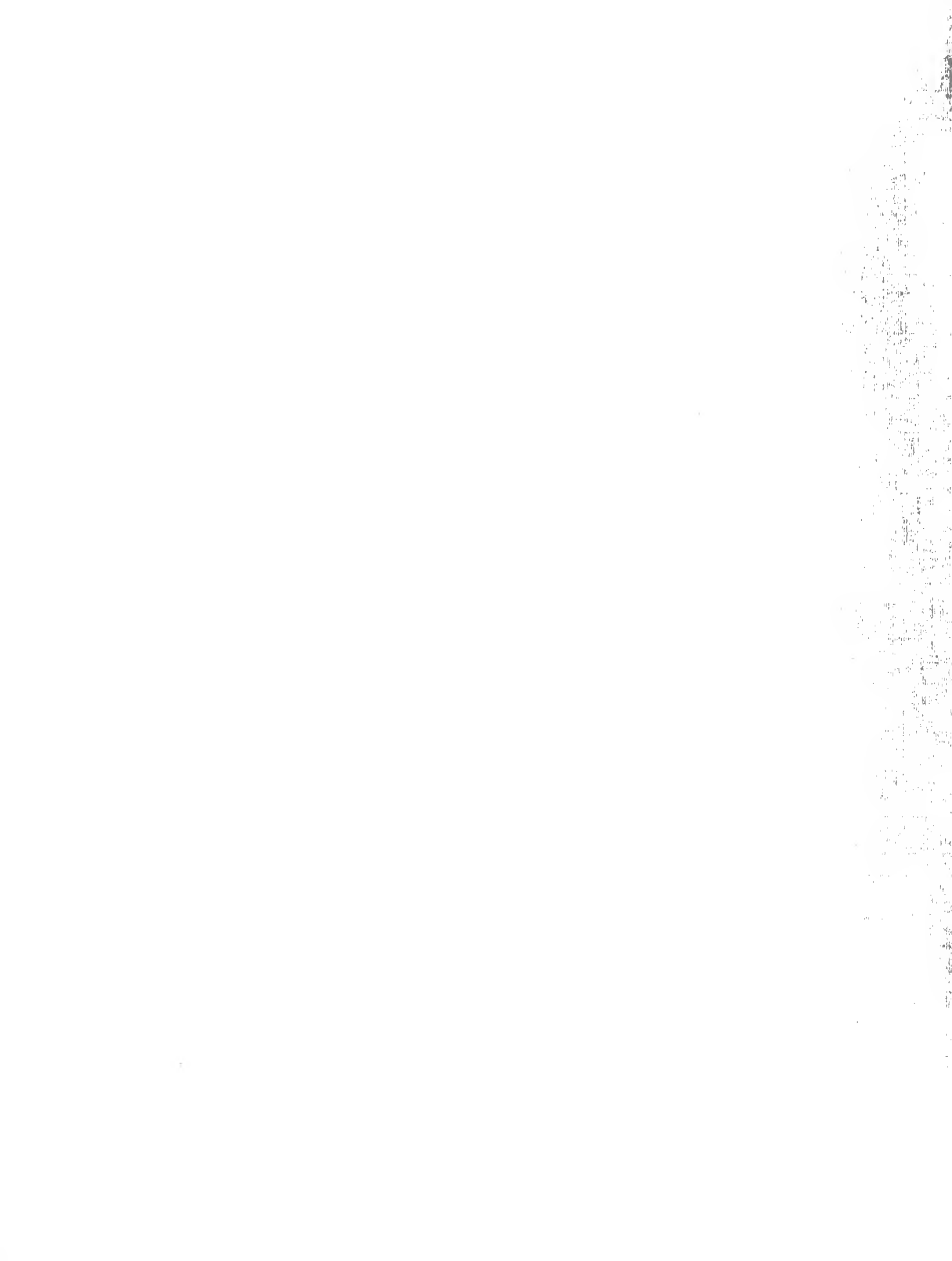




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Price Taking Behavior and Trading in Options

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Price Taking Behavior and Trading in Options

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Abstract

Equilibrium prices of options are arbitrage prices in economies in which prices are determined endogenously and all agents are price takers. We show that the price taking assumption in options' markets is unreasonable because by not being a price taker a small agent can gain much.

