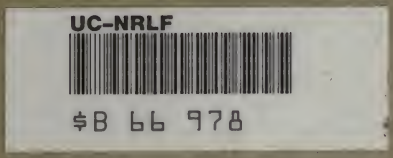

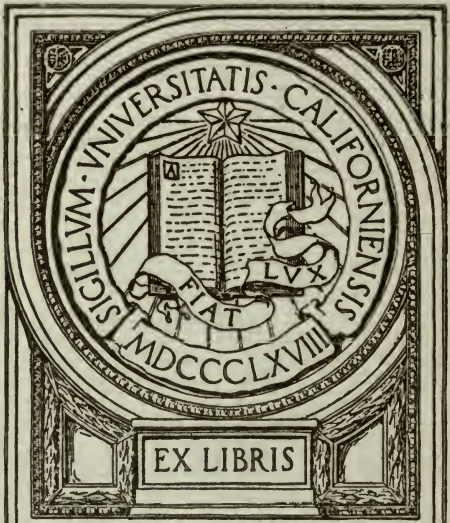

PROBLEMS IN THE TEACHING OF SECONDARY MATHEMATICS

AN ADDRESS DELIVERED BEFORE THE NEW ENGLAND ASSOCIATION OF TEACHERS OF MATHEMATICS

BY

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It is not without considerable hesitancy that I venture to address a body of teachers upon some of the great problems that confront us at the present time in the work in secondary mathematics. This hesitancy arises from several causes, prominent among them being the feeling that I shall only be "carrying coals to Newcastle." For surely these problems are already in your minds, and many of you have pondered over their significance and their solution quite as seriously as I, and no doubt with a more satisfactory issue. I hesitate, also, because I can merely state them with no attempt at solution, mindful all the time of the ancient adage referring to questions which a wise man cannot answer. But after all, there is a value in clearly stating from time to time the large questions that confront our guild, for if problems were never formulated they would never be solved, and it is upon associations like this that we must largely depend for the solution of those that I shall venture to lay before you.

I. HAS THE DAY OF SECONDARY MATHEMATICS PASSED?

The first of the great questions that confront us at the present time relates to the very existence of secondary mathematics in our curriculum. To many of us it may

seem preposterous that the question should seriously be asked. We say to ourselves that if anything is to be blotted out, let it be some language or one of the various manual arts that are from time to time exploited only to find, in most cases, an early resting place in the great educational necropolis of forgotten graves.

But the question cannot be dismissed in any such way as this. Other subjects have been seemingly as well intrenched as mathematics, and yet they have passed away. Formal logic was at one time one of the great features of a liberal education; it gave place, in the secondary school, to formal grammar; but a university course in formal logic is now a rarity, and a high-school course in formal grammar, as it was conceived of a few years ago, is almost unknown. The world seems to proceed as well without these subjects as it did when they held prominent place, and we have to face the question whether it would not get along just as well if algebra and geometry followed them into educational oblivion. The medieval Compotus was once an essential feature in the education of a learned man; it was apparently intrenched in a position of security; and yet, as I mention it to-day, half of this audience may be ignorant, and excusably so, of even the meaning of the word. Somebody at some time asked the question, "Why should an educated man need to study the Compotus?" — and the answer came in due time, "There is no reason," and the subject was soon forgotten. Somebody to-day raises the question, "Why should an educated man need to study algebra?" — and we, the teachers of mathematics, must answer. A high school in which I am interested holds its last class in Greek this year — one of the

best-known high schools in the country, a school of five hundred selected pupils, all of whom are hoping to enter college. Shall it be, a few years hence, that this same school shall be teaching its last class in geometry, compelled to drop another ancient and honored subject because it is no longer demanded?

We may say to ourselves that high-school mathematics has always existed, that it is not a college subject, and that it is absurd to talk of abolishing it. But if we say this we forget that the American high school is itself a new institution; that it has no exact parallel in other countries; that other countries select their pupils for secondary work while we seek to educate the mass; that 95 per cent of our high-school pupils do not go to college, and do not hold the intellectual standards held by the boys of our old academies; and that it is comparatively a recent demand of the American college that its candidates must offer any mathematics beyond arithmetic. So when we speak of high-school mathematics we should bear in mind that our high school has not as yet proved its worth, and is even now being weighed in the balance with rather unsatisfactory results, and that precollegiate mathematics is only a recent matter. If a pupil postpones his Greek until he enters college to-day, why should he not postpone his Latin, his algebra, and his geometry to-morrow?

Now this is not a cry of alarm for the sake of temporary effect; it is a succinct statement of the arguments that we frequently hear to-day from the general educator. Up and down this country, before many gatherings of teachers, the question is being vociferously propounded, "Why should the girl ever study algebra?" Even in associations of

teachers of mathematics the question is being asked, "Who would stand to-day for the spirit of Euclidean geometry?" And the men who ask these questions hold prominent positions; they are professors in universities, educators of influence, men whom the mass of teachers naturally look to as leaders. The problem is, therefore, a real one and one that we have to face.

