TECHNICAL REPORT

ASWEPS REPORT NO. 10

PREDICTION OF THE 400-FOOT TEMPERATURE IN THE NORTH ATLANTIC

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ABSTRACT

BT data from five ocean weather stations in the western North Atlantic were used to develop equations for time distribution of mean temperatures at the 400-foot level. Given variables in the equations are mean annual surface temperature and annual amplitude (half annual range) of surface temperature; both are plotted on individual charts. Other variables, such as reduction of mean annual temperature at 400 feet with respect to the surface, reduction of annual temperature amplitude at 400 feet with respect to the surface, and phase angle at 400 feet are given as functions of latitude. Four equations, suitable for certain areas of deep water beyond the continental shelf above 20°N in the North Atlantic, are presented.

The actual temperature at 400 feet is computed by adding the surface temperature anomaly to the mean temperature at 400 feet if anomalies at the surface and 400 feet are correlated or by adding the specific anomaly at 400 feet to the mean temperature at 400 feet if anomalies at the surface and 400 feet are uncorrelated.

TR-147

ERRATA SHEET

The following corrections should be made on the indicated pages:

Page	
1	line 19, should read "$\alpha_1, \alpha_2, \ldots =$ phase angles."
3	line 16, $\theta_{z,t}$ = temperature at depth z at time t
3	line 21, should read "$\alpha_1, \alpha_2, \ldots$ vice $\alpha_2, \alpha_2, \ldots$"
3	line 4 from bottom; "possibly vice possible".
16	Table 3, first factor should be $\lambda = \frac{\ell}{\cos\phi}$
20	Table 5, 4th column total for HOTEL should read 37
20	Table 5, 4th column total for COMBINED ATLANTIC OWS should read 238.
23	line 12, a total of 238 vice 237.
30	4th paragraph, delete "(when anomalies are correlated)".

FOREWORD

Prediction of characteristics of the thermal structure of the oceans is an important facet of the ASWEPS program. This report, describing a method of predicting temperature at the 400-foot depth, is a sequel to a previous Technical Report, TR-104, in which the author attempts to predict the depth of the thermocline. The importance of the estimation of the temperature at 400 feet was emphasized in TR-104. The present report contains important new climatic charts of the surface temperature parameter as well as new insight into behavior of temperature at depth.

This report was prepared while the author was employed by the U. S. Naval Oceanographic Office. He is presently employed by the Bureau of Commercial Fisheries. Comments and suggestions related to use of this report should be addressed to the U. S. Naval Oceanographic Office.

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I. INTRODUCTION

A. Temperature Distribution in the Upper Layers of the North Atlantic

Annual temperature cycles occur in most parts of oceans, seas, and lakes. For purposes of this discussion, a surface layer about 150 meters thick is considered. The layer may be isothermal or nearly isothermal during winter. During the remainder of the year it is stratified with a mixed layer at its surface, a seasonal thermocline (rapidly decreasing temperature) below the mixed layer, and moderately decreasing temperature below the thermocline.

The annual march of temperature in the mixed layer at any given location in the ocean has a well-defined pattern and can be approximated without much difficulty by the expression:

$$\theta_0 = \bar{\theta}_{s,a} + A_1 \cos(\omega t + \alpha_1) + A_2 \cos(2\omega t + \alpha_2) + \cdots \qquad (1)$$

where $\bar{\theta}_{s,a}$ = mean annual surface temperature at the location

$A_1, A_2 \cdots$ = amplitudes of temperature oscillation for various harmonics

ω = angular frequency equal to $\frac{2\pi}{T}$, where T = period length of the first harmonic term

t = time, and

α_1, α_2 = phase angles.

Surface temperature could always be computed by equation (1) if the annual march of temperature depended strictly on predictable factors. However, this is not the case, because nonperiodic factors are always present and actual temperature usually differs from the mean. The departure of actual surface temperature, θ_0', from θ_0 constitutes the surface temperature anomaly $\Delta\theta_0$ at a given location at a given time, thus

$$\Delta\theta_0 = \theta_0' - \theta_0$$

Equation (1) is not needed for mean surface temperature computation for most areas in the North Atlantic where surface temperature data have been collected for many years, and for which mean monthly surface temperature charts, based on sufficient data, are available. Mean daily temperatures can be interpolated from the mean monthly charts with satisfactory approximation. If equation (1) is applied for computation of mean surface temperature, amplitudes and phase angles should be either based on characteristics of the region of study or determined as functions of latitude.

The mean annual range of surface temperature in the Atlantic is about 6°F along latitude 20°N, increasing to about 16°F in the Sargasso Sea, about 20°F in the Gulf Stream, and about 30°F in coastal waters. Surface temperature anomalies cause the actual range for any one year to vary from the mean annual range. The mean annual range of surface temperature between 20° and 70°N in the North Atlantic is about 46°F. Annual temperature oscillations and average anomalies occurring at these two latitudes increase the range to about 55°F.

The annual march of temperature below the thermocline is essentially a trigonometric function of time expressed in a manner similar to equation (1). However, there will be different mean annual temperatures, amplitudes, and lag angles for each depth and location. It is not possible to compute and plot mean monthly temperature charts for subsurface levels as can be done for the surface. Sufficient data for constructing such charts will not be available for some time. Therefore, this study attempts to determine the main characteristics of temperature distribution and to devise a temperature prediction method at a level close to the thermocline but always below it during seasonal stratification and in the mixed layer in winter.

Existing BT data are satisfactory only for limited areas (near weather stations) to depths of only 400 feet; therefore required conditions for formulation of a prediction method are approached closest at 400 feet but are not optimum. In rather wide zones of tropical convergence the 400-foot level is often in the upper part of the thermocline in winter and in the lower part of the thermocline in summer. At temperate and higher latitudes, the 400-foot level is in the thermocline during the transition period in autumn. Temperature at the 400-foot level is always subject to internal wave oscillations.

The mean annual temperature range at 400 feet in the North Atlantic seldom exceeds 4°F at a given location; however, temperature anomalies at 400 feet are generally of the same magnitude as surface temperature anomalies. Therefore, temperature oscillations at 400 feet may more than double the normal annual range at a given location.

The mean annual temperature range at 400 feet for the area between 20° and 70°N is about 40°F — approximately 10°F smaller than the surface range. This range is considerably augmented by anomalies.

Temporal anomalies of temperature at 400 feet, although usually related to surface anomalies, often have quite independent characteristics and origins.

Short-term oscillations caused by internal waves at 400 feet do not occur at the surface. On the other hand, short-term oscillations caused by diurnal heating processes at the surface are imperceptible at 400 feet.

II. THEORY

A. Heat Transfer

The vertical temperature conduction in a uniform field (no advection) is expressed by the equation:

$$\frac{\partial \theta}{\partial t} - \frac{\partial}{\partial z}\left(\frac{\mu}{\rho}\,\frac{\partial \theta}{\partial z}\right) = 0 \tag{2}$$

where θ = temperature

t = time

μ = eddy conductivity, and

ρ = density.

Assuming $\frac{\mu}{\rho}$ is constant, that average temperature is a linear function of depth, and that the annual march of surface temperature is closely approximated by equation (1), the solution of equation (2) with use of notations used in reference 1 is:

$$\theta_{z,t} = \bar{\theta}_{s,a} + \ell_z + A_1 e^{-r_1 z}\cos(\omega t - \alpha_1 - r_1 z) + A_2 e^{-r_2 z}\cos(2\omega t - \alpha_2 - r_2 z) + \cdots \tag{3}$$

where $\theta_{z,t}$ = temperature at depth at time

$\bar{\theta}_{s,a}$ = mean annual surface temperature

ℓ_z = constant determining mean annual temperature at a depth z with respect to $\bar{\theta}_{s,a}$

$A_1, A_2, \cdots$ = annual surface temperature amplitudes for various harmonics

$\alpha_2, \alpha_2, \cdots$ = phase angles for various harmonics at the surface

$r_1 = \sqrt{\frac{\omega\rho}{2\mu}}$, $r_2 = \sqrt{\frac{\omega\rho}{\mu}}$ ... , and $\omega = \frac{2\pi}{T}$

where T = **fundamental period**.

The assumptions introduced in integrating equation (2) do not correspond to conditions in nature — eddy conductivity, μ , is variable with depth and time. The average temperature is not a linear function of depth, and the linear phase angle variation with depth can hardly be expected to be of the same magnitude as the exponent of amplitude variation. In addition, all these factors quite obviously vary with latitude.

Parameters assumed to be constant in equation (3) are certainly not constant but are functions of depth, time, latitude, flow, and possible several other factors. However, equation (3) can serve as a general frame, because the present problem is limited to temperature distribution at one level. At a given location these parameters would be constant at a given depth, assuming that temporal variability of the exponent would be accounted for by the cosine terms in the equation. In that case, exponent, phase angle variation, and reduction of mean annual temperature at a given level with respect to the surface must be determined individually by empirical means.

These parameters can be determined by Fourier expansion of the long-term mean temperature distribution at the surface and at 400 feet in the same location. Variability of parameters with latitude can be determined by conveniently combining two or more locations. This course is followed below.

B. Areas of Data

Derivation of practical equations for temperature distribution at 400 feet requires adequate simultaneous data at the surface and 400 feet. Obviously, only BT data are available for this purpose, and only BT traces reaching 400 feet can be applied. This requirement eliminates more than half of extant BT observations.

Data collected for the ocean weather stations are as follows:

Location	Period	Number of observations
BRAVO (56°30'N, 51°W)	1955-1960	3,563
CHARLIE (52°48'N, 35°30'W)	1945-1958	3,598
DELTA (44°N, 41°W)	1944-1958	2,251
ECHO (35°N, 48°W)	1944-1958	7,669
HOTEL (36°N, 70°W)	1944-1954	2,710

In addition to data from the ocean weather stations, data were obtained from the National Oceanographic Data Center for the following areas delineated in figure 1.

Area		Number of observations
F		290
G		442
H subarea	H_1	74
	H_2	249
	H_3	624
	H_4	322

The data collected in areas F, G, and H were assumed to be valid for the mean latitude (ϕ) of each area. The number of useful observations was considerably larger than given above; however, some large groups of data covered restricted areas in relatively short periods of time, in which case, only one or a few of the observations were used for computing the long-term mean.

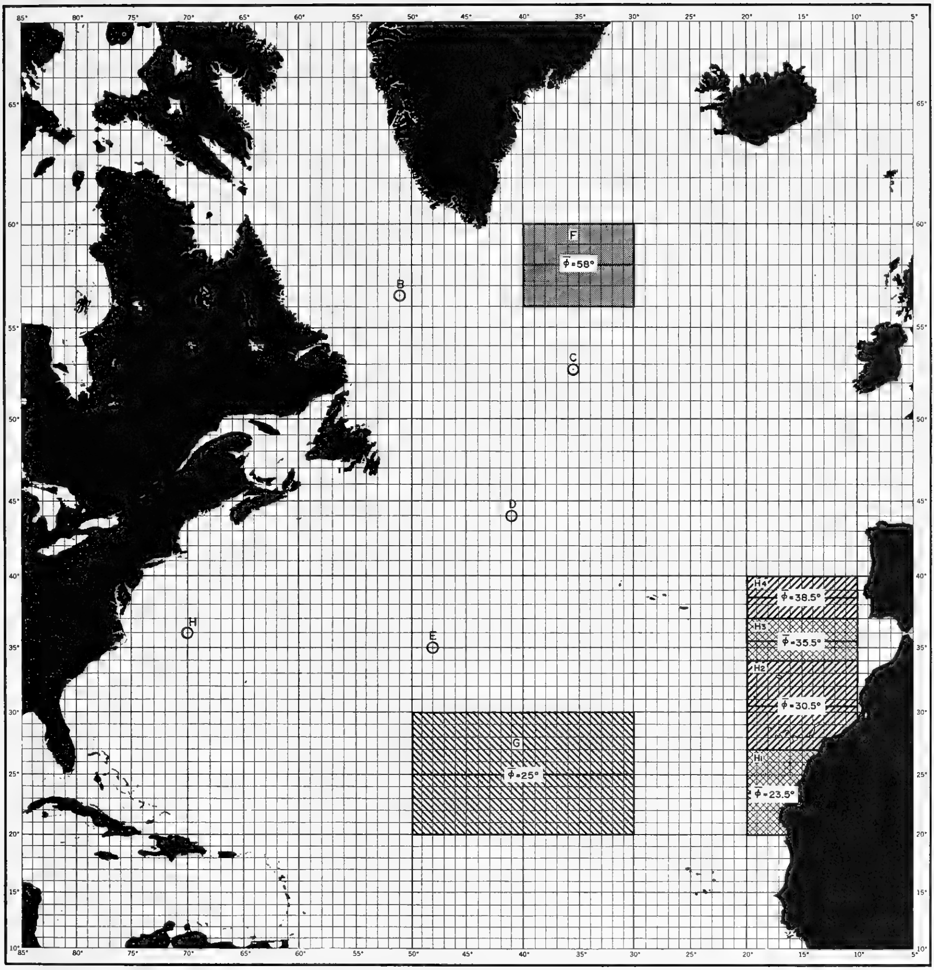

FIGURE I AREAS OF DATA

Individual BT observations are not sufficiently accurate for determining temperature distribution, because they often vary from actual values by several degrees. However, statistical parameters computed with

BT data collected over a satisfactory number of years, depending on approximation requirements, are quite accurate. Mean annual surface temperatures computed from the BT data for the locations in this study depart only 0.8°F, on the average, from mean annual surface temperatures computed from long-range data (approximately 100 years) as shown in the annual temperature chart (figure 2). This is a very good agreement when one considers the small amount of data available for this study, especially in areas F, G, and H, and allows considerable confidence in mean temperatures at 400 feet computed with the same BT data. Certainly, mean monthly temperatures may show greater variation; however, time smoothing will eliminate many inconsistencies. In general, mean annual and monthly temperatures at 400 feet computed with the BT data used in this study may be considered representative of these conditions.

C. Mean Temperature Time Series

Time-smoothed and unsmoothed mean temperature curves are shown for the surface and 400 feet at OWS BRAVO in figure 3 and at OWS ECHO in figure 4. Curves for area H mean latitudes of 23.5°N, 30.5°N, 35.5°N, and 38.5°N are shown in figure 5.

Mean temperature distributions at the ocean weather stations and along 58°N within area F have predominantly annual periods at the surface and 400 feet. The relationship between mean temperatures at the surface and 400 feet at a given location may be characterized by (a) difference of mean annual temperatures, (b) ratio of mean annual temperature amplitudes, and (c) phase lag angle. This relationship varies with latitude; mean annual temperature difference between two levels and mean annual temperature amplitudes decrease with increasing latitude. The phase lag angle, quite large in subtropical areas, approaches zero at high latitudes.

In areas G and H, mean temperature distributions at 400 feet show semiannual and shorter periods in addition to annual period. In area H there is no evidence of phase lag angle variation, but latitudinal variation of mean annual amplitudes and differences is quite conspicuous. Data were insufficient for comparing temperature distributions along two latitudes in area G; however, it is assumed that variations of mean annual temperature amplitudes and differences with latitude approach those which occur at the ocean weather stations and area H.

Normal probability functions have been applied to obtain smoothing operators for all curves. Seven discrete weights were used as ordinates of the normal curve at intervals of standard deviation, σ, so that the entire smoothing period (6σ) corresponds to 6 months. The values of weights are: 0.004, 0.054, 0.242, 0.400, 0.242, 0.054, 0.004. Rounding of weights to 3 decimals makes their sum equal to unity, so that the mean values of original and smoothed series remain unchanged. The principal weight corresponds to the ordinate of the normal curve at the mean. Since the smoothing operator is symmetrical with respect to the principal weight, the phase angle of the mean temperature distribution remains unchanged.

