt hole,
f = allowable unit fiber stress.

The above formula is good also for cast iron, using for the fiber stress a stress which is an average of the allowable tensile and compressive stresses. With cast iron the thickness obtained will be at the edge of the nut on the end of the bolt, and it may diminish to one-half this thickness at the edges. For very large washers a saving can be made by using ribs, each rib being considered as a cantilever carrying a part of the load, shear being duly taken into account. No casting should be less than $\frac{5}{8}$ in. thick and sharp corners should be avoided.

The washers at U_1 and $L_1 = \frac{25,000}{350} = 71.5$ sq. in. If extended across the chord the dimensions will be 8 in. × 9 in. and this is the best size to make them, for if square or round they will project beyond the edges, thereby decreasing the bearing area and increasing the stress on the wood. The thickness will be computed by the formula used in the case of the washers at joints U_2 and L_2.

The size of the lower chord will now be computed. The maximum tensile stress is 45,000 lbs. The allowable fiber stress is 1600 lbs. The area $= \frac{45,000}{1600} = 28.1$ sq. ins. The width will be assumed at 8 ins., from which will be subtracted 2 ins. on account of the hole for the largest vertical rod, which leaves a net width of 6 ins. The depth $= \frac{28.1}{6} = 4.7$ ins. If it will be possible to use a single piece of timber for the bottom chord we can use a 5 ins. × 8 ins. stick. It is not possible that a single stick can be obtained and we will take it for granted some splicing will be necessary, which will call for two lines of $\frac{3}{4}$-in. bolts going through the sides of the chord. This makes the thickness $4.7 + (2 \times \frac{3}{4}) = 6.2$ ins. Using commercial size timbers, this will make the chord 8 ins. wide and 7 ins. deep. It is usually best to have the depth equal or exceed the width and we will make the chord 8 ins. × 8 ins.

The maximum stress in the upper chord is 40,000 lbs. and the area $= \frac{40,000}{1200} = 33.3$ sq. ins. Assuming a width of 8 ins. and subtracting 2 ins. for the hole for the rod the depth $= \frac{33.3}{6} = 5.55$ ins. A shallow beam has a tendency to deflect unduly and if it is under compression the deflection will be increased. It will be advisable, therefore, to increase the depth and as this will decrease the breadth the minimum advisable thickness should be found.

The allowable maximum compressive fiber stress is based on a length not exceeding 15 times the least thickness. When the length is greater the fiber stress is decreased. One-fifteenth of

10 ft. = 8 ins., so this width should be maintained. The depth should not be less than this amount, which fixes the size of the top chord at 8 ins. × 8 ins. and as the piece will carry nothing but its own weight the deflection will not create anxiety, for by thus increasing the size of the piece the fiber stress becomes $\frac{40{,}000}{6 \times 8} = 834$ lbs. per square inch.

In the truss being designed the loads are small and the members are small, so there should be no difficulty in getting pieces of the dimensions here given. There will be then no additional areas to subtract for bolt holes. In trusses, however, which carry heavy loads in which several pieces must be used to form the members allowance must always be made for bolt holes. In this truss it is assumed that all the loads are concentrated at the panel points. If rafters rest on the top or bottom chords they must act as beams to carry such loads and must be designed for the stress thus caused in addition to the direct stress caused by tension in the lower chord, if the rafters rest on it, or to the stress caused by compression in the upper chord if the rafters rest on it.

For a piece acting as a combined tie and beam or acting as a combined strut and a beam use the following formula to obtain the breadth when the depth is assumed.

$$b = \frac{1}{fh}\left(\frac{6M}{h} + D\right),$$

in which b = the breadth of the piece,

f = the maximum fiber stress (compression for the upper chord, tension for the lower chord),

h = the depth of the piece,

M = the bending moment in inch pounds,

D = the total direct load (compression or tension).

In practical work, in calculating a rectangular piece, the depth may be assumed and the breadth computed to take care of M.

Add enough breadth to carry the direct load. Or, assume a breadth and design for a depth sufficient to take care of M, and add enough breadth to take care of the direct load.

Fig. 91 shows a bolted fish-plate splice used in connecting sections of a solid piece in tension. Formerly this was done on the assumption that the bolts bent and they were designed to resist the bending moment. The moment arm was equal in length to one-fourth the thickness of the middle piece, plus one-half the thickness of the outer piece, or splice pad. The load was half the total tension and each bolt carried its proportionate share. This is the method found in the majority of text books. It called for very large bolts and made a heavy, awkward-appearing connection. It is the proper method to use for pin connections (Fig. 86), the size of the pins being also figured for direct shear and the computations for end lengths and distances between bolts being computed as in the examples following.

The method for fish-plate joints used by the writer and shown in Fig. 91 is the modified fish-plate joint proposed by Mr. Dewell,

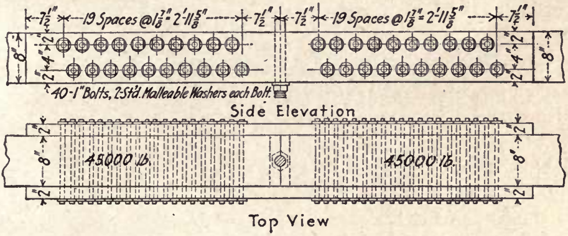


Fig. 91.

and the strength of the bolts is based on the table on page 139.

$$\text{Net area required} = \frac{45{,}000}{1600} = 28.1 \text{ sq. ins.}$$

Assume the splice pads to be 2 ins. thick and 8 ins. wide, which gives an area of 32 sq. ins. Two 1-in. bolts will occupy a space 2 ins. wide, with a bearing of 4 sq. ins., which will bring the splice pads down to the proper net size.

From the table on page 139 the strength of 1-in. bolts in double shear with 2-in. fish-plates = 2460 lbs.

Number of bolts required $= \frac{45{,}000}{2460} = 18.3$. There should be an even number of bolts and we should not use a higher stress than has been shown to be safe, so this points to the use of 20 bolts. This means 20 bolts on each side of the joint, for the entire pull must be carried into the fish-plates and then be carried by another set of bolts back from the fish-plate into the main piece on the other side of the joint. The total number of bolts therefore will be 40.

There is a transverse tension in every piece which tends to cause the wood to split along the center line of the bolts. The amount of this tension is one-tenth the longitudinal tension, and the safe allowable fiber stress is one-tenth the safe allowable longitudinal stress. The resultant is a shearing stress and must be allowed for in spacing the bolts.

The shearing stress is assumed to act at each edge of each bolt so that when a bolt passes through a 2-in. plank it exerts a shearing stress on four inches, as shown at (*c*) Fig. 90. We are now ready to space the bolts and determine the length of the splice pads.

The total shearing area in direct pull $= \dfrac{45{,}000}{120} = 375$ sq. ins.

Spacing of bolts for shear $= \dfrac{375}{20 \times 4 \times 2} =$ 2.34 ins.

Spacing required for transverse tension $= \dfrac{45{,}000 \times 0.1}{160 \times 20 \times 4} =$ .35

Adding diameter of bolts 1.00

Required spacing of bolts 3.69 ins.

Bolts will be spaced $3\frac{3}{4}$ in. staggered, with double this distance from the ends of the splice pads.

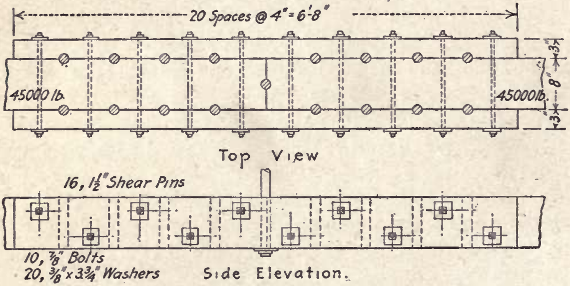


Fig. 92

By using thicker splice pads there would be required a less number of bolts and the spacing would have been closer. It is not pretended that the details here worked out are the most economical, for only the methods are shown. Each designer should work out several details. The bill of material and labor must be made out for each and unit prices applied in order to determine the least expensive detail.

All bolts should have a driving fit. This may be obtained by boring all holes from one side with a bit the right size to assure a driving fit. If any hole is large then use the next size larger bolt. In determining the working loads on bolts the washers played no part, so on bolts used for this type of fish-plate joint, standard washers may be used, drawn tight enough to insure a good bearing. "In general the fewer bolts there are to place, the less will be the cost for labor, and the more certain will be the combined action. Against these considerations must be weighed the amount of metal in the bolts and the availability of the chosen size. Stock bolts are, of course, cheaper than special sizes." (Dewell.)

In Fig. 92 is shown a shear-pin joint for tension members. This is very reliable for thoroughly seasoned timber but should not be used in green timber. Any shrinkage of the timber will allow a slip in the joint. Hardwood pins may be used, but metal is better and square bars or round are equally serviceable. Iron pipe is generally used. The pieces are cut and fitted together, then spiked to hold them in position during the fitting of the bolts. After the bolts are driven and the washers drawn tight, holes are bored and the shear pins driven. The drawing shows the shear pins vertical, but they may just as well be horizontal, if the designer fears vertical pins may fall out.

Assume the diameter of the shear pins to be $1\frac{1}{2}$ ins. and the fish-plates to be 3 ins. $\times$ 8 ins. The net section of the two plates will be $4.5 \times 8 = 36$ sq. ins. The unit tensile stress in the plates will be $\frac{45{,}000}{36} = 1250$ lbs. per square inch, which leaves the plates safe, as the allowable stress is 1500 lbs. per square inch.

The number of pins required is fixed by the bearing area on the end of the fibers in the holes, $= \frac{45{,}000}{0.75 \times 8 \times 1200} = 6.25$. Use 8 pins, for there must be an even number.

Total shearing area required $= \frac{45{,}000}{120} = 375$ sq. ins.

Spacing of pins for shear $= \frac{375}{8 \times 8} = 5.85$ ins. Adding the thickness of the pins, $5.85 + 1.5 = 7.35$ ins. Make it 8 ins. center to center.

Subtract from the thickness of the fish-plate one-half the thickness of the pins, which leaves $3 - 0.75 = 2.25$ ins. for the uncut portion, the tension acting through and considered as concentrated in the center, which is thus $3 - 1.125 = 1.875$ ins. from the shearing joint between the main piece and the fish-plates. Add to this one-half the projection of the shear-pin into the main member $= 1.875 + 0.75 = 2.625$ ins., which is the moment arm for the couple acting in the joint. The moment $= 2.625 \times 22{,}500 = 59{,}063$ in. lb., the action tending to raise one end of the plate from its seat. This is resisted by tension in the bolts.

Assuming the bolts to be set halfway between the shear pins the length of the moment (or lever) arm from the edge of the hole to the center of the bolt $= 4 - 0.75 = 3.25$ ins. The stress in the bolts $= \frac{59{,}063}{3.25} = 18{,}200$ lbs. Four bolts will be used, as shown, and the stress $= \frac{18{,}200}{4} = 4550$ lbs. per bolt. Use $\frac{7}{8}$-in. bolts, the net area of which, at the root of the threads, $= 0.42$ sq. in., which causes a stress $= \frac{4550}{0.42} = 10{,}830$ lbs. per square inch, which will be all right for a wrought iron bolt.

For developing the bolts plate washers may be designed. The tension on each bolt $= 4550$ lbs. and the area for each washer $= \frac{4550}{350} = 13$ sq. ins. Make each washer $3\frac{3}{4} \times 3\frac{3}{4}$ ins. The student can compute the thickness as an exercise by the formula on page 151. Standard round washers of equal area may of course be used.

The area of the chord will now be checked. Vertically there will be a hole $1\frac{1}{2}$ diameter (half on each side), which subtracts 12 sq. ins. Horizontally there will be two $\frac{7}{8}$-in. holes, with an area of 11.375 sq. ins., which, added to the area of the vertical holes, $= 11.375 + 12 = 23.375$ sq. ins. The gross area of the chord is 64 sq. ins. and the net area $= 40.625$ sq. ins. The fiber stress $= \frac{45{,}000}{40.625} = 1100$ lbs. per square inch. The chord has plenty of area for the maximum tension.

In Fig. 93 is illustrated an old type known as a tabled fish-plate splice. It may be considered to be reasonably effective when the entire stress can be taken by not more than two tables on either side of the chord joint. There is comparatively little secondary tension in the bolts, therefore they can act in their most efficient manner. Washers of generous size must be provided in order that the joint may be well pulled together at the time of framing and the bolts be able to hold the tables in place when the stress comes. The joint is dependent to a very large degree on the tightness with which the timbers are held in place by the bolts, and excessive shrinkage in the timber would allow the fish-plates to be overstrained. If it is not certain that the timber will be well seasoned before use, the fish-plates should be made larger than computations indicate to be necessary and it will be advisable to use about ten per cent more bolts than those provided by computations. Spikes can be toe-nailed into the fish-plates and will be a great help.

Depth of cut for table and chord: Area required for cut $= \frac{45{,}000}{1600 \times 2 \times 8} = 1.76$ ins. (make it 2 ins.)

Length of table for shear: Area required $= \frac{45{,}000}{8 \times 2 \times 120} = 22.4$ ins. (make it 23 ins.)

Size of bolts required: The stress is transmitted from the uncut portion of the chord to the uncut portion of the fish-plate past the

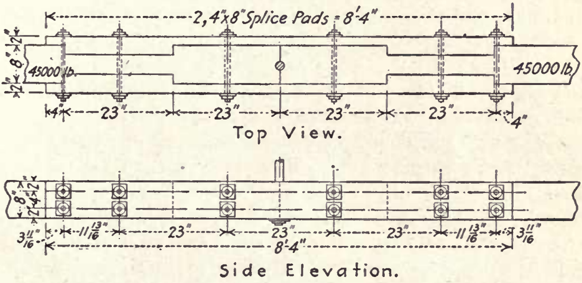


Fig. 93

joint, where it is again transmitted to the other section of the chord. The resultant stress thus travels through the center of the uncut portion of the fish-plate, which in this case is two inches thick, so the center of stress is 1 in. from the face. The resultant of the pressure on the table where it transfers stress is at the center of the cut, which in this case is 1 in. from the inner side of the fish-plate. There is a moment arm between the compressive stress on the edge of the table and the tensile stress in the uncut portion of the fish-plate which is equal to one-half the thickness of the fish-plate, in this case 2 ins. These two equal and opposite forces constitute a couple acting on the fish-plate equal to one-half the stress in the chord times one-half the thickness of the fish-plate, or, 22,500 lbs. $\times$ 2 ins. = 45,000 in. lbs. This moment must be resisted by tension in the bolts acting about the bearing face of the tables. The bolts should be placed on the vertical line through the center of the tables. Their lever arm is thus equal to one-half the table length, in this case 11.5 in. The tension on the bolts $= \frac{45,000}{11.5} = 3920$ lbs. Two bolts will be used having an area of 0.3920 sq. in., for the stress on wrought iron bolts should not exceed 10,000 lbs. per sq. in. The nearest size is found to be $\frac{5}{8}$ in. bolts, which have an area at the root of the threads = 0.202 sq. in. and the combined area of two $\frac{5}{8}$-in. bolts = 0.404 sq. in. Two $\frac{5}{8}$-in. bolts therefore will be used in the middle of each table and four more will be used as shown to bind the joint together.

In Fig. 94 is shown a steel-tabled fish-plate joint. This is a joint that requires especially good and careful inspection. It is a detail for members carrying heavy stresses. It costs considerable for materials and on account of the number of tables required the labor cost is high, for there must be very careful cutting to insure even and snug bearing for all the tables.

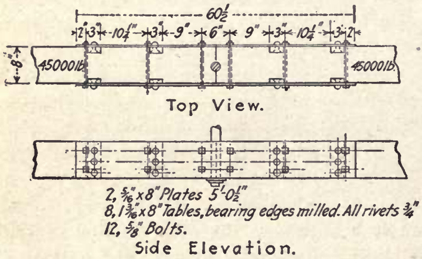


Fig. 94

Bearing area required for tables $= \frac{45,000}{12,00} = 37.5$ sq. ins.

Total combined depth of tables $= \frac{37.5}{2 \times 8} = 2.34$ in. (make it $2\frac{3}{8}$ in.).

Will use tables $1\frac{3}{16}$ in. = 8 in., requiring 8 tables in all.

Each table transmits $\frac{45,000}{4} = 11,250$ lbs. and requires three $\frac{3}{4}$-in. rivets, as determined by bearing on a $\frac{5}{16}$-in. plate (see page

190). The tables are solid pieces of steel bar $1\frac{3}{16}$ ins. thick by 3 ins. wide riveted to the $\frac{5}{16}$-in. plate used as a fish-plate. The thickness of the fish-plate is determined as follows:

$$\text{Net section of one plate required} = \frac{45{,}000}{2 \times 16{,}000} = 1.41 \text{ sq. ins.}$$

Net section of $\frac{5}{16}$ by 8-in. steel plate (deducting the three rivet holes for the table rivets) $= \frac{5}{16} \times (8 - (3 \times \frac{7}{8})) = 1.68$ sq. ins.

Size of bolts required to resist moment on tables: Moment $= 11{,}250 \text{ lbs.} \times \frac{21}{32} \text{ in.} = 7383 \text{ in. lbs.}$ Tension in bolts $= \frac{7383}{3.5} = 2110$ lbs. Use two $\frac{5}{8}$-in. bolts.

$$\text{Space between tables} = \frac{8 \times 120}{11{,}250} + 1\tfrac{1}{2} = 13\tfrac{1}{4} \text{ in.}$$

Sometimes the lower chord cannot be made of a single piece spliced by means of a fish-plate or shear pin splices, but a number of 2-in. or 3-in. planks must be used. The methods adopted for splicing such chords are illustrated in Fig. 95.

It may be assumed at the start that there should be an odd number of planks, for the center plank has so much section cut away for holes through which the verticals will pass, that it cannot be counted on as furnishing any tensile strength. Usually the middle rod is smaller than the others and about half the section of the center plank is available at this point for tensile strength, which is an additional factor of safety. The center planks are regarded merely as blocks in which a bearing is obtained for bolts.

The lower chord under consideration is so light that only three planks will be used. The center plank will be 2 ins. and the outer planks will each be 3 ins. thick. It is well to have the planks rough sawed and not finished, as the rough surfaces increase friction and also leave a small space for ventilation between the planks. The length of the truss is 60 ft. center to center of bearings, and, in the absence of data as to the end joints, we will assume two feet added, making the total length 62 ft. The outer planks must lap past each other for some considerable distance to allow space for the bolts which will be used to tie them together. Assuming that planks may be purchased 8 ins. × 38 ft., each side will have one plank 38 ft. long and one plank 62 − 38 = 24 ft. long. These lengths will be alternated so that at one end on one side there will be a 38-ft. plank and on the opposite side of the chord there will be a 24-ft. plank. This gives a lap of 14 ft.

The lap will be in the panels in which the stress is 45,000 lbs. Each outer plank will carry half the stress. We will assume two rows of $\frac{3}{4}$-in. bolts fastening the planks together and these bolts will be in double shear, for they pass through the center plank. Referring to the table on page 139 each bolt is capable of carrying 1450 lbs.

While each outer planks carries half the stress, when all bolting is done, all the stress passes through the middle splice, half coming in from each side plank and being transferred through the bolts to the plank on the other side; therefore the bolts in the middle splice must be proportioned to carry all the tension.

Number of bolts required $= \frac{45{,}000}{1450} = 31.$ Use 32 bolts.

Total shearing area $= \frac{45{,}000}{120} = 375$ sq. in.

Spacing of bolts for shear $= \frac{375}{32 \times 6 \times 2} = 0.975$ in.

Spacing required for transverse tension $= \frac{45{,}000 \times 0.1}{160 \times 32 \times 6} = 0.147$ in.

Adding diameter of bolts $= 0.75$

Required spacing of bolts $= 1.872$ in.

The spacing may be increased to any desirable amount, so before settling the matter the connections for the other planks will be taken up.

The other planks each carry one-half the tension, so all the figures above may be prorated accordingly. In each joint there will be 16 bolts. The total area required for shear $= \frac{375}{2} = 187.5$ sq. ins.

Spacing of bolts for shear $= \frac{188}{16 \times 6 \times 2} = 0.98$ in.

Spacing required for transverse tension $= \frac{22{,}500 \times 0.1}{160 \times 16 \times 6} = 0.146$

Adding diameter of bolts $= 0.75$

Required spacing for bolts $= 1.872$ in.

Before spacing the bolts determine how far the first bolts will be placed from the end of the planks. The shear on one bolt is 1450 lbs. with bearing on a thickness of 6 ins. of plank. For the interior bolts this is placed on two lines at the ends of the diameter, but for an end bolt it is best to use but one line, considering that the greatest danger is shearing on a line through the center of the bolts. The length required for shear on the end bolt

$= \frac{1450}{6 \times 120} = 2.02$ in., so the end bolts may be placed $2.02 + \frac{0.75}{2}$ $= 2.395$ ins. from the end of the plank.

In Fig. 95 is shown (with width exaggerated) the arrangement of the bolts. The spacing so carefully figured is the closest safe spacing. The bolts can be placed much farther apart if desired. In the middle splice carrying all the tension place two bolts seven inches from the ends of the planks. Space the remaining bolts 10 ins. center to center. The 14-ft. lap in the middle of the truss is then connected up so that the two outer planks act to carry

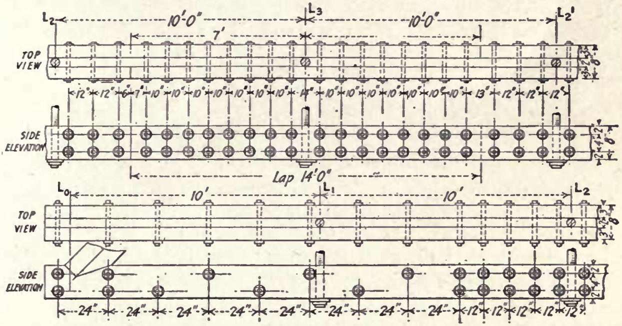


Fig. 95

half the tension from one support to the other. It now remains to bolt the other two outside planks to them in order that the rest of the tension can be carried.

There should be two bolts close to each joint, so, six inches from the end of the planks, put in two bolts. Each joint has 16 bolts in two lines, which bolts may be spaced 12 ins. center to center. For the rest of the chord put bolts at intervals of 4 ft. staggered and at the very end put two bolts through 6 ins. from the end to make a firm bearing for the end of the brace. Between the bolts, used merely to hold the planks firm, large spikes may be driven at intervals of about 12 ins. to make the whole construction more rigid.

The same principles apply when five or more planks are used. In such a case, however, it often happens that long planks may be placed in the middle of the span in such manner that all the splices will come in panels in which the stress is low, the full area of the planks being available for carrying the tension in the middle of the span where it is greatest. In any event the splicing of the bottom chord of a truss, when said chord is composed of plank, is a matter requiring a great deal of careful study. It is an easy matter to make a poor splice. The writer has seen some in which the designer calculated for one-half the stress going through a certain splice when actually all the stress went through it.

In Fig. 96 is illustrated a detail of a bottom chord in which all the tension is carried by a steel plate and underneath are two pieces of timber large enough to carry the plate and take out the sag. This is from a plan for a standard wooden highway bridge

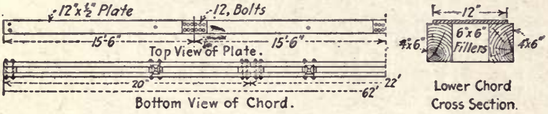


Fig. 96

designed by Hugh C. Lewis, Bridge Engineer in the State Highway Department of Utah, E. R. Morton, State Road Engineer. The figures were copied from *Engineering News*, Sept. 21, 1916. Note the spliced joint between the two timber pieces in the chord. This consists of a long splice plate, large enough to carry all the stress, between the two outer chord pieces. The use of such a splice leaves a wide air space between the two chord pieces for ventilation. When a bottom chord consists of three planks and the center plank between two vertical rods is long enough to serve as a splice piece, the two outside pieces may break joints at a section passing through the chord, as shown in this detail.

In Fig. 97 several methods are shown for making joints in a piece under compression. These joints are used in the top chords of trusses and also in columns, for the top chord of a truss is a column. The detail shown at (*a*) is the best of the lot. The ends should be carefully dressed to insure an even bearing. The detail at (*b*) is in common use and is not so good as (*a*). It has two bearing surfaces and this makes it very difficult to get an absolutely true bearing. It may fit tight at one end and not fit evenly at the other end. When the load is brought on one-half the member carries all the load until the fibers give and then the other end comes into bearing. The detail of (*c*) should never be used. It has two end bearings and in addition has the sloping face which is difficult to fit. If one, or both ends, compress through rotting or crushing of the fibers, the load is carried on the sloping face, which increases the tension on the bolts and hastens the destruction of the member. When splices are made in a top chord the joints are preferably vertical, the views shown being top or bottom, as may be desired. The number of bolts to use, and the sizes, are matters determined by judgment and experience in the three details shown.

The detail at (*d*) is one commonly used when the piece under compression is made of several pieces. The pieces should be as thick as possible, and if more than one thickness is used use the thinner pieces inside and the thicker pieces on the outside. To design such a member consider the load to be uniformly distributed so that each piece carries a load proportionate to the area. If one piece bends, part of the load it carries must be transferred to the adjoining piece by shear, so shear pins are inserted at intervals of 15 times the thickness of the thinner of the pieces. Divide the total load by the number of planes between the pieces, that is, by the number of pieces less one. This may be assumed to be shear and it is divided by the number of shear pins in one joint to determine the amount of bearing for each pin. From this the bearing area may be ascertained in the manner shown for the shear pin splice and the table fish-plate splice for tension members. The bolts are close to the pins and are designed to take tension, the amount of which is ascertained from the moment caused by the load carried by the outer pieces. A compression member, or column, carefully designed according to the above method, should be about 95 per cent as efficient as a solid piece of the same outside dimensions. The ends should be carefully dressed to insure the load being uniformly carried by all the pieces. It is advisable to use a thin sheet of lead on each end of the member.

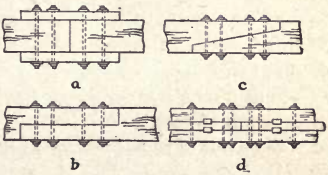


Fig. 97

For many years it has been accepted as true that if a piece under compression is made of a number of smaller pieces it will not act as a solid piece. A number of experiments were made to determine this and it was discovered that the secret lay in the connections. Pieces as ordinarily made were found to be very deficient in strength. Thin planks spiked together as thoroughly as the ingenuity of the experimenter could devise proved to be almost as strong in small specimens as solid pieces. It is not likely, however, that in actual work this amount of nailing will be done. Experiments made on rather large columns did not show up so well as experiments on smaller columns.

It is considered to be not the best practice to build up compression members of thin pieces, and when slender pieces are used they should be as few in number as possible and shear pins should be used as shown in Fig. 97 (*d*). If a number of thin planks must be used they should be spiked together by gradually building up, no expense for spikes being spared. After the piece is built up lay wide pieces across the edges and spike these pieces to the edge of each plank. These cross pieces will cover the two sides from one end to the other and serve to call each plank to the assistance of all the others in case there is any bending.

It has been stated that the compressive fiber stress given in specifications for wood is based on pieces having a length not greater than 15 times the diameter or least thickness. When the proportions adopted provide for a more slender column the following formula is used to ascertain the reduced fiber stress to be used.

$$f'' = f\left(1 - \frac{L}{80d}\right)$$

in which

f'' = reduced unit fiber stress.
f = unit fiber stress for pieces having a length not exceeding $15d$.
L = length of post.
d = diameter of round post or least dimension of rectangular post.

This formula is used in Chicago for wooden posts. There are several formulas in common use for finding the reduced stress to use for slender posts and these will be discussed in the chapter dealing with columns.

The reason it is difficult to get several pieces to work properly unless thoroughly connected by shear pins or spikes, is that the ratio of slenderness reduces the load carrying capacity in a greater degree than the load is reduced by the number of pieces. For example the allowable unit fiber stress for solid piece with dimensions equal to, or greater than, one-fifteenth the length may be 1200 lbs. per sq. in. Divide the solid piece into four slices and the ratio of slenderness for each slice (plank) is one-fourth of one-fifteenth = $\frac{1}{60}L$. By the above formula the allowable fiber stress is only 400 lbs. per sq. in. Assume that the whole load is carried on the cross section at 1200 lbs. per sq. in. provided it is a solid piece. By dividing it into four planks, each carrying one-fourth the load, it is seen the total carrying capacity of the four slender pieces is only one-fourth the total load. That is, the four pieces acting separately are each strong enough to carry one-sixteenth of the load. By nailing them together at intervals they are made to act together to some extent, and if arrangements are made to connect them together rigidly at intervals not greater than one-fifteenth the thickness of each piece the allowable fiber stress is increased. Putting bolts or spikes through several planks is not nearly so effective as spiking them together by internal nailing, that is by "piling" them, by which term is meant nailing each plank to a lower one, the spikes passing through at least three after three are assembled. Two should be spiked together and the spikes clinched. Then the "piling" is done by adding a plank first to one side and then a plank to the other side, so the first two planks form the core. To hold the two outside planks the cross pieces are nailed on the edges.

Member U_3L_2 has a length of 14.14 ft., measured from center line to center line of the chords, but somewhat less in the clear. The total length, however, will be used for convenience in proportioning the piece. Assume a piece 4 in. × 8 in., in which the ratio of least thickness to length $= \frac{14 \times 12}{4} = 40$. This is much too great. Try a 6 in. × 6 in. and the ratio $= \frac{14 \times 12}{6} = 28$, which is still large, but we will investigate and see whether the piece will do.

First find the fiber stress. This is usually done by using a straight-line formula, but other formulas are discussed in the chapter dealing with column design. The straight-line formula used for wooden columns in Chicago is as follows:

$$f' = f\left(1 - \frac{L}{80d}\right)$$

in which f' = reduced fiber stress per sq. in. in compression.
f = fiber stress used for columns having a length less than 15 times the diameter or least thickness.
L = length.
d = diameter or least thickness.

When L is in feet, d is in feet; and when L is in inches, d is in inches.

Using this formula for the case under consideration

$$f' = 1200\left(1 - \frac{14 \times 12}{80 \times 6}\right) = 1200 \times 0.65 = 780 \text{ lbs. per sq. in.}$$

The area of the piece is 36 sq. ins. and the total working strength = 36 × 780 = 28,000 lbs. The actual load it must carry is only 7190 lbs., but in ordinances and specifications the maximum ratio for the length divided by least width (ratio of slenderness) is 30 and the 6 in. × 6 in. piece barely comes within the limit. This ratio is for vertical posts, whereas sloping posts have a tendency to bend under their own weight, so something must be added for additional stiffness. If it were not for this the 4 in. × 6 in. piece would be good, as it has a safe compressive strength of 18,240 ins. considered as a vertical post. The ratio of slenderness of 40, however, is against it.

The piece U_2L_1 and the piece U_1L_0 will be made 8 in. × 8 in. without computation, for the former has only a load of 21,750 lbs. to carry and as a vertical post a 6 in. × 6 in. can safely carry 28,000 lbs. The additional stiffness secured by adding two inches to the breadth and thickness saves it from bending. To compute it we find that the safe fiber stress is 835 lbs. and it can carry as a vertical column 53,440 lbs.; therefore this size will do for the end piece.

When joint details are designed it may be discovered that some of the members must be made larger to allow for bolt holes good daps. This additional area may be added at the time, but the detailing of the truss should proceed in the order here followed, so the pieces used in the computations may be reasonably close to the actual dimensions finally adopted.

Designing Joints

In Fig. 98 is shown one method for making the joint L_0. The computations, in order, are as follows:

Depth of toe, $\theta = 45°$, therefore $n = 1200 \times 0.45 = 540$ lbs. per sq. in.

$$\text{Required area in bearing} = \frac{35{,}950}{1200} = 66.6 \text{ sq. ins.}$$

$$\text{Required depth of face} = \frac{66.6}{8} = 8.33 \text{ ins.}$$

The above operations involved finding the fiber stress in compression per sq. in., dividing the total load to find the required area and then dividing the area by the width of the chord to find the required depth of the end of the brace. This depth being normal to the angle of the brace we divide it by the secant of the angle and find that the vertical depth of cut in the chord $= 8.33 \div 1.4141 = 5.9$ ins. The depth of the cut should be such that below the point there will be enough area left in the chord to carry the tension. Neglecting the middle filler, the width of the chord is 6 ins., so the depth $= \dfrac{25{,}000}{6 \times 1600} = 2.6$ ins. Practically, the depth of the cut should not be more than one-half the depth of the chord, so another detail should be selected.

