SYSTEM

OF

Practical Arithmetick,

FOR

THE USE OF SCHOOLS,

BY THE REV. J. JOYCE.

ADAPTED TO THE

COMMERCE OF THE UNITED STATES,

BY J. WALKER.

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DISTRICT OF MARYLAND, ss.

BE IT REMEMBERED, That on the Twenty-First day of June, in the Forty-third year of the Independence of the United States of America, N. G. Maxwell of the said District hath deposited in this office the title of a Book, the right whereof he claims as Proprietor in the words following to wit:

"A System of Practical Arithmetick, for the Use of Schools, by the "Rev. J. Joyce. Adapted to the Commerce of the United States, by "J. Walker."

In conformity to an act of the Congress of the United States, entitled, "An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies during the times therein mentioned." And also to the act, entitled, "An act supplementary to an act, entitled, "An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies during the times therein mentioned," and extending the benefits thereof to the arts of designing, engraving, and etching historical and other prints."

PHILIP MOORE,
Clerk of the District of Maryland.

PREFACE TO THE ENGLISH EDITION.

IN presenting a new System of Arithmetick to the publick, some account of its plan and execution will be expected. It is hoped, that the title of the present Work will briefly explain the views of the Author, who, from his own experience, in the business of education, has long since been convinced, that, among the excellent introductory books to this useful science, no one, that he has met with, is sufficiently adapted to the occasions of common life: some are too abstruse for novices, while others are defective in such examples, as point out the application of the several rules to transactions of real business.

If the Author of this System of Arithmetick has not deceived himself, he has completely supplied these deficiencies, and he appeals without apprehension to that publick, whose candour and liberality he has already and often experienced, to decide upon this attempt to render the elementary rules of arithmetick practical and popular.

There are few children who do not experience some disgust in passing through the first four rules; occasioned, without doubt, by the paucity of examples, and by the want of interest in those that are given. The Author has therefore filled a large portion of his work with the early rules, and has illustrated them by miscellaneous questions, in which will be found much useful information, applicable in the advancing stages of life.

The modes of treating the Rule of Three, of illustrating Vulgar and Decimal Fractions, Practice, &c. &c., will best speak for themselves. But a reason may be demanded for the introduction of Logarithms, and for the particular method adopted in those parts in which the doctrine of Annuities, Reversions, Leases, &c. is illustrated.

ARITHMETICK.

ARITHMETICK *is the science which explains the various methods of computing by numbers.*

All its operations are performed by Addition, Subtraction, Multiplication *and* Division.

OF NUMERATION OR NOTATION.

When two or more figures are placed together, the first or right hand figure is taken for its simple value: the second to the left signifies so many tens: the third so many hundreds; and the fourth so many thousands; and so on, according to the following Table:

Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
5	4	3	1	2	6	9	7	8

Thus figures, besides their common value, have one which depends upon the place in which they stand when joined to others; 6 and 5 are read six and five; but if they stand together, 65, they are read sixty-five. The figure 5 on the right-hand denotes its simple value only. but the 6, from its situation, becomes ten times greater than its simple value, or sixty, therefore the two together are called sixty-five.

If there be three figures, as 978, the first figure to the right-hand denotes its simple value, as eight; the second a value ten times greater than its simple value, as seventy; and the third is a hundred times greater than its simple value, as nine hundred: the figures together are read nine hundred and seventy-eight.

In this manner, the value of each figure to the left is always ten times greater than it would be if it stood in the next place on the right; thus 6666, the first figure 6 is simply six, the next is sixty, and the third six hundred, and the fourth six thousand; the whole number is read, Six thousand six hundred and sixty-six.

The first six figures in the table, are read, One hundred twenty-six thousand, nine hundred and seventy-eight. The whole period of nine figures is thus read, Five hundred and forty-three millions, one hundred and twenty-six thousand, nine hundred and seventy-eight.

The enumeration of figures may be carried much further, according to the following Table:

Hundred thousands of billions.	Ten thousands of billions.	Thousands of billions.	Hundreds of billions.	Tens of billions.	Billions.	Hundred thousands of millions.	Ten thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
1	2	3	4	5	6	4	8	7	9	5	1	4	6	2	7	5	3

In large numbers it is common to divide them into periods of six figures each, and half periods of three figures. The foregoing three periods are read—One hundred twenty-three thousand, four hundred and fifty-six billions, four hundred eighty-seven thousand, nine hun-

dred and fifty-one millions, four hundred sixty-two thousand, seven hundred and fifty-three.*

Hence the following

RULE. *To the simple value of each figure, join the the name of its place according to the situation in the series, as hundreds, thousands, millions, billions, trillions, &c.*

EXAMPLES IN NUMERATION AND NOTATION.

Read, or write down in words, the value of the following Numbers:

Ex. 1. 19	Ex. 11. 40005	Ex. 21. 340
2. 244	12. 324060	22. 436901
3. 3045	13. 400569	23. 36945
4. 45060	14. 765	24. 9874000
5. 69305	15. 564001	25. 654328
6. 93614	16. 439762	26. 4328764
7. 564875	17. 9300044	27. 856540
8. 4500342	18. 70000021	28. 43760000
9. 5687041	19. 35000	29. 37004
10. 6843700	20. 50000000	30. 85000341

NOTE.

* *The names of the higher periods after Billions, are Trillions. Quadrillions, Quintillions, Sextillions, Septillions, Octillions, and Nonilions, each period consisting of six places of figures. The first three of every period are so many Units of it, and the latter or left hand part, so many Thousands.*—The following Table contains the whole series:

TABLE.

Nonillions,	Octillions,	Septillions,	Sextillions,	Quintillions
123,456	456,789	567,345	321 234	458,764
Quadrillions,	Trillions,	Billions,	Millions,	Units.
674,321	374,532	459,876	532,761	459,579

Ex. 31. 356074328
32. 5900007643
33. 686 0749004
34. 876430786458
35. 1000000845218
36. 34876543218764
37. 594632171834765
38. 876432851764875 89
39. 123456789001259
40. 987654321123456789

Write down the figures answering to the following Examples.

Ex. 1. Thirty-nine.
2. Four hundred and sixty-nine.
3. Two thousand and one.
4. Thirty-five thousand and twenty-eight.
5. Three hundred and seventy-six thousand.
6. One million and fifty-nine.
7. Eighty-seven millions, five hundred and eighty thousand, one hundred and nine.
8. Five hundred seventy-six millions, three hundred twenty-five thousand, three hundred and ninety-one.
9. Eight hundred millions and eigthy.
10. Three hundred and three-millions and thirty-one.

MISCELLANEOUS EXAMPLES.

Ex. 1. By a late enumeration of the people, the number of inhabitants in England is put down at nine millions, three hundred forty-three thousand, five hundred and seventy-eight; and the number found to be in London was eight hundred eighty-five thousand, five hundred and eighty-seven;—How are these numbers expressed in figures.

Ex. 2. The world was created two thousand three hundred and forty-eight years before the Deluge; three thousand two hundred and fifty-one years before the

building of Rome; four thousand and four years before the birth of Christ, and five thousand and fourteen years before the present time [1811]:—Let each of these numbers be expressed in figures.

Ex. 3. Express in words the distances of the primary planets from the Sun, which are as follow:

Mercury - - -	37,000,000	Venus - - -	66,000,000
The Earth - -	95,000,000	Mars - - -	145,000,000
Jupiter - -	493,000,000	Saturn - -	903,000,000
The Herschel -	1,813,000,000 miles.*		

FRACTIONS, or broken numbers, are expressed in the following manner:—A halfpenny is denoted by $\frac{1}{2}$; a farthing, by $\frac{1}{4}$, being the one-fourth of a penny; and three farthings by $\frac{3}{4}$, being three-fourths of a penny. Thus it appears that a fraction is any part or parts of a unit, and is expressed by two numbers separated from each other by a short line. The lower number shows how many parts the unit is divided into, and the upper figure points

---

NOTE.

* The ancient Romans, in their Notation of Numbers, made use of the following five letters: I, V, X, L, and C, which singly stood for one, five, ten, fifty, and a hundred. By repeating and combining these, any other numbers were expressed: thus II, signified *two*; III, *three*; XX, *twenty*; CC, *two hundred*, and so on. The rules for Roman Notation are as follow:

1. The *annexing* a letter of a lower value to one of a higher, increases its value, or denotes the sum of both, as VI, signifies six; XII, denotes twelve; LV, fifty-five; LXXVI, seventy-six; CLII, one hundred and fifty-two.

2. The *prefixing* a letter of a lower value, to one of a higher, shows that the value of the less is to be taken from the greater, or shows their difference: thus, I prefixed to V, or IV, is four; IX, nine; XL, forty; XC, ninety, &c.

For the sake of abbreviation, the Romans introduced these marks:—Iↀ, five hundred: CIↀ, a thousand, these, in process of time, were written D, M, so that now the D signifies five hundred, and the M, a thousand; but in the titles of many old books we find the other mode of Notation. The following table will exhibit every thing necessary to be known on this subject:

out what number of these parts are contained in the fraction: thus $\frac{3}{4}$, when standing for three farthings, shows that a penny is divided into four parts, the 3 determines the number of those parts, and we call it three-fourths of a penny.

TABLE.

I	1	LX	60
II	2	LXX	70
III	3	LXXX	80
IV, or IIII	4	XC	90
V	5	C	100
VI	6	CI	101
VII	7	CCC	300
VIII	8	Iↀ, or D	500
IX	9	IↀC, or DC	600
X	10	IↀCCC, or DCCC	800
XI	11	IↀCCCC, or DCCCC, or CM	900
XII	12	CIↀ, or M	1000
XIII	13	CIↀC, or MC	1100
XIV	14	MM, II*	2000
XV	15	Iↀↀ †, or $\overline{\text{V}}$	5000
XVI	16	IↀↀM or, $\overline{\text{VI}}$	6000
XVII	17	IↀↀMMM, or $\overline{\text{VIII}}$	8000
XVIII	18	CCIↀↀ ‡, or $\overline{\text{X}}$	10000
XIX	19	CCIↀↀM, or $\overline{\text{XI}}$	11000
XX	20	Iↀↀↀ	50000
XXI	21	IↀↀↀMM	52000
XXX	30	CCCIↀↀↀM	101000
XL	40	CIↀIↀCCC,XI,or M,DCCC,XI	1811
XLI	41		
L	50		

* The word *thousand* is often expressed by a line drawn over the top of a number: thus $\overline{\text{X}}$ signifies ten thousand and $\overline{\text{M}}$ a thousand thousands.

† The annexing ↀ to the number Iↀ, increases its value ten times; thus Iↀↀ is 5000, and Iↀↀↀ is fifty thousand.

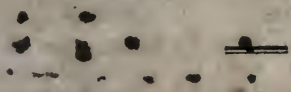
‡ The prefixing C, and at the same time annexing a ↀ to the number CIC, makes its value ten times greater; CCIↀↀ is 10,000, and CCCIↀↀↀ is 100,000.

Inches are usually divided in eighths, or eight parts, in each inch; and the fractional parts are thus expressed:

$\frac{3}{8}$ means three-eights. $\frac{5}{8}$ means five eights.
$\frac{7}{8}$ means seven-eights. $\frac{4}{8}$ means four-eights, equal to one half.

Sixteenths are likewise in common use, and we say,

$\frac{5}{16}$ five sixteenth. $\frac{11}{16}$ eleven sixteenths.
$\frac{3}{16}$ three sixteenths $\frac{15}{16}$ fifteen sixteenths.



ADDITION.

ADDITION teaches the method of finding the sum or total of several numbers.

RULE (1.) *Place the numbers under one another, so that units may stand under units, tens under tens, &c.*

(2.) *Add up the figures in the row of units: set down what remains above the even tens, or if nothing remains, a cypher, and for the tens carry as many ones to the next column.*

(3.) *Add up the other rows in the same manner, and in the last column put down the whole sum contained in it.*

Ex. 1. What is the sum of 3684, 4863, 365, 29, 56874, and 609?

3684
4863
365
29
56874
609

Answer - - - 66424 is the sum total.

PROOF. Add the numbers together in a contrary order beginning at the top instead of the bottom.

EXAMPLES.

345	8776	78329
489	6734	87293
204	5709	34650
695	9564	59417
731	3218	21004
27	4507	12345
2491	38508	293068

Ex. 1. 1234	Ex. 2. 5432	Ex. 3. 3314
3102	3241	3415
2231	2343	2510
4322	1232	3423
3413	4113	4152
2342	2000	3241
1122	3111	2324
3111	2322	4231
2322	5555	5254

Ex. 4. 4321	Ex. 5. 6543	Ex. 6. 1234
6125	2123	5654
3246	4565	3210
4350	4321	1353
5432	2345	2464
6312	6666	3210
3424	5432	4633
4301	1010	5544

Ex. 7. 7654	Ex. 8. 1357	Ex. 9. 7777
3212	2464	4343
3456	2013	6424
7654	5765	3767
3210	4324	5106
1357	1067	2007
6420	2132	7213
5234	4126	6644

Ex. 10.	Ex. 11.	Ex. 12.
1234	2345	9898
5678	6789	7676
9876	9988	4317
5432	7766	2603
1357	5544	4762
9864	3322	9437
2024	2200	6458
6809	7773	8764
8765	6499	9538
4321	3741	6749

Ex. 13.	Ex. 14.	Ex. 15.
5162	7640	49325
4876	39	24609
4008	5784	37485
3079	4304	16004
1234	9865	23348
2341	6543	32946
3468	2871	329

Ex. 16.	Ex. 17.	Ex. 18.
5432	6905	49603
5789	324	50792
1234	24	4652
5678	9	49859
9123	5068	634
4009	4981	78432
5746	5139	29764

Ex. 19.	Ex. 20.	Ex. 21.
67543	93217	8542
89678	76213	39764
56789	34567	78912
22345	89002	34567
67890	45678	91874
12932	345	43604
45764	67890	51871
85365	45632	20302

Ex. 22.	Ex. 23.	Ex. 24.
12345	12349	99887
54321	56789	44556
67854	48672	17280
58108	24	59776
49328	51403	43509
98765	46795	49312
43200	31274	56418
87219	45670	43004

Ex. 25.	Ex. 26.	Ex. 27.
764329	527648	397648
597643	476239	473465
249764	765473	247396
354673	629728	478943
576894	437649	862759
357649	276354	386475
476392	762938	928764
734629	476849	387649
562793	327649	258763

Ex. 28.	Ex. 29.	Ex. 30.
476293	587649	537649
547689	326753	764932
356743	473649	476843
827649	567326	324768
536754	478943	976439
673649	674859	267568
567937	386745	374689
645764	473659	567834
786492	768492	745687

Ex. 31.		Ex. 32.	
	527638		432999
	4 6027		763427
	538674		632942
	764327		763487
	487634		629764
	927865		394276
	732486		839467
	474288		364237
	367495		648276

MISCELLANEOUS EXAMPLES IN ADDITION.

Ex. 1. Add together the following sums; 98764, 397652, 876, 459321, 21, 80, and 76942,

Ex 2. Add 39764, 47652, 34291, 225, 48, 764871, and 10000 together.

Ex. 3. What is the sum of thirty-five thousand and four; five hundred and forty thousand, three hundred and nine; four hundred and twenty-seven; fifty thousand nine hundred and eighty; two millions and five; and seven hundred and seventy-seven?

Ex. 4. When will a child, born in 1806, be forty-nine years old?

Ex. 5. How many days are there in the first eight months of the year, when it is not leap year?

Ex. 6. How old is the world this year, 1808, supposing it was created 4004 years before the birth of Christ?

Ex. 7. A person at his death left 3287*l.* to his widow: to his eldest son he bequeathed 5250*l* and to each of five other children, he left a thousand pounds less than to the eldest son: he left also to a nephew 105*l.*, and the same sum to be divided among four distant relations: How much money did he leave behind him?

Ex. 8. The lease of my house was granted me in the year 1793, for ninety-nine years; when will it expire?

Ex. 9. How many days will there be between January the first and November the 20th, 1808, being leap year, both days inclusive?

Ex. 10. What do the following sums amount to, 1268 +8612+10018+275+919+8+550099 ?

Ex. 11. How many chapters are there in the several books of the New Testament ?

Ex. 12. How many chapters are there in the several books of the Old Testament ?

Ex. 13. How many chapters are there in the Bible, which consists of the Old and New Testaments ?

Ex. 14. In travelling from London to Bath in a post-chaise, for how many miles shall I have to pay ? The distance from London to Hounslow is 10 miles, from Hounslow to Maidenhead is 16 miles, from Maidenhead to Reading 13 miles, from Reading to Spleenhamland 16 miles, from Spleenhamland to Marlborough is 19 miles, from Marlborough to Chippenham is 19 miles, and from Chippenham to Bath is 13 miles.

Ex. 15. How far is it from London to Harwick ? To Romford are 11 miles, from thence to Ingatestone 12 miles, from Ingatestone to Chelmsford 6 miles, from Chelmsford to Colchester are 21 miles, and from Colchester to Harwick 20 miles.

Ex. 16. In travelling post to Margate I pay a shilling a mile : How many shillings shall I have paid at the end of the journey ? The distance from London to Dartford is 15 miles, from thence to Rochester is 14 miles, from Rochester to Sittingbourne is 11 miles, from Sittingbourne to Canterbury is 15 miles, and from Canterbury to Margate is 17 miles.

SUBTRACTION.

By Subtraction *we find the difference between two numbers.*

Rule (1.) *Place the lesser number under the greater, so that units may stand under units, tens under tens, &c.; begin at the right hand, and take each figure in the lower line from the figure above it, and set down the remainder.*

(2.) *If the figure in the lower line be the greater, add ten to the upper one, and then take the lower one from the sum, set down the remainder and carry one to the next lower figure, with which proceed as before.*

(3.) *When the figure in the lower line is equal to that above it, the difference is nothing, for which a cypher must be set down.*

EXAMPLES.

From - - -	874698	765087	762134
Take - - -	561436	425436	597082
Remainder	313262	339651	165052

Proof. Add the remainder to the last line, and if the sum be equal to the first, the work is right.

From - - -	658742	390076	431267
Take - - -	346121	184193	280795
Remainder	312621	205883	150472
Proof - -	658742	390076	431267

EXAMPLES FOR PRACTICE.

Ex. 1.	4867434	2. 6789491	3. 5876486	4. 3390761
	2534213	5468354	3564214	1478490
Ex. 5.	7052673	6. 9276807	7. 7231607	8. 9104008
	3860749	4859434	5987465	9031618
Ex. 9.	6734078	10. 5201832	11. 6000342	12. 1000000
	5943769	4676543	5999343	999999
Ex. 13.	4002103	14. 3874205	15. 9000123	16. 5301864
	3987654	1796432	8123456	99
Ex. 17.	7962038	18. 91111118	19. 4681035	20. 8302697
	6498100	80000009	93006	2912934
Ex. 21.	60001234	22. 71216003	23. 30061217	24. 26013032
	49993490	59876543	19996642	19125346

Ex. 25. 98743205 — 9999999

Ex. 26. 50237480 — 41926321

Ex. 27. 49764321 — 15875492

Ex. 28. 93816086 — 927908

Ex. 29. 94286730 — 32199739

Ex. 30. 92370800 — 4812719

Ex. 31. 42601304 — 22500894

Ex. 32. 27000019 — 4102094

Ex. 33. 76253922 — 344939

Ex. 34. 33861400 — 23713509

Ex. 35. 94681039 — 3041316

Ex. 36. 6901090 — 1860018

Ex. 37. 591040029 — 490300019

Ex. 38. 271216904 — 28391767

Ex. 39. 97348098 — 9290412

Ex. 40. 974689019 — 31689247

Ex. 41. 593902742 — 312003717

Ex. 42. 913062188 — 44823165

Ex. 43. 797260839 — 62310079

Ex. 44. 170909009 — 24710905

Ex. 45. 99326104 — 21281299

Ex. 46.	19390909	47. 39921090	48. 1115677333
	2109109	1937099	38103475

MISCELLANEOUS EXAMPLES IN SUBTRACTION.

Ex. 1. The invention of gunpowder was discovered in the year 1302: How long is it since to the present year, 1811?

2. What is the difference between thirty-five thousand three hundred and nine, and nine thousand and ninety-nine.

3. How much does seven hundred six thousand and four, exceed fourteen thousand nine hundred and thirty-seven?

4. How much does fifteen thousand and five want of twenty-three thousand?

5. The art of printing was discovered in the year one thousand four hundred forty-nine. How long is it since 1808?

6. Coaches were first used in England in the year 1580: How many years is it to 1808?

6. Needle making was introduced into England from India in the year 1545: How many years was that before the present king came to his throne, which was in 1760.

8. Required the answers of the three following sums; 18045—999; 2059—928; and 258764—49876.

10. How many more chapters are there in the Old Testament than in the New?

MULTIPLICATION.

MULTIPLICATION is a short method of Addition, and it teaches us to find what a number will amount to, when it is repeated a certain number of times.

RULE. *The number to be multiplied is called the Multiplicand: and the number multiplied is called the Multiplier. The number found is called the Product.*

MULTIPLICATION TABLE.

2 times or twice	3 times	4 times	5 times	6 times	7 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 4	2 6	2 8	2 10	2 12	2 14
3 6	3 9	3 12	3 15	3 18	3 21
4 8	4 12	4 16	4 20	4 24	4 28
5 10	5 15	5 20	5 25	5 30	5 35
6 12	6 18	6 24	6 30	6 36	6 42
7 14	7 21	7 28	7 35	7 42	7 49
8 16	8 24	8 32	8 40	8 48	8 56
9 18	9 27	9 36	9 45	9 54	9 63
10 20	10 30	10 40	10 50	10 60	10 70
11 22	11 33	11 44	11 55	11 66	11 77
12 24	12 36	12 48	12 60	12 72	12 84

8 times	9 times	10 times	11 times	12 times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 16	2 18	2 20	2 22	2 24
3 24	3 27	3 30	3 33	3 36
4 32	4 36	4 40	4 44	4 48
5 40	5 45	5 50	5 55	5 60
6 48	6 54	6 60	6 66	6 72
7 56	7 63	7 70	7 77	7 84
8 64	8 72	8 80	8 88	8 96
9 72	9 81	9 90	9 99	9 108
10 80	10 90	10 100	10 110	10 120
11 88	11 99	11 110	11 121	11 132
12 96	12 108	12 120	12 132	12 144

1. When the Multiplier does not exceed 12.

RULE. *Multiply every figure in the multiplicand from right to left, consider how many tens there are in each product, the remaining units set down under the figure multiplied, and carry the tens as so many ones to the next product. The last product is to be wholly set down.*

EXAMPLES.

Ex. 1. 420847	Ex. 2. 94564875	Ex. 3. 3476819
8	5	12
3366776	472824375	41721828

Thus in the first example, I say 8 times 7 are 56, in which there are five tens and six over, I put down the six, and say 8 times 4 are 32, adding the 5 from the last product, I have 37; I put down the 7, and carry the 3 for the three tens; I then say 8 times 8 are 64, and 3 are 67, 7 and carry 6; 8 times 0 is 0, but put down the 6 brought from the last product; 8 times 2 are 16, put down the 6, and then 8 times 4 are 32, and the one brought forward are 33, which as being the last product, must be set down.

EXAMPLES FOR PRACTICE.

Ex. 1. 4653245	Ex. 2. 8756894	Ex. 3. 4986587
2	3	4
Ex. 4. 3390763	Ex. 5. 7052673	Ex. 6. 9276807
5	6	7
Ex. 7. 7231607	Ex. 8. 9134908	Ex. 9. 6734078
8	9	10

Ex. 10. 5201832 × 11

Ex. 11. 6393476 × 12

Ex. 12. 3874025 × 11

Ex. 13. 83022697 × 12

Ex. 14. 5391864 × 11

Ex. 15. 4681953 × 12

Ex. 16. 98743205 × 9

Ex. 17. 50947496 × 8

Ex. 18. 49764329 × 7

Ex. 19. 5972834 × 5

Ex. 20. 5097648 × 6

Ex. 21. 5875496 × 4

Ex. 22. 5439027 × 7

Ex. 23. 9999999 × 8

Ex. 24. 8888888 × 7

Ex. 25. 9734895 × 9

Ex. 26. 9237085 × 8

Ex. 27. 5942867 × 9

This character $\times$, which is called St. Andrew's cross, is used to denote Multiplication, and when it stands between two numbers, it signifies that those numbers are to be multiplied into one another: thus $9 \times 6 = 54$, is read, nine multipl ed by six is equal to fifty-four. Again $12 \times 11 = 132$, that is 12 multiplied by 11 is equal to 132.

EXAMPLES.

Ex. 1.	528318769 × 5	Ex. 2.	956728314 × 8
Ex. 3.	825934685 × 7	Ex. 4.	486875294 × 9
Ex. 5.	496745832 × 9	Ex. 6.	683637544 × 8
Ex. 7.	578940245 × 2	Ex. 8.	759654318 × 11
Ex. 9.	987234617 × 6	Ex. 10.	867122456 × 12
Ex. 11.	716432978 × 9	Ex. 12.	687649321 × 7
Ex. 13.	795483206 × 11	Ex. 14.	779368245 × 9
Ex. 15.	91872648 × 12	Ex. 16.	986049005 × 5
Ex. 17.	85678654 × 4	Ex. 18.	390057864 × 6
Ex. 19.	894367542 × 8	Ex. 20.	765438958 × 4

II. To multiply by 10, add an 0 to the multiplicand: thus 567 × 10 is 5670; and 567 × 100 is 56700; and 6489 × 10000 = 64890000. Therefore, to multiply a given number of one denomination, by a number whose significant figures do not exceed 12, having a cypher or cyphers joined to it:

RULE. Write down the cypher or cyphers for the first part of the product towards the right hand, and then multiply every figure in the multiplicand by the significant figures of the multiplier, as in the preceding case.

Thus, 3469456 × 50 = 173472800, and 98765432 × 8000 = 790123456000, for

3469456	98765432
50	8000
173472800	790123456000

EXAMPLES.

Ex. 1.	6754328 × 70	Ex. 2.	987654329 × 800
Ex. 3.	8329674 × 110	Ex. 4.	56780943 × 120
Ex. 5.	6470078 × 9000	Ex. 6.	9237654 × 1100
Ex. 7.	7856423 × 1000	Ex. 8.	7490434 × 600

III. When the multiplier consists of several figures.

RULE. The multiplicand must be multiplied by each figure of the multiplier separately beginning with the right hand figure, and the first figure of every product must stand exactly under the figure multiplied by. Add these products together for the whole product.

To multiply by any number between 13 and 19 in one line.

RULE. Multiply the unit's figure of the multiplicand, by the right-hand digit of the multiplier; set down the unit's figure of the product, and remember what is to be carried. Multiply the *second* figure of the multiplicand; to the product, add what was to be carried, and also the first figure of the multiplicand. Then set down the unit's figure, and retain in your mind the number to be carried, as before. Multiply the third figure of the multiplicand: add the number to be carried, and also the second figure of the multiplicand, and so on; thus

74365487596
17

1264213 89132

Here I say 7 times 6 are 42; I put down the 2 and carry 4, and say 7 times 9 are 63, and 4 are 67, then add the 6, which makes 73; put down the 3, and say 7 times 5 are 35, and 7 are 42, to which add the 9, which make 51, put down 1 and carry 5, and so on, till the last figure, when I say 7 times 7 are 49, and 3 to be carried are 52, take in the 4, which make 56, put down 6, and add 7 to the 5, and set down 12.

To multiply by 21, 31, 41, &c. to 91 in one line.

RULE. Bring down the unit's figure of the multiplicand for the unit's figure of the product; multiply the same figure by the left hand digit of the multiplier, to which add the next figure on the left hand of the multiplicand, set down the unit's figure and carry the tens,

multiply the next figure of the multiplicand by the same multiplier, and so on, always observing to add the number you carry and also the first figure on the left hand of that which you multiply.

EXAMPLE.

3760942
21

78979782

Bring down the 2 then say twice 2 are 4, and 4 are 8, put down 8 and say twice 4 are 8, and 9 are 17, put down 7, and carry 1; then say twice 9 are 18 and 1 are 19, put down 9 and carry 1; next twice 0, will be 0, but the 1 you carried, and 6 make 7, put down 7; twice 6 are 12 and 7 are 19, put down 9 and carry 1; then say twice 7 are 14 and 1 are 15 and 3 are 18; put down 8 and carry 1, and lastly twice 3 are 6, and 1 are 7.

EXAMPLES.

57864329	35964827
579	846
520778961	215788962
405050303	143859308
289321645	287718616
33503446491	30426243642

PROOF. The readiest way of proving the truth of sums in Multiplication is, by *casting out the nines.*

RULE. Make a cross like that which is used to denote Multiplication: add together the figures in the multiplicand, casting out all the nines in the sum as often as they amount to 9, and put the remainder down on one side of the cross; do the same with the multiplier,

and put down the remainder on the other side of the cross Multiply the two remainders together, and casting out the nines of their product, will leave the same remainder as the nines cast out of the answer, when the work is right.

EXAMPLES.

4593267	0	7628954	1
568	0×1	857	5×2
	0		1
36746136		53402678	
27559602		38144770	
22966335		61031632	
2608975656		6538018578	

To prove the second example, I say 7 and 6 are 13; 4 above nine, (omit the 9): 4 and 2 are 6 and 8 are 14; 5 above nine, (omit the 9): 5 and 5 are 10, 1 above 9, 1 and 4 are 5: I place the 5 on the left hand of the cross, and say 8 and 5 are 13, 4 above 9; 4 and 7 are 11, 2 above 9; the 2 I put on the right hand of the cross. Now 5×2 gives 10, which is 1 above 9, I put the 1 at the top of the cross, and then cast out the 9's of the whole product, and I find the remainder is 1, which answering to the 1 at the top of the cross, leads me to conclude that the operation is right.

IV. When cyphers are intermixed with the figures in the multiplier.

RULE. Omit the cyphers, and let the first figure of each product be placed under its multiplier.

EXAMPLES.

Ex. 1. 7650329 — 690509 — 5 × 2 — 1 — 6885 961 — 38251645 — 45901974 — 4594091417461

Ex. 2. 4465348 — 7000608 — 3 — 7 × 6 — 3 — 357 32784 — 26792088 — 31257436 — 312601509315 84

Ex. 3 849275 × 706
Ex. 4. 973648 × 8005
Ex. 5. 597384 × 830004
Ex. 6. 364759 × 2709
Ex. 7. 245918 × 703006
Ex. 8. 609483 × 95007

V. When the multiplier is the product of two or more numbers in the table.

Rule. Multiply the multiplicand by one of the component parts, and that product by the other, and so on: thus if I have to multiply a given sum by 64, I find 8 × 8 = 64; instead, therefore, of multiplying by 6 and 4 in the usual way, I multiply first by 8, and then that product by 8 again.

EXAMPLES.

864392 × 64 — 8 — 5 — 6915136 — 5 × 1 — 8 — 5 — 55321088

39746285 × 168 — 7 — 3 — 278223995 — 8 × 6 — 6 — 8 — 1669343970 — 4 — 6677375880

EXAMPLES IN ALL THE CASES.

Ex. 1.	99365497	× 13
2.	54962874	× 26
3.	35729876	× 56
4.	47893062	× 48
5.	73167482	× 77
6.	8274386	× 96
7.	39745371	× 86
8.	5487962	× 357
9.	72983456	× 99
10.	3891307	× 464
11.	737394	× 4567
12.	35846	× 4682
13.	329357	× 2839
14.	58427	× 3957
15.	462875	× 6874
16.	47683	× 3456
17.	594326	× 5936
18.	87493	× 7892
19.	486752	× 4608
20.	29687	× 3579
21.	8739690279	× 397829
22.	7936820056	× 500634
23.	2576482874	× 613487
24.	9167403258	× 653000
25.	872694325	× 2900008
26	715976032	× 350706
27.	526730469	× 590734
28.	37945687	× 999999
29.	74714328	× 345627
30.	46382719	× 50000092

MISCELLANEOUS EXAMPLES.

Ex. 1. Multiply three millions thirty-nine thousand and three, by thirty-five thousand and twenty-eight.

2. Multiply six billions, six hundred thousand and sixty-five, by eight thousand and thirty-nine.

3. There are eleven hundred hackney coaches in London; suppose, on the average, each coach earns thir-

teen shillings a day, how many shillings will be expended in the hire of these carriages in a year of 365 days, Sundays being excepted ?

4. In Jamaica only there were imported, annually, not less than ten thousand eight hundred negroes from the coast of Africa: How many slaves had free-born Englishmen made in that island, between the year 1799 and the year 1807, in which the infamous traffick was abolished.

5. A boy can point sixteen thousand pins in an hour; How many will he do in six days, supposing he works eleven clear hours in a day? *See Blair's Universal Preceptor.*

6. What is the continual product of 25, 19, 705, and 999 ?

7. How many changes can be rung on twelve bells ?

8. Multiply the difference between 50487 and 30056, by the sum of 850, 9067, and 800 ?

9. The sum of two numbers is 30355, and the greater number is 25251; What is their product ?

10 The sum of two numbers is 4584, and the less is 1876; What is their product ?

11. What is the difference between twelve times fifty-seven, and twelve times seven and fifty ?

12. How many miles will a person walk in sixty-six years, supposing he travels, one day with another, six miles, and there are 365 days in a year ?

13. How many cubic feet does this room contain, which is fifteen feet long, fourteen feet wide, and thirteen feet high ?

DIVISION.

By Division, we find how often one number is contained in another of the same denomination; this is a short method of performing Subtraction.

The sum to be divided is called the *dividend ;* the figure, or figures by which we divide, is called the *divisor ;* and the result is called the *quotient.*

In this Rule, as in Multiplication, there are several distinct cases.

I. When the divisor does not exceed 12.

RULE. Write the divisor on the left hand side of the dividend, make a curve, and consider how often the divisor is contained in the first figure, or in the first two or three figures, and set the quotient under it ; and for every unit remaining after subtraction, carry TEN to the next figure of the dividend.

EXAMPLES.

Ex. 1. 4)78654328

19663582

Ex. 2. 9)85674327

9519369—6

Ex. 3. 11)10876541

988776—5

Ex. 4. 12)11272459

939371—7

In the second example, I say there are 9 nines in 85 and 4 over ; I put down the nine and carry the 4, as 40 to the 6, and the 9's in the 46, 5 times and 1 over ; put down the 5 and carry 1, as 10, and say the 9's in 17, once and 8 over ; put down the 1 and carry 8 as 80 ; 9's in 83, 9 times and two over, and so on : at the last figure there are 6 remaining, put down this beyond a small line.

It is usual, in giving the answer, to make a short line under the remainder, and place under it the divisor : thus the answer to the second sum is $9519369\frac{6}{9}$: that of the third sum is $988776\frac{5}{11}$, and that of the fourth $939371\ \frac{7}{12}$; and the three remainders are fractions, which we read six-ninths, five-elevenths, and seven-twelfths. See p. 4. and 5.

EXAMPLES.

5)7639487	7)440295	8)5678943
1527897$\frac{2}{5}$	62899$\frac{2}{7}$	7098677$\frac{7}{8}$

This character ÷, when placed between two numbers, signifies that the one is divided by the other; thus 95 ÷ 8 = 11$\frac{7}{8}$; *and we read* 95 *divided by* 8, *gives* 11 *and seven-eights over; that is, there are eleven eights in* 95, *and seven remaining.*

EXAMPLES.

Ex. 1. 5687 ÷ 7
Here 5687 ÷ 7 = 812$\frac{3}{7}$.
For 7)5687
812—3

Ex. 2. 49876 ÷ 3
49876 ÷ 3 = 16625$\frac{1}{3}$
3)49876
16625—1

Ex. 3. 87240322 ÷ 3	Ex. 4. 62304678 ÷ 4
5. 74009654 ÷ 5	6. 26730217 ÷ 6
7. 59234600 ÷ 7	8. 37026541 ÷ 9
9. 46872135 ÷ 8	10. 56438752 ÷ 7
11. 45900361 ÷ 9	12. 3256487 ÷ 8
13. 59764218 ÷ 10	14. 3532640 ÷ 11
15. 32703742 ÷ 12	16. 3335333 ÷ 12
17. 44444444 ÷ 11	18. 5598764 ÷ 12
19. 98897603 ÷ 9	20. 9330048 ÷ 8

PROOF.—The method of proving the truth of sums in Division, is to multiply the answer by the divisor, and take in the remainder, the result will be equal to the dividend.

Ex. 7959467834 ÷ 7
7)7959467834
Quotient - 1137066833—3
7
Proof - - 7959467834

8
7 × 2
8

Another method is by casting out the nines, as in Multiplication.—RULE. Cast away the nines in the divisor, and put the remainder on one side of the cross; then for the top figure multiply these two numbers together, cast away the nines, and add the excess of nines in the remainder after division, and the excess of nines in this sum will be equal to the excess of nines in the dividend, if the work is right. See the preceding example, where I put down the 7 on one side of the cross; do the same with the quotient, for the other side of the cross: the excess of nines in the quotient is 2, which I put on the other side of the cross, then I say 7 times 2 are 14, and the remainder 3 make 17, which is 8 above nine, this I put at the top of the cross, and I find that 8 is the excess above the nines in the dividend, therefore I conclude the operation is right.

II. To divide a number of one denomination, by another number whose significant figures do not exceed 12, having a cypher or cyphers joined to the right hand.

RULE. Cut off the cyphers from the divisor, and the same number of figures from the right-hand of the dividend; then divide the remaining figures of the dividend by the remaining part of the divisor, and the result is the answer.

To the remainder, if any, join those figures of the dividend, which were first cut off, and the whole will be the true remainder.

Divide 4685321 by 800; and 326441 by 1200.

8.00)46853 21	12.00)3264.41
5856—521	272—41

Of course the true answers to these sums are $5856\frac{521}{800}$, and $272\frac{41}{1200}$

EXAMPLES.

Ex. 1.	3476521 ÷ 60	Ex. 2.	8543009 ÷ 700
3.	2937648 ÷ 800	4.	90084 6 ÷ 9000
5.	5620042 ÷ 1100	6.	7641121 ÷ 500
7.	40 2079 ÷ 1200	8.	8496531 ÷ 12000
9.	7921164 ÷ 90	10.	9939216 ÷ 8000
11.	46201132 ÷ 700	12.	1234567 ÷ 120

III. To divide a given number of one denomination, by a divisor which is compounded of two or more numbers in the Multiplication Table.

RULE. Divide the given number by one of those parts, and the quotient by the other component part, and so on till each of the component parts has been used as a divisor; thus 46875815777 ÷ 105 is performed as follows: the divisor 105 is equal to 7 × 5 × 3; I therefore divide the dividend first by 7, and the quotient by 5, and this second quotient by 3.

7)46875815777

5)6696545111

3)1339309022 — 1

Answer - - 446436340 — 2

EXAMPLES.

Ex. 1.	84596543 ÷ 36	Ex. 2.	345069549 ÷ 42
3.	45897642 ÷ 56	4.	945960542 ÷ 99
5.	39200761 ÷ 66	6.	87932874 ÷ 768
7.	38426587 ÷ 550	8.	44444444 ÷ 121
9.	28476974 ÷ 720	10.	55555555 ÷ 378
11.	56342872 ÷ 132	12.	33992288 ÷ 288
13.	34765982 ÷ 144	14.	98453392 ÷ 432
15.	24853274 ÷ 512	16.	83547552 ÷ 99
17.	43838999 ÷ 343	18.	54954335 ÷ 720
19.	5555556 ÷ 729	20.	25574538 ÷ 343

IV. To divide by a number consisting of two or more digits, which number is not compounded of those in the table.

Rule (1.) Draw a curved line on the right and left of the dividend, and write the divisor on the left.

(2.) Find how many times the divisor is contained in as many figures of the dividend as are just necessary, and place the number on the right for a quotient.

(3.) Multiply the divisor by the quotient figure, and place the product under the above-mentioned figures of the dividend, subtract this product from that part of the dividend under which it stands, and bring down the next figure in the dividend, or more if necessary, to the right hand of the remainder, and proceed as before, till the whole is finished. This is called Long Division.

Ex. 5537049 ÷ 954

954)5537049(5804 Quotient.
4770...
7670
7632
3849
3816
33 Remainder. Answer $5804\frac{33}{954}$.

Here the divisor not being contained in the first three figures, I consider how often it is contained in the first four, and find it to be 5 times, the 5 I put in the quotient, and multiply the divisor by it, setting the product under the dividend. I now subtract this product, and to the remainder 767, I bring down the 0, and find that the divisor is contained 8 times in 7670, the 8 I place in the quotient, and proceed to multiply the divisor by it; the product subtracted leaves only 38; I now bring down the 4, but the divisor not being contained in 384, I put down 0 in the quotient, and bring down the 9, the remaining figure in the dividend, and proceed as before

EXAMPLES.

Ex. 1.	78654321 ÷ 76	Ex. 2.	56943278 ÷ 97
3.	68742164 ÷ 87	4.	84365487 ÷ 69
5.	77755502 ÷ 654	6.	45687403 ÷ 187
7.	53430432 ÷ 7654	8.	56943286 ÷ 429
9.	57678443 ÷ 8439	10.	58456942 ÷ 3279
11.	564320376 ÷ 3976	12.	92876487 ÷ 7392
13.	677744032 ÷ 5186	14.	46859210 ÷ 1437
15.	627432871 ÷ 4967	16.	55555555 ÷ 7777
17.	44444444 ÷ 5555	18.	888000999 ÷ 999
19.	33333333 ÷ 999	20.	111111111 ÷ 3777

Ex. 21. 487264325876 ÷ 56780909
22. 876842087621 ÷ 90956843
23. 948318296542 ÷ 56400032
24. 567843276549 ÷ 64785321
25. 877896543210 ÷ 92836058
26. 444444444444 ÷ 750000564
27. 2220003330046 ÷ 708385032
28. 54096538762 ÷ 5406057
29. 32899438654 ÷ 10010432
30. 784363254871 ÷ 99834369

MISCELLANEOUS EXAMPLES.

Ex. 1. Divide fifty millions by four thousand and seventy-nine.

2. The planet Mercury goes round the sun in 88 days, which is the length of her year, how many years of Mercury would make 50 of our years, supposing each year contained exactly 365 days?

3. It is estimated that there are a thousand millions of inhabitants in the known world: if one thirty-third of this number die annually, how many deaths are there in a year?

4. The national debt at present, cannot be less than five hundred millions sterling: how long would that be in paying off, at the rate of two millions and twenty-five pounds per annum?

5. The taxes annually collected amount to full thirty-three millions of pounds: how many poor families of six persons each would that sum support, supposing the annual expenses of the father and mother to be 20*l.*, and of each child 7*l.*?

6. My friend is to set sail to Jamaica on the first of March, 1812, the distance is reckoned to be 3984 miles from England, at what rate will he go, supposing he reaches the Island on the 10th day of April, that is, in 41 days?

7. What is the difference between the 12th part of 20,100 and the 5th part of 9110?

8. The prize of 30,000*l.* of the last Lottery became the property of 15 persons: how much was each person's share, after they had allowed 750*l.* to the office-keeper for prompt payment?

9. The sum of two numbers is 1440, the lesser is 48: what is their difference, product, and quotient?

10. The crew of a ship, amounting to 124 men, have to receive, as prize-money, 1890*l.*; but as they are to be paid off, they determined to make their commander and boatswain a present, the one of a piece of plate, value 25*l.*; the other of a whistle, which is to cost 5*l.*: how much will each receive after these deductions are made?

11. In all parts of the world a cubical foot of water weighs 1000 ounces: how many pounds are there, supposing 16 ounces make a pound?

12. A cubical foot of air weighs one ounce and a quarter, how many pounds avoirdupois of air does a room contain, which is 10 feet high, 14 feet wide, and 16 feet long?

13. Hydrogen gas, or, as it was formerly called, inflammable air, that is, the gas with which balloons are filled, is full nine times lighter than the common air which we breathe: how much less would a balloon, containing 27,000 cubical feet, weigh if filled with hydrogen gas, than if filled with common air?

14. At what rate per hour and per minute does a place on the equator move, supposing the great circle of the earth to be 25,000 miles, and the earth to turn on its axis exactly in 24 hours?

COMPOUND ADDITION.

ADDITION OF MONEY.

PENCE AND SHILLING TABLES.

Pence		s.	d.	Pence		s.	d.	Shill.		L.	s.	d.
20	are	1	8	12	are	1	0	20	-	1	0	0
25	-	2	1	18	-	1	6	25	-	1	5	0
30	-	2	6	24	-	2	0	30	-	1	10	0
35	-	2	11	30	-	2	6	35	-	1	15	0
40	-	3	4	36	-	3	0	40	-	2	0	0
45	-	3	9	42	-	3	6	50	-	2	10	0
50	-	4	2	48	-	4	0	60	-	3	0	0
55	-	4	7	54	-	4	6	70	-	3	10	0
60	-	5	0	60	-	5	0	80	-	4	0	0
65	-	5	5	66	-	5	6	90	-	4	10	0
70	-	5	10	72	-	6	0	100	-	5	0	0
75	-	6	3	78	-	6	6	110	-	5	10	0
80	-	6	8	84	-	7	0	120	-	6	0	0
85	-	7	1	90	-	7	6	130	-	6	10	0
90	-	7	6	96	-	8	0	140	-	7	0	0
95	-	7	11	102	-	8	6	150	-	7	10	0
100	-	8	4	108	-	9	0	160	-	8	0	0
105	-	8	9	114	-	9	6	170	-	8	10	0
110	-	9	2	120	-	10	0	180	-	9	0	0
115	-	9	7	132	-	11	0	190	-	9	10	0
120	-	10	0	144	-	12	0	200	-	10	0	0

UNITED STATES, OR FEDERAL MONEY.

10 Mills (m.) make 1 Cent, c.
10 Cents —— 1 Dime, d.
10 Dimes —— 1 Dollar, D. or $
10 Dollars —— 1 Eagle, E.
100 Cts. —— $ 1

ENGLISH MONEY.

4 Farthings (qrs.)	make	1	Penny, d.
12 Pence	——	1	Shilling, s.
20 Shillings	— —	1	Pound, £.

COMPOUND ADDITION is a method of collecting several numbers of the different denominations into one sum.

RULE (1.) Arrange the numbers so that those of the same denomination may stand directly under each other, and draw a line under them.

(2.) Add the numbers in the lowest denomination together, and find how many units of the next higher denomination are contained in their sum.

(3.) Write down the remainder, and carry the units to the next higher denomination, and proceed so to the end.

	L.	*s.*	*d.*
Ex.	468	19	$4\frac{1}{2}$
	123	16	$11\frac{3}{4}$
	987	12	9
	654	13	$7\frac{1}{4}$
	123	17	$4\frac{1}{2}$
	456	18	$10\frac{3}{4}$
	439	4	$6\frac{1}{2}$
	592	12	$4\frac{1}{4}$
	3847	15	$10\frac{1}{2}$

I first add together the farthings, which I find to be 14, but 14 farthings make $3\frac{1}{2}d.$ I put down the $\frac{1}{2}$ and carry the 3 to the column of pence, which I then add together, and find the sum to be 58, but by the table, 55 pence are 4*s.* 7*d.*, therefore 58 pence are 4*s.* 10*d.*, I put down the 10 and carry the 4 to the column of shillings; I now add the shillings together, and find the sum to be 115, but 115 shillings make 5*l.* 15*s.*, I put down the 15, and carry the 5 to the pounds, and proceed as in simple addition.

EXAMPLES OF MONEY.

	L.	*s.*	*d.*		*L.*	*s.*	*d.*		*L.*	*s.*	*d.*		*L.*	*s.*	*d.*
Ex. 1.	55	3	8	2.	67	2	8	3.	95	2	9	4.	49	9	11
	62	6	3		24	9	9		89	7	8		33	8	7
	96	2	1		38	2	5		72	4	3		96	12	9
	31	8	4		42	5	9		67	9	2		75	3	4
	43	7	5		78	6	6		51	8	9		51	8	9
	10	9	8		64	6	9		45	5	4		12	19	7

Ex. 5.

L.	s.	d.
58	15	9
79	5	5
61	7	10
64	16	3
32	15	10
19	12	8

6.

L.	s.	d.
42	16	9
37	15	11
73	9	9
62	10	6
29	4	4
19	17	11

7.

L.	s.	d.
92	13	$4\frac{3}{4}$
84	14	9
73	18	$4\frac{1}{2}$
69	17	10
48	15	7
35	14	$11\frac{1}{4}$

Ex. 8.

L.	s.	d.
50	19	$8\frac{1}{4}$
97	16	$7\frac{3}{4}$
35	14	2
46	16	$8\frac{1}{2}$
67	16	2
24	15	$9\frac{1}{4}$

9.

L.	s.	d.
54	17	$6\frac{3}{4}$
93	12	8
31	6	$9\frac{1}{4}$
25	10	11
76	13	10
44	6	$6\frac{1}{4}$
33	8	3

10.

L.	s.	d.
67	16	$8\frac{1}{4}$
71	13	9
84	11	$8\frac{3}{4}$
32	19	3
48	10	$4\frac{1}{4}$
55	18	$7\frac{1}{4}$
21	12	4

Ex. 11.

L.	s.	d.
18	14	$8\frac{1}{4}$
93	15	$10\frac{3}{4}$
37	6	11
78	16	$5\frac{3}{4}$
69	12	$7\frac{1}{4}$
43	8	11
12	17	$3\frac{1}{4}$

12.

L.	s.	d.
41	15	$9\frac{1}{2}$
56	10	9
62	16	$3\frac{3}{4}$
87	4	11
78	13	$7\frac{1}{2}$
92	19	$0\frac{3}{4}$
13	16	7

13.

D.	cts.	mls.
73	14	5
27	37	4
46	18	3
74	29	9
38	17	4
85	63	7

14.

D.	cts.	mls.
84	13	8
79	57	3
99	14	7
37	74	5
29	18	6
47	13	2

15.

D.	cts.	msl.
69	17	4
37	16	2
48	27	6
62	74	3
73	65	7
18	11	1

16.

Ea.	D.	d.	c.	m.
34	4	7	6	3
29	3	2	7	6
13	4	1	0	2
17	6	0	2	7
39	4	2	1	8
48	9	1	2	7

17.

Ea.	D.	d.	c.	m.
174	3	4	2	4
27	4	2	6	3
149	7	3	2	8
76	4	2	9	7
37	5	6	4	7
59	7	4	2	6

18.

L.	s.	d
46	2	$3\frac{1}{2}$
65	10	$4\frac{1}{2}$
74	0	10
81	17	$8\frac{3}{4}$
39	15	10
23	10	$8\frac{1}{2}$
19	14	$7\frac{1}{4}$

19.

L.	s.	d.
45	19	$9\frac{1}{4}$
63	17	$11\frac{1}{4}$
79	13	$5\frac{1}{4}$
46	10	$9\frac{1}{2}$
35	8	7
47	19	$10\frac{1}{2}$
19	14	6

20.

L.	s.	d.
43	17	$10\frac{1}{2}$
50	14	$6\frac{1}{4}$
72	6	$4\frac{1}{4}$
65	19	$7\frac{1}{2}$
91	5	$1\frac{1}{2}$
38	19	10
29	12	$9\frac{1}{2}$

21.

L.	s.	d.
52	18	10
67	12	$2\frac{1}{4}$
77	14	9
82	13	$10\frac{3}{4}$
98	12	$11\frac{1}{2}$
21	17	$7\frac{1}{4}$
45	12	9

22.

L.	s.	d.
77	15	$4\frac{1}{2}$
69	10	$9\frac{1}{4}$
41	0	$10\frac{1}{4}$
57	13	8
87	9	$10\frac{1}{4}$
91	16	$11\frac{3}{4}$
76	14	8

23.

L.	s.	d.
57	15	$9\frac{1}{4}$
64	9	2
76	17	$10\frac{1}{2}$
97	16	9
39	18	$11\frac{3}{4}$
45	10	10
59	17	9

24.

L.	s.	d.
446	19	$9\frac{3}{4}$
152	15	$10\frac{3}{4}$
695	12	$0\frac{1}{2}$
758	3	5
338	14	$3\frac{1}{2}$
166	19	11
279	12	$9\frac{3}{4}$

25.

L.	s.	d.
48	14	$10\frac{1}{2}$
36	13	10
74	15	$7\frac{3}{4}$
23	18	$2\frac{1}{2}$
48	9	6
81	16	$4\frac{3}{4}$
77	11	$4\frac{1}{2}$

26.

