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IMINARY DESIGN THEORY FOR POLYPHASE IMPELLERS IN UNBOUNDED FLOW

DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER



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A PRELIMINARY DESIGN THEORY FOR
POLYPHASE IMPELLERS IN
UNBOUNDED FLOW



by

B. Yim

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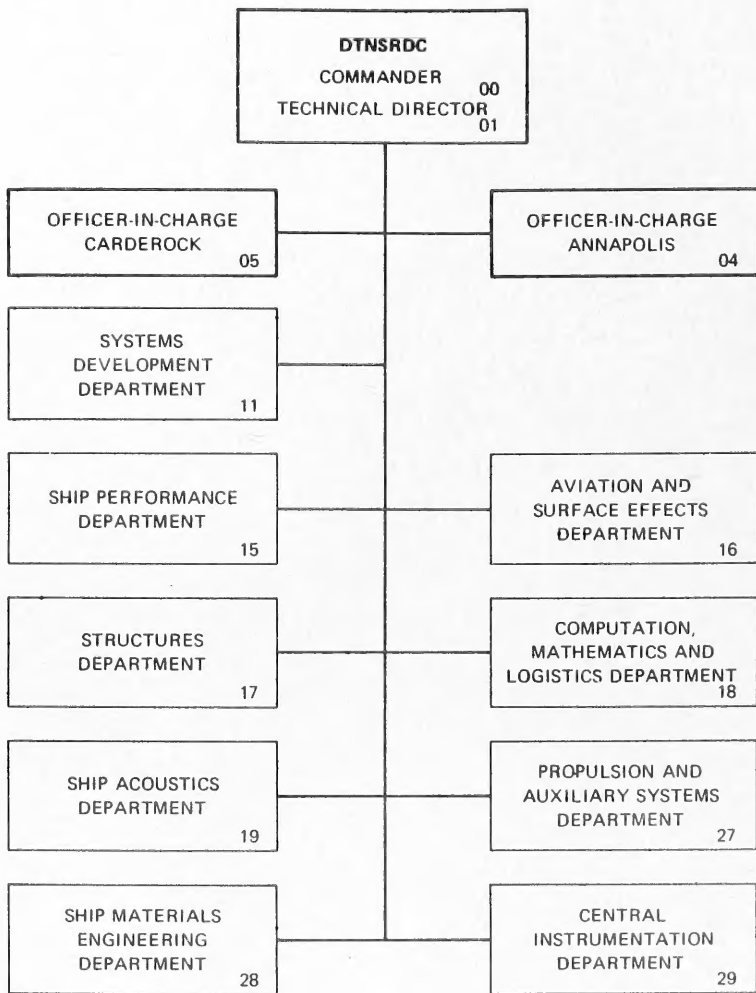
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ABSTRACT

The main role of preliminary design for supercavitating propellers is to supply the basic data for the final design, such as: the hydrodynamic pitch angle, the radial load distributions, the approximate cavity length and the distribution of cavity-source strengths which will help determine the three-dimensional cavity-source distribution. For this purpose, the effective use of supercavitating cascade theory with lifting-line theory is discussed together with influences of neighboring cavities on cavity drag, the hydrodynamic pitch angle, inflow retardation and the optimum pitch distribution of the propeller.

The computer program developed here is applied to several existing propeller models. The results show that propeller efficiency is predicted well but pitch distribution is a little larger than for the model. The results are analyzed and compared with the results of a lifting-surface design method which has been developed recently for use with the present preliminary design method.

NOMENCLATURE

c	Chord length of blade section
c_o	Camber factor
C_L	Design lift coefficient of blade section
C_{L_o}	Lift coefficient of a foil in an infinite medium
C_T	Thrust coefficient of propeller
C_P	Power coefficient
D	Diameter of propeller
d	Distance between the leading edges of neighboring blades at each section
G	Circulation distribution
J	Advance coefficient, $V/(DN)$
N	Rotation per unit time
R	Propeller radius
r	Radial coordinate divided by propeller radius
r_H	Hub radius divided by propeller radius
V	Ship speed
w	Wake fraction

w_a	Axial component of perturbation velocity
w_t	Tangential component of perturbation velocity
y	Shock-free camber-offset of blade face
Z	Blade number
α	Extra angle of attack superposed to the shock-free foil
β_1	Hydrodynamic pitch angle
γ	Stagger angle of cascade
δ	Flow exit angle of cascade
ϵ	Drag-lift ratio of a foil at the blade section
σ	Local cavitation number of design propeller
σ_∞	Cavitation number for infinite-length cavity of cascade
Ω	Angular speed of blade

INTRODUCTION

Supercavitating propellers are strong candidates for propulsors on high-speed craft^[1]. A design method for supercavitating propellers was developed first in this country by Morgan and Tachmindji^[2], with efforts still continuing^[3-7].

The main problem associated with designing supercavitating propellers as opposed to subcavitating propellers is the effect of three-dimensional cavities trailing from the propeller blades. In early work, knowledge of an isolated two-dimensional supercavitating foil was combined with subcavitating propeller-design theory^[2,3]. However, it was soon realized that blade-cavity interference between each of the supercavitating propeller blades was too large to be neglected^[3,4]. Thus, three-dimensional integrated design methods in which the cavity and the blade were considered to lie on a helical surface, were formulated by several hydrodynamicists^[5,7]. Through the medium of high-speed computers^[8,9], these efforts have shown success in the design theory for subcavitating propellers. The difficulty encountered in supercavitating propeller design is that the geometry of the cavity surface is not known a priori. For two-dimensional cavity problems the complex-variable theory could be efficiently utilized; but not for three dimensions. Numerical solutions using high-speed computers have been contemplated^[5,7]. However, so far, no practical, reliable, computer program seems to exist.

