TR-14

TECHNICAL REPORT

PRELIMINARY INVESTIGATIONS ON PREDICTING PROPERTIES OF BOTTOM PRESSURE FLUCTUATIONS

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ABSTRACT

The principal objective of this study is to determine which of the characteristics of pressure on the ocean bottom could be predicted if the sea surface wave spectrum is known. In particular, the distribution of negative pressure amplitudes, the distribution of pressure periods, and the distribution of half-periods for discrete negative amplitudes are investigated.

An analysis is made of surface wave records to obtain the inherent power at various frequencies. The resulting surface power spectrum is attenuated to the bottom using classical hydrodynamic methods. The computed power at the bottom compared closely with the total power determined from a simultaneous bottom pressure record. Given total power, the distribution of pressure amplitudes can be determined based on a Rayleigh distribution. The distribution of pressure periods cannot be obtained analytically, but empirical methods give agreeable results. A method of determining the distribution of half-periods for discrete negative pressure amplitudes is obtained for the case when this amplitude is zero. Comparison between predicted and observed pressure characteristics is presented.

FOREWORD

Recent developments in the study of ocean waves have stimulated interest in the prediction of pressure fluctuations at the ocean bottom caused by ocean waves. Although there is considerable literature on the exponential decrease in wave motion downwards from the surface, newer concepts of wave analysis at depth based on the power spectrum concept have scarcely been touched. This paper is designed to fill in some of this gap of knowledge and to provide a method for estimating bottom pressure from a given surface wave power spectrum.

This developmental paper is an interim step in the U. S. Navy Hydrographic Office's continuing program for developing suitable wave forecasting methods for use by commercial and military activities.

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A. INTRODUCTION

Since January 1954, the Navy Hydrographic Office has been engaged in the qualitative and quantitative analysis of wave and pressure data collected in 30 to 150 feet of water off the east coast of the United States. The analysis of these data is fundamental in that it has afforded an opportunity to initiate prediction techniques for determining various characteristics of bottom pressures. In some instances, the full significance of these properties have not been determined.

Many of the features of surface waves have been explained by Pierson (1952) and Neumann (1953) on the supposition that a wave record as a function of time is Gaussian. Employing the techniques of Tukey (1949) and Tukey and Hamming (1949), the theory has been further developed to explain the methods of wave analysis for estimating the power spectrum of the steady-state sea surface.

Various theories based on the analysis of noise, advanced by Rice (1944 and 1945), have been applied to surface waves. None of these theories have been carried completely or satisfactorily to the point of explaining how the surface waves are attenuated with depth. Yet the need for convenient and efficient methods of predicting properties of pressure waves has made itself increasingly felt in research and development in fields where background pressure (the variation in pressure on the ocean bottom as a result of the wave action at the surface) is important in determining an index of effectiveness for pressure gear.

In this paper some techniques are presented which have not been considered before for use in the theoretical and experimental investigations of fluctuations in pressure at the bottom of the ocean. The limitations of such procedures are discussed and some sample results are shown.

B. PRESSURE POWER SPECTRUM

It is assumed that the pressure variations at a given depth are essentially reflections of the features existing at the surface of the ocean. Consider then, the energy spectrum, namely the pressure power spectrum, formed as a result of the wave action on the surface. If the formation is governed purely by attenuation of the surface profiles, the pressure variations will be given by

$$A_p^2(\mu) = K^2 A^2(\mu) \tag{1}$$

where $A^2(\mu)$ is the power spectrum of the sea surface, K^2 is the attenuation factor, and $A_p^2(\mu)$ is the residual energy at depth d. Theoretically, a family of curves can be derived from equation (1) by varying the depth. When the depth considered is equivalent to the depth from the still-water level to the bottom, the attenuating factor is

$$K^2 = \frac{1}{\cosh^2(2\pi d/L)} \tag{2}$$

where L is the wave length associated with the spectral period, T, and d is the depth of water.

In practice, $2\pi d/L$ is obtained from a knowledge of d/L_o, where $L_o = 5.12T^2$ is the deepwater wave length. These quantities and other related factors have been tabulated by Wiegel (1948).