L.	s.	d.
92	19	$9\frac{3}{4}$
56	10	9
64	18	$7\frac{3}{4}$
38	16	3
49	15	$11\frac{1}{4}$
64	19	$3\frac{1}{4}$
92	17	$8\frac{1}{2}$

	L.	s.	d.
27.	12	14	$9\frac{1}{4}$
	93	16	$10\frac{1}{2}$
	17	12	11
	56	13	$7\frac{1}{4}$
	91	19	11
	76	14	$5\frac{1}{4}$
	14	11	3

	L.	s.	d.
28.	54	11	10
	22	19	$6\frac{1}{4}$
	61	16	$9\frac{1}{2}$
	14	17	$0\frac{3}{4}$
	58	12	$11\frac{1}{2}$
	72	10	6
	76	14	11

	L.	s.	d.
29.	414	19	9
	627	17	$11\frac{1}{4}$
	741	6	$4\frac{1}{2}$
	865	14	8
	917	6	$10\frac{3}{4}$
	347	14	$10\frac{1}{4}$
	449	13	4

	L.	s.	d.
30.	427	18	$10\frac{1}{4}$
	941	17	9
	712	19	6
	625	12	$7\frac{1}{4}$
	511	11	10
	462	10	$6\frac{1}{2}$
	383	11	$9\frac{3}{4}$

	L.	s.	d.
31.	548	11	6
	932	18	$4\frac{3}{4}$
	379	0	$6\frac{1}{4}$
	414	17	$0\frac{1}{2}$
	573	4	$5\frac{3}{4}$
	697	13	$9\frac{1}{4}$
	551	6	11

	L.	s.	d.
32.	493	2	$8\frac{1}{2}$
	347	14	$3\frac{1}{4}$
	729	19	5
	672	5	$8\frac{3}{4}$
	548	10	3
	217	12	$8\frac{1}{2}$
	974	16	$7\frac{1}{4}$
	146	5	$0\frac{1}{2}$

	L.	s.	d.
33.	412	9	$11\frac{1}{4}$
	924	19	$6\frac{1}{4}$
	750	11	$3\frac{1}{2}$
	627	19	$0\frac{3}{4}$
	438	10	$4\frac{1}{4}$
	363	2	$10\frac{1}{2}$
	221	15	8
	147	1	5

	L.	s.	d.
34.	152	15	$2\frac{1}{4}$
	255	18	$6\frac{1}{4}$
	348	12	$9\frac{1}{4}$
	410	0	10
	566	13	$1\frac{1}{4}$
	631	6	$4\frac{1}{2}$
	781	3	10
	949	16	7
	123	15	11

	L.	s.	d.
35.	504	3	$9\frac{1}{2}$
	636	19	5
	421	2	$7\frac{3}{4}$
	347	12	10
	383	7	$0\frac{1}{2}$
	848	15	$2\frac{3}{4}$
	710	0	$8\frac{1}{4}$
	483	10	$4\frac{1}{2}$
	426	19	7

	L.	s.	d.
36.	576	14	9
	613	12	$11\frac{1}{2}$
	719	13	$4\frac{3}{4}$
	914	14	$6\frac{1}{4}$
	271	10	9
	759	8	$5\frac{1}{4}$
	432	15	$3\frac{1}{2}$
	918	11	$4\frac{3}{4}$
	564	7	2

	L.	s.	d.
37.	827	18	$11\frac{3}{4}$
	550	11	$8\frac{1}{4}$
	938	9	4
	344	0	3
	615	16	$1\frac{1}{4}$
	471	2	7
	214	15	$10\frac{1}{2}$
	745	19	2
	90	9	9

	L.	s.	d.
38.	792	19	$3\frac{1}{2}$
	437	14	$9\frac{1}{4}$
	354	10	$10\frac{3}{4}$
	516	18	4
	209	13	$10\frac{1}{2}$
	524	17	$2\frac{3}{4}$
	739	6	10
	365	2	$6\frac{1}{4}$
	147	17	9

	L.	s.	d.
39.	88	16	$11\frac{1}{2}$
	26	14	$5\frac{1}{4}$
	9	7	$2\frac{1}{2}$
	36	12	$4\frac{3}{4}$
	41	18	3
	27	3	$8\frac{1}{4}$
	54	15	$11\frac{1}{2}$
	12	19	6
	20	0	10

	L.	s.	d.
40.	28	9	$4\frac{1}{4}$
	54	17	9
	6	0	11
	28	13	$5\frac{3}{4}$
	65	18	$7\frac{1}{4}$
	92	6	$4\frac{3}{4}$
	7	16	$0\frac{1}{2}$
	14	5	10
	40	0	9

	L.	s.	d.
41.	60	15	$5\frac{1}{2}$
	48	13	$1\frac{3}{4}$
	93	18	6
	7	7	$10\frac{1}{2}$
	35	19	$4\frac{1}{4}$
	73	6	$9\frac{3}{4}$
	31	17	3
	59	14	$10\frac{3}{4}$
	60	0	10

	L.	s.	d.
42.	94	1	$9\frac{1}{2}$
	88	2	$6\frac{1}{2}$
	46	5	$11\frac{3}{4}$
	29	16	$3\frac{1}{2}$
	48	12	0
	5	17	7
	61	13	$3\frac{1}{4}$
	7	14	$10\frac{1}{4}$
	12	18	$5\frac{1}{2}$

	L.	s.	d.
43.	53	11	$4\frac{1}{2}$
	6	2	8
	18	5	$5\frac{1}{4}$
	26	10	$7\frac{1}{2}$
	42	0	$4\frac{3}{4}$
	64	2	2
	71	18	$10\frac{3}{4}$
	3	14	$11\frac{1}{2}$
	80	0	$6\frac{3}{4}$

	L.	s.	d.
44.	68	19	$5\frac{3}{4}$
	84	7	$3\frac{1}{2}$
	8	6	$5\frac{3}{4}$
	25	11	$9\frac{1}{2}$
	9	13	7
	47	15	$6\frac{3}{4}$
	32	1	3
	1	18	$0\frac{1}{2}$
	2	16	4

	L.	s.	d.
45.	75	12	$8\frac{1}{2}$
	40	0	$6\frac{1}{4}$
	8	17	4
	24	19	$5\frac{3}{4}$
	59	15	$2\frac{1}{2}$
	82	6	$5\frac{3}{4}$
	7	18	$4\frac{1}{2}$
	33	2	$9\frac{1}{4}$
	8	10	$0\frac{1}{4}$

	L.	s.	d.
46.	39	14	$4\frac{1}{2}$
	97	12	$2\frac{1}{4}$
	73	15	$10\frac{1}{2}$
	6	10	$11\frac{1}{4}$
	30	2	9
	16	12	$5\frac{3}{4}$
	58	16	$1\frac{1}{2}$
	2	13	7
	82	0	$3\frac{1}{2}$
	10	10	10

	L.	s.	d.
47.	78	12	5
	17	14	$8\frac{1}{2}$
	35	0	6
	28	16	$10\frac{1}{4}$
	11	8	$3\frac{1}{2}$
	49	15	$7\frac{1}{4}$
	6	11	$4\frac{1}{4}$
	62	15	8
	5	18	$4\frac{3}{4}$
	90	0	10

	L.	s.	d.
48.	127	10	$10\frac{3}{4}$
	356	14	$9\frac{3}{4}$
	483	9	$4\frac{1}{2}$
	849	7	11
	680	18	$11\frac{1}{4}$
	774	19	$7\frac{1}{2}$
	114	6	$2\frac{3}{4}$
	251	18	$9\frac{1}{4}$
	428	15	6
	567	16	2

	L.	s.	d.
49.	515	14	$9\frac{3}{4}$
	943	17	$3\frac{1}{2}$
	623	15	$11\frac{1}{4}$
	417	19	$3\frac{1}{2}$
	338	14	10
	385	18	$11\frac{3}{4}$
	764	13	6
	453	19	$9\frac{1}{2}$
	562	18	$5\frac{3}{4}$
	223	14	2

	L.	s.	d.
50.	657	16	$10\frac{1}{2}$
	784	17	$4\frac{3}{4}$
	879	14	$3\frac{1}{2}$
	919	12	$10\frac{3}{4}$
	131	19	11
	235	7	$6\frac{1}{4}$
	496	18	$3\frac{3}{4}$
	587	9	5
	673	11	10
	820	19	4

	L.	s.	d.
51.	491	16	9
	272	15	$6\frac{1}{4}$
	889	17	$10\frac{1}{2}$
	647	19	$2\frac{3}{4}$
	398	16	7
	563	16	$10\frac{1}{4}$
	770	0	$5\frac{3}{4}$
	945	17	7
	420	13	$9\frac{1}{4}$
	150	10	0

	L.	s.	d.
52.	722	10	$9\frac{1}{4}$
	966	4	$8\frac{3}{4}$
	899	13	6
	248	16	$10\frac{1}{4}$
	532	14	9
	476	19	$7\frac{1}{4}$
	744	12	$9\frac{1}{2}$
	669	15	$7\frac{3}{4}$
	593	15	$11\frac{1}{4}$
	150	10	0

	L.	s.	d.
53.	477	16	$4\frac{1}{4}$
	395	15	$2\frac{3}{4}$
	736	5	11
	692	14	$9\frac{1}{2}$
	565	13	$5\frac{3}{4}$
	937	17	0
	441	16	$4\frac{3}{4}$
	700	18	$0\frac{3}{4}$
	672	11	11
	40	0	10

	L.	s.	d.
54.	494274	12	$9\frac{3}{4}$
	765502	6	4
	300089	2	$2\frac{1}{4}$
	402193	17	9
	375451	3	10
	269440	18	$6\frac{3}{4}$
	123428	15	10
	567865	11	$9\frac{1}{3}$
	910649	10	6

	L.	s.	d.
55.	901442	16	$10\frac{1}{2}$
	234971	5	$9\frac{1}{4}$
	567352	14	$7\frac{3}{4}$
	912261	19	$2\frac{1}{2}$
	345519	17	$9\frac{1}{2}$
	678830	12	6
	912887	19	10
	456713	10	$3\frac{1}{2}$
	891391	17	$8\frac{1}{4}$

	L.	s.	d.
56.	4567	14	$11\frac{3}{4}$
	4934	15	9
	2765	16	$10\frac{1}{4}$
	9876	19	$11\frac{1}{4}$
	3497	9	5
	1234	10	$8\frac{3}{4}$
	5678	16	10
	4376	8	9
	2794	15	$4\frac{1}{2}$
	7921	12	$10\frac{1}{4}$

	L.	s.	d
57.	3256	19	$6\frac{1}{2}$
	4397	10	$11\frac{3}{4}$
	1974	12	$9\frac{1}{4}$
	7246	8	4
	3942	15	$10\frac{3}{4}$
	4567	8	$9\frac{3}{4}$
	4567	17	$11\frac{3}{4}$
	9376	12	$8\frac{1}{2}$
	4623	2	5
	5932	5	4

	L.	s.	d.
58.	3567	12	$9\frac{1}{4}$
	7960	17	10
	1234	15	$7\frac{3}{4}$
	5678	12	$8\frac{1}{2}$
	9123	14	10
	4567	13	$11\frac{1}{4}$
	8912	17	9
	1456	9	6
	7891	10	$4\frac{3}{4}$
	2845	6	3

	L.	s.	d.
59.	1764	13	$9\frac{3}{4}$
	1805	17	4
	1764	12	7
	3459	15	11
	2946	16	$10\frac{3}{4}$
	1796	14	10
	4325	16	8
	5678	12	$11\frac{3}{4}$
	4932	14	6
	2005	9	$5\frac{1}{2}$

	L.	s.	d.
60.	2487	7	3
	5764	16	$11\frac{1}{2}$
	1234	18	2
	5678	19	$9\frac{3}{4}$
	9012	17	10
	3456	2	2
	7890	14	5
	1234	13	10
	5678	15	7
	9123	13	4

	L.	s.	d.
61.	6789	12	$5\frac{1}{2}$
	2345	13	11
	6789	16	$9\frac{3}{4}$
	4972	15	10
	3456	19	$5\frac{1}{2}$
	7891	16	$7\frac{3}{4}$
	2345	14	$11\frac{1}{2}$
	6782	12	9
	4315	11	$7\frac{3}{4}$
	2105	8	6

EXAMPLES OF WEIGHTS AND MEASURES.

TROY WEIGHT.

24 Grains (gr.) make 1 Penny wt. pwt
20 Penny wt. ——— 1 Ounce, oz.
12 Ounces ——— 1 Pound, lb.

NOTE.—By this weight are weighed Gold, Silver, Jewels, Liquors, &c.

lb.	oz.	dwts.	gr.
7684	9	16	22
1234	11	5	19
9876	8	11	22
1493	9	19	12
3587	10	10	3
2345	7	6	15
6789	9	14	21
3257	11	15	8
36271	7	1	2

In adding up the column of grains I find the sum to be 122, which I divide by 24 to bring it into pennyweights; and 122 grains make 5 pennyweights and 2 grains over; the 2 I put down, and carry the 5 to the column of pennyweights; I then add these together, and find the sum to be 101, which I divide by 20 to bring to ounces, I put down the 1 and carry 5 to the column of ounces; then adding the ounces, I find the sum 79, which, by dividing by 12, give 6 lb. 7 oz. the 7 I put down, and carry the 6 to the pounds, and proceed as in simple Addition.

	lb.	oz.	dwt.
1.	414	9	14
	617	5	13
	715	10	9
	322	7	15
	413	2	10
	514	11	15
	976	8	7

	lb.	oz.	dwt.	gr.
2.	410	9	12	19
	342	11	16	12
	912	3	14	14
	751	6	10	22
	626	10	17	16
	427	4	11	23
	123	11	17	12

	lb.	oz.	dwt.
3.	526	10	19
	712	9	17
	944	6	14
	633	10	11
	319	4	10
	247	9	12
	123	10	17

	lb.	oz.	dwt.	gr.
4.	940	10	19	15
	738	6	4	23
	614	3	17	13
	546	7	16	19
	321	10	5	22
	230	9	15	15
	946	11	19	23

	lb.	oz.	dwt.
5.	174	11	19
	74	10	13
	944	9	14
	74	11	19
	944	10	13
	74	11	3
	12	4	6

	oz.	dwt.	gr.
6.	174	19	23
	714	11	14
	714	0	18
	74	1	22
	948	2	21
	74	2	12
	301	14	4

	lb.	oz.	dwt.
7.	71	11	19
	64	8	14
	77	0	0
	14	3	11
	64	2	9
	74	6	14
	77	2	13
	105	9	12

	oz.	dwt.	gr.
8.	74	19	23
	64	14	17
	74	19	11
	66	13	9
	74	14	11
	14	10	3
	19	11	14
	13	17	5

AVOIRDUPOIS WEIGHT.

16 Drams (dr.) make 1 Ounce, oz.
16 Ounces —— 1 Pound, lb.
28 Pounds —— ¼ of a hund. qr.
4 Quarters —— 1 Hundred. Cwt.
20 Hundred —— 1 Ton, T.

NOTE.—By this weight are weighed all kinds of coarse and heavy Goods, except Gold, Silver, &c.

	lb.	oz.	dr.
1.	318	10	10
	436	9	8
	624	14	6
	419	6	15
	245	9	7
	853	11	10
	145	9	8

	tons.	cwt.	qr.	lb.
2.	416	19	2	26
	313	10	0	20
	2 1	11	3	16
	725	19	2	18
	357	14	2	25
	429	17	3	22
	235	15	2	19

	lb.	oz.	dr.
3.	539	13	15
	316	14	13
	223	12	7
	811	9	6
	700	6	14
	414	12	12
	0	0	0

	tons.	cwt.	qr.	lb.
4.	305	14	2	11
	418	18	0	0
	336	2	1	14
	119	13	3	27
	767	16	0	8
	782	9	1	16
	421	15	3	19

	tons.	cwt.	qr.
5.	174	19	3
	74	14	2
	714	13	1
	718	16	2
	734	15	2
	714	14	1
	155	0	3

	cwt.	qr.	lb.
6.	174	3	27
	724	2	24
	149	1	14
	719	2	16
	407	1	23
	149	2	17
	76	3	15

	qr.	lb.	oz.
7.	44	27	15
	74	26	14
	19	14	13
	74	12	14
	66	27	13
	74	19	10
	13	17	5

	lb.	oz.	drs.
8.	17	15	15
	27	14	11
	16	13	9
	74	14	14
	70	0	0
	64	13	10
	13	4	5

APOTHECARIES' WEIGHT.

20 Grains (gr.) make 1 Scruple, ℈.
3 Scruples —— 1 Dram, ʒ.
8 Drams —— 1 Ounce, ℥.
12 Ounces —— 1 Pound, ℔

	lb.	oz.	dr.
1.	314	8	4
	210	11	4
	766	10	2
	555	9	6
	417	8	1
	324	7	3

	oz.	dr.	sc.	gr.
2.	22	3	2	19
	56	0	1	13
	43	2	2	11
	54	7	0	17
	76	5	2	14
	45	6	1	0

	lb.	oz.	dr.	sc.	gr.
3.	646	11	4	1	19
	715	3	7	1	14
	934	3	4	0	12
	373	10	5	2	9
	216	5	1	2	16
	159	2	5	0	14

	lb.	oz.	dr.
4.	47	11	7
	94	10	6
	74	10	4
	75	9	3
	69	0	2
	57	1	2
	18	2	1
	19	3	5

	oz.	dr.	sc.
5.	149	7	2
	714	3	0
	619	2	1
	74	6	2
	162	5	2
	74	1	2
	779	6	1
	146	4	0

	dr.	sc.	gr.
6.	749	2	19
	607	1	18
	714	2	17
	400	0	0
	74	1	13
	715	2	14
	64	1	18
	16	0	10

	lb.	oz.	dr.
7.	84	11	7
	74	10	6
	37	5	4
	19	4	3
	74	1	2
	79	2	6
	19	2	4
	13	4	8

CLOTH MEASURE.

$2\frac{1}{4}$ Inches (In.)	make	1 Nail, na.
4 Nails	——	$\frac{1}{4}$ of a yard, qr.
4 Quarters	——	1 Yard, yd.
3 Quarters	——	1 Ell Flemish, E. Fl.
5 Quarters	——	1 Ell English, E. E.
6 Quarters	——	1 Ell French, E. Fr.

	yd.	qr.	nl.		E.e.	qr.	nl.		E.e.	qr.	nl.		yd.	qr.	nl.
1.	434	3	2	2.	511	4	2	3.	565	4	0	4.	543	3	2
	527	1	2		660	2	0		626	2	1		836	2	2
	613	2	3		439	4	2		724	0	1		754	2	3
	758	3	1		337	1	2		882	2	3		217	1	3
	846	1	3		854	2	3		933	0	3		725	3	2
	925	2	2		766	0	2		227	1	1		438	2	2

	E.e.	qr.	nl.		E.e.	qr.	nl.		yd.	qr.	nl.		E e.	qr.	nl.
5.	120	2	2	6.	537	0	2	7.	74	3	3	8.	77	4	3
	394	4	1		916	3	1		64	2	1		14	3	2
	110	2	0		328	3	3		74	1	3		74	2	1
	481	1	2		457	1	2		49	2	1		49	1	2
	556	4	3		646	3	2		74	1	2		74	2	1
	664	3	1		287	4	2		44	3	1		44	1	2
	779	2	3		561	2	2		16	2	3		94	0	2

LONG MEASURE.

3	Barley Corns (bc.)	make	1 Inch, in.
12	Inches	——	1 Foot, ft.
$16\frac{1}{2}$	Feet	——	1 Rod, r.
40	Rods	——	1 Furlong, fur.
8	Furlongs	——	1 Mile, m.
$69\frac{1}{2}$	Statute Miles	——	1 Degree, Deg.

ALSO,

4	Inches	make	1	Hand,
3	Feet	——	1	Yard,
$5\frac{2}{4}$	Yards	——	1	Rod, Pole, or Perch.
6	Feet	——	1	Fathom.
66	Feet	——	1	Gunter's Chain.
3	Miles	——	1	League.

EXAMPLES.

	miles,	fur.	p.	yds.		yds.	ft.	in.	b.c.		lea.	mi.	fur.	p.
1.	427	6	23	3	2.	214	2	9	0	3.	520	1	6	13
	689	5	26	5		183	2	11	2		623	1	7	27
	322	7	30	2		597	0	8	1		721	0	4	16
	510	2	38	4		649	2	7	2		826	1	3	32
	777	4	0	3		725	1	6	1		932	2	6	1
	888	3	10	4		930	1	3	0		315	1	2	28
	126	0	24	0		492	1	4	1		409	1	5	39
	412	7	39	4		291	2	10	2		376	2	7	27

	lea.	m.	fur.		fur.	p.	yds		p.	yds.	ft.		feet	in.	b.c
4.	17	2	7	5.	147	39	5	6.	177	5	2	7.	174	11	2
	14	1	6		614	37	4		714	4	1		49	10	1
	74	1	7		714	19	3		714	1	2		74	11	2
	68	2	4		674	17	1		615	0	1		64	9	1
	74	1	0		719	27	2		714	1	2		74	10	1
	69	2	1		197	19	1		719	1	1		64	11	2
	74	1	2		724	14	3		437	2	1		74	10	0
	96	2	4		604	29	5		610	4	0		94	11	2

LAND, OR SQUARE MEASURE.

144	Square Inches	make	1	Square Foot.
40	—— Rods	——	1	—— Rood.
4	—— Roods	——	1	—— Acre.

ALSO,

9	square Feet	make	1	square Yard.
$30\frac{1}{4}$	—— Yards	——	1	—— Rod.
160	—— Rods	——	1	—— Acre.
640	—— Acres	——	1	—— Mile.

EXAMPLES.

	ac.	r.	p.
1.	452	2	38
	114	1	35
	715	2	16
	430	2	35
	529	3	7
	346	1	23
	661	3	11
	214	2	35

	ac.	r.	p.
2.	982	2	24
	618	3	14
	100	1	27
	474	2	19
	363	1	31
	755	3	38
	647	0	6
	234	2	29

	ac.	r.	p.
3.	921	1	29
	604	3	32
	736	2	29
	559	3	28
	265	1	17
	427	0	30
	883	1	39
	291	3	25

	ac.	r.	p
4.	77	3	39
	64	2	37
	74	1	24
	64	2	19
	74	1	18
	64	2	17
	14	1	13
	94	3	54

	ac.	r.	p.
5.	714	3	39
	619	1	36
	714	2	27
	619	1	34
	719	2	37
	719	1	24
	615	2	14
	174	3	38

	ac.	r.	p.
6.	14	3	39
	74	1	19
	64	2	14
	74	1	18
	47	2	24
	18	1	14
	74	2	19
	74	2	24

	ac.	r.	p.
7.	174	3	39
	714	1	27
	618	2	12
	719	1	14
	734	2	11
	715	1	24
	639	2	24
	714	1	34

LIQUID MEASURE.

4	Gills (gl.),	make	1	Pint, pt.
2	Pints	——	1	Quart, qt.
4	Quarts	——	1	Gallon, gal.
63	Gallons	——	1	Hogshead, hhd.
2	Hogsheads	——	1	Pipe, P. or Butt. B.
2	Pipes or Butts	——	1	Tun, T.

	hhd.	gal.	pt.
1.	626	44	7
	753	17	1
	488	52	6
	217	13	7
	135	45	0
	497	56	2
	312	11	3
	256	0	0

	tuns,	h.	g.	qt.
2.	522	1	39	3
	257	3	34	2
	763	2	58	3
	611	3	43	1
	937	1	16	3
	238	0	31	2
	749	3	7	0
	319	2	59	3

	tuns,	h.	g.	q.
3.	148	2	25	3
	513	0	42	3
	614	1	36	1
	349	3	43	2
	416	2	56	1
	952	3	26	0
	567	1	19	3
	792	3	46	2

	tuns.	hhd.	g.
4.	714	3	62
	614	2	61
	174	1	39
	164	2	47
	274	1	49
	175	2	37
	375	1	49
	704	0	64

	hhd	gal	qt.
5.	74	41	3
	64	40	2
	74	19	1
	64	39	2
	74	40	1
	69	16	1
	17	39	2
	28	44	3

	g.	q.	p.
6.	14	3	1
	74	2	1
	39	2	1
	17	1	0
	19	2	0
	77	1	1
	39	3	1
	24	2	0

DRY MEASURE.

2 Pints (pt.) make 1 Quart, qt.
4 Quarts —— 1 Gallon, gal.
2 Gallons —— 1 Peck, pk.
4 Pecks —— 1 Bushel, bu.
40 Bushels —— 1 Load, Lo.

	bu.	pk.	gal.
1.	73	3	1
	46	2	0
	39	3	1
	48	2	0
	37	2	0
	46	1	1
	27	2	1
	39	1	1

	bu.	pk.	gal.
2.	29	2	0
	57	0	1
	38	3	1
	26	2	0
	48	1	0
	28	0	1
	76	3	1
	24	2	1

	bu.	pk.	gal.
3.	36	2	1
	99	3	1
	36	3	1
	27	2	1
	46	3	0
	27	2	1
	36	1	1
	57	2	1

	gal.	qts.	pts.		pks.	gal.	qts.		bu.	pk.	gal.	qts.
4.	56	3	1	5.	76	1	3	6.	58	3	1	3
	77	2	1		39	0	2		74	2	1	3
	64	1	0		92	1	3		63	3	0	2
	76	1	1		47	0	3		49	1	1	3
	67	2	1		36	1	2		48	2	1	3
	74	3	0		27	1	3		63	3	1	3
	62	1	1		64	0	0		75	0	0	0
	49	2	1		77	1	2		36	3	0	2

TIME.

60 Seconds (Sec.) make 1 Minute, m.
60 Minutes —— 1 Hour, h.
24 Hours —— 1 Day, d
$365\frac{1}{4}$ Days —— 1 Year. yr.
100 Years —— 1 Century, Cen.

	mo.	w.	d.	h.		w.	d.	h.	mi.		d.	h.	mi.	sec.
1.	19	2	6	19	2.	57	4	23	38	3.	62	7	47	38
	46	1	4	21		64	6	13	47		18	12	54	56
	22	3	5	9		15	3	21	19		76	21	16	49
	57	2	3	21		36	2	18	15		34	9	20	31
	62	1	6	12		78	6	9	59		90	23	31	46
	17	3	2	14		49	0	20	6		52	22	28	32
	11	3	4	16		71	5	14	48		15	4	58	23
	29	1	3	21		23	3	7	24		64	16	13	16

	yrs.	mo.	w.		mo.	w.	d.		days,	h.	m.		hrs.	min.	se.
4.	737	12	3	5.	64	3	6	6.	714	23	59	7.	647	59	59
	347	11	2		74	1	5		74	14	54		137	54	54
	618	10	1		34	2	8		94	21	55		375	56	56
	374	9	2		74	1	4		74	13	53		714	17	19
	175	1	1		63	2	1		69	12	14		615	54	54
	714	12	3		74	1	2		74	12	19		714	17	13
	615	10	1		64	2	1		37	11	17		613	34	56
	314	9	3		94	2	6		46	22	49		626	47	49

ASTRONOMY.

60 Seconds (″) make 1 Prime Minute, ′.
60 Minutes —— 1 Degree, °.
30 Degrees —— 1 Sign, S.
12 Signs or 360 Degrees —— The great circle of the Zodiack

S	°	′	″	S	°	′	″	S	°	′	″
11	24	37	41	5	3	26	25	6	9	54	36
7	12	57	21	9	5	37	56	3	29	59	7
3	25	13	17	8	24	42	59	11	26	21	19
4	29	18	29	3	9	12	15	9	24	50	40
5	16	52	43	4	8	17	41	11	18	29	27
3	19	47	51	8	26	9	8	5	13	51	46
11	29	51	36	5	16	8	27	6	7	1	9
9	18	30	30	11	20	40	50	10	12	24	36
3	4	44	44	10	9	55	37	7	21	42	56
7	25	36	51	4	22	44	56	5	23	51	46

MISCELLANEOUS EXAMPLES IN ADDITION.

1. What is the sum total, in shillings, of 54 guineas, 29 pounds, 36 guineas, and 48 pounds?

Answer, 3430 shillings.

2. Add together 16*l.* 12*s.* 2*d.*; 156*l.* 9*s.* 9¼*d.*; 20395*l* 12*s.*; 24*l.* 19*s.* 11¾*d.*; 37*l.* 6*s.* 7*d.*; 327*l.* 18*s.*; and 100 guineas. Ans. 21063*l.* 18*s.* 6*d.*

3. In collecting an account of debts owing to me, I find Mr. A. owes me 74D. 16cts.; Mr. B. 69D. 50cts.; Mr. C. 731D. 4cts.; Mr. D 38D. 37½cts.; Mr. E. 14D. 6¼cts.; what is the whole sum due to me?

Ans. 269D. 13¾cts.

4. A gentleman ordered a service of plate from his silversmith, and on receiving his bill, he finds that he had dishes and covers weighing 45 lb. 9 oz. 12 dwts.; plates weighing 70 lb. 7 oz. 16 dwts.; spoons of different sizes, and ladles, 24 lb. 9 oz. 12 dwts: waiters, 15 lb. 10 oz.; salts and castors, 4 lb. 4 oz. 3 dwts.; candlesticks, 19 lb. 11 oz. 17 dwts.; and sundry smal-

ler articles 5 lb. 3 oz. 10 dwts.; what is the weight of silver he will have to pay for? Ans. 186lb. 8oz. 10dwts.

5. A carrier brings goods to a shop keeper, viz. 8 bags of hops weighing 19 cwt. 3 qrs. 14 lb.; cheeses weighing 15 cwt. 1 qr. 21 lb.; butter weighing 12 cwt. 2 qrs.; two chests of tea weighing $1\frac{1}{2}$ cwt. each; and a sack of salt weighing 8 cwt. 2 qr. 12 lb.; how much weight will the carrier have to charge? Ans. 59 cwt. 1 qr. 19 lb.

6 The rent of my house is 50*l.* per annum; the house tax is three pound fifteen shillings; land tax 5*l.*; windows 15*l.* 12*s.* 0*d.*; poor's rates 10*l.*; lighting, watching, and street rates 3*l.* 9*s.* $3\frac{1}{2}$*d.*: how much therefore do my house and taxes stand me in per annum?

Ans. 87*l.* 16*s.* $3\frac{1}{2}$*d.*

7. The following is an estimate of the repairs wanting to my house; how much is the whole sum? Carpenter's bill 27*l.* 9*s.* $9d\frac{1}{2}$.; bricklayer's and plasterer's 17*l.* 7*s.* 6*d.*; mason's 5*l.* 5*s.*; painter's, glazier's, and plumber's, fourteen guineas; smith's, for new rails, 12*l.*; and the slater's 9*l.* 18*s.* Ans. 86*l.* 14*s.* $3\frac{1}{2}$*d.*

8. A man purchased some goods for the country; the first parcel contained 25 yds. 2 qr. 2 nl of broad cloth; the second 126 yds. 2 qrs. of serge; the third a thousand yards of green baise; and the fourth 19 yds. 3 qrs. 2 nl. of shalloon; what was the whole quantity?

Ans. 1172 yds. 0 qr. 0 nl.

9. A wine merchant, retiring from business, takes an account of the stock of wines in his cellar, and finds 5 pipes and 50 gallons of port wine, four pipes of sherry; ten pipes of Lisbon; 2 pipes of claret; of Madeira he had 36 gallons; of brandy 50 gallons; of rum two hogsheads; and of Holland, 1 hhd. and 12 gallons; what quantity of liquor did his cellar contain?

Ans. 23 pipes, 1 hhd. 22. gal.

10. A friend in Essex desired me to measure his farm, which he holds on a lease; the three fields at the back of the house measured 59 ac. 2 r. 20 p.; the large piece of ground in the valley measures 74 acres, three others measure each on an average 11 ac. 1 r. 36 p.; the field laid down in clover contains 7 ac. 3 r. 2 p. one sown with caraways, I find to be $3\frac{1}{2}$ acres; and the ground be-

longing to the garden, out-houses, &c. makes about $1\frac{1}{4}$ acres; how many acres ought he to pay for?

Ans. 180 ac. 2 rd. 10 per.

11. A merchant sends to his banker on the 2d day of the month, in money and bills, to the amount of two thousand guineas; on the fifth he sends him 900*l.* 19*s.* 4*d.*; on the eleventh he sends 500*l.*; and in the course of the remaining days of the month he sends 1515*l.* 12*s.* $11\frac{1}{2}d.$; how much therefore may he draw as occasion requires? Ans. 5016*l.* 12*s.* $3\frac{1}{2}d.$

12. A gentleman's steward received the following sums of money for rents; what was the gentleman's income? Of farmer A he received 394*l.* 12*s.* 6*d.*, of B 97*l.* 14*s.* 9*d.*, of C 175*l.* 10*s.*, of D 99*l.* 4*s.* and of E 139*l.* 12*s.* 4*d.* Ans. 906*l.* 13*s.* 7*d.*

13. A person borrows of several friends the following sums of money; of the first 500*l.*; of the second 225*l.* 12*s.*; of the third fifty guineas; of the fourth seventy guineas and 22 crowns; of the fifth he had 150*l.* 7*s.* 6*d.*; how much will he have to pay interest for?

Ans. 1007*l.* 9*s.* 6*d.*

14. A man borrowed a sum of money, and paid at different times 87 dollars, but he still owed 64 D. $37\frac{1}{2}$ cts., what was the original debt? Ans. 151D. $37\frac{1}{2}$cts.

COMPOUND SUBTRACTION,

Is the method of finding the difference between two given compound numbers.

Rule. 1. Having arranged the numbers so that the smaller may stand under the greater, subtract each number in the lower line from that which stands above it, and write down the remainders.

2. When any of the lower denominations are greater than the upper, increase the upper number by as many as make one of the next superior denomination, from which take the figure in the lower line, set down

the difference, and carry one to the next number in the lower line, and subtract as before.

Ex. Subtract 595*l.* 17*s.* 9½*d.* from 600*l.* 10*s.* 7¼*d.*

	L.	*s.*	*d.*
	600	10	$7\frac{1}{4}$
	595	17	$9\frac{1}{2}$
	4	12	$9\frac{3}{4}$
Proof	600	10	$7\frac{1}{4}$

Here I say 2 farthings from 1, I cannot, but I add 4 to the 1, because 4 farthings make a penny, and 2 from 5, and there remains $\frac{3}{4}$; I carry one to the 9; 10 from 7 I cannot, but I add 12 to 7, because 12 pence make a shilling, and 10 from 19 and there remain 9; I carry 1 to 17; and 18 from 10, I cannot, but I add 20 to the 10, because 20 shillings make a pound, and 18 from 30 and there remain 12; I now carry one to the five, and go on as in simple subtraction.

The method of proof is the same as in simple Subtraction.

EXAMPLES.

	D.	cts.	mls.		D.	cts.	mls.		D.	cts.	mls.
Ex. 1.	39	44	3	2.	76	29	4	3.	18	76	5
	27	76	2		49	13	6		9	47	6
	11	67	1		26	16	4		9	28	9

	D.	cts.	mls.		D.	cts.	mls.		D.	cts.	mls.
Ex. 4.	57	13	7	5.	62	13	7	6.	48	30	1
	49	76	9		37	4	1		24	97	2
	7	36	5		25	9	6		23	32	9

	Ea.	D.	D.	cts.	mls.		Ea.	D.	D.	cts.	mls.
Ex. 7.	67	3	7	4	6	8.	79	4	1	6	7
	39	4	2	9	7		37	6	7	4	9
	27	6	4	4	9		41	5	1	1	8

	L.	s.	d.
Ex. 9.	145	19	$9\frac{1}{2}$
	136	17	$6\frac{1}{4}$
Answer			
Proof			

	L.	s.	d.
Ex. 10.	370	17	$7\frac{3}{4}$
	369	12	$4\frac{1}{2}$

	L.	s.	d.
Ex. 11.	450	12	$6\frac{1}{2}$
	371	10	4

	L.	s.	d.
Ex. 12.	594	10	$9\frac{1}{4}$
	374	19	$5\frac{1}{2}$

	L.	s.	d.
Ex. 13.	465	12	$7\frac{1}{2}$
	349	17	$9\frac{3}{4}$

	L.	s.	d.
Ex. 14.	564	12	$2\frac{1}{2}$
	375	18	$4\frac{1}{4}$

	L.	s.	d.
Ex. 15.	371	19	$2\frac{1}{2}$
	199	17	$11\frac{3}{4}$

	L.	s.	d.
Ex. 16.	700	0	0
	376	16	$6\frac{1}{2}$

	L.	s.	d.
Ex. 17.	476	19	4
	374	12	9

	L.	s.	d.
Ex. 18.	473	18	$7\frac{3}{4}$
	291	12	$7\frac{3}{4}$

	L.	s.	d.
Ex. 19.	249	9	$9\frac{3}{4}$
	159	19	$11\frac{1}{4}$

	L.	s.	d.
Ex. 20.	376	17	7
	299	14	$4\frac{3}{4}$

	L.	s.	d.
Ex. 21.	594	0	0
	593	19	$9\frac{3}{4}$

	L.	s.	d.
Ex. 22.	796	12	$11\frac{1}{2}$
	669	8	3

	L.	s.	d.
Ex. 23.	476	17	7
	399	19	$11\frac{3}{4}$

	L.	s.	d.
Ex. 24.	399	2	2
	177	12	$7\frac{3}{4}$

	L.	s.	d.
Ex. 25.	209	18	8
	159	19	$9\frac{1}{2}$

	L.	s.	d.
Ex. 26.	500	0	0
	499	19	11

	L.	s.	d.
Ex. 27.	422	3	$6\frac{1}{3}$
	371	15	$7\frac{3}{4}$

	L.	s.	d.
Ex. 28.	224	2	$6\frac{3}{5}$
	156	6	$6\frac{3}{4}$

	L.	s.	d.
Ex. 29.	794	15	$6\frac{3}{4}$
	367	16	$4\frac{3}{4}$

	L.	s.	d.
Ex. 30.	999	0	0
	800	19	$11\frac{3}{4}$

	L.	s.	d.
Ex. 31.	764	15	$4\frac{1}{4}$
	398	12	11

	L.	s.	d.
Ex. 32.	674	6	$0\frac{3}{4}$
	249	19	$9\frac{3}{4}$

	L.	s.	d.
Ex. 33.	372	10	$6\frac{1}{4}$
	149	6	$4\frac{3}{4}$

	L.	s.	d.
Ex. 34.	649	12	$9\frac{3}{4}$
	597	19	$8\frac{1}{4}$

	L.	s.	d.
Ex. 35.	341	5	$11\frac{1}{2}$
	230	9	$4\frac{1}{2}$

	L.	s.	d.
Ex. 36.	846	9	$8\frac{3}{4}$
	375	9	$9\frac{1}{2}$

	L.	s.	d.
Ex. 37.	124	9	$10\frac{3}{4}$
	109	10	$3\frac{1}{4}$

	L.	s.	d.
Ex. 38.	90441	5	$9\frac{3}{4}$
	67217	13	10

	L.	s.	d.
Ex. 39.	438	7	10
	399	16	$9\frac{1}{2}$

	L.	s.	d.
Ex. 40.	12427	16	$11\frac{1}{4}$
	7618	14	$9\frac{3}{4}$

	L.	s.	d.
Ex. 41.	1654	12	7
	585	9	$10\frac{1}{4}$

	L.	s.	d.
Ex. 42.	14476	5	$6\frac{1}{2}$
	7614	13	$8\frac{3}{4}$

	L.	s.	d.
Ex. 43.	222	18	$9\frac{3}{4}$
	142	7	$10\frac{1}{2}$

	L.	s.	d.
Ex. 44.	96481	16	9
	3768	10	$9\frac{1}{4}$

	L.	s.	d.
Ex. 45.	164	17	$8\frac{1}{2}$
	29	2	$9\frac{3}{4}$

	L.	s.	d.
Ex. 46.	18149	14	$0\frac{1}{2}$
	17216	0	$4\frac{1}{4}$

	L.	s.	d.
Ex. 47.	417	4	$10\frac{3}{4}$
	319	11	$7\frac{1}{2}$

	L.	s.	d.
Ex. 48.	20412	13	$9\frac{3}{4}$
	19911	14	$2\frac{1}{2}$

	L.	s.	d.
Ex. 49.	425	18	9
	139	10	$9\frac{3}{4}$

	L.	s.	d.
Ex. 50.	22425	14	$9\frac{1}{4}$
	21018	8	$11\frac{3}{4}$

	L.	s.	d.
Ex. 51.	183	9	$1\frac{1}{4}$
	24	14	$10\frac{3}{4}$

	L.	s.	d.
Ex. 52.	24463	13	$11\frac{1}{4}$
	17752	16	$9\frac{1}{2}$

	L.	s.	d.
Ex. 53.	421	16	$9\frac{3}{4}$
	326	19	0

	L.	s.	d.
Ex. 54.	86476	6	$9\frac{1}{4}$
	56117	13	10

	L.	s.	d.
Ex. 55.	433	17	$2\frac{3}{4}$
	311	19	$4\frac{1}{2}$

	L.	s.	d.
Ex. 56.	28446	17	9
	19994	14	$8\frac{1}{4}$

	L.	s.	d.
Ex. 57.	194	12	$8\frac{1}{4}$
	117	12	9

	L.	s.	d.
Ex. 58.	80490	9	9
	24689	15	$10\frac{1}{2}$

	L	s.	d.
Ex. 59.	474	19	$4\frac{1}{4}$
	362	13	$7\frac{1}{4}$

	L.	s.	d.
Ex. 60.	26475	13	9
	24716	18	$11\frac{3}{4}$

	L.	s.	d.
Ex. 61.	4559	16	$9\frac{3}{4}$
	3228	9	$5\frac{1}{4}$

	L.	s.	d.
Ex. 62.	34487	15	$11\frac{1}{2}$
	31767	19	10

	L.	s.	d.
Ex. 63.	2139	7	10
	1914	13	$10\frac{1}{2}$

	L.	s.	d.
Ex. 64.	36492	7	$5\frac{3}{4}$
	20082	0	$6\frac{1}{2}$

	L.	s.	d.
Ex. 65.	3471	19	$9\frac{1}{4}$
	293	19	$9\frac{3}{4}$

	L.	s	d.
Ex. 66.	38410	14	9
	28019	19	$10\frac{1}{2}$

	L.	s.	d.
Ex. 67.	4557	18	$9\frac{1}{2}$
	3945	17	$11\frac{3}{4}$

	L.	s.	d.
Ex. 68.	601273	11	7
	462104	15	$8\frac{3}{4}$

	L.	s.	d.
Ex. 69.	5734	11	3
	559	12	7

	L.	s.	d.
Ex. 70.	424136	11	$6\frac{1}{4}$
	379126	10	$9\frac{3}{4}$

	L.	s.	d.
Ex. 71.	7860	0	0
	3271	4	7

	L.	s.	d.
Ex. 72.	441391	6	$0\frac{1}{4}$
	389091	9	$8\frac{1}{2}$

Ex. 73.

L.	s.	d.
6234	6	6
309	12	$10\frac{1}{4}$

Ex. 74.

L.	s.	d.
1414	9	$9\frac{1}{4}$
729	12	$11\frac{3}{4}$

Ex. 75.

L.	s.	d.
1173	14	$9\frac{3}{4}$
437	18	$11\frac{1}{4}$

Ex. 76.

L.	s.	d.
484760	10	9
329189	19	$9\frac{1}{4}$

Ex. 77.

L.	s.	d.
791	5	$11\frac{1}{4}$
261	19	$11\frac{1}{2}$

Ex. 78.

L.	s.	d.
14112	0	$0\frac{1}{2}$
4612	19	1

Ex. 79.

L.	s.	d.
1345	19	$9\frac{3}{4}$
345	17	$9\frac{1}{2}$

Ex. 80.

L.	s.	d.
4621	15	$9\frac{1}{2}$
394	19	$0\frac{3}{4}$

Ex. 81.

L.	s.	d.
396	19	$9\frac{1}{4}$
29	19	$9\frac{3}{4}$

Ex. 82.

L.	s.	d.
254	14	$9\frac{1}{4}$
244	19	$10\frac{1}{2}$

Ex. 83.

L.	s.	d.
1214	0	5
880	0	$8\frac{3}{4}$

Ex. 84.

L.	s.	d.
564121	10	$10\frac{3}{4}$
379178	16	$10\frac{1}{4}$

Ex. 85.

L.	s.	d.
4465	10	$9\frac{1}{2}$
304	0	$11\frac{3}{4}$

Ex. 86.

L.	s.	d.
4532	13	$9\frac{1}{4}$
4319	15	$11\frac{3}{4}$

Ex. 87.

L.	s.	d.
408	19	$4\frac{1}{4}$
254	1	$10\frac{3}{4}$

Ex. 88.

L.	s.	d.
60985	14	$4\frac{3}{4}$
1427	19	$9\frac{1}{2}$

	L.	s.	d.
Borrowed	300	0	0
Paid at different times	15	15	0
	39	7	$7\frac{3}{4}$
	76	8	1
	45	15	10
	105	0	0
Paid -	283	6	$6\frac{3}{4}$
Unpaid.	16	13	$5\frac{1}{4}$

	L.	s.	d.
Borrowed	1000	0	0
Paid at different times	177	16	$7\frac{3}{4}$
	105	0	3
	52	10	11
	246	9	$9\frac{1}{4}$
	300	0	9
Paid -	881	18	4
Unpaid -	118	1	8

Suppose a person is debtor to sundry persons in the following sums.

L.	s.	d.
7678	14	$9\frac{3}{4}$
29	17	$4\frac{1}{4}$
5	5	0
1054	12	$9\frac{1}{4}$
26	5	0
7	7	0
95	19	$9\frac{1}{2}$
39	11	3
Dr.		

And is creditor, by book-debts from different people, in the following sums.

L.	s.	d.
764	14	$9\frac{3}{4}$
39	14	4
500	0	0
859	5	$9\frac{1}{2}$
2500	0	0
5505	5	11
3000	0	0
Cr.		
Dr.		

Balance in favour of Cr.

Required the balance of this account?

Dr.			Cr.		
L.	s.	d.	L.	s.	d.
764	14	9	397	14	$11\frac{3}{4}$
397	0	$10\frac{3}{4}$	267	11	9
210	19	$9\frac{1}{2}$	720	13	$8\frac{3}{4}$
467	16	$7\frac{3}{4}$	464	16	0
371	14	9	215	12	6
564	12	$6\frac{3}{4}$	345	9	$10\frac{1}{4}$

Required the balance of this account?

Dr.			Cr.		
L.	s.	d.	L.	s.	d.
769	19	$10\frac{1}{2}$	49	12	11
643	4	4	1900	17	$9\frac{3}{4}$
248	11	7	1706	5	5
591	8	4	4	4	0
9	19	6	250	12	$8\frac{3}{4}$
300	0	0	1750	17	0

EXAMPLES OF WEIGHTS AND MEASURES.

TROY WEIGHT.

	lb.	oz.	dwt.	gr.
Ex. 1.	187	9	12	20
	169	6	14	17

	lb.	oz.	dwt.	gr.
2.	256	6	0	22
	199	9	3	20

	lb.	oz.	dwt.	gr
3.	567	4	0	0
	379	11	9	9

	lb.	oz.	dwt.	gr.
4.	254	0	0	0
	253	11	19	20

	lb.	oz	dwt	gr.
5.	675	3	0	9	6
	567	9	17	16	

	lb.	oz.	dwt.	gr
6.	423	5	15	14
	246	11	18	23

	lb.	oz	dwt.	oz.	dwt.	gr.
7.	14	11	9	8. 74	12	18
	11	10	14	71	14	17

	lb.	oz.	dwt.	oz.	dwt.	gr.
9.	175	3	10	10. 17	10	20
	159	11	14	14	11	23

AVOIRDUPOIS WEIGHT.

	tons:	cwt.	qr.	lb.	oz.	dr.
1.	72	10	3	14	10	12
	9	16	1	25	14	6

	tons.	cwt.	qr.	lb.	oz.	dr.
2.	64	15	2	15	10	9
	46	15	3	5	12	14

	tons.	cwt.	qr.	lb.	oz.	dr.
3.	25	0	0	0	0	0
	24	0	2	0	0	15

	tons,	cwt.	qr.	lb.	oz.	dr.
4.	67	2	1	4	14	2
	29	14	3	2	0	14

	tons,	cwt.	qr.	lb.	oz.	dr.
5.	36	7	1	1	1	1
	30	3	2	5	5	5

	tons.	cwt.	qr.	lb.	oz.	dr.
6.	76	3	0	0	0	4
	67	12	2	0	14	4

	tons,	cwt.	qr.	cwt.	qr.	lb.
7.	14	12	2	8. 17	1	25
	1	14	3	14	2	27

	qr.	lb.	oz.	lb.	oz.	dr.
9.	143	22	12	10. 174	11	10
	74	19	14	39	12	13

APOTHECARIES WEIGHT.

	lb.	oz.	dr.	scr.
1.	456	9	4	0
	399	4	7	2

	lb.	oz.	dr.	scr.
2.	269	8	3	2
	178	11	3	1

	lb.	oz.	dr.	scr.
3.	987	4	4	0
	379	10	5	1

	lb.	oz.	dr.	scr.
4.	564	0	0	0
	469	3	3	2

	lb.	oz.	dr.	scr.
5.	375	7	7	1
	369	4	7	2

	lb.	oz.	dr.	scr.
6.	394	2	2	0
	299	11	7	2

	lb.	oz.	dr.
7.	144	10	5
	64	11	7

	oz.	dr.	scr.
8.	27	4	1
	14	7	2

	dr.	scr.	gr.
9.	27	1	14
	14	0	19

	lb.	oz.	dr.
10.	74	10	5
	65	11	6

CLOTH MEASURE.

	yds.	qr.	n.
Ex. 1.	218	2	0
	167	1	3

	E.e.	qr.	n.
2.	46	0	0
	23	2	2

	yds.	dr.	n.
3.	567	1	1
	469	0	2

	yds.	qr.	n.
4.	459	1	2
	399	3	3

	yds.	qr.	n.
5.	174	2	1
	39	3	2

	E.e.	qr.	n.
6.	174	3	1
	49	4	2

	E.e.	qr.	n.
7.	171	1	3
	74	4	2

	E.e.	qr.	n.
8.	12	1	1
	10	4	3

LONG MEASURE.

	yds.	ft.	in.	b.c.
Ex. 1.	456	2	10	1
	379	1	11	2

	yds.	ft.	in.	b.c.
2.	669	0	0	0
	599	1	1	1

	yds.	ft.	in.	bc.
3.	267	1	1	1
	199	2	2	2

	lea.	m.	fur.	p.
4.	470	1	4	19
	279	2	7	23

	lea.	m.	fur.	p.
4.	367	0	0	0
	179	2	5	23

	lea.	m.	fur.	p.
6.	225	1	1	1
	167	2	4	4

	lea.	m.	fur.		fur.	p.	yds.		p.	yd.	ft.		ft.	in.	b.c.
7.	21	2	4	8.	14	34	5	9.	14	3	1	10.	17	11	2
	3	2	6		12	39	5		9	4	2		14	11	1

LAND MEASURE.

	ac.	r.	p.		ac.	r.	p.		ac.	r.	p.		ac.	r.	p.
Ex. 1.	456	2	25	2.	457	1	29	3.	356	0	39	4.	594	1	1
	399	0	29		264	3	39		279	3	39		259	3	17
	ac.	r.	p.		ac.	r.	p.		ac.	r.	p.		ac.	r.	p.
5.	12	0	32	6.	112	1	31	7.	12	1	25	8.	19	1	20
	1	3	14		74	2	37		10	3	39		14	2	21

WINE MEASURE.

	tuns.	hhd.	gal.	qt.	pt.		tuns,	hhd.	gal.	qt.	pt.
Ex. 1.	456	2	24	1	0	2.	257	3	10	1	1
	399	3	46	3	1		199	0	50	3	1

	tuns,	hhd.	gal.	qt.	pt.		tuns,	hhd.	gal.
3.	467	2	0	0	0	4.	27	2	54
	299	3	32	2	1		19	3	62

	hhd.	gal.	qt.		hhd.	gal.	qt.		gal.	qt.	pt.
5.	147	14	2	6.	14	1	2	7.	24	2	0
	79	3	3		12	41	3		17	0	1

DRY MEASURE.

	bu.	pks.	gal.		bu.	pks.	gal.		bu.	pks.	gal.
Ex. 1.	86	3	1	2.	59	1	0	3.	62	0	0
	46	1	0		39	3	1		24	1	1

	pks.	gal.	qts.		pks.	gal.	qts.		pks.	gal.	qts.	pts.
Ex. 4.	67	0	2	5.	28	0	1	6.	74	1	1	1
	32	1	1		12	1	3		27	1	3	1

TIME.

	d.	hr.	mim.		d.	hr.	min.	sec.		mo.	w.	d.	hr.
Ex. 1.	37	2	39	2.	74	3	12	14	3.	46	1	1	4
	29	21	49		47	21	54	36		29	3	6	21

	w,	d.	hr.	m.	s.		ysr.	m.	w.		m.	w.	d.
4.	36	0	0	0	0	5.	17	10	2	6.	147	2	3
	35	6	23	50	59		14	12	3		19	2	4

		d.	hrs.	m.			hrs.	min.	sec.
7.		167	21	50	8.		174	50	51
		19	23	54			94	59	57

MISCELLANEOUS EXAMPLES IN SUBTRACTION.

Ex. 1. I borrowed of a friend five hundred guineas, and have paid at different times, three hundred and ninety pounds six shillings and seven pence three farthings: what have I still to pay?

Answer, 134*l.* 13*s.* $4\frac{1}{4}d.$

2. A horse and his harness are worth 175 dol., but the harness is worth 47 D. $37\frac{1}{2}$ cts. I demand the value of the horse? Ans. 127 D. $62\frac{1}{2}$ cts.

3. What sum added to 150 guineas, will make up 199*l.* 9*s.* $9\frac{1}{2}d$? Ans. 41*l.* 19*s.* $9\frac{1}{2}d.$

4. At an eclipse of the sun, the moon is situated between the earth and sun: how far distant is the moon from the sun, supposing the distance between the earth and the sun 95 millions of miles, and that between the earth and moon 240 thousand?

Ans. 94760000 miles.

5. The great bell at Oxford weighs 7 tons, 11 cwt. 3 qrs. 4 lb.; that at St. Paul's 5 tons, 2 cwt. 1 qr. 22 lb.; and the great *Tom* of Lincoln weighs 4 tons. 16 cwt. 3 qrs. 16 lb.: how much heavier than these together is the great bell at Moscow, which is 198 tons?

Ans. 180 tons, 8 cwt. 3 qrs. 14 lb.

6. The Royal Exchange cost 80 thousand pounds in building; the Mansion-house 40 thousand; Blackfriars-bridge, 153 thousand; Westminster-bridge, 389 thousand; and the Monument, 13 thousand pounds; but the Cathedral of St. Paul's cost 800 thousand: how much did this cost more than all the rest? Ans. 125000*l.*

7. If my income is 367*l.* 8*s.* $4\frac{1}{2}$*d.* and my expenditure be 340 guineas: how much can I lay by?

Ans. 10*l.* 8*s.* $4\frac{1}{2}$*d.*

8. A person, by great losses, was obliged to call his creditors together: he found his whole property amount to 527*l.* 12*s.* $8\frac{3}{4}$*d.*; but he owed to one man 150*l.*; to another 300 guineas; to a third 20 crowns; to a fourth 55*l.* 8*s.* $9\frac{1}{2}$*d.*; and to a fifth 200 guineas: how much will they be losers? Ans. 207*l.* 16*s.* $0\frac{3}{4}$*d.*

9. A gentleman leaves between his two children 50,000 dollars; to the younger he leaves 17478 dollars: what was the fortune of the elder? Ans. 32522 Dollars.

10. An apprentice has served of his term of seven years, three years, two months, three weeks, four days, seventeen hours: how much longer has he to serve?

Ans. 3 yrs. 10 m. 0 w. 2 da. 7 ho.

11. From a field of $6\frac{1}{2}$ acres, I take out two gardens, one measuring $4\frac{1}{2}$ roods, and the other $2\frac{1}{4}$ roods, and a piece of ground for coach-house and stables, that measures 1 rood and 12 perches: what will be the size of the field after these pieces are taken away?

Ans. 4 ac. 1 r. 38 poles.

12. A plumber puts lead upon the different parts of my house that weighs 5 cwt. 3 qr.; and he takes away, in return, old lead weighing 2 cwt. 24 lb.: what is the difference in the weight between the new and the old lead? Ans. 3 cwt. 2 qrs. 4 lb.

COMPOUND MULTIPLICATION

Is the method of finding the amount of any given number of different denominations, by repeating it any number of times:

I. When the given multiplier does not exceed 12.

Rule. Write the multiplier under the lowest denomination of the multiplicand, multiply every number of the multiplicand by the multiplier, and bring the several products as they occur, to the next higher denomination. Write down the remainders, and carry the integers to the next product.

Ex. Multiply *L.*768 14*s.* 9½*d.* by 9.

L.	*s.*	*d.*
768	14	9½
		9
6918	13	1½

Dlls.	cts.		Dlls.	cts.	
Ex. 1. 79	14	× 3	Ex. 2. 84	62½	× 6
Ex. 3. 67	37½	× 4	Ex. 4. 79	25	× 5

Ea.	D.	d.	cts.		Ea.	D.	d.	cts.	mls.	
Ex. 5. 7	7	5	4	× 7	Ex. 6. 6	3	4	7	6	× 9

Ea.	D.	cts.		Ea.	D.	cts.	
Ex. 7. 74	3	50	× 8	Ex. 8. 29	4	43¾	× 12

	L.	s.	d.				L.	s.	d.		
Ex. 9.	3987	4	$6\frac{1}{2}$	×	2	Ex. 10.	3564	10	$7\frac{1}{2}$	×	3
11.	2987	3	$9\frac{3}{4}$	×	5	12.	2648	16	$8\frac{1}{4}$	×	5
13.	3487	12	8	×	6	14.	3498	2	$6\frac{3}{4}$	×	7
15.	5694	16	$11\frac{1}{4}$	×	8	16.	2691	18	$11\frac{1}{4}$	×	9
17.	3764	12	$8\frac{1}{2}$	×	10	18.	3465	15	$10\frac{1}{2}$	×	11
19.	4610	15	4	×	12	20.	3591	19	$9\frac{1}{2}$	×	4
21.	1456	16	10	×	12	22.	2761	14	4	×	6
23.	3420	13	$5\frac{1}{2}$	×	10	24.	4694	12	7	×	8
25	2675	19	$3\frac{3}{4}$	×	9	26.	3476	17	$8\frac{3}{4}$	×	5
27.	4675	17	$8\frac{1}{2}$	×	11	28.	4900	0	$9\frac{1}{2}$	×	7

II. When the multiplier is a composite number, and can be resolved into two or more component parts.

RULE. Multiply by its component parts successively, and the last product will be the answer.

Ex. Multiply *L*.374 10s. $11\frac{3}{4}$*d*. by 63.

L.	s.	d.	
374	10	$11\frac{3}{4}$	× 63 = 9 × 7
		9	
3370	18	$9\frac{3}{4}$	
		7	
Ans. 23596	11	$8\frac{1}{4}$	

EXAMPLES.

	L.	s.	d.				L.	s.	d.		
Ex. 1.	456	12	$9\frac{1}{2}$	×	15	Ex. 2.	436	14	$3\frac{1}{4}$	×	16
3.	784	15	4	×	18	4.	397	16	10	×	21
5.	674	18	$10\frac{3}{4}$	×	22	6.	487	16	$9\frac{1}{2}$	×	24
7.	245	10	3	×	30	8.	376	15	11	×	30
9.	246	19	$9\frac{1}{4}$	×	35	10	489	18	$8\frac{1}{4}$	×	42
11.	397	13	3	×	48	12.	369	10	2	×	54
13.	384	15	$10\frac{1}{2}$	×	56	14.	565	12	$9\frac{3}{4}$	×	63
15.	592	12	9	×	66	16.	800	9	8	×	72
17.	911	13	$2\frac{3}{4}$	×	84	18.	914	16	4	×	77
19.	397	4	$4\frac{1}{4}$	×	96	20.	374	12	$5\frac{1}{4}$	×	108
21	459	9	$9\frac{3}{4}$	×	100	22.	279	13	3	×	120
23.	376	15	4	×	121	24	347	3	9	×	132
25.	376	4	$9\frac{3}{4}$	×	144	26.	567	14	7	×	45
27.	897	16	6	×	108	28.	675	13	$3\frac{3}{4}$	×	88
29.	487	19	$11\frac{3}{4}$	×	121	30.	856	12	2	×	132

III. When the multiplier is not a composite number.

RULE. Take the composite number which is nearest to it, and multiply by the component parts, as before: then add or subtract as many times the first line, as the composite number is less or greater than the given multiplier.

(1) Multiply *L*.324 12*s*. $6\frac{1}{2}d$. by 394.

L.	*s.*	*d*		
324	12	$6\frac{1}{2}$	× 394 = 8 × 7 × 7 + 2.	
		8		
2597	0	4		
		7		
18179	2	4		
		7		
127253	16	4		
649	5	1		
127903	1	5		

The nearest composite number is 392 = 8 × 7 × 7; I accordingly multiply by these three figures, and to the product I *add* twice the original sum, which gives the true answer.

EXAMPLES.

	L	*s.*	*d.*				*L.*	*s.*	*d.*		
Ex. 1.	574	12	$6\frac{1}{2}$	×	38	Ex. 2.	387	18	$7\frac{1}{4}$	×	46
3.	325	8	4	×	58	4.	222	12	$8\frac{3}{4}$	×	68
5.	236	18	$9\frac{3}{4}$	×	78	6	136	14	5	×	94
7.	300	0	$3\frac{1}{2}$	×	273	8.	246	12	$0\frac{1}{4}$	×	359
9.	525	16	$0\frac{3}{4}$	×	412	10.	326	18	3	×	687
11.	239	9	9	×	740	12.	560	0	$2\frac{1}{4}$	×	388
13.	660	15	$4\frac{1}{4}$	×	1004	14.	407	13	1	×	1325
15.	700	0	$0\frac{3}{4}$	×	1450	16.	110	10	11	×	1208

EXAMPLES OF WEIGHTS AND MEASURES.

