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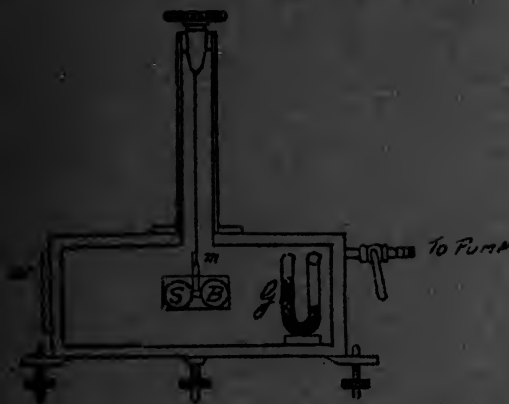


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THE PRESSURE OF LIGHT

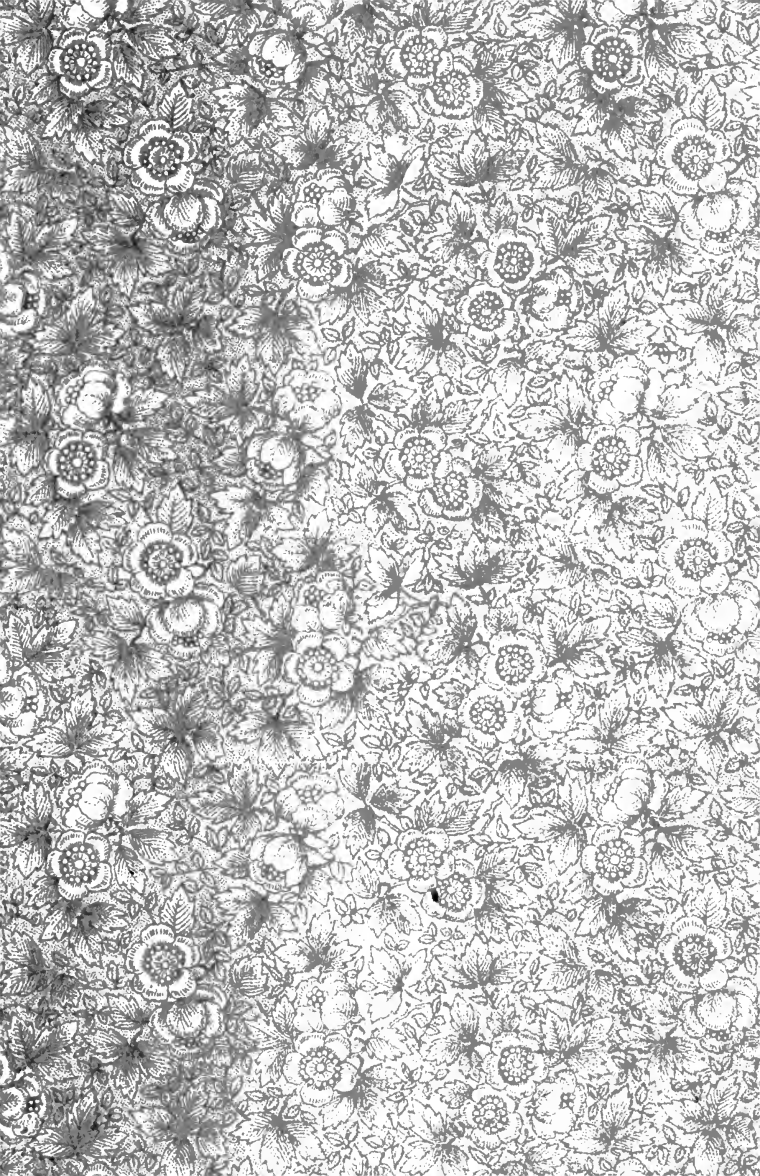


J. H. POYNTING,

Sc.D., F.R.S.



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THE PRESSURE OF LIGHT

W. O. Campbell

THE ROMANCE OF SCIENCE

THE PRESSURE OF LIGHT

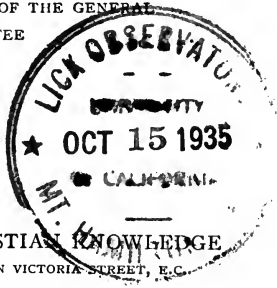
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PREFACE

IN the course of the last few years the author has lectured on the Pressure of Light in many places. Some of the lectures have already been published in full or in abstract. In this book the substance of these lectures is set forth more completely and in greater detail than was possible in any one lecture.

For readers who desire to study the mathematical calculations involved in the theory of the subject, a series of notes is appended.

July 1910.

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PRESSURE OF LIGHT

I

HOW LIGHT EXERTS PRESSURE

WHEN we see the havoc wrought on a sea-wall by a storm, it is easy to believe that ocean waves exert a pressure against the shore on which they beat. But it is not easy to think that the tiny ripples of light also press against every body on which they fall, to think that when a lamp is lighted waves of pressure are sent out from it—pressing against the source from which they start, pressing against every surface which they illuminate. Yet we now know certainly that light does exercise such pressure. It is a very minute pressure, far too small, even when it is strongest, to be felt by our bodies, and only to be detected by exceedingly sensitive apparatus.

In the following pages I shall try to give some account of the reasoning by which the existence of light-pressure was predicted, and shall then describe the experiments by which it was, many years later, actually detected and measured. I shall then point out some consequences of the

pressure which may hereafter be verified by astronomical observation.

A hundred years ago it would have been much easier to explain how light exercises pressure than it is to-day. Then almost every one believed that

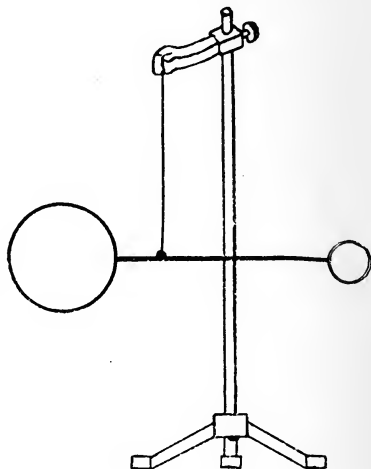


FIG. I.

light consisted of corpuscles, inconceivably small bodies, darting out at enormous speed from every glowing surface. Each molecule or atom of the surface was supposed to be a little battery of guns, keeping up a continuous fire of shot, each shot immensely smaller than the atom which fired it off. Every surface exposed to the light was regarded as

undergoing bombardment by the corpuscles, and it was quite natural to suppose that the surface was pressed back.

We may illustrate the supposed mode of action by fixing a vertical tin disc at the end of an arm and suspending the arm by a fine wire, so that it is free to turn round as in fig. 1.

Now arrange a funnel and a metal pipe, as in

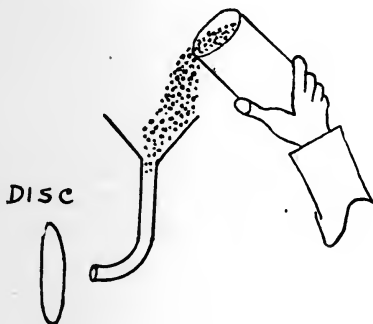


FIG. 2.

fig. 2, in which the disc is seen edgewise, and the suspension is not shown.

Pour some fine shot into the funnel, and they run down the pipe and bombard the disc, pressing it back. The shot acquire momentum. They carry this momentum and give it up to the disc when they hit it. This giving up of momentum is pressure.

In the eighteenth century, when the Corpuscular

Theory of Light flourished, many experiments were made to detect a pressure by allowing light to fall on a disc, like that in fig. 1, on a small scale, and very delicately suspended, sometimes in air, sometimes in a vacuum. Sometimes the disc was pressed back, sometimes it was drawn forward, and no observer obtained conclusive, or even consistent results.

If these early experimenters had known the Principle of the Conservation of Energy, they would have been able to calculate the value of the pressure, which they were looking for,¹ and it would have worked out on their false theory to double the actual value we now know it to have. Even this double value is far too minute to be detected by the means then attainable.

Their variable results, now an attraction now a repulsion, were due, no doubt, to two actions which are still the terror of all experimenters on the subject. When they worked in air, the light absorbed by the disc heated it. The disc in turn heated the surrounding air, which expanded and streamed up bodily, forming currents known as "convection currents"—simply minute upward gales of wind. If a flat iron plate is heated and then held in front of a lantern these currents form faint shadows on the screen and may be

¹ Note 1, p. 83.

seen rising like smoke. It depends entirely on the lie of the plate whether these rising streams of air will tend to press the plate back or to draw it forward. The action of the air currents on a disc heated by a beam of light may easily be many times greater than the pressure of the light.

When they worked in a vacuum probably another action came into play, an action discovered and investigated by Sir William Crookes, who invented a beautiful little instrument to show it, which he named the Radiometer. The radiometer in its commonest form consists of four small mica discs fixed at the four ends of a horizontal cross as in fig. 3. The cross is free to spin round on a pivot as frictionless as possible, and it is contained in a highly exhausted bulb about three inches in diameter.

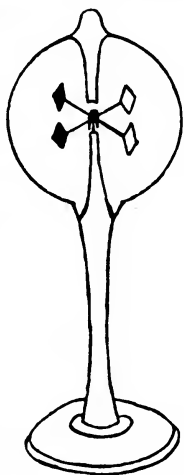


FIG. 3.

Each of the vanes is blackened on one side, and when a lighted match or candle is brought near the bulb, the blackened faces retreat from the source of light while the unblackened faces move towards it. At first it was supposed that the action might be directly due to the pressure of light, but it is easy to see that this pressure would produce

just the opposite motion. For the light falling on the unblackened surfaces is partly reflected and would therefore press not only in falling on the surface but would give, as it were, another kick back on being reflected, while the light falling on the blackened surface would press only in falling on it, for it is absorbed there and does not rebound. The unblackened surfaces would therefore retreat.

It was soon found that the action of the radiometer is due to the residual air still remaining in what we call the vacuum in the bulb. The blackened surfaces absorb the light and so become hotter than the unblackened surfaces. The molecules of air in the bulb are rushing about in all directions, and those which hit the hot black surface get a little extra energy from it and go back faster than they came up, and so give a greater kick back against it than if they rebounded at the same speed. Those which come up on the other cooler side go back with the same velocity with which they came up and do not give an increased kick back. Thus the residual air presses more against the black surface and the vanes spin round.

For reasons which we cannot enter into here, this "radiometer action," as it is termed, only comes into serious consideration when the air is very much rarefied. But no doubt it was present in the early attempts made to detect light-pressure on a disc in an exhausted vessel.

Convection currents, then, disturb the experiments when we work in air, and radiometer action disturbs them when we work in a vacuum. We shall see later how it is possible to steer between Scylla and Charybdis and reveal the true pressure due to light.

It is just a hundred years since Thomas Young killed the corpuscular theory of light and founded in its place the theory that light consists of waves, a theory soon accepted by every one. But there was no reason at that time to suppose that the waves could press, and so experiments to detect light-pressure ceased for nearly a century.

In 1873 Clerk Maxwell put forth the Electromagnetic Theory of Light, a theory now universally accepted. On this theory light still consists of waves, waves of electric and magnetic disturbance just like the waves used in wireless telegraphy, but microscopic in length instead of being yards or miles from crest to crest. He showed, too, that such waves should exert a pressure, but just half that exerted according to the discarded Corpuscular Theory. He calculated with the data he took that strong sunlight falling perpendicularly against a black surface exerts a pressure of rather less than one two-hundred-thousandth of a grain on a square inch, or rather less than one twenty-thousandth of a milligramme on a square centimeter. It only amounts to two and a half pounds weight on a square mile.

We may perhaps present Clerk Maxwell's ideas in the following way. If we rub a stick of sealing-wax with flannel it becomes negatively electrified. If we hold it near a conducting body such as the disc in fig. 1, it induces positive electrification on the surface of the conductor, and the wax and the

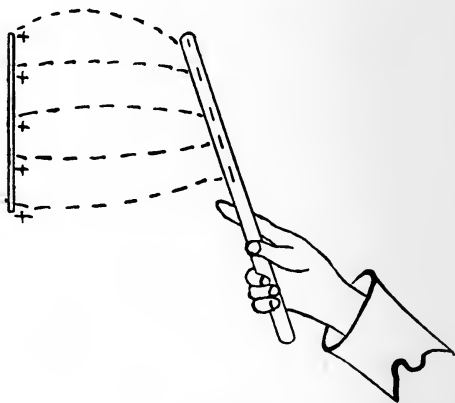


FIG. 4.

conductor tend to draw together. We have every reason to suppose that the air or medium between the two surfaces, electrified respectively positively and negatively, is modified. Perhaps the atoms join hands and form atomic chains stretching from surface to surface, and these chains tend to shorten and so pull the surfaces together. But whatever

may be the actual modification of the medium between the two electrifications we may symbolize its effect by saying that "lines of force," or "tubes of force," as in fig. 4, start from the negative on the wax to the positive on the conductor, and that these tubes of force are trying to contract and pull the bodies together. The action is like that which would occur if every tube of force were a stretched india-rubber cord with its ends fastened on the two surfaces. Imagine, then, a bundle of such stretched rubber cords. As they shorten lengthways they tend to bulge out sideways and press against each other. In somewhat similar manner the tubes of force in the modified medium, while exerting end pulls are also exerting against each other a side pressure as if they too were trying to bulge sideways. It is this side pressure of the tubes against each other or against any body on which they impinge sideways which is their essential property for our present purpose.

We may symbolize magnetic attraction in a similar way. If, for example, the north-seeking pole of a magnet is drawing towards itself a piece of iron it makes a south-seeking pole in the nearest part of the iron; and we may think of tubes of force as stretching from one pole to the other through the intervening medium. These tubes of force tend to shorten, and so to draw the magnet and iron together. We have much reason to sup-

pose that there is a spinning motion of the constituents of the medium round the line of force passing through them, perhaps a spinning of the atoms, more probably a spinning of the corpuscles round the atoms.