The functional relationship between $A_p(\mu)$ and $A(\mu)$ defines the

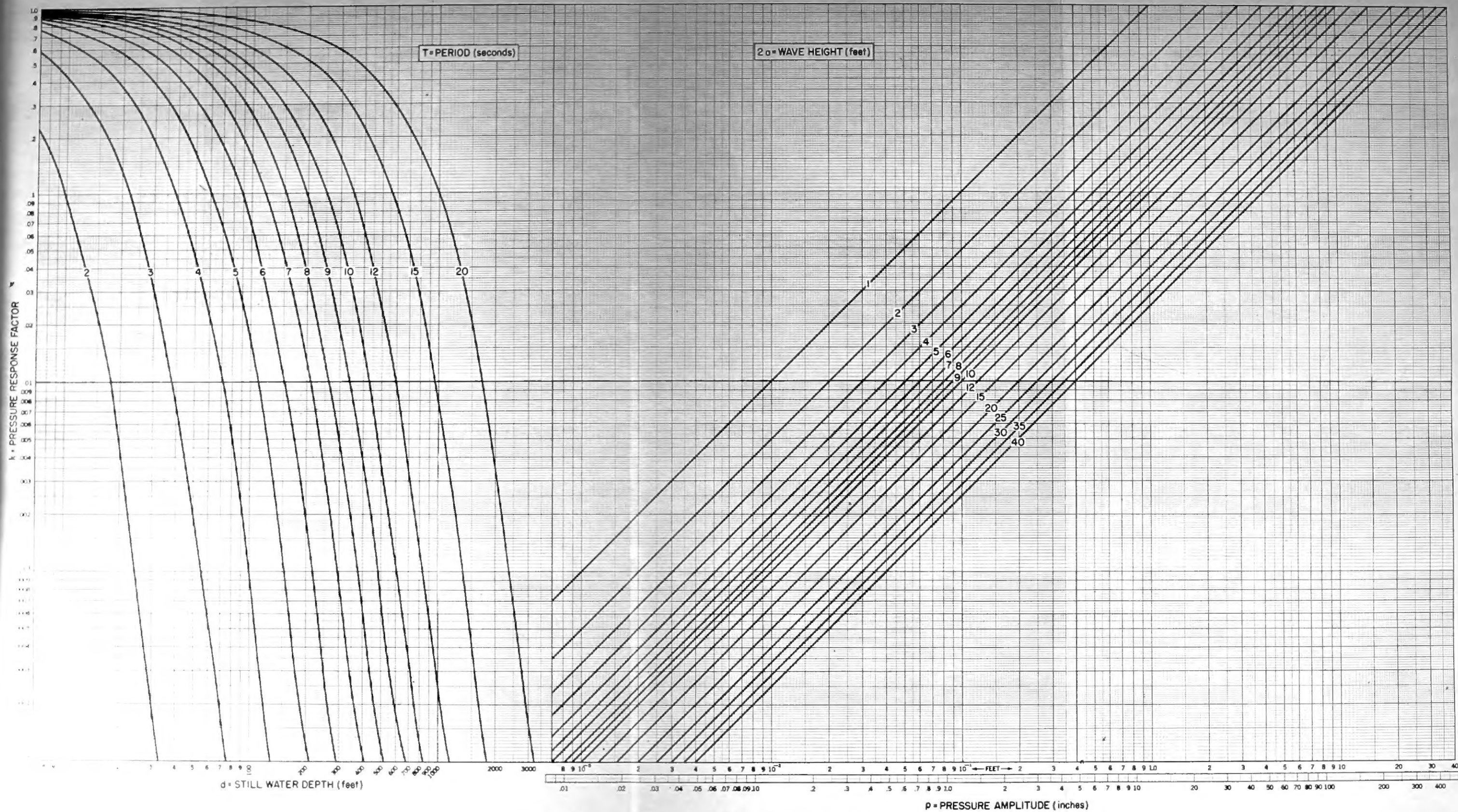


FIGURE I. PRESSURE RESPONSE FACTORS

curve of the so-called pressure response factor from which cut-off values may be determined. The cut-off value is defined as that value of μ at which there is a two-percent response of the primal wave, or the frequency at which the power is essentially zero. A set of such theoretical curves, $K = A_p(\mu)/A(\mu)$, constructed for various values of the parameters $T = 2\pi/\mu$ and d is shown in figure 1. It is to be noted that the definition of the response factor is independent whether wave height or wave amplitude is used since the linear relationship, wave height equals twice the wave amplitude, is assumed to hold.

Although a completely analytic solution of equation (1) is not possible, it can be solved numerically for any given depth. With $A^2(\mu)$ determined from a wave-staff record and K^2 given by equation (2), $A_p^2(\mu)$ can be determined.

A set of data consisting of surface wave height measurements was used to obtained a surface power spectrum $A^2(\mu_h)$. For this purpose, values $p(t_n)$ were taken off of a 20-minute wave-staff record at intervals of $\Delta t = 2$ seconds. These values were used to solve Tukey's chain of equations (Pierson, 1952) as follows:

$$Q_p = 2/(N-p) \sum_{n=1}^{N-p} a(t_n)\, a(t_{n-p}) \qquad p = 0, 1, \ldots m \tag{3}$$

$$L_h = 1/m \left(Q_0 + 2 \sum_{p=1}^{m-1} Q_p \cos(\pi ph/m) + Q_m \cos \pi h \right. \qquad h = 0, 1, \ldots m \tag{4}$$

$$U_h = .23\, L_{h-1} + .54\, L_h + .23\, L_{h+1} \tag{5}$$

$$A^2(\mu_h) = U_h\, \Delta t\, m/\pi \tag{6}$$

N is the total number of values, $a(t_n) = [p(t_n) - \bar{p}]$, considered in the analysis, $L_{-1} = L_{+1}$; $L_{n-1} = L_{n+1}$ and $\mu_h = \pi h / \Delta t\, m = 2\pi / T$.

The actual work of solving these equations was done by an IBM electronic computer at the Hydrographic Office. The apparent amplitudes, $A^2(\mu_h)$ were then transformed into true amplitudes by modifying each discrete spectral amplitude. This was done by using wave-staff correction factors determined by the University of California (1945). The surface wave record was obtained by the H. O. electric wave staff (Upham, 1955). This is a floating wave staff which requires a correction for its natural oscillatory characteristics.

The set of individually-corrected values is the surface power spectrum. To obtain the background pressure at the bottom, which is at $d = 150$ feet, it is necessary to attenuate this surface power spectrum for discrete values of the frequency μ_h . Taking $d = 150$ feet, it is possible to determine the significant range of values for $\mu_h = 2\pi / T$ from figure 1. Clearly, the frequencies range over the interval $2\pi/9$ to $2\pi/20$. With these limiting μ_h's and corresponding values of K^2 and $A^2(\mu_h)$, the pressure power spectrum follows directly from equation (1).

The pressure power spectrum so obtained is shown in figure 2. The experimental spectrum obtained from analysis of a pressure record at 150 feet, taken simultaneously with the surface record, is also shown. Quantitatively, the agreement in the significant range of frequencies between the observed and predicted values, given in table I, is considered good. The curves defining the confidence limits for the predicted spectrum are given in figure 2. These determine the ninety-percent confidence level.

