THE PROBLEM OF FLIGHT

A TEXT-BOOK OF AERIAL ENGINEERING

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The Wright Aeroplane in flight.

THE PROBLEM OF FLIGHT:

A TEXT-BOOK OF AERIAL ENGINEERING.

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PUBLISHERS' NOTE TO THE SECOND EDITION.

THE First Edition of the present work having become exhausted, occasion has been taken, in issuing a Second Edition, to incorporate extensive additions and corrections made by the Author previous to his departure for the Far East.

The formulæ given in the text have been thoroughly revised and corrected, although the absence of the Author has prevented the inclusion of the full results of the latest investigations at home and abroad. It is hoped, nevertheless, that any shortcomings in this respect will be remedied, on the Author's return, in a later edition should it be called for.

July 1910.

PREFACE TO FIRST EDITION.

In view of the fact that we now appear to be on the verge of a practical solution to this classic problem of flight, I beg to submit to the engineering profession an epitome of the knowledge at present available on the subject. In the mathematical consideration I have adopted the principle, well established in engineering practice, of omitting those factors which appear to be unimportant. The formulæ are therefore "engineering formulæ" in the strict sense of the word, *i.e.* they are not the result of a deep mathematical analysis which it is, in the majority of cases, almost impossible to apply.

I sincerely hope that the rules, in conjunction with practical experiments, will prove useful, both before and after the new and more fortunate Icarus has flown from London to Manchester.

My thanks are cordially given to the individuals and firms who have supplied blocks for use in this book, viz.: Sir H. Maxim, *The Times* (*Encyclo. Brit.*), Valentine and Thompson (*Travels in Space*), Electrical Power Storage Co., Adams Manufacturing Co., De Dion Bouton Co., Ltd., Baird & Tatlock, *Model Engineer*, and Cambridge University Press.

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May 1907.

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CHAPTER I.

THE PROBLEM OF FLIGHT.

For ages men have been trying to achieve, by mechanical means, the feat (which is so easily accomplished by bird, bat, and insect) of flying. It is only within the last hundred years (unless of course we refer to Dædalus, who has omitted to leave working drawings and a recipe for mixing a good wax) that we have been able to hope that the problem would soon be solved. The chief causes which have led to this result are as follows:—

(1) A more thorough knowledge of applied mechanics, which has enabled us to cut down weights and sizes and yet to secure adequate strength.

(2) A reduction in the weight of prime movers and an improved quality of material, so that a small weight in machinery produces a very considerable power.

(3) A careful study of the flight of birds, wind pressure, and air propulsion.

All the difficulties are not yet solved, however, so that although we have now dirigible balloons and partially successful aeroplanes, there are problems still needing attention.

At the outset it must be understood that there are two classes of air-vessels:—

(A) Lighter than air.

(B) Heavier than air.

These are different in every respect, and for many years each type has had its staunch adherents, who were convinced that theirs and no other would be the final type in aerial navigation. As a matter of fact it is exceedingly probable that both forms will survive, as each possesses peculiar advantages.

There is yet another section of expert opinion which considers no motive force necessary with the aeroplane or air-plane ; but, however this may be with favourable air currents, it certainly is true that a reserve of motive force is advantageous and in many cases an absolute necessity.

The balloon, which represents class A, dirigible or not, can rise to greater heights than there seems any prospect of the heavier machine doing. It is, moreover, easily controlled by exhausting gas or throwing away ballast. Its disadvantage is that, on account of the huge surface which is necessitated by the volume of the gas-bag, and also on account of its small inertia, the whole balloon is at the mercy of every air current. The only means we have of resisting the wind is by forming the balloon into a cigar or cylindrical form in a more or less rigid frame to which is attached machinery, a propeller and rudder. By the rudder we turn the balloon into the teeth of the wind, and by the propeller we give it a momentum in the opposite direction to that of the wind, equal or more in amount to that imparted by the wind. The rigidity required in the frame, the great stress in some parts of the balloon sac, and the weight of the machinery, all tend to reduce the efficiency of the apparatus; and, although M. Santos Dumont and others have achieved much success with the dirigible balloon, it is at present very doubtful whether any balloon can be manipulated in a high wind. This problem is considered in detail later.

As regards the heavier than air machines, these derive their upward acceleration from the reaction of the "momentum" of displaced air. Air is forced downwards by moving surfaces, and the back pressure on these surfaces is the lifting force. This is, of course, the principle upon which birds and other flying animals operate ; but, as we shall see later, the imitation of bird movements involves very complex mechanism, and it may even be doubted whether a more satisfactory result may not be obtained by applying the surfaces and the motive power in a different way.

Up to the present there are three types of surface propulsion :—

(1) Aviplane or Orthoptère, which imitates the bird.

(2) Aeroplane, which is a surface driven direct against the air.

(3) Helix or Helicoptère, which displaces a cylindrical mass of air.

The problem is greatly complicated by the curious motions of air when moved, and also by the question of balancing the whole apparatus. On the earth balancing is comparatively unimportant, as, within rather wide limits, we can obtain very large supporting reactions from the earth mass; but in the air no such reactions can occur. Neither have we the advantage which occurs in ships, where the increase of immersed volume when the ship dips tends to right her. In the airship everything depends on the vessel itself.

Then, last, but by no means least, there is always the wind.

From these ideas we can proceed to enunciate the principles of aerial locomotion.

CHAPTER II.

ESSENTIAL PRINCIPLES.

IN any appliance for aerial locomotion the following principles will hold:—

(α) The total upward thrust must at least equal the weight of the supported body.

(β) If thrust is derived from flotation, the weight of the supported body must not exceed the weight of the air it displaces.

(γ) If derived from the motion of air, the vertical component of the momentum of the air as transferred to the body must at least equal the weight of the body.

(δ) To drive against the wind (*i.e.* to advance over the earth), the horizontal component of the momentum of the air as transferred to the body must at least equal the momentum produced by the horizontal action of the wind on the body. Since the mass of the body is constant, the horizontal velocity of the aeronef must at least equal the maximum horizontal velocity of the wind when allowance has been made for the form and area of the surface exposed to the wind.

(ϵ) Momentum may be derived from the air by

(*a*) Vertically acting helices.

(*b*) Oblique planes.

(*c*) Horizontal planes which open out during the upstroke.

(ζ) Flotation may be derived from (*a*) expansion of air.

,, ,, ,, (*b*) gas lighter than air.

(η) Minimum resistance in any aeronef must be in the front and at top provided there is adequate air supply for momentum. Maximum resistance at the bottom and behind.

(θ) Sum of moments on aeronef must equal zero.

(ι) Appliance must be self-righting.

These conditions I shall constantly refer to.

M. Octave Chanute's criteria of aeronautic solutions are also useful:—

(1) Resistance and support of air.
(2) The motor, its efficiency.
(3) Instrument for propulsion.
(4) Form of supporting surface.
(5) Area of supporting surface.
(6) Materials for framing.
(7) Balancing.
(8) Dirigibility.
(9) Starting under all conditions.
(10) Alighting under all conditions.

In all machines having any measure of success, the power has been applied through the medium of a screw propeller or helix. It is therefore desirable to commence by considering this appliance in detail, both as a direct means of sustentation and also as a propeller for aeroplanes. The aviplane or flapping type, although as yet unpromising, next calls for attention, and finally the dirigible balloon, where again the propeller is called into use.

CHAPTER III.

THE HELIX AS A LIFTING AND DRIVING APPLIANCE.

THE use of a helix for adapting the momentum of displaced fluid so that a thrust is produced is now an old discovery.

The principle upon which it depends is identical with that of the ordinary screw used for fixing.

If a cord be wound on a drum so that it makes a constant angle with the axis of the drum, then the curve made by it is a "helix."

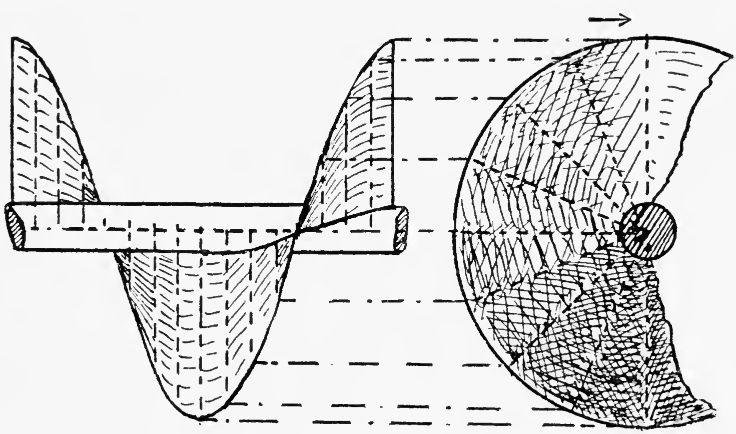


FIG. 1.—The helix.

The constant angle of the tangent to this curve as compared with a plane cutting the drum at right angles with the axis is termed the "pitch angle," the "pitch" being the distance between any two parts of the helix which occupy the same relative position in the drum (fig. 1).

If, instead of a drum, we have a small central shaft and a winding surface connecting the original helix with another of the same pitch, described on the shaft, the device is termed a "screw." If this screw be supposed to rotate in the direction shown in fig. 1, it is easy to see that the air (or any fluid in which it may be immersed) will be pushed backwards by the advance of the hinder parts of the twisted surface; or if the screw rotates in a solid, the screw itself will move forwards.

The following applications are made of this principle:—

(1) *In Solids.*—The screw or bolt for fixing or moving pieces.

(2) *In Liquids.*—The screw propeller for driving ships.

(3) *In Gases.*—The ventilating fan and the airship propeller.

In each case we have a relative motion between the surrounding mass and the helix equal to the length of the pitch per revolution, so that the velocity of displacement

$$v = pr,$$

where v is feet per second, p pitch (in feet), and r the revolutions per second.

It is usual to make the pitch equal to or rather less than the diameter of the helix.

The pitch angle is found by the following ratio:—

$$\frac{\text{pitch}}{\text{circumference of drum}} = \frac{\text{pitch}}{\pi \times \text{diameter}} = \frac{p}{\pi d} = \tan^{-1}\theta;$$

so that if the pitch equals the diameter

$$\tan\theta = \frac{1}{\pi} = \cdot 3184. \qquad \theta \therefore = 17^\circ\ 40'.$$

It should here be mentioned that the helix need not necessarily be a complete one. Several fan-shaped sectors are frequently used, the only condition being that they shall be *parts* of the helix to which they correspond. As, however, the projected area of the propeller is an important factor in the displacement, the aggregate area of these sectors in an air propeller should approach very nearly the full projected area of a complete helix, *i.e.* equal to the circle section of the drum $\left(\frac{\pi}{4}d^2\right)$, provided they are not sufficiently near together to interfere with each other's action.

If we call the projected area A, then the quantity displaced per second

$$Q = vA,$$

and the total momentum of the displaced air is $\frac{\rho Q}{g}v$, where ρ is the weight of unit volume, *i.e.* the density, and g is 32·2, the gravitational acceleration.

Summarising, we find that the thrust (the reaction of, and equal to, the momentum) is

$$T = \frac{\rho}{g} . v_0^2 A.$$

v_0 is the nett velocity (*i.e.* $v - V$) where V is the forward velocity of the airship, and v velocity of propelled air in reverse direction.

The mean value of $\frac{\rho}{g}$ is given later in formula No. (3). Comparatively few experiments have been made on this subject, but there seems to be little doubt that the thrust reaches nearly to this value. It is quite appreciable in ventilating fans of the Blackman type, and is there provided for by a special bearing.

In ships' propellers there is an appreciable, but uncertain, amount of slip over and above that due to the relative velocity of the fluid and the vessel, *i.e.* a failure of some of the fluid to be propelled through the helix, so that the effective velocity is less than $(pr - V)$. In air propellers this slip has not, to my knowledge, been measured, but it is unlikely that it would be very great at high speeds, seeing that air possesses so little inertia.

The thrust is transmitted to the framework by means of a "thrust-block," which is a bearing in which there are a number of square projecting ribs on the power shaft which fit and run in similar grooves in the block. The faces of the ribs and grooves need to be lined with special metal to obtain even and slow wear, and these "liners" should be renewable.

The framework is connected with this thrust-block in such a manner that the push from it is transmitted with fair uniformity to the vessel, which, in virtue of this force, overcomes the resistance of the air and gains acceleration.

If w is minimum weight of lifting machinery per horse-power, and c is the weight of other machinery and parts in the aeronef, then total weight

$$W = wH + c \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By condition (a) the total thrust derived from helices > W.

The excess (T − W) applied to the weight W, or rather the mass $\frac{W}{g}$, produces an upward acceleration a such that, neglecting top air resistance

$$\frac{W}{g} \cdot a = (T - W) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

According to Molesworth, the thrust T derived from a helix of projected area A and a theoretical air velocity v (= pitch × revs. per second) is as follows, when the vessel is just balanced:

$$T = kv^2A \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $k = \cdot 002288$ (varies with ρ). (Some recent experiments give a rather smaller value.)

Also, according to W. G. Walker, the horse-power required:

$$H = c_1 \frac{Q^3}{D^4} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $c_1 = \cdot 0000115$, Q = quantity, cubic feet per second, and D = diameter of helix in feet.

Also, since
$$Q = vA \quad . \quad . \quad . \quad . \quad . \quad (5)$$

(For the present we assume the motion is in still air and the vessel has little upward velocity, so that $v_0 = v$.)

$$T = kQv \text{ or } Q = \frac{T}{kv};$$

$$\therefore \quad H = c_1 \frac{T^3}{k^3 v^3 D^4},$$

and
$$T^3 = \frac{Hk^3v^3D^4}{c_1}.$$

Since $v = pr$, and taking $p = D$ (*i.e.* angle of helix $= \tan^{-1}\frac{1}{\pi}$),

$$T^3 = \frac{Hk^3r^3D^7}{c},$$

and
$$T = kr\sqrt[3]{\frac{HD^7}{c_1}} \text{ or } \frac{k}{c_1^{\frac{1}{3}}} \cdot r\sqrt[3]{HD^7} = \cdot 1014r\sqrt[3]{HD^7} \quad . \quad . \quad (6)$$

Or since, from (1)
$$H = \frac{W - c}{w},$$

$$T = \cdot 1014r\sqrt[3]{\frac{(W-c)D^7}{w}} \quad . \quad . \quad . \quad . \quad (7)$$

This formula enables us to estimate the importance of the speed, diameter and power of the helices.

It follows that the thrust

(1) Increases with the number of revolutions;

(2) Increases with the cube root of the horse-power;

(3) Increases with rather more than the square of the helix diameter;

(4) Increases with a decrease in the cube root of the weight to power ratio;

from which it would appear that the helix must be driven as fast as possible and with the maximum power. Also the helix must be as large as practicable, and the weight per horse-power has to be decreased to the maximum extent.

The last point is the rock upon which so many have broken, and it will be useful to find what must be its maximum value. Obviously this will occur when $T = W$, *i.e.* when the machine is just supported:

$$W = \cdot 1014\, r \sqrt[3]{\frac{(W-c)D^7}{w}}.$$

Let $\cdot 1014 = m$;

then $$W^3 = \frac{m^3 r^3 (W-c) D^7}{w}$$

and $$\therefore \quad w = \frac{m^3 r^3 D^7 (W-c)}{W^3} \quad . \quad . \quad . \quad . \quad (8)$$

To take a practical example: let the revolutions per second be 20 (this approaches the top limit of present obtainable speed); let $D = 4$ feet, $W = 2240$ lbs., $c = 1000$ lbs.; then we have

$$w = \frac{m^3 20^3 4^7 (2240 - 1000)}{2240^3}$$

$$\begin{array}{llrr}
\log_{10} m & = \bar{1}\cdot 0059 & \log_{10} m^3 = & \bar{3}\cdot 0177 \\
\log_{10} r & = 1\cdot 3010 & \log_{10} r^3 = & 3\cdot 9030 \\
\log_{10} D & = 0\cdot 6021 & \log_{10} D^7 = & 4\cdot 2147 \\
\log_{10}(W-c) & = 3\cdot 0934 & & 3\cdot 0934 \\
\hline
 & & & 8\cdot 2288 \\
\log_{10} W & = 3\cdot 3502 & \log_{10} W^3 = & 10\cdot 0506 \\
\hline
 & & \log_{10} w = & \bar{2}\cdot 1782
\end{array}$$

$$\text{Antilog } \bar{2}\cdot 1782 = \cdot 01508 = w.$$

From this we conclude that, with the particulars given, every pound of machinery in the prime movers must generate $\frac{1000}{15} = 66$ H.P., which is at present impossible. Actually in this case, allowing for efficiency and upward acceleration, at least 100 H.P. per lb. would be required.

This would imply that it is not yet possible to lift an aeronef by helices; but this has actually been done by Santos Dumont, Kress, Breguet, and others, with lifts varying from 5 to 80 lbs. per H.P. It will be noticed that w varies as D^7, so that we may be able to get the desired result by increasing D. Let us try. D = 10 ft.:

$$\begin{array}{ll}
\log m^3 & = \bar{3}\cdot0177 \\
\log r^3 & = 3\cdot9030 \\
\log D^7 & = 7\cdot0000 \\
\log (W - c) & = 3\cdot0934 \\
\hline
 & 11\cdot0141 \\
\log W^3 & = 10\cdot0506 \\
\hline
\log w & 0\cdot9635
\end{array}$$

$$\text{Antilog } 0\cdot9635 = 9\cdot194$$

This value for w has actually been passed, so that with D = 10 feet it would appear to be possible to use helices. H.P. in this case is 124 merely for supporting. We can now reason as to the ratio

$$\frac{\text{power}}{\text{weight of generator}} = w, \text{ when :—}$$

w Increases with cube of number of revolutions in unit time.
Increases with seventh power of diameter of helix.
Increases with nett weight of machinery.
Decreases as the cube of the weight of the car increases.

Actual values are :—

Daimler	10	lbs. per H.P.	Santos Dumont.
Mors	10	” ”	Santos Dumont.
Maxim (engine alone)	1·6	” ”	
Moy	26	” ”	(including boiler).
Antoinette petrol engine (fig. 2.)	2·2	” ”	Santos Dumont.
Wright Bros. engine	40	” ”	
Balloon machinery (electrical)	120	” ”	

I am indebted to Mr Harris Booth for the following further relation from this analysis :—

$$\frac{\text{Projected area of helix}}{\text{disc area}} = \frac{4}{\pi r}\sqrt[3]{\frac{H}{D^5}}.$$

In all these formulæ, if the vessel has an upward velocity, the value v_0 must be substituted for v where $v_0 = v - v_u$, v_u being the upward velocity.

The quantity $\frac{v - v_u}{v}$ = ratio of $\frac{\text{absolute velocity of fluid}}{\text{propeller velocity}}$, and this is termed the "slip" (usually expressed as a percentage $= \left(\frac{v - v_u}{100v}\right)$. v_u here corresponds to V, p. 8.

In all calculations where the vessel has a speed of its own, this

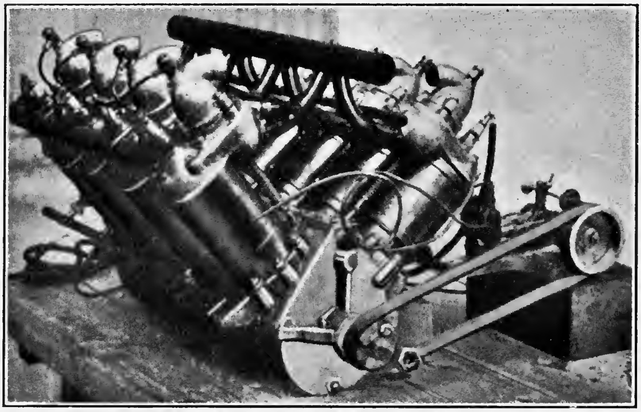


FIG. 2.—"Antoinette" 100 H.P. petrol motor (as supplied to M. Santos Dumont).

substitution must be made, the formulæ given later for the acceleration being true only when the vessel is altering its speed. So soon as it settles down to a constant speed, the air passes it at a velocity $v_u \pm v$ where v_u is the upward or forward velocity and v the actual velocity of the wind, + if against the direction of motion, and − if in the direction of motion.

It is probable that at low speeds the phenomenon of "negative" slip, previously referred to as occurring in the case of water-vessels, will appear in the propeller of the airship, the forward velocity of the ship exceeding that of the propeller. This will cause some retardation on account of the reduction of pressure at the stern. The cause of this is the eddying of the air about the propeller; but, in airships, its value will probably not be great.

In this calculation of the thrust the variable quantities are the revolutions, diameter, and power. It is assumed that the area is varied in different cases so as to agree with the expression given in the formula. Unless this is the case, energy is wasted in churning the air and overcoming friction. Mr W. Froude, in a paper on "The Elementary Relation between Pitch, Slip, and Propulsive Efficiency," has developed a simple but powerful means of computing the efficiency of marine propellers; but it depends very largely on the frictional resistance, and so cannot well be applied to air propellers. The method is briefly to consider each small part of a blade to act as a plane moving through the fluid in a direction compounded of its transverse or rotational velocity and the forward velocity of the vessel. The resistance experienced on account of displacement of the fluid is resolved into axial and transverse components, the first corresponding to the thrust, and the second, when multiplied by the radius of action, to the resisting torque. A paper by Mr Parsons to the Aeronautical Society (1908) is of importance in this connection.

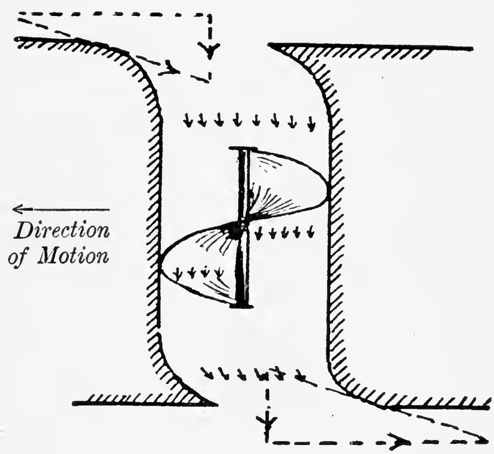


FIG. 3.—Helix shaft showing air entering and leaving with only transverse velocity.

As to the method of arranging the helices, we have to consider condition (η) as to the positions of the maximum and minimum resistances.

It may be established as a general principle that the air should enter the helix with only axial velocity and leave it similarly, *i.e.* there should be no component in the direction of horizontal motion. It would, of course, be possible to so arrange inlet and outlet that the air imparted horizontal momentum to the machine in the direction of motion; but, as this would have to be done at the expense of vertical momentum, it is certainly undesirable, as the latter is by far the more difficult to get.

It is not, moreover, desirable to have the helices themselves inclined to the direction of the horizontal motion, as the thrust acting along the axis of the helix will not then be vertical, and, unless we can so combine it with a horizontal thrust as to produce a vertical resultant, there will be an awkward turning moment to balance.

The ideal arrangement would probably be that shown in fig. 3.

Air then enters and leaves with only an axial velocity, which is employed to obtain the vertical thrust. The necessarily great diameter of the helix will probably preclude the use of this arrangement.

By reversing the entrance passages we should get an advantage in respect of the air pressure (fig. 4) thus:

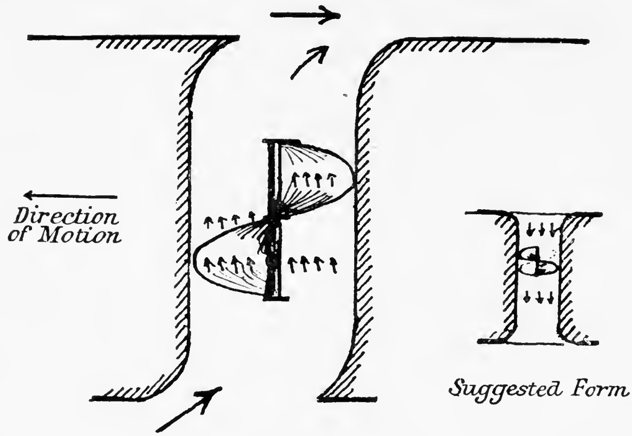


Fig. 4.—Helix shaft turned from the direction of motion, showing top vacuum and bottom pressure effect (helix stationary).

This is, however, a transitory effect, depending entirely on the horizontal motion. It would check the supply of air to the helix and retard the horizontal motion. It might be useful to be able to alter the direction of inlet and outlet during flight, so that this action could help the lifting engines; but probably the loss of supply would be more important than the increase of the bottom pressure.

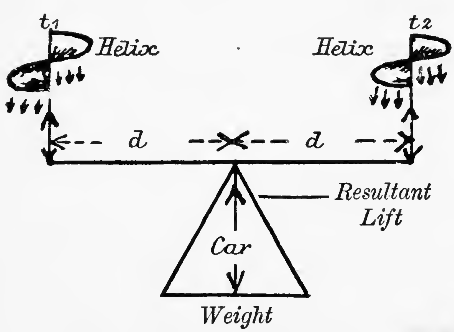


Fig. 5.—Balancing of two helices.

Another important point in connection with the use of helices and their position is the question of balancing.

The centroid of the aeronef must coincide with the centroid of the helix thrusts, or else there will be a turning moment.

The car must be so suspended and the helices or their engines must be so governed that this condition is automatically fulfilled (fig. 5).

If from any cause t_1 exceeds t_2, then we have a turning moment

$$t_1d - t_2d = d(t_1 - t_2) \quad . \quad . \quad . \quad . \quad (9)$$

which will, as the deviation increases, become

$$d \cos \theta(t_1 - t_2)$$

where θ is the angle of deviation.

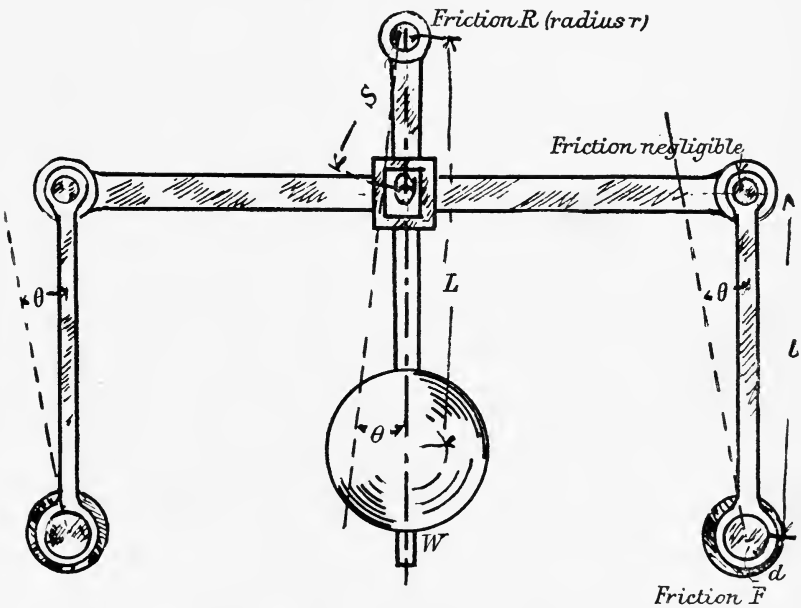


Fig. 6.—Diagram of pendulum level governor. The deflection of pendulum operates throttle valves.

If the angle θ is transmitted by suitable mechanism to the governor of the prime mover which originates the thrust t_1 so that it decreases until $= t_2$, then balance will be restored.

A heavy ball at the end of a lever moving over a sector under the influence of gravity would suffice for this and could be double-acting, increasing the thrust t_2 and decreasing t_1.

The weight of this ball (fig. 6) would be determined by the following rule:—

Weight of ball	$=$W.
Angle of deviation	$=\theta$.
Turning moment	$=$W . L $\sin\theta$.
Resisting moment of joint	$=$Rr.
" " at valves	$=$Fd.

$$\left(\frac{2\mathrm{F}d}{l}\cdot\frac{\mathrm{L}}{\mathrm{S}}+\mathrm{R}r\right)<\mathrm{W}\cdot\mathrm{L}\sin\theta.$$

Note.—Unless $l=\mathrm{S}$, the valve angles will not be the same as pendulum angle.

In connection with the governor I can foresee some difficulties will arise from the acceleration of the vessel. When stationary or travelling at constant speed the pendulum will hang vertical, but when changing speed with an acceleration a the pendulum will be inclined away from the direction of acceleration to the angle $\tan^{-1}\frac{a}{g}$ with the perpendicular, thus tending to alter the supply to the generators.

At the same time this will be an occasion when it is most required, as the wind will strike the front helix and produce a turning effect on the car. The only suggestion I can make is that another smaller pendulum should be placed alongside the large one, and when speed is being changed, the first pendulum shall be moved back by a lever through a corresponding angle.

Another difficulty will arise from lack of sensitiveness in the prime movers, due to their momentum. This will cause a decreasing oscillation, with intervals in which the helices will "hunt" or race.[1]

Efficiency of Screw Propellers.—Froude's experiments and analysis of screw motions give the following result for the maximum efficiency of a propeller:

$$\eta=\frac{\tan(\alpha-\phi)}{\tan(\alpha+\phi_1)}$$

where α is the pitch angle $=\frac{p}{\tan^{-1}\pi d}$

$$\phi=\text{slip angle (less than }10^\circ)\ \frac{v-v_u}{v}\cdot\alpha$$

$$\phi_1=\text{friction angle }\tan^{-1}\mu.$$

ϕ_1 determined by skin friction experiments. In water the efficiency is found to be about 70 per cent., with a slip of 20 per cent.

[1] See further chapter on Fittings, where balancing is discussed.

The frictional losses at the thrust-blocks can be obtained from the following considerations :—

If μ is the coefficient of friction between the surfaces when there is fair lubrication, then the total force of friction if there is a pull or thrust $T = \mu T$. This is exerted through a distance πd per revolution if d is the mean diameter of the thrust-block (internal), so that the work lost per second (n revolutions),

$$W = \mu \pi T d n \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If we call the value $\frac{\mu\pi}{550} = \nu$, then the H.P. loss if T is expressed in lbs.,

$$H = \nu T d n \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The helix is also much employed for horizontal propulsion,[1] to which it is certainly very well adapted. Its use for propelling ships is, of course, thoroughly established, and the conditions in the case of an airship are very similar.

Area and Friction.

If P is the thrust derived from the helix, and F the resistance caused by skin friction and eddies, then

$$P - F = ma \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where m is the mass $\frac{w}{g}$ of the vessel and a the acceleration.

When driving against the wind, if b is the coefficient of the surface exposed and v the velocity of the wind, the vessel is impeded by a force cpA where $p = 002288 v^2$ (c less than 1, see later),

$$ckv^2 A = ma_0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If there is motion of the vessel itself, use absolute velocity.

The forward acceleration of the aeronef will under these conditions $= a - a_0$, and in designing the helix and engines for propulsion the value of $a - a_0$ must equal the maximum velocity which will be required to be generated in one second, and the actual force of propulsion must never be less than ma_0 if the machine is not to drift with the wind. As is the case in marine practice, the ship will need to be formed with a minimum frontal resistance and will be turned towards the wind. It would be almost impossible to design a vessel capable of resisting a broadside wind pressure.

[1] The screw may be in front ("tractor"): the action is the same. There is a better air supply, but more friction against the vessel.

The value of the frictional resistance is obtainable by experiment, and according to Molesworth

$$= \cdot 0002\, D^2 v_0^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

for a cigar shape, where D is maximum diameter and v_0 the velocity in feet per second. (Sum of wind and machine's velocities.)[1]

According to the same authority the speed in feet per second derived from a horse-power H

$$= \sqrt[3]{\frac{2000000\, H}{D^2}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

It is presumable, however, that this formula is not universally correct, as it does not take into account generally the size of the helix, which certainly affects the question. It is based on a diameter for the helix $= \cdot 84$ D and a pitch equal to the diameter.