TROY WEIGHT.

	lb.	oz.	dwt.	gr.			lb.	oz.	dwt.	gr.		
Ex. 1.	187	9	12	20	×	4	Ex. 2. 256	6	0	22	×	5
3.	169	6	14	17	×	6	4. 379	11	9	9	×	7
5.	254	3	3	3	×	9	6. 253	11	4	20	×	8
7.	675	4	15	10	×	11	8. 375	0	0	17	×	12

AVOIRDUPOIS WEIGHT.

	tons.	cwt.	qr.	lb.	oz.	dr.			tons.	cwt.	qr.	lb.	oz.	dr.	
Ex. 1.	12	10	3	14	10	12	× 2	Ex. 2.	64	13	2	15	6	8	× 4
	3. 25	0	2	8	4	4	× 3		4. 46	15	3	12	4	4	× 6
	5. 75	13	0	18	6	10	× 8		6. 39	12	2	16	10	8	× 9

APOTHECARIES' WEIGHT.

	lb.	oz.	dr.	sc.				lb.	oz.	dr.	se.		
Ex. 1.	456	8	4	1	×	5	Ex. 2.	748	5	2	2	×	8
3.	534	7	6	2	×	12	4.	378	10	0	1	×	11
5.	321	5	4	1	×	10	6.	491	5	7	2	×	9

CLOTH MEASURE.

	yds.	qr.	nl.			E.e.	qr.	nl.			yds.	qr.	nl.	
Ex. 1.	210	2	1	×	4	2. 378	4	3	×	7	3. 596	3	1	× 12
4.	357	1	3	×	6	5. 738	3	2	×	9	6. 876	0	3	× 10

LONG MEASURE.

	yds.	ft.	in.	b.c.				lea.	m.	fur.	p.	
Ex. 1.	556	2	10	1	×	5	Ex. 2.	379	1	6	20	× 7
3.	369	1	9	2	×	8	4.	376	2	5	37	× 9
5.	241	2	11	1	×	10	6.	674	2	7	18	× 6

LAND MEASURE.

	ac.	r.	p.				ac.	r.	p.		
Ex. 1.	456	0	25	×	11	Ex. 2.	597	3	12	×	12
3.	371	2	18	×	4	4.	271	2	25	×	10
5.	189	3	32	×	8	6.	430	0	12	×	8

LIQUID MEASURE.

	tuns,	hhd.	gal.	qts.	p.			tuns,	hhd.	gal.	qts.		
Ex. 1.	456	3	28	2	1	× 4	2.	456	3	46	2	×	6
3.	374	2	60	3	1	× 8	4.	350	2	25	1	×	2
5.	221	1	4	1	0	× 5	6.	124	3	50	3	×	10

DRY MEASURE.

	bu.	pks.	gal.				bu.	pks.	gal.		
Ex. 1.	29	2	1	×	3	Ex. 2.	29	3	1	×	5
	3. 76	3	0	×	4		4. 27	2	1	×	6
	pks.	gal.	qts.	pts.			bu.	pks.	gal.	qts.	
5.	34	1	3	1	× 7	6.	64	2	1	2	× 8
7.	76	1	2	0	× 9	8.	37	1	1	3	× 11
9.	62	0	3	1	× 12	10.	64	3	1	2	× 12

TIME.

	w.	d.	hrs.	m.	s.				yrs.	mo.	w.	d.	
Ex. 1.	73	6	10	40	30	×	5	Ex. 2.	594	12	3	4	× 7
3.	36	4	12	15	20	×	9	4.	364	8	2	6	× 8
5.	98	5	17	13	55	×	12	6.	443	10	3	3	× 11

MISCELLANEOUS EXAMPLES.

Ex. 1. What cost 12 lb. of tea at 1 dol. 50 cts. per lb.?
Answer. 18 dolls.

2. What cost 16½ lb. of sugar, at 1*s.* 1½*d.* per. lb.?
Ans. 18*s.* 6¾*d.*

3. What is the value of 24 yards of Irish linen, at 3*s.* 6½*d.* per yard? Ans. 4*l.* 5*s.*

4. What will 79 bibles come to, at 1 dol. 12½ cts. each? Ans. 88 dol. 87½ cts.

5. What is the value of 85 gallons of brandy, at 19*s.* 9½*d.* per gallon? Ans. 84*l.* 2*s.* 3½*d.*

6. What is the weight of 28 ingots of gold, each weighing 6 lb. 7 oz. 15 dwts. 20 gr.?
Ans. 186 lb. 2 oz. 3 dwt. 8 gr.

7. What will 157 oxen cost at 15*l.* 5*s.* 9*d.* each?
Ans. 2400*l.* 2*s.* 9*d.*

8. What is the value of 576 sheep, at 1*l.* 6*s.* 3*d.* each?
Ans. 756*l.* 0*s.* 0*d.*

9. How much must I pay for 759 chaldrons of coals, at 58*s.* 6*d.* per chaldron? Ans. 2220*l.* 1*s.* 6*d.*

10. What is the value of 199 firkins of ale, at 12*s* 6*d.* per firkin? Ans. 124*l.* 7*s.* 6*d.*

11. What is the value of 245 yards of broad cloth, at 19*s.* 7*d.* per yard? Ans. 239*l* 17*s.* 11*d.*

12. What is the worth of a stack of hay, containing 75 loads, at 3*l.* 19*s.* 9*d.* per load? Ans. 299*l.* 1*s.* 3*d.*

13. What is the worth of 12½ lb. of coffee, at 25 cts. per lb? Ans. 3 dol. 12½ cts.

14. How many pounds sterling are there in 28 purses, each containing 15 guineas, 15 half-guineas, 15 seven-shilling pieces, and three crowns? Ans. 829*l.* 10*s.* 0*d.*

15. What is the weight of 1000 guineas, each guinea weighing 5 dwts. 9½ gr.? Ans. 22 lb. 5 oz. 15 dwt. 20 gr.

16. I bought at a sale 47½ dozen of port wine, at 2*l.* 5*s.* 6*d.* per dozen, how much money must I send to pay for it? Ans. 108*l.* 1*s.* 3*d.*

17. What is the value of 85 tons of iron at 18*l.* 17*s.* 9½*d.* per ton? Ans. 1605*l.* 12*s.* 3½*d.*

18. What do 79 packages of goods weigh, supposing that each package weighs 3 cwt. 3 qrs. 15 lb.?
Ans. 15 tons. 6 cwt. 3 qr. 9 lb.

19. If one ounce of gold cost 3*l.* 16*s.* 8*d*, what is the value of 436½ ounces? Ans. 1673*l.* 5*s.* 0*d.*

20. What shall I pay annually for 459 acres of land, at 2 dol. 37½ cts. per acre? Ans. 1090 dol. 12½ cts.

21. What is the price of 185 gallons of rum, at 13*s.* 6½*d* per gal.? Ans. 125*l.* 5*s.* 2½.

22. If a man spend 1 dol. 62½ cts. per day, how much does he expend in a year? Ans. 593 dol. 12½ cts.

23. How much federal money in 49*l.* sterling, allowing 4 dol. 44 cts. to a pound sterling?
Ans. 217 dol. 56 cts.

BILLS OF PARCELS.

A MERCER'S BILL.

		L.	*s.*	*d.*		*L.*	*s.*	*d.*
12 yards of silk, at	-	0	15	2	per yard			
114 Do. of flowered silk at		0	18	7½	-			
16 Do. of velvet, at	-	1	2	4	-			
12 Do. of satin, at	-	0	13	9	-			
27 Do. of brocade, at	-	0	15	7	-			
14 Do. of lustring, at	-	0	6	3	-			

A STATIONER'S BILL.

		L.	*s.*	*d*		*L.*	*s.*	*d.*
250 Reams of paper, at	-	1	2	6	per ream			
112 Do. do. at	-	2	4	6	-			
34 Do of imperial brown at		1	15	0	-			
500 Dutch quills, at	-	0	3	9	per hun.			
2500 Do. common, at	-	0	2	3	-			

A CARPENTER'S BILL.

	s.	d.		L.	s.	d.
65 cubick feet of oak, at -	4	3	per foot			
125 Do. wrought and framed, at	5	8	-			
176 Do. fir framed and moulded, at - - -	3	6	-			
15 square shed roofing, at	5	6	per square			
8 Do. hip and valley roofing, at - -	8	3				
70 feet water trunk, at -	9	10	per foot			
364 feet ovolo wainscot sashes, at - - -	0	9	-			
124 Do. do. mahogany, at	1	4	-			
10 men's labour, for 25 days, at	4	8	per day			

A BRICKLAYER'S BILL.

	L.	s.	d.		L.	s.	d.
39 rod of grey-stock brickwork, at -	13	13	0	per rod			
7 Do. in party wall, at	7	15	0	-			
105 feet of 18 inch drain, at	0	3	0	per foot			
1050 Do. of pointing old work, at - - -	0	0	$5\frac{1}{2}$	-			
1500 grey stocks, at -	0	4	6	per hun.			
125 pan-tiles, at -	0	0	$1\frac{1}{2}$	each			
45 hods of mortar, at -	0	0	7	-			
13 Do. of tarras, at -	0	4	2	-			
15 bricklayers, 25 days, at	0	4	6	per day			
12 labourers, ditto, at -	0	3	0	-			
66 load of rubbish carted away, at	0	2	6	pr load.			

A SLATER'S BILL.

		L.	s.	d.		L.	s.	d.
9	square of Westmoreland slating, at -	2	19	6	per square			
7	do. of Welsh ladies, at	1	17	4	-			
5	Do. of Welsh countess, at	1	18	3	-			
35	Do. of ripped and rubbish cleared, at	0	2	7	-			
12	slaters 7 days, at	0	4	5	per day			
6	labourers, do. at	0	2	9	-			
5050	clout nails, at	0	0	4	per hun.			

PAINTER'S ADN GLAZIER'S BILL.

		s.	d.		L.	s.	d.
1035	yards of painting 3 times in oil, at -	0	$7\frac{1}{2}$	per yard			
565	Do. do. and sand, at	1	3	-			
36	sash frames, at -	0	11	each			
432	sash squares, at -	0	$8\frac{1}{2}$	per doz.			
1265	feet of best Newcastle glass, at -	1	$7\frac{1}{2}$	per foot			
356	Do. large size, at	2	$1\frac{1}{2}$				
1000	Do. in lead work, at	1	$0\frac{1}{2}$				

COMPOUND DIVISION,

Is the method of finding how often one given number is contained in another of different denominations; or, to divide a given compound number into any proposed number of equal parts.

I. When the given divisor does not exceed 12.

RULE. Place the divisor to the left-hand of the dividend. Divide the highest denomination of the dividend by the divisor, and write down the quotient; reduce the remainder, if any, into the next lower denomination, adding to it the number which stands in that place of

the dividend, and divide as before, and so proceed to the end.

Ex. 1695*l.* 14*s.* $4\frac{1}{2}$*d.* ÷ 8.

	L.	*s.*	*d.*
8)	1695	14	$4\frac{1}{2}$
	211	19	$3\frac{1}{2}$—2
			8
Proof	1695	14	$4\frac{1}{2}$

EXAMPLES.

	D. cts.		D. cts.
Ex. 1.	74 50 ÷ 3	Ex. 2.	62 25 ÷ 4
3.	56 $37\frac{1}{2}$ ÷ 5	4.	79 $18\frac{3}{4}$ ÷ 6
5.	49 49 ÷ 7	6.	63 25 ÷ 8
	Ea. D. cts.		Ea. D. cts.
7.	43 7 $37\frac{1}{2}$ ÷ 9	8.	56 6 25 ÷ 10
9.	17 4 50 ÷ 11	10.	13 7 75 ÷ 12

	L.	*s.*	*d.*		*L.*	*s.*	*d.*
Ex. 11.	457	8	$9\frac{1}{2}$ ÷ 3	Ex. 12.	579	18	$4\frac{1}{2}$ ÷ 2
13.	396	18	$7\frac{1}{4}$ ÷ 4	14.	768	2	$6\frac{1}{2}$ ÷ 5
15.	474	12	10 ÷ 6	16.	931	14	5 ÷ 7
17.	897	16	4 ÷ 8	18.	256	17	$10\frac{3}{4}$ ÷ 10
19.	759	0	0 ÷ 9	20.	694	19	6 ÷ 12
21.	101	15	$9\frac{1}{2}$ ÷ 11	22.	496	0	0 ÷ 12
23.	900	0	0 ÷ 8	24.	500	5	5 ÷ 4
25.	800	10	2 ÷ 7	26.	270	17	$7\frac{1}{2}$ ÷ 6
27.	464	3	$9\frac{3}{4}$ ÷ 6	28.	901	1	1 ÷ 9

II. When the divisor is a composite number.

RULE. Divide by the component parts of the divisor successively, and the last quotient will be the answer.

Ex. *L*148 8*s.* $8\frac{1}{2}$*d.* ÷ 27 = 3 + 9.

	L.	*s.*	*d.*
3)	148	8	$8\frac{1}{2}$
9)	49	9	$6\frac{3}{4}$—1
	5	9	$11\frac{1}{4}$—6

$\left.\begin{matrix}\\\\\end{matrix}\right\} = \frac{1}{2}\frac{9}{7}$

The answer is 5*l.* 9*s.* $11\frac{1}{4}$*d.* $\frac{1}{2}\frac{9}{7}$.

	L.	s.	d.			L.	s.	d.	
Ex. 1	167	12	$6\frac{1}{2}$	÷ 14	Ex. 2.	769	9	$8\frac{1}{4}$	÷ 20
3.	339	15	$8\frac{1}{4}$	÷ 15	4.	594	7	6	÷ 25
5.	486	9	9	÷ 16	6.	333	10	$10\frac{1}{4}$	÷ 28
7.	987	0	$0\frac{3}{4}$	÷ 18	8.	498	9	$9\frac{1}{2}$	÷ 32
9.	439	5	$6\frac{1}{2}$	÷ 24	10.	596	12	$7\frac{1}{4}$	÷ 36
11.	379	18	7	÷ 27	12.	465	11	11	÷ 44
13.	487	9	$9\frac{3}{4}$	÷ 30	14.	564	13	$5\frac{1}{2}$	÷ 49
15.	596	4	6	÷ 33	16.	678	6	3	÷ 54
17.	854	3	$6\frac{3}{4}$	÷ 42	18.	999	9	8	÷ 56
19.	327	14	4	÷ 48	20.	564	4	6	÷ 63
21.	387	12	11	÷ 72	22.	248	3	0	÷ 84
23.	565	11	8	÷ 88	24.	505	5	$5\frac{1}{2}$	÷ 99
25.	674	18	$8\frac{1}{2}$	÷ 108	26.	564	2	2	÷ 120
27.	465	3	3	÷ 132	28.	888	8	8	÷ 144

When there are three component parts.

Ex. *L.* 1350 10*s.* 11*d.* ÷ 240 = 5 × 6 × 8.

	L.	*s.*	*d.*	
	5)1350	10	11	
	6)270	2	2 — 4	
	8)45	0	$4\frac{1}{4}$ — 2	8
	5	12	$6\frac{1}{2}$ — 1	$\frac{44}{240}$

Ex. 1. *L.* 5527 10*s.* $6\frac{1}{2}$*d.* ÷ 243. 2. 18568*l.* 12*s.* $1\frac{1}{2}$*d.* ÷ 1296.

III. When the divisor is greater than 12, and not a composite number?

Rule. The several quotients must be found by the method of Long Division, (see pp. 28 and 29), reducing the remainders to the next lower denomination, and taking in those numbers of the dividend which are of the same denomination.

Ex. Divide *L.*1350 10s. 11*d.* by 242.

L.	*s.*	*d.*	
242)1350	10	11(5	
1210			
140			
20			
242)2810(11			
2662			
148			
12			
242)1787(7			
1694			
93			
4			
242)372($\frac{1}{4}$			
242			
130			

	D.	cts.			
Ex. 1.	234	50	÷	17	
3.	427	$62\frac{1}{2}$	÷	37	
	Ea.	D.	cts.		
5.	17	3	8 $\frac{1}{2}$	÷	59
7.	723	4	35	÷	74

	D.	cts.			
Ex. 2.	627	25	÷	26	
4.	317	75	÷	43	
	Ea.	D.	cts.		
6.	127	7	$12\frac{1}{2}$	÷	68
8.	319	4	50	÷	89

	L.	*s.*	*d.*		
9.	985	18	9	÷	19
11	465	16	$4\frac{1}{2}$	÷	29
13.	565	13	3	÷	37
15.	800	8	$8\frac{1}{2}$	÷	41
17.	987	14	4	÷	46
19.	598	12	6	÷	67
21.	483	6	6	÷	73
23.	986	5	$9\frac{3}{4}$	÷	89
25.	1485	19	2	÷	107
27.	2690	12	3	÷	166

	L.	*s.*	*d.*		
10.	1001	12	$11\frac{1}{2}$	÷	23
12.	2468	13	$3\frac{1}{2}$	÷	39
14.	5746	9	6	÷	59
16.	6321	3	3	÷	61
18.	4268	12	8	÷	69
20.	4821	9	$7\frac{1}{2}$	÷	87
22.	5943	16	6	÷	97
24.	3618	4	6	÷	97
26.	4683	15	$5\frac{1}{2}$	÷	376
28.	5649	9	9	÷	439

	L.	s.	d.			L.	s.	d.	
29.	6259	11	6	÷ 215	30.	3604	10	0	÷ 509
31.	9654	7	$7\frac{3}{4}$	÷ 649	32.	6534	16	$3\frac{1}{2}$	÷ 606
33.	5942	17	$3\frac{1}{4}$	÷ 757	34.	4593	12	4	÷ 1585
35.	4628	5	9	÷ 1001	36.	5349	0	0	÷ 4786
37.	1456	16	7	÷ 3761	38.	9504	1	$1\frac{1}{4}$	÷ 8078

IV. When the divisor consists of a number not exceeding 12, with one or more cyphers.

RULE. Cut off, by a line, as many places in the pounds as there are cyphers in the divisor, and divide by short division; then reduce the remainder to the next lower denomination, as in the last rule.

Ex. Divide *L*.5645 14*s*. 4*d*. by 1200.

12.00)56 45 14 4

L. 4 — 845
20

12.00)169.14

s. 14—114
12

12.00)13.72

d. 1—172

EXAMPLES OF WEIGHTS AND MEASURES.

TROY WEIGHT.

		lb.	oz.	dwt.	gr.				lb.	oz.	dwt.	gr.	
Ex.	1.	287	9	12	20	÷ 4	Ex. 2.	356	6	0	22	÷ 5	
	3.	269	6	14	7	÷ 6	4.	379	11	9	0	÷ 7	
	5.	854	3	3	3	÷ 9	6.	355	11	4	20	÷ 8	
	7.	675	4	15	10	÷ 11	8.	775	0	0	17	÷ 12	

AVOIRDUPOIS WEIGHT.

	tons	cwt.	qr.	lb.	oz.	dr.			tons	cwt.	qr.	lb.	oz.	dr.	
1.	412	10	3	14	10	12	÷ 2	2.	664	13	1	12	6	8	÷ 4
3.	526	0	0	18	6	6	÷ 3	4.	464	0	3	27	0	3	÷ 6
5.	678	2	2	2	8	2	÷ 8	6.	591	5	0	4	3	12	÷ 9

APOTHECARIES WEIGHT.

	lb.	oz.	dr.	scr.			lb.	oz.	dr.	scr.	
Ex. 1.	591	8	4	1	÷ 5	Ex. 2.	748	5	7	0	÷ 8
3.	639	1	1	2	÷ 12	4.	392	10	6	0	÷ 11
5.	487	2	0	0	÷ 10	6.	421	4	5	1	÷ 9

CLOTH MEASURE.

	yds.	qr.	n.				E.e.	qr.	n.	
Ex. 1.	5210	2	1	÷ 4	Ex. 2.	5964	3	1	÷ 11	
3.	3976	1	2	÷ 6	4.	7645	4	2	÷ 12	
5.	4721	0	0	÷ 8	6.	3492	0	3	÷ 9	

LONG MEASURE.

	yds.	ft.	in.	b.c.			lea.	m.	fur.	p.	
Ex. 1.	5946	2	10	1	÷ 5	Ex. 2.	3795	2	7	30	÷ 7
3.	4736	1	8	2	÷ 8	4.	4965	1	3	18	÷ 9
5.	2005	0	11	2	÷ 10	6.	6743	2	6	4	÷ 6

LAND MEASURE.

	ac.	r.	p.			ac.	r.	p.	
Ex. 1.	654	2	24	÷ 11	Ex. 2.	958	3	12	÷ 12
3.	371	0	18	÷ 4	4.	379	0	25	÷ 10
5.	891	3	32	÷ 8	6.	496	1	1	÷ 8

LIQUID MEASURE.

	tuns,	hhd.	gal.	qts.	pt.	
Ex. 1.	456	3	27	2	1	÷ 4
2.	656	3	31	2	0	÷ 6
3.	594	0	30	3	0	÷ 8
4.	391	2	25	1	0	÷ 3
5.	271	0	0	2	0	÷ 6
6.	421	3	50	3	0	÷ 10

DRY MEASURE.

	bu.	pks.	gal.				bu.	pks.	gal.	
Ex. 1.	16	2	1	÷ 3	Ex. 2.	87	3	1	÷ 5	

	pks.	gal.	qts.	pts.				pks.	gal	qts.	pts.	
Ex. 3.	327	1	3	0	÷ 7	Ex. 4.	219	0	2	0	÷ 9	
5.	129	0	2	1	÷ 11	6.	99	1	3	0	÷ 12	

TIME.

	w.	d.	hrs.	m.	sec.				yrs.	mo.	w.	d.		
Ex. 1.	779	6	20	40	25	÷	5	Ex. 2.	594	12	2	4	÷	7
3.	391	4	12	16	12	÷	9	4.	954	6	3	5	÷	6
5.	913	0	4	0	5	÷	12	6.	348	10	3	3	÷	11

MISCELLANEOUS EXAMPLES.

Ex. 1. If 17 yards of cloth cost 19*l.* 3*s.* 9*d.*, what is it per yard ? Answer. 1*l.* 2*s.* $6\frac{3}{4}d.\frac{9}{17}$.

2. What is the price of one pound of sugar. if 8lb. cost nine shillings ? Ans. 1*s.* $1\frac{1}{2}d.$

3. The expenses of a journey amounting to 97*l.* 9*s.* 6*d.* are to be defrayed by six persons : how much will each have to pay ? Ans. 16*l.* 4*s.* 11*d.*

4. I have bought 12 gallons of wine for 32 dollars 50 cts. ; how much is that per gallon ?

Ans. 2 dolls. 70 cts

5. Twelve boys are to have a guinea and a half divided among them : what will be each boy's share ?

Ans. 2*s.* $7\frac{1}{2}d.$

6. A hundred and twenty-five sailors have taken 8465*l.* prize money : how much will each man be entitled to ?

Ans. 67*l.* 14*s.* $4\frac{3}{4}d.\frac{25}{125}$.

7. I have bought 144 pair of stockings for 27*l.* ; at what rate can I sell them so as to gain by each pair one shilling ? Ans. 4*s.* 9*d.*

8. What did I pay a piece for sheep, having bought 75 for 135*l.* ? Ans. 1*l.* 16*s.*

9. Cheese at 3*l.* 12*s.* 6*d.* per cwt. : how much is that per lb. ? Ans. $7\frac{3}{4}d.\frac{8}{112}$.

10. If 81 oxen cost 1781*l.* 12*s.* 6*d.* : what is the value of one ? Ans. 21*l.* 19*s.* $10\frac{3}{4}d.\frac{5}{9}$.

11. If a pipe of wine cost 95*l.* : how much is that a dozen, which contains three gallons ?

Ans. 2*l.* 5*s.* 2*d.* $\frac{108}{126}$.

12. Bought 50 dozen of wine for a hundred guineas : how much is that per bottle ? Ans. 3*s.* 6*d.*

13. Divide a thousand guineas between 23 people, and see how much it is for each ? Ans. 45*l.* 13*s.* $0\frac{1}{2}d.\frac{2}{25}$.

14. If 12 pieces of linen cloth contain 250 yards, what is the length of a single piece ?

Ans. 20 yds. 3 qr. $1\frac{4}{12}$ nail.

15. How much can I afford to spend a day, a week, and a month, if my income be 500*l.* per annum, allowing 52 weeks, or 13 months to a year ?

Ans. 1*l.* 7*s.* $4\frac{3}{4}$*d.* per day.
9*l.* 12*s.* $3\frac{1}{2}$*d.* per week.
88*l.* 9*s.* $2\frac{2}{3}$*d.* per month.

16. If 12 tea-spoons weigh 9 oz. 17 dwt. 12 gr. : what is the weight of each spoon ? Ans. 16 dwts. 11 gr.

MISCELLANEOUS QUESTIONS.

Ex. 1. It is said that Syrius, or the Dog Star, is the nearest of all the fixed stars, and that its distance is computed at 2,200,000,000,000 miles ; how many years, (each containing 365 days, 6 hours exactly,) would a cannon ball be in passing from the earth to Sirius, supposing it travelled at the rate of 480 miles per hour ?

Ans. $522853\frac{188896}{420768}$.

Ex. 2. The Planet Mercury is about thirty-seven millions of miles from the Sun ; Venus sixty-eight millions ; the Earth ninety-five millions ; Mars a hundred and forty five millions ; Jupiter four hundred and ninety-three millions ; Saturn nine hundred and eight, and the Herschel one thousand eight hundred millions of miles from the Sun : put these several distances down in figures, and add them together as a sum in Addition.

Ans. 3546.000.000

Ex. 3. How much nearer the Sun, is Mercury than Mars; and how much farther is the Herschel than the Earth ? See Ex. 2. Ans. Mercury 108 millions nearer the sun than Mars, and Herschel 1705 millions further from the Sun than the Earth.

Ex. 4. The beautiful planet Venus travels, in her annual journey round the Sun, at the rate of 75,000 miles in an hour : how many miles does she travel in one of her years, or in $228\frac{1}{4}$ days ? Ans. 410.850.000

Ex. 5. The Earth travels, in her annual course, at the rate of 68.400 miles in an hour : how many miles therefore do we move in a second ? Ans. 19

Ex. 6. There are in the Old Testament 39 books, and 929 chapters, and in the New there are 27 books, and 260 chapters: how many books and chapters are there in the Bible? Ans. 66 books, and 1189 chapters.

Ex. 7. There are 23214 verses in the Old Testament, and 7959 in the New: how much therefore do the verses in the former exceed those in the latter? Ans. 15255

Ex. 8. There are 592439 words in the Old Testament, and 181253 in the New: how many words are there in the Bible? Ans. 773692

Ex. 9. In the Old Testament there are 2,728,100 letters, and in the New there are 838.380: what are the sum and difference of these two numbers?

Ans. 3.566.480 sum, 1.889.720 difference.

Ex. 10. There are in the Bible 3.566,480 letters: how long would a person be in counting them, supposing he could count 200 in a minute? Ans. 297 hrs. 12 minutes.

Ex. 11. A printer charges $5\frac{1}{4}d.$ for every 1000 letters that he sets up: how many thousand must he set up to earn $1l.$ $15s.$ per week? Ans. 80,000

Ex. 12. If a printer set up 8500 per day, how long would he be in composing the Old Testament, and how long in composing the whole Bible? See Ex. 9 and 10.

Ans. 321 days Old Test. and $419\frac{3}{4}$ Bible, nearly.

Ex. 13. If a printer be desired to set up the Bible in Latin, how much would he earn in the business, at the rate of $5\frac{3}{4}d.$ per 1000 letters, supposing there are as many letters in the Latin as there are in the English?

Ans. $85l.$ $8s.$ $11\frac{1}{2}d.$ $\frac{4}{100}$.

Ex. 14. If there be as many letters in the Greek Testament as there are in the English, how much would a printer earn in setting it up at $8\frac{3}{4}d.$ per thousand?

Ans. $30l.$ $11s.$ $3\frac{3}{4}d.$ $\frac{8}{10}$.

Ex. 15. The name of JEHOVAH occurs 6855 times in the Old Testament: what proportion therefore does this word bear to all the other words in that book?

Ans. $86\frac{1}{2}$ nearly.

Ex. 16. The word *and* occurs in the Bible 46227 times: what proportion does that bear to the other words? See Answer to Ex. 8. Ans. 17 nearly.

Ex. 17. There are in the northern side of London 126 houses newly built, and unlet, the average rent of which is 85*l.*; and 75 houses at 50*l.* each, and 68 at 30 guineas each: what is the total annual loss of these empty houses to the proprietors? Ans. 16602*l.*

Ex. 18. There are 1100 hackney coaches in London, each of which earns on an average 18*s.* per day: how much is expended weekly, daily, and annually, on these vehicles, sundays excepted? Ans. 990*l.* per day. 5940*l.* per week. 308880*l.* per annum.

Ex. 19. What are 256 reams of paper worth, at 33*s.* 6*d.* per ream? Ans. 428*l.* 16*s.*

Ex. 20. Fifty thousand larks have been sold in a single season in London: what did they fetch, supposing they were bought at 1½*d.* each? Ans. 260*l.* 8*s.* 4*d.*

Ex. 21. The circumference of the Earth, in the latitude of London, is 15,120 miles, which is the space we pass over in 24 hours, by the diurnal motion of the earth: how much space do we pass over in a minute?

Ans. 10½ miles.

Ex. 22. Three thousand ounces of gold are imported into England annually: how many pounds and grains are imported in 50 years, at this rate, and what is the value of it at 3*l.* 18*s.* per ounce? Ans. 12.500 pounds, 72.000.000 grains, and 585,000*l.* value.

Ex. 23. To work the silver mines in South America, 40,000 negroes are imported annually: how many of these poor creatures have perished in this work during the last century? Ans. 4.000.000

Ex. 24. The duty on hops amounted, at 1½*d.* per lb. in a certain year, to 26,357*l.* 9*s.* 9*d.*: how many hops were grown that season?

Ans. 1882 tons, 13 cwt. 2 qrs. 6 lb.

Ex. 25. The battering ram employed by Titus to demolish the walls of Jerusalem, weighed 100,000 lbs.: how many tons did it contain?

Ans. 44 tons, 12 cwt. 3 qrs. 12. lb.

Ex. 26. The copper mines in the island of Anglesey produce 1500 tons annually, and those in Cornwall 4000 tons: what is the value of the whole at 9½*d.* per lb.?

Ans. 487,666*l.* 13*s.* 4*d.*

Ex. 27. Mr. Bolton coined 40,000,000 penny pieces, each weighing an ounce: how many pounds of copper were used for them: how much was the value of these in pounds sterling; and what was gained by this coinage, supposing the copper and expense of coining to be estimated at $12\frac{1}{2}d.$ per pound?

Ans. 2.500.000 lbs. 130,208*l.* 6*s.* 8*d.* 36,458*l.* 6*s.* 8*d*

Ex. 28. In the year 1794, 43,259,746 yards of Irish linen were exported from Ireland: how many packages did they make, each package containing 20 pieces, and each piece $26\frac{1}{2}$ yards? How many shirts would this linen make, at the rate of $3\frac{3}{4}$ yards per shirt?

Ans. 81.622 pack. 86 yds. $11.535.932\frac{4}{15}$.

Ex. 29. The circumference of the earth is estimated at 24,912 miles: how many barley-corns, (three of which make an inch,) would fill up this space?

Ans. 4.735.272.960

Ex. 30. The territory of the United States of America contains a million of square miles, or 640 millions of square acres: of these, about 56 millions are water: what number of acres, roods, and perches of land, do the United States contain, and how many inhabitants will they support, allowing to each $4\frac{1}{2}$ acres?

Ans. 129,777,777.

Ex. 31. There are now in England, Scotland, and Wales, 23 millions of acres of waste land: how many farms might these be divided into, allowing to each 75 acres:—and allowing 5 persons to each farm, how many souls would these waste acres support?

Ans. 306.606 farms 50 acres. 15.333.333 inha.

Ex. 32. Between the 5th of July, 1810, and the same day, 1811, there were brewed, by 12 brewers only, 939,900 barrels of porter: how much would this quantity sell for when retailed out at *5d.* per qt. allowing 36 gals. to the barrel? Ans. 2.819.700*l.*

Ex. 33. How many hours, minutes, and seconds have elapsed since the birth of Christ, which is 1808 years, supposing $365\frac{1}{4}$ days in a year? Ans. 15.848.928 ho. 950,935,680 min. 57,056,140,800 sec.

Ex. 34. It is said the Small-pox carries off in London, by death, 50 persons in a week: how many (if the disease is not checked) will it destroy in ten years?

Ans. 26.000

Ex. 35. There are about 10,540 tons of cheese imported into London annually: how much do they sell for at the average price of $7\frac{1}{2}d.$ per lb.? Ans. 737.800*l.*

Ex. 39. It is computed that there are 50,000 tons of butter annually consumed in London: what is the expense, supposing the average price $10\frac{3}{4}d.$ per lb.?

Ans. 5.016.666*l.* 13*s.* 4*d.*

Ex. 37. About 120,000 persons are employed in the cotton trade; if of these one-fourth are men, who earn 3*s.* 6*d.* a day, and one-fourth women, who earn 1*s.* 1*d.* a day, and the rest children, who earn, each, 3*s.* per week, how much is earned by manual labour in the cotton manufacture every year? Ans. 2,613,000

Ex. 38. There have been 20,000,000 lbs. of tea imported in a single year from China; what was the value of it, supposing the average price 4*s.* 9*d.* per lb.

Ans. 4,750,000*l.*

Ex. 39. The consumption of tobacco in this country is about 169,000 cwt.; how much is expended on this article at $1\frac{1}{2}d.$ per oz.? Ans. 1.577,333*l.* 6*s.* 8*d.*

Ex. 41. The consumption of milk is not less than 6,980,000 gallons annually in London; how much is expended on this article at 3cts. per pint?

Ans. 1,675,200 dollars.

Ex. 42. The iron rails round St. Paul's cost 11,202*l.* 0*s.* 6*d.*, and they weighed 200 tons and 81 lbs.; what was the iron charged per lb.? Ans. 6*d.* per lb.

Ex. 43. Westminster-bridge cost 389,500*l.* in building; how soon would it have been paid for by foot passengers, at a halfpenny each, supposing 2420 went over each day? Ans. 211 years, 241 $\frac{24}{121}$ days.

REDUCTION.

REDUCTION is the method of converting numbers from one name, or denomination, to another of the same value; and it is divided into *Reduction descending*, and *Reduction ascending*.

When numbers of a higher denomination are to be brought to a lower, it is called Reduction *descending*, and it is performed by *Multiplication*.

When numbers of a lower denomination are to be brought to a higher denomination, it is called Reduction *ascending*, and is performed by *Division*.

REDUCTION DESCENDING,

OR CONVERTING GREAT INTO SMALL.

RULE. Multiply the given number by as many of the lower denomination as make one of the higher.

Thus, in reducing 55*l.* into shillings, I multiply the 55 by 20, and the answer is 1100 shillings; in both cases the value is the same, that is, 55*l.* is equal to 1100 shillings.

REDUCTION ASCENDING,

OR CONVERTING SMALL INTO GREAT.

RULE. Divide by as many of the lower denomination as make one of the next higher.

Thus, in bringing 890 pence into shillings, I divide the number by 12, and the answer is 74 shillings and two pence over.

EXAMPLES.

Ex. 1. Reduce *L.* 29 *s.* 6 *d.* $8\frac{3}{4}$ into farthings.

20

586 shillings
12

7040 pence
4

Answer 28163 farthings.

Ex. 2. In 28163 farthings how many pounds sterling?

4)28163

12)7040—$\frac{3}{4}$

2,0)58,6—8*d.*

Ans. *L.*29 6 $8\frac{3}{4}$

Ex. 3. Reduce 37 Dimes to mills. Ans. 3700 Mills.

4. Reduce 53 dollars to cents. Ans. 5300 cents.

5. Reduce 163 eagles to dollars. Ans. 1630 dollars.

6. Reduce 74 dollars to dimes. Ans. 740 dimes.

7. Reduce 217 dollars to mills. Ans. 217000 mills.

8. Reduce 35 eagles to mills. Ans. 350000

9. Reduce 28 shillings to pence. Ans. 336 Pence.

10. Bring 56 pounds into shillings. Ans. 1120 shills.

11. Reduce 672 pence into farthings. Ans. 2688 farthings.

12. How many pence are there in 105*l.*? Ans. 25200 pence.

13. In 1000 guineas how many shillings? Ans. 21000 shillings.

14. In 4704*l.* how many pence? Ans 1128960 Pence.

15. In 3995*l.* how many farthings? Ans. 3835200 Far.

16. In 7968 guineas, how many farthings? Answer, 8031744 farthings.

17. How many farthings are there in 75 guineas?

Ans. 75600 farthings.

18. Reduce 576*l.* into farthings. Ans. 552960 far.

19. In 99*l.* how many shillings, pence, and farthings? Ans. 1980 shilings, 23760 pence, and 95040 farthings.

20. Reduce 567*l* 9*s.* 9½*d.* into farthings. Answer. 544790 farthings.

21. How many halfpence are there in 157*l.* 7*s.* 7½*d.* Ans. 75543 halfpence.

22. In 1084890 pence, how many pounds? Answer. 4520*l.* 7*s.* 6*d.*

23. In 8410896 pence, how many guineas? Answer. 33376 guineas 12*s.*

24. In 4808764 farthings, how many pounds? Ans. 5009*l.* 2*s.* 7*d.*

25. How many seven-shilling pieces are there in a thousand guineas? Ans. 3000.

26. How many groats are there in a hundred guineas? Ans. 6300 groats.

27. Bring 3110456 pence into groats. Ans. 777614 groats.

28. How many crown-pieces are there in 79*l.* 15*s.*? Ans. 319 crowns.

29. How many half-crowns are there in 85*l.* 12*s.* 6*d.*? Ans. 685 half-crowns.

30. In 769 guineas, how many sixpences? Answer. 32298 sixpences?

TROY,

OR, GOLD SMITHS' WEIGHT.

lb. oz. dwt. gr.

Ex. 1. Reduce 3 9 6 18 to grains.

12

45
20

906
24

3632
1813

21762

Ex. 2. How many pounds Troy are there in a million of grains?

4)1,000000

6)250000

2,0)41666—4 = 16 grains.

12)20833—6

173—7 Answer 173 lbs. 7 oz. 6 dwts. 16 grs.

Ex. 3. In 36 lb. 10 oz. 12 dwts. 16 grs. how many grains? Ans. 212464 grains.

4. How many pounds troy are there in 5987 penny-weights? Ans. 24 lb. 11 oz. 7 dwts.

5. In 1434 lb. 0 oz. 0 dwts. 19 grs. how many grains? Ans. 8259859 grs.

6. How many pounds are there in 45065 grains? Ans. 7 lb. 9 oz. 17 dwts. 17 grs.

7. Reduce 105 lbs. troy into grains. Ans. 604800

8. In 495 spoons, weighing 103 lbs. 1 oz. 10 dwts., how many grains? Ans. 594000

AVOIRDUPOIS,

OR GROCERS' WEIGHT.

Ex. 1. How many drams are there in 225 tons, 17 cwt. 3 qrs. 24 lb. 12 oz. 8 dr.?

tons,	cwt.	qr.	lb.	oz.	dr.
225	17.	3	24	12	8
20					
4517					
4					
18071					
28					
144572					
36144					
506012					
16					
3036074					
506013					
8096204					
16					
48577232					
8096204					
129539272					

Answer, 129539272 drams.

Ex. 2. How many tons are there in 259078544 drams?

16 { 4)259078544
4)64769636

16 { 4)16192409
4)4048102 — 1 } oz.
7)1012025 — 2 } = 9

28 { 4)144575
4)36143 — 3 = lb. 21
2.0)903.5 — 3

451 15 3 21 9

Ans. 451 tons, 15 cwt. 3 qrs. 21 lb. 9 oz.

Ex. 3. In 179 cwt. how many pounds? Ans. 20048 lb.

4. Reduce 8345 tons into quarters. Ans. 667600 qrs.

5. How many ounces are there in 4 tons, 15 cwt. 2qrs. 12 lb.? Ans. 171328 ounces.

6. In 233076 ounces of sugar, how many cwt.?
Ans. 130 cwt. 0 qr. 7 lb. 4 oz.

7. How many drams are there in 53 tons, 14 cwt. 1 qr. 12 lb. 14 oz. 8 dr.? Ans. 30804200 drams.

8. In 32384818 drams, how many tons weight?
Ans. 56 tons, 9 cwt. 1 qr. 27 lb. 3 oz. 2 dr.

APOTHECARIES' WEIGHT.

Ex. 1. How many grains are there in 2 lb. 5 oz. 4 dr. 1 scr. 17 gr. ?

lb.	oz.	dr.	scr.	gr.
2	5	4	1	17
12				
29				
8				
236				
3				
709				
20				
14197	Answer 14197 grains.			

Ex. 2. In 42591 grains, how many pounds ?

2.0)4259.1

3)2129 — 11

8)709 — 2

12)88 — 5

7 4 5 2 11

Answer - 7 lb. 4 oz. 5 dr. 2 scr. 11 gr.

Ex. 3. In 51 lb. 2 oz. of rhubarb, how many scruples ?
Ans. 14736 scrup.

4. In 234876 grains, how many pounds ?
Ans. 40 lb. 9 oz. 2 dr. 1 scr. 16 gr.

5. How many pounds are there in 1000 oz. of opium ?
Ans. 83 lb. 4 oz.

6. In 239 lb. 9 oz. 2 dr. 2 scr. 14 gr., how many grs. ?
Ans. 1381134 grs.

7. How many scruples are there in one hundred and three ounces of Peruvian bark ? Ans. 2472 scrup.

8. In 126794 grains, how many pounds ?
Ans. 22 lb. 0 oz. 1 scr. 0 dr. 14 gr.

LONG MEASURE.

Ex. 1. How many yards are there between London and Bath, the distance of which is 108 miles?

108
8

864
40

34560
$5\frac{1}{2}$

172800
17280

190080 Answer 190080 yds.

Ex. 2. In 760329 feet, how many leagues?

3)760329

253443
2
---
11)506886

4.0)4608.0 — 6 = 3
---
8)1152 — 0
---
3)144 — 0

48

Ans. 48 lea. 0 m. 0 fur. 0 p. 3 yards.

Ex. 3. How many inches are there in 1009 miles?

Ans. 63930240 inches.

4. Reduce 57 m. 4 fur. 38 p. 3 yds. 2 ft. 3 in. 1 b.c. into barley-corns? Ans. 10952578 b. corns.

5. In 100004 poles, how many inches?

Ans. 19800792 inches.

Ex. 6. In 409683 feet, how many furlongs?

Ans. 620 fur. 161 yards.

7. How often will the wheel of a coach turn round in going from London to Sheffield, or in 160 miles, supposing the circumference of the wheel to be 16 feet?

Ans. 52800 times.

8. Suppose on an average I step two feet and a half; how many steps shall I take in walking from London to Richmond, a distance of 10 miles? Ans. 21120 steps.

CLOTH MEASURE.

Ex. 1. How many inches in length are there in 156 ells English of cambrick?

156
5

780
4

3120
$2\frac{1}{4}$

6240
780

7020 Answer 7020 inches.

Ex. 2. In 1000 inches of cotton, how many yards are there?

9)1000

4)111 — 1

27 3 0 1

Ans. 27 yds. 3 qr. 0 n. 1 in.

Ex. 3. How many English ells are there in three thousand and fifty-five nails? Ans. 152 E.e. 3 qrs. 3 n.

4. In 15 yds. 2 qr. 3 n. 1 in., how many half inches?

Ans. 1131½ half inches.

5. How many inches are there in 10056 yards?

Ans. 362016 inches.

6. Reduce 546 English ells to nails. Ans. 10920 nls.

SQUARE, OR LAND MEASURE.

Ex. 1. How many yards are there in 5604 acres?

5604
4

22416
40

896640
$30\frac{1}{4}$

26899200
224160

27123360 Ans. 27123360 yds.

Ex. 2. In 6534 square feet, how many perches?

9)6534

726
4

121 121)2904(24
242

484
484

... Answer 24 perches.

Ex. 3. How many roods are there in 382 perches?
Ans. 9 roods. 22 perches.

4. In 561 acres of ground how many perch. and yds.?
Ans. 89760 per. 2715240 yds.

5. In 2967400 inches how many acres?
Ans. Not quite $\frac{1}{2}$ an acre, being only 2289 yds. $5\frac{136}{144}$.

6. How many perches are there in 997 acr. 3 rd. 10 p.?
Ans. 159610 perches.

CUBIC, OR SOLID MEASURE.

Ex. 1. In 36 solid yards, how many inches?

36
27

252
72

972
1728

7776
1944
6804
972

Answer - 1679616 inches.

3. In 1259712 solid inches, how many yards?

Ans. 27 yards.

LIQUID MEASURE.

Ex. 1. How many gallons are there in 5 pipes of wine?

5
2

10
63

Answer - 630 gallons.

Ex. 2. In 7006 pints, how many gallons?

2)7006
---
4)3503
---
875 3 Ans. 875 gal. 3 qts.

Ex. 3. In 31490 pints, how many gallons ?
Ans. 3936 gal. 1 qt.

4. In 3 tuns, 1 hhd. 49 gallons of claret, how many quarts ? Ans. 3472 quarts.

5. How many tuns of port wine are there in 46088 gallons ? Ans. 182 tuns, 3 hhd. 35 gal.

DRY MEASURE.

Ex. 1. In 79 pks. how many pts. ? Ans. 1264 pts.

2. How many bushels are there in 7649 pints ?
Ans. 119 bush. 2 pks. 1 pt.

3. How many pts. are there in 23 bush. 3 pks. 2 qts. ?
Ans. 1524 pts.

4. In 3 pks. and 1 gal. how many qts. ? Ans. 28 qts.

5. How many pks. are there in 187406 quarts ?
Ans. 23425 pks. 1 gal. 2 qts.

COMMERCIAL NUMBERS,

OR ARTICLES SOLD BY TALE.

12 articles of any kind - - -	1 dozen
13 ditto - - - - -	1 long dozen
12 dozen - - - - -	1 gross
20 articles of any kind - - -	1 score
5 score - - - -	1 hundred
6 score - - - -	1 great hundred
12 score - - - -	1 pack of wool
5 dozen skins of parchment - -	1 roll
72 words in Common la[illegible] - -	1 sheet
80 —— in the Exchequer - -	1 ditto
90 —— in Chancery - -	1 ditto
24 sheets of paper - - -	1 quire
20 quires - - - -	1 ream
21½ quires, or 516 sheets - -	1 do. printer's
2 reams - - - -	1 bundle

Folio is the largest size of books, of which,
2 leaves, or 4 pages, make a sheet.
Quarto, 4to. - 4 leaves, or 8 pages, make a sheet,
Octavo, 8vo. - 8 leaves, or 16 pages, ditto.
Duodecimo, 12 mo. 12 leaves, or 24 pages, ditto.
Octodecimo, 18mo. 18 leaves, or 36 pages, ditto.

EXAMPLES.

Ex. 1. How many long dozen are there in ten thousand oranges? Ans. 769 doz. 8 oranges.

Ex. 2. How many gross are there in one hundred and fifty thousand corks? Ans. 1041 gross, 8 doz. corks.

Ex. 3. In seventy thousand quills, how many great hundreds are there? Ans. 583 hundreds 40 quills.

Ex. 4. I have a deed containing 4 skins of parchment, and each skin contains 850 words; for how many sheets shall I have to pay the person who copies it, reckoning according to the common law charge?
Ans. 47 sheets and 16 words.

Ex. 5. The writing of an Exchequer cause occupies 315 sheets; for how many words shall I have to pay the clerk who copies it for me? Ans. 25200 words.

Ex. 6. A suit has been four years in chancery, and I wish to have a copy of all the proceedings; for how many sheets shall I pay, supposing it occupies 1264 skins of parchment, and each skin 690 words?
Ans. 9690 sheets and 60 words.

Ex. 7. How many sheets are there in 40 reams of paper? Ans. 19200 sheets.

Ex 8. How many common reams of paper are there in ten thousand printer's reams? Ans. 10750 reams.

Ex. 9. What number of sheets less are there in 500 common reams of paper, than there are in the same number of printer's reams? Ans. 18000 sheets.

Ex. 10. What number of pages are there in a folio containing 211 sheets? Ans. 844 pages.

Ex 11. What will be the difference in the number of pages, whether I print in 12mo. or 18mo., supposing my work will make fourteen sheets? Ans. 168 pages.

Ex. 12. What numbers of words are there in Dr. Gregory's Dictionary of Arts and Sciences, which contains 240 sheets 4to., and each page contains 1848 words?
Ans. 3548160 words.

Ex. 13 How many reams of paper were used in printing that Dictionary, six thousand copies having been taken off? Ans. 3000 reams.

Ex. 14. How many pens were used in writing the said Dictionary, supposing each pen to write 840 words?
Ans. 4224 pens.

TIME.

Ex. 1. In 4199 days, how many months of 28 days each, and years of 365 days each?
Ans. 149 ms. 27 days; or 11 yrs. and $\frac{1}{3}$ nearly.

2. Reduce 150 days to hours and minutes?
Ans. 3600 hours, 216000 minutes.

3. In 70 years how many days, supposing each year to consist of $365\frac{1}{4}$ days? Ans. 25567 days and $\frac{1}{2}$.

4. How many minutes, hours, and days are there in 5960034 seconds?
Ans. 99333 min. 1655 ho. or 68d. 23ho. 33min. 54s.

5. How many minutes are there in 1808 years, allowing $365\frac{1}{4}$ days make one year? Ans. 950935680 min.

6. How many seconds has a boy lived, who is 12 years, 9 months, and 13 days old, reckoning 13 lunar months of 28 days each to a year? Ans. 400291200 sec.

ASTRONOMY.

TABLE.

60 second (60″) - - -	1 minute, 1′
60 minutes - - - -	1 degree, 1°
30 degrees - - - -	1 sign
12 signs, or 360 degrees -	1 great circle.

Ex. 1. In 185 degrees how many minutes and seconds?
Ans. 11100 min. 666000 seconds.

2. How many degrees are there in five thousand and fifty-five seconds? Ans. 1 deg. 24 min. 15 sec.

3. How many seconds are there in a great circle ?
Ans. 1296000 seconds.

4. In 548056 seconds, how many signs ?
Ans. 5 signs 2 degrees 14 minutes 16 seconds.

5. How many seconds are there in 9 sig. 4 deg. 55 min. and 56 sec ? Ans. 989756 seconds.

6. In 700809 seconds, how many degrees ?
Ans. 194 deg. 40 min. 9 sec.

MISCELLANEOUS EXAMPLES.

Ex. 1. In 195 pounds, how many shillings, pence and farthings ? Ans. 3900 shil. 46800 pence 187200 far.

Ex. 2. In 77 guineas, how many shillings, pence and farthings ? Ans. 1617 shil. 19404 pence, 77616 far.

Ex. 3. How many crowns, half-crowns, shillings and sixpences, are there in 354 pounds ?
Ans. 1416 crowns, 2832 half-c. 7080 shil. 14160 sixp.

Ex. 4. In 4432127 farthings, how many pence, shillings, pounds, and guineas ? Ans. 1108031 pence $\frac{3}{4}$.
92335*s*. 11*d*. 4616*l*. 15*s*. 4396 guineas 19*s*.

Ex. 5. In 14 ingots of silver, each weighing 27 oz. 5 dwts., how many grains ? Ans. 183120 grains.

Ex. 6. In three dozen of table spoons, each weighing 2 oz. and 9 dwts., how many pounds ?
Ans. 7 lb. 4 oz. 4 dwts.

Ex. 7. In 78 bags of hops, each weighing 3 cwt. how many pounds ? Ans. 26208 lb.

Ex. 8. How many pounds and cwts. of tobacco are there in 75 hogsheads, each containing 3 cwt. 1 qr. 14 lb.?
Ans. 28350 lb. 253 cwt. 0 qr. 14 lb.

Ex. 9. In 98465 inches of broad cloth, how many yds. and ells ? Ans. 2735 yds. 5 in. ; 2188 ells, 5 in.

Ex. 10. In five thousand yards of cloth, how many nails ? Ans. 80000 nails.

Ex. 11. How many inches are there between London and Bristol, a distance of 120 miles ? Ans. 7603200 in.

Ex. 12. How many barley-corns will reach round the earth, which is a great circle of 360 degrees, and each degree contains $69\frac{1}{2}$ miles. And how many quarters of

barley would be necessary to perform this, supposing 9200 barley-corns to fill a pint measure?

Ans. 4755801600 b.c.; 1009 qrs. 5 bush. 6 pts. 8800 b.c.

Ex. 13. How often will a wheel turn in going from London to York, a distance of 198 miles, if the wheel be $2\frac{1}{2}$ yards in circumference? Ans. 139392 times.

Ex. 14. How many perches are there in a field containing 105 acres? Ans. 16800 perches.

Ex. 15. If a field of 5 acres be taken from one of 56 acres, how many square yards will remain?

Ans. 246840 sq. yards.

Ex. 16. How many pints and gallons are there in 39 hogsheads of cyder? Ans. 19656 pts.; 2457 gal.

Ex. 17. How many minutes have elapsed since the creation of the world to the present time, 1808 inclusive, supposing the world to have been created 4004 years before the birth of Christ, and each year to consist of $365\frac{1}{4}$ days? Ans. 3056879520 minutes.

AMERICAN COIN.

TABLE.

Currency	*s.*	*d.*	Fed. money,
In Maryland, Pennsylvania, Delaware and New Jersey.	7	6	make 1 Dol.
New England and Virginia.	6	0	ditto.
New York and North Carolina.	8	0	ditto.
South Carolina and Georgia.	4	8	ditto.
Canada and Nova Scotia.	5	0	ditto.

1. To reduce Maryland, Pennsylvania, Delaware and New Jersey currencies to Federal money, the value of the dollar being 7*s.* 6*d.* or 90 pence.

Rule. Reduce the given sum to pence and divide by 90, the result will be dollars, annex a cypher and continue the division for cents.

EXAMPLES.

Ex. 1. Reduce 76*l.* 14*s.* 6*d.* Maryland currency to Federal money.

L.	*s.*	*d.*
76.	14.	6.
	20	
	1534	
	12	
9,0)184140		

Ans. $ 204.60

Ex. 2. Reduce 237*l.* 17*s.* 4*d.* Pennsylvania currency to Federal money. Ans. 634 dols. 31 cts.

3. Reduce 5217*l.* 6*s* 7*d.* Delaware currency to Federal money? Ans. 13912 dols. 87 cts.

4. Reduce 673*l.* 1*s.* 2*d.* New Jersey currency to Federal money? Ans. 1794 dols. 82 cts.

5. Reduce 7*l.* 6*s.* 8*d.* New Jersey currency to Federal money? Ans. 19 dols. 55 cts.

6. Reduce 39*l.* 7*s.* 6*d.* Maryland currency to Federal money? Ans. 105 dollars.

7. Reduce 48*l.* 9*s.* 5*d.* Pennsylvania currency to Federal money? Ans. 129 dols. 25 cts.

---

2. To change Federal money to Maryland, Pennsylvania, Delaware and New Jersey currencies.

RULE. If the given sum be dollars only, multiply by 90 and the result will be pence, but if there should be cents in the given sum, multiply by 90 and cut off two figures on the right hand, the result will be in pence also, which reduce to shillings and pounds.

NOTE.

If there should be half-pence or farthings in the given sum, reduce it to the lowest denomination mentioned, and reduce also the number of pence in one dollar to the same denomination, and divide by this for the answer.

EXAMPLES.

Ex. How much Maryland currency in 76 dols. 50 cts.?

dols. cts.
76 50
90

12)6885,00

20)57,3 9

Answer - - - 28*l.* 13*s.* 9*d.*

2. Change 744 dols. into Pennsylvania currency?
Answer 279*l.*

3. In 365 dols. 25cts. how much New-Jersey currency? Ans. 136*l.* 19*s.* $4\frac{1}{2}$*d.*

4. In 7493 dollars 50 cents, how much Delaware currency? Ans. 2810*l.* 1*s.* 3*d.*

5. In 627 dollars 75 cents, how much Pennsylvania currency? Ans. 235*l.* 8*s.* $1\frac{1}{2}$*d.*

6. In 134 dollars 60 cents, how much Maryland currency? Ans. 50*l.* 9*s.* 6*d.*

7. In 1216 dollars 80 cents, how much Pennsylvania currency? Ans. 456*l.* 6*s.*

---

3. To change New England and Virginia currencies to Federal money, the value of the dollar being 6 shillings or 72 pence.

Rule.—If there be pounds and shillings only; reduce the given sum to shillings, and divide by 6; but if there be pence also, reduce the given sum to pence; then divide by 72, and the quotient will be dollars, annex two cyphers to the dividend, and continue the operation for cents.

EXAMPLES.

1. In 74*l.* 6*s.* 8*d.* New England currency, how much Federal money?

L.	*s.*	*d.*
74	6	8
20		
1486		
12		
72)17840.00(	dols.	cts.
144	247	77 Answer.
344		
288		
560		
504		
560		
504		
560		
504		
56		

or thus,

72 { 8)1784000
9)2 3000
8 247.77 cts.

2. In 64*l.* 15*s.* Virginia currency how much Federal money? Ans. 215 dollars 83 cents.

3. In 327*l.* 16*s.* 4*d.* Virginia currency, how much Federal money? Ans. 1092 dollars 72 cents.

4. In 463*l.* 12*s.* 9*d.* Virginia currency, how much Federal money? Ans. 1545 dollars 45 cents.

5. In 579*l.* 18*s.* 2*d.* New England currency, how much Federal money? Ans. 1938 dollars 2 cents.

6. In 6214*l.* 12*s.* 9*d.* Virginia currency, how much Federal Money? Ans. 20715 dollars 45 cents.

7. In 7309*l.* 13*s.* 7*d.* Virginia currency, how much Federal money? Ans. 24365 dollars 59 cents.

4. To change Federal money to New England and Virginia currencies.

RULE.—Multiply the given number of dollars by 6, and divide by 20 for pounds, or if there be cents in the question, multiply the number of cents by 72, and divide by 100, the quotient will be pence, which reduce to shillings and pounds.

EXAMPLES.

Ex. 1. Change 273 dollars 25 cents to New England currency?

27325 cts.
72

54650
100) 191275

12)19674,00

2,0)163,9 6

Answer - - - 81*l.* 19*s.* 6*d.*

2. Change 496 dollars to New England currency? Ans. 148*l.* 16*s.*
3. Change 79 dollars 50 cents to Virginia currency? Ans. 23*l.* 17*s.*
4. Change 673 dollars 60 cents to Virginia currency? Ans. 202*l.* 1*s.* 7*d.*
5. Change 762 dollars 15 cents to Virginia currency? Ans. 228*l.* 12*s.* 10*d.*
6. Change 847 dollars 75 cents to Virginia currency? Ans. 254*l.* 6*s.* 6*d.*
7. Change 1746 dols. 30 cents to Virginia currency? Ans. 523*l.* 17*s.* 9*d.*

---

5. To change New York and North Carolina currencies to Federal money, the value of the dollar being 8 shillings or 96 pence.

