

# PREDICTIONS OF THE COLLAPSE STRENGTH OF THREE HY-100 STEEL SPHERICAL HULLS FABRICATED FOR THE OCEANOGRAPHIC RESEARCH VEHICLE ALVIN 

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Out-of-roundness measurements were taken on three spherical pressure hulls which have been fabricated for ALVIN, a two-man oceanographic vehicle designed for an operating depth of 6000 ft . The results of these measurements indicated that over a critical arc length, the maximum ratio of local radius to nominal radius was 1.05 . The collapse strength of the three hulls was evaluated by means of a recently developed analysis which utilizes local geometry as determined by out-of-roundness measurements. Based on critical local geometry and available stress-strain curves, the analysis indicated that the collapse depths of the hulls ranged from approximately 15,000 to $16,000 \mathrm{ft}$. A high degree of confidence is placed in the results, and the importance of accurately measuring the shape of spherical shells is emphasized。

## INTRODUCTION

The use of the spherical shell as a pressure hull configuration is receiving a good deal of attention in the Navy for deep-depth application. The research vehicle TRIESTE, which has descended to depths of $35,000 \mathrm{ft}$, utilizes a spherical shell as a pressure hull. Other oceanographic research vehicles currently being fabricated or in the design stage also use this type of structure as a pressure hull. In addition, the ends of deeper diving submarines having basically cylindrical middle bodies will probably be closed with hemispherical shells.

Historically, the design of the spherical shell has been hampered by the lack of agreement between theory and experiments. Because of Navy interest in this type of structure, the David Taylor Model Basin engaged in an extensive experimental program to investigate this disagreement and to develop reliable design criteria for spherical shells. This program includes studies of near-perfect machined shells, machined shells with imperfections, and shells fabricated according to feasible full-scale fabrication techniques. The effects of flat spots, thin spots, mismatch, penetrations, stiffeners, and residual stresses are being studied. Although this investigation is by no means complete, a number of important conclusions
have been reached which help to explain the large disagreement between theory and early experimental work. These conclusions provide design tools for the naval architect or engineer.

Because of the Model Basin's background of research in spherical shells, the Office of Naval Research requested ${ }^{l}$ that an evaluation be made of three spherical pressure hulls being fabricated for ALVIN, a two-man oceanographic research vehicle designed for an operating depth of 6000 ft . The pressure hull of this vehicle is an HY-100 steel sphere with five large penetrations as shown in Figure 1. Although three spheres were fabricated, present plans call for utilizing only one of these spheres as the actual pressure hull for the vehicle.

In this report, a brief review of recent tests of spherical shells is given to provide a background for evaluating the effect of initial imperfections on the collapse of spherical shells. Actual measured deviations from sphericity are presented for the three ALVIN pressure hulls as finally constructed. Based on these measurements, hull thickness measurements, and available stress-strain curves, the collapse depths of the pressure hulls are calculated.

## BACKGROUND

The elastic buckling of complete spherical shells was first treated by zoelly in 1915 and is presented by Timoshenko. ${ }^{2}$ His classical buckling pressure $p_{1}$ is given by

$$
\begin{equation*}
\mathrm{p}_{1}=1.21 \mathrm{E}(\mathrm{~h} / \mathrm{R})^{2} \text { for } v=0.3 \tag{1}
\end{equation*}
$$

where $E$ is Young's Modulus,
$h$ is the shell thickness, and
$R$ is the radius to the midsurface of the shell.
Early experiments showed wide disagreement with Equation [l]. Normally, this disagreement may be attributed to initial imperfections, adverse boundary conditions, and residual stresses present in the experimental specimens. More recent tests ${ }^{3-5}$ conducted at the Model Basin on shells which more closely meet the assumption of the theory (i.e., near-perfect

[^0]