The revolutions per second are given as

$$1 \cdot 3 \times v/p \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and the value of H as

$$\cdot 0000005\, D^2 v^3 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Taking for the thrust of the helix, as before, $P = kv^2A$, and the formulæ given for the skin resistance and the wind pressure (v_0 is sum of wind and car velocities: v is nett propeller velocity $(pr - v_0)$)

$$bkv^2A - rD^2v_0^2 - ckv_0^2A = m(a - a_0)$$

when $r = \cdot 0002$; $b =$ ratio of propeller area to car sectional area.

Simplifying, we get,

$$v^2\{kA\left(b\frac{v_0^2}{v^2} - c\right) - rD^2\} = qv^2 = m(a - a_0) \quad . \quad . \quad (9)$$

from which it would appear, taking $\frac{v_0^2}{v^2}$ as fairly constant, that the forward velocity varied with the square of the theoretical velocity caused by the helix, and also with the area of the helix, so that the pitch and diameter should be as great as possible.

By so forming the prow that c and r have minimum values, we eventually get, as the maximum efficiency,

$$bkv^2A = ma \quad . \quad . \quad . \quad . \quad . \quad (10)$$

It cannot be hoped, however, that this condition will ever be reached, seeing that the helix will, of itself, offer a considerable surface upon which the wind will act when possessing a fair velocity; and we have also to consider that v_0, the value of this velocity, is not the absolute velocity of the wind as compared with the earth, but

[1] Like V on p. 12 for vertical motion.

the relative velocity compared to the aeronef, which is the sum of the absolute velocity and the velocity of the aeronef.

As before, the centroid of the machine must coincide with the centroid of the horizontal thrusts, in addition to which we must so form the prow that the centre of pressure of the wind in front is also in the same horizontal plane as the centroid of the car (fig. 7).

In the event of a disturbance of this balance (such as must inevitably occur in practice), the machine must be self-righting. This is, however, allowed for already in the consideration of the vertical helices, and the appliance described there will also serve this purpose. (See also later references to balancing.)

One further point to be considered is the alteration of angular momentum which will occur whenever the direction of the axes of rotation is altered, and consequent gyration of the vessel.

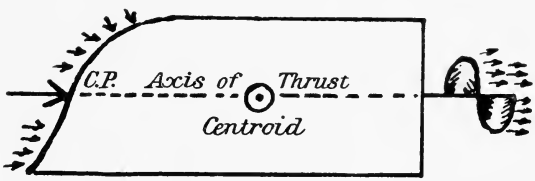


FIG. 7.—Coincidence of axis of thrust, centroid and centre of pressure.

This can be minimised considerably by making the helices in pairs with reverse rotations. Any residual effect must be controlled by the steersman, but in practice it will doubtless lead to practical difficulties, which should not, however, be insuperable.

This will to some extent serve to maintain equilibrium, as the alteration of the direction of the axis will cause a rotation to appear in the plane at right angles, which will be resisted by the action of the passing air on the sides.

Fans acting as Propellers by Jet Action (*vertical helices*).—Centrifugal air-pumps or fans (fig. 8) might possibly be useful to produce momentum.

The arrangement is very similar to that of the centrifugal pump, but a large whirlpool chamber would not be so necessary.

Assuming that the air leaves the pump with a velocity v (= that of vane tips) and the quantity is Q, then the total momentum per second would be (radial vanes)

$$\frac{\rho Q}{g} . v \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If v_r is the through velocity and A the minimum area in the passages, then the quantity $Q = v_r A$, so that the momentum (*i.e.* the force exerted) would be

$$\frac{\rho}{g} A v_r . v \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the radii inside and outside the wheel are in the ratios $r_2 : r_1$, then the velocity of the inner rim is $\frac{r_1}{r_2} . v$, so that the angle of the vanes inside is (by triangle of velocities)

$$\theta = \tan^{-1} \frac{v_r r_2}{r v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

There will be loss by compression of air and friction.

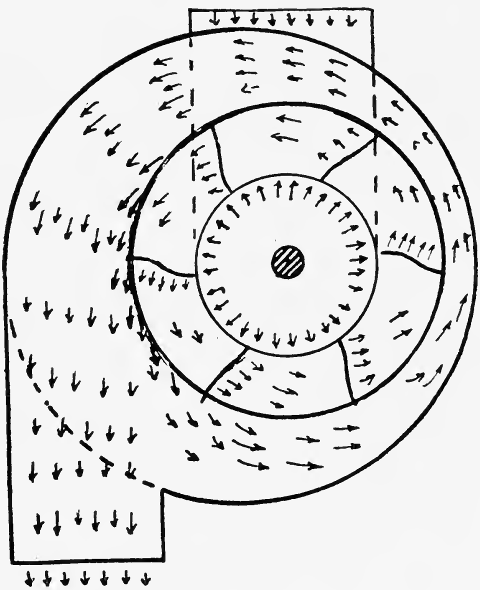


Fig. 8.—Diagrammatic section of vertical air-wheel.

Form of Propeller.—There are several forms of propellers, but the most well known are the

(1) "Elliptic" (4 blades).
(2) "Bat's-wing" (3 blades).
(3) "Fan" (2 blades).

In marine practice the elliptic blade is most used, the diameter of the boss being ·14 that of the whole propeller. It would be less for an air propeller.

The blade is one of simple helical surface, the projected width being ·23 D. The arcs which form the sides of the blades as they appear when projected in the direction of the axis have a radius ·33 D and are joined with a small circular arc at the tip. The axial dimension of the blade is usually $\frac{1}{6}$ the pitch. The pitch and the pitch angles at any particular distance from the centre can be easily found by drawing a right-angled triangle, whose perpendicular is $\frac{1}{6}$ pitch and whose base is $\frac{1}{6}.\pi \times$ diameter of the point considered. The hypotenuse makes the pitch angle with the base of this triangle (fig. 9).

The total angle of twist from the boss to the tip is, of course, equal to the difference of the pitch angle of a helix on the boss, and of the helix (same pitch) on a drum of the diameter of the propeller.

(Propellers of increasing pitch towards the tip have been found to be efficient.)

The "bat's-wing" in its axial projection appears as a segment of a circle with the tip truncated. The bounding arc has a radius of ·3 D and the square tip is 6° wide.

The "fan" type was used by Maxim in his machine, and consists of two blades tapering towards the centre. It is advantageous in the respect that it can be closely approximated to by two frame rods passing into the axis and inclined to one another at the required angle to join the two pitch angles at the centre and tip. If the distance between them is not great, the surface is almost a helical one, and can be made, as in Maxim's machine, of silk, the ribs being of steel tube.

That a helix can actually exert an *ascensional* force is, apart from mechanical principles, shown by (1) the thrust in the bearings of ventilating fans (Blackman type), and (2) the various toys which utilise this principle to make a small screw "sky," as the "spiralger," which consists of four small flat blades or feathers attached to a spindle which can be revolved by a string or by pulling a threaded block over a fixed, similarly threaded shaft. In 1768 the engineer Paucton suggested its use, and many French aeronauts (notably M. Nadas), supported the idea. In M. Jules Verne's *Albatross* it is well known to readers of fiction. M. de Landelle perfected the spiralger.

It is needless to remind engineers of the precisely similar effect in the marine screw propeller. The only difference is that the thrust is horizontal.

Mr Edison, the well-known inventor, made a series of experiments on vertically acting screws. His results were not such as he

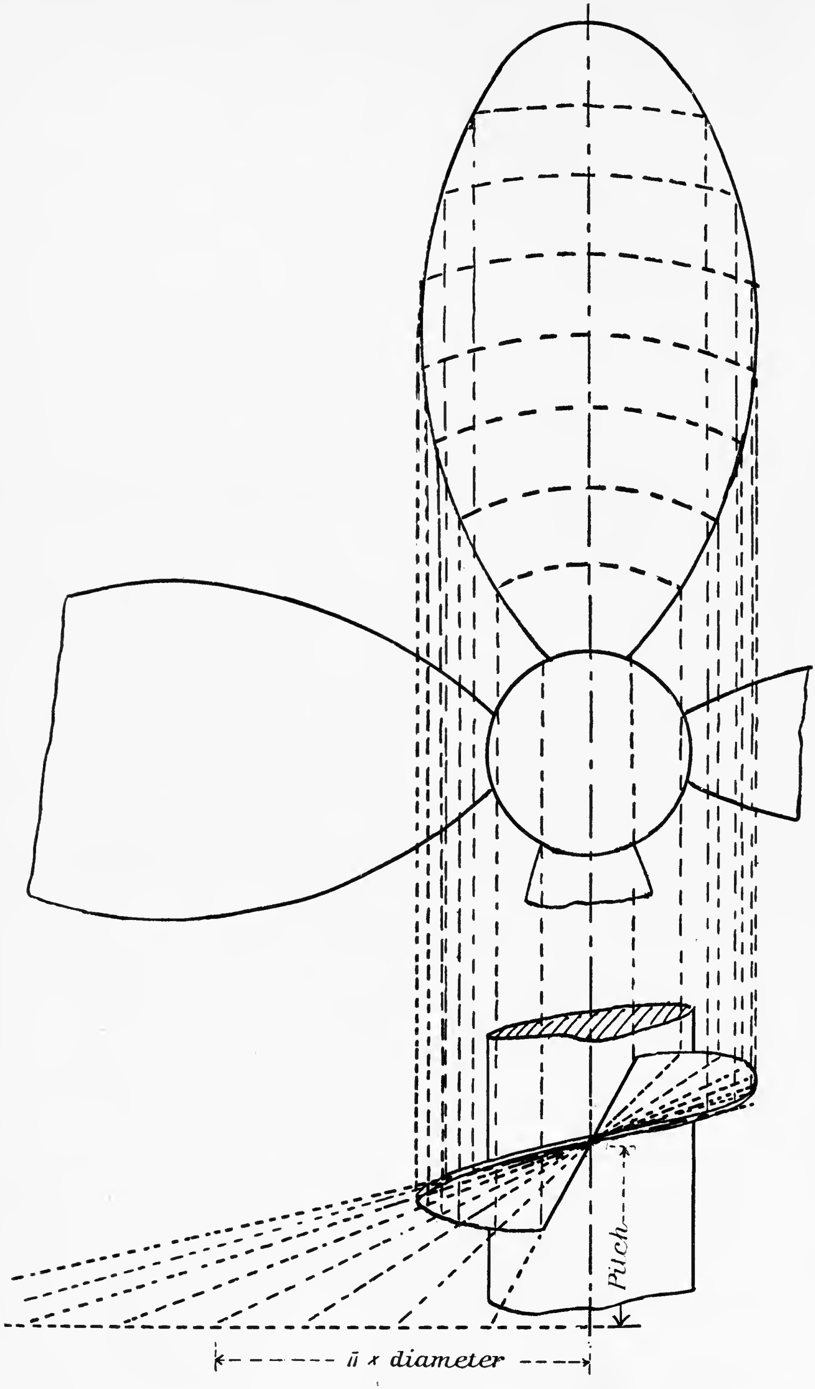


FIG. 9.—Elliptic propeller.

expected, chiefly, I think, because the prime mover at that time available was of large weight as compared with the power, and so there was but little momentum given to the air, while considerable friction at the bearings had to be overcome by the machine.

It has to be remembered, however, that there will be some waste due to the propeller itself. By friction with the blades the air acquires rotational energy, which is not only useless, but, on account of the tangential acceleration, removes from the helix some of the air whose longitudinal velocity would produce back momentum. This loss is also experienced in ships. Furthermore, it is only the relative velocity of the projected air and the surrounding air which is useful.

The thickness of the propeller blades can be ascertained by the well-known rule connecting the total bending moment with the moment of resistance at the section considered. The bending moment will be the product of the nett thrust on the blade by the distance of the centre of pressure from the axis, or

$$\mathrm{T}d = f\frac{\mathrm{I}}{y} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where d is the distance mentioned, f is the working stress (compression and tension) per square inch, I is the moment of inertia of the section, and y the distance from the centre of gravity of the section to the extreme edge (half of the depth in symmetrical sections). Thus if we have fan blades, the section of which may reasonably be considered rectangular,

$$\mathrm{T}d = f\frac{bt^3}{6t} = f\frac{bt^2}{6} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In an elliptic section the value of I is $\frac{\pi}{4}ab^3$, where a and b are the semi-axes, so that (about axis $2b$)

$$\mathrm{T}d = f\frac{\pi b^3}{4} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

A more usual section than the elliptic is one which approximates to a circular segment, the flat being the propelling side. In this and any other unsymmetric section the moment of inertia is best found graphically, taking a number of strips parallel to the neutral axis and multiplying the area of each by the square of its distance from the axis:

$$\mathrm{I} = \Sigma by^2dy \quad . \quad . \quad . \quad . \quad . \quad (4)$$

To find the actual form of the blades, it is best to project at right angles from the plan and elevation in a number of strips or sectors.

The actual form of a complete helix developed on to one plane can be found by describing two concentric circles whose radii equal those of the corresponding root and tip helices.

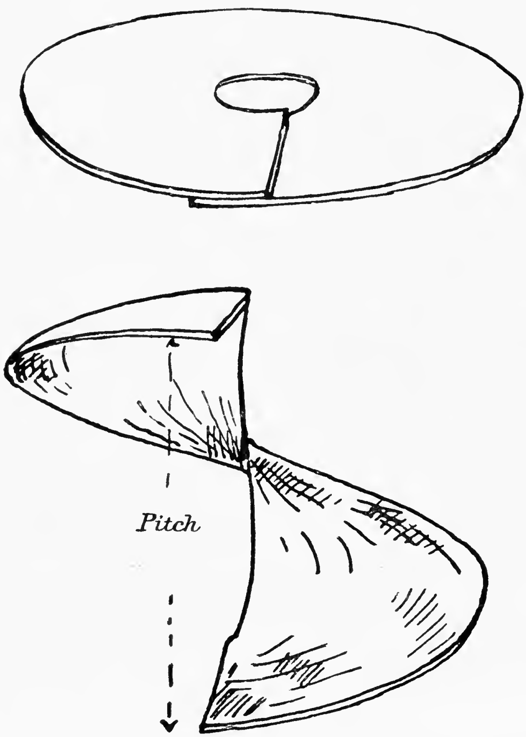


Fig. 10.—Complete helix of one revolution formed by one sheet of metal.

The actual length of each helix is found by the expression

$$L = \sqrt{\text{pitch}^2 + (\text{circumference})^2},$$

and if these lengths be stepped round the circumference, the resulting figure (an overlapping ring) will be a development of the screw surface and may be formed into the desired shape by separating the overlapping ends by the pitch-length so that the two bounding radii are parallel (fig. 10).

The considerations to be considered in the choice of a particular form of propeller are:—

(1) Amount of eddying produced.

(2) Projected (*i.e.* effective) area.

(3) Space occupied.

(4) Strength to transmit thrust.

The propeller shaft will be designed by the rule

$$D\,(\text{inches}) = k\,.\sqrt[3]{\frac{H}{N}}. \qquad (k=3 \text{ to } 4.)$$

where H is H.P. and N revolutions per minute. The investigations of Professor Greenhill show that the thrust is not of great importance in connection with a shaft. It would, however, be desirable to have a hollow section in preference to a solid one.

The liners to the thrust-block must be very smooth and well lubricated. In small vessels probably ball bearings could be successfully used.

Before leaving the subject of helices, it should be noticed that it has been held by several men who have studied the subject that the efficiency of an elastic (non-rigid) helix is vastly greater than that of the rigid helix. Professor Pettigrew goes so far as to describe this pliancy as the essential condition in flight, and affirms that wherever rigid helices or planes have been successful, there has been great loss of power. He suggests the use of an elastic two-bladed propeller of decreasing pitch.

Many considerations point to the correctness of this view, but it must also be remembered that the difficulties of constructing a symmetrically elastic helix or plane on a large scale are somewhat considerable; and, while deference should be accorded to the views of the naturalists who have devoted so much time and labour to the problem, engineering considerations would render a satisfactory rigid aeroplane or helix much more acceptable than a complex elastic one.

Furthermore, it is noteworthy that, while natural machines are of very high efficiency, they are not always the most convenient or expeditious. At present the wheel for land locomotion seems vastly more expeditious than the pedal lever (in spite of the "decapod"), and it may be that the mind can originate some contrivance as superior to the wing as the wheel is to the leg.

CHAPTER IV.

THE AEROPLANE.

THE aeroplane as now known was invented by Henson (fig. 11) in 1842.

It has long been known that an inclined surface drawn or pushed against the wind is subject to an upward thrust by the reaction of the displaced air. If this surface can be rigidly fixed to a framework capable of bearing machinery for propelling the surface (by a helix) forward through the air, and its surface and velocity are sufficient to displace air to the extent capable of producing a vertical component

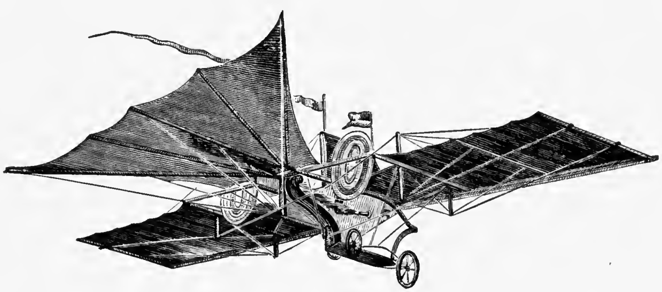


FIG. 11.—Henson's aerostat.

reaction exceeding the weight of the whole apparatus, another means of aerial locomotion is discovered.

The problem involves the following practical difficulties:—

(1) The production of a horizontal thrust capable of raising the apparatus used by means of the plane.

(2) The balancing of such a plane so that the nett turning moment on it is zero, particularly at low speeds.

(3) The arranging of the plane if possible so that its resistance to wind is less than the maximum horizontal thrust.

If we consider a simple plane pushed through the air, we notice the motions in the air shown in fig. 12 (bottom diagram).

Fig. 10a.—Curtiss Biplane.

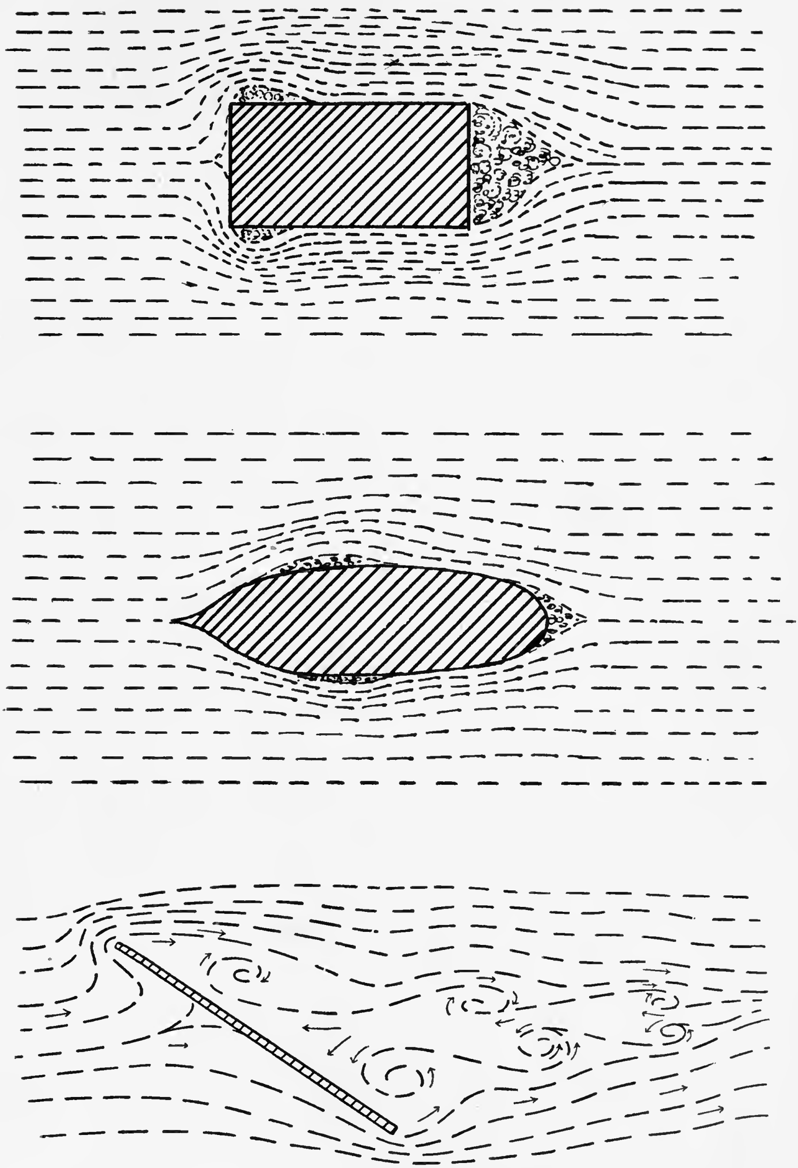


Fig. 12.—Resistance of air.

At very low speeds in any fluid there is a simple distortion of the stream lines (*i.e.* the lines which map out the direction of the flow at any point) which is amenable to the mathematical theory of Hydrodynamics (see Professor Lamb's *Hydrodynamics*, p. 94), and the physical truth of these stream-forms has been demonstrated by numerous experiments made by Professor Hele-Shaw. At usual speeds, however, there is a space at the rear of the moving surface in which the flow is discontinuous, and whirls or "eddies" of fluid occupy the gap. As the speed increases this space also increases, and the total resistance is also increased, so that there is reason to suppose that at very high velocity the resistance varies almost as the cube of the velocity.

As a perfect fluid cannot have tangential stress, the pressure caused by its loss of momentum will be exerted at right angles to the surface of the plane. Actually there is a slight tangential force.

This problem is very fully attacked by Weisbach in his *Lehrbuch der Ing. u. Masch. Mechanism*, vol. ii. pt. i., in dealing with windmills. Taking c as the velocity of the plane relatively to the wind, Q as the quantity displaced per second, ρ as the density of the air and θ as the angle of the plane, the normal force exerted against the plane

$$N = \frac{c}{g} . \sin\theta . Q\rho, \quad . \quad . \quad . \quad . \quad (1)$$

since quantity Q = velocity × area, and area = sin θ × vertical area, which we may call A, the actual area being A_0

$$N = \frac{c}{g} \sin\theta . c . A . \rho$$

where $$A = A_0 \sin\theta,$$

then $$N = \frac{c^2}{g} . \sin^2\theta . A_0 . \rho \quad . \quad . \quad . \quad . \quad (2)$$

The eddying motion of the wind behind the plane relieves the pressure there to the extent

$$\left(\frac{c^2}{2g}\right) \sin^2\theta . A_0\rho,$$

so that total normal force

$$N_{max} = 3\left[\frac{c^2}{2g} \sin^2\theta . A_0\rho\right] \quad . \quad . \quad . \quad . \quad (3)$$

Experimental proof is required of this particular value, but it is almost certain that N exceeds $\left(\frac{c^2}{g}\sin^2\theta A_0\rho\right)$. Coulomb's results agree very fairly with (3), but for small angles experimenters disagree greatly. The latest results are almost unanimous in making the variation of thrust as $\sin\theta$ and not $\sin^2\theta$. All these following expressions are divided by $\sin\theta$.

We thus get the value of the total thrust on the plane produced by the propulsion, and the vertical component of this is the lifting force.

The ratio of the horizontal component to the normal force is $\sin\theta : 1$, so that the value of the resisting force

$$R = 3\left[\frac{c^2}{2g}\sin^2\theta \,.\, A_0\rho\right] \quad . \quad . \quad . \quad . \quad (4)$$

g of course equals about 32·2, and ρ the density of air is found by the well-known rule

$$\rho = cp^{\frac{1}{\gamma}} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In this $\gamma = 1{\cdot}41$ for air and p is pressure in lbs. per sq. in. (normal value $=14{\cdot}7$ lbs.). c is a constant.

At atmospheric pressure (*i.e.* near surface of earth) the weight of air is as follows:—

Weight ρ (lbs. per cubic ft.).	Volume (cubic ft. per lb.).	Temperature (Fahrenheit).
·086	11·58	0°
·078	12·81	50°
·071	14·09	100°

Taking 50° as the average temperature, then we have, as the value of ρ compared with water,

$$\cdot078 \div 62\cdot5 = \cdot001248 = \rho_0 \quad . \quad . \quad . \quad . \quad (6)$$

The horizontal thrust has also to be considered in the resistance produced against the propelling force, so that the helix thrust

$$T > 3\left[\frac{c^2}{2g}\sin^2\theta \,.\, A_0\rho\right] \quad . \quad . \quad . \quad . \quad (7)$$

When driving against the wind, c is the relative velocity of the plane to the air, *i.e.* the sum of the plane velocity and the wind.

If we call the mere plane velocity c_0 and the wind velocity v, then

$$T > 3\left[\frac{(c_0+v)^2}{2g} \,.\, \sin^2\theta \,.\, A_0\rho\right] \quad . \quad . \quad . \quad . \quad (8)$$

From this it will be apparent that the greater the velocity the greater is the lifting force; but, unless the aeroplane is to drift with the wind, the minimum value for the helix thrust

$$T_0 = 3\left[\frac{v^2}{2g} \cdot \sin^2\theta \cdot A_0\rho\right] \quad . \quad . \quad . \quad . \quad (9)$$

If reference is made to the chapter on helices, a relation is established between the thrust T and the horse-power applied to the helix and the

$$T = \cdot 1014 r \sqrt[3]{HD^7};$$

or, in terms of weight,

$$T = \cdot 1014 r \sqrt[3]{\frac{W - c}{W} \cdot D^7}.$$

By equating these two expressions we can find the connections between weight and the reaction of the air on the plane.

Taking formula (8)

$$T = 3\left[\frac{(c_0 + v)^2}{2g} \cdot \sin^2\theta \cdot A_0\rho\right]$$

we have

$$mr\sqrt[3]{\frac{(W - c)}{w}D^7} = 3\left[\frac{(c_0 + v)^2}{2g} \cdot \sin^2\theta \cdot A_0\rho\right].$$

Cubing both sides and dividing

$$\frac{W - c}{w} = \frac{27\left[\frac{(c_0 + v)^2}{2g} \sin^2\theta \cdot A_0\rho\right]^3}{m^3 r^3 D^7} \quad . \quad . \quad . \quad (10)$$

The value of the driving thrust and the weights as compared with the area and angle of the aeroplane is in this way established, but a more satisfactory method is to balance the lifting force against the weight as follows:—

$$\text{Lift} = N \cdot \cos\theta$$

$$= \frac{3c^2}{2g} \cdot \sin\theta \cdot \cos\theta \cdot A_0\rho.$$

[*Note.*—Since the angle is usually small, for the value $\sin\theta$, $\sin^2\theta$ is used.]

Hence, for soaring, $W = N \cdot \cos\theta$, and from this c or $c + v$ the relative velocity can be computed

$$c = \sqrt{\frac{2g \cdot W}{3\sin\theta \cdot \cos\theta \cdot A_0\rho}} \text{ or approximately } \sqrt{\frac{W}{\cdot 002A_0 \cdot \sin\theta \cdot \cos\theta}}$$

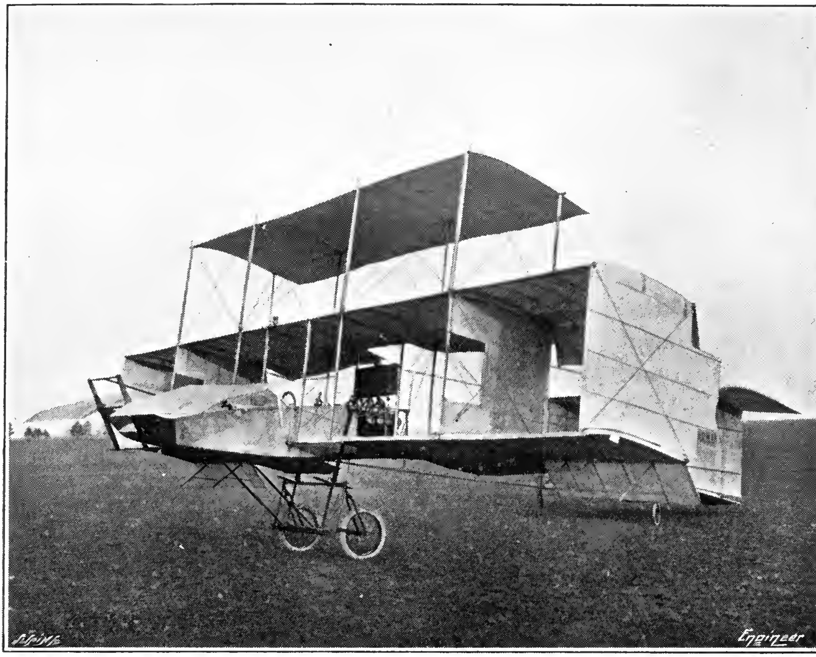


Fig. 12a.—The Farman machine.

Fig. 12b.—The Voisin type of aeroplane in flight.

Following the reasoning of Weisbach in his study of wind wheels, and calling the velocity of the aeroplane v and the velocity of the wind c,

$$N = 3\frac{(c+v)^2}{2g}\sin^2\theta \,.\, A\rho\,.$$

More attention has been given to this part of the subject than any other during recent years. It may be regarded as established that the formulæ above given are practically correct. Lord Rayleigh showed, in 1900, that the friction on the surfaces was not absolutely negligible. He gives the following rule:—

$$H = (\kappa S \sin^2 a + \mu)v^3,$$

or in the form of the preceding formulæ,

$$H = \left(\frac{3\rho}{2g} \,.\, A_0 \sin^2\theta + \mu\right)c^3;$$

μ here stands for the total friction at unit speed.

Professor Zahm has recently shown that the frictional resistance as per square foot is

$$f = 0{\cdot}00000778\, l^{\cdot 93}\, c^{1\cdot 85}$$

where l is length in feet and c is velocity as before.

Lord Rayleigh shows that the work done is a minimum when

$$\sin^2\theta = \frac{3\mu}{\left(\frac{3\rho A_0}{2g}\right)}.$$

Another very useful quantity in this connection is the lift to drift ratio, or, as it is sometimes called, "the efficiency."

Since lift $= N\cos\theta$ and drift $= N\sin\theta + \mu v^2$, we have

$$\frac{\text{lift}}{\text{drift}} = \frac{N\cos\theta}{N\sin\theta + \mu v^2} = \frac{1}{\tan\theta + \frac{\mu v^2}{N\cos\theta}}.$$

Mr Turnbull (*Aeronautical Journal*, 1908) shows that for ordinary surfaces (plane, slightly concave, or concavo-convex) this ratio has a maximum value of between 4 and 6, for an angle of incidence of about 6°.

Hence the use of the rule by many designers

$$\text{Thrust} = \frac{\text{Weight}}{4.}$$

The following formulae and tables summarise our knowledge of the state of the atmosphere so far as it affects aerostation.

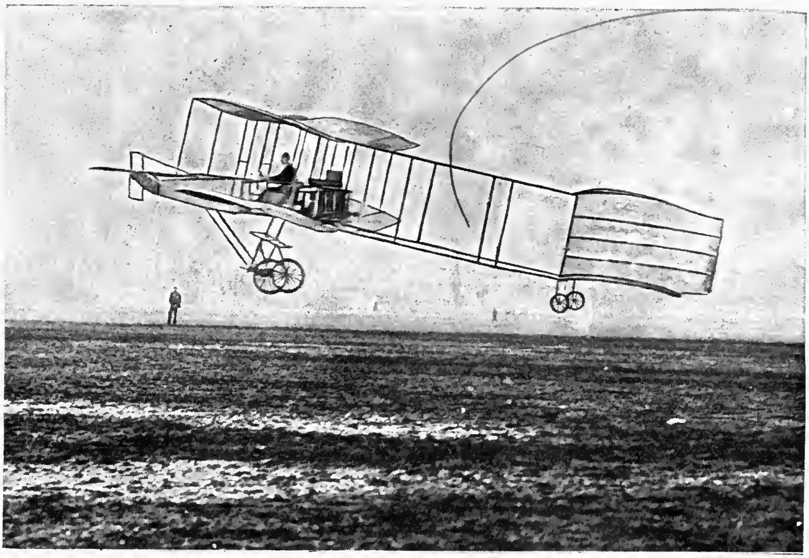


FIG. 12C.—Delagrange aeroplane.

