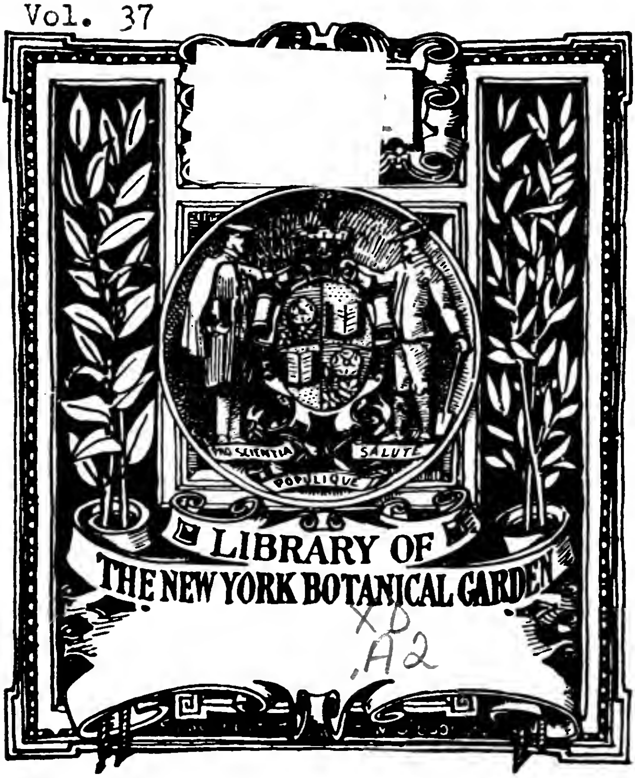
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By Theodore William Richards.

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Compressibility is a universal property of matter. It is so essential an attribute of the experimental universe that it is ascribed even to the imponderable and imaginary ether as well as to "material." The three states of matter are compressible in very varying degrees, dilute gases being compressible to a great extent, highly compressed gases and liquids to a far less extent, and solids to an extent usually even less than liquids. The first case has been studied in great detail, the last two scarcely at all.

Compressibility is simply an evidence of work done upon a system by a given pressure. If the application of considerable pressure in a system causes only a slight change of volume, it is evident that there must be other powerful influences at work. Clearly a clue as to the variation in these influences can be found in the quantitative study of the phenomena.

In all reversible cases which may be studied directly, an increase in pressure is accompanied by an increase of resistance to pressure and a diminution of volume. This depends upon the fundamental idea of equilibrium, and is a special case of the general principle sometimes named after Le Chatelier. Working backwards from this idea, one may infer with regard to any given substance at a given temperature, that it is under the influence of great pressure if its volume-change is unusually small under addition of a given pressure.

There are two conceivable causes of great compression in a substance. The pressure may be applied from the outside, or it may be due to the mutual internal attraction or affinity of the smallest particles of the substance for one another. That is, the substance may be compressed either by an outside pressure, or by the intensity of its own cohesion. The first may be typified by highly compressed gases, the second by liquids, whose small compressibility may be taken as evidence of great compression.

In solids one must consider also the directive agency which manifests itself in crystalline form and optical structure. In a few cases the "crystallogenic force" seems to be rather directive than attractive; in other cases it seems to have both properties, for considerable diminution in volume may occur. The presence of the crystal-making force complicates the phenomena and is a considerable stumbling-block in the way of the study of the internal tension of solids.

In view of these facts, it seemed to me possible that the study of compression as manifested by atomic volume under different circumstances, as well as of atomic compressibility, might afford some light as to the affinities at work. The attempt, while only just begun, has not been wholly unsuccessful.

Evidently the liquid is the most suitable state in which to study the effects of molecular and atomic compressibility. It is most suitable because the irregularities in the behavior of liquids are very great, indicating various internal stresses, and because they are nevertheless not at the mercy of the directive crystal-making tendency which superposes its own influence upon that of cohesion. The great difficulty in the subject lies in the fact that the total compressibility of a substance is usually made up of a number of parts; the molecular compressibility might be due partly to a diminishing of the so-called "free-space" between the molecules, as well as to a diminishing of the distance between the atomic centres. In words free from hypothesis, we may say that the compressibility may be made up of a chemical and a physical compressibility. When one comes to compute from compressibility the probable affinities, one is still more at a loss, — for each affinity is a mutual affair, concerning two specific substances. The immense number of variables thus introduced has discouraged most investigators, and I can find little if any hint of the significance of chemical compressibility in the literature familiar to me.*

In a case of this kind, one naturally seeks at first cases as simple as possible. A study of the volume changes which take place on mixing liquids reveals at first no apparent regularity. In some cases an expansion occurs, but more usually a contraction; sometimes heat is evolved, and at other times heat is absorbed. One law may, I think, be detected in the midst of the confusion, namely: *Similar liquids exhibit less change of volume on mixing than dissimilar ones do.* That is, where the

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* The considerations of Nordenskjöld are too seriously complicated by uncertain assumptions to have much value. (See Ostwald's Lehrbuch, I. 850 (1891), for these and similar considerations.)

affinity of a substance for itself is not unlike that of the substance for another, no great contraction or expansion occurs on mixing. Thus benzol and tuluol when mixed scarcely change in volume at all, while alcohol and water contract considerably. That is just what would be expected if affinity is the cause of contraction.

In order to use such facts it is not necessary to imagine an atomic theory adapted to them. Such a theory is ventured upon at the end of this paper, but the facts are significant without it. One only has to bear in mind that liquid and solid substances resist compression, and hence that when we find them compressed we have reason to believe that pressure has been applied upon them. It is rather a matter of common sense than a hypothetical abstract conception.

In order to present in a clear light the complications involved in the study of even a simple series of cases of chemical compression, the facts concerning the molecular volumes of several metals and their oxides are recorded and discussed below.

MOLECULAR VOLUMES OF OXIDES.

Substance.	Weight of metal combined with 16 grams oxygen.	Density of metal.	Density of oxide.	Space occupied by given weight of metal.	Space occupied by corresponding weight of oxide.	Excess of volume of oxide.
2 Ag . . .	215.86	10.56	7.521	20.55	31.55	+11.00
Hg	200.00	13.59	11.136	14.71	19.4	+ 4.7
Cu	63.6	8.95	6.40	7.10	12.4	+ 5.3
Ni	58.7	8.9	6.39	6.60	11.75	+ 5.15
Cd	112.3	8.67	6.5	12.95	19.7	+ 6.75
Zn	65.4	6.9	5.6	9.5	14.5	+ 5.0
Mg	24.36	1.74	3.4	14.0	12.0	− 2.0
2 Na . . .	46.1	0.973	2.80	47.4	22.6	−24.8
2 H . . .	2.0	0.07	1.00	28.2	18.0	−10.0
Si	14.2	2.00	2.30	7.1	13.14	+ 6.0
In compounds of carbon, according to position.						4.7 to 12.0
In liquid oxygen at −119° and 50 atm. (sp. gr. = 0.65). . .						O = 24.5 c.c.
In liquid oxygen at −181° and 1 atm.						O = 14.1 c.c.

While in the first part of this paper no atomic hypothesis is assumed, the words atomic volume, atomic weight, and atomic heat will be used in a purely material sense, as the actual constants pertaining to quantities chemically consistent.

The results recorded in this table are typical of the variety of degrees of contraction which take place when substances combine with oxygen. It is evident that in some cases the product occupies considerably more space than the metal from which it was formed, and that in others (typified by magnesium and sodium above) the oxide occupies considerably *less* space than the metal. This last remarkable circumstance at once emphasizes the absurdity of estimating the atomic volume of an element in a compound by discovering the volume-change which takes place when that element is replaced by another. Oxygen cannot be said to occupy a *minus* quantity of space, — the only possible outcome of the false assumption in this particular case. The false method gives fairly consistent results among carbon compounds only because of the great similarity of their composition. This consideration leads to the first law underlying the change of volume in chemical or physical change, namely, *The atomic volume is not a constant, but is dependent upon the environment.* This law was first suggested by Horstmann,* but he looked upon it rather as the absence of a law than as the presence of one.

If the affinity of oxygen for the metal were the only variable entering into the figures given above, it is obvious that the total contraction, the difference between the volumes of factors and product, would be at once a comparative measure of the attractive forces which produce the compression. This reasoning of course rests upon the plausible ground that a state of being which resists pressure, such as liquid oxygen or solid metal, may be compressed only by the application of pressure. In this case pressure may be supposed to be applied by the mutual affinity. But unfortunately the case is not so simple.

It is clear that in each case recorded above at least three affinities are concerned: first, the affinity of the metal for itself; second, the affinity of oxygen for itself; and third, affinity of the metal for oxygen. The second of these is constant throughout the series, hence for the present comparison it may be considered as a known quantity. Therefore each change of volume may concern at least two unknown quantities. Hence if it were possible to measure either of the two

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* Horstmann, Ostwald's Lehrbuch, I. 389 (1891).

variable affinities, an approximate idea could be obtained concerning the other from these data concerning atomic and molecular volume.

A slight uncertainty is caused also by the possible varying intensity of the "crystal-making tendency" which determines the structure of solids. The small differences caused by this uncertainty may be seen from the following typical calculation. If solid rather than liquid mercury had been chosen above, the atomic volume of the mercury would have become $\frac{200}{14.1} = 14.2$ instead of 14.7, and the excess of volume of the oxide would have been 5.2 instead of 4.7. These differences are unimportant compared with the larger values under consideration; the precise state of the solids or liquids makes less difference than one would have supposed.

Is there any direct method of determining either the mutual affinity of the two elements or the affinity of the metal for itself?

Countless attempts to measure the former have so continually resulted in failure that many chemists are inclined to deny the existence of chemical affinity. The electrometric method suggested by Ostwald * clearly measures one of the ways in which chemical affinity may accomplish work, but it is limited in application and only represents a small fraction of the possibilities. The thermal relations are complicated by well-known thermodynamic irregularities, and would be fully significant only at the imaginary absolute zero.

The direct determination of the affinity of a substance for itself is an easier matter, for many of the properties of a single substance, such as volume, compressibility, tenacity, must be associated with this affinity. Let us seek to study these relationships more closely.

If one could only be sure that all substances, when relieved of their self-affinity, would occupy the same volume, the atomic volume itself would be the simplest and most direct means of comparing this property in different substances. The smaller the actual atomic volume, the greater must be the self-affinity. Such an assumption would at first sight seem to be justified, for those elements which have the largest atomic volumes have the least inclination to remain in the elementary states. Deserting the elementary state means introducing other affinities, however; hence the assumption would be unsafe.

It has been already pointed out that compressibility, if measured over a wide range of pressures, might afford a clue to the extent of compres-

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* Ostwald, The Chemometer, Z. phys. Chem. **15**, 399 (1894).

sion already existing in any given substance. But the comparison of different substances involves the dangerous assumption that all substances would be alike compressible if freed from self-affinity, — an assumption which seems more probable than the last, but which nevertheless must be rejected. A much safer measure of the stress under which a single substance rests is the work which heat is able to do upon it. The changing of a simple substance from t° to $t^\circ + dt^\circ$ Centigrade must involve the addition to it of an amount of internal work which is represented by the rise of temperature multiplied by the heat capacity of the substance, or $C\,dt$. In a simple elementary substance, when this work does not involve the alteration of crystalline form or any other apparent change except increase in size, it seems reasonable to consider no other variables, at least as a working hypothesis. If this is the case, we may write $C\,dt = P\,dv$, in which P is the internal stress against which the heat-energy is doing work, C the molecular heat capacity, t temperature, and v volume. The stress against which this work is being done is due only to the internal stress and to atmospheric pressure (which latter may be neglected by comparison with the very large value of the former), hence the stress $= P = \frac{C\,dt}{dv}$. This can apply precisely only to infinitesimal changes, because in all probability P will vary with the volume. While it cannot be claimed that the expression just given certainly expresses a *single* pressure pitted against temperature-work, the expression certainly represents a *resultant* tendency which opposes expansion by heat, and therefore, by inference, opposes all other forms of expansion.* It is the *inward tendency,* the opposite to the driving tendency † or fugacity.‡

While then this stress, represented by the quotient of energy divided by change of volume, can hardly represent anything very definite, it must nevertheless be supposed in a general way to increase when the self-affinity increases. Hence, while giving no certain knowledge, its study may give an indication of affinity.

A typical comparison may be made of the two elements zinc and mercury. They are simple, similar, and yet widely different as to their power of holding oxygen. In each case the atomic contraction on union with oxygen is about the same. If we take as the atomic volume of

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* All the slight data which we possess upon compressibility seem to run parallel with the coefficients of expansion.

† Richards, These Proceedings, **35**, 471.

‡ Lewis.

oxygen the atomic critical volume, the contractions are as follows: 14.7 + 24.5 — 19.4 = 19.8, in the case of mercury, and 9.5 + 24.5 — 14.5 = 19.5, in the case of zinc. If the metals were originally subject to the same internal stress, we should infer from the similarity of contractions that the affinities concerned in the two cases were about equal. This inference is, however, overthrown by other facts. Both elements have about the same atomic heat capacity, hence no internal rearrangement takes place in one which is not approximated in the other. On the other hand, the increase in atomic volume for a rise of 1° of temperature exhibited by one is much greater than that exhibited by the other.

If a gram atom of one element increases more rapidly in size than the gram atom of another, it is only reasonable to suppose that the heat energy is finding less opposition in the former case. The coefficient of cubic expansion of mercury is 0.000179 at 0° C. and the heat required to raise a gram through 1° is 0.139 joule. With zinc the corresponding numbers are 0.000087 and 0.392.* The respective atomic volumes are 14.7 and 9.5. Substituting these values in the equation we obtain.

$$P_{Hg} = \frac{(200 \times 0.139)}{(14.7 \times 0.000179)} = 106{,}000 \text{ megadynes per square cm.}$$

$$P_{Zn} = \frac{(65.4 \times 0.392)}{(9.5 \times 0.000087)} = 310{,}000 \text{ megadynes per square cm.}$$

Both these pressures are very large, for a megadyne exerts on a square centimeter a pressure of almost an atmosphere. As has been said, they signify a resultant tendency which resists expansion.

It is interesting to note that these stresses agree in their indications with the comparison of boiling points and latent heats of evaporation. The boiling point of mercury is 357° C. and that of zinc about 930° C. The latent heat of evaporation of zinc is not known, but there is no reason for believing that in its case Trouton's rule is broken. Hence the criteria all indicate that zinc is harder to dissociate from itself than mercury is.

A comparison of the energy-quotients of several metals, measured in this way, may be of interest.

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* All figures not otherwise designated were taken from the tables of Landolt and Börnstein, 1894.

Metal (in order of boiling point).	Boiling point 760 m.m.	Heat capacity (mayers per gram) $= \frac{C}{\text{mol. weight.}}$	Cubic coefficient of expansion.	Energy quotient $P = \frac{Cdt}{\text{atom. expan.}}$ megadynes
Mercury . .	357° C = 630° A	0.139	0.00018	106,000
Cadmium .	770° C = 1043° A	0.23	0.000093	214,000
Sodium . .	860° C = 1133° A	1.21	0.00022	53,700
Zinc	930° C = 1203° A	0.392	0.000087	310,000
Copper . . .	unknown	0.375	0.000050	672,000
Magnesium	1100°± = 1400° A	1.02	0.000081	224,000
Lead	1400°± = 1700° A	0.126	0.000088	162,000
Silicon . .	unknown	0.7	0.0000230	755,000
Diamond .	unknown	0.5	0.0000036	4,900,000

In these figures one may find traces of many properties associated with firmness of structure or intensity of self-affinity. For example, the order of sequence of the energy-quotients agrees essentially with that of tenacity and of hardness. There is some relationship also to boiling points and melting-points, although here there are more exceptions. "Chemical affinity" is so much affected by electrical relations and by atomic volume that one would expect to find regularity only on comparing similar elements. Such comparison (zinc with cadmium, or carbon with silicon) seems to show that the energy-quotient tends to increase with diminishing atomic weight.

Having thus plausible inference, from independent sources, as to the relative values of the compressing agencies existing in metals at the ordinary temperature, it is worth while to study the correction which must be applied to the volume-change exhibited in chemical combination with another element. In zinc the self-affinity is so great (boiling point = 1200° A), and the metal is hence already so compressed, that a given further pressure causes less change in its volume than it would cause in the case of mercury. That is, the mercury contracts more than zinc when it is oxidized. Hence the difference between the volume of the oxide and the volume of the metal gives too low a value for the volume of the combined oxygen in the case of mercury.

Thus the contraction of the *oxygen* is really less in the case of mercuric oxide, although it appears to be the same.

Without going further, one can explain by means of these considerations the behavior of zincic and mercuric oxides when subjected to high temperatures. The sixteen grams of oxygen in mercuric oxide occupies a larger space than an equal weight in the case of zinc, hence one infers that it is less compressed by its affinity, hence the affinity must be less. This smaller affinity should be more easily overcome by rising temperature, a prediction which agrees with facts. *Thus there appears to be in this case a connection between the compression of substances and their tendency to combine one with another.*

The case under consideration is typical. In the case of sodium and magnesium, the affinity of the metal for oxygen is so enormous as to overcome easily the large affinity of the metal for itself, and besides this to compress both metal and oxygen together into a space smaller than that previously occupied by the metal. This fact corresponds with the great difficulty of decomposing sodic and magnesic oxides. Metallic magnesium probably has as energy-quotient a stress more than four times as great as sodium (see table on p. 10); hence the total contraction on combination with oxygen is less than in the case of sodium. Comparison with the cases of mercury and zinc will show that this small contraction does not necessarily conflict with the fact that magnesium decomposes sodic oxide at high temperatures. Again, the contraction involved in the formation of argentic oxide is very slight. In this case the large volume of oxygen is not concealed by the contraction of the metallic element, as it was in the case of mercury, for silver is not particularly compressible. Hence one can infer that the affinity of silver for oxygen is smaller than that of magnesium for oxygen, — an inference which agrees with fact. Moreover, since the relation is nearly additive, that is, neither silver nor oxygen change much in volume on combination, their combination is easily shifted, that is to say, silver oxide is easily decomposed by heat.

Of course many tables comparing the molecular volumes of solids and liquids might be drawn up, since a very great number of specific gravities have been determined. A table containing chlorides of the metals already considered may be of interest.

Here the variations in contraction are less than they were before. Chlorine evidently possesses more equally distributed affinities than oxygen does, and apparently somewhat weaker ones. The two most interesting features of this table, which may be seen without the elimi-

MOLECULAR VOLUMES OF CHLORIDES.

Substance.	Weight of metal combined with 35.5 grams of chlorine.	Density of metal.	Density of chloride.	Volume of given weight of metal.	Volume of corresponding weight of chloride.	Excess of volume of chloride above metal.
Ag	108.	10.56	5.53	10.27	45.90	+15.63
½ Hg . . .	100.	14.00	5.42	7.30	25.5	+18.2
Hg	200.	14.00	7.10	14.00	33.2	+19.2
½ Cu . . .	31.8	8.95	3.05	7.10	25.4	+18.3
½ Co . . .	28.5	9.00	2.94	3.16	21.8	18.64
½ Cd . . .	56.2	8.67	3.7	6.47	24.8	18.33
½ Zn . . .	32.7	6.9	2.753	4.75	25.0	+20.25
Mg	12.2	1.74	2.177	7.0	21.95	+15.00
Na	23.05	0.973	2.15	23.7	27.2	+ 4.2
K	39.14	0.875	1.995	45.7	37.3	– 8.4
Rb	85.44	1.52	2.21	56.1	55.0	– 1.0
H	1.01	0.07	1.27	14.1 (?)	28.9	+14.7

Combined with carbon 22.8
Liquid chlorine at –80° (boiling point, 760 mm.) (sp. gr. = 1.66) 21.5
Liquid chlorine at +80° (sp. gr. = 1.20) 29.6

nation of the self-affinities of the several metals, are the small excess in the case of silver, and the larger excess in the case of mercurous chloride. This is quite in accord with the facts; for argentic chloride is more stable than the oxide, and mercurous chloride easily splits into mercuric chloride and mercury.*

The case of the hydroxides is especially interesting.

The density of the hydroxide of zinc has not been accurately determined; indeed the data concerning cobalt, cadmium, and magnesium are not very trustworthy on account of the amorphous condition of most hydroxides. It is interesting to note that in this table, where the substances are arranged in the order of the contraction which ensues when hydroxyl combines with the metal, *should also be arranged in the electro-chemical*

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* Richards, These Proceedings, **33**, 9 (1897).

MOLECULAR VOLUMES OF HYDROXIDES.

Substance.	Weight of metal combined with 17 grams hydroxyl.	Density of metal.	Density of hydroxide.	Volume of given weight of metal.	Volume of hydroxide corresponding.	Excess of volume of hydroxide above metal.
Ag	The hydroxide is exceedingly unstable.					
½ Hg . . .	It is doubtful if the hydroxide exists.					
½ Cu . . .	The hydroxide cannot be dried without decomposition.					
½ Co . . .	28.5	9.	3.597	3.16	12.67	+ 9.51
½ Cd . . .	56.2	. . .	4.79	6.47	15.25	+ 8.78
½ Mg . . .	12.2	. . .	2.36	7.0	12.90	+ 5.90
½ Sr . . .	43.83	2.54	3.62	17.3	17.0	— 0.3
Na	23.05	0.973	2.13	23.7	18.80	— 4.9
K	39.14	0.875	2.044	45.7	27.5	—18.2
Hydroxyl in organic compounds					+12.0	
Hydroxyl in hydrogen dioxide (sp. gr. = 1.50) . .					11.4	

order. That is to say, the solution tension of a metal appears to be associated with the excess of affinity of the metal for hydroxyl over its affinity for itself, and intensity of potential seems to be associated with intensity of atomic compression. The inference to be drawn from this comparison is of course that the formation of the metallic ion in water is connected with the affinity of the metal for water, — an affinity which manifests itself even when both of the "bonds" of oxygen are filled.* Similar attraction for nitrogen or sulphur would explain cases in which the solvent does not contain oxygen.

If this is true, contraction should take place when salts are dissolved in water. This inference is amply verified by facts. In some cases the solution occupies even less space than the water alone, involving a total contraction greater than the volume of the salt itself. The best known of these cases are those of lithic, sodic, and baric hydroxides, and

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* Brühl has suggested that oxygen is the cause of dissociation, but he ascribes it rather to quadrivalence than to a general affinity.

cobalt, nickel, zinc, and magnesium sulphates,[*] but undoubtedly others exist. In a large majority of cases when an electrolyte is dissolved in water, the sum of the volumes of salt and of the solvent taken together considerably exceeds the volume of the solution. This contraction is usually ascribed wholly to the dissolved substance in dilute solutions,[†] but it seems to me that the behavior of the salts named above proves the falsity of this method of calculation. *The water as well as the salt must contract when a salt is dissolved.* So many complications are concerned in the act of the solution of an electrolyte that it is difficult to unravel the tangled clues; but the wide deviations exhibited by different substances seem to indicate that there are present overlapping contractions and expansions, the resultant of which is a smaller quantity than some of the individual influences. Such contractions and expansions are just what one would expect to find in a readjustment of affinities.

In considering the simpler case of solid non-electrolytes, one usually finds here also a contraction upon solution, although less marked than in the extreme cases named above. For this reason, one is inclined to ascribe the act of solution of all kinds primarily to the affinity of the solvent for the dissolved substance. The solution tension of a metal or salt becomes simply a balance or ratio of attractions, — the separating tendency of heat upon the dissolving phase is much assisted by the attraction from outside. This is of course no new idea. The possible method of treating mathematically these balanced influences is suggested in a recent paper on the "driving tendency" of reaction.[‡]

That electrolytic separation also should be assisted by the outside attraction for the solvent is almost a foregone conclusion. This may be inferred from the contraction shown by most electrolytes on dissolving. Hence may arise the various contact-potentials exhibited by the same substance in different solvents; for different solvents must possess different affinities. Hence also one would expect to find a much greater potential needed for the dissociation of gases than for that of dissolved substances.

The mechanism of electrolytic dissociation in gases is now usually

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[*] Thomsen, Thermochemische Untersuchungen, I. 45 (1882). MacGregor, Trans. Roy. Soc. Canada, 1890, p. 19; 1891, p. 15; Trans. Nova Scotia Inst. Nat. Sc., **7**, 368 (1890).

[†] Van't Hoff, Vorlesung. phys. theoret. Chem., III. p. 41 (1900). Drude and Nernst (Z. phys. Chem., **15**, 79 (1896)) ascribe this contraction to "Electrostriction."

[‡] Richards, Jour. Phys. Chem., **4**, 385 (1900). See specially p. 391.

explained by the aid of the ingenious hypothesis of "electrons," as amplified by J. J. Thomson and his students in the brilliant experimental researches published in the recent volumes of the Philosophical Magazine. This daring hypothesis must not be accepted without reservation, however. Some physical objections to it have been suggested by Ernest Merritt in his interesting address to the American Association for the Advancement of Science;* and other objections arise when one tries with its aid to unravel the tangle of influences involved in purely chemical action. The rejected alternative of imagining the atom as indivisible, but as capable of receiving widely varying electric charges under widely different conditions, has some advantages which the opposite hypothesis does not possess. The subject is much too large for discussion here, however. One phase of it, which bears directly upon the subject of the present paper, may receive brief notice.

The results of Thomson, Townsend, Zeleny † and others seem to indicate that the bearer of the negative electricity not only carries the high charge referred to above, but that it is very small, while the bearer of the positive electricity is very large. May it not be the atom itself which thus expands and contracts? This agrees with the verdict of the results of atomic compression given above. Change of atomic volume seems to be associated with electric stress. This assignment of electric expansibility to the atomic sphere of influence might explain other phenomena concerning the behavior of electrified gases, for example, the increase of pressure which is observed when a gas is highly charged.‡ Again, the great conductivity of a gas with adequate potential and quantity of electrical discharge § seems to indicate that then the situation must resemble that in a metal, where the spheres of stress fill the whole volume occupied by the substance. The temperature must be so high under these circumstances that the gas is probably in a condition of thermal dissociation. Hence one is inclined to refer the great conductivity to the electrical susceptibility of evenly compressed or *undistorted* atoms. The fact that pure metals conduct electricity better than alloys or compounds seems to support this conclusion. The permeability of solids to cathode rays might be explained by supposing that the smallest particles of both solid and gas are much contracted by the negative charge.

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* Proc. Am. As. Adv. Soc., **1900**, p. 49.

† Phil. Mag. [5] **46**, 120, (1898). See also Am. Chem. Journ., **25**, 340 (1901), for a résumé of this work.

‡ De la Rue and Müller, Phil. Trans., **1880**, 86.

§ Trowbridge and Richards, Phil. Mag. [5] **43**, 349 (1897).

It is with some diffidence that this paper attempts to reconcile the facts with any hypothesis, for hypotheses sometimes lead to dangerous delusions. If, however, one never forgets the essential difference between fact and hypothetical inference, a theory may afford useful suggestions for further research. The facts under discussion in the present paper seem to me to be adequately connected by none of the current conceptions concerning atoms, hence it has seemed not wholly pointless to postulate a theory which might serve better. The essential elements of this theory must be evident from the trend of the hypothetical discussion above; they are not wholly new. Since changes of atomic volume seem to be so closely associated with the most intimate properties of substance, it seems necessary to assign more importance to the atomic "sphere of influence" or the "free space" around the atomic centres than is customary. Indeed, the properties of material seem to be as much concerned with the "atomic shell" as with the "atomic centre." The two hypothetical conceptions are so closely related as to be inseparable.

Such a point of view leads to the conception of an atom as a compressible field of force possessing two attractive attributes, chemical affinity and gravitation, both of which may be concerned in chemical action. Mass may be supposed to be causally connected with gravitation. The fact that in many cases affinity diminishes with increasing atomic weight,* taken together with the Laws of Faraday and of Dulong and Petit, suggests that the two attractive forces in the atom may bear some sort of reciprocal or additive relationship to one another, — that the product or sum of the two may afford a constant basis for the vibrations of heat and electricity. This relation is often hidden by electrical attraction, which plays so important a rôle in chemical action that it is sometimes hard to distinguish the intensity of chemical affinity proper. In such an atom one can imagine that either thermal or electrical vibration might cause distention. The phenomena of electricity suggest that electricity plays around the atomic surface, while heat seems to be concerned with a more fundamental or central agitation. Light-vibration, which seems also to be intimately concerned with atomic structure, would be assumed to be a surface effect like electrical vibration.

Such an atom would be compressible under the influence of its own affinities as well as under the influence of external pressure. Permanent

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* Van't Hoff, Vorl. th. phys. Chem., III. 87 (1900). Compare also the relation of the energy-quotients of similar metals referred to on p. 10 of the present paper.

atomic distortion would accompany chemical union, and the heat of the reaction would be the outcome of the resulting decrease of internal energy. Atomic volume and atomic compressibility might limit the possibility of distortion; hence would arise a possible explanation for quantivalence, stereochemistry, and crystal form. Many other properties of material, too numerous to mention, seem to be explicable in a similar way.

It would be unreasonable to expect the hypothesis thus briefly described to correspond to all known facts. No hypothesis has ever been proposed which is wholly satisfactory; our knowledge is incommensurate with the possibilities involved. If, however, a given theory is found to explain some relationships better than other hypotheses, it may be of service in suggesting new experimental research. Such a service is of course the best one which a hypothesis can perform.

The idea discussed above has been already applied in plausible fashion to a wide range of chemical and physical phenomena. If future experimentation to be carried on here seems to warrant it, these applications may form the subject of another communication.

The object of the present paper may be summed up in a few words, as follows: It is pointed out that changing atomic volume may be used as an approximate measure of the pressure which causes it, and therefore of the affinity which causes the pressure. Some of the difficulties in the way of exact interpretation are pointed out, and hints are given as to possible modes of overcoming the difficulties.

The chief outcome of the paper is the following postulate: *The atomic volume is not constant, but a function of pressure and temperature, and probably of electric stress.*

In this connection it is pointed out that chemical affinity is possibly a reciprocal function of mass.

To explain these and many other facts, a modification of the atomic hypothesis is tentatively proposed which contends that we have no right to disregard the compressible environments around the centres of gravity and affinity.

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CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF HARVARD UNIVERSITY. — XLVII.

PRELIMINARY DIAGNOSES OF NEW SPECIES OF LABOULBENIACEAE. — IV.

By Roland Thaxter.

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Additional material illustrating the well-marked generic type described in a former paper as *Monoicomyces* renders necessary some modification of the original diagnosis, as well as the separation of several species in a second nearly allied genus, which I have called *Eumonoicomyces* (*E. Papuanus* being taken as the type), that is well characterized not only by constant differences in the structure of the peculiar antheridium, but also by reason of certain differences in gross habit which are constant in normal forms of all three of the known species, one of which, *E. invisibilis,* was formerly placed by me in *Monoicomyces.*

EUMONOICOMYCES nov. gen.

Receptacle consisting of a basal and subbasal cell; the latter producing terminally a sterile appendage and laterally a fertile branch (abnormally more than one) the axis of which is coincident with that of the receptacle from which it is not distinguished and consists of a series of superposed cells which may bear a sterile appendage, an antheridium, or an antheridium and a perithecium; the three terminal cells usually bearing these organs in the order mentioned. The antheridia consisting of a single stalk-cell, and a single, often obscure, basal cell; the body of the antheridium consisting of a series of numerous antheridial cells in four (?) vertical rows which extend obliquely inward and upward, emptying into a common cavity, and replace entirely the two tiers of wall-cells and the antheridia of *Monoicomyces*; the terminal cells growing upward directly to form four unequal sterile terminal appendages, similar to those of *Monoicomyces.*

Eumonoicomyces Papuanus nov. sp.

Nearly or quite hyaline. Basal cell of the receptacle small, usually triangular; the subbasal cell terminating in a short appendage, distinguished by a dark basal septum, and sometimes once branched. The fertile branch not differentiated from the receptacle, consisting of three, rarely two cells similar to the subbasal cell, obliquely superposed; the lowest bearing normally a short, hyaline or faintly brownish, erect, sterile appendage, similar to that of the subbasal cell; the middle cell bearing a single antheridium, and the upper an antheridium and a stalked perithecium. The antheridia rather stout, broader distally; the stalk-cell small and short; the antheridial cells very numerous — thirteen to fifteen usually visible in optical section — the terminal appendages of the usual type, short or seldom longer than the antheridium. Perithecium rather long and sometimes slender; the venter inflated; the distal portion tapering gradually and symmetrically to the blunt, nearly truncate apex; the rather short tip hardly distinguished above a slight elevation; the stalk-cell variable in length, rather slender, seldom more than half as long as the perithecium; the basal cells rather large and broad, not distinguished from the venter. Spores about 35 × 3 μ. Perithecia 80–120 × 32–40 μ, the stalk-cell 35–75 × 15 μ. Antheridia including stalk-cell and without appendages 35 × 18 μ. Total length to tip of perithecium 150–290 μ.

On all parts of a small pale species of *Oxytelus*. Ralum, New Pomerania. Berlin Museum, No. 1011.

Eumonoicomyces Californicus nov. sp.

Resembling *E. Papuanus* in general habit. Basal cell of the receptacle short, stout, geniculate, with a dark brown suffusion extending from the foot half-way up its convex margin; the subbasal cell bearing distally a long appendage consisting of a short hyaline basal cell, separated by a dark septum from a second cell above it, which is dark brown and bears two long, slender, one-celled, erect branches, brown below, becoming hyaline distally. The fertile branch not distinguished from the receptacle and consisting of three, sometimes more, very obliquely superposed cells similar to the subbasal cell: the lowest bearing a sterile appendage like that which terminates the receptacle; the middle cell usually bearing an antheridium, and the upper an antheridium and a perithecium. Antheridium short-stalked, with a more or less well-defined median constriction, resulting from an inflation of the cells which bear the terminal appendages. The latter very long, brown, extending beyond the tip of the perithecium. Perithecium short and stout, the venter inflated, the much shorter neck-like distal portion abruptly distinguished, the apex blunt, the stalk-cell usually rather short and stout. Perithecia 75 × 25 μ, the stalk-cell 20 × 18 μ. Sterile appendages, longest 150 μ. Appendages of antheridium 100 μ. Total length to tip of perithecium 150 μ.

On *Oxytelus* sp. Berkeley, California.

MONOICOMYCES Thaxter.

The characters which may be considered to separate this genus from *Eumonoicomyces* are as follows: — The stalk of the antheridium consists of two cells placed side by side; the body of the antheridium consists of two tiers of wall-cells, from each of which an inner antheridial cell is separated; the subbasal cell of the receptacle bears normally more than one heterogeneous fertile branch.

Monoicomyces Echidnoglossae nov. sp.

Subbasal cell of the receptacle somewhat smaller than the basal cell, bearing a terminal appendage the basal cell of which is as long, or nearly as long as the receptacle and often distally enlarged; the axis above it consisting of a curved series of several cells, externally opaque, black, hyaline along the inner margin, each cell giving rise from its inner side to a hyaline simple branchlet, much as in the appendage of *Laboulbenia cristata*. Fertile branches usually two, sometimes one or three, arising from the subbasal cell of the receptacle, and consisting of a single short basal cell which bears directly a perithecium (in some cases more than one) and an antheridium. Antheridium relatively large, the stalk-cells somewhat longer and narrower than the basal cells; the cells of each of the middle tiers distally more or less prominent, the rounded, almost papillate elevations thus formed from the upper tier more prominent than those from the lower tier: the distal cells proliferous externally and distally, thus forming an outer crown of shorter appendages of very unequal length, which surround the usual inner series. Perithecium becoming greatly and asymmetrically inflated below, and tapering rather abruptly to the slightly distinguished, rather short, bluntly pointed tip; the stalk-cell variably developed. Perithecia 100–125 × 45–55 μ, the stalk-cells 40–80 × 15 μ. Antheridia 75–100 μ, the sterile appendages 50–75 μ. Total length to tip of perithecium 220–250 μ.

On the inferior surface of the thorax of *Echidnoglossa Americana* Fauvel. Vera Pass, Colorado. Leconte Collection.

Monoicomyces furcillatus nov. sp.

Receptacle consisting of two small cells which are hardly distinguishable owing to a general blackish brown suffusion; producing on either side a stout blackened prolongation, the two forming a nearly symmetrical fork-like structure, the prongs of which are slightly curved inward, and slightly divergent. From near the base of these outgrowths and between them arise, apparently from single basal cells on both sides, single stalked perithecia and antheridia. The antheridia rather long and slender, their detailed structure not determinable in the types. The perithecia long and slender, straight, symmetrical, pale yellowish, slightly inflated toward the base, tapering gradually to the blunt apex. Spores about 40 × 3 μ. Perithecia 135 × 27 μ. Outgrowths from the receptacle 110 × 12 μ.

Near the tip of the abdomen of *Aleochara repetita* Sharp. Panama. Sharp Collection, No. 1095. Of the three individuals obtained one only is in fair condition, and none have antheridia in which the details of structure can be made out. Owing to the suffusion and great reduction of the receptacle it is further impossible to determine the exact origin of the remarkable fork-like outgrowths, or the other structures which arise from it. The form is a most peculiar one and recognizable without difficulty; yet, until further data are obtained concerning it, its generic position cannot be certainly determined, although it seems at least more closely allied to *Monoicomyces*, in which it is provisionally placed, than to any other known type.

Monoicomyces Aleocharae nov. sp.

Pale amber, shading to amber brown. Receptacle, together with the foot and the basal cell of the terminal appendage, forming a heart-shaped body, blackened below, bearing terminally a median, rigid, slender, almost wholly opaque, black branch, abruptly distinguished from its broad basal cell: the subbasal cell of the receptacle small, triangular when viewed sidewise, giving rise to two fertile branches, the short small basal cells of which give rise at once each to two secondary branches and an antheridium; the branchlets proliferous and forming an axis of usually three cells, the lower bearing an antheridium, and each of the two upper an antheridium and a perithecium: there being thus sixteen antheridia and eight perithecia, in fully and symmetrically developed specimens, which form a dense, spreading, fan-like tuft, the antheridia being in general posterior in position, overlapping one another between the black sterile appendage and the perithecia. Antheridium distally broadened and truncate, elongate; the stalk-cells about equal and about one half the length of the body of the antheridium or somewhat longer than this; the basal cells unequal; the cells of the two middle tiers, and their antheridia, clearly distinguishable; the terminal cells forming four unequal, rounded prominences, the upper inner angle of each cell separated by an almost vertical septum to form the four "guard cells," that terminate in papillate prominences just below which they proliferate to form the characteristic, erect, sterile appendages, all four of which do not always develop; the sterile appendages relatively short, two to three-septate, tapering to a blunt point, distinctly inflated above the slightly constricted base. Perithecium relatively large, straight or slightly curved, somewhat inflated below, tapering gradually to the rather short, moderately well distinguished tip; the apex bluntly rounded, the basal cells relatively small; the stalk-cell variably developed, its distal end usually somewhat broader than the basal cells collectively, sometimes more than half as long as the body of the perithecium. Spores about 50–55 × 4–5 μ. Perithecia 130–185 × 35–55 μ, the stalk-cell 35–100 × 18–25 μ. Antheridia 70–75 × 22 μ, its appendages 45–50 μ. Receptacle about 35 × 28 μ. Greatest general length and width of largest individual 350 × 300 μ.

On *Aleochara rufipes* Boh. Derema, Usambara, East Africa. Berlin Museum, Nos. 844 and 845.

EUHAPLOMYCES nov. gen.

Receptacle consisting of two cells, the upper bearing a free stalked antheridium and a stalked perithecium. Antheridium conical, consisting of a single stalk-cell followed by a basal cell from which is separated a group of smaller cells some of which (two or four ?) extend upward and inward to form antheridial cells: above these follow three external marginal cells, the lowest of which lies beside the antheridial cells; the uppermost succeeded by a conical chamber terminating in a pore, and extending downward along the inner sides of the marginal cells to form a cavity into which the antheridial cells empty. Perithecium resembling that of Haplomyces and having two ascogenic cells.

Euhaplomyces Ancyrophori nov. sp.

Receptacle small, the basal cell somewhat longer, nearly hyaline, tapering to the relatively small foot; the subbasal cell becoming pale amber brown. Antheridium, including its short stalk-cell, about as long as the receptacle, becoming pale amber brown, tapering to a pointed apex. Perithecium becoming pale amber brown, relatively large, thick walled, considerably and abruptly inflated above the basal cells, somewhat asymmetrical, tapering rather evenly to the blunt apex; the stalk-cell long, thick walled, slightly curved, nearly hyaline, distally somewhat broader, not distinguished from the basal cells. Spores about 40–45 × 3.5 μ. Perithecia 180–200 × 72–82 μ; the stalk-cell 110–120 × 28–30 μ. Antheridium including the stalk-cell 55–65 μ. Total length to tip of perithecium 360 μ.

On the superior surface of the abdomen of *Ancyrophorus aureus.* Dumfriesshire, Scotland. Sharp Collection, No. 1091.

Eucantharomyces Xanthophaeae nov. sp.

Perithecium (not fully mature) straw colored, somewhat asymmetrical, almost symmetrically and but slightly inflated from base to apex; the tip short, well distinguished; the lip-cells rounded, and slightly inflated, forming a knob-like termination, one of them protruding in the form of a slight tongue-like projection beyond the others: the stalk-cell about as long as the receptacle, from which it projects at an angle, being moreover turned at the same time a little to one side. The cells of the receptacle subequal, lying side by side, the basal one extending to the base of the stalk-cell of the perithecium, with which it is in contact. Appendage relatively large, the stalk-cell subtriangular, somewhat larger than the basal cell which is wholly overlapped externally by the well defined and distally somewhat inflated marginal cell; the antheridial cells in four tiers of seven, six, five and four cells respectively; the discharge-tube long and curved outward. Spores about 36 × 4 μ. Perithecia 165 × 50 μ, the stalk-cell 46 × 20 μ. The appendage to tip of discharge tube 120 μ, the antheridium proper 55 × 30 μ. Total length to tip of perithecium 290 μ.

On the right inferior margin of the prothorax of *Xanthophaea vittata* Dej., Australia. Berlin Museum, No 973.

Dichomyces bifidus nov. sp.

Basal cell slightly enlarged, pellucid, tinged with brown, about as long as broad: the lower tier, and more or less of the middle tier, opaque; the marginal cells of the latter forming a bluntly rounded, sometimes almost obsolete projection on either side, hardly extending above the venter of the short, stout, short-necked antheridia: the upper tier relatively large, more or less crescent-shaped according to the degree of lateral development, edged externally with blackish brown, more broadly below, the brown area punctate; the cells about thirty-one in the larger individuals, the marginal ones forming a rather slender series, which may curve abruptly upward nearly to the middle of the perithecia, or assume a more divergent habit; the peritheeigerous area horizontal, producing normally four perithecia, three appendages arising between the two middle ones and one between each of the others, the external cells bearing appendages as usual which vary in length. Perithecia rather long and slender, hyaline or faintly yellowish brown, conspicuously tinged with purplish brown below the perfectly hyaline tip, the anterior lip-cells forming a pointed projection, the posterior ones forming each a relatively large ear-like appendage which tapers to a pointed apex, and is slightly curved, the two diverging from one another at an angle of about 50°. Spores about 38 × 2.5 μ. Perithecium without appendages 126 × 25 μ; the appendages 14 μ. Receptacle 220–350 × 120–165 μ. Total length to tip of perithecium 300–330 μ. Appendages 20–80 μ.

On the abdomen of (?) *Philonthus* sp. Ralum, New Pomerania. Berlin Museum, No. 1013.

Dichomyces Belonuchi nov. sp.

Receptacle relatively large and long: the distal tier relatively small, consisting of from eleven to thirteen short cells, slightly suffused, the median cells little longer than the rest, the series forming slight, rounded, sometimes almost obsolete lateral projections on either side of the perithecia: the basal cell small, partly transparent: the lower and middle tiers not distinguished, uniformly opaque; a portion of the middle cell, and sometimes the tips of other cells in the middle tier, more or less translucent, the marginal cells ending in a slight rounded prominence below the base of the antheridium. Perithecia normally two, evenly suffused with pale reddish brown, rather long and slender, tapering throughout, the conformation of the lip-cells much as in *D. furciferus*. Spores about 30 × 3 μ. Perithecia 75–80 × 18–20 μ. Receptacle 108–126 × 54–58 μ. Total length to tips of perithecia 185–200 μ.

On the abdomen of *Belonuchus fuscipes* Fauvel. New Guinea. Sharp collection, No. 1090.

Dichomyces Australiensis nov. sp.

Receptacle usually rather long and narrow, the basal cell relatively large, hyaline or slightly suffused; the margins of the lower tier usually continuous with those of the middle one, the marginal cells deep blackish brown or quite opaque, the middle cell hyaline or translucent throughout, its lower third often punctate: the middle tier consisting of about nine cells, slightly suffused with pale reddish brown externally, more or less edged with deep blackish brown; the terminal cells forming a free rounded projection on either side, extending as high as about the middle of the rather large antheridia, the tips of which may reach to the bases of the perithecia: the upper tier nearly hyaline, consisting normally of from eleven to thirteen subequal cells, the terminal ones extending but slightly higher than the bases of the perithecia, which are normally two in number, rather deeply suffused with purplish brown throughout; the apex hyaline, the posterior lip-cells producing each a relatively large bluntly pointed appendage, the two diverging nearly at right angles to the axis of the perithecium, becoming slightly recurved, the distance from tip to tip about twice the diameter of the perithecium. Appendages nearly as long as the perithecia. Perithecium 60–70 × 16–20 μ, its appendages 18 μ. Receptacle 90–100 × 42–48 μ. Total length to tip of perithecium 160–170 μ.

On the superior surface of the abdomen of *Quedius ruficollis* Grav. Sharp Collection, No. 1102.

Dichomyces Mexicanus nov. sp.

General habit much like that of *D. princeps*, generally rather long and slender. Basal cell hyaline, the lower tier relatively long and narrow, broadly edged externally with black; the median cell hyaline, or only the marginal cells slightly suffused with smoky brown: the middle tier distinguished from the lower by a slight prominence, hyaline, seven to nine celled; the marginal cells protruding but slightly on either side; the antheridia brownish, short, stout, blunt pointed: the upper tier relatively very long, sometimes twice as long as the middle tier, consisting of from nine to eleven cells; the marginal cells protruding but slightly on either side, very much as in the middle tier. Perithecia normally two, about as long as the distal tier and concolorous with it, or somewhat darker, rather stout, tapering but slightly; the tip rather abruptly distinguished, broadly truncate with a slight median projection; the posterior lip-cells giving rise each to a long horizontal appendage, which becomes recurved, is bluntly pointed and somewhat narrower toward the base, the distance from tip to tip often twice the diameter of the perithecium. In a few specimens the receptacle and perithecia are somewhat evenly suffused with smoky brown. Perithecia 75–85 × 25–30 μ, the appendages 18–22 μ. Receptacle 165–200 × 55–70 μ. Total length 235–275 μ.

On the inferior surface of the abdomen of *Philonthus atriceps* Sharp. Jalapa, Mexico. Sharp Collection, No. 1112. Specimens, apparently normal, sometimes occur in which the tips of the perithecia are blunt and unmodified.

Dichomyces Homalotae nov. sp.

Form short and stout. Basal cell geniculate, more or less suffused: the lower tier more or less, sometimes wholly, suffused with reddish brown; the margins darker, more or less translucent, without contrasts, the outline somewhat uneven, the transition to the middle tier indicated by a distinct prominence: the middle tier consisting of from nine to (rarely) thirteen cells, hyaline or subhyaline, with slight lateral suffusions; the marginal cells ending in a slight hyaline rounded projection, seldom extending higher than the venter of the somewhat suffused curved antheridia: the upper tier relatively small, the cells subequal, hyaline, asymmetrical, owing to the development of but one perithecium; the appendages often equalling, or exceeding the perithecium in length. Perithecium characteristically short and stout, inflated below, sometimes oval, tapering somewhat abruptly distally, to the rather broadly truncate, or slightly rounded unmodified apex. Spores 33 × 3 μ. Perithecia 65–75 × 25–30 μ. Receptacle 70–90 × 40–55 μ. Total length 125–165 μ.

On all parts of *Homalota sordida* Marsh. Fresh Pond, Cambridge. First observed by Mr. Bullard.

Peyritschiella Xanthopygi nov. sp.

Basal cell of the receptacle very small, or hardly distinguished from the foot: the first tier consisting of three subequal cells without appendages, the middle one somewhat shorter than those on either side of it: the second tier asymmetrical, consisting of three subequal median cells, the margins of the two outer free below for nearly half their length and coincident with the margins of the tier below, the appendiculate "marginal" cells, about three to five on either side, separated from them as usual by oblique septa; the first on the right bearing the large, slender, pointed, nearly straight purplish antheridium: the upper tier consisting of about fifteen or more cells, the series distally concave, rising abruptly upward on either side above the base of the perithecium and bearing the usual appendages. Perithecium solitary at the right of the median (primary) appendage, almost symmetrically inflated from base to apex, dull purplish; the tip slightly darker, hardly distinguished; the apex truncate, sometimes slightly spreading; the lip-cells hardly projecting. Perithecia 115–150 × 34–42 μ. Receptacle 200 × 65–70 μ. Total length to tip of perithecium 310–360 μ.

On the abdomen of *Xanthopygus Solskyi* Sharp. Sharp Collection, No. 1158. Nearly allied to *P. Amazonica,* from which it differs principally in the form of the perithecium.

Chitonomyces occultus nov. sp.

Short and stout, becoming suffused with somewhat smoky amber brown. Lower portion of the receptacle deeper brown, the basal cell relatively large, broad distally; the subbasal cell broad and flattened; the lower cell of the distal portion rather large and but slightly overlapped by the subterminal cell, which may bulge slightly below the terminal cell, the latter being thus turned so as slightly to overlap the perithecium. Perithecium short and stout, its upper third or less free, darker brownish externally; the tip bent outward, tapering rather abruptly to the slightly irregular apex, its outer half or less suffused with dark brown. Spores about 22 × 2.5 μ. Perithecium 60 × 20 μ. Receptacle to tip of distal cell 90 μ. Total length to tip of perithecium 100 μ.

In the median marginal depression of the right elytron of *Cnemidotus* sp. Lake Eustis, Florida.

Chitonomyces psittacopsis nov. sp.

Nearly hyaline. Receptacle rather slender, the basal cell several times as long as the squarish subbasal cell; the cell above the latter nearly equalling it in size and separated by an oblique septum from the lowest of the marginal cells, which are all subequal; the terminal appendiculate cell of the usual form, relatively large and long, without any distinct basal enlargement; the tip of the lower appendiculate cell curved slightly outward. Perithecium relatively very large, long, slender, usually curved sidewise throughout, the upper half tapering very slightly to the curiously modified, clear black contrasting tip, which resembles the partly open beak of a parrot; a larger upper recurved mandible-like process being separated from a second, that resembles a lower mandible, by a hyaline area which includes, and extends back from, the pore; the lower lip-cells translucent, but suffused with brown in such a way as to suggest a tongue-like process projecting slightly between the "mandibles." Spores very numerous, completely filling the cavity of the perithecium, greatly attenuated, 85 × 2.5 μ. Perithecium 200 × 30 μ. Receptacle to tip of distal cell 140 μ. Total length to tip of perithecium 290–300 μ.

On the posterior legs of *Laccophilus* sp. Lake Eustis, Florida.

Chitonomyces Bullardi nov. sp.

Straw colored becoming tinged with pale amber brown. Basal cell of the receptacle monstrously developed, about as long, sometimes twice as long, as the remainder of the plant, its axis coincident with that of a distal, variably developed, blunt, tooth-like, free posterior projection, near the base of which the subbasal cell and the remainder of the plant project backward at an angle of about 45°, or less, to the axis of the basal cell, the separating septum being vertical or nearly so; the subbasal cell small and flattened: the lower marginal cell of the distal portion of the receptacle subtriangular, short and broad; the lower appendiculate cell above it relatively large; the subterminal cell larger than the lower marginal cell, curved inward so that the terminal appendiculate cell projects from it obliquely inward against the perithecium. Perithecium four fifths or more free, relatively large and stout, distinctly inflated below, tapering to the tip, which is characteristically modified through the presence of a large claw-like subterminal dark amber brown external projection, the distal half of which is somewhat abruptly recurved, like the upper mandible of a parrot, over the small hyaline incurved 4–papillate apex, which is immediately subtended on the inner side by a small, erect, dark amber brown, tooth-like process, the blunt tip of which alone is free. Appendages slender and extending to or beyond the tip of the perithecium. Spores about 20 × 2.5 μ. Perithecium average 70–75 × 30–32 μ not including the hook-like appendage, which is 25 μ to its upper margin. Receptacle: basal cell to tip of prolongation 90–220 × 15–22 μ, the portion above to tip of distal cell 48 μ.

On the right inferior anterior margin of the prothorax of *Cnemidotus 12–punctatus* Say. Glacialis Pond, Cambridge. The most singular species of the genus, discovered by Mr. Charles Bullard, to whom I take pleasure in dedicating the species.

Chitonomyces Hydropori nov. sp.

Receptacle nearly hyaline, the subbasal cell flattened, many times smaller than the basal cell, slightly inflated and distinguished from the cells above and below by slight constrictions; the two cells above subequal, the posterior somewhat broader, and separated from the lower marginal cell of the distal portion by an oblique curved septum, which overlaps its upper fourth; the subterminal marginal cell often nearly as long as the lower, the narrow upper half or more of which it overlaps. The lower appendiculate cell rather small, the upper terminal one of the typical form, relatively rather long, distinguished by a slight constriction, the appendage extending beyond the tip of the perithecium. Perithecium relatively large, its upper half or more free, distally broader, the outer margin nearly straight with a slight subterminal rounded elevation below the abruptly rounded projecting outer brownish lip-cells; the apex otherwise flat, broad, bent outward so as to be slightly oblique, the inner margin below it bulging and curved throughout. Spores 55 × 4 μ. Perithecium 98–108 × 25 μ. Receptacle to base of perithecium 80 μ, to tip of terminal cell 150 μ. Total length to tip of perithecium 185 μ.

On the mid-elytron of *Hydroporus modestus* Aubé. Cape Neddock, Maine. Mr. Bullard.

Chitonomyces Orectogyri nov. sp.

Dull purplish, the cells thick walled and marked by faint transverse striations. The basal cell of the receptacle very small and hardly distinguishable, owing to an abrupt curvature just above the foot; the subbasal cell relatively large, distally narrowed, nearly the whole upper half of its posterior margin covered by a relatively large triangular cell, from which it is separated by a nearly vertical septum; this triangular cell is in contact distally with the ascigerous cavity and the base of the lowest marginal cell; the latter is very long, extending upward, its narrow extremity ending without enlargement opposite the blackened base of the inner appendage, lying between the latter and the tip of the perithecium; the lower appendiculate cell well defined, about two thirds as long as the subterminal cell, which projects slightly above and bears the free terminal appendiculate cell, which is hyaline, about equal to the lower in length, its inner margin nearly straight, its outer margin curved abruptly inward to the base of the obliquely distinguished, blackened, narrow, erect terminal portion, from which the appendage has been broken in the types. Perithecium relatively large, of nearly equal diameter throughout; the tip broad with a bluntly rounded apex; a short erect contrasting brown prominence formed by the left posterior lip-cell, toward the base of which the inner (anterior) lip-cells are curved in a characteristic fashion, so as partly to overlap it. Spores about 75 × 5 μ. Perithecium 125 × 36 μ. Receptacle 250–270 μ. Total length to tip of perithecium 255 μ.

On the superior surface of the tip of the abdomen of *Orectogyrus specularis* Aubé. Africa. Berlin Museum, No. 806.

DIOICOMYCES nov. gen.

Male individual consisting of four superposed cells, the upper of which is a simple antheridium bearing a subterminal discharge tube.

Female individual. Receptacle ending distally in a peculiarly modified sterile cell, corresponding to the upper spore-segment: the subbasal cell producing a single perithecium laterally, and separated from the sterile terminal cell by a second small cell. Perithecium free. stalked; the ascogenic cell single, the spores more or less obliquely once-septate, and of two kinds corresponding to the sexes.

Dioicomyces Floridanus, formerly referred provisionally to *Amorphomyces*, must be transferred to this genus; since, although the male is unknown, the female has the typical characters which distinguish the genus very clearly from its near ally. *D. obliqueseptatus* on *Myrmedonia* (?) sp. must also be removed from *Amorphomyces*, on account of its obliquely septate spores, and should with little doubt be included in the present genus; although it is evident, from comparison with abundant material of the species described below, that the specimens, both females, from which the original description was made, are more imperfect than was at first supposed, and should not have been used as types. The peculiar sterile cell is present in neither of these; but, since they correspond in all other respects to the generic type, may be assumed to have been broken off. No free spores are available in either, although an examination of the spore mass within the ascus seems to show that they present the same variation in size which characterizes the species described below.

Dioicomyces Anthici nov. sp.

Male individual. Form slender, of nearly the same diameter throughout, the basal cell half the total length of the individual to the tip of the discharge tube; the third cell nearly square, the subbasal about as large as the terminal antheridial cell, which ends in a distal blunt projection; the discharge-tube arising laterally below the tip, projecting upward from a broadened base, slightly divergent from the main axis, slender, about as long, or a little longer than, the body of the antheridial cell. Length to tip of antheridial cell, including foot, 50 μ: to tip of discharge-tube 60 μ. Width 8 μ.

Female individual. Often more or less strongly curved, the terminal sterile cell bluntly pointed, slightly curved, brownish; the basal cell becoming narrower below, the upper septum convex; tinged with brown posteriorly as is the rest of the receptacle: the subbasal cell very small, subtriangular; separated from the terminal sterile cell by a somewhat smaller triangular cell. Stalk-cell of the perithecium hyaline, long, often about the same diameter throughout; the thick wall becoming gradually thicker distally: the perithecium slightly inflated, faintly brownish; the short, stout, broad, blunt tip slightly distinguished, and nearly symmetrical; the lip-cells forming an unbroken outline, without protrusions. Spores (male) 40 × 4 μ, (female) 60 × 6 μ. Perithecium 100–110 × 35–45 μ, the stalk-cell 75–115 × 18 μ. Receptacle including foot 35 × 12 μ, the sterile terminal cell 18–25 × 7–9 μ. Total length to tip of perithecium 185–220 μ.

On *Anthicus floralis* Linn. Fresh Pond, Cambridge. On *A. Californicus* Laf. California (Leconte Collection).

Dioicomyces onchophorus nov. sp.

Male individual similar to that of *D. Anthici*, slightly smaller.

Female individual. Usually strongly curved, especially at the base of the stalk-cell; similar to *D. Anthici*; the receptacle, sterile cell, and the stalk of the perithecium, relatively smaller. Perithecium dirty brown, one of the lip-cells protruding in the form of a well defined, lateral, finger-like, erect, straight, or slightly curved, blunt-tipped, concolorous process; an irregular anterior elevation or angular prominence is also more or less well defined above the middle of the perithecium. Spores (male) 35 × 4 μ, (female) 45 × 5 μ. Perithecia to tip of projection 125–140 × 40–45 μ, the stalk-cell 90 μ. Total length to tip of perithecium 210–230 μ.

Usually on the basal half or at the base of the left elytron of *Anthicus floralis* Linn. Fresh Pond, Cambridge.

Dioicomyces spinigerus nov. sp.

Male individual similar to that of *D. Anthici*, much smaller, the extremity less prominent, or almost horizontal, the discharge tube somewhat more slender, and more often erect. Total length including foot 40 × 6.5 μ; to tip of discharge-tube 47 μ.

Female individual. Receptacle relatively small, tinged with dirty yellowish, edged with brown to the tip of the small terminal sterile cell. Perithecium dirty yellowish and relatively large, considerably and more or less symmetrically inflated, above and including its basal cells, to the base of the tip, which is bent abruptly outward at right angles to the axis of the perithecium; the apex broad, blunt, the lip-cells hardly projecting: a unicellular brown, straight or slightly curved, spine-like process, which tapers to a blunt point, projects upward at an angle of about 45° from the middle of the outer (anterior) margin of the perithecium; and a slight elevation is also more or less distinct between its base and that of the tip; the stalk-cell relatively short, becoming rapidly narrower toward its base. Spores (male) 26 × 4 μ, (female) 40 × 6 μ. Perithecia including basal cells 125 × 50 μ, the spinous process 55 μ, the stalk-cell 36–40 μ. Receptacle to tip of sterile cell about 45 μ. Total length to tip of perithecium about 185 μ.

On *Anthicus floralis* Linn., with the last two species, more commonly on the inferior surface of the abdomen. Fresh Pond, Cambridge.

Teratomyces Zealandica nov. sp.

Receptacle with a distinct distal obliquity, opaque with the exception of a hyaline area just above the foot, the margins straight, the distal portion relatively narrow, the base relatively broad, the suffusion involving the bases of the appendiculate cells which are relatively numerous and narrow and more or less suffused with brownish yellow. Appendages sometimes scanty, but slightly divergent, concolorous throughout, nearly hyaline or pale yellowish; the basal cells of the larger branches relatively slender, the external branchlets and numerous beak-like cells hardly more deeply colored. Perithecia relatively large, long, rather slender, slightly inflated throughout, the blunt tip more or less abruptly distinguished; the stalk-cell very short or almost obsolete, hidden by the appendages; the basal cells relatively small and not distinguished from the body of the perithecium. Spores about 50 × 2.5–3 μ. Perithecia 150–180 × 20–28 μ, basal and stalk-cells together about 35 μ. Longest appendage 180 μ. Receptacle 75–125 × 15–18 (base) 22–30 μ (distally).

On *Quedius insolitus* Sharp. Dunedin, New Zealand. Sharp Collection, No. 1099.

Teratomyces petiolatus nov. sp.

Receptacle nearly symmetrical, almost wholly black, slender below, expanding rather abruptly distally; the appendiculate cells relatively large and long, translucent, brownish yellow, subtended by a slight enlargement. Appendages numerous, spreading, the larger ones consisting of a very large colorless or brownish basal cell, which bears a series of branchlets externally and several branches terminally; the branchlets usually short, and two-celled, the distal cell usually long, beak-like and clear purplish brown, the lower cell hyaline or light brown and in the lower branchlets usually bearing long-necked antheridia: the terminal branches with several short branchlets of a similar character. The smaller shorter appendages about the bases of the larger ones, mostly dark purplish brown, with many beak-like cells. Perithecia usually several, large, symmetrical, purplish brown; the tip short, rather narrow and abruptly distinguished; the basal cells relatively very large, forming a portion of the stalk sometimes half as long as the perithecium proper; the stalk-cell stout and elongate. Perithecia 185–225 × 45–50 μ, the basal cell 100–150 × 10 μ, the stalk-cells 180–300 μ. Receptacle about 150 μ. Appendage, longest 175, longest basal cells 110 μ.

On *Quedius* sp. Greymouth, New Zealand. Sharp Collection, No. 1103.

Teratomyces insignis nov. sp.

Receptacle usually quite opaque, long, slender; the outline unbroken and nearly straight, tapering evenly to the slightly geniculate base, which is nearly hyaline just above the foot: the margin of the suffused area distally strongly oblique, especially before maturity; the appendiculate cells small, becoming brownish. The appendages numerous, spreading, the larger ones hyaline or nearly so, consisting of a large elongate basal cell, which bears two or three small remote antheridial branches externally; and terminally, as a rule, two large branches placed side by side (one of which may be wanting) sometimes associated with one or two subterminal smaller branchlets, the basal cells of which are dark contrasting brown: the terminal branches hyaline with branchlets like those of the basal cell; the branchlets, however, more numerous, contrasting, brown, simple or branched, many having characteristic beak-like terminations, while others are blunt tipped, with oblique septa. The smaller peripheral appendages more or less crowded around the bases of the larger ones, with conspicuous and numerous beak-like terminations. The antheridia with long curved necks. Perithecia usually several, brown, long and slender, straight, very slightly inflated near the base, with a slight submedian enlargement; tapering throughout to the short, truncate, well distinguished tip: the basal cells rather small, concolorous; the group narrower than the stalk-cell and separated from it by a horizontal septum: the stalk-cell very large, usually elongate, often inflated and thick walled. Spores about 50 × 4 μ. Perithecia including basal cells 240–275 × 40 μ, the stalk-cell 150–325 × 25–35 μ. Appendages, longest 225 μ. Receptacle 100–185 × 14 (base) × 55 (distal end). Total length to tip of perithecium largest, 800 μ.

On abdomen of *Quedius* nov. sp. New Zealand. Sharp Collection, No. 1159.

ACOMPSOMYCES nov. gen.

Receptacle two-celled, bearing an antheridial branch terminally and a single perithecium laterally. Antheridium consisting of several superposed cells from which single simple antheridia are borne directly. The perithecium borne on a stalk, the lumen of which becomes continuous with that of the ascigerous cavity.

Acompsomyces Corticariae nov. sp.

Receptacle narrow below, distally enlarged, hyaline; the subbasal cell small. Basal cell of the appendage brown, distally narrowed to the base of the appendage proper, which is brown, and consists of three symmetrical cells, the upper smaller, becoming a terminal antheridium, the lower bearing several antheridia somewhat irregularly. Perithecium brown, rather abruptly distinguished from the short hyaline stalk; the tip very broad and darker; the lip-cells forming four hyaline-tipped, nearly symmetrical papillae, which terminate four corresponding ridges. Spores about 30 × 2 μ. Perithecia 90 × 26 μ, the stalk 15 μ. Receptacle 25 μ. Antheridial appendage, above stalk-cell, and including terminal antheridium, 40 μ.

On elytron of *Corticaria* sp. Berkeley, California.

STICHOMYCES nov. gen.

Receptacle consisting of two cells, the upper bearing one or more stalked perithecia laterally, and an antheridial appendage terminally. The appendage consisting of several superposed cells, the lowest sterile, or having one or two opposite lateral perithecia: those above it bearing opposite lateral branchlets distally, the series ending in a terminal sterile branch. Antheridia simple, flask-shaped, free, borne in small groups on short branchlets.

Stichomyces Conosomae nov. sp.

Dull amber brown. Receptacle and appendage undifferentiated, the basal cell of the former small, triangular in outline; the subbasal cell about as broad as long, and similar to the cells of the appendage, bearing distally and laterally a single perithecium, sometimes two, which are then paired on opposite sides of the cell, like the antheridial branchlets. Appendage consisting of five superposed subequal cells slightly longer than broad, the basal one sterile, or rarely (abnormally) producing one or two perithecia as in the subbasal cell below it: the three cells above slightly larger, the upper angles separated by oblique septa to form small cells on either side, which bear short one or few celled antheridial branchlets; the terminal cell somewhat smaller, bearing a simple terminal several-celled branch in addition to the lateral branchlets, all of which appear to be sterile. Antheridia with broad necks grouped in twos or threes. Perithecium darker brown, more or less symmetrically inflated; the tip hardly modified; the basal cells collectively broader and nearly as long as the stalk-cell. Spores 35 × 2.5 μ. Perithecia 85 × 25 μ, the stalk-cell 36 × 14 μ. Total length to tip of the appendage proper 150 μ, the terminal branch 150 μ, the antheridial branchlets about 20 μ. Total length to tip of perithecium 185–200 μ.

On *Conosoma pubescens* Payk. Belmont and Waverly, Mass. First observed by Mr. Bullard.

Rhachomyces Oedichiri nov. sp.

Receptacle strongly curved, rather short, the lower cells especially more or less suffused with clear brown, the basal cell slender, the cells of the main axis above it successively larger, about ten to twelve in all. Appendage hardly ever reaching to the tip of the perithecium; the shorter margin alone subulate and straight, the rest appressed, denser toward the base of the perithecium, where they form a tuft which does not wholly surround it, curved slightly outward, somewhat attenuated; tips abruptly recurved or subhelicoid. Perithecium somewhat inflated, hyaline, with the exception of several longitudinal dark brown marks at the tip, the base concealed by the appendages. Spores 36 × 4 μ. Perithecia 90–110 × 30–35 μ. Total length to tip of perithecium 220–250 μ. Longest appendages about 90 μ.

On *Oedichirus* nov. sp. Rio de Janeiro, Brazil. Sharp Collection, No. 1154.

Rhachomyces Glyptomeri nov. sp.

Receptacle slender, dirty translucent brown, the main axis consisting of about seven cells (below the lower of the two perithecia which are present in the type): the appendages slightly divergent, large and long, opaque brown, flexed inward near their hyaline, somewhat more slender extremities, and extending beyond the tips of the perithecia. Perithecium short-stalked, strongly curved, slightly inflated, hyaline, soiled with brownish, the tips well distinguished, blackish brown and obliquely truncate. Perithecia, including basal and stalk-cells, about 185 × 44 μ. Receptacle to base of lower perithecium 150 × 15 μ. Appendages, longest 360 μ or more.

On tip of abdomen of *Glyptomerus cavicolus* Müll. Carniola, Austria. Sharp Collection, No. 1141.

Rhachomyces Dolicaontis nov. sp.

Form elongate. Cells of the main axis of the receptacle twenty to thirty-five, more or less dirty brownish, banded with dark blackish brown below, while the more slender proximal cells are usually opaque; the axis of nearly equal diameter throughout and nearly straight above about the eighth cell; each cell containing distally one, the axis cells two, roundish or oblong brown bodies (possibly thickenings of the walls) which suggest the stigmata of an insect larva. The appendages somewhat divergent, opaque, except a narrow upper hyaline margin, short, stiff and numerous; those external more slender, slightly curved and sharply pointed; those between somewhat stouter and longer, with slightly recurved tips; those about the base of the perithecium, which they do not conceal, but slightly longer and few in number. Perithecium short-stalked, slightly more or less symmetrically inflated, dull brown, minutely punctate or granular, not uniformly suffused; the tip with darker shades, the blunt apex hyaline. Spores 66 × 5 μ. Perithecia 150–200 × 42–60 μ, including the basal and stalk-cells. Larger appendages 90–110 μ, smaller about 75 μ. Total length 600–1100 μ, the average diameter about 30–35 μ. On all parts of *Dolicaon Lathrobioides* Casteln. Cape of Good Hope, Africa. Sharp Collection. No. 1146. Berlin Museum, Nos. 833 and 842.

Sphaleromyces Quedionuchi nov. sp.

Perithecium relatively small, translucent, tinged with amber brown, straight, very slightly almost symmetrically inflated; the tip hardly distinguished; one of the lip-cells forming a blunt, terminal, irregularly curved, hyaline, sometimes abruptly distinguished projection, below the base of which arises on the inner side a tongue-like outgrowth externally and basally blackish brown, the broad rounded hyaline end of which is curved against or across the base of the terminal outgrowth; the stalk-cell small, the basal cells collectively larger, and separated from it by a very oblique septum. Basal cell of the receptacle long, black, obconical, the narrow base translucent; the subbasal cell small, nearly triangular. Appendage consisting of five very obliquely superposed cells, the two lower nearly equal, the cells above successively smaller, but equal in length; the branches which are once or twice branched and extend about to the middle of the perithecium, arising from the whole surface of their inner margins, the terminal cell soon destroyed. Spores 55 × 3 μ. Perithecia 135 × 36 μ. Basal cell of receptacle 120 μ. Appendage without branches 55 μ. Total length to tip of perithecium 290–310 μ.

On the abdomen of *Quedionuchus impunctus* Sharp. San Andres, Vera Cruz. Sharp Collection, No. 1105.

Sphaleromyces Chiriquensis nov. sp.

Almost uniformly translucent dirty amber brown. Perithecium very large and crowded with spores, long, with a very slight general inflation, the base narrower, tapering abruptly at the short tip: one of the lip-cells forming an erect, median, straight, hyaline, cylindrical or slightly inflated, nearly truncate terminal projection, which is subtended by a posterior or partly lateral, somewhat larger, spine-like, slightly divergent, deep black brown, nearly straight or slightly outcurved pointed outgrowth, its tip nearly on a level with that of the median projection: the basal cells collectively slightly larger than the short stalk-cell, and not distinguished from the base of the perithecium. Basal cell of the receptacle very large, tapering throughout from the broad distal to the narrow basal end, paler than the small, flattened, deeper brown subbasal cell. The appendage consisting of a relatively large basal stalk-cell, which is slightly longer than broad, and partly united to the stalk-cell of the perithecium; above are four short successively smaller cells, their septa slightly oblique, the three lower bearing branches as usual, which may branch once above their basal cells, the branchlets brown, erect, rigid, closely aggregated; the uppermost cell paler, with a terminal branch. Spores 50 × 2 μ. Perithecia 220–250 × 40–48 μ, to tip of median projection, the subterminal process 25 × 7 μ; the stalk-cell 35 × 25 μ. Receptacle 210 × 40 μ, the basal cell 220 μ. Total length to tip of perithecium 500–600 μ. Appendage without branches, including stalk-cell, 75 μ.

On the tip of the abdomen of *Quedius flavicaudus* Sharp. Volcan de Chiriqui, Panama. Sharp Collection, No. 1157.

Sphaleromyces Indicus nov. sp.

Perithecium relatively very long and large, yellowish, very slightly inflated toward the base, tapering very gradually to the broad, blunt tip which is subtended by a truncate, conical lateral projection; the stalk-cell relatively short. Receptacle relatively small, the two cells nearly equal, the upper bearing the stalk-cell of the perithecium terminally and the basal cell of the appendage laterally; the latter overlapping it to its base. Appendage consisting of four superposed cells, the basal (stalk-cell) small, triangular; the two cells above it larger and longer, bearing short antheridial branches from the upper inner angles; the terminal cell smaller, subconical, bearing a small terminal branchlet. Spores about 44 × 4 μ. Perithecium 290–340 × 45 μ, the stalk-cell 72 μ. Receptacle 55 μ. The appendage 125 μ.

On the upper surface of the tip of the abdomen of *Pinophilus* (near "*P. rufipennis*"). Malabar, India. Sharp Collection, No. 1151.

Corethromyces Latonae nov. sp.

Perithecium reddish brown with a purplish tinge, often straight, or externally concave, slightly inflated; the lip-cells forming a small short, slightly bent, nearly cylindrical, truncate, or papillate terminal projection, which is rather abruptly distinguished; the secondary stalk-cell, and the basal cell above it, bulging outward more or less prominently, and separated by a rather conspicuous irregular indentation: the stalk-cell small and squarish. The basal cell of the receptacle asymmetrical; its anterior margin straight and perpendicular, the posterior slightly curved and oblique; its distal margin oblique with a posterior protrusion; its slender base translucent, but otherwise opaque, the opacity involving a portion of the small flattened subtriangular subbasal cell. The appendage consisting of a series of about five successively smaller hyaline cells, the lowest greatly flattened; the series above, the distal cells of which soon disappear, often turned outward so as to become almost horizontal in position, giving rise from their inner sides to numerous hyaline branches, which may be more or less copiously branched. Spores about 35 × 2 μ. Perithecium 90–105 × 20–25 μ, the stalk and basal cells together 20–

25 μ. Receptacle 110 × 50 (distal end) × 10 μ (base). Total length to tip of perithecium 225–250 μ.

On the legs and abdomen of *Latona Spinolae* Guér. Bogota, Columbia. Berlin Museum, No. 834.

Corethromyces Stilici nov. sp.

Perithecium amber colored, with a faint brownish or reddish tinge, somewhat irregular in outline through a spiral twist in the wall-cells, which are distinguished from one another by slight furrows; slightly inflated toward the base, tapering to the broad blunt apex; the tip not at all distinguished; the basal and stalk-cells well developed, hyaline, the latter bent abruptly upward from its insertion. Basal cell of the receptacle small, hyaline on the anterior side just above the foot, but otherwise blackish brown or opaque, bulging posteriorly above the foot; distally and posteriorly proliferous to form a straight, black, blunt finger-like outgrowth, which lies external to the appendage; the subbasal cell nearly hyaline, subtriangular, separated from the basal cell by a very oblique septum. Appendage hyaline, consisting of a nearly free and nearly isodiametric stalk-cell, above which are three or four cells which produce a close tuft of hyaline branches on the inner side. Spores about 30 × 3 μ. Perithecia 80–85 × 22 μ, its stalk-cell 30 × 18 μ. Receptacle 25 μ, the outgrowth 55 × 7 μ. Appendage, including branches, 50 μ. Total length to tip of perithecium 150 μ.

On the abdomen of *Stilicus* sp., Interlaken, Switzerland. On *Stilicus rufipes Germ.*, Berlin Museum, No. 836. Europe.

Ceratomyces spinigerus nov. sp.

Bright amber brown. Perithecium paler anteriorly, about twenty-eight wall-cells in each row; narrower at the base, the lower half bulging anteriorly, tapering distally where it is rather strongly curved away from the antheridial appendage: the tip hyaline, prominent, obtuse, about half as long as the curved tooth- or spine-like one-celled deep amber brown appendage, which arises below and beside it. Basal cell of the receptacle large, long, mostly curved, broader distally, opaque; the portion above it relatively small and narrow, concolorous with the perithecium. The appendage erect, slightly divergent, stiff, long, slender, rather remotely septate, but the basal cell often broader than long, about seven-celled, tapering distally. Spores 90 × 4 μ, in one small specimen 165 × 4.5 μ. Perithecia 425–500 × 70–95 μ, the appendage 45–50 μ.

Receptacle 175–220 μ, the basal cell 150–170 μ. Antheridial appendage 200–325 μ.

On the inferior anterior margin of the thorax near the base of the right elytron of *Tropisternus apicipalpis* Cast. Jalapa, Mexico. Sharp Collection, No. 1178.

Ceratomyces procerus nov. sp.

Rather pale amber brown. Perithecium very elongate, of nearly equal diameter throughout, the wall-cells in each row more than sixty in number; the conformation at the tip similar to that in *C. confusus;* the perithecial appendage erect, short and stout, consisting of about ten cells, distally curved outward, tapering from its broad base to the bluntly pointed tip. Appendages (broken) and receptacle much as in *C. confusus.* Perithecium 800–850 × 65 μ, its appendage 125 μ. Total length to tip of perithecium more than one millimeter.

On the inferior surface of the abdomen (near the middle) of *Tropisternus* sp. San Fidelio, Brazil. Museum of Comparative Zoölogy, Cambridge, No. 1338.

Ceratomyces curvatus nov. sp.

Amber brown. Perithecium relatively large, inflated toward the base; the distal half up to the perithecial appendage of about equal diameter throughout; about forty cells, more or less, in each row of wall-cells; the configuration at the tip very similar to that in *C. confusus,* the tip itself more prominent, the apex more pointed; the perithecial appendage about nine-celled, the distal half pale, curved or recurved, broader below, shorter and stouter. Receptacle much as in *C. confusus,* the basal cell black, the further suffusion somewhat less extensive. Appendage consisting of about six or seven cells, tapering distally, rather short. Spores about 70 × 4 μ. Perithecia 500–615 × 75 μ (below) × 60 μ (distally), the appendage 150 μ. Total length to tip of perithecium 600–700 μ, to tip of antheridial appendage about 250 μ.

On *Tropisternus Caracinus* N. on inferior surface of abdomen near the tip. Caracas? Berlin Museum, No. 1057.

Ceratomyces Mexicanus nov. sp.

Dark amber brown. Perithecium with a slight submedian inflation; distally broad, the outer margin turning abruptly inward distally to the inconspicuous retracted tip, which lies close at the base of the perithecial appendage, and is externally subtended by irregular inconspicuous papillate protrusions: the basal cell of the appendage slightly divergent, several times as long as broad; the external margin straight, the inner strongly concave with a median blackish suffusion; the rest of the appendage slightly curved, about eight or nine-celled, tapering slightly and diverging strongly above the basal cell. The antheridial appendage and the receptacle much as in *C. mirabilis*. Spores 85 × 5 μ. Perithecia 400–475 × 110–125 μ, the appendage about 290 μ, its basal cell 70 × 26 and 36 μ. Total length to tip of perithecium 550–640 μ.

On the left inferior margin of the abdomen of *Tropisternus nitidus* Sharp, Sharp Collection, No. 1177, and of *T. chalybeus* Cast., British Museum, No. 772, Oaxaca, Mexico.

Ceratomyces Braziliensis nov. sp.

Dark amber brown. Perithecium somewhat inflated just above the constricted base, the upper two-thirds broad and of about the same diameter throughout; about forty-five wall-cells in each row, the tip small, short, rather narrow, abruptly hunched externally, the hyaline lips turned abruptly toward the base of the perithecial appendage, which consists of a basal cell hardly differentiated from the wall-cell below it, though somewhat longer, the portion above it erect, slender, stiff, slightly curved outward, tapering but little, the subbasal cell bearing a characteristic basal enlargement which projects toward the lip-cells and lies just above them. The appendage and receptacle much as in *C. mirabilis*. Perithecium 650 × 95 μ (basal) × 87 μ (distal). Appendage 185 μ, or more. Total length to tip of perithecium 800 μ.

On inferior thorax of *Tropisternus nitens* Cast. var. Rio de Janeiro. Sharp Collection, No. 1181.

KAINOMYCES nov. gen.

Receptacle much as in Zodiomyces, broad and flattened; consisting of a single basal cell and typical foot, above which the successive cells become variably divided by longitudinal septa into transverse cell-rows or tiers: the distal portion more or less definitely distinguished and consisting of superposed cells, the lowest of which alone become longitudinally divided, all producing laterally antheridial (?) branches: several of the tiers immediately below this appendiculate portion growing out laterally at right angles to the main axis of the receptacle on one or both sides to form "perithecial branches" consisting of superposed cells and terminated by solitary perithecia. The perithecium of peculiar form, with six wall-cells in each row in addition to the lip-cells; the base of the trichogyne persistent in the form of a peculiarly modified unicellular appendage.

It has proved impossible from an examination of the available material of this extraordinary form, to determine the character of the antheridia; yet there can hardly be any doubt as to its true position among the "Exogenae" near Zodiomyces, Euzodiomyces, and Ceratomyces, its distal appendiculate portion being evidently homologous with the "appendage" of the last-mentioned genus.

Kainomyces Isomali nov. sp.

Receptacle variably developed below the distal appendiculate portion, sometimes very broad, often much narrower: the cells above the basal cell becoming broader and flattened, and soon divided longitudinally by one or more septa, nearly hyaline and broadly edged wholly or in part below, especially on the posterior side, with contrasting brownish black, which may involve the whole of the cell, except the transverse septa; the blackened area usually characteristically indented above, and sometimes involving all but the uppermost tiers. Perithecial branches variably developed, the free portion curving upward, and consisting of from about twelve to thirty-five superposed hyaline cells, which are more or less flattened, usually separated by slight constrictions, the distal one similar to the others and followed directly by the basal cells of the perithecium. Perithecium becoming tinged with pale amber brown, usually short, stout and suboblong, often not distinguished from its basal cells; the distal end abruptly rounded, the pore subtended by a tooth-like outgrowth, half as long as and paler than the trichogynic appendage, which bears a slight resemblance to a duck's bill, is dark clear brown, somewhat narrower distally and pale tipped, broader toward the base, where it is abruptly constricted and hyaline. Spores about 30 × 3.5 μ. Perithecia 72–80 × 40–50 μ exclusive of trichogynic appendage, which measures 28–32 × 11 μ. Perithecial branch 100–253 μ. Receptacle 150–220 × 40–60 μ. Antheridial branches about 50 μ. Total length to tip of perithecium 250–460 μ.

On *Isomalus Conradti* Fauvel. Derema, Usambara, East Africa. Berlin Museum, Nos. 847–848.

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THE LAW OF PHYSICO-CHEMICAL CHANGE.

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THE LAW OF PHYSICO-CHEMICAL CHANGE.

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Introduction.

The many-sided application of thermodynamics to physical chemistry in recent years has led to a maze of mathematical expressions which is bewildering to the beginner and confusing even to the initiated. The great majority of these physico-chemical formulae are based not only upon the two laws of thermodynamics but also upon some empirical law or approximation, and are as a rule not rigorously true, but are useful in so far as the system considered does not deviate too widely from certain ideal conditions. The difficulty of treating mathematically equations which are not strictly exact is probably the chief reason for the continued separate existence of the large number of formulae which, though not identical, are tantalizingly similar in form. It seemed probable that if the present formulae could in any way be replaced by rigorously exact ones, without sacrificing concreteness or immediate applicability, then these exact equations might be so systematized that one might serve where a number of isolated equations are now in use, with a great gain in simplification. With this object in view the present investigation has been carried on, and with the unexpected success of finding a single law which is simple, exact, general enough to comprise in itself many laws and yet concrete enough to be immediately applicable to specific cases. The following development will be based upon four laws of nature and upon no other hypothesis or assumption of any kind. These laws are the following: —

1. The first law of thermodynamics.
2. The second law of thermodynamics.
3. Every gas, when rarefied indefinitely, approaches a limiting condition in which

$$Pv = RT, \tag{1}$$

if P represents pressure; v, molecular volume; R, the gas constant; T, the absolute temperature.

4. Every solution diluted indefinitely approaches a limiting condition in which

$$\Pi v = R T, \tag{2}$$

if Π represents osmotic pressure.

The present paper will discuss the laws which govern systems composed of a single, chemically simple, substance, and will be followed by a second paper in which the laws governing mixtures will be studied.

I.

Clausius' Formula Simplified.

Clausius showed that if Q represents the heat change in a reversible change, the second law of thermodynamics may be expressed by the equation

$$\frac{Q}{T} = \frac{d Q}{d T}, \tag{3}$$

which is valid for every cyclic process; moreover, that since in a cycle there is no change in internal energy, $d Q$ represents the work of the cycle, and that when the process is one in which the system undergoes a finite change of volume at constant pressure, and no other work is done,

$$d Q = d P \left(V_1 - V_2 \right),$$

where P represents the pressure and V_1 and V_2 the original and final volumes. In the specific case in which the system is composed of a liquid and its vapor we obtain the equation

$$\frac{Q}{T} = \frac{(v_1 - v_2) \, d p}{d T},$$

in which p represents vapor pressure; Q, the total heat of vaporization of one gram-molecule; and v_1 and v_2, the molecular volumes of vapor and liquid respectively. Transposing the equation gives an expression for the change of vapor pressure with change of temperature,

$$\frac{d p}{d T} = \frac{Q}{(v_1 - v_2) \, T}. \tag{4}$$

This equation of Clausius is both general and exact, but in practice it is replaced by a simpler equation, which is derived from it if two assumptions are made: First, that v_2 is negligible compared with v_1, and therefore approximately,

$$v_1 - v_2 = v_1.$$

Second, that the vapor obeys the gas law,

$$v_1 = \frac{R\,T}{p}.$$

These two equations substituted in (4) give the familiar equation,

$$\frac{d \ln p}{d\,T} = \frac{Q}{R\,T^2}. \tag{5}$$

While neither of the two assumptions made above is in any case strictly true, they differ in that the second represents a true limit as the vapor approaches the perfect gas in its behavior, but the first is always mathematically absurd, for the volume of a liquid cannot be made to approach zero even as a limit. For an exact equation, therefore, we must return to equation (4), notwithstanding its rather complicated form. There is in fact a lack of simplicity in this equation which does not appear in certain analogous expressions that will be developed in this paper. That this lack of simplicity is, however, not inherent in every exact equation for the influence of temperature on vapor pressure, but is due rather to the complex conditions for which equation (4) is proved, will be evident from the following considerations.

It is well known that at constant temperature the vapor pressure of any substance is changed by a change in the total pressure on its surface, according to the equation first obtained by Poynting,*

$$\frac{d\,p}{d\,P} = \frac{v_2}{v_1}, \tag{6}$$

in which p represents vapor pressure; P, total pressure; v_2 and v_1, molecular volumes of liquid and vapor respectively. When, therefore, the temperature of a liquid is raised, the resulting increase in vapor pressure brings an increase in the total pressure on the surface, and this in itself is a cause of further change in vapor pressure. The observed change in vapor pressure is the sum of the change due merely to temperature change and the change due to the change in total pressure upon the surface. Let us therefore determine the change in vapor pressure with change of temperature when the total pressure on the surface is kept constant by artificial means. Figure 1 represents such an arrangement. The space E D contains liquid kept at constant pressure by a piston, F. B D contains an inert insoluble gas. B C is a membrane impermeable to this gas, but permeable to the vapor of the liquid used. A B contains

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* Phil. Mag., (5) XII. 32 (1881).

this vapor alone. A change of temperature will change the vapor pressure in A B without changing the total pressure on the liquid, which is always equal to the outside pressure on F. We may simplify this arrangement by making the layer of inert gas so thin that it may be regarded together with the membrane B C merely as a single membrane, which is impermeable to the liquid but permeable to the vapor. In Figure 2 it is represented by the dotted line B. The spaces B C and A B are filled with liquid and vapor respectively, and the pistons A and C can be moved up and down so as to distribute the substance between the liquid and gaseous phases as desired. The whole is removed from the influence of gravity. Let us start with one gram-molecule of the substance, all in the liquid state, and pass through the following reversible cycle, during which the pressure, P, upon the piston, C, remains constant, while the pressure upon A is always kept equal to the vapor pressure. At first the piston A is at B; the space B C has the volume v_2. (1) The temperature is raised from T to $T + d\,T$, the pressure on A being raised at the same time from p, the original vapor pressure, to $p + d\,p$, so that none of the liquid evaporates. The piston C moves down on account of the expansion, $d\,v_2$, of the liquid. (2) All the liquid is evaporated at temperature $T + d\,T$, C moving to B, and A moving up to furnish the volume. v_1. (3) The temperature is again brought to T; the pressure on A to p. A moves down on account of the contraction $d\,v_1$. (4) All the vapor is condensed and the original condition is restored. The amounts of work done by the system in the several steps are : —

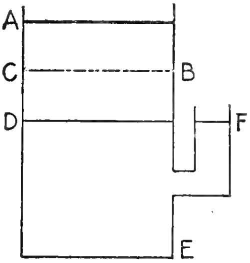

FIGURE 1.

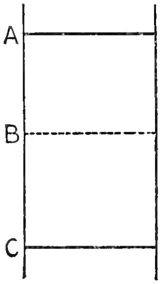

FIGURE 2.

$$W_1 = P\,d\,v_2,$$
$$W_2 = -\,P\,(v_2 + d\,v_2) + (p + d\,p)\,(v_1 + d\,v_1),$$
$$W_3 = -\,p\,d\,v_1,$$
$$W_4 = P\,v_2 - p\,v_1.$$

The total amount of work gained, the sum of these terms, is equal to the total amount of heat transformed into work, that is,

$$W_1 + W_2 + W_3 + W_4 = d\,Q = \frac{Q}{T}\,d\,T,$$

from equation (3). Adding the terms we obtain,

$$v_1\,d\,p = \frac{Q}{T}\,d\,T,$$

or writing so as to express the constancy of P,

$$\left(\frac{\partial\,p}{\partial\,T}\right)_P = \frac{Q}{v_1\,T}. \tag{7}$$

This important result may be derived directly from equations (4) and (6) and for solids as well as liquids. Since the vapor pressure is a function of the temperature, T, and the pressure on the surface, P, we may write

$$d\,p = \left(\frac{\partial\,p}{\partial\,T}\right)_P d\,T + \left(\frac{\partial\,p}{\partial\,P}\right)_T d\,P.$$

Now, in general, when only a pure substance and its vapor are present, the change in pressure on the surface of the substance is merely the change in vapor pressure, that is,

$$d\,P = d\,p.$$

Moreover, $\left(\frac{\partial\,p}{\partial\,P}\right)_T = \frac{v_2}{v_1}$, from equation (6), therefore,

$$d\,p = \left(\frac{\partial\,p}{\partial\,T}\right)_P d\,T + \frac{v_2}{v_1}\,d\,p, \text{ or}$$

$$d\,p\left(1 - \frac{v_2}{v_1}\right) = \left(\frac{\partial\,p}{\partial\,T}\right)_P d\,T, \text{ or } \frac{d\,p}{d\,T}\left(1 - \frac{v_2}{v_1}\right) = \left(\frac{\partial\,p}{\partial\,T}\right)_P.$$

Substituting for $\frac{d\,p}{d\,T}$ from equation (4),

$$\frac{Q}{T\,(v_1 - v_2)}\left(1 - \frac{v_2}{v_1}\right) = \left(\frac{\partial\,p}{\partial\,T}\right)_P, \text{ or } \left(\frac{\partial\,p}{\partial\,T}\right)_P = \frac{Q}{v_1\,T},$$

which is equation (7). We have in this equation a marked simplification of the Clausius formula with no loss of exactness. We could now, by making the single assumption that the vapor obeys the gas law, throw equation (7) into the form analogous to (5), namely,

$$\left(\frac{\partial \ln p}{\partial\,T}\right)_P = \frac{Q}{R\,T^2}.$$

Instead of using this equation we may introduce here a quantity with the aid of which it is possible to substitute for approximate equations of the type of (7) other entirely exact equations of the same form. This quantity is one whose utility I have shown in a recent paper.* It may be well to repeat and amplify the definition there given.

II.

Fugacity.

If any phase containing a given molecular species is brought in contact with any other phase not containing that species, a certain quantity will pass from the first phase to the second. Every molecular species may be considered, therefore, to have a tendency to escape from the phase in which it is. In order to express this tendency quantitatively for any particular state, an infinite number of quantities could be used, such, for example, as the thermodynamic potential of the species, its vapor pressure, its solubility in water, etc. The quantity which we shall choose is one which seems at first sight more abstruse than any of these, but is in fact simpler, more general, and easier to manipulate. It will be called the fugacity,† represented by the symbol ψ and defined by the following conditions: —

1. The fugacity of a molecular species is the same in two phases when these phases are in equilibrium as regards the distribution of that species.

2. The fugacity of a gas approaches the gas pressure as a limiting value if the gas is indefinitely rarefied. In other words, the escaping tendency of a perfect gas is equal to its gas pressure.

That these two conditions are sufficient to define a property of every substance which is not a mathematical, fictitious quantity, but a real physical quantity, capable of experimental determination in every case, must now be shown. It is obvious from the above conditions that in any case where our present methods of measurement are unable to show a deviation of the vapor of a substance from the gas law then the vapor pressure is the nearest approximation to the fugacity. In all cases the vapor pressure is an approximation to the fugacity, the approximation being nearer the nearer the vapor is to a perfect gas. When the

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* Proc. Amer. Acad., XXXVI. 145 (1900); Zeit. Phys. Chem., XXXV. 343 (1900).

† In the earlier paper this quantity was called the escaping tendency and represented by the same symbol. For the sake of brevity I have chosen to substitute the word "fugacity" for "escaping tendency" without the slightest change in the meaning of the function.

behavior of the vapor deviates perceptibly from that of the perfect gas the exact value of the fugacity may be found as follows: —

From the four laws stated in the introduction it is easy to derive the following, which is a rigorous statement of Henry's law, namely: The coefficient of distribution between a gas and its solution at constant temperature approaches a constant with increasing dilution. This constant will be designated by ρ. At infinite dilution,

$$\frac{p}{\Pi} = \rho,$$

where p is the gas pressure and Π the osmotic pressure in solution. Now p, at infinite dilution, is equal to the fugacity of the substance in the gaseous phase, and also in the solution, since the two phases are in equilibrium. Therefore,

$$\psi = \rho \Pi. \tag{8}$$

That is, the fugacity of the solute in an ideal solution is equal to its osmotic pressure multiplied by ρ. If now it is desired to find the fugacity of any molecular species X in any given phase, that phase may be brought in contact with a chosen solvent and the osmotic pressure Π_1 of the saturated solution determined. Then by diluting this solution in contact with vapor of X the limit ρ_1 of the distribution ratio may be found and so the product $\rho_1 \Pi_1$. So for another solvent we may find the product $\rho_2 \Pi_2$; for a third, $\rho_3 \Pi_3$, etc. These will all be equal except in as far as the saturated solutions deviate from the ideal solution. Practically, the product will be the same for all solvents in which X is only slightly soluble and will be the fugacity of X. Theoretically, the exact value of the fugacity is the limit approached by the product, $\rho \Pi$, as solvents are successively chosen in which X is less and less soluble.

We see, therefore, that fugacity is a real physical quantity capable in all cases of experimental determination. A complete appreciation of the meaning of this quantity is essential for the understanding of the following pages. In order, however, not to distract attention further from our main object, a further discussion of fugacity will be postponed to the last section of this paper, in which another independent method for the determination of ψ will be offered, using only such quantities as have already been determined in many cases.

The great utility of this new quantity will be shown to lie in the fact that the approximate equations containing the vapor pressure and developed rigorously except for the assumption that the vapor pressure obeys the gas law, may be replaced by exact equations of the same form or of equal simplicity containing the fugacity instead of the vapor pressure. Let us proceed to the determination of the laws according to which fugacity changes with changes in the variables upon which the condition of a substance depends, considering in the present paper only those systems which are composed of a single chemically simple substance.

III.

INFLUENCE OF TEMPERATURE AND PRESSURE ON THE FUGACITY.

Let us consider two phases of a substance at the same temperature and pressure, but not necessarily in equilibrium with each other. A solvent may be chosen in which both phases are soluble without molecular change, and to so slight an extent that the saturated solutions may be regarded as infinitely dilute. In such a case the solubility of each phase is governed by the following equation, which may be obtained directly from equations (2) and (3),

$$\left(\frac{\partial \ln \Pi}{\partial T}\right)_P = \frac{Q}{R\,T^2},$$

in which Π is the osmotic pressure of the saturated solution and Q the reversible heat of solution (that is, inclusive of the osmotic work). We may write for the two phases,

$$\left(\frac{\partial \ln \Pi_1}{\partial T}\right)_P = \frac{Q_1}{R\,T^2} \text{ and } \left(\frac{\partial \ln \Pi_2}{\partial T}\right)_P = \frac{Q_2}{R\,T^2}, \text{ or combining,}$$

$$\left(\frac{\partial \ln \dfrac{\Pi_1}{\Pi_2}}{\partial T}\right)_P = \frac{Q_1 - Q_2}{R\,T^2}. \qquad (9)$$

$Q_1 - Q_2$ may be conveniently replaced in the following way. Let one gram-molecule of the first phase be dissolved in the solvent, this solution then diluted or concentrated to the osmotic pressure Π_2, and then the gram-molecule removed as the second phase. If these three steps be done reversibly the heat absorbed in each will be respectively

$$Q_1, \quad R\,T \ln \frac{\Pi_1}{\Pi_2}, \quad -Q_2.$$

The total heat change is a function only of the conditions of the two phases, not of the path by which one passes into the other, and may be designated by $Q_{1,2}$, thus,

$$Q_{1,2} = Q_1 + R\,T \ln \frac{\Pi_1}{\Pi_2} - Q_2, \text{ or } Q_1 - Q_2 = Q_{1,2} - R\,T \ln \frac{\Pi_1}{\Pi_2}.$$

We may therefore write equation (9) as

$$\left(\frac{\partial \ln \dfrac{\Pi_1}{\Pi_2}}{\partial\,T}\right)_p = \frac{Q_{1,2}}{R\,T^2} - \frac{\ln \dfrac{\Pi_1}{\Pi_2}}{T}.$$

Since we are dealing with infinitely dilute solutions in the same solvent,

$\psi_1 = \rho\,\Pi_1$ and $\psi_2 = \rho\,\Pi_2$, therefore

$\dfrac{\psi_1}{\psi_2} = \dfrac{\Pi_1}{\Pi_2}$, and the above equation becomes

$$\left(\frac{\partial \ln \dfrac{\psi_1}{\psi_2}}{\partial\,T}\right)_p = \frac{Q_{1,2}}{R\,T^2} - \frac{\ln \dfrac{\psi_1}{\psi_2}}{T}. \qquad (10)$$

This is the desired equation connecting temperature and escaping tendency. Its form can be simplified by a slight rearrangement.

Considering the quantity $T \ln \dfrac{\psi_1}{\psi_2}$ we notice that

$$\frac{\partial\,T \ln \dfrac{\psi_1}{\psi_2}}{\partial\,T} = T\,\frac{\partial \ln \dfrac{\psi_1}{\psi_2}}{\partial\,T} + \ln \frac{\psi_1}{\psi_2}, \text{ or } \frac{\partial \ln \dfrac{\psi_1}{\psi_2}}{\partial\,T} = \frac{1}{T}\,\frac{\partial\,T \ln \dfrac{\psi_1}{\psi_2}}{\partial\,T} - \frac{\ln \dfrac{\psi_1}{\psi_2}}{T}.$$

Combining this equation with (10) gives

$$\left(\frac{\partial\,T \ln \dfrac{\psi_1}{\psi_2}}{\partial\,T}\right)_p = \frac{Q_{1,2}}{R\,T}. \qquad (11)$$

Leaving in this form for the present the equation connecting temperature and fugacity at constant pressure, let us determine the influence of pressure on the fugacity at constant temperature. I have already discussed this question in a previous paper,* but instead of using the general equation there derived it has seemed preferable to base all the reasoning of this paper directly upon the four laws stated in the introduction.

Let us consider any simple substance and a solvent, so arranged † that the pressure upon the substance in question may be altered without

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* Loc. cit.

† Several such arrangements are described in the paper just mentioned.

changing the pressure on the solvent and without preventing the substance from passing freely into or out of the solvent. The osmotic pressure of the saturated solution depends upon the pressure on the substance. If the latter is represented by P and the former by Π, then for $P + dP$ the osmotic pressure will be $\Pi + d\Pi$. We may moreover represent the molecular volume of the substance by v at pressure P, by $v - dv$ at pressure $P + dP$; the molecular volume in the solution by v' at osmotic pressure Π, by $v' - dv'$ at $\Pi + d\Pi$. If a gram-molecule of the substance at pressure P is (1) dissolved against the osmotic pressure Π, (2) its solution concentrated to $\Pi + d\Pi$, (3) removed from solution against the pressure $P + dP$ and (4) allowed to expand from $P + dP$ to P, an isothermal cycle is formed, and if each step is made reversible the total work of the cycle is zero. The work obtained in the several steps may be represented by W_1, W_2, etc.

$$\begin{aligned}
W_1 &= \Pi v' - P v, \\
W_2 &= - \Pi\, d v', \\
W_3 &= (P + dP)(v - dv) - (\Pi + d\Pi)(v' - dv'), \\
W_4 &= P\, dv.
\end{aligned}$$

Writing the sum equal to zero,

$$v\, dP - v'\, d\Pi = 0,$$

or expressing in the equation the constancy of T,

$$\left(\frac{\partial \Pi}{\partial P}\right)_T = \frac{v}{v'}. \tag{12}$$

This is an exact general equation connecting the osmotic pressure of a saturated solution and the pressure upon the pure solute. It is entirely analogous to equation (6). Since we may choose a solvent in which the solute is as slightly soluble as desired we will choose one in which the solution may be regarded as infinitely dilute. Then,

$$\Pi = \frac{R\,T}{v'},$$

from equation (2). Combining this equation with (12) we obtain

$$\left(\frac{\partial \ln \Pi}{\partial P}\right)_T = \frac{v}{R\,T}. \tag{13}$$

From equation (8), $\psi = \rho\,\Pi$. Therefore $\ln \psi = \ln \Pi + \ln \rho$, and

$$\left(\frac{\partial \ln \psi}{\partial P}\right)_T = \left(\frac{\partial \ln \Pi}{\partial P}\right)_T,$$

since ρ is constant at constant temperature. Hence equation (13) becomes

$$\left(\frac{\partial \ln \psi}{\partial P}\right)_T = \frac{v}{R\,T}. \tag{14}$$

Subtracting two such equations we obtain an equation for two phases,

$$\left[\frac{\partial \ln \frac{\psi_1}{\psi_2}}{\partial P}\right]_T = \frac{v_1 - v_2}{R\,T}. \tag{15}$$

IV.

The General Law of Fugacity.

Equations (11) and (15) show a similarity which may be made more striking by a few simple transformations. In equation (11) $Q_{1,2}$, the heat absorbed in any reversible transformation of the substance from the first to the second state is equal to the difference in entropy between the second state and the first, multiplied by the absolute temperature; that is, $\frac{Q_{1,2}}{T} = -(S_1 - S_2)$,

where S_1 and S_2 represent the entropy of the first and second states respectively.

Substituting in equation (11) and transposing the constant R, we obtain,

$$\left[\frac{\partial R\,T \ln \frac{\psi_1}{\psi_2}}{\partial T}\right]_P = -(S_1 - S_2). \tag{16}$$

In equation (15) $R\,T$ is constant, and may be transposed, bringing the equation into the form,

$$\left[\frac{\partial R\,T \ln \frac{\psi_1}{\psi_2}}{\partial P}\right]_T = v_1 - v_2. \tag{17}$$

The symmetry of equations (16) and (17) with regard to the quantities T and $-S$ on the one hand, and P and v on the other hand, is perfect. This similarity is peculiarly interesting in the light of the brilliant theory of Helm, according to which two quantities are fundamentally connected with each kind of energy, the one its intensity, the other its capacity.* Thus, for example, pressure, surface tension, electrical, potential, and temperature are considered to be the intensities concerned in energy changes in which the corresponding capacities are respectively volume, surface, quantity of electricity, and entropy. We may denote in general the intensity of any energy by I and its capacity by H. If we substitute I and H for T and S in equation (16) and for P and v in (17), the equations become identical except for the minus sign in (16). We are thus led to suspect the existence of a general equation of the form

$$\left\{\frac{\partial\, R\, T \ln \frac{\psi_1}{\psi_2}}{\partial\, I}\right\}_{I',I''\ldots} = H_1 - H_2, \tag{18}$$

and further, of the equation for a single phase,

$$\left(\frac{\partial\, R\, T \ln \psi}{\partial\, I}\right)_{I',I''\ldots} = H. \tag{19}$$

This equation would mean that if the fugacity is a function of a number of energy intensities, I, I', I'', etc., the rate of change in the quantity $R\,T \ln \psi$, with a change in one of the intensities alone, is equal to the corresponding capacity. In other words, this equation, if true, expresses a law so far reaching that it embraces every possibility of the change of state of any simple substance under all conceivable conditions. Let us examine the validity of this equation for all cases in which the escaping tendency can be shown to be influenced by the intensities of various energies.

The influence of pressure is given in equation (14), which may be written,

$$\left(\frac{\partial\, R\, T \ln \psi}{\partial\, P}\right)_T = v, \tag{20}$$

and therefore conforms to equation (19).

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* These quantities have been hitherto called the factors of energy, and their product has been written equal to the quantity of energy concerned. I believe that this part of the theory is absolutely unjustified by the facts, and that it has been the chief cause of the hostility which has been shown to a conception which is valuable in research and has proved a veritable boon in the pedagogical treatment of energetics. I hope soon in another paper to discuss this whole question, especially in the light of the results of the present paper. Meanwhile we may speak of intensity and capacity as the dimensions of energy, signifying that their product has the dimensions of energy.

The influence of temperature is expressed for two states simultaneously in equation (16), which conforms to equation (18) except for the minus sign. This slight difference might be explained away, but a much weightier difficulty confronts us when we attempt to split equation (16) into two equations, each expressing the influence of temperature upon the fugacity for a single phase, in the form,

$$\left(\frac{\partial\, R\, T \ln \psi}{\partial\, T}\right)_P = -S.$$

This equation is in general not true, notwithstanding the fact that we may choose arbitrarily the zero of entropy. If for each temperature this zero could be chosen arbitrarily it could be so chosen that the equation would be true, but as a matter of fact the entropy is in all cases a determinate function of the temperature, and the zero chosen for one temperature must be retained for all. We must conclude, therefore, either that the general equation (19) is false, or that entropy is not the capacity dimension of heat. To make the latter conclusion would appear too arbitrary were it not that other considerations lead also to the suspicion that entropy has been too hastily chosen as the capacity in question. In fact, the equation, $d\,Q = T\,d\,S$, for the heat absorbed in a reversible process, corresponding to the general equation for change of energy, $d\,E = I\,d\,H$, is the only argument for the consideration of entropy as the capacity dimension of heat. This argument would apply equally well to any other quantity, h, such that $d\,Q = \pm\, T\,d\,h$; in other words, such that $d\,h = \pm\, d\,S$. It is interesting, therefore, to determine whether there is, in fact, a quantity which fulfils this condition and also the condition

$$\left(\frac{\partial\, R\, T \ln \psi}{\partial\, T}\right)_P = h. \tag{21}$$

If a simple function can be found which satisfies these two requirements it may, I think, be accepted, at least provisionally, as the true capacity of heat energy.

The entropy of every body is a very complex function of its other variables, and even the entropy of a perfect gas is represented by the complicated equation,*

$$S = S_o + C_P \ln \frac{T}{T_o} - R \ln \frac{P}{P_o}.$$

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* See Clausius, Wärmetheorie, I. p. 214, third edition.

The value of h for a perfect gas may be found from the second of the above conditions, equation (21). For a perfect gas, according to the definition of fugacity,

$$\psi = P, \text{ and therefore}$$

$$h = \left(\frac{\partial R\, T \ln \psi}{\partial T}\right)_P = \left(\frac{\partial R\, T \ln P}{\partial T}\right)_P = R \ln P. \qquad (22)$$

We see, therefore, that the value of h which satisfies the condition of equation (21) is expressed by a far simpler function than entropy is. Let us see whether this value for the perfect gas is consistent with the other condition that,

$$d\, h = \pm\, d\, S.$$

For a perfect gas the following equations for isothermal change are familiar:

$$d\, S = \frac{d\, Q}{T} = \frac{P\, d\, v}{T} = -\frac{v\, d\, P}{T} = -\frac{R\, d\, P}{P} = -R\, d \ln P,$$

and from equation (22),

$$d\, h = R\, d \ln P, \text{ hence, for constant temperature,}$$

$$d\, h = -\, d\, S, \qquad (23)$$

and the condition is satisfied. The value $R \ln P$ satisfies both the above conditions for h in the case of a single state, the perfect gas. Moreover, every substance is capable of being brought into the state of a perfect gas isothermally by evaporation and indefinite expansion. Consequently it is easy to show that for any state of a substance either of the two conditions will define a value of h which is consistent with the other condition. Thus by the first condition, expressed now by equation (23), the difference in value of h between two states of a substance is equal to the difference in entropy and opposite in sign, that is,

$$h_1 - h_2 = S_2 - S_1.$$

If we choose as the second state the vapor of the substance at such a low pressure, P_2, that the vapor may be regarded as a perfect gas, $h_2 = R \ln P_2$, from equation (22), and the last two equations give,

$$h_1 = S_2 - S_1 + R \ln P_2, \qquad (24)$$

in which S_2 represents the entropy of the vapor at pressure P_2. This equation furnishes a complete definition of the value of h for any state. Let us see whether this value satisfies the other condition of equation (21).

Equation (16), namely,

$$\left[\frac{\partial\, R\, T \ln \frac{\psi_1}{\psi_2}}{\partial\, T}\right]_P = S_2 - S_1,$$

holds true for the two states which we have just considered, one of which is the vapor in the state of a perfect gas at the low pressure P_2. By the aid of equation (24) we may therefore write

$$\left[\frac{\partial\, R\, T \ln \frac{\psi_1}{\psi_2}}{\partial\, T}\right]_P = h_1 - R \ln P_2.$$

According to (22)

$$\left(\frac{\partial\, R\, T \ln \psi_2}{\partial\, T}\right)_P = R \ln P_2$$

and the last two equations give by addition

$$\left(\frac{\partial\, R\, T \ln \psi_1}{\partial\, T}\right)_P = h_1,$$

which is equation (21).

I think, therefore, that we are justified in considering h the true capacity dimension of heat, and in considering equation (21) the special form of equation (19) applied to heat energy. The replacement of entropy in general energy equations by the quantity h will have a further advantage on account of the much greater simplicity of the latter, the approximate value of which may be in all cases very easily determined by assuming that the vapor of the substance in question may be regarded as a perfect gas, in which case equation (24) evidently becomes

$$h = \frac{Q}{T} + R \ln p. \tag{25}$$

where Q is the total heat absorbed in the evaporation of one gram-molecule and p is the vapor pressure.*

We have now obtained equations of the form of (19) for two of the

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* This approximate equation is a special form of a general and rigorously exact equation,

$$h = \frac{Q'}{T} + R \ln \psi, \tag{25a}$$

in which ψ is the escaping tendency of the substance and Q' is the heat absorbed when one gram-molecule is allowed to evaporate irreversibly against an infinitesimal vapor pressure. Since this equation will not be used in this paper its demonstration may be postponed.

most important kinds of energy. The fugacity is also known to be a function of a third energy-intensity, namely, surface tension. Let us consider a drop of liquid containing n gram-molecules with a surface σ and a surface tension t. The change in surface of the drop with a change in its content expressed in gram-molecules, that is, $\frac{d\,\sigma}{d\,n}$, has been called the molecular surface, and we may designate it by s. If the quantity $d\,n$ is taken from the drop and added to a large mass of the liquid the process is capable of yielding work. The amount has, I think, always hitherto been written equal to $t\,d\,\sigma$, the change in surface energy. This is not strictly true. The molecular volume in the drop is not exactly equal to but always slightly less than the molecular volume in the large mass. There is therefore always a small amount of work done against the atmosphere, and the total work capable of being done by the transference of $d\,n$ gram-molecules is equal to $t\,d\,\sigma + P\,(d\,v_0 - d\,v)$, where $d\,v_0$ represents the increase in the volume of the large mass, $d\,v$ the decrease in the volume of the drop. If the transfer be made reversibly in any way the total amount of work obtained must be equal to the above. The transfer may be actually carried out reversibly as follows: Let a solvent be chosen in which the liquid in question is so slightly soluble that the solution may be regarded as an ideal one. The drop and the large mass of liquid will be in equilibrium * with the solution at two different osmotic pressures, Π and Π_o, respectively. We may now take the following steps reversibly: (1) $d\,n$ gram-molecules of the drop dissolve into its saturated solution, (2) the same amount is diluted to the osmotic pressure Π_o, and (3) passes out of solution into the large mass. The three steps yield the following amounts of work, in which $d\,v_0'$ and $d\,v'$ represent the volumes occupied by the amount $d\,n$ in solution at the osmotic pressures Π_0 and Π, respectively.

$$W_1 = \Pi\,d\,v' - P\,d\,v,$$

$$W_2 = d\,n\,R\,T \ln \frac{\Pi}{\Pi_o},$$

$$W_3 = P\,d\,v_0 - \Pi_0\,d\,v_0'.$$

The sum of these terms, written equal to the amount of work given above, gives

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* In order not to affect the surface tension of the drop, it may be separated from the solvent by its own vapor and thus pass into solution through the vapor phase.

$$P\,(d\,v_0 - d\,v) + \Pi\,d\,v' - \Pi_0\,d\,v_0' + d\,n\,R\,T\ln\frac{\Pi}{\Pi_0} = t\,d\,\sigma + P\,(d\,v_0 - d\,v).$$

Now from equation (2), $\Pi_0\,d\,v_0' = \Pi\,d\,v'$,

and, as on page 55, $\dfrac{\Pi}{\Pi_0} = \dfrac{\psi}{\psi_0}$.

Therefore

$$R\,T\ln\frac{\psi}{\psi_0} = t\frac{d\,\sigma}{d\,n} = t\,s. \tag{26}$$

This is the general equation connecting fugacity and surface tension at constant temperature and pressure. If t is variable we may differentiate, ψ_0 and s being constant, obtaining

$$d\,R\,T\ln\psi = s\,d\,t,$$

or expressing the constancy of T and P,

$$\left(\frac{\partial\,R\,T\ln\psi}{\partial\,t}\right)_{T,\,P,\ldots} = s. \tag{27}$$

This equation completely confirms the validity of equation (19) as applied to surface energy and corresponds to equations (20) and (21).

An important form of energy which we have not yet discussed is electrical energy, whose dimensions are potential, and quantity of electricity. If these be represented by π and ϵ, respectively, in any case where the fugacity is influenced by the electrical potential, we should have the equation,

$$\left(\frac{\partial\,R\,T\ln\psi}{\partial\,\pi}\right)_{T,\,P,\ldots} = \epsilon. \tag{28}$$

There are in fact a number of cases in which the potential may be shown to have an effect upon the escaping tendency, the most important being that in which the potential influences the fugacity of the ions. The following equation has been amply proved experimentally, and thermodynamically is shown to be rigorously exact on the assumption that the ions form an ideal solution.

$$\epsilon\,\pi = R\,T\ln\Pi + K,$$

in which π is the potential at which equilibrium is established between an electrode and its ions at the osmotic pressure Π, if ϵ is the charge of one gram-ion and K is at constant temperature and pressure a characteristic constant of the electrode. In other words, Π is the osmotic pressure of the ions which will be in equilibrium with the electrode when the potential π is established. Since we are discussing an ideal solution this osmotic pressure is proportional to the fugacity of the ions. That is, from equation (8), $\psi = \rho\,\Pi$, and

$$\epsilon\,\pi = R\,T \ln \psi - R\,T \ln \rho + K.$$

Differentiating at constant temperature and pressure we obtain the equation,

$$\left(\frac{\partial\, R\,T \ln \psi}{\partial\, \pi}\right)_{T, P, \ldots} = \epsilon,$$

which is equation (28).

Equations (20), (21), (27), and (28) comprise all cases in which fugacity is known to depend upon the intensity of any form of energy. The identity of these equations with equation (19) gives the highest degree of probability to the supposition that the latter equation expresses an exact law of nature and one possessing such universality as few others possess. For this equation expresses the condition for any conceivable change of state of any simple substance. Moreover, it will be shown in the paper which is to follow this, that equation (19) not only applies to chemically simple substances but, with a slight generalization in the meaning of the symbols which it contains, applies to mixtures as well, and further that it applies not merely to physical processes but also to all chemical processes,* so that this law becomes the general law of physico-chemical change.

Finally, it will be shown that the adoption of the two functions ψ and $R\,T \ln \psi$, which possess such peculiar importance, will remove many obstacles in the search for the fundamental principles of energetics, in which already so much progress has been made by the work of Helm, of Ostwald, and of other investigators. I shall therefore offer in the last section of this paper a further explanation of fugacity as a tangible, physical quantity.

V.

The Fugacity of Imperfect Gases.

The vapor pressure is determined for many substances and capable of direct or indirect determination for all. Moreover the fugacity of a sub-

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* In the further extension of this theory, analogy will be seen between the conception of fugacity and the driving tendency of chemical reaction as used by T. W. Richards (These Proceedings, **35**, 471; Jour. Phys. Chem., **4**, 385 (1900)). It is a pleasure to recall how much I owe to the many conversations full of assistance and encouragement which I had with Professor Richards during the early development of the theory of fugacity or escaping tendency.

stance is the same as that of the vapor in equilibrium with it. It is important therefore to know what relation exists in general between th fugacity of any gas or vapor and its pressure.

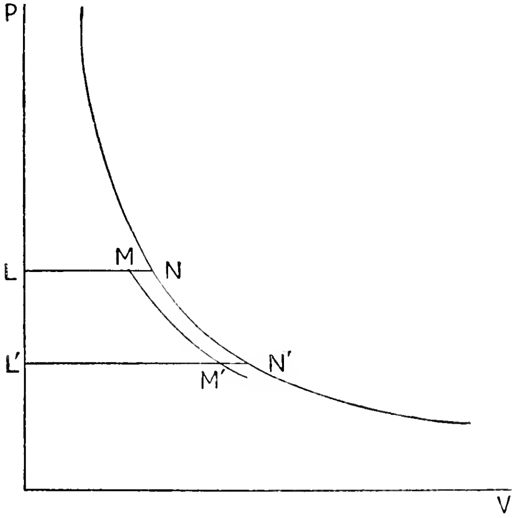

FIGURE 3.

If a section of the isothermal of any vapor is plotted on the P V diagram (Figure 3) we obtain a curve such as M M′, which, according the third law stated in the introduction, approaches asymptotically th hyperbola N N′, whose equation is,

$$P v = R T.$$

Let us determine the value of ψ for any point M of the curve. The variation of ψ with P is given by equation (20), which may be writt for constant temperature,

$$d R T \ln \psi = v \, d P.$$

Between the two points M and M′ we find by integration

$$R T \ln \frac{\psi}{\psi'} = \int_{M}^{M'} v \, d P.$$

Now if the lines of constant pressure L M N and L′ M′ N′ are drawn, $\int_{M}^{M'} v \, d P$ is equal to the area M M′ L′ L, and this is equal to the ar

L N N′ L′ minus the area M M′ N′ N. The former area is equal to $R\,T \ln \frac{P}{P'}$, and if the latter be designated by A we have the equation,

$$R\,T \ln \frac{\psi}{\psi'} = R\,T \ln \frac{P}{P'} - A. \tag{29}$$

Now if the point M′ is moved in the direction of greater volume, equation (29) holds true continuously, and therefore is true if M′ is taken at infinite volume. But at infinite volume

$$\psi' = P',$$

and therefore

$$R\,T \ln \psi = R\,T \ln P - A_{\infty}, \tag{30}$$

if A_{∞} represents the total area bounded by the line M N and the curves M M′ and N N′, each produced to infinity. This equation may be written,

$$R\,T \ln \frac{\psi}{P} = - A_{\infty}, \text{ or } \ln \frac{\psi}{P} = - \frac{A_{\infty}}{R\,T}, \text{ or } \psi = P e^{\frac{-A_{\infty}}{RT}}, \tag{31 a}$$

where e is the base of natural logarithms.

The deviation of the fugacity from the gas pressure is, therefore, dependent upon the area A_{∞}. The case that has been chosen in which the curve M M′ lies within N N′ is of course the common one. For gases of the opposite type, hydrogen and helium, the formulae will be,

$$R\,T \ln \frac{\psi}{P} = + A_{\infty} \text{ and } \psi = P e^{\frac{A_{\infty}}{RT}} \tag{31 b}$$

We see at once that for all known gases and vapors except hydrogen and helium the escaping tendency is less than the gas pressure; for these two, greater. The determination of the value of the fugacity at any pressure involves the estimation of the area A_{∞}. This must be done by integrating the most exact empirical equation of the isotherm of a gas between the pressure in question and the pressure zero. This method has the disadvantage of all extrapolation, but the value thus obtained may be checked by using a second empirical equation of another form and recalculating A_{∞}. If the two results coincide the value obtained will in all probability be very near the true value of A_{∞}.

In conclusion it may be remarked that equation (29) applies to the isothermal of all substances, not merely to gases, and can be frequently of use. For example, if it is possible to pass continuously from vapor to liquid along an isothermal, it is evident that in passing from a saturated vapor to its liquid,

$$\psi = \psi', \text{ and } P = P'$$

in equation (29). Therefore the total area A reckoned algebraically must equal zero. That is, the two areas on the two sides of the line of constant pressure P must be equal. This is the well-known principle of Maxwell.

SUMMARY.

(1) The equation of Clausius for vapor pressure is simplified.

(2) The meaning and utility of a new quantity, the escaping tendency, or fugacity, are explained.

(3) The influence of temperature and pressure upon fugacity is expressed in simple equations.

(4) A simple, general equation, which embraces every possibility of the change of state of any simple substance, is proposed.

(5) This equation rests upon the conception of the intensity and capacity dimensions of energy.

(6) This equation is verified as applied to the influence of pressure on fugacity.

(7) This equation is verified as applied to the influence of temperature, if a new quantity, instead of entropy, is regarded as the capacity dimension of heat.

(8) This equation is verified for the influence of surface tension.

(9) This equation is verified for the influence of electrical potential.

(10) A method is offered by which the fugacity may be found from the vapor pressure.

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THE VISIBLE RADIATION FROM CARBON.

BY EDWARD L. NICHOLS.

INVESTIGATIONS ON LIGHT AND HEAT, MADE AND PUBLISHED WHOLLY OR IN PART WITH APPROPRIATIONS FROM THE RUMFORD FUND.

THE VISIBLE RADIATION FROM CARBON.[*]

BY EDWARD L. NICHOLS.

Presented May 8, 1901. Received May 15, 1901.

THE law of radiation has for a long time been considered by physicists as a subject of high interest, and numerous investigations looking to the establishment of a general relation between radiation and temperature have been made both from the theoretical and the experimental standpoint. The earliest attempts to determine incandescence in its relation to temperature were made with platinum. Draper[†] in 1847 made observations upon a wire of that metal heated by an electric current. The temperatures were determined from the expansion of the wire. Zöllner[‡] in 1859 compared the light emitted by incandescent platinum with the heat evolved. E. Becquerel,[§] who made an extensive study of visible radiation from various solids at high temperatures, used thermo-elements of platinum and palladium, calibrated by reference to melting points with the air thermometer. A partial separation of the rays was effected by means of colored screens.

Becquerel found that opaque bodies, such as lime, magnesia, platinum, and carbon, at the same temperature had very nearly equal emissive powers, a conclusion vigorously contested by his contemporaries, but explained, in the light of later work, by the fact that the glowing bodies were enclosed in a long earthen tube. The conditions for ideal blackness were thus approximately fulfilled. He likewise made photometric observations upon wires electrically heated and found the light to increase much more rapidly than the emitted heat.

Although some of Becquerel's results were at fault, particularly his estimation of temperature above the melting point of gold, his work is especially noteworthy in that he employed many of the methods to which,

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[*] An investigation carried on in part by means of an appropriation from the Rumford Fund. Read at the meeting of the American Association for the Advancement of Science in New York, June 27, 1900.

[†] Draper, Philosophical Magazine, XXX. 345 (1847).

[‡] Zöllner, Photometrische Untersuchungen (1859).

[§] Becquerel, Annales de Chimie et de Physique, (3), LXVIII. 47 (1863).

in the hands of later investigators, our knowledge of the laws of incandescence is due. He established the direct proportionality of the logarithm of the intensity of radiation to the temperature and pointed out the possibility of optical pyrometry.

In 1878 Crova[*] used the Glan spectrophotometer in the comparison of various sources of light, such as candles, gas flames, the lime light, the arc light, and sunlight, and proposed an optical method for the measurement of temperatures.

In 1879[†] I published the results of a series of measurements made in this manner upon the visible radiation from platinum at various temperatures. At that time, the measurement of high temperatures by means of thermo-elements, of platinum and platinum-rhodium, or platinum-iridium, had not been developed, and the determination of the temperature from the change of resistance of the metal was, as has been previously pointed out by Siemens, a matter of great uncertainty on account of the varying performance of different samples of platinum. This difficulty, which was due to the impurities contained in the metal, has since been largely overcome, and platinum thermometry has, through the study of Callendar and others, been advanced to the position of an operation of precision, but at that time I was forced to content myself in the investigation just referred to with an expression of temperature of the glowing platinum in terms of its increase of length.

Work upon the incandescence of carbon was first taken up in a serious manner after the development of the incandescent lamp.

Schneebeli,[‡] in 1884, made some observations upon the total radiation and candle power of the Swan lamp. He made no estimation of temperatures.

In the same year Schumann[§] published his very complete spectrophotometric comparison of the various incandescent lamps in use in Germany. Lucas,[‖] in 1885, heated arc-light carbons *in vacuo*, estimated their temperature from the current employed, and measured the light given in carcels. I shall refer to his work in some detail later.

In 1887 H. F. Weber[¶] began his studies of the spectrum of the in-

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[*] Crova, Comptes Rendus, LVII. 497 (1878).

[†] Nichols, Ueber das von glühendem Platin ausgestrahlte Licht. Göttingen, 1879; also American Journal of Science, XVIII. 446 (1879).

[‡] Schneebeli, Wiedemann's Annalen, XXII. 430 (1884).

[§] Schumann, Elektrotechnische Zeitschrift, V. 220 (1884).

[‖] Lucas, Comptes Rendus, C. 1454 (1885).

[¶] Weber, Wiedemann's Annalen, XXXII. 256 (1887).

candescent lamp. He found that the first light to appear was not that of the region nearest the red end of the spectrum, but corresponded in wave length to the region of maximum luminosity, and that at these low temperatures the spectrum was devoid of color. Stenger * in the same year corroborated Weber's observations and offered what has since been received as the proper explanation of the phenomenon.

In 1889 I published in collaboration with W. S. Franklin † a series of spectrometric comparisons of incandescent lamps maintained at various degrees of brightness. No attempt was made to determine temperatures. In 1891 H. F. Weber ‡ read a paper at the Electrotechnical Congress in Frankfurt on the general theory of the glow-lamp. By means of numerous measurements through a wide range of incandescence made upon lamps with treated and untreated filaments, constants were established for his empirical formula for the relation of radiation and temperature.

The infra-red spectrum of carbon has, since the appearance of the incandescent lamp, likewise been subjected to measurement. Abney and Festing § in 1883 published curves for the distribution of energy in the spectrum of such lamps from measurements made with the thermopile. In 1894 I compared, with the help of the same instrument and a highly sensitive galvanometer, the infra-red spectra of lamps with black and gray filaments. ||

Of late years attention has been devoted especially to the problem of the law of radiation from an ideal black body, and various formulae have been proposed by means of which the rise of radiation of any single wave length upon the one hand, and of the total radiation on the other, may be expressed as a function of the temperature. Interesting as this phase of the problem is from the point of view of theoretical physics, it is perhaps even more important to know the relation between temperature and radiation for actual surfaces.

Apparatus and Outline of Method.

I propose in the present paper to describe an attempt to measure the temperature of carbon rods rendered incandescent by the passage of an

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* Stenger, Wiedemann's Annalen, XXXII. 271 (1887).

† Nichols and Franklin, Am. Jour. of Science, XXXVIII. 100 (1889).

‡ Weber, Bericht des internationalen Elektrotechniker-congresses zu Frankfurt am Main, p. 49 (1891); also Physical Review, II. 112.

§ Abney & Festing, Philosophical Magazine, (5) XVI. 224 (1883); also Proceedings of the Royal Society, XXXVII. 157 (1884).

|| Nichols, Physical Review, II 260 (1894).

electric current, and to make spectrophotometric comparisons of the visible radiation from their surfaces with the corresponding wave lengths in the spectrum of an acetylene flame.

The carbons used for this purpose were produced by the well-known process of squirting a semi-fluid carbonaceous paste through a cylindrical opening. They were straight cylindrical rods 10 cm. in length, and 2 mm. in diameter. Still larger rods would have been preferable, but I was unable to obtain any of greater diameter than the above that were capable of withstanding the temperatures to which it was necessary to heat them. The rods were mounted horizontally in a massive metal box 40 cm. in length, 20 cm. wide, and 20 cm. in height. This box, which was made especially for this investigation, had openings at the ends, through which, by means of air-tight plugs, the terminals of the carbon could be introduced. Through one of these plugs, likewise, the platinum and platinum-rhodium wires of the thermo-element, by means of which the temperature measurements were made, entered the box. In one of the vertical sides of the box was a row of five circular plate-glass windows, which could be removed for cleaning, through which the carbon could be seen and the spectrophotometric observations could be made. Other openings in the top of the box and through the opposite sides served to connect it with a mercury air pump of the Geissler type and for the introduction of manometers for the measurement of pressure. A vertical cross-section of this part of the apparatus is shown in Figure 1. Attempts

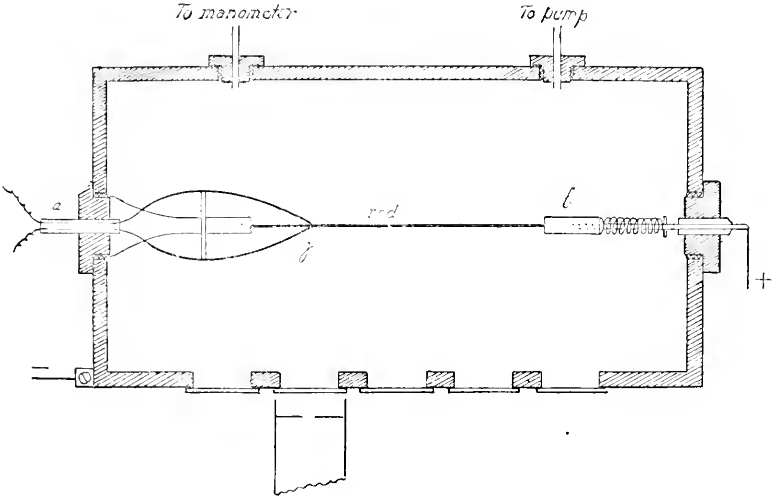

FIGURE 1.

to locate, by a variety of methods, the hot junction of the thermo-element, by means of which the temperature of the surface of the rods was to be measured, in such manner that it would assume the temperature of that surface, made it only too clear that herein lay one of the chief difficulties of the investigation. It was found that such a junction, however small its size, and however carefully it might be brought into contact with the surface of the rod, would not take even approximately the temperature of that surface; and recourse, after the failure of numerous other expedients, was had to the following plan, which although far from being free from objection, was found to be upon the whole the most reliable, and to give, when properly carried out, the most definite and satisfactory result.

By means of a drill made for the purpose from the smallest obtainable size of steel sewing-needle, a minute hole was bored radially at a point upon the surface of the rod lying within the field of view of the spectrophotometer. This hole had an approximate diameter of 0.03 cm. It extended to a depth equal to about one half the radius of the rod and was conical in form. Platinum and platinum-rhodium wires to be used for the thermo-element were drawn to a diameter of 0.016 cm., and their free ends having been laid together side by side, were fused in the flame of the oxyhydrogen blowpipe so as to form a junction. This junction, which after the action of the blowpipe took the shape of a small bead of the combined metals, was trimmed down into conical form, until it would just enter the hole in the side of the rod, care being taken that the entire junction was beneath the surface. The wires leading from this junction were next sealed into a glass tube of about 2 mm. bore, through the interior of which they were carried from end to end, care being taken that they should be nowhere in contact. They were held in place by fusing the glass around them at either end of the tube. This tube was inserted through an opening in the plug *a* (Figure 1) carrying one terminal of the rod, and there made air tight by means of cement. One end of the carbon rod was then inserted in a clamp attached to the inner face of the plug, and the wires at a distance of about 1 cm. from the junction were bent downward at right-angles, so as to bring the junction into position for insertion into the hole in the rod, and to hold it there when inserted by the slight but sufficient spring-action of the wires themselves. This arrangement of the junction and rod is indicated in Figure 2.

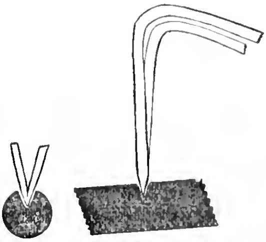

FIGURE 2.

The introduction of the thermo-element having been successfully carried out by the method just described, it was possible to insert the plug, carrying the rod and thermo-junction with it, into the end of the box and to secure it in place; after which the free terminal of the rod was introduced between the jaws of a strong clip attached to the opposite plug (*b*, Figure 1). This operation had to be performed through the open windows in the side of the box. These were then screwed rightly into place, and the box was ready for the exhaustion of the air.

This method of measuring the temperature of the surface, to be successful, involved the fulfilling of several rather difficult conditions and the application of an important correction. To bore into the material of a carbon rod carrying a current in the manner described, necessarily disturbs more or less the flow of the current; and the changes of resistance thus introduced are likely to bring about decided changes of temperature in that neighborhood. In some instances this became obvious when the rod was heated, the temperature being higher near the hole than elsewhere. Indeed, it was often possible to note this effect with the eye on account of the increased incandescence of the region in question. In all such cases the mounting was rejected. It was found possible, however, to so nearly compensate for this loss of carbon by the introduction of the platinum junction that no difference in the incandescence of the surface could be detected by the closest observation; and since differences of temperature which cannot be detected by the eye will be negligible in spectrophotometric work, this was taken as the criterion of a satisfactory mounting of the thermo-junction. Measurements were attempted only when this condition was fulfilled. It is likewise obvious that there is danger from the contact of the two wires of the thermo-junction with the sides of the hole in the rod. A branch circuit for the passage of the current is thus formed which includes the galvanometer coils, thus imperilling the integrity of the readings of the electromotive force. This could be obviated only by having the wires touch the rod at points in an equipotential surface, and the fulfilment of this condition was determined by the reversal of the current through the rod and the absence of any effect of such reversal upon the galvanometer.

Another and more serious objection to the method, and one which could only be met by the introduction of a correction, lay in the fact that even with the smallest wires which could be used for a thermo-element a certain amount of heat would be carried away by conduction through the metal: so that the junction would never reach the full temperature of the surfaces with which it was in contact. I was at first inclined to think that this correction would be a small one, but attempts to measure in a similar manner the temperature of the acetylene flame indicated that the loss of heat from this source was by no means to be neglected. These attempts are described in a subsequent section of this paper.

The numerical value of this correction was accordingly determined by direct experiment in the following manner. Thermo-elements drawn from the same pieces of wire but differing considerably in diameter were prepared. These were inserted two at a time in holes on opposite sides of a carbon rod and the rod was brought to incandescence by means of the current. The temperatures reached by these junctions were compared by means of the potentiometer, and a curve was plotted showing the relation between the cross-section of the wire in the thermo-element and the temperature of the junction. This curve, extended in the direction of decreasing cross-section, served to indicate with at least a fair degree of accuracy the temperature which would have been reached by a thermo-element of zero cross-section placed in contact with the surface to be measured. The difference between this temperature and that reached by a junction of any desired size gave the correction which was to be applied. The correction, as will be seen by inspection of the curve, Figure 3, is a very large one, amounting, even in the case of the smallest wires which it was found practicable to use, to

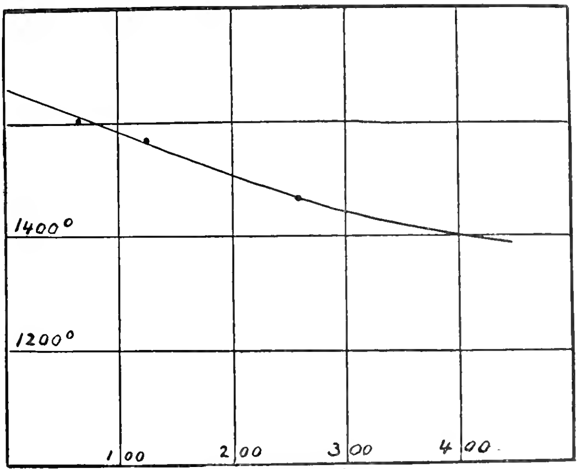

Figure 3.

about 85°. The result of the calibration agreed, however, so well with similar experiments made by placing thermo-junctions of various sizes in the non-luminous outer envelopes of the acetylene flame, of the ordinary gas flame, and of the flame of the candle, that I feel warranted in placing much dependence upon them.

The correction is not of the same size in the various cases, but the differences are such as one would expect from the nature of the flames.

This method of correcting for the loss of heat in a thermo-junction was first employed by Waggener [*] in his investigation of the temperature of the flame of the Bunsen burner. I became acquainted with his research only after the completion of my experiments.

Calibration of the Thermo-Elements.

All our estimates of very high temperatures may be said to rest in one way or another upon extrapolation. The upper limit of usefulness of the air thermometer has been found to lie in the neighborhood of 1500.° At this temperature Erhardt and Schertel, [†] in their admirable but little known research upon the melting-points of alloys of silver, gold, and platinum, were obliged to abandon direct determination; and, at about the same temperature, Holborn and Wien and Holborn and Day [‡] in their latest studies upon thermo-electric thermometry found that the indications of the air thermometer, even when constructed of the most refractory of modern porcelain, began to be erratic. We have, it is true, the investigations of Violle [§] upon the melting-points of the metals of the platinum group; but these, it must not be forgotten, are based upon an assumed value for the specific heat, and this assumption is equivalent to the extrapolation of the curve of the variation of the specific heat with temperature. The observed values, by means of which this value was determined, all lie far below those of the melting-points of the metals in question. It is necessary, therefore, in spite of the accumulation of indirect evidence of their approximate accuracy, to hold in reserve the assignment of absolute values of these melting-points until by some means as yet unthought of we shall be able to obtain direct experimental data. In the meantime, they afford us the best present available basis for a temporary scale, our confidence in the

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[*] Waggener, Wiedemann's Annalen, LVIII. 579 (1896).

[†] Erhardt and Schertel, Jahrbuch für das Hüttenwesen in Sachsen, 1879, p. 154.

[‡] Holborn and Day, American Journal of Science, VIII. 165 (1899).

[§] Violle, Comptes Rendus, LXXXIX 702, 1879.

approximate accuracy of which must rest upon the fact that the melting points for palladium, platinum, etc., as given by Violle are found to be upon what may reasonably be supposed to be an extension of the curves experimentally determined for lower temperatures by means of the air thermometer. As for the various formulae for the variation of electromotive forces of thermo-elements with the temperature, we must not lose sight of the fact that they are simply analytical expressions for experimentally determined relations, and that the extension of them to temperatures lying far beyond the experimental range is not to be regarded as more trustworthy than the extention of a curve by graphical methods.

Under these circumstances I decided to content myself with the provisional acceptance of the following values for the melting-points of gold, palladium, and platinum, namely: —

Gold,	1075° C
Palladium,	1500° C
Platinum,	1775° C,

and to ascertain as accurately as possible the electromotive force given by the thermo-elements used at these points. It was thought that by drawing a curve through them, and reading intermediate temperatures from this curve, the values obtained would be as close as our present knowledge of the subject will admit. The platinum, platinum-rhodium wire used for my elements was obtained, as has already been stated, from Heraeus in Hanau and was supposed to be of the same stock as that employed by Holborn and Wien. The fact that the electromotive force given by these thermo-elements when exposed to the temperature of melting platinum agreed very closely indeed with that obtained by extrapolation of their data seems to indicate that the metals were identical with those used by them.

Exhaustive studies at the hands of Le Chatelier,* of Barus,† and of Holborn and Wien,‡ and others have led to the conclusion that whenever thermo-elements consisting of platinum on the one hand, and of the alloys of that metal with iridium, rhodium, or any other metals of the platinum group on the other, are to be used in the measurements of

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* Le Chatelier, Comptes Rendus, CII (1866) 819; Journal de Physique (2 VI. 26 (1887); also Mesure des Temperatures Élevées (Paris, 1900), Chapter VI.

† Barus, Bulletin of the U. S. Geological Survey No. 54; also American Journal of Science, XLVIII. 336.

‡ Holborn and Wien, Wiedemann's Annalen, XLVII. 107 (1892); LVI 560 (1895).

high temperatures, it is necessary to make a thorough calibration of the individual thermo-elements involved, or at least of the set of elements manufactured from any given sample of metal. How important it is to perform such a calibration for one's self may be seen from the fact that Holman, Lawrence, and Barr* obtained an electromotive force of .0303 volts from a platinum, platinum-rhodium (10%) element at the temperature of melting platinum, whereas a similar element constructed of wire from Heraeus gave in the hands of the present writer .0182 volts at the same temperature.

Numerous more or less complicated methods of calibration involving the use of various forms of the gas thermometer have been proposed, the carrying out of which involves the use of special apparatus which is difficult of construction and laborious in operation. Fortunately it was possible in the present investigation to substitute for these a new and easy method in which the acetylene flame itself was the source of heat. This method † possesses the advantage of extreme simplicity, and it affords indications the accuracy of which leaves little to be desired.

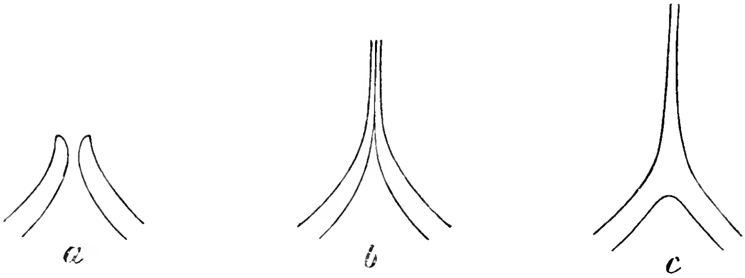

FIGURE 4.

The acetylene flame employed was of the usual flat form produced by the union of two impinging jets. There are three distinct stages observable in the form of such a flame, depending upon the pressure at which the gas is supplied to the burner. In the first, we have two separate cylindrical jets of small size (Figure 4 *a*), which, with increasing gas pressure meet without uniting, each being deflected, by impinging upon the other, into a vertical plane (Figure 4 *b*). At still higher pressures the actual union of the two jets takes place, giving the flame the structure shown in (Fig-

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* Holman, Lawrence, and Barr, J. Am. Acad. of Arts and Sciences (1895), p. 218.

† This method of calibration has been separately described in a contribution to the Lorentz Jubilee Volume. The Hague, 1900.

ure 4 *c*), in which the two cylindrical jets of gas in the process of combustion unite to form a single flat vein or envelope which constitutes the luminous portion of the flame. When this third stage is reached, there is great stability of form and position. Such a flame responds with a sharp lateral motion to air waves such as are produced by the slamming of a door, but is comparatively unaffected by slight drafts. Even in a room not essentially free from air currents the lateral motions of the flame, which may be accurately observed by throwing an enlarged image of it, viewed edgewise, upon a screen, rarely amount to more than .1 mm., and in an especially protected place, these lateral movements become entirely imperceptible. The temperature gradient in the layer of air bordering upon the luminous envelope of such a flame is very steep, but it is capable of definite determination by exploration with suitable thermo-elements, and so long as the flame remains undisturbed by lateral drafts its stability is surprising.

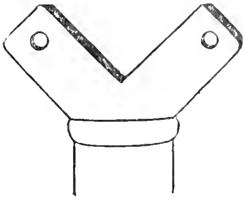

FIGURE 5.

The burner used is of a well-known form (Figure 5), and is made from a single block of steatite. It is mounted upon a horizontal bar of steel (Figure 6), along which it may be moved by means of a micrometer screw. The bar is set up in an inner room without windows, being opposite a circular opening in the wall through which the flame may be observed from without. In this opening is placed the lens of a micro-camera,

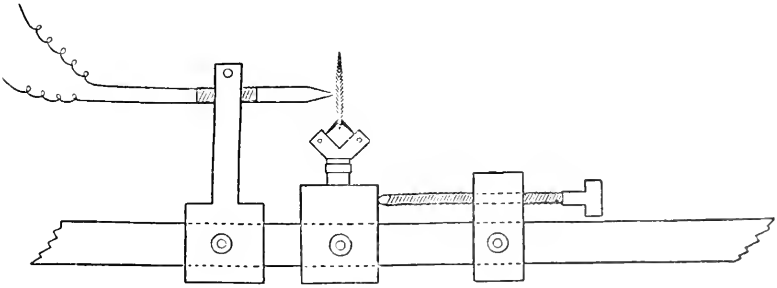

FIGURE 6.

upon the ground-glass screen of which instrument, at a distance of about two meters, an enlarged image of the flame is focussed. The platinum and platinum-rhodium wires to be tested are drawn down to a small size (diameter about 0.01 cm.), and a thermo-element is formed by cutting pieces of the platinum wire, and of the wire of the alloy to be used, about 70 cm. in length, and binding these to the opposite faces of a rectangular block of wood about 1 cm. in thickness. Beyond this block the wires project about 3 cm. They are bent toward each other until the free ends are in contact, forming a V, and these ends are then fused in the oxyhydrogen flame, forming a junction, which is subsequently trimmed down to the form shown in Figure 7. The apex of the V is cut away until the arch of fused metal joining the two wires is considerably less in thickness than the diameter of the wires themselves, the face of the junction forming a smooth plane surface.

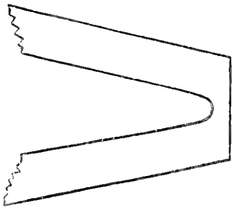

FIGURE 7.

The formation of such a junction becomes, with practice, a simple matter, and can be performed, as it is necessary to do after each observation, in a few moments. The junction is rigidly mounted upon the steel bar with the plane passing through the wires of the V vertical and the plane surface of the metal which forms the face of the junction parallel to the flat face of the acetylene flame. To the free ends of the wires are soldered the copper terminals of the galvanometer circuit, and the junctions are placed in a bath of melting ice. The support carrying the thermo-element is mounted in such a position as to bring the face of the hot junction as nearly as possible into the centre of the field of view of the camera, where it is clearly visible under the illumination of the acetylene flame, which should, at the beginning of the operation, be about 1 cm. from the junction. The micrometer screw, by means of which the flame is moved along the bar, is operated by means of a long handle with a universal joint; so that the flame can be shifted by an observer sitting opposite the ground-glass screen. For the measurement of the electromotive forces produced by the heating of the junction a potentiometer of the usual form is used. The metals the melting temperatures of which are to form points upon the calibration curve, are worked into thin foil, and from this foil strips about .03 cm. in width are cut. Such a strip is looped into the angle of the V and drawn snugly into place, the free ends being cut away until they project only about 1 mm. beyond the face of the junction. To hold this minute loop of metal in its place, it is only necessary to press the foil carefully together around the junction. The thermo-junction carrying the loop having been mounted, in the manner described, in the focus of the camera, will be clearly seen upon the ground-glass screen, the ends of the loop of metal projecting towards the flame.

The determination of the electromotive force corresponding to the melting-point is made as follows. The observer seats himself in a position where he can watch closely the image of the flame and of the thermo-element and moves the former gradually toward the junction, balancing the potentiometer approximately from time to time as the electromotive force rises with the increasing temperature.

At a definite distance from the luminous envelope of the flame, which distance depends upon the character of the metal under investigation, the projecting ends of the loop will be seen to melt. So quiet is the flame, and so well fixed the temperature gradient from its surface outward when a proper burner is used, and when the flame is placed in a locality reasonably free from air currents, that the fusion of the successive portions of the metal loop may be brought about from the end inward with the greatest nicety; and the electromotive force may be determined at each stage until the fusion has progressed to the plane coinciding with the face of the junction. Even then, in many cases, those portions of the loop of metal which lie within the angle of the junction will remain unfused, although their distance from the melted portion of the loop is only a fraction of a millimeter.

The delicacy of this operation under favorable conditions is very great, and the agreement of the successive readings of the melting-points of a given sample of metal is excellent. It is desirable to make a series of readings, leading up to the true melting-point, for the reason that when the fusion of the metal loop has progressed to that portion which lies in contact with the platinum, an alloy is almost immediately formed between the fused metal and the junction itself, which affects the thermo-electric indications of the couple. For this reason it is not possible to get consistent readings by repeating observations with a given junction. The proper procedure is to cut the wires back 2 or 3 mm. from the apex of the V after each set of readings, and to make a new junction of the proper form from the free ends thus produced. This requires but little time after the operator has gained a reasonable degree of familiarity with the method.

When the metal, the melting-point of which is desired, is platinum itself, the platinum wire of the junction begins to fuse at the same time as the loop, the platinum rhodium or platinum-iridium side remaining unmelted. The precise point at which this fusion of the platinum occurs is, however, quite as definite as in the case of metals of lower melting temperature. This method has the advantage of avoiding the use of the air thermometer and of furnaces in which fusion of the metals takes place. The amount of metal which it is necessary to melt is almost infinitesimal. The loops used in each observation weigh only a fraction of a milligram, and the operation may be repeated time after time at the will of the observer with the greatest ease. On the other hand it should be noted that the method is applicable only to such metals as will fuse before oxidation in the hot layers of the acetylene flame. It is not practicable with magnesium, aluminium, zinc, or iron, since these oxidize under the conditions of the experiment instead of fusing. For such of the metals of the platinum group as have melting-points below that of the junction itself, and for gold, silver, and copper, the method is a convenient one, and its accuracy is, I believe, fully equal to that of any other method which has thus far been employed. To guard against the deleterious influence upon the thermo-junction of the vapors of the flame, it is important to bring the latter up gradually by the slow action of the micrometer screw in the manner which I have already described. The atmosphere with which the junction is surrounded under these conditions contains an

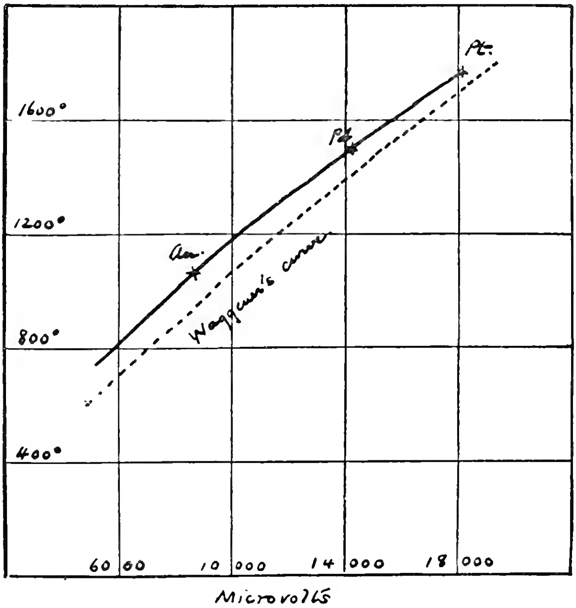

FIGURE 8.

excess of oxygen, and even where the metal to be melted is platinum itself, fusion occurs before the luminous portion of the flame, the action of which upon the thermo-electric properties of the junction is to be feared, has been reached. It is well-known that a junction, the performance of which has been vitiated by exposure to the vapors of a flame or furnace, can be restored to its original condition by immersion in an oxydizing flame. In this method of calibration the junction is continually subject to such oxidation as is necessary to preserve it. Thus one of the sources of error which it has been found most difficult to guard against in the use of the furnace is altogether avoided.

Figure 8 contains the calibration curve of the thermo-elements used in this investigation, and likewise, for purpose of comparison, a curve reproduced from Waggener's paper and extrapolated by him from data given by Holborn and Wien. It will be seen that while the curves are not identical they are of the same character, and that the differences are not greater than experience would lead us to expect in the case of different thermo-elements, even where these are from metals of the same manufacture. It is not a question of absolute electro-motive forces, but of the form of the curves, since what we need is a criterion by means of which to determine whether temperature readings based upon Violle's values for palladium and platinum are in reasonable accord with those obtained by the extension of the curve of Holborn and Wien.

The Spectrophotometer.

The spectrophotometer used was a copy of the instrument designed by Lummer and Brodhun for the Imperial Institute in Charlottenburg. It consists of a one-prism spectroscope with two collimator tubes, placed at right-angles to each other, as shown in Figure 9. Each of these tubes carries a slit the width of which is regulated by means of an accurate micrometer screw with a drum head divided into one hundred parts. By estimating tenths of a scale division, the width of the slits could be estimated to one one-thousandth of a revolution.

The essential feature of this photometer consists in the Lummer-Brodhun prism D, placed between the objective lenses of the two collimators, and the dispersing prism in such a position that the beam of light from one of the tubes is transmitted directly to the latter, while that from the other tube is bent to 90° by total reflection. The instrument was set up with collimator A in such a position that a portion of the surface of the incandescent rod lying nearest to the point at which the thermo-element had been inserted formed a field of illumination for the slit at a distance of about 25 cm. The region under observation was limited by means of a vertical diaphragm *d*, 5 mm. in width, which was mounted in a tube in front of a window of the metal vacuum box. The comparison source was the spectrum of the brightest part of an acetylene flame set up in the axis of the other collimator at a corresponding distance, and viewed through a circular aperture *c*, 5 mm. in diameter, cut in a metal screen interposed between the flame and the slit and as near the former as practicable.

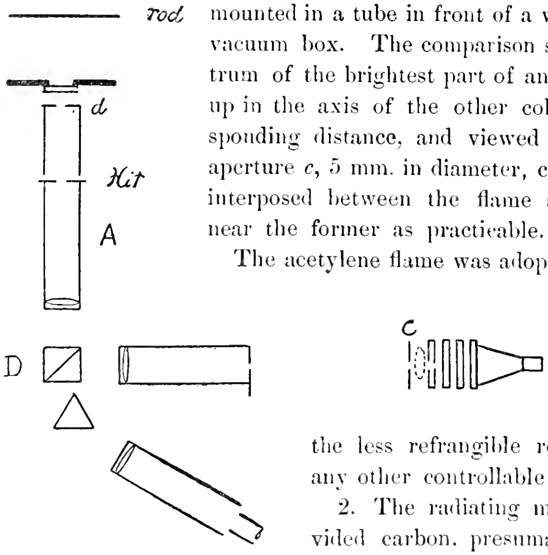

Figure 9.

The acetylene flame was adopted as a comparison standard for the following reasons: —

1. It possesses a continuous spectrum, brighter in the less refrangible regions than that of any other controllable source of light.

2. The radiating material is finely divided carbon, presumably of a character not unlike that of the surface of the untreated rod.

3. The acetylene flame is the result of the combustion of a definite fuel (C_2H_2) burning under reasonably constant conditions. It is preferable in this regard to any of the ordinary gas or candle flames in which the fuel is of an undetermined and more or less variable character.

4. When supplied with gas under constant pressure, an acetylene flame of the type used in these experiments, that, namely, obtained by means of a burner composed of a single block of steatite, is more nearly constant in its intensity and color than any other flame with which I am acquainted, with the exception of that of the Hefner lamp. It is indeed questionable whether the latter is superior to acetylene in this respect, and its comparative weakness in the blue and violet renders it very undesirable as a comparison source in spectrophotometry.

*Determination of the Temperature of the Acetylene Flame.**

Concerning the temperature of the acetylene flame, varying and incompatible statements are in existence. The temperature of combustion

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* The results of these experiments on the temperature of the comparison flame were separately communicated to the American Physical Society on February 24, 1900, and were published in the Physical Review, X. 234.

of this gas, according to Le Chatelier,* would be, when burned in air, 2100° to 2420°. Measurements with Le Chatelier's pyrometer, on the other hand, made by V. B. Lewes,† gave temperatures lower than those of ordinary gas flames. Lewes found for the obscure zone 459,° for the edge of the luminous zone 1411,° and for the region near the summit of the luminous zone 1517°. Smithells,‡ upon the appearance of the data given by Lewes, described a series of experiments for the purpose of showing that the temperature of the flame reaches, in point of fact, very much higher values than those given by that author, and that in many portions it is higher than the melting point of platinum.

It can be easily shown by inserting wires of platinum into the flat acetylene flame obtained from any one of the forms of burner usually employed, that while the thicker wires remain unmelted, those of very small diameter are readily fused. I found, for example, that a wire having a diameter of 0.0082 cm. became fused at the end with the formation of a distinct globule, before the metal had penetrated the outer luminous layer of the flame, whereas wires of 0.01 cm. or of larger diameter remained unmelted. The experiments of Waggener § show that there are portions of the flame of the Bunsen burner in which it is possible to melt platinum, while MacCrae,‖ working with a platinum-rhodium element, found for the hottest region in the Bunsen flame 1725°. It will be seen from the experiments to be described in this paper, that MacCrae's determination, which was made with wires having a diameter of 0.02 cm., is not incompatible with the observations of Waggener and others. Smithells, in the paper just cited, describes the melting of platinum wires having a diameter of 0.01 cm., in various parts of the outer sheath of a flat flame of illuminating gas. Pellissier,¶ in commenting upon Lewes's measurements, refers to experiments in which minute wires of platinum, made by Wollaston's method of silver plating, drawing, and subsequent dissolving of the silver coating, when thrust into the flame of a candle, melted instantly. I have not been able to find other printed reference to these observations and do not know with whom they originated. An attempt to repeat the experiment with a Wollaston wire having a diameter of 0.0011 cm. resulted in the ready

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* Le Chatelier, Comptes Rendus CXXI. 1144 (1895).

† Lewes, Chem. News, LXXI. 181 (1895).

‡ Smithells, Journal of the Chemical Society, LXIX. 1050 (1895).

§ Waggener, l. c.

‖ MacCrae, Wiedemann's Annalen, LV. 97.

¶ Pellissier, L'Éclairage à l'acétylene (Paris, 1897), p. 186.

fusion of the wire by the flame. An examination of the remaining portions under the microscope showed that the metal had been melted down into clean, well-rounded beads, and had not been consumed by oxidation or any other chemical reaction.

Smithells's contention that the temperature of flames cannot be obtained directly from the indications of a thermo-element because of the loss of heat by conduction and by dispersion from the surface of the latter, so that the portions submerged in the flame never arrive at the temperature of the surrounding gases, is well founded. Lewes and likewise Waggener recognized this fact, and in their measurements made use of wires of different sizes.

The apparatus which I employed for the determination of the temperature of the acetylene flame has already been described (see Figure 6). The method was similar to that used in the calibration of the thermo-elements. The electromotive force of the elements, as these were gradually brought into the flame, was measured by means of the potentiometer previously employed in the calibration of the thermo-elements and subsequently in the determination of the temperature of the carbon rods. It consisted of a sensitive galvanometer of the d'Arsonval type and an accurately adjusted resistance box containing coils ranging from 50,000 ohms to 1 ohm. A large Clark cell of the old Feussner type was mounted in series with the resistance box. The thermo-element, the galvanometer, and a subsidiary resistance of 10,000 ohms were looped around a portion of the resistance box, the ratios being varied until complete balance was secured. The electrical connections are shown in Figure 10. The type of standard cell selected for this work is subject to considerable errors from diffusion lag. It has, however, the advantage of being capable of furnishing a much larger amount of current than the small types of cell, in which diffusion lag is avoided, without appreciable loss of electromotive force. Two of these cells were placed side by side in a thick-walled inner room which

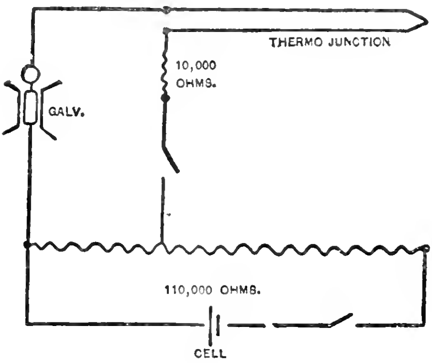

Figure 10.

had been constructed for the purpose of securing uniform temperature for the standard clock of the physical laboratory, and other similar apparatus. The range of temperature in this room fluctuated throughout the entire investigation between 18°C. and 19°C. The range was so small and the variations occurred so gradually that no changes of electromotive force of a size which it was necessary to consider in these measurements could have arisen other than those included in the usual correction for temperature.

The two cells were compared with each other from time to time by setting them in opposition to one another in circuit with a sensitive galvanometer and noting the deflection produced. It was found that although one of them was supplying current to the 100,000 ohm circuit of the potentiometer, during the times when it was necessary to close the key of that circuit, the difference of electromotive force between the used and unused cell was always very small, never more than a few hundred thousandths of a volt. At the end of the entire set of measurements, the difference was 0.00006 volts. The absolute electromotive force of these cells was checked by comparison with Clark cells of the H form and of the test-tube form, constructed in this department in 1898. As a result of these comparisons it was found that the electromotive force of the cell used in the potentiometer might be taken at 1.430 volts at 18.°

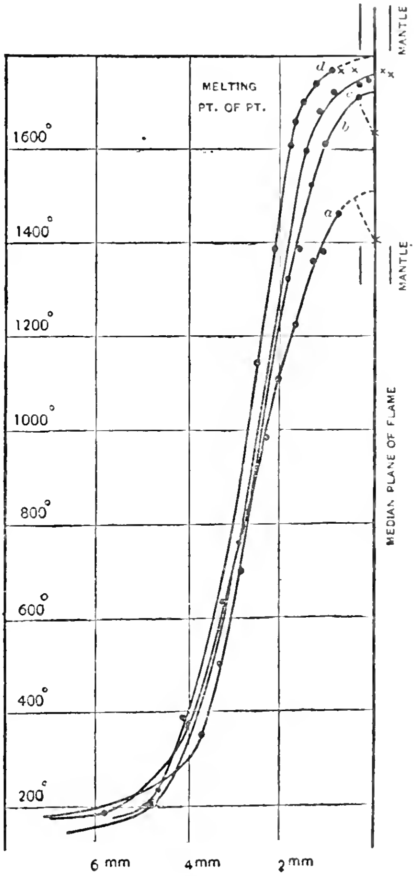

FIGURE 11.

The wires selected for the four junctions to be used in the experiment upon the acetylene flame were measured under a microscope with micrometer stage. Their diameters were as follows: —

Junction I.	Diameter 0.01996 cm.
" II.	" 0.01598 cm.
" III.	" 0.01089 cm.
" IV.	" 0.00821 cm.

Readings were first made with junction I. (diameter 0.01996 cm.). The flame was set at a distance of 6 mm. from the face of the junction, and the potentiometer was balanced. The flame was then moved stepwise nearer and nearer, and the potentiometer rebalanced at each step until the face of the junction coincided with the edge of the luminous mantle at a point just above the apex of the inner non-luminous zone.

The rise of temperature indicated by the potentiometer readings is shown in curve *a* (Fig. 11), the data for which as well as for the other curves in that figure are contained in Table II.

TABLE II.

TEMPERATURES INDICATED BY THERMO-JUNCTIONS I., II., III., AND IV. AT VARIOUS DISTANCES FROM THE MEDIAN PLANE OF THE ACETYLENE FLAME.

Junction I.		Junction II.		Junction III.		Junction IV.	
Distance.	Temp.	Distance.	Temp.	Distance.	Temp.	Distance.	Temp.
5.62 mm.	185°	. . .	. . .	5.42 mm.	165°	4.63 mm.	233°
3.91 mm.	370°	3.65 mm.	353°	4.82 mm.	183°	4.11 mm.	406°
2.85 mm.	760°	3.33 mm.	508°	3.21 mm.	657°	2.55 mm.	1168°
2.09 mm.	1128°	2.90 mm.	595°	2.03 mm.	1278°	2.12 mm.	1411°
1.66 mm.	1229°	2.30 mm.	989°	1.50 mm.	1598°	1.86 mm.	1613°
1.30 mm.	1367°	1.93 mm.	1322°	1.18 mm.	1685°	1.70 mm.	1667°
1.07 mm.	1382°	1.68 mm.	1385°	0.894 mm.	1724°	1.54 mm.	1705°
0.850 mm.	1467°	1.40 mm.	1513°	0.566 mm.	1747°	1.30 mm.	1738°
		1.09 mm.	1617°	0.238 mm.	1759°	1.025 mm.	1771°
		0.320 mm.	1715°	0.00 mm	1775°	0.780 mm.	Molten.
				– 0.29 mm.	Molten.	0.300 mm.	Molten.

The increase of temperature as the flame approaches the junction is gradual at first; but at a distance of about 0.4 cm. from the median plane, the curve suddenly becomes steep. It is probable that this distance measures the thickness of the layer of non-luminous gas which surrounds the visible flame. Outside of this region, the junction is heated almost altogether by radiation. As soon as it penetrates the column of moving gas, however, heat is brought to it principally by convection. Before the surface of the luminous mantle is reached the curve shows indications of approaching a maximum.

Upon pushing the flame still nearer to the junction so that the latter penetrated the luminous region, an accumulation of lampblack began to form upon the wire, with fall of temperature; a process so rapid that at the end of two minutes a button of carbon several millimeters in diameter is formed. This is finally torn loose from the wire by its own weight; whereupon the deposition of a new mass begins. I attempted by watching the breaking away of the carbon from the wire, which occurred at regular intervals, to determine the temperature of the wire before the coating of carbon had begun to show itself again. The highest temperature which it was possible to observe in this way was nearly one hundred degrees below that in the luminous layer, and it was obvious from the movement of the galvanometer needle that the junction was being rapidly cooled by the deposition.

Junction II. (diameter 0.01598 cm.) was now substituted for Junction I., and a similar set of readings were made. This junction, as had been anticipated, showed higher temperatures. It was found possible, owing to the small diameter and consequently high temperature of the wire, to penetrate further into the flame before the deposition of carbon began, so that measurements with the junction actually within the luminous layer could be made. The general form of the curve, as will be seen by inspection of the figure (curve *b*) is the same as that obtained with Junction I. After penetrating the luminous mantle to a small fraction of a millimeter, carbon began to gather upon this junction likewise, with lowering of temperature, as in the case of Junction I. The attempt to read temperatures immediately after the dropping of the accumulated carbon showed that the highest temperature which could thus be observed was again about one hundred degrees below the temperature of the luminous mantle. It was clear in this case, as before, from the rapid fall of temperature already going on, that this reading has no significance.

Similar readings with Junction III. (diameter 0.0108 cm.) gave a third curve of the same type as those plotted from the reading made with I.

and II., but the temperatures were higher throughout. With this junction it was found possible to penetrate to the centre of the flame without the deposition of carbon, the temperature of the wire being apparently too high to permit the formation of soot. Upon pushing through the median plane of the flame to the second luminous mantle, the junction was melted. This result was not unexpected, since the temperature of the junction at the first luminous mantle reached 1750°, so that a rise of twenty-five degrees of temperature would suffice to produce fusion. The wire when pushed through the flame in the manner just described is heated for greater and greater distances back from the junction until the losses of heat at the junction are sufficiently diminished to raise the tips of the wires to the melting-point.

With Junction IV. (diameter 0.0082 cm.), a fourth curve, similar in form to the preceding ones and with still higher temperatures, was obtained. This junction was fused at a distance of 0.075 cm. from the core of the flame, and of 0.037 cm. from the edge of the first luminous mantle. It was easy to observe in the enlarged image upon the plate of the microcamera the melting away of the platinum wire, while the platinum-rhodium alloy was still unaffected, and while contact was still unbroken. A satisfactory observation of the electromotive force of the thermo-element at the melting-point of platinum was thus obtained. This reading (0.018236 volts) differs from the value found in my calibration of the thermo-junctions used in this investigation (0.018262 volts) by a quantity of (0.000026 volts) less than the errors due to changes in the electromotive force of the standard cell. If the latter reading be taken to correspond to 1775°, the former indicates 1773°.

