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## Faculty Working Papers

PROBLEMS WITH VALIDITY OF TIME SERIES EARNINGS  
FORECAST MODELS

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#642

College of Commerce and Business Administration  
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$$\sigma_m + \tau + \sigma_z + \sigma_1$$

then, 
$$\hat{\beta}_j = \text{plim}_{n \rightarrow \infty} \hat{\beta}_j + s_j = \beta_j + \psi_j + s_j,$$

which is equation (11). Following a similar analysis of (A.6), we have

$$\hat{\alpha}'_j = \alpha_j + (\psi_j - s_j)(\bar{R}'_m + \tau + \bar{\eta} - \bar{R}'_z - A - \bar{\delta}),$$

which is equation (12).

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Summary:

The study considered a seldom mentioned set of restrictions which must be placed on the parameters of a Box-Jenkins model. Ignoring these restrictions can result in an unstable forecast model. But none of the commonly used publically available computer programs for Box-Jenkins analysis prevent the problem or warn the user when it occurs. When it does occur the researcher can: (1) constrain the parameters to fall within the acceptable region or (2) consider an alternative model. The latter may often be the better choice since the problem itself might indicate that a model is being "forced" upon a given series when an alternative model might be more appropriate.

While the example in the present study used premier models, the restrictions apply to any Box-Jenkins model. Where models are individually identified for each firm (as opposed to "preidentified" premier models) the problem also occurs and due care must be exercised to avoid it.

Finally, it was shown that a simple "rule-of-thumb" could be used to avoid the worst effects of the problem for the BR, F and GW models as studied. The rule is simply to reject models which contain any autoregressive parameters larger than 1.0 or moving average parameters larger than 1.1.



Recently forecasted earnings research has become increasingly important. This is because there has become a widespread belief that forecasted earnings is of primary importance in investment decision making. For example, Nordby [1973] found that 98% of responding financial analysts used these forecasts in decision making. In addition, the importance of predicted earnings was recently reinforced by the Financial Accounting Standards Board [1977] in their conceptual framework project.

As a result of the above, the recent accounting literature contains a number of papers utilizing time series forecasting methods.<sup>1</sup> Among these papers are a group which discuss and compare various time series models as being representative of the earnings process. These models are sometimes called premier models. One thing that has been apparently overlooked in these studies is that the roots (as discussed below) of the time series model must satisfy certain conditions. If these conditions are not met, the model will lead to a nonsensical forecast function which might even diverge to positive or negative infinity. The purpose of this paper is to discuss these conditions and their applicability in accounting research. Also we provide an example of their violation by applying them to several premier models. Finally we present a simple method of avoiding the problem.

#### The Stationarity and Invertibility Region for the Model Parameters<sup>2</sup>

All autoregressive moving average time series models can be written in the form [Box and Jenkins, 1976, p. 95]:

$$(1) \quad z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \dots + \phi_n z_{t-n} - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_L a_{t-L} - a_t$$



where  $z_t$  is the time series variable of interest,  $a_t$  is the error term and  $\phi$  and  $\theta$  represent the autoregressive and moving average parameters respectively.<sup>3</sup> This is known as the difference equation form of the model. Alternatively the difference equation form has an equivalent random shock form [Box and Jenkins, 1976, p. 95]:

$$(2) \quad z_t = a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \dots$$

or inverted form [Box and Jenkins, 1973, p. 101]:

$$(3) \quad z_t = a_t + \pi_1 z_{t-1} + \pi_2 z_{t-2} + \dots$$

Both represent an infinite series with (2) forming a weighted sum of present and past values of a "white noise" process  $a_t$  and (3) forming a weighted sum of previous values of  $z$ , plus a random shock. It is important that either (2) or (3) will diverge to infinity unless certain restrictions are placed on the parameters of (1), namely:

The roots of

$$(a) \quad (1 - X\phi_1 - X^2\phi_2 - \dots - \phi_n X^n) = 0$$

$$(b) \quad (1 - X\theta_1 - X^2\theta_2 - \dots - \theta_n X^n) = 0$$

must have absolute values greater than 1 or in the complex case each norm<sup>4</sup> must exceed 1 [Box and Jenkins, 1976, p. 74].