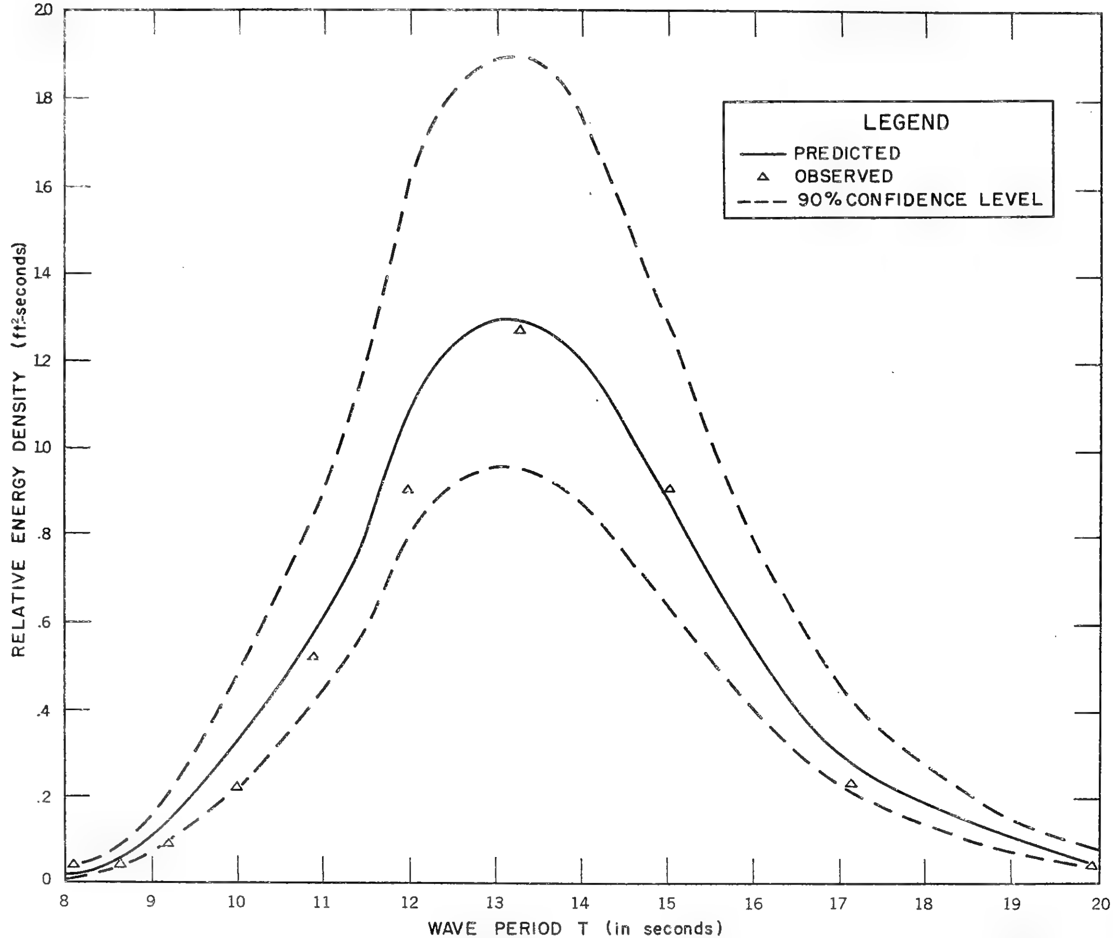


FIGURE 2. COMPARISON BETWEEN PREDICTED AND OBSERVED PRESSURE POWER SPECTRA AT A DEPTH OF 150 FEET

Table I

Predicted and Observed Values for Pressure Power Spectrum at 150 Feet

h	μ_h	T	$A_p^2(\mu_h)$ (predicted)	$A_p^2(\mu_h)$ (observed)
6	$\pi/10$	20.0	.0450	.0422
7	$\pi/8.6$	17.1	.3010	.2110
8	$\pi/7.5$	15.0	.8612	.8902
9	$\pi/6.7$	13.3	1.2964	1.2720
10	$\pi/6.0$	12.0	1.1260	.8902
11	$\pi/5.5$	10.9	.5622	.5084
12	$\pi/5.0$	10.0	.3165	.2110
13	$\pi/4.6$	9.2	.1185	.0844
14	$\pi/4.3$	8.6	.0400	.0422
15	$\pi/4.0$	8.0	.0146	.0422

That is, ninety percent of the time the spectrum will be expected to lie between the two dashed curves.

Another set of calculations was made using a relationship between the average wave length $\overline{L}$ and the average period $\overline{\overline{T}}$ of a wave system, introduced by Pierson (1952). If it is assumed that

$$\overline{L} = 2/3\,(5.12)\,\overline{\overline{T}}^{2} \tag{7}$$

holds over a narrow spectral band width, then it can be substituted in the expression

$$A_p^2(\mu) = \frac{A^2(\mu)}{\cosh^2[(\mu^2 d/g)\,\mathrm{itcoth}\,(\mu^2 d/g)]}, \tag{8}$$

in which $\mu = 2\pi/T$ and

$$K^2 = \frac{1}{\cosh^2[(\mu^2 d/g)\,\mathrm{itcoth}\,(\mu^2 d/g)]}$$

is the attenuating factor at depth d. Itcoth $(\mu^2 d/g)$ is the iterated hyperbolic cotangent of $\mu^2 d/g$, and

$$\frac{2\pi d}{L} = (\mu^2 d/g)\,\mathrm{itcoth}\,(\mu^2 d/g).$$

This factor K^2, when applied to the surface power spectrum mentioned above, determined a pressure power spectrum much smaller than the true spectrum obtained from the analysis of the pressure record. In effect, making this substitution changed the attenuating factor from

$$\frac{1}{\cosh^2 y_0} \quad \text{to} \quad \frac{1}{\cosh^2 (1.5) y_0}, \quad \text{where } y_0 = \frac{2\pi d}{L}$$