Beyond this point, it was impossible to make direct observations of temperature; but the form of this and the preceding curves were so closely allied that I felt no hesitation in extending the curve *d* to the core of the flame. This has been done by means of dotted lines in the figure. Curves *a* and *b* have been extended in the same manner. In order to form an estimate of the temperature which would have been reached by a thermo-junction of negligible cross-section, provided such a junction could have been obtained which was capable of registering temperatures above that of the melting-point of platinum, the ordinates of the four curves, *a*, *b*, *c*, and *d* were taken for the core of the flame, for the plane of the luminous mantle, for a plane distant 0.07 cm. from the core, and for a plane 0.10 cm. from the core. These readings were plotted and curves were drawn through them as shown in Figure 12; relative cross-sections of the wires being taken as abscissae, the temperatures as

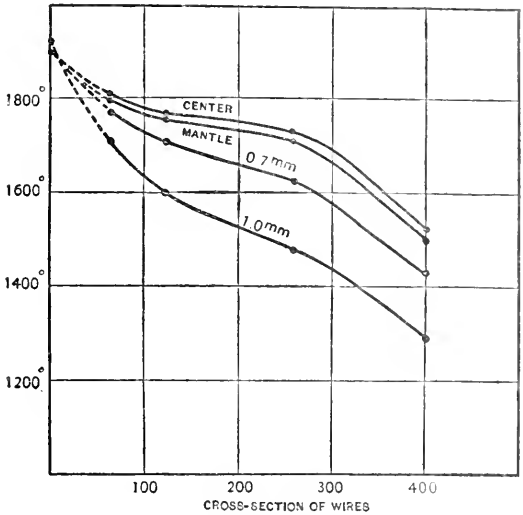

Figure 12.

ordinates. If these curves could be extended to the line representing zero cross-section, the temperatures indicated by the points in which each of them cuts that line would give the temperature of the portion of the flame to which the curve corresponds. There is a considerable element of uncertainty in extrapolation even over so short a range as this: but it is obvious from the character of the curves lying within the limits of observation, that each of them trends upward, and it seems highly probable that they all meet the line of zero cross-section at a temperature not far from 1900°. The fact that the curves cut this line at nearly the same temperature would seem to indicate that the distribution of temperatures from the centre of the flame outward for a distance of about 1 mm. is a nearly uniform one.

It would perhaps be unwise to attempt to draw any more definite conclusion from the probable trend of these curves; but I have ventured to extend them in the manner shown in the figure, so that the curve for the region 1 mm. from the centre of the flame reaches the zero of abscissae about twenty degrees above that for the centre of the flame, *i. e.* at 1920°, and the intermediate curves at temperatures lying between them. I regard this as an extreme treatment of the case, and allude to it only to indicate that, in accordance with common belief, the highest temperature may be found in the outer non-luminous layer of the flame, but that it is unlikely that the difference amounts to more than twenty degrees.

The point of intersection referred to above lies nearly one hundred degrees above the highest temperature recorded by even the smallest of the thermo-elements, and it is safe to infer that nearly all previous attempts at the measurement of flame temperatures must, for lack of correction of the error, due to loss of heat through the wire, be regarded as much too low. The junction IV. is, so far as I am aware, the smallest in cross-section that has been used in such work. With larger wires, the correction for loss of heat would be even greater, except in cases where, as in the observations made by Smithells, and by Waggener, the precaution was taken to immerse an extended portion of the wires within the flame.

Temperature of Other Flames.

For the purpose of comparison, I measured in a manner analogous to that just described, the temperature of the luminous flame of ordinary illuminating gas and the flame of a candle. The gas flame employed for this purpose was obtained from a lava tip rated at one cubic foot and giving a flat flame of the usual form. The image of this flame, when viewed upon the ground-glass screen of my camera, was found to be comparatively ill-defined and unsteady; but although the outlines of the luminous sheath were much less clearly marked than in the case of the acetylene flame, they were discernible. Owing to the continual motion of the flame, due to the small velocity of the gas issuing from the jet, no attempts were made to plot curves of temperatures outside the flame. All readings were made with the junction as nearly as possible in contact with the outer surface of the luminous sheath, at a point in the brightest portion of the flame. This position is approximately indicated by the letter x in Figure 13. The four junctions already described were mounted, one after another, in such a position that the flame could be moved up until they came into contact with the sheath at the point indicated. The temperatures of the junctions when in that position are given in the following table: —

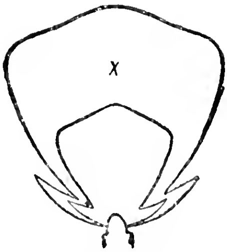

FIGURE 13.

TABLE III.

Junction	I.	1385°	Junction III.	1609°
"	II.	1484°	" IV.	1676°

These values having been plotted with relative cross-sections of the ires as abscissae, and temperatures as ordinates, were found to lie

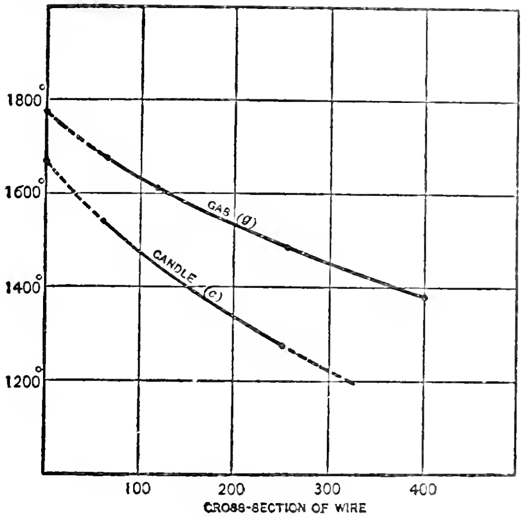

FIGURE 14.

upon a smooth curve (g) as shown in Figure 14. This curve, when extended to the line corresponding to zero cross-section, gave for the temperature of the flame 1780°, a temperature sufficient to account for the success of Smithells's experiment, already described, in which platinum wires of small diameter were melted in the outer sheath of such a flame. I found it easy, by holding a wire of the size used in junction IV. in a plane parallel to that of the flame, and moving it gradually toward the latter to verify his statement. The wire was readily melted.

It was not thought necessary to make further experiments upon this flame. The region selected was, so far as one could judge from the brightness of the luminous sheath, the hottest portion of flame. My measurements upon this region would lead to the conclusion that the *luminous sheath of ordinary gas flames is at least one hundred and twenty degrees lower than the corresponding region in the acetylene flame.* Luminous flames of ordinary illuminating gas would perhaps repay further study, but owing to the fact that such gas is an ever varying mixture and that it is burned under conditions of pressure, etc., such as to give a fluctuating character to the flame, the problem would have at best an indefinite character from which studies of acetylene are free. In the latter case we have to deal with a definite fuel, and the velocity of the jets of gas from the burner is sufficient to give a high degree of stability to the flame.

The candle would seem an even less satisfactory subject of study in these respects than illuminating gas, but the fact of the melting down of Wollaston wire, the verification of which I have briefly described in an earlier paragraph of this paper, seemed to discredit so completely the low values commonly given that I decided to redetermine its temperature by the method already described.

The fact that the flame of a candle, mounted upon a fixed stand, would move steadily downward as the material of which it was composed burned away, made it convenient, without any serious modifications of my apparatus, to explore the temperature of the luminous sheath throughout the entire length of the flame. It was only necessary for this purpose to mount a candle upon the steel bar in the position previously occupied by the acetylene flame, and when it had reached such a length that the level of the rim of the cup lay below the level of the junction, to move the candle toward the latter by means of the micrometer screw until the junction began to be submerged in the luminous sheath of the flame. It was then easy by a series of slight adjustments of the flame to explore with the junction the entire surface of the luminous sheath from base to tip, measuring temperatures from time to time, and determining the position by means of the height of the junction above the rim of the candle cup. The latter observations were readily made by means of the image of the candle upon the ground glass of the camera. Explorations of the candle flame in the manner described were made with Junctions II. and IV., and the results obtained showed a degree of consistency much greater than the fluctuating character of the source under observation had led me to expect. Both sets of observations showed a maximum of temperature in the same region: that lying just above the tip of the interior dark zone of the flame. Readings were made by watching the movements of the candle flame and securing a balance of the potentiometer at times when the face of the junction was as nearly as possible in contact with, but not deeply submerged within, the luminous layer. Whenever the wire plunged to any considerable depth beyond the luminous surface, deposition of soot occurred with lowering temperature, and it was necessary to withdraw the junction into the non-luminous regions outside and to wait until the deposit had been burned off, before proceeding with the readings. In computing the actual temperatures of the luminous sheath of the flame from these readings, I contented myself with the following rough approximation. The maximum temperatures shown by Junctions II. and IV. were plotted upon the same diagram used for the luminous gas flame. These temperatures were 1281° and 1546°; values which, as will be seen by inspection of Figure 14 (*c*), lie much below those of the corresponding readings for the luminous gas flame, but in such positions as to make it easily possible to draw through them a curve analogous in form to that obtained for the latter. Such a curve would cut the line of zero cross-section at about 1670°, which may, I believe, be taken as the approximate temperature of the hottest portions of the luminous sheath of the candle flame. Estimates of this temperature by the probably less accurate methods of drawing a straight line through the points in question and taking the point in which this line cut the line of zero cross-section to be the temperature of the flame, and estimates based upon the assumption that the true temperature is as many degrees above the temperature indicated by Junction IV. for the candle as it is for the gas flame, would lead to values respectively twenty-four degrees and forty degrees lower than that obtained by the method which I have adopted. I believe that the temperature just given (1670°) is much closer to the truth than that obtained under either of the other assumptions. Estimated temperatures for other portions of the luminous sheath were made by assuming that the correction to be applied to the readings obtained with Junction IV. would be the same in all positions. These values are given in Figure 14 which may serve in place of an ordinary table. The portions of the flame to which each reading refers are more readily indicated by giving such a diagram of the flame than in any other way.

The fact that, in the case of the acetylene flame and the ordinary gas flame, this method gives values high enough to account for the melting of platinum, but leads to an estimate of the temperature of the candle flame which is about one hundred degrees below the melting-point of that metal, would seem, at first sight, to throw the procedure into serious doubt. My experience with the method has, however, been such as to make an error of one hundred degrees in the estimation of the candle-flame temperature seem highly improbable. Messrs. Lummer and Pringsheim, in a recent communication to the German Physical Society,* give an estimate of the temperature of candle flames based upon a relation

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* Lummer and Pringsheim, Verhandlungen der deutschen physikalischen Gesellschaft, 1899, p. 214.

which they have established between the position of the maximum in the energy curve of the spectrum of a source of light and its temperature. Assuming the radiating substance in the flame to have the properties of a black body, they find this temperature in the case of the candle flame to be 1687°, a value seventeen degrees above that which I have given.

To account for the fusion of Wollaston wire in the flame of a candle, one might consider the possibility of the existence in such a flame of layers of gas the temperature of which is much above the surrounding regions, and that these layers may be so thin that it would not be possible to submerge the thermo-junction completely in them. In such a case the junction would give a value approximate to the average of the temperatures of the gases with which it was brought into contact. Before assuming this structure of the flame, which really has nothing to support it save the necessity of accounting for the apparent discrepancy which I have just pointed out, it seemed wise to consider, on the other hand, whether the melting-point of the Wollaston wire was necessarily that of pure platinum. Such wires would naturally be made of ordinary commercial metal, the melting-point of which might vary considerably from that of the purer platinum used in the determination of melting-points. It is likewise readily conceivable that in the process of drawing within the silver coating, a certain amount of silver might be worked into the pores of the platinum and not be removed by the subsequent action of the nitric acid. The determination of the melting-point of even such minute wires is fortunately a simple matter by means of the form of thermo-element used in the calibration experiments already described. It is only necessary to wrap a piece of the wire to be tested around the junction, as shown in Figure 15, to cut it off so that the end of the loop extends slightly (about 0.05 cm.) beyond the face of the junction; and having mounted the junction in the usual manner, to move the acetylene up to it by means of a micrometer screw. I performed this experiment with a piece of the same Wollaston wire which I had succeeded in melting in the candle flame, and found its melting-point, as indicated by the electro-motive force of the junction, to be 1674°. To test the question whether this very low melting-point was due to the presence of silver undissolved by the nitric acid, a piece of the same wire was left in the acid for twelve hours, after which the

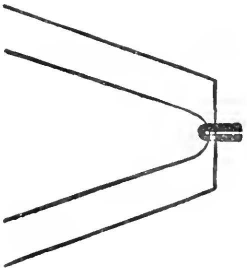

FIGURE 15.

melting-point was again tested in the manner just described. The result of this determination was 1687°. The latter reading was, I think, too high, since subsequent examination under the microscope showed that the loop of the wire behind the junction had been melted so that the junction was probably a few degrees too hot. It may safely be concluded from these determinations that the melting-point of the Wollaston wire was at least one hundred degrees lower than that of pure platinum.

Method of Checking the Constancy of the Acetylene Flame.

To secure as complete a check as possible upon the constancy of the flame, the following method, based upon the assumption that so long as the radiation from the flame remained constant, its light-giving power would not vary, was employed. A diaphragm (*d*, Figure 16) similar to that interposed between the slit and the flame, and having an aperture of the same size, and mounted on the opposite side of the latter and a thermopile *p*, was placed at a distance of about 15 cm. from this opening. A second diaphragm, *d′*, with an intervening air space, served to cut off, in large part, the radiation from the heated metal. Two thin sheets of glass forming the sides of an empty cell *c*, of the kind used in the study of absorption spectra, etc., were placed between the cone of the thermopile and the second diaphragm; so that only those rays from the flame which were transmitted by the glass fell upon the face of the pile.

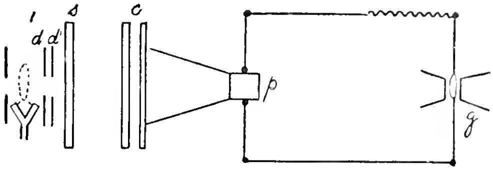

FIGURE 16.

The thermopile was connected with a sensitive d'Arsonval galvanometer *g*, the circuit being kept permanently closed; and a double metallic shutter *s*, which could be raised or lowered so as to open or close the opening in the diaphragm next to the flame, was so mounted that it could be readily operated by an observer at the telescope of the galvanometer. When a reading of the radiation from the flame was to be made, the zero point of the galvanometer was noted, and this shutter was raised during the short interval of time necessary to bring the needle, which was not strongly damped, to its first turning point. The shutter was then immediately closed in order to prevent further heating of the face of the thermopile. This throw of the galvanometer was taken as an indication of the intensity of the flame.

It was found that the thermopile would cool sufficiently within two minutes to admit of the repetition of the reading. These observations were taken by an assistant simultaneously with each setting of the spectrophotometer, the intention being to reject any spectrophotometric readings made at a time when the flame showed marked deviation from its standard intensity, and to reduce the readings to a uniform flame intensity under the assumption that for the small range of variation occurring from reading to reading, the change in the brightness of the flame would be proportional to the variations of this galvanometer reading from the mean of the whole set. In point of fact it was found that the flame rarely varied from the mean in the course of a set of observations by more than one per cent. From day to day, indeed, its intensity was usually within the limits stated above. Occasionally a larger variation was detected. None of these variations in the course of the present investigation reached values so great as to lead me to hesitate to apply the correction already referred to, and all the observations described in this paper have been reduced to a constant flame intensity by means of a correction factor obtained from the readings of the galvanometer.

Control and Measurement of the Temperature of the Carbon Rod.

The carbon rod, having been brought to the desired degree of incandescence by means of the current from a storage battery, was held at a constant temperature by varying the resistance placed in the battery circuit. The indications of the thermo-element inserted in the rod were noted by means of the potentiometer. The cells used in the measurement of the temperature of the carbon rod were the same as those employed in the calibration of the thermo-elements and in the study of the temperature of the acetylene flame.

The potentiometer having been balanced by looping the circuit containing the thermo-element around a sufficient portion of the resistance box to balance its current against that of the Clark cells, a condition which was indicated by the reduction of the galvanometer deflection to zero, the current was maintained at such a value as to hold the carbon at a constant temperature during the time necessary to complete measurements of the intensity of eight different portions of the spectrum, ranging from the extreme red to violet, with the corresponding portions of the spectrum of the flame. In order to insure the maintenance of this constant temperature in the rod, an assistant made repeated observations with the potentiometer and readjusted the resistance in the battery circuit whenever necessary. Excepting at very high temperatures, where the rod was subject to rapid disintegration, it was rarely necessary to make any adjustment during the progress of a single set of observations. Readings of the current flowing through the carbon and of the fall of potential between its ends were made at the beginning and end of each experiment.

SPECTROPHOTOMETRIC OBSERVATIONS.

It was my expectation, in planning this research, that whatever might prove true as to the character of the radiation from gray carbon, the distribution of energy in the spectrum from black carbon would change in such a manner with increasing incandescence as to become nearly or quite identical with that of the various luminous gas flames at temperatures corresponding to the temperature of the glowing carbon in those flames. I had also hoped, among other things, to be able to bring about a degree of incandescence approaching that of the acetylene flame itself, before the usefulness of the thermo-element as a means of measuring the temperature failed because of the melting of the platinum wire, and in this way to obtain a check upon my previous measurements of that flame; and at the same time to be able to determine the temperature of any given luminous flame in which the incandescent material consists of carbon particles by ascertaining the temperature of the carbon rod for which its surface had a spectrum corresponding in distribution of energy to that of the flame.

It will be seen from inspection of the curves to be discussed in a subsequent paragraph that this expectation was far from being realized, and that the distribution of energy in the spectrum of the carbon rod, instead of approaching that of the acetylene flame as the temperature of the rod increased, took on an entirely unexpected character. Even at low temperatures, that is to say up to about 1100°, the change in the spectrum was not of the comparatively simple character which had been anticipated, and shortly after passing the temperature of 1100°, unlooked for complications in the results arose. The energy in the yellow of the spectrum which from the beginning had been increasing at a relatively more rapid rate than either in the red or at the blue end, became so great as to give the distribution curve a form entirely contrary to expectation.

I was very slow to believe in the integrity of these results, and nearly a year was spent in repetitions of the measurements before I could convince myself that the phenomenon was a genuine one. Measurements taken upon a great number of different rods and at different times showed the same result, however, and I was finally forced to the conclusion that the radiation from the carbon rods showed a much more complicated law of distribution than had been anticipated, and that a sort of selective radiation occurred such as to render the establishing of any simple relationship between the curve of distribution and temperature out of the question.

The hope of being able to make direct temperature measurements up to the melting-point of platinum was also disappointed. While the carbon rods at comparatively low temperatures showed a fair degree of stability under the action of the current, they appeared to undergo a decided change of behavior at about 1400°, and before that temperature a rather rapid disintegration, showing itself by a change of resistance, manifested itself. This effect appeared to be similar to that which shortens the life of the filaments of incandescent lamps when these are subjected to a large amount of current. It appears, moreover, that at these high temperatures the carbon tends to combine with the metals of the thermo-element, affecting the electromotive force very much as the vapors in a furnace have been found to do. The thermo-elements inserted in the rod begin, in consequence of this action, to fail of their purpose. It was found that after exposure to temperatures much above 1400°, the electromotive force corresponding to even lower temperatures was considerably below the normal. I was consequently compelled to abandon the attempt to measure directly temperatures above this point, although it was possible to bring the rods to a higher degree of incandescence for a length of time sufficient to perform the spectrophotometric observations. In order to obtain at least an approximate estimate of these temperatures, I made use of the fall of potential between the terminals of the rod, and also of the current of the heating circuit; and by extending these curves, which, throughout the range of measured temperatures were found to be nearly straight, to the high temperatures which I wished to estimate, to obtain some idea, even if not an exact one, of the latter.

In expressing the results of the photometric measurements already described, I have made use of two forms of curve. One set of curves, in accordance with the nomenclature proposed in my original paper on the visible radiation from platinum, and later adopted by Paschen and other writers, I may call *isotherms*. These curves give in terms of the

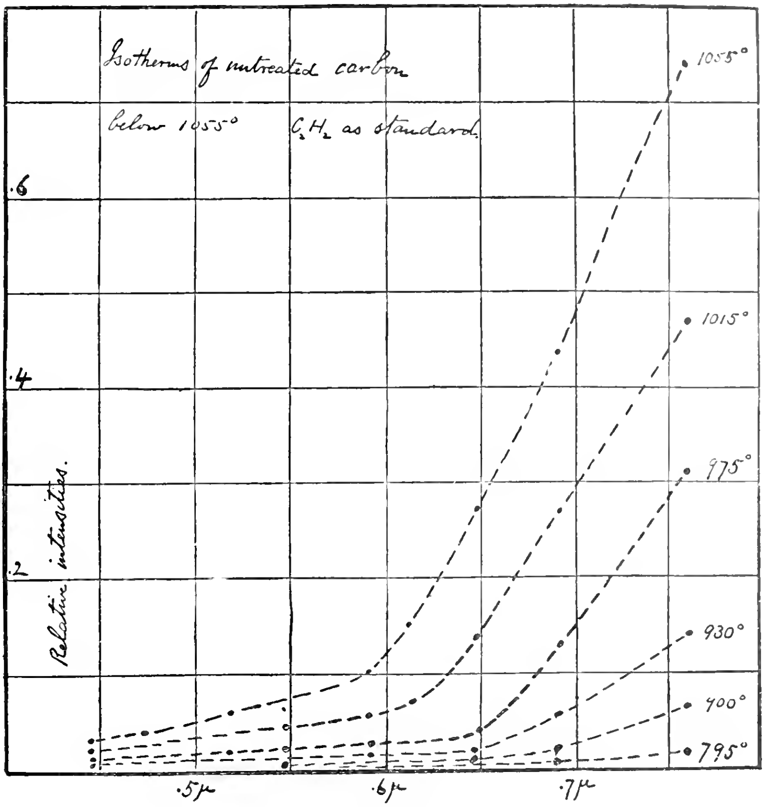

FIGURE 17.

corresponding wave lengths of the comparison source (in this case the acetylene flame), the relative distribution of energy in the visible spectrum from the carbon rods. The other curves, which I have termed *isochroms,* indicate the rise in the energy of any particular wave length of the visible spectrum, with increase of temperature. Each of these curves, taken by itself, is entirely independent of the nature of the light of the comparison source, but the absolute relation of such curves to one another can only be obtained when we know the distribution of energy in the spectrum of that source. By means of the isochroms, it is, however, possible even without this knowledge to compare the rise in intensity of any single wave length of the spectrum with increasing temperature.

The set of curves shown in Figure 17 are plotted directly from observations upon a black (untreated) carbon at temperatures ranging between 795° C and 1055° C. In this diagram abscissae are wave lengths and

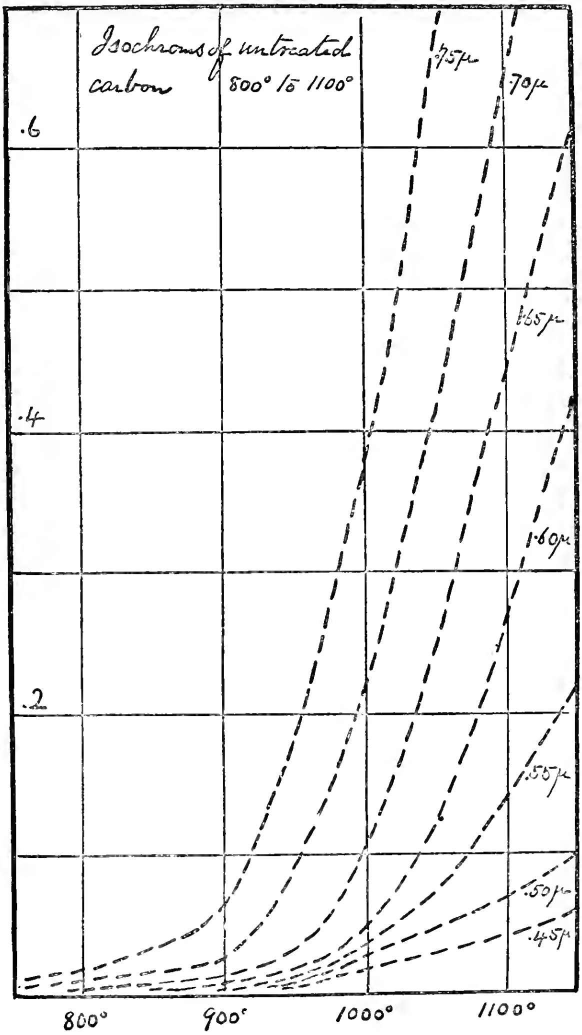

FIGURE 18.

ordinates are ratios of the brightness of the spectrum of the carbon rod in each region to that of the corresponding region in the spectrum of the acetylene flame. A noteworthy fact exhibited by means of these curves is the relatively rapid increase of intensity in the middle of the spectrum. In passing from 930° to 1055° the brightness of wave length .76 μ, increases 5.3 times; that of .70 μ, 7.2 times; that of .60 μ, 13.5 times, *and that of* .50 μ *only* 9 *times.* We have here the beginnings of a process which becomes more marked in its effects as higher temperatures are attained. From 1100° upwards it was found much more difficult to obtain satisfactory readings. The carbon rods which I had brought from Paris for this investigation would not stand prolonged heating and it was necessary to replace them frequently.

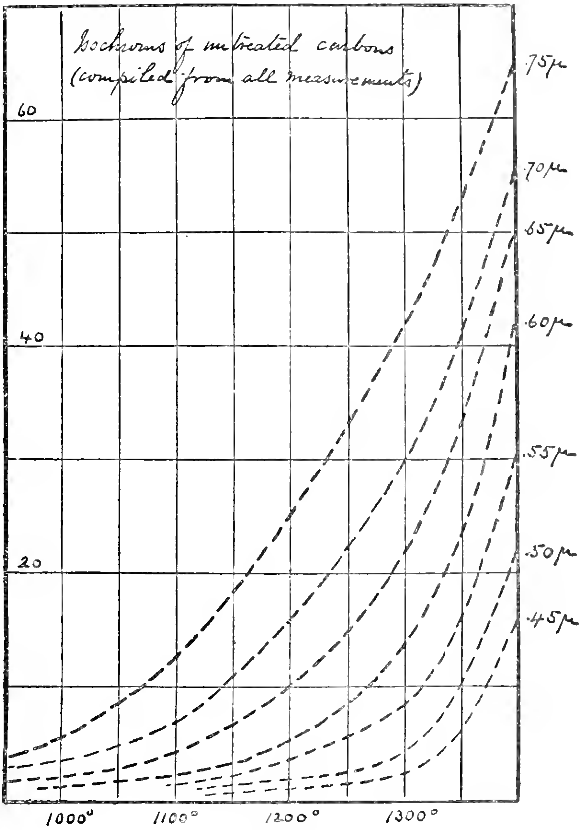

FIGURE 19.

In order to bring the observations upon the various rods to a common scale, isochroms from the readings for each rod were plotted. The general character of these curves is shown in Figure 18, in which the isochroms corresponding to the isotherms of Figure 17 are given. From the ordinate at 1000° of the isochrom for $.6\mu$, which for convenience was taken as unity for the entire set, a reduction factor was obtained by means of which all the curves for all the carbons were brought to the same scale. A new set of isochroms was then plotted for each of the wave lengths $.75\mu$, $.70\mu$, $.65\mu$, $.60\mu$, $.55\mu$, $.50\mu$, and $.45\mu$, in the drawing of which all the observations upon the rods were used. While this method did not bring the various sets of observations into perfect agreement, the results were sufficiently definite to indicate with a close degree of approximation the trend of these curves for temperatures up to 1400°. The result of this compilation for the wave lengths just mentioned is shown graphically in Figure 19. From these curves in turn, isotherms for the temperatures 900°, 1000°, 1100°, 1200°, 1300°, and 1400° were plotted. These curves are given in Figure 20. Had the law of increasing intensities throughout the spectrum with rising temperature been that anticipated at the beginning of this investigation, the trend of the isochroms would necessarily have been such as to bring all the curves together at a common point corresponding to the temperature of the acetylene flame. In other words, if the spectrum of the acetylene flame were identical throughout with that of the carbon rod at the same temperature, the isotherm of the spectrum of the rod at that temperature would be a horizontal line. It is obvious, however, that if the wave lengths of the middle of the spectrum should continue to increase faster than the red and the violet, a condition would presently be attained in which the ordinate of the isotherm would be greater in the yellow or green than at either end of the spectrum. We see indications of the approach of this condition in the diagram of isochroms (Figure 19), from which it is evident that the curves for $.65\mu$ and $.60\mu$ would cut each other and would cut the curve for $.70\mu$ at some temperature not far above 1400°: whereas the isochroms for the shorter wave lengths would not be likely to cut the curves for the red until some much higher temperature had been reached.

The curves in Figure 20 show the nature of this unexpected development of the spectrum in a somewhat different aspect. It will be seen from this figure that the growth in the extreme red so far lags behind that of the full red, and this in turn behind that of the orange, and this in turn behind that of the wave length $.6\mu$, that at 1400° the isotherm, instead of being convex to the base line throughout, actually becomes convex. I have indicated by means of lighter lines the form of curve which might have been expected had the type of isotherm which exists at lower temperatures been maintained.

Above 1400° it was found impossible to obtain consistent readings on account of the rapid disintegration of the carbon rods; but I was able to satisfy myself after repeated trials that at temperatures not far above 1500° this change in the character of the isotherms had progressed to the point at which the yellow regions of the spectrum possess an ordinate greater than that of the extreme red or of the blue or violet. At a temperature about 300° below that of the acetylene flame, then, the spectrum of the carbon rod was relatively weaker in the red, stronger in the yellow, and weaker again in the shorter wave lengths than the spectrum of the flame. There is no reason to suppose that had it been possible to heat the rods to the temperature of the flame itself the law of increase of intensity for the various wave lengths would have undergone such radical modifications to bring the two spectra at that temperature into identity.

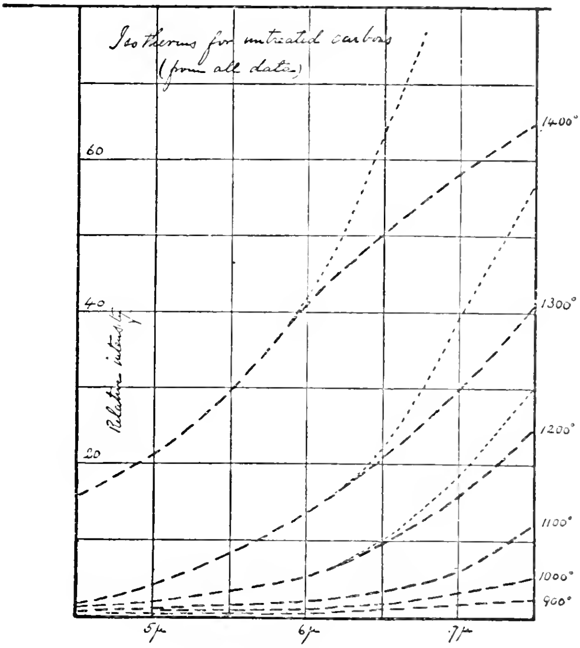

FIGURE 20.

Spectrophotometric Measurements upon Rods with Treated Surfaces.

In order to compare the radiation of rods of black surface with those the surfaces of which have acquired a gray coating by treatment in hydrocarbon vapor, rods were mounted in the usual manner, and after the exhaustion of the air from the metal box, gasoline vapor was allowed to enter until the atmosphere surrounding the rod was saturated. The rod was then brought several times to a high state of incandescence for a few seconds at a time, by which means the entire surface became coated with a gray deposit of carbon similar to that obtained by the treatment of incandescent lamp filaments. The metal box was then again pumped out and spectrophotometric measurements similar to those already described were made upon the radiation from the treated surface. It was thought that as the result of this treatment the carbon rods would stand

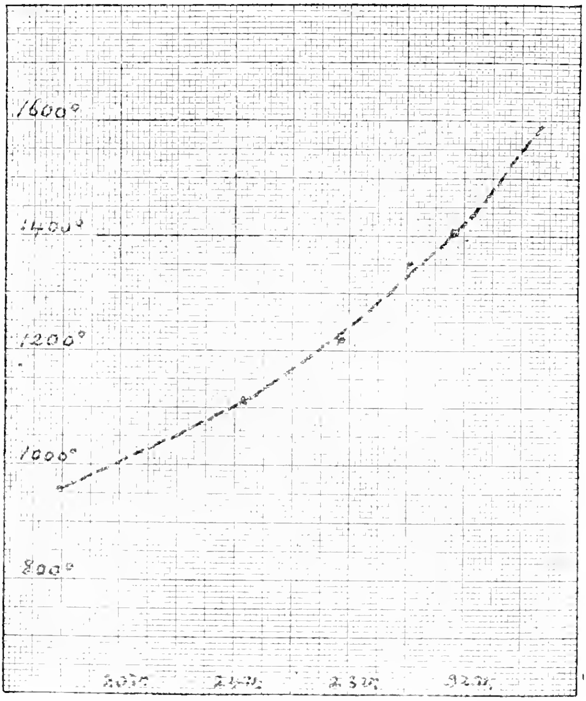

FIGURE 21.

a more prolonged exposure at high temperatures, and that thus it might be possible to extend the measurements beyond the point reached with the rods of black surface. This was found to be the case.

As has already been indicated in a previous paragraph, the indications of a thermo-junction at these high temperatures was subject to serious suspicion. I was obliged to content myself, therefore, with estimations of the temperature based upon the difference of potential between the terminals of the rod. Fortunately the relation between the electromotive force and the temperature up to 1400° was of such a character that but little error was to be feared in extrapolating. The relation between electromotive force in volts and temperature is shown in Figure 21. From this curve temperatures above 1400° were determined.

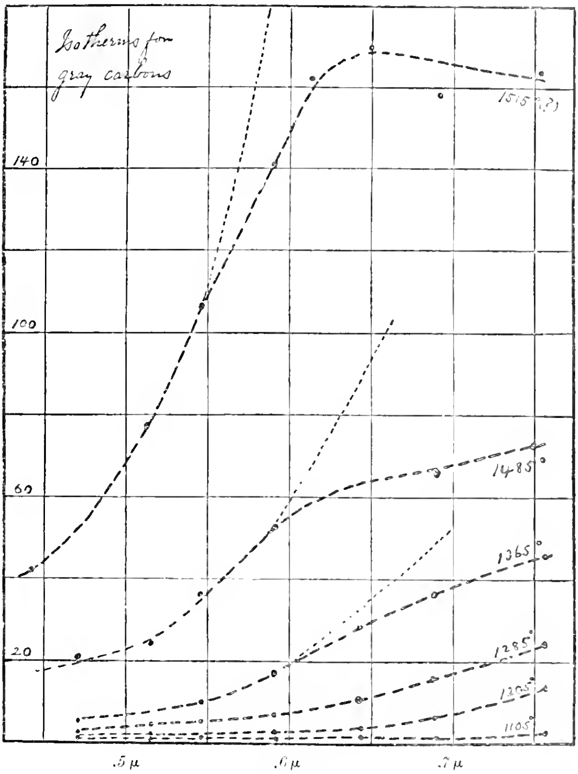

FIGURE 22.

The work upon treated carbons was confined chiefly to high temperatures, a sufficient number of readings within the range already explored with the untreated carbons being taken to show that the distribution of intensities at the lower temperatures did not differ materially from that in the spectrum of the former. The set of isotherms given in Figure 22 will suffice to indicate the general character of the results. It will be seen that in this case, as in that of the untreated carbon, the concavity of the curve between .6μ and the red end of the spectrum is well marked at 1365° ; and that at 1515° there was a well-pronounced maximum at about .65μ. The greater stability of the treated carbon made it possible to obtain consistent measurements on a number of rods at temperatures above 1500° and to establish beyond doubt the form of the curves. It is obvious that for the study of the spectrum of incandescent carbon at this and higher temperatures the conditions would be much more favorable in the case of the incandescent lamp than with rods mounted in a large vacuum chamber like that used in the present investigation. Lamp filaments in the process of manufacture are brought by thorough carbonization into a condition to withstand permanently much higher temperatures than the rods at my disposal were capable of doing. There is as yet, it is true, no direct means of determining the temperature of the lamp filament; but the curve for the relation of electromotive force to temperature (Figure 11) is of such a character as to lead us to expect that comparisons of the spectra of incandescent lamps, in which electromotive forces were used as a criterion of the degree of incandescence, would at least enable us to confirm the existence of the remarkable phenomenon brought out by the present experiments and to extend observations of it to still higher temperatures.

Mr. Ernest Blaker has, since the completion of the measurements described in this paper, compared the visible spectrum of lamps with treated filaments, and of lamps the filaments of which before exhaustion had been coated with lampblack, with the spectrum of the acetylene flame. His measurements confirm very completely those which I have described in this paper, and contribute important evidence in favor of the existence of this anomaly in the law of distribution of intensities in the spectrum of glowing carbon.

THEORETICAL ASPECTS OF THE FOREGOING DATA.

The efforts of students of radiation have of late years been directed particularly to the testing of the various formulae by means of which the mathematical physicists have attempted to express the intensity of radiation as a function of wave length and temperature. The equation reached from quite different points of view by Wien * and by Planck,†

$$I = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}},$$

in particular, has been the subject of exhaustive discussion and of experimental tests. To this end Paschen ‡ determined with the bolometer the distribution of energy in the infra-red spectra of various bodies from 15° C to 1300°. The materials thus subjected to measurement were oxide of copper, platinum, lampblack, and graphitic carbon. The range of wave lengths explored extended from 9.2μ to 0.7μ. Lummer and Pringsheim § made similar determinations upon the ideal black body, and Lummer and Jahnke ‖ finally repeated these measurements in the case of the black body and of platinum. Wanner,¶ working with Paschen, made careful spectrophotometric measurements of the visible radiation from the ideal black body. To test the applicability of the Wien-Planck formula to these measurements, the equation is given the form, —

$$\log I = \gamma_1 - \gamma_2 \frac{1}{T};$$

in which

$$\gamma_1 = \log \left(c_1 \lambda^{-5}\right),$$

$$\gamma_2 = \frac{c_2}{\lambda} \log e.$$

The isochromatic curves are then plotted with the logarithm of the intensities as ordinates and the reciprocal of the absolute temperature as abscissae. The agreement of the equation with the observations is found in the fact that isochroms thus plotted, at least as far as the work of Paschen and Wanner is concerned, always take the form of straight lines, and that the quantity c_2 computed for various wave lengths is found to be a constant. Lummer and Pringsheim, on the contrary, find in the discussion of their measurements that the constant, c_2 increases steadily with the wave length from 13,500 at $1.2\,\mu$ to 16,500 at $5\,\mu$, and 18,500 at $0.3\,\mu$. The value of c_2 computed by measurements from

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* Wien, Wiedemann's Annalen, LVIII. 662 (1896).

† Planck, Drude's Annalen, I. 69 (1900).

‡ Paschen, Wiedemann's Annalen, LVIII. 455 (1896); also LX. 662 (1897).

§ Lummer and Pringsheim, Deutsche phys. Gesellschaft, I. 23, II. 163 (1900).

‖ Lummer and Jahnke, Drude's Annalen, III. 283 (1900).

¶ Wanner, Drude's Annalen, II. 141 (1900).

Beckman at wave length 24 was found to be 24,250. Lummer and Pringsheim find, moreover, that the logarithmic isochroms, especially when extended to higher temperatures, are not straight lines, but show a slight convexity towards the $\frac{1}{T}$, axis.

Exception has also been taken to the Wien-Planck formula on the ground that it gives for infinite temperatures a finite limit to the value of the intensity, a result which Rayleigh * in a recent paper has characterized as *physically improbable.*

Rayleigh proposes the form

$$I = c_1 \, T \lambda^{-4} \, e^{-\frac{c_2}{\lambda T}}$$

but Lummer and Pringsheim find that this likewise fails to properly express their experimental results. Lummer and Jahnke propose, in view of these discrepancies, to give the equation the general form

$$I = C \, T^5 \, (\lambda \, T)^{-\mu} \, e^{-\frac{c}{(\lambda T)^{\nu}}},$$

an expression which coincides with Wien's formula for $\mu = 5$ and with Rayleigh's for $\mu = 4$. They find the measurements of Lummer and Pringsheim satisfied when μ lies between 4.5 and 5, and ν lies between .9 and 1.0. If we accept the value $\mu = 5$ and $\nu = 0.9$, this equation always leads to a finite value of intensity for infinite temperature. All other values of these quantities give infinity as the limit of intensity.

Whether logarithmic isochroms or the value of the quantity c_2, computed from measurements upon carbon rods, would aid in deciding between the various equations under discussion is a question. The data given in this paper would not lead us to class the carbon rods studied as *black bodies.* The emissive power of various forms of carbon is well-known to be smaller than that of the ideal black body, and there is no reason to suppose that it is independent of the temperature. The relative lagging behind of the intensities in the red might perhaps be taken as an indication of a tendency to approach the infinite maximum demanded by the Wien-Planck formula; but the isochrom for .76 shows that the effect, if it exists, must be looked for at some much higher temperature than that covered by these measurements. In spite of these doubts as to the applicability of the measurements on carbon rods to the problem of the

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* Philosophical Mag., XLIX. 539 (1900).

law of radiation of the ideal black body, I have plotted the various isochroms obtained in the course of this investigation in logarithmic form; absolute temperatures being taken as abscissae and the logarithm of the intensity as ordinates. These logarithmic isochroms, as will be seen from

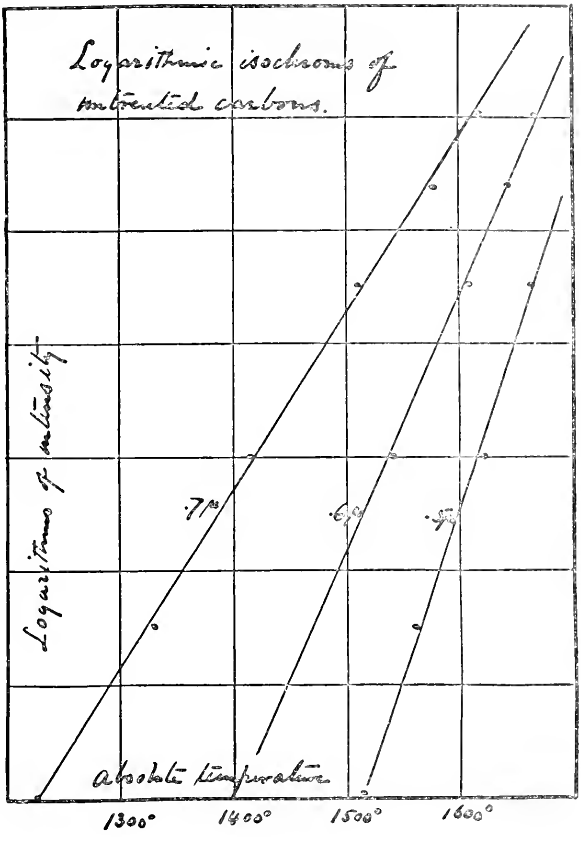

Figure 23.

Figure 23, in which three curves from Figure 19 are reproduced, are straight lines. The range of temperatures is doubtless much too small to bring out the curvature found by Lummer and Pringsheim, but the curves show clearly the change of direction with the wave length mentioned by those writers on page 222 of their paper before the German Physical Society.*

For very high temperatures no experimental data for the radiation from carbon exist excepting the measurements described by Lucas. † It has been rather the fashion to leave Lucas's work altogether out of account as being hopelessly at variance with more recent results. Kayser, ‡ for example, after giving Lucas's data, says, —

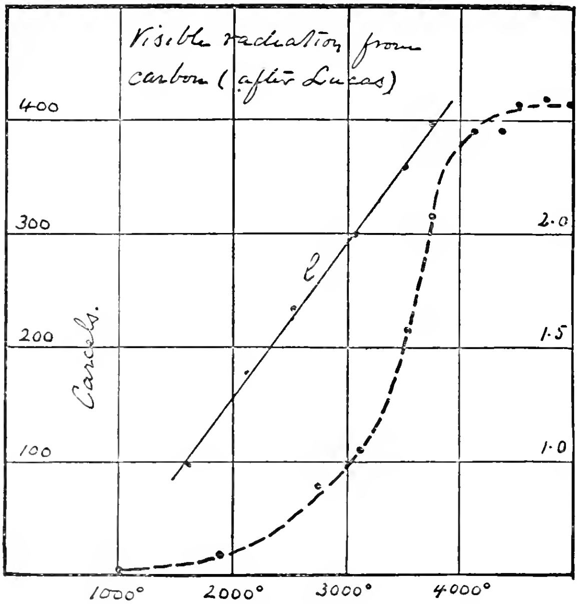

FIGURE 24.

Zu falschen Schlüssen gelangt auch Lucas, durch Versuche welche das Verdampfen der Kohle in Frage zu stellen scheinen.

His results, nevertheless, which I have given graphically in Figure 24, appear to me to be of significance. His formula for the relation of temperature to current, $t = 25\ i$, must of course be regarded as only approximately correct even at moderate temperatures. The curve for the relation between the current in a carbon and the temperature, up to about

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* Lummer and Pringsheim, Verhandl. d. Deutschen Physikal. Gesellsch (1899) p. 222.

† Lucas, Comptes Rendus, C. 1451 (1884).

‡ Kayser, Handbuch der Spectroscopie, I. 157.

1500°, does however not vary widely from a straight line. Beyond these temperatures it is a matter of extrapolation, but the same thing is true of all other attempts to estimate very high temperatures. The curve *l*, for the relation of the logarithm of the intensities and the temperatures, which I have also given in Figure 24 (between 1500° and 3750°), is in the case of Lucas's measurements nearly straight; so that in so far as this is a criterion, his curve up to this point may be said to conform to the Wien-Planck equation. It is significant that Lucas's curve shows an inflection point between 3500° and 4000°, becoming concave to the axis of temperatures. This is the temperature at which, according to nearly all the newer determinations, carbon, as in the crater of the arc, approaches its maximum condition of incandescence. At about 3750° the electrical energy developed in the rod is doubtless largely expended in the disintegration or vaporization of the carbon, so that a maximum degree of incandescence is approached. At the point at which this process begins current can no longer be taken as a measure of the temperature. The very slight falling off in the photometric measurement of intensity does not appear to me to warrant the conclusion drawn by the author that a maximum has been passed at the current value to which he assigns the temperature 4750°. The difficulty of obtaining consistent readings under conditions existing in such work would amply account for so slight a discrepancy.

Lucas's work appears, in a word, to warrant the following rather important conclusions. First, that up to about 3750° current and temperature in the case of carbon rods heated electrically are nearly proportional. We have in favor of this point two checks, — the straightness of the logarithmic curve and the fact that the inflection of Lucas's curve corresponds, as has already been pointed out, to the recognized temperature of the crater of the arc. Secondly, that for a wide range of temperatures photometric intensity, like the intensity of total radiation, follows the logarithmic law of increase. Third, that after the temperature of the crater has been attained a considerable additional increase in incandescence results from the application of further current before the maximum is finally attained. This agrees with the observations of Moissan,* that many reductions in the electric furnace which do not occur with moderate currents become possible by increase of the current strength. If, as seems proper, we ascribe the rapid approach of Lucas's curve to a finite maximum to the utilization of the energy of the cur-

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* Moissan, Comptes Rendus, CIX. 776 (1894).

rent in disintegration of the carbon, it follows that no definite temperatures can be given above the point of inflection. Lucas's measurements, therefore, cannot be said to throw any light upon the question whether the intensity of radiation of incandescent bodies reaches a finite limit as demanded by the Wien-Planck formula. The lower portion of the curve shows no approach to such a maximum. Whether the study of radiation, wave length by wave length, up to the temperature of the crater will be found to do so remains to be seen. Far beyond that temperature experiments with carbon can probably never be carried; so that the final determination of this point must probably be reached by experiments on some more refractory material.

In the prosecution of portions of this investigation I have received valuable aid from Drs. C. H. Sharp and Leopold Kann and from Mr. L. W. Hartman, to all of whom I desire to express my obligations and extend my hearty thanks.

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ON RULED LOCI IN n-FOLD SPACE.

By Halcott C. Moreno.

ON RULED LOCI IN n-FOLD SPACE.

By Halcott C. Moreno.

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The present paper is a discussion of those loci in n-fold space that can be generated by flats whose equations involve a single arbitrary parameter. The ruled loci of space of three dimensions can be represented in this way.

I. Loci derived from an $(n-1)$-flat whose Equation involves a Single Arbitrary Parameter; Developables.

1. *Description of the derived loci.*

Let us consider the loci derived from the equation

$$A = 0,$$

the equation of an $(n-1)$-flat involving a single arbitrary parameter λ. If the parameter enters rationally, we suppose it to enter to as high a degree as n, the number of ways of the space. If the parameter enters rationally to the degree m where $m < n$, the locus is of a special kind to be discussed later. As the parameter varies continuously we have a 1-fold infinite system of $(n-1)$-flats.

Two consecutive $(n-1)$-flats of the system intersect in an $(n-2)$-flat whose equations are

$$A = 0, \qquad \frac{\partial A}{\partial \lambda} = 0.$$

If from these equations we eliminate the parameter there remains a single equation of an $(n-1)$-spread, S_{n-1}, which is ruled by the 1-fold infinite system of $(n-2)$-flats.

Three consecutive $(n-1)$-flats of the system intersect in an $(n-3)$-flat whose equations are

$$A = 0, \qquad \frac{\partial A}{\partial \lambda} = 0, \qquad \frac{\partial^2 A}{\partial \lambda^2} = 0.$$

These $(n - 3)$-flats may be considered as arising from the intersection of two consecutive $(n - 2)$-flats of the system of $(n - 2)$-flats. The elimination of the parameter from these equations gives a restricted system equivalent to two independent equations. The system represents an $(n - 2)$-spread, S_{n-2}, which is ruled by the $(n - 3)$-flats.

In like manner r consecutive $(n - 1)$-flats of the system intersect in an $(n - r)$-flat whose equations are

$$A = 0, \quad \frac{\partial A}{\partial \lambda} = 0, \quad \ldots \quad \frac{\partial^{r-2} A}{\partial \lambda^{r-1}} = 0.$$

Any of these $(n - r)$-flats may be considered as arising from the intersection of two consecutive $(n - r + 1)$-flats of the system of $(n - r + 1)$-flats that are the intersections of $r - 1$ consecutive $(n - 1)$-flats of the system. The elimination of the parameter from these equations gives a restricted system equivalent to $r - 1$ independent equations. These equations represent an $(n - r + 1)$-spread, S_{n-r+1}, which is ruled by the 1-fold infinite system of $(n - r)$-flats.

The locus of the intersections of n consecutive $(n - 1)$-flats of the system is a curve, while $n + 1$ consecutive $(n - 1)$-flats do not in general have any common intersection.

We will use S_k to denote that one of the related spreads of this system that is of k ways. It is geometrically evident that each one of these spreads is a developable spread.*

Considered in this light we see that the $(n - 2)$-spread is a double spread on S_{n-1} corresponding to the cuspidal edge or edge of regression in ordinary threefold space.†

The S_{n-3} is a double spread on S_{n-2}, etc., and S_1 on S_2. We see also that S_{n-3} is a triple spread on S_{n-1}; Killing calls it doubly stationary. Finally, S_1 is an $(n - 1)$-tuple curve on S_{n-1}; it is a multiple curve on all the other spreads of the system.‡

If the equation

$$A = 0$$

contains k arbitrary parameters connected by $k - 1$ equations

$$\phi = 0, \quad \chi = 0, \ldots . \psi = 0,$$

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* Killing, Nicht-Euklidische Raumformen, p. 195 et seq.

† Puchta calls the S_{n-1} the most general developable spread in n-fold space. Puchta, Ueber die allgemeinsten abwickelbaren Räume, ein Beitrag zur mehrdimensionalen Geometrie. Wien. Berichte, CI.

‡ Killing, loc. cit.

we can, theoretically, solve these equations for $k - 1$ of the parameters in terms of the remaining one, so that this case is the same as the previous one.

The actual elimination may be avoided. Let the parameters be λ, μ, ... ν. Differentiate totally all the equations,

$$\frac{\partial A}{\partial \lambda} d\lambda + \frac{\partial A}{\partial \mu} d\mu + \ldots\ldots + \frac{\partial A}{\partial \nu} d\nu = 0$$

$$\frac{\partial \phi}{\partial \lambda} d\lambda + \frac{\partial \phi}{\partial \mu} d\mu + \ldots\ldots + \frac{\partial \phi}{\partial \nu} d\nu = 0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

From these we may eliminate the differentials,

$$B \equiv \begin{Vmatrix} \frac{\partial A}{\partial \lambda}, & \frac{\partial A}{\partial \mu}, & \ldots\ldots & \frac{\partial A}{\partial \nu} \\ \frac{\partial \phi}{\partial \lambda}, & \frac{\partial \phi}{\partial \mu}, & \ldots\ldots & \frac{\partial \psi}{\partial \nu} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial \psi}{\partial \lambda}, & \frac{\partial \psi}{\partial \mu}, & \ldots\ldots & \frac{\partial \psi}{\partial \nu} \end{Vmatrix} = 0$$

This is the equation of an $(n - 1)$-flat. The equation involves k parameters but they are connected by $k - 1$ equations. Two consecutive $(n - 1)$-flats of the system intersect in an $(n - 2)$-flat whose equations are $A = 0$, $B = 0$.

Three consecutive $(n - 1)$-flats of the system intersect in the $(n - 3)$-flat,

$$A = 0, \quad B = 0, \quad C = 0,$$

where C is the determinant B, with A replaced by B. The equation of the S_{n-1} is found by eliminating the parameters between the equations of the $(n - 2)$-flats and the equations connecting the parameters. The equations of the other spreads are derived in a similar manner. The system of related spreads is of the same character as before.

2. *Mutual relations of connected loci.*

Let us consider more in detail these connected loci. We will use F_k to denote a k-flat of the 1-fold infinite system of k-flats. Two consecutive F_{n-1}'s intersect in an F_{n-2}, three in an F_{n-3}, r in an F_{n-r}, $n - 2$ in an F_2 or plane, $n - 1$ in an F_1 or line, n in an F_0 or point. There is a 1-fold infinite system of these F_{n-2}'s which are generators of S_{n-1}, a 1-fold infinite system of F_{n-3}'s, generators of S_{n-2}, a 1-fold infinite system of lines generators of S_2, the developable surface.

Through any F_{n-2} there pass two consecutive F_{n-1}'s, through any F_{n-3} there pass three consecutive F_{n-1}'s, through any F_0, n consecutive F_{n-1}'s. Through any F_{n-3} there pass two consecutive F_{n-2}'s, through any F_{n-4} there pass two consecutive F_{n-3}'s and three consecutive F_{n-2}'s, and so on.

We may then reverse this process and start with the curve of the system. Through any two consecutive points of the curve there passes a line, an F_1, through any three consecutive points an osculating plane, an F_2, through any four consecutive points an osculating 3-flat, an F_3, through any n — consecutive points an osculating $(n - 1)$-flat, an F_{n-1}.*

That these operations may give unique results this curve must lie in the n-fold space and in no flat space of a less number of ways. If the curve lie in a k-flat, where $k \overset{=}{<} n - 1$, all the k-flats through $k + 1$ consecutive points coincide and definite $(k + 1)$-flats are not determined at all. By a theorem of Clifford, such a curve must be of an order as great as n.†

This theorem has been generalized by Veronese.‡

Let us consider any curve in n-fold space whose equations are,

$$\phi = 0, \ \chi = 0. \ . \ . \ . \ . \ \psi = 0,$$

a restricted system equivalent to $n - 1$ independent equations. The equations of the tangent at any point P' of this curve are linear equations whose coefficients are functions of the n non-homogeneous coordinates, x', y'. . . . . v'. The same thing is true of the equations of any of the osculating flats at the point P'. The osculating $(n - 1)$-flat is given by a single equation, the coefficients of which are functions of these n quantities x', y', . . . v'. If we regard these as n parameters they are connected by the equations,

$$\phi' = 0, \ \chi' = 0, \ . \ . \ . \ . \ \psi' = 0.§$$

---

* We shall say a k-flat osculates a curve if it contains $k + 1$ consecutive points of it. Killing, loc. cit.

† Clifford, Classification of Loci; Mathematical Papers, pp. 305–331.

‡ Veronese, Behandlung der projectivischen Verhältnisse der Räume von verschiedenen Dimensionen durch das Princip des Projicirens und Schneidens, Mathematische Annalen XIX.

§ $\phi' \equiv \phi \, (x', y', \ldots v')$, etc.

a restricted system equivalent to $n - 1$ independent equations. We have then the case of an $(n - 1)$-flat whose equation involves n parameters connected by $n - 1$ independent relations; this is equivalent to the case of a single equation containing one arbitrary parameter. We may, in general, consider the system of developables as given by an $(n - 1)$-flat whose equation contains a single arbitrary parameter or k parameters connected by $k - 1$ equations.*

3. *The tangent* (n — 1)*-flats that are common to* n — 1 (n — 1)*-spreads envelop a developable.*

The equation in homogeneous coördinates of any $(n - 1)$-flat may be written

$$x = \alpha y + \beta z + \ldots . + \gamma w.$$

This equation involves n independent parameters; if we connect them by any $n - 1$ independent equations we shall have the equation of an $(n - 1)$-flat that contains but a single independent parameter, so that the 1-fold infinite system of $(n - 1)$-flats represented by it envelop a developable. The tangent $(n - 1)$-flat at any non-singular point of a developable S_{n-1} contains the generating F_{n-2} through that point and touches the S_{n-1} all over this flat.† We may speak of this developable S_{n-1} as enveloped by its tangent F_{n-1}'s. If then we impose on an arbitrary $(n - 1)$-flat any conditions that give rise to $n - 1$ independent equations between the coefficients in its equation, the $(n - 1)$-flat will envelop a developable S_{n-1}.

Let
$$U = 0$$

be the equation of an $(n - 1)$-spread. The equation of the tangent $(n - 1)$-flat at any ordinary point P' is

$$x \frac{\partial U'}{\partial x'} + y \frac{\partial U'}{\partial y'} + \ldots . + w \frac{\partial U'}{\partial w'} = 0.$$

If we impose on the equation of the arbitrary $(n - 1)$-flat the conditions that it shall be this tangent $(n - 1)$-flat, the coefficients in the two equations must be proportional. We must have then

$$\frac{\frac{\partial U'}{\partial x'}}{-1} = \frac{\frac{\partial U'}{\partial y'}}{\alpha} = \ldots . \frac{\frac{\partial U'}{\partial w'}}{\gamma}.$$

From these equations by means of the equation

$$U' = 0,$$

---

* Salmon, Geometry of Three Dimensions, p. 286.

† Killing, loc. cit.

we may eliminate the coördinates of P' leaving a single equation in $\alpha, \beta, \ldots . \gamma$. For an $(n-1)$-flat to be tangent to an $(n-1)$-spread, one relation between the coefficients that enter into their equations must be satisfied. We conclude then that the $(n-1)$-flats that touch $n-1$ $(n-1)$-spreads envelop an S_{n-1}.

Let us consider only those tangent $(n-1)$-flats to an $(n-1)$-spread that touch it at the point of an $(n-2)$-spread that lies on it. Let

$$U=0$$

be the equation of the $(n-1)$-spread and let

$$U=0,\ V=0, \ldots,$$

a restricted system equivalent to two independent equations, be the equations of the $(n-2)$-spread on it. We derive now the equations

$$\frac{\frac{\partial U'}{\partial x'}}{-1} = \frac{\frac{\partial U'}{\partial y'}}{\alpha} = \ldots . \frac{\frac{\partial U'}{\partial w'}}{\gamma}$$

and

$$U'=0,\ V'=0, \ldots$$

If we eliminate the parameters from these equations there remains a restricted system equivalent to two independent equations in the coefficients $\alpha, \beta, \ldots \gamma$. For an $(n-1)$-flat to be tangent to an $(n-1)$-spread at a point of an $(n-2)$-spread on it requires two conditions between the coefficients in the equation of the $(n-1)$-flat. These two conditions may be used as part of the $n-1$ conditions that connect the coefficients of an $(n-1)$-flat that envelops a developable S_{n-1}. We have then the theorem that the $(n-1)$-flats that are tangent to ρ $(n-1)$-spreads at the points of ρ $(n-2)$-spreads that lie one on each $(n-1)$-spread, and are tangent to σ other $(n-1)$-flats, where $n-1=2\rho+\sigma$, envelop a developable.

In a similar manner for an $(n-1)$-flat to be tangent to an $(n-1)$-spread at a point of an $(n-3)$-spread that lies on it imposes three conditions on the coefficients that enter into the equation of the $(n-1)$-flat. To be tangent to the $(n-1)$-flat at a point of an $(n-4)$-flat on it requires four conditions, etc. To be tangent to an $(n-1)$-spread at a point of a curve that lies on it requires $n-1$ conditions between the coefficients, which is just sufficient to make the $(n-1)$-flat envelop a developable.

We have then the general theorem that the $(n-1)$-flats that are tangent to ρ $(n-1)$-spreads at points of ρ $(n-k)$-spreads that lie one on each, tangent to σ $(n-1)$-spreads at points of σ $(n-k+1)$-spreads that lie one on each, tangent to τ $(n-1)$-spreads at points of τ $(n-2)$-spreads that lie one on each, and finally tangent to v other $(n-1)$-spreads, where $\rho, \sigma, \ldots \tau, v$, are non-negative integers connected by the relation

$$n-1 = k\,.\,\rho + (k-1)\,\sigma + \ldots\ldots + 2\,\tau + v,$$

envelop a developable S_{n-1}.

Similar cases occur in three-fold space where we have the tangent planes that are common to two surfaces enveloping a developable surface as do the tangent planes to a surface at the points of a curve on that surface.*

4. *Some additional properties of developables; sections.*

Other properties of an S_{n-1} may be deduced by regarding it as the envelope of an $(n-1)$-flat whose equation involves a single parameter.† Through any point in space can be drawn a definite number of tangent F_{n-1}'s to the S_{n-1}. For substitute the coördinates of the point in the equation of the variable $(n-1)$-flat and there is a certain finite number of values of the parameter that satisfy the equation.

Any F_{n-1} of the system meets its consecutive F_{n-1} in a definite F_{n-2}, a generator of S_{n-1} whose equations are,

$$A = 0, \frac{\partial A}{\partial \lambda} = 0.$$

Any three consecutive F_{n-1}'s meet in a definite F_{n-3}, a generator of S_{n-2}, whose equations are,

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \frac{\partial^2 A}{\partial \lambda^2} = 0.$$

Any $n-1$ consecutive F_{n-1}'s meet in a definite line F_1, a generator of S_2, whose equations are,

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \ldots \frac{\partial^{n-2} A}{\partial \lambda^{n-2}} = 0.$$

Finally, any n consecutive F_{n-1}'s meet in a definite point of the curve of regression of S_2. The equations of the F_0 are,

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \ldots \frac{\partial^{n-1} A}{\partial \lambda^{n-1}} = 0.$$

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* Salmon, Geometry of Three Dimensions, p. 547.

† Salmon, Geometry of Three Dimensions, p. 289 et seq.

In general $n + 1$ consecutive F_{n-1}'s do not have any common intersection, for the $n + 1$ equations,

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \ldots \frac{\partial^n A}{\partial \lambda^n} = 0,$$

have no common solutions. If we regard these equations as homogeneous in the $n + 1$ coördinates we may form their resultant, and the values of the parameter that cause this determinant to vanish, give special points where $n + 1$ consecutive F_{n-1}'s intersect. These points are cusps on the curve S_1.

Reciprocally there will, in general, be a finite number of F_{n-1}'s that go through $n + 1$ consecutive points of S_1.

Veronese has shown that a curve in n-fold space has $3\,n$ singularities which are connected by $3\ (n - 1)$ relations, an extension of the Pluecker-Cayleyan characteristics of a twisted curve in three-fold space.*

In this we have assumed that the variables that enter into the equation of the enveloping $(n - 1)$-flat cannot be expressed in terms of fewer than $n + 1$ independent linear functions of the variables alone. If they could be expressed in terms of ν such linear functions, where $\nu \leqq n$, the developable S_{n-1} is a conoid with an $(n - \nu)$-way head, a case to be considered later.

The developable S_k of the series is ruled by $(k - 1)$-flats, F_{k-1}'s. The S_k, where $2 \leqq k \leqq n - 1$ can be given by means of its enveloping F_k whose equations involve a single parameter. The $n - k$ equations of the F_k must however be of the form

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \ldots, \frac{\partial^{n-k-1} A}{\partial \lambda^{n-k-1}} = 0,$$

as we have previously seen. Even the S_1 may be represented in this manner.

Any $(n - 1)$-flat $$B = 0$$

cuts the S_{n-1} in a developable $(n - 2)$-spread, for it cuts the system of F_{n-1}'s in a system of $(n - 2)$-flats that intersect consecutively in $(n - 3)$-flats. We may see this in another way. By means of this new equation we can eliminate one variable from the equation of the enveloping $(n - 1)$-flat. The resulting equation in n variables may evidently be considered as the envelope of an $(n - 2)$-spread in a new $(n - 1)$-fold space. The $(n - 1)$-flat cuts any S_k of the system in a $(k - 1)$-way

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* Veronese, loc. cit.; Killing, loc. cit. p. 197 et seq.

developable. In general any r-flat where $r \geqq n - k + 1$ cuts any S_k in a developable $(k + r - n)$-spread.

Any F_{n-1} of the system cuts the S_{n-1} in an $(n - 2)$-spread, and the F_{n-2} that it has in common with the consecutive F_{n-1} appears twice in the intersection, so that the proper $(n - 2)$-spread is of order less by two than the order of S_{n-1}. This $(n - 2)$-spread is also a developable.

An F_{n-2} is met by the consecutive F_{n-2} in an F_{n-3}; it is met by any other F_{n-2} in an $(n - 4)$-flat. In general, where $4 \leqq n$, there are a 2-fold infinite system of these $(n - 4)$-flats and their locus is an $(n - 2)$-spread which is a double spread on S_{n-1}. In the case of cones and conoids this double spread may be of fewer than $n - 2$ ways. Thus in four-fold space the planes which join a line to the successive points of an irreducible conic form a three-way developable. This developable is a conoid and the one-way head is the only multiple locus on the conoid. In three-fold space cones are the only developable surfaces that do not possess a proper double curve, if we call the cuspidal curve a double curve. In general there is a double curve distinct from the cuspidal curve. We will assume that we have the general case of a developable and not a cone or conoid. The total double spread on S_{n-1} consists in general of two parts, S_{n-2} and Σ_{n-2}, where Σ_{n-2} is the locus of the 2-fold infinite system of $(n - 4)$-flats arising from the intersection of non-consecutive F_{n-2}'s, while S_{n-2} is the locus of the 1-fold infinite system of $(n - 3)$-flats arising from the intersection of consecutive F_{n-2}'s.

Any three non-consecutive F_{n-2}'s intersect in an $(n - 6)$-flat; there are in general a 3-fold infinite system of such $(n - 6)$-flats whose locus is an $(n - 3)$-spread, a triple spread on S_{n-2}. Any $(n - 6)$-flat is the intersection of three $(n - 4)$-flats of Σ_{n-2} and any such $(n - 4)$-flat contains a 1-fold infinite system of such $(n - 6)$-flats. This 1-fold infinite system of $(n - 6)$-flats does not, in general, fill out the $(n - 4)$-flat, for this would require a 1-fold infinite system of them. The total triple spread on S_{n-1} consists in general of two parts S_{n-3} and Σ_{n-3} where Σ_{n-3} is the locus of the 3-fold infinite system of $(n - 6)$-flats. We can supply a similar mode of reasoning to the spreads of higher multiplicities on S_{n-1}. The spreads S_{n-2}, S_{n-3}, . . . are developable, but Σ_{n-2}, Σ_{n-3}, . . . are not developable.

5. *Special case where the parameter enters rationally.*

Let us illustrate this theory by the case of the developable which is the envelope of the $(n - 1)$-flat,

$$a\,t^m + m\,b\,t^{m-1} + \tfrac{1}{2}\,m\,(m - 1)\,c\,t^{m-2} + \ldots . = 0,$$

where t is a variable parameter, $a, b, c, \ldots$ are linear functions of the coördinates that are not expressible in terms of any ν linear functions of the coördinates where $\nu \overline{\overline{<}} n$, and m is an integer which is not less than n, the number of ways of the space. Two consecutive F_{n-1}'s intersect in the F_{n-2},

$$a\,t^{m-1} + (m-1)\,b\,t^{m-2} + \frac{(m-1)\,(m-2)}{2\,!}c\,t^{m-3} + \ldots . + e = 0,$$

$$b\,t^{m-1} + (m-1)\,c\,t^{m-2} + \ldots + e\,t + f = 0.$$

The elimination of the parameter from these equations gives the equation of S_{n-1}. The result is the discriminant of the original equation placed equal to zero; the order of S_{n-1} is then $2\,(m-1)$.*

Three consecutive F_{n-1}'s intersect in the F_{n-3},

$$\begin{aligned} a\,t^{m-2} + (m-2)\,b\,t^{m-3} + \ldots . &= 0, \\ b\,t^{m-2} + (m-2)\,c\,t^{m-3} + \ldots . + e &= 0, \\ c\,t^{m-2} + \ldots . + e\,t + f &= 0. \end{aligned}$$

The equations of S_{n-2} are found by eliminating the parameter from these equations. The result is a restricted system equivalent to two independent equations; the order of the system, i. e., the order of S_{n-2} is $3\,(m-2)$.†

Similarly k consecutive F_{n-1}'s intersect in the F_k, given by the k equations,

$$\begin{aligned} a\,t^{m-k+1} + (m-k+1)\,b\,t^{m-k} + \ldots . &= 0 \\ b\,t^{m-k+1} + (m-k+1)\,c\,t^{m-k} + \ldots . &= 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots + e\,t + f &= 0. \end{aligned}$$

The elimination of the parameter from these equations gives a restricted system equivalent to $k-1$ independent equations, the equations of S_{n-k+1}. The order of S_{n-k+1} is seen to be $(k+1)\,(m-k)$.

Lastly the intersection of n consecutive F_{n-1}'s is the point, F_0, given by the equations,

$$\begin{aligned} a\,t^{m-n+1} + (m-n+1)\,b\,t^{m-n} + \ldots . &= 0 \\ b\,t^{m-n+1} + (m-n+1)\,c\,t^{m-n} + \ldots . &= 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots + e\,t + f &= 0. \end{aligned}$$

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* Salmon, Higher Algebra, art. 105.

† This is the condition that the three equations have a common root; Salmon, Higher Algebra, art. 277.

The elimination of the parameter from these equations gives a restricted system equivalent to $n - 1$ independent equations, the equation of S_1 whose order is $n\ (m - n + 1)$.

We can find the equations of those exceptional points where $n + 1$ consecutive F_{n-1}'s intersect in a point, if we eliminate the parameter from the $n + 1$ equations

$$\begin{array}{l} a\, t^{m-n} + (m - n)\ b^{m-n-1} + \ldots . = 0 \\ b\, t^{m-n} + (m - n)\ c^{m-n-1} + \ldots . = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots + e\, t + f = 0. \end{array}$$

The result is a restricted system equivalent to n independent equations; it is of order $(n + 1)\ (m - n)$, which is the number of such points, cusps on S_1. We may verify this result by forming the resultant of these $(n + 1)$ equations. If we eliminate the variables from these equations we have a determinant of order $n + 1$. If we expand this result t enters to the degree $(n + 1)\ (m - n)$ so that there are $(n + 1)$ $(m - n)$ values of t that cause this resultant to vanish. These values of t give the special points in question.*

Any double point on S_{n-1} must lie on two F_{n-2}'s. We may find the equations of the total double spread on S_{n-1}, by expressing the conditions that the equations of an F_{n-2} regarded as equations in the parameter, have two roots in common. These conditions are †

$$\text{(I)} \qquad \left\| \begin{array}{llll} a, & (m-1)\ b, & \dfrac{(m-1)\ (m-2)}{2\,!}\ c, & \ldots\ldots\ldots \\ & a, & (m-1) & b, \ldots\ldots\ldots \\ & & & \ldots\ldots\ldots\ldots \\ & & & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\, e \\ & b, & (m-1) & c, \ldots\ldots\ldots \\ & & & b, \ldots\ldots\ldots \\ & & & \ldots\ldots\ldots\ldots \\ & & & \ldots\ldots\ (m-1)\ e, f \end{array} \right\| = 0,$$

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* For $n = 3$, these results agree with those of Salmon, Geometry of Three Dimensions, p. 296. Neither the results there nor these hold when the system has stationary $(n - 1)$-flats.

† Salmon, Higher Algebra, art. 275.

where there are $2(m-2)$ rows and $2m-3$ columns. This restricted system is of order $\frac{1}{2}(2m-3)(2m-4)$. The double spread represented by these equations consists of two distinct parts, S_{n-2} and Σ_{n-2}. The order of Σ_{n-2} must be,

$$\tfrac{1}{2}(2m-3)(2m-4)-3(m-2)=2(m-2)(m-3).$$

A triple point on S_{n-1} must lie on three F_{n-2}'s. We may find the equations of the total triple spread on S_{n-1} by expressing the conditions that the equations of the F_{n-2} have three common roots. These conditions are expressed by means of a rectangular system similar in form to (I), in which however there are only $2(m-3)$ rows and $2m-4$ columns. The order of the restricted system is

$$\frac{1}{3\,!}(2m-4)(2m-5)(2m-6).$$

This triple spread consists of two distinct parts, S_{n-3} and Σ_{n-3}. The order of Σ_{n-3} must be

$$\frac{1}{3\,!}(2m-4)(2m-5)(2m-6)-4(m-3)=\frac{2}{3}(m-3)(m-4)(2m-1).$$

In like manner we can find the equations of the total k-tuple spread on S_{n-1}, by expressing the conditions that the equations of the F_{n-1} have k roots in common. These conditions are expressed by means of a rectangular system similar to (I), in which, however, there are only $2(m-k)$ rows and $2m-k-1$ columns. This is a restricted system equivalent to k independent equations, of order $\frac{1}{k\,!}(2m-k-1)(2m-k-2)\ldots.(2m-2k)$. This spread consists of two parts, S_{n-k} and Σ_{n-k}; the order of the latter is

$$\frac{1}{k\,!}(2m-k-1)(2m-k-2)\ldots.(2m-2k)-(k+1)(m-k).$$

The total $(n-1)$-tuple curve on S_{n-1} is given by means of a restricted system similar to (I), in which, however, there are only $2(m-n+1)$ rows and $2m-n$ columns. We have then a restricted system equivalent to $n-1$ independent equations whose order is

$$\frac{1}{(n-1)\,!}(2m-n)(2m-n-1)\ldots(2m-2n+2).$$

The order of the curve Σ is,

$$\frac{1}{(n-1)!}(2m-n)(2m-n-1)\ldots(2m-2n+2)-n(m-n+1).^*$$

The equations of all the n-tuple points on S_{n-1} are given by means of a rectangular system similar to (I), in which, however, there are only $2(m-n)$ rows and $2m-n-1$ columns. They form a restricted system equivalent to n independent equations, whose order is

$$\frac{1}{n!}(2m-n-1)(2m-n-2)\ldots(2m-2n);$$

this is the number of n-tuple points. The number of the n-tuple points other than the cusps on S_1, are

$$\frac{1}{n!}(2m-n-1)(2m-n-2)\ldots(2m-2n)-(n+1)(m-n).$$

These points necessarily lie on Σ_1; they are either n-tuple points on Σ_1, or else they are n-tuple points on the combined curves S_1 and Σ_1. In three-fold space the double curve on the developable may have tripl points on it; it can have no double points off of the cuspidal curve.

If $m=n$, then the order of S_1 is n, and there are no cuspidal points on the curve; this is the rational normal curve of Veronese.† The order of S_{n-1} in this case is $2(n-1)$; no developable S_{n-1} can be of lower order unless it is a cone or conoid, for no curve of lower order than n can lie in the n-fold space without at the same time lying in a space of fewer than n ways.

Let us consider the case where $m=p<n$, where p is an integer. Any $p+1$ consecutive F_{n-1}'s intersect in an F_{n-p-1} whose equations are

$$A=0, \frac{\partial A}{\partial t}=0, \ldots \frac{\partial^p A}{\partial t^p}=0.$$

If we use two homogeneous parameters λ and μ instead of the single parameter t, these equations may be written

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* For $n=3$, this result agrees with that in Salmon, Geometry of Three Dimensions, p. 296.

† Veronese, loc. cit.

$$\frac{\partial^p A}{\partial \lambda^p} = 0, \frac{\partial^p A}{\partial \lambda^{p-1} \partial \mu} = 0, \ldots \frac{\partial^p A}{\partial \mu^p} = 0,$$

in which form the parameter no longer appears. Any $p + 1$ consecutive F_{n-1}'s intersect in the same F_{n-p-1} as any other consecutive $p + 1$; i. e., all the F_{n-1}'s of the system contain the same F_{n-p-1}. Any p-flat that does not meet this F_{n-p-1} cuts S_{n-1} in a developable $(p - 1)$-spread of order $2(p - 1)$. This developable $(p - 1)$-spread of order $2(p - 1)$ lying in a p-flat is exactly similar to the case in n-fold space where $m = n$. The curve at the base of this system is of order p; it is the rational normal curve of p-fold space. Hence we may derive this system by joining by lines all points of a developable $(p - 1)$-spread of order $2(p - 1)$ in a p-fold space, to all points of an $(n - p - 1)$-flat that does not meet the p-flat that contains the $(p - 1)$-spread. S_{n-1} is a conoid of $(n - 2)$-flats with an $(n - p - 1)$-way head. The generating F_{n-2}'s of S_{n-1} arise from the junction of the $(n - p - 1)$-way head with the generating $(p - 2)$-flats of the $(p - 1)$-spread. The generating F_{n-3}'s of S_{n-2} arise from the junction of the $(n - p - 1)$-way head with the system of generating $(p - 3)$-flats of the $(p - 2)$-spread, and so on. Any conoid ruled by a 1-fold infinite system of q-flats with a $(q - 1)$-way head is a developable spread, but not so if it has only an r-way head where $r \overset{=}{<} q - 2$. The latter spread is a developable only when the consecutive q-flats have $(q - 1)$-way intersection. Any conoid ruled by a 1-fold infinite system of $(n - 2)$-flats that have an $(n - 3)$-flat in common is a developable, but if they have only an $(n - k)$-flat in common where $k \overset{=}{<} 4$, the conoid may or may not be developable. The cones and conoids with a 2-fold infinite system of generators are not developables at all.

The points of intersection of two consecutive generators are not in general points of intersection of three generators. The equations of a generator may be written

$$e + (m - 1) d + \frac{(m - 1)(m - 2)}{2!} c + \ldots . = 0,$$

$$f + (m - 1) e + \frac{(m - 1)(m - 2)}{2!} d + \ldots . = 0.$$

The points of intersection of three generators of the system are given by the equations

$$\left\| \begin{array}{llll} e, & (m-1)\,d, & \dfrac{(m-1)\,(m-2)}{2}\,c, & \ldots\ldots \\ & e, & (m-1) & d, \ldots\ldots \\ & & & \ldots\ldots \\ f, & (m-1)\,e, & \dfrac{(m-1)\,(m-2)}{2}\,d, & \ldots\ldots \\ & f, & (m-1) & e, \ldots\ldots \\ & & & \ldots\ldots \end{array} \right\| = 0.$$

where there are $2\,(m-2)$ rows and $2\,m-2$ columns.

For $t = 0$ we have the particular $(n-2)$-flat

$$e = 0, \quad f = 0.$$

The next consecutive generator has for its equations,

$$\begin{aligned} e + \delta t \,.\, d &= 0 \\ f + \delta t \,.\, e &= 0 \end{aligned}$$

The intersection of the two consecutive generators is the $(n-3)$-flat whose equations are

$$e = 0, \quad f = 0, \quad d = 0.$$

This F_{n-3} does not generally lie on the total triple spread for one of the equations of that system, namely

$$\left| \begin{array}{llll} \dfrac{(m-1)\;\;(m-2)\;c}{2}, & & & \ldots\ldots \\ & (m-1)\;d, & & \ldots\ldots \\ & & e, & \ldots\ldots \\ & & & \ldots\ldots \\ (m-1) & (m-2)\;d, & & \ldots\ldots \\ & (m-1)\;e, & & \ldots\ldots \\ & & f, & \ldots\ldots \\ & & & \ldots\ldots \end{array} \right| = 0,$$

is not generally satisfied when the equations of the F_{n-3} are satisfied.

The points that satisfy both these systems of equations are evidently points on two consecutive generators and at the same time points on three generators.

If there is a linear relation between f, e, and d, then these two consecutive generators intersect in an $(n-2)$-flat, *i. e.*, they are coincident and we have a stationary generator of the system. If

$$e \equiv 0,$$

then

$$f = 0$$

is the equation of a stationary generator of the system. The equation of the developable S_{n-1} in this case is

$$\left| \begin{array}{lllll} f, & 0, & \frac{(m-1)\ (m-2)}{2\,!} & d, & \ldots\ldots \\ & f, & & 0, & \ldots\ldots \\ & & & f, & \ldots\ldots \\ & & & & \ldots\ldots\ldots \\ 0, & d, & \frac{(m-1)\ (m-2)}{2\,!} & c, & \ldots\ldots \\ & 0, & & d, & \ldots\ldots \\ & & & 0, & \ldots\ldots \\ & & & & \ldots\ldots\ldots \end{array} \right| = 0.$$

We see that f is a factor of the left member of this equation. When this factor is thrown out, the residual or proper developable is of a degree less by one than before. The orders of the multiple loci previously given are also reduced, they only holding when there are no stationary F_{n-1}'s in the system. By means of Veronese's formulae we see that when there are β stationary F_{n-1}'s the order of the λ-way developable is reduced from $(m - \lambda + 1)\ (m - n + \lambda)$ to $(n - \lambda + 1)$ $(m - n + \lambda) - (n - \lambda)\ \beta$.

6. *Tangent flats to a* p-*spread where* $2 \overline{\overline{<}}$ p.

a. *Definitions.*

We have treated up to this point the various developables that arise from a curve in n-fold space. We shall show now that similar developables do not arise from the consideration of the tangent flats of spreads of more than one way.

Let

$$U = 0$$

be the equation of an $(n - 1)$ spread of order m. We shall use the points (1), (2), $\lambda\,(1) + \mu\,(2)$ to denote the points whose coördinates are $x_1, y_1, \ldots w_1$, $x_2, y_2, \ldots w_2$, and $\lambda\,x_1 + \mu\,x_2$, $\lambda\,y_1 + \mu\,y_2, \ldots$, $\lambda\,w_1 + \mu\,w_2$, respectively, so that $\lambda\,(1) + \mu\,(2)$ represents a point on the line (12), i. e., the line joining (1) and (2). We denote the result of substituting the coördinates of the points (1) or (2) in U by U_1, and U_2 respectively. We use the symbols

$$\Delta_2\,U_1 \equiv \left(x_2\,\frac{\partial}{\partial\,x_1} + y_2\,\frac{\partial}{\partial\,x_1} + \ldots + w_2\,\frac{\partial}{\partial\,w_1}\right)\,U_1,$$

$$\Delta_2\,U \equiv \left(x_2\,\frac{\partial}{\partial\,x} + y_2\,\frac{\partial}{\partial\,y} + \ldots + w_2\,\frac{\partial}{\partial\,w}\right)\,U,$$

$$\Delta\;U_2 \equiv \left(x\,\frac{\partial}{\partial\,x_2} + y\,\frac{\partial}{\partial\,y_2} + \ldots + w\,\frac{\partial}{\partial\,w_2}\right)\,U_2,$$

$$\Delta^k{}_2\,U_1 \equiv \left(x_2\,\frac{\partial}{\partial\,x_1} + y_2\,\frac{\partial}{\partial\,y_1} + \ldots + w_2\,\frac{\partial}{\partial\,w_1}\right)^k\,U_1.$$

In the last case the operator is to be applied k times to U_1. Now $\lambda\,(1) + \mu\,(2)$ is a point on the line (12), if it is also a point of the $(n - 1)$-spread, it must satisfy the equation of the spread. Substitute the coördinates of $\lambda\,(1) + \mu\,(2)$ in U and we have

$$\lambda^m\;U_1 + \lambda^{m-1}\,\mu\,\Delta_2\;U_1 + \frac{\lambda^{n-2}\;\mu^2}{2\,!}\,\Delta^2{}_2\;U_1 + \ldots$$

$$\ldots + \frac{\mu^m}{m\;!}\,\Delta_2{}^m\;U_1 = 0.$$

The m values of $\lambda : \mu$ that satisfy this equation determine the m points where the line (12) meets the $(n - 1)$-spread. If the point (1) lies on the spread then

$$U_1 = 0.$$

If the line (12) meets the spread twice at the point (1), then

$$U_1 = 0,\;\; \Delta_2\;U_1 = 0.$$

The equation of the locus of all the lines that meet the spread twice at (1) is

$$\Delta\, U_1 = 0.$$

From the analogy of three-fold space, this locus of lines is called the tangent $(n-1)$-flat to the $(n-1)$-spread, at the point (1).[*] At each point of an $(n-1)$-spread there is in general a unique tangent $(n-1)$-flat.

A p-spread is given by the equations,

$$U = 0,$$
$$V = 0,$$
$$W = 0,$$
$$\cdots\cdot$$

a restricted system equivalent to $n-p$ independent equations. In a similar manner the equations of the locus of all lines that meet the p-spread twice at any non-singular point (1) are,

$$\Delta\, U_1 = 0,$$
$$\Delta\, V_1 = 0,$$
$$\Delta\, W_1 = 0,$$
$$\cdots\cdots$$

Since these equations are linear we may select any $n-p$ that are independent and the rest are superfluous.[†] We have then a p-flat which from analogy is called the tangent p-flat to the p-spread at the point (1). At any point of a p-spread there is in general a unique tangent p-flat.[‡]

We define a tangent r-flat at a given point of the p-spread where $r < p$ as an r-flat that lies in the tangent p-flat at that point and contains the point. If $r > p$, we define a tangent r-flat at a given point as an r-flat that contains the tangent p-flat at that point. The locus of tangent lines then to a p-spread is simply the locus of tangent p-flats to the spread. The locus of tangent planes, 3-flats, . . . , $(p-1)$-flats is this same locus. If then there are developables that arise from a p-spread, where $1 < p$ their number is not so great as $n-p-1$, for

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[*] This proof is given in Dr. Story's Lectures on Hyperspace.

[†] Some of these equations may be satisfied identically; this will be the case when (1) is a multiple point on any of the $(n-1)$-spreads, but not a multiple point on the p-spread.

[‡] Dr. Story, Lectures on Hyperspace.

the tangent lines, tangent planes, tangent 3-flats, . . . , tangent p-flats all have the same locus. The planes through two consecutive lines, the 3-flats through two consecutive planes, etc., the p-flats through two consecutive $(p-1)$-flats all have this same locus possibly of a certain multiplicity.

b. *Intersections of consecutive tangent flats.*

We shall show further that $(p+1)$-flats cannot in general be passed through two consecutive tangent p-flats, for such p-flats do not in general have $(p-1)$-flats in common. Tangent p-flats at consecutive points of a p-spread where $1 \overset{=}{<} p \overset{=}{<} \frac{n}{2}$ do intersect in points at least. Let

$$U = 0,$$

$$V = 0,$$

$$\cdots\cdot$$

a restricted system equivalent to $n-p$ independent equations be the equations of the p-spread. Let

$$P' \equiv (x', y', \ldots) \text{ and } P'' \equiv (x' + d\,x', y' + d\,y', \ldots)$$

be consecutive points of the spread. The tangent p-flats at these points are

$$\Delta\, U' \equiv x \frac{\partial\, U'}{\partial\, x'} + y \frac{\partial\, U'}{\partial\, y'} + \ldots\ldots = 0,$$

$$\Delta\, V' \equiv x \frac{\partial\, V'}{\partial\, x'} + y \frac{\partial\, V'}{\partial\, y'} + \ldots\ldots = 0,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

and

$$\Delta\, U'' \equiv \Delta\, U' + x \left(\frac{\partial^2 U'}{\partial\, x'^2}\, d\,x' + \frac{\partial^2 U'}{\partial\, x'\, \partial\, y'}\, d\,y' + \ldots\ldots \right) = 0,$$

$$\Delta\, V'' = \Delta\, V' + x \left(\frac{\partial^2 V'}{\partial\, x'^2}\, d\,x' + \frac{\partial^2 V'}{\partial\, x'\, \partial\, y'}\, d\,y' + \ldots\ldots \right) = 0,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

All of these equations being linear, only $n-p$ equations in each set can be independent. In general, $2\,(n-p)$ equations for such a value of p have no common intersection. In the present case the resultant of any $n+1$ equations of the combined systems vanishes for any consecutive points P' and P'' on the p-spread, so that no more than n equations of the combined systems can be independent. Hence tangent p-flats at consecutive points of a p-spread intersect in a point at least. Tangent planes at consecutive points of a surface in n-fold space intersect at least in points. These tangent planes do not generally intersect in lines unless the surface lies in a space of three ways. Let us take p to represent the tangent plane at any point P of the surface and take p', p'', p''', . . . to represent the tangent planes at the points P', P'', P''', . . . consecutive points of an infinitesimal closed curve about P. If p and p' intersect in a line they determine a three-flat. If the consecutive tangent planes intersect in lines, then p'' has a line in common with both p and p' and so p'' lies in this three-flat. In a similar manner it can be shown that p', p'', p''' . . . , all the tangent planes consecutive, to p lie in the same three-flat with it, i. e. a unique three-flat is determined at each point of the surface that contains the tangent plane at the point and all the tangent planes consecutive to it. Since however this three-flat is determined by any two of these tangent planes, the three-flats corresponding to P and P' any two consecutive points are the same. Take now any curve through P that lies on the surface. Since the three-flats corresponding to any two consecutive points of the curve are the same, it follows that the three-flats corresponding to all the points of this curve are the same. If we take a different curve through P the same thing is true of the points of it. The three-flats corresponding to all the points of these two curves are the same since they are all the same as the three-flat corresponding to P. From this it follows that the whole surface and all of its tangent planes lie in the same three-flat. Hence if in general all the tangent planes consecutive to any tangent plane of a surface lie in the same three-flat with it, then the whole surface lies in this three-flat.

In the same way it may be shown that if in general all the tangent planes consecutive to the tangent plane at any point of a surface lie in the same four-flat with it that the whole surface lies in this four-flat. Hence in n-fold space not only do the consecutive tangent planes of a surface not intersect in lines, but all the tangent planes consecutive to any tangent plane do not lie in the same four-flat with it.

c. *The locus of the intersections of the tangent plane at any point of a surface with the consecutive tangent planes.*

In a four-fold space let the surface be given by

$$U = 0,$$
$$V = 0,$$
$$\cdots\cdots;$$

a restricted system equivalent to two independent equations. The tangent planes at P' and P'', any two consecutive points, have for their equations

$$\Delta U' = x \frac{\partial U'}{\partial x'} + y \frac{\partial U'}{\partial y'} + \ldots . = 0,$$

$$\Delta V' = x \frac{\partial V'}{\partial x'} + y \frac{\partial V'}{\partial y'} + \ldots . = 0,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

and

$$\Delta U'' = \Delta U' + x \left(\frac{\partial^2 U'}{\partial x'^2} d\,x' + \frac{\partial^2 U'}{\partial x' \, \partial y'} d\,y' + \ldots . \right) + \ldots . = 0,$$

$$\Delta V'' = \Delta V' + x \left(\frac{\partial^2 V'}{\partial x'^2} d\,x' + \frac{\partial^2 V'}{\partial x' \, \partial y'} d\,y' + \ldots . \right) + \ldots . = 0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Let us take the first two equations in each set to be independent, then the rest are superfluous. Since P' and P'' are points of the surface,

$$U' = 0$$

$$V' = 0$$

$$U'' = U' + \frac{\partial U'}{\partial x'} d\,x' + \ldots . = 0,$$

$$V'' = V' + \frac{\partial V'}{\partial x'} d\,x' + \ldots . = 0,$$

From these three sets of equations we derive

$$x \left(\frac{\partial^2 U'}{\partial x'^2} d\,x' + \frac{\partial^2 U'}{\partial x' \, \partial y'} d\,y' + \ldots . \right) + \ldots . = 0,$$

$$x \left(\frac{d^2 V'}{\partial x'^2} d\,x' + \frac{\partial^2 V'}{\partial x' \, \partial y'} d\,y' + \ldots . \right) + \ldots . = 0,$$

$$\frac{\partial U'}{\partial x'} d\,x' + \ldots . = 0,$$

$$\frac{\partial V'}{\partial x'} d\,x' + \ldots . = 0.$$

These four equations are homogeneous in the five differentials $d\,x'$, $d\,y'$, . . . We may take one of these differentials to be zero and eliminate the other four. We have

$$\begin{vmatrix} x\,\frac{\partial^2 U'}{\partial x'^2} + y\,\frac{\partial^2 U'}{\partial x'\,\partial y'} + \ldots, & x\,\frac{\partial^2 U'}{\partial x'\,\partial y'} + y\,\frac{\partial^2 U'}{\partial y'^2} + \ldots, & -, & -, \\ x\,\frac{\partial^2 V'}{\partial^2 x'} + y\,\frac{\partial^2 V'}{\partial x'\,\partial y'} + \ldots, & x\,\frac{\partial^2 V'}{\partial x'\,\partial y'} + y\,\frac{\partial^2 V'}{\partial y'^2} + \ldots, & -, & -, \\ \frac{\partial U'}{\partial x'}\,, & \frac{\partial U'}{\partial y'}\,, & -, & -, \\ \frac{\partial V'}{\partial x'}\,, & \frac{\partial V'}{\partial y'}\,, & -, & -, \end{vmatrix} = 0.$$

This determinant and its derivatives vanish for the point P', therefore the locus is a quadratic three-way cone with its vertex at P'. This cone is intersected by the tangent plane at P' in a pair of straight lines which is the required locus. If a point $x, y, \ldots$, be taken on either of these lines, we have three independent equations just sufficient to determine the ratios of the four differentials; i. e., just sufficient to determine the consecutive point P'', so that the tangent plane at this consecutive point will intersect the tangent plane at P' in the point selected. That these two consecutive tangent planes have no further intersection may be further shown by forming the equation of the plane that goes through their common intersection and through both the points P' and P''. The equations of this plane are

$$\Delta'' V' \,.\, \Delta U' - \Delta'' U' \,.\, \Delta V' = 0,$$
$$\Delta' V'' \,.\, \Delta U'' - \Delta' U'' \,.\, \Delta V'' = 0.$$

These equations in general represent a definite plane so long as P' and P'' are not coincident.

It would be of interest to examine the motion of the point of intersection along these lines as the point P'' circles about the point P', and to see whether at any time the consecutive tangent planes intersect in one of these lines.

These lines are not inflexional tangents to the surface; lines meeting the surface in three consecutive points do not generally exist in a space of more than three ways. For such lines would have to satisfy both

$$\Delta U' = 0,$$
$$\Delta V' = 0,$$
$$\cdots\cdots$$

and

$$\Delta^2 U' = 0,$$
$$\Delta^2 V' = 0,$$
$$\cdots\cdots$$

These equations, however, in general have only the point P' counted a multiple number of times in common. In general, then, in a space of more than three ways a surface is so twisted that there are no lines that meet the surface three times at a given point. This proof is easily extended to a surface in a space of more than four ways.

d. *The spreads that arise by considering the junctions of the consecutive tangent flats.*

Consider now any surface in n-fold space. Draw the 2-fold infinite system of tangent planes. Pass a four-flat through every two consecutive planes and there is a 3-fold infinite system of four-flats, forming in general a seven-spread. Each four-flat is met by the infinity of consecutive four-flats in the same plane. We may pass six-flats through every two consecutive four-flats. There is a 4-fold infinite system of six-flats constituting a ten-spread. This system of ruled loci in no wise resembles the system of developables we derived from a curve. Starting with a surface we cannot derive a system of developables in the same manner as when we start with a curve. The same is true if we start with any p-spread where $2 \leqq p$. Only in case the p-spread lies in a $(p+1)$-flat do consecutive tangent p-flats intersect generally in $(p-1)$-flats; the only exception is in the case the p-spread is a curve.

II. LOCI DERIVED FROM AN $(n-2)$-FLAT WHOSE EQUATION INVOLVES A SINGLE ARBITRARY PARAMETER.

7. *Description of the loci.*

Let us consider next the system of loci represented by an $(n-2)$-flat whose equations involve a single arbitrary parameter. The parameter may enter rationally or irrationally. If it enters rationally we suppose it to enter to as high a degree as $\frac{n}{2}$ in each equation. Let the equations of the flat be

$$A = 0, \quad B = 0.$$

In these equations we suppose further that the linear function of the coördinates that appear as coefficients of the various powers of the parameter cannot be expressed in terms of fewer than $n+1$ linear functions of the coördinates. Eliminate the parameter from these equations and we derive the equation of an $(n-1)$-spread S_{n-1}, which is ruled by the system of $(n-2)$-flats, F_{n-2}'s.*

Two consecutive F_{n-2}'s intersect in an $(n-4)$-flat, whose equations are,

$$A=0,\ \frac{\partial A}{\partial \lambda}=0,\ B=0,\ \frac{\partial B}{\partial \lambda}=0.$$

The elimination of the parameter from these equations gives a restricted system equivalent to three independent equations. The locus is an $(n-3)$-spread ruled by the F_{n-4}'s. S_{n-3} is a double spread on S_{n-1}.

Three consecutive F_{n-2}'s intersect in an $(n-6)$-flat F_{n-6}, whose equations are,

$$A=0,\ \frac{\partial A}{\partial \lambda}=0,\ \frac{\partial^2 A}{\partial \lambda^2}=0,$$

$$B=0,\ \frac{\partial B}{\partial \lambda}=0,\ \frac{\partial^2 B}{\partial^2 \lambda}=0.$$

If we eliminate the parameter from these equations we derive a restricted system equivalent to five independent equations. The locus is an $(n-5)$-spread S_{n-5}, ruled by the F_{n-6}'s. S_{n-5} is a triple spread on S_{n-1} and a double spread on S_{n-3}.

Similarly r consecutive F_{n-2}'s intersect in an $(n-2r)$-flat F_{n-2r}, whose equations are,

$$A=0,\ \frac{\partial A}{\partial \lambda}=0,\ \ldots,\ \frac{\partial^{r-1} A}{\partial \lambda^{r-1}}=0,$$

$$B=0,\ \frac{\partial B}{\partial \lambda}=0,\ \ldots,\ \frac{\partial^{r-1} B}{\partial \lambda^{r-1}}=0.$$

On the elimination of the parameter we derive a restricted system equivalent to $2r-1$ independent equations. The locus is an $(n-2r+1)$-spread, S_{n-2r+1}, ruled by the F_{n-2r}'s. S_{n-2r+1} is an r-tuple spread on S_{n-1}; it is a multiple spread on other spreads of the system.

Two distinct cases arise according as n is odd or even. If n is odd, then $\frac{n-1}{2}$ consecutive F_{n-2}'s intersect in a line, F_1, whose equations are,

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* From now on we shall use S_k to denote the k-spread of this system.

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \ldots \frac{\partial^{\frac{n-3}{2}} A}{\partial \lambda^{\frac{n-3}{2}}} = 0,$$

$$B = 0, \frac{\partial B}{\partial \lambda} = 0, \ldots \frac{\partial^{\frac{n-3}{2}} B}{\partial \lambda^{\frac{n-3}{2}}} = 0.$$

If we eliminate the parameter from these equations we derive a restricted system equivalent to $n - 2$ independent equations. The locus is a surface S_2 ruled by the F_1's; it is an $\left(\frac{n-1}{2}\right)$-tuple surface on S_{n-1}.

Consecutive F_1's do not in general intersect for the $n + 1$ equations

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \ldots \frac{\partial^{\frac{n-1}{2}} A}{\partial \lambda^{\frac{n-1}{2}}} = 0,$$

$$B = 0, \frac{\partial B}{\partial \lambda} = 0, \ldots \frac{\partial^{\frac{n-1}{2}} B}{\partial \lambda^{\frac{n-1}{2}}} = 0,$$

have not in general any common solutions. If we regard these $n + 1$ equations as homogeneous in the $n + 1$ coördinates and form their resultant, the values of the parameter that cause it to vanish will give points where consecutive lines meet. The equations of these points may be formed by eliminating the parameter from the $n + 1$ equations, which gives a restricted system equivalent to n independent equations. These points are double points on S_2 and $\left(\frac{n+1}{2}\right)$-tuple points on S_{n-1}.

If n is even then $\frac{n}{2}$ consecutive F_{n-2}'s intersect in a point F_0, whose equations are,

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \ldots, \frac{\partial^{\frac{n-2}{2}} A}{\partial \lambda^{\frac{n-2}{2}}} = 0,$$

$$B = 0, \frac{\partial B}{\partial \lambda} = 0, \ldots, \frac{\partial^{\frac{n-2}{2}} B}{\partial \lambda^{\frac{n-2}{2}}} = 0.$$

The elimination of the parameter from these equations gives a restricted system equivalent to $n - 1$ independent equations. The locus is a curve

S_1, which is an $\left(\frac{n}{2}\right)$-tuple curve on S_{n-1}. There are not in general stationary points on S_1, for the $n+2$ equations

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \ldots, \frac{\partial^{\frac{n+1}{2}} A}{\partial \lambda^{\frac{n+1}{2}}} = 0,$$

$$B = 0, \frac{\partial B}{\partial \lambda} = 0, \ldots, \frac{\partial^{\frac{n+1}{2}} B}{\partial \lambda^{\frac{n+1}{2}}} = 0,$$

have not in general any common solutions at all.

If the equation of the $(n-2)$-flat involve k parameters connected by $k-1$ equations, the properties of the derived system of loci is the same as in the case just discussed.

8. *Mutual relations of the derived loci.*

Two consecutive F_{n-2}'s intersect in an F_{n-4}, three in an F_{n-6}, r in an F_{n-2r}, $\frac{n-1}{2}$ in an F_1, if n is odd, or $\frac{n}{2}$ in an F_0 if n is even. There is a 1-fold infinite system of each kind of flats. The F_{n-2}'s are generators of S_{n-1}, the F_{n-4}'s of S_{n-3}, the F_{n-2r}'s of S_{n-2r+1}. Let us consider the case where n is odd. Through any F_{n-4} pass two consecutive F_{n-2}'s, through any F_{n-2r} pass r consecutive F_{n-2}'s, through any F_1 pass $\frac{n-1}{2}$ consecutive F_{n-2}'s. Any F_{n-2} contains two consecutive F_{n-4}'s, three consecutive F_{n-6}'s, $\frac{n-1}{2}$ consecutive F_1's. Any F_{n-2r} contains two consecutive $F_{n-2(r+1)}$'s, any two consecutive F_{n-2r}'s determine one $F_{n-2(r-1)}$'s. We may then reverse the process and start with S_2, which lies in the space of n ways but in no flat space of a less number of ways. Through each two consecutive F_1's of this surface pass three-flats F_3's, these F_3's will generate a four-spread S_{n-4}. Through each two consecutive F_3's pass five-flats: this can be done as the F_{n-3}'s intersect consecutively in F_1's. These five-flats will generate a six-spread S_6. Finally, through each two consecutive F_{n-4}'s pass F_{n-2}'s; these F_{n-2}'s generate an $(n-1)$-spread S_{n-1}. If we start with the system of $(n-2)$-flats we come down finally to the surface, or starting with the surface we may work back to the system of $(n-2)$-flats.

If n is even, through any F_{n-4} pass two consecutive F_{n-2}'s, through any F_{n-2r} pass r consecutive F_{n-2}'s, through any F_0 pass $\frac{n}{2}$ consecutive F_{n-2}'s.

Any F_{n-2} contains two consecutive F_{n-4}'s, three consecutive F_{n-6}'s, $\frac{n}{2}$ consecutive F_0's. Any F_{n-2r} contains two consecutive $F_{n-2(r+1)}$'s and any two consecutive F_{n-2r}'s determine one $F_{n-2(r-1)}$ except in the case that $r=\frac{n}{2}$. We cannot then start with a curve and retrace our steps; two consecutive points of the curve S_1 do not determine uniquely a plane of the system. The F_2's of the system in general intersect consecutively in the points of S_1. Starting with such a system of planes we may retrace our steps. Through any two consecutive planes of the S_3 we may pass a four-flat. These four-flats are generators of S_5. Through any two consecutive F_4's we may pass six-flats; they are the generators of S_7. Finally through any two consecutive F_{n-4}'s pass $(n-2)$-flats; they are generators of S_{n-1}. We may retrace our steps only in case we do not begin with S_1.

9. *Director curves of the ruled* (n — 1)*-spread.*

Let the equation of such a ruled $(n-1)$-spread S_{n-1} be

$$\phi = 0.$$

We shall show that the equations of the generating flats of the spread may be represented by linear equations involving a single parameter. The equation in homogeneous coördinate of an arbitrary $(n-2)$-flat in n-fold space may be written

$$x = \alpha_1 z + \beta_1 s + \dots + \gamma_1 w$$

$$y = \alpha_2 z + \beta_2 s + \dots + \gamma_2 w.$$

In this form the equations of the $(n-2)$-flat, which we may call the $(n-2)$-flat AB, involve $2(n-1)$ independent arbitrary parameters. These parameters must be connected by $2(n-1)-1$ equation to make AB a generator of such an $(n-1)$-spread. We wish to connect these parameters in such a way that AB will be a generator of the S_{n-1} in question. The equations of a curve on ϕ are

$$\phi = 0,\ U_1 = 0,\ U_2 = 0, \dots\ U_{n-2} = 0.$$

If we eliminate the coördinates between these equations and the equations of AB we derive a single equation in the $2(n-1)$ parameters. This resulting equation is the necessary and sufficient condition for AB to meet the curve. In a similar way we may derive $2(n-1)-1$ such conditions and make AB meet $2(n-1)-1$ curves on ϕ. If from these $2(n-1)-1$ equations and the equations of AB we eliminate the parameters, we derive a single equation in the variables alone. It is the locus of all the $(n - 2)$-flats that can be drawn to meet the curves in question, and so it necessarily includes all the generating flats of ϕ. It includes possibly other flats besides the generators of ϕ, but in this case the general locus will break up into several components, and one component is ϕ. This is the case in three-fold space.

The spreads U_1, U_2, . . . U_{n-2} may in each case be taken to be flats; then the director curves are plane curves. These are the director curves of ϕ; any or all of these curves may be plane, or they may be twisted to any extent permitted by the space. Any $2\,n - 3$ curves in n-fold space may be taken as the director curves of a ruled $(n - 1)$-spread. In three-fold space any three curves plane or twisted may be taken as the director curves of a ruled surface. In four-fold space, any five curves plane or twisted may be taken as the director curves of a ruled three-spread. In this case the generating planes intersect consecutively in the points of a sixth curve; so in four-fold space any five curves completely determine a sixth. In five-fold space seven curves plane or twisted may be taken as the director curves of a four-spread ruled by three flats. In six-fold space nine curves determine a five-spread ruled by four-flats. Consecutive four-flats intersect in planes and these in turn intersect consecutively in points. So in six-fold space nine curves determine a tenth.

10. *Multiple loci on the ruled* (n — 1)*-spread.*

Any generator of the $(n - 1)$-spread is an $(n - 2)$-flat F_{n-2}; it is met by any other generating F_{n-2} in an $(n - 4)$-flat. If then $4 \overset{=}{<} n$ every generator is met by every other generator. If $n = 3$, any generator is met by only $m - 2$ other generators, m being the order of the surface.*

For $4 \overset{=}{<} n$, any F_{n-2} contains a single infinity of $(n - 4)$-flats where it is met by the other F_{n-2}'s. These are evidently double flats on S_{n-1}. On S_{n-1} there are in general a 2-fold infinite system of such $(n - 4)$-flats constituting a double $(n - 2)$-spread, Σ_{n-2} on S_{n-1}. In general, then, any $(n - 1)$-spread S_{n-1} ruled by $(n - 2)$-flats F_{n-2}'s has on it such a double $(n - 2)$-spread Σ_{n-2} ruled by the 2-fold infinite system of $(n - 4)$-flats. Σ_{n-4} has on it all those $(n - 4)$-flats, F_{n-4}'s that arise from the intersection of consecutive F_{n-2}'s. These F_{n-4}'s generate S_{n-3}, which therefore lies on Σ_{n-2} and forms but an infinitesimal part of it.

Any three F_{n-2}'s intersect in an $(n - 6)$-flat; there are in general a 3-fold infinite system of such $(n - 6)$-flats constituting an $(n - 3)$-

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* Salmon, Geometry of Three Dimensions, p. 427.

spread Σ_{n-3}, a triple spread on S_{n-1}. S_{n-5} lies on Σ_{n-3}, and constitutes but an infinitesimal part of it. If n is sufficiently great there is a quadruple $(n-4)$-spread Σ_{n-4} ruled by the 4-fold infinite system of $(n-8)$-flats arising from the intersections of four F_{n-2}'s. S_{n-7} lies on S_{n-5}. We can go on in this manner until we reach a limit due to the narrowness of the space. If n is odd we have finally an $\left(\frac{n-1}{2}\right)$-tuple $\left(\frac{n+1}{2}\right)$-spread ruled by the $\left(\frac{n-1}{2}\right)$-fold infinite system of lines that arise from the intersection of $\frac{n-1}{2}$ generating F_{n-2}'s. There may be further an $\left(\frac{n+1}{2}\right)$-tuple $\left(\frac{n-1}{2}\right)$-spread made up of the $\left(\frac{n-1}{2}\right)$-fold infinite system of points that are the intersection of $\frac{n+1}{2}$ generating F_{n-2}'s, an $\left(\frac{n+3}{2}\right)$-tuple $\left(\frac{n-3}{2}\right)$-spread made up of the $\frac{n-3}{2}$-fold infinite system of points that are the intersections of $\frac{n+3}{2}$ generating F_{n-2}'s, etc., but these spreads do not always occur. In special cases the Σ_{n-2}, or some component of it, may be of greater multiplicity than $\frac{n+1}{2}$. In three-fold space a ruled surface generally has on it a double curve. This double curve, or some component of it, may, however, be of greater multiplicity than two. It is to be observed that S_{n-3} lies on Σ_{n-2}. In three-fold space this means that consecutive generators of a ruled surface, if they intersect at all, must intersect in points of the double curve. If n is even we have finally an $\left(\frac{n}{2}\right)$-tuple $\left(\frac{n}{2}\right)$-spread $\Sigma_{\frac{n}{2}}$ that is made up of the $\left(\frac{n}{2}\right)$-fold infinite system of points that arise from the intersection of $\frac{n}{2}$ generating F_{n-2}'s. There may be an $\left(\frac{n}{2}+1\right)$-tuple $\left(\frac{n}{2}-1\right)$-spread $\Sigma_{\frac{n}{2}-1}$ whose points are points of intersection of $\frac{n}{2}+1$ generating F_{n-2}'s, an $\left(\frac{n}{2}+2\right)$-tuple $\left(\frac{n}{2}-2\right)$-spread $\Sigma_{\frac{n}{2}-2}$ whose points are points of intersection of $\frac{n}{2}+2$ generating F_{n-2}'s, etc., though these spreads may not always be present.

11. *Special case where the parameter enters rationally.*

Let us consider the special case where the parameter enters rationally. Let the equation of the generating $(n-2)$-flat F_{n-2} be

$$A \equiv a\,t^l + b\,t^{l-1} + c\,t^{l-2} + \ldots . = 0,$$

$$B \equiv a'\,t^m + b'\,t^{m-1} + c'\,t^{m-2} + \ldots . = 0,$$

where $a, b, c, \ldots, a', b', c', \ldots,$ are linear functions of the coördinates that cannot be expressed linearly in terms of fewer than $n+1$ linear functions of the coördinates. If we eliminate the parameter from these equations, we have the equation of the S_{n-1} ruled by the F_{n-2}'s; it is of order $l+m$. It is more convenient in what follows to use two parameters, λ and μ, that enter homogeneously into the equations.

Two consecutive generators intersect in the F_{n-4} whose equations are

$$\frac{\partial A}{\partial \lambda} = 0, \ \frac{\partial A}{\partial \mu} = 0, \ \frac{\partial B}{\partial \lambda} = 0, \ \frac{\partial B}{\partial \mu} = 0.$$

The elimination of the parameter from these equations gives a restricted system equivalent to three independent equations the locus is S_{n-3}, whose order is

$$2\,(l-1) + 2\,(m-1) = 2\,(l+m-2).$$

The order is found by expressing the conditions that the four equations have a common root. The locus of the intersections of three consecutive F_{n-2}'s is a locus of F_{n-6}'s; the equations of this locus are found by eliminating the parameters from the equations,

$$\frac{\partial^2 A}{\partial \lambda^2} = 0. \ \frac{\partial^2 A}{\partial \lambda\,\partial \mu} = 0, \ \frac{\partial^2 A}{\partial \mu^2} = 0,$$

$$\frac{\partial^2 B}{\partial \lambda^2} = 0, \ \frac{\partial^2 B}{\partial \lambda\,\partial \mu} = 0, \ \frac{\partial^2 B}{\partial \mu^2} = 0.$$

This gives a restricted system equivalent to five independent equations; it represents S_{n-5}, whose order is $3\,(l+m-4)$.

The r-tuple spread S_{n-2r+1} on S_{n-1} is represented by the equations that result from eliminating the parameters from the equations,

$$\frac{\partial^{r-1} A}{\partial \lambda^{r-1}} = 0, \ \frac{\partial^{r-1} A}{\partial \lambda^{r-2}\,\partial \mu} = 0, \ldots, \frac{\partial^{r-1} A}{\partial \mu^{r-1}} = 0,$$

$$\frac{\partial^{r-1} B}{\partial \lambda^{r-1}} = 0, \ \frac{\partial^{r-1} B}{\partial \lambda^{r-2}\,\partial \mu} = 0, \ldots, \frac{\partial^{r-1} B}{\partial \mu^{r-1}} = 0.$$

The equations then are of S_{n-2r+1} form a restricted system equivalent to $2r-1$ independent equations whose order is $r\,(l+m-2r+2)$. As we have seen, there are two cases according as n is odd or even. If n is odd we come down finally to an $\left(\frac{n-1}{2}\right)$-tuple surface S_2. The equations of S_2 are found by eliminating the parameters from the equations

$$\frac{\partial^{\frac{n-3}{2}} A}{\partial \lambda^{\frac{n-3}{2}}} = 0,\ \frac{\partial^{\frac{n-3}{2}} A}{\partial \lambda^{\frac{n-5}{2}}\,\partial \mu} = 0,\ \ldots,\ \frac{\partial^{\frac{n-3}{2}} A}{\partial \mu^{\frac{n-3}{2}}} = 0,$$

$$\frac{\partial^{\frac{n-3}{2}} B}{\partial \lambda^{\frac{n-3}{2}}} = 0,\ \frac{\partial^{\frac{n-3}{2}} B}{\partial \lambda^{\frac{n-5}{2}}\,\partial \mu} = 0,\ \ldots,\ \frac{\partial^{\frac{n-3}{2}} B}{\partial \mu^{\frac{n-3}{2}}} = 0.$$

The equations of S_2 form a restricted system equivalent to $n-2$ independent equations, whose order is $\frac{n-1}{2}\,(l+m-n+3)$.

There are also $\left(\frac{n+1}{2}\right)$-tuple points F_0's on S_{n-1}, though in general $\frac{n+1}{2}$ consecutive F_{n-2}'s do not intersect. If we form the resultant of the $n+1$ equations

$$\frac{\partial^{\frac{n-1}{2}} A}{\partial \lambda^{\frac{n-1}{2}}} = 0,\ \frac{\partial^{\frac{n-1}{2}} A}{\partial \lambda^{\frac{n-3}{2}}\,\partial \mu} = 0,\ \ldots,\ \frac{\partial^{\frac{n-1}{2}} A}{\partial \mu^{\frac{n-1}{2}}} = 0,$$

$$\frac{\partial^{\frac{n-1}{2}} B}{\partial \lambda^{\frac{n-1}{2}}} = 0,\ \frac{\partial^{\frac{n-1}{2}} B}{\partial \lambda^{\frac{n-3}{2}}\,\partial \mu} = 0,\ \ldots,\ \frac{\partial^{\frac{n-1}{2}} B}{\partial \mu^{\frac{n-1}{2}}} = 0,$$

we have a determinant of the $(n+1)$-st order, in which the parameters enter to the degree $\frac{n+1}{2}\,(l+m-n+1)$. There are then $\frac{n+1}{2}\,(l+m-n+1)$ values of the parameters that cause this determinant to vanish, and so this is the number of points F_0. We can find the equations of these points by eliminating the parameters from these $n+1$ equations. The result is a restricted system equivalent to n independent equations. The order of the system is $\frac{n+1}{2}\,(l+m-n+1)$. This is another proof of the number of points F_0 on S_{n-1}.

In case n is even we have finally the $\left(\frac{n}{2}\right)$-tuple curve whose equations are found by eliminating the parameters from the equations,

$$\frac{\partial^{\frac{n-2}{2}} A}{\partial \lambda^{\frac{n-2}{2}}} = 0, \frac{\partial^{\frac{n-2}{2}} A}{\partial \lambda^{\frac{n-4}{2}} \partial \mu} = 0, \ldots, \frac{\partial^{\frac{n-2}{2}} A}{\partial \mu^{\frac{n-2}{2}}} = 0,$$

$$\frac{\partial^{\frac{n-2}{2}} B}{\partial \lambda^{\frac{n-2}{2}}} = 0, \frac{\partial^{\frac{n-2}{2}} B}{\partial \lambda^{\frac{n-4}{2}} \partial \mu} = 0, \ldots, \frac{\partial^{\frac{n-2}{2}} B}{\partial \mu^{\frac{n-2}{2}}} = 0.$$

The order of the restricted system is $\frac{n}{2}(l + m - n + 2)$, the order of S_1.

We find the equation of the double spread Σ_{n-2} on S_{n-1}, by imposing on the equations of the generating F_{n-2} the conditions that they have two common roots in the parameter. These conditions are,*

$$\text{(II)} \qquad \left\| \begin{array}{llll} a, & b, & c, & \ldots\ldots \\ & a, & b, & \ldots\ldots \\ & & & \ldots\ldots\ldots \\ a', & b', & c', & \ldots\ldots \\ & a', & b', & \ldots\ldots \\ & & & \ldots\ldots\ldots \end{array} \right\| = 0 \quad ,$$

where there are $l + m - 2$ rows and $l + m - 1$ columns. This is a restricted system equivalent to two independent equations; the order of the system is $\frac{1}{2}(l + m - 1)(l + m - 2)$. On Σ_{n-2} must be S_{n-3}. We find the equations of Σ_{n-3} by expressing the conditions that the equations of the generating flat have three common roots in the parameter.† The result is a restricted system similar in form to (II), in which, however, there are only $l + m - 4$ rows and $l + m - 2$ columns. This restricted system is equivalent to three independent equations, and its order is $\frac{1}{6}(l + m - 2)(l + m - 3)(l + m - 4)$.

The equations of Σ_{n-r} are found by expressing the conditions that the equations of the generating $(n - r)$-flat have r roots in common. By an extension of the previous method we derive a restricted system of the same form as (II), in which, however, there are only $l + m - 2(r - 1)$ rows and $l + m - (r - 1)$ columns. This is a restricted system equiva-

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* Salmon, Higher Algebra, Art. 275.

†Ibid., Art. 285.

lent to r independent equations, the order of the system is $\frac{1}{r\,!}(l + m - r + 1)(l + m - r) \dots (l + m - 2r + 2)$.

Whether n is odd or even we have finally a curve Σ_1 of multiplicity $n - 1$, whose equations are found by expressing the conditions that the equations of the generating $(n - 2)$-flat have $n - 1$ roots in the parameter in common. We derive a restricted system of the same form as (II) in which however there are $l + m - 2(n - 2)$ rows and $l + m - (n - 2)$ columns. The order of this system is $\frac{1}{(n - 1)\,!}$ $(l + m - n + 2)(l + m - n + 1) \dots (l + m - 2n + 4)$. This curve has n-tuple points on it whose equations are found by expressing the conditions that the equations of the generating $(n - 2)$-flat have n roots in common. We again have a restricted system of the same form as (II), in which, however, there are $l + m - 2(n - 1)$ rows and $l + m - n + 1$ columns. The order of this system is $\frac{1}{n\,!}(l + m - n + 1)(l + m - n)$ $\dots (l + m \,.\, 2n + 2)$, which is the number of points in question. For $n = 3$ these formulae for the order agree with those given in Salmon.*

A very special case is where the parameter enters only linearly in one of the equations of the generating $(n - 2)$-flat. Let the equations of the flat be

$$A \equiv a\,t + b = 0,$$

$$B \equiv a'\,t^m + b'\,t^{m-1} + \dots = 0,$$

where we make the same suppositions regarding a, b, a', b', ..., as before. The S_{n-1} in this case is a ruled spread with m sheets through the $(n - 2)$-flat, whose equations are

$$a = 0,\ b = 0\,;$$

it has no other multiple locus on it at all. Consecutive generating F_{n-2}'s of the system intersect in the flat, whose equations are,

$$a = 0,\ b = 0,\ B = 0, \frac{\partial B}{\partial t} = 0.$$

All the F_{n-4}'s of the system lie in the same $(n - 2)$-flat; they generate a developable $(n - 3)$-spread S'_{n-3} in this flat. S'_{n-3} is the section by this flat of the developable $(n - 1)$-spread enveloped by the $(n - 1)$-flat B. Consecutive generating F_{n-2}'s of S_{n-1} intersect in generating F_{n-4}'s of

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* Salmon, Geometry of Three Dimensions, p. 428.

S'_{n-3}. By means of an $(n-3)$-way developable lying in an $(n-2)$-flat and two arbitrary curves we can generate a ruled $(n-1)$-spread by taking all the $(n-2)$-flats that can be drawn through the enveloping $(n-3)$-flats of the developable so as to meet both curves.

We have seen that the section of an $(n-1)$-way developable by an $(n-1)$-flat gave an $(n-2)$-way developable of the same nature, so here the section of an $(n-1)$-spread ruled by $(n-2)$-flats by an $(n-1)$-flat gives an $(n-2)$-spread of the same nature as the $(n-1)$-spread.

III. LOCI DERIVED FROM AN $(n-k)$-FLAT WHOSE EQUATIONS INVOLVE A SINGLE ARBITRARY PARAMETER.

12. *Description of the derived loci.*

We shall complete the general theory by considering the locus of the 1-fold infinite system of $(n-k)$-flats, where $2 \leqq k$ whose equations all contain a single arbitrary parameter. Let the k equations of the flat be

$$A = 0,\ B = 0,\ \ldots,\ C = 0.$$

The equations of the locus of these F_{n-k}'s are found by eliminating the parameter from these equations. The result is a restricted system equivalent to $k-1$ independent equations.

The locus is an $(n-k+1)$-spread S_{n-k+1} ruled by the F_{n-k}'s. Any two consecutive F_{n-k}'s intersect in an $(n-2k)$-flat F_{n-2k} whose equations are

$$A = 0,\ \frac{\partial A}{\partial \lambda} = 0,\ B = 0,\ \frac{\partial B}{\partial \lambda} = 0,\ \ldots.$$

If we eliminate the parameter from these equations, we derive a restricted system equivalent to $2k-1$ independent equations. The locus is an $(n-2k+1)$-spread S_{n-2k+1} ruled by the F_{n-2k}'s; it is a double spread on S_{n-k}.

Any three consecutive F_{n-2k}'s intersect in an $(n-3k)$-flat F_{n-3k} whose equations are,

$$A = 0,\ \frac{\partial A}{\partial \lambda} = 0,\ \frac{\partial^2 A}{\partial \lambda^2} = 0,\ B = 0,\ \frac{\partial B}{\partial \lambda} = 0,\ \ldots$$

The elimination of the parameter from these equations gives a restricted system equivalent to $3k-1$ independent equations. Their locus is an $(n-3k+1)$-spread ruled by the F_{n-3k}'s. S_{n-3k+1} is a triple spread on S_{n-k+1}.

The equations of the locus of the intersections of r consecutive F_{n-k}'s are found by eliminating the parameter from the equations

$$A = 0, \frac{\partial A}{\partial \lambda} = 0, \ldots \frac{\partial^{r-1} A}{\partial \lambda^{r-1}} = 0,$$

$$B = 0, \frac{\partial B}{\partial \lambda} = 0, \ldots\ldots\ldots\ldots$$

This gives a restricted system equivalent to $r k - 1$ independent equations. The locus is an $(n - r k + 1)$-spread ruled by the F_{n-rk}'s, it is an r-tuple spread on S_{n-k+1}.

There are k cases according as $n \equiv 0 \pmod{k}$, $n \equiv 1 \pmod{k}$, . . . , $n \equiv k - 1 \pmod{k}$. In the first case we come finally to a curve S_1 which is an $\binom{n}{k}$-tuple curve on S_{n-k+1}. In the second case we come down finally to a system of lines F_1's which are generators of a ruled surface S_2. In the last case we come down finally to a k-spread ruled by $(k - 1)$-flats. There are on S_k in general special points where two consecutive F_{k-1}'s intersect.

13. *Multiple loci on the spread; mutual relations of the system of spreads.*

S_{n-k+1} has on it in general multiple loci that arise from the intersection of non-consecutive F_{n-k}'s. Any F_{n-k} intersects every other F_{n-k} in an $(n - 2 k)$-flat; there is in general a 2-fold infinite system of such $(n - 2 k)$-flats constituting a double $(n - 2 k + 2)$-spread Σ_{n-2k+2} on S_{n-k+1}. Evidently S_{n-2k+1} lies on Σ_{n-2k+2}. Any three F_{n-k}'s intersect in an $(n - 3 k)$-flat; there is a 3-fold infinite system of such $(n - 3 k)$-flats, they constitute in general a triple $(n - 3 k + 3)$-spread Σ_{n-3k+3} on S_{n-k+1}. S_{n-3k+1} lies on Σ_{n-3k+3}. Any r consecutive F_{n-k}'s intersect in an $(n - r k)$-flat; there is an r-fold infinite system of such $(n - r k)$-flats in general, constituting an r-tuple $(n - r k + r)$-spread Σ_{n-rk+r} on S_{n-k+1}, on which lies S_{n-rk+1}.

Finally the locus of the intersection of any a F_{n-k}'s where a is the greatest integer in $\frac{n}{k}$ is an a-tuple $[n - a (k - 1)]$-spread $\Sigma_{n-a(k-1)}$ on S_{n-k+1}; it is ruled by the a-fold infinite system of $(n - a k)$-flats.

The question arises, When, in general, do these double loci cease to exist? The double spread is in general an $(n - 2 k + 2)$-spread Σ_{n-2k+2}. To have a continuous locus of double points we must generally have

$$n - 2\,k + 2 \geqq 1 \text{ or } k \leqq \frac{n+1}{2}.$$

For values of k that satisfy this condition there is in general a continuous locus of double points. If

$$n - 2\,k + 2 = 0, \text{ or } k = \frac{n+2}{2}$$

there is in general only a finite number of double points on the locus. If

$$n - 2\,k + 2 < 0, \text{ or } k > \frac{n+2}{2}$$

there are in general no double points on the locus.

If there enter into the equations of the generating $(n - k)$-flat ρ parameters connected by $\rho - 1$ equations the properties of the system of related loci will be similar to those of the system just described.

Any two consecutive F_{n-k}'s intersect in an F_{n-2k} while through any F_{n-2k} pass two consecutive F_{n-k}'s. Any three consecutive F_{n-k}'s intersect in an F_{n-3k} while through any F_{n-3k} pass two consecutive F_{n-2k}'s and three consecutive F_{n-k}'s. Any two consecutive F_{n-rk}'s determine in general one $F_{n-k\,(r-1)}$. An exception may occur if $r = a$ the greatest integer in $\frac{n}{k}$. Thus, if $n \equiv 0 \pmod{k}$, two consecutive points of S_1 do not determine a $(k+1)$-flat where $2 \leqq k$.

If $n \equiv 1 \pmod{k}$, two consecutive lines of S_2 do not determine a $(k+1)$-flat, except in the case $k = 2$. In the last case, however, where $n \equiv k - 1 \pmod{k}$, two non-intersecting $(k-1)$-flats do determine a $(2\,k - 1)$-flat. Only in this last case can we retrace the steps if we come down to the last spread. We can always retrace the steps if we do not come down to this last case.

14. *Director spreads of the ruled spread.*

The equation in homogeneous coördinates of any $(n - k)$-flat, $2 \leqq k$, may be written

$$\begin{aligned} x &= \alpha_1\, s + \beta_1\, t + \ldots. + \gamma_1\, w, \\ y &= \alpha_2\, s + \beta_2\, t + \ldots. + \gamma_2\, w, \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ z &= \alpha_k\, s + \beta_k\, t + \ldots. + \gamma_k\, w. \end{aligned}$$

In this form the equations of the flat contain $k\,(n - k + 1)$ independent parameters. These parameters must be connected by $k\,(n - k + 1) - 1$ equations for this $(n - k)$-flat to be a generator of such a ruled $(n - k + 1)$-spread. Any curve is given by the equations

$$\phi = 0,$$

$$\chi = 0,$$

$$\cdots\cdot$$

a restricted system equivalent to $n - 1$ independent equations. If we eliminate the coördinates between the equations of the flat and curve, we derive a restricted system equivalent to $k - 1$ independent equations in the parameters alone. These are the conditions that must be satisfied for the $(n - k)$-flat to meet the curve. In a similar way we may derive a restricted system equivalent to $k - p$ independent equations in the parameters alone which are the necessary and sufficient conditions for the $(n - k)$-flat to meet a certain p-spread where $1 \leqq p \leqq k - 1$. We may have then curves, surfaces, . . . , or p-spreads where $1 \leqq p \leqq k - 1$ for the director loci of a ruled $(n - k + 1)$-spread. The numbers of loci of each kind that must be taken are λ, μ, . . . ν, ρ, namely, non-negative integers chosen to satisfy the equation

$$\lambda\,(k - 1) + \mu\,(k - 2) + \ldots . + \nu\,.\,2 + \rho\,.\,1 = k\,(n - k + 1) - 1.$$

If we consider a group of one or more points as a director locus of the spread, we have to select integers to satisfy

$$\kappa\,.\,k + \lambda\,(k - 1) + \ldots . + \rho\,.\,1 = k\,(n - k + 1) - 1.$$

We may apply this to special cases. The director loci of a ruled surface in three-fold space are three curves. We may take one curve and a group of κ points, in which case the ruled surface is reducible and has for its components κ cones whose vertices are the κ points and whose common base is the curve in question. In four-fold space the director loci of a ruled surface may be five surfaces, three surfaces and one curve, or one surface and two curves. The ruled surface in each case consisting of all the lines that can be drawn to meet all the director loci. In the same space the director loci of a three-spread ruled by planes may be taken to be five curves.

If the director loci be all taken on any S_{n-k+1}, then the locus of all the $(n - k)$-flats that can be drawn to meet these director loci will include as one of its components the S_{n-k+1} in question; it may or may not have other components.

There are several special cases illustrative of these methods that can be worked out in still greater detail. Some of these I hope to make the subject of another paper.

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THE ARC SPECTRUM OF HYDROGEN.

By O. H. Basquin

With Two Plates.

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THE ARC SPECTRUM OF HYDROGEN.

By O. H. Basquin.

Presented by C. R. Cross. Received June 8, 1901.

The Problem.

The arc spectra of those elements which are gases at ordinary temperatures and pressures have not been extensively studied. Their spark spectra, however, are easily obtained, and were among the first to be investigated. The general impression prevails, therefore, that these elements do not possess arc spectra. On the other hand practically all the so-called "hot stars" and all the "new stars" possess the more important lines of the hydrogen spectrum. Although our knowledge of what is going on in the arc and in the spark is very crude and unsatisfactory, yet it is, to the average mind, much easier to imagine a star as being in a condition similiar to that of the arc, rather than in one similar to that of the electric spark. It has seemed worth while, therefore, to search for the more important lines of hydrogen in the arc spectrum. This is the problem of the following investigation.

Historical.

Liveing and Dewar* examined the carbon arc in an atmosphere of hydrogen and saw "the fairly bright" C line of hydrogen, also "a faint diffuse band" at the position of the F line of hydrogen. They obtained these two lines also by allowing small drops of water to fall into the arc in air.† They found the F line usually obscured by continuous spectrum, becoming visible at intervals only, when, from some variation in the working of the arc, the continuous spectrum was less brilliant. Crew and Basquin ‡ incidentally noticed these two lines of hydrogen while working with the rotating metallic arc in an atmosphere of this gas.

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* Proc. Roy. Society, **30**, 156 (1880).

† Ibid., **35**, 75 (1883).

‡ Proc. Amer. Acad., **33**, 18 (1898).

APPARATUS.

In searching for these lines I have employed the rotating metallic arc,[*] which enables one to use chemically pure electrodes having little or no chemical reaction with the gas employed. In this arc, then, one may expect the gas to give off its characteristic radiations with greater intensity than in one where the gas may enter into chemical compounds before a temperature is reached at which it becomes luminous. This arc enables one also to select such metals as do not have strong lines in the neighborhood of the lines sought for, while in the spectrum of the carbon arc there are few spaces not already occupied by lines of carbon or of an impurity.

In the rotating arc, one electrode, either a disc or a rod of metal, rotates upon an axis, making about 700 rotations per minute, while the other electrode has a slow movement of translation toward the axis of rotation. The rotation not only prevents the excessive heating and welding together of the electrodes, but it throws the hot gases to one side, so that the arc has the appearance of a small fan. The part of the flame thus separated from the poles is very free from continuous spectrum.

In the apparatus used in these experiments the arc is enclosed in a brass box, or "hood," having a volume of about 1½ litres and being comparatively gas-tight. The light from the arc issues through a long brass tube closed with a lens at the outer end; the lens thus forms part of the wall of the hood, but is so far removed from the arc that it receives comparatively little of the deposit sometimes formed inside the hood, and hence remains clean.

A stream of gas enters the hood at one stop-cock and leaves it at another; a third cock is provided for attachment to a manometer. Although the hood is not absolutely gas-tight, the purity of the gas inside was preserved, in these experiments, partly by the small excess of pressure inside the hood above that outside, and partly by the fresh supply of pure gas constantly running through the hood. The hydrogen used was generated electrolytically, and varied in quantity from 10 to 15 litres per hour.

The spectra have been examined both visually and photographically by means of a small plane grating spectroscope and by means of a large concave grating spectroscope.

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[*] Crew and Tatnall, Phil. Mag., **38**, 379 (1894).

Observations of Hydrogen Lines.

The arc spectra of the following metals in hydrogen have been examined: Aluminium, copper, magnesium, coin-silver, sodium, tin, and zinc. With the exception of sodium the arc of each metal shows to the eye very clearly the H_α and H_β lines of hydrogen, and in most of them the H_γ line comes out with the small instrument very clearly, and indistinctly with the large one. The H_δ line shows only rarely, and then to the eye rather indistinctly. The H_α line is quite sharp and well defined, unless the electric current through the arc is unusually great; it has much the same appearance as the zinc line at 6363. The other three are always broad, hazy, and ill-defined.

On the photographs taken with the large spectroscope H_β and H_γ usually show very plainly, always excepting the spectrum of metallic sodium, while H_δ shows in spectra of tin, silver, and copper. On photographs taken with the small spectroscope these lines show more sharply, on account of the very much smaller dispersion, and the photographs of tin show the next hydrogen line, H_ϵ quite clearly. Not having found the hydrogen lines in the metallic sodium arc (using copper as stationary electrode), I tried it in dry hydrogen also, thinking that in some way the water vapor might have affected the appearance of the hydrogen lines, but I have been unable to detect any of the hydrogen lines in that arc in any way.

None of these lines excepting H_α is sharply defined. A wide space in the middle of each line has fairly uniform intensity, shading off gradually and uniformly to each side. The following table gives a rough estimate of widths, in Ångström units, of these lines as they appear on the photographic plates, the middle of the shading being taken as the edge of the line.

Line.	Maximum width.	Minimum width.	Mean width.
H_α	6	4	5
H_β	65	13	31
H_γ	44	14	26
H_δ	32	12	20
H_ϵ	faint, same general width.		

It will be noticed that these lines, with the exception of H_α, are excessively wide, and I think it is for this reason alone that I have been unable to photograph the still weaker hydrogen lines of Balmer's series.

They may appear upon the plates, but are so wide and so faint that they cannot be detected upon the general shading of the plates.

That these lines are not merely spark lines introduced into these arc spectra by the supposed spark at the breaking of the current through the rotating arc is shown, first, by the fact that they were first observed in the carbon arc, and, second, by the fact that I have seen H_α and H_β quite clearly in the magnesium metallic arc, when the poles were not rotating. The lines produced in the stationary arc have much the same character as in the rotating arc, but there is a large amount of continuous spectrum, appearing as a background, in the case of the stationary arc, so that it would be difficult to photograph the hydrogen lines in this way.

These lines in the arc seem to be due to hydrogen, and not to water vapor coming from the hydrogen generators.* This is shown by the following two experiments: (1) I passed the stream of hydrogen through concentrated sulphuric acid and phosphorus pentoxide; and even after the stream of dry gas had been running through the hood for three hours, I found the H_α line as bright as it was in the damp hydrogen coming directly from the generators. (2) In place of the current of dry hydrogen, I passed through the hood a stream of air bubbling through warm water, so that this air was charged with moisture to about the same degree as the moist hydrogen coming directly from the generators. In this case I was not able to detect the faintest trace of the H_α line. Magnesium poles were used in both the above experiments.

Other Methods.

I have examined some of these metals in commercial ammonia gas, such as is used in refrigeration. In this gas the hydrogen lines come out with nearly the same intensity as in hydrogen when copper or aluminium electrodes are used; no hydrogen lines are seen in the sodium arc in ammonia, although the arc works well, and when tin electrodes are used in ammonia a black dust collects in the atmosphere about the arc to such an extent as to shut off practically all the light within thirty seconds after starting the arc. From the standpoint of convenience and safety, the ammonia gas is much to be preferred to hydrogen.

The copper arc in coal gas shows the H_α line very clearly, but the other hydrogen lines are not distinguishable on account of the multitude of comparatively strong carbon lines which the coal gas furnishes in this part of the spectrum.

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* Trowbridge, Phil. Mag., **50**, 338 (1900).

Following the suggestion of Liveing and Dewar, above referred to, I have tried the rotating metallic arc in air, playing a very small jet of water upon the rotating electrode. In this manner the silver arc works rather more poorly than usual, and resembles a rapid series of small explosions. The hydrogen lines come out clearly, but are rather weaker and more diffuse than in the hydrogen atmosphere.

The copper arc works well in an atmosphere of steam, much better than in hydrogen. The hydrogen lines are nearly, if not quite, as strong in steam as in hydrogen. The electrodes of the arc are slightly oxidized and have very beautiful colors. In making this experiment a slight alteration was necessary in the hood of the arc. The window through which the light issues is usually as far away from the arc as possible, but it was moved for this experiment so as to be as close to the arc as possible. It was placed at the inner end of a brass tube projecting into the hood, in order that the heat of the surrounding steam and hot air, as well as that of the arc itself, might prevent condensation of steam upon the surface of the window.

CHEMICAL ACTION IN THE ARC IN HYDROGEN.

Historical.

Crew and Basquin [*] have sought to eliminate the radiations due to chemical causes in the electric arc by using chemically pure metallic electrodes and enclosing the arc in an atmosphere of hydrogen or nitrogen. They interrupted the current through the arc about 110 times per second and examined the light of the arc while the current was null. They found in the rotating metallic arc *in air* "a luminous cloud" persisting for several thousandths of a second after the current through the arc had ceased, but they found no such luminous effect in an atmosphere of hydrogen or nitrogen. This seems to show that the cloud is due to chemical action going on in the gases after the electric current has stopped, and that in hydrogen the chemical action is too feeble to be noticed in this way.

Liveing and Dewar[†] found a magnesium "line" at 5210, making its appearance in the arc spectrum only upon the introduction of hydrogen or coal gas into the arc. Professor Crew [‡] gives a number of lines appearing in the iron arc in hydrogen and not appearing in the arc in air.

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[*] Proc. Amer. Acad., **33**, 18 (1898).

[†] Proc. Roy. Society, **30**, 96 (1880).

[‡] Phil. Mag., **50**, 497 (1900).

HYDROGEN-METAL FLUTINGS.

With the exception of tin, every metal thus far examined in the rotating metallic arc in hydrogen gives a characteristic set of spectrum lines which are not found in the arc in air. Inasmuch as compounds of hydrogen with some metals are known, I have, in lieu of a better hypothesis, supposed that these lines are due to such compounds formed in the arc. No new isolated lines, surely due to hydrogen, have been found. The following description takes up the metals in the order of the relative intensities of these flutings.

TIN.

No fluting has been discovered due to a combination of tin and hydrogen. There are four lines of intensity $\frac{1}{2}$ on Rowland's scale, at approximately 3715, 3841, 4245, and 4386, which have not yet been identified. These may be weak tin lines not listed, or weak impurity lines. The deposit which is formed in the hood enclosing the arc is very small in amount and of a greenish color, and consists of very small globules. If this deposit is heated upon platinum foil in a Bunsen flame it quickly glows, and thereafter has a slate color; and if this powder is placed in hydrochloric acid it dissolves when heat is applied and gives off bubbles of gas. If the dark powder, after the first heating, is reheated on foil in the flame, it glows again, apparently at a higher temperature than before, and then becomes a very white powder, both of which experiments go to show that the original powder is not metallic tin but is possibly some combination of tin and hydrogen.

COIN SILVER.

This metal gives a delicate fluting with first head at 3333.86 and running toward longer wave lengths. There are only about fifty lines in this fluting, and they have an average intensity rather less than $\frac{1}{2}$ on Rowland's scale.

COPPER.

This metal gives a rather open fluting, having the head at 4279.77 and running toward the longer wave lengths. The number of lines in this fluting is about sixty, and they are individually stronger than those of the coin-silver fluting. This fluting makes its appearance also when an atmosphere of ammonia or of steam is used. The deposit formed inside the hood is rather small in amount and of a brown color. The following table gives the wave lengths of the hydrogen-copper flutings: —

Wave lengths.	Intensity.	Remarks.	Wave lengths.	Intensity.	Remarks.
4279.77	2	head.	4332.98	1−	
4280.72	1		4335.20	1+	
4281.25	1+	ghost of 4275 ?	4339.80	1−	
4281.85	1+		4341.98	1+	
4282.48	½		4347.06	1−	
4283.38	1+		4349.13	1+	
4284.15	½		4354.59	1−	
4285.26	1+		4356.73	1+	
4287.58	1+		4364.68	1+	
4290.25	1+		4373.01	1+	
4293.45	1+		4381.70	1+	
4294.86	1−		4382.92	½	hazy
4296.98	1+		4384.74	1−	
4298.55	1−		4390.	——	very indistinct.
4300.92	1+		4390.85	1+	
4302.63	1+		4400.30	1+	
4305.24	1+		4405.04	1−	
4307.07	1+		4410.12	1+	
4309.98	1		4413.09	½	
4311.89	1+		4420.42	1+	
4315.12	1		4421.59	1−	
4317.07	2	slight shading toward	4430.94	1	
4320.68	1−	[blue.	4436.48	1	
4322.74	1		4447.18	½	
4324.59	1+		4453.30	1	
4326.61	1		4458.03	1	
4328.77	1+		4465.01	1	
4331.38	½	hazy.	4477.15	1	

Aluminium.

The aluminium arc in hydrogen gives a beautiful fluting with first head at 4241.26 and running toward longer wave lengths. This fluting appears equally well in an atmosphere of ammonia. The following table gives the wave lengths and intensities of the principal lines: —

Wave lengths.	Intensities	Remarks.	Wave lengths.	Intensities.	Remarks.
4241.26	3	1st head.	4248.09	2	
4241.75	3		4249.68	2	
4242.41	2		4250.34	1	
4243.10	2		4251.44	2	
4243.94	3	wide.	4253.26	2	
4245.32	4		4255.22	2	
4246.58	3		4257.35	1+	
4247.58	1		4259.71	3	wide, 2d head.

Wave lengths.	Intensities.	Remarks.
4261.18	3	
4261.77	3	
4262.59	3	
4263.50	3	
4264.58	3	
4265.80	3	
4267.24	3	
4268.86	3	
4270.68	3	
4272.72	3	
4274.98	5	impurity here.
4277.70	4	impurity here.
4280.67	4	
4283.94	4	
4287.30	2	3d head?
4287.75	3	
4289.91	3	
4290.68	2	
4292.01	2	
4294.31	3	
4296.99	2	
4298.10	3	
4302.08	3	
4302.65	1	
4306.34	3	
4310.82	3	
4315.57	3	
4320.63	3	
4326.00	5	impurity superposed.
4331.91	2	
4338.37	2	
4345.34	1	
4353.38	2	4th head.
4354.13	1	
4355.17	1	
4356.64	1	
4361.30	1	
4362.21	1	
4363.50	1	
4365 18	2	
4367.21	2	
4368.	½	
4369.67	2	
4371.49	½	
4372.54	1	
4375.18	½	
4375.97	1	
4379.19	½	
4379.90	½	
4388.23	1	
4393.42	1	
4399.19	1	

MAGNESIUM.

The magnesium arc in hydrogen gives the three flutings discovered by Liveing and Dewar * in the magnesium-hydrogen spark, with first heads at 5618, 5210, and 4849, and running toward the shorter wave lengths. The fluting at 5210, which is the one showing the plainest on my photographs, is made up of such very fine lines near the heads that the principal head appears like a line by itself; but farther away from the heads the lines seem to become stronger and to overlap one another, so that many of these lines are much stronger than the head itself and their distribution seems quite irregular. I mention this more particularly because it is characteristic of the hydrogen-zinc and hydrogen-sodium flutings described below. I have noticed that in the spark, the intensity of the magnesium flutings is greatly increased with respect to that of the "b" group by the introduction of inductance in series with the capacity

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* Proc. Roy. Society, **32**, 189 (1881).

shunted about the induction coil. The deposit in the hood enclosing the magnesium arc in hydrogen is quite plentiful, has a dark slate color, decomposes water at ordinary temperature, giving alkaline reaction, and oxidizes rapidly on heated platinum.

ZINC.

The zinc arc in hydrogen gives a collection of lines between 4300 and 4050, having an average intensity from 2 to 4, and not found in the arc in air. This appears to be a set of flutings of complicated structure having heads less distinctly marked than usual and running toward the shorter wave lengths. The semi-opaque deposit formed in the atmosphere of the hood is so considerable that a current of not more than about four amperes can be used. This deposit is dark brown in color, gives alkaline reaction in water, but does not decompose it enough to form bubbles even when heated. It dissolves completely in sulphuric acid, forming a clear solution, and rapidly oxidizes on heated platinum.

SODIUM.

The sodium spectrum was obtained by using metallic sodium as the cooler rotating electrode and copper as the stationary one. As above mentioned, there is not the slightest trace of any of the hydrogen lines to be detected in this spectrum either visually or on the photographs, but there is a strong series of lines between 5000 and 3800, resembling the hydrogen-magnesium series in character. This is probably a complicated fluting of heads less clearly marked than usual and running toward the shorter wave lengths. A compound of sodium and hydrogen is already well known. The formation of the semi-opaque deposit in the atmosphere of the hood is so considerable that the arc can be run only about five minutes at a time. I have not tried the sodium arc in air.

The sodium spectrum obtained in hydrogen is itself quite interesting. All the sodium lines given by Kayser and Runge * come out very clearly, but the principal interest centres about the D lines, which are very intense, and so wide as to cover all the region between them. When observed visually their reversals change in width quite rapidly. At first these reversals may be quite narrow black lines, and then they quickly widen and blot out the whole of the bright field between them. The width of the two lines taken together is about 150 Ångström units, though the photographic plates are stained for a much greater width.

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* Kayser & Runge, *Weid. Ann.*, **41**, 302 (1890).

The strongest copper lines show only very faintly, the weaker ones not at all.

Correlation of Effects.

In the metals arranged in the order given above (tin, silver, copper, magnesium, aluminium, zinc, and sodium) the following relations hold roughly: —

(1) The set of lines characteristic of the spectrum of each metal in an atmosphere of hydrogen is stronger than that of the *preceding* metal of the series; (2) the hydrogen lines appearing in the spectrum of the metallic arc of each metal are stronger than in that of the *succeeding* metal of the series; (3) the general working of the metallic arc is worse for the metals at the first of the series than for those at the end. Briefly stated, the intensities of the hydrogen lines coming out in the spectra of various metals are roughly inversely proportional to the intensities of the characteristic flutings of those metals.

GENERAL EFFECTS OF THE HYDROGEN ATMOSPHERE.

Historical.

Liveing and Dewar * found the carbon arc to work badly in hydrogen, and to give spectral lines of different relative intensities than in air. Professor Crew † has given quantitative measurements of the changes of intensities for the metallic arc spectra of magnesium, zinc, and iron.

The general effects of the hydrogen atmosphere may be summarized thus: —

(1) The arc works poorly in hydrogen. (2) The intensity of the whole spectrum is greatly reduced in hydrogen. (3) Those metallic lines which belong to the series of Kayser and Runge are uniformly reduced in intensity. (4) Other lines are reduced in intensity but not uniformly. (5) Certain lines supposed to belong to the spark spectrum make their appearance in the arc in hydrogen.

Discussion.

The radiations of the electric arc are generally admitted to be due to three causes, — electrical, chemical, and thermal. The chemical cause must depend upon the electrical cause in some way, for the chemical cause

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* Proc. Roy. Society, **33**, 480 (1882).

† Phil. Mag., **50**, 497 (1900).

cannot originate the arc, and the chemical cause follows the electrical in point of time, as is shown by the "luminous cloud" of Crew and Basquin above referred to. The thermal cause also must depend upon the electrical cause in some way. It probably depends upon it directly, but in any event, it is a function of it through the chemical cause, for all chemical reactions either take in heat or give off heat.

Let us consider two arcs which are alike except that a larger current runs through the first than through the second. Since the secondary causes of radiation go hand in hand with the electrical cause we may expect the first arc to have a spectrum which is uniformly brighter from one end to the other than that of the second arc. With the exception of a slight variation probably due to conduction losses, this is just what is always observed and confirms the secondary character of the chemical and thermal causes of radiation. If these causes were not dependent upon the electrical cause, we might possibly get an arc which would give only a flame spectrum or an arc which would give only a spark spectrum.

Let us now suppose that we run the same current through both the similar arcs, and suppose that in some way we reduce the chemical action going on in the second arc. What difference may we expect to observe in them?

A reduction of the chemical action necessarily involves a reduction of the temperature of the arc, because the chemical reaction in the arc in air is exothermic. We have then an arc of lower temperature. If it is a stationary arc it will be shorter and will go out more frequently. If it is rotating it will have a smaller flame and work more poorly. All of which is amply verified by experiments in hydrogen.

But we may expect this reduction of chemical action to have certain effects upon the spectrum. If all the lines of the spectrum of this arc were functions of the *electrical cause alone*, then there would be *no reduction* in intensity of any part of the spectrum when the chemical action is reduced. Professor Crew estimates from 5 to 100 times as the reduction in intensity caused by the hydrogen atmosphere. The electrical cause alone can account, then, for only a small part of the radiation. The secondary causes play very important parts.

If all the lines of the spectrum of this arc were the *same function* of the causes of radiation, then all the lines of the spectrum would be *uniformly* reduced in intensity upon the reduction of chemical action. Experiment shows this hypothesis to be too broad, but the lines belonging to the series of Kayser and Runge are uniformly reduced in intensity, so that it is *probable* that these lines are all the same function of the causes of radiation.

Of the other lines, those which are reduced more in intensity than the series lines, must be less intimately related to the electrical or thermal causes of radiation than are the series lines.

Let us agree that the average intensity of the spectrum of the arc in hydrogen is only one fifth of its intensity in air, and let us agree that the electrical cause of radiation remains practically constant with constant current and voltage although the general intensity of the arc is greatly reduced by the hydrogen atmosphere, then it follows that of the total radiation, that fraction which must be attributed to the electrical cause alone, is relatively five times as great in hydrogen as it is in air. Any line, therefore, which is a function of the electrical cause alone, should have in hydrogen five times the relative intensity that it has in air. It seems quite likely that this may account for the appearance in hydrogen of numerous strong spark lines, not found in the arc in air.

The appearance of the spark lines in hydrogen is not confined to the rotating arc; the magnesium spark line at 4481 appears clearly in the stationary metallic arc in hydrogen but not in air. The above explanation for the appearance of these lines makes it probable that the electrical cause of radiation is not zero in either atmosphere.

In the rotating arc the current is interrupted about twenty-five times per second when the rotating electrode is a rod, instead of a disc, of metal, and this spark at the breaking of the current may account, in part, for the appearance of these spark lines in hydrogen. But we may inquire why this spark should partake any more of the nature of the true spark in hydrogen than in air. The reduction of the chemical action in the arc reduces the temperature and conductivity of the gases between the poles in hydrogen, and it occurred to me that this action may affect the appearance of the spark lines in either of two ways: —

1. It may be that a gas which is in the hot condition of the arc in air cannot give off spark lines; the arc spectrum may be characteristic of this condition of the gas and may have nothing to do with electrical action, and so, in this state, would give off only arc lines if a spark were passed through it.

2. It may be that the conductivity of the gases in air is reduced so slowly at the breaking of the current in the rotating arc that the voltage of break never rises high enough to make a true spark.

In either of these cases, in hydrogen, the hot gases are largely absent, owing to reduction of chemical action, and give opportunity for the spark to appear.

In order to test the first suggestion I arranged an electrical circuit as shown in the diagram. The dynamo furnishes a direct current of 110 volts, and when the switch was closed the current simply passed through the arc and the resistance in series. The arc was stationary, one electrode was carbon and the other a zinc rod. The induction coil used is a duplicate of the one designed by Professor Rowland to give a short spark but a very powerful discharge; an alternating current of 110 volts, 6 amperes, was run through the primary, without an interrupter. The condenser used has a capacity of $\frac{1}{50}$ microfarad. It will be noticed that the spark can take place only by passing in succession the two gaps marked "arc" and "spark." The spectroscope is adjusted to observe phenomena at "arc" gap.

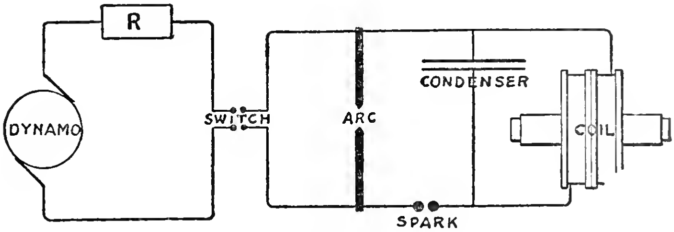

FIGURE 1.

In performing this experiment I first turned on the spark and set the cross-hairs of the eyepiece of the 10-ft. concave grating upon the zinc spark line at 5895, between the D lines of sodium. The spark was turned off and the arc turned on. The spark lines no longer appeared, but came out instantly when the spark was again started along with the arc; both arc and spark were now running through the gap marked "arc" and the spectroscope showed both arc and spark lines. Now while both currents were on, the arc current was turned off; the arc spectrum disappeared, but the spark spectrum persisted with apparently the same intensity as before and without an interval of darkness.

This experiment shows that the first suggestion is not true; that the arc spectrum is not characteristic of the condition of the gases in the arc, and makes it highly probable that the electrical cause of radiation is not zero.

In order to test my second suggestion above, I short-circuited the spark gap shown in Figure 1. The spark line appeared as before in the spark, but disappeared as soon as the arc current was made; the arc and the spark discharges were both passing through the arc as before; I had simply cut out the "spark" gap, but the spark line could not be seen when both currents were on. Now when both currents were on I broke the arc circuit, and nothing at all could be seen in the spectroscope; neither the arc nor the spark lines remained, although the spark current was still passing. After remaining at the eyepiece of the spectroscope about one second I began to see traces of the spark lines, and then they soon came out with their usual brightness, and the spark discharge which had been silent during that second of darkness assumed its usual noisy character.

This experiment shows that the gases of the arc do not furnish enough resistance to the passage of a high voltage alternating current to cause the discharge to assume the character of a spark for a full second after the breaking of the arc current. This seems to confirm the second suggestion above, to the effect that the conductivity of the gases decreases so slowly on the breaking of the arc current in air as to give rise to no very high voltage, and so accounts for the non-appearance of the spark lines in the rotating arc in air.

These two experiments throw an interesting light upon the nature of the spark. The spark at the arc gap in these experiments seems to be due to neither the current nor to the voltage, but to some kind of an impulse furnished by the sudden rush of electricity across the auxiliary "spark" gap.

In the second experiment, above described, the spark lines do not all seem to come out at the same time. I hope in the near future to be able to arrange an automatic apparatus for making and breaking the currents and an adjustable occulting-screen which will enable one to photograph the spectrum of the spark at definite intervals of time after the arc current is broken. A series of these photographs will probably furnish an interesting story of the development of the spark spectrum.

Physical Laboratory,
Northwestern University.

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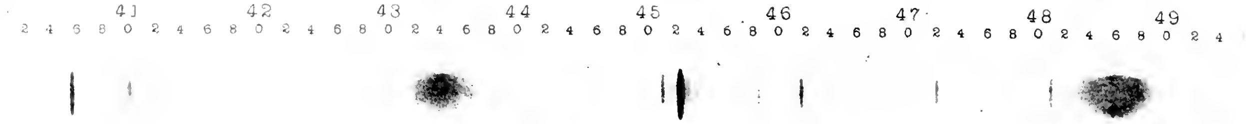
BASQUIN — THE ARC SPECTRUM OF HYDROGEN

PLATE I



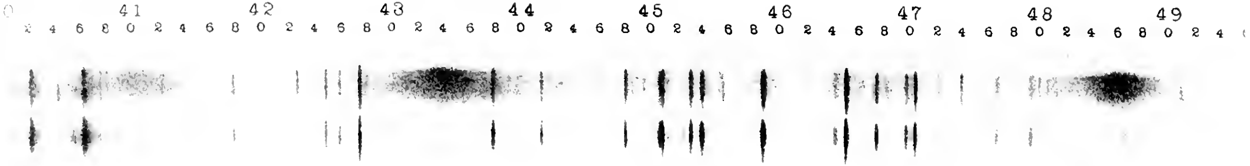


Fig 2

Fig 3

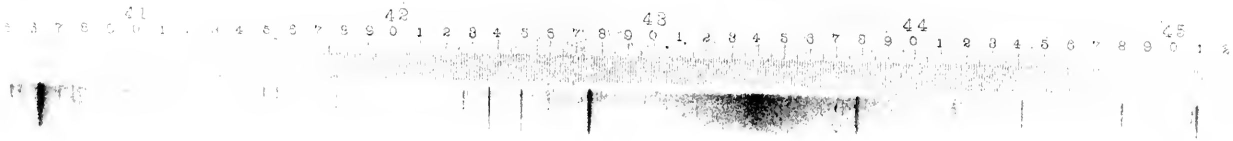


Fig 4

THE HELIOTYPE PRINTING CO., BOSTON

BASQUIN — THE ARC SPECTRUM OF HYDROGEN

PLATE II

Fig. 1

Fig. 2

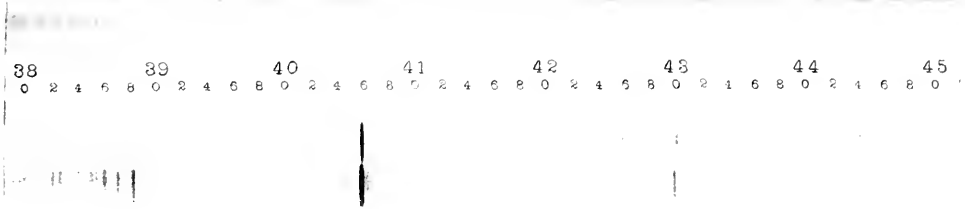


Fig. 3

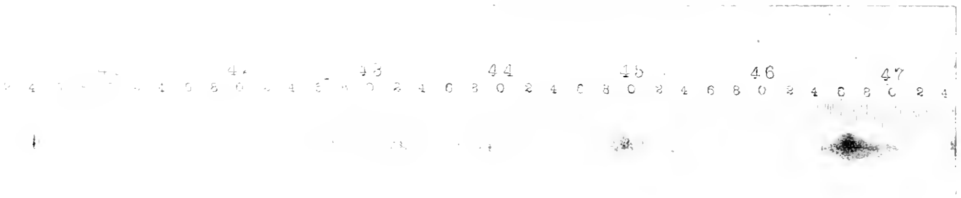


Fig 4

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

THE STANDARD OF ATOMIC WEIGHTS.

By Theodore William Richards.

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THE STANDARD OF ATOMIC WEIGHTS.

By Theodore William Richards.

Received July 27, 1901.

The long continued discussion concerning the relative advantages of hydrogen and oxygen as standards of the numerical values of chemical combining weights seems to need yet another word. In spite of the fact that an international committee has decided by a large majority in favor of oxygen, the opposing arguments have not been put to rest.

The latest paper on this subject is by Erdmann,* the well known champion of the old unit value for hydrogen and the new value for every other atomic weight. The paper consists mainly of a partial reply to an earlier paper by Brauner.† The weight of the argument in these papers seems to be distinctly on Brauner's side, but it is not my purpose to recapitulate all the arguments which these gentlemen and others have advanced.‡ I wish rather to call attention to a few points which do not seem to have received the attention which they deserve.

The first of these concerns the question of fact. What element has served as the actual standard of comparison in a plurality of cases? The question is easily answered by referring to Clarke's valuable compilation.§

Evidently hydrogen in combination has been weighed accurately only in the cases of water and the ammonium salts. The atomic weights of zinc, aluminum, iron, nickel, cobalt, and gold have been determined by

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* Zeitschrift für anorg. Chem., 27, 127 (1901).

† Zeitschrift für anorg. Chem., 26, 186 (1901)

‡ A recent recapitulation of many of the arguments on each side may be found in the report of the American Chemical Society's branch of the International Committee, published in the Journal of the American Chemical Society, February, 1901, p. 44 of the Proceedings.

§ F. W. Clarke, A Recalc. of the At. Weights, Smithson. Misc. Coll., The Constants of Nature, Part V. (1897).

measuring or weighing the hydrogen which they displace or to which they correspond, but the results of different experimenters are far from concordant. All other elements beside these eight have been referred to hydrogen only with the assistance of oxygen.

On the other hand, oxygen has been used as the direct standard of reference in countless cases. The determination of oxygen in the chlorates, bromates, and iodates may be considered as the starting-point for the calculation of Ag, K, Na, Cl, Br, and I, and through them of very many others. Into this remarkable series of experiments, executed in great measure by Stas, the value of hydrogen enters only in the case of ammonic salts. If the atomic weight of nitrogen were certain, we should indeed have here a direct basis of comparison, but unfortunately the value for this element may be as much as 0.05 per cent, or even more, in error. The direct practical determination of the exact composition of ammonia gas, either by analysis or synthesis, has not yet been accomplished. The value for nitrogen depends largely upon the analysis or synthesis of nitrates, thus making oxygen the essential standard of reference in this case also. The other elements which have been determined more or less accurately by reference to oxygen are as follows: H, C, Cu, Ca (through the carbonate), Pb (through the nitrate), Zn, Cd, Hg, Tl (through the nitrate), Sn, P, As, Sb, Bi, Mo, U, W, Se, Te, Mn, Fe, Ni, Co. If one adds to these all those which are connected less directly with oxygen through the halogen and silver values and the sulphates, all the chemical elements are included in the list. Thus an overwhelming majority of elements is referred more directly to oxygen than to hydrogen.

Erdmann points out in his recent paper that there are possible causes of error in some of the methods used by Stas for the analysis of chlorates. Unfortunately he does not touch upon the very important question of the percentage effect of these causes of error. It is undoubtedly true that in these cases, as well as in every other case, absolute accuracy was not attained. No analytical method is wholly free from the possibility of error, and hence it is vain to expect that any table of atomic weights should be perfectly trustworthy. When the accuracy of Stas has been exceeded in actual fact, it will be time to forsake his results for the newer values.

Erdmann suggests that silver be chosen as the standard of reference, and the suggestion is one which has some advantages. On the other hand the tendency which this metal has to absorb oxygen has cast a suspicion over some of the work in which it was used. A further objection to silver lies in the fact that it cannot be directly used in the demonstration of Avogadro's rule. Moreover, one is in doubt as to the value to assign to this element, supposing that it should be selected as the standard. According to Erdmann's earlier arguments, logically followed out, one should make silver 100.000, but this would cause hydrogen to be less than unity. If silver is taken as 107.11, hydrogen would be 1.000 at the present time, but what it might be in the future no one can predict, since hydrogen is compared with silver at present only in a roundabout fashion. Hence each of these assumptions would bring with it a further disadvantage besides that attending the immediate inconvenience of using new values.

The most important argument used by the minority is the pedagogic one. It is contended that the uneven value for hydrogen, 1.0075, complicates the explanation of the very important rule of Avogadro. If this were true, it would indeed be worthy of consideration, but according to my experience there is no difficulty in the matter.

For some time I have abandoned the comparison of specific gravities as a means of demonstrating Avogadro's rule. I have used instead the *densities* of gases and vapors, — that is, the actual weights of a litre of the several substances at 0° C, or at 273° C or at 546° C. This seems to be a more successful method, probably because density has concrete dimensions, and is not a numerical abstraction as specific gravity is. The student at once comprehends the equation of ratios $\frac{1.97}{1.43} = \frac{x}{32}$. If the exact experimental values for the densities of the two gases are given, the solution of this equation gives the student not only the observed molecular weight of carbon dioxide, but also an insight into the extent of the actual deviations from Avogadro's rule. Since the introduction of this method of presentation, I have had far less trouble, and far more successful examination results, than were formerly obtained. The student usually learned by heart the old rule, "The molecular weight equals twice the specific gravity," without understanding it. Because the density-method would serve equally well with any gas used as a standard, the pedagogic argument against H = 1.0075 seems to me illusory.

The argument just discussed has led the Committee of the German Chemical Society to an action which seems to me exceedingly unfortunate, — namely, the publication of *two* tables of atomic weights. This action has already been criticised by Küster and others. Either table alone, supported by suitable weight of opinion, would have been vastly better than two. The mistake is especially to be regretted because the eminent committee in question has previously acted with so much wisdom and ability.

It seems to me that by far the most important questions which have been raised in the whole discussion are the questions of uniformity and permanence of usage. These were indeed the prime objects of the foundation of the German Committee in the first place. Nothing could be more destructive to accurate calculation than a changeable standard of measurement; and yet this very uncertainty marks the present state of affairs.

I cannot but think that every one should accept the standard of reference upon which any considerable majority of representative chemists agree, since the matter is rather a question of convenience than a question of principle. In the first place I preferred O = 16.000 primarily because so much valuable work, both in analytical and in physical chemistry, has already been calculated upon this basis, and because of the effect of a possible change in the oxygen-hydrogen ratio. At present a still more important reason for preferring this standard exists, namely the action of the International Committee, consisting of some of the most prominent chemists of many countries, appointed for the express purpose of voting upon this question. This Committee, by a large majority, decided to call oxygen exactly 16.000. I cannot avoid the belief that until a yet more representative body of chemists is appointed by international co-operation, or until the present committee reconsiders its vote in parliamentary fashion, the present verdict of this committee should rule the chemical world. Unless chemists are prepared to accept such a ruling, the appointment of an international committee is a waste of time.

Representative government in civil affairs would be impossible if the minority refused to act in accordance with the decision of the majority. Does not the same principle apply to scientific rulings? Of course intelligent *discussion* is always desirable — the restriction applies to action and not to speech. Before the action of the International Committee the situation might have been called one of scientific barbarism, but at present it may be called one of scientific rebellion.

Formerly new determinations of atomic weights made at Harvard were expressed in publication both upon the basis O = 16.000 and upon the basis O = 15.879, because the question had not been decided by representative vote. In future, out of respect to the action of the International Committee, only the former standard will be used in this

Laboratory. If an adequate internationally representative body of chemists should in the future decide that some other standard is better, immediate change of practice will be made to suit the new decision. One regrets that so much time should have been spent in discussing a matter which involves no fundamental principle, but is simply a question of form and of convenience.

The subject matter of the present paper may be summed up in the following sentences. It is pointed out that oxygen has actually served as the experimental standard of reference in a great majority of cases, that a great bulk of valuable work has already been published on the basis O = 16.000, and that the use of this standard involves no important didactic difficulties. It is further contended that the decision of the representative International Committee is in itself an important reason for adopting this standard, and that uniformity of usage is more important than any of the special advantages claimed by either side in the discussion.

Seal Harbor, Mt. Desert, Maine,
July 22, 1901.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR. — No. 127.

STUDIES ON THE REACTIONS OF LIMAX MAXIMUS TO DIRECTIVE STIMULI.

By Peter Frandsen.

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By Peter Frandsen.

Presented by E. L. Mark. Received September 3, 1901.

CONTENTS.

Introduction.

The following studies were made at Harvard University during the fall and winter of 1898–99. The problem was proposed by Dr. C. B. Davenport and the investigation carried on under his immediate direction. I wish here to acknowledge my indebtedness to his many suggestions and helpful criticisms throughout the year. In connection with the preparation of the manuscript for publication, I am under obligation to Dr. E. L. Mark for many kindnesses.

The behavior of any organism toward artificial stimulation is probably always largely dependent on its normal environmental conditions. The long action of those conditions, assisted, perhaps, by the animal's own efforts, conscious or unconscious, to adapt itself to them, finally results in certain habits and instincts. The process of adaptation being extremely slow, organisms are strongly averse to great or sudden changes in their environment and incapable of adjusting themselves to them. As a rule, then, we should expect animals to seek those conditions of light, heat, moisture, and other physical and chemical influences which are most in accordance with those to which they are normally subjected.

The most easily observed responses of animals are naturally those which find their expression in locomotion. The number of stimuli which may influence locomotion are, of course, numerous, but of these a certain limited number play much the larger part. If we had an accurate knowledge of the relative weight of these different forces, we might predict with certainty the path any animal would follow under certain given conditions. An experimental study of the different stimuli ought at least to enable us to find out which ones do operate, and perhaps to establish certain general laws regarding them and the biological tendencies which impel the animal to respond.

The present paper is a study of the locomotor responses of the slug Limax maximus to three kinds of stimuli, — those of touch, gravity, and light. In connection with these studies new problems have constantly arisen, some of which have been cursorily considered, many others merely alluded to, so that the work is far from being complete.