Wind Pressures and Velocities.—(*Note.*—Maximum observed pressure in Great Britain = 55 lbs. per sq. foot, Dr Nicol, Glasgow Observatory. *See*, however, p. 37 *re* Forth Bridge.)

Lbs. per sq. foot.	Feet per second.	Feet per minute.	Miles per hour.
p	v		
·005	1·47	88	1
·123	7·33	440	5
·492	14·67	880	10
3·075	36·6	2200	25
12·3	73·3	4400	50
24·108	102·7	6160	70
49·2	146·6	8800	100

$p = kv^2$ lbs.
$k =$ ·002288 (see below).

Direction of wind in England in days per 1000
(Kämtz).

N.	82
N.E.	111
E.	99
S.E.	81 ← *min.*
S.	111
S.W.	225 ← *max.*
W.	171
N.W.	120
	1000 days.

Variation with Height.—The wind pressure increases in the higher zones of the atmosphere. According to Stevenson, the following expression, other conditions being the same, indicates the ratio of increase:—

$$P = p\sqrt{\frac{H+72}{h+27}}$$

P = pressure at height H
p = " " h.

Also Kempe gives

$$v = 1347{\cdot}4\sqrt{\frac{H-h}{h}},$$ where v is velocity at different zones.

Relative Resistance of Submerged Bodies at velocity of 1 *foot per second* (Beaufoy):—

Mid-Section Circular.

Bow.	Stern.		
Square	Conical	·958	Cone angle 77°.
Square	Square	1·0067	
Semispheric	Conical	·2706	Cone angle 77°.
Square	Semicircular	·8883	
Conical	Conical	·2836	Front cone 56°, stern 71°.
Semicircular	Square	·2332	
Conical	Conical	·2848	Front 63°, stern 71°.
Semicircular	Semicircular	·1349	

With a square mid-section these coefficients are increased in a ratio between 1·5 and 2·0.

Pressure on Surfaces.—If the wind moves parallel to the plane of section, the following ratios are found to obtain:

square, 1·00; hexagonal, ·75; octagonal, ·65; circular, ·5.

(Used for chimney design.)

Pressure on Oblique Surfaces.—Lord Rayleigh's formula for the pressure on an oblique surface when N is the pressure which would result from the speed if plane were rectangular to the direction of motion, and N_0 is the actual normal force on the plane:

$$N_0 = \frac{N(4+\pi)\sin\theta}{4+\pi\sin\theta}, \text{ or } \frac{N\,.\,2\pi\sin\theta}{4+\pi\sin\theta},$$

θ being the angle of deviation of the plane from the direction of motion. Froude increases this in ratio 1·86.

This value may be used for calculating the turning effect of the rudder, the total value being

$$N_0 A \times \Delta.$$

Other values are given in this chapter for P_0.

Professor H. Adams on Wind Pressures gives the following collection of rules:—

$N =$ pressure, lbs. per sq. ft.
$v =$ velocity in ft. per sec.
$N = kv^2$.
$N_0 =$ lbs. per sq. ft.
$A =$ area sq. ft.
$\Delta =$ distance from c.p. to axis of rotation.

(1)	$k =$ ·001154	Professor C. A. Carus Wilson.
(2)	·001378	J. A. F. Aspinall, M.I.C.E.
(3)	·00233	Smeaton and Rouse.
(4)	·002502	Hawksley.
(5)	·002737	D. Kinnear Clark.
(6)	·0017	Du Buat.
(7)	·00167	Professor Langley. (Important.)

(1) and (2) are rational; (3) to (5) empirical.

Professor Carus Wilson gives $N = \frac{\rho}{2g}\,.\,v^2$ where ρ is weight per cubic foot (about 0·0807 lb.).

This is only half the result obtained by the momentum method and comparison with empirical results.

The momentum method, which I have previously employed, gives $N = \cdot 0054\ V^2 (= V$ miles per hour$) = \cdot 002517\ v^2$, which is near the mean value of Nos. (3), (4), and (5).

Impulse of the Wind.

TABLE OF RESULTS OF DIFFERENT AUTHORITIES (from Weisbach) (vertical surfaces).

Mariotte:

$$N = 1{\cdot}73 \frac{c^2}{2g} \cdot A\rho.$$

Woltmann:[1]

$$N = \frac{4}{3} \frac{c^2}{2g} \cdot A\rho.$$

Hutton:

$$N = 1{\cdot}86 \frac{c^2}{2g} \cdot A^{1 \cdot 1}\rho.$$

Weisbach:

$$N = 3 \frac{c^2}{2g} \cdot A\rho.$$

The number of experiments made on this subject is enormous, and I have elsewhere tabulated over thirty values for this constant. It is certainly more than Professor Carus Wilson's value, and it is almost equally certainly less than twice this value. M. Eiffel's recent experiments have shown that it varies with the velocity, but for aeronautical purposes I have come to the conclusion that a value between ·0015 and ·002 may be safely used.

Experiments on large areas generally show great local variations, and apparently there is a decrease of the average pressure with a large area, due to lack of uniformity in the structure of the wind.

Professor Henry Adams gives the following value for the greatest pressure produced on a stationary surface, centre of gravity of which is h feet above the ground and the width w feet:

$$\log p = 1{\cdot}125 + 0{\cdot}32 \log h - 0{\cdot}12 \log w.$$

If surface is inclined $\theta°$ to wind, the natural pressure is $p \sin \theta$ and effective pressure in direction of wind $p \sin^2 \theta$.

This formula indicates:

(1) Initial constant value for maximum wind pressure.

(2) Increase as about cube root of height.

(3) Decrease as about the eighth root of the width.

For a width of 5 feet a height of 150 feet (*e.g.*) the pressure is then 54·6 lbs. per sq. foot.

[1] This result is based on resistance only, and not impulse.

Professor Adams also gives the following constants for different forms of surface exposed to the wind, reference being made in each case to the maximum cross-section :—

Plane	1·00
Cylinder	$\frac{\pi}{4}$
Sphere	$\frac{\pi}{8}$

(These results do not agree with the ratios employed in chimney construction which have already been given.)

Professor Adams gives resistance of small planes with variable inclinations as proportionate to $\sin^2\theta$, and of large ones as $\sin\theta$.

He quotes Dr Hutton's experiments as giving resistance ($=N\sin\theta$)

$$R = ap \sin\theta^{1\cdot84\cos\theta}$$

where a is area of surface, p pressure in lbs. per sq. inch.

Also from E. F. Etchell, on surfaces with θ from 10° to 60°, normal pressure $= p\sin^2(1\cdot2\,\theta + 18°)$.

Mr Hunter on Wind Pressures.—Much valuable information on the subject of the effect of air in motion on obstructing surfaces has been collected by Mr Adam Hunter, M.I.C.E.

Since the Forth Bridge was finished in 1890, records of maximum wind pressures have been taken. The highest recorded pressure equals about 65 lbs. per sq. foot.

It is clearly established that the average pressure on large areas is less than that recorded on small ones, and according to Captain Bixby, M.Am.Soc.C.E., "the value of the wind pressure in gusts on large bridge surfaces may be taken at 60 per cent., and the average steady wind on a bridge at 36 per cent. of the maximum of small plate-pressure anemometer records of neighbourhood." Also that "wind velocities may be taken at 70 to 90 per cent. of ordinary cup rotary anemometer records converted into wind pressure by formula $p = \cdot0043\,V^2$ when V is velocity in miles per hour.

The Bidston Observatory, near Liverpool, records, during twenty years (1868 to 1888), indicate that the following velocities are attained :—

83 miles per hour once in sixteen years,
83 to 64 miles about once a year,
64 to 54 miles about twice a year,
less than 54 miles at shorter intervals;

which are computed to represent pressures of

33 to 29 lbs. per sq. ft. twice in sixteen years (*sic*),
29 „ 18 „ „ once a year,
18 „ 15 „ „ twice a year,
15 lbs. at shorter intervals.

By the rule $N = \cdot 0043\, V^2 = \cdot 00201\, v^2$ where v = velocity in feet per second.

Experiments made at the National Physical Laboratory (*Proceedings I.C.E.*, vol. clvi.) give $N = \cdot 0027\, V^2 = \cdot 00137\, v^2$.

The diversity in these constants is most extraordinary.

Table of Velocities obtained from Experiments by Stevenson (quoted by Mr A. Hunter, A.M.I.C.E., A.M.Am.Soc.C.E., in paper read before J.I. Eng., December 1906).

Feet above Ground.	5	9	15	25	52
Velocities (miles per hour)	4 7 13 19 26	6 17 23 28 32	6 18 25 31 34	6·5 21 30 35 37	7·5 23 32 40 43
Average .	13·8	21·2	22·8	25·9	29·2

Mr Hunter quotes the results obtained at the National Physical Laboratory with building and roof models (slope 30°), air velocity 10 miles per hour.

Position.	Rates of Pressure.	
At ground level. . .	179	on vertical surface.
At centre	182	
At eaves level . . .	168	

Indicating an increase with height and then diminution on account of the roof.

He suggests the following rule for large vertical areas for maximum pressures :—

30 lbs. per sq. ft. up to 300 sq. ft.
29 „ „ „ 400 „
28 „ „ „ 500 „

and so on, decreasing 1 lb. per 100 sq. feet to a minimum of 20 lbs.

He further thinks that the building should resist a turning moment producible by steady pressure of 50 lbs. per sq. foot.

Hutton, Robins', and Smeaton's Results.—The researches of Messrs Hutton and Robins led to the following important conclusions as to the resistance offered to a plane:—

(1) Resistance varies as the area of the surface, but increases in a slightly greater ratio with large surfaces. In other words, $N \propto A^{1+\frac{1}{n}}$.

(2) Round and sharp ends offer less resistance than flat (compare Beaufoy's results).

(3) Two solids with same fronts are not equally resisted unless the hinder parts are similar.

(4) The resistance varies with the actual relative velocity and not the apparent velocity.

(All these conclusions have been arrived at by other experimenters, and appear elsewhere.)

Hutton and Smeaton's empirical formula for the resistance of an inclined plane is

$$R \propto V^2(\sin\theta)^{1\cdot842\cos\theta}, \text{ as on p. 35,}$$

and, since $R = N \sin\theta$, as a final formula we have

$$P = kAV^2(\sin\theta)^{1\cdot842\cos\theta - 1};$$

and as for small angles $\cos\theta$ approaches unity, this gives for such angles $P = AV^2 \sin\theta$, nearly or more accurately $AV \sin\theta^{0\cdot8}$.

(A here should include $\frac{\rho}{g}$, the mass per unit volume, to make the expression have a value in lbs. pressure.)

Duchemin's rule has recently been found very satisfactory, and is

$$P = kv^2A \times \left(\frac{2\sin\theta}{1+\sin^2\theta}\right). \quad \text{(Very important rule.)}$$

The values for the ratio between pressures on normal and inclined surfaces may be tabulated as follows:—

(1) $P_\theta = P_{90}\kappa \,.\, \sin\theta$. ($\kappa$ = about 2.)

(2) $P_\theta = P_{90} \sin\theta^{1\cdot84\cos\theta - 1}$ . . . . (Hutton)

(3) $P_\theta = P_{90}\dfrac{(4+\pi)\sin\theta}{4+\pi\sin\theta}$ . . . . . (Rayleigh)

$$(4)\quad P_\theta = P_{90}\frac{2\sin\theta}{1+\sin^2\theta} \quad . \quad . \quad . \quad . \quad . \quad \text{(Duchemin)}$$

$$(5)\quad P_\theta = P_{90}\frac{2\sin\theta(1+\cos\theta)}{1+\cos\theta+\sin\theta} \quad . \quad . \quad . \quad \text{(De Louvrié)}$$

$$(6)\quad P_\theta = P_{90}\sin^2(1{\cdot}2\theta+18^0) \quad . \quad . \quad . \quad \text{(Etchell)}$$

$$(7)\quad \left.\begin{aligned} P_\theta &= P_{90}\cdot\frac{\theta^0}{30^0} \text{ up to } 30^0 \\ P_\theta &= P_{90} \text{ from } 30^0 \text{ to } 90^0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad \text{(Eiffel)}$$

TABLE OF DENSITIES AT DIFFERENT HEIGHTS.

(*Cosmos*, April 1893, "La pratique des ascensions aérostatiques.")

Height in Metres.	Reciprocal of Density.
12,900	5
18,400	10
29,500	40
42,300	200
49,700	500

The following authorities have also contributed to our knowledge:—

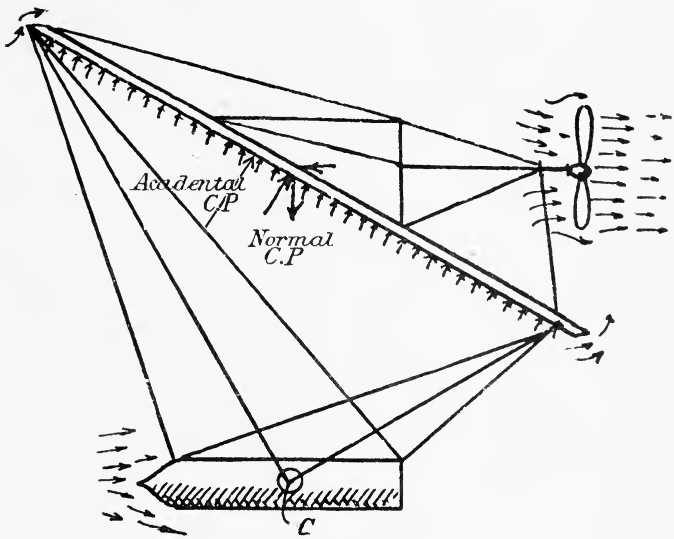
Coriolis, Coulomb, Smeaton, Burg, Borda, Rouse, Poncelet, Euler, Vince, Crelle, Thibault, Hagen, Joëssel, Renard, Canovetti, Langley, and Dines.

Balancing.—Having settled the question of area and thrust by the preceding rules, we must next consider the balancing, which is a very difficult problem to solve.

Presuming that the air will strike the plane fairly uniformly, we might conclude that the centre of pressure is the centre of gravity of the plane; but, as a matter of fact, the eddying of the air will probably cause the position of this point to vary considerably, and this is the chief cause of the difficulty. Also at small angles the c.p. is considerably forward of the c.g., as much as ·3 of the width when θ is small, but below a certain critical angle (about 20°) the displacement decreases.

It is not very convenient to have the propelling mechanism immediately behind the centroid of the plane, but if this is not done, there will be a couple tending to upset the whole apparatus, and also we have to consider the effect of the force acting vertically. If the direction of this passes through the general centroid, equilibrium will be obtained, but not otherwise.

In other words, the point at which all the forces (or rather their resultants) meet is the position for the centroid. This condition is



Note.—C.P. will be in front of the geometrical centre.

FIG. 13.—Aeroplane with suspended load, showing state of unstable equilibrium.

conveniently satisfied if we have two planes and the motor between them. (*Note.*—Distance from c.p. to c.g. $=(0{\cdot}3-0{\cdot}3\sin\theta)$ length.)[1]

If the planes are of equal areas and parallel (fig. 14), the forces

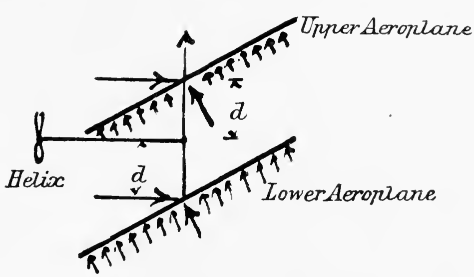


FIG. 14.—Balance of aeroplane system (two planes set at same angle).

on them, being respectively proportional to the areas, are also equal, and the c.g. and the thrust must be equidistant from both, *i e.*,

$$d_1 = d_2 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

[1] Joëssel's very important rule.

If the areas or angles are different (fig. 15), then the thrusts

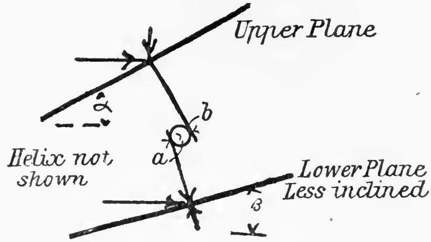


FIG. 15.—Balancing of aeroplanes, planes unequally inclined.

N_1 and N_2 acting about the centroid must be equal in their turning effects, *i.e.*,

$$N_1 a = N_2 b,$$

and
$$N_1 = \frac{N_2 b}{a} \text{ or } N_2 = \frac{N_1 a}{b} \quad . \quad . \quad . \quad . \quad (12)$$

The areas will need to be proportioned accordingly.

If there are a number of planes, then the condition of equilibrium is that

$$\Sigma(Nx) = 0 \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where N is the normal force and x the distance of its line of direction from the centroid.

The thrust must almost pass through the centroid, or, if there are n_1, n_2 thrusts, and their distances from the centroid are l_1, l_2, l_3, etc., then

$$\Sigma(nl) = 0 \quad . \quad . \quad . \quad . \quad . \quad (14)$$

It will not be safe to say that

$$\Sigma(Nx) = \Sigma(ln),$$

as the values of both N and n will be subject to variation. It will be preferable to have more than one thrust n, so that by a balance weight (such as is described in the chapter on helices) the supply to each can be controlled. In this way, if the aeroplane deviates from the normal position, the thrusts will be altered and produce a righting moment. It will be impossible to make this occur immediately, so that vibration is unavoidable; but this should be damped out by the resistance of the planes. Gyrostatic balancing is also possible.

With regard to the arranging of the planes so that they present a minimum resistance to the wind, it must be realised that this very arrangement involves loss of lifting effect. To drive against the wind we must rely on getting a momentum from the driving helices sufficient to overcome the relative wind pressure.

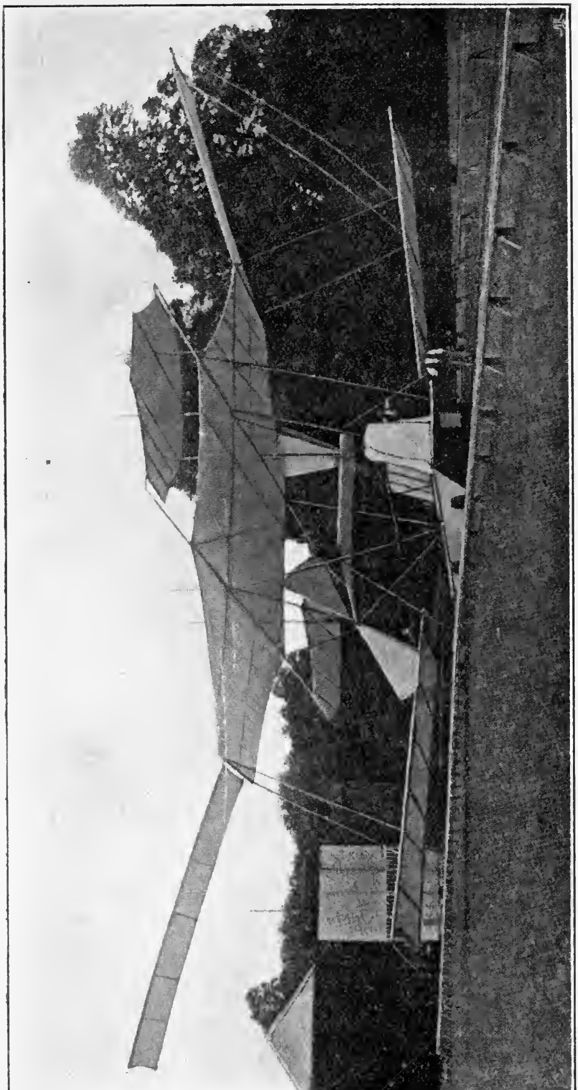


FIG. 15A.—Sir Hiram Maxim's Steam Flying Machine.

Sir Hiram Maxim's aeroplane (Fig. 15A) is a good example of the arrangement, consisting as it does of so many planes. There is a drawing of this in the South Kensington Mechanical Museum (branch of the Victoria and Albert), where may also be seen one of the large propellers and the engine (fig. 16), which has been cut as fine as possible. The balance is, however, deficient.

The design of the framework of the aeroplane involves several important considerations. It is obvious that the planes are subject to a considerable tension, and that, if there is to be no bending in

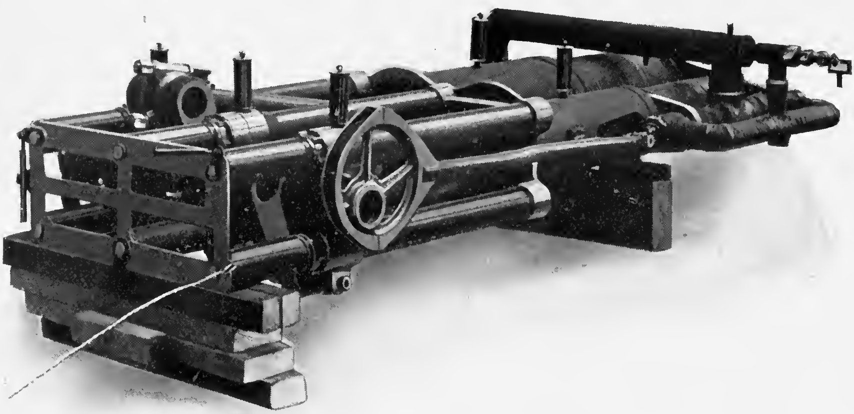


FIG. 16.—Engine of Maxim's aeroplane.

them, the frame pieces must be strong and numerous. Thus if a small plane of area A is stayed at the outer edge by two rods (one on each side), and N is the thrust, there is a turning moment about the centre point $=\frac{N}{4}\cdot\frac{b}{2}$ which has to be balanced by a tension T_1 in the rod, so that

$$T_1 r_1=\frac{Nb}{8}$$

$$T_1=\frac{Nb}{8r_1} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Similarly on the other side there is the balance

$$T_2 r_2=\frac{Nb}{8}.$$

There will be a horizontal tension in the joint at end of strut

$$=T_1 \cos \alpha + T_2 \cos \beta \quad . \quad . \quad . \quad . \quad (16)$$

and a vertical push

$$=T_1 \sin \alpha + T_2 \sin \beta \quad . \quad . \quad . \quad . \quad (17)$$

This will be opposed to the up-thrust N cos θ, so that the force in rod OR equals

$$N \cos \theta - (T_1 \cos \alpha + T_2 \cos \beta) \quad . \quad . \quad . \quad (18)$$

In a similar way all other rods and struts will need to be designed, using the maximum working stresses that can be allowed. Weight for weight steel is as strong as aluminium, so that using good steel rod, breaking at, say, 30 tons in the inch, and using a

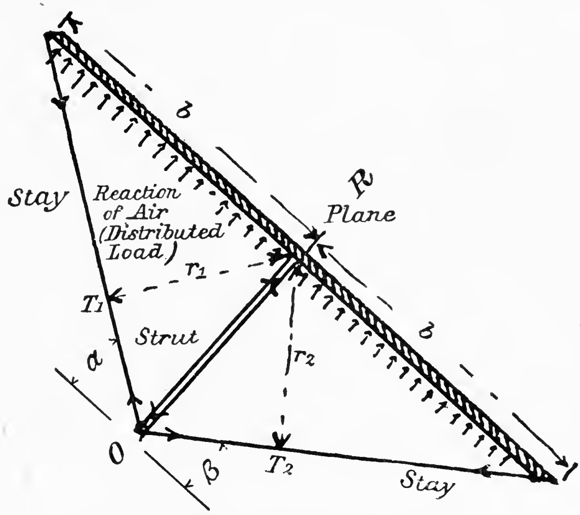


FIG. 17.—Diagram showing stresses in stays of aeroplane.

factor of safety of 5, we can allow 6 tons per square inch. The dimensions must, of course, be cut down to the greatest possible extent. Most machines consist of one longitudinal lattice-framed box girder, the planes being supported on transverse similar girders. The steering surfaces are carried on extensions of these frames. A slight concavity is desirable in the planes, as the eddying behind the plane is thereby increased (as in fig. 18), and there is a greater diversion of the stream-lines. The head resistance cannot be so well diminished, but the lift is greater.

It is, moreover, very likely that such an arrangement causes greater stability in the position of the centre of pressure, which would decrease the vibration of the car.

The relation between the horizontal and vertical velocities being determined by the angle of the plane, so that

$$\frac{\text{vertical displacement}}{\text{horizontal displacement}} = \tan^{-1}(\pi - \theta), \qquad . \quad . \quad (19)$$

by altering the plane we can alter the relative velocities, decreasing the lift and gaining horizontal velocity, or *vice versâ*. This is not exactly true for small angles. (See Lord Rayleigh's paper referred to in Bibliography.)

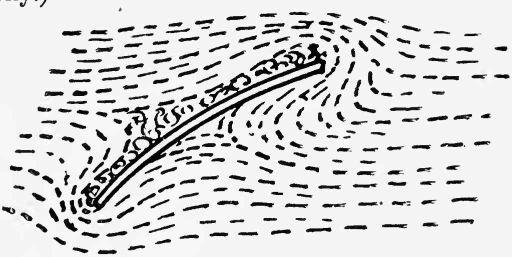


FIG. 18.—Resistance of air to curved aeroplane ("aerocurve").

In all aeroplanes it is desirable that the surfaces should be so controllable that they can make a very small angle with the horizontal. The advantages of this are twofold:—

(1) At certain heights and relative velocity to the air it will be possible to support the plane on the air as a soaring bird does, and at the same time reduce the propelling thrust.

(2) In the event of any accident to the propelling mechanism, a nearly horizontal plane will support the vessel and will whirl down slowly to the ground.

This effect can be seen well if one drops a stiff card from a high place. Last year I made several experiments in this way from a height of about 200 feet. Pieces of stout cartridge paper were allowed to descend. They went down in easy curves without any overturning, and alighted on the earth at distances varying from 200 yards to half a mile. The same thing is seen in the descent of an unweighted kite. (See Appendix, *re* plunging.) It is necessary, however, that the c.g. should lie in front of the centre of area of the plane or planes, if there is to be no final overturning.

It is probable that the safety of heliconefs could be considerably enhanced by the use of such planes, whether permanently or only temporarily in action.

The stability appears to be at a maximum when there is a slight curvature (compare Du Bois Raymond), as then there is the maximum frictional resistance to the air. This curvature is very noticeable in the wings of birds. (See previous page.) Mr Phillips has made many experiments with "aerocurves," and gets an increase of lift amounting to 50% or more with cissoidal curves, the cusp being in front and the versive of the curve about ·1 of the chord. Farman and others put the cissoid the other way on.

Seeing that the efficacy of the aeroplane depends on the velocity with which it moves against the air, it follows that in its simple form it cannot exert any initial ascensional force, and it has been the practice in most experiments to give it an initial velocity by making it descend a slope. The only alternative to this is to fit it with a motor which can give it a forward velocity by friction against a surface: in other words, a locomotive or automobile. The helices will usually be able to propel it along the ground. If this is permanently fitted to the car, its weight would be a great difficulty; and if it were not, only one flight would be possible. We may therefore summarise the question of starting an aeroplane by saying that there are three means available:—

(1) Gravitational acceleration.

(2) Locomotive attachment.

(3) Some lifting device, such as a heliconef, balloon, aviplane—*or* a natural height.

Under the last we have to consider that any aerial arrangement which will lift the aeroplane might just as well be used throughout. As regards height there is the objection that sufficient horizontal velocity might not be attainable during the fall; although, of course, on account of the area of the planes, the descent would be slow. Then again there would be danger of whirling against the side of the precipice or framing from which the descent is made.

Altogether the first method seems the most feasible, although it is certainly unsatisfactory, as considerable space and little friction are required. Sir Hiram Maxim's machine, it will be remembered, was started on guide rails, wheels being fitted to the aeroplane. There is, however, yet another objection to this. If, at the moment of the desired horizontal velocity being reached, there is any adverse air current, the plane will again descend and probably leave the rails.

The acceleration down a plane inclined to the angle θ and with a coefficient of friction $\mu = \tan\phi$ is

$$a = g\frac{\sin(\theta - \phi)}{\cos\phi} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The acceleration will not, however, reach this full value, as there will be air resistance to the planes; so that the total force acting at any moment will be

$$mg\frac{\sin(\theta - \phi)}{\cos\phi} - kv^2\mathrm{A}\sin^2\psi \quad . \quad . \quad . \quad (2)$$

where A is the area of the plane and ψ its angle. (v and k are as before.)

This result is somewhat complicated from the fact that if the actual acceleration is a_0, then $v = a_0 t$, and a_0 is indeterminate from the equation.

We can, however, disregard the back thrust at the commencement of the run and can also find the value of v, at which the gravitational acceleration is neutralised by equating the two terms and solving for v.

Maxim found the lifting moment occurred when the linear velocity was between 30 and 36 miles per hour, but this, of course, depends on the areas of the planes and weight of the contrivance.

If we have a mechanism which will rotate the planes so that just as the critical velocity attained is known by the equation

$$v = tg\frac{\sin(\theta - \phi)}{\cos\phi} \quad . \quad . \quad . \quad . \quad (3)$$

then we do away with the thrust opposing the force of gravity and introduce the lift at the most opportune moment.

Some such controllable rotations will also be an essential feature of a good aeroplane, so that this arrangement should in most cases be possible.

If, as there probably will be, wheels are fitted to the plane, then μ will be the observed coefficient of friction, including the rolling friction at the flanges and the sliding friction at the journals.

In the event of an automobile attachment, the effective H.P. of this arrangement must be such that the machine can be propelled along the ground at a velocity v such that the vertical component of the momental thrust is equal to the whole weight, and, as before, the plane would need to be turned so as to produce a minimum of resistance.

In the case of a drop there will, of course, be a maximum value for the downward velocity $=gt$ or $\sqrt{2gh}$, although the vertical thrust against the planes will never permit this value to be attained.

An approximate value for a_0, the retarded gravitational acceleration, in the case of directly downward fall, is found by the following reasoning:—

$$(W-T)=\text{force}=m(g-a)=ma_0 \quad . \quad . \quad . \quad (4)$$

since $$T=kv^2A \quad \therefore a=\frac{kv^2A}{m}$$

and $$a_0=g-\frac{kv^2A}{m}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

but $$v^2=a_0t,$$

$$\therefore ma_0=mg-mka_0{}^2t^2A \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and $$a_0=\frac{g}{1+kt^2A} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In further considering the resistance which is opposed to the passage of a plane through the air, it must be remembered that the displaced fluid is not wholly forced aside, but tends also to be compressed forwards, a wave of such condensation being propagated at the rate of about 1100 feet per second. Behind the moving body the air rushes in with a maximum velocity of about 1300 feet per second, and as the space is not instantaneously filled with air at normal pressure, the total resistance to the body is further increased.

If the velocity forward is less than 1100 feet per second, the air may be considered as of normal density in front of the surface; and behind the body there will be a partial vacuum of greater or less rarity, according as the velocity is near to the velocity of the air rushing into a space of low pressure.

From this it appears that the resistance to a plane, although ordinarily varying as the square of the velocity, will yet further increase; and so it is found that with a velocity of about 1700 feet per second the back pressure is upwards of three times that which the ordinary rules give. In view of the high speeds which will probably be obtainable when the problem of aviation has been solved, this indicates a very important although apparently paradoxical result,

4

viz.—at very high speeds less thrust and lifting power are necessary than at low ones, and so the area of the bearing surfaces can be diminished considerably, the effect being most marked with small angles of inclination.

It must be understood that this applies only to thin-edged planes.

Mr Curtis gives an algebraic proof of this somewhat startling result, which was first clearly demonstrated by Professor Langley.