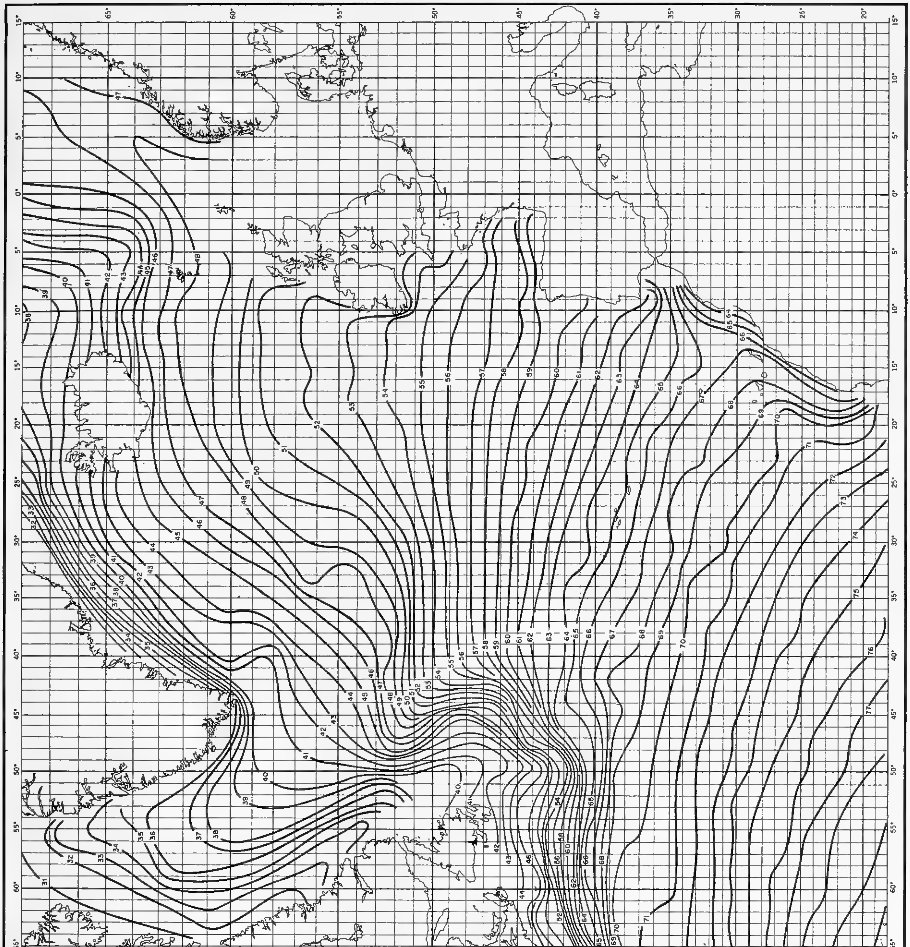

FIGURE 2 MEAN ANNUAL SEA SURFACE TEMPERATURE IN THE NORTH ATLANTIC (° F)

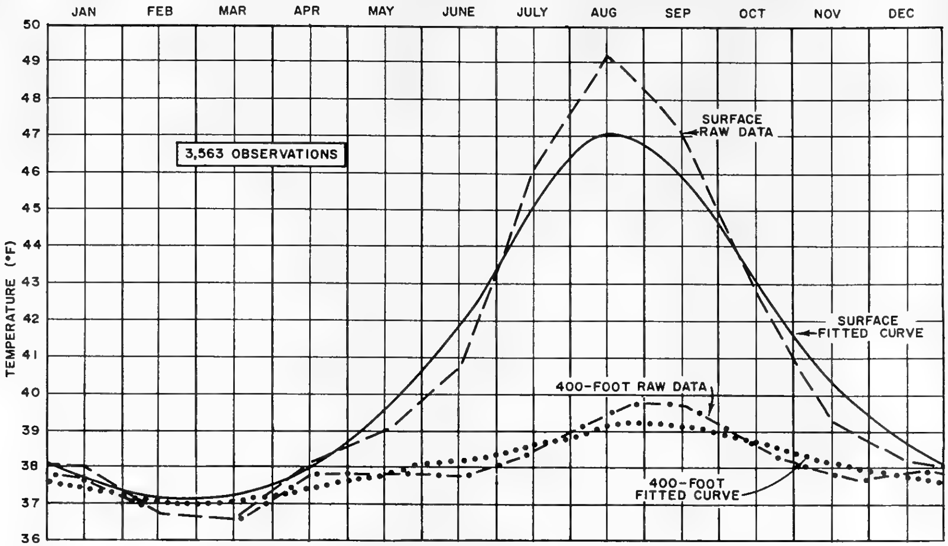

FIGURE 3 MEAN TEMPERATURE AT SURFACE AND 400 FEET AT OWS BRAVO

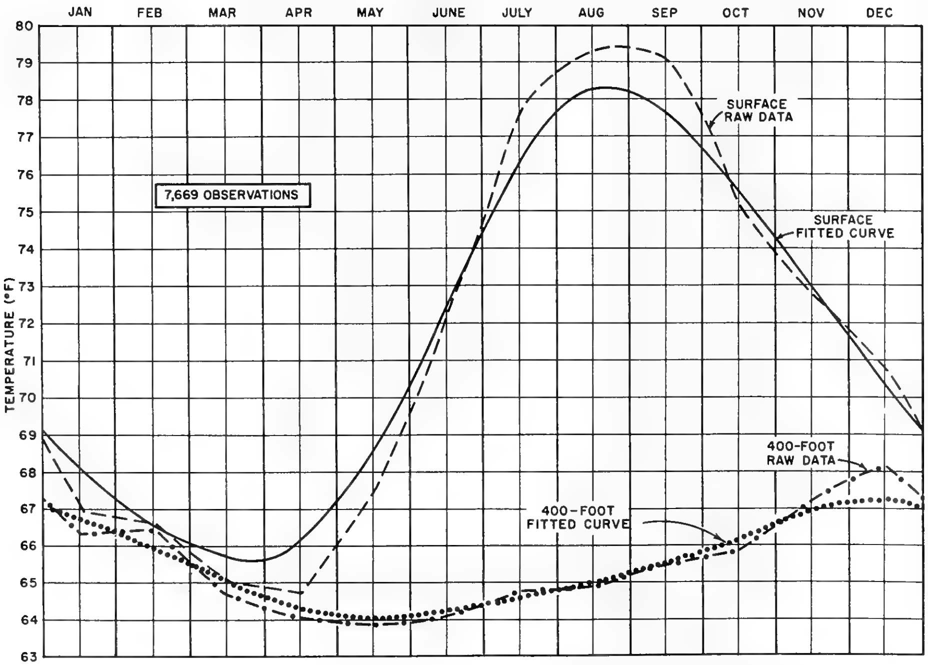

FIGURE 4 MEAN TEMPERATURE AT SURFACE AND 400 FEET AT OWS ECHO

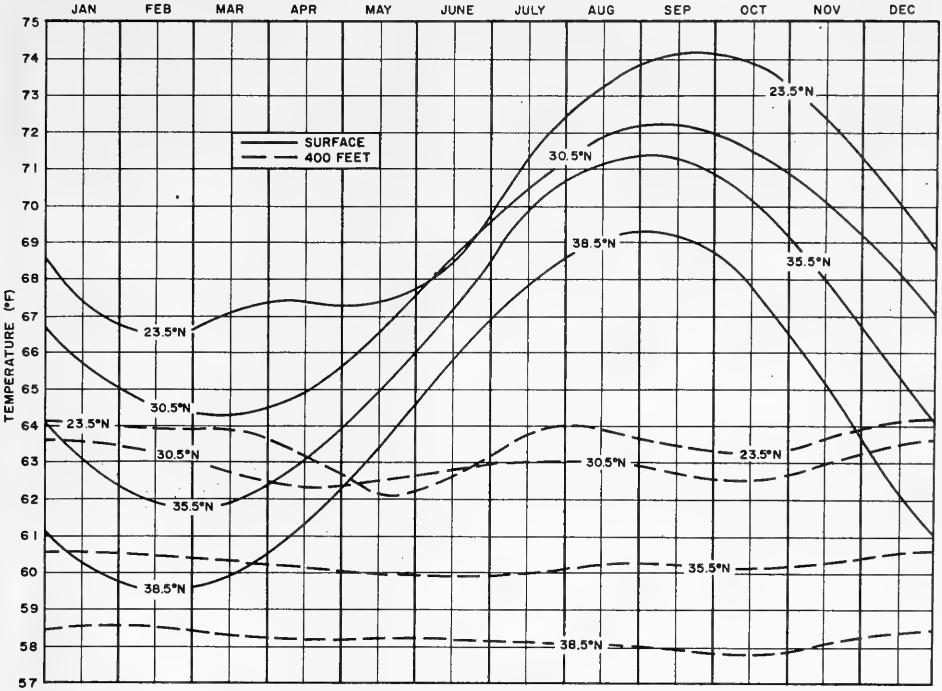

FIGURE 5 MEAN TEMPERATURE AT SURFACE AND 400 FEET IN AREA H

The theoretical frequency response (ratio between amplitudes of smoothed and unsmoothed waves) for the yearly curve (T = 12 months, f = 1/12) of this type of smoothing function **(reference 2) is:**

$$R(f) = e^{-2\pi^2 \sigma^2 f^2} = 0.88$$

Frequency responses of the surface curves at the 5 ocean weather stations agree closely with theoretical response and range between 0.79 and 0.88.

Frequency responses at 400 feet range between 0.47 and 0.78. The lowest value, which occurred at OWS CHARLIE, corresponds approximately to the theoretical frequency response for f = 1/5 and tends to indicate presence of "noise" with periods of several months. Such noise may occur in the data through concentration of large numbers of observations taken during periods of strong temperature anomalies. Such inconsistencies would be attenuated or nearly eliminated by use of long-term data. In this study the data periods are quite short; however, some areas may be more exposed to this type of error in data than others. Such "noise" distorts low amplitude annual waves more than it does high amplitude waves. The unsmoothed mean annual wave amplitude for OWS CHARLIE is 0.63°F. The mean for the other four stations is 2.74°F, ranging from 1.52° to 3.95°F with a mean frequency response of 0.73.

D. Mean Temperature Distribution Equations

Time-smoothed mean temperature distributions at the surface and 400 feet provide general information on distributions and their variability with latitude. Further exploitation of these distributions permits derivation of empirical models similar to equation (3) for various types of water in the North Atlantic. For this purpose smoothed curves were expanded in Fourier series of the type:

$$\theta(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} \left(a_n \sin n\omega t + b_n \cos n\omega t \right)$$

The constants a_o, a_n and b_n were computed for four harmonics. The amplitude and phase angle for each harmonic were determined by:

$$A_n = \sqrt{a_n^2 + b_n^2} \quad \text{and} \quad \alpha_n = -\tan^{-1}\left(\frac{a_n}{b_n}\right)$$

Since an equation is desired for one level only (below the seasonal thermocline at 400 feet), the parameters will not be functions of depth z. Phase angles at 400 feet are designated as $\alpha_{n,400}$ and are obtained directly from expansion of the 400-foot curves. Because phase angles at 400 feet were not computed in terms of surface phase angles, the phase lag factors may be discarded. With these simplifications equation (3) may be rewritten:

$$\theta_{400} = \bar{\theta}_{s,a} + \ell + Ae^{-r_1} \cos\left(\omega t - \alpha_{1,400}\right) + Ae^{-r_2} \cos\left(2\omega t - \alpha_{2,400}\right) + \cdots \quad (4)$$

where $\bar{\theta}_{s,a}$ is again the mean annual surface temperature, and A is the annual temperature amplitude of the smoothed curve for the surface. This general amplitude was substituted for individual amplitudes of each harmonic of the surface curve, because individual exponents of each harmonic at 400 feet were computed in terms of this general amplitude as well as in terms of the individual amplitudes of each harmonic of the 400-foot curve.

The parameter ℓ is obtained from $\frac{a_o}{2}$ values at the surface and at 400 feet from corresponding expansions

$$\ell = \left(\frac{a_o}{2}\right)_{400} - \left(\frac{a_o}{2}\right)_{surf} \quad (5)$$

The term $\left(\frac{a_o}{2}\right)_{400}$ is less than $\left(\frac{a_o}{2}\right)_{surf}$; therefore ℓ is negative.

The exponents for each harmonic are obtained by

$$r_1 = \ln\left(\frac{A}{A_{1,400}}\right), \; r_2 = \ln\left(\frac{A}{A_{2,400}}\right) \cdots \quad (6)$$

After the ℓ and r parameters were computed for all ocean weather stations being considered, variation of the parameters with latitude was determined by grouping the stations with similar oceanographic regimes.

CHARLIE and ECHO were combined as stations without permanent currents, and DELTA and HOTEL were paired as stations lying within permanent currents. This grouping appears to be justified by the latitudinal change of parameter ℓ . The relationship is quite simple.

$$
\begin{aligned}
\lambda &= \frac{\ell}{\cos\phi} = -7.52 \text{ for OWS CHARLIE} \\
\lambda &= \frac{\ell}{\cos\phi} = -7.36 \text{ for OWS ECHO} \\
\lambda &= \frac{\ell}{\cos\phi} = -5.16 \text{ for OWS DELTA} \\
\lambda &= \frac{\ell}{\cos\phi} = -5.86 \text{ for OWS HOTEL}
\end{aligned}
\qquad (7)
$$

A more elaborate function of latitude could be devised to arrive at identical values of coefficients λ for each group. However, despite deficiencies and limitations of the data from which the time smoothed distributions were obtained, the above values agree sufficiently to permit acceptance of a simple cosine function. The mean value of each group was accepted to be a generalized coefficient, λ . Parameter ℓ, valid at a given location, could then be replaced by $(\lambda \cos\phi)$ in the general equation for areas with similar oceanographic regimes (as has been done by combining ocean weather stations).

A simple sine function of latitude also yields satisfactorily close values of coefficients for at least the exponents of the first harmonics.

$$
S_1 = \frac{-r_1}{\sin\phi}, \quad S_2 = \frac{-r_2}{\sin\phi} \cdots \qquad (8)
$$

In general, mean values of S were accepted for each group of combined stations and for all harmonics. Scattering of S values can be expected to increase with increasing order of harmonics.

A summary of factors for the CHARLIE-ECHO area is given in table 1.

TABLE 1

MEAN TEMPERATURE DISTRIBUTION FACTORS AT 400 FEET FOR OWS CHARLIE-ECHO

Factor	CHARLIE	ECHO	Accepted Factor	$\phi = 58^{\circ}N$ (Area F, Figure 1)
$\lambda = \frac{\ell}{\cos \phi}$	-7.52	-7.36	$\bar{\lambda} = -7.44$	-4.94
$S_1 = \frac{-r_1}{\sin \phi}$	-3.64	-2.44	$S_1 = -2.44$	-2.22
$S_2 = \frac{-r_2}{\sin \phi}$	-4.21	-5.36	$\bar{S}_2 = -4.78$	-2.86
$\alpha_{1,400}$	$\boxed{88.0^{\circ}}$ 96.4°	$\boxed{27.0^{\circ}}$ 26.0°	$2.1\,\phi \sin \phi$	$\boxed{103.5^{\circ}}$ 102.2°
$\alpha_{2,400}$	$\boxed{131.0^{\circ}}$ 130.7°	$\boxed{356.0^{\circ}}$ 356.0°	$441 \sin (\phi - 35.5^{\circ})$	$\boxed{170.0^{\circ}}$ 136.3°

Factors for latitude 58°N, area F, are also given. Part of the latter area appears to lie in water similar to that of the CHARLIE-ECHO area, and another part appears to lie in water similar to that of the DELTA-HOTEL area, so that area F lies approximately in a transition zone. Therefore, properties of the temperature distribution at 58°N may approach those of either area. Data being very limited, factors for 58°N could not be used for determination of accepted values and are included in table 1 only for interpretation of the dubious factors of the CHARLIE-ECHO area.

The accepted factors (column 4, table 1) are based on values for OWS CHARLIE and ECHO. λ is a mean of two stations. S_1 is very large for CHARLIE because of extremely low frequency response in the smoothing process. The S_1 value for latitude 58°N agrees closely with the S_1 value for ECHO. This agreement favored acceptance of the ECHO value for the general equation without modification, because the S_1 factor for CHARLIE does not appear reliable.

The phase angles have been derived directly from expansion of the 400-foot curve. Values of these angles were made positive, and by simple manipulations the functions $2.1\,\phi \sin \phi$ and $441 \sin (\phi - 35.5^{\circ})$

were obtained for $\alpha_{1,400}$ and $\alpha_{2,400}$. The fitted values with these functions are shown in boxes (table 1) above the value computed from corresponding expansions.

By substituting the accepted values in equation (4), an empirical model for CHARLIE-ECHO conditions is obtained.

$$\bar{\theta}_{400} = \bar{\theta}_{s; a} - 7.44 \cos \phi + Ae^{-2.44 \sin \phi} \cos\left(\omega t + 2.1 \phi \sin \phi\right) + Ae^{-4.78 \sin \phi} \cos \left[2 \omega t + 441 \sin (\phi - 35.5)\right] \quad (9)$$

Equation (9) was limited to two harmonics, because constants a_3 , a_4 , b_3 , and b_4 were quite small and the corresponding harmonics could be neglected.

The second data area includes BRAVO, DELTA, and HOTEL. The accepted parameters are based mainly on DELTA and HOTEL. Parameters for BRAVO, though quite consistent in most cases, have been used only for reference and comparison, because BRAVO data extend over a period of only 5 years, and because the station borders on CHARLIE-ECHO type of conditions. A summary of factors for the DELTA-HOTEL area is shown in table 2.

TABLE 2

MEAN TEMPERATURE DISTRIBUTION FACTORS AT 400 FEET FOR OWS DELTA-HOTEL

Factor	DELTA	HOTEL	Accepted Factor	BRAVO
$\lambda = \frac{\ell}{\cos \phi}$	-5.16	-5.86	$\bar{\lambda}$ = -5.51	-5.35
$S_1 = \frac{-r_1}{\sin \phi}$	-1.71	-1.58	S_1 = -1.64	-1.98
$S_2 = \frac{-r_2}{\sin \phi}$	-2.82	-4.00	S_2 = -4.00	-4.88
$\alpha_{1,400}$	[106.3°] 109.8°	[90.0°] 90.4°	153 sin ø	[127.0°] 117.7°
$\alpha_{2,400}$	324.0°	166.0°	166.0°	131.0°

Accepted factors λ and S_1 are mean values of DELTA and HOTEL. S_2 at DELTA is quite small and is probably unreliable, because cold and warm water masses often interchange in this area in summer. This boundary effect produces an unrealistically large semiannual term. The BRAVO factor of S_2 tends to substantiate the HOTEL value; therefore the S_2 value of HOTEL was used as the accepted factor. The same uncertainty existed in choice of phase angle values. The phase angle for the second harmonic at DELTA is not very realistic and was apparently distorted by frequent mass advection. The phase angle for the second harmonic at BRAVO does not provide clear indication of variability of this angle with latitude; therefore the phase angle of HOTEL was used as the accepted factor. The assumption that the angle $\alpha_{2,400}$ varies little with latitude is somewhat corroborated by the relatively small variation of $\alpha_{1,400}$ with latitude.

Substitution of the accepted parameters in equation (4) yields

$$\bar{\theta}_{400} = \bar{\theta}_{s,a} - 5.51 \cos \phi + Ae^{-1.64 \sin \phi} \cos\left(\omega t + 153 \sin \phi\right) + Ae^{-4.00 \sin \phi} \cos\left(2\omega t + 166°\right) \quad (10)$$

Equation (10) is for the DELTA-HOTEL type of oceanographic conditions and applies mainly in areas with latitudinal mass transport.