The hydrodynamic design method of subcavitating propellers may be divided into two steps: preliminary or performance design and final or lifting surface design. Preliminary design employs three-dimensional lifting-line theory^[10] to find the hydrodynamic pitch angle β_1 and the vortex distribution G along the blade span that will supply the required thrust for the design speed. The hydrodynamic pitch angle is the angle of inflow velocity at the propeller plane. From this angle, the angle of attack of the blade is measured. The hydrodynamic pitch angle defines a basic helical surface where the vortex distribution will be located in the lifting-surface theory for the final design^[8]. Chordwise pressure and thickness distributions are obtained from two-dimensional airfoil theory. In designing supercavitating propellers, it seems to be natural to consider the same two steps as those for subcavitating propellers. Thus, the first problem will be preliminary design without which there cannot be the final design. As previously mentioned, the preliminary design method of Tachmindji and Morgan^[2] has not been found to be accurate enough for use in the final design. In addition, since final design is extremely complicated^[5,7], it is to be hoped that a preliminary design method will help reduce the complexity of the final design.

To improve the existing preliminary design method for supercavitating propellers, several problem areas need to be considered:

1. Influence of neighboring cavities on cavity drag, which will affect the thrust and torque of the propeller.
2. Influence of cavities on the hydrodynamic pitch angle.
3. Inflow retardation caused by blades and cavities.
4. Effect of cavity on the optimum pitch distribution.

These problem areas will be discussed in the following sections.

The main role of preliminary design is to supply the basic data for the final design, such as: the hydrodynamic pitch angle β_i , the radial load distributions, the approximate cavity length and the distribution of cavity-source strengths which will help determine the three-dimensional cavity-source distribution. For this purpose the effective use of supercavitating cascade theory with lifting line theory is discussed. Inputs for the preliminary design of supercavitating propellers include cavitation number, leading-edge cavity thickness, and camber shape. In addition, the same information is required as for subcavitating propellers. The minimum leading-edge cavity thickness is supplied from a blade strength analysis. The minimum cavity length at each blade section is assumed to be 1.5 chord lengths.

The computer program developed here is applied to several existing propeller models. The results show that propeller efficiency is predicted well but pitch distribution is a little larger than for the model. The results are analyzed and compared with the results of a lifting surface design method which has been developed recently for use with the present preliminary design method.

LIFTING LINE THEORY

Consider the flow field of a supercavitating propeller rotating with a constant angular velocity in an otherwise uniform flow. If fluid viscosity is neglected, the flow is irrotational, and can be computed from appropriate vortex and source distributions. The analysis is based on linear theory in which the blade angle of attack and the camber are considered to be small so that the cavity is thin. The singularity distribution may be, therefore, located on a basic helical surface, near or in the blade and cavity. The vortex distribution is proportional to the load distribution on the blade. Since there is no load outside of the projection of the blade on the basic surface, the vortex distribution for the supercavitating propeller is not different essentially from that for the subcavitating propeller. Thus, the basic feature of lifting line theory for the preliminary design of a supercavitating propeller is not different from that of a subcavitating propeller. The theory, developed by Lerbs^[10], has been programmed, and is widely used. Because both cavity and blade-friction drag alter the thrust and torque of the propeller due to the vortex distribution on the blades in inviscid flow, the calculation has to be iterated to find the lifting line vortex distribution in the cavity flow required to produce the required thrust.

The main changes in the present lifting line program are the addition of an option for computation of the optimum distribution of hydrodynamic pitch, which will be discussed later, and consideration of cavities from supercavitating cascades. These changes enable a full consideration of blade cavity interferences to be incorporated while maintaining requirement on the minimum leading edge cavity thickness to produce a cavity with a length 50 percent longer than the blade chord.

BLADE CAVITY INTERFERENCE

Results of experiments^[11,12] conducted on supercavitating propellers designed by the method of Tachmindji and Morgan^[2] indicated that, in general, the propellers were underpitched. One reason for this was probably inadequate treatment of blade cavity-interference effects^[2-4]. Tulin^[4] suggested using either supercavitating cascades or a cavitating foil above a free surface in two dimensions instead of an isolated two-dimensional cavitating foil, to calculate the cavity drag of the blade section of the propeller. Scherer et al^[13] also considered a cascade model for the design of a supercavitating

propeller. It is of interest to point out that the cascade corrections^[14,15] were originally used in subcavitating propeller design methods before the lifting-surface theory was fully developed. Both theoretical and experimental data for supercavitating cascades which would be useful for the design of supercavitating propellers are very scarce. The cascade section theory and its associated computer program developed by Yim^[16-21] are used here to aid in designing blade shape.

It must be recognized, however, that a supercavitating cascade is not a perfect model for blade-section design principally because of the disparity of the cavitation numbers between the cascade and the propeller^[4]. That is, in a cascade, the cavitation number can never be smaller than the infinite cavity cavitation number σ_∞ which is always larger than zero, while a ventilating propeller has zero cavitation number. Even for supercavitating propellers, the cavitation number σ near the blade tip, in general, is smaller than σ_∞ . Yet, the drag-lift ratio of a meaningful supercavitating foil, influenced by neighboring cavities, is considered to be properly analyzed in supercavitating cascade theory, and approaches the drag-lift ratio of a blade in the infinite medium near the blade tip. In addition, the cascade effect is very sensitive to the shock-free entry angle and to the relation between the leading-edge cavity thickness and the cavitation number^[21]. These also seem to be important blade cavity interferences. The effect of the cavitation number should be considered again in the final design^[22]

SUPERCAVITATING CASCADE THEORY

A two-dimensional supercavitating cascade theory is applied to each blade section. Each section has a different hydrodynamic pitch angle β_i and blade chord length c . The distance d between the neighboring leading-edges of propeller blades at a blade section is

$$\frac{d}{c} = (2\pi r)/(Zc) \quad (1)$$

where r is the radial coordinate and Z is the number of blades. This ratio, d/c represents the solidity of the cascade at the blade section. The stagger angle is

$$\gamma = \frac{\pi}{2} - \beta_i \quad (2)$$

The parameters d/c and γ are typical of cascades. The angle β_i is not known initially. Thus the geometrical advance angle β is used first and then the approximate β_i is used for the second iteration.