If T is the power exerted, R the resistance, and V the velocity, and we take $R = W \tan \alpha$, where W is the weight of the plane and α its angle (a small one), and differentiate the equation

$$T = VW \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad (1)$$

as follows:

$$\frac{dT}{dV} = \left(W \tan \alpha + V \sec^2 \alpha \frac{d\alpha}{dV}\right) \quad . \quad . \quad . \quad (2)$$

then it is obvious that $\frac{dT}{dV}$ (the rate of increase of the power with regard to the velocity) is controlled by $\frac{d\alpha}{dV}$. Experiment, however, shows that V increases as α decreases, so that $\frac{d\alpha}{dV}$ is negative, and therefore the second term of the right-hand side of the equation is also negative, and the greater V is, the less is $\frac{dT}{dV}$.

I have dealt with Langley's work in the Appendix, but I would here point out that it is the *soaring* flight V which increases as α increases, not necessarily the velocity of translation.

Although this result is very interesting, we have to consider that planes alone would probably be insufficient for the practical needs of conveyance, and although M. Drzwiski and others have indicated a very small angle (about 2°), low speeds and massive bodies would probably necessitate a larger angle and a consequently larger horizontal component force. There can be no doubt, from ordinary experiences with kites, that a plane worked steadily against air currents (whether or not produced by its own motion) will do good work, even if large angles of elevation are used, although, of course, the resistance is greater. (Farman's and the Voison machines generally use rather large angles.)

The reason for this lies in the fact that the smaller the angle the less the *quantity* of air displaced, and therefore the less is the momentum derived from it.

While it is undeniable that at small angles we have a most satisfactory proportion between the lift and the horizontal resistance, the former increasing and the latter decreasing, yet only the total of the two can have an appreciable value at low velocities, if the area presented transverse to the direction of motion is adequate.

I have given full particulars of the relation between weight and plane area, but it will be as well to notice the two following formulæ which have been obtained:—

Hannel's Formula:

$$x = y \, . \, \log 500.$$

x = kilogrammes of weight, y = width of plane in metres.

Hartings' Rule: Area of plane (one side of machine),

$$A = 5 \text{ to } 10 \times W^{\frac{2}{3}}.$$

A = area in square centimetres and W = weight in grammes.

As De Lucy's exhaustive figures indicate, however, there is no absolute rule. Seeing that speed of wing is quite as important as area, this result might of course be anticipated.

M. Chanute computes the H.P. at about 5·87 H.P. per ton, and finds that a pigeon exerts 10 H.P. per ton.

Professor Langley's extensive experiments have thrown much light on the subject, and are discussed in the Appendix.

Professor W. H. Dines' extensive experiments give much information and generally corroborate the results given above save that the pressures are by him found to be slightly less.

Reference has been made to the necessity for constant balance. Professor Bryan and Captain Ferber have studied this question analytically, and have come to the conclusion that—

(1) There is, for every aeroplane, a certain critical velocity below which oscillation may occur to a dangerous extent.

(2) Above this velocity (which depends on the weight and dimensions of the aeroplane) stability is ensured if (*a*) the longitudinal moment of inertia about a transverse axis through the centre of gravity is such that the radius of gyration (metres) does not exceed

$$\sqrt{\frac{\text{weight in kilos}}{37 \times \text{lateral spread in metres}}},$$

(*b*) the centre of gravity of the whole machine falls ahead of the general centre of area of the planes, the exact position being found by experimental glides.

(3) Lateral stability is ensured by the use of a keel or a dihedral arrangement of the main planes, the total effective lateral area being at least 10 times the head resistance due to causes other than the lifting thrust, and the said keel being behind and well above the centre of gravity.

CHAPTER V.

ORNITHOPTÈRES.

It has been shown by various students, including Wenham, Marey, Borelli, Mouillard, Ader, and others, that the mere downward displacement does not, as a rule, produce sufficient momentum to cause ascent; but that slow flight by turning the wings towards the direction of motion (an aeroplane effect) gives a vertical thrust which provides support. The famous experiments of Lilienthäl and Pilcher have shown that this is an extremely important factor, and

Fig. 19.—Beetle in flight.

that when once sufficient momentum has been obtained to give the start forward, propulsion or the adaptation to the wind currents will provide nearly all the ascensional force needed.

Professor Langley's aerodrome demonstrates the possibilities of this effect to a remarkable degree. A description of the various experiments with it will be found in *Travels in Space*, by Valentine and Thompson, but I do not find that the general conclusions at all militate against the hypotheses which I give below.

The attention of would-be aëronauts has long been directed to the flight of birds and insects, and it will perhaps be useful to roughly examine the mechanics of a beetle's flight.

It must be assumed that the upward thrust of the air is produced by the downward action of the wings. The weight of a large beetle (I purposely choose the heaviest and direct flying type) is about $\frac{1}{120}$ lb.

Taking the effective area of each wing (*i.e.* the difference of the projected areas in up- and down-strokes) as $A = \frac{3}{4}$ square inch, and V the downward velocity as $n \times l$ where n is the number of strokes per second and l the mean length of the strokes (1 inch), we have T the thrust $= kV^2A$. To balance in the air T must at least equal the weight $\left(\text{cub. } k = \cdot 002288, \text{ say, } \frac{1}{400}\right)$.

$$W = kV^2A = kl^2n^2 \times 2A$$

$$\frac{1}{120} = \frac{1}{400} \cdot \frac{1}{12} \cdot \frac{1}{12} \cdot n^2 \cdot \frac{3}{288}$$

$$n^2 = \frac{400 \times 12 \times 12 \times 288}{3 \times 120}$$

$$n = 215 \text{ beats per second.}$$

This corresponds to an audible note a little below the middle C, and everyone is familiar with the musical notes produced by the flight of insects.

This reasoning is made on an assumed vertical flight, but in the case of ordinary flight there is forward as well as upward motion, so that we have an aeroplane action as well as a simple downward one.

Let us now take the case of the bee, in which the weight is about $\frac{1}{500}$ lb., stroke about $\frac{1}{2}$ inch, and area $\frac{1}{288}$.

$$\frac{1}{500} = \frac{1}{400} \cdot \frac{1}{24} \cdot \frac{1}{24} \cdot n^2 \cdot \frac{1}{288}$$

$$n^2 = \frac{400 . 24 . 24 . 288}{500}$$

$$n = 342.$$

This corresponds to a note just above the middle C (256). In ascending, this speed will, of course, be exceeded. Naturalists are familiar with the gradual raising of the pitch of the note until it ceases to be audible. This point of inaudibility corresponds to a frequency of upwards of 12,000 per second, or 720,000 beats per minute. It should, however, be noticed that the sound is not always produced by the wings.

This will indicate to what an extent natural flying appliances have evolved.

Observation of birds (particularly gulls) indicates, however, that such high velocities are not essential. I have frequently seen gulls supporting themselves with not more than three beats per second, delivered with a firm quick down-stroke and slow up-stroke. Taking the weight at 2 lbs.,

$$2 = \frac{1}{400} . V^2 A.,$$

A is about $1\frac{1}{2}$ sq. ft.

$$\therefore V^2 = \frac{2}{3} \cdot \frac{200}{1} = \frac{400}{3} = 133$$

$$V = 11{\cdot}52 \text{ ft. per sec.}$$

Taking three strokes per second, this indicates that the sum of the mean wing velocity and that of the air current (if any) must equal 12 feet. This is the actual distance moved through, if there is no air current, so that the stroke would be about 2 feet per wing (actually the velocity is not uniform, but greatest a little after the commencement of the down-stroke). If the instantaneous wing velocities are such that their mean value exceeds 12 feet, the stroke need not be quite so long (figs. 20–23).

In Mr Shipley and Professor MacBride's *Text-Book of Zoology* (Cambridge Press), the following description is given of a bird's flight:—

"A bird, when it is in the air, like any other heavy body, is continually falling; the blow of the wing has therefore not only to effect a forward impulse, but also an upward one sufficient to compensate for the distance the bird has fallen between two strokes. These impulses are, of course, derived from the elastic reaction of the air compressed by the down-stroke of the wing. When the wing is expanded it is slightly convex above and concave beneath. This arises from the fact that the quill feathers are attached to the upper edge of the webbed limb and project gently downwards and backwards, so that there is a space left which is bounded by the quills and in front by the bones and web of the limb. Now if this space had a symmetrical shape, the air would be compressed in such a way that the resultant impulse would be directly upward; but it is not symmetrical, for its roof has a very steep slope in front and a very gentle one behind, and the air is compressed in such a way that an oblique reaction results, a reaction which we can resolve by the

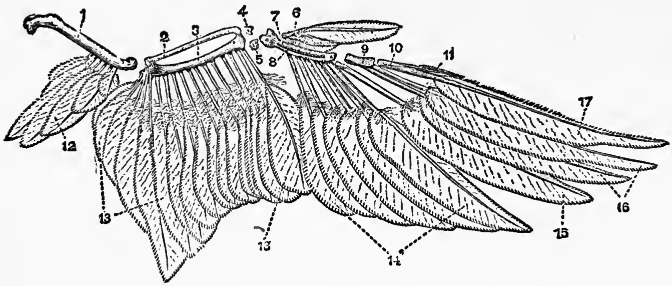


FIG. 20.—Ventral (under) view of left wing of duck, disarticulated and without covert feathers.

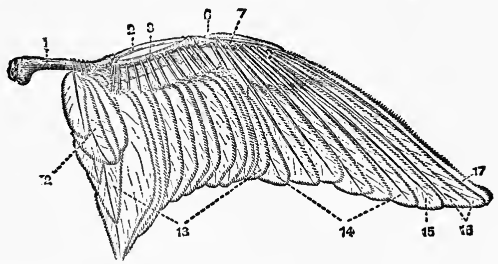


FIG. 21.—Dorsal (back) view of right wing of duck (covert feathers removed).

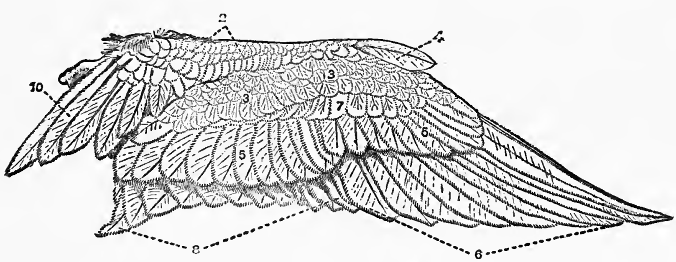


FIG. 22 —Ventral view of left wing of duck.

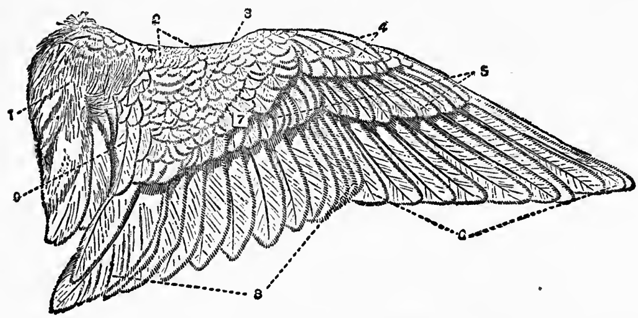
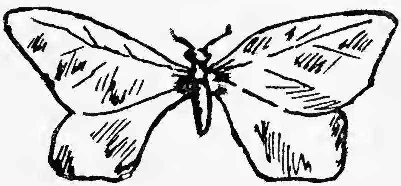


FIG. 23.—Dorsal view of right wing of duck.

parallelogram of forces into an upward and an onward one. So much for the flight of a bird in still air. The air is, however, very rarely still, and the currents which exist are never quite horizontal, but generally inclined slightly upwards, since the lowest layer is checked by friction against the ground, and the birds which are good flyers can, by inclining their wings at the proper angle, obtain quite sufficient support from the play of the current against the wing without exerting themselves to any great extent. This is called soaring, and can be beautifully seen in the flight of the seagull. In this manœuvre the birds are assisted by the tail."

I am inclined to think that too much importance is here attached to the elasticity of the air. The reaction against the wing is rather more a question of momentum than elasticity. Although it is true that a sudden impulse produces a compression and consequent back



Wings Vibrate Transversely

FIG. 24.—Butterfly in flight.

pressure (in accordance with Boyle's law), this pressure can only have a large value when the air is confined. The only resistance to displacement is the pressure of the surrounding atmosphere, except in the direction of the wing, and the air will rather tend to escape than to be compressed. This is, however, a problem which can be attacked only by experiment, as the curious air currents produced will not be determinable by any simple mathematical treatment. With the concave plane, however, it is certain that compression will somewhat add to the forward thrust.

The flying apparatus of the class of insects including the butterfly and moth gives another example of the aviplane.

The weight W $=$ about $\frac{1}{2000}$ lb.
Wing area $=$ about 2 sq. inches each.
Length of stroke $=$ about 1 inch.

$$\frac{1}{2000} = \frac{1}{400} \cdot \frac{1}{12} \cdot \frac{1}{12} \cdot n^2 \cdot \frac{2}{144}.$$

This corresponds to a frequency of 47 per second, which is more than is usually observed.

The wing area varies, according to M. de Lucy, between the following limits:—

49 sq. feet per lb. (gnat).
0·44 „ „ (Australian crane).

Enough has been said to show the relation between the strokes, area, and reach of the wing, but we have to consider also the fact that the wings are so tilted that there is a minimum resistance in the direction of the resultant momentum (*i.e.* compounded of the vertical and forward thrust).

We thus find that the flight of a bird or insect consists of two component actions on the down-stroke and two on the up-stroke.

(1) Downward thrust, from which is obtained the ascensional force.

(2) Forward thrust, which is combined with (1), so obtaining the aeroplane effect.

(1) and (2) together constitute the down-stroke which provides the lift.

(3) Rotation of wing from an inclined plane into a nearly horizontal plane.

(4) Upward movement of limb.

(3) and (4) together form the up-stroke, which, by drawing against the air current of the down-stroke, gives forward propulsion.

In the case of a bird's flight on the up-stroke, the feathers are thought to be partially separated, so that, if there is any part of the wing traversed by the air, the resistance will be as small as possible.

On the question of balancing, the bird well illustrates the principle I have repeatedly referred to of the coincidence of the centroid with the axis of the thrusts. A flying bird extends its neck forwards and spreads the tail so that the whole body is balanced about a point midway between the wings. The wings are readily adjustable, so that the turning effect of the air-thrust in one balances that from the other, and horizontal equilibrium is also maintained by moving the tail feathers.

There are thus two couples acting on the bird, and the algebraic sum of them should be zero.

In plan we have the forces shown in fig. 25, the arrows indicating the position of the resultant thrust on each wing and O the centroid.

In elevation we have the same two forces passing diagonally through the plane of the centroid, and also a zero moment of weight about the centroid. By altering the wing area and plane the bird can momentarily effect a deviation from the line of motion, and by modifying the frequency can ascend or descend.

All the flying actions proceed in all probability from the reflex action of the lower nerve centres, so that, unless the bird requires

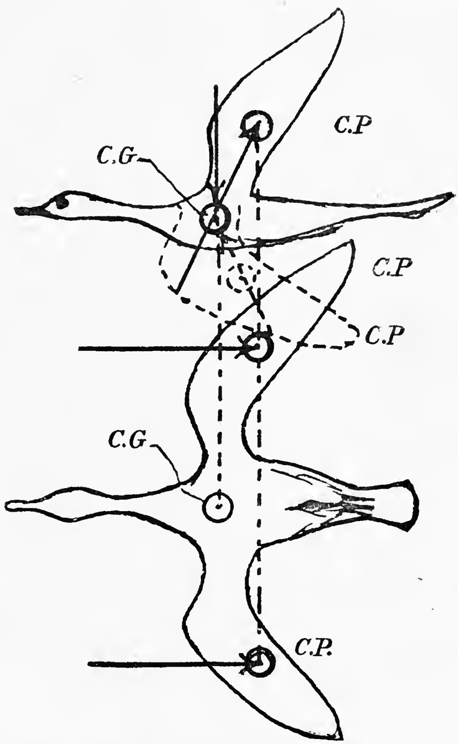


Fig. 25.—Under and side views of flapping bird, showing the coincidence of the resultant thrust with the centroid.

to alter its direction or velocity, no mental effort is needed until it begins to become exhausted.

Professor J. Bell Pettigrew, M.D., F.R.S., made a series of very elaborate researches into the flight of birds, the results of which were summarised in two papers read by him in 1867 before the Royal Institution and the Linnean Society. They were published in the Proceedings of these societies, and in his book *Animal Locomotion.* The points he established were chiefly the following:—

(1) The wings act downwards and forwards in the lifting stroke and upwards and backwards in the propelling stroke.

(2) The wing tip, wing roots, and the centre of gravity of the body each describes a sine curve during flight, the curve of the tip being half a period different in phase from that of the body, and the root curve about a quarter of a period.

Hence there is a helical motion in the wing both in regard to the locus of the points in their forward motion and also laterally, so

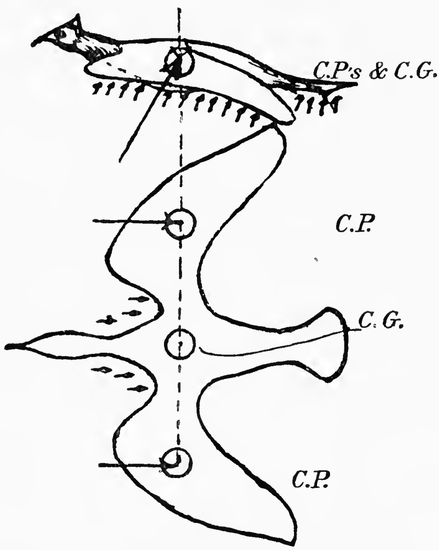


FIG. 26.—Under and side views of a soaring bird, showing coincidence of resultant of thrusts with centre of gravity.

that the wing may be said to screw and unscrew during flight, thus getting a maximum grip of the air.

The following diagram (fig. 27) of a period indicates well the wing changes:—

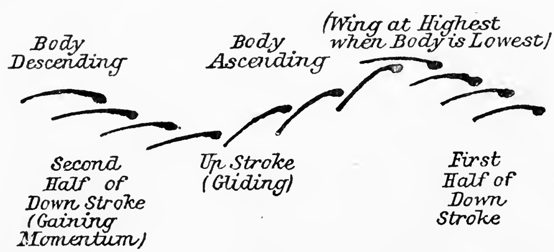


FIG. 27.—Wing cycle (according to Professor Pettigrew). (Modified.)

(3) The effect of wing largely depends on the resilience of the surfaces and their ribs.

These conclusions have been, to some extent, substantiated by Professor E. J. Marey, whose results appear in the *Revue de Cours Scientifique de la France et de l'Étranger*, 1869, and are given below.

One of the most important points considered to be established by Professor Pettigrew is the value of elasticity and the helical action of the wings of birds, and his following remarks on the subject are very noteworthy:—

"That the wing twists upon itself structurally, not only in the insect, but also in the bat and bird, anyone may readily satisfy himself by a careful examination: and that it twists upon itself during its action I have had the most convincing and repeated proofs. The twisting in question is most marked in the posterior or thin margin of the wing, the anterior or thick margin performing more the part of an axis. As a result of this arrangement the anterior or thick margin cuts into the air quietly, and as it were by stealth, the posterior one producing on all occasions a violent commotion, especially perceptible if a flame be exposed behind the vibrating wing. Indeed it is a matter for surprise that the spiral conformation of the pinion and its spiral mode of action should have eluded observation so long; and I shall be pardoned from dilating upon the subject when I state my conviction that it forms the fundamental and distinguishing feature in flight, and must be taken into account by all who seek to solve this most involved and interesting problem by artificial means."

The above quotation from *Animal Locomotion* synthesises the professor's conclusions, and I insert it here as indicating the rigid uniplanar aeroplane to be not necessarily the last word on the subject. I have described above the motion of a bird's wing as studied by the same expert. I think it is almost entirely agreed among aviators that some measure of elasticity and helical motion in aeroplanes will vastly enhance their practicability, so that his results are important.

In connection with aviplanes it is important to notice a result obtained by Sir G. Cayley many years ago, viz.:—

"In very acute angles with the current, it appears that the centre of resistance does not coincide with the centre of its surface but is considerably in front of it. As the obliquity of the current decreases these centres approach and coincide, when the current becomes perpendicular to the plane: hence any heel of the machine backwards or forwards removes the centre of support behind or before the point of suspension." I referred to this in the last chapter, and gave a value for the displacement.

This is a consideration which will vitally affect the stability of any system of aviplanes, as the reaction will tend to produce a turning moment upon the vessel with a consequent backward heel. This heel will continue until, on account of it, the centre of pressure has returned to a position where stability is obtained.

If there are two planes, only slightly inclined, with the supported weight between them, the displacement of the centre of pressure could be allowed for by a jockey-weight travelling forwards so as to shift the centroid to a position corresponding with the resultant upthrust. This is done in M. José Weiss' new artificial bird.[1]

The actual motion which occurs in an aeroplane is best studied by observing the motion of a kite. The kite string, when pulling the kite against an air current (already existing or produced relatively by drawing the kite against the air), acts in much the same manner as the thrust of a screw behind an aeroplane would do, and the motion of the kite may roughly be considered as compounded of a sliding motion up the inclined plane (of air) on which the surface is acting, and the downward acceleration due to the weight of the supported body.

A similar motion will be assumed by the aviplane, but it is affirmed by Professor Pettigrew that there is only an appreciable buoyancy when the plane attacks the air at a variety of angles and possesses considerable elasticity throughout. Whether this is an absolute essential to flight (many experimenters do not seem to think it is), practical experience only can tell. It is, however, certain that a more or less unchanging plane can buoy up a considerable weight, but there is little doubt that an elastic and curved surface is more efficacious than a rigid and plane one.

If a non-rigid plane is used the front edge must be the strongest, and the extent of the plane laterally should be considerable. It must possess considerable elasticity in all directions, and preferably be slightly twisted on itself, so that the sections of the surface are helices.

The back edge can be rendered very pliable by having the

[1] In those I have seen, made by this inventor, tractors, not propellers, are used, but the general arrangement is shown in the illustration (fig. 31A).

material of decreasing thickness or making the stays upon which it is framed of decreasing section towards the edge.

A vast amount of useful and interesting information on the subject of flight is contained in Professor Marey's work on *Animal Mechanism.* He contests Professor Pettigrew's conclusion as to the analogy of the wing to the screw propeller, and while admitting that the wing does rotate during its motion, contends that the angular displacement does not warrant its comparison with a screw propeller. Furthermore there is a reverse rotation during the return stroke, which would invalidate the action of a screw propeller. Nevertheless,

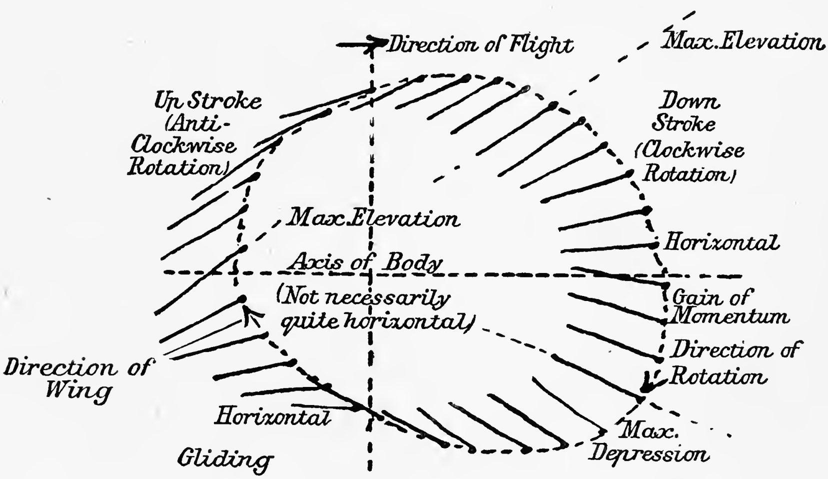


FIG. 28.—Wing cycle, according to Professor Marey. (Modified.)

the results he obtains as to the paths of the wings of birds and insects appear to agree fairly well with those of Pettigrew (fig. 28). Marey's experiments were all made with recording apparatus, and are therefore more reliable.

A careful comparison of the two sets of results does not show much appreciable difference, and I think we may fairly say that these two experimenters, although they approached the subject from diverse points of view, have together firmly established a knowledge of the motions of wings.

The chief difference between the two lies in the large importance attributed to elasticity of wing and ligament by Pettigrew, which is not referred to at any length by Marey, although he allows that it is largely responsible for the change of angle (torsion) in the plane of wing.

In regard to Professor Marey's experiments, it cannot be denied that they are far more scientific in method than those of Pettigrew. As far as possible every cause of error was eliminated, and actual records of the various motions were taken by him with pneumatic and electric apparatus. Professor Pettigrew, on the other hand, seems to rely almost entirely on observation, which, on account of the great velocity of the wing, is very difficult.

The apparatus used by Marey was of such a character as to measure each motion independently of others, and the only flaw that appears anywhere is in that he seems to overlook the irregularity of angular transmission through the Cardan or Hooke's joint. This would not, however, be of any great importance for the small angles measured, and I think it can be fairly said that he and Pettigrew have exhausted the subject from an experimental point of view.

The whole principle of flight appears to lie in the fact that a curved surface tends to glide in the direction of curvature, or, in the limit, an inclined surface slides in the direction of inclination.

Hartings (in *Archives Néerlandaises*, vol. xiv. p. 1869) taking

a — area of surface (sq. centimetres)
p — weight of body (grammes),

finds that for each wing approximately

$$\frac{\sqrt{a}}{\sqrt[3]{p}} = 2{\cdot}25,$$

or 5·0 for the two wings, and we thus fix a relation between weight and wing area. Mullenhoff and Weiss have arrived at similar rules.

To conclude these researches into natural flight, we may collect the chief points:—

Borelli—" Wing acts on air like a wedge."
Strauss-Durkheim—" Wing acts as an inclined plane."
Liais—" Effective in lifting on both up- and down-stroke."
Count d'Esterns—" Support possible without motion of wing."
De Lucy—" Definite relation between weight and area."
Hartings—" Square root of area varies as cube root of weight."

Other quotations of similar character could be given, but these represent what may be considered the axiomatic truths indispensable to any understanding of the subject of natural flight, which must always be a far less simple matter to comprehend than at first sight it would appear.

Penaud, Marey, and Pettigrew—" An effective artificial wing can be formed acting on the same principles as a bird's wing."

A man does not possess either the physical or the mental energy necessary to produce these motions by any mechanical appliance, and any means of aerial locomotion on the aviplane principle will need to be driven by some prime mover, but can soar on an air current (Lilienthäl).

A simple fan-wheel (fig. 29), the blades of which rotated in a

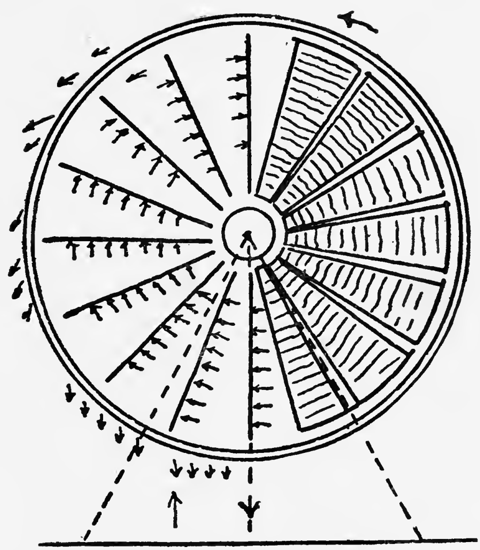


Fig. 29.—Elevation of vertical fan, the blades of which exert a downward force in the air.

vertical plane in the upward motion and turned horizontal in the downward motion, would serve to produce an upward thrust. The resistance to the air would be as before. The tipping could easily be arranged by guides and $T=kV^2A$, although it is probable, if the fans were close together, the value of k would be less than $\frac{\rho}{g}$, by reason of the back momentum from the following fan.

This momentum would have its resultant practically at the centroid of each fan, and would act along a semicircle concentric with the wheel, the base of the semicircle being vertical. The resultant upward thrust could approximately bear to the total thrust the proportion $\frac{1}{\pi}$, so that, if there are n fans each of area A and half act in the air at once, the total upward thrust would be

$$T = \frac{kV^2 \cdot nA}{\pi} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the normal thrust against the fans being

$$P = kV^2 nA \quad . \quad . \quad . \quad . \quad . \quad (2)$$

V, the velocity, would be the mean velocity of each fan, *i.e.* the velocity of its c.g. The angular velocity of the wheel would be $\omega = \frac{V}{r}$ if r is the radius from the centre of gravity of the fans to the axis; so that the torque on the shaft is

$$Pr = kV^2 rnA \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the work, excluding the frictional resistance at the bearings, is

$$W = kV^3 nA \text{ or } kr^3\omega^3 nA \quad . \quad . \quad . \quad . \quad (4)$$

which, divided by 550 ft. lbs. per second, gives the H.P. the wheel must develop after all internal losses. It would be necessary, of course, to find what value k reaches with different degrees of closeness of the vanes.

The conditions assumed are ideal, and it is doubtful if such great values would be obtained for T. Moy's aerial steamers were designed on this principle, but were underpowered.

The horizontal thrust of this appliance is zero, and, if we combine it with horizontally acting helices, and shield the wheels from the horizontal action of the wind, a possibly practical contrivance could be made.

It would probably be necessary to have at least four of these wheels arranged in a rectangle, and with level governors controlling their speeds in two directions, thus :—

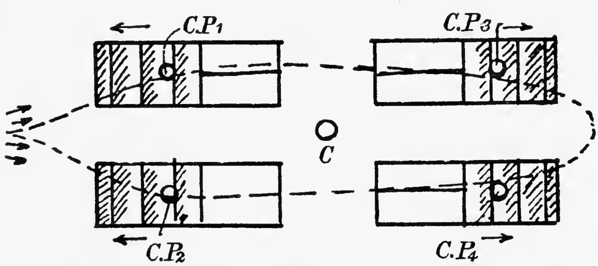


FIG. 30.—Plan of fans for airship, showing condition of equilibrium.

In this way each wheel in a pair would balance the other and produce the desired condition of zero-moment.

The centroid for equal wheels would be equidistant from the four centres, and to ensure the vertical balance, the axis of the drawing helix, or the direction centroid of the helices, if more than one, would have to pass through this general centroid.

An aviplane (*i.e.* a plane which moves like a bird's wing, *pteron* or *aïla*, would be a more suitable word), resembling even more closely a natural wing than the one just described, has been a favourite with many inventors. It consists of a rectangular or triangular membrane, plane or slightly convex above, stretched on a frame which is hinged at the inner edge. By means of rods an oscillatory motion is given, the tips describing an arc about the hinge. By constructing this plane to the correct angle, it has been hoped that

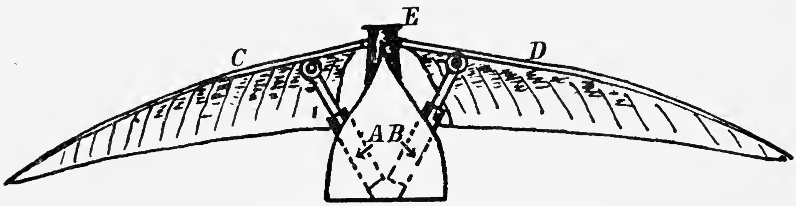


Fig. 31.—End view of mechanical bird or "aviplane." At E there are controllable guide cams which depress and rotate the wing ribs.

flight would be possible. I am not aware that an appliance of this kind (except Major Moore's "Flying Fox," which only hops) has been in use of recent years, with modern sources of power, but even if arranged in the most efficient manner, there must be a difficulty in balancing it.

Fig. 31 shows the manner in which I suggest the motions can be arranged.

A and B are two reciprocating engines which alternately depress and raise the wings C and D, of which the vertical projections are seen.

The guides are so arranged that the plane descends forwards and then at the end of the stroke commences to slowly rotate so that near the end of the upstroke it is horizontal. Better results would be gained by using elastic ribs to the plane (see previous description of bird flight).