A spinning body like the Earth tends to draw together along the axis of spin, and to press out sideways at the Equator. Similarly the spinning medium round a magnetic line of force tends to

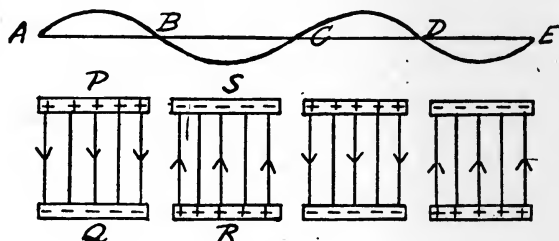


FIG. 5.

draw together lengthways and to bulge out sideways, and so we have side pressure of the tubes of magnetic force against each other just as we have it with tubes of electric force, though probably it is now produced in a different way.

Now let us see what electric and magnetic conditions we must imagine in a train of light waves.

Imagine that we could travel on with the waves and at their speed, so that we could always have the same waves under observation, and let us sym-

bolize them in the usual conventional way by the curve ABCDE in fig. 5.

The actual electric condition between A and B will be the same as that between a positively electrified plate P and a negatively electrified plate Q, while that between B and C will be the same as that between a positively electrified plate R and a negatively electrified plate S. CD will be like AB, and DE like BC.

Of course there is nothing corresponding to the electrified plates in the actual waves, but the medium in their course is modified just as the medium between the successive pairs of plates is modified. The length of a wave includes two oppositely affected halves, *i.e.* stretches from A to C, and for ordinary light a wave is somewhere about a fifty-thousandth of an inch long.

These waves are travelling on, say from left to right, and the changed condition of the medium symbolized by the lines of force is being propagated on from point to point in their course. But to propagate a condition we must have motion in the machinery which propagates it.

This motion of the machinery is supplied by the spinning round the magnetic lines of force, which must accompany the electric lines to make their propagation possible. We know from various experiments that the magnetic lines must be at right angles to the electric lines and also at right

angles to the direction of propagation, that is, perpendicular to the plane of the paper in fig. 5.

If we held a horse-shoe magnet with its plane perpendicular to AB, as in fig. 6, then the magnetic lines in the part of the medium between A and B would have the same direction as those between the poles of the magnet when its north pole was in front. The lines between B and C would have the same direction as those of the magnet with its south pole in front, and so on.

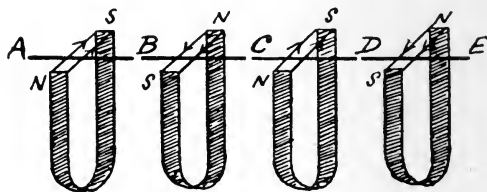


FIG. 6.

Thus we have in these electro-magnetic waves two sets of tubes, electric and magnetic respectively, at right angles to each other and to the line of propagation. Both kinds of tube tend to bulge as it were sideways, tend to press against each other, and tend to press against any surface from which they issue, and against any surface on which they fall.

Clerk Maxwell showed that on his theory the pressure thus exerted against a square centimetre

by a beam of light was numerically equal to the energy in a cubic centimetre of the beam.

This, then, is Maxwell's explanation of light-pressure on his electro-magnetic theory. The light consists of tubes of electric force and tubes of magnetic force, which are rushing sideways along the beam and pressing sideways against each other and against any surface which the beam strikes.

But though we all now believe in the electro-magnetic theory, and can hardly conceive that it should be superseded, we must be warned by the fate of the corpuscular theory, and recognize that the electro-magnetic theory may have to go, if some other still unimagined theory should account for observed facts more completely and more correctly.

It is interesting then to know that whatever kind of waves we imagine, so long as they have the properties which we observe in light, these waves must press against the surface from which they start, and they must press against the surface on which they strike. They must, in fact, carry momentum with them just as surely as if they were moving particles on the old corpuscular theory. This was first pointed out by Bartoli in 1875, and the proof was put in a precise and simple form by Sir Joseph Larmor.

The fundamental idea of the proof is, that a train of waves is somewhat like a compressed

spiral spring. The waves contain energy. If we compress them into a shorter length we have to put more energy into them, somewhat as we have to put more energy into the spiral spring when we crush it up. The ends of the spring are pressing outwards, and if we compress it we do work against the outward pressure, and so we put extra energy into the spring. Similarly when we compress waves into a shorter length we put more energy into them, and therefore they, like the spring, must be pressing outwards at each end or must exert a pressure against any surface which is moved so as to crush them up.

Wave energy is of two kinds, that due to the shape of the waves and that due to the motion of the material waving.

As an illustration of the energy due to shape and its dependence on length of wave, we will take the special case of zigzag waves on a stretched cord, which we will suppose of india-rubber.

Let AB, fig. 7(a), be the cord, stretched to some extent. Let C be the middle point, and let D and E be the middle points of AC and CB. Suppose that D is drawn up a small distance DM, and that E is drawn down an equal small distance EN, as in fig. 7(b), so that C remains in the original line. Then work is done in pulling D and E out of position, or energy is put into the cord in changing its shape into the zigzag wave form ADCEB.

The length of the wave formed is AB . Now let the cord return to its straight form, as in (a). Bisecting AD in F , DC in G , etc., let us again pull it out; but this time, as in fig. 7(c), forming two waves in the length AB . Let F be drawn up through $FP = DM$ in (b), and let G be drawn down by an equal amount, and similarly with H and K . Then D, C, E remain in the original line. The pull

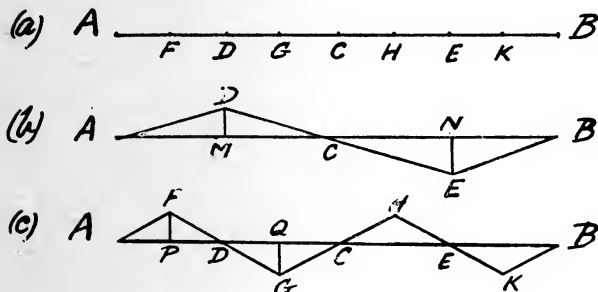


FIG. 7.

required on F in (c) is greater than the pull on D in (b), for the slope of the cord is greater, and its tension is more inclined to FP . If, as we suppose, the displacement is only small, the slope in (c) is double that in (b) and the pull along FP is double that along DM . Then the work done at F in (c) is double that done at D in (b). But in (c) there are twice as many points drawn aside. With twice as many points and double work for each, we must put in four times as much work in all; or, where

we halve the length of wave, keeping the height and depth the same, we require four times as much energy in the same length to change the shape. It is easily seen that with three waves in the same length, we should have nine times as much energy, and generally the energy in given length for waves of the same height is in inverse proportion to the square of the wave-length.

The dependence of the energy of motion on

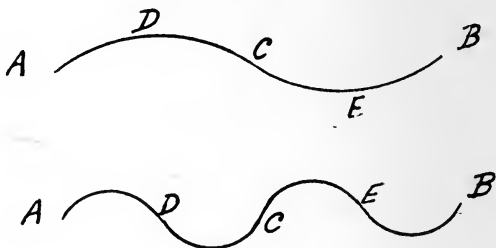


FIG. 8.

the length of wave is seen by considering fig. 8, where two trains of waves of equal height and depth are represented. The lower train has half the wave length of the upper train. Let them be supposed to travel at equal speed from left to right. Any particle, such as D, moves up and down as the waves pass by it and through an equal excursion in each case. But the particle has to make twice as many excursions in the same time in the lower train, and must therefore move with an average

velocity twice as great as that of a particle in the upper train. But the energy of motion is proportional to the square of the velocity. Then the energy in the lower train is four times as great as that in the upper train. Or, again, for waves of the same height the energy in given length is in inverse proportion to the square of the wave-length. Since, then, both kinds of energy follow the same law, the total energy is in inverse proportion to the square of the wave-length, when the height and depth remain the same.

Now let us see how waves may be crushed up or be opened out. When a source is giving out waves and is at the same time moving, the length of the waves is altered, shortened if it is following them up, lengthened if it is moving backwards from them. This effect was first pointed out by Doppler in 1842. It is easily observed in the sound emitted by a locomotive, whistling as it passes you. The note heard is higher when it is coming towards you than when it has passed and is moving away from you. The effect is noticeable even in the hoot of a motor. The reason is easily seen from fig. 9. Let the upper part of the figure represent a stationary engine sending out waves of sound forward and backward from C. Let us consider four waves in each direction. These will be of the same length and will travel at the same speed, so that an observer at A will

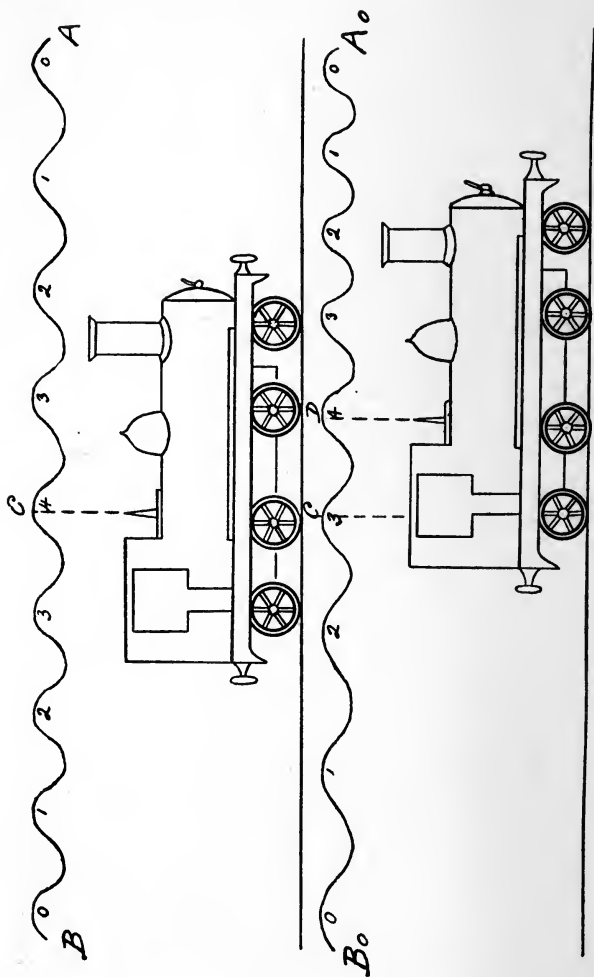


FIG. 9.

receive the same number per second as an observer at B. The two observers will, therefore, hear notes of the same pitch, since pitch depends solely on the number of waves entering the ear per second. In the lower part of the figure let the engine be travelling from left to right as it whistles. Suppose that at one moment it is at C and a moment later while it has sent out four waves let it have moved to D. The first wave will have travelled to A in one direction and to B in the other the same distances as before, for the velocity of the waves is not altered by the travel of the engine. But the fourth wave will have just issued when the engine is at D, so that in the direction of motion the four waves are crushed up into the shorter length AD and the observer at A receives more per second, while, behind, they are opened out and the observer at B receives fewer per second. The pitch of the note is therefore higher at A and lower at B.

The Doppler effect is easily observed by putting a pitch-pipe into one end of a rubber tube several feet long. The experimenter blows through the open end and whirls the tube in a horizontal circle round his head, the pitch-pipe moving round the circumference of the circle. A hearer outside the circle notes a rise in pitch as the pipe approaches him and a fall as it recedes, though to the experimenter the pitch is the same all the time.

Another interesting illustration of the effect is

afforded by two tuning-forks of exactly the same pitch mounted on resonance boxes. If they are both sounded and are both at rest, no beats are heard, since they are in unison. But if, while they are sounding, one of the forks is moved towards an observer, he hears beats, for the waves of the moving fork are shortened and he receives more per second. Though the forks are still emitting the same number of waves per second the hearer is receiving more from one than from the other: the two notes to him are not the same and they beat.

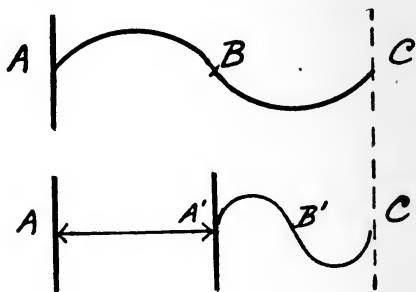


FIG. 10.

Let us now apply this "Doppler Principle" to a source emitting light, or, more generally, radiation of the light kind, whether it is within the range sensible by the eye or not.

Consider the cases shown in fig. 10.

In the upper part of the figure suppose that the source A is at rest and that it is emitting waves of

the length ABC. In the lower part of the figure suppose that the same source is moving forwards, and, to take a very extreme case for simplicity of calculation, suppose that it moves at half the speed of the waves, so that while the front of the wave travels to C the back is just issuing from A'. The height and depth of the waves will be the same, for they depend only on the temperature of the source, which is the same in the two cases. We must suppose, too, that the energy given out by the light sources is the same in the two cases. But in the latter case the energy in the wave is twice as great in the half length, since it is four times as great in an equal length. We can only account for this extra energy by supposing that the wave presses like a compressed spring against the surface from which it starts, and that we supply the extra energy in moving the source forwards against the pressure.

If the source is moving backwards it is easily seen that the waves will contain less energy than that emitted from the light source and that the difference between that emitted from the source and that existing in the medium is to be accounted for by the work done by the wave pressure in pushing the source back.¹

When the theory is worked out fully it is found that the pressure is greater when the source is moving forward than when it is at rest, and that

¹ Note 2, p. 84.

when it is moving back it is less than when it is at rest. When it is at rest the pressure is equal to the energy in a unit length of the beam.