The term "geotaxis" has been used to designate the influence of gravity on locomotion. Interesting and careful studies have been made on the geotaxis of numerous Protista by Schwarz ('84), Aderhold ('88), Massart ('91), and Jensen ('93). These investigations clearly show a geotactic response in the unicellular organisms studied. The kind of response varies according to other conditions, such as those of light, heat, density of medium, chemical influences, etc., and may also differ in individuals of the same genus under apparently like conditions. Massart ('91, pp. 161–162) found that, when a number of Spirilla were put into a vertical tube, one group collected in the upper part and another at the lower part. He also found (p. 164) that Chromulina woroniniana was negatively geotactic — that is, moved upward, or in a direction opposite to that of the pull of gravity — at 15° to 20° C., but positively geotactic at 5° to 7° C. Jensen's work also showed the important influence of other agents in modifying geotaxis. Loeb ('88, pp. 7–8) found that cockroaches preferred the steepest side of a box whose four sides were inclined at different angles; that is, they are negatively geotactic. He also discovered that a number of other Metazoa were geotactic.

In a certain way, the present paper is a continuation of a recent study made by Dr. C. B. Davenport and Miss Helen Perkins on geotaxis in the slug. Davenport and Perkins ('97, p. 105) discovered that the intensity of the animal's geotactic response was directly proportional to the sine of the angle of deviation from the vertical, and hence "varied directly as the active component of gravity." In the third section of their paper, the question, "What determines whether the head end of the slug shall be directed up or down?" was raised and considered. The results showed that certain individuals appeared to have a fairly marked positive geotaxis, for, when placed on an inclined glass plate, such animals swung the head-pole of the axis toward the earth; but others showed as strongly marked a tendency to move away from the earth, and a few seemed indifferent as to whether they went up or down. Their experiments showed further that there was, apparently, no inherent tendency in individual animals to move either to the right or to the left, so that the difference in geotactic response could not be explained as due to differences of an inherent tendency of this kind. The effect of a slight initial impulse given to the head of the animal indicated that the thigmotactic, or contact, stimulus imparted to the animal in handling might, to some extent, modify its response to the stimulus of gravity. But Davenport and Perkins did not reach any definite, satisfactory answer to the main question.

It was to test their observations by a larger number of experiments, and, if confirmed, to explain them by further experimentation, that the present investigation was undertaken. In the first place, I wished to find out whether certain individuals, if put on an inclined glass plate, always responded to the pull of gravity by directing the head end up and moving away from the earth, and whether certain other individuals always did the contrary. If this proved to be true, then it was my main problem to seek the reason for it. Is the force which makes some slugs go up, others down, and still others indifferent to the attraction of gravity, a purely accidental one, — is it a physical force, or is it what we may call a psychical peculiarity, which varies in different individuals and in the same individual at different times? As a preliminary to the main problem, I first made a series of experiments on the animal's thigmotaxis, — its response to contact- and pressure-stimuli. By virtue of its thigmotaxis, an animal moves either toward or away from the agent which comes in contact with it, just as its geotaxis is expressed in a movement toward or away from the earth, in response to the attraction of gravity.

I. THIGMOTAXIS.

Material and Methods. — The animal used in all the following experiments was Limax maximus, which is fairly abundant in the greenhouses about Cambridge. Material was obtained from several different greenhouses and kept in a large closed tin box, the bottom of which was covered with moss kept moist, so as to afford an environment as much like the customary one as possible. Fresh cabbage leaves constituted the animal's main food. The cannibalistic tendencies of the slug, together with an unavoidable deterioration due to repeated handling, necessitated a frequent renewal of the animals.

The methods used in the experiments were simple. The slug was placed on a circular glass plate set horizontally in the bottom of a cuboidal wooden box which was made impervious to light and covered with a thick, black cloth. Precautions were taken to avoid thermal and chemical influences by keeping the box at as equable a temperature as possible and by wiping the plate free from slime before each test. The tests were made only when the animal had definitely oriented itself and was moving ahead in a straight line. Two series were made. In the first series the dorsal tentacle was touched gently with the forefinger. The box was then immediately covered with the black cloth. Observations were made after the lapse of 20 to 30 seconds and the position of the animal noted. The right and left tentacles were touched alternately.

Results. — The following Table (I.) gives the results of a number of experiments on ten different animals.

TABLE I.

Response to Thigmotactic Stimulation of the Tentacles.

Animal No.	Number of Trials.			Total Number of Trials.
	—	+	0	
1	7	2	3	12
2	11	3	3	17
3	8	3	1	12
4	4	3	3	10
5	7	2	0	9
6	6	0	2	8
7	10	2	2	14
8	16	4	2	22
9	18	1	5	24
10	22	1	4	17
Totals . .	99	21	25	145

The column headed with the minus sign shows the number of times the animal responded by moving *away* from the source of stimulus; the one headed with the plus sign, the number of times it moved *towards* that source; and the zero column, the number of times there was *no* response. I found that the animal would respond very definitely and precisely to stimuli two or three times in succession by immediately retracting the tentacle touched and moving away from the stimulating influence. After the third trial, however, it either refused to change its direction of locomotion or else moved directly towards the source of the stimulus. If a respite of a few seconds before the next stimulation was then permitted, the animal would again give a precise negative response for two or three trials, and then, as before, it desisted. Out of the total 145 tests, there was a negative response in two thirds of the trials. The remaining trials — one third of the whole — were about equally divided between the positive responses and refusals to respond at all. Sometimes five or six tests were made in quick succession, so that the total negative response is rather less than it would have been if a rest had been given in each case after three tests. Out of the 21 cases of direct positive response, 15 were cases where the right tentacle was touched, and the remaining 6 were due to stimulation of the left tentacle. Similar, but more marked, differences between the results of stimulating the right and the left tentacles were observed in other experiments. This suggests that either the right tentacle may be less sensitive to stimuli, or that its counterirritancy may be more readily aroused. There is, however, a third possible cause. The animal may have an innate tendency to go to the right, and, if so, this tendency may diminish to some extent the force of the stimulating agent when it impinges on the right side of the animal, and correspondingly increase the response when the stimulus is directed upon the left side of the animal. Something further will be said about this point in a later part of the paper.

A few thigmotactic experiments were next made on the sides of the animal posterior to the head. The right and left sides were touched alternately. The results are given in Table II.

Phenomena like those observed in stimulating the tentacles are seen here, and they also agree with similar observations by Davenport and Perkins ('97, p. 109.) After two or three trials, the animal begins to show resistance, and if the finger is held against its side, will sometimes try to displace the finger by pushing against and curling the body around it. The frequency of the negative response is here somewhat less marked than in the preceding experiments, which is as we should expect, owing to the greater sensitiveness of the tentacles as special tactile organs. In these experiments every one of the minus and zero results was due to stimulation of the right tentacle.

TABLE II.

Response to Thigmotactic Stimulation of the Sides of the Body.

Animal No.	Number of Trials.			Total Number of Trials.
	—	+	0	
1	11	3	3	17
2	8	6	2	16
3	17	5	4	26
Totals . .	36	14	9	59

These facts clearly prove that, under ordinary circumstances, the slug is negatively thigmotactic. In our consideration of the animal's responses to other stimuli, we shall have to take this into account, as causing occasional vagaries, and therefore endeavor to eliminate it as much as possible from the experiments.

II. Geotaxis.

What determines whether the head end of the slug shall be directed up or down?

Methods. — The same apparatus was used as in the preceding experiment. A circular glass plate was employed so that the animal could be rotated into any desired position without the necessity of its being handled. The plate was set in a box at an angle of about 45° with the horizon. In each test the animal was so placed on the plate that the long axis was horizontal, different sides being directed downward in different trials. At first the experimentation consisted mostly of watching the animals in order to obtain some clue for further work. Later, rough sketches of the pigment patterns of the individual animals were made, so that it was possible to identify individuals with certainty; the same animal could then be subjected to experiments at different times and the difference in results noted. The methods used in working out particular questions will appear as these questions are considered. As the same number of experiments were not made on each animal studied, I have, for the sake of comparison, estimated in each case the geotaxis in per cents. This percentage is obtained by dividing the number of positive or negative responses by the total number of responses. The nearer the geotaxis percentage approaches 100 the more precise has been the kind of response. No fixed time was allowed to elapse between successive tests, but in each test the observation was made at an interval of from 30 to 60 seconds after covering the box.

Operations and Results. — The first question investigated was whether particular animals exhibited a decisive positive or negative geotaxis. A number of tests were, therefore, made on each of several selected individuals. The results obtained were like those of Davenport and Perkins ('97, p. 108); that is, certain animals showed a very marked positive geotaxis; others, an equally decided negative tendency; and a few, perhaps one animal out of 12 or 15 where 10 or more tests were made, were apparently geotactically indifferent. The occasional irregularities in the responses of individual animals were easily seen to be due to influences other than pure gravity, such as jarrings of the plate, influence of contact in putting the animal on the plate, and to the influence of light admitted in lifting the cover of the box. Frequently, upon the raising of the cloth to make an observation, the animal would retract its tentacles, as if dazzled by the sudden inflow of light, and at the next observation it would be seen to have altered its response.

Naturally, this question next arose, Is the response the same on different days? In Table IV. (p. 195) are given the results with a number of animals experimented on to test this point. These are numbers 2, 7, 8, 22–25, 27. Number 2 was positively geotactic on two days and negative on another day. A similar variation is seen in the case of slugs 7 and 22. In the case of all the rest, however, there is a very marked constancy. The ninth (last) column in the table indicates the condition of the animals at the time of experimentation. We see from this that on the days of different response, the animals were in somewhat unlike conditions, which may account for the irregularity of response. The significance of this will be dealt with later. The important matter here is, that the animals, when in the same condition and under the same circumstances, have a fairly constant geotaxis from day to day. One of the most marked cases is that of number 24. This animal was experimented on at different times for a period of three weeks. During this period, it was always active and in good condition, and, as the table shows, at all times, exhibited nearly the same percentage negative geotaxis. At the last trial made, it responded irregularly, and so slowly, — at one time not changing its position for thirty minutes, — that I had to give up the attempt to obtain a series. This was often the case with other individuals after a few definite responses.

Tests were then made on the geotaxis of the same individuals at different times of the same day. Considering the slug's normal environment, it would not be surprising if, for instance, it should show an upward tendency in the evening and a downward geotaxis in the daytime. Its nocturnal habits and dislike of daylight might give it a different geotactic instinct at night from that of the daytime. I insert here a table (III.) giving the results of a few experiments bearing on this point. As the table shows, the response is pretty constant at different times of the same day. The one exception is number 2. That it was negative on one evening at 7 P. M., may be explained by the fact that its condition was not good. Moreover, on another evening at the same time the animal had become positively geotactic.

TABLE III.

Geotaxis of three Individuals at Different Times in the Day.

Animal No.	Time of Day.	Number of Trials. +	Number of Trials. −	% Geotaxis.	Condition of Mucus.
1	8.00 A.M.	5	2	+71.4	Good
	1.30 P.M.	6	3	+66.6	Rather Dry
	8.30 P.M.	7	4	+63.6	Tail Dry
2	1.30 P.M.	16	1	+94.1	Good
	7.00 P.M.	9	18	−66.6	Rather Dry
	7.00 P.M.	12	5	+70.5	Fair
	10.00 P.M.	6	3	+66.6	Fair
3	7.00 A.M.	3	8	−72.7	Fair
	1.30 P.M.	0	5	−100.	Fair

From the observations recorded in Tables III. and IV., it is plain that the geotactic response is not due to purely accidental factors, but can be explained only by some marked difference between the individual animals. The first thought is that differences in response are due to a difference in size, and the facts seem to give some support to that explanation. Most of the positively geotactic individuals were found among the small and medium-sized animals, and nearly all the negative animals were of large size. Moreover, the few indifferent individuals were of medium size. This, however, was not an invariable rule. Small animals were sometimes negatively geotactic and, occasionally, a large slug would migrate earthward.

A second, clearly important, factor is the condition of the animal's mucus. As shown by the preceding experiments, animals, positively geotactic when normal, became negatively geotactic when lacking in an abundance of sticky slime; *e. g.* animal 2, Table III., and animals 2 and 7, Table IV. On the other hand, in one instance (22 b), a negative animal, when extremely sticky, went downwards. Abundant, sticky mucus is evidently connected with a downward migration, and dryness seems to force the animal to take an upward direction. But these facts are not enough to explain all responses. For sometimes two animals of nearly the same size and in equally good condition gave different geotactic responses. We must look for other differences. It will, however, be necessary first to refer briefly to the form and external appearance of the slug.

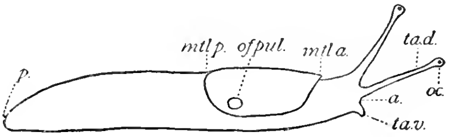

FIGURE 1.

Outline of Limax maximus. *mtl. a.*, anterior edge of mantle; *mtl. p.*, posterior edge of mantle; *a.* to *mtl. p.*, anterior region of body; *mtl. p.* to *p.*, posterior region; *oc.*, eye; *ta. d.*, dorsal tentacle; *ta. v.*, ventral tentacle; *of. pul.*, pulmonary orifice.

The slug, if we except the respiratory opening on the right side of the body, is externally bilaterally symmetrical. It has no external shell. There are two pairs of tentacles, — a dorsal pair bearing the eyes and a smaller ventral pair. The mantle extends from the neck, ventrally, to near the edge of the foot. Posteriorly, it forms a prominent fold, as indicated in the figure, which may be used to separate the body into an anterior and a posterior region. Observations of the animal reveal that it has very different degrees of control over these two regions of the body. In locomotion, the head end of the body, back as far as the respiratory opening, is freely swung about from side to side and determines the axis of orientation of the animal. Over the posterior region, the animal seems ordinarily to have very imperfect control. The relation between the two regions is crudely that of a span of horses to a chain of wagons which they are pulling. When the horses change direction, the wagons come only slowly around into position one after the other, and there is likely to be some slipping in the process, especially if it takes place on a down grade. In watching the slug, I saw that the adhesion of the anterior region appeared considerably greater than that of the posterior. When the animal gets dry, it does so first at the posterior region. The tip of the tail is the part first to lose its clinging power, and it may curl up dorsally as a result of the drying process. If an animal which is thus beginning to deteriorate in its supply of mucus be put on a glass plate and the plate raised into a vertical position, the slug will move along and desperately cling to the plate with the anterior part of its body. The posterior region will gradually swing downward as a result of the pull of gravity, and, in consequence, the animal's head will eventually be directed upward. From this, we are justified in concluding that the same principle will operate, although to a considerably less degree, in the animal's normal condition. A hasty examination showed that there was a good deal of variation in the proportions of the two regions in different individuals. As a crude and easy way of estimating these proportions, I measured the length in millimeters of the anterior region from the tip of the head to the posterior fold of the mantle, and similarly the length of the posterior region from that fold to the tip of the tail.*

The results from 27 animals thus measured are given in Table IV.

The individuals (Table IV.) are arranged in a series, beginning with those in which the two regions are most nearly of the same length and ending with those in which the disproportion is greatest. In animal No. 1, the length of the anterior region is 83.3 per cent (column 8) of the posterior; that is, the ratio is almost one to one. In No. 25, the anterior region is only 45 per cent as long as the posterior, or less than half its length.

The fifth column in the table gives the geotaxis of individuals in per

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* The measurements were made when the animal was extended and moving across the plate. The amount of elongation varies a good deal, but the regions retain pretty closely their relative proportions.

TABLE IV.

Summary of Geotactic Results.

Animal No.	Series of Observations on Different Days.	Number of Trials.		% Geotaxis.	Length of Anterior Region in mm.	Length of Posterior Region in mm.	Ratio of Ant. to Post. Region in per cents.	Condition of Animal.
		+	−					
1	. . .	10	0	+100.	20	24	83.3	Good.
2	a	6	3	+ 66.6	18	22	82.	Fair.
2	b	5	13	− 72.2	18	22	82.	Dry.
2	c	12	5	+ 70.5	18	22	82.	Good.
3	. . .	6	0	+100.	6.5	8	81.	Good.
4	. . .	6	1	+ 85.7	17	21	81.	Good.
5	. . .	5	2	+ 71.4	24	30	80.	Good.
6	. . .	7	1	+ 87.5	11	15	73.	Good.
7	a	0	4	−100.	26	40	65.	?
7	b	9	6	+ 60.	26	40	65.	Rather dry.
7	c	7	3	+ 70.	26	40	65.	Slow.
7	d	0	2	−100.	26	40	65.	Tail slips.
8	a	12	6	+ 66.6	?	?	?	Good.
8	b	17	1	+ 94.4	?	?	?	Good.
9	. . .	0	6	−100.	21	33	63.6	Tail slips.
10	. . .	2	8	− 80.	20	32	62.	Mucus watery.
11	. . .	14	3	+ 82.3	21	34	61.8	Good.
12	. . .	6	0	+100.	27	44	61.	Good.
13	a	5	12	− 70.5	17	25	61.	Active.
13	b	3	8	− 72.7	17	25	61.	Active.
14	. . .	9	3	+ 75.	20	33	60.5	Good.
15	. . .	1	8	− 88.8	24	40	60.	Fair.
16	. . .	0	10	−100.	28	50	56.	Good.
17	. . .	6	14	− 70.	30	55	54.5	Good.
18	. . .	8	3	+ 72.7	30	56	53.5	Extrem'ly sticky.
19	. . .	0	8	−100.	23	43	53.5	Fair.
20	. . .	1	5	−83.3	17	32	53.	Good.
21	. . .	4	12	− 75.	21	40	52.5	Good.
22	a	7	8	− 53.3	41	79	52.	Sticky.
22	b	8	5	+ 61.5	41	79	52.	Very sticky.
23	a	5	9	− 64.2	18	36	50.	Good.
23	b	6	10	− 62.5	18	36	50.	Good.
23	c	0	5	−100.	18	36	50.	Good.
23	d	0	4	−100.	18	36	50.	Good.
24	a	3	15	− 83.3	27	54	50.	Good.
24	b	3	14	− 82.3	27	54	50.	Good.
24	c	2	19	− 90.5	27	54	50.	Good.
24	d	4	19	− 82.6	27	54	50.	Good.
24	e	3	14	− 82.3	27	54	50.	Good.
25	a	7	17	− 70.8	21	44	48.8	Good.
25	b	4	14	− 77.7	21	44	48.8	Good.
25	c	0	12	−100.	21	44	48.8	Dry.
26	. . .	3	15	− 83.3	?	?	45.	Good.
27	a	0	14	−100.	32	71	45.	Good.
27	b	1	5	− 83.3	32	71	45.	Good.

cents. The table includes those animals which were fairly active in response but does not give individuals obviously unable to respond because of a lack of slime secretion. The positively geotactic animals, with two exceptions, are all found in the upper half of the table and almost all the negative animals in the lower half. Supposing other conditions the same, we can say that those animals in which the ratio of anterior to posterior regions is as 2 : 3, or greater, will be positively geotactic. Those between the ratios of 2 : 3 and 3 : 5 will be more uncertain in their geotaxis, which will depend largely on the combination of other conditions. Finally, those in which the ratio is less than 3 : 5 will almost invariably be negatively geotactic. The nearer one gets to the extremes, the greater the accuracy of prediction. This prediction, it is understood, applies only to animals tested on the glass plate.

An examination of the ninth column shows that the few cases of negative geotaxis occurring in the positive half of the table are probably due to a deficiency in the second most important factor affecting the geotaxis; namely, the condition of the slime secretion of the animal. This secretion may be deficient either (1) in quantity, as in the case of slug 2 b; or (2) in quality, as was the case with slug 10. Of the two cases of positive geotaxis occurring in the negative half of the table, the first, that of slug 18, is easily explained as due to an extraordinary tenacity of the mucus. Moreover in this, and more markedly in the case of slug 22 b, the slugs were very large and rather slow in their movements. Slug 22 b, instead of moving ahead actively, like most slugs when in good condition, often swung its head toward the earth without any foreward movement, and hence did not give the pull of gravity the most favorable opportunity to work on the posterior region of its body. This connects itself with a general observation on all the animals. When active, they are usually very precise and uniform in their responses. If stupid, slow, and averse to movement, — a condition in which the best of them sometimes get, — they will either obstinately refuse to move, or else, keeping the posterior region firmly fixed, will swing the head end toward the earth. Sometimes such a slug will slowly move in a circle, first down then up, and finally curl itself up, like a dog by the fireplace, and apparently go to sleep. This peculiarity may be connected with the food conditions of the animals, as will be shown in a set of experiments to be given later on.

The two most important factors in determining the geotaxis of individual slugs are, therefore: first, the proportion of the anterior (mantle-covered) and posterior (uncovered) regions of the body; secondly, the character of the slime secretion of the animal. If accurate measurements were made of the two regions of the body, we might obtain exactly the relative weights of these two factors. By means of a spring balance, the effectiveness of the mucus in counteracting gravity could be ascertained with a fair degree of accuracy. A large number of such observations in connection with geotactic tests might, finally, enable us to state precisely what combination of the two factors — weight of regions and strength of mucus — would be necessary to make an animal positively or negatively geotactic. I have made no such calculations, and it would perhaps not be worth the trouble. The suggestion is instructive, however, as indicating the possibilities of predicting, with a certain degree of exactness, a phenomenon which seems at first sight to be entirely haphazard. Perhaps perfect mathematical exactness would, however, never be possible in this case, for, as I shall show a little later, other factors of importance probably enter in to modify the results. However, these too are not out of the reach of precise definition.

Certain slugs are negatively geotactic because gravity pulls the posterior region of the body down faster than it does the anterior region. Since in all slugs the posterior region somewhat exceeds in length the anterior, we should expect all animals to respond in the same way, provided gravity acted in only a mechanical way. But about the same number of slugs go down as go up. Therefore, there must be some other factor, such as an inherent tendency, impelling these positive slugs to seek the earth. But if so, is it not probable that *all* slugs have this inherent tendency to move towards the earth, the tendency being obscured in the negative slug by the superior force of the mechanical difficulties to be overcome? The fact that positive slugs, when deficient in means of resisting the pull of gravity, — that is, when dry, — assume a negative geotaxis, shows that the inherent tendency *is* sometimes obscured. If this hypothesis is true, then we ought to be able, by diminishing the force of gravity, or better, by increasing the animal's powers of resisting the disproportionate pull on the posterior region, to make the negative animals become positive. Similarly, if this mechanical difficulty of adhesion is the cause of negative geotaxis, we ought, by increasing it, to be able to compel positive animals to become negative. The first end may be attained by substituting for the glass plate a wooden one, which will presumably offer the animal a better chance of adhesion. The second end may be reached by substituting for the glass plate one which has been coated with vaseline or a similar substance. Both ends may also be attained, to a certain extent, by increasing or decreasing the angle of inclination of the plate. An examination of the tables given by Davenport and Perkins ('97, p. 103) shows that the largest average number of negative responses occurred when the glass plate was vertical; that is, when the mechanical difficulties were greatest. There is a gradual decrease in this average (and a corresponding increase in the average number of positive responses), as the angles of inclination of the plate with the horizon were diminished from 90° to 60°, 45°, and 30° successively. At the still smaller inclinations of 22½°, 15°, 7°, and 0° (*i.e.*, horizontal), however, there is on the whole an *increase* in the average number of negative responses, though this is quite irregular. Since the proportion of anterior to posterior region of the animals experimented on is not known, we cannot tell how far this factor may have been the cause of this irregularity in the sense of the response.

I have made a few experiments by varying for the same individual the angle of inclination of the plate. The animals were all in good condition throughout the experiments. The results — given in Table V. — show a decided increase in negative geotaxis with increase in the angle of inclination.

TABLE V.

PER CENT OF GEOTAXIS AT DIFFERENT ANGLES OF INCLINATION OF THE SUPPORT.

Animal No.	Angle of Inclination.	Number of Trials.		% Geotaxis.	Condition of Animal.
		+	−		
1	45°	8	2	+ 80.	Good.
1	90°	0	14	−100.	Good.
2	45°	7	1	+ 87.5	Good.
2	70°	8	3	+ 72.7	Good.
3	45°	2	8	− 80.	Good.
3	90°	0	10	−100.	Good.

The most striking case is the complete reversal of geotaxis, seen in the first animal experimented on.

Still more conclusive results were obtained by the substitution of wood or vaselined glass surfaces for the clean glass plate. In order to make sure that the animal's power to hold on varied with different surfaces, and to determine approximately the relative strength of the adhesion, Dr. Davenport suggested the use of a delicate spring balance, such as are used in weighing letters. The animal was placed on a horizontal glass plate. When it had oriented itself, and was moving forward, the pan of a letter balance was held against the side of the animal and gradually increased pressure exerted until the animal was made to slip along the plate. The maximum reading (in ounces) on the indicator was noted. Then the same animal was placed on a wooden plate and a similar test made under like conditions of movement and activity. The same was done on the vaselined plate. A number of such tests were made on each individual. In order to avoid possible differences in results due to a gradual deterioration in the condition of the animal, the sequence of the surfaces was varied in the successive sets (three) of trials so that each surface was once employed for the first experiment of a set. This method proved fairly satisfactory and gave in some instances very striking results.

TABLE VI.

Amount of Pressure required to dislodge the Slug from Different Horizontal Surfaces.

Animal No.	Wood.	Glass.	Vaselined Glass.
1	1.8 ounces	1.5 ounces	.23 ounces
2	1.25 ounces	.67 ounces	.34 ounces
3	3.16 ounces	2.16 ounces	1.55 ounces
4	4.33 ounces	2.55 ounces	1.55 ounces
5	3. ounces	1.16 ounces	.50 ounces
6	5.7 ounces	3 50 ounces	1.52 ounces

The results recorded for each individual are the averages of three trials on each of the surfaces used. The table shows a considerable difference in the degree of adhesion to the different surfaces. In the last four cases the animals were all very large. They were in excellent condition, having just been captured, and secreted a sticky slime in large quantities.

After being on the vaselined surface, there was a noticeable decrease in the power to hold on to the glass or wood, due probably to the vaseline which still adhered to the animal. Regarding these cases as typical of all slugs, we can say that the wooden surface affords a condition nearly twice as favorable as that of the glass plate for the exhibition of an internal tendency. The vaselined surface, on the contrary, is only about half as favorable as the glass plate; that is, it doubles the obstacles. As a general rule, owing to the irregularities of other influences, the difference between the different surfaces would be, probably, somewhat less. For active, well-conditioned animals, however, we have no hesitation in concluding that the ratios obtained from these cases are fairly representative.

Having thus established the fact that the character of the surface does modify the animal's power to attach itself, I next give a table (VII.) showing the results of a series of experiments on twelve different individ-

TABLE VII.

GEOTAXIS OF THE SLUG ON DIFFERENT SURFACES.

Animal No.	Ratio of Anterior to Posterior Parts in %.	Plate at Inclination of 45°.								
		Wooden Plate.			Glass Plate.			Vaselined Glass Plate.		
		No. of Trials.		% Geotaxis.	No. of Trials.		% Geotaxis.	No. of Trials.		% Geotaxis.
		+	–		+	–		+	–	
1	61.	9	8	+ 53.	0	5	–100.	0	5	–100.
2	?	5	0	+100.	6	5	+ 54.5	0	9	–100.
3	52.	5	0	+100.	1	3	– 75.	0	5	–100.
4	47.	7	0	+100.	1	8	– 88.8	0	slips	0
5	76.	5	2	+ 71.4	5	1	+ 83.3	1	3	– 75.
6	50.	7	3	+ 70.	2	8	– 80.	0	slips	0
7	66.6	9	1	+ 90.	1	9	– 90.	0	slips	0
8	83.3	5	0	+100.	10	0	+100.	4	6	– 60.
9	56.	9	1	+ 90.	0	10	–100.	0	0	0
10	61.	5	1	+ 83.3	5	1	+ 83.3	1	5	– 83.3
11	53.	8	0	+100.	1	5	– 83.3	0	slips	0
12	53.5	6	0	+100.	7	2	+ 77.7	2	slips	0

uals. The geotaxis of each animal was tested on three different surfaces, — the glass plate, a circular wooden plate, and a glass plate coated with vaseline. Care was taken to have other conditions as nearly as possible the same. Circular plates were employed so that the animal could be rotated into a horizontal position without being touched by the hand. In several cases a series was made on an animal using the glass surface; the animal was then transferred to a wooden plate and the same number of trials made; the same individual was then put back on the glass plate and as many more tests were made; finally, it was returned to the wooden plate and an equal number of observations made. The same thing was tried alternating between glass and vaselined surfaces.

The second column shows what per cent of the length of the posterior region of the animal's body its anterior region is, as previously defined. A comparison of the columns "% Geotaxis" under the different conditions at once shows, in nearly every case, a marked difference in the geotactic response with the three kinds of surfaces. The same number of trials was not always made on a given animal under the different conditions, so that the comparisons are not always on exactly the same basis. The results, however, prove pretty conclusively that all animals have an inherent tendency to move toward the earth. On the glass plate, the animals moving upward and downward are about equal in number, the rea-ons for which we have already given. On the wooden plate, which affords the best of the three surfaces for adhesion, all the animals have become positive. A vaselined surface offers still greater difficulties to positively geotactic responses; it compels the positively geotactic animals to become negative (Nos. 2, 5, 8, 10). Some animals are utterly unable to adjust themselves to this extraordinary condition, especially if not endowed with the power of secreting excellent mucus. These animals either vainly cling with the anterior end of the body to the plate, while the posterior region slips downward, thus directing the animal up, or they roll off the plate altogether as soon as it is placed in an inclined position. For this reason some of the animals negatively geotactic on the glass plate gave no geotactic response when they were placed on the vaselined surface. These facts, then, conclusively answer in the affirmative our second question. All slugs have a tendency to move toward the earth.

Now the question naturally comes up, Can we not assist this tendency in those animals which are negatively geotactic on a glass surface by bringing some other stimulus — light, for example — to bear upon them? This slug is negatively phototactic to strong light, as the third part of this investigation will show. By exposing the animals to strong light, can we not make the desire for darkness coöperate with the inherent positive geotactic tendency to such an extent that the two together will overcome all mechanical difficulties and cause the animal to move downward? The following table (VIII.) answers this question in the affirmative.

TABLE VIII.

Geotaxis of Slug on Glass Plate at an Angle of 45° influenced (1) by Gravity alone, and (2) by Gravity and Strong Light.

Animal No.	Size.	Gravity alone.			Gravity + Influence of Strong Light.		
		No. of Trials.		% Geotaxis.	No. of Trials.		% Geotaxis.
		+	–		+	–	
1	Big	0	17	–100.	8	8	+ 50.
2	Big	0	14	–100.	7	5	+ 58.3
3	Big	1	7	– 87.5	2	2	± 50.
4	Big	0	5	–100.	2	4	+ 66 6
5	Medium	3	9	– 75.	4	4	± 50.
6	Small	0	5	–100.	0	5	–100.

These experiments were carried on in the evening. The animal was first tested on a glass plate at an angle of 45° in the dark, in the ordinary way. Then it was placed on a horizontal glass plate and strong lamp light thrown directly upon it for a few seconds. In most cases it immediately gave a negative response to the light. When definitely oriented, the plate was again placed in the box at an angle of 45° and the box covered with a black cloth. Two or three geotactic observations were then taken, and the animal again exposed to strong light. The exposure to light was repeated about three times in the course of ten observations. The table shows that the influence of light has been to change a condition of strong negative geotaxis to one of indifference. The only exception is No. 6, which seemed little affected by the light. I hope to make a fuller study of the combined action of light and gravity later.

It has been said that all slugs have an innate tendency to move toward the earth. Now, this tendency is probably due to the environment and habits of the animal. The slug, we know, is nocturnal in its habits. In the nighttime, it is actively moving about in search of food. In the daytime, it is inactive and seeks concealment, which is of course accomplished by moving toward the earth. In hunting for food, it must naturally do some climbing. These facts lead us to expect a possible difference between the geotactic response of the nighttime and that of the daytime. My experiments in this matter, however, gave inconclusive results. But the animals experimented on were not in their normal environment. There was no light and little change in temperature to assist the instinct, if it exists, in divining night from day. Moreover they did not have to seek food, for it was constantly supplied them. Such being the case, the instinct of concealment would be the main environmental influence on the animal, and this impels it toward the earth.

These experiments have shown, then, that when the mechanical conditions are favorable, most animals exhibit a positive geotaxis. This is as we should expect. There were, however, a few exceptions. A few animals went up when all the factors enumerated seemed to point to the probability of a downward movement, and there were also a few animals which went down when the mechanical difficulties were such as should have impelled them upward. As previously noted, the upward-moving animals sometimes displayed an unusual amount of activity, and the exceptional cases of positive geotaxis in the negative group were those of animals usually slow and stupid. As the effort was constantly made to select only fairly active animals in good condition for producing mucus, there were not many of these exceptions. Knowing the habits of the animal, we may naturally associate its activity with its food condition.

The question then comes up, Does the state of the animal's nutrition affect its tendency to move toward the earth? Does a poorly nourished animal respond to the stimulus of gravity differently from a well-nourished individual? To get an answer to this question, four animals were put into a small box which contained nothing but moist earth. The slugs were kept there for three days, and a series of geotactic tests was then made upon them. Two of the four individuals were inactive, and so unsatisfactory in response that no series was obtained. The other two were rather restless, but precise in response. All the animals were then returned to the box and supplied with fresh cabbage leaves. The next morning another series of geotactic stimuli was given. The rather meagre results given in Table IX. are perhaps not worth very much, since only one individual (No. 1) out of the four responded well in both cases.

The ratios given in the second column (Table IX)indicate that slugs Nos. 1 and 2 belong with those of the positive half of Col. 8, Table IV. I unfortunately neglected to control these experiments by observing the geotaxis *before* the animals were deprived of food. In

TABLE IX.

COMPARISON OF RESPONSES OF INDIVIDUALS WHEN POORLY NOURISHED AND WHEN WELL NOURISHED.

Animal No.	Proportionate Ratio of Anterior to Posterior Region.	Poorly nourished.			Well nourished.		
		No. of Trials.		% Geotaxis.	No. of Trials.		% Geotaxis.
		+	−		+	−	
1	70.	3	9	−75.	7	2	+77.7
2	60.	1	8	−88.8	4	1	+80.
3	56.	0	0	0	12	6	+66.6

both instances (Nos. 1 and 2) the animals were rather dry, and they were not noticeably different in this respect after being well fed. No. 2 was less active and less precise in response after it had had plenty of food. I think these experiments too few to warrant laying much stress upon them, but I have given them here because they at least point in the direction of what we might reasonably expect, since the natural desire of the animal to escape from its narrow prison and the impulse to seek food would both tend to make it go up, if given the opportunity.

Another element which may alter the slug's inherent geotaxis is probably the state of fear. This element may be combined with the impulse to seek food, as is perhaps the case in the instances just given, or it may operate by itself. Animals which had just been captured were always kept in a small tin box. The captured animals would thrust themselves between the box and lid, which was not perfectly tight, in their endeavors to escape, and they had to be frequently pushed back. When they were transferred to the large box mentioned at page 187, it was always found that they had all collected in the upper part of the smaller box. This may have been solely for the purpose of getting air, but such animals put on a glass plate were exceedingly active and restless, and usually exhibited a decided negative geotaxis. I have not made any careful set of experiments to find out whether these negatively geotactic animals afterwards became positive. In one instance, I confined over night in a small flower-jar a slug (not a freshly captured one) which had shown a very decided positive geotaxis. In the morning it was found at the top of the jar, and, when placed on a glass plate, showed great activity, as though it sought to escape. In every one of the tests which I then made, it responded negatively. From these few observations, it would seem that fear, by impelling the animal to escape from captivity, may alter its geotatic response. Such freshly captured slugs, moreover, which seem unusually restless and excited, respond more capriciously to the stimulation of light, as some later experiments will show.

Summary of Part II.

The results of the foregoing experiments warrant the following conclusions: —

1. On an inclined *glass plate*, all slugs give a geotactic response.

2. Certain slugs give a decided positive, others a markedly negative geotactic response; a few are somewhat indifferent.

3. The geotaxis of animals kept in confinement does not vary much on different days, nor at different times on the same day.

4. The occasional vagaries in the responses of individual animals are to some extent due to thigmotactic and phototactic influences.

5. The different geotactic response, on a *glass plate*, of different individuals is due mainly to two factors: (*a*) The quantity and quality of the slime secreted, which is a very important factor; (*b*) the relative proportions of the length of the anterior and the posterior regions of the animal's body. All the conditions being the same, it is this factor which "determines whether the head end will be directed up or down."

6. If the ratio of length of anterior to posterior region of body is 2 : 3, or more, and the mucus is of good quality and sufficient quantity, the slug will be positively geotactic.

7. If the ratio is 3 : 5, or less, the animal will usually migrate upward, and the nearer the ratio approaches 1 : 2 the more apt is the slug to respond negatively.

8. In a small number of individuals, in which the ratio lies between 2 : 3 and 3 : 5, the response will depend largely on the condition of the mucus and coöperation of other factors.

9. All slugs have a natural tendency to move towards the earth. This tendency is masked in the animals which are negatively geotactic on a *glass plate* by the greater pull of gravity on the disproportionately larger and heavier posterior region of the animal.

10. The general downward tendency may vary normally at different times of the day, owing to the animal's habit of remaining in concealment in the daytime and feeding at night.

III. PHOTOTAXIS.

The influence of light on the direction of locomotion has been very generally noticed among organisms, even the mostly widely separated. The swarm spores of many algae, desmids, and other lowly organized plants, are as truly responsive to light stimuli as are crustaceans or vertebrates. According to the character and direction of the stimulating light rays, two kinds of light responses have been distinguished. Phototaxis is the response with reference to the *direction* of the rays of light. The organism moves in the path of the ray, either positively (toward) or negatively (away from it). The response to different *intensities* of light from which the directive force of the rays has been eliminated is known as photopathy. A photopathic animal is one that selects, out of a series of uniformly increasing intensities of light, a limited field of a certain intensity.

Some animals, like butterflies and fresh-water Entomostraca, are strikingly positively phototactic to diffuse daylight; others, such as the earthworm and the leech, are as pronouncedly negative. The kind of response (positive or negative) may be different in closely allied forms and in different stages of development of the same species. For example, butterflies are attracted by strong sunlight, while moths are repelled by it. The adult house fly is positively phototactic to daylight; its larva, negatively (Loeb, '90, pp. 69–77, 81–83).

The phototactic sense has been shown in some forms to change with different intensities of light. Thus, Famintzin ('67) found that swarm spores positively phototactic to a certain intensity of light became negative to a light of greater intensity. The same phenomenon has been found true of various flagellates, desmids, diatoms, oscillariae, etc. Wilson ('91, p. 414) found that Hydra fusca was attracted by diffuse daylight and repelled by strong sunlight. Finally, the moth's liking for candlelight and aversion to daylight is well known. The fact that many organisms are photopathic — that is, have a preference for light of a certain intensity — makes it probable, in connection with these observed variations in phototactic responses, that, for most organisms, there is an optimum intensity to which they will respond positively. This optimum will vary widely in different species, probably according to the habits and the usual environment of the species. Inhabitants of sunny pools or the open air will have an optimum of relatively high intensity; those which dwell in the ground or in shady places will have a correspondingly lower optimum. May it not be that every organism will respond positively to a certain range of light intensities and negatively to another range of intensities which is greater? The nature of the phototaxis may sometimes be gradually changed by organisms becoming acclimated to new conditions. Verworn ('89, pp. 47–49) found that a culture of the diatom Navicula brevis, which ordinarily is negatively phototactic to very weak light, became positively phototactic when reared for several weeks near a window. Groom und Loeb ('90, pp. 166–167) found that young Nauplius larvae of Balanus which were at first positively phototactic to daylight became negatively phototactic later in the day, probably as the result of the accumulated effects of this exposure.

The character of the light responses, as was the case with geotaxis, depends also to a certain extent on other external conditions, such as those of temperature, the states of density and pressure, and various chemical influences. Polygordius larvae, when gradually cooled from 16.5° C. to 6° C., were found by Loeb ('93, pp. 90–96), to change from a negative to a positive phototaxis. Like results were obtained by him from Copepoda. When the temperature was raised from 6° C. to 16° C., the animals again became negative. Increasing the density of sea-water by the addition of sodium chloride produced a change from a negative to a positive response, thus acting like diminished temperature. Engelmann ('82, pp. 391–392) showed the apparent phototactic response of chlorophyllaceous ciliates to be really a chemotactic attraction for oxygen, which chlorophyll can produce only in the light. These facts make it important in any study of light response to consider other possible influences, and above all to take account of the strength of the stimuli used.

Davenport and Perkins ('97) found that the slug (Limax maximus) responded with marked precision to the varying stimuli of gravity at different angles of inclination of the glass plate. The precision of response varied correlatively with the force of gravity. In fact, the parallelism was almost perfect. The question naturally rises, Is there a similar parallelism between other stimuli and their responses?

A very little experimentation shows that the slug is extremely sensitive to light. We have already seen how light may enter in to modify the action of gravity. Casual observation shows that the response is in most cases negative, — the animal moves away from the source of light. Owing to its method of locomotion, the slug is easily experimented on. It moves slowly and deliberately. In regard to its responses to light, the following questions suggest themselves: (1) Are all individuals negatively phototactic to artificial light? (2) Does the precision of response vary correlatively with the intensity? (3) Within what limits of intensity is the animal responsive? (4) Does the kind of response vary at different intensities? (5) Is there a difference in the sensitiveness to light of the two sides of the animal's body? (6) In what part, or parts, of the animal's body does the sensitiveness reside? (7) How does the animal move when in the dark and deprived of all stimulating influences? These various problems came up gradually as the work progressed and were considered in turn. Other interesting studies have suggested themselves in the course of the investigation, but there has not been time to go much beyond a consideration of the questions above proposed. The experiments performed were all phototactic; that is, they were studies of the response of the slug to the direct rays of light.

Methods. — The methods used were simple. For light, the standard candle and the ordinary small Christmas candle, of a one fourth candle power, were employed. The candle was placed in a box 50 cm. (20 inches) high and having a bottom 12.5 cm. (5 inches) wide and 20 cm. (8 inches) long. It could be raised or lowered to any desired position by means of an adjustable stage inside the box. A circular opening in the middle of one of the broad sides of the box 2 cm. ($\frac{4}{5}$ inches) in diameter permitted the light to pass out. This opening was covered by a piece of oiled paper, so as to give a well-defined uniform source of light. During the experiment the box was closed by a lid. The intensity of the light was varied by altering the distance between the box and the animal. Additional thicknesses of paraffined paper were also employed when it was desired to greatly diminish the intensity of the light. The animal was put on a circular glass plate which rested horizontally on a support, and the box was raised so that the centre of the light opening was in the same horizontal plane as the body of the animal. The movement of the slug from its original position was measured in degrees in the following manner. A circle of the same size as the glass plate was described on a sheet of thin paper and divided by radii into 72 sectors of 5° each. This sheet was pasted to the under side of a second circular glass plate (of the same size as the first), on which also a heavy base line was drawn, corresponding with a diameter of the circle. This second plate was so placed that the centre of the source of light was on a line perpendicular to the base line at its middle point. The slug was put on the first glass plate, which could be rotated so as to bring the animal into any desired position with reference to the base line. The experiments were carried on in a dark room provided at one end with a hinged window which could be easily and quickly thrown open. The window was covered with a thick, black cloth, so that, when closed, external light was almost completely shut off. Unfortunately, it was impossible, owing to the position and nature of the room used, entirely to equalize all conditions. The temperature was not the same from day to day and varied somewhat in different parts of the room. Generally, it was so hot and close that it was necessary to leave an opening between the sashes, and this of course created a slight draft and produced irregularities of temperature. No account was taken of the varying humidity of the atmosphere, a factor which may have somewhat influenced the animal's locomotion. Moreover, as the room was not perfectly light-tight, there were feeble light stimuli in addition to the artificial ones used. However, all these imperfections were but slight, and, since they entered more or less into all the experiments, could not greatly alter the relation between the results, which was the main thing sought in the investigation. Other unestimated possible influences were the nutrition of the animal and such slight thigmotactic stimuli as could not well be avoided.

The strength of the different intensities of light used was measured by moving a piece of paper, the centre of which was oiled, between a light of known intensity and the light whose intensity it was desired to know, until the oiled spot on the paper was not distinguishable from the rest of the paper. The distance from this point to each source of light was then measured. Since the intensity varies inversely as the square of the distances, it is an easy matter to calculate the relative strengths. This method is accurate enough for all ordinary purposes.

Operations and Results. — In beginning any experiment, the slug, as soon as it had definitely oriented itself, was rotated into such a position that the axis of its body coincided with the base line, and its head was at the centre of the disk. The window was then immediately closed and the time noted. At the expiration of 45 seconds, the window was thrown open and the animal's position instantly noted. The extent of positive or negative migration was at first ascertained by finding the length of the arc stretching from the base line to the radius which was *parallel with the axis of the slug's body.* Any movement into the half of the circle toward the source of light was called positive; any movement into the other half, negative. It would occasionally happen that an animal would at first move into the positive half of the circle and then turn away from the light. In this case the axis of orientation made a negative angle with the base line, although the animal itself lay in the positive half of the circle. Later, in the course of the experiments, the positive or negative movement of the animal was measured by taking the radius which passed *midway between the two tentacles,* without regard to the position of the body axis. A comparison of the two methods showed but little difference in the results. The animals only occasionally made these irregular responses, first in a plus and then a minus direction. As a rule, the migration was unequivocal after the head end had oriented itself to the stimulus. Experiments were made with 18 different intensities of light, each constituting a "series." Six successive observations were made on each individual (3 with the right side exposed; 3 with the left), and from 8 to 14 animals were employed in each "series." *i. e.*, at each intensity of light, making a total of from 48 to 84 observations at each candle power used. A summary of the results for each of 18 such "series" is given in Table X.

The first column gives the number of the series; the second, the intensities of light. This intensity is expressed in terms of the standard candle power at a distance of one meter. The next column (3) shows the total positive migration of the (8 to 14) animals experimented with. Column 4 similarly gives the total negative migration. Column 5 represents the average arithmetical angular deviation from the original position due to phototactic stimuli, effected in a period of 45 seconds by all the slugs, without regard to the positive or negative character of the individual phototaxis. This average was obtained by adding together the average phototactic responses (whether plus or minus) of each *individual* of the series and dividing the result by the number of animals. The average plus or minus phototactic response (algebraic average) for each series (column 6) was obtained by getting the difference between the sums of all the plus and all the minus movements of each *series* and dividing this difference by the number of tests (observations) made. Column 7 gives the number of positively phototactic animals in each series; column 8, the number of negative animals; column 9, the number of indifferent animals; and column 10, the total number of individuals employed in each series. The sequence of the series is not the same as that of the experiments, but is based on gradually diminishing light intensities. I did not determine the possible influence of the heat of the candle for each of the series, but in one series of experiments in the dark (18*b*), a candle, covered (to shut out the light) with an opaque paper of the same thickness as the paraffined paper, was left burning at a distance of 30 cm. (intensity .676 C. P.).

A casual glance at the table at once answers the first of the questions proposed in the statement of the problems (pp. 207–208). All slugs are not negatively phototactic. At the strongest intensity of light used, two animals exhibited a positive phototaxis, — they moved toward the stimu-

TABLE X.

RESPONSES OF THE SLUG TO LIGHT.

1	2	3	4	5	6	7	8	9	10
No. of Series.	Intensity of Light.	Total Phototactic Migration in Degrees.		Average Response in Degrees in a Period of 45 Minutes.		No. of Animals.			
		+	−	Arithmetrical Sum.	Algebraic Sum.	+	−	0	Total.
1	.676	330	2155	45.5	−38.	2	6	0	8
2	.382	625	2772	40.	−25.5	2	12	0	14
3	.169	440	2430	27.5	−25.5	2	11	0	13
4	.042,4	625	1330	26.	−11.7	2	8	0	10
5	.010.5	250	1165	17.6	−15.	2	8	0	10
6	.004,7	830	1140	16.1	− 5.1	3	7	0	10
7	.001,69	405	760	9.1	− 6.	3	6	1	10
8	.000,754	695	595	13.	+ 1.4	7	5	0	12
9	.000,424	1145	895	17.	+ 3.5	6	4	2	12
10	.000,260	823	345	14.5	+ 7.9	7	3	0	10
7*a*	.001,69	365	480	4.6	+ 1.7	4	7	0	11
8*a*	.000,754	845	345	11.8	+ 8.3	7	3	0	10
9*a*	.000,424	985	130	14.7	+14.2	9	1	0	10
10*a*	.000,260	740	435	11.	+ 4.2	8	4	0	12
11	.000,022	1395	55	22.3	+22.3	10	0	0	10
12	.000,009,6	630	515	8.6	+ 2.	7	3	0	10
13	.000,003,35	865	255	13.	+10.	8	2	0	10
14	.000,002,00	800	170	10.5	+10.5	9	0	1	10
15	.000,001,26	850	415	11.1	+ 7.2	7	3	0	10
16	.000,000,185	1375	145	24.	+20.5	7	3	0	10
17	.000,000,018,8	445	370	8.9	+ 1.	6	4	0	10
18*a*	Darkness.	1440	1200	3.6	+ 1.2	10	8	2	20
18*b*	" with candle heat.	475	635	8.7	− 3.	3	6	0	10

lating light rays. Here, then, arises another problem, similar to the one treated of in the first part of this paper, viz., What determines whether a particular slug shall be positively or negatively phototactic? In the first series of experiments — in fact throughout this whole set — the animals used were about equally divided between large, small, and medium-sized individuals. The two positive animals in series 1 were both of large size. They were very active. The only peculiarity wherein they seemed to differ from other individuals was in the unusually sticky character of the slime. Whether there is any correlation between this fact and the liking for strong light, I am not prepared to say. It is possible — and certain observations seem to indicate that it is highly probable — that the food conditions of the animals have some influence on their responses to light, as they were shown to have on their responses to gravity. The psychic state of the animal is also to some extent, I think, a factor. Freshly caught slugs when put on a glass plate sometimes acted as if in great fear. They displayed unusual activity and were very erratic in their movements. If forcibly checked or held, they made strenuous efforts to escape. The great activity of the positive individuals indicates a possible state of fear. One animal in particular seemed highly abnormal. Several times it moved directly toward the circular field of light and even placed its tentacles against the oiled paper which covered the opening. This was the only individual in the whole course of the experiments which exhibited a response like that of moths. No definite set of experiments was planned or carried out in regard to this matter.

As we run down column 5, we see that the average arithmetical response varies quite strikingly at the different intensities. The first seven series show a gradual decrease in the average response as the strength of the light is diminished. Although not so regular, there is also a gradual decrease in the degree of negative response on the part of these seven groups of animals, as shown by the average algebraic sums of their responses (column 6).

Owing to the constant dying off and deterioration of the stock, it was found impossible to use the same set of animals in all the different series of experiments. Moreover, this was not desirable, for the reason that an animal which is constantly experimented on gradually loses its sensitiveness, and thus its responses become untrustworthy. Not knowing the factors which determine the kind of phototaxis, it was of course impossible to make a uniform selection in this respect. We see, however, that the number of negative animals (column 8) is less at the weaker intensities than at the stronger. When we come to series 8 of the table, we meet with a new condition of affairs. Instead of a still further decrease in the amount of deviation, there is a sudden slight increase, from 9.1° to 13°, and a reversal in phototaxis for the series from an average response of —6° to + 1.4°. The number of positive individuals has increased from 3 to 7. It was because of this striking change that it was thought best to repeat this series and the three succeeding ones on another set of animals. The absolute positive or negative migration was this time taken without regard to the position of the body axis. Series 7*a*, 8*a*, 9*a*, and 10*a* are hence taken at the same intensities as 7, 8, 9, and 10 respectively. These repeated series indicate as strongly as the first set that an intensity of .001,69 C. P. very nearly marks the lower limit of negative phototaxis in the slug. Somewhere near a candle power of .000,754, lies an intensity which attracts about as many animals as it repels and in about the same degree. That is, the average phototaxis (algebraic sum) is zero. Below this intensity, there is more attraction than repulsion, and hence there is an average increase of migration toward the light. The table shows that the average positive response increases to some extent correlatively with the diminution of the light intensity, up to a certain point. This point, according to the results here obtained, is the intensity of .000,022 C. P., where the average movement toward the light, in a period of 45 seconds, was through an angle of 22.3°. As we go below this intensity, there is again a falling off in the strength of the positive response, which diminishes, however, with a good deal of irregularity until absolute darkness is reached. These facts will become more apparent from the study of their graphic portrayal in the curve here given.

The continuous line represents the curve as plotted from the results of Table X., column 6; the dotted line, the curve of responses as one may assume theoretically it would have been, could all of the conditions other than intensity of light have been equalized. The abscissae here represent the logarithms of the intensities of light + 10. Beginning with darkness on the left end, there is a constant increase of intensity as we move toward the right. The sines of the angles of response are marked off on the ordinates. Remembering that the left represents a region of weak intensity and the right a region of strong light, that all points above the line x x′ are points of positive response and all points below it of negative response, we can understand the significance of the curve. In the region of strong light, the curve lies far below the line x x′, but gradually rises toward and finally crosses it, as the light

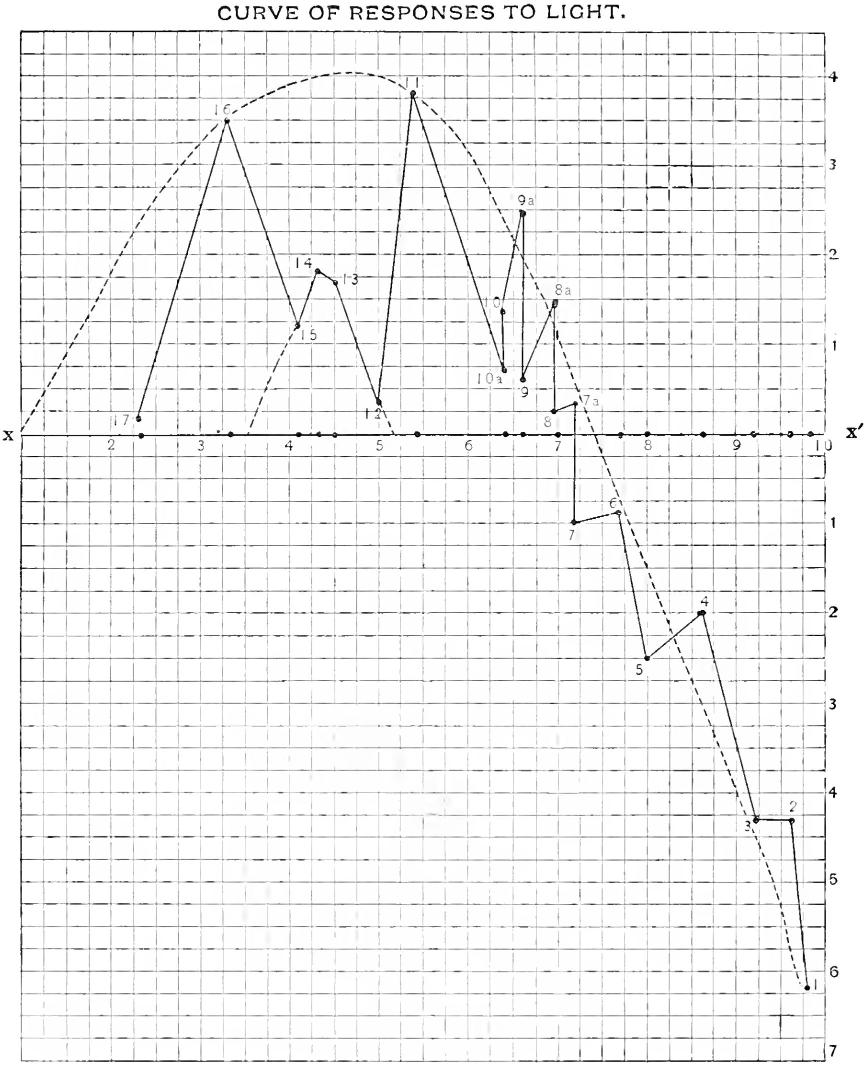


FIGURE 2.

Curve of Responses to Light. *Abscissæ* are logarithms of light intensities plus 10; *ordinates* are sines of angles of responses multiplied by 10.

diminishes in strength. Then there is a gradual increase in positive reaction, which reaches its height in a response of +22.°3 at a .000,022 C. P., and then falls toward the zero line as we approach darkness. There is some irregularity in the negative region, but on the whole the rise is gradual. In the region of positive response, there is a considerable lack of regularity, especially marked by the interpolation of one series (12) of very low response between the two series of greatest response. These series intermediate between Nos. 11 and 16 represent later experiments than the two series bearing those numbers. Having obtained such a marked positive response at two widely separated intensities of light, it was thought desirable to get other intermediate series. Hence, the order of the series as arranged in the table, on the basis of gradually diminishing light intensities, does not, as already stated, represent the order in which the series were obtained in my experiments. While the slugs, thus far, had, on the whole, been in good active condition, they were not so in these intermediate series. Although a fresh supply was obtained, all the animals seemed much more stupid and irresponsive than usual. Some of them refused to move, when put on a plate, and many of those that did, responded in a very half-hearted way. The cause of this unusual lack of activity, I could not discover. It may be that a slight change in the food of the animals, which I made at this time, was partly responsible. At any rate, instead of obtaining responses intermediate in amount between those of series 11 and 16 as might have been expected, the results were as have been given. Series 12 was the last one taken. In this, the animals were noticeably more stupid and irresponsive than in any of the preceding experiments. It is very evident from these results, I think, that the precision of response will vary to some slight extent from day to day. The negative responses — those to strong intensities of light — will not be as variable at different times as the positive responses — those to weaker stimuli — as the curve shows. The varying thermal conditions of the room, already mentioned, may have been in part a cause of this irregularity. Furthermore, an animal that has had plenty of food is likely to be stupid and slow in movement and is more apt than a hungry one to seek darkness and concealment. On the other hand, a hungry, active slug will probably exhibit positive phototaxis in a most marked and sometimes abnormal degree, as was the case occasionally with the positive animals at the strongest light intensities. Besides this individual variation, there is, I think, a general variation for all slugs from time to time, for reasons imperfectly known, which will find its expression in curves of different heights.

Thus the less responsive animals of the intermediate but later series mentioned fall into a less prominent curve, as is indicated by the shorter dotted line in the diagram. The curve of positive response approaches, but never actually reaches, the zero line. Even in darkness there is a slight positive migration. This series (No. 18*a*) represents the average of two series of experiments, one of 54 and the other of 66 determinations, each taken at different times during the investigation. This slight positive response — speaking of it as positive with reference to the position of the source of light in the preceding series (17) — may be independent of conditions of light and due to several causes. As mentioned before, the thermal conditions of the room were not uniform, consequently the positive response may have been a response to heat. The movement was away from the window and hence might be explained as a negative response to the repeated inflowing of daylight, when the window was thrown open to make observations. In the last few experiments an opaque screen was put up between the animal and the window. In these cases the average of the responses was slightly negative, so there is some reason to suppose that it was in part the position of the window in the previous experiment that determined the slight positive migration. The actual phototactic responses to the candle light in the positive half of Table X. would then be the observed responses minus this small positive movement in the dark. The actual negative responses to the strong intensities would be the observed responses plus this increment. In series 18*b* the box was placed at a distance of 30 cm. (C. P. 0.676) with the light burning, but the opening was covered with a piece of black paper to shut out the influence of the light while leaving that of heat. The small average response of —3.0 may possibly be regarded as a thermotactic one, and, if so, will have to be deducted from the negatively phototactic response to this intensity of light. For intensities less than the 0.676 C. P., the response to the heat would be correspondingly less.

We can now answer the second and fourth questions (pp. 207–208) by saying, — that the precision of the phototactic response does, on the whole, vary correlatively with the intensity of the light, and that the kind of phototaxis (positive or negative) is not the same for different intensities of light. The slug gives a negative phototactic response to strong light, a positive one to weak intensities, and is neutral to an intensity somewhere between the extremes.

A few individuals were tested successively at different light intensities in order to find out with what precision an individual's phototaxis might vary with a change of intensity.

TABLE XI.

Responses of Individuals to Different Intensities of Light.

Animal No.	Intensity.	Response.	Intensity.	Response.	Intensity.	Response.
1	.382 C. P.	−36.°	.169 C. P.	−34.°	.067 C. P.	−27.°5
2	.382 C. P.	−39.°	.169 C. P.	−14.5°	.067 C. P.	−10 °
3	.382 C. P.	−42.°	.169 C. P.	−31.°	.067 C. P.	−23.°

In all these cases, there is seen to be a gradual diminution in the degree of response as the intensity of light diminishes. Again, from an animal which responded negatively to a certain intensity of light, a positive response could be got by weakening the light sufficiently (Nos. 2 and 3, Table XII.), and a positive animal could be made to give a negative response by using stronger light (No. 1, Table XII.), as the following instances show.

TABLE XII.

Responses of Individuals to Different Intensities of Light.

No.	Intensity.	Response.	Intensity.	Response.	Intensity.	Response.	Intensity.	Response.
1	.382 C. P.	+41.°	Strong Light.	−22.°				
2	.676 C. P.	−15.°	.0124 C.P.	+35.°				
3	.169 C. P.	−37.°	.0188 C.P.	− 2.°5	.0067 C.P.	−32.°	.0047 C.P.	+36.°

No. 3, Table XII., shows a less regular response than any of the other animals. From a response of −37° it drops to one of −2.5°, and, under the influence of a still lower intensity of light, it again rises to a marked negative response of −32.° At a still lower intensity, it gives a striking positive response of +36°. Here, however, we have well illustrated in particular individuals the law laid down for all slugs, — that they are negatively phototactic to strong intensities of light, the precision of response varying correlatively with the intensity of the stimulus; that to weak intensities they are positive; and that to a certain intermediate intensity they are neutral.

A glance at the intensity column (Table X.) shows that the slugs are responsive to a very wide range of intensities. They would probably continue to respond negatively to still stronger light, until the light became strong enough to kill the animal. They respond positively to a light (series 16) less than one three millionth part as intense as the strongest intensity experimented with. The response to the weakest intensity used (series 17) is less than the positive migration in the dark. Hence we cannot speak of this as a phototactic response. This attenuation of light was so weak that I could not be sure I saw it myself, and had constantly to reassure myself by approaching it. The slug is evidently sensitive to a very minute degree of light.

Where does the slug's sensitiveness reside? The first and most natural answer is, that the eyes are the important organs. The matter was tested on five different individuals. The normal phototactic response was first taken with a .676 candle power. Then the dorsal tentacles, bearing the eyes, were snipped off with scissors and the animal again experimented on. The results are given in Table XIII.

TABLE XIII.

Effect of Amputation of Tentacles.

Animal No.	Normal Phototactic Response.	Response after Amputation of Dorsal Tentacles.	Ventral Tentacles also Amputated.
1	−70.°	+41.°	
2	−26.°	− 3.°	
3	−44.°	−29.°	+7.°
4	−53.°	+16.°	
5	−65.°	+ 6.°	

As soon as the operation was performed, the stumps were retracted, as the tentacles are when stimulated by touching, or by strong light. After a moment or two, the animal again rolled out the stumps and began moving forward in perfectly normal fashion, as though nothing had happened. The only observable difference was a perhaps slightly increased activity. This table (XIII.) shows a striking effect of the amputation on the phototactic response. In some cases, the animal deviated but very little either positively or negatively from its original position, but kept on moving ahead in a straight line. In other cases, the amputation seemed to cause a change from a strongly negative to a more or less positive response. In the case of animal No. 3, removal of the eyes did not seem to altogether prevent, though it considerably reduced, the negative response. Thereupon, the ventral tentacles were also amputated and the result then was a slight positive response. Since there is probably some shock to the nervous system by the amputation, these results ought to be corroborated by other experiments where the eyes are covered over with some substance to shut off the rays of light. This, I have not yet succeeded in doing satisfactorily.

The experiment of removing only one of the ocular tentacles was tried on two different animals with the following interesting results.

TABLE XIV.

Comparison of Effect of Amputation of Right and Left Dorsal Tentacles.

Animal No.	Normal Phototactic Response.	Response after Amputation of	
		Right Tentacle.	Left Tentacle.
1	−70.°	−27.°	
2	−55.°		+3.°

In the case where the right tentacle was removed, the animal still responded negatively with considerable precision. Amputation of the left tentacle, in the case of No. 2, on the other hand, resulted in a slight positive phototaxis. While these two cases by themselves have little, if any, significance, taken in connection with facts now to be discussed, they seem to indicate a greater degree of sensitiveness to strong light on the part of the left side of the animal's body than the right.

It will be remembered that our thigmotactic experiments pointed to a possible asymmetry in the sensitiveness of the right and left tentacles of the slug. Do we find a similar asymmetry in the responses to light? Table XV. gives the responses of right and left sides respectively for the 18 series. Column 1 gives the number of the series, column 2 the intensities of light, columns 3 and 4 the total angular migrations in a positive and negative direction for the series when the right side was exposed to the light, and the fifth column the algebraic average (positive or negative) phototactic response of the right side. Similarly, the next three columns, 6, 7, and 8, give the responses of the left side. Column 9 represents the total movement of the series in degrees to the right. This result was obtained by adding the total positive responses of the right side (column 3)

TABLE XV.

Comparison of Responses of Right and Left Sides to Light.

1	2	3	4	5	6	7	8	9	10	11
No. of Series.	Intensity of Light.	Responses of Right Side in Degrees.			Responses of Left Side in Degrees.			Total Movement in Degrees to		Total No. of Animals.
		+	—	Average Photo-taxis + or —	+	—	Average Photo-taxis + or —	Right.	Left.	
1	.676	305	770	—19.	25	1385	—57.	1690	795	8
2	.382	250	1056	—19.	375	1716	—32.	1966	1431	14
3	.169	425	595	— 4.3	15	1835	—46.8	2260	610	13
4	.042,4	255	355	— 3.3	370	975	—20.	1230	725	10
5	.010,5	65	730	—22.	185	435	— 8.	500	915	10
6	.004,7	295	805	—17.	535	335	+ 6.7	630	1340	10
7	.001,69	250	500	— 8.3	155	260	— 3.9	510	655	10
8	.000,754	280	330	— 1 4	415	265	+ 4.1	545	745	12
9	.000,424	530	645	— 3.	615	250	—10.	780	1260	12
10	.000,260	435	245	+ 6.3	388	100	+ 9.6	535	633	10
7*a*	.001,69	250	210	+ 1.2	115	270	— 5.	520	325	11
8*a*	000,754	410	165	+ 8.1	435	180	+ 8.5	590	600	10
9*a*	.000,424	380	75	+10.	605	55	+17.	435	680	10
10*a*	.000,260	560	230	+ 9.	180	205	— 0.7	765	410	12
11	.000,022	955	0	+31.7	440	55	+12.7	1010	440	10
12	.000,009,6	160	275	— 3.8	470	240	+ 7.6	400	745	10
13	.000,003,35	460	120	+11.7	405	135	+ 9.	595	525	10
14	.000,002,0	410	90	+10.7	390	80	+10.3	490	480	10
15	.000,001,26	395	320	+ 2.5	455	95	+12.	490	775	10
16	.000,000,185	915	40	+29.	460	105	+12.	1020	500	10
17	.000,000,018,8	210	215	— 0.2	235	155	+ 2.6	365	450	10
18*a*	Darkness	1220	240	+16.	220	1050	—13.8	2270	460	20
18*b*	“ with candle heat	155	495	—10.	320	140	+ 7.	295	815	10
Totals		9570	8506		7808	10321		19891	16314	
“ less 18*a* & 18*b*		8195	7771		7268	9131		17326	15039	

and the total negative responses of the left side (column 7), — these responses being necessarily right-hand movements. The total movement in degrees to the left (column 10) was likewise obtained by adding the total negative responses of the right side and the positive responses of the left side. Column 11 gives the total number of animals used in each series.

In the region of negative phototaxis, the total positive and negative angular migrations, and the average negative phototaxis of all the series (1–7, inclusive) when the right and left sides respectively were turned toward the light, were as follows.

TABLE XVI.

SUM OF THE RESPONSES OF RIGHT AND LEFT SIDES WHEN PHOTOTAXIS IS NEGATIVE.

Side turned toward Light.	Total Angular Migration.		Average Negative Phototaxis.
	+	–	
Right.	1845.°	4811.°	13.°1
Left.	1660.°	6941.°	23.°4

This shows on the whole a less sensitive right side, or, to put it differently, a more marked negative phototaxis of the left side. How is it when the animals become positively phototactic? Table XVII. gives the average positive response of the right and left sides for series 8 to 18, including series *7a*, *8a*, *9a*, and *10a*.

TABLE XVII.

SUM OF RESPONSES OF RIGHT AND LEFT SIDES WHEN PHOTOTAXIS IS POSITIVE.

Side turned toward Light.	Total Angular Migration.		Average Positive Phototaxis.
	+	–	
Right.	6350.°	2960.°	7.°68
Left.	5608.°	2190.°	7.°75

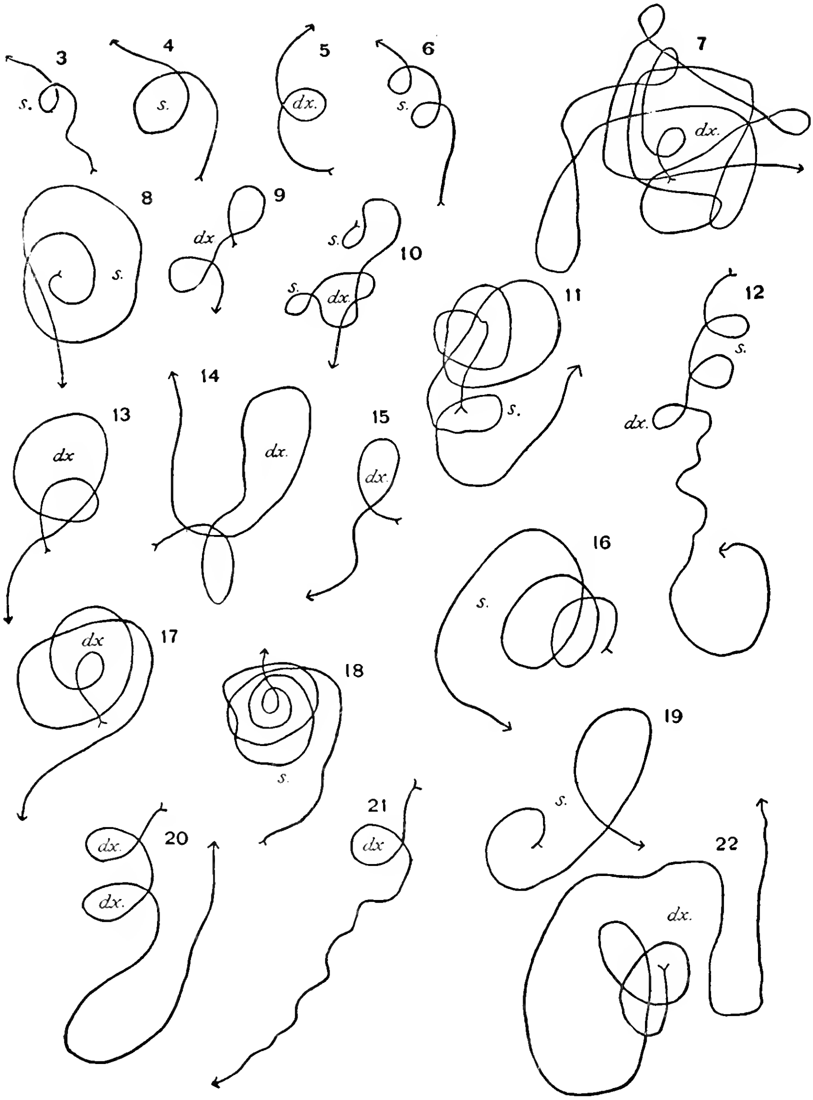
Here an asymmetrical response is less strongly marked. The left side, however, appears on the average to be somewhat more strongly attracted toward the light. The results prove that the asymmetry in response of the right and left sides cannot be wholly due to a tendency to move toward the right, for, if this were so, we should expect an average positive response of the right side as much greater than that of the left side, as the average negative response of the left is greater than that of the right side, for both these would mean a greater movement to the right. These facts curiously suggest that the right and left sides are attuned to slightly different intensities of light. Is this possibly due to ancestral habits of life in which environment, acting unequally on the two sides, produced this difference?

The results obtained for the right and left sides from the experiments in darkness (series 18*a*) are rather puzzling. If the responses are due to some uncontrolled directive stimuli of the kind already suggested, it would seem that the two sides had given opposite responses. As these experiments represent two series taken at different periods, it is the more surprising that they should both show this peculiarity. Again, in the responses to weak candle heat (series 18*b*) the left seems to have been positively, and the right side negatively affected. So far as is known, there was no unequal operation of stimuli on the two sides.

Related to this matter is the question, — Is there any tendency on the part of all slugs to move either to the right or to the left? Individuals were noticed which seemed to have a marked tendency to continue moving toward the right, and there were others which seemed to be as strongly biassed toward the left. Not many seemed entirely indifferent. The total movement of all the slugs in the region of negative response (series 1–8, Table XV.) toward the right side was 8786° (col. 9), and to the left 6471° (col 10). In the positive region (series 8–18, Table XV.), the total migration toward the right side was 8540° (col. 9), and toward the left 8568° (col. 10). Thus, there seems to have been considerably less migration toward the left in the range of negative responses, but only a slightly greater movement toward the left in the region of positive response. In all the 17 series, there was a migration towards the right of 17,326°, and towards the left of 15,039°. That is, there appears on the whole to have been a slightly greater average movement for all slugs toward the right than there has been toward the left. What do we find to be the case with the animals experimented on in the dark? Out of the 120 determinations made on 20 animals in the dark (series 18*a*), the amount of right-hand movement was 2270° and the left-hand movement only 460°. That is, there was nearly five times more migration toward the right than there was toward the left. In series 18*b*, however, there seems to have been a marked preponderance of movement toward the left. From the foregoing experiments, it seems pretty clear that there is a difference in the sensitiveness of the right and left sides. There is also some indication of a slightly greater average tendency to move to the right. But a further study of the undirected movements of slugs in the dark is needed.

Studies have been made by several observers on the undirected movements of a number of different animals, chiefly ants and other insects. In all animals experimented on, there appears to be a tendency to travel in loops or constantly widening spirals. Man, when he loses his way, travels in a circle. Some interesting observations have been made by George and Elizabeth Peckham ('98, pp. 211–219) on the sense of direction in the solitary wasps. When the wasp starts out from its nest, it flies quite around it and gradually circles farther and farther away in a constantly enlarging spiral, sometimes recrossing its path a number of times. The authors' observations show that this action is to enable the wasp to familiarize itself with its surroundings, so that it can find its way home when it so desires. The similar phenomenon observed in other insects, such as ants, is, no doubt, for the same purpose. Davenport ('97, pp. 278–279) in his experiments on Amoebae found that, when their movement was undirected by any external stimulus, they tended to travel in curious spiral loops. Pouchet ('72, pp. 227–228) made observations on the movement of larvae of Musca (Lucilia) caesar in the dark. There is a striking contrast between the paths given by him of the undirected movements and those made in response to the stimulus of light. The tendency to travel in a gradually widening spiral has also been observed by the writer in young frog and toad larvae — before the development of mouth and eyes — when they are dislodged from the support to which they are clinging.

Most of the following experiments on the slug were made in a room about 12 feet square. The floor was sometimes covered with cardboard or paper, but in other experiments was left bare. Heavy curtains were hung in front of the windows and light shut out as completely as possible. The experiments were conducted at night, and the temperature of the room was nearly, if not quite, constant. A slug was put on the floor in the centre of the room and left to itself for two or three hours, sometimes longer. By means of the mucous secretion, which hardened into white, shiny flakes, the exact path of the animal could, in most cases, be easily followed. This path was roughly reproduced by pencil on paper. A number of these paths are given in Figures 3-22, much reduced from the actual space covered. The series here given includes all the animals experimented on, with the exception of three individuals which did not



Figures 3-22.

Much reduced copies of the tracks made by slugs (Limax maximus) in the dark. *dx.*, right-handed loops; *s.*, left-handed loops.

give any characteristic paths. Two of the three moved only a short distance in wavy lines without recrossing their paths, and were in poor condition, for they did not go far, and shortly died. One extremely active little individual moved ahead in a straight line quite across the floor, a distance of eight or ten feet. With these few exceptions, it will be seen that there is a very marked tendency to travel in loops. In general, the loops varied in size from a couple of inches in diameter to two feet and sometimes more. The animal generally makes a circle soon after starting out, and then may travel for some distance before again recrossing its tracks. The individuals which did the most looping also showed a tendency, by gradually swinging away from the starting point, to make larger and larger circles. Nos. 7, 8, 11, 13, 14, 16, 17, 19, and 22 all showed this tendency. The smaller individuals usually make the smaller loops, but this is not always the case. Although the paths made by different animals have a very different appearance, they all show the same general looping tendency. It will at once be noticed that all curves are not in the same direction. Some are right-handed loops, others are left-handed, and two cases, Nos. 10 and 12, contain loops of both right and left hand character, or at least indicate a tendency to the formation of such loops. As a rule, however, the individual shows a marked constancy in the character of the loops made. Disregarding the two cases in which there were both right and left hand loops, we have ten individuals with a tendency to circle to the right and eight individuals with just as marked a tendency to circle to the left. This does not indicate a very great preponderance of individuals travelling to the right. If the total space travelled over by all individuals be considered, I think it might show, on the average, a more marked swerving to the right than does a counting of right and left circling individuals, but I have not measured the distances carefully enough to speak confidently on this point. The evidence thus far accumulated in regard to an asymmetrical response of the right and left sides to artificial stimuli points to a greater sensitiveness of the left side, which is perhaps correlated with a *slight* average tendency to move toward the right side more than to the left.

SUMMARY OF PART III.

These studies on the light responses of Limax maximus seem to establish the following points: —

(1) The animals are markedly phototactic.

(2) There are individual differences in phototaxis, as there are in geotaxis.

(3) To strong light, slugs, on the average, give a strong negative response.

(4) The degree of response gradually diminishes with the reduction in the strength of the stimulus.

(5) There is a certain strength of light which appears neither to repel nor attract the slug. This may be said to be a neutral stimulus.

(6) Reduction of the intensity of the light beyond the neutral point changes the phototaxis from negative to positive.

(7) The positive response becomes stronger up to a certain degree of intensity.

(8) It then gradually diminishes with decreasing intensity until absolute darkness accompanied by no response is reached.

(9) Slugs are responsive to light stimuli covering a wide range of intensities.

(10) The principal organ of response is probably the eye.

(11) The response is unsymmetrical on the part of the right and left sides of the animal's body. The right side is not as sensitive to stimuli as is the left. On the whole the right side moves through a slightly greater arc in a period of 45 seconds than does the left.

(12) In the dark, other directive stimuli being eliminated, the slug tends to travel in a spiral of gradually increasing radius, though almost invariably producing one or more loops. Some slugs make right-hand loops, others left-hand ones; there is a slightly greater tendency to right-hand circling.

These responses of the slug to touch, gravity, and light-stimuli emphasize the fact that it is an animal's normal environmental conditions which chiefly determine its general response to artificial stimuli. The variations in precision and character of this general response are mainly dependent on certain internal factors, such as the food conditions of the animal, its fear of an enemy, and desire to escape captivity.

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THE ALGAE OF JAMAICA.

By Frank Shipley Collins.

THE ALGAE OF JAMAICA.

By Frank Shipley Collins.

Presented October 9, 1901. Received October 15, 1901.

The earliest reference to the algae of Jamaica, and very nearly the earliest reference to the algae of America, appears to be by Sloane; * in the chapter on submarine plants 43 species are named and described, among which, however, are a few aquatic phanerogams, and a considerable number of corals; of the remainder most are too vaguely described to be now identified, but by the help of the plates, we can give with fair certainty the modern names for four.

Vol. I. p. 57, Pl. XX. Fig. 2, *Corallina opuntioides, ramulis densioribus, et filis magis sinuatis atque corrugatis,* is Halimeda Opuntia. P. 58, Pl. XX. Fig. 3, *Corallina major, nervo crassiore fuciformi, internodia breviora nectente,* White Bead Bandstring *dicta,* is Cymopolia barbata. P. 61, Pl. XX. Fig. 9, *Fucus minimus denticulatus triangularis,* is Bryothamnion triangulare. P. 58, Pl. XX. Fig. 6, *Fucus marinus vesiculas habens membranis extantibus alatas,* is Turbinaria trialata.

P. 58, Pl. XX., *Corallina minima capillacea,* is probably our present Corallina capillacea, but neither plate nor description is characteristic enough to make this certain. P. 51, Pl. XVIII., *Corallium album pumilum nostras,* seems to be some species of Lithothamnion. The other descriptions are too uncertain to hazard any identifications.

A few algae are mentioned by Browne,† apparently mostly copied from Sloane; some plants undoubtedly belonging to the genus Sargassum are mentioned, and from the description of the great floating masses, S. bacciferum is undoubtedly meant, but it is probable that other species are included under this name.

Lunan ‡ gives seven species of algae, as follows, p. 157–158:

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* A voyage to the Islands Madera, Barbados, Nieves, S. Cristophers and Jamaica, by Hans Sloane, M.D., London, 1707.

† The Civil and Natural History of Jamaica, by Patrick Browne, M.D., 1756.

‡ Hortus Jamaicensis, by John Lunan, 1814.

Fucus turbinatus = Turbinaria trialata.
" natans = Sargassum bacciferum, at least in part.
" acinarius.
" vesiculosus.
" triqueter.
Ulva pavonia = Padina sp.
" Lactuca.

After this date, except for an occasional reference in some general work, we find nothing until Murray's West India list.* In this are included references to Sloane and Browne, and several species are added from specimens in the British Museum, collected by Chitty; in a few cases, however, these are species so little to be expected in tropical regions, that it seems as if there must have been some displacement of labels. The total number of Jamaica species mentioned in Murray's list is surprisingly small, if we consider the size of the island, and that it has been so long a comparatively thickly settled English colony. It would be hardly fair to compare it with the Mazé & Schramm Guadeloupe list,† for it is not improbable that half the species in the latter, certainly more than half the new species, will ultimately be relegated to synonymy or to the catalogue of indeterminables. As an instance of this, see the genus Gracilaria; 57 species are given by Mazé and Schramm under Gracilaria and Plocaria; 15 of these are species whose previously known distribution would lead one to expect them in Guadeloupe; of 5, the previous record would make their occurrence here unlikely; the remaining 37 are new species, with scanty description or none at all. Any one at all familiar with Gracilaria will recognize what this means.

But as compared with Puerto Rico, for which Hauck's list ‡ gives 92 species against 31 Jamaica species in Murray's list, the disproportion is so great that it might seem as if there must be some special conditions at Jamaica to impoverish the marine flora.

Within the past few years the writer has had the opportunity of examining three collections of algae from this island, that show quite conclusively that this is not the case, and that there is every reason to

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* Catalogue of the Marine Algae of the West Indian Region, by George Murray. Journal of Botany, Vol. XXVII. p. 224. 1889.

† Algues de la Guadeloupe. 2d Edition. Mazé & Schramm, Basse Terre, 1870–77.

‡ Meeresalgen von Puerto-Rico, von F. Hauck. Engler's Botanische Jahrbücher, Vol. IX. p. 30, 1888.

suppose that the flora of the island is in no way inferior to similar regions.

The first collection was made by Mrs. Cora E. Pease of Malden, Mass., and her sister, Miss Eloise Butler of Minneapolis, Minn. In July, 1891, they collected at Port Antonio and points in its vicinity; and some collecting was done at other ports, where the steamer touched for a few hours. In 1894 Morant Bay was visited in July, with a visit to Borden and Annotto Bay the first of August, followed by Orange and Hope Bays and Port Antonio, where the greater part of August was spent. In June, 1900, short visits were made to Ora Cabessa, Rio Novo, Runaway Bay, and Rio Bono; June 21 to 27 was spent at Montego Bay; June 29 to July 1 at Kingston; and the time to July 18 was spent at Manchioneal, Port Morant, Hope Bay, Port Antonio, St. Ann's Bay, and Port Maria, in the order named.

The second collection was made by the late Dr. J. E. Humphrey, in March and April, 1893, mostly at or near Kingston, but also near Port Antonio; a few specimens in Dr. Humphrey's herbarium were collected by R. P. Bigelow at Kingston in July, 1891. In 1897 Dr. Humphrey made a second visit to Jamaica; on August 16 he was attacked by the island fever, and died two days later. Among the collections made that year is a large amount of material of shell boring algae, of which he hoped to make a thorough study on his return; unfortunately no one has been able to take up this task, and only such notes as Dr. Humphrey made at the time of collecting have been available for this list.

The third collection, received when this paper was practically ready for publication, was made near Kingston, May 3, 1901, by Dr. J. E. Duerden, who at that time was collecting corals for the Museum at Kingston. By the kindness of Dr. William Fawcett, Director of the Museum, arrangements were made whereby two large cans of algae preserved in formalin were forwarded to the writer. Of the 47 species which were included, six were not represented in the other and larger collections.

In the following list the abbreviation P. & B. has been used for the first named collection, H. for the second; where the specimens had a number in the Humphrey herbarium, the number is given here; notes on station, depth of water, etc., have been copied; and Dr. Duerden's name is given for the third collection. Of one species, not included in either of these collections, I have received specimens from F. Borgesen, collected by O. Hansen.

Many Jamaica algae have been distributed in the two sets of exsiccatae, Phycotheca Boreali-Americana, issued by Collins, Holden and Setchell, and Phykotheka Universalis, issued by Hauck and Richter: references to these are given under the respective species, with the abbreviations P. B.-A. and P. U., and the numbers.

The Humphrey collection includes 25 fresh water algae, the Pease and Butler collection 9; only two species are common to both. If we compare the marine species * in these two collections, we find that of the whole number, 215, only 72 occur in both; 143 are found in one and not in the other. A natural inference from this would be that the field was by no means exhausted, and that more species might be expected.

In Murray's list four species are given, which are omitted here: Gymnogongrus furcellatus, Phyllophora Brodiaei, Liagora viscida, and Plocamium coccineum, the first on the authority of Wright, the others of Chitty. Probably a misplacement of labels has occurred.

Tables have been prepared, comparing the marine flora of Jamaica with the floras of New England, Great Britain, the northern coast of Spain, the coast of Morocco, the Canary Islands, and Puerto Rico, lists having been published of these regions of sufficient extent to make a comparison of interest.†

Some of these regions having been more thoroughly explored than others, too much importance should not be given to the total number of species in any region; the relative proportion of the different classes is of more weight, while the number of species common to two regions

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* In making up these statistics, named varieties and forms have been counted the same as species.

† The data of these tables are from the following works: —

Preliminary List of New England Marine Algae, by F. S. Collins, Rhodora, Vol. II. p. 41, 1900.

A Revised List of the British Marine Algae, by E. M. Holmes and E. A. L. Batters, Annals of Botany, Vol. V. p. 63, 1892.

Note Préliminaire sur les Algues Marines du Golfe de Gascogne, par C. Sauvageau, Journal de Botanique, Vol. XI., 1897.

Les Algues de P.-K.-A. Schousboe, par E. Bornet, Memoires de la Société Nationale des Sciences Naturelles de Cherbourg, Vol. XXVIII. p. 165, 1892.

Plantes Cellulaires des Iles Canaries, par C. Montagne, Paris, 1840.

Crociera del Corsaro alle Isole Madera e Canarie; Alghe, per Antonio Piccone, Genova, 1884.

Contributions à la Flore Algologique des Canaries, par Mlle. A. Vickers, Annales des Sciences Naturelles, Series 8, Botany, Vol. IV., 1897.

Meeresalgen von Puerto-Rico, von F. Hauck, Engler's Botanische Jahrbücher, Vol. IX. p. 30, 1888.

In addition to the published lists of the Canary Islands, some species have been included from the collection of the author.

indicates the affinities of the floras. The tables are useful merely as showing general tendencies, not exact relations. Exactness would be possible only when the districts compared had been explored and studied to the same extent, with the same care and under the same conditions, a thing practically impossible.

Table No. I. shows the distribution, in the districts named, of each species found in Jamaica; Table No. II. summarizes by classes the total number of species for each of the seven regions, — it represents less the probable richness of each region, than the extent to which it has been explored. A tolerable test of thoroughness of exploration is often found in the proportion which the Schizophyceae bear to the whole number. Being insignificant, usually microscopic plants, they are quite overlooked by the non-scientific collector. Where the knowledge of a region depends on collections made by a non-scientific collector, or by a collector who, however competent in other departments, is not specially an algologist, the red algae constitute a larger, the blue-green a smaller proportion of the whole.

The Puerto Rico collection, and in great part the Canary collection, were made by non-algologists; the Morocco was made by a skilled algologist, but before much was known of the lower algae, or microscopes perfected so that they could be suitably studied. The Biscay collection was the work of one man, a trained algologist, studying the plants on the spot; while the lists for New England and Great Britain cover the most thoroughly studied parts of the world, and the work of generations of botanists. The proportion of Schizophyceae, as shown by Table No. III., follows these conditions fairly well. In the New England list it is exceptionally large, as that list included a number of species, normally fresh water, which were found growing with marine forms, but which usually would not be included in a marine flora. The totals in all parts of the Great Britain list are increased by the fact that in that list the naming of forms is carried out more fully than in any of the others; the percentage, however, is but little affected by this.

It is noticeable that in the first five floras, which might be grouped as warm water floras, the red algae constitute over half the whole list, while in the two northern they are less than half, New England, the most arctic in character though not in latitude, having only 37 per cent. Puerto Rico and Jamaica, the most southern, have the highest percentage of green algae, 27 and 28, respectively, they being in the region of the Siphonaceous plants. The Canaries have less of this element, but more than the region farther north. The low percentage of green algae in the Biscay region is noticeable, but not easy to account for. The high percentage of brown algae in New England and Great Britain is due to their northern latitude, these plants becoming increasingly prevalent as we go from the equator to the poles; in actual arctic waters they constitute the most conspicuous element of the flora.

Table No. IV. shows the number of species common to the flora of Jamaica and the other floras respectively; No. V. shows the per cent of each class of the Jamaica flora which is found in each of the other floras; No. VI. the per cent of each of the others found in Jamaica. A thoroughly explored country shows a larger per cent in No. V., a smaller per cent in No. VI. than a region less known, but certain general deductions can be made. The Puerto Rican flora is closely allied to the Jamaican, 69 per cent being common to the latter; further exploration would probably increase rather than reduce this. The Canaries come next, and it is noticeable that the percentage in Table No. V. is nearly the same in green, brown, and red algae. In Table VI., which is perhaps the one best showing the relationships, the common elements in the European floras grow regularly less as the distance increases, only 8 per cent of the flora of Great Britain being found in Jamaica.

The Schizophyceae seem to vary least in different regions, the other classes coming, Chlorophyceae, Rhodophyceae, Phaeophyceae, the common per cent of the latter being surprisingly small outside of Puerto Rico and the Canaries.

It is worth noting that Jamaica and the Canaries have 66 species in common, being 30 per cent of the former and 24 per cent of the latter; while New England and Great Britain, at about the same distance, have 258 in common, being 60 per cent for the former, 35 for the latter. This merely illustrates the general rule that beginning almost identical, in the Arctic Ocean, the floras of the two shores of the Atlantic diverge increasingly as we go south. There are, however, a few species common to Jamaica and the Canaries which have not apparently been found on the mainland of either continent; these probably represent an actual communication between the two.

Of the 34 fresh water algae, all but 2 are found in Europe, quite in conformity with the rule that the fresh water algae of the two continents, though separated by salt water, in which they cannot exist, are much more alike than the marine algae, inhabiting the two shores of the Atlantic.

The island of Jamaica is situated in the Caribbean Sea, between lat. 17.40 and 18.30 N. and between long. 76.10 and 78.28 W. from Greenwich. The land vegetation is distinctly tropical in character, though the high land of the interior, and the steady sea breezes of the eastern coast, make the climate more comfortable than might be expected from the latitude. The marine flora is also of a tropical character, as is shown by the number of species of the Dictyotales, and of green algae of the Caulerpaceae, Codiaceae, Valoniaceae, and Dasycladaceae, as also by the absence of any representative of the Laminariaceae. Coral abounds all along the shore, and the coral reefs are often richly overgrown with algae.

The following notes by Mrs. Pease give an idea of the character of the shore and the conditions for collecting algae; occasionally throughout the list that follows similar notes by Mrs. Pease on special localities or forms will be inserted, enclosed, like this, in quotation marks.

" The island of Jamaica is scalloped with beautiful little bays or harbors, and a description of one will apply to nearly all of them. The semicircular shores of these bays, about which the little villages cluster, are usually low for only a very short distance back from the water; then they rise abruptly into steep hills or mountains. From one to several small rivers empty into each of these bays; the shores are often of 'tufa,' a porous rock, very trying to a pedestrian, but sometimes relieved by little stretches of sandy beach. . . .

" At Port Antonio, which was visited at each of our trips, the harbor is varied by having a small island lying at its entrance, and by a bold point of land running out to break the shore into two little scallops instead of one, one of the bays being barred by a coral reef, the other having a very narrow channel for the entrance of vessels. This reef was the best collecting ground at this place; the water was shallow for quite a distance, and on jagged rocky bottom, the water about waist deep, we found a very luxuriant growth. Caulerpa clavifera grew like little clusters of green grapes, in big soggy masses; there were great clumps of the encrusted algae, Halimedas, Amphiroas, Galaxauras, Cymopolias, etc.; these continued up towards the shore, and with them upon the rocks were those green, warty, potato-ball-like Dictyosphaerias, Padina, Colpomenia sinuosa, and Anadyomene stellata. Still nearer the shore, the water flattened out to nothing, and the bottom was sand, like powdered shell. Corallina still grew here, but the others dropped out, and Caulerpa ericifolia and C. plumaris covered the bottom, as club mosses grow in the woods. We searched here in vain for a long time for Penicillus, and only at our last visit I noticed, in water barely deep enough to cover them, peculiar little mounds in the sand; brushing off the tops of these revealed the Penicillus capitatus, as abundant as seedling evergreens in a neglected Maine pasture lot. Not far from here, on a stone wall at the edge of a gentleman's garden, the ribbon Ulva, U. fasciata, streamed out into the water, quite filling it for a distance of about a meter. It grew here, on a very limited area, on each of our visits, but we found it nowhere else on the island. . . .

"Morant Bay is larger, and has a comparatively long stretch of sandy beach, but the surf comes in so heavily that seaweeding is very difficult. Annotto Bay is somewhat unusual, the land for some distance from the sea being low and swampy, with sluggish rivers entering the sea by several mouths, but the sandy pebbly shores retained the usual beautiful curve. Montego Bay has a group of small atolls overgrown with mangrove trees, surrounded with shallow water. Kingston has a fine large harbor, enclosed by a long, narrow, sandy arm. On the outside of this, deep water species were often washed ashore. . . .

"The conditions under which one must collect algae in the tropics are somewhat different from those for collecting in the North, where we have the rise and fall of the tide at intervals of a few hours, alternately laying bare and covering the algae on the rocks. At Jamaica many weeds grow on rocks so situated as to be alternately bared and covered by the wash of the waves at intervals of a few minutes. Many of the Polysiphonias, Gelidiums, Gracilarias, etc., are generally found under these conditions. Padina and the Galaxauras occur at these stations, but the finest growth of Padina that we saw was at Montego Bay, from a road passing over a bluff, directly on the edge of the sea, down into which one could look and see Padina growing like a field of gray morning-glory blossoms set upon stones in the shallow, rather quiet water. Near by were patches of Zonaria variegata, like red-brown morning glories.

"Much of our collecting was done from boats, rowed by two or three strong, experienced boatmen. We would be rowed out to the shallow places overgrown with grass, the water even there being to our waists, then jump from the boat into the water, and fish about for seaweeds. We always wore bathing suits and boys' thick hip rubber boots. On the reefs or by the ledges the waves were often strong enough to take us off our feet. Then we would cling closely together, one holding on to the other, while the latter plunged for the seaweeds. Even then we would sometimes be washed away from our footing. The boatmen would be busy keeping the boat from the rocks, and stood ready to assist us back into the boat, often with great difficulty. Most of the Caulerpas were collected in this way, at places some distance from the shore. Even when the plants grew near land, often the shores were so precipitous that one to reach them must use a boat."

In the list that follows, the arrangement practically follows that of Die Natürlichen Pflanzenfamilien of Engler and Prantl, but the names of orders, families, etc., are not given; these are shown later in Table I., giving the comparison of the marine flora of Jamaica with the floras of other regions; the few fresh water algae are included in their appropriate positions in the general list, and the fact of their being fresh water plants is noted by a star prefixed to the name.

General List.

Chroococcus turgidus (Kuetz.) Naeg. Among various fresh water algae, forming a scum on a small roadside brook at the base of a cliff, near the baths, Bath, July, 1900, P. & B. P. B.-A., No. 751. Among marine algae, near Kingston, Duerden.

*Gloeocapsa quaternata (Breb.) Kuetz. Roadside, Bath, July, 1900, P. & B.

Chroothece Richteriana Hansg. Among other algae, in small quantity, Montego Bay, P. & B.

Xenococcus Schousboei Thuret. On Spermothamnion Gorgoneum, Kingston, July, 1900, P. & B.

*Oscillatoria anguina Bory. In still water, Roaring River, near St. Ann's Bay, March, 1893, H.

O. Corallinae (Kuetz.) Gomont. In a pellicle on coral rock, Port Antonio, March 27, 1893, H. Among other algae, near Kingston, Duerden.

*O. formosa Bory. In still water, Roaring River, near St. Ann's Bay, March, 1893; Castleton, April, 1893, H.

*O. princeps Vauch. In mats in stream, St. Ann's Bay, March, 1893, H; Bath, July, 1900, P. & B.

O. princeps forma purpurea n. f. Trichomes and stratum a bright purple, otherwise like type. Forming a stratum on a roadside brook, near the baths, July, 1900, P. & B. P. B.-A., No. 753.

*O. proboscidea Gomont. In a pool by "Wag Water," and in stream from reservoir, Castleton, April, 1893, H.

*O. tenuis Ag. In company with O. princeps forma purpurea, Bath, July, 1900, P. & B.

*Phormidium Retzii (Ag.) Gomont. In tufts on plants, Rio Cobre, Bog Walk, April, 1893, H.

Lyngbya aestuarii (Mert.) Liebm. In mats on stones, Kingston, April, 1893, H; Port Antonio, July, 1891, P. & B. Near Kingston, Duerden.

L. confervoides forma violacea n. f. In company with L. majuscula, Manchioneal Bay, July, 1900, P. & B. Differing from the type only in color.

L. majuscula Harv. Forming a film on marine algae, Port Antonio, March, 1893, H. Same locality, July, 1891, P. & B. Forming extensive tufts on muddy bottom, near the mouth of a small stream, Manchioneal Bay, July, 1900, P. & B.

*L. putalis Mont. Morant Bay, July, 1900, P. & B.

*L. versicolor (Wartm.) Gomont. Marine Garden, Kingston, H. P. B.-A., No. 54.

Symploca hydnoides Kuetz. var genuina Gomont. On rocks in shallow water, in small patches, not abundant, Montego Bay and Manchioneal Bay, 1900, P. & B.

S. hydnoides var. fasciculata (Kuetz.) Gomont. With var. genuina, P. & B.

*Plectonema Nostocorum Bornet. Among Gloeocapsa quaternata, Bath, July, 1900, P. & B.

*P. Wollei Farlow. Morant Bay, Aug., 1894, P. & B. Roaring River, H. P. B.-A., No. 55.

*Schizothrix coriacea (Kuetz.) Gomont. In tufts on sides of lily tanks, Botanic Garden, Castleton, April, 1893, H.

*S. Mexicana Gomont. On rock in "Wag Water," Castleton, April, 1893, No. 399, H.

Microcoleus chthonoplastes (Fl. Dan.) Thuret. In turfs of algae, St. Ann's Bay, March, 1893, H.

M. tenerrimus Gomont. In company with M. chthonoplastes, March, 1893, H.

*M. vaginatus (Vauch.) Gomont. On moist rock, Rio Cobre, Bog Walk, April, 1893, H.

*Nostoc commune Vauch. In crusts on sandy soil, Constant Spring, April, 1893, No. 365, H.

*N. microscopicum Carm. On steps into reservoir, Constant Spring, April, 1893, No. 361, H. The specimens are sterile, so that the determination is somewhat in doubt.

*N. verrucosum Vauch. On rocks in "Wag Water," Castleton, April, 1893, H. No. 362, H., from trough in running water, Castleton, April, 1893, is probably the same species.

*Cylindrospermum muscicola Kuetz. On sides of basin, Constant

Spring; on sand at edge of river, Castleton, April, 1893, No. 364, H.

Hormothamnion enteromorphoides Grunow. In shallow water, St. Ann's Bay; on coral reef, Navy Island, July 25, 1897, H. P. B.-A., No. 56. Near Kingston, Duerden.

*Scytonema Arcangelii Born. & Flah. On moist rocks by spring, Castleton, April, 1893, H.

S. conchophilum Humphrey ms. In old conch shell, Port Antonio, March, 1893, H. Kingston, June, 1897, H; Producing slight, gray, pustular roughenings of outside of shell, Mastigocoleus testarum occurring on inside of same shell.

Filaments 5–8 μ diam., irregularly branched, branches single or geminate, tips rounded, cells two thirds to two times as long as broad, 2.7–4.5 μ diam., pale bluish when separate. Heterocysts globose or slightly elongated, 5 μ diam., rarely two or three together, intercalary. Sheath rather thin, deep yellow, homogeneous; when old, rough outside, hyaline and thin at growing tips. J. E. Humphrey.

*S. crispum (Ag.) Bornet. On sides of trough, Constant Spring; in basin, Kingston, April, 1893, H. P. B.-A., No. 60.

*S. densum (A. Br.) Bornet. In turfs, moist places, Port Antonio, April, 1893, H.

*S. Hofmanni Ag. On steps of Court House, Port Antonio, April, 1893, H.

*S. Javanicum (Kuetz.) Bornet. On flower-pot in garden, Castleton, April, 1893, H.

*S. ocellatum (Dillw.) Thuret. On old palm stems, Castleton, April, 1893, H.

*Hapalosiphon fontinalis (Ag.) Bornet. On rock, "Wag Water," Castleton, April, 1893, H.

Mastigocoleus testarum Lagerh. In old shells, Kingston, 1897, H.

Calothrix aeruginea (Kuetz.) Thuret. On Dasya arbuscula, Montego Bay, June, 1900, P. & B.

C. confervicola (Roth) Ag. On various algae, Port Antonio, March, 1893, H.

C. Contarenii (Zan.) Born. & Flah. On wreck on beach, Port Morant, March, 1893, H.

*C. fusca (Kuetz.) Born. & Flah. Among Gloeocapsa quaternata, Bath, 1900, P. & B.

*C. Juliana (Meneg.) Born. & Flah. On stones in stream, Roaring River, St. Ann's Bay, March, 1893, H.

C. pilosa Harv. On Bostrychia tenella, Port Antonio, Aug., 1894, P. & B.

Dichothrix penicillata Zan. On Cymopolia barbata, Port Maria, H. On Dictyota dichotoma, P. & B. P. B.-A., No. 62.

*Gloeotrichia natans (Hedw.) Rab. Under Nymphaea leaves, Botanic Garden, Castleton, April, 1893, H.

*Spirogyra decimina (Muell.) Kuetz. Manchioneal, July, 1900, P. & B.

The spores agree with this species, and as far as can be judged from dried specimens, the vegetative characters. A sterile Spirogyra from Bath has the same dimensions of cells, but cannot be specifically determined.

Ulva fasciata Delile. In dense masses just below water mark, but only in one limited locality, Port Antonio, July, 1891, P. & B. P. B.-A., No. 221. Near Kingston, Duerden.

U. Lactuca var. rigida (Ag.) Le Jolis. Port Antonio, Aug., 1894; Kingston, Montego Bay, June, 1900, P. & B. Near Kingston, Duerden.

Enteromorpha erecta (Lyng.) J. Ag. Port Antonio, April, 1892, P. & B.

E. flexuosa (Wulf.) J. Ag. Port Antonio, July, 1891; Runaway Bay, July, 1900; washed ashore, Manchioneal Bay, July, 1900, P. & B. Near Kingston, Duerden.

E. intestinalis (L.) Link. Port Antonio, washed ashore, July, 1894, P. & B.

E. prolifera (Muell.) J. Ag. Runaway Bay, Montego Bay, Manchioneal, on stones; also in fresh water at Bath, on stones in river, 1900, P. & B.

*Stigeoclonium tenue (Ag.) Rab. No. 366, H., locality not given.

Diplochaete solitaria n. g. & sp. Frond epiphytic, consisting of a single cell, with thick, transparent wall, and bright green contents, spherical or flattened, the outline as seen from above round or slightly oval; two hairs arising from each cell, usually opposite, and from points on the under surface quite near the edge. Cell 25–30μ diameter, half this diameter being occupied by the wall; hairs 4–6μ diameter, slightly tapering, straight. On Laurencia obtusa, near Kingston, Duerden.

This minute plant was observed on a specimen of Laurencia, after it had been mounted for the herbarium, so that nothing is known as to its development, but it seems so distinct from any described genus of the Chaetophoraceae as to require a new name.

Pringsheimia scutata Reinke. On Laurencia obtusa, near Kingston, Duerden.

*Mycoidea parasitica Cunningham. On leaves of various plants, Roaring River, March, 1893, Nos. 324 & 325; Bath, 1897, H. P. B.-A., No. 763.

Chaetomorpha brachygona Harv. Port Antonio, July, 1891; Manchioneal Bay, Rio Bono, 1900, P. & B. Forming dense mats on bottom of Kingston Harbor, April, 1893, No. 369, H. Near Kingston, Duerden. Hardly distinct from C. cannabina of Europe.

C. clavata (Ag.) Kuetz. Washed ashore, Port Antonio, P. & B. St. Ann's Bay, March, 1893, No. 329, H. A rather slender form.

C. aerea (Dillw.) Kuetz. Washed ashore, Port Antonio, Aug., 1894, P. & B.

C. Linum (Fl. Dan.) Kuetz. Kingston Harbor, Aug., 1891, R. P. Bigelow. Manchioneal, in company with C. brachygona, Morant Bay, June, 1900, P. & B.

The plant from Morant Bay has very moniliform filaments, up to .4 mm. diameter, the cell wall thin, color light green, articulations one to two diameters; perhaps a distinct species.

C. Linum var. brachyarthra Kuetz. Port Antonio, July, 1891, P. & B.

C. Melagonium (Web. & Mohr.) Kuetz.? Growing in mud near the mouth of a river, Manchioneal, July, 1900, P. & B. Quite like the northern form usually known as C. Picquotiana, but possibly not distinct from C. Linum.

Cladophora fascicularis Kuetz. Port Antonio, July, 1891; Montego Bay, Manchioneal, 1900, P. & B.; Port Antonio, Feb., 1893, No. 179, H. Generally distributed, usually growing on pebbles in mud in shallow water.

C. crystallina (Roth) Kuetz. Ora Cabessa, June, 1900, P. & B.

C. fuliginosa Kuetz. In turfs, Port Maria, No. 298, H. Morant Bay, Annotto Bay, etc., P. & B. Apparently common everywhere; usually known as Blodgettia confervoides.

C. Hutchinsiae (Dillw.) Kuetz. Port Antonio, July, 1891, P. & B.

C. intertexta n. sp. Filaments 200–350μ diam., articulations one to three diameters, usually one and one half to two; sparingly branched, branches naked or with short, usually secund ramuli; terminal cells blunt, rounded. Tufts densely matted, prostrate.

The plant forms dense masses on the bottom of pools, creeping over the coral sand and broken shells; the upright branches are usually simple, and the plant resembles an entangled mass of some coarse Chaetomorpha rather than a Cladophora, but occasionally the free branches have a series of secund, two or three-celled ramuli, issuing one from each articulation. In the entangled mass more branching of this character will be found, also long normal branches in no definite order. The habit of C. intertexta is much like that of C. repens (J. Ag.) Harv., but the filaments are two or three times as large as in that species, and the color is a light green, somewhat whitish in drying, instead of the dull olive green of C. repens; the latter has, moreover, a vaguely dichotomous branching, and articulations many times — according to Harvey, even twenty times — the diameter. C. herpestica (Mont.) Kuetz. has filaments of about the same size as C. intertexta, but it has long articulations, up to fifteen diameters, and irregular branching, with the upper branches fasciculate.

Found along the shore near Manchioneal, July, 1900, P. & B. P. B.-A., No. 818.

C. trichocoma Kuetz. Manchioneal, July, 1900, P. & B.

Gomontia polyrhiza (Lagerh.) Born. & Flah. In old shells, coral and bones, Kingston, 1897, H.

Bryopsis Harveyana J. Ag. In tufts on stones, Kingston Harbor, April, 1893, No. 367, H.

B. pennata Lamour. In tufts on rocks, Apostles' Battery, Kingston Harbor, April, 1893; Port Maria, March, 1893, No. 297, H. A single specimen, Port Morant, July, 1900, P. & B.

Caulerpa cupressoides var. typica Weber. On sandy bottom, Navy Island, Port Antonio, March, 1893, No. 188, H.; Port Antonio, P. & B. P. B.-A., No. 79.

C. cupressoides var. Turneri Weber. Port Antonio, P. & B. P. B.-A., No. 765.

C. cupressoides var. mamillosa (Mont.) Weber. Among eel-grass, at about one meter depth, Montego Bay, July, 1900, P. & B. Including forma typica and forma nuda. P. B.-A. No. 765. Near Kingston, Duerden.

C. cupressoides var. ericifolia (Turn.) Weber. Port Antonio, July, 1891, P. & B.

C. pinnata forma Mexicana (Sond.) Weber. Montego Bay, July, 1900, P. & B.

C. plumaris forma longiseta (J. Ag.) Weber. Forming dense mats in mud in shallow water, Port Antonio, July, 1891, P. & B. P. B.-A., No. 27. Near Kingston, Duerden; very luxuraint, the erect fronds 20 cm. high.

C. plumaris forma brevipes (J. Ag.) Weber. Port Antonio, July, 1891; Montego Bay, July, 1900, among eel-grass at about one meter depth, P. & B. P. B.-A., No. 766. P. U., No. 672. Near Kingston, Duerden.

C. prolifera (Forsk.) Lamour. Washed ashore, not common, Port Morant, July, 1900, P. & B.

C. racemosa var. clavifera (Turn.) Ag. Port Antonio; Port Morant, at about one meter depth, July, 1900, P. & B. In tufts on rocks, Kingston, April 8, 1893, No. 370, H. P. B.-A., No. 767.

C. racemosa var. clavifera forma macrophysa (Kuetz.) Weber. On coral reef, Port Antonio, 1894 & 1900, P. & B. Near Kingston, Duerden, passing insensibly into var. clavifera. P. B.-A., No. 870.

C. taxifolia (Vahl) Ag. Washed ashore, Port Morant, July, 1900. Annotto Bay, 1894, P. & B. Chitty. P. B.-A., No. 768.

C. verticillata J. Ag. In tufts on coral rocks, Port Antonio, Feb. 27, 1893, No. 181, H. Near Kingston, Duerden.

C. verticillata forma charoides (Harv.) Weber. Kingston, June, 1900, P. & B. Forming fine moss-like mats in soft mud near Mangrove swamp, at depth of about one meter. Near Kingston, Duerden.

Penicillus capitatus Lam. Port Antonio, Montego Bay, Manchioneal, nearly buried in coral sand, 1900, P. & B. Port Maria, No. 294, H. Sloane. P. B.-A., No. 271. P. U., No. 523. Near Kingston, Duerden.

P. dumetosus (Lamour.) Decsne. Annotto Bay, washed ashore, Manchioneal, July, 1900, P. & B. Specimen without locality, H. P. B.-A., No. 769.

" Penicillus dumetosus grew in some abundance in a pool near Manchioneal. The pool was narrow, with precipitous tufa walls, which towards the sea closed over the pool in an arch, through which the waves broke heavily. The Penicillus grew among eel-grass, in muddy soil, covered by a coating of powdered shell and coral. With it were P. capitatus, Avrainvillea longicaulis, and Halimedas. The P. dumetosus looked like miniature groves of carefully trimmed evergreen trees, gray green in color."

Rhipocephalus Phoenix (Ell. & Sol.) Kuetz. Port Morant, a single specimen washed ashore, July, 1900, P. & B.

Avrainvillea longicaulis (Kuetz.) Murray & Boodle. Montego Bay, June, Manchioneal, July, 1900, P. & B. P. B.-A., No. 770.

Avrainvillea nigricans Decsne. Singly in shallows, Port Maria, March 17, 1893, No. 270, H. Manchioneal, July, 1900, P. & B. P. B.-A., No. 771.

"Avrainvillea longicaulis at Montego Bay grew imbedded in mud among eel-grass in shallow water, near a small island consisting of mangrove swamp. It was discovered by the sense of feeling as we were dredging in the mud among the eel-grass roots for Caulerpa. We were continually feeling through the thick soles of our rubber boots a sensation as of stepping on drowned kittens. It proved to be the curious fleshy fronds of Avrainvillea, somewhat resembling a downy, dirty, swollen Udotea, often full of worms and other small animals. Avrainvillea grew also at Manchioneal, in an enclosed salt water pool, in eel-grass with Penicillus dumetosus, rooted in a clean bottom of powdered shells and coral; but on the rocks bordering the pool was another species, A. nigricans, with short stems, and tops not so flabellate, resembling in shape our stemmed puff-balls."

Udotea conglutinata (Sol.) Lamour. Closely set on bottom, Port Maria, March 17, 1893, No. 269, H.

U. flabellata Lamour. On sandy bottom. Port Antonio, March 3, 1893, No. 202; Port Maria, March 17, 1893, No. 268, H. On muddy bottom, Port Antonio, July, 1894; washed ashore, Morant Bay, P. & B.

Halimeda Opuntia (L.) Lamour. In dense tufts, Port Maria, March, 1893, H. Port Antonio, July, 1891, P. & B. Near Kingston, Duerden. Sloane. Growing similarly to the preceding species.

H. tridens (Ell. & Sol.) Lamour. In tufts, St. Ann's Bay, March 23, 1893; Port Maria, March 17, 1893, H. Port Antonio, July, 1891, growing in shallow water, in soil composed of broken shells and coral. Near Kingston, Duerden.

It is impossible to distinguish H. incrassata (Ell.) Lamour from H. tridens. In any considerable collection typical forms of each and a series of intermediate forms are to be found.

H. Tuna (Ell. & Sol.) Lamour. In dense tufts, shallows, Port Antonio, March 10, 1893, No. 235, H.

Codium adhaerens (Cabr.) Ag. Port Antonio, Aug., 1894, P. & B. Specimen without locality, No. 293, H.

C. tomentosum (Huds.) Stack. In immense tufts, Port Maria, March 17, 1893, No. 266, H. Port Antonio, July, 1891; Kingston, July, 1900, P. & B. Near Kingston, Duerden. Washed ashore in large quantities, nearly everywhere. P. B.-A., No. 168.

Valonia aegagropila Ag. On rocks in shallows, Port Maria, March 20, 1893, No. 296, H. Montego Bay, July, 1900, on rocks in shallow water, P. & B. P. B.-A., No. 772.

V. ventricosa J. Ag. On rocks in shallows, Port Antonio, March 11, 1893; Port Maria, March 20, 1893, No. 295, H. On rocks in shallow rough water, Montego Bay, June, 1900, P. & B. "Fronds smooth and transparent, as if made of thin green glass."

V. verticillata Kuetz. On rocks in shallow water, Port Morant, Manchioneal, July, 1900, P. & B.

Siphonocladus membranaceus (Ag.) Bornet. Growing in mats on rocks, near shore, Port Antonio, Aug., 1894; Runaway Bay, June, 1900, P. & B. Near Kingston, Duerden.

S. tropicus (Crouan) J. Ag. Washed ashore, Morant Bay, July, 1894, P. & B.

Dictyosphaeria favulosa (Ag.) Decsne. On rocks in shallows, Port Antonio, March 3, 1893, Nos. 205 & 271, H. On coral reef, Port Antonio, July, 1891, P. & B. P. B.-A., No. 124.

Chamaedoris annulata (Lam.) Mont. Washed ashore, Morant Bay, July, 1894, P. & B.

Microdictyon umbilicatum (Velley) Zan. In dense tufts, Port Antonio, Feb. 27, 1893, No. 174, H.

Anadyomene stellata (Wulf.) Ag. In tufts on rocks, Port Antonio, Feb. 27, 1893, H. Similar localities, Port Antonio, July, 1891; Kingston, Port Morant, July, 1900, P. & B. P. B.-A., No. 169.

Acetabularia crenulata Lamour. Port Antonio, Annotto Bay, Aug., 1894; Rio Novo, June, 1900, P. & B. Near Kingston, Duerden. P. B.-A., No. 125.

"At Annotto Bay Acetabularia and Dasycladus grew in water nearly to our shoulders, not very rough, on cobble stones, the two species growing together like minute forests covering the stones."

Dasycladus clavaeformis (Roth) Ag. In tufts on rocks, Port Maria, Apr. 19, 1893, No. 285, H; Annotto Bay, with the preceding species; on pebbles washed ashore, St. Ann's Bay, 1900, P. & B. P. B.-A., No. 170.

Botryophora occidentalis (Harv.) J. Ag. In salt pools, Palisadoes, Kingston Harbor, April 10, 1893, No. 386, H. Port Antonio, Aug., 1894, P. & B.

Neomeris dumetosa Lamour. Kingston Harbor, on mangrove roots, July, 1900, P. & B. "Looking like small green worms."

Cymopolia barbata (L.) Lamour. In tufts on stones, St. Ann's Bay and Port Maria, March, 1893, H. On coral reef, Port Antonio, Annotto Bay, 1891 & 1894, washed ashore; Kingston, Port Morant, 1900, P. & B. Near Kingston, Duerden. P. B.-A., No. 28. P. U., No. 674. Sloane.

Many specimens agree with the description of C. Mexicana J. Ag., but all intermediate forms occur, and often the same individual will agree with one species in one part of the frond, with the other in other parts.

E. Mitchellae Harv.? Kingston, March, 1893, Nos. 141, 142, 372, H. Not exactly like the type of this species, the plurilocular sporangia being longer and sometimes clavate. Possibly E. Duchassaingianus Grunow.

Striaria attenuata (Ag.) Grev. Montego Bay, June, 1900, washed ashore on sandy beach, P. & B.

S. attenuata var. ramosissima (Kuetz.) Hauck. With the type, June, 1900, P. & B.

Colpomenia sinuosa (Roth) Derb. & Sol. On coral rocks, Port Antonio, March 8 and 23, 1893, Nos. 153 and 212; Port Maria, March 17, 1893, No. 273, H. Annotto Bay to Port Antonio, in shallow water, Aug., 1894, P. & B.

Hydroclathrus cancellatus Bory. On coral rocks, Port Antonio, Feb. 10, 1893, No. 234, H.

Cutleria sp. A single specimen, attached to a frond of Udotea flabellata, seems to be the Aglaozonia form of some Cutleria, but in the absence of fruit it is indeterminable. The frond consists of radiating articulate filaments, united laterally, and varying much in diameter.

Turbinaria trialata Kuetz. Washed ashore, Port Antonio, March 8, 1893, No. 211; in tide pools, Port Maria, March 16, 1893, No. 249, H. Washed ashore, Port Antonio, July, 1891; Montego Bay, July, 1900, P. & B. P. B.-A., No. 774. T. vulgare, Sloane, is undoubtedly this species.

Sargassum bacciferum (Turn.) Ag. Washed ashore, Port Maria, March 18, No. 248, H. Sloane, Chitty.

S. lendigerum (L.) Kuetz. Washed ashore, Port Antonio, July, 1891, P. & B. In tufts in tide pools, Port Maria, March 17, 1893, No. 292, H.

S. platycarpum Mont. Washed ashore, Port Antonio. July, 1891, P. & B. Same locality, March 8, 1893, No. 210, H. P. B.-A., No. 775.

S. vulgare Ag. Washed ashore, Port Maria, March 18, 1893, No. 247, H. The references to Sloane and Chitty are doubtful, and some other form may have been referred to under this name.

S. vulgare forma ovata n. f. Washed ashore, Montego Bay, June, 1900, P. & B. P. B.-A., No. 776. Leaves thick, dark, ovate to suborbiculate, coarsely and sharply, sometimes doubly toothed, usually slightly oblique at the base. The branching is dense, the leaves numerous and of form and thickness mentioned above; otherwise it agrees with typical S. vulgare.

S. vulgare var. foliosissimum (Lamour.) J. Ag. Washed ashore, Port Antonio, July, 1891, P. & B.

Spatoglossum Schroederi (Mert.) J. Ag. Two specimens only, washed ashore on sandy beach with high surf, near lighthouse, Kingston harbor, July, 1900, P. & B. Chitty.

Stypopodium lobatum (Ag.) Kuetz. Washed ashore, Port Maria, March 10 and 19, Nos. 231 and 286; St. Ann's Bay, March 23, 1893, No. 311, H. Annotto Bay, July, 1891; Montego Bay, June, 1900, P. & B. P. B.-A., No. 777.

"Stypopodium lobatum grew in magnificent clumps of two sorts, one with the frond narrowly divided and heavily marked with dark bars, making the plant resemble bunches of turkey feathers; the other with fronds of broader divisions and not so prominently barred. The first mentioned form grew deeper down in the water, so deep as to have to be pulled off by the boatmen by means of a long handled boat-hook. The two forms were plainly distinguished as they grew in the water."

Gymnosorus variegatus (Lamour.) J. Ag. Kingston, Montego Bay, 1900, P. & B. P. B.-A., No. 778.

"Gymnosorus variegatus grew with Padina, which it resembled in manner of growth, being in shape like clusters of short-stemmed morning glory flowers. It formed a covering to the rocks nearer shore than the Stypopodium, the water being about knee deep. G. variegatus is reddish brown in color, Padina gray, Sargassum and Turbinaria rich yellow brown; Dictyota a darker brown with less yellow; Stypopodium generally grayish brown with dark markings. The contrasting colors were very rich in the water."

Padina Durvillaei Bory. On rocks, Port Antonio, Feb. 28, 1893, No. 173, H. Port Antonio, July, 1891; Ora Cabessa, Montego Bay, 1900, P. & B. Near Kingston, Duerden. The P. Pavonia of Murray and earlier lists is probably this species.

Dictyopteris delicatula Lamour. In tufts on rocks, Port Maria, March 19, 1893, H. Washed ashore, Annotto Bay, Aug., 1894; Hope Bay, Kingston, 1900, P. & B. P. B.-A., No. 485.

D. Justii Lamour. Washed ashore, Port Antonio, July, 1891; Morant Bay, Annotto Bay, Aug., 1894; Kingston, 1900, P. & B. In tufts on rocks, Port Maria, March 17, 1893, No. 264, H. Chitty.

D. plagiogramma Mont. Annotto Bay, July, 1894, washed ashore, P. & B. Chitty.

Dictyota Bartayresiana Lamour. Washed ashore in mats, Port Antonio, March, 1893, Nos. 154, 194, 229, H. Port Antonio, July, 1891; on rocks in shallow water, Kingston, Montego Bay, Manchioneal, 1900, P. & B. Near Kingston, Duerden. P. B.-A., No. 579. Found in both broad and narrow forms, at nearly all the localities, often appearing like two distinct species.

D. cervicornis Kuetz. Washed ashore, Port Antonio, Aug., 1894, P. & B. Near Kingston, Duerden.

D. ciliata Ag. In tufts on rocks, Port Maria, March 16, 1893, Nos. 246 and 287; Port Antonio, March 10, 1893, No. 230, H. Washed ashore, Montego Bay, Ora Cabessa, Manchioneal, 1900, P. & B. P. B.-A., No. 779. All three kinds of fruit are represented in the specimens distributed in the Phycotheca Boreali-Americana, the plants being collected at the same time. All are similarly arranged, occupying the whole of the fertile segments, except a narrow strip at the margin. The male plants are mostly old and battered, as if the antheridia were produced somewhat earlier in the season than the other kinds of fruit.

"Dictyota ciliata at Montego Bay, June 23, 1900, grew on boulders near a precipitous rocky shore in water more than waist deep. It formed large round clumps. The water being very clear here, the hairs on the edge of the frond were so conspicuous as to easily distinguish in the water this form from other Dictyotas. The rocks in this locality were beautifully draped with the Dictyota, robust plants of Turbinaria in large thick masses, a Sargassum with rounded leaves, and Stypopodium in magnificent clumps."

D. dentata Lamour. Washed ashore, Port Maria, March 17, 1893, No. 265, H. Port Antonio, July, 1891, P. & B. On rocks in rough water, one meter or more deep. P. U., No. 669. Some specimens have the tips of the branches so finely divided as to seem ciliate.

D. dichotoma (Huds.) Lamour. Kingston Harbor, July, 1891, R. P. Bigelow. On rocks, Port Antonio, July, 1891; Montego Bay, June, 1900, P. & B. Chitty.

D. divaricata Lamour. In various places, 1900, P. & B. Near Kingston, Duerden. Connected by intermediate forms with D. Bartayresiana.

D. fasciola (Roth) Lamour. Washed ashore, Port Antonio, July, 1891; Rio Novo, June, 1900, P. & B.

Dilophus alternans J. Ag. Port Antonio, July, 1894, P. & B.

D. Guineensis (Kuetz.) J. Ag. On flat rocks washed by the waves, in company with Gelidium rigidum, Montego Bay, Rio Novo, June, 1900, P. & B.

Dictyerpa Jamaicensis n. g. & sp. Frond filiform, 1–3 mm. diam. up to 2 dm. long; consisting of two layers of cells, an inner layer of large, colorless, cylindrical cells, about three diameters long, symmetrically arranged; an external monostromatic layer of brown rectangular cells from one to three diameters long, in distinct longitudinal series. Branching di- or trichotomous, with occasional irregularly placed lateral branches, mostly at wide angles, each branch ending in a large, depressed-hemispherical cell, by whose division the growth of the branch proceeds. Tufts of very fine, rust-colored or colorless confervoid rhizoidal filaments at irregular intervals on the frond. Fructification? Washed ashore, Manchioneal, July, 1900. P. B.-A., No. 780.

Though evidently belonging to the Dictyotaceae, this plant differs from any genus of the family yet described, in having the frond terete throughout. Many Dictyotaceae have prostrate rooting filaments from which the erect fronds arise, but in all species found in Jamaica this prostrate growth is quite insignificant in comparison with the plant in question. It was found washed ashore in two places, in considerable quantity, and in no case shows any indication of fructification, or of producing erect flattened fronds. It may seem hazardous to give it a generic name, but as it is a plant of quite distinct habit, and cannot be now identified with any named form, it seems to require at least a provisional name.

As washed up on the beach, it appeared like rolled and twisted strings. The dried plant is quite black in color, and under a hand lens shows closely set constrictions, probably due to the large interior cells being of uniform length, and terminating at the same level, as in the frond of Polysiphonia. These constrictions are lost when the frond is remoistened.

Goniotrichum Humphreyi Collins. On woodwork of wreck, St. Ann's Bay, March 24, 1893, No. 316, H. P. B.-A, No. 421.

“Frond filamentous, solid, gelatinous, occasionally forking or dividing into several branches, the terminal portion consisting of a single series of cells; the older part containing numerous cells, irregularly placed near the surface of the filament; lateral branches abundant, simple, issuing nearly at a right angle, composed of a single series of cells.” This description is copied from the label of P. B.-A., No. 421.

G. elegans (Chauv.) Le Jolis. Among other algae, on Laurencia obtusa, near Kingston, Duerden.

Chantransia Saviana (Menegh.) Ardiss. Among other algae, on Laurencia obtusa, near Kingston, Duerden.

Liagora Cheyneana Harv. Washed ashore, Port Maria, March 17, 1893, No. 281; Port Antonio, March, 1893. No. 186, H.

L. decussata Mont. Washed ashore, Hope Bay, July, 1891, and Aug., 1894, P. & B. Very abundant in 1894. P. B.-A., No. 89. The finest species of the genus, with fronds in shape of a fir tree, sometimes over a meter in length. Apparently confined to the islands on the two sides of the Atlantic.

L. elongata Zan. Hope Bay, July, 1891 ; Montego Bay, July, 1900, P. & B.

L. pulverulenta Ag. Washed ashore, Manchioneal, July, 1900, P. & B.

L. valida Harv. In large tufts, Port Maria, March 17, 1893, No. 283 ; Port Antonio, March 10, 1893, No. 240, H. Hope Bay, Orange Bay, Montego Bay, 1891 and 1900, P. & B. Under No. 687, P. B.-A., a form was distributed as L. tenuis, which it now seems better to regard as L. valida. It is difficult to see how the two species can be distinguished, when one has a large number of specimens. Harvey's name, being the older, must be maintained.

Galaxaura cylindrica (Sol.) Decsne. Port Antonio, Morant Bay, Manchioneal and elsewhere, common, P. & B. Near Kingston, Duerden. Sloane. Chitty. P. B.-A., No. 134.

G. lapidescens (Sol.) Lamour. In large tufts, Port Antonio, March 10, 1893, No. 239, H. Annotto Bay, Port Antonio, July, 1891 ; Montego Bay, on rocks, June, 1900, P. & B. Chitty. Not so common as other species of the genus.

G. marginata (Ell. & Sol.) Lamour. On stones at tide-mark, Port Antonio, March 10, No. 145 ; March 21, No. 241. H. Port Antonio, Annotto Bay, Montego Bay, Manchioneal, 1900, P. & B. Common, growing very densely on rocks.

G. obtusata (Ell. & Sol.) Lamour. Port Antonio, July, 1891 ; Port Maria, July, 1900, P. & B., in company with other species of the genus.

G. rugosa (Sol.) Lamour. In large tufts, Port Antonio, March, 1893, No. 131, H. Port Antonio, July, 1891 ; Rio Novo, Rio Bono, Montego Bay, 1900, P. & B. Near Kingston, Duerden. P. B.-A., No. 133. P. U., No. 510. Sloane. Usually washed ashore on beaches.

Wrangelia Argus Mont. Montego Bay, June, 1900, forming soft mats on rocks, P. & B. Specimen without locality, H.

Gelidium coerulescens Crouan. Port Antonio, July, 1891 ; July, 1900, P. & B. P. B.-A., No. 783.

By the kindness of Dr. Bornet this plant has been compared with authentic specimens from Guadeloupe, and it is the plant referred to by Mazé & Schramm, Algues de Guadeloupe, p. 199. Whether it is the plant of Kuetzing, Tab. Phyc., Vol. XVIII. Pl. 56, from New Caledonia, is not certain.

G. crinale (Turn.) J. Ag. Port Antonio, July, 1900, with G. coerulescens, P. & B.

G. rigidum (Vahl) Ag. Port Antonio, July, 1891; Montego Bay, June, 1900, P. & B. P. B.-A., No. 784. Appears to be the form known as var. radicans (Bory) J. Ag.

G. supradecompositum Kuetz. Morant Bay, July, 1894, P. & B. No. 227, no locality, H.

The identification of this form is from a specimen from Fajardo, Puerto Rico, received from Hauck. If G. crinale were taken in a broad sense, it might include this form.

Catenella Opuntia var. pinnata (Harv.) J. Ag. Manchioneal, July, 1900, P. & B. Forming a thin greenish coating on small stones in shallow water, on muddy bottom near the mouth of a small river. P. B.-A., No. 792.

Agardhiella tenera (J. Ag.) Schmitz. Morant Bay, July, 1894; Montego Bay, June, 1900, P. & B.

Solieria chordalis (Ag.) J. Ag. Washed ashore, Port Antonio, July, 1891. P. & B.

Eucheuma echinocarpum Aresch. Montego Bay, a few small plants, June, 1900, P. & B.

Gracilaria Blodgettii Harv. Washed ashore, Montego Bay, June, 1900, P. & B.; only a few specimens, some of which show a tendency to pass into G. confervoides.

G. caudata J. Ag. Port Antonio, Aug., 1894, P. & B.

G. cervicornis (Kuetz.) J. Ag. Washed ashore, Morant Bay, July, 1894; Manchioneal, July, 1900, P. & B. Near Kingston, Duerden. P. B.-A., No. 787. Some of the plants are quite like Mediterranean specimens of G. armata. The Florida plant described as G. armata by Harvey in the Nereis Boreali-Americana seems to be different, and has not been found in Jamaica.

G. compressa (Ag.) Grev. Annotto Bay, Aug., 1894, P. & B.

G. confervoides (L.) Grev. On small stones, St. Ann's Bay, March 23, 1893, No. 312, H. Washed ashore, Borden, July, 1894; Montego Bay, Manchioneal, 1900, P. & B. Near Kingston, Duerden. Common and variable.

G. cornea J. Ag. Washed ashore, Rio Bono, June, 1900, P. & B.

G. Curtissiae J. Ag. Washed ashore, Annotto Bay, Aug., 1894, P. & B.

G. damaecornis J. Ag. Annotto Bay, Aug., 1894; Manchioneal, July, 1900, P. & B. P. B.-A., No. 788.

G. divaricata Harv. In short tufts, Navy Island, Port Antonio, March, 1893, Nos. 155 and 228, H. Port Antonio, July, 1891; Port Morant, Rio Bono, June, 1900, P. & B. P. B.-A., No. 789. Generally distributed but nowhere common.

G. Domingensis Sond. Manchioneal, June, 1900, P. & B. Found only in a very limited station, in large tufts on rocks about one meter depth, in rough water; very luxuriant plants, showing beautiful shades of violet.

By J. G. Agardh this is considered as merely a form of G. multipartita var. polycarpa. Imperfectly developed specimens have some resemblance to that variety, but well developed plants are quite different; the habit reminds one rather of Laurencia pinnatifida. All three kinds of fruit were found in the Manchioneal specimens, the cystocarps and tetraspores as usual in this genus, the antheridia in crypts, as described by Thuret for G. confervoides. The description of G. Krugiana in Hauck's Puerto Rico list is quite suggestive of some of these specimens.

G. ferox J. Ag. Washed ashore, Morant Bay, July, 1894, P. & B.

G. multipartita (Clem.) J. Ag. Port Antonio, July, 1891; Port Morant, Montego Bay, Ora Cabessa, Manchioneal, 1900, P. & B. No. 380, no locality, H. Near Kingston, Duerden. Chitty. P. B.-A., No. 885.

G. Wrightii (Turn.) J. Ag. Annotto Bay, Aug., 1894; Montego Bay, June, 1900, P. & B. A few plants only.

The fresh frond is very stout and densely branched, and not at all compressed; it shrinks much in drying, and herbarium specimens give the idea of a flattened frond.

Hypnea divaricata Grev. In large tufts on rocks in shallow water, Montego Bay, Manchioneal, 1900, P. & B.

H. musciformis (Wulf.) Lamour. On stones at tide mark, Port Antonio, March, 1893, Nos. 147 and 223; St. Ann's Bay, March 24, 1893, No. 320, H. Near Kingston, Duerden. Common everywhere, P. & B. Chitty.

H. Valentiae (Turn.) Mont. Annotto Bay, Aug., 1894, P. & B.

The species is here taken in the same sense as by Hauck, Hedwigia, 1887, Heft 1, to include H. nidifica J. Ag. and H. fruticulosa Kuetz.; forms corresponding to both of these occur at Annotto Bay.

Cordylecladia irregularis Harv. Annotto Bay, Aug., 1894, P. & B. Near Kingston, Duerden.

Some of the plants from each locality have tetraspores, which appear not to have been previously reported. They are arranged much as in C. erecta, except that they are at the ends of short lateral branches, instead of terminal on the larger branches; the modified portions of the branches being ovate or subspherical rather than lanceolate. One of the Kingston specimens has cystocarps, which are spherical and external on the branches, as in other species of the genus.

Cordylecladia Peasiae n. sp. Fronds slender, filiform, arising from a more or less distinct crustaceous base, dichotomously divided, with occasional scattered or secund ramuli, usually quite short. Tetraspores cruciate, in the somewhat swollen and darkened tips of the branches and ramuli, immersed in the cortical layer. Cystocarps globular, sessile along the main branches. Color purplish brown, changing into whitish or greenish; substance rigid.

Somewhat resembles C. erecta, which is, however, a smaller plant, much less branched, and having the receptacles for tetraspores larger and of different shape. C. conferta and C. Andersoniana have the tetraspores in densely tufted special lateral branches. C. irregularis is stouter, with hollow stems and with oval or subspherical lateral branches for the tetraspores. In C. furcellata the tetraspores are borne in branches resembling the vesicles of Chrysymenia uvaria. C. heteroclada has a flat frond, and C. Huntii is unrecognizable from the description of Harvey.

Manchioneal, July, 1900, P. & B. P. B.-A., No. 791.

Chrysymenia halymenioides Harv. Washed ashore, Morant Bay, July, 1894, P. & B.

Champia parvula (Ag.) Harv. Montego Bay, Port Maria, 1900, P. & B.

Caloglossa Leprieurii (Mont.) J. Ag. Among Bostrychia, just above water level, Port Antonio, July, 1900, P. & B.

Asparagopsis Delilei (Ag.) Lamour. In tree-like tufts, Navy Island, March 10, 1893, H.

Laurencia cervicornis Harv. Annotto Bay, Aug., 1894; washed ashore, Kingston, July, 1900, P. & B.

L. implicata J. Ag. Morant Bay, July, 1900, P. & B.

L. obtusa (Huds.) Lamour. In tufts on rocks, Kingston Harbor, Apr. 8, 1893, No. 376; no locality, No. 224, H. Port Antonio, July, 1891; on rocks, Montego Bay, June, 1900, P. & B. Near Kingston, Duerden. Chitty.

L. papillosa (Forsk.) Grev. In tufts on rocks, Kingston Harbor, Apr. 8, 1893, H. Port Antonio, Kingston, Montego Bay, Manchioneal, Port

Maria, P. & B. Near Kingston, Duerden. Closely covering ledges in rather shallow water, also washed ashore. Chitty.

L. perforata Mont. Densely carpeting rocks in shallow water, Montego Bay, July, 1900, P. & B. P. B.-A., No. 794.

L. tuberculosa var. gemmifera (Harv.) J. Ag. Washed ashore, Morant Bay, Annotto Bay, 1894; Ora Cabessa, July, 1900, P. & B.

Chondria Baileyana Harv. Hope Bay, July, 1900, P. & B. No. 336, no locality, H.

C. dasyphylla (Woodw.) Ag. Washed ashore, Port Antonio, July, 1891; Montego Bay, June, 1900, P. & B.

C. tenuissima (Good. & Woodw.) Ag. Washed ashore, on sandy beach, Montego Bay, June, 1900, P. & B.

Acanthophora Thierii Lamour. Common on rocks in Kingston Harbor, Port Maria, Nos. 176, 195, 278, 377, H. Port Antonio, July, 1891, P. & B. Near Kingston, Duerden.

Digenea simplex (Wulf.) Ag. In tufts on rocks, Port Maria, March 16, 1893, No. 252; on stones in shallows, St. Ann's Bay, March 30, 1893, No. 334, H. Washed ashore, Orange Bay, 1894; Manchioneal, July, 1900. P. & B. Near Kingston, Duerden.

Polysiphonia cuspidata J. Ag. In tufts on piles at beach, Port Maria, March 16, 1893, No. 251; on stones in shallow water, St. Ann's Bay, March 30, 1893, No. 335, H. Port Antonio, Aug., 1894, covering rocks in shallow water; Manchioneal, Port Morant, 1900, P. & B.

P. ferulacea Suhr. In dense tufts on rocks and eel-grass, Rio Novo, June, 1900, P. & B. Near Kingston, Duerden, a slender, long-jointed form.

P. Havanensis Mont. On mangrove roots, Port Antonio, March 8, 1893, No. 214; on other algae, Kingston Harbor, Apr. 8, 1893, Nos. 374b, 375, H. Washed ashore, Montego Bay, Port Antonio, 1900, P. & B. Near Kingston, Duerden.

P. Havanensis var. Binneyi (Harv.) J. Ag. Port Antonio, July, 1891, P. & B.

P. Pecten-Veneris Harv. On other Florideae, Port Maria, March 17, 1893, No. 276, H.

P. secunda (Ag.) Zan. On other algae, Kingston Harbor, Apr. 8, 1893, No. 374, H. Washed ashore, Borden, Morant Bay, 1894, P. & B.

P. subulata (Ducl.) J. Ag. Washed ashore, Montego Bay, June, 1900, P. & B.

Only two specimens collected of this species, which has not before been reported from America. These agree well with specimens from the Mediterranean. The range of this species, as previously known, has been from the English Channel to Spain, the northern shore of the Mediterranean and the Adriatic.

Lophosiphonia obscura (Ag.) Falk. Covering stones in shallow water, Manchioneal, July, 1900, P. & B.

Bryothamnion triangulare (Gmel.) Kuetz. In great tufts in pools, Port Maria, March 16, 1893, Nos. 254 and 277, H. Washed ashore, Annotto Bay, Aug., 1894; Ora Cabessa, June, 1900, P. & B. Chitty. P. B.-A., No. 95.

B. Seaforthii (Turn.) Kuetz. Washed ashore, Port Antonio, July, 1891; Kingston, July, 1900, P. & B.

Bostrychia tenella (Vahl) J. Ag. Port Antonio, on rocks reached only by spray, July, 1891, and 1894; Manchioneal, similar locality, July, 1900, P. & B. P. B.-A., No. 796.

B. Mazei Crouan. In dense tufts on rock, Port Antonio, Feb. 23, 1893, No. 158, H.

B. Moritziana var. intermedia J. Ag. On rocks, shore of island, Port Antonio, Aug., 1894, P. & B.

"The Bostrychias grew upon rocks and ledges, usually above water, but dashed by spray."

Murrayella periclados (Ag.) Schmitz. On mangrove roots, Port Antonio, March 8, 1893, No. 215; in dense tufts on wood, St. Ann's Bay, March 24, 1893, H. Manchioneal, July, 1900, P. & B. P. B.-A., No. 795.

Amansia multifida Lamour. Washed ashore, Morant Bay, Annotto Bay, July, 1894; Rio Bono, Rio Novo, Kingston, 1900, P. & B. P. B.-A., No. 94. P. U., No. 708.

Dasya arbuscula (Dillw.) Ag. Washed ashore, Montego Bay, July, 1900, P. & B.

D. Gibbesii Harv. Washed ashore, Port Antonio, Aug., 1894, P. & B.

D. mucronata Harv. Washed ashore, Morant Bay, July, 1894, P. & B.

Heterosiphonia Wurdemanni (Bailey) Falk. On Gelidium rigidum, No. 276, H. Annotto Bay, Aug., 1894, P. & B.

Dictyurus occidentalis J. Ag. Annotto Bay, Aug., 1894; Kingston, near the lighthouse, July, 1900, P. & B. Always washed ashore, never in large quantity, usually only a fragment here and there. P. B.-A., No. 797.

Halodictyon mirabile Zan. Washed ashore, St. Ann's Bay, March 30, 1893, H.

Spermothamnion Gorgoneum (Mont.) Bornet. On Codium tomentosum. Port Antonio, Aug., 1894; Kingston, July, 1900, P. & B. Port Antonio, Feb. 27, 1893, No. 175 a, H. P. B.-A., No. 441.

"Both cystocarps and polyspores have been found in Jamaica specimens; in the former the spores have thick cell walls and are arranged as in Spermothamnion; the involucre is only slightly developed. The polyspores are quite numerous, in an ovate or subspherical mass, occupying not more than half the diameter of the large, hyaline sporangium." Note from label of P. B.-A., No. 441.

S. Turneri var. variabile J. Ag. On Bryothamnion Seaforthii, Kingston, July, 1900, P. & B.

Callithamnion byssoideum var. Jamaicensis Collins. In dense tufts on rocks, Port Antonio, Feb. 27, No. 170, H. P. B.-A., No. 443.

"This plant has the divided cystocarps, with conical lobes, characteristic of C. byssoideum: antheridia and tetraspores also agree; but the habit is strikingly different, everything being condensed, the branches relatively shorter and stouter, and very densely set, the terminal ramuli often arranged more like C. corymbosum. It may possibly be the same as C. Hypneae Crouan in Mazé & Schramm, Algues de Guadeloupe; the name must be considered as provisional, awaiting comparison with authentic specimens of the latter." Note from the label of P. B.-A., No. 443.

C. corymbosum (Eng. Bot.) Lyng. On Codium tomentosum, Port Antonio, Aug., 1894, P. & B.

Haloplegma Duperryi Mont. Washed ashore, Morant Bay, Annotto Bay, Orange Bay, 1894; Kingston, July, 1900, P. & B. Only a few fragments at each place.

Crouania attenuata (Bonnem.) J. Ag. On Cryptonemia crenulata, Morant Bay, July, 1894, P. & B. In small tufts, Navy Island, March 10, 1893, H.

Antithamnion Butleriae n. sp. Fronds erect, ecorticate, simple or with a few branches, which may be dichotomous, alternate, or occasionally opposite, diameter near base about 30μ, cells 3–6 diameters, walls thick. The lower portion of the frond or branch is naked; above that each cell bears normally a pair of ramuli, issuing at about two-thirds the height of the cell; the lowest ramuli are simple, subulate, of from two to six cells about as long as broad; sometimes by the suppression of a ramulus the branching is apparently alternate; farther up the frond these ramuli are compounded with similar smaller subulate ramelli, appearing first on the lower side of the ramulus. The upper pinnae have from each cell of the rachis a pair of ramelli which touch each other laterally, so that the pinna forms a continuous triangle. At the tips of the branches the cells are much shorter than those below, and the triangular compound pinnae are in contact, giving a linear outline to the whole. Color a rich rose. On Bryothamnion Seaforthii, Kingston, July, 1900, P. & B.

From A. pteroton (Schousb.) Bornet it differs in the more densely branched pinnae, with ramelli on both sides, or on the lower only. From Ptilothamnion micropterum (Mont.) Bornet it differs by the absence of the apparent bifurcation of the pinna. Callithamnion microptilum Grunow has much shorter articulations in the main branches, and less dense pinnules, which also are alternately more and less developed, as in some species of Ptilota. In the absence of fruit it is impossible to determine that the plant in question may not, when fruit is found, have to rather bear the name of Ptilothamnion Butleriae.

Spyridia aculeata Kuetz. Washed ashore, St. Ann's Bay, March 30, 1893, No. 337; in tufts, Port Antonio, March 10, 1893, No. 228, H.

S. filamentosa (Wulf.) Harv. In dense tufts, Port Antonio, March 10, 1893, No. 222, H. Port Morant, Kingston, Montego Bay, Manchioneal, P. & B. Probably common everywhere. Chitty.

Ceramium byssoideum Harv. Washed ashore, Port Antonio, July, 1891, P. & B.

C. clavulatum Ag. Port Maria, Nos. 275 and 301; Port Antonio, No. 183, H. Morant Bay, Manchioneal, Kingston, Montego Bay, P. & B. Common everywhere and very variable.

C. fastigiatum Harv. Washed ashore, Port Antonio, July, 1891; Ora Cabessa, Rio Bono, Rio Novo, June, 1900.

C. gracillimum Harv. On rocks, Apostles Battery, Kingston Harbor, Apr. 10, 1893. H.

C. nitens (Ag.) J. Ag. Washed ashore, Port Antonio, July, 1891; Manchioneal, Montego Bay, 1900, P. & B.

C. tenuissimum (Lyng.) J. Ag. On eel-grass, St. Ann's Bay, March 24, 1893, No. 318, H. Port Antonio, July, 1891; Manchioneal, Montego Bay, 1900, P. & B. P. B.-A., No. 798. The Montego Bay specimens are small, connecting the type with the following variety.

C. tenuissimum var. pygmaeum (Kuetz.) Hauck. On Laurencia obtusa, near Kingston, Duerden. P. B.-A., No. 896. A very small form, hardly visible to the naked eye, but in full tetrasporic fruit.

Halymenia Floresia (Clem.) Ag. Washed ashore, Montego Bay, June, 1900, P. & B.

Grateloupia filicina (Wulf.) Ag. Morant Bay, on rocks washed by the waves, but not really under water, July, 1894; Rio Bono, Rio Novo, July, 1900, P. & B. In tufts on wood, St. Ann's Bay, March 24, No. 419; Kingston Harbor, Apr. 8, 1893, No. 381, H.

"The Grateloupia gathered in 1900 was lying in coarse, black, dry, rigid tangle on the beach, totally unlike the Grateloupia found in 1894 at Morant Bay, growing on a big boulder on shore washed by heavy surf. At the latter locality, when the water was over the plants they floated out like fine, greenish-brown hair; as the water receded the plants fell back on to the rock, covering it like a soft jelly. From the habit of the two forms, one would never suspect that they were the same species."

G. dichotoma J. Ag. Near Kingston, Duerden. Fronds broader than usual in this species as found in the Mediterranean or at the Canaries, but otherwise the same.

G. prolongata J. Ag. Near Kingston, Duerden. Agreeing well with Agardh's description, and with the form from California which passes under this name.

Cryptonemia crenulata J. Ag. Morant Bay, Annotto Bay, and coast towards Port Antonio, washed ashore and growing on "sea-fans," July and Aug., 1894; Kingston, July, 1900, P. & B.

Cruoriella Armorica Crouan. On stones and shells, Annotto Bay, July, 1891, P. & B.

Peysonnellia Dubyi Crouan. On corals, Port Maria, March 17, No. 283; Port Antonio, Feb. 23, 1893, No. 161, H.

P. rubra (Grev.) J. Ag. On rocks, Port Maria, March 19, 1893, No. 291, H.

Hildenbrantia Prototypus Nardo. On coral rock, Port Antonio, Feb. 23, 1893, No. 161; Port Maria, March 20, 1893, No. 300, H.

Melobesia farinosa Lamour. On Dictyota, etc., Port Antonio, July, 1891, P. & B. On various algae, near Kingston, Duerden.

M. Lejolisii Rosanoff. On various algae and eel-grass, P. & B.

M. membranacea Lamour. On various algae, P. & B.

M. pustulata Lamour. On Gracilaria Domingensis, P. & B.

Lithothamnion incrustans Phil. On rocks, Port Maria, March 16, 1893, No. 258, H. Montego Bay, July, 1900, P. & B.

L. Lenormandi (Aresch.) Foslie. On shells, Port Antonio, P. & B.

Amphiroa charoides, Lamour. Port Antonio, July, 1891, P. & B. In tufts on bottom, Port Antonio, March 2, 1893, H.

A. debilis Kuetz. Port Antonio, July, 1891, P. & B. In tufts on rocks, Port Antonio, Feb. 27, No. 177 ; Kingston Harbor, Apr. 8, 1893, No. 382, H. Near Kingston, Duerden.

A. fragilissima Lamour. Growing like a moss on coral reef and sand near shore, in shallow water, Port Antonio, July, 1891, P. & B.

Murray gives this species on authority of a specimen by Sloane, but as he also refers to Farlow, Anderson & Eaton, No. 15, it is probable that Sloane's specimen is rather A. debilis. The plant distributed under No. 15 was originally labelled A. fragilissima, but a revised label was afterwards issued, as A. debilis.

Corallina capillacea Harv. Annotto Bay, Aug., 1894, P. & B. In dense tufts, Kingston Harbor, Apr. 8, No. 383 ; Port Maria, March 17, 1893, H. P. B.-A., No. 150.

C. Cubensis Mont. Annotto Bay, Aug., 1894, P. & B. In dense tufts, Port Maria, March 16, 1893, No. 256, H.

C. pumila (Lamour.) Kuetz. On Turbinaria trialata, Port Antonio, July, 1891 ; on Stypopodium lobatum, Montego Bay, June, 1900, P. & B. P. B.-A., No. 799.

C. rubens L. In dense tufts, Port Maria, March 16, 1893, No. 257, H. On rocks, Port Morant, July, 1900, P. & B. P. B.-A., No. 800. Sloane. Chitty.

C. subulata Ell. & Sol. Kingston, Feb., 1896, O. Hansen. Sloane.

TABLE I.

Comparison of Marine Floras of Jamaica and Other Regions.

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
CLASS SCHIZOPHYCEAE.						
Family **Chroococcaceae.**						
Chroococcus turgidus						+
Chroothece Richteriana						
Family **Chamaesiphonaceae.**						
Xenococcus Schousboei			+	+	+	+
Family **Hormogoneae.**						
Oscillatoria Corallinae				+	+	+
Lyngbya aestuarii	+		+		+	+
confervoides f. violacea						
majuscula	+	+	+	+	+	+
Symploca hydnoides		+	+		+	+
" var. fasciculata						
Microcoleus chthonoplastes					+	+
tenerrimus						
Hormothamnion enteromorphoides		+				
Scytonema conchophilum						
Mastigocoleus testarum				+	+	+
Calothrix aeruginea					+	+
confervicola		+	+	+	+	+
Contarenii				+		
pilosa						
Dichothrix penicillata						
CLASS CHLOROPHYCEAE.						
Family **Ulvaceae.**						
Ulva fasciata						
Lactuca var. rigida		+	+	+	+	+
Enteromorpha erecta			+		+	+
flexuosa			+			
intestinalis		+	+	+	+	+
prolifera	+		+		+	+
Family **Chaetophoraceae.**						
Diplochaete solitaria						
Family **Mycoideaceae.**						
Pringsheimia scutata					+	+

TABLE I. — *continued.*

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Family **Cladophoraceae.**						
Chaetomorpha brachygona	+					
clavata						
aerea		+	+	+	+	+
Linum	+	+	+		+	+
" var. brachyarthra						
Melagonium f. typica					+	+
Cladophora crystallina		+			+	
fascicularis	+					
fuliginosa						
Hutchinsiae			+	+	+	+
intertexta						
trichocoma					+	
Family **Gomontiaceae.**						
Gomontia polyrhiza				+	+	+
Family **Bryopsidaceae.**						
Bryopsis Harveyana						
pennata						
Family **Caulerpaceae.**						
Caulerpa cupressoides var. typica						
" var. Turneri						
" var. mamillosa						
" var. ericifolia						
pinnata f. Mexicana	+					
plumaris f. longiseta	+					
" f. brevipes						
prolifera		+	+			
racemosa var. clavifera		+				
" " " f. macrophysa						
taxifolia	+					
verticillata						
" f. charoides						
Family **Codiaceae.**						
Penicillus capitatus	+					
dumetosus	+					
Rhipocephalus Phoenix						
Avrainvillea longicaulis						
nigricans						
Udotea conglutinata						
flabellata	+					
Halimeda Opuntia	+					
tridens	+					
Tuna	+	+	+			
Codium adhaerens	+	+	+	+	+	
tomentosum	+	+	+	+	+	

TABLE I. — *continued.*

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Family **Valoniaceae.**						
Valonia aegagropila		+				
ventricosa						
verticillata						
Siphonocladus membranaceus		+				
tropicus		+				
Dictyosphaeria favulosa						
Chamaedoris annulata						
Microdictyon umbilicatum		+				
Anadyomene stellata		+				
Family **Dasycladaceae.**						
Acetabularia crenulata	+					
Dasycladus clavaeformis		+	+			
Botryophora occidentalis						
Neomeris dumetosa						
Cymopolia barbata	+	+				
CLASS PHAEOPHYCEAE.						
Family **Ectocarpaceae.**						
Ectocarpus Mitchellae		+		+	+	+
Family **Striariaceae.**						
Striaria attenuata					+	+
" var. ramosissima						
Family **Encoeliaceae.**						
Colpomenia sinuosa	+	+	+			
Hydroclathrus cancellatus						
Family **Fucaceae.**						
Turbinaria trialata	+					
Sargassum bacciferum		+				+
lendigerum						
platycarpum	+					
vulgare	+	+				
" var. foliosissimum	+					
" f. ovata						
CLASS DICTYOTALES.						
Family **Dictyotaceae.**						
Spatoglossum Schroederi						
Stypopodium lobatum	+	+				
Gymnosorus variegatus		+				
Padina Durvillaei	+					
Dictyopteris delicatula	+					

TABLE I. — *continued.*

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Family **Dictyotaceae.** — *continued.*						
Dictyopteris plagiogramma	+					
Justii	+					
Dictyota Bartayresiana						
cervicornis						
ciliata						
dentata	+					
dichotoma		+	+	+	+	
divaricata						
fasciola		+				
Dilophus alternans						
Guineensis						
Dictyerpa Jamaicensis						
CLASS RHODOPHYCEAE.						
Family **Bangiaceae.**						
Goniotrichum Humphreyi						
elegans			+	+	+	+
Family **Helminthocladiaceae.**						
Chantransia Saviana		+				
Liagora Cheyneana						
decussata		+				
elongata		+				
pulverulenta		+				
valida		+				
Family **Chaetangiaceae.**						
Galaxaura cylindrica	+	+				
lapidescens		+				
marginata						
obtusata	+					
rugosa	+	+				
Family **Gelidiaceae.**						
Wrangelia Argus		+				
Gelidium coerulescens						
crinale		+	+	+	+	+
rigidum						
supradecompositum	+					
Catenella Opuntia var. pinnata						
Family **Rhodophyllidaceae.**						
Agardhiella tenera	+					+
Solieria chordalis			+	+		
Eucheuma echinocarpum						

TABLE I. — *continued.*

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Family **Sphaerococcaceae.**						
Gracilaria Blodgettii						
caudata						
cervicornis	+					
compressa			+		+	
confervoides	+	+	+	+	+	
cornea						
Curtissiae						
damaecornis						
divaricata						
Domingensis						
ferox						
multipartita	+		+	+	+	+
Wrightii	+					
Hypnea divaricata						
musciformis	+	+	+	+		+
Valentiae						
Family **Rhodymeniaceae.**						
Champia parvula	+	+	+	+	+	+
Cordylecladia irregularis						
Peasiae						
Chrysymenia halymenioides						
Family **Delesseriaceae.**						
Caloglossa Leprieurii	+					+
Family **Bonnemaisoniaceae.**						
Asparagopsis Delilei		+				
Family **Rhodomelaceae.**						
Laurencia cervicornis						
implicata						
obtusa		+	+	+	+	
perforata		+				
papillosa	+	+				
tuberculosa var gemmifera						
Chondria Baileyana						+
dasyphylla	+	+	+	+	+	+
tenuissima		+	+	+	+	+
Acanthophora Thierii	+					
Digenia simplex						
Polysiphonia cuspidata	+					
ferulacea		+				
Havanensis	+					
" var. Binneyi						
Pecten-Veneris						

TABLE I. — *continued.*

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Family **Rhodomelaceae**. — *continued.*						
Polysiphonia secunda		+	+			
subulata			+		+	
Lophosiphonia obscura			+	+	+	
Bryothamnion Seaforthii	+					
triangulare	+					
Bostrychia tenella						
Mazei						
Moritziana var. intermedia						
Murrayella periclados						
Amansia multifida	+					
Dasya arbuscula		+	+	+	+	
Gibbesii						
mucronata						
Heterosiphonia Wurdemanni		+	+			
Dictyurus occidentalis						
Halodictyon mirabile			+			
Family **Ceramiaceae**.						
Spermothamnion Gorgoneum		+				
Turneri var. variabile		+	+	+	+	+
Callithamnion byssoideum var. Jamaicensis						
corymbosum		+		+	+	+
Haloplegma Duperryi						
Crouania attenuata			+		+	
Antithamnion Butleriae						
Spyridia aculeata			+			
filamentosa	+	+	+	+	+	+
Ceramium byssoideum						
clavulatum	+	+	+			
fastigiatum					+	+
gracillimum			+	+	+	
nitens						
tennissimum	+			+	+	+
" var. pygmaeum	+					
Family **Grateloupiaceae**.						
Halymenia Floresia		+	+			
Grateloupia filicina	+		+	+	+	
dichotoma		+	+	+	+	
prolongata						
Cryptonemia crenulata	+					
Family **Squamariaceae**.						
Cruoriella Armorica						
Peysonnellia Dubyi				+	+	
rubra	+	+	+			

TABLE I. — *continued.*

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Family **Corallinaceae.**						
Hildenbrantia Prototypus			+	+	+	+
Melobesia farinosa	+	+		+	+	+
Lejolisii					+	+
membranacea	+	+	+	+	+	+
pustulata		+	+	+	+	+
Lithothamnion incrustans		+		+		
Lenormandi				+	+	+
Corallina capillacea						
Cubensis						
pumila	+					
rubens	+	+	+	+	+	
subulata	+					
Amphiroa charoides						
debilis						
fragilissima	+					

TABLE II.

SUMMARY OF MARINE FLORAS, ARRANGED BY CLASSES.

	Jamaica.	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Schizophyceae	19	2	7	24	34	66	75
Chlorophyceae	62	25	62	59	33	130	88
Phaeophyceae Dictyotales	29	16	61	75	80	193	118
Rhodophyceae	114	49	156	237	173	346	153
Total	224	92	286	395	320	735	434

TABLE III.

Percentage by Classes in each Flora.

	Jamaica.	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Schizophyceae	8	2	3	6	11	9	17
Chlorophyceae	28	27	21	15	10	18	20
Phaeophyceae Dictyotales	13	17	21	19	25	26	26
Rhodophyceae	51	54	55	60	54	47	37

TABLE IV.

Common to Jamaica in other Floras.

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Schizophyceae	2	4	5	6	9	10
Chlorophyceae	17	17	13	7	14	10
Phaeophyceae Dictyotales	11	8	2	2	3	3
Rhodophyceae	33	36	31	27	29	21
Total	63	65	51	42	55	44

TABLE V.

Percentage of Jamaica Flora common to other Floras.

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Schizophyceae	11	22	26	32	47	53
Chlorophyceae	28	29	22	12	23	16
Phaeophyceae Dictyotales	38	27	7	7	10	10
Rhodophyceae	29	31	27	23	25	18
Total	28	30	23	19	25	19

TABLE VI.

Percentage of other Floras common to Jamaica.

	Puerto Rico.	Canaries.	Morocco.	Biscay.	Great Britain.	New England.
Schizophyceae	100	57	21	18	14	13
Chlorophyceae	68	29	23	21	11	11
Phaeophyceae Dictyotales	69	13	3	3	2	3
Rhodophyceae	72	22	13	15	9	14
Total	69	24	14	11	8	10

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
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MODIFICATIONS OF HEMPEL'S GAS-APPARATUS.

By Theodore William Richards.

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Received October 26, 1901. Presented November 13, 1901.

The object of this paper is the description of some simple devices which make possible the accurate analysis of gases with a minimum of special apparatus.

I. Absorbing Pipette.

The essential feature of Hempel's method is the use of simply constructed vessels distinct from the measuring burette for the purpose of absorbing successively the various constituents of a gaseous mixture. Hempel used for this end a modification of Ettling's gas pipette, which answers the purpose admirably; but of course many other combinations of apparatus might be used. The simplest is perhaps a bulb or wide tube inserted over liquid contained in a bottle. In order to prevent the access of air into this bulb from below, it is well to make the lower part of the tube somewhat narrow, and to bend it upward. If desired, the capillary serving to admit the gas may be bent downwards and then upwards, as it is in the Hempel pipette; but with intelligent use of the pinchcock this precaution is not necessary. A satisfactory form of the apparatus is illustrated in Figure 1.

Fifty cubic centimeters is quite enough gas for analysis, if a suitably narrow burette is used for measurement, hence the receiving bulb of the pipette (A) need not exceed seventy-five cubic centimeters in capacity. The bottle (C) should be capable of holding one hundred and fifty cubic centimetres in this case.

The "compound pipette" of Hempel may be imitated by the addition at B of another bottle containing water and a levelling funnel, or the same object may be attained merely by connecting to the outlet B a flexible rubber bulb, such as a child's toy balloon.

For solids, the stem D of the pipette may be made of wider tubing, closed at the bottom with a perforated stopper. A small tube bent upwards may be inserted in this perforation, if especial precaution against incoming air is desired.

An explosion-pipette could be made of similar apparatus, with the addition of a stopcock just below the bulb A and the usual conducting wires.

The pipette for fuming acid might be made with a ground-glass joint instead of a stopper to connect bulb with bottle. In that case the bottle should be provided with a suitable side tube on the neck, bent upwards.

The method of using these pipettes will be understood without difficulty by any one familiar with the Hempel apparatus.

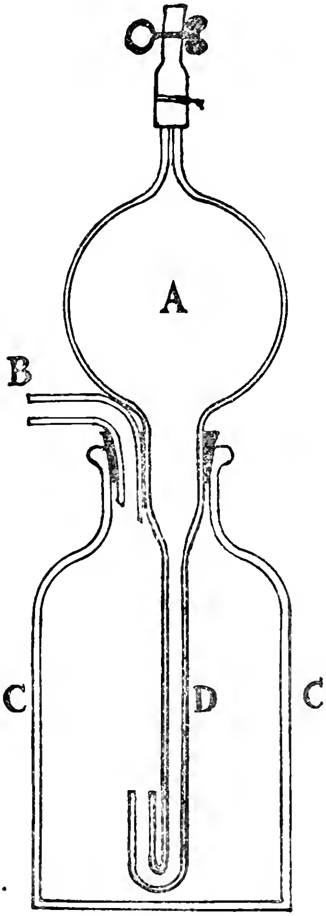

Figure 1.

II. Measuring Apparatus.

The most serious cause of error in Hempel's ordinary apparatus is due to the possible change of temperature. This is considerably greater than the probable error in reading; for a single degree Celsius causes an error of 0.5 per cent of the total volume of gas measured under ordinary conditions, while the volume is easily read within 0.05 per cent. Hence, unless much greater care than usual is taken to preserve constant temperature, the reading of the volume is unnecessarily precise. But Hempel's ingenious arrangements for maintaining constant conditions in a 100 c.c. burette are so large as to be inconvenient for students' use in cramped quarters.

For these reasons I have often used somewhat smaller volumes, which may be surrounded with an envelope of water without producing thereby an unwieldy combination. An ordinary 50 c. c. burette, inverted and provided with a levelling bulb or funnel, answers very well as a measuring instrument. The burette may even be used in its usual position, if it is provided above with a smooth rubber stopper with a single hole for the capillary connecting-tube. Of course the stopper is always pushed precisely into a definite position, indicated by a carefully made mark on the burette. There is little risk of displacing this stopper if it is firmly wired into place. In any case of course the ungraduated space at the upper extremity must be carefully calibrated. An especially made 50 c.c. instrument, graduated all the way to the capillary tube at the top, is more convenient, although no more accurate than the inverted burette. For convenience in cleaning, it is well not to have both ends of the burette drawn down to small diameter. The small size of the burette makes it easily possible to provide the water jacket which is so essential for accurate work, and both burette and pipette may be supported upon the ordinary iron ring stand.

III. Practical Operation.

Of course the precautions usually necessary in gas analysis must be used in all the operations with this apparatus. For example, due time must be allowed for the running down of the liquid from the moistened walls. Again, care must be taken that the same amount of gas, at definite pressure (as small an amount as possible) is always left in the connecting capillary tubes. In order to make sure that no air-bubbles are caught, it is well to draw out the ends of the tubes in the manner illustrated in the diagram, which indicates two successive stages of the glass blowing, as well as the finished and connected nipple. The object of blowing the small bulbs is to render the bore of the portions drawn out as large as that of the rest of the tube.

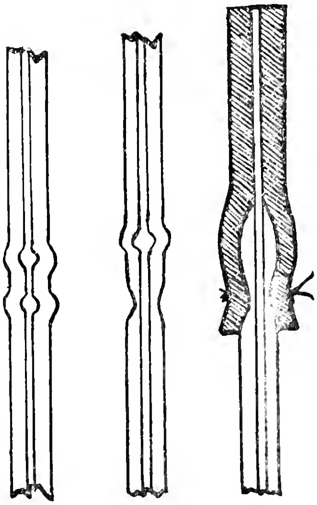

Figure 2.

While the apparatus thus constituted was devised primarily for use in an emergency, it has several advantages over the Hempel apparatus. It dispenses with the necessity of calibrating the whole length of a new burette, it is very inexpensive, and it occupies but little space. Each student may possess a complete set of apparatus, and every one knows the value from a pedagogic standpoint of such a possibility. A further advantage lies in the fact that the pipette is easy to fill and to clean; and a precipitate in the liquid is not apt to clog its working. The short straight capillary brings an obvious gain of speed in transferring. Moreover, because of this speed, and the fact that the pressure during transferrence is always from the outside inward, the danger of loss by leakage is considerably less than it is with Hempel's apparatus. It is well known that in a rubber tube an internal pressure may cause leakage, while an external pressure tends to stop small outlets by causing the rubber tube to be pressed more closely together.

On the other hand, the calculation is less obvious, because the volume taken is not just a hundred cubic centimeters; and somewhat more care must be used to prevent the access of air into the pipette from below while shaking. A little practice enables one to shake thoroughly the liquid in the bulb without much agitation in the bottle if the movement is hinged about the point D; hence the danger is slight. Another slight difficulty is the possible leakage of the absorbent around the stopper of the pipette bottle, — an unpleasant occurrence which has no effect upon the accuracy of the method.

In presenting for general use any new instrument one must record its practical working in the laboratory. Everybody knows that plausible

ANALYSIS OF KNOWN MIXTURES OF AIR AND CARBON DIOXIDE.

Volume CO_2 taken.	Volume Air taken.	Volume Air found.	Error.
c. c.	c. c.	c. c.	c. c.
16.95	32.02	32.01	−0.01
18.45	32.21	32.12	−0.09
12.20	42.20	42.20	±0.00
20.00	32.00	32.10	+0.10
14.90	37.60	37.60	±0.00
13.00	34.50	34.48	−0.02
16.50	36.50	36.55	+0.05

Excess of positive over negative errors, 0.03.

inventions do not always stand the test of indiscriminate use. Accordingly a large class in gas analysis has been asked to use the new devices, with favorable outcome.

The pipette and burette were tested as follows. A definite amount of air was run into the burette, and the volume measured with the usual care. Pure carbon dioxide was then run in from a generator, and the gain in volume was noted. This known mixture of air and carbon dioxide was run over into the new pipette, and after suitable shaking the residual air was returned to the burette and measured.

These figures, taken at random from among the results of the class, agree with one another as well as could be expected; and since the positive deviation balances the negative, there is no constant error. No trouble was experienced as to manipulation.

I am much indebted to Mr. Bisbee, the assistant, and to the gentlemen of the class in gas analysis, for their kindness in carrving out the practical trial of the apparatus.