The behavior of these functions is shown in figure 3. The results reported

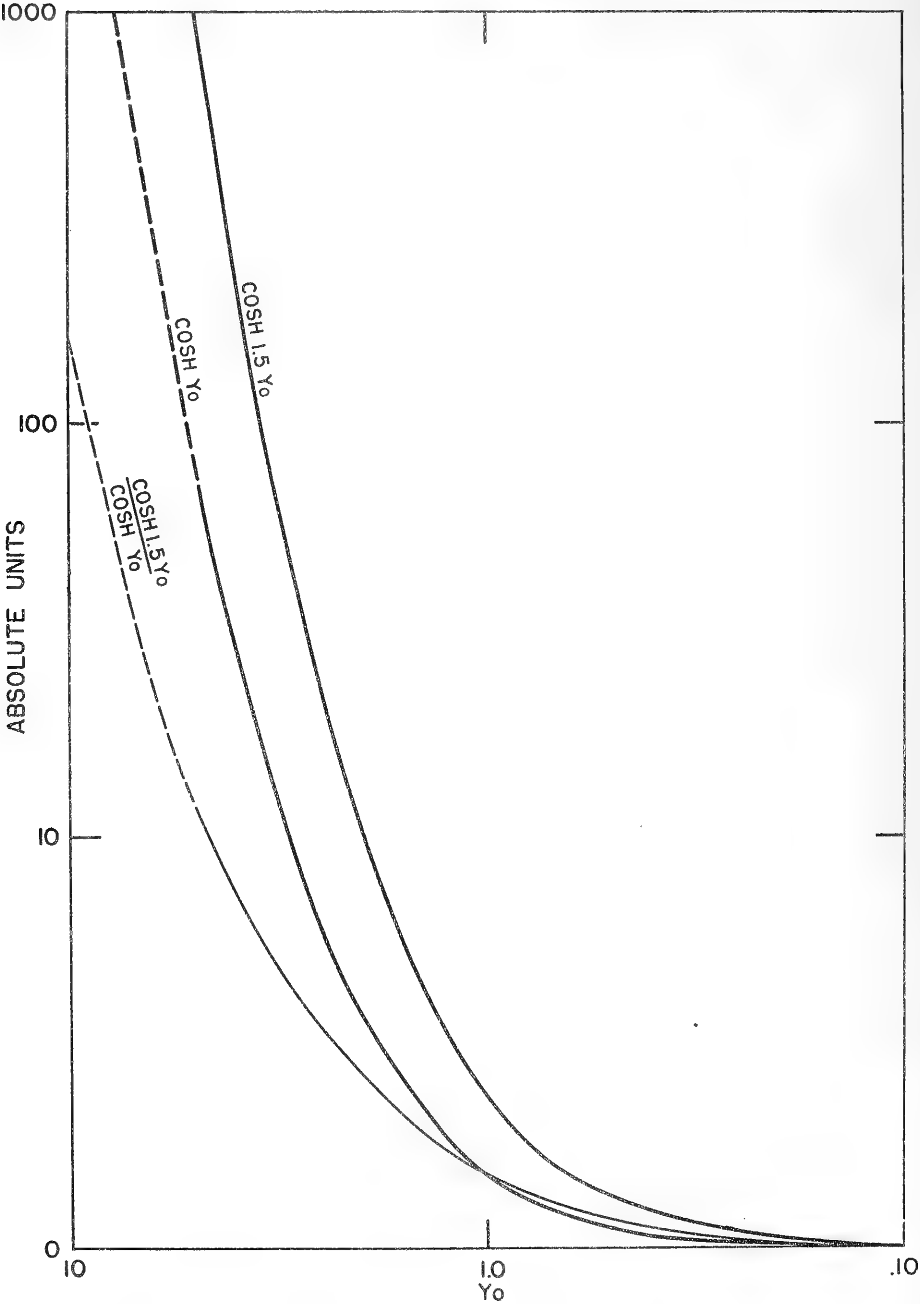


FIGURE 3. BEHAVIOR OF COSH FUNCTION

here indicate that the use of the average , $\overline{L}$, is not valid in this instance. It was considered worthwhile to give these results as an indication of what had been found for one particular case.

In the following sections, the methods of predicting and analyzing power spectra will be described. The three characteristics of bottom pressure fluctuations of general interest will be discussed in some detail. These are the distribution of negative wave amplitudes , p_s , the distribution of wave periods , T, and the distribution of a discrete negative wave amplitude, Δp, with half-periods, t . All distributions are best expressed in ogive curves. These ogive curves and various other properties of pressure fluctuations will be determined and compared with observed results.

C. PREDICTION OF POWER SPECTRA

For a given wind velocity, v , when fetch and duration are considered unlimited, the so-called co-cumulative power spectra are defined and a number of such $E(f)$ curves are given by Neumann (1953). These curves define the power spectrum for each wave system with total inherent energy

$$E = .242 \ (v/10)^5 \ ft^2 \quad . \qquad (9)$$

When either fetch or duration of wind is a limiting factor, the energy in the system also is obtainable from Neumann's curves.

Basically, E is proportional to the summation of the amplitudes squared over the entire range of frequencies, and is equivalent to $Q_0 = \sum U_h$ given, respectively, by equations (3) and (5).

For all practical purposes, the power spectrum is given by

$$A^2(f) = \frac{d\,E(f)}{df} \qquad (10)$$

where $E(f)$ is nondecreasing and the largest value is given by equation (9). Hence, once $E(f)$ is known, the power spectrum of the surface follows directly from the operations indicated in equation (10). The adopted values of $A^2(f)$ over the significant band of frequencies can be attenuated to a given depth as outlined previously. The result will be the pressure power spectrum. The procedure for analyzing this spectrum to obtain some characteristics of pressure on the bottom of the ocean will be described next.

D. ANALYSIS OF PRESSURE POWER SPECTRUM

Writing equation (10) as $A_p^2(f)\,df = d\,E_p(f)$ and integrating both sides of the equation results in

$$\int_f A_p^2(f)\,df = E_p(f). \qquad (11)$$

$E_p(f)$ can be described as the area under the power curve for a give range of frequencies. More explicitly, $E_p(f_a)$ is the area under $A_p^2(f)$ from $f=\infty$ to $f=a$. If it is assumed that $E_p(f)$ starts with the value zero at $f=b$ the cut-off frequency, then $E_{p\,max}(f)$ will not differ appreciably from the sum of the amplitudes squared, obtained by integrating over the entire range of frequencies from $f=0$ to $f=\infty$. Thus $E_{p\,max}(f)$ is the total area under $A_p^2(f)$ over the significant band of frequencies. It should be noted that the expression $E_p(\mu)$ may be obtained by replacing f by its equivalent value $\mu/2\pi$.

The total area under the power spectrum may be obtained in a number of ways. The simplest method, of course, is to use a planimeter. The method more frequently followed, however, consists of dividing the frequency/axis into a set of intervals and determining the value of the power curve at the midpoint of each interval. The product of this value and the length of the interval summed over the entire range of frequencies is approximately $E_{p\max}(\mu)$

Figure 4 shows the experimental data $A_p^2(\mu)$ at a depth of 150 feet. The associated $E_p(\mu)$ curve was determined by integrating over the spectrum with a planimeter. The total area was found to be $.259\ ft^2$ over the significant band of frequencies, which approximates closely the value $E_{p\max}(\mu) = .264\ ft^2$ already obtained by machine analysis. The numerical value $E = E_{p\max}(\mu)$ is the quantity used in forecasting the wave amplitudes. Once E is known, the ogive curve representing the number of waves with amplitudes greater than a specified amplitude, P_s , can be obtained from table II, if it is assumed that the values therein hold for all wave records.

Using $E = .264\ ft^2$, i.e. $\sqrt{E} = .514\ ft.$ the cumulative distribution of waves with amplitudes greater than P_s inches was determined for the power curve shown in figure 4. Figure 5 shows the comparison of the predicted curve and the curve obtained from a hand analysis of the original wave record.

The next step is to consider the distribution of waves with periods greater than a specified period, T . Theoretically, very little has been done along this line because of the variation of periods from wave to wave. However, Rice (1944 and 1945) has derived an expression for the expected number of zeros per second which may occur in a random time series. It is assumed that the number of times the record crosses the mean line is the number of zeros; thus the expected number of zeros per second is twice