In using supercavitating cascade theory there are two ways to select blade shapes: one is from a given mode of chordwise pressure distributions^[16-20] and the other is from a given mode of foil shapes^[21]. The present program can handle either of the two approaches although the pressure mode is restricted^[16,18] to special mathematical forms. The details of cascade theory and the associated program are explained in References 17, 18, and 20 for the pressure mode and in Reference 21 for the foil mode. In practice, the foil mode is better for designing a supercavitating propeller.

It is well known that the thrust and power coefficients of propellers are influenced by the viscous drag/lift ratio ϵ as shown by the following equations^[23].

$$C_T = 4Z \int_{r_n}^1 G(r) \left[\frac{r}{\lambda} - \frac{w_t}{V} \right] (1 - \epsilon \tan \beta_i) dr \quad (3)$$

$$C_p = \frac{4Z}{\lambda} \int_{r_n}^1 r G(r) \left[(1 - w) - \frac{w_a}{V} \right] (1 + \epsilon/\tan \beta_i) dr \quad (4)$$

where ϵ includes the cavity drag effects; r is a nondimensional radial coordinate; $r_n < r < 1$ with r_n the nondimensional hub radius; and $\lambda = V/(\Omega R)$ with ship speed V , angular speed Ω , and propeller radius R .

Since ϵ is very sensitive to section cascade parameters^[16,21], it has to be obtained from supercavitating cascade theory. In addition, ϵ is, in general, a function of the cavitation number and the section lift coefficient. Thus, for a different lift coefficient, ϵ has a different value. However, the lifting-line propeller design program does not supply the lift coefficient distribution as an input but rather as an output. The lift coefficient distribution is obtained by iteration, as for subcavitating propellers. Thus, as usual, all the physical quantities such as pressure p , velocity u , drag-lift ratio ϵ , cavitation number σ , etc, are normalized by lift coefficient C_L . Then even if C_L changes, the actual values can be obtained by multiplication of C_L . However, σ not σ/C_L is given as an input. Therefore σ/C_L changes with C_L while all the physical quantities normalized by C_L are functions of σ/C_L , not σ alone^[21]. In addition, even if σ/C_L were known, the cavity problem with finite cavity length should be solved by iteration because the geometry of the problem associated with the cavity length is not known a priori. Thus the design of a supercavitating propeller involves double iterations.

To circumvent this difficulty, the supercavitating cascade problem is divided into two parts^[21]: one is the problem of infinite cavity length and the other is that of finite cavity correction. Then, if the solution for the infinite cavity length is obtained once at each blade section it can be used repeatedly for each iteration for a different value of C_L . Fortunately the finite cavity correction is simple^[17,21] in a linear design theory where the load distribution is fixed, and can be readily computed during each iteration for a different cavity length.

In the problem for the infinite-cavity cascade^[21] the basic camber shape, such as two-term camber in an infinite medium, is given as an input. Then the shock-free angle is found at each section. The shock-free angle is very sensitive to cascade parameters^[21]. To this basic camber, which has its own shock-free angle, an angle of attack and a point drag are combined to meet the desired lift and leading-edge cavity thickness. There are three options at this stage: (1) the amount of camber is given (2) the amount of angle of attack is given, and (3) no point drag is given. The method of hydrofoil airfoil correspondence^[16] and the Fast Fourier Transform Technique^[21] are used to compute the drag-lift ratio, the normal velocity on the foil and cavity, the foil cavity shape, and the pressure distribution.

For the finite cavity correction, a simple superposition method^[17] is used for many different values of cavity lengths assuming that the load distribution on the foil is exactly the same as the case of infinite cavity. Therefore, the cavity drag, the angle of attack, and the cavity thickness for a given load distribution decrease when the cavity length decreases, although for a fixed foil shape the opposite results are obtained. However, by a proper correction, the minimum leading-edge cavity thickness is maintained. The cavity length corresponding to σ/C_L is obtained by interpolation.

The local cavitation number is smallest at the blade tip of the propeller; much larger at the hub and is the function of camber shape, load distribution and the given leading-edge cavity thickness. As shown in Reference 21, σ/C_L is a linear function of the leading-edge cavity thickness. Also, near the hub σ/C_L varies very little even when the cavity length changes^[21]. That is, near the hub where the solidity is large, the lift coefficient is proportional to σ and almost independent of cavity length.

Since σ/C_L varies considerably from the blade tip to the propeller hub, sometimes σ/C_L may be too large to have a stable cavity near the hub. If the supercavitating propeller must have a stable cavity at every section, the foil shape should be designed accordingly. Although the leading-edge cavity thickness is prescribed such that enough blade thickness can be accommodated inside the cavity, the thickness may have to be larger than the strength analysis requires in order to have a stable cavity at least as long as 1.5 chord lengths. When this situation applies, the leading-edge cavity thickness is computed for the 1.5-chord-length cavity length (see Appendix A), and the leading-edge point drag^[18] is superposed to supply the leading-edge thickness without changing the load distribution.

When the given cavitation number σ is smaller than σ_∞ for a minimum leading-edge cavity thickness, all the physical quantities corresponding to σ_∞ are taken as an approximation in order to have a consistent application of the cascade theory. This situation occurs near the blade tip where σ and σ_∞ are both smallest, but the physical quantities for a given foil with an infinite cavity seem to differ very little between the two cases near the blade tip where the solidity is small^[21].

INFLOW ANGLE INFLUENCED BY CAVITIES

Blade-cavity interference effects may have to be considered not only for cavity drag but also for induced velocity on the blades. The induced velocity determines the angle of inflow velocity or the hydrodynamic pitch angle β_i . The angle β_i determines the regular helical surface where the trailing vorticity is usually located^[10]. For a subcavitating propeller, the angle β_i is determined completely by trailing vortices on the helical surface. The inflow velocity at the propeller plane is the vector mean velocity of the velocities far ahead and far behind the propeller. The Kutta-Joukovsky theorem for the propeller^[24] holds with the vector mean velocity and not with the velocity far upstream as in two-dimensional theory. The finite blade thickness, which can be represented by a source distribution, can influence neither the flow velocity at infinity nor the vector mean velocity because the source effect on the perturbation decays very rapidly with the distance. The finite-cavity thickness can be represented also by a source distribution. Therefore, in a supercavitating propeller with finite cavity, the cavity cannot influence the hydrodynamic pitch angle directly when the vortex and cavity source are superposed for the propeller representation.

