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PRICE STICKINESS AND ELASTIC OUTPUT AS EFFECTS OF IMPERFECT INFORMATION IN A MONETARY EQUILIBRIUM

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Abstract. An economy with individuals who produce output with labor is subjected to a one-time change in the aggregate money stock, the magnitude and expected waiting time of which are known. Individuals cannot observe the aggregate quantity of money however, and must therefore infer the time of occurrence of the shock from observations of market-specific prices. Because their information is incomplete, they erroneously respond to aggregate nominal changes as if they were market-specific real changes. An aggregate real response ensues as predicted by Phelps (1969) and as previously modelled by Lucas (1972).

In contrast to Lucas's approach, I use a transactions-oriented model of money. Money growth is effected neither through proportional transfers nor through lump sum transfers. There is an intertemporal substitution effect induced by the money growth mechanism. When this effect is netted out of the equilibrium, there is residual fluctuation that can be ascribed to imperfect information.

1. INTRODUCTION

The giant axon of the squid is much larger than ordinary nerve cells. Its size enables physiologists to isolate it from the squid, immerse it in a saline solution, and insert voltage probes into it. When shocked at one end, an electrochemical impulse, the so-called action potential, propogates along the axon. Its speed of propogation, the pattern of voltages as it passes the probes, and the ion exchanges that propogate the impulse and recover in preparation for new impulses can be monitored. Such laboratory experiments capture only the axon's impulse process, not its interaction with other nerves and with the muscles of the living squid. But it is the very fact that it can be studied in isolation from the distractions of the squid's body that makes understanding possible.

What follows is a study of an abstract economy's action potential, isolated from distracting influences. The shock is a sudden change in the supply of money. The action potential is the economy's response to the shock with real output. Isolating the shock reveals two mechanisms that respond to the shock: an intertemporal substition effect and a signal extraction effect. The signal extraction effect is the same effect found by Robert E. Lucas, Jr. in his 1972 article, "Expectations and the Neutrality of Money". Replicating the effect in a single shock model makes possible the separate analysis of the intertemporal substitution effect and signal extraction effects. They are synergistic.

Recall the Phelps (1969) island paradigm that Lucas used to motivate the signal extraction effect:

> I have found it instructive to picture the economy as a group of islands between which information flows are costly: To learn the wage paid on an adjacent island, the worker must spend the day travelling to that island to sample its wage instead

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of spending the day at work. [...] Producers on each island are in pure competition in the labor market as well as in the interisland product markets. Each morning, on each island, workers "shape up" for an auction that the determines the market-clearing money wage and employment level.

[...] Now let aggregate demand fall. If the decline of derived demand for labor were understood to be general and uniform across islands, money wage rates (and with them prices) would fall so as to maintain employment and the real wage rate. [...] But suppose workers on every island believe the fall of demand is at least partially island-specific, owing to their island's individual product mix. It is natural then to postulate, with Alchian, that workers' expectations of money wage rates elsewhere (on other islands) will "adapt" less than proportionally to the unforseen fall of money wage rates. ...*Effective* labor supply thus shifts leftward at every real wage rate: real wages rise, and profit maximizing output and employment fall. [Phelps, 1969, pp. 6-7.]

Lucas's paper incorporates several features in order to model this paradigm when the decline in aggregate demand is due to a monetary contraction: it is fully market clearing, it is dynamic, individuals have imperfect asymmetric information about the economy's state with prices as their information source, and it incorporates money demand explicitly.

The model I present here is an attempt to imitate the Lucas-Phelps program— a dynamic general equilibrium asymmetric information model of monetary shocks —using a transactions-based demand for money. Such models are difficult to solve because each individual's state variables must be accounted for in the ongoing equilibrium, and the state variables include potentially complicated and heterogeneous information vectors. An equilibrium is a sequence of distribution functions, not of scalar prices. What makes a solution possible here are two radical simplifications. The first is of individuals' utility functions. Utility is linear, which decouples marginal utility from the level of consumption. It turns out that concave utility is a totally unnecesary facet of a business cycle model. The second simplification is of the nature of the monetary shock. By limiting the shock to a single occurence, and limiting the uncertainty about the shock to its time of occurrence, the model's structural complexity can be reduced enough to analyze the response to the shock, with the bonus that comparative dynamics, using full-information benchmark equilibria, are possible.

The framework I use is a linearized version of Lucas's (1978) model of an exchange economy with a cash in advance constraint. There is a continuum of infinitely lived individuals and a continuum of markets. Each individual participates in a market in each period, both as a producer and seller, and as a buyer. Within each market there is a continuum of individuals with demand shocks as in Lucas (1978). But in addition individuals share a common taste shock within the market: this can be interpreted as the common weather (relevant in, say, a market for taxicab rides), or a common fad. Each individual lands randomly in a new market each period, and faces a new local market price. This models the fact that individuals diversify in consumption, and do not sample all prices of all goods simultaneously; rather, they sample one good and one price at a time.

In Lucas's (1972) model the supply and demand decisions were undertaken by different individuals in each period and market: the old demanded goods, and the young supplied them. In economies with infinitely-lived individuals, it is not as easy to separate the supply and demand facets of individual behavior. The model here assumes individuals are both suppliers and demanders in each period, and since they have both sides of the market, they can more sharply distinguish nominal from real shocks. A central task of the model will be to demonstrate whether the two-sidedness affects the extent and sign of the residual effect of the unobserved shock. Because the cash-in-advance transactions construct induces effectively separate timing of production and consumption decisions, the residual effect is the same.

My modelling strategy is similar to that of Grossman and Weiss (1982). They use a money in the utility function formulation: they impose an extraneous lag between labor supply decisions and the output resulting from them. The lag adds an investment aspect to labor. They also presume shocks to money demand that are exogenous. These are somewhat like the taste shocks I impose here, but it is a third shock, to money supply, where the model here departs from the Grossman-Weiss model. Both models ultimately depend on the same facet however: that there be too few prices in the model for individuals to discern from those prices the relevant aggregate nominal quantity. They argue that when information is costly, as in Grossman and Stiglitz (1980), this paucity and noninvertibility of prices will be the equilibrium. Hahm (1987) shows that this idea works in an extension of Lucas's (1972) overlapping generations model.

The central finding of the imperfect information equilibrium is that a monetary shock that is imperfectly detected raises output above its full information level, affirming Lucas's findings. The structure of the economy makes it clearer that this is because individuals are using prices as a signal or means of communication. An information externality is therefore associated with prices. The conclusion analyzes this further.

In the following section the detailed structure of the economy is presented, and the maximum problem facing individuals is stated. Individual policy functions are derived in section 3. In section 4 some assumptions are made about the equilibrium in order to characterize the value function for the individual's problem. In section 5 resource constraints are combined with these demand and supply conditions to write down the equilibrium conditions for each local market. In section 6 these conditions are united to yield an equilibrium for the full-information post-shock economy. The properties of the price function there are used to anchor the analysis of the price and output effects of the shock in the full-information economy in section 7. In section 8 the necessary conditions for the imperfect-information equilibrium are set out. In section 9, the effect of the assumptions about the information structure on the equilibrium are detailed, and in section 10 the price and output effects of the shock are derived for the imperfect information equilibrium. Section 11 concludes by integrating the findings and speculating about welfare and policy implications.

2. THE ECONOMY'S STRUCTURE

The economy evolves over discrete time. There is a continuum of markets indexed by market-specific taste shocks distributed over the real line. Within each market there is a continuum of individuals in each period. Each individual can consume purchases made with cash within the market, and each produces with labor in the same market. Within a market, individuals share a common component of a shock to their taste for consumption of that market's good in that period. They also have individual-specific taste shocks, and no individual can separately observe the individual and market-wide component of the shock.

Within each market, individuals are indexed by their individual-specific shocks and by the initial distribution of nominal money they hold. Individuals also produce output with labor according to a production function with a marginal productivity of labor that varies stochastically across individuals in a manner similar to the taste shocks. Individuals who produce output collect money income from its sale in markets distinct from the market in which they buy goods. Because of the cash-in-advance constraint individuals face, this money income cannot be spent until the ensuing period. [2.1]

In the next period, individuals are randomly assigned to new markets in their role as consumers. The individuals within each market are thus spread out over all the markets in the next period. The sampling of individuals is done in such a way that the initial distribution of nominal balances across individuals within each market is identical across markets. Viewed from the perspective of the local market, the individuals in a market have a completely heterogeneous history of price observations from the past.

The individual's problem. Each individual solves a maximum problem that results from the following recursion in each period:

$$V(M, p, \theta, \gamma, \phi, \tau, \Omega) = \max_{c, \ell, M'} \{\theta c - \gamma \ell + \beta E\left[V(M', p', \theta', \gamma', \phi', \tau', \Omega')|\Omega\right] \tag{2.1}$$

subject to the constraints

$$c + pM' \leq pM + (1-\tau)\phi\ell \tag{2.2}$$

$$0 \leq c \leq pM \tag{2.3}$$

$$0 \leq M' \tag{2.4}$$

$$0 \leq \ell \leq \bar{\ell}. \tag{2.5}$$

The notation is as follows. c is real consumption of the market good; θ is the taste shock; ℓ is labor supplied, with maximum possible labor $\bar{\ell}$; γ is the relative distaste for labor; M is nominal balances carried over from the previous period; M' is nominal balances acquired in the current period; ϕ is the current marginal productivity of labor for the individual; τ is

[2.1] As in the Lucas (1972) model, lending markets are completely absent here. Having such markets would have two effects. First, individuals with high productivity or low hunger would intertemporally substitute their labor into the future via lending (and conversely, borrow under the opposite pattern of shocks). This would intensify fluctuations of output within local markets. Second, since the use of money as a consumption-smoothing asset would diminish, price fluctuations within markets would be affected. Whether such fluctuations would be intensified or damped is not clear, nor are the informational effects on aggregate fluctuations.

the tax or subsidy on output; p is the purchasing power of money in the market where the individual buys and sells goods; β is the common discount factor; V is the value function; Ω is the information set of the individual.

The constraints have the following interpretations. First, (2.2) states that expenditure on consumption and new money balances must not exceed old balances and labor income. Second, (2.3) is the cash-in-advance constraint; consumption must be paid for in cash carried over from the previous period. Third, (2.4) prevents using money as a borrowing medium. This is the most severe restriction in the model, because in the presence of so much heterogeneity, many lending opportunities exist that are not cleared by a market. [2.2] Fourth, (2.5) restricts labor to finite bounds.

Because the utility function is linear, the solution will consist of a list of corner solutions characterized by inequalities. Individuals will work either not at all or the maximum possible, $\bar{L}$, they will consume either nothing or spend all their real balances, and they will have four possible outcomes of next-period nominal balances: zero, no change, current wages, or savings plus wages. The objective is simple, so the list of inequalities defining each case is short. Structuring the problem this way has two advantages: first, the analytic form of the value function can be stated explicitly, and second, the corner conditions partition the supports of the distributions of the stochastic shocks in a tractable way. Instead of having complicated tastes and a simple economic structure as in static general equilibrium theory, there are simple tastes and a complicated economic structure. The supply and demand functions that emerge from aggregating individual supply and demand functions have the properties usually associated with them: demand functions are smooth and downward sloping functions of prices.