Black and Scholes (1973) considered the problem of option pricing in an economy with two securities. In their model the prices of these securities are given exogenously by

$$(1) \quad B(t) = e^{rt}, \quad S(t) = e^{at} + b W(t)$$

where $\{W(t); 0 \leq t \leq T\}$ is a standard Brownian Motion. In this economy it is assumed that traders can trade continuously without transaction costs.

The Black and Scholes result implies that a generalized option (contingent claim) that pays at date $t = T$, $f(S(T))$ dollars, has a unique, reasonable price. One can show that a trader can form a portfolio of the two securities, change this portfolio continuously in the interval $[0, T]$, in such a way that buying an additional amount of one security is financed by selling an amount of the other security of equal value (i.e., the trading strategy used is self financing), so that the payoff at date $t = T$ is $f(S(T))$ dollars with probability one. The value of this portfolio at $t = 0$ is called the arbitrage price of the claim at date $t = 0$.

A trader in this economy will agree to sell any number of options at prices which are a bit higher than their arbitrage price, knowing that a sure profit can be made by duplicating the cash flow of the option at date $t = T$ by trading continuously in the securities B and S . Therefore if options are traded in this economy their arbitrage prices will be their equilibrium prices.

In the Black and Scholes model it is assumed implicitly that the trading in options does not affect the prices of the securities. Since, in this model, the basic securities prices are given exogenously, this assumption is reasonable. However to study the reasonableness as well as the implications of this assumption, a model in which both option prices and security prices are determined endogenously is needed.

In this paper we examine the arbitrage method in an economy in which securities prices are the result of the agents efforts to maximize their utility. An example of this type of economy is described by Kreps (1979) or Harrison and Kreps (1979). We shall consider in this paper an economy which is a special case of the economy described by Kreps (1979).

In Kreps (1979) and Harrison and Kreps (1979) an economy in which equilibrium prices of the securities can be given by (1) was developed. In their model all agents are assumed to be price takers. This assumption implies that trading in options does not affect the prices of the securities and also that the equilibrium prices of the options are their arbitrage prices. The assumption that agents are price takers is very strong and inconsistent since it is assumed that the traders are, in all other respects, very sophisticated. Hence one would expect that if prices can be manipulated by options trading these agents will in fact manipulate them.

In this paper we consider the problem of whether, in the Harrison-Kreps economy (when a large number of options can be traded), it is reasonable to assume that agents are price takers believing that prices of the basic securities are determined by (1). In effect we study whether price taking behavior is reasonable in the sense that by not accepting the "equilibrium" security prices a small agent can gain a lot or even corner the market and thus, in effect, make a pronounced change in the "equilibrium" securities prices.

In order to study this question we assume that all agents but one believe that no matter what options they sell, the prices of the basic securities do not change. They are therefore willing to sell any option at a little higher than its arbitrage price on the belief that they can make arbitrage profits.

It will be shown that there are certain arbitrarily inexpensive options such that the one remaining agent, by buying one of these options at date $t = 0$ at its arbitrage price, (or a bit higher), can gain a lot, to the detriment of those who sold him this option, by not behaving as a price taker after date $t = 0$. If all agents anticipate this, no agent would sell options at their arbitrage prices, and all agents would want to buy certain options at their arbitrage price (or slightly higher).

The key to this argument is that a single agent, investing a small amount of funds, can reap enormous profits, if this agent does not take prices as given. Thus assuming price-taking in a "large-but-finite" economy of this sort is unwarranted. (A similar sort of result is obtained by Hart (1979). Hart shows that when short-selling is permitted in securities markets, firms that are small but not infinitesimally small can have large effects on the economy. Hence "price taking" behavior is not appropriate in his model, as well.) Since price taking is shown to be unreasonable, one wonders what is a reasonable way to define equilibrium for such economies. This question is beyond the scope of this paper and will not be discussed.