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THE PARAMETRIC REPRESENTATION OF THE NEIGHBORHOOD OF A SINGULAR POINT OF AN ANALYTIC SURFACE.

BY C. W. M. BLACK.

THE PARAMETRIC REPRESENTATION OF THE NEIGHBORHOOD OF A SINGULAR POINT OF AN ANALYTIC SURFACE.

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INTRODUCTION.

A. — Outline of Kobb's Treatment of the Problem.

The problem of the representation, by a finite number of parametric formulae in two variables, of the neighborhood of a singular point of an algebraic surface is considered and alleged to be solved in an article "Sur la théorie des fonctions algébriques de deux variables,"* by Gustav Kobb. A brief outline of Kobb's method follows: —

1. *Treatment of the Original Singular Point.* 1) Let the equation of the surface be written in the form

$$F(x, y, z) = 0,$$

where F is a function of the three independent variables x, y, z analytic in the point $x = a$, $y = b$, $x = c$. The function F is transformed by means of a change of axes to the form

$$\Phi(\xi, \eta, \zeta) \equiv (\xi, \eta, \zeta)_m + (\xi, \eta, \zeta)_{m+1} + \ldots\ldots = 0 \qquad \text{(a)}$$

where the expression $(\xi, \eta, \zeta)_n$ is a homogeneous polynomial of degree n, the resulting surface (a) having the singular point considered at the origin, while the function $(\xi, \eta, \zeta)_m$ is of a form convenient for later treatment.

2) By the quadratic transformation

$$\xi = \tau\zeta, \qquad \eta = \sigma\zeta,$$

$$\left.\begin{aligned} \Phi(\xi, \eta, \zeta) &= \zeta^m\left[(\tau, \sigma, 1)_m + \zeta(\tau, \sigma, 1)_{m+1} + \ldots\ldots\right] \\ &= \zeta^m\left[\phi(\tau, \sigma) + \zeta\chi(\tau, \sigma) + \ldots\ldots\ldots\right] \\ &= \zeta^m \bar{\Phi}(\tau, \sigma, \zeta) \end{aligned}\right\} \qquad \text{(b)}$$

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* Journal de mathématiques pures et appliquées, 4th Series, Vol. VIII. (1892), p. 385.

and the neighborhood of the original point is represented by the neighborhood of the curve

$$\phi(\tau, \sigma) = 0, \qquad \zeta = 0, \tag{c}$$

on the surface

$$\bar{\Phi}(\tau, \sigma, \zeta) = 0.$$

3) The neighborhood of the curve (c) is included in the domains of a finite number of points which are

a. regular points of the curve (c), the domain of each being represented by a single power series

$$\tau = p(\sigma, \zeta); \tag{d}$$

b. critical points of the curve (c), the domain of each being represented by an equation of the form

$$\tau^{\mu} + p_1(\sigma, \zeta)\,\tau^{\mu-1} + \ldots\ldots + \bar{p}_{\mu-1}(\sigma, \zeta)\,\tau + \bar{p}_{\mu}(\sigma, \zeta) = 0; \tag{e}$$

c. points at an infinite distance on the curve (c), the domain of each being represented by an equation of form (d) or (e) in the variables τ_1, σ_1, η, where

$$\frac{\tau}{\sigma} = \tau_1, \qquad \frac{1}{\sigma} = \sigma_1, \qquad \zeta\sigma = \eta.$$

4) The selection of the points in 3) depends upon the character of the curve

$$\phi(\tau, \sigma) = 0.$$

a. If ϕ is irreducible, all points of class 3) *b* are first taken, then all points of class 3) *c*, these being regular; finally a finite number of points of class 3) *a*. Here, all the points selected, if singular, are of order less than m.

b. If ϕ is reducible, but contains no multiple factors, the same selection of points holds as in *a*, but there may occur a singular point of order m.

c. If ϕ contains multiple factors, all critical points of the curves corresponding to any factor, together with all points of intersection of two different factors, are first taken, then all points of class 3) *c*, these being possibly singular; finally, a finite number of regular points of the several curves corresponding to the different factors of ϕ, these last points being possibly singular points of the surface. In this case, there may occur a number of singular points of order m.

2. *Treatment of Points Determined in* 1. The same treatment as in 1 is applied to each of these points and to each of the corresponding

resulting points in turn, so long as they are singular. If after a finite number of such processes, all the resulting points are regular, then by combining the results it is assumed that the neighborhood of the original point is represented by the domains of a finite number of regular points, and so by a finite number of parametric formulae as desired.

3. *Proof that a Finite Number of the Processes of* 1 *will be Sufficient to make all Points in* 2 *Regular.* Starting with the surface

$$f(u, v, w) = 0\,, \tag{f}$$

in which the singular point considered is at the origin, the transformations in 1, 1) and 2) are combined in the form

$$\left.\begin{aligned} u &= (\alpha_1 \tau + \beta_1 \sigma + \gamma_1)\zeta \\ v &= (\alpha_2 \tau + \beta_2 \sigma + \gamma_2)\zeta \\ w &= (\alpha_3 \tau + \beta_3 \sigma + \gamma_3)\zeta \end{aligned}\right\} \tag{g}$$

We can assume that

$$\gamma_2 \neq 0\,, \qquad \gamma_3 \neq 0$$

by making, if necessary, upon $f(u, v, w)$ a suitable homogeneous linear transformation. Then the next set of transformations, in 2, can be expressed in the form

$$\left.\begin{aligned} \tau &= (\alpha_1' \tau_1 + \beta_1' \sigma_1 + \gamma_1')\zeta_1 \\ \sigma &= (\alpha_2' \tau_1 + \beta_2' \sigma_1 + \gamma_2')\zeta_1 \\ \zeta &= (\alpha_3' \tau_1 + \beta_3' \sigma_1 + \gamma_3')\zeta_1 \end{aligned}\right\} \tag{h}$$

in which $\gamma_3' \neq 0$,* and the later sets of transformations are of the same type with the corresponding γ_3's:

$$\gamma_3'' \neq 0\,, \qquad \gamma_3''' \neq 0\,, \ldots\ldots \text{ etc.}$$

So we consider a succession of transformations of type (g), which give a succession of surfaces with multiple points each of order m. These transformations will combine in the form

$$\left.\begin{aligned} u &= [\gamma_1 \gamma_3' \gamma_3'' \ldots\ldots \gamma_3^{(r)} + (\tau_r, \sigma_r, \zeta_r)]\,\zeta_r = [\Gamma_1 + (\tau_r, \sigma_r, \zeta_r)]\,\zeta_r \\ v &= [\gamma_2 \gamma_3' \gamma_3'' \ldots\ldots \gamma_3^{(r)} + (\tau_r, \sigma_r, \zeta_r)]\,\zeta_r = [\Gamma_2 + (\tau_r, \sigma_r, \zeta_r)]\,\zeta_r \\ w &= [\gamma_3 \gamma_3' \gamma_3'' \ldots\ldots \gamma_3^{(r)} + (\tau_r, \sigma_r, \zeta_r)]\,\zeta_r = [\Gamma_3 + (\tau_r, \sigma_r, \zeta_r)]\,\zeta_r \end{aligned}\right\} \tag{i}$$

where the symbol $(\tau_r, \sigma_r, \zeta_r)$ represents in the expression in which it occurs all of the variable terms, and $\Gamma_2 \neq 0$, $\Gamma_3 \neq 0$.

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* To secure this, Kobb makes unwarranted use of a quadratic transformation, which, however, might be replaced by a homogeneous linear transformation. He also overlooks one class of transformations which will arise (see 4).

Next, as $f(u, v, w)$ can be supposed to be irreducible, we have a relation of the form

$$L(u, v, w) f(u, v, w) + M(u, v, w) \frac{\partial}{\partial u} [f(u, v, w)] = \chi(v, w)$$
$$= (v, w)_\lambda + (v, w)_{\lambda+1} + \dots\dots + (v, w)_n \neq 0. \qquad \text{(j)}$$

Now it is shown that the first member of equation (j) becomes divisible by $\zeta_r^{(m-1)(r+1)}$ after the substitutions (i), and the establishment of an upper limit for the power of ζ_r which can then be taken out as a factor of the function resulting from $\chi(v, w)$, will secure a corresponding limit for r, as is needed to finish the proof.

B. — Critique of Kobb's Analysis.

We now show in what respects Kobb's method and proof are at fault. Some of these errors are noted in a memoir "Sulla riduzione delle singolarità puntuali delle superficie algebriche dello spazio ordinario per trasformazioni quadratiche." by Beppo Levi.*

4. Kobb overlooks in his succession of transformations of type (g) the occurrence of transformations which arise from 1, 3), c. These are equivalent to

$$\left.\begin{aligned} \xi &= \tau_1 \eta \\ \eta &= \eta \\ \zeta &= \sigma_1 \eta \end{aligned}\right\}.$$

and here the number corresponding to γ_3' of (h) is zero; so that the proof, even if correct in other respects, would fail to cover all the cases involved.†

5. Without specific discussion of several unwarranted assumptions of Kobb,‡ we show by an example the failure of his proof for the upper limit of the exponent of the power of ζ_r to be taken out as a factor of $\chi(v, w)$ in (j) under the substitution (i). Let the given surface be

$$f = u^2 - 2uw - v^2 + 2vw + uvw - vw^2 - uw^2 + w^3 = 0. \qquad \text{(k)}$$

Here,

$$\chi(v, w) = (4 + w^2)(w - v)^2.$$

The curve

$$\phi(u, v) = u^2 - 2u - v^2 + 2v = 0$$

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* Annali di matematica, Series 2, Vol. XXVI. (1897), p. 219.

† Cf. Levi, l. c., p. 224.

‡ Cf. Levi, l. c., pp. 225–6.

has a singular point at

$$u = 1, \qquad v = 1.$$

So the first transformation is

$$\left.\begin{aligned} u &= (\tau + 1)\,\zeta \\ v &= (\sigma + 1)\,\zeta \\ w &= \zeta \end{aligned}\right\} \tag{l}$$

which, applied to (k), gives

$$\zeta^2 (\tau^2 - \sigma^2 + \tau\sigma\zeta) = 0 \tag{m}$$

and

$$\chi(v, w) = \zeta^2 \sigma^2 (\zeta^2 + 4). \tag{n}$$

Now the set of transformations to which Kobb is naturally led in this case is the following: — *

$$\left.\begin{array}{lll} \tau = \tau_1 \zeta_1 & \sigma = \sigma_1 \zeta_1 & \zeta = \zeta_1 \\ \tau_1 = \tau_2 \zeta_2 & \sigma_1 = \sigma_2 \zeta_2 & \zeta_1 = \zeta_2 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \end{array}\right\}$$

whence

$$\tau = \tau_r \zeta_r^{\,r}, \qquad \sigma = \sigma_r \zeta_r^{\,r}, \qquad \zeta = \zeta_r.$$

But this substitution in (n) gives

$$\chi(v, w) = \zeta_r^{\,2r+2} \sigma_r^{\,2} (\zeta_r^{\,2} + 4)$$

in which the exponent of ζ_r increases indefinitely with r.

6. In the case in which the curve

$$\phi(\tau, \sigma) = 0$$

has multiple factors, the regular points of such factors taken in 1, 4) c are possibly singular points of the surface, whose domains are represented by equations of form (e). When a further quadratic transformation is applied to such a point, we are not warranted in assuming that the resulting developments will represent the whole of the domain of the point considered.† Kobb makes this assumption in proposing to

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* This set, combined with the transformation (l), possesses all the properties required by Kobb in (g), (h), and (i); its appearance here invalidates his proof. It can easily be shown, moreover, that the most general set of transformations which he could use in this case would produce the same condition as shown here.

† The development about the point first considered, to begin with, is a relation *im Kleinen;* it becomes, however, on passing to the later transformations, a relation

use in 2 only the set of points determined in 1. We are not warranted, either, in assuming that, when a reduction of singularity arises from the appearance of a term of lower degree in a different variable from that with reference to which the first development is derived, the resulting development will hold throughout the same region as the first development. As an example consider the surface

$$\tau^2 + \sigma\zeta - \zeta = 0.$$

Regarded as a development for τ, its coefficients converge for all finite values of σ and ζ; but when we develop for ζ,

$$\zeta = \frac{\tau^2}{1 - \sigma},$$

and the resulting series converges only when

$$|\sigma| < 1.$$

7. From geometrical considerations we should not expect the quadratic transformation used to resolve the singularity in all cases. In ordinary space the transformation

$$\xi = \tau\zeta, \qquad \eta = \sigma\zeta,$$

will transform in a one-to-one manner, without change of the ζ coördinate, all points except those in the $\zeta = 0$ plane. Now in the surface from (m),

$$\tau^2 - \sigma^2 + \tau\sigma\zeta = 0,$$

all points in the ζ-axis are singular, and whatever the reduction that may be secured for the origin, there will be within the neighborhood of the origin points whose singularity is not reduced. The same considerations would be seen to apply if we had any space curve as a singular line.

Levi, in the article previously mentioned, does not attempt a proof of the entire proposition, but directs his work toward establishing by geometrical considerations the reduction of the singularity, making exception, however, of certain cases,* which are closely related to the one considered in 7.

Having thus considered the failure of Kobb to establish the proposition even for the general case of an algebraic surface, we shall, in the

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im Grossen, the limit to the number of points taken being determined by finding the extent of the domain of each; while the developments about the later points give relations *im Kleinen*, as far as the first point is concerned.

* Cf. Levi, l. c. p. 227. Cf. also a second paper by Levi, Atti R. Acc. Sci. Torino, Vol. XXXIII., 5 Dec, 1897.

present article, supply the deficiency, and treat at once the more general case of an analytic surface, *i. e.*, the case that the function $F(x, y, z)$ is not merely a polynomial, but is any analytic function which vanishes at the point $(a, b, c.)$

§ 1

A. — The Fundamental Theorem.

1. The theorem, the proof of which forms the subject of this article, is the following.

Theorem: *Let $F(x, y, z)$ be a function such that*

1) *$F(x, y, z)$ is analytic in the three independent variables in the neighborhood of the point $x = a$, $y = b$, $z = c$;*

2) $F(a, b, c) = 0$;

3) $\left(\frac{\partial F}{\partial x}\right)_{(a,\,b,\,c)} = \left(\frac{\partial F}{\partial y}\right)_{(a,\,b,\,c)} = \left(\frac{\partial F}{\partial z}\right)_{(a,\,b,\,c)} = 0$;

then we can represent all values of (x, y, z) satisfying the equation

$$F(x, y, z) = 0$$

and lying in the neighborhood of the point (a, b, c):

$$|x - a| < \delta, \qquad |y - b| < \delta, \qquad |z - c| < \delta,$$

by a finite number of parametric formulae of the following type:

$$\left.\begin{aligned} x &= \phi_\rho(u, v) \\ y &= \psi_\rho(u, v) \\ z &= \chi_\rho(u, v) \end{aligned}\right\} \qquad \rho = 1, 2, \ldots\ldots \bar{\rho}, \tag{A}$$

where ϕ_ρ, ψ_ρ, χ_ρ are analytic in the arguments (u, v) throughout a certain region; further for each set of values of (x, y, z), the values $(0, 0, 0)$ excepted, there corresponds for at least one value of ρ a pair of values (u, v) lying within the region in which the functions ϕ_ρ, ψ_ρ, χ_ρ are considered, and for any value of ρ for which this is the case, there corresponds no second pair of values. To the set of values $(0, 0, 0)$ corresponds at least one, and in general an infinite number of pairs of values (u, v) for every value of ρ.

2. *Explanation of Symbols.* The symbol $(x, y, z, \ldots\ldots)_n$ indicates, in the expression in which it appears, the total collection of terms of degree n in the arguments taken together, which belong to that expression.

A functional sign expressed by means of a letter will always represent an analytic function.

The symbol $E\,(x, y, z, \ldots\ldots)$ will always represent a function which is analytic at the point $(0, 0, 0 \ldots\ldots)$ and for which $E\,(0, 0, 0 \ldots\ldots) \neq 0$. If written with a subscript, as $E_r\,(x, y, z, \ldots\ldots)$ it represents a particular function of the class; if without a subscript, it represents a general function of the class; so that two functions $E\,(x, y, z, \ldots\ldots)$ both expressed by the same symbol, need not be equal to each other.

B. — The Transformations.

3. The equation

$$F\,(x, y, z) = 0$$

can be transformed to the form

$$\Phi\,(\xi, \eta, \zeta) = (\xi, \eta, \zeta)_m + (\xi, \eta, \zeta)_{m+1} + \ldots\ldots = 0$$

where

1) $m \geqq 2$.

2) the polynomial

$$(\xi, \eta, 1)_m = \phi\,(\xi, \eta)$$

contains the term ξ^m.

3) the points in which the curves corresponding to the irreducible factors of $\phi\,(\xi, \eta)$ cut the line at infinity shall be distinct from each other and from the point in which the line $\xi = 0$ cuts that line.

To do this, we first make the transformation

$$x = u + a, \qquad = v + b, \qquad z = w + c,$$

thus obtaining

$$F\,(x, y, z) = f\,(u, v, w) = (u, v, w)_m + (u, v, w)_{m+1} + \ldots\ldots$$

Here, $m \geqq 2$. the singularity now being at the origin. Next we make a linear homogeneous transformation with non-vanishing determinant,

$$\left.\begin{aligned} u &= \alpha_1 \xi + \beta_1 \eta + \gamma_1 \zeta \\ v &= \alpha_2 \xi + \beta_2 \eta + \gamma_2 \zeta \\ w &= \alpha_3 \xi + \beta_3 \eta + \gamma_3 \zeta \end{aligned}\right\} \qquad (1)$$

with the result:

$$f\,(u, v, w) = \Phi\,(\xi, \eta, \zeta) = (\xi, \eta, \zeta)_m + (\xi, \eta, \zeta)_{m+1} + \ldots\ldots = 0.$$

For this equation, conditions 2) and 3) can be secured. as is readily seen by a proper choice of the coefficients in transformation (1).

The surface $\Phi = 0$ corresponds in the neighborhood considered, point for point, to the surface $F = 0$, and thus it is only necessary to prove the theorem for $\Phi = 0$.

We may assume that of the irreducible factors* of Φ there are none of degree lower than m vanishing at the point $(0, 0, 0)$, for otherwise each of such factors could be treated separately by the methods here used, and the results combined. This provision excludes the case in which one of the variables has equal roots for all values of the other two in the neighborhood of the point $(0, 0, 0)$.

4. The quadratic transformation

$$\xi = \bar{\xi}\,\zeta \qquad\qquad \eta = \eta \tag{2}$$

reduces $\Phi(\xi, \eta, \zeta)$ to the form

$$\begin{aligned}\Phi(\xi, \eta, \zeta) &= \zeta^m\,[\phi(\bar{\xi}, \bar{\eta}) + \zeta\,\chi(\bar{\xi}, \bar{\eta}, \zeta)] \\ &= \xi^m\,\bar{\Phi}(\bar{\xi}, \bar{\eta}, \zeta)\end{aligned} \tag{3}$$

where, an arbitrarily large positive number Γ having been chosen at pleasure, δ can be so determined that the function $\bar{\Phi}$ will be analytic when

$$|\bar{\xi}| < \Gamma, \qquad |\bar{\eta}| < \Gamma, \qquad |\zeta| < \delta.$$

Equation (3) follows at once from the intermediate form

$$\Phi(\xi, \eta, \zeta) = \zeta^m\,[(\bar{\xi}, \bar{\eta}, 1)_m + \zeta\,(\bar{\xi}, \bar{\eta}, 1)_{m+1} + \zeta^2\,(\bar{\xi}, \bar{\eta}, 1)_{m+2} + \ldots\ldots].$$

We now proceed to the proof that the function $\bar{\Phi}(\bar{\xi}, \bar{\eta}, \zeta)$ is analytic within the above limits.

$$\text{Let } \Phi(\xi, \eta, \zeta) = \Sigma A_{ijk}\,\xi^i\,\eta^j\,\zeta^k, \qquad i + j + k \geqq m,$$

and suppose it to be convergent when

$$|\xi| < h, \qquad |\eta| < h, \qquad |\zeta| < h \qquad h > \delta_1.$$

Then, for the general term, we have

$$|A_{ijk}|\,\delta_1{}^{i+j+k} < M,$$

M being a positive constant.

By transformation (2)

$$\Phi(\xi, \eta, \zeta) = \Sigma A_{ijk}\,\bar{\xi}^i\,\bar{\eta}^j\,\zeta^{i+j+k}$$

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[*] For the definition and the fundamental properties of the irreducible factors of an analytic function of several variables, which vanishes in a point, cf. Encyclopädie der mathematischen Wissenschaften, II. B. 1, Nr. 45.

and

$$\bar{\Phi}(\bar{\xi}, \bar{\eta}, \zeta) = \Sigma A_{ijk} \bar{\xi}^i \bar{\eta}^j \zeta^{i+j+k-m}$$

Now choose δ so that $\Gamma \delta \leqq \delta_1$.

Then, when

$$|\bar{\xi}| = \Gamma_1, \qquad |\bar{\eta}| = \Gamma_1, \qquad |\zeta| = \delta, \qquad \Gamma_1 > 1,$$

the absolute value of the general term of series $\bar{\Phi}$ becomes

$$\begin{aligned} &\leqq |A_{ijk}| \,.\, \Gamma_1^{i+j} \,.\, \delta^{i+j+k-m} \\ &\leqq |A_{ijk}| \,.\, \delta_1^{i+j} \,.\, \delta^{k-m} \\ &< \frac{|A_{ijk}| \,.\, \delta_1^{i+j+k}}{\delta^m} \\ &< \frac{M}{\delta^m} \end{aligned}$$

Accordingly, the series

$$\Sigma A_{ijk} \bar{\xi}^i \bar{\eta}^j \zeta^{i+j+k-m}$$

is convergent when

$$|\bar{\xi}| < \Gamma_1 \qquad |\bar{\eta}| < \Gamma_1 \qquad |\zeta| < \delta.\text{*}$$

and it represents an analytic function for these values of the arguments.

5. The family of lines tangent to the surface $\Phi(\xi, \eta, \zeta) = 0$ at the point $(0, 0, 0)$ forms a cone that cuts the plane $\zeta = 1$ in the curve $\phi(\xi, \eta) = 0$. If the line $\eta/\zeta = \beta$, $\xi/\zeta = \alpha$, (α and β being finite) is one of this family, then the point $\bar{\xi} = \alpha$, $\bar{\eta} = \beta$, $\zeta = 0$ of the surface $\bar{\Phi}(\bar{\xi}, \bar{\eta}, \zeta) = 0$, (3) is at most a singular point of order m of that surface, and its neighborhood corresponds to a portion of the neighborhood of the singular point of the original surface $\Phi(\xi, \eta, \zeta) = 0$. In fact, cut the surface

$$\Phi(\xi, \eta, \zeta) = 0$$

by the plane

$$\eta - \beta \zeta = 0\,.$$

Then the curve of intersection C will have a multiple point at $(0, 0, 0)$ and the equations of the tangents to C at $(0, 0, 0)$ will be

$$\left.\begin{aligned} \eta - \beta \zeta = 0 \\ \xi - \alpha_\sigma \zeta = 0 \end{aligned}\right\} \qquad \sigma = 1, 2, \ldots\ldots s \leqq m.$$

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* Cf. Stolz, Allgemeine Arithmetik, Vol. I. p. 293.

Now, the transformation (2) being made, the points of the region

$$T: \qquad |\xi| < \delta, \qquad |\eta| < \delta, \qquad |\zeta| < \delta,$$

which lie in the neighborhoods of the lines

$$\left.\begin{array}{l} \eta - \beta\,\zeta = 0 \\ \xi - a_\sigma \zeta = 0 \end{array}\right\} \qquad \sigma = 1, 2, \ldots\ldots s,$$

can, with the exception of the point (0, 0, 0), be transformed in a one-to-one manner on the neighborhoods of the points (0, 0, 0) of a set of surfaces

$$g_\sigma\,(\xi_\sigma, \eta_1, \zeta) = 0\,, \qquad \sigma = 1, 2, \ldots\ldots s,$$

the coördinates being connected by the relation

$$\left.\begin{array}{l} \xi = \zeta\,(\xi_\sigma + a_\sigma) \\ \eta = \zeta\,(\eta_1 + \beta) \end{array}\right\} \tag{4}$$

By the *neighborhood* of the above line is here meant the set of points (ξ, η, ζ) which satisfy the condition

$$|\xi - a_\sigma \zeta| \leqq \epsilon\,|\zeta|\,, \qquad |\eta - \beta\,\zeta| \leqq \epsilon\,|\zeta|, \qquad |\zeta| < \delta.$$

To deal with the points for which a, β would be infinite, cut the surface

$$\Phi\,(\xi, \eta, \zeta) = 0$$

by the plane

$$\zeta = 0\,.$$

The equations of the tangents to the curve of intersection at (0, 0, 0) are

$$\left.\begin{array}{r} \xi - a_\tau\,\eta = 0 \\ \zeta = 0 \end{array}\right\} \qquad \tau = 1, 2, \ldots\ldots t \leqq m\,.$$

By means of a transformation corresponding to (2),

$$\xi = \bar{\bar{\xi}}\,\eta\,, \qquad \zeta = \bar{\bar{\zeta}}\,\eta,$$

the points of T which lie in the neighborhoods of the lines

$$\left.\begin{array}{r} \xi - a_\tau\,\eta = 0 \\ \zeta = 0 \end{array}\right\} \qquad \tau = 1, 2, \ldots\ldots t \leqq m,$$

can, with the exception of the point (0, 0, 0), be transformed in a one-to-one manner on the neighborhoods of the points (0, 0, 0) of the set of surfaces

$$g_\tau\,(\xi_\tau, \eta, \bar{\bar{\zeta}}) = 0, \qquad \tau = 1, 2, \ldots\ldots t \leqq m,$$

the coördinates being connected by the relations

$$\left.\begin{aligned} \xi &= \eta\,(\xi_\tau + a_\tau) \\ \zeta &= \eta\,\bar{\bar{\zeta}} \end{aligned}\right\} \tag{5}$$

By the neighborhood of the line

$$\left.\begin{aligned} \xi - a_\tau\,\eta &= 0 \\ \zeta &= 0 \end{aligned}\right\}$$

is here meant the set of points (ξ, η, ζ) which satisfy the condition

$$|\xi - a_\tau\,\eta| \leqq \epsilon\,|\eta|, \qquad |\zeta| \leqq \epsilon\,|\eta|, \qquad |\eta| < \delta.$$

The singularities of the surfaces

$$g_\sigma\,(\xi_\sigma, \eta_1, \zeta) = 0,$$
$$g_\tau\,(\xi_\tau, \eta, \bar{\bar{\zeta}}) = 0$$

at the points $(0, 0, 0)$ are at most of order m. Their further properties will be considered later.

Let G be an arbitrarily chosen (large) positive quantity, δ a second suitably chosen positive quantity: then any point of T, for which

$$|\xi| < G\,|\zeta|, \qquad |\eta| < G\,|\zeta|, \qquad 0 < |\zeta| < \delta,$$

is carried by the transformation (4) into one of the neighborhoods above considered on the surfaces $g_\sigma = 0$. If $\zeta = 0$, but ξ, η do not both vanish, then the point (ξ, η, ζ) is carried by (5) into one of the neighborhoods considered on the surfaces $g_\tau = 0$.

In (3), the function $\phi\,(\xi, \eta)$ contains the term $\bar{\xi}^m$ by 3, 2). Apply the transformation

$$\bar{\eta} - \beta = \eta_1, \tag{6}$$

whence (3) takes the form

$$\Phi\,(\xi, \eta, \zeta) = \zeta^m\,[\phi_1\,(\bar{\xi}, \eta_1) + \zeta\,\chi_1\,(\bar{\xi}, \eta_1, \zeta)].$$

In $\phi_1\,(\bar{\xi}, \eta_1)$, take out all terms not containing η_1, so that

$$\phi_1\,(\bar{\xi}, \eta_1) = \prod_{\sigma=1}^{s}\,(\bar{\xi} - a_\sigma)^{\mu_\sigma} + \eta_1\,\psi_1\,(\bar{\xi}, \eta_1), \quad \mu_1 + \ldots\ldots + \mu_s = m.$$

Then make the transformation

$$\bar{\xi} - a_\sigma = \xi_\sigma \tag{6'}$$

and we have

$$\begin{aligned} \Phi\,(\xi, \eta, \zeta) &= \zeta^m\,[\xi_\sigma^{\mu_\sigma} \prod_{\sigma^1=1}^{s}{}'\,(\xi_\sigma + a_\sigma - a_{\sigma'})^{\mu_{\sigma'}} + \eta_1\,\psi\,(\xi_\sigma, \eta_1) + \zeta\,\chi\,(\xi_\sigma, \eta_1, \zeta)] \\ &= \zeta^m g_\sigma\,(\xi_\sigma, \eta_1, \zeta) = 0 \end{aligned} \tag{7}$$

where g_σ has a term in $\xi_\sigma^{\mu_\sigma}$ free from η_1 and ζ, since $a_\sigma - a_{\sigma'} \neq 0$. So there are near the point (0, 0, 0) μ_σ values of ξ_σ satisfying the equation $g_\sigma = 0$ for every pair of values of η_1 and ζ in the neighborhood of the point $\eta_1 = 0$, $\zeta = 0$. Now, for any such set of values of ξ_σ, η_1, ζ, different from the set (0, 0, 0), satisfying the equation $g_\sigma = 0$, there is a corresponding set of values of ξ, η, ζ satisfying the equation $\Phi(\xi, \eta, \zeta) = 0$, their coördinates being connected by the relations (2), (6), and (6′), which are equivalent to the required relation (4). Also by considering the other factors of $\prod_{\sigma=1}^{s} (\xi - a_\sigma)^{\mu_\sigma}$, we get $(s - 1)$ other equations of form (7), the corresponding coördinates being connected by relations of form (4).

No two points (ξ, η, ζ), (ξ', η', ζ') of T (distinct from (0, 0, 0)), derived from points $(\xi_\sigma, \eta_1, \zeta_1)$ $(\xi_{\sigma'}, \eta_2, \zeta_2)$ lying respectively in the neighborhoods of the singularities which are given by two distinct equations

$$g_\sigma = 0, \qquad\qquad g_{\sigma'} = 0,$$

can be the same. For suppose

$$\begin{aligned} \xi &= \xi' = \zeta_1 (\xi_\sigma + a_\sigma) = \zeta_2 (\xi_{\sigma'} + a_{\sigma'}) \\ \eta &= \eta' = \zeta_1 (\eta_1 + \beta) = \zeta_2 (\eta_2 + \beta) \\ \zeta &= \zeta' = \zeta_1 \qquad\qquad\;\; = \zeta_2 \end{aligned}$$

Then we must have

$$\xi_\sigma + a_\sigma = \xi_{\sigma'} + a_{\sigma'},$$

$$\xi_\sigma - \xi_{\sigma'} = a_{\sigma'} - a_\sigma,$$

and, by taking the neighborhoods of the singularities in question sufficiently small, we can insure that the difference $\xi_\sigma - \xi_{\sigma'}$ is less in absolute value than the difference $a_{\sigma'} - a_\sigma$. In a similar manner it is shown that, if the equation $g_\sigma = 0$, regarded as an equation in ξ_σ, has equal roots for all values of η_1, ζ in the neighborhood of the point $\eta_1 = 0$, $\zeta = 0$, the equation $\Phi = 0$ must also have equal roots at the corresponding points, and this case has been excluded. So as each equation $g_\sigma = 0$ has near the point (0, 0, 0) μ_σ values of ξ_σ, in general distinct, for each pair of values of η_1 and ζ, and as $\sum_{\sigma=1}^{s} \mu_\sigma = m$, the collection of equations

$$g_\sigma = 0, \qquad\qquad \sigma = 1, 2, \ldots\ldots s,$$

has within sufficiently small limits as many different roots as the equation $\Phi = 0$, and thus represents the latter equation within the corresponding limits, i. e., when

$$|\zeta| < \delta, \qquad |\eta_1| < \epsilon, \qquad \xi_\sigma| < \epsilon,$$

or

$$|\xi - a_\sigma \zeta| < \epsilon |\zeta|, \qquad |\eta - \beta\zeta| < \epsilon |\zeta|, \qquad |\zeta| < \delta.$$

Next we consider points for which $\zeta = 0$, but ξ, η are not both zero. For these we use the transformation

$$\xi = \bar{\bar{\xi}}\,\eta, \qquad \zeta = \bar{\bar{\zeta}}\,\eta. \tag{8}$$

Then, by the same method of treatment as above, putting $\bar{\bar{\zeta}}$ for $\bar{\eta}$ and η for ζ, and taking $\beta = 0$, we derive a set of surfaces

$$g_\tau\,(\xi_\tau, \eta, \bar{\bar{\zeta}}) = 0, \qquad \tau = 1, 2, \ldots\ldots t \leqq m,$$

on which are mapped all points of the original neighborhood for which

$$|\eta| < \delta_1, \qquad \left|\frac{\zeta}{\eta}\right| < \epsilon_1,$$

and so all points for which

$$|\eta| < \delta_1, \qquad |\zeta| < \epsilon_1 |\eta|, \qquad |\xi - a_\tau \eta| < \epsilon_1 |\eta|.$$

Here, we have a function corresponding to $\phi\,(\bar{\xi}, \bar{\eta})$:

$$\bar{\phi}\,(\bar{\bar{\xi}}, \bar{\bar{\zeta}}) = (\bar{\bar{\xi}}, 1, \bar{\bar{\zeta}})_m$$

Now, for the infinite roots of

$$\phi\,(\bar{\xi}, \bar{\eta}) = 0,$$

we put the equation into the form

$$\left(\frac{\xi}{\eta}, 1, \frac{0}{\eta}\right)_m = 0.$$

So the equation

$$\bar{\phi}\,(\bar{\bar{\xi}}, \bar{\bar{\zeta}}) = 0$$

is such that its roots for $\bar{\bar{\xi}} = 0$ are the same as the ratios of the infinite roots of the equation

$$\phi\,(\bar{\xi}, \bar{\eta}) = 0,$$

and by 3, 3) these ratios are all finite.

C. — THE NUMBER OF THE NEIGHBORHOODS, $t_1, t_2, \ldots\ldots t_\nu$, REQUIRED TO REPRESENT T IS FINITE.

6. In the foregoing paragraph it has been shown that the neighborhood of each tangent line to the surface $\Phi = 0$, at the singular point can be mapped on the neighborhood of a (regular or singular) point of the surface $g_\sigma = 0$. We now proceed to show that the whole neighborhood

$$T: \qquad |\xi| < \delta, \qquad |\eta| < \delta, \qquad |\xi| < \delta$$

can be covered by the neighborhoods of a finite number of such lines. We distinguish two cases: —

CASE I. — The polynomial $\phi(\bar{\xi}, \bar{\eta})$ has no multiple factors.
CASE II. — This polynomial has multiple factors.

THEOREM: *The neighborhood T can be completely covered by a finite number of regions $T_1, T_2, \ldots\ldots T_\nu$, which overlap each other and which are mapped respectively on the following regions $t_1, t_2, \ldots\ldots t_\nu$:*

In Case I: 1) the region t_i, $i = 1, 2, \ldots\ldots \kappa$, consists of the neighborhood of a singular point of the surface $g^{(i)} = 0$;

2) the extent of each of the neighborhoods $t_1, t_2, \ldots\ldots t_\kappa$ having been arbitrarily determined, the regions t_j, $j = \kappa + 1, \ldots\ldots \nu$, then consist of regular regions of surfaces $g^{(j)} = 0$.

In Case II: 1) the region t_i, $i = 1, 2, \ldots\ldots \kappa$, consists of the neighborhood of a singular point of the surface $g^{(i)} = 0$;

2) the extent of each of the neighborhoods $t_1, t_2, \ldots\ldots t_\kappa$ having been arbitrarily determined, the regions t_j, $j = \kappa + 1, \ldots\ldots \nu$, then consist of regions of surfaces $g^{(j)} = 0$ defined as follows: omitting the index j throughout, we write

$$g(\xi_\sigma, \eta_1, \zeta) = [\xi_\sigma^r + p_1(\eta_1, \zeta)\,\xi_\sigma^{r-1} + \ldots\ldots + p_r(\eta_1, \zeta)]\,E(\xi_\sigma, \eta_1, \zeta),$$

where $p_e(\eta_1, \zeta)$ is analytic throughout a region

$$|\eta_1| < h, \qquad\qquad |\xi| < \delta.$$

Here r, for a given value of j, is a positive integer satisfying the relation $1 \leqq r \leqq m$.

CASE I. — The polynomial $\phi(\bar{\xi}, \bar{\eta})$ contains no multiple factors.

Here, the equation

$$\phi(\bar{\xi}, \bar{\eta}) = 0$$

can have multiple values of $\bar{\xi}$ only for a finite number of values of $\bar{\eta}$, these being the values for which the equations

$$\phi = 0, \qquad \frac{\partial \phi}{\partial \bar{\xi}} = 0$$

have common roots, and by the condition 3, 3) none of these values of $\bar{\eta}$ become infinite.

Now we consider all such values of $\bar{\eta}$

$$\bar{\eta} = c_r, \qquad r = 1, 2, \ldots\ldots l,$$

for which the equation, considered as an equation in $\bar{\xi}$,

$$\phi(\bar{\xi}, \bar{\eta}) = 0$$

has multiple roots. Deal with each of these as in 5, c_r taking the place of β in (6); then, in equation (7), some of the μ_σ's will, in general, be greater than unity, i. e. some of the equations $g_\sigma = 0$ will have for the lowest terms in ξ_σ alone exponents greater than 1. For such as have their $\mu_\sigma = 1$, there are regular points. The others will afford singular points unless they have terms of the first degree in either η_1 or ζ. Surround these points by neighborhoods

$$|\xi_\sigma| < \delta, \qquad |\eta_1| < \delta, \qquad |\zeta| < \delta,$$

i. e.

$$|\bar{\xi} - a_\sigma| < \delta, \qquad |\bar{\eta} - c_r| < \delta, \qquad |\zeta| < \delta,$$

which are to be considered later.

Now let $\bar{\eta} = b$ be any value for which the equation

$$\phi(\bar{\xi}, \bar{\eta}) = 0$$

has not equal roots. Then the equations $g_\sigma = 0$ of (7) each have a term in ξ_σ to the first degree, free from η_1 and ζ, and thus the points of the surface $g_\sigma = 0$ lying in the neighborhood of the point $\xi_\sigma = 0$, $\eta_1 = 0$, $\zeta = 0$, can be represented by a power series

$$\xi_\sigma = p(\eta_1, \zeta).$$

So, in this case, we have m developments

$$\xi_\sigma = \bar{p}_\sigma(\eta_1, \zeta), \qquad \sigma = 1, 2, \ldots\ldots m,$$

and, by using the relations (4), we have

$$\xi = p_\sigma(\eta, \zeta), \qquad \sigma = 1, 2, \ldots\ldots m.$$

It is readily seen that the function

$$\xi = p(\eta_1, \zeta)$$

is analytic within the region

$$|\eta_1| < h - e, \qquad |\zeta| < \delta_1 \div 0,$$

where h is the distance to the nearest value of η_1 for which the equation corresponding to

$$\phi(\bar{\xi}, \bar{\eta}) = 0$$

has a critical point, e is a positive number which can be taken arbitrarily small and, having been chosen, determines an upper limit, not zero, for δ_1. In fact, ξ_σ is a continuous function of the two independent variables η_1, ζ within this region; furthermore, for any fixed value of ζ such that $|\zeta| < \delta_1$, ξ_σ is an analytic function of η_1 throughout the region $|\eta_1| < h - e$; and, similarly, for any fixed value of η_1 such that $|\eta_1| < h - e$, ξ_σ is an analytic function of ζ throughout the region $|\zeta| < \delta_1$.*

Also consider the surfaces

$$g_\tau(\xi_\tau, \eta, \bar{\bar{\zeta}}) = 0$$

in 5. Here also we have m regular points of surfaces, and as a result m functions of the form

$$\xi_\tau = q(\eta, \bar{\bar{\zeta}}).$$

These, by the same method of proof as above, are seen to be analytic when

$$|\bar{\bar{\zeta}}| < h_1 - e_1, \qquad |\eta| < \delta_2,$$

where h is the nearest point in the $\bar{\bar{\zeta}}$-plane for which the equation

$$\bar{\phi}(\bar{\bar{\xi}}, \bar{\bar{\zeta}}) = 0$$

has multiple roots for ξ, i. e. the smallest value of $\bar{\bar{\zeta}}$ for which the equation

$$(\bar{\bar{\xi}}, 1, \bar{\bar{\zeta}})_m = 0$$

has equal roots for $\bar{\bar{\xi}}$. But this is the smallest value of $\frac{1}{\eta}$ for which the equation

$$\left(\frac{\xi}{\eta}, 1, \frac{1}{\eta}\right)_m = 0$$

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* Cf. Briot et Bouquet's Théorie des fonctions elliptiques, § 28. The proof of continuity there given for polynomials in two variables will apply with very slight modifications to analytic functions of any number of variables. Cf. further Jordan's Cours d'analyse, I. § 206, § 258.

regarded as an equation in ξ/η, has equal roots. Thus $\frac{1}{h_1}$ is the largest value of $\bar{\eta}$ for which the equation

$$\phi(\bar{\xi}, \bar{\eta}) = 0$$

has a critical point. So the functions are analytic and give all points of the original neighborhood for which

$$|\bar{\bar{\zeta}}| = \left|\frac{\zeta}{\eta}\right| < h_1 - e_1, \qquad |\eta| < \delta_2,$$

or for which

$$\left|\frac{\eta}{\zeta}\right| > \frac{1}{h_1 - e_1} = \frac{1}{h_1} + \frac{e_1}{h_1(h_1 - e_1)} = h_2 + e_2, \qquad \left(h_2 = \frac{1}{h_1}\right),$$

$$|\zeta| = |\bar{\bar{\zeta}}|\,|\eta| < \delta_2(h_1 - e_1) = \delta_3\,,$$

thus securing the limits

$$|\eta| < \delta_2\,, \qquad |\zeta| < \delta_3\,, \qquad |\eta| > (h_2 + e_2)\,|\zeta|,$$

where h_2 is the distance to the furthest point in the $\bar{\eta}$-plane for which the equation

$$\phi(\bar{\xi}, \bar{\eta}) = 0$$

has a critical point, and if e_2 is first chosen arbitrarily small, δ_3 can be determined not zero.

Now consider the neighborhoods of the critical points of the curve

$$\phi(\bar{\xi}, \bar{\eta}) = 0.$$

In these, however small we take the $\bar{\delta}$, all the remainder of a circle in the $\bar{\eta}$-plane including all the values for which the curve $\phi = 0$ has critical points can be covered with circles such as were determined for the domains of the regular points above. these circles overlapping the circles about the singular points and not reaching out to these points in any case. Let the radius of the large circle be G where

$$G > 1\,, \qquad G \geqq h_2 + e_2\,.$$

Then, if we take for δ_4 the smallest value of any δ_1 or $\bar{\delta}$, the developments within these circles together with the neighborhoods of the set of new singular points will represent all points of the original neighborhood for which

$$|\eta| < G\delta_4\,, \qquad |\zeta| < \delta_4\,.$$

Finally, taking for δ the smallest of the three quantities δ_2, δ_3, δ_4, the whole set of functions thus determined will represent all points of the original neighborhood for which

$$|\eta| < \delta, \qquad |\zeta| < \delta.$$

The new set of singular points may or may not be all of degrees lower than m, but if they are we have simplified the problem; we have *reduction*, as we shall say, borrowing a term frequently used in the theory of algebraic invariants of a linear transformation; and if not, the further treatment will be considered later.

D. — An Example.

Before taking up Case II, however, we consider an example in which the degree is reduced by one quadratic transformation, and the parametric representation (A) is at once secured.

Let the surface be

$$\Phi(\xi, \eta, \zeta) = \xi^2 + \eta^2 - \zeta^2 - \xi\,\zeta^2 = 0.$$

The transformation

$$\xi = \bar{\xi}\,\zeta, \qquad \eta = \bar{\eta}\,\zeta,$$

secures for the equation corresponding to (3)

$$\bar{\Phi}(\bar{\xi}, \bar{\eta}, \zeta) = \bar{\xi}^2 + \bar{\eta}^2 - 1 - \bar{\xi}\,\zeta = 0.$$

Here

$$\phi(\bar{\xi}, \bar{\eta}) = \bar{\xi}^2 + \bar{\eta}^2 - 1$$

and the critical points are

$$\bar{\xi} = 0, \qquad \bar{\eta} = 1,$$
$$\bar{\xi} = 0, \qquad \bar{\eta} = -1.$$

Let

$$\xi_1 = \bar{\xi}, \qquad \eta_1 = \bar{\eta} - 1,$$

and we have

$$\xi_1{}^2 + \eta_1{}^2 + 2\,\eta_1 - \xi_1\,\zeta = 0.$$

Hence

$$\eta_1 = -1 + \sqrt{\xi_1(\zeta - \xi_1) + 1}. \tag{a}$$

Also let

$$\xi_2 = \bar{\xi}, \qquad \eta_2 = \bar{\eta} + 1,$$

and we have

$$\eta_2 = 1 - \sqrt{\xi_2(\zeta - \xi_2) + 1}. \tag{b}$$

In (a) and (b), only that branch of the radical is taken which becomes $+1$ for zero values of the arguments.

Again, we make the transformation

$$\xi = \bar{\bar{\xi}}\eta, \qquad \zeta = \bar{\bar{\zeta}}\eta,$$

and derive the surface

$$\bar{\bar{\xi}}^2 + 1 - \bar{\bar{\zeta}}^2 - \bar{\bar{\xi}}\bar{\bar{\zeta}}^2\eta = 0.$$

Here

$$\bar{\phi}(\bar{\bar{\xi}}, \bar{\bar{\zeta}}) = \bar{\bar{\xi}}^2 + 1 - \bar{\bar{\zeta}}^2$$

and for the value $\bar{\bar{\zeta}} = 0$ we have the roots

$$\bar{\bar{\xi}} = i, \qquad \bar{\bar{\xi}} = -i.$$

Let

$$\xi_3 = \bar{\bar{\xi}} - i,$$

and we have the surface

$$\xi_3{}^2 + 2\,i\,\xi_3 - \bar{\bar{\zeta}}^2 - \bar{\bar{\zeta}}^2\eta\,\xi_3 - i\,\bar{\bar{\zeta}}^2\eta = 0,$$

whence

$$\xi_3 = \frac{\bar{\bar{\zeta}}^2\eta - 2\,i}{2} + \tfrac{1}{2}\sqrt{\bar{\bar{\zeta}}^4\eta^2 + 4\,\bar{\bar{\zeta}}^2 - 4}\,. \tag{c}$$

In a similar way, from the other root,

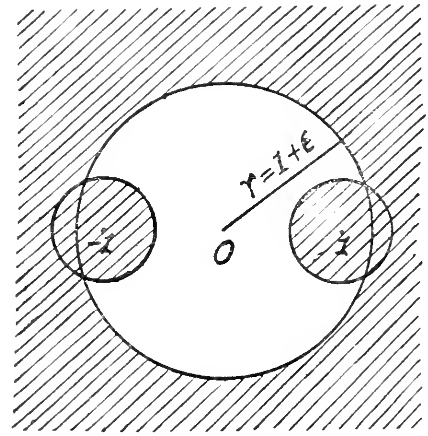
$$\xi_4 = \frac{\bar{\bar{\zeta}}^2\eta + 2\,i}{2} - \tfrac{1}{2}\sqrt{\bar{\bar{\zeta}}^4\eta^2 + 4\,\bar{\bar{\zeta}}^2 - 4}\,. \tag{d}$$

In (c) and (d), for the radical is taken only that branch which becomes $+\,2\,i$ for zero values of the arguments, and the function is seen to be analytic for sufficiently small values of η when

$$|\bar{\bar{\zeta}}| = \left|\frac{\xi}{\eta}\right| < 1 - \epsilon_1;$$

and similarly when

$$|\bar{\eta}| = \left|\frac{\eta}{\xi}\right| > 1 + \epsilon.$$



Thus, in the η-plane, we have by the formulas (a), (b), (c), (d) covered two small circles about the points 1 and -1 corresponding to developments (a) and (b), and all of the region outside of a circle of radius $(1 + \epsilon)$, corresponding to developments (c) and (d). We must now obtain further formulas so as to fill up the remaining unshaded region.

Consider the point

$$\bar{\xi} = -1, \qquad \bar{\eta} = 0.$$

Let $\bar{\xi} = \xi_5 - 1$ and we have

$$\xi_5^2 - 2\xi_5 + \bar{\eta}^2 - \xi_5\zeta + \zeta = 0,$$

whence

$$\xi_5 = \frac{\zeta + 2}{2} - \tfrac{1}{2}\sqrt{\zeta^2 - 4\bar{\eta}^2 + 4}. \qquad \text{(e)}$$

In the same way, about the point

$$\bar{\xi} = 1, \qquad \bar{\eta} = 0,$$

we have the function

$$\xi_6 = \frac{\zeta - 2}{2} + \tfrac{1}{2}\sqrt{\zeta^2 - 4\bar{\eta}^2 + 4} \qquad \text{(f)}$$

In (e) and (f), for the radical we take only that branch which becomes $+2$ for zero values of the argument, and for sufficiently small values of ξ the functions are analytic when

$$|\bar{\eta}| < 1 - \epsilon_2.$$

Again, consider the point

$$\bar{\xi} = 2\sqrt{1 - i}, \qquad \bar{\eta} = 1 + 2i.$$

Let

$$\bar{\xi} = \xi_7 + 2\sqrt{1 - i}, \qquad \bar{\eta} = \eta_7 + 1 + 2i,$$

and we have

$$\xi_7^2 + 4\sqrt{1 - i}\,\xi_7 + \eta_7^2 + 2(1 + 2i)\eta_7 - \xi_7\zeta - 2\sqrt{1 - i}\,\zeta = 0,$$

whence

$$\xi_7 = -\frac{4\sqrt{1 - i} - \zeta}{2} + \tfrac{1}{2}\sqrt{16 - 16i + \zeta^2 - 4\eta_7^2 - 8(1 + 2i)\eta_7}. \qquad \text{(g)}$$

For the corresponding point

$$\bar{\xi} = -2\sqrt{1 - i}, \qquad \bar{\eta} = 1 + 2i,$$

we have the formula

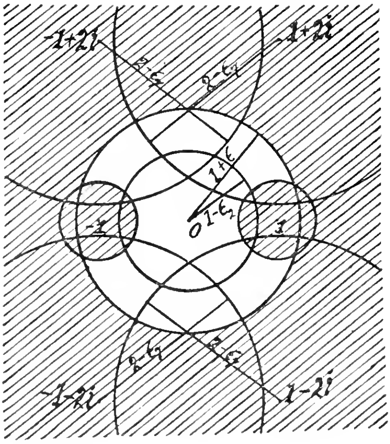
$$\xi_8 = \frac{4\sqrt{1 - i} + \zeta}{2} - \tfrac{1}{2}\sqrt{16 - 16i + \zeta^2 - 4\eta_8^2 - 8(1 + 2i)\eta_8}. \qquad \text{(h)}$$

In (g) and (h), for the radical we take only the branch which becomes $+4\sqrt{1 - i}$ for zero values of the arguments, the same value of the radical $\sqrt{1 - i}$ being taken in all cases. These functions are analytic for sufficiently small values of ζ when

$$|\eta_7| = |\eta_8| < 2 - \epsilon_7.$$

Also, considering the corresponding points of

$$\phi(\bar{\xi}, \bar{\eta}) = 0$$



for which

$$\bar\eta = -1 + 2i,$$
$$\bar\eta = 1 - 2i,$$
$$\bar\eta = -1 - 2i,$$

we have evidently similar regions for each. Then, by taking the ϵ's all small enough, we cover the whole unshaded remainder of the $\bar\eta$-plane by five circles within each of which there is a development as required.

The sets of parametric formulae, derived by using the intermediate transformations, are

$$\left\{\begin{array}{llll} \xi = uv, & \eta = v\sqrt{u(v-u)+1}, & \zeta = v & \text{from (a)} \\ \xi = uv, & \eta = -v\sqrt{u(v-u)+1}, & \zeta = v & \text{" (b)} \end{array}\right.$$

$$\left\{\begin{array}{llll} \xi = \frac{v}{2}\left(u^2 v + \sqrt{u^4 v^2 + 4u^2 - 4}\right), & \eta = v, & \zeta = uv & \text{" (c)} \\ \xi = \frac{v}{2}\left(u^2 v - \sqrt{u^4 v^2 + 4u^2 - 4}\right), & \eta = v, & \zeta = uv & \text{" (d)} \end{array}\right.$$

$$\left\{\begin{array}{llll} \xi = \frac{u}{2}\left(u - \sqrt{u^2 - 4v^2 + 4}\right), & \eta = uv, & \zeta = u & \text{" (e)} \\ \xi = \frac{u}{2}\left(u + \sqrt{u^2 - 4v^2 + 4}\right), & \eta = uv, & \zeta = u & \text{" (f)} \end{array}\right.$$

$$\left\{\begin{array}{ll} \xi = \frac{u}{2}\left(u + \sqrt{16 - 16i + u^2 - 4v^2 - 8(1+2i)v}\right), & \\ \qquad \eta = u(v + 1 + 2i), \quad \zeta = u & \text{" (g)} \\ \xi = \frac{u}{2}\left(u - \sqrt{16 - 16i + u^2 - 4v^2 - 8(1+2i)v}\right), & \\ \qquad \eta = u(v + 1 + 2i), \quad \zeta = u & \text{" (h)} \end{array}\right.$$

with three more sets similar to (g) and (h).

CASE II. — The polynomial $\phi(\bar\xi, \bar\eta)$ contains multiple factors.

Here, any points which are common to two different irreducible factors of $\phi(\bar\xi, \bar\eta)$, or are critical points of a single irreducible factor, will be critical points of the curve

$$\phi(\bar\xi, \bar\eta) = 0,$$

and all such points, a finite number in all, will be treated like the critical points of the previous case. But also any point on a multiple factor is a critical point of the curve, and further treatment is needed for such points.

Suppose $\bar{\xi} = \alpha$, $\bar{\eta} = \beta$ is a regular point of a factor $\phi_1(\bar{\xi}, \bar{\eta})$ of multiplicity r, i. e. of the irreducible factor whose rth power is equal to $\phi_1(\bar{\xi}, \bar{\eta})$ and not a point of any different factor of $\phi(\bar{\xi}, \bar{\eta})$. Then, in the corresponding equation of form (7), g_σ will contain a term ξ_σ^r as the lowest term in ξ_σ^r free from η_1 and ζ, and by Weierstrass's Theorem * we can develop the function about the point in the form

$$\left[\xi_\sigma^r + p_1(\eta_1, \zeta)\,\xi_\sigma^{r-1} + \ldots\ldots + p_r(\eta_1, \zeta)\right] E(\xi_\sigma, \eta_1, \zeta) = 0. \quad (9)$$

These functions

$$p_\lambda(\eta_1, \zeta), \qquad \lambda = 1, 2, \ldots\ldots r,$$

are shown by a method similar to that used for the functions in Case I to be analytic within a region

$$|\eta_1| < h - e, \qquad |\zeta| < \delta_1,$$

where h is the distance to the nearest value of η_1 which gives a point of intersection of two different irreducible curves corresponding to factors of $\phi(\bar{\xi}, \bar{\eta})$, or to a critical point of one of the irreducible curves.

Now none of the excepted points can be at infinity, on account of the provision in 3, 3). So the points on the surfaces $g_\tau = 0$ in 5 will also afford developments of order (9), and by the method of Case I, we have a similar region for the convergence of the coefficients of the different powers of ξ_τ in the polynomial, i. e. the exterior of a circle including all of the excepted points.

Accordingly, in this case also, we represent the neighborhood of the original singular point by a finite number of neighborhoods of new singular points together with a finite number of functions, some of which are now not analytic for the values of the arguments considered, but satisfy equations of the form

$$\xi_\sigma^r + p_1(\eta_1, \zeta)\,\xi_\sigma^{r-1} + \ldots\ldots + p_r(\eta_1, \xi) = 0. \quad (11)$$

For the further treatment of these functions, we shall establish an auxiliary theorem in § 2.

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* See Picard's Traité d'analyse, Vol. II. p. 241.

Any point in T can be carried by a suitable transformation into a point on one of the surfaces $g_\sigma = 0$ or $g_\tau = 0$. Let G be an arbitrarily chosen (large) positive quantity; then any point of T for which

$$|\xi| < G|\zeta|, \qquad |\eta| < G|\zeta|, \qquad |\zeta| < \delta,$$

is carried by the transformation (4) into one of the neighborhoods considered on the surfaces $g_\sigma = 0$.

If $\zeta = 0$, but ξ, η do not both vanish, then the point (ξ, η, s) is carried by (5) into one of the neighborhoods considered on the surfaces $g_\tau = 0$.

§ 2.

A. — A Lemma.

1. The treatment of the multiple curves of Case II depends on the following

Lemma. — *Given an analytic surface*

$$\Phi(x, y, z) = \phi(x, y) + z\Psi(x, y, z) = 0, \tag{α}$$

where

$$\phi(x, y) = 0$$

is a multiple curve; let $\phi(x, y)$ have the form in the neighborhood of the point $x = 0$, $y = 0$,

$$\phi(x, y) = [x + p(y)]^m E(x, y), \tag{β}$$

where $p(y)$ is analytic at the point $y = 0$. and $p(0) = 0$. The function $\Psi(x, y, z)$ shall be analytic at the point $(0, 0, 0)$, but shall not be divisible by $x + p(y)$ at that point. Consider a region for which $|y| < h$, and let h be chosen

a) less than the radius of convergence of the Taylor's series which represents the function $p(y)$ developed about the point $y = 0$, and

b) sufficiently small, so that the points $(x = p(y), y)$ will lie in the region in which $E(x, y)$ is analytic and different from zero. Then the part of the neighborhood of the curve

$$x + p(y) = 0, \qquad z = 0,$$

which lies on the surface

$$\Phi(x, y, z) = 0$$

can be transformed, by means of quadratic transformations of the type

$$x = x'z,$$

on a finite number of regions $\tau_1, \tau_2, \ldots\ldots \tau_\nu$, *which fall into two categories:* —

1) *the region* $\tau_i\ (i = 1, 2, \ldots\ldots \kappa)$ *is the neighborhood of a singular point of order* $\leqq m$;

2) *each of the neighborhoods of* 1) *having been determined arbitrarily small, the region* $\tau_j\ (i = \kappa + 1, \ldots\ldots \nu)$ *is then a regular piece of an analytic surface, represented in its whole extent by a single set of parametric formulae of the type* (A).

By the neighborhood of the curve

$$x + p(y) = 0, \qquad z = 0,$$

is meant the set of points (x, y, z) *satisfying the relation*

$$|y| < h, \qquad |z| < \delta, \qquad |x + p(y)| < \epsilon.$$

B. — PROOF OF THE LEMMA.

2. To prove the lemma we begin by expressing equation (α) by means of (β) in the form

$$\Phi(x, y, z) = [x + p(y)]^m E(x, y) + z\Psi(x, y, z) = 0, \qquad (\gamma)$$

and then making the transformation

$$x + p(y) = x_1, \qquad (\delta)$$

thus obtaining the equation

$$\Phi(x, y, z) = \Phi_1(x_1, y, z) = x_1^m E(x_1, y) + z\psi_1(x_1, y, z) = 0. \qquad (\gamma')$$

Here, the function $E(x_1, y)$ is analytic and different from zero in the neighborhood of any point $x_1 = 0$, $y = y_0$, $(|y_0| < h)$, which corresponds to the neighborhood of the point $x_0 = p(y_0)$, y_0, and hence $E(x_1, y)$ is analytic throughout a region including in its interior the region

$$|x_1| < \epsilon, \qquad |y| < h,$$

if the positive quantity ϵ is suitably chosen. A similar remark applies to the analytic character of the function $\psi_1(x_1, y, z)$, and hence $\Phi_1(x_1, y, z)$ is an analytic function of its three arguments throughout a region including in its interior the region

$$|x_1| < \epsilon, \qquad |y| < h, \qquad |z| < \delta.$$

Now express equation (γ') in the form

$$\Phi_1(x_1, y, z) = \Sigma p_{rs}(y) x_1^r z^s + F(x_1, y, z) = 0, \qquad (\epsilon)$$

where

$$0 < r + s = m_1 \leqq m,$$

m_1 being the lowest degree of any term in x_1 and z together, and $F(x_1, y, z)$ including all terms of degree higher than m_1 in the two variables x_1, z. Each coefficient $p_{rs}(y)$ may be divisible by a power of y, y^l. In that case, however, m_1 must be less than m, for the term in x_1^m is present in $\Phi_1 \; (x_1, y, z)$.

By means of a transformation with non-vanishing determinant,

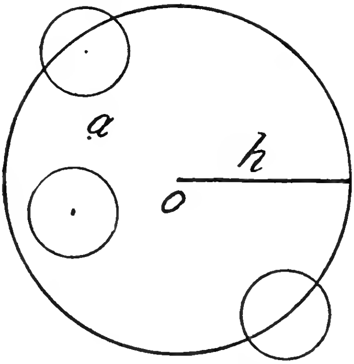
$$\left.\begin{aligned} x_1 &= a_1 x_2 + \beta_1 z_2 \\ z &= a_2 x_2 + \beta_2 z_2 \end{aligned}\right\}, \qquad (\zeta)$$

Φ_1 can be thrown into the form:

$$\Phi_1(x_1, y, z) = \Phi_2(x_2, y, z_2) =$$

$$q_0(y) x_2^{m_1} + q_1(y) x_2^{m_1-1} z_2 + \ldots\ldots + q_{m_1}(y) z_2^{m_1} + F_2(x_2, y, z_2) = 0 \quad (\eta)$$

where $q_0(y) \not\equiv 0$.



Consider first the points of the circle $|y| \leqq h$ at which $q_0(y) = 0$, if such exist. Each one of these points y_i, $(i = 1, 2, \ldots\ldots \kappa)$ is a singular point of $\Phi_2 = 0$ of order not greater than m, and its neighborhood

$$|x_1| < \epsilon, \quad |y - y_i| < \epsilon, \quad |z| < \delta$$

may be chosen arbitrarily small. Surround each of these points in the circle $|y| = h$ by a circle of arbitrarily small radius ϵ'. We now proceed to consider the region about an arbitrary point a of the circle $|y| \leqq h$ not lying in any of the regions just cut out. Let

$$y_2 = y - a$$

and let Φ_2 then be written in the form

$$\Phi_2(x_2, y, z_2) = \bar{\Phi}_2(x_2, y_2, z_2) =$$

$$\bar{q}_0(y_2) x_2^{m_1} + \bar{q}_1(y_2) x_2^{m_1-1} z_2 + \ldots\ldots + \bar{q}_{m_1}(y_2) z_2^{m_1} + F_2(x_2, y_2, z_2)$$
$$= [x_2^{m_1} + r_1(y_2) x_2^{m_1-1} z_2 + \ldots\ldots + r_{m_1}(y_2) z_2^{m_1}] E(y_2) + \bar{F}_2(x_2, y_2, z_2)$$
$$= 0. \qquad (\theta)$$

3. Apply to the function $\bar{\Phi}_2$ the quadratic transformation

$$x_2 = x_3 z_2.^*$$

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* Here, for the first time, a quadratic transformation of the type that transforms but a single variable is employed. Such transformations do not occur in Kobb's analysis. They appear to be indispensable.

Let the result be written as follows: —

$$\bar{\Phi}_2(x_2, y_2, z_2) = z_2^{m_1}\Phi_3(x_3, y_2, z_2) =$$

$$z_2^{m_1}\{[x_3^{m_1}+r_1(y_2)x_3^{m_1-1}+\ldots\ldots+r_{m_1}(y_2)]E(y_2)+z_2F_3(x_3,y_2,z_2)\}=0.(\kappa)$$

From this last equation we deduce the following theorem: —

All points of the surface $\Phi = 0$ *in the neighborhood of the curve*

$$\phi(x, y) = 0, \qquad z = 0,$$

are mapped upon a finite number of new neighborhoods which are

1) *neighborhoods of singular points of degree* $\leqq m$, *which neighborhoods may be taken arbitrarily small;*

2) *neighborhoods of new multiple curves on surfaces constituted like the surface* $\Phi(x, y, z) = 0$ *of the lemma, the values of* m *thus arising never exceeding the original* m *of the lemma.*

By the same kind of reasoning as in § 1, 5, we show namely that for any one of the above values of a, the corresponding value of y_2 being in or on the circle of convergence of the Taylor's development about the point $z_2 = 0$ of the function

$$r_\lambda(y_2), \qquad \lambda = 1, 2, \ldots\ldots m_1,$$

all points of the surface $\Phi_2 = 0$ in the neighborhood of the curve

$$\phi(x, y) = 0, \qquad z = 0,$$

are represented by points in the neighborhoods of points of the curve

$$x_3^{m_1} + r_1(y_2)x_3^{m_1-1} + \ldots\ldots + r_{m_1}(y_2) = 0, \qquad z_2 = 0,$$

on the surface $\Phi_3 = 0$, i. e. if such a value of y is b, so that the corresponding value of y_2 is $(b - a)$, and if the roots of the equation

$$x_3^{m_1} + r_1(b-a)x_3^{m_1-1} + \ldots\ldots + r_{m_1}(b-a) = 0 \qquad (\mu)$$

are $a_1, a_2, \ldots\ldots a_{m_1}$ then points of the surface $\Phi_2 = 0$ for which

$$|x_2| < \delta, \qquad |z_2| < \delta, \qquad y = b,$$

are connected with the points of the surface (κ) by the relation

$$x_2 = z_2(x_\sigma + a_\sigma), \qquad y_2 = b - a, \qquad \sigma = 1, 2, \ldots\ldots m_1.$$

Further, if we limit y_2 to a circle not reaching out to the nearest point for which $\bar{q}_0(y_2)$ vanishes, we have an upper limit for a_σ as a root of the equation (μ), and thus by taking z_2 and x_σ small enough we can make x_2 as small as we please. Then the transformations (δ) and (ζ) still secure a limit for the values of x and z, and thus we have represented a neighborhood of the curve

$$\phi(x, y) = 0, \qquad z = 0,$$

on the surface

$$\Phi(x, y, z) = 0$$

as required.

Now, however small the neighborhood we shut off about the points in the region $|y| < h$ for which $q_0(y)$ vanishes, since the results established above would hold also in a circle of radius $h_1 > h$, but still less than the radius of convergence of the series for $p(y)$ in (β), we can fill up the remainder of the circle of radius h with circles within which $q_0(y)$ does not vanish, these circles overlapping at all points the boundaries of the excepted neighborhoods and not reaching up to the excepted points. Within each of these circles we have a development of type (κ). Consider one of these new circles. We want to consider the neighborhood of the curve

$$\phi_3(x_3, y_2) = x_3^{m_1} + r_1(y_2)\, x_3^{m_1-1} + \ldots\ldots + r_{m_1}(y_2) = 0. \qquad (\nu)$$

If this is a multiple curve of the m_1-th order and $m_1 < m$, we have reduction. Moreover, if $m_1 = m$, but

$$x_3^{m_1} + r_1(y_2)\, x_3^{m_1-1} + \ldots\ldots + r_{m_1}(y_2) \neq [x_3 + p_3(y_2)]^{m_1},$$

we also have reduction. We need consider, therefore, only the case that

$$\left.\begin{aligned} x_3^{m_1} + r_1(y_2)\, x_3^{m_1-1} + \ldots\ldots + r_{m_1}(y_2) = [x_3 + p_3(y_2)]^{m_1}, \\ m_1 = m, \end{aligned}\right\} \qquad (\nu')$$

and show that this case can repeat itself at most but a finite number of times.

4. Suppose the function $\phi_3(x_3, y_2)$ has the form (ν'). Apply to the surface $\Phi_3(x_3, y_2, z_2) = 0$, (κ), the transformation

$$x_3 + p_3(y_2) = x_4,$$

and reduce the result to the form

$$\Phi_4(x_4, y_2, z_2) = x_4^{m_1} E(y_2) + z_2 F_4(x_4, y_2, z_2) = (0). \qquad (o)$$

If any term in $z_2 F_4(x_4, y_2, z_2)$ is of degree in x_4 and z_2 together less than m_1, it appears at once that we have a line of lower order. So we assume there are no such terms. Also, as the coefficient of $x_4^{m_1}$ does not vanish identically in y_2 (in fact, not at all) no transformation of type (ζ) is needed, and after collecting all terms of the m_1-th order we make at once the substitution

$$x_4 = x_5 z_2 ,$$

and proceed in the same manner as before if the degree is not reduced. For convenience, we suppose the succession of multiple curves of the same order to begin with that on the surface $\Phi = 0$, and use a notation independent of that hitherto employed.

Our successive transformations are of the type

$$\left.\begin{array}{ll} x - p_1(y) = \bar{x}_1 & \bar{x}_1 = x_1 z \\ x_1 - p_2(y) = \bar{x}_2 & \bar{x}_2 = x_2 z \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ x_{\nu-1} - p_\nu(y) = \bar{x}_\nu & \bar{x}_\nu = x_\nu z \end{array}\right\} \qquad (\pi)$$

whence

$$x = p_1(y) + p_2(y) z + p_3(y) z^2 + \ldots\ldots + p_\nu(y) z^{\nu-1} + x_\nu z^\nu . \qquad (\rho)$$

Develop the function Φ in (γ) by Weierstrass's Theorem:

$$\begin{aligned} \Phi(x, y, z) &= [x^m + q_1(y, z) x^{m-1} + \ldots\ldots + q_m(y, z)]\, E(x, y, z) \\ &= F(x, y, z)\, E(x, y, z). \end{aligned}$$

From (ρ) we derive the relation:

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial x_\nu} \frac{\partial x_\nu}{\partial x} = \frac{1}{z^\nu} \frac{\partial F}{\partial x_\nu}. \qquad (\sigma)$$

The succession of transformations (π) so long as it does not reduce the degree in x and z, takes out of the F factor at each step the factor z^ν, since, on account of the constant term in the E factor, no power of z could come out of it. So, after the ν transformations (π), we have

$$\begin{aligned} F(x, y, z) &= z^{\nu m} F_\nu(x_\nu, y, z) = \\ z^{\nu m} &[x_\nu^{\,m} + q_{1\nu}(y, z) x_\nu^{\,m-1} + \ldots\ldots + q_{m\nu}(y, z)], \end{aligned} \qquad (\tau)$$

and by (σ)

$$\frac{\partial F}{\partial x} = z^{\nu(m-1)} \frac{\partial F_\nu}{\partial x_\nu}.$$

Now we may consider F as having no multiple factors vanishing at the point $(0, 0, 0)$. So we have the relation

$$L(x, y, z) F + M(x, y, z) \frac{\partial F}{\partial x} = R(y, z) = z^\lambda R_1(y, z) \qquad (\upsilon)$$

where

$$R(y, z) \neq 0, \qquad R_1(y, 0) \neq 0.$$

Substitute in equation (ν) for x from (ρ), using for F and $\dfrac{\partial F}{\partial x}$ their values as derived above, and we have

$$z^{\nu m} L_\nu(x_\nu, y, z) F_\nu + z^{\nu(m-1)} M_\nu(x_\nu, y, z) \frac{\partial F_\nu}{\partial x_\nu} = z^\lambda R_1(y, z).$$

The left side of this equation is divisible by $z^{\nu(m-1)}$ and so the right side must be also.

$$\therefore \qquad \nu(m-1) \leqq \lambda,$$

and we have an upper limit for ν, the number of transformations which leave the order of the multiple line unchanged.

The securing of the regions of class 2) in 1, follows from the reduction just proved. If for all multiple curves of order n or less the lemma is assumed to hold, this reduction establishes it for all curves of order $n+1$, since by it the neighborhoods are represented by those of lower order. But we know it to be true for curves of the first order, and so by mathematical induction we establish it for curves of all orders.

5. *The neighborhoods of singular points in* 3, *if they are of the m-th order can be taken along the curve*

$$\phi(x, y) = 0, \qquad z = 0,$$

on the surface

$$\Phi(x, y, z) = 0.$$

In fact, the first lot of points excepted, those for which in equation (η) $q_0(y)$ vanishes, are along the line

$$x_2 = 0, \qquad z_2 = 0,$$

which is connected with the original curve by the one-to-one transformations (δ) and (ζ). Also so long as the multiple curve does not break up into simpler curves, the neighborhoods correspond, and when this reduction takes place we can cut out the neighborhoods of the points common to all of the resulting curves by cutting out neighborhoods along the original curve for the same values of y.

C. — The Reduction of the Original Singularity.

The transformations hitherto considered, when applied to the original surface $\Phi(\xi, \eta, \zeta) = 0$, make it possible to map the neighborhood of the point (0, 0, 0) of that surface on a finite number of regions which are of two classes : —

1) neighborhoods of singular points of transformed surfaces ;

2) regular pieces of transformed surfaces.

The pieces of class 2) lead at once to representation by means of parametric formulae of type (A). The singular points of class 1) are all of lower order than the original singularity except in one case, and it is this case that it remains to consider in §§ 3, 4. The case can present itself at the outset only if the polynomial $(\xi, \eta, \zeta)_m$ is the product of m linear factors in ξ, η, ζ, all vanishing for a single set of values of the arguments ξ, η, ζ not all zero. Geometrically, the tangent cone, $(\xi, \eta, \zeta)_m = 0$, of the surface $\Phi(\xi, \eta, \zeta) = 0$ at the point $(0, 0, 0)$ consists of m planes having a common line of intersection. It is found necessary to distinguish two sub-cases according to whether the planes themselves are not all coincident, or are all coincident.

To sum up, then, we already have reduction in all cases except when we are led to singular points in class 1) of the particular type just described.

§ 3.

A. — The Singular Points of Special Type.

1. In the special case in which the function $(\xi, \eta, \zeta)_m$ is composed of m linear factors, each vanishing for all points on a common line, it is possible to reduce the singularity by means of a finite succession of quadratic transformations together with certain additional transformations.

We consider two cases: —

Case A. — The m linear factors of $(\xi, \eta, \zeta)_m$ are not all equal.

Case B. — The m linear factors of $(\xi, \eta, \zeta)_m$ are all equal.

2. Case A. — $(\xi, \eta, \zeta)_m$ is composed of m linear factors not all equal.

The surface can be expressed in the form

$$\Phi(\xi, \eta, \zeta) = (\xi, \eta)_m + (\xi, \eta, \zeta)_{m+1} + \ldots\ldots = 0 \tag{13}$$

where $(\xi, \eta)_m$ contains terms in both ξ^m and η^m.

If the surface were in a form

$$f(u, v, w) = (u, v, w)_m + (u, v, w)_{m+1} + \ldots\ldots = 0$$

with the condition that the m linear factors of $(u, v, w)_m$ all vanish for the line

$$u = \alpha w, \qquad v = \beta w,$$

we could make the transformation

$$\xi = u - \alpha w, \qquad \eta = v - \beta w, \qquad \zeta = w,$$

and all the resulting linear factors would have to vanish when

$$\xi = 0, \qquad \eta = 0,$$

and so not contain ζ.

Also by a linear homogeneous transformation in ξ and η we can secure the presence of terms in ξ^m and η^m, and in such case every linear factor of $\phi(\xi, \eta)$, which here is $(\xi, \eta)_m$ itself, will contain ξ and thus secure condition 3) of § 1, 3.

B. — QUADRATIC TRANSFORMATIONS.

3. The succession of surfaces and corresponding quadratic transformations which are applied to the new singular points as found, so long as they do not reduce the degree, can be written in the form

$$\left.\begin{array}{rll}
\Phi(\xi, \eta, \zeta)=\zeta^m\Phi_1(\xi_1, \eta_1, \zeta), & \xi=\zeta(\xi_1+\gamma_1), & \eta=\zeta(\eta_1+\delta_1), \\
\Phi_1(\xi_1, \eta_1, \zeta)=\zeta^m\Phi_2(\xi_2, \eta_2, \zeta), & \xi_1=\zeta(\xi_2+\gamma_2), & \eta_1=\zeta(\eta_2+\delta_2), \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\Phi_{\nu-1}(\xi_{\nu-1}, \eta_{\nu-1}, \zeta)=\zeta^m\Phi_\nu(\xi_\nu, \eta_\nu, \zeta), & \xi_{\nu-1}=\zeta(\xi_\nu+\gamma_\nu), & \eta_{\nu-1}=\zeta(\eta_\nu+\delta_\nu).
\end{array}\right\} \quad (14)$$

Apply to the surface (13) the transformation

$$\xi = \xi_1 \zeta, \qquad \eta = \eta_1 \zeta,$$

and we have

$$\begin{aligned}
\Phi(\xi, \eta, \zeta) &= \zeta^m[(\xi_1, \eta_1)_m + \zeta(\xi_1, \eta_1, 1)_{m+1} + \ldots\ldots] \\
&= \zeta^m[(\xi_1, \eta_1)_m + \zeta\psi(\xi_1, \eta_1, \zeta)] \\
&= \zeta^m\Phi_1(\xi_1, \eta_1, \zeta).
\end{aligned} \quad (15)$$

As we assume the transformation does not reduce the degree of the singular point, there can be no term of degree less than m in the part $\zeta\psi(\xi_1, \eta_1, \zeta)$ and as all terms of this contain ζ, when we put the expression in the form

$$\Phi_1(\xi_1, \eta_1, \zeta) = (\xi_1, \eta_1, \zeta)_m + (\xi_1, \eta_1, \zeta)_{m+1} + \ldots\ldots \quad (16)$$

we will secure reduction by another quadratic transformation unless $(\xi_1, \eta_1, \zeta)_m$ is the product of m linear factors with a common line of intersection. In this case the factors cannot be all equal, for then $(\xi_1, \eta_1, 0)_m$ would have its linear factors all equal, but these are the factors of $(\xi_1, \eta_1)_m$. Also the common point of intersection of the lines in which the plane $\zeta = 1$ cuts the planes corresponding to these factors is at a finite distance. We have now the conditions 2) and 3) of § 1, 3, and are ready to apply the transformations

$$\xi_1 = \bar{\xi}_1 \zeta, \qquad \eta_1 = \bar{\eta}_1 \zeta,$$

giving

$$\Phi_1(\xi_1, \eta_1, \zeta) = \zeta^m [(\bar{\xi}_1, \bar{\eta}_1, 1)_m + \zeta(\bar{\xi}_1, \bar{\eta}_1, 1)_{m+1} + \ldots\ldots] = 0. \quad (17)$$

Now if $\bar{\xi}_1 = \gamma_2$, $\bar{\eta}_1 = \delta_2$, is the common point for which the m factors of $(\bar{\xi}_1, \bar{\eta}_1, 1)_m$ vanish, then the substitution

$$\xi_2 = \bar{\xi}_1 - \gamma_2, \qquad \eta_2 = \bar{\eta}_1 - \delta_2,$$

gives a group of mth degree terms in ξ_2 and η_2 exactly corresponding to the terms of $(\xi, \eta)_m$. So in the successive collection of terms of the mth degree, the terms of $(\xi, \eta)_m$ are always carried over with merely a change of subscript, and thus we never introduce the condition of m equal linear factors. Accordingly so long as the degree of the singular point is not reduced, the intermediate transformations are of the type

$$\begin{aligned} \xi_\mu &= \bar{\xi}_\mu \zeta, & \eta_\mu &= \bar{\eta}_\mu \zeta, \\ \xi_{\mu+1} &= \bar{\xi}_\mu - \gamma_{\mu+1}, & \eta_{\mu+1} &= \bar{\eta}_\mu - \delta_{\mu+1}, \end{aligned}$$

thus securing the succession of transformations (14).

4. The succession of transformations in 3 will lead to the relation

$$\xi^{m\nu} L(\xi_\nu, \eta_\nu\, \zeta)\bar{\Phi}_\nu + M(\xi_\nu, \eta_\nu, \zeta)\,\zeta^{(m-1)\nu} \frac{\partial \bar{\Phi}_\nu}{\partial \xi_\nu} = R(\eta, \xi) \not\equiv 0 \quad (18)$$

where

$$\Phi_\nu(\xi_\nu, \eta_\nu, \zeta) = \bar{\Phi}_\nu(\xi_\nu, \eta_\nu, \zeta)\, E(\xi_\nu, \eta_\nu, \zeta).$$

Combining transformations (14) we have the relations

$$\left.\begin{aligned} \xi &= \gamma_1 \zeta + \gamma_2 \zeta^2 + \ldots\ldots + \gamma_\nu \zeta^\nu + \zeta^\nu \xi_\nu \\ \eta &= \delta_1 \zeta + \delta_2 \zeta^2 + \ldots\ldots + \delta_\nu \zeta^\nu + \zeta^\nu \eta_\nu \end{aligned}\right\} \quad (19)$$

$$\Phi(\xi, \eta, \zeta) = \zeta^{m\nu} \Phi_\nu(\xi_\nu, \eta_\nu, \zeta)$$

As Φ contains both ξ^m and η^m terms, we can develop by Weierstrass's Theorem.

$$\left.\begin{aligned} \Phi(\xi, \eta, \zeta) &= [\xi^m + p_1(\eta, \zeta)\xi^{m-1} + \ldots\ldots + p_m(\eta, \zeta)]\, E_1(\xi, \eta, \zeta) \\ &= \bar{\Phi}(\xi, \eta, \zeta)\, E_1(\xi, \eta, \zeta) \\ \Phi(\xi, \eta, \zeta) &= [\eta^m + q_1(\xi, \zeta)\eta^{m-1} + \ldots\ldots + q_m(\xi, \zeta)]\, E_2(\xi, \eta, \zeta) \\ &= \bar{\Psi}(\xi, \eta, \zeta)\, E_2(\xi, \eta, \zeta). \end{aligned}\right\} (20)$$

As the function $E_1(\xi, \eta, \zeta)$ contains a constant term, when the first transformation of (14) is made, the factor ζ^m must come out of the Φ, and a similar result is true for all of the succeeding transformations. So in the first part of (14) we could write $\bar{\Phi}$ for Φ, $\bar{\Phi}_\mu$ for Φ_μ, $(\mu = 1, 2, \ldots\ldots \nu)$, where the $\bar{\Phi}$'s are derived successively in the same way as the Φ's. At each stage the $\bar{\Phi}$ factor must contain all the terms of lowest degree in the corresponding Φ (except for a constant multiple), and no lower terms; for, otherwise, either there would be lower terms in the product by the corresponding E factor on account of its constant term, or the required terms would not be present.

Now, by (19),

$$\frac{\partial \bar{\Phi}}{\partial \xi} = \frac{\partial \bar{\Phi}}{\partial \xi_\nu} \frac{\partial \xi_\nu}{\partial \xi} = \frac{1}{\zeta_\nu} \frac{\partial \bar{\Phi}}{\partial \xi_\nu}. \tag{21}$$

Also

$$\bar{\Phi} = \zeta^{m\nu} \bar{\Phi}_\nu\,; \tag{22}$$

and, combining with (21), we have

$$\frac{\partial \bar{\Phi}}{\partial \xi} = \zeta^{(m-1)\nu} \frac{\partial \bar{\Phi}_\nu}{\partial \xi_\nu}. \tag{23}$$

But as Φ has no multiple factors vanishing at (0, 0, 0) (see § 1, 3), we have the relation

$$L(\xi, \eta, \zeta)\bar{\Phi} + M(\xi, \eta, \zeta) \frac{\partial \bar{\Phi}}{\partial \xi} = R(\eta, \zeta) \not\equiv 0. \tag{24}$$

Then, substituting for ξ and η from (19) on the left side of equation (24) and using the relations (22) and (23), we have the required relation (18).

5. If ν is taken large enough the transformations (14) will lead to the relations

$$L_\nu(\xi_\nu, \eta_\nu, \zeta)\bar{\Phi}_\nu + M_\nu(\xi_\nu, \eta_\nu, \zeta) \frac{\partial \bar{\Phi}_\nu}{\partial \xi_\nu} = \zeta^{q_1}[\eta_\nu + \omega_1(\zeta)]^{r_1} E(\eta_\nu, \zeta), \tag{25}$$

$$P_\nu(\xi_\nu, \eta_\nu, \zeta)\Psi_\nu + Q_\nu(\xi_\nu, \eta_\nu, \zeta) \frac{\partial \bar{\Psi}}{\partial \eta_\nu} = \zeta^{q_2}[\xi_\nu + \omega_2(\zeta)]^{r_2} E(\xi_\nu, \zeta), \tag{26}$$

where

$$\Phi_\nu(\xi_\nu, \eta_\nu, \zeta) = \bar{\Phi}_\nu(\xi_\nu, \eta_\nu, \zeta) E_1(\xi_\nu, \eta_\nu, \zeta) = \bar{\Psi}_\nu(\xi_\nu, \eta_\nu, \zeta) E_2(\xi_\nu, \eta_\nu, \zeta). \tag{27}$$

We consider the effect of the transformations (14) on R in (18). Express it in the form

$$R(\eta, \zeta) = \zeta^p[(\eta, \zeta)_n + (\eta, \zeta)_{n+1} + \ldots\ldots] = \zeta^p S(\eta, \zeta),$$

where

$$S(\eta, 0) \not\equiv 0.$$

If $(\eta, \zeta)_n$ contains no term in η, the first transformation of (14) will allow the factor ζ^n to be taken out of S, leaving behind a constant term, and thus securing the form (25) at once with

$$q_1 = p + n, \qquad r_1 = 0.$$

Next, suppose $(\eta, \zeta)_n$ does contain terms in η, but no term in η^n, i. e. we can express it in the form

$$(\eta, \zeta)_n = \eta^r \zeta^s (\eta, \zeta)_{n-r-s},$$

where $(\eta, \zeta)_{n-r-s}$ contains terms in both η^{n-r-s} and ζ^{n-r-s} and $s > 0$. Then if any transformation

$$\eta = \zeta(\bar{\eta} + \delta)$$

is applied, there can be divided out of $(\eta, \zeta)_n$ the factor ζ^n leaving behind as the term of highest degree one in $\bar{\eta}^{n-s}$. This cannot be cancelled with any term from another part $(\eta, \zeta)_{n+l}$, for any term from this would have as a factor ζ^l after the ζ^n has been divided out. As long, then, as the η variable does not enter to the highest degree in the expression corresponding to $(\eta, \zeta)_n$ if $n > 0$, the degree of the S factor is decreased with each transformation, while the exponent of ζ outside may be increased. Accordingly, by a finite number of transformations, we reduce the S factor either to an E function or to an expression in which the η variable enters to the highest degree in the collection of terms of lowest order. In the former case we have the form required. In the latter case, suppose for convenience that this condition holds for the function $S(\eta, \zeta)$. By Weierstrass's Theorem we develop in the form

$$\begin{aligned} S(\eta, \zeta) &= [\eta^n + r_1(\zeta)\eta^{n-1} + \ldots\ldots + r_n(\zeta)]E(\eta, \zeta) \\ &= T(\eta, \zeta)E(\eta, \zeta). \end{aligned} \tag{28}$$

Consider the n factors of $T(\eta, \zeta)$,

$$T(\eta, \zeta) = \prod_{\lambda=1}^{n}[\eta + s_\lambda(\zeta)]. \tag{29}$$

If the factors are not all equal, pair them off, so that in each pair there will be two different factors, leaving a number of equal factors:

$$\{[\eta + s_{t_1}(\zeta)][\eta + s_{u_1}(\zeta)]\} \ldots\ldots \{[\eta + s_{t_h}(\zeta)][\eta + s_{u_h}(\zeta)]\}[\eta + s_\nu(\zeta)]^l. \tag{30}$$

Now, for each pair,

$$N_k = [\eta + s_{t_k}(\zeta)][\eta + s_{u_k}(\zeta)],$$

we have the relation

$$N_k + P_k(\eta, \zeta)\frac{\partial N_k}{\partial \eta} = L_k(\zeta) \neq 0, \tag{31}$$

since the two factors are unequal. Then, by the same reasoning as used in 4, the succession of transformations (14) which leaves the degree of T unchanged will secure for equation (31) a form

$$\zeta^{2\nu} N_{k\nu}(\eta_\nu, \zeta) + \zeta^\nu P_{k\nu}(\eta_\nu, \zeta) \frac{\partial N_{k\nu}}{\partial \eta_\nu} = L_k(\zeta) = \zeta^\lambda E(\zeta).$$

The left side of the equation is divisible by ζ^ν, and so the right side must be,

$$\therefore \qquad \nu \leqq \lambda,$$

and we have an upper limit for ν, the number of transformations which leave the factor N_k of the second degree, and as a result leave the function T of the nth degree. So, unless the function $T(\eta, \zeta)$ in (28) is composed of n equal factors of form

$$[\eta + s(\zeta)]^n, \tag{32}$$

the transformation of (14) will finally reduce its degree. Then, by applying the same reasoning to the resulting function, we see that finally the function corresponding to $S(\eta, \zeta)$ either becomes an E function or has besides the E factor a factor of form (32), thus securing the form (25) if we divide out the factor $\zeta^{(m-1)\nu}$.

The condition (26) is secured by using on the second equation in (20) the same kind of reasoning as applied in 4 and 5. Then we take for ν the larger of the two values required to secure conditions (25) and (26).

C. — Further Transformations.

6. A transformation

$$\left.\begin{aligned} \xi_\nu &= \bar{\xi}_\nu - \omega_2(\zeta) \\ \eta_\nu &= \bar{\eta}_\nu - \omega_1(\zeta) \end{aligned}\right\} \tag{33}$$

applied to the surface

$$\Phi_\nu(\xi_\nu, \mu_\nu, \zeta) = 0$$

in 5 will secure a form in which the singularity will be reduced by either

1) a further succession of transformations as in 3,

2) the method of the Lemma, § 2.

Let us consider here the case in which either r_1 or r_2 in (25) and (26) is zero. Then in one of the equations a further succession of transformations of type (14) will not change the power of ζ as a factor on the right; and if there are μ such further transformations, the reasoning of 4 shows that the left side becomes divisible by $\zeta^{(m-1)\mu}$. So we have either

$$(m-1)\mu \leqq q_1 \qquad \text{or} \qquad (m-1)\mu \leqq q_2$$

and thus an upper limit for μ, the number of transformations which leave the order of the singular point unchanged.

Now, to consider the transformation (33), we see that it is a one-to-one transformation by which the surface remains analytic near the origin. $\omega_1(\zeta)$ and $\omega_2(\zeta)$ contain no constant term, for otherwise the factor

$$\eta_\nu + \omega_1(\zeta) \qquad \text{or} \qquad \xi_\nu + \omega_2(\zeta)$$

could be combined with the E factor. Then the transformation (33) leaves the E factors still E factors, and the factors vanishing at the origin still vanishing there. Also, it is easily seen that this transformation leaves the terms of type $(\zeta, \eta)_m$ still in the part $(\xi_\nu, \eta_\nu, \zeta)_m$. Further, if the function $\bar{\Phi}_\nu(\xi_\nu, \eta_\nu, \zeta)$ goes over into $X(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)$, we have

$$\frac{\partial \bar{\Phi}_\nu}{\partial \xi_\nu} = \frac{\partial X}{\partial \xi_\nu} = \frac{\partial X}{\partial \bar{\xi}_\nu}\frac{\partial \bar{\xi}_\nu}{\partial \xi_\nu} = \frac{\partial X}{\partial \xi_\nu}$$

and similar conditions hold for the partial derivative with reference to η_ν. Accordingly, if by the transformation (33) $\Phi_\nu(\xi_\nu, \eta_\nu, \zeta)$ goes over into $\Omega(\xi_\nu, \eta_\nu, \zeta)$ we replace equations (25), (26), and (27) by

$$L_\nu(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)\, X(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta) + M_\nu(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)\frac{\partial X}{\partial \bar{\xi}_\nu} = \zeta^{q_1}\, \bar{\eta}_\nu{}^{r_1} E(\bar{\eta}_\nu, \zeta), \quad (34)$$

$$P_\nu(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)\, H(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta) + \bar{Q}_\nu(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)\frac{\partial H}{\partial \bar{\eta}_\nu} = \zeta^{q_2}\, \bar{\xi}^{r_2} E(\bar{\xi}_\nu, \zeta), \quad (35)$$

$$\Omega(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta) = X(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)\, \bar{E}_1(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta) = H(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)\, \bar{E}_2(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta). \quad (36)$$

Now, in a further succession of transformations of type (14) on the surface $\Omega(\xi_\nu, \eta_\nu, \zeta) = 0$, if there enters either a γ or a δ not 0, then on the right side of equation (34) or (35) the only factor remaining outside of the E factor is a power of ζ, and we must finally have a reduction as shown above. So it is only in the case in which all the γ's and δ's of the later transformations are 0 that we are not already sure of reducing the singularity. Now if in $\Omega(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)$ there is any term of degree less than m in $\bar{\xi}_\nu$ and $\bar{\eta}_\nu$ combined, such a succession of transformations must reduce this term to a degree less than m and thus reduce the singularity.

For suppose such a term to be $a\bar{\xi}_\nu^f \bar{\eta}_\nu^g \zeta^h$, where $f + g < m$. Then, by a succession of ρ transformations such as defined, we have

$$\bar{\xi}_\nu = \zeta^\rho \xi_{\nu+\rho}, \qquad \bar{\eta}_\nu = \zeta^\rho \eta_{\nu+\rho},$$

(derived from form of (19) when all γ's and δ's are 0). Substituting this in the expression above we get

$$a\,\xi_{\nu+\rho}^f\, \eta_{\nu+\rho}^g\, \zeta^{h+\rho(f+g)}$$

But we must divide out of this $\zeta^{m\rho}$, so that we have left the term

$$a\,\xi_{\nu+\rho}^f\, \eta_{\nu+\rho}^g\, \zeta^{h+\rho(f+g-m)}.$$

This term could not combine with any other derived in a similar way, for if we had another term $b\bar{\xi}_\nu^f \bar{\eta}_\nu^g \zeta^k$, we should get

$$b\,\xi_{\nu+\rho}^f\, \eta_{\nu+\rho}^g\, \zeta^{k+\rho(f+g-m)}$$

and this would not combine with the other unless $k = h$. Now, if the degree of the singular point is not reduced, we must have for the sum of the exponents

$$f + g + h + \rho\,(f + g - m) \geqq m$$

$$\text{or } (\rho + 1)\,(m - f - g) \leqq h,$$

$$\text{and as } m > f + g$$

$$\rho + 1 \leqq \frac{h}{m - f - g}$$

thus securing an upper limit for ρ, the number of transformations which leave the term and the singular point of the mth order.

So it is only in the case in which all terms of $\Omega(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)$ are of degree not less than m in $\bar{\xi}^\nu$ and $\bar{\eta}_\nu$ together that we do not have a reduction of singularity by the succession of transformations of type (14). But, in this exceptional case, we have the conditions of the Lemma of § 2, where in equation (θ) we take

$$\bar{\xi}_\nu = x_2, \qquad \bar{\eta}_\nu = z_2, \qquad \zeta = y_2,$$

the singular line being

$$\bar{\eta}_\nu = 0, \qquad \bar{\xi}_\nu = 0.$$

There is in $\Omega(\bar{\xi}_\nu, \bar{\eta}_\nu, \zeta)$ a term in $\bar{\xi}_\nu^m$, and so the expression $\bar{q}_0\,(y_2)$ does not vanish when $y_2 = 0$. Accordingly, within a neighborhood about this point, we can break up the singularity by the methods of

§ 2. Further, since the expression $(\bar{\xi}_\nu, \bar{\eta}_\nu)_m$ is not composed of m equal factors, the part

$$\bar{q}_0(0)\, x_2^{m_1} + \bar{q}_1(0)\, x_2^{m_1-1} z + \ldots\ldots + q_{m_1}(0)\, z_2^{m_1}$$

from (θ) which corresponds to (ξ_ν, η_ν) is not composed of m equal factors, and the resulting curve in (κ)

$$x_3^{m_1} + r_1(y_2)\, x_3^{m_1-1} + \ldots\ldots + r_{m_1}(y_2) = 0$$

has not m equal roots when $y_2 = 0$. So a single transformation of the kind in § 2, 3, reduces the singularity in the neighborhood considered here.

7. The neighborhood of the original singular point is mapped upon a finite number of neighborhoods of simpler points.

At every stage the function $(\xi_\mu, \eta_\mu, \zeta)_m$ contains the terms of the type $(\xi, \eta)_m$ found in the original equation (13). So there is but one singular point of the m-th order in the finite region of the η_μ-plane. Further, the equation $(\xi_\mu, 1, \zeta)_m = 0$ for the value $\zeta = 0$ cannot have m equal roots since $(\xi, \eta)_m$ is not a perfect m-th power of a linear factor. Accordingly, the transformation corresponding to (8) in § 1, 4, cannot produce a singular point of the m-th order. So, at each step, the neighborhood of the singular point is represented by a number of regions as in § 2, C, in which but one of the points of class 1) is of the mth order. Further, the extra transformations (33) carry the neighborhood of the singular point over into that of the new point. So, by combining all the representations, as the singularity is finally reduced, we have the original neighborhood mapped upon a finite number of regions as in § 2, C, in which all points of class 1) are of order lower than m.

§ 4.

A. — THE SINGULAR POINTS OF SPECIAL TYPE (*continued*).

1. CASE B. — The m linear factors of $(\xi, \eta, \zeta)_m$ are all equal.

The surface can be expressed in the form

$$\begin{aligned}\Phi(\xi, \eta, \zeta) &= [\xi^m + p_2(\eta, \zeta)\,\xi^{m-2} + \ldots\ldots + p_m(\eta, \zeta)]\, E(\xi, \eta, \zeta) \\ &= X(\xi, \eta, \zeta)\, E(\xi, \eta, \zeta) = 0, \qquad (37)\end{aligned}$$

where, in X, ξ^m is the only term of degree m.

If it were in the form

$$f(u, v, w) = (\alpha u + \beta v + \gamma w)^m + (u, v, w)_{m+1} + \ldots\ldots = 0,$$

as one of the three numbers, α, β, γ, is not zero, suppose $\alpha = 0$.

Then by a linear homogeneous transformation

$$\left.\begin{aligned}\bar{u} &= \alpha u + \beta v + \gamma w\\ \bar{v} &= v\\ \bar{w} &= w\end{aligned}\right\}$$

we secure the form

$$\begin{aligned} f(u, v, w) &= \bar{f}(\bar{u}, \bar{v}, \bar{w})\\ &= \bar{u}^m + (\bar{u}, \bar{v}, \bar{w})_{m+1} + \ldots\ldots \end{aligned}$$

By Weierstrass's Theorem we can express this in the form

$$f(\bar{u}, \bar{v}, \bar{w}) = \left[\bar{u}^m + \bar{p}_1(\bar{v}, \bar{w})\,\bar{u}^{m-1} + \ldots\ldots + \bar{p}_m(\bar{v}, \bar{w})\right] E(\bar{u}, \bar{v}, \bar{w}). \qquad (38)$$

Now, in the expression

$$\bar{p}_\lambda(\bar{v}, \bar{w}), \qquad \lambda = 1, 2, \ldots\ldots m$$

there is no term of degree less than $\lambda + 1$, for otherwise on account of the constant term in the E factor, there would have to be present in $\bar{f}$ a term of degree $\leqq m$ containing $\bar{v}$ or $\bar{w}$.

Make in (38) the transformation

$$\left.\begin{aligned}\bar{u} + \tfrac{1}{m}\bar{p}_1(\bar{v}\ \bar{w}) &= \xi\\ \bar{v} \qquad\qquad &= \eta\\ \bar{w} \qquad\qquad &= \zeta\end{aligned}\right\}$$

As $\bar{p}_1(\bar{v}, \bar{w})$ contains no term of degree less than 2, by the consideration above, f goes over into form (37).

B. The Quadratic Transformation.

2. The transformation

$$\xi = \bar{\xi}\zeta, \quad \eta = \bar{\eta}\zeta,$$

applied to $\Phi(\xi, \eta, \zeta)$ secures the form

$$\Phi(\xi, \eta, \zeta) = \zeta^m \bar{\Phi}(\bar{\xi}, \bar{\eta}, \zeta) = \zeta^m[\bar{\xi}^m + \zeta\bar{\Psi}(\bar{\xi}, \bar{\eta}, \zeta)]. \qquad (39)$$

Here the curve $\phi(\bar{\xi}, \bar{\eta}) = 0$ becomes $\bar{\xi}^m = 0$, and so, applying the Lemma of § 2 to a circle in the $\bar{\eta}$-plane however large, we have within it but a finite number of singular points to treat further. But one such circle is needed, for by taking it large enough we can deal with all of the $\bar{\eta}$-plane outside of that circle by the transformation

$$\xi = \bar{\bar{\xi}}\eta, \qquad \zeta = \bar{\bar{\zeta}}\eta.$$

So we need to consider for further treatment only a finite number of points along the line $\bar{\xi} = 0$, and the point at infinity.

3. The quadratic transformations to be used are of two types

$$1)\quad \xi_\mu = \xi_{\mu+1}\zeta_\mu, \qquad \eta_\mu = (\eta_{\mu+1} + \delta_{\mu+1})\zeta_\mu, \tag{40}$$

$$2)\quad \xi_\mu = \xi_{\mu+1}\eta_\mu, \qquad \zeta_\mu = (\zeta_{\mu+1} + \epsilon_{\mu+1})\eta_\mu. \tag{41}$$

In a succession of transformations of type (14) we see that $\gamma_1 = 0$, since the first set of points is taken on the line $\bar{\xi} = 0$. Further, suppose after the substitution $\bar{\eta} - \delta_1 = \eta_1$ in $\bar{\Phi}$ of (39) the expression

$$(\bar{\xi}, \eta_1\, \zeta)_m$$

contains terms besides the $\bar{\xi}^m$; then it cannot be composed of m equal linear factors, for that would require a term containing ξ^{m-1}; but no such term can arise from the factor X of (37), and, on the other hand, it could not be the product of a term from X by a non-constant term of the E factor, for then, on account of the constant term of the E factor, there would have to be present in $\bar{\Phi}$ a term of degree lower than m. So as soon as the function corresponding to ϕ of $\bar{\Phi}$ contains more than the mth power of the ξ variable, the function corresponding to $(\xi, \eta, \zeta)_m$ is no longer the product of m equal linear factors, and we have one of the cases treated earlier.

The same considerations apply to the transformations corresponding to type 2), since, when the transformation which deals with the infinite region is introduced, the first one of that order is of form

$$\xi = \bar{\bar{\xi}}\eta, \qquad \zeta = \bar{\bar{\zeta}}\eta.$$

Accordingly, the most general succession of transformations here is one in which groups of types 1) and 2) alternate. We shall call them the ζ and η types respectively, and when a change is made from one type to the other, we shall speak of it as a *reversal of type.*

We shall treat the subject in two cases, first supposing that there is no reversal of type in the succession of transformations used, and later supposing that reversals of type occur.

C. — SUCCESSION OF QUADRATIC TRANSFORMATIONS IN WHICH THERE IS NO REVERSAL OF TYPE.

4. After a sufficient number of quadratic transformations the surface can be reduced to the form

$$\bar{\Phi}_\nu(\xi_\nu, \bar{\eta}_\nu, \zeta) = \bar{X}_\nu(\xi_\nu, \bar{\eta}_\nu, \zeta)\, E(\xi_\nu, \bar{\eta}_\nu, \zeta) =$$
$$[(\xi_\nu^m + \bar{\eta}_\nu^{p_2}\zeta^{q_2} E(\bar{\eta}_\nu, \zeta)\,\xi_\nu^{m-2} + \ldots + \bar{\eta}_\nu^{p_\nu}\,\zeta^{q_\nu} E(\bar{\eta}_\nu, \zeta)]\, E(\xi_\nu, \bar{\eta}_\nu, \zeta), \tag{42}$$

while all later transformations can be taken of the type

$$\xi_\mu = \xi_{\mu+1}\zeta, \qquad \eta_\mu = \eta_{\mu+1}\zeta. \tag{43}$$

After ν transformations of type (40), since there can be no interchange of terms among the coefficients of the different powers of the ξ variables in the X factor of (37), the surface will take the form

$$\Phi_\nu(\xi_\nu, \eta_\nu, \zeta) = X_\nu(\xi_\nu, \eta_\nu, \zeta)\,E(\xi_\nu, \eta_\nu, \zeta) = \\ [\xi_\nu^m + p_{2\nu}(\eta_\nu, \zeta)\,\xi_\nu^{m-2} + \ldots\ldots + p_{m\nu}(\eta_\nu, \zeta)]\,E(\xi_\nu, \eta_\nu, \zeta) = 0. \tag{44}$$

Now by the same reasoning as used for the function R in § 3, 5, if ν is taken large enough, the coefficients of the powers of ξ_ν in X_ν will all be of the type

$$p_{s\nu}(\eta_\nu, \zeta) = \zeta^{q_s}[\eta_\nu + \nu_s(\zeta)]^{r_s}E(\eta_\nu, \zeta),$$
$$s = 2, 3, \ldots\ldots m.$$

For any one of the functions

$$\eta_\nu + \nu(\zeta)$$

there is a determinate succession of transformations of type

$$\eta_\mu = \zeta(\eta_{\mu+1} + \delta_{\mu+1})$$

which will leave it of the same degree after the ζ is divided out, all others reducing the degree at once, i. e., if

$$\eta_\mu + \nu(\zeta) = \eta_\mu + a_1\zeta + a_2\zeta^2 + \ldots\ldots,$$

we must take

$$\eta_\mu = \zeta(\eta_{\mu+1} - a_1),$$
$$\eta_{\mu+1} = \zeta(\eta_{\mu+2} - a_2),$$
$$\vdots$$
$$\text{etc.}$$

So, unless the factors

$$\eta_\nu + \nu_s(\zeta), \qquad s = 2, 3, \ldots\ldots m$$

are all equal, we must have finally some coefficient of a power of ξ_ν with the η_ν present only in the E factor, and by taking ν large enough we come to a point where all the factors

$$\eta_\nu + \nu_s(\zeta), \qquad s = 2, 3, \ldots\ldots m,$$

are equal, some of them possibly having zero exponents.

Then we use the transformation

$$\eta_\nu + \nu_s(\zeta) = \bar{\eta}_\nu \tag{45}$$

and arrive at the form (42) required. Now any further transformation of type (40) in which the δ is not zero will leave the η variable present only in the E factors, so that the general term (after the first) of the function X_ν is of type

$$\zeta^{q_s} E(\bar{\eta}_\nu, \zeta)\, \xi_\nu^{m-s}, \qquad s = 2, 3, \ldots\ldots m.$$

Suppose, after this, there are ρ transformations of type (40). Then the corresponding term after the factor $\zeta^{m\rho}$ has been divided out is

$$\zeta^{q_s + (m-s)\rho - m\rho} E(\eta_{\nu+\rho}, \zeta)\, \xi_{\nu+\rho}^{m-s}$$

and if this is of degree not less than m, as it must be if we are not to secure reduction, we have

$$m - s + q_s - \rho s \geqq m$$

or

$$\rho \leqq \frac{q_s - s}{s},$$

thus securing an upper limit for the number of transformations of type (40) which do not give reduction of singularity. Accordingly, after the form (42) is reached, it is only when all later δ's are zero that we are not sure of reduction.*

5. A sufficient number of transformations of type (43) applied to (42) secures either

1) reduction of singularity, or
2) the condition that for some term (the rth) of the X factor

$$\left.\begin{array}{l} \dfrac{p_r}{r} \leqq \dfrac{p_s}{s} \\[2ex] \dfrac{q_r}{r} \leqq \dfrac{q_s}{s} \end{array}\right\} \quad s = 2, 3, \ldots\ldots m.$$

If, for any term

$$\bar{\eta}_\nu^{p_r} \zeta^{q_r} E(\bar{\eta}_\nu, \zeta)\, \xi_\nu^{m-r}, \qquad p_r < r,$$

a transformation of type (43), after the factor ζ^m has been divided out, yields

$$\eta_{\nu+1}^{p_r} \zeta^{q_r + p_r - r} E(\eta_{\nu+1}, \zeta)\, \xi_{\nu+1}^{m-r},$$

decreasing the exponent of ζ by $r - p_r$. This decrease takes place at every such transformation, and thus the exponent of ζ must finally be

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* We do not need to consider the possibility of having all the coefficients of the powers of ξ_ν lower than the m-th vanish, for then the function X_ν would have m equal factors ξ_ν and this case has been excluded.

reduced to a value q'_r less than $r - p_r$, in which case the sum of the exponents of the three variables,

$$p_r + q'_r + m - r,$$

is less than m and reduction ensues. So it is only in the case in which for every term

$$p_s \geqq s, \qquad s = 2, 3, \ldots\ldots m,$$

that we are not sure of reduction. Suppose the number of transformations after this point to be n. Then we get for the new exponent of ζ

$$q_s + n(p_s - s).$$

Now by taking n large enough we can make the quotient

$$\frac{n(p_s - s) + q_s}{s}$$

have the lowest value for the term in which $\frac{p_s - s}{s}$ is lowest, while if this is the same for two or more terms, we can make the fraction above lowest for the one in which $\frac{q_s}{s}$ is lowest. Accordingly, by a finite number of transformations of type (43) we secure the condition that $\frac{p_r - r}{r}$ and so $\frac{p_r}{r}$ is lowest in the same term in which $\frac{q_r}{r}$ is lowest.

6. A succession of transformations of type

$$\xi_\mu = \xi_{\mu+1}\zeta, \tag{46}$$

followed by a succession of type

$$\xi_\mu = \xi_{\mu+1}\eta, \tag{47}$$

secures the surface with condition 5, 2) in the form

$$\left.\begin{array}{l} \Phi_\rho(\xi_\rho, \eta_\rho, \zeta) = X_\rho(\xi_\rho, \eta_\rho, \zeta)\, E(\xi_\rho, \eta_\rho, \zeta) \\ = [\xi_\rho^m + \eta_\rho^{p_2}\zeta^{q_2} E(\eta_\rho, \zeta)\,\xi_\rho^{m-2} + \ldots + \eta_\rho^{p_m}\zeta^{q_m} E(\eta_\rho, \zeta)]\, E(\xi_\rho, \eta_\rho, \zeta) \\ \text{where for some particular term in } X_\rho, \text{ the } r\text{th}, \\ \qquad\qquad p_r < r, \qquad q_r < r. \end{array}\right\} \tag{48}$$

Consider the surface (42) with the condition 5, 2), the sth term being

$$\bar{\eta}_\nu^{p_s}\zeta^{q_s} E(\bar{\eta}_\nu, \zeta)\,\xi_\nu^{m-s},$$

and suppose we apply to the surface n transformations of type (46), dividing out each time the factor ζ^m. The resulting term is

$$\bar{\eta}_\nu^{p_s}\zeta^{q_s - ns} E(\bar{\eta}_\nu, \zeta)\,\xi_{\nu+n}^{m-s}.$$

If n is taken large enough, the exponent

$$q_s - ns$$

is made less than s, so that we have

$$s > q_s - ns \geqq 0,$$

or

$$\frac{q_s}{s} - 1 < n \leqq \frac{q_s}{s}.$$

So the term for which $\frac{q_s}{s}$ is least is among those first reached in which the new exponent of ζ is lower than s.

In the same way we show that, by applying a succession of transformations of type (47), the term for which $\frac{p_s}{s}$ is least is among the first lot reached for which the new exponent of $\bar{\eta}_\nu$ is less than s. But, by condition 2) in 5, $\frac{p_s}{s}$ and $\frac{q_s}{s}$ were least in the same term. So we secure the surface in form (48).

7. A further succession of quadratic transformations of type (43) as applied to the surface in form (48) will reduce the singularity. This follows at once by the reasoning in the first part of 5.

D. — SUCCESSION OF QUADRATIC TRANSFORMATIONS IN WHICH THERE ARE REVERSALS OF TYPE.

8. A succession of transformations in which there is a sufficient number of reversals of type will secure a surface of type (42).

If there is but a finite number of reversals, after the last one we are in the same position as at the start in 4, and the succession of transformations which follows, not having any reversal of type, will enable us to secure the condition derived by the method of 4. So we need here to consider only the possibility of an indefinitely large number of reversals of type.

In equation (37) consider any one of the coefficients

$$p_r(\eta, \zeta) = \zeta^\lambda \bar{p}_r(\eta, \zeta) = \zeta^\lambda[(\eta, \zeta)_{n_r} + (\eta, \zeta)_{n_r+1} + \cdots\cdot]$$

where

$$\bar{p}_r(\eta, 0) \not\equiv 0.$$

A transformation of type (40) will give for $\bar{p}_r$ a function from which we take out the factor ζ^{n_r}, the other factor being of degree less than n_r unless the part $(\eta, \zeta)_{n_r}$ has n_r equal linear factors. For, if

$$(\eta, \zeta)_{n_r} = \prod_{\rho=1}^{n_r} (a_\rho \eta - \beta_\rho \zeta)$$

and not all the linear factors are equal (or linearly dependent), then the substitution

$$\eta = \zeta(\eta_1 + \delta_1)$$

gives

$$\zeta^{n_r} \prod_{\rho=1}^{n_r} (a_\rho \eta_1 + a_\rho \delta_1 - \beta_\rho)$$

and leaves an absolute term in any factor for which

$$a_\rho \delta_1 \mp \beta_\rho,$$

thus securing in the product of the factors terms of degree less than n_r. Also the degree might be lowered on account of terms in some later part as $(\eta, \zeta)_{n_r+k}$. But, if all the factors of $(\eta, \zeta)_{n_r}$ are equal (or linearly dependent) and δ_1 is taken so as to satisfy the condition

$$a_\rho \delta_1 = \beta_\rho, \qquad \rho = 1, 2, \ldots\ldots n_r,$$

then after the factor ζ^{n_r} is divided out, we have left but one term in $\eta_1^{n_r}$, which cannot cancel with any term from another part of the function, as all later terms have as a factor some power of ζ. Accordingly a succession of transformations of type (40), if it does not reduce the degree of the part not divisible by ζ, must leave a term in $\eta_\nu^{n_r}$. Now when the reversal of type is first made, the ϵ of (41) is zero, as is seen by considering the use of transformation (8) § 1, 5. Then we take out a factor $\eta_\nu^{n_r}$ and leave a constant term. So a succession of transformations which contains reversals of type must reduce the degree of the function p_r (possibly to zero), except for factors taken out which are powers of the η and ζ variables. Accordingly, by a succession of transformations containing a sufficiently large number of reversals of type, the coefficient p_r must be reduced to the type

$$\eta^{p_r} \zeta^{q_r} E(\eta, \zeta).$$

9. All further transformations to be considered may be taken of the types

$$\xi_\mu = \xi_{\mu+1} \zeta, \qquad \eta_\mu = \eta_{\mu+1} \zeta, \tag{49}$$

$$\xi_\mu = \xi_{\mu+1} \eta, \qquad \zeta_\mu = \zeta_{\mu+1} \eta. \tag{50}$$

For if a transformation of type (40) or (41) in which the δ or ϵ is not zero were used, we should have in all the coefficients of X_ν in (42), outside of the E factor, only powers of one variable. Suppose it to be ζ; then, by means of a succession of transformations of type (46), we can reduce some term to a form in which the exponent of ζ is less than r, and thus secure a reduction of singularity.

10. A sufficiently long succession of transformations of types (49) and (50), applied to surface of type (42), unless it first secures reduction of singularity, will secure the condition that, for some term (the rth),

$$\left.\begin{array}{l} \dfrac{p_r}{r} \leqq \dfrac{p_s}{s} \\[2ex] \dfrac{q_r}{r} \leqq \dfrac{q_s}{s} \end{array}\right\}, \qquad s = 2, 3, \ldots\ldots m.$$

Consider the two terms

$$\eta^{p_s}\zeta^{q_s}E(\eta, \zeta)\,\xi^{m-s}, \qquad\qquad \eta^{p_t}\zeta^{q_t}E(\eta, \zeta)\,\xi^{m-t}$$

Any transformation of type (49) leaves the p_s and p_t unchanged, and increases the

$$\begin{array}{l} q_s \text{ by } p_s - s, \\ q_t \text{ `` } p_t - t. \end{array}$$

Any transformation of type (50) leaves the q_s and q_t unchanged, but increases the

$$\begin{array}{l} p_s \text{ by } q_s - s, \\ p_t \text{ `` } q_t - t. \end{array}$$

Represent

$$\left.\begin{array}{l} \dfrac{q_r - r}{r} \text{ by } \mathrm{K}_r, \\[2ex] \dfrac{p_r - r}{r} \text{ `` } \Pi_r, \end{array}\right\} \qquad r = 2, 3, \ldots\ldots m, \qquad (51)$$

So, for each transformation of type (49) the K_r is increased by the Π_r, and for each transformation of type (50) the Π_r is increased by the K_r. We shall show that finally we must have one of two conditions

$$\begin{array}{ll} a)\ \Pi_s \geqq \Pi_t, & \mathrm{K}_s \geqq \mathrm{K}_t, \\ b)\ \Pi_s \leqq \Pi_t, & \mathrm{K}_s \leqq \mathrm{K}_t. \end{array}$$

Suppose, at any stage, neither of these conditions holds, and we have, for example,

$$\Pi_s > \Pi_t, \qquad \mathrm{K}_s < \mathrm{K}_t. \qquad (52)$$

Then, for a transformation of type (49), supposing the new K's to be K_s', K_t', we have

$$K_s' = K_s + H_s,$$
$$K_t' = K_t + H_t,$$

and so

$$K_t' - K_s' = K_t - K_s - (H_s - H_t) < K_t - K_s.$$

Also, for a transformation of type (50), if the new H's are H_s' H_t', we have

$$H_s' = H_s + K_s,$$
$$H_t' = H_t + K_t,$$

and

$$H_s' - H_t' = H_s - H_t - (K_t - K_s) < H_s - H_t.$$

So when a condition of type (52) holds, any transformation applied will reduce the difference of either the H's or K's, if in fact it does not change the sign of the difference. Further, the reduction is each time by a value not infinitesimal, for it is at least $1 / st$, as is seen by considering the values of K_r and H_r in (51). So the succession of transformations of whatever kind must finally reduce the difference of either the H's or the K's to zero, or change its sign, and then we secure either condition *a*) or *b*).

When one of these conditions has once been secured, any further transformation will not change it; for, in condition *a*), a transformation of type (49) will add at least as much to the K_s as to the K_t, and so retain the inequality of the same order, and similar conditions are seen to hold in the other cases. Also, as one of the conditions *a*) or *b*) must hold finally, whatever the pair of values s and t, we shall have some value as r such that

$$H_r \leqq H_s, \qquad K_r \leqq K_s, \qquad s = 2, 3, \ldots\ldots m.$$

from which follows the required condition

$$\left.\begin{aligned} \frac{p_r}{r} &\leqq \frac{p_s}{s} \\ \frac{q_r}{r} &\leqq \frac{q_s}{s} \end{aligned}\right\} \qquad s = 2, 3, \ldots\ldots m.$$

11. The method of 6, applied to the surface resulting from the treatment of 10, will secure the result of 6. It may be that already either $p_r < r$ or $q_r < r$, but in such a case the number of transformations of

type (46) or (47) can be considered zero, while in the other case we have exactly the initial conditions of 6, the result of which then can be secured in any case whatever.

12. In the case of surface (48) any succession of transformations of types (49) and (50) will finally reduce the degree of the singular point.

Consider the term

$$\eta_\rho^{p_r} \zeta^{q_r} E(\eta_\rho, \zeta) \xi_\rho^{m-r}.$$

Any transformation of type (49) adds to the exponent of ζ, $p_r - r$, and as $p_r < r$, the exponent of ζ is reduced. In the same way we see that any transformation of type (50) reduces the exponent of the η variable. So in any case, by virtue of the reduction of degree, we must have finally either

$$p_r < r - q_r \qquad \text{or} \qquad q_r < r - p_r,$$

in either of which cases the sum of the exponents of the three variables

$$(m - r) + p_r + q_r$$

is less than m, and we have reduction of the singularity.

§ 5.

PARAMETRIC REPRESENTATION OF THE NEIGHBORHOOD OF THE ORIGINAL SINGULAR POINT.

We have shown that in all cases T, the neighborhood of a singular point, can be mapped upon a finite number of regions $t_1, t_2, \ldots\ldots t_\nu$ as defined in § 2, C. Apply a properly chosen transformation to each point of class 1) and repeat the operation on each set of resulting points of the same class as they are formed. We have proved also that after a finite number of operations all the resulting points of class 1) are of order lower than m. Then, by continuing the process, it follows that, after a finite number of transformations, all points of class 1) must disappear, and so we shall have left only regions of class 2). Each of these regions admits of representation by means of a finite number of sets of parametric formulae of type (A).

Classify all the singular points which present themselves in groups as follows: —

In the first group, place the original point; in the second, all singular points derived from it by the first quadratic transformation, together with whatever auxiliary transformations accompany it; these points correspond to the singular points of the curves that represent the irreducible factors of $\phi(\xi, \eta)$, to the points of intersection of two such curves, and to the points of class 1) in § 2, 1. In the third group place all singular points derived in a similar way from those of the second group, etc.

Suppose n to be the number of the last group in which there are singular points. From what we have proved, n must be finite.

The neighborhood of a point in the nth group is represented by the neighborhoods of a finite number of regular points, together with a finite number of regular regions, and so by a finite number of parametric formulae of type (A). The neighborhood of a point in the $(n-1)$st group is represented by the neighborhoods of a finite number of points of the nth group, together with a finite number of regular regions, however small the neighborhoods of the singular points are taken; but as the neighborhood of any point in the nth group is represented by a finite number of parametric formulae of type (A), the same follows for any point of the $(n-1)$st group, using the intermediate transformation to get the parametric formulae.

This reasoning can be carried on until the original singular point is reached, since the mapping of the neighborhood of the original point upon a finite number of regions of classes 1) and 2) applies to each of the later singular points also, and then furnishes the step by which we know that we can always pass from the $(\nu+1)$st to the νth group.

Thus we have the coördinates ξ, η, ζ of the surface

$$\Phi(\xi, \eta, \zeta) = 0$$

expressed in parametric formulae of the desired type, the parameters being in general coördinates of points of some simple surface. Then by using the intermediate transformations connecting x, y, z with ξ, η, ζ, we represent the first set of coördinates in the desired form.

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CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF HARVARD UNIVERSITY. — XLVIII.

A PRELIMINARY ENUMERATION OF THE SOROPHOREÆ.

By Edgar W. Olive.

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A PRELIMINARY ENUMERATION OF THE SOROPHOREÆ.

By Edgar W. Olive.

Presented by Roland Thaxter. Received November 9, 1901.

Owing to unavoidable delay in the publication of a monograph of the Acrasieæ and their allies which the writer has in preparation and for which figures have already been drawn, the following preliminary synopsis, which includes all the known forms and which will be supplemented as soon as possible by the more extended paper, has seemed advisable. This investigation was undertaken some years since at the suggestion of Professor Thaxter, and a majority of those species that I have myself studied have been kept under observation in pure cultures for a long period, so that the constancy of the characters distinguishing them has been definitely determined. As far as I am aware only one member of the group has been heretofore reported from America, although certain of them are very abundant in laboratory cultures. Of the European representatives several remain unknown except through the original diagnoses, which are unfortunately, in a majority of cases, meagre and unaccompanied by figures.

A comparison of the conditions presented by the individuals which constitute the so-called fructifications of these organisms indicates that the term *spore* cannot be properly applied to them in all cases. In the genera Sappinia and Guttulinopsis the individuals, even in mature fructifications, are merely slightly contracted and hardened, secreting no definite wall. At germination such resting individuals, therefore, gradually assume the form of a vegetative amœba without casting off a spore wall of any kind. In order to distinguish these bodies from true spores, such as occur in a majority of the genera, as well as from the transitory resting conditions of isolated vegetating amœbæ which were first characterized as "microcysts" by Cienkowsky, the term *pseudospore* is here employed, since it expresses with sufficient exactness the actual conditions.

It will be noted further that in characterizing the Acrasieæ as a whole, emphasis has been laid on the fact, usually overlooked in accounts of these organisms, that the vegetative stage ends before the pseudoplasmodium condition begins. The latter, therefore, is a phenomenon connected not with vegetation but with fructification, and is by no means homologous with the plasmodium of true Myxomycetes; nor is it comparable to the vegetative net-plasmodium of the Labyrinthuleæ.

I have followed Zopf, moreover, in characterizing as a "net-plasmodium" the peculiar form of association occurring in the Labyrinthuleæ, although it appears to be doubtful whether, in all cases at least, the condition thus distinguished represents a true fusion, or whether the relation is merely one of contact.

SOROPHOREÆ Zopf.

Amœbæ of the usual irregular myxamœba form or more or less regular and spindle-shaped, never possessing a swarm spore stage, forming either a pseudoplasmodium or a net-plasmodium; resting bodies borne in sessile or stalked sori, which are either naked or imbedded in a gelatinous matrix.

ACRASIEÆ Van Tieghem.

Saprophytic, usually coprophilous, organisms, having two definitely recurring stages, — a vegetative period, in which independent myxamœbæ crawl about by means of amœboid movements and undergo multiplication by division; and a fructifying period, in which the myxamœbæ typically aggregate into colonies called pseudoplasmodia and form either spores or pseudospores, held together by a mucus substance, and borne in stalked or sessile naked masses, or sori.

SAPPINIACEÆ.

Myxamœbæ comparatively large, with lobose pseudopodia. The resting stage consisting either of a single encysted individual or of many individuals encysted in masses at the ends of projections of the substratum.

This group is included here only provisionally, since the amœbæ normally become encysted singly, thus forming microcysts, and do not show the characteristic phenomenon of aggregation, or colony formation. The aggregations which, it is true, often occur at the distal ends of small projections above the surface of the substratum, are not due to any chemotactic stimulus such as must be assumed to cause the formation of true pseudoplasmodia, but, although they may perhaps suggest the possible beginnings of such conditions, are probably accidental, resulting rather from a tendency of the amœbæ to seek drier situations at the period of fructification.

SAPPINIA Dangeard (1896).

Characters of the order.

Sappinia pedata Dangeard.

Le Botaniste, 5 Ser. p. 1–20. 5 Figs in text. 1896.

Amœbæ forming resting conditions of three kinds: "*amibes pédicellées*," in which they are transformed into a pear-shaped body without definite wall raised above the substratum by a stalk of about equal length; "*kystes pédicellés*," in which they are similarly modified but which form a definite wall about the oval body; and "*spores*," in which groups of individuals become encysted at the ends of projections from the substratum.

On dung of horse, cow, dog. France; Russia; Massachusetts; Indiana.

At least two species of this genus appear to be common on various kinds of dung in this country, but owing to the fact that Dangeard gives no measurements I have been uncertain which of them should be referred to *S. pedata*. In both forms resting bodies comparable to the aggregated "*spores*" occur, as well as "*amibes pédicellées*," although I have not as yet observed the definitely walled "*kystes*" which Dangeard appears to distinguish from them.

The larger and more frequent of the American species, which I have assumed to belong to *S. pedata*, has the following measurements: stalk of the "*amibes pédicellées*" 30μ–125μ, head 30μ–60μ long; rounded individuals ("*spores*") of the aggregations 20μ–50μ in diameter.

GUTTULINACEÆ Zopf.

Myxamœbæ either limax-shaped, without pseudopodia, or of the ordinary form with rounded or lobose short pseudopodia. The sori, irregular in shape or spherical, sessile or stalked, consisting of either spores or pseudospores.

GUTTULINOPSIS nov. gen.

Myxamœbæ having lobose pseudopodia. Sori sessile or stalked, composed of pseudospores, those of the stalk usually slightly elongated.

Guttulinopsis vulgaris nov. sp.

Sori usually stalked, sometimes sessile, about 150μ–500μ in height × 150μ–400μ broad. Fructifications varying in color from whitish to dirty yellowish according to the character of the substratum and the dryness of the sorus. Pseudospores usually irregularly spherical, about 4μ–8μ in diameter.

On dung of horse, cow, pig, mouse, etc. Cambridge, Mass; Alabama; Indiana; Maine; Porto Rico.

This form, which has conspicuous fructifications so large that they may be readily seen with the naked eye, has been met with very frequently on fresh cultures of various kinds of dung. Although *Guttulina aurea* Van Tieghem may prove to be identical with the above species, the fact that, according to the original description, it possesses resting bodies which are characterized as "spores," having a golden yellow color, renders it improbable that the two forms are the same.

Guttulinopsis stipitata nov. sp.

Sori yellowish white, long stalked, the stalk composed of individuals similar to those of the head. Sorus about 1 mm.–1.2 mm. high; the stalk about 800μ long, the head 250μ in diameter. Pseudospores spherical, 3μ–5μ in diameter.

On dung of dog. New Haven, Conn.

This species, the largest representative of the genus, has been met with but once, and is founded on a mounted specimen and dried material collected at New Haven some years ago by Dr. Thaxter.

Guttulinopsis clavata nov. sp.

Sori yellowish white when young, comparatively long-stalked, the stalk composed of a column of slightly elongated individuals surrounded by mucus. The stalk-cells held within the peripheral mucus adhere together after the deliquescence of the pseudospores of the head, forming at the apex a rounded or conical columella of elongated adherent cells. Sorus about 400μ–800μ in height, the stalk about 170μ–250μ long, the head 100μ–400μ in diameter. Pseudospores of the head somewhat broadly oval, 3μ–4μ × 6μ – 7μ, or spherical, then 4μ–5μ in diameter; those of the stalk about 3μ–5μ × 7μ–10μ.

On dung of dog. Cambridge, Mass.; Indiana.

This distinct species is frequently met with in fresh cultures of the dung on which it has its habitat. The base of the stalk is often imbedded in an abundant mucus, which is especially noticeable when it swells after being placed in water.

GUTTULINA Cienkowsky (1873).

Myxamœbæ limax-shaped, without pseudopodia. Sori irregular in shape or spherical, sessile or stalked, composed of spores which have a definite protective cell-wall. The cells of the stalked forms somewhat differentiated in shape.

Guttulina rosea Cienkowsky.

Trans. 4th Session of Russ. Nat. at Kazan, 1873.

"Sori short-stalked and rose-colored; head 700μ long, supported upon a stalk of about equal length. Spores of the head spherical; those of the stalk closely laid and wedge-shaped."

On dead wood. Russia.

Known only from the original description above quoted.

Guttulina protea Fayod.

(*Copromyxa protea* Zopf.)

Bot. Zeit., 11, p. 167–177. 1 Plate. 1883.

Sori 1–3mm. high, sessile or short-stalked, of somewhat irregular form, yellowish white, with crystalline lustre. Spores $9\mu \times 14\mu$; hyaline, colorless or slightly yellowish, more or less oblong or oval, bean-shaped, or almost triangular in outline.

On dung of horse and cow. Germany.

This form, which is known only from Fayod's original description, is retained under its original name, notwithstanding the fact that it has been separated by Zopf under the name *Copromyxa* on the ground that the "myxamœbæ undergo no differentiation into stalk and head cells, whereas in Cienkowsky's form, there is a slight differentiation." The fact that certain species of *Guttulinopsis* show both stalked and sessile forms in the same culture diminishes the importance of the stalk as a character of generic value and justifies the resumption of the original name given by Fayod.

Guttulina aurea Van Tieghem.

Bull. de la Soc. Bot. de France, XXVII. p. 317. 1880.

"*Guttulina aurea* has its fruit pedicelled and resembles closely *G. rosea*, but differs in color. The spores spherical, 4μ–6μ, golden-yellow. Upon dung of horse." France.

Guttulina sessilis Van Tieghem.

Bull. de la Soc. Bot. de France, XXVII. p. 317. 1880.

"Fruit sessile; a simple droplet of pure white, resting directly on the substratum. Spores oval, colorless, aggregated in a sphere and cemented, as in the preceding species, by a gelatinous substance; $4\mu \times 8\mu$. On the integument of beans in a state of decay." France.

Guttulina aurea and *G. sessilis* are known only from the original descriptions above quoted.

DICTYOSTELIACEÆ Rostafinski.

Myxamœbæ possessing slender elongated pseudopodia. Sori consisting of spherical masses of spores or of a chain of spores; stalked, the stalks composed of distinct parenchyma-like cells with cellulose walls.

ACRASIS Van Tieghem (1880).

Spores concatenate, terminating an erect simple filament, consisting of a single row of superposed cells.

Acrasis granulata Van Tieghem.

Bull. de la Soc. Bot. de France, XXVII. p. 317. 1880.

Spores spherical, with a slightly roughened or granular wall, having a cuticularized external portion of deep violet color; 10μ–15μ in diameter, often unequal in the same chain, the chain varying much in the number of component spores and cells.

On a culture of beer yeast. France.

Known only from the original description.

DICTYOSTELIUM Brefeld (1869).

Sori stalked; the stalk simple or only occasionally bearing irregularly disposed branches; luxuriant fructifications frequently gregarious. Sori spherical, or subglobose.

Dictyostelium mucoroides Brefeld.

(*Ceratopodium elegans* Sorokin.)

Abh. d. Senck. Nat. Ges., VII. p. 85–108. Pl. I–III. 1869.

Sorus and stalk white, or when old, yellowish; the fructifications varying in height from 2–3 mm. to 1 cm. or more. Spores oval or elongated ellipsoid, 2.4μ–$3\mu \times 4\mu$–6μ.

On the dung of various animals, such as horse, rabbit, dog, guinea pig, grouse, etc. Also found on cultures of yeast, paper, fleshy fungi, etc., in a state of decomposition. Germany, Russia, common in America.

This very common species is extremely variable in the size of its spores and fructifications. The limits of the spore measurements as given by Brefeld in his original description have been therefore somewhat increased.

Dictyostelium sphærocephalum (Oud.) Sacc. and March.

(*Hyalostilbum sphærocephalum* Oudemans.)

Aanw. Myc. Nederl., IX.–X. p. 39. Pl. IV. 1885.

Sorus white; when old, yellowish or greenish-white. Stalk frequently very long and luxuriant, varying from 2 mm. to 1.5 cm. Spores oval, rarely spherical, or sub-inequilateral, 3μ–5μ × 5μ–10μ.

Dung of mouse, (common), rat, bird, toad, deer, turtle, muskrat, etc. Belgium; Cambridge and Boston, Mass.; New Hampshire; Florida; Pennsylvania; Liberia.

In the above description the limits of the measurements of spores and of the length of stalks are greater than those given by Marchal, by whom the maximum length of the spore is stated as 8μ and that of the stalk as 5mm. The measurements of the fructifications are certainly more variable than indicated by Oudemans. This species was founded by Marchal from the fact that the spores differed in size from those of *Dictyostelium mucoroides*, which he states to be only about one-half as large. As will be seen by the measurements given above, this difference is by no means as great as indicated; and, although the present arrangement is retained for the present, it may prove desirable to unite these two variable species.

Dictyostelium roseum Van Tieghem.

Bull. de la Soc. Bot. de France, XXVII. p. 317. 1880.

"Spore mass spherical, of a bright rose color. Spores elongated oval, 4μ × 8μ. On the dung of various animals; especially on rabbit dung, in company with *Pilobolus microsporus*." France.

Dictyostelium lacteum Van Tieghem.

Bull. de la Soc. Bot. de France, XXVII. p. 317. 1880.

"The mass of spores forms a milk-white drop at the summit of a stalk which I have always seen composed of a single row of cells. Spores colorless, spherical, very small, 2μ–3μ in diameter. This form has been met with several times on decaying agarics." France.

Neither of the two preceding forms have been found in American cultures, hence the writer can add nothing to our knowledge concerning them.

Dictyostelium brevicaule nov. sp.

Sorus white; stalks 1–3 mm. high. Spores oval, 3μ–4μ × 4μ–7μ or rarely spherical and 3μ–4μ in diameter.

Dung of sheep and goat. Cambridge, Mass.

A small, erect fructification, quite constant in the possession of a short rather rigid stalk bearing a sorus of comparatively large size and very different in aspect from the long, luxuriant, frequently flexuous, fructifications of *D. mucoroides* and *D. sphærocephalum.* Throughout the four years that this species has been kept growing in laboratory cultures, it has retained its original distinct characters.

Dictyostelium purpureum nov. sp.

Sorus and stalk purplish or violet; when mature, almost black. Spores oval, rarely somewhat inequilateral, 3μ–5μ × 5μ–8μ.

Dung of mouse, toad, cow, horse, sheep, muskrat. Cambridge, Mass.; Indiana; Florida.

This distinct species, well-marked by its color, was collected in August, 1897, in Crawfordsville, Indiana, on mouse dung cultures, and in October of the same year by Dr. Thaxter in Eustis, Florida, on toad dung. Both forms have been cultivated ever since in the laboratory, with no particular precautions as to the dissemination of the spores, and it is not impossible that the fructifications which appeared at Cambridge on sub-strata other than the two just mentioned represent laboratory escapes.

Dictyostelium aureum nov. sp.

Mature sori light to golden yellow, 1.5mm.–4mm. high. Spores oval, or frequently inequilateral, 2.5μ–3μ × 5μ–8μ.

Mouse dung from Porto Rico.

This species, communicated by Dr. Thaxter, is quite well defined through the color of its fructifications, but especially so by its myxamœbæ and its manner of growth. It matures very slowly on a horse dung decoction or on other media especially favorable for the rapid development of the common species; while the myxamœbæ, instead of possessing the usual form with elongated, sharp pseudopodia, are in general irregularly lobed and nodulated, even when growing under normal conditions. Such irregular shapes are similar to those assumed by the myxamœbæ of other species when they are growing under such abnormal conditions as are furnished by an insufficient water supply.

POLYSPHONDYLIUM Brefeld (1884).

Sori spherical, borne terminally on primary and secondary stalks, the latter branching in whorls from the main axis; the fructification occasionally simple as in *Dictyostelium*. Whorls varying in number from 1–10, and the number of branches in each whorl from 1–6.

Polysphondylium violaceum Brefeld.

Schimmelpilze, VI. p. 1–34. Pl. I, II. 1884.

Sori and stalks purplish or dark violet, varying in height from about ½cm.–2cm.; sori about 50μ–300μ in diameter. Spores elongated oval, 2.5μ–5μ × 6μ–8μ.

On dung of horse, bird, sheep, toad, muskrat. Italy, Maine, New Hampshire, Massachusetts, Florida.

The limits of spore measurements as given by Brefeld have been increased here as in other instances. The form growing on bird dung, brought by Prof. F. O. Grover from Center Ossipee, N. H., and the Massachusetts form on the dung of muskrat, seem to correspond very closely to the type description. The spores of the Maine and Florida forms are somewhat smaller, while the general aspect of the fructifications is different in that they are more delicate and less luxuriant and the sori have a less diameter than those of the type. These differences, however, seem hardly more than varietal.

Polysphondylium pallidum nov. sp.

Sori and stalks white, the sori about 50μ–80μ in diameter. Spores oval, 2.5μ–3μ × 5μ–6.5μ, or occasionally spherical, about 7μ–8μ in diameter.

On dung of ass, rabbit, muskrat. Liberia, Africa; Arlington and Stony Brook, Mass.

This delicate species is well characterized by the small size of its sori. In an interesting specimen, found by Mr. A. F. Blakeslee on muskrat dung, luxuriant fructifications showed that some of the branches themselves bore several whorls of branchlets. That this doubly verticillate character was not constant, however, was proved by growing the form on a sterilized nutrient medium, on which the fructifications showed simply the normal method of branching.

Polysphondylium album nov. sp.

Sori and stalks white, the sori 100μ to 200μ in diameter. Spores oval, 2.5μ–3μ × 4μ–5.6μ.

On dung of toad from Eustis, Florida.

Although the two forms above described have some features in common, their gross characters are such as to justify their being placed in separate species. The sori of *P. album* are not only larger but are usually more numerous in a whorl, hence its fructifications are more conspicuous; moreover, the stalks of this species are rather constantly weak at the base, so that the fructifications lie close to the substratum in a characteristic fashion.

CŒNONIA Van Tieghem (1884).

Sorus globular, borne at the summit of a stalk which is dilated into a sort of cupule, in which the sorus is supported.

Cœnonia denticulata Van Tieghem.

Bull. de la Soc. Bot. de France, XXXI. p. 303–306. 1884.

Sorus yellowish; stalk colorless, 2–3 mm. high, having a dilated base and expanding at the summit into a cupule which is finely toothed at its edges; each peripheral cell of the stalk bearing a tooth or papilla on its exposed side. Spores 6μ–8μ in diameter, with yellowish cell walls.

On decaying beans. France.

This remarkable form, so far as I am aware, has not been met with since it was originally described by Van Tieghem.

LABYRINTHULEÆ Cienkowsky.

Organisms having two definitely recurring stages, — a vegetative stage in which spindle-shaped or rarely spherical amœbæ, bearing usually bipolar filiform pseudopodia singly or in tufts, may be either isolated or combined by the union of the pseudopodia into colonies forming net-plasmodia; and a fructifying stage, in which aggregations of individuals, comparable to pseudoplasmodia, form spores borne in stalked or sessile sori.

Saprophytic or parasitic organisms living on dung, or on algæ in fresh or salt water.

LABYRINTHULA Cienkowsky (1867).

Amœbæ spindle-shaped, colorless, or colored by means of yellow fat bodies. Spores borne in formless masses, producing one to four amœbæ at germination.

The species of this genus have thus far been observed only by the authors cited.

Labyrinthula vitellina Cienkowsky.

Archiv. f. mikros. Anat., III. p. 274, Taf. 15-17. 1867.

Amœbæ containing orange-red coloring matter, which turns blue with iodine. Spores oval or spherical, 12μ in diameter, producing four amœbæ at germination.

Living on sea-weeds growing on piles in Odessa harbor, Russia.

Labyrinthula macrocystis Cienk.

Archiv. f. mikros. Anat., III. p. 274, Taf. 15–17. 1867.

Colorless or feebly yellowish. Spores spindle-shaped, 18μ–25μ long, imbedded in a hyaline substance; the contents producing four amœbæ at germination.

Living on algæ growing on piles at a higher elevation than *L. vittelina,* only submerged by the surf. Russia.

Labyrinthula Cienkowskii Zopf.

Beiträge zur Phys. u. Morph. niederer Organismen, II. p. 36–48, Taf. IV, V. 1892.

Sori colorless, naked. Spores at germination producing only one or at most two amœbæ.

Living in fresh water, parasitic on Vaucheria. Germany.

DIPLOPHRYS Barker (1868).

Amœbæ spindle-shaped or nearly spherical, with yellowish oil globules. Fructification (in *D. stercorea*) a definite stalked or sessile sorus.

Diplophrys Archeri Barker.

Quart. Jour. Mic. Sci., VII. p. 123. 1868.

Individuals nearly spherical or broadly elliptical, 4μ–5μ in diameter, bearing at almost opposite poles a tuft of filiform pseudopodia; the protoplasm containing an oil-like refractive globule of an orange or amber color. Fructification unknown.

Living in fresh water. Ireland, Germany, Pennsylvania and New Jersey (Leidy).

In this provisional arrangement, I have followed Cienkowsky in referring this species to the Labyrinthuleæ, although I regard it as improbable whether *Diplophrys Archeri* and *D. stercorea* should be included in the same genus. The aggregations of the vegetating amœbæ of *D. Archeri* seem to be an association of the young in groups, the colonies being formed by successive division of the individuals; and there is nothing definite known concerning a resting stage.

Diplophrys stercorea Cienkowsky.

Archiv. f. mikr. Anat., Bd. XII. p. 44. Pl. VIII. 1876.

Individuals lens- or spindle-shaped, about 4μ–6μ long, bearing at both ends several pseudopodia, almost bilaterally symmetrical. In the interior a nucleus, one or two contractile vacuoles and a yellow pigment body. Both the isolated and united individuals of the net-plasmodium finally becoming aggregated to form without change of shape pseudospores borne in sori, which are usually stalked, sometimes sessile.

On dung of horse, cow and porcupine. Russia; Cambridge, Mass.; Intervale, New Hampshire.

This species has been met with twice in American cultures, and so far as I am aware, with the exception of *D. Archeri*, is the only representative of the Labyrinthuleæ which has been found in this country.

A form, which is probably the resting condition of *Chlamydomyxa labyrinthuloides* Archer, has been found growing in the cells of sphagnum, at Kittery, Maine, by Professor Thaxter. As Archer and others have pointed out, however, it is very doubtful whether this peculiar organism should be included in the Labyrinthuleæ.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE DECOMPOSITION OF MERCUROUS CHLORIDE BY DISSOLVED CHLORIDES: A CONTRIBUTION TO THE STUDY OF CONCENTRATED SOLUTIONS.

By Theodore William Richards and Ebenezer Henry Archibald.

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Introduction.

Long ago Miahle observed that a concentrated solution of common salt acts upon calomel with the formation of small amounts of mercuric chloride.* Many years afterwards, one of us,† without knowing of his work, rediscovered this reaction, and found that the fluctuations in the potential of the "normal calomel electrode" of Ostwald, are due to its disturbing influence. At that time it was shown that the reaction is much diminished by dilution, and hence that a decinormal solution is far better as an electrolyte than a normal solution. The "decinormal electrode," thus recommended for the first time, has since come into common use.

It was shown also that neither light nor oxygen are important causes in effecting the decomposition, but that the reaction is much furthered by increase of temperature. No attempt was made at the time to fathom the matter, but a suggestion was made that the reaction might be due to the catalytic action of the ionized chlorine of the dissolved chloride.

The investigation of the problem which was at that time promised has now been continued, and the object of this paper is to show that while the second condition of this suggestion seems probable, the first does not hold. Another example is thus afforded of the frequently recurring circumstance of the removal of a reaction from its classification among catalytic phenomena after better acquaintance with its nature.

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* Miahle, J. Pharm., **26**, 108; Ann. Chim. et Phys. (3), **5**, 177 (1842).

† Richards, These Proc., **33**, 1 (1897); Z. phys. Ch., **24**, 39.

The method employed was to treat calomel with solutions of chlorides of various concentrations for varying times, and to determine the extent of the reaction by determining the amount of mercury dissolved.

PREPARATION OF MATERIALS.

Mercury already very pure was thoroughly freed from the possible presence of substances with greater solution-tension by treatment with sulphuric acid and potassic dichromate, and subsequent spraying through ten per cent nitric acid. Calomel was resublimed at as low a temperature as possible, and thoroughly washed with water and with the solution to be used in each particular case. One of us had previously shown that the source of the calomel is immaterial.* Sodic chloride was precipitated by pure hydrochloric acid from a saturated solution of the so-called "chemically pure" salt. It was then twice recrystallized from water, and thoroughly dried to drive off any possible traces of acid. Pure calcic nitrate was made by many recrystallizations; this was converted into carbonate, and the carbonate converted again into chloride. Several recrystallizations freed this chloride from every trace of the nitrate or of ionized hydrogen. Baric chloride was crystallized first from a solution strongly acid with hydrochloric acid, and subsequently from aqueous solutions by precipitation with pure alcohol. It also was wholly neutral to methyl orange. Cadmic chloride was made by dissolving the pure metal in pure acid and recrystallizing twice. The salt was dried thoroughly in order to make certain of the absence of ionized hydrogen, which is less easily detected in this case. Hydrochloric acid itself was purified by redistillation, the purest acid of commerce serving as the starting-point.

APPARATUS AND METHOD OF ANALYSIS.

It was necessary to digest the mixtures for long periods of time at a constant temperature. For this purpose they were placed in large test-tubes of sixty cubic centimeters capacity arranged to rotate transit-fashion in an Ostwald thermostat after the manner suggested by Schröder.† In the case of the weaker solutions several of these tubefuls were used for each analysis, but with the stronger solutions fifty cubic centimeters sufficed. The tubes were corked with rubber stoppers

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* Richards, loc. cit.

† Richards and Faber, Am. Ch. J., **21**, 168 (1899). The thermometer used to register the temperature was of course suitably verified.

which had previously been boiled with dilute alkali and scrupulously rubbed and washed. Into each tube was placed a large excess of calomel, about a decigram of mercury, and fifty cubic centimeters of one of the solutions of chlorides.

After a slight shaking, the settled precipitate was always covered upon standing with a layer of gray partially reduced material, which settled more slowly and hence gave more opportunity for reduction. When the equilibrium was completed by prolonged shaking, this gray material was mixed evenly throughout, and no longer appeared on the surface of the precipitate. Thus the absence of a gray film on settling was a rough guide to the completion of the reaction.

After five or six hours of agitation in the thermostat at 25.° ± 0.05° one of the tubes was opened, its contents filtered, and the dissolved mercury determined analytically. At intervals of an hour successive tubes were similarly treated, and after seven or eight hours no change was found in any case. Evidently a state of equilibrium is soon attained, and the reaction cannot be called catalytic. The values given below are of course the values corresponding to this maximum.

In this paper no evidence is given concerning the size of the grains of calomel. Ostwald * has recently shown that this may be an important factor in determining the concentration of a saturated solution, and hence in fixing the basis of the present equilibrium. Concerning this point it need only be said that while the *absolute* extent of solubility may vary with the size of the grains, the *relative results*, upon which alone the conclusions of this paper are founded, are not affected. This is the case because the same preparation of calomel was used in every instance. Moreover, since the calomel was sublimed and since it is notoriously difficult to powder, the individual diameters could not have been very small, hence a value approximating that corresponding to a flat surface must have been obtained.

A number of experiments indicated that the mercury salt thus dissolved was in the mercuric rather than in the mercurous state. The visible deposition of mercury during the reaction is alone almost enough to prove this. Moreover, neither permanganate nor bichromate suffered more than the faintest trace of reduction upon addition to a solution which contained much dissolved mercury. The minute trace of decolorization which was observed was no greater than that produced by a solution of mercurous chloride in pure water. On the other hand, small

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* Zeitschr. phys. Chem., **34**, 495 (1900).

amounts of stannous chloride gave plentiful white precipitates of calomel.

In all cases except that of cadmium, the mercuric salt in solution was determined as sulphide. The black precipitate produced by hydrogen sulphide was collected on a Gooch crucible, washed with alcohol, carbon disulphide, and again with alcohol, and finally dried at 100°. Satisfactory agreement between parallel analyses, which were almost always made in duplicate, was obtained. In the tenth-normal solutions of sodic chloride the amount of mercuric chloride was too small to be collected, hence it was determined colorimetrically by comparison with known solutions of similar dilution.

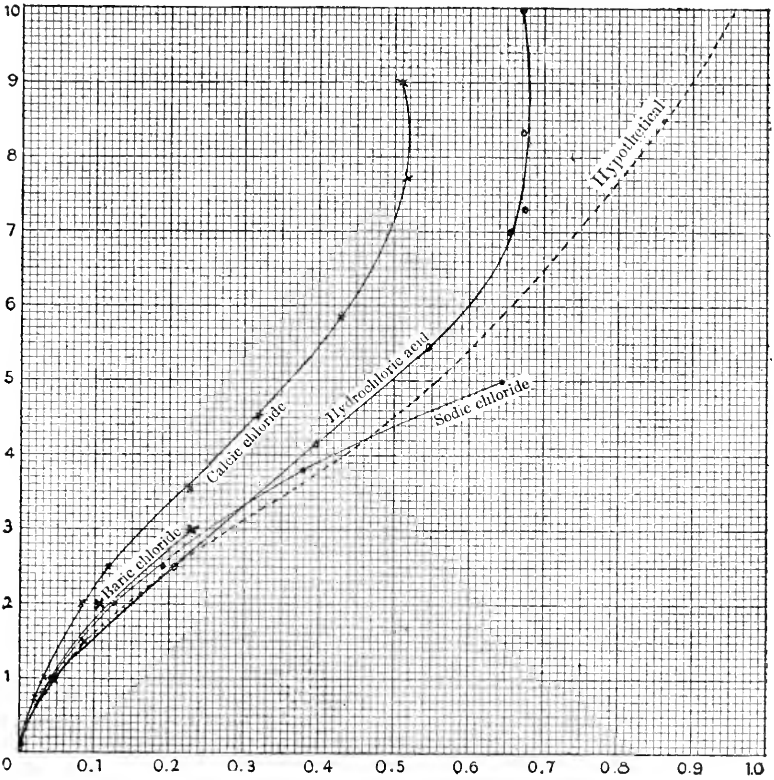
The following table explains itself. The last-column contains an arbitrary ratio which is an index of the changing relationship between the amounts of mercuric chloride formed and the amounts of sodic chloride present. The values in the third column were calculated from those in the second; and the values in the fifth column from those in the third and fourth.

Mercuric Chloride found in Solutions of Sodic Chloride.

No. of Exp.	Wt. of Solution taken.	Vol. of Solution.	Wt. of HgS found.	Wt. of $HgCl_2$ in 1 Litre of Solution.	c Mean Wt. of $HgCl_2$ in 1 Litre.	C Conc. of NaCl Solution in Equiv. Grams.	$1000\,\frac{c}{C}$ Milligrams $HgCl_2$ for every Mol. NaCl.
	grm.	c. c.	m. g.	grm.	grm.	equiv.	
1	. . .	. . .	. . .	0.0041	0.0041	0.10	41.0(?)
2 a	64.5	62.0	2.2	0.041	0.041	1.00	41.5
b	66.1	63.5	2.3	0.042			
3 a	65.9	61.1	6.8	0.130	0.129	2.00	64.5
b	80.3	74.5	8.2	0.128			
4 a	75.4	68.8	11.4	0.194	0.194	2.50	77.6
b	83.0	75.7	12.6	0.194			
5 a	73.8	64.6	21.1	0.382	0.380	3.80	100.0
b	80.3	70.3	22.8	0.379			
6 a	58.7	49.4	27.2	0.642	0.643	5.00	128.6
b	69.7	58.8	32.5	0.644			

These facts, together with similar facts concerning solutions of three other chlorides, are represented in the accompanying diagram. Evidently the first parts of the four curves are very similar in tendency, but as the highest concentrations are reached, the curves develop individuality.

Mercuric Chloride found in various Solutions.



The ordinates represent equivalent concentrations of the solvent chlorides, and the abscissae represent grams of mercuric chloride per litre of solution. The data for baric, calcic, and hydric chlorides are to be found on pages 352, 353, and 354.

Manifestly some particular property of the several solutions must be responsible for the reaction; and since the reaction results in raising the quantivalence of the mercury, it may be concluded that the particular property in question is the tendency of some molecular species already in the solution to combine with mercuric chloride.

This conclusion concerning the action of the substances on *mercurous* chloride is reinforced by the facts concerning the extent to which *mercuric* chloride is dissolved by solutions of various chlorides. Solutions of sodic chloride dissolve amounts of mercuric chloride which increase with the amounts of common salt present, until the saturation point is reached, while solutions of hydrochloric acid dissolve a maximum of mercuric chloride at a concentration of acid of seven times normal, remaining almost constant in action upon further concentration.*

The parallelism between the tendency of these soluble chlorides to dissolve mercuric chloride on the one hand, and their tendency to decompose mercurous chloride on the other hand, is thus rather striking.

In addition to the four chlorides given in the tables, cadmic chloride was used in a special series of experiments. The solution after digestion with calomel was analyzed by immersing in it a roll of clean copper gauze, which was dried and weighed, and then ignited in hydrogen and weighed again. Preliminary experiments showed this to be a convenient and sufficiently accurate method of determining mercury in the presence of cadmium.

Although solutions of 2, 4, and 8 times normal were used, in no case

MERCURIC CHLORIDE FOUND IN SOLUTIONS OF BARIC CHLORIDE.

No. of Exp.	Wt. of Solution taken.	Vol. of Solution.	Wt. of HgS found.	Wt. of $HgCl_2$ in 1 Litre of Solution.	c Mean Wt. of two Det. of $HgCl_2$ in 1 Litre Solution.	C Conc. of $BaCl_2$ Solutions in Equiv. Grams.	$1000\frac{c}{C}$ Milligrams $HgCl_2$ for every $\frac{1}{2}$ Mol. $BaCl_2$.
	grm.	c. c.	m. g.	grm.	grm.	equiv.	
1 *a*	106.5	97.8	3.6	0.043	0.044	1.00	44.0
1 *b*	112.0	103.0	3.9	0.045			
2 *a*	101.8	89.8	6.7	0.087	0.088	1.50	58.6(?)
2 *b*	120.3	106.1	8.0	0.088			
3 *a*	94.5	80.2	7.3	0.106	0.107	2.00	53.5
3 *b*	112.8	95.7	8.8	0.107			
4 *a*	131.2	103.8	20.5	0.231	0.231	3.00	77.0
4 *b*	106.8	84.5	16.7	0.231			

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* Homeyer and Ritsert, Pharm. Ztg., **33**, 738, quoted by Comey, Dict. of Solubilities, 227 (1896).

Ditte, Ann. Chim. phys., (5) **22**, 551; Engel., ibid. (6), **17**, 362. See Comey, as above.

could a trace of mercury be detected in the solution. Moreover, no gray precipitate of reduced mercury was ever observed when the cadmium solution was shaken with calomel in the first place. One infers that there is not in dissolved cadmic chloride any considerable concentration of a molecular species capable of combining with mercuric chloride.

This conclusion is quite in accordance with the fact that the temperature-coefficient of the potential of the calomel electrode with solutions of cadmic chloride exhibits none of the irregularities observed when other chlorides are used.*

Mercuric Chloride found in Solutions of Calcic Chloride.

No of Exp.	Wt. of Solution taken.	Vol. of Solution.	Wt. of HgS found.	Wt. of $HgCl_2$ in 1 Litre of Solution.	c Mean Wt. from two Det. of $HgCl_2$ in 1 Litre Solution.	C Conc. of $CaCl_2$ Solution in Equiv. Grms. ($\frac{1}{2}$ $CaCl_2$).	$1000 \frac{c}{C}$ Milligrams $HgCl_2$ for every $\frac{1}{2}$ Mol. $CaCl_2$.
	grm.	c. c.	m. g.	grm.	grm.	equiv.	
1	. . .	. . .	. . .	0.022	0.022	0.72	31.6
2	. . .	. . .	. . .	0.033	0.033	1.00	33.0
3 a	80.2	75.4	5.2	0.082	0.081	2.00	40.5
b	75.6	69.3	4.7	0.079			
4 a	75.4	68.2	6.9	0.118	0.118	2.50	47.2
b	78.4	70.9	7.2	0.118			
5 a	59.4	51.6	10.3	0.232	0.231	3.52	65.6
b	61.5	53.5	10.8	0.230			
6 a	92.2	76.5	21.0	0.320	0.322	4.64	69.3
b	99.4	82.5	22.8	0.323			
7 a	57.4	46.2	17.0	0.429	0.430	5.85	73.4
b	67.0	53.9	19.9	0.431			
8 a	48.7	37.0	16.4	0.518	0.518	7.80	66.4
b	76.4	58.1	25.8	0.519			
9 a	50.0	36.8	16.1	0.511	0.510	9.00	56.6
b	47.5	34.9	15.2	0.509			

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* Richards, These Proceedings, **33**, 1 (1897).

Interpretation of Results.

There are two possible interpretations of the phenomena under discussion. According to one, the undissociated mercuric chloride may be supposed to combine with the undissociated part of the electrolyte, forming an undissociated double salt, while according to the other, the undissociated mercuric chloride may be supposed to combine with the chlorine ion to form a complex ion. The following considerations attempt to decide which of these is more probable.

Mercuric Chloride found in Solutions of Hydrochloric Acid.

No. of Exp.	Wt. of Solution taken.	Vol. of Solution.	Wt. of HgS found.	Wt. of $HgCl_2$ in 1 Litre of Solution.	c Mean Wt. of $HgCl_2$ in 1 Litre of Solution.	C Conc. of HCl Solution in Equiv. Grams.	$1000\frac{c}{C}$ Milligrams $HgCl_2$ for every Mol. HCl.
	grm.	c. c.	m. g.	grm.	grm.	equiv.	
1	. . .	. . .	. . .	0.034 0.034	0.034	0.83	41.0
2	. . .	. . .	. . .	0.048 0.048	0.048	1.00	48.0
3 a	132.0	126.7	22.4	0.206	0.207	2.50	83.0
3 b	179.0	171.8	30.6	0.208			
4 a	74.5	69.7	23.9	0.400	0.399	4.15	96.1
4 b	87.8	82.0	28.0	0.398			
5 a	85.2	78.1	36.6	0.548	0.548	5.48	100.0
5 b	75.8	69.5	32.6	0.548			
6 a	85.4	76.8	42.9	0.653	0.654	7.00	92.8
6 b	90.4	81.3	45.5	0.655			
7 a	82.6	73.8	42.7	0.676	0.675	7.30	92.3
7 b	95.6	85.5	49.3	0.673			
8 a	70.0	61.8	35.4	0.669	0.670	8.31	80.6
8 b	95.8	84.6	48.6	0.671			
9 a	115.0	99.6	57.7	0.672	0.673	10.00	67.3
9 b	123.0	106.5	61.4	0.674			

The shape of the first section of the curves, where the concentration of the mercury present increases at a greater rate than does the corresponding amount of electrolyte, suggests at first that the undissociated part of the latter is the portion concerned in the reaction; but the curve representing a power of the concentration of the ionized chlorine has of course a similar tendency.

Hence the general shape of the curve is an insufficient basis for decision between the two hypotheses.

The fact that strong solutions of cadmic chloride have little or no influence on mercurous chloride supports the latter of the two hypotheses, since concentrated cadmic chloride solutions contain but a very small concentration of ionized chlorine.

More direct light upon the question is obtained by the measurement of electrolytic conductivity. According to the first hypothesis, which demands the presence of an undissociated double salt, the conductivity of salt solution should be considerably decreased by the addition of mercuric chloride. As a matter of fact, we found that the dissolving of mercuric chloride to saturation in a twice normal solution of common salt diminished but slightly the conductivity of the solution. The work of Le Blanc and Noyes* furnishes similar results concerning hydrochloric acid; and moreover these investigators showed by the catalysis of methyl acetate that the concentration of the hydrogen ion was undiminished by the addition of mercuric chloride. Hence the new compound is to be considered as highly ionized.

Yet further evidence is to be obtained by referring to the specific conductivities of strong solutions of the chlorides studied.† Here we find that while the conductivities of solutions of sodic and baric chlorides increase with the concentration as far as they may be followed, those of calcic and hydric chlorides exhibit maxima at a concentration about six times normal. The agreement between these maxima and those exhibited by our own curves at seven times normal is close enough to suggest an essential relation between the cause of conductivity and the cause of Miahle's reaction.

The evidence thus furnished is all consistent in indicating that the nature of the reaction is the addition of $HgCl_2$ to the chlorine ion, with the formation of a complex ion. This conclusion agrees with that of Le Blanc and Noyes, based upon other data.

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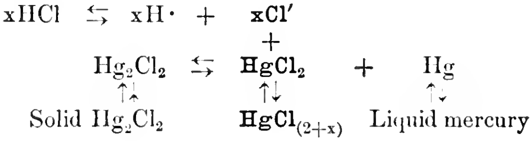
* Le Blanc and Noyes, Zeitschr. phys. Chem., **6**, 389, seq. (1890).

† See Kohlrausch and Holborn (1898), Leitvermögen d. Electrol., pp. 145–154.

It remains now to detect the mechanism of the reaction. The work of Le Blanc and Noyes led them to believe that in dilute solutions containing an excess of the soluble electrolyte the new ion is bivalent, being formed by the reaction $2Cl' + HgCl_2 = HgCl_4''$. It will be shown that our own evidence supports this conclusion also.

The reaction with which we are concerned may perhaps be written thus: —



$$\begin{array}{ccccccc}
x\mathrm{HCl} & \leftrightarrows & x\mathrm{H}^{\cdot} & + & x\mathrm{Cl}' & & \\
 & & & & + & & \\
 & & \mathrm{Hg_2Cl_2} & \leftrightarrows & \mathbf{HgCl_2} & + & \mathrm{Hg} \\
 & & \uparrow\downarrow & & \uparrow\downarrow & & \uparrow\downarrow \\
 & & \text{Solid } \mathrm{Hg_2Cl_2} & & \mathbf{HgCl}_{(2+x)} & & \text{Liquid mercury}
\end{array}$$

The ion $HgCl_{(2+x)}$ will of course be the bearer of x negative charges of electricity. The above expression does not attempt completeness, but strives merely to represent the most essential features of the reaction in the simplest possible form.

The first conclusion to be noted is that the concentration of the uncombined but dissolved mercuric chloride will be constant, since it is formed by a reaction involving two precipitates. Hence the concentration of the ion $HgCl_{(2+x)}$ should vary as the concentration of the chlorine ion raised to the x*th* power.

It is immediately clear that x must be more than unity, for in the less concentrated solutions the concentration of the mercury present increases faster than that of the dissolving chloride, while the concentration of the ionized chlorine is supposed to increase less rapidly than the latter.

By taking $x = 2$ we obtain much more satisfactory agreement. If we assume that the concentration of the ions present is proportional to the specific conductivity,* we find that for solutions as far as twice normal the calculated curve agrees almost precisely with the actual amounts of mercury found. The specific conductivity of a twice normal solution of hydrochloric acid is 0.505, while that of a normal solution is 0.295. The squares of these numbers are respectively 0.255 and 0.087, two values which are very nearly proportional to the weights 148 and 48 milligrams of mercury per litre which were actually found to be dissolved from calomel by twice normal and by normal solutions of hydrochloric acid respectively.

With more concentrated solutions the results of this calculation agree

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* The possible dangers of this assumption are well known. It is made here simply in default of more certain knowledge.

less and less satisfactorily with the facts, the amount of mercury actually found always exceeding the calculated amount. Evidently this disagreement may be due to the fact that some of the new complex acid remains in the undissociated state; the calculation considers only the ion, while the mercury weighed in analysis constituted the sum total. The exact calculation of the amount undissociated is impossible for two reasons; in the first place, the mode of dissociation of such a tri-ionic compound as H_2HgCl_4 is uncertain; and in the next place, we have no data for the extent of the dissociation of the compound beyond the strength of a normal solution.

In spite of this double uncertainty, it is possible to make an approximate calculation. This is sufficient to show that in a general way the argument is sound. The approximate calculation is based upon the fact that so far as the extent of dissociation of the complex acid is known, it is equal to that of hydrochloric acid at the same concentration.[*] On making the assumption that this relation holds in very strong as well as in moderately strong solutions, and waiving entirely the uncertainty as to the possible existence of the half-way ion $HHgCl_4'$, the proportion of the

APPROXIMATE CALCULATION OF THE TOTAL AMOUNT OF MERCURY.

Concentration of Hydrochloric Acid Solution or Gram-Equiv. per Litre.	κ = specific Conductivity of Hydrochloric Acid x.[†]	κ^2.	$a = \frac{\Lambda_v}{\Lambda_\infty}$.	$\frac{385\,\kappa^2}{a}$.	Milligrams Mercury found in 1 Litre Solution.[‡]
1	0.295	0.087	0.796	42	48
2	0.505	0.255	0.672	148	148
3	0.645	0.417	0.568	282	263
4	0.727	0.530	0.48	427	383
5	0.760	0.580	0.40	569	495
6	0.762	0.582	—	—	—
7	0.745	0.554	0 28	765	654
10	0.635	0.420	0.17	955	672

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[*] Le Blanc and Noyes, loc. cit.

[†] These figures were obtained by graphic interpolation from the figures of Kohlrausch and Holborn, Leitvermögen d. Electrol., p. 154 (1898).

[‡] By interpolation.

undissociated complex may be calculated by simply multiplying the supposed concentration C of the ionized part of the complex by $\frac{1-\alpha}{\alpha}$, when α is the degree of dissociation of the acid. The total concentration of the mercury present would then be $C + C\,\frac{1-\alpha}{\alpha} = \frac{C}{\alpha}$. But if the new ion has the formula $HgCl_4''$, its concentration should be proportional to the square of the specific conductivity, κ, according to our previous reasoning. That is to say, $\frac{C}{\alpha} = k\,\frac{\kappa^2}{\alpha}$.* This equation is tested in the following table, by taking a value for the constant k which best satisfies the early part of the curve — namely 385.

The bearing of these rather discrepant figures is best seen by plotting the results. The curve which depicts the relation of the quantity $\frac{385\,\kappa^2}{\alpha}$ to the concentration of the hydrochloric acid is indicated by a dotted line in the diagram on page 351. While with great concentrations it deviates considerably from the curve representing the amount of mercuric chloride formed by hydrochloric acid, it is nevertheless of the same general character. Considering the many uncertainties, including the doubt concerning the equation $\alpha = \frac{\Lambda_v}{\Lambda_\infty}$, which interfere with its exact determination, the agreement is indeed as close as one has a right to expect.

Corresponding curves, with about the same degree of agreement, may be calculated for the other chlorides. It is perhaps worth while to call attention to the fact that the amount of mercury found in the most dilute solution studied, the tenth normal solution of sodic chloride, although very small, is too great to correspond to the theoretical value. The excess of about three milligrams per litre above the requirement of theory may well be due to dissolved calomel, which possesses a slight but unknown solubility of its own.†

All these arguments, reinforcing the conclusions which Le Blanc and Noyes reached from a different series of facts, seem to indicate that as

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* $k\,\frac{\kappa^2}{\alpha} = k'\,\frac{\kappa}{V}$ because $\alpha = \frac{\kappa V}{\Lambda_\infty}$ and $k' = k\,\Lambda_\infty$. The more complex form is retained because its meaning is the more obvious.

† The work of Kohlrausch and Rose (Zeitschr. phys. Chem. **12**, 241) is not conclusive concerning this solubility, since the behavior of calomel on solution is too little known. Their results seemed to indicate that the solubility amounted to three or four milligrams per litre.

nearly as the present means can determine, the reaction which we have been studying is to be thus represented in its simplest form: —

$$Hg_2Cl_2 + 2Cl' = Hg + HgCl_4''.$$

It is of interest to classify the equilibrium under consideration according to the Phase Rule of Willard Gibbs. Looked at from this point of view, we may speak of the system as consisting of four components, — water, soluble salt, mercury, and mercuric chloride. It is clear, therefore, that when we have together the four phases, — mercury, mercurous chloride, solution, and vapor, — at a fixed temperature, a single condition of freedom remains to be fixed in order to fix the system. The concentration of the ionized chlorine seems to supply this sixth ($n + 2$d) condition, determining the fixed points in the tables.