But we must not deceive ourselves by thinking that we can successfully meet it by mere opprobrium. We not infrequently hear it asserted that the general educator is usually a man of a low degree of scholarship, that his range of culture is limited, that he was taught Latin so poorly that he believes it can never be taught in any other way, and that he rarely stands for any intellectual ideals; but we must remember that, even if the assertion has some truth, enough people of this type might easily create a Zeitgeist that would not down by any such formula as "Weave a circle round him thrice." These men who attack the ancient culture have been rather recklessly called educational muckrakers, men who seek only the bad and judge everything by that; they have been hysterically denominated pedagogical anarchists, men who destroy without rebuilding; and they have been frequently looked upon as intellectual iconoclasts, those who in their zeal to destroy idols are willing that all the beauties of art, which their dull vision fails to see, should go to the scrap heap. In all these assertions there may be some grain of truth, but no one gains anything in an argument by giving expression to such an attitude of mind. Such an expression merely brings the countercharge that those who hold other views are reactionaries, *laudatores temporis acti*, unprogressive,

and selfish clingers to their little jobs. The epithet of "old fogy" is as weighty as that of "muckraker."

When we calmly consider the question, we find that it relates to the value of algebra and geometry for the democracy that America, in distinction from the rest of the world, is trying to educate in the high school. What does democracy want of mathematics? And in our America of the dollar we find that the question is often reduced to that of the immediate utility of algebra and geometry. The potential utility does not seem to enter into the consideration of the type of reformer that seems to speak the loudest upon the subject. And here appears to be the real point at issue: one side demands the immediately useful, while the other stands for that which it claims to be potentially so. Can we, therefore, justify our secondary mathematics on the potential side? For surely no one would for a moment claim that the teaching of the immediately practical part of algebra to a mechanic would require more than a month, or that the immediately practical part of demonstrative geometry exists at all, taking these words in their usual popular significance. Such, then, is the first of the large problems that seem to loom up before us. Call it a claim for mental discipline if you please — this is a mere question of fashionable or unfashionable phraseology; it is a claim for serious attention to a vital issue in education.

II. WHY SHOULD NOT ALL MATHEMATICS BE ELECTIVE?

The general educator is usually found to concede that mathematics should be taught in our high schools, but he is frequently heard to assert that it should be elective.

Many teachers of mathematics, perhaps most of them, would personally welcome such a change, since the pleasure of teaching is largely increased if the learner takes the subject *con amore*. But of late a new type of educator has appeared, the one who proposes to weigh in psychological scale the intellect of youth and to guide it aright. You have it in your own part of the country to-day in the phase of vocational guidance, and in this work so many excellent people are seriously engaged that we are certain to see it become an important phase of modern education. Let the boy who gives promise in science begin his specialization early, say those who seek to guide the youth in a scientific manner, and let the one who takes to Latin bend his energies there. Let there be scientific tests to show whether or not the particular individual can hope for success in the particular vocation — a worthy effort and one that will produce good results. But there are not wanting those who will be less scientific, and who will assert that one who, by virtue of his surroundings and family, is destined to be a hewer of wood, should early come to like to hew, and should be taught chiefly the nobility of labor with the hand. That we may realize some of the dangers that beset those who seek to guide the youth aright, and who may feel called upon to sidetrack all that is not immediately practical, let me tell you some advice that I myself have given within a few years past in cases like these, and lay before you the problem that I had to face.

Not long ago there came to me a father who wished to train his boy for trade in a seaport town, and who asked my advice as to the proper education to give him. The problem seemed simple. The community was not an

educated one; it lived off its little shipping industry; the boy was destined to small business and to small reward; he gave no promise of anything better, and the advice was, therefore, unhesitatingly offered that the only mathematics he needed was arithmetic through the sixth grade.

Another parent asked me a little later about his son. The boy was of the ordinary type and would probably follow his father's occupation, that of a sculptor. What mathematics would it be well for him to take? I suggested a little study of curves, some geometric drawing, and the modeling of the common solids — a bit of vocational guidance that seemed to me then and seems even yet particularly happy.

A third boy happened to be with me on a steamer and I took some interest in talking with him and with his mother. They lived in a city of no particular note, at any rate at that time, and the boy was going into the selling of oil within a few years. The profits of the Standard Oil Company appealed to the family, and I advised him to learn his arithmetic well and get into business as soon as he could.