We can, at this point, assuming the computed depth of cut is correct, proceed to find the length of chord projection for shear and find the center line of support; merely to show in detail the necessary computations. For this purpose the depth of the vertical cut in the chord will be assumed to be four inches.

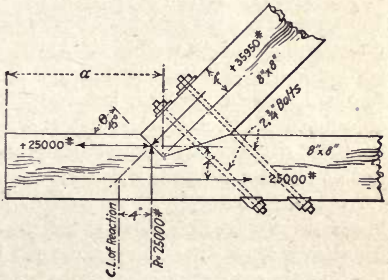


Fig. 98

Forces are assumed to be concentrated, or to act along the center lines of stressed members. In the end piece the compression is all acting on the square end face, 4 ins. deep × 8 ins. wide and the center lines of the forces are as shown by the heavy lines. Below the end piece the uncut area of the chord carries all the tension, and this acts in the center, as shown by the heavy arrow.

The two forces form a couple with a moment arm of 4 ins. The moment = $4 \times 25{,}000 = 100{,}000$ in. lbs.

If the center line of the support is placed under the vertical line dropped from the center of the end face the bending moment just found will exert a tendency to open the joint, because of the pull around the lower edge. The reaction therefore should come under the intersection of the center line drawn through the face of the end piece and the line through the center of area of tension, as illustrated in Fig. 98. The exact position may be computed by dividing the moment just found, by the reaction, $\frac{100{,}000}{25{,}000} = 4$ ins. which is the distance required to the left. It is merely a coincidence that in this particular example the reaction equaled the tension in the end panel of the lower chord.

It remains to find the chord projection for shear, the length a. The width of the chord is 8 ins. and the allowable shearing stress is 120 lbs. per sq. in., with a total compression load of 25,000 lbs.

$$a = \frac{25{,}000}{120 \times 8} = 26 \text{ ins.} = 2 \text{ ft. } 2 \text{ ins.}$$

We have discovered that the center line of the reaction should be 4 ins. to the left of the center of the bearing area on the end brace. This is 5.42 ins. to the left of the lowest point of the end brace, where it is set into the chord, and there is consequently a projection, $26 - 5.42 = 20.58$ ins. beyond the center of the supporting wall or column. This may be concealed by corbelling out the brickwork of the wall, but very often such a projection is objectionable. Designers who do not think clearly will often place the support under the very end of the projecting piece, and this sets up bending moments which will cause the chord to break.

Assume, for example, in the present instance that the center line of the support is 6 ins. from the end of the projecting chord. This leaves 20 ins. to the deepest cut and 18.58 ins. to the vertical line from the center of the face of the brace. The bending moment $= 18.58 \times 25{,}000 = 464{,}500$ in. lbs. The vertical moment arm between the compression and tension area is 4 ins. and the area of the tension side below the cut $= 8 \times 5.17 = 41.36$ sq. ins.

The tensile stress $= \dfrac{464{,}500}{4 \times 41.36} = 2807$ lbs. per sq. in., and the allowable stress is 1600 lbs. per sq. in., therefore the center line of the support cannot be so far from the end of the brace. The proper position, in order to keep all forces in equilibrium, is 5.42 ins. left of the bottom point of the end piece. The tension caused by moving the support farther to the left must be added to the tension in the chord; so the actual stress, if the support is away at the end, $= (41.36 \times 2807) + 25{,}000 =$ 141,098 lbs.

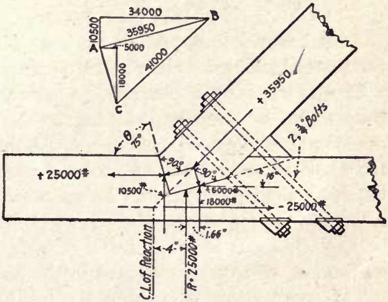


Fig. 99.

In Fig. 99 is shown another method of forming the joint, the toe cut not being perpendicular (normal) to the line of thrust. The angle at the toe is 75 degrees and the angle of the sloping bottom of the cut is 16 degrees; for the angle between the surfaces may vary, it not being necessary to have the lower point a right angle. Computations will be made to obtain the area of the pressed surface and the allowable and actual pressures on the surfaces. The computations for obtaining the projecting length of the chord for shear will not be made, for it is like the work done in the detail shown in Fig. 98.

Depth of toe: $\theta = 75$ degrees. $n = 1200 \times 0.75 = 900$ lbs. per sq. in.

Required area in bearing $= \dfrac{35{,}950}{900} = 40$ sq. ins.

Required depth of toe $= \dfrac{40}{8} = 5$ ins.

This depth is too great for the lower chord, for if it is used the depth of the chord must be increased, which will increase the weight and be an uneconomical proceeding. We will therefore abandon this type of joint for the truss, but the computations will be carried through, merely as a problem, in order to show how the two bearing surfaces affect the position of the center line of the support.

Pressure on inclined bed: $\theta = 16$ degrees. $n = 1200 \times 0.16 = 192$ lbs. per sq. in. This is below the allowable safe pressure across the grain, which is 350 lbs. per sq. in. which we will use.

$$\text{Required area in bearing} = \frac{18{,}000}{350} = 51.5 \text{ sq. ins.}$$

Actual area $= 10.5 \times 8 = 84$ sq. ins. The area therefore is more than sufficient for its component of pressure.

The student is to observe the effect the distribution of the pressure on two bearing faces has on the depth of the cut. It is obvious that when the cut is normal to the stress in the brace the whole thrust must be taken on the toe of the post and none is taken by the inclined face. Actually there is a small component normal to the center line of the brace, but it is so small that it can be neglected. It is probably taken care of by friction of the toe on the cut, or by slight tension in the bolts.

The small diagram in the upper left-hand corner of Fig. 99 is perhaps self-evident, but will be explained. The load travels down the brace until it reaches a point opposite the center of the inclined face, when it divides, part going to the inclined face and part to the toe. This is drawn to scale on the line AB. The line AC is parallel to the line of the toe. The other lines require no explanation, for the values are marked on the diagram and also on the drawing.

The vertical component of the inclined face is 18,000 lbs. and the vertical component of the toe is 10,500 lbs., the distance between them being 4.5 in. The center of gravity $= \frac{10{,}500}{18{,}000+10{,}500}$ $= 0.369 \times 4.5 = 1.66$ in. from the right component.

The bending moment of the couple in the chord $= 4 \times 25{,}000 = 100{,}000$ in. lbs. This divided by the reaction gives the distance the center line of the support must be shifted to the left to insure equilibrium. The reaction in this case merely happens to equal the tension in the lower chord, so the center line of the reaction must be 4 ins. to the left of the position above found, or, $4 + 1.66 = 5.66$ ins. to the left of the center of the lower bed on which the brace rests.

Fig. 99 purposely contains as few lines as possible in order to avoid confusing the student. In both details there are two inclined bolts with cast-iron washers to keep the pieces in position, so all the bearing areas will have a proper contact. No method is known whereby the stress can be computed in these bolts. The size is fixed by judgment based on experience. For a truss with such light loads as the one under review, ⅝-in. bolts might be used, but designers generally prefer to use nothing smaller than a ¾-in. bolt, this being good practice. Since they carry a very small stress, if any, it is not necessary to cut a seat in the lower chord, which would weaken it, but cast iron washers of the form shown are used, they being seated in the timber about half an inch.

The detail in Fig. 99 is not good, for the reason that the sloping face on the 16-degree angle cannot be accurately cut on account of the depth to which the piece must go into the chord. The back edge of the sloping face should meet the top of the chord. These details must be worked out on the drawing board to a large scale in addition to being computed. The computations and drawing to scale must go together. In all the end joint details a block is inserted through which the bolts go. This is to diminish as much as possible the effect of secondary stresses set up by the bolts. In Fig. 99 the slope of the bed might be carried to the top of the chord without weakening it and a block fitted in the space, but it increases the labor cost. The designer understands, of course, that he should make several designs and choose the one that costs the least and will do the work.

In Fig. 100 is illustrated a very common form of joint and it is not a good one. This joint is used in an attempt to get rid of the long end projection caused by designing to resist shear. The end of the brace is dapped into the top of the chord merely to hold it in place. The diagonal bolts take all the stress and the shear is resisted close to the bottom of the chord. Note the line diagram in the upper left-hand corner of

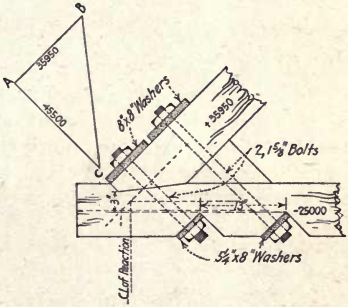


Fig. 100

the figure. The line AB is parallel with the brace and is drawn to a scale to represent the load. The line AC is parallel to the direction of the bolts. The line BC is perpendicular to the bed cut into the top of the chord. The line BC intersects the line AC at C. The line AC to scale gives the tension in the bolts. This is found to be 45,500 lbs.

Deciding to use two bolts the tension in each = 22,750 lbs., which calls for two $1\frac{5}{8}$-in. bolts. The holes must be filled so there is no reason to use bolts with upset threads, and the size of the bolt is determined by the area at the root of the thread. The shear $= \frac{25,000}{2 \times 8 \times 120} = 13$ ins., therefore the clear distance between the edges of the cuts to receive the bolts cannot be less than 13 ins. The distance from the edge of the first cut to the end of the chord must be equal to or exceed 13 ins., for each bolt washer is assumed to carry half the total shear.

The allowable pressure on the side of the wood is 350 lbs. per sq. in. and the washers on the face of the end brace are dimensioned as follows: $\frac{22,750}{8 \times 350} = 8.1$ ins. Make them 8 in. $\times$ 8 in. The thickness may be found by the rules mentioned earlier. The washers on the lower end can be smaller, for they bear on the wood at an angle. The allowable pressure, the angle being 45 degrees, $= 1200 \times 0.45 = 540$ lbs. per sq. in. The width of the washers $= \frac{22,750}{8 \times 540} = 5.25$ ins. The vertical cut into the bottom of the chord to permit these washers to bear properly is $3\frac{3}{4}$ ins. deep, which leaves a space of 3 ins. between the top of the cut and the bottom of the brace seat for the tension in the chord. Counting the width as being 6 ins. the strength of the chord $= 3 \times 6 \times 1600 = 28,800$ lbs., so the chord is safe. When this form of joint is detailed so part of the load is carried by thrust against the chord and part is carried by the bolts, the weaker detail will give way before the other is brought into play, as they do not act together. Therefore an end joint should be designed so all the load is carried

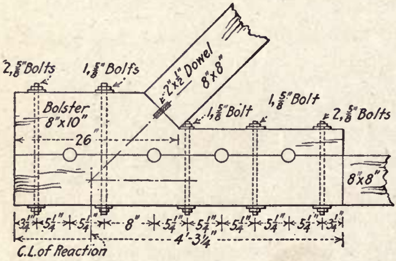


Fig. 101

by the thrust of the brace against the chord, or it must all be carried by the bolts.

In Fig. 101 is illustrated a low-cost end joint. A bolster is placed on top of the lower chord and pins are used to transfer the stress by shear, 2-in. pins being used. There may be unequal shrinkage in the chord and bolster which will interfere with the proper action of the round pins, so it is better to omit the pins and use a bolster large enough to permit it to lock into the top of the chord as a tabled fish-plate. The vertical bolts carry one-half the tension and the method to use in figuring the size of the bolts has been presented.

This joint is designed as follows: Using an 8 in. × 10 in. bolster the depth below the bottom of the cut will be 4.34 ins. Deduct 8 sq. ins. for the half pins and $\frac{3}{4} \times 4.34$ ins. for the bolt holes, leaving an area of 23.47 sq. ins. The tensile stress will be $\frac{25{,}000}{23.47} = 1065$ lbs. per sq. in., and the allowable safe fiber stress is 1600 lbs. per sq. in. Therefore the 8 in. × 10 in. bolster is O.K.

$$\text{Uncut projection for shear} = \frac{25{,}000}{8 \times 120} = 26 \text{ ins.}$$

The compression on the round pins will be taken at 800 lbs. per sq. in. The required number of 2-in. pins $= \frac{25{,}000}{8 \times 800} = 3.9$. Use 4 pins.

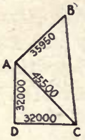
The thickness of bolster back of the brace required for tension $= \frac{25{,}000}{8 \times 1600} = 2$ ins. The thickness is 4.34 ins. less 1 in. for the pins = 3.34 ins., so this is O.K.

$$\text{The stress per pin} = \frac{25{,}000}{4} = 6250 \text{ lbs.}$$

$$\text{The clear space between pins} = \frac{6250}{8 \times 120} = 6.5 \text{ ins.}$$

Sometimes a detail similar to that shown in Fig. 100 is used, but instead of two bolts one is used. In the case considered this bolt will be $2\frac{1}{4}$-in. diameter. Instead of making a triangular cut in the bottom of the chord to form a bearing surface for the washer, a casting is used at the bottom for the lower end of the bolt. It is very common to use such washers without computing the size necessary, and in many existing trusses this detail is weak. The computations are as follows, referring to Fig. 102 (*a*):

In the diagram in the upper left-hand corner the tensile stress in the bolt is 45,500 lbs. This at the bottom of the chord is converted into a horizontal and a vertical component, shown in the lines AC and CD. First taking the value given by the line AD we find the depth of cut necessary for the vertical component.



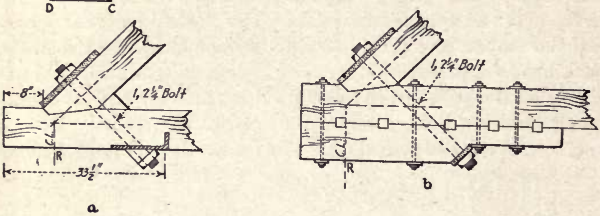


Fig. 102.

Depth of cut $= \dfrac{32{,}000}{8 \times 1200} = 3.34$ ins. (make it $3\frac{1}{2}$ ins.).

The bottom of the plate must be large enough to allow a bearing of 350 lbs. per sq. in. against the side of the wood, the width of the plate being 8 ins.

Length of plate $= \dfrac{32{,}000}{8 \times 350} = 11.4$ ins. (make it $11\frac{1}{2}$ ins.).

The thickness of this plate must be found by assuming the edges projecting beyond the collar around the bolt, as cantilevers uniformly loaded. The thickness of the sloping leg in the chord must be figured in the same way. The plate on top of the bracer at the upper end of the bolt must have enough area to keep the bearing down to 350 lbs. per sq. in.

The plate must be set far enough from the end of the chord to prevent shearing.

Length for shear $= \dfrac{32{,}000}{8 \times 120} = 33.4$ ins. (make it $33\frac{1}{2}$ ins.).

The center of the bolt should be as nearly as possible at the center of the plate, which helps to fix the position of the bolt through the chord and brace, taking into consideration the size of washer at the upper end. Insert a hardwood block in the angle.

In Fig. 102 (*b*) is a detail used to prevent the cutting of the bottom chord for washers and to avoid the expense of the special casting shown at (*a*). The position of the bolt is fixed by the size of the washer at the upper end and by the necessity for having the uncut portion of the block long enough to resist failure by shear. The pins are shown square, but they may be round if desired. From the examples given the student should have no difficulty in designing a detail such as this. Supply enough pins for bearing and enough space between them for shear. The vertical bolts are in tension, but of course this tension is greatly reduced by the stiffness of the block, the thinner part of which must have area enough to carry the tension in the end panel of the chord. The lower washer must have area enough to keep the pressure to the limit imposed by the angle at which the pressure is delivered to the wood.

In Fig. 103 is illustrated a cast-iron shoe. It may have slightly different details, this being true of every design here illustrated. The form shown is a rather common type. There are no diagonal bolts, so the pressure on the top of the chord is not uniform. A toe projecting in front of the brace is provided to take care of this. The depth of the lugs is fixed by the end bearing strength of the wood. The thickness of the lugs is fixed by their resistance to shear and bending, for they are short cantilevers and are so designed. The spacing of the lugs is governed by the shearing strength of the timber with the grain. The first lug is usually placed directly under the end of the batter post. The arrangement of the ribs fitting into the end of the batter post is much a matter of judgment. Cast iron is a brittle metal and it is best to have the least thickness more than half an inch. The thickness to use will usually be determined by the size of the casting; a large heavy casting requiring larger individual parts, should have all the parts thick, because a fall will be more apt to injure it than a smaller casting.

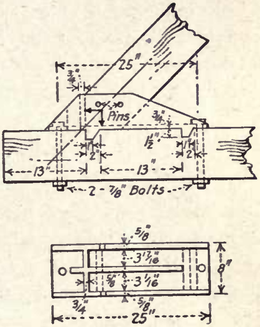


Fig. 103.

Proceeding with the design. The wood at the end of the batter post is not held by bolts, nor confined in a shoe, therefore a reduced stress will be used.

Depth of toe, $\theta = 45°$. $n = 540$ lbs. per sq. in.

Required area for end bearing $= \frac{25,000}{540} = 46.4$ sq. ins.

Required depth of vertical cut $= \frac{46.4}{8} = 5.8$ ins.

Required area of horizontal cut, $\theta = 45°$. $n = 540$ lbs. per sq. in. Use vertical component of the diagonal load, which for 45 degrees is one-half.

$$\text{Area} = \frac{35,950}{2 \times 540} = 33.3 \text{ sq. ins.}$$

$$\text{Length of horizontal cut} = \frac{33.3}{8 - (3 \times \frac{5}{8})} = 5.5 \text{ ins.}$$

In this example the angle is 45 degrees and the vertical cut and horizontal cut will be equal, each being $8 \times 0.707 = 5.66$ ins. The vertical cut, for the assumed fiber stress, should be 5.8 ins., which indicates that the casting should be designed with the two bearing surfaces forming an angle differing enough from 90 degrees to keep the stress within the limits fixed. If the design is not altered the stress on the end area $= \frac{25,000}{5.66 \times 8} = 552$ lbs. per sq. in., which is an increase of only 4 per cent, so will be allowed to stand.

The maximum unit bearing pressure of the shoe on the lower chord must not exceed the allowable bearing stress on the side of the fibers. Draw a horizontal line from the mid-height of the toe to intersect with the diagonal line meeting the point of the toe. From this intersection drop a vertical line to represent the vertical component of the thrust. The diagonal line represents the diagonal thrust and the horizontal and vertical components, respectively, are shown as heavy arrows.

The distance from the vertical component to the front edge of shoe (point of maximum pressure) scales 8.5 ins. and this length will be called a. Next find the length of the shoe. The lugs each carry one-half the shear, or 12,500 lbs.

The area for bearing $= \frac{12,500}{1200} = 10.4$ sq. ins. The depth of the lug $= \frac{10.4}{8} = 1.3$ ins. (make it 1.5 ins.).

$$\text{Length required for shear} = \frac{12,500}{8 \times 120} = 13 \text{ ins.}$$

This fixes the clear distance between lugs at 13 ins. and the front lug will be set not less than 13 ins. from the end of the lower chord. The end of the shoe will extend 2 ins. beyond the rear lug. This makes the total length of the shoe, $L = 25$ ins.

Let a = distance from front lug to toe of shoe.
L = length of shoe.
q = maximum pressure in lb. per sq. in. at front edge of toe.

$$P = \frac{\text{vertical reaction}}{\text{width of chord}} = \frac{35{,}950}{2 \times 8} = 2242 \text{ lbs.}$$

$$\text{Then } q = 2\left(2 - \frac{3a}{L}\right)\frac{P}{L} = 2\left(2 - \frac{3 \times 8.5}{25}\right) \times \frac{2242}{25} = 176 \text{ lbs.}$$

which is well within allowable stress.

The thickness of the lugs is determined by treating them as cantilevers. The depth is 1.5 ins. loaded uniformly with 12,500 lbs.

$$\text{The bending moment} = \left(\frac{0.75 + 1.5}{2}\right) \times 12{,}500 = 14{,}080 \text{ in. lbs.}$$

The moment of resistance of a rectangular section, $M_r = Rbd^2$.

The compressive stress of cast iron is 10,000 lbs. per sq. in. and the tensile stress is 3000 lbs. per sq. in. A mean of these values may be used in determining the resisting moment, but it is better to use the tensile stress, and $R = 3000 \div 6 = 500$. Then

$$d = \sqrt{\frac{14{,}080}{500 \times 8}} = 1.88 \text{ in. (make it 2 ins.).}$$

Moment of rotation of lugs = bending moment on lug = 14,080 in. lbs. Tension in bolt back of lug $= \frac{14{,}080}{(2 + \frac{3}{4}'')} = 5120$ lbs. Use one $\frac{7}{8}$-in. bolt.

Figures 104, 105, and 106 are reproduced from an article by Henry D. Dewell in *Western Engineering* for September, 1916, the fourth in the valuable series referred to.

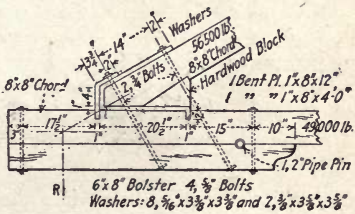


Fig. 104.

The computations for Fig. 104 are as follows:

Area required for bearing between upper and lower chord $= \frac{28{,}125}{285} = 99$ sq. ins. (Mr. Dewell used side bearing stress of 285 lbs. per sq. in.)

Depth of lugs = $\frac{49,000}{2 \times 1600 \times 8}$ = 1.915 ins. Use 2-in. lugs.

Thickness of lugs for bending:

Bending moment on one lug = 24,500 lbs. × 1.5 = 36,750 in. lbs.

Thickness of lug assumed to be 1 in.

Required section modulus = $\frac{36,750}{25,000}$ = 1.465.

Required thickness of lug = 1.05 ins. Use 1-in. plate as assumed.

Length required for shear between lugs = $\frac{49,000}{2 \times 150 \times 8}$ = 20.4 ins.

Use 1 ft. 8.5 ins.

Depth of toe = $\frac{49,000}{1600 \times 8}$ = 3.84 ins. Use 4 ins.

Bearing stress of 1600 lbs. per sq. in. is used, as the timber fibers are confined and therefore capable of taking full end compression.

Stress in bolster = horizontal component of stress in two $\frac{3}{4}$-in. bolts = 4830 lbs. × 2 × 0.5 = 4830 lbs.

No. of shear pins required = $\frac{4830}{800 \times 8}$ = 0.75. Use one 2-in. pin.

In the cast-iron shoe shown in Fig. 103 the projection of the toe limited the bearing stress on the top of the chord. In the form shown in Fig. 104, since the line of thrust intersects the shoe practically at the end of the toe, the inclined bolts will be called into play to prevent a rotation of the shoe, which would greatly increase the toe pressure. The bolts, therefore, must be always tight in order to secure an approximately uniform vertical pressure.

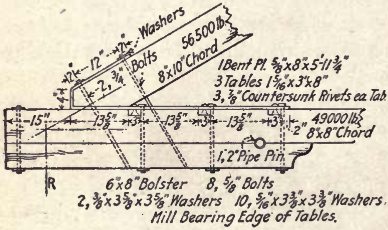


Fig. 105.

This form of joint is considered very good because it is simple in action and comparatively easy to frame into the timber. Both lugs must have an even bearing against the wood and this is a hard thing to secure, but if the inspection is good the work will be all right. When there is more than one bearing surface this objection always arises. If the fitting is not properly done the joint can be shimmed with thin metal shims.

Mr. Dewell makes the following comments: "This shoe can be used only for stresses requiring not more than two lugs, hence its field of application is limited. Another defect is that the forge work is difficult with the thickness of plate used. Especially is this true of the bending of the end of the inner plate to form the inner lug. Incidentally, this detail forms a good example of the consideration of actual unit working stresses as compared with purely theoretical values, as mentioned in the first article of this series. With a 2-in. depth of lug, the bearing pressure against the ends of the fibers is assumed to be 1600 lbs. per sq. in. On account of the fillet formed in bending the plate, the actual bearing area will be decreased and the actual unit working stress will probably be around 1800 lbs. per sq. in."

In Fig. 104 and Fig. 105 the sizes of the two diagonal bolts are determined by judgment and experience. They are not susceptible of computation. The vertical bolts are found by computation. When any computation is omitted in any of the examples it is for the reason that the student is assumed to know how to make it. Every detail must be investigated according to the principles and methods illustrated.

Fig. 105 is a modification of Fig. 104. Steel tables rivetted to the plate are substituted for the lugs used in Fig. 104. The forge work is less; any number of tables may be used; and the main plate may be reduced to a thickness determined by consideration of shear and tension alone. In Fig. 104 the plate thickness is determined by the thickness required of the lug to prevent it straightening under load. No table should be placed under the foot of the batter post, for the seat for the table is usually cut a little deeper than the table, so the full bearing area under the post will not be obtained with a table under it.

The computations for Fig. 105 are as follows:

Depth of toe as in Fig. 104, 4 in.

Area required for bearing between upper and lower chord $\frac{28{,}125}{285} = 99$ sq. ins.

A 10-in depth will, therefore, be required for the upper chord, giving an area of 8×13 ins. = 104 sq. ins.

Depth of tables (assuming 3 used) $= \dfrac{49{,}000}{3 \times 8 \times 1600} = 1.275$ ins. Use $1\frac{5}{16} \times 3$ ins.

Assuming three rivets in each table, stress in each rivet $= \dfrac{49{,}000}{9} = 5450$ lbs. Use three $\frac{7}{8}$-in. rivets in each table.

Thickness of plate for bearing against rivets $= \frac{3}{8}$ in.

Thickness of plate for shear $= \dfrac{49{,}000}{10{,}000 \times 8} = 0.614$ in.

Thickness of plate for tension $= \dfrac{49{,}000}{16{,}000 \times (8 \text{ in.} - 2.8 \text{ in.})} = 0.59$ in. Make plate $\frac{5}{8}$ in. thick.

Moment of rotation of tables $= \dfrac{1.3125 \text{ in.} + 0.625 \text{ in.}}{3} \times \dfrac{49{,}000}{3}$ $= 15{,}800$ in. lbs.

Stress in bolts $= \dfrac{15{,}800}{3.5} = 4520$ lbs.

Add stress due to pin in bolster $= \frac{1}{4} \times \frac{1}{2} \times 800$ lbs. $\times 8$ ins. $= 800$ lbs.

Total stress in two bolts $= 5320$ lbs. Use two $\frac{5}{8}$-in. bolts.

Using two $\frac{3}{4}$-in. diagonal bolts, the horizontal component in the bolster will be as in Fig. 104, requiring one pin.

Distance required between tables for shear $= \dfrac{49{,}000}{3 \times 8 \times 150} =$ 13.6 ins. Use $13\frac{5}{8}$ ins.

Fig. 106 illustrates a practically perfect joint, except that it is not cheap. The chord and batter post stresses are transmitted to the gusset plates by means of lag screws acting in shear. There is no eccentricity of stresses and consequently no secondary stresses. With good inspection the lag screws will fit closely. The holes in the steel plates for the lag screws are drilled $\frac{1}{32}$ in. larger diameter than the diameter of the screws. The plates should be fitted to the timber and the holes marked, after which they are bored

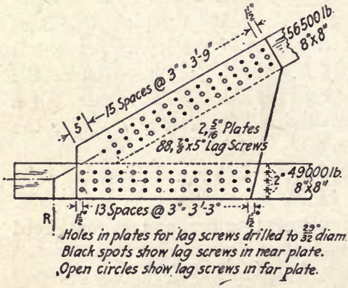


Fig. 106.

in the wood to a diameter a trifle less than the diameter of the shank of the screws at the base of the threads. The lag screws are to be screwed, not driven, into place. Bolts should not be used, for it would be next to impossible to fit the plates so the holes will all be in line. To make a fit the bolts would require some bending and thus much of their value in shear would be lost.

"This type of end detail is well suited to trusses of an A shape, resting upon posts. The side plates in such cases may be extended to engage the top of the post, and thus to give considerable stiffness to the building frame." — Dewell.

Computations for Fig. 106.

$$\text{Number of lag screws in upper chord} = \frac{56{,}500}{1200} = 47.$$

$$\text{Number of lag screws in lower chord} = \frac{49{,}000}{1200} = 41.$$

Thickness of plate $= \frac{5}{16}$ in.

Intermediate Joints in Trusses

Intermediate joints in trusses must follow the general rule for joints in wood, that the carpenter work must be as simple as possible.

The condition must be satisfied that the center lines of all members must meet at a common point. In nearly all joints of the types shown in Fig. 107 and Fig. 108 it often happens that when all the center lines meet at a common point the hole for the rod will cut away a part of the strut, or the toes of the struts will bear against the rods. Sometimes this condition cannot be avoided if the strut is to be dapped into the chord. Quoting again from Dewell: "If it so happens that the rod has not a driving fit in the chord, which condition will usually exist, especially with an upset rod and a deep chord, the toe of the strut will have bearing against the chord for only part of its width. The result of this condition will be that the actual bearing area may not be over one-half of what was assumed in design, and the unit-bearing stress may consequently be double the allowable."

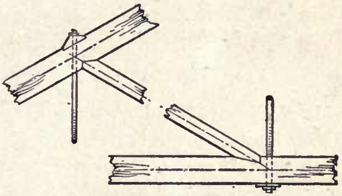


Fig. 107.

Two methods of framing intermediate joints are shown in Fig. 107 and Fig. 108. They are very common and yet violate the principle that the carpenter work should be simple. When the strut is not normal to the member it abuts against, the two surfaces of the indent must be separately investigated and the bearing pressure found, for each. The unit bearing pressure having been found the minimum bearing area must then be determined by methods already given. It involves considerable "cut and try" work. It is also imperative that the *exact* angles used must be marked on the drawings so the carpenters can make the joints in the field and secure the conditions assumed in the design. The angles of cuts having been found so the bearing is correct on each face, the depth of each cut is fixed by the bearing stress on the ends of the fibers, at the assumed angles. In Fig. 107 the cuts are not normal, the stress actually acting along the center line of the strut, or so nearly along the center line that the moment due to eccentricity may be neglected. In Fig. 108 the cuts are normal and the total thrust is assumed to act over the face of the normal cut. The unit stress on the fibers of the chord is found as shown in the design of the cut for the end brace. The depth of the cut is then found. At the upper end the normal (right angle) cut is on the lower side of the strut and at the lower end it is on the upper side. Draw lines through the centers of these normal areas, parallel with the top and bottom of the strut. The eccentricity is the distance between these center lines.

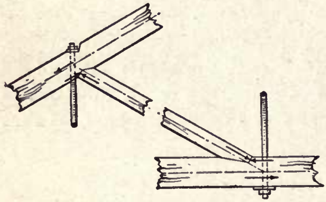


Fig. 108

Multiply the thrust by the eccentricity in inches and get the bending moment in inch pounds. This has a tendency to make the end of the strut move on the face of the normal cut and "jump out." It must be resisted by the friction of the wood on the face of the cut. Divide the eccentric moment by the length of the strut in inches and this gives the force to be developed by friction. Assuming the coefficient of wood against wood, for sliding friction, to be 0.2, multiply the direct thrust by 0.2 and obtain the resistance the wood will offer against being forced out of the cut by the bending moment. Nails and spikes offer resistance against being pulled out, so if the ends of the strut are "toe nailed" this additional resistance will be good. It seldom happens that the eccentric moment divided by the length of the strut will give an amount exceeding the direct thrust multiplied by the coefficient of friction, but if it does then spikes or bolts must be used to hold the toe in place. Carelessness in keeping all joints tight reduces the effect of friction, and decay in the joint also seriously affects it. In the joints illustrated there is often a serious loss in the efficiency of the upper and lower chords because of the depth of the indent. Details tending to reduce cutting into chords should be favored.