When a long cavity is considered, the infinite-cavity cascade theory shows that the downstream flow deflection, or exit angle, is sizable. Thus, the long cavity may induce a sizable contribution on the inflow angle. This effect can be seen in the considerable increment of shock-free angle of two-term-camber foil in cascade^[21]. Therefore it may be worth trying in the final design a pitch angle of blade cavity different from a wake angle β_i where the trailing vortices are located^[9].

For supercavitating propellers with long cavities the problem of determining β_i is more complicated than for subcavitating propellers because the inflow angle is influenced by flow retardation as well as a cascade effect in addition to the vortex induced velocities. Fortunately, the effect of flow retardation tends to cancel the cascade effect^[4,22]. In practice, even a long cavity does not extend very far downstream while the trailing vortices extend to infinity. Therefore the present approach is to consider the cavity essentially finite and to employ a cascade theory to approximate the inner flow of the propeller, considering that β_i is influenced only by the vortices. For subcavitating propellers the use of two-dimensional airfoil theory requires determination of an angle of attack with respect to β_i . Two-dimensional theory may be considered then as an inner flow theory which is imbedded in propeller theory.

Likewise, when a cascade model is applied to a blade section of a supercavitating propeller, the problem is reflected in how to match the inflow velocity of the propeller and the cascade velocity field. That is, the hydrodynamic pitch angle β_i must be matched to the vector mean of the velocities far upstream and far downstream of the cascade. From momentum considerations in the cascade^[4], the far downstream velocity angle δ is

$$\delta = -(c C_L \cos \gamma)/(2d) \tag{5}$$

which is a function of C_L only. Therefore, the vector mean velocity is deflected from the velocity from far upstream at an approximate angle^[4] of $\delta/2$. This direction of the cascade velocity has to coincide with the direction of the hydrodynamic pitch angle β_i of the propeller. In this way, the cascade effect is fully manifested in the angle of attack of the blade. That is, the angle of attack becomes much larger than when an isolated supercavitating foil is considered.

OPTIMUM DISTRIBUTION OF HYDRODYNAMIC PITCH

The optimum distribution of hydrodynamic pitch for a supercavitating propeller was recently found^[23]. The optimum-pitch relation can be adopted as an option in the present design program. Namely, instead of using

$$(r/\lambda) \tan \beta_i = c_1 \quad (6)$$

the corresponding new relation can be used^[23],

$$(r/\lambda) \tan \beta_i = \left\{ c_1 - \left(\epsilon + G \frac{d\epsilon}{dG} \right) r/\lambda \right\}^{1/2} / \left\{ 1 + c_1 \left(\epsilon + G \frac{d\epsilon}{dG} \right) \lambda/r \right\}^{1/2} \quad (7)$$

where $\epsilon(G)$ is the drag-lift ratio derived from supercavitating cascade theory as a function of circulation G , and c_1 is a constant which allows a specified thrust requirement to be met.

INFLOW RETARDATION

The performance of propellers is highly dependent on the advance coefficient $J = V/(DN)$, where D is the diameter and N is revolutions per unit time, or $J/\pi = \lambda$. It is well known that if J becomes small, the angle of attack with respect to the inflow velocity grows, and accordingly, the cavity thickness increases. Thus, investigations have found that the velocity retardation in front of the supercavitating propeller results in a considerable change in performance^[7,25,26]. This can be a serious problem, due to the local perturbation velocities caused by three-dimensional cavity sources in a rotating helical surface of the propeller. This effect is qualitatively different from that of the two-dimensional cascade approximation of blade sections^[25]. The former is a near-field effect with respect to the outer flow of a propeller and the latter is purely an inner flow effect without consideration of the outer flow. An axisymmetric model was used^[25,26] to evaluate the flow retardation. However, this can be more properly handled by the lifting surface theory of supercavitating propellers^[22].

COMPUTER PROGRAM

A flow chart for the computer program is shown in Figure 1. Essentially the computer program is a combination of the lifting-line design program for subcavitating propellers and the appropriate supercavitating cascade theory. After reading the inputs, such as the propeller geometry, design speed, revolutions per minute, and average wake fraction, flow field calculations for a supercavitating cascade having infinite cavity length at each section are performed and saved for later use. Then the first approximation of the hydrodynamic pitch angle β_i , is used to calculate the radial lift distribution and the section lift coefficients C_L by lifting line theory^[10]. For this calculation there are two options; (1) $\tan \beta_i(r)$ is proportional to any given set of $\tan \beta_i(r)$ and (2) the optimum pitch relation, Equation (7) is satisfied. Then the computed value of C_L is used to calculate σ/C_L which is used to find the cavity-length. Then, the infinite cavity drag-lift ratio is corrected, and the thrust (or power) is computed and compared with the design thrust (power). If the value of the thrust is not within a specified error band, of the design thrust, a new β_i is obtained according to the Newton-Raphson rule to compute a new thrust as previously described. This process is iterated until the design thrust is obtained; usually two or three iterations are required. The final outputs are the power (thrust) coefficient, the efficiency, the nose-tail line angle of attack with respect to β_i , the pitch distribution,

