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It is the object of this paper to describe some measurements of acoustic impedance and to offer a technique for such measurements in the laboratory.

Definitions of Acoustic Impedance and Acoustic Impedance Density: If a small thin rigid disk, or circular diaphragm AB, Figure 1, fitting without edge friction, in a smooth cylindrical tube TT, T′T′, of S sq. cm. internal area, be actuated, through the central rod, by a simple alternating mechanical force, or vibromotive force (vmf.) F root-mean-square dynes, directed along the axis of the tube; then the effect of the force, neglecting the friction of the air on the walls of the tube, will be to set up an alternating velocity $\dot{x}$ cm. per second, in the

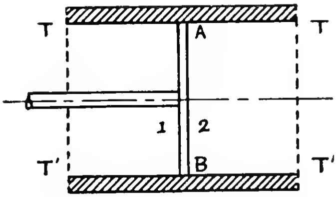

Figure 1. Diagram of Longitudinal Section of an Acoustic Tube with Vibrating Disk.

air or other fluid contained in the tube. The maximum cyclic amplitude of vibration $\pm x_m$ cm. about the quiescent position, is supposed to be very small. The fluid filling the tube on each side of the disk may be compressible, as in the ordinary case of an air tube. The maximum cyclic velocity of the fluid $\dot{x}_m$ in contact with the disk, must be the same as the maximum cyclic velocity of the disk itself. In the case of a rigid disk, this velocity would be the same at all parts of its surface. The magnitude and phase of this velocity for a given vmf. will depend upon the impedance to vibration of the column of fluid on each surface. This impedance to vibration velocity, of the fluid in a tube at a driving section, may be called the *acoustic impedance* of the fluid, at each of the two surfaces of the disk.

Acoustic impedance is the planevector sum of *acoustic resistance* and *acoustic reactance.* Acoustic resistance absorbs and dissipates the energy of acoustic vibration. Acoustic reactance cyclically stores and releases, without dissipation, the energy of acoustic vibration. If we denote the acoustic impedance of the fluid on sides 1 and 2 of the disk by $\mathbf{z}_1$ and $\mathbf{z}_2$ respectively; then the vector root-mean-square velocity of vibration at the disk surface will be:

$$\dot{x} = \frac{F}{\mathbf{z}_1 + \mathbf{z}_2} = \frac{F}{\mathbf{z}} \qquad \text{rms. kines,[1] or cm. per sec.} \angle \qquad (1)$$

where $\mathbf{z}$ is the total acoustic impedance to velocity of the fluid in the tube, at the disk,

or $$\mathbf{z} = \frac{F}{\dot{x}} \qquad \begin{array}{l}\text{acoustic absohms} \angle \\ \text{or dynes per kine} \angle\end{array} \qquad (2)$$

F being taken as of standard phase.

If we employ the C. G. S. system of units throughout, the acoustic impedances $\mathbf{z}_1$ and $\mathbf{z}_2$ may be expressed in vector acoustic absohms. An absohm in acoustic resistance is therefore such a total resistance to vibration at the surface of a disk, subjected to a simple harmonic vibromotive force of 1 rms. dyne, as will cause a vibrational velocity to be produced of 1 cm. rms. per second, in phase with the vmf. An acoustic impedance is, however, a planevector quantity, having both a size and a slope, and is expressible in complex numbers. If the slope of $\mathbf{z}$ is $+\beta^\circ$, the phase of the velocity will be retarded, by (1), through β° behind the vmf. Acoustic impedance is presented by the air in contact with a vibrating telephone diaphragm, and measurements made of the impedance offered by the air to a telephone diaphragm have shown the need for a definition of this quantity and of its unit.

If the tube, on its two sides 1 and 2, is perfectly symmetrical with respect to the disk AB, the two impedances $\mathbf{z}_1$ and $\mathbf{z}_2$ must be equal at least in size if not in slope; but whether equal or not, their sum will be equal to a total vector impedance $\mathbf{z}$ acoustic absohms $\angle$.

The impedance $\mathbf{z}$ will manifestly increase as the area of the tube is increased. Neglecting friction at the walls of the tube, the increase will be in direct proportion to the area. If S be the area of the tube in sq. cm., then the acoustic impedance per sq. cm., or *acoustic im-*

1 The kine is a name of the C. G. S. unit of linear velocity, or cm./sec., as proposed at a B. A. meeting.

pedance density, will be $\mathfrak{z} = \frac{\mathbf{Z}}{S}$ vector acoustic absohms per sq. cm.

Or, if we employ a vibromotive pressure intensity $p = \frac{F}{S}$ vector maximum cyclic dynes per sq. cm. to actuate the disk; then p being taken with standard phase:

$$\mathfrak{z} = \frac{p}{\dot{x}} \qquad \frac{\text{acoustic absohms}}{\text{sq. cm.}} \angle \qquad (3)$$

or

$$\dot{x} = \frac{p}{\mathfrak{z}} = \frac{p}{\mathfrak{z}_1 + \mathfrak{z}_2} \qquad \text{rms. cm. per sec.} \angle \qquad (4)$$

The dimensions of acoustic impedance, as derived from (2), are MT^{-1}, or mass per unit of time (force divided by velocity). The dimensions of acoustic impedance density, as derived from (3) are $ML^{-2}T^{-1}$ or MT^{-1}/L^2.

The acoustic impedance of a fluid at the surface of a vibrating diaphragm is therefore the opposition to the development of vibrational velocity at that surface under impressed vmf. Although when so defined, acoustic impedance involves the existence of a diaphragm or mechanical surface, at which the impedance is produced; yet acoustic impedance may be conceived of as occurring at an imaginary surface, such as the cross section of an acoustic tube, and without the interposition of a diaphragm. Moreover, acoustic impedance is not confined to a tube, but may occur in a region of any shape.

We have hitherto assumed that the velocity $\dot{x}$ was the same at all parts of the vibrating disk or diaphragm. In any actual flexible diaphragm, however, such as a telephone-receiver diaphragm, the vibrational displacement x, and velocity $\dot{x}$, will be different at different distances from the center of the disk. If we take any elementary area dS of the surface of the disk, at which the velocity is $\dot{x}$ rms. kines, the power of this motion is

$$dP = \dot{x}^2 \mathfrak{z}\, dS \qquad \text{abwatts or ergs per sec.} \angle \qquad (5)$$

where $\dot{x}$ is taken as of standard phase and zero slope. Of this power, the real component is dissipated, and the imaginary component is cyclically stored and released. The total power delivered to the disk, including both sides, will be

$$P = \int_0^S \dot{x}^2 \mathfrak{z}\, dS = \mathfrak{z} \int_0^S \dot{x}^2\, dS \qquad \text{abwatts} \angle \qquad (6)$$

ζ being assumed constant over the entire surface of the diaphragm.

This power may be expressed as

$$P = \underline{\dot{x}^2} \zeta S \qquad \text{abwatts} \qquad (7)$$

where

$$\underline{\dot{x}^2} = \frac{1}{S} \int_o^s \dot{x}^2 \, dS \qquad (\text{rms. kines})^2 \qquad (8)$$

or the vector mean velocity square $\underline{\dot{x}^2}$ is the integrated mean value of $\dot{x}^2$ over the entire surface. Under these conditions, the acoustic impedance of the tube at the disk including both of its sides, will be

$$\mathbf{z} = \frac{F}{\underline{\dot{x}}} \qquad \text{acoustic absohms} \angle \qquad (9)$$

The mean square ratio $\left(\frac{\underline{\dot{x}}}{\dot{x}_o}\right)^2 = \left(\frac{\underline{x}}{x_o}\right)^2$ of average to maximum central velocity or amplitude has been called the mass factor [2] of the diaphragm denoted by $\frac{m}{M}$.

Total Mechanic Impedance on a Diaphragm: When a telephone diaphragm is set in vibration under an impressed vmf., due to a simple harmonic alternating current in the coils, the maximum cyclic velocity of the diaphragm over the poles depends upon the total mechanic impedance to motion. The vector vmf. F may be taken as proportional to the exciting current I rms. absamperes flowing through the coils.[3]

$$F = A I \qquad \text{rms. dynes} \angle \qquad (10)$$

Here A is the vector force factor of the instrument, and ordinarily has a slope β°, of about -30°, so that the vmf. F has the same frequency as I, and lags in phase behind I by this angle β°. The total mechanic impedance $\mathbf{z}'$ of the diaphragm limits the max. cyclic velocity to

$$\dot{x} = \frac{F}{\mathbf{z}'} = \frac{F}{\mathbf{z}_d + \mathbf{z}} \qquad \text{rms. kines} \angle \qquad (11)$$

or

$$\mathbf{z}' = \frac{F}{\dot{x}} \qquad \text{total mechanic absohms} \angle \qquad (12)$$

2 Bibliography 9, page 477.
3 Bibliography 7, 8 and 9.

$\mathbf{z}'$ contains an internal mechanic impedance $\mathbf{z}_d$, and an external acoustic impedance $\mathbf{z}$, including both surfaces of the diaphragm. If the receiver is operated *in vacuo*, the external acoustic impedance $\mathbf{z}$ disappears, and the total mechanic impedance on the diaphragm is reduced to $\mathbf{z}_d$.

The rms. velocity $\dot{x}$ of a telephone receiver diaphragm, and also the impressed vmf. F, can be measured electrically in the laboratory by mapping out the motional impedance diagram, and measuring the amplitude of vibration at resonance. Consequently, by means of (12) the total mechanic impedance on the disk can be evaluated. If we measure the internal mechanic impedance, we can then find $\mathbf{z}$ the acoustic impedance, and ascertain how it varies under different conditions. The measurements described in this paper offer a starting point in this direction.

Brief History of the Method: In preceding researches on the motional impedance of telephone receivers, the receiver has been usually regarded as a motor, and the mechanic resistance of the receiver diaphragm as a motor load, partly due to internal frictions, and partly to external acoustic wave emission. In a few cases, the mechanic resistance was varied, either by operating the receiver in a partial vacuum, or by attaching to the diaphragm a disk vane immersed in a damping fluid.[4] In the research here described, however, the results obtained have led to changing the function of the tested receiver from a reciprocating motor to a reciprocating generator of sound, with the object of studying the reaction, or change of acoustic load, on this generator, produced by sound reflection and absorption. It is believed that the method here described is susceptible of numerous applications in the science of acoustics. Although the definition of acoustic impedance offered in this paper is believed to be new; yet the concept can be discerned in the writings of various acousticians.[5]

Outline Theory of the Method: It has been shown in preceding publications, that if A is the force factor of a telephone receiver in dynes per absampere $\angle$, a complex or planevector quantity having the lagging slope β°, and representing the electro-magnetic pull upon the diaphragm per rms. absampere of exciting current I, taken at standard phase, Z′ is the motional impedance of the receiver as measured at the impressed frequency, and with a given acoustic load (absohms

4 Bibliography 9, pages 460–462.

5 Bibliography 15. Since delivering this paper the authors' attention has been directed to Dr. Drysdale's recent Kelvin lecture (Bibliography 20) where this theory is developed.

$\angle$), $\mathbf{z}'$ is the mechanic impedance of the diaphragm at the same frequency and load (dynes per kine $\angle$ or mechanic absohms $\angle$). Then $\dot{x}$ the rms. vibrational velocity of the diaphragm over either pole of the receiver is

$$\dot{x} = \frac{\mathrm{A\,I}}{\mathbf{z}'} \qquad \text{rms. kines } \angle \qquad (13)$$

If the mechanical impedance $\mathbf{z}'$ is reactanceless, as happens at the impressed frequency of apparent resonance, it degrades from a complex quantity to the real quantity r'', or mechanic resistance. In this case, the phase of the velocity $\dot{x}$ will be the same as that of the product (AI). Taking the phase of I as standard, or its slope as zero, that of A is $-\beta°$; so that $\dot{x}$ will then lag $\beta°$ behind I in phase. In general, however, with mechanic reactance present, the phase of $\dot{x}$ will be displaced from that of (AI) by the slope of $\mathbf{z}'$.

Again,

$$Z' = \frac{\mathrm{A}\dot{x}}{\mathrm{I}} = \frac{\mathrm{A}^2}{\mathbf{z}'} \qquad \begin{array}{l}\text{electric absohms } \angle \\ \text{or C.G.C. magnetic} \\ \text{units of resistance } \angle\end{array} \qquad (14)$$

This means that in the motional impedance circle of the receiver, at any impressed frequency, the vector motional impedance Z', as obtained from electrical measurements with a Rayleigh bridge, is equal to the square of the complex force factor A, divided by the total mechanic impedance $\mathbf{z}'$ at that frequency. At the frequency of apparent resonance, this becomes

$$Z'_0 = \frac{\mathrm{A}^2}{r''} \qquad \text{electric absohms } \angle \qquad (15)$$

since $\mathbf{z}'$ degrades at resonance into the total mechanical resistance r''. The slope of the motional impedance Z' will then be the same as the slope of A^2, or $-2\beta°$. Z'_0 is thus the diameter of the motional impedance circle, in the ordinary simple case, where the telephone is tested for its motional impedance.

From (14) it follows that, at any impressed frequency, the total mechanic impedance $\mathbf{z}'$ is

$$\mathbf{z}' = \frac{\mathrm{A}^2}{Z'} = \mathrm{A}^2\,\mathrm{Y}' \qquad \begin{array}{r}\text{dynes per kine } \angle \\ \text{or mechanic absohms } \angle\end{array} \qquad (16)$$

Consequently, if we divide the complex constant A^2 by the measured motional impedance Z', we obtain the size and slope of the total mechanic impedance of the diaphragm at this frequency.

The total mechanic impedance $\mathbf{z}'$ is made up of several mechanic impedances; namely,

(1) The mechanic impedance of the diaphragm

$$\mathbf{z}_d = r_d + j\left(m_d\omega - \frac{s_d}{\omega}\right) \qquad \text{mechanic absohms } \angle \quad (17)$$

(2) The virtual mechanic impedance of the diaphragm [6] due to its motion in the permanent magnetic field

$$\mathbf{z}_v = j\frac{pA}{\omega} = r' + \frac{js'}{\omega} = \frac{p|A|}{\omega}\sin\beta + j\frac{p|A|}{\omega}\cos\beta$$

$$\text{mechanic absohms } \angle \quad (18)$$

The real part of $\mathbf{z}_v$, although varying inversely with ω, is ordinarily taken as constant to a first approximation, the working range in ω being usually small.

(3) The acoustic impedance $\mathbf{z}_1$ of the air in the chamber behind the diaphragm, and which varies, in some manner similar to (17), as the impressed frequency is varied.

(4) The acoustic impedance $\mathbf{z}_2$ of the air in front of the diaphragm, and which constitutes the load to be varied.

Consequently,

$$\mathbf{z}' = \mathbf{z}_d + \mathbf{z}_v + \mathbf{z}_1 + \mathbf{z}_2 \qquad \text{mechanic absohms } \angle \quad (19)$$

Keeping the impressed frequency constant, $\mathbf{z}_d$, $\mathbf{z}_v$, and $\mathbf{z}_1$ will remain constant; but $\mathbf{z}_2$ can be made to vary by changing the acoustic load. The changes in the electric impedance, measured on changing the acoustic load, enable the latter to be computed. Moreover, by observing the changes in Z' when the frequency is varied, if allowance can be made for the changes thereby produced in $\mathbf{z}_d$, $\mathbf{z}_v$ and $\mathbf{z}_1$, the variations in the acoustic impedance $\mathbf{z}_2$ may also be computed.

Measurement of Acoustic Impedance of the Air Column in a Tube of Varied Length: A straight smooth fibre tube 87.8 cm. long, 3.1 cm. internal diameter, and 3.9 cm. external diameter, was supported horizontally, by metallic strips packed with felt, over a wooden base BB, Figure 2. The tube TT was brought into contact with the cover of a regular form of bipolar telephone receiver R, as is shown in detail by Figure 3. The axis of the tube was in line with the axis of the receiver. A hard rubber piston PP slides freely in the tube, and can

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6 Bibliography 16.

be set at any desired position within it, by means of the projecting brass rod *r*. The telephone receiver R is clamped, by means of a metallic strip, on to a wooden pedestal rising from the base BB.

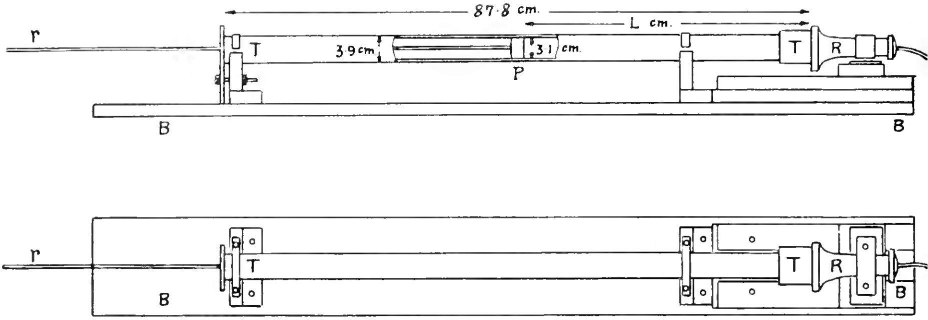

FIGURE 2. Side Elevation and Plan of Experimental Tube.

The telephone receiver used in the tests here described was a bipolar Western Electric Company's instrument of a well known pattern, provided with a composition cap or cover. This cover had its opening enlarged, as is shown in Figures 2 and 3, so as to connect more easily with the acoustic tube TT.

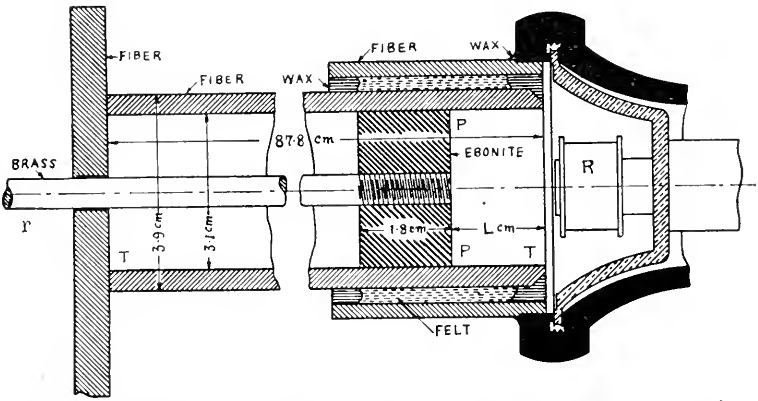

FIGURE 3. Longitudinal Section of Air Tube and Telephone Receiver fastened to the Same.

The electric impedance of the receiver was measured in the Rayleigh bridge shown in Figure 4. The adjustable-frequency source S, was a

Vreeland oscillator in some of the tests, and a triode vacuum-tube oscillator in others. The resistance R was adjusted to give an alternating current of 3 rms. milliamperes to the bridge. In some tests, this current was raised to 4 milliamperes. Half this current passed through the receiver Z, and the other half through the adjustable anti-inductive resistance *r*, and variometer *l*. The two arms *a* and *b* of the bridge are of equal anti-inductive resistances. The current-detector, or galvanometer, was a pair of medium-resistance head telephones.

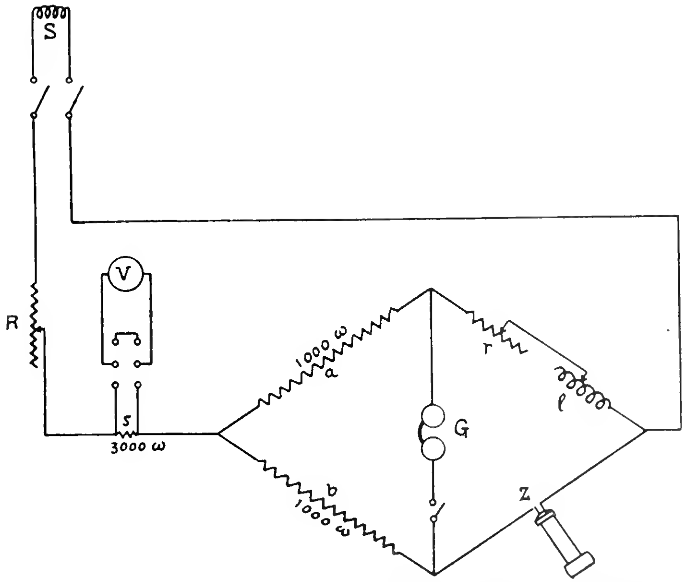

FIGURE 4. Electric Connections of Rayleigh Bridge for measuring the Motional Impedance of the Telephone Receiver Z.

The measurements were all made with approximately constant *a-c.* strength through the measured receiver Z. The measured impedance of the receiver did not, however, vary materially over a small range in current strength; so that it was not important to keep the current strength very nearly constant. The current was ascertained and adjusted by means of a static voltmeter V, bridged across an anti-inductive resistance *s* of 3000 ohms. In some of the tests, a thermojunction and *d-c.* galvanometer were substituted for *s* and V.

Technique of Measurements: A series of measurements is first made of the apparent resistance R and inductance L of the receiver **z**, Figure 4, over a fairly considerable range of frequency; so as to include the resonant frequency f_0 of the receiver, which is allowed to vibrate in the free air of the testing room. This series of measurements of R and L at varied frequency, but constant testing-current strength, is then repeated without delay, with the diaphragm damped, or prevented from vibrating. This damping may be effected in several ways, that have been described in preceding publications.[7] A convenient procedure, however, has been found to consist in forming a T of paraffin wax, and melting the extremities of this with a hot wire, so as to press gently, but adherently, on the center of the diaphragm, and also across a diameter of the receiver cap, as is indicated in Figure 5. By this means, the diaphragm is prevented from vibrat-

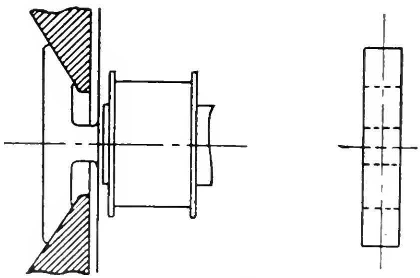

FIGURE 5. Diagram indicating the mode of clamping a Receiver Diaphragm by the application of a Wax Bridge or T.

ing without altering the normal airgap. When great precision is not desired, a still swifter and simpler method of damping the receiver, is to insert a round plug of wood, wax, or other solid material, into the hole in the receiver cap, so as to plug this hole without touching the diaphragm. This method of damping the receiver has been called "acoustic damping." It permits of some vibration of the diaphragm, especially towards higher impressed frequencies; but the cushioning of the air, by the inserted plug, prevents the vibration from exceeding a small fraction of that which occurs with the plug withdrawn. In other words, the acoustic impedance $\mathbf{z}_2$ of such a plug and air cushion is very large.