Out of the store of my memory I recall a curious lad whom I came to know through my sympathy with the family. The mother was a poor woman, and she took the boy, when little more than a baby, over to Riverside Park one day when there was a naval parade. A drunken sailor, having had a fight with a group of hoodlums, rushed through the crowd of spectators and slashed right and left with a knife. In the excitement the boy, in his mother's arms, was horribly cut in the face. When I knew them he was about ten years old, unable to speak plainly, and

already a misanthrope through his affliction. I advised the mother to give the boy a vocational education, telling her that through the use of the hands he would satisfy his desire for motor activity, and that this would compensate him for the loss of verbal fluency and would tend to make him more contented with his lot. In this advice I feel that I would have the approval of educational circles.

And finally, out of this series of experiences, let me recall the case of a boy whom I came to know through a noble priest who found him one morning, an infant a few days old, on the steps of his church. We talked over the best thing to do for such a foundling, one who, at the time I knew him, was in the primary grades. He showed no great promise, he was without family recognition, and his only chance, apparently, was in the humbler walks of life. I recommended a vocational school where he could quickly prepare for the shop or the lower positions of trade, and the good priest approved my plan at the time, although he finally followed quite a different course.

It is apparent, however, that I have here spoken in parables. Perhaps you already recognize the boys, and perhaps you feel how sadly I blundered in my counsel. For the first of these whose cases I have set before you felt a surging of the soul a little later, and this was recognized in time, and he became one of the Seven Wise Men of Greece—Thales the philosopher, he who introduced the scientific study of geometry into Greece. The second felt a similar struggle of the soul, and his parents recognized my poor counsel in time to save him and to give to the world the founder of its first university—Pythagoras of Samos. The third boy, for whom only the path of commerce

seemed open, and this in a town only just beginning to be known, was the man who finally set the world's first college-entrance examination, the one who wrote over the portal of the grove of Academos the words, "Let no one ignorant of geometry enter here" — Plato, the greatest thinker of all antiquity. The fourth, the hopeless son of poverty, maimed, sickly, with no chance beyond that of laboring in the shop for such wage as might by good fortune fall to his lot, became the greatest mathematician of his day — always the stammerer (Tartaglia), but one whom Italy has delighted to honor for more than three centuries. And the last one of the list, the poor foundling on the steps of St. Jean-le-Rond in Paris, became D'Alembert, one of the greatest mathematicians that France, a mother of mathematicians, ever produced.

Shall we, then, advocate the selection of those who are to study mathematics and close the door to all the rest? Are we so wise that we can foresee the one who is to like the subject, or succeed in it? Have we so adjusted the scales of psychology that we can weigh the creases in the brain, or is there yet invented an X-ray that will reveal to us the fashioning of the cells that make up its convolutions?

Of course it will at once be said that these illustrations that I have given are interesting, but that they are unfairly selected; that those boys gave earlier promise in mathematics than I have said. It will be asserted that I should have taken the case of the stupid boy, the one who did not like school, the one who liked to play with little wind wheels, who liked to fight, who actually did run away from school, and who stood near the bottom of the class in mathematics. Such a case would be a fair one, one in

which we could safely say that prescribed algebra and geometry are out of place. And I suppose we must agree to this and confess that the argument from the historical incidents that I have mentioned was unsound. Let us rather take this case that I have just described, and let us see to whom the description applies. I need hardly tell you who this boy is; he is well known to you; he is well known to the world; and long after every educational reformer has passed into oblivion his name will stand forth as one of England's greatest treasures, for it is the name of Sir Isaac Newton.

But again I have been unfair, perhaps. I should have taken positively hopeless cases, for such can surely be found. I should have taken some illiterate man, one who does not learn to read until he is nearly out of his teens, or else some man who shows no promise in mathematics by the time he reaches manhood, or some one who by the time he is thirty is to show no aptitude in the science. It is so easy to theorize! But let us have care, for the men whom I have now described are Eisenstein, Boole, and Fermat. Take them away and where is your theory of invariants, your modern logic of mathematics, and the greatest genius in theory of numbers that the world has ever seen?

But I am wandering afield, and I fear I may be interpreted to question the modernizing of our educational work. I thoroughly favor scientific, vocational guidance as undertaken by a small number of our best scholars in the field. With equal zeal do I favor industrial education when it is not so narrow as to condemn a boy to some particular groove in life, and I earnestly hope that we shall so guide our youth that every boy and girl will leave school fitted to

do something well. I do not believe that any thoughtful educator wishes to guide a pupil in a narrow path nor keep from him the chance that the world owes him. It seems right, however, to set the problem clearly before us: Can we safely say that we may close the door of mathematics to any boy? Should he not be given the chance? If he fails, that ends it, but if he succeeds, the world is the winner in the lottery. Of course this does not answer the question as to what this chance should be; it is quite possible that it should not be our present algebra; it is even possible that it should be merely some form of mensuration that masks under the time-honored name of geometry; it may conceivably be some emasculated form of fused mathematics that has none of the logic of geometry and none of the beauty of algebra, although I do not believe it; and it may even be some form of technical shop mathematics that appeals to but few pupils because of its very technicalities. This is the part of the problem to be solved. But that the door of mathematics of some substantial character shall not be opened, and opened after arithmetic has been laid aside as the leading topic, seems unthinkable.