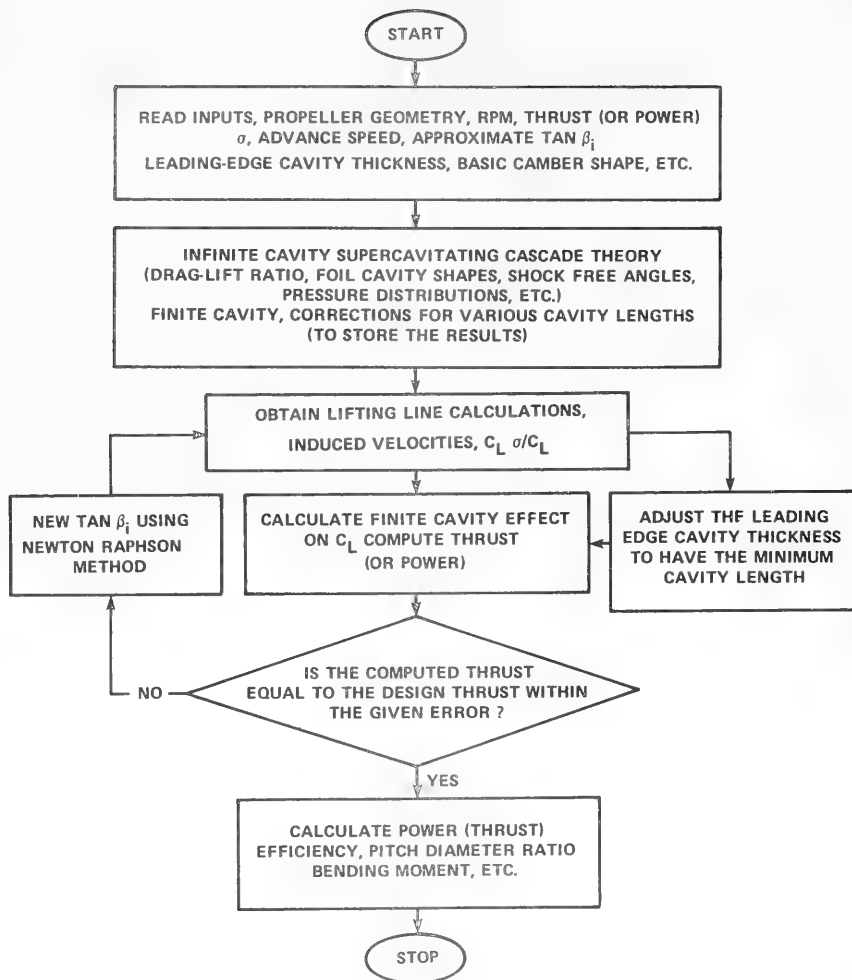


Figure 1 – Flow Chart of Computer Program of the Preliminary Design of Supercavitating Propeller

and the section drag-lift ratio, in addition to all of the cascade section data such as the foil cavity shape, the pressure distribution, and the leading-edge thickness. The cascade section data are also to be used for the lifting-surface theory of supercavitating propellers as a leading term of the three-dimensional cavity solution.

The computer execution time on CDC 6700 is about 80 seconds to produce all of the information for preliminary propeller design.

NUMERICAL RESULTS AND DISCUSSIONS

The preliminary design program is fully implemented when it is used with the final lifting-surface design program^[22]. These programs although tested for existing supercavitating propellers, have not been used for designing a new supercavitating propeller. Experimental evaluations of propellers designed by the new theory are planned for the near future. Among many propeller models tested at DTNSRDC, two propellers, DTNSRDC Models 3770 and 3870, were chosen for comparison. The former propeller has three blades and a low advance coefficient and the latter has four blades and a high advance coefficient. Experiments showed that both propellers had smooth cavities. The experimental results and the previous design calculations are reported in Reference 3.

The design and performance characteristics of the two propellers are listed in Table 1. The predicted efficiencies of the lifting-line designs are very close to the measured efficiencies, although they are related to the distribution of the leading-edge cavity thickness which is determined from the cavity length predicted by cascade theory and a blade strength analysis. When the design leading-edge cavity thickness is given, there are two design approaches used here in the preliminary design process to meet the lift and the leading-edge conditions: Case 1 – specify the camber factor per unit C_L

Table 1 – Design and Performance Characteristics of Supercavitating Propellers

	Propeller	3770	3870
Experiment	Z	3	4
	P/D (0.7)	0.786	1.243
	E A R	0.508	0.727
	ℓ/D (0.7)	0.351	0.344
	J	0.44	0.834
	σ	0.617	0.45
	k_t	0.075	0.115
	η	52.0	59.4
k_t	by Venning & Haberman ³ Preliminary Design	0.1004	0.1402
	(Lifting-Line Theory) Lifting-Surface Design	0.075	0.115
	Case 1	0.073	0.114
	Case 2	0.073	0.114
η	by Venning & Haberman ³ Preliminary Design	54.1	64.0
	Case 1	50.1	58.2
	Case 2	50.5	56.7
	Lifting-Surface Design Case 1	47.8	58.7
	Case 2	50.4	58.9

which multiplies the camber of a two-term foil in an infinite medium, and then adjust the angle of attack and the point drag to meet the conditions of the given lift and leading-edge cavity thickness; Case 2 - find the camber and the angle of attack to meet the conditions without introducing a point drag. If the specified leading-edge thickness is not large enough to produce a cavity longer than 1.5 chord lengths, a point drag is added to produce a 1.5-chord cavity length without changing the lift coefficient. If the given camber factor is too large the angle of attack may be negative, i.e. less than the shock-free angle of attack. In this case, the given camber of the shock-free foil is reduced to meet the given lift coefficient without the negative angle of attack.

The pitch distributions for Models 3770 and 3870 are shown in Figures 2-4 for various leading-edge cavity thicknesses which are shown in Figures 5 and 6 along with the various design thrust coefficients C_T for Case 1 and Case 2. In the figures, a group of four numbers or symbols (A, B, C, D) indicates: A = camber, B = thrust coefficient, C = efficiency, and D = leading-edge cavity thickness distribution, as shown in Figures 5 and 6. Radial sections that have finite-length cavities