(Cf. models made by Penaud, illustrated in the *Encyclopædia Britannica*, 9th edition, article "Flight.")

Owing to the shifting of the centre of pressure, longitudinal balancing will be very difficult, and lateral balancing almost impossible.

The thrust can be computed as before. The only suggestion I can make as to balancing is that a number of small helices should be employed which shall mutually govern each other by a level, as before, their difference of thrusts acting against the difference of thrusts that may occur on the aviplane. I am inclined to think, however, that the whole arrangement is needlessly complicated,[1] and that a solution of the problem is more probably to be found in the simple use of aeroplanes or helices or both.

[1] *I.e.*, having in view the fact that the motive force does not, as is the case with animals, have to be applied in a very limited space.

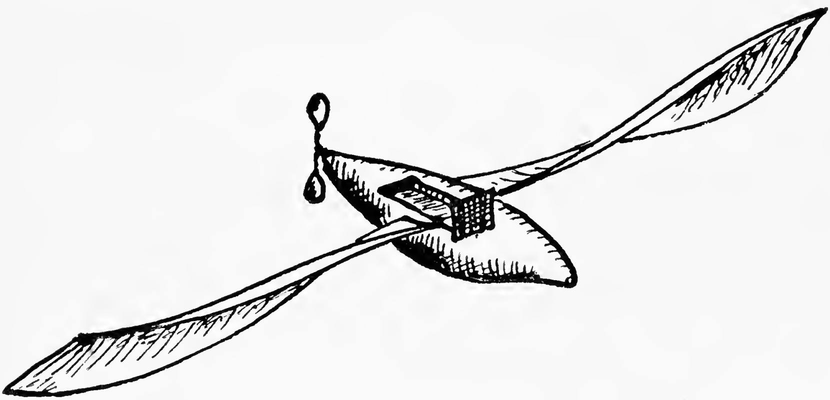


FIG. 31A.—Weiss aviplane.

CHAPTER VI.

DIRIGIBLE BALLOONS.

THE difficulty of manipulating and balancing flying machines heavier than air early led to research into the subject of flotation by means of light gases enclosed in an envelope, to which was attached a car for the aeronaut. The Jesuit Francis Lana in 1670 proved the possibility of aeronautics by considering the relative pressures of gases, but there was no practical result until the Montgolfier brothers made their fire balloon in which a quantity of heated air, in virtue of its expansion, was employed to lift a small car.

The fire balloon is now extinct, as the advance of chemical science has enabled us to obtain, fairly easily, gases which far exceed expanded air in their buoyancy.

The principle upon which this buoyancy depends is known as that of Archimedes, and is stated as principle (β) in Chapter II.

By this we have the following equation when the balloon is just about to move:

$$W+V\frac{\rho}{n}=w+V\rho \quad . \quad . \quad . \quad . \quad (1)$$

where W = weight of balloon, envelope, car, and aeronauts.
V = volume of balloon.
ρ = density of air.
n = density of gas as compared with air.
w = weight of air displaced by car and aeronauts and envelope of balloon.

If we call the weight of the gas in the balloon M, then we can write this equation in the following manner:

$$W+M=w+nM \quad . \quad . \quad . \quad . \quad . \quad (2)$$

from which we find that

$$M = \frac{W - w}{n - 1} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$V = \frac{W - w}{\rho} \cdot \frac{n}{n - 1} \quad . \quad . \quad . \quad . \quad (4)$$

thus obtaining the volume of gas required. If the volume of the gas-bag, car, aeronauts, etc. $= v$, then $w = v\rho$; so that (4) may be written

$$V = \frac{W - v\rho}{\rho} \cdot \frac{n}{n - 1} \quad . \quad . \quad . \quad . \quad (5)$$

The following practical rules are used in this connection:—

(*a*) One cubic foot of air near level of sea weighs $\frac{1}{13}$ lb.

(*b*) One cubic foot of hydrogen weighs $\frac{1}{14}$ of the weight of one cubic foot of air at the same pressure (*i.e.* $n = 14$).

(*c*) 1000 cubic feet of coal gas will lift 40 lbs.

The ascensional force of a gas n times lighter than air is, of course,

$$A = \rho\left(1 - \frac{1}{n}\right)$$

where ρ is the density (weight in lbs. per cubic foot).

If then this force $\left(\text{total} = V\rho\left(1 - \frac{1}{n}\right)\right.$ practically) is applied to the weight of the balloon $(W - w)$, the upward acceleration follows from the mechanical law $P = ma$, *i.e.*,

$$V\rho\left(1 - \frac{1}{n}\right) = \left(\frac{W - w}{g}\right)a,$$

or

$$a = \frac{V\rho\left(1 - \frac{1}{n}\right)}{W - w} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

We next have to consider the differences due to height and temperature. It is here necessary to remind ourselves of Boyle's law:

$$p \propto \frac{1}{v}, \text{ where } p \text{ is pressure and } v \text{ is volume.}$$

Also Gay-Lussac's law:

$p \propto \theta$, where p is pressure and θ is temperature ("absolute").

When air expands without losing its heat

$$pv^{\gamma} = \text{constant},$$

or

$$\rho = cp^{\frac{1}{\gamma}}$$

where γ is the ratio

$$\frac{\text{specific heat at constant pressure}}{\text{specific heat at constant volume}} = 1{\cdot}414 \text{ for air.}$$

This condition ($\rho = cp^{\frac{1}{\gamma}}$) is termed the state of Convective Equilibrium, and observation has shown that it is fairly maintained in the atmosphere. It is also found that the temperature decreases in the same ratio as the height. From a consideration of the equation $\rho = cp^{\frac{1}{\gamma}}$, it is calculated that the thermometer falls (see Greenhill's *Hydrostatics*)

1° C. in 336 feet.
1° F. in 186 feet.

It will then appear that if the balloon is completely inflated on the ground and is prevented from rising by N lbs. of ballast, neglecting the weight w,

$$W + N = VA.$$

The removal of the ballast will cause the balloon to rise up a part of the air when the density is only ρ_0, the ratio of ρ_0 to ρ being

$$\frac{\rho_0}{\rho} = \frac{W}{VA} = 1 - \frac{N}{VA} \quad . \quad . \quad . \quad . \quad (7)$$

If $\rho_0 - \rho$ corresponds to $\theta°$ C., then the height $= \theta°. 336$ feet.

To rise to an additional height h, where there is further diminution of density a, ballast b must be thrown out.

$$b = VA\left(\frac{a}{\rho}\right) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

As mentioned in Chapter I., in order to obtain a minimum resistance to the air, the balloon is made cigar-shaped, *i.e.* the figure of revolution made by a segment turning about its chord, and

Molesworth gives the following proportions in terms of the middle diameter :—

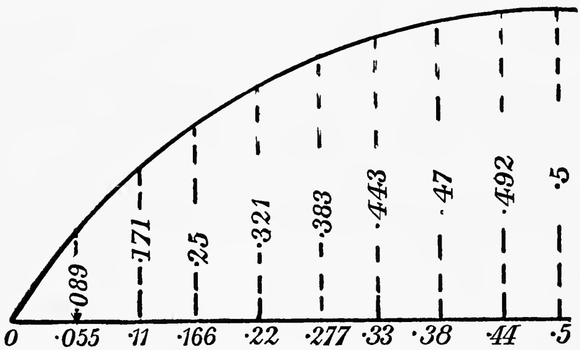


FIG. 32.—Method of setting out balloon sections.

Cylindrical forms with rounded ends are also employed in Dr Barton's and Santos Dumont's airships, but the general balance of opinion is in favour of the pointed ends.[1]

The following rule is used to determine D, the mid-diameter :—

$$D = \sqrt[3]{\frac{W}{\cdot 5236\rho\left(1 - \frac{1}{n}\right)}} \qquad . \quad . \quad . \quad . \quad . \quad (9)$$

from which it also follows that

$$W = \cdot 5236\, D^3 \rho\left(1 - \frac{1}{n}\right) \qquad . \quad . \quad . \quad . \quad . \quad (10)$$

The resistance to propulsion is of the now well-known form

$$R = kV^2A,$$

where A is the mid-section, and $A = \frac{\pi}{4} . D^2$;

so that we have the rule $\quad R = f . D^2V^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$

where f is ·0002 and V is speed in feet per second.

The rules already given for helix propulsion apply again.

The axis of thrust must pass through the centroid, which will usually be nearly the base of the gas-bag, if there is a semi-rigid frame; if not, through the car.

By using fore and aft propellers the thrust can more conveniently be controlled, and by using propellers in lateral pairs with governors level can be maintained.

[1] An air-bag in the balloon which can be filled from a fan in the car is always now employed, since it serves to maintain the form of the gas envelope and also to control the lift by increasing the total weight.

The helices should have an aggregate effective diameter approaching that of the balloon.

The weight of balloon material (varnished silk) is about $\frac{1}{40}$ lb. per square foot, and the silk cordage is used.[1]

If the balloon is not to lag behind the car (this will cause an alteration in the position of the centroid), a stiff frame will be required, which can be made partly of steel rods and stay ropes. The aggregate area of these rods must be sufficient to resist a vertical stress equal to the weight of the car, and have a shearing resistance equal to the total air resistance at the maximum relative velocity.

The gas used is either (A) fairly pure hydrogen, or (B) coal gas.

The ascensional forces from each have been given. Hydrogen may be generated by passing steam over red-hot iron and drying the effluent. Sodium acting in water also produces about 610 times its own volume of hydrogen; but, being very expensive, would only be used in an emergency.

Electrolysis of water will produce hydrogen at the positive pole, but the quantity is small, being ·010384 milligramme per coulomb (coulombs = amperes × seconds).

The quantities producible from the former two processes can be found by the following chemical formulæ:—

Steam on Iron:

$$3\,Fe + 4H_2O = Fe_3O_4 + 4H_2.$$

As the atomic weights are: Fe = 56, H = 1, and O = 16, then 168 parts by weight of iron produce 8 parts by weight of hydrogen, when acting on 72 parts by weight of steam.

If the iron is placed in a fire-clay or wrought-iron tube heated in a furnace and steam passed through it, hydrogen will issue at the far side.

Sulphuric Acid on Iron:

$$H_2So_4 + Fe = H_2 + FeSo_4.$$

The gas must be dried and scrubbed before use in tanks.

Sodium in Water:

$$Na + H_2O = NaHO + H,$$

so that, since Na has an atomic weight of 23, each 23 parts by weight of metallic sodium liberate 1 part by weight of hydrogen. Great care is required with this process, as the liberated hydrogen

[1] Modern dirigibles such as the Zeppelin and Lebaudy airships have longitudinal or transverse braced steel frames in the envelope to give rigidity and maintain the form.

is in a chemically "free" state when dissociated, and will fire with atmospheric oxygen if the latter is not excluded.

Ballast or "délestage" is also needed for a balloon, and may consist of sand in bags or pig iron. Some discretion is certainly necessary in liberating it, both with regard to the alteration produced in the weight, and also with regard to the results likely to happen on the ground. Sand, if slowly thrown away when there is an air current, will be dissipated in the air, and will not produce any injurious result on the earth. Special sand distributors can now be purchased.

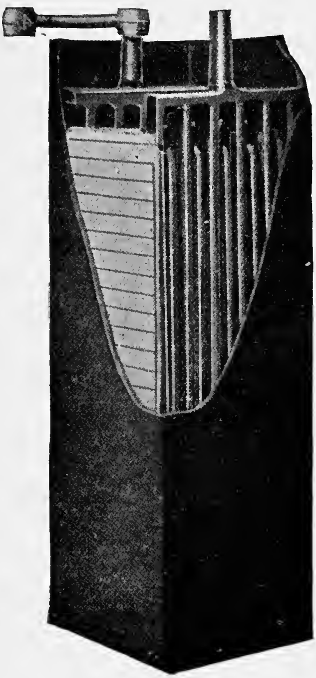


FIG. 33.—E.P.S. Accumulator (traction type).

The steering of balloons may be effected by the differential action of helices or by rudder planes.

Rudder planes may be trapezoid or triangular, and suspended on one of the rods connecting the car to the balloon frame. The mechanics of steering are referred to in the next chapter.

For the purposes of balancing and descent balloons have been constructed in sections separated by gas-tight partitions, each communicating with the other by a tube with valves.

These valves are controlled by cords from the car, so that each section may be wholly or partially exhausted to the air, or the gas in one may be admitted to another at a lower pressure.

This arrangement also serves as a safety appliance in the event of the balloon bursting, as it could rarely happen that all the sections would fail, and the unexploded ones could give a partial buoyancy and prevent disaster.

The machinery and the accommodation for aeronauts is usually in the base of the car, although, if possible, the motor should be on the driving shaft, which, if there is a quasi-rigid frame, has to pass through the centroid, and will then be at a higher level.

It is desirable that the car should be covered so as to present a minimum resistance to the wind somewhat in the manner described in the next chapter.

Small aeroplanes revolvable about horizontal axes are also used for controlling the ascent in the French and English military dirigibles.

I am inclined to think the use of the open car with numerous irregular surfaces (including the bodies of the aeronauts) has to some extent been the cause of the poor success of the dirigible balloon. Efforts have been concentrated to forming the balloon of a minimum resistance, but the car has in many cases been neglected, and certainly when it is not running level its impedance will be great. Even simple rods and stays cause considerable air resistance. The prime mover for the helices is either a heat engine (steam or oil) or an electric motor.

I give below some particulars of the weight and fittings for electric motors, but it is quite well realised now that this is the heavier source of energy; but its advantages in other respects are very numerous (such as sensitiveness, safety, starting, torque, etc.).

The secondary cells, which accumulate chemical energy in such a form that it can be reconverted into current, are generally of the Electric Power Storage Company's type, with an E.M.F. of two volts.

Accumulators (fig. 33).—The E.P.S. secondary cell is principally used for generating current for electric motors (continuous type).

The relative values of the different cells supplied by the Company can be estimated from the following table :—

K. High discharge type. L. Medium type.
WS. Higher discharge (short periods).
P. Large power cell. C. Lighting.
T. Propulsion.

The class T would appear to be the most generally suitable, and the following particulars will be useful :—

No. of Plates.	Material of Box.	Acid.	Charge.	Discharge.	Capacity.	Height.	Weight complete.
		lbs.	amperes	amperes	ampere hours	inches	lbs.
15	Teak	14	24–28	1–30	95	$13\frac{1}{8}$	52
	Ebonite	14	24–28	1–30	95	$12\frac{3}{4}$	42
23	Teak	22	38–42	1–50	145	$13\frac{1}{8}$	79
	Ebonite	22	38–42	1–50	145	$12\frac{3}{4}$	66

Remembering that 746 watts per second = 1 H.P., and that the cells are run at about 2 volts, we get for the 23 plate "T" cells 100 watts per cell available for about $2\frac{1}{2}$ hours (cells must not be worked right out), so that 8 cells would be required per H.P. working $2\frac{1}{2}$ hours. This would also be subject to the voltage at which the motor was designed to be driven, so that, if the cells were arranged in series to drive a 50-volt machine, at least 25 cells would be necessary with a total power per second of 2500 watts (at 50 amperes), or a little over 3 H.P., lasting $2\frac{1}{2}$ hours. For a longer time the number of cells must be increased proportionately, and a glance at the last column shows the result of this on the weight to be carried.

The motor would need to be axle-hung (*i.e.* direct coupled to the driving shaft) and with multipolar armature. Starting rheostat, switches, and friction clutch would complete the set.

The efficiency of a well-designed motor is about 90 per cent., so that we have the following train of energy loss:—

Charging machine, which supplies energy to	
Secondary cells, to amount of, say,	100 per cent.
Connections	(loss practically nil).
Motor	loss about 10 per cent.
Shaft journals	2 or 3 per cent. further loss.
Helix	unknown loss due to slip.

$$\left(\frac{V-v}{V} \pm x\right)$$

V = propeller velocity (*pr*).
v = vessel's velocity.
x = variable extra slip.

It will thus be seen that the weight of electrical machinery (which even at the lowest limit comes to upwards of 1 cwt. per H.P.) involves considerable increase in the ascensional force required. (Weight of motor per H.P. on a 100 H.P. machine is about 44 lbs.)

For this reason, until some lighter and more convenient means of generating electrical energy can be discovered, petrol engines will be most suitable for propulsion.

(The crux of the problem of reducing the weight of accumulators lies chiefly in the substitution of some lighter metal than lead for the plates.)

In connection with the power necessary for propulsion, the figures of Captain Renard will doubtless be useful.

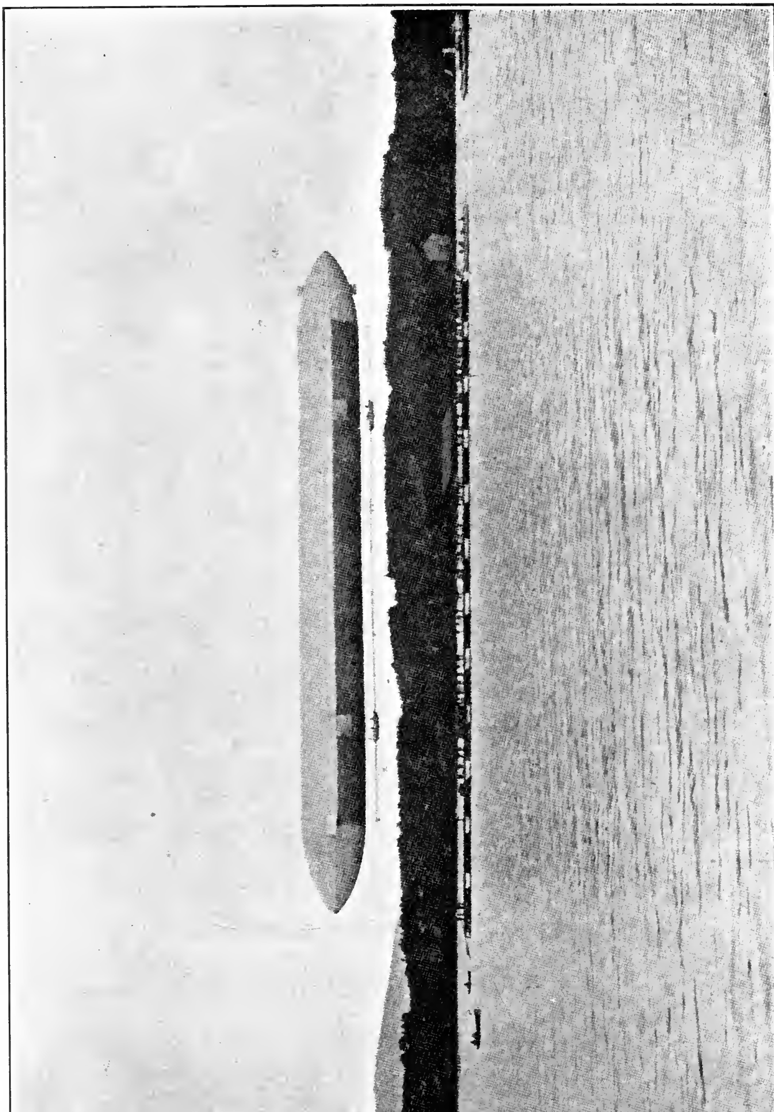


Fig. 33a.—The Zeppelin dirigible balloon sailing above Lake Constance.

Taking R = resistance, U = power, U′ work on screw shaft, D = diameter of propelled body (assumed circular in section), and v the velocity, in metric units, the following relations are given:

$$R = 0{\cdot}01685\, D^2V^2 \quad . \quad . \quad . \quad . \quad (1)$$
$$U = 0{\cdot}01685\, D^2V^3 \quad . \quad . \quad . \quad . \quad (2)$$
$$U' = 0{\cdot}0326\, D^2V^3 \quad . \quad . \quad . \quad . \quad (3)$$

(*I.e.* efficiency of machinery, 50 per cent.)

An analogous formula for the resistance in English engineers' units is Pole's

$$R = 0{\cdot}000195\, d^2v^2 \quad . \quad . \quad . \quad . \quad (1)$$

Where d is diameter in feet of propelled body, v is velocity in feet per second. (Compare Molesworth's rule, Chapter III.)

Captain Renard estimates the power needed for a balloon 33 feet in diameter, to travel 22 miles per hour, at 45 H.P.

The trials which were made with Renard and Kreb's balloon seem to indicate the accuracy of their calculations, and recent experiments in France (1906) with the large balloon "La Patrie" have further demonstrated the feasibility of propelling balloons against moderate winds.

Sir H. Maxim, in his extensive experiments, found that the friction against the surfaces of propelled bodies was almost imperceptible, and this result accords with Professor Unwin's figures given elsewhere.

Professor Thurston confirms these conclusions from his own experiments, so that we need only consider as important the head resistance as given by Molesworth or Renard. On this point, however, see next chapter.

CHAPTER VII.

FORM AND FITTINGS OF AERIAL VESSELS.

THE form of the ship, once the minimum of accommodation has been determined, depends to a very great extent on the resistance.

Experiments and marine practice show that the resistance to the motion of ships arises from three causes:

(1) The skin friction.

(2) The formation of eddies.

(3) The formation of waves.

The last cause disappears in the airship, depending as it does on the existence of a surface of separation between two fluids of different densities. Any energy which would in a ship be expended in this direction will in the case of an airship appear as eddy resistance.

The resistance to the motion of a ship in water is well illustrated by an experiment in which a cylinder or parallelepiped is drawn through water (see fig. 12).

The fluid in the immediate proximity of the moving body is most affected, and, disregarding the waves formed (on the surface only), we have:

(*a*) Skin friction due to the rubbing on the surface.

(*b*) Eddying near the angles, due to the deviation of the stream lines, the centrifugal acceleration of the water being expended in the formation of small whirls.

In a vessel of curved form the middle illustration (fig. 12) shows the result.

The eddying here arises principally from bluntness of the prow, and, according to marine experience, rarely exceeds 8 per cent. of the skin resistance.

The skin friction depends on the areas of cross-section, and, as already mentioned, $=fAv^n$ where n is approximately 2 and v is the relative velocity of vessel and fluid.

Such experiments as have been made indicate that a similar law obtains in the case of air; but, of course, the coefficient f will be less (depending chiefly on the viscosity of the fluid). Its value has already been given for the cigar shape (fig. 33A), and does not vary much for any gently curved form.

The minimum eddy resistance is said to be obtained when the areas are such that the bounding envelopes have sine-curve sections. I am informed by a naval expert that this is well borne out in practice.

Resistance of Ships.—Froude's name is coupled with this subject, his experiments being classic.

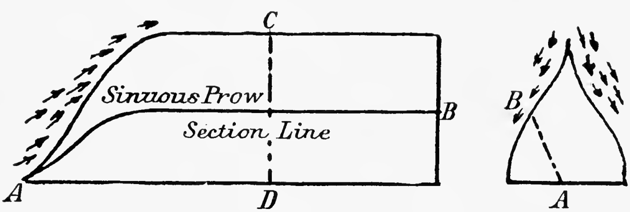


FIG. 34.—Form for prow of airship.

The following rules have been established by him connecting the resistance of a ship with that of a similarly formed model:—

$$\text{Resistance of ship} = \text{resistance of model} \left(\frac{\text{dimension of ship}}{\text{dimension of model}}\right)^3;$$

or, if the ship is n times the linear dimensions of the model,

$$R = rn^3 \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

The velocity of ship must be considered as $=$ velocity of model $\left(\frac{\text{dimension of ship}}{\text{dimension of model}}\right)^{\frac{1}{2}}$;

or

$$V = v\sqrt{n} \qquad . \quad . \quad . \quad . \quad . \quad (2)$$

The standard formulæ connecting the power (H), skin resistance (S), velocity (V), cross-section (A), and displacement (D) are:

For cross-section and skin resistance combined

$$H = \alpha V^3(\beta A + \gamma S) \qquad . \quad . \quad . \quad . \quad (3)$$

For sectional area alone

$$H = \delta V^3 A \qquad . \quad . \quad . \quad . \quad (4)$$

For skin resistance alone

$$H = \epsilon V^3 S \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For displacement alone

$$H = \zeta V^3 \sqrt[3]{D^2} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where α, β, γ, δ, ϵ, and ζ are empirically determined constants.

From Froude's researches it would appear, however, that H does not vary absolutely as V^3, but more nearly as $H \alpha V^3 + C$ where C is a constant.

Also the resistance R in the case of water does not appear to vary absolutely as V^2, but rather as V^n, where n is a little less than 2. (1·85 to 1·9). (Experiments with air give a very similar result. See page 46, Chapter IV.)

From trials made with the ships *Iris* and *Greyhound*, the ratio

$$\frac{\text{I.H.P.}}{\text{effective H.P.}} = \text{about } 2{\cdot}5.$$

It would then appear that the best form for the body of the vessel which is to have a maximum of upward and forward velocity is one with a cuspoidal or sinuous form in front and above, the section lines in all planes directed upwards or forwards being sine curves.

The stern of the vessel will need to be eased away to admit air to the helices, if these are in the same horizontal axis as the vessel, much in the same way as the stern of a ship.

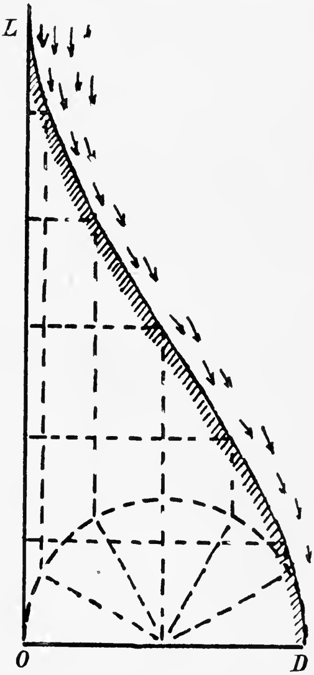


FIG. 35.—Diagram showing method of drawing sine curve for profile of vessel.

To design prow or ridge of vessel set off the length of the prow OL, and the semidiameter OD. Describe a semicircle on OD, and divide into any number of equal parts. Divide OL into the same number. Erect perpendiculars through the semicircle divisions from OD and on OL from its divisions. The intersections give the curve.

If this form is approximated to, any alterations in the general form will make little difference in the resistance, provided the surface is smooth.

I am not aware that vessels have been constructed in this manner, but it would certainly appear to be the most suitable.

If helices are used (contrary to the dreams of Jules Verne and W. Griffiths, there should be no long masts, as they would be subject to great bending and shearing stresses), they should be at or below the level of the ridge (I cannot call it a "keel"). To prevent the helices being impeded by the current, shields will be required with funnels above and below, the body being eased round these shields. See Chapter III. It should be mentioned that experiments with screw-shafts on ships show that a large clearance round the screw is desirable. Fig. 36 will show the arrangement suggested for the heliconef.

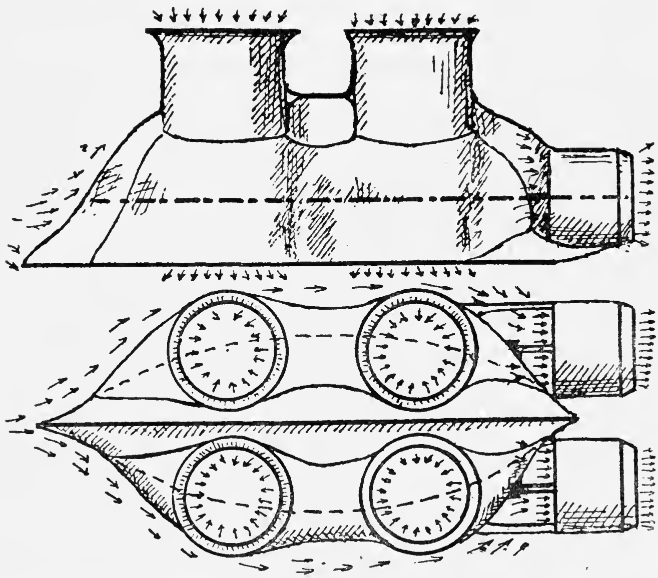


FIG. 36.—Plan and elevation of suggested hull of airship, with minimum resistance.

(*N.B.*—Any multiple of 2, more than 4, can be used as number of shafts; the longer the vessel the less is the relative resistance.)

For the aeroplane an arrangement somewhat as in fig 37 will comply with the requirements, although a more stable arrangement is shown in fig. 38.

Two helices arranged in a lateral pair are fixed at A and two in a vertical pair at B. By one of the level governors described, the speeds of these are modified so that the sum of moments is zero.

At this point some general considerations of balancing appliances will be useful.

Note on Multiple Surfaces.—I have endeavoured to show how the equilibrium of the aeroplane depends on the coincidence of the thrust with the centroid, but it is necessary to note two further details, in regard to the pairing of surfaces and manipulation.

(1) If the car is not rigidly attached to the planes, but simply swung thereto, it will tend to hang under the point of support, but will be retarded or accelerated by the rate of change of velocity of the whole device. Such an arrangement would for some positions

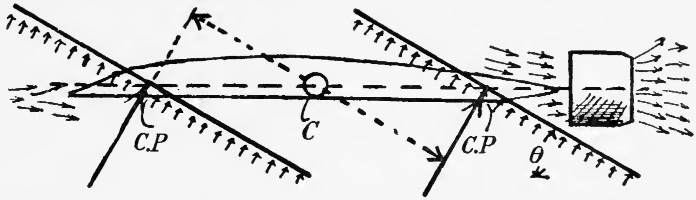


FIG. 37.—Duplex aeroplane showing equipoise about the centroid.

be stable, but the moment the centre of pressure ceased to coincide with the axis of suspension, a turning moment would be introduced on the plane with consequent irregularity of motion and the plane could only with difficulty be controlled.

(2) When a turning moment is exerted on an aeroplane, owing to the varying velocities of different points on the planes, increasing with their remoteness from the axis of rotation, there will appear

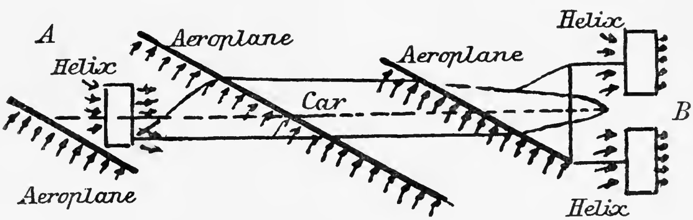


FIG. 38.—Arrangement of triple aeroplane.

complex aerial reactions, so that the plane will tend to sweep through curves in a vertical plane with a *reverse* rotation to that which would be caused by the turning moment, and this action would only cease when the air currents were non-effective.[1] Then, of course, the rotation would occur in the proper direction, and the plane would fall, but not vertically. With a balanced system, on the other hand, a vertical fall would occur when speed was sufficiently reduced. Experiment would doubtless show that considerable facilities for vertical steering are obtainable by shifting the centroid, as in No. 4 below.

[1] See Appendix, *re* Plunging.

There appear to be the following types of balancing apparatus available, each of which possesses advantages and disadvantages:—

(1) The pendulum (described under helix).

(2) The fin for transverse balance.

(3) The plane for longitudinal balance.

(4) The jockey-weight.

(5) The gyrostat.

I would here mention that it would seem to be quite impossible to regulate the level by hand control. Mind, hand, and mechanism would not be rapid enough, nor would it be possible to always gauge the righting force required.

(1) The pendulum I have already described and mentioned its disadvantages, which are (α) angle due to acceleration, and (β) non-sensitiveness of prime movers. (α) would not be of any consequence in transverse balancing.

(2) The fin is a vertical plane hung to a rod above or below the car, which is maintained in position by the wind pressure. When the car sways so that an angle is made between the central plane of section and the fin, the angular displacement is transferred to the throttles of the governors, probably by a small relay engine.

(3) The plane for longitudinal balance is a counterpoised horizontal plane which transfers its rotative angular motion in the same way to the vertical sources of propulsion. The rotation of a similar plane about a horizontal axis would serve for transverse balancing.

(4) The jockey-weight. This is a weight which, by means of a small relay engine, moves so as to produce a righting moment. (There could be two, one at either end, which by a leading screw with right- and left-handed threads of long pitch would move both at once, one in and the other out.)