Equations (9) and (10) can be applied in all cases where oceanographic conditions may be generalized and approximated to one of the two types of conditions represented by the equations. However, two special areas considered in this study do not approximate the conditions for equations (9) and (10). One (area G, figure 1) lies in a subtropical convergence zone, and the other (area H, figure 1) lies in an upwelling region. More such special areas could probably be located; however, none have a similar degree of independence and distinctness.

Special conditions in the subtropical convergence zone are created mainly by the relative shallowness of the seasonal thermocline in winter. Not only is it shallow (about 300 to 500 feet), but also it has a rather large gradient. The thermocline is more permanent than seasonal and oscillates about the 400-foot level. The phase angle at 400 feet is turned by about 180°. The maximum temperature at 400 feet occurs in spring rather than in fall or early winter as expected (figure 6). During summer, this water is apparently cooled more from below by some mechanism (probably by admixture from deeper strata) than it is heated from above. Temperature of the mixed layer in winter does not fall below the temperature at 400 feet in summer. As the mixed layer thickness increases and the interface increases in depth, the temperature at 400 feet is raised by the higher temperature of the mixed layer.

Area G does not cover the entire subtropical convergence but is considered to represent a typical portion of it. Data were not sufficient for computing parameters along two latitudes; however, since the cosine function was found to apply for the mean annual temperature parameter λ in the rest of the ocean, it was also assumed to be applicable in this area. The same assumption was made for the sine function in the exponent. Phase angles were held constant throughout the area. Possible latitudinal variations are considered to be small. Setting computed factors from Fourier expansion in equation (4) and extending to four harmonics results in:

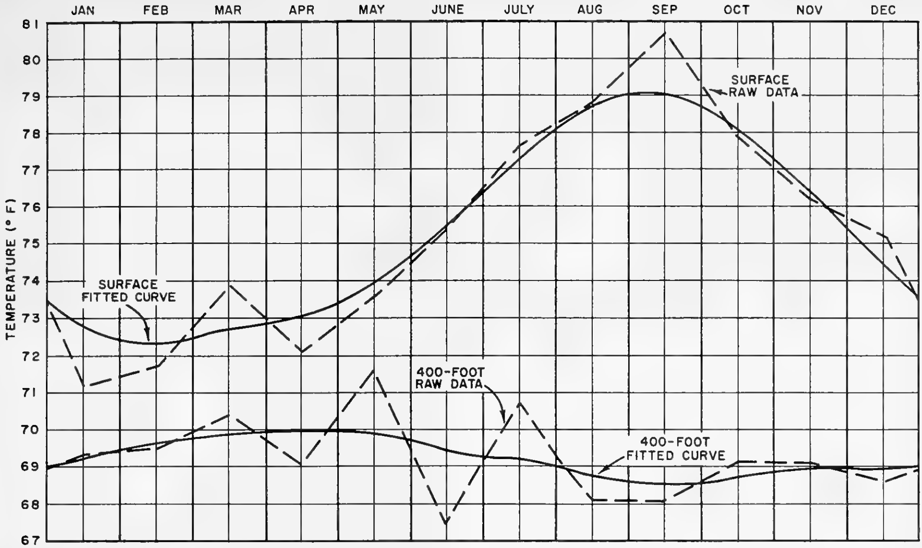

FIGURE 6 MEAN TEMPERATURE AT SURFACE AND 400 FEET IN AREA G

$$\begin{aligned}\bar{\theta}_{400} = \bar{\theta}_{s,a} - 6.65 \cos\phi &+ Ae^{-3.91 \sin\phi}\cos(\omega t + 270°) + \\ &Ae^{-8.23 \sin\phi}\cos(2\omega t + 130°) + \\ &Ae^{-9.42 \sin\phi}\cos(3\omega t + 350°) + \\ &Ae^{-9.42 \sin\phi}\cos(4\omega t + 284°)\end{aligned} \qquad (11)$$

An annual period predominates in this equation; however, since coefficients in the exponents of further harmonics did not differ much from one another, the four harmonics were taken to approximate the original distribution.

In the upwelling region, area H, surface temperature distribution shows a predominant annual period similar to that of the entire North Atlantic; however, mean temperature distribution at 400 feet has a peculiar shape. Semiannual and shorter periods are strongly developed. Although data are sparse, this distribution can be considered quite real because approximately the same pattern is repeated along all four latitudes in area H. The summary of factors computed from the corresponding Fourier expansions along the four latitudes is given in table 3.

The factors along each latitude are satisfactorily consistent. The accepted factor (means) computed for each latitude is presumed to yield good approximation for the entire upwelling area. The only regular variation of phase angle with latitude is detected in the third harmonic. Regularity of the increase is probably accidental, because values of other angles do not show orderly progression with latitude. Since scattering is not exceedingly large, accepted mean factors for the four angles along individual latitudes were applied throughout area H.

Substituting the accepting factors in equation (4) and extending to four harmonics yields:

$$\bar{\theta}_{400} = \bar{\theta}_{s,a} - 7.18 \cos\phi + Ae^{-4.97\sin\phi}\cos(\omega t + 335.8°) + Ae^{-5.42\sin\phi}\cos(2\omega t + 324.4°) + Ae^{-7.3\sin\phi}\cos(3\omega t + 142.3°) + Ae^{-8.92\sin\phi}\cos(4\omega t + 317.6°) \quad (12)$$

TABLE 3

MEAN TEMPERATURE DISTRIBUTION FACTORS AT 400 FEET FOR THE UPWELLING AREA H

Factor	MEAN LATITUDE $\bar{\phi}$				Accepted Factor (means)
	ϕ = 23.5°	ϕ = 30.5°	ϕ = 35.5°	ϕ = 38.5°	
$\lambda = \frac{-\ell}{\cos\phi}$	-7.03	-6.17	-7.73	-7.81	-7.18
$S_1 = \frac{-r_1}{\sin\phi}$	-4.97	-4.99	-5.03	-4.89	-4.97
$S_2 = \frac{-r_2}{\sin\phi}$	-5.02	-4.32	-6.55	-5.81	-5.42
$S_3 = \frac{-r_3}{\sin\phi}$	-5.97	-8.96	-7.36	-6.92	-7.30
$S_4 = \frac{-r_4}{\sin\phi}$	-9.06	-8.50	-9.27	-8.83	-8.92
$\alpha_{1,400}$	331.3°	341.8°	354.8°	314.5°	335.8°
$\alpha_{2,400}$	297.9°	343.7°	304.1°	352.0°	324.4°
$\alpha_{3,400}$	116.2°	132.9°	144.4°	175.6°	142.3°
$\alpha_{4,400}$	345.0°	312.7°	282.3°	330.4°	317.6°

The mean temperature distribution at 400 feet in area H differs distinctly from that in most parts of the North Atlantic. In general the annual period predominates in the North Atlantic at the surface and at 400 feet. The semiannual period in area H is almost as important as the annual period. Temperature distribution at the Pacific OWS NOVEMBER is very similar to temperature distribution in area H and appears to be quite typical for the North Pacific. A general equation approximating temperature distribution at 400 feet in the North Pacific may resemble equation (12), although regional differences would certainly have to be considered.

Equations (9) through (12) do not express actual temperature distribution but are only temporal and spatial approximations of the mean distribution at 400 feet. Equations (9) and (10) are considered satisfactory because they are based on numerous data restricted to individual points. Equations (11) and (12) are considered tentative, because they are based on limited data collected in large areas and are applicable only to given latitudes. If values could be computed with these equations for a sufficient number of points in the North Atlantic, charts of mean monthly temperatures at various levels could be plotted. Such charts could serve as normal temperature distributions for computing temporal anomalies at 400 feet in the same way that the temporal anomalies at the surface can be computed from mean monthly surface charts.

III. TEMPERATURE ANOMALIES

An instantaneous temperature at a given location at any time can be interpreted as a sum of the long-term mean and the difference between the mean and actual temperature,

$$\theta = \bar{\theta} + \Delta\theta$$

where $\Delta\theta$ is the temporal anomaly. The mean and the actual temperatures must be known in order to compute the anomaly. Conversely, the instantaneous temperature can be predicted from the mean temperature if the anomaly could be predicted. Mean surface values can be interpolated from the mean monthly charts, and mean temperatures at 400 feet can be computed with equations (9) through (12) or interpolated from charts based on these equations. If mean values are considered sufficiently accurate for practical application, the rest of the problem is reduced to prediction of anomalies.

The first logical step would be to study surface anomalies and to formulate analytical, empirical, or statistical ways to predict them. The second part of the problem would be establishment of a relationship between anomalies at the surface and 400 feet and a method of predicting the anomaly at 400 feet in terms of the surface anomaly.

Surface anomalies have been studied on several occasions (3,4), but results were inconclusive. Their causes and formation processes are complicated. No prediction system exists at this time; however, once the surface anomaly is established, it extends over large areas and usually persists for a considerable time (several weeks or months). If values of surface anomalies can be obtained for the entire ocean or a portion of it from observed data, it may be assumed without much risk of significant error that the anomalies will be applicable to 5- or 10-day periods. The problem then becomes one of establishing a relationship between anomalies at the surface and 400 feet.

A detailed study of the relationship would require a large amount of data collected over a period of many years at many points. The only locations for which reasonable amounts of data are available are the five North Atlantic ocean weather stations. These data are limited to periods of 5 to 16 years. Consequently, determination of the relationship cannot be exact and complete and will only permit approximations satisfactory for limited applications.

The range of temperature errors is rather large in individual BT's; therefore, correlation of mean anomaly values at the surface and 400 feet would produce better results. The monthly mean temperature anomaly is the difference between the mean for a given month and the mean monthly value based on long-term data. Large errors in individual observations are smoothed during computation of monthly means. Since the change in anomalies is rather slow, monthly mean anomalies can be considered representative of individual anomalies.

Mean anomalies at the surface and 400 feet can be considered as normally distributed random variables. Table 4 shows the mean and standard deviations of anomalies at the individual ocean weather stations and the same parameters computed for the five combined North Atlantic ocean weather stations. The means and standard deviations of anomalies based on all data are given in column 4. Since many of the values included in these means are insignificant in correlation of the anomalies at the surface and 400 feet, only the values exceeding 0.5°F have been used. These values are listed in column 5.

The true nature of small anomaly differences between the two levels is not certain. The main difficulty in interpretation arises from the fact that BT error varies between the two levels and that the same instrument is often used during a large part of one month at one station. Thus, relationship of small differences of anomalies with different signs at the two levels cannot be determined, because the instrumental error remains in the monthly mean value. The author (5) has shown that 81 percent of instrumental errors between the two levels are less than 0.5°F. Exclusion of anomalies less than 0.5°F permits more conclusive computations. In a total of 405 pairs of surface and 400-foot anomalies at the five North Atlantic ocean weather stations, 238 pairs exceeded 0.5°F.

A further attempt to identify data representing reliable linkage between anomalies at the surface and 400 feet resulted in the values listed in column 6, table 4. This column includes only those observations in which anomalies are greater than 0.5°F and of the same sign at both levels. In the total of 238 cases, 201 pairs met these conditions.

IV. CORRELATION

Linear correlation coefficients computed for the same three groups of data in table 4 are shown in table 5. Values in the first column were computed from all available observations. The correlations at all North Atlantic stations, except CHARLIE, agree closely. The value at CHARLIE does not improve substantially by limiting computation to anomalies greater than 0.5°F; however, by using pairs of anomalies exceeding 0.5°F and with the same sign, the value becomes alined with those of the other stations. Some anomalies of different origin at the two levels are eliminated by excluding anomalies with different signs in the computations. However, the remaining anomalies of different origin but coincident in sign constitute a deficiency in computations, because all anomalies are treated as though they were of the same origin. Therefore the third column of correlation coefficients in table 5 can be considered minimal for anomalies of the same origin. With 201 pairs of anomalies for the combined North Atlantic stations the correlation coefficient is 0.835. With z-transformation to normal distribution, 95 percent confidence limits are 0.778 and 0.879 for $\bar{n} = 40$ (mean number of pairs for individual stations). All correlation coefficients of the individual stations exceed the lower limit. All five stations are located in the western North Atlantic and the number of points is extremely small; however, since their distribution is well balanced with respect to the main surface water masses in the North Atlantic, the computed coefficients may be considered fairly representative of the entire North Atlantic.

TABLE 4

MEANS AND STANDARD DEVIATIONS OF MONTHLY TEMPERATURE ANOMALIES AT SURFACE AND 400 FEET

STATION	LEVEL	Statistical Parameter	$\Delta\theta$ (all obs)	$\Delta\theta >$ 0.5°F	$\Delta\theta >$ 0.5°F and of the same sign at the surface and 400 feet	Number of Obs
BRAVO	Surface	$\overline{\Delta\theta}$	-0.09	+0.14	+0.05	53
		S	1.54	2.01	2.07	
	400 feet	$\overline{\Delta\theta}$	+0.06	+0.15	+0.29	
		S	1.05	1.46	1.43	
CHARLIE	Surface	$\overline{\Delta\theta}$	+0.16	+0.38	+0.22	85
		S	1.82	2.05	1.86	
	400 feet	$\overline{\Delta\theta}$	+0.15	+0.26	+0.42	
		S	1.39	1.65	1.72	
DELTA	Surface	$\overline{\Delta\theta}$	-0.13	-0.25	-0.42	89
		S	3.13	3.34	3.55	
	400 feet	$\overline{\Delta\theta}$	-0.21	-0.41	-0.30	
		S	3.00	3.16	3.29	
ECHO	Surface	$\overline{\Delta\theta}$	-0.20	-0.21	-0.55	104
		S	1.81	2.19	2.31	
	400 feet	$\overline{\Delta\theta}$	-0.02	-0.30	-0.34	
		S	1.44	1.63	1.72	
HOTEL	Surface	$\overline{\Delta\theta}$	+0.31	+1.00	-0.04	74
		S	1.37	1.64	1.17	
	400 feet	$\overline{\Delta\theta}$	-0.66	-1.27	-0.94	
		S	1.92	2.19	2.34	
COMBINED ATLANTIC OWS's	Surface	$\overline{\Delta\theta}$	+0.03	-0.02	-0.24	405
		S	2.13	2.54	2.63	
	400 feet	$\overline{\Delta\theta}$	-0.14	-0.34	-0.21	
		S	1.95	2.34	2.44	
Total obs		n	405	242	201	
PACIFIC OWS NOVEMBER	Surface	$\overline{\Delta\theta}$	+0.01	+0.11	0	134
		S	1.98	1.96	1.99	
	400 feet	$\overline{\Delta\theta}$	-0.13	-0.31	-0.11	
		S	1.87	2.33	2.18	

$\Delta\theta$ = Monthly mean temperature anomaly in °F.
$\overline{\Delta\theta}$ = Mean of the monthly mean temperature anomalies in °F.
S = Standard deviation in °F.

TABLE 5

LINEAR CORRELATION OF MONTHLY MEAN TEMPERATURE ANOMALIES BETWEEN SURFACE AND 400 FEET

STATION	$\Delta\theta$ (all Observations)		$\Delta\theta \geq$ 0.5°F		$\Delta\theta \geq$ 0.5°F and of the same sign at the surface and 400 feet	
	r	n	r	n	r	n
BRAVO	0.755	53	0.789	26	0.903	24
CHARLIE	0.300	85	0.379	47	0.793	36
DELTA	0.761	89	0.820	68	0.872	64
ECHO	0.506	104	0.611	60	0.844	45
HOTEL	0.550	74	0.629	41	0.791	32
COMBINED ATLANTIC OWS	0.596	405	0.686	242	0.835	201
NOVEMBER	0.398	134	0.384	75	0.532	67

$\Delta\theta$ = Monthly mean temperature anomaly in °F
r = Linear correlation coefficient
n = Size of the sample (number of pairs)

Correlation coefficients based on all pairs of monthly mean anomalies were computed with zero to 6 months' lag for all stations and are shown in figure 7. The correlations are best for zero lag and drop quite rapidly; however, most stations show an increase at 4 months' lag. The correlation coefficients computed for the combined stations also show this increase as well as about half the correlation value at 1 month's lag. The large decrease indicates rather limited persistence; however, satisfactory persistence for a few weeks can be inferred from these curves. Lag correlations were not computed for anomalies exceeding 0.5°F; persistence in this group would probably be considerably greater.

V. ANOMALY RATIO

If the anomalies at both levels have the same sign and are not too different in magnitude, they are considered to be correlated. Will the magnitudes of the anomalies be the same at both levels when anomalies are correlated? If the magnitudes are not expected to be the same, what are their ratios and are the ratios constant or variable?