The stochastic shocks are distributed as follows. First, the taste shocks have two components:

$$\theta = \delta\Delta \tag{2.6}$$

where δ is the individual-specific component of the shock and Δ is the market-specific component of the shock, with independent distributions

$$\begin{aligned} \delta &\sim F(\cdot), \qquad 0 < \underline{\delta} \le \delta \le \bar{\delta} \\ \Delta &\sim \Phi(\cdot), \qquad 0 < \Delta \le \infty. \end{aligned} \tag{2.7}$$

Productivity shocks have no market-specific components, although in principle they could be added. Thus within each market all producers have individual-specific productivity shocks, but unlike the demand shocks, the distribution of these shocks is the same for all markets. The productivity shocks are distributed as follows:

$$\phi \sim G(\cdot), \qquad 0 < \underline{\phi} \le \phi \le \bar{\phi} \tag{2.8}$$

[2.2] Nevertheless there is good reason to believe that not much is being missed by this omission. Taub (1988a, b) finds that both assets reflect the mechanics of an insurance contract in which there is private information, and that neither intrinsically dominates the other using a framework like the one here. But Levine (1986) using a setting like that of Scheinkman and Weiss (1983) finds the reverse.

The Δ-shocks play a key role. Because they are market-wide (but not economy-wide) they have the effect of shifting the market demand curve, much as a rainstorm shifts the local demand function for taxicab rides to the right. If there are no market-wide productivity shocks and no monetary shocks, as the market supply curve is traced out the resulting price movements yield unambiguous information about the state of market demand. But if there are unperceived monetary shocks this information is partly masked.

Information sets. The individuals in the benchmark economy have full information about the economy's state. In the perfect information economy, individuals are unable to observe any aggregate variables contemporaneously, and they do not know the market-specific taste shock, Δ. They can, however, observe the market-specific price to infer the market-specific shock and the aggregate state. This somewhat restrictive definition of information is a sacrifice at the altar of tractability, but has the advantage of revealing more clearly the role of prices in transmitting information. [2.3] [2.4] Thus the information set, Ω, is defined as follows:

$$\Omega' = (p'; \Omega). \tag{2.9}$$

Current information thus consists of previous observations and current observations of the relative price of goods bought. [2.5]

The money supply process. It is the usual practice in money growth models to use seignorage to finance real lump sum distributions, or to distribute nominal money supply increases directly to individuals via lump sum transfers. [2.6] If monetary contractions are the focus, neither of these strategies will work here because of the nonnegativity constraints on consumption, labor, and nominal money. With the economy comprised of stochastically heterogeneous individuals, there will always be some individuals unable to provide their requisite share of a lump-sum tax. Randomizing the tax across individuals would not help: only individuals with positive realizations of income could then be taxed. Individuals with zero income would avoid the tax, vitiating its lump sum character. There is instead a simple proportional tax on output here. An increase in the money stock will be used to

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[2.3] A more complete model would allow individuals to use other information that they implicitly use in solving their maximum problems:

$$\Omega' = (\theta', p', (1 - t')\phi'; \Omega).$$

Current information would thus consist of previous information and current observations of the individual's taste shock which has information about the market's common taste shock, the relative price of goods bought, and a statistic with information about the seignorage-financed subsidy.

[2.4] If individuals were permitted to become informed at some finite cost, some would nevertheless choose not to pay the cost but instead to use price information alone. Hahm (1987) works out the mechanics of this idea by combining the models of Lucas (1972) and Grossman and Stiglitz (1980).

[2.5] A more realistic model would have individuals producing in a market separate from the one where they purchase consumption. This would increase their information—they would observe two prices instead of one. This would complicate the information sets, but the role of prices in transmitting information would be unaffected.

[2.6] See for example Brock (1975).

purchase real output in each market and this output will be distributed by a subsidy to production, τ, sufficient to use up the seignorage. Because individuals cannot observe the subsidy directly, they will respond to positive nominal shocks—which will raise apparent relative productivity by lowering τ—through their effect on apparent productivity as well as through observations of relative prices in their shopping market.

In both the full information and imperfect information models, a money supply shock occurs at a random time with an autonomous waiting time distribution—it is a Poisson process. The probability of the shock occurring in any period, if it has not already occurred, is λ. The increase in the money supply is fixed and known to all individuals in advance—in other words, the stochastic process governing the money supply is known, as in Lucas (1972). Thus in the imperfect information equilibrium, individuals do not solve an estimation problem, only a signal extraction problem.

Individuals do not observe this one-time increase in the money supply in the imperfect information economy. Rather, they infer it from their observation of prices. If the shock were never revealed directly, one would expect the economy to discover asymptotically that the shock has occurred. Assuming that the monetary shock is fully revealed by an announcement one period after it occurs eschews this difficulty. Before the shock, and during the period of the shock, however, individuals are presumed unable to observe the money supply. This requires them to filter the information available to them in these periods, and preserves the imperfect information aspects of the model. The period of the shock will then be of central interest: in that period the money supply rises but individuals cannot perceive this directly. They must extract a signal from prices.

Endogenous distributions. There will be equilibrium distribution functions of money and information both within and across markets. Within markets, individuals will enter the market with a distribution of nominal balances, Ψ, using Lucas's (1978) notation. At the end of each period individuals will have traded goods and money, resulting in an end-of-period distribution of money that is market-dependent. Each market is indexed by by its local demand shock, Δ, and the local relative purchasing power, p, will be a function of Δ since there are no other market-specific shocks. The end-of-period distribution of nominal balances will be dependent on the local demand shock, or equivalently on the local price:

$$M' \sim \Upsilon(\cdot|\Delta).$$

The next-period beginning of period nominal balances is thus

$$\Psi(M') = \int_0^\infty \Upsilon(M'|\Delta)d\Delta.$$

Posterior distributions on the economy's state. The economy's state, n, is indexed by the monetary shock. There are three possible states, because the shock is announced one period after it occurs:

$$n = \begin{cases} -1 & \text{pre-shock} \\ 0 & \text{shock in current period} \\ 1 & \geq 1 \text{ periods since shock.} \end{cases}$$

Since individuals will not be able to observe the state directly, and hence cannot distinguish between states $n = -1$ and $n = 0$, they will form a posterior distribution of the state conditioned on their current information:

$$n \sim H(\cdot|\Omega).$$

This posterior will be updated with new information. Finally, individuals will base their current behavior on their predictions of their own future states, including the economy's aggregate state. The distribution of prices will depend on this state, and predictions of prices will depend on the prior distribution over these states.

3. SOLUTION OF THE INDIVIDUAL'S PROBLEM

Since the contemporaneous return to consumption is potentially unbounded, the function-analytic techniques used by Lucas (1978) cannot be translated intact to this setting, but the simple structure of utility makes it possible to posit an explicit form for the value function. Dynamic programming can be used to show existence and uniqueness even when there is no explicit candidate for the value function. Here, there is an explicit candidate, but more work must be done to be assured it is the value function. The techniques outlined in Stokey and Lucas (1989, pp 243-247) can then be used to affirm that it is the value function. The explicit form for this conjectured value function is affine in nominal balances:

$$V(M, \theta, p, \phi^*, \Omega) = A_0(\theta, p, \phi^*, \Omega) + A_M(\theta, p, \phi^*, \Omega)M. \tag{3.1}$$

with the notation

$$\phi^* = (1 - \tau)\phi.$$

Because of this form the contraction mapping that naturally arises out of the solution is an ordinary nonlinear difference equation.

The individual's maximum problem can be stated as a Lagrangian:

$$\max_{c,\ell,M'} \{\theta c - \gamma\ell + \beta(E(A_0) + E(A_{M'})M')$$

$$+\lambda_0(\phi^*\ell + pM - pM' - c) + \lambda_1(pM - c) + \lambda_2 M' + \lambda_3 c + \lambda_4 \ell + \lambda_5(\bar{\ell} - \ell)\}. \tag{3.2}$$

The first order conditions are:

$$(c) \qquad \theta - \lambda_0 - \lambda_1 + \lambda_3 = 0$$

$$(\ell) \qquad -\gamma + \lambda_0\phi^* + \lambda_4 - \lambda_5 = 0$$

$$(M') \qquad \beta E(A_{M'}) - p\lambda_0 + \lambda_2 = 0.$$

The corresponding policy functions are given in the following table:

	c	ℓ	M'	θ	ϕ^*
(i)	0	0	M	$\theta < \beta EA'_m/p$	$\gamma p/\beta EA'_m$
(ii)	0	$\bar{\ell}$	$M + \phi^*\bar{\ell}/p$	$<$	$\geq$
(iii)	pM	0	0	$\geq$	$<$
(iv)	pM	$\bar{\ell}$	$\phi^*\bar{\ell}/p$	$\geq$	$\geq$

The states correspond to regions on the plane, illustrated in Figure 1. In state (i) individuals are not hungry for consumption and not productive, so they consume nothing and do not work. In state (ii) they are not hungry for consumption but their labor is temporarily productive, so they work, acquiring nominal income to be spent in the future. In state (iii), they are hungry but not productive, so they do not work but they spend all the money in their possession on consumption. Finally, in state (iv), they are both hungry and productive.

The dividing line between hunger and non-hunger, and that between productivity and nonproductivity, is determined by the expected future value of an endogenous variable, $\beta EA'_m$, the discounted expected future value of an endogenous variable, $\beta EA'_m$, the discounted expected marginal value of money. As the expected marginal value increases, individuals tend to decrease their consumption and to increase their labor supply. They decrease their consumption (more precisely, there are more individuals who consume nothing) because they wish to acquire real balances to spend in the future, when purchasing power is higher. They also work harder to increase their current cash income for the same reason. Thus intertemporal substitution effects are captured by this single parameter.

The response of consumption to changes in purchasing power is the inverse of the response of labor. This is a consequence of the cash in advance constraint. When current purchasing power, p, is high relative to its future value, it makes sense to spend money in anticipation of its future decline in value. But current-period nominal income cannot be spent until the following period, when purchasing power declines, because of the cash-in-advance constraint.