(5) The gyrostat (fig. 39) has already been used to balance the torpedo. It consists of a mass of considerable inertia rotating in gimbals. It is found to exert a considerable force, opposing any tendency to deviate the axis of revolution, and a torque then appears tending to twist the framework in a plane at right angles both to the axis of rotation and the deviating force ("precession"). The transmission of this torque by levers to the controlling mechanism would serve to right the car.

[1] Or by allowing the horizontal frame to precess, a constraint will be exercised on the vessel. This can be increased by assisting the precession mechanically.

In all these cases the governor may (1) act directly on the supply, valves, or shafts of the prime movers, or (2) rotate or displace some plane or rudder to produce an opposing torque by the wind pressure. In the latter case some work must be done by the governor, which would possibly reduce its effectiveness. The plane (No. 3) is, of course, analogous to the controlling tail feathers of a bird.

In connection with the gyrostat it may be useful to note the

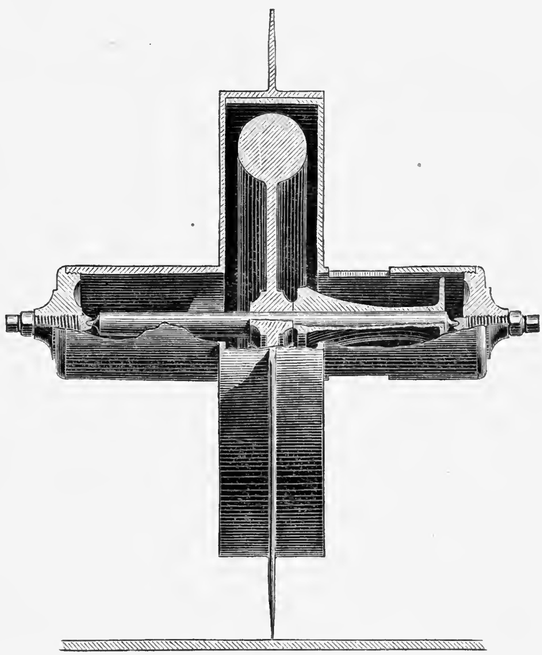


FIG. 39.—Gyrostat. The fly-wheel (shown in section) is kept revolving by small motor. When its axis is deflected a torque appears in frame at right angles to axis.

following formula connecting a, the angular velocity of precession resulting from the endeavour to turn the axis of the gyrostat whose revolving mass has a moment of inertia I and an angular velocity Ω. The total angular momentum is $\mathrm{I}\Omega$, and since torque = angular momentum × angular velocity

$$\mathrm{T} = \mathrm{I}\Omega a \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It is to be noticed that helices themselves have some gyrostatic action, and it is possible that the means of balancing might be to some extent directly derived from their rotation.

We must next consider the question of steering. This may be accomplished by

(A) Rudder planes.

(B) Unequal helix thrusts.

The rudder plane is the means more usually employed, although in twin-screw steamships the use of one propeller for rapid turning is familiar.

The action of a rudder plane may be analysed for our purpose in the following manner :—

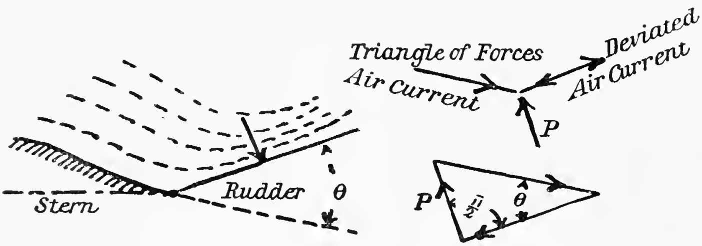


FIG. 40.—Action of rudders.

The air is deviated by the rudder plane through an angle θ.

Taking the air as approaching parallel to the axis of the vessel, and calling its momentum P, the momentum resultant normal to the rudder is $P . \sin\theta . = F$.

The volume of air approaching P, if the area is A, is $VA . \sin\theta$; where V is the relative velocity. The momentum is therefore $\frac{\rho}{g} . V . A \sin\theta$, so that the normal momentum (=force) is about

$$F = \frac{\rho}{g} . V . A \sin^2\theta \quad . \quad . \quad . \quad . \quad (2)$$

(Note, however, that for small angles the value of F will exceed this. See Chapter IV.)

This force may be considered as acting at the centre of pressure of the plane, so that, if r is the distance from the centre of pressure to the centroid of the vessel, then the force F produces a motion of translation as well as rotation, unless the c.p. of the rudder happens to coincide with the centre of oscillation (*i.e.*, $r = \frac{k^2}{c}$ is the radius of gyration). This certainly is the most desirable position for the rudder. In this case there will only be rotation such that the angular acceleration

$$a = \frac{I}{F} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where I equals the moment of inertia (mass-moment of the second degree $\Sigma\left\{\left(\frac{w}{g}\right)r^2\right\}$).

This would be best obtained by experiment, as the calculation would be extremely complex, whereas by suspending the car I can easily be found.

When altering the helix thrusts by modifying the number of revolutions per unit time, if T_1 is the thrust of one helix and T_2 the thrust of another, the lateral distance between each and the centroid being d_1 and d_2, then the nett turning moment is $T_1d_1 - T_2d_2$, and

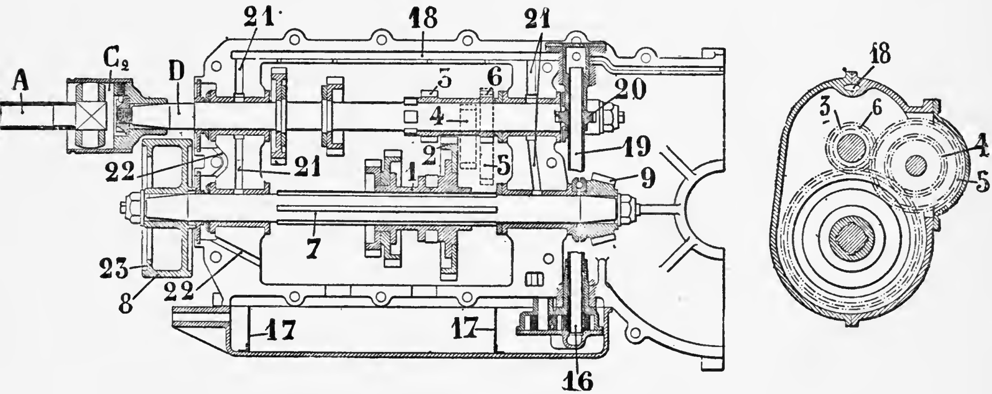


FIG. 41.—Section of change-speed gear box. (De Dion Bouton Co.)

the angular acceleration will follow from the rule (3), modified as follows:

$$a = \frac{I}{T_1d_1 - T_2d_2} \quad . \quad . \quad . \quad . \quad (4)$$

The engines by which the helices are driven (for, whether simple helices or aeroplanes are used, the energy must be transmitted through a helix) will doubtless, under present circumstances, be of the petrol type with electric ignition. The weight of the engine in lbs. per H.P. has now been reduced to 2·2 in some makes (see table of the values of "w," page 11), and the following fittings will be required in addition to the complete engine set (including petrol tank, carburettor, engine, crank, shafts, and fly-wheels):—

(1) Friction clutches with operation levers.

(The level governor could control these levers to some extent, but considerable discretion would be necessary.)

(2) Change-speed gearing (fig. 41) with a wide range: a type which

did not involve stopping the helices would be necessary (*i.e.*, it must be on engine side of clutch).

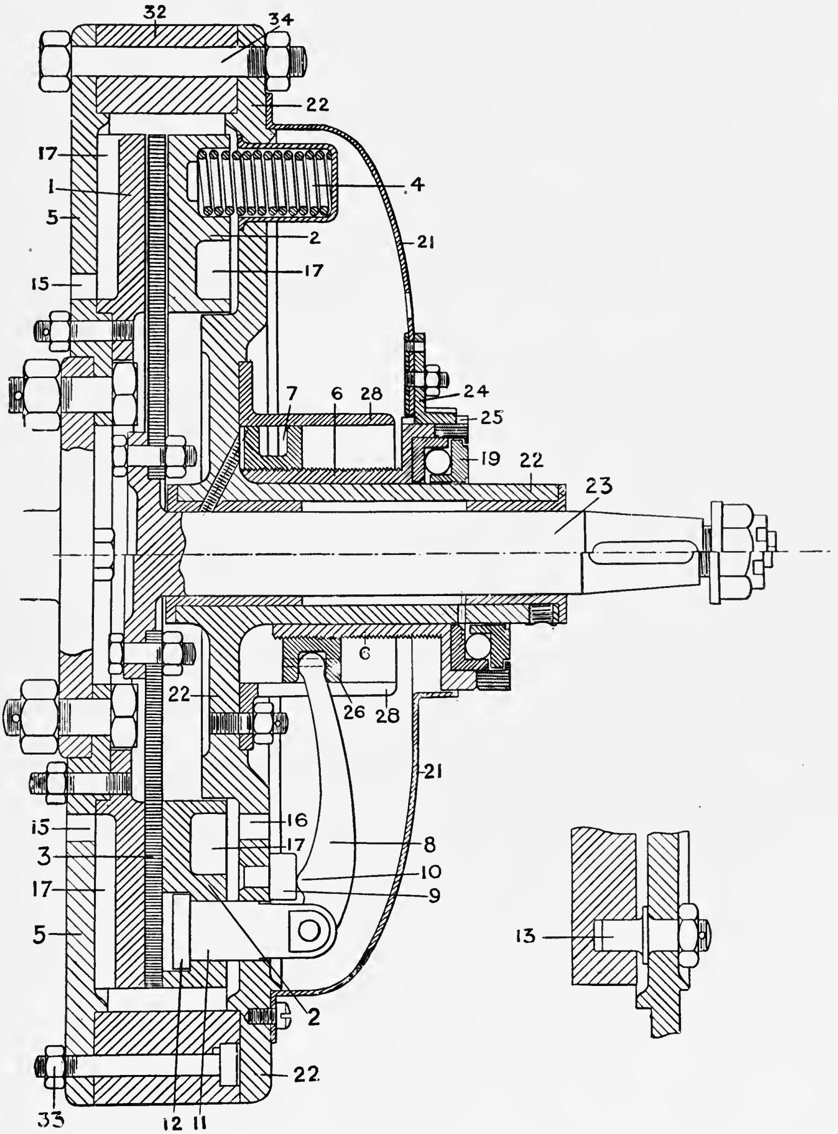


Fig. 42.—Section of plate clutch. (De Dion Bouton Co.)

(3) Valve controlling gear, including throttles, carburettor, feed-pipe, and levers to the exhaust and admission tappets.

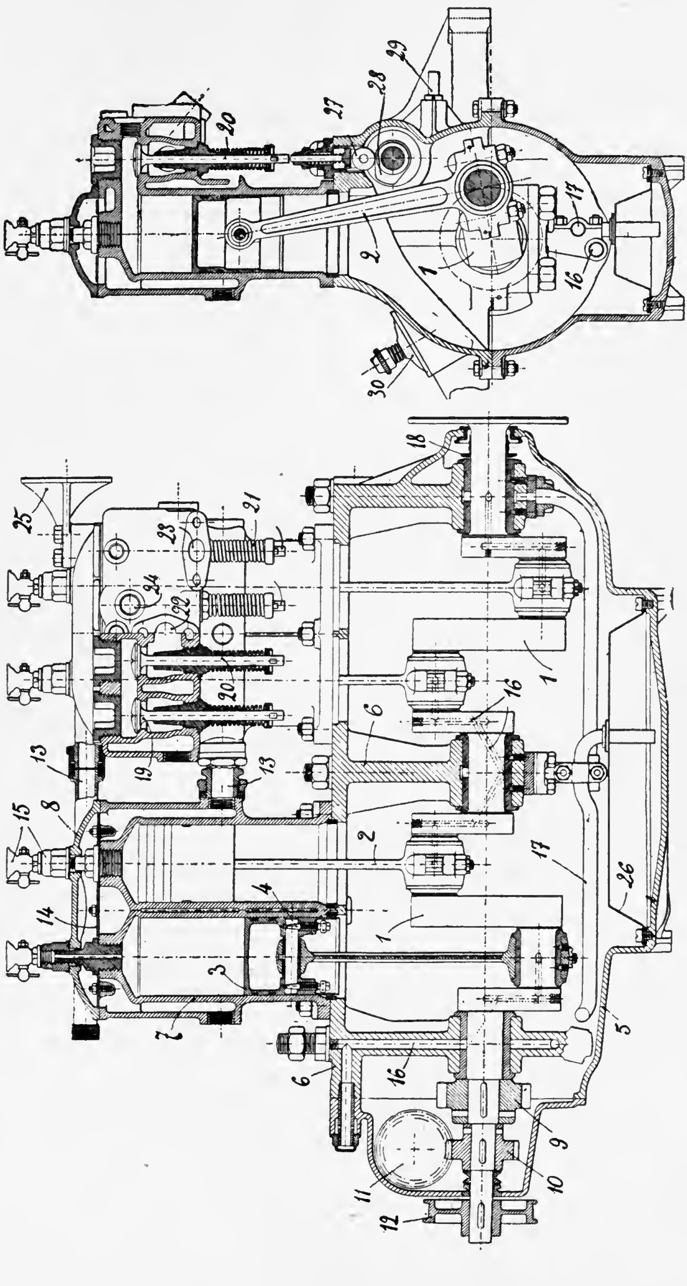


FIG. 43.—Section of 4-cylinder petrol motor. (De Dion Bouton Co.)

Starting Gear.—Reversal of motion can be obtained by the use of change speed gearing, which is altered for the backward motion, so that its torque is reversed and transmitted by the clutch (fig. 42).

Reversible wings to the helices can also be used for this purpose, but the rigidity would probably be impaired.

Magneto-ignition would be used in preference to secondary cells.

List of principal makers:—De Dion Bouton, Puteaux; J. Julien, Panhard et Levassor, Antoinette Co., Dalifol, Aster, Renault, Daïmler, Mors, Dufaux Frères, Esnault-Pelterie.

Any indirect coupling would have to be accomplished by chain or wheel gearing, as at the speeds attained belt gearing would be of poor efficiency on account of the centrifugal force.

Compound engines with three or more cylinders would be necessary (fig. 43), so as to do away with any balancing of the momenta of the moving parts, and I am inclined to think that for vertical helices triple-cylinder engines with the pistons trunked and connected to one crank pin would be most convenient, and certainly would occupy less useful space.

The thrust-block of the vertical helix shafts should be near the ridge of the car, so that the tension in the shafts may be kept in a short length between the helix and the thrust-block.

Safety appliances would consist of automatic parachutes attached above the helices, and possibly aeroplane attachments. In the case of the simple aeroplane vessel accidental stoppage of the driving helix would be extremely dangerous; but if the aeroplane could be rotated until horizontal, the downward velocity would be considerably diminished; and, possibly, in the interval, if the stoppage was not due to any serious cause, the helix could be restarted. When experience has enabled us to find the exact worth of the helix, possibly spare ones with independent engines would be available; but at present there seems little prospect of so much extra weight and resistances being portable. Small balloons might also be carried eventually.

Fig. 44.—Aneroid barometer.

Of the various instruments required for navigating purposes the following is a partial list:—

(1) Barometer (aneroid) for air pressure (fig. 44).

(2) Thermometer for temperature.

(3) Anemometer to measure velocity of air currents (fig. 45).

(4) Wind vane to show direction of resultant velocity.

(5) Nautical sextant.

(6) Mariners' compass (preferably divided on the degree system).

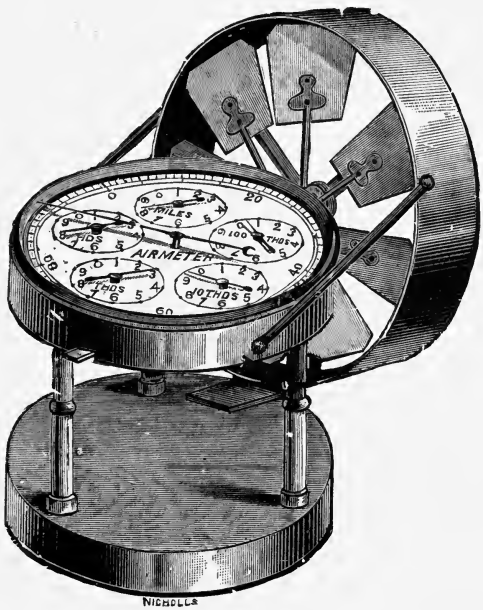


FIG. 45.—Anemometer.

(7) Speed counter for engines.

(8) Accelerometer to find change of speed.

Among other arrangements which I would suggest as useful are the following, especially when larger vessels can be constructed:—

(1) Air-tight interior, into which the air is pumped when great heights are reached, to restore the pressure to which our lungs are accustomed.

(2) The use of aluminium sheets for covering the vessel, with steel frames (mild steel, such as is generally used for constructional work). There is no advantage in using aluminium frames, as the sections have to be larger than steel. Aluminium bronze is the only

alloy which approaches steel in strength, and its weight is practically the same. On the other hand, for the covering sheet aluminium is admirable, being only about $\frac{1}{3}$ the weight of steel. Corrugated aluminium for helices and aeroplanes has been found to be very effective.

With regard to the use of artillery on airships, the recoil will be a most important consideration. Unless the axis of the gun passes through the centroid of the vessel, there will be a rotation; and even if it does so pass, unless the gun is fired in the direction of motion, the motion of translation will not be at all likely to improve the stability of the vessel.

On the other hand, explosives could be dropped quite easily from the car, and by attaining certain heights and velocities a fair degree of accuracy in directing missiles would be possible. There would certainly be an economy of ammunition and a maximum of effect.

The enormous advantage of the airship for surveying and meteorological purposes need not be referred to.

In the use of shields for helices the discharge through the tubes will be subject to the same low degree of friction as external surfaces.

If we call the normal velocity of efflux v, then we have the following coefficients reducing

$$v \text{ to } V_e$$
$$V_e = kv.$$

The discharge from an orifice is at a maximum when the external pressure is ·5 to ·6 the pressure within the reservoir.

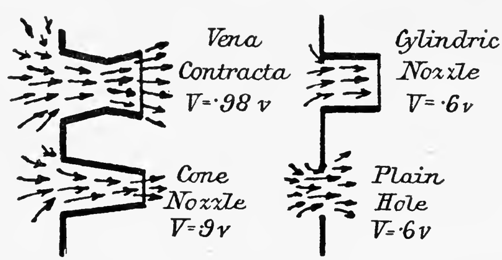


Fig. 46.—Discharge through orifices.

According to Professor Unwin, the resistance to the flow of air in pipes is $\cdot 000005\,SV^2$ where S is the skin area (feet) and V the velocity in feet per second.

Statical Stability of Balloons.—Although I have already indicated the general conditions of equilibrium for the various vessels, there must also be considered the problem of disturbance. If equilibrium is maintained during and after disturbance, then it is said to be stable.

The crux of the problem is well stated by Lord Kelvin and Professor Tait in their *Treatise on Natural Philosophy*, § 292, vol. i.:—

"If there is just as much work resisted as performed by the

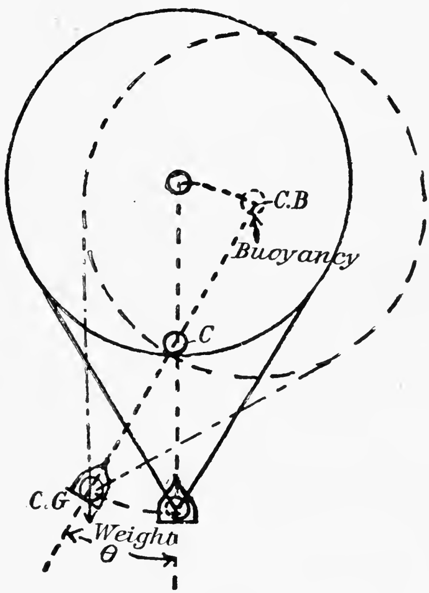


FIG. 47.—Lateral stability of aerostat.

applied and internal forces in any possible displacement, the equilibrium is neutral, and not unless. If in every possible infinitely small displacement from a position of equilibrium they do less work among them than they resist, the equilibrium is thoroughly stable, and not unless. If in any, or in every, infinitely small displacement they do more work than they resist, the equilibrium is unstable."

Presupposing the design is such that with no additional force there is equilibrium, then air vessels will be subject to the following disturbing forces:—

(1) Lateral or unequal wind pressure.

(2) Unbalanced forces, arising from displacement of masses

(aeronauts !) in vessel, or by the accidental or designed alteration of surfaces or supporting forces.

Taking first the case of the balloon, as being simpler, and considering lateral disturbance, such as would arise from lateral wind pressure or the moving of an aeronaut to the edge of the car, where there is a lateral force F, the whole mass will rotate (if rigid) about the centroid, and heeling, will proceed until

$$\mathrm{FL} . \cos \theta = \mathrm{W} . \mathrm{L} \sin \theta \quad . \quad . \quad . \quad . \quad (1)$$

L is distance to centroid, W = weight of car.

Considering an additional force $\delta\mathrm{F}$, we have an additional angle of heel $\delta\theta$,

$$\mathrm{L} . (\mathrm{F} + \delta\mathrm{F})[\cos(\theta + d\theta)] = \mathrm{W} . \mathrm{L} . [\sin(\theta + d\theta)].$$

$\mathrm{F} + \delta\mathrm{F}$ may be considered $= \mathrm{F}$, so that substituting for the constants FL and WL respectively κ and λ,

$$\kappa . [(\cos(\theta + d\theta)] = \lambda[\sin(\theta + d\theta)].$$

Expanding we have,

$$\kappa(\cos\theta . \cos d\theta - \sin\theta . \sin d\theta) = \lambda(\sin\theta . \cos d\theta + \sin d\theta . \cos\theta);$$

and since in the limit $\cos\delta\theta = 1$ and $\sin\delta\theta = 0$, during the change of θ the turning arm of the wind increases at the rate $-\sin\theta$ (*i.e.* decreases), and the turning arm of the weight increases at the rate $\cos\theta$.

Hence, as F increases, so its arm decreases, while W remains constant and its arm is increasing, so that in any infinitely short period of time the work done by W is more than that done by F, and hence there is stable equilibrium.

It would thus follow that the balloon is in all essential respects laterally stable, the centre of buoyancy being above the centre of gravity (the centre of buoyancy is the centre of gravity of the displaced air).

Regarding the longitudinal stability (fig. 48) the same reasoning applies when there is no motion against the wind; but when there is a helix, we have a continuous turning couple which has to be balanced.

If the axis of the helix does not pass through the centroid of the air resistance (*i.e.* the centre of pressure), there will be a permanent inclination backwards when travelling. If we give the car a prejudice forwards so that the centre of gravity is no longer equidistant from the ends of the balloon, a reverse couple will be produced which, if sufficient, will induce a horizontal position, but this position (not equilibrium) will be unstable.

$$Pd = Wr.$$

If P passes through the centroid (metacentre), then $L = 2d$

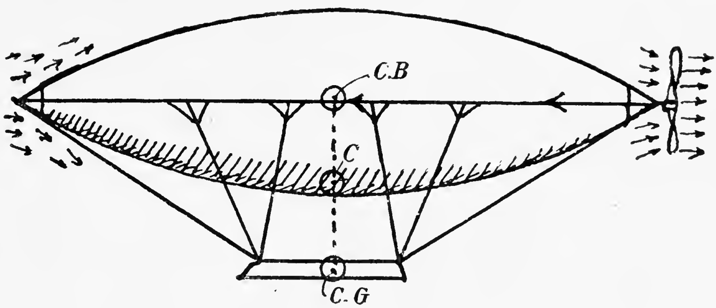


FIG. 48.—Longitudinal stability of aerostat.

(when the balloon is symmetrical), and the forward position of c.g. will be indicated

$$\tan^{-1}\left(\frac{Pd}{2W}\right) = \theta \quad . \quad . \quad . \quad . \quad (2)$$

Of course, when not accelerating, if there is a rigid frame the car will

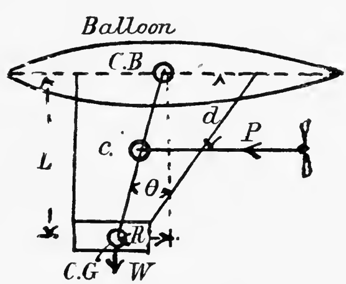


FIG. 49.—Longitudinal stability of balloon.

deflect by the angle θ, so that the helix in the line of the balloon is the preferable arrangement.

The stability of the air-vessel of any momentum type is a far more serious matter.

Considering the helicoptere first, let us take the lateral conditions. We have the following forces to consider:—

(1) The helix thrusts.

(2) The weight (=helix thrusts).

(3) The disturbing (lateral) force.

Noticing that a central helix gives us a force with no moment about the centre of gravity, we see there is nothing to resist heeling, except a slight increase in the surface presented to the air on the far side from the wind.

From this it will at once appear that the form is essentially unstable.

The same reasoning applies universally to flying machines

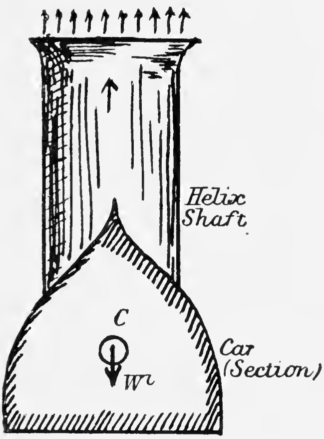


Fig. 50.—Stability of single-shaft heliconef without lateral force.

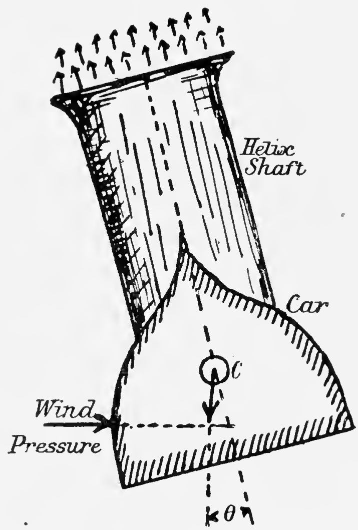


Fig. 51.—Instability of single-shaft heliconef when lateral force is applied.

heavier than air. Since the thrust is derived from the momentum of the air, it must have no moment about the c.g. This is the Gordian knot that has troubled so many inventors.

The only solution is the use of some means of splitting up and varying the thrust so that it may adjust its resultant to always comply with these conditions. Taking the case of the heliconef again (note incidentally that the vertical thrust is reduced and a horizontal component introduced by the inclination: this horizontal force has, however, no turning effect, but will tend to push the vessel laterally), let us have a pair of helices such as I have already referred to.

The axes are equidistant from the centre of gravity, and, as before, there is no righting moment. If, however, when heeling commences, we have a quick adjustment which increases the speed of the far helix, then the thrust will produce a righting moment. The result will be a pendulum swing, which we must damp out by increasing the air resistance laterally as much as possible. Longitudinal planes on edge would do this. It is also to be noticed that if the form of the sides is such that when a uniform gust of wind strikes the vessel its

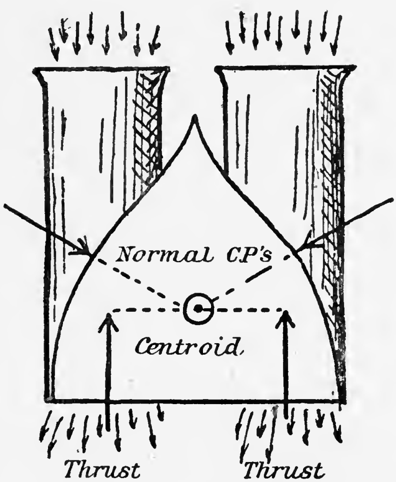


FIG. 52.—Section of aeronef.—Two helix shafts arranged symmetrically about the centroid wind-pressure resultants passing through the same point.

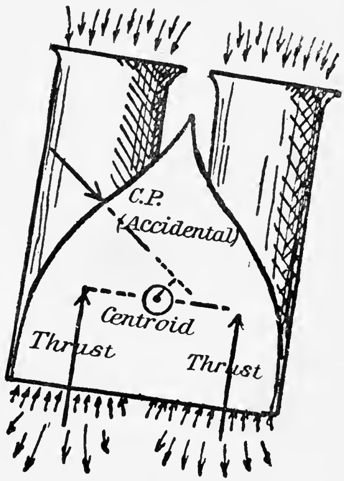


FIG. 53.—Section of aeronef, showing disturbance of equilibrium through the shifting of centre of pressure. Righting moment to be produced by increase of the right-hand helix thrust.

normal momentum component shall have a resultant passing nearly through the c.g., the turning will be at a minimum. This, however, it is not possible to be always sure of, but it should be aimed at as far as practicable.

Taking now the aeroplane and, similarly, the aviplane, the thrust must (if it is not to produce a spin) go through the c.g., but here we are far less certain of the position of the centre of pressure (or even the direction of the force at any moment). I have already mentioned my opinion that it is most desirable to have the car practically at the centre of the planes. Only in this way is it possible to assure oneself of the even approximate permanence of balance.

The equilibrium in this case is neutral (*i.e.* transitory). To ensure its duration we must, directly an inclination commences, so alter the planes that a greater turning effect is produced on the side which is sinking.

Thus, taking a front elevation, if we have planes on either side balance will normally occur as shown in fig. 54.

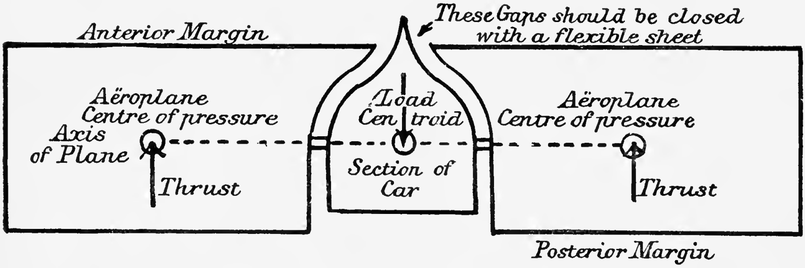


FIG. 54.—Lateral balance of aeroplane.

Let us suppose, by reason of a local current, the wind pressure on the right plane exceeds that on the left, or that the centre of pressure on the right shifts outward from the centre of area. The whole appliance will then tip downwards to the left, and the left plane must therefore rotate to an angle less inclined to the horizontal, so that its upward component may be greater.[1] In this way, with immediate readjustment, it would be possible to maintain a level position. This is actually done by birds and other flying animals by alteration of angle or areas. In the aviplane the balance is similar. The paddle wheel or vertical fan I have suggested could have its speed modified like that of the helices, and the guide rails which control the motion of the wings in the second bird type might be controlled by a link motion and so a greater or more rapid deflection could be effected automatically.

Dynamical Stability.—The methods suggested above only include the idea of a gradually increasing wind pressure balanced by a correspondingly increasing righting moment. In practice, however, we must also consider the effect of gusts, and by mechanical principles we have the following rule connecting the deflection caused by such a gust with that which would appear under a steady pressure.

"Any method of applying force which, during the deflection, is

[1] The Wright Bros. patent, I since find, includes a device to achieve this result.

calculated to do the same work, will cause the same deflection' (Perry, *Applied Mechanics*).

If now the righting effect, however produced, varies with the angle of heel, then we may write, as the condition of equilibrium under *steady* pressure,

$$F = M = \phi(\theta) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where F is the wind moment and ϕ the function which applied to θ (the angle of deflection) gives us the righting moment (M).