III. DOES THE GIRL NEED TO STUDY ALGEBRA?

A third question that seems at the present time to agitate the educational interests in some parts of the country relates to the study of algebra by the girl. This carries with it the corollary that no mathematics whatever, beyond mere computation, is to be required of at least half the force that controls the world.

How this meets the views of the emancipated woman I do not know. I assume that she would say, if asked, that

if algebra and geometry are good for the boy, save in the narrowest technical sense, they are good for the girl also. I should think she would say that if mathematics is the one subject that makes us understand our infinitesimal nature in the infinite about us; if it is the one science that has had the most to do with banishing the superstition that comes from simply looking at the heavens with lackluster eye; if it brings a mental uplift that no other science brings and lets us see what seems the nearest to exact truth of anything that we meet; and if we find at every turn the mathematical invariant, — this timely symbol of the unchangeable that presents itself just as the youth is passing into manhood, — I should think that she would say that if mathematics brought these things in its train it is worth while for the girl if it is worth while for the boy.

But to me there is a more serious side to the problem. Mathematics, in one form or another, is going to continue to be taught in the schools. It will gradually change to meet the future demands of the times as it has always done in the past. So far as we can see there will be a mathematics that is immediately practical for those who are not hoping for any intellectual leadership, and there will be the mathematics that I have described as potentially useful for those who are not content with remaining in the lower intellectual class. But in any case there will be mathematics, and the boy will study it. The question as it relates to the girl is, as I have intimated, a more far-reaching one. When this was an agricultural country the father directed the education of the children. In the long winter evenings he had the time and inclination to help those of his household who at that season of the year were taking what was termed

their schooling. The mother had other duties that filled her time, and, moreover, was not herself well enough educated to give much assistance in the matter of study. But America has changed. With our urban population at least, the father no longer has control of the education of the children. In our manufacturing centers he is busy in the shop, and his hours are no longer limited by the light of day. On the other hand, the urban mother no longer weaves and spins, no longer helps in the fields, no longer preserves fruit, and has almost forgotten how to make bread. She has more time for the higher life, and it is she, rather than the husband, who gives the direction, the help, and the encouragement in the education of the son as well as the daughter. The father may need some algebra in his trade, for if he reads the artisans' journals he must know how to manipulate formulas, but the mother must have a general knowledge of the subjects that children study if she is to be the sympathetic director of their intellectual activities. It is the woman quite as much as the man who needs a broad education at the present time.

When I hear some man who is skilled in dialectics attack the teaching of algebra to the girl because it does not enter into her immediate life, I am led to wonder if he would have women learn anything whatever except the work of the household and the club. The classics of the world appeal to her immediate needs and interests quite as little as mathematics, and hence these would be dropped. Indeed, we hear not infrequently of some school principal who proposes the epoch-making plan of studying literature only from current magazines. If we agree to drop all noble literature, all knowledge of history except what comes from

the newspapers, all study of good music and art except by those who expect to be musicians or painters or designers, all science except such as relates to the simple chemistry of foods, all astronomy, all physics, everything except what concerns the science of good physical living, then we may let mathematics go with the rest. Perhaps this is not what these agitators mean — one never knows, for they themselves never seem to have any exact ideas upon the subject. Theirs it seems to be to agitate, but never to formulate, and yet they serve a purpose in the economy of education as a yeast microbe does in the economy of nature. I refer here not to the scientific psychologist who is seriously working on the great problem of education, but to the man of brilliant speech who prostitutes his powers of persuasion to cast doubt on whatever fails to appeal to his immediate fancy.

Hence the problem seems to me not so much to decide whether or not the girl should study algebra, as to decide how we shall so teach the subject to her that she will know of its beauties, of its purposes, and of the feeling of mastery that comes from its pursuit. Such a problem may well occupy the attention of associations like the one I have the honor to address.

IV. WHAT SHALL BE THE MATHEMATICS OF OUR TECHNICAL SCHOOLS?

A fourth problem that is thrust upon us by modern conditions relates to the mathematics of our technical schools. While we do not hear such vociferous assertions as we did a short time ago about the fact that "the doctrine of formal discipline has been exploded," — a resonant but rather uncertain phrase that was fashionable for a time, — it is

neverthless quite axiomatic that the mathematics of a technical school should aim at something in addition to general culture or power. In a school of mechanics we need the mathematics of mechanics, and so for other special fields. And yet, as we look over courses of study for our agricultural schools, for example, we find nothing but arithmetic taught in one college, while mathematics through the calculus is taught in another; and in the high schools the confusion is equally apparent. And what is true for agricultural schools is quite as true for other technical institutions. We have not even begun to think seriously about solving the problem for our high schools, and the same thing can be said for our industrial schools of a more elementary character. Mere technical mathematics alone has never succeeded, and the nature of the general mathematics that is best suited to develop the power to handle the problems that confront the foreman who is erecting a skyscraper has never yet been determined. Here, then, is another serious problem that meets us when we try to settle upon the best course in mathematics for the technical schools of our country. These schools are hardly started in America as yet, although in Europe they are well established; and if we may judge from world experience, their success is to depend in no small degree upon the quality of mathematics that will enter into their curricula.