7 Bibliography 7, 9.

The "damped impedance" of the receiver is subtracted vectorially from the "free impedance," for a number of successive frequencies. The differences, or "motional impedances" of the receiver are plotted on a separate diagram. The ordinary motional-impedance diagram of a receiver, thus tested in the open, is very nearly a circle passing through the origin; i. e., the well known motional-impedance circle. The motional-impedance circle of the tested receiver is given in Figure 6. Its diameter is $158 \diagdown 55^\circ.6$ ohms; so that the angle $\beta^\circ = -\frac{55^\circ.6}{2} = -27^\circ.8$.

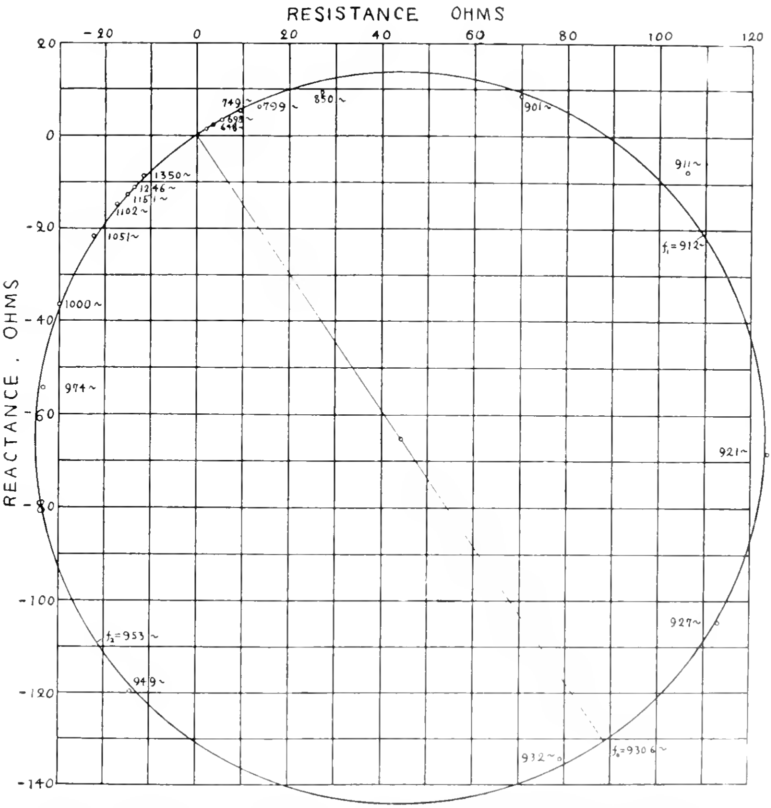

FIGURE 6. Motional Impedance Circle of Telephone Receiver Used in the Tests.

The *d-c.* resistance of this receiver at 20°C is 86.4 ohms; so that the motional impedance per unit of *d-c.* resistance, $Z'_0 \, R_1$ for this instrument is [8] $1.83 \angle 55^{\circ}.6$. The angle β°, according to theory, is the angle which the locus of damped impedance makes with the vertical or X axis, in the neighborhood of the resonant frequency. The frequency of apparent resonance is $f_{o} = 930.6 \sim$ at 20°C. The quadrantal frequencies are $f_1 = 912 \sim$ and $f_2 = 953 \sim$; so that $\Delta = 41\pi = 128.8$ and the sharpness of resonance $\Lambda_{o} = 2\pi f_{o}/\Delta = 45.4$.

The next step is to connect the receiver to its acoustic load. In the case of Figure 2, the load is the cylindrical column of air in the tube TT. The resistance and inductance of the telephone are now meas-

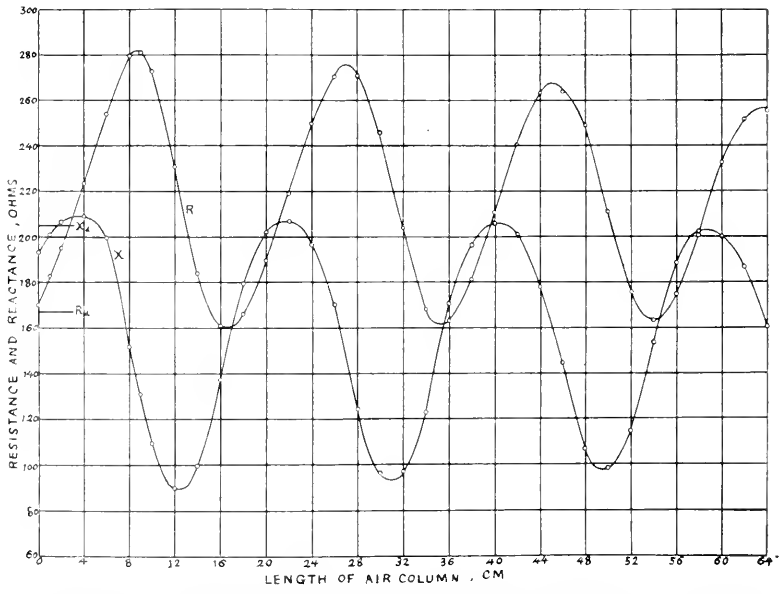

FIGURE 7. Curves of Apparent Resistance and Reactance of Receiver when attached to Air Column of successively varied Length.

ured at successive positions of the piston P, holding the frequency f, and alternating-current strength, constant. The values of receiver resistance and inductance so measured, are found to fluctuate periodically as the length L of the air column in the acoustic load changes. Figure 7 gives a pair of wavy curves in one test, at $f = 921 \sim$ and I = 1.575 milliamperes rms. in the receiver.

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[8] Bibliography 16, page 527.

It is important that the frequency and the temperature in such a test should be held as nearly constant as possible. The frequency should be near to the resonant frequency f_0. If the frequency is remote from the resonant frequency, the mechanic reactance of the diaphragm is likely to be so large that the addition of the acoustic load to be measured will not affect the vector motional impedance materially; whereas at resonance, a small change in added acoustic load will be likely to affect the motional impedance considerably.

The variations in R and X indicated in Figure 7, represent corresponding variations in the electric impedance $Z = R + jX$. These are due to variations in the acoustic impedance $\mathbf{z}_2$, which is the subject of investigation. The curves of R and X in Figure 7, represent total apparent resistance and reactance, in the receiver, at varying air-column lengths. The values R_d and X_d indicated on the left-hand side of the figure, are respectively the damped resistance and reactance of the receiver at this frequency. With reference to the impedance $Z_d = R_d + jX_d$, as origin, the values of motional impedance Z', corresponding to the successive air-columns, are plotted vectorially in Figure 8. The numbers marked on this spiral are air-column lengths in cm. It will be seen that at L = 0 cm., or with the piston within 1.5 mm. of the diaphragm, the motional impedance was $12.5 \diagdown 75^\circ.4$ ohms. This value was nearly repeated with L = 19 cm. At L = 10 cm., however, the motional impedance had increased to $142.8 \diagdown 42^\circ.1$ ohms. The curve is a slowly contracting spiral, with a pitch of 18.7 cm., which is half a wave length at 921 ∾; since the velocity of sound at 20°C being 34,430 cm. per sec., the wave length $\lambda = 34{,}430/921 = 37.38$ cm.

In order to utilize the motional-impedance diagram of Figure 8 for determining the acoustic impedance of the load in front of the receiver diaphragm, we may refer to equations (16) and (19). Equation (16) shows that we must find Y', the vector reciprocal of the motional impedance, and multiply by the square of the vector constant A of the receiver, in order to derive the total mechanic impedance $\mathbf{z}'$ on the diaphragm. This means that we must invert the diagram of Figure 8, or find the locus of the reciprocal spiral.

If we operate upon the graph of motional admittance by the vector factor A^2, where $A = 5.138 \times 10^6 \diagdown 27^\circ.8$ (the value measured for this particular receiver)[9]; i.e., multiplying each vector admittance

[9] In order to determine the value of the vector force factor A, it is necessary to measure the amplitude of receiver diaphragm vibration at resonance, as has been described in preceding papers, Bibliography 9.

by $26.4 \times 10^{12} \nabla 55°.6$, we obtain the total mechanic impedance graph of Figure 9. Here the curve is carried from L = 0 cm. to L = 62 cm. Thus in Figure 8, the vector 0–18 measures $26 \times 10^9 \nabla 92°.6$ absohms. The reciprocal of this is $Y' = 0.03846 \times 10^{-9} \angle 92°.6$ abmhos. Multiplying this reciprocal by $A^2 = 26.4 \times 10^{12} \nabla 55°.6$,

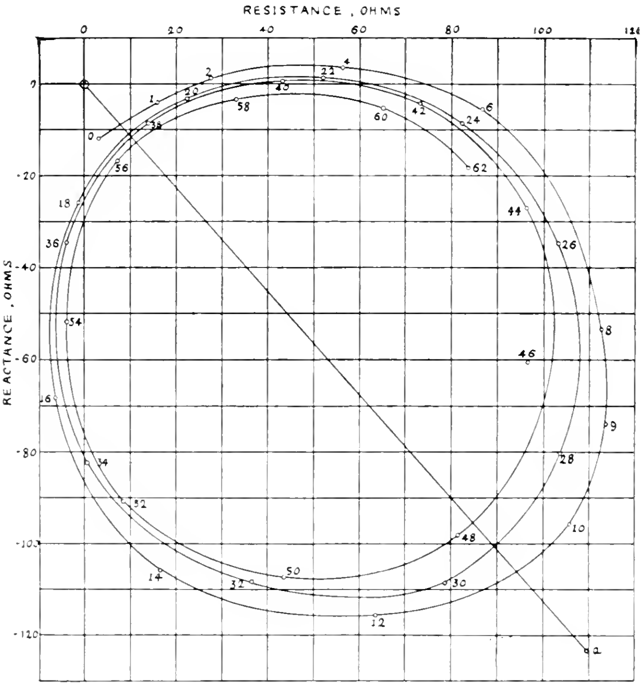

FIGURE 8. Motional Impedance Diagram of Receiver at Constant Impressed Frequency ($921 \sim$) with varying length of Attached Air Column.

we obtain $1.015 \times 10^3 \angle 37°$ dynes per kine. This is the vector total mechanic impedance, at 18 cm. of air-tube length. It will be observed that the curve is a spiral, with the same direction of rotation as in Figure 8, but the relative positions of observation points about the axis have been changed. In Figure 9, the straight line A B C D E is the axis of the acoustic impedance spiral.

Analysis of Figure 9 indicates that the acoustic impedance OA = $160 \nabla 7°.5$ mechanic absohms is the total internal mechanic impedance $(\mathbf{z}_d + \mathbf{z}_v + \mathbf{z}_1)$ at the impressed frequency of the measurement (921 ∾). This leads to the inference that if the acoustic impedance in the tube and in front of the receiver diaphragm could be completely removed, the mechanic impedance left in and behind the diaphragm would be the vector impedance OA. This would correspond to a vector motional impedance $Oa = 165 \nabla 48^\circ.1$ on Figure 8. Oa is therefore the inferred motional impedance for the case of removal of all acoustic load from the front of the diaphragm. The addition of the acoustic load of the air column in the tube is a vector

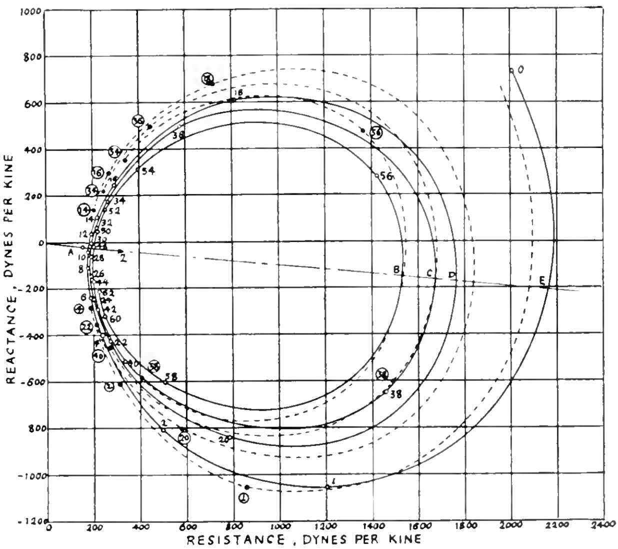

FIGURE 9. Graph of Total Mechanic Impedance on Receiver Diaphragm obtained by inverting the graph of Figure 8 and multiplying by A^2. Heavy curve connects observation values. The dotted curves represent the computed values.

with its origin at A, Figure 9. The spiral locus 0 1 2 4. . . .62 is the spiral of vector impedance of the tube acoustic conductor, with its distant end sealed at distances of 0 1 2 4. . . 62 cm. from the diaphragm, or more strictly from a plane 1.5 mm. in front of the diaphragm, where the tube may be considered as beginning. This spiral impedance conforms fairly well with the familiar expression

$$\mathbf{z}_0 \tanh \delta_A \qquad \text{dynes per kine} \angle \qquad (20)$$

where z_0 is the "surge impedance" of the tube, or the acoustic impedance offered to this frequency by an indefinitely great length of the tube, and δ_A is the "position angle" at the sending end A of the tube as the length of the same is varied. The value deduced for $\mathbf{z}_0$ is $187 \diagdown 5^\circ.2$, or A Z on Figure 9. This point Z is thus the inferred inner terminus of the spiral E D C B if the tube were prolonged indefinitely. The hyperbolic angle δ_A appears to be

$$\delta_A = \{L\,(0.000706 + j\,\underline{0.107}) + (0.0974 + j\,\underline{0.97}\,\}\ \text{hyps}\ \angle \qquad (21)$$

In the electric-conductor analogy, if the tube were stopped with a perfectly reflecting air-tight plug, the position angle at the point of stoppage would be $j\frac{\pi}{2}$ hyperbolic radians or $j\,\underline{1}$; i.e., one imaginary quadrant,[10] or 90°. This imperfection of the plug apparently alters this to $0.0974 + j\,\underline{0.97}$.

The dotted spiral in Figure 9 indicates the computed locus according to (21). The agreement between the heavy spiral of observations and the dotted spiral of formula (21) is seen to be fairly satisfactory. At the time when the observations were made, the theory of the analogy between the impedance of an acoustic-tube conductor and that of an electric-line conductor had not been reached. It has been pointed out, however, by Fleming [11] and Fitzgerald [12] that an analogy exists between the waves along air tubes and electric waves.

According to this analogy, therefore, the plugged tube of varied length offered an acoustic impedance similar to the electric impedance of a uniform alternating-current line, of varied length, put to ground in each case at the distant end through a certain leak, or high resistance. At each quarter wave of tube lengthening, the impedance of the tube crosses the axis of $\mathbf{z}_0$ and reaches alternately a high and a low value. Thus at C = 37.7 cm., the impedance is a maximum:

$$\begin{aligned}\mathbf{z}_{37.7} &= 187 \diagdown 5^\circ.2\ \tanh\,\{(0.0266 + j\,\underline{4.03}) + (0.0974 + j\,0.97)\}\\ &= 187 \diagdown 5^\circ.2\ \tanh\,(0.124 + j\,\underline{5})\\ &= 187 \diagdown 5^\circ.2\ \coth 0.124 = 187 \diagdown 5^\circ.2 \times 8.106 = 1517 \diagdown 5^\circ.2,\\ &\qquad\qquad \text{acoustic absohms}\ \angle\end{aligned}$$

The quarter wave length is 9.35 cm.; so that at L = 0.30, 19.0, 37.7, 56.4 cm., the acoustic impedances should be maxima, and should fall on the axis of $\mathbf{z}_0$ at E, D, C and B respectively; while at L = 9.65,

10 Bibliography 11 and 13.
11 Bibliography 12.
12 Bibliography 2.

28.35, 47.05 cm., the impedance should be a minimum, and should fall on the same axis, nearer to A. All these requirements are fairly well met in the diagram of Figure 9.

It is thus possible, by making purely electrical measurements of the motional impedance of a telephone receiver, to arrive at the corresponding variations in acoustic impedance, when the acoustic load is varied, at constant impressed frequency. In the case of a simple air tube of varied length, the acoustic impedance appears to follow substantially the same quantitative behavior as the electric impedance of an alternating-current line of correspondingly varied length.

In the case of an electric line, we can always determine the values of the line angle θ and the surge impedance z_0, at a point, by successively grounding and freeing the line at the distant end, and measuring the corresponding impedance Z_{gA} and Z_{fA} at the home end A. The ratio of these two impedances is $\tanh^2\delta_A$ and their product is z_0. In the case of a tube or acoustic line, we can approximately free the line, by plugging the tube at the far end with a smooth hard plug, which is designed to serve as a perfect sound reflector; but no means are at present available for "grounding" the acoustic line. With the tube wide open at the distant end, there is still an appreciable acoustic impedance in and beyond the open end. If the tube opens into a room or walled space, this terminal acoustic impedance will, in general, possess some reactance due to reflections from the walls: but if the tube terminates in the open air, the terminal acoustic impedance is likely to have only a small reactive component or slope. If the amount of the terminal acoustic impedance σ of a tube opening out of doors, could be ascertained from the geometry of the tube, it would be readily possible to compute θ and $\mathbf{z}_0$ from the acoustic impedance Z_{fA} with the distant end plugged, and $Z_{\sigma A}$ with the distant end open to the free air. Until σ can be satisfactorily predetermined, it will be necessary to measure θ and $\mathbf{z}_0$ by indirect electrical methods, and to determine σ from the differences between Z_{fA} and $Z_{\sigma A}$. Such investigations might lead to the experimental determination of σ, the open-end terminal acoustic impedance of a tube of given dimensions.

Effects of different Apertures at the Far End of the Tube on the Acoustic Impedance at the Sending End: Figure 10 shows several motional impedance diagrams for the case of a fixed length of the same air-tube (87.8 cm.), each diagram being taken at constant impressed frequency, and with different sizes of circular aperture in the plug at the distant end of the tube. The tube was closed with a flat slab of fiber 0.62 cm. thick. Circular holes marked Nos. 1, 2, 3, and 4 were cut in different parts of this slab, with the following diameters; No. 1, 0.6 cm.; No. 2, 0.95 cm.; No. 3, 1.3 cm.; No. 4, 1.6 cm., and No. 5, full tube aperture (3.1 cm.); i.e., with slab removed. No. 0 represents the case of a complete closure of the tube by the unpierced slab.

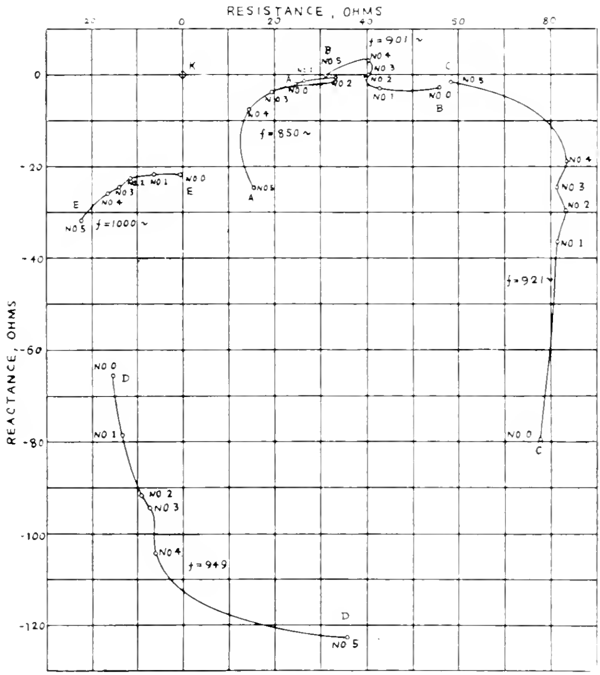

FIGURE 10. Motional Impedance of Receiver at Constant Impressed Frequencies, attached to Air-Tube of Constant Length (87.8 cm.) with apertures of different diameters at far end.

The origin of the diagram is at K, Figure 10. At the constant frequency of 850 ∾, the successive motional impedances of the receiver attached to the tube, with successive terminal orifices 0–5 were K–O, K–1 . . . K–5. The ends of these vectors are connected by the curve AA. Repeating the motional-impedance measurements at the successive steady frequencies of 901, 921, 949 and 1000 ∾, the corresponding vector loci with different terminal apertures are marked on the curves BB, CC, DD, and EE.

It is evident that the motional-impedance variations, due to varying the distant-end orifice of the tube, are greatest on the long curves CC and DD, apparently because the frequencies pertaining to them (921∾ and 949∾ respectively) lie nearest to the resonant frequency of the receiver, when loaded with the tube under these conditions. For sensitiveness in the measurement of tube terminal effects in the receiver, the frequency of 850∾ was too low, and that of 1000∾ too high.

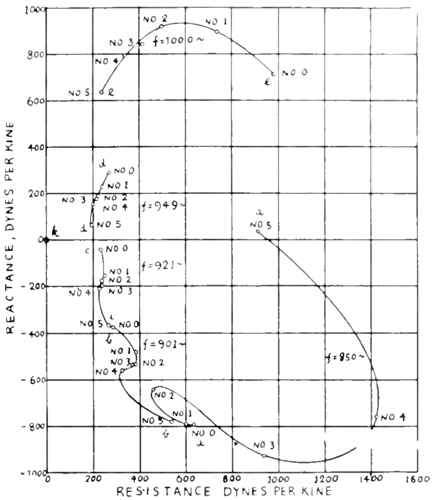

FIGURE 11. Total Mechanic Impedance at Receiver Diaphragm by Inversion from Figure 10 and Multiplying by A^2.

The mechanic impedance at the receiver diaphragm, in the various measurements recorded in Figure 10, are presented in Figure 11, after inversion and multiplication by A^2. These mechanic impedances are vectors, expressible in mechanic resistance and reactance, as dynes per kine, or mechanic absohms, as may be preferred. The origin in Figure 11 is at *k*. The vector graph A A of Figure 10, becomes converted into the vector graph *a a* of Figure 11, B B into *b b*, and so on.

It will be observed that the most sensitive graphs C C and D D of Figure 10 give rise to graphs *c c* and *d d* in Figure 11, revealing only small changes in mechanic resistance, but relatively large changes of mechanic reactance, with change of tube aperture. On the other hand, *e e* at 1000∾ includes more change in mechanic resistance

(230 to 980 dynes per kine) than in mechanic reactance (j 630 to j 920 dynes per kine). This indicates that the changes in acoustic load, effected by altering the tube aperture at the distant end, may preponderate in mechanic resistance, or in mechanic reactance, depending upon the impressed frequency, and the system of stationary sound waves set up in the tube.