RULE.—If the lowest denomination mentioned in the given sum be shillings, reduce it to that denomination

and divide by 8; but if the lowest denomination be pence, reduce the given sum to pence and divide by 96, the quotient will be dollars; bring down two cyphers and continue the operation for cents.

EXAMPLES.

Ex. 1. In 74*l.* 16*s.* New York currency how much Federal money?

L.	*s.*
74	16
20	
8)1496	

187 dollars. Ans.

2. In 29*l.* 17*s.* New-York currency how much Federal money? Ans. 74 dolls. 62½ cts.

3. In 365*l.* 7*s.* 4*d.* New-York currency how much Federal money? Ans. 913 dols. 41 cts.

4. In 497*l.* 16*s.* 10*d.* North Carolina currency how much Federal money? Ans. 1244 dols. 60 cts.

5. In 563*l.* 12*s.* 6*d.* New-York currency how much Federal money? Ans. 1409 dols. 6 cts.

6. In 728*l.* 13*s.* 9*d.* New-York currency how much Federal money? Ans. 1821 dols. 71 cts.

7. In 3674*l.* 8*s.* 7*d.* North Carolina currency how much Federal money? Ans. 9186 dols. 7 cts.

---

6. To change Federal money to New-York and North Carolina currencies.

Rule—Multiply the given number of cents by 96, and divide the product by 100, the quotient will be pence, which reduce to shillings and pounds, or if there be dollars only in the question, multiply them by 8 and divide by 20 for pounds, the remainder, if any, will be shillings.

EXAMPLES.

Ex. 1. Reduce 49 dols. 50 cts. to New York currency.

```
          cts.
          4950
            96
        ------
         29700
1,00)   44550
        ------
     12)4752.00
        ------
     2,0)39.6
        ------
Ans. L19 16
```

2, Reduce 246 dols. to North Carolina currency.
Ans. 98*l*. 8*s*

3. Reduce 418 dols. 75 cts. to New-York currency.
Ans. 167*l*. 10*s*.

4. Reduce 672 dols. 25 cts. to North Carolina currency.
Ans. 268*l*. 18*s*.

5. Reduce 847 dols. 60 cts. to North Carolina currency.
Ans. 339*l*. 0*s*. 9*d*.

6. Reduce 1184 dols. 40 cts. to North Carolina currency.
Ans. 473*l* 15*s*. 2*d*.

7. Reduce 2756 dols. 50 cts. to New-York currency.
Ans. 1102*l*. 12*s*.

---

7. To change South Carolina and Georgia currencies to Federal money, the value of the dollar being 4*s*. 8*d* or 56 pence.

Rule.—Reduce the given sum to pence, then divide by 56 and the quotient will be dollars, bring down two cyphers and continue the operation for cents.

EXAMPLES.

Ex. 1. In 69*l.* 15*s.* 6*d.* South Carolina currency, how much Federal money ?

L.	*s.*	*d.*
69	15	6
20		
1395		
12		
56)16746(299		03
112		
554		
504		
506		
504		
200		
168		
32		

D. cts.

2. In 864*l.* 17*s.* 2*d.* South Carolina currency, how much Federal money ? Ans. 3706 dols. 53 cts.

3. In 927*l.* 16*s.* 9*d.* Georgia currency, how much Federal money ? Ans. 3976 dols. 44 cts.

4. In 673*l.* 12*s.* 8*d.* Georgia currency, how much Federal money ? Ans. 2887 dols.

5. In 763*l.* 18*s.* 1*d.* Georgia currency, how much Federal money ? Ans. 3273 dols. 87 cts.

6. In 111*l.* 11*s.* 11*d.* Georgia currency, how much Federal money ? Ans. 478 dols. 26 cts.

7. In 3106*l.* 17*s.* 4*d.* Georgia currency, how much Federal money ? Ans. 13315 dols. 14 cts.

---

8. To change Federal money to South Carolina and Georgia currencies.

RULE.—Multiply the given number of cents by 56, and divide by 100. the quotient will be pence, which reduce to shillings and pounds.

EXAMPLES.

Ex. 1. How much Georgia currency, in 216 dols. 50 cts. ?

cts.
21650
56

129900
1,00) 10825 0

12)12124,00

20)101,0 4

Ans. *L.* 50 10 4

2. How much South Carolina currency in 467 dols. 25 cts. ? Ans. 109*l.* 0*s.* 6*d.*

3. How much South Carolina currency in 762 dols. 30 cts. ? Ans. 177*l.* 17*s* 4*d.*

4. How much Georgia currency in 939 dols. 70 cts. ? Ans. 219*l.* 5*s.* 3*d.*

5. How much Georgia currency in 1000 dols. ? Ans. 233*l.* 6*s.* 8.

6. How much Georgia currency in 2172 dols. 50 cts. ? Ans 506*l.* 18*s.* 4*d.*

7. How much Georgia currency in 9999 dols 99 cts. ? Ans. 2333*l.* 6*s.* 7*d.*

---

9. To change Canada and Nova Scotia currencies to Federal money, the value of the dollar being 5 shillings or 60 pence.

RULE—Reduce the given sum to pence and divide by 60, the quotient will be dollars; or if the lowest denomi-

nation be shillings, reduce to shillings and divide by 5, the quotient will be dollars; annex two cyphers and continue the operation for cents.

EXAMPLES.

Ex. 1. Reduce 87*l*. 16. 4*d*. Canada currency to Federal money?

L.	*s.*	*d.*
87	16	4
20		
1756		
12		
6,0)2107,60		

Ans. $351 26 cts.

2. Reduce *L.* 24 *s.* 14 to Federal money?

	20
5)494.00	

Ans. $98 80 cts.

3. Reduce 827*l*. 15*s*. Nova Scotia currency, to Federal money? Ans 3311 dols.

4. Reduce 268*l*. 12*s*. 3*d*. Canada currency, to Federal money? Ans. 1074 dols 45 cts.

5. Reduce 719*l*. 9*s*. 2*d*. Canada currency, to Federal money? Ans. 2877 dols. 83 cts.

6. Reduce 672*l*. 10*s*. 10*d*. Canada currency, to Federal money? Ans. 2690 dols 16 cts.

7 Reduce 926*l* 11*s*. 11*d*. Canada currency, to Federal money? Ans 3706 dols. 38 cts.

8 Reduce 5119*l*. 19*s*. 7*d*. Canada currency, to Federal money? Ans. 20479 dols. 91 cts.

10. To change Federal money to Canada and Nova Scotia currencies.

RULE—Multiply the given number of cents by 60, and divide the product by 100, the quotient will be pence, which reduce to shillings and pounds; or if there be dollars only in the question multiply them by 5, divide by 20 for pounds, and the remainder will be shillings.

EXAMPLES.

Ex. 1. In 68 dols. 50 cts. how much Nova Scotia currency?

cts.
6850
60
1,00)———
12)4110,00
———
2,0)34,2 6
———
Ans. *L.*17 2 6

2. In 124 dols. 25 cts. how much Canada currency?
Ans. 31*l.* 1*s.* 3*d.*

3. In 7648 dols. how much Canada currency?
Ans. 1912*l.*

4. In 867 dols. 35 cts. how much Canada currency?
Ans 216*l.* 16*s.* 9*d.*

5. In 1714 dols. 75 cts. how much Nova Scotia currency?
Ans. 428*l.* 13*s.* 9*d.*

6. In 6179 dols. 20 cts. how much Canada currency?
Ans. 1544*l.* 16*s.*

7. In 4444 dols. 44 cts. how much Canada currency?
Ans. 1111*l.* 2*s.* 2*d.*

PROPORTION,

OR

THE RULE OF THREE.

This rule is called the Rule of Three, because by three numbers being given we find a fourth; and it is either the Rule of Three *Direct* or *Inverse.*

THE RULE OF THREE DIRECT

teaches, from three given numbers to find a fourth, which shall have the same proportion to the second, as the third has to the first; that is, if the *first* be *greater* than the *third*, the *second* will be *greater* than the *fourth*; and, if the *first* be *less* than the *third*, the *second* will be *less* than the *fourth.*

RULE 1. STATE THE QUESTION: that is, place the given numbers so that the first and third may be of the same kind, and the second the same as the number required.

2. Bring the first and third numbers into the same denomination, and the second into the lowest denomination mentioned.

3. Multiply the second and third numbers together, and divide the product by the first, and the quotient will be the answer, in the same denomination as that in which the second number was left.

Ex. 1. What is the value of a pipe of wine, if 5 gallons cost 4*l*. 17*s*.

gal. *L*. *s*. pipe.
5 : 4 17 :: 1
20 2
97 2
63
126
97
882
1134
5)12222
2.0)244 4 — 2
12
122.4——
5)24
4 — 4
4
5,16
— 1

Ex. 2. If I can buy 27lb. of sugar for 1*l.* 13s. how much can I purchase for thirty guineas?

L. s lb. guineas.
1 13 : 27 : : 30
20 21
33 630
27
4410
1260
lbs.
33)17010(515
165
51
33
180
165
15
16
oz.
33)240(7
231
9

Ex. 3. What is the value of 28 ells of cloth, if 4 ells cost 18s.?

ells. shill. ells.
4 : 18 : : 28
18
224
28
4)504
2.0)12.6
6.6

Ex. 4. If six yards of cloth cost 24 shillings, what will 81 yards cost? Ans. 16*l.* 4*s.*

Ex. 5. If 8 gallons of wine cost 17 dollars 25 cents, what is the value of 35 gallons? Ans. 75 dols. 46 cts.

Ex. 6. If 5lb. of potatoes cost 4*d.*, what is the worth of 1 cwt. on the same terms? Ans. 7*s.* $5\frac{5}{6}$*l.*

Ex. 7. If 5lb. of old iron cost 3*d.*, how many can I buy for 40*s.*? Ans. 7 cwt. 0 qrs. 16 lb.

Ex. 8. If 10 English ells of cloth cost 11 dols. 11 cts., what is the value of 5 pieces, each containing 26 yards? Ans. 115 dollars 54 cents.

Ex. 9 If 16 yards of muslin cost 10 guineas, how many ells can I buy for 45*l.* Ans. 54 ells $4\frac{60}{210}$qrs.

Ex. 10. If I can purchase 25 books for 30 dollars, how many can I have for 75 dols. 60 cts.? Ans. 63 books.

Ex. 11. If a servant's wages be 25 guineas a year, how much has he to receive for 87 days' service?

Ans. 6*l.* 5*s.* $1\frac{1}{2}$*d.* $\frac{43}{73}$.

Ex. 12. If a servant receive three guineas and a half for 20 weeks service: how long ought he to remain in his place for 12 guineas? Ans. 68 weeks 4days.

Ex. 13. If I pay half a crown for 4lb. of cheese: how much can I have for three crowns and nine-pence?

Ans. 25lb. $3\frac{3}{10}$ oz.

Ex. 14. If 2 lb. 4 oz. of honey cost 3*s.* 9*d.*: what is the value of 28 lb.? Ans. 2*l.* 6*s.* 8*d.*

Ex. 15. If 3 lb. of sugar cost $37\frac{1}{2}$ cents, what will 1 cwt. amount to? Ans. 14 dollars.

Ex. 16. If a dozen of wine glasses cost 10*s.* 6*d.*, what is the value of 500? Ans. 21*l.* 17*s.* 6*d.*

Ex. 17. If I can buy 3 pair of shoes for 7 dollars 50 cts. what must I pay for 17 pair? Ans. 42 dols. 50 cts.

Ex. 18. If a cwt. of tobacco cost 8 guineas; what is the value of 7,000,000 of lbs.? Ans. 525,000*l.*

Ex. 19. If 6 lb. of soap cost 1 dollar $37\frac{1}{2}$ cents. what is the value of 1 cwt. Ans. 25 dols. 66 cts.

Ex. 20. If I pay 39 shillings per cwt. for lead; how much will it cost to cover the roof of a building with lead that weighs 5505 lb.? Ans. 95*l.* 16*s.* 11*d.* $\frac{4}{112}$.

Ex. 21. I want to know how much I have to pay for a cistern 990 lbs. at the rate of 2*l.* 2*s.* per cwt., the plum-

ber agreeing to allow me at the rate of 1*l.* 14*s.* per cwt. for the old lead, which weighs 458 lb.?

Ans. 11*l.* 12*s.* $2\frac{3}{4}$*d.*

Ex. 22. If a journeyman can earn 9 dollars 50 cents, in 6 days, how much will he earn in 305 days?

Ans. 482 dollars 91 cents.

Ex. 23. The brazen statue of Apollo, that was erected by Chares, at Rhodes, weighed 720,000 lbs.: how much did the old brass sell for at four guineas per cwt.?

Ans. 27,000*l.*

Ex. 24. If I pay 1*l.* 7*s.* for 18 gallons of porter, how much shall I expend in that article in a year, if my family drink nine gallons of it every week?

Ans. 35*l.* 2*s.*

Ex. 25. If I buy 14 gallons of brandy for 35 dollars, how much must I pay for 4 hogsheads? Ans. 630 dols.

Ex. 26. If I buy, at Sheffield, 6 razors for 8*s.* 6*d.*; how much will I have to pay for twelve dozen at the same rate? And, how much can I sell them for, so as to gain by the bargain $2\frac{1}{2}$*d.* on each razor?

Ans. 10*l.* 4*s.* cost, 1*l.* 10*s.* gain.

Ex. 27. In building an out-house 5050 bricks have been used; how much do they come to at 4*s.* 6*d.* per hundred? Ans. 11*l.* 7*s.* 3*d.*

Ex 28 It requires 32 bricks to pave 9 square feet: how many bricks will be wanted for the pavement of a cellar 24 feet long, and 19 feet wide Ans. $1621\frac{1}{3}$ bricks.

Ex. 29. It requires 144 Dutch clinkers to pave 9 square feet: how many will be wanted for a court 35 feet long, and 29 feet wide, and how much will they come to at 5*s.* 6*d.* per hundred? Ans. 16240 clinkers, and 44*l.* 13*s.* $2\frac{21}{25}\frac{60}{100}$.

Ex. 30. It requires sixty persons six days to manufacture a pack of wool into cloth: how much wool will they work up in a year, supposing they work 5 days in each week? Ans. $43\frac{1}{3}$ packs.

Ex. 31. Six children of different ages will earn in five days, at spinning wool, 5*s.* 9*d.*, and the mother will earn 1*s.* 4*d.* per day: how much will they all earn in a year, allowing that they work, one week with another, $5\frac{1}{2}$ days per week? Ans. 35*l.* 10*s.* $2\frac{3}{8}$*d.*

Ex. 32. At some large iron founderies, they can run off 6000 lbs. of iron in twenty-four hours: how many tons weight will they cast in a year, allowing them to work 298 days, and 16 hours each day?

Ans. 532 tons, 2 cwt. 3 qr. 12 lb.

Ex. 33. By a patent machine for making combs, the teeth of two combs can be cut in three minutes: how many can be manufactured in 28 days, if the machine is worked at the rate of eight hours a day?

Ans. 8960 combs.

Ex. 34. What is the price of a carpet that measures 15 feet each way, at 7*s*. 6*d*. for every 9 square feet?

Ans. 9*l*. 7*s*. 6*d*.

Ex. 35. If 13 cwt. of sugar cost 150 dollars, how much must I pay for 15 casks of the same, each weighing 4 cwt. 2 qr. 12 lb.? Ans. 797 dols. 39 cts.

Ex. 36. How much flour can I purchase for 1087 dols. 50 cts., at 12 dols. and 50 cts. per barrel?

Ans. 87 barrels.

Ex. 37. If candles sell for 2 dols. $12\frac{1}{2}$ cts. per dozen lb.; how much will 250 lb. cost? Ans. 44 dols. 27 cts.

Ex. 38. If mould candles cost 12*s*. 6*d*. per dozen: how many pounds can I purchase for fifty guineas?

Ans. 84 dozen.

Ex. 39 The best mottled soap is bought at 4*l*. 6*s*. per cwt.: for how much must it be sold per lb., so as to allow a profit of one penny on each pound?

Ans. $10\frac{1}{4}d$. nearly.

Ex. 40. If I buy $6\frac{1}{2}$ yards of Irish cloth for 5 dols. 75 cts.: how much must I pay for 8 pieces, each containing 26 yards? Ans. 184 dols.

Ex. 41. If 40 yards of Irish cloth will make 12 shirts: how many may be made out of 4 pieces, each containg 26 yards? Ans. $31\frac{8}{20}$ shirts.

Ex. 42. If 12 gallons of brandy pay 3 dollars duty: how much must be paid for 37 hhds. each containing 63 gallons? Ans. 582 dols. 75 cts.

Ex. 43. The average price of sugar, exclusive of duty was, Aug. 21, 1805, 2*l*. 11*s*. $9\frac{1}{4}d$. per cwt.: I demand the value of the 9,999,360 lbs. that were imported into London the preceding week? Ans. 231105*l*.

Ex. 44. The average price of tallow was, on the same day, 4*s.* 2*d.* per stone of 8lb.: what is the worth of 276 tons, imported the preceding week? Ans. 16.100*l.*

Ex. 45. What will 31218 gallons of Port wine, sell for at 7 dols. $12\frac{1}{2}$ cts. per dozen, supposing each dozen to contain 3 gallons? Ans. 74142 dols. 75 cts

Ex. 46. What is the value of 115 seal-skins, at 3*s.* 6*d.* per lb. supposing the skins to weigh, one with the other, 9 ounces each? Ans. 11*l.* 6*s.* $4\frac{3}{4}d\frac{9}{16}$.

Ex. 47. How many quills can I have for 156 dols. 25 cts. at $62\frac{1}{2}$ cts. per hundred? Ans. 25,000

Ex. 48. How much brown Holland can I buy for ten guineas, if I pay 5*s.* 9*d.* for four yards and a quarter?

Ans. $155\frac{5}{23}$ yards.

Ex. 49 Suppose a person save out of his income 5*s.* 6*d.* per week: how long will he be laying by 100*l.*

Ans. 363 weeks $4\frac{1}{2}$ days nearly.

Ex. 50. I want to know the height of a tree, by means of the length of its shadow; I set up a straight stick that measures, above the ground 3 feet 4 inches; the shadow of this is 5 feet 2 inches, and the shadow of the tree, at the same moment I find to be 79 feet 10 inches?

Ans. 51 feet, $6\frac{4}{65}$.

Ex. 51. What is the height of a steeple, whose shadow is 148 feet 4 inches, when a shadow 5 feet 3 inches long is projected from a staff 6 feet 4 inches?

Ans. 178 feet, $11\frac{19}{65}$.

Ex. 52. If I pay 4*s.* 9*d.* for a hundred of pens: how many shall I get for 100*l.* Ans. $42105\frac{5}{19}$.

Ex. 53. If my income is 450*l.* per ann.: how much may I spend in 73 days, supposing I mean to lay by 50 guineas at the year's end? Ans. 79*l* 10*s.*

Ex 54. What is the value of 57 yards of muslin, at the rate of $87\frac{1}{2}$cts. per Ell English? Ans. 39 dols 90 cts.

Ex. 55. How many Ells English of cloth can be bought for 73 dols. $87\frac{1}{2}$ cts.: at the rate of 1 dollar 50 cts per yard? Ans. 39 E. E. 2 qrs.

Ex 56. I have a tankard that weighs 2 lb. 3 oz. that cost 10*l.* 2*s.* 6*d.*; how much, at the same rate, will a service of plate cost that weighs 125lb. 9oz.?

Ans. 565*l.* 17*s.* 6*d.*

Ex. 57. How much must be paid for 74 yards of cloth, at 37½ cts. per Ell Flemish? Ans. 37 dols.

Ex. 58. What was paid for 22,275 bushels of flour, at 4*l.* 7*s.* the sack of five bushels? Ans. 19,379*l.* 5*s.*

Ex. 59. What is the value of 6 casks of raisins, each weighing 3 cwt. 2 qrs. 14 lb. at 5*l.* 10*s.* 6*d.* per cwt.? Ans. 120*l.* 3*s.* 4½*d.*

Ex. 60. How much must I give for a gold snuff-box that weighs 8 oz. 9 dwts., at the rate of 4*l.* 3*s.* 9*d.* per oz? Ans. 35*l.* 7*s.* 8½*d.*

Ex. 61. How much tax must I pay for 586 dollars, at 6¼ cts. per dollar? Ans. 36 dols. 62½ cts.

Ex. 62. A Bankrupt has but 1020 dollars to pay debts to the amount of 3225 dollars; how much can he pay in the dollar? Ans. 31 cts. 7 mills nearly.

Ex. 63. A merchant failing, his assignees find effects and good debts to the amount of 3335*l.*; but he owes 4225*l.*; the expences attending his bankruptcy will be 212*l.* 9*s.*: how much, therefore, will he pay in the pound? Ans. 14*s.* 9*d.* $\frac{1787}{4225}$.

Ex. 64. An honest tradesman, through unforeseen misfortunes, is obliged to call his creditors together; he finds his debts to be 4226*l.* and he can pay 14*s.* 6*d.* in the pound: how much has he still left? Ans 3136*l.* 7*s.*

Ex. 65. Hops are remarkably cheap, and I have 100*l.* to spare: how many can I purchase at 3*l.* 15*s* 6*d.* per cwt. Ans. 26 cwt. 2 qrs. nearly.

Ex. 66. If 10 lbs. of tea are worth 10 dols. 25 cts.; how much of the same sort can I purchase for 38 dols. 43¾ cts.? Ans. 37 lb. 8 oz.

Ex. 67. What must I pay for the carriage by the canal, from Manchester to Etruria, 705 tons, 5 cwt. of goods, at 15*s.* per ton; and what is the difference between this and the land carriage, at 2*l.* 15*s.* per ton?

Ans. I must pay 528*l.* 18*s.* 9*d.*, for carriage by Canal, and 1410*l.* 10*s.* difference in the price of carriage.

Ex. 68. What weight of goods can be carried on the the canal between Manchester and Birmingham for 85*l.* at the rate of 1*l.* 10*s.* per ton: and how much can be carried the same distance, by land-carriage, at 5*l.* per ton? Ans. 56⅔ tons by water, and 17 tons by land.

Ex. 69. The clothing of a regiment of 760 men comes to 3050*l.* how much is that per man? Ans. $4l.\frac{40}{760}$.

Ex. 70. What may a man spend per week, whose income is 2000*l.* per annum, supposing 52 weeks in a year? Ans. 38*l.* 9*s.* $2\frac{3}{4}d.\frac{4}{52}$.

Ex. 71. If by selling fine Irish cloth at 5 dollars per ell, I gain 108 dollars; how much shall I gain if I sell it at 6 dols. 25 cts. per ell? Ans. 135 dollars.

Ex. 72. If sugar that cost 9 cts. per. lb. be sold at 3 lb. for $37\frac{1}{2}$ cts. what will be the gain by selling 1 cwt.?

Ans 3. dols. 92 cts.

Ex. 73. I purchase 5 pieces of Holland, each containing 36 yards, at 4*s.* 9*d.* per yard: how much shall I gain by selling it at 6*s.* 2*d.* per ell English?

Ans. 1*l.* 13*s.* profit.

Ex. 74. Two persons part at the same time for the same place, the one travels north 24 miles a day, and the other 21 miles a day south: when will they be 1000 miles asunder? Ans. $22\frac{2}{4}$ days nearly.

Ex. 75. If a pack of wool weighs 3 cwt. 2 qrs. 7 lb., what is it worth at 21*s.* 6*d.* per tod of 14 lbs.?

Ans. 30*l.* 12*s.* 9*d.*

Ex. 76. The rents of a parish amount to 1750*l.*, and a rate for the poor is wanted of 65*l.* 7*s.* 6*d.*: what is that per pound? Ans. 9*d.* nearly.

THE RULE OF THREE INVERSE.

This rule, like the last, teaches, from three given numbers, to find a fourth, which fourth number shall bear the same proportion to the second, as the first has to the third. Thus, if the question be, If 10 men can mow a certain field in 6 days, how soon can it be done by 20 men? The answer will evidently be in 3 days, because double the number of men will certainly do the same work in half the time; the proportion will therefore stand, 10 men : 6 days : : 20 men : 3 days; and 3 bears the same proportion to 6, that 10 does to 20; that is, the fourth number bears the same proportion to the second, that the first does to the third.

Rule.—State the question, and when necessary, reduce the terms as before. Multiply the first and second terms together, and divide the product by the third term; the quotient is the answer in the same denomination as the second term; thus in the foregoing example,

$$\frac{10\times 6}{20} = 3 \text{ days.}$$

Ex. 1. If 15 reapers can cut down a field of corn in 4 days, in how long time will the same work be performed by 40 men?

15 : 4 : : 40

4

4.0)6.0

$1\frac{1}{2}$ day.

Ex. 2. If the penny loaf weighs 4 ounces when flour is 4*s.* per peck, how much must it weigh when flour is 5*s.* 4*d.* per peck ? Ans. 3 ounces.

Ex. 3. A person lent me 240 dollars for 8 months ; in return for his kindness, how much ought I to lend him for 18 months ? Ans. 106 dollars 66 cents.

Ex. 4 How many men must be employed to finish a canal in 12 days, which 5 could perform in six weeks, or 36 days ? Ans. 15 men.

Ex. 5. If 24 pioneers can make a trench in 12 days, what length of time would the same work employ 9 men ? Ans. 32 days.

Ex. 6. The floor of a chapel 96 feet in length, and 70 feet in breadth, is to be covered with matting 2 feet six inches broad : how many yards will it require ? Ans. 2688 feet.

Ex. 7. If a person travel 12 hours a day, and finish his journey in three weeks ; how long would the same journey take him, if he travelled only 9 hours a day at the same rate ? Ans. 4 weeks.

Ex. 8. If the town and garrison of Bhurtport, containing 22,400 persons, have provisions to last three weeks, how many inhabitants must Holkar send away, so as to make the provisions last 7 weeks, which is as long as General Lake can carry on the siege ? Ans. 12,800 persons.

Ex. 9 If a besieged garrison have 4 months provisions, at the rate of 18 ounces per man per day ; how long will they be able to hold out, if each man is allowed only 12 ounces per day ? Ans. 6 months.

Ex. 10. If there are in a garrison provisions sufficient for 1500 men 10 weeks, which, on account of the rains, is seven weeks longer than the siege can last ; how many soldiers may be brought to defend the place for three weeks, without lessening the quantity of food to any individual ? Ans 3500 soldiers.

Ex 11 If 9 plasterers can finish the inside of a chapel in 10 days ; how long will it take 4 men, supposing the other 5 sent away on a new job ? Ans. $22\frac{1}{2}$ days.

Ex. 12. If $3\frac{1}{2}$ yards of broad cloth, $1\frac{3}{4}$ wide will make a suit of clothes ; how much will be necessary of cloth only $\frac{3}{4}$ wide ? Ans. 8 yards $0\frac{2}{3}$.

Ex. 13. If 52 clerks in the bank are sufficient to make up the books in a certain office in 15 days, how many clerks would be required to do the same work in 6 days?

Ans. 130 clerks.

Ex. 14. If the carriage of $15\frac{1}{2}$ cwt. for 60 miles, came to 7*s*. 9*d*.; how far can I have carried $3\frac{3}{4}$ cwt. for the same sum? Ans. 248 miles.

Ex. 15. If 24 men can finish a piece of work in 16 hours; how many men will it require to do the same work in 12 hours? Ans. 32 men.

Ex. 16. If 12 inches in length, and 12 inches in breadth, make a square foot; what length of board, 8 inches broad, will be equal to the same measure.

Ans. 18 inches.

Ex. 17. If 220 yards in length, and 22 in breadth, make an acre; what must be the breadth when the length is 120 yards? Ans. 40 yards.

Ex. 18. If 5 horses can be maintained when oats are 18*s*. per quarter; how many can be supported at the same cost, when they are 30 shillings per quarter?

Ans. 3 horses.

Ex. 19. If 250 dollars gain 12 dollars at interest in 12 months; what principal will gain an equal sum in 5 months? Ans. 600 dollars.

Ex. 20. There are two rooms in the floors of which there are an equal number of square feet; the length of the one is 50 feet, and its breadth is 42: but the breadth of the other is 48 feet; what is its length? Ans $43\frac{36}{48}$.

Ex. 21. The cock to a large water-tub will empty it in 36 minutes; how many such cocks will empty it in $4\frac{1}{2}$ minutes? Ans. 8 cocks.

Ex. 22. The sides of a room are found to measure 138 feet in length, and the height is 14 feet 6 inches; how much paper, 2 feet 3 inches wide, will cover it; and what is the value of it at 9*d*. per yard?

Ans. 296 yds. 1 ft. 4 in. 11*l*. 9*s* 4*d*.

Ex. 23. If 50 cows can be kept in a field 17 days; how long will the same pasture feed 70 cows?

Ans. $12\frac{1}{7}$.

THE DOUBLE RULE OF THREE.

THE Double Rule of Three teaches, from five given numbers to find a sixth. Three of the numbers contain the suppositions, and the remaining two are terms of demand.

RULE (1.) Put the terms of supposition one above another in the *first* place, except that which is of the same nature with the term sought, which put in the *second* place.

(2.) Place the terms of demand one above another in the third place, in the same order as the terms of the supposition were put in the first place.

(3.) The first and third term in every row will be of the same nature, and must be reduced to one denomination; and the middle term must be brought to the lowest denomination mentioned.

(4.) Examine each stating separately, using the middle term as common to both, in order to know if the proportion be *direct* or *inverse*. When it is direct mark the first term with an asterisk, and when it is inverse, mark the third term with an asterisk.

(5.) Multiply the numbers together which are marked for a divisor, and those which are not marked for a dividend, and the quotient will be the answer.

Ex. 1. If 12 persons spend 160*l.* in 4 months: how much will 32 persons expend in 8 months?

persons. *L.* persons.
12 : 160 : : 32
months months.
4 : : : 8

or,

$12 \times 4 : 160 : : 32 \times 8$

$$\frac{32 \times 8 \times 160}{12 \times 4} = 853l.\ 6s.\ 8d.$$

Ex. 2. If a garrison of 600 men have provisions for 5 weeks, allowing each man 12 ounces per day; how many can be maintained 10 weeks by the same quantity, if each man is limited to 8 ounces a day?

weeks. men. weeks.
5 : 600 : : 10
oz. oz.
12 : : : 8

or,

$5 \times 12 : 600 : : 10 \times 8$

$$\frac{5 \times 12 \times 600}{10 \times 8} = 450 \text{ men, Ans.}$$

Ex. 1. If 15 pecks of wheat will last a family of 9 persons 22 days, in how many days will six persons consume 20 pecks?

pecks. days. pecks.
15 : 22 : : 20
persons. persons.
9 : : : 6

or,

$9 \times 22 \times 20$

$$\frac{}{15 \times 6} = 44 \text{ days.}$$

Ex. 4. If 6 pioneers can dig a ditch 34 yards long in 10 days; how many yards may be dug by 20 men in 15 days? Ans. 170 yards.

Ex. 5. If 1050 soldiers consume 250 quarters of corn in 6 months; how many soldiers will 960 quarters serve 4 months? Ans. 6048 men.

Ex. 6. If a cask of beer last 8 persons 14 days: how many casks will serve 2 persons 365 days?

Ans. $6\frac{29}{56}$ casks.

Ex 7. If 10 men in 6 weeks earn 500 dollars; how many weeks must 15 men work to earn 1000 dols.

Answer, 8 weeks.

Ex 8. Suppose I walk 66 miles in 4 days, of eight hours each day: how many days, of 14 hours each, shall I be in going from London to York, or 196 mites.

Ans. $6\frac{52}{66}$, almost 7 days.

Ex. 9. If three boats take 6000 herrings in 8 days: how long will 600 boats be in taking 20,000 barrels, each containing 700 herrings? Ans. $93\frac{3}{8}$

Ex. 10. If, against a general mourning, 6 tailors can make 10 suits of clothes in 4 days: how many suits can 600 men make in the 7 days which occur before the mourning is wanted? Ans. 1750 suits.

Ex. 11. If 12 mantua-makers can make 27 mourning dresses in 4 days: how many persons would be required to make 189 dresses in 8 days?

Ans. 42 mantua-makers.

Ex. 12 If 3000 copies of a History of America, each containing 11 sheets, require 66 reams of paper: how much paper will 5000 take, if the work be extended to $12\frac{1}{2}$ sheets. Ans. 125 reams.

Ex. 13. As 12 inches in length, 12 in breadth, and 12 in thickness, make a solid foot: what length of plank, which is 7 inches broad and 3 inches thick will make the same? Ans. $82\frac{2}{7}$ inches.

Ex 14. If 450 tiles, each 12 inches square, will pave my cellar: how many tiles must I have, if the tiles are 9 inches long and 8 broad? Ans. 900 tiles.

Ex. 15. If the expence of 3 persons on a tour for 5 months be 123*l.* 8s.: what will 2 persons spend in 9 months? Ans. 148*l.* 1*s.* 7*d.*

Ex. 16. If 12 ounces of wool make $2\frac{1}{2}$ yards of very fine cloth, 6 quarters wide: how much wool would be required to 150 yards, 4 quarters broad?

Ans. 480 ounces.

Ex. 17. If 300 dollars gain 15 dollars interest in a year, in what time will 900 dollars gain 180 dollars.

Ans. 4 years.

Ex. 18. If an iron bar 4 feet long, 3 inches broad, and $1\frac{1}{4}$ inch thick, weigh 36 lbs.: how much will a bar weigh that is six feet long, 4 inches broad, and 2 inches thick?

Ans. $115\frac{1}{5}$ lbs.

MISCELLANEOUS QUESTIONS ON ALL THE FOREGOING RULES.

Ex. 1. What three numbers are those, the first of which is 105, the second $\frac{2}{3}$ds of the first, and the third 67 less than the first and second together?

Ans. first, 105, second 70, third, 108.

Ex. 2. A gentleman left his eldest daughter 1000 guineas more than the youngest, and to three other daughters he left 7000 guineas between them, which was equal to the sum left to the youngest and eldest together: what was each child's fortune. Answer, Eldest, 4000, youngest, 3000, three other daughters, 7000.

Ex. 3. What is the difference in value between five times five and twenty guineas, and five times twenty-five guineas? Ans. 80 guineas.

Ex. 4. What was the value of a prize taken by 25 sailors, besides officers, so that each sailor received 19*l.* 9*s.* 9*d.*, and the officers received as much as the sailors?

Ans. 974*l.* 7*s.* 6*d.*

Ex. 5. A prize valued at 13,177*l.* 10*s.*, after the officers have had their share, is to be divided among 525 sailors: what would each man have to take?

Ans. 25*l.* 2*s.* each man's share.

Ex. 6. What is a fourth proportional to the numbers 6, 9, and 24? Ans. 36 fourth proportional.

Ex. 7. What is the value of 4 packs of cloth, each pack containing 4 parcels, each parcel 10 pieces, and

each piece 26 yards, at the rate of 12 dols. 50 cts. for 3 yards? Ans. 17333 dols. 33 cts.

Ex. 8. How many yards of paper, 3 quarters wide, will be sufficient for a room 48 yards round, and four yards high: and what is the value of the paper, at the rate of 18s. per piece of 24 yards?

Answer, 256 yards, worth 9*l.* 12*s.*

Ex. 9. If 100 dollars gain 5 dollars in 12 months, what will 75 dollars gain in 9 months?

Ans. 2 dols. 81¼ cts.

Ex. 10. If 48 cannon consume, in 3 days, 288 barrels of powder, how much will be spent in 15 days, when 144 cannon are to be supplied? Ans. 4320.

Ex. 11. Fifteen people joined to purchase a lottery ticket, for which they gave three shillings less than eighteen guineas: if it came up a prize of 30,000 guineas, what did each man receive, and what was his gain? Ans. 2100*l.* each man's share, and 2098*l.* 15*s.* each man's gain.

Ex. 12. A tobacconist bought two parcels of tobacco, which weighed 9 cwt. 2 qrs, for a hundred guineas, the difference of the parcels in weight was 3 qrs. 12 lb., and in value eight guineas: what was their weight and values? Ans. one parcel. 5 cwt. 0qr. 20 lb. the other parcel 4 cwt. 1 qr. 8 lb., cost 54 guineas, and 36 guineas.

Ex. 13. The clothing of 100 charity children came to 211*l.*, of which 135*l.* was expended on 60 boys: what was paid for the 40 girls, and how much did the clothes of each child cost? Ans. girls clothing 76*l.*, price of each boy's clothes, 2*l.* 5*s.* ditto girl's clothes 1*l.* 18*s.*

Ex. 14. A great grazier left to his four sons 220 oxen and 1200 sheep: I demand the value of each son's legacy, supposing the oxen worth 18 guineas each, and the sheep 39 shillings each? Ans. 1624*l.* 10*s.* each son's legacy.

15. What number is that which, multiplied by 384, will give a product of 3,018,248? Answer, 7847

Ex. 16. What is gained by the sale of 456 yards of cloth, that was bought at the rate of 7 dols. 25 cts. per yard, and sold at the rate of 11 dols. 50 cts. per yard?

Ans. 1853 dols.

Ex. 17. If 9 printers can set up the New Testament in 22 days, in what time could it be done if 15 were employed? Answer, $13\frac{1}{3}$ days.

Ex. 18. If 8 men will earn on an average 84 dols. in 6 days, how much can 15 men earn in 27 days?

Ans. 708 dols. 75 cts.

Ex. 19. When the quotient is 1083, and the divisor 555, what is the dividend, if there be a remainder of 79? Answer, 601144.

Ex. 20. The silk mill at Derby winds off 73,726 yards of silk every time the great wheel goes round, which is thrice in a minute: how many yards will it wind in a year, allowing that it works every day, except Sunday, 15 hours, and how many skeins will be made, supposing 960 yards go to the skein?* Ans. 62,305,842,600 yards made in a year, and $64901919\frac{3}{8}$ the number of skeins.

Ex. 21. In the partition of some waste lands in the west of England, A had $59\frac{1}{2}$ acres, B $76\frac{1}{4}$ acres, C 110 acr. 2r. 12 per., D. 15 acres, and E. 39 acr. 0r. 12 per., but these, taken together, were but one-fifth of the whole: how many acres were divided, and what was the value of the whole, supposing each acre worth 15*l.* 9*s.* 6*d.*? Ans. 1502 acr. 0r. 0p. land divided, 23243*l.* 9*s.* 0*d.* value of the land.

Ex. 22. An Island in the West Indies contains 42 parishes, and every parish 76 houses, and each house at the rate of $5\frac{1}{2}$ white persons; besides these, there were 65 negroes to each of 54 plantations: how many people were there on the whole island? Ans. 21066 persons.

Ex. 23. In the club mentioned in the Spectator (No. 9.), there were 15 persons, weighing together 3 tons: how many pounds, ounces, and drams, Avoirdupois, did each man weigh? Ans. 448 lb. 7168 oz. 114688 dr.

Ex. 24. The British possessions in Hindostan contain 212,406 square miles, and the population is estimated at fourteen millions: how many inhabitants are there to a square mile? Ans. 66 persons nearly.

Ex. 25. If 9 lb. of tea cost 7 dollars 20 cts., what is the worth of 4 chests each weighing 1 cwt. 2 qrs.?

Ans. 537 dollars 60 cents.

* 365—52 equal 313 the number of working days in a year.

Ex. 26. What shall I give for a farm containing 256 acres, for which I am to pay at the rate of 95 dollars for 4 acres? Ans. 6080 dollars.

Ex. 27. What will it cost a young man to come into a farm, for the lease of which he is to pay 1000 guineas; for 22 horses he is to pay at the rate of 18 guineas each; for crops in the ground 354*l.*; for 210 bushels of wheat he is to pay 4*l.* 10*s.* per 8 bushels; the household furniture is appraised to him at 298 guineas, and for farming utensils of all kinds he is to pay 196*l.*?

Ans. 2446*l.* 16*s.* 6*d.*

Ex. 28 The revenue collected in Hindostan by the British, is reckoned at 3,400,000*l.*, how much is that from each inhabitant, supposing they amount to 14 millions?

Ans. 4*s.* 10½*d.*

Ex. 29. The number of negroes in Jamaica is estimated at 250,000, and of whites 20,000, how many slaves are there to a single white man, and what do the planters reckon their property worth in the article of slaves only, supposing each to be worth 93 guineas?

Ans. 12½ slaves, 24,412,500*l.*

Ex. 30. The population of the United States is estimated at six millions and a half, and the number of slaves still existing in that free country is reckoned to be 697,697, how many free people are there to one slave?

Ans. 9⅓ nearly.

Ex. 31. The extent of China Proper is equal to 1,397,999 square miles, and the population is estimated at 333,000,000, how many inhabitants are there to a square mile? Ans. 238 nearly.

Ex. 32. In Spain each person pays 10 shillings to government for protection; in France, under the old government, each paid 20*s.* for protection; and in England we pay full three guineas each for the same advantages, how much is the revenue of the three governments, supposing the population of Spain to be 10¼ millions; of France, at the period referred to, 25 millions; and of England and Wales 9,343,173? Ans. 59,555,994*l.* 19*s.*

Ex. 33. The population of London, Westminster, and Southwark, is 864,865, that of Paris 547,756, how

much does the population of London exceed that of Paris? Ans. 317.109.

Ex. 34. How many minutes and seconds have elapsed since the birth of Christ, or 1808 years*?

Ans. 950,935,680 min. 57,056,140,800 sec.

Ex. 35. How long would it require to count five hundred millions sterling, supposing a person were to reckon 150*l.* in a minute, and were to be employed 10 hours each day, and six days a week, till he had finished the job? Ans. 926 weeks, nearly.

Ex. 36. How many barley-corns will reach round the earth, supposing the length to be 25,200 miles?

Ans. 4,790,016,000

Ex. 37. How many seven-shilling pieces are there in a thousand pounds? Ans. 2857 seven-shil. 1 shil.

Ex. 38. A French franc is worth 10*d.*, how many francs are there in 100*l.* Ans. 2400 Francs.

Ex. 39. If 8 men can mow 18 acres in 4 days, how many men will be required to mow 50 acres in six days? Ans. $14\frac{22}{27}$

Ex. 40. A balloon has moved at the rate of 6492 feet in a minute, how long would it have been sailing round the earth at the same rate, supposing the circumference of the earth to be 25,200 miles?

Ans. 14 days 5 hours 35 min. 22 sec.

Ex. 41. How much oftener will the small wheel of a coach turn than the large one, between London and Bristol, or 120 miles, if the former be 10 feet 8 inches in circumference, and the latter 18 feet 4 inches?

Ans. 24840.

Ex. 42. If my income be 250*l.* per annum, and I have foolishly expended 15*s.* per day, how much shall I be in debt at the year's end, and what may I expend per day the following year, so as to have ten guineas in hand at the conclusion of it? Ans 23*l.* 15*s.* debt, 11*s.* $9\frac{3}{4}$*d.* spend.

Ex. 43. It is said the impositions of hackney-coachmen, by overcharges, are equal to one-fourth of what they earn; now, if they earn each on an average 18*s.* per day, and there be 1100 employed 313 days in a year, I demand the amount of their overcharges in a year?

Ans. 77467*l.* 10*s.*

* Allowing 365¼ days in one year.

Ex. 44. There were at Vauxhall gardens on the Prince of Wales birth-day, 1805, 10,059 persons; the admission money was 3*s.* each; now, supposing each person to spend 3*s.* more, the half of which was profit to the proprietor, what would he clear by the night, allowing that the incidental expenses were 250*l.*?

Ans. 1258*l.* 17*s.*

Ex. 45. If $3\frac{1}{4}$ yards of cloth will make a shirt, how much of the same stuff will be wanted to make two shirts for each man of a regiment, consisting of 855 men?

Ans. $5557\frac{1}{2}$ yards.

Ex. 46. In November, 1800, 276,334 five-pound bank notes were issued; in December 2,626,700; and in the January following 2,769,160; what was the nominal value of the notes issued in these three months; and what was the cost of white rags, from which they were made, supposing each ounce of rag might be manufactured into twenty five-pound notes, and the rags to be worth 8*d.* per lb.? Ans. 28,360.970 nominal, 590*l.* 17*s.* $0\frac{1}{2}$*d.*

Ex. 47. Two persons depart from London to York on the same day; the one walks 19 miles a day, the other only $15\frac{1}{2}$ miles; how far distant will they be from one another after ten days travelling, and when will each get to York, which is 197 miles from London?

Ans. 35 miles distant, he who goes 19 miles a day will complete his journey on the 11th. day, while the other will not complete his journey till the 13th. day.

Ex. 48. The population of the world is estimated at a thousand millions of human beings; if the face of the earth be re-peopled every 33 years, how many persons are born and die in a year, week, day, and minute?

Ans. 30,303,030 year, $582,750\frac{30}{52}$ week, 83250 day.
3468 nearly in an hour, 58 nearly a min.

Ex. 49. The field opposite my house will serve 50 cows forty days; how long will it afford 220 with equal feed? And. 9 days and a fraction.

Ex. 50. If 10 persons expend 250 dols. in 4 months; how much ought 3 persons to expend in 12 months?

Ans. 225 dollars.

FRACTIONS.

A Fraction is the part, or parts of a whole, or of any whole quantity expressed by unity, and is expressed by two figures, with a line drawn between them, as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$.

The upper figure of a fraction is called the numerator, and the under one the denominator.

The denominator shews how many parts the unit is divided into, and the numerator, how many of these parts are to be taken: thus $\frac{3}{4}$, or three-fourths, shews that the whole is divided into four parts, and that three of those parts are to be taken: and $\frac{5}{8}$, or five-eighths, shew that the whole is divided into eight parts, and that five of these parts are taken.

There are four sorts of fractions, simple and compound, proper and improper.

A simple fraction has only one numerator and denominator, as $\frac{1}{3}$, or $\frac{3}{8}$.

A compound fraction consists of two or more parts, and is known by the word *of* placed between them, as $\frac{3}{4}$ of 6: or $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{9}{10}$.

A proper fraction is, when the numerator is less than the denominator.

An improper fraction is, when the numerator is equal to, or greater than, the denominator.

A mixed number is formed from an integer and a fraction joined together, as $8\frac{5}{6}$.

A complex fraction is one that has a fraction or a mixed number for its numerator, or denominator, or both.

REDUCTION OF FRACTIONS.

THE method of changing fractions from one form to another, without altering their value, is called Reduction. $\frac{24}{48}=\frac{12}{24}=\frac{6}{12}=\frac{3}{6}=\frac{1}{2}$. Reduction serves to prepare fractions for Addition, Subtraction, Multiplication, and Division.

CASE 1. To reduce fractions to their least terms.

RULE. Divide the terms of the given fraction by any number, which will divide them both without a remainder, and the quotients will be the terms of a new fraction, equal in value to the given fraction. Repeat the operation, till the terms of the reduced fraction are divisible only by 1.

Ex. 1. Reduce $\frac{3136}{3584}$ to its lowest terms.

$$8\overline{)}\,\frac{3136}{3584}=\frac{392}{448}\text{, and }8\overline{)}\,\frac{392}{448}=\frac{49}{56}\text{, and }7\overline{)}\,\frac{49}{56}=\frac{7}{8}$$

Reduce the following fractions to their lowest terms.

Ex. 1	Ex. 2.	Ex. 3.	Ex. 4.	Ex. 5.
$\frac{32}{120}=\frac{4}{15}$	$\frac{208}{684}=\frac{52}{171}$	$\frac{156}{72}=\frac{17}{9}$	$\frac{156}{336}=\frac{13}{28}$	$\frac{360}{708}=\frac{30}{59}$

Ex. 6.	Ex. 7.	Ex. 8.	Ex. 9.	Ex. 10.
$\frac{384}{1152}=\frac{1}{3}$	$\frac{5184}{6012}=\frac{432}{501}$	$\frac{2475}{2880}=\frac{55}{64}$	$\frac{4032}{4806}=\frac{224}{267}$	$\frac{3105}{15705}=\frac{69}{267}$

Reduce $\frac{2\times3\times4\times5}{3\times4\times7\times8}$ to its lowest terms.

$$\frac{2\times3\times4\times5}{3\times4\times7\times8}=\frac{10}{56}=\frac{5}{28}$$

$$\frac{3\times8\times9\times2}{4\times3\times14\times36}$$

Reduce to the lowest terms.

$$\frac{3\times8\times9\times2}{4\times3\times14\times36}=\frac{3\times2\times4\times9\times2}{4\times3\times2\times7\times4\times9}=\frac{2}{28}=\frac{1}{14}$$

Ex. 11. $\frac{3\times4\times15\times4}{5\times6\times24\times3}=\frac{1}{2}$

Ex. 12. $\frac{10\times27\times30\times12}{15\times9\times55\times30}=\frac{24}{55}$

Case II. To find the greatest common measure of a fraction.

Rule. Divide the greater term by the less, and this divisor by the remainder, then the last divisor will be the greatest common measure of both terms of the fraction.

Ex. What is the greatest common measure of the fraction $\frac{918}{1998}$?

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918)1998(2
   1836
   ----
  162)918(5
      810
      ---
   108)162(1
      108
      ---
Ans. 54 greatest C. M.  54)108(2
                           108
                           ---
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$$\frac{918}{1998}=\frac{17}{37}$$

What are the greatest common measures of the following fractions?

Ex. 1.	Ex. 2.	Ex. 3.
$\frac{270}{306}$ Ans. 18	$\frac{1080}{1224}$ Ans. 72	$\frac{720}{1736}$ Ans. 8
Ex. 4.	**Ex. 5.**	**Ex. 6.**
$\frac{3108}{3552}$ Ans. 444	$\frac{9600}{16800}$ Ans. 2400	$\frac{14960}{18320}$ Ans. 80

CASE III. To reduce an improper fraction to an equivalent, whole, or mixed number.

RULE. Divide the numerator by the denominator, and the quotient will be the integer, or mixed number required: thus $\frac{35}{8}=4\frac{3}{8}$, and $\frac{45}{9}=5$.

Reduce the following improper fractions to their proper terms.

Ex. 1. $\frac{29}{8}=3\frac{5}{8}$. Ex. 2. $\frac{57}{7}=8\frac{1}{7}$. Ex. 3. $\frac{69}{8}=8\frac{5}{8}$. Ex. 4. $\frac{75}{12}=6\frac{1}{4}$. Ex. 5. $\frac{96}{16}=6$. Ex. 6. $\frac{101}{13}=7\frac{10}{13}$.

Ex. 7. $\frac{850}{24}=35\frac{5}{12}$. Ex. 8. $\frac{9764}{556}=17\frac{78}{139}$. Ex. 9. $\frac{5640}{450}=12\frac{8}{15}$. Ex. 10. $\frac{889}{3}=296\frac{1}{3}$.

CASE IV. To reduce a mixed number to an equivalent improper fraction.

RULE. Multiply the whole number by the denominator of the fraction, to the product add the numerator, for a new numerator, under which place the denominator;

Thus $4\frac{3}{8}=\frac{35}{8}$, and $296\frac{1}{3}=\frac{889}{3}$.

Reduce the following mixed numbers to their equivalent improper fractions.

Ex. 1. $3\frac{5}{8}=\frac{29}{8}$. Ex. 2. $8\frac{5}{8}=\frac{69}{8}$. Ex. 3. $6\frac{5}{12}=\frac{77}{12}$.

Ex. 4. $7\frac{10}{13}=\frac{101}{13}$ Ex. 5. $18\frac{7}{9}=\frac{169}{9}$ Ex. 6. $435\frac{11}{16}=\frac{6971}{16}$

Ex. 7. $378\frac{5}{15}=\frac{1135}{3}$ Ex. 8. $499\frac{3}{15}=\frac{2496}{5}$

Ex. 9. $54\frac{2}{33}=\frac{1784}{33}$ Ex. 10. $67\frac{24}{57}=\frac{3853}{57}$

CASE V. To reduce a compound fraction to an equivalent simple one.

RULE (1.) If any of the proposed quantities be integers, or mixed numbers, reduce them to their proper terms.

(2.) Multiply all the numerators together for a new numerator, and all the denominators for a new denominator, and then reduce the fraction to its lowest terms.

Reduce $\frac{4}{5}$ of 3 of $7\frac{5}{6}$ to a simple fraction.

$$\text{Operation} \quad \frac{4}{5} \times \frac{3}{1} \times \frac{47}{6} = \frac{2 \times 2 \times 3 \times 47}{5 \times 1 \times 2 \times 3} = \frac{94}{5}.$$

The fraction $\frac{94}{5}$ is already in its lowest terms, because no figure higher than the unit will divide both terms of the fraction without a remainder.

Ex. 1. $\frac{7}{9}$ of $\frac{6}{5}$ of 5 of $\frac{3}{4} = \frac{7}{2}$. Ex. 2. $\frac{3}{6}$ of 4 of $5\frac{3}{7} = 12\frac{3}{7}$

Ex. 3. $\frac{5}{11}$ of 8 of $7\frac{9}{9}$ of $12 = 33\frac{6}{11}$.

Ex. 4. $\frac{4}{13}$ of $\frac{3}{18}$ of 12 of $9\frac{3}{9} = 6\frac{10}{117}$.

Ex. 5. $\frac{7}{16}$ of 10 of $\frac{12}{8}$ of $18\frac{2}{3} = 122\frac{1}{2}$. Ex. 6. $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{4}$ of $\frac{6}{10} = \frac{3}{8}$.

CASE VI. To reduce fractions of different denominators to others of equal value, having a common denominator.

RULE. (1.) Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators for a common denominator.

Reduce $\frac{3}{5}$, $\frac{7}{9}$, $3\frac{2}{3}$, and 3, to a common denominator.

Operation, $\frac{3}{5}$, $\frac{7}{9}$, $\frac{11}{3}$, $\frac{3}{1}$.

New numerators.

$3 \times 9 \times 3 \times 1 = 81$

$7 \times 5 \times 3 \times 1 = 105$

$11 \times 5 \times 9 \times 1 = 495$

$3 \times 5 \times 9 \times 3 = 405$

New denom.

$5 \times 9 \times 3 \times 1 = 135$

Answer, $\frac{81}{135}$, $\frac{105}{135}$, $\frac{495}{135}$, $\frac{405}{135}$.

Ex. 1. Reduce $\frac{2}{3}$, $\frac{5}{4}$, and $\frac{3}{4}$ to a common denominator.

Ans $\frac{32}{48}$, $\frac{60}{48}$, $\frac{36}{48}$.

2. Reduce $\frac{6}{7}$, $\frac{5}{9}$, $\frac{4}{5}$, and $\frac{7}{8}$, to a common denominator.

Answer, $\frac{2160}{2520}$, $\frac{1400}{2520}$, $\frac{2016}{2520}$, $\frac{2205}{2520}$.

3. Reduce $\frac{2}{7}$, $\frac{3}{5}$, $\frac{6}{7}$, and 3, to a common denominator.

Answer, $\frac{70}{245}$, $\frac{147}{245}$, $\frac{210}{245}$, $\frac{735}{245}$.

4. Reduce $\frac{4}{15}$, $\frac{1}{10}$, 8, and $11\frac{1}{2}$, to a common denominator.

Answer, $\frac{80}{300}$, $\frac{30}{300}$, $\frac{2400}{300}$, $\frac{3450}{300}$.

5. Reduce $\frac{3}{11}$, $\frac{1}{2}$, $\frac{2}{7}$, 4, and $2\frac{1}{3}$, to a common denominator.

Answer, $\frac{210}{770}$, $\frac{385}{770}$, $\frac{220}{770}$, $\frac{3080}{770}$, $\frac{1694}{770}$.

6. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{2}{7}$, and $\frac{15}{20}$, to a common denominator.

Answer, $\frac{140}{280}$, $\frac{168}{280}$, $\frac{80}{280}$, $\frac{210}{280}$.

7. Reduce $\frac{7}{8}$, $\frac{5}{9}$, $\frac{1}{4}$, $\frac{2}{5}$, and 7, to a common denominator.

Answer, $\frac{1260}{1440}$, $\frac{800}{1440}$, $\frac{360}{1440}$, $\frac{576}{1440}$, $\frac{10080}{1440}$.

8. Reduce $\frac{4}{10}$, $\frac{4}{5}$, $\frac{3}{7}$, and $\frac{9}{16}$, to a common denominator.

Answer, $\frac{448}{560}$, $\frac{240}{560}$, $\frac{315}{560}$.

9. Reduce $\frac{1}{6}$, $\frac{3}{8}$, $\frac{1}{7}$, and $\frac{4}{11}$ of 9, to a common denominator.

Answer, $\frac{616}{3696}$, $\frac{1386}{3696}$, $\frac{528}{3696}$, $\frac{12096}{3696}$.

(2.) To find the *least* common denominator.

Set down the denominators of the given fractions in a line, and divide as many of them as possible, by any number which will leave no remainder, and set down the quotients, and the undivided numbers below. Repeat the operation till there be no two numbers which can be divided without a remainder. Then the product of all the divisors, and the quotients in the last lines will give the least common denominator. Divide this least common denominator by each of the given denominators separately, and multiply the quotients by their several numerators, their products will be the new numerators.

Reduce $\frac{3}{5}$, $\frac{7}{9}$, $\frac{11}{3}$, $\frac{3}{1}$, to the *least* common denominator.

3)5. 9. 3. 1

5, 3, 1, 1

———, then $3\times5\times3\times1\times1=45$, is the common denominator, and 45 divided by the given denominators, 5, 9, 3, 1, give 9, 5, 15, 45; these multiplied by the given numerators, give 27, 35, 165, 135, for new numerators, and the fractions will stand $\frac{27}{45}$, $\frac{35}{45}$, $\frac{165}{45}$, $\frac{135}{45}$.