At the seven times normal point the concentration of the mercury dissolved seems to attain almost a constancy, being no longer increased by further addition of soluble electrolyte. According to the Phase Rule, such a phenomenon might be caused by the appearance of a new phase. This new phase would of course be one which would remove hydrochloric acid from the solution; hence its presence or absence is easily discovered.

As a matter of fact, we found that after continued shaking with calomel, hydrochloric acid having an original concentration of 9.22 normal was reduced only to 9.20 normal. This is quite too small a difference to be due to the formation of a new phase; it must be ascribed either to adsorption by the calomel or to analytical error.

Hence the constancy of mercury dissolved is to be ascribed to conditions within the solution, and not to the appearance of a new phase.

Since the reaction seems to be effected primarily by the action of the chloride ion, it might be used to determine the concentration of the chloride ion, — or in the corresponding cases, that of the bromide or iodide ion. Especially would the case be applicable to the ionized chlorine because here the amount of mercury dissolved is too small to affect seriously other equilibria existing in the solution. Of course, with very dilute solutions the solubility of mercurous chloride itself would have to be taken into account.

This tendency of mercuric chloride to add to the chloride ion is a highly interesting circumstance. Other similar phenomena are being more and more frequently reported.* The tendency of cadmium to form

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* Cushman, Zeitschr. für anal. Chem., **34**, 368 (1895).

a similar complex ion is well known; it has even been used by Cushman under Sanger's direction as a means of separating cadmium from other metals. In this case the complex ion was formed simply by adding an excess of sodic chloride, which prevents cadmium from being precipitated by hydrogen sulphide. Upon dilution the sulphide of cadmium begins to be precipitated, owing to the splitting apart of the ion in dilute solutions according to the law of "mass" action.

The same tendency has been used to explain the otherwise incomprehensible migration values of cadmium salts. Very recently Noyes has shown that probably a similar ion, $BaCl_4''$,* exists in baric chloride solutions; and the migration values of concentrated calcic and magnesic chloride solutions lead one to infer that in these cases yet a greater concentration of $CaCl_4''$ and $MgCl_4''$ may exist.

It is interesting to note that the decomposition of the mercurous halide is carried to a much greater extent under similar conditions in the case of the bromide than in that of the chloride,† and yet further in the case of the iodide. This may be due simply to the greater solubilities of mercurous bromide and iodide, but besides this cause there may exist a greater affinity of the molecule for the ion. The study of the migration values of cadmium salts seems to show that the iodide has a much greater tendency to add to ionized iodine than the chloride has to add to ionized chlorine; and it is probable that the same relation exists in the case of mercury.

The facts recorded above show that an accurate quantitative analysis of a mercurous salt by precipitation with a soluble chloride is not to be expected, unless the chloride is added only in very slight excess, and then the solubility of mercurous chloride itself must be considered. When, however, a large excess of mercuric salt is present, as for example in the recent work of Ogg,‡ it is obvious that the disturbing effect of the side-reaction must be much hindered, according to the law of "mass" action.

It is possible that the medicinal action of calomel is due to the small but definite concentration of mercuric complex salt produced by common salt or hydrochloric acid in the alimentary canal. In any case, one is disposed to recommend cautious medicinal use of other chlorides in connection with calomel.

Preliminary experiments with sulphates showed that with these salts

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* A. A. Noyes, J. Am. Chem. Soc., **23**, 37–57 (1901).

† Richards, loc. cit.

‡ Ogg, Zeitschr. phys. Chem., **27**, 291 (1898).

the tendency to form complex compounds is much less than that exhibited by chlorides; hence the Latimer-Clarke and Weston cells are not much affected by this type of side-reaction.

The results of the present paper may be stated briefly as follows: —

1. The action of dissolved chlorides upon calomel is not catalytic, but results in the establishment of a definite equilibrium.

2. With equivalent solutions, less concentrated than five times normal, hydrochloric acid and sodic chloride have about equal tendencies to effect the reaction; baric chloride has less tendency, calcic chloride still less, and cadmic chloride no appreciable tendency.

3. The extent of the reaction in solutions not too concentrated is approximately a simple function of the square of the concentration of the chloride ion. This relation, taken in connection with a number of other considerations, points to the existence of a highly ionized complex $HgCl_4''$ in the solution, and thus confirms the work of Le Blanc and Noyes.

4. If approximate allowance is made for the probable concentration of undissociated complex salt present, all the figures, even as far as ten times normal solutions, seem to be explicable.

5. The suggestion is made that the reaction may be of use as a means of determining the concentration of the chlorine ion.

6. The corresponding reactions are much less marked with sulphates, but much more so with bromides and iodides.

7. Caution is needed when using mercurous chloride as a means of determining mercury in quantitative analysis.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

A NEW INVESTIGATION CONCERNING THE ATOMIC WEIGHT OF URANIUM.

By Theodore William Richards and Benjamin Shores Merigold.

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By Theodore William Richards and Benjamin Shores Merigold.

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Introduction.

Our knowledge of uranium dates from the year 1789, when it was first recognized as an element by Klaproth. It can by no means, therefore, be classed with the new elements, nor is it of great rarity. Nevertheless, comparatively few determinations of the atomic weight of this element have been made, and of these, one only has been carried out with the degree of accuracy necessary in work of this kind. During the fifty years following the discovery of uranium a number of atomic weight determinations were made by Berzelius, Arfvedson, Schönberg, Marchand, and Rammelsberg. This early work is now of historical interest only, for the results vary widely, and in some cases are of such a nature as scarcely to be considered quantitative, in the modern sense of the word. For example, Rammelsberg obtained results varying from 184 to 234, calculated upon the modern basis.

In 1841 Peligot discovered that the substance then known as uranium was not an element, but an oxide. This discovery, while it did not impair the value of the analytical work previously done, necessitated a recalculation of the numerical value of the atomic weight. The new value was 120, and this remained practically unchanged during the next thirty years. When the periodic classification of the elements was first suggested, uranium, with the atomic weight 120, was one of the elements for which there was no place. From a study of the properties of uranium and its compounds, Mendeléeff declared that the atomic weight

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* The greater part of the work described in this paper was presented to the Faculty of Arts and Sciences of Harvard University by B. S. Merigold, as a thesis for the degree of Ph.D., in June, 1901.

of uranium was probably 240 instead of 120.* The question was not definitely settled until Zimmermann, in 1885, carried out the suggestions of Mendeléeff, and by specific heat and vapor density determinations confirmed the higher value.†

Owing to the wide variations in the published results, the atomic weight of uranium has long been considered one of the least satisfactorily determined of the atomic weight values. A glance at the results thus far obtained is sufficient to show the need for further work in this line. A complete résumé of the older work upon the subject is to be found in Clarke's recent work on the atomic weights. ‡ The following table summarizes those investigations which seem to possess even a little quantitative value: —

LESS INACCURATE PREVIOUS WORK ON THE ATOMIC WEIGHT OF URANIUM.

$O = 16.000$

1841	Péligot §— Analysis of Green Chloride	240. ±
1842	Ebelmen ‖ — Combustion of Oxalate	238. ±
1843	Wertheim ¶ — Double Acetate of Sodium and Uranium	239. ±
1846	Péligot ** — Combustion of Oxalate and Acetate . .	240. ±
1886	Zimmermann †† — Reduction of Oxide, U_3O_8 to UO_2 .	239.6
1886	Zimmermann ‡‡ — Ignition of Double Acetate . . .	239.5

The work of Ebelmen, Wertheim, and the early work of Péligot is necessarily of little weight in assigning a probable value to the atomic weight of uranium. In some cases the material used was impure, and in others the methods of analysis were faulty. Consequently it is not surprising to find differences of whole units in the individual determinations of each series.

Péligot's later determinations from the oxalate is perhaps the best of the early work. His material was carefully purified, and his method is far preferable to the work of Ebelmen and Wertheim. By combustion

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* Annalen der Chemie u. Pharmacie, Supp. Vol. **8**, 178 et. seq.

† Annalen der Chemie u. Pharmacie, **216**, 1.

‡ A Recalculation of the Atomic Weights, by F. W. Clarke, Smithson. Misc. Coll., Constants of Nature, Part V. (1897), 263.

§ Compt. Rend. **12**, 735. Ann. Chim. Phys. (3) **5**, 5 (1842).

‖ J. prkt. Chem. **27**, 385 (1842).

¶ Ibid., **29**, 209 (1843).

** Compt. Rend., **22**, 487 (1846).

†† Ann. d. Chem., **232**, 299 (1886).

‡‡ Ibid.

analysis he determined the ratio between uranium oxide and carbon dioxide. Thus he eliminated the error involved in weighing a crystallized salt which would probably contain more or less included water. The principal sources of error are the questionable use of combustion analysis in atomic weight investigations, and the possibility of unoxidized carbon remaining in the uranium oxide. His best results vary from 239.4 to 241.1.

The work of these chemists, though a great improvement over the attempts of Rammelsberg and the other early workers, leaves much to be desired, and does little more than give an approximate idea of the probable value of the atomic weight of uranium.

Zimmermann's investigation of the ratio between the oxides UO_2 and U_3O_8 was much more carefully carried out, and is the only work thus far published that is worthy of serious consideration. Using carefully purified material, and giving attention to detail, Zimmermann oxidized the lower oxide by means of a stream of oxygen, and observed the gain in weight. His results for the atomic weight varied from 239.49 to 239.76, an extreme difference of 0.27, or 0.11 per cent. The average was about 239.6. The chief probable cause of error in this method is the difficulty which is always experienced in forming a more voluminous solid from a less voluminous one. Uranous oxide has a specific gravity of 10.2, while the "Uranoso-uranic" oxide has a specific gravity of only 7.3. The great increase of volume which occurs when the higher oxide is formed must tend to protect particles of the lower oxide from the action of the oxygen. Hence the gain in weight will be too small, and the apparent atomic weight of the metal too large.* It is clear that a very small deficiency in the weight of the higher oxide must cause a great increase in the apparent atomic weight.

Moreover, any incompleteness in the reduction by which the lower oxide was prepared, or any retention or occlusion of gases within this oxide, would also tend to raise the apparent atomic weight. Hence one is inclined to believe, even without further evidence, that Zimmermann's result for uranium must be too high.

A new determination of the atomic weight of uranium has recently been made by J. Aloy.† The method employed differs materially from any previously used in uranium work. The values obtained are derived

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* Compare Richards and Baxter, These Proceedings **34**, 351 (1898). Ztsch. anorg. Chem. **21**, 251 (1899).

† Comptes Rendus, **132**, 551 (1901). This work is discussed rather fully here, since it is too recent to have been included in Clarke's book.

from the ratio between the weight of nitrogen and that of uranous oxide contained in crystallized uranyl nitrate. Uranyl nitrate was purified by repeated crystallization. A quantity of the pure nitrate, the weight of which need not be known, was put into a boat, and the boat surrounded by a section of platinum tube, to prevent loss of material. The whole was placed in a combustion tube between spirals of reduced copper. One end of the combustion tube was connected with a carbon dioxide generator, and the other with an absorption apparatus containing a concentrated solution of potash.

After sweeping the air out of the apparatus with a current of carbon dioxide, the nitrate was heated so long as evolution of nitrogen occurred, the temperature being finally raised to red heat. The reduced copper was kept at red heat throughout the operation. When it was certain that no more nitrogen was evolved, the green oxide remaining in the boat was reduced by hydrogen to uranous oxide and weighed. The nitrogen was transferred to a measuring tube reading to tenths of a cubic centimeter. From the ratio of the weight of this volume of nitrogen to the weight of the oxide, the atomic weight is calculated.

The following are the results of the eight determinations given : —

ATOMIC WEIGHT OF URANIUM.

$N = 14.04$

Volume of nitrogen,	15.25 cc.	33.5	38.0	52.5	81.25	125.0	151.2	165.0
Atomic wt. of uranium,	239.3	239.4	239.6	239.5	239.4	239.5	239.4	239.4
				Average				239.4

This method has the merit of simplicity, and does not involve the weight of the crystallized salt. There are, however, several sources of possible constant error that have not been taken into account. When crystallized uranium nitrate is heated, it first melts in its water of crystallization. As in all similar cases, it requires the very greatest care to prevent spattering while the crystal water is being driven off. It was undoubtedly as a precaution against loss of material in this way that Aloy used his platinum tube. By the time the crystal water is expelled, the fused mass has hardened into a solid cake, changing in color from yellow to orange, and finally to the green of urano-uranic oxide, U_3O_8.

This method of preparing the green oxide from pure uranyl nitrate was used in the work to be described in the following pages. It was *invariably* found, however, that during the decomposition of the dried nitrate, and the subsequent oxidation, the oxide first produced forms a protecting crust, as it were. This prevents, or at least very materially retards, the decomposition of the material within the interior, *even when the temperature is maintained for several hours at red heat.* On the outside, the material had the appearance of being completely converted to oxide. On powdering the lumps, however, and again heating, there was *in every case* a further evolution of nitric fumes. Moreover, nitrogen itself is often retained by oxides prepared in this way.* It seems thus extremely probable that the quantities of nitrogen measured by Aloy were in every case too small. Obviously, until this point is definitely settled, Aloy's results must be regarded with more or less suspicion.

It has been pointed out that reduction is usually much more complete than oxidation.† During the reduction of an oxide, there is formed, perhaps, by the removal of a portion of the oxygen, a kind of skeleton framework, giving to the remaining substance a porous structure which enables the reducing gas to penetrate farther into the interior of the mass, until reduction is complete. Owing to this action, it is probable that when the green oxide of uranium is finally reduced by hydrogen, all the nitrogen is expelled, and the final product is pure uranous oxide. Consequently, the weight of uranous oxide used in the calculation is probably nearly correct, the principal error being in the volume of nitrogen.

Aside from this special objection to the use of this method in its application to uranium, there is the general objection to the use of such a method where great accuracy is desired. The exact measurement of small quantities of gas offers considerable opportunity for error, especially, when, as in this case, the gas is first to be transferred from the collecting to the measuring apparatus. When the volume or weight of a gas is involved in an atomic weight investigation, it is customary to work with as large volumes as possible, thus reducing to a minimum the effect of the errors inevitably connected with the measurement of the gas. The exact measurement of a volume no larger than 165 cubic centimeters, even, — the largest volume measured by Aloy, — is a matter of considerable experimental difficulty, while with the smaller volumes, 15, 33, and 38 cubic centimeters, errors of at least 0.1% are

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* Richards and Rogers, These Proceedings, **28**, 200 (1893); also Richards, Ibid. **33**, 399 (1898).

† Richards and Baxter, loc. cit.

not only possible, but extremely probable. A difference of one tenth of one per cent in the volume of nitrogen makes a difference of 0.3 in the value of the atomic weight. The errors of collection and transference of the gas are more likely to result in reading too small rather than too large volumes, giving too high values for the atomic weight.

From these considerations, it is evident that Aloy's results are at least somewhat doubtful. Aloy gives notice of his intention to apply this method to the determination of other atomic weights, but it is to be hoped that before doing so he will clear up some of the doubtful points in connection with the process. As carried out in this investigation, the method certainly is not a valuable addition to the methods of atomic weight determination.

From the earlier results Clarke computed the value 239.6, while the German Committee recommend 239.5. Both figures are practically identical with Zimmermann's figures.

The investigation herein described was undertaken with the hope that by increasing the experimental basis of our knowledge of the subject, we might be able to reduce to somewhat narrower limits our present uncertainty in regard to the real value of this constant.

Preliminary Work upon the Preparation, Properties, and Methods of Analysis of Some Uranium Compounds.

In view of the well known advantages of the halogen compounds for accurate analysis, when these compounds can be prepared and weighed in a state of purity, — it seemed desirable to use a halogen compound as the basis of a determination of the atomic weight of uranium.

Of the four chlorides of uranium known to exist, none can be prepared in a state of purity that is beyond question. Green uranous chloride, UCl_4, which results from passing dry chlorine over a mixture of uranium oxide and carbon at red heat, is easily converted to the pentachloride, UCl_5, by further action of chlorine at high temperatures. There can be no positive evidence that the green chloride would not contain some of the pentachloride, and if the attempt is made to prepare the pentachloride from the green chloride, it is equally difficult to be sure that the conversion is complete. The trichloride, UCl_3, is made by reducing the tetrachloride with hydrogen, and here again it is difficult to be sure that the tetrachloride is completely reduced. Uranyl chloride, UO_2Cl_2, cannot be prepared in the dry state.

It is extremely probable, then, that any of the chlorides will contain larger or smaller quantities of a higher or lower chloride. It may be observed, in this connection, that Zimmermann used the chlorides in his vapor density determinations, and his analyses show good agreement. This does not show conclusively, however, that his material was free from small, but fairly constant quantities of higher or lower chlorides as impurities.

On the other hand, bromine forms with uranium only three distinct compounds: the tribromide, UBr_3; uranous bromide, UBr_4; and the oxybromide, or *uranyl* compound, UO_2Br_2. The tribromide can be produced only from the tetrabromide by the action of reducing agents. Uranyl bromide, UO_2Br_2, has been certainly formed only in solution, resulting in hydrated crystals. It has never been definitely obtained in anhydrous form. Zimmermann made many attempts to form the pentabromide, corresponding to the pentachloride, by passing bromine at high temperatures over sublimed uranous bromide. Every attempt gave negative results, showing that at temperatures up to the subliming point of uranous bromide higher bromides cannot exist. Since higher bromides are non-existant under the conditions prevailing in the formation of the tetrabromide, the objections to the use of the tetrachloride are not applicable in the case of uranous bromide. The investigations of Zimmermann* have shown that the tetrabromide can be formed in an apparently definite state. It seemed probable, therefore, from the literature on the subject, that in uranous bromide we had a compound well suited to the purposes of our investigation.

The method of preparation followed at first was essentially that described by Zimmermann.* In an apparatus constructed wholly of glass, a mixture of dry nitrogen and bromine vapor was passed over a mixture of the green oxide of uranium, U_3O_8, and pure carbon. The air was first thoroughly swept out of the apparatus by a current of nitrogen, and the oxide was heated to a high temperature. When the bromine vapor was passed in, uranous bromide formed, and sublimed in brilliant crystalline plates of a brownish color. After cooling in a current of nitrogen, the sublimate was transferred to a weighing bottle. At this point, however, unexpected difficulties arose, owing to the rapid oxidation of the bromide. Uranous bromide is extremely deliquescent, and forms with water and oxygen the oxybromide, with liberation of hydrobromic acid. Consequently, when exposed to the moist air of the laboratory even for the short time required for removing the sublimate from the combustion tube, the bromide loses its brilliant lustre, and

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* Annalen der Chemie, **216**, 3.

assumes a dull, greenish yellow appearance, due to formation of the oxy-salt. If not protected from further action of moist air, the salt liquifies completely in a surprisingly short space of time.

In an attempt to change the coating of oxybromide back to the normal salt, recourse was had to the method which has been used successfully in many atomic weight investigations carried on in this laboratory. The salt was transferred to a platinum boat and placed, with a weighing bottle of suitable size, in a glass bottling apparatus.* A stream of dry hydrobromic acid gas was then passed over the bromide at a temperature just below the subliming point of the salt. This treatment, however, fails to restore the original brilliant appearance of the freshly sublimed bromide. The yellow color of the oxybromide still remains. Apparently the oxybromide, once formed, cannot, by this method, be reduced to the normal uranous bromide.

In the previous investigations upon zinc, magnesium, nickel, and cobalt, in which this method of converting oxy-salts to the normal compounds has been used, the presence of even minute quantities of oxy-salt was made known by the opalescence of the solutions on account of the insolubility of these salts. With uranium, however, this method of detecting the presence of uranyl bromide cannot be used, for the oxybromide of uranium is even more soluble than uranous bromide.

The analysis of uranous bromide presents further difficulties. All uranous salts reduce silver nitrate. When a solution of silver nitrate, slightly in excess of the calculated amount, is added to a solution of uranous bromide, the silver bromide first precipitated is probably mixed with metallic silver; for if the silver bromide is filtered off, and the filtrate set aside, finely divided metallic silver soon separates. If a large excess of silver nitrate is added to the uranous bromide, a brilliant purple precipitate is obtained. It is possible that the precipitate may be a mixture of finely divided metallic silver and argentic bromide, or perhaps of normal argentic bromide and the long sought sub-bromide. Although this is an interesting phenomenon, it was not considered advisable to interrupt the research at this period for the length of time necessary for an investigation. The addition of nitric acid prevents the formation of this colored precipitate, but owing to the danger of the loss of bromine, this is not an advisable expedient. Of course it is possible to determine the bromine by first precipitating the uranium and adding silver nitrate to the filtrate, but this introduces

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* For a description of this apparatus, see These Proceedings, **32**, 59.

a complexity of operations incompatible with the degree of accuracy requisite in an atomic weight investigation.

On account of these formidable difficulties in the preparation and analysis of pure uranous bromide, it was thought best to search for some compound which offered fewer obstacles. It will be seen that this search was vain, although it required many months.

In view of the great tendency of uranous bromide to oxidize, under ordinary conditions, the use of uranyl bromide seemed to offer the simplest solution of the problem. Anhydrous uranyl bromide has never been prepared in a pure state. In the preparation of uranous bromide, if the nitrogen used contains a little oxygen, or if traces of moisture are present, there is formed, in addition to the uranous bromide, a yellow powder, very different in appearance from the brown color of finely divided uranous bromide. This powder has been assumed by various investigators to be the oxybromide. Owing to the fact that it is always mixed with uranous bromide, an analysis has never been obtained.

There seemed to be, however, some basis for belief that under suitable conditions of temperature, moisture, and oxygen supply, it might be possible to obtain anhydrous uranyl bromide entirely free from the uranous compound. With this end in view, the green oxide, without any admixture of carbon, was heated in a stream of bromine, also in a current of hydrobromic acid. In each case there was apparently no action whatever other than a partial and gradual reduction to the black oxide. This slight reducing action is probably not due to the gases used, in the sense of being peculiar to them, for Zimmermann has shown that this reduction takes place whenever the green oxide is heated in a current of inactive gas such as nitrogen or carbon dioxide.*

Both moist and dry gases were used. Mixtures of these gases and air were also tried, at different temperatures. The green oxide was then reduced by hydrogen to uranous oxide, UO_2, and this was then treated with various combinations of dry and moist bromine vapor, hydrobromic acid, and air, at various temperatures. Again the results were negative. Under these conditions the bromine did not combine to the slightest extent with the uranium. Since combination fails to take place, even in the presence of considerable quantities of oxygen, there is naturally some cause to doubt that the light colored powder above mentioned is really an oxybromide. Possibly it is, after all, uranous bromide in a different state of aggregation.

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* Loc. cit. See also Richards, These Proceedings, **33**, 423 (1898).

The hydrated uranyl bromide is more easily obtained. The green oxide was reduced by hydrogen to uranous oxide, suspended in water, and heated with bromine on the steam bath. After driving off the excess of bromine, uranyl bromide remains in solution. The solution may be evaporated to the consistency of a thick syrup, and even under the best conditions the yield of crystals is very small. Moreover, it is almost impossible to wash the crystals free from the mother liquor, since they are extremely soluble in water and alcohol, and ether decomposes the compound, setting free bromine. Hence uranyl bromide was abandoned.

Of the iodine compounds of uranium, the iodate alone seemed promising. This compound has been prepared and described by A. Ditte,* who assigns to it the anhydrous formula $UO_2(IO_3)_2$. The iodate was prepared by us as follows: —

To a solution of uranyl nitrate, containing much nitric acid, was added a solution of iodic acid, prepared by warming finely powdered iodine with nitric acid of specific gravity 1.50. Both solutions were heated to boiling before mixing. Uranyl iodate is precipitated as a yellow, finely crystalline salt, but slightly soluble in water at ordinary temperatures. At 100°, however, if some nitric acid is added, it is possible to obtain a solution containing ten grams of iodate to the litre. On cooling, 2.5 to 3.0 grams of iodate crystallize out. By recrystallizing a few times, in sufficiently large vessels, it is possible to obtain a compound in a high state of purity.

The method of preparation described above is that recommended by Ditte. Although Ditte's course of procedure was carried out as exactly as possible, the compound obtained differed from that which he describes. Instead of being anhydrous, it contained one molecule of water. Inasmuch as Ditte's statement of the amount of nitric acid which he used is extremely vague, different concentrations were tried, from a solution slightly acid up to one containing twenty-five per cent of strong nitric acid. In every case the hydrated compound was obtained. Ditte did not recrystallize his compound, but our recrystallized product was identical with that which was only once precipitated. The analysis given is the average of ten concordant analyses of material prepared from both hot and cold solutions. Both recrystallized iodate and that precipitated only once are represented. The method of analysis is described below.

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* Annales de Chimie et de Physique, 6th Series, **21**, 158 (1890).

ANALYSIS OF URANYL IODATE.

	Found.	Calculated for $UO_2(IO_3)_2H_2O$.
Uranous oxide	42.54%	42.34 %
Iodic acid	54.84	54.84
Water (by difference)	2.62	2.82
	100.00%	100.00%

In determining the composition of the iodate, a weighed quantity of the substance was used, and the percentage composition by weight calculated in the usual manner. For an atomic weight determination, however, any method which involves the original weight of a salt crystallized from solution as a factor in the calculation must of course be avoided on account of the ever present possibility of included mother liquor. It was necessary, then, to determine directly the ratio of iodine to uranium, or to uranium oxide. To determine the uranium, advantage was taken of the behavior of the iodate on ignition. When heated, the iodate is decomposed, water, oxygen, and iodine being given off, leaving uranium oxide. The process was carried on in an ordinary combustion tube of hard glass, a current of dry air being passed through the tube. Since Zimmermann has shown that the green oxide undergoes partial reduction at high temperature unless in an atmosphere of oxygen,* a stream of oxygen was finally passed through the tube. The oxide was then cooled in an atmosphere of oxygen. Treated in this way, the decomposition of the iodate is not complete. Some iodine always remained in the oxide, even when the heat was maintained for three hours at a temperature just below the softening point of the combustion tube. To correct for this amount of iodine, the oxide was weighed, dissolved in dilute nitric acid, and the iodine precipitated as argentic iodide. The amount of iodine found in this way varied from 0.1% to 1.0% of the total iodine, according to the duration of the period of ignition.

Iodine was determined in another sample of material exactly similar to that used for the uranium. The method was, briefly, reduction of the iodate by sulphurous acid, and precipitation with silver nitrate. Stas has shown that silver iodate can be converted completely and without loss into silver iodide by the use of sulphurous acid,† and the same

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* Annalen der Chemie u. Pharmacie, **232**, 287 (1886).

† Untersuchungen über die Gesetze der chemischen Proportionen über die Atomgewichte u. ihre gegenseitigen Verhältnisse, J. S. Stas. Aronstein's translation, p. 69.

method applies equally well to uranium iodate. The iodate was suspended in 200 c.c. of water acidified with 20 c.c. sulphuric acid, cooled in ice to 0°, and pure sulphur dioxide was passed in until the solution smelled strongly of this reagent. The flask was then removed from the ice and shaken occasionally. From three to four hours is required before complete reduction takes place and the last traces of iodate go into solution. When completely reduced, silver nitrate is added, and heated to 60° in order to cause the more coherent deposition of the precipitate.* Thus it was found possible to convert the insoluble iodate into soluble iodide without loss of iodine.

In this way the ratio of uranium oxide to iodine may be determined, regardless of the presence of occluded water in the iodate used, provided that the amount of water occluded be exactly the same in each of the samples. It would obviously be more satisfactory to determine both uranium and iodine in the same sample, provided a sufficiently simple method could be found.

The following method was found to fulfil the required conditions fairly well. A quantity of the iodate was placed in a boat in a combustion tube, to one end of which was attached, by a ground glass joint, a weighed U-shaped tube. The free end of this tube was drawn out and fused to a smaller tube which dipped into a solution of sulphurous acid. On heating the iodate in a stream of air and oxygen, the salt was decomposed and the iodine was carried over and condensed in the U-tube, which was packed in ice. The small quantity of iodine vapor not condensed was collected in the sulphurous acid and precipitated as silver iodide. The heating was continued for an hour after no more iodine could be seen coming off. The end of the U-tube was then sealed by fusing off the small tube, and the other end was closed by a ground glass stopper immediately after disconnecting from the combustion tube. In this way about ninety-nine per cent of the total iodine was weighed directly as free iodine. Of course the small amount of iodine remaining in the oxide after ignition had to be determined separately, as already described. By this method the amount of iodine found was practically identical with that found by the sulphurous acid method.

In determining the iodine present in the oxide after ignition, it has been assumed that the iodine is present as iodide. Although it is hard

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* When silver iodide is precipitated in the presence of sulphurous acid, the supernatant liquid does not become clear enough to filter even after several days, unless heated to 60°.

Vide Stas, "Untersuchungen," p. 69.

to believe that at the temperature employed any of the iodine can exist as iodic acid, it is impossible to prove the point experimentally. The uncertainty in regard to this point renders the use of the method inadvisable where the greatest possible accuracy is desired. Hence none of these analyses have any significance as a basis for computing the atomic weight of uranium.

Besides the bright yellow, slightly soluble iodate, we prepared a paler yellow, more soluble, and more highly hydrated salt, which suffers transition quickly into the earlier compound at a high temperature and more slowly at a low temperature. Double iodates with sodium and potassium were also prepared. Some of our observations were inconsistent with the published record concerning the subject; but in spite of our desire to clear up the uncertainty and to study the rather interesting transition phenomena, we abandoned the iodates because none of them gave promise of a precise basis for the determination of the desired atomic weight.

The next compound investigated was the oxalate, which has the composition $UO_2C_2O_4 \cdot 3H_2O$. Owing to the comparatively slight solubility of this compound it can be obtained in a state of great purity by a few crystallizations.

The best method of analysis is that of dry combustion, the carbon dioxide being absorbed in potash in the usual manner. The uranium is left in the combustion tube as the green oxide, U_3O_8, and consequently can be compared directly with the weight of carbon dioxide obtained. This obviates the necessity of using the weight of the oxalate as a factor in the calculation of the atomic weight, and so eliminates the error due to included water. As already mentioned, this method has been used by Ebelmen and Péligot in their determination of the atomic weight of uranium. There is in this method a possible source of error, difficult of detection and correction, but none the less dangerous, in the possibility that the uranium oxide may after combustion still retain traces of carbon. Moreover, it became evident, after a few analyses had been made, that combustion analysis, as ordinarily conducted, is an exceedingly questionable method where great accuracy is desired. The great difficulty in obtaining absolute "blanks" is well known. Our experience amply confirmed the observations of Mabery,* Auchy,† and others in regard to

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* Inaccuracies in the Determinations of Carbon and Hydrogen of Combustion, C. F. Mabery, Journal Am. Chem. Soc., **20**, 510 (1898).

† George Auchy, Journal Am. Chem. Society, **20**, 243 (1898).

the loss of water and possibly of carbon dioxide from the ordinary form of potash bulbs. We also found a single sulphuric acid tube entirely insufficient to absorb all the water. Clearly, then, if we were to use this method, an elaborate investigation of the form of apparatus, method of procedure, and limits of error, was absolutely imperative. The use of the oxalate, however, did not seem sufficiently promising to warrant the necessary expenditure of time.

After thus investigating the uranium compounds which seemed likely to furnish a suitable basis for an atomic weight determination, anhydrous uranous bromide, in spite of its disadvantages, seemed most likely to fulfil the necessary requirements. As already mentioned, this compound oxidizes with the greatest ease on exposure to moist air. It was necessary, therefore, to devise apparatus which should preclude any possibility of bringing the sublimed bromide in contact with the air of the laboratory until it had been collected and weighed. After much experimenting with different forms of apparatus, the following method was adopted.

Preparation and Collection of Pure Uranous Bromide.

The mixture of urano-uranic oxide and carbon was placed in a porcelain boat within the larger of two "telescoping" porcelain tubes. The portion of the tube containing the oxide was heated in a Fletcher furnace, and after thoroughly sweeping out the apparatus with dry nitrogen, a mixture of dry nitrogen and bromine vapor passed over the oxide. The sublimed bromide collected near the inner end of the smaller porcelain tube. The very efficient and elaborate desiccating apparatus which served so well in the work on the atomic weights of cobalt and nickel, was very kindly given by Dr. Baxter for use in this investigation.* This apparatus, with slight modifications, was used for drying the nitrogen and bromide, and was connected by a ground glass joint with the porcelain combustion tube.

With this apparatus traces of air diffused through the annular joint between the porcelain tubes, forming a coating of oxide on the inner tube.† In the case of uranium, the oxide is found to be copiously mixed with the sublimate also. This diffusion of air takes place even when the outer end of the inner porcelain tube is nearly closed, thus making a considerable outward current within the tubes.

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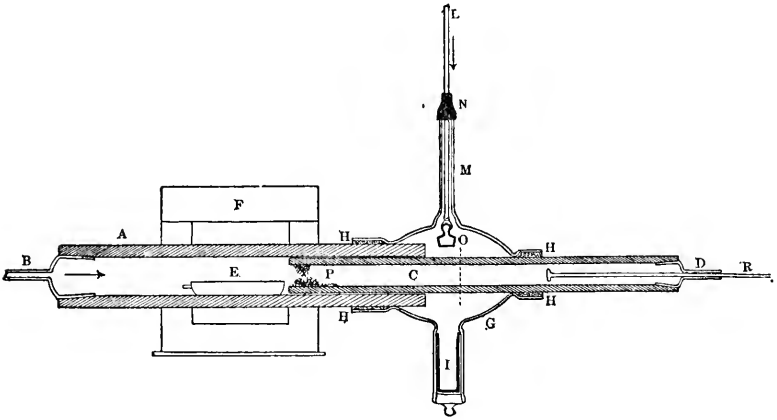
* For a full description of this apparatus see There Proceedings, **33**, 124 (1897).

† In the case of cobalt and nickel this oxide was easily removed by subsequent treatment, but in the present case removal was impossible.

In order to obviate the difficulty and exclude air a glass jacket was slipped over the joint between the tubes. The construction and use of this jacket will be made clear by reference to the accompanying drawing.



SECTION OF SUBLIMING AND BOTTLING APPARATUS.

A, outer porcelain tube fitted with ground-glass joint B; C, inner porcelain tube with ground-glass stopper D; E, boat containing oxide and carbon; F, furnace; G, glass jacket; H, H, H, H, packing of asbestos wool; I, weighing bottle; L, tube for admitting nitrogen, sliding within tube M through rubber connection N, and carrying at its end stopper O of weighing bottle; P, sublimate; R, rod for removing sublimate.

The jacket was drawn down at the ends, so as to fit the porcelain tubes A and C as well as possible, and the spaces between the tubes and the jacket were packed tightly with asbestos wool. This packing makes a joint sufficiently tight to withstand a pressure equal to that of eight or ten centimeters of water. The jacket was provided with a long tube, M, within which slid a second tube, L, connection being made by means of the short piece of rubber tubing, N. To the end of the inner tube was attached, by platinum wires, the stopper, O, of the weighing bottle. The outside diameter of L was very little less than the inside diameter of M, thus leaving very little space between the walls of the two tubes. For this reason, and also on account of the length of the tube M, — about fifteen centimeters, — there was little danger of bromine diffusing up in sufficient quantities to attack the rubber connection, N. Even if this were the case there could be no possibility of contamination of the sublimate thereby, since there was always a constant outward pressure of bromine during the sublimation. The outer end of L was connected with the nitrogen supply of the desiccating apparatus. All glass joints and stop-cocks were lubricated with syrupy phosphoric acid.

The method of procedure was as follows: In the porcelain boat, E, was placed an intimate mixture of urano-uranic oxide and pure carbon, the carbon being about twenty per cent of the weight of the mixture, thus insuring a large excess of carbon. The apparatus was then thoroughly swept out by nitrogen, which enters at B and L simultaneously. After the air was completely expelled, the combustion tube was gradually raised to a high temperature by the blast lamp. Heating in a current of nitrogen was then continued for three hours at least, sometimes longer, in order to insure complete removal of all traces of air and moisture. During this and subsequent operations, the outlet of the stopper D of the inner tube was nearly closed by asbestos wool, thus maintaining a constant and considerable pressure within the apparatus, and hindering the diffusion of air. After this preliminary heating in nitrogen, bromine vapor was passed in through B. During the first trials of the apparatus it was our practice to keep a slow current of nitrogen passing in at L during the sublimation. This kept the jacket entirely free of bromine, a very slow current of nitrogen being sufficient to keep any bromine from passing between the walls of the porcelain tubes. It was found, however, that traces of air diffused through the permeable asbestos packing, and were of course carried into the combustion tube by the current of nitrogen, forming on the inner tube a coating of oxide, and contaminating the sublimate. In order to avoid this, the nitrogen was shut off from L sometime before turning on the bromine. After turning on the bromine, the jacket slowly filled with dilute bromine vapor. While the greater part of the sublimate collected within the inner tube, a little collected between the walls of the two tubes, almost sealing the annular space. This sublimate, which collected on the outside of the inner tube, is a valuable indicator of the condition of the sublimate within. In the presence of mere traces of oxygen the lustrous brown color of the uranous bromide gives place to a dull yellow color easily distinguishable. Comparatively small quantities of oxygen form a coating of black oxide. When the sublimation is conducted according to the method described, the outside of the inner tube is free from any traces of the supposed oxybromide or of oxide, thus showing that no appreciable quantity of moist air could have reached the innermost portions of the sublimate. The best proof of the purity of the sublimate is of course found in the agreement of analyses of substance formed under various conditions of bromine supply.

After the bromine had been run for about one and a half hours, the sublimate was cooled for three hours in a current of nitrogen. When the tubes were thoroughly cold, nitrogen was finally passed into the jacket through L, in order to sweep out any traces of bromine that might still remain. The inner tube, containing the sublimate, was then carefully drawn out until the inner end reached a position over the mouth of the weighing bottle, indicated in the diagram by the dotted line. This can be done without seriously disturbing the asbestos packing, a rapid current of perfectly dry nitrogen being admitted meanwhile through L. By means of the glass rod, R, the sublimate was pushed out of the tube and dropped into the weighing bottle, I. The tube L, carrying the stopper, was then pushed down and the stopper inserted. The stopper was held by the platinum wires so lightly that after pushing it into place the tube L could be withdrawn, leaving the stopper inserted in the bottle.

Thus uranous bromide was sublimed, collected, and bottled up in an atmosphere of dry nitrogen ready for weighing, without once coming in contact with the air of the laboratory. That the apparatus is effective for the purpose intended, and capable of producing material of constant composition, was shown by the first rough analyses of uranous bromide, which yielded 57.41, 57.41, and 57.42 per cent bromine respectively. These analyses were made with material that had not been purified, but served to show the constancy of composition of the sublimate; for not only was the length of time occupied in the sublimation varied, but in one case the sublimate was cooled in bromine instead of in nitrogen. Of course if an appreciable amount of an oxygen compound were formed, by diffusion of air or moisture, there would almost certainly be discrepancies in the results, since it is hardly conceivable that under the varying conditions exactly the same quantities of oxy-salt should be formed each time.

Because the specific gravity of uranous bromide was unknown, the following determinations were made: 2.0328 grams of the salt displaced on one occasion 0.3332 gram of kerosene at 21°, and at another trial 0.3322 gram. The kerosene had been redistilled, and only the high boiling portion was used. The density of the kerosene at 21°, referred to water at 4°, was 0.7919. Hence the specific gravity of the uranous bromide was (1) 4.830 and (2) 4.846, giving as the mean 4.838. This value was used in reducing the observed weights of bromide to the vacuum standard.

During the weighing in the final analyses, the bromide of uranium was still surrounded by an atmosphere of pure dry nitrogen in the tightly stoppered weighing bottle. Since this bottle had been full of dry air when it was first weighed, a small correction had to be applied on this account. The difference in weight between 6.70 cubic centimeters (the interior volume of the weighing bottle) of air and the same volume of nitrogen at 20° C. is 0.000265 gram. Of this nitrogen a gram of uranous bromide displaced $\frac{1}{4.84} = 0.206$ cubic centimeters, or 0.24 milligram, while the brass weights used in weighing the bromide displaced 0.145 milligram of air. Hence in vacuum a gram of uranous bromide would weigh $0.265 + 0.24 - 0.145 = 0.36$ milligram more than the observed weight, while two grams would weigh $0.265 + 2(0.24 - 0.145) = 0.46$ more than the observed weight. All the weights given in the tables are corrected in this way to the vacuum standard.

Methods of Analysis.

By the use of these devices we were able to prepare and weigh pure uranous bromide in a definite state. There still remained, however, the problem of devising a suitable method of analysis. As previously mentioned, all uranous compounds reduce silver nitrate. making impossible the usual method of procedure in halogen determinations.

The method of precipitating the uranium and determining bromine in the filtrate involves too much danger of loss of material in the multiplicity of operations. The most satisfactory solution of the problem seemed to be to oxidize the compound to the uranyl salt, provided this could be done without loss of bromine. Nitric acid is of course effective as an oxidizing agent, but the oxidation is accompanied by loss of bromine. After much experimenting, hydrogen dioxide was found to be the most suitable oxidizer. From neutral solutions of uranium compounds, hydrogen dioxide precipitates a hydrated peroxide of uranium. If the solution is slightly acid, this precipitation is prevented and the uranous compound completely oxidized to the uranyl state. The weighed sample of uranous bromide was dissolved in considerable water — at least 400 cubic centimeters of water to each gram of bromide. The bottle containing the bromide was opened by means of a suitable glass fork. either below the water or just above the surface, so that it could be instantly submerged, and thus avoid loss of hydrobromic acid by the action of moist air. The calculated volume of a standard solution of pure hydrogen dioxide was then diluted to about 100 c.c., one cubic centimeter of pure dilute sulphuric acid was added, and the mixture was slowly run into the solution of uranous bromide. The green color of the uranous salt soon changes to the yellow color characteristic of uranyl compounds. On adding the first few cubic centimeters of the dilute hydrogen dioxide solution, a greenish white precipitate came down. Addition of more of the acid dioxide solution redissolved it, and the resulting solution was perfectly clear. This peculiar hydrolytic action is due to the acid, and not to the hydric dioxide, for the same reaction occurs if dilute sulphuric acid alone is added to the solution.

The explanation of this interesting phenomenon, which is just the opposite of what might have been expected, is, undoubtedly, that the bromide is already hydrolyzed to a great extent by merely dissolving in water. The hydrate is probably in solution in the colloidal state. Evidence of this is found in the fact that if the clear aqueous solution of uranous bromide is allowed to stand exposed to the air. a hydrate gradually separates, giving to the solution a cloudy, murky appearance. After two or three days this precipitate disappears, giving place to a clear yellow solution of oxybromide and hydrobromic acid. The addition of sulphuric acid coagulates the colloid before it can all be converted into uranyl salt.

In order to be sure that no bromine or hydrobromic acid is lost by this method of oxidation, the following experiment was made. 0.5 gram of bromide was dissolved in 250 c.c. of water, 50 c.c. of dilute sulphuric acid (1 : 10) was added, and the hydrogen dioxide solution was run in. This was done in a closed flask, similar in construction to a gas washing bottle. A current of air was drawn through the bottle and then through starch solution containing potassium iodide to see if bromine is liberated. Not the slightest trace of blue color appeared in the starch solution, even after adding a large excess of hydrogen peroxide and allowing it to stand over night. A test for hydrobromic acid was sought in a similar way, by drawing the air through a solution of silver nitrate, again with negative results, as was to have been expected. These experiments show conclusively that uranous bromide can be oxidized completely by hydrogen dioxide, without loss of bromine.

Silver nitrate, in moderately concentrated solutions, is not acted upon by a three per cent solution of hydrogen peroxide. Consequently a considerable excess of the latter reagent could do no harm. Nevertheless care was taken never to add more than the calculated amount of hydrogen dioxide. Moreover, the solution of hydrogen dioxide used contained only one per cent of this reagent, and this was diluted ten times before adding to the bromide solution, thus reducing to a minimum the possibility of too vigorous oxidation, with consequent liberation of bromine.

After the oxidation, bromine was precipitated by pure silver nitrate in the usual manner. This precipitation was conducted in an Erlenmeyer flask fitted with a ground glass stopper. The silver bromide was collected on a Gooch crucible, and dried in an electrically heated drying oven. Of course the asbestos shreds carried away in washing the silver bromide were collected by passing the filtrate and wash water through a fine filter, and their weight was added to that of the silver bromide. The bromine determination was carried on in orange colored light.

It was found in the work upon cobalt and nickel that the porcelain tube is attacked by bromine vapor at the high temperature employed during the sublimation, with the result that sodium bromide was always present in the sublimate. In these investigations this impurity was determined by the reduction of the bromide to the spongy metallic state by means of hydrogen, and extraction by water.* A somewhat similar method was tried with uranium. Since hydrogen reduces uranous bromide only to the tri-bromide, the bromide was ignited in a current of air and the resulting oxide leached with water. It was found to be impossible to oxidize the bromide completely. A little uranous bromide invariably remained and was washed out with the alkali. Both dry and moist air was tried, also ignition in steam, but in every case uranium was washed out in considerable quantity.

Precipitation of the uranium by hydrogen dioxide was next tried, but it was found impossible to precipitate the uranium completely. The rather unsatisfactory method of determining the sodium in the filtrate from the bromine precipitation, or in a new sample of uranous bromide as nearly similar as possible, after removing the uranium with ammonium sulphide, appeared to be the only available method. The filtrate and wash waters from the bromine precipitation were evaporated in platinum to small bulk, and the uranium and excess of silver precipitated by pure colorless ammonium sulphide. This reagent precipitates uranium completely. The filtrate was then evaporated to dryness, the ammonium salts expelled by ignition, and the residual sodic nitrate converted to the sulphate and weighed as such. Of course these operations were all conducted in platinum vessels. This method of work is not wholly satisfactory, on account of the complexity of operations involved, but it seems to be the only practical method.

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* These Proceedings, **34**, 329, 359 (1899).

Purification of Materials.

As the source of uranium, commercial "chemically pure" uranium acetate was used.[*] This was first converted to the chloride on account of the greater solubility of this compound, — by precipitation as ammonium uranate and redissolving in dilute hydrochloric acid. To the hot and slightly acid solution, pure sulphuretted hydrogen was added to saturation. The free acid was then neutralized with ammonic hydroxide, a slight excess of the alkali was added, and more sulphuretted hydrogen was run in. In this way some uranyl sulphide was precipitated, in order to sweep down with it any colloidal sulphides of the higher groups which might otherwise escape removal. The excess of sulphuretted hydrogen was boiled off, and after standing over night the supernatant liquid was decanted through a washed filter.

The next step depended upon the fact that uranium remains in a solution of the double carbonate of ammonium and uranium, in the presence of an excess of ammonium sulphide, while all the other members of the aluminum and iron groups are thrown down by this reagent. Consequently ammonic hydrate and ammonium carbonate in slight excess were added to the filtrate, forming the double carbonate. If the solutions are concentrated, the double carbonate is precipitated when more than a slight excess of ammonic carbonate is used. This happened in some cases, when it was necessary to redissolve the precipitate in dilute hydrochloric acid and again add ammonic carbonate in more dilute solution. About fifty grams of carbonate per litre was found to give the best results. Ammonic hydroxide was then added to the hot solution, and sulphuretted hydrogen in excess. After standing over night the solution was filtered. In several of the more concentrated solutions, a considerable quantity of the salt crystallized out. These crystals were worked up separately, as they were probably purer than the solution. On boiling the solution to decompose the excess of ammonium sulphide, some of the ammonic carbonate was decomposed, causing the precipitation of some uranium sulphide. This precipitate was discarded, as it might have contained iron, or other analogous metals which had previously escaped precipitation. Dilute hydrochloric acid in slight excess was added, and the carbon dioxide was expelled by boiling. The free acid was then almost neutralized with pure ammonic hydroxide, and

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[*] This method of uranium purification, with some modifications and additions, is similar to that employed by Zimmermann. Annalen der Chemie u. Pharmacie, **232**, 299.

pure ammonic sulphhydrate added in excess. The color of the resulting precipitate of uranium sulphide varies greatly with the temperature. In warm solution it was at first reddish brown, while that precipitated in the cold varied from bright red to brownish yellow. On washing, all turn black, the sulphide being decomposed into uranous oxide and sulphur. After thorough washing the resulting mixture of oxide and sulphur was ignited in a porcelain dish, the green urano-uranic oxide being the product.

The oxide was then dissolved in a platinum dish in redistilled nitric acid, evaporated, and recrystallized from nitric acid solution. Uranyl nitrate does not crystallize well from aqueous solution, but it was found that if a little nitric acid is added, it crystallizes readily in fairly large monoclinic prisms. This recrystallization was repeated ten times from acid solution, and finally twice from aqueous solution. Finally the pure nitrate was converted to the oxide by ignition in platinum. A second sample, used in the preliminary series, was prepared by repeated fractionation of the mother liquors of the first sample.

Since this work was carried out, Sir William Crookes * has published the account of several methods by which he was able to prepare specimens of uranyl nitrate which were not radio-active. The radio-activity of uranium has hitherto been supposed to be characteristic of this element. Crookes has shown, however, that this is not the case, but that the active element can be separated by treatment with ether, by fractional crystallization, or by treatment with excess of ammonium carbonate. Unfortunately none of the pure oxide prepared for this investigation remained, hence it is impossible to test directly its radio-activity. Since two of Crookes's methods were used in purifying our material, viz. the ammonium carbonate treatment and fractional crystallization, it is highly improbable that our oxide was radio-active. In repeating Crookes's work with nitrate made from some of the same material used in preparing our best nitrate, it was found that a sample of the *fifth* crystallization gave no trace of action on twenty-four hours exposure to a quick photographic plate. The material used in this experiment had not been submitted to the ammonium carbonate treatment. When it is considered that the material used for our atomic weight determinations was first put through the carbonate process, — in itself sufficient to remove the radio-active element, — and then was recrystallized *twelve times* as nitrate, it would seem that our pure oxide must have been free from all radio-active material.

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* Proceed. Lond. Royal Soc., **66**, 409 (1900).

There is another phase of this subject that deserves to be considered, namely, the possible effect of radio-active matter, even if present, upon the atomic weight value. The purest specimen of radium or "polonium" yet obtained has consisted of a mixture containing probably little more than fifty per cent of the active element, as nearly as could be estimated. This highly impure material, however, possesses 8,000 times the radio-activity of uranium. The radio-active power of the pure material is undoubtedly very much greater than that of the impure mixture. Consequently the quantity of ratio-active substance necessary to give to uranium the comparatively slight degree of activity that it possesses must be exceedingly minute. Giesel has recently shown * that a quantity of radium so small that it cannot be detected by sulphuric acid is sufficient to affect a photographic plate. Crookes also says on this point, "Considering my most active UrX does not contain sufficient of the real material to show in the spectrograph, yet is powerful enough to give a good impression on a photographic plate in five minutes, what must be its dilution in compounds which require an hour, a day, or a week to give an action?" † Even in the ordinary active uranium compounds it is most unlikely that the active element — if indeed it is an element — could possibly be present in quantity sufficient to exert any influence whatever upon the atomic weight of uranium.

Pure carbon was obtained by ignition of sugar. Large, clear crystals of the best "rock candy" of commerce were ground up in a porcelain mortar and ignited at low heat in a platinum dish as long as organic gases were given off. The resulting charcoal was then powdered in an agate mortar and ignited in a hard glass combustion tube; first in a stream of pure, dry nitrogen, and finally in a stream of bromine vapor. In this way the carbon was freed from any impurities which might, if present, be acted upon during the sublimation and contaminate the sublimate. Owing to the presence of undecomposed carbohydrates, or possibly of water, most of the bromine was converted into hydrobromic acid. Heating in bromine was continued until acid fumes ceased to be given off. Finally, the carbon was again heated in a current of dry nitrogen. Five grams of carbon, thus prepared, left no visible or weighable residue after combustion in oxygen.

The method of bromine purification was essentially identical with that used in many other atomic weight investigations in this laboratory, and has

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* Berichte der deutschen chemischen Gesellschaft, **33**, 3569 (1900).

† Proceed. Lond. Royal Soc., **66**, 422 (1900).

been proved by long experience to be the most efficient and satisfactory. Commercial, "pure" bromine was partially freed from chlorine by shaking with a fifteen per cent solution of potassic bromide. One fourth of the bromine was then converted to calcic bromide by running it slowly into milk of lime in the presence of a large excess of ammonia. The calcic bromide solution was filtered and concentrated by evaporation, and the rest of the bromine was added to it. A little zinc oxide was then added, and after standing over night the bromine was distilled, nearly free from chlorine. Most of the iodine is removed as zinc iodate. After redistilling the bromine, in order to remove any calcium bromide that may have spattered over in the first distillation, it was converted into hydrobromic acid by slowly dropping it into a mixture of red phosphorus and hydrobromic acid. The red phosphorus was at first washed free from chlorides. The hydrobromic acid, containing some free bromine, was distilled. The free bromine liberates any iodine which may have escaped the zinc oxide. The first portion of the distillate, containing free bromine and iodine, and organic matter, was rejected, and so was the last portion, which may have contained traces of arsenic. The hydrobromic acid was then converted into bromine by distilling over pure manganese dioxide previously treated with sulphuric acid and washed. One half the bromine is obtained by the manganese dioxide alone. As soon as no more bromine comes off, a little redistilled sulphuric acid is added, and the rest of the bromine was obtained. It was then redistilled several times, rejecting the first and last portions, and finally dried over pure phosphorous pentoxide.

The silver precipitation also presents no new features, except, perhaps, its somewhat unusual thoroughness. Partially purified silver was dissolved in nitric acid, diluted, and precipitated with pure hydrochloric acid. After thorough washing the chloride was reduced by invert sugar and sodic hydrate which had been purified by electrolysis. The metallic silver was thoroughly washed, dissolved in nitric acid, and again precipitated as chloride and reduced. It was then dried and fused on charcoal; the lumps of silver were cleaned with sand, dissolved in pure nitric acid, diluted to a volume of two litres, and again precipitated with pure hydrochloric acid. The resulting chloride was then digested on the steam bath with aqua regia, washed, and once more reduced by invert sugar and sodic hydrate. After drying, it was fused on pure sugar charcoal. The buttons of silver were cleaned with sand, and then purified electrolytically, a small portion being dissolved in nitric acid to serve as the electrolyte, and the rest serving as anode material. The crystals of electrolytic silver were then dried over potash and fused in vacuo on a boat of pure lime. The buttons of silver thus obtained were treated with nitric acid to remove the surface, dried, and kept over potash. A second sample was obtained by fusing in vacuo electrolytic silver which had been prepared from the silver bromide obtained in Dr. Baxter's work upon cobalt, which was known to be very pure.

Hydric dioxide was purified as follows: To a solution of the ordinary commercial peroxide prepared for medicinal use, was added a solution of baric hydroxide, which had been purified by recrystallization. The precipitated baric dioxide was washed until a nitric acid solution of the same showed no trace of halogen. It was then added to pure dilute sulphuric acid, and the resulting solution of hydric dioxide was filtered and distilled in a partial vacuum. The solution thus obtained showed no trace of halogen, and left no visible residue on evaporation in platinum.

Ammonium sulphide was made from pure ammonia, which had been redistilled in platinum, and pure sulphuretted hydrogen. It left no visible residue on evaporation in platinum.

Hydrochloric and nitric acids were redistilled in a platinum still, and throughout the work platinum vessels were used wherever possible.

Water was twice redistilled, once over alkaline potassic permanganate, and again over acid potassic sulphate from a Jena glass flask, a block-tin condenser and Jena glass receiver being used.

THE RESULTS OF THE ANALYSES OF URANOUS BROMIDE.

The method of analysis has been already fully described.

The analyses recorded in the first series were made by adding an excess of silver nitrate to the solution of uranyl bromide. From the ratio of the observed weights of uranous bromide to argentic bromide, the molecular weight of uranous bromide was calculated, that of argentic bromide being assumed to be 187.885. From the results obtained from this preliminary series the weight of silver necessary to precipitate the bromine in one gram of uranous bromide was calculated. In the subsequent determinations the exact weight of silver required was weighed out, as nearly as possible, and dissolved in pure nitric acid with suitable precautions to avoid loss. The exact end point was reached by standard hundredth normal solutions of argentic nitrate and hydrobromic acid, by means of the nephelometer.* After determining the end point a slight excess of argentic nitrate was always added, and the weight of the total

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* Richards, These Proceedings, **30**, 385 (1894). Z. anorg. Chem., **8**, 269 (1895).

argentic bromide determined. Thus from each of these analyses two distinct ratios were obtained as a basis for the calculation of the molecular weight of uranous bromide, — the ratio of uranous bromide to argentic bromide, and that of uranous bromide to silver.

As would naturally be expected from the complexity of operations involved, determinations of the sodium in the filtrates from the argentic bromide gave unsatisfactory results. The large quantity of filtrate and wash waters had to be evaporated to small bulk, the uranium precipitated, and the sodium determined in the residue. It seemed advisable to make a series of separate analyses for sodium only, and use the average percentage of sodium found as a constant correction. This method was used in the work upon cobalt and nickel.*

Accordingly three alkali determinations were made, *wholly in platinum,* the material not coming in contact with glass at any time except during the original collection and weighing of the sublimed bromide. The sublimate was dissolved in pure water, in a platinum dish, and the uranium was precipitated with pure ammonium sulphide. The ammonium sulphide was freshly prepared for each analysis, wholly in platinum. It left no residue on evaporation in platinum. The precipitated sulphide was digested on the water bath to expel most of the excess of ammonium sulphide, filtered through a platinum funnel, and the filtrate and wash water evaporated to small bulk in a platinum dish. The sodium bromide was then converted to sodium sulphate and weighed. The following table contains the data and result: —

No.	Weight of Uranous Bromide.	Weight Sodic Sulphate obtained.	Equivalent Weight of Sodic Bromide.	Per cent Sodic Bromide.
	grams.	gram.	gram.	
1	1.656	0.00092	0.00133	0.081
2	2.629	0.00143	0.00207	0.079
3	1.407	0.00121	0.00175	0.124
			Average	0.095

The average of these three determinations, 0.095, per cent, is practically identical with the amount of sodic bromide found in the cobalt and nickel work, which was 0.10 per cent. The porcelain tubes used in this inves-

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* These Prcoeedings, **34**, 339, 365 (1899).

tigation were of the same manufacture as those used in the nickel and cobalt work, and since the method of preparation of the three bromides was practically the same, probably the quantity of sodium extracted from the tubes by the action of the hot bromine vapor was the same, — *on the average*, — in all three cases, and not far from 0.10 per cent. Consequently, in calculating the following results, this value was used as a constant correction. The effect of applying the correction is to raise the calculated atomic weight about two tenths of a unit. Of course by this method the quantity of sodic bromide calculated will vary somewhat from the exact quantity present, *in individual determinations.* The *average* result, however, will undoubtedly vary but little from the result obtained if the alkali could be determined in each sample. It certainly is very much nearer the truth than the results to be obtained by the cumbersome method of determining the alkali in the filtrate from each precipitation of argentic bromide.

Analysis No. 2 was rejected on account of contamination of the uranous bromide by shreds of asbestos from the packing of the jacket, and No. 4 was not used because the combustion tube cracked during sublimation, rendering probable the formation of some oxybromide. The silver required in analysis No. 6 was determined for practice preparatory to the final series, being 0.9087 gram when all corrections were applied. It is not included in the table, since its nature was essentially preliminary. As usual, all weighings were reduced to the vacuum standard. While all

THE ATOMIC WEIGHT OF URANIUM.

O = 16.000; Ag = 107.93; Br = 79.955.

FIRST SERIES (PRELIMINARY). $UBr_4 : 4AgBr$.

No. of Analysis.	Total Weight of Uranous Bromide + Sodium Bromide in vacuo.	Weight of Uranous Bromide corrected for NaBr.	Total Weight of Silver Bromide in vacuo.	Weight of Silver Bromide corrected for NaBr.	Parts of Uranous Bromide equiv. to 100 parts Argentic Bromide.	Atomic Weight of Uranium.
	grams.	grams.	grams.	grams.	grams.	
1	2.20795	2.2058	2.97391	2.9699	74.272	238.36
3	1.44321	1.4418	1.94272	1.9401	74.316	238.69
5	1.40639	1.4050	1.89355	1.8910	74 299	238.56
6	1.17607	1.1749	1.58396	1.5818	74.276	238.39
			Average		74.289	238.50

SECOND SERIES. $UBr_4 : 4AgBr$.

No. of Analysis.	Weight of Uranous Bromide + Sodic Bromide in vacuo.	Wt. of Uranous Bromide corrected for Sodic Bromide.	Total Weight of Silver Bromide in vacuo.	Weight of Silver Bromide corrected for NaBr.	Parts of Uranous Bromide equiv. to 100 parts Argentic Bromide.	Atomic Weight of Uranium.
	grams	grams.	grams.	grams.	grams.	
7	1.80174	1.7999	2.42588	2.4226	74.296	238.54
8	1.06723	1.0662	1.43713	1.4352	74.290	238.50
9	1.85698	1.8551	2.50009	2.4967	74.302	238.59
				Average	74.296	238.54

THIRD SERIES. $UBr_4 : 4Ag$.

No. of Analysis.	Weight of Uranous Bromide with all Corrections.	Weight of Silver in vacuo (not corrected for Sodic Bromide).	Weight of Silver with all Corrections.	Wt of Uranous Bromide corresponding to 100 grams Silver.	Atomic Weight of Uranium.
	grams.	grams.	grams.	grams.	
10 (7)	1.7999	1.39365	1.3918	129.322	238.49
11 (8)	1.0662	0.82559	0.8245	129.315	238.46
12 (9)	1.8551	1.43617	1.4342	129.347	338.60
				Average	238.52

Average of all determinations 238.52
Average of six final determinations 238.53

the weighings were actually made to the hundreths of a milligram the final corrected data are rounded off to the nearest tenth of a milligram, since the deviations of the results show that the hundredths could have had no significance.

The extreme difference between the highest and the lowest values in the preliminary series is 0.33 unit, in the second series 0.09 unit, and in the third series 0.14 unit. At first sight these variations seem large, but their relative magnitude appears smaller when the great molecular weight of uranous bromide, 558.34, is taken into consideration. Thus the extreme percentage error of the preliminary series is 0.06, while those of the last two series are only 0.016 and 0.024 per cent respectively.

The so-called "probable error" of the average atomic weight computed from the six analyses numbered 7 to 12 inclusive, if each is given the same weight, is 0.015. That is, according to the theory of least squares, the atomic weight of uranium should be between 238.515 and 238.545.

The magnitude of the maximum deviations in these two final series is, moreover, about as large as would have been expected from known analytical uncertainty. The observed variation in the amount of sodic bromide, for which a constant correction had to be applied, would account for three quarters of it, and the rest, corresponding to less than the tenth of a milligram in the weighings, might easily be due to unavoidable errors of weighing or manipulation.

Further evidence of the trustworthiness of the figures is to be found in the comparison of the amounts of silver used in analyses 10, 11, and 12, with the corresponding amounts of argentic bromide, found in analyses 7, 8, and 9. This comparison is given in the following table, which gives the weights of silver corresponding to 100.000 parts of argentic bromide.

Weight of AgBr in vacuo.	Weight of Ag in vacuo.	Quotient × 100 = per cent of Silver in Argentic Bromide.
grams. 2.42588	grams. 1.39365	57.449
1.43713	0.82259	57.447
2.50009	1.43617	57.445
	Average	57.447
	Stas found	57.445

The result not only verifies the mechanical work, but affords evidence that the precipitate must have been pure argentic bromide. Clearly, then, the analysis is as accurate as need be. Further repetition of the process might reduce the so-called "probable error," but could not change the average by a significant amount. In the present state of the question, the method seems to have been carried as far as expediency demands.

It is worth while to inquire whether or not the method may conceal some source of constant error beyond the reach of the experimental precautions detailed above. Such an error could hardly have occurred during the analysis; for every step of this procedure was verified by confirmatory evidence. If a flaw existed, it must have been in the purity of the original substance. Since the observed atomic weight is lower than the former results, it is important to examine into only those possible irregularities which could have had the effect of lowering the apparent value.

The probable impurities tending to lower the atomic weight are, first, sodic bromide; second, hydrobromic acid; third, free bromine; fourth, uranic pentabromide; and fifth, an unknown metal with a lesser equivalent. The first impurity was found to be present, its amount was determined, and a suitable correction was applied. The second could not have been formed during the sublimation of the uranous bromide, because compounds of hydrogen were scrupulously excluded. If formed by the action of water after the sublimation, the atomic weight would have appeared too high — for moist uranous bromide emits hydrobromic acid instead of absorbing it. The third impurity, free bromine, could hardly have been imprisoned or absorbed by the sharply crystalline salt to any appreciable extent, since the concentration of the bromine vapor in the issuing gases was but small.

The evidence in regard to the absence of pentabromide is fairly conclusive, although somewhat indirect. All attempts by many investigators to form this compound have failed, in spite of the recognized existence of the corresponding chlorine compound. It seemed possible, however, that while this compound is not formed at high temperatures, lower temperatures might permit the addition of the extra bromine. Accordingly the preparations used in Analyses 7, 8, 10, and 11, were cooled in a current of dilute bromine vapor, instead of in pure nitrogen. The presence of a comparatively small amount of pentabromide would make a very decided difference in the quantity of bromine found. Hence the essential agreement of the average result of these analyses, 238.50, with the average result of all the others, 238.52, is good evidence of the absence of uranium pentabromide.

With regard to the fifth possible impurity nothing can be said except to point out the many operations involved in the purifications. These seem to point toward probable purity; but it is nevertheless to be regretted that lack of time prevented the analysis of many different fractions of material, prepared in varying ways.

The presence of oxybromide would of course cause low bromine analyses, and too high an apparent atomic weight. Therefore this possible cause of error need not be considered, even if the oxybromide had ever been made in the absence of water. In the light of all these considerations, there would seem to be no good reason to question the purity of our bromide.

On comparing the result of this investigation, 238.53, with that of Zimmermann's, 239.59 (the only previous work worthy of serious consideration), the difference of over a unit seems at first to be one of great magnitude. The percentage difference (0.45%) is however smaller than many a difference which often has been passed by unheeded in small atomic weights, such as those of magnesium or aluminum. This point illustrates the difficulty of obtaining results with high atomic weights which can satisfy the cursory reader.

Nevertheless, such a difference is far too great to pass unchallenged. It seems highly probable that the greater part of it is due to the previously discussed sources of inaccuracy in Zimmermann's method, — especially to the difficulty of wholly re-oxidizing the lower oxide. The failure to oxidize half a per cent of the uranous oxide, involving an error in the weight of only 0.017 per cent of the total weight of the substance, would account for the discrepancy.

Hence it seems not unlikely that the atomic weight of uranium is really as low as 238.53. Nevertheless, the question cannot be looked upon as conclusively settled. Certainty can be obtained only by the application of a new method, radically different from the two just compared. Our experience of nearly four years of varied work seems to indicate that the search for such method will not be an easy one. The many degrees of quantivalence of uranium and the unsuitable properties of its compounds combine to render the problem one of unusual difficulty. When face to face with a problem of this kind one cannot but admire Stas's wisdom in selecting chiefly univalent elements with powerful affinities in order to prove the constancy of the atomic weights.

The result of our analyses of uranous bromide may be summed up in the following words: If oxygen is taken as 16.000, and bromine as 79.955, the atomic weight of uranium appears to be not far from 238.53.

Cambridge, Mass., U. S. A. 1897–1901.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE SIGNIFICANCE OF CHANGING ATOMIC VOLUME.

II. — *THE PROBABLE SOURCE OF THE HEAT OF CHEMICAL COMBINATION, AND A NEW ATOMIC HYPOTHESIS.*

By Theodore William Richards.

Investigations on Light and Heat made and published wholly or in part with Appropriations from the Rumford Fund.

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THE SIGNIFICANCE OF CHANGING ATOMIC VOLUME.

II. — THE PROBABLE SOURCE OF THE HEAT OF CHEMICAL COMBINATION, AND A NEW ATOMIC HYPOTHESIS.

By Theodore William Richards.

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I. Presentation of the Facts.

In a paper first presented to the American Academy of Arts and Sciences in May, 1900, then revised and printed in the Proceedings a year later,[*] certain interesting facts concerning the significance of changing atomic volume were pointed out and emphasized. It was shown that the contractions and expansions occurring in liquids and solids during chemical reaction are related to the affinities concerned, as nearly as we can estimate those affinities. A greater affinity seems to produce a greater contraction, if the compressibilities concerned are equal. It seemed possible that this idea might have very fundamental and far reaching applications as to matters of fact, and might lead moreover to a somewhat new conception of the atomic hypothesis.

Many such applications have already been tested with plausible results. The complete detailing of the ramifications of this idea would need the compass of a book ; in the present paper the attempt will be made merely to sketch the relations of a single side of the question.

In the paper already referred to the suggestion was made that the heat of chemical reaction might be traceable to the work done by chemical affinity in compressing the substances concerned. The discussion below will show the close relationship which exists between these facts.

The most serious difficulty in the way of determining the relationship is the extreme scarcity of data concerning compressibility. Obviously

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[*] These Proceedings, **37**, 1 (June, 1901).

the compressibility of a *compound* contains too many possible variables to form at once the certain basis of exact reasoning; and among *elements* only mercury, lead, copper, and iron in the uncertain form of steel, seem to have been even crudely studied.* The problem is moreover complicated by the fact that the coefficient of compressibility diminishes as the pressure increases.

The work which is needed in order to compress a given substance to a given extent can only be computed accurately when the varying compressibility through the whole range is known; and since the pressures involved in the present question are clearly many thousands of atmospheres, the precise solution of the problem seems to be a distant matter, although by no means impossible.

By a process of approximation some light may be obtained, however. If one selects a single series of compounds, such as the chlorides, it is obvious that a large part of the compressibility throughout the series should correspond to the compressibility of the chlorine. In those cases where the compressibility of the metal is smallest, the change of volume would be due almost solely to the compression of the non-metal.

In view of these considerations, the first approximation should be obtained by comparing the actual contractions taking place during the formation of amounts of substance containing the same weight of chlorine with the heat evolved in each case. The starting point in each case is liquid chlorine, having a molecular volume of about 50 (or an atomic volume of about 25) at 20°. The heat of formation of the chloride is usually given in tables of data as starting from chlorine gas, under atmospheric pressure; hence the latent heat of evaporation and expansion of the chlorine should be subtracted from the usual values in order to institute a precise comparison.† However, these quantities cannot be large in proportion to the heat of combination with the metal, and they

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* Landolt and Börnstein, Phys. Chem. Tab., pp. 268, 278 (1894). Unless otherwise stated, all data used in this paper were taken from this admirable book of tables.

† The latent heat might be approximately calculated from the data of Knietsch (Landolt and Börnstein, p. 80 (1894)) as follows:—

$$Q = \frac{R\,T^2 dP}{P\,dT} = \frac{8.32 \times (293.5)^2 \times 0.19}{6.62 \times 1} = 20{,}500 \text{ joules, or } 20.5$$

kilojoules, between 20° and 21° C, for the evaporation of one gram-molecule. The wide deviations from the gas-law exhibited by chlorine render the calculation very uncertain. It is enough, however, to show that the value is relatively small. The heat absorbed on expansion must also be in doubt on account of the same deviations.

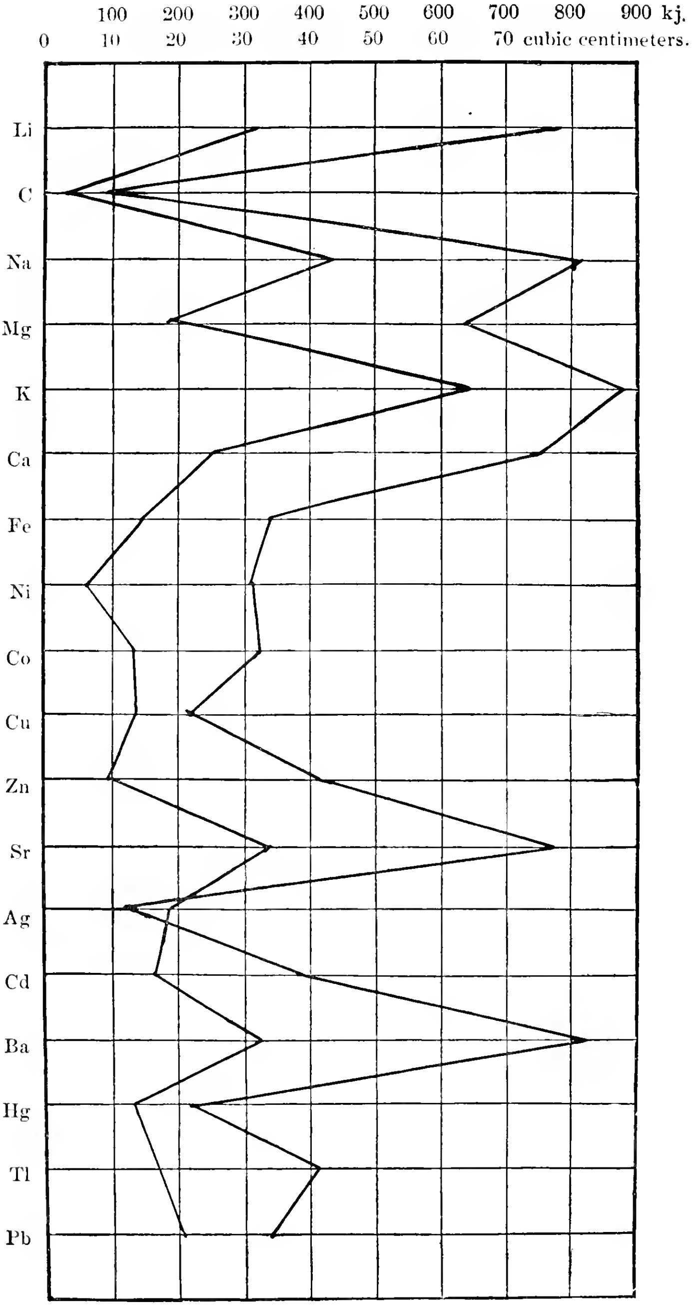
would apply equally in each case; hence in the first approximation the usual values for the heats of combination may be given without affecting the argument.

The table of data herewith collected compares the contraction which takes place when two gram-atoms of chlorine combine with some other substance, and the heat evolved during the operation.

COMPARISON OF CONTRACTION WITH HEAT OF FORMATION INVOLVED IN THE SYNTHESIS OF CHLORIDES.

Metal.	Atomic Volume of Metal.	Atomic Vol. Metal + n times Atomic Vol. Chlorine.	Molecular Volume of Chloride.	Difference or Contraction.	Contraction corresponding to 2 Atoms Chlorine.	Heat of Formation corresponding to 2 Atoms Chlorine. (Kilojoules.
Lithium .	11.9	36.9	20.9	16.0	32.0	784
[Carbon^{IV}] .	3.4	103.4	96.5	6.9	3.5	99
Sodium . .	23.7	48.7	27.2	21.5	43.0	816
Magnesium	13.3	63.3	43.9	19.4	19.4	632
Potassium .	45.5	70.5	37.8	32.7	65.4	872
Calcium .	25.3	75.3	50.0	25.3	25.3	760
Iron^{II} . .	7.1	57.1	42.6	14.5	14.5	343
Nickel . .	6.7	56.7	50.4 (?)	6.3	6.3	312
Cobalt . .	6.7	56.7	44.2	12.5	12.5	320
Copper . .	7.1	57.1	44.1	13.0	13.0	216
Zinc . . .	9.5	59.5	49.6	9.9	9.9	407
Strontium .	34.5	84.5	51.9	32.6	32.6	772
Silver . .	10.3	35.3	26.5	8.8	17.6	123
Cadmium .	13.0	63.0	46.5 (?)	16.5	16.5	390
Barium . .	37.0	87.0	53.9	33.1	33.1	815
Mercury^{II} .	14.7	64.7	50.0	14.7	14.7	223
Thallium .	17.2	42.2	34.1	8.1	16.2	406
Lead . . .	18.2	68.2	48.1	20.1	20.1	346

The parallelism of the heat of reaction and the contraction which results from it, is obvious from the table and the accompanying diagram,


100
200
300
400
500
600
700
800
900 kj.
0
10
20
30
40
50
60
70 cubic centimeters.
Li
C
Na
Mg
K
Ca
Fe
Ni
Co
Cu
Zn
Sr
Ag
Cd
Ba
Hg
Tl
Pb

which represents graphically the results recorded in the table. The elements are arranged in the order of their atomic weights, and both sets of data are drawn as abscissae, because this method of treatment will facilitate later comparison, and because it obviates certain irregularities due to periodicity. Each division stands for ten cubic centimeters of contraction on the left hand curve, and a hundred kilojoules of heat-energy on the right hand curve.

The correspondence is obviously too close to be the result of chance. One is forced to believe that a fundamental relationship exists between the two phenomena.

In these curves the compressibility is ascribed wholly to chlorine, and that of the other substance is neglected; but when the latter is large, it also must enter into the problem. Unfortunately our data concerning compressibility are unusually limited; but approximate calculations, based upon such as are known or may be guessed, show that at least some of the irregularities in the parallelism may be ascribed to this source.

We may thus formulate the following law: *The work needed for the compression involved in the formation of one solid or liquid by the combination of two others is approximately proportional to the heat evolved.*

While the general tendency of the law is manifest, and a correction for individual compressibilities would undoubtedly make it more so, there are nevertheless several exceptions to be explained. These may arise from several causes; in the first place, many specific gravities of solids are known only approximately; * in the next place, it is important that the same modifications of each substance should enter into each calculation. A plausible explanation has been found even for the exceptionally wide deviation exhibited by argentic chloride; but this point will not be dwelt upon now, since it is being submitted to the test of experiment.

The relation may be further illustrated by a table giving the data for a few bromides, and of course many other data might also be given. In order to eliminate as much as possible the contraction of the metal, it is well to choose for comparison a common non-metal possessing a comparatively large coefficient of compressibility, hence both chlorine and bromine serve well.

As a final example, the case of a single metal combining with several

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* See Richards, These Proceedings, **31**, 163 (1895); also Ostwald, Zeitschr. phys. Chem. **3**, 143 (1889).

non-metals may be cited. Potassium is chosen in this last case because it is probably among the most compressible of metals.

COMPARISON OF CONTRACTION WITH HEAT OF FORMATION INVOLVED IN THE SYNTHESIS OF THE BROMIDES.

Metal.	Atomic Volume of Metal.	Atomic Vol. Metal + n times Atomic Vol. Bromine.	Molecular Volume of Bromide.	Difference or Contraction.	Contraction corresponding to 2 Atoms Bromine.	Heat of Formation corresp'd'g to 2 Atoms Bromine.
Sodium .	23.7	49.2	34.2	15.0	30.0	718
Potassium	45.5	71.0	44.2	26.8	53.6	796
Calcium .	25.3	76.3	60.1	16.2	16.2	648
Zinc . . .	9.5	60.5	53.4*	7.1	7.1	318
Strontium.	34.5	85.5	58.4*	27.1	27.1	659
Cadmium .	13.0	64.0	56.9	7.1	7.1	315
Barium .	37.0	88.0	62.2*	25.8	25.8	711

COMPARISON OF CONTRACTION WITH HEAT OF FORMATION INVOLVED IN THE SYNTHESIS OF POTASSIC HALIDES.

Halogen.	Atomic Volume of Halogen.	Sum of At. Vols. of Metal and Halogen.	Molecular Volume of Salt.	Difference or Contraction.	Heat of Formation of 2 Mols.
Chlorine .	25.0	70.5	37.8	32.7	850±†
Bromine .	25.5	71.0	44.2	26.8	796
Iodine . .	25.7	71.2	53.8	17.4	670

When the more obvious experimental errors have been considered, two important questions at once suggest themselves: Does this proportionality signify equality, or is some of the energy of compression stored as potential energy and not manifested as heat? Again, if this equality exists, is it always exact, or is it modified by subordinate secondary effects?

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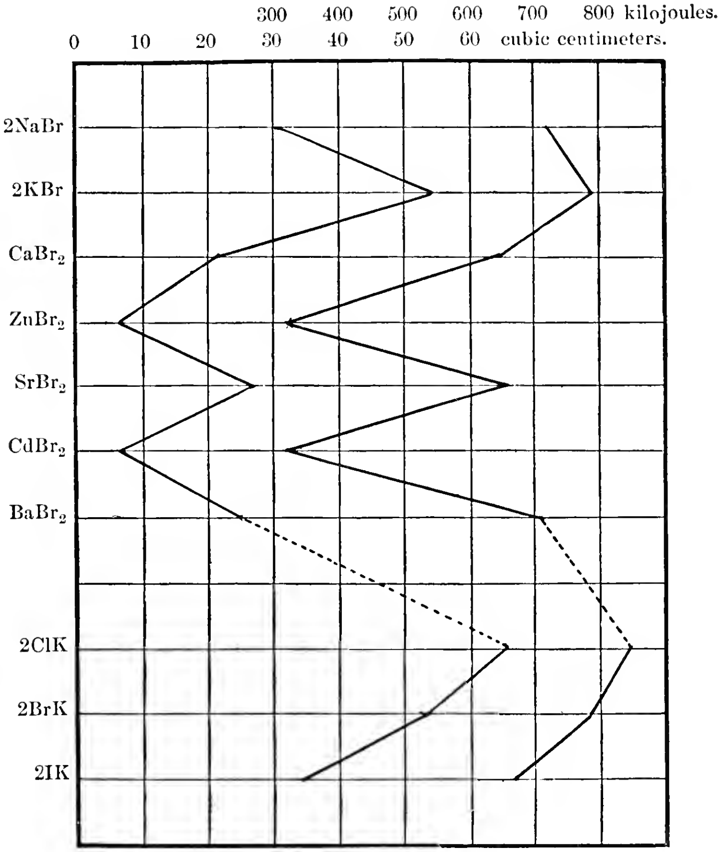
* These values are calculated from very accurate determinations of specific gravity made recently in this Laboratory. See These Proceedings, **31**, 163 (1895).

† Approximately corrected for heat of evaporation and expansion.

These questions cannot be answered at present. The total amount of work done in any case cannot be computed without a knowledge of the compressibility of the substances involved throughout the total range of volume, as has already been said. Unfortunately no suitable data exist capable of satisfying the conditions of the problem. Before long I hope to present such data, and to formulate answers to both questions; for the present the following unsatisfactory approximation is suggested as being better than nothing.



From the study of many allied data I have been able to form an approximate evaluation of the compressibilities of sodium and chlorine.

If one accepts these guesses, and imagines that the compressibilities decrease with decreasing volume according to the usual approximate law, one arrives at the conclusion that an amount of work equivalent to the heat of combination of sodium and chlorine would correspond to a change of volume in the system not far from the observed change of volume. The outcome is complicated by the fact that even in elements, but especially in compounds, there may be superposed several grades of compressibility. This can be explained hypothetically as follows: When the molecule is composed of two atoms, the highly compressed portion of each atom at the point of chemical union should have a much smaller coefficient of compressibility than the slightly compressed remainder of the molecule. If the molecule is polymerized, there will probably be yet other grades of compressibility in the various parts. The only object of a calculation so uncertain as this is to show that the heats of formation are of the same order of magnitude as the work involved in the compression.

In spite of the inevitable difficulties in the way of interpretation — difficulties which seem to be inherent in the problem — the presumption is strong that the *chief source of the heat of chemical combination is the work performed in compressing the material.* Since the heat of reaction is known to represent only approximately the free energy of the reaction, while the compression may really represent the affinities at work, one would hardly expect the relation to be exact. The generalization is a question of fact; it does not necessarily involve any atomic hypothesis, and can be regarded as uncertain only on account of the uncertainty of the data at present accessible. It is my intention to carry out the experimentation necessary to place the law on a more stable basis.

In the same way any other manifestation of attraction or affinity, such as cohesion or adhesion, should have a compressing effect and therefore evolve heat. The superficial and limited nature of these phenomena would ordinarily prevent any appreciable rise in temperature. In some cases, however, as in the adsorption of liquids and gases by porous material exposing a large surface, such a heating effect has been actually observed. Thus the essential difference between water of crystallization and adsorbed water is that the former penetrates the mass, while the latter is merely superficial.

It is obvious, moreover, that the same considerations apply to solidification and change of allotropic form. For example, liquid phosphorus, yellow phosphorus, and red phosphorus have at 44° the atomic volumes

17.66, 17.1, and about 14.1 respectively. The first small contraction is attended with an evolution of 0.65 kilojoules, and the second larger one with the evolution of 114 kilojoules of heat energy. In those cases where there is a transition from a more compressible union to a stabler, less compressible one, involving more work of compression, solidification would involve increase of volume, as in the case of water.

II. A Plausible Interpretation.

It becomes now an interesting question to determine, if possible, the mechanism by which this work is converted into heat. One is reminded at once of the compression of a gas, where the work of compression reappears quantitatively as heat energy. But the compression under consideration differs from the other in detail, because in the present case the attraction of the two substances for one another seems to be the cause of their mutual compression; and this mutual compression takes place not from the outside, but throughout the whole substance.

Those who shun the atomic hypothesis and consider substance only in the mass, will rest contented without further attempt at interpretation; but those who hold that the hypothesis is a useful tool, to be thrown aside when newer invention has devised a better one, will be tempted to go further.

The case, considered hypothetically, seems to be this: When two different atoms possessing mutual affinity approach one another, they are drawn closer than they can be to their respective fellows, and in the process evolve heat. The "repulsion" which is often supposed to surround an atom, and prevent it from touching any other, seems to be partially overcome by the potential energy of affinity. But of what nature is this "repulsion"? Ordinarily it is assumed to be due to the frequent impacts of a hard atom in the centre of the space; but no evidence is afforded of the existence of a free space. Indeed, it seems inconceivable that solids should retain their structure, or should be capable of retaining gases or liquids, if they are so loosely built up. A pile of sand would be stable compared to such a fabric.

The present research points to quite a different interpretation of the facts. The space occupied by a solid seems to have a *chemical* significance as well as a physical one; it seems, indeed, to be as essential a property of the material as any other property. Since the significance of the total volume is a chemical one, the "free space" around each individual atom must also have a chemical as well as a physical significance.

In other words, we have no right to imagine that the space is "free" or that there is a hard particle in the centre; the shell is as essential an attribute of the atom as the centre. But how are we to account for heat vibration, if the atom is supposed to fill the whole space? This question is important; but before answering we must consider some of the consequences of this form of compression.

Let us imagine two highly elastic spheres; for example, two very thin-walled india-rubber balls filled with gas. Imagine these to be drawn together by a powerful attraction resident throughout themselves. When they come in contact, each will compress the other and evolve heat in the process. They will remain bound together and distorted, unless some force separates them. If the shell of an atom is elastic and compressible, it is only reasonable to suppose that the interior is also. In that case the whole substance of both of two combining atoms will suffer distortion from the mutual attraction of every part of their substance; and the concentration of those constituents in each atom which cause the affinity will thus be increased in the half nearest the other atom. The supposition that the affinity comes from within will cause here an essential divergence from the actual conditions in two balls filled with gas, in which the gas is distributed equally throughout. As a consequence, the opposite half which is not combined will lose some of it attractive constituents, and should then have less tendency to unite with the new substances than it had before its union with the first atom. This plausible influence agrees with the well-known facts of "false equilibrium" and the nascent state; in fact, it would account in general for the permanence of slightly stable compounds.

By the process of hypothetical reasoning given above, one concludes that the *whole substance* of the atom may be elastic. In that case heat vibration might consist simply in alternate condensation and rarefaction of the medium within the shell, started by the momentum of impact. This would continue indefinitely, unless the vibration were imparted to other substances possessing less. Such internal rarefaction and condensation might well tend to distend the atom if any portion of the atom were held by another.

Thus, it is evident that there is no difficulty in imagining internal vibration in an atom which is packed on all sides closely with other atoms, or in explaining the mechanism of the thermal expansion of solids and liquids upon that basis. The chief reason for imagining a small hard particle with a large free space around it is therefore removed.

Two other reasons for retaining the conception of the old atom may be urged; one, the continuity of the liquid and gaseous state, and the other, the porosity of solids.

In answer to the first, attention may be called to the fact that the continuity of the liquid and gaseous condition exists actually only at the critical pressure; below that point they are, as a matter of fact, discontinuous and very different. Perhaps the critical pressure is simply the point where the gas molecules at the critical temperature are pressed into actual contact. The compressibilities of very compressed gases are, in fact, of the same order of magnitude as those of liquids.

Porosity is usually only manifest under very great pressure, which might be enough to compress the atoms into smaller space, and thus open orifices which previously did not exist.

From these considerations it seems to me that the new kinetic conception of the solid and liquid state has no disadvantages which the old conception does not possess, while it has many advantages which the old theory has not.

But it is not the intention of the present paper to enter into the detail of so large a question. I hope that in the next few years I may be permitted to study and report upon the possible consequences of the significance of changing atomic volume.

In the preceding paper and the present one, the following phenomena have been suggested as capable of a new and plausible interpretation if atoms are considered as capable of altering their volume through a wide range; namely, the heat of chemical reaction, adsorption, adhesion, and cohesion; ordinary solution; electrolytic solution; electrolytic dissociation; the passage of electricity through solids, liquids and gases; the nature of cathode rays (and probably also X rays and radium); the laws of Faraday and Dulong and Petit; false and true equilibrium; heat capacity and thermal expansion; quantivalence; stereo-chemistry and crystal form; and the critical phenomena.

Following papers will be devoted to a development, quantitative where possible, of these applications, as well as of many others. Unless further study reveals discrepancies, which have hitherto been concealed, I expect to be able to show: —

1. That the conception is not inconsistent with the two laws of energy.
2. That it conflicts with none of the quantitative conclusions of the atomic hypothesis, nor with the kinetic theory of gases, if heat be assumed to be due to mechanical energy operating upon atomic inertia.
3. That it is able to interpret the actual deviations of gases from the gas law better than any other theory, retaining the essential import of

the equation of van der Waals, and modifying this equation only as regards the changeability of a and b.

4. That it is consistent with the varying specific heats of substances in the solid, liquid, and gaseous states.

5. That with the help of this theory such physical properties as tenacity, ductility, malleability, and coefficient of expansion assume for the first time a conceivable consistency.

6. That upon it may be based a definition of the essential influences of chemical change and equilibrium.

7. That the variable compressibility of atoms furnishes a plausible explanation for many of the phenomena of quantivalence, including even the feeble affinities holding water of crystallization and other so-called molecular combinations.

8. That it explains all the tridimensional relations of material, such as stereochemistry and crystal form, at least as well as any other theory.

9. That with the proviso that electrical energy is a rhythmic manifestation of energy, — tending to repel itself and therefore to keep upon the surface of material which is susceptible to it, and hence to expand a free atom. — many of the electrical and magnetic phenomena of matter become more conceivable.

10. That the effect of light in hastening the attainment of chemical equilibrium, and the possibility of storing and emitting light energy possessed by material, may be interpreted in a similar way.

11. That the careful consideration of all these and other facts leads to a somewhat new conception of the relation between gravitation and chemical affinity, as well as between matter and luminiferous ether. This conception involves simply an antithesis of contracting and expanding tendencies, and is thus founded entirely upon an energetic basis.

12. That the idea is capable of throwing light upon the periodic system, and the genesis and permanence of the elements.

13. That it may be applied even to such astrophysical problems as the cause of the sun's heat.

This is a large program; some of it is already in manuscript, and more must await further exact experiment. The program is given here only to call attention to the wide possibilities of the consistent introduction of the conception of atomic compressibility into chemistry and molecular physics.

The present paper is only one step in the direction indicated. It is nevertheless an important step, for it adds approximate quantitative evidence to the previously given qualitative evidence concerning the significance of changing atomic volume.

III. Summary.

The contents of the paper may be divided into two parts: In the first part is set forth an approximate generalization which rests upon facts alone. This part of the paper can be overthrown only by the proof that the facts upon which it rests are erroneous. In the second part of the paper a plausible hypothetical interpretation of the facts is given. This part of the paper stands ready to share the fate of all hypotheses, — namely, to retire into oblivion if it is not capable of aiding the discovery of truth.

In brief, the chief points touched upon may be summed up as follows: —

I. (*a*) It has been shown that the contraction exhibited during chemical combination is in many cases approximately proportional to the heat evolved.

(*b*) Upon correcting the results for known differences of compressibility, the approximation becomes closer.

(*c*) An approximate calculation of the work which would probably be involved by the compression of a gram-atom each of sodium and chlorine into the space occupied by a gram-molecule of salt showed this work to be of the same order of magnitude as the actual heat of formation.

(*d*) From these facts and calculations the inference is drawn that the heat of chemical reaction is chiefly due to the energy required for the compression which takes place in the reaction.

(*e*) Possible corrections are pointed out.

(*f*) An explanation is given upon the same basis of the mechanism of the heat of adsorption, adhesion, and change of allotropic form.

II. (*a*) While the evidence is not exact, it affords a strong presumption in favor of the hypothesis of compressible atoms. The possibly far-reaching effect of this simple and plausible hypothesis upon chemical theory is pointed out.

(*b*) There is given a list of the especially prominent aspects of the question which will form the subjects of immediate experimental and theoretical study in this Laboratory.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

ON THE ACCURACY OF THE IMPROVED VOLTAMETER.

By Theodore W. Richards and George W. Heimrod.

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Introduction.

In a recent preliminary paper * it was shown that the disturbing influences in the common silver "voltameter" (or better, *coulometer* †) are due to the concentrated liquid which falls from the anode. In order to avoid the inaccuracy thus caused, it was suggested that the anode be surrounded by a fine-grained porous cup, which is capable of preventing this heavy liquid from reaching the kathode.

The weight of silver deposited by a given current in such a voltameter was found to correspond very closely to the amount of copper deposited at the same time in a copper voltameter shielded as much as possible from all discoverable sources of error; hence it seemed probable that the new voltameter gives the true value of the electrochemical equivalent of silver.

In a matter so important as this, however, it seemed advisable to obtain much more information concerning the constancy and trustworthiness of the new instrument, as well as to discover if possible the mechanism of the phenomena which rendered the older form untrustworthy. The investigation described below was undertaken with these objects.

I. The Constancy of the Porous Cup Voltameter.

The first problem was to determine if two instruments in series would always give identical results; in other words, to find if the new voltameter is always consistent with itself.

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* Richards, Collins, and Heimrod, These Proceedings, **35**, 123 (1899).

† The word "voltameter" was devised before electrical dimensions were understood. It is moreover too much like the universally used and suitable word "voltmeter." Now that the former instrument is placed upon a firm basis of accuracy, it may appropriately receive also an accurate name; and it is hoped that the new word "coulometer" may replace wholly the anachronism.

Nine such duplicate experiments were made. The first of these was a crude trial, and need not be recorded; the eight others are given in the following table.

The apparatus employed was precisely like that described in the previous paper. For the sake of easy reference, the description is repeated below.

Small cylinders of Pukal's porous ware (Berlin), suitable for osmotic pressure experiments, were used to enclose the anode in order to prevent the heavy anode-solution from reaching the kathode. These vessels were 50 millimeters high and 20 in diameter; their walls were not much over one millimeter in thickness. Their impurities were removed by boiling with nitric acid and thorough washing with water. Before being used they should be carefully searched and tested for cracks or imperfections. They were suspended in the solution by means of a platinum wire hung upon a glass hook, which insulated the wire from the electric connections. By means of a siphon, or a small pipette with a rubber top, the liquid within the cup was always kept at a lower level than that without, so as to prevent outward filtration.

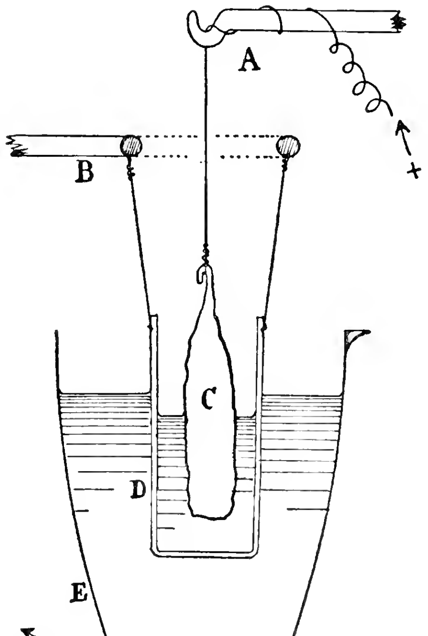

Figure 1. — Porous Cup Voltameter ($\frac{2}{3}$ actual size).

A, glass hook for supporting anode. B, glass ring for supporting porous cup. C, silver anode. D, porous cup. E, platinum kathode.

The kathodes consisted of large crucibles weighing only 60 grams, although they were capable of holding 120 cubic centimeters; they were provided with lips. A crucible exposes a smaller surface of liquid to the impurities of the atmosphere, and gave in our experiments a more evenly distributed deposit than a bowl.

The anodes were bars 5 × 1 × 1 centimeters of the purest silver, supported by silver wires and not enclosed in filter paper; and the electrolyte usually contained ten grams of pure, freshly prepared argentic nitrate in a hundred cubic centimeters of solution.

The manipulation was simple. The platinum crucibles were cleaned, dried at 160°, and weighed after three or four hours' cooling in a large desiccator kept in the balance-room. In order to prevent leakage during the electrolysis, the several stands were insulated by separate glass plates, and all the connections were air lines. The apparatus was protected, as in the earlier experiments with copper, by a miniature house with walls of fine cotton cloth, which helped to exclude dust. When the current was broken, the electrolyte was removed, the silver was rinsed twice with water, a third filling with water was allowed to stand in the crucible for two or three hours, and a fourth one remained in it over night. The wash-waters were collected and filtered if the silver showed any tendency to break off. In such cases a Gooch crucible was employed to collect the particles; and a very small filter, afterwards burned, served to catch the minute flakes of asbestos detached from the mat. On the next morning the crucibles were washed once more, rinsed twice with pure alcohol, and finally dried and weighed as before. This method of treatment gave opportunity for the diffusion of mother liquor from the intricate recesses of the crystallized mass, while it did not run the risk of dissolving silver which may attend the use of boiling water for washing.

As has been said, the crucibles were dried at 160°. It was subsequently shown, in agreement with the results of Lord Rayleigh and Mrs. Sidgwick, that a red heat is needed to drive off all the included liquid from the silver crystals; but since the amount included is fairly constant, this fact does not interfere with the availability of the uncorrected data for the present purpose of comparing one weight of silver with another.

Weighings were made upon the balance which served for the weighings in the earlier work upon copper, — one which has served also for many determinations of atomic weights. Its results with small objects may be depended upon to within $\frac{1}{50}$ milligram. All weighings were made by double substitutions, a similar vessel being used as a tare, and the weights were of course carefully standardized. Since the question concerned merely the comparison of silver with silver, the results were not at first corrected to the vacuum standard.

The results show that the average difference between the weights of the silver deposited in two crucibles placed in series was less than the tenth of a milligram, or only about four parts in one hundred thousand.

Considering the size of the platinum vessels weighed, this average agreement is all that could be expected; hence the test of the constancy of the apparatus seems to have been satisfactory.

TABLE I.

TEST OF THE CONSTANCY OF THE POROUS CUP VOLTAMETER.

No. of Experiment.	Voltameter I. Weight of Silver.	Voltameter II. Weight of Silver.	Difference.	Percentage Difference.
	grams.	grams.	milligram.	per cent.
2	2.43744	2.43749	0.05	0.002
3	2.69691	2.69713	0.22	0.008
4	2.36193	2.36196	0.03	0.001
5	2.14900	2.14913	0.13	0.006
6	1.65485	1.65490	0.05	0.003
7	2.31480	2.31500	0.20	0.009
8	2.22258	2.22260	0.02	0.001
9	2.67264	2.67268	0.04	0.002
	Mean difference		0.09	0.004

There is of course nothing in this table to show whether the figures represent the weight of silver which ought to have been deposited by the quantities of electricity employed. It may be that the error of the old voltameter was merely reduced, and that a small constant error still remained. The most probable cause of such a remaining error seemed to be the possible diffusion or migration of the heavy anode-liquid through the bottom of the porous cup, in spite of the fact that it was continually removed by means of a pipette or siphon. In order to prevent this, the bottom and a few millimeters of the sides of a porous cup were filled with melted paraffin, which effectually sealed the pores. The upper part of the sides only served to allow the passage of the electricity. A tenth comparison showed that a current which deposited 1.83370 grams of silver in this cup deposited 1.83375 grams in the ordinary porous cup coulometer. This difference is no greater than a possible experimental error; hence we may conclude that the effect of the diffusion is imperceptible. It will be shown later that the substance which causes the chief irregularity of Lord Rayleigh's voltameter is probably a heavy complex ion; hence it is not surprising that both the migration-rate and the diffusion-rate of the impurity is small. On the other hand, when the porous cup is too coarse-grained or too large, or when the anode solution is allowed to rise too high and thus filter through, the effect of the diffusion begins to be manifest. The same error begins to show itself when the viscosity of the solution is diminished by increasing temperature, as we showed in the preceding paper.

If now the formation of ionized silver at the anode is attended by such disturbing side reactions, it is reasonable to assume that a remedy may be found in the use of an anode of some other metal. For this purpose zinc seemed to offer peculiar advantages; it possesses only one degree of quantivalence, and has so great a solution-tension as to avoid the possibility of contaminating the deposit of silver at the kathode.

A zinc rod (so-called "C. P.") served as the anode in the following two experiments, and it was surrounded by a ten per cent solution of zincic nitrate prepared from the same material by solution in nitric acid (standing for a week over zinc), filtration, and crystallization. The kathode solution consisted of a ten per cent solution of argentic nitrate, as usual.

TABLE II.

THE EFFECT OF A ZINC ANODE.

No. of Experiment.	Wt. of Silver in Ordinary Porous Cup Voltameter.	Wt. of Silver in Voltameter with Zinc Anode.	Difference.	Percentage Difference.
	grams.	grams.	milligram.	per cent.
11	2.69702	2.69688	−0.14	−0.005
12	2.36195	2.36209	+0.15	+0.006
	Average error	. . . .	0.01	0.001

A peculiar reaction was observed during this electrolysis. The zinc rod was covered with a copious white flaky precipitate, and a marked test for nitrite was observed in the supernatant solution.* Thus the ionization of the zinc is attended with the formation of basic salt and

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* See also Senderens, Comp. Rend., **104**, 504; also Ber. d. d. ch. Ges., **20**, 197 R (1887).

zinc nitrite. The NO_3' ion must have been decomposed into NO_2' and oxygen. This same reaction takes place when silver serves as an anode in its nitrate solution, although to a much smaller extent.

In spite of the irregularities just described, the deposition on the kathode proceeded in a perfectly regular manner, and the figures show that as much silver was deposited in one cell as in the other.

Still another means of testing the porous cup voltameter was found in its comparison with a device which eliminates the porous cup wholly, but which nevertheless keeps the anode solution quite away from the cathode. This device consists in placing the anode at the bottom of a tall beaker filled with a concentrated solution of argentic nitrate (200 grams of the salt in a litre of solution), and arranging the kathode in the upper part of the vessel.* The anode solution becomes heavier and remains around the anode, while the kathode solution becomes lighter and rises to the surface. In order to prevent this dilution around the kathode from diminishing too much the concentration of the contiguous liquid, it is well to sink the kathode at least two centimeters below the surface. A circular disk of platinum wire gauze,† six centimeters in diameter, was used as the kathode, since many holes in gauze permitted the ready escape upward of the impoverished electrolyte. The gauze was bent around a stout circular platinum wire, and the disk was stiffened by four radial wires, and was hung rigidly from the centre. The vertical distance between this kathode and the anode was about seven centimeters. The anode consisted of a plate of pure silver, and its platinum connecting wire was protected from the solution by an enclosing glass tube.

The chief trouble encountered in manipulating the voltameter thus constructed is the danger of losing fine crystals of silver from the flexible gauze. In the two experiments described below every precaution was taken to avoid this source of error, and it is believed that no appreciable weight was lost. Another disadvantage of the gauze is the fact that metals deposited upon it are very apt to include minute quantities of electrolyte because of the interstices arising from its woven structure. Even silver deposited in a crucible contains some included mother liquor, and that deposited on the gauze contains much more. In the two experiments given below, the first deposit on the gauze lost 0.42 milligram on gentle ignition in a large porcelain crucible, and the

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* Merrill, Phys. Rev., X, 169 (1900).

† Paweck, Zeitsch. für Berg. u. Hüttenwesen, **46**, 570 (1898); Winkler, Ber. d. d. ch. Ges., **32**, 2192 (1899).

second lost 0.72 milligram, while the two crucible deposits lost respectively 0.20 and 0.24 milligram. These losses, accompanied by audible decrepitation, must have been due to retained electrolyte.

In the table the weights of the ignited precipitates are given.

TABLE III.

Comparison of Porous Cup Voltameter with Wire Gauze Voltameter.

No. of Experiment.	Weight of Silver in Porous Cup Voltameter.	Weight of Silver deposited on Gauze.	Difference.	Percentage Difference.
	grams.	grams.	milligram.	per cent.
13	2.10326	2.10344	+0.18	+0.009
14	2.31237	2.31234	−0.03	−0.001
	Average error		+0.08	+0.004

The gauze kathode thus showed an average surplus of less than a tenth of a milligram. But even this slight error is explicable, for it is clear that the argentic nitrate held by the electrolyte must have left silver nitrite or silver behind on heating. If we assume that the temperature of ignition was enough wholly to decompose the electrolyte, the average loss of 0.57 milligram would correspond to a residue of about 0.1 milligram, while the corresponding residue from the weaker solution used in the porous cup voltameter could not have exceeded 0.02 milligram. The difference between these two figures is exactly equal to the observed difference between the gauze voltameter and the porous cup voltameter, so that the two may be said to give precisely identical results.

An important point connected with this experiment is the fact that the kathode surface available for deposition on the gauze had an area of less than half that on the inside of the large crucible. Hence the current density in the gauze voltameter must have been over twice that in the standard.

There has thus been accumulated a convincing array of evidence indicating that the porous cup voltameter affords a means of depositing the amount of silver which really corresponds to the quantity of electricity sent through it. The numerical averages may be summed up in a brief table as follows : —

Average deviation of two porous cup voltameters in series = $\pm \frac{4}{100,000}$
Difference caused by sealing bottom of cup = $- \frac{3}{100,000}$
Difference caused by use of zinc anode = $\pm \frac{1}{100,000}$
Difference (corrected) between gauze voltameter and cup voltameter ± 0

The agreement of these results is as close as could be expected, since the discrepancies do not exceed the possible experimental error. With Lord Rayleigh's method, when two precisely similar voltameters are compared, Kahle * and Rodger and Watson † have shown that an accuracy of 6 or 7 parts in 100,000 can be obtained. On the other hand, the least variation of size of kathode or anode, or of any other condition, causes large deviations which may amount to ten times as large an error. In our experiments given above, the most radical changes of method were introduced, without affecting the results.

Among the efficient forms of apparatus described above, the porous cup voltameter with a silver anode is the most convenient. Hence for the further purposes of this paper it will be chosen as the standard method.

II. The Separate Effect of each Anode Irregularity.

It is obvious from the study of earlier work that more than one irregularity exists at the anode in a silver cell; and the separation and identification of the individual effect of each irregularity became a matter of considerable interest. The outcome was instructive as an example of the multitude of hidden minor influences which so often modify the obvious outcome of chemical experiment.

Qualitative testing revealed not only acid, but also nitrite, in the anode liquid; and in those cases where the anode is very small, some experimenters have indicated the formation of highly oxidized compounds of silver. Moreover, the singular crystalline silver dust which forms around the anode demands an explanation. In order to solve the problem, of course an obvious available method was to introduce artificially each impurity in turn into the pure liquid around the kathode in the porous cup voltameter, and study its effect on the gain in weight of the kathode.

The first impurity to be investigated was the nitrite. In order to pre-

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* Wied. Ann. N. F., **67**, 22 (1899).

† Phil. Trans., **186** A, 633 (1895).

pare the nitrite, we had recourse at first to a method used by Proust.* He has found that on boiling an argentic *nitrate* solution with finely divided silver, the *nitrite* is produced in quantity. In repeating this experiment, powdered silver reduced from purest silver chloride by the Stas method was boiled in a ten per cent argentic nitrate solution. Nitrite was indeed formed, but a very fine film of crystallized metallic silver was formed on the surface; a complication which seemed to point towards the existence of a reaction similar to the solution of copper in cupric sulphate. But it was found that pure silver nitrite in neutral silver nitrate solution likewise deposits a fine silver mirror on exposure to the light; hence the silver in both cases must be supposed to result simply from the decomposition of the nitrite. The solution boiled with silver was filtered through a Gooch crucible, and after cooling was employed in a voltameter with a porous cup. The solution containing the nitrite deposited 2.27945 grams of silver, while pure argentic nitrate in another standard voltameter deposited 2.27944 grams, a difference of only 0.01 milligram. (Exp. 15.)

Evidently the nitrite present had no effect at the kathode; and the liquid in an ordinary voltameter could hardly contain more nitrite than this solution which had been boiled with metallic silver. In order to pursue the matter further, however, we prepared silver nitrite from pure potassium nitrite and silver nitrate.† Pure potassic hydrate was neutralized with nitric acid; the nitrate was re-crystallized and fused in a silver crucible, and the resulting mixture of nitrate and nitrite was extracted

TABLE IV.

STANDARD *vs.* VOLTAMETER WITH SOLUTION SATURATED WITH $AgNO_2$.

No. of Experiment.	Type of Voltameter containing Nitrite.	Weight of Silver in Standard.	Weight of Silver in Voltameter containing $AgNO_2$.	Difference.	Percentage Difference.
		grams.	grams.	milligrams.	per cent.
16	Standard.	2.27944	2.28011	0.67	0.030
17	Filter paper volt.	2.30276	2.30539	2.63	0.114

* Journ. de Physique, March, 1806, 211; also Nicholson's Journal, **15**: 378. This reference has evidently been lost, since no text-book, including Dammer, gives it, although all mention Proust's observation. After a long search through the journals published in Proust's days, the reference was rediscovered.

† Victor Meyer, Liebig's Ann., **171**, 23 (1874).

with boiling water. The great bulk of nitrate may then be removed by one crystallization. If to the mother liquor is added a solution of argentic nitrate, the argentic nitrite will precipitate at once as a thick yellow paste. This is washed and re-crystallized from hot water, until the color has changed to white. The pure nitrite was dissolved in a nitrate solution to saturation, and this was employed, *first*, with a porous cup (16), and *second*, with a paper-wrapped anode (17).

The results show that a saturated solution of nitrite really has the effect of increasing the kathode deposit.

Since the increase due to a paper-wrapped anode over the weight found with a porous cup would have been from 0.04 to 0.08 per cent, the nitrite caused an increase of about the same amount in each case. But this increase happens only when the solution is *saturated* with nitrite; hence it is interesting chiefly as a limiting effect, and can hardly be important in solutions of nitrite as dilute as those formed spontaneously around the anode. The formation of nitrite is evidently the result of the breaking up of the NO_3^- ion into the nitrite ion NO_2^- and oxygen, and the latter is probably taken up by the silver in forming one of the oxidized compounds to be discussed later.

It is not at all surprising that this side reaction should take place to a small extent. The current is normally carried from the anode to the solution by the formation of the silver ion from the metal; but a slight tardiness in this reaction (which might be named "physico-chemical inertia") would result in assistance from the anions in the neighborhood. They would seek to adjust the potential by discharging their negative electricity on the anode. Of course the most plentiful anion in the vicinity is the nitrate ion; its deionization would make possible the formation of the *nitrite* ion and oxygen, which might at once oxidize the silver plentifully present.* The reaction might be written thus: —

$$NO_3 + 3Ag = Ag^+ + NO_2^- + Ag_2O.$$

Thus the electrolysis of a strong solution of argentic nitrate might be predicted to result in the neutralization of a previously acid solution — a prediction which agrees with the fact discovered by Rodger and Watson † with thirty per cent solutions of argentic nitrate. It is possible that a higher oxide also would be found if the anode were small.

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* The probable presence of silver in supersaturated solution around the anode will be shown later.

† Rodger and Watson, Phil. Trans., **186** A, 631 (1895).

But Kahle * found that in weaker solutions acid is produced instead of being removed, and we have verified his results. Clearly this must be due to yet another irregularity. When the solution is dilute and neutral, oxygen and hydroxyl ions are both present in appreciable amount, according to modern electrochemical interpretation. Their greater ease of deionizatian would compensate for their relatively small concentration, and traces of negative electricity might be carried out of the solution through their agency with the formation again of argentic oxide, or even oxygen gas. The reaction would, however, leave an excess of ionized hydrogen (acid) in solution, a state of affairs not paralleled in the case of the nitrite. This would explain the phenomena in question.

But would not the argentic oxide at once dissolve in the simultaneously formed acid, and thus form argentic nitrate again? Or, in other words, does argentic oxide form with silver an oxide-complex of any degree of stability? Hellwig † in a recent paper has shown that the weak silver ion in the presence of the strong NO_3^- ion tends to strengthen itself by taking up a molecule of some other undissociated substance, as AgCl, AgI, AgBr, or AgCN. He has proved also that the solution actually contains ions like Ag_2I^+, since on electrolysis the iodine accumulates at the kathode, and disappears from the anode.

In order to find if argentic oxide could in a similar way associate itself with the silver ion, we boiled very pure argentic oxide with a concentrated solution of argentic nitrate, and filtered the solution hot. Upon being diluted with cold water this solution yielded a white precipitate, which turned gray upon standing. The precipitate was soluble in dilute nitric acid, hence it could not have been an argentic halide; besides, every precaution had been taken to exclude the halogens. The compound precipitated on dilution must, therefore, be silver hydroxide or a basic salt; and a basic complex must have existed in solution.

It is by no means inconceivable that this complex, although finally decomposed by acid, should not yield at once to its action. In the meantime the acid, diffusing at a far more rapid rate than the heavy complex, would have partly left the immediate neighborhood of the anode, and hence the heavy solution around the latter would fall with its basic load to the bottom of the kathode vessel. There the complex ion (possibly Ag_3O^+) would be capable of transferring electricity as well as any other ion, and upon deionization would deposit over three times the weight of material

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* Kahle, Wied. Ann., **67**, 1 (1899).

† Zeitschr. anorg. Chem., **25**, 157 (1900).

contained in the silver ion carrying the same quantity of electricity. Part of this material would be in the form of oxide, and would therefore cause dark spots on the surface of the kathode, — spots which have actually been noticed by other experimenters.

This interpretation serves to explain also the very high results obtained by Lord Rayleigh in the presence of argentic acetate. The possibility of forming slightly dissociated acetic acid would prevent any considerably accumulation of ionized hydrogen, and the oxide-complex would grow in concentration without hindrance. This explanation seems more plausible than the provisional one suggested by Lord Rayleigh, — namely, that the acetate itself might be carried down with the silver.

In order to submit these interpretations to the test of experiment, electrolyses were conducted with solutions saturated with argentic oxide. Kahle, Patterson and Guthe, and others, have likewise carried out such electrolyses, showing that the deposit is as a matter of fact too great; but our trial is easier to interpret, because by means of the porous cup all anode complications were excluded, and the result of experiment gives the effect of argentic oxide alone.

Three trials were made, in which a given current of 0.25 ampere as usual was run first through a standard porous cup voltameter, and then through a cell precisely similar except that the kathode solution in the latter was saturated with the argentic oxide. The following table contains the results: —

TABLE V.

STANDARD *vs.* STANDARD SATURATED WITH Ag_2O.

No. of Experiment.	Weight of Silver in Standard.	Weight of Ag Standard with Ag_2O.	Difference.	Percentage Difference.
	grams.	grams.	milligrams.	per cent.
18	2.30276	2.30449	1.73	0.075
19	2.34799	2.34887	0.88	0.037
20	2.21379	2.21489	1.10	0.050
			Mean	0.055

The solution after boiling with silver oxide thus really contains, therefore, some ion heavier than Ag^+. Since this has been formed directly from the oxide, it probably contains Ag_2O, and may be assumed to have the formula already given, Ag_3O^+. Thus the preceding interpretation is confirmed.

The next question which arises concerns the permanence of this complex in the presence of acid. In order to test this, a solution of argentic nitrate was saturated with argentic oxide, and then treated with a slight excess of nitric acid. After a short time, perhaps an hour, electrolyses were made with this solution in series with the standard, as usual.

TABLE VI.

STANDARD *vs.* STANDARD SATURATED WITH Ag_2O, BUT AFTERWARDS ACIDIFIED.

No. of Experiment.	Weight of Ag in Standard.	Weight Ag from Sol. with $Ag_2O + HNO_3$.	Difference.	Percentage Difference.
	grams.	grams.	milligrams.	per cent.
21	2.34799	2.34836	+0.37	+0.016
22	2.21379	2.21361	–0.18	–0.008

The results are somewhat less regular than usual, but clearly most if not all of the oxide-complex had been removed by the acid. Thus, while the complex is capable of existence in a neutral solution, the speed of its reaction with acid results in its decomposition in a short time, as would be expected.

It is possible that this oxide-complex is not the only one capable of being formed at the anode. Kahle, Sulc,* Mulder and Heringa † and others, present evidence showing that with a small anode, where both silver and nitrate ions would be less available for transferring electricity, a highly oxidized compound having some such formula as Ag_7NO_{11} may be formed. This compound is capable of dissolving in acids, forming a brown solution; and it may be responsible for the colored rings which Kahle has noticed from old acid solutions. The fact that after boiling with metallic silver such solutions cease to yield colored rings is evidence that the foreign compound is a highly oxidized substance.

In spite of the fact that the nitrite, the oxide-complex, and the per-

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* Sulc, Z. anorg. Chem., **12**, 89, 180 (1896) ; **24**, 305 (1900).

† Mulder and Heringa, Ber d. d. ch. Ges., **29**⁴, 583 (1896).

oxide-complex, may explain many of the irregularities observed during the electrolysis, they cannot explain them all. The chief questions remaining to be answered concern the cause of the high results which are still to be obtained when all the preceding causes of irregularity have been eliminated, as well as the mechanism of the formation of the plentiful "anode dust."

A number of facts point to the conclusion that some other complex compound exists in the electrolyzed liquid which is capable of depositing metallic silver upon a silver surface. Among others is the well known fact that a pure silver kathode receives a larger deposit with a given current than a platinum kathode in the old Lord Rayleigh voltameter. It seemed to be worth while to test once more this relation, in order to confirm the results of Lord Rayleigh, Kahle, and others, and also to discover if a pure argentic nitrate solution in the porous cup voltameter would give like results. The following tables record the results of our experiments. In the first place we repeated Kahle's experiments, using an anode protected only by filter paper.

TABLE VII.

FILTER PAPER VOLTAMETER ON PLATINUM AND ON SILVER.

No. of Experiment.	Weight of Deposit on Platinum.	Weight of Deposit on Silver.	Difference.	Percentage Difference.	Weight of Anode.
	grams.	grams.	milligram.	per cent.	grams.
23	2.26680	2.26672	−0.08	−0.003	2
24	2.17215	2.17250	+0.35	+0.016	4.5
25	2.18071	2.18100	+0.29	+0.013	6.7
26	2.11134	2.11162	+0.28	+0.013	8.8
			Mean	+0.010	

There is an undeniable surplus when the deposit is made on silver. The main question now arises, — Is this effect due to the anode solution, or is it an irregularity which would come equally from pure argentic nitrate? The answer to the question is easily determined by means of our porous cup; a comparison of deposits made in a standard voltameter on a silver and a platinum kathode gave the following results: —

TABLE VIII.

Standard Method on Platinum and on Silver.

No. of Experiment.	Weight of Deposit on Platinum.	Weight of Deposit on Silver.	Difference.	Percentage Difference.
	grams.	grams.	milligram.	per cent.
27	2.69700	2.69674	0.26	−0.009
28	2.25769	2.25770	0.01	+0.000

The only difference is now in the opposite direction; and this was due to known experimental error. In experiment 27 a small loss of silver particles in the wash-water from the silver cell produced the difference of 0.009 per cent. It is highly probable that but for this accident, the deposit on silver would have been equal to that on platinum, as it is in No. 28. These results permit us to draw two conclusions. First, it is not the greater inclusion of silver salt in the crystals which increases the total weight when the kathode is silver. Otherwise 27 and 28 should have grown heavier in the same ratio. Secondly, it is the anode solution again which is responsible.

The increase in the deposit on a silver surface indicates the existence of silver in the solution in a supersaturated state; and this existence shows that there must be present some complex gradually dissociating, with metallic silver as one of its products. If this is the case, we should expect to find that an oxidizing environment would be capable of removing this cause of inaccuracy, while substituting another easily removed by nitric acid. As a matter of fact, Schuster and Crossly * have shown that deposits made in vacuo are heavier than when made in air; again those made in an atmosphere of air are heavier than when made in oxygen. Of course it is understood that in all three cases the anode was only wrapped in filter paper. The solution usually contained fifteen per cent of silver nitrate, but sometimes as much as thirty per cent. They used the solution over and over again, thereby accumulating the irregular compounds. Under reduced pressure (about "1⅛ inch"), the deposits exceeded those made in air by about 0.04 per cent, while the latter exceeded those in oxygen by 0.04 per cent more. Myers,† who repeated these experiments, found the difference between deposits in air and in vacuo to be as much as 0.10 per cent for 20–40 per cent solutions.

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* Proc. Roy. Soc., **50**, 356 (1892).

† Wied Ann., **55**, 291 ff. (1895).

In an atmosphere of nitrogen an excess of .05 per cent in the deposit was observed. Our own experiments in the same direction verify the results of these experiments, and need not be recorded here.

Putting aside for the moment the question concerning the nature of the irregular compound which can thus be partly oxidized out of existence, it seemed worth while to discover if an oxidizing agent in the solution could remove this compound to such an extent as to cause the filter paper voltameter to yield accurate results. The only practical oxidizer for this purpose is hydrogen peroxide. A fairly strong solution was prepared from pure crystalline barium peroxide and dilute sulphuric acid. The excess of acid was removed with barium hydroxide, and the solution was filtered. In this solution the usual amount of silver nitrate was dissolved, and this was used in the large bowl as well as in the lipped crucible, both anodes being wrapped in filter paper. In the large bowl black crystals of argentic peroxide, or Ag_7NO_{11}, were soon formed which bridged across to the kathode. While the result in the bowl thus became useless, the crucible showed no such disturbance, but yielded nevertheless a deficit of .14 per cent on comparison with a standard. This must have been due to a side reaction, especially since the kathode was found covered with small gas bubbles, which were probably oxygen. It is possible that negative electricity was carried from the kathode to the solution by the ionizing of a trace of oxygen. Better results were obtained after the hydrogen peroxide had been diluted to one-tenth its former strength; these are recorded below: —

TABLE IX.

STANDARD *vs.* FILTER PAPER VOLTAMETER CONTAINING H_2O_2.

No. of Experiment.	Standard.	Filter paper Voltameter + H_2O_2.	Difference.	Percentage Difference.
	grams.	grams.	milligram.	per cent.
30	1.78593	1.78648	0.55	0.031
31	1.78593	1.78642	0.49	0.027
32	1.83375	1.83406	0.31	0.017
		Mean		+0.025

The usual difference of from .04 to .08 per cent is thus reduced to .025 per cent; therefore hydrogen peroxide seems to eliminate a part of the usual disturbance. But in view of the fact that even a deficit of 0.14 per cent can be obtained, not much importance can be attached to these results, since it is impossible to say how much is due to the oxidizing action, how much to the disturbing influence capable of causing an actual deficiency.

These indirect methods of determining the presence of a complex yielding metallic silver being somewhat unsatisfactory, recourse was had to a more direct method. It seemed highly probable that the anode solution ought to be able to deposit silver without the help of the galvanic current. In order to show this, a porous cup voltameter was set up in the usual manner, except that the anode was closely wrapped in filter paper to retain the fine crystal powder which always separates from it. A current of 0.25 ampere was sent through the voltameter, and every ten minutes a portion of the clear anode solution was taken from the bottom of the porous cup by means of a small pipette, and quickly transferred to a small weighed platinum crucible.

The crucible had been previously coated with silver in order to establish equilibrium more quickly in case a compound existed in the solution which tended to deposit silver.

After one hour's standing, the liquid was removed and the crucible was washed and dried, as a deposit from electrolysis would have been. The increase in weight of the crucible must represent the deposit from the anode solution.

TABLE X.

GAIN IN WEIGHT OF SILVER IN CONTACT WITH ANODE SOLUTION.

No.	Increase in Weight. Milligram
33	0.35
34	0.08
35	0.25
36	0.63
Mean	0.33

The weight of the same crucible did not change perceptibly when allowed to remain in contact with a solution of silver nitrate of like concentration, through which no current had previously been passed. The above increase in weight shows beyond a doubt, therefore, that the anode solution is capable of depositing on a silver surface either silver or some compound of this metal which must have been formed at the anode.

The most striking evidence that a compound exists around the anode which is capable of depositing pure silver is the existence of the "anode dust." This consists of a fine powder, more or less closely adhering to the anode. Examination with the microscope indicates that this powder consists of minute crystals, which have every appearance of being metallic silver. Rodger and Watson * analyzed the air-dried powder, and found as a matter of fact that the metal is essentially pure. The contrary conclusions of Myers † and others may have been based upon results obtained with small anodes, where argentic peroxide may have been formed.

In our experience the weight of this dust is approximately proportional to the area of the silver anode, with a given current. It seems highly probable, then, that the silver at first tends to separate from the anode as a polymerized ion, perhaps Ag_3^+, according to the common principle that an unstable compound often forms the bridge between two stable conditions. ‡ The greater portion of this complex ion would be expected to break up at once into the normal argentic ion and metallic silver ($Ag_3^+ = Ag^+ + 2Ag$), the latter forming the "anode dust." The last traces of the complex might, however, persist for some time, and give rise to all the phenomena seeming to be due to the existence of supersaturated silver in the solution.

The argument has been so protracted that it is perhaps worth while to recapitulate the way in which this interpretation would explain the irregularities not to be attributed to the nitrite and oxycomplexes.

This complex ion of polymerized silver undoubtedly unloads silver at a lower potential (*i. e.* more easily) than the simple silver ion. Hence the larger the kathode surface exposed, the greater part will the complexes take in the carrying of the current, and the larger will be the deposit of silver. This consequence of the theory agrees with the experience of all experimenters. Moreover, since the complexes are unstable, and continually tending to decompose, there must be always in solution a trace of molecular unionized silver, which, being supersaturated, will deposit on contact with solid silver. If the platinum bowl has been previously lined with silver, this extra deposition will begin almost immediately; while if it has not been thus lined, an appreciable silver surface will have to be formed before the relieving of the supersaturation will begin to take place. This reasoning explains the invariable excess of the deposit upon a silver kathode over and above the amount deposited

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* Phil. Trans., **186** A, 632 (1895).

† Wied. Ann., **55**, 295 (1895).

‡ Ostwald, Z. phys. Chem., **22**, 307 (1897).

on one of platinum by the same current when only filter paper is used to protect the kathode. The results of Kahle and others seem to indicate that the presence of acid, which prevents the formation of the simple oxycomplex, is favorable to the formation of the ion $Ag_3{}^+$. This is not surprising, since the oxycomplex is probably formed at the expense of some of the silver which would otherwise remain in the polymerized condition. The fact that the kathode deposit in the common voltameter consists of comparatively few large crystals, while the porous cup voltameter yields a host of evenly distributed small crystals, is also explained by this interpretation. Solutions having a tendency to supersaturation always tend to deposit large crystals, for obvious reasons. When the absence of acid increases the number of available hydroxyl ions, the formation of the silver-complex is less considerable; but the oxide-complexes then begin to affect the result. In concentrated solutions of the nitrate, this ion also enters into the irregularities. Thus the various irregularities are not necessarily coexistent; circumstances determine which one shall play the most important part.

There seems, then, to be concordant evidence of conflicting tendencies at work, some oxidizing and some reducing; some tending to cause the dissolving of too much silver at the anode, and some to cause the dissolving of too little. It seemed worth while to test the complicated conclusion by determining accurately the loss of weight of silver at the anode, in order to obtain a last ray of light upon the question. The disintegration of the anode renders the determination of the loss somewhat difficult; but by carefully collecting all the silver powder left in the porous cup (when no filter paper is used) on a Gooch crucible, and adding this weight to the weight of the coherent part of the anode, fairly good results may be obtained. The following table records a series of such determinations. In each case the current strength amounted to about 0.25 ampere. The experiments are arranged below in the order of current density.

In some cases the anode loses more than the ideal amount, in other cases less. Such results can only be explained by the assumption of several causes of inaccuracy, and the four which we have discussed seem capable of explaining all the changes. But it is not worth while to trace out every possible variation; enough has been said to emphasize the great complexity of the side reactions which interpenetrate a process apparently so simple, and at the same time to permit those readers who are especially interested to work out the combinations for themselves.

TABLE XI.

Loss at Anode compared with Gain at Kathode.

No. of Determ.	Weight of Anode.	Loss of Anode, corrected for Silver Powder.	Deposit on Kathode.	Difference in Weight.	Percentage Difference.
	grams.	grams.	grams.	milligrams.	per cent.
37	6	2.43854	2.43744	+1.10	+0.045
38	9	2.60603	2.60420	+1.83	+0.070
39	12	3.05185	3.04996	+1.89	+0.062
40	13	1.76332	1.76283	+0.49	+0.028
41	28	2.44485	2.44599	−1.14	−0.047
42	31	2.43613	2.43744	−1.31	−0.054
43	33	2.60399	2.60420	−0.21	−0.008

In the porous cup voltameter all the anode reactions which constitute the most serious causes of inaccuracy are safely eliminated by keeping the contaminated liquid within the porous cup. It is obvious that this device, or some other accomplishing the same end, should always be used when accuracy is desired.

III. The Purity of the Silver Deposit.

An important question remains to be answered, namely, is the deposit thus obtained perfectly pure silver, or does it contain traces of included mother liquor?

That impurities in the solution, such as copper, or any of the common metals occurring with silver, do not affect silver deposit to any great extent has been shown by Lord Rayleigh. Even if the solution actually turns green from the copper dissolved at the anode, not a trace of copper can be detected in the deposit. We used on one occasion commercial silver nitrate with an anode of sterling silver wrapped in paper, and found that the difference between this and the standard was about .024 per cent, or only about .02 per cent smaller than a similar deposit with the purest silver. Metals of greater solution tension than silver have therefore no important effect on the weight of silver, although they may change the structure of the silver deposit. Of course they had always been excluded in this work.

On the other hand, the deposit, in common with most crystals, may retain small quantities of solution or wash-water. Lord Rayleigh seems to be the only one who has taken this possible source of error into account. He heated the crucibles to incipient redness, after they had been dried at 130° to 160°, and weighed. A loss of about .014 per cent was thus found. Richards and Collins, in looking for an explanation of the cause of discrepancies in the atomic weight of copper, had found by analysis the silver deposit to contain about 0.01 per cent of impurity.

For our purpose the direct method of Lord Rayleigh seemed better than the indirect analytical one. The deposits, which had been dried thoroughly at 160° and weighed, were heated over an alcohol lamp to constant weight. Care was taken to heat the whole crucible evenly, and to use as high a temperature as possible without the formation of an

TABLE XII.

LOSS OF WEIGHT OF SILVER DEPOSITS ON HEATING.

No. of Experiment.	At 150°.	At Incipient Redness.	Difference —0.10 mg.	Percentage Loss.
	grams.	grams.	milligram.	per cent.
44	1.97875	1.97859	0.06	0.003
45	1.97946	1.97907	0.29	0.015
46	1.98032	1.97998	0.24	0.012
47	1.69330	1.69284	0.36	0.021
48	1.69351	1.69307	0.34	0.020
49	1.69471	1.69411	0.50	0.030
50	2.10356	2.10326	0.20	0.010
51	2.06825	2.06764	0.51	0.025
52	2.31268	2.31234	0.24	0.010
53	2.06714	2.06663	0.41	0.020
54	1.64322	1.64266	0.46	0.028
55	1.64212	1.64166	0.36	0.022
			Mean	0.018

alloy, — although several times this could not be prevented. For heating the deposits on platinum gauze (see Table IX), a small oven was constructed from a large porcelain crucible, covered by a platinum funnel. The platinum disc was supported by a wire reaching through the tube of the funnel. In this case, the silver in the platinum crucibles with which that on the gauze was to be compared, was heated in the oven also, in order to expose both to the same temperature. Since the figures of this comparison are given in Table IX, it is necessary only to tabulate here the loss observed in crucibles when heated directly. Of course allowance has been made for the very slight hygroscopic loss (0.10 milligram) which a platinum crucible without silver deposit would have undergone. The silver films were usually those remaining from some of the preceding determinations.

This percentage loss is slightly higher than that given by Lord Rayleigh, and still larger than that determined indirectly by Richards and Collins. It is evident that the amount of included mother liquor varies according to the rate and mode of deposition, and it is quite possible that different average amounts were really included in the several investigations. The inclusion is probably chiefly in recesses in the platinum kathode. The differences in included liquid given in the above table are of the same order as the differences in the uncorrected weights of silver given at first;* hence we may ascribe at least a part of the differences in the early table to inclusion of mother liquor.

All this evidence unites in indicating that even under the best conditions the silver does not exceed a purity of 99.99 per cent; and in applying a correction, one should obviously use the value found in the particular investigation under review.

IV. THE ATOMIC WEIGHT OF COPPER.

Having thus clear light upon the various errors of the silver voltameter, it became a matter of great interest to recur to the original question which started the whole investigation, namely, the quantitative accuracy of Faraday's law.

Accordingly, a voltameter like that used by Richards and Collins† — a modified form of Lord Rayleigh's instrument — was compared with a standard porous cup voltameter, neither precipitate being ignited. The

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* See page 418.

† These Proceedings, **35**, 133 (1899).

eighteen results, including three given in the last paper, are recorded below : —

TABLE XIII.

Comparison of Porous Cup with modified Lord Rayleigh Voltameter.

No. of Experiment.	Current Strength.	Weight Ag in Standard (Porous Cup).	Weight Ag in Filter Paper Voltameter.	Difference.	Percentage Difference.
	amperes.	grams.	grams.	milligrams.	per cent.
A. 37		1.94124	1.94267	1.43	+0.074
A. 38		1.76283	1.76425	1.42	+0.080
A. 39		3.04996	3.05270	2.74	+0.090
56	0.25	2.26624	2.26680	0.56	+0.024
57	0.25	2.17289	2.17215	0.26	+0.012
58	0.25	2.17896	2.18071	1.75	+0.080
59	0.25	2.11095	2.11134	0.39	+0.019
60	0.25	2.14906	2.14974	0.68	+0.032
61	0.25	2.09580	2.09648	0.68	+0.033
62	0.25	2.09580	2.09650	0.70	+0.033
63	0 25	1.65487	1.65520	0.33	+0.020
64	0 25	2.09756	2.09840	0.84	+0.040
65	0.25	2.09756	2.09834	0.78	+0.037
66	0.25	2.02063	2.02100	0.37	+0.018
67	1.25	2.02063	2.02144	0.81	+0.040
68	0.45	2.31490	2.31568	0.78	+0.034
69	0.12	2.22259	2.22343	0.84	+0.038
70	1.00	2.67266	2.67364	0.98	+0.037
			Mean		+0.041

The comparison of the deposits thus shows that when the anode is wrapped in paper, the deposit is on the average greater by 0.041 per cent. This average difference is smaller than that given in the previous paper, but it is probably more accurate, because it comprehends so many determinations. The wide deviations between the individual determinations illustrate the uncertainty of a voltameter in which the anode is merely wrapped in filter paper.

When to this difference is added the amount (0.018 per cent) caused by the included mother liquor, it is obvious that the weight of silver observed in the experiments upon Faraday's law made by Richards and Collins must have been 0.059 per cent too heavy. This would cause the observed electro-chemical atomic weight of copper (63.563 *) to be too small by the same percentage. Correcting for this error, the atomic weight of copper calculated from the results of the experiments upon Faraday's law becomes 63.601, while the most probable value found in purely chemical ways is 63.604. †

The agreement is as close as the probable accuracy of the electrolytic determinations. Thus good experimental evidence is furnished, showing that Faraday's law holds rigorously true in aqueous solution at ordinary temperatures. Apparent deviations are simply due to the disturbing effect of side reactions.

V. The Electrochemical Equivalent of Silver.

It becomes now an important matter to determine, if possible, a correction which might be applied to the methods of earlier physical experiments upon the electrochemical equivalent of the ampere. Such correction must at best be an unsatisfactory expedient; the only really satisfactory method of proceeding would be to repeat the work wholly, using the new voltameter as a chemical measure of the current. But such a proceeding involves an expenditure of time not now at our disposal; hence it seems not wholly fruitless to attempt the correction of the older results.

The series of comparisons of the standard with the filter paper voltameter just given (p. 422) will hardly serve for the purpose, since the latter voltameter changes in its indications with every change of form; and the two comparisons with Lord Rayleigh's form, given in the previous paper, form too small a basis upon which to make so serious a correction. Hence another series of these experiments was made, in which the porous cup voltameter was compared directly with a voltameter

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* This result was obtained by extrapolation for a copper kathode of zero area. It had a "probable error" of 0.004, and possibly contained a source of error tending to make it slightly too large.

† Richards, These Proceedings, 26, 293 (1891).

made exactly according to Lord Rayleigh's directions. These are given below, together with the two determinations given in the last paper.

TABLE XIV.

Comparison of Porous Cup with Original Rayleigh Voltameter.

No. of Experiment	Weight of Silver in Standard.	Weight of Silver in Lord Rayleigh's Form.	Difference.	Percentage Difference.
	grams.	grams.	milligrams.	per cent.
A. 40	3.09629	3.09918	2.89	+0.093
A. 41	2.53256	2.53439	1.83	+0.072
71	2.16429	2.16538	1.09	+0.050
72	2.09580	2.09653	0.73	+0.035
73	1.65487	1.65549	0.62	+0.038
		Mean		+0.058

This is 0.017 per cent more than the average of the preceding series. Probably a mean of the average of the two series, or +0.050 per cent, represents as nearly as possible the correction to be applied to Lord Rayleigh's voltameter. This value is not only an average of averages, involving twenty-three determinations, but is also very nearly the mean between the two extreme results 0.012 and 0.093. It may probably be relied upon to within 0.01 per cent of the total weight of the silver.

It finds support in some results given in Kahle's* paper. He made a comparison between an ordinary voltameter and one in which the anode solution was constantly siphoned off and thus prevented, more or less perfectly, from reaching the kathode. The solution in both voltameters was strongly acid, but equally so. The siphon voltameter deposited, in good agreement with the above results, 0.05 per cent less silver than the ordinary voltameter.

The fact, however, that the extremes vary from .012 per cent to .093 per cent indicates that unless great care is taken in the way in which the anode is wrapped, in the strength of the current and in the size of the anode, the depositions in the ordinary voltameter according to Lord Rayleigh are untrustworthy.

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* Wied. Ann. N. F., **67**, 30 (1899).

In order to correct Patterson and Guthe's results, it became necessary to repeat comparisons of the standard with the voltameter containing old solution saturated with oxide, as used by them.

TABLE XV.

Standard *vs.* Patterson and Guthe's Method.

No. of Experiment.	Date.	Amp.	Weight of Silver in Standard.	Weight of Silver in P. & G.	Difference.	Percentage Difference.
			grams.	grams.	milligrams.	per cent.
A. 43	6-11-99		1.89800	1.90238	4.38	0.230
A. 44	6-18-99		2.55012	2.55460	4.48	0.176
74	3- 6-01	0.25	2.08330	2.08492	1.62	0.078
75	3-14-01	0.25	2.09756	2.09951	1.95	0.094
76	3-27-01	0.25	2.02063	2.02217	1.54	0.077
77	4- 1-01	0.45	2.31490	2.31734	2.44	0.106
78	4- 4-01	0.12	2.22259	2.22344	0.85	0.039
79	5- 8-01	1.00	2.67266	2.67527	2.61	0.098
				Mean		0.112

This result is perplexing, and much lower than the average computed from the first two determinations, which was given in the preceding paper. It indicates that the Patterson and Guthe method gives results 0.06 per cent higher than those given by Lord Rayleigh's method, while Patterson and Guthe's own comparisons give a difference of 0.11 per cent.* Evidently the saturated-oxide method is more variable in different hands even than Lord Rayleigh's. Perhaps the safest number to use in the correction is the average of both, 0.085 per cent above the Lord Rayleigh method, or 0.135 per cent above the porous cup method.

We are now in a position to make an approximate correction for the effect of the contaminating anode liquid in each of the more important investigations which bear upon the electro-chemical equivalent of silver. Of these, those of Lord Rayleigh, Fr. and W. Kohlrausch, K. Kahle,

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* Phys. Review, **7**, 280. Kahle (Wied. Ann. **67**, 32, also Brit. Ass't. A. Sc. 1892, 148), found 0.05 per cent, but his solutions were probably fresher.

and Patterson and Guthe have attracted most attention. Since the first three investigations used a voltameter of the original Lord Rayleigh type, a correction of —0.05 per cent should be applied to each. Moreover, Kohlrausch and Kahle did not heat their deposits to redness; hence an additional reduction of about 0.016 per cent* is necessary. Finally, Kohlrausch deposited the silver on a silver kathode, while Lord Rayleigh and Kahle made their determinations with platinum kathodes — a correction which leads to a further reduction of .01 per cent for Kohlrausch's value, or 0.076 in all. Patterson and Guthe, on the other hand, deposited the silver on platinum, but used old solutions saturated with silver oxide. Such solutions may have yielded about 0.135 more silver than the standard. When the correction for heating is added to this the total reduction becomes 0.15 per cent. Thus we are led to the following table: —

TABLE XVI.

THE CORRECTED ELECTROCHEMICAL EQUIVALENT OF SILVER.

(1)	Lord Rayleigh and Mrs. Sidgwick,†	0.0011179	—0.050%	0.0011173
(2)	Fr. & W. Kohlrausch,‡	0.0011183	—0.076	0.0011175
(3)	Kahle,§	0.0011183	—0.066	0.0011176
(4)	Patterson & Guthe,‖	0.0011192	—0.150	0.0011175
		Average		0.0011175

The greatest deviation from this average is 0.02 per cent, a remarkable agreement considering the variety of physical method used by the experimenters. Lord Rayleigh and Kahle used an electro-dynamometer and Kohlrausch an accurate tangent galvanometer for the calculation of the current, while Patterson and Guthe made themselves entirely free from the acceleration of gravity or the strength of the magnetic field by means of an absolute electro-dynamometer. Hence for the present the great bulk of evidence seems to favor the value 0.0011175, the mean of four entirely independent determinations, as the true electrochemical equivalent of silver. Our data also give the electrochemical equivalent of copper in the cupric condition as 0.00032929 gram per ampere per second.

The number of coulombs attached to one gram equivalent of any electrolyte is therefore 96,580.

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* The average of Lord Rayleigh's results and ours.

† Phil. Trans., **175**, 411 (1884).

‡ Wied. Ann. N. F., **27**, p. 1 (1886).

§ Wied. Ann. N. F., **67**, 1 (1899).

‖ Phys. Review, **7**, 257 (1898).

A few more points may be touched upon here, which follow directly from the new value of the equivalent. A great number of physical instruments have been standardized on the basis of a somewhat higher electrochemical equivalent of silver, 0.001118. Will they be affected by the lowering of this number? Evidently not, since if the value corresponding to a given mode of deposition is applied throughout, whenever this method is used, no constant error can result. Thus our low value cannot be employed when the anode is unprotected, and the deposit not heated to redness.

Therefore, as was shown in our last paper, the discovery of a constant error in the silver voltameter cannot help the discrepancy which exists between the electrical and mechanical methods of determining Joule's equivalent.

It is to be hoped that in the future, however, all experimenters will use some method, such as ours, in which the anode complications are excluded. Obviously even the present condition of electrical science demands a more precise electrochemical definition of the ampere than that now prescribed.

The present research seems to define the practical unit of current strength no less accurately than the practical unit of electro-motive force has been defined. Thus in a laboratory provided with pure chemicals, each of these units may be established without outside help, and with their assistance a standard ohm may be produced without comparison with any other standard ohm.

VI. SUMMARY.

The results of the prolonged investigation may be summed up as follows: —

1. The electrochemical equivalent of silver as determined by the Lord Rayleigh voltameter is too high by at least 0.05 per cent.

2. The true rate of deionization of silver can be determined by the use of a porous cup which prevents the solution at the anode from reaching the kathode. Results of great consistency and accuracy are then obtainable.

3. The porous cup does not introduce any new source of error, for without it the same low results may be obtained when the anode is placed below the kathode.

4. At higher temperature the complications grow larger.

5. The main disturbing factor is a complex silver ion formed at the anode and carried over to the kathode, where it decomposes, thereby

increasing the deposit of silver. Most of this polymerized material decomposes at once, however, forming the silver dust at the anode.

6. The hydroxyl ion discharges at the anode, forming silver oxide and probably so-called peroxide. Ionized hydrogen is thus developed.

7. Dissolved gases affect the deposit whenever they react with the complex ions.

8. Nitrite is formed at the anode, but has probably not much effect on the weight of the deposit.

9. The deposited silver always contains included solution, varying in amount from 0.01 per cent to 0.04 per cent according to circumstances.

10. A new name, *coulometer*, is proposed, to replace the old and unsuitable designation *voltameter*.

11. The true electrochemical equivalent of silver is probably 0.0011175 milligram per coulomb.

12. Therefore, 96580 coulombs are associated with one gram equivalent of any electrolyte.

13. The electrochemical equivalent of cupric copper is 0.00032929; therefore the electrochemical atomic weight of copper (63.601) is in close agreement with the chemical value (63.604).

14. Faraday's law is thus verified for two kathions more exactly than ever before.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series. — No. XXII.

By M. L. Fernald.

I. The Northeastern *Carices* of the Section *Hyparrhenae.*

II. The Variation of some Boreal *Carices.*

With Five Plates.

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By M. L. Fernald.

Presented May 8, 1901. Received January 31, 1902.

I. — THE NORTHEASTERN CARICES OF THE SECTION HYPARRHENAE.

The Carices of Koch's subgenus *Vigneae*, with its sections *Acroarrhenae* and *Hyparrhenae* of Fries, have always perplexed the systematist, and by the general student they have as a rule been ignored or vaguely referred to such characteristic species as *Carex straminea* or *C. echinata*. Recently, however, the generally widening interest in systematic botany has brought together in *Carex*, as in other groups, a large mass of material; and an attempt to identify these specimens has made it necessary to study in great detail the minuter but tolerably constant characteristics of the fruiting plants.

In general, the classification of Carices has always been based upon characters in the inflorescence; and although the detailed study of the perigynia (or utriculi) has been the final resort of the specialist, an attempt has been made in our manuals to separate species as much as possible upon the more obvious characters of the inflorescence. Thus *Carex scoparia* is described in the two current manuals as having the spikelets (spikes) "all contiguous or bunched" or "usually aggregated;" while in only one of these works is Boott's var. *minor* given recognition, and there as a mere dwarf variety. Yet in plants which are undoubtedly *C. scoparia* the spikelets are often scattered, forming a loose moniliform spike; and the northern plant described by Boott as var. *minor* has a distinct range and unique habitat, while its minute thick-bodied perigynia distinguish it at a glance from the more southern species with which it has been associated.

The case of *Carex scoparia* is only one of many in which the attempt to rely upon superficial characteristics has led us to confuse plants which are genetically very distinct. Consequently, as stated, an attempt has been made to get at a more satisfactory basis for classification by studying the characteristics of the perigynia, which, naturally, are subject to less variation than is the superficial aspect of the inflorescence as a whole. But since variations in texture and nerving, which are perfectly evident upon comparison of specimens, are extremely difficult to render clear in descriptions, it has been found advisable to employ as the primary basis of division, at least in the groups here discussed, the actual or proportional measurements of the perigynia or the achenes. Even this method of careful measurement may sometimes prove misleading, but in most species the perigynia vary within certain clearly defined limits, and it is only the very exceptional individual which will not fit the system here proposed. And, although in rare cases a species thus presents perplexing forms in which the perigynia are not characteristic, many attempts to classify the members of this group have convinced the writer that by actual measurement alone can we safely identify plants of such strong outward resemblance as *Carex straminea*, *C. scoparia* and *C. tenera*, or *C. alata* and *C. albolutescens*.

As a result of these studies it has been found desirable to treat many plants in a manner somewhat different from that in any current synopses of the genus, and in some cases a study of the original descriptions and specimens has brought the writer to conclusions very different from those generally accepted by American caricologists. Some of these points are of slight significance, others of fundamental importance; and, since it is inadvisable to complicate the synoptic treatment of the species with detailed discussions as to the identity and synonymy of different forms, the more important questions may be here discussed.

Carex scoparia, Schkuhr, presents little difficulty, as the original figure is unmistakable. The species has, however, been made to harbor plants of very different aspect; and a study of the fruiting characters shows these to fall into three groups with marked and constant characteristics. *C. scoparia*, itself, has the perigynium very thin and scale-like, with the wings so strongly developed as to minimize the apparent thickness of the body. This plant in its different forms is of broad range south and west of the Gulf of St. Lawrence.

The other two species which have been included with *Carex scoparia* have the narrower subulate or elongate-lanceolate perigynia so little winged as quite to lack the scale-like character seen in that species. The best known of these two plants is the form described by Francis Boott as *C. scoparia*, var. *minor*. The material from which Boott's plate was drawn was collected by Tuckerman at the base of the White Mountains; and since it is necessary to distinguish the plant by a new specific name (*minor* having been used too often as a varietal name to be eligible) and since there is already a *Carex Tuckermani*, it is a pleasure to commemorate the explorations and generous services of the Crawford family, familiar to a long generation of visitors to the White Mountains. This plant with which their name now becomes associated is common in northern New England and about the Great Lakes, thence extending far northward.

The other plant with narrow thick perigynia is more puzzling. In the dark brown color of its broad scales it is unlike the other forms which have been referred to *Carex scoparia*. In fact, by different students it has been referred with doubt to *C. tribuloides*, *C. leporina*, and *C. foenea* as well. Yet in its perigynium it resembles only Boott's *C. scoparia*, var. *minor*. This tall dark-spiked plant, which is common in the region of Orono, Maine, has been collected by Professor Lamson-Scribner and by the writer, but it seems to be unknown from other regions. This fact immediately suggests that it may be an introduced form, but a careful search through Old World material and descriptions fails to show anything to which it can be referred. It is, therefore, here treated as a local species, taking the name of the town from which all our material has been collected.

One other form of the *scoparia* group should be specially mentioned since, by an unfortunate misinterpretation, it has already caused needless confusion. This is *Carex scoparia*, var. *moniliformis*, Tuckerman. A specimen in the Gray Herbarium from Tuckerman himself, is without question a slender-spiked form of *C. scoparia*. The variety was so treated by Francis Boott, in whose table 368 it is well represented. Yet in his Preliminary Synopsis of the genus Professor Bailey treated it without question as identical with his own *C. tribuloides*, var. *reducta*; and Professor Britton, following his lead, has since made the new combination, *C. tribuloides*, var. *moniliformis* (Tuckerman) Britton, for a plant very different from that to which the varietal name was originally applied.

Carex tribuloides, Wahl., has been clearly treated by Professor Bailey.[1] *C. Bebbii*, Olney, however, which by him is reduced to a variety of that species, seems to be as well marked as other members of the subgenus, and it is here given equal rank with them. In its shorter, broader, and

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[1] Mem. Torr. Cl., I. 54.

thicker perigynia it is more nearly related to *C. straminea* and its allies. So, likewise, *C. cristata*, Schwein., is reinstated as a species, since its tolerably constant habit and its shorter, firmer perigynia place it as near *C. straminea* as to *C. tribuloides*.

The diverse plants which have been treated by various authors, now as distinct species, now as forms of *Carex straminea*, fall into groups which are, in the main, fairly free from complexity. The attempt to separate these forms by color-characters has naturally led to much confusion, for plants which in bright sunlight have a strongly marked ferrugineous tendency, in shade are often quite green. The shape, size, nerving, and texture of the perigynia, however, show that almost without exception the species proposed by Willdenow, Schkuhr, Torrey, Schweinitz, Dewey, and other early students of the group were based on permanent characters. To treat all these well marked and constant forms as varieties of one species is adding confusion rather than clearness to our interpretation of the genus, especially when several of them are as closely related to other well recognized species.

The identity of Willdenow's *Carex straminea* was settled by Professor Bailey[1] in 1889, and a recent examination of Willdenow's material by Dr. J. M. Greenman has verified Professor Bailey's conclusions. *C. albolutescens*, Schweinitz, is now well understood, as are likewise *C. mirabilis*, Dewey, *C. tenera*, Dewey, *C. Bicknellii*, Britton (*C. straminea*, var. *Crawei*, Boott), and *C. alata*, Torrey. But *C. festucacea*, Schkuhr, *C. straminea*, var. *brevior*, Dewey, and *C. foenea*, var. *β*, Boott, seem to have been less clearly understood.

Schkuhr's *Carex festucacea*, according to the original description, was a plant with about eight spikelets subapproximate or in a loosely cylindric spike, and the species is so represented in Schkuhr's figure. It is likewise well represented by Dr. Boott, who apparently had a clear conception of the species, in his table 386. Schkuhr's *C. straminea*, which we now know to be different from Willdenow's plant of that name, was an extreme form of *C. festucacea* with fewer spikelets, and until recently it passed as the type of the species; i. e., *C. straminea* (*typica*) of Boott and others. This plant, however, was called by Dewey *C. straminea*, var. *brevior*, and under that name it has been treated by Professor Bailey. He includes with it, though, the *C. festucacea* of Schkuhr, a plant which, though closely related, is of rather marked appearance and of more limited range. More recently Dr. Britton, in

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[1] Mem. Torr. Cl., I. 21.

restoring to specific rank *C. festucacea*, has included in it Dewey's *C. straminea*, var. *brevior*, and in the Illustrated Flora he figures the latter plant under the former name. But the late Dr. Eliot C. Howe, in his admirable treatment of the New York Species of Carex, has recognized both plants, thus following the general treatment of Francis Boott and other earlier writers and at the same time clearing the names *festucacea* and *brevior* from the confusion which has recently surrounded them.

Carex foenea, var. *β* of Boott has had a peculiarly unsettled history. When Francis Boott described and figured the plant as a variety of *C. foenea*, the latter name applied to *C. albolutescens*, Schweinitz, not to the true *C. foenea* of Willdenow. It was Boott's opinion, then, that the slender brown-spiked plant of the interior was a phase of what we now know without much doubt to be *C. albolutescens.* In the fifth edition of the Manual Dr. Gray took up *C. foenea*, var. *β* as *C. foenea*, var. (?) *ferruginea;* and later the plant was distributed by Olney as a variety of Dewey's *C. tenera* (*C. straminea*, var. *aperta*, Boott). In his Preliminary Synopsis in 1886, Professor Bailey reduced it to synonymy under *C. straminea*, Schkuhr (not Willd.), and later in his Critical Studies of Types he treated this plant along with *C. festucacea*, Schkuhr, and *C. straminea*, var. *Crawei*, Boott (*C. Bicknellii*, Britton) as identical with *C. straminea*, var. *brevior*, Dewey (*C. straminea*, Schkuhr, not Willd.). Subsequently, however, he has taken out of his *C. straminea*, var. *brevior*, two plants, which he treats as parallel varieties, var. *Crawei*, Boott, and var. *ferruginea* (*C. foenea*, var. *β*, Boott); and at the same time he has discussed as a species *C. albolutescens*, Schweinitz (*C. foenea* of authors, not Willd.). This course has greatly cleared the group from its former confusion; but it is unfortunate that while separating *C. albolutescens* specifically Professor Bailey should have attached *C. foenea*, var. *β* to the slender usually flexuous-spiked *C. straminea*, whose identity he had already so carefully worked out. *C. foenea*, var. *β* in its stiff habit, its strongly appressed broad-ovate perigynia, and the texture of its leaf-sheaths, is quite unlike that species, but is very close to *C. albolutescens* with which it had been placed by Francis Boott. In these characters, likewise, it is equally close to *C. alata*, Torr., while its perigynia and the occasional awn-tips of the scales are so like those of the latter species as to place it nearer to that than to the former plant.

The two species, *Carex foenea*, Willd., and *C. adusta*, Boott, have already been discussed and very clearly settled by Professor Bailey.[1] But his own *C. foenea*, var. *perplexa* has proved very puzzling to students of the group. In the original description of this variety at least two distinct species are referred to, while the words "head erect or nearly so" have proved misleading for a plant with more flexuous spikes (heads) than ordinarily occur in the type of the species.

Dr. J. M. Greenman has kindly compared with Willdenow's original material various plants passing in America as *Carex foenea*, and he has furnished the writer with detailed camera-drawings from Willdenow's material. From these comparisons there seems no doubt that the original *C. foenea* was, as Professor Bailey has already stated, the smallest form of the species, with 4 to 9 spikelets in a suberect linear-cylindric spike. This is the plant subsequently described by Tuckerman as *C. argyrantha* and figured by Boott in his table 382, fig. 2.

Professor Bailey's *Carex foenea*, var. *perplexa* was based on Boott's table 380 and, a portion of table 382 (presumably fig. 1), upon Olney's *C. albolutescens* (Exsicc. fasc. 1, no. 8), as well as his *C. albolutescens*, var. *sparsiflora* (fasc. V. no. 11). Now, the perigynia of good *Carex foenea* are strongly and conspicuously nerved on both faces, and the spikelets are pale green or silvery brown. The first part of var. *perplexa* (Boott's table 380) shows a perigynium quite nerveless or only faintly short-nerved on the inner face; the second component (table 382, fig. 1) is the characteristic large form of *C. foenea* with crowded spikes of large spikelets; the third (*C. albolutescens* of Olney) is, as represented by two sheets in the Gray Herbarium, a form between the large state and the small typical *C. foenea*; while the fourth component (*C. albolutescens*, var. *sparsiflora*, Olney — at least the New Brunswick plant) in habit as well as in the nerveless inner face of the perigynium closely matches the first cited plate (Boott's table 380). From the fact that var. *perplexa* was proposed as a variety of *C. foenea* it is probable that its author had in mind the coarse form represented by Boott's table 382, fig. 1, and in the present treatment of the group it has seemed advisable to retain that name for the large plant.

Olney's *Carex albolutescens*, var. *sparsiflora* is represented in the Olney Herbarium by two different plants. One of these, from Oregon, is the dark-spiked form of *C. praticola* which has been described as *C. pratensis*, var. *furva*, Bailey. The other, from Kent Co., New Brunswick, the northeastern plant which is identified with Boott's table 380, is much more closely related to *C. adusta*, Boott, than to *C. foenea*, Willd.

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[1] Mem. Torr. Cl., I. 24.

From the former species it differs constantly in its more slender habit and flexuous elongated spikes of clavate-based spikelets, as well as in smaller achenes. It is a plant of broad range from Labrador to British Columbia, creeping south to the coast of New England and the mountains of New England and New York. Since its varietal name, *sparsiflora*, is preoccupied in the genus, another specific name is here proposed in reference to the characteristic color of the mature inflorescence.

The other large group of the *Hyparrhenae* which has been treated by recent authors as the subsection *Elongatae* contains plants of two markedly different tendencies. One group is characterized by strongly divergent thin-edged perigynia which are spongy at base. The other group has ascending plump or plano-convex perigynia which are rarely thin-edged and are without conspicuously spongy bases. Mr. Theodor Holm, who has recently studied some of the members of the first group, includes with them *Carex gynocrates* and *C. exilis*, which by most other authors have been placed in the *Dioicae*. The texture and aspect of the perigynia seem to justify the treatment proposed by Mr. Holm and formerly for *C. exilis* by Francis Boott;[1] and for the group thus constituted Mr. Holm suggests the name *Astrostachyae*.[2] The other group, with ascending blunt-edged perigynia, may well retain the subsectional name *Elongatae*, since the characteristic species, *C. elongata*, *C. brunnescens* (*C. Gebhardii*), *C. canescens* (*C. curta*), etc., were originally included in it by Kunth.

Mr. Holm, in the paper cited, takes exception[3] to Professor Bailey's recent treatment[4] of *Carex echinata*, *C. sterilis*, and *C. scirpoides*, on the ground that that author had been more controlled by the original specimens of Willdenow and of Schkuhr than by the original diagnoses. That Willdenow's original descriptions do not accord well with Professor Bailey's conclusions there can be no doubt; and when we are told by Professor Bailey that *C. sterilis* and *C. scirpoides* are identical, and when he says "the figures of both *C. sterilis* (fig. 146) and *C. scirpoides* (fig. 180) in Schkuhr's 'Riedgräser' are unequivocal,"[5] we find it indeed difficult to understand his observations. An examination of Schkuhr's figures shows his *C. sterilis* (fig. 146) to be a coarse plant with sharp-pointed ovate scales and broad-ovate cordate perigynia with distinct beak shorter than the body. Schkuhr's *C. scirpoides* (fig. 180), on the other hand, is represented with broad-oblong or elliptical blunt

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[1] Boott, Ill., I. 17.

[2] Holm, Theo., Am. Jour. Sci., Ser. 4, XI. 205-223.

[3] Holm, l. c., 212.

[4] Bailey, Bull. Torr. Cl., XX. 422.

[5] Bailey, l. c., 424.

scales and deltoid-ovate obscurely short-beaked perigynia. These figures of Schkuhr's agree very well with his descriptions. Furthermore, they agree equally well with Willdenow's diagnoses, for these latter were essentially the same as Schkuhr's. Professor Bailey further states that *C. sterilis* and *C. scirpoides* are identical with the common American plant which he had formerly treated as *C. echinata*, var. *microstachys*, a plant with lanceolate or narrowly ovate slender-beaked perigynia; and for this aggregate he takes up the name *C. sterilis*. After thus bunching three very different species as *C. sterilis*, he separates from "our so-called *Carex echinata*" two plants, *C. atlantica* and *C. interior*, with "ample specific characters."

Through the kindness of Dr. J. M. Greenman the writer has been able to examine camera-drawings of Willdenow's original material; while from Professor Carl Mez he has received fragments from the original material of Schkuhr. The drawings of the Willdenow material of both *Carex sterilis* and *C. scirpoides*, and the Schkuhr specimens of *C. scirpoides* agree with the original diagnoses. Dr. Greenman has, further, compared critically specimens sent him of the different American forms with Willdenow's plants and with authentic specimens of *C. stellulata*, Gooden. (*C. echinata*, Murray). The identification thus made of these forms, leads to a conclusion very different from that published by Professor Bailey. These results may best be stated by discussing separately the three clearly cut species which have been so unfortunately confused.

Carex echinata, Murray (*C. stellulata*, Gooden.). This species was long considered a boreal plant of broad range, and it was so treated by Torrey, Tuckerman, Dewey, Carey, and other early students of American *Carices*. Francis Boott distinctly implied that the European species occurs in British America, saying: "I have not seen specimens which I can satisfactorily refer to the European *C. stellulata*, south of the British provinces of North America."[1] Yet Professor Bailey has interpreted this to mean that "Francis Boott questioned if the American plant is the same as the European *C. stellulata* (or *C. echinata*);" and in "eliminating the European species from our flora," he says: "Definite specific characters of separation are obscure, and yet I am convinced that they exist. The American plant is habitually taller than the European, the scales are sharper and usually longer, the perigynia are more strongly nerved and more attenuated or conical,

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[1] Boott, Ill., I. 56.

and above all, it is far more variable. . . . There are probably no species common to both countries, except those which are hyperboreal and occur through the Arctic regions of both hemispheres, being found in Greenland."[1]

Then Professor Bailey defines his conception of the "habitually taller" American plant with "sharper" scales, etc., etc., including in it forms varying from the low slender *Carex stellulata*, var. *angustata*, Carey, with "*narrowly-lanceolate perigynia* tapering into a long . . . beak,"[2] to the tall (often nearly 1 m. high) coarse *C. sterilis*, Willd., with broad-ovate perigynia, and the slender *C. scirpoides*, Schkuhr, with thick scarcely beaked often nerveless deltoid-ovate perigynia and elliptic blunt scales. The two latter constituents of this aggregate apparently do not occur outside North America and if they are included with the other American representative of *C. echinata* as one species, it is of course easily said that the American plant is taller or shorter, coarser or more slender than the European; and certainly a species so constituted is "far more variable."

When, however, we eliminate from the complex *Carex sterilis* of Professor Bailey's treatment the true *C. sterilis* and *C. scirpoides*, there is left a plant characterized by slender culms and leaves, the perigynia barely half as broad as long, and tapering to a slender conspicuous beak which is often nearly as long as the body. This is the *C. echinata* or *C. stellulata* of American authors and it includes as formal variations the very slender var. *angustata*, Carey (*C. echinata*, var. *microstachys*, Boeckeler), and the tall *C. sterilis*, var. *excelsior*, Bailey, while a very coarse variation with rather better defined characteristics is *C. echinata*, var. *cephalantha*, Bailey.

This American species with the narrow perigynia has been compared many times by the writer with European *C. echinata* in a vain attempt to find some point of distinction. Specimens collected by Godet at Lignières on the River Cher in central France are inseparable from Mertens' material from Sitka, and, again, Japanese specimens collected by Chas. Wright and by Maries are identical in their slender perigynia with Newfoundland plants. In order, however, to test still further the specific value of the American plant a portion of Allen's Labrador material was forwarded to Dr. Greenman at Berlin, and he was asked to compare it, along with other American forms, with Willdenow's types

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[1] Bailey, Bull. Torr. Cl., XX. 423.

[2] Carey in Gray, Man. 541.

and with other authentic European specimens of the group. In reply Dr. Greenman writes of this specimen:

"No. 4. Differs from the original *C. sterilis*, Willd., in the following characters: (*a*) narrower, more gradually acuminate and longer beaked perigynium; (*b*) more oblong achene, which is less narrowed at the base. To me, however, your No. 4 is a *perfect* match for *Carex stellulata* in herb. Willdenow, and for European *C. echinata*, Murr. I am quite unable to make any distinction between them. The perigynial characters are exactly the same."

Extreme difficulty is experienced, then, in attempting to distinguish the American *Carex echinata* from Old World material. The range of the American plant, too, from Labrador to Alaska, and southward in the mountains, immediately places the species in the hyperboreal flora from which Professor Bailey, at least by inference, would exclude it. In view of these two facts there seems, then, as Mr. Holm has already indicated, good reason to consider both the American and the European plant *C. echinata*, Murr.

Carex sterilis, Willd. This plant has already been sufficiently defined in the discussion of Willdenow's original description and of Schkuhr's figure. The writer has, however, examined with much care camera-drawings of Willdenow's material made by Dr. Greenman and fragments of Schkuhr's material generously sent by Professor Carl Mez. The Willdenow plant, which alone is of final importance, proves to be identical with the large species of the Atlantic seaboard recently described as *C. atlantica*. The fragment sent by Professor Mez from the Schkuhr herbarium is, however, from cultivated material, and is only a form of *C. echinata* with narrow perigynia quite unlike those shown in Schkuhr's figure and in the Willdenow plant as further shown by Dr. Greenman's report of his critical comparisons in the Willdenow herbarium.

Besides No. 4, the Labrador *Carex echinata*, two other forms were sent to Dr. Greenman for comparison with *C. sterilis*. No. 1 is *C. echinata*, var. *cephalantha*, Bailey, collected by Dr. C. B. Graves at Waterford, Connecticut, May 27, 1896. No. 2 is characteristic *C. atlantica*, Bailey, collected by Dr. G. G. Kennedy at Ponkapog, Canton, Massachusetts, July 12, 1899. Of these two plants Dr. Greenman writes:

"No. 1. This differs from *C. sterilis*, Willd., in the following characters: (*a*) longer inflorescence, more remote and slightly longer spikelets; (*b*) longer and more prominently beaked perigynium; (*c*) achene less narrowed at the base.

"No. 2. I am quite unable to distinguish this plant from the original of *C. sterilis*, Willd. It has the same broad-ovate, short-acuminate or short-beaked perigynium, and the same achenial characters, that is, the achene is rather conspicuously narrowed below. The characters of the inflorescence are the same, except as to color. The Willdenow plant is more brownish: this, however, may be due, at least to a certain extent, to age."

From Willdenow's original description, from Schkuhr's description and figure, and from Dr. Greenman's examination and drawings of the Willdenow plant, there seems no question, then, that *Carex atlantica*, Bailey, is the true *C. sterilis*, Willd.

Carex scirpoides, Schkuhr. The characters of this species, likewise, are sufficiently stated in the discussion of Schkuhr's and Willdenow's characterizations. Material from the Schkuhr herbarium received through Professor Mez is identical with camera-drawings made by Dr. Greenman from Willdenow's plant. These accurately agree, also, with Schkuhr's fig. 180. This species, was, furthermore, correctly interpreted by Sartwell, Carey, and Boott, and it is well represented as *C. stellulata*, var. *scirpoides* in Boott's Illustrations, t. 146.** Sartwell's No. 36 and Boott's plate are the only exact citations given by Professor Bailey for his *C. interior*, and his description of the so-called new species accords well with those of Willdenow and of Schkuhr. In distinguishing *C. interior* from *C. scirpoides*, Bailey says that the former has "greenish-tawny spikes," while the latter is "fulvous;" and he furthermore describes Schkuhr's *C. scirpoides*, "as the plate plainly shows," with "long-beaked broad-winged perigynia." How such a statement and such conclusions could have been made is very puzzling. There can be no question, however, that the figure of Schkuhr's *C. scirpoides* as interpreted by Dewey, Schweinitz, Torrey, Sartwell, Carey, Francis Boott, Holm, and other students of the genus, is the same as Boott's table 146** upon which, in part, *C. interior* was founded.