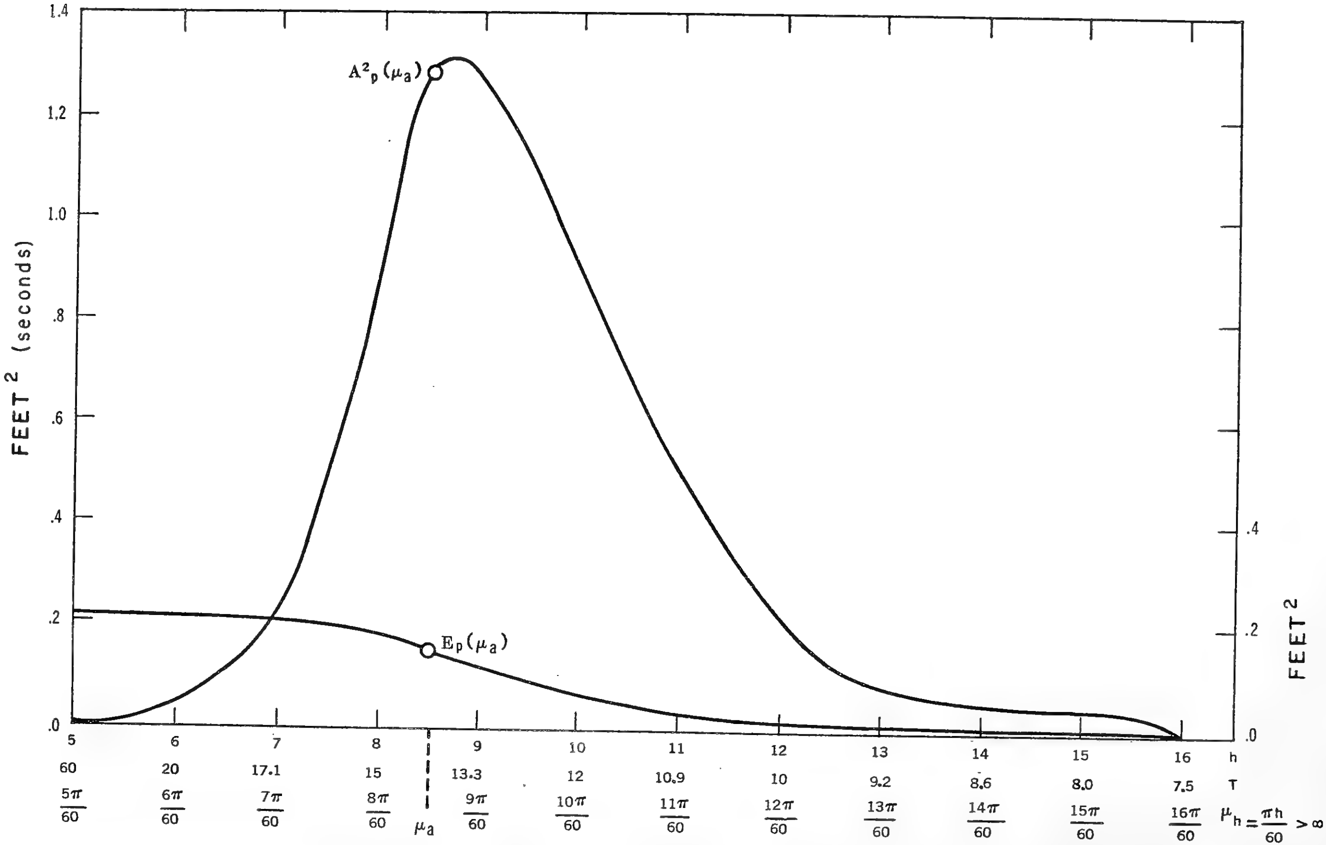


FIGURE 4. PRESSURE POWER SPECTRUM AND ASSOCIATED Ep(μ) CURVE AT 150 FEET

Table II

Theoretical Values to Be Used in Predicting Wave Amplitude Distributions

Percent of waves with amplitudes $\geq P_s$	P_s (inches of water)
100	$0.00\sqrt{E}$
90	$3.84\sqrt{E}$
80	$5.64\sqrt{E}$
70	$7.20\sqrt{E}$
60	$8.52\sqrt{E}$
50	$9.96\sqrt{E}$
40	$11.52\sqrt{E}$
30	$13.20\sqrt{E}$
20	$15.24\sqrt{E}$
10	$18.24\sqrt{E}$

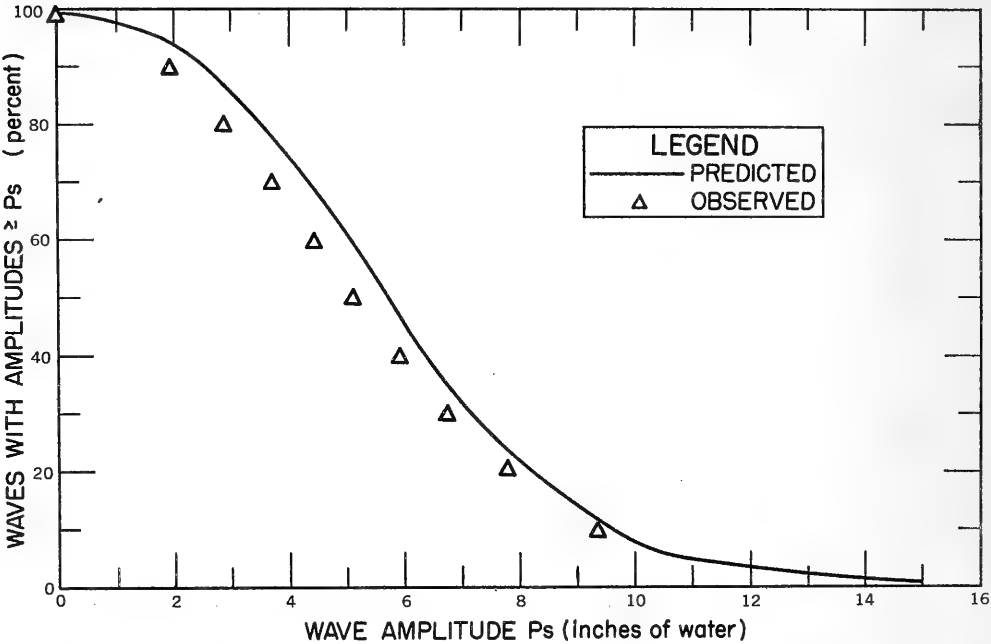


FIGURE 5. COMPARISON BETWEEN PREDICTED AND OBSERVED AMPLITUDE DISTRIBUTIONS

the expected number of waves per second. This may be expressed as follows:

$$\text{Expected number of zeros per sec.} = 2\left[\frac{\int_0^\infty f^2 A^2(f)\,df}{\int_0^\infty A^2(f)\,df}\right]^{1/2}, \quad (12)$$

from which is obtained the

$$\text{Expected number of waves per sec} = \left[\frac{\int_0^\infty f^2 A^2(f)\,df}{\int_0^\infty A^2(f)\,df}\right]^{1/2}, \quad (13)$$

Substituting $f = 1/T$, equation (13) becomes

$$\text{Expected number of waves per sec.} = \left[\frac{\int_0^\infty T^{-4} A^2(T)\,dT}{\int_0^\infty T^{-2} A^2(T)\,dT}\right]^{1/2}. \quad (14)$$

The reciprocal of this expression gives the average period

$$\tilde{T} = \left[\frac{\int_0^\infty T^{-2} A^2(T)\,dT}{\int_0^\infty T^{-4} A^2(T)\,dT}\right]^{1/2}. \quad (15)$$

Neumann (1953) obtained an empirical spectrum for a fully-developed wind-generated sea,

$$A^2(\mu) = c\,\mu^{-6}\, e^{-2(g/v\mu)^2},$$

where v is the wind velocity. If Neumann's spectrum is applicable, equation (15) can be written