Another set of measurements was made with the same telephone receiver and fiber tube plugged at various tube lengths, but with a row of holes, like those of a flute, bored in the tube wall. The diameter of each hole was 6.5 mm., and their distance apart, between centers, 4 cm. Removable plugs were inserted in these holes. The motional impedance of the receiver was measured, at four steady frequencies between 900∾ and 1000∾, when one plug was removed at successive holes along the line. The motional impedance was found to be sensitively affected by the position of the particular plug removed. The results, although interesting, have not been analyzed, and are therefore not presented here in quantitative detail. They indicate, however, that the acoustic impedance of the air column in a tubular musical instrument, such as a flute, undergoes marked variations when the instrument is manipulated, as in playing. These changes of acoustic impedance might be measured, in the manner described, with respect to a telephone-receiver diaphragm as the source of sound.

Motional Impedance and Acoustic Impedance of a Telephone Receiver pressed against the Ear: When a telephone receiver is exposed to the air in a large room, the total mechanic impedance on its diaphragm is the total internal impedance ($\mathbf{z}_d + \mathbf{z}_r + \mathbf{z}_1$) plus the external impedance $\mathbf{z}_2$ of the air in front of the diaphragm, including the cushion of air under the cap. The ordinary motional impedance circle of the instrument is measured under these conditions. When, however, the telephone is pressed against the ear, the external acoustic impedance $\mathbf{z}_2$ is that of the air in the ear cavity, as well as in the cap of the instrument. The motional impedance of the instrument under these conditions, approximates to that actually presented in the average telephonic use of the device as a receiver of speech.

Figure 12 gives the motional impedance circle O A B C of a bipolar receiver, No. 143–2, similar to that used in the tests recorded in preceding figures. The frequency of apparent resonance was $f_0 = 969$∾ when the motional impedance was $177 \angle 44^\circ.2$ ohms. This receiver, when held against the ear of a listener, as in the ordinary conversational use of the instrument, gave rise to the smaller graph O D E F, which departs very clearly from the strictly circular form. The exact shape of this graph O D E F is not regarded as of great importance; because, if the instrument was placed against the ear with less or more pressure, or if it was centered differently on the ear; or if some other ear was selected, the shape of the graph would be likely to be altered appreciably. To a first approximation, however, the graph O D E F may be regarded as an approach to the dotted circular motional impedance O G E H, which may be described as the motional circle of

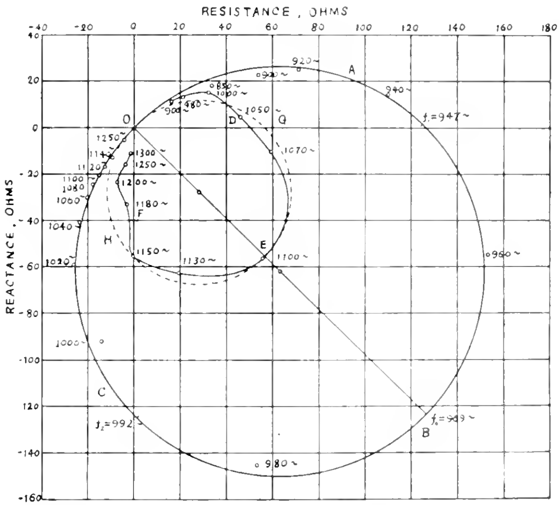

FIGURE 12. Motional Impedance Graphs of Receiver in Room and pressed against Listener's Ear.

reference, with the instrument applied to this ear. It will be observed that the diameter of this circle of approximation is $78.5 \diagdown 44^{\circ}$ ohms. The diameter O E has, therefore, substantially the same slope as the free-air motional impedance diameter O B; but has been reduced in length somewhat more than 50 per cent. The frequency of apparent resonance has also been changed from 969∾ to 1100∾, by applying the receiver cap to the ear.

Figure 13 gives the corresponding mechanic impedance graphs for this receiver and ear. The straight line *a b c*, parallel to the reactance axis, corresponds to the motional impedance circle O A B C of Figure 12, taken in the free air of a room. This means that the total mechanic impedance of the receiver free to the room, was a constant mechanic resistance of 149 dynes per kine, plus or minus a mechanic reactance, depending on the impressed frequency. At 969 ∾, the frequency of apparent resonance, this reactance disappears, leaving the resistance of 149 dynes per kine as the residual mechanic impedance.

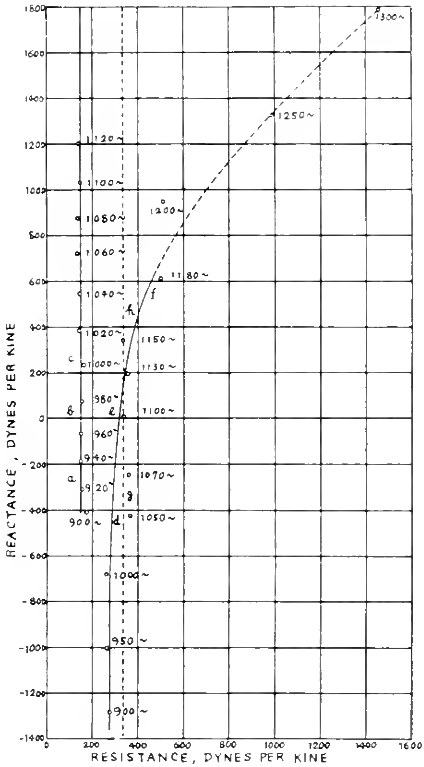

FIGURE 13. Mechanic Impedance of Receiver in Room and when pressed against Listener's Ear, by inversion from Fig. 12, and multiplying by A^2.

It should be observed that any motional-impedance circle of a telephone receiver, passing through the origin of coördinates, corresponds to such a straight-line graph of total mechanic impedance parallel to the reactance axis, no matter what the depression angle of the motional-impedance diameter may be. The application of the factor A^2, in formula (16), cancels out the diametral depression angle.

In other words, all motional-impedance circles of telephone receivers are convertible into rectilinear mechanic impedance graphs of constant mechanic resistance, neglecting the influences which are known to distort the motional impedance graph from the purely circular form.[13]

If the motional-impedance circle of approximation O G E H of Figure 12 is accepted for the receiver applied to the listener's ear; then the dotted straight line *g e h*, Figure 13, would represent the corresponding mechanic impedance graph, where the mechanic resistance is 335 dynes per kine, and zero reactance is found near 1100 ∾. The curved line *d e f* corresponds more nearly to the actually recorded motional-impedance graph O D E F, Figure 12. The particular shape of this curved graph *d e f* may not, however, be relied upon.

Figure 13 indicates therefore that, at least to a first approximation, the acoustic resistance of the listener's ear in this measurement was 86 dynes per kine. At higher frequencies, this resistance appears to increase.

Experiments were also made with an artificial ear,[14] composed of paraffin wax, and in form modelled closely after an anatomical model human ear. A motional-impedance graph made for the same telephone receiver, applied to this artificial ear, agreed fairly well in form with the graph for the actual ear O D E F, Figure 12. The corresponding approximate circle had a diameter of 98 ⦣ 44° ohms; or was somewhat larger than the circle O G E H of Figure 12, corresponding to a somewhat lower mechanic resistance. It may be noted that a receiver, when applied to a model ear in a solid substance like paraffin wax, does not fit to it as closely as when applied to the human ear. The imperfect fit would tend to lower the acoustic resistance.

With the receiver applied face downwards upon a flat slab of glass, a motional impedance graph was a closed curve through the origin roughly circular in form. The approximating equivalent circle had a diameter and depression angle about the same as when the measure-

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13 These deviating influences are the variation in the force factor A, with impressed frequency, and the variation in r' the component of hysteretic frictional resistance of the diaphragm, with change of impressed frequency; see Bibliography 16.

14 It is believed that an "artificial ear," consisting of a flat slab of some suitable substance selected experimentally, on which a receiver is laid face down, has been used for some years in the telephone laboratories of the Western Electric Co. in New York.

ment was made with the receiver open to the air of the room. The frequency of apparent resonance was, however, raised to 1090 ∾.

Application of the Method to the Tuning of Telephone Receivers for increasing their Sensibility: Referring to Figures 7 and 8, it will be seen that for the given impressed frequency of 921 ∾ on the receiver connected to the fiber tube, varying the length of the tube through a range of 9.4 cm., or one quarter of a wavelength, altered the motional impedance from a maximum to a minimum, or *vice versa*, and also altered the maximum cyclic velocity of the diaphragm in the ratio of more than 8:1. From this it will be evident that when such a tube is inserted between the telephone cover and the listener's ear, by the insertion of an ear tube into the plug P, Figure 2, the loudness of the sound emitted by the receiver, under steady excitation, can be adjusted by altering the tube length. In other words, the tube length can be tuned to maximum sensibility of the receiver. This principle may have useful applications in measurements with the telephone receiver as a detector. Observations made in this way have shown that this method of tuning is applicable over a certain range of impressed frequency. Thus, if the impressed frequency is 900 ∾, and the receiver has an apparent resonant frequency, under its actual acoustic load, of 1000 ∾, an inserted adjustable tube may enable the resonant frequency to be reached at 900 ∾.

Possible Applications of the Method to Architectural Acoustics: When the free impedance of a telephone receiver is measured with the axis of the instrument horizontal, and with its cover facing a wall of the room at not too great a distance, it is found that, near the frequency of apparent resonance, the electric impedance of the instrument is affected by its distance from the wall. Holding the frequency steady, the electric impedance of the receiver is found to vary if the instrument is moved nearer to, or farther from, the wall which it faces. This means that the acoustic impedance, at the diaphragm, of the air in front of the instrument, is affected by the distance from the wall in relation to the wave length of the emitted sound. The amount of variation of electric impedance, or the sensibility of the measurement to small mechanical displacements, depends upon the sharpness of resonance of the diaphragm. It seems likely that this method of measurement is capable of being applied to architectural acoustics; as, for example, in the measurement of the sound-reflection coefficient of draperies. For such purposes, a telephone receiver should be selected with a small total mechanic resistance in its diaphragm, and with a small damping constant Δ; i.e., with a large sharpness of resonance.[15] Such an instrument would have a relatively large motional-impedance circle, and a small change of impressed frequency would carry the vector from one quadrantal point, through resonance, over to the other. Relatively feeble reflected sound waves, falling on the diaphragm, would thus appreciably affect the motional impedance, and enable the corresponding acoustic impedance variation to be measured.

Summary of Results.

(1) Acoustic resistance, reactance and impedance are defined. A technique for their measurement is described.

(2) Acoustic impedance is offered to any disk or diaphragm steadily vibrating in a fluid.

(3) Acoustic impedance can be measured at a telephone-receiver diaphragm, but can be considered as existing in a fluid at any surface that can be drawn, and across which sound is steadily transmitted.

(4) When the amplitude of vibration of a diaphragm is not uniform over its surface, the acoustic activity over its surface can be evaluated in terms of the mean square amplitude over the surface.

(5) Acoustic impedance at the diaphragm of a telephone receiver, carrying an alternating current, can be determined by measuring changes in the motional impedance of the receiver under changes of acoustic load.

(6) The total mechanic impedance at a receiver diaphragm, under any steadily impressed a-c. frequency and acoustic load, is the reciprocal of its motional impedance, multiplied by A^2, the square of the vector forcefactor of the instrument.

(7) The mechanic impedance graph, under varied frequency, of any receiver whose motional impedance diagram is a circle through the origin, consists of an infinite straight line parallel to the reactance axis.

(8) By applying a telephone receiver to one end of an air tube, or to the mouth of any air chamber, the acoustic impedance of the tube or chamber can be determined in this condition, by measuring its effect upon the motional impedance in the neighborhood of the resonant frequency.

(9) In the case of a long straight cylindrical air-tube with smooth walls, attached to the cap or cover of a receiver, the stationary sound-

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15 Bibliography 7, page 133 and Fig. 11, where the receiver had a resonant sharpness of about 320.

wave system set up in the tube, at a given frequency, appears to correspond to the stationary electromagnetic wave system set up in a certain equivalent alternating-current line conductor. The quantitative relations in the two cases present remarkable analogies.

(10) The plugging of the air tube at its distant end in the last mentioned case, corresponds to freeing the a–c. electric conductor at its distant end. On the other hand, although the electric conductor can be grounded at its distant end, no corresponding means appear to be available for reducing to zero the acoustic impedance of the open end of the tube.

(11) Applying a telephone receiver to the ear of a listener roughly doubled the total mechanic resistance at the receiver diaphragm, and also increased its apparent elastic constant.

(12) A telephone receiver may be tuned to maximum response or vibratory displacement, over a certain range of impressed frequency, by altering the length of an air column between the receiver and the listener's ear.

(13) The method of measuring acoustic impedance electrically appears to be applicable to architectural acoustics.

(14) Tubular musical instruments, such as flutes, evidently possess acoustic impedances that are variable under manipulation, and that are susceptible of measurement under the particular conditions described.

APPENDIX.

On the Analogies between Sound-Wave Transmission along a Uniform Acoustic Tube Conductor, and Electromagnetic-Wave Transmission along a Uniform Electric Line Conductor.

We may assume that when a telephone receiver, actuated as an acoustic generator, sings a pure tone steadily into a long smooth straight tube of S sq. cm. internal cross section, arranged as in Figure 2, the sound waves travelling along the tube are plane waves, and that the effects on them of friction at the walls of the tube may be neglected. Even if the actual conditions do not conform closely with these postulates, the results thus arrived at may safely be regarded as first approximations.

The distance of any cross-section of the tube, measured along the axis, from the generator diaphragm may be expressed as **l** cm. The clamping-circle diameter of the receiver diaphragm may first be supposed to be equal to the internal diameter of the tube. The very small vibratory displacement of the air, in a sound wave, from its normal quiescent position, measured parallel to the axis, in the direction of increasing **l**, may be denoted by x cm. Since x, in the assumed case of a pure tone and simple harmonic vibration, may be taken as the projection, at time t seconds, on the tube axis, of the rotating plane vector

$$x = x_m \epsilon^{j\omega t} \qquad \text{instantaneous cm.} \angle \qquad (22)$$

where ω is the impressed angular velocity of rotation, and x_m is the maximum cyclic vibratory displacement, the vibratory velocity of a transverse layer of air in the tube, also measured along the axis, and in the direction of increasing **l**, is

$$\frac{\partial x}{\partial t} = \dot{x} = j\omega x_m \epsilon^{j\omega t} = j\omega x = \dot{x}_m \epsilon^{j\omega t} \qquad \text{inst. kines or cm./sec.} \angle \qquad (23)$$

The velocity $\dot{x}$ is also a planevector quantity, capable of being dealt with by the rules of complex arithmetic. Likewise, the vibratory acceleration will be

$$\frac{\partial^2 x}{\partial t^2} = \ddot{x} = j\omega \dot{x}_m \epsilon^{j\omega t} = j\omega \dot{x} = -\omega^2 x_m \epsilon^{j\omega t} = -\omega^2 x \qquad \text{inst. kines/sec. or cm./sec.}^2 \angle \qquad (24)$$

These equations indicate that in the steady state, the acceleration is in leading quadrature to the velocity, which, in turn, is in leading quadrature to the displacement.

The linear mass, or mass of air normally occupying 1 cm. length of tube will be

$$\mathbf{m} = S\rho \qquad \frac{\text{gm.}}{\text{linear cm.}} \qquad (25)$$

where ρ is the density of the quiescent air in the tube at its actual temperature and its normal pressure intensity p_0 dynes per sq. cm.

The fundamental dynamic equation expressing the instantaneous acceleration of a thin layer of air $\partial\mathbf{l}$ cm. long, and having a mass $\mathbf{m}\,\partial\mathbf{l}$ grams is

$$\mathbf{m}\,\ddot{x}\,\partial\mathbf{l} = -\,\partial F \qquad \text{dynes}\angle \qquad (26)$$

where $+\,\partial F$ is the excess of total pressure on the far side of the layer above that on the near side; or

$$\mathbf{m}\,\ddot{x} = S\rho\ddot{x} = -\,\frac{\partial F}{\partial\mathbf{l}} \qquad \frac{\text{dynes}}{\text{linear cm.}}\angle \qquad (27)$$

and[16]

$$\rho\ddot{x} = -\,\frac{\partial p}{\partial\mathbf{l}} \qquad \frac{\text{dynes/sq. cm.}}{\text{linear cm.}}\angle \qquad (28)$$

where p is the excess of the pressure intensity p_0 dynes per sq. cm. on the far side of the layer above that on the near side.

It is also a well known acoustic condition [17] that

$$\ddot{x} = v^2\,\frac{\partial^2 x}{\partial\mathbf{l}^2} \qquad \frac{\text{kines}}{\text{sec.}}\angle \qquad (29)$$

where v is the velocity of transmission of sound in the tube, along its axis, in kines.

From (26) and (24)

$$\partial F = -\,\dot{x}\,(j\mathbf{m}\omega)\,\partial\mathbf{l} \qquad \text{rms. dynes}\angle \qquad (30)$$

This expresses the relation between a small difference of alternating pressure ∂F across an elementary length $\partial\mathbf{l}$ of the tube conductor, and the simultaneous vibratory velocity $\dot{x}$ of the element. It corresponds to the well known relation between the difference of alternating

16 Lamb's Hydrodynamics, p. 458, Eq. (3).
17 " " p. 458, Eq. (6).

electric pressure of emf. ∂E, across an elementary length ∂l of a line conductor, and the simultaneous alternating current I in the element

$$\partial E = -I\,(j\mathfrak{L}\omega)\,\partial l \qquad \text{rms. volts}\,\angle \qquad (31)$$

where $\mathfrak{L}$ is the linear inductance of the conductor in henrys per km., and the linear resistance r of the conductor is ignored. Here ω is the angular velocity of the impressed electric pressure, and l is the length of line in km., measured outwards from the generating end.

Again, from (27) and (29), we have

$$-\frac{\partial F}{\partial \mathbf{l}} = \mathbf{m}v^2\,\frac{\partial^2 x}{\partial \mathbf{l}^2} \qquad \frac{\text{dynes}}{\text{linear cm.}}\,\angle \qquad (32)$$

Integrating $$F = -\mathbf{m}v^2\,\frac{\partial x}{\partial \mathbf{l}} + \text{constant} \qquad \text{dynes}\,\angle \qquad (33)$$

The constant of integration vanishes, and may be dropped, because F the pressure deviation from the normal pressure F_0 over a cross-section of the tube, vanishes when $\dfrac{\partial x}{\partial \mathbf{l}} = 0$.

Hence, $$F\,\partial\mathbf{l} = -\mathbf{m}v^2\,\partial x \qquad \text{cm.-dynes}\,\angle \qquad (34)$$

Differentiating with respect to time:

$$\frac{\partial F}{\partial t}\,\partial\mathbf{l} = \dot{F}\,\partial\mathbf{l} = -\mathbf{m}v^2\,\partial\dot{x} \qquad \frac{\text{cm.-dynes}}{\text{sec.}}\,\angle \qquad (35)$$

or

$$j\omega\,F\,\partial\mathbf{l} = -\mathbf{m}v^2\,\partial\dot{x} \qquad \frac{\text{cm.-dynes}}{\text{sec.}}\,\angle \qquad (36)$$

and

$$\partial\dot{x} = -\frac{j\omega}{\mathbf{m}v^2}\,F\,\partial\mathbf{l} \qquad \text{rms. kines}\,\angle \qquad (37)$$

The quantity $\mathbf{m}v^2$ may be replaced by $\mathbf{s}$, the total elastic force resisting compression, expressed in ergs per cm. of displacement over the section; so that the instantaneous differential increase in vibrational velocity over an element of length $\partial\mathbf{l}$ is

$$\partial\dot{x} = -F\left(j\frac{\omega}{\mathbf{s}}\right)\partial\mathbf{l} \qquad \text{rms. kines}\,\angle \qquad (38)$$

The quantity $\mathbf{s}$ is the normal adiabatic elastic force $Sp_0' = S\gamma p_0$, of the medium over the section, γ being the ratio of specific heats, and p_0

the quiescent pressure intensity in the medium. The last equation corresponds to the well known equation of differential increase in alternating current ∂I, over an element of conductor length ∂l; namely

$$\partial I = - E(jc\omega)\,\partial l = - E\left(j\frac{\omega}{s}\right)\partial l \qquad \text{rms. amperes} \angle \qquad (39)$$

where c is the linear capacitance of the conductor in farads per km., and s is its reciprocal. In this case, the linear leakance g of the conductor is ignored.

Since the complete electric conductor equations for (31) and (39), including r and g, are

$$\partial E = - I(r + j\mathfrak{L}\omega)\partial l = - Iz\,\partial l \qquad \text{rms. volts} \angle \qquad (40)$$

and

$$\partial I = - E\left(g + j\frac{\omega}{s}\right)\partial l = - Ey\,\partial l \qquad \text{rms. amperes} \angle \qquad (41)$$

we may reasonably assume that the corresponding equations for the acoustic conductor, including frictional resistance along the tube and losses due to imperfect elasticity are, from (30) and (38)

$$\partial F = - \dot{x}(\mathbf{r} + j\mathbf{m}\omega)\,\partial \mathbf{l} = - \dot{x}\,\mathbf{z}\,\partial \mathbf{l} \qquad \text{rms. dynes} \angle \qquad (42)$$

and

$$\partial \dot{x} = - F\left(\mathbf{g} + j\frac{\omega}{\mathbf{s}}\right)\partial \mathbf{l} = - F\mathbf{y}\,\partial \mathbf{l} \qquad \text{rms. kines} \angle \qquad (43)$$

In electric conductors, the linear dissipation constants r and g are accepted as constant at any single impressed frequency, whatever values may pertain to E and I; but they are known to change when the impressed frequency is varied over a wide range. Similarly, in acoustic tubes, the linear dissipation constants **r** and **g** are perhaps constant at any single frequency, although they may vary when the frequency is varied. Numerous measurements will have to be collected before the acoustic dissipation constants of a tube can be determined with precision.

The known equations of electric propagation along uniform lines are applicable, following the above theory, to acoustic propagation along uniform tubes, when hydrostatic difference of pressure F is substituted for electric difference of pressure E, vibratory displacement x for alternating quantity q, vibratory velocity $\dot{x}$ for alternating current I, linear mass **m** for linear inductance $\mathfrak{L}$, linear frictional resistance **r** for linear electric resistance r, linear acoustic elasticity loss **g** for linear electric leakance g, and elastic force **s** for s, the reciprocal of the linear capacitance.