4. FUNCTIONAL EQUATIONS FOR THE VALUE FUNCTION

Substituting the policy functions derived in the previous section into (1.1) yields a recursive functional equation in the value function. Because of the nature of the utility function, there are four separate equations:

$$
\begin{array}{lll}
(i) & A_0^1 + A_m^1 M = & + \beta(EA'_0 + EA'_m) \\
(ii) & A_0^2 + A_m^2 M = & - \gamma\bar{\ell} + \beta(EA'_0 + EA'_m(M + \phi^*\bar{\ell}/p)) \\
(iii) & A_0^3 + A_m^3 M = \theta pM & + \beta EA'_0 \\
(iv) & A_0^4 + A_m^4 M = \theta pM & - \gamma\bar{\ell} + \beta(EA'_0 + EA'_m\phi^*\bar{\ell}/p)
\end{array}
\qquad (4.1)
$$

with the four states defined as in Table 3.1. Rather than solve this in matrix form, it is convenient to collapse these four equations into a single equation by integrating across hunger and productivity states as follows. Define

$$\hat{A}_0 \equiv \int_{\underline{\phi}}^{\tilde{\phi}} \int_{\underline{\delta}}^{\tilde{\delta}} A_0^1 dF(\delta) dG(\phi) + \int_{\tilde{\phi}}^{\bar{\phi}} \int_{\underline{\delta}}^{\tilde{\delta}} A_0^2 dF(\delta) dG(\phi)$$

$$+ \int_{\underline{\phi}}^{\tilde{\phi}} \int_{\tilde{\delta}}^{\bar{\delta}} A_0^3 dF(\delta) dG(\phi) + \int_{\tilde{\phi}}^{\bar{\phi}} \int_{\tilde{\delta}}^{\bar{\delta}} A_0^4 dF(\delta) dG(\phi) \tag{4.2}$$

and

$$\hat{A}_m \equiv \int_{\underline{\phi}}^{\tilde{\phi}} \int_{\underline{\delta}}^{\tilde{\delta}} A_m^1 dF(\delta) dG(\phi) + \int_{\tilde{\phi}}^{\bar{\phi}} \int_{\underline{\delta}}^{\tilde{\delta}} A_m^2 dF(\delta) dG(\phi)$$

$$+ \int_{\underline{\phi}}^{\tilde{\phi}} \int_{\tilde{\delta}}^{\bar{\delta}} A_m^3 dF(\delta) dG(\phi) + \int_{\tilde{\phi}}^{\bar{\phi}} \int_{\tilde{\delta}}^{\bar{\delta}} A_m^4 dF(\delta) dG(\phi) \tag{4.3}$$

where

$$\tilde{\delta} \equiv \frac{\beta E A'_m}{p\Delta}; \qquad \tilde{\phi} \equiv \frac{\gamma p}{(1-\tau)\beta E A'_m} \tag{4.4}$$

and with $\tilde{\phi}$ arising from the inequalities in Table 3.1 and from (2.10): $\phi^* \equiv (1-\tau)\phi \leq \gamma p / \beta E A'$, or $\phi \leq \gamma p/(1-\tau)\beta E A'$.

Thus the focus will be on parameters that are averages of the four combinations of individual-specific hunger and productivity shocks. Combining the equations in (4.1), write the single equation.

$$\hat{A}_0 + \hat{A}_m M = \beta E \hat{A}'_0 + \beta E \hat{A}'_m M F(\tilde{\delta}) G(\tilde{\phi}) - \gamma \bar{l} F(\tilde{\delta})(1 - G(\tilde{\phi}))$$

$$+ \beta E \hat{A}'_m M F(\tilde{\delta})(1 - G(\tilde{\phi})) + \beta E \hat{A}'_m F(\tilde{\delta}) p^{-1} (1-\tau) \bar{l} \int_{\tilde{\phi}}^{\bar{\phi}} \phi dG(\phi)$$

$$+ \Delta p M G(\tilde{\phi}) \int_{\tilde{\delta}}^{\bar{\delta}} \delta dF(\delta) + \Delta p M (1 - G(\tilde{\phi})) \int_{\tilde{\delta}}^{\bar{\delta}} \delta dF(\delta)$$

$$- \gamma \bar{l} (1 - F(\tilde{\delta}))(1 - G(\tilde{\phi})) + \beta E \hat{A}'_m (1 - F(\tilde{\delta})) p^{-1} (1-\tau) \bar{l} \int_{\tilde{\phi}}^{\bar{\phi}} \phi dG(\phi).$$

Gathering common terms and equating the coefficients of M and constant terms yields the equations

$$\hat{A}_0 = \beta E \hat{A}'_0 - \gamma \bar{l}(1 - G(\tilde{\phi})) + \beta E \hat{A}'_m p^{-1} (1-\tau) \bar{l} \int_{\tilde{\phi}}^{\bar{\phi}} \phi dG(\phi) \tag{4.5}$$

$$\hat{A}_m = \beta E \hat{A}'_m F(\tilde{\delta}) + \Delta p \int_{\tilde{\delta}}^{\bar{\delta}} \delta dF(\delta) \tag{4.6}$$

The right hand side of (4.5) is a function of both $\hat{A}'_0$ and $\hat{A}'_m$; but that the right hand side of (4.6) is a function only of $\hat{A}'_m$, the future averaged marginal value of money. Thus (4.6) can be solved separately. In order to save notation, define $A \equiv \hat{A}_m$, and rewrite (4.6) as

$$A = p\Delta\mathcal{F}(\tilde{\delta}) \tag{4.7}$$

with the shorthand definition

$$\mathcal{F}(x) \equiv xF(x) + \int_x^{\bar{\delta}} \delta dF(\delta).$$

The production side of the economy does not appear explicitly in this equation. This separation of the demand and supply sides will facilitate the solution.

To solve equation (4.7) it is necessary to be explicit about the expectation operator that is implicit on the right hand side of the equation. There are three separate equilibria in which the expectation must be calculated. The first is a stationary economy in which there are no aggregate fluctuations. The second is the full information economy in which there are two sub-states: before and during the shock. The third is the incomplete information equilibrium.

Stationary full-information economy. First, suppose there is full information in an economy that will never be shocked (or one which has already been shocked). We wish to find a way to calculate $E(A)$ to correspond to $E(A')$ in (4.7). Examining (4.7), the current period value of A depends on the stochastic parameters Δ and p. But purchasing power, p, will be a function of Δ:

$$p = \pi(\Delta, 1),$$

explicitly accounting for the fact that $n = 1$. Integrating (4.7) over the contemporaneous shock Δ—that is, integrating over markets cross-sectionally—is equivalent to taking the prior expectation of A because of the stationarity of the economy. This yields a difference equation in the average of A, denoted by $\overline{A}$:

$$\overline{A} = \int_0^{\infty} \Delta\pi(\Delta, 1)\mathcal{F}\left(\frac{\beta E\overline{A}'}{\Delta\pi(\Delta, 1)}\right)d\Phi(\Delta). \tag{4.8}$$

This will be analyzed further in section 6.

Pre-shock full-information economy. Because there is full information, individuals know whether the monetary shock has occurred in the current period or not. The expectation is conditioned on this state. The conditional probability of the next-period state of the economy, $\nu(\cdot|m)$, is straightforward:

$$\nu(n|-1) = \begin{cases} 1-\lambda, & n = -1 \\ \lambda, & n = 0 \end{cases};$$

and

$$\nu(n|m) = \begin{cases} 0, & n \neq m+1, m \geq 0, \\ 1, & n = m+1, m \geq 0. \end{cases}$$

The expectation is therefore

$$E(\overline{A}'|n = -1) = \sum_{n'=-1}^{0} \int \overline{A}'(n')d\Phi(\Delta')\nu(n'|-1).$$

with $E(\overline{A}')$ identical to the stationary full-information economy case.

Asymmetric information, state-dependent economy. If there is individual-specific information, things are not so simple. The reason is that the expectation of the future value of $\overline{A}$, $\overline{A}'$, is conditioned on current information. Finding the current value of $\overline{A}$ means integrating over the previous period's information. Thus

$$E\overline{A}' = E(\overline{A}'|\Omega).$$

Now $\overline{A}'$ is a function of the next-period information set, Ω', which is in turn a function of Δ', which is exogenous, and future price, which depends on both Δ' and on the state of the economy. Thus if $p = \pi(\Delta, n)$, then

$$E(\overline{A}'|\Omega) = \sum_{n=-1}^{1} \sum_{n'=-1}^{1} \int \overline{A}'(\Omega')d\Phi(\Delta')\nu(n'|n)h(n|\Omega). \tag{4.9}$$

The calculation of the conditional probability distribution, $H(\cdot|\Omega)$, is one of the central difficulties of the model. This distribution expresses individuals' subjective evaluations of the aggregate state of the economy, which they cannot observe directly. It is updated via a Bayesian mechanism. Thus

$$h(n|\Omega') = \frac{J(\Omega'\backslash\Omega|n)h(n|\Omega)}{\sum_{m=-1}^{1} J(\Omega'\backslash\Omega|m)h(m|\Omega)}$$

where $\Omega'\backslash\Omega$ denotes the increment to the information set, $\{p'\}$, and $J(\cdot|n)$ denotes the conditional likelihood of the increment to the information, $\Omega'\backslash\Omega$. This will be analyzed in detail in section 9.

The next step is to analyze the equilibrium properties of the stationary post-shock economy and use these as a foundation on which to recursively analyze the effects of the shock on both the full-information and incomplete-information economies.

5. LOCAL SUPPLY AND DEMAND FUNCTIONS

Within each market there is a continuum of demanders, each with the same market taste shock, Δ, but independent individual-specific shocks, δ, and each with individual-specific initial money balances, M, with distribution Ψ. The market demand function must therefore aggregate over individual demands parametrized by these individual quantities. In addition, the monetary authority adds to the market demand by spending seignorage.

This seignorage is in turn acquired from taxing all markets at the same rate and then selling the proceeds for money with the market regardless of the local shock. If a monetary shock expands the money supply—which will be assumed through the rest of the analysis—then the monetary authority uses the increase money to buy output and the real seignorage is used for a subsidy of production. Thus real demand in market Δ is:

$$q_{\Delta}^{D} = \int_{\beta E\overline{A}'/p\Delta}^{\bar{\delta}} \int_{0}^{\infty} pMd\Psi(M)dF(\delta) - \tau \int_{\gamma p/(1-\tau)\beta E\overline{A}'}^{\bar{\phi}} \phi\bar{l}dG(\phi). \tag{5.1}$$

The interpretation of this equation is as follows. Within each market, endexed by its demand shock, Δ, individuals who are hungry spend all their money balances on real consumption. Since they face a cash-in-advance constraint, they cannot spend money they are currently earning from labor on output, so this component of individual behavior can be ignored. On the other hand, the monetary authority is adding money to the economy by subsidizing production (τ negative) or removing it by taxing production and burning the money (τ positive). If for example the money supply is increasing, the monetary authority spends freshly printed money in market Δ, and uses the real quantities to subsidize production in all markets evenly (i.e. at the same rate τ). This translates into an increased demand for goods in market Δ. The demand function is now simpler:

$$q_{\Delta}^{D} = p(\Delta)(1 - F(\tilde{\delta})) \int_{0}^{\infty} Md\Psi(M) - \tau\bar{l} \int_{\tilde{\phi}}^{\bar{\phi}} \phi dG(\phi). \tag{5.2}$$

where the limits of integration have the obvious definitions from (5.1).