It has been said that all forces should act through the center lines of members. All the detailing is done with this in mind. Due to careless detailing, or, if the detailing has been good, then due to careless framing, any variation in the relation of web members meeting in a panel point may increase secondary stresses to a dangerous amount. The horizontal component of the diagonal thrust acts through the lower chord on a line intersecting the center of bearing of the thrust. The tension in the chord acts on the center line through the uncut portion of the chord. There is a moment developed by the vertical distance between these two lines of action. The vertical component of the thrust acts through the center of the face of the cut. This forms a couple with the tension in the vertical rod. There is a moment developed by the distance between the center of the rod and the line of action of the vertical component of the thrust in the strut. In wooden trusses the secondary stresses are seldom important except when very high unit stresses are used, but we cannot afford to neglect consideration of the possibility of neglect to keep the joints tight and the possibility of rotting setting in. It is very little trouble to investigate the effect of secondary stresses and provide additional material, or to redesign the joint to reduce the secondary stresses to a minimum. It is at least important that secondary

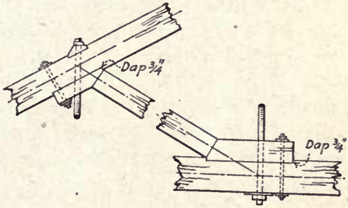


Fig. 109.

stresses should be investigated when high unit stresses are used.

The advantages of the type of joint shown in Fig. 109 are best summed up in the words of Mr. Dewell:[1] "In this joint, the strut has a full bearing on the butt block, and the butt block, in turn, utilizes the total width of the chord for bearing. Also, the detail takes advantage of the full bearing pressure in end compression of the butt block on the chord, resulting in a minimum depth of cut into the chord. Nearly all the cuts are normal, and the others are simple. All the cuts can be easily and accurately laid out and made by the carpenter. The length of the butt block can be adjusted to fit all conditions of possible interference with other connections. Its minimum length is determined by longitudinal shear. The bolt through the end of the butt block holds the block securely in its socket. Whether there is any actual tension in the bolt depends upon the length of the butt block. This can be determined at once by inspection. If the line of the thrust of the strut falls within the base of the block, there can be no tension in the joint. However, it is well to provide at least a $\frac{5}{8}$-in. bolt to bind the joint together thoroughly." In another place, Mr. Dewell says: "The detail of Fig. 109 is seldom used; nevertheless it is the most consistent and logical in principle and the simplest of construction of the three types shown." In the foregoing remarks the author heartily concurs. In too many cases draftsmen, not entitled to be termed designers, merely butt opposing diagonals against one another with no provision for transmitting the component of the diagonal stress to the chord. Designers must never forget that all forces can be assumed to act along lines: these lines intersect lines in other members and the force is then divided and goes in two directions. The main force is termed the resultant and the other forces the components. This will be discussed fully in the chapter on Graphic Statics.

Pin Connections

A pin connection is sometimes an economical connection. It may be used with either wooden or metal frames. The pieces connected

[1] *Western Engineering*, Oct. 1916, p. 386.

by the pin must have enough bearing area to prevent crushing. This being attained the pin is designed to resist shear and bending.

Pin connections require a minimum of material in the members. The cost of fabrication with pin connected trusses is not high. Such trusses cost less than rivetted trusses. The joint is flexible, if the pin does not rust, for all forces meet on the axis of the pin. It is theoretically a perfect joint and for many years was favored by American bridge engineers for the reasons given. European engineers always favored the rivetted joint because of its rigidity and all joints were designed to take care of eccentric stresses. Under heavy traffic it was found that the pin holes wore badly and thus the trusses became too flexible. When pins rusted into place eccentric stresses were set up and frequently the members were too small to take care of them. The pin-connected joint at the present time is not high in favor with bridge engineers, but it is all right for roof trusses.

Referring to Fig. 110 the bearing area is found by assuming the whole load to rest on a strip having a length equal to the combined thicknesses of the pieces connected, with a width equal to the diameter of the pin. Sometimes, for example in the case of a built-up member, an extra thickness of steel is rivetted to the side of a member in order to obtain increased bearing area at the pin hole. This is often cheaper than to increase the thickness of the metal in the member throughout the whole length.

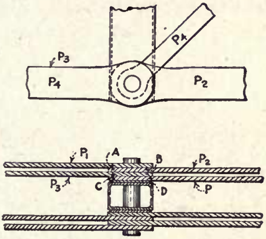


Fig. 110.

The shear on the pin seldom determines the thickness, the bearing area and bending stresses being usually of greater importance. The shear, however, should in all cases be investigated. The number of joints, that is the divisions between the pieces, will be one less than the number of pieces. Divide the sum of the loads on the pin by the number of joints to obtain the shear on each joint, if the pieces are of equal thickness. If they are not equally thick the shear on each joint will be equal to the sum of the reactions of the pieces on either side of the joint. If the pin is found to be too small to carry the shear the diameter must be increased, which will have the effect of reducing the unit stress in bearing.

To determine the flexure in pins the following formula is used for the resisting moment:

$$M = \frac{f\pi d^3}{32} = \frac{fAd}{8}$$

in which

M = moment of forces for any section through the pin.
f = allowable unit fiber stress in bending.
π = 3.1416.
d = diameter of pin.
A = cross-sectional area of pin.

The load in every member must be reduced to the horizontal and vertical component loads, and must be considered as acting in each member along the center line, so that the point of application of each horizontal and vertical component is at the center of bearing of the corresponding member. This means that if two $\frac{1}{2}$-in. bars are side by side the moment arm $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ in. The horizontal forces are equal on both sides of the pin, otherwise there would not be equilibrium. Similarly the vertical force downward is equal to the upward acting vertical force.

The bending moment (to which the resisting moment must be equated) is as follows:

$$M = \sqrt{(Mh)^2 + (Mv)^2}.$$

in which M = resulting bending moment in inch pounds.
(Mh) = maximum moment of all horizontal stresses.
(Mv) = maximum moment of all vertical stresses.

In designing pin joints no two adjacent bars should pull in the same direction, unless they shall by so doing reduce the bending moment. The joint must be symmetrically arranged to avoid torsion. Diagonal ties should be placed close to the vertical member and the horizontal ties should preferably be on the outside. Sometimes packing pieces are required between the members that carry stress, but these packing pieces merely lengthen the moment arm between adjacent members. The joint being symmetrical the computation stops at the center piece.

In determining the horizontal moments take one-half the sum of the thickness of adjacent bars for the moment arm between these two bars. The moment between the first two bars is equal to the load on the outer bar times the moment arm. The moment between the second and third bars is equal to the moment just found plus the difference between the loads on the first and second bars times the moment arm between the second and third. The moment between the third and fourth bars is equal to the last moment plus the difference between the load on the first bar, less the sum of the loads on the second and third, times the moment arm between the third and fourth bars. In determining the vertical moment multiply the vertical load by the distance between centers of the vertical member and the most distant inclined member. If there are a number of inclined members then proceed as in computing the horizontal moments, using the vertical loads.

Referring to Fig. 110 the loads on the members are designated as P_1, P_2, P_3 and P_4. The moment arms are designated as A, B, C, and D, being in inches. The resulting moments are designated by M_a, M_b, M_c, and M_d.

At P_2 the moment $= M_a = P_1 \times A$.

At P_3 the moment $= M_b = M_a + (P_1 - P_2) \times B$.

At P_4 the moment $= M_c = M_b + (P_1 - P_2 + P_3) \times C$.

The members P_4 are inclined and the center piece is vertical. The vertical moment is equal to the vertical load multiplied by the arm D. The vertical member is made of two channels and the other members are eye bars.

Fig. 111 is from the 1913 edition of the Carnegie Pocket Companion. A pin has to carry a load of 64,000 lbs.: required the size at 24,000 lbs. fiber stress, assuming the distance between points of support to be 5 ins.

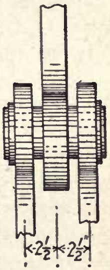


Fig. 111.

Bending moment $= 64{,}000 \times 5 \div 4 = 80{,}000$ in. lbs. This it is seen considers the center load as concentrated and allows nothing for the distribution of the load over a part of the span. The size of the pin may be obtained from the table on page 219 in that book. Looking in the column headed by 24,000 find the nearest (larger) resisting moment, which is 80,900 in. lbs. In the first column at the left is the diameter, 3¼ in.

The size and diameter of the pin may also be found from the expression, $M = fAd \div 8$.

First divide f by 8 = 24,000 ÷ 8 = 3000.

$$\text{Then } M = 3000\ Ad$$

$$\text{and } Ad = \frac{M}{3000}.$$

$$A = 0.7854\ d^2, \text{ therefore } Ad = 0.7854\ d^3.$$

$$d^3 = \frac{80{,}000}{0.7854 \times 3000}.$$

$$\text{and } d = \sqrt[3]{\frac{80{,}000}{0.7854 \times 3000}} = 3\tfrac{1}{4} \text{ in.}$$

Many tension members in steel work are made of round rods or rectangular eye bars. The ends are fastened to the frame by means of pins passing through loops or yokes or eyes. The area of the main part of the member is found by dividing the total tension by the allowable fiber stress. The thickness of the loop, the yoke or the eye is determined by the required bearing area on the pin. If the enlarged section on the end to receive the pin is welded to the member the stress used should be low to allow for imperfections in the welding. If the members are purchased from the mills already welded they should be purchased under very rigid specifications. The use of clevises, turnbuckles and sleeve nuts permits tension members to be lengthened and adjusted for length.

In the Carnegie Pocket Companion, 1913 edition, all the information the designer needs about the sizes of screw threads, bolts, eye bars, loop rods, clevises, turnbuckles and sleeve nuts is found on pages 112 to 122 inclusive, 215, 218, 219, 223.

Similar information is found on pages 322 and pages 331 to 357 inclusive in the 1914 edition of the Cambria Steel Hand Book. In the 1916 edition of Jones & Laughlin, Standard Steel Construction, this information is on page 246 and on pages 255 to 268 inclusive. The Lackawanna Steel Company Hand Book contains similar information on pages 339 to 363 inclusive.

Rivets and Rivetting

A rivet is a piece of metal which connects together two or more pieces of metal. In structural work rivets are made of soft steel. A head is formed on one end of a rivet when it is made and when used the rivet is heated and the surplus length projecting beyond the plates is formed into a head by means of presses in the shop or by hammers in the field.

A rivet should never be used in tension when it is possible to avoid so using it. In very exceptional cases a rivet may have to be so used and then the allowable tensile stress should not exceed 8000 lbs. per sq. in. The body of the rivet when used in tension may be amply large, but the thickness of the head must be investigated to determine whether it will be sheared by the pull on the rivet, this shearing being on a circle having a diameter equal to the diameter of the body of the rivet. The head of the rivet must be thick enough to withstand the shear.

The reason for not using rivets in tension is that the unequal heating and cooling during the process of fabrication of a member which is rivetted together sets up in the rivet expansion and contraction stresses of unknown amount. It has happened many times that rivet heads were snapped off when cooling and they snap off sometimes in extremely hot weather and in extremely cold weather. With such stresses existing in rivets it is manifestly dangerous to further impose on them a direct tensile stress, for the rivet heads may be on the verge of snapping off and any slight additional load may cause them to go. It is best to use bolts in joints in which rivets would be subjected to tension when field rivets are driven and if shop driven rivets are under tension the stress should be very low. Shop driven rivets are pressed into place and all the conditions in the shop make it likely that the work is uniform. It is impossible, however, to secure proper conditions in the field, for the heat cannot be controlled and many rivets are burned. They are thrown through the air and driven after some cooling has taken place. The hammering may be uneven and the rivets may not be hot enough when driven to be forced to completely fill the hole.

Rivets are assumed to act entirely in shear and all computations for rivetted joints are based on this assumption. There can be no doubt that friction is a big factor in rivetted joints, the rivets in shrinking drawing the plates together and holding them in contact so that friction between the plates assists the shearing strength of the rivets. The assistance obtained from friction is neglected in computations and merely increases the factor of safety of the joints.

Rivets may fail by bending. This effect, however, is not

SHEARING AND BEARING VALUE OF RIVETS FOR QUIESCENT LOADS AS USED IN BUILDINGS

Diam. of Rivet in Inches		Area of Rivet	Single Shear at 10,000 lbs. per sq. in.	Bearing Value for Different Thicknesses of Plate at 20,000 lbs. per square inch (= Diameter of Rivet × Thickness of Plate × 20,000 lbs.)										
Fraction	Decimal			$\frac{1}{4}''$	$\frac{5}{16}''$	$\frac{3}{8}''$	$\frac{7}{16}''$	$\frac{1}{2}''$	$\frac{9}{16}''$	$\frac{5}{8}''$	$\frac{11}{16}''$	$\frac{3}{4}''$	$\frac{13}{16}''$	$\frac{7}{8}''$
$\frac{3}{8}$	.375	.1104	1,100	1,880										
$\frac{7}{16}$	.4375	.1503	1,500	2,190	2,730									
$\frac{1}{2}$	.5	.1963	1,960	2,500	3,130	3,750								
$\frac{9}{16}$	.5625	.2485	2,490	2,810	3,520	4,220	4,920							
$\frac{5}{8}$	.625	.3068	3,070	3,130	3,910	4,690	5,470							
$\frac{11}{16}$	.6875	.3712	3,710	3,440	4,300	5,160	6,020	6,880						
$\frac{3}{4}$	.75	.4418	4,420	3,750	4,690	5,630	6,560	7,500	8,440					
$\frac{13}{16}$	.8125	.5185	5,190	4,060	5,080	6,090	7,110	8,130	9,140	10,160				
$\frac{7}{8}$	.875	.6013	6,010	4,380	5,470	6,570	7,660	8,750	9,840	10,940				
$\frac{15}{16}$	.9375	.6903	6,900	4,690	5,860	7,030	8,200	9,380	10,550	11,720	12,890			
1	1.	.7854	7,850	5,000	6,250	7,500	8,750	10,000	11,250	12,500	13,750	15,000		
$1\frac{1}{16}$	1.0625	.8866	8,870	5,310	6,640	7,970	9,300	10,630	11,950	13,280	14,610	15,940	17,270	
$1\frac{1}{8}$	1.125	.9940	9,940	5,630	7,030	8,440	9,840	11,250	12,660	14,060	15,470	16,880	18,280	19,690
$1\frac{3}{16}$	1.1875	1.1075	11,080	5,940	7,420	8,910	10,390	11,880	13,360	14,840	16,330	17,810	19,300	20,780

important except in very long rivets holding several plates. The action is then similar to that on a pin and is investigated similarly. It may be advisable then to use bolts or pins.

Rivets are much cheaper than bolts, otherwise bolts would be used, for they are safe in bending and in tension as well as in shear. An objection, however, to bolts is that it is difficult to screw the nuts tight and keep them from becoming loose under vibration. When bolts are used where rivets, if used, might be in tension, means must be provided for keeping the nuts tight. Even the best nut locks require frequent inspection.

The accompanying table from the Jones & Laughlin Hand Book gives the value of rivets in plates of different thicknesses. The values used are common and the steel handbooks all contain tables for other values.

In *single shear* rivets connect two plates, so there is one joint on which the plates may slide, precisely like the blades of shears. In *double shear* rivets connect together three, or more, plates so there are at least two joints on which the plates may slide. When three plates are used the middle plate is assumed to be pulling out from between the two outer plates.

In the table the thickness of the plates connected by rivets is given together with the bearing value and the shearing value. Figuring shear at 10,000 lbs. per sq. in. the value of the rivet in shear is constant, no matter what the thickness of the plate. The bearing value of the plate is 20,000 lbs. per sq. in. and a comparison of the figures shows immediately when the value of a rivet is determined by bearing and when by shear.

The strength in double shear is of course just twice that in single shear. The bearing on the plate is determined by the thickness of a single plate, of two adjacent plates in which the stresses are opposite. If two or more adjacent plates are fastened together and act as one plate, then the plate thickness is the combined thickness of the plates.

In the table the strength of the rivetted joint is determined by the shearing value of the rivet above the heavy line in some of the columns. Below the heavy lines the strength is fixed by the bearing value of the plate.

To determine the number of rivets to use first select the size of rivet. The thickness of the plate is determined when designing the member. Looking in the table, in the column headed by thickness of plate, opposite the size of rivet will be found the bearing value. In the column headed by shear find the shearing value of the rivet. Use the smaller value. Divide the total load by this selected value and obtain the number of rivets. If the rivets are in double shear double the shearing value given in the table and compare it with the bearing value, using the smaller amount. The rivets selected in this way are safe against destruction by shearing across and the edges of the holes through the plate will not crush. Fig. 112 illustrates single and double shear.

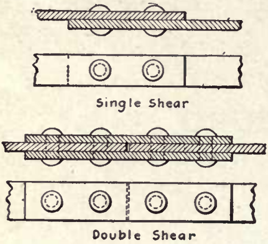


Fig. 112.

Let f = bearing value on plate in lbs. per sq. in.
t = thickness of plate in inches.
v = shearing value of rivet in lbs. per sq. in.
d = diameter of rivet.

Then

Shearing value in single shear $= 0.7854 \times d^2 \times v$.
Shearing value in double shear $= 2 \times 0.7854 \times d^2 \times v$.
Bearing value $= d \times t \times f$.

When driven, rivets are assumed to completely fill the holes, and, therefore, in compression pieces no reduction in area is made for the rivet holes. Tensile stress cannot, however, be transmitted through the rivets and the area occupied by rivet holes weakens the piece in tension. If the member is narrow and one hole is drilled, or punched through it, the piece must be increased in width or thickness to make up for the area removed by the hole. Thus, if in a bar 4 ins. wide and $\frac{1}{2}$ in. thick a hole $\frac{3}{4}$ in. diameter is made, the bar will be increased in width by $\frac{3}{4}$ in., or the thickness will be increased, provided the width must be maintained. The thickness is increased as follows: The width left is 3.25 ins. The area removed is $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ sq. in. The thickness to be added equals $0.3750 \div 3.25 = \frac{5}{32}$ in., making the total thickness $\frac{21}{32}$ in. There may be one hole through the plate or there may be several. If the holes are in line the cross-sectional area of one hole is deducted. If the holes are in two lines the cross-sectional area of two holes will be deducted. Similarly three holes will be deducted for three lines, etc. It means that a strip of metal equal to the width of the rivet hole is ignored provided all the rivets are driven within the strip, or if the rivets are driven in two or more strips then these strips are ignored in making computation for strength in tension.

A structural designer will determine the number of rivets in the manner described. He may then arrange them in a group

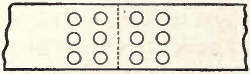


Fig. 113.

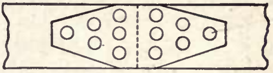


Fig. 114.

similar to that shown in Fig. 113, for structural designers do not always study joint efficiency. A boiler maker, on the contrary, would study the joint in order to obtain the maximum efficiency and his detail would resemble Fig. 114. There is no reason why a structural designer should not obtain the highest possible efficiency in designing rivetted joints.

Assume that the tension in the joint is 30,000 lbs. and each rivet carries 5000 lbs. In Fig. 113 three rivet holes must be deducted, which means additional material added to the member, for only the material left after the holes are made can carry tension. The rivets carry shear at an assumed unit shearing stress. The holes, however, cut out an area which carries tension at an assumed unit tensile stress.

In Fig. 114 the end rivet carries one-sixth of the stress as shear. The plate on a section through the hole carries the total stress. The diameter, therefore, of the rivet hole is one-seventh of the total width of the plate. On a section through the two rivets the plate loses in each hole one-seventh of the total width, but as it had enough area to carry seven-sixths of the load the cutting of two holes leaves enough area to carry five-sevenths of the load.

The plate with an area sufficient to carry five-sevenths of the load then reaches the section near the end where there are three rivet holes. Each hole has a diameter of one-seventh the width of the plate and the three holes leave four-sevenths of the area intact.

At the single rivet one-sixth of the stress is carried in shear, while the plate can carry the whole. At the line of the two rivets one-half (three-sixths) of the stress is carried by shear while the remainder of the plate can carry five-sevenths of the load. At the line of the three rivets all the stress can be carried by shear while the plate has area enough remaining to carry four-sevenths of the load. Therefore, by this arrangement of rivets it is necessary to deduct only the width of one hole, whereas if the rivets are arranged as in Fig. 113 it will be necessary to deduct the width of three holes.

When an arrangement is made such as that shown in Fig. 114 the splice plates will be a little longer and possibly a little thicker than the plates used in an arrangement such as that shown in Fig. 113. This is, however, offset by the fact that when material is added to overcome the cross-sectional area cut out by holes, the material is added to the area of the member throughout its whole length. The saving effected by a design like Fig. 114 is two rivet holes when compared with the design shown in Fig. 113.

The efficiency principle is applicable only to splices and in connections of rivetted trusses. There are many cases in which the efficiency principle must be disregarded, except as it affects the sizes of rivets used.

A rivetted joint fails by shearing of the rivets or by the metal between the rivets giving way. The distance between centers of rivets is termed the pitch. It is fixed partly by considering the shearing strength of the metal. It is fixed partly by arbitrary specifications. It is fixed partly by the requirement that the section between rivet holes must be fully as strong as the rivet.

Referring to the rivets shown in Fig. 113 and Fig. 114. The plate if it tears will tear at the weakest point. This may be on the vertical line joining centers of the rivets and may be on a zigzag line joining centers of adjacent lines of rivets. Experiments apparently indicate that rupture is as likely to occur on the zigzag line as on the vertical line, the cross-sectional net area determining this matter. By net area is meant the area measured between edges of holes. Some specifications require that the net area on the zigzag line exceed the square area by 30 per cent, but general rules should never be followed, except when required by specifications. It is poor policy for designers to follow arbitrary rules when they are competent to investigate, and have the time to investigate, the conditions of a particular case. Much poor designing and unsafe designing can be traced to blind observance of rules with apparent disregard of reasoning.

Standards

Fabrication standards are fixed by the types and capacities of the tools and machines with which a fabricating shop is equipped. A designer should know the standards of the shop in which his steel work will be fabricated and endeavor to arrange his detailing accordingly. This will insure the lowest possible cost for his client.

Standards for rivet spacing will be found on pages 212, 213, and 214, Carnegie, 1913 ed.: 328, 329 and 330, Cambria, 1914 ed.; 246 to 259 incl., Jones & Laughlin, 1916 ed.; 336, 337 and 338, Lackawanna, 1915 ed. The student is advised to study carefully a number of other pages in these books, dealing with the subject of rivets. Tables for the pitch (the distance between rivets) of rivets in angles are based on the angle developed. That is, the angle bent flat as if it were a narrow plate. This is an important thing to remember in detailing.

All manufacturers have standard beam connections based on developing the full strength of the beam on the shortest span on which it will carry the maximum load without failure by crippling or shear. When loading conditions are not severe some expense can be saved by designing connections to fit the case. When no details are shown for beam connections the standard connections are understood. It is well, however, to make a note to this effect on the drawings and avoid controversy. Refer also to the particular standards wanted. Standard beam connections are given on page 207, Carnegie, 1913 ed.; 42 to 50 incl., Cambria, 1914 ed.; 88, 89, Bethlehem, 1911 ed.; 323 to 331 incl., Lackawanna, 1915 ed.; 126 to 129 incl., Jones & Laughlin, 1916 ed.

It is time now for the student to procure sets of specifications for detailing structural steel. Some of the handbooks contain such specifications and all contain valuable data for specifications. The following specifications are recommended for purchase and study:

Standard Specifications for Structural Steel, Timber, Concrete and Reinforced Concrete. By John C. Ostrup. Sold by the U. P. C. Book Company, Inc., New York, for $1.00

General Specifications for Structural Work of Buildings. By C. C. Schneider. Sold by the publisher of this book for 75 cts.

Specifications and Tables for Steel Framed Structures. Prepared by the American Bridge Company. Distributed by the New York and Chicago offices free of charge.

Building Code recommended by The National Board of Fire Underwriters. Address the officers of the Board, 76 Williams Street, New York, N. Y. Similar information is to be had in the building ordinances of all large cities. Small details, however, of the sizes of members and spacing of rivets, etc., can only be had in specifications similar to the first three mentioned.

A number of standard specifications for structural work are sold and a list can be obtained from any large publishing and bookselling concern.

Secondary Stresses in Framed Structures

Secondary stresses in framed structures are due, primarily, to faulty details. In the general design of a framed structure it is assumed that all forces meet at a common point, which is the intersection of the axes through the centers of gravity of the members forming the joint. In the case of a pin connected truss, with the pin clean and the joint in first-class condition, this assumption is very nearly met. In the case of rivetted joints the direction of each member is rigidly fixed and when the structure deflects under load all members are placed in double curvature. This condition of secondary stress is accentuated by faulty joints, the resulting stresses with carefully studied joints being often negligible. With faulty joints a structure may fail because of secondary stresses.

The effect of faulty design can best be shown by an example and Fig. 115 shows a joint in the top chord of a Warren truss. Taking *A* as the center of moments the total bending moment, due to eccentricity, is $35{,}600 \times 7.5 = 267{,}000$ in. lbs. This moment is apportioned among the four members meeting at the joint in accordance with their relative rigidities, which is found by dividing their Moment of Inertia (to be found in the steel handbooks) by one-half the length of the member, all in inches.

To understand this question of relative rigidity assume that a bending moment is set up at a joint where all the members are rigidly connected and that the other ends of the members are likewise rigidly connected. The members are bent at their ends in opposite directions, thus setting up a double curvature, with a point of contraflexure, or point of zero moment in the middle of the length. The members may be considered as beams fastened at the joint with the middle point the free end. All the members, therefore, resist the bending moment in proportion to their relative rigidities.

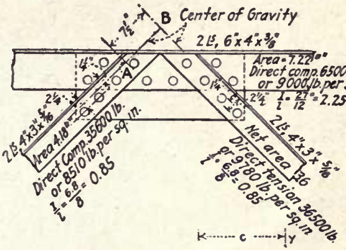


Fig. 115.

The angular displacement of the joint is the same for all the members meeting at the joint. The angular displacement at the joint then is the deflection of the middle point of any member divided by the half length of that member. This can be demonstrated by an expression in which appears the modulus of elasticity, the bending moment, the moment of inertia, the half length and the angular displacement. Dropping all the factors common to all the members there are left only the Moment of Inertia and the half length, so it is readily seen that the total bending moment is divided among the several members in proportion to their respective rigidities, that is in proportion to $\frac{I}{l}$. The Moment of Inertia is in inches and the half length should properly be in inches. However fewer figures are used and the work simplified by dividing the Moment of Inertia by the total length of the member in feet. The proportionate values are the same. This will be illustrated.

Let the total length of the chord between joints be 12 ft. and the depth between centers of gravity be 6 ft. The length of the diagonals will be 8.24 ft. but in the division we will use only 8 ft. The Moment of Inertia of the top chord is 27, the value for one angle being 13.5, as shown on page 148, Carnegie, 1913 ed., and on page 113, Jones & Laughlin, 1916 ed., similar values being given in the other standard steel handbooks. Similarly the Moment of Inertia for the braces is 6.8. Dividing, $\frac{27}{12} = 2.25$, and $\frac{6.8}{8} = 0.85$. Suppose the 8 and the 12 are divided by 2, to get the half-length, and then multiplied by 12 to reduce the half-length to inches, it is plain to see that this is equivalent to multiplying the 8 and 12 by 6, so the result of the division in each case will be one-sixth as large as before, but the proportion is unaltered. This illustration has been worked out because it explains the appearance of many expressions which often cause the amateur considerable trouble. A man accustomed to reasoning as he figures will often introduce many short cuts into formulas and expressions which he alone will understand, but the reasons for which can be readily worked out by any other equally competent man.

Returning to Fig. 115, add together the values of $\frac{I}{l}$ for the two chord members and the two web members, the sum being $(2 \times 2.25) + (2 \times 0.85) = 6.19$. The moments are now distributed as follows:

$$\frac{2.25 \times 267{,}000}{6.19} = 97{,}000 \text{ in. lbs. for the chord.}$$

$$\frac{0.85 \times 267{,}000}{6.19} = 36{,}700 \text{ in. lbs. for the web members.}$$

The maximum fiber stress in the members induced by these moments is as follows:

For chords, $f = \frac{My}{I} = \frac{97{,}000 \times 4}{27} = 14{,}400$ lbs. per sq. in. The 4 in. is the distance from the top of the angles of the chord members to the center of gravity axis, parallel to it, of the rivets.

For Web Members, $f = \frac{My}{I} = \frac{36{,}700 \times 2.25}{6.8} = 12{,}300$ lbs. per sq. in. The 2.25 ins. is the distance from the back of the angle to the center line of the rivets.

Compare the stresses due to the eccentricity caused by failing to have the lines through the center of gravity of the members meet at a common po nt, with the stresses used in the design and marked on Fig. 115. It will be seen that the secondary stresses are about fifty per cent greater than the direct stresses and will cause the failure of the joint ultimately, for the effect of the direct and eccentric stresses is the sum of the two.

In proportioning the web member carrying tension the area of one rivet hole is deducted from the area of the member. In the web member subjected to compression the whole area is taken, for the rivet is assumed to fill the hole. The resulting direct stresses are shown in the figure. Provided the rivets are driven on the line of the center of gravity of the member the ascertained direct stresses will be realized. If the rivets are driven to one side of the axis of the center of gravity an eccentricity will be developed which may very seriously increase the stress. Where angles are used to resist direct stress, and connected through one leg only, the gauge line for the rivets should be set in as close to the back of the angle, or as near to the center of gravity axis as possible. This matter is of fundamental importance and yet it is habitually disregarded in structural work.

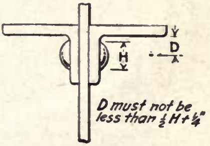


Fig. 116

The rivet clearance for machine driving is shown in Fig. 116. It is customary to use so-called "standard gauges" for angles, pitching the rivets from the back of the angle a distance somewhat greater than the half width of the leg. In the case of the web members shown in Fig. 115 the dimension D would be $\frac{15}{16}$ in. Adding to this the thickness of the outstanding leg, we obtain $1\frac{1}{4}$ ins. as the permissible gauge of these angles. This coincides exactly with the center of gravity axis of the angles and if the rivets were so placed the fiber stress due to eccentricity of the line of rivets would be entirely eliminated. The method to use in figuring the stress due to eccentricity in the line of rivets is given in detail in another chapter.

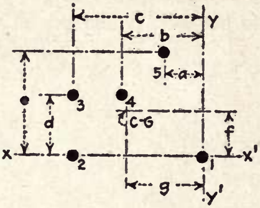
To avoid eccentricity as much as possible in a group of rivets the forces should act through the center of gravity of the group. The best way to obtain the center of gravity is by using the method of moments, as shown in Fig. 117.

Having found the center of gravity of a group of rivets find the axis through the center of gravity of the member connected to the plate or angle forming the connection. Extend this axis through the group of rivets. If it does not pass through the center of gravity of the group it can be replaced by a force equal in amount and parallel, which will pass through the center of gravity. There will be a moment developed equal to the load multiplied by the perpendicular distance between these two lines of action.

Each rivet in the group carries an amount of direct stress as shear equal to the total load divided by the number of rivets.

STATICAL MOMENT ABOUT AXIS xx'

Rivet	Area	Arm	Moment
1	*A*	*o*	*Ao*
2	*A*	*o*	*Ao*
3	*A*	*d*	*Ad*
4	*A*	*d*	*Ad*
5	*A*	*e*	*Ae*



Total Area = $5A$
Total M = $A(2d + e)$

$$f = \frac{A(2d + e)}{5A}$$

STATICAL MOMENT ABOUT AXIS yy'

Rivet	Area	Arm	Moment
1	*A*	*o*	*Ao*
2	*A*	*c*	*Ac*
3	*A*	*e*	*Ac*
4	*A*	*b*	*Ab*
5	*A*	*a*	*Aa*

Total Area = $5A$
Total M = $A(a + b + 2c)$

$$g = \frac{A(a + b + 2c)}{5A}$$

Fig. 117.— Method of Moments to find Center of Gravity of Group of Rivets.

If the direct application of the load causes a bending moment, then each rivet has an added stress due to bending moment.

The stress in any rivet due to bending moment varies directly as its distance from the center of gravity of the group of rivets, and its resisting moment varies as the square of this distance.

Fig. 118 represents an angle connection for the end of a 10-in. I-beam weighing 25 lbs. per lineal foot. The reaction from the load carried on the beam is 13,720 lbs. delivered to the leg outstanding from the beam, the other leg being connected to the web of the beam. Finding the center of gravity of the three rivets and multiplying we get an eccentric bending moment,

$$M = 13{,}720 \times 3.25 = 56{,}290 \text{ in. lbs.}$$

The shear on each rivet due to direct stress is $\frac{13,720}{3} = 4573$ lbs.

Each rivet has also some stress due to bending moment.