Table I gives a comparison between electric and acoustic quantities for the case of negligible losses in transmission.

Table II gives some comparative data for electric and acoustic conductors, still assumed as having no losses; but freed at the distant end.

Table III gives similar comparative data for the general case of conductors having linear losses of known values.

Surge-Impedance Density: In regard to Table I, it may be noted that the acoustic surge impedance density has no counterpart in the ordinary theory of electric lines.

The value of the surge impedance density, by (49), is $\sqrt{\rho\, p'_0}$ where ρ is the density of quiescent air at 0°C and the standard pressure of 10^6 bars or dynes per sq. cm. The value of $p'_0 = \gamma p_0$ may be taken as 1.41×10^6 bars, and $\rho = 1.276 \times 10^{-3}$. Consequently $\mathfrak{z} = \sqrt{17.99 \times 10^2} = 42.4$ mechanic absohms per sq. cm. of tube cross-section, under the standard condition of 1 megabar pressure (10^6 dynes per sq. cm.) and 0°C. If the actual pressure of the quiescent air in the tube is p_0 bars, and the actual temperature is t° C., the surge impedance density becomes

$$\mathfrak{z} = \frac{42.4\sqrt{p_0 \times 10^{-6}}}{\sqrt{1 + 0.00366\, t}} \qquad \frac{\text{mechanic absohms}}{\text{sq. cm.}} \angle \qquad (70)$$

or for values not exceeding 25° C.,

$$\mathfrak{z} = \frac{42.4 \times \sqrt{p_0} \times 10^{-3}}{1 + 0.00183\, t} \qquad \frac{\text{mechanic absohms}}{\text{sq. cm.}} \angle \qquad (71)$$

A rigid disk diaphragm of S sq. cm. on each face, vibrating with uniform amplitude over its surface, so as to generate plane waves in the tube indefinitely long, would develop on each face a surge resistance of

$$\mathbf{z}_0 = \frac{42.4\, S \sqrt{p_0} \times 10^{-3}}{1 + 0.00183\, t} \qquad \text{mechanic absohms} \angle \qquad (72)$$

In the case of the receiver used in the reported tests, the diameter of the diaphragm inside the clamping ring was 4.96 cm., the corresponding surface area being 19.3 sq. cm. The surge impedance on this surface at 20°C. and 1 megabar pressure, if the diaphragm faced an indefinitely long tube of the same cross-sectional area without acoustic

TABLE I

Properties of Electric and Acoustic Lines with negligible Losses $r = g = 0$, $\mathbf{r} = \mathbf{g} = 0$

Quantity	Electric		Acoustic		
Linear hyperbolic angle	$\alpha = j\omega\sqrt{\mathfrak{L}c} = j\omega\sqrt{\frac{\mathfrak{L}}{s}} = j\frac{\omega}{v}$ $= j\frac{2\pi}{\lambda} = j\alpha_2$	$\frac{\text{hyps}}{\text{km}}\angle$	$\alpha = j\omega\sqrt{\frac{\mathbf{m}}{\mathbf{s}}} = j\frac{\omega}{v} = j\omega\sqrt{\frac{\rho}{p_0'}}$ $= j\frac{2\pi}{\lambda} = j\alpha_2$	$\frac{\text{hyps}}{\text{cm}}\angle$	(44)
Hyperbolic angle of conductor	$\theta = \mathrm{L}\alpha = j2\pi\frac{\mathrm{L}}{\lambda} = j\mathrm{L}\alpha_2$	hyps $\angle$	$\theta = \mathrm{L}\alpha = j2\pi\frac{\mathrm{L}}{\lambda} = j\mathrm{L}\,\alpha_2$	hyps $\angle$	(45)
Wave length	$\lambda = \frac{2\pi}{\alpha_2} = \frac{2\pi}{\alpha}$	km	$\lambda = \frac{2\pi}{\alpha_2} = \frac{2\pi}{\alpha}$	cm	(46)
Velocity of propagation	$v = \frac{\omega}{\alpha_2} = \frac{\omega}{\alpha} = \sqrt{\frac{s}{\mathfrak{L}}} = f\lambda$	$\frac{\text{km}}{\text{sec}}$	$v = \frac{\omega}{\alpha_2} = \frac{\omega}{\alpha} = \sqrt{\frac{p_0'}{\rho}} = \sqrt{\frac{\mathbf{s}}{\mathbf{m}}} = f\lambda$	kines	(47)
Surge impedance	$z_0 = \sqrt{\frac{\mathfrak{L}}{c}} = \sqrt{\mathfrak{L}s} = \frac{s}{v}$	ohms	$\mathbf{z}_0 = \sqrt{\mathbf{ms}} = \frac{\mathbf{s}}{v} = \mathrm{S}\mathfrak{z} = \mathrm{S}\sqrt{p_0'\rho}$	mechanic absohms	(48)
Surge impedance density			$\mathfrak{z} = \frac{\mathbf{z}_0}{\mathrm{S}} = \frac{\mathbf{m}v}{\mathrm{S}} = \sqrt{\frac{\rho\mathbf{s}}{\mathrm{S}}} = \sqrt{p_0'\rho}$	$\frac{\text{dynes}}{\text{kine cm}^2}$	(49)
Impedance ratio for P and A	$\frac{Z_P}{Z_A} = \frac{\tanh\delta_P}{\tanh\delta_A}$	numeric $\angle$	$\frac{Z_P}{Z_A} = \frac{\tanh\delta_P}{\tanh\delta_A}$	numeric $\angle$	(50)
Power at A	$\mathrm{P}_A = \mathrm{E}_A\,\mathrm{I}_A$	watts $\angle$	$\mathrm{P}_A = \mathrm{F}_A\,\dot{x}_A$	abwatts $\angle$ [18]	(51)
Power at A, when L = ∝	$\mathrm{P}_A = \mathrm{E}_A\,\mathrm{I}_A = \frac{\mathrm{E}_A^2}{z_0} = \mathrm{I}_A^2 z_0$	watts $\angle$	$\mathrm{P}_A = \mathrm{F}_A\,\dot{x}_A = \frac{\mathrm{F}_A^2 v}{\mathbf{s}} = \dot{x}_A^2\,\mathbf{m}v$ $= \mathrm{S}pkv = \mathrm{S}\dot{x}^2\rho v$	abwatts $\angle$ [18]	(52)

TABLE II.

Negligible Losses and distant end free. $r = g = 0$, $\mathbf{r} = \mathbf{g} = 0$, $\delta_B = j\frac{\pi}{2} = j1$

Quantity	Electric		Acoustic		
Impedance at Point P	$Z_P = \sqrt{\mathfrak{L}s}\tanh j\,(L_2\alpha_2 + 1)$	ohms ∠	$Z_P = \sqrt{\mathbf{ms}}\tanh\,(jL_2\alpha_2 + j\,1)$	mech.	(53)
	$= -j\sqrt{\mathfrak{L}s}\cot\left(2\pi\frac{L_2}{\lambda}\right)$	"	$= -j\sqrt{\mathbf{ms}}\cot\left(2\pi\frac{L_2}{\lambda}\right)$	absohms ∠	(54)
	$= -j\sqrt{\mathfrak{L}s}\cot\,(L_2\alpha_2)$	"	$= -j\sqrt{\mathbf{ms}}\cot\,(L_2\alpha_2)$		(55)
Impedance ratio for P and A	$\frac{Z_P}{Z_A} = \frac{\cot\,(L_2\alpha_2)}{\cot\,(L\alpha_2)}$	numeric	$\frac{Z_P}{Z_A} = \frac{\cot\,(L_2\alpha_2)}{\cot\,(L\alpha_2)}$	numeric	(56)
Current / Velocity " " "	$\frac{I_P}{I_A} = \frac{\sin\,(L_2\alpha_2)}{\sin\,(L\alpha_2)}$	"	$\frac{\dot{x}_P}{\dot{x}_A} = \frac{\sin\,(L_2\alpha_2)}{\sin(L\alpha_2)}$	"	(57)
Voltage / Pressure " " "	$\frac{E_P}{E_A} = \frac{\cos\,(L_2\alpha_2)}{\cos\,(L\alpha_2)}$	"	$\frac{F_P}{F_A} = \frac{\cos\,(L_2\alpha_2)}{\cos\,(L\alpha_2)}$	"	(58)
Current / Velocity at end A	$I_A = \frac{jE_A}{\sqrt{\mathfrak{L}s}\cot\,(L\alpha_2)}$	amperes ∠	$\dot{x}_A = \frac{j\,F_A}{\sqrt{\mathbf{ms}}\cot\,(L\alpha_2)}$	kines ∠	(59)
Voltage / Pressure at end B	$E_B = \frac{E_A}{\cos\,(L\alpha_2)}$	volts	$F_B = \frac{F_A}{\cos\,(L\alpha_2)}$	dynes ∠	(60)

TABLE III.

Properties of Electric and Acoustic Lines when Losses are not negligible

Quantity	Electric		Acoustic		
Linear hyperbolic angle	$\alpha = \sqrt{yz} = \sqrt{(r + j\ell\omega)\left(g + j\frac{\omega}{s}\right)}$	$\frac{\text{hyps}}{\text{km}}\angle$	$\alpha = \sqrt{\mathbf{yz}} = \sqrt{(\mathbf{r} + j\mathbf{m}\omega)\left(\mathbf{g} + j\frac{\omega}{\mathbf{s}}\right)}$	$\frac{\text{hyps}}{\text{cm}}\angle$	(61)
	$= \alpha_1 + j\alpha_2$	"	$= \alpha_1 + j\alpha_2$	"	(62)
Hyp. Angle of conductor	$\theta = \text{L}\alpha = \text{L}(\alpha_1 + j\alpha_2) = \theta_1 + j\theta_2$	hyps $\angle$	$\theta = \text{L}\alpha = \text{L}(\alpha_1 + j\alpha_2) = \theta_1 + j\theta_2$	hyp $\angle$	(63)
Wave length	$\lambda = \frac{2\pi}{\alpha_2}$	km	$\lambda = \frac{2\pi}{\alpha_2}$	cm	(64)
Velocity of propagation	$v = \frac{\omega}{\alpha_2}$	$\frac{\text{km}}{\text{sec}}$	$v = \frac{\omega}{\alpha_2}$	$\frac{\text{cm}}{\text{sec}}$	(65)
Surge impedance	$z_0 = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + j\ell\omega}{g + j\omega/s}}$	ohms $\angle$	$\mathbf{z}_0 = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}} = \sqrt{\frac{\mathbf{r} + j\mathbf{m}\omega}{\mathbf{g} + j\omega/\mathbf{s}}}$	$\frac{\text{dynes}}{\text{kine}}\angle$	(66)
Impedance at point P	$Z_P = z_0 \tanh \delta_P$	ohms $\angle$	$Z_P = \mathbf{z}_0 \tanh \delta_P$	"	(67)
Impedance ratio for P and A	$\frac{Z_P}{Z_A} = \frac{\tanh \delta_P}{\tanh \delta_A}$	numeric $\angle$	$\frac{Z_P}{Z_A} = \frac{\tanh \delta_P}{\tanh \delta_A}$	numeric $\angle$	(68)
Current / Velocity } ratio " "	$\frac{I_P}{I_A} = \frac{\cosh \delta_P}{\cosh \delta_A}$	"	$\frac{\dot{x}_P}{\dot{x}_A} = \frac{\cosh \delta_P}{\cosh \delta_A}$	"	(69)
Voltage / Pressure } ratio " "	$\frac{E_P}{E_A} = \frac{\sinh \delta_P}{\sinh \delta_A}$	"	$\frac{F_P}{F_A} = \frac{\sinh \delta_P}{\sinh \delta_A}$	"	(70)

losses, and if the vibration amplitude were uniform over the whole diaphragm, would be 789 $\angle$ 0° acoustic absohms. The mass factor of the diaphragm, as measured electrically, with its usual cover opening into the air, was 0.263; so that if this mass factor had been maintained under the geometrical conditions indicated in Figure 3, the acoustic impedance on the outer surface of the diaphragm would be 789 $\angle$ 0° × 0.263 = 208 $\angle$ 0° mechanic absohms. The actual analysis, however, of the results with the tube attached as in Figure 3, made the observed surge impedance on the front surface 187 $\angle$ 5°.2 mechanic absohms. The discrepancy between the computed and observed value of $\mathbf{z}_0$ may be attributed to three influences (1) acoustic losses in the tube; (2) the sudden change in tube sectional area from 19.3 to 7.55 sq. cm. close to the diaphragm, and (3) a probable change of mass factor m/M under the actual conditions of Figure 3.

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List of Symbols employed.

A	Force factor of a telephone receiver (dynes/absampere ∠).
$a = a_1 + ja_2$	Linear hyperbolic angle of an electric or acoustic conductor $\left(\frac{\text{hyps}}{\text{km.}} \angle\right)$ or $\left(\frac{\text{hyps}}{\text{cm.}} \angle\right)$
β°	Slope of force factor, or of a vector quantity (circular degrees).
c	Linear capacitance of electric conductor (farads/km.).
$\gamma = 1.41$	Ratio of specific heats of air or other gas (constant pressure to constant volume) (numeric).
Δ	Apparent damping constant of a telephone receiver (hyps/second).
δ_P, δ_A	Position angles at point P and at sending end A, of electric or acoustic conductor (hyps ∠).
E, E_P, E_A, E_B	Electromotive force in a circuit, at a point P on a conductor, at sending end A, and at receiving end B (rms. volts ∠).
$\epsilon = 2.718\ldots$	Napierian base (numeric).
F	Vibromotive force on a diaphragm; also at any surface across which vibration occurs (rms. dynes ∠).
F_P, F_A, F_B	Vibromotive force over section of acoustic conductor at P, at sending end A or far end B (rms. dynes ∠).
f	Impressed frequency (cycles per second).
f_1, f_2	Lower and upper quadrantal frequencies of telephone receiver (cycles/sec.).
f_0	Frequency of apparent resonance of a receiver (cycles/sec.).
g	Linear leakance of electric conductor (mhos/km.).
g	Coefficient of linear resilience loss of acoustic conductor (mech. absohms/cm.).
$\theta = \theta_1 + j\theta_2$	Angle subtended by an electric or acoustic conductor (hyps ∠).
I	Current strength in a telephone receiver (rms. absamperes ∠); also current strength along an electric conductor (rms. amperes ∠).
$j = \sqrt{-1}$	
$\kappa = \frac{p}{p'_0}$	Compression, or ratio of rms. diminution in volume to quiescent volume (numeric).
L	Length of electric or acoustic conductor (km. or cm.).
L_2	Distance from point P to far end of an electric or acoustic conductor (km. or cm.).
$\mathfrak{L}$	Linear inductance of electric conductor (henrys/km.).
l	Length of an electric conductor (km.).
l	Length of an acoustic conductor (cm.).
λ	Wavelength on electric or acoustic conductor (km. or cm.).
M	Total mass of a vibrating diaphragm within its clamping circle (gm.).

m, m_d	Equivalent mass of a vibrating diaphragm within its clamping circle (gm.). Equivalent mass of diaphragm in absence of air (gm.).
$\mathbf{m}$	Linear mass of air or other gas in acoustic conductor (gm./cm.).
P	Power at a point on an electric or acoustic conductor (watts or abwatts $\angle$).
p	Ratio of equivalent displacement turns to exciting turns in a telephone receiver (absamperes cm.).
$p = \frac{F}{S}$	Acoustic alternating pressure (deviation from quiescent hydrostatic pressure) at a cross section of an acoustic conductor (dynes/sq. cm. $\angle$).
$p_\circ$	Hydrostatic pressure in quiescent air of acoustic conductor (dynes/sq. cm.).
$p'_\circ = \gamma p_\circ$	Adiabatic hydrostatic pressure in quiescent air of tube (dynes/sq. cm.).
$\pi = 3.141 \ldots$	(numeric).
q	Quantity of electricity in an a–c. line conductor (rms. coulombs).
R_1	Electric resistance of a telephone receiver as measured with a continuous current (absohms).
R, R_d	Apparent resistance of a telephone receiver to alternating currents when free and damped (absohms).
r	Mechanic resistance of a diaphragm excluding effects of magnetic hysteresis (mech. absohms).
r''	Total mechanic resistance of a diaphragm at apparent resonance (mech. absohms).
r'	Effective mechanic resistance of a diaphragm due to magnetic hysteresis (mech. absohms).
r	Linear resistance of electric conductor (ohms km.).
$\mathbf{r}$	Linear resistance of acoustic conductor (mech. obsohms/cm.).
ρ	Density of quiescent air or other gas in acoustic conductor (gm./c.c.).
S	Surface area on one side of a diaphragm or cross-sectional area of an acoustic conductor (sq. cm.).
$s = \frac{1}{c}$	Reciprocal of linear capacitance of electric conductor (daraf-km.).
$\mathbf{s} = S\, p'_\circ$	Resilience force of acoustic conductor over its entire cross-section (dynes).
s_d	Stiffness coefficient of a telephone receiver diaphragm (dynes/cm.).
s'	Apparent change in mechanic stiffness of a diaphragm due to its vibration in a permanent magnetic field (dynes/cm.).
σ	Electric or acoustic load at the B end of a long conductor (ohms $\angle$ or mech. absohms $\angle$).
t	Elapsed time from epoch (seconds).
t°	Temperature in gas within acoustic conductor (deg. Cent.).
$\Lambda_\circ$	Oscillatory sharpness of resonance (numeric).
v	Velocity of transmission in electric conductor (km. sec.) and in acoustic conductor (cm./sec.).
X, X_d	Reactance of a telephone receiver free and damped respectively (absohms).

x	Displacement of a diaphragm from its quiescent position or of a thin layer of air in a tube from its quiescent position (rms. cm.).
$\dot{x}$	Vibrational Velocity of a diaphragm or of a thin layer of air in a tube (rms. cm./sec. ∠).
$\ddot{x}$	Vibrational acceleration of a diaphragm or of a thin layer of air in a tube (rms. cm./sec.^{2} ∠).
$x_m, \dot{x}_m, \ddot{x}_m$	Maximum cyclic sizes of vibrational displacement, velocity and acceleration (cm., kines, kines/sec.).
$\underline{x}, \underline{\dot{x}}$	Root mean square values of displacement and velocity taken over entire surface of flexible vibrating diaphragm (cm. ∠, kines ∠).
$x_o, \dot{x}_o$	Root mean square values of displacement and velocity at center of vibrating telephone diaphragm (cm. ∠, kines ∠).
$Y' = 1/Z'$	Electric motional admittance of telephone receiver (ohms ∠).
y	Linear admittance of electric conductor (ohms/km. ∠).
$\mathbf{y}$	Linear admittance of acoustic conductor (mech. absohms/cm. ∠).
Z, Z_d	Impedance of a telephone receiver, free and damped (absohms ∠).
Z_P, Z_A	Impedance at point P and at sending end of electric line (ohms ∠) or for acoustic line (mech. absohms ∠).
$Z' = Z - Z_d$	Motional impedance of telephone receiver (absohms ∠).
Z'_o	Maximum or diametral value of receiver motional impedance (absohms ∠).
$\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}$	Acoustic impedance on sides 1 and 2 of a diaphragm and their vector sum (acoustic absohms ∠).
$\mathbf{z}'$	Total mechanic impedance on a diaphragm (mech. absohms ∠).
$\mathbf{z}_d$	Internal mechanic impedance of a diaphragm (mech. absohms ∠).
$\mathbf{z}_v$	Virtual mechanic impedance of a diaphragm due to its vibration in a permanent magnetic field (mech. absohms ∠).
z	Linear impedance of electric conductor (ohms/km.).
$\mathbf{z}$	Linear impedance of acoustic conductor (mech. absohms/cm. ∠).
$z_o, \mathbf{z}_o$	Surge impedance of electric and acoustic conductor (ohms ∠, mech. absohms ∠).
$\mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}$	Acoustic impedance densities on sides 1 and 2 of a diaphragm, and their vector sum (acoustic absohms/sq. cm. ∠).
$\omega = 2\pi f$	Impressed angular velocity (radians/sec.).
∠	Sign indicating a planevector quantity or complex number.
rms	Root mean square.
ab or *abs*	Prefix signifying a C.G.S. unit.
a-c	Alternating-current.
d-c	Direct-current.

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By Louis Bell.

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The visibility of celestial objects, as of objects in general, depends on brightness contrast with the background. The more general light in the field of vision the more must come from a particular object to bring it within range of the eye. The average total brightness of the sky has been shown by Yntema (Gron. Pub.# 22) and van Rhijn (Ap. J. **50**, 374) to be 0.13 or 0.14 of a 1st magnitude star per square degree. Considering the naked eye as an optical instrument of about 5 mm. aperture, one finds equality of the image with this background at about $6^m.5$, which suggests that the contrast required for visibility is fully $0^m.5$. With telescopes the case is less favorable since 35 to 40 per cent. of the light is generally lost in objective and ocular, substantially half in the latter. The loss by reflection goes chiefly outside the instrument as regards the objective. The loss in the ocular is chiefly scattered on diaphragms and other surfaces close to the eye thus lighting up the field, and part actually passes directly to the eye. The result is a material loss of visibility in faint objects due to the enhanced brightness of the field, and sometimes the appearance of false images.

This investigation is directed at the magnitude and distribution of the reflected components of the light and particularly those which are within the ocular, too often constructed without regard to the effect of its reflections on the field viewed by it.

For the present purpose a ghost in an optical instrument may be defined as an image, real or virtual, formed by the reflection of light from the surfaces of the system. It is commonly far out of focus so that it merely forms a hazy spot in the field, or diverging still more widely merely serves to scatter a small amount of light over the whole field. Under certain conditions it may, so to speak, materialize as a fairly bright image, in which case it becomes a nuisance. Most visible ghosts are referable to the ocular of the instrument, whence the desirability of considering the ocular, which has been grossly neglected.

Any beam of light incident normally on a lens system suffers reflection at each refracting surface according to the perfectly well known law established by Fresnel. In any lens system of free surfaces $I' = I(1 - K)^n$ where I' is the transmitted light and k is Fresnel's factor $\left(\frac{\mu - \mu'}{\mu + \mu'}\right)^2$. The transmitted light is then, for unity incident light, $(1 - k)^n$. For practical purposes k varies from a scant 4% in light crown glass to 6% or a little more in dense flint, averaging perhaps 5% in lenses as they are found in practice. The total light thus reflected, and lost to the incident ray as such, evidently increases rapidly with the number of surfaces, and runs about as follows:—

TABLE

Kind of lens	Number surfaces.	Tr.	Ref.	No. 2nd order Ghosts
Single lens	2	.90	.10	1
Double (ordinary objective)	4	.81	.19	6
Triple objective (Cooke, Heliar, etc.)	6	.73	.27	15
Uncemented pair achromats (Ross, Celor, etc.)	8	.65	.35	28
5 lens objective (Beck Isostigmar)	10	.59	.41	45

Measured transmissions are often several per cent. below these figures.