Figure 1 - ALVIN Pressure Hull
shells) lend considerable support to Zoelly's equation. Tests of small, near-perfect machined hemispherical shells which had ideal boundaries and which failed at stress levels below the proportional limit have given experimental pressures ranging from 70 to 90 percent of the classical buckling pressure. The tests indicated that the classical buckling coefficient of 1.21 may be attainable for the ideal spherical shell. However, the tests also demonstrate that for small, almost unmeasurable imperfections, the buckling coefficient falls off very rapidly to about 70 percent of the classical value. Based on these results, the Model Basin recommended ${ }^{3-6}$ that the following formula be used to predict the collapse strength of near-perfect spherical shells whose initial departures from sphericity are less than $21 / 2$ percent of the shell thickness:

$$
\begin{equation*}
p_{3}=0.84 \mathrm{E}\left(\mathrm{~h} / \mathrm{R}_{\mathrm{o}}\right)^{2} \text { for } v=0.3 \tag{2}
\end{equation*}
$$

where $R_{0}$ is the radius to the outside surface of the shell. Initially perfect shells may buckle at pressures approaching 43 percent greater than the pressure given by this empirical equation. However, it appears unrealistic at this time to rely on this additional strength due to the difficulty in measuring the initial contours of most practical shells to the degree of accuracy required.

Based on the results of the elastic buckle specimens, an empirical formula was also developed which adequately predicted the collapse of nearperfect machined hemispherical shells which had ideal boundaries and which failed at stress levels above the proportional limit. ${ }^{3}$ This formula may be expressed as

$$
\begin{equation*}
p_{E}=0.84 \sqrt{E_{s} E_{t}}\left(h / R_{o}\right)^{2} \text { for } v=0.3 \tag{3}
\end{equation*}
$$

where $E_{s}$ is the secant modulus and $E_{t}$ is the tangent modulus. For stress levels below the proportional limit, Equation [3] reduces to Equation [2]. From simple equilibrium, the average stress may be expressed as

$$
\begin{equation*}
\sigma_{\mathrm{avg}}=\frac{\mathrm{p} \mathrm{R}_{\mathrm{o}}^{2}}{2 \mathrm{hR}} \tag{4}
\end{equation*}
$$

Equation [3] can then be solved by a trial and error process using the stress-strain curve for the material used in the test specimen. The
agreement between this empirical equation and the model tests is shown in Figure 2. Equation [3] therefore provides a baseline for predicting the elastic or inelastic collapse of a near-perfect, initially stress-free, deep spherical shell with ideal boundaries.

Tests were also conducted at the Model Basin to determine the relationship between unsupported arc length and the elastic and inelastic collapse strength of machined shallow spherical caps with clamped edges. Although previous data in the literature showed wide disagreement in experimental results, these tests followed a very definite pattern. The test results for the inelastic case, which is of particular interest, are plotted in Figure 3. The ordinate is the ratio of the experimental collapse pressure to the empirical pressure, and the abscissa is the nondimensional parameter $\theta$ defined as

$$
\begin{equation*}
\theta=\frac{0.91 \mathrm{~L}_{\mathrm{a}}}{\sqrt{\mathrm{Rh}}} \text { for } v=0.3 \tag{5}
\end{equation*}
$$

where $L_{a}$ is the unsupported arc length of the shell. The results are plotted in families of curves which basically represent varying degrees of stability; shells with the lowest values of $\mathrm{p}_{\mathrm{E}} / \mathrm{p}_{1}$ are the most stable. The results demonstrate that for $\theta$ values greater than approximately 2.2 , the effect of clamping the edges diminishes as the shells become more stable.

Although all of the test results discussed thus far have been for near-perfect models, they do provide the basis for the analysis of spherical shells with initial imperfections. Equations [3] and [4] adequately predict the collapse of near-perfect models. Figure 3 indicates that the collapse of a spherical shell with a $\theta$ value greater than about 2.2 is relatively independent of the boundary conditions. Thus the collapse strength of shells with initial imperfections depends primarily on the local geometry over a critical arc length. These two observations form the basis of the imperfection analysis developed in Reference 6. In this analysis the general form of Equations [1] and [4]becomes

$$
\begin{equation*}
\mathrm{p}_{\mathrm{l}}^{\prime}=1.21 \mathrm{E}\left(\frac{\mathrm{~h}_{\mathrm{a}}}{\mathrm{R}_{\mathrm{l}}}\right)^{2} \tag{6}
\end{equation*}
$$