V. THE PROBLEM OF OUR BACKWARD AMERICAN MATHEMATICS

There were in the mind of those who initiated the work of the International Commission on the Teaching of Mathematics three large problems. The first was that of

leading each nation to take stock of its own work in presenting mathematics to its youth, that it might have before it a kind of moving picture of its teaching, from the kindergarten through the university. The second was that of informing other nations of this work, to the end that all might profit by the success and failure of each. And the third problem was that of looking beyond the confines of one's own land and seeing what the rest of the world is doing. In the United States upward of a dozen reports were issued, telling the story of our own work. But there remains the third problem, that of looking abroad and seeing wherein other nations are surpassing us, and then of finding the causes and the remedy. This is the problem of the immediate future, and it is proper on this occasion to set forth one of its phases.

When we compare, year for year, the work in mathematics here and abroad, we are struck by the fact that we in the United States are not only not the leaders, but in nearly every respect we are distinctly behind the other prominent countries of the world. At the end of our seventh grade we are about a year behind, and at the end of our twelfth school year we are about two years behind, other great educational nations in the teaching of mathematics. You say this to a professional pedagogue of the general type and he will make all sorts of apologies. He will say, "Oh, this is America, and we have different problems,"—as if that were any excuse. He may lay it to climatic conditions, to the necessity for assimilating a million immigrants a year, to the paucity of teachers in a new country, to the brief tenure of office of women teachers and even of men, to our shifting population, to our democracy of education,

to the greater breadth of our curriculum, or to any one of dozens of other causes. But he is a rare educator who will come out and assert that we have a soft pedagogy that often dominates our elementary school, a sweet but mushy pedagogy that brings a maximum of temporary pleasure with a minimum of intellectual attainment. Perhaps the reason that he does not say this is that it is not true. Perhaps we are wiser than our European neighbors in selecting topics for our schools that are better adapted to train to good citizenship. If so, the results certainly do not show it as yet. And when one sees the vigor, contentment, good spirit, and comradeship that are found in this generation in so many of the schools that represent the influence of Pestalozzi in certain of the countries abroad, and then sees the intellectual progress that these schools foster, he must question, if he is open-minded, the wisdom of those who have directed the American policy. We are behind; bad as the European arithmetic is, ours is worse; backward as the European boy may be in his algebra, ours is more so; faulty as may be his attainment in geometry, that of our boy or girl is still more so. I know of plenty of schools in which boys at the end of the twelfth school year have a good working knowledge of trigonometry, a fair command of the basal principles of analytics, and enough ability in the calculus to meet the demands of a pretty good course in analytic mechanics, but they are not American schools. I join you in excusing ourselves; I know the standard explanations by heart; I am even willing to join with every American, of the type satirized by our foreign friends, in asserting our claim to having the longest river, the

biggest lake, the tallest skyscraper, the wealthiest men, and the most abject poverty to be found anywhere. But after all this boasting is over, I have to confess to myself that, in the teaching of the science in which this association and I are interested, America is behind, definitely and unquestionably behind, and I seek not for a dozen causes so much as for one good remedy.

When we attempt to free ourselves from our insular habit, and turn our attention to learning from the experience of the rest of the world instead of from the theories of the lecture room, we find that the work of the first two years is generally more definite in other countries than in ours. The idea that arithmetic shall be merely incidental in these years is not held abroad, and, indeed, has little scientific standing even here, although it has numerous advocates. Six years are elsewhere generally deemed sufficient to cover the essentials of arithmetic, the subject thereafter being reviewed and applied along with algebra and geometry. Instead of beginning formal algebra as a new topic in the ninth school year as with us, the subject is introduced by easy steps in the sixth and seventh grades, so that the pupil is initiated by slow degrees into the advanced stage. Instead of withholding geometry until the tenth school year, and then suddenly springing the demonstrative phase upon the pupil, this subject is introduced along with algebra in the elementary grades. In other words, instead of devoting the seventh and eighth grades to a business arithmetic that is often too difficult for the children, a simple initiation is given into the algebra of the formula and equation, into geometric drawing and the simpler demonstrations, and into such higher arithmetic as

is within the grasp of the pupils of these grades. There is thus an intelligent preparation for formal algebra and demonstrative geometry that is quite lacking with us. Moreover, in the eleventh and twelfth school years there are often opportunities to take courses in trigonometry, analytics, the calculus, and mechanics, and sometimes mathematical astronomy, that are almost never found in our schools, and that have proved their worth by the results attained.