Components of a common origin of anomalies at both levels may be

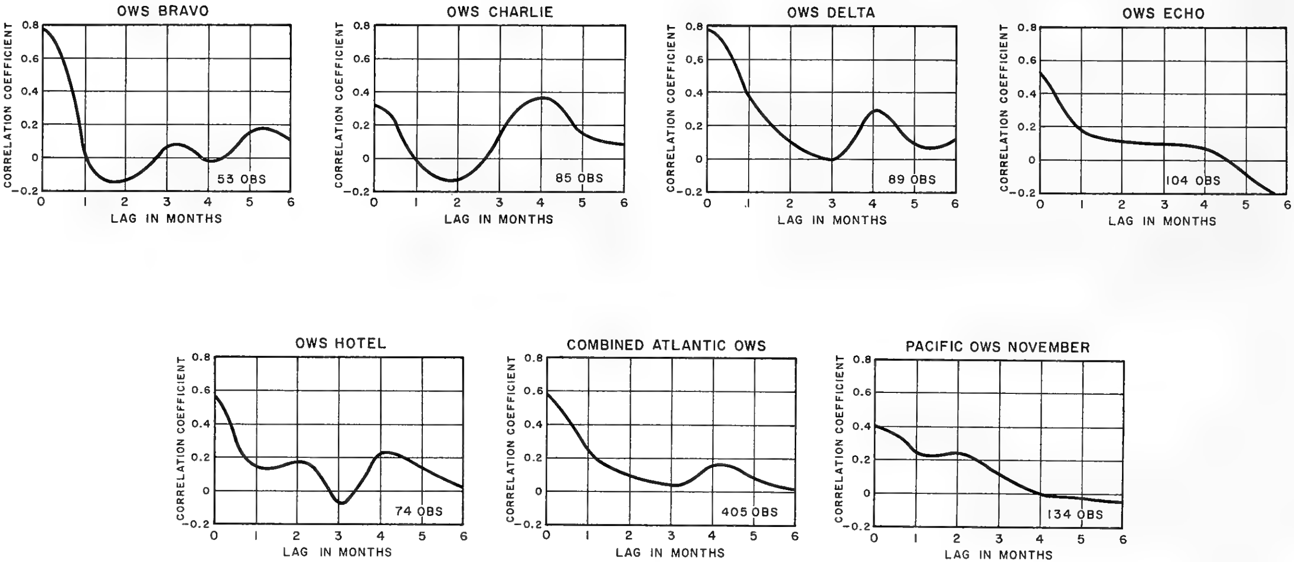

FIGURE 7 LAG CORRELATIONS BETWEEN TEMPERATURE ANOMALIES AT SURFACE AND 400 FEET AT OCEAN WEATHER STATIONS

somewhat different in strength at each level. For example, upwelling may be more intense at the surface than at 400 feet or vice versa. Forces tending to modify anomalies of the same origin may also be of different intensity. For example, originally negative anomalies at both levels will be modified by heating in summer at different rates. Consequently, anomalies cannot be expected to be of the same magnitude at both levels even if they are correlated; differences will usually exist, and coinciding magnitudes will only be accidental.

Owing to sparsity of data at individual ocean weather stations or lack of data for certain months, analysis of individual anomaly ratios is difficult. A workable distribution which covers at least all months can be achieved only by combining the data for all 5 Atlantic stations. These data are shown in the last columns of table 5. After grouping of positive and negative anomalies, mean values at each level were computed for individual months. The ratio of each pair of mean anomalies was then computed for each month. The monthly ratio distribution was time smoothed by the normal smoothing function. The smoothed ratios for negative and positive anomalies are plotted against time in figure 8. Although the annual range of ratio variations for positive anomalies is somewhat larger than that for negative anomalies, both curves show

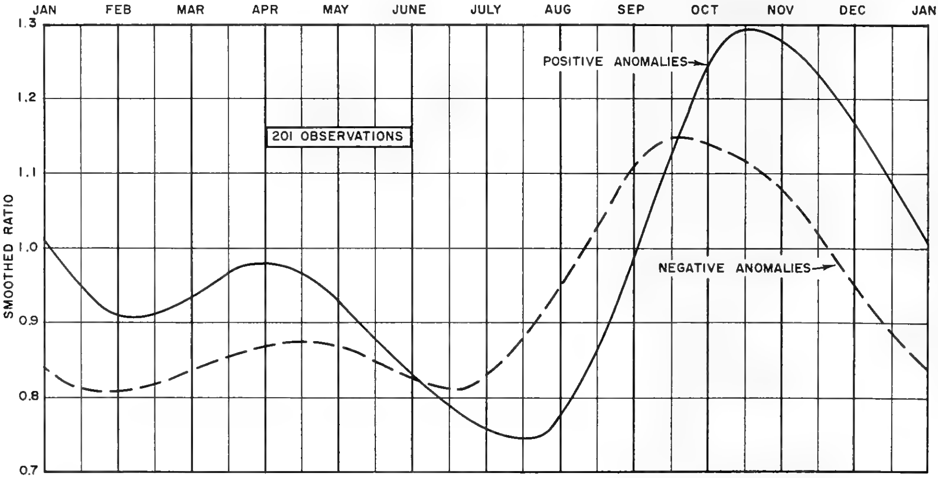

FIGURE 8 MEAN RATIOS OF MONTHLY MEAN TEMPERATURE ANOMALIES (400 FT/SURFACE)

remarkable similarity. The summer minimum in the positive curve may be caused by excessive heating of the mixed layer. This heating effect does not extend through the thermocline. The autumn maximum may result from temperature increase at 400 feet caused by intense mixing by autumn storms. The secondary minimum and maximum of the positive curve in spring and the ratio distribution of negative anomalies are not as easily explained.

VI. CORRELATED AND UNCORRELATED ANOMALIES

About 16 percent of the pairs of anomalies are reversed; i.e., if positive at the surface, they are negative at 400 feet and vice versa. It appears logical to assume that cases of reversed anomalies must be of different origins and are not correlated. There are 37 such pairs of anomalies in a total of 237 pairs greater than 0.5°F in the combined data of the Atlantic stations. Monthly distribution of the 37 uncorrelated observations with differing signs is shown in figure 9. Total distribution (figure 9a) shows that about 73 percent of all uncorrelated observations with differing signs occurred in the summer months from June through September, and 93 percent of these were positive at the surface and negative at 400 feet (figure 9b). The distribution of anomalies negative at the surface and positive at 400 feet is shown in figure 9c.

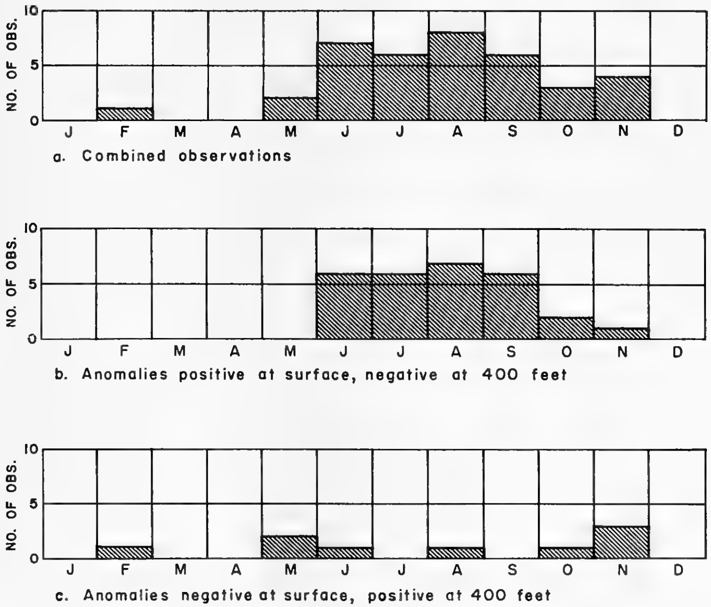

FIGURE 9 DISTRIBUTION OF UNCORRELATED PAIRS OF ANOMALIES OF DIFFERENT SIGNS AT SURFACE AND 400 FEET AT NORTH ATLANTIC OCEAN WEATHER STATIONS

Uncorrelated anomalies often exist in areas of vertical boundaries where frequent advections at both levels may cause strong anomalies of different origin. However, the extent of such abnormal areas is small in comparison with the entire area of the ocean, and the resulting uncorrelated anomalies are normally produced by different causes than tongue-like advection in boundary areas.

The number of uncorrelated pairs of anomalies does not provide a sufficient base for firm conclusions; however, monthly weather charts for the 37 cases indicate two principal causes of reversed anomalies at two levels. The first is horizontal advection at one level or different horizontal advection at both levels, and the second is surface heating. The basic annual anomaly is probably established in winter in the entire mixed layer, which usually extends below 400 feet. If positive anomalies exist at both levels in winter, they will be present at both levels after the thermocline exists only above the 400-foot level. Later, if prolonged strong westerly winds develop, the mean southward transport of water in the mixed layer seems to produce a negative anomaly at the surface while a positive anomaly is maintained below the thermocline at 400 feet. If negative anomalies originally exist at both levels, prolonged strong easterly winds may produce a positive anomaly at the surface, and the negative anomaly remains at 400 feet.

The second main cause of uncorrelated anomalies (surface heating) occurs in the upper mixed layer in areas of prolonged weak variable winds and calms. Studies of surface anomalies (3,4) established no correlation between heating processes and surface anomalies; however, subsurface conditions were not considered. The mixed layer thickness must be an important factor; and the results would probably have been different, if thermocline depths had been included in the computations. Obviously, a shallow mixed layer is heated more easily than a deep mixed layer. Increased heat in the mixed layer is not increasingly conducted to the 400-foot level owing to increased stability of the thermocline. Admixture of cool water from deeper strata may cause the anomaly at 400 feet to become less positive or more negative. This change of value results from stirring caused by internal waves and horizontal flow. Reversed anomalies of this origin seem to occur more frequently in areas south of the North Atlantic Current than they do to the north.

A typical example of a large positive surface anomaly produced by heating in summer during periods of very limited mechanical mixing is shown in figure 10. The three BT's were taken at 1-hour intervals within the same 1-degree square (50°N,10°W). Note the change of the very thin mixed layer of about 12 feet in the first BT to a thermocline extending from the surface to about 200 feet 2 hours later in the third BT. The surface anomaly is 5.3°F, and the anomaly at 400 feet is 0.5°F. The 400-foot anomaly was computed with the mean obtained with equation (9).

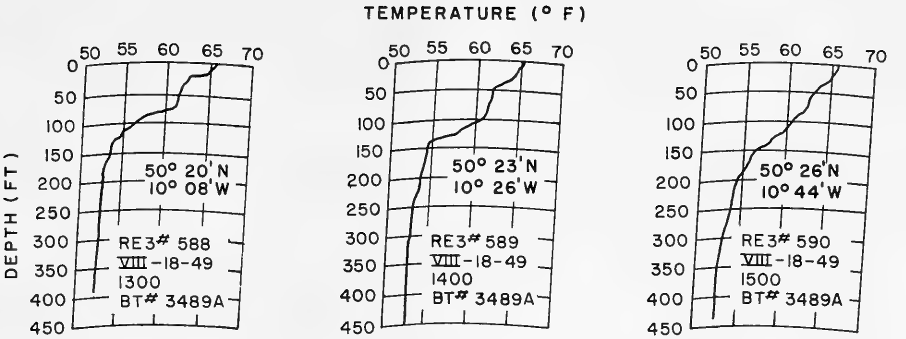

FIGURE 10 TYPICAL EXAMPLE OF LARGE POSITIVE SURFACE TEMPERATURE ANOMALY DUE TO HEATING

Data from all of the Atlantic stations show that 84 percent of surface and 400-foot anomalies generally have the same sign. The remainder are of different sign and may be considered to be uncorrelated or of different origins. However, an equal amount of the anomalies of the same sign at both levels may also be considered uncorrelated; that is, they may be of different origins. Uncorrelated anomalies of the same sign may be detected by examining the data for large differences in magnitude of anomalies at the two levels. Such detection is difficult owing to unsatisfactory establishment of an objective criterion for a large difference of anomalies at the two levels. The only available basis for such a criterion is the annual distribution of 400-foot and surface anomaly ratios. Figure 8 shows that the maximum annual range of the ratios (0.55) equals 42 percent of the maximum annual value of the ratio (1.3). By adding 0.5°F, the likely error between temperature measurements at two levels (table 1, reference 5), we obtain significance limits of

$$\pm \ell = 0.42\,\Delta\theta + 0.5 \qquad (13)$$

where $\Delta\theta$ is the larger value in a pair of anomalies. If the significance limits are exceeded, the difference of ratios is considered large. For example, if the surface anomaly is the larger of a pair and is equal to 3.0°F, the significance limits are ± 1.76°F (equation 13), resulting in limits of 1.24° and 4.76°F (3.0°F ± 1.76). Any 400-foot anomaly between these limits would be considered correlated; an anomaly beyond these limits would be considered uncorrelated. Of 201 pairs, 24 pairs are uncorrelated anomalies with the same sign, so that 12 percent were assumed to be uncorrelated. This percentage closely approximates the 16 percent of uncorrelated anomalies of different signs. Some of the uncorrelated anomalies with coinciding signs (probably 2 to 4 percent) may have smaller differences than the significance limits determined by equation (13). The significance limits may be high, because they are based on the entire range of variation. However, it should be remembered that sharp peaks in the ratio distribution have been eliminated by smoothing. A wide margin of safety is desirable; therefore, significance limits determined by equation (13) are more realistic by being large.

In summary, correlated pairs of surface and 400-foot anomalies are defined as those which have the same sign and do not differ in magnitude by more than 42 percent of the larger value of a pair plus 0.5°F. All other pairs are considered uncorrelated.

The third set of correlation coefficients in table 5 were computed according to this definition with elimination of only part of the uncorrelated anomalies. If uncorrelated pairs determined by equation (13) had also been excluded, the correlation coefficients would be considerably larger. About 70 percent of anomalies at both levels in the North Atlantic are correlated; the remainder are uncorrelated.

VII. PREDICTION TECHNIQUE

A. Mean Temperature at 400 Feet

Mean temperature at 400 feet can be computed by use of equations (9), (10), (11), or (12), depending on the area under study. In order to apply the equations, $\theta_{s,a}$, the mean annual surface temperature, and A, the annual amplitude of mean monthly surface temperature at the given point, must be known.

Mean annual surface temperatures, computed with more than 100 years of data, are plotted in figure 2 at intervals of 1°F except for the Gulf Stream where 2°F intervals were used to prevent crowding. The annual amplitude of mean monthly surface temperature (half annual range) is plotted in figure 11 at 0.5°F intervals.

Computations of mean temperatures are rather time consuming; therefore, graphic representations of the equations are given in figures A-1 through A-4 (appendix A). These nomograms contain four parts for each equation. In the first part, the harmonics are computed with the amplitude, A, factored out. For example, the graph in the upper left of figure A-1 represents:

$$K_1 = e^{-2.44 \sin\phi} \cos(\omega t + 2.1\phi \sin\phi) + e^{-4.78 \sin\phi} \cos\left[2\omega t + 441 \sin(\phi - 35.5°)\right]$$

The nomograms are used as follows:

A straightedge place horizontally from a point representing a given latitude and time in the upper left of each nomogram intersects the annual amplitude in the upper right of the nomograms ($K_2 = K_1 A$). A straightedge placed vertically from this point intersects the given latitude in the lower right ($K_3 = K_2 - 7.44 \cos\phi$). Horizontally from

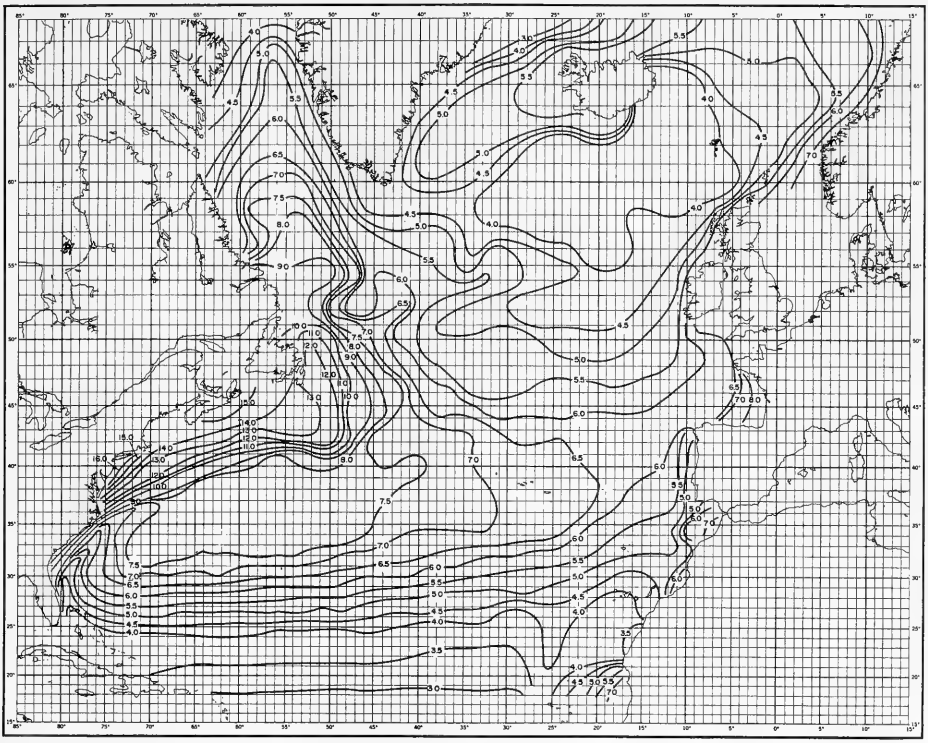

FIGURE 11 ANNUAL AMPLITUDE OF MEAN MONTHLY SEA SURFACE TEMPERATURE IN THE NORTH ATLANTIC (°F)

this point, the mean annual surface temperature ($\bar{\theta}_{s,a}$) in the lower left graph is intersected. Vertically below this point is the predicted mean temperature at 400 feet, $\bar{\theta}_{400}$. For example, given latitude 35°N, 15 May, annual surface amplitude of 7.0°F, and mean annual surface temperature of 70.0°F, $\bar{\theta}_{400}$, is 62.0°F.

A problem arises in choice of an equation and its corresponding nomogram. This is not an easy problem in many cases. The approximate areas and boundaries of the 4 equations are shown in figure 12. The boundaries are based partly on test results and partly on the isopleths which represent zonal temperature anomaly distribution for August.