Supply of goods in market Δ. The supply of goods in market Δ is straightforward. It is

$$q_{\Delta}^{S} = \int_{\gamma p(\Delta)/(1-\tau)\beta E\overline{A}'}^{\bar{\phi}} \phi\bar{l}(1-\tau)dG(\phi) \tag{5.3}$$

In an economy with complete information, $E\overline{A}'$ will be independent of Δ.

Distribution function of money. The end-of-period distribution of nominal balances, $\Upsilon(\cdot|\Delta)$, is determined by the four separate combinations of taste and productivity shocks. Thus,

$$\begin{aligned} \Upsilon(M'|\Delta) &= F(\tilde{\delta})G(\tilde{\phi})\Psi(M') && (i)\\ &+F(\tilde{\delta}) \int_{\tilde{\phi}}^{\bar{\phi}} \Psi(M' - (1-\tau)\phi\bar{l}/p)dG(\phi) && (ii)\\ &+(1 - F(\tilde{\delta}))G(\tilde{\phi}) && (iii)\\ &+(1 - F(\tilde{\delta}))(G(M'p/(1-\tau)\bar{l}) - G(\tilde{\phi})). && (iv) \end{aligned} \tag{5.4}$$

where the numeration corresponds to Table 3.1. The cases are as follows. In case (i), individuals are neither hungry nor productive, so they simply keep their beginning-of-period balances and do not acquire more cash from the sale of output. In case (ii), individuals are not hungry, and so keep their beginning of period balances, and also acquire additional balances from the sale of output. Thus the integration is over holdings of real balances from the sale of output. Thus the integration is over holdings of real balances such that the sum of these two does not exceed M' : $M \leq M' - (1-\tau)\phi^\star \bar{l}/p$. In case (iii), individuals are hungry but not productive. They take in no new balances and spend all that they have. All individuals in this state will revert to zero real balances. Finally, in case (iv), individuals are hungry and productive. Therefore they spend all beginning of period balances, and acquire cash from the sale of output. This yields the pair of inequalities $M' \geq \phi^\star \bar{l}/p$, and $\phi \geq \tilde{\phi}$.

Simplifying the above expression slightly yields

$$\Upsilon(M'|\Delta) = F(\tilde{\delta})G(\tilde{\phi})\Psi(M') + F(\tilde{\delta})\int_{\tilde{\phi}}^{\overline{\phi}} \Psi(M' - (1-\tau)\phi\bar{l}/p)dG(\phi)$$

$$+(1-F(\tilde{\delta}))G(M'p/(1-\tau)\bar{l}). \tag{5.5}$$

The beginning-of-period distribution of money holdings is derived by sampling from the end-of-period distributions evenly across markets:

$$\Psi(M') = \int_0^\infty \Upsilon(M'|\Delta)d\Phi(\Delta)$$

$$= \int_0^\infty \{F(\tilde{\delta})G(\tilde{\phi})\Psi(M') + F(\tilde{\delta})\int_{\tilde{\phi}}^{\overline{\phi}} \Psi(M' - (1-\tau)\phi\bar{l}/p)dG(\phi)$$

$$+(1-F(\tilde{\delta}))G(M'p/(1-\tau)\bar{l})\}d\Phi(\Delta). \tag{5.6}$$

This equation corresponds to the recursive equation for the distribution Ψ in Lucas (1978) and Taub (1988). The mixing of individuals across markets is accomplished here. It permits the analysis to "start fresh" each period in each market. Subsequently it is not necessary to be concerned with the structure of Ψ, and it is possible to write

$$\overline{M} = \int_0^\infty M d\Psi(M).$$

In an equilibrium, the supply and demand functions must be equal within each Δ-market. Equating (5.2) and (5.3) yields

$$p(\Delta)(1-F(\tilde{\delta}))\int_0^\infty M d\Psi(M) = \int_0^\infty \int_{\gamma p/(1-\tau)\beta E\overline{A}'}^{\overline{\phi}} \phi\bar{l}dG(\phi)d\Phi(\Delta). \tag{5.7}$$

This is a functional equation in p, holding $E\overline{A}'$ fixed. Taking $E\overline{A}'$ as given, it is possible to solve for p; see Figure 2. Because p is purchasing power, the demand and supply functions have the opposite of their conventional slopes.

Money-market equilibrium. Equilibrium in the money market is achieved when end-of-period nominal balances equal the existing stock of money in the hands of individuals plus the new stock that is used for the subsidy within each market:

$$\overline{M}' = \overline{M} - \tau\bar{l}\int_{\gamma p/(1-\tau)\beta E\overline{A}'}^{\bar{\phi}} \phi\bar{l}\, dG(\phi). \tag{5.8}$$

The equation of exchange. The right hand term of (5.7), when integrated over all the Δ-markets, is the aggregate output of the economy. The term $1-F$, which is the fraction of individuals who are hungry in the market, is the local velocity. Thus the equation of exchange for this economy is

$$\int_0^\infty \overline{M}(\Delta)(1-F(\tilde{\delta}))d\Phi(\Delta) = \int_0^\infty p(\Delta)^{-1}\int_{\phi}^{\bar{\phi}} \bar{l}\phi\, dG(\phi)d\Phi(\Delta). \tag{5.9}$$

Both output and prices are averaged on the right hand side of this equation. Dividing out aggregate output yields the price level:

$$P \equiv \int_0^\infty p(\Delta)^{-1}\int_{\phi}^{\bar{\phi}} \bar{l}\phi\, dG(\phi)d\Phi(\Delta) \Big/ \int_0^\infty \int_{\tilde{\phi}}^{\bar{\phi}} \bar{l}\phi\, dG(\phi)d\Phi(\Delta).$$

This is different from the average price of output across markets. Economy-wide velocity is also an average:

$$V \equiv \int_0^\infty \overline{M}(\Delta)(1-F(\tilde{\delta}))d\Phi(\Delta) \Big/ \int_0^\infty \overline{M}(\Delta)d\Phi(\Delta)$$

6. EQUILIBRIUM IN THE STATIONARY FULL-INFORMATION ECONOMY

The next step is to prove the existence of and characterize the equilibrium in the stationary full-information — that is, post-shock — economy. The equilibrium can be found by considering just two equations in the marginal utility of money, $\overline{A}$, and the purchasing power of money function, $\pi(\cdot, 1)$:

$$\overline{A} = \int_0^\infty \Delta\pi(\Delta, 1)\mathcal{F}\left(\frac{\beta E\overline{A}'}{\Delta\pi(\Delta, 1)}\right)d\Phi(\Delta) \tag{4.8}$$

and

$$p(\Delta)(1-F(\tilde{\delta}))\int_0^\infty M d\Psi(M) = \int_0^\infty \int_{\gamma p/(1-\tau)\beta E\overline{A}}^{\bar{\phi}} \phi\bar{l}\, dG(\phi). \tag{5.7}$$

Reasonable properties from $\overline{A}$ follow from (4.8) holding $\pi(\cdot, 1)$ fixed, and conversely properties for $\pi(\cdot, 1)$ follow from (5.7) holding $\overline{A}$ fixed. This generates insight for solving both equations simultaneously.

A simplification that sidesteps many complications is the assumption that there is no money growth after the shock, and hence $\tau = 0$ in the stationary full-information economy. The goods market equilibrium equation then becomes

$$\pi(\Delta, 1)(1 - F(\beta E\overline{A}'/\Delta\pi(\Delta, 1)))\overline{M} = \int_0^{\infty} \int_{\gamma\pi(\Delta,1)/(1-\tau)\beta E\overline{A}'}^{\overline{\phi}} \phi \bar{l}\, dG(\phi). \tag{6.1}$$

The expected marginal utility of money. Consider equation (4.8) first. First of all a unique stationary marginal utility of money exists, holding constant the price function.

PROPOSITION 6.1. *Holding $\pi(\Delta, 1)$ fixed, there exists a unique solution to (4.8).*

PROOF: The proofs of all propositions are in the Appendix. ∎

Solution of the price function. Now analyze equation (6.1). Holding constant $E\overline{A}$ yields an equation in $\pi(\cdot, 1)$, and the existence of $\pi(\cdot, 1)$ as a function of Δ is immediate via standard supply-demand arguments. Formally,

PROPOSITION 6.2. *In the stationary full-information equilibrium, with F and G continuous and differentiable, holding constant $E\overline{A}$, and equilibrium price function $\pi(\Delta, 1)$ exists such that: (i) $\pi(\Delta, 1)$ is continuous; (ii) $\pi > 0$; (iii) π is differentiable and decreasing in Δ; (iv) the elasticity of price is*

$$\eta \equiv \frac{d\pi/\pi}{d\Delta/\Delta} = \frac{\overline{M}\pi\delta F'(\tilde{\delta})}{\overline{M}\pi[\delta F'(\tilde{\delta}) + 1 - F(\tilde{\delta})] + \bar{l}\tilde{\phi}^2 G'(\tilde{\phi})} \tag{6.2}$$

with

$$|\eta| < 1;$$

and (v) π is differentiable and increasing with respect to $E\overline{A}'$.

The fractional price elasticity accords with intuition: some of this increase in demand in a market should result in increased output, and some should raise the price of output.

The next step is to integrate the properties of EA' and $\pi(\Delta, 1)$ to show the existence of an equilibrium.

PROPOSITION 6.3. *There exist EA' and $\pi(\Delta, 1)$ such that (6.1) and (6.2) are satisfied, and therefore an equilibrium exists.*

Market-specific demand shocks. Suppose there is a market-wide demand shock, Δ. The effect can be represented intuitively as seen in Figure 1b. The increased demand raises the price (and equivalently lowers the purchasing power of money), and this lowers the threshold of supply. More individuals have ϕ-shocks that exceed the threshold and supply rises. The rise of Δ, by reducing the consumption threshold, increases demand for output. Because of the fractional price elasticity in equation (6.2), this effect dominates the opposing effect of the increase of price.

7. EQUILIBRIUM WITH A MONEY SHOCK UNDER FULL INFORMATION

Now consider the economy that receives a one-time increase in the quantity of money, of known magnitude but at a random time. When the shock occurs, it is fully and contemporaneously observed by all individuals in the economy. There will be three states.

n, of the economy: pre-shock ($n = -1$), during-shock ($n = 0$), and post-shock ($n = 1$). In each state, there will be three equilibrium conditions: the marginal utility of money condition, the market-clearing condition, and the money-growth finance constraint. Because the states are sequential, the early states will depend on the later states, but not vice versa. The analysis will therefore work backwards from the later states.