Figure 2 - Experımental Buckling Data for Machined Deep Spherical Shells with Ideal Boundaries


Figure 3 - Experimental Inelastic Buckling Data for Machined Shallow and Deep Spherical Shells with Clamped Edges

$$
\begin{align*}
& p_{3}^{\prime}=0.84 E\left(\frac{h_{a}}{R_{l_{0}}}\right)^{2}  \tag{7}\\
& p_{E}^{\prime}=0.84 \sqrt{E_{s} E_{t}}\left(\frac{h_{a}}{R_{I_{0}}}\right)^{2}  \tag{8}\\
& \sigma_{a v g}^{\prime}=\frac{p\left(R_{1_{0}}\right)^{2}}{2 h_{a} R_{I}} \tag{9}
\end{align*}
$$

where $h_{a}$ is the average thickness over a critical arc length,
$\mathrm{R}_{1}$ is the local radius to the midsurface of the shell over a critical arc length, and
$\mathrm{R}_{\mathrm{I}_{0}}$ is the local radius to the outside surface of the shell over a critical arc length.
These equations may be used to predict the collapse strength of both nearperfect shells and shells with initial imperfections. In Reference 6 it is assumed that the critical arc length $\mathrm{L}_{\mathrm{c}}$ may be determined by:

$$
\begin{equation*}
\mathrm{L}_{\mathrm{c}}=\frac{2.2}{0.9 \mathrm{I}} \sqrt{\mathrm{R}_{\mathrm{I}} \mathrm{~h}_{\mathrm{a}}} \tag{10}
\end{equation*}
$$

The primes in Equations [6] through [9] simply indicate that the local geometry is used to calculate the pressures and stresses.

Thus, for the case of the spherical shell with imperfections, it is necessary to determine the local radius over a critical arc length. .* The simplest way to do this is to express the local radius in terms of deviations from a nominal radius or in terms of out-of-roundness. Figure 4 shows the assumed relationship between deviations from a nominal radius d, out-of-roundness $\Delta$, and local radius $R_{1}$ over a critical arc length. With a given out-of-roundness the ratio of $R_{1} / R$ may be calculated in terms of $\Delta / h_{a}$ and $h_{a} / R$ from geometry. The results of these calculations are presented graphically in Figure 5.

To verify the validity of the analysis given above, models with
*'In this analysis it is assumed that the critical arc length is associated with a $\theta$ value of 2.2 .
known "flat spots" and "thin spots" were machined and tested. A sketch of these models is shown in Figure 6. The ratio of $h / R_{0}$ and the angle $\phi$ were varied systematically. A detailed description of the models and analysis of the results is presented in Reference 6. Briefly, the collapse of these models was predicted using the local geometry over a critical arc length as described above. A comparison of the experimental results and the pressure predicted by Equation [8] is presented in Figure 7. The ordinate is the ratio of $p_{\exp }$ to $p_{l}^{\prime}$, and the abscissa is the ratio of $p_{E}^{\prime}$ to $p_{1}^{\prime}$. It is apparent that this procedure adequately predicted the collapse of these shells. The apparent experimental scatter for the less stable shells $\left(\mathrm{p}_{\mathrm{E}}^{\prime} / \mathrm{p}_{1}^{\prime \prime} \approx 0.7\right)$ can be explained on an individual basis. The collapse pressures of those models which fall below the line were affected by secondary moments. The curves shown in Figure 3 indicate that secondary moments significantly reduce the collapse pressures of the less stable shells. These effects diminish as the shells become more stable. For those models which lie above the line, the experimental buckling pressure was more than 70 percent of the classical pressure. Recent tests indicate that it may be possible to achieve 100 percent of the classical pressure.