The zonal temperature anomaly for a given location is the difference between the mean monthly temperature for that location and the mean monthly temperature computed with all data collected in the ocean along the latitude

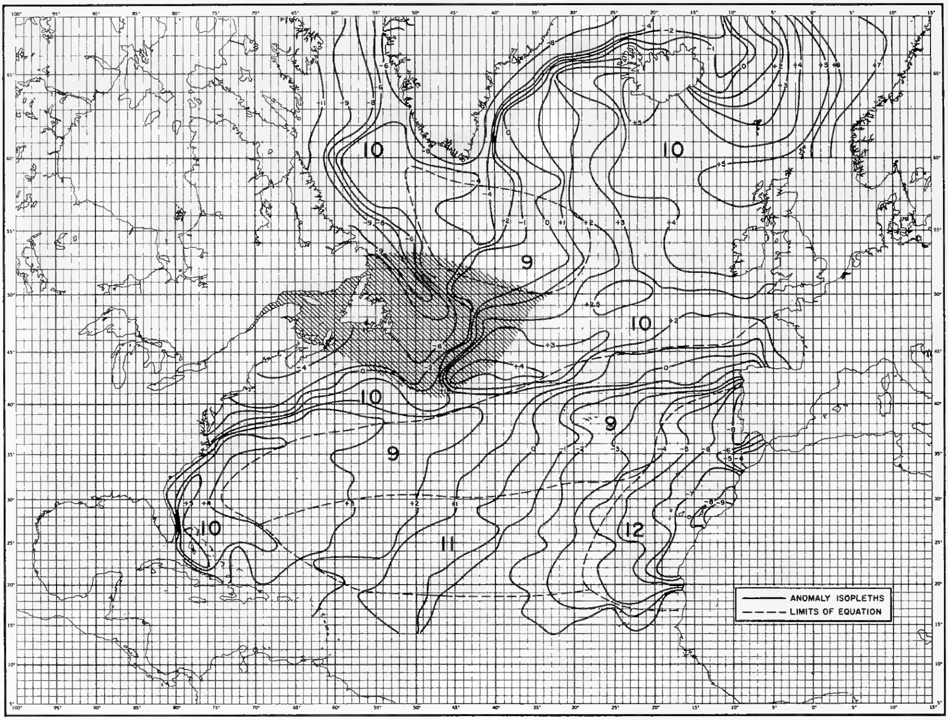

FIGURE 12 ZONAL ANOMALY OF SEA SURFACE TEMPERATURE IN AUGUST WITH APPROXIMATE AREAS FOR USING EQUATIONS (9), (10), (11), AND (12)

of the location. One can expect the zonal anomaly distribution to be helpful in identification of main surface water masses. The distribution reveals approximate areas of individual water types, although not as distinctly as desired. Positions and variability of boundaries given in figure 12 are only approximate. The boundaries can be expected to oscillate periodically and nonperiodically.

In general equation (9) is suitable for temperate areas where weak and moderate zonal anomalies occur. Equation (10) applies to areas with strong zonal anomalies, which usually occur in waters with northward or southward flow. The northern boundaries of equation (11) seem to concur with inflections of weak and moderate anomalies from a southwesterly trend; similar inflection occurs along the southern boundary. The boundaries of equation (12) were designed approximately along -3° to -4°F zonal anomaly isopleths.

The zonal anomaly distribution in February is shown in figure 13. The isopleths shift considerably in some areas, but the water masses are basically the same as shown in figure 12; therefore, zonal anomalies for other months were not plotted. The two charts (February and August) satisfy the purpose of illustrating general distribution of water types.

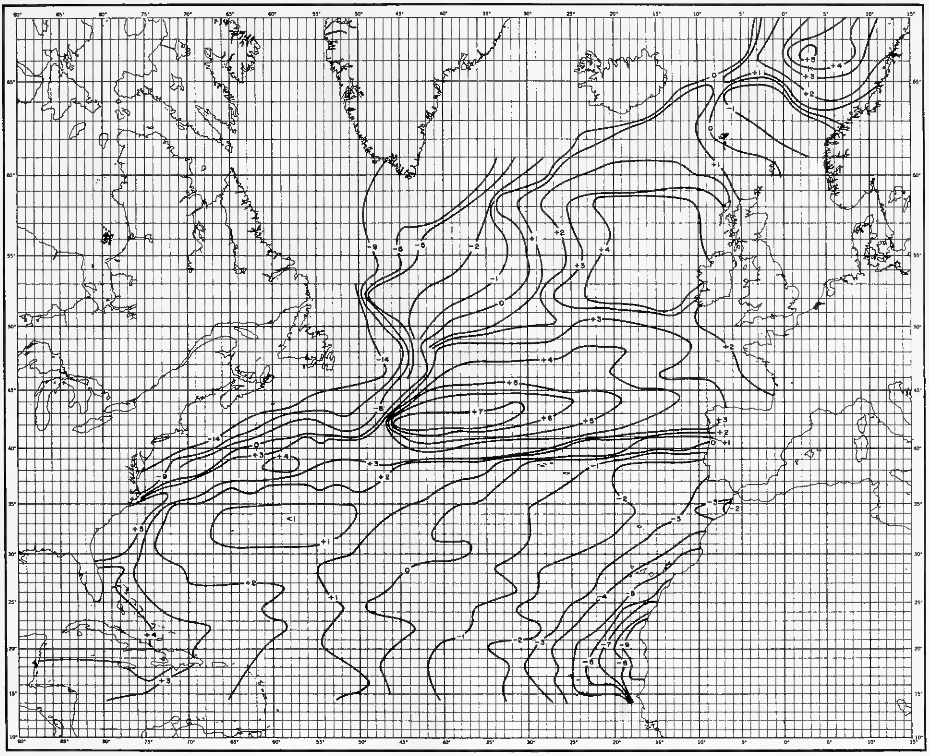

FIGURE 13 ZONAL ANOMALY OF SEA SURFACE TEMPERATURE IN FEBRUARY (°F)

In boundary areas, the surface transition between warm and cold water masses is often quite marked. This is not true in subsurface waters at the usual depths of the seasonal thermocline. Extensive tongues of cold and warm water penetrate each other over wide transition areas. Distribution of temperature and other variables in such boundary areas is extremely complicated, and satisfactory prediction cannot be expected for the time being. However, some periodicity of interchanging masses may be detected as more data become available. One of the most renowned of such areas is approximately indicated by the shaded portion of figure 12.

Equations (9) through (12) apply to certain water masses. Along boundaries between the masses, interpolation would probably give the best results. If mean temperatures at 400 feet were computed for the entire North Atlantic, computations in the boundary areas could overlap. Inconsistencies could then be smoothed out.

B. Determination of the Anomaly at 400 Feet

Mean monthly surface temperature charts are presented in appendix B. These charts are based on long-term data, with numerous observations taken by 1-degree squares. Observations in the Labrador Sea, around Greenland, and northeast of Iceland are very limited. Consequently, the isotherms in these areas are not too reliable. The mean monthly temperature for any given time and location can be interpolated from two charts. The surface temperature anomaly can be computed from observed surface temperature values and appropriate mean monthly temperature charts. The anomaly at 400 feet, determined from the surface anomaly and by use of figure 8, is added algebraically to the mean temperature computed with the appropriate equation. The result is the predicted temperature at 400 feet.

The exact reliability of anomaly ratios in figure 8 is unknown. However, the mean error of prediction in 53 tests (735 individual predictions) in the eastern and northern part of the North Atlantic was 1.06°F by use of figure 8 and 1.17°F without use of figure 8. The ratio seems to reduce prediction error. The reduction is not apparent from one or two predictions, because the prediction error is sometimes increased through application of the ratio correction. However, the statistical evidence given above supports use of the ratio.

Use of the corrected surface anomaly to predict temperature at 400 feet is successful about 70 percent of the time (when anomalies are correlated). A successful prediction is one in which the prediction error of temperature at 400 feet is small or moderate.

Large errors occur when anomalies at the surface and 400 feet are uncorrelated. There is no way of knowing when anomalies are uncorrelated if only surface temperatures are available. If mean monthly temperature charts for 400 feet were computed by equations (9) through (12), the areas of uncorrelated anomalies could be detected from daily BT observations taken by various ships in the charted area. The charts would serve as bases for computation and interpretation of anomalies at 400 feet in the same manner that mean monthly surface temperature charts are used for computing surface anomalies.

BT observations would pinpoint daily locations of uncorrelated anomalies. Superposition of observations from several days or weeks would outline areas of uncorrelated anomalies. Further superposition should reveal more definite areas of uncorrelated anomalies and the nature of their variations. Two anomaly charts are required for optimal prediction: one for surface anomalies over the entire North Atlantic and one for uncorrelated anomalies at 400 feet.

If areas of anomalies do not change too rapidly, weekly charts should be sufficient for determining surface anomalies and uncorrelated anomalies at 400 feet. Such charts covering periods of several years would yield valuable data for further study of the possible periodicity and causes of uncorrelated anomalies at 400 feet and most likely would also yield some clues leading to a better understanding of surface anomalies.

C. Prediction Example

The following example illustrates the prediction technique at 52°N,15°W on 3 August 1951. Surface temperature, based on 3 BT's, was 61.0°F. Mean annual temperature taken from figure 2 is 54.1°F. Annual temperature amplitude (A) taken from figure 11 is 4.5°F. The location, according to figure 12, is in a region in which equation (10) and figure A-2 apply. Following the procedure outlined on page 26 by entering the upper left of the nomogram with latitude 52° and time 3 August, $K_1 = 0.22$ is obtained. Entering the second graph with $K_1 = 0.22$ and $A = 4.5$, $K_2 = 1.0$. Descending from this point to latitude 52° in the third graph, $K_3 = -2.45$ on the left margin. From the K_3 value and mean annual temperature at the surface, $\bar{\theta}_{s,a} = 54.1$°F, the mean temperature at 400 feet ($\bar{\theta}_{400}$), 51.7°F, is obtained in the lower left corner of the nomogram.

Mean temperature interpolated from the mean monthly charts for July and August (figures B-7 and B-8) is 59.1°F. Since the observed surface temperature is 61.0°F, the surface anomaly is 1.9°F. The anomaly ratio from figure 8 is 0.75. The surface anomaly multiplied by the ratio yields a 400-foot anomaly of 1.4°F. Addition of this anomaly to the mean temperature at 400 feet, 51.7°F, gives the predicted temperature at 400 feet (θ_{400}), 53.1°F. The observed temperature at 400 feet computed from the 3 BT's was 51.9°F. The prediction error (E) is 1.2°F.

As shown by this example, some of the mean temperatures at 400 feet agreed closely with observed temperatures; however, anomaly corrections generally reduced prediction errors, especially when the anomaly values were negative.

In warm seasons (spring, summer, and early autumn), it is better to disregard large positive anomalies in the North Atlantic (except in the Labrador Sea). Instead, it is better to use the uncorrected mean temperature at 400 feet obtained from the nomogram. About 95 percent of test predictions during warm seasons were more accurate without anomaly correction when the surface temperature anomaly exceeded an arbitrary value of 2.5°F. This rule does not apply during cold seasons and does not apply at any time in the Labrador Sea. Any other value could be chosen as the criterion; however, in summer, large positive surface anomalies are generally uncorrelated with anomalies at 400 feet. Therefore, the prediction is more successful when the large positive anomalies are disregarded.

D. Prediction

Prediction of the temperature at 400 feet can be made several days in advance by using the surface temperature observed at the time of prediction. In the absence of significant advection at 400 feet, prediction can be made up to a week in advance without serious error. If predicted surface temperature is used instead of observed temperature, the prediction at 400 feet is applicable to the same prediction period.

Success of each prediction depends on accuracy of the mean temperature and anomaly. Accuracy of the mean temperature depends on the number of observations available and on the period of time which they cover. Mean temperatures at 400 feet used in this report are based on theoretical deductions made from data collected within a few very limited areas. However, the results of these theoretical deductions may be rather accurate in the areas of the data, and the mean temperatures obtained with equations (9) through (12) are valid in the vicinity of the five ocean weather stations in the western North Atlantic and in areas G and H (figure 1). The more the calculations depart from these areas, the greater the likelihood of significant errors in mean temperatures.

Since greater reliability of equations (9) and (10) was expected to occur in the western part of the temperate North Atlantic where the data were collected, prediction tests were carried out mainly in the eastern and northern North Atlantic.

Accuracy of mean temperatures computed with equations (9) through (12) cannot be determined, because prediction errors include other more significant components. The only indication of reliability of these values is shown by the distribution of negative and positive errors. The distribution of errors in 748 predictions is shown in figure 14. The distribution appears to be normal with a mean very close to zero and tends to substantiate computation of mean temperatures at 400 feet with the equations. In certain areas the prediction may be biased. Errors in mean temperature computations are probably much smaller than errors due to inaccurate anomaly evaluation.

Frequency distribution of absolute values of errors used in figure 14 is shown in figure 15. All predictions used as a basis for these figures were made in water masses for which the equations were intended, that is, transition zones between different water masses were avoided.

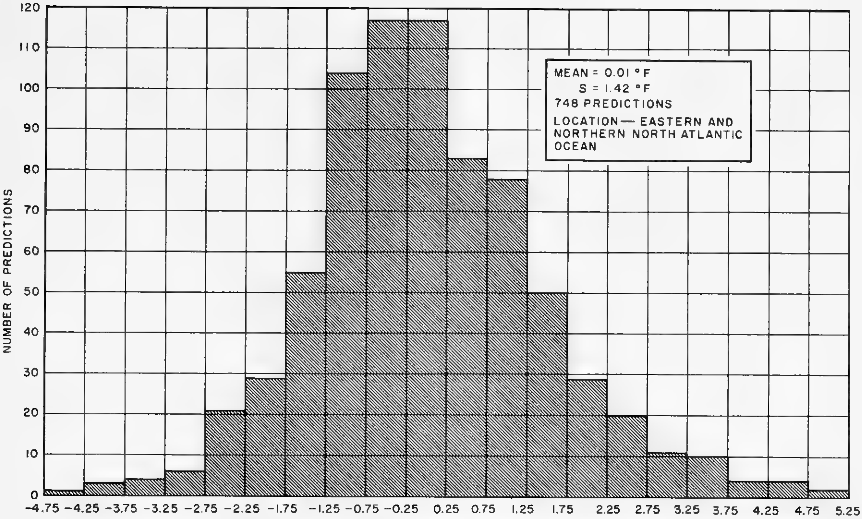

FIGURE 14 FREQUENCY DISTRIBUTION OF ERRORS OF PREDICTION (° F)

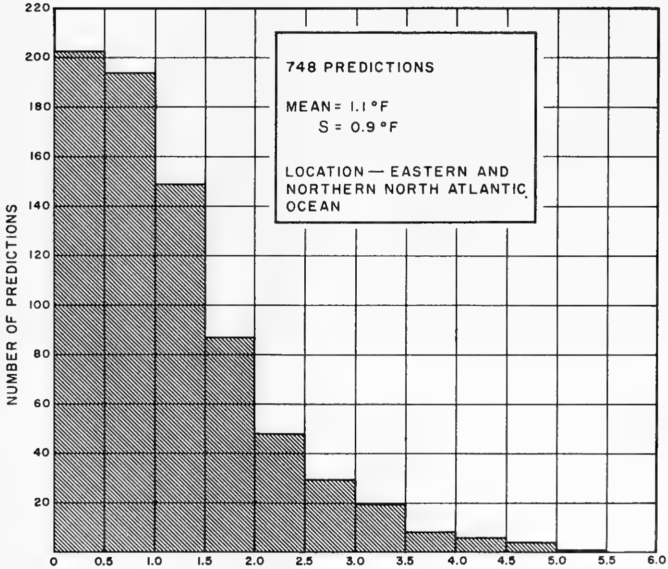

FIGURE 15 FREQUENCY DISTRIBUTION OF ABSOLUTE ERRORS OF PREDICTION (° F)

The mean of the distribution of absolute values of errors is 1.1°F, and the standard deviation is 0.9°F. For any sample of 10 predictions the standard error is 0.283°F, and the 95-percent confidence limits are 0.47 and 1.73°F. If the mean of the 10 predictions in a given area exceeds the upper confidence limit of 1.73°F, either the anomalies in the area are uncorrelated or the sample lies in a transition zone.

Six tests involving 81 individual predictions in transition areas resulted in much larger errors with a mean value of 1.92°F. The tests were made without consideration for uncorrelated anomalies, although anomalies are known to be uncorrelated a large percentage of the time. No doubt the approximation would be better and predictions would be considerably more accurate if uncorrelated anomalies were considered. The prediction error is probably larger in areas of intense frontal interchange, such as the shaded area in figure 12.

Prediction with this system may be unsatisfactory in many instances but may be useful when requirements are not too exact. Evaluation of individual prediction errors is also uncertain, especially when the prediction is verified with a BT observation which may be as erroneous as the prediction. Statistical parameters of many prediction errors may be accepted with more confidence.

VIII. CONCLUSIONS

1. The prediction system presented in this study attempts to provide a means for determining the temporal distribution of mean temperature at 400-foot depths beyond the continental shelf in the North Atlantic.

2. Actual temperature at 400 feet can be obtained by algebraic addition of the temperature anomaly and the mean temperature at 400 feet. The anomaly at the 400-foot level can be obtained by relation to the surface temperature anomaly, if the anomalies at both levels are correlated. If the anomalies are uncorrelated, the 400-foot anomaly could be determined from special charts, if enough data were available to permit construction of such charts.

3. Establishment of more accurate relationships between the surface temperature and the 400-foot temperature may provide a means for evaluating the approximate stability factor in the thermocline.

4. If the mixed layer thickness and the 400-foot temperature are predicted with reasonable approximation, a satisfactory composite trace of the vertical temperature distribution could be constructed.

ACKNOWLEDGEMENT

The author wishes to express appreciation for assistance rendered by various NAVOCEANO colleagues, especially to Mr. C. M. Winger who programmed the data for construction of the nomograms presented in appendix A and to Messers M. D. Burkhart and J. B. Hazelworth for analysis of the mean monthly temperature charts presented in appendix B.