Since, then, the waves are pressing back against the source, the source is pressing forward against them. Or it is giving them forward momentum,

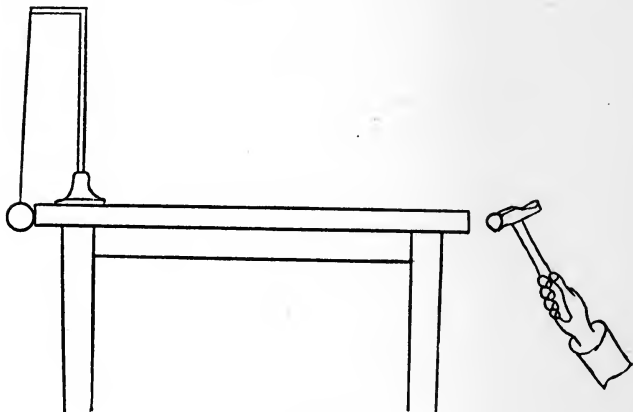


FIG. II.

another way of expressing exactly the same idea. It is giving the waves it sends out momentum, just as certainly as if they were a rush of corpuscles, and the waves carry this momentum forward into space. We need not suppose that the waving material rushes forward. It hands on the momentum from layer to layer. We may illustrate this point by hanging a ball so as just to touch one

edge of a long table as in fig. 11. If we give a sharp blow with a hammer to the opposite edge a wave of pressure travels through the table, a wave of momentum, put in by the hammer, which is given up by each part of the table to the next and which finally arrives at the ball and presses it out.

Let us now follow the course of a limited train of waves travelling out perpendicularly from a source A to a receiving surface B, fig. 12. When the waves start from A, as at *a*, they are pressing



FIG. 12.

against A, and A is pressing against them, that is, A is pouring forward momentum into them. They rush out carrying the momentum with them, and when they are clear of A, *i.e.* when A has ceased to pour out momentum, it is no longer pressed. The momentum is now being carried through mid space as at *c*. Let the waves finally strike the surface B. If they are absorbed there, that is, if B is a quite black surface, the waves cease and as they cease they deliver their momentum up to B. As they take the same time to pass into B that they took to issue from A, and as they give up in that time all the momentum which they received

they must press against B in being absorbed as much as they pressed against A in being emitted. The pressure against B is therefore equal to the energy per cubic centimetre in the beam.

But if B is a reflector, a perfect reflector let us suppose, the reflected waves press back just as much as the incident waves and so the pressure is doubled. Or, to put it in another way, the incident waves bring up momentum in the direction AB and give it to B. B must give up momentum to the reflected waves in the direction B to A, which has the same effect as if it received momentum in the direction A to B. That is, it receives a double dose of momentum and the pressure is doubled. Since we have twice as much energy in the space just outside B where reflection is going on, as when we have only the single beam, the pressure is still equal to the energy per cubic centimetre.¹

We can now see what the pressure amounts to. In full sunlight outside our atmosphere the sunlight received on a square centimetre of a black surface would suffice to raise 1 gramme of water about 2.5° C. per minute² or about 0.0417° C. per second according to recent experiments. This is equi-

¹ Note 3, p. 85.

² There is still great uncertainty as to the value of this heating effect—the "Solar Constant." We shall take it throughout our calculations as 2.5 calories per sq. cm. per minute.

valent mechanically to $0.0417 \times 4.2 \times 10^7$ ergs = 1.75×10^6 ergs. But this energy is spread over a column 1 sq. cm. in area and 3×10^{10} cm. long, the length that light travels in one second, so that the energy in 1 cubic centimetre is $1.75 \times 10^6 \div (3 \times 10^{10}) = 5.8/10^5$ ergs. The pressure exerted by the sunlight on the absorbing surface is therefore about $6/10^5$ dynes, or, say, six one-hundred-thousandths of a milligramme weight.

II

EXPERIMENTS ON THE PRESSURE OF LIGHT FALLING PERPENDICULARLY ON A SURFACE

WHEN Maxwell put forward his theory of light-pressure and calculated the exceedingly small value due even to full sunlight, he remarked that probably "a much greater energy of radiation might be obtained by means of the concentrated rays of the electric lamp. Such rays falling on a thin metallic disc, delicately suspended in a vacuum, might perhaps produce an observable mechanical effect."¹

Twenty-seven years later Professor Lebedew of Moscow described to the Congrès International de Physique² an experiment in which he detected the pressure in precisely this way, and in which he found a value agreeing very fairly with that given by Maxwell's theory.

In a large glass globe 20 cm. or 8 inches in diameter, among other arrangements of discs,

¹ *Electricity and Magnetism*, § 793.

² *Rapports*, vol. 2, p. 133. A fuller account is given in *Ann. du Physik*, vi. 433, Nov. 1901.

that shown in fig. 13 was used. This was suspended by a fine glass thread TH. HH was a glass rod to which was fixed a mirror M, which reflected a scale into a telescope to give the position of the arrangement. On cross arms were fixed discs of platinum foil each 0.5 cm. in diameter, and each with its centre 1 cm. from the vertical axis of suspension. The upper pair were 0.1 mm. thick and the lower pair 0.02 mm. thick. The discs on the right hand were covered on both sides with a layer of platinum black, while those on the left hand were polished. The globe was exhausted as far as possible by means of a Sprengel pump, and then the beam from an arc lamp was directed on to one of the discs, and the consequent retreat of the disc was observed by means of the motion of the scale reflected into the telescope. The value of the force required to push the disc back by the observed amount was determined by observing the times of vibration of the system when loaded with a body of known size and weight and when unloaded—the method always used to find the force needed to twist a fibre or wire through any angle.

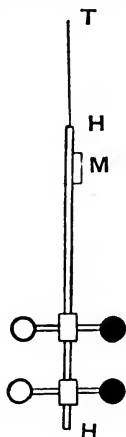


FIG. 13.

With the high degree of exhaustion used there was probably no bodily convection of the air in

contact with the disc. But if any convective action still remained it was sought to eliminate it by directing the beam first on to the front face and then on to the back face of the disc, and taking the difference of the two displacements. For the convection currents produced would depend on the rise in temperature of the disc, and also on the difference of temperature of the two sides. The rise in temperature would be very considerable, but with the very thin discs used the difference of temperature of the two sides would be negligibly small. The convective action would, therefore, be due practically to the rise in temperature of the disc, and would be the same in magnitude and direction on whichever side the beam fell. Its effect would depend on the "lie" of the disc; and if the disc could be set absolutely vertical, it would probably vanish, for the air would simply stream up equally against two vertical sides. But this absolutely vertical arrangement is practically unattainable. If the light presses the disc back a distance P , and if convection moves it C , when the light falls on the front face we observe $P + C$, and when the light falls on the back face we observe $-P + C$, so that the difference between the two positions of the disc is $2P$, and C is eliminated.

The radiometer action depends on the small difference of temperature of the two sides of the disc. This difference is greater with the thicker

disc than with the thinner—five times greater, since the disc is five times thicker. If, for instance, the deflection is 18 divisions with the black disc, 0.1 mm. thick, and 13 divisions with the black disc, 0.02 mm. thick, we get a reduction of 5 divisions for a reduction of .08 in thickness. A further reduction of .02 would lessen the deflection by $5 \times \frac{.02}{.08} = \frac{5}{4} = 1.25$. That is, an exceedingly thin disc should give a deflection of $13 - 1.25 = 11.75$ divisions, and an exceedingly thin disc should have the same temperature on the two sides, and give no radiometer action. In this way, by observations with the same beam, first on the thicker, and then on the thinner black disc, the radiometer action was eliminated.

On the bright discs the radiometer action was too small to be measurable.

On the black discs the beam was absorbed, and gave its full pressure $P = E$. On the bright discs the beam was partly reflected, and the reflected beam was also pressing against the surface. The fraction of light reflected was determined by some subsidiary experiments. Suppose it to be r , then the pressure was $P(1 + r)$.

To test the theory of light-pressure it was necessary to measure the energy E in a cubic centimetre of the beam. One way in which this was done is illustrated in fig. 14. A blackened copper block C,

of known size and weight, had a small hole bored in it, and a small thermometer was inserted. A screen with a hole D, just the size of a disc, was placed in front, and the beam was directed on to D, and passing through, fell on and warmed C.

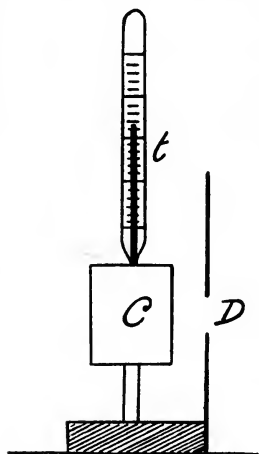


FIG. 14.

The rise in temperature in a given time showed how much energy the beam brought up in that time, and so the energy brought up in a second was determined.

If V is the velocity of light, and if H is the heat developed in C per second, measured in ergs, then H is the energy in length V of the beam, and H/V is that in a length of one centimetre. The force on a black disc, the size of the hole D , should therefore be H/V . The heat

measurements agreed with the force measurements within about one-fifth of the whole.

While Professor Lebedew was making this experiment, Professors Nichols and Hull were also experimenting on the subject, and in 1903¹ they published the full account of their work. Their

¹ *Proc. American Academy of Arts and Sciences*, vol. xxxviii. p. 559. April 1903.

method was somewhat like that of Lebedew, but it differed in very important details. Two circular microscope cover glasses, CD (fig. 15), each 12.8 mm. in diameter, and 0.17 mm. thick, were hung up in a partially exhausted glass vessel by a quartz fibre. They were silvered on the front side and brilliantly polished. m was a small mirror, in which the reflection of a scale could be viewed by a telescope.

A beam of light was directed on to a disc, and the force which it exerted was measured by the deflection of the scale in the telescope.

Nichols and Hull made use of the fact not yet explained, that when the pressure of the air in the vessel is reduced to 2 or 3 cm. of mercury, say somewhere about an inch, the convection

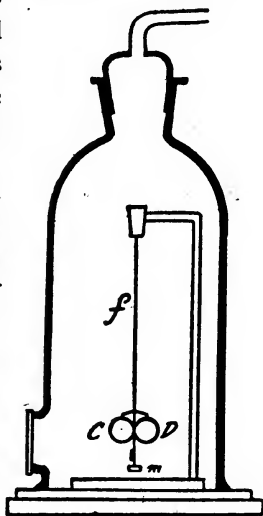


FIG. 15.

effects are enormously reduced. Further, these effects take time to develop, for the discs take time to heat up. They therefore put the beam on to the disc for 6 seconds only, one quarter of the time of a complete torsional swing of the suspended system. The light-pressure has its full value in-

stantly, and keeps that value through the whole time of exposure. The convection effect only gradually creeps up to its full value, and has not become serious in 6 seconds. From the deflection produced by the 6 seconds incidence of the beam its full effect with long-continued incidence could be calculated.

To eliminate the radiometer action the beam was first directed on to the front face of the disc. That face would be slightly the hotter, and the radiometer action would join with the light pressure in pushing the disc back. The beam was then directed on to the back face of the disc, and the light pressure was reversed. The beam, passing through the transparent glass, would still be incident on the silver on the front. This would still therefore be the warmer face, and the radiometer action would be in the same direction as before. That is, it would now be opposed to and lessen the apparent effect of the light-pressure. The mean of the two deflections observed would therefore give the light-pressure, and eliminate the radiometer action.

The beam was almost totally reflected by the silver, and so the pressure was nearly twice as great as if the beam had been absorbed. The actual reflecting power of the silver was determined, and its slight defect from total reflection was allowed for.

To determine the energy in the beam it was

allowed to fall on a blackened silver disc of known size and weight, and the rise in temperature was determined by a thermo-electric method, into which we shall not enter. The rise in a given time gave the heat developed, and thence the energy per cubic centimetre of the beam could be determined.

Allowance was made for departure in various ways from the ideal of the perfectly reflecting disc, and ultimately Nichols and Hull found that the pressure observed agreed with the value of the energy per c.c. in the incident beam within less than one per cent.

When we consider the minuteness of the force to be measured and the magnitude of the disturbances, we must recognize this experiment as one of the finest achievements of our time.

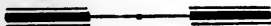


FIG. 16.

Professor Hull has made some interesting experiments,¹ in which the receiving disc was enclosed between two parallel glass plates, fixed respectively in front and behind the disc, with a small interval between, as shown in fig. 16. The radiometer action was then eliminated. For if a molecule gave an extra kick back against the absorbing disc, it rushed forward and gave an extra kick

¹ *Physical Review*, XX. May 1905.

forward against the front plate. These two kicks neutralized each other. Another action, due to emission of particles from the absorbing surface itself when heated, was at the same time eliminated, for particles thus shot out would give equal impulses, back against the disc and forward

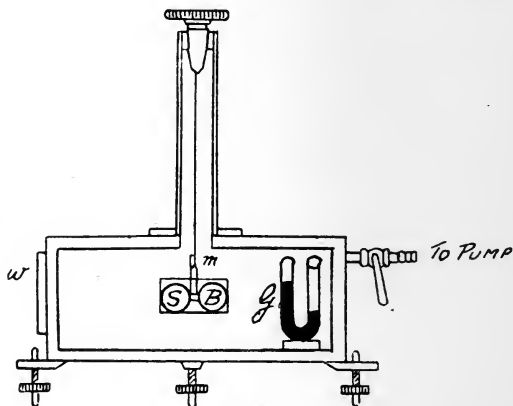


FIG. 17.

against the plate. Hull found that with such an arrangement the light-pressure was measurable even at a pressure of 70 mm. of mercury. It is interesting to note that Sir William Crookes¹ had found that radiometer action nearly ceased even in full sunshine with a similar arrangement of an enclosed disc.

¹ *Phil. Trans.* 170, 1879, p. 88, § 389.

This principle of enclosure may be used to show the pressure of light to an audience. Fig. 17 represents an arrangement which I have found to work very well.

A silvered disc *S* and a blackened disc *B* are fixed inside a thin mica rectangular box suspended by a quartz fibre in a metal case with plate glass front; *w* is a side window through which a beam of light is sent on to a mirror *m* and reflected thence on to a scale. The pressure is reduced by a pump to about 1 or 2 cm. of mercury indicated by a small barometer *G*. When the light is directed for a short time on to *S*, *S* is pushed back. When it is directed on to *B* for the same time, *B* is pushed back, but not so much as *S*.

III

EXPERIMENTS ON THE PRESSURE OF LIGHT AGAINST THE SOURCE FROM WHICH IT ISSUES. THE RECOIL FROM LIGHT

WE have seen that theory leads us to suppose that waves of light carry momentum—forward momentum—with them as surely as if they were particles shot out from the source, and that they receive this momentum from the source. The loss of momentum by the source should be manifested as a back pressure against it. In fact, just as a gun recoils from a bullet which it sends forth, so a luminous body should recoil from the waves of light which it pours out.