The work done is, under steady wind moment,

$$F\theta = \int \phi(\theta) \, . \, d\theta \quad . \quad . \quad . \quad . \quad (2)$$

If now the wind, instead of being applied slowly and steadily, suddenly reaches the value F, then the vessel will heel over until the work done by F in moving through the angle θ_1 is equal to the work done by the gradually increasing righting moment through that same angle, or, in symbolic expression

$$F\theta_1 = \int_0^{\theta_1} \phi(\theta) \, . \, d\theta \quad . \quad . \quad . \quad . \quad (3)$$

Since $\phi(\theta)$ will in most cases have a maximum value for some particular angle (less than 90°), when θ_1 reaches a certain high value any increase in F will cause the work done by F to be greater than that which can be performed by the righting torque for the same angle, and the vessel will capsize.

This will be best understood by a diagram. Let the base line (fig. 55) indicate the angle of heel from 0 to π (180°) and the vertical ordinates the righting moment.

We will assume that the righting moment (M) is a sine function (it will not necessarily be so in the case of an airship), so that it has a maximum value at $\frac{\pi}{2}$. Assuming that the torque F of the wind does not alter as the vessel heels (this is the most severe condition possible), if steadily applied the vessel will not overturn until F reaches nearly the value M_{max}. This would give $\frac{\pi}{2}$ as the "angle of vanishing stability." In the case of ships the righting moment is usually only a function of the type $\phi(\theta) = W\frac{l}{k} \sin k\theta$, so that the maximum value is obtained at the angle $\frac{\pi}{k}$ (k always exceeds 2). A ship will not right herself past this angle. (If $k=4$, then $\frac{\pi}{4}=45°$.)

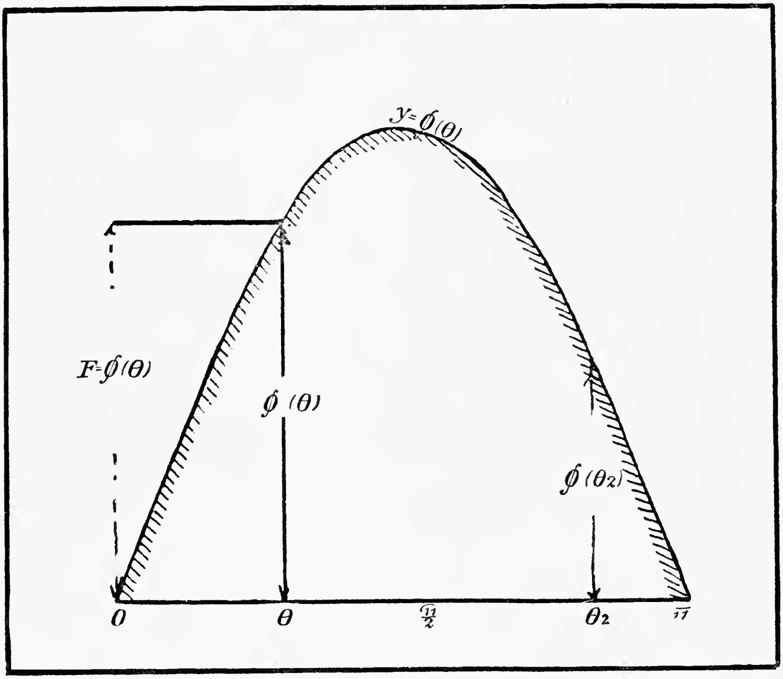


FIG. 55.—Curve of "static stability" showing equality of wind moment steadily applied and the righting moment.

So much for the steady wind.

If now the wind is suddenly applied of the value F, the work it does, instead of being an integral including the growth of F through the angle θ, will simply be the product of F and a new angle θ_1, so that the integral for the righting moment's work reaches the same value. This is shown on the curve, where F, although it has only a value$=M_\theta$, on account of its sudden full value does the same work as M_{θ_1}, so that the area of the rectangle$=$area of the part of the parabola up to θ_1 (fig. 56).

In this way we can find θ_1.

Now consider the ultimate dynamical heel permissible. Repeating the diagram, let us suppose that F_0 is such a value of the wind's moment that the area of the parabola is not equal to that of the rectangle until the value θ_2 is reached (notice that F_0 corresponds to a statical moment on the right side of the angle of vanishing stability): then any increase in F_0 will cause its work to exceed that available from the righting moment and the vessel must overturn.

If the value of $\phi(\theta)$ is known (as it must be from the arrange-

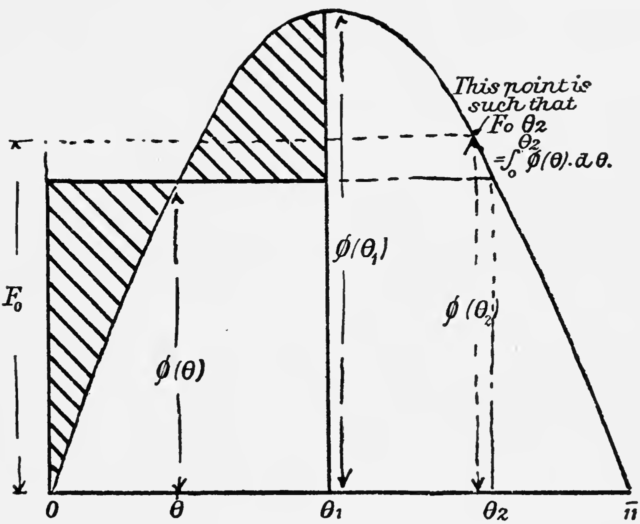


FIG. 56.—Curve of "static stability" showing new value of righting moment when wind moment is suddenly applied. (The shaded areas are equal.)

ment of the governing device, or in the case of flotation by the position of the centres of buoyancy and gravity), then it will be noticed that the ultimate heel which can occur dynamically is of such an angle θ_2 that the final righting moment $\phi(\theta_2)=F_0$.

In all practical cases it will be more convenient to do the problem by graphic methods. Integration is performed by taking the area of the curve indicating the function to be integrated up to the ordinate considered. Thus, for the curve already drawn, we shall have an integral as shown by the ascending curve.

As an example of the manner in which $\phi(\theta)$ can be determined, we will suppose that T, the thrust of a helix, whose prime mover is controlled by a ball governor, varies as the H.P., and that the H.P. varies with the admission area of the throttle valve. (This of course would have to be more accurately determined by experiment.) Also let us assume that the governor is such that the said admission increases as the sine of n times the angle; then the righting moment $M \propto \sin n\theta$, or $= \kappa \sin n\theta$.

For convenience in integration we use $\kappa = \frac{\lambda}{n}$, so that

$$M = \frac{\lambda}{n} \sin n\theta \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

and the angle of vanishing stability for a static moment $= \frac{\pi}{n}$.

Oscillations.—In marine practice the heeling of a vessel is considerably prolonged, and in some cases accelerated by the periodic

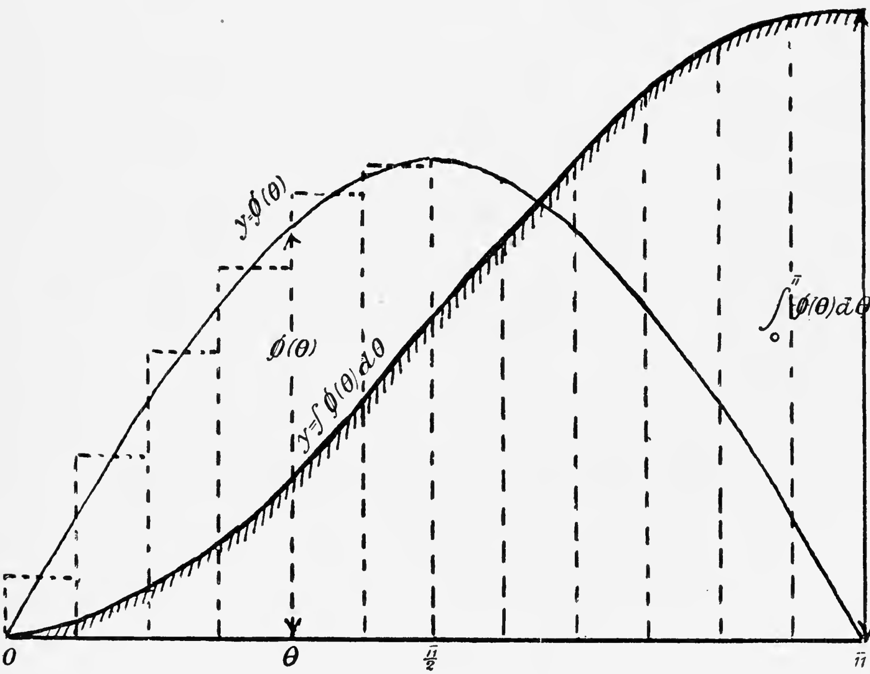


Fig. 57.—Curves of "static stability" and its integral "dynamic stability."

motion of the vessel. This is termed "rolling." First a heel takes place from the lateral force. The righting moment brings the ship back, but causes it to pass the position of equilibrium by its kinetic energy. There is again this process from the other side, and it continues in alternate directions until the energy has been absorbed by the water. I think it is safe to prophesy that this will be an even more important factor in air-vessels, and it will probably be useful to consider the mechanical rules concerning periodic motion.

It is shown in mechanics that the periodic time for an oscillation is as follows:

$$\text{Time} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} \quad . \quad . \quad . \quad (1)$$

or for angular motion (substituting torques for forces and angular displacements for linear),

$$T = 2\pi\sqrt{\frac{\bar{\theta}}{\ddot{\theta}}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where θ is angular displacement and $\ddot{\theta}$ is angular acceleration.

In applying this to an airship, θ is the actual angle of heel (in the plane of which the motion is being considered) and $\ddot{\theta}$ is known from the formula

$$\ddot{\theta} = \frac{T}{I} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where T is the righting torque and I is the second mass moment ("Moment of Inertia") of the vessel about the centroid $[I = \Sigma(mr^2)$ or $Mk^2)]$.

Hence

$$T = 2\pi\sqrt{\cdot\frac{\theta \, . \, I}{T}} \quad . \quad . \quad . \quad . \quad (4)$$

We shall rely for the removal of this oscillation on the damping effect of the air acting on the various surfaces. The displacement will thus, after the first gust has caused it, gradually diminish. The frequency $\left(\frac{2\pi}{T}\right)$ will also not be so great, but will only have a value to be found from the equation:—

$$\frac{2\pi}{T_0} = \sqrt{\left(\frac{4\pi^2}{T^2}\right) + \left(\frac{a^2}{4\pi^2}\right)} \quad . \quad . \quad . \quad . \quad (5)$$

where a is a constant depending on the fluid which damps the vibrations. (In air a = about 0·7.) T_0 is original periodic time.

The maximum value of the displacement θ will now be for each oscillation :

$$\theta_0 = \theta\epsilon^{-at} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where θ_0 is the maximum displacement, θ is the initial displacement, ϵ the Naperian base, a the aforesaid constant, and t the time since damping commenced.

In the event of the gusts of wind occurring at such intervals that their periodic time is the same as that of the swing of the vessel, most dangerous oscillations will be produced, and it will probably capsize. When violent swinging indicates this, the vessel must be turned so as to present a section to the gusts having a different periodic time.

A similarly disastrous result will occur if heavy masses are moved about the vessel with the same periodic motions.

In connection with the subject of oscillation, and also the stability of the vessel, it will be most important to obtain the values of the moment of inertia about a longitudinal and tranverse axis through the centroid before any important ascent is undertaken.

The theory of dynamic stability has been studied in some detail by Professor Bryan (*Proc. Roy. Soc.*, vol. lxxiii.), and Captain Ferber (*Revue d'Artillerie*, August, October, and November 1905), and some of their practical conclusions have been referred to in the chapter on aeroplanes. The method employed is rather beyond the scope of this book, but it may be interesting to note that it consists of (1) formulating the equations of motions in terms of the velocities; (2) adding small increments to the velocities, simplifying and solving the simultaneous equations by a determinant so as to obtain a general equation of stability; and (3) if the coefficients of this equation satisfy certain conditions as to sign and combination, the device is stable, and not otherwise.

The particular results of the method are to place limits (inferior and superior) in the velocities, dimensions, weight, and moments of inertia.

APPENDICES.

APPENDIX A.

PERIODICITY OF WING FLIGHT. (After Marey.)

	Name.	Cycles per Second.
Insect.	Common fly	330
	Drone fly	240
	Bee	190
	Wasp	110
	Humming-bird moth (*Macroglossa*)	72
	Dragon fly	28
	Butterfly (*Pieris rapæ*)	9
Bird.	Sparrow	13
	Wild duck	9
	Pigeon	8
	Moor buzzard	$5\frac{3}{4}$
	Screech owl	5
	Buzzard	3

Velocity during Cycle.

Name.	Total Duration of Cycle, $\frac{1}{100}$ths of second.	Ascent.	Descent.
Duck	11·6	5	6·6
Pigeon	12·5	4	8·5
Buzzard	32·5	12·5	20

APPENDIX B.

TABLE OF AREA OF WING-TO-WEIGHT RATES.

(Square feet per lb. From M. de Lucy's results.)

Gnat	48·77	Stag beetle (male)	3·75
Dragon fly (small)	29·59	Rhinoceros beetle	3·11
Lady bird	22·06	Swallow	4·87
Dragon fly (common)	21·6	Sparrow	2·72
Daddy long legs	14·82	Turtle dove	2·13
Bee	5·22	Pigeon	1·27
Meat fly	5·605	Stork	0·97
Drone	5·053	Vulture	0·82
Cockchafer	5·16	Australian crane	0·41
Stag beetle (female)	4·65		

APPENDIX C.

NOTE ON THE CONSTRUCTION OF A FLEXIBLE WING OR SCREW.

PROFESSOR PETTIGREW describes the methods he adopted of forming artificial wings with canes, and overlapping layers of fabric, in *Animal Locomotion*, but it may be doubted whether such construction would be satisfactory for large machines either from the point of view of strength or of durability.

Steel tube, such as is now used for the frames of bicycles and automobiles, could well be used for the framing, arranged telescopically, diminishing from a maximum section at the root of the blade to a minimum

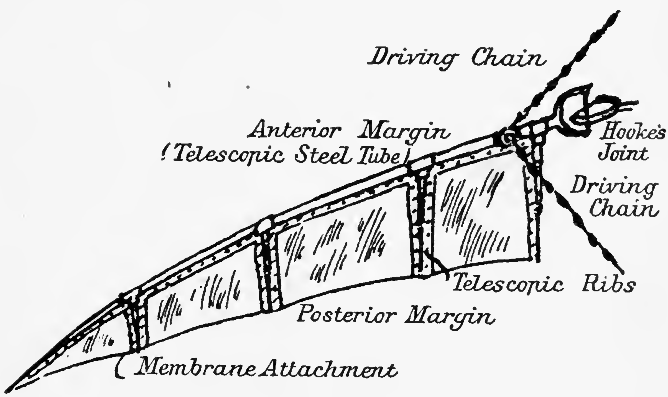


FIG. 58.—Artificial wing.

at the tip. Welded into V-junctions springing from this could be similar shorter lengths of telescopic tubing forming ribs (like the phalanges of a bat), the membrane fabric being stretched between these. This fabric could be of oiled silk, possibly with a light and flexible wire foundation (asbestos fabric would be desirable). To attach it to the ribs and anterior margin I would suggest sheathing of the same material to be fixed over the ribs in two pieces, so that the free edges come on either side of the rib, and the sheets of fabric be fixed between these with rivets or clips. The edge into which these clips are attached should be strengthened with a hem or extra thickness, as likewise should be the posterior edge of the blade. The elasticity of the ribs, both transversely and in the plane of the blade, would permit helical strain when air was being displaced. A Hooke's joint at the root of the wing would allow for universal motion, the actual direction being determined by the points of application and motion of the motive chains or rods. Pettigrew suggests the use of flexible bands at the root.

APPENDIX D.

The Maxim Flying Machine.

Specification.

Extent of aeroplanes	126 ft.
Total area	4000 sq. ft.
Angle of planes	72° from vertical.
Weight	8000 lbs.
Power	363 H.P.
Thrust	2000 lbs.
Width	200 ft.
Speed with no weight	30 miles per hour
Lifting speed (observed)	36 ,, ,,

Two 20 ft. propellers. Weight of engines 600 lbs. (figs. 16, 59).

Professor S. P. Langley's Aerodrome (fig. 60).

Power	1·5 H.P.
Weight of engine	7 lbs.
Weight of aerodrome	12 lbs.
Length across planes	14 ft.
Velocity	20 miles per hour.

Four rectangular aeroplanes.
Two propellers.

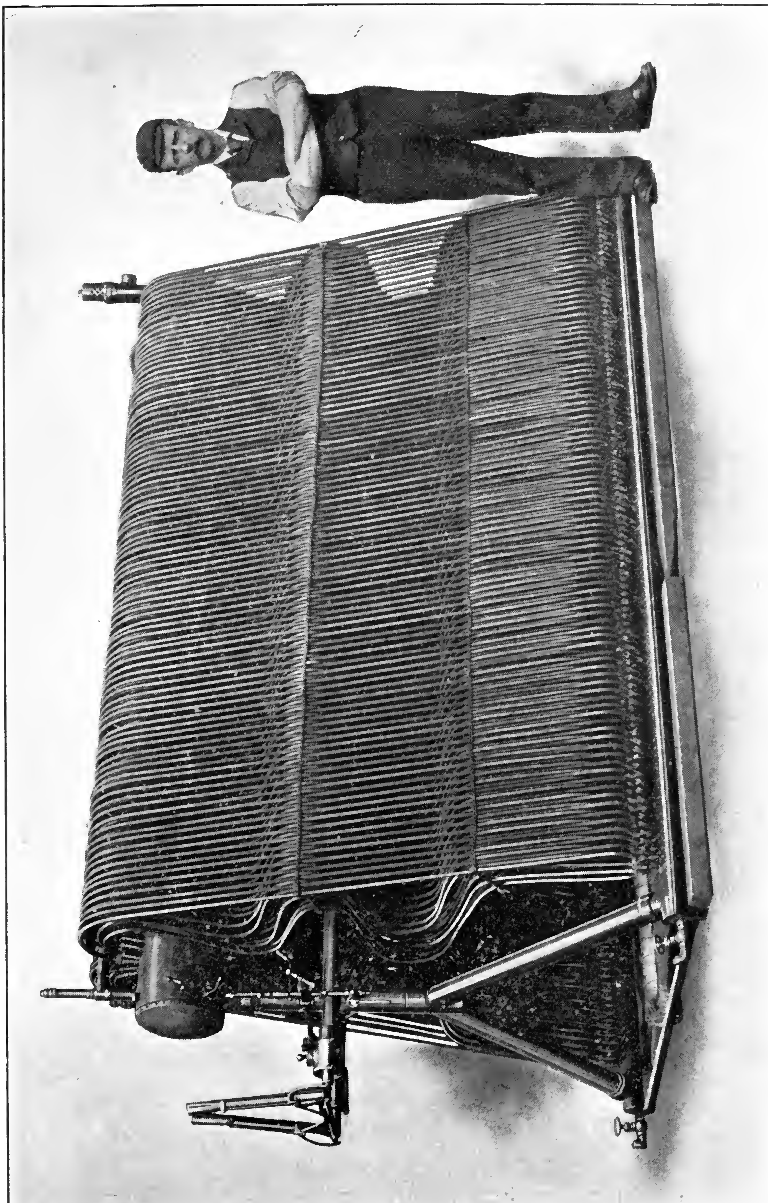


FIG. 59.—Boiler for Maxim's aeroplane.

APPENDIX E.

THE MATHEMATICAL THEORY OF THE BALLOON.

THE following analysis from Dr Glaisher's article on "Aeronautics" in the *Encyclopædia Britannica* will possibly interest academic students, but it has little practical value, including, as it does, many exceedingly small and uncertain quantities, and not allowing for convective equilibrium.

Let M = mass of balloon, gas, car, and appurtenances.
V_0 = capacity of expanded envelope.
v_0 = volume of gas at atmospheric pressure introduced at starting.
v = volume (less than v_0) occupied by same gas at height x
ρ_0 = density of gas in balloon on earth.
ρ = density at height x.
σ_0 = density of air at earth.
σ = density at height x.
u = initial upward velocity of balloon (usually zero).
u_0 = vertical velocity at height x.

CASE I.—*Balloon partly filled and ascending from Earth.*

Equation of motion:

$$M u\frac{du}{dx} = \sigma v g' - M g' - \lambda u^2 \epsilon^{-\frac{g}{k}x}$$

(change of momentum) (air displaced) (balloon) (air resistance)

$$g' = g\frac{a^2}{(a+x)^2}$$

where a is radius of earth ($g' = g$ for practical purposes).

Under isothermal conditions, by Boyle and Marriotte's law σv is constant ($= \sigma_0 v_0$). Let $\sigma_0 v_0 - M = C$, $\frac{2\lambda}{M} = \alpha$, $\frac{2cg}{M} = \beta$, $\frac{g}{k} = n$, $\frac{\alpha}{n} = m$.

Equation to motion is now

$$\frac{du^2}{dx} + \alpha\epsilon^{-nx} = \beta\frac{a^2}{(a+x)^2}.$$

Integrating this as a linear differential equation of the first order (multiplying by $\epsilon^{-nx} = X$) we have the following:—

$$u^2\epsilon^{-mX} = \beta a^2\int \epsilon^{-mX}\frac{dx}{(a+x)^2}$$

$$= \beta a^2\left\{-\frac{\epsilon^{-mX}}{a+x} + \alpha\int\frac{e^{-mX-nX}dx}{a+x}\right\}$$

$$= \beta a^2\left\{-\frac{\epsilon^{-mX}}{a+x} + \alpha\int\frac{dx}{a+x}\left(\epsilon^{-nx} - m\epsilon^{-2nx} + \frac{m^2}{1.2}\epsilon^{-3nx}\ \ldots\ldots\right)\right.$$

$$= \beta a^2\left[-\frac{\epsilon^{-mX}}{a+x} + \alpha\int \epsilon^{na}\mathrm{E}i(-na-nx) - m\epsilon^{2na}\mathrm{E}i(-2na-2nx)\right.$$

$$\left. + \frac{m^2}{1.2}\epsilon^{3na}\mathrm{E}i(-3na-3nx) - \ldots\ldots\right\}\Bigg] + C.$$

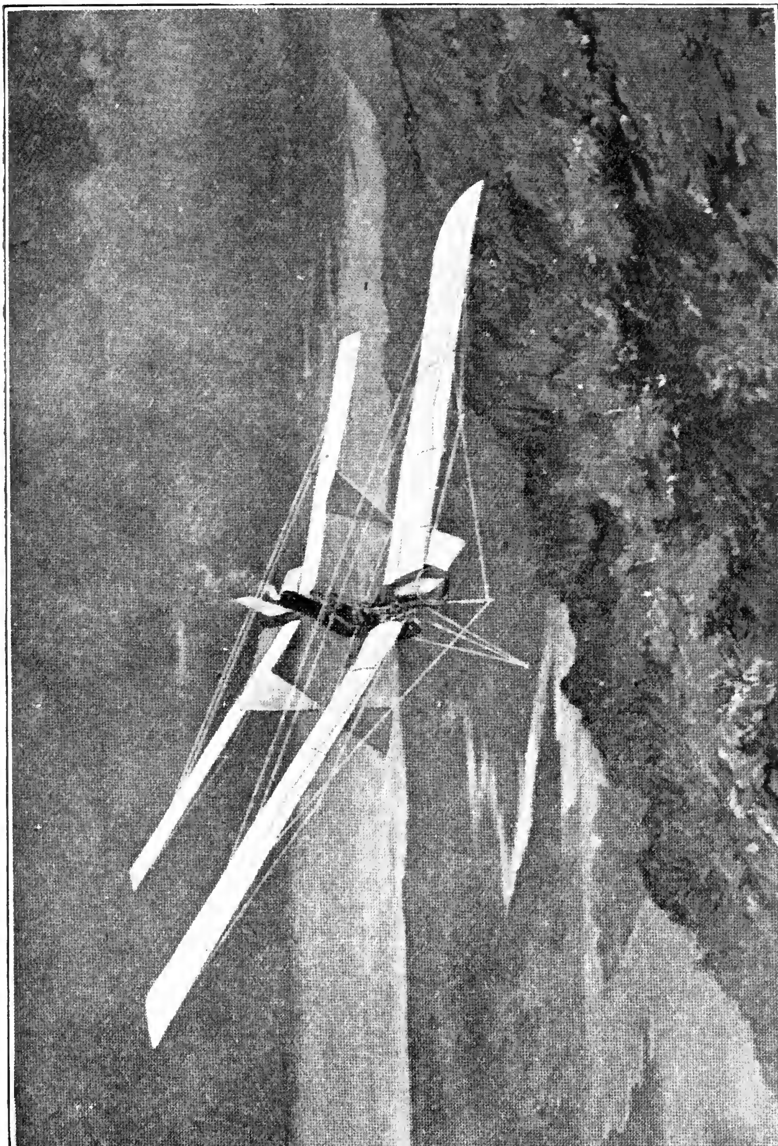


FIG. 60.—Langley's aerodrome.

Let $x=0$, so that $u=u_0$, and we have

$$u_0^2\epsilon^{-m}=\beta a^2\Big[-\frac{\epsilon^{-m}}{a}+a\Big\{\epsilon^{na}\mathrm{E}i(-na)-m\epsilon^{2na}\mathrm{E}i(-2na)$$

$$+\frac{m^2}{1.2}\epsilon^{3na}\mathrm{E}i(-3na)-\ldots\Big\}\Big]+\mathrm{C},$$

whence by subtraction

$$u^2\epsilon^{-m\mathrm{X}}-u_0^2\epsilon^{-m}=\beta a^2\Big[\frac{\epsilon^{-m}}{a}-\frac{e^{-m\mathrm{X}}}{a+x}$$

$$+a\Big\{\epsilon^{na}\mathrm{E}i(-na-nx)-m\epsilon^{2na}\mathrm{E}i(-2na-2nx)$$

$$+\ldots\ldots-\epsilon^{na}\mathrm{E}i(-na)+m\epsilon^{2na}\mathrm{E}i(-2na)-\ldots\Big\}\Big],$$

and therefore

$$u^2=u_0^2\epsilon^{-m(1-\mathrm{X})}+\beta a^2\Big[\frac{\epsilon^{-m(1-\mathrm{X})}}{a}-\frac{1}{a+x}$$

$$+a\epsilon^{m\mathrm{X}}\Big\{\epsilon^{na}\mathrm{E}i(-na-nx)-\epsilon^{na}\mathrm{E}i(-na)$$

$$-m\epsilon^{2na}\mathrm{E}i-(2na-2nx)+m\epsilon^{2na}\mathrm{E}i(-2na)$$

$$+\frac{m^2}{1.2}\epsilon^{3na}\mathrm{E}i(-3na-3nx)-\frac{m^2}{1.2}\epsilon^{3na}\mathrm{E}i(-3na)+\ldots\Big\}\Big].$$

The value $\mathrm{E}i\,.\,x$ which is substituted is the exponential integral

$$\int_{-\infty}^{x}\frac{e^x}{x}.\,dx.$$

Values for this have been tabulated, and appear in *Phil. Trans.*, 1870, pp. 367–388.

It must be remembered that temperature is not considered, and that this equation is only true before the balloon is full.

Generally g is constant and $\lambda=0$, so that

$$\frac{du^2}{dx}=\beta \text{ and } u^2-u_0^2=\beta x,$$

from which we get the well-known fact that the balloon rises till full.

CASE II.—*Balloon started full (from ground or after passing through the Case I).*

Equation of motion:

$$\Big\{\mathrm{M}-v_0(\rho_0-\rho)\Big\}u\frac{du}{dx}=g\frac{a^2}{(a+x)^2}\Big\{v_0\sigma-\mathrm{M}+v_0(\rho_0-\rho)\Big\}-\lambda u^2\epsilon^{-nx},$$

or $$\Big\{\mathrm{M}-v_0\rho_0(1-\epsilon^{-nx})\Big\}u\frac{du}{dx}=g\frac{a^2}{(a+x)^2}\Big\{v_0\sigma_0\epsilon^{-nx}-\mathrm{M}+v_0\rho_0(1-\epsilon^{-nx})\Big\}$$
$$-\lambda u^2\epsilon^{-nx}.$$

(Neglect $v_0\rho_0(1-e^{-nx})$ the escaping gas, as compared with M.)

Then $$\tfrac{1}{2}\,\mathrm{M}\frac{du^2}{dx}+\lambda\epsilon^{-nx}u^2=\frac{ga^2}{(a+x)^2}\left\{v_0\sigma_0\epsilon^{-nx}-\mathrm{M}\right\}$$

or $$\frac{du^2}{dx}+\alpha\epsilon^{-nx}u^2=\frac{\gamma a^2}{(a+x)^2}\left\{v_0\sigma_0\epsilon^{-nx}-\mathrm{M}\right\}\qquad\left[\gamma=\frac{2g}{m}\right]$$

(Multiply by $\epsilon^{-m\mathrm{X}}$ and integrate.)

$$u^2\epsilon^{-m\mathrm{X}}=v_0\sigma_0\gamma a^2\int\frac{\epsilon^{-nx-m\mathrm{X}}dx}{(a+x)^2}-\mathrm{M}\gamma a^2\int\frac{\epsilon^{-mx}dx}{(a+x)^2}+\mathrm{C}$$

$$=v_0\,\sigma_0\,\gamma a^2\left\{-\frac{\epsilon^{-nx-m\mathrm{X}}}{a+x}+\int\frac{dx}{a+x}\left(-n\epsilon^{-nx-m\mathrm{X}}+\alpha\epsilon^{-2nx-m\mathrm{X}}\right)\right\}$$

$$-\mathrm{M}\gamma a^2\left\{-\frac{\epsilon^{-m\mathrm{X}}}{a+x}+\alpha\int\frac{\epsilon^{-m\mathrm{X}-nx}dx}{a+x}\right\}+\mathrm{C}$$

$$=v_0\sigma_0\gamma a^2\left[-\frac{\epsilon^{-nx-m\mathrm{X}}}{a+x}-n\left\{\epsilon^{na}\mathrm{E}i(-na-nx)\right.\right.$$

$$\left.-m\epsilon^{2na}\mathrm{E}i(-2na-2nx)+\ldots\right\}+\alpha\left\{\epsilon^{2na}\mathrm{E}i(-2na-2nx)\right.$$

$$\left.\left.-m\epsilon^{3na}\mathrm{E}i(-3na-3nx)+\ldots\right\}\right]-\mathrm{M}\gamma a^2\left[-\frac{\epsilon^{-nx}}{a+x}\right.$$

$$+\alpha\left\{\epsilon^{na}\mathrm{E}i(-na-nx)-m\epsilon^{2na}\mathrm{E}i(-2na-2nx)\right.$$

$$\left.\left.+\ .\right\}\right]+\mathrm{C}.$$

(C determined by putting $x=0$, when $u=u_0$.)
u_0 is not 0 except when balloon starts from earth quite full.

The mathematical treatment when convective equilibrium has to be considered is not possible with such exhaustive detail. I have given in the chapter on balloons the simple method which is fairly accurate.

APPENDIX F.

Langley's Experiments in Aeroplanes.

In regard to the great variation found in the experimental and analytical determination of pressures on planes, it should be noted that Professor Langley's researches are perhaps the most thorough and scientific of any that have been made on this subject. They are described in full, together with particulars of the apparatus employed, in the *Smithsonian Contribution to Knowledge*, No. 801, entitled "Experiments in Aerodynamics."

The following are the chief points established by these researches :—

(1) The value of κ in $P=\kappa v^2$ is equal to ·00166 in feet, lbs., seconds, units.

(2) The ratio of the pressure on an inclined surface to that on a vertical surface is *not* expressed by $\sin^2\theta$, but approximates to Duchemin's rule

$$P_o = P_n \frac{2 \sin \theta}{1+\sin^2}.$$

(3) A plane moving with a high velocity does not fall in accordance with the parallelogram of velocities, but, if the speed is sufficiently great, the fall may be protracted indefinitely.