(i) *Marginal utility of money.* The marginal utility of money equations have the form of (4.8). The post-shock equilibrium is identical to that analyzed in section 6. The other equations are as follows.

$$(n = 0): \qquad \overline{A}(0) = \int_0^{\infty} \Delta\pi(\Delta, 0)\mathcal{F}(\tilde{\delta})d\Phi(\Delta) \tag{7.1}$$

$$\tilde{\delta} = \beta\overline{A}(1)/\Delta\pi(\Delta, 0); \tag{7.2}$$

$$(n = -1): \qquad \overline{A}(-1) = \int_0^{\infty} \Delta\pi(\Delta, -1)\mathcal{F}(\tilde{\delta})d\Phi(\Delta) \tag{7.3}$$

$$\tilde{\delta} = \beta((1-\lambda)\overline{A}(-1) + \lambda\overline{A}(0))/\Delta\pi(\Delta, -1); \tag{7.4}$$

The $n = 0$ equation reflects the fact that the next period's state is known to be state $n = 1$, and hence $E(\overline{A}') = \overline{A}(1)$, even though the current state is $n = 0$. The last equation reflects the fact that in the pre-shock state the next-period state may again be the pre-shock state, $n = -1$, with probability $1 - \lambda$, or the shock may arrive with arrival probability λ, with state $n = 0$. If the price functions $\pi(\Delta, 1)$, $\pi(\Delta, 0)$, and $\pi(\Delta, -1)$ are known, then these three equations will yield solutions for $\overline{A}(1)$, $\overline{A}(0)$, and $\overline{A}(-1)$ by solving the equations sequentially in the order 1, 0, -1, and successively substituting the results into the next equation.

(ii) *Market clearing.* Once again, there are three equations, one for each state of the economy. The during- and pre-shock market-clearing equations are as follows:

$$(n = 0): \qquad \pi(\Delta, 0)(1 - F(\tilde{\delta}))\overline{M}(0) = \overline{l}\int_{\tilde{\phi}}^{\overline{\phi}} \phi dG(\phi). \tag{7.5}$$

$$\tilde{\phi} = \gamma\pi(\Delta, 0)/(1 - \tau(0))\beta\overline{A}(1); \tag{7.6}$$

$$(n = -1): \qquad \pi(\Delta, -1)(1 - F(\tilde{\delta}))\overline{M}(-1) = \overline{l}\int_{\tilde{\phi}}^{\overline{\phi}} \phi dG(\phi). \tag{7.7}$$

$$\tilde{\phi} = \gamma\pi(\Delta, -1)/(1 - \tau(0))\beta((1-\lambda)\overline{A}(-1) + \lambda\overline{A}(0));; \tag{7.8}$$

with $\tilde{\delta}$ defined as in the marginal utility of money equations in each state.

(iii) *Money-growth finance condition.* Because the money supply change only at the time of the shock, the following conditions hold:

$$(n = -1): \qquad \overline{M}(-1)\text{given}; \tag{7.9}$$

$$(n = 0): \qquad \overline{M}(0) = \overline{M}(-1) - \tau\bar{l}\int_0^\infty \int_{\gamma\pi(\Delta,0)/(1-\tau(0))\beta\overline{A}(1)}^{\bar{\phi}} \phi dG(\phi)d\Phi(\Delta); \tag{7.10}$$

$$(n = 1): \qquad \overline{M}(1) = \overline{M}(0). \tag{7.11}$$

Purchasing power and output effects of the money shock. In period zero, under full information, individuals anticipate transiting into the stationary state, $n = 1$, in the following period. Therefore as of period zero, $E\overline{A}' = \overline{A}(1)$, a constant independent of period-zero events, and it is possible to calculate the effect of the shock on purchasing power using the equilibrium condition (5.1) alone. It is not necessary to incorporate the money-market equilibrium condition, (7.10), because at time zero, the money supply has already grown.

PROPOSITION 7.1. *A full-information equilibrium exists.*

PROPOSITION 7.2. *(i) The during-shock purchasing power is lower than the post-shock purchasing power. (ii) The during-shock output is higher than the post-shock output.*

Thus the effect of an increase in the money supply will be to increase output in the period of the monetary shock relative to its future value. This increased output has two causes. The first is the direct effect on output from the seignorage-financed subsidy, which increases output. The second effect is an intertemporal substitution effect: the decline in purchasing power anticipated as of period zero discourages production in period zero. The direct subsidy effect overwhelms the intertemporal substitution effect in the derivative

$$d\tilde{\phi}/d\tau = \frac{\tilde{\phi}}{1-\tau} + \frac{\tilde{\phi}}{\pi(\Delta,0)}\frac{d\pi}{d\tau}.$$

The first term, the direct effect of the subsidy, is positive. The second term is the price effect and is negative but smaller in magnitude than the subsidy effect.

Marginal utility of money effect. Since the price function is affected by the money shock, so will the marginal utility of money in state zero. This has no direct effect on the state-zero equilibrium but does affect the pre-shock state. From equation (7.3),

$$\frac{d\overline{A}(0)}{d\tau} = \frac{d}{d\tau}\int_0^\infty \Delta\pi(\Delta,0)\mathcal{F}(\tilde{\delta})d\Phi(\Delta)$$

$$= \int_0^\infty \Delta\{\frac{d\pi}{d\tau}\mathcal{F}(\tilde{\delta}) + \pi(\Delta,0)\mathcal{F}(\tilde{\delta})\frac{-\tilde{\delta}}{\pi}\frac{d\pi}{d\tau}\}d\Phi(\Delta)$$

$$= \int_0^\infty \Delta\frac{d\pi}{d\tau}\int_{\tilde{\delta}}^{\bar{\delta}} \delta dF(\delta)d\Phi(\Delta) < 0. \tag{7.12}$$

If there is a shock that increases the quantity of money,

$$\overline{A}(0) > \overline{A}(1),$$

that is, the marginal utility of money is higher in the period of the shock than in the post-shock equilibrium because of the anticipated decline in purchasing power. This effect will later be central to demonstrating the output effects of the shock under imperfect information.

Pre-shock price and output effects. The anticipation of the shock will affect the pre-shock economy. The effects are harder to calculate because the marginal utility of money equation and the equilibrium equation must be brought into play simultaneously.

PROPOSITION 7.3. *Relative to post-shock levels, the anticipation of a positive monetary shock (i) reduces pre-shock purchasing power and (ii) reduces pre-shock economy-wide output.*

Thus a positive monetary shock induces not only a short run increase in output in the period of the shock, but the long run post shock output level will be higher than the pre-shock output. It would be incorrect, however, to conclude that the post-shock output is unusually high: in fact is the the pre-shock output that is unnaturally low. Individuals know that the purchasing power of money has a positive chance of falling, and therefore the pre-shock marginal value of money is lower than the post-shock value. Fearing the decline in value of their nominal wages, individuals work less hard in the pre-shock state than in the post-shock state as a result, but attempt to spend down their current money balances and increase consumption in anticipation of the decline in purchasing power. The effect is illustrated in Figure 1c. Relative to the post-shock equilibrium, the low expected marginal utility of money and the relatively high purchasing power in the pre-shock equilibrium lowers the ratio $\beta\overline{A}/p$.

The permanent rise in output observed with the money shock would not be evidence of true output-stimulating effects of monetary shocks. It would be just as correct to say that the anticipation of money shocks reduces output. In observing such a full-information economy in the aggregate without identifying restrictions about individual markets and preferences, there would be the paradox that easily measured short run increases in the money supply increase output, but difficult-to-measure declines in long run output result from such an environment.

8. IMPERFECT INFORMATION WITH REVELATION AFTER ONE PERIOD

Now consider the equilibrium in which the money shock is identical to that in the full information economy, but individuals observe only the price in their market. All individuals are fully informed one period after the shock, in state $n = 1$, but before the shock ($n = -1$) and during the shock ($n = 0$) individuals are uninformed about whether they are in state -1 or state 0. The uninformed alter the subjective probability of the state from their observations of local purchasing power. Because the information is imperfect, the posterior probability of the state is a weighted average of the two possible states.

As in the full information economy, there are three sets of equations that characterize the equilibrium: the marginal utility of money conditions, the market clearing conditions, and the money growth finance conditions. In addition, the posterior probability of each state must be analyzed. The post-shock equations are identical to those of section 6. The pre- and during-shock equations are as follows (the subscript U denotes "uninformed"):

(i) *Marginal utility of money.* The marginal utility of money for individuals in the pre- and during-shock states is

$$\overline{A}_U(-1) = \int_0^\infty \Delta\pi_U(\Delta,-1)\mathcal{F}(\tilde{\delta}(-1))d\Phi(\Delta), \tag{8.1}$$

$$\overline{A}_U(0) = \int_0^\infty \Delta\pi_U(\Delta,0)\mathcal{F}(\tilde{\delta}(0))d\Phi(\Delta), \tag{8.2}$$

with

$$\tilde{\delta}(-1) = \beta E(\overline{A}'|\pi_U(\Delta,-1)/\Delta\pi_u(\Delta,-1), \tag{8.3}$$

$$\tilde{\delta}(0) = \beta E(\overline{A}'|\pi_U(\Delta,0)/\Delta\pi_u(\Delta,0), \tag{8.4}$$

and with the definitions

$$E(\overline{A}'|\pi_U(\Delta,-1)) = h(-1|\pi_U(\Delta,-1))((1-\lambda)\overline{A}(-1)+\lambda\overline{A}(0))+h(0|\pi_U(\Delta,-1))\overline{A}(1) \tag{8.5}$$

$$E(\overline{A}'|\pi_U(\Delta,0)) = h(-1|\pi_U(\Delta,0))((1-\lambda)\overline{A}(-1)+\lambda\overline{A}(0))+h(0|\pi_U(\Delta,0))\overline{A}(1) \tag{8.6}$$

where $h(n|\pi)$ is the posterior probability of state n conditional on purchasing power. Observe that the form of the expected marginal value of money is identical in both states: any difference will arise only from the different level of purchasing power influencing the subjective probability of the current state.

(ii) *Market-clearing.* The uninformed market-clearing conditions are as follows:

$$(n=-1): \qquad \pi_U(\Delta,-1)(1-F(\tilde{\delta}(-1)))\overline{M}(-1) = \overline{I}\int_{\tilde{\phi}(-1)}^{\overline{\phi}} \phi dG(\phi), \tag{8.7}$$

$$(n=0): \qquad \pi_U(\Delta,0)(1-F(\tilde{\delta}(0)))\overline{M}(0) = \overline{I}\int_{\tilde{\phi}(0)}^{\overline{\phi}} \phi dG(\phi), \tag{8.8}$$

where $\tilde{\delta}(-1)$ and $\tilde{\delta}(0)$ are defined as before and

$$\tilde{\phi}(-1) = \gamma\pi_U(\Delta,-1)/\beta E(\overline{A}'|\pi_U(\Delta,-1), \tag{8.9}$$

$$\tilde{\phi}(0) = \gamma\pi_U(\Delta,0)/(1-\tau(0))\beta E(\overline{A}'|\pi_U(\Delta,0), \tag{8.10}$$

(iii) *Money-growth finance condition.* Because the money supply changes only at the time of the shock, the following conditions hold:

$$(n=-1): \qquad \overline{M}(-1)\text{given}; \tag{8.11}$$

$$\overline{M}(0) = \overline{M}(-1) - \tau(0)\overline{I}\int_{\tilde{\phi}_U(0)}^{\overline{\phi}} \phi dG(\phi) \tag{8.12}$$

$$\overline{M}(1) = \overline{M}(0) \tag{8.13}$$

In order to analyze the equilibrium further, it is necessary to spell out the updating mechanism in more detail.