The most satisfactory and most direct method of experiment would no doubt consist in suspending a disc, black on one side and silvered on the other, in as perfect a vacuum as possible. Inside the body of the disc should be a coil of wire. An electric current should be introduced by the suspension to heat the coil. The heat would be given out as radiation almost altogether by the black surface, and hardly at all by the silver sur-

face. The black surface should be pushed back by the issuing radiation. But the experimental difficulties in the way of this direct method appear to be quite insuperable.

Dr. Barlow and the author have made an experiment¹ showing the back pressure in a rather less direct way, the disc being heated by allowing a beam of light to fall upon it. The temperature of the disc rose till it attained the steady state in which the energy emitted as radiation was equal to the energy absorbed. The effect was therefore due in part to the pressure of the incoming radiation, in part to the pressure of the outgoing radiation, and the two parts of the effect had to be disentangled.

The nature of the action to be looked for may be explained by considering ideal cases. Let us suppose in the first case that a beam with energy P per cubic centimetre is allowed to fall normally on a very thin disc, perfectly black or fully absorbing on both sides; and let the disc be suspended in a perfect vacuum so as to be quite free from air disturbances. The disc is heated and rises in temperature till it gives out as much energy as it receives. If it is very thin it is practically at the same temperature on the two faces, and it gives out half the energy from each face. The pressures of the issuing radiations are therefore equal and

¹ *Proc. Roy. Soc. A.*, vol. lxxxiii. p. 534, 1910.

opposite, and produce no effect. We have, therefore, only the pressure P of the incident radiation.

In the second case let us take a disc black on the front face and perfectly reflecting on the back face, and let us send the same beam on to the front face. When the disc has reached a steady temperature the energy of the radiation which it emits is equal to that of the radiation which it receives. The back surface being a perfect reflector does not send out any radiation. All the energy pours out from the front surface. If it were given out along the normal only it would produce pressure P , equal to that of the incident beam, and the total pressure would be $2P$. But it is sent out in all directions, and is distributed in the different directions in the same way as the light from a white-hot surface. It can be shown that the pressure is thereby reduced to $\frac{2}{3}P$, so that the total pressure of incident and emitted radiation is $\frac{5}{3}P$.

If there were no back pressure of, no recoil from, the issuing radiation, the pressure on the two discs would be P in each case. What, then, is to be looked for to prove the existence of back pressure is a greater value in the second case than in the first.

In the experiment four discs were used, the front and back surfaces being respectively black and black, black and silver, silver and silver, silver and black. Supposing that the black was perfectly

absorbing, and that the silver was perfectly reflecting, and that the energy in the beam was P per cubic centimetre, the pressures on these should be—

B B	B S	S S	S B
P	$\frac{5}{3}P$	$2P$	$2P$

But the black surface reflected a slight amount which was estimated at 5%, and the silver surface did not reflect the whole but an amount which was estimated at 95%. We assumed that the black radiated out 0.95 as much as a full radiator, and that the silver radiated out 0.05 as much as a full radiator. With these values it can be shown that the pressures should be—

B B	B S	S S	S B
1.05 P	1.62 P	1.95 P	1.92 P

Again, though we suspended the discs in a flask evacuated as far as possible, there was still a small radiometer action due to the residual gas. For the front face of a disc was always a little higher in temperature than the back face, higher by the amount necessary to carry through from front to back the energy emitted at the back. The difference in temperature was greatest with the black and black disc, and with that disc the radiometer action was the greatest. It tended to press the disc back from the source.

Each disc was made of two thin circular plates

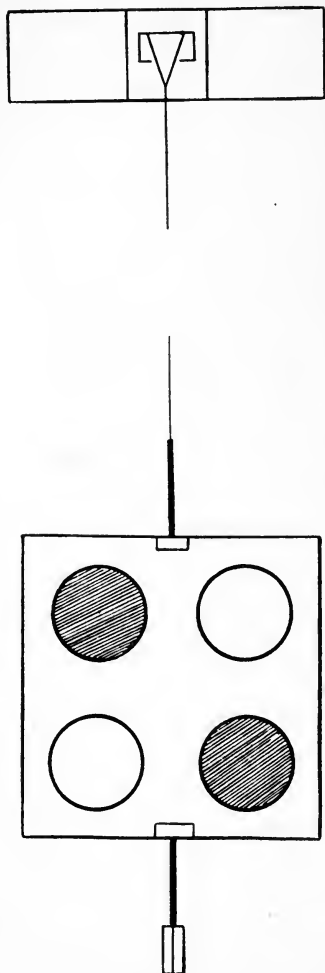


FIG. 18.

of glass 1.2 cm. in diameter and 0.1 mm. thick, with a layer of asphaltum also about 0.1 mm. thick between. To make a disc the asphaltum was placed on one plate which was heated till the asphaltum melted and then the other plate was pressed upon it. The silver surfaces were made by depositing silver from a silver cathode in an exhausted receiver on to the outside surface of the disc.

The discs were then fixed in holes in a plate, fig. 18, the centres of the discs being at the corners of a 2 cm. square and the plate was suspended in a flask 16 cm. in diameter by a quartz fibre 9 cm. long, from a spring collar

fitting in the neck. It is not necessary to describe here in detail how the flask was exhausted. It is sufficient to say that it was filled with dry oxygen and exhausted many times, and that finally it was sealed off at S, fig. 19, when exhausted, and that the bulb C, containing charcoal,

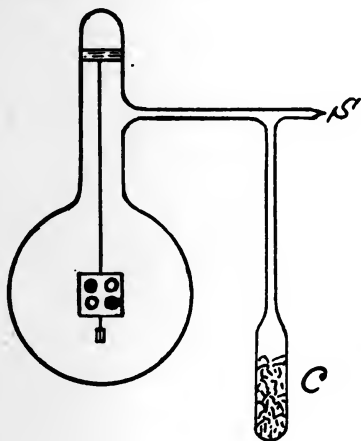


FIG. 19.

was surrounded by liquid air kept boiling at low pressure for some hours before and during an experiment. The residual oxygen was then nearly all absorbed by the charcoal, and the vacuum, as the behaviour of the apparatus showed when light was directed on to the discs, was exceedingly high.

The plan of the apparatus is shown in fig. 20. S was an Ediswan 50-volt focus lamp, run steadily at 60 volts. L_1 was a lens, and L_2 a second lens, which threw an image of L_1 on whichever disc was being used. B and C were a lamp and scale, a

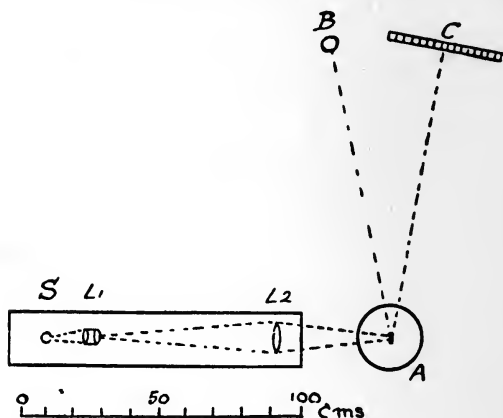


FIG. 20.

mirror below the mica plate reflecting an image of B on to C.

The force corresponding to an observed deflection on the scale C was calculated in the usual way from the time of swing of the suspended system, first unloaded and then loaded with a known mass.

The energy of the incident beam was measured by allowing it to fall on a blackened silver plate of

known weight and observing the rate at which the temperature of the silver rose, the device used by Nichols and Hull (*ante*, p. 41). The energy was such that the incident beam alone falling on a perfectly black surface should have produced a deflection of 13·6 divisions. In the table below the first line gives the nature of the disc, and the second line gives the ratios of the deflections calculated on the supposition that the black reflects 5 per cent. and that the silver reflects 95 per cent. The third line gives the deflections which might have been expected if there had been no other forces than light-pressure. They are obtained by multiplying 13·6 by the ratios. The fourth line gives the deflections actually observed.

Discs	B B	B S	S S	S B
Calculated ratios . . .	1·05	1·62	1·95	1·92
Calculated deflections .	14·3	22·0	26·5	26·1
Observed deflections .	16·1	22·3	28·7	28·0

The excess of 16·1 over 14·3 with the Black-Black disc is no doubt due to a small radiometer action still remaining. The nearness of the observed and calculated deflections of the Black-Silver disc appears to give conclusive evidence of the back pressure of the radiation issuing from its front surface.

IV

EXPERIMENTS ILLUSTRATING THE CARRIAGE OF MOMENTUM BY A BEAM OF LIGHT

SEVERAL experiments have been made by Dr. Barlow and the author to illustrate the carriage of momentum by a beam of light,¹ and recently Prof. Lebedew² has described an experiment

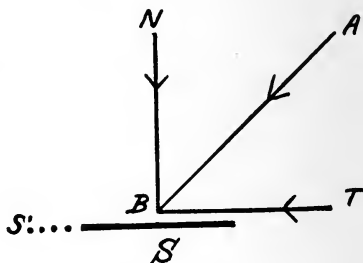


FIG. 21.

showing the absorption of momentum by a gas which absorbs light. I shall now describe these experiments.

(1) When a beam of light falls obliquely on an

¹ *Phil. Mag.* IX, 1905, p. 169 and p. 393. *Nature*, vol. 75, Nov. 22, 1906, p. 90.

² *Ann. der Physik*, Bd. 32, 1910, p. 411.

absorbing surface it exerts a force which has a component along the surface.

Let a beam of light or radiation AB fall on the absorbing surface S , fig. 21. Then it has momentum in the direction AB . Let the length of AB represent the momentum which it brings in one second. Resolve AB into NB perpendicular to the surface and TB along it. If S cannot be pushed back, NB will have no visible effect. But if S can slide on its own plane TB will push it towards S' .

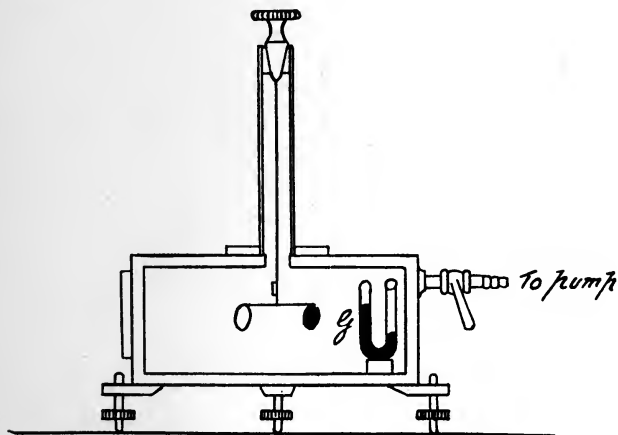


FIG. 22.

To test this two discs of glass, one blacked, the other silvered, each about 2 cm. in diameter were fixed at the ends of a thin glass rod 5 cm. long,

the discs being perpendicular to the rod. They were hung by a quartz fibre in a glazed case as shown in fig. 22. Attached to the rod was a mirror by which the reflexion of a scale could be viewed in a telescope and so the position of the rod could be determined.

The case was then exhausted till the pressure was 1 or 2 cm. of mercury, and a horizontal beam of light from a Nernst lamp was directed on to

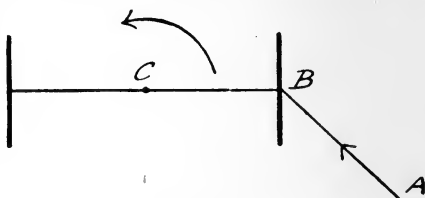


FIG. 23.

the black disc at 45° to its normal as shown in plan in fig. 23.

The disc was pushed and the rod was turned round in the direction of the arrow. The energy in the beam was measured by directing it on to a blackened silver plate of known weight and by observing its rate of rise of temperature. The torque corresponding to a given deflection of the rod was found in the usual way and so the actual torque exerted was measured and could be compared with that which the momentum in the beam should exert as calculated from its energy.

Thus in a particular experiment the observed torque was 21×10^{-6} cm. dynes, while the torque calculated from the energy was 22×10^{-6} cm. dynes. But there was doubtless considerable gas action and the close agreement is probably accidental. All that can be said is that the calculated and observed effects agreed within a few per cent.¹

When the beam was directed on to the silvered disc, the deflection, as might be expected, was much less. For the reflected beam took away the momentum parallel to the surface which the incident beam brought up.

To obtain consistent results this form of apparatus requires very accurate construction and adjustment. For if the black disc is not accurately vertical, if the normal through its centre does not pass exactly through the axis of suspension, and if the incident beam is not perfectly uniform all over the disc, the disturbing forces due to convection and radiometer action may easily turn the suspended system more than the light-force and quite possibly in the opposite direction.

Another form of the experiment was adopted which more easily gave definite and consistent results. A blackened disc of mica, about 5 cm. in

¹ The values here given and those in the succeeding experiments are the results of a redetermination of the various constants and a revision of the calculations, and are not quite in accordance with the values given in the papers cited in the footnote above.

diameter, was suspended in a case with glass sides, by a quartz fibre, so that the disc was horizontal. The pressure was reduced to 1 or 2 cm. of mercury.

A beam of light, AB, was directed, as in fig. 24, at 45° on to a small area B, near the circumference of the disc, the beam AB being in the plane through the normal BN, and perpendicular to the radius OB. The light force parallel to the surface

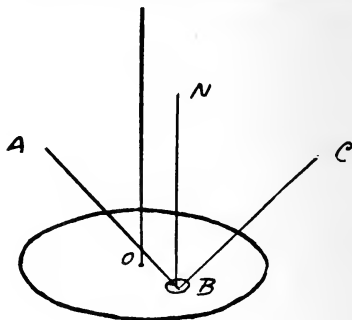


FIG. 24.

tended to push the disc round, through an angle which we will call L . But the disc was heated by the beam so that convection currents and radio-meter action came into play. Unless the disc were perfectly horizontal, a perfection which could not be attained in practice, these would also turn the disc round by an amount and in a direction which would depend on the slope of the disc at B . Let us call this angle of turn D . The total angle of twist was therefore $D + L$, and this was

observed by a telescope viewing the reflection of a scale in a mirror on the suspended system.