Reduce $\frac{3}{3}$, $\frac{5}{4}$, $\frac{2}{5}$, $\frac{4}{6}$, and $\frac{3}{8}$, to the least common denominator.

3)3, 4, 5, 6, 8
4)1, 4, 5, 2, 8
2)1, 1, 5, 2, 2
1, 1, 5, 1, 1

The least denominator is, accordingly,
$3\times4\times2\times5=120$;
$120\div3, 4, 5, 6, 8=40, 30, 24, 20, 15$
40×2; 30×3; 24×2; 20×4; 15×3,
for new numerators; therefore the fractions required are $\frac{80}{120}, \frac{90}{120}, \frac{48}{120}, \frac{80}{120}, \frac{45}{120}$.

CASE VII. To reduce a fraction of one denomination to the fraction of another denomination of equal value.

RULE. (1.) When it is from the less to a greater denomination, "Multiply the denominator by all the denominations from that given to the one sought."

Thus, to reduce $\frac{3}{4}$ of a penny to a fraction of a pound, the answer will be $\frac{3}{4\times12\times20}=\frac{3}{960}$.

(2.) When it is from a greater to a less denomination, "Multiply the numerator by all the denominations, from that given to the one sought."

Thus, to reduce $\frac{6}{7}$ of a pound to the fraction of a farthing, $\frac{6\times20\times12\times4}{7}=\frac{5760}{7}$.

Ex. 1. Reduce $\frac{288}{8}$ of a farthing to the fraction of a pound. Answer, $\frac{288}{8640}$.

2. Reduce $\frac{5}{9}$ of a penny to the fraction of a shilling. Answer, $\frac{5}{108}$.

3. Reduce $\frac{3}{8}$ of a pound to the fraction of a farthing. Answer, $\frac{3840}{8}$.

4. Reduce $\frac{4}{15}$ of a pound to the fraction of a penny. Answer, $\frac{960}{15}$.

5. Reduce $\frac{11}{14}$ of a pound to the fraction of a farthing. Answer, $\frac{10560}{14}$.

6. Reduce 3 shillings to the fraction of a pound. Answer, $\frac{3}{20}$.

7. Reduce $\frac{4}{5}$ of a dwt. to the fraction of a lb. Troy.
Answer, $\frac{1}{540}$.
8. Reduce $\frac{5}{7}$ of a cwt. to the fraction of an ounce.
Answer, 1280 oz.=80lb.
9. Reduce $\frac{7}{8}$ of a week to the fraction of an hour.
Answer, $\frac{1176}{8}$.
10. Reduce $\frac{3}{4}$ of a mile to the fraction of a yard.
Answer, 1320 yards.
11. Reduce $\frac{7}{8}$ of a pipe to the fraction of a gallon.
Answer, $\frac{441}{4}$.
12. Reduce ^{1} cent to the fraction of a dollar
Ans. $\frac{1}{200}$ dollar.

CAES VIII. To find the *value* of a fraction in numbers of inferior denomination.

RULE. Multiply the integer, or its value in the next lower denomination, by the numerator, and divide by the denominator:

Thus, the value of $\frac{3}{5}$ of a pound is equal to $\frac{3 \times 20}{5}=12$ shillings, and $\frac{2}{3}$ of a shilling equal to $\frac{2 \times 12}{3}=\frac{5}{}$ 8 pence.

Ex. 1. What is the value of $\frac{5}{9}$ of a pound? Ans. 11s.$1\frac{1}{3}d$.
2. What is the value of $\frac{7}{8}$ of a shilling? Ans. $10\frac{1}{2}d$.
3. What is the value of $\frac{9}{16}$ of half a crown?
Ans. 18*d*.
4. What is the value of $\frac{3}{4}$ of a lb. Troy?
Ans. 9 ounces.
5. What is the value of $\frac{9}{14}$ of a cwt.? Ans. 72 lb.
6. What is the value of $\frac{5}{9}$ of a mile?
Ans. $977\frac{7}{9}$ yards.
7. What is the value of $\frac{3}{7}$ of a cwt.? Ans. 48 lbs.
8. What is the value of $\frac{5}{12}$ of a dollar? Ans. $41\frac{2}{3}$cts.
9. What is the value of $\frac{6}{7}$ of a hogshead of wine?
Ans. 54 gallons.

CASE IX. To reduce a complex fraction to an equivalent simple fraction.

RULE. If the numerator or denominator, or both, be whole or mixed numbers, reduce them to improper fractions; and multiply the denominator of the lower fraction into the numerator of the upper, for a new numerator, and the denominator, of the upper fraction into the numerator of the lower for a new denominator.

Thus, $\frac{4}{\frac{7}{8}} = \frac{\frac{4}{1}}{\frac{7}{8}} = \frac{4\times8}{7\times1} = \frac{32}{7}$. And $\frac{\frac{4}{10}}{5} = \frac{\frac{4}{10}}{\frac{5}{1}} = \frac{4}{50}$.

And $\frac{5\frac{7}{8}}{8} = \frac{\frac{47}{8}}{\frac{8}{1}} = \frac{47}{64}$. And $\frac{9}{3\frac{2}{7}} = \frac{\frac{9}{1}}{\frac{23}{7}} = \frac{63}{23}$. And again $\frac{5\frac{1}{4}}{3\frac{3}{7}} = \frac{\frac{21}{14}}{\frac{24}{7}} = \frac{147}{96}$. No other varieties can occur.

Ex. 1. Reduce $\frac{3\frac{1}{4}}{4}$ to a simple fraction. Ans. $\frac{13}{16}$

2. Reduce $\frac{\frac{3}{4}}{\frac{2}{5}}$ to simple fraction. Ans. $\frac{15}{8}$.

3. Reduce $\frac{3\frac{11}{16}}{19\frac{7}{9}}$ to a simple fraction. An. $\frac{84}{445}$.

4. Reduce $\frac{15\frac{2}{5}}{56}$ to a simple fraction. Ans. $\frac{77}{265}$.

Ex. 5. Reduce $\frac{\frac{3}{8}}{9}$ to a simple fraction. Ans. $\frac{1}{24}$

6. Reduce $\frac{7\frac{1}{8}}{9\frac{2}{3}}$ to a simple fraction. Ans. $\frac{171}{232}$

7. Reduce $\frac{5}{\frac{4}{7}}$ to a simple fraction. Ans. $\frac{35}{4}$

8. Reduce $\frac{4}{19\frac{7}{9}}$ to a simple fraction. Ans. $\frac{36}{173}$

ADDITION OF FRACTIONS.

RULE. Reduce mixed numbers to improper fractions, and compound or complex fractions to simple ones, and bring them all to a common denominator. Add all the numerators together, and write the sum over the common denominator.

Ex. Add $\frac{3}{5}$, $\frac{2}{3}$, $5\frac{1}{2}$, and $\frac{1}{4}$ together; which is thus performed: $\frac{3}{5}$, $\frac{2}{3}$, $\frac{11}{2}$, $\frac{1}{4}$.

$3 \times 3 \times 2 \times 4 = 72$
$2 \times 5 \times 2 \times 4 = 80$
$11 \times 5 \times 3 \times 4 = 660$
$1 \times 5 \times 3 \times 2 = 30$
$5 \times 3 \times 2 \times 4 = 120$

Therefore $\frac{72}{120} + \frac{80}{120} + \frac{660}{120} + \frac{30}{120} = \frac{842}{120} = 7\frac{2}{120} = 7\frac{1}{60}$ which is the answer.

This may be performed by bringing the given fractions to the least common denominator.

Thus, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{11}{2}$, $\frac{1}{4}$, then $\frac{2)5, 3, 2, 4,}{5, 3, 1, 2,}$, and the new denominator $= 60$; the fractions will be $\frac{36}{60} + \frac{40}{60} + \frac{330}{60} + \frac{15}{60} = \frac{421}{60} = 7\frac{1}{60}$.

Ex. 1. Add $\frac{4}{9}$, $\frac{3}{5}$, and $\frac{6}{7}$ together. Ans. $1\frac{284}{315}$

2. Add $\frac{3}{5}$, $\frac{7}{8}$, and $\frac{9}{2}$ together. Ans. $5\frac{39}{40}$

3. What is the sum of $\frac{3}{5}$, $\frac{4}{7}$, and $4\frac{1}{2}$? Ans. $5\frac{47}{70}$

4. Add together $3\frac{5}{7}$, $4\frac{3}{8}$, and $\frac{2}{5}$. Ans. $8\frac{137}{280}$

5. Add $\frac{2}{7}$, $\frac{6}{8}$, $2\frac{3}{4}$, and $5\frac{1}{2}$ together. Ans. $9\frac{9}{56}$

6. What is the sum of $7\frac{2}{5}$, $3\frac{1}{2}$, and $\frac{6}{4}$. Ans. $11\frac{51}{90}$

7. What is the sum of $\frac{3}{7}$ of a guinea, $\frac{3}{8}$ of a shilling, and $\frac{6}{9}$ of a penny? Ans. *0l.* 9*s.* $5\frac{1}{3}$*d.*

8. What is the sum of $\frac{2}{3}$ of a pound, $\frac{4}{7}$ of a shilling, and $\frac{7}{12}$ of a penny? Ans. *0l.* 5*s.* $0\frac{3}{4}$*d.* $\frac{2}{21}$

Ex. 9. What is the sum of $\frac{3}{4}$ of a guinea, $\frac{2}{3}$ of a shilling, and $\frac{9}{10}$ of a penny? Ans. 16s. $5\frac{9}{10}d.$

10. If I have $\frac{3}{8}$ of a coasting vessel, and purchase another share of $\frac{2}{11}$, what part of her will belong to me?
Ans. $\frac{49}{88}$

11. Add $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a mile together.
Ans. 1320 yds. 2 feet.

12. What is the sum of $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot and $\frac{2}{3}$ of an inch? Ans. 2 ft. 9 in. 2 b.c.

13. Add $\frac{1}{5}$ of a lb. troy to $\frac{3}{8}$ of an ounce.
Ans. 2 oz. 15 dwts. 12 grs.

14. What is the sum of $\frac{3}{4}$ of an eagle and $\frac{3}{5}$ of a dollar? Ans. 8 dols. 10 cts.

15. Add $\frac{5}{9}$ of dollar to $\frac{5}{7}$ of a cent? Ans. $56\frac{27}{63}$ cts.

SUBTRACTION OF ...TIONS.

RULE.—...uce the given fractions to the same denominator as in Addition, then subtract the lesser numerator from the greater, and under the difference place the common denominator.

Ex. Take $\frac{3}{9}$ from $\frac{5}{12}$: and $\frac{9}{18}$ from $\frac{11}{16}$.

$$\left.\begin{matrix} 5 \times 9 \\ 3 \times 12 \\ \hline 9 \times 12 \end{matrix}\right\} \text{Therefore} \quad \frac{45-36}{108} = \frac{9}{108} = \frac{1}{12} \text{ Answer.}$$

$$\left.\begin{matrix} 11 \times 15 \\ 9 \times 16 \\ \hline 15 \times 16 \end{matrix}\right\} \text{Therefore} \quad \frac{165-144}{240} = \frac{21}{240} = \frac{7}{80}.$$

Ex. 1. From $\frac{7}{8}$ take $\frac{3}{5}$. Ans. $\frac{11}{40}$.
2. From $\frac{11}{12}$ take $\frac{4}{7}$. Ans. $\frac{29}{84}$.
3. From $\frac{15}{11}$ take $\frac{7}{13}$. Ans. $\frac{118}{143}$.

4. From $\frac{12}{5}$ take $\frac{4}{7}$. Ans. $\frac{64}{35}$.
5. From $9\frac{3}{4}$ take $4\frac{7}{8}$. Ans. $4\frac{7}{8}$.
6. From $12\frac{1}{2}$ take $\frac{2}{3}$ of 17. Ans. $1\frac{1}{6}$.
7. From $\frac{5}{8}$ of a shilling take $\frac{1}{60}$ of a pound. Ans. $3\frac{1}{2}d$.
8. From $\frac{3}{4}$ of a pound take $\frac{7}{11}$ of a pound. Ans. $\frac{5}{44}$.
9. From 1 take $\frac{7}{18}$. Ans. $\frac{11}{18}$.
10. From 1 take $\frac{2}{5}$ of $\frac{4}{9}$. Ans. $\frac{11}{15}$.
11. From 12 take $\frac{5}{9}$. Ans. $11\frac{4}{9}$.
12. From 10*l.* take $\frac{5}{9}$ of a pound. Ans. 9*l.* 8*s.* $10\frac{3}{9}d$.
13. From $\frac{5}{8}$ of a pound take $\frac{3}{10}$ of a pound. Ans. 6*s.* 6*d.*
14. From $\frac{2}{3}$ of a pound take $\frac{1}{2}$ of $\frac{5}{8}$ of a shilling. Ans. 13*s.* $0\frac{1}{4}d$.
15. From $\frac{2}{3}$ of 6 dollars take $\frac{2}{5}$ of 5 dollars. Ans. 2 dols.
16. Subtract $33\frac{1}{3}$ cts, from $8\frac{4}{9}$ dollars. Ans. 8 dols. $11\frac{1}{9}$ cts.

MULTIPLICATION OF FRACTIONS.

Rule. Reduce mixed numbers to improper fractions, and compound fractions to simple ones; multiply all the numerators together for a new numerator; and all the denominators for a common denominator.

Ex. Multiply $3\frac{5}{8}$, $\frac{3}{4}$, and $\frac{5}{6}$ of 8 together.

$$\frac{29}{8} \times \frac{3}{4} \times \frac{5}{6} \times \frac{8}{1} = \frac{29 \times 3 \times 5 \times 8}{8 \times 4 \times 3 \times 2} = \frac{29 \times 5}{4 \times 2} = \frac{145}{8}$$

$= 18\frac{1}{8}$, the answer.

Ex. 1. Multiply $\frac{3}{11}$ by $\frac{4}{9}$; and $\frac{3}{9}$ by $\frac{3}{16}$. Ans. $\frac{1}{16}$ and $\frac{3}{48}$.
2. What is the product of $\frac{7}{8}$, $\frac{4}{5}$, and $3\frac{1}{2}$? Ans. $2\frac{9}{20}$.
3. What is the product of 57 by $\frac{9}{11}$? Ans. $46\frac{7}{11}$.
4. What is the product of $7\frac{2}{5}$ multiplied by 35? Ans. 259
5. What is the product of $\frac{5}{9}$, $\frac{3}{7}$, $12\frac{1}{2}$, and $\frac{3}{5}$ of 10? Ans. $17\frac{1}{2}$.

6. What is the continued product of $\frac{7}{8}$, $\frac{3}{5}$, $5\frac{1}{9}$, and 6? Ans. $16\frac{1}{10}$.

7. What is the product of $\frac{3}{7}$ of $\frac{5}{9}$, $\frac{1}{3}$ of $\frac{14}{15}$? Ans. $\frac{28}{567}$.

8. What is the product of $\frac{2}{31}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{4}$, $\frac{7}{8}$, $\frac{8}{9}$ and $\frac{9}{10}$? Ans. $\frac{7}{20}$.

9. How many yards are there in $5\frac{1}{4}$ pieces of Irish, each containing $26\frac{1}{4}$? Ans. $137\frac{13}{16}$.

10. How many pounds are there in $8\frac{1}{2}$ cheeses, each containing $25\frac{3}{4}$ lb. Ans. $218\frac{7}{8}$.

DIVISION OF FRACTIONS.

RULE. Reduce the fractions, as in Multiplication; then invert the divisor, and proceed as in Multiplication: thus $\frac{3}{5}$ to be divided by $\frac{3}{9}$.

$$\frac{3}{5} \div \frac{3}{9} = \frac{3}{5} \times \frac{9}{3} = \frac{27}{15} = \frac{9}{5}.$$

Ex. Divide $\frac{3}{8}$ of $4\frac{3}{5}$ by $\frac{3}{7}$ of $\frac{1}{4}$.

$$\frac{3}{8} \times \frac{23}{5} \div \frac{3}{7} \times \frac{1}{4} \text{ or } \frac{3 \times 23}{8 \times 5} \div \frac{4 \times 7}{7} = \frac{3 \times 23}{4 \times 2 \times 5} \times \frac{4 \times 7}{3}$$

$= \frac{161}{10} = 16_{10}$ the answer.

EXAMPLES.

Ex. 1. Divide $\frac{15}{17}$ of 12 by $\frac{4}{3}$. Ans. $7\frac{16}{17}$.

2. Divide $\frac{7}{11}$ of 8 by $\frac{3}{13}$. Ans. $22\frac{2}{33}$.

3. Divide $\frac{25}{6}$ by $\frac{11}{15}$. Ans. $5\frac{15}{22}$.

4. Divide $\frac{2}{9}$ of 54 by $\frac{3}{5}$. Ans. 20.

5. Divide $\frac{4}{7}$ of 12 by $3\frac{2}{3}$. Ans. $1\frac{67}{77}$.

6. Divide $\frac{4}{5}$ of 36 by $3\frac{1}{4}$. Ans. $8\frac{56}{65}$.

7. Divide $\frac{1}{3}$ of 4 by $\frac{5}{4}$ of 2. Ans. $\frac{8}{15}$.

8. Divide $11\frac{3}{7}$ by $\frac{7}{4}$ of $\frac{2}{3}$. Ans. $19\frac{29}{49}$.

9. Divide $\frac{7}{8}$ of $\frac{2}{5}$ of 5 by $\frac{4}{7}$ of $\frac{1}{5}$. Ans. $15\frac{5}{16}$.

10. Divide $\frac{7}{11}$ of $\frac{3}{5}$ by $\frac{2}{17}$ of 5. Ans. $\frac{357}{1100}$.

11. What number multiplied by $\frac{2}{5}$ will give $9\frac{1}{2}$? Ans. $\frac{95}{4}$.

2. What number multiplied by $\frac{5}{13}$ of 3 will give 56? Ans. $44\frac{4}{5}$.

13. What number multiplied by $\frac{5}{8}$ of $\frac{2}{3}$ of 15 will produce $\frac{3}{5}$ of 4 ? Ans. $\frac{48}{125}$.

14. From 5 subtract $\frac{3}{4}$ of $\frac{2}{7}$ of 4, and divide the remainder by 4. Ans. $1\frac{1}{28}$.

15. What is a person's share of a prize of *L.*20,000 $\frac{4}{5}$ths of which is to be divided among 13 persons ? Ans. 1230*l.* 15*s.* $4\frac{1}{2}$*d.* $\frac{6}{13}$

PRACTICE.

PRACTICE is a method of finding the value of any quantity of goods, from the price of an integer being given.

ALIQUOT PARTS of any number or quantity, are such as will exactly divide it without leaving a remainder: thus 7 and 4 are aliquot parts of 28, 4 pence is an aliquot part of a shilling, and 5 shillings is an aliquot part of a pound.

TABLES OF ALIQUOT PARTS.

Aliquot parts of a *L.*

s.	*d.*	
10	0	= $\frac{1}{2}$
6	8	= $\frac{1}{3}$
5	0	= $\frac{1}{4}$
4	0	= $\frac{1}{5}$
3	4	= $\frac{1}{6}$
2	6	= $\frac{1}{8}$
2	0	= $\frac{1}{10}$
1	8	= $\frac{1}{12}$
1	4	= $\frac{1}{15}$
1	3	= $\frac{1}{16}$
1	0	= $\frac{1}{20}$

Parts of a shil.

d.	
6	= $\frac{1}{2}$
4	= $\frac{1}{3}$
3	= $\frac{1}{4}$
2	= $\frac{1}{6}$
$1\frac{1}{2}$	= $\frac{1}{8}$
1	= $\frac{1}{12}$

Parts of a sixpence.

$\frac{3}{4}$	= $\frac{1}{8}$
$\frac{1}{2}$	= $\frac{1}{12}$

Parts of 3 pence.

q.	
3	= 1
4	= $\frac{1}{4}$
1	= $\frac{1}{6}$
2	= $\frac{1}{8}$
1	= $\frac{1}{12}$

Parts of a penny.

$\frac{1}{2}$	= $\frac{1}{2}$
1	= 1
4	= 4

I. When the price is less than a penny.

RULE. Divide the quantity by the aliquot parts in a penny, and the quotient by 12 and 20.

Ex. What is the value of 7853 yards of tape, at $\frac{3}{4}$ per yard?

$\frac{1}{2}$ |	$\frac{1}{2}$ |	7853
$\frac{1}{4}$ |	$\frac{1}{2}$ |	$3926\frac{1}{2}$
		$1963\frac{1}{4}$
		$12)5889\frac{3}{4}$
		$2,0)49.0\ \ 9\frac{3}{4}$

Answer, *L.*24 10 $9\frac{3}{4}$

EXAMPLES.

		L.	*s.*	*d.*
Ex. 1.	4567 at $\frac{1}{4}$ per yd. Answer,	4	15	$1\frac{3}{4}$
2.	6784 at $\frac{1}{2}$ per lb.	14	2	8
3.	3976 at $\frac{3}{4}$	12	8	6
4.	7655 at $\frac{1}{2}$ per yd.	15	18	$11\frac{1}{2}$
5.	7486 at $\frac{3}{4}$ per lb.	23	7	$10\frac{1}{2}$
6.	9984 at $\frac{1}{2}$	20	16	0
7.	6327 at $\frac{3}{4}$ per yd.	19	15	$5\frac{1}{4}$
8.	5934 at $\frac{1}{4}$ per lb.	6	3	$7\frac{1}{2}$
9.	7585 at $\frac{1}{2}$	15	16	$0\frac{1}{2}$
10.	4767 at $\frac{1}{4}$ per yd.	4	19	$3\frac{3}{4}$
11.	6493 at $\frac{3}{4}$ per lb.	20	5	$9\frac{3}{4}$
12.	5388 at $\frac{3}{4}$	16	16	9

II. When the price is an aliquot part of a shilling.

RULE. Divide the given number by the aliquot part, and this quotient by 20: the answer will be in pounds.

Ex. What is the value of 2785 lbs. of salt at 4*d.* per lb.?

4*d.*	$\frac{1}{3}$	2785
		2.0)92.8 4

Answer, *L.*46 8 4

		L.	*s.*	*d.*
Ex. 1.	3764 at 2*d.* Answer,	31	7	4
2.	5943 at 3*d.*	74	5	9
3.	4953 at $1\frac{1}{2}$*d.*	30	19	$1\frac{1}{2}$
4.	5943 at 4*d.*	99	1	0
5.	3987 at 3*d.*	49	16	9
6.	5964 at 1*d.*	24	17	0
7.	5684 at 4*d.*	94	14	8
8.	2705 at 2*d.*	22	10	10
9.	3456 at 2*d.*	28	16	0
10.	5924 at $1\frac{1}{2}$*d.*	37	0	6
11.	5964 at 2*d.*	49	14	0
12.	5215 at 4*d.*	86	18	4

III. When the price is pence and farthings, and no aliquot part of a shilling.

RULE. (1.) Find what aliquot part of a shilling is nearest to the given price, and divide the proposed number by it. (2) Consider what part the remainder is of this aliquot part of the given price, and divide the former quotient by it, &c. (3) Add the several quotients together, and the answer will be in shillings, which divide by 20 to bring into pounds.

Ex. What is the value of 4277 yds., at $10\frac{3}{4}$*d.* per yd.?

6	$\frac{1}{2}$	4277	
3	$\frac{1}{2}$	2138	6
$1\frac{1}{2}$	$\frac{1}{2}$	1069	3
$\frac{1}{4}$	$\frac{1}{6}$	534	$7\frac{1}{2}$
		89	$1\frac{1}{4}$
		2,0)383.1	$5\frac{3}{4}$

Answer, *L.*191 11 $5\frac{3}{4}$

		L.	s.	d.
Ex. 1.	4784 at $1\frac{1}{4}$	Answer, 24	18	4
2.	5964 at $1\frac{3}{4}$	43	9	9
3.	4659 at $2\frac{1}{4}$	43	13	$6\frac{3}{4}$
4.	1765 at $2\frac{1}{4}$	16	10	$11\frac{1}{4}$
5.	4305 at $2\frac{3}{4}$	49	6	$6\frac{3}{4}$
6.	3694 at $3\frac{1}{4}$	50	0	$5\frac{1}{2}$
7.	7641 at $2\frac{1}{2}$	79	11	$10\frac{1}{2}$
8.	9875 at $6\frac{1}{2}$	267	8	$11\frac{1}{2}$
9.	5476 at $10\frac{3}{4}$	245	5	7
10.	3592 at $3\frac{1}{2}$	52	7	8
11.	3046 at $6\frac{3}{4}$	85	13	$4\frac{1}{2}$
12.	3214 at $11\frac{1}{2}$	154	0	1
13.	8764 at $3\frac{3}{4}$	136	18	9
14.	5921 at $7\frac{1}{4}$	178	17	$3\frac{1}{4}$
15.	5178 at $9\frac{1}{2}$	204	19	3
16.	9714 at $4\frac{1}{2}$	182	2	9
17.	5643 at $8\frac{1}{2}$	199	17	$1\frac{1}{2}$
18.	4932 at $10\frac{1}{4}$	210	12	9
19.	8934 at $5\frac{1}{4}$	195	8	$7\frac{1}{2}$
20.	2458 at $9\frac{3}{4}$	99	17	$1\frac{1}{2}$
21.	8764 at $11\frac{3}{4}$	429	1	5
22.	5687 at $5\frac{3}{4}$	136	5	$0\frac{1}{4}$
23.	1435 at $10\frac{1}{2}$	62	15	$7\frac{1}{2}$
24.	5842 at $7\frac{1}{4}$	176	9	$6\frac{1}{2}$
25.	5943 at $9\frac{1}{2}$	235	4	$10\frac{1}{2}$
26.	1876 at $2\frac{3}{4}$	21	9	11
27.	4316 at $7\frac{3}{4}$	139	7	5
28.	1956 at $8\frac{3}{4}$	71	6	3
29.	4235 at $5\frac{1}{2}$	97	1	$0\frac{1}{2}$
30.	1327 at $9\frac{1}{2}$	52	10	$6\frac{1}{2}$
31.	2748 at 11	125	19	0
32.	9374 at $7\frac{1}{4}$	283	3	$5\frac{1}{2}$
33.	4285 at $11\frac{1}{4}$	200	17	$2\frac{1}{4}$
34.	1594 at $3\frac{1}{2}$	23	4	11
35.	5632 at 5	117	6	8
36.	1114 at $5\frac{1}{2}$	25	10	7

IV. When the price is more than one shilling, and less than two.

RULE. Let the given number stand for shillings, and work for the pence and farthings as before.

Ex. What is the value of 1187 quartern loaves, at 1s $1\frac{3}{4}d.$ each?

$1\frac{1}{2}$	$\frac{1}{6}$	1187
$\frac{1}{4}$	$\frac{1}{6}$	148 $4\frac{1}{2}$
		24 $8\frac{3}{4}$
		2,0)136.0 $1\frac{1}{4}$

Answer L.68 0 $1\frac{1}{4}$

		s.	d.		L.	s.	d.
Ex.	1. 3456 at 1		$2\frac{1}{2}$	Answer,	208	16	0
	2. 4870 at 1		$5\frac{1}{2}$		355	10	10
	3. 5792 at 1		$8\frac{1}{2}$		494	14	8
	4. 2632 at 1		$3\frac{3}{4}$		172	14	6
	5. 4092 at 1		$7\frac{1}{4}$		328	4	3
	6. 2596 at 1		10		237	19	4
	7. 4735 at 1		$7\frac{1}{4}$		325	10	$7\frac{1}{2}$
	8. 3724 at 1		$9\frac{1}{2}$		333	12	2
	9. 3451 at 1		$6\frac{3}{4}$		269	12	$2\frac{1}{4}$
	10. 7321 at 1		$7\frac{3}{4}$		602	9	$1\frac{3}{4}$
	11. 5928 at 1		11		568	2	0
	12. 6542 at 1		$8\frac{3}{4}$		565	12	$2\frac{1}{2}$
	13. 8465 at 1		$9\frac{1}{2}$		758	6	$5\frac{1}{2}$
	14. 4371 at 1		$3\frac{1}{2}$		282	5	$10\frac{1}{2}$
	15. 8937 at 1		$3\frac{3}{4}$		586	9	$9\frac{3}{4}$
	16. 1234 at 1		11		118	5	2
	17. 5629 at 1		$1\frac{3}{4}$		322	9	$10\frac{3}{4}$
	18. 4516 at 1		2		263	8	8
	19. 5678 at 1		$2\frac{3}{4}$		348	19	$2\frac{1}{2}$
	20. 9272 at 1		$4\frac{3}{4}$		647	0	$9\frac{1}{4}$
	21. 5461 at 1		7		432	6	7
	22. 8234 at 1		$5\frac{1}{2}$		600	7	11
	23. 5928 at 1		$10\frac{1}{2}$		555	15	0
	24. 8750 at 1		5		619	15	10

V. When the price is any number of shillings under 20.

RULE. (1.) If the price is an even number, multiply the given quantity by half the said number, doubling the first figure to the right hand for shillings, and the rest are pounds. (2.) If the price is an odd number, find for the greatest even number, as before, to which add the $\frac{1}{20}$th of the given number for the odd shilling, and the sum is the answer.

Ex. What is the value of 3456 yards of cloth, at 18*s.* per yard?

3456
9

Ans. *L.* 3110 8

Ex. What is the value of 2592 yards of second cloth, at 11*s.* per yard?

$1 = \frac{1}{20}$ | 2592
5

1296 0
129 12

Answer *L.* 1425 12

EXAMPLES.

		s.		*L.*	*s.*
Ex. 1.	5075 at	2	Answer	597	10
2.	4374 at	3		656	2
3.	5916 at	4		1183	4
4.	7591 at	5		1897	15
5.	6743 at	6		2022	18
6.	9430 at	8		3772	0
7.	5734 at	10		2867	0
8.	5946 at	11		3270	6
9.	3004 at	7		1051	8
10.	2935 at	13		1907	15
11.	4392 at	14		3074	8
12.	5931 at	19		5634	9

		s.		L.	s.
Ex.	13. 4917 at	18	Answer	4425	6
	14. 3271 at	9		1471	19
	15. 9515 at	17		7917	15
	16. 2514 at	16		2011	4
	17. 1392 at	10		696	0
	18. 5462 at	19		5160	8

VI. When the price is shillings and pence.

RULE. (1.) If they are an aliquot part of a pound, divide the quantity by that part, and the quotient is the answer. (2.) If they are not an aliquot part, multiply by the shillings, and take parts for the pence.

Ex. What is the value of 2769 yards of Irish, at 3s. 4*d*. per yard ?

3s. 4*d*. | 2769

Answer *L*. 461 10*s*.

Ex. What is the value of 3758 yards of muslin, at 12*s*. 9*d*. per yard ?

6 | ½ | 3756
| | | 12
3 | ½ | 45072
| | | 1878
| | | 938

2.0)4788.9

Answer *L*. 2394 9*s*.

EXAMPLES.

		s.	d.		L.	s.	d.
Ex.	1. 8943 at	2	0	Answer	894	6	0
	2. 3532 at	4	0		706	8	0
	3. 8671 at	7	6		3251	12	6
	4. 2524 at	3	9½		478	10	2
	5. 5971 at	5	10		1741	10	10
	6. 5460 at	7	6		2047	10	0

		s.	d.		L.	s.	d.
Ex. 7.	3764 at	10	0	Answer	1882	0	0
8.	5638 at	8	11		2513	12	2
9.	3745 at	9	11		1856	17	11
10.	8756 at	15	10		6931	16	8
11.	3942 at	4	5		870	10	6
12.	2475 at	16	8		2062	10	0
13.	5642 at	18	$4\frac{1}{2}$		5183	11	9
14.	1764 at	5	8		499	16	0
15.	5931 at	17	6		5189	12	6
19.	9143 at	6	8		3047	13	4
17.	7189 at	3	7		1288	0	7
18.	4604 at	19	6		4488	18	0

VII. When the price is pounds and shillings, or pounds, shillings, pence, and farthings.

RULE. Multiply the quantity by the pounds, and work the rest by the foregoing rules.

Ex. What is the value of 5428 hogsheads of ale, at 4*l.* 12*s.* per hogshead?

5428
4 12

21712
3256 16

Answer, *L.* 24968 16

Ex. What is the value of 2714 cwt. of sugar, at 3*l.* 12*s.* $9\frac{1}{2}$*d.* per cwt.?

10*s.*	$\frac{1}{2}$	2714			
		8			
		8142			
2*s.* 6*d.*	$\frac{1}{4}$	1357			
3*d.*	$\frac{1}{10}$	339	5	0	
$\frac{1}{2}$	$\frac{1}{6}$	33	18	6	
		5	13	1	

Answer, *L.* 9877 16 7

		L.	*s.*	*d.*	*L.*	*s.*	*d.*
Ex.	1. 5674 at	5	17	6	33334	15	
	2. 6431 at	4	8	4	28403	11	8
	3. 3416 at	5	11	$6\frac{3}{4}$	19054	17	6
	4. 4931 at	9	4	0	45365	4	
	5. 3146 at	10	12	9	33405	11	6
	6. 4316 at	10	19	$6\frac{1}{2}$	47377	1	10
	7. 5648 at	12	13	0	71447	4	
	8. 1436 at	10	10	6	15113	18	
	9. 1346 at	3	13	4	4935	6	8
	10. 2714 at	18	9	0	50073	6	
	11. 9614 at	4	14	6	45426	3	
	12. 5789 at	7	7	7	42717	19	11
	13. 1590 at	12	12	0	20034		
	14. 6341 at	8	18	6	56593	8	6
	15. 4803 at	9	9	$9\frac{3}{4}$	45583	9	$5\frac{1}{4}$
	16. 3465 at	8	15	0	80318	15	
	17. 7182 at	11	12	10	83610	9	0
	18. 1604 at	4	11	10	7865	0	8

VIII. If there be a fraction in the given quantity.

RULE. Work for the whole number, according to the preceding rules, to which add $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{8}$, &c. of the price, according to the nature of the question.

Ex. What is the value of $5354\frac{3}{4}$ cwt. of soap, at 4*l.* 4*s.* 8*d.* per cwt.?

4*s.*	$\frac{1}{5}$	5354	$\frac{3}{4}$
		4	
		21416	
8*d.*	$\frac{1}{6}$	1070	16
		178	9 4
		3	3 6

Answer, *L.*22668 8 10

$\frac{1}{2}$	$\frac{1}{2}$	4	4 8
$\frac{1}{4}$	$\frac{1}{2}$	2	2 4
		1	2 2

*L.*3 3 6

		L.	*s.*	*d.*
Ex.	1. $4562\frac{1}{4}$ at	3	15	$9\frac{1}{2}$
	2. $6744\frac{1}{2}$ at	9	9	$10\frac{1}{2}$
	3. $2654\frac{3}{4}$ at	7	15	4
	4. $7394\frac{3}{4}$ at	12	8	$8\frac{1}{4}$
	5. $4651\frac{1}{4}$ at	5	12	10
	6. $3749\frac{1}{2}$ at	16	9	5

	L.	*s.*	*d.*
Ans.	17289	0	$6\frac{1}{4}$—2
	64030	11	$11\frac{1}{4}$
	20618	11	2
	91949	1	$10\frac{1}{2}$
	26240	16	$0\frac{1}{2}$
	61757	7	$9\frac{1}{2}$

			L.	s.	d.		L.	s.	d.
Ex.	7. 3875	at	8	18	$6\frac{3}{4}$.	Ans.	34596	9	$8\frac{1}{4}$
	8. $4365\frac{3}{8}$	at	11	11	11		50624	10	$2\frac{1}{4}$
	9. $9724\frac{1}{4}$	at	6	16	$4\frac{1}{2}$		66307	4	7
	10. $3618\frac{1}{2}$	at	4	4	$6\frac{3}{4}$		15426	6	$3\frac{1}{4}$

TABLES OF ALIQUOT PARTS.

Aliquot part of a ton. cwt. qr. lb.				Aliquot parts of a cwt. qrs. lb.				Aliquot parts of a qr. of cwt. lb.		Aliquot parts of a lb. oz.	
10	0	0	$= \frac{1}{2}$	2	0	$= \frac{1}{2}$		14	$= \frac{1}{2}$	8	$= \frac{1}{2}$
5	0	0	$= \frac{1}{4}$	1	0	$= \frac{1}{4}$		7	$= \frac{1}{4}$	4	$= \frac{1}{4}$
4	0	0	$= \frac{1}{5}$		16	$= \frac{1}{7}$		4	$= \frac{1}{7}$	2	$= \frac{1}{8}$
2	3	12	$= \frac{1}{7}$		14	$= \frac{1}{8}$		$3\frac{1}{2}$	$= \frac{1}{8}$	1	$= \frac{1}{16}$
2	2	0	$= \frac{1}{8}$		8	$= \frac{1}{14}$		2	$= \frac{1}{14}$		
2	0	0	$= \frac{1}{10}$		7	$= \frac{1}{28}$		$1\frac{3}{4}$	$= \frac{1}{16}$		
1	0	0	$= \frac{1}{20}$					1	$= \frac{1}{28}$		

IX. When the given quantity is of several denominations.

RULE. Multiply the given price by the highest denomination, as in Compound Multiplication, and take parts of the price for the inferior denominations of the given quantity.

Ex. What is the value of 22 cwt. 3 qr. 21 lb. of hops, at 4*l*. 18*s*. 6*d*. per cwt. ?

		L.	s.	d.	
2 qr.	$\frac{1}{2}$	4	18	6	Here, for the 22 cwt., I multiply by 11 and by 2; then I take parts for the 3 qrs. 21 lb., according to the preceding table, and by case VIII.
				11	
		54	3	6	
				2	
		108	7	0	= value of 22 cwt.
1 qr.	$\frac{1}{2}$	2	9	3	= ditto 2 qrs.
14lb.	$\frac{1}{8}$	1	4	$7\frac{1}{2}$	= ditto 1 qr.
7 lb.	$\frac{1}{2}$		12	$3\frac{3}{4}$	= ditto 14 lb.
			6	$1\frac{3}{4}$-1	= ditto 7 lb.
Ans.		*L.* 112	19	4 — 1	

cwt. qrs. lb. dolls. cts.

Ex. 1. 8 2 14 at 20 50 per cwt.
Ans. 176 dolls. $81\frac{1}{4}$ cts.

2. 16 1 21 at 14 80 per cwt.
Answer, 243 dolls. $27\frac{1}{2}$ cts.

3. 37 3 22 at 12 11 7 per cwt.
Answer, *L.* 477 6 8.

4. 73 2 $10\frac{1}{2}$ at 3 16 9 per cwt.
Answer, *L.* 282 8 $3\frac{3}{4}$.

5. 38 1 16 at 2 12 6 per cwt.
Answer, *L.* 100 15 $7\frac{1}{2}$.

6. 33 2 8 at 39 3 8 per cwt.
Answer. *L.* 1315 8 $9\frac{3}{4}$.

7. 84 3 14 at 12 11 8 per cwt.
Answer, *L.* 1068 0 $2\frac{1}{2}$.

L. s. d.

8. 56 tons, 4 cwt. 2 qrs. 0 lb. at 58 7 6 per ton.
Answer, *L.* 3282 2 $8\frac{1}{2}$.

9. 39 tons, 12 cwt. 1 qr. 14 lb. at 25 12 8 per ton.
Answer, *L.* 1015 11 2.

10. 124 tons, 16 cwt. 2 qr. 16 lb. at 12 18 7 per ton.
Answer, *L.* 1615 19 6 nearly.

11. 16 lb. 8 oz. 12 dr. - - at 4 3 6 per lb.
Answer, *L.* 69 1 $7\frac{3}{4}$.

12. 25 lb. 12 oz. 4 dr. - - at 8 12 6 per lb.
Answer, *L.* 222 4 $6\frac{3}{4}$.

13. 35 lb. 4 oz. 12 dwt. - - at 11 9 9 per lb.
Answer, *L.* 406 9 $3\frac{1}{2}$.

14. 48 lb. 8 oz. 16 dwt. - - at 14 4 4 per lb.
Answer, *L.* 692 16 $5\frac{3}{4}$.

15. 25 lb. 6 oz. 5 dwt. - - at 15 3 9 per lb.
Answer, *L.* 387 11 $11\frac{1}{4}$.

16. 18 yds. 2 qr. 3 nails - - at 0 16 8 per yd.
Answer, *L.* 15 11 $5\frac{1}{2}$.

17. 55 yds. 3 qr. 2 nails - - at 1 3 9 per yd.
Answer, *L.* 66 7 $0\frac{1}{4}$.

18. 15 acr. 3 rd. 24 per. - - at 38 3 6 per acr.
Answer, *L.* 606 19 $7\frac{3}{4}$.

19. 25 acr. 1 rd. 4 per. - - at 22 50 0 per acr.
Answer, 568 dolls. $68\frac{3}{4}$ cts.

20. 39 acr. 2 rd. 18 per. - - at 33 25 0 per acr.
Answer, 1317 dolls. $11\frac{9}{16}$ cts.

TARE AND TRET.

TARE AND TRET are a set of practical rules for deducting certain allowances, made by wholesale dealers in selling their goods by weight.

GROSS WEIGHT is the whole weight of goods, including package, or whatever contains them.

NEAT WEIGHT is what remains after all allowances are made.

TARE is an allowance to the buyer, for the weight of the package, and is either at so much per barrel, chest, &c, or at so much per cwt., or at so much for the whole.

TRET is an allowance of 4lb. in every 104lb. for waste, dust, &c., or the $\frac{1}{26}$ part of the whole.

CLOFF is an allowance, after Tare and Tret are deducted, of 2lb. upon every 3 cwt. that the weight may hold good when sold by the retail.

SUTTLE is when only part of the allowance is deducted from the gross. Thus, after the tare is deducted from the gross, the remainder is called tare suttle.

CASE I. When the tare is so much for the whole.

RULE. From the gross weight subtract the tare, and the remainder will be the neat weight required.

Ex. What is the neat weight of 25 barrels of indigo, weighing 116 cwt. 2 qr. 14 lb., allowing 2 cwt. 3 qr. 12 lb. tare?

	cwt.	qr.	lb.
	116	2	14
	2	3	12
Answer -	113	3	2 neat weight.

Ex. 1. What is the neat weight of 55 barrels of figs, weighing 35 cwt. 2 qr. 15 lb., tare being allowed at 1 cwt. 1 qr. 24 lb.? Ans. 34 cwt. 0 qr. 19 lb.

Ex. 2. What is the neat weight of 20 casks of Russian tallow, weighing 74 cwt., tare being allowed at 2 cwt. 2 qr. 5 lb.? Ans. 71 cwt. 1 qr. 23 lb.

CASE II. When the tare is at so much per barrel, chest, &c.

RULE. (1.) Multiply the tare by the number of hogsheads, barrels, chests, &c. subtract the product from the gross, and the remainder will be the neat weight required: or

(2.) Subtract the tare of each parcel from the given weight, and multiply by the number of parcels.

Ex. What is the neat weight of 8 hhds. of tobacco, each weighing 4 cwt. 2 qr. 24 lb. gross, tare being allowed at 2 qrs. 4 lb. per hhd.?

	cwt.	qr.	lb.		qr.	lb.	
	4	2	24		2	4	
			8			8	
Gross weight	37	2	24			4	1 4 Tare.
	4	1	4				
Answer -	33	1	20 neat weight.				

Ex. 1. What is the neat weight of 25 frails of Malaga raisins, each weighing 2 cwt. 3 qrs. 12 lb., when the tare upon each frail is 17 lb.? Ans. 67 cwt. 2 qr. 15 lb.

Ex. In 79 barrels of figs, each weighing 1 cwt. 12 lb. and tare 9 lb. per barrel, what is the neat weight?
Ans. 81 cwt. 0 qr. 13 lb. neat weight.

Ex. 3. What is the neat weight of 24 hhds. of tobacco, the weight of each being $4\frac{1}{2}$ cwt., and tare 67 lb. per hhd.? Ans. 93 cwt. 2 qr. 16 lb. neat weight.

Ex. 4. In 18 casks of currants, each weighing 6 cwt. 1 qr. 12 lb. and tare 61 lb per cask, what is the neat weight? Ans. 104 cwt. 2 qr. 14 lb. neat weight.

CASE. III. When the tare is at so much per cwt.

RULE. Take the aliquot part or parts of the whole gross weight that the tare is of a cwt, as in Practice, and subtract the result from the gross weight.

Ex. What is the neat weight of 24 barrels of figs, each weighing 3 cwt. 2 qrs. 12 lb. and tare 12 lb. per cwt.?

cwt. qr. lb.
3 2 12 × 24 = 6 × 4
6

				lb.		cwt.	qr.	lb.		
21	2	16		8	$\frac{1}{14}$	86	2	8		
		4								oz.
				4	$\frac{1}{2}$	6	0	20	9	
Gr. wt. 86	2	8	oz.			3	0	10	$4\frac{1}{2}$	
Tare, 9	1	2	$13\frac{1}{2}$			9	1	2	$13\frac{1}{2}$	
Ans 77	1	5	$2\frac{1}{2}$ nt. wt.							[tare.

Ex. 1. What is the neat weight of 21 barrels of potash, each barrel weighing 1 cwt. 3 qr. 8 lb., tare being 10 lb. per cwt.? Ans. 34 cwt. 3 qr. $9\frac{1}{2}$ lb. neat wt.

Ex. 2. What is the neat weight of 35 barrels of anchovies, each weighing 1 qr. 12 lb., tare at 14 lb. per cwt.?
Ans. 10 cwt. 3 qr. 21 lb. neat weight.

Ex. 3. Required the neat weight of 15 hhds. of tobacco, each weighing 4 cwt. 2 qrs. 12 lb., tare at 20 lb. per cwt.? Ans. 56 cwt. 3 qr. 2 lb. neat weight nearly.

Ex. 4. What is the value of 26 hogsheads of tobacco, at 8*l.* 8*s.* per cwt. each hogshead weighing $4\frac{1}{2}$ cwt., and the allowance for tare being 13 lb. per cwt.?

Ans. 868*l.* 14*s.* 6*d.*

CASE IV. When there is an allowance both of tare and tret.

RULE. Find the tare by the last rule, subtract it from the gross weight, the remainder or suttle, divided by 26, gives the tret, which being subtracted from the suttle, gives the answer.

Ex. What is the neat weight of 15 casks of tallow, each weighing 6 cwt. 2 qr. 12 lb., tare being 12 lb. per cwt. and tret as usual?

cwt. qr. lb.
6 2 12 × 15 = 5 × 3.
5

				lb.		cwt.	qr.	lb.	
33	0	4		8	$\frac{1}{14}$	99	0	12	
		3							
Gross wt. 99	0	12		4	$\frac{1}{2}$	7	0	8	— 12
Tare - 10	2	12				3	2	4	— 6
26)88	2	0				10	2	12	— 18
3	1	17	— 6						

Answer 85 0 10 neat weight.

Ex. 1. In 18 cwt. 1 qr. 6 lb. gross, tare 63 lb., and tret as usual, how much neat?

Ans. 17 cwt. 0 qr. 7 lb. neat weight.

Ex. 2. In 14 casks of raisins, each 2 cwt. 14 lb. gross, tare 18 lb. per cwt., and tret as usual, what is the neat weight? Ans. 24 cwt. 0 qr. 1 lb. neat weight.

Ex. 3. In 9 chests of sugar, each weighing 8 cwt. 2 qr. 10 lb., tare 14 lb. per cwt., and tret as usual, what is the neat weight? Ans. 64 cwt. 3 qr 24 lb. nt. wt.

CASE V. When cloff is allowed,

RULE. Subtract the tare from the gross, and the tret from the tare suttle; then divide the tret suttle by 168, and the result will be the Cloff, which being subtracted from the last suttle, gives the neat weight required.

Ex. What is the neat weight of 19 cwt. 1 qr. 2 lb. gross, tare 3 cwt. 3 qr. 22 lb., and tret and cloff at the usual rate?

	cwt.	qr.	lb.
Gross -	19	1	2
Tare -	3	3	22
	26)15	1	8
Tret -		2	10
Tret suttle	14	2	26
Cloff -		9	13 oz.
Ans. cwt.	14	2	16 3 neat weight.

cwt. qr. lb.
4)14 2 26÷168=4×6×7
6)3 2 20 8 oz.
7)2 12 12
9 13$\frac{1}{7}$

Ex. 1. What is the neat weight of 224 cwt. 3 qr. 20 lb. of tobacco, tare being 25 cwt. 3 qr., tret and cloff as usual. Ans. cwt. 190 1 14 neat weight.

Ex. 2. In 14 hhds. of tobacco, each weighing 5 cwt. 3 qr. 17 lb. gross, tare 11 lb. per cwt., and tret and cloff as usual, what is the neat weight?

Ans. cwt. 70 2 2 neat weight.

Ex. 3. What is the neat weight of 15 casks of currants, each weighing 5½ cwt. gross, tare 35 lb. per cask, tret and cloff usual? Ans. cwt. 74 1 14 neat weight.

Ex. 4. In 9 chests of sugar, each containing 7 cwt. 2 qr. 12 lb. gross, tare 13 lb. per cwt., tret and cloff as usual, what is the neat weight, and what is the value of it at 9½*d.* per lb.?

Ans. cwt. 57 3 14 nt. wt 256*l.* 11*s.* 7*d.*

DECIMAL FRACTIONS.

1. Decimal, or Decimated Fractions, are such as always have 1 with one or more cyphers for their denominators. The denominators are never expressed, being understood to be 10, 100, 1000, &c., according as the numerators consist of 1, 2, or 3 figures: thus, instead of $\frac{2}{10}$, $\frac{24}{1000}$, $\frac{211}{1000}$, the numerators only are written, with a dot or inverted comma before them, as 2; .24; 211.

2. If a decimal consists of only one figure, one is supposed to be divided into ten equal parts, and the decimal represents as many of those parts as the decimal figure expresses; thus, .7 means seventenths of an unit: If it consist of two figures, one is supposed to be divided into 100 equal parts, of which the decimal represents as many as the figure expresses: thus, .65 means sixty-five hundredths of an unit.

3. Cyphers to the right-hand of decimals cause no difference in their value, for .5; .50; .500, are decimals of the same value, being each equal to $\frac{1}{2}$; that is, $.5 = \frac{5}{10}$; $.50 = \frac{50}{100}$; $.500 = \frac{500}{1000}$; but if the cyphers are placed on the left-hand of decimals, they diminish their value in a ten-fold proportion, thus .3; .03; .003, are 3-tenths, 3-hundredths; 3-thousandths; and answer to the vulgar fractions $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, respectively.

4. A whole number and decimal is thus expressed, 85.74 which is

equal to $85\frac{74}{100} = \frac{8574}{100}$ and $45.04 = 85\frac{4}{100} = \frac{8504}{100}$, &c.

REDUCTION OF DECIMALS.

CASE I. To reduce a vulgar fraction to a decimal of an equal value.

RULE. Divide the numerator of the fraction, increased by a cypher, or cyphers, by the denominator, and the quotient will be the decimal sought.

Reduce $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, to decimals of the same value.

$\frac{1}{2} = \frac{1^0}{2} = .5$. $\frac{1}{4} = \frac{1 \cdot 00}{4} = .25$. $\frac{1}{8} = \frac{1 \cdot 000}{8} = .125$.

$\frac{1}{16} = \frac{1 \cdot 0000}{16} = .0625$.

The cyphers added to the numerators are separated from the original figures by a dot, to shew that they are borrowed for the sake of forming the decimal.

Ex. 1. What decimal expressions answer to the following vulgar fractions, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{2}{9}$, $\frac{11}{15}$?

Ans. $\frac{3 \cdot 000}{8} = 375$. $\frac{5}{8}^{000} = .625$. $\frac{7}{8}^{\cdot 000} = .875$. $\frac{2}{9}^{\cdot 00} = .222$, &c. $\frac{11}{15}^{000} = .733$, &c.

Ex. 2. Required the equivalent decimals of the fractions, $\frac{5}{25}$, $\frac{9}{16}$, $\frac{3}{4}$, $\frac{7}{11}$, $\frac{9}{18}$.

Ans. $\frac{5 \cdot 0}{25} = .2$. $\frac{9}{16}^{0000} = .5625$. $\frac{3}{4}^{\cdot 00} = .75$. $\frac{7}{11}^{\cdot 000} = 0363$, &c. $\frac{9}{18} = \frac{1}{2} = .5$.

Ex. 3. What is the decimal that answers to $\frac{1}{64}$?

Ans. $\frac{1}{64} = \frac{1 \cdot 000000}{64} = .015625$.

Ex. 4. What are the decimals answering to the fractions $\frac{5}{128}$, $\frac{15}{256}$, and $\frac{45}{2304}$?

Ans $\frac{5}{128} = .0390625$. $\frac{15}{256} = .05859375$. $\frac{45}{2304} = .019531$, &c.

Ex. 5. What decimal expressions answer to $\frac{1}{3}$, $\frac{2}{99}$, and $\frac{41}{333}$?

Ans. $\frac{1}{3} = .333$, &c. $\frac{2}{99} = .020202$, &c. $\frac{41}{333}$. $= .123123123$, &c.

CASE II. To reduce numbers of different denominations to their equivalent decimal values.

RULE. (1) Write the given numbers under each other for dividends, proceeding from the least to the greatest.

(2) Place on the left side of each dividend, for a divisor, the number that will bring it to the next superior denomination. (3) Begin with the uppermost number, and set down the quotient of each division, as decimal parts, on the right hand of the dividend next below it, and so proceed to the last quotient, which is the decimal required.

Ex. Reduce 12*s.* $3\frac{3}{4}$*d.* to the decimal of a pound.

4	3 *qrs.*	I divide the $\frac{3}{4}$ by 4, supplying cy-
12	3*d*..75	phers to the 3 by the imagination ;
20	12*s*..3125	the quotient is .75, which is placed
		by the side of the 3*d.*, and then di-
	.615625 }	vide the 3.75 by 12; the quotient.
	decimal of a *L.* }	.3125, I set by the side of the 12*s*,
		and divide by 20, which gives

.615625 for the answer : that is, if a pound were divided into 1,000,000 parts, the 12*s.* $3\frac{3}{4}$*d.* would be 615625 such parts, in the same manner as if a penny were divided into 100 parts, $\frac{3}{4}$ would be equal to 75 such parts.

Ex. 1. Reduce 8*s.* $4\frac{1}{2}$*d.* to the decimal of a pound.

Answer, .41875.

2. What decimal of a pound is 15*s.* $5\frac{3}{4}$*d.* ?

Answer, .77395833, &c.

3. What decimal of a pound is 4*s.* $6\frac{1}{4}$*d.* ?

Answer, .22604166, &c.

4. Reduce 18*s.* 6*d.*, 8*s.* 2*d.*, and 5*s.* to decimals of a pound.

1st. Ans. .925. 2d. Ans. .40833, &c. 3d. Ans. .25.

5. Reduce 5 oz. 6 dwts. 8 gr. troy, to the decimal of a pound. Answer, .443055.

6. Reduce 3 qrs. 7 lb. 8 oz. avoirdupois, to the decimal of a cwt. Answer, .816964.

7. Reduce 2 qrs. 1 n. to the decimal of a yard.

Answer, .5625.

8. Reduce 3 pks. 1 gal. 2 qts. to the decimal of a bushel ? Answer, .9375.

Case III. To find the value of any given decimal in terms of the integer. This is the reverse of the last case.

RULE. Multiply the decimal by the number of parts in the next less denomination, and cut off as many places to the right-hand, as there are places in the given decimal, and so proceed through each denomination.

Ex. What is the value of .615625 of a pound?

.615625
20
12.312500
12
3.7500
4
3.00

It may be observed, that as cyphers to the right do not alter the value in decimals, they are omitted in each step of the operation.

Answer, 12s. $3\frac{3}{4}d.$

Ex. 1. What is the value of .625 of a shilling?
Answer, $7\frac{1}{2}$ pence.

2. What is the value of .1275 of a pound?
Answer, 2s. $6\frac{1}{2}d.$ 4.

3. What is the value of .575 of a cwt.
Answer, 2 qr. 8 lb. 6 oz. 6 dr.—4.

4. What is the value of .875 of a bushel?
Answer, 3 pks. 1 gal.

ADDITION OF DECIMALS.

RULE. (1.) Arrange the numbers under each other, according to their several values. (2.) Find the sum as in Addition of whole numbers, and cut off, for decimals, as many figures to the right as there are decimals in any one of the given numbers.

Ex. What is the sum of 23.45, 7.849, 543.2, 8.6234 and 253.004 ?

23.45
7.849
543.2
8.6234
253.004

Answer, 836.1264

Ex. 1. What is the sum of 37.035, 4.26, 598.034, 9.3076, 4.321, and 5 ? Answer, 657.9576.

2. Find the value of 39.33, 4.2056, .98735, 46.287, 3.7491, and 8.004. Answer, 97.56305.

SUBTRACTION OF DECIMALS.

RULE. Arrange the numbers according to their value; subtract as in whole numbers, and cut off for decimals, as in Addition.

Ex. Subtract 35.87043 from 132.005.

132.005
35.87043

Answer, 96.13457

Ex. 1. What is the difference between 104.326 and 74.05 ?
Answer, 30.276

Ex. 2. Find the difference between 394.832 and 148.0076.
Answer, 246.8244

Ex. 3. From 372.971 take 270.30041.
Answer, 102.67059

MULTIPLICATION OF DECIMALS.

Rule. Multiply as in whole numbers, and cut off as many figures from the product as there are decimals in the multiplier and multiplicand.

Ex. Multiply .025 by .045 : also 4.82 by 3.53.

.025	4.82
.045	3.53
125	.1446
100	2.410
	14.46
.001125	
	17.0146

In the first instance, there being but four figures in the product, and six decimals in the multiplier and multiplicand, two cyphers must be added to the left hand of the product.