The necessary resisting moment is found as follows:

$M = A(x^2 + y^2 + z^2)$

in which

M = resisting moment

A = stress in a rivet due to bending.

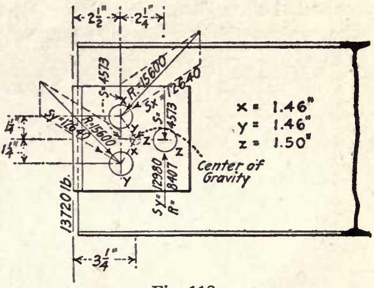


Fig. 118

x, y and z represent, respectively, the moment arm for the rivets bearing these letters.

Transposing,

$$A = \frac{M}{x^2 + y^2 + z^2} = \frac{56,290}{6.513} = 8650 \text{ lbs.}$$

The stress in each rivet due to bending is equal to this figure, multiplied by the distance of the rivet from the center of gravity.

Stress on x = 8650 × 1.46 = 12,640 lbs.
Stress on y = 8650 × 1.46 = 12,640 lbs.
Stress on z = 8650 × 1.50 = 12,980 lbs.

Adding the direct shearing stress to these figures, by the parallelogram of forces drawn in Fig. 118 the resultant stress on rivets x and y is 15,600 lbs., as shown.

The web thickness of a 10-in., 25-lb. I-beam is 0.31 in. The bearing area of a $\frac{3}{4}$-in. rivet is, therefore, 0.31 × 0.75 = 0.2325 sq. in.

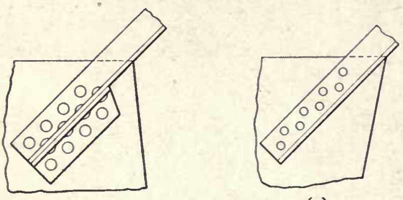
15,600 lbs. divided by 0.2325 = 62,100 lbs. per sq. in. bearing stress on web of beam. This is more than three times the amount shown as permissible in the rivet table, where the allowable stress in bearing is 20,000 lbs. per sq. in. The connection here shown is, therefore, not good for a reaction of 13,720 lbs.

In Fig. 119 at (*a*) is shown an angle connected by both legs to the gusset plate. Only one hole is deducted, for the holes are staggered to preserve the net section. This staggering is done also, when not necessary to preserve the net section, in order to permit of driving the rivets in two legs with plenty of "clearance."

When an angle is connected by both legs the total area of the angle, less the area of the holes, is taken. When an angle is connected through one leg, as at (*b*), the load will be eccentric. The problem is then complicated, but an easy practical solution is to use only the area of one leg.

Problem. An angle in tension has to carry a load of 50,000 lbs., using $\frac{3}{4}$-in. rivets.

$$\text{Area of member} = \frac{P}{f} = \frac{50{,}000}{16{,}000} = 3.12 \text{ sq. ins.}$$



(a) (b)

Fig. 119

If connected by the two legs try a $5'' \times 5'' \times \frac{3}{8}''$, the area of which is 3.61 sq. ins. (Carnegie, p. 146.) The area of the hole is $\frac{7}{8}'' \times \frac{3}{8}'' = \frac{21}{64}'' = 0.328$ sq. in. The net area is $3.61 - 0.328 = 3.282$ sq. ins. Notice that the diameter of the hole is $\frac{1}{8}''$ greater than the diameter of the rivet, giving $\frac{1}{16}''$ clearance. This rule is general for all rivets.

If connected by one leg it is assumed that this leg will carry all the stress. Assume a thickness of $\frac{1}{2}''$, and as the area is 3.12 sq. ins. divide and add, to the width thus obtained, the diameter of the hole. Then $3.12 \div 0.5 = 6.24'' + 0.875'' = 7.115''$. This is plainly not suitable, for it does not fit any standard angle. Try a thickness of $\frac{5}{8}''$ and we get $3.12 \div 0.625 = 5'' + 0.875 = 5.875''$. This is nearly six inches, so we will use a $6'' \times 3\frac{1}{2}'' \times \frac{5}{8}''$ angle.

The practice of using $\frac{1}{4}$-in. and $\frac{5}{16}$-in. gusset plates in roof trusses is very common, yet considerations of economy, as well as efficiency, would seem to indicate the use of thick plates. The plates should be of such thickness that the bearing value of a rivet in the plate is about equal to the value of the rivet in double shear. This would reduce the number of rivets in a joint considerably and reduce the size of the plate correspondingly. The slight increase in the weight of the plates is apt to be more than offset by the reduction in the number of rivets. The use of thicker places and fewer rivets also measurably reduce the secondary bending stresses in the members due to fixity of their ends. The author never uses a plate as thin as $\frac{1}{4}$ in. The least thickness of any member or plate in a truss should be $\frac{5}{16}$ in., for some metal should be provided to offset the wasting effect of rust. It is well enough to consider that the metal will always be inspected and kept painted, but we know that painting is neglected for shamefully long periods of time.

In Fig. 120 (*a*) is shown a joint in a riveted Pratt truss that is of common occurrence. Here the axes of the members are con-

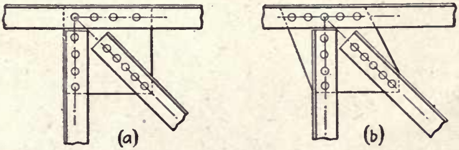


Fig. 120

current, but the rivet connections through the chord are eccentric to the intersection of the lines of stress, and a bending moment results. The proper construction of this joint is shown at (*b*) and the student is advised to perform the calculations for the two joints and determine for himself the amounts of the eccentric stresses. The truss for which joints were designed in wood can be designed now for steel and the two details in Fig. 120 can be assumed for this truss.

Fig. 121 (*a*) shows the heel of a roof truss. It is a common detail, but that does not make it desirable or proper. It merely shows the power of example and illustrates the proneness of draftsmen to copy blindly. The three forces acting at the heel, namely, the compression in the rafter, the tension in the bottom chord, and the column, or wall, reaction are non-concurrent. A bending moment results which induces large fiber stresses in the members. The method to follow in determining the amounts of the stresses was illustrated in the case of the Warren truss.

Fig. 121 (*b*) is, likewise, an improper detail unless the heel plate is thick enough to resist the bending moment between the point of intersection of the three forces and its attachment to the members. The plate should also be planed or chipped flush with the backs of the angles of the bottom chord when it is not possible to get sufficient rivets immediately over the column to transmit the total reaction into the plate.

Fig. 121 (*c*) show an efficient and proper detail for the heel of a truss.

Fig. 122 (*a*) shows a detail of a knee brace connection to a column which is not uncommon in mill building construction.

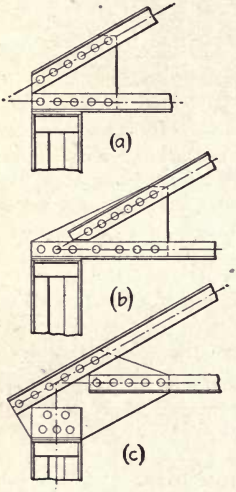


Fig. 121

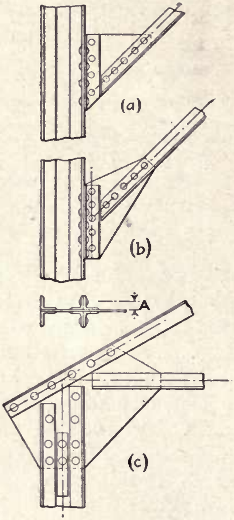


Fig. 122

This detail is open to the same criticism as the other eccentric connections already discussed. It is especially to be condemned in view of the fact that the knee brace is subject to tension, as well as compression, and when the knee brace is in tension the entire stress must be resisted by two rivet heads.

Fig. 122 (*b*) shows the proper detail for this connection. The gauge *A*, for the rivets connecting the knee to the column flange, should be as small as possible, and the thickness of the connection angles should be such that their moment of resistance at the rivets is equal to the bending moment. This bending moment is equal to one-half the horizontal component of the stress in the knee brace, multiplied by *A*.

Fig. 122 (*c*) is a detail known as a knuckle plate connection.

It is sometimes used, in lieu of a knee brace, in order to economize head room and to avoid obstructing the crane trolley travel. The knuckle plate should never be used as a substitute for the knee brace in a building high enough for a crane.

The greater part of the material and all but three of the illustrations in the section on "Secondary Stresses in Framed Stresses" was taken from a paper bearing that name presented by E. W. Pittman before the Engineers Society of Western Pennsylvania. In several paragraphs the exact language was used. The paper was published in the Proceedings of the Society (Pittsburgh, Pa.) for February, 1909. The student should read the whole of the paper and the discussion following it. In the December, 1916, issue of the *Journal* appeared a valuable paper by E. W. Pittman entitled "Factors Affecting Cost of Structural Steel Work," and a paper by George H. Danforth entitled "Some Items Affecting Cost of Structural Steel Work," with discussion by a number of men of wide experience. Back numbers of the *Journal* cost fifty cents each.

Reference was made in an earlier chapter to Smoley's Parallel Tables of Logarithms and Squares, which are indispensable to structural designers in computing the lengths and bevels of truss and frame members. During the month of January, 1917, appeared Smoley's Parallel Tables of Slopes and Risers, with Ready Solution of Right Triangles. This second book also will be one that structural designers will not willingly go without, once they learn its value.

The design of compression members will be dealt with in the chapter on Columns and Structures. In Chapter VI will be taken up also the design of members subjected to both tension and compression.

The "One-book Man" is not a broad man. Even with the best possible explanations an author does not always succeed in getting the reader to thoroughly grasp his ideas. It is, therefore, an excellent plan to do some collateral, parallel reading when studying, in order to obtain the methods of working of more than one person. The author believes students will receive a great deal of benefit by starting at this point to study the following books.

"Bridge and Structural Design." Thomson, $2.

"Structural Engineering." Husband & Harby, $2.50.

"Typical Steel Railway Bridges." Thomson, $2.

The books named abound in worked examples of detailing. The authors had in mind men of limited mathematical attainments.

Not all the students will become, or wish to become, structural detailers. To become a first-class commercial detailer will require a great deal of practical experience and the following books should be studied and kept as works of reference.

"Steel Structures." Morris, $2.25.

"Structural Engineers' Handbook." Ketchum, $5.

The only really adequate book on the design of modern high steel frame buildings, popularly called "sky-scrapers," is "Steel Construction," by H. J. Burt, $2.25. It is written in a simple manner for the instruction of correspondence school students. The books by Morris and Ketchum are of college grade. The publishers of this book can supply all the books mentioned.

CHAPTER VI

Graphic Statics

THE student knows that a line can be drawn to represent a force, because forces act through the center of gravity of a body and this is a point. A line is a succession of points, or is the path of travel of a point. A line representing a force indicates by its direction the direction in which the force acts. When drawn to a scale the length may represent the amount of the force in any selected unit,—pounds, tons, etc.

In earlier chapters some simple graphical methods were presented for obtaining bending moments and shear on beams, but to solve more complicated problems, and to make even those shown a little more simple, reciprocal diagrams must be used. The reciprocal diagram method for making computations is known by the name of Graphic Statics.

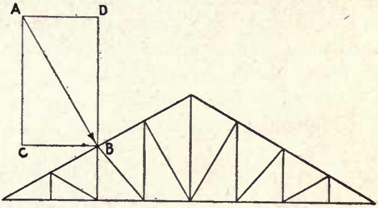


Fig. 123.—Parallelogram of Forces.

In Fig. 123 is shown the Parallelogram of Forces. The line *AB* shows the pressure of the wind against a roof truss. All forces act normally to the surface pressed, so the wind always blows perpendicularly to the slope of a roof. It may be resolved into a vertical force represented by *BD* and a horizontal force represented by the line *BC*.

The vertical force as weight may be added to the weight of the roof and any loads that may come upon it, and the members of the truss be proportioned to carry all the loads. The horizontal force may be assumed to act to push the roof off the support and is a measure of the force to be resisted by bolts which tie the truss to the supports.

All forces may be resolved into components. The force *AB* in Fig. 123 might be considered as produced by two given forces, *BC* and *BD*. That is, if the horizontal force had been given, the line *CB* could have been drawn. The vertical force having been given, the line *BD* could have been drawn, it being assumed that the two forces meet at one point, *B*. From *D* draw a horizontal line and from *C* draw a vertical line to intersect it, at *A*. The diagonal line *AB* is the *resultant* of the two forces.

The *resultant* is the line required to close a figure and the other forces are *components* of the *resultant*.

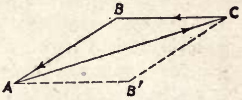


Fig. 124. — Triangle of Forces.

In Fig. 124 it is shown that it is not necessary to complete the parallelogram. Let *AB* and *AB'* be two forces acting at the point *A*. Draw them to scale and from *B'* draw a line parallel to *AB*. From *B* draw a line parallel to *AB'*. The two lines will meet at *C*. The line *AC* is the resultant. The force *AB'* is the force *BC*. The resultant would have been obtained if we had merely drawn *AB* and then from *B* drawn *BC* to represent the force *AB'*. The figure *ABC* is known as the Triangle of Forces. It is formed by drawing the forces end to end and obtaining the resultant by drawing a closing line.

In order to obtain *equilibrium* it is necessary that all the forces acting on any body meet at a common point or be parallel. If all non-parallel forces do not meet at a point then there will be a moment arm which will cause rotation. When all the forces are parallel equilibrium is obtained by supplying a resultant large enough to balance the forces acting in opposite directions.

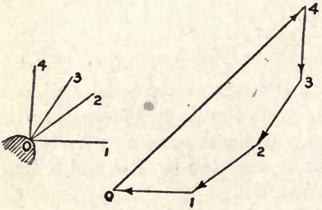


Fig. 125. — Polygon of Forces.

In Fig. 125 are four forces acting at the point *O*, of a body. The polygon of forces is shown at the right and all the lines are drawn to scale. The line *O*4 drawn to close the polygon represents the weight which the body must have to resist being moved by the forces. It represents also the direction in which the body will move if not heavy enough to resist the push.

This polygon could have been drawn as follows, and the student is advised so to draw it in order to get a clear idea of the matter;

First, taking the forces 1 and 2, construct a parallelogram and obtain the resultant. Using this resultant as a force form a parallelogram with it and force 3. The resultant thus obtained will be combined as a force with force 4. The final resultant obtained will be the force $O4$. On the figure here shown the resultant of 1 and 2 will be a line $O2$. The resultant of $O2$ and force 3 will be a line $O3$. Then the resultant of force 4 and $O3$ will be $O4$.

Action and *reaction* are equal when equilibrium is to be preserved. The arrow points on the force lines show the direction in which each force acts. The resultant measures the force necessary to preserve equilibrium; therefore, the direction is against the general direction of all the forces. The arrow point on the resultant indicates this, and the result is that the closed figure has the arrow points so arranged that the forces can be followed consecutively. This indicates equilibrium, and if the arrow points do not indicate a consecutive line of travel it is evidence that equilibrium does not exist. The polygon should not be closed, or there is some mistake in the construction.

The resultant is the force required to maintain equilibrium, or it may be a force which can replace all the other forces. In Fig. 123 is an example of where two forces are substituted for one force. The vertical and horizontal components may be assumed to replace the diagonal force acting against the roof. Here there is, strictly speaking, no resultant considered. A certain force has been resolved into two components.

To obtain a resultant all the forces are arranged to form a triangle or polygon and the closing line is the resultant. To resolve a force into two forces acting at any angle draw the resultant to scale. From one end draw a line of indefinite length in the direction of one component. From the other end, on the same side, draw another line of indefinite length in the direction of the second component. The lines will intersect and the lengths thus fixed will represent to scale the amounts of the components. The student is advised to study carefully the difference between resultant forces and component forces. A force may have any number of components, but in a system of framing the number will be fixed by the number of members meeting at a joint.

The designer of buildings will usually deal only with parallel loads. The direct loads on a roof act vertically at the joints. The wind load will also act at the joints, but not vertically, this, however making no difference in the method of treatment, for the wind loads on the joints will be parallel, although not vertical. In an earlier chapter the wind effect on roofs was assumed to be a vertical load, but when finding the stresses in a roof truss by graphic statics one diagram is drawn for the vertical loads and a separate diagram for the effect of wind. Stresses are tabulated for each system of loading and added. There are several "combined" methods for making a single diagram serve for the vertical loads and wind loads, but they are not easy to remember and the separate diagrams cannot be forgotten once they are mastered.

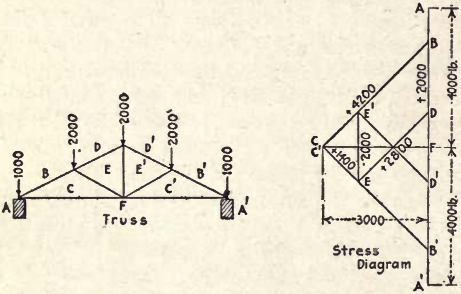


Fig. 126. — Forces in King Truss by Graphic Statics.

In Fig. 126 is shown a king truss with the reciprocal diagram. The loads are marked at each joint. The two end loads are carried directly by the walls, so do not affect the stresses in the members of the truss. The spaces between members are lettered. A vertical line is drawn on one side of the truss. This is usually on the right side, but it may be on the left if most convenient. Beginning at the top, all the loads are set off as shown. The loads are between the lettered portions of the members, so the spaces on the vertical load line drawn to scale represent the loads between the letters. After setting off all the loads the lines of the truss members are transferred by means of triangles, or parallel rule, to connect with the load line. This gives the sloping lines meeting on the line *CF*.

The construction here described is for a truss uniformly loaded with equal reactions. The amount of the reaction at each end is shown on the right of the load line. The loads are set off vertically to scale and when the construction of the stress (reciprocal) diagram is completed the lengths of the lines are measured by the same scale and the amounts of the stresses found.

The character of the stress in each member must be determined and this can only be done by some study and by following the loads. It is a help, at first, to make a free-hand diagram for each joint and place arrow heads on each line, remembering that if the frame is to be in equilibrium it must be possible to start at one joint and follow the lines clear around the figure to the starting point. The student is advised to make the figures now to be described, free hand, and move the pencil as he follows the directions.

The kind of stress is to be found. Start with the joint between *B* and *D*, sketching it on the stress diagram. Here there is a vertical load of 2000 lbs., from *B* to *D* on the reciprocal (stress) diagram. Move from *D* to *E* and the arrow point is seen to point to the joint. The stress is compressive. Move from *E* to *C*. The arrow point is towards the joint, so the stress is compressive. Move from *C* to *B*. The arrow point is towards the joint and the stress is compressive.

Take the joint at the foot of the truss, between *A* and *B*. This should have been taken first, but the second joint gave the best illustration for a beginner. Start from *B* on the load line and move to *C*. This is towards the joint, so the stress is compressive. Move from *C* to *F* on the load line. This is away from the joint, so the stress is tensile.

It has been determined that the stress in *DE*, in *D′E′*, in *CE*, and in *C′E′*, is compressive. A vertical member connects the upper and lower joint. Assuming this member to be cut, will the joints be spread apart or will they close up? If they will be spread apart, then the action of the forces at the joints will cause tension in the member. If they will close up, then the member will be in compression. Applying this reasoning, it is seen that the stress in *E E′* is tensile. The character of stresses in all members of trusses may be determined by such reasoning.

The stress in the vertical member may, however, be determined by the first method. On the load line take the load of 2000 lbs. between *D* and *D′*. Go from *D′* on the load line to *E′* and as the arrow point is towards the joint the stress is compressive.

From E' to E the arrow point is away from the joint, so the stress is tensile. From E to D the arrow point is towards the joint, so the stress is compressive.

The truss is symmetrical with symmetrical loads, so having obtained the stresses and their character in the members on one side of the center, as well as in the center vertical member, all the required information has been found.

In Fig. 127 is shown a Queen truss with its reciprocal diagram. The truss is symmetrical with symmetrical loading, so the end reactions are equal. It is only necessary to draw one half of the stress diagram, for there is no center vertical member. *ABCIA*

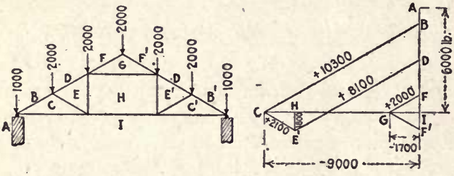


Fig. 127. — Forces in Queen Truss by Graphic Statics.

is the reciprocal for the joint between *A* and *B*. *BCEDB* is the reciprocal for the joint between *B* and *D*. *ICEHI* is the reciprocal for the joint *CEHI*. *DFGHED* is the reciprocal for the joint *EGH*. *FF'GF* is the reciprocal of the joint between *F* and *F'*. The point on the load line is below the point *I*, a distance equal to *FI*.

In the examples shown no account has been taken of wind. The reactions are equal and when wind is considered as acting normal to the surface of the roof the reactions are unequal. Therefore until the graphical method for obtaining reactions with unequal loading is shown, we will assume the wind to be reduced to a load acting vertically. For slopes not exceeding 30 degrees from the horizontal this will not make much difference and is fairly common practice. To avoid confusing the student vertical loading will be considered first and after illustrating the work on the forms of trusses in common use the general method for obtaining stresses caused by wind will be taken up.

The word "stress" has been used because it is so commonly used in connection with the members of a truss. It is not strictly correct. "Force" is the proper word and forces act on the members. These forces set up strains in the members. A strain is the amount of deformation caused by the action of an external force and all internal stresses are caused by strains. From this point the word "force" will be used instead of the word "stress."

The loads on the panel joints are found by assuming an area equal to the width of a panel and a length equal to the spacing between trusses. This is based on each truss carrying the load on a distance measured halfway between adjacent trusses and each panel in a truss carrying a load on a width measured half way between adjacent verticals. Half the load on the end panels of a truss is carried by the wall.

Fig. 128 shows a truss with a middle vertical and two side verticals, together with one-half the force diagram. The con-

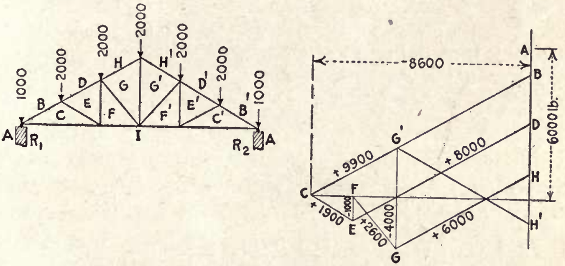


Fig. 128.—Forces by Graphic Statics in Queen Truss with Center Rod.

struction is not different from the trusses already illustrated, but attention is called to the fact that the side verticals, in the reciprocal diagram, end at the tie rod. The middle vertical, on the contrary, ties the truss together — speaking in a general way — so it is measured from G to G'. The student should be able without trouble to trace the reciprocals on Fig. 128.

Fig. 129 illustrates a truss with a cambered tie rod. Notice that the loads are plotted in order on the reciprocal diagram and the sloping lines transferred as before. From the middle point there are two reaction lines, representing the camber in the tie rod, instead of one horizontal line. The side verticals end on the lines representing the tie rod, but the middle vertical goes across the open space to tie the two main rafters together in the top chord and carry the stress to the tie rod. If this vertical line were not drawn in this way there would be a vertical line with a gap in it and the letter *m* and the letter *l* would be repeated. Since there is but one member with these letters there can be but one line in the reciprocal diagram having these letters at the ends.

In Fig. 129 is used for the first time the most orderly arrangement of letters to designate truss members. Beginning at the left the letters run consecutively along the outside of the truss to the extreme right and continue in the same consecutive manner back to the left. When the loads are laid off on the vertical load line the letters run in regular order. This is a convenient way

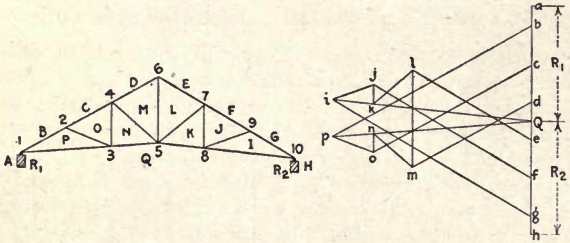


Fig. 129.—Truss with Cambered Tie Rod.

when the truss is unsymmetrically loaded and is, therefore, convenient when it is symmetrically loaded and but one-half of the reciprocal diagram is shown. Capital letters are used on the truss diagram and the corresponding small letters shown on the reciprocal diagram. The joints are numbered, in order that the joint referred to may be described readily with the fewest words. This system for numbering joints is used generally for trusses having non-parallel chords. Trusses with parallel chords have the joint numbered with the letter *U* prefixed to the number on the upper chord and with the letter *L* prefixed to the number on the lower chord. The student is advised to use the method of lettering and numbering shown in Fig. 129.

In Fig. 130 is shown a simple Fink truss with cambered tie rod. The Fink truss is said to have been invented by Albert Fink and is an early form of metal truss. In Europe it is known as the Belgian truss and is commonly supposed there to have been the invention of a Belgian engineer. In some English text books it is called a truss. Such happenings are common in engineering. Men educated in mathematics and applied mechanics, when set to solve a problem, are apt to solve it in pretty much the same way, although separated

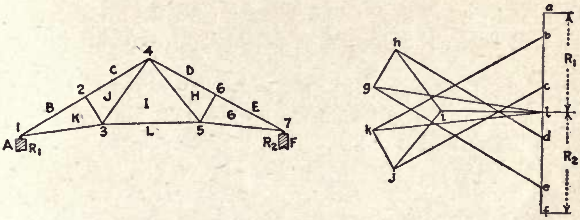


Fig. 130. — Fink Truss with Cambered Tie Rod, Load on Upper Chord.

by oceans and continents and perhaps ignorant each of the other. Patent attorneys say that certain patents are applied for at about the same time by several men, the one first making an application receiving the patent, and the others are left with a firm belief that some one must have told him of their work.

The truss with the cambered tie rod and diagonal, instead of vertical, ties is treated like the truss in Fig. 129, but the diagonal ties meet as shown at a point on the horizontal center line.

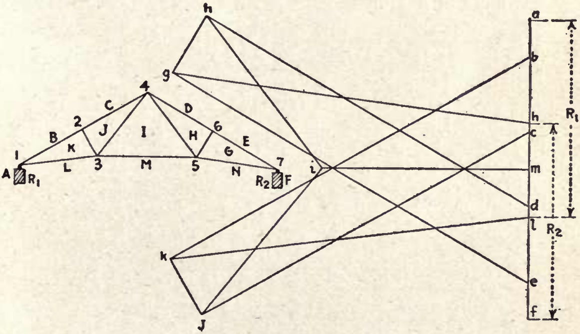


Fig. 131. — Truss with Load on Upper and Lower Chords.

In Fig. 131 is shown a truss with loads on the lower chord as well as on the upper chord. First the loads on the upper chord are laid off on the vertical load line. The loads on the lower chord are added and one-half the total gives the reaction for each end. The reactions are laid off as shown, overlapping on the load line. The cambered tie rods are transferred to meet these points and extended to intersect with the rafter lines. The horizontal part of the tie rod is drawn from the middle point of the load line. The diagram is not hard to construct if care is taken.

The drawing for a truss with a horizontal tie rod, or horizontal lower chord, is similar. First the loads on the top chord are set off on the vertical load line. Then the loads on the lower chord are added, the reactions obtained and set off and horizontal lines drawn from the points marking the amount of the end reactions.

The Fink truss is economical because the struts are short and most of the members are in tension. Partial loading cannot cause maximum stresses in the members as it will in other common forms of trusses. It is a difficult roof to frame when the slope is slight, so it should be used only for pitches exceeding 27 degrees. The form shown in Figs. 130 and 131 is the most simple one, a common form of the Fink truss being illustrated in Fig. 132 and Fig. 133.

In *Building Age* for May, 1916, Mr. Harry B. Wrigley, Allentown, Pa., presented the following method for dealing with the web members of a Fink truss, *without substitution or change.* So far as the author knows, this method has never before been presented in print, so Mr. Wrigley may be justified in claiming it to be original with himself.

In the graphical analysis of forces in a Fink, or Belgian, truss, a difficulty is encountered at joint 5, supporting the load *CD*, Fig. 132; for after determining the forces in the members meeting at joints 1, 2, and 3 there remain three unknown forces at joint 5, namely, in members *DP*, *PO*, and *ON*. A similar difficulty is met with at joint 4, where there are three unknown forces, namely, in members *NO*, *OR*, and *RK*.

It is a well-known fact that in order to construct a polygon of forces in equilibrium, acting in the same plane, through the same point, all conditions but two must be known.

In Fig. 132 (*a*) is shown the truss, with the force diagram at (*b*). To illustrate the new method consider the left half of the truss as shown at (*c*) and lay off the load line *abcdek*, and reactions kr' and $r'a$, in the usual manner; then construct the stress diagram for the web members as shown by the dotted lines at (*b*), taking the joints in numerical order.

It is evident that the panel loads to the right of load *EF* do not affect the web members to the left of this load; therefore, having determined the web forces, complete the load line *aj*, and reactions *jk* and *ka*, at (*b*), to obtain the forces in the chord members. The diagram *lmnopqr* should be made identical with

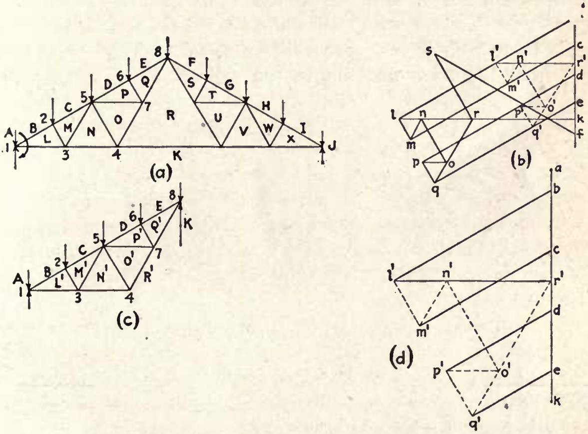


Fig. 132. — The Wrigley Solution of the Fink Truss Problem.

the dotted diagram *l′m′n′o′p′q′r′*. The above method applies as well to diagrams for wind loads.

In Fig. 133 three methods are shown for drawing the reciprocal diagram for a Fink truss. The Barr method is to proceed with the construction until the line is drawn from *v* to *u*. Then on the truss draw a dotted line from joint 4 to joint 6. On the reciprocal diagram from the point *u* draw a line towards *t*, of indefinite length. Transfer from the truss diagram the line from 4 to 6, meeting, on the reciprocal diagram, the line *ut* at *d′*, and terminating at *r*, on the line *er*. From *r* draw the line *rs* and the remainder of the diagram is then readily drawn. This is applicable for cases of unequal loading and for wind.

A method which is correct when the reactions are equal and the joints are equally loaded, is to draw the line, on the reciprocal diagram, from *w* to *v* and then from *s* to *r*. With unequal loading this method must not be attempted. It cannot be used for wind loads.

A third method is known as the "Moment solution." A truss is merely a framed beam. Find the maximum bending moment and divide by the depth of the truss at the point of maximum moment. In the figure the maximum moment is at the middle of the span, where the rise is a maximum. Dividing the moment by the depth gives the force in the tie rod *QX*. Set this off to scale from *x* on the reciprocal diagram, to *q*. Transfer the line *QR* from the truss diagram to the reciprocal diagram, from *q*

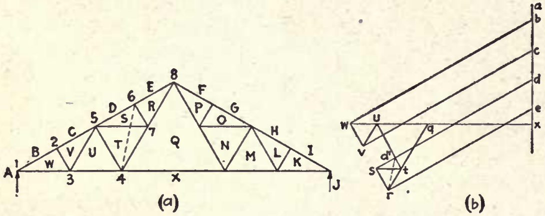


Fig. 133. — Three Solutions of the Fink Truss Problem.

to *r*. This fixes the point *r* and from here the reciprocal diagram may be completed. This method is applicable for cases of unequal loading and for wind.

Unequal Loading

In Fig. 134 is shown a beam loaded at several points with loads varying in amount. The problem is to find the reactions. Draw the beam to scale and place the loads on it at the proper points. To the right draw a vertical line and set off the loads to scale.

Set off a point, *O*, at any distance, but the best position is one which will make the two lines drawn from it to the end of the load line about equal in length. This would make the pole distance, horizontally, about half the length of the load line. The scale will be chosen so the diagram will not be too large, provided the scale used will give all the necessary information. From the points on the load line indicating the loads, draw lines to the pole.