It would be a sad error if we should conclude from such a statement that our work is all bad and the foreign work all good. The human tendency that Horace satirized, of looking on our neighbor's possessions as better than our own, must always be recognized. The fact is that we have in America an excellent course in algebra and geometry, better in some respects than those found abroad. Our arithmetic work, too, has many features of superiority. But our deficiency seems to lie in three features: our dawdling over early primary arithmetic, our neglect of the initial stage of algebra and geometry in the seventh and eighth grades, and our failure to offer advanced electives in the last two years of our high school.

I am well aware of the difficulties to be met in applying the remedy, and it is unnecessary to dwell upon them here. For example, I know the gap between democracy and aristocracy in education, but I don't believe it to be as wide as many people think. I wish to set the problem before you rather than to attempt to point out the slow steps by which the solution may be effected. If an ideal is kept before our people we will all gradually move toward it, and this gradual trend is wiser and safer than any attempt suddenly to attain

results that seem to us desirable. The problem is to preserve the serious, orderly mathematics that we have, while adopting such good features as the rest of the world may suggest to us.

VI. THE QUESTION OF PARALLELISM

A glance at the foreign schools, such as has been given, suggests another problem that we are reasonably certain to meet in the near future — the one of rearranging our curriculum. America is one of the few countries in which algebra and geometry are commonly taught in tandem fashion. The required work in most of our high schools is one year of algebra followed by one year of geometry, and this followed by an elective course in algebra, and this by one in solid geometry. The rest of the world in general does not pursue such a plan. It carries its algebra and geometry separately, as we do, but it carries them side by side, say from the seventh grade through our high-school period. It may definitely assign two days of the week to algebra, two to geometry, and one to arithmetic; or it may make some other arrangement, but in any case the arrangement is based upon the idea of parallelism. I do not believe that America is ready for this at the present time. Indeed, I think that it never will be ready unless it adopts the plan of a six-year high school, beginning with the seventh grade, or goes even farther in following the European arrangement. But there are many arguments in favor of the scheme in case such an administrative change is made. At any rate, the plan succeeds everywhere else, and because it does not succeed under our present conditions is no reason for believing that we shall not have to face the problem in the changing conditions that are likely to be met in the near future.

VII. ARE WE MAKING MATHEMATICS INTERESTING?

Among all of the experiments that have been made in teaching mathematics in this country, not much that is strikingly new has been evolved, nor much that seems permanent. We have had in general education a great many bubbles to prick, as witness the failure of the type of manual training that was a few years ago asserted to be a panacea, the fading away of the doctrines of concentration and correlation, the semioblivion into which the culture epoch theory has passed, and the fate of the Grube method. Likewise in the teaching of mathematics we have our bubbles, bubbles blown with the enthusiasm of youth and pricked with the experience of years. Sometimes the bubble is a geometry syllabus, sometimes an impossible fusion of mathematical topics as far apart as Latin and chemistry, sometimes a "ratio method," sometimes a wild adoption of the graph, and sometimes a form of practical mathematics that is so technical that it repels pupils and teachers alike. And hence it is not without hesitancy that I suggest what very likely may develop into another bubble — the problem of making mathematics more interesting in and for itself.

Although no two sets of educational statistics ever seem to prove exactly the same thing, all statistics that touch upon the subject tend to show that mathematics ranks well up in the scale of pupils' interests. The latest set that I have happened to see placed it third or fourth from the top on a scale of twelve, with the vocational training, that was to have been our salvation, down at the bottom of the whole list. However taught, mathematics always has in it

the game element. You play the game, and you win if you really set about to do so, and when you win you have a definite result. We start, then, in the teaching of mathematics, with this great advantage.

But the question arises, Are we making enough of this matter of interest? If we give a couple of pages of dull, dry, algebraic formalism, with nothing contributed by the teacher to enliven it, are we doing our best? If not, what more can we do?

The problem is not one to be solved in a moment. But let me suggest that the history of mathematics is not being used as wisely as it might be in our classes. I do not think it is presented to best advantage in the form of disconnected notes in a textbook, but as outside material to be brought into class, to be the subject of a moment's inspiring talk by the teacher, it has unquestionable value. And so it is with the recreations of mathematics, a subject on which we have considerable available literature. Are we using this material as we should? Is it feasible in our schools to establish mathematical clubs, such as Mr. Newhall describes in his monograph in the Commission report on mathematics in the secondary schools? I have myself ventured to suggest in a recent number of the *Teachers College Record*, under the title "Number Games and Number Rhymes," a few possibilities, and to incorporate some material on the subject in my work on the Teaching of Arithmetic. Certain it is that there is an opportunity for serious work here, and that some good may be accomplished if we do not go to the extreme of making a mere bubble out of the effort. There would not be so much heard about the immediately practical in mathematics if we