This last case includes prism glasses and ordinary terrestrial telescopes. No account of loss from absorption as such is here included. Absorption ordinarily amounts to between 1 and 2% for 1 cm. thickness following the usual exponential law for greater thicknesses. It does not bear any particular relation to the index of refraction until one comes to very dense flints when the absorption is specific in the blue and violet. More light is lost in a single reflection than in several cm. of glass. For our present purpose no account is taken of loss from imperfect polish, and dust, which may produce some scattering and obstruction of light.

Now the ghosts, visible as such, or spread over the whole field, have an aggregate intensity equal to the total light lost by reflection, and they may be divided into two series, odd and even with respect to the number of reflections. All the odd series are directed toward the incident ray and in themselves produce no visible effect, save as they may be reflected from the inner side of diaphragms. In some instances they may become objectionable, since the diaphragm is likely to be as good or better a reflector than a glass surface. The coefficient of reflection generally is nearer 10 than 5 % for it is a very dense and perfect black indeed that falls below 5%. The ghosts of even order, however, furnish light traveling with the incident ray and consequently the even order ghosts bring light directly into the field and may become distinctly visible.

In general the total of the reflected light is divided into ghosts of the odd and even series in the approximate ratio $k : k^2$. If P be the total reflected light, then any 1st order ghost must be less than $\frac{P}{n}$, any 2nd order less than $\frac{2kP}{n(n-1)}$.

More precisely the ghosts of the first order odd series have intensities kI, $kI(1-k)^3$, $kI(1-k)^5$ and their sum is, putting $t = (1-k)$

$$\Sigma i = kI\,(1 + t^3 + t^5 - - - + t^{2n-1})$$

while the second order ghosts, even series, have an approximate total intensity $k^2\,It^n$, since each ray concerned has to pass at least n surfaces, in average somewhat more.

Intensities of Individual Ghosts.

Since ghosts of the 3rd order have coefficients in k^3 and pass forward they may practically be neglected, and those of the 4th order likewise, having, even if in focus, magnitudes at least 14–15 m. less than the primary image from which they are derived.

Of the n ghosts of the 1st order none can cause trouble save indirectly, as already shown.

There are $\frac{n(n-1)}{2}$ 2nd order ghosts and the maximum brightness of any one ghost in focus must be at least 7.5 m. below its primary. In ordinary objectives and photographic doublets the 2nd order ghosts are 7.5 to 8.0 mg. below the primary even when in focus. The difference can readily be measured by bringing the principal ghost to the area of its primary by changing focus and then using a photographic wedge to measure the relative intensities. The difference in brilliancy comes out at approximately 7–8 m.

Cemented surfaces may practically be left out of account since for ordinary pairs $\left(\frac{\mu - \mu'}{\mu + \mu'}\right)^2$ does not exceed 0.0009 and in a ghost of the 2nd order this small quantity must be multiplied kt^{n-2}, since there is one free reflection and 2 surfaces are abolished by cementing. The available light in a ghost from a cemented doublet is then

$$.0009 \times .05 \times .90 = .00004$$

i.e., such a ghost is 11 mg. fainter than its primary, and is barely discernible even under laboratory conditions.

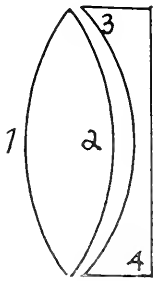

FIG. 1.

Taking up now the typical case of a telescope objective with reference to ghosts, we may, in Figure 1, sketch the form of common shape, numbering its surfaces 1, 2, 3, 4. It will give 6 second order reflections from the following combinations of surfaces

2 – 1
3 – 1
4 – 1
3 – 2
4 – 2
4 – 3

Of these the first four commonly give real images somewhere along the axis, the last two correspond to virtual images in the direction of the incident light, the light being spread to the rear as is usual where a diverging system does the reflecting. The first 3 reflections obviously come to focus relatively near the objective in lenses of ordinary form, and thus merely scatter light of which some actually enters the visible field. The remaining reflection 3–2 is between surfaces optically almost parallel so that after its second reflection the ray is nearly parallel to the transmitted ray incident on 3, and comes to about the same focus.

It is the ghost thus produced which is occasionally troublesome in ordinary objectives, and frequently so in photographic doublets of the Petzval construction. Under any conditions where two consecutive surfaces are of substantially equal curvature the ghost should be looked out for, whether the surfaces are those of an air space or are formed by a lens element of very slight refraction. The exact position of this ghost from the intermediate surfaces of an objective may be quickly found graphically. From any point of surface 2 draw a line to the focus of the front lens considered by itself. The intersection of this line with surface 3 at once gives the angle of incidence there from which the reflected ray can be traced backward and then through the rear lens to its new focal point. If one traces a ray in fact through any combination an inspection of the angles of incidence at the various surfaces will generally show at once where any possible danger from ghosts lies. In dealing with a completed lens a very brief inspection with a bright light in the field or the exposure of a photographic plate will tell the story. In a simple doublet all 6 reflections can be photographed and in a triple or quadruple combination most of the ghosts will show although some may be lost by overlapping.

Effect of the Ghosts.

In an ordinary objective the result of the 2nd order reflections is slightly to brighten the field, except that the air space ghost may appear as a spot. In the photographic case the effects are more conspicuous and where, as in uncemented 4-lens combinations, the total intensity of the reflections may amount to something over 2% of the total light, the clarity of the image may be materially impaired and in photographing nebulae false images may appear. Such systems give pictures distinctly less crisp and brilliant than does the single cemented landscape lens. In the visual case a visible ghostly image requires a surface brightness greater than the background of the sky, in other words comparable with the star disc of the *minimum visibile* in the instrument used.

As the 2nd order ghost in focus is about 7–8 m. fainter than the primary it must be thrown out of focus enough to dim it below practical visibility. In the objective the angle α between the ray incident on surface 3 and its return after two reflections is

$$\alpha = 2\,(i - i')$$

where i' is the angle of reflected incidence on surface 2. Calling this Δi and noting that the deviation of a ray is for small refractions constant, the variation of focal length ΔF is approximately

$$\Delta F = F\frac{\sin \Delta i}{\sin i}$$

The situation is just as if the original ray incident on 3 were tilted through an angle Δi swinging the line to focus along with it.

It should be noted that where $(i - i')$ is large, as for large values of $\varrho_2\ \varrho_3$, ΔF increases, so that for a given focal length high absolute curvature, as in some flint-ahead combinations and objectives of Fraunhofer form, gives larger variation of focus, while in Gaussian objectives where $\varrho_2\ \varrho_3$ are both large and widely different, the ghosts are widespread and the field exceptionally dark, as in the Princeton 9''.6 glass, designed by Professor Young.

In actual practice, objectives of ordinary glasses computed to meet the sine condition give ϱ_2 and ϱ_3 differing by 2–3% which is more than enough to wipe out the visible ghost.

Since, *caeteris paribus*, $(i - i')$ depends on the ratio between ϱ_2 and ϱ_3, roughly,

$$\Delta F = F\frac{2\,\Delta \varrho}{\varrho_3}$$

Whence a difference of 1% between ϱ_2 and ϱ_3 will shift the focus about 2% back or forward according as $\varrho_2 < \varrho_3$ or the reverse. Thus is a telescope of 100 cm. focus and 10 cm. aperture a ghost image of 1'' radius has a diameter of 0.01 mm. The intersect of its cone of light at 2 cm. from focus would be 2 mm. and the surface intensity would be $\frac{1}{40000}$, i.e. 11 mg. below the original, or at least 18 mg. below the primary, a negligible figure even allowing for the greater visibility of an enlarged area.

It is in oculars that troublesome ghosts are most frequently met, sometimes distinctly visible, often brightening the field.

Consider a simple field-lens, A, Figure 2, of radii r, r' receiving substantially parallel light which is reflected from its rear surface. It is subjected to 2 refractions and one reflection and comes to focus as if the lens were replaced by a concave mirror of focus f defined by

$$\frac{1}{f} = \left(\frac{\mu}{r'} - \frac{\mu - 1}{r}\right)$$

For $r = \infty$, the common form of field lens, and common crown glass, $f = \frac{1}{6} F$ approximately, F being the ∞ focus of the lens. This defines the position of the 1st order ghost. The 2nd order ghost due to 2 reflections and one refraction lies at a closely similar distance, f' to the rear. The loci can be readily found by holding a large lens at a fair distance from a lamp and looking for the ghosts with a slip of ground glass. The position of these 2nd order ghosts is not greatly changed by distributing the total curvature over both sides of the lens, while with thick lenses the ghosts lie closer in toward the vertices, and in achromats further out.

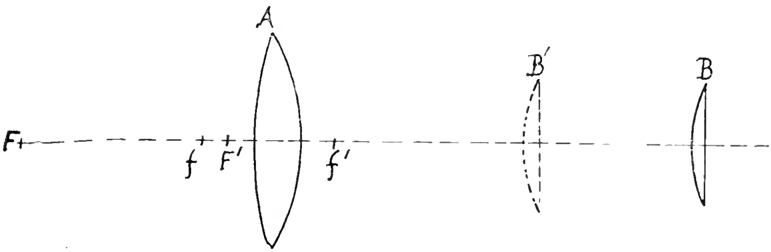

FIG. 2.

It is this 2nd order field-lens ghost that is liable to cause the most trouble. In a 2-lens ocular some 10% of the whole incident light is scattered behind the stop, which may be a large quantity compared with the object under scrutiny, and seriously affect the dark adaptation of the eye. A variable stop, could often be used to advantage.

The ghost focussed at f' lies inside the focus F' of the ocular and is visible as a somewhat blurred spot the more conspicuous the nearer to focus. In a Ramsden ocular of the theoretical form $f = f' = F$ the ghost is very obtrusive.

For example, in the rather thick field lens of a big positive ocular I found the rear ghost at less than $\frac{1}{10}$ the focal length back of the rear vertex. And a very little experimenting with a pocket lens will enable one to realize that such a ghost is not out of focus enough to count. As the ocular is shortened by moving the eye lens B to the conventional separation of $\frac{2}{3} f$ the ghost goes considerably out of focus and appears as a large hazy spot increasing as B moves toward B'. An ocular with sliding fit between field and eye lens gives a very instructive view of the way the ghost comes to approximate focus as the lenses are separated, especially if one half-silvers a surface of the field lens, when the effects are very brilliant.

The best place to study ghosts is in an ordinary prism glass where a prism surface lies just ahead of the flat face of the field lens with three other optically parallel faces in reserve. Three ghosts are conspicuously bright. One, the ordinary field-lens ghost, one from the reflection between the front of field lens and prism face, and the third by reflection from back of field lens and prism face. Besides these at least two others rather faint and dodging quickly about in the field can be made out, chargeable to the adjacent rear prism faces.

With respect to oculars themselves the general rule applies regarding the number and effect of reflecting surfaces, with the same proviso that the 2nd order ghosts are the ones seen, while the 1st order may cause troublesome reflections from the mount.

The fact is that oculars have received very scant treatment at the hands of practical opticians despite the fact that half the light lost in the telescope is generally lost in the ocular, and that it is the seat of no inconsiderable trouble from stray light. It is not the loss of light in the image that hurts, but the effect of the light lost. What is the use of an objective of the highest corrections producing a perfect primary image if that image is magnified by an eye piece of indifferent definition and appears shrouded in a haze of scattered light?

The ordinary ocular has a strongly curved field, somewhat distorted and imperfectly corrected for color. Despite the fact that very material improvements have been made, mostly to meet the severe requirements of microscopic vision, telescope makers stick quite closely to the old Huyghenian and Ramsden forms, chiefly perhaps because these are composed of simple plano-convex lenses and are therefore easy and cheap to make. There is a tendency, too, toward struggling for an abnormally wide field at the expense of definition. Nearly all eye pieces, and particularly these mentioned, have a notably curved field or else are viciously astigmatic in the outer zone, and these faults increase with the angle of view. The eye can take in above 40° but not much more than half this angle is anywhere nearly simultaneously sharp and anything materially over 40° is only attained by peering around the edges of the field.

Fortunately the only cases in which wide field is really useful are those in which the magnifying power is relatively low, which enables the accommodation of the eye to help out. The curvature of the image derived from Petzval's equation is

$$\frac{1}{R_o} - \frac{1}{R_i} = \frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} + \text{etc.} = \frac{1}{\Sigma \mu f}\text{; when } R_o \text{ is large, } \frac{1}{R_i} = \Sigma \frac{1}{\mu f}.$$

Where R_o is the radius of curvature of object, R_i of image, μ, as usual, the index of refraction, and f_1, f_2, etc. the focal lengths of the equivalent thin lenses obtained in the ordinary way. It hence is obvious that the radius of curvature of the sharp image in a single crown glass lens is practically $\frac{3}{2}f$ while in a 2-lens combination widely separated, as in the common forms, it is about $\frac{3}{4}f$. Other oculars lie between these limits. Evidently highly refracting glasses relieve the situation a little, but they are rarely used, being rather harder to get than common crown. Since the easy range of accommodation of the eye except in early youth is not much over 2 diopters the extent to which it can compensate the curvature of the image is rather unsatisfactory, except in low power eyepieces. Conrady (M. N. *78*, 445) in discussing this subject has shown that for a total field of 40° the sharp field fails in ordinary eyepieces for focal lengths under an inch, while a carefully designed achromatic combination, e.g., #4, 5, 9, Figure 3, will reduce this figure to about $\frac{1}{2}$ in., and he further shows a very interesting possibility in the construction of anastigmatic eyepieces to give at once a field at once flat and free from marginal astigmatism. It is to be hoped that this masterly optical investigator will carry out his own suggestions, but even without anastigmats it is quite feasible to get greatly improved eyepieces of moderately large field and with interference from reflections greatly reduced.

The freest of all from reflections are, of course, the single lenses, of which the more common forms are shown in numbers 1 to 5 of Figure 3. The simplest of them is the ordinary plano-convex lens such as is obtained by removing the field lens of a Huyghenian ocular. It gives somewhat increased light but a very small sharp aperture, not over 10°. A little better field and similar illumination is given by the solid eyepiece commonly known under the name of Coddington, although due to Sir David Brewster. It is derived from a glass sphere by removing a thick equatorial belt and then cutting an equatorial groove for a stop, down to a diameter of something less than half the radius of the sphere. Such an ocular of crown glass is of focus about three halves the radius of the sphere and the field is rather larger than that of a simple lens, but badly chromatic toward the edges.

The next step in improvement of results from the single lens is the simple achromat, #3, such as is made for an eyepiece by Cooke and others. When well designed and preferably of high index glasses it gives a field of from 15° to as high as 20°, free of color, but not fully orthoscopic. Next, one comes to the triple achromats, of which #4 is a typical example, as made by Zeiss, Steinheil, and others. The field of such a triplet is usually good for 20° to 30°, colorless, quite flat

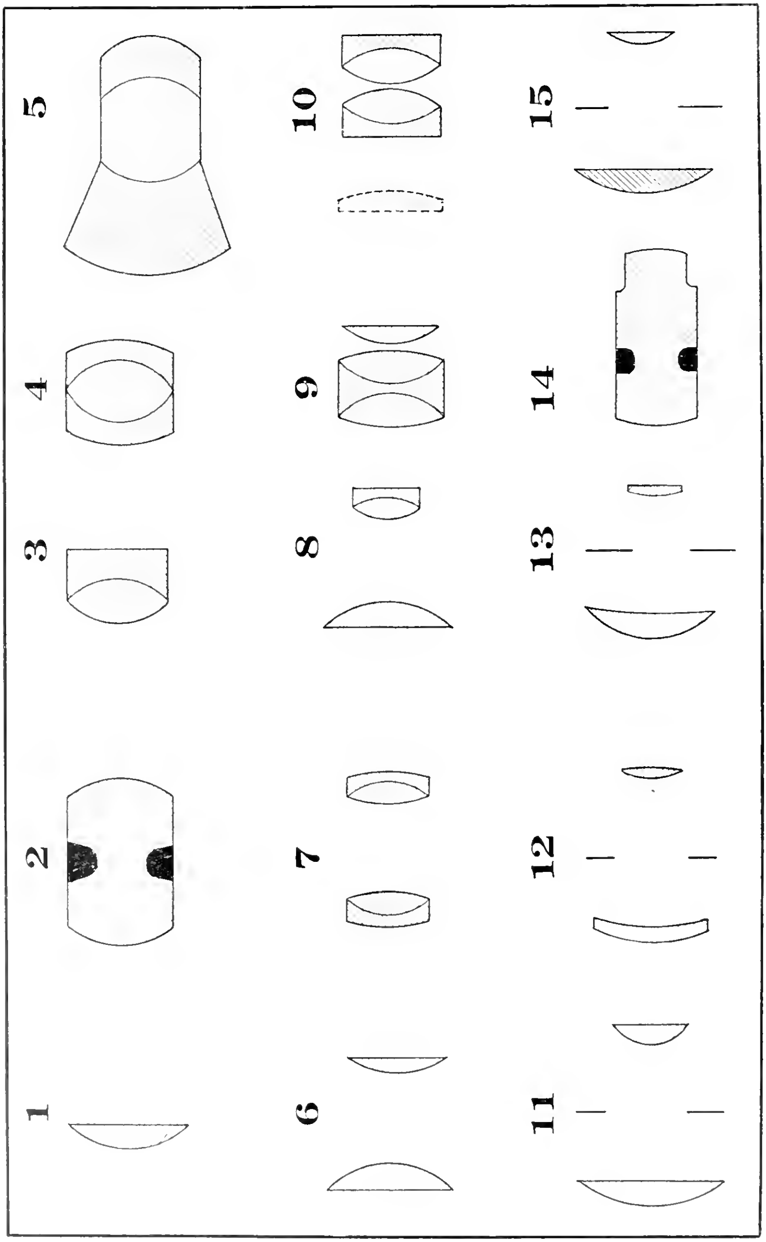

Fig. 3.

and orthoscopic. It is a form which when properly designed makes one of the most useful of eyepieces, beautiful in definition and remarkably free of ghosts.

A modification, #5, is the so-called monocentric eyepiece of Steinheil, in which advantage is taken of thickness of one of the components to secure better flatness of field and also orthoscopy. Thickness gives an additional disposable factor for this purpose in $\frac{d\,i}{d\,h}$ with respect to the last surface. This is a device practically very useful, and employed both in other triplets and in photographic lenses. The monocentric eyepiece has a field of about 32°, remarkably flat and orthoscopic, and like other forms of triplet has a fairly good eye distance. All these oculars are practically free of ghosts.

The positive doublets are commonly troublesome as respects ghosts. As already mentioned, the Ramsden, #6, even at its common separation gives a large hazy ghost as well as some residual color. It has an ordinary field of about 35°. Its achromatic form, #7, has a slightly better aperture, is more nearly orthoscopic and is quite free of color. The ghost is much less troublesome, for the reason already mentioned, that it lies further out of focus.

The ordinary Kellner eyepiece, #8, is the worst of those in common use with respect to ghosts, although otherwise valuable, since the field runs to as high as 45° and the orthoscopy and color are both excellent. The trouble comes from the field lens being so nearly in the focus of the eye lens. Sometimes, indeed, the eye lens focus lies within rather than outside the plane surface of the field lens. Just what the original form of Kellner was is somewhat dubious, but probably it had a double convex field lens which would not, however, improve it in the matter of ghosts.

Two other forms of positive doubtlets are worth mentioning, as they are not only unusually free of ghosts but possess singularly valuable properties. One of them is the so-called orthoscopic ocular of Zeiss and Steinheil #9, consisting of a thick triple achromat with a plano-convex eye lens almost touching it. The field is about 40°, is quite free of visible ghosts for obvious reasons, and is quite flat and orthoscopic, with exquisite definition and a fairly good eye distance. The other form, #10, is a type of long relief ocular much used in artillery sights. It consists of two simple achromats with practically equi-convex crowns placed close together with the crowns almost touching. It is used either with or without a field lens of relatively long focus. When so mounted the field is over 40°, beautifully flat and orthoscopic with the finest of definition clear up to the edge. The focus of the field lens is so long that no ghost is noticeable, while without the field lens the slightly reduced field is still of high quality. The eye lenses being large allow a relief practically equal to the focal length, and sometimes more. It is a combination of which good astronomical use might be made in finders. These two oculars, #9 and #10, are so much superior optically to the ordinary forms that it is a pity they are not more used.

Finally we come to the group of so called negative oculars, all substantially ghost free, and modifications of the Huyghenian type. The freedom from visible ghosts results from the relatively considerable separation of the field and eye lens whereby the 2nd order ghost from the former falls much beyond the focus of the latter and merely scatters a little light over the field. No. 11 is the ordinary Huyghenian with ratio between the focal lengths of field and eye lens of 3:1 to 2:1, giving fields generally of 40° to 45° a little distorted toward the edges, with pretty efficient pseudo-achromatism and generally good definition. No. 12 is the Airy form with the lenses in the ratio of 3:1 as before, but with the field lens a rather strongly curved meniscus, and the eye lens crossed. There is a distinct gain in marginal field and a generally slightly better performance at wide angles.

No. 13 is the Mittenzwey eyepiece, manufactured by Steinheil and others. It is quite similar to the Airy except that it is often made on the 2:1 ratio, the front meniscus is less curved, and the eye lens planoconvex. I can say from experience that these eyepieces are most admirable. The definition is good and they give a preposterously big field, near 50°. This is perhaps the most practical form of eyepiece where the maximum possible field is required, the optical properties being excellent and the meniscus curves being within the bounds of easy construction.

No. 14 is the solid Huyghenian eyepiece of the late R. B. Tolles which he frequently made both for telescopes and microscopes. It is practically a Huyghenian eyepiece with curvature ratios of $1\frac{1}{2}$: 1 between field and eye lens, the eye end being of course convex. A groove stop, in diameter about $\frac{1}{2}$ the focal length, is cut around the long lens at about $\frac{1}{3}$ its length from the field end, practically in the focus of the eye end, of which the radius is substantially half the focal length. This ocular seems to have passed out of use with the death of its original constructor, but it possesses remarkable qualities in that it is quite free of reflections, gives a wide and beautifully sharp field and the brilliancy which accompanies the single lens. Its field is nearly as big as that of the Mittenzwey eyepiece. I recently tried out a $\frac{1}{4}''$ ocular of this kind beside two Mittenzwey oculars and found it distinctly superior to them in brilliancy and at least equal in definition of any eyepiece I have ever seen. It seems to be really a most admirable form, particularly for small telescopes where it is necessary to save all the light possible without sacrifice of other qualities.