The same beam was then directed on to the same area B along CB at 45° on the other side of the normal BN. The heat absorbed was the same and therefore convection and radiometer action might be supposed to give the same turn D and in the same direction. But the horizontal light force L was reversed, so that the angle now observed was $D - L$. The difference between the two observed angles was $2L$, double that due to the light force in a single beam. The effect of the disturbing forces was thus eliminated, and the elimination was verified by directing the beam on to different points near the circumference, when nearly the same value of $2L$ was obtained.

At first the experiment was tried with air at low pressure in the case. Then hydrogen was used instead, and the results obtained were much more consistent. The following table gives the observed torque in a series of experiments at various pressures, and the torque calculated from the energy in the beam—

IN HYDROGEN.		
Pressure in cm. of mercury.	Observed torque in 10^{-6} cm. dynes.	Calculated torque in 10^{-6} cm. dynes.
1.8	6.0	5.3
1.4	6.5	6.2
1.6	6.3	5.4
1.25	6.2	5.0

The gas action was so much less and so much more regular in hydrogen that it was found possible to obtain evidence for the tangential light force even at atmospheric pressure. The following table gives a series of values obtained for the deflection in scale divisions due to the light force alone, the angle we have denoted by L —

IN HYDROGEN.

Pressure in cm. of mercury.	Scale divisions deflection due to light force.
0.04	5.9
0.09	6.1
0.20	5.2
0.44	5.4
0.74	5.5
1.2	5.7
2.1	5.2
3.2	5.1
6.0	5.5
10.4	5.4
22.4	5.2
47.7	5.2
73.6	4.4

(2) When a beam of light is shifted parallel to itself, it exerts a torque tending to turn the system by which it is shifted.

Let us begin with a mechanical illustration. If

a bent brass tube ABCDE (fig. 25, plan), is suspended by its middle point C so as to be in a horizontal plane, and if a stream of air is blown through the tube by a pump P, the stream has to turn two corners, B and D. As it turns each corner it presses outwards against the corner, and so we get two forces turning the tube round in the direction of the arrow.

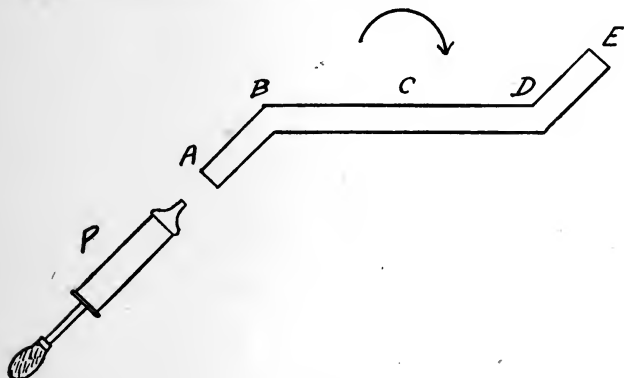


FIG. 25.

We may regard the action thus. The air possesses momentum, and the stream of momentum is shifted from the line AB into the line DE, parallel to AB. We should get the same effect by a force P_1 acting at B against the motion, and destroying the momentum along AB, and by an equal force P_2 acting at D along DE, giving the momentum again in the new line DE, fig. 26.

P_1 , P_2 constitute a counter-clockwise couple or torque, acting on the air, and there must be an equal and opposite couple acting on the tube, turning it round clockwise.

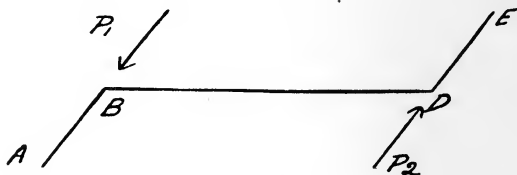


FIG. 26.

Since a beam of light is just as much a stream of momentum as a blast of air, if we shift a beam of light parallel to itself we must exert a couple or torque upon it, and the light will react with an equal and opposite torque on the system by which the change is effected.

An experiment to show this was made by suspending two small glass prisms, with refracting angle 34° , at the ends of a rod 3 cm. long, hung up by a quartz fibre in an exhausted case. The plan is shown in fig. 27, C being the point of suspension. The sloping side of each prism was 2.15 cm. long and 1.6 cm. high. The ray AB was refracted along BD, and then emerged along DE, turning a corner in each prism. It exerted a force outwards against each corner, and the suspended system turned round in the direction

indicated by the arrow. One set of readings, made under good conditions, gave a mean deflexion of 3.3 scale divisions, corresponding to a torque of 20×10^{-6} cm. dynes. The energy of the beam was measured, and after allowing for reflexions, it should have produced a torque of 20.3×10^{-6} cm. dynes. Other observations of the torque were not so near that calculated from the energy, though always of the right order.

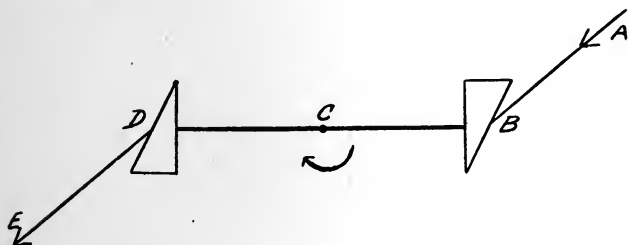


FIG. 27.

Another smaller pair of prisms was then used, with the same refracting angle of 34° , but with sloping side only 1.35 cm. long and 1.05 cm. high. It was hoped that with this lighter system and a smaller and presumably more uniform beam, the results would be more consistent, but the hope was not justified, as the following observations in four experiments show. The torque was too small for accurate measurement.

	Observed torque in 10^{-6} cm. dynes.	Calculated torque in 10^{-6} cm. dynes.
1	7.1	7.1
2	7.6	7.1
3	4.6	3.0
4	8.8	5.3

The effect of shifting a beam parallel to itself was tested by another experiment. A rectangular

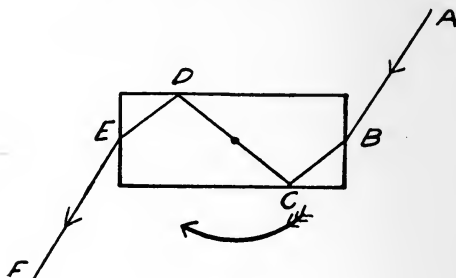


FIG. 28.

block of glass 3 cm. \times 1 cm. \times 1 cm. was suspended in the exhausted case with its long axis horizontal, and a beam of light was passed through it in a horizontal plane, as shown in fig. 28 plan, the beam emerging along EF parallel to its line of incidence AB.

Again the block was turned round in the direction of the circular arrow, but as the fibre was rather coarse the deflexion was very minute. To render the effect more evident the beam was

only sent in intermittently. The system was made to vibrate torsionally in the horizontal plane. In each vibration, while B was moving from the source the beam was sent in, and while B was moving towards the source the beam was shut off. The swings then gradually increased always. Then the beam was sent in while B was moving towards the source and was shut off while B was moving away from it. The swings then gradually decreased always. The observed torque was of the order expected from the energy in the beam, but the effect was so small that accurate measurements could hardly be made. The experiment is to be regarded as qualitative rather than quantitative.¹

(3) Professor Lebedew's Experiment on Absorbing Gases.

We may understand the principle of the experiment by imagining that a beam of light with energy E per cubic centimetre, passes through a chamber ABCD (fig. 29) containing an absorbing gas, and having transparent windows AB, CD. We suppose that the beam just fills the chamber. If the gas absorbs a fraction a of the energy of the beam it absorbs the same fraction a of the momentum carried by the beam. Since the momentum brought up per second per sq. cm. is E , that

¹ Note 4, p. 87.

absorbed is aE . This absorbed momentum must be neutralized by the window CD pressing against the gas more than the window AB. Or the pressure of the gas close to CD exceeds that close to AB by aE .

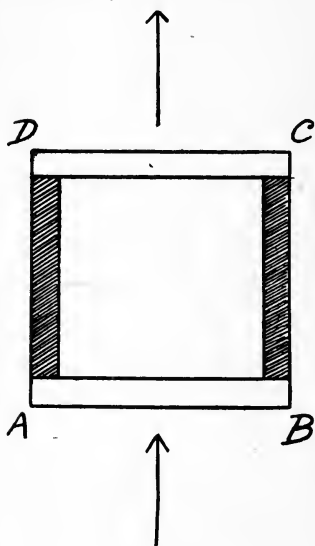


FIG. 29.

What is needed, then, is the measurement of E and of a on the one hand, and the measurement of the difference of pressure p of the gas at the two ends of the chamber on the other hand, and we should expect to find that $p = aE$.

To measure E the same plan was followed as in the experiment described on p. 38, the beam falling on a small calorimeter in which the rate of rise of temperature was observed.

To determine a , the fraction of E absorbed by the gas, a thermo-electric junction of platinum and constantan was placed in front of the window AB and another similar junction was placed beyond the window CD. The second junction was less heated than the first, and from the two

heating effects *a* could be found. The windows were plates of fluorite made as transparent to the beam as possible by putting a thick fluorite plate in front of the source of the beam, a Nernst lamp. This plate at once sifted out the rays which

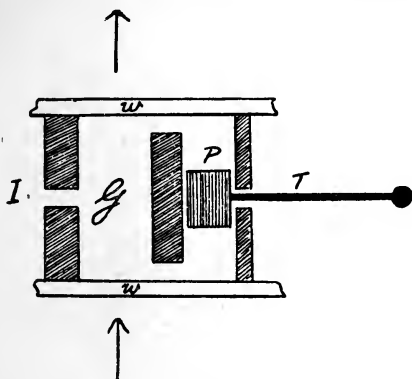


FIG. 30.

would be absorbed by the windows and so left them transparent to the beam as it actually arrived.

To measure *P* the apparatus was arranged somewhat as shown in plan in fig. 30 and in elevation in fig. 31. *G* was the gas chamber, 7 mm. long as traversed by the beam, and of rectangular section 4 mm. \times 3 mm., cut through a brass block; *w, w* were the fluorite windows. *I* was an inlet for the gas. The two ends of the gas

chamber communicated with a side cylindrical chamber bored through the block and 3.25 mm. in diameter. In this, nearly closing it, was a small metal piston P, 2.85 mm. in diameter. This piston was at one end of a torsion rod suspended from T by a quartz fibre, and on the rod was a mirror m

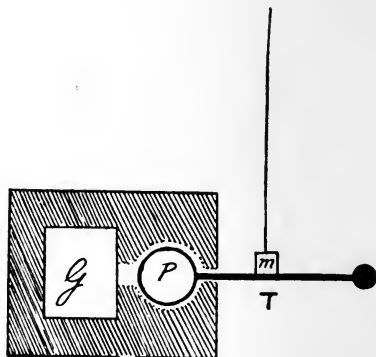


FIG. 31.

which the deflection was read by a telescope on a scale 5 metres away. The rod passed through a small hole in the side of the cylindrical cavity, just large enough to give it sufficient play. The whole apparatus represented in the figures was enclosed in an outer gas-tight case.

If the piston had fitted the cylindrical cavity exactly and had been entirely free from friction, when the beam passed through G there would have

been a difference of pressure p (that to be determined) on the two ends of the piston, and it would have been pushed along until the twist of the fibre introduced a torque equal and opposite to that due to p . But of course this is an impossible ideal, and to avoid sticking friction the piston had free play in the cavity and the gas streamed past it to a small extent and circulated round the two chambers. The difference of pressure on the two faces was therefore somewhat less than p . The pressure lost was determined by a subsidiary experiment into which we shall not enter.

The value of 1 scale division deflection was found in the usual way from the time of vibration of the torsion rod loaded and unloaded.

The gases chosen were Carbon dioxide (CO_2), Methane (CH_4), Ethylene (C_2H_4), Acetylene (C_2H_2), Propane (C_3H_8), and Normal-Butane (C_4H_{10}), all highly absorbing. With the absorption of light the gas in the chamber became hotter, and slightly hotter, no doubt, in front than at the back. There was also local heating owing to the need of concentrating the beam on the gas instead of making it parallel. Thus there was a tendency to the formation of convection currents. To reduce these each gas was diluted, usually with an equal volume of hydrogen. Hydrogen is practically transparent, but is comparatively a good conductor, and it served to equalize the temperature in different

parts of the gas chamber. In the mixtures so formed a ranged from 0.005 to 0.02.

The observed pressure differences were of the order of a millionth of a dyne, and different observations with the same gas agreed with each other within about 10 per cent. The values calculated from aE , when corrected in various ways into which we cannot enter, agreed usually within about 20 per cent. with the observed values.

When we realize how exceedingly minute is a pressure of a millionth of a dyne per centimetre—a million millionth of an atmosphere—we can only admire the experimental skill which succeeded in making consistent measurements and thereby proving that there is at least an approximate agreement between theory and experiment.

V

THE PRESSURE OF LIGHT IN ASTRONOMY. SOME POSSIBLE CONSEQUENCES

THE forces due to light pressure are so small, and the disturbances due to the air are, in comparison so great, that here on the surface of the Earth and immersed in its atmosphere, we cannot expect to find any recognizable results of the pressure except in carefully made laboratory experiments.

Out in the space, however, in which the planets roll round the sun, where the rarity of any matter present must be vastly greater than that in any so-called vacuum which we can produce, light-pressure may have undisturbed play, and may produce very considerable results.

We cannot expect to detect any effect on the large bodies in our system. For instance, the whole pressure of the sunlight falling on the Earth, if it were entirely absorbed, would amount only to about 74,000 tons weight.¹ This appears a big

¹ I use here and elsewhere the energy of sunlight at the distance of the Earth as 2.5 calories per minute per sq. cm.

force, but it is a mere nothing compared with the pull of the Sun on the Earth due to its gravitation, a force 47 million million times as great. While the Sun, then, is pushing against the Earth with its light, it is pulling it in with its gravitation enormously more.