In addition to the machined models with imperfections, models of HY-80 steel fabricated according to feasible large-scale fabrication techniques are being studied. ${ }^{7}$ The local geometry is determined from thickness and sphericity measurements and is used to determine the stresses and collapse pressures. Although the tests have not been completed, the preliminary results indicate good agreement with the analysis presented above.

The model tests and results just described have been presented briefly in order to provide a background for the evaluation of the ALVIN pressure hulls. A more detailed description of the test results is given in the references cited.

## DESCRIPTION OF ALVIN PRESSURE HULL

The ALVIN pressure hull (see Figure 1) is composed of two spun HY100 steel hemispheres. The inside contour of each hemisphere was machined

where outward deviations from nominal radius are positive and inward deviations from nominal radius are negative.

Figure 4 - Assumed Relationship between Out-of-Roundness $\Delta$ and Local Radius $\mathrm{R}_{1}$


Figure 5-Relationship between $R_{1} / R$ and $\Delta / h_{a}$


Figure 6a - Machined "Flat Spot" Models


Figure 6b - Machined "Thin Spot"' Models

Figure 6 - Machined Models with Imperfections


Figure 7 - Experimental Buckling Data for Machined Models with Imperfections
to size and the two hemispheres were welded to form the complete sphere. The outside contour was machined after welding. Five large penetrations were made in the completed hull and the reinforcement inserts were welded in place. After all welding, the hull was stress relieved. Three hulls, designated Hull No. 1, Hull No. 2, and Hull No. 3, were fabricated.

Typical compressive stress-strain curves for the HY-100 steel material used in the fabrication of the hulls are shown in Figure 8. These curves were obtained from tests of specimens machined from the material removed for the penetrations. Specimen coupons were obtained from the material removed from the hatch and from a viewing port (Window No. 3). Thus, the curves of Figure 8 represent each of the hemispheres used in manufacturing the hulls.* Other representative material characteristics are presented in Figure 9.

Hull thickness measurements provided by Hahn and Clay are presented in Figure 10. These measurements were made by vidigage after the welds had been ground.

## MEASUREMENTS OF INITIAL IMPERFECTIONS

Initial imperfections and fabrication tolerances were important considerations in the design of this vehicle. A maximum radius of curvature of 45 in . was specified as a reasonable fabrication tolerance. After fabrication, it was necessary to take out-of-roundness measurements to determine the maximum radius of curvature over a critical arc length. This information is valuable not only in evaluating the ALVIN pressure hulls but also in determining feasible fabrication tolerances for similarly fabricated vehicles.

The instrument shown in Figure 11 was designed to take sphericity measurements from a common point for the completed ALVIN hulls. Basically,
*Six specimens were tested for each hull. The total variation in yield strength for each hull was only 4 percent.
*** he hull thickness measurements presented in Figure 10 were used in all strength calculations. No allowances were made for corrosion.


Figure 8 - Typical Compressive Stress-Strain Curves for HY-100 Steel Material Used in Fabrication of ALVIN Pressure

Hulls


Figure 9 - Representative Material Characteristics

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Figure 10 －Hull Thickness Measurements HATCH

Figure 10a－Hull No． 1

|  |  | $\left.\begin{gathered} \frac{0}{0} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \dot{0} \end{gathered} \right\rvert\,$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & \frac{0}{3} \\ & \stackrel{y}{3} \\ & \stackrel{5}{5} \\ & \stackrel{y}{3} \\ & \dot{c} \end{aligned}$ |  |  |  |  | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ \vdots \end{array}\right\|$ |  |  |  |  |  |  |
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Figure 10b－Hull No． 2