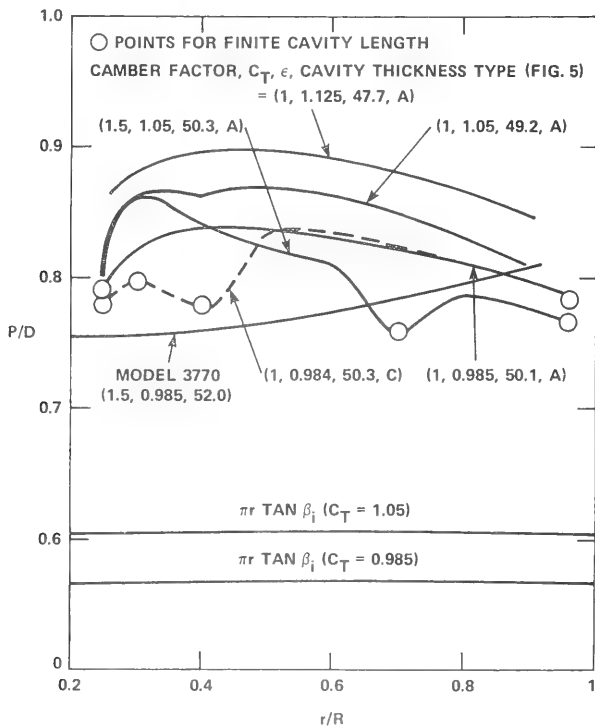


Figure 2 - Pitch Distributions of Lifting Line Design for Supercavitating Propeller Model 3770, Case 1

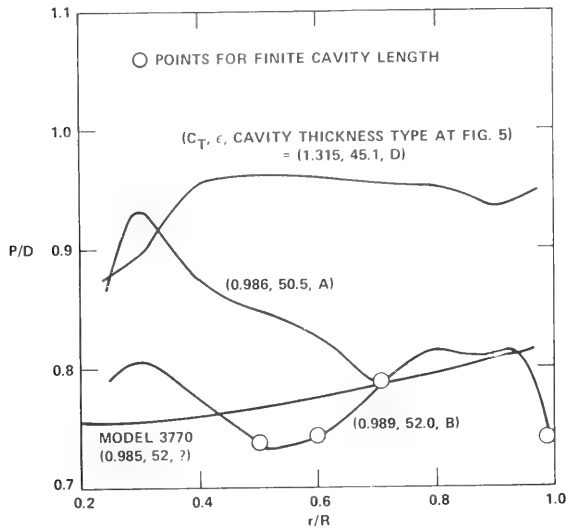


Figure 3 - Pitch Distributions of Lifting Line Design for Model 3770, Case 2

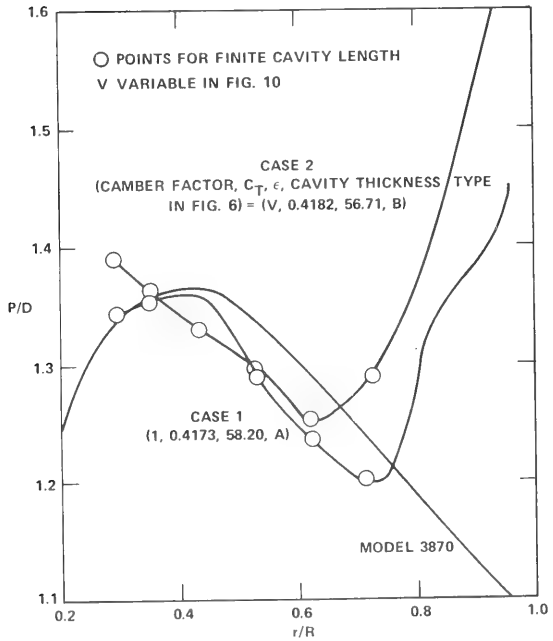


Figure 4 - Pitch Distributions for Model 3870

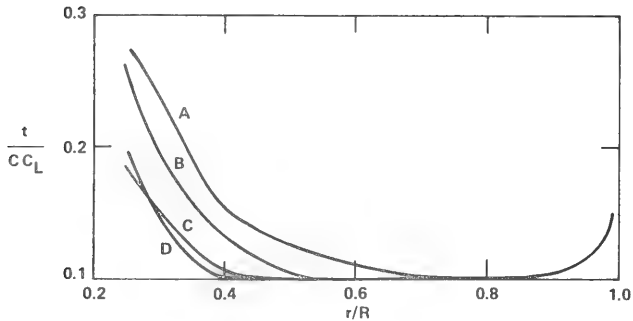


Figure 5 - Leading-Edge Cavity Thickness Distributions
at $x/c = 0.1$ for Model 3770

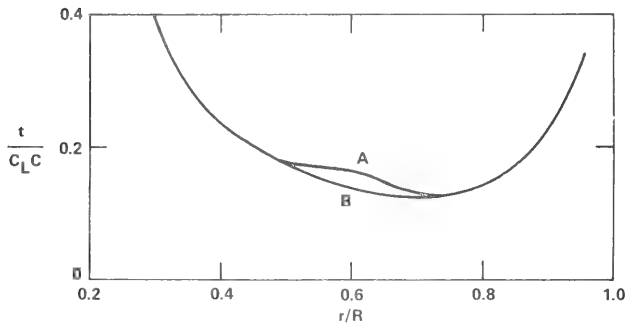


Figure 6 - Leading-Edge Cavity Thickness Distributions
at $x/c = 0.1$ for Model 3870

in the cascade theory are marked by small circles on the curves. The pitch distributions for a finite cavity propeller have a considerable variation along the span of the blade because the short cavity effect on the angle of attack is much larger than the influence of the infinite-cavity cavitation number on the leading-edge cavity thickness and on the angle of attack. The short cavity decreases both the angle of attack and the cavity drag. Thus, when the leading edge is chosen such that each section has an infinite cavity, the pitch distribution is smooth but the efficiency becomes lower as is shown in Figures 2-4 and 7. In any case, the pitch distribution is, in general, larger than those of the models. This may indicate that the effect of flow retardation is significant.

From the pitch distribution, the angle of attack with respect to the hydrodynamic advance angle β_i , can be figured easily. An example of the angle-of-attack distribution determined from the lifting-line design method is shown in Figure 8 along with that of Model 3770. The camber distributions c_0/C_L for propellers 3770 and 3870 are shown in Figures 9 and 10 respectively. The value of c_0/C_L is a measure of the camber assuming that $c_0 y/C_{L0} - \alpha x$ is the actual foil shape without the point drag, having the lift coefficient C_{L0} of the supercavitating foil in the infinite medium, the additional nose-tail-line angle of attack α , to the shock-free camber y . When the camber shape is given such as by the two-term camber, y is the value of the foil shape which is at the shock-free angle in the blade section. Therefore, c_0/C_L decreases when α is large, but increases when the cascade effect is large with a fixed value of α .