This might be an explanation for the fact that the option pricing model fails to predict prices of options that are well out of the money in real markets. The reason suggested in this paper for the occurrence of this phenomenon is that the behavior of real world traders for well out of the money options is not approximated well by the price taking assumption.

In Section 2 we describe an economy which is a special case of the economy described by Kreps. In Section 3 we construct our example. In Section 4 some concluding remarks are made.

2. The Economy

Consider an economy with one consumption good that can be consumed only at time $t = 1$, with two securities that are actually state contingent claims on the consumption good at $t = 1$.

A probability space (Ω, \mathcal{F}, P) is given, and $w \in \Omega$ represents a state of the world.

At time $t = 1$, the securities pay

$$P_1(1, w) = e^r, \quad P_2(1, w) = e^{a+b W(1, w)},$$

where r , a , and b are constants, and $W = \{W(t); 0 \leq t \leq 1\}$ is a standard Brownian motion on (Ω, \mathcal{F}, P) . Denote by \mathcal{F}_t the σ algebra generated by $\{W(u); 0 \leq u \leq t\}$ and assume that $\mathcal{F}_1 = \mathcal{F}$. Each agent is informed at time t which sets $B \in \mathcal{F}_t$ contain the true state of the world.

There are N identical agents who consume at time $t = 1$. At time $t = 0$ each agent holds $1/N$ of both securities.

Assume prices of the securities at time t , if the state is w , are given by

$$P_1(t, w) = e^{rt}, \quad P_2(t, w) = e^{at + b W(t, w)} \quad (2)$$

Assume agent i can hold at time t , $q_j^i(t, w)$ of security j , as long as he is using admissible self-financing trading strategies, as defined in Harrison and Kreps (1979).

Agent i will choose an admissible self-financing trading strategy (q_1^i, q_2^i) to maximize

$$\int U(q_1^i(1, w)e^r + q_2^i(1, w)e^{a+b W(1, w)}) dP(w),$$

over all the admissible self financing trading strategies, where U , the agent's utility function, is a strictly concave function defined on the real line. We assume further that there exists a function $g(h)$ such that $g(h) \rightarrow 0$ as $h \rightarrow 0$ and

$$h U(-h^{-1}g(h)) \rightarrow -\infty, \text{ as } h \rightarrow 0.$$

Assume also that prices (2) are equilibrium prices in the sense that

$$\sum_{i=1}^N q_j^i(t,w) = 1 \quad j = 1,2 \quad 0 \leq t \leq 1$$

Harrison and Kreps (1979) and Kreps (1979) show that if the equilibrium prices of the basic securities are given by (2), then the equilibrium prices of any options that are traded (in zero net supply) will be their arbitrage prices.

We will show that if a large number of options are traded, then the assumption that agents are price takers, believing that the prices of the basic securities are determined by (2), is unreasonable.

3. An Example

Let $C(x, P_2, t)$ be the Black-Scholes prices at time t of the European call option with striking price x (that pays $(P_2(1) - x)_+$ at $t=1$). Assume that options with striking prices x , $x+h$ and $x-h$ are traded, where x and h are further described below.

Suppose agents 2, 3, ..., N agree at time $t = 0$ to trade options with agent 1 in exchange for the appropriate "hedge portfolios" of the stock and the bond. If agent 1 pays a bit more bond for the transaction, the other agents will be happy to make this transaction because they think they will make arbitrage profits.

Agent 1 buys from all other agents $g(h)h^{-2}$ options with striking prices $x+h$ and $x-h$, and agent 1 sells $2g(h)h^{-2}$ options with striking price x . Thus agent 1 purchases the contingent claim

$$g(h)h^{-2}((P_2(1) - x+h)_+ - 2(P_2(1) - x)_+ + (P_2(1) - x - h)_+).$$

The payoff of this claim as a function of $P_2(1)$ is depicted in figure 1. The "price" of this package of options (at the Black-Scholes prices) is

$$g(h) h^{-2} (C(x+h, P_2, 0) - 2C(x, P_2, 0) + C(x-h, P_2, 0)),$$

or, approximately, $g(h) \frac{\partial^2}{\partial X^2} C(x, P_2, 0)$. This is small if h is small.