| Wall Thickness Readings |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Location | Near Side | Far Side | Location | Near Side | Far Side | Comments |
| 1 | 1.345 | 1.331 | 29 | 1.307 |  |  |
| 2 | 1.360 |  | 30 | 1.318 |  | No. 2 on Feld |
| 3 | 1.345 | 1.330 | 31 | 1.319 |  | No. 3 on Weld |
| 4 | 1.333 |  | 32 | 1.321 |  |  |
| 5 |  | 1.330 | 33 | 1.336 | 1.318 |  |
| 6 |  | 1.336 | 34 | 1.336 | 1.321 |  |
| 7 | 1.325 | 1.337 | 35 | 1.321 | 1.323 |  |
| 8 | 1.325 | 1.358 | 36 | 1.317 | 1.323 | No. 8 on Weld |
| 9 |  | 1.350 | 37 | 1.315 |  | No. 9 on Weld |
| 10 |  | 1.321 | 38 | 1.327 |  |  |
| 11 | 1.355 |  | 39 |  | 1.340 | No. 11 on Weld |
| 12 | 1.358 | 1.333 | 40 |  | 1.345 | No. 47 on Weld |
| 13 | 1.341 | 1.332 | 41 |  | 1.355 | No. 13 and 41 on Weld |
| 14 | 1.356 |  | 12 | 1.320 | 1.334 | No. 14 on Weld |
| 15 | 1.34 .5 | 1.320 | 43 | 1.346 |  |  |
| 16 | 1.335 | 1.340 | 44 | 1.344 |  |  |
| 17 | 1.328 |  | 45 | 1.324 |  |  |
| 18 | 1.327 |  | 46 | 1.329 |  |  |
| 19 |  | 1.319 | 47 | 1.341 |  |  |
| 20 |  | 1.357 | 48 | 1.347 |  | No. 20 on Wela |
| 21 |  | 1.336 | 49 | 1.336 |  |  |
| 22 |  | 1.336 | 50 | 1.335 |  |  |
| 23 | 1.333 |  | 51 |  | 1.342 | No. 51 on Weld |
| 24 | 1.326 |  | 52 |  | 1.356 | No. 52 on Weld |
| 25 | 1.341 |  | 53 | 1.353 |  | No. 25 and 53 on Weld |
| 26 | 1.347 |  | 54 |  | 1.348 | No. 25 and 54 on Weld |
| 27 | 1.341 |  | 55 |  | 1.344 | No. 55 on Weld |
| 28 | 1.322 |  | 56 | 1.348 |  | No. 56 on Weld |


Figure 10c - Hull No. 3


Figure 11 - Instrument Used in Measuring Departures from Sphericity
the instrument is designed to measure the distance from a central point inside the sphere to any point on its inside surface. This central point should coincide closely with the center of the sphere. It is possible to take the readings obtained from the instrument and then, utilizing the high-speed computer facilities of the Applied Mathematics Laboratory at the Model Basin, to calculate the theoretical location of the center of the sphere relative to the actual position of the instrument. Departures from sphericity may then be calculated by a computer relative to the theoretical center of the sphere. This was not done for the ALVIN spheres since it was possible to locate the instrument with sufficient accuracy (within 0.050 in.$)$ by taking readings on the Ames dial gage and adjusting the position of the instrument by means of the vertical adjustment screw and the horizontal adjustment holes shown in Figure ll. Small variations between the theoretical center of the sphere and the position of the instrument would have a negligible effect on the local curvature over a critical arc length. The instrument can rotate 360 deg about the horizontal pivot points, and the adjustable arm can rotate 180 deg about the vertical pivot point.

Over 1200 measurements were taken on each hull to ensure that at least five readings were taken over a critical arc length. Additional readings were taken in the areas of the welds to locate points of maximum deviations. The Ames dial gage was calibrated for each hull by the use of inside micrometers. Deviations were then measured and computed relative to the nominal inside radius.

## RESULTS OF MEASUREMENTS

Measured deviations from the nominal inside radius are plotted in the form of contour maps in Figure 12. Inward deviations are plotted as minus contours. The radial scale used in all drawings is constant. The solid circles in the figures represent a complete hemisphere, and the radial scale may be determined by measuring the diameter of this circle. This measured diameter represents one-half the inside circumference of the sphere. This is done for accuracy in analyzing the data. As is the case with the problem of mapping, however, the scale in all other directions varies, depending on the distance from the center of the plot and the

Figure 12 - Deviations from Sphericity*


Figure 12a - Hun Number 1, Inside View Looking Forward
NOTE: Contours are plotted in intervals of 5 mils. Minus contours indicate inward deviations, i.e., $\mathbf{- 1 0}$ indicates distance from center of sphere is 39.630 in . For local geometry in areas marked by Roman Numerals, see Table 1.