would show our pupils the interest in the subject *per se*, and the meaning of the science in the larger life about us. I wonder if we ourselves ever stop to think of the effect of blotting out of existence every book or manuscript, say even on so important a subject as the general science of education, and also every book or manuscript on mathematics. In the former case the schools would open next week as they opened this; the teaching would go on as before; the world would know no difference save in a few institutions for the training of teachers, and even they might conceivably be the better off. But in the case of mathematics every great engineering project in the world would be stayed; the skyscraper would not be planned; the next ocean leviathan of steel could not be begun; the banking of the country would halt; all safe navigation would cease; the science of artillery would need to be begun anew; astronomy would stand aghast; mechanics would have again to frame its laws; and civilization would send out the hurry call for intellects to repair the damage. The question, then, as to what would happen if mathematics were taken away, might well be suggested to a mathematics club in a high school, and this is a type of dozens of other questions that would probably add to the interest that the pupils take in the study. Imagine the joy of a healthy-minded pupil when he first reads "Flatland," or "Another World," or Hill's "Geometry and Faith," or when he comes to know Ball's "Mathematical Recreations," or White's "Scrap Book of Mathematics"!

Sometimes, however, I feel that we get little thanks for such an attempt. I know a certain prominent educator who in private conversation is always sane and thoughtful,

but who now and then seems to fall into grave error in his public utterance. He is reported as having recently remarked, in one of his somewhat ill-considered outbursts against mathematics, that no teacher had ever yet told his daughter why she studied the subject. Of course it is quite probable that no one ever told her why she studied anything. He admitted, however, that she was very fond of the subject, but he raised the question as to whether this is any justification for teaching it to her or to any one else. I know of no one who would make any such claim, but I am equally certain that it is an advantage to have the subjects that we teach made as attractive as possible without taking from them the sterner qualities that they may possess. And so, at the risk of being opposed because I take such a commonplace position, I suggest as one of the problems of our time the effort to make mathematics still more interesting, independent of its higher purposes or its narrow field of immediate applications.

VIII. THE FUNCTION PROBLEM

It is perhaps well to inject into this discussion a problem that is a little more mathematical than the ones that have been suggested. For this purpose I select one of which a great deal has been said in Europe during the past five or ten years, and one of which we are soon to hear. I refer to the introduction of the function concept early in our work in mathematics, making of it a kind of unifying principle of the elementary science. I have not the time to refer to its recent history; to the rise of the idea of introducing the concept into elementary work, due largely to the efforts of the French engineers; to its elaboration

by men like Tannery; to its cool initial reception by the French teachers; to the subsequent efforts of Klein, and to its recent success in Germany. Suffice it to say that there seems to be good reason for bringing before the pupil, as soon as he begins the study of algebraic symbols, the idea of function. This idea naturally enters into the study of the simplest formula, it is the essence of the equation, it is the major part of mensuration and trigonometry, and in the mathematics that follows it plays a part of ever-increasing importance. The question is, What shall be its fate in America? Shall we receive it with Western enthusiasm, exploit it as we did graphs, go to an extreme from which we must recede as we did in that case, and finally, after much waste of energy, come down to a sane use of this valuable aid to the study of mathematics? Or shall we first ask ourselves the question as to why we study the subject, what we propose to accomplish by it, where we can really use it to advantage, and what are the extremes to be avoided? If we take the latter course, we shall reach the sane stage much more quickly than by the former route. Already the danger has appeared, and so it is well that we consider this as one of the problems demanding our serious attention.

IX. THE PROBLEM OF ALGEBRA AND GEOMETRY

It would be too much to expect you to listen to any extended statement of the problem of our present algebra and geometry. That such a problem exists is patent to us all. That conditions are not ideal we would all agree. It is not the best type of education that begins the work in algebra with mere formalism, nor the work in geometry

with a difficult demonstration. But while we agree to this statement, and object to the wretched plan that was at one time followed, quite as strongly as our colleagues in the general field of education can possibly object to it, we must recognize the fact that we are already improving the situation very rapidly, quite as rapidly, indeed, as seems safe. Practically all of our better algebras to-day have introductory chapters that set forth the uses of the science in a simple and interesting manner, and the problems that they offer are becoming more real from year to year. Our better textbooks in geometry no longer begin with a proposition to be demonstrated, but, like the algebras, have their introductory chapters that show the significance of the subject. This steady progress is accompanied, to be sure, by many amateurish efforts and experiments that are foredoomed to failure, but the great body of common-sense teachers is not disturbed by these eccentricities and is steadily progressing to a better understanding of the problem and to its certain solution.