Finally I note in #15 a modification of an ordinary Huyghenian ocular which I have recently tried with reference to improving the color correction. It is well known of course that the objectives of telescopes are usually over-corrected for color in order to compensate for the under-achromatism of the eye combined with its eyepiece. This over-achromatization can, of course, compensate accurately only one particular degree of under-correction, that which corresponds to son e selected power, or more precisely, to some selected aperture of the eye, the achromatic error diminishing with the diameter of the emergent pencil from the ocular. The point chosen is usually that which corresponds to about .5 mm. diameter in the emergent pencil. For all lower powers the objective is under-corrected, for higher powers over-corrected.

Therefore it will be of distinct advantage to have the lower oculars somewhat over-corrected and the higher ones under-corrected. Now it is easy to apply under-correction to a telescope ocular in practically the same way that it is applied to the compensating ocular used with apochromatic microscope objectives. Several years ago I had applied the scheme in correcting the residual color in gun sights, and recently tried out the same principle by using, in a Huyghenian of 2:1 ratio, a field lens of highly dispersing flint. The obvious result is to pull down the focus for the blue end of the spectrum, initially too long on account of over-correction.

I have tested this particular ocular of 5 mm. equivalent focus on a much over-corrected $3\frac{1}{2}''$ telescope and a $3''$ of about the usual degree of over-correction with the expected result of cutting down the blue outstanding light, and reducing the in-focus image of an artificial star to the normal hue of the secondary spectrum. The plan gives every appearance of working well, at least for objectives of moderate focal length, and it would seem possible by following it out very materially to improve the color corrections otherwise attainable. Since one cannot achromatize for all powers the best plan would seem to be to achromatize for the most generally useful power, and, since the oculars are far the least expensive part of the telescope outfit, to improve the color correction through modification of these, under-correcting the high powers and over-correcting the low ones. Inasmuch as the eye loses in acuity very rapidly at apertures below 1 mm. it would seem advisable to correct objectives for a rather lower power than is now customary.

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ELECTRICAL RESISTANCE UNDER PRESSURE, INCLUDING CERTAIN LIQUID METALS.

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CONTENTS.

Introduction.

In an earlier investigation [1] I have determined the effect of pressure on the resistance of a number of the metallic elements, and have considered the significance of the results for theories of metallic conduction. The results then obtained suggested a number of important questions which could not then be answered because the data had not been determined. In particular, the effect of pressure on the resistance of only one liquid metal had been measured, mercury, and the comparative effect of pressure on the resistance of the same metal in the solid and the liquid state was not known for any substance. Furthermore, the effect of pressure on none of the alkali metals was known; it is just here that the largest effects would be expected and the most significant results for theory. The extension of the measurements to these substances demanded certain changes and improvements in the technique. I was soon able to make measurements on the alkali metals, and to my very great surprise found that the resistance of lithium increases under pressure, whereas that of the other alkali metals sodium and potassium decreases greatly. It was obvious that our pictures of the mechanism of conduction are not yet so perfect that we can reason by analogy even from one element to another closely related in the periodic table. The importance of making actual measurements on as many of the elements as possible, particularly the rarer ones, was obvious.

In the present work I have therefore attempted to extend the previous measurements to as many new elements as the new resources of my technique, and the availability of the elements themselves has made possible, with especial reference to the question of the resistance of the metals liquid within a moderate temperature range. Furthermore, a number of commercial alloys have been employed in other work, and rough measurements of the effect of pressure on the resistance of them have been necessary; it has been an easy matter to make the measurements precise enough to be included here. Of course the entire question of the effect of pressure on the resistance of the alloys is a most complicated one, and these isolated results cannot as yet have much significance.

The results given here are for eighteen new elements and six commercial alloys. In addition, better results than were possible in the previous work are here given for tungsten and magnesium. Better results on tungsten were possible because of the availability of purer material, and for magnesium the new technique makes possible the elimination of contact resistance, which introduced very large error in the previous work.

Experimental Method.

The metals measured in the previous investigation could all be made into wire of small diameter, and therefore high resistance, and nearly all of them could be soldered. The resistance was measured on a Carey Foster bridge. It was essential to accuracy that the resistance be so high and the contacts so good that the relative changes in contact resistance under pressure were negligible. Very few of the metals of this investigation satisfy this condition. For the measurement of the effect of pressure on the resistance of these, some sort of potentiometer method is indicated. The resistance of the contacts then is without effect, and relatively small changes on small resistances can be measured accurately.

The application of a potentiometer method demands four leads to the pressure chamber, two current leads, and two potential leads. Three of these leads must be insulated from the pressure chamber; the fourth may be grounded on the walls. An insulating plug carrying three leads was used in this work. It is an obvious modification of the single terminal plugs previously used. It may be mentioned that the modification of the design of the plug suggested on page 641 of the paper referred to, making possible the use of the ring packing, has been in use now for a number of years, and was used with the new three terminal plug. The chief difficulties encountered in the three terminal plug were the mechanical difficulties of securing the accuracy required in getting three terminals into a small space. The fine insulated stems were made of piano wire 0.032 inches in diameter, held into a head at the top by a special wedge grip. Each of these stems with its head was tested before assembling with a tension of 270 pounds, corresponding to a pressure of over 20,000 kg/cm^2. Without a preliminary test there is danger that the stem will pull out of the head under pressure. The mica washers used for insulation of the plug were punchings, made a tight fit for the hole, which was 0.209 inches diameter. They were forced into place in the plug, and drilled concentrically for the stem with suitable jigs after they were in final position. Later the mica washers have been replaced with pipe-stone washers with more satisfactory results. The chief difficulty encountered with this plug has been in using it near 100°. Here the rubber insulation gives out under the action of the petroleum ether with which pressure is transmitted, and the insulation has to be renewed much more frequently than with the larger plug.

The method of electrical measurement is a null substitution method. In series with the material under pressure is a second resistance, part of which consists of a slide wire. By means of a variable slider, it is possible to tap off a variable part of this second resistance. A throw-over switch enables either the potential terminals of the pressure coil or the variable part of the series resistance to be connected to the indicating galvanometer. This galvanometer is a Leeds and Northrup high sensitivity moving coil instrument. At the scale distance used its sensitiveness was 10^{-9} volts; its sensitiveness could be decreased with appropriate shunts.

The measurements are made by adjusting the various resistances so that there is no change of deflection on operating the throw-over switch. The resistance of the pressure coil may then be computed from the known values of the other resistances. In practise, sensitiveness and speed of operation are increased by throwing into the galvanometer circuit another e.m.f. approximately equal and opposite to the potential difference across the pressure coil, so that the actual deflection is approximately zero. This balancing e.m.f. should be variable over a wide range and should be fairly constant. To produce it, I used the apparatus previously used in measuring thermal e.m.f. under pressure, tapping across the former pressure terminals. The refinements of that apparatus were not necessary, but it was easier to use apparatus already at hand than to construct new. The apparatus by which the variable balancing e.m.f. was applied is indicated by VE in Figure 1.

The details of the connections are shown in Figure 1. When adjustments are made we know that the potential drop around R (the pressure coil, which is usually a small fraction of an ohm) is equal to that about R_3 plus r_1. This latter drop of potential may be computed from the known values of R_1, R_2, R_3, r_1, and r_2, and gives

$$R = \frac{R_1(R_3 + r_1)}{R_1 + R_2 + R_3 + r_1 + r_2}.$$

The resistance r_1 (and r_2 accordingly) is the only resistance varied during a pressure run; the other resistances R_1, R_2, and R_3 being appropriately chosen and then kept constant during each run. Since $r_1 + r_2 = r$ (a constant) we have, for any one run,

$$\Delta R = C\,\Delta r_1$$

or, putting $R = R_0 + \Delta R$, and writing r_{10} for the initial value of r_1 corresponding to $R = R_0$

$$\frac{\Delta R}{R_0} = \frac{\Delta r_1}{R_3 + r_{10}} = \frac{\Delta l}{L_3 + l_0},$$

where L_3 is the length of the slide wire having the resistance R_3, and l_0 and Δl are the actual readings of the slide wire. This arrangement therefore, gives immediately in terms of slide wire settings the proportional change of resistance under pressure. If R_3 is kept constant for the runs on the same metal at different temperatures, making the necessary adjustments in passing from one temperature to another by changing only R_1 and R_2, the readings give directly a comparison of the pressure coefficients at different temperatures without demanding a knowledge of how the resistance itself varies with temperature. This was the procedure followed in this work; the pressure coefficients

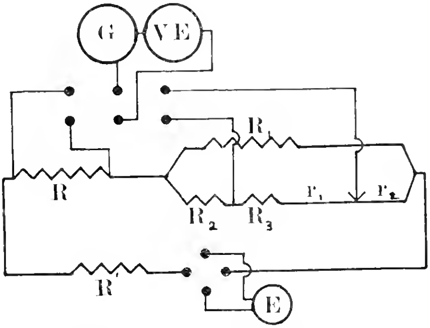


FIGURE 1. The electrical connections by which resistance is measured. The resistances are so adjusted that the potential drop across R is equal to that across $R_3 + r_1$.

are therefore independent of any error in the temperature coefficient of resistance at atmospheric pressure.

In actual construction, all parts practical were made of copper. The resistances R_1 and R_3 were fixed coils dipping in mercury cups in $\frac{3}{4}$ inch copper bars. The apparatus was so designed that all terminals were within a few inches of each other, in order to avoid thermal effects. The slide wire was of manganin. The slider was also of manganin, and was attached by soldering to a flexible many-stranded manganin wire making connection at the other end to a copper block surrounded by the bars containing the other terminals. By soldering the flexible manganin lead to the slider very close to the point of contact with the slide wire, thermal electromotive forces in this part of the circuit were very largely avoided.

The effect of parasitic electromotive forces in the circuit was eliminated by taking the mean of readings with direct and reversed potentiometer current. A reversing switch was supplied for this purpose, as also in the circuit of the balancing variable e.m.f. The only requirement in the potentiometer current is that it shall remain constant during the substitution of the pressure coil for the slider. The substitution was made rapidly by a double throw switch, so that an ordinary dry cell was entirely good enough as the source of the potentiometer current.

The function of the resistance R^1 (Figure 1) was merely to provide an additional adjustment by which the deflection of the galvanometer might be easily made null. An ordinary decade box was good enough for this.

The usual procedure was as follows. A preliminary examination indicated the order of the pressure effect. R_3 was then so chosen that the motion of the slider would be of the order of 40 cm. for the entire pressure range, the total length of the slide wire being 60 cm. R_1 and R_2 were then so chosen, depending on the actual resistance of the pressure coil, that the galvanometer deflection was unaltered on substituting the pressure coil for the slide wire. The following coils for R_1 and R_3 usually gave sufficient range of choice; two 0.5 ohm coils, two 1's, and one each of 2, 3, 4, 10, 20, and 30 ohms. For R_2 a plug box from 0.1 to 1000 ohms was used. R^1 was then set at about 2000 ohms, or less if the parasitic e.m.f.'s. were troublesome, and the balancing e.m.f. adjusted to give no deflection. These adjustments were preliminary to the application of pressure. The adjustments after each change of pressure were, first, an adjustment of R^1 to give again approximately null deflection when the pressure coil is in circuit, second, setting of the slider to give no change in deflection on substitution, third, reversal of the potentiometer current and the balancing e.m.f. and resetting of the slider for no change of deflection, and fourth, change of the potentiometer current and the balancing e.m.f. back to their original directions and again setting the slider. By taking an odd number of readings the effect of any slow change of the parasitic e.m.f. due to dissipation of the heat of compression was eliminated. The potentiometer current was usually so chosen that the difference of slider setting for the two directions of the current occasioned by the parasitic e.m.f. was less than 1 cm.

The coils were all compared with standards and proper corrections applied. The slide wire was calibrated and corrections applied for lack of uniformity. In general, all the precautions of manipulation and construction previously employed in measurements of resistance or thermal e.m.f. under pressure were observed here also.

The methods of computation were essentially the same as those which have been described in great detail in the previous paper. Any slight modifications were entirely obvious, and it is not necessary to describe them further.

A description in detail of the results obtained for the separate substances now follows.

Detailed Data for Individual Substances.

Lithium. Two distinct series of measurements were made on this substance, at two different times. The first series was on the pressure coefficient of the solid, in the spring of 1919. It was found that the pressure coefficient is positive, and large. This result was so surprising in view of the high compressibility of the metal, and its close relation to sodium and potassium, both of which have a very high negative coefficient, that a correlation of this positive coefficient with other properties of lithium was desirable. For instance, does lithium expand in freezing, like bismuth, and if so, is this connected in any way with the positive coefficient? But on looking up the data I could not find that the melting data for lithium had ever been determined. It was not even known whether lithium expands or contracts on melting. The second series of measurements, in the winter of 1919–20, was concerned with the effort to obtain some of the missing data. In particular, it was desirable to find the volume relations on melting, and to find whether the pressure coefficient of the liquid is positive as well as that of the solid. I had already found that the positive coefficient of solid bismuth changes to negative on melting.

The first series of measurements, on the resistance of the solid, was made on lithium from Merck, prepared a number of years ago, but kept under oil in sealed glass since then. A chemical analysis by Mr. N. S. Drake showed 0.7% Al, and a trace of Fe. Sodium, if any, could not be determined because only one gram of the lithium was available for analysis. The method of preparation by electrolysis should not allow much impurity of sodium if ordinary care is exercised.

The lithium was extruded cold through a steel die into wire 0.030 inches in diameter, and wound bare onto a bone core. Connections were made at the ends with spring clips. So far as I am aware, no previous measurements have been made on the properties of bare lithium wire, but because of danger of oxidation, the lithium has been usually enclosed in a glass envelope. Measurements in glass, such as that of temperature coefficient of resistance for example, are not entirely free from objection, because of the constraining action of the glass walls. The error introduced by effects of this sort is presumably not large, but it is nevertheless gratifying to be able to avoid it.

A somewhat special technique is necessary to handle the bare wire successfully. It must, of course, be protected at all times from direct contact with the air. This was accomplished at first by extruding it directly into melted white vaseline, and winding it directly from the pot of vaseline onto the bone core, which was mounted for slow rotation by hand. The wire, in passing from the pot of vaseline to the bone core, becomes covered with a capillary film of vaseline, which rapidly solidifies in the air, forming a perfectly protecting coating. Later, however, the vaseline was replaced with a mixture of "Nujol," which is a carefully refined heavy hydrocarbon, prepared by the Standard Oil Co. and sold as a remedy for constipation, and refined paraffine, melted together in such proportions as to have about the consistency of vaseline. This mixture was suggested to me by Dr. Conant of the Chemistry Department. This change was made necessary by the chemical action of the vaseline. The vaseline of commerce is not a substance of standardized properties; the first can of vaseline which I tried was without appreciable chemical action, but the second was unpleasantly corrosive in its action. The mixture of Nujol and paraffine was much more satisfactory, the lithium remaining bright for days.

The liquid transmitting pressure must also be chosen with care in order to avoid chemical action. At first I used commercial kerosene which had been standing in contact with sodium for several weeks. This was not satisfactory, however, the kerosene gradually turning yellow at room temperature in contact with the sodium, and at higher temperatures the reaction is much accelerated. The transmitting medium finally used was a mixture of Nujol and "petroleum ether" in different proportions, depending on the temperature of the work. At 0° nearly pure petroleum ether must be used in order to avoid freezing under pressure. Another source of chemical action, besides the transmitting medium, is the bone core on which the wire is wound. It is necessary to drive the water as completely as possible out of the core by prolonged heating to 130° or so, but under these conditions the bone becomes very brittle and must be handled with extreme care. Even after every precaution had been taken, some chemical action still remained. The action tends to cease at the higher pressures. Error from chemical action was eliminated as far as possible by taking the means of readings with increasing and decreasing pressure, and at the higher temperatures by never releasing the pressure to atmospheric, but obtaining the zero reading from an extrapolation of the readings at higher pressures, where the chemical action is slower. The technique in handling sodium was the same as that used for lithium. It is a curious fact that although the chemical action of the mixture of Nujol and paraffine at atmospheric pressure is considerably less on the lithium than on the sodium, at higher pressures the reduction of the action is considerably greater in the case of sodium, so that the zero shift after a run at higher pressures and temperatures was greater in the case of lithium than sodium.

Runs were made on the effect of pressure on the resistance of the bare wire at 0°, 25°, 50° (partial run), 75°, and 96°. The difference between readings with increasing and decreasing pressure decreased uniformly from zero to the maximum pressure, instead of being almost entirely confined to the zero reading, as was the case with sodium. The zero shifts were 7% of the total effect at 0°, 5.5% at 25°, 7.8% at 75° and 18% at 96°. The run at 50° was not completed because of accident. In spite of the large zero shifts, the mean of the readings with increasing and decreasing pressure ran smoothly, and should be only little affected by the chemical action.

The temperature coefficient of resistance at atmospheric pressure was obtained from a coil of bare wire similar to that of the pressure measurements. In order to avoid as much as possible the effect of chemical action, four thermostats were kept running simultaneously at 0°, 25°, 50°, and 75°. The coil was immersed in a well of Nujol which had previously come to the temperature of the bath. After a reading at one temperature the coil was transferred in a few seconds to the bath at the next temperature, and readings made after a fixed constant interval. Seventeen minutes proved to be sufficient for acquiring complete thermal equilibrium. Readings were made successively from 0° to the maximum and back to 0° again. The mean of the ascending and descending readings should be free from error from chemical action. The zero shift after the run was 2.8% of the total effect, against 5.4% for sodium. The average coefficient between 0° and 100° was 0.00458. Bernini [2] found for lithium in glass the mean value 0.00457 between 0° and 177.8°. He found the relation between temperature and resistance to be linear. I found the resistance to increase more rapidly at the higher temperatures;

this would mean an average coefficient between 0° and 177.8° even larger than 0.00458.

The independent pressure runs at different temperatures did not fit as smoothly together as they frequently do, but the pressure effect varied irregularly from one temperature to another. Within the limits of error a dependence of pressure coefficient on temperature could not be established, and in the finally smoothed results the pressure coefficient is assumed independent of temperature. This demanded a maximum readjustment of the observed readings of 2.0% at 75°.

TABLE I.

Resistance of Lithium.

Pressure kg/cm²	Resistance 0°	25°	50°	75°	100°
0	1.0000	1.1044	1.2122	1.3280	1.4580
1000	1.0069	1.1120	1.2206	1.3372	1.4681
2000	1.0140	1.1199	1.2292	1.3466	1.4784
3000	1.0212	1.1278	1.2379	1.3561	1.4889
4000	1.0285	1.1359	1.2468	1.3655	1.4996
5000	1.0360	1.1442	1.2558	1.3758	1.5105
6000	1.0436	1.1525	1.2651	1.3859	1.5216
7000	1.0514	1.1612	1.2745	1.3963	1.5329
8000	1.0591	1.1700	1.2842	1.4069	1.5446
9000	1.0675	1.1789	1.2940	1.4177	1.5564
10000	1.0757	1.1880	1.3039	1.4285	1.5683
11000	1.0841	1.1973	1.3142	1.4397	1.5806
12000	1.0927	1.2068	1.3246	1.4513	1.5932

Average coefficient 0 to 12000 kg. + 0.0_5772.
This coefficient is independent of temperature.

The values of the resistance of the solid at 25° intervals of temperature and 1000 kg. intervals of pressure are shown in Table I. The pressure coefficient is seen to be positive, as already noted. Furthermore, the pressure coefficients, both instantaneous and absolute, increase with increasing pressure. This we would not expect, but it seems to be the normal type of behavior for substances with positive coefficient.

The second series of measurements was made with another sample of lithium, much larger in amount, which I obtained through the kindness of Dr. A. W. Hull of the Research Laboratory of the General Electric Co. The purity was not known. It was prepared by electrolysis of the fused chloride with graphite electrodes; the chloride was pure, so that any impurities were introduced in the electrolysis. There were inclusions in it of some slag-like substance which had to be cut out as well as possible. This specimen was used for a determination of the melting curve. The method was that of the discontinuity of volume, which I have previously used in determining melting or transition curves, and has been fully described elsewhere.[3] The volume of the specimen was about 5 c.c. It was placed in an iron container, and pressure transmitted to it with Nujol. It was evidently somewhat impure, for the corners of the melting curve were

TABLE II.

Melting Curve of Lithium.

Pressure kg/cm²	Temperature
0	178.4°
1000	182.1
2000	185.5
3000	188.8
4000	191.8
5000	194.6
6000	197.1
7000	199.4
8000	201.6

considerably rounded, melting being detectible at least 1000 kg. before the end of melting. This corresponds to a spreading of melting at constant pressure over a temperature range of 3.5°. What the depression of the freezing point is at the conclusion of melting, that is, how much the observed curve should be raised in order to correct for the impurity, it is not possible to state from the data. Points on the freezing curve were obtained at three temperatures. A curve was drawn through these points and the data from the smooth curve are given in Table II. The curve and the observed points are also shown in Figure 2. The points lie on the curve as drawn, but it is seen that there may be some uncertainty about the extrapolation to atmospheric pressure. The value of the melting temperature at atmospheric pressure given in the Table was extrapolated from the three observed points on the assumption that $\frac{d^2\tau}{dp^2}$ is constant.

The most important result of these melting observations is that the melting curve is normal, pressure and temperature rising together. This means that the solid contracts on freezing, as is normal. The positive pressure coefficient of resistance of the solid therefore need not be due to the peculiar mechanism that it is in the case of bismuth.

The changes of volume on melting are very small, and because of the rounding of the corners, very hard to determine accurately. The error was too great to establish any regular variation of the change along the melting curve. The best mean value for the fractional increase of volume on melting is 0.006.

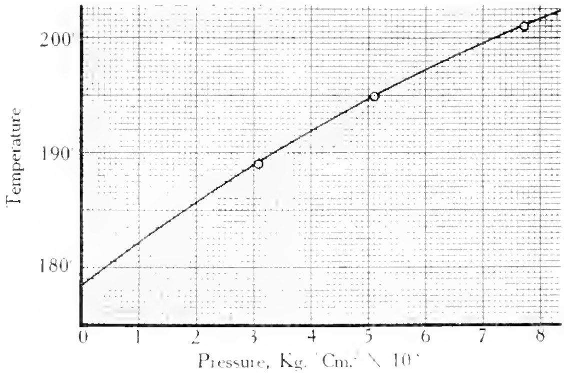


FIGURE 2. The melting curve of lithium.