Ex. 1. Multiply 76.43 by .875 : also .897 by .452.
Answers, 66.87625—.405444

Ex. 2. Multiply 324.004 by .7872
Answer, 255.0559488

Ex. 3. What is the product of 9.57 and .074 ?
Answer, .70818

Ex. 4. Multiply .643 by .389
Answer, .250127

When the number of decimals in the multiplicand is large, and it is not wished to carry the operation to more than a certain number of decimals in the product, it is done by the following Rule, which I shall illustrate by an example.

Rule. Having arranged the multiplicand, count as many figures from the decimal point, as you intend to keep decimals in the product, and make a * over the last of these, under which, after you have inverted the multiplier, place the units figure of the multiplier thus inverted, and the others in their proper order. Then multiply each figure of the inverted multiplier, beginning, as usual

at the right hand and set down the respective products so that the right hand figures may fall in a straight line under one another. In multiplying, no attention is to be paid to the figures on the right hand of that which you multiply by, unless it be with the two preceding figures, to find what number should be carried.

Ex. Required the product of 1.570796, multiplied by 26.3719, with four places of decimals in the product. This, in the usual method, would yield ten places of decimals: by contraction it is thus performed.

*
1.570796
9.17562

314159 = product with 2 regard being had to 2 × 6
94247 = — — 6 — — — 6 × 9
4712 = — — 3 — — — 3 × 7
1099 = — — 7 — — — —
15 = — — 1 — — — —
14 = — — 9 — — — 9 × 57

41.4246

We will now work the example in the common way.

1.570796
26.3719

14|137164
15|70796
1099|5572
4712|388
94247|76
314159|2

41.4248750324

From this it will appear plain, why in the contracted form the multiplier is inverted: the last product here being the first there. In the contracted form, the units place is 6; it would however be 8, if the 2 were carried from the 27, obtained in the next line by Addition.

Ex. 2. Multiply 128.678, by 38.24 so as to have but one place of decimals.

Common method.	Contracted method.
128.678	128 678 *
38.24	42.83
514712	38603
257356	10294
1029424	257
386034	51
4920.64672	4920.5

DIVISION OF DECIMALS.*

RULE. (1.) Divide, as in whole numbers, and cut off as many figures in the quotient, as the decimal places in the dividend exceed those of the divisor. (2.) If there be not figures enough in the quotient, the deficiency must be supplied by prefixing cyphers. (3.) If there be a remainder, or there be more decimal places in the divisor than in the dividend, cyphers may be affixed to the dividend, and the quotient carried on to any extent.

Divide 1.7154 by 1.5; and .37046 by 16.

1.5)1.7154	16).37046
1.1436	.02315375

In the first example, by supplying a single cypher there is no remainder left; but in the second I must supply three cyphers to obtain an even answer; and I find the quotient has one figure less than there are decimals in the dividend so supplied. I must therefore prefix a cypher to the quotient found.

NOTE.

* The Contracted method of Divison may be thus performed.

RULE. Having determined how many places of whole n here will be in the quotient, if any, which is easily know section; if there are none, then consider of what v

Ex. 1. Divide 25.64 by 3.645. Ans. 7.0343 nearly.
2. Divide 4752 by .9587. Ans. 4956.$7\frac{1171}{9587}$.
3. Divide .865439 by .156. Ans. 5.5477 nearly.
4. Divide 79 by 3965. Ans. .01992, &c.
5. Divide 33.64472 by 882. Ans. .038146, nearly.
6. Divide .218 by 7.435. Ans. .0293, &c.
7. Divide 76.42 by 58. Ans. 1.317, &c.
8. Divide 88 by .88. Ans. 100.

NOTE.

first figure in the quotient will be, and proceed as in common Division, only omitting one figure of the divisor at each operation; viz. for every figure of the quotient dot off one in the divisor, remembering to carry for the increase of the figures cut off, as was done in Multiplication.

Ex. Let it be required to divide 23.41 by 7.9863.

Contracted method.	Common method.
7.9863)23.4100(2.9312	7.9863)23.4100(2.9312
..... 15.9726	15.9726
Here it must be observed, that in each of the subtractions except the first, unit must be carried to the first figure, as would be the case in the usual course.	
.74374	.74374 0
71876	71876 7
.2497	.2497 30
.2395	2395 89
.101	.101 410
79	79 363
.21	.21 5470
15	15 4726
.5	.5 5744

REDUCTION OF DECIMALS.

To change the currencies of the different states to Federal money, and Federal money to currency by decimals.

1.—To reduce Maryland, Pennsylvania, Delaware, and New Jersey currencies to Federal Money.

* Rule. Reduce the given sum to the decimal of a pound, and divide by .375 the quotient will be the answer.

EXAMPLES.

Ex. 1. Reduce 76*l.* 14*s.* 6*d.* Maryland currency to Federal money ?

12)6

2,0)14.5

.375)76.725(204.6 or 204 dols. 60 cts.
75 0

1725
1500

2250
2250

Ex. 2. Reduce 237*l.* 17*s.* 4*d.* Pennsylvania currency to Federal money ? Ans. dols. 634.3111 &c.

Ex. 3. Reduce 673*l.* 1*s.* 2*d.* New Jersey currency to Federal money ? Ans. dols. 1794.8222 &c.

Ex. 4. Reduce 7*l.* 6*s.* 8*d.* New Jersey currency to Federal money ? Ans. dols. 19.5555 &c.

* Note. As 7*s.* 6*d.* of this currency make a dollar, reduce it to the decimal of a pound, and it will be .375*l.* the divisor given in this rule.

2.—To change Federal money to Maryland, Pennsylvania, Delaware, and New Jersey currencies.

RULE. Multiply the given sum by .375 and the product will be pounds, which reduce to shillings and pence.

EXAMPLES.

Ex. 1. How much Maryland currency in $76.50?

76.50
·375

38250
53550
22950

L. 28.68750
20

13.75000
12

9.00000

Ans. 28*l.* 13*s.* 9*d.*

Ex. 2. Change 744 dols. into Pennsylvania currency? Ans. 279*l.*

3. Change 365.25 dols. into Pennsylvania currency? Ans. 136*l.* 19*s.* 4½*d.*

Ex. 4. Change 627.75 dols. into Maryland currency? Ans. 235*l.* 8*s.* 1½*d.*

8.—To change New England and Virginia currencies to Federal money.

* RULE. Reduce the given sum to the decimal of a pound, and divide by .3, the quotient will be the answer.

NOTE.

* As 6 shillings of this currency make one dollar, reduce 6 shillings to the decimal of a pound, and it will be .3, the divisor given in the rule.

EXAMPLES.

Ex. 1. In 74*l.* 6*s.* 8*d.* New England currency, how much Federal money?

12)8

2,0)6.6′

.3)74.333′ &c.

$ 247.77′ &c.

Ex. 2. In 64*l.* 15*s.* Virginia currency, how much Federal money? Ans. dols. 215.833 &c.

Ex. 3. In 327*l.* 16*s.* 4*d.* Virginia currency, how much Federal money? Ans. dols. 1092.722 &c.

Ex. 4. In 463*l.* 12*s.* 9*d.* Virginia currency, how much Federal money? Ans. dols. 1545.45833 &c.

4.—To change Federal money to New England and Virginia currencies.

Rule. Multiply the given sum by .3, and the product will be pounds, which reduce to shillings and pence.

EXAMPLES.

Ex. 1. Change 273.35 dols. to New England currency?

273.25

.3

81.975

20

19,500

12

6,000 Ans. 81*l.* 19*s.* 6*d.*

Ex. 2. Change 496 dols. to New England currency? Ans. 148*l.* 16*s.*

3. Change 79.50 dols. to Virginia currency? Ans. 23*l.* 17*s.*

4. Change 673.60 dols. to Virginia currency? Ans. 202*l.* 1*s.* 7.2*d.*

5. To change New-York and North-Carolina currencies to Federal money.

* Rule. Reduce the given sum to the decimal of a pound, and divide by .4, the quotient will be the answer.

Examples.

Ex. 1. In 74*l.* 16*s.* New-York currency how much Federal money?

2,0)16

.4)74.8

$187 Ans.

Ex. 2. In 29*l.* 17*s.* New-York currency, how much Federal money? Ans. $74.625

Ex. 3. In 365*l.* 7*s.* 4*d.* New-York currency, how much Federal money? Ans. $913.4166, &c.

Ex. 4. 497*l.* 16*s.* 10*d.* North Carolina currency, how much Federal money? Ans. $1244.604166, &c.

6.—To change Federal money to New-York and North Carolina currencies.

Rule. Multiply the given sum by .4 and the product will be pounds, the decimal parts of which reduce to shillings and pence.

Examples.

Ex. 1. Reduce $49 50 to New-York currency?

49.50
.4

19.800
20

16.000 Ans. 19*l.* 16*s.*

---

* Note. As 8 shillings of this currency make one dollar, reduce 8 shillings to the decimal of a pound, and it will give .4 the divisor given in the rule.

Ex. 2. Reduce $246 to North-Carolina currency?
Ans. 98*l.* 8*s.*

Ex. 3. Reduce $418.75 to New-York currency?
Ans. 167*l.* 10*s.*

Ex. Reduce $847.60 to New-York currency?
Ans, 339*l.* 0*s.* 9.6*d.*

7.—To change South-Carolina and Georgia currencies to Federal money.

* Rule. Reduce the given sum to the decimal of a pound, multiply by 30 and divide the product by 7; the quotient will be the answer.

Examples.

Ex. 1. In 69*l.* 15*s.* 6*d.* South-Carolina currency, how much Federal money?

12)6

2,0)15.5

69.775

30

7)2093.250

299.035'71428'

Ex. 2. In 864*l.* 17*s.* 2*d.* South Carolina currency, how much Federal money?
Ans. $3706.535714285'.

---

* Note.—As reducing the currency of these states to the decimal of a pound, would produce a circulating decimal, I have formed this rule on the principle of Vulgar Fractions.

4*s.* $8d = \frac{56}{240}$ or $\frac{7}{30}$ of a pound, consequently the proportion will stand thus $\frac{7}{30}l.$: 1*doll.* :: pounds : dollars, or as 7 is to 30 so are pounds to dollars, agreeably to the rule.

Ex. 3. In 927*l.* 16*s.* 9*d.* Georgia currency, how much Federal money?

Ans. $3976.446428571

Ex. 4. In 673*l.* 12*s.* 8*d.* Georgia currency, how much Federal money?

Ans. 2886.99 or 2887 dollars.

8.—To change Federal money to South-Carolina and Georgia currencies.

Rule. Multiply the given sum by 7, and divide that product by 30, the quotient will be the answer in pounds; the decimal parts of which reduce to shillings and pence.

Examples.

Ex. 1. How much Georgia currency in $216.50?

$216.50
7

3,0)151,5.50

50.516′ circulates
20

10.33′ &c.
12
3.99′ &c. Ans. 50*l.* 10*s.* 4*d.*

Ex. 2. How much South-Carolina currency in $467.25? Ans. 109*l.* 0*s.* 6*d.*

Example 3. How much South-Carolina currency in $762.3? Ans. 177*l.* 17*s.* 4.8*d.*

Example 4. How much Georgia currency in $939.7? Ans. 219*l.* 5*s.* 3.19′*d.*

---

9. To change Canada and Nova-Scotia currencies to Federal money.

* RULE. Reduce the given sum to the decimal of a pound; and divide by .25 the quotient will be the answer?

EXAMPLES.

Ex. 1. Reduce 87*l.* 16*s.* 4*d.* Canada currency to Federal money?

12)4.

2,0)1,6.3

.25)87.816'($351.26' &c.
75

128
125

31
25

66
50

166
150

16

or thus,

.25 { .5)87.816'
.5)17563'

$351.26' &c.

Ex. 2. Reduce 827*l.* 15*s.* Nova-Scotia currency to Federal money? Ans. $3311.

Ex. 3. Reduce 268*l.* 12*s.* 3*d.* Canada currency to Federal money? $1074.45.

Ex. 4. Reduce 719*l.* 9*s.* 2*d.* Canada currency to Federal money? Ans. $2877.83'

10. To change Federal money to Canada and Nova-Scotia currencies.

* Note.—5 shillings of this currency make one dollar. The divisor in this rule is obtained by reducing this sum to the decimal of a pound.

RULE. Multiply the given sum by .25, and the product will be pounds, the decimal part of which reduce to shillings and pence.

EXAMPLES.

Ex. 1. In 68.5 dols. how much Nova Scotia currency?

68.5
.25
3425
1370
17.125
20
2.500
12
6.000 Ans. 17*l.* 2*s.* 6*d.*

Ex. 2. In $124.25 how much Canada currency?
Ans. 31*l.* 1*s.* 3*d.*

Ex. 3. In $7648 how much Nova Scotia currency?
Ans. 1912*l.*

Ex. 4. In $867.35 how much Nova Scotia currency?
Ans. 216*l.* 16*s.* 9*d.*

Note.—The shortest method of working the examples in this currency, is to multiply the given number of pounds by 4 for dollars, and to reduce dollars to pounds divide by 4.—This number is used because 5 shillings are $\frac{5}{20}$ or $\frac{1}{4}$ of a pound.

INVOLUTION.

INVOLUTION is a method of raising numbers to higher powers.

A *power* is the product arising from multiplying any given number into itself once, or oftener: thus, $3 \times 3 = 9$ is the second power of 3, and it is denoted in this manner 3^2.

The number denoting the power is called the *index*, or *exponent* of that power: thus, in 3^2 the 2 is the index or exponent.

The third power of 4 is $4^3 = 4 \times 4 \times 4 = 64$

The fourth power of 3 is $3^4 = 3 \times 3 \times 3 \times 3 = 81$

The sixth power of 5 is $5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 15625$.

The third power of $\frac{1}{4}$ is $\frac{1}{4}^3 = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$.

The fifth power of .03 is $.03^5 = .03 \times .03 \times .03 \times .03 \times .03 = .0000000243$.

EXAMPLES.

Ex. 1. What is the sixth power of 6? Ans. 46656

2. What is the eighth power of 7? Ans. 5764801

3. What is the fourth power of $\frac{3}{5}$? Ans. $\frac{81}{625}$

4. What is the fifth power of $\frac{7}{8}$? Ans. $\frac{16807}{32768}$

5. What is the third power of .25? Ans. .015625

6. What is the fourth power of .05?

Ans. .00000625

7. What is the third power of .305 ?

Ans. .028372625

8. What is the ninth power of 9 ?

Ans. 387420489

9. What are the squares of 3 and 6; 5 and 10 6 and 12; 2, 4, 8, and 16 ?

Ans. $3^2 = 9$ $5^2 = 25$ $6^2 = 36$
$6^2 = 36$ $10^2 = 100$ $12^2 = 144$
$2^2 = 4$ $4^2 = 16$ $8^2 = 64$ $16^2 = 256$

10. What are the cubes of 3 and 6; 5 and 10; 6 and 12; 2, 4, 8, and 16 ?

Ans. $3^3 = 27$
$6^3 = 216 = 8 \times 27$

$5^3 = 125$
$10^3 = 1000 = 8 \times 125$

$6^3 = 216$
$12^3 = 1728 = 8 \times 216$

$2^3 = 8$
$4^3 = 64 = 8 \times 8$
$8^3 = 512 = 8 \times 64$
$16^3 = 4096 = 8 \times 512$

EVOLUTION.

EVOLUTION is the method of extracting roots.

The root of any number, or power, is such a number, as being multiplied into itself once, or oftener, produces that power: thus 3 is the square root of 9, because 3 multiplied into itself gives 9: 4 is the cube root of 64, because 4 multiplied into itself twice, gives 64. The roots are denoted by indices, or exponents, in this manner:

The cube root of 125 is $\sqrt[3]{125} = 5$.

The square root of 81 is $\sqrt{81} = 9$.

The fifth root of 243 is $\sqrt[5]{243} = 3$.

Ex. 1. What are the square roots of 49 and 64?
Answer, 7. 8.

2. What are the cube roots of 216, 343, 512, and 729? Answer, 6. 7. 8. 9.

3. What are the fourth roots of 625, 2401, and 4096? Answer, 5. 7. 8.

4. What are the fifth roots of 3125 and 32768? Answer, 5. 8

To extract the square root.

RULE. (1.) Divide the given number into periods of two figures each, by placing a dot over units, another over hundreds, and so on. (2.) Find the greatest square in the first period, and set its root on the right-hand, as a quotient figure in division. (3.) Subtract the square thus found, and to the remainder annex the succeeding period for a new dividend. (4.) Double the root for a divisor, and examine how often it is contained in the dividend, exclusive of the place of units, and put the

result into the quotient and in the units place of the divisor. (5) Multiply the divisor thus increased by the new quotient figure, and subtract the product from the dividend. (6.) Bring down the next period, find a divisor as before, by doubling the figures already in the root, and proceed as before.

The rule will be rendered clear by the following examples:

What are the square roots of 16777216 and 43046721 ?

```
  .  .  .  .
16777216(4096
16
---
809). .7772
      7281
     ----
8186)49116
     49116

  .  .  .  .
43046721(6561
36
---
125)704
    625
    ---
1306)7967
     7836
     ----
13121)13121
      13121
```

EXAMPLES.

Ex. 1. What is the square root of 117649 ?
Answer, 343.

2. What is the square root of 262144 ?
Answer, 512.

3. What is the square root of 531441 ?
Answer, 729.

4. What is the square root of 1679616 ?
Answer, 1296.

Ex. 5. What is the square root of 5764801 ?
Answer, 2401.

6. What is the square root of 1073741824 ?
Answer, 32768.

7. What is the square root of 119550669121 ?
Answer, 345761.

8. What is the square root of 20 ?
Answer, 4.4721, &c.

9. What is the square root of 300 ?
Answer, 17.3205, &c.

10. What is the square root of 1000 ?
Answer, 31.622, &c.

11. What is the square root of $\frac{1}{2}$: $\frac{6}{36}$; $\frac{25}{144}$?
Answer, .7071, &c. .4082, &c. .4166.

12. What is the square root of .25 ?
Answer, .5.

MISCELLANEOUS EXAMPLES.

Ex. 1. A gentleman desirous of making his kitchen garden, which is to contain 4 acres, a complete square, I demand what will be the length of the side of the garden ? Ans. 139 yards.

Ex. 2. Six acres of ground are to be allotted to a square garden ; but for the sake of more wall for fruit, there is to be a smaller square within the larger, which is to contain 3 acres, I demand the length of the sides of each square ? Ans. outer 170.41 yds. inner 120$\frac{1}{2}$nearly.

Ex. 3. What is the mean proportional between 12 and 75 ? Ans. 30.

Ex. 4. How long must a ladder be to reach a window 30 feet high, when the bottom stands 12 feet from the house ? Ans. 32.31 feet.

To extract the cube root.

I. Rule. (1) Find, by trials, the nearest cube to the given number, and call it the assumed cube. (2) Say as twice the assumed cube added to the given number, is to twice the number added to the assumed cube, so is the root of the assumed cube to the root required nearly.

What is the cube root of 27455 ?

Here the nearest root that is a whole number is 30, the cube of which is 27000 : therefore I say,

As $27000 \times 2 + 27455 : 27455 \times 2 + 27000 :: 30$
or $81455 : 81910 :: 30 : 30.1675$.

It is evident that the true root, omitting the last two figures, is somewhere between 30.16 and 30.17, the former being too little, the latter something too large. By taking the root thus found 30.16, as the assumed cube, and repeating the operation, the root will be had to a still greater degree of exactness.

Ex. 1. What is the cube root of 15625 ? Ans. 25

2. What is the cube root of 140608 ? Ans. 52

3. What is the cube root of 444194947 ? Ans. 763

4. What is the cube root of the difference between 140608 and 14625 ? Ans. 50.13 nearly.

II. Rule. (1) Separate the given number into periods of three figures each, beginning from units place ; then from the first period subtract the greatest cube it contains, put the root as a quotient, and to the remainder bring down the next period for a dividend. (2) Find a divisor by multiplying the square of the root by 300, see how often it is contained in the dividend, and the answer gives the next figure in the root. (3) Multiply the divisor by the last figure in the root. Multiply all the figures in the root by 30, except the last, and that product by the square of the last. Cube the last figure in the root. Add these three last found numbers together, and subtract this sum from the dividend ; to the remainder bring down the next period for a new dividend, and proceed as before.

Ex. 5. What is the cube root of 444194947 ?

444194947(763 Answer.
343

$7 \times 7 \times 300 = 14700)'01194$
95976

$76 \times 76 \times 300 = 1732800)5218947$
14700 = divisor
6

88200
7560 = 7×30×36
216 = 6×6×6

95976

5218947

1732800 = divisor
3

5198400
20520 = 76×30×9
27 = 3×3×3

5218947

Ex. 6. What is the cube root of 46656 ? Ans. 36

7. What is the cube root of 65939264 ? Ans. 404

8. What is the cube root of 3, carried to 2 places of decimals ? Ans. 1.44

9. What is the cube root of $\frac{8}{512}$? Ans. $\frac{1}{4}$

10. What is the cube root of $\frac{125}{1728}$? Ans. $\frac{5}{12}$

11. What is the cube root of .729 ? Ans. .9

12. What is the cube root of .003375 ? Ans. .15

MISCELLANEOUS EXAMPLES.

Ex. 1. What is the length of one side of a vessel, which contains 13824 solid inches ? Ans 24 inches.

Ex. 2. In a cubical building that measures 2744 feet, what is the length of a side ? Answer, 14

ARITHMETICAL PROGRESSION.

When a series of numbers increases or decreases by some common excess, or common difference, it is said to be in arithmetical progression, such as 1, 3, 5, 7, 9, &c., and 12, 10, 8, 6, 4, &c.

The numbers which form the series are called the terms of the progression; of these the first and last are called the extremes.

The first term is called - -	a
The last term is called - -	z
The number of terms is called -	n
The common difference is called -	d
The sum of all the terms is called	s

Any three of these terms being given, the others may be easily found.

I. When the first term a, and the last term z, and the number of terms n, are given, to find the sum of all the terms, s.

Rule. Multiply the sum of the extremes by the number of terms, and divide by 2, the quotient is the answer: or

$$\overline{a+z} \times \frac{n}{2} = s.$$

Ex. 1. What is the sum of an arithmetical series, whose first term is 5, last term 29, and the number terms 7.

$$\text{Here } s = \overline{5+29} \times \frac{7}{2} = 34 \times \frac{7}{2} = \frac{238}{2} = 119, \text{ the answer.}$$

Ex. 2. The first and last terms of a series are 3 and 111, and the number of terms 37 : what is the sum ?
Ans. 2109

Ex. 3. How many strokes do the clocks of Venice strike in 24 hours, where they strike from 1 to 24 ?
Ans. 300

Ex. 4. The first and last terms of a series are 1 and 1000, and the number of terms 100 : required the sum.
Ans. 50050

Ex. 5. If 100 stones are placed in a right line, exactly a yard asunder, and the first, one yard from a basket, what length of ground will a man go over, who gathers them up, one by one, returning with each to the basket ?
Ans. 5 miles and 1300 yards.

Ex. 6. What must a man give for 54 timber trees, for which he pays 5 shillings for the first, and 20*l.* for the last, and the prices of the others being in arithmetical progression ? Ans. 546*l.* 15*s.*

Ex. 7. A butcher buys a drove of oxen, consisting of 32 ; for the first he pays 15*s.*; and for the others he is to pay in arithmetical progression, so that for the last he is pay 38*l.*: what will they all come to. Ans. 620*l.*

Ex. 8. A horse-dealer sends to a fair 63 horses of various kinds and worth, which he is willing to dispose of according to the principles of arithmetical progression, demanding 3*l.* only for the first, provided he had 53*l.* for the last: how much did he receive for the whole, and what was the average value of each horse ?

Ans. 1764*l.* for the whole—28*l.*, average price of each horse.

II. The first and last terms, a and z, and number of terms being given, to find the common difference d.

Rule. The difference of the extreme terms divided by the number of terms less 1, will be the common difference sought :

$$\text{or } \frac{a \sim z}{n-1} = d.$$

Ex. 1. What is the common difference of an arithmetical progression, whose extremes are 8 and 200, and the number of terms 17 ?

$$d = \frac{8 \backsim 200}{16} = \frac{200 - 8}{16} = \frac{192}{16} = 12.$$

Ex. 2. When the extremes of an arithmetical progression are 6 and 57, and the number of terms 18, what is the common difference ? Ans. 3

Ex. 3. A gentleman gives at Christmas, among his 25 poor neighbours, a sum of money in arithmetical progression : to the least needy he gives 5 shillings, and to the poorest, with a very large family, he gives five guineas : what was the common difference ? Ans. 4*s.* 2*d.*

Ex. 4. A traveller is out on his journey a month, of which he travels 25 days ; on the first he rides 7 miles, and on the last, having little to do, he comes 43 miles : how much was the daily increase of his travelling, and how many miles did he ride in the whole ?

Answer $1\frac{1}{2}$ miles increase—625 the number of miles travelled.

III. The extreme terms a and z, and common difference d being given, to find the number of terms v.

Rule. Divide the difference of the extremes by the common difference, and the quotient increased by unity is the number sought : or $\frac{a \backsim z}{d} + 1 = n$.

Ex. 1. When the extremes are 4 and 106, and the common difference is 3, what is the number of terms ?

$$\frac{4 \backsim 106}{3} + 1 = \frac{106 - 4}{3} + 1 = \frac{102}{3} + 1 = 34 + 1 = 35 = v.$$

Ex. 2. If the least term be 6, the greatest 216, and the common difference 5, what is the number of terms ? Ans. 43

Ex. 3. What debt can be paid, and in what time, supposing I agree to lay by 3*s.* the first week, 7*s.* the next, 11*s.* the third, and so on in arithmetical progression, till the last saving be four guineas ? Ans. 46*l* 4*s* $4\frac{1}{2}d$. = the debt to be paid.—$21\frac{1}{4}$ weeks = the time.

Ex. 4. I set out for Hastings, which is 69 miles from this place, and I walk the first day 4 miles, the second 7, increasing every day by 3 miles, and on the last 19 miles: how many days will the journey take?

Ans. 6, the number of day's journey.

In addition to the above, the learner may commit to memory the following facts on the subject:

1. If three numbers are in arithmetical progression, the sum of the extremes is equal to double the mean term; as, 6, 9, 12, where $6+12=2\times9=18$.

2. If four numbers be in arithmetical progression, the sum of the two extremes is equal to the sum of the means; as 5, 8, 11, 14, where $5+14=8+11=19$.

3. When the number of terms is odd, the double of the middle term will be equal to the sum of the extremes: or of any other two means equally distant from the middle term; as 3, 8, 13, 18, 23, 28, 33, where $3+33=2\times18=13+23=8+28$.

GEOMETRICAL PROGRESSION.

A GEOMETRICAL PROGRESSION is a series of numbers, the terms of which gradually increase or decrease by the constant multiplication or division of some particular number; as 1, 3, 9, 27, 81, 243, &c., or 64, 32, 16, 8, 4, 2, 1, $\frac{1}{2}$, &c.

In the *first* case, the series is *increasing* by the constant multiplication of 3; in the *second*, it is a *decreasing* series by the constant division of 2. It is evident that both series may be carried on for ever.

The number by which the series is constantly increased or diminished is called the *ratio*.

The first term is called - - - - a
The last term is called - - - - z
The number of terms is called - - n
The ratio is called - - - - - - r
The sum of all the terms is called - s.

Any three of these terms being given or known, the others may be determined.

1. Given the first term a, the last term z, and the common ratio r, to find the sum s.

RULE. Multiply the last term by the ratio, and from the product subtract the first term, and the remainder divided by the ratio, less one, will give the sum of the series; or $\frac{z \times r - a}{r-1} = s$.

Ex. 1. The first term of a series in geometrical progression is 5, the last term is 3645 and the ratio 3: what is the sum?

Here $s = \frac{3645 \times 3 - 5}{3-1} = \frac{10935-5}{2} = \frac{10930}{2} = 5465$.

For the terms are 5, 15, 45, 135, 405, 1215, and 3645; which, being added together, make 5465.

Ex. 2. The first and last terms of a geometrical series are 4 and 3294172, and the common ratio is 7: what is the sum? A. 3843200

Ex. 3. The first and last terms of a geometrical progression are 4 and 262144, and the ratio 4: what is the sum? A. 349524

I. Given the first term a, the number of terms n, and the ratio r, to find the last term z.

The last term may be obtained by continual multiplication; but as that, in a long series, is a tedious process, we shall give the following rule:

1. When the first or least term is equal to ratio.

RULE. Write down some of the leading terms of the geometrical series, over which place the arithmetical series 1, 2, 3, 4, &c., as indices;* find what figures of

* When the natural numbers 1, 2, 3, 4, 5, &c., are set over a geometrical series, they are called *indices* or *exponents*, and they shew the distance of any term from unity, or from the first term: thus, in the series 2^1, 4^2, 8^3, 16^4, 64^5, 128^6, &c., 1, 2, 3, &c. are the indices, and shew the distance of any term; the series from the first term, index 5, for instance, shews that 64 is the fifth term in the series.

these indices added together will give the index of the term wanted in the geometrical series; then multiply the numbers, standing under such indices, into each other, and their product will be the term sought.

Ex. 1. What is the last term of a geometrical series having 13 terms, of which the first is 2, and the ratio 2?

Here the series, with their indices, will stand thus:

$$2^1,\ 4^2,\ 8^3,\ 16^4,\ 32^5,\ 64^6,\ \&c.$$

The number of terms being 13, the index to the last term will be 13 equal to the indices $2+5+6$, which figures standing over 4, 32, and 64, shew that these last are to be multiplied together, and the product is the term sought; thus $4\times32\times64=8192$.

Ex. 2. What is the last term of the series having 9 terms, of which the first is 3, and the ratio 3?

Answer, 19683.

Ex. 3. What did the last of 12 oxen cost, the first of which was sold for 3*s.*; the second for 9s. and so on.

Answer, 26572*l.* 1*s.*

2. When the first term a, of the series, is *not* equal to the ratio r.

Rule. Write down the leading terms of the series, and place their indices over them, beginning with a cypher, add together the most convenient indices to make an index less one than the number expressing the place of the term sought; then multiply the numbers standing under such indices, into each other, dividing the product of every two by the first term in the geometrical series; the last quotient is the term required.

Ex. 1. What is the last term of the series, whose first term is 4, ratio 3, and numbers of terms 15?

$$4^0,\ 12^1,\ 36^2,\ 108^3,\ 324^4,\ 972^5,\ 2916^6,\ \&c.$$

The number of terms being 15, the index sought must be 14 equal to $6+5+3$, under which stand the terms 2916, 972, and 108, then

$$\frac{2916\times972}{4}=708588, \text{ and } \frac{708588\times108}{4}=19131876=$$

$=$ last term.

Ex. 2. The first term of a geometrical series is 2, the number of terms 12, and the ratio 5, required the last term ? Answer, 97656250

Ex. 3. The first term of a geometrical series is 1, the ratio 2, and the number of terms 25, what is the last term, and also the sum of all the terms ?

Answer, 16777216 last term 33554431 sum.

Ex. 4. The first term of a series is 5, the ratio 3, and the number of terms 16, what is the last term, and the sum of the terms ?

Answer, 71744535 last term 107616800 sum.

Ex. 5. A hosier sold 12 pair of stockings, the first pair at 3*d.*, the second 9*d.*, and so on in geometrical progression: for what did he sell the last pair, and how much had he for the whole ?

Answer, 531441 last term, 3321*l.* 10*s.* sum.

Ex. 6. What would a horse fetch, supposing it was sold on condition of receiving for it one farthing for the first nail in his shoes, a halfpenny for the second, one penny for the third, and so on, doubling the price of every nail to 32, the number in his four shoes ?

Answer, 4473924*l.* 5*s.* $3\frac{3}{4}d.$

Ex. 7. A husbandman agreed to serve his master during hay-time and harvest, or five-and-forty clear days, provided he would give him a barley-corn only for the first day's work, 3 for the second, 9 for the third, and so on in geometrical proportion; what would he have to receive in money for his labours, supposing there were half a million of grains in a bushel, and each bushel was worth 4*s.* ?

Ans. 590.862.541.310.166*l.* 14*s.* $9\frac{1}{2}d.$ $\frac{696323}{500000}$.

The following facts may be committed to memory:

1. If three numbers are in geometrical progression, the product of the extremes is equal to the square of the mean; as 3, 9, 27, here $3 \times 27 = 9 \times 9 = 81$.

2. If four numbers are in geometrical progression, the product of the extremes is equal to the product of the means; as 2, 4, 8, 16; here $2 \times 16 = 4 \times 8 = 32$.

3. If the series contain an odd number of terms, the square of the middle term is equal to the product of the adjoining extremes, or of any two terms equally distant from them; as 3, 9, 27, 81, 243; here $27^2 = 3 \times 243 = 9 \times 81 = 729$.

INTEREST.

INTEREST, is the sum of money paid, or allowed for the loan or use of some other sum, lent for a certain time, according to a fixed rate.

The sum lent, and on which the interest is reckoned is called the PRINCIPAL.

The sum per Cent. agreed on as interest, is called the RATE.

The principal and interest added together, is called the AMOUNT.

Interest is distinguished into SIMPLE and COMPOUND.

SIMPLE INTEREST, is that which is reckoned on the principal only, at a certain rate for a year, and at a proportionately greater or less sum, for a greater or less term: thus, if 5*l.* is the rate of interest of 100*l.* for one year, 10*l.* is the interest for two years, 20*l.* for four years, and so on.

RULE. (1) Multiply the principal by the rate, and divide the product by 100, and the quotient is the interest for one year.

Thus the interest of 250*l.*, at 5 per cent. is $\frac{250 \times 5}{100} =$ 12*l.* 10*s.*

(2) Multiply the interest for one year by the number of years, and the product is the interest for the same:

Thus the interest of 250*l.* for 7 years is 12*l.* 10*s.* × 7 = 87*l.* 10*s.*

(3) If parts of a year be given, they must be worked for by the aliquot parts of a year, as in Practice, or by the Rule of Three Direct.

Ex. 1. What is the interest of 853*l.* 10*s.* for 4 years and 8 months, at 5 per cent. per annum?

853	10	6 | ½	42	13	6	= interest for one year.
	5	2 | ⅓			4	
L. 42.	67 10		170	14	0	
	20		21	6	9	
			7	2	3	
shill. 13.	50					
	12	*L.*	199	3	0	
pence 6.	00					

Answer, 199*l.* 3*s.* 0*d.*

To find the *amount*, I must add the principal to the interest. In this example, the amount is equal to 853*l.* 10*s.* + 199*l.* 3*s.* = 1052*l.* 13*s.*

Ex. 2. What is the amount of 142*l.* 10*s.* for four years and 52 days at 4½ per cent?

L. 142	10	*L.*	*s.*	*d.*	
	4½	6	8	3	= interest for one year.
				4	
570	0				
71	5	25	13	0	= interest for four years.
L. 6.41	5				
	20				
shill. 8.25		To find the interest for the 52 days, I say,			
	12				
pence 3.00					

				days.	*L.*	*s.*	*d.*	days.
				If 365	: 6	8	3	:: 52
					20			
L.	*s.*	*d.*			128			
25	13	0			12			
0	18	$3\frac{1}{4}$						
26	11	$3\frac{1}{4}$	=interest		1539			
142	10	0	=principal		52			
169	1	$3\frac{1}{4}$	=amount		3078			
					7695			
								12

365)80028(219$\frac{1}{4}$

18 $3\frac{1}{4}$=interest for 52 ds.

Ex. 3. What is the interest of 461*l.* at 4 per cent. for 5 years? Ans. 92*l.* 3*s.* $11\frac{1}{2}$*d.*

Ex. 4. What is the interest of 230*l.* 15*s.* for $6\frac{1}{2}$ years, at 5 per cent. per annum? Ans. 74*l.* 19*s.* $10\frac{1}{2}$*d.*

Ex. 5. What is the amount of 225*l.* for 7 years, at $3\frac{1}{2}$ per cent. per annum? Ans. 280*l.* 2*s.* 6*d.*

Ex. 6. How much shall I have to receive at the end of 5 years for 350*l.* supposing $4\frac{1}{2}$ per cent. be allowed as interest? Ans. 428*l.* 15*s.*

In most computations relating to simple interest, the work is shortened, if the interest of 1*l.* for a given term is known, as the interest of any other sum for the same term will then be found by only multiplying by the given sum.

The interest of 1*l.* for a year must be in the same proportion as the interest of 100*l.* to its principal; therefore, at 5 per cent, we say, as 100*l.* : 5*l.* :: 1*l.* : .05*l.* Hence the interest of 1*l.* for one year,

	L.
At 3 per cent. is - - -	,03
$3\frac{1}{2}$ - - - - -	,035
4 - - - - -	,04
$4\frac{1}{2}$ - - - - -	,045
5 - - - - -	,05

Ex. 7. What is the interest of 540 dollars for 1 year, at 6 per cent. per annum? Ans. $32.40

Ex. 8. What is the interest of $275.50 for 3 years at $5\frac{1}{2}$ per cent. per annum? Ans. $45.45c. 7.5m.

Ex. 9. What is the interest of $1034.25 for 4 years at $6\frac{1}{4}$ per cent. per annum? Ans. $258.56c. 2.5m.

Ex. 10. How much will 750 dollars, amount to in $7\frac{1}{2}$ years at $5\frac{3}{4}$ per cent. per annum?

Answer, $1073.43cts.7.5 mills.

The Interest of One Pound for any number of Years.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent	5 per Cent.
10	,3	,35	,4	,45	,5
20	,6	,7	,8	,9	1,0
30	,9	1,05	1,2	1,35	1,5
40	1,2	1,4	1,6	1,8	2,0
50	1,5	1,75	2,0	2,25	2,5
60	1,8	2,1	2,4	2,7	3,0
70	2,1	2,45	2,8	3,15	3,5
80	2,4	2,8	3,2	3,6	4,0
90	2,7	3,15	3,6	4,05	4,5
100	3,0	3,5	4,0	4,5	5,0

The 365th part of the yearly interest is always considered as the proper interest for a day, and its multiples as the interest for any number of days; thus, at 5 per cent. the interest for a day is $\frac{.05}{365} = .0001369$; and the interest for 12 days, at the same rate, is $.0001369 \times 12 = .0016428$. Hence by means of the following table, all calculations at 5 per cent. Simple Interest are easily performed, for any number of days.

days	Interest	days	Interest.	days	Interest.	days	Interest.
1	,0001369	26	,0035616	51	,0069863	76	,0104109
2	,0002739	27	,0036986	52	,0071232	77	,0105479
3	,0004109	28	,0038356	53	,0072602	78	,0106849
4	,0005479	29	,0039726	54	,0073972	79	,0108219
5	,0006849	30	,0041095	55	,0075342	80	,0109589
6	,0008219	31	,0042465	56	,0076712	81	,0110958
7	,0009589	32	,0043835	57	,0078082	82	,0112328
8	,0010958	33	,0045205	58	,0079452	83	,0113698
9	,0012328	34	,0046575	59	,0080821	84	,0115068
10	,0013698	35	,0047945	60	,0082191	85	,0116438
11	,0015068	36	,0049315	61	,0083561	86	,0117808
12	,0016438	37	,0050684	62	,0084931	87	,0119178
13	,0017808	38	,0052054	63	,0086301	88	,0120547
14	,0019178	39	,0053424	64	,0087671	89	,0121917
15	,0020547	40	,0054794	65	,0089041	90	,0123287
16	,0021917	41	,0056164	66	,0090411	91	,0124657
17	,0023287	42	,0057534	67	,0091780	92	,0126027
18	,0024657	43	,0058904	68	,0093150	93	,0127397
19	,0026027	44	,0060274	69	,0094520	94	,0128767
20	,0027397	45	,0061643	70	,0095890	95	,0130137
21	,0028767	46	,0063013	71	,0097260	96	,0131506
22	,0030137	47	,0064383	72	,0098630	97	,0132876
23	,0031506	48	,0065753	73	,0100000	98	,0134246
24	,0032876	49	,0067123	74	,0101369	99	,0135616
25	,0034246	50	,0068493	75	,0102739	100	,0136986

RULE. Multiply the figures corresponding with the number of days by the sum :

Thus, if the interest of 75*l.* for 61 days be required: I find opposite to 61, the number .0083561, which multiplied by 75, gives .6267075 of a pound, which reduced, is 12*s.* 6¼*d.*

Ex. 1. What is the interest of 155*l.* for 49 days ?
Ans. 1*l.* 0*s.* 9¾*d.* nearly

COMMISSION AND BROKERAGE.

COMMISSION is an allowance of a certain sum per cent. to a correspondent or agent, for buying and selling goods for his employer, or to a banker for drawing bills and managing accounts.

BROKERAGE, though of a different name, is of the same nature as COMMISSION.

Ex. 1. A salesman at Smithfield, in the course of a year, sells for his correspondents 1120 loads of hay, at the average price of 5*l.* 10*s.* per load ; and 620 loads of straw, at 55*s.* per load : I wish to know the commission money, at 2½ per cent ? Answer, 176*l.* 19*s.* 3*d.*

1120 × $5\frac{1}{2}$ = 5600 + 560 = *L.*6160 = what the hay sold for.

620 × $2\frac{1}{2}$ $\frac{1}{2}$ = 1240 + 310 + 155 = 1705 = what the straw sold for.

1705

*L.*7865 × $2\frac{1}{4}$ = 15730 + 1966.25 = 176.9625

20

19.25

12

3.00

Answer, 176*l.* 19*s.* 3*d.*

Ex. 2. A Manchester manufacturer allows his agent in London $4\frac{1}{4}$ per cent. for goods sold by him; in the course of the year 1807 he sold to the amount of 15,400*l.*, what was his commission for that year, and how much was the agent's clear gains, supposing his losses on the year's account, by bad debts, amounted to 225*l.* 10*s.* 6*d.*?

Ans. 654*l.* 10*s.* 0*d.* Com. 428*l.* 19*s.* 6*d.* clear gains

Ex. 3. A Liverpool merchant sells goods in a year, for his American correspondents to the amount of 144,454*l.* 10*s.*, on which he reckons his clear gains at the rate of $\frac{3}{8}$ per cent., what is his income on this one concern?

Answer, 541*l.* 14*s.* 1*d*

Ex. 4. What is the commission of $1026.50, at $3\frac{3}{4}$ per cent? Ans. 38 dolls. 49 cts. 3.75 mls.

Ex. 5. A bookseller in London allows his agent in America 5 per cent. commission; what does he pay him for the remittance of 8540*l.* 15*s.* 9*d.*?

Answer, 427*l.* 0*s.* $9\frac{1}{4}$*d.*

Ex. 6. What is the brokerage of $1210, at $\frac{1}{4}$ per cent.? Answer, 3 dolls. 2 cts. 5 mls.

Ex. 7. What is the claim of a broker at $3\frac{3}{8}$ per cent. on $1550.50.? Ans. 52 dolls. 32 cts. 9.375 mls.

Ex. 8. What is the commission on $1000 at $\frac{5}{8}$ per cent.? Answer, 6 dolls. 25 cts.

Ex. 9. What have I to pay my broker for the sale of goods to the amount of 9950*l.* 9*s.*, at $1\frac{1}{4}$ per cent.?

Answer, 124*l.* 7*s.* $7\frac{1}{4}$*d.*

Ex. 10. What will the commission of a country banker amount to on 12314*l.* 8*s.* 9*d.*, at $\frac{1}{8}$ per cent.?

Answer, 15*l.* 7*s.* $10\frac{1}{4}$*d.*

Ex. 11. What is the brokerage of 1526*l.* 13*s.* 6*d.*, at $1\frac{1}{2}$ per cent.? Answer, 22*l.* 18*s.*

DISCOUNT.

Discount is an allowance made for advancing money on securities before they are due. The present worth of any sum, due sometime hence, is such, as if put to interest for that time at the rate per cent. given, would amount to the given sum.

Rule.—As the amount of 100*l.* or dollars, at the rate and time given is to 100: so is the given sum to the present worth. The present worth taken from the given sum will be the rebate or discount.

or thus, for the discount;

As the amount of 100*l.* or dollars, at the rate and time given, is to the interest of the same sum at the same rate and time, so is the given sum to the discount required.

Ex. 1. What is the present worth and discount of 620 dollars, due 4 years hence at 6 per cent. per annum discount?

6
4

24 Interest of $100 at 6 per cent. for 4 years.
100

124 Amount of $100 for 4 years at 6 per cent.

124 : 100 :: 620
100

$

124)62000(500
620

00

$620
500 present worth.

$120 discount.

Proof.
500
6

30.00
4

120.00
500

$620

or thus;

124 : 24 :: 620
24

2480
1240

$

124)14880(120
124

248
248

620
120 discount.

500 present worth.

Ex. 2. What is the discount of $718.75 for 5 years at 5 per cent. per annum? Ans. 143 dols. 75 cts.

Ex. 3. What is the present worth of 1092*l.* 13*s.* due 5 years hence at 6 per cent. per annum?

Answer, 840*l.* 10*s.*

Ex. 4. What is the present worth of 284 dols. 28 cts. due 8 months hence at $4\frac{1}{2}$ per cent. per annum?

Answer, 276 dollars.

Ex. 5. What is the discount of 250*l.* 10*s.* 6*d.* due 2 years and 4 months hence, at $6\frac{1}{4}$ per cent per annum?

Answer, 31*l.* 17*s.* $8\frac{2}{5}$*d.*

Ex. 6. What is the present worth of 1000*l.* due 3 years and 7 months hence, at $5\frac{3}{4}$ per cent. per annum?

Answer, 829*l.* 3*s.* 2*d.* $\frac{578}{5789}$

Ex. 7. What is the present worth of 640*l.* 10*s.* due 10 years and 2 months hence, at $4\frac{1}{2}$ per cent. per annum discount? Answer, 439*l.* 9*s.* 0*d.* $\frac{156}{583}$

Ex. 8. What is the discount of 740 dols. 50 cts. due $7\frac{1}{2}$ years hence, at $6\frac{1}{2}$ per cent. per annum?

Answer, 242 dols. 68 cts. $\frac{58}{119}$

Ex. 9. What is the present worth of 500 dollars, one half payable in 6 months, and the other half in 8 months, discount at 6 per cent. per annum?

Answer, 483 dols. 10 cts. $\frac{410}{1339}$

Ex. 10. What difference is there between the interest of 600 dollars for 1 year and 9 months at 6 per cent. per annum, and the discount of the same sum at the same rate and for the sametime? Ans. 5 dols. 98 cts. $\frac{142}{221}$

NOTE.

Discount in business is generally reckoned in the same manner as common interest.

When the sum is not very large, and the time short, the difference between the discount and the interest is a mere trifle; but when the sum is large and the time considerable, their difference then becomes essential, and the sum should be calculated on correct discount principles.

PROFIT AND LOSS

Is a rule that discovers what is gained or lost on the prime cost in the purchase and sale of goods, and it teaches how to to fix the price of their goods so as to gain so much per cent.

Questions in this rule are performed by the Rule of Three Direct, upon this principle, that quantities, or sums of money, which gain or lose at the same rate, are to one another as their gains or losses.

Ex. 1. A tallow chandler has this day purchased mottled soap, at 102*s.* 6*d.* per cwt., at how much per lb. must he retail it out to gain 10 per cent. profit ?

L.		*L.*		*s.*	*d.*
100	:	110	::	102	6 ÷ 112
20		102	6		
2000		220			
		1100			
		55			
2,000)11,275					
		5.6375			

L. 5.6375 and $\frac{5.6375}{112}$ = 1*s.* $\frac{75}{112}$ = 1*s.* 0$\frac{3}{4}$*d.* nearly.

Ex. 2. How much per cent. is gained at the rate of 2*d.* in a shilling ? Answer 16*l.* 13*s.* 4*d.*

Ex. 3. If 3 dollars be gained in selling at 21 dollars, at what rate per cent is that ? Answer 16$\frac{2}{3}$ per cent.

Ex. 4. Three pounds of tobacco are bought at 5*s.* 9d. and sold for 7*s.* 6*d.*, what is the gain upon the sale of what cost 100*l.* Answer 30*l.* 8s. $8\frac{1}{4}d.$

Ex. 5. Bought cheese at 3*l.* 3s. per cwt., and sold it again at $10\frac{1}{2}d.$ per lb.: what is the gain per cwt. supposing the loss in weight to be 4lb. per cwt.

Answer, *L.* 1 11 6 gain per cwt.

Ex, 6. Bought silk stockings at $4 25 per pair, what must they be sold for to gain 20 per cent profit?

Answer, $5.10.

Ex. 7. If 375 yards of cloth be sold for 290*l.* and there be 20 per cent. profit, what did it cost per yard?

Answer, 12*s.* $10\frac{6}{7}d.$

Ex. 8. If 90 English Ells of Cambric cost 120 dolls. for how much must I sell it per yard to gain 18 per cent?

Answer, $1 $25\frac{13}{15}$.

Ex. 9. A plumber sold 5 fother of lead, for 102*l* 2s.6*d.* (the fother being $19\frac{1}{2}$ cwt.), and gained after the rate of 12*l.* 10s. per cent.: what did it cost him per cwt.?

Answer, 18s. $7\frac{1}{4}d.$

Ex. 10. Bought 218 yards of cloth, at the rate of 8s. 6*d.* per yard, and sold it for 10s. 4*d.* per yard: what was the gain of the whole? Answer, 19*l.* 19s. 8*d.*

Ex. 11. Paid 69*l.* for one ton of steel, which is retailed at 8*d.* per lb., what is the profit or loss by the sale of 12 tons? Answer *L.* 68 gain.

PARTNERSHIP

Is a general rule, by which merchants, &c., trading in company with a joint stock, are enabled to ascertain each person's particular share of the gain or loss, in proportion to his share in the stock.

This rule divides itself into two parts, viz. 1. Partnership without regard to time: and 2. Partnership with me.

I. Partnership without Time.

Rule. "As the whole stock is to the whole gain or loss, so is each man's share in the stock to his share of the gain or loss."

Ex. 1. Two merchants embark in business, the one puts in as capital *L.*5550, and the other *L.*3420, and they gain in the first year *L.*1260, what is each man's gain?

*L.*5550
3420

8970 = joint stock.

8970*l.* : 1260*l.* : : 5550*l.* : 779*l.* 12*s.* nearly; of course the profits of the other are 1260*l.*—779*l.* 12*s.*=480*l.* 8*s.*

Ex. 2. Three persons trade together: A puts in 100*l.*; B 150.; C 200*l.*; and they gain 900*l.*: what is each man's gain? Ans. A 200—B 300—C 400.

Ex. 3. A, B, and C, enter into partnership; A puts in 3640*l.*, B 4820*l.*, and C 5000*l.*, and they gained 8670*l.*: what is each man's share in proportion to his stock?
Ans. A 2344*l.* 13*s.* nearly—B 3104*l.* 14*s.*—C 3220*l.*13*s.*

Ex. 4. Four merchants. B, C, D, and E, make a stock; B put in 2270*l.*, C 3490., D 1150*l.* and E 4390; in trading they gained 4280*l.* I demand each merchant's share of the gain? Ans. B 859*l.* 16*s.* nearly—C 1321*l.* 17*s.* 6*d.*—D 435*l.* 11*s.* 6*d.* nearly—E 1662*l.* 15*s.*

Ex 5. Three persons, D, E, and F, join in company; D's stock was 3750*l.*, E's 2800*l.*, and F's 2500*l.*, and at the end of 12 months they gained 3420*l.*; what is each man's particular share of the gain?
Ans. D 1417*l.* 2*s.* 6½*d.*—E 1058*l.* 2*s.* 5*d.*—F 944*l.* 15*s.* ½*d.*

II. Partnership with Time.

Rule. As the sum of the product of each man's money and time is to the whole gain or loss, so is each man's product to the share of the gain and loss.

Ex. 1. Two persons lay out 1500*l.* in trade, in the proportion of 3 to 2: that is, A put in 900*l.*, and B 600*l.*; A leaves his money in the concern, 9 months, and B does not want his for 12 months: what profits belong to each, supposing they gain 250*l.*?

*L.*900 × 9 = 8100
600 × 12 = 7200

15300

15300 : 250 :: 8100
250

153.00)2025000(132*l.* 7*s.*

Ans. A's share of profit *L.*132 7 0
B's - - - - 117 13 0

*L.*250 0 0

Ex. 2. A puts into a concern 2080*l.* for 2 months, B 970*l.* for 5 months, and C 400*l.* for 15 months; they gain among them 650*l.*; what must each receive for his share of profit?

Ans. 180*l.* 3*s.* A's profit nealy.; 210*l.* B's profit.; 259*l.* 17*s.* C's profit?

Ex. 3. Three merchants join in company for 18 months: D put in 500*l.* and at 5 months' end took out 200*l.*; at 10 months' end put in 500*l.*, and at the end of 14 months takes out 130*l.*; E puts in 400*l.*, and at the end of 3 months 270*l.* more; at 9 months he takes out 140*l.*, but puts in 100*l.* at the end of 12 months, and withdraws 99*l.*, at the end of 15 months. F put in 900*l.*, and at 6 months took out 200*l.*; at the end of 11 months puts in 600*l.*, but takes out that and 100*l.* more at the end of 13 months. They gained 200*l.* I desire to know each man's share of the gain?

Ans. 57*l.* nearly D's gain; 59*l.* 7*s.* 5*d.* E's gain; 83*l.* 12*s.* 7*d.* = F's gain.

ALLIGATION

Teaches to mix things of different values, so as to ascertain the price of the mixture. There are two cases in this rule.

I. To find the mean value of a mixture composed of several quantities of different values.

RULE. Multiply each quantity by its respective value, and divide the sum of the products by the sum of the quantities.

Ex. 1. A tea-dealer mixes $3\frac{1}{2}$ cwt. of tea, at 9*s.* per lb., with 2 cwt., at 7*s.* and $4\frac{1}{4}$ cwt. at 5*s.* 6*d.*, at how much per lb. can he sell the whole mixture?

$3\frac{1}{2} \times 112 = 392$
$2 \times 112 = 224$
$4\frac{1}{4} \times 112 = 476$

and

$392 \times 9 = 3528$
$224 \times 7 = 1568$
$476 \times 5\frac{1}{2} = 2618$

1092 | 7714 (7*s.* $0\frac{3}{4}$*d.* $\frac{84}{1092}$

7644

. . 70

Answer - 7*s.* $0\frac{3}{4}$*d.*

Ex. 2. What is a lb. of sugar worth which is compounded of 3 cwt. at 46*s.*; 2 cwt. at 59*s.*; $1\frac{1}{2}$ cwt. at 84*s.*; and 56 lb. at 60*s.*? Answer, $6\frac{1}{4}$*d.* $\frac{176}{784}$.

Ex. 3. What is the average earnings of workmen, 4 of whom earn 10 dollars each per week; 8 earn 9 dollars each; and 12 will get only 6 dolls. 50 cts. each?

Answer, 7 dolls. $91\frac{2}{3}$ cts.

Ex. 4. A tobacconist mixes 80 lb. of tobacco at 20*d.* per lb.; 150 lb. at 2*s.* 3*d.* per lb.; and 40 lb. at 3*s.* 10*d.* per lb.; what will be the value of the mixture per oz.?

Answer, $1\frac{3}{4}$*d.* nearly.

II. To find how much of different things of different values, must be taken, in order to make a mixture of a certain mean value.

RULE (1). Set down the names of the things to be mixed, together with their prices; then, finding the difference between each of these, and the proposed price of the mixture; place these differences in an alternate order, and they will shew the proportion of the ingredients.

Ex. 1. Orange wine, at 9*s.* per gallon, is to be mixed with raisin wine at 6*s.* per gallon; what will be the proportions, so as to sell the mixture at 7*s.* per gallon?

		Proposed price.	
Orange -	9*s.*	7*s.*	1
Raisin -	6*s.*		2

A mixture therefore of these wines in the proportion of one orange to two raisin, will be the answer.

Ex. 2. A spirit at 16 shillings, and another at 12 shillings per gallon, are to be mixed with low wines at 6 and 5 shillings, in order to produce a mixture worth 9 shillings per gallon; what must the quantities of each be?

Spirit, -	16	3	The answer is, 3 gallons at 16*s.*,
Ditto, -	12	4	4 at 12*s.*: 7 at 6*s.*; and 3 at 5*s.*;
Wine, -	6	7	will make a mixture that may be
Ditto, -	5	3	sold for 9 shillings per gallon: for

$$3 \times 16 = 48$$
$$4 \times 12 = 48$$
$$7 \times 6 = 42$$
$$3 \times 5 = 15$$

17 153 and $\frac{153}{17} = 9s.$ Proof.

Ex. 3. A tea-dealer would mix four sorts of tea together, viz. at 4*s.*, 4*s.* 6*d.*, 5*s.* 6*d.*, 6*s.*, and 7*s.* per lb.; in order that he may sell the whole mixture at 5*s.* 6*d.* per lb., what proportion of each will he use?

Ans. $1\frac{1}{2}$ lb. at 4*s.*; $\frac{1}{2}$ lb. at 4*s.* 6*d.* 1 lb. at 6*s.*; and $1\frac{1}{2}$ lb. at 7*s.*; and as much as you please at 5*s.* 6*d.*

Ex. 4. How much coffee at 48 cts., 42 cts., 27 cts. and 24 cts. per lb. will compose a mixture worth 30 cts. per lb.

	lb.	*cts.*		*lb.*	*cts.*
Ans.	6	at 48		$\frac{1}{2}$	at 48
	3	— 42	or	$\frac{1}{4}$	— 42
	12	— 27		1	— 27
	24	— 18		$1\frac{1}{2}$	— 24

OR THUS.

	lb.	*cts.*		*lb.*	*cts.*
Ans.	1	at 48		$\frac{1}{2}$	at 48
	2	— 42		1	— 42
	6	— 27		3	— 27
	4	— 24		2	— 24*

III. When the prices of all the things to be mixed are given, likewise where the quantity of one, and the mean rate are also given, to find the several quantities of the others.