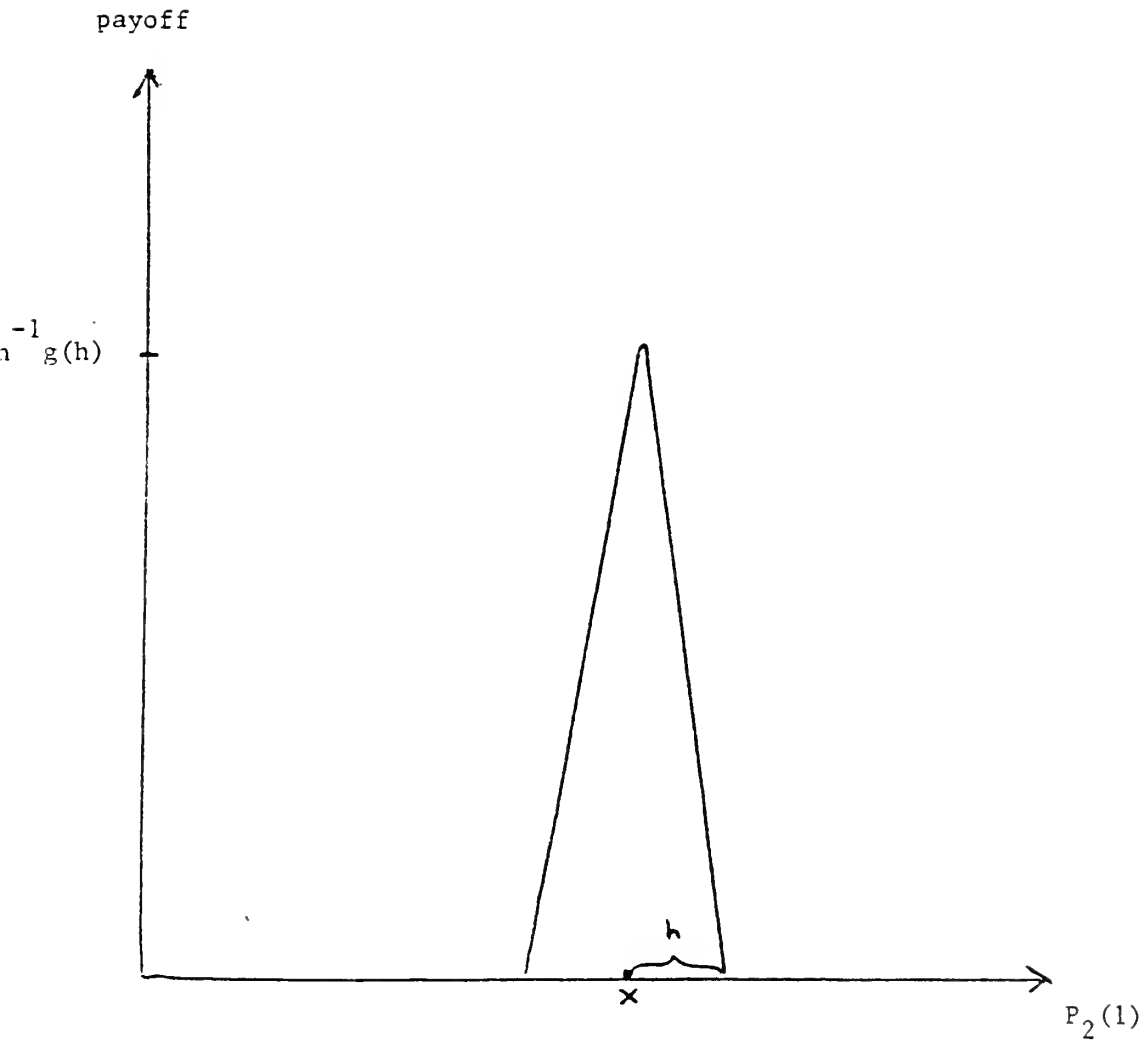


Figure 1: payoff of the contingent claim at date $t=1$

The payment to agents 2, 3, ..., N at $t = 0$ includes

$$g(h)h^{-2} \left(\frac{\partial}{\partial P_2} (C(x-h, P_2, 0) - 2C(x, P_2, 0) + C(x+h, P_2, 0)) \right)$$

of security 2. This is approximately

$$g(h) \frac{\partial^3}{\partial P_2 \partial x^2} C(x, P_2, 0) ,$$

and is also small if h is small. Thus agent 1, in exchange for the "tent" depicted in figure 1, has given up a small amount of both the stock and the bond.