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NOMENCLATURE

A	Annual temperature amplitude of the smoothed surface curve
A_1, A_2	Amplitudes of temperature oscillation for various harmonics
ℓ_z	Parameter determining mean annual temperature at a depth z with respect to the surface
n	Size of the sample (number of pairs of anomalies)
r	Linear correlation coefficient
r_1, r_2	Exponents determining reduction of annual temperature amplitude at 400 feet with respect to the surface
S	Standard deviation in $^{\circ}F$
S_1, S_2	Coefficients of exponent determining reduction of annual temperature amplitude at 400 feet with respect to the surface
α_1, α_2	Phase angles for various harmonics at the surface
$\Delta\theta$	Monthly mean temperature anomaly in $^{\circ}F$
$\bar{\theta}_{s,a}$	Mean annual surface temperature
$\theta_{z,T}$	Temperature at depth z at time T
$\bar{\theta}_{400}$	Predicted mean temperature at 400 feet
λ	Coefficient of mean annual temperature reduction at 400 feet with respect to the surface
μ	Eddy conductivity
ρ	Density
ω	Angular frequency $\left(\frac{2\pi}{T}\right)$, where T = period length of the first harmonic term.

APPENDIX A

NOMOGRAMS FOR DETERMINING MEAN TEMPERATURES AT 400 FEET

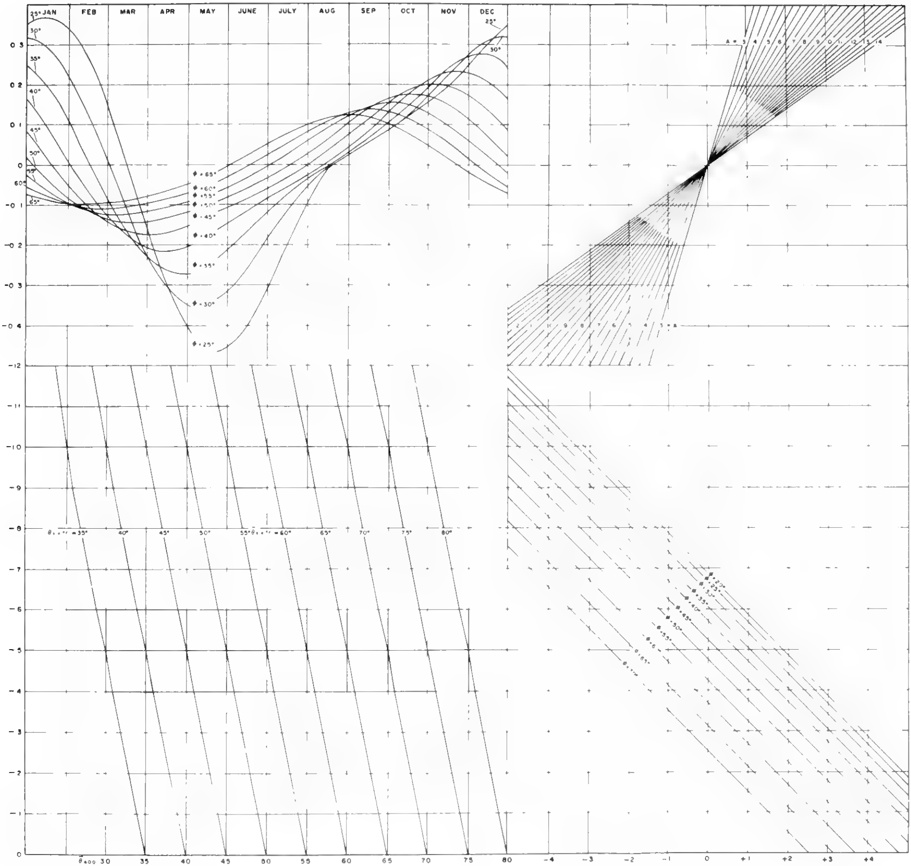

FIGURE A-1 NOMOGRAM FOR DETERMINING MEAN TEMPERATURE AT 400 FEET FOR CHARLIE-ECHO CONDITIONS BY EQUATION (9)

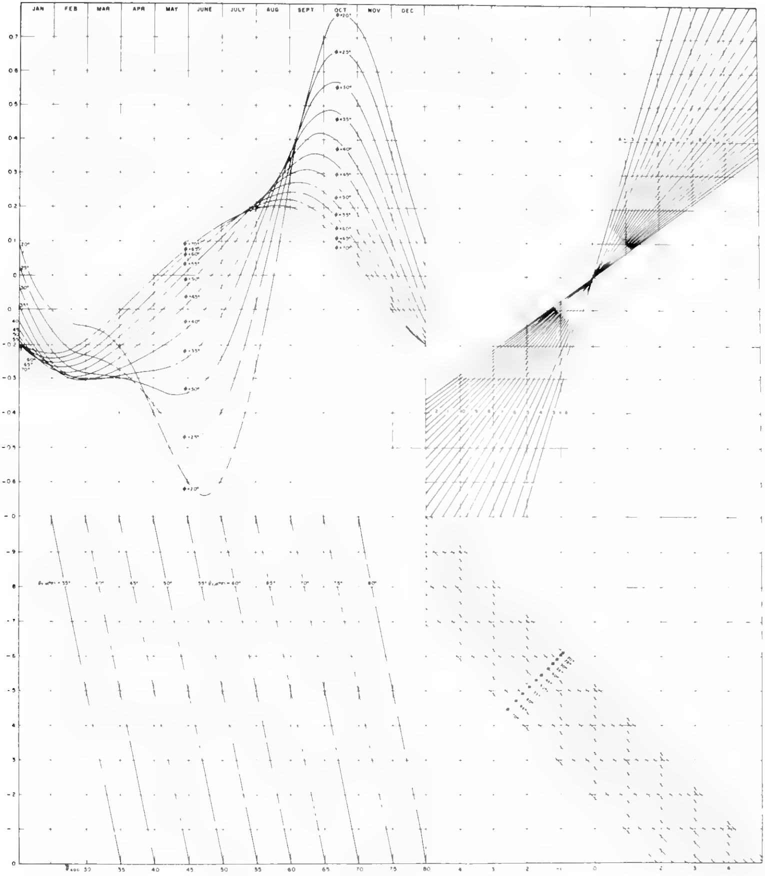

FIGURE A-2 NOMOGRAM FOR DETERMINING MEAN TEMPERATURE AT 400 FEET FOR DELTA-HOTEL CONDITIONS BY EQUATION (10)

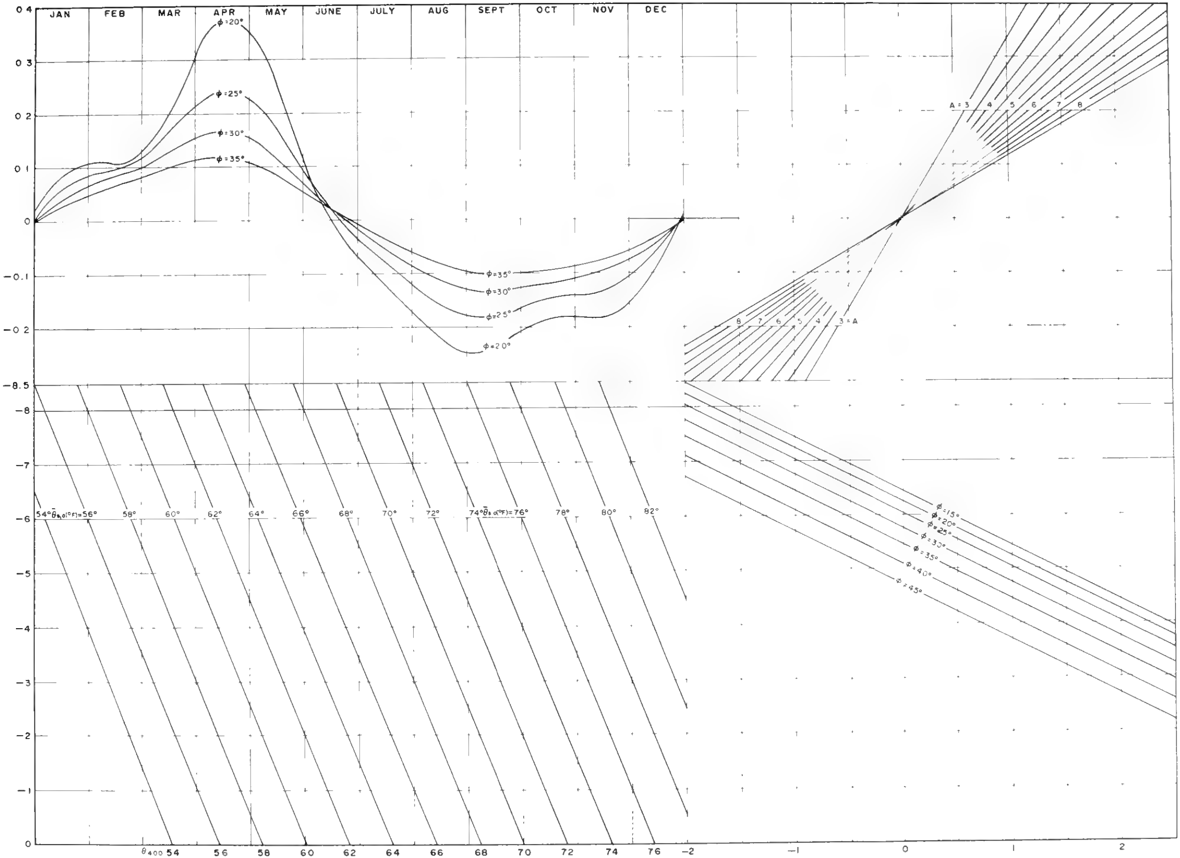

FIGURE A-3 NOMOGRAM FOR DETERMINING MEAN TEMPERATURE AT 400 FEET FOR AREA G BY EQUATION (11)

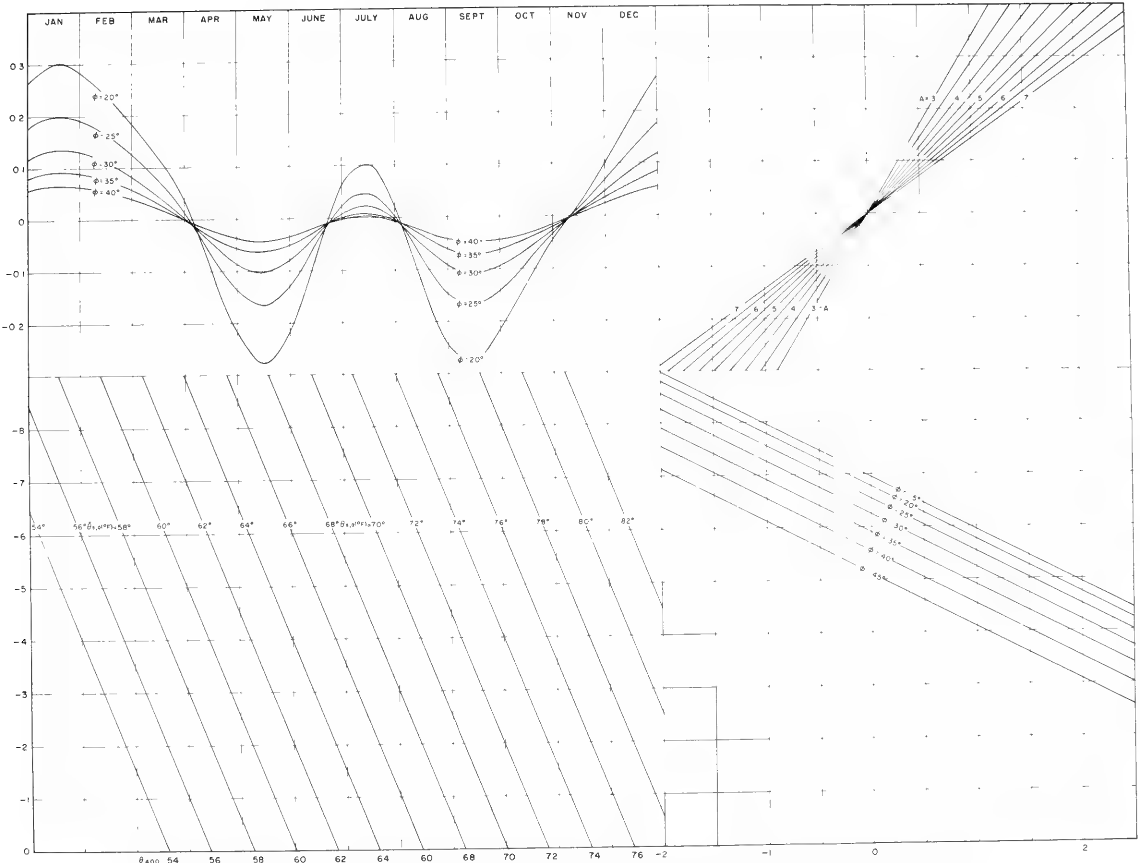

FIGURE A-4 NOMOGRAM FOR DETERMINING MEAN TEMPERATURE AT 400 FEET FOR AREA H BY EQUATION (12)