Rule. (1). Take the difference between each price and the mean rate as before. (2). As the difference of that thing, whose quantity is given, is to the rest of the differences severally; so is the quantity given to the several quantities required.

Ex. 1. A rectifier of compounds has 200 gallons of spirit that he can sell for 12*s.* 6*d.* per gallon, but he means to mix it with three other kinds of spirit at 13*s.* 4*d.*, at 15*s.*, and 18*s.* 4*d.*, per gallon, in order that he may sell the whole at 14*s.* 2*d.* per gallon; how much must he use of each?

I reduce the several prices to pence, which stand as follows:

170	150 —	50	
	160)	10	50 : 10 : : 200 : 40
	180)	10	50 : 10 : : 200 : 40
	220 —	20	50 : 20 : : 200 : 80

* Note.—A variety of answers can be obtained to these questions, by linking them different ways, they may also be made infinite by multiplying or dividing any result by one common number.

The answer is ; to 200 gallons, at 12*s.* 6*d.*, must be added 40 at 13*s.* 4*d.*, 40 at 15*s.*, and 80 at 18*s.* 4*d.* ; the truth of which is proved thus ;

200	at 12	6 =	125	0	0
40	at 13	4 =	26	13	4
40	at 15	0 =	30	0	0
80	at 18	4 =	73	6	8
360			255	0	0 and $\frac{255}{360}$ = 14*s.* 2*d.* Proof.

Ex. 2. A grocer has 100 lb. of tea worth 4*s.* per lb. which he means to mix with others at 12*s.* 3*d.*, 10*s.*, and 6*s.* per lb. ; in order to sell the whole at 8*s.* how much of each must be used ?

Ans. 100 lb. at 4*s.* ; 100 lb. at 12*s.* 3*d.* ; 200 lb. at 10*s.* ; and 212½ lb. at 6*s.*

IV. When the price of each thing is given, also the quantity and the mean rate, to find how much of each sort will make that quantity.

Rule. (1). Take the difference between each price and the mean rate as before : then (2). As the sum of the differences is to each particular difference, so is the quantity given to the quantity required.

Ex. 1. A wine merchant means to mix 860 gallons of wine to sell for 8*s.* a gallon, out of other wines that he already sells for 12*s.*, 9*s.* 6*s.*, and 5*s.* per gallon, how much must he take of each ?

8	12	3	10 : 3 :: 860 :	258
	9	2	10 : 2 :: 860 :	172
	6	1	10 : 1 :: 860 :	86
	5	4	10 : 4 :: 860 :	344
Sum of differences =		10		860

The answer is 258 gallons at 12*s.* ; 172 at 9*s.* ; 86 at 6*s.* ; and 344 at 5*s.* per gallon, may be mixed and sold at 8*s.* per gallon.

Ex. 2. A goldsmith has four sorts of gold, viz. of 24, 10, 18, and 15 carats fine, wishes 125 oz. of the fineness of 17 carats, how much will he want of each sort?

Ans. 14 oz. 16 dwt. $11\frac{5}{17}$ gr. of 24.; 7 oz. 8 dwt. $5\frac{1}{17}$ gr. of 10.; 51 oz. 17 dwt. $15\frac{9}{17}$ gr. of 18.; 51 oz. 17 dwt. $15\frac{9}{17}$ gr. of 15.?

Ex. 3. A drug grinder has bark worth 16s. per lb., some at 10s., and some at 4s.; but he is desirous of making up two parcels, viz. one containing a cwt. at 9s., and the other 84 lb. at 12s.; what proportions of each must be used?

Ans. $43\frac{1}{13}$ lb. at 16s.; $8\frac{8}{13}$ lb. at 10s.; $60\frac{4}{13}$ lb. at 4s. for 112 lb.; 48 lb.; 12 lb.; 24 lb. for 84 lb.?

POSITION.

Position, or as it is sometimes called, the Rule of False, is a rule, that by means of any supposed numbers, others that are true, and that answer to the terms of the question, are found. There are two kinds of Position, viz. Single and Double.

Single Position is performed, by using a supposed number, and working with it as the true one, till the real number is found.

Rule. Take any number and perform the work with it, as if it were the right number: then say, As the result of this work is to the position, so is the result in the question to the number required.

Ex. 1. A person counting some guineas, being asked how many he had, replied: "If you had as many, and as many more, and half as many, and one quarter as many, you would have 264." How many had the person who was counting his gold?

By way of supposition, I take 80 as the number; then, by the terms of the question, it will be

	80		96
As many more,	80	220 : 264 : : 80	96
Half as many,	40	80	48
$\frac{1}{4}$th. as many,	20	———	24
	220	220)21120(96Ans.	264 Proof.

Ex. 2. A person after spending $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{6}$, of his money, finds he had 500*l.* left, what was his original property?

I take a number divisible by 2, 4, and 6, for the supposition, viz. 60.

Suppose	60	60 — 55 = 5, therefore	Proof.
	—	As 5 : 60 : : 500	$\frac{1}{2}$ = 3000
$\frac{1}{2}$	30	60	$\frac{1}{5}$ = 1500
$\frac{1}{4}$	15	———	$\frac{1}{6}$ = 1000
$\frac{1}{6}$	10	5)30.000	———
	55	Answer, *L*.6.000	5500
			500 rem.

Ex. 3. Three persons bought goods at Baltimore, which cost 600 dollars. The first person was to have a third part more than the second, and the third a fourth part more than the first; what was each man's share?

Ans. $200 first person's share, $150 second share, $250 third share.

Ex. 4. In a leaky vessel there were three pumps of different capacities; the first would empty the hold of the ship in 20 minutes, the second would require double that time, and the third would not perform the business in less than an hour; how long would all three together take in doing it? Answer, 11 minutes nearly.

DOUBLE POSITION.

QUESTIONS in this rule are resolved by making suppositions of two numbers, which *may* both prove false; in that case the errors are made to correct each other.

RULE. (1.) Place each error against its respective position, and multiply them cross ways. (2.) If the errors are alike, that is, both greater or both less than the given number, take their difference for a divisor, and the difference of their products for a dividend. But if unlike, take their sum for a divisor, and the sum of their products for a dividend, the quotient will be the answer.

EX. 1. Three persons have obtained the 20,000*l.* prize in the lottery, and it is to be so divided, that the second is to have 600*l.* more than the first, and the third 800*l.* more than the second, what is each person's share?

Suppose the first had	5000	Suppose the first had	5600
Then the second had	5600	The second had	6200
and the third had	6400	The third had	7000
	17000 too little by	3000	18800 too little by 1200.

3000 5000 } that is, { 3000 × 5600 = 16800000
× 1200 5600 } { 1200 × 5000 = 6000000

Difference of Products - 10800000 = dividend.

3000 — 1200 = 1800 (diff. of errors) for a divisor.

Therefore, $\frac{10.800.000}{1800} = L.6000$

5000
6600
7400
L. 20.000 Proof.

Ex. 1. A gentleman at Christmas, wished to give several poor families 5 shillings each, but he found he had 16*s.* 8*d.* too little; he then gave them 3*s.* 6*d.* each, and found he had 4*s.* 4*d.* left, how many families were there? Answer, 14 families.

Ex. 3. A person purchased a house and land, together with a carriage and horses, for 15 000 dollars; he paid 4 times the price of the carriage and horses for the land, and 5 times the price of the land for the house, what was the value of each separately?

Ans. $600 carriage and horses, $2,400 land, $12,000 house.

COMPOUND INTEREST AND ANNUITIES.

Compound Interest, or interest upon interest, is that which is paid not only for the use of the money lent, but also for the use of the interest as it becomes due.

There are two methods of working Problems in this Rule, viz. by Common Arithmetic; and by Decimals; I shall give examples under each.

1. *By Common Arithmetic.*

Rule. 1. Find the amount of the given principal for the time of the first payment by simple interest. (2.) Consider this amount as the principal for the second payment, the amount of which is to be calculated as before, and so on through all the payments to the last, still reckoning the last amount as the principal for the next payment.

Ex. 1. What is the amount of 550*l.* for three years, at 5 per cent. compound interest?

20)550	0	0	given principal.
27	10	0	first year's interest.
20)577	10	0	second year's principal.
28	17	6	second pear's interest.
20)606	7	6	third year's principal
30	6	$4\frac{1}{2}$	third year's interest.
Answer -	636	13	$10\frac{1}{2}$

Ex. 2. What is the amount of 400*l.* for four years, at 5 per cent. compound interest? Ans. 486*l.* 4*s.* $0\frac{1}{2}$*d.*

Ex. 3. What is the compound interest of 600 dols. for 5 years at 5 per cent per annum? Ans. $165.7689375

II. By Decimals.

Rule. 1. Find the amount of 1*l.* for a year, at the given rate per cent. 2. Involve the amount thus found, to such a power as is denoted by the number of years. 3. Multiply this power by the principal or given sum, and the product will be the amount required. 4. Subtract the principal from the amount, and the remainder will be the interest.

Ex. 1. What is the compound interest of 550*l.* for 3 years, at 5 per cent. per annum?

1.05 = amount of 1*l.* for a year, at 5 per cent.;
Then $1.05 \times 1.05 \times 1.05 = 1.157625$, and
$1.157625 \times 550 = 636.69375$ = amount,
$636.69375 - 550 = 86.69375 = 86l.\ 13s.\ 10\frac{1}{2}d.$

Ex. 2. What is the amount of 100 dols. for 4 years, at 6 per cent. per annum, compound interest?

Answer, $126.247696

Ex. 3. What is the compound interest of 620*l.* for 5 years, at 5 per cent.? Answer, 171*l.* 5*s.* 10*d.*

A TABLE.

Shewing the Sum to which 1*l.* or $1 Principal will increase at 5 per cent. Compound Interest, in any number of years not exceeding a hundred.

Yrs	Amount.	Yrs.	Amount.	Yrs.	Amount.	Yrs.	Amount.
1	1.05	26	3.555672	51	12.040769	76	40.774320
2	1.1025	27	3.733456	52	12.642808	77	42.813036
3	1.157625	28	3.920129	53	13.274948	78	44.953688
4	1.215506	29	4.116135	54	13.938696	79	47.201372
5	1.276281	30	4.321942	55	14.635630	80	49.561441
6	1.340095	31	4.538039	56	15.367412	81	52.039513
7	1.407100	32	4.764941	57	16.135783	82	54.641488
8	1.477455	33	5.003188	58	16.942572	83	57.373563
9	1.551328	34	5.253347	59	17.789700	84	60.242241
10	1.628894	35	5.516015	60	18.679185	85	63.254353
11	1.710339	36	5.791816	61	19.613145	86	66.417071
12	1.795856	37	6.081406	62	20.593802	87	69.737924
13	1.885649	38	6.385477	63	21.623492	88	73.224820
14	1.979931	39	6.704751	64	22.704667	89	76.886061
15	2.078928	40	7.039988	65	23.839900	90	80.730365
16	2.182874	41	7.391988	66	25.031895	91	84.766883
17	2.292018	42	7.761587	67	26.283190	92	89.005227
18	2.406619	43	8.149666	68	27.597664	93	93.455488
19	2.526950	44	8.557150	69	28.977548	94	98.128268
20	2.623297	45	8.985007	70	30.426425	95	103.034676
21	2.785962	46	9.434258	71	31.947746	96	108.186410
22	2.925260	47	9.905971	72	33.545134	97	113.595730
23	3.071523	48	10.401269	73	35.222390	98	119.275517
24	3.225099	49	10.921333	74	36.983510	99	125.239293
25	3.386354	50	11.467399	75	38.832685	100	131.501257

1. To find by means of the table what any sum will amount to in a given number of years.

RULE. Multiply the number in the table, opposite to the term of years, by the sum, and the product will be the answer.

EX. 1. To what sum will 500*l.* amount to in 44 years, at 5 per cent. compound interest?

Opposite to 44 in the table I find 8.557150, this I multiply by 500, and the answer is 4278*l.* 11*s.* 6*d.*

Ex. 2. What will 350*l.* amount to in 25 years, at 5 per cent. compound interest?

Answer, 1185*l.* 4*s.* $5\frac{3}{4}$*d.* nearly.

Ex. 3. A prudent young man marries at the age of 22; the fortune which he has with his wife is 2500*l.*, half of which he readily gives into the hands of trustees to be accumulated at 5 per cent. compound interest; what will it amount to, supposing he lives 32 years, which he may reasonably expect?

Answer, 5956*l.* 3*s.* $6\frac{1}{4}$*d.*

Ex. 4. The year 1808, is that in which the late Mr. Pitt calculated there would be four millions surplus to be applied to the payment of the national debt of England: I demand how much this single four millions will accumulate in half a century, at 5 per cent. compound interest? Answer, 45,869,596*l.*

(*See other questions on this subject after the next table.*)

II. To find the number of years in which a given sum will increase to another given sum, in consequence of being improved at Compound Interest.

Rule. Divide the latter sum by the former, and the sum in the table which is nearest to the quotient will shew the terms required.

Ex. 1. In what time will 200*l.* increase to 1500*l.*, if improved at 5 per cent. compound interest?

$\frac{1500}{200} = 7.5$. The nearest number in table I. to 7.5 is 7.391988, opposite to which is 41, the number of years. Of course 200*l.* in a little more than forty-one years would, by being accumulated at compound interest, at 5 per cent., amount to 1500*l.*

Ex. 2. In what time will 100*l.* increase to 500*l.* same rate of interest? Ans. 33 years

Ex. 3. In what time will 860*l.* increase to 1[illegible]

Ans. between 50 an[illegible]

Ex. 4. In how long would five millions be in paying the national debt, which in January, 1806, was upwards of 580 millions? Ans. between 97 and 98 years.

Ex. 5. Admiral Rainier left, in 1808, 25,000*l.* towards paying off the national debt, when will it have accumulated to a million at 5 per cent. compound interest? Ans. 76 years, nearly.

TABLE II.

Shewing the sum to which 1*l.* *per annum* will increase at 5 per cent. Compound Interest, in any number of years not exceeding a hundred.

Yrs.	Amount.	Yrs	Amount	Yrs.	Amount.	Yrs.	Amount.
1	1,0000	26	51,1135	51	220,8154	76	795,4864
2	2,0500	27	54,6691	52	232,8562	77	856,2607
3	3,1525	28	58,4026	53	245,4990	78	879,0738
4	4,3101	29	62,3227	54	258,7739	79	924,0274
5	5,5256	30	66,4388	55	272,7126	80	971,2288
6	6,8019	31	70,7608	56	287,318	81	1020,7903
7	8,1420	32	75,2988	57	302,7157	82	1072,8298
8	9,5491	33	80,0638	58	318,8514	83	1127,4713
9	11,0 66	34	85,0670	59	335,7940	84	1184,8448
10	12,5779	35	90,3203	60	353,5837	85	1245,0871
11	14,2068	36	95,8363	61	372,2629	86	1308,3414
12	15,9171	37	101 6281	62	391,8760	87	1374,7585
13	17,7130	38	107,7095	63	412,4698	88	1444,4964
14	19,5986	39	114,0950	64	434,0933	89	1517,7212
15	21,5786	40	120,7998	65	456,7980	90	1594,6073
16	23,6575	41	127.8398	66	480,6379	91	1675,3377
17	25,8404	42	135,2317	67	505,6698	92	1760,1045
18	28,1328	43	142,9933	68	531 9583	93	1849.1098
19	30,5390	44	151,1430	69	559,5510	94	1942,5653
20	33,0659	45	159,7002	70	588,5285	95	2040,6935
21	35,7192	46	168.6852	71	618,9549	96	2145,7282
22	38,5052	47	178,1194	72	650,9027	97	2251,9146
23	41,4305	48	188,0254	73	684,4478	98	2365,6103
24	44,5020	49	198,4267	74	719,6702	99	2484,7859
25	4 ,7271	50	209,348	75	756,6537	100	2610,0252

I. To find in what time a given annuity will amount to a given sum at compound niterest.

RULE. Divide the given sum by the given annuity, and the number in the table nearest to the quotient will be the answer.

Ex. 1. A person owes 1000*l.* and resolves to appropriate 20*l.* per annum, to be accumulated at 5 per cent. per ann. compound interest, in how many years will the debt be paid ?

$\frac{1000}{20} = 50$. The nearest number in table II. preceding page, to 50 found, is 51.1135, and the number answering to this is 26, so that in less than 26 years a debt of 1000*l.* would be extinguished by laying by, and accumulating, at compound interest, annually 20*l.* per annum. If the rate of interest had been 6 per cent. 24 years would have paid the debt, but at 4 per cent. it would have taken between 28 and 29 years.

Ex. 2. How long will 75 guineas a year be in accumulating to 2000*l.* at the same rate ?

Ans. in somewhat less than 17 years.

Ex. 3. In what time will an annuity of 25*l.* amount to 3375*l.*, at the same rate ?

Ans. in little more than 43 years.

Ex. 4. How long will the national debt, left at the time of Mr. Pitt's death, viz. 581 millions, be in paying off, supposing five millions annually be appropriated for that purpose, and the rate of compound interest 5 per cent. ? Ans. in less than 40 years.

Ex. 5. The national debt was, at Midsummer 1807, 756 millions of pounds, out of which the commissioners had redeemed 117 millions and a half, how long would the remainder take in paying off, if eight millions be applied annually, at the rate of 5 per cent. compound interest for the purpose ? Ans. 33 years.

II. To find how much a given annuity will amount to in a given term, at 5 per cent. compound interest.

RULE. Multiply the given annuity by the number in the table standing opposite to the given term of years.

Ex. 1. I can lay by 50*l.* per annum with its interest; that is, I can appropriate 50*l.* a year to be accumulated at 5 per cent. compound interest, how much shall I have saved if I live 21 years?

Opposite to 21 years I find 35.7192, which multiplied by 50, gives 1785.9600. Answer, 1785*l.* 19*s.* 2*d.*

Ex. 2. How much will an annuity of 35*l.* amount to in 83 years? Answer, 39461*l.* 9*s.* $10\frac{3}{4}$*d.*

Ex. 3. To what sum will an annuity of 100 guineas amount in 19 years, at 5 per cent. compound interest?
Answer, 3206*l.* 12*s.*

Ex. 4. To what sum will 60 dollars per annum amount to in 25 years, at 5 per cent. compound interest?
Answer, $2863, 62 cts. 6 mls.

III. The PRESENT VALUE of an annuity is that sum which, if improved at compound interest, would be sufficient to pay the annuity.—For this the following table is adapted.

TABLE III.

Shewing the present Value of an Annuity of 1*l.* for any number of Years not exceeding 100, at 5 per cent. per annum, Compound Interest.

Yrs	Value.	Yrs.	Value.	Yrs.	Value.	Yrs.	Value.
1	,952381	26	14,375185	51	18,338977	76	19,509495
2	1,859410	27	14,643034	52	18,418073	77	19,532853
3	2,723248	28	14,898127	53	18,493403	78	19,555098
4	3,545950	29	15,141074	54	18,565146	79	19,576284
5	4,329477	30	15,372451	55	18,633472	80	19,596460
6	5,075692	31	15,592810	56	18,698545	81	19,615677
7	5,786373	32	15,802677	57	18,760519	82	19,638978
8	6,463213	33	16,002549	58	18,819542	83	19,651407
9	7,107822	34	16,192904	59	18,875754	84	19,668007
10	7,721735	35	16,374194	60	18,929290	85	19,683816
11	8,306414	36	16,546852	61	18,980276	86	19,698873
12	8,863252	37	16,711287	62	19,028834	87	19,713212
13	9,393573	38	16,867893	63	19 075080	88	19,726869
14	9,898641	39	17,017041	64	19,119124	89	19,739875
15	10,379658	40	17,159086	65	19,161070	90	19,752262
16	10,837770	41	17,294368	66	19,201019	91	19,764059
17	11,274066	42	17,423208	67	19 239066	92	19,775294
18	11,689587	43	17,545912	68	19,275301	93	19,785994
19	12,085321	44	17,662773	69	19,309810	94	19,796185
20	12,462210	45	17,774070	70	19,342677	95	19,805891
21	12,821153	46	17,880066	71	19,373978	96	19,815134
22	13,163003	47	17,981016	72	19,403788	97	19,823937
23	13,488574	48	18,077158	73	19,432179	98	19,832321
24	13,798643	49	18,168722	74	19,459218	99	19,840306
25	14 093945	50	18,255925	75	19,484 70	100	19,847910

To find the present value of an annuity for a term of years.

Rule. Multiply the number in the table opposite to the given term of years, by the sum, and the product is the answer.

Ex. 1. What is the present value of an annuity of 126*l.* for 21 years?

In the table opposite 21 is 12.821153; this multiplied by 126, gives 1615.465278 = 1615*l.* 9*s.* 3*d.*

Ex. 2. What is the present value of an annuity of 75 dollars for 12 years, at 5 per cent. ?

Answer, $664 : 74.39.

Ex. 3. What present sum is equivalent to a nett rent of 45*l.* per annum for 84 years, allowing interest of money at 5 per cent. ? Answer, 885*l.* nearly.

CHANCES.*

Question I.—Suppose a counter, having a black and a white face, be thrown up, to see which will be uppermost, after the counter has fallen to the ground, and if the white face appear uppermost, a person is to have 5 shillings, what is the chance, or probability, that he will be entitled to the five shillings ?

Solution. Since either the black or the white face must be uppermost, there is an equal chance for the appearance of either face, of course the chance, or the probability, may be expressed by $\frac{1}{2}$, or a bystander ought to give him 2*s.* 6*d.* for his chance of getting the five shillings.

Question II.—Suppose there are three counters put into a bag, one red, another white, and a third black; out of which, if a person blindfolded take the red he is to have 5 shillings, I demand the value of the chance, or what is the probability of his drawing the red counter ?

* It is meant only to give so much of the doctrine of chances, as shall enable the pupil to understand upon what ground the doctrine of Annuities, &c. depends. To illustrate this part of the subject, recourse will be had to some familiar instances, which may seem, at first sight, to lead to gaming; but it is believed, that the facts adduced must, if properly considered, deter young persons from this pernicious and destructive vice, which is too much encouraged by the almost perpetual drawing of state lotteries.

Solution. He has evidently one chance out of three, and therefore the probability may be valued at $\frac{1}{3}$, and another person inclining to purchase his chance, ought to give for it the $\frac{1}{3}$d of 5 shillings, or 1*s.* 8*d.*

In the former case, the chances for the event's happening and failing are equal, and each being equal to $\frac{1}{2}$, the certainty is reckoned as 1, or unity.

In this last case, there is one chance for the event's happening, and two for its failing: in other words the chance for its happening is $\frac{1}{3}$, and for its failing $\frac{2}{3}$: here, again, the chances for the happening and failing are equal to unity, because $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$.

Question III.—Suppose there are five counters, two white and three black, out of which, when mixed, a person blindfolded is to draw one of the white, and in that case is to be entitled to 5*s.*, what is his chance for so doing, and what is his expectation worth?

Solution. It is plain here are five chances in the whole, of which there are two only out of five for taking a white counter, and the other three for taking a black one; therefore the probability of winning may be expressed by the fraction $\frac{2}{5}$, and of missing $\frac{3}{5}$, and he might sell his expectation of the five shillings for $\frac{2}{5}$ths of that sum, that is, for two shillings.

Ex. 1. At the conclusion of the last state lottery, when there were only five tickets left in the wheel, there were two prizes of 50*l.* each, and three blanks, what was the value of one of those tickets?

Answer, 20*l.*

Ex. 2. What is the value of one ticket when only five are left in one wheel, and in the other there is one prize of 100*l.* and four blanks?

Answer, 20*l.*

Ex. 3. What chance has the holder of a single lottery ticket of a prize, when there are three blanks to a prize?

Answer, 4 to 1.

Question IV.—What is the probability of throwing an ace with a single die, in one trial ?

Solution. There are six faces to a die, of which one only is the ace, therefore the probability of throwing an ace with a single die in one trial is expressed by $\frac{1}{6}$; and the probability of not throwing an ace is $\frac{5}{6}$: here, as before, the chances for not throwing the ace, and that for throwing, are together equal to unity.

Question V.—What is the probability of throwing an ace in four throws ?

Solution. We must consider the probability of failing in the four throws. The probability of missing the first time will be $\frac{5}{6}$; so it is the second, third, and fourth times; therefore the probability of missing in all four throws will be $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{625}{1296}$; which subtracted from unity or 1, gives $\frac{1296-625}{1296} = \frac{671}{1296}$, which is the probability of throwing it once or oftener in four turns; therefore the odds of throwing an ace in four times, is as 671 to 625, or rather more than an even chance.

The probability in three throws will be,

$$1 - \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = 1 - \frac{125}{216} = \frac{216-125}{216} = \frac{91}{216}$$

Here the odds is against throwing the ace in three throws, as 91 is less than 125.

Question VI. In two heaps of cards, one containing the 13 diamonds, the other the 13 spades, placed promiscuously, what is the probability that, taking one card at a venture, out of each heap, I shall take out the two aces ?

Solution. The probability of taking the ace out of the first heap is $\frac{1}{13}$; the probability of taking the ace out of

the second heap is also $\frac{1}{13}$, therefore the probability of taking out both aces is $\frac{1}{13} \times \frac{1}{13}$, or $\frac{1}{169}$, which subtracted from 1, gives $\frac{168}{169}$, of course the chances against me are as 168 to 1: in other words, I may *expect* to do this once in 169 attempts.

On similar principles the *expectation of life* is found. It is known by accurate observation, that of 46 persons aged 40 years, one will die every year, till they are all dead in 46 years; therefore half 46, or 23 years, will be the expectation of life of a person 40 years of age. That is, the number of years enjoyed by them all, will be just the same as if every one of them had lived 23 years, and then died. The same reasoning applies to all other ages, which leads us to a more particular consideration of the subject.

EXPECTATION OF LIFE.

From the Bills of Mortality in different places, tables have been constructed which shew how many persons, upon an average, out of a certain number born, are left at the end of each year to the extremity of life. From such tables, which, as we have seen, are founded on the doctrine of Chances, the probability of the continuance of a life, of any proposed age is known.

TABLE I.

Shewing the Probabilities of the Duration of Human Life, deduced from the Register of Mortality at Northampton.

Age	Persons living.	Dec. of Life	Age	Persons living.	Dec. of Life.	Age.	Persons living	Dec. of Life.
0	11650	3.00	33	4180	75	66	1552	80
1	8650	1367	34	4085	75	67	1472	80
2	7283	502	35	4010	75	68	1392	80
3	6781	335	36	3935	75	69	1312	80
4	6446	197	37	3860	75	70	1232	80
5	6249	184	38	3785	75	71	1152	80
6	6065	140	39	3710	75	72	1072	80
7	5925	110	40	3685	76	73	992	80
8	5815	80	41	3559	77	74	912	80
9	5735	60	42	3482	78	75	832	80
10	5675	52	43	3404	78	76	752	77
11	5623	50	44	3326	78	77	675	73
12	5573	50	45	3248	78	78	602	68
13	5523	50	46	3170	78	79	534	65
14	5473	50	47	3092	78	80	469	63
15	5423	50	48	3014	78	81	406	60
16	5373	53	49	2936	79	82	346	57
17	5320	58	50	2857	81	83	289	55
18	5262	63	51	2776	82	84	234	48
19	5199	67	52	2694	82	85	186	41
20	5132	72	53	2612	82	86	145	34
21	5060	75	54	2530	82	87	111	28
22	4985	75	55	2448	82	88	83	21
23	4910	75	56	2366	82	89	62	16
24	4835	75	57	2284	82	90	46	12
25	4760	75	58	2202	82	91	34	10
26	4685	75	59	2120	82	92	24	8
27	4610	75	60	2038	82	93	16	7
28	4535	75	61	1956	82	94	9	5
29	4460	75	62	1874	81	95	4	3
30	4385	75	63	1793	81	96	1	1
31	4310	75	64	1712	80			
32	4235	75	65	1632	80			

CASE I. To find, by this Table, the expectation of any single life.

RULE. Divide the sum of all the living in the table, at the age whose expectation is required, and at all great-

er ages, by the sum of all that die annually at that age, and above it, or, which is the same thing, by the number in the table of the living at that age, and half unity, or .5 subtracted from the quotient will be the expectation required.

Ex. 1. What is the expectation of a life at 60?

The sum of the living at the age of 60 and upwards, by the table, is 27947, which divided by 2038, the number of living at that age, gives 13.71, from which subtract .5, and the expectation of a life at 60 is equal to 13.21, or 13 years 11 weeks nearly.

Ex. 2. What is the expectatin of a life 70 years of age, one of 80, and one of 90?

Ans. life of 70 is 8 years 31 weeks—life of 80 is $4\frac{3}{4}$ years—life of 90 is 2 years, 20 weeks, and 5 days.

Case II. To find the probability that a given life shall continue any number of years, or attain a given age.

Rule. Make the number in the table, opposite to the proposed age, the numerator of the fraction, and for the denominator take the number opposite the present age.

Ex. 1. What is the probability that I, who am 45, shall live to 60?

The number against 60 = 2038 } Therefore the chances in
The number against 45 = 3248 } my favour are 20 : 12
nearly, or as 5 : 3.

For, since the probability of living is equal to $\frac{2038}{3248}$, the chance of dying during that period is

$$1 - \frac{2038}{3248} = \frac{3248 - 2038}{3248} = \frac{1210}{3248}.$$

The denominators being the same, the chance of life is to the probability of dying as 2038 to 1210, or as 20 to 12, or as 5 to 3 nearly.

Ex. 2. What is the probability that a person aged 21, as lattain to 54 ?

Ans. $\frac{2530}{5060}$ chance of living.

Ex. 3. What is the probability that a person aged 15 should live till 70 ?

Ans. $\frac{1232}{5423}$ chance of living.

Ex. 4. What chance has a person aged 70 of living 10 years longer ?

Ans. $\frac{469}{1232}$ chance of living.

From the foregoing table is formed

TABLE II.

Shewing the expectation of Human Life at every Age according to the Probabilities found by Table I.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
0	25,18	25	30,85	50	17,99	75	6,54
1	32,74	26	30,33	51	17,50	76	6,18
2	37.79	27	29,82	52	17,02	77	5,83
3	39,55	28	29,30	53	16,54	78	5,48
4	40,38	29	28,79	54	16,06	79	5,11
5	40,84	30	28 27	55	15,58	80	4,75
6	42,07	31	27,76	56	15,10	81	4,41
7	41,08	32	27,24	57	14,63	82	4,09
8	40,79	33	26,72	58	14,15	83	3,80
9	40,36	34	26,20	59	13,68	84	3,58
10	39,78	35	25,68	60	13,21	85	3,37
11	39,14	36	25,16	61	12,75	86	3,19
12	38.49	37	24,64	62	12,28	87	3,01
13	37,83	38	24 12	63	11,81	88	2,86
14	37,17	39	23,60	64	11,35	89	2,66
15	36,51	40	23,08	65	10,88	90	2,41
16	35,85	41	22,56	66	10,42	91	2,09
17	35,20	42	22,04	67	9,96	92	1,75
18	34 58	43	21,54	68	9,50	93	1,37
19	33,99	44	21,03	69	9,05	94	1,05
20	33,43	45	20,52	70	8,60	95	0,75
21	32,90	46	20,02	71	8,17	96	0,50
22	32,39	47	19,51	72	7,74		
23	31,88	48	19,00	73	7,33		
24	31,36	49	18,49	74	6,92		

To find the expectation of any given life.

RULE. Seek in the table the given age, and opposite to it is the expectation.

Thus, the chance of life to an infant just born is 25.18, or rather more than 25 years; to a person of 45 years of age 20,52, as we have found before, and to a person of 69, just 9 years.

Upon these tables is founded the doctrine of

LIFE ANNUITIES.

LIFE ANNUITIES are annual payments to continue during any life or lives. These are generally purchased or sold for a present sum of money.

"The present value of a life annuity" is the sum that would be sufficient, (allowing for the chance of life failing, which has been considered in the preceding pages) to pay the annuity without loss.

If money bore no interest, the value of an annuity of 1*l* would be equal to the expectation of life. Thus, Table II. the value of an annuity for a life of 20 years of age, if money bore no interest would be equal to nearly 33 years and a half purchase; that is 33*l*. 10*s*. in hand for each life, would be sufficient to pay to any number of such lives 1*l*. per annum.

If money is capable of being improved by being put out to interest, the sum just mentioned would be more than the value, because it would be more than sufficient to pay the annuity; and it will be as much more than sufficient as the interest is greater. As an example.

If money can be improved at 5 per cent. compound interest, the half of 33*l*. 10*s*., or 16*l* 15*s*., will, as we have seen, in little more than 14 years, produce the 33*l*. 10*s*. required.

It must not however be supposed, that 16*l*. 15*s*. is the true value of an annuity of 1*l*. during a life of 20. The

value of an annuity certain for a term equal to the expectation, always exceeds the true value, because, in a number of life annuities, many of the payments would not be to be made till a much more remote period than the term equal to the expectation.

Upon this principle the following table is computed, from which it appears that the present value of an annuity of 1*l.* on a life of 20 years of age, is equal to 14*l.* and a small fraction only; that is, 14*l.* in hand for each life, improved at compound interest, will be sufficient to pay to any number of such lives 1*l.* per annum.

TABLE I.

Shewing the Value of an Annuity of 1*l*, on a Single Life, at every Age, according to the probabilities of the Duration of Human Life at Northampton, reckoning interest at 5 per cent.

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
Birth.	8 863	25	13.567	50	10,269	75	4,744
1 year	11.563	26	13.473	51	10,097	76	4,511
2	13.420	27	13.377	52	9,925	77	4,277
3	14.135	28	13.278	53	9,748	78	4,035
4	14.613	29	13.177	54	9,567	79	3,776
5	14 827	30	13.072	55	9,382	80	3,515
6	15.041	31	12.965	56	9,193	81	3,263
7	15.166	32	12.854	57	8,999	82	3,020
8	15.226	33	12 740	58	8,801	83	2,797
9	15.210	34	12.623	59	8,599	84	2,627
10	15.139	35	12.502	60	8,392	85	2.471
11	15.043	36	12.377	61	8,181	86	2,328
12	14.937	37	12.249	62	7,966	87	2,193
13	14.826	38	12.116	63	7,742	88	2,080
14	14.710	39	11.979	64	7,514	89	1,924
15	14.588	40	11,837	65	7,276	90	1,723
16	14.460	41	11,695	66	7,034	91	1,447
17	14.334	42	11,551	67	6,787	92	1,153
18	14.217	43	11,407	68	6,536	93	0,816
19	14.108	44	11,238	69	6,281	94	0,524
20	14.007	45	11,105	70	6,023	95	0,238
21	13 917	46	10,947	71	5,764	96	0,000
22	13.833	47	10,784	72	5,504		
23	13.746	48	10,616	73	5,245		
24	13.658	49	10,443	74	4,990		

To find the value of an annuity for a person of any given age.

RULE. Multiply the number in the table against the given age, by the sum, and the product is the answer.

Ex. 1. What should a person, aged 45, give to purchase an annuity of 60*l.* per annum during life, interest being reckoned 5 per cent ?

The value in the table against 45 years is 11.105, and this multiplied by 60 gives the answer, 666*l.* 6*s.*

Ex. 2. A person aged 69 years would purchase an annuity of 200*l.* for life, what must he pay for it in ready money at the same rate of interest ?

Answer, 1256*l.* 4*s.*

Ex. 3. A merchant marries a lady aged 28, whose fortune for life is 300*l.* per annum, being desirous of converting the same into money, what ought he to have for it, allowing interest 5 per cent. ? Answer, 3983*l.* 8*s.*

Ex. 4. What is the value of an annuity of 200 dollars during the life of a person aged 25 years ?

Answer, $2713 40 cts.

Ex. 5. What is the value of 50*l.* per annum, payable during the life of a person aged 41 years ?

Answer, 584*l.* 15*s.*

Ex. 6. What is the value of a clear annuity of 75*l.* during the life of an old man aged 76 ?

Answer, 338*l.* 6*s.* 6*d.*

Ex. 7. What is the value of a landed estate during the life of a person aged 38, produing nett 30*l.* 9*s.* per annum ? Answer, 368*l.* 18*s.* 7½*d.*

Ex. 8. What is the life interest of a person aged 53, in 1250*l.* 3 per cent. Consols worth ?

Answer, 365*l.* 11*s.*

Ex. 9. A gentleman aged 60, who receives an annuity of 150*l.* per annum, for life, out of a freehold estate,

wishes to exchange his life for that of his wife, aged 32: what ought to be required of him for so doing?

Answer, 669*l*, 6*s*.

Ex. 10. A person having an annuity of 100*l*. during a life of 37 years, agrees to exchange it for an equivalent annuity during a life of 45; what annuity should be granted him? Answer, 110*l*. 6*s*.

Ex. 11. What annuity will 100*l*. purchase during the life of a person aged 28? Answer, 7*l*. 10*s*. 7*d*.

Ex. 12. A parish means to raise a sum of money for building a workhouse, by life annuities; at what ages should they grant 7, 8 and 9 per cent.?*

Ans. To persons of 18, 35 and 45 years of age.

Ex. 13. What is the difference in value between an annuity of 40*l*. during a lifeof 36, and an annuity certain for 20 years?†

Answer, 3*l*. 8*s*. 2*d*. nearly.

Ex. 14. What annuity should be granted to a person aged 57 during his life, for 2,000*l*. five per cent. stock, which is now at $99\frac{5}{8}$? Answer, 221*l*. 8*s*.

NOTES.

* Questions of this sort are answered by dividing 100*l*. by the rates per cent., and opposite to the numbers in the table that are nearest the quotient, are the required ages: thus, to find at what age a life annuity of 9 per cent. should be granted, $\frac{100}{9} = 11.111$: the nearest number in the table is 11.105, by the side of which is 45, hence, to ages of 45, an annuity of 9 per cent. may be granted.

TABLE II.

Shewing the Value of an Annuity during the joint continuance of Two Lives, according to the probabilities of Life at Northampton, reckoning interest at 5 per cent.

Ages.	Value.	Ages.	Value.	Ages.	Value.	Ages.	Value.
5-5	11,984	15-35	10,655	30-30	10,255	45-70	5,195
5-10	12,315	15-40	10,205	30-35	9,954	45-75	4,206
5-15	11,954	15-45	9,690	30-40	9,576	45-80	3,197
5-20	11,561	15-50	9,076	30-45	9,135	50-50	7,522
5-25	11,281	15-55	8,403	30-50	8,596	50-55	7,098
5-30	10,959	15-60	7,622	30-55	7,999	50-60	6,568
5-35	10,572	15-65	6,705	30-60	7,292	50-65	5,897
5-40	10,102	15-70	5,631	30-65	6,447	50-70	5,054
5-45	9,571	15-75	4,495	30-70	5,442	50-75	4,112
5-50	8,941	15-80	3,372	30-75	4,365	50-80	3,140
5-55	8,256	20-20	11,232	30-80	3,290	55-55	6,735
5-60	7,466	20-25	10,989	35 35	9,680	55-60	6,272
5-65	6,546	20-30	10,707	35-40	9,331	55-65	5,671
5-70	5,472	20-35	10,363	35-45	8,921	55-70	4,893
5-75	4,362	20-40	9,957	35-50	8 415	55-75	4,006
5-80	3,238	20-45	9,448	35-55	7,849	55-80	3,076
10-10	12,665	20-50	8,861	35-60	7,174	60-60	5,888
10-15	12,302	20-55	8,216	35-65	6,360	60-65	5,372
10-20	11,906	20-60	7,463	35-70	5,382	60-70	4,680
10-25	11,627	20-65	6,576	35-75	4,327	60-75	3,866
10-30	11,304	20-70	5,532	35-80	3,268	60-80	2,992
10-35	10,916	20-75	4,424	40-40	9,016	65-65	4,960
10-40	10,442	20-80	3,325	40-45	8,643	65-70	4,378
10-45	9,900	25-25	10,764	40-50	8,171	65-75	3,665
10-50	9,260	25-30	10,499	40-55	7,654	65-80	2,873
10-55	8,560	25-35	10,175	40 60	7,015	70-70	3,930
10-60	7,750	25-40	9,771	40-65	6,240	70-75	3,347
10-65	6,803	25-45	9,301	40-70	5,298	70-80	2,675
10-70	5,700	25-50	8,739	40-75	4,272	75-75	2,917
10-75	4,522	25-55	8,116	40-80	3,236	75-80	2,381
10-80	3,395	25-60	7,383	45-45	8,312	80-80	2 018
15-15	11,960	25 65	6,515	45-50	7,891	85-85	1,256
15-20	11,585	25-70	5,489	45-55	7,311	90-90	0,909
15-25	11,324	25-75	4,396	45-60	6,822		
15-30	11,020	25-80	3,308	45-65	6,094		

CASE I. To find the value of an annuity on the longest of two single lives.

RULE. From the sum of the values of the single lives, subtract the value of their joint continuance, and the remainder will give the value of the longest of the lives.

Ex. 1 What is the value of the longest of two lives aged 10 and 15?

Table I. { The value of a life at - - 10 = 15.139
— — — — - - - 15 = 14.588

29.727

* Table II. The value of the joint continuance of two lives of - - 10 and 15 = 12.302

Value of the longest of the two lives 17.425

Therefore an annuity of 100*l.* a year upon the longest of two lives, one 10 and the other 15, would be worth nearly 17 years and a half purchase, or more accurately, 1742*l.* 10*s.*

Ex. 2. What is the value of an annuity on the longest of two lives whose ages are thirty and forty?

Answer, 1533*l.* 6*s.*

CASE II. To find the value of an annuity on three joint lives.

RULE. Take the value of the two elder, and find the age of a single life equal to that; then find the value of the joint lives of this now found, and the youngest.

Ex. 1. Let the three lives be 20, 30, and 40.

The value of the joint continuance of the two eldest; viz. of 30 and 40 (by Table II.) is equal to 9.576, which answers to a single life (by Table I.) of 54. Now, the value of the joint lives of 20 and 54 by Table II., or the ages which come nearest, viz. 20 and 55, is 8.216* for the value sought: hence an annuity of 40*l.* on three joint lives would be worth about 328*l.* 12*s.*

Ex. 2. To find the value of 3 joint lives of the ages 15, 30, and 45. Answer, 8.403.

NOTE.

* The numbers 9.576 and 8.216, are not quite accurate, because the limits of this book do not admit of a table giving the combinations of all ages.

Ex. 3. What is the value of an annuity of 150*l.* on the joint continuance of three lives of the ages 50, 60, and 70? Answer, 587*l.* 14*s.*

CASE III. To find the value of the longest of any three lives.

RULE. From the sum of the values of all the single lives subtract the sum of the values of all the joint lives, combined two and two. To the remainder add the value of the three joint lives, and the sum will be the value of the longest of the three lives.

Ex. 1. What is the value of the longest of three lives, whose ages are 20, 30, and 40?

Table I.
Value of a life of 20 = 14.007
— — — 30 = 13.072
— — — 40 = 11.837
38.916

Value of two joint lives of 20 and 30 = 10.707
— — — — — — 20 and 40 = 9.937
— — — — — — 30 and 40 = 9.576
38.916
30.220
30.220

8.696+8.216 (the value of the joint lives found in Ex. 1. Case II.) = 16.912 = the value of the longest of the three lives.

Ex. 2. What is the value of the longest of three lives, whose ages are 15, 30, and 45? Ans. 17.126.

Ex. 3. What is the value of an annuity on the longest of three lives, whose ages are 50, 60, and 70? Answer, 12.300.

EXAMPLES FOR PRACTICE.

Ex. 1. What is the present value of an annuity of 50*l.*, on the joint lives of two persons, each 30 years of age? Answer, 512*l.* 15*s.*

Ex. 2. What is the present value of an annuity of 65*l.*, during the joint lives and the life of the survivor, of a man aged 45, and his wife aged 35 ?

Answer, 954*l.* 12*s.* nearly.

Ex. 3. What is the value of a lease producing 27*l.* 13*s.* per annum, on the longest of two lives aged 60 and 45 ? Answer, 350*l.* 9*s.*

Ex. 4. What is the value of an annuity of 40*l.* on two joint lives of 70 and 5 years ? Ans. 218*l.* 17*s.* 7*d.*

Ex. 5. What is the value of an annuity of 50*l.* on the longest of two lives of 70 and 5 years ?

Answer, 768*l.* 18*s.*

CASE IV. To find the value of an annuity on a given life for any number of years.

RULE. Find the value of a life as many years older than the given life as are equal to the term for which the annuity is proposed. Multiply this value by 1*l.* payable at the end of this term, and also by the probability that the life will continue so long. Subtract the product from the present value of the given life, and the remainder multiplied by the annuity will be the answer.

Ex. 1. What is the value of an annuity of 50*l.* per annum, for 14 years, on a life of 35 ? 35+14 = 49.

The value of a life of 49 (14 years older than the given life, by Table I.) - - - - - = 10.443

The value of 1*l.* payable at the end of 14 years (Table) - - - - - - - - = .505068

The probability that a life of 35 will continue 14 years (Table and the 2d Case.) - - - - - - - - - = $\frac{2936}{4010}$

$10.443 \times .505068 \times \left(\frac{2936}{4010}\right) .7322 = 3.861$, which, subtracted from 12.502, the value of a life of 35, Table I. gives 8.641 ; and $8.641 \times 50 = 432l.$ 1*s.*

Ex. 2. What is the value of an annuity of 80*l.* per annum for 20 years provided a person aged 45 live so long ? Answer, 785*l.* 13*s.* 7*d.*

TABLE.

Shewing the present Value of 1*l.* to be received at the end of any number of years, not exceeding 100; discounting at 5 per Cent. Compound Interest.

Yrs.	Value	Yrs.	Value.	Yrs.	Value.	Yrs.	Value.
1	.952381	26	.281241	51	.083051	76	.024525
2	.907029	27	.267848	52	.079096	77	.023357
3	.863838	28	.255094	53	.075330	78	.022245
4	.822702	29	.242946	54	.075743	79	.021186
5	.783526	30	.231377	55	.068326	80	.020177
6	.746215	31	.220359	56	.065073	81	.019216
7	.710681	32	.209866	57	.061974	82	.018301
8	.676839	33	.199873	58	.059023	83	.017430
9	.644609	34	.190355	59	.056212	84	.016600
10	.613913	35	.181290	60	.053536	85	.015809
11	.584679	36	.172657	61	.050986	86	.015056
12	.556837	37	.164436	62	.048558	87	.014339
13	.530321	38	.156605	63	.046246	88	.013657
14	.505068	39	.149148	64	.044044	89	.013006
15	.481017	40	.142046	65	.041946	90	.012387
16	.458112	41	.135282	66	.039949	91	.011797
17	.436297	42	.128840	67	.038047	92	.011235
18	.415521	43	.122704	68	.036235	93	.010700
19	.395734	44	.116861	69	.034509	94	.010191
20	.376889	45	.111297	70	.032866	95	.009705
21	.358942	46	.105997	71	.031301	96	.009243
22	.341850	47	.100949	72	.029811	97	.008803
23	.325571	48	.096142	73	.028391	98	.008384
24	.310068	49	.091564	74	.027039	99	.007985
25	.295303	50	.087204	75	.025753	100	.007604

In order to find the present worth of any sum which is to be received at the end of a certain number of years.—Multiply the number in the table opposite to the term of years, by the sum, and the product will be the answer.

Ex. 1. What is the present value of 750*l.* to be received at the expiration of 9 years?

The number in the table even with 9 years is .644609, which is to be multiplied by 750.

	.644609
	750
	3223045
	4512263
	483.45675
	20
	9.1350
	12
	1.620
	4
Answer, 483*l.* 9*s.* 1½*d.*	2.48

Ex. 2. What is the present value of 574*l.* 10*s.* 6*d.*, to be received 15 years hence? Ans. 276*l.* 6*s.* 11*d.*

Case V. To find the value of a given sum payable at the decease of a person, whenever that shall happen. That is, to find the value of an assurance of any given sum on the whole duration of life.

Rule. Subtract the value of the life from the perpetuity. Multiply the remainder by the product of the given sum into the rate, and this last product divided by 100*l.* increased by its interest for a year, will give an answer in a single present payment. This payment divided by the value of the life will give the answer in annual payments during the continuance of life.

Ex. 1. What ought I, who am now 45, to pay to assure on my life *L.*1000; that is, what ought I to pay annually, to insure to my children at my decease *L.*1000, allowing money at 5 per cent.?

The value of a life of 45, by Table, p. 236, is 11,105; and the perpetuity is $\frac{100}{5} = 20$. Therefore, by the rule; 20—11.105 = 8.895, which multiplied by 5000, gives 44475; this, divided by 105, or $\frac{44475}{105} = 423l.\ 11s.\ 5d.$, equal the answer in a single present payment. Therefore $\frac{423l.\ 11s.}{11.105} = 38l.\ 2s.\ 10d.$ nearly, in annual payments continued during life.

Ex. 2. Let the life be 20: the sum *L.*100, and the rate 5 per cent.?

The value of a life of 30 is, by Table, p. 236, equal to 13.072, and the perpetuity 20. Therefore, 20 — 13.072 = 6.928, which, multipled by 500, gives 3464, which divided by 105, or $\frac{3464}{105} = 33l.$ nearly, being the sum to be paid in a single payment; and $\frac{33}{13\ 072} = 2l.\ 10s.\ 6d.$ nearly, in annual payments continued during life.

If the interest of money be supposed 4 per cent., then the value of a life of 30 is equal 14.68,* and the perpertuity is equal $\frac{100}{4} = 25$. Therefore 25 — 14.68 = 10.32. This multiplied by 400*l.* = 4128. And $\frac{4128}{104} =$ 39*l.* 14*s.* nearly; and $\frac{39l.\ 14s.}{14.68} = 2l.\ 14s.$

NOTE.

* This is taken from a table not in this book. See Price's Reversionary Payments, and Morgan's Doctrine of Annuities, &c

Hence it appears, that when the values are required in a single payment, the difference in the rate per cent. is considerable, though but trifling when made in annual payments during life. In this question, if money be improved at 5 per cent., the value of the single payment would be 33*l.*: but at 4 per cent. it would be 39*l.* 14*s.*, which is one-fifth more in the latter case than in the former: but, when the value is paid in annual sums during life; at 5 per cent., each payment is 2*l.* 10*s.* 6*d.*, and at 4 per cent. it is 2*l.* 14*s.*, making a difference of 3*s.* 6*d.* per annum, being an increase of less than one fourteenth.

If the first of the annual payments is to be made immediately, then the single payment is to be divided by the value of the life, with unity added to it, so that at 5 per cent. it will be $\frac{33}{14.072} = 2l.\ 6s.\ 11d.$ nearly; and at 4 per cent. it will be $\frac{39l.\ 14s.}{15.68} = 2l.\ 9s.\ 4\frac{1}{2}d.$

Ex. 3. Let the life be 25, the sum 1000*l.*, and the rate 5 per cent. Answer, 21*l.* annually.

Ex. 4. Let the life be 60, the sum 1000*l.*, and the rate 5 per cent. Answer, 59*l.* nearly.

CASE VI. To determine the value of an annuity certain on a given life for any number of years.

RULE. Find the value of a life as many years older than the given life as are equal to the term for which the annuity is proposed. Multiply this value by 1*l.* payable at the end of this term, and also by the probability that this life will continue so long. Subtract the product from the present value of the given life, and the remainder multiplied by the annuity will be the answer.

Ex. 1. Let the annuity be 50*l.* the age of the given life 30 years, and the term proposed 15 years; interest 5 per cent.

The value of a life of 45, or 15 years older than the given life, by Table, page 236, = 11.105. The value of 1*l.* payable at the end of 15 years is, by table, page 243, = .481: and the probability that the life of 30 will exist so long, is by Table, page 232 = $\frac{3248}{4385}$ = .74 nearly. Therefore 11.105 × .481 × .74 = 3.953. And the present value of the given life, by the Table, page 236, = 13.072: therefore 13.072 — 3.953 = 9.119, and this multiplied by 50 = 455*l.* 19*s.*

Had the interest been only 4 per cent. the value would have been about 490*l.*: that is, in the one case 455*l.* 19*s.*, and in the other 490*l.*, by a person who would insure an annuity of 50*l.* per annum for 15 years certain, which depends on the contingency of the life of a person aged 30.

Ex. 2. Let the annuity be 40*l.* the age of the given life 40, and the term proposed 20 years.

Answer, 402*l.* 16*s.* 10*d.*

Case VII. To find the value of a given sum payable at the decease of a person, should that happen within a given term. In other words: What ought a person to give for having his life assured to him for a certain term?

Rule. From the value of an annuity certain for the given term, subtract the value of the life for the same term, and reserve the remainer. Multiply the value of 1*l.* due at the end of the given term, by the perpetuity, and also by the probability that the given life shall *fail* in the given term. The product is to be added to the reserved remainder, and the sum multiplied by the given sum: this last product divided by the perpetuity increased by unity, gives the value in one present payment.

Ex. 1. A merchant at Liverpool, aged 30, expects to realize a considerable property in the next 15 years; but as he may die before he can accomplish his views, he is willing to insure on his life, during that period, the sum of 5000*l.*, what must he pay for the same?

The value of annuity certain for 15 years, by Table, p. 227, is equal to 10.379; and by example, page 247, the value of an annuity certain for 15 years on a life of 30 = 9.119; therefore 10.379—9.112 = 1.26 = reserved remainder.

The value of 1*l.* to be received at the end of 15 years, by Table, page 243, = .481; and the probability that a life of 30 shall fail in 15 years, is $\frac{1142}{4385} = .26$:* and the perpetuity is $\frac{100}{5} = 20$. Therefore, $.481 \times .26 \times 20 = 2.5$, and this added to the reserved remainder 1.26 = 3.76, which multiplied by 5000, the given sum, and divided by 21 (the perpetuity increased by unity) is equal 895*l.* 5*s.* nearly, the value required in a single payment. That is, a person of 30 must give 895*l.* 5*s.* to secure to his heirs 5000*l.* supposing he dies within 15 years. Or he must pay annually during the 15 years, if he live so long, 985*l.* 5*s.* divided by 9.119, or 98*l.* 3*s.* 4*d.*† for the same security.

NOTES.

* The probability of life's failing, is always equal to the probability of its continuing, subtracted from unity. Thus the probability of a life of 30 continuing 15 years, is by table, p. 232, $= \frac{3248}{4385} = .74$, and the probability of its failing $1 = \frac{3248}{4385} = \frac{4485-3248}{4385} = \frac{1137}{4385} = .26$.

See Chances, p. 230.

† The payments are supposed to be made at the end of every year. But in all assurances, the first premium is paid immediately, and the remaining ones at the be-

If money can be improved at 4 per cent. only, then the sum to be paid at once will be 929*l.* 4*s.* 2*d.*, and the annual payments will be 101*l.* nearly.

Ex. 2. If I live 7 years, I shall receive 2000*l.*; what must I give to insure my life for that period, being now 46 years of age?

Ans. 51*l.* for each annual payment for 7 years, if he live so long.

CASE VIII. To explain, by examples, the mode of granting annuities by the British Government established in the year 1808.

[The following examples are deduced from the tables printed and circulated by Government, and which may be had, gratis, at the Office, Bank Buildings, Royal Exchange, London.]

Ex. 1. By the tables it appears, that for every 100*l.* stock in 3 the per cent. consolidated annuities, will be given annually for life, to a person of 46 years, 5*l.* 11*s.** If, therefore, a person of that age transfer 1000*l.* stock, he will receive an annuity for life of 55*l.* 10*s.* But he will receive interest 30*l.* and keep his capital; and to insure 660*l.* at the Equitable, or Royal Exchange Offices, he must pay rather more than 4 per cent.; that is, he must

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NOTES.

ginning of ever year after; hence the proper divisor will be the value of the life for one year less than the given term added to unity, or, in this case, the value of a life for 14 years. And generally: the divisor for determining the annual payments must be increased by unity, whenever it is proposed that the first payment should be made immediately. See p. 246.

* Supposing stocks to be at 66, which they are at present.

pay between 26 and 27*l.* annually, during life, to insure to his heirs at his death the 660*l.* which he transfers to Government: he will of course be a loser by the transfer, of between one and two pounds per annum. It is therefore obvious, that no one, when stocks are at 66, can join in the plan held out by Government, who is not willing to give up his capital.

Ex. 2. When stocks are at 60, he will receive for 1000*l.* stock, 52*l.* 10*s.*; and to insure 600*l.* must pay more than 24*l.* to insure his life, and will of course be a loser of 1*l.* 10*s.* per annum.

Ex. 3. When stocks are at 80, as they may be, he will receive for the 1000*l.* stock 62*l.*; but, to insure 800*l.* he must pay annually rather more than 32*l.*; in this case there will be his interest left, and he will he neither gainer nor loser.

These examples will suffice for the whole.

REVERSIONS.

Reversions, or Reversionary Annuities, are those which do not commence till after a certain number of years, or till the decease of a person, or some other future event has happened.

Case I. To find the present value of an annuity for a term of years, which is not to commence till the expiration of a certain period.

Rule. Subtract from the value of an annuity for the whole period, the value of an annuity to the time when the reversionary annuity is to commence.

Ex. 1. What is the present value, at 5 per cent. compound interest, of 80*l.* per annum for 24 years, commencing at the end of 8 years? 24+8=32.

The present value of an annuity (Table, p. 227,) for 32 years, is 15.802677, and the value of one for 8 years is 6.463213, therefore,

15.802677
6.463213

9.339464 ×80=747.15712=747*l.* 3*s.* 1½*d.*

Ex. 2. What is the present value of an annuity of 55*l.* for 15 years, to commence at the end of 15 years?
Answer, 274*l.* 12*s.*

Ex. 3. What is the present value of an annuity for 49 years, to commence at the end of 47 years?
Answer, Something more than a year and half's purchase.

CASE II. To find the value of an annuity certain for a given term, after the extinction of any life or lives.