Through the load points on the beam drop vertical lines and transfer the rays from the polar diagram to form the equilibrium polygon at (*b*). The broken line forming the bottom of the equilibrium polygon must begin at one reaction line and close on the other. The two ends are connected by the line *oo*. This line is transferred to the polar diagram, as shown by the dotted line. The distance on the load line from the top to an intersection with the closing line *oo*, is the amount of the left reaction and from the point *o*, on the load line, to the bottom of the load line, is the right reaction, all as shown at (*c*).

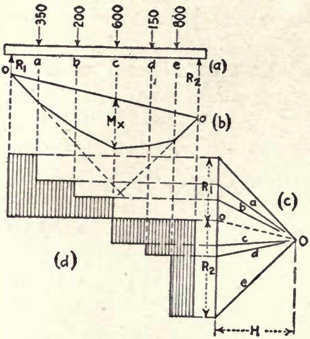


Fig. 134. — Moment, Shear and Reactions for Beam.

The polar distance, *H*, at (*c*) is measured with the scale used in setting off the loads, for it is a force. The vertical distances on the equilibrium polygon, from the closing line to the broken bottom line, are measured with the scale used in drawing the beam. The vertical depth of the equilibrium polygon, at any point, multiplied by the polar distance gives the bending moment at that point. If the vertical distance is in feet and the polar distance is in pounds, the moment is in pound feet.

At (*d*) is shown the shear diagram, for which it is believed no explanation is necessary.

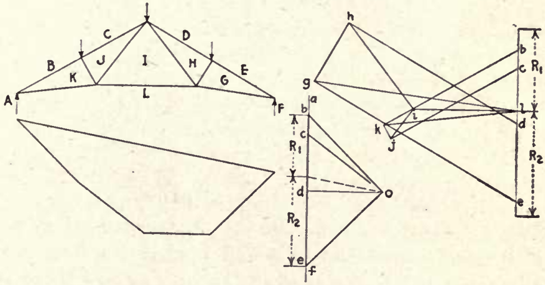


Fig. 135. — Roof Truss Unequally Loaded.

In Fig. 135 is shown a roof truss having unequal loads on the panel points. A polar diagram is drawn first, to obtain the reactions. The load line is then set off for the reciprocal diagram and the reactions are scaled off on it. In this case the load line must show all the loads and must be complete whereas in trusses symmetrically loaded it is necessary to draw only one-half the reciprocal diagram.

In Fig. 136 the treatment of a Warren truss loaded on the top

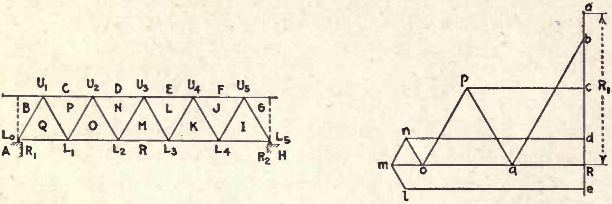


Fig. 136. — Warren Deck Truss.

chord is given. The vertical dotted lines at the ends are in compression and deliver their loads direct to the abutments. The horizontal line to the first joint in the upper chord delivers part of its load as a reaction, to the upper joint and part to the vertical post.

Notice carefully the treatment of the Warren through truss (loaded on lower chord), as shown in Fig. 137.

In Fig. 138 is shown the reciprocal diagram for a Howe truss loaded on the lower chord and in Fig. 139 is shown a Howe truss

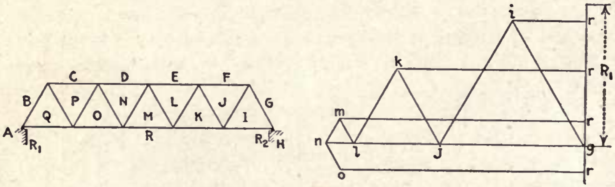


Fig. 137. — Warren Through Truss.

loaded on the upper chord. In the through truss the middle vertical carries a load, but in the deck truss it carries no load, as shown by the two letters at the end of the reciprocal diagram. All reciprocal diagrams must close and when a line must be omitted in order to make a diagram close, put the extra letters at the joint where there is an apparent jumble and letter the next joint in order. Reciprocal diagrams show plainly when there is a redundancy of members and also show when a member should

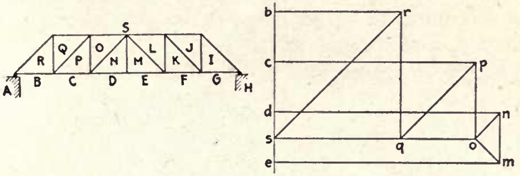


Fig. 138. — Howe Through Truss.

be added. The student is advised to make reciprocal diagrams for odd panel trusses of the Warren, Howe and Pratt types after studying this chapter, as an exercise. Odd panel trusses have been purposely omitted in order to give the student an oppor-

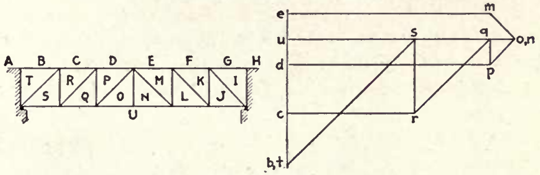


Fig. 139. — Howe Deck Truss.

tunity to exercise his brain in studying the problem. It will help to make the complete diagram and run the lines from the two ends, so if there is any trouble encountered it will be caught in the middle and can be readily solved. In some trusses there

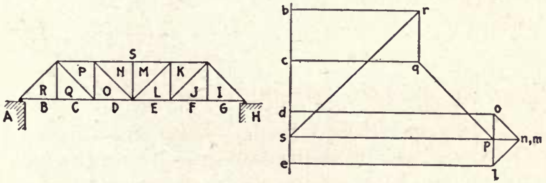


Fig. 140. — Pratt Through Truss.

may be members that are not stressed except under moving loads. Determine this positively. Take nothing for granted.

When reciprocal figures do not close there is either another member required or the work of the draftsman is poor.

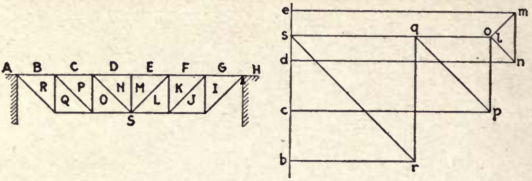


Fig. 141. — Pratt Deck Truss.

The diagrams for the Pratt truss, shown in Fig. 140 and Fig. 141 are readily followed.

In Fig. 142 and Fig. 143, of bowstring trusses, the student should observe how nearly uniform the stress is in the chord for each panel. The web stresses are very small, and, as the arch

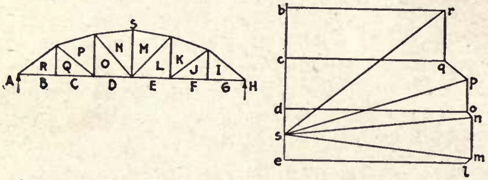


Fig. 142. — Through Bowstring Truss and Girder Stringer.

more nearly approaches a parabola the lower will be the stresses in the web members and the more nearly equal will be the stress in each panel length of the chords. The student should make diagrams for Pratt, Howe, Whipple and Warren trusses with parallel chords and compare these with the same framing of web

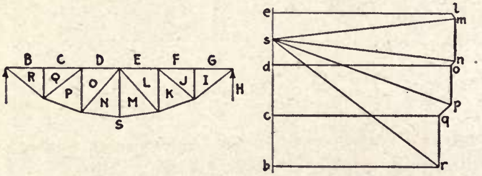


Fig. 143. — Deck Bowstring Truss.

members in bowstring trusses of the same span, carrying the same loads.

A form of truss used for exhibition halls, drill halls, etc., is shown in Fig. 144 with Whipple framing of web members, and in Fig. 145, with the web members framed as in a Warren truss.

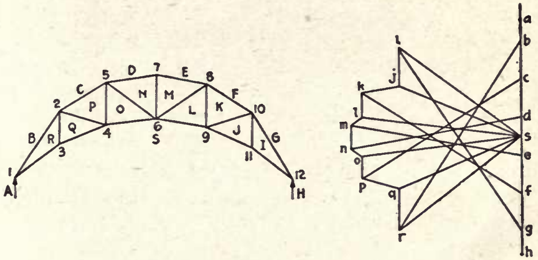


Fig. 144.—Crescent Roof Truss. Whipple Framing.

On the load line the loads are set off vertically to scale and from each point a line is drawn parallel with the top chord. The loads being symmetrical it is really necessary to draw but one-half of the force diagram. With trusses having as complicated a framing as these curved, crescent shape trusses, it is difficult to transfer all the lines from the truss diagram to the force diagram and have them truly parallel. In such cases it sometimes pays to

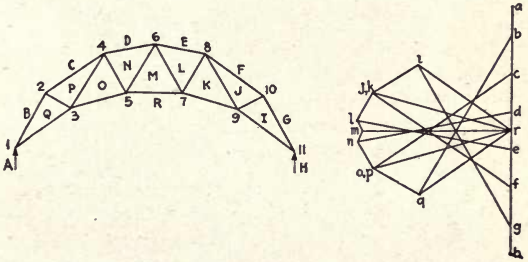


Fig. 145.—Crescent Roof Truss. Warren Framing.

compute the angles of slope by trigonometry and set the lines off with a protractor or by using a table of chords.

The hog-back truss shown in Fig. 146 is in common use and sometimes the upper chord is curved instead of straight. No difficulty should be encountered in obtaining the stresses in such a truss, for it is merely necessary to classify the web framing and follow the methods given for that special framing.

The shed-roof truss (Fig. 147) is so called because it slopes one way, like the roof of a lean-to shed. In this type it is necessary to set off the whole load line, as though the roof had unequal reactions. Notice the difference in the stresses in the member *QR* and the member *OP*.

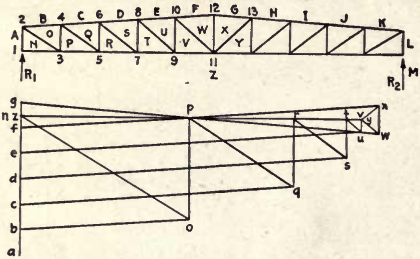


Fig. 146. — Hog Back Truss.

The scissors truss shown in Fig. 148 is a very common truss, a favorite with many builders and draftsmen. Setting off the loads on the load line it is very quickly discovered that to make the force diagram close it is necessary to commence with the load on joint 3, instead of either joint 2 or joint 5. Completing the diagram, which has to be done by drawing the dotted lines *aj* and *fj*, it is discovered that all the members are in compression. The dotted lines *aj* and *fj* represent the thrust of the truss against the walls or tops of the buttresses. The dotted line *oj* represents the tension required to resist the thrust, consequently the

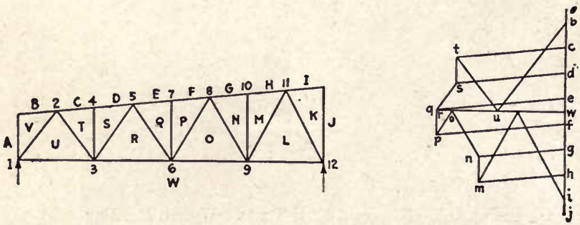


Fig. 147. — Shed Roof Truss.

pull in a rod which can be run from joint 1 to joint 6 and convert the diagonal thrust into a vertical reaction.

A vertical rod may be used to connect joints 3 and 4, and this will render the rod from 1 to 6 unnecessary and will convert the truss into one having vertical reactions. This rod will also change the diagram and render it possible to start from either end of the load line and project the members. This the student is advised to do as an exercise. If the reactions from roof trusses are not vertical, walls will be forced out and the trusses will sag.

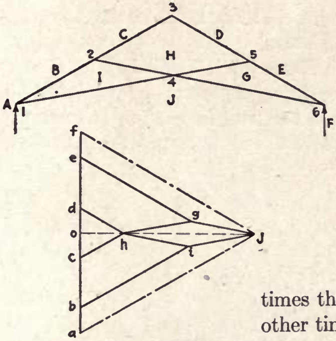


Fig. 148.—Scissors Truss.

A step forward from the scissors truss gives the truss with curved tie, shown in Fig. 149. This is very often seen in churches. Sometimes the framing is exposed and at other times a ceiling is attached to the curved tie.

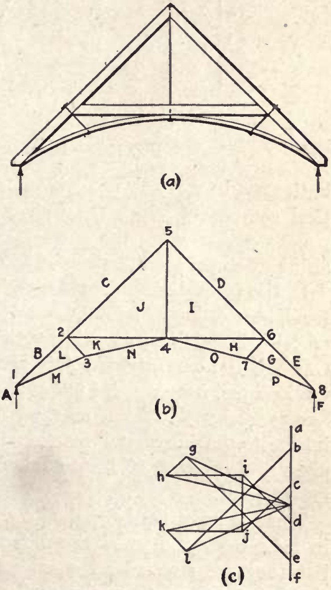


Fig. 149.—Truss with Curved Tie.

The curved tie is usually a T iron which is bolted to the rafters at the ends. The upright leg of the T is set into the horizontal brace and into the rafters as well. Though the tie is curved the pull is straight from joint 4 to joint 3 and joint 7 and is straight from 3 to 1 and from 7 to 8. At (*b*) is shown the truss diagram and at (*c*) is shown the force diagram. Between the joints the T must have enough stiffness to retain the curved form into which it is rolled, or bent.

The hammer-beam truss, shown in Fig. 150, is a handsome truss much used for churches and stately halls, with exposed rafters. The curved members *SW* and *NW* have no stress except under wind. Another form of this roof dispenses with the vertical tie *PQ* and the analysis resembles that for the scissors truss. Nothing is gained by doing without the vertical tie, but, on the contrary, the roof is bettered by having the tie.

In this truss all that portion above joints 5 and 10 is treated as a separate roof, resting on the frame represented by joints 1, 5, 10, 14. The dotted lines from 5 to 1 and from 10 to 14 represent in direction the thrust of the roof and it is necessary to have

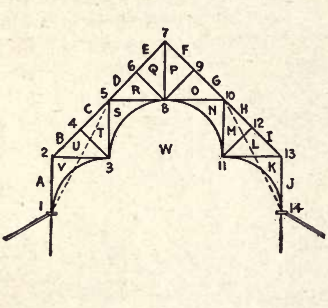


Fig. 150.—Hammer Beam Truss.

buttresses to carry it, or divide it at the lower joints into a vertical component which the wall will carry and a diagonal component acting along the roof of an aisle in the building. The aisle roof in turn will deliver its load to walls or buttresses.

In the force diagram, to the right, the loads are laid off vertically, beginning with joint 4, represented by the load *bc*, the loads on joint 2 and joint 13 going vertically into the wall or column. One-half the load at joint 4 is carried by the lower joint, so on the load line from a point midway between *b* and *c* draw a dotted line parallel with the line on the truss diagram from 5 to 1. This intersects the horizontal reaction line at *w*, and the length *xw* gives the amount of horizontal thrust on the support at 5th lowest joint.

There is no stress in *rw* and *ow*. These members are stressed in tension by the small upper truss of which they are the tie, but they are stressed an equal amount in compression, due to the thrust against the walls, and one stress neutralizes the other.

Wind Force

It was stated that when the slope of a roof is less than 30 degrees it is customary to assume the wind load as acting horizontally. When the slope is greater than 30 degrees the wind is an important matter and the exact amount and direction must be considered. When the forces in a roof are treated graphically the best practice is to obtain the exact forces caused by wind, no matter what the slope of the roof.

A number of formulas for wind are in use, but the most modern is that of Duchemin. It is based on very careful experiments and is considered the most reliable wind pressure formula now in use.

Let P = horizontal wind pressure in pounds per square foot.

P_n = wind pressure normal to the surface of the roof, in pounds per square foot.

A = angle of the surface of the roof, with the horizontal, expressed in degrees.

then $P_n = P \dfrac{2 \sin A}{1 + \sin^2 A}$.

All designers like simple straight-line formulas, so the following is used by a number of men. It gives values somewhat lower than those given by the Duchemin formula, but agrees fairly well with some experiments. For roofs having a slope exceeding 45 degrees the full horizontal pressure is used. When the angle is less than 45 degrees, the straight-line formula is

$$P_n = P(A \div 45).$$

A number of years ago Professor Karl Pearson proposed that the pressure on a roof, normal to the surface, be taken as equal in pounds per square foot to the slope of the roof expressed in degrees, up to a maximum of the number of pounds horizontal pressure, after which the normal pressure should be equal to the horizontal pressure. For example, when the angle of the roof with the horizontal is 20 degrees, the normal pressure will be 20 lbs. per square foot. Expressed as a formula, using a maximum of 50 lbs. per square foot, the horizontal pressure used in early days, it appears

$$P_n = \frac{P \times A}{50}$$

and the maximum angle of slope will be 50 degrees. Professor Ricker a few years ago proposed a similar formula, using 30 lbs. pressure and 30 degrees maximum slope, to accord with modern practice.

The horizontal wind pressure is fixed in specifications. It is usually taken as 30 lbs. per square foot against vertical flat surfaces. The following modifications are made for surfaces not flat:

Cylindrical chimney, 67 per cent of horizontal pressure.

Octagonal chimney, 71 per cent.

Rectangular building of large size, 80 per cent.

Concave side of shallow cylinders, channels and cups, 115 to 130 per cent. For deep cups and concave side of spheres, 130 to 170 per cent. (Ketchum.)

When the wind pressure against a roof is reduced to the normal pressure a stress diagram may be drawn after finding the reactions. The pressure normal to the surface acts at each joint the same as any load for which the roof may be designed. The effect the wind force on the roof will have on the walls or columns determines the stresses in the roof members.

There are three general cases:

1. The roof may be fastened to the support at both ends.
2. The roof may be attached to one end support and the other end may rest on a plate and be free to move.
3. The roof may be attached to one end support and the other end may rest on rollers.

With Case 2 and Case 3

(a) The wind may come from the attached end.

(b) The wind may come from the free end.

With Case 1 the reactions cannot be vertical and the horizontal thrust causes bending in the roof support, whether it be a wall or a column. The reactions are parallel to the resultant wind pressure.

With Case 2 the reaction at the fast end is parallel to the resultant wind pressure and the reaction at the "free" end makes an angle with the vertical equal to the coefficient of friction between steel and steel, about 18 degrees.

With case 3 the reaction at the free end is vertical.

The cases selected for illustration are very simple. The matter is simple. To give a number of force diagrams showing the effect of wind on a number of forms of roof trusses would make it appear complicated. A simple truss is sufficient. The principles are as easy to grasp as any of the work in graphic statics and the earnest student can go through all the trusses illustrated in this chapter and make diagrams for the effect of wind on each truss.

First obtain the direction of the resultant of the wind and the directions of the reactions due to wind. Draw a triangle representing this. From the intersection where the reactions meet, draw a reaction line, representing the forces in amount and direction on the lower chord. On the inclined wind resultant set off the wind load on each joint and from this load line draw lines parallel to the members of the truss and complete the force diagram. The force diagram for wind differs from that in which vertical loads are considered, merely by having the load line inclined and not vertical. All the other lines are parallel to the truss members.

Tables of stresses must be made for roof trusses when all loads are separately considered. Such a table will have a number of columns ruled on lined paper. Each system of loading will have two columns, one for tension and one for compression. The columns are as follows, from left to right:

1. Designation of members between joints.
2. Dead load on top chord (+) and (−).
3. Snow load (+) and (−).
4. Wind load (+) and (−).
5. Uniform load on lower chord (+) and (−).
6. Trolley, or other, moving loads (+) and (−).
7. Total loading (+) and (−).

Each member shown in the last column to have both tensile and compressive forces to resist is designed accordingly.

In Fig. 151 at (*a*) is shown a roof truss and the graphical method for finding the reactions and the wind stresses, with the two ends of the truss secured to the supports. First, referring to (*a*), the reactions are found by multiplying the length of the slope on one side by the distance between trusses, to obtain the area acted upon by the wind. This area is multiplied by the wind pressure per square foot. It acts at the center of area, as shown by the arrow.

To obtain the reactions graphically, prolong the line of the wind resultant through the truss, and the length, *ab*, represents to scale the amount of the wind force. At the left end, R_1, lay off the length *cd*, equal to *ab*. Connect the right end, R_2, to *d* with a straight line, intersecting *ab* in *e*. Then the length *ae*

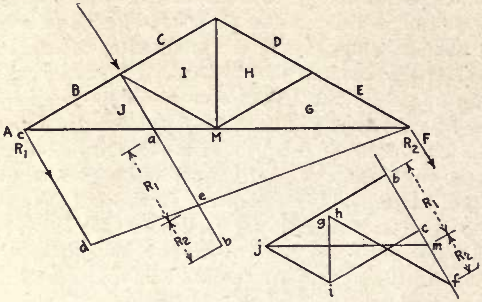


Fig. 151.— Wind on Roof with Ends Fast.

represents the left reaction and the length *be* represents the right reaction. The wind is assumed to be blowing from the left, but, for this case, the members are dimensioned to be strong enough to resist the wind from either side. The reactions will then in amount be equal to the larger reaction. At (*b*) is shown the force diagram for the wind.

In Fig. 152 is shown a method for computing the reactions when both ends of the roof are fast (secured). The reaction lines are drawn parallel to the wind resultant. The wind acts at the center of one side of the roof. The distances x and y are measured normal to the resultant. Multiply the wind by the length y and divide by the length x. This gives R_2, which is subtracted from the total wind force to obtain R_1. The problem is seen to be that of a beam carrying a single concentrated load.

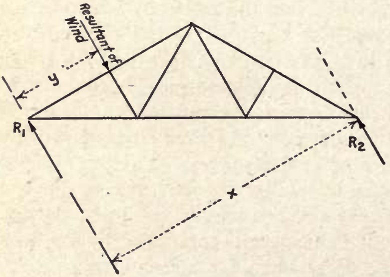


Fig. 152.— Inclined Reactions from Wind.

In Fig. 153 the roof is assumed to rest at one end on rollers, in order to take care of temperature changes, which, in trusses secured at both ends, often cause tremendous changes in the stresses. The reaction at the free end is vertical and the wind is from the fast end. At the free end drop a vertical line. Through the center of area on the windward side of the roof draw a line, normal to the slope, downward to an intersection with the vertical reaction line. The point of intersection is then connected to the fast end by a line, which gives the direction of resultant R_1.

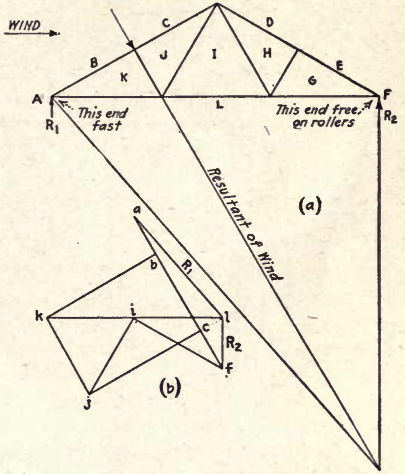


Fig. 153.—Wind Pressure on Roof-wind on Fast Side.

At (*b*) is the force diagram. First draw a load line parallel to the wind resultant and lay off the amount of wind at each end joint and at each joint on the truss. From the ends draw lines parallel to the two reaction lines. This forms a triangle *alf*, the side *fl* being equal to R_2, and the side *al* being equal to R_1. The remainder of the diagram is readily drawn, all the lines on the windward side being parallel to the members of the truss, with the load line (the wind) inclined.

In Fig. 154 is shown the method to use when the wind is blowing from the free end towards the fast end. No explanation is required for this figure if the explanations given for Fig. 153 are understood.

The free end of a truss may rest on steel plates instead of rollers. The only difference between this method and that when the free end rests on rollers is that the reaction under the free end is inclined at an angle of 18 degrees from the vertical away from the wind, this being practically the angle of friction of steel on steel. Trusses are of course designed for the maximum stresses, and with the majority of trusses the maximum stresses occur with the wind from the fast side. Analyze the truss with wind from either side and then proportion each member for the greatest force it is expected to resist, the two sides of the truss being alike.

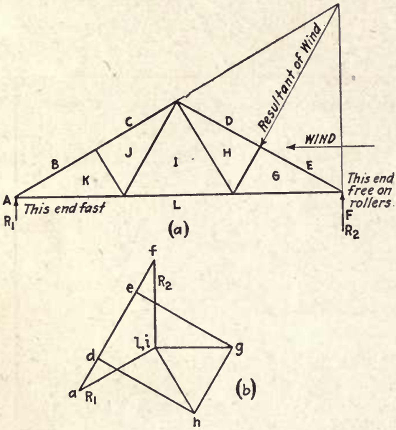


Fig. 154.— Wind Pressure on Roof-Wind on Free Side.

Concentrated Loads

Sometimes a roof truss must be designed to carry a trolley at some joint. The designer does not always know in advance on which panel the trolley will be carried, the owner of the building wishing to be free to change such things at pleasure. Instead of a trolley it may be a shaft for machinery, or a heavy pipe.

The method to pursue in such a case is to design the truss for the dead load, which will include the allowance for snow if any, then design for wind, then make diagrams for the concentrated loading at each joint where it is liable to come. This brings up the question of maximum and minimum stress and reversal of stress.

Maximum and Minimum and Reversed Stresses

Specifications usually state the safe allowable unit stress for all materials, but seldom give the stresses to use for members subjected to changing stresses and reversal of stress. It is customary in such cases to use a "Range Formula."

Let f = unit stress specified, and which will be used for dead load, or for total quiescent load.

S_m = maximum load on the member.

S_p = minimum load on member.

then

$$\textit{Working Stress} = \frac{f}{1.5}\left(1 + \frac{S_p}{2 \times S_m}\right).$$

When one load is compressive and the other is tensile replace the positive (+) sign by a negative (−) sign.

Example. The maximum and minimum loads in a certain member are respectively 105,000 lbs. tensile and 49,000 lbs. tensile. What is a proper working stress?

Ans. $\textit{Working stress} = \frac{16{,}000}{1.5}\left(1 + \frac{49{,}000}{2 \times 105{,}000}\right) = 13{,}200$ lbs. per sq. in.

Example. The maximum and minimum loads are respectively 105,000 lbs. tension and 23,000 lbs. compression. What is the proper unit working stress?

Ans. $\textit{Working stress} = \frac{16{,}000}{1.5}\left(1 - \frac{23{,}000}{2 \times 105{,}000}\right) = 9600$ lbs. per sq. in.

The reduced stress found by the above range formula is used for members in tension. The compressive stress is determined by an appropriate column formula, but it cannot exceed the range stress.

Snow Load

The snow load is always included with the dead load. It varies with the latitude as well as with the slope of the roof. In Fig. 155 is shown the snow load to use according to the recommendation of Professor Ketchum in "The Design of Steel Mill Buildings" in 1903.

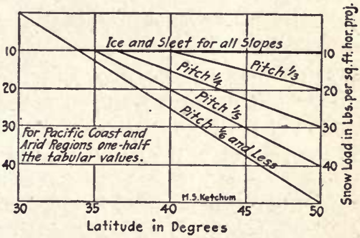


Fig. 155.—Snow Load on Roofs for Different Latitudes

English text books state that an allowance of 5 lbs. per sq. ft. of horizontal projection is common for Great Britain. Ketchum recommends the minimum ice and sleet load for all slopes of roofs, plus the recommended snow load; for a high wind may succeed a heavy sleet. Not all engineers use the snow load in addition to the wind load, arguing that a high wind will blow away the snow. The possibility, however, of a high wind following sleet, which cannot be blown away, must be considered. A minimum of 10 lbs. per sq. ft. should be used except in localities mentioned on Fig. 155. A sleet storm may follow a heavy snowstorm, and, in its turn, be followed by a heavy wind.

Wind on a Curved Roof

In Fig. 156 is shown the graphical method to follow in obtaining the reactions for a roof having curved chords. First find the inclination of each panel of the top chord in degrees and find the normal component of the wind on each slope. Multiply the area of each panel by the normal force of the wind on the panel and set this off at each end of the panel (*i.e.*, at each joint) and complete each parallelogram of forces. Draw the dotted lines, representing the resultant for each parallelogram.

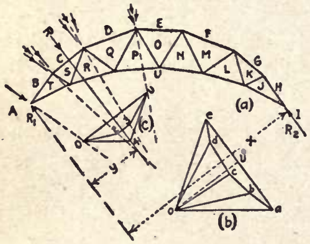


Fig. 156 — Reactions for Wind on Curved Roof

At (*b*) draw the polar diagram. The line *ed* is parallel with the resultant at the joint *DEP*. The line *dc* is parallel with the resultant at the joint *CDR*, etc. The length of each line is equal to the amount of the resultant wind force at the joint through which the resultant passes. Connect the points *ae* and the direction of the resultant wind force on one side of the roof is found, and its amount.

At (*c*) draw the equilibrium polygon. The line *o2* is parallel with the line *ob* of the polar diagram. Similarly the line *23* is parallel with the line *oc* and the line *3u* is parallel with the line *od*, of the polar diagram. The line *ou* is transferred to the polar diagram as the resultant closing the equilibrium polygon. The line *oe* on the polar diagram is transferred to the equilibrium polygon as the line *u4* and the line *oa* is transferred as the line *o4*. The intersection of these lines fixes the location of the resultant wind force, which acts normally to the lie *ou*.

The roof is assumed to be fast at the supports, so the resultant wind force multiplied by y and divided by x gives the amount of R_2. If one end of the roof is free the reactions are found as in Fig. 153 and Fig. 154, after finding the amount and location of the resultant as just shown.

The remainder of the process for ascertaining the forces in the members is exactly as shown in other cases, the only difference being that the load line is parallel to the resultant of the wind pressure. The diagram will appear to be complicated but it only needs care and patience to make it right. Some of the lines on the truss diagram are very short and it may be advisable to plot them with a protractor, or compute their direction by using tables, as it is difficult to transfer a short line and draw a long line parallel with it.

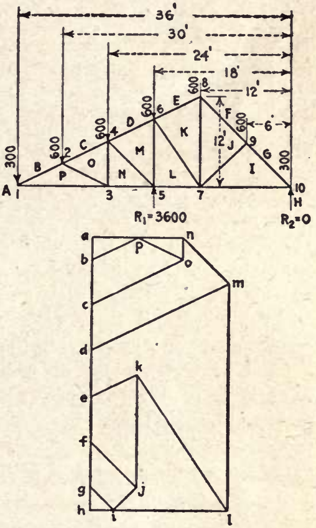


Fig. 157 — Cantilever Truss

Cantilever Trusses

In Fig. 157 is shown a cantilever truss that appears to be a favorite with examiners, for it is found in many examination papers. In this particular example the right reaction is zero. Sometimes this roof is shown with the left support under joint 7. Sometimes the loads are varied so there is a negative reaction on the right, which means the end of the truss must be tied down. Sometimes there is a small positive reaction at the right, which is ignored if the forces are small in the members. This roof is for a grand stand. In a cantilever truss the loads at the ends are used, whereas they are ignored in ordinary trusses because there they are carried directly by the walls.

The reactions are computed as follows, for the truss shown,

$$R_1 = \frac{(300 \times 36) + 600(30 + 24 + 18 + 12 + 6)}{18} = 3600 \text{ lbs.,}$$

which is equal to the total load on the truss, therefore the truss is exactly balanced on the support at joint 5. The author has examination papers in which this roof truss appears with supports at the two ends and also at the right end and (in turn) each vertical, which gives four different combinations. Some of the problems require the wind load to be computed while others assume all loads as vertical. With four different ways to support the truss, with and without wind and these various conditions varied by varying the amount of vertical load at each joint it is readily seen why examiners like this truss. Students preparing for architects' license examinations should work it with a number of changes in all the conditions.

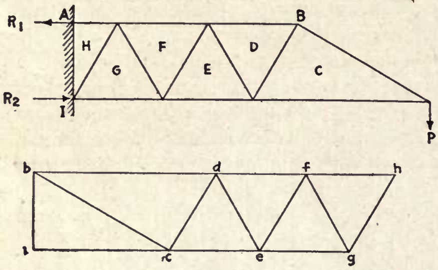


Fig. 158 — Braced Cantilever with Concentrated Load at End

Fig. 158 represents a braced cantilever carrying a single concentrated load at the end. The dead load is neglected and the frame therefore is weightless.