9. THE POSTERIOR DISTRIBUTION OF THE AGGREGATE STATE.

Before and during the shock, individuals will attempt to infer the state of the economy from their observations of the price of output, p. For prior probabilities of the state $h(n|\Omega_{-1})$, the updating formula for the prior is

$$h(n|\Omega) = h(n|p;\Omega) = \frac{\Pr(p|n)h(n|\Omega_{-1})}{\sum_{m=-1}^{1}\Pr(p|m)h(m|\Omega_{-1})}.$$

Because the shock is a one-time shock and there is revelation after the shock, an individual knows that he is not in the post-shock state if it has not occurred. Thus the priors are the raw likelihoods $1-\lambda$ and λ of the two states $n=-1$ and $n=0$, and the updating formula has the time-independent form

$$h(-1|p) = \frac{\Pr(p|-1)(1-\lambda)}{\Pr(p|-1)(1-\lambda)+\Pr(p|0)\lambda}.$$

and

$$h(0|p) = \frac{\Pr(p|0)\lambda}{\Pr(p|-1)(1-\lambda)+\Pr(p|0)\lambda}. \tag{9.1}$$

Consider the probability that the price is in a small interval, $(p, p+\epsilon)$. This is just the probability that Δ is in $(\Delta_1(0), \Delta_2(0))$ if $n=0$, and the probability that Δ is in $\Delta_1(-1), \Delta_2(-1))$ if $n=-1$, weighted by the probabilities of those two states. These latter probabilities are

$$\Pr\{\Delta : \Delta \in (\Delta_1(-1), \Delta_2(-1))\} = \int_{\Delta_1(-1)}^{\Delta_2(-1)} d\Phi(\Delta),$$

$$\Pr\{\Delta : \Delta \in (\Delta_1(0), \Delta_2(0))\} = \int_{\Delta_1(0)}^{\Delta_2(0)} d\Phi(\Delta).$$

These probabilities can be approximated:

$$\Pr\{\Delta : \Delta \in (\Delta_1(-1), \Delta_2(-1))\} \cong \Phi'(\Delta_1(-1))(\Delta_2(-1)-\Delta_1(-1)),$$

$$\Pr\{\Delta : \Delta \in (\Delta_1(0), \Delta_2(0))\} \cong \Phi'(\Delta_1(0))(\Delta_2(0)-\Delta_1(0)).$$

An approximation for the difference of the prices is:

$$\epsilon = (\Delta_1(0)-\Delta_2(0))\pi'(\Delta_1(0),0) = (\Delta_1(-1)-\Delta_2(-1))\pi'(\Delta_1(-1),-1). \tag{9.2}$$

Using Bayes's rule,

$$\Pr(n=-1|(p,p+\epsilon)) = \frac{\Pr((p,p+\epsilon)|n=-1)\Pr(n=-1)}{\Pr((p,p+\epsilon)|n=-1)\Pr(n=-1)+\Pr((p,p+\epsilon)|n=0)\Pr(n=0)}$$

$$= \frac{\Phi'(\Delta_1(-1))(\Delta_2(-1) - \Delta_1(-1))(1-\lambda)}{\Phi'(\Delta_1(-1))(\Delta_2(-1) - \Delta_1(-1))(1-\lambda) + \Phi'\Delta_1(0))(\Delta_2(0) - \Delta_1(0))\lambda}.$$

Substituting for the ratio $(\Delta_2(0) - \Delta_1(0))/(\Delta_2(-1) - \Delta_1(-1))$ from (9.2) and letting ϵ grow small yields the posterior probability

$$h(n = -1|p) = \frac{(1-\lambda)\Phi'(\Delta_1(-1))/\pi'(\Delta_1(-1), -1)}{(1-\lambda)\Phi'(\Delta_1(-1))/\pi'(\Delta_1(-1), -1) + \lambda\Phi'(\Delta_1(0))/\pi'(\Delta_1(0), 0)}$$

and similarly for $\Pr(n = 0|p)$.

The definition of the posterior $h(\cdot|p)$ uses derivatives of purchasing power but depends on values of Δ corresponding to observed price. To use the posterior in the definition of expected marginal utility of money (8.3) and (8.4), define the inverse function of purchasing power:

$$D(p, n) \equiv \Delta \text{ such that } \pi(\Delta, n) = p.$$

Then write

$$h(n = -1|\pi(\Delta, m)) = \frac{N(-1)}{N(-1) + N(0)}.$$

where

$$N(-1) \equiv (1-\lambda)\Phi'(D(\pi(\Delta, m), -1))/\pi'(D(\pi(\Delta, m), -1), -1);$$

$$N(0) \equiv \lambda\Phi'(D(\pi(\Delta, m), 0))/\pi'(D(\pi(\Delta, m), 0), 0),$$

and similarly for $h(0|p)$.

It is possible to calculate the derivative of the price function in terms of itself and Δ. The starting point is the elasticity analysis of the price function in (6.2), which says

$$\pi'(\Delta) = -\frac{\pi}{\Delta}\eta.$$

and no derivatives appear in the formulation of π', and the posterior is thus defined in levels of purchasing power:

$$h(-1|\pi(\Delta, m))$$

$$= \frac{\frac{(1-\lambda)\Phi'(D(\pi(\Delta,m),-1))D(\pi(\Delta,m),-1)}{\pi(\Delta,m)\eta(-1)}}{\frac{(1-\lambda)\Phi'(D(\pi(\Delta,m),-1))D(\pi(\Delta,m),-1)}{\pi(\Delta,m)\eta(-1)} + \frac{\lambda\Phi'(D(\pi(\Delta,m),0))D(\pi(\Delta,m),0)}{\pi(\Delta,m)\eta(0)}}. \tag{9.6}$$

It is apparent that the posterior is a continuous function of $\pi(\Delta, m)$ and of $E\overline{A}'$. This fact is used in the proof of existence in the next section.

10. PRICE AND OUTPUT EFFECTS UNDER IMPERFECT INFORMATION

The effect of the anticipated but unobserved money shock will now be calculated. Individuals qcquire imperfect information about two things simultaneously: the economy's aggregate state and local demand, by observing a single statistic, price. Because price does double statistical duty, it performs both less than perfectly.

PROPOSITION 10.1. *If Φ', F, F', and G' are continuous, there exist unique equilibrium $\overline{A}(-1)$ and $\overline{A}(0)$.*

PROPOSITION 10.2.

$$\pi_I(\Delta, 0) < \pi_{I^*}(\Delta, 0) < \pi_{I^*}(\Delta, -1) < \pi_I(\Delta, -1).$$

COROLLARY 10.3. *(i) The during-shock output is higher under imperfect information than under full information. (ii) Pre-shock outpout is lower under imperfect information than under full information.*

Thus, because it is mistaken for a market-specific demand shock, the monetary shock has the same output-increasing effect, but the effect is economy-wide. This is just the same mechanism proposed by Phelps (1969), and found by Lucas (1972). This is not the whole story, though. In the full information case there are other effects at work. First, before the shock, the "natural rate" of output is lower than in the post-shock state because in the pre-shock state individuals recognize the positive probability that the value of their earnings will be diluted by a money shock. Second, contemporaneous with the shock, output is lower than it would be under a pure subsidy because of the anticipated decline in purchasing power in the subsequent period.

The results can be appreciated by examining Figure 5. The squeezing together of the price functions under imperfect information is a form of *price stickiness*. The stickiness by reducing price differences, reduces the very information value of the prices. It retards recognition of the shock in the following sense:

> ... imagine that some event occurs which would, if correctly perceived by all, induce an increase in prices generally. Sooner or later, then, this adjustment will occur. Initially, however, more traders than not perceive a relative price movement, possibly permanent, in their favor. As a result, employment and investment both increase. Through time, as price information diffuses throught the economy, these traders will see they have been mistaken. In the meantime, however, the added capacity [and output] retards price increases generally, postponing recognition of the initial shock. [Lucas, 1977; bracketed term added.]

Because the uninformed will respond with more output than if they were informed due to price stickiness, output will respond more elastically, and therefore rise higher, than it would in the full information equilibrium. This means that for a given rise in the quantity of money, the subsidy rate is higher, even of the informed, and this raises output still more. Thus the intertemporal substitution effect is present here, and even interacts synergistically with the imperfect information effects. But even in the absence of the intertemporal substitution effect—even if money were superneutral—the imperfect information effect would persist. Output responds more to a monetary shock under imperfect information than under full information.

The intuition of price stickiness might be connected to the intuition of asymmetric information. In asymmetric information models such as insurance models, a central organization attempts to elicit information from heterogeneous individuals about their state. The necessity of eliciting information makes the task, such as insurance provision, imperfect. Here, the information transmission process is reversed: individuals wish to infer the aggregate state by observing a local statistic, price. But again, because price must do two things at once—clear markets and provide information—it cannot do both perfectly.

11. CONCLUSION.

There have been many simplifications here. I "cooked" the model to obtain the intuitively reasonable results. The main discovery is that when individuals use price information for consumption and production decisions, a positive monetary shock can induce them to increase outpout beyond their response were they fully informed. The differential response of consumption and production arises from the effective lag between production and consumption induced by the cash-in-advance constraint. The response arose in the model simultaneously with a Tobin-Mundell effect that is, with the economy responding positively to inflation even when fully informed, due to intertemporal substitution of labor and consumption. Viewed only in the aggregate, it would be impossible to distinguish these effects.

The Sargent and Wallace (1973) observation that anticipation of the shock has effects before the shock has been affirmed. Judging from results of Friedman (1969) and others about the optimum deflation, the increased output from the Tobin-Mundell effect will not be welcome, because it induces a pre-shock output decline. The rise in output noted during and after the shock is thus entirely misleading: it should be interpreted as a reversion to a beneficial condition from a deleterious one in which inflation was anticipated.

The deepest result here is the additional output response generated under imperfect information, when prices convey information. Because individuals become partly informed by observing prices, they garner an external benefit from the market activities of others, a benefit in the form of useful information. Incorporating this benefit would surely complicate the assesment of welfare.

Why is this information externality expressed through nominal prices? It must be that the alternative of a non-monetary system is worse in some way. Suppose the attempt were made to internalize the externality via policy. In the simple framework used here, an extreme increase in the money supply will result in all prices rising so much that all individuals in all markets become close to perfectly informed but with the deleterious side effect of decreased pre-shock output. Surely there is nothing here to suggest that expansionist monetary policy is called for: the classical solution of an externality is either a stronger definition of property rights or a tax or a subsidy. It is hard to imagine how property rights to prices could be helpful to an economy, and it is not clear what should be subsidized. The only clear policy implication seems to be to avoid the fluctuations that mask the information in prices in the first place. In other words: Let the money supply grow at k percent per year immutably.