The name *Carex scirpoides*, Schkuhr, so long attached to this plant, was published in 1805, but it cannot, unfortunately, be retained for the species, since in 1803 Michaux published *C. scirpoidea*, the well known dioecious plant of extreme boreal and alpine regions. The next clearly defined name for the plant seems to be *C. interior*, although, as originally intended by its author, that name was supposed to apply to a species very distinct from *C. scirpoides*. Tuckerman, it is true, published in his Enumeratio Methodica the name *C. stellulata*, var. *scirpina*, citing *C. scirpoides*, Schkuhr, as a synonym. On a preceding page, however, in an unfortunate endeavor to latinize one of Michaux's names, he had substituted *C. scirpina* for *C. scirpoidea*, Michx., not *C. scirpoides*, Schkuhr. This unfortunate citation of "C. scirpina" as a pure synonym of Michaux's *C. scirpoidea* attaches to that name a decided element of indefiniteness. It is, therefore, wiser to take for the plant of Schkuhr and of Willdenow the more clearly defined name, *C. interior.*

One other plant of the *Astrostachyae* has been the source of much confusion in the treatment of New England species of this group. Unlike *Carex echinata*, *C. sterilis*, and *C. interior*, the perigynia of this plant are broadest at the middle, thence tapering to a narrow base. In aspect the plant is strikingly like the largest form of *C. canescens*, but its thin-edged strongly recurved perigynia place it clearly in the *Astrostachyae.* The species is not uncommon from eastern Massachusetts to Delaware and central New York, and in New England herbaria it has recently passed variously as *C. atlantica*, *C. interior*, *C. canescens*, var. *vulgaris*, *C. sterilis*, var. *excelsior*, &c. From notes left by the late William Boott it is apparent that he recognized in some of Chas. Wright's Connecticut material an undescribed form, but evidently he never described the plant. A portion of the original material of the late Dr. Eliot C. Howe's *Carex seorsa*, generously furnished the writer by Professor C. H. Peck, agrees in every regard with the perplexing New England plant, and under that name the species should now be known.

The members of the *Elongatae*, as here interpreted, offer less difficulties than the other species of the *Hyparrhenae*, and special discussion is needed only of the forms which have been at various times associated with *Carex canescens.* These plants present two marked forms in their perigynia: in one plant, *C. arcta*, the perigynium is broadest at the rounded or subcordate base; while in *C. canescens* and *C. brunnescens* (*C. vitilis*, Fries) the perigynium is nearly elliptic in outline, being broadest near the middle.

Carex arcta of Francis Boott was originally published by him as *C. canescens*, var. *polystachya*, but in his latest treatment of the plant he considered it a distinct species. As stated, its perigynial character is very constant. Furthermore, its rather limited strictly American range and unique habit quickly separate it from most forms of *C. canescens.* *C. canescens*, var. *oregana*, Bailey, said to differ from var. *polystachya* in having the "head larger and more dense . . . becoming brownish," has identical perigynia with that plant, and the spikes (heads) are green or brownish, as are those of the eastern plant, a character dependent on age and exposure to light.

Carex canescens, L., is characterized by its glaucous color and strongly appressed-ascending elliptic pointed perigynia tapering very gradually to the short beak. Another plant, *C. brunnescens*, Poir. (*C. canescens*, var. *alpicola*, Wahl., *C. canescens*, var. *vulgaris*, Bailey), is usually bright green, and the few loosely spreading-ascending perigynia are rather abruptly contracted to a definite serrulate-based beak. This plant is common in dry soils throughout the boreal sections of America and Europe; while the glaucous *C. canescens* is a species of very wet situations. Under various names, *C. vitilis*, Fries, *C. Gebhardii*, Hoppe, etc., *C. brunnescens* has been treated as a species, and as often again as a variety of *C. canescens*. An examination of much material shows its characters to be essentially constant, and, though the plant superficially resembles small forms of *C. canescens*, its claim to specific rank rests upon a number of definite characters.

When *Carex arcta* and *C. brunnescens* are removed from *C. canescens*, there remains a species characterized by its glaucous foliage and appressed scarcely beaked perigynia. This species presents in America three noteworthy variations. The true *C. canescens*, L., of northern Europe has the spikes 2.5 to 5 cm. long, of 4 to 7 oblong-cylindric to narrowly obovoid spikelets 0.6 to 1 cm. long. This plant occurs in Arctic America coming south to northern New England and New York, the Rocky Mts., and Vancouver. Rare in the eastern United States and Canada, the typical form of *C. canescens* has been misinterpreted by recent American students, although the species was very clearly discussed by Francis Boott. The American plant which has passed as true *C. canescens* is, however, strikingly different in aspect, and consequently the typical plant has more than once been published as a local American variety — var. *dubia*, Bailey, and var. *robustina*, Macoun.

Another form of *Carex canescens* common to northern Europe and America is var. *subloliacea*, Laestadius. In this plant the spike is usually rather shorter than in typical *C. canescens*, the less approximate globose or short-oblong few-flowered spikelets are only 4 to 7 mm. long, and the smaller perigynium is nearly or quite smooth. In its smooth perigynium this plant approaches *C. heleonastes*, which, however, has larger spikelets and perigynia and quite lacks the distinctive glaucous aspect of *C. canescens*. The var. *subloliacea*, which is commoner in northern New England than is the true *C. canescens*, also simulates *C. brunnescens*; but it is very canescent and the perigynia otherwise as in true *C. canescens* are essentially smooth, while in the greener *C.*

brunnescens they are distinctly beaked, of more membranous texture, and usually with serrate margins.

The commonest form of *Carex canescens* in North America is the plant mentioned without name by Francis Boott and figured by him in his Illustrations, IV. table 496. This unique American form, which in essential characters is like true *C. canescens*, differs in its elongated inflorescence, 5 to 15 dm. long, at least the lower spikelets very remote. The plant seems to have been generally treated by American authors as typical *C. canescens*, and no published name is available for it.

The following synopsis presents the characters and ranges of the northeastern *Hyparrhenae* as now understood by the writer. In its preparation he has studied the material in the Gray Herbarium and the herbarium of the New England Botanical Club; as well as the hundreds of sheets in the herbarium of the Geological Survey Department of Canada, kindly placed at his disposal by Mr. James M. Macoun; those of the Olney Herbarium of Brown University, made accessible to him by Mr. J. Franklin Collins; and a series from the Fairbanks Museum at St. Johnsbury, Vermont, rich in forms of the *scoparia* group, specially accumulated by the director, Dr. T. E. Hazen, for detailed study, and then generously forwarded to the writer. He has also been greatly assisted by the use of material from the private herbaria of the Honorable J. R. Churchill; President Ezra Brainerd; Doctors C. B. Graves, J. V. Haberer, G. G. Kennedy, and C. W. Swan; and Messrs. Luman Andrews, C. H. Bissell, Walter Deane, E. L. Rand, W. P. Rich, and E. F. Williams. The identification of dubious species of Willdenow and of Schkuhr has been facilitated by the coöperation of Dr. J. M. Greenman while at the Royal Botanical Museum in Berlin, and by Prof. Carl Mez of the University of Halle; and authentic material of the late Dr. E. C. Howe's *Carex seorsa* has been generously furnished by Prof. C. H. Peck.

HYPARRHENAE, Fries. Staminate flowers scattered or at the base of the uniform spikelets (only in exceptional individuals and in the often dioecious *C. gynocrates* and *C. exilis* the entire spikelet staminate).

Key to Species.[1]

* Perigynia with thin or winged margins.

+ Perigynia ascending, the tips only sometimes wide-spreading or recurved, not spongy at base, the margins winged at least toward the beak.

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[1] The perigynial characters are here based on study of mature plants. In general the perigynia at the tip of the spikelet are less characteristic than those nearer the middle; and, if possible, the latter alone should be used in critical comparisons.

+ + Bracts wanting or setaceous, if broad at most twice as long as the spike.
= Plant strongly stoloniferous; culms rising from an elongated rootstock: perigynium firm, 5 to 6 mm. long . . . . . (4) *C. siccata.*
= = Plant not strongly stoloniferous; culms solitary or in stools.
a. Perigynia less than 2 mm. broad.
1. Perigynia 5 mm. or more long.
○ Perigynia 7 to 10 mm. long: spikelets oblong-cylindric, pointed, 1.5 to 2.5 cm. long . . . . (1) *C. muskingumensis.*
○ ○ Perigynia shorter (or, when exceptionally 7 mm. long, in shorter spikelets).
+ Perigynia half as broad as long, plump, nerveless or obscurely short-nerved on the inner face . . . . (21) *C. aenea.*
+ + Perigynia one-third as broad as long.
× Perigynia thin and scale-like, scarcely distended over the achenes, distinctly nerved on the inner face, and prominently exceeding the subtending scales.
§ Leaves at most 3 mm. wide: spikelets 3 to 9, glossy brown or straw-colored, pointed.
Spike oblong-ovoid or subcylindric, with ascending approximate spikelets . . . . (2) *C. scoparia.*
Spike moniliform . . (2) *C. scoparia*, var. *moniliformis.*
Spike short-globose or broad-ovoid, the spikelets crowded and divergent . (2) *C. scoparia*, var. *condensa.*
§ § Leaves more than 3 mm. wide: spikelets 8 to 14, green or dull brown, blunt . . . . (3) *C. tribuloides.*
(For vars. see below.)
× × Perigynia firm, obviously distended over the achenes, nerveless or obscurely nerved on the inner faces, equalled by the subtending scales . . . . (7) *C. praticola.*
2. Perigynia less than 5 mm. long.
○ Perigynia thin and scale-like, scarcely distended over the achenes: leaves 3 to 8 mm. broad.
+ Perigynia with appressed tips.
Spike oblong, the spikelets approximate . (3) *C. tribuloides.*
Spike moniliform, the spikelets scattered (3) *C. tribuloides*, var. *turbata.*
+ + Perigynia with spreading tips: spike flexuous (3) *C. tribuloides*, var. *reducta.*
○ ○ Perigynia firm, obviously distended over the achenes.
+ Perigynia elongate-lanceolate or subulate, less than one-third as broad as long, at most 1.4 mm. broad.
× Tips of perigynia conspicuously exceeding the lance-subulate dull scales.
Culms 1 to 4 dm. high: leaves 1 to 2.5 mm. wide: spikelets 3 to 7 mm. long . . . . . . (5) *C. Crawfordii.*

Culms taller: leaves broader: spikelets 8 to 11 mm. long (5) *C. Crawfordii*, var. *vigens*

× × Tips of perigynia equalled by the ovate bluntish glossy dark scales . . . . (6) *C. oronensis.*

+ + Perigynia broader, nearly or quite half as broad as long.

× Tips of perigynia distinctly exceeding the subtending scales.

§ Leaves 2.5 mm. or more wide.

□ Spikelets compactly flowered, the mature perigynia with recurved or spreading tips concealing the scales . . . . (8) *C. cristata.*

□ □ Spikelets with ascending or slightly spreading perigynia; scales apparent.

△ Mature perigynia greenish or pale straw-colored, in loose spikelets: spikes more than 2.2 cm. long (if shorter, with dark chestnut scales).

Spikelets approximate in ovoid or oblong spikes.

Scales pale, not strongly contrasting with the perigynia . . . . (10) *C. mirabilis.*

Scales dark chestnut, strongly contrasting with the perigynia . . (10) *C. mirabilis*, var. *tincta.*

Spikelets scattered in a moniliform spike (10) *C. mirabilis*, var. *perlonga.*

△ △ Mature perigynia brown, in dense spikelets: spikes at most 2.2 cm. long: scales pale brown (17) *C. Bebbii.*

§ § Leaves narrower.

Spike stiff, with crowded closely flowered spikelets (17) *C. Bebbii.*

Spike flexuous and moniliform, or at least with the loosely flowered spikelets scattered . (11) *C. straminea.*

× × Tips of perigynia equalled by the subtending scales.

§ Spike stiff and erect, or at least with the spikelets approximate.

Spike brown or ferrugineous . . . . (19) *C. leporina.*

Spike brownish white . . . . (20) *C. xerantica.*

§ § Spike flexuous, or at least with the lower spikelets remote.

□ Perigynia nerveless or minutely short-nerved on the inner face.

Mature perigynia straw-colored or pale brown, one-third as broad as long . . . . . . (7) *C. praticola.*

Mature perigynia olive-green or bronze, one-half as broad as long . . . . (21) *C. aenea.*

□ □ Perigynia with strong ribs the length of the inner face: spike silvery green . . . . (18) *C. foenea.*

b. Perigynia 2 mm. or more broad.

1\. Tips of the perigynia distinctly exceeding the subtending scales.

- ○ Perigynia thin and scale-like, barely distended over the achenes, one-fourth to one-third as broad as long.
  - Perigynia 7 to 10 mm. long . . . . . (1) *C. muskingumensis.*
  - Perigynia shorter . . . . (2) *C. scoparia.* (For vars. see above.)
- ○ ○ Perigynia firmer, obviously distended over the achenes, nearly or quite half as broad as long.
  - + Perigynia lance-ovate, about half as broad as long.
    - × Leaves 2.5 mm. broad, or broader . . . . (10) *C. mirabilis.* (For vars. see above.)
    - × × Leaves narrower.
      - § Perigynia distinctly about 10-nerved on the inner faces, 4 to 6 mm. long.
        - Spikelets 8 to 12 mm. long: perigynia 4.8 to 6 mm. long . . . . (12) *C. tenera.*
        - Spikelets 5 to 8 mm. long: perigynia 4 to 5 mm. long (12) *C. tenera*, var. *invisa.*
      - § § Perigynia 3- to 5-nerved on the inner faces, mostly less than 4 mm. long.
        - Perigynia with ascending inconspicuous tips (11) *C. straminea.*
        - Perigynia with divergent conspicuous tips (11) *C. straminea*, var. *echinodes.*
  - + + Perigynia with broad-ovate to orbicular bodies.
    - × Spike moniliform and flexuous, with mostly clavate-based spikelets.
      - Spikelets brownish-white; of close-appressed obscurely beaked firm perigynia . . . . (14) *C. silicea.*
      - Spikelets ferrugineous; the abrupt slender beaks of the perigynia with conspicuous loosely ascending or spreading tips . . . . (12) *C. tenera*, var. *Richii.*
    - × × Spike stiff (or, if flexuous, with brown or ferrugineous spikelets).
      - § Perigynia 5.6 to 7.7 mm. long, very thin, scale-like, almost transparent: scales blunt . . . (13) *C. Bicknellii.*
      - § § Perigynia less than 5.6 mm. long, firm and opaque (when exceptionally longer in *C. alata*, with aristate scales).
        - □ Scales long-acuminate or aristate: perigynia 4 to 5.5 mm. long: achenes oblong.
          - △ Spike green, or finally dull brown: scales lance-subulate: perigynia obovate, 2.8 to 3.7 mm. broad, abruptly narrowed at base . . . . (15) *C. alata.*
          - △ △ Spike dark brown or ferrugineous: perigynia 2.3 to 2.8 mm. broad.
            - Spikelets closely approximate: scales ovate-lanceolate: perigynia ovate, tapering gradually to the beak . . . . (15) *C. alata*, var. *ferruginea.*

Spikelets scattered in a flexuous spike: scales lanceolate: perigynia orbicular, abruptly slender-beaked . . . . . (12) *C. tenera*, var. *Richii.*

□ □ Scales blunt or at most acutish.

Spikelets gray-green or finally dull brown, with strongly appressed-ascending very firm perigynia 3.5 to 4 (very rarely 4.5) mm. long (9) *C. albolutescens.*

Spikelets straw-colored or ferrugineous, with spreading-ascending perigynia 4 to 5.5 mm. long.

Spike of 5 to 10 mostly distinct spikelets (16) *C. festucacea.*

Spike of 3 to 6 approximate spikelets (16) *C. festucacea*, var. *brevior.*

2. Tips of perigynia equalled by the subtending scales.

○ Spike stiff and erect, or at least with approximate spikelets.

\+ Spike whitish or gray-green.

× Perigynia lance-ovate, 4 to 4.8 mm. long, nerveless on the inner faces, golden-yellow at base . . (20) *C. xerantica.*

× × Perigynia broad-ovate to suborbicular.

Perigynia strongly ribbed the length of the inner faces, 2 mm. broad . . . . (18) *C. foenea.*

Perigynia nerveless or faintly nerved on the inner faces, broader . . . . (9) *C. albolutescens.*

\+ + Spike bronze or ferrugineous.

Perigynia distinctly concave on the usually nerved inner faces: achene 1 mm. broad . . . . . (19) *C. leporina.*

Perigynia flat or convex on the usually nerveless inner faces, very plump: achene 2 mm. broad . (22) *C. adusta.*

○ ○ Spike flexuous, at least the lowest spikelets remote.

\+ Perigynia nerveless or only faintly short-nerved on the inner faces.

Perigynia ovate-lanceolate, one-third as broad as long: achene 1 mm. broad . . . . (7) *C. praticola.*

Perigynia ovate, half as broad as long: achene 1.5 mm. broad . . . . (21) *C. aenea.*

\+ + Perigynia distinctly nerved on the inner faces.

× Perigynia 2.8 to 4.4 mm. long, at most 2.4 mm. broad, 7- to 13-ribbed on the inner faces, abruptly beaked.

Spike of 4 to 9 spikelets 6 to 10 mm. long: perigynia 2.8 to 4 mm. long . . . . (18) *C. foenea.*

Spike of 6 to 15 spikelets 10 to 17 mm. long: perigynia 3.5 to 4.4 mm. long . . . (18) *C. foenea*, var. *perplexa.*

× × Perigynia 4 to 5.3 mm. long, 2.5 to 3 mm. broad, 3- to 5-nerved on the inner faces, obscurely broad-beaked (14) *C. silicea.*

⧺ ⧺ Bracts leaf-like and much prolonged, the lowest 1 to 2 dm. long: spikelets crowded: perigynia subulate . . . . (23) *C. sychnocephala.*

+ + Perigynia horizontally spreading or reflexed when mature, spongy at base, with thin but scarcely winged margins.

++ Spikelets solitary and terminal, pistillate or staminate, or with the flowers variously scattered.

Stoloniferous; the filiform culms at most 3 dm. high, from filiform rootstocks . . . . (24) *C. gynocrates.*

Not stoloniferous; the wiry culms 2 to 7 dm. high, in caespitose stools (25) *C. exilis.*

++ ++ Spikelets 2 to several.

= Perigynia broadest at base: beak rough or serrulate.

a. Perigynia at most half as broad as long, finally yellowish, with slender beak nearly equalling the body: scales pointed.

1. Perigynia ovate, 3 or 4 mm. long.

○ Spikelets at most 12-flowered.

Spike 1 to 3 cm. long, the 2 to 6 spikelets subapproximate (26) *C. echinata.*

Spike 2 to 6 cm. long, the 2 to 4 spikelets very remote, the terminal with a clavate base 0.5 to 1 cm. long (26) *C. echinata*, var. *ormantha.*

○ ○ Spikelets with more flowers.

Leaves 1 to 2.5 mm. broad: spikelets scattered, 12- to 20-flowered: perigynia less than half as broad as long (26) *C. echinata*, var. *excelsior.*

Leaves 2 to 4 mm. broad: spikelets mostly approximate, 15- to 40-flowered; perigynia half as broad as long (26) *C. echinata*, var. *cephalantha.*

2. Perigynia lanceolate or ovate-lanceolate, 2.5 to 3 mm. long: spike of 2 to 6 approximate spikelets (26) *C. echinata*, var. *angustata.*

b. Perigynia more than half as broad as long, firm, brownish or dark green, the beak one-fourth to one-half as long as the body.

1. Scales sharp-pointed: leaves 2.5 to 4.5 mm. broad: spike 1.5 to 3.5 cm. long; spikelets 15- to 50-flowered: coarse plant (27) *C. sterilis.*

2. Scales blunt: leaves narrower: spike 1 to 2 cm. long; spikelets 5- to 15-flowered: slender plants.

Leaves 1 to 2 mm. broad: perigynia faintly nerved or nerveless on the inner faces . . . . (28) *C. interior.*

Leaves narrower: perigynia strongly nerved (28) *C. interior*, var. *capillacea.*

= = Perigynia broadest near the middle, less than 2 mm. broad, very thin and conspicuously nerved, with short smooth beak: spikelets remote . . . . (29) *C. seorsa.*

* * Perigynia not thin-winged, ascending from the first, plano-convex.

+ Perigynia 4 mm. or more long, long-beaked.

Spikelets lanceolate, in a loosely linear-cylindric spike: perigynia 1 to 1.3 mm. broad, strongly nerved: scales oblong: leaves 1 to 2.5 mm. broad . . . . (33) *C. bromoides.*

Spikelets ovate, in flexuous spikes, the lowest very remote: perigynia 1.6 to 1.9 mm. broad, faintly nerved or nerveless: scales ovate: leaves 2 to 5 mm. broad . . . . (34) *C. Deweyana.*

++ ++ Perigynia less than 4 mm. long.

++ Perigynia 2 mm. or more long.

= Perigynia with serrulate beaks or margins.

a. Spike elongate, from linear-cylindric to oblong.

1\. Perigynia ovate, broadest at base: spikelets mostly or all approximate in an oblong-cylindric spike . . . (30) *C. arcta.*

2\. Perigynia broadest near the middle.

○ Plant glaucous: leaves 2 to 4 mm. broad: spikelets with many appressed-ascending glaucous obscurely beaked perigynia.

Spikelets 6 to 10 mm. long, approximate, or the lowest rarely 1.5 cm. apart: perigynia 2.3 to 3 mm. long (31) *C. canescens.*

Spikelets 4 to 7 mm. long, subapproximate or remote: perigynia about 2 mm. long (31) *C. canescens,* var. *subloliacea.*

Spikelets 6 to 12 mm. long, remote, the lowest 2 to 4 cm. apart . . . . (31) *C. canescens,* var. *disjuncta.*

○ ○ Plant green, not glaucous: leaves 1 to 2.5 mm. broad: spikelets with few loosely spreading dark green or brown distinctly beaked perigynia . . . . . . (32) *C. brunnescens.*

b. Spike subglobose, of 2 to 4 closely approximate subglobose loosely flowered silvery spikelets: perigynia oblong, beakless, nerved, 3 to 3.4 mm. long . . . . (35) *C. tenuiflora.*

= = Perigynia smooth throughout.

a. Spike whitish, silvery-green or pale brown, not ferrugineous nor dark brown.

1\. Spike elongate, at least the lower spikelets scattered.

Uppermost spikelet divaricate-pedunculate, the lowermost subtended by a long leaf-like bract: perigynia more than 3 mm. long . . . . (36) *C. trisperma.*

Spikelets continuous in a linear-cylindric loose spike, bractless or only short-bracted: perigynia 2 to 3 mm. long (31) *C. canescens.*

(For vars. see above.)

2\. Spike subglobose, of 2 to 4 closely approximate subglobose loosely flowered spikelets: perigynia beakless, 3 mm. or more long . . . . (35) *C. tenuiflora.*

b. Spike ferrugineous or dark brown.

1\. Terminal spikelet with conspicuous clavate base: perigynia abruptly beaked: culms smooth (or harsh only at tips).

○ Spikelets distinct; the lowest 4 or 5 mm. thick; the terminal 1 to 1.8 cm. long: perigynia pale, about equalled by the yellowish-brown blunt scales . . . . . . (38) *C. norvegica.*

○ ○ Spikelets approximate; the lowest less than 4 mm. thick.

Plant weak and lax: leaves involute, 0.5 to 1.5 mm. broad: perigynia pale, equalled by the ferrugineous acutish scales (39) *C. glareosa.*

Plant stiff and upright: leaves flat, 1 to 3 mm. broad: perigynia brown or reddish. exceeding the fuscous obtuse scales . . . . (40) *C. lagopina.*

2. Terminal spikelet without conspicuous clavate base: perigynia obscurely beaked, brown-tinged, exceeding the blunt scales: culms sharply angled, harsh and stiff: leaves flat, erect (41) *C. heleonastes.*

++ ++ Perigynia at most 1.5 mm. long, oblong-cylindric, plump, nerveless, beakless or with a very short broad truncate beak: culms wiry: spike linear-cylindric, dull brown . . . . (37) *C. elachycarpa.*

SYNOPSIS OF SPECIES.

Ovales, Kunth. *Perigynia ascending* or slightly spreading (when horizontally spreading, always with winged margins), *with thin or winged margins*, mostly *with concave inner faces* when mature.

§ OVALES proper. Bracts, when present, setaceous, or, if broader, only once to twice longer than the spike.

* Mature perigynia one-fourth to one-third (.24 to .36) as broad as long.

+ Perigynia extremely thin and scale-like, barely distended over the achenes.

++ Perigynia 7 to 10 (average 8.3) mm. long.

1. C. MUSKINGUMENSIS, Schweinitz. — Figs. 1, 2. — Culms 1 m. or less tall, very leafy: the loose flat *leaves subcordate* at their junction with the loose green sheaths; those of the sterile shoots crowded and almost distichous: spike oblong, of 5 to 12 appressed-ascending *oblong-cylindric pointed spikelets 1.5 to 2.5 cm. long.* — Ann. Lyc. N. Y. i. 66; Dewey, Am. Jour. Sci. x. 281; Bailey in Gray, Man. ed. 6, 620; Britton in Britton & Brown, Ill. Fl. i. 355, fig. 861. *C. arida*, Schwein. and Torr. Ann. Lyc. N. Y. i. 312, t. xxiv. fig. 2; Carey in Gray, Man. 545; Boott, Ill. i. 20, t. 54; Boeckeler, Linnaea, xxxix. 112; Bailey, Proc. Am. Acad. xxii. 147; Macoun, Cat. Can. Pl. ii. 129. *C. scoparia*, Torr. Ann. Lyc. N. Y. iii. 394, in part, not Schkuhr. *C. scoparia*, var. *muskingumensis*, Tuck. Enum. Meth. 8, 17. — Meadows, swamps, and wet woods, OHIO to MANITOBA and MISSOURI. July, August.

++ ++ Perigynia at most 6.5 (very rarely 7) mm. long.

= Perigynia 5 to 6.5 (average 5.7) mm. long.

2. C. SCOPARIA, Schkuhr. — Figs. 3, 4. — Culms 0.2 to 1 m. high, mostly slender and erect: *leaves narrow* (*at most 3 mm. wide*), shorter than the culm: *spike* oblong-ovoid to subcylindric, *of 3 to 9 straw-colored or brownish mostly shining and ascending* ovoid *pointed spikelets* 0.5 to 1.5 cm. long. — Schkuhr in Willd. Sp. iv. 230, & Riedgr.

Nachtr. 20, t. Xxx. fig. 175; Dewey, l. c. viii. 94; Schwein. & Torr. l. c. 313; Torr. l. c.; Carey, l. c.; Boott, l. c. iii. 116, t. 368, in part; Bailey, l. c. 148, & in Gray, l. c.; Macoun, l. c. 131; Britton, l. c. 356, fig. 863; Howe, 48 Rep. N. Y. Mus. Nat. Hist. 42. *C. leporina,* Mich. Fl. ii. 170, not L. *C. lagopodioides,* var. *scoparia,* Boeckeler, l. c. 114. — Low grounds or even dry open woods, NEWFOUNDLAND to SASKATCHEWAN and OREGON, and southward. May–August.

Var. MONILIFORMIS, Tuck. Spikelets scattered in a slender *moniliform spike,* the lowest usually remote. — Enum. Meth. 8, 17; Boott, Ill. l. c. t. 368, in part. *C. tribuloides,* var. *reducta,* Bailey, Proc. Am. Acad. xxii. 147, as to syn., in part. *C. tribuloides,* var. *moniliformis,* Britton, l. c. as to syn., in part. — Range of species, but infrequent.

Var. **condensa**. — Fig. 5. — *Spikelets spreading, crowded* in a short globose or broad-ovoid head. — NEW HAMPSHIRE, Randolph, July 23, 1897 (*E. F. Williams*): VERMONT, Westmore, July 26, 1894 (*E. F. Williams*); Rutland, July 14, 1899 (*W. W. Eggleston*): MASSACHUSETTS, Tewksbury, July 21, 1858, Medford, July 26, 1865, Mystic Pond, Aug. 9, 1868, and July 20, 1873 (*Wm. Boott*): RHODE ISLAND, Providence, July 19, 1871 (*S. T. Olney*): CONNECTICUTT, Griswold, June 16, 1899 (*C. B. Graves*, no. 150): NEW YORK, Jefferson Co. (*Crawe*); Fulton Chain Lakes, August, 1895 (*J. V. Haberer*): ONTARIO, Courtland, June 26, 1901 (*John Macoun,* Herb. Geol. Surv. Can., no. 26,631).

= = Perigynia 3.7 to barely 5 (average 4.5) mm. long.

3. C. TRIBULOIDES, Wahlenb. — Figs. 6, 7. — Culms loose and usually tall, 0.3 to 1 m. high, sharply trigonous: *leaves soft and loose, 3 to 8 mm. broad,* numerous; the upper often nearly or quite overtopping the culm; *those of the sterile shoots crowded and somewhat distichous: spike* oblong, *of 8 to 14 obovoid ascending more or less crowded gray-green or dull brown spikelets* 7 to 12 mm. long: *perigynia with appressed tips.* — Köngl. Acad. Handl. xxiv. 145, and Fl. Lapp. 250; Bailey, Proc. Am. Acad. l. c., in Gray, l. c., & Mem. Torr. Cl. i. 54; Macoun, l. c. 130; Britton, l. c. fig. 862; Howe, l. c. 41. *C. lagopodioides,* Schkuhr in Willd. l. c., & Riedgr. Nachtr. 20, t. Yyy, fig. 177; Dewey, l. c. 95; Schwein. & Torr. l. c.; Carey, l. c.; Boott, Ill. l. c. t. 370; Boeckeler, l. c. 113. *C. scoparia* var. *lagopodioides,* Torr. Ann. Lyc. N. Y. iii. 394; Tuck. ll. cc. — Swales and rich open woods, particularly in alluvial soil, NEW BRUNSWICK to SASKATCHEWAN, and southward. June–Sept.

Var. TURBATA, Bailey. Spikelets remote, forming a *moniliform spike.* — Mem. Torr. Cl. i. 55, & in Gray, Man, l. c. — *C. lagopodioides*, var. Boott, l. c. 117, t. 371, fig. 1. — Range of species.

Var. REDUCTA, Bailey. — Fig. 8. — *Spike* usually *flexuous*, at least the *lowest spikelets scattered : perigynia with loosely spreading or recurved tips.* — Proc. Am. Acad. l. c., Mem. Torr. Cl. i. 56, & in Gray, l. c.; Macoun, l. c.; Howe, l. c. 42. *C. cristata*, Kunze, Car. t. 44, fig. *g*; Boott, l. c. 117, in part, t. 373; not Schwein. *C. lagopodioides*, var. *moniliformis*, Olney, Exsicc. fasc. ii. no. 8; Bailey, Bot. Gaz. x. 380. *C. tribuloides*, var. *moniliformis*, Britton, l. c., not *C. scoparia*, var. *moniliformis*, Tuck. — GULF OF ST. LAWRENCE to NOVA SCOTIA, NEW ENGLAND, NEW YORK, IOWA, and western ONTARIO; ascending in the White Mts. to 1,385 m. altitude.

+ + Perigynia firm, not scale-like, obviously distended over the achenes.

++ Plant strongly stoloniferous; culms rising from an elongated rootstock.

4. C. SICCATA, Dewey. — Figs. 9 to 11. — Culms slender, 1 to 6 dm. high; *leaves stiff, 1 to 3 mm. wide : spike of 3 to 7* approximate or scattered, *glossy brown spikelets*, the *staminate and pistillate flowers variously mixed or in distinct spikelets :* perigynia 5 or 6 mm. long, 2 mm. broad, usually with distinct serrulate wings. — Am. Jour. Sci. x. 278, t. F. fig. 18; Hook. Fl. Bor.-Am. ii. 212; Torr. l. c. 391; Carey, l. c. 539; Boott, Ill. i. 19, t. 52; Boeckeler, l. c. 134; Bailey, Proc. Am. Acad. l. c. 147, & in Gray, l. c. 619; Macoun, l. c. 114; Britton, l. c. 355, fig. 860; Howe, l. c. 47; Meinsh. Acta Hort. Petrop. xviii. 319. *C. pallida*, C. A. Meyer, Mém. Acad. St. Petersb. i. 215, t. 8. *C. Liddoni*, Carey, l. c. 545, not Boott. — Dry or sandy soil, VERMONT to BRITISH COLUMBIA and ALASKA, south to MASSACHUSETTS, CONNECTICUT, NEW YORK, OHIO, MICHIGAN and westward. May–July.

++ ++ Plant not strongly stoloniferous, culms solitary or in dense stools.

= Perigynia at most 1.4 mm. wide, elongate-lanceolate or subulate, 3.5 to 4 (rarely 4.5) mm. long.

a. Tips of perigynia conspicuously exceeding the lance-subulate scales : plant comparatively low, in dense stools.

5. C. Crawfordii. — Figs. 12, 13. — Very *slender*, *1 to 3 dm. high ;* the narrow (1 to 2.5 mm. wide) leaves ascending, often equalling or exceeding the culms: *spike dull brown*, oblong or ovoid, *often subtended by an elongate-filiform bract ;* the 3 to 12 oblong or narrowly ovoid

ascending *spikelets 3 to 7 mm. long, approximate:* the linear-lanceolate perigynia plump at base, about 1 mm. wide. — *C. scoparia*, var. *minor*, Boott, Ill. iii. 116, t. 369; Gray, Man. ed. 5, 579; Bailey in Gray, Man. ed. 6, 621; Howe, l. c. 43. — Dry or rocky soil, or open woods. NEWFOUNDLAND, Whitbourne, Aug. 15, 1894 (*Robinson & Schrenk*, no. 94): PRINCE EDWARD ISLAND, Tignish, July 20, 1888 (*J. Macoun*, Herb. Geol. Surv. Can. no. 30,382): NEW BRUNSWICK, Nepisiquit Lakes, July, 1884 (*J. Brittain*, Herb. Geol. Surv. Canada, no. 30,377): QUEBEC, Rivière du Loup, Aug. 2, 1896, Lake Edward, Aug. 21, 1896, Tadousac, Aug. 26, 1896 (*Ezra Brainerd*); Roberval, July 27, 1892 (*G. G. Kennedy*): MANITOBA, Lake Winnipeg, July 29, 1884 (*John Macoun*, Herb. Geol. Surv. Can., no. 30,307, in part): ASSINIBOIA, Cypress Hills, June 25, 1894 (*J. Macoun*, Herb. Geol. Surv. Can., no. 7,461): SASKATCHEWAN, Carleton House and Bear Lake (*Sir John Richardson*): ATHABASCA (*Sir John Richardson*, Herb. Geol. Surv. Can. no. 30, 396): MAINE, Van Buren, July 25, 1893 (*M. L. Fernald*, no. 163); St. Francis, Aug. 7, 1893, Farmington, July 8, 1896 (*M. L. Fernald*); Beech Mt., Mount Desert Island, Aug. 20, 1890, Somesville, July 5, 1891, Southwest Harbor, Aug. 1, 1892, Little Cranberry Isle, July 10, 1894, Seal Harbor, July 5, 1897 (*E. L. Rand*); Gilead, Aug., 1897 (*Kate Furbish*): NEW HAMPSHIRE, Randolph, July 23, 1897 (*E. F. Williams*); near Crawfords, July 6, 1878, Mt. Washington, July 29, 1887, Franconia, July 6, 1878 (*E. & C. E. Faxon*); Crawford Notch, Aug. 24, 1891, Aug. 13, 1897, and Lebanon, July 22, 1890 (*G. G. Kennedy*): VERMONT, Mt. Mansfield, July 24, 1884 (*C. W. Swan*), Sept. 9, 1897 (*E. Brainerd*); Willoughby, July 21, 1896 (*G. G. Kennedy*); Middlebury, July 11, 1896, Ripton, July 19, 1898 (*E. Brainerd*); Rutland, July 1, 1899 (*W. W. Eggleston*): MASSACHUSETTS, Malden and Revere, June 21, 1879 (*H. A. Young*); Chelsea, July 19, 1891 (*W. P. Rich*): MICHIGAN, Houghton, Sept. 15, 1871 (*H. Gillman*); Keweenaw Co., Sept., 1888 (*O. A. Farwell*).

Var. **vigens**. — Fig. 14. — Stouter throughout: culms 3 to 6 dm. high: *leaves 2.5 to 3 mm. broad: spikelets mostly greener, 8 to 11 mm. long, densely crowded* in a broad-ovoid to globose head. — Thickets and damp gravelly soil. NEW BRUNSWICK, Campbellton, July 20, 1880 (*R. Chalmers*, Herb. Geol. Surv. Can. no. 30,363): QUEBEC, Gaspé, Aug. 1, 1882 (*John Macoun*); Rivière du Loup, July 20 and Aug. 4, 1896, Lake Edward, Aug. 21, 1896 (*Ezra Brainerd*): ONTARIO, Eastmans Springs, Sept. 16, 1892 (*J. Macoun*, Herb. Geol. Surv. Can. no. 30, 386); Cache Lake, July 11, 1900 (*John Macoun*): SASKATCHEWAN, plains, Aug. 1, 1872 (*J. Macoun*): BRITISH COLUMBIA, Nelson, Kootenay Lake, July 3, 1890 (*J. Macoun*, Herb. Geol. Surv. Can., no. 30, 393): MAINE, St. Francis, Aug. 9, 1893, Sherman, Aug. 23, 1897 (*M. L. Fernald*): NEW HAMPSHIRE, Randolph, Aug. 2, 1897 (*E. F. Williams*); Mt. Washington, July 28, 1861 (*Wm. Boott*); Mt. Pleasant House, July 31, 1897 (*W. Deane*): VERMONT, Burlington, July 13, 1896 (*E. Brainerd*): MICHIGAN, Keweenaw Co., Aug., 1890 (*O. A. Farwell*).

b. Tips of perigynia mostly equalled by the ovate blunt or acutish scales: plant tall, forming loose stools.

6. **C. oronensis.** — Figs. 15, 16. — Culms *tall* and erect, 0.5 to 1 m. high, sharply angled and harsh above: leaves smooth, 2.5 to 4 mm. broad, much shorter than the culms: *spike oblong-cylindric*, *erect*, of 3 to 9 ascending *dark brown rhomboid-ovoid pointed spikelets* 0.5 to 1 cm. long: *scales* mostly *glossy brown, with pale scarious margins: perigynia appressed, about 4 mm. long, 1.3 mm. broad,* very narrowly winged above. — Dry fields, thickets, open woods, and gravelly banks. MAINE, Orono, about 1870 (*F. Lamson-Scribner*), June 28, 1890, June 30, 1891, July 3, 1897 (M. L. Fernald).

= = Perigynia 1.5 to to 2 mm. broad, ovate-lanceolate, 4.5 to 6.5 (average 5) mm. long.

7. C. PRATICOLA, Rydberg. — Figs. 17, 18. — Culms smooth and slender, 3 to 6 dm. high, overtopping the smoothish flat (2 to 3.5 mm. broad) leaves; *spike slender, flexuous, moniliform, the 3 to 7 silvery brown mostly remote pointed spikelets few-flowered,* 7 to 1.7 mm. long, mostly *long-clavate at base;* perigynia nerveless or minutely short-nerved on the inner face, equalling the ovate-lanceolate acutish or blunt scales. — Mem. N. Y. Bot. Gard. i. 84; Britton, Man. 226. *C. pratensis*, Drejer, Rev. Crit. Car. Bor. 24; Fl. Dan. xiv. 8, t. 2368; Bailey, Proc. Am. Acad. xxii. 147; Britton, in Britt. & Brown, l. c. 354. fig. 858; not Hose. *C. adusta*, var. *minor*, Boott in Hook. Fl. Bor.-Am. ii. 215, & Ill. iii. 119, t. 383. *C. Liddoni*, in part, of authors, not Boott. — Open woods, clearings, and prairies, LABRADOR to SASKATCHEWAN and BRITISH COLUMBIA, south to NOVA SCOTIA, Aroostook County, MAINE, LAKE SUPERIOR, and NORTH DAKOTA; also in GREENLAND. June–Aug.

* * Mature perigynia distinctly more than one-third (.44 to .75) as broad as long.

+ Perigynia one-fifth to one-third (.19 to .34) as thick as broad (rarely thicker in *C. mirabilis*).

++ Mature perigynia 3 to 4 mm. long (very rarely longer in *C. mirabilis* and *C. albolutescens*).

= Mature perigynia with roseate-spreading tips.

8. C. CRISTATA, Schweinitz. — Figs. 19 to 21. — *Culms* 1 m. or less high, *harsh above: leaves soft and flat, 3 to 7 mm. broad*, often equalling the culms, *sheaths loose: spike* usually *dense, linear-cylindric or oblong*, of 6 to 15 *globose closely flowered* greenish or dull-brown *spikelets* 0.5 to 1 cm. long. — Ann. Lyc. N. Y. i. 66; Schwein. & Torr. Ann. Lyc. N. Y. i. 315, t. 24, fig. 1; Dewey, l. c. 44; Boott, l. c. 117, in part; Gray, Man. ed. 5, 579; Boeckeler, l. c. 115; Howe, l. c. 41. *C. lagopodioides*, var. *cristata*, Carey, l. c. 545. *C. straminea*, var. *cristata*, Tuck. l. c. 9, 18. *C. tribuloides*, var. *cristata*, Bailey, Proc. Am. Acad. xxii. 148, in Gray, Man. ed. 6, 620, & Mem. Torr. Cl. i. 55; Macoun, l. c. 130. *C. cristatella*, Britton, l. c. 357, fig. 865. — Swales and wet woods, western NEW ENGLAND to PENNSYLVANIA, "VIRGINIA," MISSOURI, SASKATCHEWAN, and BRITISH COLUMBIA. June–Aug.

= = Mature perigynia with ascending tips.

a. Plant stout and stiff: spikes stiff and upright: the gray-green mostly approximate spikelets with appressed firm perigynia.

9. C. ALBOLUTESCENS, Schweinitz. — Figs. 22 to 24. — Culms 2 to 8 dm. high: *leaves* erect, long-pointed, *pale green*, 2 to 5 mm. wide, shorter than the culms: spike linear-cylindric to subglobose, with or without elongated bracts, of 3 to 30 (sometimes compound) conic-ovoid to subglobose spikelets 0.6 to 1 cm. long: *perigynia 2 to 3 mm. broad, rhombic-ovate to suborbicular, with a short deltoid firm greenish tip.* — Ann. Lyc. N. Y. i. 66; Bailey, Bull. Torr. Cl. xx. 422 (incl. var. *cumulata*); Britton, l. c. 359, fig. 873; Howe, l. c. 43. *C. foenea*, Ell. Sk. ii. 533; Schwein. & Torr. l. c. 315; Carey, l. c. 546; Boott, l. c. 118 (excl. vars.), t. 375; not Willd. *C. straminea*, var. *foenea*, Torr. Ann. Lyc. N. Y. iii. 395; Bailey, Proc. Am. Acad. xxii. 150, & in Gray, Man. ed. 6, 622; Macoun, l. c. 132. *C. straminea*, var. *intermedia*, Gay, Ann. Sci. Nat., ser. 2, x. 364. *C. leporina*, var. *bracteata*, Liebmann, Mex. Halv. 76. *C. straminea*, var. *chlorostachys*, Boeckeler, l. c. 118. *C. straminea*, var. *cumulata*, Bailey, Mem. Torr.

Cl. i. 23, & in Gray, l. c. — Damp or even very dry soil, principally on the coastal plain, NEW BRUNSWICK to FLORIDA, TEXAS, MEXICO, and CENTRAL AMERICA; rarely inland to Bear Mt., Livermore, MAINE (*Kate Furbish*); Mt. Monadnock, alt. 925 m., NEW HAMPSHIRE (*R. M. Harper*); Taghkanick Range, Columbia Co., NEW YORK (*L. H. Hoysradt*); also from LAKE HURON to MANITOBA. July–Sept.

b. Plant not very stiff: the bright green or brownish spikelets with spreading or ascending (not appressed) perigynia.

1. Leaves 2.5 to 6 mm. wide: culms 0.3 to 1.5 m. high.

10. C. MIRABILIS, Dewey. — Figs. 25, 26. — *Culms very loose and smooth;* leaves soft and thin, the sheaths rather loose: *spikelets* 4 to 12, greenish, subglobose or ovoid, 5 to 9 mm. long, *mostly approximate in an oblong spike;* perigynia with divergent tips. — Am. Jour. Sci. xxx. 63, t. Bb, fig, 92; Boott, l. c. 117 (under *C. cristata*), t. 374; Howe, l. c. 46. *C. straminea*, var. *mirabilis*, Tuck. l. c. 9, 18; Bailey, Proc. Am. Acad. xxii. 150, & in Gray, Man. ed. 6, 621; Britton, l. c. 358. *C. festucacea*, var. *mirabilis*, Carey, l. c. 545. *C. cristata*, Kunze, Car. t. 44, figs. *a*, *e*, and *f* (colored), not Schwein. *C. cristata*, var. *mirabilis*, Gray, Man. ed. 5, 580. *C. lagopodioides*, var. *mirabilis*, Olney, Exsicc. fasc. ii, no. 9. *C. tribuloides*, var. *cristata*, Macoun, l. c. 130, in part, not Bailey. — Dry banks, open woods, or even moist copses, central MAINE to MANITOBA, south to NORTH CAROLINA and MISSOURI. June, July.

Var. **perlonga**. — Fig. 27. — *Spikelets scattered in a moniliform spike.* — NEW HAMPSHIRE, dry thicket, Barrett Mt., New Ipswich, June 5, 1896 (*M. L. Fernald*): VERMONT, Little Notch, July 9, 1901 (*E. Brainerd*): MASSACHUSETTS, Stoneham, June 5, 1887 (*F. S. Collins*); Oak Island, Revere, July 5, 1891 (*W. P. Rich*); Beaver Brook Reservation, July 6, 1894 (*C. W. Swan*); Sharon, June 17, 1896 (*W. P. Rich*): CONNECTICUT, dry open woods, Southington, June 17, 1900 (*C. H. Bissell*): NEW YORK, Binghamton, June 29, 1871 (*Wm. Boott*); Sacondago River (*J. V. Haberer*): MICHIGAN, Grosse Isle, June 30, 1867 (*Wm. Boott*); open swales, Lansing, June 8, 1886 (*L. H. Bailey*, no. 283, in part): ILLINOIS, Marion Co. (*M. S. Bebb*).

Var. **tincta**. *Spike of 3 to 7 ovoid approximate brown-tinged spikelets: scale brown* with a pale margin. — NEW BRUNSWICK, banks of St. John River, July 4, 1899 (*J. Macoun*, Herb. Geol. Surv. Can. no. 22): MAINE, Fort Kent, June 16, 1898 (no. 2158), Masardis, June 6, 1898 (no. 2159), Ashland, June 13, 1898 (no. 2160), Fort Fairfield, July 12, 1893 (no. 165), Foxcroft, June 25, 1894, Dover, June 28, 1894, Orono, July 6, 1891, — all coll. *M. L. Fernald;* Sangerville, July 17, 1896 (*G. B. Fernald*, no. 176): NEW HAMPSHIRE, between Marshfield and Fabyans, July 6, 1878, Bethlehem, June 20, 1887 (*E. & C. E. Faxon*); Whitefield, July 3, 1896 (*W. Deane*): VERMONT, St. Johnsbury, June 21, 1901 (*T. E. Hazen*, no. 206). Resembling northwestern forms of the polymorphous *festiva* group, but not satisfactorily referable to any of them.

2. Leaves 0.5 to 2 mm. wide: culms 3 to 7 dm. high: spikelets remote or at least distinct in a moniliform or linear-cylindric spike.

11. C. STRAMINEA, Willd. — Figs. 28, 29. — *Culms very slender*, smooth except at summit: *spikelets* 3 to 8, yellow-brown, or rarely green, ovoid or subglobose, 4 to 8 mm. long, *usually forming flexuous spikes: perigynia with ascending inconspicuous tips; the inner faces 3- to 5-nerved or nerveless.* — Willd. in Schkuhr, Riedgr. 49, t. G, fig. 34; Bailey, Mem. Torr. Cl. i. 21, & in Gray, Man. ed. 6, 621; Britton, l. c. fig. 868; Howe, l. c. 44. *C. straminea*, var. *minor*, Dewey, Am. Jour. Sci. xi. 318, t. N, fig. 45; Torr. l. c. 395. *C. festucacea*, var. *tenera*, Carey, l. c. 545. *C. straminea*, var. *tenera*, Boott, l. c. 120, t. 384 (except perigynia from Olney); Gray, Man. ed. 5, 580; Macoun, l. c. 132. — Meadows, or occasionally on dry banks or in open woods, NEW ENGLAND to BRITISH COLUMBIA, KENTUCKY and ARKANSAS. June–Aug.

Var. echinodes. — Fig. 30. — *Tips of the slightly longer perigynia divergent and conspicuous.* — ONTARIO, Wyoming, June 24, 1901 (*J. Macoun*, Herb. Geol. Surv. Can., no. 26,624): MICHIGAN, Detroit, July 20, 1867 (*H. P. Sartwell*), June 26, 1870, and June 22, 1873 (*Wm. Boott*): IOWA, Ames, 1872, Spirit Lake, June 21, 1881 (*J. C. Arthur*). Superficially resembling *C. tribuloides*, var. *reducta.*

⊷ ⊷ Mature perigynia more than 4 mm. long (very rarely shorter in exceptional individuals of *C. tenera*, var. *invisa*, and *C. festucacea*, var. *brevior*).

= Perigynia elongate-ovate, about half as broad as long (suborbicular in var. *Richii*).

12. C. TENERA, Dewey. — Figs. 31, 32. — Culms slender and flexuous, sharply angled, smooth except at summit, 3 to 9 dm. high: leaves shorter than or rarely exceeding the culms, very ascending, 1 to 2.5 mm. broad: *spike slender, moniliform* (or on late culms more or less congested), *of 3 to 9 broadly ovoid brownish spikelets 8 to 12 mm. long*, with

or without subtending elongated bracts: *perigynia* ascending or rarely spreading, *distinctly about 10-nerved on either face, 4.8 to 6 (average 5.2) mm. long:* scales lance-attenuate or aristate. — Am. Jour. Sci. viii. 97, & ix. t. C, fig. 9; Britton, l. c. fig. 870. *C. straminea*, var. *aperta*, Boott, l. c. 120, t. 385; Gray, Man. ed. 5, 580; Bailey, Proc. Am. Acad. xxii. 152, & in Gray, Man. ed. 6, 622; Maconn, l. c. 133; Howe, l. c. 45. *C. tenera*, var. *major*, Olney, l. c. no. 15. *C. straminea*, var. *tenera*, Bailey, Bot. Gaz. x. 381, & Mem. Torr. Cl. v. 94. — Brackish or fresh marshes, mostly near the coast, GULF OF ST. LAWRENCE to DELAWARE and IOWA; also in BRITISH COLUMBIA, Yellow Head Pass (*Spreadborough*, Herb. Geol. Surv. Can. no. 20,871). June–Aug.

Var. INVISA, Britton. — Figs. 35, 36. — Lower; with *spikelets 5 to 8 mm. long*, and perigynia 4 to 5 (average 4.5) mm. long. — Britton, l. c. 358. *C. straminea*, var. *invisa*, W. Boott, Bot. Gaz. ix. 86; Bailey, Proc. Am. Acad. xxii. 152, & in Gray, Man. ed. 6, 622; Howe, l. c. — Range of the species and too often intergrading with it; mostly in dry soil or even in pure sand.

Var. **Richii**. — Figs. 33, 34. — *Perigynia* 4 to 5 mm. long, *with suborbicular bodies* abruptly contracted to slender *conspicuous loosely ascending or spreading beaks.* — MASSACHUSETTS, Reading, June 14, 1883 (*C. E. Perkins*); Fresh Pond, Cambridge, June 8, 1887 (*W. Deane*); near Spot Pond, and north end of Doleful Pond, Stoneham, May 30, 1894, near Bear Hill, Stoneham, June 5, 1894 (*Wm. P. Rich*); Amherst (*E. Tuckerman*): CONNECTICUT, Newington, May, 1879 (*Chas. Wright*). In its elongate loose brown spikes and subulate- or awn-tipped narrow scales clearly an extreme form of *C. tenera*, although the perigynia when well developed suggest those of *C. festucacea.*

= = Perigynia with broadly ovate to suborbicular bodies, more than half as broad as long.

a. Perigynia 5.7 to 7.7 mm. long.

13. C. BICKNELLII, Britton. — Figs. 37 to 40. — Culms comparatively stout, 4 to 9 dm. high, smooth except at summit: leaves ascending, rather short and firm, 2 to 4.5 mm. broad: *spike of 3 to 7 silvery brown or greenish* ovoid obovoid or subglobose *approximate or slightly remote spikelets* 8 to 14 mm. long: *perigynia ascending, the tips becoming conspicuous, broadly wing-margined, when mature almost translucent and with about 10 nerves on either face.* — Britton, l. c. 360, fig. 874. *C. straminea*, var. *Crawei*, Boott, l. c. 121, t. 388; Bailey, Bull. Torr. Cl. xx. 422; Howe, l. c. *C. straminea*, var. *Meadei*, Boott, l. c. t. 389; Gray,

Man. ed. 5, 581. *C. straminea*. var. *brevior*, Bailey, Mem. Torr. Cl. i. 22, in part, not Dewey. — Dry or rocky soil, eastern MASSACHUSETTS to MANITOBA, NEW JERSEY, OHIO, and ARKANSAS. May–July.

b. Perigynia at most 5.5 mm. long.

1. Spikelets whitish or silvery-brown, mostly scattered in a flexuous moniliform spike.

14. C. SILICEA, Olney. — Figs. 41, 42. — *Culms slender, stiff*, smooth except at summit, 3 to 8 dm. high: *leaves erectish*, shorter than or equalling the culms, *usually glaucous*, 2 to 4.5 mm. wide, often becoming involute: *spike of 3 to 12 usually remote conic-ovoid usually clavate spikelets* 1 to 1.5 cm. long: *perigynia strongly appressed, firm and opaque*, 4 to 5 mm. long, 2.2 to 3 mm. broad, *short-beaked, broad-winged, the body distinctly 3- to 5-nerved on the inner, 6- to 12-nerved on the outer face.* — Proc. Am. Acad. vii. 393; Bailey, Mem. Torr. Cl. i. 24, & in Gray, Man. ed. 6, 621; Britton, l. c. 358, fig. 869; Howe, l. c. 44. *C. straminea*, var. *moniliformis*, Tuck. l. c. 9, 17; Bailey, Proc. Am. Acad. xxii. 151; Macoun, l. c. 133. *C. adusta*, Carey in Gray, Man. ed. 2, 516, not Boott. *C. foenea*, var. γ, Boott, l. c. 118, t. 377. *C. foenea*, var. (?) *subulonum*, Gray, Man. ed. 5, 580. *C. straminea*, var. *silicea*, Bailey, Carex Cat. 4. — Sand and rocks near the sea, PRINCE EDWARD ISLAND to NEW JERSEY. June–Aug.

2. Spikelets green or brownish, approximate or only slightly remote in a mostly upright spike (*C. tenera*, var. *Richii*, with moniliform flexuous spikes might be looked for here).

○ Sheath of the leaf green and strongly nerved nearly or quite to the narrow subchartaceous auricle: perigynia appressed-ascending: achenes mostly oblong.

15. C. ALATA, Torr. — Figs. 43, 44. — Culms rather stout, smooth except at summit, 0.5 to 1 m. high: *leaves* mostly short and *harsh*, 2.5 to 4.5 mm. wide: *spike oblong or ovoid, of 3 to 8 compact green* or finally dull-brown *conic-ovoid to oblong spikelets* 8 to 15 mm. long: *perigynia firm and opaque, orbicular or obovate*, 4.3 to 5.5 mm. long, *2.8 to 3.7 mm. broad, broad-winged, very faintly nerved or nerveless, much broader than the lance-subulate usually rough-awned scales.* — Ann. Lyc. N. Y. iii. 396; Boott. l. c. 118, t. 378; Gray, Man. ed. 5, 581; Britton, l. c. 359, fig. 872; Howe. l. c. 45. *C. straminea*, var. *alata*, Bailey, Carex Cat. 4, Proc. Am. Acad. xxii. 150 & in Gray, Man. ed. 6, 622. — Marshes and wet woods, NEW HAMPSHIRE to MICHIGAN and FLORIDA, mostly near the coast. June, July.

Var. **ferruginea.** — Figs. 45, 46. — Slender: *the 3 to 5 irregularly clustered spikelets tawny or ferrugineous* from the first: *perigynia ovate*, 4 to 5 mm. long, *2.3 to 2.8 mm. broad: scales lance-ovate, mostly awnless.* — *C. foenea*, var. *β*, Boott, l. c. 118, t. 376. *C. foenea*, var.? *ferruginea*, Gray, Man. ed. 5, 580. *C. tenera*, var. *suberecta*, Olney, Exsicc. fasc. ii. no. 16. *C. straminea*, var. *ferruginea*, Bailey, Bull. Torr. Cl. xx. 421.— Ohio to Michigan, Illinois, and Iowa.

○ ○ Sheath with a thin barely nerved or nerveless pale band extending down from the membranaceous auricle: perigynia spreading-ascending: achenes suborbicular.

16. C. festucacea, Schkuhr. — Figs. 47, 48. — *Culms stiff, 0.5 to 1 m. high: leaves stiff*, erect, shorter than the culms, 2 to 4 mm. wide: *spike* narrowly oblong, rarely ovoid, *of 5 to 10 distinct or rarely approximate subglobose or broadly ovoid-conic yellow-brown or green-brown ascending spikelets* 7 to 12 mm. long: *perigynia* broad-ovate to suborbicular, 4 to 5.5 mm. long, 2.7 to 3.5 mm. broad, *strongly 7- to 15-nerved on the outer*, nerveless or faintly nerved on the inner *face*: scales blunt. — Schkuhr in Willd. Sp. iv. 242, & Riedgr. Nachtr. 23, t. Www. fig. 173; Dewey, Am. Jour. Sci. viii. 96; Schwein. & Torr. l. c. 316; Torr. l. c. 394; Carey, l. c. 545; Britton, l. c. 359, in part. *C. straminea*, var. *festucacea*, Tuck. l. c. 9, 18; Boott, l. c. 120, t. 386; Macoun, l. c. 132; Bailey, Mem. Torr. Cl. v. 94, in part; Howe, l. c. *C. straminea*, Bailey, Proc. Am. Acad. xxii. 149, in part, not Willd. *C. straminea*, var. *brevior*, Bailey, Mem. Torr. Cl. i. 22, in part, not Dewey. — Dry or rocky soil, Maine to Manitoba and Pennsylvania. June–Aug.

Var. **brevior.** — Figs. 49 to 51. — *Lower* (rarely more than 0.6 m. high), *and more slender: spikelets 3 to 6, approximate or subapproximate.* — *C. straminea*, Schkuhr, Riedg. Nachtr. 23, t. Xxx. fig. 174; Schwein. & Torr. l. c. 314; Carey, l. c. 546; Torr. l. c. 395; Boott, l. c. 121, t. 387; Bailey, Proc. Am. Acad. xxii, 149, in part; not Willd. in Schkuhr, Riedgr. 49, t. G. fig. 34, & in herb. *C. straminea*, var. *brevior*, Dewey, Am. Jour. Sci. xi. 158; Bailey, Mem. Torr. Cl. i. 22, in part, & in Gray, Man. ed. 6, 622; Howe, l. c. *C. straminea*, var. *Schkuhrii*, Gay, Ann. Sci. Nat. ser. 2, x. 363; Tuck. l. c. 8, 17. *C. straminea*, var. *typica*, Gray, Man. ed. 5, 580; Macoun, l. c. 131. *C. festucacea*, Britton, l. c. 359, in part (including fig. 871), not Schkuhr.— Commoner than the species, extending to British Columbia, Arkansas, &c. May–July.

+ + Perigynia two-fifths to one-half (.40 to .50) as thick as broad.

++ Tips of the perigynia distinctly exceeding the scales: spikes short, compact, ovoid or short-oblong, brown: perigynia 3 to 3.5 mm. long.

17. C. Bebbii, Olney. — Figs. 52, 53. — Culms rather slender, 2 to 6 dm. high, smooth except at tip: *leaves* mostly shorter, *ascending but not stiff*, 1.75 to 4.5 mm. wide: *spikes 1 to 2 cm. long*, of 3 to 12 globose or ovoid-oblong ascending *spikelets 5 to 8 mm. long: perigynia* narrowly ovate, *mostly dull brown and loosely ascending, faintly few-nerved or nerveless, 1.5 to 2 mm. broad:* scale oblong, bluntly acuminate. — Exsicc. fasc. ii, no. 12, as *nomen nudum*. *C. tribuloides*, var. *Bebbii*, Bailey, Mem. Torr. Cl. i, 55 & in Gray, Man. ed. 6, 620; Britton, l. c. 356; Howe, l. c. 42; Cratty, Bull. Lab. Nat. Hist. Univ. Ia., iv. 359, t. 8. — Low ground, Newfoundland to western Massachusetts, central New York, Illinois, the Rocky Mts., British Columbia, and northward. June–Aug.

++ ++ Tips of the perigynia nearly or quite equalled by the scales: perigynia more than 3.5 mm. long (sometimes shorter in the *slender-spiked silvery green C. foenea*).

= Perigynia with strong nerves the entire length of the inner face (very rarely nerveless).

a Spike loose and elongated, green or silvery-brown.

18. C. foenea, Willd. — Figs. 54, 55. — *Culms slender and lax*, smooth except at tip, *3 to 9 dm. high: leaves soft and loose, pale green or glaucous*, mostly shorter, 2 to 4 mm. broad: *spike linear-cylindric or moniliform, erect or flexuous, of 4 to 9* globose or ovoid clavate-narrowed appressed-ascending *spikelets 6 to 10 mm. long: perigynia ovate, 3 to 4 mm. long*, 1.8 to 2.2 mm. broad, appressed-ascending, finally a little spreading. — Enum. 957; Bailey, Mem. Torr. Cl. i. 25, & in Gray, Man. ed. 6, 621; Macoun. l. c. 377; Britton l. c. 357, fig. 867; Howe, l. c. 43. *C. argyrantha*, Tuck. in Herb. distr. (1859). *C. adusta*, Boott, l. c. 119, in part, t. 382, fig. 2, not Boott in Hook. Fl. Bor.-Am. ii. 215. *C. albolutescens*, var. *argyrantha*. Olney, Exsicc. fasc. i. no. 9. *C. adusta*, var. *argyrantha*, Bailey, Carex Cat. 2. — Dry woods and rocky banks, Maine to British Columbia and Maryland. July.

Var. perplexa, Bailey. — Figs. 56, 57. — *Coarser*, and often taller: *spikes heavier, mostly nodding, the 6 to 15 spikelets larger, 1 to 1.7 cm. long, the terminal ones often crowded: perigynia 3.5 to 4.4 mm. long.* — Mem. Torr. Cl. i. 27, in part, & in Gray, Man. Ed. 6, 621; Britton,

l. c.; Howe, l. c. 44. *C. adusta*, Boott, Ill. iii. 119, in part, t. 381, 382, fig. 1; Gray, Man. ed. 5, 580; Macoun, l. c. 129, in part (excl. syn.) — Commoner than the species. NEWFOUNDLAND to MANITOBA and VIRGINIA. June–Aug.

b. Spike with approximate or subapproximate brown or ferrugineous spikelets.

19. C. LEPORINA, L. — Figs. 58 to 60. — Culms stiff and ascending, 2 to 8 dm. high: leaves mostly short and rather firm, 1.5 to 4 mm. broad: spike from subglobose to cylindric, of 3 to 6 obovoid to oblong-ovoid ascending spikelets 8 to 1.4 mm. long: perigynia 3.8 to 4.5 mm. long, 1.8 to 2.3 mm. broad, ascending. — Sp. 973, & Fl. Suec. ed. 2, 326 (excl. cit. Fl. Lapp.); Wahl. Fl. Lapp. 228; Reich. Ic. Fl. Germ. viii. t. 211; Anders. Cyp. Scand. 63, t. 4, fig. 26; Boott, l. c. iv. 190; Bailey, Proc. Am. Acad. xxii. 152, & in Gray, Man. ed. 6, 622; Britton, l. c. 356, fig. 861; Meinsh. Acta Hort. Petrop. xviii. 324. *C. ovalis*, Good. Trans. Linn. Soc. ii. 148; Eng. Bot. t. 306; Vahl. Fl. Dan. vii. t. 1115; Host, Gram. i. 39, t. 51; Willd. l. c. 955; Schkuhr, l. c. 20, t. B, fig. 8. — EUROPE and ASIA: NEWFOUNDLAND, shores of Quiddy Viddy Lake, Aug. 2, 1894 (*Robinson & Schrenk*): NOVA SCOTIA, Yarmouth, July 22, 1896 (*E. Brainerd*): MAINE, low, rocky pasture, South Berwick, June 23, 1898 (*J. C. Parlin*, no. 959); hillside pastures, East Parsonsfield, July 4, 1900 (*J. F. Collins & M. L. Fernald*): NEW HAMPSHIRE, dry hillsides, Alstead, July 9, 1901 (*M. L. Fernald*); Gap Mt., Troy, June 13, 1898 (*E. L. Rand & B. L. Robinson*, no. 508): MASSACHUSETTS, Essex Co., Aug. 23, 1881 (*W. P. Conant*); Long Island, Boston Harbor, July 6, 1871, July 1, 1873 (*Wm. Boott*); Nobscot Hill, Framingham, June 14, 1901 (*M. L. Fernald*); Purgatory Swamp, Dedham, June 23, 1878 (*E. & C. E. Faxon*): NEW YORK, slopes of Bald Mt., north of Fulton Chain, Herkimer & Hamilton Cos., Aug. 12, 1895 (*J. V. Haberer*, no. 1103) NEW JERSEY, ballast ground, Camden, 1878 (*Isaac Burk*). Doubtless introduced at the latter station, but perhaps indigenous northward.

= = Inner face of perigynia nerveless or only slightly nerved at base (exceptional individuals of *C. leporina* might be sought here).

a. Ellipsoidal spikelets brownish-white: the appressed perigynia golden-yellow at base.

20. C. XERANTICA, Bailey. — Figs. 61, 62. — *Culms stiff*, scabrous above, 3 to 6 dm. high: *leaves short, mostly near the base*, 2 or 3 mm. broad: *spike linear-cylindric, of 3 to 6 distinct ascending spikelets* 8 to

13 mm. long: perigynia 4 to 4.8 mm. long, 2 to 2.3 mm. broad. — Bot. Gaz. xvii. 151; Britton, l. c. 355, fig. 859. — Open prairies, western MANITOBA and adjacent ASSINIBOIA. July.

b. Obovoid spikelets brownish or ferrugineous: the loosely ascending perigynia dark green or brown when mature.

1. Spike loose and flexuous; spikelets mostly long-clavate at base, the lowest remote: achene 1.5 (1.3 to 1.7) mm. broad.

21. **C. aenea.** — Figs. 63 to 66. — *Culms* smooth and wiry, but *more or less flexuous at tip*, 0.25 to 1.2 m. high: leaves much shorter, rather soft and flat, 2 to 4 mm. broad: *spike loosely cylindric or moniliform*, of 3 to 12 *spikelets 0.8 to 2.5 cm. long* (in luxuriant plants often peduncled or compound): perigynium 4 to 5 mm. long, 1.9 to 2.7 mm. broad. — *C. adusta*, Boott, l. c. iii. 119, in part, t. 380, not Boott in Hook. Fl. Bor.-Am. ii. 215. *C. albolutescens*, var. *sparsiflora*, Olney, fasc. v. no. 11, in part (as *nomen nudum*), not *C. sparsiflora* Fries. *C. adusta*, var. *sparsiflora*, Bailey, Carex Cat. 2 (as *nomen nudum*)? *C. foenea*, var. *perplexa*, Bailey, Mem. Torr. Cl. i. 27, as to syn., in part. *C. foenea*, var. *sparsiflora*, Howe, l. c. 44. — Open woods, dry banks, or rarely in low ground. LABRADOR, without station, Aug. 23, 1896 (*Spreadborough*, Herb. Geol. Surv. Can. no. 13,354): UNGAVA, East Main R., 1892 (*A. H. D. Ross*, Herb. Geol. Surv. Can. no. 30,582): NEWFOUNDLAND, Grand Lake, Bay of Islands, Aug. 6, 1897 (*A. C. Waghorne*): QUEBEC, Rivière du Loup, July 23, 1861 (*Wm. Boott*), Aug. 2, 1896 (*E. Brainerd*): Calumet, June, 1891 (*J. M. Macoun*, Hb. Geol. Surv. Can. no. 16,535): NEW BRUNSWICK, Kent Co. (*J. Fowler*, in Olney, Exsicc. fasc. v. no. 11, in part): MAINE, Fort Fairfield, 1881 (*Kate Furbish*); Milford, June 30, 1864 (*J. Blake*); Orono, July 7, 1892, July 3, 1897, June 8, 1901 (*M. L. Fernald*); Mt. Desert Island, numerous stations (*Rand*, *Faxon*, *Williams et al.*): NEW HAMPSHIRE, Franconia, June 23, 1888 (*E. & C. E. Faxon*); Crawford Notch, July 16, 1895 (*G. G. Kennedy*): VERMONT, East Mt., Middlebury, June 23, 1882, Moosalamoo Mt., Salisbury, July 5, 1901 (*E. Brainerd*); MASSACHUSETTS, Mt. Wachusett, June 27, 1878 (*Wm. Boott*): NEW YORK, base of Stony Creek Mt., June 29, 1899 (*Rowlee, Wiegand & Hastings*): ONTARIO, near Michipicoten, July 26, 1869, Cache Lake, July 12, 14, 1900 (*J. Macoun*); Lake Victoria, Sept. 12, 1901 (*E. Brainerd*): MICHIGAN, Isle Royale (*H. Gillman*): ATHABASCA, Methy Portage (*Sir John Richardson*, fide Boott, Ill. t. 380): ALBERTA, Banff, Rocky Mts., July 10, 1891 (*J. Macoun*, Herb.

Geol. Surv. Can. no. 16, 536): British Columbia, Beaver Creek, Selkirk Mts., July 13, 1885 (no. 10,797); Kicking Horse Lake, Aug. 11, 1890 (no. 30,603); Revelstoke, May 19, 1890 (no. 30,604) — *J. Macoun*, Herb. Geol. Surv. Can.

2. Spike dense and stiff, erect; spikelets full and rounded at base, mostly approximate: achene 2 (1.8 to 2 1) mm. broad.

22. C. adusta, Boott. — Figs. 67 to 69. — *Culms* smooth, *stiffly erect*, 2 to 8 dm. high: leaves usually shorter, 2 to 5 mm. broad: *spike ovoid to cylindric, usually subtended by a stiff rather prominent bract*, of 3 to 15 simple or compound *spikelets 6 to 12 mm. long:* perigynia 4 to 5 mm. long, 2 to 3 mm. broad. — Boott in Hook. Fl. Bor.-Am. ii. 215, & Ill. iii. 119, in part, t. 379; Bailey Mem. Torr. Cl. i. 24, & in Gray, Man. ed. 6, 621; Britton, l. c. 357, fig. 866. *C. albolutescens*, var. *glomerata*, Olney, Exsicc. fasc. v. no. 10. *C. adusta*, var. *glomerata*, Bailey, Carex Cat. 2, Bot. Gaz. ix. 139, & Proc. Am. Acad. xxii. 149. *C. pinguis*, Bailey, Bull. iii. Geol. and Nat. Hist. Surv. Minn. 22; Macoun, l. c. 129. — Dry woods, rocky banks and recent clearings, Newfoundland to Mount Desert Island, Maine, west to Minnesota, Assiniboia, Saskatchewan, and Keewatin. June–Sept.

§§ Cyperoideae. Bracts leaf-like and much prolonged, forming a conspicuous involucre.

23. C. sychnocephala, Carey. — Figs. 70, 71. — Culms smooth, 2 to 6 dm. high: *leaves soft*, ascending, 2 to 4 mm. wide; *bracts unequal, the lowest longest, 1 to 2 dm. long: spikelets* 4 to 10, oblong, 8 to 15 mm. long, *forming a dense ovoid or oblong spike: perigynia lance-subulate*, 5 mm. long, barely 1 mm. wide, firm, slightly nerved or nerveless. — Am. Jour. Sci. Ser. 2, iv. 24, & in Gray, Man. 545; Boott, Ill. i. 46, t. 118; Bailey, Proc. Am. Acad. xxii. 153; Macoun, l. c. 121; Britton, l. c. 360, fig. 875; Howe, l. c. 46; Cratty, Bull. Lab. Nat. Hist. Univ. Ia., iv. 363, t. 9. *C. cyperoides*, Dewey, Am. Jour. Sci. Ser. 2, iii. 171, not L. — Meadows, ditches, and wet sandy soil, locally from central New York to the Ottawa River (Canada), Iowa, Saskatchewan, and British Columbia. July, Aug.

Astrostachyae, Holm. Monoecious or dioecious, the spikelets often purely staminate or purely pistillate, or with the flowers variously mixed. Bract not sheathing, if present short and filiform. *Perigynia horizontally spreading or reflexed at maturity, spongy at base*, glabrous, nervose, distinctly pointed or beaked, *with thin margins* and bidentate apex.

* Spikelets terminal and solitary (rarely one or two secondary ones below): plants usually dioecious.

+– Culms filiform or setaceous, solitary or few from filiform creeping stoloniferous rootstocks.

24. C. GYNOCRATES, Wormskiold. — Figs. 72 to 77. — *Culms* 0.6 to 3 dm. high, *mostly exceeding the setaceous leaves:* spikelets 0.5 to 2 cm. long, some staminate and linear or linear-lanceolate, with oblong mostly blunt-pointed scales; others staminate above, with 1 or more pistillate flowers below; others oblong, strictly pistillate, with 6 to 12 *rather plump subterete, but thin-edged strongly nerved conic-beaked perigynia.* — Wormsk. in Drejer, Rev. 16; Fries, Mant. iii, 134, & Sum. 222; Anders. Cyp. Scand. 71, t. 3, fig. 8; Kunze, Car. 123, t. 31, fig. 1; Carey, in Gray, Man. ed. 2, 509; Boott, Ill. iv. 143, t. 459, 460; Bailey, Proc. Am. Acad. xxii. 142, & in Gray, Man. ed. 6, 617; Macoun, l. c. 109; Howe, l. c. 49 (incl. var. *monosperma*, Peck); Holm, l. c. 209. *C. Redowskiana*, Bailey, Mem. Torr. Cl. v. 89; Britton, l. c. 340, fig. 815; not C. A. Meyer, according to Meinshausen, Acta Hort. Petrop. xviii. 305. *C. dioica*, Schwein. & Torr. l. c. 293; Dewey, l. c. Ser. 1, x. 283; Carey, in Gray, Man. 537; not L. *C. monosperma*, Macoun, in Bailey, Carex Cat. 3, *nomen nudum*. *C. alascana*, Boeckeler, Engler's Bot. Jahrb. vii. 277, acc. to Bailey. — Swamps and bogs, LABRADOR to ALASKA, south mostly in *Thuya* swamps to Restigouche Co., NEW BRUNSWICK; Aroostook and Piscataquis Cos., MAINE; Herkimer, Yates and Genessee Cos., NEW YORK; Alleghany Co., PENNSYLVANIA; and Alcona and Oscoda Cos., MICHIGAN; in the Rocky Mts. to COLORADO: also in northern EUROPE and ASIA. June, July.

+– +– Culms stouter, rigid, forming strongly caespitose stools without stolons.

25. C. EXILIS, Dewey. — Figs. 78 to 83. — *Culms wiry*, 2 to 7 dm. high, usually much exceeding the *filiform stiff leaves:* spikelets mostly solitary, 1 to 3 cm. long, staminate, or pistillate, or with the flowers variously situated: *perigynia* ovate-lanceolate, *with serrulate thin margins, strongly convex on the outer, flattish and few-nerved or nerveless on the inner face.* — Am. Jour. Sci. xiv. 351, t. Q, fig. 53; Carey, l. c. 538; Boott, Ill. i. 17, t. 47; Bailey, Proc. Am. Acad. xxii. 142, & in Gray, Man. ed. 6, 617; Macoun, l. c. 111; Britton, l. c., 340, fig. 816; Howe, l. c. 38; Holm, l. c. 207. *C. exilis*, var. *squamacea*, Dewey, l. c. fig. 54. *C. exilis*, var. *androgyna*, Dewey, in Wood, Class-book, ed.

1861, 750. — Bogs and meadows near the coast, or on the coastal plain, locally from LABRADOR and NEWFOUNDLAND, to NEW JERSEY: also summit of Smoky Mt., Cape Breton, NOVA SCOTIA; Crystal, MAINE; Bristol and Peacham, VERMONT; Essex and Onondaga Cos., NEW YORK; Mer Bleue, ONTARIO; Calumet, MICHIGAN; and reported from Hennepin and Crow Wing Cos., MINNESOTA. May–Aug.

* * Spikelets 2 to several, the staminate flowers mostly at their bases; plants very rarely dioecious.

+ Perigynia broadest at the rounded or subcordate base; the beak rough or serrulate.

++ Perigynia .40 to .50 as broad as long, the slender beak conspicuous, often nearly as long as the body: scales pointed.

26. C. ECHINATA, Murray. — Figs. 84 to 88. — *Culms rather wiry*, 1 to 4 dm. high: leaves shorter than or equalling the culms, 1 to 2.5 mm. wide: *spike* linear-cylindric, *1 to 3 cm. long*, *of 2 to 6 subapproximate* or slightly remote subglobose or oblong *3- to 12- flowered spikelets: perigynia finally yellowish*, *narrowly ovate*, early ascending, later wide-spreading, faintly nerved or nerveless on the inner face, *3 to 4 mm. long*, one-third or one-half exceeding the ovate pointed brownish scale. — Prodr. 76; Boeckeler, Linnaea, xxxix. 124; Bailey, Proc. Am. Acad. xxii. 142; Mem. Torr. Cl. i. 57, & Bull. Torr. Cl. xx. 424; Macoun, l. c. 126; Richter, Pl. Eur. i. 150; Holm, l. c. 212. *C. muricata*, Huds. Fl. Ang. 406 (1778): Leers, Fl. Herb. 200, t. 14. fig. 8; not L. *C. Leersii*, Willd. Prodr. 28. *C. stellulata*, Gooden. Trans. Linn. Soc. ii. 144; Schkuhr, Riedgr. 45, t. C, fig. 14; Host, Gram. i. 41, t. 53; Schwein. & Torr. l. c. 317; Reich. Ic. Fl. Germ. viii. 9, t. 214, fig. 560; Carey in Gray, Man 544; Boott, Ill. i. 55. *Vignea stellulata*, Reich. Fl. Exc. 57. *C. sterilis*, Gray, Man. ed. 5, 578; Bailey, Bull. Torr. Cl. xx. 424; Britton, l. c. 350, fig. 844: Howe, l. c. 38; not Willd. — Open low ground, LABRADOR and UNGAVA to ALASKA, south to Garrett Co., MARYLAND, OHIO, MICHIGAN, SASKATCHEWAN, and Humboldt Co., CALIFORNIA: also in EUROPE and ASIA. June–Aug. Extremely variable, passing by numerous transitions to the following more marked extremes.

Var. **ormantha.** — Fig. 89. — *Spikes* 2 to 6 cm. long, *of 2 to 4 very remote 3- to 9-flowered spikelets, the terminal one with a clavate base 0.5 to 1 cm. long:* perigynia as in the species, spreading or slightly ascending, mostly twice as long as the scales. — *C. echinata*, W. Boott, in Wats. Bot. Cal. ii. 237, in part. — RHODE ISLAND, Providence, 1846

(*Geo. Thurber*); Connecticut, without locality (*Chas. Wright*); Southington, June 5, 1898 (*C. H. Bissell*); Waterford, May 29, 1889 (*C. B. Graves*): Oregon, without locality, 1871 (*E. Hall*, no. 582): California, bogs along Strawberry Creek, El Dorado Co., alt. 1,815 m., July 18, 1897 (*E. Brainerd*, no. 160 [type]); Big Trees, Calaveras Co. (*Bolander*, no. 2324); Santa Rosa (*J. M. Bigelow*); Bluff Lake, San Bernardino Mts., alt. 2,280 m., June, 1895 (*S. B. Parish*, no. 3702).

Var. **excelsior.** — Fig. 90, 91. — *Tall* and slender, 0.3 to 1 m. high: spike 3 to 5.5 cm. long; *spikelets 3 to 9, distinct*, only the lowermost remote, *12- to 20-flowered*, at first oblong-cylindric, with the perigynia ascending, later subglobose, with strongly reflexed perigynia one-third longer than the scales. — *C. sterilis* β, Boott, Ill. i. 56, t. 146.* *C. sterilis*, var. *excelsior*, Bailey, Bull. Torr. Cl. xx. 424; Howe, l. c. — Newfoundland to Michigan and North Carolina.

Var. cephalantha, Bailey. — Figs. 92 to 94. — *The coarsest form*, 3 to 7 dm. high: *leaves 2 to 4 mm. broad: spike* cylindric or slightly moniliform, *3 to 7.5 cm. long; the 4 to 8 broad-oblong spikelets approximate or slightly remote* (rarely 1 cm. apart), *15- to 40-flowered: perigynia ovate, one-half as broad as long*, wide-spreading or reflexed. — Mem. Torr. Cl. i. 58, & in Gray, Man. ed. 6, 618. *C. sterilis*, Boott, Ill. i. 55, t. 146. *C. sterilis*, var. *cephalantha*, Bailey, Bull. Torr. Cl. xx. 425; Britton, l. c.; Howe, l. c. 39. *C. sterilis*, var. *aequidistans*, Peck in Howe, l. c. — Newfoundland to North Carolina, Michigan, and British Columbia.

Var. angustata, Bailey. — Figs. 95 to 97. — *Extremely slender* or almost setaceous, 1 or 2 dm. high (in shade often taller): *leaves 0.5 to 1.5 mm. wide: spike 0.75 to 2.5 cm. long; the 6 or fewer 3- to 15-flowered spikelets approximate* (or slightly remote in shade): the divaricate *perigynia lance-ovate or lanceolate, 2.5 to 3 mm. long*, twice exceeding the scales. — Mem. Torr. Cl. i. 59, & in Gray, Man. ed. 6, 618. *C. stellulata*, var. *angustata*, Carey in Gray, Man. 544. *C. sterilis*, var. *angustata*, Bailey, Bull. Torr. Cl. xx. 425; Howe, l. c. — Nova Scotia to Connecticut, Lake St. John, Quebec, Illinois, and Michigan.

++ ++ Perigynia about .70 as broad as long, the beak short, one-fourth to one-half as long as the body.

= Tall: leaves 2.5 to 4.5 mm. broad: perigynia 2 to 3 mm. broad: scales sharp-pointed.

27. C. sterilis, Willd. — Figs. 98 to 100. — *Coarse*, 1 m. or less high: leaves flat, shorter than or equalling the culms: spike 1.5 to 3.5

cm. long; *the 3 to 6 subglobose or oblong-cylindric densely 15- to 50-flowered olive-green spikelets* crowded or distinct: the *thick strongly many-nerved perigynia broad-ovate, 3 to 3.5 mm. long,* squarrose or with recurved tips. — Sp. iv. 208; Schkuhr, Riedgr. Nacht. 3, t. Mmm, fig. 146. *C. stellulata,* var. *sterilis,* Carey in Gray, Man. 544. *C. stellulata,* var. *conferta,* Chapman, Fl. 534. *C. echinata,* var. *conferta,* Bailey, Carex Cat. 2, Proc. Am. Acad. xxii. 143, Mem. Torr. Cl. i. 58, & in Gray, Man. ed. 6, 618; Macoun, l. c. 126. *C. atlantica,* Bailey, Bull. Torr. Cl. xx. 425; Britton, l. c. 350. — Bogs and damp pine-barrens, near the coast from NEWFOUNDLAND to FLORIDA, rarely inland on cold bogs, at Lake St. John, QUEBEC (*G. G. Kennedy*); Squapan, Aroostook Co., and northern flank (near Bell Camp) of Mt. Katahdin, MAINE (*Fernald*); Adirondack Mts., Essex Co., NEW YORK (*Knieskern*); and Mt. Sorrow, Valley Forge, PENNSYLVANIA (*C. E. Smith*). June, July.

= = Low: leaves 0.5 to 2 (very rarely 2.5) mm. broad: scales blunt.

28. C. INTERIOR, Bailey. — Figs. 101 to 105. — *Slender,* 1.5 to 5 dm. high; the *leaves 1 to 2 (rarely 2.5) mm. broad,* shorter than or exceeding the rather stiff culms: spike 1 or 2 cm. long; *the 2 to 5 spikelets* all fertile, all sterile, or variously mixed, usually subglobose, *4 or 5 mm. in diameter, the terminal long-clavate at base, 5- to 15-flowered: perigynia firm, plump,* olive-green or -brown, more or less nerved or almost nerveless, *broadly deltoid-ovate, obscurely short-beaked* and with slightly thickened margin, *2.3 to 3.2 mm. long, 1.5 to 2 mm. broad,* finally wide-spreading or recurved, much exceeding the oblong or ovate *blunt scales.* — Bull. Torr. Cl. xx. 426; Britton, l. c. fig. 846; Howe, l. c. 39. *C. scirpoides,* Schkuhr, Riedgr. Nacht. 19, t. Zzz, fig. 180; Willd. Sp. iv. 237; Schwein. & Torr. l. c. 317; Dewey, Am. Jour. Sci. viii. 96; not *C. scirpoidea,* Michx. *C. stellulata* γ, Torr. l. c. 392. *C. stellulata,* var. *scirpina,* Tuck. Enum. Meth., 9, not *C. scirpina,* Tuck. l. c. 8. *C. stellulata,* var. *scirpoides,* Carey in Gray, Man. 544; Boott, Ill. i. 56, t. 146.** *C. echinata,* Boeckeler, Linnaea, xxxix. 124, in part, not Murray. *C. norvegica,* E. P. Sheldon, Bull. Torr. Cl. xx. 284, & Minn. Bot. Studies, i. 224, not Wahl. In damp or wet soil, NEW BRUNSWICK to RUPERT LAND and VANCOUVER ISLAND, south to FLORIDA and ARIZONA. Commonest northward and in the interior. May–July.

Var. CAPILLACEA, Bailey. Stiff, *culms almost setaceous; leaves about 0.5 mm. broad,* often involute: *perigynia strongly nerved.* — Bull. Torr. Cl. xx. 426; Howe, l. c.; Britton, l. c. 351. — NEW HAMPSHIRE to NEW YORK, NEW JERSEY, and PENNSYLVANIA.

+ + Perigynia broadest near the middle, tapering to a narrow base and a smooth beak.

29. C. SEORSA, E. C. Howe. — Figs. 106 to 109. — *Culms soft*, in loose stools, 3.5 to 6.5 dm. high: *leaves* shorter, *soft, pale*, 2 to 4 mm. broad: spikes 2.5 to 7 cm. long, of *2 to 6 mostly remote* subglobose or oblong 6- to 20-flowered green *spikelets* 3.5 to 7 mm. long, the terminal one usually with a long-clavate base, the lower often subtended by a setiform bract: *perigynia very thin and conspicuously nerved, elliptic-ovate, with a very short smooth beak* and a *narrow substipitate base*, 2.7 mm. long, 1.9 mm. broad, wide-spreading or recurved, much exceeding the acutish scales. — 48 Rep. N. Y. Mus. Nat. Hist. 40. *C. canescens*, var. *vulgaris*, Deane, Met. Park Fl. 95, not Bailey. — Wet woods and swamps, from Middlesex Co., MASSACHUSETTS to Suffolk and Oneida Cos., NEW YORK, south to DELAWARE. May, June.

Elongatae, Kunth. Spikelets remote or approximate in a simple elongated or short inflorescence. Staminate flowers at the base of the spikelets. *Perigynia ascending* when mature, glabrous, *ovate to oblong or lanceolate, plano-convex*, beaked or beakless, *not thin-winged.*

* Perigynia more or less roughened or serrulate on the upper edges (sometimes smooth in exceptional forms of *C. canescens*; and by exception obscurely toothed in rare individuals of *C. tenuiflora*).

+ Perigynia broadest at the rounded or subcordate base.

30. C. ARCTA, Boott. — Figs. 110 to 113. — *Pale green* or somewhat glaucous: *culms very soft, in loose stools*, 1.5 to 6 dm. high, often overtopped by the *soft flat leaves 2.5 to 4 mm. broad: spike oblong-cylindric*, of 5 to 13 ovoid or oblong *closely approximate or slightly remote spikelets* 6 to 11 mm. long: *perigynia ovate, with a rather definite beak*, strongly nerved on the outer, faintly on the inner face, 2 to 3 mm. long, 1.2 to 1.5 mm. broad, somewhat exceeding the acute, often brown-tinged, scales. — Ill. iv. 155, t. 497; Macoun, l. c. 124; Britton, l. c. 352, fig. 850. *C. canescens*, var. *polystachya*, Boott in Richards. Arct. Exped. ii. 344; Bailey, Proc. Am. Acad. xxii. 144, Mem. Torr. Cl. i. 75, & in Gray, Man. ed. 6, 619. *C. Kunzei*, Olney, Proc. Am. Acad. viii. 406 (excl. syn.). *C. canescens*, var. *oregana*, Bailey, Mem. Torr. Cl. i. 75. — Wet woods, alluvial thickets and swales, from the larger river-valleys of MAINE and QUEBEC, Lake Champlain, VERMONT, and the Adirondack Mts., NEW YORK to Lake Nipigon, ONTARIO, and BRITISH COLUMBIA, south to MICHIGAN, MINNESOTA, and the coast and mountains of WASHINGTON and OREGON. June–Aug.

+ + Perigynia broadest near the middle.

++ Perigynia 2 to 3 mm. long, fully half as broad.

= Plant glaucous: spikelets oblong-cylindric to ovoid; the strongly appressed-ascending pale perigynia slightly roughened or smooth above, tapering gradually to the short obscure beak.

31. C. canescens, L. — Figs. 114, 115. — *Culms soft, in loose stools*, 1.5 to 6 dm. high: *leaves soft and flat*, shorter than or exceeding the culms, *2 to 4 mm. broad: spike 2.5 to 5 cm. long*, of 4 to 7 oblong-cylindric to narrowly obovoid *appressed-ascending approximate* or slightly remote *spikelets 0.6 to 1 cm. long, the lowermost rarely 1.5 cm. apart: perigynia glaucous*, ovoid-oblong, usually serrulate toward the short-pointed tip, *2.3 to 3 mm. long, 1.3 to 1.7 mm. broad*, more or less nerved on both faces, somewhat exceeding the ovate pointed scale. Sp. ii. 974; Oeder, Fl. Dan. ii. 8, t. 285; Lightf. Fl. Scot. ii. 550; Reichb. Ic. Fl. Germ. viii. 7, t. 206, fig. 546; Anders. Cyp. Scand. 57, t. 4, fig. 39; Boott, Ill. iv. 154, in part; W. Boott ex Rothrock in Wheeler, Rep. 277; Ett. & Pok. Phys. Pl. Aust. vi. t. 515; Richter, Pl. Eu. i. 151. *C. brizoides*, Huds. Fl. Ang. 349, not L. *C. elongata*, Leers, Fl. Herb. 197, t. 14, fig. 7; Olney ex Wats. Bot. King Rep. 365; Bailey in Coulter, Man. Rocky Mt. Reg. 394, in part; not L. *C. cinerea*, Pollich, Pl. Palat. ii. 571. *C. Richardi*, Thuill. Fl. Par. (1799) 482. *C. curta*, Good. Trans. Linn. Soc. ii. 145; Host, Gram. i. 37, t. 48; Schkuhr, Handb. iii. 347, t. 287C, fig. 13; Eng. Bot. xx. t. 1386; Kunth, Enum. ii. 403. *C. lagopina*, Olney ex Wats. Bot. King Rep. 365, in part, not Wahl. *C. canescens*, var. *dubia*, Bailey, Bot. Gaz. ix. 119 & Proc. Am. Acad. xxii. 143. *C. canescens*, var. *robustina*, Macoun, l. c. 376. — Northern Europe. In wet places, seen from the following American stations — Labrador, (*Spreadborough* hb. Geol. Surv. Can. no. 13,372): Rupert Land, Lake Mistassini (*J. M. Macoun*, hb. Geol. Surv. Can. no. 30,511): Maine, Fort Kent, Island Falls, and Foxcroft (*M. L. Fernald*, nos. 2143, 2144, 2145): New Hampshire, Mt. Washington and Mt. Lafayette (*E. & C. E. Faxon*); Crawfords (*E. F. Williams*): Vermont, Ripton (*Ezra Brainerd*): Ontario, Belleville & Lake Nipigon (J. Macoun, hb. Geol. Surv. Can. nos. 30,513, & 30,512): Michigan, Alma (*C. A. Davis*): Colorado, Twin Lakes (*J. Wolff*, no. 1017); Bob Creek, alt. 3,230 m. (*Baker, Earle & Tracy*, no. 693): Montana, Grasshopper Valley (*S. Watson*, no. 435): Wyoming, without station (*Parry*, no. 278); Centennial Hills (*A. Nelson*, no. 1730); Beaver Lake (*A. & E. Nelson*, no. 6130):

UTAH, Bear River Cañon, alt. 3,080 m. (*S. Watson*, nos. 1231^a & 1233); Alta, Wahsatch Mts., alt. 2,460 m. (*M. E. Jones*, no. 1273): ALASKA, Ounalaska (*J. M. Macoun*, hb. Geol. Surv. Can., no. 30,514); Sitka (*Mertens*): ALBERTA, Lake Louise (*E. Brainerd*): BRITISH COLUMBIA, Revelstoke (nos. 19 & 30,526), Comox (no. 371), Port Henly (no. 20,500), Mt. Mark, Vancouver Isl. (no. 30,515), Beaver Creek, Selkirk Mts. (no. 30,519), Dead Man River (no. 30,522), — *John Macoun*, hb. Geol. Surv. Can.; Ilgacho Brook (*Dawson*, hb. Geol. Surv. Can. no. 30,518); Lulu Island, Fraser River Delta (*R. B. Dixon*): WASHINGTON, upper valley of the Nesqually (*O. D. Allen*, no. 163). May–Aug.

Var. SUBLOLIACEA, Laestad. — Figs. 116, 117. — Smaller, the short-oblong or subglobose *spikelets 4 to 7 mm. long: perigynia* smaller, *barely 2 mm. long*, smooth throughout. — Nov. Act. Soc. Sci. Ups. xi. 282; Andersson, Cyp. Scand. 57; Boott, l. c.; Bailey, Mem. Torr. Cl. i. 66; Richter, l. c. 152. *C. lapponica*, Lange, Linnaea, xxiv. 539. *C. canescens* in part, of Am. authors. — LAPLAND. In America specimens examined from UNGAVA, Ungava Bay (*L. M. Turner*): HUDSON BAY (*Sir John Richardson*): NEW BRUNSWICK, South Tobique Lakes (*G. U. Hay*, no. 55); Petitcodiac (*J. Brittain*, hb. Geol. Surv. Can. no. 30,510): NOVA SCOTIA, Halifax (*J. Macoun*): MAINE, Orono and Southport (*M. L. Fernald*): NEW HAMPSHIRE, Mt. Washington (*Asa Gray*, et al.); Mt. Monadnock — ledges toward summit (*W. P. Rich*): VERMONT, Willoughby Lake (*W. Boott, G. G. Kennedy*); summit of Mt. Mansfield (*E. Brainerd*); bog, Wallingford, alt. 675 m. (*E. Brainerd*): MASSACHUSETTS, Sharon (*W. P. Rich*); Washington, Berkshire Co. (*W. Boott*): NEW YORK, Fairfield (*A. Gray*); Pen Yan (*Sartwell*, no. 32); Oriskany Swamp (*Kniesken*); tamarack swamps, Herkimer Co. (*J. V. Haberer*): ONTARIO, Ottawa (*J. Fletcher*, hb. Geol. Surv. Can., no. 7408); Hastings Co. (*J. Macoun*): MICHIGAN, Flint (*D. Clark*); Lansing (*L. H. Bailey*, no. 262); Alma (*C. A. Davis*): BRITISH COLUMBIA, Mts. east of Adams Lake (*Dawson*, hb. Geol. Surv. Can., no. 30,520): WASHINGTON, Seattle (*C. V. Piper*, no. 1106).

Var. **disjuncta** — Figs. 118 to 120. — Tall and lax, 3 to 8 dm. high: *spike elongated, flexuous, 0.5 to 1.5 dm. long;* the 5 to 8 oblong-ovoid to cylindric *spikelets 6 to 12 mm. long, all but the terminal remote, the lowermost 2 to 4 cm. apart:* perigynia as in the species, serrulate above. — *C. canescens* of most Am. authors. *C. canescens*, form, Boott, Ill. iv. 154, t. 496. The common form in eastern America found in most swamps or on wet shores from NEWFOUNDLAND to MICHIGAN, OHIO and PENNSYLVANIA. The following numbered specimens belong here — PRINCE EDWARD ISLAND, Brackley Point (*J. Macoun*, hb. Geol. Surv. Can. no. 30,509): NEW BRUNSWICK, Serpentine River (*Hay*, no. 84); Chipman (*Wetmore*, hb. Geol. Surv. Can. no. 30,507): NOVA SCOTIA, Boylston (*C. A. Hamilton*, hb. Geol. Surv. Can., no. 25,443); Baddeck (no. 20,805), Sable Island (nos. 22,076 & 23,071), Truro (no. 30,506) — *J. Macoun*, hb. Geol. Surv. Can.: MASSACHUSETTS: Framingham (*E. C. Smith*, no. 628): CONNECTICUT, Southington (*L. Andrews*, no. 590): ONTARIO, Cache Lake (*J. Macoun*, hb. Geol. Surv. Can., no. 22,036).

= = Green, not glaucous: spikelets subglobose to short-oblong, few-flowered: the loosely spreading dark green or brown perigynia serrulate at the base of the distinct beak.

32. C. BRUNNESCENS, Poir. — Figs. 121 to 124. — *Very slender* and lax: culms 1.5 to 7 dm. high: *leaves soft, flat, 1 to 2.5 mm. wide*, shorter than or equalling the culms: *spike 1 to 6 cm. long*, of 3 to 6 more or less remote or approximate *spikelets 3 to 7 mm. long: perigynia* 2 to 2.7 mm. long, 1 to 1.5 mm. broad, *with distinct slender beaks, loosely spreading when mature.* — Suppl. iii. 286; Britton, l. c. 351, fig. 848. *C. curta*, var. *brunnescens*, Pers. Syn. ii. 539. *C. canescens*, var. *alpicola*, Wahlenb. Fl. Lapp. 232; Bailey, Proc. Am. Acad. xxii. 143, & in Gray, Man. ed. 6, 618; Macoun, l. c. 124; Howe, l. c. 37. *C. Gebhardii*, Hoppe Car. Germ. 30. *Vignea Gebhardi*, Reichb. Fl. Exc. 58. *C. canescens, β*, Torr. Ann. Lyc. N. Y. iii. 393. *C. Persoonii*, Lange, Flora, xxv. (1842), 748; Reichb. Ic. Fl. Germ. viii. 7, t. 206, fig. 547. *C. canescens*, var. *sphaerostachya*, Tuck. Enum. Meth. 10, 19; Carey in Gray, Man. 544. *C. vitilis*, Fries, Mant. iii. 137; Anders. Cyp. Scand. 58, t. 4, fig. 38; Boott, Ill. iv. 219; Fl. Dan. xvii. t. 2973. *C. Buckleyi*, Dewey, Am. Jour. Sci. xlviii. 143, t. Dd, fig. 104. *C. sphaerostachya*, Dewey, l. c. xlix. 44, t. Ee, fig. 110. *C. canescens*, var. *vitilis*, Carey in Gray, Man. ed. 2, 514. *C. canescens*, var. *brunnescens*, Boott, l. c. 220 (*nomen nudum*); Bailey. Mem. Torr. Cl. v. 74. *C. canescens*, var. *vulgaris*, Bailey, Bot. Gaz. xiii. 86, Mem. Torr. Cl. i. 66, v. 74, & in Gray, Man. ed. 6, 618; Macoun, l. c. 123; Howe, l. c. 37. *C. brunnescens*, var. *gracilior*, Britton, l. c. 350. — Open woods and dry, rocky banks, NEWFOUNDLAND and LABRADOR to BRITISH COLUMBIA, south to IDAHO, MICHIGAN, and mostly in the mountains to NORTH CAROLINA. Also in GREENLAND and northern

EUROPE. June–Aug. On alpine summits becoming more rigid and browner than in sheltered situations.

++ ++ Perigynia 4 to 5.5 mm. long, distinctly less than half as broad.

= Leaves very narrow (1 to 2.5 mm. broad): spikelets lanceolate: perigynia 1 to 1.3 mm. wide.

33. C. BROMOIDES, Schkuhr. — Figs. 125, 126. — *Very slender* and lax, *green*, scarcely glaucous, the culms 3 to 8 dm. long, mostly exceeding the soft flat leaves: *spike loosely subcylindric*, 2 to 5.5 cm. long, *of 2 to 6 approximate or slightly scattered spikelets 0.5 to 2 cm. long:* beak of the *perigynium* one-half to two-thirds as long as the *strongly nerved* body, slightly exceeding the *oblong pointed scale.* — Riedgr. Nachtrag. 8, t. Xxx, fig. 176; Willd. Sp. iv. 258; Schwein. & Torr. Ann. Lyc. N. Y. i. 300; Torr. l. c. 391; Carey in Gray, Man. 539; Chapm. Fl. 533; Boott, l. c. ii. 82, t. 227; Bailey, Proc. Am. Acad. xxii. 146; Macoun, l. c. 114; Britton, l. c. 354, fig. 857; Howe, l. c. 47. — Rich low woods and swamps, NOVA SCOTIA, southern NEW BRUNSWICK and central MAINE to western ONTARIO and MICHIGAN, south to FLORIDA and LOUISIANA.[1] May–July.

= = Leaves broader (2 to 5 mm. broad): spikelets ovoid: perigynia 1.6 to 1.9 mm. wide.

34. C. DEWEYANA, Schweinitz. — Figs. 127, 128. — Very lax, *glaucous*, the culms 2 to 12 dm. long, much exceeding the soft, flat leaves: *spike flexuous*, 2 to 6 cm. long, *of 2 to 5* (in very luxuriant individuals rarely 6 or 7) *3- to 12-flowered spikelets 5 to 12 mm. long*, the upper subapproximate or scattered, *the lowest very remote, usually subtended by an elongate slender bract:* beak about one-half as long as the *obscurely nerved or nerveless body of the perigynium*, somewhat exceeding the *ovate acuminate or short-cuspidate* pale *scale.* — Ann. Lyc. N. Y. i. 65; Dewey, Am. Jour. Sci. ix. 62, t. 3, fig. 11; Schwein. & Torr. l. c. 310; Torr. l. c. 392; Carey, l. c. 544; Boott, l. c. i. 27, t. 70; W. Boott in Wats. Bot. Calif. ii. 236; Bailey in Coulter, Man. Rocky Mt. Reg. 394, & Proc. Am. Acad. xxii. 146; Macoun, l. c. 124; Britton, l. c. fig. 856; Howe, l. c. 36. *C. remota*, Richards. in Frankl. 1st Journ. ed. 2, App. 35, acc. to Boott, not L. — Rich open woods and banks, NOVA SCOTIA and

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[1] Californian and other northwestern specimens referred here seem much better placed with the stouter broader-leaved *C. Bolanderi*, Olney.

QUEBEC to ATHABASCA and BRITISH COLUMBIA, south to PENNSYLVANIA, MICHIGAN, NEW MEXICO, and WASHINGTON.[1] May–Aug.

* * Perigynia entirely smooth at the tip (exceptional forms of *C. canescens* might be looked for here; and very rare individuals of *C tenuiflora* might be sought in the preceding section).

+ Perigynia oblong or ovate-oblong.

++ Perigynia 3 to 4 mm. long, nerved: culms weak, almost capillary: spikelets 2 to 4, loose, silvery-green or silvery brown.

= Spikelets closely approximate in a small usually bractless terminal cluster: perigynia beakless.

35. C. TENUIFLORA, Wahlenb. — Figs. 129, 130. — Lax, the culms 2 to 6 dm. long, mostly exceeding the very narrow (0.7 to 2 mm. broad) pale green leaves: *spikelets subglobose, 3- to 10-flowered: perigynia 3 to 3.4 mm. long*, 1.5 to 1.7 mm. broad, with the bluntish scarcely beaked tip smooth or rarely with one or two teeth, about equalled by the ovate or ovate-oblong white scale. — Kongl. Vet. Acad. Handl. xxiv. 147, & Fl. Lapp. 232; Schkuhr, Riedgr. Nachtr. 17, t. Eeee, fig. 187; Anders. Cyp. Scand. 59, t. 4, fig. 36; Hook. Fl. Bor.-Am. ii. 214; Torr. l. c. 392, 443; Carey, l. c. 543; Boott, Ill. iv. 144, t. 463; Fl. Dan. Suppl. 13, t. 167; Bailey, Proc. Am. Acad. xxii. 145; Macoun, l. c. 122; Britton, l. c. 352, fig. 851 (as to habital drawing); Howe, l. c. — Cold bogs among the mountains, SCANDINAVIA. Bogs and wet mossy woods, local, from eastern UNGAVA to western KEEWATIN and MANITOBA; south to Westmoreland and Victoria Cos., NEW BRUNSWICK; southern Aroostook, Penobscot and Oxford Cos., MAINE; Hampshire Co., MASSACHUSETTS; Oneida Co., NEW YORK; Ingham Co., MICHIGAN; Milwaukee Co., WISCONSIN; Chisago and Hennepin Cos., MINNESOTA: also on Elbow River, ALBERTA, and near Victoria, BRITISH COLUMBIA (*Macoun*, hb. Geol. Surv. Can. nos. 25,571 & 30,517).

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[1] The California material which has been referred here is *C. Bolanderi*, Olney, differing in its less acutely angled culm, longer spikes of more approximate usually more numerous lance-cylindric many-flowered spikelets, the lowest with or without a short bract. The northwestern *C. Bolanderi*, var. *sparsiflora*, Olney (*C. Deweyana*, var. *sparsiflora*, Bailey) is a distinct species, probably *C. laeviculmis*, Meinshausen, Acta Hort. Petrop. xviii. 326, in its small short-beaked strongly nerved finally spreading thin-edged perigynia much nearer related to the eastern *C. seorsa* than to the members of the *Elongatae*.

= = Spikelets remote, the uppermost strongly divaricate-pedunculate; the lowermost subtended by a long leaf-like bract: perigynia beaked.

36. C. TRISPERMA, Dewey. — Figs. 131, 132. — Culms almost filiform, 2 to 7 dm. long, usually much overtopping the soft narrow (0.5 to 2 mm. wide) leaves: *the 2 or 3 spikelets, 2- to 5-flowered:* the finely many-nerved *perigynia 3.3 to 3.8 mm. long,* 1.6 to 1.8 mm. broad, slightly exceeding the ovate-oblong pale obtuse to mucronate-acuminate scale. — Am. Jour. Sci. ix. 63, t. 3, fig. 12; Hook. Fl. Bor.-Am. ii. 213; Schwein. & Torr. l. c. 311; Carey, l. c. 543; Boott, l. c. i. 29, t. 74; Bailey, Proc. Am. Acad. xxii. 144; Macoun, l. c. 122; Britton, l. c. 353, fig. 855; Howe, l. c. 35. — Mossy woods and bogs, NEWFOUNDLAND and LABRADOR to SASKATCHEWAN, south to northern PENNSYLVANIA, OHIO, MICHIGAN, and NEBRASKA (according to Webber), and in the mountains to Garrett Co., MARYLAND. Ascending to 770 m. in the New England mountains. June–Aug.

++ ++ Perigynia 1.2 to 1.5 mm. long, nerveless, with a very short broad truncate beak, or beakless: culms wiry: spikelets 3 to 5, closely flowered, in a greenish-brown or straw-colored linear spike.

37. C. **elachycarpa.** — Figs. 133, 134. — Tufted, the *stiff slender culms* 3 or 4 dm. high, *strongly scabrous above,* longer than the soft narrow (1 to 2 mm. broad) green leaves: *spike 0.5 to 1.5 cm. long; the appressed ascending narrowly ovoid approximate or slightly remote spikelets 3 to 6 mm. long: perigynia oblong, plump, smooth and nerveless, subtruncate at base, shorter than the* oblong-ovate acuminate *dull-brown, green-ribbed scales.* —MAINE, wet sandy river bank, Fort Fairfield, June 29, 1899 (*M. P. Cook, E. L. Shaw & M. L. Fernald*). A unique plant, in maturity strongly suggesting an immature slender form of *C. echinata,* or the little-known *C. helvola,* Blytt, which, however, have very different perigynia.

+ + Perigynia broadly elliptic to suborbicular: spikes mostly tinged with brown.

++ Terminal spikelet with conspicuous clavate sterile base: perigynia rather abruptly contracted to the slender beak.

= Spikelets mostly distinct, the lowest 4 or 5 mm. thick.

38. C. NORVEGICA, WILLD. — Figs. 135, 136. — *Glaucous* and *freely stoloniferous; culms smooth and soft,* 1 to 4.5 dm. high, mostly overtopping the soft flat rather narrow (1 to 2.5 mm. broad) leaves: spike

1.5 to 5.5 cm. long, of *2 to 6 ovoid or broad-oblong spikelets; the lower 5 to 12 mm. long, the terminal*, including the clavate sterile base, *1 to 1.8 cm. long: perigynia* pale, faintly nerved, 2.5 to 3.3 mm. long, 1.6 to 2 mm. broad, *conic-rostrate*, usually abruptly contracted to a substipitate base, about equalled by the *yellowish brown orbicular to ovate blunt scales.* — Willd. ex. Schkuhr, Riedgr. 50, t. S, no. 66, & Spec. iv. 227; Wahlenb. Kongl. Vet. Acad. Handl. xxiv. 146, & Fl. Lapp. 233, t. 15, fig. 3; Anders. Cyp. Scand. 61, t. 4, fig. 29; Goodale in Holmes, Prelim. Rep. Nat. Hist. & Geol. Me. (1861), 128, & Proc. Portland Soc. Nat. Hist. i. 135; Gray, Man. ed. 3, Addend. xcvii; Boott, l. c. iv. 214; Fl. Dan. Suppl. 13, t. 103; Bailey, Proc. Am. Acad. xxii. 145; Macoun, l. c. 125; Britton, l. c. 351, fig. 849 (as to habital sketch). — Brackish marshes, northern SCANDINAVIA. Damp usually brackish soil, coast of southern LABRADOR: Anticosti Island, and Kamouraska, Saguenay, Rimouski, and Gaspé Cos., QUEBEC: locally southward along the coast in NEW BRUNSWICK at Shediac, Westmoreland Co., and Back Bay, Charlotte Co. (*J. Brittain*, herb. Geol. Surv. Can. nos. 30,421 & 30,420); Whale Cove, Grand Manan and Fryes Island (*Hay*): NOVA SCOTIA, Baddeck, Cape Breton and Truro (*J. Macoun*, herb. Geol. Surv. Can. nos. 20,846 & 30,422); Boylston (*C. A. Hamilton*, herb. Geol. Surv. Can. no. 25,521): MAINE, Little Cranberry Isle (*Redfield*); Wells (*Blake*): reported from Alaska.[1] June–Aug.

= = Spikelets approximate at the tip of the culm, the lowest 2.5 to 4 mm. thick.

a. Plant weak and lax, with filiform or involute leaves.

39. C. GLAREOSA, Wahlenb. — Figs. 137, 138. — *Culms acutely angled, mostly curved*, scabrous at tip, 1 to 3 dm. high, once and a half or twice exceeding the *flaccid narrow (0.5 to 1.5 mm. broad) leaves:* spike oblong to obovoid, 0.7 to 2 cm. long, *with 2 to 4 appressed-ascending obovoid spikelets; the lower 4 to 9 mm. long, 3 or 4 mm. thick, the terminal* larger, including the slender sterile base, *6 to 11 mm. long: perigynia pale*, elliptic or ovate, acute at base, *with narrowly conic beak*, faintly nerved or nerveless, 2.5 to 3 mm. long, 1.1 to 1.9 mm. broad, nearly or quite equalled by the *ferruginous white-edged ovate acutish scales.* — Kongl. Vet. Acad. Handl. xxiv. 146, & Fl. Lapp. 230; Willd.

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[1] Prof. Conway MacMillan has courteously forwarded me the Minnesota specimens referred to *C. norvegica* by Mr. E. P. Sheldon (Bull. Torr. Cl. xx. 284, & Minn. Bot. Studies, i. 224), and they prove to be *C. interior*, Bailey.

Spec. iv. 251; Schkuhr. Riedgr. Nachtr. 24, t. Aaa, fig. 97; Anders. l. c. 62, t. 4, fig. 31; Torr. l. c. 396; Dewey, Am. Jour. Sci. Ser. 2, iv. 344; Boott, l. c. 153, t. 494; Fl. Dan. xiv. 8, t. 2430; Bailey, Proc. Am. Acad. xxii. 146; Macoun, l. c. 127; Britton, l. c. 353, fig. 854; Meinshausen, Acta Hort. Petrop. xviii. 325. — Arctic regions of both hemispheres, extending south in America along the coast of LABRADOR to QUEBEC, Bonne Esperance (*Allen*). Watsheeshoo (*St. Cyr*, hb. Geol. Surv. Can. no. 16,524), and Tadousac (*Kennedy*), Saguenay Co.; Pointe des Monts (*Bell*) and Grand Etang (*Macoun*, hb. Geol. Surv. Can. no. 30,413), Gaspé Co.: also on the coast of ALASKA. June–Aug.

b. Plant stiff and upright, with flat leaves.

40. C. LAGOPINA, Wahlenb. — Figs. 139, 140. — *Culms obtusely angled, mostly erect*, smooth except at tip, 1 to 4 dm. high, more or less exceeding the *narrow* (*1 to 3 mm. wide*) *leaves*: spike from cylindric to globose, 1 to 2.5 cm. long, with *3 to 6 ascending spikelets mostly larger than in the last*: *perigynia brown or reddish-brown*, from elliptic-lanceolate to broadly obovate, *rather abruptly beaked*, 2.5 to 3.8 mm. long, 1.5 to 1.9 mm. wide, exceeding the *ovate obtuse white-margined fuscous scales*. — Kongl. Vet. Acad. Handl. xxiv. 145, & Fl. Lapp. 229; Gay, Ann. Sci. Nat. Ser. 2, xi. 177; Drejer. Rev. 25; Anders. l. c. 63, t. 4, fig. 28; Reichenb. l. c. t. 204, fig. 543; Torr. l. c. 393; Boott, Ill. iv. 189; W. Boott in Wats. Bot. Calif. ii. 233; Bailey in Coulter, Man. Rocky Mt. Reg. 395, & Proc. Am. Acad. xxii. 145; Britton, l. c. 353, (fig. uncharacteristic); Meinsh. l. c. *C. leporina*, L. Spec. 973, in part (cit. Fl. Lapp.); Oeder, Fl. Dan. ii. 9. t. 294; Willd. Spec. iv. 229; Schkuhr, Riedgr. Nachtr. 17, in part (excl. t. Fff, fig. 129); Host, Gram. iv. 45. t. 80; Eng. Bot. Supp. iii. t. 2815. *C. Lachenalii*, Schkuhr, Riedgr. 51, t. Y. fig. 79. *C. approximata*, Hoppe, ex DC. Fl. Fr. vi. 290. *C. parviflora*, Gaud. Etr. Fl. 84. acc. to Boott. *C. furva*, Webb, Iter Hisp. 5. — Arctic and alpine regions of EUROPE and ASIA: GREENLAND: ARCTIC AMERICA, rarely south to Mt. Albert, Gaspé Co., QUEBEC, the mountains of COLORADO, and northern CALIFORNIA. June–Aug.

++ ++ Terminal spikelet ovoid or subglobose, not conspicuously clavate at base: perigynia tapering gradually to the tip: culms sharply angled and harsh, upright, the 2 to 5 spikelets crowded at the tip: leaves flat.

41. C. HELEONASTES, Ehrh. — Figs. 141, 142. — *Culms* 1.5 to 3.5 cm. high, *stiff*, usually overtopping the *erect narrow* (*1 or 2 mm. wide*)

leaves: the *globose or ovoid spikelets 4 to 8 mm. long: perigynia* 2.5 to 3.5 mm. long, 1.2 to 1.7 mm. broad, *brown tinged*, mostly exceeding the *ovate blunt scales.* — Ehrh. in L. f. Suppl. 414; Wahlenb. Kongl. Vet. Acad. Handl. xxiv. 146, & Fl. Lapp. 230; Schkuhr, Riedgr. 51, t. Ii, fig. 97; Hoppe & Sturm, Car. Germ. t. 6; Hook. Fl. Bor.-Am. ii. 214; Reichenb. Ic. Fl. Germ. viii. t. 204, fig. 542; Anders. Cyp. Scand. 62, t. 4, fig. 30; Boott, Ill. iv. 152, t. 489; Fl. Dan. Suppl. t. 31; Bailey, Proc. Am. Acad. xxii. 145; Macoun. l. c. 127; Britton, l. c. 352, fig. 852. *C. leporina*, Schkuhr, Riedgr. Nacht. t. Fff, fig. 129, not L. *C. Carltonia*, Dewey, Am. Jour. Sci. xxvii. 238, t. U. fig. 64; Torr. l. c. 393. *C. marina*, Dewey, l. c. xxix. 247, t. X, fig. 74; Torr. l. c. — Bogs and mossy places, arctic and alpine EUROPE. Very locally in America: examined from the following stations: — KEEWATIN, York Factory (*Sir John Richardson*): SASKATCHEWAN, Norway House and Carlton House (*Richardson*): ALBERTA, Lake Louise (*Ezra Brainerd*, no. 172): BRITISH COLUMBIA, Glacier (*Ezra Brainerd*); Kicking Horse Lake (*J. Macoun*, hb. Geol. Surv. Can. nos. 28; 49; 30,410; 30,411; 30,412). July, Aug.

II. — THE VARIATIONS OF SOME BOREAL CARICES.

CAREX AQUATILIS.

C. AQUATILIS, Wahlenb., Kongl. Acad. Handl. xxiv. 165. — Plants 3 to 9 dm. high; leaves 4 to 7 mm. broad: spikelets[1] slender; the pistillate 1.5 to 5.5 cm. long, 3 to 4.5 mm. thick, the lowermost often long-attenuated and remotely flowered at base: scales dark, subacute, hardly equalling or barely exceeding the perigynia. — Northern EUROPE, GREENLAND. In North America from the Shickshock Mts., Gaspé, QUEBEC,

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[1] The inflorescences of *Carex* are simple or compound spikes, racemes, or panicles; and, since in other genera of *Cyperaceae*, as *Cyperus* and *Scirpus*, the ultimate spicate divisions of the inflorescence are called *spikelets*, that term is here adopted, for the sake of uniformity and clearness, for these ultimate spicate divisions of the inflorescence of *Carex*. The species in which there is a solitary simple inflorescence (or true spike), as *C. gynocrates* and *C. exilis*, are few in comparison with those in which the inflorescence has more than one such division. From the occurrence in those plants, however, of occasional secondary divisions of the inflorescence, the term *spikelet* seems not inappropriate to the normal inflorescence of such species.

to Bear Lake, Mackenzie & British Columbia, south to Maine, Vermont, central and western New York, and Utah. The Scandinavian material examined has been referred to the true *C. aquatilis* by Andersson, Fries, Laestadius, and Wickström, and it agrees well with Lange's representation of the plant in Flora Danica, Supplement, t. 33. This is the plant of broadest range in America. Many extreme variations have been described by European authors. The identity of these is too often obscure, but some of the forms recognized by Mr. Arthur Bennett in Great Britain (Jour. Bot. xxxv. 248) are found to occur also in America. As extreme variations these plants may well be distinguished, though many transitional specimens occur which render their ready separation difficult. The best marked forms are the following:

Var. elatior, Bab. Man. Brit. Bot. 341; Bennett, l. c. 249. — Robust, 0.9 to 1.5 m. high: leaves 5 to 8 mm. broad: pistillate spikelets stout and heavy, 3.5 to 8 cm. long, 5 to 8 mm. thick: scales dark, blunt or acuminate, about equalling or slightly exceeding the perigynia. — Maine. Fort Fairfield and Orono (*M. L. Fernald*, nos. 136, in part, 395): New York, Pen Yan & Junius (*Sartwell*); Dexter (*G. Vasey*); Jefferson Co. (*Crawe*); Niagara Falls (*W. Boott*): Ohio (*Sullivant*): Michigan, Pêche Isle, Detroit River (*C. F. Wheeler*): Manitoba, English River (*Sir John Richardson*).[1] I have been unable to see authentic specimens of Babington's plant, but from his description and the note of Mr. Bennett, it seems probable that our large form should be referred there. The material from Orono (where the once abundant plant has been exterminated by the "improvement" of the meadow) has been described as a hybrid, *C. aquatilis* × *stricta*, Bailey, Bot. Gaz. xvii. 153; but there was little besides the local occurrence of the plant to suggest hybrid origin. The same very large form is shown in Crawe's New York material, as well as in Richardson's English River plant, and it is closely matched by Boott's plate 542, drawn from New York specimens.

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[1] Richardson's plant probably came from the river rising in Lake Sal and flowing into Lake Winnipeg from the southeast. The name *English River* has been applied to a district between the Saskatchewan and Athabasca Lake, and it was long used for the upper portion of Churchill River (emptying into Hudson Bay). This larger northern river, however, was consistently spoken of by Richardson in his Arctic Searching Expedition (1852), p. 62, &c., as Missinippi or Churchill River, while to the more southern river flowing from Lake Sal he applied the name English River (p. 362).

Var. VIRESCENS, Anders. Cyp. Scand. 46; Bennett, l. c. — Scales pale and short, mostly hidden by the closely imbricated perigynia, thus giving the spikelets a pale green color. — Northern EUROPE. The only American specimens seen are from MICHIGAN, without locality (*Michigan State Collection* in herb. Gray); near Alma (*C. A. Davis*). Material from Pownal, VERMONT, closely approaches this variety, but has longer darker scales.

Var. CUSPIDATA, Laest. ex Fries, Bot. Not. (1843) 104; Bennett, l. c. — Spikelets slender, 3 or 4 mm. thick: scales cuspidate, distinctly exceeding the perigynia. — Northern EUROPE. QUEBEC, Grand Etang, Gaspé (*J. Macoun*): NEW JERSEY, Camden (*C. F. Parker*). The Gaspé plant is a perfect match for Lapland material from Nylander, but the New Jersey specimen shows a nearer approach to typical *C. aquatilis*.

Var. EPIGEJOS, Laest. Kongl. Vet. Akad. Handl. (1822) 339; Bennett, l. c. — Very slender: the leaves 2 to 3.5 mm. broad: spikelets at most 5 cm. long, 2 to 4.5 mm. thick; scales dark and blunt. — Northern EUROPE, GREENLAND. NEWFOUNDLAND (*La Pylaie*); Packs Harbor (*A. C. Waghorne*, no. 35): LABRADOR, L'Anse au Loup (*J. A. Allen*): QUEBEC, Mont Louis, Cape Rosier, and Madaline River, Gaspé (*J. Macoun*, nos. 23, 27, 31). The material examined matches well Scandinavian material from Ahlberg. It is also identical with plants from Lapland distributed by Andersson as var. *sphagnophila*. The latter variety, however, is said by Andersson to differ from var. *epigejos* in its pale not dark scales.

CAREX PILULIFERA and C. COMMUNIS.

Carex pilulifera, L., a common species of Europe, presents three rather marked tendencies. The original plant of Linnaeus was apparently the common form with the pistillate spikelets subapproximate or slightly remote at the tip of the somewhat curved culm. This form with the lower spikelets sometimes 1 cm. apart, is represented in the Gray Herbarium by specimens from many parts of northern and central Europe. In this plant the perigynium is 2.5 to 3.5 mm. long, tipped by a short bidentate beak less than 1 mm. in length. Another phase of the plant, evidently rare in Europe, has larger more scattered spikelets, the lower often subtended by a conspicuous leafy bract; and the larger perigynia more ellipsoid or with the longer beak equalling the stipitate spongy basal portion and thus giving the perigynia a symmetrical spindle-form. This larger plant was described by Lange as var. *longibracteata* and later figured by him in Flora Danica, xvii. t. 3050; and again it has been described by Ridley and figured in Jour. Bot. xix. 97, t. 218, as var. *Leesii*. A third European form, var. *pallida*, Peterm., as shown by Reichb. Ic. Fl. Germ. viii. 26, t. 240, has the densely flowered spikelets closely approximate in an ovoid or subglobose head.

In studying this European species in connection with the well known American plant which has recently been called *C. communis*, Bailey, the writer has been baffled in every attempt to find constant distinguishing characters to separate the plants of the two continents. The form of the plant most common perhaps in America is apparently rare in Europe (var. *longibracteata*, Lange; var. *Leesii*, Ridley), but it passes by absolutely promiscuous variations into a small form which can be distinguished in none of its characters from the smaller tendency of the European *C. pilulifera*.

By early caricologists the American plant was supposed to be *Carex varia*, Muhl., and under that name it passed until in 1889 Professor Bailey showed that Muhlenberg's plant was the more slender species described by Dewey as *C. Emmonsii*. In place of the misapplied name, *C. varia*, Professor Bailey proposed for the plant which had long borne that name the new appellation *C. communis*, giving no suggestion that the plant has close affinity to the common *C. pilulifera* of Europe. To earlier students, however, the separation of the American and European plants of this group had presented many perplexities. Drejer stated in his Revisio that he could find no distinctions either in the descriptions or specimens: "Forsitan nostra planta rectius cum *C. varia* Mühlenb. conjungitur; quo modo autem *C. variam* a *C. pilulifera* distinguam, neque ex descriptione neque ex speciminibus eruere possum."[1] Schlechtendahl discussing specimens in the Willdenow herbarium which he took for *C. varia* was unable to point out any character to separate it from *C. pilulifera*: "Species haec vero simillima *C. pululiferae* et uti nobis fere videtur eadem."[2] Whether Drejer and Schlechtendahl had true *C. varia* of Muhlenberg or the coarser plant which so long passed under that name is not perfectly clear, although it is probable that Schlechtendahl at least had the true *C. varia*.[3] This plant, the true *C. varia* (*C. Emmonsii*, Dewey) is readily distinguished from *C. pilulifera* by its much more slender habit, very narrow leaves and smaller-bodied longer-beaked perigynia.

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[1] Drejer, Rev. Crit., 55.

[2] Linnaea, X. 262.

[3] See Bailey, Mem. Torr. Club., I. 40.

The coarse American plant, *C. communis*, Bailey, which until recently passed as *C. varia*, presents, however, less definite marks of specific distinctness. The most careful analysis of the characters which are supposed to separate *C. communis* (*C. varia* of authors) from *C. pilulifera* was published by Francis Boott, who inclined to regard the two species as separable. In his discussion of *C. pilulifera*, Boott said: "A *C. varia*, Muhl. [*C. communis*, Bailey], differt spicis confertis, plurifloris, subinde apice masculis, e viridi-purpureo variegatis; perigyniis enerviis, rostello semper recto breviore bidentato; basi styli persistente abrupte compresso-deflexa; culmo incurvo, basi vaginis foliorum pallide ferrugineis tecto; foliis viridibus."[1] In discussing *C. varia* [*C. communis*, Bailey] he said: "A *C. pilulifera* differt inflorescentia laxa; spicis plus minus remotis, laxifloris, saepe paucifloris; perigyniis subinde nervatis, rostro nunc excurvato, bifido; basi styli persistente recta; vaginis foliorum purpureis."[2]

When we analyze these supposed differences in the light of old specimens and the abundant modern ones which have accumulated since the publication of Dr. Boott's work, certain traditional marks of separation fail. The large form of the American plant figured by Boott (t. 288) as *C. varia*, and treated by Bailey as *C. communis* and by Britton as *C. pedicellata*, has the spikelets more remote than in the common European form of *C. pilulifera*; but a comparison of this plate with Lange's illustration of his *C. pilulifera*, var. *longibracteata* (Fl. Dan. xvii. t. 3050) and the figure of *C. pilulifera*, var. *Leesii* (Jour. Bot. xix. t. 218), shows that the rarest form of the European plant is not to be distinguished by the crowding of the spikelets from our larger form of C. *communis*. If, furthermore, we compare Boott's *C. varia*, var. *minor* (t. 289), a common plant in America, with the smaller European specimens of *C. pilulifera* with slightly remote spikelets, no constant difference can be found to separate them. The plant in America passes by innumerable transitions to the coarsest form (var. *longibracteata*), as shown in the large middle specimen in Boott's t. 289, but in its extreme form, as shown by the smaller specimens in that plate, the spikelets are often subapproximate. A comparison of this plate as well as scores of American specimens such as Eggleston's no. 434 from Middlebury, Vermont; Brainerd's material from Mt. Mosalamoo, Vermont; no. 4897b of the Biltmore Exsiccatae from Craggy Mt., North Carolina; Bailey's material of June 13, 1888, from West Harrisville, Michigan, and his no. 187 from Lansing;

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[1] Ill., II. 96.

[2] Ibid. 98.

Wheeler's specimens from Grand Ledge, Michigan; Macoun's 1876 material from Quesnelie, British Columbia, with specimens of *C. pilulifera* from Berne, Switzerland (*Seringe*); Stockholm, Sweden (*Andersson*); Finland (*Simming*); the Grosser Pfalzberg, Austria (*Halácsy*, no. 1064), and St. Petersburg, Russia (*Turczaninow*); shows conclusively that the remoteness of the spikelets is not to be relied upon in separating our smaller American material from the European plant. In the accompanying tabulation of measurements from European specimens and the smaller form of the American plant it will be seen that in the length of the inflorescence and the number, length and remoteness of spikelets essentially identical conditions are found, although the European material shows a tendency to a reduction in the length of the rachis between spikelets, thus passing to the short-headed var. *pallida*, while the American plant varying toward the elongated variety *longibracteata* shows a natural lengthening of the rachis.

Dr. Boott laid stress upon the more abundantly flowered spikelets of *C. pilulifera*, but an examination of the European material shows that this character is maintained only in the extreme specimens with unusually full spikelets. In the others many spikelets are found bearing less than ten flowers while not a few have only four or five. The presence or absence, in the American or the European plant, of staminate flowers at the tips of the pistillate spikelets is likewise a character upon which little reliance can be placed. Both Goodenough[1] and Dr. Boott[2] noted this tendency in European specimens and in a sheet of Austrian material it is very conspicuous. In America likewise this tendency to androgynous spikelets occurs, but it seems to be quite as unusual as in Europe.

The pale or castaneous scales of *Carex communis* were emphasized by Dr. Boott as opposed to the purple scales of *C. pilulifera*. Students of American Carices, however, are all familiar with specimens of *C. communis* from sunny or open situations in which the scales are quite as purple (or rather maroon) as in *C. pennsylvanica*; and many specimens of European *C. pilulifera* show quite as little color in the scales as do the commoner plants of America.

The basal nerves supposed to distinguish the perigynium of *C. communis* from that of *C. pilulifera* are also quite as often wanting as present; and although Dr. Boott laid stress upon this character in his comparative note, he described the perigynia of *C. communis* (his *C. varia*) as "enerviis vel basi plus minus nervatis pallidis." The length,

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[1] Trans. Linn. Soc., II. 191.

[2] Ill., II. 96.

TABLE OF MEASUREMENTS OF EUROPEAN *Carex pilulifera* AND THE SMALLER FORM OF AMERICAN *C. communis*.

European Specimen.	Collector.	Length of Inflorescence in mm.	Length of staminate Spikelet in mm.	Number of pistillate Spikelets.	Length of pistillate Spikelet in mm.	Distance in mm. between lowest Spikelets.	Length of Perigynia in mm.	Length of Beak in mm.
Strömbacka, Sweden . .	Laurén	12–18	6–8	2–3	4.5–6	4.5–7	3.4	0.6
Finland	Simming	16–19	6–7	3–4	4	8	2.8	0.8
Stockholm, Sweden . .	Andersson	17–22	10–11	2–3	7	5–7	2.7	0.7
Halifax, England . . .	Leyland	13–22	7–10	2–3	5.5	5–10	3.0	0.7
Dresden, Germany . . .		20	9–13	3	4–6	7.5	2.9	0.7
Halle, Germany	A. Schulz	18–23	8–9	4	4–8.5	9	3.0	0.8
Berne, Switzerland . .	Seringe, no. 1238	17–22	10	2–3	4.5–7	3.5–7	3.0	0.8
Upsala, Sweden	Angström	14–26	9–16	2–4	5–9	3–9	2.8	0.7
Kyffhauser (Mt.), Germ'ny		17–22	11	3	6–8	6	3.0	0.7
Grosser Pfalzberg, Austria	Halácsy, no. 1064	14–26	7–18	1–3	3–6	3.5–6.5	2.7	0.9
Salzburg, Austria . . .	Hoppe	26–32	11–13	4–5	6–11	6–9	3.0	0.7
St. Petersburg, Russia .	Turczaninow	18–23	9–11	2	5–7	6–9	2.9	0.7
Upsala, Sweden	Tuckerman	25	10	4	6–8	9	3.0	0.8
Snowdon, Wales . . .	J. Ball	18	6.5	3	6	6	2.9	0.8
Extremes in Europe . .		12–32	6–18	1–5	3–11	3–10	2.7–3.4	0.6–0.9
American Specimen.								
Keweenaw Co., Mich. . .	Farwell, no. 653	10–13	4	2–3	4	5–6	3.3	0.8
Alcona Co., Mich. . . .	Bailey	15–19	4–8	2	5–7	7–15	3.2	0.8
Burlington, Vt.	Jones & Eggleston	11–23	6–13	1–2	6–8.5	7–8	3.2	0.8
Quesnelle, Brit. Columbia	Macoun	15–23	9–10	2–3	5–6	5–10	3.0	0.7
Grand Ledge, Mich. . .	Wheeler	18–19	8	3	5–6	6–9	3.3	0.8
Mt. Mosalamoo, Vt. . .	Brainerd	17–23	8	3	4–6	7–12	2.4	0.8
East Mt., Middlebury, Vt.	Eggleston, no. 434	17–24	9–11	2–3	5–6.5	5–13	2.6	0.9
Willoughby Mt., Vt. . .	Faxon	13–26	6.5–10	1–2	4–6	8–12	3.0	0.7
Lake Memphremagog, Q'b.	Faxon	13–29	3.5–9.5	2–4	4–8.5	7–11	3.0	0.9
Craggy Mt., No. Carolina	Biltmore Herb., no. 4897[b]	21–31	9–16	2–3	4–8	7–11	2.8	0.7
Orono, Me.	Fernald	23–36	6–13	3–4	4–9	11–12	3.1	1.0
Franconia, N. H. . . .	Faxon	24–39	8–11	3–4	4–9	7–15	3.3	0.8
Milwaukee, Wis. . . .	Lapham	30–35	14	3	4–7	9	3.0	0.9
Lansing, Mich.	Bailey, no. 187	25–39	13–18	2	4–8	11–14	3.2	0.8
Extremes in America . .		10–39	3.5–18	1–4	4–9	5–15	2.4–3.3	0.7–1.0

bending, and orifice of the beak show likewise considerable variation in Old World specimens, all of which can be matched by our plant, while the curving of the base of the style is a tendency not infrequent in American as well as European specimens. On the other hand, the straight style supposed to characterize the American plant is clearly represented by Lange in his plate of *C. pilulifera*, var. *longibracteata.*

The deeper purple coloring of the lower sheaths of the American plant, a character much emphasized by authors, is not a satisfactory distinction. The color in the American plant is usually conspicuous and is pronounced by Mr. F. Schuyler Mathews a dilute maroon with no true purple tendency, but rather fading in the older sheaths to chestnut. Mr. Mathews, who has likewise examined the sheaths of European specimens, finds the same red present in them. This color of the sheaths generally fades with age, yet in specimens collected by John Ball on Snowdon, by Andersson at Stockholm in 1860, by Laurén at Strömbacka in 1855, and by Tuckerman at Upsala in 1841 or 1842, show quite as conspicuous a red as the average American plant.

The bright green color of the leaves of *C. pilulifera* has likewise been maintained as a character separating that plant from the American *C. communis.* From dried specimens alone it is impossible to make this distinction apparent, although the fresh plant may sometimes show a brighter color than is often seen in *C. communis.* Yet in the American plant the leaves vary from a weak to a deep green, and in Bailey's var. *Wheeleri*, which is certainly inseparable from European specimens of *C. pilulifera*, the leaves were originally described as "bright green."

The length of the staminate spikelet and the breadth of the leaves, two characters upon which stress is sometimes laid, were not emphasized by Dr. Boott. An examination of the accompanying table of measurements of the inflorescence will show that the length of the staminate spikelets is thoroughly inconstant and not concomitant with other characters. In fact, both short and long staminate spikelets are often found on the same individual, as shown by Halácsy's no. 1064 of the Austro-Hungarian Exsiccatae (staminate spikelets from 7 to 18 mm. long), by Fernald's no. 151 from Maine (spikelets 6 to 13 mm. long), and a Faxon plant from Franconia, New Hampshire (spikelets from 8 to 14 mm. long). The variations in the breadth of the leaf, likewise, are very great on both continents. The young leaves at the fruiting season are naturally much narrower than the old and weather-beaten ones, which, unfortunately, are too often torn away in the preparation of attractive specimens. Measurement of the breadth of these older leaves where present shows in the American plant a variation from 2 to 5.5 mm. and in the European from 2 to 4.5 mm. These measurements, however, include the largest American form, in which all the parts are conspicuously more developed than in the smaller American and the apparently identical European plant.

The length of the lower bract, emphasized in the descriptions of *C. pilulifera*, var. *longibracteata* and var. *Leesii*, seems to the writer an unfortunate character to make prominent. In America, at least, this elongation of the bract accompanies no other definable character. It is a purely vegetative development which may occur either in the large form (*C. varia* [typical] of Boott's Ill. t. 288) or in the smaller *C. communis*, var. *Wheeleri* with shorter inflorescence and more approximate spikelets.

This study of the European *Carex pilulifera* and the American *C. communis* (*C. varia* of many authors) has led to the following conclusions. The form of *C. pilulifera* of Europe with the pistillate spikelets subapproximate or slightly remote, the lowest from 0.5 to 1 cm. apart, is also common in America, where the plant has passed generally as *C. varia*, var. *minor*, Boott; *C. communis*, Bailey, and *C. pedicellata*, Britton, in part; or *C. communis*, var. *Wheeleri*, Bailey (*C. pedicellata*, var. *Wheeleri*, Britton). Another European form, the large *C. pilulifera*, var. *longibracteata*, Lange, is rare in Europe, but in America is represented by the large extreme which has passed as *C. varia* and later as *C. communis* and *C. pedicellata*. The American plants, then, should be called

C. PILULIFERA, L. Culms 1 to 5 dm. high, usually overtopping the leaves: inflorescence 1 to 3.5 cm. long, the lowest spikelet subtended by a short and narrow or sometimes elongated broad bract: staminate spikelet from green to chestnut-brown or maroon, sessile or stalked, 3.5 to 20 mm. long; pistillate spikelets 1 to 5, loosely flowered, 4 to 11 mm. long, sessile or short-pedicelled, subapproximate or slightly remote, the lowest rarely 1.5 cm. apart: perigynia hairy, obscurely 3-angled, 2.5 to 3.5 mm. long, the body plump, obovoid or subglobose, with a more or less elongated spongy nerveless or slightly nerved stipitate base; the beak broad, bidentate, rarely 1 mm. long, nearly or quite equalled by the green brown or reddish-brown ovate acuminate scale. — Sp. 976; Gooden. Trans. Linn. Soc. ii. 190; Schk. Riedgr. 78, t. I, fig. 39; Andersson, Cyp. Scand. 30, t. 7, fig. 82; Reichb. Ic. Fl. Germ. viii. t. 260; Boott, Ill. ii. 96, t. 283. *C. filiformis*, Pol. Pl. Palat. ii. 581; Vahl, Fl. Dan. vi. t. 1048; not L. *C. Bastardi-*

ana, DC. Fl. Fr. vi. 293. *C. varia*, Authors, incl. Boott, Ill. l. c. 97, in part, not Muhl. *C. varia*, var. *pedicellata*, Dewey, Am. Jour. Sci. xi. 163, in part. *C. varia*, var. *minor*, Boott, l. c. t. 289. *C. communis*, in part, and var. *Wheeleri*, Bailey, Mem. Torr. Cl. i. 41. *C. pedicellata*, in part, and var. *Wheeleri*, Britton, Mem. Torr. Cl. v. 87. 88. — In dry soil, NEW BRUNSWICK to BRITISH COLUMBIA, NORTH CAROLINA, OHIO and WISCONSIN: common in EUROPE. Passing gradually to

Var. LONGIBRACTEATA, Lange. Coarser: the inflorescence often 5 to 8 cm. long, the usually fuller and longer pistillate spikelets remote, the lowest 1.5 to 4 cm. apart: perigynia larger, more ellipsoid or spindle-form, with longer beak. — Haandb. Dansk. Fl. 621, & Fl. Dan. xvii. 12, t. 3050; Kneucker, Allgem. Bot. Zeitschr. (1898) 128. *C. varia*, Authors, in part, incl. Boott, l. c. t. 288, not Muhl. *C. varia*, var. *pedicellata*, Dewey, l. c., in part. *C. saxumbra*, F. A. Lees, Jour. Bot. xix. 25. *C. pilulifera*, var. *Leesii*, Ridley, Jour. Bot. xix. 98, t. 218. *C. communis*, Bailey, l. c. in part. *C. pedicellata*, Britton, l. c. in part. — NEW BRUNSWICK to IOWA and GEORGIA: rare and local in northern EUROPE.

CAREX PENNSYLVANICA.

Carex pennsylvanica, Lam., is one of the widest-distributed of the North American Carices, and as one of the earliest-flowering it is perhaps better known to the general botanist than any of the other species. In the length and breadth of its leaves, the comparative height of its culm, etc., the plant shows considerable variation, and many formal varieties have been based upon these characters. But since they are all of a purely vegetative nature, often produced in a colony of the species by changes of ecological conditions, none of these variations seem to the writer of sufficient constancy to merit recognition as more than trivial forms. The color of the spikelets, also, a character too commonly relied upon to separate *C. pennsylvanica* from the closely related *C. pilulifera*, L. (*C. communis*, Bailey), is not to be accepted as final, since *C. pennsylvanica*, ordinarily characterized by dark reddish brown scales, may often have them pale or even straw-colored when growing in deep shade. Furthermore, *C. pilulifera* in northern Europe as well as in America is frequently found with dark red scales, especially when growing in very sunny or exposed situations. The simplest means of distinguishing *C. pennsylvanica* from its nearest common ally is in its stoloniferous character; for when well developed the plant produces conspicuous elongated stolons, while *C. pilulifera* (*C. communis*) is caespitose, with short assurgent basal shoots. As may be implied, varieties of *C. pennsylvanica* based upon color of the spikelets are quite as inconstant as are those based upon the length or breadth of the leaf, or other purely vegetative tendencies. In the character of its perigynia, however, *C. pennsylvanica* presents three marked variations which, from the material examined, seem to belong to well marked geographic areas. These forms of the plant are:

C. PENNSYLVANICA, Lam. Dict. iii. 388. Strongly stoloniferous; the slightly caespitose small stools with reddish bases: leaves soft, comparatively narrow, 1.5 to 3.5 mm. broad, 0.5 to 5 dm. long, shorter than, equalling, or often exceeding the slender culms: pistillate spikelets 1 to 4, globose or ovoid, loosely flowered, approximate or more or less remote, the lowest rarely peduncled, often subtended by a narrow leafy bract: scales usually maroon or red-tinged, rarely pale: perigynia from subglobose to obovoid, puberulent, the short bifid beak one-fourth to one-fifth as long as the body: staminate spikelet clavate, 1 to 2 cm. long, sessile or short-stalked, usually reddish, rarely straw-colored. — In dry or sandy soil from Cumberland Co., MAINE, to ALBERTA, south to GEORGIA and NEW MEXICO. It is impossible to say from the original description whether this or the following variety was intended by Lamarck, but the commonest form of the species has been accepted as typical since it was so considered by Boott, Kunze, and other classic writers on the genus. The varieties and forms described by Peck (46 Rep. N. Y. Mus. Nat. Hist. 51; 48 Rep. 76) appear to be vegetative states due largely to different degrees of light and exposure.

Var. **lucorum**. Perigynium puberulent or glabrate, with a conspicuous slender beak nearly or quite as long as the body. — *C. lucorum*, Willd. Enum. Pl. Berol. Suppl. 63; Kunze, Car. 153, t. 39; Boott, Ill. ii. 98, t. 291, in part. — MAINE to MICHIGAN and "ARCTIC AMERICA," and in the mountains to NORTH CAROLINA. MAINE, Orono, May 31, 1890, June 4, 1898 (no. 2006) — *M. L. Fernald;* Cambridge (*F. S. Bunker*); Glassface Mt., Rumford, July 13, 1890 (*J. C. Parlin*): NEW HAMPSHIRE, Barrett Mt., New Ipswich, June 5, 1896 (*M. L. Fernald*): VERMONT, Chipman Hill, Middlebury, May 30, 1897, Burlington, June 16, 1898 (*E. Brainerd*); Pownal, May 29, 1898 (*J. R. Churchill*): MASSACHUSETTS, Spot Pond, Stoneham, May 29, 1855, Malden, June 11, 1861, Medford, May 21, 1865. Blue Hills, Milton, June 3, 1870 (*Wm. Boott*); Purgatory Swamp, Dedham, May 26, 1878 (*E. & C. E. Faxon*); Wilmington, May 14, 1899 (*E. F. Williams*):

RHODE ISLAND, Cumberland (*S. T. Olney*): CONNECTICUT, Southington, June 4, 1899 (*C. H. Bissell*); Fairfield, June 23, 1901 (*E. H. Eames*, no. 168): MICHIGAN, Detroit, May 22, 1864, June, 1860, May 9, 1858 (*Wm. Boott*): VIRGINIA, Harper's Ferry, May 7, 1881 (*John Donnell Smith*): NORTH CAROLINA, Broad River, May, 1841 (*Rugel* according to Kunze, l. c.). The long slender beak of the perigynium and its essentially northern and montane range suggest that further knowledge of the plant may show it to be well distinguished from *C. pennsylvanica*. No other character has yet been found by which it can be recognized, and occasional individuals show transitions in the elongation of the beak.

Var. VESPERTINA, Bailey, Mem. Torr. Cl. i. 74. Rather coarser than the species: the usually very dark staminate spikelet peduncled: perigynia more coarsely hairy, almost hirsute. — The northwestern form, from the Cascade Mts. of BRITISH COLUMBIA to OREGON and VANCOUVER ISLAND.

CAREX UMBELLATA.

Like *C. pilulifera* and *C. pennsylvanica*, *C. umbellata*, Schkuhr, presents considerable variation in the length and breadth of its leaves and in the length of its culms and peduncles. As in those species, likewise, these purely vegetative characteristics in *C. umbellata* seem to accompany no fixed characteristic of the perigynia, nor any special geographic areas; and too often the long-peduncled spikelets of the so-called var. *vicina* may be found on portions of a clump which is otherwise good *C. umbellata*. As in the related species just discussed, however, *C. umbellata* presents at least two geographic tendencies seemingly characterized by constant differences in the perigynia. A third form, of which we as yet know too little, has the perigynia glabrous, thus breaking through one of the distinguishing marks of the *Montanae*.

Carex umbellata is related on the one hand to *C. nigro-marginata*, and on the other to *C. deflexa*. From these two it is usually distinguished without difficulty, but occasional specimens occur which are perplexing. The writer has found that in such cases the best means of distinction between *C. umbellata* and *C. nigro-marginata* is offered by the thickness of the perigynia. In *C. nigro-marginata* the mature perigynia vary from 1.3 to 1.6 mm. in thickness, while in mature *C. umbellata* they are from 1.7 to 2.4 mm. thick. From doubtful forms of *C. deflexa*, *C. umbellata* may best be distinguished by an examination of the scales. In *C. umbellata* the scales are nearly or quite as long as the subtended perigynia, while in *C. deflexa* they are distinctly shorter.

The most marked tendencies of *C. umbellata* are

C. UMBELLATA, Schkuhr, Riedgr. Nachtr. 75, t. Www, fig. 171 (*C. umbellata*, var. *vicina*, Dewey, Am. Jour. Sci. xi. 317 & x. t. D, fig. 13). Low and conspicuously caespitose, forming dense mats: leaves rather stiff, 0.5 to 4.5 dm. long, 1 to 4.5 mm. wide: culms mostly very short and crowded at the base of the leaves, or some elongated, rarely even to 2 dm., and bearing both staminate and pistillate, or staminate spikelets alone: pistillate spikelets 1 to 4, ovoid or oblong, 0.5 to 1 cm. long, sessile or on short or occasionally elongate-capillary peduncles: perigynia plump, stipitate or substipitate, puberulent, 3.2 to 4.7 mm. long; the slender beak nearly or quite as long as the ellipsoid-ovoid to subglobose or pyriform body, and about equalled by the ovate acuminate green or purple-tinged scale: staminate spikelets subsessile or peduncled, 6 to 12 mm. long. — Dry sandy or rocky places, PRINCE EDWARD ISLAND to central MAINE, west to SASKATCHEWAN and BRITISH COLUMBIA, and south to NEW JERSEY, DISTRICT OF COLUMBIA, and INDIAN TERRITORY.

Var. **tonsa**. Similar, but with the perigynia glabrous or merely puberulent on the angles of the long beak. — MAINE, Streaked Mt., Hebron, June 2, 1897 (*J. A. Allen*): CONNECTICUT, rocky wooded slope of Lantern Hill, North Stonington, May 30, 1901 (*C. B. Graves*). A plant with identical glabrous perigynia is figured in Boott, Ill. ii. t. 293, from specimens collected at Methy Portage, ATHABASCA, by *Sir John Richardson*. This and the New England plant represent a tendency unusual in the *Montanae*.

Var. BREVIROSTRIS, Boott, Ill. ii. 99, t. 294. Perigynia rather smaller, the broad beak short, about one-third as long as the plump short-hairy body. — The commonest form from SASKATCHEWAN to VANCOUVER ISLAND, south in the mountains to CALIFORNIA and NEW MEXICO: also MAINE, Fort Kent, Ashland, Masardis, Island Falls and Foxcroft (*M. L. Fernald*, nos. 2111, 2112, 2113, 2114, 2115); summit of Sargent Mt., Mount Desert Island (*E. & C. E. Faxon*): NEW HAMPSHIRE, Mt. Willard, and Bald Mt., Franconia (*E. & C. E. Faxon*).

CAREX VAGINATA and C. SALTUENSIS.

C. VAGINATA, Tausch, Flora (1821) 557 (*C. vaginata*, var. *alto-caulis*, Dewey, Am. Jour. Sci., Ser. 2, xli. 227. *C. saltuensis*, Bailey, Mem. Torr. Cl. i. 7. *C. altocaulis*, Britton, in Britton & Brown, Ill. Fl. i. 326, fig. 773). The American plant was long considered by Francis Boott and other caricologists identical with the European; but in 1866 the New York plant was distinguished by Dewey, on account of its tall slender culm, narrow leaves and loose spikelets as var. *alto-caulis*. In 1889, however, Professor Bailey raised the American plant to specific rank as *C. saltuensis*, separating it from the European *C. vaginata* "by its much more slender and less caespitose habit, narrower leaves and less conspicuous sheaths, its alternately-flowered spikes, and its much smaller, less inflated, and conspicuously nerved perigynium." And Dr. Britton, following Professor Bailey's lead in treating the plant as strictly American, has taken up for it Dewey's varietal name as *altocaulis* (not *alto-caulis*).

That American specimens from the deep swamps of western New York, Ontario and Michigan are more slender than some European specimens there can be no doubt; but in northeastern Maine, where the plant is a common species of arbor-vitae swamps, it varies greatly in these characters. Individuals growing in excessive shade are naturally taller and more slender than those in bright light; and the spikelets vary indiscriminately from the slender alternate-flowered tendency supposed to characterize the American plant to the dense-cylindric form said to distinguish the European.

The height of the European plant, too, is often as great as that of the American, while our own plant sometimes fruits when scarcely 2 dm. high (Mt. Albert, Quebec — *Allen*: Blaine, Maine — *Fernald*). A specimen from Christiania collected by Blytt is 5 dm. high, while the extreme height given by Dr. Britton for his *C. altocaulis* is 2 feet (6 dm.).

The breadth of the leaf, likewise, is as variable on one continent as on the other. Both Dewey and Bailey have maintained that the European plant is broader-leaved; yet a specimen from Fries collected in Jemtland (Sweden) has leaves from 1.5 to 1.75 mm. wide, while the broadest leaves seen on the European plant are those of a Lapland specimen (5 mm. wide) from N. J. Andersson. In the American plant the leaves vary from 1.5 mm. wide (Blaine, Maine) to 5 mm. (Montreal).

The variation in the density of the spikelet in the American plant has been already mentioned. In Europe the same variation occurs, specimens from Jemtland (*Ahlberg*), Lapland (*Andersson*) and Finland (*Lehmann*) having the spikelets as loosely flowered as in the most extreme American form.

Nor are the differences assigned by Professor Bailey to the perigynia maintained in mature specimens. Young individuals of the American as well as the European plant have the nerves poorly developed, but in mature fruit no difference is apparent between plants from Christiania, Norway, and Aroostook Co., Maine.

The sheath, said by Professor Bailey to be "less conspicuous" in the American plant, is 4 cm. long, by 2.7 mm. wide in one of Macrae's Montreal specimens, fully as conspicuous as in the best developed European material. There is, then, no reason why the American *Carex saltuensis*, Bailey (*C. altocaulis*, Britton) with no constant vegetative or morphological character and with a broad range from northern Labrador to the Mackenzie River, northern New England, New York, the Great Lakes and the upper Rocky Mts., should be treated as distinct from *C. vaginata* of Greenland, northern Europe and Asia.

CAREX CAPILLARIS.

C. CAPILLARIS, L. Sp. 977. The Linnaean plant was the low plant of the Scandinavian mountains, described as a span high. This plant, well represented in the Gray Herbarium by European specimens from Andersson, Holmgren, Hoppe, Lehmann, Tuckerman, and others, varies in height from 3 to 25 cm., the spikelets being subapproximate or scarcely remote, the lower at most 2 cm. apart. This dwarf plant occurs likewise in Greenland and northeastern Asia. It has been examined from the following regions in America — LABRADOR, Dead Islands (*J. A. Allen*): NEWFOUNDLAND, without locality (*La Pylaie*); Middle Arm, Bay of Islands (*A. C. Waghorne*): QUEBEC, dry stony ground, near summit — 1,150 m. — Mt. Albert (*J. A. Allen*): MAINE, Mt. Kineo (*T. C. Porter et al*): NEW HAMPSHIRE, Mt. Washington (*Wm. Oakes et al*): COLORADO, Rocky Mts., alt. 3,385 m. (*E. L. Greene* in Exsicc. Olney); South Park (*J. Wolfe*, no. 1059); Clear Creek, Georgetown, alt. 2,615 m. (*H. N. Patterson*, no. 144, in part): UTAH (*S. Watson*, no. 1261): WYOMING, La Plata Mines (*E. Nelson*, no. 5260).

Var. ELONGATA, Olney, in herb. & in Rothr. Prelim. Rep. Wheeler Pl. 53 (as *nomen nudum*). Tall, 2 to 6 dm. high, forming loose stools: pistillate spikelets remote, often 6 or 8 cm. apart. — Mossy woods and sphagnum-swamps. RUPERT LAND, Lake Mistassini (*J. M. Macoun*): NEWFOUNDLAND, Coal River, Bay of Islands (*A. C. Waghorne*, no. 24): QUEBEC, Ste. Anne des Monts and Little Metis (*J. A. Allen*): NEW BRUNSWICK, Drury's Cove, St. John (*Wm. Boott*): MAINE, Fort Fairfield (nos. 140, 2029), Blaine (no. 2028), Mars Hill — *M. L. Fernald*: NEW YORK, Otter Creek, near Cortland (*S. N. Cowles*):

ONTARIO, Bruce Co. (*J. Macoun*): MICHIGAN, Point de Tour (*Wm. Boott*); Port Huron (*C. K. Dodge*): SASKATCHEWAN (*Bourgeau*): ASSINIBOIA, Assiniboine River (*J. Macoun*): ALBERTA, Bow River (*J. Macoun*): COLORADO, Rocky Mts., alt. 2460 m. (*E. L. Greene* in Exsicc. Olney); Twin Lakes (*J. Wolfe*, no. 1060 [type]); Clear Creek (*Parry*, no. 386, *Patterson*, no. 144, in part): UTAH, Aquarius Plateau (*L. F. Ward*, no. 484): IDAHO, Lake Pend d'Oreille (*Sandberg, MacDougal & Heller*, no. 751). A plant confined in the East to arborvitae swamps at low altitudes, and in its tall lax habit and very distant spikelets hardly suggesting the dwarf alpine *C. capillaris* with approximate spikelets. Somewhat similar specimens in the Gray Herbarium from Salzburg, Austria, suggest that the same form may be present in Europe.

INDEX TO SPECIES.

EXPLANATION OF PLATES.[1]

PLATE I.

Carex muskingumensis: Fig. 1, spike; Fig. 2, perigynium.

C. scoparia: Fig. 3, spike; Fig. 4, perigynium.

C. scoparia, var. *condensa:* Fig. 5, spike.

C. tribuloides: Fig. 6, spike; Fig. 7, perigynium.

C. tribuloides, var. *reducta:* Fig. 8, spike.

C. siccata: Figs. 9, 10, spikes; Fig. 11, perigynium.

C. Crawfordii: Fig. 12, spike; Fig. 13, perigynium.

C. Crawfordii, var. *rigens:* Fig. 14, spike.

C. oronensis: Fig. 15, spike; Fig. 16, perigynium.

C. praticola: Fig. 17, spike; Fig. 18, perigynium.

C. cristata: Fig. 19, spike; Figs. 20, 21, perigynia.

C. albolutescens: Figs. 22, 23, spikes; Fig. 24, perigynium.

PLATE II.

C. mirabilis: Fig. 25, spike; Fig. 26, perigynium.

C. mirabilis, var. *perlonga:* Fig. 27, spike.

C. straminea: Fig. 28, spike; Fig. 29, perigynium.

C. straminea, var. *echinodes:* Fig. 30, spike.

C. tenera: Fig. 31, spike; Fig. 32, perigynium.

C. tenera, var. *Richii:* Fig. 33, terminal spikelet; Fig. 34, perigynium.

C. tenera, var. *invisa:* Figs. 35, 36, spikes.

C. Bicknellii: Figs. 37, 38, spikes; Figs. 39, 40, perigynia.

C. silicea: Fig. 41, spike; Fig. 42, perigynium.

C. alata: Fig. 43, spike; Fig. 44, perigynium.

C. alata, var. *ferruginea:* Fig. 45, spike; Fig. 46, perigynium.

PLATE III.

C. festucacea: Fig. 47, spike; Fig. 48, perigynium.

C. festucacea, var. *brevior:* Figs. 49, 50, spikes; Fig. 51, perigynium.

C. Bebbii: Fig. 52, spike; Fig. 53, perigynium.

C. foenea: Fig. 54, spike; Fig. 55, perigynium.

C. foenea, var. *perplexa:* Fig. 56, spike; Fig. 57, perigynium.

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[1] The plates illustrating this synopsis were prepared by Mr. F. Schuyler Mathews from characteristic specimens. The figures of the spikes represent life-sized individuals, while those showing the *inner* faces of the perigynia are four times as large as in nature.

C. leporina: Figs 58, 59, spikes; Fig. 60, perigynium.
C. xerantica: Fig. 61, spike; Fig. 62, perigynium.
C. aenea: Figs. 63, 64, spikes; Figs. 65, 66, perigynia.
C. adusta: Fig. 67, spike; Figs. 68, 69, perigynia.
C. sychnocephala: Fig. 70, spike; Fig. 71, perigynium.

PLATE IV.

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C. exilis: Figs. 78, 79, 80, 81, 82, spikes; Fig. 83, perigynium.
C. echinata: Figs. 84, 85, 86, 87, spikes; Fig. 88, perigynium.
C. echinata, var. *ormantha:* Fig. 89, spike.
C. echinata, var. *excelsior:* Figs. 90, 91, spikes.
C. echinata, var. *cephalantha:* Figs. 92, 93, spikes; Fig. 94, perigynium.
C. echinata, var. *angustata:* Figs. 95, 96, spikes; Fig. 97, perigynium.
C. sterilis: Figs. 98, 99, spikes; Fig. 100, perigynium.
C. interior: Figs. 101, 102, 103, spikes; Figs. 104, 105, perigynia.
C. seorsa: Figs. 106, 107, spikes; Figs. 108, 109, perigynia.

PLATE V.

C. arcta. Figs. 110, 111, 112, spikes; Fig. 113, perigynium.
C. canescens: Fig. 114, spike; Fig. 115, perigynium.
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C. canescens, var. *disjuncta:* Figs. 118, 119, spikes; Fig. 120, perigynium.
C. brunnescens: Figs. 121, 122, spikes; Fig. 123, 124, perigynia.
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C. Deweyana: Fig. 127, spike; Fig. 128, perigynium.
C. tenuiflora: Fig. 129, spike; Fig. 130, perigynium.
C. trisperma: Fig. 131, spike; Fig. 132, perigynium.
C. elachycarpa: Fig. 133, spike; Fig. 134, perigynium.
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C. glareosa: Fig. 137, spike; Fig. 138, perigynium.
C. lagopina: Fig, 139, spike; Fig. 140, perigynium.
C. heleonastes: Fig. 141, spike; Fig. 142, perigynium.

Fernald — Carex § Hyparrhenae.

Plate I.

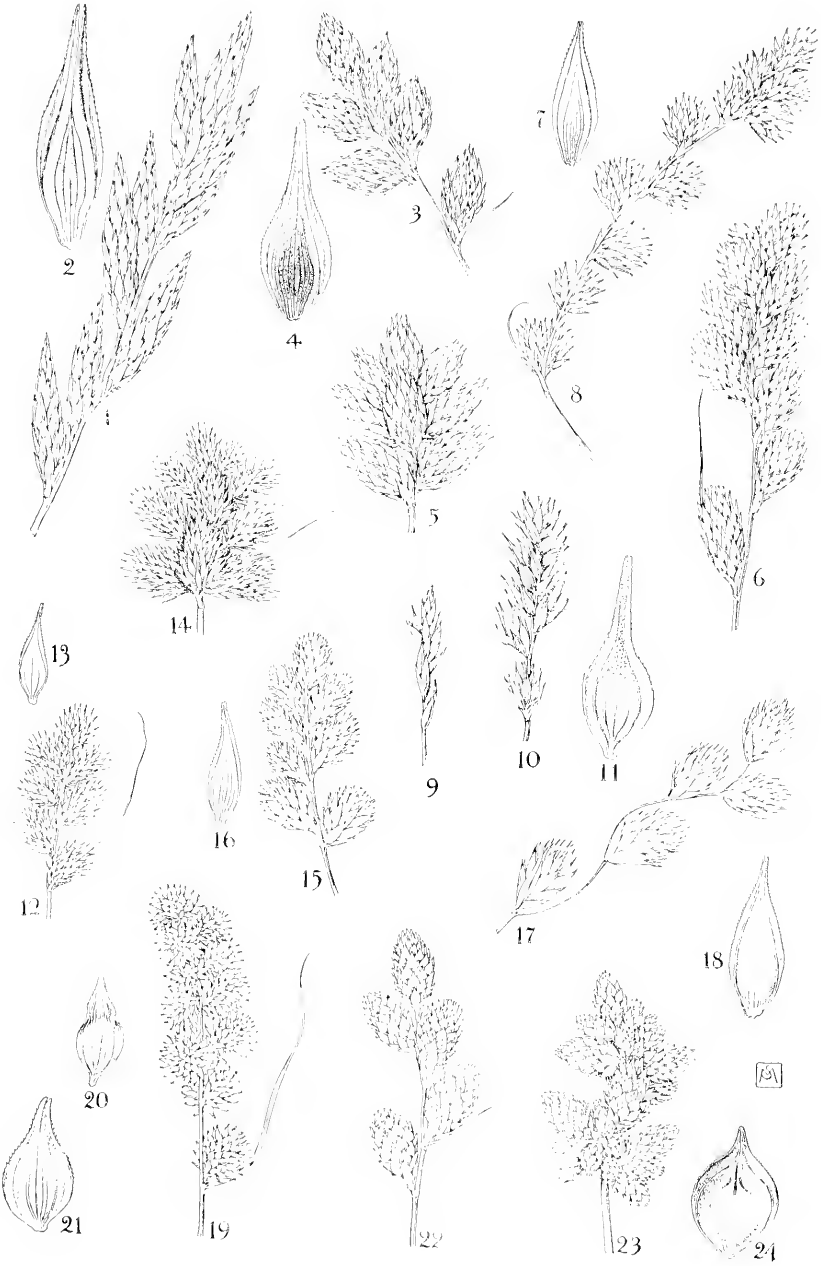


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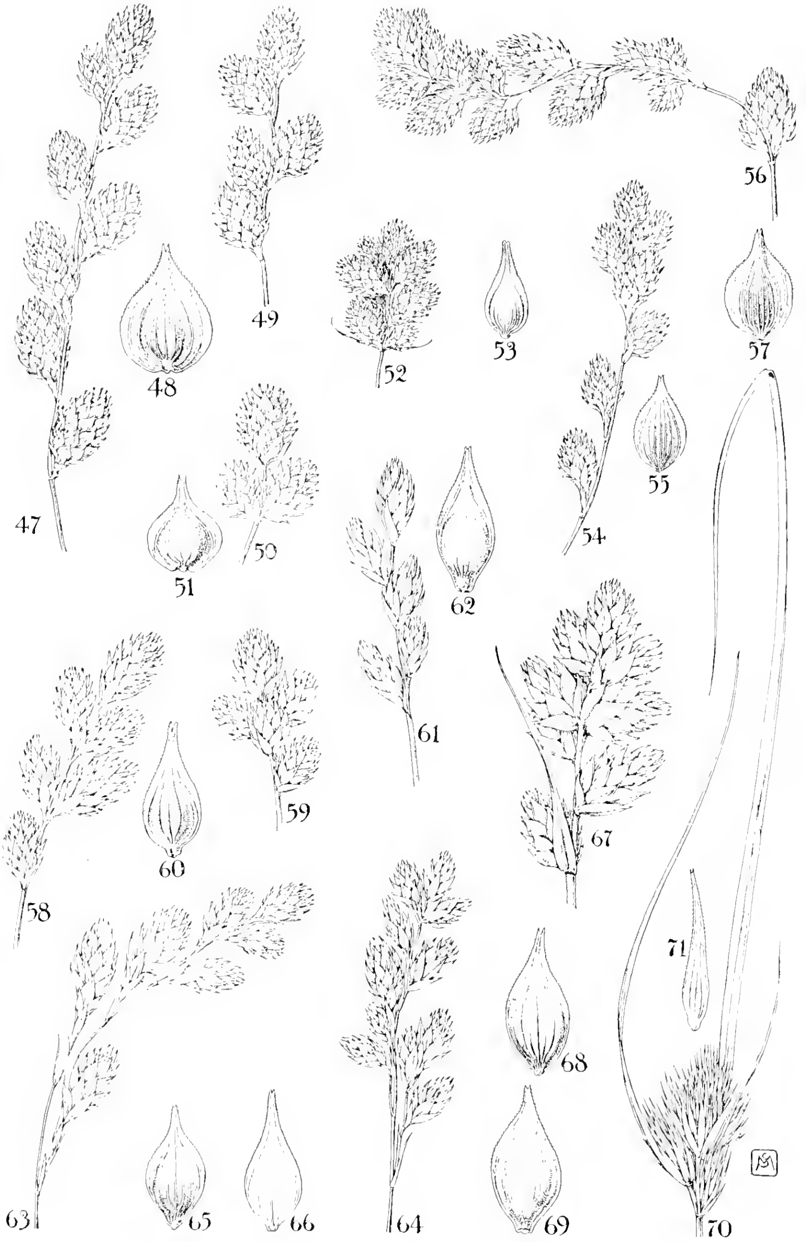
Plate III.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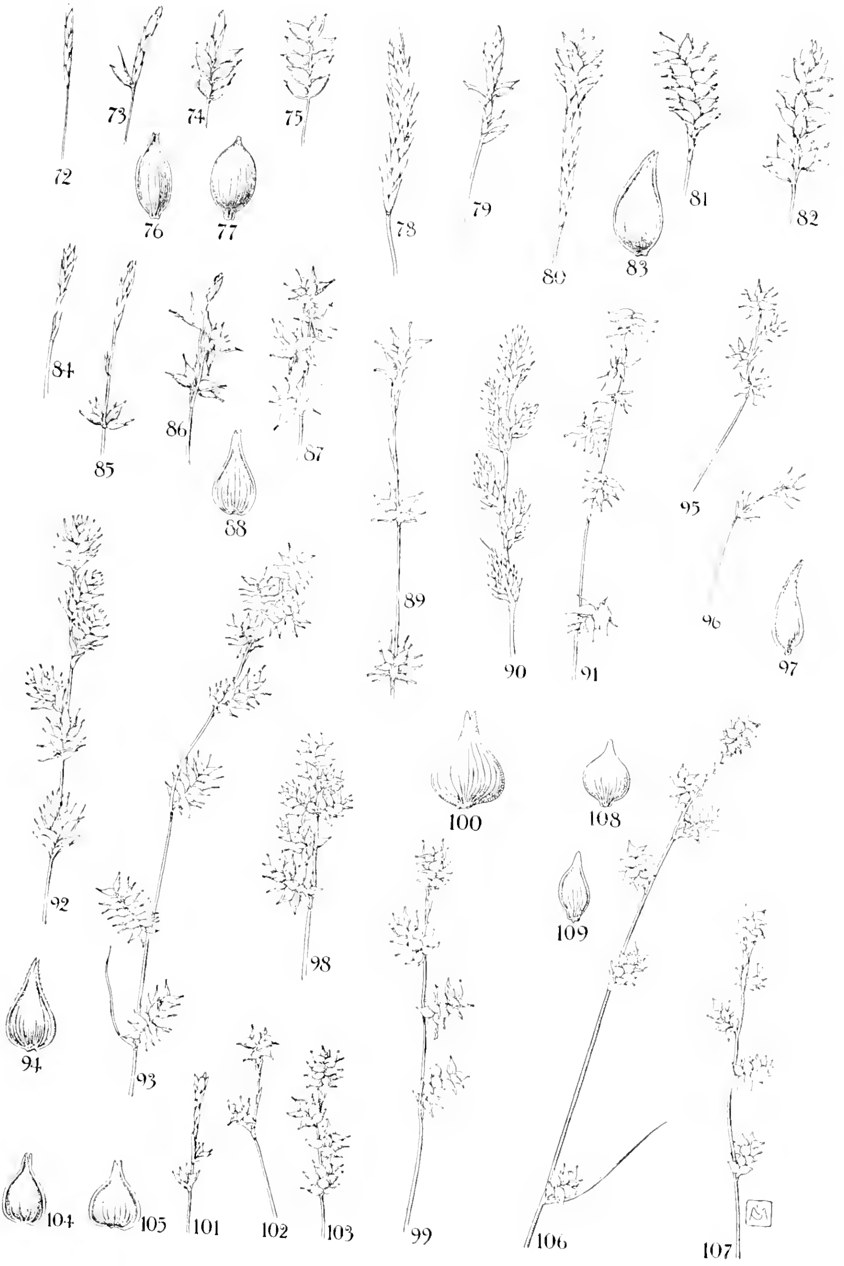
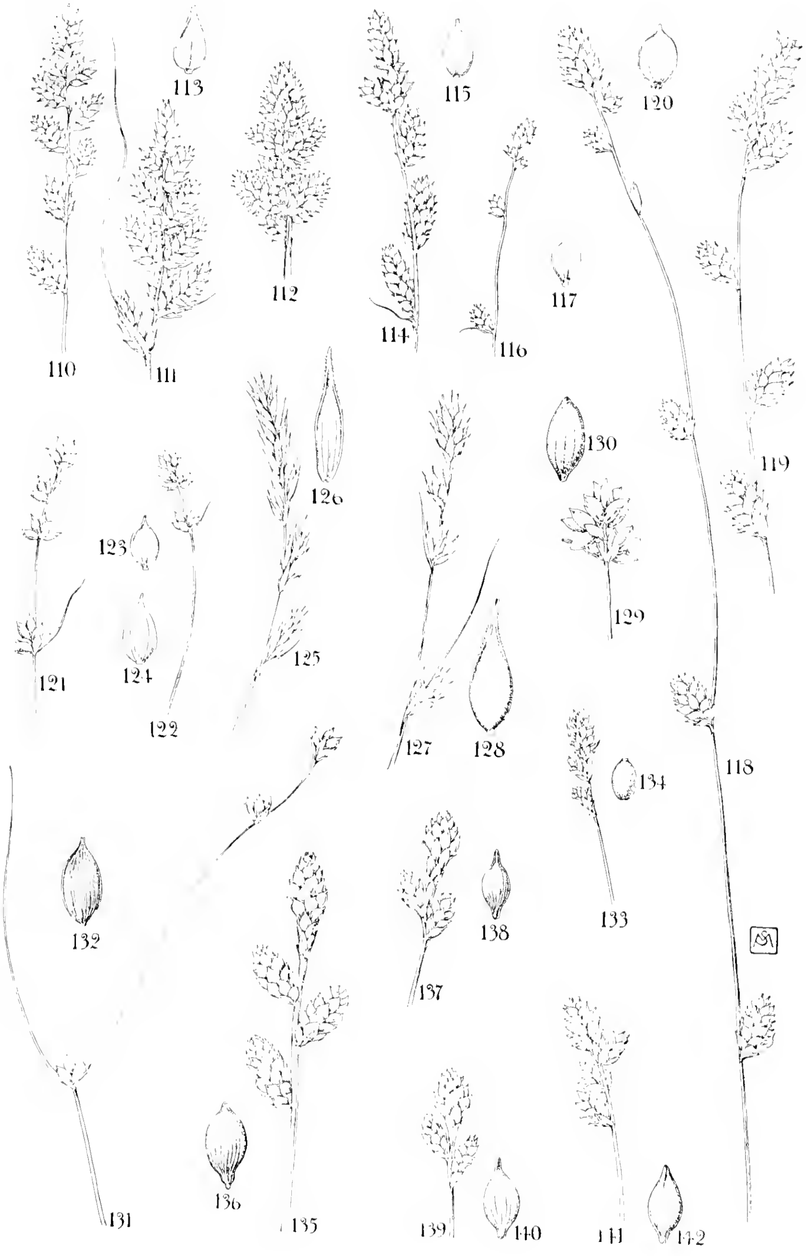
Fernald — Carex § Hyparrhenae.