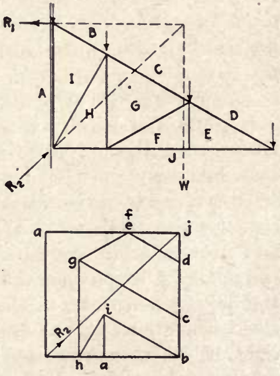


Fig. 159 — Braced Cantilever Loaded on Joints

Fig. 159 represents a braced cantilever with a load at each joint. This illustrates the general method for dealing with cantilever trusses. On the truss diagram the loads are shown at the joints. Multiply each load by the horizontal distance from the support. Add the products. Divide by the sum of the loads, to obtain the position of the center of gravity of all the loads. Continue the horizontal reaction line (R_1) to an intersection with the vertical dotted line through the center of gravity. Draw the diagonal line (R_2) to show the direction of the reaction at the bottom chord support.

To draw the stress diagram first lay off on a vertical load line the sum of the loads. From the upper end draw a horizontal line and from the lower end draw a diagonal line parallel to the inclined reaction. The point of intersection, *j*, on the force diagram fixes the amount of each reaction. From *j* drop a vertical load line on which set off each joint load and close the diagram at the bottom. The rest of the diagram is evident. The vertical *fe* is not stressed but is merely used to carry the weight of the lower chord in the end panel.

Accuracy in Drawing

In graphic statics everything depends on the care with which the work is done. The pencils used should be very sharp and the lines as thin as possible. The lines in the force diagram must positively be parallel with the lines on the truss diagram. The work checks when the reciprocal diagrams close and if they do not close the work must be carefully searched for errors. A useful check is to determine some stresses analytically by taking moments. The scale should be one that will not require too large a sheet of paper and will allow a reading of one hundred pounds. For roof trusses of usual spans the scale can be twenty thousand pounds per inch.

The foregoing presentation of the subject of graphic statics covers the subject only so far as roof trusses are concerned. It may be applied to any braced structure and other applications will be shown in the following chapter. The principles are simple and any student who works faithfully through the examples given should have no hesitancy in attempting to analyze graphically the forces in any braced frame.

Continuous Beams

The continuous beam is not used much in steel buildings but is used in all reinforced concrete buildings. All methods for dealing with the continuous beam are based on the assumption that the supports are at the same level and remain there. If one support settles the stresses are increased enormously. It is customary to use the "Three-moment Theorem" in dealing with continuous beams, but it is rather involved and a great many men do not take the trouble to investigate carefully the moments on continuous beams. The following graphical method was proposed many years ago by T. Claxton Fidler in "A Practical Treatise on Bridge Construction." Another graphical method is demonstrated in DuBois "Graphical Statics" and Church's "Mechanics of Engineering."

The Fidler method is known as the "Method of Characteristic Points." Refer to Fig. 44 on page 49, where the effect of restrain-

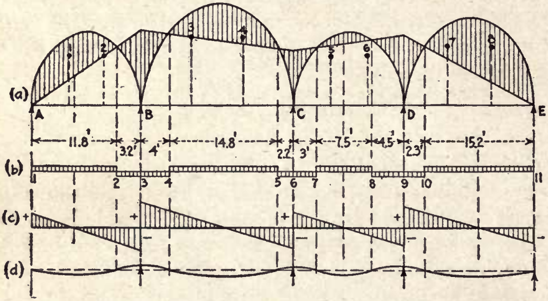


Fig. 160 — A Graphical Treatment of Continuous Beams. Fidler method

ing the ends of a uniformly loaded beam is discussed. To extend this to a continuous girder uniformly loaded compute the bending moment on each span separately as though it were simply supported at the ends. The bending moment = $WL \div 8$ and a parabola must be drawn on each span with the middle ordinate equal to the moment, as in Fig. 160.

Divide each span into three equal parts and at the third points erect vertical lines. Make each vertical line equal to two-thirds the height of the parabola within which it is situated. Draw a circle around the end of the line, the characteristic point being in the center of the circle. These characteristic points in Fig. 160 are numbered from *1* to *8* inclusive.

The end spans, of the series here shown, rest freely on the outer end supports, hence characteristic points *1* and *8* are disregarded.

If the end spans were restrained at the outer ends, the broken base line would pass through points *1* and *8*. Starting from *A* a line is drawn upward to the vertical line through support *B*. This broken line should pass above point *2* and below point *3*, or it should pass below point *2* and above point *3*, or it should pass through these points. In the present case it passes through them.

Passing through point *3* the line passes below point *4* and strikes the vertical line through support *C*. It then passes above point *5* and point *6* and below point *7* to close on support *E*.

The broken line must be fixed by trial in all cases. It must pass *below* (*or above*) one point and *above* (*or below*) the adjacent point in the adjoining span to the vertical line through the supports. If the spans are equal the broken base line passes as far below one point as it passes above the adjacent point on the adjoining span. When the spans are unequal the line passes above or below inversely as the length of the span. That is, on the shorter span the vertical space is greater than it is on the longer span; in proportion to the lengths of the spans.

In fixing the position of the broken line the author puts over the drawing a sheet of tracing paper on which to mark the several trial lines. When the final line is selected the points on the vertical lines through supports are pricked in with a needle, the tracing paper is taken off, and the line drawn. The line can have only one position.

In the figure the moment diagram is shown at (*a*). The broken line is a base line from which to scale the bending moments. All the shaded portion within the parabola on each span represents positive moment. All the shaded portion outside the parabola between it and the vertical line through supports represents negative moment. At (*b*) is shown graphically the system of cantilever and simply supported beams into which a continuous beam over several supports is divided. The ends of the cantilevers are at the point of contraflexure, the curved line at (*d*) showing the deflection of the beam to an exaggerated scale.

At (*c*) is the curve for shears and also reactions. The reaction is always equal to the shear. The shear is zero at the point of maximum bending moment, or, rather, it passes through zero at this point, the sign for shear being positive (+) above the base line and negative (−) below the line. The reaction on either side of the support is equal to the shear on the same side. The total reaction on any support is the sum of the positive and negative shear.

To compute the shear and reactions proceed as follows: Shear at A (+) = half the uniform load on span *1* to *2*. The reaction is equal to the shear.

Shear on left of B (−) = total load on the span, less left reaction.

Shear on right of B (+) = load on cantilever *3* to *4*, plus half the load on the suspended span *4* to *5*.

Reaction on support B = sum of the + and − shear, as found above.

Shear on left of C (−) = half the load on span *4* to *5*, plus the load on the cantilever *5* to *6*.

Shear on the right of C (+) = load on cantilever *6* to *7*, plus half the load on the suspended span *7* to *8*.

Reaction on support C = sum of the + and − shear.

Shear on the left of D (−) = half the load on suspended span *7* to *8*, plus the load on the cantilever *8* to *9*.

Shear on the right of D (+) = load on cantilever *9* to *10*, plus half the load on the suspended span *10* to *11*.

Reaction on support D = sum of the + and − shear.

Shear and reaction at E = half the load on the span *10* to *11*.

Check the results by using the formulas on page 52.

The method for obtaining moments, shears, and reactions by the use of "characteristic points" may be used for any number of spans, equal or unequal, all spans loaded or some carrying a live and dead load and others carrying only a dead load.

CHAPTER VII

Columns and Structures

A PIER is made of brick, stone, or concrete. That is, it is a masonry post and because it is not safe to permit any bending stress it must be limited in height. A concrete pier reinforced with steel may develop into a slender column.

The Chicago Building Ordinance provides that no masonry pier can have a height exceeding 12 times the least thickness. When the height is less than 6 times the least thickness the allowable unit compressive stress is that fixed in the ordinance. When the height exceeds 6 times the least thickness the fiber stress must be reduced by the following formula:

$$f = c\left(1.25 - \frac{H}{20D}\right),$$

in which f = reduced unit compressive stress;

c = unit compressive stress mentioned in the ordinance;

H = height in feet;

D = least width, or thickness, in feet.

The distinction between posts and columns is seldom definitely drawn. It may be said that a post is solid and short. A column is long and may be hollow or of some shape other than round or rectangular.

Specifications vary with the ideas of the men who write them and great differences exist between specifications and building ordinances the country over. The statements made in this section are therefore not to be taken as meeting the requirements of the leading designers but are presented merely as examples of how such things are regulated in some places.

In Chicago the maximum length of timber posts cannot exceed 30 diameters, or 30 times the least thickness. Hereafter when diameter is mentioned in connection with columns and posts it is understood to mean also the least thickness, if the column is rectangular and not round. Timber posts, or columns, cannot be used in buildings over one hundred feet in height, nor in buildings of a greater height than twice the width, for wooden posts are not continuous and, therefore, cannot be relied upon in case of heavy winds to stiffen the building.

The allowable unit compressive stress for wood is for short blocks only, in which the length does not exceed twice the diameter. The unit stress on wooden posts is found by the following formula:

$$f = c\left(1.00 - \frac{L}{Cd}\right),$$

in which f = reduced fiber stress;

c = unit compressive stress mentioned in ordinance;

L = length in feet;

d = diameter, or least thickness, in feet;

C = a constant, which is 80 in Chicago and has values ranging from 60 to 100 in other cities.

The above formula is called in some places the "Straight-line formula" and in other places the "Winslow formula," from Benjamin Winslow, who is credited with being the originator.

Wooden posts should be solid. A number of experiments made on wooden posts built up of thick planks spiked side by side showed that the strength of such posts is not the sum of the strength of the planks. Each plank when loaded on the end tends to deflect as though it were a long slender column. Some experiments made for Mr. Dewell in California on short models gave better results than any other recorded experiments, but his built-up posts were better made than is apt to be the case with full-size posts. If a wooden post is built up shear pins must be put between the planks and bolts must go through the other way. Sometimes plates on the edges securely screwed to each of the planks will make them act together.

Only very short posts fail by crushing under load. The usual failure in posts and columns is caused by bending, which crushes the fibers on the concave side and frequently causes tension in the convex side. For long columns it is necessary to use a formula for reducing the allowable compressive stress for short blocks. The stress is progressively reduced as the length of the column is increased until finally a point is reached beyond which the "slenderness ratio" is so great that the column may be unsafe. The "slenderness ratio" is $\frac{l}{d}$ for masonry piers and wooden posts and it is $\frac{l}{r}$ for metal columns. The factor "r" is the "*radius of gyration.*"

Before describing this important factor the general question of column formulas may well be touched on. The Euler formula is intended for such long slender columns that it is not in practical use, being of value to investigators and mathematicians in studying the effect of loads applied at the end of pieces like piston rods.

At least a century ago Tredgold proposed a general form for column formulas and this was later modified by Professor Gordon, so it appeared as follows:

$$f = \frac{c}{1 + k\left(\frac{L}{d}\right)^2},$$

in which f = reduced unit fiber stress,

c = allowable compressive unit stress,

k = a constant,

L = length,

d = diameter, or least thickness.

The constant "k" depended not only upon the material *but on the shape of the section.*

With the Gordon formula it was necessary to make innumerable experiments and thus obtain constants. It would be necessary to make columns of many sizes and of every imaginable shape, built up in every conceivable way, and test them to destruction in order to be able to design similar columns.

Professor Rankine, who succeeded Gordon as Professor of Civil Engineering in the University of Glasgow, modified the Gordon formula by substituting the radius of gyration for the diameter. A great many writers refer to the Gordon formula when they mean the Rankine formula, and others refer to the Rankine formula as the Gordon-Rankine. There appears to be considerable confusion as to what constitutes the difference, and some men do not appear to realize that there is any difference.

The Rankine formula is essentially a modification of the Euler formula by combining the underlying principles of that formula, which dealt with a thread, with the Gordon formula which took into account the fact that a column had thickness as well as length. The Rankine formula is known in Germany as the

Schwarz formula, an engineer of that name on the continent of Europe having developed it independently of Rankine about the same time. A number of other men proposed the same, or a similar, form for the expression but Rankine and Schwarz obtained the best publicity in advance of their colleagues.

In the Rankine formula the constant "k" of the Gordon formula, which was fixed by the shape and the material, becomes the constant "a" which is fixed by the material alone. The constant is modified by the method of supporting the ends of the column:

Rankine formula for flat ends (fixed in direction):

$$f = \frac{c}{1 + a\left(\frac{l}{r}\right)^2}.$$

For rounded ends (direction not fixed) multiply a by 4.

For hinged ends (position fixed but direction not fixed) multiply a by 2.

For one end flat and the other round multiply a by 1.78.

In the Rankine formula the compressive fiber stress was the breaking strength and the reduced stress was the reduced breaking strength, which was divided by a factor of safety to obtain the safe working stress. The same result is obtained by dividing the breaking strength by the factor of safety. For example Rankine used $c = 70{,}000$ and $a = \frac{1}{20{,}000}$. The stress f was divided by the factor of safety 5. To-day $c = 14{,}000$ and $a = \frac{1}{20{,}000}$ as before, but f is the safe unit stress without further operation.

The values of the breaking stress and the empirical constants to use in the Rankine formula were experimentally determined by Christie and Hodgkinson many years ago as follows:

Hard steel, $c = 70{,}000$ lbs. per sq. in. $a = \frac{1}{20{,}000}$.

Mild steel, $c = 48{,}000$ lbs. per sq. in. $a = \frac{1}{30{,}000}$.

Wrought iron, $c = 36{,}000$ lbs. per sq. in. $a = \frac{1}{36{,}000}$.

Cast iron, $c = 80{,}000$ lbs. per sq. in. $a = \frac{1}{6400}$.

Timber, $c = 7{,}200$ lbs. per sq. in. $a = \frac{1}{3000}$.

The recommended factors of safety were as follows: 10 for timber; 5 for metal under moving load; 4 for metal under quiescent load. The constants were for ratios of $\frac{l}{r}$ between 20 and 200, and are, therefore, not reliable for longer columns.

The Chicago building ordinance limits the extreme length of cast iron columns to 70 times the least radius of gyration. The length of rolled steel compression members cannot exceed 120 times the least radius of gyration, but the limiting ratio of struts for wind bracing may be 150 times the least radius of gyration. See some of the specifications recommended for study and compare them with the provisions above quoted.

Radius of Gyration

The radius of gyration was once humorously referred to as a happy thought in terminology as it is not a radius and has nothing to do with gyration. It is a term used by mathematicians and students of mechanics of materials to describe a factor used in the design of compression members in structures. It is actually the square root of the moment of inertia of a section divided by the area, or,

$$r = \sqrt{\frac{I}{A}},$$

in which r = radius of gyration,
I = moment of inertia,
A = area of section.

The moment of inertia and the area being in inches, the radius of gyration is in inches.

The would-be humorist was wrong in his statement, for the radius of gyration may be shown to be a radius and it has actually to do with gyration.

Each cross section has two radii of gyration, one perpendicular to the axis y and the other perpendicular to the axis x. In using a formula the least radius is chosen, except when it may be safe to use the greater. Assume that the mass rotates (bends) about the given axis. If the column bends, some resistance will be offered by the section, which is assumed to be a mass moved by the rotation (bending) of the column about the axis chosen.

Assuming the section to be a rotating body, there is some kinetic energy developed, and in order to find the amount it is first necessary to determine the point through which the kinetic energy acts, or determine what is essentially a center of gravity. The moment of inertia is the sum of all the small units of a section multiplied by the square of the distance of each unit from the axis. Divide this by the area and extract the square root and the radius of gyration is found to be the root mean square of the distances of all the separate units of the mass from the axis. It is actually a radius from the axis to the center of an imaginary ring in which is assumed to be concentrated the mass of the section. If this is not plain the author offers his apologies, for he cannot make it any plainer without wandering off into a mathematical demonstration which would defeat the objects aimed at in writing this book. It is of little consequence, however, as it is enough to accept the judgment of eminent men who have worked the matter out satisfactorily.

Straight-line Formula

The Euler formula applies only to the thread in the vertical axis of a very long and very slender column and the Rankine formula is also to a large extent a theoretically correct formula, the value of which is seriously affected by faults in workmanship and design, as well as by defects in materials. It is a laborious formula to use and engineers like simple formulas. In making tests of full-size columns it was found by plotting the results that the Rankine formula gives rather low stresses for columns having a slenderness ratio under 80, and somewhat high stresses for a slenderness ratio over 150. A straight line drawn through the points, on the sheet on which the results of the tests were plotted, in such a way that it passed through the center of mass of the points, resulted in the formula:

$$f = 16{,}000 - 70\frac{l}{r},$$

which is known as the "Straight-line" formula for steel columns. It is used in Chicago, where the formulas for wrought iron and cast iron are as follows:

$$\text{wrought iron, } f = 12{,}000 - 60\frac{l}{r}$$

$$\text{cast iron, } f = 10{,}000 - 60\frac{l}{r}$$

but in no case is the maximum stress permitted to exceed that fixed in the ordinance, 14,000 for steel; 10,000 for wrought iron; and 10,000 for cast iron. For steel columns filled with, and encased in, concrete extending at least three inches beyond the outer edge of the steel, where the steel is calculated to carry the entire live and dead load, the allowable stress per square inch on the steel is determined by the following formula,

$$f = 18{,}000 - 70\frac{l}{r},$$

but cannot exceed 16,000 lbs.

The student is referred to the following pages in the standard steel handbooks: Carnegie (1913), 251 to 282 incl., 327 to 329 incl. Cambria (1914), 192 to 276 incl., 394–5. Jones & Laughlin (1916), 176 to 217 incl., 281 to 283 incl. Lackawanna (1915), 205 to 288 incl., Bethlehem (1911), 8, 43 to 55 incl., 70 to 87 incl., 97 and 98. In addition to the information contained on the pages mentioned there are tables of the radius of gyration of pieces having different shapes and of pieces in combination, such as angles back to back, etc.

In Fig. 162 the curves show the allowable fiber stresses permitted in the larger American cities and given in various steel handbooks, etc. Some of the curves are for modifications of the original Rankine formula with the theoretically correct curve according to that formula. Others give values according to various straight-line formulas. Speaking generally the first figure in the straight-line formula gives a fair idea of the factor of safety intended by the man responsible for the expression. Assuming a maximum strength of 64,000 lbs. per sq. in. for structural grade steel, the factor of safety when the formula starts with 16,000 lbs. is 4; for 19,200 lbs. it is 3.333, etc. This is modified again by the slope of the curve (even straight lines being called curves in graphical work). Notice that there is a top limit when the curve goes horizontally, this being 14,000 lbs. for Chicago.

To use the chart determine the ratio of slenderness within which the column length is fixed. Assume a section by trial and determine the radius of gyration in inches. Divide the length in inches by the radius of gyration to obtain the slenderness ratio which must be within the limit decided upon. If it is past the limit the work must be done over with another assumed section. If the slenderness ratio is within the limit find it at the bottom of the chart and go vertically upward to the curve representing the formula used. From this intersection proceed horizontally to the left, where the proper fiber stress will be found. Divide the load by this fiber stress and obtain the required cross-sectional area of the column. If it agrees with, or is less than, the area of the assumed column section the assumed section may be used and the designing of the details proceeded with. Another

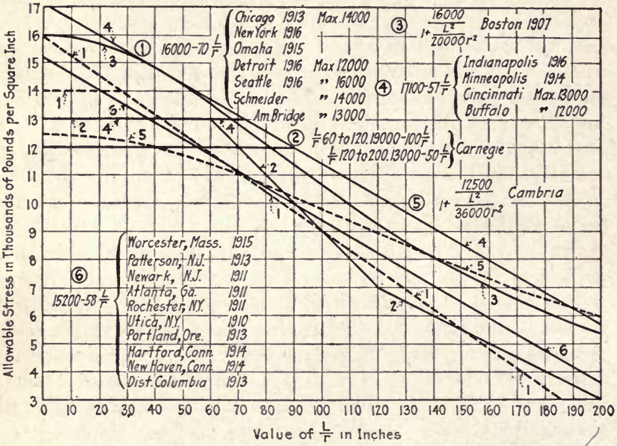


Fig. 162 — Steel Column Formulas Used in the United States

way is to multiply the cross-sectional area of the assumed column section by the fiber stress and if this gives a carrying capacity equal to, or greater than, the load the section may be used. If it is less, then another section must be assumed and the work gone through again.

The steel handbooks contain tables of columns intended to save the designers much of the above work. In using these tables the designer will notice that values of the carrying capacities, for the columns are given with the lesser and with the greater radius of gyration. Be careful in using the tables to see which value is used. Either value may be used in computing the effect of eccentric loads, depending upon which side the load comes. The smaller radius of gyration is used in determining the unit stress for concentric loading. However, there are cases when the use of the larger radius of gyration may be permitted. If a column is built into a wall of first-class masonry so that it cannot bend in the weaker direction the larger radius of gyration may be used. A casing of a few inches of concrete is not enough to satisfy the requirement that the column be stayed in the weaker direction. The supported length of a column is the length used in the formulas. If a column is supported in the weaker direction by adequate bracing the supported length is the distance between the attached ends of the stays, and the column may be designed with the smaller radius of gyration combined with the shorter lengths, or it may be designed with the larger radius of gyration combined with the greater length. When possible the weaker dimension of the column should be turned in the direction of the closer supports. Even when the least radius of gyration is chosen the column should be so placed in the structure that the heavier loads come on the longer axis.

The effect of eccentric loading is taken care of by increasing the size of the column. The tendency of the column to bend is determined by the slenderness of the section and it can bend sideways to the load, this being the reason for using the least radius of gyration regardless of the direction from which the load may come to the column. In the column tables in the steel handbooks the total load is generally given, together with a statement as to which radius of gyration is used in computing the strength of the column. The tables are computed by one of the several formulas plotted in Fig. 162.

In assuming column sections the formulas given do not take into account the various methods for attaching the principal parts together. From the result of experiments it is believed safe to use the allowable fiber stress by formula for columns with solid web plates, as for example plate and angle columns. For laced columns use about seventy-five per cent and for columns fastened by batten plates use about fifty per cent of the fiber stress given by formula.

Fig. 163 appeared in *Engineering News,* in 1913 in an article by O. von Voigtlander on Approximate Radii of Gyration. The use of the table saves a great deal of labor on the part of the designer when he can know in advance the outside dimensions of his columns or struts. It is not necessary to know this accu-

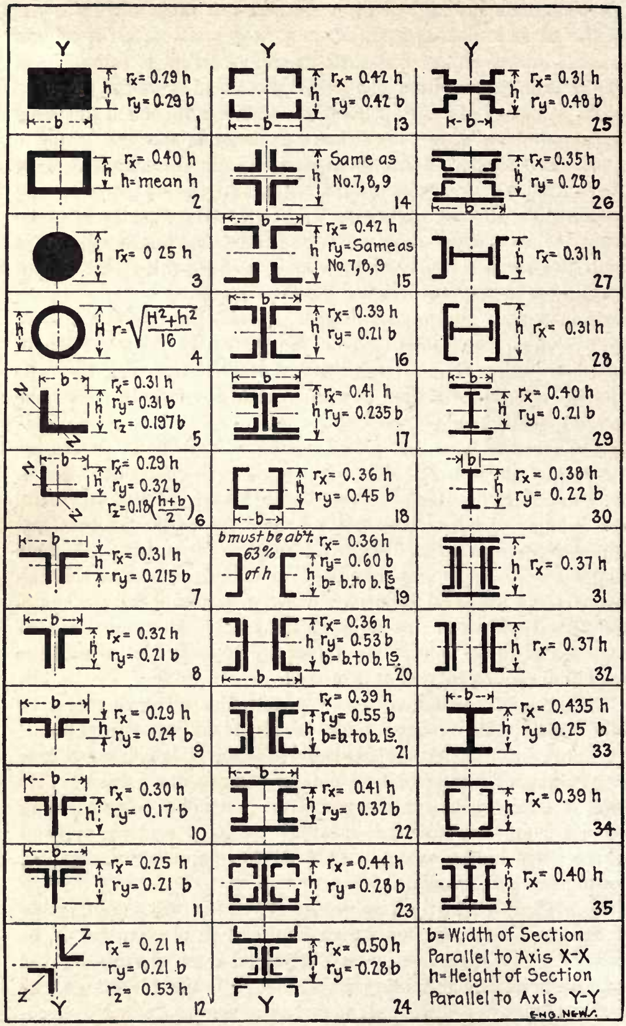


Fig. 163—Approximate Radii of Gyration "r"

rately, but usually it can be determined in advance just what maximum size is permissible. In the figure the meaning of the letters used is plain, but attention must be called to 19 in which "*b*" is the distance back to back of the channels and it must be not less than 63 per cent of the nominal size of the channel. The procedure is to assume the form of section and the extreme dimensions. Then apply the rules given in Fig. 163 and thus get an approximate value for the radius of gyration. Proceed as before and when the allowable fiber stress is found proceed to get the area and then select the plates and shapes to make the selected section. When it has been designed find the exact radius of gyration and test for the fiber stress.

Wrought iron columns are seldom used, for the material is hard to obtain and steel is stronger pound for pound. Wrought iron and steel columns are usually two or three stories long. Column splices should be so arranged that not more than one-half the total number of columns splice at any one floor level. All connections between columns, girders, and beams should be rivetted. Theoretically it is best to vary the sizes of columns from story to story, but it is less expensive with steel and wrought iron columns to have them not less than two stories long, of the same size, for the extra amount of material often costs less than the labor required to change sizes at each floor. Cast iron columns and wooden columns are never more than one story in length and the practical impossibility of making rigid connections at floor levels limits the use of cast iron and wood for columns to low buildings, for they offer poor resistance to wind.

No column is free to turn as though the end were round or as if it bore against a pin. Such conditions do not arise in building construction, although they may be nearly attained in bridges. The columns in massive buildings are sometimes considered as fixed at the ends, but mass implies positive rigidity. In sheds and low mill and shop structures columns are not considered as fixed at the ends unless specially massive foundations are used for the purpose of assuring such a condition, something seldom done. The majority of engineers advocate the assumption of two rounded ends for all cases short of positive fixety, as there are so many secondary stresses, experiments showing that columns tested to destruction fail in detail rather than as a whole.

Fig. 164 illustrates the four conditions affecting the end loading of columns. At (*a*) is shown the column with pin-joints at top and bottom. This is the standard case assumed for all column formulas, a modification being a flat-ended column, the bolts at the ends of which are intended to merely hold it in position and are not strong enough to resist much tension. The *l* in the formulas is the total unsupported length of the column.

At (*b*) is a column fixed at the ends to maintain both position and direction. The *l* to use is one-half the unsupported length.

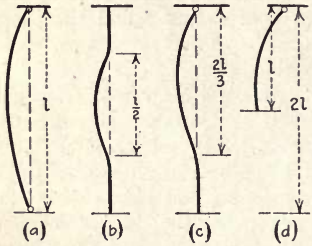


Fig. 164 — Methods of Fixing Columns

At (*c*) one end is fixed (in position and direction) and the other end is a pin, or hinged, end, fixed in position but not in direction. The *l* to use is two-thirds the unsupported length.

At (*d*) the lower end of the column is fixed in position and direction but the upper end is free to move laterally, differing from (*a*) and (*c*) in which the upper end of the column is vertically over the lower end. The column shown at (*d*) bends in a simple curve which is one-half that of a column double the length, as shown by the lower dotted end. Obviously the *l* to use is twice the actual length of the column.

The temptation to consider a greater degree of fixity than is actually obtained is great, and all designers must be warned against yielding to it. Probably the only fixed columns in a building are those on the ground floor of a high building with massive foundations. Above the ground floor there is bound to be some vibration and swaying, especially in a wind.

To Proportion Struts or Compression Members

Every strut is designed as a column with rounded or hinged ends. Nothing is deducted for rivet holes, as the rivets are assumed to fill them. First select the form of strut from the many illustrated in Fig. 163, decide on the radius of gyration, and be careful with angles to use two, back to back, even when the computations show one to be amply strong. Steel compression members in trusses are apt to contain considerable excess material because they are usually composed of angles, but the excess material is often of considerable advantage when wind is considered. The consideration of holes in tension members leads to an excess of material, and the effect of the radius of gyration is similar in compression members.

Eccentric Loads on Columns

The column formulas heretofore considered are based on a load acting vertically and applied at the upper end of the vertical axis of the column. This is termed concentric loading. The only tendency to bend is that caused by the fibers being too strong to crush or tear until after considerable side bending takes place.

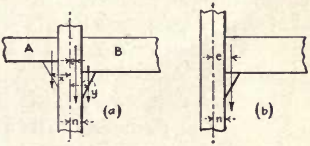


Fig. 165 — Eccentric Loads on Columns

An eccentric load is one applied at some distance off the center of the column and acting vertically. This is illustrated in Fig. 165. At (*a*) two loads are shown carried on opposite sides of the column. Each reaction is assumed to act vertically at the middle of the bracket on which the beam rests. From the reaction of A to the axis of the column the distance is x and from the reaction of B to the axis of the column the distance is y. The center of gravity of the two loads is found by the principle of moments, and the distance of the center of gravity from the axis of the column is e, the eccentricity of the loads.

$$e = \frac{(Ax) + (By)}{A + B}.$$

This eccentricity is always on the side of the heavier load.

At (*b*) the eccentricity is the distance from the center of the bracket support to the center line (axis) of the column. In both cases the load is the total eccentric load, which, multiplied by the eccentricity, causes a bending moment in the column. This bending moment increases the compression on the side of the column on which the eccentricity exists and causes tension on the far side. Sometimes this tension may be great enough to overcome the compression caused by the direct load, in which case the column will fail. All columns are loaded eccentrically and suggested methods for dealing with this condition are not always correct.

The effect of an eccentric load is assumed to disappear at each story height. That is, the tension, or additional compression, caused in any story length of column by eccentric loading is assumed to be a maximum at the mid-length of the column on that story and to be zero at the ends.

With a concentric load the unit compressive stress over the cross-sectional area of the column is

$$f = \frac{W}{A},$$

in which f = unit compressive stress in pounds per square inch,
W = concentric load in pounds,
A = area of cross section in square inches.
Let e = eccentricity in inches,
n = distance from axis of column to extreme fiber in the direction of the eccentric load,
P = eccentric load = A or $A + B$.

The bending moment due to the eccentric load is

$$M = Pe,$$

and the extreme fiber stress due to the combination of direct and eccentric load is

$$f = \frac{W}{A} \pm \frac{Mn}{I},$$

in which I = moment of inertia in direction of bending.

The positive sign (+) is used to obtain the compression and the negative sign (−) is used to obtain the tension. The compressive stress cannot exceed the safe allowable stress determined by a column stress reduction formula and the tensile stress is not considered.

The foregoing is an approximation only, but is satisfactory when the flexural stress due to eccentric loading is not large. The majority of designers use only seventy-five per cent of the moment due to eccentricity and reduce the eccentric load to an equivalent concentric load by the following expression:

$$W_e = 0.75\left(\frac{Pen}{r^2}\right),$$

in which W_e = the equivalent eccentric load,
r = radius of gyration.

With the both formulas it is necessary to select a column section and obtain the values of e, r and n. There is little difference in the values of r and n between columns of nearly the same size, so one computation for the value of W_e will generally be sufficient. An experienced designer can usually select a trial size so nearly right that but one approximation will be necessary.

Having obtained W_e it is necessary to add to it the direct concentric load W and the eccentric load P; thus $W_eWP = W_t$ the total equivalent concentric load, and the uniform compressive fiber stress becomes $f = \frac{W_t}{A}$.

When something better than a very close approximation (good enough for ninety-five per cent of columns) is wanted, the following formula by Professor J. B. Johnson may be used.

$$f_b = \frac{Mn}{I \frac{PL^2}{10E}},$$

in which f_b = unit flexural stress in pounds per square inch,
L = length of the piece in inches,
E = modulus of elasticity of the material.

The Johnson formula is used for beams subjected to bending as well as to direct compression or tension; and, also, to struts and ties eccentrically loaded in addition to having a concentric load to carry. The unit flexural stress must be added, algebraically, to the direct stress due to the concentric load, and the sum of the two cannot exceed the safe fiber stress of the column as determined by a column stress reduction formula.

For columns, ties and struts, the load P acts parallel to the piece and $M = Pe$. For beams the moment M is the bending moment caused by the dead load of the beam plus whatever additional transverse load there may be on it. Therefore, for beam subjected to direct tension or compression in addition to cross bending the P is really W, as used in the expression $\frac{W}{A}$, and is not an eccentric load, but is the direct concentric tension or compression.