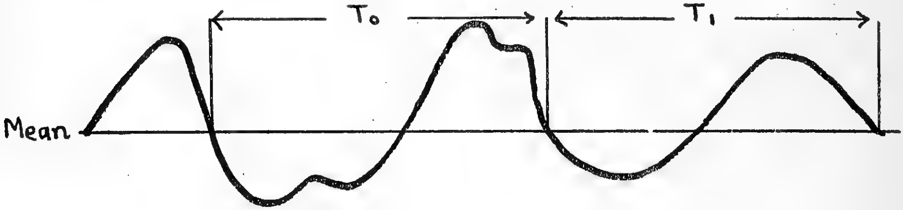
$$\tilde{T} = \left[\frac{\int_0^\infty T^4 e^{-a^2T^2}\,dT}{\int_0^\infty T^2 e^{-a^2T^2}\,dT}\right]^{1/2}, \quad (16)$$

where

$$a = g / \sqrt{2}\,\pi V \; .$$

This expression is valid for surface waves, where theoretically all periods between 0 and ∞ are possible. To extend this concept to depth it is necessary to transform the surface power spectrum to a pressure power spectrum and, considering the filtering action of depth, evaluate the integrals from the cut-off period $T = T_b$ to $T = \infty$.

First, to reiterate what is meant by a wave period, consider a time-series record of pressure variations on the bottom of a body of water due to the passage of surface wind waves.



The time it takes for the record to complete a cycle (downcross to downcross) is called a period T_i $(i = 0, 1, 2 \ldots .)$. The average wave period at a given depth (bottom) will be represented by

$$\tilde{T}_b = \left[\frac{\int_{T_b}^{\infty} T^{-2} A^2(T) \operatorname{sech}^2(2\pi d/L)\, dT}{\int_{T_b}^{\infty} T^{-4} A^2(T) \operatorname{sech}^2(2\pi d/L)\, dT} \right]^{1/2} . \qquad (17)$$

In general, it is not possible to solve this equation analytically, but a numerical solution will be valid.

The wave measurements represented by figure 4 were made at a depth of

150 feet, where periods less than 7 seconds are not represented. Therefore, the limits of integration for equation (17) are 7 and ∞. Thus, the average wave period given by equation (17) is $T_b = 12.75$ seconds. This closely approximates the value $\widetilde{T} = 12.70$ seconds already obtained from a direct analysis of the wave data.

Assuming a gamma-type distribution, prediction curves for wave period distributions can be determined from the mean period, the standard deviation, and a measure of the skewness. This type of analysis has been done previously by Putz (1952). In fact, Putz gives relationships in terms of the average period for both the standard deviation S_t and the skewness α_3. These expressions are:

$$S_t = 0.313\,\overline{T} - 0.759 \quad ,$$

$$\alpha_3 = -0.249\,\overline{T} + 2.795 \quad .$$

A comparison of these empirical prediction curves with the period distributions from several pressure records is illustrated in figure 6 and figure 7.

The period for Putz's prediction curves has been defined in a different manner than the definition used in this report. Putz has defined a wave period as twice the time it takes for the record to complete a half cycle (trough to peak). This difference in measuring periods will have great effect in some cases. It is believed that the good agreement for the records shown is due to the relatively deep depths at which Putz's measurements were made for the empirical curves. As will be shown later, at these depths the ratio of the number of wave maxima to the number of waves approaches one.