Plate IV.



Fernald — Carex § Hyparrhenae.

Plate V.



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CONTRIBUTIONS FROM THE HARVARD MINERALOGICAL MUSEUM. — X.

APATITE FROM MINOT, MAINE.

By John E. Wolff and Charles Palache.

With a Plate.

CONTRIBUTIONS FROM THE HARVARD MINERALOGICAL MUSEUM. — X.

APATITE FROM MINOT, MAINE.

By John E. Wolff and Charles Palache.

Presented December 11, 1901. Received February 7, 1902.

In the summer of 1901, while prospecting for tourmaline or other gem minerals on the farm of Mr. P. P. Pulsifer in Minot, Maine, a pocket was opened in the granite containing the material here described. It was first brought to our notice by Mr. C. L. Whittle, formerly of this Department, and the whole was subsequently acquired by the Harvard Mineralogical Museum.

This find is noteworthy for the unusually rich purple color of the crystals, and the purity, crystalline perfection, and abundance of the material, which comprises about two thousand loose crystals or fragments of crystals with a total weight of over a kilogramme, and about a dozen large groups of crystals on the matrix. Of the loose crystals about three hundred show at least one perfect termination, five hundred are slightly less perfect, and the rest imperfect or fragmentary.

Paragenesis.

The apatite was found in a single cavity in pegmatitic granite, the walls of which appear to have been lined with crystals of quartz, orthoclase, and lepidolite, with which in smaller amounts were albite, muscovite, and cookeite.

The quartz crystals range from small dimensions up to a height and thickness of 15 cm. They show the common quartz forms only, the positive and negative unit rhombohedrons and the prism, and are notable chiefly as presenting in a very striking manner the evidence of two periods of growth. Wherever broken and whether large or small, the crystals show a core of glassy, light to dark smoky quartz; surrounding this is a surface layer of white opaque quartz from 1 to 3 mm. in thickness, crystallographically continuous with the smoky quartz, but on many of the faces, especially those of the rhombohedrons, composed of a multitude of small parallel crystals which give the surface a pitted appearance.

A thin section cut across this boundary showed under the microscope that the outer opaque layer was in crystallographic continuity with the inner part, but the line between them was sharp, the opaque layer containing very abundant liquid and obscure solid inclusions and showing faintly a division into fibres or columns perpendicular to the surface. The appearance pointed to a second period of quartz deposition rather than to an alteration of quartz previously formed. The evidence of the secondary deposition of the white quartz was rendered stronger by the occurrence in several places between the two layers of a thin deposit, not more than 1 mm. thick, of tiny muscovite crystals, or of a double layer of muscovite and cookeite. The apatite crystals are often deeply embedded in the white quartz and seem to have developed in part *pari passu* with this material; but at no place does the apatite appear to penetrate the smoky quartz.

The lepidolite is in part in confused lamellar aggregates, in part in quite definite hexagonal prismatic crystals with somewhat rounded basal terminations. The interior of these crystals is of the characteristic pale lilac color of lepidolite, but their surfaces are everywhere covered by a uniform layer of pale greenish-white muscovite about 1 mm. in thickness. The cleavage of the lepidolite and of the bordering muscovite is absolutely continuous, but the boundary between them is sharp and plane, showing that the muscovite represents, not an alteration of lepidolite, but a later parallel growth of the new and isomorphic mineral, a sort of secondary enlargement. Sharply bounded lepidolite crystals without the muscovite border are sometimes enclosed in the smoky quartz, showing that these two minerals were of contemporaneous growth.

The orthoclase, a pale flesh-colored variety, and the albite, colorless, in thin plates showing albite twinning, are small in amount, and their relations to the smoky quartz show that they belonged to the same period of growth with it.

Cookeite occurs quite abundantly on some of the specimens as crusts or clumps of scales or platy crystals of a greenish-white color. It is similar in appearance to the muscovite, but is slightly darker in color, less pearly in lustre, and readily distinguished by its reactions before the blowpipe. The cookeite appears to have been formed at several periods of mineral growth in the cavity. It is seen occasionally as above stated in thin layers between the outer white quartz layer and the coating of muscovite; more frequently it forms an irregular layer on the lepidolite-muscovite crystals, showing, however, no parallelism with them; and it is rarely included in, and in small amount deposited upon, the apatite crystals. In no case does the cookeite appear to have been formed at the expense of any of the minerals previously formed in the cavity, which are perfectly fresh and free from alteration.

The apatite is implanted upon the quartz crystals and upon the lepidolite-muscovite crystals or the cookeite which covers them. As stated above the apatite is embedded at times in the white quartz layer in which it leaves sharp moulds when broken out, it having maintained its crystal form despite the interference of the quartz.

Finally a third generation of quartz in minute crystals is found associated with the cookeite, and rarely implanted upon the apatite crystals.

To briefly recapitulate the facts relating to the paragenesis of this deposit we may say that we find:

First, the crystallization of the smoky quartz, lepidolite, orthoclase, and albite, the normal constituents of the granite, to form the walls of the cavity.

Second, the crystallization of muscovite, coating smoky quartz crystals in part and the lepidolite crystals wholly.

Third, the crystallization of cookeite, coating muscovite, wholly or in part.

Fourth, the simultaneous crystallization of apatite and white quartz, the latter confined to enlargement of smoky quartz crystals.

Fifth, a second period of cookeite formation, accompanied by a final deposition of quartz.

CRYSTALLOGRAPHY.*

The apatite crystals are in general of pronounced prismatic habit, the average size being about 1 cm. in height and 0.5 cm. in diameter. Crystals larger than this are, however, common, the largest measuring nearly 3 cm. in height and diameter. Crystals smaller than the average, which are also numerous, tend to assume a more or less rounded habit by nearly equal development of prismatic and terminal planes.

The crystals are generally so implanted upon a terminal face that one end has developed freely, and the fact that over three hundred loose crystals with complete single termination and prism zone were

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* By C. Palache.

obtained from the collection shows how prevailing is this habit of growth. Occasionally the attachment to the matrix is by a prism plane, and then both terminations are developed.

The forms observed were as follows, the letters used being those of Dana:

c (0001), *m* ($10\bar{1}0$)), *a* ($11\bar{2}0$), *h* ($21\bar{3}0$), *z* ($30\bar{3}1$), *y* ($20\bar{2}1$), *x* ($10\bar{1}1$), *r* ($10\bar{1}2$), *w* ($70\bar{7}3$), *s* ($11\bar{2}1$), μ ($21\bar{3}1$), μ_1 ($3\bar{1}\bar{2}1$).

Four crystals were carefully measured on the two-circle goniometer and the same forms found on all. The results of measurement of the better developed forms agreed so well among themselves that it seemed worth while to calculate the axial ratio from the better readings, and this was done, using the forms *y*, *x*, *r*, and *s*. The following table shows the average angle to the base from each of these, the ratio calculated for each crystal, and the average ratio obtained:

	Angle from 0001 to 2021.	No. of Faces.	d.*		Angle from 0001 to 1012.	No. of Faces.	d.*
Cryst. 1 . .	59° 29′	5	3′	Cryst. 1 . .	23° 00′	6	1′
" 2 . .	$59^\circ\ 29\frac{3}{5}'$	5	2′	" 2 . .	23° 00′	5	5′
" 3 . .	$59^\circ\ 30\frac{1}{3}'$	6	3′	" 3 . .	23° 00′	3	0′
" 4 . .	$59^\circ\ 28\frac{1}{5}'$	5	3′	" 4 . .	$22^\circ\ 59\frac{3}{4}'$	4	3′
	Angle from 0001 to 1011.				Angle from 0001 to 1121.		
Cryst. 1 . .	40° 18′	6	3′	Cryst. 1 . .	55° 45′	6	1′
" 2 . .	40° 19′	5	5′	" 2 . .	55° 46′	6	3′
" 3 . .	40° 19′	5	6′	" 3 . .	55° 46′	4	3′
" 4 . .	40° 18′	5	2′	" 4 . .	55° 45′	6	4′

Crystal 1, from 23 measurements, $p_0 = 0.848307$
Crystal 2, from 21 measurements, $p_0 = 0.848739$
Crystal 3, from 18 measurements, $p_0 = 0.848753$
Crystal 4, from 20 measurements, $p_0 = 0.848148$
Average from 82 measurements, $p_0 = 0.848476$ or $a : c = 1 : 0.734800$
Angle calculated from $p_0 = 0.848476$, 0001 to $20\bar{2}1$ 59° 29′ 22″
0001 to $10\bar{1}1$ 40 18 50
0001 to $10\bar{1}2$ 22 59 19
0001 to $11\bar{2}1$ 55 45 59

Two types of combinations may be distinguished among these crystals. One of these is represented in figure 1, and consists essentially of the

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* d is the difference in minutes between largest and smallest readings for faces of any form.

prism of the first order and the base, the edges modified by narrow planes of the forms *a*, *s*, *r*, *x*, and *y*. Crystals of this type are not uncommon and often show double terminations. They merge, however, by slight gradations into the second type, more characteristic for the locality, shown in figures 2 and 3. Here the pyramidal planes become more prominent and the most notable feature is the simultaneous occurrence of the right and left third order pyramids, giving the appearance of the normal dihexagonal pyramid.

The different forms may be characterized as follows:

c (0001) always present, generally large, brilliant, and plane giving perfect reflections.

m ($10\bar{1}0$) always present, generally dominant, brilliant, and generally plane but sometimes faintly striated vertically.

a ($11\bar{2}0$) generally present but narrow and commonly dull from deep striation, the striae vertical and bounded by faces of adjoining planes of *m*. Occasionally the striations stop abruptly in the centre or near the boundaries of a face as shown in figure 3, or they may be wholly lacking, in which case the face is brilliant and gives good reflections.

h ($21\bar{3}0$) rarely developed and then narrow as shown in figure 4. Surface plane, not involved in striations on *a*.

r ($10\bar{1}2$), *x* ($10\bar{1}1$), and *y* ($20\bar{2}1$) all nearly always present with all their faces, in varying proportions and often large, faces always brilliant and free from striations, giving perfect reflections.

w ($70\bar{7}3$) observed but once as a line face in the zone between *y* and *m*.

z ($30\bar{3}1$) generally present only as a deeply striated face, sometimes very large as in figure 4, giving no reflection but determined by its zonal relation to μ and μ_1. The striae bounded by faces parallel to adjoining planes of *m* and *y*. Narrow faces of *z* giving faint reflections sometimes present on the edges of the striae nearest to *m*.

s ($11\bar{2}1$) always present with brilliant faces, often large.

μ ($21\bar{3}1$) and μ_1 ($3\bar{1}\bar{2}1$) are both present on many crystals, but vary widely in size, quality, and regularity of development. Generally the faces of both are dull and the forms are then indistinguishable. On some crystals their faces are brilliant and reflecting but grooved or pitted, and a constant difference in the character of these markings was found by which, when they were not too far developed, the two forms could be distinguished. On μ the markings ordinarily take the form of sharp grooves parallel to the intersection of *m* and μ as shown in figures 2 and 3. The grooves seem to be in a way continuations of the striae on the faces of z, for they never extend beyond the intersection of μ with that face, and are absent if z is not developed. The grooves are bounded by faces parallel to adjacent planes of s and of m. Very often they stop short in the middle of the face as shown in figure 3.

On μ_1 the markings are in the form of irregular pits or curving grooves, sometimes showing approximate parallelism to the intersection of m and μ_1 but with an irregularity giving them a character wholly different from the lines on μ. No constant difference could be observed in the brilliancy of the reflecting portions of faces of the two forms, nor in their relative size. Both are irregular in their occurrence on individual crystals, lacking nearly always some of their faces. As shown in the figures, both may present on the same crystal faces of very unequal size which in some cases are so large as to dominate the termination of the crystal.

The occurrence of third order pyramids in apparently holohedral combination has been observed on apatite from various localities, notably Knappenwand, Tyrol,* Ala, Piedmont,† and Elba.‡ But in none of the crystals described does there appear to have been any observable difference between the faces of the right and left forms by which they could be distinguished.

Reference has been made in the preceding pages to striations which appear quite constantly on certain faces of the apatite. They are a striking feature of the crystals and the attempt has been made to reproduce them in the drawings. Their most pronounced development was on the largest crystal of the collection, which is reproduced in figure 4; the striations on the faces of z and of a were almost equally strong and gave the crystal a curiously tetragonal aspect when inspected casually. On both of these forms the striations are doubtless growth forms, the result of oscillatory combination, on a of adjacent faces of m, and on z of planes of m and y. The markings on the faces of μ and μ_1 seem to have a different character, however. The irregularity of their development, appearing on some faces as mere grooves or pits, on others invading the whole face and reducing it to a dull surface, indicates that they are rather the result of etching by some agent which has attacked the crystals after they were formed.

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* C. Klein, Neues Jahrb. Miner., 1871, 485; 1872, 121.

† G. Struever, Att. Acc. Torino, **3**, 125, 1867; **6**, 363, 1871; Rendic. R. Acc. Lincei, Roma, 1899, **8** (1), 427–434.

‡ E. Artini, Rendic. R. Acc. Lincei, Roma, 1895, **4** (2), 259.

CHEMICAL COMPOSITION. *

The material for analysis was taken from the deep purple clear crystals, which were broken free from any adhering gangue and carefully examined with the lens; while the microscope confirmed the purity of the mineral. The method followed was essentially that used

	A.	B.	Ratios.		J.
P_2O_5	41.30	41.58	0.2928	0.2928	39.84
$(FeAl)_2O_3$ . .	0.71	0.71	0.0044	1.0029	Al_2O_3 2.02 FeO 0.62
MnO	0.85	0.86	0.0121		0.22
CaO	53.43	53.79	0.9605		53.36
MgO	0.70	0.70	0.0173		0.25
K_2O	0.27	0.27	0.0028		0.52
Na_2O	0.36	0.36	0.0058		0.42
H_2O	0.29	0.29	†0.0323	0.1586	0.48
Cl	abs.	. . .	. . .		1.82
F	2.38	2.40	0.1263		1.03
Loss at 320° .	0.04	0.04			
	100.33	101.00			100.58
Less O = F .	1.00	1.00			0.90
	99.33	100.00			99.68
Sp. gr., 3.159 at 20° C.					

A. Apatite from Minot, Maine.
B. Calculated to 100.
J. Apatite from Ceylon, Jannasch and Locke, loc. cit.

$$\begin{array}{rcccc} \text{Hence,} & P_2O_5 & : & RO & : & F\text{-}OH, \\ & 1 & : & 3.42 & : & 0.57 \\ & 1.5 & : & 5.13 & : & 0.85 \\ \text{or,} & \multicolumn{5}{l}{Ca_5P_3[F.OH]O_{12}.} \end{array}$$

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* By J. E. Wolff.

† Calculated as OH = 0.55 per cent OH.

by Jannasch and Locke,* namely solution in nitric acid with addition of mercuric oxide, precipitation with ammonia and determination of phosphoric acid, most of the lime and the other bases in the precipitate, while the rest of the lime and the alkalies were determined in the first filtrate. Water was determined directly by fusion with plumbic oxide mixed with potassium di-chromate.

Fluorine was determined by the method of Fresenius, that is by heating the finely powdered mineral, mixed with previously ignited quartz, in a flask with strong sulphuric acid and absorbing the SiF_4 in weighed tubes with the prescribed precautions. From the total weight obtained there was subtracted a correction for the general gain in weight of the absorption tubes due to the action of the air current on the rubber connections; etc., which had been previously determined by experiment. The process was continued for five hours or to a constant weight. Chlorine was absent.

The mineral was soluble without residue in nitric acid. At about 320° C., the purple color disappears and the mineral becomes colorless or faintly yellow; this change is accompanied by some decrepitation, by phosphorence, and the production of a vapor (in part water?) which is deposited in drops on the walls of the tube; there is also a petroleum-like odor. The loss of weight accompanying this change was determined by gently heating three grammes of the mineral in a bulb tube in a current of dry air, weighing, and heating again cautiously in the current of air to complete decolorization, and determining the loss of weight.

OPTICAL PROPERTIES.†

For the determination of the indices of refraction one of the best clear crystals was used, having a deep purple color and a brilliant basal plane. The determination was made with the Abbé crystal refractometer by the differential method ‡ and for this purpose a glass prism was selected having the index $n_{Na} = 1.6326$, for which the boundary of total reflection was carefully determined and the telescope clamped. The apatite crystal was then placed with its base on the glass hemisphere of the apparatus and the angular difference in the boundaries for ω and ϵ determined by the millimeter screw reading to six seconds. The boun-

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* Zeit. anorg. Chemie, **7**, p. 154; also Jannasch, Praktischer Leitfaden d. Gewichts Analyse, p. 259.

† By J. E. Wolff.

‡ C. Viola, Zeit. Krystall., **30**, p. 438, and **32**, p. 311.

dary lines were sharp and the readings generally good. From the average of a large number of readings the following values were obtained:

$$\omega_{Na} = 1.63353 \qquad \omega - \epsilon = 0.00191 \qquad \epsilon_{Na} = 1.63162$$

$$\omega_{Li} = 1.63067 \qquad \omega - \epsilon = 0.0020 \qquad \epsilon_{Li} = 1.62865$$

The crystal was then heated to 320° C. or until decolorized and the indices again determined as follows:

$$\omega_{Na} = 1.63346 \qquad \epsilon_{Na} = 1.63165 \qquad \omega - \epsilon = 0.00181$$

The change in the bi-refringence and in both indices is within the limits of error.

The pleochroism is strong and the ray vibrating parallel to $\dot{c}$ (ϵ) reddish purple, perpendicular to $\dot{c}$ (ω) deep violet blue. In converging light the thick clear crystals show on the basal plane a marked bi-axial character with the vertical axis the acute (negative) bisectrix and a division of the base into six sectors, in each of which the axial plane is parallel to a prism of the second order (or perpendicular to a lateral axis). These sectors come out clearly with the sensitive tint of the gypsum plate; while some are almost perfect, others merge together and overlap at the centre of the crystal. The angle of the optic axes in one of these sectors was measured in the optic angle apparatus:

$$2E_{Na} = 20°,$$

but it appears to vary in different sectors of the same crystal. These phenomena of apparent orthorhombic symmetry were described and figured by Mallard[*] for the violet apatite from Schlaggenwald, but appear to be even more distinct in the Maine apatite. While driving off the coloring matter destroys the pleochroism the anomalous bi-axial characters are not affected.

CONCLUSION.

The Minot apatite is a pure fluor-apatite with a fluorine content lower than that necessary for the formula $Ca_5F(PO_4)_3$ and indicating Groth's formula $Ca_5P_3(F.OH)O_{12}$ as also deduced by Jannasch from the analysis quoted above. Rammelsberg[†] explained the low content of fluorine (and chlorine) in certain apatites as due to removal of these elements

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[*] Annales des Mines, VII. 10, 1876, p. 147.

[†] N. J. M., 1897, **2**, p. 38.

by a process of alteration, and therefore, according to him, all such apatites are altered. The freshness of the Minot material makes such a supposition inapplicable here.

The axial ratio of the Minot apatite is the largest and the birefringence the lowest recorded for the species. It was interesting to see what data existed for a comparison between fluorine or chlorine content and the axial ratio, specific gravity, and birefringence.

The normal angle *c* to *x* and also the specific gravity of apatite have been held by numerous observers to diminish with increasing chlorine content.

G. Rose (Ref. 3 below), the first to discover the fluorine and chlorine in apatite, stated as the result of his studies that the angles and specific gravity were alike in all apatites of like composition, but that the reverse of this statement was not proved.

Von Kokscharow (Ref. 2) extended this statement, holding that the normal angle *c* to *x* of all chlorine-containing apatite was somewhat less than that of pure fluor-apatite.

The analyses of Pusyrewsky (Ref. 18) seemed to confirm this view, and he further maintained that the specific gravity regularly decreased with increasing chlorine.

Von Kokscharow (loc. cit.) incorporated the results of the last writer with his measurements and published a table showing the relations of the three values, but without comment.

Baumhauer (Ref. 4) was the next to investigate the subject, and his measurements, analyses, and specific gravity determinations seemed to support the supposed relations. His table has been republished by several authors with slight modifications (Dana, Syst., 1892, 764, and Weibull, Ref. 20 below).*

In order more fully to test the matter a table has been prepared and is given below, arranged according to increasing values of the angle *c* to *x* or of the axial ratio, and showing for all occurrences for which accurate crystallographic data existed, the specific gravity, birefringence, and chlorine and fluorine content so far as such data could be found.

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* In Baumhauer's table and in all later tables of the same kind the locality Schlaggenwald is given with *c* to *x* 40° 20′, the largest value for this angle observed on apatite. Reference to the description of this occurrence by Schrauf (Ref. 8 below) showed that the measured crystals were poorly adapted to measurement, having curved faces; and the average axial ratio calculated from all the measurements gave a value much lower, about 40° 17′. This locality was therefore omitted from the table below.

TABLE OF PHYSICAL AND CHEMICAL CHARACTERS OF APATITE.

	Locality	Angle 0001 to 10$\bar{1}$1			c	Sp. Gr.	ω−ε	Cl	F	Ref.
		°	′	″						
1	Comba di Compare Robert, Piedmont	40	4		0.7281	. . .	. . .	. . .	. . .	1
2	Achmatowsk . . .	40	6	21	0.7294	3.120	. . .	0.51	. . .	2 & 18
3	Laacher See . . .	40	6	21	0.7294	3.202	. . .	. . .	. . .	3
4	Rothenkopf, Tirol .	40	10	46	0.7313	3.1495	. . .	0.085	. . .	4
5	Zillerthal, Tirol	. . .			. . .	. . .	. . .	absent	1.51	5
	" "	. . .			. . .	. . .	.00435	. . .	. . .	6
6	Tirol	. . .			. . .	. . .	.0044	. . .	. . .	7
7	Ala, Piedmont . .	40	10	46	0.7313	. . .	. . .	. . .	. . .	4
8	Kirjabinsk	40	13	30	0.7325	3.126	. . .	trace	. . .	2
9	Jumilla, Spain . .	40	11	48	0.7318	. . .	. . .	. . .	. . .	8
	" " . .	40	13	37	0.7326	3.235	. . .	0.557	. . .	2
	" " . .	. . .			. . .	. . .	.00448	. . .	. . .	9
	" " . .	. . .			. . .	. . .	.0042	. . .	. . .	19
	" " . .	. . .			. . .	. . .	. . .	0.24	1.98	5
	" " . .	. . .			. . .	. . .	. . .	0.47	3.54	10
10	Knappenwand, Tirol	40	15	26	0.7333	3.153	. . .	0.028	. . .	4
	" "	. . .			. . .	. . .	. . .	0.03	3.63	10
11	Tirol (talc schist) .	. . .			. . .	. . .	. . .	0.20	3.58	10
12	Sulzbachthal, Tirol .	. . .			. . .	. . .	.0026	. . .	. . .	7
13	Berg Blagodat . .	40	16	10	0.7337	3.132 3.200	. . .	0.21	. . .	2
14	Nordmarken . . .	40	16	10	0.7337	. . .	. . .	. . .	. . .	11
15	St. Gotthard . .	40	17		0.7340	3.197	. . .	0.028	. . .	2 & 3
	" . .	40	17		0.7340	3.2154	. . .	. . .	. . .	4
16	Tavetsch	40	17		0.7340	. . .	. . .	. . .	. . .	12
17	Floitenthal . . .	40	17		0.7340	. . .	. . .	. . .	. . .	12
18	Schwarzenstein . .	40	17		0.7340	. . .	. . .	trace	. . .	4
19	Vestanå (Mn. apatite)	40	17	20	0.7341	3.225	.0042	trace	3.74	20
20	Hiddenite Mine, N. C.	40	17	45	0.7343	. . .	. . .	. . .	. . .	13
21	Turkistan	40	18	10	0.7345	3.199	. . .	trace	3.64	14
22	Tokowaja, Urals . .	40	18	22	0.7346	3.201 3.212	. . .	0.01	4.20	2 & 18
23	Ehrenfriedersdorf	40	18	22	0.7346	3.211	. . .	absent	. . .	3
	"	. . .			. . .	. . .	. . .	absent	2.27	5
24	Pisek	40	18	25	0.7346	3.091	. . .	trace	3.56	15
25	Elba	40	18	48	0.7348	. . .	. . .	. . .	. . .	16
26	Minot, Me.	40	18	50	0.7348	3.159	.0020	absent	2.38	
27	Zwiesel (Mn. apatite)	. . .			. . .	3.169	. . .	absent	2.15	17
28	Ilmen Mts.	. . .			. . .	3.216	. . .	trace	3.97	18
29	Sudjanka River . .	. . .			. . .	3.178	. . .	0.109	4.02	18

1 G. Boeris, Atti. della R. Acc. Sc. di Torino, **34**, 609, 1899.

2 Kokscharow, Mat. zu Min. Russ., V. 86, 1866.

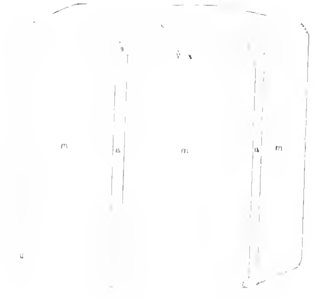
3 G. Rose, Pogg. Ann., IX. 206, 1827.
4 H. Baumhauer, Zeit. f. Kryst., **18**, 31, 1890.
5 Hoskyns-Abrahall (Inaug. Diss. 1889), Abs. Zeit. f. Kryst., **21**, 389.
6 Heusser, Pogg. Ann., **87**, 468, 1854.
7 K. Zimányi, Zeit. f. Kryst., **22**, 331, 1893.
8 Schrauf, Ber. Acad. Wien, **62** (2), 745, 1870.
9 " " " " **42**, 111, 1862.
10 Carnot, Bull. Soc. Franc. Mineral., **19**, 135, 1896.
11 Flink, Bihang t. K. Sv. Vet. Akad. II. Stockholm, **12** (2) No. 2, 42, 1886.
12 Schmidt, Zeit. f. Kryst., **7**, 551, 1883.
13 Hidden & Washington, Zeit. f. Kryst., **14**, 299, 1888.
14 Jeremejew & Nikolajew, Zeit. f. Kryst., **11**, 389, 1886.
15 Vrba, Zeit. f. Kryst., **15**, 464, 1889.
16 Artini, Rendic. R. Acc. Lincei, Roma, **4** (2), 259, 1895.
17 Sandberger (Hilger), N. J. Min. 1885, **1**, 171.
18 Pusyrewsky, Verh. k. k. Mineral. Gesell. St. Petersburg, 1859–1860 (cited by Baumhauer, No. 4 above).
19 Latterman, Rosenbusch, Mik. Phys., I. 409, 1892.
20 { Weibull, Geol. För. Forh., Stockholm, **20**, 63, 1898.
{ " " " " " **8**, 492, 1886.

One point brought out by the preparation of this table is the lack of studies of apatite in which on the same material all these characters have been determined. It is also to be noted that no crystallographic data whatever appear to have been secured on what could fairly be called a chlor-apatite, the highest chlorine content in the table being about 0.5 per cent only. Without such data it does not seem that the theory of Pusyrewsky and Baumhauer that axial ratio decreases with chlorine content can be considered as established. Moreover exceptions to that rule may be noted in the table, notably Nos. 5, Zillerthal, and 9, Jumilla. On the other hand it seems fairly safe to accept the statement that an apatite with large angle c to x (40° 17′ or more) will be practically free from chlorine.

The table shows that absolutely no definite relation exists between the specific gravity and the chlorine content. Nos. 2 and 9, from Achmatowsk and Jumilla, having about 0.5 per cent chlorine, have specific gravity respectively 3.12 and 3.235; while the chlorine-free varieties have specific gravities from 3.09 to 3.22, or practically the same range. Observations on the birefringence are few, but so far as they go do not point to a definite relation to the chlorine content.

HARVARD MINERALOGICAL LABORATORY,
December, 1901.

Wolff and Palache Apatite.



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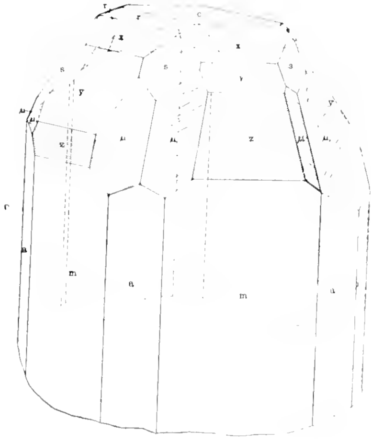

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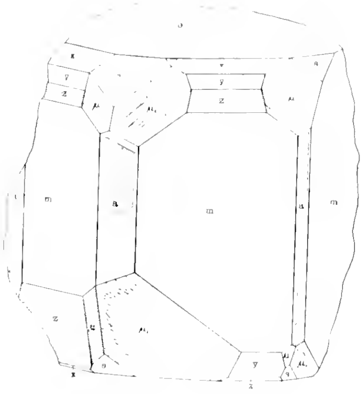

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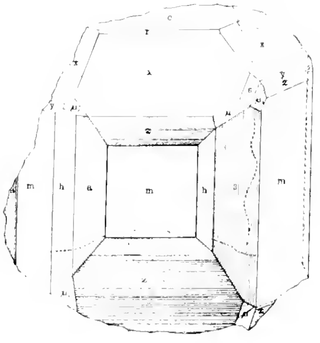

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Proceedings of the American Academy of Arts and Sciences.

Vol. XXXVII. No. 19. — March, 1902.

CONTRIBUTIONS FROM THE HARVARD MINERALOGICAL MUSEUM. — XI.

A DESCRIPTION OF EPIDOTE CRYSTALS FROM ALASKA.

By Charles Palache.

With a Plate.

CONTRIBUTIONS FROM THE HARVARD MINERALOGICAL MUSEUM.—XI.

A DESCRIPTION OF EPIDOTE CRYSTALS FROM ALASKA.

By Charles Palache.

Presented by John E. Wolff, January 8, 1902. Received February 7, 1902.

The epidote crystals described in this paper were sent to the Harvard Mineralogical Laboratory for crystallographic study by Mr. W. C. Hart of Manitou, Colorado, to whom we wish to express our thanks for the generous supply of material placed at our disposal.

This material is from a new locality for the mineral and is remarkable for the size and unusual habit of the crystals; it therefore seemed well worthy of description.

According to Mr. Hart the epidote is found at Sulzer, Prince of Wales Island, Alaska. It is in the close vicinity of a body of copper ore and is further associated with garnet, albite, magnetite, and quartz. The country rock of the region is limestone, which is cut by numerous igneous dykes, and it seems probable that the deposit is the result of contact metamorphism of the limestone by the dyke rocks, resembling closely in this respect the epidote occurrence with copper ore in the Seven Devils Mts. in Idaho.*

The specimens at hand consist of several loose crystals and a magnificent cluster of large crystals implanted on massive epidote. The only associated mineral is quartz in small clear crystals of later formation than the epidote.

The epidote is very dark green to greenish black in color, but oil-green and translucent in thin crystals or where bruised or cracked. The larger crystals are in the form of nearly square tables, which measure as much as 5.5 cm. each way and 3 cm. in thickness. In the smaller crystals the tabular habit is less pronounced and the mineral sometimes assumes the ordinary prismatic habit parallel to the $\bar{b}$ axis. The crystals are not

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* Compare Am. J. Science, VIII, 1899, 299.

infrequently doubly terminated. In the large group to which reference was made about twenty of the tabular crystals are found on a surface measuring about 15 by 20 cm., several of the crystals over 3 cm. on an edge, and attached by an edge in such fashion as to present an appearance altogether foreign to epidote. The crystals are frequently twinned according to the ordinary law for epidote, twinning plane the orthopinacoid.

Measurement of numerous crystals proved the presence of some twenty-six forms as shown in the following list. The smaller crystals were measured on the two-circle goniometer and most of the faces gave excellent reflections. For the large crystals contact measurements were used, and these, with a study of zonal relations made determinations of forms fairly certain. A few forms were thus found that were not observed on the smaller crystals. Two forms new for epidote are marked with an asterisk.

c (001), b (010), a (100), u (210), z (110), o (011), e (101),
i ($\bar{1}02$), N ($\bar{3}04$), r ($\bar{1}01$), l ($\bar{2}01$), n ($\bar{1}11$), α ($\bar{2}12$), F ($\bar{4}54$),
Z ($\bar{2}32$), Φ ($\bar{3}53$), φ ($\bar{1}21$), δ ($\bar{1}41$), E ($\bar{1}51$), q ($\bar{2}21$), O ($\bar{5}44$),*
j ($\bar{7}55$),* X ($\bar{3}22$), y ($\bar{2}11$), R ($\bar{4}11$), Ψ ($\bar{4}13$).

Of these forms *c*, *a*, *b*, *u*, *r*, and *n* are nearly always present and define the habit of most crystals. *z*, *e*, *o*, and *q* are also found on many crystals; the remaining forms are rare and subordinate in development. The forms may be characterized as follows:

c (001) always present, bright and unstriated, often broad.

b (010) always present, always dull and striated parallel to intersection with *n*; generally has narrow faces.

a (100) always present; the largest face on tabular crystals; bright but often striated faintly horizontally.

u (210) always present, generally with large bright faces.

z (110) generally quite subordinate to *u* but frequently present as a narrow face and always pitted and dull.

o (011) bright face, generally small.

e (101), i ($\bar{1}02$), N ($\bar{3}04$), l ($\bar{2}01$) are infrequent forms in orthodome zone, generally narrow, bright and unstriated when they do occur. This zone is remarkably poor in forms and free from striation as compared with most crystals of epidote.

r ($\bar{1}01$) always present, broad, generally striated lightly and less brilliant than *c* with which it is easily confused on the crystals.

n ($\bar{1}11$) always present, often large, brilliant, and striated. The only form found in the re-entrant angles rarely found on twin crystals.

a ($\bar{2}12$) present on one crystal only (figure 7) as a large, fairly bright face.

F ($\bar{4}54$), Z ($\bar{2}32$), Φ ($\bar{3}53$), *φ* ($\bar{1}21$), δ ($\bar{1}41$), and E ($\bar{1}51$) are pyramids of the zone [$\bar{1}11$ to 010]. Unimportant forms but several of them sometimes present on single crystals as shown in the drawings. F ($\bar{4}54$), a rare form for epidote, has been reported heretofore only from Persberg, Sweden* by Flink; he determined it by a single bright face in two zones, no angles being given. It was found with a single face on each of two crystals and was measured as follows:

	Measured.		Quality.	Calculated.		
	φ	*ρ*		*φ*	*ρ*	
$\bar{4}54$	−160° 40′	67° 21′	fair	−160° 43′	67° 18′	(010 as pole)
	128° 01′	29° 33′	fair	128° 18′	29° 27′	(normal position)

Φ and δ were determined solely by contact measurements; Z and *φ* by contact measurements and zonal relations.

q ($\bar{2}21$) often present, sometimes large, always dull.

O ($\bar{5}44$), j ($\bar{7}55$), X ($\bar{3}22$), y ($\bar{2}11$), and R ($\bar{4}11$) are pyramids of the zone [$\bar{1}11$ to 100]. Of these forms *y* alone is common; *O* and *j* are new to epidote and *X* and *R* are rare.

O was measured on three crystals (two of them twins) with six faces as follows:

	Measured.		Quality.	Calculated (010 as pole).	
$\bar{5}44$	*φ*	*ρ*		*φ*	*ρ*
Crystal No. 4,	−42° 04′	38° 30′	poor	42° 07′	39° 33′
" " 5,	42 57	39 11	fair	42 07	39 33
	137 06	38 58	fair	137 53	39 33
" " 6,	−42 13	39 20	good	−42 07	39 33
	−137 37	39 19	good	−137 53	39 33
	136 48	39 18	fair	137 53	39 33
Average $-\phi' = \pm\phi$ or $180° - \phi$	−42° 07′	39° 33′		−42° 37′	39° 06′

The agreement of measured and calculated angles is not very close, but the form seems assured.

j was measured on two twin crystals with three faces as follows:

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* Bihang t. K. Sv. Vet. Akad. H. Stockholm, **12**, No. 2, 1886.

	Measured.		Quality.	Calculated.	
$\bar{7}55$	ϕ	ρ		ϕ	ρ
	37°41′	42° 17′	fair	37° 40′	42° 11′
	142 07	41 44	poor	142 20	42 11
	143 16	42 42	bad	142 07	42 11
Average ϕ or 180° $-\phi$ =	37° 26′	42° 14′		37° 40′	42° 11′

The agreement between measured and calculated angles is here fairly satisfactory and the form seems assured.

X ($\bar{3}22$) has been reported only once on epidote from Elba by A. Artini * who measured a single bright face agreeing well with its calculated angle.

It was found here on a single crystal with one measurable face, but was noted frequently as a dull face in the zone [$\bar{2}21$ to $\bar{1}01$].

	Measured		Calculated (010 as pole)	
	ϕ	ρ	ϕ	ρ
$\bar{3}22$	142° 00′	66° 08′	141° 47′	66° 20′

y ($\bar{2}11$) and *R* ($\bar{4}11$) were the commonest forms of this series and were well determined by measurement on several crystals.

Ψ ($\bar{4}13$) present only on one crystal (figure 7) as a small face.

Several of the above forms are lacking in the Winkeltabellen of Goldschmidt, and as the various values there given have been calculated for each of them they are given in the following table, which also includes two forms for which certain values were found to be incorrectly stated in the Tabellen:

No	Letter	Symb.	ϕ	ρ	ξ_0	η_0	ξ	η	x'	y'	d'
			° ′	° ′	° ′	° ′	° ′	° ′			
†54	Φ	$\bar{3}53$	. . .	. . .	. . .	. . .	. . .	67 34	. . .	. . .	. . .
‡79	Ψ	$\bar{4}13$	$\overline{63}$ 34	53 31	50 27	31 02	$\overline{46}$ 04	20 58	$\bar{1}$.2113	0.6019	1.3525
83	F	454	$\overline{19}$ 17	67 18	$\overline{38}$ 18	66 06	$\overline{17}$ 44	60 33	$\bar{0}$.7897	2.2570	2.3914
84	O	$\bar{5}44$	$\overline{31}$ 29	64 43	$\overline{47}$ 53	61 01	$\overline{28}$ 11	50 27	$\bar{1}$.1060	1.8057	2.1176
85	j	$\bar{7}55$	$\overline{35}$ 39	65 46	$\overline{52}$ 20	61 01	32 07	47 49	$\bar{1}$.2955	1.8057	2.2224
86	X	$\bar{3}22$	$\overline{38}$ 13	66 29	54 53	61 01	34 34	46 05	$\bar{1}$.4220	1.8057	2.2983

The drawings show the extremely variable habit of the crystals.

Figure 1 represents perhaps the commonest type, a tabular twin crystal, the two individuals entirely symmetrical to the twinning plane

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* Mem. Acad. Lincei. **4**, 380, 1887.

† Correction, Winkeltabellen, p. 130, line 32 from above, col. 10.

‡ Correction, Winkeltabellen, p. 131, line 12 from above, whole line.

and so developed as to present no re-entrant angles. Here as in all the twin crystals the reversal of direction of the striations on 010 in the twinned crystal is the easiest means of recognizing the composite nature of the group.

Figures 2 and 2a are orthographic projections of the same crystal on the orthopinacoid and the clinopinacoid respectively. They represent the largest crystal studied in natural size, and show how irregularly the two twinned crystals are sometimes united — in this case an imperfect penetration having taken place. The upper surface of this crystal is bounded by cleavage planes parallel to *c* where the crystal was broken from its matrix.

The remaining figures (3 to 7) are orthographic projections on the clinopinacoid.

Figure 3 is another tabular crystal in which the larger portion of the crystal is a single individual. Rarely such tabular crystals are untwinned.

Figures 4 and 5 are two very symmetrical twin crystals, both prismatic parallel to the axis $\bar{b}$ and doubly terminated. They show many of the less common forms and the re-entrant angle between two faces of *n* which is not common on these crystals.

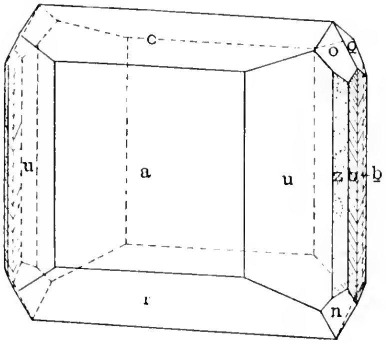
Figure 6 is a type of the untwinned crystal, prismatic parallel to axis $\bar{b}$, the usual epidote habit. It is a left-hand termination. Many of the smaller crystals are of this habit with varying development of the planes of *n* and *u*.

Figure 7 is a small crystal of prismatic habit but quite unlike any other found in its terminal planes.

In conclusion it may be said that this Alaska epidote ranks among the finest occurrences of American crystallized minerals, and is only surpassed in the size, beauty, and complexity of its crystals by the epidote from the Knappenwand in the Tyrol.

MINERALOGICAL LABORATORY, HARVARD UNIVERSITY,
January, 1902.

Palache. — Epidote Crystals.



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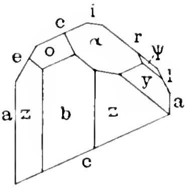

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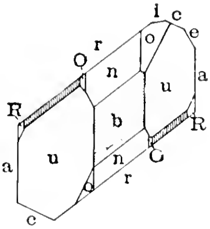

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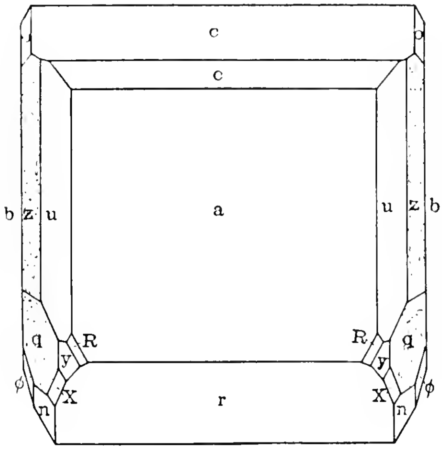

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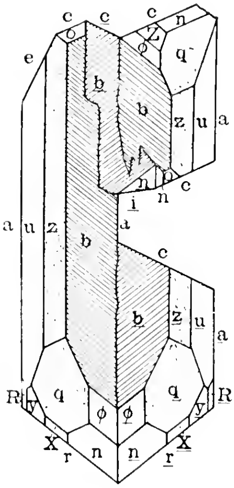

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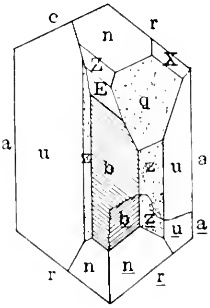

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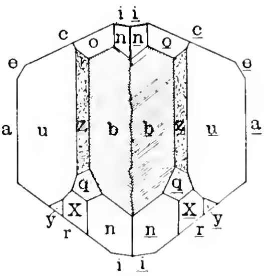

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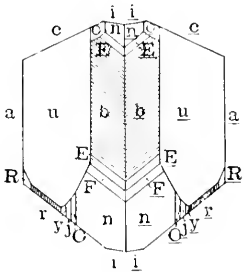

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Proceedings of the American Academy of Arts and Sciences.

Vol. XXXVII. No. 20. — March, 1902.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF CASE SCHOOL OF APPLIED SCIENCE. — XL.

ON THE SPECIFIC HEAT AND HEAT OF VAPORIZATION OF THE PARAFFINE AND METHYLENE HYDROCARBONS.

By Charles F. Mabery and Albert H. Goldstein.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF CASE SCHOOL OF APPLIED SCIENCE. — NO. XL.

ON THE SPECIFIC HEATS AND HEAT OF VAPORIZATION OF THE PARAFFINE AND METHYLENE HYDROCARBONS.

By Charles F. Mabery and Albert H. Goldstein.

Received February 10, 1902. Presented February 12, 1902.

Since 1819, when Dulong and Petit, on the basis of their work on thirteen of the chemical elements, announced the law that atoms of all elementary bodies possess the same capacity for heat, or that the specific heats of the elements vary inversely as their atomic weights, the specific heats of the elements have been important physical constants. With some exceptions, the constant 6.54 represents the product of the atomic weight into the specific heat. Later work showed that this law could be extended to compounds. In 1831, Neumann discovered that compounds, with analogous composition have the same specific heat. Or in a series of compounds with analogous composition the specific heat varies inversely with the molecular weight. These laws apply to approximately forty elements and to solids only at temperatures much below their melting points. The specific heats of many organic compounds have since been determined, and although no law has been stated, it is evident that, in certain homologous series of organic compounds, a condition exists in some of these series analogous to the law of Neumann. For instance, determinations have been made on a few of the alcohols with the following results:

Ethyl alcohol	0.680
Iso-propyl alcohol	0.5286
Iso-butyl alcohol	0.5078
Iso-amyl alcohol	0.4932

That the variations on certain homologous series so far as they have been observed do not conform to a general law, is shown by the following series, in which the specific heats increase with increase in molecular weights:

Methyldichloracetate	0.3202
Ethyldichloracetate	0.3384
Propyldichloracetate	0.3506

In general, the data on specific heats of organic compounds are meagre and not concordant. For the same substance the results of different observers seldom agree in the third decimal place; they often do not agree in the second, and even in the first place (see tables of Landolt and Börnstein). These variations are probably due to the fact that specific heat is materially affected by impurities in the substance, and the temperatures at which it is taken. Then, furthermore, the details of the determinations demand very careful attention to insure concordant results. In the determinations of the hydrocarbons to be described in this paper, it was found that impurities depressed the specific heats very considerably.

Although the paraffine series of hydrocarbons offers the best field for study of an homologous series, very little has been done in the direction of ascertaining the specific heats of these bodies. In a study of distillates separated from Pennsylvania petroleum, by Bartoli and Stracciati,* the specific heats of the following hydrocarbons were determined:

Hexane	C_6H_{14}	.5042	16°–37°
Heptane	C_7H_{16}	.4869	16°–37°
Octane	C_8H_{18}	.5111	12°–19
Decane	$C_{10}H_{22}$	.5057	14°–18°
Tetradecane	$C_{14}H_{30}$	.4995	
Hexadecane	$C_{16}H_{34}$	.4963	15°–22°

The inference derived from these results was that the specific heats of these hydrocarbons were remarkably constant.

No data could be found relating to the specific heats of the methylene hydrocarbons C_nH_{2n}, nor those of the series still poorer in hydrogen. With the greatly extended uses of crude petroleum and refined distillates for fuel, in gasoline and oil engines, accurate information concerning specific heat and heat of vaporization is greatly desired.

Having in hand a great variety of hydrocarbons of the various series, which have been prepared in as pure form as possible, and numerous inquiries having been received for more complete information concerning these physical properties, it seemed advisable to make some determinations of these constants.

The hydrocarbons of the series C_nH_{2n+2}, were obtained from Pennsylvania petroleum, those of the other series from California petroleum.

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* Gazz. Chim., 1885, 417–445.

Each hydrocarbon had been separated by a long process of distillation, and purified, by treatment with sulphuric acid, sodic hydrate, and drying over sodium.

The determinations were made in a Bunsen ice calorimeter, to which was attached a capillary side tube 70 centimeters long, and the bore of such size that 1 centimeter contained 0.0579 grams of mercury. To maintain the temperature at 0°, the calorimeter was placed in a jar of ice-water with an excess of ice. and this jar was placed in another jar and the space between filled with cotton. The calorimeter was filled with air-free water and dry mercury, and the inside film of ice, 2 to 3 millimeters thick, was formed by evaporation of ether by an air blast within the inner tube. Approximately 2 grams of the hydrocarbon was placed in a small glass stoppered tube of thin glass, whose heat equivalent was determined. The tube and hydrocarbon were heated to 50°, in a larger tube placed in a beaker of water, kept at this temperature for at least 15 minutes, then transferred by a thread to the calorimeter. With care this could be done without loss of heat by radiation. The contraction of the mercury column was from 15 to 25 centimeters.

This method can be used for the paraffine hydrocarbons from C_6H_{14} to $C_{16}H_{34}$, the limit at which the hydrocarbons remain wholly liquid at 0°.

After obtaining the water constant of the apparatus, three to six determinations of each hydrocarbon were made at the temperatures 0° and 50°, with results given in the following table:

	Boiling Points.	Specific Heat.
C_6H_{14}	68	.5272
C_7H_{16}	91	.5005
C_7H_{16}	98	.5074
C_8H_{18}	125	.5052
C_9H_{20}	151	.5034
$C_{10}H_{22}$	162	.4951
$C_{10}H_{22}$	172	.5021
$C_{11}H_{24}$	195	.5013
$C_{12}H_{26}$	214	.4997
$C_{13}H_{28}$	226	.4986
$C_{14}H_{30}$	242	.4973
$C_{15}H_{32}$	260	.4966
$C_{16}H_{34}$	275	.4957
Commercial Gasoline		.5135
Crude Ohio Petroleum		.4951

The following table gives the specific heats obtained from the series of methylene hydrocarbons :

	Boiling Points. °	Specific Heat.
C_6H_{12}	68	.5062
C_7H_{14}	98	.4879
C_8H_{16}	119	.4863
C_9H_{18}	135	.4851
$C_{10}H_{20}$	160	.4692
$C_{11}H_{22}$	190	.4819
$C_{12}H_{24}$	212	.4570
$C_{13}H_{26}$	232	.4573
$C_{14}H_{28}$	244	.4531
$C_{15}H_{30}$	263	.4708

It appears from these results that there is a uniform decrease in specific heat with increase in molecular weight. Furthermore the normal hydrocarbons, such as heptane, C_7H_{16}, B. P. 98°, and decane, $C_{10}H_{22}$, B. P. 172°, have higher specific heats than their isomers, such for example as isoheptane, C_7H_{16}, B. P. 91°, and isodecane, $C_{10}H_{22}$ B. P. 162°.

The same variation also appears in the methylene series, with high values for certain members that probably indicate different structural relations.

It is further interesting to observe the materially lower values given by the methylene hydrocarbons as compared with the values for the paraffine hydrocarbons. Whether this be due to greater compactness in the methylene molecule or to some quality of its ring structure, it would be interesting to ascertain.

Perhaps the falling off in specific heat with increasing molecular weight will appear to better advantage when arranged as ordinates on a curve with the molecular weights as abscissae. Only those compounds are given on the curve that are known to be normal, although, of course, this is not known with reference to the higher members. The different values of the isomers heptane and decane is shown on the shorter curve.

This uniform decrease in specific heat with increasing molecular weight in the series C_nH_{2n-2}, suggest a constant relation analogous to the law of Neumann.

If the constant K be expressed in terms of the specific heat multiplied by the molecular weight and the product divided by the number of atoms in the molecule, the specific heats found for the hydrocarbons of this series give the following values for the constant:

Hydrocarbon.	Mol. Wt.	Sp. Heat.	No. Atoms.	K.
C_6H_{14}	86	.5272	20	2.26
C_7H_{16}	100	.5074	23	2.21
C_8H_{18}	114	.5052	26	2.21
C_9H_{20}	128	.5034	29	2.22
$C_{10}H_{22}$	142	.5021	32	2.23
$C_{11}H_{24}$	156	.5013	35	2.23
$C_{12}H_{26}$	170	.4997	38	2.23
$C_{13}H_{28}$	184	.4986	41	2.24
$C_{14}H_{30}$	196	.4973	44	2.23
$C_{15}H_{32}$	210	.4966	47	2.24
$C_{16}H_{34}$	224	.4957	50	2.23

The constant, therefore, for the homologous series of hydrocarbons C_nH_{n2+2}, is 2.23.

A similar curve drawn for the specific heats of the methylene hydrocarbons, so far as they were determined, show also a regular variation.

The constant K, for the methylene hydrocarbons calculated from the determinations, shows a somewhat higher mean value than that of the paraffine hydrocarbons:

Hydrocarbon.	Mol. Wt.	Sp. Heat.	No. Atoms.	K.
C_6H_{12}	84	.5062	18	2.26
C_7H_{14}	98	.4879	21	2.28
C_8H_{16}	112	.4863	24	2.37
C_9H_{18}	126	.4851	27	2.27
$C_{11}H_{22}$	154	.4819	33	2.25
$C_{15}H_{30}$	210	.4708	45	2.20

The values for the specific heats of both hexane and hexamethylene are higher than should be expected from the results on the other members of the series. These hydrocarbons were well purified, except it seems probable that the distillates contained certain proportions of both.

Determinations were also made of the specific heats of a series of hydrocarbons separated from the high boiling portions of Pennsylvania petroleum. This series is under examination to establish its composition and relations to series, separated from heavy petroleums from other fields. These hydrocarbons were cooled to —10°, and filtered to remove so far as possible the solid hydrocarbons with which they are associated in Pennsylvania petroleum. They have been shown to be members of the series C_nH_{2n}. A more detailed description of these bodies will be presented in a subsequent paper, which is now in preparation.

The following values were obtained for the specific heats of these hydrocarbons:

	B. P. °	Sp. Heat
$C_{16}H_{32}$	173	.4723
$C_{18}H_{36}$	202	.4723
$C_{20}H_{40}$	223	.4706
$C_{23}H_{46}$	260	.4612
$C_{24}H_{48}$	272	.4586

From data obtained with the paraffine hydrocarbons, there is no opportunity to compare the specific heats, although the sudden drop from the last member examined of the series C_nH_{2n+2}, $C_{16}H_{34}$, specific heat .4957, to the first member of the last series above in which the solid hydrocarbon $C_{16}H_{32}$ had been removed by cooling and filtration, specific heat .4723, shows a marked difference in structure. This relation appears in the longer curve in the table.

Comparing the last member of the series C_nH_{2n} from California oil $C_{15}H_{30}$, specific heat .4708, with the hydrocarbon $C_{16}H_{32}$, specific heat .4746, it appears that the series C_nH_{2n} from Pennsylvania petroleum is a continuation of the same series from California petroleum. Unfortunately the higher members of the series from California petroleum were not available for comparison. But results already obtained as to the composition, specific gravity, refractive index, &c., are in accord with the specific heats. The specific heat was also determined in several hydrocarbons of the series C_nH_{2n-2}, and the series C_nH_{2n-4}, which had been separated from Texas petroleum with the following results:

SERIES C_nH_{2n-2}.

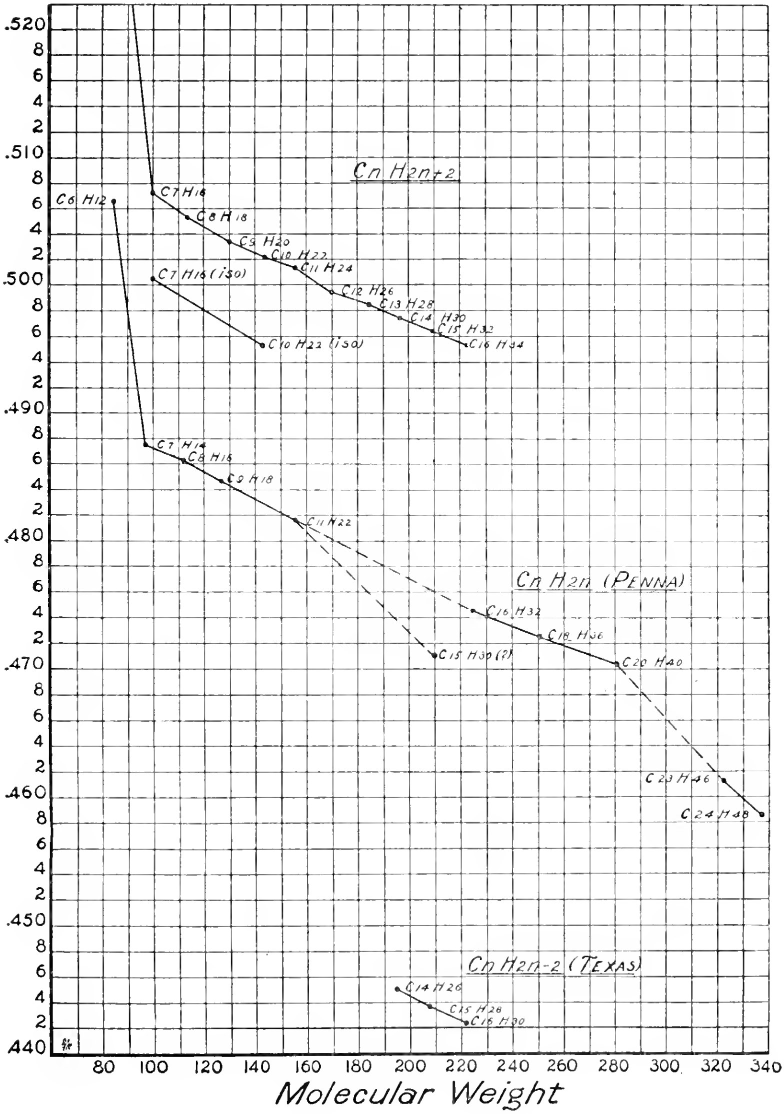
	B. P. 50 mm. °	Sp. Heat.	K.
$C_{14}H_{26}$	127	.4447	2.15
$C_{15}H_{28}$	142	.4439	2.15
$C_{16}H_{30}$	162	.4426	2.14

SERIES C_nH_{2n-4}.

	B. P. 50 mm.	Sp. Heat.
$C_{21}H_{38}$	218°	.4560
$C_{25}H_{46}$	273°	.4650

The latter results cannot be accepted as reliable, for the quantities of the hydrocarbons were very small, and the oils began to crystallize at 0°. There is no doubt that the specific heats of these hydrocarbons are smaller than those of the preceding series.

To convey a clear idea of the variation in hydrocarbons in the same series, and also to show the differences between the different series, the results are brought together on the following coördinate table:



The regular variation in members of the series C_nH_{2n+2}, appears in its curve. and the differences in what are known to be isomeric forms in the same series. It is further interesting to observe the continuation of the curve representing the members of the series C_nH_{2n} from the curve containing the members of the series C_nH_{2n} from California petroleum. The lower values in the series C_nH_{2n-2} from Texas petroleum form a characteristic curve near the base of the table.

The specific heat was determined in the following crude oils from various fields:

	Specific Gravity.	Specific Heat.
Pennsylvania	0.8095	.5000
Berea Grit	0.7939	.4690
Japanese	0.8622	.4532
Texas (Lucas well)	0.9200	.4315
Russian	0.9079	.4355
Wyoming	0.8816	.4323
California	0.9600	.3980
Texas	0.9466	.4009
Ohio		.4951
Commercial Gasoline		.5135

These values show that the specific heat of the crude oils is an important property from a practical point of view. It also appears that there is no close agreement between specific heat and specific gravity. Pennsylvania oil stands at the head, and Berea Grit with a much larger proportion of volatile constituents is next. Of the heavier oils it appears in general that the specific heats are much lower, but with no definite relation.

It would be interesting to ascertain the specific heats of the less volatile constituents of petroleum from different fields, including the solid hydrocarbons. This would require observations at different temperatures, and it would add to the interest of the data if all determinations could be made within a wide range of temperatures.

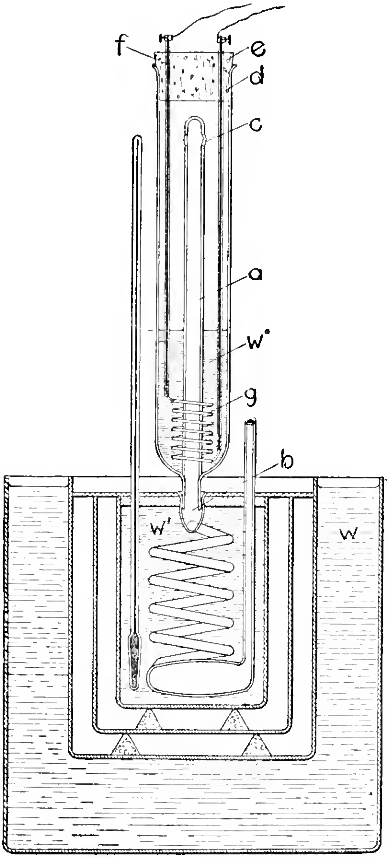
HEATS OF VAPORIZATION OF HYDROCARBONS OF THE PARAFFINE AND METHYLENE SERIES.

Since it had been found that a boiling point constant can be calculated from the absolute boiling point, and latent heat of evaporation, which may be used as a basis of a method for the determination of molecular weights, more complete knowledge of the latent heat of evaporation or heat of vaporization of liquids at their boiling points has become of great importance. From a practical point of view, the greatly extended use of crude petroleum and its constituents can only be economically continued with the aid of further information concerning the heats of vaporization. Numerous inquiries from persons interested in these directions attest an appreciation of further knowledge on this subject.

In 1885 the following law was proposed by Dudley, on the heats of vaporization of members of a homologous series.*

"In any homologous series the heat of vaporization in a unit of volume of the vapor, under the same conditions of temperature and pressure, is proportional to the density and also to the absolute boiling point." This generalization was based on data selected from determinations of the heats of vaporization of the formiates, acetates, propionates, butyrates, isobutyrates, alcohols, and aromatic hydrocarbons.

With numerous members of the different series of hydrocarbons at hand, it was our intention to determine the latent heat of the series C_nH_{2n+2}, C_nH_{2n}, C_nH_{2n-2}. But the only apparatus available was constructed of glass, which would not withstand the high temperatures necessary in determinations of the hydrocarbons with high boiling points, and the time was too limited to permit of the construction of a metallic apparatus.



For the apparatus used, we are indebted to the kindness of Professor Kahlenberg of the University of Wisconsin, who allowed us to use the

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* Journ. Am. Chem. Soc., Vol. XVII., No. 12.

form devised by him, before he had published his description which appeared in the Journal of Physical Chemistry, April, 1901.

This apparatus is an ingenious modification of Berthelot's method, in which the substance is heated and volatilized by means of an electric current within the body of the liquid, thus avoiding errors due to external heating. Through the kindness of Professor Kahlenberg and Professor Trevor we are able to show in this connection the form of this apparatus.

In the publication referred to, the apparatus is described as follows: "The retort consists of a tube 17 centimeters long and 5.5 centimeters in diameter, into the bottom of which is fused a tube which fits into a condenser with a ground glass joint. At the other end of the inner tube are two large lateral openings. Glass tubes pass through the cork at the top, and into these are fused the ends of the spiral of platinum wire. This spiral consists of about 40 centimeters of fairly stout platinum wire, to the ends of which are welded short heavy pieces of platinum rod, and these rods are in turn fused into the glass tubes. Long, rather heavy copper wires pass down into the glass tubes, at the bottom of which they are connected with the ends of the platinum rods by means of a few drops of mercury. The calorimeter is covered with a heavy piece of asbestos board and the retort is enclosed in asbestos and cotton batting."

A current from eight to ten amperes, regulated by a rheostat, gave sufficient heat to vaporize from 20 to 30 grams of the oil in from 5 to 6 minutes. The calorimeter, about 2500 cubic centimeters capacity, was made of thin nickel-plated sheet copper. It was elliptical in form, to conform to the shape of the condenser, and was provided with a copper stirrer. The capacity of the calorimeter was reduced, as shown in the figure, by bending closer together the glass tubes leading from the body of the condenser. Temperatures were taken on a Beckman thermometer.

The water equivalent of the calorimeter, condenser, stirrer, and thermometer were found to be 185 grams, practically the same value as the equivalent calculated from the weights and specific heats of the parts of the apparatus.

As mentioned above, this form of the apparatus is limited in this work by the fact that when oils of boiling points higher than 125° are volatilized, the sudden change in temperature at the water line of the condenser is so great that glass will not stand it.

The following results were obtained with a few members of the series C_nH_{2n+2}, as the mean of several observations:

	Boiling Point. °	Heat of Vaporization in Calories.
Hexane, C_6H_{14}	68	79.4
Heptane, C_7H_{16}	98	74.
Octane, C_8H_{18}	125	71.1

Determinations were also made on the methylene hydrocarbons that could be volatilized in this form of apparatus :

	Boiling Point. °	Heat in Calories.
Hexamethylene, C_6H_{12}	68–70	87.3
Dimethylpentamethylene, C_7H_{14}	90–92	81.
Methylhexamethylene, C_7H_{14}	98	75·7
Dimethylhexamethylene, C_8H_{16}	118–119	71.7

These results indicate a rapid falling off in latent heat, with increase in molecular weight. It is to be regretted that we had not the metallic condenser, which would have enabled us to carry these observations up to include the less volatile hydrocarbons of both series. Advantage will be taken of the earliest opportunity to continue this work.

The hydrocarbons used in the work described in this paper were prepared with assistance granted by the Academy from the C. M. Warren Fund for Chemical Research.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOOLOGY AT HARVARD COLLEGE, UNDER THE DIRECTION OF E. L. MARK. — No. 129.

CERTAIN SENSE ORGANS OF THE PROBOSCIS OF THE POLYCHAETOUS ANNELID RHYNCHOBOLUS DIBRANCHIATUS.

By Adele Oppenheimer.

With Six Plates.

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By Adèle Oppenheimer.

Presented by E. L. Mark, April 13, 1898. Received February 15, 1902.

The proboscis of Rhynchobolus dibranchiatus was described by Ehlers ('64–68, p. 670) as "short, thick, club-shaped, with small egg-shaped papillae (compare Plate 1, Fig. 1), and was divided by him (p. 678) into two parts, the "Rüsselröhre," or sheath of the proboscis, and the "Kieferträger," or bearer of the jaws. Before eversion the "Rüsselröhre" is anterior to the "Kieferträger," but when the proboscis is everted (Fig. 1) the latter is anterior. The "Keiferträger" may be subdivided, as Ehlers suggested, into three regions, which in the non-everted state are respectively anterior, middle, and posterior: (1) the anterior has none of the small egg-shaped papillae; (2) the middle region is that supporting the four jaws; and (3) the posterior is, as a rule, not everted, it is the region of the four glands (*gl.*) of the jaws and the remainder of the proboscis following the glands. The boundary between "Kieferträger" and "Rüsselröhre" is marked, according to Ehlers, by the place of attachment to the proboscis of four partial diaphragms, called by him "Lappen" (Fig. 1, *lmn.*).

When cross sections of the everted proboscis are made in the region of the four partial diaphragms (Fig. 2), one encounters in succession in passing from the surface toward the centre (1) a cuticula (*cta.*); (2) an epithelial layer (*e'th.*); (3) a connective-tissue layer (*tis. co'nt.*), in which are embedded eighteen longitudinal nerves (*n. lg.*), and a nerve plexus; (4) a region composed of eighteen longitudinal muscles (*mu. lg.*); (5) a sheet of circular muscles (*mu. crc.*); (6) a fascia or peritoneum (*pi'tn.*)

lining the body-cavity; then in the body-cavity the four partial diaphragms; and finally that part of the proboscis which has not been everted. This consists of nearly the same kinds of layers arranged in the reverse order, namely a peritoneum, circular muscles, longitudinal muscles, nerves, connective tissue, and cuticula.

The epithelial layer directly beneath the cuticula is not mentioned as such by Ehlers. Since it apparently undergoes an interesting metamorphosis, it is worthy of further study.

From the underlying connective-tissue layer eighteen projections of connective tissue pass radially inward between the eighteen longitudinal muscles to the region of the circular muscles. Where the radial projections are continuous with the outer circular portion of the connective tissue the eighteen longitudinal nerves (*n. lg.*) are seen cut crosswise. (Compare Pl. **2**, Fig. 10.)

Concerning the structure of these longitudinal nerves I have nothing to add to what Ehlers ('64–68, p. 696) has already pointed out. They are evidently surrounded by a protecting connective tissue, within which lie what are apparently nerve fibres. In preparations fixed in vom Rath's picric-osmic-platinic chloride-acetic mixture, the nervous plasm is flocculent and has shrunken away from the nerve sheath.

From these longitudinal nerves, fibres pass out (Pl. **2**, Fig. 10) to form the peripheral nerve plexus, which is embedded in the connective tissue occupying the space between the longitudinal muscles and the cuticula. Other nerve fibres (*n. r.*) starting from the plexus pass radially inward, skirting the longitudinal muscle (Fig. 10); yet apparently they do not innervate the muscles, for I have seen no nerve fibre pass through the sheath enclosing the muscle. Still other radial nerve fibres (*n. r.'*) can be followed from the longitudinal nerves passing through the middle of the radial connective-tissue projections toward the centre of the sections as far as to the membrane immediately superficial to the circular muscles (Pl. **1**, Fig. 2; Pl. **2**, Fig. 10). In the anterior region, where the four partial diaphragms, the "Lappen" of Ehlers, are attached to the wall of the proboscis, radial nerve fibres occupying the same relative position as those marked in other regions *n. r.'* can be traced into these four pendent structures. Ehlers says concerning these "Lappen" (p. 686): "By means of a fold it [the fascia which invests the surface of all these parts] forms the four 'Lappen,' which are attached at the boundary between 'Rüsselröhre' and 'Kieferträger;' these 'Lappen' therefore possess the fine tense membrane on both surfaces; between lies a fibrous tissue, which is apparently identical with the subcuticular tissue of the sheath of the proboscis, with which, moreover, it is evidently continuous. This tissue . . . consists of a fibrous network, in the meshes of which lie ganglion cells." Further on (p. 696) he says: "The ganglion cells between the leaves of these 'Hautlappen' lie in a single layer and are surrounded by strands of fibres, so that they lie as it were in the meshes of such a net made up of bundles of fibres; however, it seems to me very doubtful whether these strands of fibres which make the meshes are all of nervous nature; on the contrary I believe that the greater mass of this fibrous tissue is identical with that which lies under the chitinous cuticula of the 'Rüsselröhre' and forms the sheath of the longitudinal nerves."

If I understand Ehlers correctly (he has no figures showing these histological conditions), I do not entirely agree with him concerning the structure of the "Lappen." Within the peritoneum I find connective tissue, ganglion cells, and also cells not mentioned by Ehlers (Pl. **2**, Figs. 7, 8). These last have an epithelial character; they form, indeed, the main bulk of the lobe, as appears both in material prepared in the vom Rath mixture and in two haematoxylin preparations made from material fixed respectively in corrosive sublimate and in sublimate-acetic. The "Fasergewebe" of Ehlers I consider nervous in large part. Almost all of the fibres (Fig. 8) surround, not the ganglion cell, as one might infer from his description, but its nucleus, and pass out at one pole of the cell body to the longitudinal nerves of the proboscis.

Finally, nerve fibres from the longitudinal nerves and from the peripheral nerve plexus can be traced out peripherally into the small papillae which are thickly distributed over the surface of the "Russelröhre."

Through the kindness of Mrs. Margaret Lewis Nickerson, who suggested to me the subject of the present paper, I was able to begin my study of the distribution of the sensory papillae of the proboscis on a preparation of the cuticula already made by her. The cuticula had been prepared by a method which was first employed by Mrs. Nickerson. All my subsequent preparations of the cuticula of other individuals were secured by the same method, which was as follows: The worm, after being narcotized in a mixture of sea-water and alcohol, was placed in a ten per cent solution of common salt until it was evident that its skin was loosened from the body. A cut was then made through the cuticula along a longitudinal line of the body, and the animal placed in tap-water. After the salt had been thoroughly washed out, the worm was cut transversely into pieces short enough for the cuticula to be mounted conveniently on a slide. The cuticula was next peeled off with needles and floated upon glass slides. These preparations were ready for study as soon as they were dry.

The whole surface of the proboscis, except the part which is most anterior in the usual state of eversion (Fig. 1), is covered with conical or thimble-shaped papillae, which are arranged on the summit of transverse folds (Pl. 5, Figs. 32, 33). In general the axes of the papillae are perpendicular to the surface of the proboscis, or are directed outward and either slightly backward or slightly forward. The rows of papillae are as a rule separated from each other by regular intervals, but sometimes there is an anastomosis (Fig. 32) of the folds from which these organs project. The folds follow one another closely, and there are one or two rows of papillae to each fold. At the posterior part of the everted proboscis the transverse rows are divided into eighteen longitudinal groups (Fig. 33); the interspaces correspond to the position of the eighteen longitudinal nerves. Otherwise the arrangement and frequency of these organs is the same from the anterior to the posterior end of the proboscis, and there is no other evidence of special grouping in any part.

The papillae are more or less ovoid or conical. On a proboscis about $5\frac{1}{2}$ mm. in diameter at the anterior end, they were found to be about 80 μ in height and about 35 μ in diameter at the thickest part.

The cuticula of the proboscis passes over each papilla, but is here reduced to about two-thirds the thickness it has elsewhere. The cuticula of the posterior face of each papilla is coarsely corrugated. The ridges are most clearly seen in preparations of removed cuticula (Pl. 6, Fig. 34), or in sections stained in Kleinenberg's haematoxylin (Pl. 1, Fig. 6; Pl. 3, Fig. 13). Though varying in number in different papillae, the ridges show considerable regularity of form and arrangement, for the outlines produced by them are always rather sharply bent in a region corresponding with the middle of the posterior face of the papillae, so that the surface view of that face shows a series of V-shaped outlines, like the longitudinal section of a nest of funnels, the apices of the V's being directed toward the base of the papilla. Sometimes, however, there is an anastomosis of the folds (Pl. 3, Fig. 13). Ehlers (p. 679) says of this species of Rhyncholbolus that the cuticula of the papillae has "fine folds, which, like those of the gills, occur in spiral lines, surrounding the papilla, or more rarely, standing out as sharply projecting ridges." Concerning the gills he says (p. 676): "The chitinous covering possesses at fairly regular intervals furrows which pass around the circumference spirally; their significance probably consists in their laying the gill into definite folds when it collapses and withdraws into the parapodial pouch." Whatever may be the condition in the case of the gills, the furrows of the papillae do not encircle those organs, for I have found that they exist on the posterior face of the papilla only. That the function of the furrows of the papillae is similar to that suggested by Ehlers for those of the gills, namely to determine the place of folding when the organs are retracted, may well be questioned, for there is no evidence that the papillae are ever retracted; there are no muscles to effect contraction, nor have I ever found the organs in a retracted condition.

The papillae have been studied in sections fixed in a mixture of corrosive sublimate and acetic acid and subsequently stained in Kleinenberg's haematoxylin; in sections fixed in corrosive sublimate and stained in iron haematoxylin; in preparations fixed in vom Rath's ('95, p. 282) picric-osmic-platinic chloride-acetic mixture (to which tap-water was sometimes added); and in methylen-blue preparations. The sections stained in iron haematoxylin I prepared, through the kindness of Professor Lloyd, in the laboratory of the Teachers College, Columbia University.

The living substance of the papillae appears to consist of either four or five cells, which are, to judge from the nuclei, of two kinds. Two of the nuclei (Pl. **1**, Fig. 3; Pl. **3**, Fig. 16, *nl. ba.*) found in the papillae are basal in position and larger than the others; the remaining two or three (*nl. ax.*) are nearer the apex of the papilla and also usually more nearly axial in position (Pl. **1**, Figs. 3, 4; Pl. **2**, Figs. 9*a*, 9*b*, 11; Pl. **3**, Figs. 16, 17; Pl. **4**, Figs. 26, 28, 30). The boundaries of the two cells to which the two basal nuclei belong cannot be made out by any process that I have employed.

In preparations made with vom Rath's mixture, the protoplasmic contents of the papilla are distinctly vacuolated. The vacuoles are also seen with nearly equal distinctness in the methylen-blue preparations, but not quite so clearly in sections stained with iron haematoxylin or with Kleinenberg's haematoxylin. The vacuoles are merely clearer, usually roundish, regions, which stand out distinctly, in contrast to the deeply stained granular or fibrous surrounding substance, and are quite variable in size, as is to be seen in Pl. **3**, Figs. 18, 20; Pl. **4**, Figs. 22, 25, 29. I believe that some of the more elongated vacuoles and the clusters of the more rounded ones in the region of the central nuclei (Figs. 22, 29), and perhaps a lighter coloring of the axial region of the papilla (Pl. **1**, Fig. 4; Pl. **2**, Fig. 11), gave rise to the following opinion expressed by Ehlers (p. 679): "There lies under the chitinous covering a thin sheet of finely granular substance, *which in the papilla appears to surround a narrow cavity*, and there is connected with this sheet a thick layer of fibrous tissue."

Connective-tissue fibres pass from the connective tissue of the proboscis into the papillae (Pl. **1**, Fig. 4; Pl. **2**, Fig. 11; Pl. **3**, Figs. 12, 19, 20; Pl. **4**, Fig. 27); as a rule, these could not be traced more than half-way to the apex of the papilla, but sometimes the contents of the papilla, in great part or entirely, looked fibrous (Pl. **1**, Fig. 4; Pl. **3**, Figs. 12, 15, 19). These fibres of the papilla are, as Ehlers says, in close connection with a finely granular substance. There is a particularly dense and deeply stained layer of this finely granular substance immediately under the cuticula (Pl. **1**, Figs. 3, 4; Pl. **2**, Fig. 11; Pl. **3**, Fig. 16; Pl. **4**, Fig. 30); it surrounds not a cavity, but a central region in which there is a little granular substance and in which there are many vacuoles. At one point of the base of the papilla, where the connective tissue enters (Pl. **1**, Fig. 4; Pl. **2**, Fig. 11), and again at one point near the apex, apparently in the region of the sensory termination of the papilla (Pl. **4**, Fig. 30*c*), there is a break in the dense layer of finely granular substance.

Of the two basal nuclei (*nl. ba.*) one is near the anterior, the other near the posterior face of the papilla (Pl. **2**, Fig. 9*b*). They are spheroidal or ellipsoidal, and contain small irregularly scattered chromatin granules in large numbers; but in preparations stained in haematoxylin (Pl. **1**, Fig. 3; Pl. **2**, Figs. 9*b*, 11; Pl. **3**, Figs. 16, 17) they appear less deeply colored than the remaining nuclei.

The more distal nuclei (*nl. ax.*) are more elongated, being ellipsoidal or spindle-shaped. They present an elliptical outline whether seen in sections perpendicular to the axis of the proboscis (Pl. **1**, Figs. 3, 4; Pl. **2**, Fig. 11; Pl. **4**, Fig. 30*c*), in longitudinal sections of the proboscis passing through the axis of the papilla (Fig. 28), or in sections perpendicular to the axis of the papilla (Pl. **2**, Fig. 9*a*; Pl. **3**, Figs. 16, 17). The outline may be more or less pointed at one end, and is more nearly circular in the sections perpendicular to the axis of the papilla than in those parallel to the axis. The deeply staining granulations of the distal, or axial, nuclei are larger and not less numerous than those of the basal nuclei; and it is perhaps for this reason that the first-named nuclei appear more deeply stained than the basal ones. The granulations of the axial nuclei are also more evenly distributed. Both kinds of nuclei have a clearly defined nuclear membrane. In the preparations fixed in sublimate-acetic and stained in Kleinenberg's haematoxylin, I have seen a nucleolus in the basal nucleus only, and here only occasionally (Pl. 3, Fig. 16; Pl. 4, Fig. 24). Sometimes, though rarely, there are in a basal nucleus *two* larger granulations (Pl. 2, Fig. 11; Pl. 4, Fig. 30*c*), which may perhaps be entitled to rank as nucleoli. In preparations stained in iron haematoxylin and in those fixed in vom Rath's mixture the nucleolus is regularly seen with great distinctness near the centre of the basal nucleus (Pl. 2, Fig. 9*b*; Pl. 4, Fig. 26–28). The nucleolus is not infrequently surrounded by a light area.

From the different effects produced on the two kinds of nuclei by haematoxylin and by methylen blue, it is fair to conclude that the cells to which the basal nuclei belong are very different from those of the apical nuclei, and that they have nothing to do directly with the nervous system. They are evidently indifferent subcuticular cells, which probably have the same functions as the cover cells of more complicated sensory organs.

The central elongated nuclei found in haematoxylin preparations, judging from their position, evidently correspond to the two or three spindle-shaped cell bodies which appear in methylen-blue preparations.

"I have not succeeded," says Ehlers (p. 690), "in finding proof positive that there are nerves in the fibrous tissue which enters the papilla from the common subcuticular layer." What Ehlers was unable to find, I have, by the use of improved histological methods, succeeded in demonstrating with entirely satisfactory clearness. The spindle-shaped cells are evidently nerve cells of sensory function. For, on the one hand, the basal end is connected with one of the eighteen longitudinal nerves of the proboscis by a nerve fibre passing to that nerve, either directly or, through the intervention of the peripheral nerve plexus, indirectly; and on the other hand the peripheral end tapers toward the apex of the papilla, where it terminates in a sensory structure, the precise nature of which it is difficult to make out.

Each of the sensory cells of the papilla has the form of an elongated spindle tapering at its free end to a delicate fibre-like structure, and continuous at its basal end with a fibre traceable to a nerve trunk. This spindle-shaped enlargement, or cell body, lies in the axis of the papilla and about midway between its base and apex. An exception to this rule regarding the position of the cell body is seen in Figure 20 (Pl. 3), where the cell seems to have a basal position. I am, however, in doubt as to whether the sensory cells in this case are actually basal in position, or whether the appearance may not be due to an accidental staining of parts adjacent to the nerve fibres, — a sort of extravasation, — accompanied by a failure to stain on the part of the real cell body and the more distal portions of the sensory cell. The spindle-shaped enlargement is sometimes stained uniformly, but more often the staining is irregular and blotchy; in some cases a nucleus is to be distinguished near the middle of the cell body in the widest part of the spindle, which it almost completely fills. In one case (Pl. 3, Fig. 14) the nucleus was sharply differentiated from the cell body, which was not at all blotchy, but distinctly fibrous and sparsely granular.

From the distal end of the spindle-shaped cell body there passes off a fibre that, I believe, breaks up into a number of fibrils, each of which seems to me to end in a disc (Pl. 1, Fig. 5; Pl. 3, Fig. 14). In Figure 31 (Pl. 4), the fibrils are quite clearly recognizable; in Figures 25 and 29 (Pl. 4), though distinguishable, they are not so evident. The terminal discs (Pl. 3, Fig. 18; Pl. 4, Figs. 25, 29) may, it is true, be artefacts; but the frequency of their occurrence and the similarity of their appearance seem to me to be arguments against that supposition. Sometimes the blue is deposited in great amount around this bunch of fibrils (Pl. 3, Figs. 12, 15, 18; Pl. 4, Fig. 29), but in other cases it has failed entirely to stain the portion of the sensory cell that is distal to the spindle-shaped enlargement. On the other hand, there are cases in which the peripheral part of the distal portion of the sense cell has been differentiated by staining in haematoxylin (Pl. 2, Fig. 11, not well brought out in the figure). In the case in which I have seen fibrils with their terminal discs most distinctly (Pl. 3, Fig. 14), the discs at the ends of the fibrils are at the surface of the papilla outside the cuticula; in other preparations, the fibrils seem not to pass through the cuticula, but to end at its deep surface. It is probable that in most cases the cuticula has been artificially separated from the protoplasmic mass of the papilla, and that normally the fibrils pass to the surface of the papilla.

The connection of the cell body with one of the eighteen longitudinal nerves of the proboscis is often to be traced on a single thick section. The process which the cell body sends centripetally either joins a longitudinal nerve directly, or enters the peripheral nerve plexus, which in turn joins the longitudinal nerve (Pl. 2, Fig. 10; Pl. 3, Fig. 19). The basal end of each of the two or three cell bodies of the papilla seen in methylen-blue preparations (Pl. 3, Figs. 12, 14, 20; Pl. 4, Figs. 23, 25, 29) is prolonged into a slender nerve fibre. While the fibre belonging to one of the cells of a papilla bends to the left when it joins the nerve plexus, that belonging to another cell of the same papilla may bend to the right, as is to be seen in Figures 10 (Pl. 2), 12, and 19 (Pl. 3). Occasionally the fibres twist around each other, and there is sometimes to be found an appearance which suggests anastomosis of these fibres, but focusing shows that in a great number of such cases the fibres cross without touching each other; in still other cases (Pl. 3, Figs. 12, 19) the blue staining is not confined to the fibres, and this makes the following out of the fibres more difficult.

The condition shown in Figure 15 (Pl. 3), which seems to be an exception to the rule that the basal end of each spindle-shaped cell body tapers into a nerve fibre, is probably the result of the well-known capriciousness of methylen-blue staining. In no case have I seen a nerve fibre arise from an abruptly rounded basal end of one of these sensory cells, but the cell body seems always to taper gradually into the nerve fibre. There are, however, quite a number of cases in which the inner end of the cell body does not simply taper into a single nerve fibre, but in which it is prolonged into a few processes which ultimately unite to form the fibre (Pl. 4, Figs. 23, 25).

These nerve fibres on their way to the longitudinal nerves often show at intervals those characteristic swellings, or varicosities, which have been so frequently figured in recent works on nerve fibres treated either by the methylen-blue or the Golgi methods.

SUMMARY.

The papillae of the proboscis of Rhyncholbolus are sensory organs. They are considered to be sensory on the following grounds: —

1. The papillae are well differentiated organs.

2. They are found over almost the entire surface of the everted proboscis.

3. They are elevated above the surrounding surface.

4. The cuticula which passes over each papilla is reduced to about two-thirds the thickness it has elsewhere on the proboscis.

It should be mentioned that the cuticula of the posterior face of each papilla is coarsely corrugated, but the significance of this wrinkling is unknown.

5. There are two or three spindle-shaped cells in a papilla, each of which terminates — either below the cuticula or more probably at the very apex of the papilla — in what is clearly a sensory structure, and each of these cells tapers gradually at its base into a nerve fibre. These

nerve fibres are connected either directly or indirectly — through the intervention of a peripheral nerve plexus — with the eighteen longitudinal nerves of the proboscis.

6. There are two basal nuclei that belong to cells which probably have the function of cover cells.

It remains to be said that there enter each papilla besides nerve fibres, connective-tissue fibres. These latter are found in close connection with a finely granular substance, of which there is a particularly dense and deeply staining layer immediately under the cuticula. Standing out in contrast to the deeply stained granular or fibrous surrounding substance are the clear, generally rounded vacuoles.

If there is any differentiation in function between papillae, it is not correlated with any pronounced difference in structure.

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EXPLANATION OF PLATES.

ABBREVIATIONS.

coel.	Coelom, body-cavity.
cta.	Cuticula.
cta. + e'th.	Cuticula and epithelium.
gl.	Gland.
gna.	Jaw.
lmn.	Lemniscus.
mu. crc.	Circular muscle.
mu. lg.	Longitudinal muscle.
n. crc.	Circular nerve.
nl. ax.	Axial nucleus.
nl. ba.	Basal nucleus.
n. lg.	Longitudinal nerve.
n. r.	Radial nerve fibre skirting longitudinal muscle.
n. r.'	Radial nerve fibre passing directly to the membrane superficial to the circular muscles.
pap.	Papilla.
pr'tn.	Peritoneum.
tis. co'nt.	Connective tissue.

In many figures not only the papilla is shown, but also a portion of the underlying parts.

PLATE 1.

FIG. 1. Longitudinal section of the everted proboscis showing: (1) the sheath of the proboscis; (2) the bearer of the jaws and its subdivision; and (3) the lemniscus (*lmn.*), which marks the boundary between (1) and (2).

Narcotized in a mixture of sea-water and alcohol; fixed in Müller's fluid; stained with Beale's ammonia carmine. × *circa* 11.

FIG. 2. Cross section of the partially everted proboscis in the region of the four lemnisci (*lmn.*), showing, among other things, a diagrammatic representation of the papillae and the connection of their sensory cells with the circular and the longitudinal nerves, and also the nerve fibre (*n. r.'*) passing to the membrane which invests the circular muscles.

Chloroform, methylen blue, Bethe's ammonium molybdate for invertebrates. × 14.5.

FIG. 3. Papilla from a cross section of the proboscis, showing two "basal" and three "axial" cell nuclei.

Sea-water and alcohol, sublimate-acetic, Kleinenberg's haematoxylin. × 675.

FIG. 4. Longitudinal section of a papilla, from a cross section of the proboscis, showing the two axial nuclei and one of the two basal nuclei, also fibrous structures entering the base of the papilla. Treatment the same as in Fig. 3. × 585.

FIG. 5. Longitudinal section of a papilla, from a sagittal section of the proboscis, showing two sensory axial cells with peripheral sensory termination and prolongation of the basal end of each into a slender nerve fibre.

Chloroform, methylen blue, Bethe's ammonium molybdate for invertebrates. × 650.

FIG. 6. Papilla from cross section of proboscis viewed from behind, showing the corrugations of the cuticula on the posterior face of the papilla, and in optical section the two zones of living substance together with one of the basal nuclei.

Sea-water and alcohol, sublimate-acetic, Kleinenberg's haematoxylin. × 585.

PLATE 2.

Fig. 7. Portion of cross section of proboscis, showing structure of lemniscus.
Sea-water and alcohol, vom Rath's mixture. × 200.

Fig. 8. Part of Fig. 7 enlarged. × *ca.* 400.

Figs. 9*a*, 9*b*. Sections of a papilla perpendicular to its axis. Figure 9*a* represents the more distal of the two sections, and shows the form and position of the three axial nuclei; Figure 9*b* shows the two basal nuclei. The anterior face of the papilla is directed toward the top of the plate in both cases.

Sea-water and alcohol, corrosive sublimate, iron haematoxylin.

Fig. 10. Portion of the cross section of an everted proboscis, showing one of the eighteen longitudinal nerves (*n. lg.*) cut crosswise, the peripheral nerve plexus, the union of the centripetal processes from the sense cells with the longitudinal nerve (in the case of the third papilla from the upper margin of the Figure, one of the two nerve fibres bends to the left when it enters the nerve plexus, the other to the right), a radial nerve (*n. r.*) following the surface of the longitudinal muscle (this is sketched in from an adjacent section), and another radial nerve (*n. r.'*) passing directly to the membrane which is immediately superficial to the circular muscles.

Chloroform, methylen blue, Bethe's ammonium molybdate for invertebrates. × 145.

Fig. 11. Papilla from a cross section of proboscis showing one of the basal and one of the axial nuclei; there are two large granulations in the basal nucleus.

The differentiation of the distal portion of the sense-cell is not well shown.

Sea-water and alcohol, sublimate-acetic, Kleinenberg's haematoxylin. × 460.

OPPENHEIM [illegible]

PLATE 3.

Figs. 12, 14, 15, 18, 19, 20. Preparations made by use of chloroform, methylen blue, and Bethe's ammonium molybdate for invertebrates.

Figs. 13, 16, 17. Prepared by use of sea-water and alcohol, sublimate-acetic, Kleinenberg's haematoxylin.

Fig. 12. Papilla from cross section of proboscis, showing connective-tissue fibres passing into the papilla; deep coloration of terminal fibrils; the nerve fibres bending in opposite directions where they enter the nerve plexus. × 680.

Fig. 13. Papilla from cross section of proboscis, showing corrugations of posterior face of papilla, and the outline of one of the basal nuclei. × 460.

Fig. 14. Papilla from sagittal section of proboscis; the two sensory (axial) cells, their peripheral terminations, and their proximal nerve-fibre prolongations stained blue.

The nucleus of one of the sensory cells more deeply stained than the cell body. Cuticula distended and detached from substance of the papilla by treatment. × 710.

Fig. 15. Papilla from cross section of proboscis, showing deeply stained axial body, from which a single peripheral, deeply stained process extends to the apex of papilla, where it terminates in a specialized and stained area; the contents of the papilla in great part fibrous. × 1020.

Fig. 16. Somewhat oblique cross sections of two papillae from a cross section of the proboscis. In one papilla are two basal nuclei and a part of one of the axial nuclei; in the other the three axial nuclei cut crosswise. × 670.

Fig. 17. Cross sections of two papillae from a cross section of proboscis. In one are seen two axial nuclei, each surrounded with a clear area; in the other a basal nucleus and portions of two axial (?) nuclei. × 670.

Fig. 18. Papilla from cross section of proboscis. The two sensory cells are stained throughout; their distal prolongations have a more or less spiral course and terminate in a cluster of discs at the apex of the papilla. Vacuoles large. × 715.

Fig. 19. Papilla from cross section of proboscis, showing that where the centripetal fibres from two sensory cells meet the nerve plexus, one bends to the right, the other to the left. × 725.

Fig. 20. Papilla from cross section of proboscis, showing the basal position of the sensory cell body (?); the basal end of each sensory cell is prolonged into a slender nerve fibre. × 682.5.

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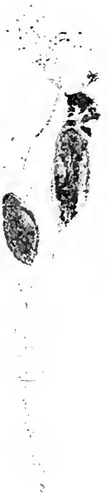

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PLATE 4.

Figs. 21–23, 25, 27, 29, 30. Longitudinal sections of papillae from cross sections of proboscis.

Figs. 21–23, 25, 29. Preparations made by use of chloroform, methylen blue, Bethe's ammonium molybdate for invertebrates.

Figs. 24, 30. Preparations made by use of sea-water and alcohol, sublimate-acetic, Kleinenberg's haematoxylin.

Fig. 21. Three sensory cells, two showing peripheral fibres and terminations. × 715.

Fig. 22. Papilla showing a row of axial vacuoles.
Sea-water and alcohol, Müller's fluid, Beale's ammonia carmine.

Fig. 23. The nuclei of the two sensory cells distinguishable from the cell body by their deeper stain. Peripheral and proximal fibres stained. × 710.

Fig. 24. Basal nucleus of a papilla showing a large single nucleolus. × 670.

Fig. 25. Highly vacuolated papilla, fibrils and discs of the sensory termination stained blue, the deep ends of each of the sensory cells prolonged into a few processes, which unite to form the single nerve fibre. × 700.

Fig. 26. Cross section of a small papilla, showing a nucleolus in each basal nucleus.
Sea-water and alcohol, vom Rath's mixture. × 680.

Fig. 27. Papilla showing one of the basal nuclei with large nucleolus, and the passage of connective-tissue fibrils into the papilla.
Sea-water and alcohol, vom Rath's mixture.

Fig. 28. Papilla from sagittal section of proboscis, showing three axial nuclei and two basal nuclei.
Sea-water and alcohol, sublimate, iron haematoxylin.

Fig. 29. Papilla from cross section of proboscis, showing numerous small vacuoles, fibrils and discs of sensory termination. The basal end of each sensory cell is prolonged into a slender nerve fibre. × 730.

Figs. 30*a*–30*d*. Four successive sections from a single papilla.
Fig. 30*b* shows one of the basal nuclei; Fig. 30*c*, the other basal nucleus and the two axial nuclei.
In the region of the apex of the papilla, there is an interruption in the cortical layer of finely granular substance, not well shown, and the region is traversed by fine fibres. × 585.

Fig. 31. Fibrils from the peripheral termination of a sensory cell.

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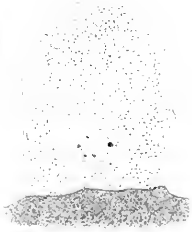

c

d

PLATE 5.

Fig. 32. From a photograph of the cuticula of the proboscis stripped by maceration (consult text, p. 555) and mounted on glass slide. The part of the figure nearest the top of the plate is toward the anterior end of the everted proboscis. To show the arrangement of the papillae in transverse rows. × 22.5.

Fig. 33. From a photograph of a preparation similar to that of Fig. 32, showing the appearance of the cuticula and attached papillae near the posterior end of the everted proboscis. Nine of the eighteen longitudinal columns of papillae are shown. × 18 5.

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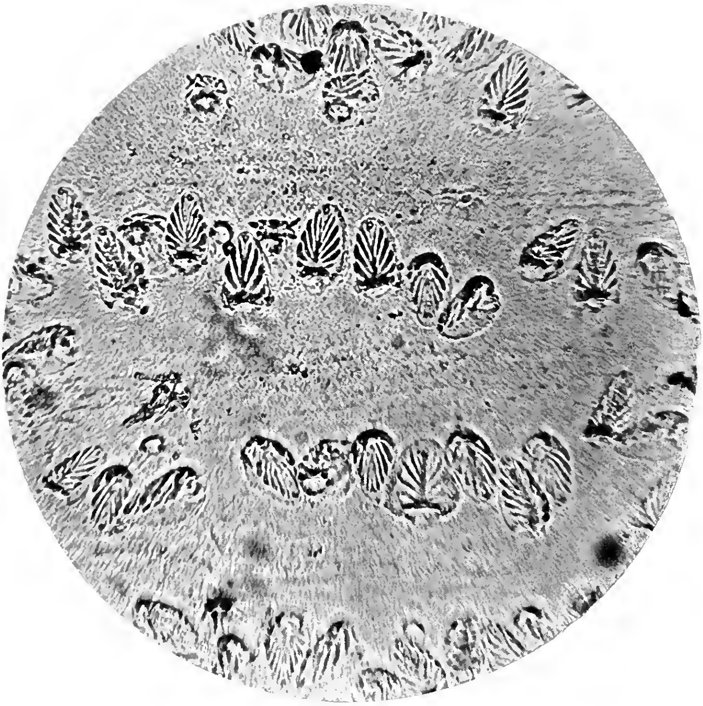
PLATE 6.

FIG. 34. Highly magnified view of portions of four transverse rows of papillae, to show the corrugations of the flattened and dried papillae, and the circular wall and pit of the cuticula at the apex of the papilla, marking the position of the termination of the sensory cells. × 110.



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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF CASE SCHOOL OF APPLIED SCIENCE. — XLI.

THE COMPOSITION OF PETROLEUM.

By Charles F. Mabery.

ON THE HYDROCARBONS IN PENNSYLVANIA PETROLEUM WITH BOILING POINTS ABOVE 216°.

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Received March 24, 1902. Presented April 9, 1902.

In a former paper * the principal hydrocarbons in Pennsylvania, Ohio, and Canadian petroleum below 216°, were shown to have the composition represented by the general formula C_nH_{2n+2}. Concerning the constituents with higher boiling points very little is known. Pelouze and Cahours † collected distillates to which they gave the following formulas: —

216°–218°	236°–240°	255°–260°	280°
$C_{13}H_{28}$	$C_{14}H_{30}$	$C_{15}H_{32}$	$C_{16}H_{34}$

From paraffine the following hydrocarbons have been separated (Beilstein's Handbook): —

212°–215°	230°–235°	252°–255°	273°–275°
$C_{13}H_{28}$	$C_{14}H_{30}$	$C_{15}H_{32}$	$C_{16}H_{34}$

So far as I am aware, these are the only allusions to the composition of Pennsylvania petroleum in these portions. It appears that the distillations were made under ordinary atmospheric pressure in presence of air with no attempts to avoid decomposition under these conditions. Then, furthermore, as I have suggested in a former paper, the high specific gravity of the distillates separated by Pelouze and Cahours indicate that their distillates were obtained from Canadian petroleum. It does not appear that any attempts have been made to ascertain the molecular weights of the individual hydrocarbons. Indeed this has been

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* These Proceedings, XXXII. 121.

† Ann. Chim. Phys. (4), 1, 5 (1864).

possible only within recent years since the freezing and boiling point methods for the determination of molecular weights have been known. Then it is not possible to determine the vapor densities of these hydrocarbons, because, as I have recently ascertained, even in vacuo small quantities of the hydrocarbons such as are used in vapor density determinations, undergo serious decomposition; and this occurs even in oils that have been distilled many times over in bulk in vacuo. In the earlier work, it was evidently assumed that a few distillations under atmospheric pressure were sufficient to collect the individual hydrocarbons within the limits of their boiling points, sufficient at least to afford reliable data as to their composition. A glance at the small differences in percentage composition is sufficient to show that it is not possible by analysis alone of products even well purified to distinguish between homologous members of a series, although such analysis may define the series.

But the chief difficulty is to obtain each hydrocarbon uncontaminated by any admixture of its homologues or by products of decomposition. This is well illustrated by the experience of Markownikoff in separating the hydrocarbons in the Russian oil, who found it impossible to collect distillates closer than limits of five degrees on account of decomposition. That the same is true perhaps in a less degree in distillates from Pennsylvania oil is evident whenever distillation is made of the higher portions under atmospheric pressure. The rank odor is evidence of cracking. Yet the constituents with higher boiling points are under the influence of vapor tension as much as the constituents with lower boiling points, and consequently require as prolonged distillation for complete or approximately complete separation. In my experience it is only possible to obtain even an approximate separation by exclusion of air and depression of boiling points.

In this manner the higher hydrocarbons may be distilled any number of times with no appreciable decomposition. The only limit is the patience of the operator. But the stability of these hydrocarbons is evidently dependent on the influence of mass. Since as mentioned above, while distillation of any considerable quantity of the oil may be carried on indefinitely, a limited quantity cannot be volatilized even in vacuo without decomposition. Many attempts to determine the vapor density of the hydrocarbons in Pennsylvania and California petroleum by volatilization in vacuo according to the method of Lunge and Neuberg have failed on account of cracking, even so far as the separation of sooty carbon from the members with high boiling points.

In September, 1896, I set out to ascertain the composition of the principal hydrocarbons in Pennsylvania petroleum above 216° so far as they can be separated by distillation on a laboratory scale. Through the courtesy of the Standard Oil Company, I procured a barrel of crude oil from Oil City and this material has been used to separate the hydrocarbons that will be described in this paper. That this oil was an approximately average specimen of Pennsylvania petroleum, appears from its properties. A determination of the specific gravity of the crude oil at 20° gave 0.8095. A combustion of the oil dried over sodium gave the following percentages of carbon and hydrogen: Carbon, 85.80; Hydrogen, 14.04. Eight hundred grams distilled in the ordinary way in the following proportions: —

50°–150°	150°–200°	200°–250°	250°–300°	+300°
166	88	83	100	337

Fifty-six kilos of the crude oil was distilled in quantities of 10 litres each in a porcelain still, collecting under atmospheric pressure to 200°, and within limits of 10° under a vacuum of 50 mm. to 300°, then within limits of 5°, and finally within limits of 2°. After eight distillations the following proportions collected: —

	124°–126°	136°–138°	156°–158°	174°–176°	188°–199°
Grams,	125	145	240	205	240

199°–201°	210°–212°	226°–228°	242°–244°
225	335	150	130

Since the weights of these fractions represent all that came from the original crude oil, it is possible to gain a very general idea of the proportion of the hydrocarbons which are contained in the crude oil. But such estimation must be only approximate from the fact that any distillation however thorough gives only an approximate separation, and a considerable portion of any hydrocarbon must be contained in the intermediate distillates.

The percentages of the weights are as follows: —

$C_{13}H_{28}$	$C_{14}H_{30}$	$C_{15}H_{32}$	$C_{16}H_{34}$	$C_{17}H_{36}$
124°–126°	136°–138°	156°–158°	174°–176°	188°–199°
0.22	0.26	0.43	0.37	0.43 per. cent.

$C_{18}H_{38}$	$C_{19}H_{40}$	$C_{20}H_{42}$	$C_{21}H_{44}$
199°–201°	210°–212°	230°–232°	242°–244°
0.40	0.60	0.27	0.23 per. cent.

In the intervals between these fractions the weights were much smaller.

After the eighth distillation each fraction was agitated at first with common concentrated sulphuric acid until the acid was not much colored and then several times with fuming sulphuric acid. That the distillates consist almost entirely of the principal hydrocarbon is shown by the slight change in specific gravity by the acid treatment:—

156°–158°	174°–176°	182°–184°	
0.805	0.8064	0.8100	Original distillate.
0.7992	0.8031	0.8093	After treatment with acid.

There was some loss in weight of the fraction by the acid treatment, but it was doubtless due for the most part to solution of the principal hydrocarbons in the acid. The weights of the fractions before and after treatment were as follows:—

156°–158°	174°–176°	182°–186°
240	215	205
195	190	155

This solvent action has been observed in other oils with high boiling points in continuous treatment with fuming sulphuric acid, which caused a gradual loss without changing materially the specific gravity. After the purification with the acid, distillation was continued through a Hempel column filled with glass beads or broken glass, under 50 mm. within limits of one degree, until the hydrocarbons collected in considerable quantities. After the thirtieth distillation, the hydrocarbons came together within the following limits:—

124°–126°, 142°–143°, 158°–159°, 173°–174°, 189°–190°, 198°–199°.

Even after nearly continuous distillation of sixteen months, these products showed very little indication of decomposition. Leaks in the apparatus immediately cause decomposition, as shown by a disagreeable odor, and the appearance of the distillates. So long as air is excluded from the hot vapors there is no danger of decomposition. But as we found in attempting to ascertain the boiling points under atmospheric pressure, a single distillation in air causes a very rank odor of decomposition.*

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* The difference in stability of the constituents of different petroleums is shown by their behavior when air comes in contact with the hot vapors. In accidents that have occurred during distillation, letting in air on the hot vapors, in the case of Pennsylvania petroleum the still becomes filled with dark vapors, but in a

In determining the boiling points of these hydrocarbons under atmospheric pressure, 70 grams of the fraction 124°–126° distilled as follows, under 760 mm. and with the mercury column all within the vapor : —

	224°–225°	225°–226°	226°–227°	227°–228°	228°–229°
Grams,	4	28	26	3	3

Colored residue, 6

The portions between 225° and 227° collected almost entirely between 225°.5 and 226°.5.

The fraction 142°–143° nearly all distilled at 237°–238° atmospheric pressure, the fraction 158°–159° at 256°–257°, the fraction 173°–174° at 274°–275°, the fraction 188°–189° at 288°–289°, and the fraction 198°–199° at 300°–301°. The hydrocarbon dodecane $C_{12}H_{26}$ was identified in the fraction 214°–216°.*

TRIDECANE, $C_{13}H_{28}$.

The next homologue, tridecane, was sought for in the series of distillates that collected between 215° and 235°. After carrying these fractions eight times through a Hempel bead column a larger portion, 200 grams, collected at 221°–222°. This portion was distilled twenty-four times, when 70 grams collected, as shown above. The specific gravity of the distillate before further treatment was 0.7866, and after thorough agitation with fuming sulphuric acid, 0.7834. A combustion gave the following values for carbon and hydrogen : —

0.1506 grm. of the oil gave 0.4690 grm. CO_2 and 0.2028 grm. H_2O.

	Calculated for $C_{13}H_{28}$.	Found.
C	84.78	84.94
H	15.22	14.96

The molecular weight of this oil as determined by the Beckman method at the freezing point of benzol, in the hands of different workers, corresponded to that of $C_{13}H_{28}$.

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distillation of Russian petroleum, air accidently admitted caused such a violent explosion that the thermometer was sent violently across the room and broken against the wall.

* These Proceedings, XXXII. 138.

I. 1.1209 grm. of the oil and 36.3043 grms. benzol gave a depression of 0°.82.

II. 1.2502 grm. of the oil and 33.3631 grms. benzol gave a depression of 1°.015.

III. 1.496 grm. of the oil and 32.9131 grms. benzol gave a depression of 1°.205.

Calculated for $C_{13}H_{28}$.	Found. I.	II.	III.
184	184.5	181	184.9

In still further confirmation of the formula of tridecane, the index of refraction was determined, and its molecular refraction calculated. The index was found to be 1.4354 at 20°, and the molecular refraction as follows: —

Calculated for $C_{13}H_{28}$.	Found.
61.94	61.44

Monochlortridecane, $C_{13}H_{27}Cl$. — In the preparation of the chlorine derivative of tridecane, chlorine was allowed to act on the hydrocarbon in screened sunlight, over water. With care to avoid an excess of chlorine, the product consisted for the most part of the monochloride. It was fractioned in vacuo under 12 mm. several times until considerable of the oil collected at 135°–140°. This fraction gave as its specific gravity at 20°, 0.8973. A determination of chlorine gave a percentage required for the monochloride: —

0.1920 grm. of the oil gave 0.1273 grm. AgCl.

	Required for $C_{13}H_{27}Cl$.	Found.
Cl	16.23	16.39

The molecular weight as determined at the freezing point of benzol corresponded to the same formula: —

0.4585 grm. of the oil and 18.35 grms. benzol gave a depression of 0°.568.

Calculated for $C_{13}H_{27}Cl$.	Found.
218.5	216

A determination of the index of refraction confirmed the same formula; the index as determined was 1.451, corresponding to the molecular refraction: —

Calculated.	Found.
65.71	66.67

TETRADECANE, $C_{14}H_{30}$.

The fraction 142°–143°, collected after the twenty-fourth distillation, gave as its specific gravity, after drying over sodium, 0.7848. After agitating with concentrated sulphuric acid, washing, and drying over sodium, it gave 0.7847. After treatment with fuming sulphuric acid it gave 0.7814; this determination, like the others, was made at 20°.

A determination of the molecular weight of this fraction purified with fuming sulphuric acid gave the following result: —

I. 1.1049 grm. of the oil and 36.8505 grms. benzol gave a depression of 0°.735.

II. 1.052 grm. of the oil and 35.970 grms. benzol gave a depression of 0°.718.

Calculated for $C_{14}H_{30}$.	Found. I.	Found. II.
198	199.9	199.5

A combustion gave the following percentages of carbon and hydrogen:

0.1502 grm. of the oil gave 0.4698 grm. CO_2 and 0.2024 grm. H_2O.

	Calculated for $C_{14}H_{30}$.	Found.
C	84.84	85.02
H	15.16	14.96

This specimen was purified with common concentrated acid; another portion purified with fuming sulphuric acid gave slightly different proportions: —

0.1458 grm. of the oil gave 0.4532 grm. CO_2 and 0.1970 grm. H_2O.

	Calculated for $C_{14}H_{30}$	Found.
C	84.84	84.76
H	15.16	15.02

The boiling point of this fraction under 760 mm. was 236°–238°. On account of decomposition when the oil is distilled in air, it is difficult to determine the boiling point with great precision.

A determination of the index of refraction of this hydrocarbon gave 1.4360, which corresponds to the following molecular refraction: —

Calculated or $C_{14}H_{30}$.	Found.
66.54	66.36

Monochlortetradecane, $C_{14}H_{29}Cl$. — Since only small quantities of the purified hydrocarbons were available for the study of the chlorides, great care was necessary to avoid too high chlorination. Most of the hydrocarbons gave only sufficient of the chlorine derivative to verify its formula by the percentage of chlorine. The chlorine product obtained from tetradecane was fractioned in vacuo until it collected in larger quantities at 150°–153° under 20 mm. A determination of chlorine gave the following result: —

0.1966 grm. of the oil gave 0.1245 grm. AgCl.

	Calculated for $C_{14}H_{29}Cl$.	Found.
Cl	15.25	15.65

Its specific gravity at 20° was found to be 0.9185. The quantity of product was not enough for other determinations. Another portion of the chlorine product collected at 175°–180°, 17 mm., which gave as its specific gravity at 20°, 1.032. A determination of chlorine gave the value required for the dichloride: —

0.1937 grm. of the oil gave 0.2125 grm. AgCl.

	Calculated for $C_{14}H_{28}Cl_2$.	Found.
Cl	26.55	27.12

A determination of molecular weight at the freezing point of benzol confirmed the dichloride: —

1.3407 grm. of the oil and 19.81 grms. benzol gave a depression of 1°.255.

Calculated for $C_{14}H_{28}Cl_2$.	Found.
267	264.3

PENTADECANE, $C_{15}H_{32}$.

The specific gravity of the fraction 158°–159° (50 mm.) dried over sodium was found to be 0.8054 at 20°. After treatment with concentrated sulphuric acid it gave 0.7939, and after thorough treatment with fuming sulphuric acid it gave 0.7896.

The molecular weight was determined by the Beckman method: —

I. 1.050 grm. of the oil and 35.9775 grms. benzol gave a depression of 0°.675.

II. 1.3946 grm. of the oil and 23.2679 grms. benzol gave a depression of 1°.37.

Calculated for $C_{15}H_{32}$.	Found. I.	Found. II.
212	211	212

Combustion I. was made of the unpurified distillate dried over sodium; and combustion II., of the oil after treatment with fuming sulphuric acid: —

I. 0.1440 grm. of the oil gave 0.4500 grm. CO_2 and 0.1919 grm. H_2O.
II. 0.1608 grm. of the purified oil gave 0.5002 grm. CO_2 and 0.2198 grm. H_2O.

	Calculated for $C_{15}H_{32}$.	Found. I.	Found. II.
C	84.92	85.21	84.87
H	15.08	14.80	15.20

In determining the boiling point of pentadecane under atmospheric pressure, it distilled almost completely at 256°–257°.

A determination of the index of refraction gave 1.4413, from which the molecular refraction was calculated: —

Calculated for $C_{15}H_{32}$.	Found.
71.15	70.49

Dichlorpentadecane, $C_{15}H_{30}Cl_2$. — With the small quantity of the hydrocarbon at hand, we did not succeed in limiting the action of chlorine to the formation of the monochloride. Fractioned in vacuo under 13 mm. the chlorinated product collected for the most part at 175°–180°. This product gave as its specific gravity at 20°, 1.0045. A Carius determination for chlorine gave the following percentage: —

0.1411 grm. of the oil gave 0.1462 grm. AgCl.

	Calculated for $C_{15}H_{30}Cl_2$.	Found.
Cl	25.28	25.63

The molecular formula was established by a determination of its molecular weight: —

1.4308 grm. of the oil and 18.53 grms. benzol gave a depression of 1.336.

Calculated for $C_{15}H_{30}Cl_2$.	Found.
281	283.2

HEXADECANE, $C_{16}H_{34}$.

The heap that collected at 174°–175°, 50 mm., after the thirtieth distillation gave as its specific gravity at 20°, 0.8000. After treatment with concentrated sulphuric acid it gave 0.7964, and after treatment with fuming sulphuric acid, 0.7911. It distilled almost completely at 275°–

276° under 760 mm., barometric pressure. In determining the molecular weight of this fraction at the freezing point of benzol, it gave the formula $C_{16}H_{34}$: —

I. 1.1507 grm. of the oil and 35.27 grms. benzol gave a depression of 0°71.

II. 1.1833 grm. of the oil and 35.63 grms. benzol gave a depression of 0.715.

Calculated for $C_{16}H_{34}$.	Found. I	II.
226	225.2	227.6

Determinations of carbon and hydrogen were made in the unpurified distillate (I.), in a portion purified with concentrated sulphuric acid (II.), and in a third portion purified with fuming sulphuric acid (III.), with the following results: —

I. 0.1477 grm. of the oil gave 0.4600 grm. CO_2 and 0.1973 grm. H_2O.
II. 0.1454 grm. of the oil gave 0.4522 grm. CO_2 and 0.1986 grm. H_2O.
III. 0.1454 grm. of the oil gave 0.4516 grm. CO_2 and 0.1971 grm. H_2O.

	Calculated for $C_{16}H_{34}$.	Found. I.	II.	III.
C	84.96	84.94	84.80	84.60
H	15.04	14.84	15.18	15.06

The index of this hydrocarbon was found to be 1.4413, from which the molecular refraction was calculated: —

Calculated for $C_{16}H_{34}$.	Found.
75.750	75.555

Dichlorhexadecane, $C_{16}H_{33}Cl_2$. — The chlorine product obtained from hexadecane collected for the larger part at 205°–210°, under 16 mm. Its specific gravity was 1.0314 at 20°. A determination of chlorine gave a value required for the dichloride: —

0.1477 grm. of the oil gave 0.1525 grm. AgCl.

	Calculated for $C_{16}H_{32}Cl_2$.	Found.
Cl	24.61	24.44

This formula was also confirmed by its molecular weight: —

0. 5019 grm. of the oil and 18.21 grms. benzol gave a depression of 0°.457.

Calculated for $C_{16}H_{32}Cl_2$.	Found.
295	295

A portion of the original distillate 174°–175° was cooled to −10°, which caused the separation of a crystalline mass. It was filtered cold, but the solid remaining formed but a small part of the original oil.

The specific gravity of the filtered oil 0.8005, was slightly higher than the unfiltered distillate. The quantity of the solid was not sufficient for analysis or further examination. No further examination was made of the filtered oil, for it was evident that the small amount of solid hydrocarbon could not change the composition, nor other constants, especially since, as shown above, the original distillate has the composition of the series C_nH_{2n+2}.

HEPTADECANE, $C_{17}H_{36}$.

The fraction which collected at 188°–190° after the 42d distillation gave as its specific gravity at 20° after drying over sodium, 0.8017. After agitation with sulphuric acid it gave 0.8019, and after purification with fuming sulphuric acid, 0.8000. Under 760 mm. pressure this oil distilled almost entirely at 288°–289°, with very little residue above 289°. The small residue was badly colored from decomposition. Evidently it would not be possible to distill this oil continuously under atmospheric pressure in presence of air without serious decomposition.

The formula of this distillate was established by two determinations of its molecular weight and by analysis.

I. 1.4294 grm. of the oil and 25.7086 grms. benzol gave a depression of 1°.17.

II. 1.4382 grm. of the oil and 25.6785 grms. benzol gave a depression of 1°.18.

Calculated for $C_{17}H_{36}$.	Found. I.	Found. II.
240	241.2	240.9

Analysis I. was made of the unpurified oil dried over sodium, and Analysis II. after purification with fuming sulphuric acid.

I. 0.1534 grm. of the oil gave 0.4778 grm. CO_2 and 0.2044 grm. H_2O.

II. 0.1491 grm. of the oil gave 0.4641 grm. CO_2 and 0.2014 grm. H_2O.

	Calculated for $C_{17}H_{36}$.	Found. I.	Found. II.
C	84.96	84.94	84.87
H	15.04	14.80	15.01

Monochlorheptadecane, $C_{17}H_{35}Cl$. — The chlorine product from heptadecane collected in considerable quantity at 175°–177°, 15 mm. Its specific gravity at 20° was found to be 0.8962. The percentage of chlorine corresponded to the monochloride: —

0.1510 grm. of the oil gave 0.0807 grm. AgCl.

	Calculated for $C_{17}H_{35}Cl$.	Found.
Cl	12.92	13.21

On cooling a portion of the distillate 188°–189° to —10°, it formed a pasty mass from which a small amount of a crystalline solid was obtained by filtration. The solid after crystallization from ether and alcohol melted at approximately 10°. The amount of solid was not sufficient for purification or examination. The filtered oil gave as its specific gravity at 20°, 0.8035, slightly higher than the distillate before filtration. Since the original distillate showed the composition of the series, C_nH_{2n+2}, it did not seem worth while to make any further examination of the filtered oil.

OCTODECANE, $C_{18}H_{38}$.

After the twenty-sixth distillation larger quantities of distillates collected between 198° and 204°, mostly at 199°–200° (50 mm). The distillate 199°–200° distilled for the most part, although with considerable colored residue and bad odor, at 300°–301°, under 760 mm. After drying over sodium its specific gravity was 0.8054, after agitation with sulphuric acid, 0.8035, and after purification with fuming sulphuric acid, 0.8017, at 20°.

Its molecular weight was ascertained by the Beckman method at the freezing point.

I. 0.9963 grm. of the oil and 36.4129 grms. benzol gave a depression of 0°.53.

II. 0. 9926 grm. of the oil and 23.2544 grms. benzol gave a depression of 0°.84.

Calculated for $C_{18}H_{38}$.	Found.	
	I.	II.
254	252.7	254.2

Combustion I. was made of the unpurified distillate dried over sodium; combustion II., of the oil after purification with concentrated sulphuric acid; and combustion III., after treatment with fuming sulphuric acid.

I. 0.1423 grm. of the oil gave 0.4435 grm. CO_2 and 0.1915 grm. H_2O.
II. 0.1513 grm. of the oil gave 0.4702 grm. CO_2 and 0.2054 grm. H_2O.
III. 0.1524 grm. of the oil gave 0.4727 grm. CO_2 and 0.2064 grm. H_2O.

	Calculated for $C_{18}H_{38}$.	Found. I.	II.	III.
C	85.06	85.02	84.76	84.59
H	14.94	14.96	15.09	15.05

It is evident from the slight change in specific gravity after purification, and the percentages of carbon and hydrogen, that the original distillate consisted to a large extent of octodecane.

Monochloroctodecane, $C_{18}H_{37}Cl$. — The product obtained by the action of chlorine on octodecane, collected in greater part at 185°–190°, under 15 mm., and this fraction gave as its specific gravity at 20°, 0.9041. The percentage of chlorine corresponded to the monochloride: —

0.1482 grm. of the oil gave 0.0782 grm. AgCl.

	Calculated for $C_{18}H_{37}Cl$.	Found.
Cl	12.35	13.05

The results given above were obtained with the hydrocarbon that was liquid at ordinary temperatures. When it was found that crystals separated from this distillate at 3°, and that it became pasty at 0°, it was cooled to —10°, when it became so thick it filtered only slowly. The solid after filtering was melted and again cooled and filtered, after which it was perfectly white. It was then crystallized from ether and alcohol, after which it melted at 20°. It was estimated that twenty per cent of the original oil separated as the solid hydrocarbon on cooling. It was difficult to separate the solid completely on account of the great solvent action of the oil.

The filtered and pressed solid melted at 20°, and after crystallization from ether and alcohol and from gasoline, the melting point could not be raised. Krafft * gave 28° as the melting point of octodecane, which he obtained from stearic acid; but his octodecane boiled at 214°.5 under 50 mm. pressure. The specific gravity of the solid hydrocarbon was

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* Ber. deutsch. chem. Gesellsch., XV. 1703 (1882).

0.7830 at $\frac{20°}{20°}$, and 0.7816 at $\frac{20°}{40°}$. Krafft gave 0.7768 as the specific gravity of $C_{18}H_{38}$ at 28°. A determination of the molecular weight of the purified hydrocarbon confirmed its formula: —

1.1003 grm. of the oil and 19.65 grms. benzol gave a depression of 0.941.

Calculated for $C_{18}H_{37}Cl$.	Found.
288.5	291.5

This molecular weight, showing that the hydrocarbon boiling at 300° is octodecane, does not agree with Krafft's conclusion as to the formula of the hydrocarbon obtained from stearic acid. In heating stearic acid with hydriodic acid Krafft assumed that all the oxygen is removed, leaving intact the carbon of the carboxyl, with the formation of octodecane. But when this work was done the only means of verifying the formula was by analysis, which was sufficient to determine the series, but not the individual members of the series. While the results of Krafft's combustions gave almost exact values for the formula $C_{18}H_{38}$, the size of the molecule could not be determined. Krafft looked upon the hydrocarbon boiling at 303° as having the formula $C_{17}H_{36}$.

The specific gravity of the oil after cooling and filtration was somewhat higher than before, 0.8110 at 20°, and higher than the specific gravity of the filtered solid octodecane, 0.7830. The molecular weight of the filtered oil was the same as before filtration.

0.9904 grm. of the substance and 16.10 grms. benzol gave a depression of 1°.184.

Calculated for $C_{18}H_{38}$.	Found.
254	254.6

A combustion of the liquid hydrocarbon showed some change in the proportions of carbon and hydrogen: —

0.1483 grm. of the substance gave 0.4636 grm. CO_2 and 0.1954 grm. H_2O.

	Calculated for		Found.
	$C_{18}H_{36}$.	$C_{18}H_{38}$.	
C	85.70	85.06	85.25
H	14.30	14.94	14.64

While there is a narrow difference in calculated percentages between the two formulae, the percentages found, together with the higher specific gravity, indicate that the filtered oil was a mixture of the two series C_nH_{n2} and H_nC_{2n+2}.

A determination of the index of refraction, which was found to be 1.4435, and the molecular refraction, correspond more nearly to the formula $C_{18}H_{36}$: —

Calculated for $C_{18}H_{36}$	Calculated for $C_{18}H_{38}$.	Found.
82.90	84.96	82.60

A combustion of the solid hydrocarbon gave proportions required for the series C_nH_{2n+2} : —

0.1564 grm. of the substance gave 0.4883 grm. CO_2 and 0.2083 grm. H_2O.

	Calculated for $C_{18}H_{38}$.	Found.
C	85.06	85.15
H	14.94	14.80

The position in the series was shown by its molecular weight: —

I. 1.9475 grm. of the solid and 25.21 grms. benzol gave a rise of 0°.7734.

II. 1.9475 grm. of the solid and 25.28 grms. benzol gave a rise of 0°.7830.

Calculated for $C_{18}H_{38}$.	Found. I.	Found. II.
254	256	253.2

The formula of octodecane was further confirmed by its index of refraction. The index was found to be, at 20°, 1.440, which corresponds to the molecular refraction : —

Calculated for $C_{18}H_{38}$.	Found.
84.96	84.53

NONODECANE, $C_{19}H_{40}$.

In the eighth distillation under 50 mm., 335 grams collected at 210°–212° with much smaller weights on either side. After continuing the distillation twenty-seven times, a portion of the fraction 210°–212° was purified with fuming sulphuric acid; before this treatment the oil gave as its specific gravity at 20°, 0.8274, and afterward, 0.8122. In determining its molecular weight by the freezing point method the following values were obtained: —

I. 1.1039 grm. of the oil and 39.7462 grms. benzol gave a depression of 0°.575.

II. 1.1418 grm. of the oil and 36.2175 grms. benzol gave a depression of 0°.505.

Calculated for $C_{19}H_{40}$.	Found. I.	 II.
268	269.5	268

Determinations of carbon and hydrogen gave values for the series C_nH_{2n+2}: —

I. 0.1530 grm. of the oil gave 0.4771 grm. CO_2 and 0.1994 grm. H_2O.

II. 0.1591 grm. of the oil gave 0.4976 grm. CO_2 and 0.2132 grm. H_2O.

	Calculated for		Found.	
	$C_{19}H_{38}$.	$C_{19}H_{40}$.	I.	II.
C	85.70	85.08	85.04	85.29
H	14.30	14.92	14.48	14.89

Analysis I. was made of the oil before purification, and Analysis II. afterward.

A determination of the index of refraction gave 1.4522, which corresponds to the following molecular refraction: —

Calculated for $C_{19}H_{40}$.	Found.
89.55	88.68

The results on the composition of the distillate 210°–212° were obtained on the purified distillate without cooling to separate the solid hydrocarbon. On cooling a portion of this distillate to —10°, filtering cold and pressing the solid, 5 grams of the solid hydrocarbon, and 30 grams of the liquid hydrocarbon were obtained; the solid hydrocarbon, therefore, formed a small part of the original distillate. The specific gravity of the filtered oil was 0.8208 at 20°. The specific gravity of the distillate before cooling as shown above was 0.8122 at 20°.

After crystallization from ether and alcohol, the specific gravity of the solid hydrocarbon was 0.7725, $\frac{30^\circ}{30^\circ}$, and 0.7781 at $\frac{30^\circ}{40^\circ}$.

A determination of its molecular weight gave a value required for the hydrocarbon $C_{19}H_{40}$.

1.4011 grm. of the solid and 26.66 grms. benzol gave a rise of 0°.496.

Calculated for $C_{19}H_{40}$.	Found.
268	271.6

The melting point of the solid was found to be 33°–34° corresponding to the melting point, 32°, that Krafft found for the solid hydrocarbon distilling at, 226°.5, 50 mm.

A combustion of the oil filtered from the solid hydrocarbon gave percentages of carbon and hydrogen required for $C_{19}H_{38}$.

0.1495 grm. of the oil gave 0.4715 grm. CO_2 and 0.1928 grm. H_2O.

Calculated for $C_{19}H_{38}$.	Found.
85.70	86.00
14.30	14.33

A determination of the molecular weight confirmed the formula: —

2.5445 grms. of the oil and 24.63 grms. benzol gave a rise of 0°.994.

Calculated for $C_{19}H_{38}$.	Found.
266	267

The formula was further verified by its index of refraction.

It gave the index 1.4515, corresponding to the molecular refraction: —

Calculated for $C_{19}H_{38}$.	Found.
87.46	87.51

With the distillate 212°–214°, 50 mm., the limit is reached of the solid hydrocarbons whose molecular weight can be determined by the freezing point method on account of the crystallization of the hydrocarbon before the benzol freezes. The molecular weights of solid higher members were determined by the boiling point method.

The results described in this paper defining the physical properties and formulae of the hydrocarbons separated from Pennsylvania petroleum differ in several essential particulars both from the hydrocarbons obtained by Krafft by decomposition of the monobasic acids with high molecular weights, and those formerly reported as among the constituents of Pennsylvania oil. It has been shown that the hydrocarbon at 196° is undecane, $C_{11}H_{24}$, and not dodecane as found by Pelouze and Cahours. Likewise the hydrocarbon at 216° proved to be dodecane and not tridecane. Since there was no method for ascertaining the molecular weights of these bodies at the time when they were separated by Pelouze and Cahours, and elementary analysis could only determine the series, it was only by analogy that the homologous members of the series could be guessed at. Since 182° was accepted at the boiling point of undecane, naturally dodecane and tridecane should fall into the heaps at 196° and 216°.

The boiling point of tridecane has been variously stated at 219°, 216° and 212°–215°. But neither of these temperatures can be accepted as the boiling point of tridecane since its molecular weight is found to be that of the hydrocarbon boiling at 225°–226°.

The hydrocarbon tetradecane boils at 236°–238°, the same boiling point as was assigned to the tridecane separated by Pelouze and Cahours, but the specific gravity of tetradecane, 0.7812 at 20° is essentially lower than that found by Pelouze and Cahours, 0.809.

Pentadecane, boiling point 156°–157°, is nearly the same in boiling point as the hydrocarbon separated by Pelouze and Cahours, boiling point 260°, although its specific gravity, 0.7896 at 20°, is much lower than they found, — 0.825 at 19°.

The boiling point of hexadecane is not very different from that given by Pelouze and Cahours, but its specific gravity is considerably lower. It does not differ in boiling point materially from hexadecane which Krafft obtained by heating palmitic acid, nor from that of hexadecane, boiling point 278°, obtained by Zincke from normal octyl iodide.* Since, however, the oils obtained by freezing out the solid hydrocarbons have specific gravities considerably higher than those of the original distillates, and apparently belong to another series, as shown by analysis and refraction indices, it is possible that the solid hydrocarbons held in solution in the oils have their boiling points depressed in the fractional distillation by which they were separated from the main body of the crude oil. Still, the molecular weights of the solid hydrocarbons correspond to definite formulas; for instance, from the distillate 300°–301°, atmospheric pressure, octodecane was separated, in a practically pure form.

The less volatile portions of Pennsylvania petroleum consist of several series of hydrocarbons. The series C_nH_{2n} is liquid even at low temperatures, of higher specific gravity, and another is composed of solid hydrocarbons, of the series C_nH_{2n+2}.

In a former paper † it was shown that the high values assigned by Pelouze and Cahours as the specific gravity of the distillates separated by them from petroleum, indicated that their hydrocarbons were separated from Canadian petroleum. The same inference is supported by the high specific gravity of the hydrocarbons separated by Pelouze and Cahours boiling above 216°, as compared with the specific gravity of the hydrocarbons separated from Pennsylvania petroleum described in this paper.

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* Ann. Chem. u. Pharm. 152, 15.

† These Proceedings, XXXII. 171.

After establishing a homologous series by analysis, evidently the only means available for Pelouze and Cahours to determine the molecular size of the hydrocarbons was to assume that a hydrocarbon constituted the chief portion of any distillates that collected in unusually large amounts, and to compute the series in unbroken order from the lower members.

Several attempts were made to ascertain the presence of the hydrocarbon $C_{20}H_{42}$; but the distillates between the limits 215° and 225° were small, and in none of them did the molecular weight obtained correspond to this formula.

HENEICOSANE, $C_{21}H_{44}$, AND LIQUID HYDROCARBON, $C_{21}H_{42}$. FROM DISTILLATE 230°–232°, 50 MM.

Larger quantities of distillates amounting to 200 grams collected at 230°–232°, for the most part at 230°–231°. The specific gravity of the unpurified distillate was 0.8321; after purification it gave 0.8230. A combustion of the purified oil gave the following percentages of carbon and hydrogen: —

0.1540 grm. of the oil gave 0.4813 grm. CO_2 and 0.1968 grm. H_2O.

	Calculated for		Found.
	C_nH_{2n}	$C_{21}H_{44}$.	
C	85.70	85.14	85.23
H	14.30	14.86	14.32

There was evidently some loss in this analysis, but the percentages are more satisfactory for the formula $C_{21}H_{42}$. It will be shown that this distillate was a mixture of a solid hydrocarbon with an oil of higher specific gravity. Its molecular weight was determined at the freezing point of benzol.

I. 1.4807 grm. of the oil and 25.9125 grms. benzol gave a depression of 0°.949.

II. 0.6845 grm. of the oil and 19.48 grms. benzol gave a depression of 0°.579.

Calculated for		Found.	
$C_{21}H_{42}$	$C_{21}H_{44}$.	I.	II.
294	296	295	297.4

The index of refraction of this hydrocarbon was found to be 1.4608, corresponding to the molecular refraction: —

Calculated for $C_{21}H_{42}$.	Found.
96.66	96.91

The molecular weight and molecular refraction show that this distillate is composed of a twenty-one carbon hydrocarbon, and the combustion and high specific gravity point to the series C_nH_{2n}.

A portion of the oil was cooled to −10°, filtered cold, the solid well pressed in filter paper, and crystallized from ether and alcohol. The ready solubility of these solid hydrocarbons in ether and insolubility in alcohol afford an easy means of purification. The melting point of the purified solid was 40°–41°.

A combustion gave the following values for carbon and hydrogen: —

0.1353 grm. of the substance gave 0.4237 grm. CO_2 and 0.1778 grm. H_2O.

	Calculated for $C_{21}H_{44}$.	Found.
C	85.13	85.39
H	14.87	14.72

The filtered oil gave as its specific gravity at 20°, 0.8424. The percentages of carbon and hydrogen were ascertained by a combustion: —

0.1498 grm. of the oil gave 0.4724 grm. CO_2 and 0.1898 grm. H_2O.

	Calculated for $C_{21}H_{42}$.	Found.
C	85.70	85.98
H	14.30	14.08

Its molecular weight was determined at the freezing point of benzol.

0.9466 grm. of the oil and 21.01 grms. of benzol gave a depression of 0°.737.

Calculated for $C_{21}H_{42}$.	Found.
294	299

DOCOSANE, $C_{22}H_{46}$, AND LIQUID HYDROCARBON, $C_{22}H_{44}$.
FROM DISTILLATE 240°–242°, 50 MM.

After the thirtieth distillation, 150 grams collected at 240°–242°, which gave as its specific gravity before purification 0.8341. After purification with fuming sulphuric acid its specific gravity was 0.8262. Combustions gave the following percentages of carbon and hydrogen: —

I. 0.1538 grm. of the oil gave 0.4800 grm. CO_2 and 0.1992 grms. H_2O.

II. 0.1560 grm. of the oil gave 0.4874 grm. CO_2 and 0.2024 grm. H_2O.

III. 0.1362 grm. of the oil gave 0.4257 grm. CO_2 and 0.1788 grm. H_2O.

	Calculated for		Found.		
	$C_{22}H_{44}$	$C_{22}H_{46}$.	I.	II.	III.
C	85.70	85.16	85.09	85.21	85.25
H	14.30	14.84	14.40	14.42	14.59

The molecular weight was determined as follows: —

I. 0.8367 grm. of the oil and 20.38 grms. benzol gave a depression of 0°.642.

II. 2.5442 grms. of the oil and 21.91 grms. benzol gave a rise in boiling point of 0°.9566.

Calculated for	Found.	
$C_{22}H_{44}$.	I.	II.
308	313.4	311.3

The index of refraction was found to be 1.454 and the molecular refraction: —

Calculated for $C_{22}H_{44}$.	Found.
101.27	100.7

The distillates in the vicinity of 240°, 50 mm., deposited no solid on standing at ordinary temperatures, but higher fractions all deposited solids. When cooled to 0°, the fraction 242°–254° became nearly solid. After further cooling to —10°, the solid was filtered in a funnel surrounded with salt and ice, pressed in filter paper and crystallized from ether and alcohol. The solid melted at 43°, and further purified from gasoline, at 44°. Krafft gave 44°.4 as the melting point of the hydrocarbon $C_{22}H_{46}$. Its specific gravity at 60° was found to be 0.7796. A combustion gave proportions of carbon and hydrogen required for the series C_nH_{2n+2}.

0.1521 grm. of the solid gave 0.4721 grm. CO_2 and 0.2021 grm. H_2O.

	Calculated for		Found.
	$C_{22}H_{44}$	$C_{22}H_{46}$.	
C	85.70	85.16	85.13
H	14.30	14.84	14.86

The quantity of the hydrocarbon was not sufficient for a determination of its molecular weight.

The filtered oil gave as its specific gravity at 20°, 0.8296, a value somewhat higher than that obtained before filtration. A combustion gave the following percentages of carbon and hydrogen : —

0.1505 grm. of the oil gave 0.4717 grm. CO_2 and 0.1937 grm. H_2O.
0.1411 grm. of the oil gave 0.4419 grm. CO_2 and 0.1819 grm. H_2O.

	Calculated for $C_{22}H_{44}$.	Found.	
C	85.70	85.49	85.41
H	14.30	14.31	14.32

These proportions correspond to the formula $C_{22}H_{44}$, which is supported by the high specific gravity.

The molecular weight of the filtered oil was also determined: —

1.0713 grm. of the oil and 19.60 grms. benzol gave a depression of 0°.858.

Calculated for $C_{22}H_{44}$.	Found.
308	312

The wide difference in specific gravity between the solid and liquid hydrocarbons at 240°–242°, 50 mm., point to different series. While the percentages of carbon and hydrogen given by analysis could not alone be depended on to prove the different series, the results of combustion with specific gravity are sufficient. The differences in theoretical composition of the two series are 0.5 of one per cent for carbon and for hydrogen. In combustions conducted under the most favorable conditions and with the greatest care, the different series may be shown in well purified materials. But with so many determinations, and the extreme care in details of the method, while the percentages obtained are sufficiently close to indicate the series, the results are not in all cases as close to the calculated percentages as should be reached in the greater precision of a few analyses.

TRICOSANE, $C_{23}H_{48}$, AND LIQUID HYDROCARBON, $C_{23}H_{46}$. FROM DISTILLATE 258°–260°, 50 MM.

After the nineteenth distillation, 175 grams collected at 258°–260°, 50 mm., for the most part at 260°–261°, which deposited a considerable quantity of solid hydrocarbon on standing. The specific gravity of the unpurified distillate decanted from the solid was as follows : —

60°, 0.8341 70°, 0.8320 80°, 0.8310

The oil was agitated several times with concentrated sulphuric acid until the acid was not much colored, and washed with sodic hydrate and water, and finally with salt brine. On account of the high specific gravity of the oil some heavy solution such as brine or calcic chloride was necessary to separate the water and oil. The oil was dried for examination over calcic chloride and metallic sodium.

A combustion of the oil gave the following percentages of carbon and hydrogen: —

0.1508 grm. of the oil gave 0.4711 grms. CO_2 and 0.1945 grm. H_2O.

	Calculated for		Found.
	$C_{23}H_{46}$.	$C_{23}H_{48}$.	
C	85.70	85.18	85.21
H	14.30	14.82	14.33

A part of the carbonic dioxide was evidently lost, but the results point to the series C_nH_{2n}, which represents the composition of the oil, much the larger part of the distillate.

A portion of the distillate was cooled to —10°, and filtered cold to separate the crystalline solid. The solubility of the solid hydrocarbon seemed to diminish rapidly with lower temperatures, consequently a small proportion of the solid remains in the oil below —10°. After pressing and crystallizing from ether and alcohol, the solid melted at 45°. Krafft's hydrocarbon, $C_{23}H_{48}$, melted at 47°.7. Two determinations of its specific gravity at 60° gave (1) 0.7894, (2) 0.7900.

A combustion of the solid gave results for the series C_nH_{2n+2}: —

0.1515 grm. of the substance gave 0.4710 grm. CO_2 and 0.1989 grm. H_2O.

	Calculated for $C_{23}H_{48}$.	Found.
C	85.20	85.06
H	14.80	14.64

A determination of its molecular weight at the boiling point of benzol was made: —

1.1208 grm. of the substance and 23.08 grms. benzol gave a rise of 0°.412.

Calculated for $C_{23}H_{48}$.	Found.
324	327

The filtered oil gave as its specific gravity at 20°, 0.8569. It gave percentages of carbon and hydrogen required for the series C_nH_{2n}: —

0.1504 grm. of the oil gave 0.4711 grm. CO_2 and 0.1945 grm. H_2O.

	Calculated for $C_{23}H_{46}$.	Calculated for $C_{23}H_{48}$.	Found.
C	85.70	85.18	85.41
H	14.30	14.82	14.36

The molecular weight was determined at the freezing point of benzol:

3.464 grms. of the oil and 26.06 grms. benzol gave a rise of 1°.0475.

Calculated for $C_{23}H_{46}$.	Found.
322	325

The index of refraction was found to be 1.4714, from which was calculated the molecular refraction: —

Calculated for $C_{23}H_{46}$.	Found.
105.87	105.31

TETRACOSANE, $C_{24}H_{50}$, AND LIQUID HYDROCARBON, $C_{24}H_{48}$.
FROM DISTILLATE 272°–274°, 50 MM.

After the nineteenth distillation, 150 grams collected at 272°–274°, 50 mm., that became partly solid on standing. The decanted oil gave as its specific gravity 0.8448 at 20°. A part of the distillate was then cooled to —10°, and the solid filtered cold under pressure.

The distillate was purified by treatment with successive portions of fuming sulphuric acid, until the acid was nearly colorless, then washed with sodic hydrate and a concentrated solution of calcic chloride, and dried over fused calcic chloride and sodium. Nearly one third of the volume was removed in purification. The purified oil then gave as its specific gravity at 20°, 0.8598, and at higher temperatures: —

60°, 0.8375	70°, 0.8366	80°, 0.8354

A combustion of the purified oil gave percentages of carbon and hydrogen required for the series C_nH_{2n}.

0.1539 grm. of the oil gave 0.4769 grm. CO_2 and 0.2000 grm. H_2O.

	Calculated for $C_{24}H_{48}$	Calculated for $C_{24}H_{50}$.	Found.
C	85.70	85.21	85.35
H	14.30	14.79	14.54

These percentages evidently support the formula $C_{24}H_{48}$.

A determination of the molecular weight of the liquid hydrocarbon gave at the boiling point of benzol a result required for $C_{24}H_{48}$.

2.0681 grms. of the oil and 25.78 grms. benzol gave a rise of 0°.6096.

Calculated for $C_{24}H_{48}$.	Found.
336	337.4

The index of refraction of the oil was found to be 1.4726, from which the following molecular refraction was calculated: —

Calculated for $C_{24}H_{48}$.	Found.
110.47	109.75

Specific gravity of the oil at $\frac{20^\circ}{40^\circ}$, 0.8582.

The solid separated by filtration melted at 48°. Krafft's hydrocarbon, $C_{24}H_{48}$, melted at 51°. It gave as its specific gravity the following values: —

60°,	0.7902	70°,	0.7893	80°,	0.7875

With water at 4° these results reduce to 0.7742. The specific gravity given by Krafft to tetracosane at 4° was 0.7784, the same as that of the solid hydrocarbon which he separated from shale oil. The solid products from Pennsylvania oil show only slight variations in specific gravity. But the heavy oils show a decided increase in specific gravity with increase in molecular weight.

A combustion gave percentages of carbon and hydrogen required for the series C_nH_{2n+2}.

0.1433 grm. of the solid gave 0.4479 grm. CO_2 and 0.1895 grm. H_2O.

	Calculated for		Found.
	C_nH_{2n}.	$C_{24}H_{50}$.	
C	85.70	85.23	85.25
H	14.30	14.77	14.70

The molecular weight was determined at the boiling point of benzol.

Calculated for $C_{24}H_{50}$.	Found.
338	337

To ascertain whether the solid which separated at ordinary temperatures was identical with what remains in solution, a portion of the oil was cooled to 0°, filtered at the same temperature, and the filtrate cooled to —10° and filtered under pressure at the same temperature. The three solids were carefully purified by crystallization from ether and alcohol and their melting points taken. The solid separated at ordinary temperatures melted at 48°, that separated at 6° melted at 51°–52°, and that separated at —10° melted at 51°–52°.

It is therefore evident that this distillate consists chiefly of one solid hydrocarbon, $C_{24}H_{50}$, and that the oil remaining liquid at —10° belongs to a different series.

PENTACOSANE, $C_{25}H_{52}$, AND LIQUID HYDROCARBON, $C_{26}H_{52}$. FROM DISTILLATE 280°–282°, 50 MM.

After the fifteenth distillation, 100 grams collected at 280°–282°, 50 mm., which deposited a larger quantity of solid hydrocarbon than the lower distillates. This solid was separated from the oil and the latter was then cooled to 0° and filtered at the same temperature.

The specific gravity of the filtered oil at 20° was 0.8580.

A combustion gave the following values: —

0.1593 grm. of the oil gave 0.4997 grm. CO_2 and 0.2107 grm. H_2O.

	Calculated for $C_{26}H_{52}$.	Found.
C	85.70	85.55
H	14.30	14.67

The formula was established by its molecular weight, determined at the boiling point of benzol: —

3.9867 grms. of the oil and 30.19 grms. benzol gave a rise of 0°.936.

Calculated for $C_{26}H_{52}$.	Found.
364	362

A determination of the index of refraction gave 1.4725, which corresponded to the molecular refraction: —

Calculated for $C_{26}H_{52}$.	Found.
119.87	119.12

After crystallization from ether and alcohol, from which it separated in crystalline plates, the solid hydrocarbon gave percentages of carbon and hydrogen required for the series C_nH_{2n+2}.

0.1534 grm. of the substance gave 0.4787 grm. CO_2 and 0.2006 grm. H_2O.

	Calculated for $C_{25}H_{52}$.	Found.
C	85.25	85.09
H	14.75	14.53

A determination of its molecular weight verified the formula: —

1.7583 grm. of the substance, and 24.39 grms. benzol gave by the boiling point method a rise of 0°.5231.

Calculated for $C_{25}H_{52}$.	Found.
352	353.4

Melting point of the solid hydrocarbon, 53°–54°.

Hexacosane, $C_{26}H_{54}$, and Liquid Hydrocarbon, $C_{27}H_{52}$. From Distillate 292°–294°, 50 mm.

After the fifteenth distillation, 100 grams collected at 292°–294°, 50 mm., which deposited a considerable quantity of solid crystalline hydrocarbon. These crystals were filtered, pressed, and purified by crystallization from ether and alcohol. Melting point, 58°. The specific gravity of this hydrocarbon was determined as follows: —

60°, 0.7977	70°, 0.7956	80°, 0.7943

A combustion of the solid gave the following percentages of carbon and hydrogen: —

0.1508 grm. of the substance gave 0.4709 grm. CO_2 and 0.2033 grm. H_2O.

	Calculated for $C_{26}H_{54}$.	Found.
C	85.24	85.17
H	14.76	14.98

The molecular weight was ascertained at the boiling point of benzol.

1.2754 grm. of the substance and 24.2827 grms. benzol gave a rise of 0°.416.

Calculated for $C_{26}H_{54}$.	Found.
366	364

On cooling the original distillate to —10°, it formed a thick pasty mass. It was filtered under pressure, keeping cold. The filtered crystals were pressed, and crystallized from ether and alcohol. Melting point, 58°. The solids pressed out from the fractions from 288° to 302°, 50 mm., showed very slight variations in melting points.

288°–290°,	56°.	300°–301°,	59°–60°.
294°–296°,	58°.	302°–304°,	59°–60°.

The oil filtered under pressure was very thick and viscous. Its specific gravity at 20° was 0.8688. A combustion gave the following percentages of carbon and hydrogen: —

0.1500 grm. of the oil gave 0.4750 grm. CO_2 and 0.1812 grm. H_2O.

	Calculated for $C_{27}H_{52}$.	Found.
C	86.17	86.36
H	13.83	13.43

Its molecular weight at the boiling point of benzol corresponded to the formula $C_{27}H_{52}$.

3.6506 grms. of the oil and 25.80 grms. benzol gave a rise of 0°.964.

Calculated for $C_{27}H_{52}$.	Found.
376	376.2

The index of refraction corresponded to the same formula.

The index was found to be 1.4722, and the molecular refraction: —

Calculated for $C_{27}H_{52}$.	Found.
122	121.4

OCTOCOSANE, $C_{28}H_{58}$, AND LIQUID HYDROCARBON, $C_{28}H_{54}$. FROM DISTILLATE 310°–312°, 50 MM.

After the tenth distillation 75 grams collected at 310°–312°, from which a considerable quantity of crystals collected above the oil on standing. The oil separated from the crystals was then cooled to —10° and filtered cold under pressure. The solid was pressed and purified by crystallization from ether and alcohol. Melting point 60°. Its specific gravity was determined as follows: —

70°,	0.7945	80°,	0.7927	90°,	0.7911

A combustion gave the following percentages of carbon and hydrogen: —

0.1508 grm. of the substance gave 0.4703 grm. CO_2 and 0.2032 grm. H_2O.

	Calculated for $C_{28}H_{58}$.	Found.
C	85.28	85.07
H	14.72	14.97

The molecular weight at the boiling point of benzol corresponded to the formula $C_{28}H_{58}$.

3.070 grms. of the solid hydrocarbon and 26.21 grms. benzol gave a depression of 0°.7538.

Calculated for $C_{28}H_{58}$.	Found.
394	399

The very thick oil separated by filtration gave as its specific gravity at 20, 0.8694. A combustion gave percentages of carbon and hydrogen required for the series, C_nH_{2n-2}.

0.1500 grm. of the oil gave 0.4729 grm. CO_2 and 0.1836 grm. H_2O.

		Calculated for		Found.
	$C_{28}H_{58}$	$C_{28}H_{56}$	$C_{28}H_{54}$.	
C	85.28	85.70	86.02	85.96
H	14.72	14.30	13.98	13.60

The molecular weight was found at the boiling point of benzol.

I. 2.6792 grms. of the oil and 29.85 grm. benzol gave a rise of 0°.5826.
II. 1.9196 grms. of the oil and 27.98 grm. benzol gave a rise of 0°.4459.

Calculated for $C_{28}H_{54}$.	Found.	
	I	II.
392	396	394.4

The index of refraction was found to be 1.480, which corresponds to the molecular refractions: —

Calculated for $C_{28}H_{56}$.	Found.
127	126.78

It will be observed that the two liquid hydrocarbons last described have been shown to belong to a series C_nH_{2n-2}. Results already obtained but not yet published indicate that the same series of hydrocarbons constitute the less volatile portions of Canadian petroleum, and probably also of Ohio petroleum. Results already published show that the less volatile distillates from California and Texas petroleum are composed of the same series and other series still poorer in hydrogen.

The unexpected appearance of the series C_nH_{2n-2} in Pennsylvania petroleum suggests a closer relationship between this petroleum and the heavier oils from other fields, such as those in Texas and California, than was suspected. To gain further information concerning the heavier portions of Pennsylvania oil, we allowed three kilos of the sample from which the hydrocarbons described in this paper were prepared, to evaporate spontaneously in the air in a strong draught, but with no application of heat. At the end of thirty days there remained one kilo that gave as its specific gravity 0.8620, practically the same as that of $C_{28}H_{54}$, 0.8694. A combustion gave percentages of carbon and hydrogen required for the series C_nH_{2n-2}. A distillation showed that 65 per cent of the residual oil was composed of hydrocarbons above $C_{15}H_{30}$. The composition of these hydrocarbons will be ascertained in connection with the study of natural and commercial paraffine, which is now in progress.

This interesting relation, and others between the numerous petroleums from different oil fields that have been examined in this laboratory will be presented more at length in a later paper.

As a general summary of the results described in this paper, hydrocarbons have been identified as shown in table on opposite page.

It appears that the liquid hydrocarbon $C_{28}H_{54}$ in fraction 310°–312° has the same number of carbon atoms as the solid hydrocarbon octocosane in the same fraction. In this respect the hydrocarbons in this fraction differ from those in the two preceding fractions, in each of which the liquid hydrocarbon is one carbon higher than the solid constituent. Determinations of the molecular weights of hydrocarbons with high carbon content can only be made by the boiling-point method; and even with the greatest care in manipulation, this method is somewhat uncertain for high molecular weights of solid hydrocarbons, for the reason that the rise in boiling points diminishes with the increase in molecular weight. With oils there is less difficulty. For instance, the molecular weight, 376.2, of the liquid hydrocarbon $C_{27}H_{52}$, given on page 592, is one of five closely concordant determinations by different persons. As an illustration of the care necessary in details, heating with a lamp supplied with gas from the laboratory mains is so irregular on account of variation in gas pressure that the gas must be supplied from a tank under water

Name.	Symbol.	Boiling Point.	Melting Point
Tridecane	$C_{13}H_{28}$	226°	
Tetradecane	$C_{14}H_{30}$	236°-238°	
Pentadecane	$C_{15}H_{32}$	256°-257°	
Hexadecane	$C_{16}H_{34}$	274°-275°	
Heptadecane	$C_{17}H_{36}$	288°-289°	10°
Octodecane	$C_{18}H_{38}$	300°-301°	20°
Nonodecane	$C_{19}H_{40}$	210°-212°, 50 mm.	33°-34°
Heneicosane	$C_{21}H_{44}$	230°-231°, "	40°-41°
Hydrocarbon, liquid at −10°	$C_{22}H_{44}$	240°-242°, "	
Docosane	$C_{22}H_{46}$		44°
Hydrocarbon, liquid at −10°	$C_{23}H_{46}$	258°-260°, "	
Tricosane	$C_{23}H_{48}$		45°
Hydrocarbon, liquid at −10°	$C_{24}H_{48}$	272°-274°, "	
Tetracosane	$C_{24}H_{50}$		48°
Hydrocarbon, liquid at −10°	$C_{26}H_{52}$	280°-282°, "	
Pentacosane	$C_{25}H_{52}$		53°-54°
Hydrocarbon, liquid at −10°	$C_{27}H_{52}$	292°-294°, "	
Hexacosane	$C_{26}H_{54}$		58°
Hydrocarbon, liquid at −10°	$C_{28}H_{54}$	310°-312°, "	
Octocosane	$C_{28}H_{58}$		60°

pressure. Much attention has been given to these determinations, especially by Messrs. O. J. Sieplein and R. P. Cushing.

The preparation of the distillates described in this paper was begun December 1, 1896, by Mr. A. S. Kittelberger, who distilled 56 kilos of Pennsylvania crude oil. The distillations were later continued by different assistants. The following gentlemen have also aided in the purification, examination, and analysis of these hydrocarbons: Messrs. Shaw, Latimer, R. P. Cushing, Dr. E. J. Hudson, and O. J. Sieplein. To the latter especially is due the analysis and identification of the chlorine derivatives of the hydrocarbons.

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RECORDS OF MEETINGS.

Nine hundred and twenty-fifth Meeting.

May 8, 1901. — Annual Meeting.

Vice-President Thayer in the chair.

The Corresponding Secretary read letters from Frank S. Collins, accepting Fellowship; from Franklin P. Mall, acknowledging his election as Associate Fellow; from Vl. Markovnikoff, thanking the Academy for its message of congratulation; from the Johns Hopkins University, announcing the death of Professor Henry A. Rowland; from the Royal Lyncean Academy, of Rome, announcing the death of its President, Professor Angelo Messedaglia; from the Royal Academy of Sciences of Turin, announcing the death of Professor Giulio Bizzozero; from V. Pissaroff, Vice-President of the Ural Society of Natural Sciences of Ekaterinburg, announcing the proposed establishment of an ophthalmological hospital, and asking for gifts of works or instruments; from Arthur MacDonald, of Washington, regarding the establishment of a Psycho-Physical Laboratory in the Department of the Interior; from S. C. Mastick, secretary of the committee on the modification of the Federal legacy tax, announcing that the desired changes have been incorporated in the Statutes of the United States and that the committee has adjourned *sine die*.

The Chair announced the death of William Stubbs, of Oxford, Foreign Honorary Member in Class III., Section 3.

The Corresponding Secretary presented the Annual Report of the Council.

The Treasurer presented his annual report, of which the following is an abstract: —

GENERAL FUND.

Receipts.

Balance from last year		$259.50
Investments	$5,344.80	
Assessments	915.00	
Admission fees	20.00	
Sale of publications	101.01	6,380.81
		6,640.31

Expenditures.

General expenses	$2,461.49	
Publishing expenses	2,438.82	
Library expenses	1,922.33	
Expenses of moving	5.00	
		$6,827.64
Balance		187.33
		$6,640.31

RUMFORD FUND.

Receipts.

Balance from last year		$1,775.93
Investments	$2,640.81	
Sale of publications	35.00	2,675.81
		$4,451.74

Expenditures.

Researches	$916.00	
Medals	327.00	
Publishing	266.22	
Library	374.44	
Miscellaneous	11.25	1,894.91
Income invested during the year and transferred to capital account		10.75
Balance		2,546.08
		$4,451.74

WARREN FUND.

Receipts.

Balance from last year	$994.57
Investments	1,332.97
	$2,327.54

Expenditures.

Investigations	$600.00	
Income invested during the year and transferred to capital account	451.25	$1,051.25
Balance		1,276.29
		$2,327.54

BUILDING FUND.

Receipts.

Balance from last year	$539.52
Investments	783.02
	$1,322.54

Expenditures.

Income invested during the year and transferred to capital account	$942.50
Balance	380.04
	$1,322.54

The following reports were presented: —

REPORT OF THE RUMFORD COMMITTEE.

At the Annual Meeting of the Academy held May 9, 1900, the amount of $1,000 was put at the disposal of the Rumford Committee for the furtherance of research.

From this sum grants have been made as follows: —

Oct. 10, 1900: Two hundred dollars to Dr. Charles E. Mendenhall, of Williams College, in aid of his investigations upon a hollow bolometer.

Oct. 10, 1900: Five hundred dollars to Professor George E. Hale, of the Yerkes Observatory, in aid of his researches in connection with the application of the radiometer to a study of the infra-red spectrum of the chromosphere.

March 13, 1901: Three hundred dollars to Professor Arthur A. Noyes, of the Massachusetts Institute of Technology, in aid of his research upon the effect of high temperature on the electrical conductivity of aqueous salt solutions.

It was furthermore voted by the Committee at its meeting of April 12, 1901, to recommend to the Academy the appropriation of the sum of five hundred dollars from the income of the Rumford Fund to Professor Theodore W. Richards of Harvard University, in aid of his research upon the Thomson-Joule Free Expansion Experiment, which recommendation was favorably acted upon by the Academy.

The Rumford Committee has given much consideration to the question introduced by Professor E C. Pickering, of the feasibility of co-operation among the various committees in this country having in charge the administration of funds devoted to research. It was voted by the Committee that the Chairman be authorized to represent the Committee in such consideration of the subject as might be brought about. Several entirely informal conferences have taken place among representatives of such committees, and it is hoped that some general understanding may be reached which will be helpful to those engaged in research.

The following recommendations have been voted by the Committee, and are now presented to the Academy for its consideration.

Oct. 10, 1900, it was voted that the Committee recommend to the Academy the appropriation of one hundred dollars from the income of the Rumford Fund to aid in the cataloguing of the books in the Academy Library.

March 13, 1901, it was voted to recommend to the Academy that a replica in bronze of each Rumford Medal hereafter awarded by the Academy be struck off and preserved in the Hall of the Academy.

April 12, 1901, it was voted that the Committee recommend to the Academy the appropriation of one hundred and fifty dollars from the income of the Rumford Fund for the purchase and binding of the usual periodicals for the current fiscal year.

At the same meeting it was voted that the Committee recommend to the Academy the appropriation of the sum of one hundred and fifty dollars from the income of the Rumford Fund for the purchase and binding of books on light and heat, said works to be purchased upon the recommendation of the Rumford Committee.

At the same meeting it was furthermore voted that the Chairman of the Committee recommend to the Academy the appropriation from the income of the Rumford Fund of one thousand dollars for the immediate needs of the Committee in the furtherance of research.

A wish has frequently been expressed for a complete list of persons to whom the Rumford Premium has been awarded, and to meet this desire such a list is appended to the present report.

Papers embodying the results of researches conducted wholly or in part by the aid of grants from the Rumford Fund as follows have been printed during the past year in the Proceedings of the Academy.

"On the Thermal Diffusivities of Different Kinds of Marble," by B. O. Peirce and R. W. Willson.

"On the Thermal and Electrical Conductivity of Soft Iron," by Edwin H. Hall.

"False Spectra from the Rowland Concave Grating," by Theodore Lyman.

"A Study of Growing Crystals by Instantaneous Photomicrography," by Theodore W. Richards and Ebenezer H. Archibald.

In accordance with a vote of the Committee passed Nov. 10, 1897, all persons receiving grants from the Rumford Fund are expected to present an annual report of the progress of their work. In response to the usual request such reports have been received from the following persons, regarding their several researches as stated below: —

Mr. Arthur L. Clark, on the molecular properties of vapors in the neighborhood of the critical point.

Professor Henry Crew with Mr. O. H. Basquier, on electric arc spectra.

Professor Edwin B. Frost, on the spectroscopic determination of the radial velocities of stars.

Professor Edwin H. Hall, on the thermal properties of iron.

Professor George E. Hale, on the application of the radiometer to the study of the infra-red spectrum.

Professor Frank A. Laws, on the thermal conductivity of metals.

Professor Edward L. Nichols, on the visible radiation from carbon, accompanied by a paper for presentation to the Academy embodying the result of his researches.

Mr. C. E. Mendenhall, on the hollow bolometer.

Professor Edward C. Pickering, on the determination of the light of very faint stars.

Professor Theodore W. Richards, on (a) the photographic study of growing crystals; (b) the transition temperatures of salts as fixed points in thermometry; (c) the experimental study of the Joule-Thomson Effect.

Professor Wallace C. Sabine with Mr. Theodore Lyman, on the study of false spectra from the Rowland Concave Grating.

The Committee has devoted much time to the consideration of the desirability of an award of the Rumford Premium. The claims of several meritorious candidates have been investigated and discussed at length. As a result of these deliberations the Committee voted unanimously on Feb. 13, 1901, for the first time, and on March 13, 1901, for the second time, that the Academy be recommended to award the Rumford Premium to Elihu Thomson for his inventions in electric welding and lighting.

CHAS. R. CROSS, *Chairman.*

Awards of the Rumford Premium.

May 28, 1839. ROBERT HARE, of Philadelphia, for his invention of the compound or oxyhydrogen blowpipe.

June 1, 1862. JOHN ERICSSON, of New York, for his improvements in the management of heat, particularly as shown in his caloric engine of 1855.

May 30, 1865. DANIEL TREADWELL, of Cambridge, for improvements in the management of heat, embodied in his investigations and inventions relating to the construction of cannon of large calibre, and of great strength and endurance. Presented November 14, 1865.

June 12, 1866. ALVAN CLARK, of Cambridge, for his improvements in the manufacture of refracting telescopes as exhibited in his method of local correction. Presented February 26, 1887.

May 25, 1869. GEORGE HENRY CORLISS, of Providence, for his improvements in the steam engine. Presented January 11, 1870.

June 6, 1871. JOSEPH HARRISON, Jr., of Philadelphia, for his mode of constructing steam-boilers, by which great safety has been secured. Presented January 9, 1872.

May 27, 1873. LEWIS MORRIS RUTHERFURD, of New York, for his improvements in the processes and methods of astronomical photography. Presented March 10, 1874.

May 25, 1875. JOHN WILLIAM DRAPER, of New York, for his researches on radiant energy. Presented March 8, 1876.

May 25, 1880. JOSIAH WILLARD GIBBS, of New Haven, for his researches in thermodynamics. Presented January 12, 1881.

May 29, 1883. HENRY AUGUSTUS ROWLAND, of Baltimore, for his researches in light and heat. Presented February 14, 1884.

May 25, 1886. SAMUEL PIERPONT LANGLEY, of Allegheny, for his researches in radiant energy. Presented May 11, 1888.

May 29, 1888. ALBERT ABRAHAM MICHELSON, of Cleveland, for his determination of the velocity of light, for his researches upon the motion of the luminiferous ether, and for his work on the absolute determination of the wave-lengths of light. Presented April 10, 1889.

May 26, 1891. EDWARD CHARLES PICKERING, of Cambridge, for his work on the photometry of the stars and upon stellar spectra. Presented January 13, 1892.

May 8, 1895. THOMAS ALVA EDISON, of Orange, N. J., for his investigations in electric lighting. Presented May 13, 1896.

May 11, 1898. JAMES EDWARD KEELER, of Allegheny, for his application of the spectroscope to astronomical problems, and especially for his investigations of the proper motions of the nebulae, and the physical constitution of the rings of the planet Saturn, by the use of that instrument. Presented June 14, 1899.

May 10, 1899. CHARLES FRANCIS BRUSH, of Cleveland, for the practical development of electric arc lighting. Presented March 14, 1900.

May 9, 1900. CARL BARUS, of Providence, for his various researches in heat.

REPORT OF THE C. M. WARREN COMMITTEE.

The C. M. Warren Committee recommends to the Academy the following appropriations from the income of the C. M. Warren Fund: —

To Professor C. F. Mabery, Case School of Applied Science, Cleveland, Ohio, four hundred dollars for use in his researches on petroleum.

To Professor A. A. Noyes, Massachusetts Institute of Technology, three hundred dollars for use in his investigation of a systematic procedure for the qualitative analysis of the rare metals.

To Professor Charles H. Herty, Athens, Georgia, one hundred and forty-five dollars for use in his research on platinum and allied metals.

C. L. JACKSON, *Chairman.*

REPORT OF THE COMMITTEE OF PUBLICATION.