But if the size of the body acted on is less, the proportion of light-pressure to gravitative pull becomes greater. For let us imagine that the earth were divided into equal spheres, each with

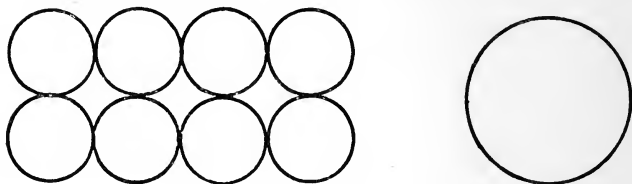


FIG. 32.

half the radius of the actual earth. There would be eight such spheres. If these eight spheres were set facing the sun, as in fig. 32, they would experience in all the same total gravitative pull, but they would expose in all twice the surface to his light and would experience twice the light-pressure. The gravitative pull would, therefore, be only $23\frac{1}{2}$ million million times the light-pressure. If each of these small spheres were again divided into eight equal spheres of half the radius the surface would again be doubled, and the gravitative pull

would be $11\frac{3}{4}$ million million times the light-pressure. Or, the ratio of light-pressure to gravitative pull on each sphere increases in proportion as the radius diminishes. If we imagine the subdivision continued, when the earth was subdivided into equal spheres, each with a radius one 47 million millionth of the radius of the earth, the total light-pressure would equal the total gravitative pull, and if each sphere had the same mean density as the earth, viz. 5.5, the equality would hold for each sphere separately. The radius of each would then be about 13.5 millionths of a centimetre. If the radius were still further reduced, sunlight-pressure would exceed gravitative pull and the sun would push the sphere from it.

If the push on a particle exceeds the pull at any one distance from the sun, it will exceed it in the same proportion at all distances, for both gravitation pull and light push vary inversely as the square of the distance, and as the particle is driven away they both lessen together. If we suppose the sphere to have the density of water, $5\frac{1}{2}$ times less than that of the earth, pressure equals pull when the radius is 5.5 times greater; that is, when it is about 75 millionths of a centimetre—about the length of a wave of deep red light.¹

Absorbing spheres of the density of water and of this radius would be neither attracted to nor

¹ Note 5, p. 90.

repelled from the sun. Smaller spheres would be repelled, and finally would be driven away altogether. If they reflected back some of the light, the repulsion would be thereby increased, and a drop of water of radius a hundred thousandth of a centimetre would be repelled probably ten or twelve times as much as it would be attracted. But the law of inverse proportionality to the radius does not hold when the radius becomes small compared with a wave-length of the repelling light. Certain diffractive effects, pointed out by Swartzschild¹ come into play, and at a certain radius, not far from that last given, the ratio of pressure to pull begins to fall off, and falls off rapidly.

If, then, there are dust particles in the solar system a hundred thousandth of a centimetre across, and not much more dense than water, they will be vigorously repelled, and will be driven right out of our system.

The formation of comets' tails, which are almost always directed away and nearly straight away from the sun, has been ascribed to light-pressure.

Euler used light-pressure to explain them long ago, but without any clear demonstration that light exerted pressure. A few years after Maxwell had published his theory of the pressure, Fitzgerald² revived the explanation, applying it on the

¹ *Kgl. Bayer, Ak. d. Wess*, XXXI. 293 (1901).

² *Scientific Writings*, p. 108.

assumption that the tail was gaseous. But this is just a case in which light pressure cannot account for the formation of a tail at all, as indeed Fitzgerald recognized later.¹ For no gas will absorb from sunlight enough momentum to acquire the motion observed in the stretching of the tail.²

Soon after Fitzgerald's suggestion Lebedew³ investigated the pressure on small absorbing particles, and showed that if the particles were small enough, repulsion would overcome attraction, and that the motions observed in some comets' tails could be thus accounted for, on the supposition that they consisted of particles of the requisite smallness. Arrhenius⁴ discussed the question in greater detail, and sought to explain the self-luminosity of the tail, which certainly coexists with light reflected from the sun, by certain electrical actions emanating from the sun.

Observation shows that the head of a comet on the side facing the sun presents the appearance—it may be only an appearance—of jets of matter spouting forward in various directions with nearly the same velocity, and then turning round and streaming out behind very much as a fountain spouts up drops which rise a little way and then stream

¹ *Scientific Writings*, p. 531.

² Note 6, p. 92.

³ *Ann. der Phys. u. Chem.*, XLV., 1892.

⁴ *Lehrbuch der Komuschen, Physik*, 1903, or *Worlds in the Making* (1908), chap. iv.

downwards. If this is what really happens with comets, if the front or "envelope" on the sun side of the nucleus is really the boundary to which the fountains reach, it is possible to calculate the ratio to gravitative pull of the repulsive force which first destroys the forward velocity and then pushes the matter back to form the tail. In some comets the repulsion is forty, in some twenty times the pull, while in some it is only slightly in excess. Some comets have several tails with apparently a different ratio of push to pull in each. In cases such as these light-pressure might account for what is observed if we could suppose that, as the comet approaches the sun, the nucleus sends out clouds of equal-sized dust, or, maybe, of equal-sized droplets, and all with nearly the same velocity. But in Morehouse's comet, 1908, Mr. Edington¹ finds that the apparent paths of the streaming matter would require repulsion hundreds of times greater than the attraction, and it is hardly possible that the pressure of light could account for this.

The light-pressure theory, then, will not suffice to explain the motions which are in some cases apparently observed. But it is a fascinating theory, and the only theory which seeks to explain the formation of the tails of comets on definite lines. Any electrical explanation is at present vague; and though we may be almost sure that the self-

¹ *Monthly Notices R.A.S.*, March 1910,

luminosity in the tail is due to electric action, the nature of the action is only as yet a matter of speculation. Perhaps we shall find some time that both light-pressure and electric action come into play.

Leaving the formation of comets' tails as an enigma yet unsolved, let us turn to another class of bodies, large compared with the particles which are repelled and yet minute compared with the planets, a class of bodies which reveal their abundant presence in the solar system when they enter our atmosphere and perish as shooting-stars. From the amount of light which they give out in burning up they must as a rule be small, many of them hardly more than specks of matter.

We will suppose that one of these bodies is travelling round the sun nearly in a circle and at the distance of the earth. Then it is to all intents and purposes a minute planet. Let it be black so that it absorbs all the sunlight falling on it. If it reflects some of the light, the first effect which we are going to discuss will be rather greater than for a black body.

Let us suppose that it is 1 cm. in radius, and of the density of the earth, viz. $5\frac{1}{2}$. The push of the sunlight against it will lessen the total pull on it by about 1 in 74,000. This implies that it does not need to move round with quite such a high velocity as the earth to keep it from falling

into the sun. The velocity is less by 1 in $2 \times 74,000$, and the year is therefore greater for it by 1 in 148,000 of our year, or by about 210 seconds, say $3\frac{1}{2}$ minutes. If the particle has a radius $1/1000$ of a centimetre the effect, being 1,000 times greater, will lengthen out its year by about 58 hours, or nearly $2\frac{1}{2}$ days.

If, then, there were a group of particles of diameters from a centimetre downwards, and so thinly scattered that their mutual actions could be neglected, they would gradually get sorted out, the larger ones going ahead of the smaller ones, and ultimately the particles would form a ring round the sun.

This would be still true if the particles were moving in an elliptic orbit instead of the simple circular orbit which we have supposed.

If the orbit is elliptical, another effect of light-pressure comes in, which we may term the "Doppler Reception Effect."

Let fig. 33 represent the orbit. As a particle is moving through P, say towards the apse A, and is so lessening its distance from the sun, it is meeting the stream of momentum flowing from the sun, and is therefore receiving more momentum each second and experiencing a greater sunlight pressure than if it were at rest or were moving in a circular orbit. But when the particle has passed the apse A and is moving, say at Q, away from

the sun, it is moving to some extent with the stream of momentum issuing from the sun, and so is receiving less momentum each second and experiencing less sunlight pressure than if it were at rest or were moving in a circular orbit. We may regard these forces, due to excess and defect of

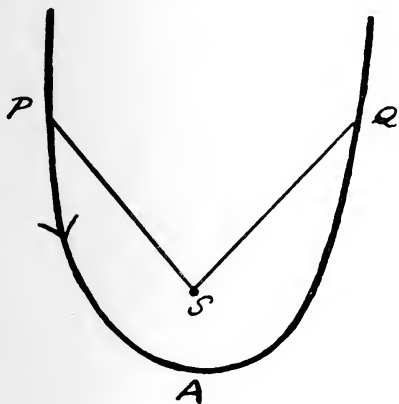


FIG. 33.

momentum received, as extra forces superposed on the inverse square force. At such a point as *P* the extra force is along *SP* and is a resistance to the shortening of *SP*, and at such a point as *Q* the extra force is along *QS* and is a resistance to the lengthening of *SQ*. The net result is a resistance to the change of distance from the sun, a tendency to make the elliptic orbit less elliptic and more circular.

If a group of particles of different sizes is moving in an ellipse, the ellipse tends to become more circular for all, but the effect in a given time is greater for the smaller particles, and there is a sorting action, reducing the ellipticity for them more rapidly than for the larger particles.

There is a third effect of light-pressure on a small particle which we may term the "Doppler Emission Effect," and this must be manifested as a force which always opposes the motion of the particle. The particle is heated by the sun on the side exposed to sunlight, and if it is small enough the heat is conducted through to all parts rapidly enough to make it all at practically the same temperature. If it is at the earth's distance from the sun, and if it absorbs all the radiation falling on it, it will be at a temperature nearly the same as the mean of the Earth's surface, say about 15° C. It is sending out as much radiation as it receives when at this temperature. But the waves it sends out in the direction towards which it moves are shorter than those which it sends out sideways; and these again are shorter than those which it sends out backwards. This is seen from fig. 34, where A, B, C, D are successive positions of the particles and W_A, W_B, W_C, W_D the positions at a given instant of the waves sent out by the particle when it is at A, B, C, D respectively. As shown in the first chapter, there is more energy in the shorter waves

in front than in the longer waves behind, and therefore a greater pressure in front than behind; and the difference between them is a force directly opposing the motion.¹ This force is found to be proportional to the solar radiation, proportional to the cross section of the particle and proportional to its velocity. Its effect is that it is always drawing energy out of the particle. The particle is, in

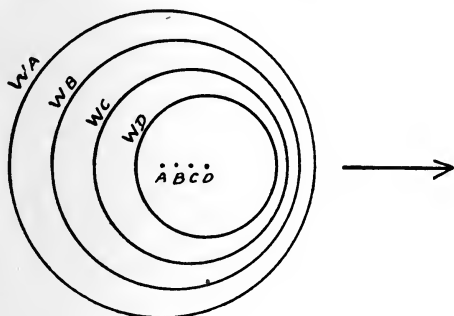


FIG. 34.

fact, always sending out more energy of radiation than it receives from the sun, its expenditure is greater than its income, and it is always drawing on its capital, its own store of energy, to make both ends meet.

When a body moving round the sun is subjected to a small resisting force taking out energy, it does not, as we might at first thought expect, move more

¹ Note 7, p. 95.

slowly. It falls inwards a little towards the sun, and so yields up some of its potential energy. But the potential energy thus yielded up is more than that which the resisting force takes out, and the balance is converted into energy of motion, and the particle moves faster than it did. So that we have as the result of this resistance to motion an increase in the velocity, but in an ever-lessening orbit.

Calculation shows that a sphere of the density of the earth, black so that it absorbs all the sun's radiation falling on it, and 1 cm. radius, say the size of a marble, will fall in towards the sun, if at the earth's distance from it, about 820 metres in the first year. In successive revolutions, each taking less time than the last, it will fall in less, but in successive periods equal to our year it will fall in more; and if it can be supposed to move in a nearly circular spiral, always shortening its distance by the same law, it will reach the sun in about 90,000,000 years.

With smaller particles the action is more rapid and a particle $\frac{1}{1000}$ cm. in radius will reach the sun from the distance of the earth in 90,000 years.

To sum up these effects of light-pressure, we have, first, the repulsion of exceedingly minute particles which may be driven out of our system altogether if they are sufficiently minute. On

larger bodies for which gravitative pull exceeds light-pressure we have three kinds of effect—

1. A lengthening of their period of revolution.
2. A tendency to make the orbit more circular.
3. A drawing in, in a spiral, ending sooner or later in the sun.

On bodies as large as the planets these effects are inappreciable in any time which we need consider ; but on bodies of radius from 1 cm. downwards, the effects might be measurable. One result at once appears. Our system is now full of such bodies, if we are right in assuming that many of the shooting stars are small particles. Whatever age we ascribe to the Sun it must be vastly longer than the time required to have drawn all such particles originally contained in his system into himself. The supply must therefore be continually renewed, and we are led to at least a probable conclusion that it is renewed from the space outside our system. There is some ground for supposing that we draw some, at least, of the comets from outside, and if we suppose that they consist of clouds of such particles, we can see that the actions which we have here considered would gradually lead to their disintegration, even if no other action tended in the same direction. There would be in the first place a tendency of the smaller particles to lag behind in the orbit. Then a tendency for all the particles to move in

less and less elliptic orbits, the smaller particles yielding sooner to this tendency than the larger. And all would tend to lessen their orbits and finally to pass into the sun. There are very good grounds for the supposition that some of the periodic meteor showers are comets which have been disintegrated and spread along their orbits. We may perhaps make the additional supposition that we witness examples of the further disintegration into orbits differing widely from the original ones in the casual meteors which flash across the sky and which cannot be assigned to any known group.

But the end of all must be the same. The Sun cannot tolerate dust. With the pressure of his light he drives the finest particles altogether away from his system. With his heat he warms the larger particles. They give out this heat again and with it some of that energy which enables them to withstand his attraction. Slowly he draws them to himself, and at last they unite with him and end their separate existence.

NOTES

NOTE 1, p. 12.