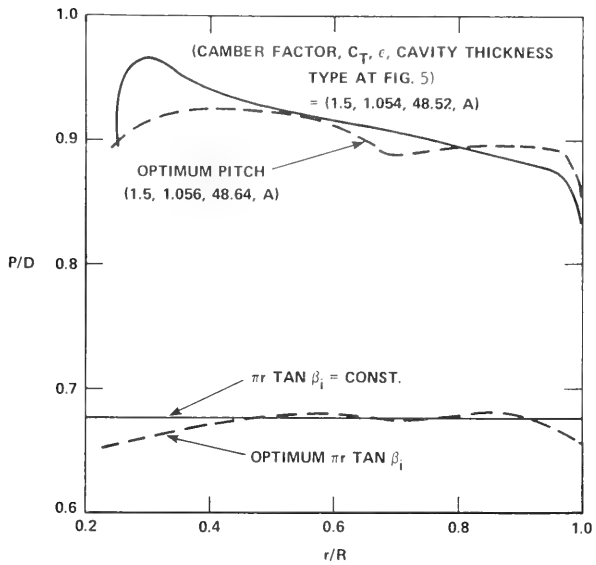


Figure 7 – Comparison of Optimum Variable Pitch and Constant Pitch in Model 3770 (Case 1)

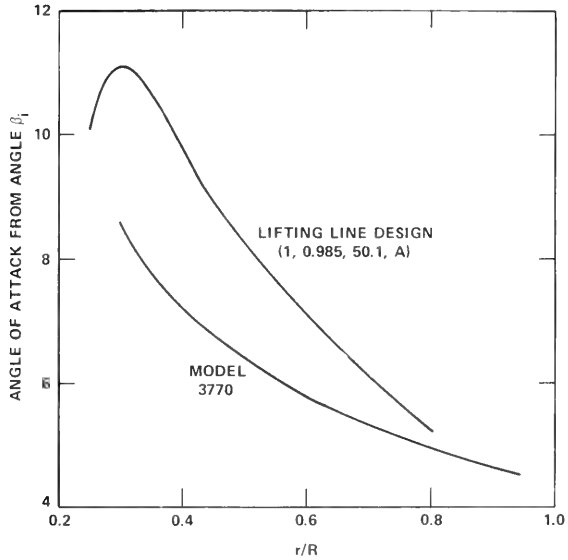


Figure 8 – Angle of Attack from Angle β_1

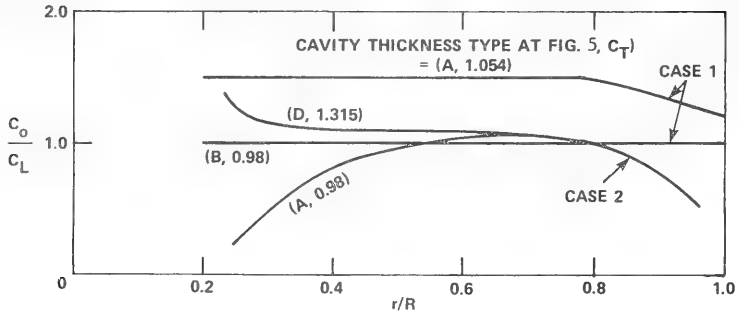


Figure 9 – Camber Distributions for Model 3770

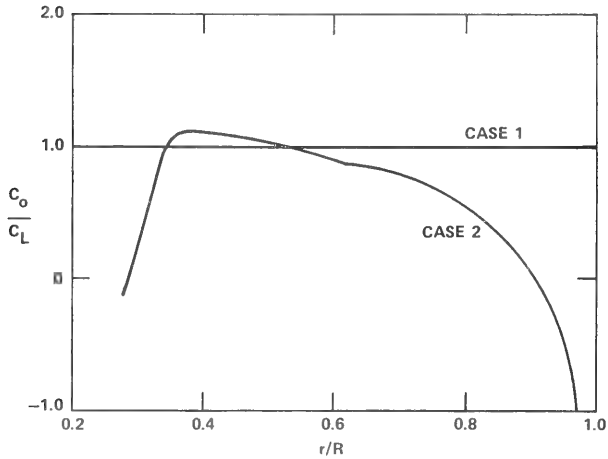


Figure 10 – Camber Distributions for Model 3870

According to lifting-surface theory^[22], only a small correction is required to the cavity source distribution for the infinite-cavity cascade; in the case of finite cavities the correction is comparatively large. The efficiency of the lifting surface is also very close to the efficiency predicted by lifting-line theory. The pitch and camber distributions computed by lifting surface theory are a little smaller than those of lifting-line theory because of the effect of flow retardation. However, for the supercavitating propeller designed with a short cavity, the corrections to the cascade cavity source, pitch, and camber are quite large.

It must be recognized in the old design of Propeller 3770, that angle is added to the angle of attack of the two-dimensional supercavitating foil in the infinite medium as a lifting surface effect. However, the increment of angle of attack is actually due to the cascade effect; the three-dimensional effect is manifested as a decrement of angle of attack because of the effect of flow retardation.