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Proof of Proposition 6.1. Define the function $f(\delta) = \Delta\pi(\Delta, 1)$. Then write the equation as

$$\overline{A} = \int_0^\infty f(\Delta)\mathcal{F}(\beta\overline{A}/f(\Delta))d\Phi(\Delta). \tag{A.1}$$

(Observe that $\overline{A} = E\overline{A}'$ because the Δ's are not serially correlated.) Now $\overline{A}$ is a number, not a function. Assume that $f(\Delta)$ is a positive bounded continuous function. Intuitively, $\pi(\Delta, 1)$ should be a decreasing function of Δ (remember Δ is a demand shock and p is purchasing power). Thus the derivative of f is at this point ambiguous.

The following useful properties of $\mathcal{F}$ are straightforward to demonstrate:

$$\text{(i)} \qquad \mathcal{F}'(x) = F(x);$$

$$\text{(ii)} \qquad \mathcal{F}(0) = \int_{\underline{\delta}}^{\overline{\delta}} \delta dF(\delta);$$

$$\text{(iii)} \qquad \mathcal{F}(x) = x, \quad x \geq \overline{\delta}.$$

Figure 3 plots $f(\delta)\mathcal{F}(\beta\overline{A}/(\delta))$ as a function of $\overline{A}$, holding Δ fixed. If f is not too small then this graph will cross the 45 degree line at a unique point.

Next, define $\overline{f}$ as

$$\overline{f} = \int_0^\infty f(\Delta)d\Phi(\Delta) < \infty.$$

Then

$$\int_0^\infty (f(\Delta)/\overline{f})d\Phi(\Delta)$$

is a probability measure. Thus the equation for $\overline{A}$ becomes

$$\overline{A} = \overline{f}\int_0^\infty (f(\Delta)/\overline{f})\mathcal{F}(\beta\overline{A}/(\delta))d\Phi(\Delta). \tag{A.2}$$

That is, the right hand side of (A.2) is a convex combination of the graphs generated by the $f\mathcal{F}$ terms. Since all these graphs share the same properties, the convex combination will share these properties. (See Figure 4.) But therefore there is a fixed point, $\overline{x}$. ∎

Proof of Proposition 6.2. First, observe that π is clearly nonnegative:

$$\pi = \frac{1}{\overline{M}(1-F)}\overline{l}\int_{\gamma\pi(\Delta,1)/\beta E\overline{A}'}^{\overline{\omega}} \omega dG(\omega) \geq 0.$$

Next observe that $\pi \neq 0$. Suppose the contrary. The equilibrium condition would then be

$$0 = \overline{l}.$$

a contradiction. Similarly, as π approaches $\bar{\phi}\beta E\overline{A}'/\gamma$, the equilibrium conditioncannot be satisfied.

To show continuity, just note that because F is continuous by assumption, so is π. This shows (i) and (ii).

To show (iii), use the implicit function theorem to calculate the derivative, using the differentiability of F and G. After some algebra, this yields

$$\frac{d\pi}{d\Delta} = -\frac{\pi}{\Delta}\eta < 0, \tag{A.3}$$

where η is the elasticity defined in (6.2). This monotonicity implies that $\pi(\Delta, 1)$ is measurable, and therefore the product $f(\Delta) = \Delta\pi(\Delta, 1)$ is measurable (Taylor, 1966, pp. 103-105).

Property (v) follows directly from differentiating the equilibrium condition:

$$\frac{d\pi}{d(E\overline{A}')} = \frac{\overline{M}\pi\delta F'(\tilde{\delta}) + \bar{I}\tilde{\phi}^2 G'(\tilde{\phi})}{\overline{M}\pi[\delta F'(\tilde{\delta}) + 1 - F(\tilde{\delta})] + \bar{I}\tilde{\phi}^2 G'(\tilde{\phi})} \tag{A.4}$$

completing the proof. ■

Proof of Proposition 6.3. From (6.2), calculate

$$\frac{d}{d\Delta}(\Delta\Pi(\Delta, 1)) = \pi + \Delta\pi'(\Delta) = \pi(1 + \eta) > 0. \tag{A.5}$$

From Proposition 6.2 (v), an increase in $E\overline{A}'$ increases $\pi(\Delta, 1)$, and from (A.5) this increases f, which increases the right hand side of (A.1). The mapping in (A.1) can be written $\overline{A} = T[\overline{A}]$, and clearly T maps from the real line into itself. The real line is a Banach space under the absolute value norm. In order to show that T is a contraction, then by the theorem of Blackwell (1965), the following properties suffice.

(i) *Monotonicity:* $A > B$ implies $T[A] > T[B]$. Since T is a function, it suffices to show that T has a positive derivative. Recalling that $\mathcal{F}' = F$,

$$\frac{d}{dA}\int_0^\infty f(\Delta)\mathcal{F}(\beta A/f(\Delta))d\Phi(\Delta)$$

$$= \int_0^\infty \{f'(\Delta)\mathcal{F}(\beta A/f(\Delta)) + F(\beta A/f(\Delta))(1 - (Af'(\Delta)/f(\Delta))\beta\}d\Phi(\Delta)$$

where the prime denotes differentiation with respect to A. Using the definition of $\mathcal{F}$, this yields

$$\int_0^\infty \{f'(\Delta)[\tilde{\delta}F(\tilde{\delta}) + \int_{\tilde{\delta}}^{\bar{\delta}} \delta dF - \tilde{\delta}F(\tilde{\delta})] + \beta F(\beta A/f(\Delta))\}d\Phi(\Delta)$$

$$= \int_0^\infty \{f'(\Delta)\int_{\tilde{\delta}}^{\bar{\delta}} \delta dF + \beta F(\beta A/f(\Delta))\}d\Phi(\Delta)$$

From (A.5), $f'(\Delta) > 0$, and therefore T' is positive.

(ii) *Discounting*: The mapping has the discounting property if for a constant, C, there exists a bound, λ, such that $T[A+C] \leq T[A] + \beta C$. Thus,

$$\int_0^\infty f(\Delta)\mathcal{F}(\beta(A+C)/f(\Delta))d\Phi(\Delta)$$

$$\leq \int_0^\infty f(\Delta)\{\mathcal{F}(\beta A/f(\Delta)) + [\sup \mathcal{F}']\beta C/f(\Delta)]\}d\Phi(\Delta)$$

where the supremum is taken over the domain of $\mathcal{F}$, the real line. But $\mathcal{F}' = F \leq 1$, so

$$\leq \int_0^\infty f(\Delta)\{\mathcal{F}(\beta A/f(\Delta)) + \beta C/f(\Delta)\}d\Phi(\Delta)$$

$$\leq \int_0^\infty f(\Delta)\{\mathcal{F}(\beta A/f(\Delta))\}d\Phi(\Delta) + \beta C.$$

This completes the proof. ∎

Proof of Proposition 7.1: The existence of the during-shock equilibrium is immediate since $E(\overline{A}'|n=0) = \overline{A}(1)$ whose existence was proved in Proposition 6.3. However in the pre-shock state, $E(\overline{A}'|n=-1) = (1-\lambda)\overline{A}(-1) + \lambda\overline{A}(0)$. Thus $\overline{A}(0)$ is fixed, but $\overline{A}(-1)$ is endogenous. Multiplying both sides by $(1-\lambda)$ and then adding $\lambda\overline{A}(0)$ to both sides yields

$$T[(1-\lambda)\overline{A}(-1) + \lambda\overline{A}(0)]$$

$$= (1-\lambda)\int_0^\infty f(\Delta)\mathcal{F}(\beta((1-\lambda)\overline{A}(-1) + \lambda\overline{A}(0))d\Phi(\Delta) + \lambda\overline{A}(0).$$

with $f(\Delta) = \Delta\pi(\Delta, -1)$. Since the addition of a constant to the transformation does not alter the monotonicity and discounting properties, the convex combination $(1-\lambda)\overline{A}(-1) + \lambda\overline{A}(0))$ exists and hence so does $\overline{A}(1)$. This completes the proof. ∎

Proof of Proposition 7.2: It is most convenient to get at the effect by varying the tax rate, τ. When this tax increases, the money supply decreases in period zero. The effects of money growth, then, will have the opposite sign of the effect of taxes. Implicitly differentiating (7.5),

$$d\pi(\Delta, 0)/d\tau = -\frac{\pi(\Delta, 0)}{1-\tau}\frac{\bar{T}\bar{\phi}^2G'(\bar{\phi})}{\overline{M}\pi[\bar{\delta}F'(\bar{\delta}) + 1 - F(\bar{\delta})] + \bar{T}\bar{\phi}^2G'(\bar{\phi})} < 0.$$

This derivative is calculated holding the post-shock, rather than the pre-shock money supply fixed. Therefore the effect on purchasing power is relative to the post-shock stationary economy. Observing the change in purchasing power forward through time, then, an increase in the tax rate increases purchasing power over time: this corresponds to a decline in the quantity of money. Conversely, an increase in the quantity of money will produce a decline in purchasing power between state zero and state one.

The output effect is also straightforward to calculate. Aggregate output is the weighted average of each market's output:

$$y = \int_0^\infty \bar{l} \int_{\tilde{\phi}}^{\bar{\phi}} \phi dG(\phi) d\Phi(\Delta).$$

Thus

$$\frac{dy}{d\tau} = -\int_0^\infty \bar{l}\tilde{\phi}G'(\tilde{\phi})\frac{d\tilde{\phi}}{d\tau}d\Phi(\Delta).$$

We have

$$\frac{d\tilde{\phi}}{d\tau} = \frac{d(\gamma\pi(\Delta,0)/(1-\tau)\beta\overline{A}(1))}{d\tau} = \frac{\tilde{\phi}}{1-\tau} + \frac{\tilde{\phi}}{\pi(\Delta,0)}\frac{d\pi}{d\tau}$$

$$= \frac{\tilde{\phi}}{1-\tau}\left(1 - \frac{\bar{l}\tilde{\phi}^2G'(\tilde{\phi})}{\overline{M}\pi[\delta F'(\tilde{\delta}) + 1 - F(\tilde{\delta})] + \bar{l}\tilde{\phi}^2G'(\tilde{\phi})}\right) > 0.$$

Thus the output effect is

$$\frac{dy}{d\tau} = -\int_0^\infty \bar{l}\frac{\tilde{\phi}^2}{1-\tau}G'(\tilde{\phi})\frac{\overline{M}\pi[\delta F'(\tilde{\delta}) + 1 - F(\tilde{\delta})]}{\overline{M}\pi[\delta F'(\tilde{\delta}) + 1 - F(\tilde{\delta})] + \bar{l}\tilde{\phi}^2G'(\tilde{\phi})}d\Phi(\Delta)$$

$$= -\int_0^\infty \bar{l}\frac{\tilde{\phi}^2}{1-\tau}G'(\tilde{\phi})\eta d\Phi(\Delta) < 0.$$

This completes the proof. ■

Proof of Proposition 7.3: First observe that

$$dM(-1) > 0,$$

that is, holding the post-shock money supply constant, an increase in the tax rate increases the pre-shock money supply.