RULE. Subtract the value of the life or lives from the perpetuity,* and reserve the remainder. Then say, as the perpetuity is to the present value of the annuity certain, so is the reserved remainder, to the number of years purchase required.

NOTES.

* PERPETUITY, is the number of years purchase to be given for an annuity which is to continue for ever; and it is found by dividing 100*l.* by the rate of interest; thus,

Ex. 1. What is the value of an annuity certain for 14 years, to commence at the death of a person aged 35, allowing 5 per cent. ?

The value of a life of 35 (Table, p. 236) = 12.502; this subtracted from 20, the perpetuity, leaves 7.498 = reserved remainder. Then, as 20: 9.898† :: 7.498: 3.7107 = number of years purchase.

NOTES.

allowing 5 per cent., the perpetuity is 20 years, or $\frac{100}{5} = 20$; and at the rates most usually adopted, the perpetuity is as follows :

At 3 per cent. $\frac{100}{3} = 33.33$, &c.

$3\frac{1}{2}$ ditto $\frac{100}{3.5} = 28.57$, &c.

4 ditto $\frac{100}{4} = 25.$

$4\frac{1}{2}$ ditto $\frac{100}{4\,5} = 22.22$, &c.

5 ditto $\frac{100}{5} = 20.$

6 ditto $\frac{100}{6} = 16.66$, &c.

7 ditto $\frac{100}{7} = 14.28$, &c.

8 ditto $\frac{100}{8} = 12.5.$

These are the number of years purchase to be given for a perpetual annuity, on the supposition that it is receivable yearly: but, as annuities are more commonly received half-yearly, and the interest of money likewise paid half-yearly; in this case the perpetuity will be somewhat greater or less than the above, as the periods at which the annuity is payable are more or less frequent than those at which the rate of interest is here supposed payable.

† The value of an annuity certain for 14 years. Table·

Ex. 2. A and his heirs are entitled to an annuity of *L*.100 certain for 25 years, to commence at the death of a cousin aged 45 years; what can A sell his interest in this annuity for? Answer, 626*l*. 16*s*.

Case III. To find the value of an annuity for a term certain; and also for what may happen to remain of a given life after the expiration of this term.

Rule. Find the value of a life as many years older than the given life, as are equal to the term for which the annuity certain is proposed. Multiply this value by 1*l*. payable at the end of the given term, and also by the probability that the given life will continue so long. Add the product to the value of the annuity certain for the given term, and the sum will be the answer.

Ex. 1. What is the value of an annuity of 60*l*. for 14 years, and also for the remainder of a life now aged 35, after the expiration of that term? $35+14=49$.

The value of a life aged 49 (Table 1, page 236.) - - - - - - - - = 10.443
The value of 1*l*. payable at the end of 14 years (Table, page 243) - - = .505068
The probability that the life will exist so long, (Table, page 232.) - = $\frac{2936}{4010}$

Therefore, $10{,}443 \times .505068 \times \frac{2936}{4010} = 3.861$; this added to 9.898, the value of an annuity certain for 14 years, (see Table, page 227.) = 13 759, the number of years purchase; and $13.759 \times 60 = 825l.\ 10s.\ 9\frac{1}{2}d.$

Ex. 2. What is the value of an annuity of 75*l*. for 10 years, and also the remainder of a life now aged 24, after the expiration of that term? Ans. 1070*l*. 5*s*.

Case IV. To find what annuity can be purchased for a given sum, during the joint lives of two persons of given ages, and also during the life of the survivor, on condition that the annuity shall be reduced one-half at the extinction of the joint lives.

RULE. Divide twice the given sum by the sum of the value of the two single lives, and the quotient will give the annuity to be paid during the joint lives, one-half of which is therefore the annuity to be paid during the remainder of the surviving life.

Ex. 1. A man and his wife, aged 35 and 27, are desirous of sinking 2000*l.* in order to receive an annuity during their joint lives, and also another annuity of half the value during the remainder of the surviving life: what annuities ought to be granted them?

The value of a life of 27 } Table I. p. 236 { =13.377
The - - - - - - - - - - 35 } { =12.502

25.879

Therefore, $\frac{4000 \text{ (twice the sum)}}{25.879} = 154l.\ 11s.\ 3d. =$

annuity during their joint lives: and 77*l.* 5*s.* 7½*d.* annuity during the life of the survivor.

Ex. 2. A single man, aged 60, possessed of 1500*l.* is desirous of purchasing with it an annuity for himself and his sister, aged 40, during their joint lives, with one of half the value, during the remainder of the life of the survivor, at the death of either: what will be the value of the annuities?

Answer, 148*l.* 6*s.* annuity during joint lives, and
74*l.* 3*s.* do for the survivor.

Ex. 3. A man possessed of 1000*l.* which he will sink in the same way, and for the same purposes, during the joint lives of himself and father; the age of the one is 55, of the other 80: what annuities can be given for it?

Answer, 155*l.* annuity during joint lives, and
77*l.* 10*s.* do for the survivor.

V. To find the value of the expectation of a perpetual annuity, provided one person of a given age survives another of a given age.

(1.) *If the Expectant be the elder.*

RULE. Find the value of an annuity on two equal joint lives, whose common age is equal to the age of the oldest of the two proposed lives; subtract this value from the perpetuity, and take half the remainder: then say,

As the expectation of the duration of life of the younger,
Is to that of the elder:
So is the half remainder to a fourth proportional:
which will be the number of years purchase, if the expectant is the older.

(2.) *If the Expectant be the younger.*

Add the value found, as above, to that of the joint lives, and let the sum be subtracted from the perpetuity, and the remainder is the answer.

Ex. 1. What is the value of B's expectation, (aged 30,) of an estate 50*l.* per annum, provided he survive A aged 20?

Value of two joint lives, aged 30, (Table II. p. 239) =10.255, the difference between which and 20, (the perpetuity,) is 9.745, the half of which is 4.872: therefore,

As 33.43 { The expectation of A Table p. 234. } : 28.27 { Expectation of B. }
:: 4.872 : 4.119 = 205*l.* 19*s.*

Ex. 2. What is the value as above, when B is 20, and A 30?

Then, to 4.119, just found, add [p. 239)
10.707, value of the joint lives (Table II.

14.826; this subtracted from 20, the perpetuity and the remainder, 5.174 × 50 = 258*l.* 14*s.* is the true answer.

EXAMPLES FOR PRACTICE.

Ex. 1. What is the difference in the value of an annuity of 20*l.* certain for 30 years, and an annuity of the same amount on the longest of two lives, aged 25 and 40?

Answer, *L.*5 4 $4\frac{1}{2}$ difference.

Ex. 2. What is the value of an estate of 150*l.* per annum held on the longest of two lives, aged 40 and 50, subject to the payment of an annuity of 14*l.* to a life of 62, and another annuity of 18*l.* to a life of 65?

Answer, 1847*l.* 16*s.* value

Ex. 3. What is the present worth of 2000*l.* to be received at the decease of a person aged 65?

Answer, 1272*l.* 8*s.* present worth.

Ex. 4. What is the present value of 36*l.* a year, being the third part of a farm in Essex, after the death of a person aged 54 years?

Answer, 375*l.* 11*s.* 9*d.* present value.

Ex. 5. What is the present value of a reversionary annuity of 252*l.* 3*s.* 8*d.* during the life of a person aged 24, in case he survives his brother, aged 34?

Answer, 1539*l.* 5*s.* 9*d.* present value.

Ex 6. What should be the consideration to be paid at the death of a person aged 85, for 1000*l.* now advanced to a person aged 25, in case the latter survives the former?

Answer, 1193*l.* 6*s.*

Ex. 7. What is the value of the reversion of 91*l.* per annum forever, after the death of a person aged 53?

Answer, 932*l.* 18*s.* 7*d.* value.

Ex. 8. A person aged 52, is entitled to 800*l.* at the death of another aged 76, provided the former survives the latter; what is its present worth?

Answer, 522*l.* 0*s.* 9*d.*

Ex. 9. What is the present value of an annuity on the longest of two lives, now aged 25 and 30, the annuity not to commence till 14 years hence?

Answer, 854*l.* 19*s.* 1*d.*

LEASES.

A LEASE is a conveyance of any lands and tenements, made, in consideration of rent, or of a present sum of money, for life, or for a term of years.

The purchaser of a Lease may be considered as the purchaser of an annuity equal to the rack-rent of the estate; its value must therefore be calculated on the same principles as that of an annuity.

The sum paid down for the grant of a lease is so much, as being put out to interest will enable the landlord to repay himself the rack-rent of the estate, or the yearly value of his interest therein.

The value of the lease depends on the length of the term, and the rate of interest which the landlord can make of his money.

The value of leases at 5 per cent. compound interest may be found in the Table page 227.

Thus, the value of a lease for 14 years, of a farm worth 150*l.* per annum, is by that table, $9.898641 \times 150 = 1484l.\ 15s.\ 11d.$

Ex. 1. What ought to be given for a lease of 26 years of an estate of 18*l.* per annum clear annual rent, in order that a purchaser may make 5 per cent of his money?

Answer 258*l.* 15*s.*

Ex. 2. A friend has just purchased the lease of a house for 54 years, for which he gave 570*l.* and he is to pay a ground-rent of 1*l* per annum: how much ought the house to let for, allowing 5 per cent. interest only?

Answer 30*l.* 12*s.* 4*d.*

Leases are generally calculated at a higher rate of interest; we shall therefore insert the following

TABLE,

Shewing the Number of Years Purchase that ought to be given for a Lease, for any Number of Years not exceeding 100, at 6, 7, and 8 per cent. interest.

Yrs.	6 per cent.	7 per cent.	8 per cent.	Year.	6 per cent.	7 per cent.	8 per cent.
1	.9433	.9345	.9259	51	15,8130	13,8324	12,2532
2	1.8333	1,8080	1,7832	52	15,8613	13,8621	12,2715
3	2 6730	2,6243	2,5770	53	15,9009	13,8898	12,2884
4	3.4651	3,3872	3,3121	54	15,9499	13,9157	12,3041
5	4.2123	4,1001	3,9927	55	15,9905	13,9399	12,3186
6	4 9173	4,7665	4,6228	56	16,0288	13,9265	12,3320
7	5.5823	5,3892	5,2063	57	16,0649	13,9837	12,3444
8	6 2097	5,9712	5,7466	58	16,0989	14,0034	12,3560
9	6 8016	6,5152	6,2468	59	16,1311	14,0215	12,3669
10	7.3600	7,0235	6,7100	60	16,1614	14 0391	12,3765
11	7 8868	7,4986	7,1389	61	16,1900	14,0553	12,3856
12	8 3838	7,9426	7,5360	62	16,2170	14,0703	12,3941
13	8.8526	8,3576	7,9037	63	16,2424	14,0844	12,4020
14	9 2949	8,7454	8,2442	64	16,2664	14,0976	12,4092
15	9.7122	9,1079	8,5594	65	16,2891	14,1099	12,4159
16	10.1058	9,4466	8,8513	66	16,3104	14,1214	12,4222
17	10 4772	9,7632	9,1216	67	16,3306	14,1321	12,4279
18	10.8276	10,0590	9,3718	68	16,3496	14,1422	12,4333
19	11 1581	10,3355	9,6035	69	16,3676	14,1516	12,4382
20	11 4699	10,5940	9,8181	70	16,3845	14,1603	12,4428
21	11 7640	10,8355	10,0168	71	16,4005	14,1685	12,4470
22	12.0415	11,0612	10,2007	72	16,4155	14,1762	12,4509
23	12 3033	11,2721	10,3710	73	16,4297	14,1834	12,4546
24	12 5503	11,4693	10,5287	74	16,4431	14,1901	12,4579
25	12 7833	11,6535	10,6747	75	16,4558	14,1963	12,4610
26	13 0031	11,8257	10,8099	76	16,4677	14,2022	12,4639
27	13.2105	11,9867	10,9351	77	16,4790	14,2076	12,4666
28	13 4061	12,1371	11,0510	78	16,4896	14,2127	12,4691
29	13.5907	12,2776	11,1584	79	16,4996	14 2175	12,4713
30	13 7648	12,4090	11,2577	80	16,5091	14,2220	12,4735
31	13 9290	12,5318	11,3497	81	16,5180	14,2261	12,4754
32	14 0840	12,6465	11,4349	82	16,5264	14,2300	12,4772
33	14 2302	12,7537	11,5138	83	16,5343	14,2337	12,4789
34	14 3681	12,8540	11,5869	84	16,5418	14,2371	12,4805
35	14.4982	12,9476	11,6545	85	16,5489	14,2402	12,4819
36	14 6209	13,0352	11,7171	86	16,5556	14,2432	12,4833
37	14.7367	13,1170	11.7751	87	16,5618	14,2460	12,4845
38	14 8460	13,1935	11,8288	88	16,5678	14,2486	12,4856
39	14 9490	13,2649	11,8785	89	16,5734	14,2510	12,4867
40	15.0462	13,3317	11,9246	90	16,5787	14,2533	12,4877
41	15.1380	13,3941	11,9672	91	16,5836	14,2554	12,4886
42	15.2245	13,4524	12,0066	92	16,5883	14,2574	12,4894
43	15 3061	13,5069	12,0432	93	16,5928	14,2592	12,4902
44	15 3831	13,5579	12,0770	94	16,5969	14,2610	12,4909
45	15.4558	13,6055	12,1084	95	16,6009	14,2626	12 4916
46	15.5243	13,6500	12,1374	96	16,6046	14,2641	12,4922
47	15 5890	13,6916	12,1642	97	16,6081	14,2655	12,4928
48	15.6500	13,7304	12,1891	98	16,6114	14,2668	12,4933
49	15 7075	13,7667	12,2121	99	16,6145	14,2680	12,4938
50	15 7618	13,8007	12,2334	100	16,6175	14,2692	12,4943

CASE 1. To find the sum that ought to be given for lease.

RULE. Look in the table against the number of years for which the lease is to continue, and on the line even with it, under the given rate of interest, is the number of years purchase that ought to be given for the same.

Ex. What sum ought to be given for the lease of an estate of 17 years, of the clear annual rent of 75*l.* allowing the purchaser to make 7 per cent. interest of his money?

Answer, 9.7632×75—732.24=732*l.* 4*s.* 9½*d.*

Ex. 2. What must be given for a lease of 21 years, at the clear annual rent of 50 guineas, allowing 8 per cent. for money? Answer, 525*l.* 17*s.* 9*d.*

Ex. 3. What is the worth of a lease of 83 years of an estate of 78*l.* per annum, interest being 6 per cent.?

Answer, 1289*l.* 13*s.*

Ex. 3. What sum ought to be given for a lease of 69 years, of a farm of 150*l.* per annum, the purchaser being allowed 6 per cent. for his money? Ans. 2455*l.* 10*s.*

Ex. 5. What sum ought to be given for the lease of 46 years, of an estate estimated at 200*l.*, but which is charged with the payment of a reserved rent of 70*l.* 15*s.* besides taxes and incidental expenses to the amount of 49*l.* 12*s.* annually; allowing the purchaser 6 per cent. interest for his money? Answer, 1236*l.* 9*s.* 9*d.*

Ex. 6. What sum ought to be given for the ground rent of a house of 15*l.* per annum, for 18 years, allowing the purchaser 8 per cent? Ans. 140*l.* 11*s.* 6*d.*

CASE II. To find the annual rent corresponding to any given sum paid for a lease.

RULE. Divide the sum paid for the lease by the number of years purchase that are found against the given term, and under the rate of interest intended to be made of the purchase money, the quotient will be the annual rent required.

Ex. 1. I am asked 1500*l.* for a 40 years lease, to what annual rent is that equivalent, allowing 6 per cent. for money?

$$\text{Answer, } \frac{1500}{15.046} = 99l.\ 13s.\ 11d. \text{ nearly.}$$

Ex. 2. If I sell the lease of my house, which has 81 years to run, for 800 guineas, at what rent will the purchaser stand, who will have a ground rent of 5*l.* 5*s.* per annum to pay likewise, allowing 7 per cent?

Answer, 64*l.* 5*s.*

Case III. To find the number of years purchase given for a lease that cost a certain sum of money.

Rule. Divide the sum paid for the lease by the clear annual rent of the estate for which it is given, and the quotient will be the number of years purchase required.

Ex. 1. The lease of a house, at the clear annual rent of 116*l.* was sold for 1630*l.*, what number of years purchase was given for it?

$$\frac{1630}{116} = 14 \text{ years, 0 months, 2 weeks, 4 days.}$$

Ex. 2. How many years purchase did the lease of a house sell for which cost 800*l.* and the rent was 60 guineas? Answer, 12 years, 8 month, 12 days.

FREEHOLDS.

Case I. To find the gross sum which ought to be paid for a freehold estate.

Rule. (1) "Multiply the number of years purchase by the annual rent." Or, (2) "Multiply the annual rent by 100, and divide the product by the rate of interest which it is proposed to make of money; the quotient will be the sum required."

Ex. What ought I to give for a freehold, the rent of which is 75*l.* per annum, supposing I mean to make 4 per cent. of my money?

By the 1st. Rule, the answer is $25 \times 75 = 1875l.$

By the 2d. — — — — $\frac{75 \times 100}{4} = 1875l.$

If I had wanted 5 per cent. for my money, the answer would have been - 1st. $20 \times 75 = 1500l.$

2d. $\frac{75 \times 100}{5} = 1500l.$

But if I were contented with 3 per cent., then I might afford to give for it 2500*l.*

$$\frac{75 \times 100}{3} = 2500l.$$

CASE II. To find the clear annual rent which a freehold ought to produce, so as to allow the purchaser a given rate of interest for his money?

RULE. Multiply the sum paid for the same, by the given rate per cent., and divide by 100, the quotient will be the annual rent required.

Ex. A person has given 3000 guineas for a freehold-estate, and wishes to let it so as to have $4\frac{1}{2}$ per cent. for his money, what must be the annual rent?

Answer, 141*l.* 15*s.*

CASE III. To find the value of a freehold, to be entered upon after a certain term.

RULE. Subtract the value of that certain term, from the value of the perpetuity, and the difference will be the true value.

Ex. 1. What sum should be given for the reversion of a freehold after 14 years, allowing interest 6 per cent., and the clear annual rent 85*l.*

Value of a lease of 14 years, Table, page 258, = 9.295; which subtracted from 16.667, the perpetuity, leaves 7.372; and this multiplied by 85*l.* gives the value = 626*l.* 12*s.* $4\frac{3}{4}$*d.*

Ex. 2. What ought I to give for the reversion of a freehold worth 120*l.* per annum; but a lease of which is sold for 5 years to come, supposing interest 5 per cent.

Answer, 1880*l.* 10*s.* 5*d.* nearly.

RENEWAL OF LEASES.

CASE I. To ascertain what fine should be given for the renewal of any number of years lapsed in a lease originally granted for 24 years.

RULE. This is done by means of the following

TABLE,

For Renewing any Number of Years lapsed in a lease for Twenty-one Years.

Years.	3 per Ct.	4 per Ct.	5 per Ct.	6 per Ct.	8 per Ct.	£.11.564 per Cent.
1	,538	,439	,359	,294	,199	,100
2	1,091	,895	,736	,606	,413	,213
3	1,661	1,870	1,132	,936	,645	,338
4	2,249	1,863	1,547	1,287	,895	,477
5	2,854	2,377	1,983	1,658	1,165	,633
6	3,477	2,911	2,441	2,052	1,457	,806
7	4,119	3,466	2,922	2,469	1,773	1,000
8	4,780	4,043	3,428	2,911	2,113	1,216
9	5,461	4,644	3,958	3,380	2,481	1,457
10	6,162	5,269	4,515	3,877	2,878	1,726
11	6,885	5,918	5,099	4,404	3,307	2,026
12	7,629	6,594	5,713	4,962	3,770	2,361
13	8,395	7,296	6,358	5,554	4,270	2,734
14	9,185	8,027	7,035	6,182	4,810	3,151
15	9,998	8,787	7,745	6,847	5,394	3,616
16	10,835	9,577	8,492	7,552	6,024	4,135
17	11,698	10,399	9,275	8,299	6,705	4,713
18	12,586	11,254	10,098	9,091	7,440	5,359
19	13,502	12,143	10,962	9,931	8,234	6,079
20	14,444	13,068	11,869	10,821	9,091	6,882
total	15,415	14,029	12,821	11,764	10,017	7,779.

Ex. 1. What ought to be given as a fine for the renewal of 15 years lapsed, or expired in a lease for 21 years, allowing the tenant 5 per cent. interest, and estimating the clear and improved rent at 60 guineas per annum?

Against 15 in the table, and under 5 per cent., is 7.745, and this multiplied by 63*l.* gives $487.935 = 487l.\ 18s.\ 8\frac{1}{4}d.$

If the interest agreed on had been 6 or 8 per cent., the answers would have been

$$6.847 \times 63 = 431l.\ 7s.\ 2d.$$
$$\text{Or, } 5.394 \times 63 = 339l.\ 16s.\ 5d.$$

Ex. 2. What ought to be given to a landlord for adding seven years to a lease, of which fourteen years are unexpired, allowing the tenant 6 per cent. interest for his money, and the improved rent to be 60*l.* per annum?

Answer, 148*l.* 2*s.* $9\frac{1}{2}d.$

CASE II. To ascertain the value of the fine which ought to be paid for renewing a given number of years in any lease.

RULE. The value for renewing an additional term, or for adding any number of years to the unexpired part of an old lease, is equal to the difference between the value of the lease for the whole term, and the value of the unexpired part.

Ex. 1. What ought to be given for the addition of seven years to a lease, of which 13 are unexpired; allowing 6 per cent. for money?

The whole term for which the new lease is to be granted is 20 years; therefore, Table, under 6 per cent, and

against 20 is 11.469, and

against 13 is 8.852; therefore this last subtracted from the former will leave 2.617 for the number of years' purchase which ought to be given for the renewal.

Ex. 2. What should be given for the completing a 60 years' lease, of which a tenant has an unexpired term of 15 years, allowing him 7 per cent. for his money?

Answer, 4.932 year's rent.

Ex. 3. I have a house for a lease of 48 years, but I wish to extend the lease to 97 years: how much must I pay for it, supposing the house worth 50*l.* per annum, and the interest 8 per cent.? Ans. 15*l.* 4*s.*

It will be seen by working Ex. 2. of Case I, by this rule, that the answer will be precisely the same by both methods: for the whole term for which the new lease is granted is 21 years: the value of a lease for this term is, by Table, 11.764, and the value of the 14 years' lease yet to come is 9.295; this subtracted from the other, gives 2.469, as before, which, multiplied by 60, and the answer is 148*l.* 2*s.* 9½*d.*

The following table will comprehend the cases that most frequently occur at the rate of 5 and 6 per cent.

TABLE,

For Renewing, with one Life, the Lease of an Estate held on Three Lives.

Life put in.	Age of lives in possession.	5 pr. Ct.	6 pr. Ct.	Life put in.	Age of lives in possession.	5 pr. Ct.	6 pr. Ct.
	30—30	1,741	1,305		40—75	3,943	3,076
	30—40	2,035	1,521		50—50	3,289	2,536
	30—50	2,431	1,832		50—60	3,910	3,039
	30—60	2,838	2,160		50—70	4,546	3,579
	30—70	3,277	2,535	15	50—75	4,816	3,819
	30—75	3,402	2,571		60—60	4,692	3,678
	40—40	2,397	1,792		60—70	5,780	4,627
	40—50	2,916	2,204		60—75	6,034	4,849
	40—60	3,451	2,637		70—70	7,125	5,805
10	40—70	3,914	3,032				
	40—75	4,264	3,273		30—30	1,404	1,079
	50—50	3,563	2,723		30—40	1,673	1,284
	50—60	4,206	3,242		30—50	2,019	1,557
	50—70	4,873	3,819		30—60	2,363	1,831
	50—75	5,174	4,062		30—70	2,813	2,218
	60—60	5,023	3,911		30—75	2,845	2,241
	60—70	6,161	4,917		40—40	2,027	1,558
	60—75	6,452	5,142		40—50	2,467	1,908
	70—70	7,556	6,124		40—60	2,943	2,293
				20	40—70	3,358	2,641
	30—30	1,572	1,191		40—75	3,615	2,873
	30—40	1,857	1,407		50—50	3,010	2,841
	30—50	2,227	1,699		50—60	9,607	2,828
	30—60	2,600	1,996		50—70	4,208	3,337
15	30—70	3,052	2,381		50—75	4,474	3,576
	30—75	3,127	2,408		60—60	4,347	3,433
	40—40	2,224	1,687		60—70	5,386	4,338
	40 50	2,701	2,067		60—75	5,636	4,558
	40—60	3,205	2,474		70—70	6,695	5,489
	40—70	3,641	2,839				

RULE. The years' purchase in the table, multiplied by the improved annual value of the estate, beyond the rent payable under the lease, gives the fine to be paid for putting in the new life.

Ex. What must be given to put in a life of 10 years, when the ages of those in possession are 40 and 50, allowing 6 per cent. for money?

Ans. 2.204, or not quite $2\frac{1}{4}$ years' purchase.

If the life to be added be 15 years, the answer would be 2.067, or very little more than 2 years' purchase. And,

If the life to be added be 20 years, the answer would be 1.908, or less than 2 years' purchase.

PERMUTATIONS AND CONBINATIONS.

The Permutation of quantities is the changing or varying the order of things.

The Combination of quantities is the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order in which they stand.

Case I. To find the number of changes that can be made of any given number of things, all different from each other.

Rule. Multiply all the terms one into another, and the last product will be the number of changes required.

Ex. 1. How many changes can be rung on 12 bells?

$1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11\times12 = 479{,}001{,}600.$

Ex. 2. How many days can eight persons be placed in a different position at a dinner table?

Answer, 40320.

CASE II. Any number of different things being given, to find how many changes may be made out of them, by taking a given number of quantities at a time.

RULE. Multiply the number of things given, by itself less 1, and that product by the same number less 2, diminishing each succeeding multiplier by an unit, till there are as many products, except one, as there are things taken at a time the last product will be the answer.

Ex. 1. How many changes can be rung with 4 bells out of 12 ?

$$12\times 12-1\times 12-2\times 12-3=12\times 11\times 10\times 9=11880.$$

Ex. 2. How many changes can be rung with 5 bells out of 10 ? Answer, 30240.

Ex. 3. What number of words, containing each 6 letters, can be formed out of the 24 letters in the alphabet, supposing any 6 to form a word ?

Answer, 96909120.

CASE III. To find the combinations of a less number of things out of a greater, all different.

RULE. Take the series 1, 2, 3, 4, &c. up to the less number of things, and multiply them continually together for a divisor : then take a series of as many terms, decreasing each by an unity, from the greater number of things, and multiply them continually together for a dividend. Divide the latter product by the former, and the quotient will be the answer.

Ex. 1. How many combinations can be made of 10 things out of 100 ?

$$1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9\times 10$$

(the number to be taken at a time)$=3,628,800$

$$100\times 99\times 98\times 97\times 96\times 95\times 94\times 93\times 92\times 91$$

(the same number of terms taken from 100)

$=62,815,650,955,529,472,000.$

and $\frac{62815650955529472000}{3628800}=17310309456440.$

Ex. 2. How many combinations can be made of 3 letters out of the 24 letters in the alphabet?

Answer, 2024 combinations required.

Ex. 3. A club of 21 persons agreed to meet weekly, five at a time, so long as they could, without the same five persons meeting together, how long would the club exist? Answer, 391 years.

Case IV. To find the compositions of any number, in sets of equal numbers, the things or persons themselves being different.

Rule. Multiply the number of things in every set continually together, and the product is the answer.

Ex. 1. There are three parties of cricketters, in each eleven men, in how many ways can 11 of them be chosen, one out of each?

Answer, $11 \times 11 \times 11 = 1331$.

Ex. 2. In how many ways can the four suits of cards be taken, four at a time? Ans. 28561.

Ex. 3. There are four parties of whist players; in one there are 6, in the second, 5, in the third 4, and in the fourth 3 persons, how often can the set differ with these persons? Ans: 360.

EXCHANGE.

By Exchange is meant the bartering, or exchanging, the money of one place for that of another, by means of an instrument in writing, called *a bill of exchange*.

Exchanges are carried on by merchants and bankers all over Europe, and are transacted on the Royal Exchange of London, the Royal Exchange of Dublin, the Exchange of Amsterdam, and those of the principal cities of this country and the continent.

When an exchange is mentioned between two places, one place gives a determined price, to receive an undetermined one.

The determined price is called *certain:* thus,

London gives a pound sterling, which is a certain price, to receive from Paris a number of francs, more or less, to be paid or received there. Again London gives 100*l.* which is a certain price, to Dublin and other parts of Ireland, for an uncertain number of pounds, shillings and pence Irish, to be paid or received there, viz. from 105*l.* to 115*l.* Irish, as the exchange may be.

The undetermined price is called *uncertain,* because it is always subject to variation: for instance,

London pays an uncertain price to Spain, as a number of pence sterling, to receive a dollar which is certain in exchange.

The *real money* of a state signifies one piece or more, of any kind of metal coined, and made current by public authority, as guineas, shillings, &c. of England.

The *imaginary money* is chiefly used in keeping accounts, as pounds sterling, for which there is no coin to answer.

The *par of exchange* is the quantity of the money, whether real or imaginary, of one country, which is equal in value to a certain quantity of the money of another; thus,

100*l.* sterling is equal in value to 108*l.* 6*s.* 8*d.* Irish: and 100*l.* sterling is worth 140*l.* of the currency in the West Indies, and equal to 166*l.* 13*s.* 4*d.* currency of the United States.

The *course of exchange* is the value agreed upon by merchants and others and is continually fluctuating above or below the par of exchange, according as the demand for bills is greater or less.

Agio denotes the difference in Amsterdam and other places, between current money, and the exchange or bank-money, the latter being finer than the former.

Usance is a certain space of time allowed by one country to another for the payment of bills of exchange. Bills are either payable at sight, or at a certain number of days after sight: at *usance, double usance.* or *half usance.* At *one, two,* &c. usance means at *one, two, &c.*

months' *date*. Half usance is 15 days, be the month what it may.

Days of Grace are a certain number of days allowed for the payment of bills of exchange, after the expiration of the term specified in such bills, and are variable in different countries. In England three days are allowed.

RULES for finding what quantity of the money of one country will be equal to a given quantity of the money of another according to a given course of exchange.

CASE I. When the course of exchange is given, how much money of one country answers to a certain sum of another, as of Great Britain?

RULE. As the given course of exchange, is to one pound sterling, so is the given sum in foreign money, to its corresponding value in sterling money.

Ex. 1. How much sterling money can I have for 2035 Flemish shillings, when the course of exchange is 37 shillings for 1*l.*?

Here I say, As 37 : 1 :: 2035 : 55 = pounds sterl.

Ex. 2. How much sterling money can I get for 4086 florins, 4 stivers, 6 penings banco, supposing 1*l.* is worth 38 schillings and 2 grotes?*

schil. gr.	*L.*		florins st. p.
38 2	1	: :	4086 4 6
12			40
458			163440 grotes
			8 grotes = 4 stivers
			$\frac{8}{4}$ of a grote = 6 penings

458)163448$\frac{8}{4}$(356*l.* 17*s.* 6*d.* Ans.

NOTE.

			s.	d.
* 8 penings	make 1	grote, or penny =	0	0 54
2 grotes	—— 1	stiver - - - =	0	1.09
12 grotes	—— 1	schilling - - =	0	6 56
20 schillings	—— 1	pound Flemish =	10	11.18
40 grotes	—— 1	guilder or Florin =	1	9.8

Ex. 3. What sterling money will 293*l.* 10*s.* 6*l.* Irish fetch, when the exchange is 114*l.* Irish for 100*l.* sterling?

114*l.* : 100*l.* : : 293*l.* 10*s.* 6*d.* : 257*l.* 9*s.* 6½*d.*

Ex. 4. Dublin remits to London 826*l.* 13*s.*, what must be received there, exchange being 110*l.* per cent?

Answer, 751*l.* 10*s.*

Ex. 5. Jamaica remits to London 287*l.* 0*s.* 10½*d.* currency, what must be received for it, exchange being 135*l.* per cent.?

Answer, 212*l.* 12*s.* 5*d.*

Case II. Given the course of exchange, to bring any quantity of sterling money into the money of another country.

Rule. As 1*l.* sterling is to the course of exchange, so is the given sum, in sterling money, to its corresponding value in foreign money.

Ex. 1. How much Flemish money will 233*l.* 6*s.* 8*d.* sterling be worth, when the exchange is 34*s.* per 1*l.* sterling?

1*l.* : 34*s.* : : 233*l.* 6*s.* 8*d.* : 396*l.* 13*s.* 4*d.* Answer.

Ex. 2. How much Flemish money must be given for 628*l.* 10*s.* sterling when the exchange is 33*s.* 8*d.* per *L.* sterling. Answer, 1057*l.* 19*s.* 6*d.*

Case III. To reduce the currency of any state into bank or exchange money.

Rule. As 100, with the agio added to it, is to 100, so is any given sum current to its value in bank money.

Ex. 1. How much bank money can a merchant in Amsterdam have for 5550 guilders, when the agio is 4½ per cent.?

$$104\tfrac{1}{2} : 100 : : 5550 : 5311\ \frac{5}{104.5}$$ Answer.

Ex. 2. How many florins bank will 3000 currency purchase, agio being $6\frac{1}{4}$ per cent.?

Answer, 2823 florins, 21 grotes, 1 penning.

Case IV. To reduce bank money into currency.

Rule. As 100 is to 100, with the agio added to it, so is the bank money to the currency.

Ex. 1. How much currency can I have in Venice for 1500 ducats bank, when the agio is 15 per cent.?

100 : 115 : : 1500 : 1725

Ex. 2. How much currency can I have for 5000 bank florins, agio being 8 per cent.

Answer, 5400 florins.

IRELAND.

Accounts are kept in Ireland as in England, viz. in pounds, shilling, and pence.

The par of exchange in Ireland is 108*l.* 6*s.* 8*d.*; that is, 108*l.* 6*s.* 8*d.* Irish is equal in value to 100*l.* sterling; or 1*s.* 1*d.* Irish, is equal to one shilling English.

The course of Exchange varies from 105*l.* to 115*l.* according to the balance of trade. See page 269.

Ex. 1. London remits to Dublin 300*l.* sterling, what must be received for it, exchange being 106*l.* Irish per cent., and also when it is 112 per cent.?

Here I say, As 100 : 106 : : 300 : 318*l.* Answer.
and 100 : 112 : : 300 : 336*l.* Answer.

Here it is evident, that when England remits the *certain* price to another country, the higher the exchange, the greater advantage is derived by England;

for when the exchange is 106, she will receive 418*l.* for her 300*l.*, and when it is 112*l.*, she will receive 336*l.* for the same sum.

Ex. 2. Dublin remits to London 700*l.* Irish, what is it equal to when the exchange is 108*l.*, and also when it is 110*l.*?

Here Dublin remits the *certain*, and London gives the uncertain price, and I say,

As 108 : 100 :: 700 : 648*l.* 2*s.* 11½*l.* Answer.
110 : 100 :: 700 : 636*l.* 7*s.* 3*d.* Answer.

Here Dublin is gainer when the exchange is low, because, in that case 700*l.* purchases 648*l.* 2*s.* 11½*d.*, and in the other it purchases only 636*l.* 7*s.* 3*d.*

Ex. 3. London remits to Dublin 545*l.* 10*s.* sterling, for how much Irish must London be credited, exchange being 110½?

Answer, 602*l.* 15*s.* 6½*d.*

Ex. 4. Dublin remits to London 900*l.* 15*s.*, how much sterling must be received, exchange being 112*l.*?

Answer, 804*l.* 5*s.* nearly.

Ex. 5. I purchase sundry books in Dublin, for which I give as follows:

For the first - - -	*L.* 0	9	6	Irish,
second - - - -	0	18	0	
third - - - -	0	5	6	
fourth - - - -	1	5	0	

what are they worth in English money?*

Answer, 2*l.* 13*s.* 6½*d.* nearly.

AMERICAN STATES.

RULE.—As the value of one dollar of the given State currency, is to the value of one dollar of the required State currency, so is the currency given in the question, to the sum required.

* The exchange being at par.

EXAMPLES.

Ex. 1. How much Maryland currency must I have for 3500*l.* of New York currency?

s.		*s.*	*d.*		*L.*
8	:	7	6	: :	3500
		12			20
		90			70000
					90
					8)6300000
					12)787500
					2,0)6562,5

Answer, *L.* 3281 5

Ex. 2. In 576*l.* 10*s.* New England currency, how much South Carolina? Answer, 448*l.* 7*s.* $9\frac{1}{3}d.$

Ex. 3. Bring 6274*l.* 5*s.* South Carolina Currency, to North Carolina currency? Answer, 10755*l.* 17*s.* $1\frac{5}{7}d.$

Ex. 4. How much Canada currency in 5000*l.* New York? Answer, 3125*l.*

Ex. 5. In 464*l.* 7*s.* 8*d.* Pennsylvania currency, how much South Carolina? Answer, 288*l.* 18*s.* 11*d.* $\frac{4}{45}$

Ex. 6. In 694*l.* 13*s.* 4*d.* Maryland currency, how much New York currency?

Answer, 740*l.* 19*s.* $6\frac{2}{3}d.$

Ex. 7. Exchange for South Carolina currency 1000*l.* Canada currency? Answer. 933*l.* 6*s.* 8*d.*

Ex. 8. How much Georgia currency in 426*l.* 12*s.* 4*d.* New Jersey? Answer, 265*l.* 9*s.* 0*d.* $\frac{4}{45}$

AMERICA AND THE WEST INDIES.

Accounts are kept in these places as in England, in pounds, shillings, and pence.

Ex. 1. London remits to Barbadoes 945*l.* 17*s.* sterling, how much currency will this amount to, when the exchange is 140 currency?

Answer, 1324*l.* 3*s.* 9½*d.*

Ex. 2. Sir Francis Baring writes word, that he has received for me a remittance of one quarter's dividend on 4000 dollars, at 5½ per cent. interest and the exchange is 164 per cent. what has he to pay me?

In this case the regular interest is 55 dollars, which at 4*s.* 6*d.* each, when exchange is at par, or at 166*l.* 13*s.* 4*d.*, would be 12*l.* 17*s.* 6*d.*, but the exchange is 164; therefore I say,

As 164 : 166*l.* 13*s.* 4*d.* :: 12*l.* 7*s.* 6*d.* : 12*l.* 11*s.* 6*d.* Ans.

Or by decimals,

164 : 166.66, &c. :: 55 : 55.899 dollars = 12*l.* 11*s.* 6.*d*

The following is a *Table* of the *Course of Exchange*, taken with slight variations from the Monthly Magazine for the 1st of May, 1808.

COURSE OF EXCHANGE.

	April 5.		April 12.	
Hamburgh gives	34.5	——	34 6	for 1*l.*
Altona - gives	34.7	——	34.7	for do
Amsterdam gives	35.5.2U.	——	35.4.2.U.	do
Ditto, sight gives	34.9	——	34.8	for do
Paris, 1.d. gives	23.13	——	1.d.24.0	for do[rials
Leghorn receives	49¾ pence	——	49¾	for 1 pezza of 8
Naples ditto	42 ditto	——	42	for 1 ducat
Genoa ditto	45 ditto	——	45½	for 1 pezza
Lisbon ditto	60 ditto	——	60	for 1 milrea
Oporto ditto	65 ditto	——	65	for 1 milrea
Madrid ditto,	38¾ doEff	——	—	for 1 dollar
Palermo ditto	92 per oz.	——	92	per oz.
Dublin ditto	110¼*l.*	——	110	for 100
Agio of Bank of Holland	6½ per cent		6¾ per cent.	

This table, in addition to what is gone before, will afford an opportunity of explaining every thing that a man of business will wish to be acquainted with.

On the 5th of April, the exchange between Hamburgh and London was at the rate of 34 schillings, 5 grotes for a pound sterling; that is, if a merchant in London sell a bill on Hamburgh for 500*l.*, he would be paid for it $34.5 \times 500 = 17208$ schillings, 4 grotes; but on the 12th, such bill would have fetched $34.6 \times 500 = 17250$ schillings. Here, the higher the exchange the greater the advantage to England; for the merchant, in this instance, gains 41 schillings, 8 grotes, by the rise in the exchange.

For Altona, the course of exchange is the same on both days, viz. the *L.* is worth 34 schillings, 7 grotes; and for Amsterdam, the course of exchange falling, the merchant in London would be a loser, who put off his market from the 5th to the 12th.

In this case 35.5 2U. means, that a pound sterling is worth, on the 5th 35 schillings, 5 grotes, allowing it to be payable at two months' date: but if it is payable at sight, it is then worth only 34 schillings 9 grotes. This difference, which on a bill of 100*l.* is equal to 34 schillings 4 grotes, is instead of the interest of money for the interval.

The course of exchange rose between London and Paris from the 5th to the 12th of April. On the first of these days 1*l.* was at 1.d., that is, at one day's sight, worth 23.13, or 23 francs, and 13 cents.; but on the 12th its value was 24 francs.

Leghorn receives $49\frac{3}{4}$ pence for 1 pezza of 8 rials, that is, a bill of exchange of 5000 pezza would be worth 4*s.* $1\frac{3}{4}$*d.* multiplied by 5000, or 1036*l.* 9*s.* 2*d.* A Naples ducat was worth 3*s.* 6*d.*: a Genoa pezza 3*s.* 9*d.*: a milrea of Lisbon 5 shillings, one of Oporto 5*s.* 5*d.*

Madrid receives 38¾*d.* *Eff.* for 1 piastre of 8 rials,* that is, a Spanish piastre of exchange was worth 3*s.* 2¾*d.*

A species of paper money, denominated *vales rials*, is circulated in Spain, the value of which, independently of interest on them, is this:—Vales rials for 600 dollars are worth 9035 rials, 10 maravedies of *vellon*,† that is, as 34 maravedies is equal to one rial, 1 dollar payable in this sort of paper is worth 15 rials, 2 maravedies. The paper is transferable by indorsement; and, by law, should be received in payment according to the nominal value; but as it experiences depreciation, it is necessary in drawing on Spain for effective money, to insert the words "*payable in effective*" in the body of the bill, which might otherwise be payable in vales rials: hence the word *Eff.* in the table, which is an abridgment of "*in effective*".

NOTES.

* In some parts of Spain they reckon by silver money, which is of two kinds, viz. old and new plate, the former is the most valuable: thus the piastre of exchange consists of 8 rials old plate, or of 10 rials new plate, the rial being at the par of exchange worth little more than 5¼*d.*

† The copper money of Spain is called vellon.

In Madrid, and the principal places of Spain, accounts are kept in piastres (called also *dollars*) rials, and maravedies; and sometimes in ducats.

TABLE.

				s.	*d.*
34 maravedies	make	1 rial	=	0	5⅜
8 rials		1 piastre	=	3	7
375 maravedies		1 ducate	=	4	11⅜

Hence the piastre at par is 3*s.* 7*d.*, and the ducate at par 4*s.* 11¼.; but the course of exchange of the piastre varies from 35 to 45 pence.

Palermo 92 pence per oz. In Sicily exchanges are made per onza by the ounce of Silver, for which on the day referred to Palermo, received 92 pence, or 7*s.* 8*d.**

Dublin 110¼ for 100*l.* that is, at the date of the table there would have been given on the exchange of London a bill on Dublin for 110*l.* 5*s.* for 100*l.* sterling. See page 275.

By the agio of the Bank of Holland is meant, as we have seen, page 269, the difference between cash and bank money, which, by the table, is on the 5th of April, 6½, or 6*l.* 10*s.* per cent.; that is, 106*l.* 10*s.* currency must be given for 100*l.* bank, and so in proportion.

Exchange between London and other Places in this Country.

The several cities, towns, &c. in Great Britain, exchange with London for a small premium in favour of London, as from ½ to 1 or 1½ per cent. The premium is more or less according to the greater or less distance, and according to the demand for bills.

Ex. York draws on London for 560*l.* 10*s.* exchange being ¾ per cent.; how much money must be paid at York for the bill?

½	¹/₂₀₀	560	10	0
¼	½	2	16	0½
		1	8	0¼
		*L.*564	14	0¾

To avoid paying the premium, which in some cases, would not be just, it is the usual practice to take the bill payable a certain number of days after date. On this principle, interest being 5 per cent., 73. days are equivalent to 1*l.* per cent. because $\frac{365}{5} = 73$.

NOTE.

* The Sicilian ounce is 600 grains, and the monies are regulated by the following Table:

10 grains - make - 1 carlin,
2 carlins - make - 1 tarin,
30 tarins - (600 gr.) - 1 ounce.

A crown (seudo) is equal 240 grs., therefore 5 crowns =2 ounces.

Ex. A friend at Exeter has received for me 68 guineas, in which he is no ways interested, and having no means of sending the money but by a bill of exchange, he agrees with his banker to draw it 30 days after date, rather than pay the premium of $\frac{1}{2}$ per cent., is my friend, or the banker, the gainer, allowing 5 per cent. ?

Answer, the banker loses 1*s.* 2*d.* of his usual profit.

EXAMPLES FOR PRACTICE.

Ex. 1. How much currency will 6630 guilders, bank-money, be worth in Holland, agio, being $8\frac{1}{4}$ per cent. ?

Answer, 7176 guilders, 39 grotes.

Ex. 2. What is the agio of 3310 guilders, $6\frac{1}{4}$ per cent.?

Answer, 206 guilders, 35 grotes.

Ex. 3. A London merchant draws on Amsterdam for 1564*l.* sterling; how many pounds Flemish, and how many guilders will that amount to, exchange being 34 schil. 8 gro. per *L.* sterling. See table, page 270.

Answer, 2710 18 8=pounds Flemish—16265 24 guilders.

Ex. 4. How much sterling money will pay a Portuguese bill of exchange of 1654 ⌢ 372 millreas; that is, of 1654 millreas and 372 reas, exchange being $65\frac{1}{2}$ pence sterling per millrea ?* Answer, 451*l.* 10*s.* $1\frac{1}{4}d.$ $\frac{464}{1000}$.

NOTE.

*In Portugal accounts are kept in reas and millreas, the latter being equal to 1000 of the former; and they are distinguished from each other by some such mark as that in the question.

The millrea, in exchange with this country, is at par $67\frac{1}{2}$ sterling or 5*s.* $7\frac{1}{2}$ sterling, and the course usually runs from 5*s.* 3*d.* to 5*s.* 8*d.*

TABLE—Par in sterling.

				s. d. f.
		1 rea	=	0 0 0.27
400 reas	make	1 crusade	=	2 3 0
1000 reas	make	1 millrea	=	5 7 $\frac{1}{2}$

The reas being the thousandth parts of the millreas, are annexed to the integer, and the work proceeds as in decimals.

Ex. 5. How many Portuguese reas will 750*l.* sterling amount to, exchange being $64\frac{5}{8}$ per millrea ?

Answer, 2785 milr. 299 reas $\frac{5}{6}\frac{2}{4}\frac{1}{6}\frac{2}{2}\frac{5}{5}$.

Ex. 6. A Spanish merchant imports from Seville, goods to the value of 1081 piastres, 6 rials : how much sterling money will this amount to, exchange being, on the day of payment, $41\frac{1}{2}$ pence per piastre ? See Table, page 277. ? Ans. 187*l.* 1s. $0\frac{1}{2}d.$

Ex. 7. I want to purchase goods at Cadiz, and for this purpose pay into a Spanish house 1000*l.* : how much value, in piastres, may I expect, exchange being 3s. $6\frac{1}{2}d.$ per piastre ? Answer, $5647\frac{2}{4}\frac{5}{2}\frac{5}{5}$ piastres,

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ARBITRATION OF EXCHANGES.

The course of exchange, between nation and nation, naturally rises or falls, as we have seen, according as the circumstances and balance of trade may happen to vary. To draw upon, and to remit money to foreign places, in this fluctuating state of exchange, in the way that will turn out most profitable is the design of arbitration.

Arbitration of Exchange, then, is a method of finding such a rate of exchange between any two places, as shall be in proportion with the rates assigned between each of them and a third place.

By comparing the par of exchange thus found, with the present course of exchange, a person is enabled to find which way to draw bills or remit the same to most advantage.

Arbitration of exchange, is either *simple* or *compound*.

In *simple* arbitration, the rates of exchange from one place to two others are given, by which is found the correspondent price between the said two places, called the arbitrated price.

An example or two will make the subject clear.

Ex. 1. If exchange between London and Amsterdam be 34 schil. 9 grotes per *L.* sterling, and if exchange between London and Genoa be 45 pence per pezza what is the par of arbitration between Amsterdam and Genoa:

Here $1l.=240$ pence: therefore, as

$$240d. : 34s.\ 9\ gr. : : 45d. : 78\tfrac{45}{240}\ gr.$$

Answer, 78 Flemish grotes, or pence per pezza Genoa

Ex. 2. If exchange from London to Amsterdam 35s. 9*d.* per *L* and if exchange from London to Paris be 32*d.* per crown, what must be the rate of exchange from Amsterdam to Paris? Ans. 54*d.* Flemish per crown.

Ex. 3. If exchange from Paris to London be 32*d.* per crown, and if exchange from Paris to Amsterdam be 54*d.* Flemish per crown, what must be the rate of exchange between London and Amsterdam, in order to be on a par with the other two? Answer, 33 9 per *L.*

Ex. 4. Amsterdam exchanges on London at 35 schil. 5 grotes per *L.* sterling; and the exchange between London and Lisbon is 60 pence per millrea, what is the exchange between Amsterdam and Lisbon?

Ans. 106.25 grotes.

The course of exchange being given, and the par of arbitration found, we obtain a method of drawing and remitting to advantage.

Ex. 5. If exchange from London to Paris be 32 pence sterling per crown, and to Amsterdam 405 Flemish per *L.*, and if I learn that the course of exchange between Paris and Amsterdam is fallen to 52 pence Flemish per crown: what may be gained per cent., by drawing on Paris and remitting to Amsterdam?

By Ex. 2, the par of arbitration between Paris and Amsterdam is 54*d.* Flemish per crown; then

d. cr. *L.* cr.
32 : 1 :: 100 : 750 drawn at Paris.

cr. *d.*Fl. cr. *d.*Fl.
1 : 52 :: 750 : 39000 credit at Amsterdam.

*d.*Fl. *L.* *d.*Fl. *L.* *s.* *d.*
405 : 1 :: 39000 : 96 5 11 to be remitted.

therefore 100*l.*—96*l.* 5*s.* 11*d.* = 3*l.* 14*s.* 1*d.*=gain per cent.

If the course of exchange between Paris and Amsterdam be at 56 Flemish per crown, instead of 52; and if I would gain by the negociation, I must draw on Amsterdam and remit to Paris; thus,

L. *d.*Fl. *L.* *d.*Fl.
1 : 405 :: 100 : 40500 drawn at Amsterdam.

*d.*Fl. cr. *d.*Fl. cr.
56 : 1 :: 40500 : 723 credit at Paris.

cr. *d.* cr. *L.* *s.*
1 : 32 :: 723 : 96 8

therefore 100*l.*—96*l.* 8*s.* = 3*l.* 12*s.* gain per cent.

COMPOUND ARBITRATION.

In Compound Arbitration, the rate of Exchange between three or more places is given, to find how much a remittance passing through them all will amount to at the last place: or to find the arbitrated price, or par of arbitration, between the first and last place.

Examples of this kind may be worked by several successive statings in the Rule of Three, or according to the following Rules:

(1) Distinguish the given rates, or prices, into antecedents and consequents, placing the antecedents in one column, and the consequents in another, with the sign of equality between them.

(2) The first antecedent, and the last consequent to which an antecedent is required, must be of the same kind.

(3) The second antecedent must be of the same kind with the first consequent, and the third antecedent of the same kind with the second consequent, &c.

(4) Multiply the antecedents together for a divisor, and the consequents together for a dividend, and the quotient will be the answer required.

Ex. If a merchant in London remit 500*l.* sterling to Spain by way of Holland, at 35 shillings Flemish per pound sterling, thence to France at 58 pence per crown, thence to Venice at 10 crowns for 6 ducats, and thence to Spain at 360 mervadies per ducat; how many piastres of 272 mervadies will the 500*l.* amount to in Spain?

1*l.*	=	35*s.* or 420*d.* Fl.
58*d.*	=	1 crown
10 cr.	=	6 ducats
1 duc.	=	360 mervadies
272 mer.	=	1 piastre
How many piastres	=	500*l.*

Omitting the units, we have by the rule,

$$\frac{420 \times 6 \times 360 \times 500}{58 \times 10 \times 272}$$ and this fraction reduced to its lowest terms, gives $$\frac{21 \times 3 \times 45 \times 500}{29 \times 17} = \frac{1417500}{493} =$$ 2875¼ piastres, which is the answer.

By the Rule of Three we should have said,

1*l.*	:	420*d.*	: :	500*l.*	:	210000*d.*
58*d.*	:	1 cr.	: :	210000*d.*	:	3620 cr.*
10 cr.	:	6 duc.	: :	3620 cr.	:	2172 duc.
1 duc.	:	360 mer.	: :	2172 duc.	:	781920 mer.
272 mer.	:	1 pias.	: :	781920mer:		2875½ pias.

If the course of direct exchange to Spain were 42½ pence sterling, then 500*l.* remitted would only amount to 2823½ piastres, of course 2875½—2823½, gives 52, which is the number of piastres gained by the negotiation.

DUODECIMALS.

DUODECIMALS, or *Cross Multiplication*, is made use of by artificers in measuring their several works, and is performed by means of the following table;

12''''	fourths	-	make 1	third.
12'''	thirds	-	- 1	second.
12''	seconds	-	- 1	inch.
12'	inches	-	- 1	foot.

Glaziers, Masons, and others, measure by the square foot.—Painters, Paviors, Plasterers, &c., by the square yard.—Slating, tiling, flooring, &c. by the square of 100 feet.—Brickwork is measured by the rod of 16½ feet, the square of which is 272¼.

RULE. (1) Arrange the terms of the multiplier under the same denomination of the multiplicand. (2) Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and write the result of each under its respective term, observing to carry one for every twelve. (3) Multiply, in the same manner, by the inches, and set the result of each term one place re-

NOTE.

* The fractions are omitted, and on that account the answer by this method will not be quite accurate.

moved to the right-hand of those in the multiplicand.* (4) Multiply then by the seconds, setting the result of each term two places removed to the right-hand of those in the multiplicand.

Multiply 9 ft. 4 in. 8 sec. by 5 ft. 8 in. 6 sec.

9	4	8			
5	8	6			
46	11	4			
6	3	1	4′′′		
	4	8	4	0′′′′	
53	7	1	8	0	

Ex. 1. How much must I pay for a slab of marble 7 ft. 4 in. long, and 2 ft. 1 in. 6 sec. broad, at the rate of 7*s.* per square foot?

Answer, 5*l.* 9*s.* 1*d.*

Ex. 2. What will be the expence of glass for a window that measures, in the clear 10 ft. 6½ in. in height, and 4 ft. 9 in. in width, at 1*s.* 9*d.* per foot?

Answer, 4*l.* 7*s.* 6*d.*

Ex. 3. How much will a room cost in painting, at 9½*d.* per yard; the sides are 18 ft. 10 in. by 10 ft. 3 in. and the two ends are 16 ft. 6 in. by 10 ft. 3 in.?

Answer, 3*l.* 3*s.* 8½*d.*

Ex. 4. What shall I have to pay for statuary marble about my fire-place, at 14*s.* per foot; the hearth measures 6 ft. 4 in. by 2 ft. 3 in., the three fronts are each 4 ft. 2 in. by 8 in., and the mantle-piece slab is 6 ft. by 9 in.? Answer, 18*l* 19*s.*

Ex. 5. What will the paving of a court-yard come to, at 1*s.* 2*d.* per foot, the yard being 74 feet long, and 56 ft. 8 in. wide? Answer, 244*l.* 12*s.* 2½*d.*

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* Feet multiplied into feet give feet.
Feet multiplied into inches give inches
Feet multiplied into seconds give seconds.
Inches multiplied into inches give seconds.
Inches multiplied into seconds give thirds.
Seconds multiplied into seconds give fourths.

Ex. 6. How much shall I have to pay for slating a house, consisting of two sloping sides, each measuring 24 ft. 5 in. by 15 ft. 9 in. at the rate of 41*s.* per square of 100 feet? Answer, 15*l.* 18*s.* 7*d.*

Ex. 7. What will the tiling of 10 houses come to, the roof of each house consisting of two sides, each 18 feet by 14, and the price of tiling at 28*s.* per square?
Answer, 70*l.* 11*s.* $2\frac{1}{2}$*d.*

Ex. 8. How many square rods are there in a brick wall 44 ft. 6 in. long, and 7 ft. 4 in. high, and $2\frac{1}{2}$ bricks thick?* Answer, 2 rods nearly.

Ex. 9. If an oblong garden be 254 ft. 6 in. long, and 184 ft. 8 in. wide, what will a wall cost 10 ft. 6 in. high, and $2\frac{1}{2}$ bricks thick, at 15*l.* 15*s.* per square rod?
Answer, 888*l.* 6*s.*

Ex. 10. How much shall I have to pay for the plate-glass of four windows; each window consists of 16 panes, and each pane measures $20\frac{1}{2}$ inches by $15\frac{3}{4}$ inches at 9*s.* 6*d.* per foot? Answer, 68*l.* 3*s.* 3*d.*

NOTE.

* Bricklayers value their work at the rate of a brick and a half, or three half bricks thick; and if the wall be more or less than this, it must be reduced to that thickness by the following rule:—"Multiply the measure found by the number of half bricks, and divide by three:" thus, if the wall be $2\frac{1}{2}$ bricks thick, I multiply by 5, and divide the product by 3.

Ex. If the wall be 50 feet long, and 9 high, and 2 bricks thick, it will be $50 \times 9 \times \frac{4}{3} = 600$ feet; and $\frac{600}{272\frac{1}{4}} = 2\frac{1}{4}$ square rods nearly.