The effect of pressure on the resistance of the liquid was measured on the same sample as that of the melting curve, the sample from Merck having been used up by the chemical analysis. Considerable difficulty was met in devising a suitable method. It is well known that melted lithium attacks glass, and this I verified by my own experience. Previous experimenters have been able to measure the resistance of the liquid in a glass capillary by protecting the interior surface of the capillary with a film of oil. This introduced error because of the space occupied by the film of oil, and is not adapted to use under pressure, the oil film being penetrated after a few compressions. The method finally adopted was to surround the lithium with a capillary of an alloy of high specific resistance, which is not attacked by the lithium. For this purpose I used the allow known under the trade name of "#193 Alloy," manufactured by the Driver Harris Co. The specific resistance is about 50 times that of copper. I succeeded in drawing this into a fine capillary 0.045 inches outside diameter, and 0.032 inside diameter. An attempt to treat Nichrome similarly did not meet with such ready success, it being much harder to draw. The capillary was plugged at one end with a pin of iron, and at the other end an iron cup was silver soldered, no silver solder coming in contact with the lithium. Four copper leads were soldered to the outside of the capillary, two at each end, for use as current and potential leads, and the resistance was measured in the regular way by the potentiometer method. The capillary was filled by melting the lithium into it in vacuum. The resistance of the capillary was from five to ten times the resistance of the lithium which filled it, varying with the pressure and temperature. Preliminary measurements were made on the resistance of the capillary when empty, and on its temperature and pressure coefficients. These values for the capillary are given for themselves under the heading #193 Alloy. The temperature and pressure coefficients are both small compared with those of lithium.

The capillary and the lithium inside it constitute two resistances in parallel. The various resistances are connected by the relation

$$\frac{1}{R_0} = \frac{1}{R_c} + \frac{1}{R_L}$$

where R_L is the resistance of the lithium filling the capillary, R_c is the resistance of the capillary alone, and R_0 is the observed resistance of the capillary and lithium in parallel. By differentiating this expression with respect to the pressure we obtain

$$\frac{1}{R_L}\frac{dR_L}{dp} = \frac{R_L}{R_0}\left(\frac{1}{R_0}\frac{dR_0}{dp}\right) - \frac{R_L}{R_c}\left(\frac{1}{R_c}\frac{dR_c}{dp}\right).$$

From the pressure coefficient as observed, and the pressure coefficient of the capillary separately determined, it is therefore possible to obtain the pressure coefficient of the lithium alone which fills the capillary. The term involving the pressure coefficient of the capillary is seen to be small, so that this coefficient need not be known with great accuracy, and the ratio of the resistance of the lithium to the observed resistance may be found with any accuracy desired, so that the results should not have appreciably any larger error than the observed resistances themselves.

It is obvious that the temperature coefficient of resistance may be found in a way precisely similar to the pressure coefficient.

Readings were made on the resistance of the liquid as a function of pressure at 202.5° and 237.4°. The pressure range of the lower temperature run was 8000 kg., since the melting curve restricts the domain of existence of the liquid, but at the higher temperature the pressure range was the entire 12000 kg. In order to avoid chemical action as far as possible, pressure was not released entirely to zero, but the minimum was about 1000 kg., and the results were extrapolated to zero. Measurements were also made on the solid at 171.6° to 8000 kg. (pressure was not raised higher for fear of distorting the capillary), and on the resistance of the solid as a function of temperature at atmospheric pressure down to 0°.

The most important result is that the pressure coefficient of the liquid is positive like that of the solid, reversing the behavior of bismuth. At the two temperatures the relation between pressure and resistance was linear within the limits of error of the measurements. At the lower temperature the maximum departure of any observed point from a straight line was 2% of the total effect, and at the higher temperature it was 1.3%. The coefficient is $+ 0.0_5927$, independent of temperature to the last figure. The coefficient of the liquid is seen to be slightly larger than that of the solid. The correction for the capillary brought the observed value from 0.0_5700 to 0.0_5927.

The temperature coefficient of the liquid, corrected for the capillary, was such as to give between 202° and 237° an increase of 0.00145 of the resistance at 202° for every degree rise of temperature. This is less than the reciprocal of the absolute temperature, giving, when multiplied by the absolute temperature, 0.689. The coefficient of the solid was found to be markedly higher than the reciprocal of the absolute temperature. This is again an example of the fact that the temperature coefficient of the liquid is in general less than that of the solid. Bernini [2] found for the liquid between 180° and 200° a coefficient equal to 0.00077 of the resistance at 200°, considerably lower than the value above.

No correction has been applied to the above values for the compressibility or thermal expansion of the capillary, since these are not known. We have seen that the temperature correction is in general slight. If the compressibility of the alloy is the same as that of pure iron, which is a not unplausible assumption, the coefficient of the specific resistance will be about 2% less than the value above.

The pressure coefficient of the solid lithium was measured in the capillary at 171°. The most important result of this measurement was the verification of the positive coefficient. The numerical value of the coefficient cannot be accepted, however, because of the restraining effect of the capillary. A similar effect had already been met in the case of Gallium. The results for the resistance of the solid were very irregular, there being deviations of 10% from the mean curve. The value found for the coefficient was $+0.0_4159$, nearly twice the value found at low temperatures for the bare wire. It is quite possible that part of this large difference is real, since we in general expect the coefficient to become larger at the higher temperatures. It was not possible to definitely state any variation of the coefficient within the range 0° to 100° for the bare wire, since chemical action cut down the accuracy of the measurements. It is also possible that some of the difference may be due to difference of the material, this latter specimen not being so pure. The pressure coefficient of the solid in the capillary was also measured at 94°, but the irregularities of the data were much greater than at 171°. One would expect the effect of constraint to become greater farther from the melting point. The best value of the coefficient at 94° was 25% higher than at 171°, but the accuracy was so low that it is not at all certain that there was any real difference.

The temperature coefficient of the solid in the capillary was also measured between 0° and 171°. Here again we should not expect the results to be very accurate because of the effects of constraint. The mean coefficient over this range was 0.0039. This is considerably less than for the unconstrained wire; part of the difference may be due to greater impurity.

The change in the resistance on melting at atmospheric pressure could be computed from the measurements on solid and liquid separately. The specific resistance of the liquid is thus found to be 1.68 times that of the solid at the melting point at atmospheric pressure. The only other published value is by Bernini,[2] who found 2.51. Bernini's value of the melting point was 177.84°, somewhat lower than the value given above, so that it is not probable that the difference is to be ascribed to greater purity of his sample. I also made an attempt to find the variation of the ratio of specific resistance of solid and liquid along the melting curve, but this cannot be very accurate, because of the constraining effect of the capillary. Using the value found for the pressure and temperature coefficients of the solid in the capillary, and assuming that the temperature coefficient is not affected by pressure (which has been proved to be true by direct measurement in most cases), the resistance of the solid was extrapolated to the melting curve at 202.5° and 8430 kg., and at this point the ratio of resistance of liquid to solid found to be 1.49. The accuracy is not high, and probably the only conclusion justified is that the change of the ratio along the melting curve is not large.

Sodium. The material was from the same sample as that previously used in determinations of the change of melting point under pressure.[4] No chemical analysis has been made, but the sharpness of the melting is evidence of the high purity. Two series of measurements of the resistance were made. The first, in the spring of 1919, was on the resistance of the solid below the melting point at atmospheric pressure. The second series, in the fall and winter of 1919–20, was on the resistance of both solid and liquid at temperatures above 97.6°.

For the measurements of the resistance of the solid below 97.6° the sodium was extruded into wire through a steel die, and the wire wound bare on a bone core. As in the case of Lithium, I am not aware that measurements have been made previously on the resistance of the bare wire, but the material has been enclosed in a glass envelope. The details of manipulation were essentially the same as for lithium, except that sodium is much softer, and consequently harder to handle. Two sizes of wire were used, at first 0.015 inches in diameter, and later 0.030. The larger size is considerably easier to handle, and the resistance is high enough to give the requisite accuracy. The initial resistance was of the order of 0.5 ohms for the small wire, and 0.2 ohms for the larger wire. Connections were made to the sodium wire with spring clips, as with lithium. The springs were helical springs of piano wire, 0.009 inches diameter, wound on a mandrel 0.06 inches diameter. The springs were pulled out so as to separate the spires, and the sodium wire dropped in between the spires. The wire was protected between the spires by wrapping round it a covering of gold or silver foil 0.001 or 0.002 inches thick; otherwise the soft wire is pinched off by the spring. Due to chemical action, the resistance at the spring contacts sometimes became so high as to make trouble. Contact could always be restored by passing through the contact a current from a small magneto.

Experiments were made on six different samples before a complete set of readings was obtained. Repetitions were necessary because of the effect of corrosion, and also because of accidents to the insulation of the three-terminal plug. In addition to complete runs at 0°, 25°, 50°, and 75°, one fragmentary run was made at 0°, and three at 50°.

These fragmentary runs have been given due weight in the final results.

No preliminary pressure seasoning of the wire was attempted. This is usually unnecessary for soft metals, and in this case was undesirable because of the effect of chemical action. The amount of chemical action may be estimated from the amount of the change of the zero after a run. At 0° the zero change was 5.6% of the total pressure effect; at 25° 8.1%; at 50° 1.9%, and at 75° at 1000 kg. 4.7%. The smaller effects at 50° and 75° are because the Nujol mixture was used at these temperatures to transmit pressure instead of kerosene. Since the readings with increasing and decreasing pressure were made at uniform time intervals, the mean zero should contain little error from corrosion. Aside from the zero displacements, the points at high pressures lay very regularly on smooth curves. At 0° the greatest departure of any point from a smooth curve was 1.6%; at 25° 1.3%, at 50° 0.8%, and at 75° 1.0%.

The final results were obtained by first smoothing independently the results at each temperature, and then smoothing the runs at each temperature so as to give smooth temperature differences. The maximum adjustment in this temperature smoothing was at 50° and 3000 kg., where an increase in the observed readings of 1.2% of the total effect was necessary.

The temperature coefficient at atmospheric pressure was obtained from a coil of bare wire similar to that of the pressure measurements. The details of the measurements were exactly the same as for lithium. The two readings at 0° differed by 5.4% of the total temperature effect. The relation between temperature and resistance can be expressed by a second degree equation in the temperature. The results at even temperature intervals are included in Table III. The resistance of the solid at 100° (melting point 97.62°) may be extrapolated from readings between 0° and 75° and gives as the average temperature coefficient between 0° and 100° 0.005465. Northrup [5] gives for the temperature coefficient of sodium in glass the value 0.0053, obtained by a linear extrapolation of values between 20° and 93.5°. Bernini [6] gives for the same temperature range (0° to 100°) 0.00428. As already remarked, there seem to be no previous values on the unconstrained metal.

The measurements on the resistance in the domain of both liquid and solid above 97.6° were made with the sodium enclosed in a glass capillary. The details were exactly the same as for potassium. In point of time the measurements of potassium were made first, and the

TABLE III.

Resistance of Sodium.

Pressure kg/cm²	0°	20°	40°	60°	80°	100°	120°	140°	160°	180°	200°
0	1.0000	1.0950	1.1971	1.3078	1.4227	2.2381	2.3835	2.5399	2.7054	2.8849	3.0725
1000	.9376	1.0248	1.1181	1.2168	1.3196	1.4257	2.181	2.326	2.476	2.639	2.808
2000	.8830	.9641	1.0513	1.1111	1.2340	1.3283	2.018	2.147	2.283	2.433	2.580
3000	.8348	.9192	.9926	1.0762	1.1608	1.2151	1.877	1.995	2.120	2.257	2.397
4000	.7921	.8647	.9409	1.0192	1.0971	1.1739	1.226	1.866	1.982	2.106	2.235
5000	.7550	.8235	.8955	.9692	1.0419	1.1123	1.147	1.755	1.861	1.975	2.092
6000	.7218	.7865	.8517	.9240	.9926	1.0590	1.077	1.178	1.755	1.861	1.968
7000	.6922	.7538	.8183	.8844	.9495	1.0118	1.017	1.112	1.662	1.758	1.857
8000	.6658	.7243	.7862	.8492	.9100	.9695	0.964	1.053	1.578	1.667	1.758
9000	.6422	.6980	.7569	.8167	.8758	.9320	0.917	1.002	1.505	1.588	1.670
10000	.6211	.6741	.7305	.7878	.8443	.8986	0.875	0.956	1.033	1.517	1.593
11000	.6022	.6528	.7070	.7618	.8163	.8690	0.841	0.919	0.991	1.455	1.525
12000	.5854	.6332	.6854	.7381	.7908	.8425	0.810	0.885	0.954	1.401	1.464

description of the capillary and the method of filling it will be found under potassium. Seven runs were made with the sodium in glass, with several different capillaries. These runs were at 134.0°, on both solid and liquid, at 143.6° on the liquid only, at 163.1° on the liquid only, at 165.0° on both liquid and solid, and at 171.8°, 197.1° and 198.1° on the liquid only. In addition, readings were made at atmospheric pressure from which the change of resistance on melting and the temperature coefficient of resistance of the liquid could be obtained. These runs were of varying degrees of excellence. Those at 165° and 197° showed zero shifts of only 0.5% and 0.3%, and were given the greatest weight in computing the resistance of the solid. In general the observed points lay very smoothly, and there was little difference between the results with increasing and decreasing pressure, indicating little direct effect due to the constraint exerted by the walls of the capillary.

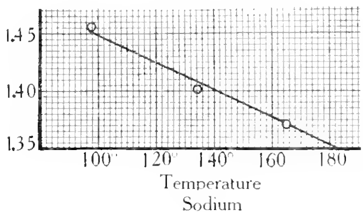


FIGURE 3. The ratio of the resistance of liquid to solid sodium as a function of temperature along the melting curve.

The ratio of resistance of liquid to solid was observed at three temperatures; the observed points are shown in Figure 3. The ratio decreases somewhat with increasing pressure and temperature along the melting curve. In reducing the observed values to smoothness, it was assumed that the ratio varies linearly with temperature, and is given by the line shown in the figure.

The values of resistance as a function of pressure and temperature from 0° to 200° and to 12000 kg. are shown in Table III. This table falls into two parts; the first part, including the values through 100°, are relative values of the "observed" resistance, being derived from measurements on the bare wire. Above 100° the values listed are relative values of the specific resistance, the observations having been made on the sodium in a glass capillary, and corrections applied for the compressibility and thermal expansion of the glass. For the linear expansion the value 0.0_58 was assumed, and for the cubic compressibility Amagat's figure 2.2×10^{-6}, with his temperature coefficient of compressibility of 10% for 100°. In order to reduce the part of the table below 100° to relative specific resistances or that above 100° to relative "observed" resistances, it would have been necessary to have known the compressibility of sodium over this range of pressures and temperatures, and this has not yet been determined experimentally. From the differences of the pressure coefficients in the two parts of the table, however, it is possible to get some idea of the magnitude of the compressibility. Thus it will be found that the mean coefficient of "observed" resistance between 5000 and 12000 kg. at 100° becomes consistent with the mean coefficient of specific resistance over the same range if the compressibility is 0.00002. Richards found for the initial compressibility at 20° the value 0.000015. The difference between these two values does not mean an impossibly large temperature effect.

In Figure 4 the isothermals of resistance against pressure have been drawn for a number of temperatures. The values are taken from the table and have the same discontinuity at 100° as the values of the table. In fact, this discontinuity is quite evident in the figure. The change of resistance with pressure is seen to be large, larger than for any other metal which I have measured except potassium. Under 12000 kg. the change of resistance of the solid is of the order of 40% of the initial resistance. The mean coefficient of the liquid is larger, the decrement being about 50% for the same pressure range. The initial coefficient of the liquid varies little with temperature, but the initial coefficient of the solid increases markedly with rising temperature.

The pressure coefficient of resistance of sodium has not been previously measured, so there are no other values for comparison, but other observers have measured the temperature coefficient of solid and liquid and the ratio of resistance of liquid to solid at atmospheric pressure. The values of Northrup and Bernini for the coefficient of the solid have been already quoted. It is to be noticed that the values of Northrup and Bernini are for the specific resistance, since their materials were enclosed in a rigid container, whereas my coefficient is of the "observed" resistance, and was obtained on the bare solid. The coefficient of specific resistance is greater than that of "observed" resistance by the linear expansion. Taking as the linear expansion of sodium 0.000069, my value of the coefficient of the "observed" resistance would give 0.00552 for the coefficient of the specific resistance. This value is seen to be much higher than that

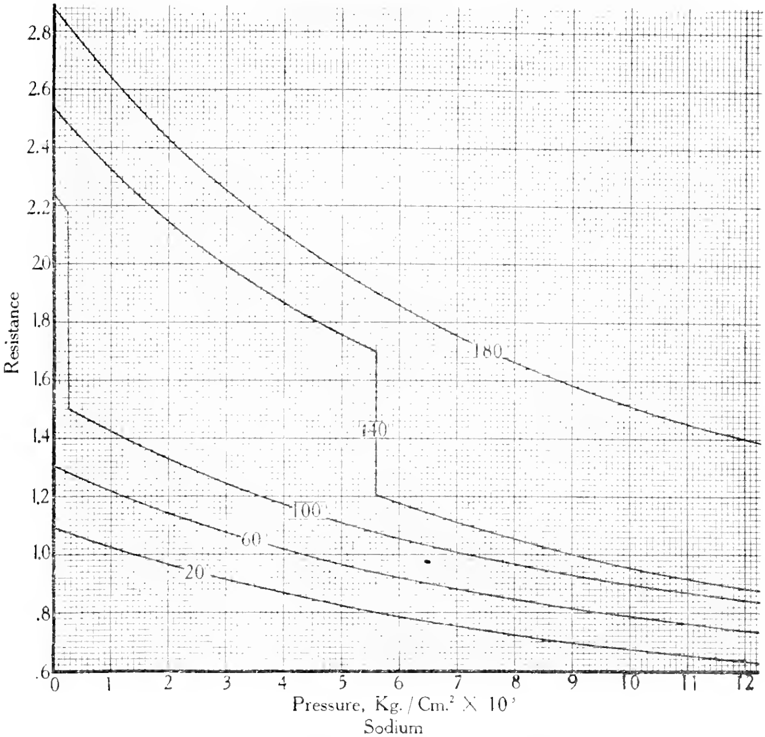


FIGURE 4. Relative resistances of sodium at constant temperature as a function of pressure.

of Bernini, and appreciably higher than that of Northrup. Doubtless the freedom from constraint of the bare wire is in large part responsible. As the solid sodium expands in the glass container it will exert a pressure on the walls, which will have the effect of decreasing the resistance, and making the temperature coefficient appear too low.

For the temperature coefficient of the liquid, Bernini [6] found between 100° and 110° 0.00279 of the resistance at 100°. Northrup's [5] mean value between 100° and 140° is 0.00330 of the resistance at 100°. My mean value between 100° and 120° is 0.00325, slightly lower than the value of Northrup. It is not yet established whether the purer liquid metal usually has a higher temperature coefficient than the impurer or not.

For the ratio of the resistance of liquid to that of solid at the melting point Bernini found 1.337, and Northrup 1.44. The value which I found above was 1.451, in good agreement with that of Northrup. The agreement of my high pressure values with that found at atmospheric pressure makes it probable that in my work the formation of cavities did not cause any essential error at atmospheric pressure.

It will be noticed that throughout, my values are considerably closer to those of Northrup than Bernini. It is probable that in spite of his efforts, Bernini did not succeed in eliminating the effect of cavities, or that there was some other source of consistent error.

Potassium. Material from the same lot was used as that used previously for the measurements of the coördinates of the melting curve.[4] The purity was high, as shown by the sharpness of freezing. Resistance measurements under pressure were made on both the liquid and the solid. The mechanical softness of potassium, and its much greater chemical activity made it infeasible to use the solid in the form of bare wire, as had been possible with sodium and lithium. The metal had to be contained, therefore, in a glass capillary. This is to be regretted, but its much greater softness makes any error introduced by the restraining action of the glass much less than in the case of the other metals. Special examination was made of the magnitude of the error introduced in this way by measuring the difference between the resistance under increasing and decreasing pressure at 28°. Very noticeable differences between the ascending and descending curves were found, corresponding to a maximum difference of mean pressure in the metal and the surrounding liquid of 100 kg. The maximum discrepancy occurred at the highest pressure, where it would be expected that the viscosity of the metal would be the greatest. The pressure difference of 100 kg. was estimated from the differences of resistance, assuming that the stresses in the metal had the same effect on resistance as a hydrostatic pressure. Of course there actually were stresses in the metal of a shearing nature, and it is exceedingly unlikely that the effect of such stresses is equivalent to a hydrostatic pressure, so that it is probable that the glass capillary was called on to support a stress difference of considerably more than 100 kg. The mean of readings with ascending and descending pressure were taken as the correct value. There were, however, considerable irregularities, doubtless due to the irregular response of the glass capillary to the heterogeneous strains in it. At higher temperatures the differences between the readings with ascending and descending pressure became much less than at 28°, as one would expect, but it was evident that some of the irregularities remained at all temperatures. In view of the probably much greater magnitude of the effect at still lower temperatures, no readings were attempted at temperatures lower than 28°.

The capillary containing the potassium was in the form of a U, with open cups at the two ends. Into each cup, two fine platinum wires were sealed for the current and potential leads, and measurements were made by the potentiometer method. The capillary was filled by melting the potassium into it in high vacuum. The filling was accomplished by sealing one of the cups, and connecting to the other cup a succession of bulbs communicating with each other through narrow necks. The potassium was placed in the most remote of the bulbs, and the apparatus was exhausted with a diffusion pump, heating the bulbs and capillary to remove occluded gases. The glass was now sealed off from the pump, and the melted potassium run in succession from one bulb to the next. In this way the scum of oxide was removed. Previous work had shown that further purification of this particular specimen of potassium, as by distilling, was superfluous. When the melted potassium reached the cup of the capillary, into which it could not enter because of capillary action, illuminating gas was admitted to the farther bulb, driving the melted potassium before it and thus completely filling the capillary. The seal of the other cup was now broken, and the capillary mounted as soon as possible in the pressure apparatus under Nujol, the open ends of the capillaries being protected from oxidation with Nujol and paraffine paste during mounting.