APPENDIX B

MEAN MONTHLY SEA SURFACE TEMPERATURE CHARTS - NORTH ATLANTIC

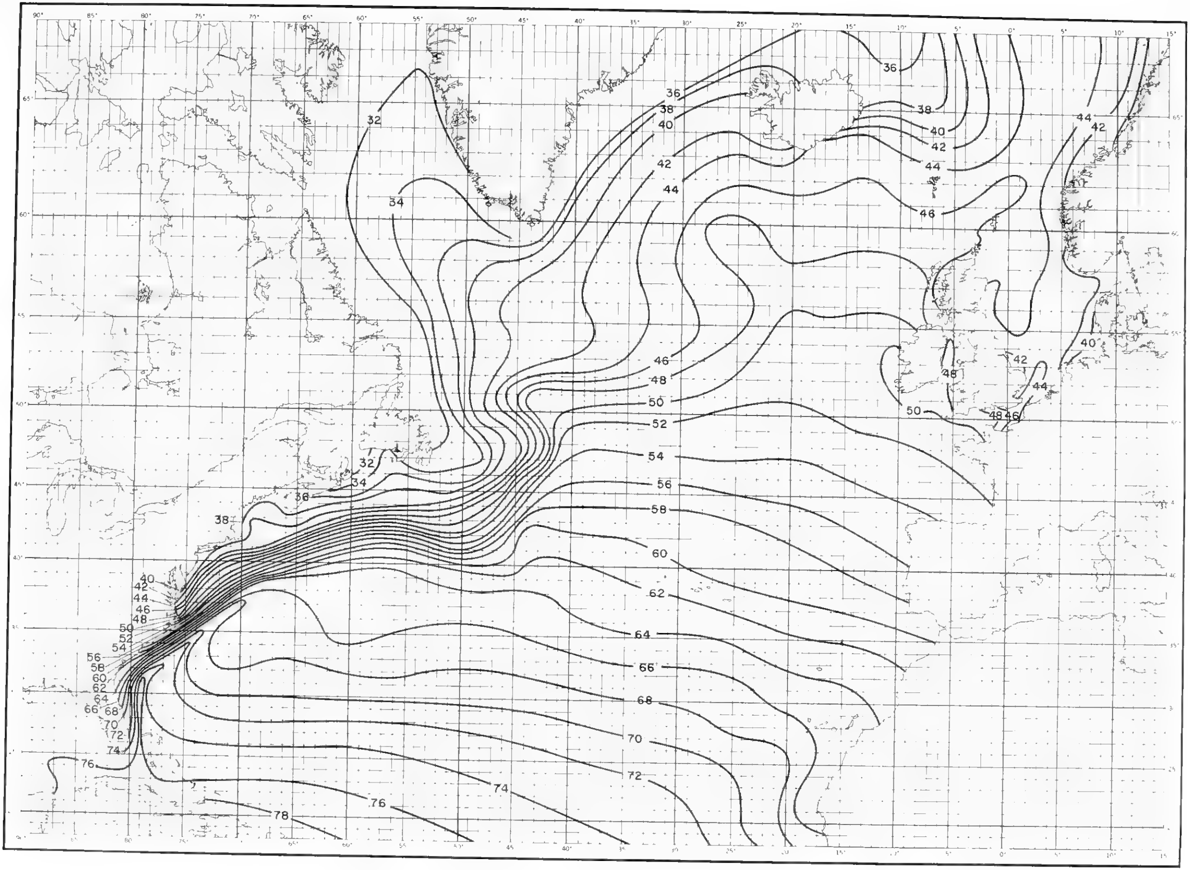

FIGURE B-1 MEAN MONTHLY SEA SURFACE TEMPERATURE — JANUARY

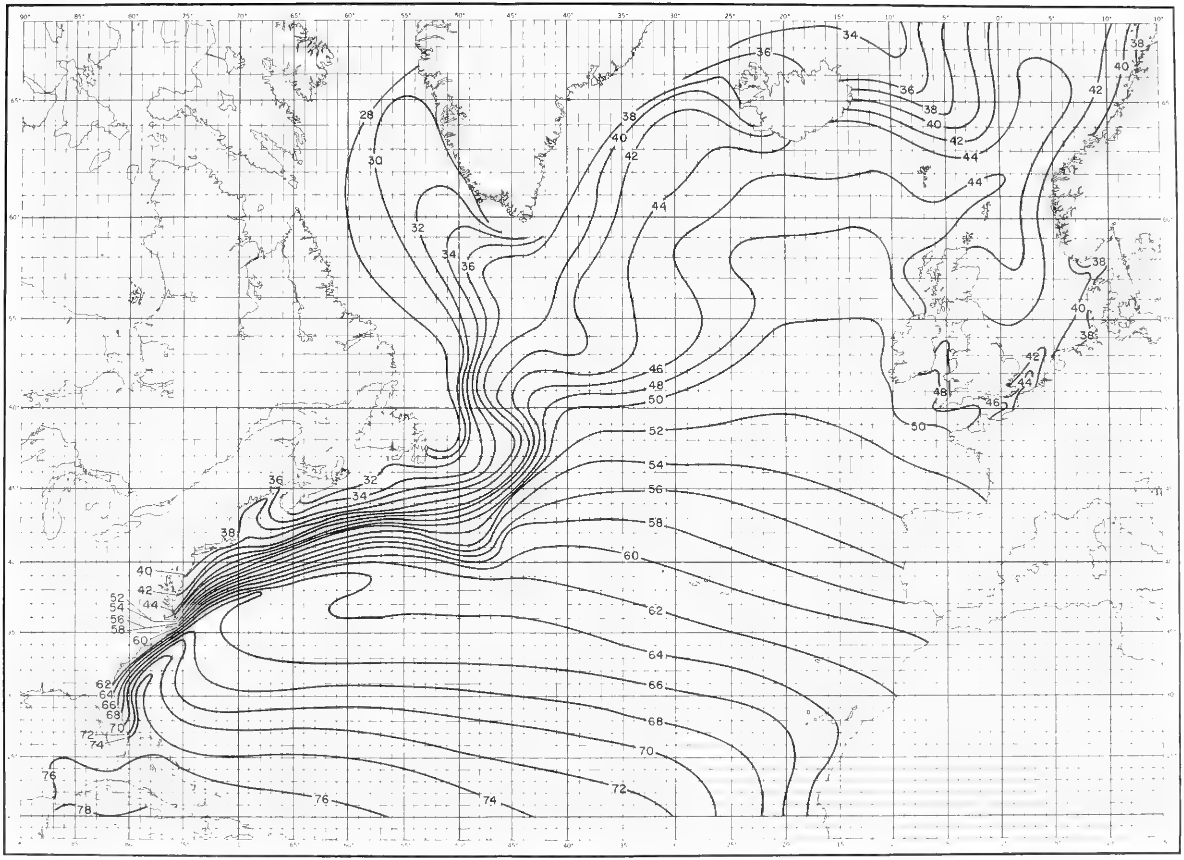

FIGURE B-2 MEAN MONTHLY SEA SURFACE TEMPERATURE — FEBRUARY

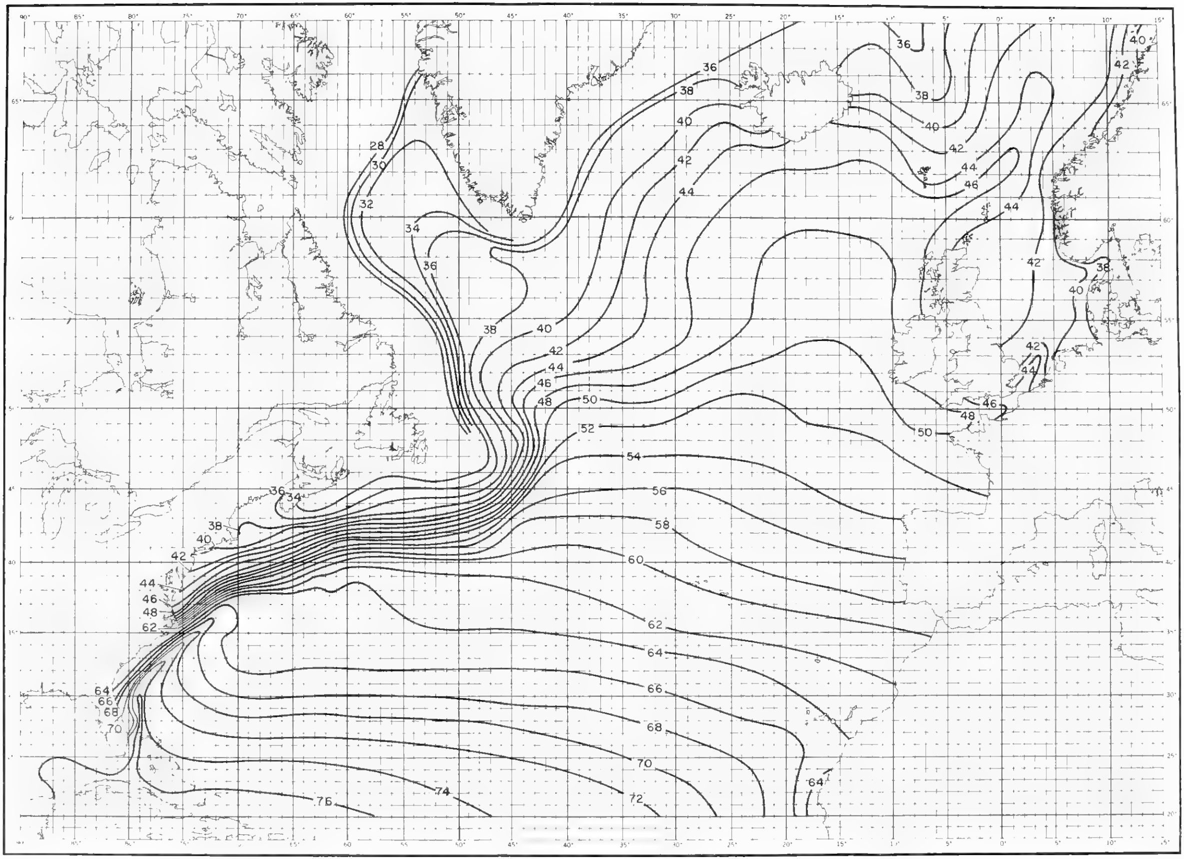

FIGURE B-3 MEAN MONTHLY SEA SURFACE TEMPERATURE – MARCH

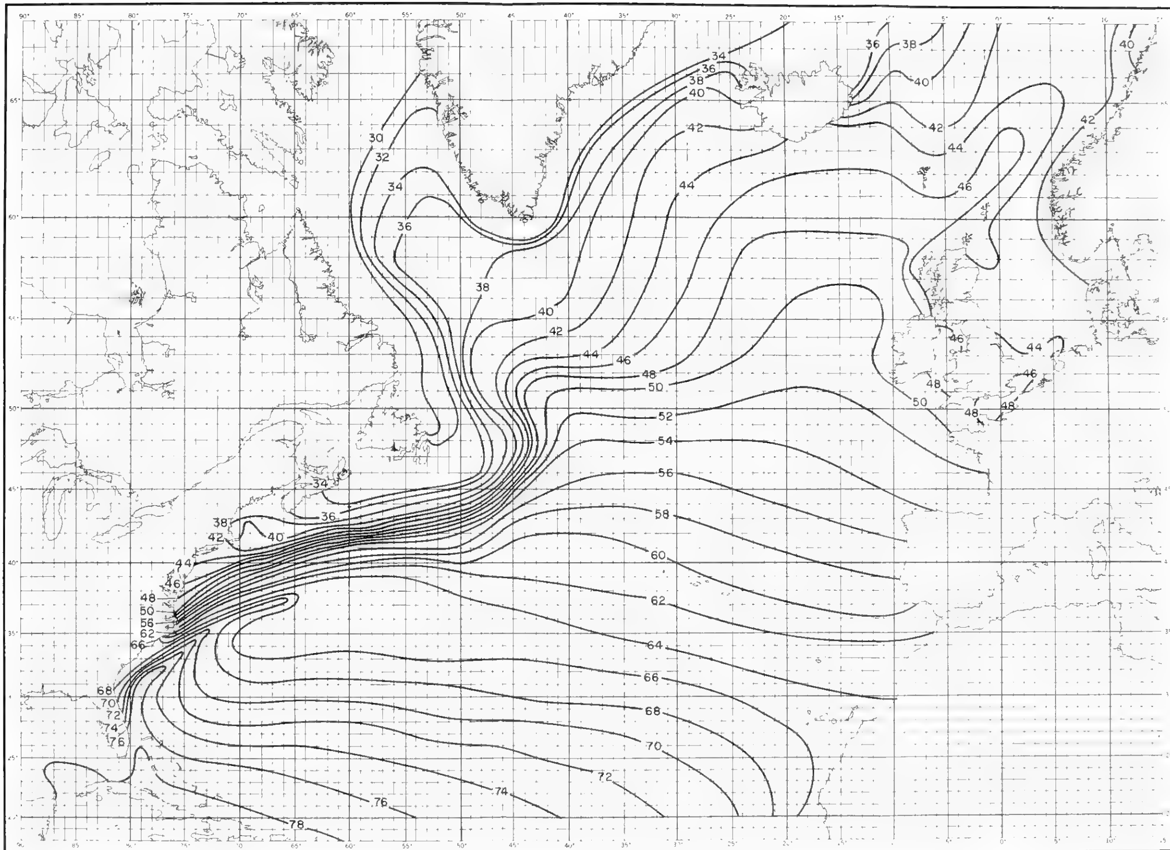

FIGURE B-4 MEAN MONTHLY SEA SURFACE TEMPERATURE—APRIL

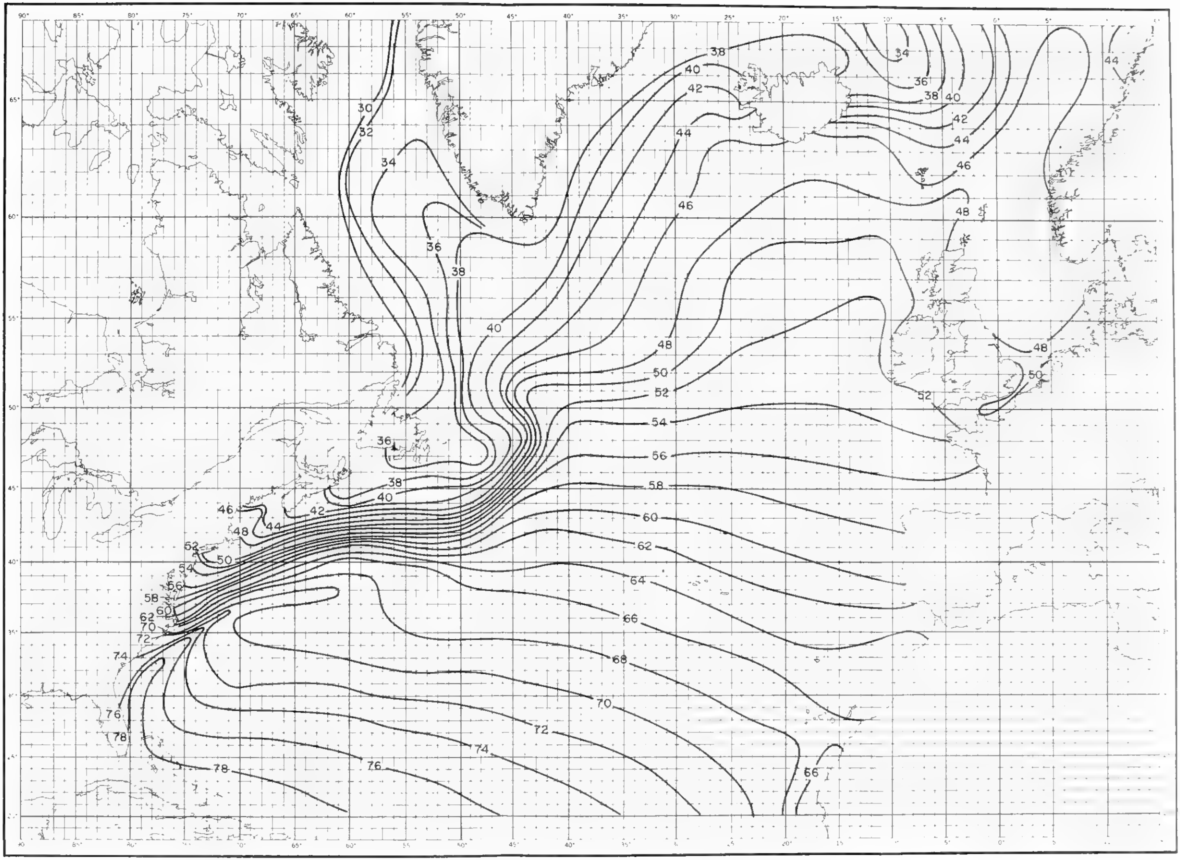

FIGURE B-5 MEAN MONTHLY SEA SURFACE TEMPERATURE— MAY

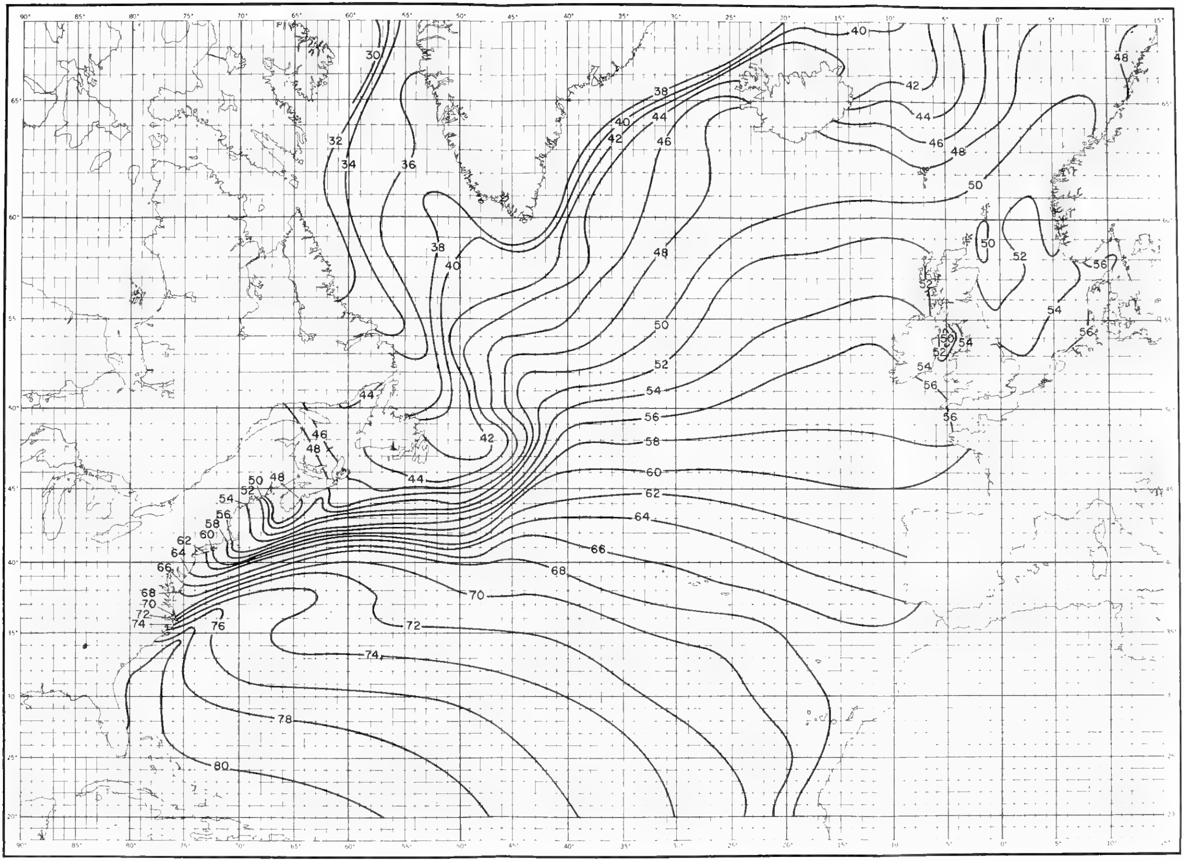

FIGURE B—6 MEAN MONTHLY SEA SURFACE TEMPERATURE — JUNE