The beams carried by wooden columns and steel columns rest on brackets attached to the columns. There can, therefore, be no uncertainty as to the amount of eccentricity, e. Concrete columns are cast integrally with beams and slabs, so considerable uncertainty often exists in the minds of draftsmen as to the amount of eccentricity. The author assumes that the load from the beam is delivered to the column in a uniformly varying amount from the face of the column to the vertical axis. The vertical load acting through the center of gravity makes the moment arm for each beam equal to one-third the width, measured from the center of the column, that is, two-thirds of half the width. Multiply each load by this arm, add the products, and divide by the sum of the loads, which will give the eccentricity measured from the center of the column.

When columns are connected by girders then the deflection in the frame will vary with the relative rigidities of the connected members and this will fix the stresses at the connections.

Wind Bracing for Columns and Frames

In the handbook of the Passaic Rolling Mill Co., a number of years ago the whole subject of wind bracing in buildings was disposed of with the presentation of Fig. 166 and Fig. 167. In the "Bethlehem Handbook" (1908) of which only the one edition, now out of print, was issued the same figures and formulas were given.

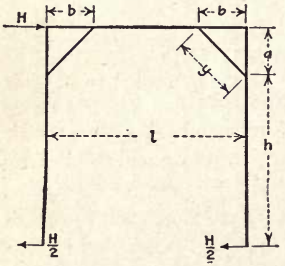


Fig. 166—Case I of Portal Framing

In the following formulas columns are considered as fixed at both ends. If columns are not fixed at the ends substitute $2h$ for h, everywhere in the formulas. All members are constructed to resist tension (−) and compression (+).

H = total horizontal force at top of frame.

$$\text{Stress in the knee braces} = \pm H\left(\frac{1}{2} + \frac{h}{4a}\right)\frac{y}{b}.$$

$$\text{Stress in the columns} = \pm H\left(a + \frac{h}{2}\right)\frac{1}{l}.$$

$$\text{Stress in the girder} = \pm H\left(1 + \frac{h}{4a}\right).$$

$$M_b \text{ on the columns} = H + \frac{h}{4}.$$

$$M_b \text{ on the girder} = H\left(\frac{1}{2} - \frac{b}{l}\right)\left(a + \frac{h}{2}\right).$$

$$\text{Stress in } AB = \pm H\left(1+\frac{h}{4a}\right).$$

$$\text{Stress in } CD = \pm H\left(\frac{1}{2}+\frac{h}{4a}\right).$$

$$\text{Stress in diagonals} = \pm H\left(\frac{a}{2}+\frac{h}{4}\right)\frac{y}{la}.$$

$$\text{Stress in columns} = \pm H\left(a+\frac{h}{2}\right)\frac{1}{l}.$$

$$M_b \text{ on columns} = H \times \frac{h}{4}.$$

Diagonal bracing is the cheapest in tall buildings, but it is not possible always to use it on account of window or other openings. It is, therefore, necessary in most cases to use girders at floor levels, as illustrated in Fig. 166, or trusses, as illustrated in Fig. 167. The figures show but one frame and apply to single story buildings. It is easy to say that a high building may be considered to be a series of such frames superimposed and side by side. Difficulties arise when it is necessary to apportion the wind load on each story and on each column.

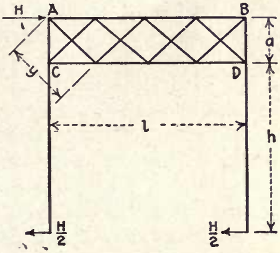


Fig. 167 — Case II of Portal Framing

It is assumed that at each floor level the stiff floors will distribute the load to the columns and it is only in the vertical frames that attention must be paid to proportioning of members to resist wind.

Every building having a height twice the width must be proportioned to resist wind and the pressure per square foot of the wind is fixed by the ordinance followed by the designer. If, there is no building ordinance to be followed then the usual requirement is 30 lbs. per sq. ft. of *exposed* surface.

The designer has a choice of lines through which the wind force may be carried to the foundations. Look up the specifications and see just how the stresses are fixed when wind is included. Then test each line of columns to see if they can carry wind as well as the gravity loads. If they can do this the building is stable against wind. If they will not do it the designer must fix on some lines of columns, with their connecting floor girders, which must be designed as a frame to carry the wind loads. If, by slightly deepening the girders on all lines of columns the matter can be accomplished, then all the lines of columns may be called into service. Usually, however, the necessity for making openings through interior walls requires that a minimum of depth be used in girders and that the sizes of columns must be a minimum. When it is decided to keep the area between columns as free as possible from obstructing beams, ties, and struts, it will be necessary to select a few lines of columns parallel with the wind, which, with their connecting girders, trusses, or ties, will be designed as frames to resist the force of the wind. Wall columns are usually chosen and the spandrel beams are deepened, because, being in the walls, they cannot be in the way of partitions or alterations in the interior of the building. When necessary to strengthen framework across the interior of a building it is usual to do it on the wall lines of light wells.

For wind alone the columns of a building may be considered to have two fixed ends, except the columns supporting the roof. If the footings are not designed to resist the additional force of the wind the lower columns are not considered fixed. It is hardly probable that the foundations will not be so designed. Assuming the columns to be fixed and the trusses or girders connecting the columns to be strongly attached to them, there will be a point of contraflexure in each column and in each girder. For convenience in designing this point of contraflexure is taken to be in the middle. It is not an accurate assumption, but it is safe and lessens the time required for computation and simplifies the work. There is moment only at the ends, and, so far as the wind force is concerned, the columns and girders can be hinged at the points of contraflexure. This really means that each column consists of two cantilevers extending upward and downward from the floor beam with a length equal to half the story height; and each girder consists of two cantilevers extending to the right and left of the column and with a length equal to one-half the span.

For each story the total amount of wind on the story is assumed to be concentrated at the middle of the column on. This force is horizontal shear. Assuming the tall building to be a vertical cantliever beam the wind loads are the loads (horizontal shear) in each story. Adding these loads from the top down the total shear at each story height is found, precisely as shear is determined for any cantilever beam.

This shear is distributed across the frame in the direction of the wind by dividing the total shear at any floor by twice the number of panels. (Number of columns less 1 = number of panels.) Each end column carries the amount thus found and each intermediate column carries double this amount, because the end columns support only one-half a panel and each intermediate column supports a full panel.

For the top story the formulas in relating to Fig. 166 or Fig. 167 may be used. The direct stress in the column is added to the concentric load in that column, but is not carried to the floor below. The bending moment in the column is treated as a moment due to eccentric loading.

For each story below the top the moment for each column is equal to the total horizontal shear on that column multiplied by the story height. The wind force is assumed to act at the mid-height and as the column is practically a cantilever with a length equal to half the story height the proper length to use for this condition of a column fixed at the bottom and free at the upper end is twice the actual length.

The bending moment in a girder is in all cases the mean between the bending moments in the column below and above the girder. It is independent of the span. The moments in columns and girders are at the ends, the force in the middle being shear. The bending moments in girders must be provided for by adding haunches to them instead of using simple brackets. Brackets on which girders rest are assumed to resist vertical shear by the rivets which connect them to the columns. They should be designed, when possible, so none of the rivets will be in tension. Gusset plates and brackets for connecting wind-bracing girders to columns are in compression below, or above the girder and are in tension above, or below, the girder. It is, therefore, necessary to use a very low tensile stress in the rivets, which it is certain they can withstand, and this it will be found calls for a great number of rivets. A tensile stress of 6000 lbs. per sq. in. is used in such cases.

Fig. 168 is a graphical representation of the moments in some of the columns and girders of a frame. The moment is a maximum at the end and varies uniformly, so it is only necessary to plat each moment on opposite sides of the members and draw a straight line connecting the end lines. The shear is constant, as it is a concentrated load.

The author has here presented the method he uses in designing frames of buildings to resist wind. There are several other methods in use, and not all engineers will agree with the method here given. It is, however, simple and agrees well with such meager knowledge as we now possess of the actual force of the wind on tall buildings. It is probably in more common use than any other method. Another method is to assume the building frame as a vertical cantilever beam loaded at the mid-point of each story with the wind as a concentrated load. The total moment is found for the beam at each floor level and this is divided among the lines of columns proportionately to their distance from the neutral axis, which is assumed to be midway between the exterior columns. The bending moments in columns and girders are equal at the respective floor levels.

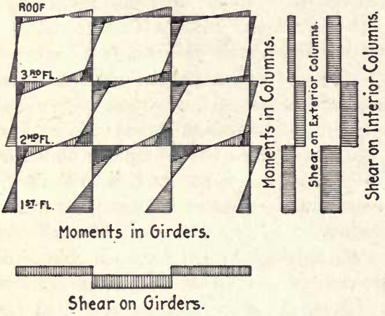


Fig. 168 — Graphic Representation of Moments and Shears in Frame of a Building

In Fig. 169 is shown the method of bracing frames by means of diagonal ties, known as sway bracing. It is assumed that the bracing is provided only between the exterior columns and the first line of interior columns. The wind load is applied at each floor level. The two lines of columns become, respectively, the upper and lower chord of a cantilever truss, the floor beams or girders are the verticals, and the diagonal ties carry shear. Such a method

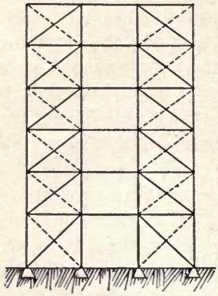


Fig. 169 — Diagonal Wind Bracing in Tall Buildings

causes no bending in the columns but does add a direct load. It adds a direct load to the floor beams or girders, which must be investigated. The forces may be ascertained analytically or graphically. It is usually best to put in counters, as indicated by the dotted lines.

Loads on Columns in Buildings

The dead weight of a building is a constant matter. Live loads vary from time to time. Just what proportion of live load should be carried to columns is not settled, but it is common practice to design the columns supporting the roof for the full dead and live roof load. The columns supporting the floor next to the roof are designed to carry the load transmitted to them by the roof columns together with the dead load of the floor and part of the live load. In Chicago only 85 per cent of the live load on the top floor is carried to the columns. In other cities 90 per cent is used and some engineers use 95 per cent of the live load. It is only on the top floor columns that any difference of opinion exists. On all columns below the top floor the live load is reduced progressively 5 per cent per floor until the reduction amounts to 50 per cent of the live load. From this floor, only 50 per cent of the live load on each floor is carried to the column, together with the total dead load. Using Chicago requirements and assuming a ten-story building with 100 lbs. per sq. ft. live load on each floor; roof live load 30 lbs., etc., the load per square foot carried to the columns will be as follows:

Columns under	Loads per sq. ft. to columns for each floor			Total load per sq. ft. on column
	Dead	Live	Total	
Roof	60	30	90	90
10th floor	75	85	160	250
9th "	75	80	155	405
8th "	75	75	150	555
7th "	75	70	145	700
6th "	75	65	140	840
5th "	75	60	135	975
4th "	75	55	130	1105
3rd "	75	50	125	1230
2nd "	75	50	125	1355
1st "	75	50	125	1480

The loads are tabulated as in the above table and the area of floor served by a column is multiplied by the total load per square foot shown in the last column, in order to determine the column load at each floor.

The area of floor served by interior columns is equal to the space enclosed between four columns. Side wall columns serve an area equal to one-half this and corner columns serve an area one-fourth that of the space between four columns. In other words loads go to the nearest support. The wall columns carry the additional dead load of the walls, each column carrying a length of wall measured midway between adjacent columns, that is, a panel length.

For the load permitted on various soils and for the amount of live load to be carried to foundations, with and without piling, consult the various specifications mentioned and the steel handbooks.

The distribution of load on footings is not treated adequately in many textbooks. The dead load is constant and the live load is variable. If the total load, dead plus reduced live load, is used in proportioning the footings the footings under the interior columns will be entirely too large if the building stands vacant. The footings under the walls, however, will continue to settle and old buildings with humps in the floors over the girders were no doubt so designed that the footings under all columns were proportioned for the total dead and live load, or total dead and reduced live load.

A common method is to use for interior columns the allowable soil load and for wall columns a soil load about 500 lbs. per sq. ft. less, and then proportion the footings for the total load brought down as illustrated. Some men make the soil load for the exterior columns one-third less than that for the interior columns and design the footings for the total dead plus the reduced live load.

Mr. Schneider recommends the following method: Proportion the footing under the column carrying the maximum live load. Divide the total load by the allowable soil pressure and obtain the square feet required to carry the load. Divide the dead load by the area thus found and obtain a reduced soil pressure per square foot. Using this reduced soil pressure design the rest of the footings for the dead load only. This is very conservative but may well be used for reinforced concrete buildings, as all the beams, girders, and slabs are designed as continuous. Any settlement will be bad and generous foundations will prevent settlement.

The Schneider method calls for expensive foundations, and Mr. Daniel E. Moran suggests using one-half the *probable maximum* live load, instead of the full live load advised by Mr. Schneider. Mr. Moran says: "The maximum probable load is the load which in the opinion of the designer will actually come upon the footings, and is to be determined by a study of the conditions which will obtain when the building is occupied. For instance, in a schoolhouse the number of children in each class room and the weight of desks, chairs, etc., may be determined with considerable accuracy and these loads will make the maximum probable live load. As a further illustration, in many schoolhouses there is an assembly room which is only used when the class rooms are vacant, and consequently if class room loads are used assembly room loads should be omitted or *vice versa,* the greater one of these loadings to be used for the probable load." See on this point *Engineering News,* March 6, 1913, and April 3, 1913.

The author was taught, thirty years or more ago, to proportion the loads as follows: the dead plus the reduced live loads were carried down and the footing under the *corner* column carrying the least dead load was designed for the dead and live load. The allowable soil pressure was multiplied by the per cent of dead load brought down to this footing, to obtain a "soil factor." The soil factor was divided by the percentage of dead load brought down for each column and thus was obtained a new allowable soil pressure for each column. With these allowable soil pressures as thus determined for each footing the footings were designed for the dead and live load. This method is really practically the same as that proposed by Mr. Schneider, except that the column loaded the most heavily with dead load is the critical column, whereas with the Schneider method the column loaded the most heavily with live load is the critical column. The method so long used by the author is preferred by him, but instead of the corner column carrying the least dead load he selects that column on which the live load is not less than 15 per cent. This will be sufficient for many buildings. When there is much machinery in a building and the building is occupied by large numbers of employees for eight hours and is closed for sixteen hours each day, that column is selected on which the live load is from 20 to 25 per cent, to allow for the constant machine load. The student can see that there is considerable room for the exercise of judgment in the matter, provided the fact is recognized that to design all footings for the sum of the dead and live loads is wrong.

The load on walls is assumed to be one foot long. The load on one lineal foot of wall is divided by the allowable soil pressure per square foot and the width l of the footing found, Fig. 170. It is then stepped. An old rule was to draw a line upward, at an angle of 60 degrees from the end of the footing to the lower corner of the wall and form steps so the line touched the inner corners. By assuming a safe fiber stress for the masonry and computing the offsets as projecting cantilevers the thickness and projections of the steps may be computed. The following formula is used generally by designers; referring to Fig. 170:

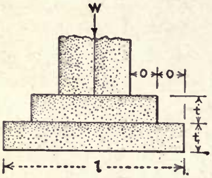


Fig. 170 — Stepped Masonry Footing

o = offset in inches,

t = thickness in inches,

p = allowable soil pressure in pounds per sq. in.,

s = safe unit tensile stress in the material,

= 30 lbs. per sq. in. for 1–3–5 concrete,

= 60 lbs. per sq. in. for 1–2–4 concrete.

Various authorities give values for stone and for brick laid in cement mortar. The values for stone cannot be used unless the stone projects less than one-half its length beyond the step above. This is to provide for true cantilever action. If built in this way s = 80 to 130 lbs. per sq. in. for limestone, the same for sandstone and 180 lbs. per sq. in. for granite. Hard-burned brick laid up in cement mortar in good bond by a first-class mason is considered to be good for 40 lbs. per sq. in. The author advises the use of concrete.

$$o = 4t\sqrt{\frac{3s}{p}}.$$

The formula is derived as follows:

$$M_b = \frac{po}{144} \times \frac{o}{2} = \frac{po^2}{288},$$

for p is in pounds per square feet and o is in inches, so it is necessary to reduce p to pounds per square inch. The load is uniformly distributed along the cantilever $o = po$, and the force acts through the center of gravity $= \frac{o}{2}$. The moment of resistance for a rectangular section $= \frac{sbt^2}{6}$, but, since $b = 1$ (the unit width) the equation becomes

$$M_b = M_r = \frac{po^2}{288} = \frac{st^2}{6}.$$

Dividing, $\frac{po^2}{48} = st^2$.

Multiplying, $po^2 = 48st^2$.

Dividing $o^2 = \frac{48st^2}{p} = \frac{3 \times 16st^2}{p}$.

Extracting the square root, $o = 4t\sqrt{\frac{3s}{p}}$.

The assumption is made in this case that the projection, or offset, is a cantilever with the maximum moment at the face of the support. This is true in the case of all stepped footings for walls. When the wall is relatively thin, the projection of the footing being long, the maximum moment is assumed to be in the center of the footing under the center of the wall.

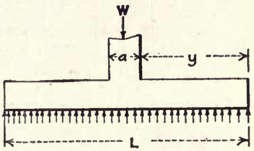


Fig. 171 — Bending in Wall Footing

The formula for footings such as grillage beams and reinforced concrete, referring to Fig. 171, is

$$M = \frac{Wy}{4}.$$

The load is assumed to be uniformly distributed over the pressed area. The formula may be derived in two ways.

First. Assume one-half the load to be carried on one projecting end having a length $= \frac{L - a}{2} = y$

$$M_b = \frac{W}{2} \times \frac{y}{2} = \frac{Wy}{4}.$$

Second. Assume the slab to be a freely supported beam with a span length $= L$ and loaded in the middle with a load W occupying a width a.

$$M_b = \frac{WL}{4} - \frac{Wa}{8} = \frac{2WL}{8} - \frac{Wa}{8} = \frac{W(L-a)}{8} = \frac{Wy}{4},$$

since $$(L - a) = 2y.$$

Not all authorities agree that the above formula for the design of footings is correct, the contention being that the maximum moment is at the face of the wall, and not under the center. The following formula is used for this assumption:

$$M_b = \frac{Wy}{2y + a} \times \frac{y}{2} = \frac{Wy^2}{2(2y + a)}.$$

To design a wall footing let w = weight per lineal foot of wall and divide this by the safe soil load per square foot. This will give the width L of the footing. The bending moment being obtained the total thickness will be t and the total offset is o, in the formula for stepped footings. This will give the thickness at the face of the wall and it may be stepped off by dividing it into any number of steps each with a thickness, t, and solving for o by the formula for each step.

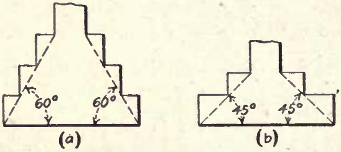


Fig. 172—Design of Stepped Footings

Such footings are seldom so designed. The usual way is to ascertain the width and then from the edge of the bottom of the wall draw a line at an angle of 45 degrees, or 60 degrees, with the horizontal until the horizontal distance separating the lines is equal to the spread of the footing. Steps are then drawn to touch this line, as shown in Fig. 172.

The real use made of the formulas for bending moment on footings is to determine the size of I beams to use in a grillage foundation, or the thickness and reinforcement for a reinforced concrete footing. The design of grillage footings is given in the steel handbooks. A reinforced concrete footing is designed as a slab one foot wide, the thickness being fixed both by shear and bending.

Column footings differ from wall footings in being square or rectangular. If the footing is square the area is found by dividing the total load by the allowable soil pressure. The square root of this area gives the length of each side. If the side of the column is longer than the end and it is desired to proportion the ends and sides of the footing in the same ratio the following procedure is adopted.

Let L = the long side,

b = the short side,

A = the area of the footing in square feet.

Then $\frac{L}{b} = a$, which is the ratio of the length and breadth of the column area and of the area of the footing, that is,

$$L = ab,$$
$$A = bL = b + ab = ab^2,$$
$$b = \sqrt{\frac{A}{a}}.$$

Having obtained the length and breadth of the footing it is assumed that there are four cantilever beams projecting from

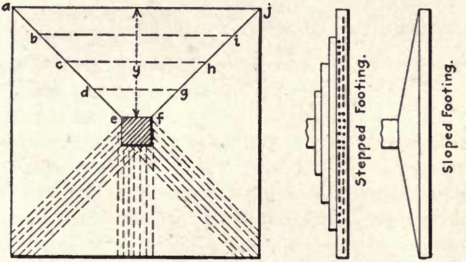


Fig. 173 — The Design of Column Footings

the column. Each has a width at one end equal to the width of the column base and a width at the other end equal to the length of the side. The beams are thus wedge shaped with the maximum moment at the narrow end. Each beam carries one-fourth of the total load. For convenience the beam may be considered to be divided into four strips, as shown in Fig. 173.

Referring to the figure and assuming that in this case one-fourth the load is carried on each beam, then the beam *ajfe* carries $\frac{W}{4}$. Each strip carries a part of this load in proportion to the area of the strip. The strips may be of equal width, as shown, or they may be varied in order to have equal loads on the strips.

Each load is assumed to be concentrated at the center of gravity of each strip and the projection y is assumed to be the length of a cantilever beam. The moment across the beam on the lines *bi, ch, dg,* and *ef,* can be readily ascertained and the shears can also be found on these lines. In this example we are assuming a reinforced concrete beam. The thickness of the footing on each line can be found in the usual way for the design of a concrete beam, or slab, both for bending stress and for shear. The steel may be proportioned and the logical method for arranging the steel would be to have it in four layers, two normal to the sides of the slab and two diagonal, as indicated by the dotted lines in the lower part of Fig. 173.

Another method, used by the author, is to take the expression $M = \frac{Wy}{4}$, used for wall footings, and assume that the footing being square and there being eight projections instead of two, the expression should be $M = \frac{Wy}{16}$, and use this moment to design a reinforced concrete beam having a width equal to the column base on top of the footing. The beams are considered as being so arranged that two are normal to the sides of the footing and two are diagonal. They are designed as reinforced concrete beams and as merged so that while each layer of steel carries the tension for the beam it represents, the concrete is stressed in compression from all directions, which makes it safe and increases its resistance to shear.

Reinforced concrete footings may be stepped or sloped on top. If sloped the forms must be well anchored down, for the concrete will have a tendency to cause them to float. The author obtained the best results with concrete footings by stepping them. The steps are formed by frames of boards. The first step is cast to the proper level and the frame for the next step placed on it, when it becomes firm enough to carry the weight of the next step without bulging up around the edges of the form. If the concrete is mixed to the proper consistency there will never be any trouble with this bulging and the steps can be poured quickly. The proper consistency for concrete is that of a soft tooth paste. It should never be thin enough to pour into a form. For reinforced concrete it should be thin enough to flow very sluggishly so it will surround the reinforcement, but it should never be so thin that the aggregates have a tendency to separate. Get in touch with the Portland Cement Association, Chicago, Ill., when reliable information on concrete is required. The Association has a well equipped laboratory where all questions affecting the use of portland cement and the manufacture and use of concrete are investigated.

Column Brackets and Bases

Brackets on steel columns for carrying beams and girders are rivetted to the columns. For light loads they are simple shelf angles. For heavy loads they consist of plates stiffened with angles. Details are given in the steel handbooks.

Post caps for wooden columns are illustrated in a book entitled "Heavy Timber Mill Construction Buildings" distributed free of cost by the National Lumber Manufacturers Bureau, Chicago, Ill. Mill construction has its place and every architectural designer should have a copy of the book. Some of the statements therein should be modified and the student is advised to obtain from the Portland Cement Association, Chicago, Ill., a bulletin entitled "Why Build Fireproof," written by the author, and thus obtain a glance at both sides of the question. The book on mill construction illustrates cast iron and steel post caps for carrying beams and girders. The girders do not rest on top of the posts, for the carrying power of the posts would be reduced thereby. It was formerly the custom to use wooden bolsters, on which to rest the girders and beams, in order to shorten the span. The side bearing strength of wood is much lower than the strength with the fibers. When the direct load comes down the post with a bearing stress, say, of 1100 lbs. per square inch, and rests on the side of a bolster the bearing stress is reduced at once to 235 pounds per square inch, or more, depending upon the wood. Bolsters to-day are used only under roof girders. They are better in case of fire than cast iron or steel but so greatly reduce the carrying power of the posts that they are not economical. Post caps come in a number of different shapes. To investigate the strength of a post cap obtain the moment of inertia of the cross section of the carrying portion. Then find the fiber stress by the method shown on page 67. The bending moment is the moment caused by the reaction from the beam acting at half the length of the projection from the face of the column.

Cast iron brackets are cast on the side of cast iron columns. There is really no rational method for designing them. They should be at least as thick as the shell of the column. For light loads one bracket is used under the shelf and for wide beams two, or more, brackets are used to avoid eccentricity in the loading. The depth of the brackets at the face of the column must be enough to prevent shearing. There is of course a bending moment created by the reaction from the beam. This bending moment divided by the depth of the bracket gives the tension in the shelf where it is attached to the shell of the column. This tension must be divided by the length of the edges of the shelf and multiplied by the thickness of the metal in the shell to determine the shearing stress which acts to tear the shelf away from the shell. This shearing stress divided by the safe allowable shear in the metal must be less than the tension, or the thickness of the shelf must be increased. Such computations are merely checks but should not be neglected.

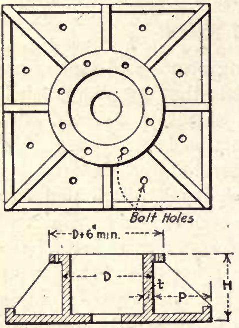


Fig. 174— Cast Iron, or Steel, Ribbed Column Base

In Fig. 174 is illustrated a ribbed cast iron base cap for a column. No known rational method exists for determining the stresses, so these bases are made according to empirical rules. The thickness of all the parts should be not less than the thickness of the shell of the column. All parts should have the same thickness to avoid danger of casting cracks. A small fillet should be used in every angle.

The projection at the top should be not less than three inches wide, so bolts can be used with plenty of clearance for the heads. When the bottom projection P is greater than six inches, ribs should be used. The height H should never be less than the projecting P and the diameter of the base under the column should be equal to that of the column. The number of ribs should never be less than eight, and an empirical rule for fixing the number of ribs is that the space between ribs at the circumference of the column should never be greater than twice the thickness of the shell.

Cast steel bases are better than cast iron bases and built-up steel bases are a great deal better than cast steel bases. Cast steel bases are designed in the same way as cast iron bases. Built-up steel bases are illustrated in the steel handbooks.

When the projection P of a cast iron, or cast steel, base is not greater than six inches a plate may often be used to advantage. The formula to use in the design of such a plate is given on page 152, where it is used to design a washer under the head of a bolt. Mention is there made of ribs acting as cantilevers and if the student wishes to attempt to design bearing plates and bases of cast iron according to formulas, he is advised to procure from the Engineering Experiment Station, University of Illinois, Urbana, Ill., a copy of the bulletin on the design of cast iron column bases and bearing plates. He is advised also to procure a copy of *Bulletin* No. 67 on the design of reinforced concrete footings.

The size of a column is fixed by the compressive strength of the material. The column rests on concrete, stone or brick footings, which have a lower strength in compression, so it becomes necessary to enlarge the lower end in order not to overstress the masonry. It is most convenient, in the majority of cases, to set the spread base before erecting the column, so bases are made to which the columns are bolted. The area of the bottom of the base is obtained by dividing the load by the bearing strength of the masonry.

Eccentric Loads on Footings

In Fig. 175 is illustrated a common case of eccentric loading on the footing of a wall. The formula to use is given on pages 100 and 101. The direct load divided by the area of the footing gives the pressure per square foot on the soil. A vertical line is drawn through the center of gravity of the footing and a vertical line is drawn through the center of gravity of the wall. The horizontal distance e is the moment arm. Multiply the load in pounds by the moment arm, e, in feet and use the bending moment in the formula. In the formula h is used as the depth of the member. In the case of the footing h is the width, l shown in the figure. The resulting fiber stress cannot exceed the safe allowable bearing pressure on the soil. A positive (+) result indicates compression and a negative (−) sign indicates uplift. The two shaded diagrams illustrate the action. The upper one shows the load per square foot over the footing, provided the load is applied through the center of gravity. The lower diagram shows the effect of the bending moment to increase the compression on one side and lessen it on the other.

If the sum of the two does not exceed the safe allowable pressure at one toe and there is no uplift (tension) on the other toe the footing will be safe. The rule to assure safety is known as the "Middle-third rule" in which the resultant pressure to prevent overturning must be kept within the middle third of the base. The middle-third rule has been considerably overworked. What it really amounts to is a statement that if the resultant of all pressures brought to bear on a footing base is kept within the middle third the average stress will not exceed one-half the maximum and there will be no tension.

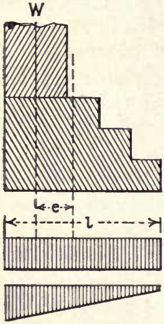


Fig. 175—Eccentric Load on Wall Footing

The condition shown in Fig. 175 is not always possible to avoid, for foundations must be kept within lot lines. The remedy apparently is to so construct the footing that a line may be drawn at an angle of thirty degrees, with the vertical, from the edge of the wall to the lower edge of the footing and keep within the footing. When the load brought down by the wall reaches the footing it will spread out and thus the center of effort of the load will not be directly under the center of the wall, but will be somewhat nearer the center of gravity of the footing. This may be the case if the wall is not in excavation. If it is in excavation the load is no doubt partly distributed to the earth on the outside of the wall, so the center of effort of the load is actually under the center of the wall, provided the load passes as readily through the masonry as it does through the earth on the side.

The footing may be of solid concrete, with a depth fixed by the sixty degree line, so that it will not distort under load. If this is the case, then as soon as the earth under the heavily pressed edge gives way the entire bottom of the footing will come into bearing and relieve the stress on the soil. The same effect should possibly be secured by using a lighter footing of concrete heavily reinforced so it cannot bend. Something also may be gained by having the inner toe deeper than the outer instead of keeping the base level.

The best remedy is to drive piling to help the earth carry any excessive load. In several of the specifications mentioned, rules are given for the use of piles. A pile acts by the bearing of the lower end of the pile on the soil into which it is driven, plus the friction of the pressed soil on the surface of the pile. The minimum distance center to center of piles should not be less than three feet, except under unusual conditions. Driving piles too closely together often results in an actual lessening of the carrying capacity.

To find the center of gravity of a stepped footing such as that shown in Fig. 175 multiply the area of each strip by half the distance from the outer edge. Add the results and divide by the total area. This gives the horizontal distance to the center of gravity. The distance from the bottom is found similarly by dividing the area into strips by vertical lines and multiplying each area by half the depth. It is the method of moments, already explained for irregular sections and for bending moments on rivets. The center of gravity is not essential in the footing problem, but it is essential to obtain the horizontal distance to a vertical line through the center of gravity.

Eccentric Loads on Column Base

A column exposed to the force of wind acting to push it to one side will put an eccentric load on the base. There are two cases.

I. The column hinged to the base. The horizontal force in this case acts at the base of the column. The vertical force acts vertically through the center of the base and the resultant of the horizontal and vertical forces should be kept within the middle third if possible.

II. The column is fastened to the base sufficiently to make the base practically a part of the column, or at least develop a bending moment $= H \times \frac{l}{2}$, in which

H = horizontal thrust,
l = length of column.

In Case I the horizontal force acts at the top of the footing, as shown in Fig. 176. In Case II it acts halfway up on the column. It is important to remember these distinctions. The distance x on the bottom of the footing should in all cases be not more than $\frac{1}{6}$ the base, to keep the average stress within one-half the maximum and insure that there is no tension, or uplift, on the other side. This "middle third theory" is merely a statement. Provided the maximum soil pressure is not greater than the allowable safe pressure the resultant can be outside the middle third. The statement is frequently made that when the resultant passes beyond the middle third the structure is in danger of being overturned. The fact is that whenever the resultant of a horizontal and a vertical force passes through a footing at any point off center the overturning tendency is present. To be safe it is necessary to see that no undue load is placed on the soil. The "middle third" theory is a safe one to follow, but it should not be followed blindly.

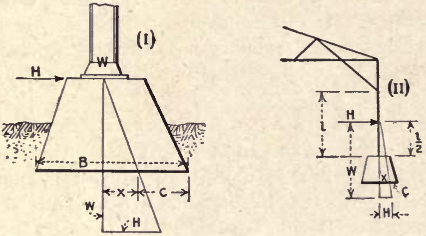


Fig. 176 — Eccentric Load on Column Bases

Let H = horizontal force,

h = distance at which H acts above bottom of base,

W = vertical load on footing,

then $x = \dfrac{Hh}{W}$.