(4) The work done in soaring (*i.e.* just supporting a plane) is much greater than that put out at high speeds, so that the greater the speed the less the power required, the surplus being performed by the weight of the plane. These results are all so apparently paradoxical that, but for the research having been exceedingly thorough, inquirers might have some hesitation in accepting them, but it is fairly well agreed that the results are correct. I find, however, that Mr Frost reported to the Aeronautical Society of Great Britain having carried out similar experiments, and although he corroborates in the main Professor Langley's conclusions, yet he had great difficulty with the inertia of the air and wind, and this possibly may render the figures subject to slight revision. He has made an aviplane with artificial feathers. Messrs Wenham and Browning confirmed Duchemin's formula many years ago.

With regard to the fourth point, which is one that has been enlarged upon considerably in regard to future developments, it must be remembered that it is first necessary to attain such a speed as will give support to the plane. This is known from the following formula:

$$W = \frac{2 \sin \theta}{1+\sin^2 \theta} . \cos \theta \kappa v^2 A,$$

i.e.

$$v = \sqrt{\frac{W(1+\sin^2 \theta)}{2 \sin \theta . \cos \theta . \kappa . A}}.$$

Also the advantages of power economy will only make themselves apparent when the kinetic energy needed by the body has been imparted to it. Furthermore, the resistances of bodies other than planes are not subject to such great diminution, so that the car and mechanism will always present a continually increasing resistance.

Professor Langley's own experiments in aviation, although successful in the case of the model aerodrome, have been unfortunate with the larger machines; and although this result is in a large degree attributable to lack of funds, he would not appear to have said the last word as to the root principles.

Personally, I am firmly convinced that the success or otherwise of an aeroplane does not so much depend on the angle or arrangement of the plane as upon the efficiency of the balancing and propelling appliances. No aeroplane can make a successful flight which does not answer to every pulsation in the air or irregularity in the ascensional and propulsive efforts. This, and this alone, is the crux of the problem.

In view, however, of the importance of these experiments, I have based upon them another value for the angle at which planes should be used.

The pressure formula in full is as follows:

$$P_n = \frac{2 \sin \theta}{1 + \sin {}^2\theta} \cdot \kappa v^2 \text{ per square foot.}$$

The vertical component of this is

$$\frac{2 \sin \theta \,.\, \cos \theta}{1 + \sin^2 \theta} \kappa v^2 = \text{lift per square foot,}$$

and the horizontal component

$$\frac{2 \sin \theta \,.\, \sin \theta}{1 + \sin^2 \theta} \cdot \kappa v^2 = \text{resistance to propulsion.}$$

In order to get the most efficient plane, the lift should have a maximum value at the same time as the resistance has a minimum, *i.e.* when

$$\frac{2 \sin \theta \,.\, \cos \theta - 2 \sin \theta \,.\, \sin \theta}{1 + \sin^2 \theta} \text{ is a maximum.}$$

A simple solution to this (for which I am indebted to my colleague, Mr T. Worrall, M.Sc.) is as follows:

$$y = \frac{2 \sin \theta \,.\, \cos \theta - 2 \sin^2 \theta}{1 + \sin^2 \theta} = \frac{\sin 2\theta - 2 \sin^2 \theta}{1 + \sin^2 \theta}$$

$$\frac{dy}{d\theta} = \Big[(2 \cos 2\theta - 4 \sin \theta \cos \theta)(1 + \sin^2 \theta)$$

$$- 2 \sin \theta \,.\, \cos \theta (\sin 2\theta - 2 \sin^2 \theta)\Big] \div F(\theta).$$

The numerator for a maximum value must equal zero, so we have

$$(2 \cos 2\theta - 2 \sin 2\theta)\left(1 + \frac{1 - \cos 2\theta}{2}\right) - \sin 2\theta(\sin 2\theta - 1 + \cos 2\theta) = 0,$$

or $$\cos (2\theta - \sin 2\theta)(3 - \cos 2\theta) - \sin 2\theta(\sin 2\theta + \cos 2\theta - 1) = 0$$

$$3 \cos 2\theta - 3 \sin 2\theta - \cos^2 2\theta + \cos 2\theta \sin 2\theta - \sin 2\theta \cos 2\theta$$

$$- \sin^2 2\theta + \sin 2\theta = 0$$

$$3 \cos 2\theta - 2 \sin 2\theta = 1$$

$$9(1 - x^2) = 1 + 4x + 4x^2 \text{ where } x = \sin 2\theta$$

$$x = -\frac{2}{13} \pm \frac{10{\cdot}4}{13} = \frac{8{\cdot}4}{13}, \text{ or } -\frac{12{\cdot}4}{13}$$

$$= {\cdot}6461, \text{ or } -{\cdot}954$$

$$\therefore \quad 2\theta = 40^\circ\ 50', \text{ or } 252^\circ\ 34' \text{ or } 287^\circ\ 26'.$$

The first value is the only practicable one, so that $20^\circ\ 25'$ is the angle.

APPENDIX G.

Note on the Longitudinal Stability of Aeroplanes.

Assuming that Duchemin's formula for the reaction on an inclined plane and Joëssel's formula for the displacement of the centre of pressure are established (they have been to all intents and purposes), we are in a position to discuss the phenomenon of "plunging" which occurs when kites or aeroplanes descend. The importance of the problem is obvious.

When the aeroplane is proceeding at or above soaring speed (say v), the centre of pressure will be at a distance d in front of the centre of gravity of each plane depending on the angle of inclination. So long as soaring is maintained the plane will be stable, but as soon as descent commences the centre of pressure will go back. If the plane is only slightly inclined to the horizontal, the new position of the centre of pressure will coincide with the centre of gravity of each plane, and as this was not the position of the resultant force before, a turning moment will be introduced. If a is the downward acceleration (less than g), t the time of descent, and the plane is only slightly inclined (to an angle θ with the horizontal), the turning moment will be

$$\mathrm{P}a = \kappa a^2 t^2 \,.\, \mathrm{AL}\,[0{\cdot}3 - 0{\cdot}3 \sin\theta].$$

The greatest value this can have is therefore $0{\cdot}3(\kappa g^2 t^2 \mathrm{AL})$, and provision must be made by jockey weights, gyroscope, or displaced planes to supply an equal and opposite turning moment. (A is area and L length of aeroplane.)

APPENDIX H.

On the Safety of Aeroplanes.

In connection with the practical utility of flying machines, the question of safety is a very important one, and it may serve some useful purpose to consider the particular risks run by aeroplanes with a view to determining in what manner the danger may be reduced to a minimum.

We may classify these under the following headings:—

(1) Risks in connection with stability.
(2) Risks in connection with aerial disturbance.
(3) Risks in connection with alighting.
(4) Risks in connection with structural strength.

In addition to these there is the chance of collision with hills, trees, etc., during flight, but this will, as in the case of marine navigation, be a matter depending on the experience of the aviator with the machine he is operating.

Stability.

With regard to the question of stability, it is fairly well recognised now that an aeroplane can be constructed so as to be automatically stable under the ordinary conditions of flight. Mathematical criteria have been provided by Professor Bryan (*Proc. Roy. Soc.*, 1903), Captain Ferber (*Revue d'Artillerie*, November 1905), and F. W. Lanchester (*Aerodonetics*). It is necessary that stability should be assured both in regard to oscillations about a transverse axis through the centre of gravity (longitudinal stability), oscillations about a longitudinal axis (lateral stability), and deviations about a vertical axis (directional stability). Only Captain Ferber and Mr Lanchester have considered all these kinds of stability, and the latter also finds another form ("rotational stability") must be dealt with.

Captain Ferber's rules are as follow :—

(1) *For Lateral and Directional Stability.*

There must be a keel or fin above and behind the centre of gravity of the machine, whose area is ten times that of a surface of normal resistance equivalent to the head resistance of the machine, situated behind the centre of gravity (presumably from the c.g. to the centre of pressure) at a distance

$$-\frac{\eta}{5}\frac{C}{A}\sqrt{\frac{S}{s}}.$$

C is the moment of inertia (kg. metre^2) about the longitudinal axis.
A is the moment of inertia about the transverse axis.
η is the height of the centre of the surface above the longitudinal axis through the c.g. (metres).
S is the area of the main surfaces (sq. metres).
s is the equivalent area above referred to (sq. metres).

(2) *For Longitudinal Stability.*

The weight must bear a certain relation to the lateral spread of the wings ("envergure") and the radius of gyration about the transverse axis.

$$P > 37B_1^2E,$$

where P is the weight in kilogrammes,
B_1 is the radius of gyration (metres) about the transverse axis,
E is the spread of the wings (tip to tip, metres).

There is also a condition as to the position of the centre of gravity with regard to the supporting surfaces. This point must be ahead of the centre of area of those surfaces, and Captain Ferber says (Conséquence XVIII.): "When the condition (as above) of stability is satisfied, one can make a first trial under good conditions by arranging the projection of the centre of gravity to fall in successive positions along the length of the wing" (*i.e.* the best position should be found by trial, shifting the c.g. along the longitudinal axis).

Mr Lanchester's mathematical criteria are more precise, but probably have not such a wide application as Captain Ferber's. His principal rule is that for longitudinal stability.

$$\Phi = \frac{4l\mathrm{H}_n{}^2 \tan \gamma}{\mathrm{I}\left(\frac{1}{\mathrm{K}} + \frac{1}{c\mathrm{C}\rho\epsilon a\beta}\right)} > 1.$$

In this formula l is the length (feet) from the centre of pressure on the main surfaces to the centre of pressure on a tail or balancer. H_n is the potential of the machine in feet $= \mathrm{V}_n{}^2/2g$, where V_n is the normal speed (Ferber's "vitesse de régime"), γ is the gliding angle ($\tan \gamma =$ resistance $\div$ weight), I is the moment of inertia (ft.-lbs.^{2}) about the transverse axis through the centre of gravity.

K = W (poundals) $\div$ V^2, where W is the weight (poundals) and V is the velocity in feet per sec. (equals V_n normally), c, C, ρ, ϵ are constants whose product (for a tail plane whose breadth to length dimensions are in a 10 to 1 ratio) is about $\frac{1}{10}$, a is the area in square feet of the tail plane, and β is the angle in radians made by the aerofoils (supporting surfaces) with the flight-path.

Mr Lanchester considers that a value $\phi = 2$ is sufficient to ensure perfect longitudinal stability, so that to produce a simpler formula we might write

$$\frac{l\mathrm{V}_n{}^4 \tan \gamma}{g^2\mathrm{R}^2\left(\frac{\mathrm{V}^2}{g} + \frac{10\mathrm{W}}{a\beta}\right)} = 2,*$$

where R is the radius of gyration (feet).

The rules given by Lanchester for the other varieties of stability involve too much explanation to be given here, and his book should be referred to. He adopts fins or keels, like Captain Ferber.

From a practical point of view it is probable that if a new machine is designed with a rough approximation to the above rules, and is modified after *long* trial guides, it will be automatically stable under ordinary conditions of flight, and it must next be considered whether it will be automatically stable under *all* conditions—*i.e.* in the presence of maximum aerial disturbance.

Stability in Turbulent Air.

It is a well-known fact that the wind is very irregular both in direction and velocity. Owing probably to interfering currents, obstacles, and cyclonic motion, a gale would seem to consist of a mass of drifting eddies or vortices. From the late Professor Langley's *Internal Work of the Wind* it can be shown that the velocity of the wind varies about 50 per cent. above and below its mean value, with a periodicity from 2 seconds or so to 30 or 40 seconds. The direction of the wind may almost instantaneously change 10° or 20° in azimuth and 5° or 6° in altitude. (See Moedebeck's *Pocket-Book of Aeronautics*, p. 391.) Simultaneous observations of periodicity, velocity, and change of direction would enable a rough idea to be obtained as to size of the vortices, but apparently no such observations have been made. The Forth Bridge and other anemometer readings show that the maximum unit pressure on large surfaces is considerably less (about two-thirds) than that

* There would seem to be some imperfection in this formula, having regard to its lack of reference to the position of the centre of gravity.

on small surfaces, although in a *steady* current of uniform structure the unit pressure is greater on large surfaces. (See author, *The Force of the Wind*, and letters in *Engineering*, 1908.)

According to Lanchester (*Aerodonetics*, p. 74), it is desirable that the gust velocity (*i.e.* the maximum velocity relative to the air in which the machine is travelling) shall not exceed $0·33V_n$, where V_n is the normal velocity of the machine or $V_n = 3 \times$ the gust velocity. Assuming the maximum gust velocity is 30 miles an hour (it is actually more than this on occasions), we find that V_n should be about 100 miles per hour to be immune from gusts. It is obvious that unless V_n does considerably exceed the gust velocity, the periodic variation in lift and resistance[1] will cause oscillations which, if they synchronise with the natural oscillations of the machine, may be excessively dangerous.

If we suppose that the wind consists of a general drift whose velocity is roughly the average velocity of the wind combined with an eddying motion consisting of rotations about approximately horizontal axes, then the actual velocity of the wind will vary from $V + v$ to $V - v$, where V is the average velocity of the wind and v is the orbital velocity of the rotating air; v then will be the relative motion of the air in the eddy to general mass, and the aeroplane will be subject to an increase or decrease of relative velocity when traversing the top and bottom of the eddies equal to v. This will involve an alteration of potential equal to $\pm (v^2 \pm 2V_n v/2g)$. Again, when crossing the diameter of the eddy it will be subject to a torque due to the descent and ascent of the revolving air. It will be at present impossible to give definite values to the effects so produced since the size of the eddies is unknown, and this will affect the area acted upon by the horizontal stream in the eddy or the torque due to the vertical streams. Such a manner of considering the matter is defective in the following respects: (1) The average velocity observed is not the true average, since this is computed from anemometer (*i.e.* pressure) readings, and is really the root of the mean square; (2) the orbital velocity in the vortices is not constant, but increases in inverse ratio to the radius of the part of the eddy considered; and (3) eddies cannot be cut in the manner implied, but will be deformed or displaced with a more or less indeterminate reaction. However this may be, there can be no doubt that the smaller the ratio $v^2 \pm 2V_n v/V_n$ is, the less is the effect on the machine, and this ratio can only be reduced by increasing V_n, the normal speed of the aeroplane. It must not be supposed that the actual effect is represented by this ratio, since there has been no account taken of the masses of air in the eddy and sweep of the aeroplane respectively, but it will be roughly proportionate to this ratio, *i.e.* to

$$\pm \frac{v^2 \pm 2V_n v}{V_n} = \pm \left(\frac{v^2}{V_n} \pm 2v\right).$$

We thus arrive at the conclusion that for an aeroplane to be safe in a turbulent wind the normal velocity must be as high as possible. The author does not absolutely agree with Lanchester's values, since these are based on the assumption that the mass in the eddies is at least equal to that handled by the aeroplane. This assumption is, of course, on the right side,

[1] On Lanchester's basis of least resistance, the total resistance is almost independent of the velocity, so that only the lift will vary.

but it may be unnecessarily so. Mr Herring, in *L'Aéro-Mécanique* (No. 1, 1908), refers to the device employed by him on the Chanute gliders by which the surfaces accommodate themselves to the intensity of the relative wind and balance is maintained. This would certainly appear to be useful, and further experiment is necessary to demonstrate the applicability of the method.

The Wright Bros. are understood to have acquired sufficient experience to enable them to warp the main surfaces and operate the governors so that balance may be maintained in an unsteady current; but as far as the author's information goes, they have not made flights in winds where the gust velocity was comparable with the natural velocity of the machine.

The question of turbulence is also of importance in regard to soaring, and here again the question may be raised whether the eddies are sufficiently large to provide enough mass. In *Aerodonetics*, sec. 160, p. 303, Lanchester says "on the basis that the whole energy of the peripteral area is available" and proceeds to deduce an equation for the energy derivable from turbulence. The author submits that although such a method may be available for bird flight it is extremely doubtful whether the extent of the turbulence is sufficient to ensure enough energy being captured by a large machine. Captain Ferber (*Revue d'Artillerie*, November 1905, p. 104) takes a very pessimistic view of wind fluctuation as a means of producing "soaring" flight. He says: "C'est l'explication la meilleure du vol à voile; mais elle place un peu l'oiseau planeur dans la situation du joueur qui livre son capital aux fluctuations du tapis vert, si la bonne ondulation ne se présente pas à temps il risque de perdre une précieuse partie de son altitude." This is probably going just too much in the opposite direction as far as bird flight is concerned, since the periodicity of the pulsations is fairly regular and "soaring" is only explicable on such a basis, but Captain Ferber's remarks *may* apply to the case of a large flying machine having a far greater ratio of its dimensions to those of the aerial vortices.

Alighting.

We now come to a very serious difficulty—one which has even induced many students of the subject to prefer helicopter types of machine. A machine necessarily possesses a certain normal velocity necessary for sustentation, and consequently when descending, unless there is a general wind of precisely opposite amount (relative to the earth), it will have a considerable velocity relative to the earth, and the chances of a bad collision are great. We may rule out of court the difficulty of very uneven ground, since this will have to be avoided by the aviator as rocks are by the sailor, and assume that the surface is moderately plane. In the first place, we will also assume that it is not only plane but horizontal, and that the air has no appreciable velocity relative to the earth. The machine descends at the gliding angle $\gamma = \tan^{-1}\left(\frac{\text{Resistance}}{\text{Weight}}\right)$. This may also be expressed roughly as

$$\frac{\gamma}{2} = \tan^{-1}\sqrt{\frac{s}{S}},$$

where S is the plan area of the main supporting surfaces and *s* is the equivalent area of the head resistances (including those of the main surfaces).

This rule is one of Captain Ferber's, and differs from the preceding one (due to Chanute) in that the head resistance is considered perpendicular to the axis of the machine, which would not be horizontal during horizontal flight in the Ferber standard type. Certainly the Chanute rule is simpler to employ. However this may be, it is well known from practical experience that γ, the gliding angle, cannot be reduced much below 8° or 10°, and it is in many cases more than this. Ferber machine No. 5 had a gliding angle of 13°. The gliding angles of the Wright and Voisin machines are between 7° and 8°, so that we may write $\tan\gamma = \frac{1}{8}$ to $\frac{1}{6}$. Using the latter as the more unpropitious form, we have the relative velocity of the machine perpendicular to the surface of the earth is $\frac{V}{6}$, and the momentum to be destroyed is $\frac{WV}{192}$. In the absence of experimental information as to the time of impact, it will be better to consider the kinetic energy loss. The mass W/32 has a unit mass kinetic energy $V^2/36 \times 2$, so that the kinetic energy concerned in the vertical impact is $\frac{WV^2}{32 \times 36 \times 2} = \frac{WV^2}{2304}$ foot-lbs. (W in lbs., V in feet per second). This must eventually be lost, but at the moment of impact will be converted to strain energy. If the machine is provided with alighting springs capable of storing this energy, then the force of impact is reduced to a minimum, and should not have any appreciable effect on the machine. This means that the stiffness (half the safe load) on the springs multiplied into the length (vertical) available for compression must equal the above kinetic energy

$$FL = \frac{WV_2}{2304}.$$

Rules will be found in all text-books on applied mechanics for the design of springs of any type to satisfy these requirements. If the spring is stressed up to the elastic limit the quantity FL is the resilience. If there are n vertical springs, all of which come into action during vertical impact,[1] then the resilience of each must be equal to $FL/n = WV^2/2304n$. In Perry's *Applied Mechanics*, p. 631, it is shown that this is equal in the case of a spiral spring to

$$\frac{\pi f^2 d^2 l}{16N},$$

where f is the shearing stress (lbs. per sq. inch), d is the diameter of the wire (inches), l is the total length of wire in the spring (inches), and N is the modulus of rigidity (tons per sq. inch). The greatest force acting on the aeroplane framing is 2F, and the compression of the spring is (as above mentioned) L.

(Note that in using the resilience formula the energy must be converted to *inch-lbs.* by multiplying by 12.)

The magnitude of F can, of course, be decreased by increasing L, or be distributed by increasing n, so that the pressure on the machine frame can be easily reduced to a reasonable amount.

The above case of alighting on a fairly smooth and horizontal surface is

[1] This implies maximum lateral stability.

almost the most favourable one that can occur in practice, the only superior one being the case where the machine alights on a surface which is inclined in the same direction as the gliding gradient and to about the same amount. In the first case, the machine will continue to travel along the ground with a velocity parallel to the surface of an amount $V \cot \gamma$, which will be gradually extinguished by the frictional retardation, or may be more rapidly destroyed by canting the steering surfaces.

If the ground be inclined in the reverse direction to the gliding gradient (say to an angle θ) or the wind be rising towards the machine (relative to the earth) at an angle θ, the ground being horizontal, then the velocity normal to the surface of the ground is $V \tan(\gamma + \theta)$, and a greater kinetic energy has to be absorbed by the springs.

If the ground be irregular, then, in addition to the above effects, there will be rapid extinction of the forward velocity, involving very considerable strain energy in the frame of the machine if no provision is made for this purpose. Probably it will be well to employ springs with a greater resilience than is above specified, and incline their axes slightly forward so that the shock on the frame is diminished.

The whole question of alighting springs depends on the same principle as that of buffer design in connection with rolling stock.

An important point in connection with alighting is the possibility of momentarily diminishing γ at the time of impact. If the lift can be suddenly increased without a corresponding increase in the resistance, then the angle γ may be very small. We must, however, carefully distinguish between this effect and that of a mere torque which rotates the machine so that its axis is (say) parallel with the ground, since in the latter case the centre of gravity will continue to travel in the same direction until the effect of the new attitudes of the supporting surfaces has modified the lift and resistance. The skin friction on supporting surfaces will not vary much with their position, so that this source of resistance may be regarded as negligible. The aerodynamic head resistance of a steering surface will vary as $\sin^2 \beta$, where β is the attacking angle, while the lift varies as $\sin \theta \cos \beta$. The rate of change of the latter function is much greater than that of the former for small values of β, so that if the steering surface is sufficiently large (in ratio to the main supporting surfaces) the lift will be momentarily increased by deviating them through a small clockwise angle to a greater extent than the resistance, and the angle γ will, as required, be decreased, and consequently the energy to be absorbed at impact will likewise decrease. This probably is the secret of successful landing, and will necessarily depend on the ability of the aviator to judge the velocity, gliding angle, and effect of the steering surfaces.

Structural Risks.

With regard to this question, the author does not propose to deal with the matter, since it involves the whole question of the design of aeroplanes. A paper submitted by him to the Institution of Civil Engineers (Ireland) in 1909 considers the question of structural design. It is sufficient here to say that the framing must be constructed in accordance with the principles of strength of materials so as to be capable of sustaining the dead load, aerodynamic load, motor and propeller thrusts and vibrations, and the force of impact due to a clumsy landing. At present it would seem doubtful whether the frames can be sufficiently strongly designed to resist the force of extraordinary impacts, although heavy high-speed machines can be constructed much more strongly than small ones if proper designs are followed, because the supporting force per unit area of surface is much larger.

APPENDIX I.

The Centre of Pressure on Aeroplanes.

During recent years a body of knowledge concerning the behaviour of streams of fluid flowing past an obstacle has gradually been built up, and although the theoretical aspect of the subject is extremely deficient, yet there are certain features which have an intimate relation to the design of aeroplanes. Among these there is none more important than the position of the point at which the pressure upon the plane may be regarded as wholly acting, *i.e.* the centre of pressure. It is well known that this point rarely coincides with the centre of area, so that any plane which is suspended or fixed about an axis passing through the latter point is subject to a turning moment whose value depends on

(1) The resultant pressure on the whole surface.
(2) The distance between the centre of pressure and the centre of area.

These two quantities the author calls P and Δ. The value of the turning moment is obviously the product of the two, *i.e.* $P\Delta$.

Among many designers the rules of Colonel Duchemin for P and Captain Joëssel for Δ are the most widely accepted, and are as follows:—

$$P = \kappa A V^2 \frac{2 \sin \theta}{1 + \sin^2 \theta}$$
$$\Delta = (0{\cdot}3 - 0{\cdot}3 \sin \theta) l,$$

where κ is the pressure on a plane of unit area at unit velocity moving normal to its plane, θ is the inclination between the plane and its direction of motion, A is the area, V is the velocity, and l the length of the plane in the direction of motion.

Although it is now known that neither of these rules is exactly true in all cases, we may, as a preliminary, combine them as follows. Calling A, V, and l constant, we may write:—

$$P = \alpha \frac{\sin \theta}{1 + \sin^2 \theta}$$

and $$\Delta = \beta (1 - \sin \theta).$$

Then $$P\Delta = \alpha\beta \, . \, \frac{\sin \theta (1 - \sin \theta)}{1 + \sin^2 \theta} = \alpha\beta \, . \, \psi(\theta).$$

It will be found that this function of θ increases rapidly with θ, reaching a maximum at about 30°, and then decreasing at first rapidly and then slowly, becoming zero at 90° (see Tables, p. 122). In this way we are able to estimate the turning moment on the plane for any angle. We also know that it varies with the area and the square of the velocity.

Having thus obtained a general notion as to the turning effect, we may examine some of the other rules which have been given for P and Δ.

The existence of Δ was, the author believes, first pointed out by Sir George Cayley in *Nicholson's Journal* in November 1809. Subsequent practical determinations have been made by Hagen,[1] Joëssel,[2] Kummer,[3] Langley,[4] Phillips,[5] Turnbull,[6] and Lanchester.[7] The experimental work on stream-lines by Dr Hele-Shaw[8] has great value in this connection, and Kirchhoff's[9] and Lord Rayleigh's[10] theoretical work needs mention.

The more important of the values for Δ are as follows:—

Joëssel . . $\Delta = 0{\cdot}305l(1 - \sin\theta)$

Hagen . . $\Delta^2 = (l - \Delta)^2 \dfrac{\theta^\circ}{90^\circ}$

Lord Rayleigh $\Delta = \dfrac{\;}{4} \cdot \dfrac{\cos\theta}{4 + \pi\sin\theta} \cdot l.$

Kummer found that in planes where $b<l$ (b = breadth across direction of motion), under a certain critical angle Δ is less than Joëssel's value, and above this angle Δ is greater. The angle is about 30°. For planes where $l<b$ the reverse condition holds good.

Langley confirms Joëssel's results for square planes, and Kummer's results for oblong ones.

Turnbull finds that Joëssel's law is not true under an angle of 18° (26° for an oblong plane $b = 2l$), but that there is a reversal, Δ becoming smaller with θ, so that at 0° $\Delta = 0$ instead of $0{\cdot}305l$. From 0° to 18°, Δ varies with θ. He also gets interesting results for aero-curves.

Phillips has invented a special form of aero-curve, and finds the value of $\Delta = 0{\cdot}33\ l$ when the chord of the curve is horizontal.

Professor Lamb (see Dr Hele-Shaw's paper to the Inst. Nav. Arch., 1898) finds $\Delta = \frac{l}{2}\cos\theta$, and also deduces that the central stream-line is a hyperbola, which, when $\theta = 45°$, meets the plane at right angles. Dr Hele-Shaw has invented a very simple geometrical method of finding the point, and his experiments confirm Professor Lamb's purely theoretical result.

Mr Lanchester deduces that a ballasted aeroplane can be made stable on account of Δ, and Dr Hele-Shaw, in a recent lecture to the Royal Society of Arts, shows this to be the case in some instances, but that this conclusion is generally true is not admitted.

Mr Spratt has also made experiments on aero-curves[11] which agree with Mr Turnbull's.

[1] "Hydromechanics," *Encyclop. Brit.* (ninth edition).
[2] *Génie Maritime.* Mem., 1870.
[3] *Berlin Akad. Abhandlungen*, 1875–6.
[4] *Experiments in Aerodynamics.*
[5] *Engineering*, 1885.
[6] *Physical Review*, 1907.
[7] *Aerodynamics.*
[8] *Trans. Inst. Nav. Arch.*, 1898.
[9] *Vorlesungen über Mathematische Physik.*
[10] *Phil. Mag.*, Dec. 1876.
[11] *Pocket-Book of Aeronautics*, by H. W. L. Moedebeck.

Taking a general review of the experimental results, the author finds the following conclusions may be arrived at :—

(1) *On square planes* Joëssel's law is true from 18° to 90°, but from 0° to 18° there is an inversion, the value of Δ varying directly as θ. (Turnbull.)

(2) *On rectangular planes* the same law holds good, but the constants are smaller. (Turnbull, Kummer, and Langley.) On a 2 : 1 pterygoid plane Δ is 20 per cent. smaller than on a square plane. (Turnbull.) On planes having the breadth in a greater ratio to the length there is a further decrease in Δ, having a minimum about 30 per cent. below Joëssel's value.

(3) *On aero-curves* (convex side upward) there is a negative value for Δ from 0° to about 13°, and a value of Δ about 50 per cent. of that on a similar *plane* for higher angles. (Spratt and Turnbull.) This applies only to circular curves. With a quasi-cissoid curve the value of Δ is always positive (*i.e.* from 0° to 90°), and is about $0{\cdot}3 \times$ length when θ is 0°. (Phillips.)

(4) *On aero-curves* with reflex curvature, the c.p. has a front edge position from 0° to 10°, thereafter decreasing in the ratio $(1 - \sin\theta)$. We can almost write

$$\Delta = 0{\cdot}5(1 - \sin\theta)l$$

for this curve.

It will be understood that the concave downward side is forward. (Turnbull.)

Mr Turnbull has some interesting results also on aero-curves with the concave side upward, which do not, however, call for notice here.

In considering the effect of an eccentric or counterpoised plane it is necessary to notice that the turning moment produced by an eccentric weight varies with the cosine of the angle of inclination, so that the net turning moment *about the central axis* of the plane is

$$P\Delta - W\cos\theta,$$

where P is the total normal pressure,
Δ the displacement of the c.p. from the c. area,
W the eccentric weight,
θ the angle of inclination with the horizontal.

It is usual to suspend the plane eccentrically, *i.e.* the axis of suspension contains the centre of pressure corresponding to the angle at which the plane is generally running. Under these circumstances the turning moment will be much reduced, but at 0° there is a maximum plunging moment.

[TABLES.

TABLES TO APPENDIX I.

I.

Table of displacements of c.p. on a square plane expressed as fractions of its length. Values of $\kappa = (0·3 - 0·3 \sin \theta)$. (Joëssel's Law.)

θ	κ	θ	κ	θ	κ	θ	κ
0°	0·3	8°	0·2582	16°	0·2173	24°	0·1780
1°	0·2957	9°	0·2531	17°	0·2123	25°	0·1732
2°	0·2885	10°	0·2479	18°	0·2073	30°	0·1500
3°	0·2843	11°	0·2428	19°	0·2023	40°	0·1072
4°	0·2791	12°	0·2376	20°	0·1974	50°	0·0700
5°	0·2738	13°	0·2325	21°	0·1924	60°	0·0402
6°	0·2687	14°	0·2274	22°	0·1876	70°	0·0181
7°	0·2634	15°	0·2224	23°	0·1828	80°	0·0046

II.

Turnbull's revised values of κ (from 0° to 20°).

θ	κ	θ	κ	θ	κ	θ	κ
		5°	0·0621				
0°	0	6°	0·0744	11°	0·1366	16°	0·1980
1°	0·0124	7°	0·0868	12°	0·1488	17°	0·2020
2°	0·0248	8°	0·0995	13°	0·1612	18°	0·2050
3°	0·0372	9°	0·1118	14°	0·1736	19°	0·2020
4°	0·0497	10°	0·1242	15°	0·1860	20°	0·1974

III.

$$\kappa \,.\, \phi(\theta) = \frac{2 \sin \theta}{1 + \sin^2 \theta} (0·3 - 0·3 \sin \theta) \,.\, (0·00166)$$

(Duchemin) (Joëssel) (Langley)

(Turning moments on square plane at unit velocity and unit dimensions.)

θ	$\phi(\theta)$	θ	$\phi(\theta)$	θ	$\phi(\theta)$
0°	0	30°	0·000199	60°	0·000066
10°	0·000139	40°	0·000162	70°	0·000031
20°	0·0002	50°	0·000112	80°	0·000008

Turnbull's experiments show that for 10° $\kappa \,.\, \phi(\theta) = 0·000069$.

APPENDIX J.

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