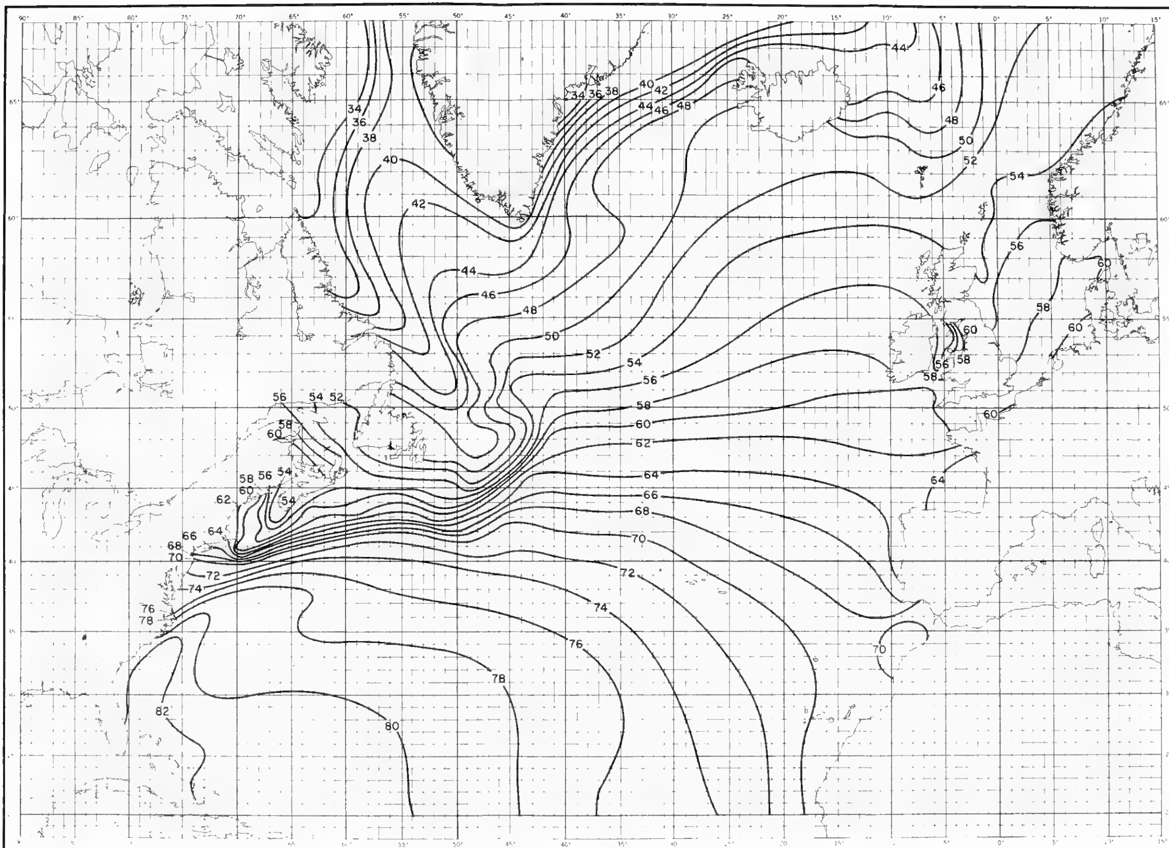

FIGURE B-7 MEAN MONTHLY SEA SURFACE TEMPERATURE — JULY

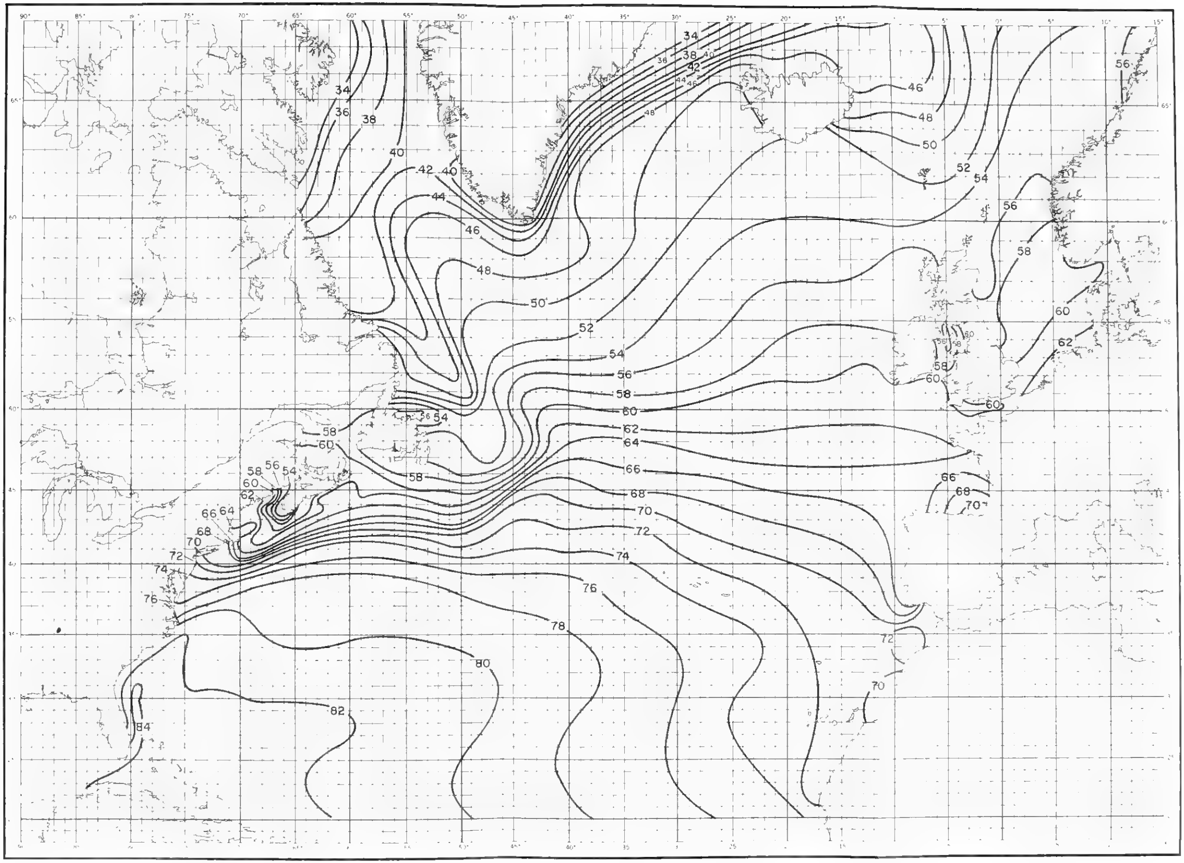

FIGURE B-8 MEAN MONTHLY SEA SURFACE TEMPERATURE — AUGUST

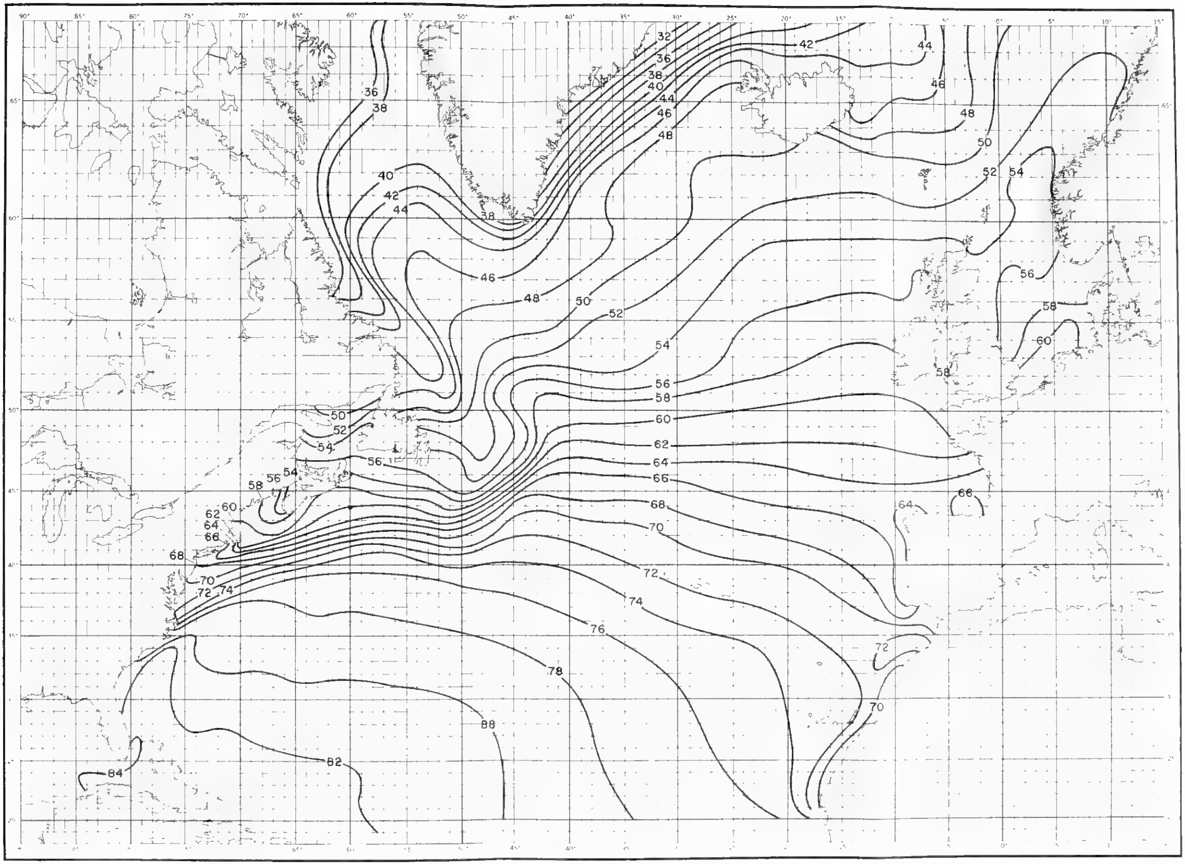

FIGURE B–9 MEAN MONTHLY SEA SURFACE TEMPERATURE—SEPTEMBER

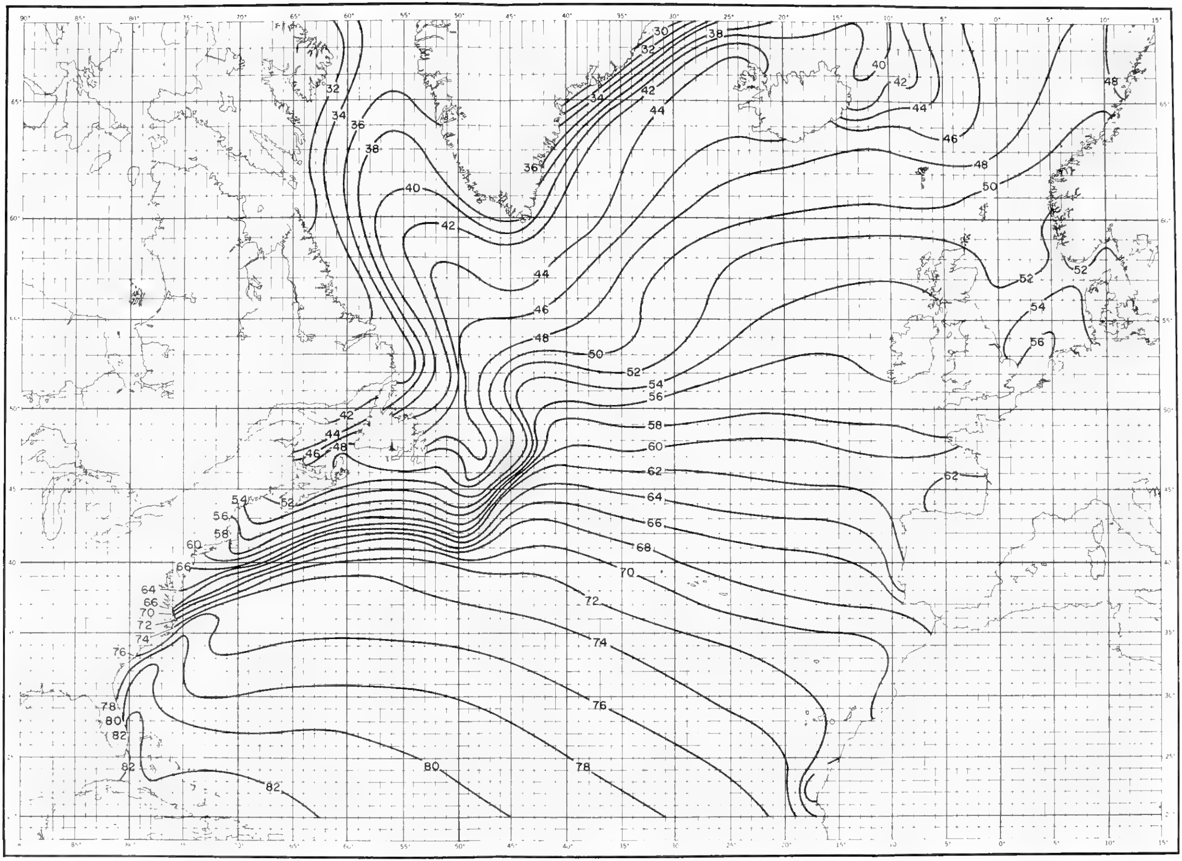

FIGURE B-10 MEAN MONTHLY SEA SURFACE TEMPERATURE – OCTOBER

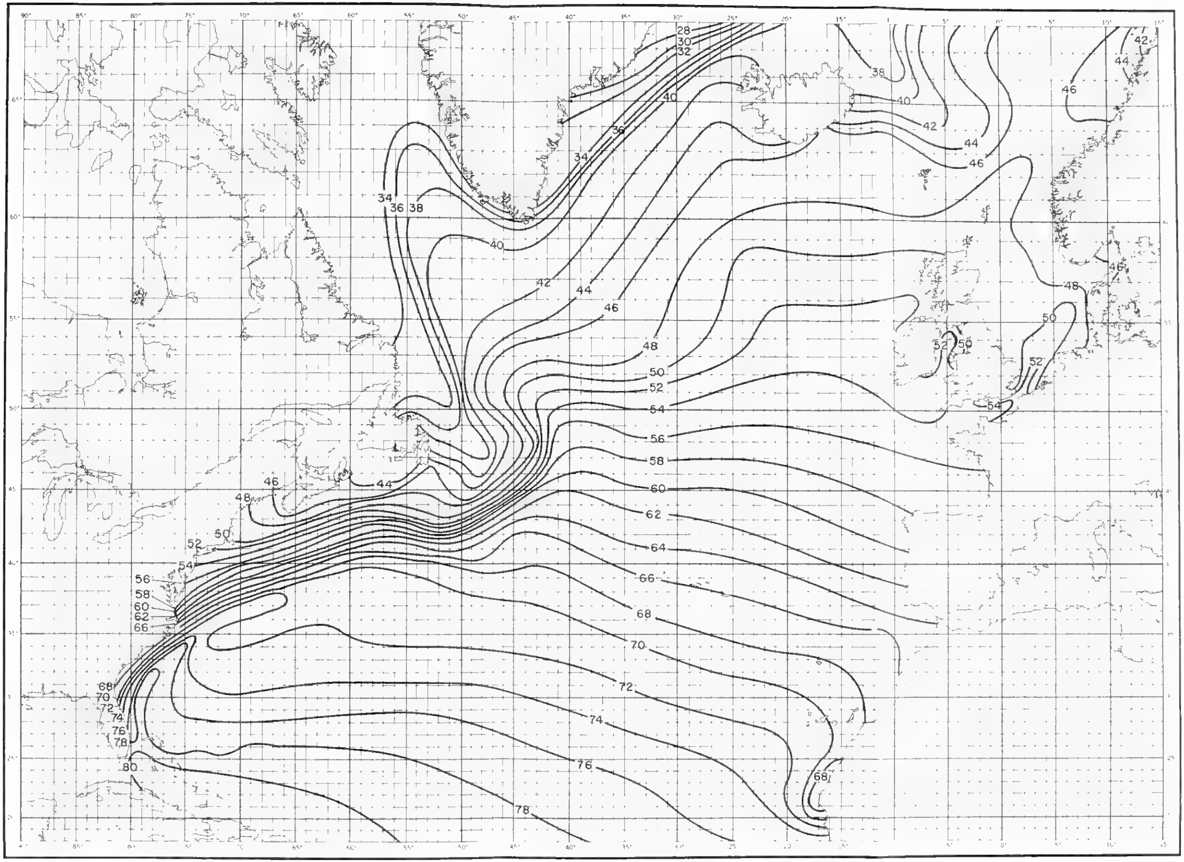

FIGURE B—11 MEAN MONTHLY SEA SURFACE TEMPERATURE —NOVEMBER

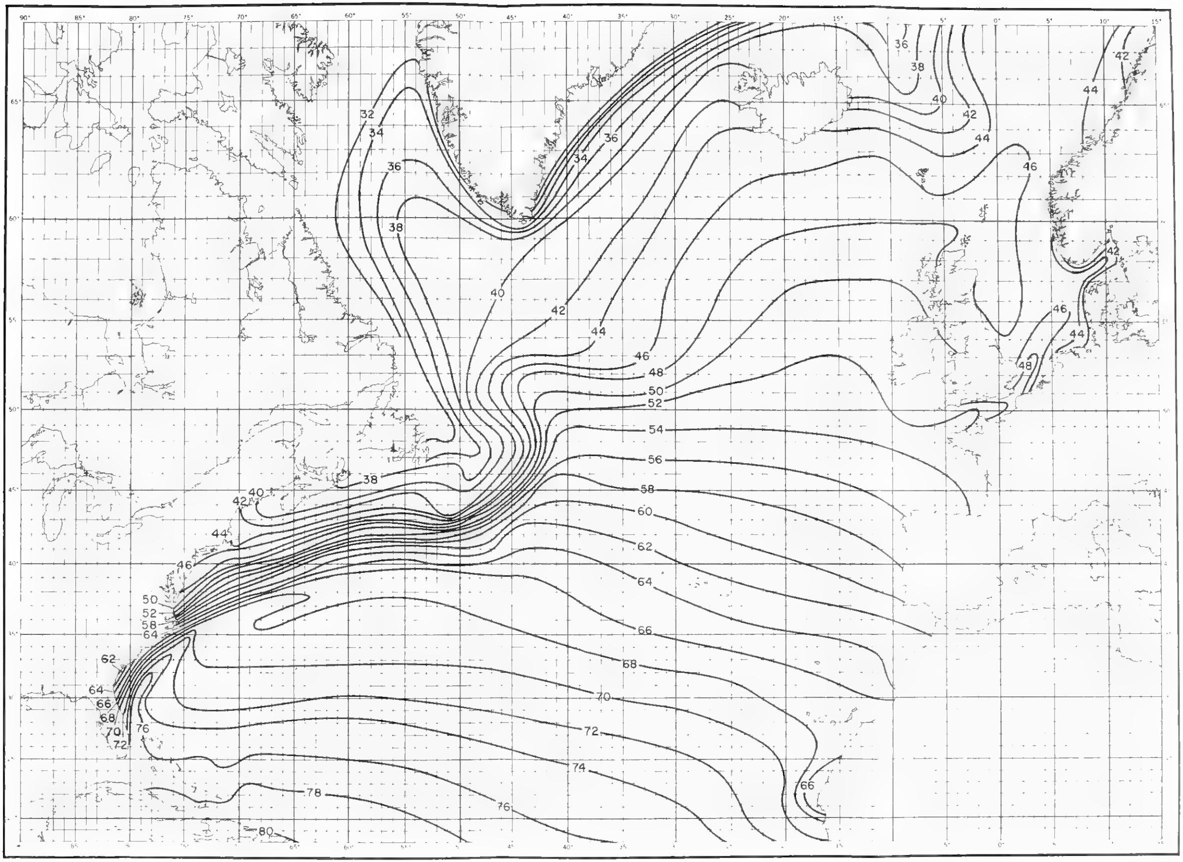

FIGURE B-12 MEAN MONTHLY SEA SURFACE TEMPERATURE—DECEMBER

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