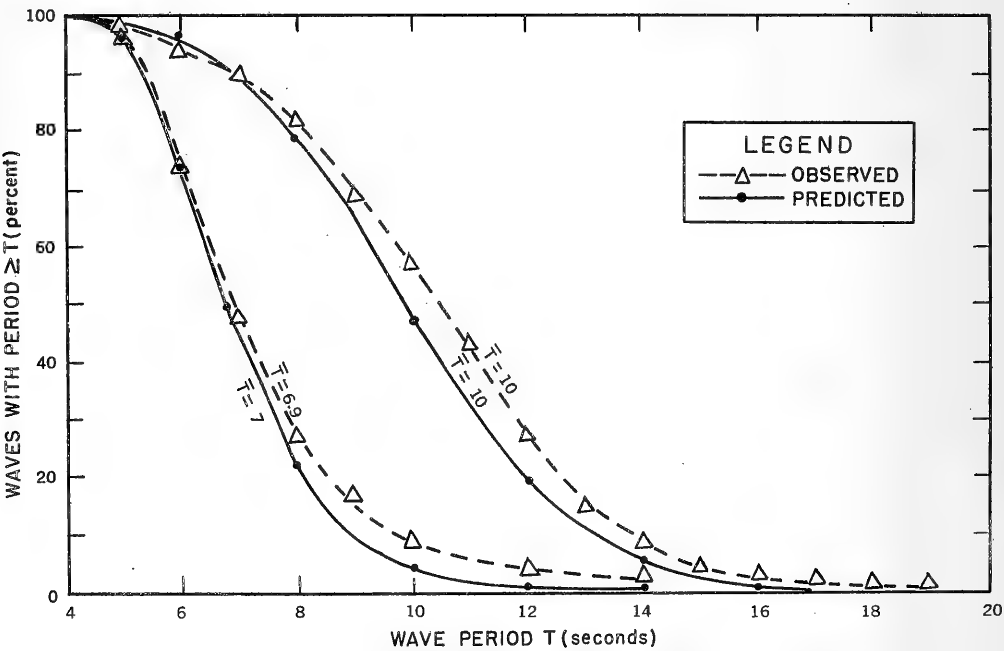


FIGURE 6. PREDICTED AND OBSERVED OGIVE CURVES FOR WAVE PERIODS

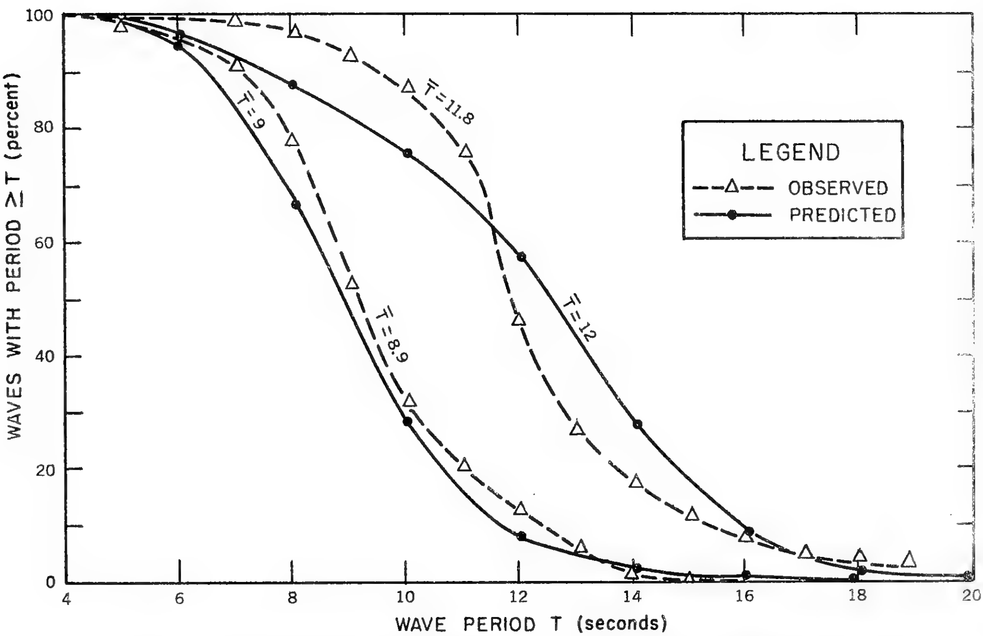


FIGURE 7. PREDICTED AND OBSERVED OGIVE CURVES FOR WAVE PERIODS

For the example cited, in which the measurements were taken at a depth of 150 feet, it was found that Putz's relationships were not valid. In fact, S_t given by the empirical relationship was almost exactly the square of the standard deviation computed from the wave record, and α_3 was found to be three times larger than the value obtained from the original wave record. Since α_3 was small in absolute value, it was considered worthwhile to fit the normal distribution (the symmetric member of the gamma-type distributions) to the wave-period distribution. A survey of the results indicates that the approach was valid, and the agreement is in order as shown in figure 8. This observation is a qualitative one and it should be emphasized that quantitative conclusions are not given much weight at this time. However, it appears from analyses of additional pressure records that both the standard deviation and the skewness coefficient are relatively constant for pressure variations with average periods of nine seconds or more.

Next, to show the behavior of the ratio of the number of wave maxima to the number of waves, it is necessary only to obtain the

$$\text{Expected number of wave maxima per sec} = \left[\frac{\int_0^\infty f^4 A^2(f)\,df}{\int_0^\infty f^2 A^2(f)\,df}\right]^{1/2} \qquad (18)$$

and form the ratio of this to equation (13), with $A^2(f)$ replaced by $A_p^2(f)$. The ratio becomes

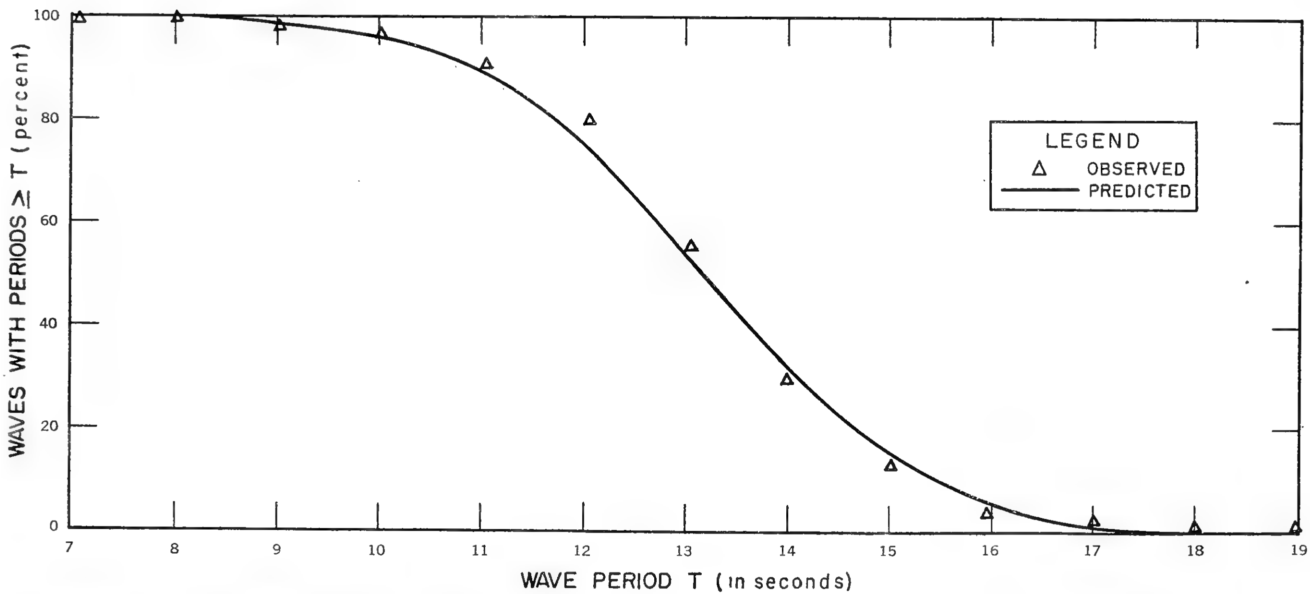


FIGURE 8. COMPARISON BETWEEN PREDICTED AND OBSERVED DISTRIBUTIONS OF WAVE PERIODS

$$R_f = \left[\frac{\left(\int_0^\infty f^4 A_p^2(f)\, df\right)\left(\int_0^\infty A_p^2(f)\, df\right)}{\left(\int_0^\infty f^2 A_p^2(f)\, df\right)^2} \right]^{1/2} \tag{19}$$

The higher frequencies are essentially damped out due to the filtering action of depth. Therefore, in a pressure record the limits of equation (19) go from 0 to b where b varies according to the depth. Furthermore, from the analysis of power spectra it appears that the lower limit can be chosen also. Thus, in effect the requirements can be fulfilled for an ideal band pass filter whose pass band extends from f_a to f_b.

$$R_f^2 = \frac{9}{5} \left[\frac{(f_b^5 - f_a^5)(f_b - f_a)}{(f_b^3 - f_a^3)^2} \right] \tag{20}$$

or in terms of T,

$$R_T^2 = \frac{9}{5} \left[\frac{(T_a^5 - T_b^5)(T_a - T_b)}{(T_a^3 - T_b^3)^2} \right] \tag{21}$$

Choosing $f_a = .05$, the graph of equation (20) showing R_f for various values of f_b is given in figure 9. The choice of f_b is necessarily a function of the pressure recorder and the depth. The choice of f_b with depth is not exact, but since there is probably some maximum amplitude of a wave for a given period, $1/f_b$, there is some maximum f_b which will be associated with it. This f_b maximum is not realistic for a great many sea conditions, however, and thus the choice of $f_{b\,max}$ remains a question.