LIGHT-PRESSURE ON THE CORPUSCULAR THEORY

LET a beam of light, supposed to consist of corpuscles moving with velocity V , be incident perpendicularly on a completely absorbing, that is, a quite black surface. Let m be the mass of the corpuscles in a cubic centimetre. Then the mass coming up to and entering a square centimetre of the surface in one second is that in a column V centimetres long and 1 sq. cm. cross section. The total mass entering is therefore mV . As it has velocity V the momentum entering per second is mV^2 . But this momentum entering per second is the pressure P per sq. cm.

Then

$$P = mV^2.$$

The energy of translation per cubic centimetre is $\frac{mV^2}{2}$,

so that the pressure is twice the energy of translation.

If the experimenters of the eighteenth century had known of the relation between heat and mechanical energy, they would have measured the heat received per second, and expressing this in mechanical units they would have equated it to the energy of translation

$\frac{mV^2}{2} \times V$ in a length V of the beam. Dividing by

V , the known velocity of light, they would have obtained

$\frac{mV^2}{2}$ or half the pressure they were looking for.

If the corpuscles could be supposed to have spinning energy as well as translational energy, and equal to it in amount, and if further they could be supposed to give up this spinning energy on absorption, the total energy per c.c. would be mV^2 , or would be equal to the pressure.

NOTE 2, p. 29.

THE PRESSURE DUE TO WAVES ISSUING NORMALLY
FROM A SURFACE

We may obtain the value of the pressure thus—

Let V be the velocity of light, and let E be the

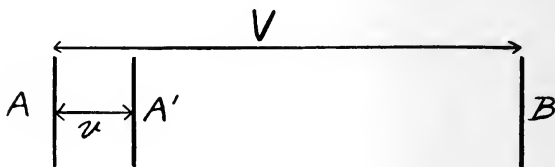


FIG. 35.

energy per cubic centimetre emitted in a beam issuing perpendicularly from a square centimetre when the source is at rest. Now let the source be moving forward with velocity v , as in fig. 35, where A represents the source at the beginning of the second, B the point reached by the waves from it at the end of the second, and A' the position of the source at the end of the second. If λ_1 is the wave-length for the source at rest, λ_2 that for the moving source, the number of waves occupying V in the first case are crushed into $V - v$ in the second case, or

$$\frac{\lambda_1}{\lambda_2} = \frac{V}{V - v}$$

We assume that the heights or amplitudes are the same. The energies in equal lengths, being inversely as the

squares of the wave-lengths, are therefore in the ratio

$$\frac{\lambda_2^2}{\lambda_1^2} = \frac{(V - v)^2}{V^2}$$

So that the energy in the length $V - v$ in the second case is

$$\frac{EV^2}{(V - v)^2} (V - v) = \frac{EV^2}{V - v}$$

The extra energy put in is therefore

$$\frac{EV^2}{V - v} - EV = \frac{EVv}{V - v}$$

This is put in by the work done by moving A forward through v against the pressure P .

Then
$$Pv = \frac{EVv}{V - v}$$

or
$$P = \frac{EV}{V - v}$$

If $v = 0$, *i.e.* if the source is at rest,

$$P_0 = E = \text{energy in length 1.}$$

If v is positive, *i.e.* if the source is moving forwards, P is greater than P_0 , and if v is negative P is less than P_0 . Neglecting squares and higher powers of v/V we have

$$P = P_0 \left(1 + \frac{v}{V} \right)$$

NOTE 3, p. 32.

THE PRESSURE OF A BEAM INCIDENT NORMALLY ON A PERFECTLY REFLECTING SURFACE

Let V be the velocity of light, and let E be the energy per cubic centimetre in the beam. Let the receiving surface be moving towards the source with velocity v . Let it be at A at the beginning of a second (fig. 36)

and at B at the end of the second. Then during that second it receives the radiation in the length, $CA = V + v$.

The radiation reflected will be crowded up into the length, BD, where $AD = V$, and BD therefore $= V - v$.

If λ_1 is the wave-length of the incident beam, and λ_2 that of the reflected beam, there are the same number of wave-lengths in CA and in BD. Then

$$\frac{\lambda_2}{\lambda_1} = \frac{V - v}{V + v}$$

The perfect reflection requires that the resultant disturbance at the reflecting surface shall always be zero.

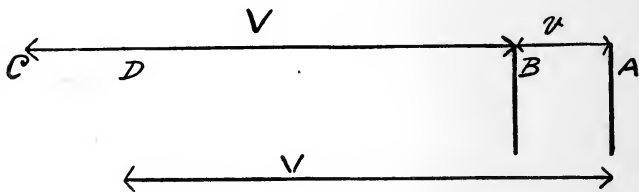


FIG. 36.

Hence the amplitudes in the incident and reflected waves must be equal and at the surface opposite. If, then, E' is the energy per cubic centimetre in the reflected train,

$$\frac{E'}{E} = \frac{\lambda_1^2}{\lambda_2^2} = \left(\frac{V + v}{V - v} \right)^2$$

The energy in length, $V + v$, of the incident train cross-section 1 is

$$E(V + v)$$

That in length $V - v$ of the reflected train is

$$E'(V - v) = \frac{E(V + v)^2}{V - v}$$

The increase in energy is

$$\frac{E(V + v)^2}{V - v} - E(V + v) = E \frac{V + v}{V - v} \cdot 2v$$

This can only be accounted for by supposing that there is a pressure P by the waves against the surface, so that work Pv is done against them in moving the surface forward, and we have

$$Pv = E \frac{V + v}{V - v} \cdot 2v$$

or
$$P = 2E \frac{V + v}{V - v}$$

If $v = 0$,

$$P = 2E$$

or the pressure is equal to the sum of the energies per cubic centimetre in the direct and reflected trains.

If the surface is moving,

$$E + E' = 2E \frac{V^2 + v^2}{(V - v)^2}$$

$$\text{and } P = \frac{V^2 - v^2}{V^2 + v^2} (E + E')$$

$$= \left(1 - \frac{2v^2}{V^2}\right) (E + E') \text{ approx.}$$

NOTE 4, p. 63.

THE LIGHT FORCE AT A REFRACTING SURFACE

In the experiment with the rectangular block of glass, the turning moment is, in all probability, due to the forces introduced at the points of total internal reflection at C and D (fig. 28).

The forces at the points of entry and emergence, B and E, can, on a very probable assumption, be shown to be

normal to the surface. They therefore pass through the axis of suspension, and have no moment round it.

Theory shows that in a vacuum the momentum per length V of a beam, where V is the velocity of light, is equal to the energy per length 1 . This is fully confirmed by experiment, which shows also that it holds in gases at quite appreciable pressures.

Let us assume that it holds, too, in a solid medium, such as glass. Then we can prove as follows that the

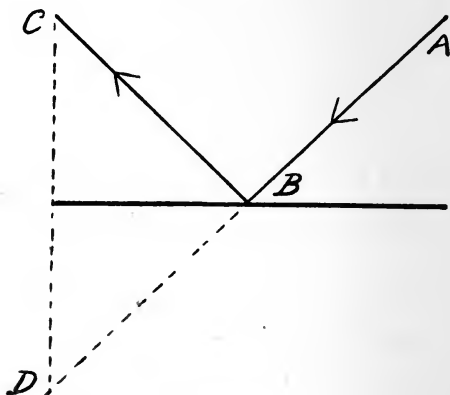


FIG. 37.

force at the point of refraction is a pull outwards on the refracting medium.

Let us consider separately the reflected and refracted parts of a beam incident obliquely on a refracting medium. Let AB (fig. 37) be reflected along BC . Let AB represent the momentum per second brought to B by the part of the incident beam which is reflected. Then BC equal to AB will represent that in the reflected beam. Produce AB to D , making BD equal to AB or BC .

Then momentum represented in magnitude and direction by DC must be added to AB to convert it to BC , though it acts through B .

Then a force, CD , acts through B on the beam, and an equal and opposite force, DC , acts on the medium at B , that is, we have a normal pressure inwards.

Now let A_1B_1 (fig. 38) represent the momentum per second brought to B_1 by the part of the incident beam which is refracted along B_1C_1 .

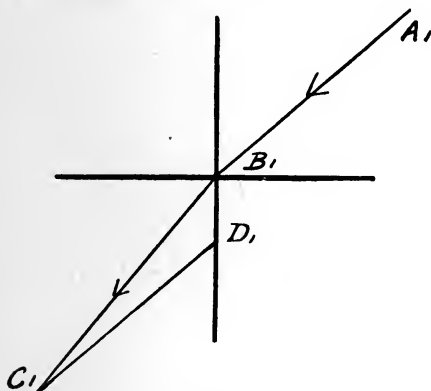


FIG. 38.

If E is the energy in length 1 of A_1B_1 , and if E' is that in length 1 of B_1C_1 , the equality of energy passing along the two beams gives

$$EV = E'V'$$

where V and V' are the velocities in the two media.

But if M and M' are the momenta per second carried by the two beams, our assumption gives

$$M = E \text{ and } M' = E'$$

Then

$$MV = M'V'$$

or since $V/V' = \mu$, the refractive index,

$$M' = \mu M$$

Draw $B_1C_1 = \mu A_1B_1$. Then B_1C_1 represents M' .

Draw C_1D_1 parallel to B_1A_1 , meeting the normal at B_1 in D_1 .

We have

$$\frac{B_1C_1}{D_1C_1} = \frac{\sin B_1D_1C_1}{\sin D_1B_1C_1} = \frac{\sin i}{\sin r} = \mu$$

Then

$$B_1C_1 = \mu D_1C_1$$

or,

$$D_1C_1 = A_1B_1.$$

The momentum $D_1C_1 = A_1B_1$ is converted into momentum B_1C_1 by the addition of momentum B_1D_1 .

Then B_1D_1 represents the force acting on the beam of light, and D_1B_1 represents the equal and opposite force acting on the refracting medium, a force outwards along the normal.

We see also that in the prism experiment, fig. 27, the torque is on this theory produced by two pulls outwards on the glass at D and E.

NOTE 5, p. 71.

THE PRESSURE OF SUNLIGHT AGAINST AN ABSORBING SPHERE COMPARED WITH THE GRAVITATION PULL

The total pressure of light against an absorbing sphere radius a is $\pi a^2 S/V$, where S is the stream of solar energy per second, and V is the velocity of light, equal to 3×10^{10} cm./sec.

The value of S at the earth's distance b , is 2.5 calories per minute, or 1.75×10^6 ergs per second. Therefore the value of the total pressure at any distance r is

$$\pi a^2 \frac{1.75 \times 10^6 b^2}{3 \times 10^{10} r^2}$$

The pull of the sun on the sphere is

$$\frac{4}{3} \pi a^3 \rho \frac{GM}{r^2}$$

where ρ is its density, G is the gravitation constant, and M is the mass of the sun.

If light repulsion is n times gravitation pull,

$$\begin{aligned} \frac{1.75 \times 10^6}{3 \times 10^{10}} &= n \frac{4}{3} a \rho \cdot \frac{GM}{b^2} \\ &= n \frac{4}{3} a \rho \times 0.59 \end{aligned}$$

for $\frac{GM}{b^2}$ is the acceleration of the earth in its orbit, and is equal to 0.59 cm./sec.^2 ;

whence $a \rho = \frac{3}{4n} \cdot \frac{1}{10^4}$ nearly.

If $n = 1$ and $\rho = 5.5$ we get

$$a = \frac{13.5}{10^6} \text{ cm.}$$

If $n = 1$ and $\rho = 1$ we get

$$a = \frac{75}{10^6} \text{ cm.}$$

We see that a would be inversely proportional to n if the repulsion continued according to the same law. But, as stated in the text, diffraction comes into play when a is very small, and the ratio of light pressure to gravitation pull diminishes when a is reduced below a certain value. The maximum ratio is probably when a is about $\frac{7.5}{10^6} \text{ cm.}$ for $\rho = 1$.

NOTE 6, p. 73.

THE AMOUNT OF MATTER WHICH CAN BE PUSHED OUT
BY THE PRESSURE OF SUNLIGHT

Consider a square centimetre area perpendicular to the rays from the sun. The momentum streaming through it per second is

$$E = \frac{S}{V} \cdot \frac{b^2}{r^2}$$

where S is the solar constant in ergs per sec. per sq. cm. at the earth's distance b from the sun and r is the distance of the square centimetre from the sun.

Let the square centimetre be the origin of matter which experiences light-pressure equal to n times the gravitation pull. Let the matter be supposed to be all within a short distance behind the square centimetre. The maximum amount which can be repelled is that which absorbs all the sunlight and all its momentum. Let it be m .

The gravitation pull on m is

$$P = \frac{GMm}{r^2}$$

where G is the gravitation constant and M is the mass of the sun.

If $E = nP$

$$\frac{S}{V} \cdot \frac{b^2}{r^2} = n \frac{GM}{r^2} m$$

whence

$$m = \frac{1}{n} \cdot \frac{S}{V} \cdot \frac{1}{\frac{GM}{b^2}}$$

But $S = 1.75 \times 10^6$ ergs per sq. cm. per sec.

$V = 3 \times 10^{10}$ cm. per sec.

$$\frac{GM}{\rho^2} = 0.59 \text{ cm./sec.}^2.$$

Then $m = \frac{1}{n} \cdot \frac{1}{10^4}$ very nearly

If then $n = 1, m = \frac{1}{10^4}$

Or fully absorbed sunlight can only balance the pull on $\frac{1}{10^4}$ gramme of matter close behind each square centimetre. If the sunlight is partly reflected then something less than twice this amount of matter can have push balancing pull.

If the absorbing matter is scattered at different distances behind the square centimetre, the amount of matter which can be balanced will be increased. For suppose it at double the distance. The cross section of the cone, with vertex at the sun and the square centimetre as base, will at the double distance have four times the area, and four times the matter can thus be balanced. But we confine the investigation to the case in which the matter is not far behind the square centimetre in comparison with the distance r , the case to which the tails of comets, at any rate, roughly correspond; and the correspondence is the closer in that the density must decrease rapidly as we recede from its head. With constant acceleration outwards, half the matter is in the first quarter of the tail. We see at once that no gas can be repelled. For there is no gas known in which the absorption of a layer of mass 10^{-4} gm. per sq. cm. at all approaches completeness.