Next, calculate the marginal utility of money:

$$\frac{dA(-1)}{d\tau} = \int_0^\infty \{\Delta\frac{d\pi}{d\tau}\int_{\tilde{\delta}}^{\bar{\delta}} \delta dF(\delta) + \beta F(\tilde{\delta})[(1-\lambda)\frac{d\overline{A}(-1)}{d\tau} + \lambda\frac{d\overline{A}(0)}{d\tau}]\}d\Phi(\Delta).$$

Gathering terms,

$$\frac{dA(-1)}{d\tau} = \frac{\int_0^\infty \Delta\frac{d\pi(\Delta,-1)}{d\tau}\int_{\tilde{\delta}}^{\bar{\delta}}\delta dF(\delta)d\Phi(\Delta) + \beta\lambda\frac{d\overline{A}(0)}{d\tau}\int_0^\infty F(\tilde{\delta})d\phi(\Delta)}{1 - \beta(1-\lambda)\int_0^\infty F(\tilde{\delta})d\phi(\Delta)} \quad (A.6)$$

Noting that $E\overline{A}' = (1-\lambda)\overline{A}(-1) + \lambda\overline{A}(0)$, it will be useful to calculate the derivative of this quantity:

$$\frac{dE\overline{A}'}{d\tau} = (1-\lambda)\frac{d\overline{A}(-1)}{d\tau} + \lambda\frac{d\overline{A}(0)}{d\tau}$$

$$= \int_0^\infty \{(1-\lambda)\{\Delta \frac{d\pi(\Delta,-1)}{d\tau} \int_{\tilde{\delta}}^{\bar{\delta}} \delta dF(\delta) + \beta F(\tilde{\delta})[(1-\lambda)\frac{d\bar{A}(-1)}{d\tau}\}$$

$$+\lambda\{\Delta \frac{d\pi(\Delta,0)}{d\tau} \int_{\tilde{\delta}}^{\bar{\delta}} \delta dF(\delta)\}\}d\Phi(\Delta). \quad (A.7)$$

The next step is to calculate the derivative of purchasing power. Implicitly differentiating the equilibrium condition,

$$\frac{d\pi(\Delta,-1)}{d\tau} = -\frac{\pi(\Delta,-1)(1-F(\tilde{\delta}))\frac{d\bar{M}(-1)}{d\tau}}{\bar{M}(-1)[\delta F'(\tilde{\delta})+1-F(\tilde{\delta})]+\bar{T}\tilde{\phi}^2 G'(\tilde{\phi})/\pi}$$

$$+\frac{\pi\bar{M}(-1)\delta F'(\tilde{\delta})+\bar{T}\tilde{\phi}^2 G'(\tilde{\phi})}{\pi\bar{M}(-1)[\delta F'(\tilde{\delta})+1-F(\tilde{\delta})]+\bar{T}\tilde{\phi}^2 G'(\tilde{\phi})}\frac{\pi}{E\bar{A}'}\frac{dE\bar{A}'}{d\tau} \quad (A.8)$$

This equation, combined with (A.7), yields solutions for the derivatives of the marginal utility of money and purchasing power with respect to the tax. Writing (A.8) in compact notation,

$$\frac{d\pi(\Delta,-1)}{d\tau} = -\kappa_0 + \kappa_1 \frac{\pi}{E\bar{A}'}\frac{dE\bar{A}'}{d\tau}$$

where κ_0 and κ_1 are positive. Substitute this into (A.7) to solve for $dE\bar{A}'/d\tau$:

$$\frac{dE\bar{A}'}{d\tau}$$

$$= \frac{\int_0^\infty \{(1-\lambda)\Delta\frac{-\kappa_0}{1+\frac{\kappa_1}{\pi}}\int_{\tilde{\delta}(-1)}^{\bar{\delta}} \delta dF(\delta) + \lambda\Delta\frac{d\pi(\Delta,0)}{d\tau}\int_{\tilde{\delta}(0)}^{\bar{\delta}} \delta dF(\delta)\}d\Phi(\Delta).}{1-(1-\lambda)\int_0^\infty\{\beta F(\tilde{\delta})+\Delta\kappa_1\frac{\pi}{E\bar{A}'}\int_{\tilde{\delta}(-1)}^{\bar{\delta}} \delta dF(\delta)\}d\Phi(\Delta)} \quad (A.9)$$

Since $d\pi(\Delta,0)/d\tau < 0$, this expression is negative if the following inequality holds:

$$(1-\lambda)\int_0^\infty \{\beta F(\tilde{\delta}) + \Delta\kappa_1 \frac{\pi}{E\bar{A}'}\int_{\tilde{\delta}(-1)}^{\bar{\delta}} \delta dF(\delta)\}d\Phi(\Delta) < 1. \quad (A.10)$$

The left hand side can be written

$$\beta(1-\lambda)\int_0^\infty \{F(\tilde{\delta}) + \frac{\kappa_1}{\tilde{\delta}}\int_{\tilde{\delta}(-1)}^{\bar{\delta}} \delta dF(\delta)\}d\Phi(\Delta).$$

Since κ_1 is a positive fraction,

$$\leq \beta(1-\lambda)\int_0^\infty \{F(\tilde{\delta}) + \frac{1}{\tilde{\delta}}\int_{\tilde{\delta}(-1)}^{\bar{\delta}} \delta dF(\delta)\}d\Phi(\Delta)$$

$$= \beta(1-\lambda)\int_0^\infty \frac{\Delta\pi}{\beta E\overline{A}'}\{\tilde{\delta}dF(\tilde{\delta}) + \int_{\tilde{\delta}(-1)}^{\tilde{\delta}} \delta dF(\delta)\}d\Phi(\Delta)$$

$$= (1-\lambda)\frac{\overline{A}(-1)}{\beta E\overline{A}'} \leq 1.$$

Therefore $d(E\overline{A}')/d\tau < 0$, and $d\pi(\Delta, -1)d\tau < 0$. That is, the effect of a rise in the money supply will be to raise the price level over its pre-shock level, in accord with intuition.

The output effect can now be calculated. The response of the labor supply threshold is

$$\frac{d\tilde{\phi}}{d\tau} = \tilde{\phi}\left(\frac{1}{\pi}\frac{d\pi}{d\tau} - \frac{1}{E\overline{A}'}\frac{dE\overline{A}'}{d\tau}\right)$$

$$= \tilde{\phi}\left(\frac{-\kappa_0}{\pi} + \frac{\kappa_1 - 1}{E\overline{A}'}\right)$$

Since κ_1 is a fraction, and $dE\overline{A}'/d\tau$ is negative, this expression is positive. Therefore

$$\frac{dy(-1)}{d\tau} = -\int_0^\infty \overline{\gamma}\tilde{\phi}G'(\tilde{\phi})\frac{d\tilde{\phi}}{d\tau}d\Phi(\Delta) \leq 0.$$

This completes the proof. ■

The following lemma is needed in the proof of Proposition 10.1.

LEMMA . *For a fixed posterior distribution, $h(-1)$, there exist unique $\overline{A}(-1)$ and $\overline{A}(0)$ under imperfect information.*

PROOF: Define

$$T[A] = (1-\lambda)\int_0^\infty f(\Delta,-1)\mathcal{F}(\beta\frac{h(-1)A + h(0)\overline{A}(1)}{f(\Delta,-1)}d\phi(\Delta)$$

$$+\lambda\int_0^\infty f(\Delta,0)\mathcal{F}(\beta\frac{h(-1)A + h(0)\overline{A}(1)}{f(\Delta,0)}d\phi(\Delta).$$

Each of the summands has the monotonicity and discounting properties demonstrated in the proof of Proposition 6.3. Thus the convex combination, T, has these properties. Thus if $A > B$, then $T[A] > T[B]$, and $T[A+C] \leq T[A] + \beta(\max_\Delta h(-1))C \leq T[A] + \beta C$. Thus T is a contraction mapping, and a unique A exists. Then

$$\overline{A}(-1) = \int_0^\infty f(\Delta,-1)\mathcal{F}(\beta\frac{h(-1)A + h(0)\overline{A}(1)}{f(\Delta,-1)}d\phi(\Delta)$$

and

$$\overline{A}(0) = \int_0^\infty f(\Delta,0)\mathcal{F}(\beta\frac{h(-1)A + h(0)\overline{A}(1)}{f(\Delta,0)}d\phi(\Delta)$$

This completes the proof. ■

Proof of Proposition 10.1: First use the convex combination $A = (1-\lambda)\overline{A}(-1)+\lambda\overline{A}(0)$. The posterior is determined by a mapping $\nu : \mathbf{R} \rightarrow \mathcal{H}$, with elements $h \in \mathcal{H}$, the space of posterior probability functions. Inspection of the formula for $h(-1|\pi)$ shows ν to be continuous under the assumption that $\Phi'(\cdot)$, $F(\cdot)$, $F'(\cdot)$, and $G'(\cdot)$ are continuous.

Given a posterior, $h(-1|\pi)$, there is a unique A by the lemma above. Thus there is a mapping $\mu : \mathcal{H} \rightarrow \mathcal{H}$, with $h \in \mathcal{H}$. This mapping is clearly continuous if $\mathcal{H}$ is normed by the sup norm.

The composite mapping $\mu\nu : \mathbf{R} \rightarrow \mathbf{R}$ is thus continuous. It remains only to show that $\mu\nu$ maps into a compact set. Clearly $\mu\nu(A) \geq 0$. Claim: If $A \leq \overline{A}(1)$, then $\mu\nu(A) \leq \overline{A}(1)$. Thus $\mu\nu$ maps the convex set $[0, \overline{A}(1)]$ into itself. By Brower's fixed point theorem [Hutson and Pym, p. 205, Th. 8.1.1] there exists a fixed point. ■.

Proof of Proposition 10.2: (ii) Examining (8.7) and (8.8), the equations are identical for the same π, except for the tax term and the higher money supply in the $n = 0$ equation. Having shown in the full information case that the decreased purchasing power caused by the rise of the money supply outweighs the increase in purchasing power brought about by the production subsidy, (ii) follows.

(i) Observe that $E(\overline{A}'|\pi)$ is a convex combination of marginal values of money in the post shock state, and the marginal utility of money in states zero and -1. But in the during-shock state, the subsidy increases the marginal utility of money, and in the pre-shock state, the lower quantity of money raises the purchasing power of money. Thus, $[(1-\lambda)\overline{A}(-1) + \lambda\overline{A}(0)] > \overline{A}(1)$, and therefore

$$E(\overline{A}('|\pi) = h(-1|\pi)[[(1-\lambda)\overline{A}(-1) + \lambda\overline{A}(0)] + h(0|\pi)\overline{A}(1) > \overline{A}(1).$$

Therefore (i) holds. A similar argument establishes (iii). ■

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