#### Examples

Consider a special case of the model proposed by Griffin [1977] and Watts [1975] for modeling quarterly earnings per share:

$$z_t = (1 - .1182B)(1 - .31027B^4)a_t$$

where  $B$  is the backshift operator such that  $B^n a_t = a_{t-n}$ . This model

can be converted to the form (1) by expansion of its two factors by simple algebraic multiplication. This gives approximately

$$\begin{aligned} z_t &= [1 - .1182B + (.1182B)(.31027B^4) - .31027B^4]a_t \\ &= [1 - .1182B + .0012139B^5 - .31027B^4]a_t \\ &= a_t - .1182a_{t-1} - .31027a_{t-4} - (-.001239a_{t-5}) \end{aligned}$$

we therefore solve for the roots of:

$$1 - .1182X - .31027X^4 + .001239X^5 = 0$$

giving 3 real and 2 complex roots namely

$$1) 5.4966, 2) -1.3398, 3) 1.3398i, 4) -1.3398i, \text{ and } 5) 1.3398$$

Since the roots 1), 2), and 5) are real and have absolute values greater than one, and the norms of 3) and 4) are both equal to 1.795 and are greater than one, the model meets the above validity test.

#### Application to a Sample of Firms for Some Models Commonly Used in the Literature

Several models have been considered in the literature as useful representatives of the quarterly earnings per share process. These are (1) a consecutively and seasonally differenced first order moving average model (Griffin [1977] and Watts [1975]), (2) a seasonally differenced first order autoregressive model (Foster [1977]), and (3) a seasonally differenced first order autoregressive and seasonal moving average model (Brown and Rozeff [1978]). In Box and Jenkins notation [1976, Chapter 9] these are designated as (0,1,1) (0,1,1), (1,0,0) (0,1,0), and (1,0,0) (0,1,1) respectively and will henceforth be referred to as the GW, F and BR models.

To demonstrate empirically the applicability of the root criterion, the above 3 models were estimated using primary EPS (earnings per share before extraordinary items) for a sample of 267 firms.<sup>5</sup> The models were estimated with forty-eight quarters of data beginning with the first quarter of 1962 and ending the last quarter of 1973. The models were reestimated 16 additional times with one new quarter of data being added each time. Four forecasts were made for each estimation.

Table 1 presents the mean absolute relative forecast error<sup>6</sup> for those models having at least one root whose norm is less than .88. Although any root with a norm less than 1 is a problem, norms near the borderline will probably not result in drastic deterioration of the expectation function. Thus a cutoff somewhat below 1 was used. A lower cutoff would show more divergence; a higher cutoff less divergence. As can be seen, the series with models whose roots have norms that fall below the cutoff perform worse than series with norms above the cutoff in comparison to individually identified (and valid) Box-Jenkins models for the same series. This is the case even for the short forecast horizon used here. Note the general pattern of increasing difference as forecast horizon increases for both BR and F.

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Table 1 about here  
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The GW model doesn't exhibit as much differentiation. Although the forecasts from the models below the cutoff are relatively poorer than the models above the cutoff, the differences are smaller and do not exhibit the pattern of increasing differences with greater forecast horizon.<sup>7</sup> Further investigation of this behavior led to the discovery

that all 42 BR models which had roots below the cutoff had those "invalid" roots in equation (a) above related to the autoregressive parameter in the model. Similarly all problems with the F model are in the same equation (a) since the F model contains no moving average parameters. This leads to the potential conclusion that violation of the root interior for equation (a) above is probably more likely to lead to deterioration of forecast accuracy than violation of the root criterion for equation (b).