The Publishing Committee begs leave to report that there have been issued during the last academic year five numbers of Volume XXXV. of the Proceedings and the first twenty-eight numbers of Volume XXXVI., aggregating 719 pages and 11 plates. Besides this a small edition of Volume XXXIV. was reprinted, at a cost of $180, to replace losses by fire in the bindery. Four numbers of the current Proceedings (62 pages and 4 plates) were printed at the cost of the Rumford Fund ($266.22). The total expenditure for printing falling on the General Fund was $2438.82. The appropriation was $2400, and the return from sales $101.01, leaving an unexpended balance of $62.19. The Committee recommends for the coming year an appropriation of $2400, the same as in the last.

For the Committee,
SAMUEL H. SCUDDER, *Chairman.*

REPORT OF THE COMMITTEE ON THE LIBRARY.

The two most important matters relating to the Library have been the installation of a steel stack for folios by the Massachusetts Historical Society, on the same terms as the other stacks were furnished, and the commencement of a new card catalogue of subjects and authors, for which $200 was appropriated last year. About 1300 cards, covering nearly the whole of the works on mathematics and astronomy, have been typewritten at a total cost of $70.62.

The reappropriation of $100 and an appropriation of $100 from the income of the Rumford Fund is requested to continue this work and to purchase a catalogue case.

The accessions during the year have been as follows:

	Vols.	Parts of vols.	Pams.	Maps.	Total
By gift and exchange	473	2027	296	5	2798
By purchase — General Fund .	28	717			745
By purchase — Rumford Fund .	36	340			376
Total	537	3084	296	5	3919

Last year the total number of accessions was 3224.

28 volumes and 717 parts of volumes were bought with the appropriation from the income of the General Fund at an expense of $339.52; 340 parts of volumes were bought with the appropriation from the income of the Rumford Fund for $101.48; 36 volumes of the "Fortschritte der Physik," needed to complete the set to date, for which a special appropriation was made from the income of the Rumford Fund, have been purchased at an expense of $202.66; 698 volumes were bound at an expense of $925.41, of which $861.11 was charged to the General Fund and $64.30 to the Rumford Fund.

. . . .

A. LAWRENCE ROTCH,
Librarian and Chairman of the Committee on Library.

On the recommendation of the Rumford Committee, it was

Voted, To appropriate from the income of the Rumford Fund —

One hundred dollars ($100) to aid in the cataloguing of the books in the Academy Library.

One hundred and fifty dollars ($150) for the purchase and binding of periodicals.

One hundred and fifty dollars ($150) for the purchase and binding of books on light and heat, said works to be purchased upon the recommendation of the Rumford Committee.

One thousand dollars ($1000) for the immediate needs of the Committee in the furtherance of research.

Voted, That a replica in bronze of each Rumford Medal hereafter awarded be struck off and preserved in the Hall of the Academy.

Voted, To award the Rumford Premium to Elihu Thomson for his inventions in electric welding and lighting.

On the recommendation of the C. M. Warren Committee, it was

Voted, To appropriate from the income of the C. M. Warren Fund —

Four hundred dollars ($400) to Professor C. F. Mabery, of Cleveland, Ohio, for use in his researches on petroleum.

Three hundred dollars ($300) to Professor A. A. Noyes, of Boston, for use in his investigation of a systematic procedure for the qualitative analysis of the rare metals.

One hundred and forty-five dollars ($145) to Professor Charles H. Herty, of Athens, Georgia, for use in his research on platinum and allied metals.

On the recommendation of the Committee on Publication, it was

Voted, To appropriate from the income of the General Fund twenty-four hundred dollars ($2400) for publications.

On the recommendation of the Committee on the Library, it was

Voted, To appropriate from the income of the General Fund one hundred dollars ($100) to continue the catalogue of the Library and to purchase a catalogue-case.

On the recommendation of the Committee of Finance, it was

Voted, To appropriate from the income of the General Fund two thousand dollars ($2000) for general expenses.

Voted, That the assessment for the ensuing year be five dollars.

On the recommendation of the committee to whom certain alterations of the Statutes were referred at the meeting of December 12, 1900, it was

Voted, To amend the Statutes as follows: —

Ch. I., Sec. 1, first sentence. "The Academy consists of Resident Fellows, Associate Fellows, and Foreign Honorary Members."

Ch. I., Sec. 2. "The number of Resident Fellows shall not exceed two hundred. Only residents in the Commonwealth of Massachusetts shall be eligible to election as Resident Fellows, but resident fellowship may be retained after removal from the Commonwealth. Each Resident Fellow shall pay an admission fee of ten dollars and such annual assessment, not exceeding ten dollars, as shall be voted by the Academy at each annual meeting. Resident Fellows only may vote at the meetings of the Academy."

Ch. I., Sec. 3. "The number of Associate Fellows shall not exceed one hundred, of whom there shall not be more than forty in either of the three classes of the Academy. Associate Fellows shall be chosen from persons residing outside of the Commonwealth of Massachusetts. They shall not be liable to the payment of any fees or annual dues, but on removing within the Commonwealth they may be transferred by the Council to resident fellowship as vacancies there occur."

Ch. I., Sec. 4. Omit "And" at the beginning of last sentence.

Ch. II., Sec. 2, first sentence. "At the Annual Meeting of 1901, nine Councillors shall be elected by ballot, one from each class of the Academy to serve for one year, one from each class for two years, and one from each class for three years; and at annual meetings thereafter three Councillors shall be elected in the same manner, one from each class, to serve for three years; but the same Fellow shall not be eligible for two successive terms."

Ch. II., Sec. 2, second sentence. Change "These" to "The." At end of this sentence add: "Five members shall constitute a quorum."

Ch. V., Sec. 5. "The Committee of Publication, of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Memoirs and the Proceedings shall be intrusted."

Ch. V., Sec. 6. "The Committee on the Library, of the Librarian *ex officio*, and three other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

Ch. VI., Sec. 1. Omit the second sentence.

Ch. VI., Sec. 2, third sentence. "He shall notify the meetings of the Academy, apprise officers and committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy."

Ch. VI., Sec. 3. "The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them calculated to promote its interests."

Ch. VII., Sec. 2. Omit the words "by order of the President or presiding officer."

Ch. VII., Sec. 3. "The Treasurer shall keep separate accounts of the income and appropriation of the Rumford Fund and of other special funds, and report the same annually."

Ch. VII., Sec. 4. Omit the words "on such securities as the Academy shall direct."

Ch. VIII., Sec. 1. "It shall be the duty of the Librarian to take charge of the books, to keep a catalogue of them, to provide for the delivery of books from the Library, and to appoint such agents for these purposes as he may think necessary. He shall make an annual report on the condition of the Library."

Ch. VIII., Sec. 2. "The Librarian, in conjunction with the Committee on the Library, shall have authority to expend such sums as may be appropriated, either from the General or Special Funds of the Academy, for the purchase of books and for defraying other necessary expenses connected with the Library."

Ch. VIII., Sec. 3. "To all books procured from the income of the Rumford Fund, or other special funds, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured."

Ch. VIII., Sec. 5. Omit "And" at beginning of second sentence.

Ch. VIII., Sec. 7. "The Librarian shall have custody of the Publications of the Academy and shall distribute copies among the Associate Fellows and Foreign Honorary Members at their request. With the advice and consent of the President, he may effect exchanges with other associations."

Ch. X., Sec. 2, first sentence. "Candidates for election as Resident

Fellows must be proposed by two Resident Fellows of the section to which the proposal is made, in a recommendation signed by them, and this recommendation shall be transmitted to the Corresponding Secretary, and by him referred to the Council for nomination."

Ch. X., Sec. 2, second sentence. Change "seven" to "five."

Ch. X., Sec. 3. Abbreviate first sentence, as follows: "The nomination and election of Associate Fellows may take place in the manner prescribed in reference to Resident Fellows."

Ch. X., Sec. 6. Change first word ("each") to "a majority of any."

Under Rumford Premium, change "a gold and silver medal" to "a gold and a silver medal."

The annual election resulted in the choice of the following officers and committees for the academic year 1901–02: —

Alexander Agassiz, *President.*
John Trowbridge, *Vice-President for Class I.*
Alpheus Hyatt, *Vice-President for Class II.*
James B. Thayer, *Vice-President for Class III.*
William M. Davis, *Corresponding Secretary.*
William Watson, *Recording Secretary.*
Francis Blake, *Treasurer.*
A. Lawrence Rotch, *Librarian.*

Councillors.

Harry M. Goodwin, for one year.	Class I.
Charles R. Sanger, for two years.	
George F. Swain, for three years.	
George H. Parker, for one year.	Class II.
Theobald Smith, for two years.	
Robert DeC. Ward, for three years.	
William Everett, for one year.	Class III.
A. Lawrence Lowell, for two years.	
Denman W. Ross, for three years.	

Member of Committee of Finance.

Eliot C. Clarke.

Rumford Committee.

ERASMUS D. LEAVITT,	AMOS E. DOLBEAR,
EDWARD C. PICKERING,	ARTHUR G. WEBSTER,
CHARLES R. CROSS,	THEODORE W. RICHARDS,

THOMAS C. MENDENHALL.

C. M. Warren Committee.

CHARLES L. JACKSON,	LEONARD P. KINNICUTT,
SAMUEL CABOT,	ARTHUR M. COMEY,
HENRY B. HILL,	ROBERT H. RICHARDS,

HENRY P. TALBOT.

The Chair appointed the following standing committees: —

Committee of Publication.

SAMUEL H. SCUDDER, SETH C. CHANDLER,
CRAWFORD H. TOY.

Committee on the Library.

HENRY W. HAYNES, SAMUEL HENSHAW.
THEODORE W. RICHARDS.

Auditing Committee.

HENRY G. DENNY, WILLIAM L. RICHARDSON.

The following gentlemen were elected members of the Academy: —

George Frisbie Hoar, of Worcester, as Resident Fellow in Class III., Section 1 (Philosophy and Jurisprudence).

John Fritz, of Bethlehem, Pennsylvania, as Associate Fellow in Class I., Section 4 (Technology and Engineering).

Thomas Chrowder Chamberlin, of Chicago, as Associate Fellow in Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe), in place of the late George Mercer Dawson.

Ferdinand Freiherr von Richthofen, of Berlin, as Foreign Honorary Member in Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe).

Adolph Engler, of Berlin, as Foreign Honorary Member in

Class II., Section 2 (Botany), in place of the late Jacob Georg Agardh.

Angelo Celli, of Rome, as Foreign Honorary Member in Class II., Section 4 (Medicine and Surgery).

Gaston Paris, of Paris, as Foreign Honorary Member in Class III., Section 4, in place of the late Charles Jacques Victor Albert, Duc de Broglie.

The Treasurer proposed an amendment to Chapter V., Section 2, of the Statutes. This proposition was referred to a committee consisting of the Treasurer and James B. Ames.

James B. Thayer read an obituary notice of John E. Hudson.

Clarence J. Blake read an obituary notice of his father, John H. Blake, and F. W. Putnam gave an account of the archaeological work of Mr. Blake.

The following papers were presented by title: —

"On Ruled Loci in n-Fold Space." By Halcott C. Moreno. Presented by W. E. Story.

"The Possible Significance of Changing Atomic Volume." By T. W. Richards.

"The Visible Radiation from Carbon." By Edward L. Nichols.

Contributions from the Gray Herbarium of Harvard University. New Series. — No. XXII. I. "The Northeastern Carices of the Section Hyparrhenae;" II. "Notes on the Variations of Certain Boreal Carices." By M. L. Fernald.

Contributions from the Cryptogamic Laboratory of Harvard University. — XLVII. "Preliminary Diagnoses of New Species of Laboulbeniaceae." — IV. By Roland Thaxter.

Nine hundred and twenty-sixth Meeting.

October 9, 1901. — Stated Meeting.

The Academy met at the house of the President, Cambridge.

The President in the chair.

The Corresponding Secretary read letters from Theodore Lyman, accepting Resident Fellowship; from George E. Hale, W. W. Keen, E. H. Moore, C. O. Whitman, acknowledging election as Associate Fellows; and from Sir Lauder Brunton,

A. V. Dicey, A. Engler, Henry Jackson, R. Koch, Müller-Breslau, Gaston Paris, Poincare, Fr. Richthofen, acknowledging election as Foreign Honorary Members. He also read letters from the President and Fellows of Yale University, inviting the Academy to be represented at the celebration of the two hundredth anniversary of the founding of Yale College; from the Natural History Society of Nuremberg, inviting attendance at the celebration of its one hundredth anniversary; and from a committee of the Anthropological Section of the American Association for the Advancement of Science, announcing that the thirteenth session of the International Congress of Americanists would be held at New York in 1902, and inviting the Academy to appoint a representative to the General Committee of the Congress.

On the motion of E. S. Morse, it was

Voted, To authorize the President to appoint delegates in response to these invitations.

The Chair announced the following deaths: —

Truman Henry Safford, of Class I., Section 1, and John Fiske, of Class III., Section 3, Resident Fellows.

Joseph LeConte, of Class II., Section 1, Associate Fellow.

Friherre Adolf Erik Nordenskiöld, of Class II., Section 1, Félix Joseph Henri de Lacaze-Duthiers, of Class II., Section 3, and Friedrich Herman Grimm, of Class III., Section 3, Foreign Honorary Members.

On the motion of the Recording Secretary, it was

Voted, To meet, on adjournment, on the second Wednesday in November.

The following gentlemen were elected members of the Academy: —

Henry Smith Pritchett, of Boston, to be a Resident Fellow in Class I., Section 1 (Mathematics and Astronomy).

William Townsend Porter, of Boston, to be a Resident Fellow in Class II., Section 3 (Zoölogy and Physiology).

George Wharton Pepper, of Philadelphia, to be an Associate Fellow in Class III., Section 1 (Philosophy and Jurisprudence), in place of the late William Mitchell.

The President made a few remarks on the condition and prospects of the Academy.

The Rumford Medals were presented to Carl Barus and Elihu Thomson.

The President gave an account of the Albatross Expedition to the Tropical Pacific.

The following paper was read by title: —

"The Algae of Jamaica," by Frank S. Collins.

Nine hundred and twenty-seventh Meeting.

NOVEMBER 13, 1901. — ADJOURNED STATED MEETING.

The Academy met at the house of James Ford Rhodes.

VICE-PRESIDENT J. B. THAYER in the chair.

The Corresponding Secretary read a letter from Mrs. Cooke, presenting to the Academy a bronze bas-relief of her husband, the late Josiah Parsons Cooke, President of the Academy.

Voted, That the Academy gratefully accept this gift and that the Corresponding Secretary be instructed to inform Mrs. Cooke to that effect.

A letter from the National Society of Natural and Mathematical Sciences of Cherbourg, requesting sympathetic souvenirs on the occasion of the fiftieth anniversary of its establishment, was referred to the Council.

Letters were also read from W. T. Porter, accepting Resident Fellowship; from George Wharton Pepper, acknowledging election as Associate Fellow; from A. Mislawsky, of Ekaterinburg, acknowledging the congratulations of the Academy on the occasion of the fiftieth anniversary of his medical service; from the Nobel Committee of the Royal Academy of Sciences of Sweden, soliciting suggestions for the award of the Nobel Prize in 1902.

Percival Lowell read a biographical notice of the late Augustus Lowell.

William Everett read an essay entitled "The Malignity of Dante."

A paper entitled "The Parametric Representation of the Neighborhood of a Singular Point of an Analytic Surface," by C. W. M. Black, was presented by title.

Nine hundred and twenty-eighth Meeting.

DECEMBER 11, 1901.

The Academy met at the Massachusetts Institute of Technology.

The CORRESPONDING SECRETARY in the chair.

In the absence of the Recording Secretary, G. F. Swain was elected Secretary *pro tempore.*

The Chair announced the death of Joseph Henry Thayer, Resident Fellow in Class III., Section 2.

The following papers were read: —

"Some Results from the Last Opposition of Mars." By Percival Lowell.

"The Atharva Veda and its Significance for the History of Hindu Tradition and Hindu Medicine." By Charles R. Lanman.

The following papers were presented by title: —

"The Standard of Atomic Weights." By T. W. Richards.

"Modifications of Hempel's Gas-apparatus." By T. W. Richards.

"A New Determination of the Atomic Weight of Uranium." By T. W. Richards and B. S. Merigold.

"The Decomposition of Mercurous Chloride by Dissolved Chlorides: a Contribution to the Study of Concentrated Solutions." By T. W. Richards and E. H. Archibald.

"Apatite from Minot, Maine." By John E. Wolff and Charles Palache.

Nine hundred and twenty-ninth Meeting.

JANUARY 8, 1902. — STATED MEETING.

The CORRESPONDING SECRETARY in the chair.

A letter was read from A. Celli, acknowledging his election as Foreign Honorary Member; also, a circular inviting attendance at the Thirteenth Session of the International Congress of Americanists, in New York, in October, 1902.

The Chair announced the death of Aleksandr Onufrijevic Kovalevsky, Foreign Honorary Member in Class II., Section 3.

The following gentlemen were elected members of the Academy: —

Harry Ellsworth Clifford, of Newton, to be a Resident Fellow in Class I., Section 2 (Physics).

Theodore Hough, of Boston, to be a Resident Fellow in Class II., Section 3 (Zoölogy and Physiology).

Francis Henry Williams, of Boston, to be a Resident Fellow in Class II., Section 4 (Medicine and Surgery).

Morris Hicky Morgan, of Cambridge, to be a Resident Fellow in Class III., Section 2 (Philology and Archaeology).

Edmund Beecher Wilson, of New York, to be an Associate Fellow in Class II., Section 3 (Zoölogy and Physiology), in place of the late George Mercer Dawson.

Julius Hann, of Vienna, to be a Foreign Honorary Member in Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe).

Edwin Ray Lankester, of London, to be a Foreign Honorary Member in Class II., Section 3 (Zoölogy and Physiology), in place of the late Félix Joseph Henri de Lacaze-Duthiers.

Victor Alexander Haden Horsley, of London, to be a Foreign Honorary Member in Class II., Section 4 (Medicine and Surgery).

Friedrich Delitzsch, of Berlin, to be a Foreign Honorary Member in Class III., Section 2 (Philology and Archaeology), in place of the late Friedrich Herman Grimm.

Samuel Rawson Gardiner, of Sevenoaks, to be a Foreign Honorary Member in Class III., Section 3 (Political Economy and History), in place of the late William Stubbs.

The Corresponding Secretary announced that Thomas C. Mendenhall had removed from the Commonwealth and that his name had again been placed in the list of Associate Fellows.

Upon the recommendation of the Council, it was

Voted, To transfer Percival Lowell, Resident Fellow, from Class III., Section 4, to Class I., Section 1.

Upon the recommendation of the committee on amending the Statutes, it was

Voted, To amend the first sentence of Chapter V., Section 2, of the Statutes to read as follows: —

"The Committee of Finance, to consist of the President, Treasurer, and one Fellow chosen by ballot, who shall have full control and management of the funds and trusts of the Academy, with the power of investing or changing the investment of the same at their discretion."

A. Lawrence Lowell read a paper entitled, "Party Votes in Parliament, Congress, and the State Legislatures."

The following papers were presented by title: —

Contributions from the Gray Herbarium of Harvard University. New Series. — No. XXIII. "A Revision of the Galapagos Flora." By B. L. Robinson.

"The Probable Source of the Heat of Chemical Combinations." By Theodore William Richards.

"A Description of Crystals of Epidote from Alaska." By Charles Palache. Presented by John E. Wolff.

John E. Wolff exhibited a specimen of apatite from Minot, Maine.

Nine hundred and thirtieth Meeting.

FEBRUARY 12, 1902.

In the absence of the regular presiding officers, the chair was taken by CHARLES R. CROSS.

Letters were received from Theodore Hough and Morris H. Morgan accepting Fellowship; from Edmund B. Wilson, acknowledging his election as Associate Fellow; and from the National Society of Natural and Mathematical Sciences of Cherbourg, acknowledging congratulations on the occasion of its fiftieth anniversary.

The Chair announced the following deaths: —

Alpheus Hyatt, Vice-President for Class II.

Clarence King, Associate Fellow in Class II., Section 1.

Karl Weinhold, Foreign Honorary Member in Class III., Section 2.

In accordance with the Statutes, the following Councillors were appointed a committee to nominate a candidate for the office of Vice-President for Class II.: —

HARRY M. GOODWIN, of Class I.
GEORGE H. PARKER, of Class II.
WILLIAM EVERETT, of Class III.

The following communications were presented: —

"Experiments on Forms of Least Resistance to Passage through Air." By Samuel Cabot. Remarks on this subject were made by Messrs. Davis, Atkinson, Webster, Clayton, and the Recording Secretary.

"What Science has not yet Accomplished in the Art of War." By Edward Atkinson.

The following paper was presented by title: —

"On the Specific Heat and Heat of Vaporisation of the Paraffine and Methylene Hydrocarbons." By Charles F. Mabery and Albert H. Goldstein.

Nine hundred and thirty-first Meeting.

MARCH 12, 1902. — STATED MEETING.

VICE-PRESIDENT TROWBRIDGE in the chair.

Letters were read from Friedrich Delitzsch, S. R. Gardiner, Julius Hann, Victor Horsley, E. Ray Lankester, acknowledging their election as Foreign Honorary members.

The Chair announced the following deaths: —

James Bradley Thayer, Vice-President for Class II.

Samuel Rawson Gardiner, Foreign Honorary member in Class III., Section 3.

The vacancy occasioned by the death of Alpheus Hyatt was filled by the election of

HENRY P. WALCOTT, *Vice-President for Class II.*

The following gentlemen were elected members of the Academy: —

Heinrich Oscar Hofman, of Boston, to be a Resident Fellow in Class I., Section 4 (Technology and Engineering).

Thomas Augustus Jaggar, Jr., of Cambridge, to be a Resident Fellow in Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe).

Edward Henry Strobel, of Cambridge, to be a Resident Fellow in Class III., Section 1 (Philosophy and Jurisprudence).

Herbert Putnam, of Washington, to be an Associate Fellow in Class III., Section 4 (Literature and the Fine Arts).

The Chair appointed the following Councillors to serve as Nominating Committee: —

William Everett, of Class III.
George H. Parker, of Class II.
Harry M. Goodwin, of Class I.

On the motion of the Recording Secretary, it was

Voted, To rescind Standing Vote 9, "The Annual Meeting and the other stated meetings shall be holden at eight o'clock P. M."

The following papers were read: —

"Biographical Notice of the late Horace E. Scudder." By Thomas W. Higginson.

"Biographical Notice of the late Joseph H. Thayer." By Crawford H. Toy.

"The Formation of River Terraces." By William M. Davis.

"The Spectra of Gases at High Temperatures." By John Trowbridge.

The following papers were presented by title: —

"Experiments on the Effect of Freezing and other Low Temperatures upon the Viability of the Bacillus of Typhoid Fever, with Considerations regarding Ice as a Vehicle of Infectious Disease." By William Thompson Sedgwick and Charles-Edward A. Winslow.

"Statistical Studies on the Seasonal Prevalence of Typhoid Fever in Various Countries and its Relation to Seasonal Temperature." By William Thompson Sedgwick and Charles-Edward A. Winslow.

Nine hundred and thirty-second Meeting.

April 9, 1902. — Stated Meeting.

The Academy met at the house of Robert Amory.

The Corresponding Secretary in the chair.

The following papers were read: —

"Account of the Ninth Jubilee Celebration of the University of Glasgow." By William G. Farlow.

"Biographical Notice of the late John Fiske." By A. McF. Davis.

The following paper was presented by title: —

Contributions from the Case School of Applied Science. — XLI. "On the Hydrocarbons in Pennsylvania Petroleum with Boiling Points above 216°." By Charles F. Mabery.

Nine hundred and thirty-third Meeting.

MAY 14, 1902. — ANNUAL MEETING.

VICE-PRESIDENT TROWBRIDGE in the chair.

The Corresponding Secretary read letters from Henry P. Walcott, accepting his election as Vice-President for Class II.; H. E. Clifford, H. O. Hofman, T. A. Jaggar, Jr., Edward H. Strobel, accepting Resident Fellowship; Herbert Putnam, acknowledging election as Associate Fellow; Madame Cornu, announcing the death of her husband, Alfred Cornu; the University of Oxford, inviting the Academy to send a representative to the commemoration of the 300th anniversary of the opening of the Bodleian Library, on October 8 and 9, 1902; the Royal University of Christiania, announcing the celebration, in September next, of the 100th anniversary of the birth of Nicolaus Henricus Abel and inviting the attendance of delegates.

On the motion of the Recording Secretary, it was

Voted, To appoint Herbert Putnam, Associate Fellow, to represent the Academy at the Bodleian Library commemoration.

The annual report of the Council was read by the Corresponding Secretary.

The Treasurer presented his annual report, of which the following is an abstract: —

GENERAL FUND.

Receipts.

Balance (Deficit), April 30, 1901			$187.33
Assessments	$990.00		
Admission fees	150.00		
Sale of publications	138.78	$1,278.78	
Income from investments		5,735.58	7,014.36
			$6,827.03

Expenditures.

General expenses		$2,786.94
Publishing	$2,095.59	
Library	1,576.68	
Catalogue	95.40	3,767.67
Balance, April 30, 1902		272.42
		$6,827.03

RUMFORD FUND.

Receipts.

Balance, April 30, 1901		$2,546.08
Income from investments	$2,514.17	
Sale of publications	5.00	2,519.17
		$5,065.25

Expenditures.

Researches	$1,800.00	
Medals	474.00	
Publishing	417.51	
Library	183.12	
Catalogue	70.00	
Miscellaneous	23.66	$2,968.29
Income invested and transferred to capital acc't.		1,715.00
Balance, April 30, 1902		381.96
		$5065.25

Warren Fund.

Receipts.

Balance, April 30, 1901	$1,276.29
Income from investments	329.43
	$1,605.72

Expenditures.

Investigations	$845.00
Balance, April 30, 1902	760.72
	$1,605.72

Building Fund.

Receipts.

Balance, April 30, 1901	$360.04
Income from investments	309.51
	$689.55

The following reports were also presented: —

Report of the Librarian.

The card-catalogue, commenced in 1900, has been continued by the Assistant Librarian, who has type-written during the past year 2770 cards, the total number of cards now being upwards of 4000. Most of the works on general science, mathematics, astronomy, physics, optics, heat and electricity are now catalogued. A catalogue-case was purchased for $70. Of the $200 appropriated, the total amount expended on account of the catalogue was $164.40, of which $95.40 was charged to the General Fund and $70 to the Rumford Fund. The same appropriation that was made last year is requested for continuing this work, namely: $100 from the income of the General Fund and $100 from the income of the Rumford Fund.

The accessions during the year have been as follows: —

	Vols.	Parts of Vols.	Pams.	Maps.	Total.
By gift and exchange	496	2486	384	2	3368
By purchase — General Fund	18	665			683
By purchase — Rumford Fund	1	256			259
Total	515	3406	384	2	4310

Last year the total number of accessions was 3919.

At the request of the Rumford Committee, 21 volumes on light and heat, for the purchase and binding of which $150 was appropriated from the income of the Rumford Fund, have been ordered but not paid for.

. . . .

The expenses charged to the Library were as follows: — Miscellaneous, which includes expenses in no way relating to the Library, $439.95; Binding, $604.70; Subscriptions, $532.03, making a total of $1450.38. The usual appropriation of $1500 from the income of the General Fund is requested in addition to the customary appropriation from the income of the Rumford Fund, namely $150.

A. Lawrence Rotch, *Librarian.*

Boston, May 14, 1902.

Report of the Rumford Committee.

At the Annual Meeting of the Academy held May 8, 1901, the sum of $1000 was placed at the disposal of the Rumford Committee, to be expended at its discretion in aid of researches in light and heat.

The following grants have been voted: —

Nov. 13, 1901. One hundred dollars to Professor Henry Crew of Northwestern University, in aid of his research on the order of appearance of the different lines of the spark spectrum.

Nov. 13, 1901. Three hundred and fifty dollars to Professor R. W. Wood of Johns Hopkins University, in aid of his researches on the anomalous dispersion of sodium vapor.

Nov. 13, 1901. Sixty-five dollars to Professor A. G. Webster of Clark University, in payment of the cost of fluorite plates purchased for use in a research on the distribution of energy in various spectra.

Feb. 12, 1902. Three hundred dollars to Professor Ernest F. Nichols of Dartmouth College, for the purchase of a spectrometer in furtherance of his research on resonance in connection with heat radiations.

April 9, 1902. Three hundred dollars as it is or may become available to Professor Arthur A. Noyes of the Massachusetts Institute of Technology, in aid of his research upon the effect of high temperatures upon the electrical conductivity of aqueous solutions.

At the meeting of February 12, 1902, it was voted to authorize the Librarian to purchase certain books upon light and heat as specified in a list transmitted to him by the Committee.

The following papers, embodying the results of researches aided by appropriations from the Rumford Fund, have been printed in Volume XXXVII. of the Proceedings of the Academy: —

"The Visible Radiation from Carbon," by Edward L. Nichols.

"The Arc Spectrum of Hydrogen," by O. H. Basquin.

"The Probable Source of the Heat of Chemical Combination and a New Atomic Hypothesis," by Theodore William Richards.

Reports of the progress of unfinished researches which have been aided by grants from the Rumford Fund have been received from the following persons: Messrs. Arthur L. Clark, Henry Crew, Edwin B. Frost, George E. Hale, Frank A. Laws, Charles E. Mendenhall, Ernest F. Nichols, Arthur A. Noyes, Edward C. Pickering, Theodore W. Richards, Wallace C. Sabine, R. W. Wood.

The following recommendations have been voted by the Committee, and are now presented to the Academy for its consideration.

On February 12, 1902, it was voted to ask the Academy to appropriate the sum of seven hundred and fifty dollars from the income of the Rumford Fund to be expended for the construction of a mercurial compression pump designed by Professor Theodore W. Richards, and to be used in his research on the Thomson-Joule Effect.

At the meeting of April 9, 1902, it was voted to request the Academy to appropriate the sum of one thousand dollars from the income of the Rumford Fund, for the immediate needs of the Committee in furtherance of research, and also to appropriate the sum of one hundred and fifty dollars from the same source for the purchase and binding of periodicals.

At the meeting of May 14, 1902, it was voted to ask the Academy to reappropriate from the income of the Rumford Fund the unexpended balance of the amount granted at the last annual meeting, for the purchase and binding of books on light and heat for the Library.

The Committee has considered at length the question of an award of the Rumford Premium, and at the meeting of April 9 it was unanimously voted for the first time, and at the meeting of May 14 it was unanimously voted for the second time, to recommend to the Academy that such award be made to Professor George E. Hale, Director of the Yerkes Observatory, for his investigations in Solar and Stellar Physics, and in particular for the invention and perfection of the Spectroheliograph.

Chas. R. Cross, *Chairman*.

Report of the C. M. Warren Committee.

The Committee in charge of the C. M. Warren Fund has the honor to report that Professor Mabery and Professor A. A. Noyes have reported satisfactory progress in the work supported by grants from the Fund. Professor Herty, owing to an unforeseen change of occupation, is unable to continue his work on platinum, and will return the money granted him.

The Committee recommends the following grants from the C. M. Warren Fund for this year: —

C. F. Mabery, of Cleveland, $300.00 for the continuation of his researches on Petroleum.

A. A. Noyes, of Boston, $300.00 for the continuation of his work on the qualitative analysis of the rare elements.

H. O. Hofman, of Boston, $160.00 for a research on the decomposition of sulphate of zinc.

C. L. Jackson, *Chairman.*

Report of the Committee of Publication.

The Publishing Committee begs leave to report that there have been issued during the last academic year one number of Vol. XXXVI. and twenty-one numbers of Vol. XXXVII. of the Proceedings, aggregating 648 pages and 15 plates.

Three numbers of the current volume (82 pp. and 2 pl.) were printed at the cost of the Rumford Fund ($417.51). The expense of printing falling on the General Fund was $2095.59; the appropriation was $2400, and the returns from sales $138.59, leaving an unexpended balance of $443. The Committee recommends for the ensuing year the usual appropriation of $2400. An expensive memoir, closing Vol. XII., is in press.

For the Committee,

Samuel H. Scudder, *Chairman.*

Boston, May 14, 1902.

On the recommendation of the Committee of Finance, it was

Voted, To make the following appropriations from the income of the General Fund for expenditures during the ensuing year: —

For general expenses, ...	$2400
For the library, ...	1500
For cataloguing, ...	100
For publishing, ...	2400

On the recommendation of the Rumford Committee, it was

Voted, To make the following appropriations from the income of the Rumford Fund: One thousand dollars ($1000) for the immediate needs of the Committee in furtherance of research; seven hundred and fifty dollars ($750) to be expended for the construction of a mercurial compression pump designed by Theodore W. Richards, and to be used in his research on the Thomson-Joule Effect; one hundred and fifty dollars ($150) for the purchase and binding of periodicals.

Voted, To re-appropriate the unexpended balance of the amount granted at the last annual meeting for the purchase and binding of books on light and heat for the Library.

On the recommendation of the C. M. Warren Committee, it was

Voted, To make the following grants from the income of the C. M. Warren Fund: (1) To C. F. Mabery of Cleveland, three hundred dollars ($300) for the continuation of his researches on petroleum. (2) To A. A. Noyes of Boston, three hundred dollars ($300) for the continuation of his work on the qualitative analysis of the rare elements. (3) To H. O. Hofman of Boston, one hundred and sixty dollars ($160) for a research on the decomposition of sulphate of zinc.

On the motion of the Corresponding Secretary, it was

Voted, That the assessment for the ensuing year be five dollars ($5).

The annual election resulted in the choice of the following officers and committees: —

ALEXANDER AGASSIZ, *President.*
JOHN TROWBRIDGE, *Vice-President for Class I.*
HENRY P. WALCOTT, *Vice-President for Class II.*
JOHN C. GRAY, *Vice-President for Class III.*
WILLIAM M. DAVIS, *Corresponding Secretary.*

William Watson, *Recording Secretary.*
Francis Blake, *Treasurer.*
A. Lawrence Rotch, *Librarian.*

Councillors for Three Years.

Arthur G. Webster, of Class I.
Edward L. Mark, of Class II.
Arlo Bates, of Class III.

Member of Committee of Finance.

Eliot C. Clarke.

Rumford Committee.

Erasmus D. Leavitt,
Edward C. Pickering,
Charles R. Cross,
Amos E. Dolbear,
Arthur G. Webster,
Theo. W. Richards,
Elihu Thomson.

C. M. Warren Committee.

Charles L. Jackson,
Henry B. Hill,
Arthur M. Comey,
Samuel Cabot,
Leonard P. Kinnicutt,
Robert H. Richards,
Henry P. Talbot.

The following standing committees were appointed by the Chair: —

Committee of Publication.

Seth C. Chandler, of Class I., Edward L. Mark, of Class II.,
Crawford H. Toy, of Class III.

Committee on the Library.

William F. Osgood, of Class I., Samuel Henshaw, of Class II.,
Henry W. Haynes, of Class III.

Auditing Committee.

Henry G. Denny, William L. Richardson.

The following gentlemen were elected members of the Academy: —

Arthur James Balfour, of London, to be a Foreign Honorary Member in Class III., Section 1 (Philosophy and Jurisprudence), in place of the late Charles Russell, Baron Russell of Killowen.

William Edward Hartpole Lecky, of London, to be a Foreign Honorary Member in Class III., Section 3 (Political Economy and History), in place of the late Samuel Rawson Gardiner.

On the recommendation of the Rumford Committee, it was

Voted, To award the Rumford Premium to George Ellery Hale, of the Yerkes Observatory.

James Barr Ames read a biographical notice of the late James Bradley Thayer.

A biographical notice of the late Alpheus Hyatt, by Alpheus S. Packard, was read by the Corresponding Secretary.

The following papers were presented by title: —

"The Influence of Atmospheres of Nitrogen and Hydrogen on the Arc Spectra of Iron, Zinc, Magnesium and Tin, compared with the Influence of an Atmosphere of Ammonia." By Royal A. Porter. Presented by Charles R. Cross.

"On the Multiple Points of Twisted Curves." By John N. Van der Vries. Presented by William E. Story.

"Regular Singular Points of a System of Homogeneous Linear Differential Equations of the First Order." By Otto Dunkel. Presented by Maxime Bôcher.

Contributions from the Cryptogamic Laboratory of Harvard University. — L. "Preliminary Diagnoses of New Species of Laboulbeniaceae." — V. By Roland Thaxter.

Contributions from the Cryptogamic Laboratory of Harvard University. — LI. "On Cauloglossum transversarium (Bosc) Fries." By J. R. Johnston. Presented by Roland Thaxter.

"On the Ionization of Soils." By Amon Benton Plowman. Presented by George Lincoln Goodale.

Contributions from the Gray Herbarium of Harvard University. New Series. — XXV. I. "Flora of Cocos Island of the Pacific." II. "Diagnoses and Synonymy of some Mexican Spermatophytes." By B. L. Robinson.

"On the Dibromdinitrobenzols derived from Paradibrombenzol." Second paper. By C. Loring Jackson and D. F. Calhane.

"On the Colored Substances derived from Nitro-compounds." Fourth paper. By C. Loring Jackson and R. B. Earle.

"On Certain Derivatives of Picric Acid." By C. Loring Jackson and R. B. Earle.

"On Symmetrical Dinitrobenzolsulphonic Acid." By C. Loring Jackson and R. B. Earle.

"On Certain Derivatives of 1, 2, 3-tribrombenzol." By C. Loring Jackson and A. H. Fiske.

A TABLE OF ATOMIC WEIGHTS

OF SEVENTY-SEVEN ELEMENTS.

Compiled in April, 1902, from the most Recent Data.

BY THEODORE WILLIAM RICHARDS.

Name.	Symbol.	Atomic Weight.	Name.	Symbol.	Atomic Weight.
Aluminium	Al	27.1	Molybdenum	Mo	96.0
Antimony	Sb	120.0	Neodymium	Nd	143.6
Argon	A	39.92	Neon	Ne	19.94
Arsenic	As	75.0	Nickel	Ni	58.71
Barium	Ba	137.43	Niobium	Nb = Cb	94.
Beryllium	Be = Gl	9.1	Nitrogen	N	14.04
Bismuth	Bi	208.	Osmium	Os	190.8
Boron	B	11.0	Oxygen (standard)	O	16.000
Bromine	Br	79.955	Palladium	Pd	106.5
Cadmium	Cd	112.3	Phosphorus	P	31.0
Cæsium	Cs	132.88	Platinum	Pt	195.2
Calcium	Ca	40.13	Potassium	K	39.14
Carbon	C	12.001	Praseodymium	Pr	140.5
Cerium	Ce	140.	Rhodium	Rh	103.0
Chlorine	Cl	35.455	Rubidium	Rb	85.44
Chromium	Cr	52.14	Ruthenium	Ru	101.7
Cobalt	Co	59.00	Samarium ?	Sm	150.
Columbium	Cb = Nb	94.	Scandium	Sc	44.
Copper	Cu	63.60	Selenium	Se	79.2
"Didymium"	Nd + Pr	142.±	Silicon	Si	28.4
Erbium	Er	166.	Silver	Ag	107.93
Fluorine	F	19.05	Sodium	Na	23.05
Gadolinium	Gd	156. ?	Strontium	Sr	87.68
Gallium	Ga	70.0	Sulphur	S	32.065
Germanium	Ge	72.5	Tantalum	Ta	183.
Glucinum	Gl = Be	9.1	Tellurium	Te	127.5 ?
Gold	Au	197.3	Terbium ?	Tb	160.
Helium	He	3.96	Thallium	Tl	204.15
Hydrogen	H	1.0076	Thorium ?	Th	233. ?
Indium	In	114.	Thulium ?	Tu	171. ?
Iodine	I	126.85	Tin	Sn	119.0
Iridium	Ir	193.0	Titanium	Ti	48.17
Iron	Fe	55.88	Tungsten	W	184.
Krypton	Kr	81.7	Uranium	U	238.5
Lanthanum	La	138.5	Vanadium	V	51.4
Lead	Pb	206.92	Xenon	X	128.
Lithium	Li	7.03	Ytterbium	Yb	173.
Magnesium	Mg	24.36	Yttrium	Yt	89.0
Manganese	Mn	55.02	Zinc	Zn	65.40
Mercury	Hg	200.0	Zirconium	Zr	90.6

NOTE.

The accompanying table of atomic weights is but little changed since last year. Cæsium is made 132.88 instead of 132.9; calcium, 40.13 instead of 40.1; iron, 55.88 instead of 55.9; hydrogen, 1.0076 instead of 1.0075; and nickel, 58.71 instead of 56.70. The value for cæsium is due to some work, as yet unpublished, of Richards and Archibald, and that for calcium is increased in accuracy because the recent investigation of Hinrichsen* supports the less recent Harvard value.† The other very small changes are due simply to slight differences in the interpretation of data already well known. The decimal might have been omitted from palladium, because this element may still be a whole unit in doubt; but it has been retained as a compromise.

The differences between the present table, that of the German Committee,‡ and that of F. W. Clarke,§ are diminishing year by year. Nevertheless to as many as twenty-eight elements out of the seventy-seven are given values in these three tables differing among themselves by over one tenth of a per cent; namely, the atomic weights of antimony, bismuth, cerium, columbium, fluorine, gadolinium, germanium, helium, hydrogen, lanthanum, magnesium, mercury, neon, osmium, palladium, platinum, potassium, samarium, scandium, selenium, tantalum, tellurium, thorium, thulium, tin, titanium, uranium, and zirconium. To this list of uncertain elements should be added erbium, gallium, glucinum, indium, terbium, tungsten, ytterbium, upon which the three tables agree only because of lack of data upon which to base a disagreement. Thus nearly half of the elements are still in doubt by at least one part in a thousand. This circumstance is not so much a reproof to the many earnest workers upon the subject, as an evidence of the great difficulty of some of the problems involved.

Three of the elements given in the list above should not properly be included among the uncertain values, namely, hydrogen, magnesium, and potassium. The first finds its way into the list because of the disregard of significant figures by the German Committee, and the second chiefly because Clarke has included in his calculation work upon magnesic oxide undoubtedly erroneous on account of the presence of included gases.‖ The case of potassium is somewhat peculiar; for in spite of the great wealth of data concerning this element, Clarke assigns to it the value 39.11, while the German Committee chooses 39.15. The low value is chiefly due to very unsatisfactory data concerning potassic iodide. To me it seems that the most recent work of Stas is far more satisfactory than his earlier work or than the work of any one else, hence the value 39.14 has been assigned to potassium in the present table since its first publication. Careful analyses by E. H. Archibald and myself confirm this conclusion.

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* Hinrichsen, Zeitschr. phys. Chem., **39**, 311 (1901).

† Richards, Journ. Am. Chem. Soc., **22**, 72 (1900), also **24**, 374 (1902).

‡ Landolt, Ostwald, and Seubert, Extra insertion in Berichte d. d. ch. Ges. 1902. Heft 1.

§ F. W. Clarke, Journ. Am. Chem. Soc., **24**, 201 (1902).

‖ Richards and Rogers, These Proceedings, **28**, 209 (1893).

AMERICAN ACADEMY OF ARTS AND SCIENCES.

Report of the Council. — Presented May 14, 1902.

BIOGRAPHICAL NOTICES.

Augustus Lowell	Percival Lowell.
Truman Henry Safford	Arthur Searle.
Horace Elisha Scudder	Thomas Wentworth Higginson.
Joseph Henry Thayer	C. H. Toy.
John Fiske	Andrew McFarland Davis.
James Bradley Thayer	James Barr Ames.

REPORT OF THE COUNCIL.

The Academy has lost sixteen members by death since the annual meeting of May 8, 1901: Six Resident Fellows, — John Fiske, Alpheus Hyatt, Truman Henry Safford, Horace Elisha Scudder, James Bradley Thayer, Joseph Henry Thayer; two Associate Fellows, — Clarence King, Joseph LeConte; eight Foreign Honorary Members, — Marie Alfred Cornu, Samuel Rawson Gardiner, Friedrich Herman Grimm, William Edward Hearn, Aleksandr Onufrijevič Kovalevsky, Félix Joseph Henri de Lacaze-Duthiers, Friherre Adolf Erik Nordenskiöld, Karl Weinhold.

AUGUSTUS LOWELL.

Augustus Lowell was born in Boston, Jan. 15, 1830. His father was John Amory Lowell and his mother Elizabeth (Putnam) Lowell, daughter of Hon. Samuel Putnam of Salem. Both the Lowell and the Putnam families were early settlers in the new world, the former landing in Newburyport in 1639, the latter in Salem in 1630. Mr. Lowell thus came of Puritan stock on both sides. Otherwise the parts of his inheritance differed, for the Lowells were Norman by descent — the name, originally Lowle, dating from the conquest — while the Putnams, originally Puttenham, were apparently Saxon. He inherited the qualities of his name. Mentally he was the son of his father; as a matter of fancy as much as of fact, his mother's share in him being chiefly physical. For while in feature he looked like her, in mind he not only resembled his father but looked up to him with a very unusual amount of reverence and esteem. The feeling doubtless was born of the fact and is noteworthy because of the common belief that capable men have had capable mothers. Yet not only in his case but in the case of his father, grandfather, and great-grandfather before him, the capacity followed the name. Indeed the family has proved a singular instance of prepotence in the male line, while the temperament has been as strikingly a maternal gift.

In Boston and in its immediate neighborhood his boyhood was spent. Of the winter delights of town as seen through youthful eyes we are given a glimpse in a letter written at the time to his friend, Mr. Augustus Peabody. Chief among them it would seem was coasting on the Common, and in the epistle we are informed of the existence of two coasts there: "one the big boys' coast and the other the small boys' coast;" "but," the writer adds to fire the ambition of his friend and so induce him to come up for a visit, "the big boys do coast on the small boys' coast and the small boys do coast on the big boys' coast." The rounded accuracy of this statement, devoid of even the least suspicion of the elliptical, testifies conclusively to the writer's time of life.

His father had inherited the family country place in Roxbury, which then was country indeed, innocent of bricks and mortar, of city streets and of course of railroads. Horses and carriages made sole means of outside communication. Partly from necessity, therefore, partly for pleasure, Mr. John Amory Lowell every day drove into town to his business and with him he took his son to attend the Boston Latin School. This school, so named from teaching "small latin and less greek," was then the popular school for boys of the place. To it in consequence went many well-known men, among them his lifelong friends, Mr. George A. Gardner and Mr. Thornton K. Lothrop. The "small latin" was hardly such in quantity, if one may judge by report of the approved Latin grammar of the day. Indeed education would seem to have consisted of the learning by heart — pathetically so called in such connection — of a mass of rules and their elephantine exceptions, sufficient to stagger even a Roman into speaking something else. At all events, of the son's labors at that institution of learning the sole document extant is of the *lucus a non* kind: a petition to his Honor the Mayor and Chairman of the School Committee to allow the boys the first day of May as a holiday in which they might "enjoy the beauties of nature and a recreation and relaxation from school labors." Mr. Lowell appears heading the interesting document, which was couched as convincingly as possible by a classmate.

By nature the place in Roxbury was beautiful, though one would never divine it to-day. Shorn of its fine old trees, even pared of its hills, the land is possessed now by a brewery and tenement houses. But in those days it was otherwise, as fading photographs show, and its garden was both a delight and a name. For Mr. John Amory Lowell had two pastimes, algebra and botany. His spare moments were devoted to one or the other of these pet pursuits. When he was not setting himself problems he was puttering over plants. And he did both to some effect.

His algebraic propensities won him local reputation as a mathematician, and a manuscript volume upon the same, still in the family's possession, is both curious and interesting reading. As a botanist he was known not only at home but abroad, and was on terms of correspondence, not to say criticism, with botanists of his day. His botanical care was not confined to the living; in his studies he collected a fine herbarium which received fully as much of his attention, and attracted attention from others. The son inherited both paternal proclivities, but both rather as deep-seated mental characteristics than as current mental traits. Mathematics he neither cared for, nor was proficient in, but he derived from his father that logical exactness of mind which is their basis. The botany bore greater fruit. His tastes for plants, including both trees and flowers, proved a very deep-seated passion. Doubtless fostered in part by his father's familiarity with shrubs — though as a boy he showed no marked symptoms of botanic zeal — the love of growing things later became his most pronounced avocation.

In 1846 Mr. Lowell entered Harvard College where he spent the four years enjoined for a degree and was duly graduated in 1850. It was not then more than it is now the fashion to study, and he took his parchment void of invidious distinction. Indeed his recollections do not seem to have been specially academic, as one of the most vivid of them had to do with a certain midnight ride for illicit purposes to the Watertown arsenal. His rank in his class, if I am right, was sixteenth, just below what was at the time the ΦBK line. He was not therefore a member of that deservedly distinguished society of learning, but it is significant of his subsequent standing in the community that on the fiftieth anniversary of his graduation he was elected into it as an honorary member, an honor he never lived to receive or even know of, as unknown to the election committee he was on his deathbed at the time.

In college he was neither dissipated nor lazy. His course was much like that of all his fellows, and is distinguished from the commonplace only by a comical dream with which his ancestors saw fit to favor him later on the subject. I say his ancestors advisedly as will shortly appear, and I repeat the dream partly because of its touch of humor, of which he was always fond, and partly because of its psychologic import. The gusto with which he related it at the time proves the censure implied to have been undeserved, but the atavism betrayed by it makes it worth recording.

It was the family tradition that at college its scions should be students, a traditional devoir handed down from father to son, though I am not aware that the fathers always followed it themselves as religiously as they inculcated it upon the sons. In consequence of his supposed neglect of this precept, it was perhaps not unnatural that his ancestors should disapprove and should show their disapproval. This they did in the only way in their power — by means of a dream. For dreams are really reversions to type and are in consequence very interesting things. When we dream it is the atavic paths of which we are conscious. We think again the thoughts of our progenitors.

The occasion of this visitation was the going up of his second son for the entrance examinations, and the paternal mind was naturally full of the subject. With the unimpeachable authority of dreams he was suddenly made aware one night that he had not done all he might in college. Profoundly stirred by the thought, the singleness of which made it pass for truth, he decided after due and weighty consideration — lasting at least a tenth of a second — to enter the university once more and go over the course again. The fact that he was middle-aged, married, and had a large family only made the resolve seem, after the manner of dreams, the more meritorious. On the strength of his already holding a degree, the college faculty consented to admit him without examination. He was thus enabled triumphantly to get in. His action caused some comment, chiefly commendatory, such as follows an unusually pious deed. He thus became, against his will, something of a cynosure. So the first year glided by till with a speed peculiarly their own the annual examinations were upon him and with them the eyes of the community. Then, and somehow not till then, did he realize, to his consternation, that he had done nothing and was quite unprepared to pass. The situation was beyond words. At this awful moment he woke, — to the pleasing consciousness that his son, not he, would have to pass them on the morrow.

Just before his graduation in 1850 his father, who was not very well, decided to go abroad with his family, including his son Augustus, in the event of needing his help. Mr. Lowell stayed with his father till the spring of 1851. In Paris he was joined by his friend and classmate, Mr. Lincoln Baylies, and there at the same time was John Felton, brother of the then president of the college, with whom the two young men foregathered. John Felton was something of a character and a good deal of a man, with fiery red hair on the outside of his head and much genial wit and wisdom within it. Under his guidance, philosophy, and friendship the two young men passed an interesting and not unprofitable winter, frequenting the theatres to pick up French. Labiche was then in his prime. In the spring the two classmates went off to travel in Germany and Switzerland, and returned by themselves in the autumn to the United States.

On getting home he began his career in State Street, going into the counting-room of Bullard & Lee, East Indian merchants, to learn the business. His quickness of body as well as of mind here procured him a questionable distinction. From his father he had inherited considerable athletic ability, and it was soon discovered in the office that he was fleet of foot. In consequence he was promoted to the post of messenger, with the duty of carrying the foreign business letters to the mail. Now Mr. Lee was addicted to lengthy epistles, to extreme peculiarity in completing them, and to never finishing on time. As the mail was inconsiderate of their importance, he eagerly embraced Lowell's pedestrian possibilities. In consequence it soon became the regular thing for young Lowell to be seen standing, watch in hand, waiting while Mr. Lee completed his last page, folded the foolscap down methodically with his large thumb, and elaborately sealed it. Meanwhile the minutes slipped by with the young man calculating if he still had time to catch the post. It was midsummer and hot. Nevertheless the human Mercury was kept standing within, regardless of how its metallic namesake stood without. Finally when only running at his topmost speed would suffice to get the letter in he would hint that there were but six minutes more before the mail closed. "How many did you make it in last time, Augustus?" Mr. Lee would ask. "Five and three-quarters, sir, but I had to get it in the back way." "I think you can do it this time then." And he did.

This little episode occurred as regularly as mail day. After it had been cheerfully going on for some months, Mr. Bullard, who had been abroad, came home and one afternoon happened in on it. He said nothing at the time; but when Lowell, hot and breathless, had returned once more successful he called him into his private office. "Does what I saw this afternoon occur often, Lowell?" he said to him. "Every mail day, sir," the young man answered. "It shall not occur again," he said. And it never did. With tact equal to his considerateness, Mr. Bullard, on the ground — if I am right — of preferring to do it himself, from that day took the foreign correspondence into his own hands. Perhaps — after Balzac's phrase — this episode may be put between leads and given the air of a thought: the young man who goes quickly will go far.

After two years spent with Bullard & Lee Mr. Lowell's father thought it advisable the young man should learn another line of business, — one in which the family was interested. Francis C. Lowell, the elder, who was the founder of the cotton manufactures of New England and after whom Lowell — their chief seat — was named, was the uncle of Mr.

Lowell's father. In consequence the father had come himself into connection with them, and it seemed well that the son should do likewise. He was therefore sent to Lowell to become practically acquainted with the running of the mills. The house in which he boarded was kept by a woman who was destined through her own exertions to no little notoriety later on. She had a sister who had a beautiful voice. This voice was one of the few alleviations of the place to the boarders, and the same voice, more ably than considerately exploited by the boarding-house keeper, proved the family's making. For the boarding-house keeper was so successful in her management that she soon became the proprietress of the Revere House in Boston, and next emerged by the help of the voice at her entertainments into one of the chief lights of Newport and New York society. Such in a nutshell was the career of Mrs. Paran Stevens.

After passing a year at the mills, Mr. Lowell in 1853 became engaged to and in 1854 married Katharine Bigelow Lawrence, the youngest daughter of the Hon. Abbott Lawrence, then recently returned from his post at the Court of St. James. Mr. Lawrence was as closely identified with the then nascent cotton manufactures of New England as was Mr. John Amory Lowell. Mr. Augustus Lowell thus found himself doubly involved in them, first by birth and then by marriage. For the two centres of the industry were the towns of Lowell and Lawrence, the one named as I have said after his father's uncle, the other after his father-in-law. On his engagement Mr. Lawrence put him in with J. M. Beebe, Morgan & Co. Thus for the years preceding and following his marriage he was busy learning the details of what was to make Massachusetts' mercantile greatness, her manufacturing interests. With one exception, from this period to the end of his life, he was always associated in one way or another with the Lowell and Lawrence mills. He was successively treasurer, that is, the executive head, of more than one of them, and president of many others.

The exception occurred some time after Mr. Lawrence's death, which happened in 1855, when Mr. Lowell entered into business ventures of his own, forming a partnership with Mr. Franklin H. Story for the purpose of engaging in the East Indian trade. For some years this trade was profitable, but the firm was brought to a close by the panic of 1857, for though the firm did not suffer the East Indian trade did. The friendship remained, and among the pleasantest incidents of the writer's boyhood was the acquaintance of this genial gentleman. By a coincidence he died only about a week before his former partner.

In 1864 the health of his wife necessitated his taking her and his family abroad. They sailed for England in May, and for the next two years and a half lived in Europe; the summers spent in travelling, the winters in Paris. To one so temperamentally prone to a busy life at home, this existence was no sinecure. With a wife at the point of death as it was thought and four young children, Mr. Lowell had his hands full. For a long time Mrs. Lowell did not gain at all. Indeed it was only during the second summer, under the treatment of a country doctor fortuitously encountered in the Austrian Tyrol, that she began to mend. It is instructive, if tardy, to perceive now, in view of the widespread professional ignorance on the subject, that what Mrs. Lowell was suffering from was nervous exhaustion, — a disease, this, which it may be noted incidentally, Faraday, Darwin, Huxley, and Parkman all suffered from without knowing it.

Three little episodes may serve to mark these years of a search after health. The first summer the wanderers happened to be at Bonchurch in the Isle of Wight when the action between the "Kearsarge" and the "Alabama" took place just across the channel off the coast of France. In the second they were among the first to go to that nook in the Austrian Salzkammergut, the village of Ischl, since become well known and popular. In the third and last they were lodged at Schwalbach near Wiesbaden, when that little watering place suddenly became one of the seats of war, and thereupon was occupied alternately by the two opposing forces, the invading Prussians and the native Hessians. Usually evacuation considerately took place before occupation set in; but once by accident the two interfered and a battle occurred between the rear guard of the one army and the advance scouts of the other under the very windows of the hotel. The Hessians, who had been quartered in the town, had heard of the proposed Prussian advance and had at once started to evacuate the place. But they were a little too Teutonically slow. The invaders, although Prussians and *landwehr* at that, were, quite to their own surprise, too quick for them; a belated squad of Hessians had got only halfway up the hill on its way out when the Prussian cavalry was heard cantering into the town. There was no time to go on unseen when fortunately a friendly wood pile by the side of the road offered its shelter. Instantly the squad deployed behind it and waited. Five minutes later three cavalrymen cantered past the hotel, their pistols pointed at the windows as they went by, and started unsuspiciously up the hill. The spectators in the secret stood waiting the surprise. Just as the dragoons got abreast of the wood pile the squad deployed out and fired.

One dragoon fell on the spot, a second turned like a flash and leaped his horse over an embankment twenty feet to a road below, while the third wheeled in his tracks and came galloping wildly down the street again. All which served to relieve the watering place dulness.

By the autumn of 1866 Mrs. Lowell was so far recovered that Mr. Lowell was able to return with her to the United States. It was many years before he left it again.

He now took an office next his father's, and became gradually connected, on the one hand, with the manufacturing interests which his father controlled, and on the other with the many trusts his father managed. During Mr. John Amory Lowell's subsequent absences in Europe the care of these things devolved upon his son, and with the former's increasing years the care became more and more permanent. In 1875 he was chosen treasurer of the Boott Cotton Mills. This office he held for eleven years. About the same time he was elected to succeed his father on the board of the Massachusetts Hospital Life Insurance Company, — familiarly known as the Life Office, State Street's oldest, staidest, and most famous institution, whose real business has but a bowing acquaintance with its name, — and later was put upon its executive committee. Of the corporation of the Provident Institution for Savings, — another financial landmark, not so deceptively named to the uninitiated, — he was likewise made a member, and eventually became its president, succeeding the Mr. Lee of epistolary fame. At this date too he began his long career upon the board of the Boston Gas Light Company, then so ably managed by Mr. Greenough, a career which ended more than twenty years later in the negotiations he conducted as its president when it became necessary to sell the property, which he did for two and a quarter times all it had ever cost. In addition to holding the offices above mentioned he was treasurer of the Merrimack Manufacturing Company, June 20–October 29, 1877; president of the Massachusetts Cotton Mills; of the Massachusetts Mills in Georgia; of the Pacific Mills; of the Merrimac Manufacturing Company, 1887–8, 1892 to death; of the Boott Cotton Mills; of the Lowell Bleachery; of the Lowell Machine Shop; of the Glendon Iron Company; and a director of the Everett Mills; of the Middlesex Company; of the Lawrence Mills; of the Lowell Manufacturing Company; of the Suffolk National Bank; of the Cranberry Iron Company; of the Plymouth Cordage Company; besides being a trustee of the Union Trust Company of New York. This long list means even more than it usually would; for Mr. Lowell was a director who did direct. In every concern into which he entered he very soon took a leading part. Never seeking a place, his ability was such that he found himself forced into position after position of responsibility. Indomitable, he was always selected to do what others agreed ought to be done but were averse to doing. For Mr. Lowell knew no such thing as shirking in the discharge of duty. He disliked the disagreeable as much as any one, but he was not weak. Of the financial position he held in the down-town community it is enough commentary that seven bonds of treasurers of great corporations were found in his tins at his death, deposited with him as president.

Such were the business concerns with which he was connected. But side by side with them he gave much time and thought to matters of more public interest. For many years he was a trustee of the Boston Eye and Ear Infirmary. Not simply one in name, for to him and to Mr. Brown its management was for a long time chiefly due.

Ex-officio he was a trustee of the Boston Art Museum for twenty years, and a trustee of the Lowell Textile School for the four years preceding his death. Of purely public functions he once performed one, that of member of the Boston School Committee in 1857–58, and from the echoes of this which have reached the writer it would seem that politics played as objectionable a part in what should have been above them then as now.

Before going abroad he had had a summer place at Beverly, but attributing the loss of a child there to unhealthiness of the shore he sold it. On coming home he cast about for a country-place where he could live the year round, as being alike beneficial for his wife and his children. He found it in Brookline. His children were still young, and he took to repeating the experience of his own boyhood, driving them and himself into town every day to school and to business respectively. Out of it, beyond business hours, his life was now quite bucolic. The place he had bought possessed already a fine garden and two greenhouses. In them he centred his affections, greenhouse and garden dividing the year between them. Two hot-houses of grapes helped to shield the latter, which lay in a hollow open to the south. Natural embankments enclosed it on the east and west, and a raised roadway, shut off from view, made artificial protection on the north. Clipped evergreens stood for sentinels along a terraced path, ending in an arbor which fringed one side of it, and a corresponding row faced them upon the slope opposite. In this sheltered spot he spent much of his time. Pruning his shrubs, tying up his plants, and attending generally to the welfare of his flowers, he was almost as much of an inhabitant of the place as they. It was a world in which he found infinite satisfaction. His roses were his chief delight. And fine they were — no finer than the feeling with which he showed them off. But nothing vegetal was alien to him. He would point out with almost as much zest, punctuated by a wink, a foreign thorn-tree, which flanked the avenue, a platted mass of thorns a foot long, the despair of squirrels and cats.

His botany was of the old-fashioned kind. He did not pursue it as a science, but cultivated it as an art. His plants were rather pets than subjects for vivisection. Philosophically he was not concerned with their genealogy or relationship and disbelieved Darwinism to the day of his death. But in his intercourse with them he knew the life and the merits or demerits of each, and took pleasure in their thriving with something like affectionate interest. He behaved like a distant relative, the while stoutly denying that he was one. Indeed the relation did not seem so very distant, for he was never tired of attending to them, and took a paternal pride in their introduction to others. He would conduct you to view some bush at the moment in flower, and point out in what lay its peculiar praiseworthiness with the care of long acquaintance. Pretty much every tree upon his place — and it included some rare ones — was personally known to him. And if you strolled round with him he would talk fine print about each with you. He was constantly importing new plants and then watching them succeed. Though he made no parade of knowledge or of success, he not infrequently had plants which knew no rival in the neighborhood. A contrast this side of his life made with that of his morning down-town, where he played so prominent a part in the active affairs of men.

The long list of business offices held by him might lead one to infer that his time in the city must have been fully occupied by them alone. But he was much too busy a man for such to be the case. With all his industrial and financial concerns he found time for an equal employment in educational affairs. His ability was of the executive kind, which was as vital to the one as to the other. It thus came about that side by side with his business, and almost hand in hand with it, so practical was he in his workings, went another employment — usually only on speaking terms with the first. and then those of a beggar — the conduct of educational concerns. Busy as Mr. Lowell was with purely business affairs, he was equally engaged in matters of mind. Partly the accident of birth, partly the possession of ability, placed him in positions of authority in two important educational institutions: the Lowell Institute in the first place, and the Massachusetts Institute of Technology in the second.

Of the first of these he became the trustee in 1881, on the death of his father. Even before this, however, much of the work had fallen to him. The Lowell Institute is too well known to need description, but one phase of it will bear mention in connection with the man who for so long was its trustee. Most institutions of learning live by begging. If they happen to be possessed of presidents who are past masters in the art, they thrive; if not so blessed, they languish. That a president should be an able intellectual director is unfortunately not so pressing a demand as that he should be a persistent, importunate, and successful beggar. In view of this fact deficits in college finances have lost their terror and surpluses are unknown, a sympathetic public being with confidence relied on to stand in the gap. Now the peculiarity of the Lowell Institute has been not only that it is not dependent upon alms-giving but that it has thriven and grown without it. Although on the one hand it has paid larger salaries than any college or kindred institution to the teacher, it has asked no fee whatever of the taught. Yet despite this liberality on both sides, its funds have more than quadrupled in amount. Part of this increase has been due to the wise terms of the endowment, part to the like wisdom of the two successive trustees. Kindred wisdom it has been in both senses, for by a provision of the testator the trustee must be of the testator's family if a fit person exist of the name. How fit Mr. Lowell was for the post this able result of his administration of the finances attests.

But besides being its financial head, Mr. Lowell was its intellectual body and its executive arm as well. For the Institute is a one man power, an absolute dictatorship. Mr. Lowell was president, corporation, and treasurer all together. And the success he made of it shows again the wisdom of such a rule, provided only the ruler be fit. Of his capacity as financier the property speaks; of his ability in general administration the list of lecturers before the institution sufficiently betokens. At the time the Institute was founded lectures were a popular form of instruction, and the object of the testator was to secure for the people of Boston lectures by the most eminent men at home and abroad, and to give these to the public free of charge. His wish has been well carried out. On the roster of the books are to be found a majority of the names which are known the world over, and almost every one of those to whose possessors distance or age or language did not prove an impassable bar. America, Europe, even Asia have contributed to the list. Some of these men came more than once; and many of them became well known personally to Americans. But the fact connected with them which speaks most for the institution and its trustee is that well-nigh without exception each came originally at his instigation. Almost all the famous foreigners in science, literature, or art who have been in this country have owed their personal introduction to it to the trustee of the Lowell Institute. Since from over seas these lecturers came, simply as a bond between countries the Institute has played no unimportant part.

Mr. Lowell's tie to science was thus rather indirect than direct, but it was none the less intimate if in a different way. By virtue of his office he was brought personally in contact with the scientists of his day, and in a most pleasant and withal domestic manner. For besides meeting them at the lectures, of which he always attended the opening one and oftener than not the whole course, he was in the habit of entertaining the lecturers during their stay in Boston at his house at dinner, sometimes more than once. Many is the memorable evening he passed in consequence with men who have made the world what it is. Such personal knowledge of a man is as invaluable as it is invigorating. Even in an estimate of the mind a side light of no mean value is shed on it by intercourse with the personality. The man proves a footnote to his own writings. This advantage of glosses on the text Mr. Lowell possessed; and in various aspects in as much as he was thrown with these men in diverse relations. Intercourse of the sort he enjoyed more or less for nearly half a century. For, as I have said, before he became trustee he had been acting for his father, and even before that had met the lecturers at his father's house. During the second half of the nineteenth century he had thus been familiar, not only with the century's best thought, but with most of its best thinkers. And he passed away just as the century itself was drawing to a close.

Coincident with holding this responsible post in educational matters of a general character Mr. Lowell filled a second position of a more direct kind and not less important. For quite as long a term as he managed the Lowell Institute was he associated with the government of the Massachusetts Institute of Technology. Entering the corporation of that institution in the early seventies, he very soon took a leading part in its policy. From that time the conduct of its affairs had been intimately connected with him, much more so than the public is cognizant of. For Mr. Lowell never put himself forward, having an innate aversion to unnecessary publicity. Even on the few occasions when it was indispensable for him to appear, he only did so, as those in his confidence are aware, after great reluctance.

Mr. Lowell was identified with this phenomenally successful institution almost from its start. The Massachusetts Institute of Technology was founded in 1861, chiefly through the instrumentality of Prof. William B. Rogers. To the same eminent mind it owed its early success. Measure of the man's executive ability in the first place, its success was in the long run the sign of his forethought in founding it. A school of technology was exactly what the American genius had demanded for many years in vain. It seems strange that no one should have heeded this unmistakable cry of nature before; but men are prone to being thus strangely deaf, till an interpreter arises. For a century the American has been noted for his innate inventiveness and general ingenuity, and has been equally noted for the untrained character of his craft. In some things this did well enough, but in the higher branches it left a good deal to be desired. To supplement natural aptitude with proper training was thus the one thing needful. To think of it was so simple a matter as to require a master mind for the thought. It was a piece of educational acumen of the highest order. And it has borne its inevitable result. But though it was destined to great and permanent success it would be contrary to common sense to suppose that the move was fully appreciated, from the very start. On the contrary, had it not been for its founder the institution would probably have gone under.

After Mr. Rogers' death much came to devolve upon Mr. Lowell; and since then, that is for the last quarter of a century, the policy of the Institute has been intimately associated with him. Elected a member of the corporation in 1873, he was chosen a member of the executive committee in 1883, and was kept upon it to the day of his death. During his term of service were chosen four presidents, and I need only mention the name of General F. A. Walker, who was the longest incumbent, to suggest how wisely made these choices were. But the work of the committee did not end with the selection of the executive; as its name implies, it was itself a part of that executive and its function was continual. As the senior member of the committee Mr. Lowell's force was felt in every portion of the policy pursued. Not a measure was passed which had not been influenced by his opinion. His judicious advice was fully appreciated by General Walker. Indeed the two men were natural complements to each other, General Walker with his brilliant, engaging personality, and Mr. Lowell with his uncommon judgment and invincible determination.

The position taken by the Institute under this leadership is well known. The institution has quadrupled in size, and what is far more important, has more than quadrupled in prestige. It is recognized to-day not only as the first, but as easily the first, school of technical arts in this country. To it now flock students from the farthest portions of these United States: from Oregon and Texas, from Illinois and Ohio, as well as from New York and Massachusetts. And as graduates they go back again to help develop the country. If any such institution may fairly be called national the Massachusetts Institute of Technology is the one.

Nor is this all. Not confined to the limits of this continent, its fame has successfully invaded lands across the sea. It is not long since Sir Robert Ball informed the writer that it was in advance of anything of the kind in Great Britain; a belief which he had years before acted on by sending his son to it, who is now practising in England. The belief would seem to be spreading; for in June, 1901, examinations for admission to it were held in London. Its rank would seem even to be recognized at home, which means that it probably is of some importance, as the American believes firmly in the *ignota pro magnifico*. The post-graduate course, pursued by the ranking men of the U. S. Naval Academy at foreign institutes heretofore, is in future to be taken at the Institute. It has been the custom of the Academy since 1883 to send the first few scholars of the highest grade, the construction department, abroad to finish their education. At first it was Greenwich they went to, till the British Government ludicrously enough became sensitive to the cadets outstripping their own students and forbade them. Then the Navy sent men to the University of Glasgow, and lastly to the École Polytechnique in Paris, where the recent ones have all graduated. In future it will be in Boston. Evidently the United States Government is convinced of the primacy of the Institute.

What Mr. Lowell's share in this success was may best be gathered from an episode which occurred about a twelvemonth before his death. Feeling himself worn by a painful trouble which he had had for years, he was minded in a moment of acute access of it to give up active work. Accordingly he sent in his resignation to his colleagues of the corporation. They refused to accept it, and the committee did their best to persuade him to reconsider his determination; but in vain. Whereupon a memorial was drawn up, signed by every member of the corporation accessible at the time, protesting against his resignation, and begging him not to withdraw his services from the institution. Such unanimous spontaneity of appreciation in a body of forty odd members is not common. That he was profoundly touched by this mark of confidence and esteem needs no saying.

Of the American Academy of Arts and Sciences he was made a member in 1886. He was first the treasurer and then the vice-president. On the death of Professor Cooke, deeming it fitting that the post of president should be filled by a man of science, he secured the election of Agassiz. When the change into sections was made he became the vice-president of his section, — jurisprudence and literature. He was also a member of the American Association for the Advancement of Science from 1898; of the Massachusetts Historical Society in 1900; of the Colonial Society of Massachusetts from 1898. He died on June 22, 1901.

Such, in brief, was what Mr. Lowell did. Quite as important is what he was. For the man was always behind his measures, as the whole includes the part. His actions were but parcel of himself. Not always is this the case. Some men become noteworthy for what they do, while being notorious for what they are. But with him the act was outcome of the man. He said what he meant and meant what he said. In this unity lay one element of his force. To those with whom he came in contact this oneness with one's self made itself felt. To the world at large, which sees the works but not the workings, his hand in matters which he had brought about often escaped notice. For a certain ingrained aversion to publicity prevented him from putting himself forward. Nothing, however, restrained him from pushing his measures. In consequence, many as were the acts one can point to in his unusually active life, those which actuated others without appearing themselves were more; in consequence also, the world remained in ignorance of the motive cause. For he acted for results; and what is to take effect does not need to make it.

Effect indeed was the very opposite of what Mr. Lowell was in thought or word or deed; and very refreshing it is, like a cool breath of pure air in the artificial heat and closeness of a crowded room, to consider such a character in these days of blatant, forth-putting mediocrity. When to seem is at a premium, and to be at a discount, it is invigorating to turn to a life which owed nothing to adventitious or meritricious aid; a life which not only was fine, but escaped the soiling consequent upon too much mental fingering by the world at large. To be generally in evidence means a loss of that delicacy of distinction, if it means nothing more, which is for so much in beauty of character. But it means usually very much more; it leads inevitably to a substitution of superficiality for solidity, of appearance for reality, of a sinking to a level of one's audience instead of a rising superior to applause. To say that a man owed nothing to effect is to say of him the best that can possibly be said. The natural forces with which we daily come in contact owe nothing to such cause;

on the contrary they stir us all the deeper, if we stop to think, for the very fact that they do not stir us without such thought. We are impressed the more by what seems superior to the impression it makes.

There is, too, another merit in the absence of effect — a gain in effectiveness. It is the greatest compliment to a man's ability that he should succeed without seeming to do so, because it shows that all his force has been massed upon the one strategic point. We are all familiar with this when it is done of intent aforethought.

As potent is the principle when the self-effacement is unconscious. The one obliteration differs from the other only in being instinctive instead of being thought out; and the one is as telling as the other. However it be brought about, the fact that the self is effaced is proof that the work has been done well. For it shows that the result has been brought to pass with the least expenditure of force. Personality causes friction, and evidence of self is therefore proof that force has been uselessly employed. The fact that a man has succeeded in having his idea prevail without forcing himself along with it is sign of the best kind of work.

Now this was the case with Mr. Lowell. It was so because of an unusual combination of characteristics, a singular wedding of energy in deed with dislike of its external trappings.

To an exceptional extent, therefore, Mr. Lowell's distinction lay in character. Three qualities he possessed to an unusual degree, qualities each rare enough as it is: will, ability, and integrity. He was, in the first place, a combination of force and ability as simple and as uncommon as success, which is its immediate consequence. The one is but the necessary premiss to the other's conclusion. If a man be originally possessed of the first he is sure eventually to possess the second. Schopenhauer's definition of the world as all will and representation certainly holds of one part of it, — the affairs of men. If the affairs consist rather in the dealing with men than nature the representation takes the form of words, and may be paraphrased as first the skill to put a thing convincingly and then the will to put it through. Mr. Lowell combined the two qualities.

Will he possessed to the full. He was noted for his determination. To his lot, in consequence, fell many necessary and thankless tasks. He likewise escaped many empty honors. For where he went he worked. No one ever thought of preferring him to a post merely *honoris causa*. For people knew that in getting him they got not a figure-head, but a man who was certain to make himself felt; not because he tried to do so, but because it was in him to do it. He entered concerns not by the postern gate of popularity, but by the portal of inevitableness. He was chosen because he was necessary. And he stayed for the same reason.

Now will is pure force, uncomplexioned, the mere dynamic outcome of the idea. Its effectiveness to any particular end depends, therefore, upon the character of the idea whose explosive force it is. With Mr. Lowell the idea owed its carrying power to two characteristics: judiciousness in itself and judiciousness in its presentation. In the first place he was apt to be right, that is, to be wise. His judgment of things within his own field was excellent. It was essentially sound. His was that uncommon sense-possession, the possession of common sense. Instinctively his mind worked correctly. It was the exact opposite of the mind of the crank, which may often hit off a brilliant conception, but which is too unsafe to be trusted. With him no one idea ever usurped the right of way to the exclusion of others. Each had its due effect; which fundamental balance makes the only safe foundation for superstructure.

In the next place he was as shrewd as he was sound. He had a keenness for the essential point which almost assured success in advance. Insisting upon what was vital, he waved less important issues to the other side. In this consists the consummation of the art of commerce with one's kind. An instance of the combined breadth and shrewdness of his business insight occurs to my mind. "When I lease a building," he once said to me, "I ask a good price of the tenant and then do all the little repairs he wants. The price makes its impression but once; the perquisites repeatedly, and the latter impressions stand nearer to the falling due of the lease."

Backing up his judgment was his excellence of exposition. His ideas were the more telling for being well told. His words were few and to the point. In a twinkling he would dissect a situation, and with equal terseness suggest its remedy. With ability for audience this had immediate effect; with mediocrity it was rather his tact that told. His logic was too accurate for popular approval, which prefers the coloring of emotion to the lines of thought. For very few men care for truth as they care for their feelings. And Mr. Lowell's forte was not the silver tongue of eloquence, but the more golden gift of statement. He could put a point so that it pierced perception instantly.

Lastly, there was about his advocacy of his measures an impersonality as potently as it was subtly persuasive. It was not that the ideas themselves were what one would call impersonal, but that the idea appeared by itself with so little of that aura of the personal, which in human affairs the man unconsciously throws around it, as to appear to stand alone. For in Mr. Lowell's case it was as if he were but the mouthpiece of the idea, so heartily did he identify himself with it, and yet so single was his intent. It was the idea he thought of, not of himself. Such a condition tends in a twofold way to conviction; first, by the sincerity of the pleading, and secondly, by the absence so far as is humanly possible, of the antagonism roused by personality.

Recognition of his ability followed any knowledge of him; it did not, as with some men, precede it. Those qualities compounded of sociability and forth-puttingness, however unintentional, which make for instant distinction among one's fellows, were not his by nature. His abilities were solid, not showy. Nor was it his bent to go out of his way in the road we all travel to make a new path. He neither courted position nor shirked it. When it once fell to him he became as it were the office. Nothing was ever done by him for his own sake, however incidentally. He seemed simply to embody his trust. In intent he was singularly single. Indeed, in describing his action I find it difficult to convey the combination of self-obliteration and of self-sufficiency in its best sense, which he was. For the character is uncommon. One often witnesses self-abnegation. But it is usually wedded to weakness. Or, on the other hand, one sees strength associated with self-seeking. Few men are essentially impersonal enough to strive strenuously for the thing in itself, as if it were a person. He did.

This was perhaps the stranger that his mental makeup was not of the abstract but of the distinctly concrete kind. In practical, not in theoretical matters, he was great. Widely read as he was he never seemed to care to theorize. He enjoyed highly the theories of others, when they did not collide with the puritanism which, as I have said, he inherited doubly distilled. Even this was perhaps as much due to the society in which he had been brought up. He was educated before the modern movement in thought took place, and Boston of sixty years ago was even behind the rest of the world in this stirring of the waters of stagnation. Not in knowledge nor in intellect; it was in cast of mind he differed. His preference was for action. Of this he never tired. To recreation he was less given. Such as he took was of a serious kind. He was a member of the Wednesday Evening Club, of the Thursday Evening Club, and of a class dinner club; but clubs which consist but of a local habitation and a name he never cared to join. Loafing and he were strangers.

Will and the power of representation were, as I have said, two of his attributes. But the second of these should, though it often does not, include a quality which is itself fundamental to all character, and which

Mr. Lowell possessed to the utmost — the quality of honesty. In these days, when successful financial operations so often depend upon will and *mis*representation, it is no small thing to say of a successful man of affairs that he was conspicuously honest. When to steal enough is to steal with credit, it is cheering to see business triumph attendant on unimpeachable integrity. And this was typically true of him. Honest he was by essence. Verity was of the very fibre of his being.

Nor is it only of the grosser form of that attribute which has usurped the generic name of honesty of which I would speak, but of that finer sense of fair dealing which we include under the appellation of a just man. His uprightness was perfectly well known. No adversary ever questioned that. A tribute to the fact once came in an amusing manner to Mr. Lowell's ears in one of the latter years of his life. He was passing through a railway station in Boston one afternoon when he chanced to overhear two men unknown to him discussing his character. It was his own name that caught his attention. "Augustus Lowell," said one, "is a hard man, but he is absolutely honest." "Yes," said the other, "he is emphatically that." It is not often that one overhears a bit of one's own obituary during one's life, nor is made privy to concurrent testimony on the subject from both sides of a discussion. As to the hardness imputed to him, it had no foundation in fact, though it was often attributed to him by people who knew him only from the outside. A cast of countenance which looked stern when in repose, and which was purely a matter of feature, was chiefly responsible for the reputation. He was quite aware of the look himself, as well as of that to which it was due. As a matter of fact he was very tender-hearted, singularly so for a man of his determination. Few suspected him of the kindnesses he was constantly doing, so unostentatiously were they performed, and almost no one credited him with the affection he felt.

The complexion of his character — for will is an uncomplexioned force — may be described in one word: exactness. Accuracy of statement and honesty of purpose are both but facets of a crystallization of thought. A man who sees clearly must be honest by instinct if he be not dishonest by intent. There is with him no limbo of self-deception. Much of the untruth current in the world is due to an initial haziness of conception subsequently seized upon and distorted to its own ends by passion, without disquiet to the perpetrator, because unrecognized as distortion by him. Mr. Lowell was essentially exact. His nature therefore imposed honesty. He saw much too correctly either to jumble or to juggle with his thoughts.

Important as the qualities he possessed are to the making of a man, they are no less so to the making of a community. And in any constitutional country no small part of the value of a man lies in his value as a citizen. Indirect as well as direct his influence may be, and with universal suffrage the former is apt to be the case with the best men. To be determined, discerning, and honest does not, unfortunately, in our system of supposed political equality, lead to purely civic distinction. For the choice of a popular suffrage cannot rise above its source. But if the qualities do not lead to civic distinction for their possessor they do something as enduring, — they tend to raise to his level the community of which he forms a part. For without the first attribute, nothing is possible; without the second, foolishness; without the third, knavery. The apathy of most of us, the crankiness of a few, and the financial trickery of others, are the several results of the absence of these qualities.

Too strong a personality to be generally popular, recognition of such a character is slow. For we are all prone to praise what we like. Only when distance does away with personal perspective do men, like hills, reveal their height.

Posterity gives the final judgment. For posterity judges of a man's worth unhaloed by the engaging lack of it, and sets the seal of its appreciation upon those who have contributed to the world's advance and incidentally to posterity's own existence. To make for this advance is the best any man can do, and to this end to be determined, discerning, and honest is one of the surest means. If a man possess these attributes he will not have lived in vain.

PERCIVAL LOWELL.

TRUMAN HENRY SAFFORD.

TRUMAN HENRY SAFFORD was born January 6, 1836, at Royalton, Vermont. The course of his life was determined by a phenomenal capacity for the mental solution of arithmetical problems, which began to display itself when he was only six years old. This faculty, which might under easily conceivable circumstances have been wasted in mere display for the amusement of the curious, fortunately attracted the attention of judicious and eminent men, and thus secured for him the advantages of a thorough education. He graduated at Harvard College in the class of 1854, which he joined at the beginning of its Junior year. As a boy he had computed an almanac, and given other evidences of interest in astronomy, and capacity for it; and immediately after his graduation he obtained employment at Harvard College Observatory, where he continued for nearly twelve years. He married Elizabeth M. Bradbury, of Cambridge, in March, 1860, "on six hundred dollars a year," as he once told the writer of this notice; for astronomy has never been a promising road to riches for young Americans insufficiently endowed with the practical turn of mind generally regarded as characteristic of their countrymen. He was elected Fellow of the Academy, Nov. 13, 1861.

Safford's position at Cambridge, if not pecuniarily advantageous, offered him in some other respects greater advantages than, perhaps, he could secure in later life; for he had here comparatively few hindrances to the undisturbed development of his scientific abilities. Accordingly, the results of his work soon began to make him widely and favorably known in astronomical circles. One of the most generally interesting of these investigations related to the orbital movement of Sirius. Many years before, the observed want of uniformity in the proper motion of this star had led astronomers to the belief that it formed one of a system of bodies revolving about a common centre of gravity; its companion, or companions, as the case might be, being too faint to be visible, at least with the existing instrumental means. Still more recently, the character of the supposed revolution of Sirius had been discussed by means of its right ascensions, as observed at different times. Safford now undertook a similar discussion of its observed declinations, and after combining the result of this work with that previously found, on the supposition that only one disturbing body occasioned the observed effects, was able to indicate its direction from Sirius at the time, in excellent agreement with the actual place of the companion discovered almost simultaneously by the younger Alvan Clark.

A catalogue of the declinations of five hundred and thirty-two stars, intended for use in the government survey of the lake region, was prepared by Safford during his connection with Harvard College Observatory, and probably marks the beginning of the geodetic work which occupied a large part of his time in later years.

In 1863 he received the formal title of Assistant Observer; and two years later, upon the death of Professor G. P. Bond, he was placed in charge of the Observatory. At this time, he completed and prepared for publication Professor Bond's researches on the nebula of Orion, which appeared as Volume V. of the Observatory Annals. Volume IV. of the same series is also the work of Safford. The first part, dealing with the preparation of a list of fundamental stars for transit observations, was published in 1863. By means of these fundamental stars, the right ascensions of five hundred and five stars were determined by observations in the years 1862 to 1865; the second part of the volume, containing the result of this work, appeared in 1878.

In 1866 Safford was appointed director of the Dearborn Observatory at Chicago, which had recently been provided with the large refracting telescope, by means of which, while still in its maker's hands, the companion of Sirius had been discovered. This position, with which was connected a professorship of astronomy in the University, seemed to offer the fairest prospect of permanent and congenial employment to its occupant; but the disastrous fire which destroyed so large a part of Chicago in 1871 deprived the Observatory of the financial support upon which its activity depended. Professor Safford, accordingly, now found it necessary to maintain his family by geodetic work connected with the government surveys. He had undertaken the observation of one of the zones of stars distributed among various observatories under the general system arranged by the Astronomische Gesellschaft; but this, and other pieces of work begun at Chicago, were now necessarily laid aside.

In 1876, however, Professor Safford was restored to his favorite pursuits by appointment to the chair of astronomy at Williams College. In this position, teaching required much of his time, and of course largely impeded his attention to scientific investigation; he also acted as librarian of the College, and was at times engaged in other business connected with its administration. It is not probable that he felt the work of instruction to be a burden; on the contrary, he took great interest in the subject of pedagogy, which he studied theoretically as well as practically. Notwithstanding all hindrances to the pursuit of strictly astronomical research, he accomplished much in that direction during the years spent at Williamstown, devoting himself largely, as before, to the subject of accuracy in the determinations of the positions of fixed stars. One of the principal results of this work was the publication (in the Proceedings of this Academy, Volume XIX.) of a catalogue of the mean right ascensions of one hundred and thirty-three stars near the north pole; but many other articles in scientific periodicals, particularly the Monthly Notices of the Royal Astronomical Society, attest Professor Safford's perseverance and success in scientific work during his later years.

He died June 13, 1901, at Newark, New Jersey, where he was residing at the time with one of his sons. A stroke of paralysis, three years previously, had put an end to his activity in science. His widow, with four sons and a daughter, survives him.

ARTHUR SEARLE.

HORACE ELISHA SCUDDER.

It is a merit of the American Academy of Arts and Sciences that it does not limit itself to one form of intellectual pursuits, as do the merely historical or scientific societies or even some which share the name of Academy. It also has the merit that it is ready to recognize the various subdivisions of each pursuit, and has a place of honor for every such department. Intellectual self-respect is to be found only in honoring every form of work in its place. It has been generally felt, I think, that no disrespect was shown to our late associate, John Fiske, when the New York *Nation* headed its very discriminating sketch of him with the title "John Fiske, Popularizer;" and in speaking of another late associate who has left us, I should feel that I showed no discourtesy, but on the contrary, did him honor in describing him as Horace Elisha Scudder, Literary Workman. I know of no other man in America, perhaps, who so well deserved that honorable name; no one, that is, who if he had a difficult piece of literary work to do could be so absolutely relied upon to do it carefully and well. Whatever it was, compiling, editing, arranging, translating, indexing, — his work was uniformly well done. Whether this is the highest form of literary distinction is not now the question. What other distinction he might have won if he had shown less of modesty or self-restraint, we can never know. It is certain that his few thoroughly original volumes show something beyond what is described in the limited term, workmanship. But that he brought simple workmanship up into the realm of art is as certain as that we may call the cabinet-maker of the middle ages an artist.

Mr. Scudder was born in Boston on October 16, 1838, the son of Charles and Sarah Lathrop (Coit) Scudder; was a graduate of Williams College and after graduation went to New York, where he remained for three years engaged in teaching. It was there that he wrote his first stories for children, entitled "Seven Little People and Their Friends" (New York, 1862). After his father's death he returned to Boston and thenceforward devoted himself almost wholly to literary pursuits; prepared the "Life and Letters of David Coit Scudder" his brother, a missionary to India (New York, 1864); edited the "Riverside Magazine" for young people during its four years' existence (from 1867 to 1870); and published "Dream Children" and "Stories from My Attic." Becoming associated with Houghton, Mifflin and Company he edited for them the Atlantic Monthly Magazine from 1890 to 1898, preparing for it also that invaluable index, so important to bibliographers; he also edited the "American Commonwealth" series, and two detached volumes, "American Poems" (1879) and "American Prose" (1880). He published also the "Bodley Books" (8 vols. Boston, 1875 to 1887); "The Dwellers in Five Sisters' Court" (1876); "Boston Town" (1881); "Life of Noah Webster" (1882); "A History of the United States" for schools (1884); "Men and Letters" (1887); "Life of George Washington" (1889); "Literature in School" (1889); "Childhood in Literature and Art" (1894), besides various books of which he was the editor or compiler only. He was also for nearly six years (1877–82) a member of the Cambridge School Committee; for five years (1884–89) of the State Board of Education; for nine years (1889–98) of the Harvard University visiting committee in English literature; and was at the time of his death a trustee of Williams College, Wellesley College, and St. John's Theological School, these making altogether a quarter of a century of almost uninterrupted and wholly unpaid public service in the cause of education. Since May 28, 1889, he was a member of this Academy, until January 11, 1902, when he died. This is the simple record of a most useful and admirable life, filled more and more, as it went on, with gratuitous public services and disinterested acts for others.

As a literary workman, his nicety of method and regularity of life went beyond those of any man I have known. Working chiefly at home, he assigned in advance a certain number of hours daily as due to the firm for which he labored; and he then kept carefully the record of these hours, and if he took out a half hour for his own private work, made it up. He had special work assigned by himself for a certain time before breakfast, an interval which he daily gave largely to the Greek Testament and at some periods to Homer, Thucydides, Herodotus, and Xenophon; working always with the original at hand and writing out translations or commentaries, always in the same exquisite handwriting and at first contained in small thin note-books, afterwards bound in substantial volumes, with morocco binding and proper lettering. All his writings were thus handsomely treated, and the shelves devoted to his own works, pamphlet or otherwise, were to the eye a very conservatory and flower garden of literature; or like a chamberful of children to whom even a frugal parent may allow himself the luxury of pretty clothes. All his literary arrangements were neat and perfect, and represented that other extreme from that celebrated collection of De Quincey in Dove Cottage at Grasmere, where that author had five thousand books, by his own statement, in a little room ten or twelve feet square; and his old housekeeper explained it to me as perfectly practicable "because he had no bookcases," but simply piled them against the walls, leaving here and there little gaps in which he put his money.

In the delicate and touching dedication of Scudder's chief work "Men and Letters" to his friend Henry M. Alden, the well known New York editor, he says: "In that former state of existence when we were poets, you wrote verses which I knew by heart and I read dreamy tales to you which you speculated over as if they were already classics. Then you bound your manuscript verses in a full blue calf volume and put it on the shelf, and I woke to find myself at the desk of a literary workman." Later, he says of himself, "Fortunately, I have been able for the most part to work out of the glare of publicity." Yet even to this modest phrase he adds acutely: "But there is always that something in us which whispers *I*, and after a while the anonymous critic becomes a little tired of listening to the whisper in his solitary cave, and is disposed to escape from it by coming out into the light even at the risk of blinking a little, and by suffering the ghostly voice to become articulate, though the sound startle him. One craves company for his thought, and is not quite content always to sit in the dark with his guests."

The work in which he best achieves the purpose last stated is undoubtedly the collection of papers called by the inexpressive phrase "Men and Letters;" a book whose title was perhaps a weight upon it and which yet contained some of the very best of American thought and criticism. It manifests even more than his "Life of Lowell" that faculty of keen summing up and epigrammatic condensation which became so marked in him that it was very visible, I am assured, even in the literary councils of his publishers, two members of which have told me that he often, after a long discussion, so summed up the whole situation in a sentence or two that he left them free to pass to something else. We see the same quality for instance in his "Men and Letters," in his papers on Dr. Mulford and Longfellow. The first is an analysis of the life and literary service of a man too little known because of early death, but of the rarest and most exquisite intellectual qualities, Dr. Elisha Mulford, author of "The Nation" and then of "The Republic of God." In this, as everywhere in the book, Mr. Scudder shows that epigrammatic quality which amounted, whether applied to books or men, to what may be best described as a quiet brilliancy. This is seen, for instance, when in defending Mulford from the imputation of narrowness, his friend sums up the whole character of the man and saves a page of more detailed discussion by saying, "He was narrow as a cañon is narrow, when the depth apparently contracts the sides" (page 17). So in his criticism called "Longfellow and His Art." Scudder repeatedly expresses in a sentence what might well have occupied a page, as where he says of Longfellow, "He was first of all a composer, and he saw his subjects in their relations rather than in their essence" (page 44). He is equally penetrating where he says that Longfellow "brought to his work in the college no special love of teaching," but "a deep love of literature and that unacademic attitude toward his work which was a liberalizing power" (page 66). He touches equally well that subtle quality of Longfellow's temperament, so difficult to delineate, when he says of him: "He gave of himself freely to his intimate friends, but he dwelt, nevertheless, in a charmed circle, beyond the lines of which men could not penetrate" (page 68). These admirable statements sufficiently indicate the rare quality of Mr. Scudder's work.

So far as especial passages go, Mr. Scudder never surpassed the best chapters of "Men and Letters," but his one adequate and complete work as a whole is undoubtedly, apart from his biographies, the volume entitled "Childhood in Literature and Art" (1894). This book was based on a course of Lowell lectures given by him in Boston, and is probably that by which he himself would wish to be judged, at least up to the time of his admirable "Biography of Lowell." He deals in successive chapters with Greek, Roman, Hebrew, Mediaeval, English, French, German, and American literary art with great symmetry and unity throughout, culminating, of course, in Hawthorne and analyzing the portraits of children drawn in his productions. In this book one may justly say that he has added himself, in a degree, to the immediate circle of those half dozen great American writers whom he commemorates so nobly at the close of his essay on "Longfellow and his Art," in "Men and Letters." "It is too early to make a full survey of the immense importance to American letters of the work done by half a dozen great men in the middle of this century. The body of prose and verse created by them is constituting the solid foundation upon which other structures are to rise; the humanity which it holds is entering into the life of the country, and no material invention, or scientific discovery, or institutional prosperity, or accumulation of wealth will so powerfully affect the spiritual well-being of the nation for generations to come" (p. 69).

If it now be asked what prevented Horace Scudder from showing more fully this gift of higher literature and led to his acquiescing, through life, in a comparatively secondary function, I can find but one explanation, and that a most interesting one to us in New England as illustrating the effect of immediate surroundings. His father, so far as I can ascertain, was one of those Congregationalists of the milder type who, while strict in their opinions, are led by a sunny temperament to be genial with their households and to allow them innocent amusements. The mother was a Congregationalist, firm but not severe in her opinions; but always controlled by that indomitable New England conscience of the older time which made her sacrifice herself to every call of charity and even to refuse, as tradition says, to have window curtains in her house, inasmuch as many around her could not even buy blankets. Add to this the fact that Boston was then a great missionary centre, that several prominent leaders in this cause were of the Scudder family and the house was a sort of headquarters for them, and that Horace Scudder's own elder brother, whose memoirs he wrote, went as a missionary to India, dying at his post. Speaking of his father's family in this memoir, he says of it, "In the conduct of the household, there was recognition of some more profound meaning in life than could find expression in mere enjoyment of living; while the presence of a real religious sentiment banished that counterfeit solemnity which would hang over innocent pleasure like a cloud" (Scudder's Life of David Coit Scudder, p. 4). By one bred in such an atmosphere of self-sacrifice, that quality may well be imbibed; it may even become a second nature, so that the instinctive demand for self-assertion may become secondary until a man ends in simply finding contentment in doing perfectly the appointed work of every day. If we hold as we should that it is character, not mere talent, which ennobles life, we may well feel that there is something not merely pardonable, but ennobling in such a habit of mind. Viewed in this light, his simple devotion to modest duty may well be to many of us rather a model than a thing to be criticised.

THOMAS WENTWORTH HIGGINSON.

JOSEPH HENRY THAYER.

JOSEPH HENRY THAYER was born in Boston, November 7, 1828. He graduated from Harvard in 1850, spent one year (1854–55) in the Harvard Divinity School, graduated from the Andover Theological Seminary in 1857, and was minister of the Crombie Street Church in Salem from 1859 to 1864; a part of this time, from September, 1862 to May, 1863, he served as Chaplain of the Fortieth Infantry Regiment of Massachusetts Volunteers. His career as teacher began in 1864, when he became Professor in the Andover Theological Seminary. Resigning his chair in 1882, he came to Cambridge, was Lecturer in the Harvard Divinity School for the year 1883–84, and in 1884, on the death of Ezra Abbot, succeeded him as Bussey Professor of New Testament Criticism and Interpretation; this position he held up to 1901. He was a member of the Harvard Corporation from 1877 to 1884. He was elected a Fellow of the American Academy of Arts and Sciences March 9, 1887, and, though not an active member, was always deeply interested in the work and fortunes of the Academy. Other societies to which he belonged are the Archaeological Institute of America, the American Oriental Society, and the Society of Biblical Literature. He received the degree of A.M. from Harvard, the degree of S.T.D. from Yale, Harvard, and Princeton, and the degree of Litt.D. from Dublin.

Dr. Thayer chose as his field of study the grammar and lexicography of the New Testament, and his distinguished services in this department have been universally recognized in Europe and America. He brought to his task wide learning, patience in investigation, minute accuracy in details, and critical acumen. His "Greek-English Lexicon of the New Testament" will long remain a manual for students and a monument of erudition and industry. The statement on the title-page, that it is a "revised and enlarged translation" of a German lexicon (Grimm's Wilke), hardly conveys a correct impression of its character. In fact the increase of the breadth and precision of definitions, the verification ot references, the addition of further references, and the construction of the New Testament text from the best manuscript authorities, entailed an amount of labor almost equivalent to the production of an independent lexicon. This breadth of research and exactitude of statement characterized all his scientific work — his articles in the Bible Dictionaries of Smith and Hastings, his translation of the New Testament Greek grammars of Winer and Buttmann, and his work on the Revised Version of the New Testament. To this last he gave many years of labor, as a member of the American Committee collaborating with the English Committee, and as principal editor of the American Version (the English Version with the changes introduced by the American Committee), which by agreement with the English Committee was published last year. His reading in his chosen field was wide and critical. He found time amid pressing professional and editorial duties to keep up with the enormous mass of New Testament literature that every year produced in Europe and America, and to form well-defined opinions as to its value.

He was not only singularly precise in details, he had a marked capacity for organization. He conceived large plans, and worked them out with patience and success. As early as 1864 he announced his purpose to translate Grimm — he completed the translation in Cambridge in 1885. It is mainly to him that we owe the establishment of the American School of Oriental Research in Jerusalem. Year after year he set forth the desirableness and the feasibility of such a school, and by unwearied exertions secured the indorsement of the Society of Biblical Literature and of the American Oriental Society, and the coöperation and financial support of a number of colleges, and of the Archaeological Institute of America. The school went into operation in the year 1900, and seems certain to give an impulse to Oriental study in this country, and to increase our knowledge of Oriental (especially Semitic) life, ancient and modern.

Dr. Thayer was an enthusiastic teacher, ever ready to give sympathy and time to his students. He was exacting in his demands, had small patience with negligence, and refused to lower his standards on any personal grounds, such as lack of previous preparation, or sickness; but he knew how to encourage and assist backward students, and to stimulate all by his own sense of the requirements of scholarship. He held firmly to the traditional New England standard of a minister's outfit, insisting on the necessity of Hebrew and Greek for the preacher. This point was the subject of debate in the Harvard Divinity Faculty for years, and the final decision made it possible for a student to take the degree of Bachelor of Divinity without a knowledge of Hebrew or Greek, the Faculty reserving the right, however, to pass on every individual case. In point of fact, it is true, in the past thirty years at least, only one man without Greek had received the degree, and he was a Japanese, from whom critical study of the Chinese classics was accepted in lieu of Greek. But Dr. Thayer, seeing that the Hebrew requirement was practically given up, believed there was danger that the Greek requirement would go the same way. Against this disposition to dispense with the original languages of the Bible he set his face steadfastly; he lost no opportunity to protest against what he regarded as a lamentable lowering of the standard of ministerial learning. When the question was finally decided, he, of course, accepted in good faith the action of the Faculty. Accept it cordially he could not: he was not an easy-going man, willing to fall in gracefully with the opinions of the majority; on the contrary, he took things very seriously, and, in matters that interested him, expressed himself pointedly. To the last he never spoke of the attitude of the Faculty toward the Hebrew and Greek requirements without a word of emphatic distrust and condemnation.

His thinking was notably clear-cut — he could not abide haziness. This trait, which is prominent in his scholarly work, appears also in his theological views. He was not intolerant of other men's opinions; he only held tenaciously to his own opinions, and claimed the right to define his position precisely. When he found, in 1882, that he could not subscribe the Andover Creed as it was then interpreted by the governing boards, he resigned his professorship in the Seminary — a sundering of old ties that gave him great pain. His own creed was distinct, yet catholic; he held firmly to certain principles and facts that he believed to be fundamental, and among these he gave a prominent place to scientific truth and personal experience.

Born and brought up in Boston, his traditions and training were those of New England, modified, however, by travel in foreign countries, and by a wide knowledge of men and things. He was a scholar and a man of affairs, a Puritan and a man of the world. In personal intercourse he showed an engaging frankness and friendliness, and the same devotion that appears in his scholarly undertakings manifested itself in his relations with his friends, for whom he was always ready to do the uttermost. He was fortunate in retaining his physical soundness and vigor up to a few months before his death. His erect carriage, alert step, and cheery manner gave him, even in his last years, a remarkably youthful appearance, and his bodily alertness was in keeping with his mental activity. His literary career extended over forty years, apparently without diminution of interest. He had the great happiness of seeing his main undertakings brought to a successful completion — the Greek lexicon, the revision of the English New Testament, and the establishment of the Jerusalem School.

At the close of the year 1900–01 he resigned his position in Harvard, and was made Professor Emeritus. The following summer he spent in Europe, and, returning to America, died in Cambridge after a short illness, November 26, having not long before passed his seventy-third birthday.

C. H. Toy.

JOHN FISKE.

On the 4th of July, 1901, John Fiske, philosopher, lecturer, and historian, died at Gloucester. On the morning of the fifth, hundreds of obituary notices of this distinguished man were read in the daily newspapers from Maine to Texas, from the Atlantic to the Pacific, and even across the water in the capital of Great Britain, by a public familiar, through his ministrations on the platform, with his giant form and ruddy countenance. These preliminary notices were followed at a later date by biographical and critical articles treating of his career, more finished in style and more analytical in character, in reviews and magazines; in weekly, monthly, and quarterly publications. Many of these were characterized by a familiarity with the details of Mr. Fiske's early life, unusual under such circumstances, but easily to be accounted for, since his biography had been partially written during his lifetime by two competent authors.

The first of these sketches, and in some respects the more complete of the two, was published by Edwin D. Mead, in the "Christian Register," in a series of papers occasioned by an address by Mr. Fiske before the Concord School of Philosophy in 1886. The second was by the late Horace E. Scudder, and appeared in a sort of introduction to one of the editions of "The War of Independence." The striking similarity of these biographies extends even to the language used, and indicates a common origin. It is certain that Mr. Fiske himself furnished the material for Mr. Mead's sketch, and there can be but little doubt that he did the same by Mr. Scudder. This will fully explain the points of coincidence, and will also give to both the authoritative character, which neither in words claims, of being practically autobiographical.

From these sketches we learn that on the 30th of March, 1842, there was born in Hartford, Connecticut, to Edmund Brewster Green and Mary Fiske Green, a son named by them Edmund Fiske Green, the greater part of whose child life was spent in Middletown, Connecticut. This Edmund Fiske Green was our John Fiske, his name having been changed during boyhood to that borne by his maternal grandfather.

At an early age the wonderful precocity of the child foreshadowed the marvellous attainments of his later years. His education was carried on first in the lower schools at Middletown and later at Stamford. Then he returned to Middletown and was placed in a private school, after which he went to Cambridge. Meantime he seems to have browsed in a library in the family mansion, and to a great degree taught himself much that is acquired with difficulty by persons of ordinary intellect even when assisted by the best of masters.

In his "Dutch and Quaker Colonies," Mr. Fiske says of James Logan: "He was an infant prodigy; at the age of twelve his attainments in Greek, Latin, and Hebrew had attracted much notice, and he afterward obtained distinction in modern languages, mathematics, physics, and natural history." The story of Logan's precocity is fairly eclipsed by Fiske's own record, but what he says of Logan shows us what his dispassionate judgment was as to his own childhood career. Fiske's biographers recapitulate his progress from year to year. It is needless to give in full detail the story of his prodigious acquisitions. Suffice it to say, that when six years old he began the study of Latin, and at the age of seven he amused himself by reading Cæsar, and found entertainment in such authors as Rollins and Josephus, and in the perusal of Goldsmith's Greece. The taste for history thus disclosed led him on to the works of other authors, and before he was eleven years old he had not only devoured many histories of divers peoples, but had from memory filled a quarto blank-book of sixty pages with chronological tables of events between 1000 B. C. and 1820 A. D. By the time he was thirteen he had read the greater part of the writings of about a dozen Latin authors, the work thus accomplished being in fact more than would be required in that line of a graduate at Harvard. Meantime, mathematics had not been neglected. Beginning with algebra at the age of eight, he had, by the time he was thirteen, gone through Euclid. plane and spherical trigonometry, surveying and navigation, and analytical geometry, and had made a good start in differential calculus.

Until he had mastered Latin sufficiently to make use of a Greek lexicon in which the meanings were given in Latin, he could not take up Greek, a lexicon of this description being the only one at his command. So trifling a discouragement as that did not long delay him. As soon as he felt competent to make use of the means at hand, he entered upon the study of Greek, and even before he obtained a modern lexicon he made considerable progress in his knowledge of the language. With the facility for study gained through the acquisition of a suitable key to the meanings of the words, he reached such proficiency, at the age of fifteen, that he could read Plato and Herodotus at sight.

He began his philosophical studies at the age of eleven with Locke's "Essay of the Understanding," and at fourteen himself wrote an essay on the habitability of the planets, in which he made the point that

Jupiter and Saturn, owing to their great size and slow refrigeration, are in a much earlier stage of development than Venus, Mars, and the Earth.

His taste for philology led him to attack the modern languages at the age of fifteen. He began with German; took up Spanish, in which he kept a diary; conquered French; and then attacked Italian. At the end of six months he had read the whole of Giuccardini, with parts of Ariosto and Petrarch. He then turned his attention to Portuguese.

We have followed him as a boy down to the time when he is about to leave home to go to Cambridge. What had college to offer him in the way of instruction? It is true that in much of the work he had performed he had been without a master, and of course there was much that he might still learn, but clearly the regular curriculum would practically be merely review work for him. Nevertheless, he looked forward with yearning to the time he should spend at Harvard, knowing that he could discover avenues in which the extraordinary mental activity which had impelled him along this wonderful path of study could find exercise.

We are told that until he was sixteen "he averaged twelve hours study daily for twelve months in the year." With the qualifications which will naturally suggest themselves this statement would seem probable, yet this boy who could cope with problems which present difficulties to the ordinary collegiate student, and whose learning at fifteen years of age far exceeded in many directions the standard which we should set for a cultivated man of maturity, found time for other occupations than delving in books. He taught himself to play upon the piano; participated in out-of-door sports, and took pleasure in walking, riding, and boating upon the Connecticut. He was much interested in church and oratorio music, was a member of the church choir, and his fondness for choral music, then developed, is said to have abided by him throughout life. We do not find evidence that works of fiction had much attraction for him as a boy. Later in life, we know that he was fond of novels, and that the characters portrayed by the masters of fiction were as real to him as the heroes with whom he met in history. His reading at this time must have been controlled by his surroundings, and what the libraries at his command furnished we can conjecture from the list of his acquirements. He gives us a hint of what there was at hand for him to read, in addition to what might be termed "useful books," in the following: "I remember," he says in one of his essays, "that when I was about ten years old, a favorite book with me was one entitled 'Criminal Trials of all Countries by a Member of the Philadelphia Bar.' I read it and read it, until forbidden to read such a grewsome work, and then I read it all the more."

He also tells us that he had access to a few scientific books owned by a strange character in Middletown, a sort of hermit; a dabbler in biology and geology, who led a solitary life; immersed, apparently, in studies and speculations concerning things far above his stage of cultivation. In the curious den — the library, workshop, and probably living room also — of this friendly recluse, among stuffed birds, mounted animals, strange creatures preserved in alcohol, specimens of fossil footprints from the Connecticut sandstone, and a few books on the subjects in which the owner was interested, the learned boy was admitted as a privileged guest, and here he talked with his strange companion concerning the surrounding objects, and from his host young Fiske borrowed such of the books as he cared to read.

The future author of "Outlines of a Cosmic Philosophy" and "Through Nature to God," was at this time a teacher in the Sunday-school and was active at prayer-meetings. What it cost him to reach the frame of mind which could put forth these works is substantially set forth in his Cosmic Philosophy. "A person," he says, "is educated in an environment of Presbyterian theology, accepting without question all the doctrines of Calvinism. By and by his environment enlarges. Facts in science or in history, methods of induction, canons of criticism present themselves to his mind as things irreconcilable with his old creed. Hence painful doubts, entailing efforts to escape by modifying the creed to suit new mental exigencies. Hence eager study and further enlargement of the environment, causing fresh disturbance of equilibrium and renewed doubt, resulting in further adaptation. And so the process continues, until, if the person in question be sufficiently earnest and sufficiently fortunate, the environment enlarges so far as to comprehend the most advanced science of the day, and the process of adaptation goes on until an approximate equilibrium is attained between the order of conception and the order of phenomena, and scepticism, having discharged its function, exists no longer, save in so far as it may be said to survive in the ingrained habit of weighing evidence and testing one's hypotheses." Elsewhere, and this time speaking in the first person singular, he refers to his early religious opinions as being based upon the fear of the "burning hell with which my childish imagination had been unwisely terrified."

He entered the sophomore class at Harvard in 1860 at the age of eighteen, and was graduated in 1863. His study of the modern languages, which as we have seen already comprehended nearly all those in use in Eastern Europe, was followed by an attack on the ancient tongues, Hebrew and Sanskrit; the former before he entered college, the latter after he reached Cambridge. While in college he is said to have worked from twelve to fifteen hours each day, during vacations as well as terms, his time being divided between comparative philology, ancient and modern history, and modern literature. His philological studies at this period comprehended the Icelandic, Gothic, Danish, Swedish, Dutch, and Roumanian tongues, and an attack on the Russian.

"He was but a lad of seventeen," says one of his eulogists, "when Darwin's great work appeared and aroused in him the zeal that determined his mental activity for more than a score of years." Mr. Mead, in his sketch, gives a long list of the authors whose books were read in prosecution of the study thus kindled, and adds that Fiske's training was that of a literary character even when he studied science. It is perhaps unnecessary to recapitulate the names of these writers. Every page of the Cosmic Philosophy bears evidence of Fiske's extensive researches at this time, and apart from the fact that he is avowedly preaching the doctrines of Spencer, it is clear that the scientific work upon which his reasoning is based does not claim to be original. He had not prosecuted laboratory researches in chemistry or biology; he had not gained his knowledge of astronomy at the observatory; he simply made skilful use of that which was done by others, never claiming for himself more than was his due.

While still an undergraduate he published two papers. The first, in 1861, was entitled Mr. Buckle's Fallacies; the second, in his senior year, was an essay on the Evolution of Language. The latter is said to have attracted the attention of Mr. Spencer, and thus laid the foundation for the intimate friendship which afterwards existed between Fiske and himself.

After his graduation, Mr. Fiske entered the Harvard Law School, and in 1865 took his degree of LL.B. In 1864, while a member of the Law School, he was admitted to the Suffolk bar, and in September of that year he married Abby Morgan Brooks of Petersham. After receiving his degree from the Law School, he opened an office in Boston and entered upon the practice of his profession. It is said that his prospects at the bar were fairly good, but he found professional work distasteful, and in about a year abandoned his office. In thus closing the door to a possible success in the profession which he had chosen, and taking upon himself the chance of supporting his family through the precarious channels of literary contributions to newspapers and magazines, there is a touch not only of the simple faith and optimism of youth, but of the Bohemian indifference to money-matters characteristic of the John Fiske whom we knew in later years. His confidence in himself was apparently justified by the result, for by some means or other, then and ever after, he was able to keep the wolf away from the door, and in an easy and comfortable style of living to support his family. It is evident, however, that at a later period he realized the boldness of the step then taken. "Literature as a profession," he said to an interviewer a few years ago, "looked as precarious in that generation as it does to you in this, but by the time I was four years out of college I managed by constant labor to earn enough by my pen to keep house and support a small family. . . . I wrote at first for the magazines and newspapers . . . upon science and philosophy and literature, and I sometimes wrote political leaders. . . . I earned more by my review work and historical and literary studies than I thought was possible when I stood upon the brink; but an intellectual revolution will be necessary before my experiences and that of my generation can be repeated by the young men who are looking towards literature to-day."

In 1868, he published a little book called "Tobacco and Alcohol. It does pay to Smoke — The Coming Man will Drink Wine." In this he criticised the hasty and unscientific writings of James Parton on the same subject, and as a reviewer states, "clearly developed" "the fundamental principle that everything in diet and medication depends on the dose."

He was appointed, in 1869, as Lecturer on Positive Philosophy at Harvard, which place he filled for two years. During the second half of 1869 he was also an Instructor of History, and from 1872 to 1879 he was Assistant Librarian. In 1885 he received the appointment as Professor of American History at Washington University, St. Louis. The duties of this position were fulfilled by the delivery there of occasional courses of lectures. During 1895–96 he was Lecturer at Harvard on the Campaigns of the Civil War west of the Alleghanies, and was also during 1896–97 Lecturer on Colonial Virginia and other Southern Colonies. He was elected an Overseer of Harvard in 1879, again in 1885, and a third time in 1899. He took his A.M. at Harvard in course, and in 1894 received the honorary degree of LL.D. The same year the University of Pennsylvania gave him the degree of Litt.D. He was a

Fellow of the Academy and a Member of the Massachusetts Historical Society.

The character of the thoughts which occupied his mind for nearly twenty years after his graduation is shown by the publications which rapidly followed. In 1872 we have "Myths and Myth Makers;" in 1874, "Outlines of Cosmic Philosophy;" in 1876, "The Unseen World and Other Essays;" in 1879, "Darwinism and Other Essays;" in 1884, "Excursions of an Evolutionist and the Destiny of Man viewed in the Light of his Origin;" and in 1885, "The Idea of God as affected by Modern Knowledge."

It will be noticed that during his career as an Instructor at Harvard his time was divided between philosophy and history. It is generally understood that a professorship there would have been grateful to him. In that event, if he had found a place in the philosophical department, we should probably never have had from his pen his contributions to American History. Two reasons have been assigned for his failure to secure this appointment, — each of which may have had weight. One was the attack upon Harvard by the religious press after the publication of his Cosmic Philosophy, and the other was his iconoclasm. Harvard had its idols. Of these Agassiz was one, and him the aggressive young evolutionist did not spare.

His position as Assistant Librarian was not worthy of him, nor was the work congenial. He therefore resigned from the library corps. He had previously, as we have seen, cut adrift from the law. In which of the two fields of literary labor, philosophy or history, for which he was specially fitted, was there the best chance for a young man with the growing responsibilities of a family on his hands to find the means of support? Such, to a person glancing at his career, would seem to have been the problem which was submitted to him when he severed his connection with Harvard. Yet, if we may accept his own statement, the wonderful amount of learning displayed in the pages of his Cosmic Philosophy was simply acquired as a formative process by way of preparation for his future historical work. "The absorbing and overmastering passion for the study of history," he says, "first led me to study evolution in order to obtain a correct method."

Professor Royce, whose analysis of Fiske's contributions to philosophical and religious discussions is very thorough and far reaching, gives him credit for being entirely in earnest in making this statement. "Any critic," he says, "who lacks his [Fiske's] range of reading must be easily tempted to regard his literary activities as too miscellaneous, and so must in some measure fail to understand in what degree he had his vast resources of imagination under control. Any judge whose human sympathies are narrower than his must find it a baffling task to look for the unity of interest, of opinion, and of ideal which in his mind bound together the many undertakings that marked his career, and the various stages of development through which his thought passed." The critic who had Fiske's range of reading is probably not to be found among us, but if we accept the proposition that he had historical work in view during all the time of this preliminary study in so many fields, still we can safely state that the precise form in which he proposed to put forth his labor was not determined until after he met John Richard Green in London, and talked with him about the "Short History of the English People" which Green was then planning. "I heard him," says Fiske, "telling about his scheme, and I thought it would be a very nice thing to do something of the same sort for American history."

This meeting with Green could not have taken place until 1879. It is plain, therefore, that if he relied upon his own capacity to support his family when he left the Harvard Library, it must have been through literary labor. He had been invited in 1878, while still connected with the Library, to deliver six lectures in the Old South Meeting House Course. This service was performed in 1879, and in June of the same year he was invited by Huxley to lecture before the University College in London. The acceptance of this invitation was fraught with great results. His lectures before the Harvard students were characterized by President Eliot: the first set, as "interesting and inspiring;" the later lectures, as "graphic and stimulating." The Old South lectures demonstrated his power with the public. The London lectures, before a radically different audience, corroborated this conclusion, and his visit brought him in friendly contact with the great body of distinguished men in England who were then busy investigating Darwin's "Theory of Development" and Spencer's "Doctrine of Evolution." Here, too, he met Green and had his mind turned definitely towards specific work in the field of American history. Circumstances thus determined that it was to be through lectures and writing American history that he was to earn his living, a determination which necessarily involved serious limitations as to the time which he could devote to research and which materially influenced the quality of his work.

His success as a lecturer in London led to his being called there again in 1880, when he delivered his three lectures on "American Political Ideas" at the Royal Institute. These he repeated at the Philosophical

Institute of Edinburgh and again in London. He was, indeed, invited to deliver them at the Sorbonne, but the invitation came too late.

His historical publications appeared in the following chronological order. The first was "American Political Ideas," in 1865; he was one of the editors of "Appleton's Cyclopædia of American Biography, 1887–1889" (his selection being in part due, undoubtedly, to his reputation as an historical student); "The Critical Period of American History," in 1888; "Washington and His Country," a book for the young, in 1889; "The War of Independence," a book of the same character, in 1889; "Beginnings of New England," in 1889; "Civil Government in the United States," a school book, in 1890; "American Revolution," in two volumes, in 1891; "Discovery of America," also in two volumes, in 1892; "History of the United States," for schools, 1894; "Old Virginia and Her Neighbors," in two volumes, in 1897; "Dutch and Quaker Colonies in America," in two volumes, in 1899.

Throwing out school books and volumes for the young, we have in the above series ten volumes, written as monographs, and published entirely without regard to their chronological succession, yet each intended as a contribution towards a complete history. Concerning this method of treatment he himself said: "I found myself dwelling upon special points, and insensibly without any volition on my part, it [the history] has been rather taking the shape of separate monographs. But I hope to go on that way until I cover the ground with these separate books." It is not unlikely that Parkman's example may have influenced him in this respect. His enthusiastic admiration for that great and popular writer of history shines forth from every page of the charming essay which he wrote on Parkman's life and works. The condensed form of "Beginnings of New England," containing as it does only the essentials for the development of the theme, suggests the process of digestion and careful elimination which characterizes Parkman's works. Besides the ten historical volumes mentioned, Fiske also published in 1900 a monograph on the "Mississippi Valley in the Civil War," and it is stated that a "History of the United States" will be issued in three volumes posthumously.

Mr. Fiske's works naturally divide themselves into two classes, and these divisions are practically chronological, thus representing the subjects to which his mind turned at different periods of his life. The brief period between the two, when he first took up lecturing and for a few years published only essays and magazine articles, indicates, in all probability, merely a time of study and preparation for future work.

Mr. Scudder says that the impulse toward American history was given by the preparation for the first course of Old South lectures, which were concerned especially with the Colonial period. When Fiske settled down deliberately to his life-work, he found that he could make the lectures subservient to his publications. He describes his method of doing this as follows: " I look it up or investigate it and then write an essay or lecture on the subject. That serves as a preliminary statement either of a large subject or of special points. It is a help to me to try to state the case. I never publish anything after this first statement, but generally keep it with me for, it may be, some years, and possibly return to it several times." While the general proposition is undoubtedly true that the preparation of historical work in tentative form, and the frequent recurrence to it under the stimulus of new studies and varying conditions of mind are of great assistance to the historian, still it must have been true that the great draft upon Mr. Fiske's time and strength occasioned by his lecture tours seriously affected the character of his work. "Fiske's lectures were a drag upon him," says Professor Hart, "because they were so good. Even big men have a limited stock of vitality, and he put into his lectures a power which ought to have gone into investigation. For years together, he appeared as a lecturer, more than a hundred times annually, besides numerous lectures abroad. So far as this work was a needed support for a man with a rising family, it was simply a misfortune; so far as it took the place of equally well paid literary work it was a mistake."

If we turn to the prefaces of his several publications we can there see how much of his time was occupied with these lectures, and we can also learn from the same source how familiar his form must have become to the lecture-going people of the entire country. Yet while his time was thus occupied, the old topics with which his name was associated earlier in life asserted their control over him, and found vent in essays or addresses upon occasions. In 1900 he published a volume entitled "A Century of Science;" following this came "Through Nature to God." The last address which he delivered, "Life Everlasting," was issued by his publishers after his death. This was made possible because Fiske rarely changed a word after he had once put his thoughts on paper.

His great fondness for music was not only evident to those who knew him well, but crops out in his books. He enjoyed the skilful performance of a symphony by an orchestra, and was also capable of interpreting it. To him there was not only harmony and rhythm and melody and the perfection of mechanical execution in the rendering of the music, but there was some underlying sentiment expressed by the composer which was conveyed to his mind. "When I look upon Parkman's noble life," he says, "I think of Mendelssohn's Chorus, 'He that shall endure to the end,' with its chaste and severely beautiful melody, and the calm, invincible faith which it expresses." Were it not that one cannot conceive how he found time to do it, it would occasion no surprise to learn that he composed a mass as well as several songs.

Mr. Fiske was a large man, and at the time of his death he was very corpulent. He enjoyed good health, borrowed no troubles, and was the type of a vigorous, happy human being, full of affection for his family and of good-will towards his fellow-men. He was absolutely independent and unconventional in his habits, both mentally and physically. The humorous description which he gave of his mode of life thoroughly illustrates this. "I always sit in a draught when I find one," he said, "wear the thinnest clothes I can find, winter and summer; catch cold once in three or four years, but not severely; and prefer to work in a cold room 55 to 60 degrees. Work the larger part of each twenty-four hours, and by day or night indifferently. Scarcely ever change a word once written; eat when hungry; rarely taste coffee or wine or smoke a cigar, but drink two or three quarts of beer a day and smoke a pipe all the time when at work; never experienced the feeling of disinclination for work and therefore never had to force work." The indifference which he expresses to night or day he brings forth in his essay on Chauncey Wright. "At two o'clock in the morning," he says, "he [Wright] would perhaps take his hat and saunter homeward with me by way of finishing the subject; but on reaching my gate a new suggestion would turn us back, — and so we would alternately escort each other home, perhaps a dozen times, until tired Nature asserted her rights, and the newly opened vistas of discussion were regretfully left unexplored." This quotation from Fiske's own works brings him before us as a willing disputant. It must, however, be taken with a grain of salt. If he discussed questions orally with persons from whom he differed in opinion, he selected his opponent. He could not under ordinary circumstances be dragged into an oral discussion.

As a lecturer, his manner of delivery was described as "simple, direct, sincere, and in a way appealing. He talked to his audience in a manner to make them feel that he was talking with them. He had a certain eloquence, which was engaging rather than stirring."

His reviewers concur in saying that his Cosmic Philosophy was more than a mere exposition of Spencer's doctrine. Fiske not only made clear that which was confused, but he added new propositions. Among these was his chapter on the prolongation of human infancy, a doctrine of great significance and a contribution of importance to the general argument. Its value was recognized by his fellow evolutionists, and he himself repeatedly referred to it in his works, claiming with evident pride it was his and his alone. Most of his biographers find in his later works devoted to religious topics a softened tone which they attribute to a change of views. He himself maintained that he was consistent. Perhaps he was affected and made less aggressive by the change of opinion then going on. There can be no doubt that the public of to-day can read the vigorous attacks of the young evolutionist upon traditional faiths and ingrained prejudices with less feeling than was provoked by them when they were first delivered. On the other hand Fiske may have been unconsciously borne upon the wave of scholarship whose "philosophical, idealistic trend," according to Professor Munsterberg, is "only swelling to-day, but whose highest point may be ten or twenty years hence." At any rate such a sentence as this — "I believe in the immortality of the soul, not in the sense in which I accept the demonstrable proofs of science, but as a supreme act of faith in the reasonableness of God's work" — could not have found place in the pages of Cosmic Philosophy. Fiske may not have changed his doctrines, but he certainly modified his manner of expressing them. He combined, according to Professor Royce, "the child's love of the unseen and mysterious with the modern sceptical student's scorn for superstition." These characteristics pervade both his early and late works.

Fiske quotes from Humboldt, "Nous avons consideré le style comme expression de caractère, comme reflet de l'interieur de l'homme." There can be no doubt that Fiske's publications reveal the personality of the author to the reader. We can easily see, through the lines, the image of the good-natured, straightforward, genial man, whose intellectual honesty leads him to say what he thinks, and whose sense of humor impels him to enliven with a jest even those pages which are devoted to the most abstruse subjects. The weary student of philosophy experiences relaxation from the strain upon his attention consequent upon his effort to follow the argument, when he is told that "the waves of motor energy which the human organism absorbs in whiffs of tobacco smoke are but a series of pulsations of transformed sunlight." The reader, perplexed by the abstruse speculations quoted from some learned philosopher, finds relief in the assertion that the troublesome paragraph is regarded by Mr. Fiske as "sheer nonsense," or that the whole of a certain system of philosophy is "made up of tawdry rhetoric, quite innocent of observation or induction." It is a satisfaction to learn that an objectionable Spaniard is a "green-eyed, pitiless, perfidious, old wretch." It is refreshing to have such positive opinions occasionally expressed concerning books, as the following: "For perverse ingenuity in creating difficulties where none exist, this book is a curiosity in the literature of psychology. From long staring at mare's nests the author had acquired a chronic twist in his vision." The most ardent protectionist could not fail to be amused at the vigorous attacks on his favorite doctrine with which the several volumes on American History are interspersed. Lovers of "Alice in Wonderland" will recognize upon the pages of Fiske's books their old acquaintance, the Jabberwok, and readers of the "Arabian Nights Entertainment" will find that several familiar genii do service by way of illustration or to make some point. Characters from Cervantes, Scott, Lowell, Dickens, and Charles Reade intrude themselves upon the reader, generally with the claim that they already know him and therefore the form of an introduction may be dispensed with. One thing is noticeable, and that is the absence of quotations from our favorite poets. "Hudibras" and "The Biglow Papers" attract him; the quaint attempts at verse of some of our early American writers evidently amuse him; but poetry as such does not appeal to him. On the other hand humor always does, and we find him gravely quoting Diedrich Knickerbocker, with the warning of course that he is dealing with fiction, but nevertheless accepting Irving's burlesque descriptions as representative of his conception of the persons therein characterized. The mention of large oysters in Virginia recalls to Fiske an anecdote of Thackeray, with which his reader is assumed to be familiar. "We remember Thackeray," he says, "when we encounter oysters so large that Basil Ringrose has to cut them into quarters." The detection of an error on the part of a famous writer leads to the following foot-note: "Aliquando dormitat bonus Homerus." No reader of the Discovery of America but will understand this. By such means, Fiske lures the reader on, and entices him over passages in his books which might otherwise prove dull. His simple, direct, and lucid style; his obvious purpose to deal honestly with facts; his pronounced opinions upon points not free from doubt in the minds of many students; his discrimination in sifting out the events which are significant; his sagacity in measuring the proportion of their relative importance; even his open advocacy of those whose career appealed to him no matter what the opinion of others, all combined to secure the approval of a large reading public, and thus earned for him the honorable title which has been conferred upon him since his death, "Popularizer of useful knowledge." — In its restricted application to the field of history, this epithet was adopted by Colonel Higginson in some remarks before the Massachusetts Historical Society in February, and was repeated by him with emphatic recognition of the honor thereby intended to be conferred, at the March meeting of the Academy.

Fiske's whole life was, in the words of Mead, "a noble illustration of resolute intellectual integrity." "Only another John Fiske," says Professor Royce, "if such a being were possible — a man as widely read as he was, and with a soul as sweetly humane in sentiment, as clear in vision, as free from pettiness, as childlike in faith in what it had once accepted, and yet as keen in critical intelligence regarding what it rejected as was his soul — only such a man could estimate adequately Fiske's beneficent life-work and his manifold mental accomplishments."

In conclusion let me say, that in accepting the appointment to write Mr. Fiske's memoir, I did so with the full consciousness of my unfitness for the task, if knowledge of the subjects discussed in what the London "Times" terms the bewildering variety of his publications, were to be made the basis of one's qualifications. To find a memorialist up to this standard might be difficult even in the Academy. It seemed to me, therefore, that all that could be expected of any person would be to throw upon the screen a composite picture, made up from contributions by Fiske himself and by the various writers who have furnished biographies of his life and criticisms of his works. This is what I have striven to do.

ANDREW MCFARLAND DAVIS.

JAMES BRADLEY THAYER.

A Massachusetts man by ancestry, birth, and training, James Bradley Thayer, our late vice-president, represented by the simplicity of his life, his scholarly tastes and achievements, his practical good sense, his public spirit, and generous sympathies, the highest type of the New Englander. He was born January 15, 1831, in Haverhill, where his father exercised a wide and wholesome influence as a journalist. He entered Harvard College at the age of seventeen, having fitted himself for the examinations after his fourteenth year, like his brother before him, without the aid of a teacher. He ranked high in his class and was the class orator. After an interval spent in teaching he entered the Harvard Law School in 1854. Here he gave proof of his literary and legal ability by winning, in his second year, the class prize for an essay on the "Law of Eminent Domain." It is interesting to note that his first legal essay, which was printed at once in the leading law periodical of the day, was upon a topic in Constitutional Law, one of the two branches of law in which he afterward acquired his great distinction.

An incident in his career at the Law School exhibited the character of the man. The Harvard Corporation had appointed Judge E. G. Loring to a professorship in the Law School. But the Board of Overseers, on account of the Judge's decision, sending back to slavery the fugitive slave Anthony Burns, refused to confirm this appointment. The Southerners and their sympathizers in the Law School moved in their parliament a vote of censure upon the Overseers. The motion was opposed on various parliamentary grounds, but finally the majority determined to put the vote through in disregard of orderly procedure, and the Clerk was directed to call the roll of yeas and nays. Mr. Thayer, who was Clerk, rose, and in a quiet but impressive manner declined to be a party to this unparliamentary action, resigned his office, and walked away from his desk. The motion was ultimately carried, but Mr. Thayer's calm, dignified rebuke of their proceedings robbed the victory of well-nigh all its glory even in the minds of the victors.

For nearly twenty years Mr. Thayer was active in the practice of his profession, residing during the greater part of this time in Milton, where he was conspicuous for his public-spirited interest in all that affected the welfare of the town.

In 1874 he was appointed a professor in the Harvard Law School. He had previously declined the offer of a professorship in the English Department of the College. Although his rare gift for thoughtful, graceful, and effective writing could not have failed to make him highly successful as a professor of English, his decision not to give up his chosen profession was doubtless a wise one. Certainly it was a fortunate one for the Law School and the law.

Wherever the Harvard Law School is known, he has been recognized for many years as one of its chief ornaments. When, in 1900, the Association of American Law Schools was formed, it was taken for granted by all the delegates that Professor Thayer was to be its first President. No one can measure his great influence upon the thousands of his pupils. While at the School they had a profound respect for his character and ability, and they realized that they were sitting at the feet of a master of his subjects. In their after life his precept and example have been, and will continue to be, a constant stimulus to genuine, thorough and finished work, and a constant safeguard against hasty generalization or dogmatic assertion. His quick sympathy, his unfailing readiness to assist the learner, out of the class-room as well as in it, and his attractive personality, gave him an exceptionally strong hold upon the affections of the young men. Their attitude towards him is well expressed in a letter from a recent graduate of the School, who describes him as "one of the best known, best liked, and strongest of the Law Professors."

During the early years of his service he lectured on a variety of legal topics, but Evidence and Constitutional Law were especially congenial to him, and in the end he devoted himself exclusively to these two subjects, in each of which he had prepared for the use of his classes an excellent collection of cases. Evidence was an admirable field for his powers of historical research and analytical judgment. He recognized that our artificial rules of evidence were the natural outgrowth of trial by jury, and could only be explained by tracing carefully the development of that institution in England. The results of his work appeared in his "Preliminary Treatise on the Law of Evidence," a worthy companion of the masterly "Origin of the Jury," by the distinguished German, Professor Brunner. His book gave him an immediate reputation, not only in this country, but in England, as a legal historian and jurist of the first rank. An eminent English lawyer, in reviewing it, described it as "a book which goes to the root of the subject more thoroughly than any other text-book in existence."

Although he published no treatise upon Constitutional Law, he achieved, by his essays, by his collection of Cases, and by his teaching, a reputation in that subject hardly second to his rank in Evidence. To the few who knew of it, President McKinley's wish to make Professor Thayer a member of the present Philippines Commission seemed a natural and most fitting recognition of his eminence as a constitutional lawyer, and if he had deemed it wise to accept the position offered to him, no one can doubt that the appointment would have commanded universal approval.

It is greatly to be deplored that he was not permitted to give to the world the additional contributions to legal literature, which the vigor of his powers and his known purposes led us to expect from him. That he did not realize these purposes earlier was due to his very virtues. His wide range of interests, his constant service in helping other writers in their work, and above all his passion for perfection in his own work, explain why the message he might have given remains incomplete. The pathetic interest of high hopes unfulfilled attaches to a memorandum found among his papers, and written last September.

> "Sept. 15
> For next year.
>
> Have a single plan to put through. Without that the small everyday matters eat up all the time. They easily may, for they can be done either well enough or *perfectly*.
>
> That plan must be the 2nd volume of Evidence.
>
> For the year following, a small Vol. on Const. Law.
>
> For the time following that, the works, writings and life of Marshall —
> *and then an End.*"

The relations of the law professors are probably closer than those of any other department of the University. No one who has not known, as his colleagues have known, the charm of his daily presence and conversation, and the delightful quality of his vacation letters, can appreciate the deep and abiding sense of the irreparable loss they have suffered in the death of Professor Thayer.

In our great grief we find our chief comfort in the thought of his simple and beautiful life, greatly blessed in his home and family, rich in choice friendships, crowned with the distinction that comes only to the possessor of great natural gifts nobly used, full of happiness to himself, and giving in abundant measure happiness and inspiration to others.

JAMES BARR AMES.

There have been no resignations during the year. One Resident Fellow, formerly an Associate, having again made his residence outside of Massachusetts, has been restored to Associate Fellowship.

New members elected are: Resident Fellows, 9; Associate Fellows, 3; Foreign Honorary Members, 5.

The roll of the Academy therefore now includes 200 Resident Fellows, 100 Associate Fellows, and 71 Foreign Honorary Members.*

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* By the death of a Resident Fellow, and by the election of new members at the annual meeting of May 14, 1902, the roll stands at date of publication 199 Resident Fellows, 100 Associate Fellows, and 73 Foreign Honorary Members.

American Academy of Arts and Sciences.

OFFICERS AND COMMITTEES FOR 1902-03.

PRESIDENT.

Alexander Agassiz.

VICE-PRESIDENT.

Class I	Class II.	Class III.
John Trowbridge,	Henry P. Walcott,	John C. Gray.

CORRESPONDING SECRETARY.

William M. Davis.

RECORDING SECRETARY.

William Watson.

TREASURER.

Francis Blake.

LIBRARIAN.

A. Lawrence Rotch.

COUNCILLORS.

Class I.	Class II.	Class III.
Charles R. Sanger,	Theobald Smith, *Terms expire* 1903.	A. Lawrence Lowell,
George F. Swain,	Robert De C. Ward, *Terms expire* 1904.	Denman W. Ross,
Arthur G. Webster,	Edward L. Mark, *Terms expire* 1905.	Arlo Bates,

COMMITTEE OF FINANCE.

Alexander Agassiz, Francis Blake, Eliot C. Clarke.

RUMFORD COMMITTEE.

Erasmus D. Leavitt, Edward C. Pickering, Charles R. Cross,
Amos E. Dolbear, Arthur G. Webster, Theodore W. Richards,
Elihu Thomson.

C. M. WARREN COMMITTEE.

Charles L. Jackson, Samuel Cabot, Henry B. Hill,
Leonard P. Kinnicutt, Arthur M. Comey, Robert H. Richards,
Henry P. Talbot.

COMMITTEE OF PUBLICATION.

Seth C. Chandler, of Class I., Edward L. Mark, of Class II.,
Crawford H. Toy, of Class III.

COMMITTEE ON THE LIBRARY.

A. Lawrence Rotch,
William F. Osgood, of Class I., Samuel Henshaw, of Class II.,
Henry W. Haynes, of Class III.

AUDITING COMMITTEE.

Henry G. Denny, William L. Richardson.

LIST

OF THE

FELLOWS AND FOREIGN HONORARY MEMBERS.

(Corrected to May 14, 1902.)

RESIDENT FELLOWS. — 199.

(Number limited to two hundred.)

Class I. — *Mathematical and Physical Sciences.* — 82.

Section I. — 20.

Mathematics and Astronomy.

Solon I. Bailey,	Cambridge.
Maxime Bôcher,	Cambridge.
William E. Byerly,	Cambridge.
Seth C. Chandler,	Cambridge.
Gustavus Hay,	Boston.
Percival Lowell,	Boston.
Henry Mitchell,	Nantucket.
William F. Osgood,	Cambridge.
James Mills Peirce,	Cambridge.
Edward C. Pickering,	Cambridge.
William H. Pickering,	Cambridge.
Henry S. Pritchett,	Boston.
John Ritchie, Jr.,	Roxbury.
John D. Runkle,	Cambridge.
Edwin F. Sawyer,	Brighton.
Arthur Searle,	Cambridge.
William E. Story,	Worcester.
Henry Taber,	Worcester.
O. C. Wendell,	Cambridge.
P. S. Yendell,	Dorchester.

Section II. — 23.

Physics.

A. Graham Bell,	Washington, D.C.
Clarence J. Blake,	Boston.
Francis Blake,	Weston.
Harry E. Clifford,	Newton.
Charles R. Cross,	Brookline.
Amos E. Dolbear,	Somerville.
A. W. Duff,	Worcester.
H. M. Goodwin,	Roxbury.
Edwin H. Hall,	Cambridge.
Hammond V. Hayes,	Cambridge.
William L. Hooper,	Somerville.
William W. Jacques,	Newton.
Frank A. Laws,	Boston.
Henry Lefavour,	Williamstown.
Theodore Lyman,	Brookline.
Benjamin O. Peirce,	Cambridge.
A. Lawrence Rotch,	Boston.
Wallace C. Sabine,	Boston.
John S. Stone,	Boston.
Elihu Thomson,	Swampscott.

John Trowbridge,	Cambridge.
A. G. Webster,	Worcester.
Robert W. Willson,	Cambridge.

SECTION III. — 22.

Chemistry.

Samuel Cabot,	Boston.
Arthur M. Comey,	Cambridge.
James M. Crafts,	Boston.
Charles W. Eliot,	Cambridge.
Henry B. Hill,	Cambridge.
Charles L. Jackson,	Cambridge.
Walter L. Jennings,	Worcester.
Leonard P. Kinnicutt,	Worcester.
Charles F. Mabery,	Cleveland, O.
Arthur Michael,	Boston.
George D. Moore,	Worcester.
Charles E. Munroe,	Wash'gton, D.C.
John U. Nef,	Chicago, Ill.
Arthur A. Noyes,	Boston.
Robert H. Richards,	JamaicaPlain.
Theodore W. Richards,	Cambridge.
Charles R. Sanger,	Cambridge.
Stephen P. Sharples,	Cambridge.
Francis H. Storer,	Boston.
Henry P. Talbot,	Newton.
Charles H. Wing,	Ledger, N. C.
Edward S. Wood,	Boston.

SECTION IV. — 17.

Technology and Engineering.

Eliot C. Clarke,	Boston.
Heinrich O. Hofman,	Jamaica Plain.
Ira N. Hollis,	Cambridge.
L. J. Johnson,	Cambridge.
Gaetano Lanza,	Boston.
E. D. Leavitt,	Cambridge.
William R. Livermore,	Boston.
Hiram F. Mills,	Lowell.
Cecil H. Peabody,	Brookline.
Alfred P. Rockwell,	Manchester.
Andrew H. Russell,	Manilla.
Peter Schwamb,	Arlington.
H. L. Smyth,	Cambridge.
Charles S. Storrow,	Boston.
George F. Swain,	Boston.
William Watson,	Boston.
Morrill Wyman,	Cambridge.

CLASS II. — *Natural and Physiological Sciences.* — 66

SECTION I. — 14.

Geology, Mineralogy, and Physics of the Globe.

H. H. Clayton,	Milton.
Algernon Coolidge,	Boston.
William O. Crosby,	Jamaica Plain.
William M. Davis,	Cambridge.
Benj. K. Emerson,	Amherst.
O. W. Huntington,	Newport, R. I.
Robert T. Jackson,	Cambridge.
T. A. Jaggar, Jr.,	Cambridge.
William H. Niles,	Cambridge.
John E. Pillsbury,	Boston.
Nathaniel S. Shaler,	Cambridge.
Robert DeC. Ward,	Cambridge.
John E. Wolff,	Cambridge.
J. B. Woodworth,	Cambridge.

SECTION II. — 11.

Botany.

F. S. Collins,	Malden.
Geo. E. Davenport,	Medford.
William G. Farlow,	Cambridge.
Charles E. Faxon,	Jamaica Plain.
Merritt L. Fernald,	Cambridge.
George L. Goodale,	Cambridge.
John G. Jack,	Jamaica Plain.
B. L. Robinson,	Cambridge.
Charles S. Sargent,	Brookline.
Arthur B. Seymour,	Cambridge.
Roland Thaxter,	Cambridge

SECTION III. — 25.

Zoölogy and Physiology.

Alexander Agassiz,	Cambridge.
Robert Amory,	Boston.

James M. Barnard,	Milton.
Henry P. Bowditch,	Jamaica Plain.
William Brewster,	Cambridge.
Louis Cabot,	Brookline.
William E. Castle,	Cambridge.
Samuel F. Clarke,	Williamstown.
W. T. Councilman,	Boston.
Charles B. Davenport,	Chicago, Ill.
Harold C. Ernst,	Jamaica Plain.
Edward G. Gardiner,	Boston.
Samuel Henshaw,	Cambridge.
Theodore Hough,	Boston.
John S. Kingsley,	Somerville.
Edward L. Mark,	Cambridge.
Charles S. Minot,	Milton.
Edward S. Morse,	Salem.
George H. Parker,	Cambridge.
William T. Porter,	Boston.
James J. Putnam,	Boston.
Samuel H. Scudder,	Cambridge.
William T. Sedgwick,	Boston.
James C. White,	Boston.
William M. Woodworth,	Cambridge.

SECTION IV. — 16.

Medicine and Surgery.

Samuel L. Abbot,	Boston.
Edward H. Bradford,	Boston.
Arthur T. Cabot,	Boston.
David W. Cheever,	Boston.
Frank W. Draper,	Boston.
Thomas Dwight,	Boston.
Reginald H. Fitz,	Boston.
Charles F. Folsom,	Boston.
Frederick I. Knight,	Boston.
Samuel J. Mixter,	Boston.
W. L. Richardson,	Boston.
Theobald Smith,	Jamaica Plain.
O. F. Wadsworth,	Boston.
Henry P. Walcott,	Cambridge.
John C. Warren,	Boston.
Francis H. Williams,	Boston.

CLASS III. — *Moral and Political Sciences.* — 51.

SECTION I. — 9.

Philosophy and Jurisprudence.

James B. Ames,	Cambridge.
Horace Gray,	Boston.
John C. Gray,	Boston.
G. Stanley Hall,	Worcester.
Geo. F. Hoar,	Worcester.
Francis C. Lowell,	Boston.
Josiah Royce,	Cambridge.
Jeremiah Smith,	Cambridge.
Edward H. Strobel,	Cambridge.

SECTION II. — 21.

Philology and Archæology.

William S. Appleton,	Boston.
Charles P. Bowditch,	Jamaica Plain.
Lucien Carr,	Cambridge.
Franklin Carter,	Williamstown.
Joseph T. Clarke,	Boston.
Henry G. Denny,	Roxbury.
William Everett,	Quincy.
J. W. Fewkes,	Washington, D.C.
William W. Goodwin,	Cambridge.
Henry W. Haynes,	Boston.
Charles R. Lanman,	Cambridge.
David G. Lyon,	Cambridge.
Morris H. Morgan,	Cambridge.
Bennett H. Nash,	Boston.
Frederick W. Putnam,	Cambridge.
Edward Robinson,	Boston.
F. B. Stephenson,	Boston.
Crawford H. Toy,	Cambridge.
John W. White,	Cambridge.
John H. Wright,	Cambridge.
Edward J. Young,	Waltham.

SECTION III. — 10.

Political Economy and History.

Charles F. Adams,	Lincoln.
Edward Atkinson,	Brookline.
Andrew McF. Davis,	Cambridge.
Ephraim Emerton,	Cambridge.

A. C. Goodell,	Salem.
Henry C. Lodge,	Nahant.
A. Lawrence Lowell,	Boston.
James F. Rhodes,	Boston.
Charles C. Smith,	Boston.
F. W. Taussig,	Cambridge.

SECTION IV. — 11.

Literature and the Fine Arts.

Francis Bartlett,	Boston.
John Bartlett,	Cambridge.
Arlo Bates,	Boston.
George S. Boutwell,	Groton.
J. Elliot Cabot,	Brookline.
T. W. Higginson,	Cambridge.
George L. Kittredge,	Cambridge.
Charles G. Loring,	Boston.
Charles Eliot Norton,	Cambridge.
Denman W. Ross,	Cambridge.
Barrett Wendell,	Boston.

ASSOCIATE FELLOWS. — 100.

(Number limited to one hundred. Elected as vacancies occur.)

Class I. — *Mathematical and Physical Sciences.* — 38.

Section I. — 14.

Mathematics and Astronomy.

Edward E. Barnard,	Williams Bay, Wis.
S. W. Burnham,	Chicago.
George Davidson,	San Francisco.
Fabian Franklin,	Baltimore.
Asaph Hall,	Goshen, Conn.
George W. Hill,	W. Nyack, N.Y.
E. S. Holden,	New York.
Emory McClintock,	Morristown, N.J.
E. H. Moore,	Chicago.
Simon Newcomb,	Washington.
Charles L. Poor,	New York.
George M. Searle,	Washington.
J. N. Stockwell,	Cleveland, O.
Chas. A. Young,	Princeton, N. J.

Section II. — 8.

Physics.

Carl Barus,	Providence, R.I.
J. Willard Gibbs,	New Haven.
G. E. Hale,	Williams Bay, Wis.
S. P. Langley,	Washington.
T. C. Mendenhall,	
A. A. Michelson,	Chicago.
Ogden N. Rood,	New York.
E. L. Nichols,	Ithaca, N. Y.

Section III. — 8.

Chemistry.

T. M. Drown,	So. Bethlehem, Pa.
Wolcott Gibbs,	Newport, R.I.
Frank A. Gooch,	New Haven.
S. W. Johnson,	New Haven.
J. W. Mallet,	Charlottesville, Va.
E. W. Morley,	Cleveland, O.
J. M. Ordway,	New Orleans.
Ira Remsen,	Baltimore.

Section IV. — 8.

Technology and Engineering.

Henry L. Abbot,	Cambridge.
Cyrus B. Comstock,	New York.
W. P. Craighill,	Charlestown, W. Va.
John Fritz,	Bethlehem, Pa.
F. R. Hutton,	New York.
George S. Morison,	New York.
William Sellers,	Edge Moor, Del.
Robt. S. Woodward,	New York.

Class II. — *Natural and Physiological Sciences.* — 33.

Section I. — 12.

Geology, Mineralogy, and Physics of the Globe.

Cleveland Abbe,	Washington.
George J. Brush,	New Haven.
T. C. Chamberlin,	Chicago.
Edward S. Dana,	New Haven.
Walter G. Davis,	Cordova, Arg.
G. K. Gilbert,	Washington.
J. Peter Lesley,	Milton, Mass.
S. L. Penfield,	New Haven.
J. W. Powell,	Washington.
R. Pumpelly,	Newport, R.I.
A. R. C. Selwyn,	Vancouver.
Charles D. Walcott,	Washington.

SECTION II. — 6.

Botany.

L. H. Bailey,	Ithaca, N. Y.
D. H. Campbell,	Palo Alto, Cal.
J. M. Coulter,	Chicago.
C. G. Pringle,	Charlotte, Vt.
John D. Smith,	Baltimore.
W. Trelease,	St. Louis.

SECTION III. — 9.

Zoölogy and Physiology.

Joel A. Allen,	New York.
W. K. Brooks,	Lake Roland, Md.
F. P. Mall,	Baltimore.
S. Weir Mitchell,	Philadelphia.
H. F. Osborn,	New York.
A. S. Packard,	Providence, R.I.
A. E. Verrill,	New Haven.
C. O. Whitman,	Chicago.
E. B. Wilson,	New York.

SECTION IV. — 6.

Medicine and Surgery.

John S. Billings,	New York.
W. S. Halsted,	Baltimore.
W. W. Keen,	Philadelphia.
William Osler,	Baltimore.
Wm. H. Welch,	Baltimore.
H. C. Wood,	Philadelphia.

CLASS III. — *Moral and Political Sciences.* — 29.

SECTION I. — 7.

Philosophy and Jurisprudence.

James C. Carter,	New York.
Joseph H. Choate,	New York.
Melville W. Fuller,	Washington.
William W. Howe,	New Orleans.
Charles S. Peirce,	Milford, Pa.
G. W. Pepper,	Philadelphia.
T. R. Pynchon,	Hartford, Conn.

SECTION II. — 7.

Philology and Archæology.

Timothy Dwight,	New Haven.
B. L. Gildersleeve,	Baltimore.
D. C. Gilman,	Baltimore.
T. R. Lounsbury,	New Haven.
Rufus B. Richardson,	Athens.
Thomas D. Seymour,	New Haven.
A. D. White,	Ithaca, N.Y.

SECTION III. — 6.

Political Economy and History.

Henry Adams,	Washington.
G. P. Fisher,	New Haven.
H. E. von Holst,	Chicago.
Henry C. Lea,	Philadelphia.
H. Morse Stephens,	Ithaca.
W. G. Sumner,	New Haven.

SECTION IV. — 9.

Literature and the Fine Arts.

James B. Angell,	Ann Arbor, Mich.
L. P. di Cesnola,	New York.
H. H. Furness,	Wallingford, Pa.
R. S. Greenough,	Florence.
Herbert Putnam,	Washington.
Augustus St. Gaudens,	Windsor, Vt.
John S. Sargent,	London.
E. C. Stedman,	Bronxville, N. Y.
W. R. Ware,	New York.

FOREIGN HONORARY MEMBERS.—73.

(Number limited to seventy-five. Elected as vacancies occur.)

Class I.—*Mathematical and Physical Sciences.*—23.

Section I.—7.

Mathematics and Astronomy.

Arthur Auwers,	Berlin.
George H. Darwin,	Cambridge.
H. A. E. A. Faye,	Paris.
Sir William Huggins,	London.
H. Poincaré,	Paris.
Otto Struve,	Karlsruhe.
H. C. Vogel,	Potsdam.

Section II.—5.

Physics.

Ludwig Boltzmann,	Vienna.
Oliver Heaviside,	Newton Abbot.
F. Kohlrausch,	Berlin.
Lord Rayleigh,	Witham.
Sir G. G. Stokes, Bart.,	Cambridge.

Section III.—6.

Chemistry.

Adolf Baeyer,	Munich.
Marcellin Berthelot,	Paris.
J. H. van't Hoff,	Berlin.
D. Mendeleeff,	St. Petersburg.
Sir H. E. Roscoe,	London.
Julius Thomsen,	Copenhagen.

Section IV.—5.

Technology and Engineering.

Sir Benjamin Baker,	London.
Lord Kelvin,	Largs.
Maurice Lévy,	Paris.
H. Müller-Breslau,	Berlin.
W. Cawthorne Unwin,	London.

Class II.—*Natural and Physiological Sciences.*—27.

Section I.—7.

Geology, Mineralogy, and Physics of the Globe.

Sir Archibald Geikie,	London.
Julius Hann,	Vienna.
Albert Heim,	Zurich.
Sir John Murray,	Edinburgh.
Freih. v. Richthofen,	Berlin.
Henry C. Sorby,	Sheffield.
Heinrich Wild,	Zurich.

Section II.—6.

Botany.

E. Bornet,	Paris.
A. Engler,	Berlin.
Sir Joseph D. Hooker,	Sunningdale.
W. Pfeffer,	Leipsic.
H. Graf zu Solms-Laubach,	Strassburg.
Eduard Strasburger,	Bonn.

SECTION III. — 7.

Zoölogy and Physiology.

Sir Michael Foster,	Cambridge.
Carl Gegenbaur,	Heidelberg.
Ludimar Hermann,	Königsberg.
A. von Kölliker,	Würzburg.
H. Kronecker,	Bern.
E. Ray Lankester,	London.
Elias Metschnikoff,	Paris.

SECTION IV. — 7.

Medicine and Surgery.

Sir T. L. Brunton,	London.
A. Celli,	Rome.
V. A. H. Horsley,	London.
R. Koch,	Berlin.
Lord Lister,	London.
F. v. Recklinghausen,	Strassburg.
Rudolf Virchow,	Berlin.

CLASS III. — *Moral and Political Sciences.* — 23.

SECTION I. — 5.

Philosophy and Jurisprudence.

A. J. Balfour,	Prestonkirk.
Heinrich Brunner,	Berlin.
A. V. Dicey,	Oxford.
F. W. Maitland,	Cambridge.
Sir Frederick Pollock, Bart.,	London.

SECTION II. — 7.

Philology and Archæology.

Ingram Bywater,	Oxford.
F. Delitzsch,	Berlin.
W. Dörpfeld,	Athens.
Sir John Evans,	Hemel Hempstead.
H. Jackson,	Cambridge.
J. W. A. Kirchhoff,	Berlin.
G. C. C. Maspero,	Paris.

SECTION III. — 4.

Political Economy and History.

James Bryce,	London.
Theodor Mommsen,	Berlin.
Sir G. O. Trevelyan, Bart.,	London.
W. E. H. Lecky,	London.

SECTION IV. — 7.

Literature and the Fine Arts.

E. de Amicis,	Florence.
Georg Brandes,	Copenhagen.
F. Brunetière,	Paris.
Jean Léon Gérôme,	Paris.
Rudyard Kipling,	Rottingdean.
G. Paris,	Paris.
Leslie Stephen,	London.

STATUTES AND STANDING VOTES.

STATUTES.

Adopted May 30, 1854: *amended September* 8, 1857, *November* 12, 1862, *May* 24, 1864, *November* 9, 1870, *May* 27, 1873, *January* 26, 1876, *June* 16, 1886, *October* 8, 1890, *January* 11 *and May* 10, 1893, *May* 9 *and October* 10, 1894, *March* 13, April 10 *and May* 8, 1895, *May* 8, 1901, *and January* 8, 1902.

CHAPTER I.

OF FELLOWS AND FOREIGN HONORARY MEMBERS.

1. The Academy consists of Resident Fellows, Associate Fellows and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, viz.: Class I. The Mathematical and Physical Sciences; — Class II. The Natural and Physiological Sciences; — Class III. The Moral and Political Sciences. Each Class is divided into four Sections, viz.: Class I., Section 1. Mathematics and Astronomy; — Section 2. Physics; — Section 3. Chemistry; — Section 4. Technology and Engineering. Class II., Section 1. Geology, Mineralogy, and Physics of the Globe; — Section 2. Botany; — Section 3. Zoölogy and Physiology; — Section 4. Medicine and Surgery. Class III., Section 1. Philosophy and Jurisprudence: — Section 2. Philology and Archæology; — Section 3. Political Economy and History; — Section 4. Literature and the Fine Arts.

2. The number of Resident Fellows shall not exceed two hundred. Only residents in the Commonwealth of Massachusetts shall be eligible to election as Resident Fellows, but resident fellowship may be retained after removal from the Commonwealth. Each Resident Fellow shall pay an admission fee of ten dollars and such annual assessment, not exceeding ten dollars, as shall be voted by the Academy at each annual

meeting. Resident Fellows only may vote at the meetings of the Academy.

3. The number of Associate Fellows shall not exceed one hundred, of whom there shall not be more than forty in either of the three classes of the Academy. Associate Fellows shall be chosen from persons residing outside of the Commonwealth of Massachusetts. They shall not be liable to the payment of any fees or annual dues, but on removing within the Commonwealth they may be transferred by the Council to resident fellowship as vacancies there occur.

4. The number of Foreign Honorary Members shall not exceed seventy-five; and they shall be chosen from among persons most eminent in foreign countries for their discoveries and attainments in either of the three departments of knowledge above enumerated. There shall not be more than thirty Foreign Members in either of these departments.

CHAPTER II.

OF OFFICERS.

1. There shall be a President, three Vice-Presidents, one for each Class, a Corresponding Secretary, a Recording Secretary, a Treasurer, and a Librarian, which officers shall be annually elected, by ballot, at the Annual Meeting, on the second Wednesday in May.

2. At the Annual Meeting of 1901, nine Councillors shall be elected by ballot, one from each Class of the Academy to serve for one year, one from each Class for two years, and one from each Class for three years; and at annual meetings thereafter three Councillors shall be elected in the same manner, one from each Class, to serve for three years; but the same Fellow shall not be eligible for two successive terms. The nine Councillors, with the President, the three Vice-Presidents, the two Secretaries, the Treasurer, and the Librarian, shall constitute the Council. Five members shall constitute a quorum. It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections. With the consent of the Fellow interested, they shall have power to make transfers between the several Sections of the same Class, reporting their action to the Academy.

3. If any office shall become vacant during the year, the vacancy shall be filled by a new election, and at the next stated meeting, or at a meeting called for this purpose.

CHAPTER III.

OF NOMINATIONS OF OFFICERS.

1. At the stated meeting in March, the President shall appoint from the next retiring Councillors a Nominating Committee of three Fellows, one for each class.

2. It shall be the duty of this Nominating Committee to prepare a list of candidates for the offices of President, Vice-Presidents, Corresponding Secretary, Recording Secretary, Treasurer, Librarian, Councillors, and the Standing Committees which are chosen by ballot; and to cause this list to be sent by mail to all the Resident Fellows of the Academy not later than four weeks before the Annual Meeting.

3. Independent nominations for any office, signed by at least five Resident Fellows and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call for the Annual Meeting, which shall then be issued not later than one week before that meeting.

4. The Recording Secretary shall prepare for use, in voting at the Annual Meeting, a ballot containing the names of all persons nominated for office under the conditions given above.

5. When an office is to be filled at any other time than at the Annual Meeting, the President shall appoint a Nominating Committee in accordance with the provisions of Section 1, which shall announce its nomination in the manner prescribed in Section 2 at least two weeks before the time of election. Independent nominations, signed by at least five Resident Fellows and received by the Recording Secretary not later than one week before the meeting for election, shall be inserted in the call for that meeting.

CHAPTER IV.

OF THE PRESIDENT.

1. It shall be the duty of the President, and, in his absence, of the senior Vice-President present, or next officer in order as above enumerated, to preside at the meetings of the Academy; to summon extraordinary meetings, upon any urgent occasion; and to execute or see to the execution of the Statutes of the Academy. Length of continuous membership in the Academy shall determine the seniority of the Vice-Presidents.

2. The President, or, in his absence, the next officer as above enumerated, is empowered to draw upon the Treasurer for such sums of money as the Academy shall direct. Bills presented on account of the Library, or the Publications of the Academy, must be previously approved by the respective committees on these departments.

3. The President, or, in his absence, the next officer as above enumerated, shall nominate members to serve on the different committees of the Academy which are not chosen by ballot.

4. Any deed or writing to which the common seal is to be affixed shall be signed and sealed by the President, when thereto authorized by the Academy.

CHAPTER V.

OF STANDING COMMITTEES.

1. At the Annual Meeting there shall be chosen the following Standing Committees, to serve for the year ensuing, viz.: —

2. The Committee of Finance, to consist of the President, Treasurer, and one Fellow chosen by ballot, who shall have full control and management of the funds and trusts of the Academy, with the power of investing or changing the investment of the same at their discretion. The general appropriations for the expenditures of the Academy shall be moved by this Committee at the Annual Meeting, and all special appropriations from the general and publication funds shall be referred to or proposed by this Committee.

3. The Rumford Committee, of seven Fellows, to be chosen by ballot, who shall consider and report on all applications and claims for the Rumford Premium, also on all appropriations from the income of the Rumford Fund, and generally see to the due and proper execution of this trust.

4. The C. M. Warren Committee, of seven Fellows, to be chosen by ballot, who shall consider and report on all applications for appropriations from the income of the C. M. Warren Fund, and generally see to the due and proper execution of this trust.

5. The Committee of Publication, of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Memoirs and the Proceedings shall be intrusted.

6. The Committee on the Library, of the Librarian ex officio and three other Fellows, one from each class, who shall examine the Library, and make an annual report on its condition and management.

7. An Auditing Committee of two Fellows, for auditing the accounts of the Treasurer.

CHAPTER VI.

OF THE SECRETARIES.

1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are received; and at each meeting he shall present the letters which have been addressed to the Academy since the last meeting. Under the direction of the Council for Nomination, he shall keep a list of the Resident Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council.

2. The Recording Secretary shall have charge of the Charter and Statute-book, journals, and all literary papers belonging to the Academy. He shall record the proceedings of the Academy at its meetings; and after each meeting is duly opened, he shall read the record of the preceding meeting. He shall notify the meetings of the Academy, apprise officers and committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy. He shall post up in the Hall a list of the persons nominated for election into the Academy; and when any individual is chosen, he shall insert in the record the names of the Fellows by whom he was nominated.

3. The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them calculated to promote its interests.

CHAPTER VII.

OF THE TREASURER.

1. The Treasurer shall give such security for the trust reposed in him as the Academy shall require.

2. He shall receive officially all moneys due or payable, and all bequests or donations made to the Academy, and shall pay such sums as the Academy may direct. He shall keep an account of all receipts and expenditures; shall submit his accounts to the Auditing Committee; and shall report the same at the expiration of his term of office.

3. The Treasurer shall keep separate accounts of the income and appropriation of the Rumford Fund and of other special funds, and report the same annually.

4. All moneys which there shall not be present occasion to expend shall be invested by the Treasurer, under the direction of the Finance Committee.

CHAPTER VIII.

OF THE LIBRARIAN AND LIBRARY.

1. It shall be the duty of the Librarian to take charge of the books, to keep a correct catalogue of them, to provide for the delivery of books from the Library, and to appoint such agents for these purposes as he may think necessary. He shall make an annual report on the condition of the Library.

2. The Librarian, in conjunction with the Committee on the Library, shall have authority to expend such sums as may be appropriated, either from the General, Rumford or other special Funds of the Academy, for the purchase of books, and for defraying other necessary expenses connected with the Library.

3. To all books in the Library procured from the income of the Rumford Fund, or other special funds, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured.

4. Every person who takes a book from the Library shall give a receipt for the same to the Librarian or his assistant.

5. Every book shall be returned in good order, regard being had to the necessary wear of the book with good usage. If any book shall be lost or injured, the person to whom it stands charged shall replace it by a new volume or set, if it belongs to a set, or pay the current price of the volume or set to the Librarian; and thereupon the remainder of the set, if the volume belonged to a set, shall be delivered to the person so paying for the same.

6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

7. The Librarian shall have custody of the Publications of the Academy and shall distribute copies among the Associate Fellows and Foreign Honorary Members, at their request. With the advice and consent of the President, he may effect exchanges with other associations.

CHAPTER IX.

OF MEETINGS.

1. There shall be annually four stated meetings of the Academy; namely, on the second Wednesday in May (the Annual Meeting), on the second Wednesday in October, on the second Wednesday in January, and on the second Wednesday in March. At these meetings only, or at meetings adjourned from these and regularly notified, shall appropriations of money be made, or alterations of the statutes or standing votes of the Academy be effected.

2. Fifteen Fellows shall constitute a quorum for the transaction of business at a stated meeting. Seven Fellows shall be sufficient to constitute a meeting for scientific communications and discussions.

3. The Recording Secretary shall notify the meetings of the Academy to each Fellow residing in Boston and the vicinity; and he may cause the meetings to be advertised, whenever he deems such further notice to be needful.

CHAPTER X.

OF THE ELECTION OF FELLOWS AND HONORARY MEMBERS.

1. Elections shall be made by ballot, and only at stated meetings.

2. Candidates for election as Resident Fellows must be proposed by two Resident Fellows of the section to which the proposal is made, in a recommendation signed by them, and this recommendation shall be transmitted to the Corresponding Secretary, and by him referred to the Council for nomination. No person recommended shall be reported by the Council as a candidate for election, unless he shall have received a written approval, signed at a meeting of the Council by at least five of its members. All nominations thus approved shall be read to the Academy at a stated meeting, and shall then stand on the nomination list during the interval between two stated meetings, and until the balloting. No person shall be elected a Resident Fellow, unless he shall have been resident in this Commonwealth one year next preceding his election. If any person elected a Resident Fellow shall neglect for one year to pay his admission fee, his election shall be void; and if any Resident Fellow shall neglect to pay his annual assessments

for two years, provided that his attention shall have been called to this article, he shall be deemed to have abandoned his Fellowship; but it shall be in the power of the Treasurer, with the consent of the Council, to dispense (*sub silentio*) with the payment both of the admission fee and of the assessments, whenever in any special instance he shall think it advisable so to do.

3. The nomination of Associate Fellows may take place in the manner prescribed in reference to Resident Fellows. The Council may in like manner originate nominations of Associate Fellows, which must be read at a stated meeting previous to the election, and be exposed on the nomination list during the interval.

4. Foreign Honorary Members shall be chosen only after a nomination made at a meeting of the Council, signed at the time by at least seven of its members, and read at a stated meeting previous to that on which the balloting takes place.

5. Three fourths of the ballots cast must be affirmative, and the number of affirmative ballots must amount to eleven to effect an election of Fellows or Foreign Honorary Members.

6. A majority of any section of the Academy is empowered to present lists of persons deemed best qualified to fill vacancies occurring in the number of Foreign Honorary Members or Associate Fellows allotted to it; and such lists, after being read at a stated meeting, shall be referred to the Council for Nomination.

7. If, in the opinion of a majority of the entire Council, any Fellow — Resident or Associate — shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his Fellowship; and provided that a majority of two thirds of the Fellows at a stated meeting, consisting of not less than fifty Fellows, shall adopt this recommendation, his name shall be stricken off the roll of Fellows.

CHAPTER XI.

Of Amendments of the Statutes.

1. All proposed alterations of the Statutes or additions to them, shall be referred to a committee, and, on their report at a subsequent meeting, shall require for enactment a majority of two thirds of the members present, and at least eighteen affirmative votes.

2. Standing votes may be passed, amended, or rescinded, at any

stated meeting, by a majority of two thirds of the members present. They may be suspended by a unanimous vote.

CHAPTER XII.

OF LITERARY PERFORMANCES.

1. The Academy will not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its publications.

STANDING VOTES.

1. Communications of which notice had been given to the Secretary shall take precedence of those not so notified.

2. Resident Fellows who have paid all fees and dues chargeable to them are entitled to receive one copy of each volume or article printed by the Academy, on application to the Librarian personally or by written order, within two years from the date of publication. And the current issues of the Proceedings shall be supplied, when ready for publication, free of charge, to all the Fellows and members of the Academy who desire to receive them.

3. The Committee of Publication shall fix from time to time the price at which the publications of the Academy may be sold. But members may be supplied at half this price with volumes which they are not entitled to receive free, and which are needed to complete their sets.

4. Two hundred extra copies of each paper accepted for publication in the Memoirs or Proceedings of the Academy shall be placed at the disposal of the author, free of charge.

5. Resident Fellows may borrow and have out from the Library six volumes at any one time, and may retain the same for three months, and no longer.

6. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

7. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy, except by special leave of the Librarian.

8. Books, publications, or apparatus shall be procured from the income of the Rumford Fund only on the certificate of the Rumford Committee that they, in their opinion, will best facilitate and encourage the making of discoveries and improvements which may merit the Rumford Premium.

9. A meeting for receiving and discussing scientific communications may be held on the second Wednesday of each month not appointed for stated meetings, excepting July, August, and September.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Benjamin, Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of said fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a premium to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

INDEX.

Note. For index to the species of Carex, see pp. 510–512.