[^1]

Figure 12b - Hull Number 1, Inside View Looking Aft


Figure 12c - Hull Number 2, Inside View Looking Forward
*


Figure 12d - Hull Number 2, Inside View Looking Aft


Figure 12e - Hull Number 3, Inside View Looking Forward


Figure 12f - Hull Number 3, Inside View Looking Aft
orientation with the radial direction．The location of the penetration and girth welds are shown in Figure 13 to the same scale as the contour plots．

## DISCUSSION

To determine the maximum local radius of curvature，it is necessary to obtain the maximum out－of－roundness $\Delta$ over a critical arc length．This value may be obtained using the contour maps（Figure 12）and the scales presented in Figure 14．These scales have been drawn to overcome the mapping problem．Each scale contains two curves；the inner curve covers an arc length of 16.6 in．，the outer curve an arc length of 19.4 in．The critical arc length may be calculated by Equation［10］．Assuming a thick－ ness of 1.33 in ．and a ratio $R_{1} / R$ of $I_{0} 05$ ，the critical arc length is 18.0 in．，which is midway between the curves presented in Figure 14．Areas of maximum out－of－roundness may be determined by placing Figure 14 over Figure 12 and rotating the scales．The values for the deviations from sphericity $\delta_{A}$ and $\delta_{C}($ see Figure 4）are determined across any＂diameter＂of a scale at a point midway between the two curves．The deviation $\delta_{B}$ is determined at the center of each scale．The location of points with relatively large values of out－of－roundness $\Delta$ may be found in this manner．

Using the hull thickness measurements presented in Figure 1.0 and the curves presented in Figure 5，further refinements can be made in determining the critical arc length．A sample calculation sheet is shown in Figure 15．Values of $\Delta, h_{a}$ ，and $R_{1} / R$ were determined for several locations on each of the three hulls．Some of these values are shown in Table l． The maximum ratio of $R_{l} / R$ for the three hulls was 1.05 ．This demonstrates the accuracy with which these hulls were fabricated and provides a guide－ line for reasonable tolerances for similarly fabricated steel vehicles． For example，Figure 5 indicates that a steel hull $23 / 4$ in．thick and an out－of－roundness $\Delta$ of 0.107 in。would also have a ratio $R_{1} / R$ of 1.05 。 It is important to remember that the out－of－roundness is defined over a critical arc length。

The collapse depths of the three ALVIN pressure hulls were cal－ culated using a critical local geometry。 A minimum ratio of hull thickness

Figure 13 - Location of Penetrations and Girth Welds **


Figure 13a-Inside view looking forward

[^2]

Figure 13b - Inside view looking aft


Figure 14 - Arc Length Scales*
*The surface enclosed by the solid circle shown represents a hemisphere unfolded into a flat surface whose radial scale remains constant.

Figure 14 is also reproduced on transparent film for use as an overlay on Figure 12 and enclosed in a back-cover pocket.

TABLE 1
Local Geometry of the Three ALVIN Pressure Hulls

| Hull | Location (See Figure 12) | $\begin{gathered} \mathrm{h}_{\mathrm{a}} \\ \text { (See Figure 10) } \end{gathered}$ | $\mathrm{R}_{1} / \mathrm{R}$ |
| :---: | :---: | :---: | :---: |
| 1 | I | 1.35 | 1.04 |
|  | II | 1.36 | 1.03 |
|  | III* | 1.32 | 1.05 |
|  | IV | 1.35 | 1.05 |
|  | V | 1.35 | 1.04 |
|  | VI | 1.35 | 1.05 |
| 2 | I | 1.34 | 1.03 |
|  | II | $1.33{ }^{\text {\%** }}$ | 1.03 |
|  | III* | 1.34 | 1.04 |
|  | IV | 1.34 | 1.03 |
|  | V | 1.36 | 1.03 |
|  | VI | 1.34 | 1.03 |
| 3 | I | 1.33 | 1.03 |
|  | II | 1.34 | 1.04 |
|  | III | 1.35 | 1.04 |
|  | IV | 1.34 | 1.04 |
|  | $v^{*}$ | 1.32 | 1.03 |
|  | VI | 1.34 | 1.04 |
| *Location of minimum value of $h_{a} / R_{1} I_{0}$ |  |  |  |