As $h \rightarrow 0$, therefore, even if agent 1 does not trade after $t = 0$, the expected utility from this portfolio will be, at worst, a bit less than before the options' transaction.

Agents 2 through N expect to make money by continuously trading with agent 1, in order to carry out the Black-Scholes hedge. But suppose agent 1, after time 0, announces that he will no longer engage in such trades. If agent 1 stays out of the market after date $t = 0$, the other agents must then trade among themselves. As time passes and securities' prices change, the other agents try to buy and sell securities in order to hedge their portfolio. But since they are identical, nobody will sell. If agent 1 never trades with them again, they wind up (at time 1) with a little extra of both the stock and the bond, and they are short the "tent" depicted in figure 1. The extra stock and bond raises their utility slightly, but the "tent" lowers their expected utility by a term on the order of $h U(-g(h)h^{-1})$. That is, with probability on the order of h , they must make good on the "tent," lowering their consumption by this amount. For small enough h , our assumption on U ensures that they view this as a precarious position indeed. What will happen at this

point we cannot say precisely, because agent 1 finds himself in the position of a monopolist. There are clearly gains to trade between agent 1 and all the rest, and as agent 1 holds the key to those gains, we expect that he will extract much of the surplus. That is, agent 1 will at some time before $t = 1$ trade again, but on very favorable terms.

This example can be altered by assuming that only agent 2 agrees to sell (or buy) every option to (from) agent 1 at its arbitrage price. Agent 1 will sell the claim (with a very small h) to agent 2 and then will get out of the market. If securities prices change agent 2 would like to buy and sell securities in order to hedge his portfolio. Since agent 1 is out of the market the demand for these securities will be different than the demand in the case in which agent 1 is trading. So prices will change. Agent 2 will observe he can not hedge this portfolio using the arbitrage method. He will then be willing to buy back the claim at a loss. The other agents can gain a little by selling agent 2 some of the claims at a high price. However, they cannot sell too much because the moment they sell this claim they would be in the same trouble as agent 2. Agent 1 is the only one in the economy who can sell large quantities of this claim, and thus can gain much. This example can also be generalized to the case in which the agents are not identical and also instead of one agent of type 1 and one agent of type 2 there are several agents of both kinds.

The conclusion is that an agent who buys the above claim, with h small enough, can gain. Moreover, it is unimportant from whom he buys. Therefore, if everything else remains the same, all agents would like to buy this claim at its arbitrage price while nobody would be willing to sell at that price. We do not know what the market equilibrium will be but we do know what it will not be.

4. Conclusion

The prices (2) of the basic securities together with the arbitrage prices of the options are an equilibrium in an economy of the kind described by Kreps in which all agents are price takers. The price taking assumption is usually made in the case in which a small agent cannot gain very much by not being a price taker. This is certainly not the case here.

Although the example in Section 3 is pathological (when h is very small), it demonstrates, in an extreme case, what may happen in less pathological cases. Prices (2) of the basic securities together with the arbitrage prices of the options, may be reasonable equilibrium prices if only a small number of options are allowed to be traded. However if many options are traded it seems that the equilibrium of price takers is no longer reasonable and an equilibrium which takes into account the possibility of manipulation is needed.

In Section 3 we could have used out of the money call options instead of the option described there. There is evidence that in real options' markets the prices of options that are very well out of the money are much higher than predicted by the Black Scholes model. The sort of "market manipulation" described in this paper may provide an explanation for this phenomena.

The example of this paper is not unique to prices given by (1) but can be generalized to prices given by much more general stochastic processes. Furthermore a similar example can be constructed for a discrete time model like the one given in Cox, Ross and Rubinstein (1979).

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