Knowing that most researchers are not anxious to solve potentially complex equations, an analysis of ways of simplifying the validity testing was conducted. For the F and BR models, the root of equation (a) has a norm of  $\frac{1}{|\phi_1|}$  where  $\phi_1$  is the autoregressive parameter. For the BR model the root of equation (b) must also be considered, but that root is simply  $4\sqrt{\frac{1}{|\phi_4|}}$  where  $\phi_4$  is the moving average parameter. Thus for either of these models any parameters with absolute value greater than one will lead to a violation of the root criterion. Unfortunately solution of equation (b) for the GW model is not as simple. Therefore an attempt was made to find a more readily applied guideline. For this (fairly large) sample it turns out that a simple rule of rejecting all GW models with either parameter having absolute value greater than 1.1 gives satisfactory discrimination on forecast accuracy even though some of the models "rejected" would pass the root validity test. Thus, as shown in the last section of Table 1, it seems that this simple "rule-of-thumb" has some usefulness. We conclude by summarizing the "rule-of-thumb" as follows: Reject models which contain any autoregressive parameters larger than 1.0 or any moving average parameters larger than 1.1.

### Conclusion

The study considered a seldom mentioned set of restrictions which must be placed on the parameters of a Box-Jenkins model. It was demonstrated that ignoring these restrictions can result in an unstable forecast model. Also to the writers' knowledge none of the commonly used publically available computer programs for Box-Jenkins analysis prevent the problem or warn the user when it occurs. There are at least two actions which the researcher can take when it does occur: (1) constrain the parameters to fall within the acceptable region or (2) consider an alternative model. The latter alternative may often be the better choice since the problem itself might indicate that a model is being "forced" upon a given series when an alternative model might be more appropriate.

Also it should be emphasized that while the example in the present study used premier models, the restrictions apply to any Box-Jenkins model. In the case where models are individually identified for each firm (as opposed to "preidentified" premier models) the problem also occurs and due care must be exercised to avoid it.

Finally, it was shown that a simple "rule-of-thumb" could be used to avoid the worst effects of the problem for the ER, F and GW models as studied. The rule is simply to reject models which contain any autoregressive parameters larger than 1.0 or moving average parameters larger than 1.1.

Table 1

Demonstration of the Effect of Model Invalidity  
on Mean Absolute Percentage Forecast Error\*

		Number of quarters ahead forecasted					
		1	2	3	4	N	
M	BR	p>.88	-.0206	-.0154	-.0065	-.0105	4497
		p<=.88	.0749	.2504	.3218	.5087	42
O	F	p>.88	.0118	.0225	.0270	.0234	4517
		p<=.88	.0874	.2235	.4822	.6838	22
E	GW	p>.88	-.0059	.0067	.0223	.0247	4408
		p<=.88	.0567	.0713	.0835	.0329	131
S		$\theta < 1.1$	-.0054	.0063	.0198	.0214	4466
		$\theta > 1.1$	.0754	.1460	.2844	.2434	73

\*  $\frac{|\text{Prediction} - \text{Actual}|}{\text{Actual}} - \frac{|\text{BJ Prediction} - \text{Actual}|}{\text{Actual}}$

each limited to 3.0

p = norm of minimum root of characteristic equation

$\theta$  = maximum moving average parameter value

NOTES

<sup>1</sup>For examples of this type of research see a summary given by Collins and Hopwood [1980]. Also see Foster [1977] and Lookabill [1976].

<sup>2</sup>The reason for the terminology "stationarity and invertibility" is complex and is not discussed in this paper. The interested reader should consult Chapter 3 of Box and Jenkins [1976].

<sup>3</sup>In practice  $z_t$  might be a differenced series (i.e., a series of changes).

<sup>4</sup>The norm of a complex number is analagous to the absolute value of a real number since both represent a measure of distance from zero. For a complex number  $a + bi$ , the norm is  $\sqrt{a^2 + b^2}$ .

<sup>5</sup>The firms met the following criteria:

- a. Their fiscal year ended on December 31 throughout the period 1962-1978.
- b. Their quarterly primary EPS were available on the COMPUSTAT quarterly industrial tape for the entire period.
- c. They were listed on the New York Stock Exchange.

<sup>6</sup>In order to control for the possibly different difficulty of predicting different series, the numbers shown are the mean of absolute relative forecast error for the stated model minus the absolute relative forecast error of the forecast from an individually identified Box-Jenkins model for that same observation. Thus, the negative numbers indicate the stated model had lower absolute relative error than the Box-Jenkins models for the same set of forecasts.

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