**Assumed thickness; no values given.
( $h_{a}$ ) to outside local radius ( $\mathrm{R}_{I_{0}}$ ) was obtained for each pressure hull. The local geometries for these critical areas are presented in Table 2. Using these values of $h_{a}$ and $R_{1_{0}}$ in Equations [8] and [9], the collapse depths $p_{E}^{\prime}$ were calculated. The results of these calculations indicate that Hulls No. 1, 2, and 3 will collapse at depths of $15,800,16,100$, and 15,100 ft, respectively. The lower collapse depths for Hull No. 3 may be attributed to its relatively low yield strength (see Figure 8).

A high degree of confidence can be placed on these collapse depths for a number of reasons. First and primarily, the shape of the hull has been measured accurately. ${ }^{*}$ Previous tests have indicated that the collapse strength of a spherical shell can be predicted accurately if the critical local geometry is known. Second, the analysis presented in this report is conservative for stable shells; i.e., for those shells whose empirical inelastic buckling pressure is considerably lower than the classical elastic buckling pressure. The average ratio of $p_{E}^{\prime}$ to $p_{1}^{\prime}$ for the three ALVIN hulls is approximately 0.20 (see Table 2). Figure 5 indicates that the analysis is conservative in this range. The reason for this is that a Poissons ratio of 0.3 was used in the plastic range and the three-dimensional Hencky-Von Mises effect of the pressure was neglected. In addition, buckling coefficients higher than 0.84 are possible. Third, the effect of secondary moments, which are not considered in the analysis, is negligible. This is shown by the experimental results plotted in Figure 3 which indicate that even for clamped edges, the effect of secondary moments is very slight for stable shells with $\theta$ values of 2.2 or greater.

The viewing port and access hatch inserts should not adversely affect the collapse strength of the pressure hulls. Reference 8 reports tests of two $0.286-s c a l e$ hemispherical models of ALVIN, each penetrated by a single viewing port. Strains were measured on the penetration insert as well as on the adjacent portion of the spherical shell. These strains and the calculated experimental stresses in and around the penetration inserts indicated that the design of the penetration inserts was adequate.
*Obviously this confidence would be destroyed if significant changes occur in the shape of the hulls due to any additional penetrations or welding.

TABLE 2
Critical Local Geometry and Calculated Collapse Depths for the Three ALVIN Pressure Hulls

| Hull | $\mathrm{h}_{\mathrm{a}}$ | $\mathrm{R}_{1_{0}}$ | $\mathrm{p}_{1}^{\prime}$ | $\mathrm{p}_{\mathrm{E}}^{\prime}$ | $\mathrm{p}_{\mathrm{E}}^{\prime} / \mathrm{p}_{1}^{\prime}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| psi | psi | Collapse <br> $\mathrm{p}_{\mathrm{E}}^{\prime}$ <br> ft |  |  |  |  |
| 1 | 1.32 | 43.0 | 34,200 | 7000 | 0.20 | 15,800 |
| 2 | 1.34 | 42.6 | 36,000 | 7160 | 0.20 | 16,100 |
| 3 | 1.32 | 42.2 | 35,600 | 6710 | 0.19 | 15,100 |

Figure 15 - Sample Calculation Sheet for Determining Ratio of $R_{1} / R$

Equation [10] $L_{c}=\frac{2.2}{0.91} \sqrt{\mathrm{R}_{1} \mathrm{~h}_{\mathrm{a}}}$
Assume $h_{a}=1.33$ and $R_{1} / R=1.05$
Then $\mathrm{L}_{\mathrm{c}}=18.0 \mathrm{in}$. (Midway between curves)