Let us suppose that the matter consists of opaque absorbing particles, and, for the sake of illustration, suppose $n = 10$, a value probably existing in some comets' tails.

Then

$$m = \frac{1}{10^5}$$

if the whole momentum is absorbed. But it is an exceedingly high estimate to suppose that $\frac{1}{100}$ of the sunlight is stopped. The mass repelled even on this high estimate of absorption cannot exceed

$$m = \frac{1}{10^7}$$

Behind a square kilometre or 10^{10} sq. cm. there cannot be more than

$$\frac{10^{10}}{10^7} = 10^3 \text{ grammes}$$

or 1 kilogramme of matter.

We thus obtain a superior limit to the amount of matter in a comet's tail on the light-pressure theory. If we suppose that the tail is 10^7 kilometres long, and that it consists of absorbing spheres of density 1 and radius 10^{-5} cm. each, and if we suppose that $n = 10$, the maximum number ν of particles behind a square centimetre is given by

$$\nu \cdot \frac{4}{3} \pi 10^{-15} = 10^{-7},$$

$$\text{or } \nu = \frac{3}{4\pi} 10^8$$

As the number of c.c. behind 1 sq. cm. in a length of 10^7 kilometres is 10^{12} , the number of particles per c.c. N is

$$N = \frac{\nu}{10^{12}} = \frac{3}{4\pi 10^4}$$

or about 25 per cubic metre.

NOTE 7, p. 79.

THE RESISTING FORCE ON A SPHERE MOVING ROUND
THE SUN AND RADIATING R PER SQ. CM. PER SECOND
AND THE REDUCTION OF ITS ORBIT ROUND THE SUN
—THE DOPPLER EMISSION EFFECT

The investigation¹ is divided into three parts: (1) The pressure on a very small area moving along its own normal; (2) the tangential stress on a very small area moving in its own plane; (3) the application of (1) and (2) to a moving sphere

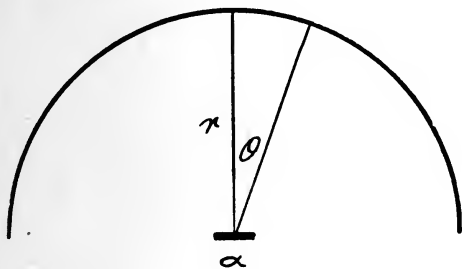


FIG. 39.

The principle of the investigation for (1) and (2) consists in finding the momentum in the space between two hemispherical surfaces containing the disturbance emitted in a short time which we may without loss of precision take as the unit time. The resultant of this momentum reversed is the pressure on the area at the time of emission. The stream of energy is normal to the hemispherical surfaces, and we may therefore apply the normal stream method of Note 2, which gives the momentum density

¹ An investigation of the Doppler emission effect was given by the author in *Phil. Trans.*, 202, 1903, p. 546, but, as pointed out in a note on p. 550, the result obtained was double the true value. The method here given appears to be more direct and satisfactory.

$M = E/V$ where E is the energy density and V is the velocity of light.

(1) Pressure on an area moving along its normal and emitting R per sq. cm. when at rest.

Let a be unit area at the centre of a large hemisphere radius r , fig. 39.

If a is at rest the energy density at the hemisphere is

$$E_0 = \frac{R \cos \theta}{\pi r^2 V}$$

for the energy flow given by this is

$$\int_0^{\pi/2} E_0 V \cdot 2\pi r^2 \sin \theta d\theta = R$$

Let the wave-length of the radiation emitted when a is at rest be λ_0 .

Now give a a velocity v along its normal. The velocity in direction θ is $v \cos \theta$, and therefore the wave-length in that direction is

$$\lambda = \lambda_0 \frac{V - v \cos \theta}{V}$$

Assuming that the amplitude is unaltered, the energy density E at the surface of the sphere is as in Note 2 :

$$\begin{aligned} E &= E_0 \frac{\lambda_0^2}{\lambda^2} = \frac{E_0 V^2}{(V - v \cos \theta)^2} \\ &= E_0 \left(1 + \frac{2v \cos \theta}{V} \right) \text{ neglecting } \frac{v^2}{V^2} \\ &= \frac{R \cos \theta}{\pi r^2 V} \left(1 + \frac{2v \cos \theta}{V} \right) \end{aligned}$$

The momentum density is E/V , or

$$M = \frac{R \cos \theta}{\pi r^2 V^2} \left(1 + \frac{2v \cos \theta}{V} \right)$$

If ABC, fig. 40, is the hemisphere of disturbance which left α_0 at a given previous instant, and if DEF is that which left α a second later, the distance between the two hemispheres in direction θ is

$$V - v \cos \theta$$

The momentum between the surfaces per unit area of one of them is

$$M (V - v \cos \theta)$$

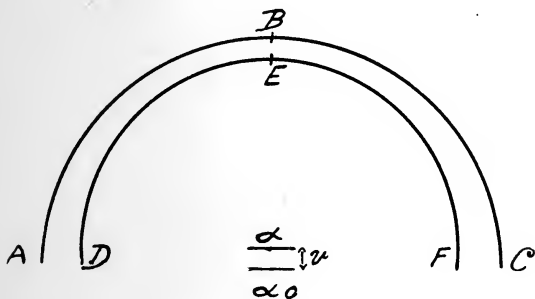


FIG. 40.

This was emitted from α in one second. Resolving along the normal and integrating for the hemisphere

$$\begin{aligned} P &= \int^{\frac{\pi}{2}} M(V - v \cos \theta) 2\pi r^2 \sin \theta \cos \theta d\theta \\ &= \int^{\frac{\pi}{2}} \frac{2R \cos^2 \theta \sin \theta}{V} \left(1 + 2 \frac{v}{V} \cos \theta\right) \left(1 - \frac{v}{V} \cos \theta\right) d\theta \\ &= \frac{2R}{3V} + \frac{1}{2} \frac{Rv}{V^2} \text{ to the first power of } \frac{v}{V} \end{aligned}$$

(2) Tangential stress on an area moving in its own plane.

Let A (fig. 41) be a point towards which α is moving with velocity v .

Let aN be the normal meeting in N the hemisphere through A with a as centre. Let P be a point on the hemisphere at θ from N and at ψ from A . Let $ANP = \phi$.

The velocity of a towards P is $v \cos \psi$, so that the wavelength is altered from λ_0 to λ where

$$\lambda = \lambda_0 \left(1 - \frac{v \cos \psi}{V} \right)$$

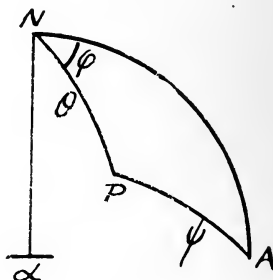


FIG. 41.

But $\cos \psi = \sin \theta \cos \phi$.

Then
$$\lambda = \lambda_0 \left(1 - v \frac{\sin \theta \cos \phi}{V} \right)$$

The energy density at P is

$$E = \frac{E_0 \lambda_0^2}{\lambda^2} = \frac{R \cos \theta}{\pi v^2 V} \left(1 + \frac{2v}{V} \sin \theta \cos \phi \right)$$

The distance between two hemispheres one second apart is $V - v \cos \psi = V - v \sin \theta \cos \phi$. The momentum between them per unit area resolved in the direction of motion is

$$\frac{E}{V} \cos \psi (V - v \sin \theta \cos \phi)$$

Substituting $\sin \theta \cos \phi$ for $\cos \psi$ and the value found for E , and integrating over the hemisphere from $\theta = 0$ to $\theta = \frac{\pi}{2}$ and from $\phi = 0$ to $\phi = 2\pi$ we get the tangential stress

$$\iint \frac{R \cos \theta}{\pi r^2 V} \sin \theta \cos \phi \left(1 + \frac{v}{V} \sin \theta \cos \phi \right) r^2 \sin \theta d\theta d\phi$$

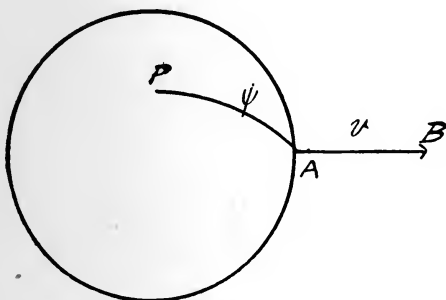
$$= \frac{Rv}{4V^2}$$


FIG. 42.

(3) Force on a sphere radius a moving with velocity v .

Let A (fig. 42) be the end of the diameter along which the sphere is moving. A point P making ψ with A has velocity $v \cos \psi$ along the normal at P and velocity $v \sin \psi$ along the tangent at P which passes through AB.

The normal velocity gives a force along BA,

$$\int_0^\pi \frac{Rv}{2V^2} \cos^2 \psi \cdot 2a^2 \sin \psi d\psi$$

for the term $\frac{2R}{3V}$ disappears on integration,

The tangential velocity gives a force along BA

$$\int_0^\pi \frac{Rv}{LV^2} \sin^2 \psi 2\pi a^2 \sin \psi d\psi$$

Adding them together and integrating, the total force along BA is

$$\frac{4}{3} \pi a^2 \frac{Rv}{V^2}$$

We can now find the diminution of the orbit of a spherical absorbing particle moving in a nearly circular orbit round the sun.

If the solar energy absorbed per second per sq. cm. is S , this is taken in through a cylinder of cross section πa^2 . It is emitted at rate R over the whole surface $4\pi a^2$.

Then $4\pi a^2 R = \pi a^2 S$

or $R = \frac{1}{4} S$

The resisting force is therefore

$$\frac{1}{3} \pi a^2 \frac{Sv}{V}$$

If the density of the sphere is ρ its mass is $\frac{4}{3} \pi a^3 \rho$, and the negative acceleration is

$$\frac{1}{4} \cdot \frac{1}{a\rho} \cdot \frac{Sv}{v^2}$$

If m is the mass of the particle, the energy taken out per second is

$$\text{Force} \times \text{velocity} = \frac{m}{4a\rho} \frac{Sv^2}{V^2}$$

Let us suppose that the orbit is so nearly circular that we may put

$$\frac{v^2}{r} = \text{acceleration to Sun} = \frac{GM}{r^2}$$

or

$$v^2 = \frac{GM}{r}$$

Then the energy taken out is $\frac{m}{4a\rho} \frac{S}{V^2} \frac{GM}{r}$ per second.

In one second let the velocity change from v_1 to v_2 and the distance from r_1 to r_2 .

The kinetic energy lost is

$$\frac{mv_1^2 - v_2^2}{2} = \frac{GMm}{2} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

really a gain, since $r_1 > r_2$.

The potential energy lost is

$$GMm \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

The total loss is therefore

$$\frac{GMm}{2} \frac{r_2 - r_1}{r_1 r_2}$$

As $r_2 - r_1$ is very small we may put $r_1 r_2 = r^2$ in the denominator and $r_2 - r_1$ the diminution of r per second is

$$-\frac{dr}{dt}$$

Equating to the loss through resistance

$$-\frac{GMm}{2r^2} \frac{dr}{dt} = \frac{GMm}{r} \cdot \frac{1}{4a\rho} \cdot \frac{S}{v^2}$$

If we put S in terms of its value S_e at the Earth's distance b

$$S = \frac{S_e b^2}{r^2}$$

or omitting the suffix and remembering that S is now our terrestrial Solar constant

$$-2r \frac{dr}{dt} = \frac{1}{a\rho} \frac{S b^2}{V^2}$$

If in this we put $a = 1$ as in the text; $\rho = 5.5$; $r = b = 14.8 \times 10^{12}$ cm. the earth's distance from the sun;

$S = 1.75 \times 10^6$; $V = 3 \times 10^{10}$ we obtain as the diminution of r in one second

$$\frac{2.61}{10^3} \text{ cm.}$$

The diminution in one year—one revolution at distance b —is obtained by multiplying by 31.5×10^6 the value of a year in seconds, and we get

$$820 \text{ metres.}$$

If we suppose that the orbit remains so nearly circular that we can use the equation

$$-\frac{2rdr}{dt} = \frac{1}{a\rho} \frac{Sb^2}{V^2}$$

let us integrate from $t = 0$ to $t = t$ and $r = b$ to $r = r_t$. Then

$$(b^2 - r_t^2) = \frac{1}{a\rho} \frac{Sb^2}{V^2} t$$

If r_t is the radius of the sun, r_t^2/b^2 may be neglected, and the time taken to reach the sun is

$$= a\rho \frac{V^2}{S}$$

Dividing by 31.5×10^6 and using the values $a = 1$, $\rho = 5.5$ as before

$$t = 90,000,000 \text{ years.}$$

The time is proportional to $a\rho$, so that for a sphere radius 0.001 cm. and of density 5.5

$$t = 90,000 \text{ years.}$$

If the density is that of water these times are reduced respectively to 16,000,000 and 16,000 years.

The methods here adopted are obviously at the best approximate, but the diminution at the distance of the Earth in a single year agrees with that found from the

exact equation given by the author in *Phil. Trans.*, loc. cit., when corrected according to the note at the end of that paper.

Mr. E. B. Wilson has investigated "The Revolution of a Dark Particle about a Luminous Centre" (*Annals of Mathematics*, Second Series, vol. viii, No. 3, April 1907) taking into account the three effects discussed in the text and integrating the equations of motion.

He finds that the fall inwards during the first revolution for a particle of density 5.5 and radius 0.01 cm. will be about 80 kilometres, agreeing with the value found above. Its change of eccentricity will be about 1 in a million, and it will in all probability make less than 10^7 revolutions.

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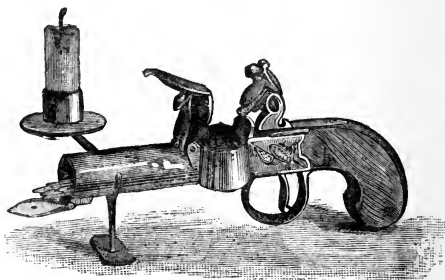
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