A comparison of the optimum pitch distribution obtained from Equation (7) and the pitch distribution obtained from the normal condition $r \tan \beta_i = \text{constant}$ is shown in Figure 7. The

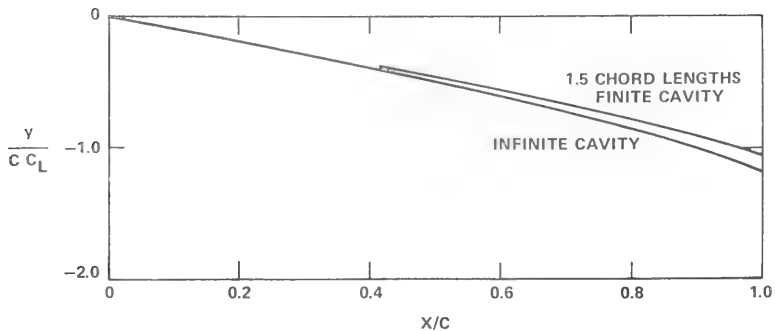


Figure 11 – Finite Cavity Effects on Foil Shapes
 $r/R = 0.53$, Model 3870

efficiencies of the two different supercavitating propellers are almost the same although the efficiency of the propeller with the optimum pitch is a little better. In other examples the efficiency due to the optimum pitch distribution is also only slightly improved.

The pressure sides of the blades on Propeller Models 3770 and 3870 have the two-term camber shape. The actual leading-edge cavity thickness of the models were not measured. Figure 11 shows that the short cavity effect is considerable in changing the foil shape, especially near the trailing-edge. If the point drag is added at the leading edge the foil shape changes again. There are lifting-surface effects to pitch and camber which must be combined. Therefore, without an actual design and experimental evaluation, there can not be a detailed evaluation of the present theory; only an approximate comparison of the resulting behaviors can be made. Yet the results of the present theory seem to indicate that proper use of supercavitating cascade theory is a correct procedure to use in the preliminary design of a supercavitating propeller.

In the usage of the supercavitating cascade theory, the following points are reemphasized.

1. The angle of shock-free entry for a cascade is considerably different from that of a single foil in an infinite medium.
2. The leading-edge cavity thickness is one of the important design parameters, and has a linear relationship with the infinite-cavity cavitation number.
3. Separate treatment of the finite-cavity effect simplifies the application of supercavitating cascade theory to the propeller blade section design.
4. The results of infinite-cavity cascade theory are acceptable for preliminary design near the blade tip where the actual cavitation number is smaller than the infinite-cavity cavitation number.

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APPENDIX A
Leading-Edge Cavity Thickness for a Given Cavity Length

According to Reference 17 the solution for an finite-cavity cascade can be obtained by superposition of the infinite cavity solution

$$u_1 - iv_1 \quad (A1)$$

and the finite cavity correction

$$u_2 - iv_2 \quad (A2)$$

That is, the solution is

$$u - iv = u_1 + u_2 - i(v_1 + v_2) \quad (A3)$$

The value at infinity downstream is represented as

$$\begin{aligned} u_1 &= a \cos \gamma \bar{C}_L / (4\pi K) \\ v_1 &= -(a^2 \bar{C}_M + a \sin \gamma \bar{C}_M) / (4\pi K) \end{aligned} \quad (A4)$$

where \bar{C}_L and \bar{C}_M represent the lift and moment coefficients, respectively, in the transformed plane; a is the transformation constant^[17]; and $K = (a^2 d \cos \gamma) / (2\pi)$.

The values at infinity downstream for the finite cavity correction are

$$\begin{aligned} u_2 &= -\frac{1}{\pi} (\sigma/2 - u_3) \left(\tan^{-1} \frac{-\cos \gamma}{a_1 a - \sin \gamma} - \tan^{-1} \frac{-\cos \gamma}{-a_3 a - \sin \gamma} \right) \equiv -U(\sigma/2 - u_3) \\ v_2 &= -\frac{1}{2\pi} (\sigma/2 - u_3) \log \frac{a_1^2 a^2 - 2a_1 a \sin \gamma + 1}{a_3^2 a^2 + 2a_3 a \sin \gamma + 1} \equiv -V(\sigma/2 - u_3) \end{aligned} \quad (A5)$$

where

$$\begin{aligned} u_3 &= u_1 + u_2 \\ v_3 &= v_1 + v_2 \\ u_3 &= v_3 \tan \gamma \end{aligned} \quad (A6)$$

and a_1 and a_3 are the corresponding values of the upper and lower cavity lengths in the transformed plane.

Therefore, \bar{C}_L/\bar{C}_M can be represented in terms of the given finite cavity parameters σ , a_1 , and a_3 .

When the cavity length for the given design cavitation number and the given leading-edge thickness is too small, a new leading-edge thickness which produces the required cavity length can be obtained as follows. When the cavity length is known, the coordinates of the corresponding cavity end point in the transformed plane, a_1 and a_3 , are known. From Equations (A4) through (A6), $\sigma/2$ can be represented in terms of \bar{C}_L/\bar{C}_M , a_1 and a_3 .

$$\sigma/2 = \frac{1}{V} \left\{ -1 + V \tan \gamma \right\} \frac{a}{4\pi K} \left(\frac{1}{V \tan \gamma - U} \right) \cdot \left\{ Ua \bar{C}_M + (U \sin \gamma + V \cos \gamma) \bar{C}_L \right\} - \frac{1}{4\pi K} (a^2 \bar{C}_M + a \sin \gamma \bar{C}_L) \quad (A7)$$

Thus

$$\frac{\bar{C}_L}{C_M} = \left[\frac{\sigma}{2\bar{C}_M} V - \left\{ (-1 + V \tan \gamma) B U a - \frac{a^2}{4\pi K} \right\} \right] / \left\{ (-1 + V \tan \gamma) B (U \sin \gamma + V \cos \gamma) - \frac{a \sin \gamma}{4\pi K} \right\} \quad (A8)$$

where

$$B = \frac{a}{4\pi K} \left(\frac{1}{V \tan \gamma - U} \right)$$

The cavitation number for infinite cavity has the relation with \bar{C}_L/\bar{C}_M from Equation (A4)

$$\frac{\sigma_\infty}{2\bar{C}_M} = a \cos \gamma \bar{C}_L / (\bar{C}_M 4\pi K) \quad (A9)$$

Since σ_∞/\bar{C}_M is a linear function of the leading-edge cavity thickness, the new leading-edge cavity thickness corresponding to the new value of σ_∞/C_L can be obtained. Because the load distribution is already satisfied by the camber and the angle of attack, the leading-edge thickness should be augmented by the additional point drag which does not contribute to the lift.

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