If not more than two percent of an elemental wave were recorded by a pressure recorder, then using figure 1, the minimum periods to be expected in a pressure record would be as follows:

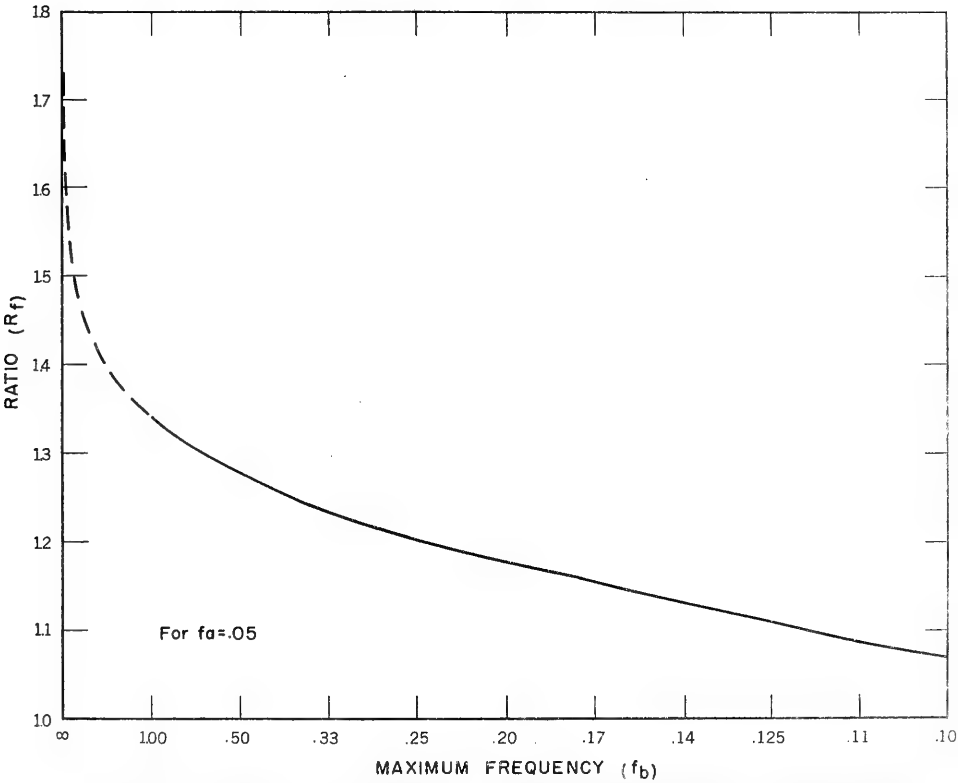


FIGURE 9. RATIO OF EXPECTED NUMBER OF MAXIMA TO EXPECTED NUMBER OF WAVES AS A FUNCTION OF THE MAXIMUM FREQUENCY f_b

Depth	Minimum Period
30 feet	3 seconds
90 feet	5 seconds
150 feet	7 seconds

These values determine $f_{b\,max.}$ and consequently R_f from figure 9 as given in table III.

Table III

R_f Determined for Minimum T

$T_{min.}$	$f_{b\,max.} = 1/T_{min.}$	R_f
3	.333	1.24
5	.200	1.18
7	.143	1.13

The ratios of the number of wave maxima to the number of waves were determined from the original records mentioned above, and the results are indicated as follows:

Depth	No. of Records	Average Ratio
30 feet	13	1.30
90 feet	12	1.13
150 feet	8	1.09

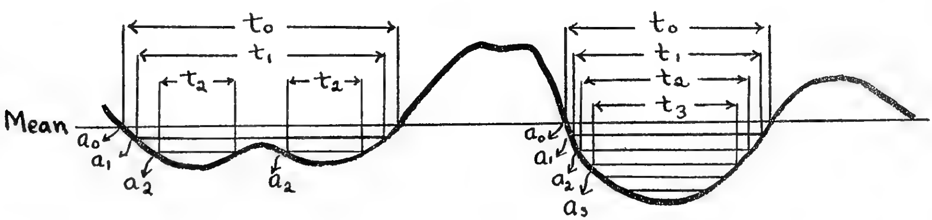
It is interesting to note that for a purely wind-generated sea, one in which all heights and periods are possible,

$$R_T = \left[\frac{\left(\int_0^\infty T^4 e^{-a^2T^2} dT\right)\left(\int_0^\infty e^{-a^2T^2} dT\right)}{\left(\int_0^\infty T^2 e^{-a^2T^2} dT\right)^2} \right]^{1/2} \qquad (22)$$

$$= \left\{ \frac{\Gamma(5/2)\ \Gamma(1/2)}{[\Gamma(3/2)]^2} \right\}^{1/2} = \sqrt{3} = 1.73 \quad .$$

That is, a random time series of the sea surface will have 73 percent more maxima than waves. This value also has been found by Pierson (1954).

Another important property of pressure fluctuations is represented by the ogive curve which expresses the distribution of waves with a discrete negative amplitude, Δp_i, and negative half-period, t_i. Consider a sample from a pressure record,



then, the ogive curve Δp_o is the negative half-period distribution of waves with amplitudes $a_o \geqslant 0$, and the ogive curve Δp_1 is the negative half-period distribution of waves with amplitudes $a_1 \geqslant 1$ (inches of water), etc. Due to the asymmetry of the record, a discrete T_o does not exactly equal $2t_o$ but in a sufficiently long record, $2\bar{t}$

should be nearly equal to $\overline{T}$.

The total number of cases for computing the Δp versus t curves is not equal to the total number of cases for computing the period, T, curves. The number of wave maxima are used in the first, and the number of waves are used in the latter. However, the Δp versus t curves can be drawn as a percentage of either the number of wave maxima or the number of waves, since the ratio of the two is presumably known. This ratio for a given depth can be determined from figure 9.

The Δp_0 curve is obtained from the ogive curve representing the distribution of waves with respect to period, and in fact is equal to

$$\frac{\text{percent of waves with period} \geq T}{R_f}, \quad \text{where}$$

$T = 2t$ and R_f is the ratio of the number of wave maxima to the number of waves. The curves for Δp_1, Δp_2, etc., can not be deduced at this time, as there is no apparent relationship between a discrete amplitude and a half-period distribution.

E. REMARKS

It has been established that some of the characteristics of bottom pressures can be predicted. However, these forecasting techniques are based on several basic assumptions which must be considered in any application of the principles. The first assumption is that the sea surface spectrum is known, either predicted from wind conditions or measured. Secondly, it has been considered that the bottom is rigid and that there will be no correction necessary to the basic attenuation factor

$$K^2 \doteq \frac{1}{\cosh^2(2\pi d/L)} \cdot$$

In these techniques, fluctuations of the ocean bottom due to microseisms, cataclysms, etc., have not been considered. The results shown appear to be excellent, but it remains to be seen how the theory will stand after further tests. A large quantity of data is presently being taken and comparative analyses between surface waves and associated pressure fluctuations will be carried out.

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