X. THE PROBLEM OF SCIENTIFIC EDUCATION

It would not be right to close without some reference to a problem that has recently come to the front in educational circles, fostered to no small extent by the labors of my distinguished colleague Professor Thorndike, and carried on by a number of his former students. I refer to the attempt to find scientific sanction for what we do or think we should do in the classroom. We proceed at present as a result of what we believe to be the best experience of the race, and this in itself is a scientific method. But may we not accelerate this experience and improve

more rapidly? It is the belief of most educators of scholarly rank that it is possible to find out the causes of any unnecessary retardation in the advance of the mass of pupils, and we should certainly encourage all scientific attempts in this direction so far as they relate to the work in our field. If the trained investigator can show us where we can best begin informal algebra, informal geometry, and formal mathematics; if he can find a general norm for the drill work that shall give the best results; if he can tell us what we may safely demand of the average boy and girl; and if, through the examination of a sufficient number of cases, he can standardize the work in mathematics in our high schools, we shall all be his debtors. To solve this problem he must take account of a large number of factors, and his efforts will not meet with success until these factors are all considered; but meanwhile we should recognize the problem and give our encouragement to its scientific solution.

XI. THE PROBLEM OF OUR DUTY

Addresses like the present one are ephemeral; they reach no conclusion, and they are not likely to provoke very serious thought. And yet they serve a purpose if they select from the wide range of questions of the day a few that seem to demand special attention. As I said at the outset, I have no definite solutions of any of these problems, or rather I do not believe that, by laying before you such as I have, so much will be gained as by stating the problems themselves. The answer to all such questions will follow in the general agreement of the large, silent, thoughtful body of American teachers. But it would not be right that I

should close merely with the enunciation of a few questions, accompanied by remarks that are more or less rambling. I therefore wish to suggest one more problem, that of our present duty, and to indicate what seems to me its solution.

I believe it to be our duty to stand solidly against the lowering of the standard of mathematics that shall make of it only a science that is immediately instead of potentially practical. I believe that we should open the door of the great field of algebra and geometry to every boy and every girl in our country. If they fail, let them substitute some other science for this, but offer them the chance. If we can extend our high school downward to include the seventh grade, the offer may be made in the seventh and eighth grades, and this may be better than to require the algebra and geometry of to-day. But to close the door of opportunity in any one of the great branches of knowledge is a crime, whether it be in biological science, in music, in language, in mathematics, in history, or in any other field of world importance. I also believe it to be our duty to favor the extension of high-school mathematics downward to the seventh grade at least, taught by the high-school teachers, which teachers should in the near future be trained in physics as well as our own science. With this should come the extension of electives upward, to include courses in analytics, calculus, and mechanics. To say that this is impossible is to say that the American youth and the American teacher lack the abilities of their foreign colleagues.

I believe it to be our duty to encourage in every way a proper industrial education, but to insist that serious, thoughtful mathematics shall have its place, and shall not become merely a collection of a few rules of thumb. I feel

that we should guard against making the function concept ridiculous as we, in some quarters, made the graph an absurdity; but that we should accept it for what it is really worth and use it wherever in our teaching, to use the jargon of the pedagogue, it "functions." I conceive, furthermore, that an association like this is unanimous in the belief that we should "hold fast to that which is good," not allowing ourselves to give up the serious study of algebra because it has not always been well taught in the past, nor the serious study of demonstrative geometry because some one has, for the millionth time, slaughtered Euclid. The possible improvements that are suggested by the problems that I have ventured to lay before you may be effected with only a natural and gradual change of the science as we have it; they demand no cataclysm, least of all the cataclysm that would follow if some of our reformers had their way. When we see a man claiming that he speaks for the 95 per cent of high-school pupils in demanding the abolition of real mathematics, we will do well to listen to his objections, trying to remove any just cause for his complaints. But if we find that he is merely the advocate of the nonintellectual, and that he would take from us all the idealism that we have, then we have a right to put forward the claims of the boy and girl who really wish to learn and in whose souls idealism is beginning to take a start. But even more than this, we have the duty to do our best to bring this idealism into the schools for the 95 per cent of whom we hear so much, and this I believe we shall do effectively only when our high-school work begins earlier than at present, and we put some introductory and practical work in the seventh and eighth grades.

And above all, it seems to me to be our duty to stand for the interest of mathematics for its own sake, for setting forth its beauty of symmetry, for voicing its poetry, for living its religion, and for exalting it because of the truth that it sets forth so clearly and because of the invariant properties that characterize it in every branch. So far as possible, all this should grow out of the child's experiences and needs; not merely out of his external experiences and needs in the narrow sense of the workshop, but also out of those which are more vital and of which he is conscious within. It is only by being imbued with such feelings and ambitions that we can bring our pupils to love the subject and to feel the great mental uplift that comes from its study. May the statement of the few problems that I have set before you assist us all to maintain this spirit in the future as I am sure we have tried to maintain it in the past.