Hull No. I - Location I (See Figure 12a)


25 deg Scale
Inner curve covers arc length of 16.6 in. Outer curve covers arc length of 19.4 in.

$$
\begin{aligned}
& \Delta=\frac{\mathrm{a}+{ }^{\delta} \mathrm{c}}{2}-\delta_{\mathrm{B}}(\text { see Figure } 4) \\
& \Delta=\frac{-14-10}{2}-(-54)=42 \mathrm{mils}=0.042 \mathrm{in} . \\
& \frac{\Delta}{\mathrm{ha}}=\frac{0.042}{1.33}=0.032
\end{aligned}
$$

$$
\frac{\mathrm{R}_{1}}{\mathrm{R}}=1.04 \text { (See Figure 5) }
$$

From Figure $10 \mathrm{~h}_{\mathrm{a}}=1.35$
Assume $R_{1} / R=1.04$
$\mathrm{L}_{\mathrm{C}}=18.0^{\prime \prime} \Delta=0.042$
$\frac{\Delta}{\mathrm{ha}}=\frac{0.042}{1.35}=0.031$
$R_{1} / R=1.04$ (Agrees with assumed value)

Normally these stresses were less than the membrane stresses in the spherical hull. Any mismatch which might occur on the three prototype hulls would cause very local bending stresses in an area reinforced by the increased thickness of the penetration insert. Since these stresses are normally less than the membrane stresses or are very local, they should not adversely affect the strength of the hulls.

Throughout the analysis, reference has been made to available stress-strain curves. It has been assumed that the stress-strain curves presented in Figure 8 are representative of the material used in the hull. As additional data becomes available, they should be compared with the data used in this analysis. If significant differences are reported from other sources the predicted collapse depths can easily be adjusted using the geometry presented in Table 2 and Equations [8] and [9]. It should be mentioned that preliminary data on the weld material indicate that the strength of the welds may be 5 percent lower than that of the hull material. In view of the stability of these hulls, it is not felt that this would affect the collapse of these spheres since this is a very local effect.

Although the results of this analysis differ little from the results obtained using the nominal geometry of the spherical pressure hulls, it should be emphasized that this is only because the hulls have been fabricated so accurately. For example, if the hull had been built to an out-of-roundness tolerance of $\pm 1 / 8 \mathrm{in} .$, the collapse depths for the three hulls would range from 12,600 to $13,400 \mathrm{ft}^{*}$. Further, if normal boiler code tolerances of $\pm 1$ percent of the outside diameter were obtained in fabrication this range would be reduced even further, that is, the collapse depths would range from 6000 to $6400 \mathrm{ft}^{*}$. Thus the effect of neglecting initial imperfections and the importance of determining the exact shape of spherical pressure hulls is evident.
${ }^{*}$ For these calculations it is assumed that the maximum allowable out-ofroundness $\Delta$ ( 0.25 in . and 1.64 in. ) occurs over the critical arc length.

1. On the basis of measured deviations from sphericity, hull thickness measurements made by the fabricators and material stress-strain curves available at this time, collapse depths of the ALVIN Pressure Hulls No. l, 2 , and 3 were calculated as $15,800,16,100$, and $15,100 \mathrm{ft}$, respectively. 2. The maximum local radius of curvature for the three hulls was only 5 percent greater than the nominal radius.
2. Local geometry and not nominal geometry must be used to determine the collapse strength of a spherical shell. The collapse strength of a spherical shell with the same geometry as ALVIN, fabricated to boiler code tolerances may be as low as 40 percent of the strength which was predicted for the ALVIN spheres.

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Figure 14 - Arc Length Scales


[^0]:    ${ }^{\mathrm{I}}$ References are listed on page 37.

[^1]:    *The surface enclosed by the solid circle shown represents a hemisphere unfolded into a flat surface whose radial scale remains constant.

[^2]:    *The surface enclosed by the solid circle shown represents a hemisphere unfolded into a flat surface whose radial scale remains constant.