Let b = length of base (that is, the dimension at right angle to the force H, the dimension B being in the direction of the force H).

p = pressure on soil in pounds per sq. ft., all the dimensions being expressed in feet and the weight and wind force in pounds;

then, when the resultant falls within the middle third,

$$p = \frac{W}{bB^2}\,(B \pm 5x).$$

When the resultant falls beyond the middle third

$$p = \frac{2w}{3bc}.$$

The student should study the formation of the last three formulas. The first one means that a force H, acting with an arm h, tends to overturn a body having a weight W. It is, therefore, necessary to find the length of an arm x, through which W acts to resist the overturning moment Hh.

In the second the distance x cannot be greater than one-sixth of B. The total weight is distributed over an area Bb. With these hints the student should attempt to construct the formulas as an exercise.

Attaching Column Bases to Footings

Column bases are attached to footings by bolts. A horizontal force, such as wind, develops a bending moment in the column where it is attached to the footing. Referring to Fig. 177, divide the bending moment by the distance x between the center lines of the bolts in the direction of the force. This gives the pull on the bolts. The distance is measured between centers of bolts instead of from the leeward edge of the plate to the center line of the windward bolts, to avoid bending the edge of the plate.

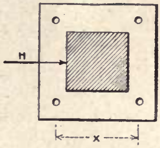


Fig. 177 — Foundation Bolts for Columns

The pull in the bolts is divided by the allowable tensile fiber stress, to obtain the required bolt area. Dividing this by two gives the area of one bolt, and the circumference is readily found when the diameter is known. Dividing the circumference in inches by 50 lbs. gives the total bond resistance per lineal inch of bolt. Dividing the uplift on one bolt by this amount, the length of bolt is obtained.

The area of footing is determined by the bearing value of the soil. The depth must be great enough to make the footing of the weight required and also furnish area for embedment of bolts. The weight of the footing is fixed by the requirement that it be heavy enough to anchor the structure, or so much of the structure as may be carried by the column which rests on the footing.

The horizontal force acting on the column is applied to the column at the proper height. The force multiplied by the height exerts an overturning moment. Dividing this moment by the width of the building the weight of the foundation is obtained.

This is illustrated in Fig. 178. The direct weight W is made up of the weight carried to the foundation by the column, plus the overturning moment exerted by the force H acting on the windward column. The force H, acting on the leeward column is an eccentric load on the footing, as already described. The student must remember that each column carries part of the total horizontal thrust.

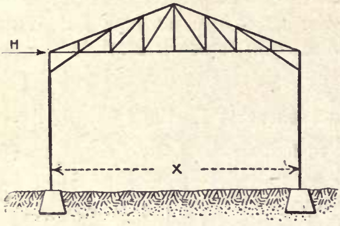


Fig. 178 — Foundations under Columns

Owing to the uncertainty of just where the force will act because the attachment of the columns to the base may not be rigid, there is some uncertainty as to the exact amount of force exerted on footings. If the columns are fixed (hinged) in such a way that they bend at the top of the footing, the only force exerted by wind on a footing will be that transmitted by the windward column as a direct vertical load. If the column is rigidly attached then the leeward column adds an eccentric load.

If the columns are rigidly attached the force H acts at a height equal to one-half the column length. If there is a knee brace some men assume the length l to be the distance from the base to the lower end of the knee brace. To be safe it is best to consider the column length to be measured to the bottom chord of the truss.

If the columns are poorly connected, or hinged, to the footing, the total wind force on the windward column is assumed to act at a height equal to the full column length, plus one-half the height of the roof truss.

For taller buildings than here illustrated the total force of the wind is assumed to act at half the height of the building, or if only the upper portion of the building is exposed, at half the height of the exposed portion, measured from the ground, the full amount of wind being figured only on the exposed portion. This moment divided by the width of the building gives the weight which must be opposed to resist overturning. Determine the direct weight coming on the footing and if it is not enough increase the depth of the footing so it will have enough weight.

Cantilever Footing

The subject of cantilever footings is very simple, although a number of students seem to find it difficult. In Fig. 179 the column on the right is against a wall, or property line, and the footing must be kept within the limits of the property. The first line of interior columns must, therefore, help out the wall columns. First find the proper size of footing under each line of columns. The outside footing is arranged so the outer face is even with the lot line. The load P, coming down the wall column, acts at the center of the column and the distance to the center of the footing gives the moment due to eccentric loading, thus $M = Pa$.

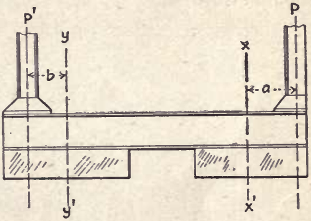


Fig. 179 — Cantilever Footings

Divide this by the load in the interior column to obtain the eccentricity b. The footing under the interior column is then so located that the center of the footing will be distant b feet from the center line of the column. Thus,

$$b = \frac{Pa}{P'}.$$

To transfer the loads from the columns to the centers of the respective footings the columns rest on a girder having a resisting moment $M_r = M_b = Pa = P'b$. The girder must also be designed for deflection, as a cantilever at each end, and for shear which is usually very high.

"Foundations of Bridges and Buildings," by Jacoby and Davis, price $7.50, is the best book on the subject with which the writer is acquainted. The student is advised to consult it if he needs more information on this important subject.

Stresses in Towers

A tank tower, or any braced tower or pole, may be designed as a vertical cantilever beam or truss.

The vertical load, consisting of the weight of the water, tank, and framework, is transmitted directly to the foundations through the columns, each carrying its proportionate share of the load. This vertical load is not considered in the graphical stress diagram used to determine the force due to wind, so must be added to the wind forces after the stress diagram is constructed and scaled. The struts and ties carry no part of the tank and water load, being used to take care of the wind load and to divide the column into intermediate lengths so the columns may be as

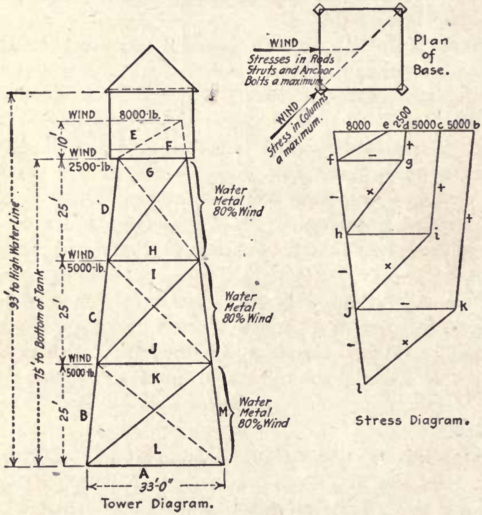


Fig. 180 — Stresses in Water Tower

small as possible, each column being considered as having a length equal to the height of one bay.

The plan of the base in Fig. 180 shows that the maximum stresses in the rods and struts occur when the wind is blowing against the side of the tower. The maximum stresses in columns occur when the wind is blowing diagonally across the tower and are 0.707 of the amounts obtained from the diagram. The wind force is not the result of a constant load, so in a number of specifications only 80 per cent of the maximum wind force is considered.

The wind acts at the top through the center of gravity of the filled tank, and at the joints on the side of the tower. In the tower diagram dotted lines are shown, transmitting the wind from the tank to the frame. In the stress diagram the uplift is measured from f to l. The tower loads are all positive (compression). In the tower diagram dotted counters are shown. These are not stressed with the wind coming from the left, but are stressed with the wind coming from the right. Each bay, therefore, is braced with cross braces designed both for tension and compression. The stress diagram shown is only one method for drawing such diagram.

Wind acts against the side of a square or rectangular building over the whole area. Against an octagonal structure the width is measured on the longest possible diagonal, and the area of the height multiplied by this diagonal is multiplied by 0.707, for some of the force of the wind is lost against the sloping faces. Against a cylindrical structure more of the force is lost on the curved surface, so the diameter is multiplied by the height and the product by 0.667. In old text books the statement is frequently met with that the pressure of the wind against a cylindrical surface is one-half that against a square having a side equal to the diameter of the cylinder. Exact mathematical analysis shows it to be two-thirds instead of one-half.

The design of foundations under water towers is similar to the design of foundations under the columns of buildings. The weight of the water and tank carried down by each column, added to the weight of the foundation, must be sufficient to oppose the overturning effect of the wind, the uplift. It is necessary to make a stress diagram, assuming the tank to be empty. It can be assumed that the columns are rigidly braced so there will be no bending to set up an eccentric load in the foundation blocks. The bolts anchoring the column footings to the columns must be strong enough to lift the weight of the blocks.

The Design of Chimneys

A chimney is subjected to direct stress and also to a bending moment caused by wind. The resultant stress on the leeward edge must not exceed the safe allowable stress.

The formula is $f = \frac{W}{A} \pm \frac{Mc}{I}$,

in which W = weight of stack above section considered,

A = area of ring in square inches,

M = bending moment in inch pounds,
c = distance from resultant through section to leeward edge, in inches,
I = moment of inertia.

The quantity c is of importance because the chimney shaft is hollow. With a solid symmetrical section the distance to the most stressed fiber is measured from the center of area, that is, the center of gravity. In a hollow ring or square the pressure varying uniformly from zero on the windward edge to a maximum on the leeward edge is not an average at the neutral axis of the section, but is an average at a point a trifle beyond. The distance from the center of area to the center of pressure will be termed q.

Let D = outer diameter of a round hollow shaft or outside length of a square hollow shaft.
d = inner diameter of a round hollow shaft or inner length of a square hollow shaft.

then, for a round hollow shaft $q = \frac{D^2 + d^2}{8D^2}$,

and for a square hollow shaft $q = \frac{D^2 + d^2}{6D^2}$.

Approximately values of q are $\frac{1}{3}$ for square and $\frac{1}{4}$ for round shafts.

$$c = \frac{D}{2} - q.$$

The chimney for analysis is divided into a number of sections and the horizontal area found at each section. The weight of the chimney above any section is ascertained and divided by the area of the ring to get the direct pressure. The vertical area of the chimney above the section is obtained and multiplied by the factor 0.707 if octagonal and 0.667 if cylindrical. The height to the center of this area, measured from the section, is a moment arm, by which the wind moment is obtained. Then find c and apply the formula. The compressive fiber stress should not exceed the maximum allowed for the material and for good brick in cement mortar there can be some tension on the windward side not exceeding one-tenth the compressive stress, provided the tension is not more than one-fifteenth of the allowable compressive stress in the material. With lime mortar tension is not permissible.

The material is generally brick and the maximum allowable compressive stress is fixed by building ordinances or in the specifications governing the design.

For a rough check of a design

Let P = the total wind pressure in pounds,
h = height in feet to the center of gravity of the shaft,
W = weight of shaft above section,
D = outer dimension,

then, for round shafts $hP = \frac{WD}{4}$, (Approx.)

and, for square shafts $hP = \frac{WD}{3}$. (Approx.)

Having tested the shaft at a number of joints, approximately twenty-five feet apart, redesigning the walls if found to be unstable, or if the maximum pressure exceeds the safe allowable pressure the final test is made at the base, on top of the foundation.

The spread base must be designed to carry the shaft without exceeding the safe allowable soil pressure. It may be square, octagonal, or round. It is subjected to a direct load, which is the weight of the shaft. The weight of the lining is neglected for brick and concrete chimneys. It is subjected to an eccentric load due to the moment caused by the wind. Then,

$$p = \frac{W}{A} \pm \frac{M}{2I},$$

in which p = soil pressure in pounds per square foot,
M = moment in foot pounds,
I = moment of inertia in square feet,

and the distance to the most stressed fiber is one-half the length, or width, of the footing.

$$I, \text{ for hollow circular shaft} = \frac{W(D^4 - d^4)}{64}.$$

$$I, \text{ for hollow square shaft} = \frac{D^4 - d^4}{12}.$$

$$I, \text{ for square section} = \frac{D^4}{12}.$$

$$I, \text{ for circular section} = \frac{WD^4}{64}.$$

A concrete chimney is frequently more economical than a brick chimney because the maximum compressive stress is greater than for brick and the tension can be equal to nf_c, in which

n = ratio of deformation between steel and concrete,

f_c = maximum unit compressive stress in the concrete.

The wind may come from any direction therefore, the reinforcement must be equally spaced around the circumference of the shaft. The shaft may then be designed by trial. Assume a certain steel area and assume it be in the form of a thin ring of steel. Find the moment of inertia and ascertain how much direct load and bending load it will carry (see page 67). The steel may be assumed to have a value of 12,000 lbs. per sq. in. A concrete shell is designed to carry the direct load and wind moment the steel cannot carry, and when the combined concrete section and steel section is found, which will carry the direct load and wind moment, the steel is placed in the middle of the thickness of the concrete ring in the form of rods or bars.

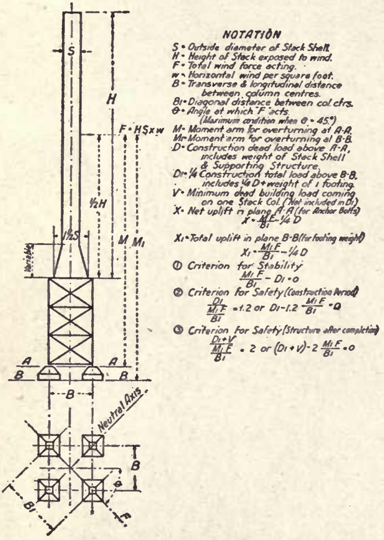


Fig. 181 — Formulae for Self-supporting Steel Stacks

Each section is designed in this manner and when the base is reached the vertical bars are run into it a sufficient length for anchorage.

Self-supporting steel stacks are often carried on columns and the proper formulas to use for this condition are shown in Fig. 181. These stacks may be anchored to concrete foundations by means of bolts, or they may be on girders and anchored by a number of bolts to rings riveted to the girders. To determine the size of the bolts the chimney is assumed as tending to overturn about one edge of the bolt circle. The principle is that used in the case of bolts fastening column bases to footings.

Brick stacks may usually start with a thickness of nine inches for the top twenty-five feet and increase half a brick in each twenty-five feet down. This is merely a rule by which to determine trial thicknesses. A general rule for the top thickness is as follows

$$t = 3 + 0.4 + 0.005H,$$

in which t = thickness in inches of upper course (neglecting ornamentation).

d = clear inside diameter at top in feet,

H = height of stack in feet.

The thickness of metal in steel stacks is governed by durability as well as by strength. The stack is a hollow circular cantilever beam (see page 67) in which the weight of the metal is of relatively small importance, the wind being the largest force. It is usual to start with plates $\frac{3}{8}$ in. thick at the top of the stack if not lined and some designers use $\frac{1}{4}$ in. plates at the top for lined stacks. Some designers increase the thickness by $\frac{1}{16}$ in. each 30 or 40 feet, while others increase by $\frac{1}{8}$ in. At each 30 or 40 feet the section is investigated and the thickness of the plate fixed by the fiber stress.

To insure tight joints the rivet spacing is not less than 2.5 times the rivet diameter, or more than 16 times the thickness of the plate. Usually the rivet spacing is investigated and determined only for the lowest tier of plates of any thickness. The rivets are in shear due to bending moment as well as ordinary shear.

Not enough data is available for reinforced concrete chimneys to fix trial thicknesses, as for brick and steel stacks. The least thickness should be six inches. The average increase in thickness is approximately at the rate of one inch in 50 ft. for trial sections.

Tanks and Retaining Walls

Fig. 182 shows curves for ascertaining the pressure in bins and tanks. They were computed by the author when he was Chief Engineer of the Fireproof Construction Bureau, Portland

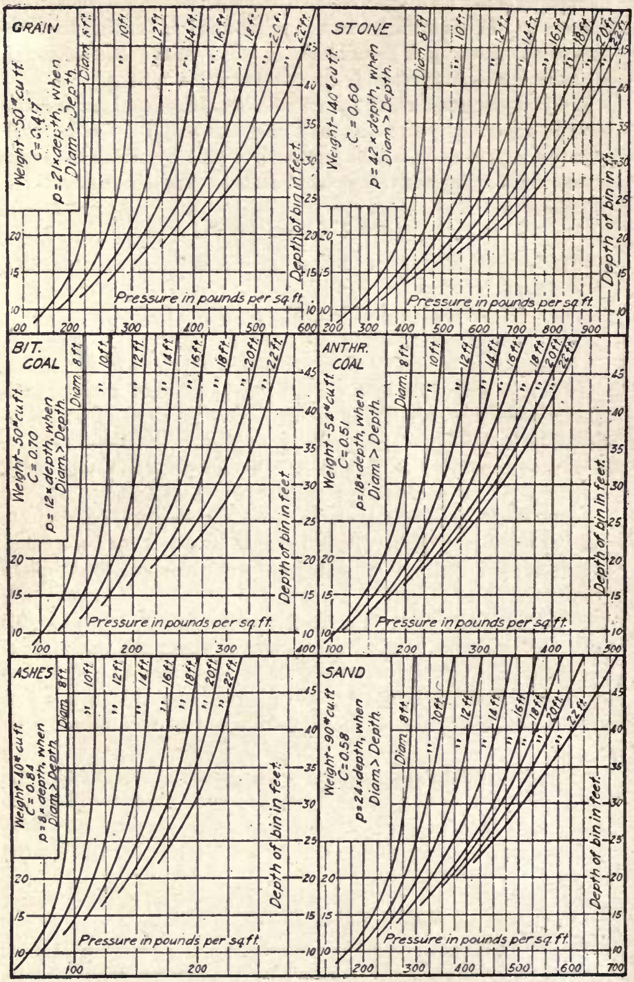


Fig. 182 — Curves for Designing Tanks

Cement Association, and have been printed in booklets and several periodicals.

These curves were computed by the Janssen formula and checked with other curves and tables. The pressures are for walls of concrete. For walls of steel multiply by 1.20 and for walls of wood multiply by 0.95.

When the depth of a tank is less than the diameter the surface of the slope of repose of the material will pass through the top without intersecting the opposite wall. The pressure in such a case is similar to that exerted by a fluid and the expression for pressure at any depth is

$$P = md,$$

in which m = a constant,
d = depth in feet,
P = pressure in pounds per square foot.

The constant m for water is 31.25, no matter how deep the tank or what its diameter. The constants for common materials are shown in Fig. 182. They represent one-half the weight of an equivalent fluid, that is a fluid which at any depth exerts the same pressure as the material considered.

When granular materials are confined in deep bins the opposite sides come into play as soon as the depth exceeds the diameter. The friction of the material against the walls causes the walls to carry some of the load, whereas with a fluid the pressure is always normal to the surface pressed. For deep bins the pressure at any depth in a bin or tank may be read directly from the curves.

When the depth of the bin is less than the diameter the total pressure against a vertical strip one foot wide is

$$H = Md^2,$$

in which H = total horizontal pressure.

The overturning moment $M = H \times \frac{d}{3}$.

The foregoing formulas are used to design square and rectangular tanks and bins, and retaining walls, holding water or granular dry materials.

In a circular tank the horizontal pressure is coverted into tension in the circumference.

$$T = W \times \frac{D}{2},$$

in which T = circumferential tension in a strip one foot deep,

D = diameter in feet,

W = the weight of one cubic foot of a fluid.

It is customary to take one-half the weight of the fluid, or equivalent fluid, and multiply it by the diameter. This is the constant m for each material shown in Fig. 182.

The circumferential tension divided by the allowable fiber stress in the material gives the number of square inches required. If the tank is to be of steel the thickness of the plate is found by dividing the area by 12, the depth of the strip. The proper allowance must then be made for rivet holes.

If the tank is to be of reinforced concrete the steel area must be sufficient to carry all the tension. Sometimes cracks will open in concrete walls, and if the concrete is relied on to carry part of the stress, the tensional strength of the concrete is lost with the first crack and the steel immediately carries this additional stress.

When there are no cracks in the concrete it does carry part of the load, so the thickness of the shell is fixed by assuming that the strength of the concrete in tension is 150 lbs. per sq. in. Dividing the total stress by 150 the concrete area is found and dividing this by 12 the thickness of the shell is fixed. It should never be less than four inches when first class experienced workmen are employed and six inches is a safe enough minimum to use for all tanks.

The shell of the tank as thus designed will carry double the tension, part of which is carried by the steel and part by the concrete.

Let A = total area = $A_c + A_s = A_c + nA_c$,

in which A_c = area of concrete in square inches,

A_s = area of steel in square inches,

n = ratio of deformation between steel and concrete.

The thickness of the concrete multiplied by 12 is the total area from which must be subtracted the area of the steel, leaving A_c, the net concrete area. The area of the steel is multiplied by n and added to the net concrete area, this giving the area, A, in the formula. Dividing the total stress by A the average unit stress in tension is obtained and this is the stress on the concrete. Multiplied by n it gives the actual unit tensile stress in the steel. This stress is very low but the instant a crack appears in the concrete, thus reducing the section, the steel stress is increased by an amount equal to n times the area of concrete in the face of the crack, times the average unit stress. If the crack extends through the wall the steel carries all the tension.

All granular materials have an angle of repose, which is as follows: Sand, 25° to 30°, gravel and broken stone, 30° to 40°; ashes, 25° to 30°; coal, 30° to 45°; grain 28°. The angle of repose is generally designated in formulas by ϕ.

The factor k fixes the ratio of lateral to vertical pressure and according to the theory of the ellipse of stress,

$$k = \frac{1 - \sin \phi}{1 + \sin \phi}.$$

It has been determined experimentally for several materials and has a value of 0.6 for grain.

The weight carried on the bottom of any bin is the total weight of the material, minus the weight carried by the walls. The foundations are designed under the walls by taking the total weight transferred to the walls, plus the weight of the walls, plus wind force. If columns and girders are used under the floor of the bin, part of the weight on the bottom is carried to the walls as reaction.

Materials have an angle of repose which is an angle of slope assumed by the surface of the material when piled. There is also an angle of friction, the tangent of which is known as the coefficient of friction. The coefficient of friction of the grains of material on each other is the factor k. The coefficient of friction of the material against a surface confining it is determined experimentally, and the factor C in Fig. 182 is the coefficient of friction of the various materials against concrete. In determining the weight carried by the walls it is necessary to consider k and C.

The weight carried by the wall on a strip one foot wide for any depth is, approximately,

$$wR\left(h - \frac{R}{kC}\right),$$

in which w = weight of material per cubic foot,

R = hydraulic radius = $\frac{\text{area}}{\text{perimeter}}$ in feet,

h = depth of bin in feet,

k = ratio of lateral to vertical pressure,

C = coefficient of friction.

The above expression is only approximate, as the entire expression is very complex. The result, however, is correct within such a small per cent that it is safe to use it. The load on a vertical strip one foot wide multiplied by the circumference in feet gives the total vertical weight carried by the walls, the remainder being carried by the bottom. The weight on the bottom is not uniform, being in the form of an ellipsoid, the bending moment for which will be $M = \frac{2}{3} + \frac{WD^2}{7}$ provided the attachment of the bottom to the sides is good.

The curves may be used for round or square tanks. In a round tank the pressure is that on a square foot on the circumference. For square tanks it is the pressure per square foot of perimeter. It may be used for rectangular tanks in which the length is not more than 1.5 times the breadth by dividing 4 times the area of the rectangular tank by the perimeter. This gives the diameter of an equivalent circular tank, or the side of an equivalent square tank, by means of which from Fig. 182 can be obtained the pressure to use with the dimensions of the rectangular tank.

Hoppered bottoms are used for bins as a rule but are somewhat expensive when made of concrete, on account of the formwork. A common practice for bins having tunnels underground is to make a flat bottom and pile cinders, or damp sand, on it with the surface sloping towards the discharge hole. The surface is then covered with concrete several inches thick, generally reinforced.

The pressure against a retaining wall, and the overturning moment, may be obtained by formula, using the constants for equivalent fluid pressure. That method, however, is good only for a wall retaining a fill level with the top of the wall. It is not applicable to a surcharged wall, that is, one holding a fill which extends above the top of and slopes to the wall. The graphical method shown in Fig. 182 is a development of the Coulomb theory of a "maximum wedge." According to this theory the fill will not slip forward until the surface is steeper than the natural angle of repose. When it starts to slip it breaks on a line approximately halfway between the angle of repose and the vertical, the wedge ahead of this line alone exerting an overturning pressure on the wall.

In Fig. 183 the line AE represents the surface slope at the angle of repose ϕ. The line EI is the surface of the fill, the angle *x* being the angle of surcharge. The length of the line *AE* is fixed by the intersection of the angle of repose drawn from the bottom of the wall and the angle of surcharge drawn from the top of the wall. From the middle point of the line *AE* a semicircle *ABDE* is drawn.

The angle *C* is the angle of friction of the filling against the back of the wall. The line *IH* is drawn at an angle with the back of the wall equal to the sum of the angle of repose and the angle of friction *C* to an intersection with the line *AE*.

From *A* as a center, with a radius = *AB*, describe an arc cutting *AE* at *F*. Draw *FJ* parallel with the line *IH*. With radius

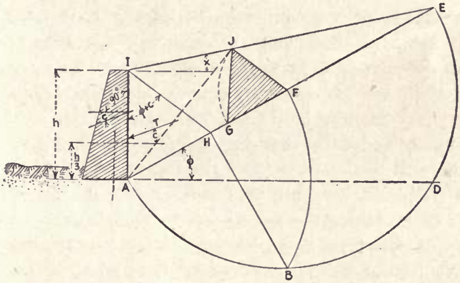


Fig. 183 — Graphical Method for obtaining Pressure against Retaining Wall

FJ describe an arc intersecting the line *AE* at *G* and draw the triangle *FJG*.

The line *AJ* is the cleavage line of the material and the area of the triangle *FJG* multiplied by the weight of a cubic foot of the material will give the pressure against a strip of the wall one foot long. This pressure is considered to be concentrated at a point above the base of the wall equal to one-third the height, the height being measured from the bottom of the wall, and not from the surface of the ground. The direction of this thrust *T* is not determined, authorities not agreeing. Some assert that it is parallel with the slope of the surcharge and some that it makes an angle, *C*, with the back, while others count it as a horizontal thrust. In the figure it is drawn parallel with the angle of friction. A larger moment is obtained by considering the thrust as horizontal and a wall designed to resist this horizontal thrust has a larger factor of safety than one designed to resist a thrust at an angle. The author designs retaining walls on the assumption of a horizontal thrust.

The resultant of the weight of the wall and the thrust must pass through the base at such a point as will keep the toe pressure within the allowable soil pressure per square foot. It is considered best to keep the resultant within the middle third of the wall. In the figure the thrust is shown as meeting a vertical line through the center of gravity at a point one-third the height above the base. This makes the thrust line strike the wall a trifle above the one-third point. With a horizontal thrust the application is exactly one-third above the base.

The diagram here given is independent of the shape of the wall. In fact a single line representing the back of the wall could have been drawn just as well. The diagram merely gives the amount of thrust, its direction and the point of application. A separate diagram may be drawn from the wall if the work is to be graphical, and only the thrust line from this figure will be required.

For a well-built concrete wall not reinforced the width of the base can be one-third the height for ordinary earth. For a brick or well-built cut stone wall with cement mortar joints the bottom width can be one-third the height + 1. For an ordinary stone or brick wall the thickness of the base should be at least one-half the height. Such empirical rules make it very easy to draw plans for walls. With reinforced concrete walls it is necessary to know the pressure and overturning moment so the wall may be designed to resist definite forces.

A reinforced concrete retaining wall is built in the shape of a capital *L*. The weight of the earth on the outstanding rear leg is counted as part of the weight of the wall, the back of the wall being vertical and extending upward from the rear end of the slab. The coefficient of friction is $k = \frac{1 - \sin \phi}{1 + \sin \phi}$, and the angle corresponding to this is used in the graphical analysis instead of the angle *C*.

The thrust acting at one-third the height tends to overturn the wall. The vertical front face of the wall must, therefore, be designed as a cantilever beam to resist this moment, anchoring into the base. The small toe in front is extended to widen the base and bring the toe pressure within the safe maximum pressure. This projecting toe is designed as a cantilever beam resisting the upward pressure. The slab in the rear is designed as a cantilever beam to carry the weight of the earth on it which forms a part of the wall.

Sometimes the wall has counterforts along the back at regular intervals, acting as ties. The spacing of these counterforts varies from an interval equal to the height of the wall for walls under fifteen feet in height to one-third the height for walls thirty feet in height, and proportionately for walls more than fifteen and less than thirty feet high. These counterforts have rods running in them from the top of the wall to the back edge of the bottom slab, to reinforce them as cantilever girders carrying the front slab. The front slab is designed as a slab with a span equal to the distance center to center of counterforts, and is reinforced horizontally.

The wall designed as a cantilever has vertical reinforcement in the front wall and the reinforcement in the bottom slab and toe is normal to the face of the wall. The counterforted wall has vertical and sloping steel in the counterforts, but the slab reinforcement is all parallel with the length of the wall.

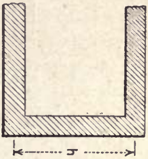


Fig. 184 — Restrained Wall

In Fig. 184 is shown, diagrammatically, a retaining wall tied at the top and bottom. This may occur in the case of an area wall pressing against a sidewalk at the top and against a heavy floor at the bottom. It may occur as a wall pressing against a foundation, or floor at the bottom, and having a long girder, or waling, along the top held by ties to deadmen.

The maximum bending moment is at a point = 0.58 h. It is then $M = 0.64ph^3$,

in which p = pressure in pounds per square foot of a fluid. For grain, $p = 42$; for stone, $p = 48$; for bituminous coal, $p = 24$; for anthracite coal, $p = 36$; for ashes, $p = 16$; for sand, $p = 48$; for earth, $p = 32$ (average).

The maximum bending moment caused by water is

$$M = 4h^3.$$

The fluid weights are not actual weights but merely represent the weight that must be possessed by a fluid which would exert the same pressure against a wall as the material to which it corresponds. The formulas for fluid pressure are more simple than those in which a number of factors must be used, so the material is assumed to act as a fluid having a weight per cubic foot very much less than the actual weight of the material.

In Fig. 185 three problems in the design of lintels over openings are shown. At A the lower opening is spanned by a lintel which carries a load indicated by the shaded triangle, which is equilateral. The reason this triangular load is carried is that the brick work bond will have strength enough to assume a form resembling an arch. The bending moment due to a triangular load is

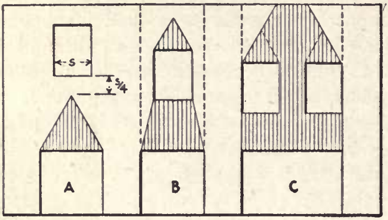


Fig. 185 — Lintels over Openings

$$M = \frac{wL^2}{6}.$$

The point of the arch over the lower opening is below the bottom of the upper opening a depth equal to, or greater than, one-fourth the span of the upper opening. In the upper opening there is a lintel to carry the coping wall. A sixty degree line drawn from each upper corner will intersect the coping wall; therefore the lintel must be figured to carry all the load above it, within the shaded area.

At B is shown another case. The sides of an equilateral triangle will intersect the bottom of the upper opening so it is common to assume the sloping lines at the side to connect the corners of the openings as shown. The triangle over the top of the upper opening is more than one-fourth the span below the top of the wall, so as some arch action can take place the lintel is assumed to carry only the triangular portion of the wall. The author would not so design the two lintels. The lintel over the bottom opening would be designed to carry all the load between the dotted vertical lines.

At C a similar condition is found. The lintel over the lowest opening should be designed to carry all the load between two vertical lines extended from the upper corners to the top of the wall. It is not safe to assume the sloping form of the broken wall opening unless there is a bridging across sufficient to form an arch. That is why the arch is not assumed to act unless the wall across the break has depth enough to insure it acting as a beam to span the opening.

In the three cases, therefore, the lower lintels must be designed to carry the area of wall shown between the dotted lines, and the lintels for the upper openings are designed to carry the small shaded triangular areas at B and C and the rectangular area shown at the top at A.

A few things pertaining to the design of buildings have not been discussed because they are to be found in the steel and lumber handbooks, without which no designer can work. With the assistance of those books the student should have no difficulty in handling all the ordinary problems arising in the design of structures. It is hoped that the application of the MOMENT has been treated so consistently throughout this book that the student will have no difficulty in analyzing any problem that may come up in his work.

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