Five runs were made, with two capillaries. The first filling gave measurements of the resistance of the solid alone at 28.6° and 54.2°. The second filling was used at higher temperatures, 95.7°, 132.2° and 167.0°, and gave measurements on both solid and liquid. For the second set of runs a special apparatus had to be used by which the insulating plug was kept cold in a third cylinder. This was first used in the measurements on liquid bismuth, and will be found described under that metal. The same apparatus was also used in the high temperature measurements on lithium and sodium. In addition to the pressure runs, the same fillings of the capillaries were used to give the temperature coefficient of resistance by varying the temperature at atmospheric pressure.

The resistances as measured were smoothed to uniform temperature and pressure intervals, choosing a temperature interval of 35° as being closest to the greatest number of the actual readings. In this smoothing the irregularities introduced by the glass capillary were apparent. The individual readings seldom showed irregularities of much more than the sensitiveness of the measurements, but there were consistent departures between readings with increasing and decreasing pressure, and consequent uncertainty as to the correct result. Sometimes an adjustment of as much as 2% in the total resistance was necessary to bring the runs for different temperatures into smooth register with each other. Because of the extreme largeness of the coefficient, a change of 2% in the resistance itself usually means a much smaller percentage change in the decrement of resistance. Since it was the decrement which was measured, the actual measured quantities

TABLE IV.

Resistance of Potassium.

Pressure kg/cm²	25°	60°	95°	130°	165°
0	1.128	1.307	2.387	2.724	3.040
1000	.941	1.079	1.911	2.222	2.568
2000	.799	.911	1.586	1.842	2.176
3000	.692	.786	.880	1.567	1.853
4000	.615	.696	.777	1.346	1.586
5000	.554	.623	.693	1.183	1.374
6000	.503	.563	.624	.685	1.195
7000	.458	.511	.564	.618	1.050
8000	.420	.464	.508	.551	.928
9000	.387	.425	.463	.500	.829
10000	.358	.389	.420	.450	.481
11000	.333	.358	.383	.407	.432
12000	.310	.330	.350	.369	.389

seldom had to be adjusted by as much as 2%. In making the adjustments, the observed temperature coefficients of both solid and liquid at atmospheric pressure was accepted as most probably correct and retained with only immaterial smoothing. Also the observed change of resistance on passing from the liquid to the solid at various temperatures was accepted as essentially correct. With these as fixed data in the table of resistances, the other measured values for the decrement of resistance were adjusted to smoothness with as little change as possible.

The resistances as thus smoothed are shown in Table IV. The values tabulated are relative values of the specific resistance, taking as unity the resistance of the solid at atmospheric pressure at 0°. The observed values have been corrected for the expansion and compressi-

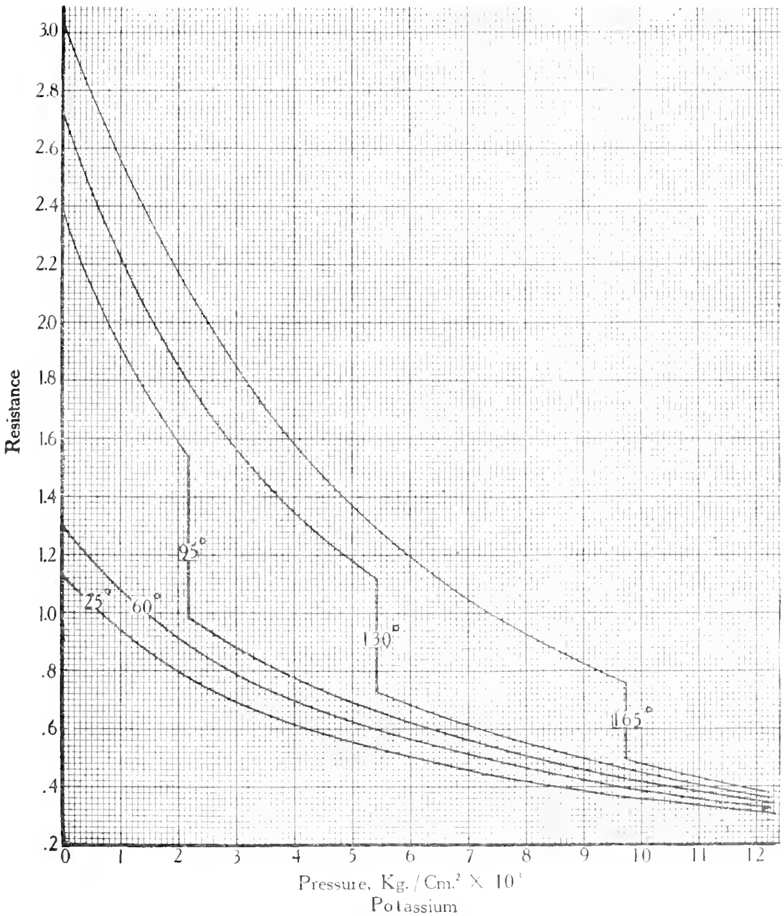


FIGURE 5. Relative resistances of potassium at several constant temperatures as a function of pressure.

bility of the glass, assuming for the linear expansion 0.0_57, and for the linear compressibility 0.0_68. The corrections so introduced are much smaller than usual, because of the very large pressure and temperature coefficients of this substance. The correction for the thermal expansion of the glass is perceptible as a change of one unit in the last figure in only one part of the table, and the maximum correction for the compressibility is 0.96% of the corresponding resistance. It is to be noticed that the table contains relative specific resistances, and not relative "observed" resistances, as is the case for all metals which are measured in wire form. It is also to be noticed that the table contains the resistances of both solid and liquid; the domain of the solid is separated from that of the liquid by a heavy line.

The data of the table have been plotted in Figure 5. The first and most striking result shown by the table and the figure is the great magnitude of the effect; it is not equalled by any other metal, and is exceeded only by black phosphorus among the substances which I have measured. At 25° the resistance of the solid at 12000 kg. is only 27.5% of its resistance at atmospheric pressure. At the higher temperatures, where change of phase occurs, the resistance at 12000 may be as little as 13% of the atmospheric value. At the melting temperature at atmospheric pressure the coefficient of the liquid is greater than that of the solid, but the coefficient of the liquids shows a quite marked decrease with rising temperature.

TABLE V.

POTASSIUM.

Relative Values of Specific Resistance on the Melting Curve.

Temperature Centigrade	Pressure kg/cm²	Resistance Liquid	Resistance Solid
62.5°	0	2.059	1.320
95.0°	2200	1.540	0.990
130.0°	5430	1.128	0.728
165.0°	9710	0.772	0.498

In Table V are shown the resistances, also in terms of the resistance of the solid at 0° at atmospheric pressure as unity, at the melting points. Within the limits of error, the ratio of specific resistance of the liquid to that of the solid at the melting point is nearly independent of the pressure and temperature of melting. The experimental values found were 1.664 and 1.56 at 62.5° and 0 kg., 1.493 at 95.7° and 2260 kg., 1.550 at 132.2° and 5650 kg., and 1.550 at 167.0° and 10000 kg. The higher of the two values at atmospheric pressure was not satisfactory because the capillary was not filled as well as usual, there being obviously minute cavities between the surface of the metal and the glass, and that result was accordingly discarded. The remaining results show that the ratio is almost exactly constant, and in computing the tables the following values were used; 1.56 at 0 kg. 1.555 at 2200 kg., 1.550 at 5430 kg., and 1.550 at 9710 kg.

The value for the ratio of the resistance of liquid to that of solid at atmospheric pressure may be compared with that of other observers. Bernini [7] has found 1.392, and Northrup [5] 1.53. It is to be seen that my value agrees much better with that of Northrup. This entire question of the ratio of the resistance of the liquid to that of the solid is still in a most unsatisfactory state experimentally, and results by different observers disagree by much more than can be accounted for by errors of measurement or by impurity of the materials. All values with which I am acquainted, both for potassium and other metals as well, have been obtained from measurements of the liquid in a glass capillary, and from measurements of the solid in the same capillary after freezing. The resistance of the solid is without doubt likely to be largely in error because of strains and because of cavities formed during freezing. Matthiesen,[8] in his early work, found discrepancies in measurements of the resistances of the solid of as much as 5% which he traced to this cause. What is more, after the solid has once been formed, and is again melted to the liquid, there may be cavities between the surface of the liquid and the glass. Bernini has found large discrepancies in the resistance of the liquid due to this cause. Furthermore, in most experiments, there has been a film of oil between the surface of the metal and the glass; irregular capillary effects in this film will introduce error. Much of the previous work should be repeated with increased precautions. Measurements on the liquid should be made with a capillary completely filled in vacuum, and the liquid should not be allowed to freeze. The specific resistance of the solid should be obtained from independent measurements of the bare metal, preferably extruded to ensure complete freedom from cavities. Measurements on both solid and liquid should be made over a wide enough temperature range to allow unquestionable extrapolation to the melting point. No measurements should be given much confidence which show premature rounding of the corners of the melting curve. It can now be stated with confidence that all such premature rounding is due to impurity; in the early days of this kind of measurement there was room for honest question whether the absolutely pure substance would show premature rounding or not.

While it cannot be claimed that the measurements above meet all these requirements, it is evident that any error from cavities must be negligible when freezing takes place under thousands of atmospheres, and as far as purity goes, the potassium used above never showed any preliminary rounding, thus bearing out the observations on the same material on the melting curve. The melting of Bernini's sample was not abrupt, and took place at 62.04°, nearly 0.5° below that of mine. Northrup records the value 63.5° for the melting point of his specimen; this is so high that it seems that it must be due to errors in temperature measurement.

The value which I found for the temperature coefficient of resistance at atmospheric pressure is considerably lower than that of either Bernini or Northrup. The first found 0.00601 between 0° and 50°, and the latter found 0.0058, in terms of the resistance at 0°C. between 20° and 50°. I found the relation to be linear, and the value of the coefficient, corrected for the expansion of the glass, 0.00512 between 0° and 50°. My value for the liquid is, on the other hand, larger than that of Bernini. I find for the mean coefficient between 95° and 130° 0.00403 of the resistance at 95°, and Bernini gives between 90° and 100° 0.00358 of the value at 95°. Northrup's value, reduced to fractional parts of the resistance at 95°, is 0.00342.

In using the Table, caution should be employed not to force it beyond its accuracy. In particular, too much importance should not be attached to the variation of the differences with temperature. If at any time in theoretical work it should be important to know exactly the variations of pressure and temperature coefficients with pressure and temperature for small ranges of pressure, this work should be repeated with apparatus capable of measuring pressures with greater sensitiveness, and the pressures should not be pushed so high as to introduce irregularities in the glass. In this work the smallest pressure steps were 1000 kg., and no readings were made between 0 and 1000 kg. Where the changes are so large in an interval of 1000 kg. it is entirely possible that some essential detail of behavior may have been overlooked, or smoothed out in constructing the Table.

Magnesium. In the previous paper on the resistance of metals under pressure it was possible to give only very rough values for the pressure coefficient of magnesium. Difficulty was previously found in making good connections because of the impossibility of soldering magnesium and because of the very high resistance of the film of oxide on the surface. With a potentiometer method of measuring resistance, however, this was no longer a difficulty, and accordingly the attempt was made to get more accurate values.

Measurements were made with two samples of magnesium. The first was from the same piece as that on which measurements of resistance and thermal e.m.f. have been already published, and was a contiguous length from the same spool as the e.m.f. sample. It was originally obtained from Eimer and Amend, of commercial quality. The method of manufacture of magnesium, however, is such that impurities are not likely to get into it, and it is a matter of experience that commercial magnesium is of higher absolute purity than most commercial metals. The second sample I owe to the kindness of Dr. MacKay of the Research Laboratory of the General Electric Co. It had been especially purified by him by distillation in vacuum. This was extruded hot to wire of about 0.020 inches diameter, the same in dimensions as the other specimen. Both specimens were mounted in the same way for the measurements, by winding bare on a bone core. Contact was made with spring clips. The resistance at the contacts was so high that error might be introduced because of fluctuations of the potentiometer current, unless the precaution were taken to brighten the wire with sandpaper immediately under the clips just before assembling the apparatus. With this precaution no trouble was experienced from contact resistance.

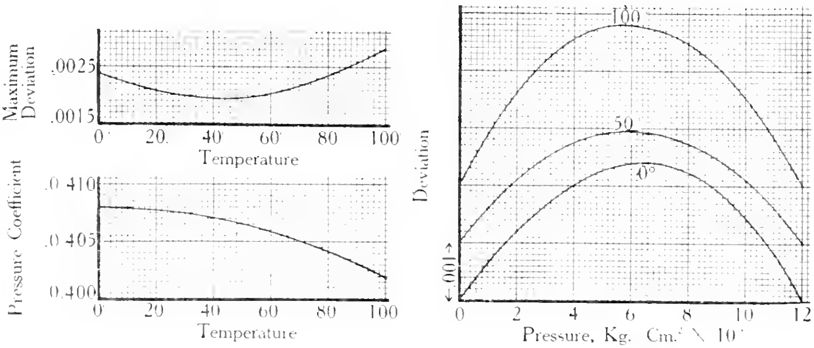
The measurements on the first sample of magnesium were made just after the apparatus had been constructed, and before all points in the best handling of it had been settled, so that there were a number of incomplete runs. In all, there were ten runs on this sample, five of them complete. The incomplete runs were given due weight in the final results. The maximum deviation of the individual readings from regularity was of the order of 1%. It will not pay to reproduce the results in detail, because this sample was presumably less pure than the second, and the results are somewhat different.

Three runs were made on the second and purer sample, at 0°, 51° and 95°, after three preliminary applications of 3000 kg. at room temperature to season. The results were smoothed and a Table constructed in the regular way. The readings ran regularly. At 0° the zero displacement after the run was 1.6% of the total effect, at 51° it was 1.2%, and at 95° 0.85%. These displacements are also essentially the same as the maximum departures from a smooth curve of any of the other observed points. The results are exhibited in Table VI and Figure 6

TABLE VI.

MAGNESIUM.

Temp °C.	Resistance	Pressure Coefficient At 0 kg.	Pressure Coefficient At 12000 kg.	Pressure Coefficient Average 0–12000	Maximum Deviation from Linearity	Pressure of Maximum Deviation
0°	1.0000	-0.0_5477	-0.0_5311	-0.0_54080	.00238	6500
50	1.1975	462	348	4065	196	5900
100	1.3900	473	341	4018	283	5600



Magnesium

FIGURE 6. Results for the measured resistance of magnesium. The deviations from linearity are given as fractions of the resistance at 0 kg. and 0°C. The pressure coefficient is the average coefficient between 0 and 12000 kg.

in the same way as the results of the previous paper. The average pressure coefficient decreased with rising temperature, which is unusual. The relation between resistance and pressure departs from linearity in the usual direction, but it is unusual that the relation is more nearly linear at 50° than at either 0° or 100°. The pressure of maximum departure from linearity moves progressively towards lower values at higher temperatures.

It is interesting to compare these values with those found for the first and presumably less pure sample. The temperature coefficient of the first sample was 0.00412, against 0.00390 for the second, and the relation between temperature and resistance was linear between 0° and 100°. Here we have the unusual case of the presumably purer substance with a smaller temperature coefficient. Recent observations of Holborn [9] make it likely, however, that the behavior of aluminum is similar, and it is not surprising if the behavior of magnesium and aluminum should be the same. The average pressure coefficients of the first sample between 0 and 12000 kg. were -0.0_5446, 441, and 436 at 0°, 50°, and 100° respectively. This is seen to be about 10% higher than that of the second sample. The two samples agree, however, in having a coefficient which decreases with rising temperature. The maximum departures from linearity of the impurer sample were also nearly the same as for the purer, the maximum deviations being 0.0023, 0.0026, and 0.0029 at the three temperatures respectively. The deviations from linearity of the first sample were symmetrical with respect to pressure, in each case the relation between pressure and resistance being representable by a second degree equation, and the pressure of maximum deviation being at 6000 kg. The first sample differs from the second in not showing a closer approach to linearity at 50° than at the other two temperatures.

These new and more accurate results differ considerably from the rough value given for the first sample; it was stated in the previous paper that the average coefficient for the first sample was probably -0.0_555.

Calcium. This material was obtained from the Research Laboratory of the General Electric Co. through the kindness of Dr. Langmuir. An analysis by Mr. N. S. Drake by the method of differences showed not more than 0.1% total impurity, the error of the measurements being 0.1%. Qualitative analysis showed no detectible impurity except a trace of iron, too small to determine quantitatively.

The calcium was furnished in the form of a solid ingot about 1 inch in diameter. Pieces of the appropriate size were cut from it with a hack saw, and were formed into wire 0.013 inches in diameter by extrusion through a steel die. An extrusion pressure of the order of 10000 kg per cm² was required. A phenomenon shown to a more or less pronounced degree by all metals during extrusion was particularly prominent with calcium. If the extrusion pressure is pushed too high, or sometimes for no apparent reason, the wire will suddenly break and the metal spit out of the extrusion block in long gulps or small pieces. This spitting forth reaches explosive violence in the case of calcium, and the fine dust into which the issuing wire breaks takes fire spontaneously. The explosive sound, the flash of fire and the pungent smoke are likely to be rather terrifying on first experience. For nearly all metals the effect can be greatly reduced by a proper design of the die; the best shape of the die varies from metal to metal.

In spite of the satisfactoriness of the chemical analysis of this sample, it must be recognized that there may perhaps have been in this specimen minute impurities which possibly may have exerted an appreciable effect on the electrical properties. It has been the experience of the General Electric Co. that different samples of calcium, with no perceptible chemical difference, offer very different resistances to extrusion. The effect may not be chemical, and its explanation is entirely obscure. There is no way of knowing whether the specimens difficult of extrusion are more or less likely to be impure, and no connection seems to have been looked for between extrusion and electrical characteristics. The specimen used by me was difficult of extrusion; we cannot now make any use of this fact, but if in the future the cause of the variations should be found, it is well to have the characteristics of this specimen recorded.

Temperature had to be raised to 400° or higher for successful extrusion. Although extrusion would take place at lower temperatures, the wire so formed was brittle, and could be handled only with difficulty. In any event, the best wire that could be formed had to be handled with care. It broke if bent more than once at a sharp angle, and was entirely different in its properties from sodium. Extrusion took place directly into the air, but as the wire exuded from the die, it was wound immediately onto a spool covered with a protecting paste of Nujol and paraffine. Chemical action by the air is much slower than in the case of sodium and lithium, but is no less complete, the wire eventually crumbling away into dust after standing several weeks. Even the Nujol and paraffine does not act as a complete protection, but there is either direct action or else slow diffusion of the air through the protective coating. In order to avoid error from this effect, the wire must be mounted in the pressure apparatus and measurements made as soon after extrusion as possible, while it is still bright. Unless this procedure is followed effects from the high resistance of the surface film are much more troublesome than for any other metal I have tried, not excepting magnesium. With fresh wire, however, such effects become vanishingly small with proper manipulation. The contact resistance may always jump spasmodically under increases of pressure, but with fresh wire the contact can always be sufficiently restored by the momentary passage of a current from a small magneto.

The wire was wound as usual bare on a bone core, and connections made with spring clips, using a protective coating of gold foil at the point of contact.

Runs were made on two samples and at five temperatures, 0°, 25°, 50°, 75° and 96°. The runs on the first sample were terminated by the failure of the insulation of the three-terminal plug. The transmitting medium was Nujol and petroleum ether. Chemical action was never entirely absent, but as in the case of sodium, it was mostly confined to low pressures. It increases rapidly with rising temperature, and at 96° was so rapid that 1500 kg. was the lowest pressure at which readings were attempted. Unlike sodium and lithium, calcium shows pronounced seasoning effects of pressure, and the runs at 25° and 75°, which were those of the initial application of pressure to the two samples respectively, were much less regular than the subsequent runs. Because of the necessity of obtaining readings rapidly because of chemical action, preliminary seasoning applications of pressure were omitted, and the initial runs were included in the final results. At 25° the total zero shift (initial application) was 4.2% of the total pressure effect; at 0° 0.5%; at 50° 3.9%; at 75° (initial application) 6.9%; and at 96° (zero taken from 1500 kg.) 3.3%. At 25° and 75° the individual points lie very closely on two smooth curves, different for increasing and decreasing pressure. The incomplete seasoning shows itself in a sequence of readings like that of an open hysteresis loop. The open end of this loop at atmospheric pressure has the width given above by the zero displacements. At the other temperatures the readings also show a tendency to hysteresis effects, but the departure from the mean is much less. At 0° the maximum departure of any single point from the smooth curve representing the mean of the points with increasing and decreasing pressure is 1.2% of the total effect; at 50°, 1.5%; and at 96°, 1.5%.

The temperature coefficient of resistance at atmospheric pressure was determined by the same method and at the same time as the readings for sodium and lithium. The "observed" resistances, which are the mean of points with ascending and descending temperature, at four temperatures (0°, 25°, 50°, and 75°) all lie within the limits of error (one part in 7000) on a second degree curve. The total shift of the zero during the run, presumably due to chemical action, was 1.6% of the total temperature effect. The value given in the table for the resistance at 100° was obtained from the second degree curve by extrapolation. The mean coefficient between 0° and 100° determined in this way is 0.003327.

The temperature coefficient of resistance of Calcium at atmospheric pressure has been previously measured by Northrup,[11] who found between 0° and 100° the value 0.00246, and by Swisher,[10] who found the relation between temperature and resistance to be linear between 0° and 600°, and the coefficient to be 0.00364 of the value at 0°. An examination of Swisher's results shows that there were considerable irregularities, and that within his limits of error it is not possible to say whether his coefficient between 0° and 100° is greater or less than my value above. The low value of Northrup is probably due to impurity in his sample.

In smoothing the experimental results for the most probable final values, the usual procedure was followed. The results at each temperature were first smoothed independently, and then readjusted so as to give smooth temperature differences. The effect of insufficient seasoning in the runs at 25° and 75° was shown by the greater readjustments necessary at these temperatures. The maximum readjustment necessary was 0.0% at 0°, 4.6% at 25°, 1.4% at 50°, 1.8% at 75°, and 0.4% at 96°.

TABLE VII.

Calcium.

Pressure kg/cm²	Resistance 0°	25°	50°	75°	100°
0	1.0000	1.0748	1.1552	1.2402	1.3327
1000	1.0107	1.0859	1.1668	1.2522	1.3454
2000	1.0217	1.0974	1.1789	1.2651	1.3589
3000	1.0330	1.1092	1.1912	1.2780	1.3725
4000	1.0447	1.1213	1.2039	1.2914	1.3865
5000	1.0569	1.1340	1.2171	1.3051	1.4008
6000	1.0696	1.1473	1.2309	1.3193	1.4157
7000	1.0827	1.1609	1.2451	1.3341	1.4311
8000	1.0963	1.1751	1.2599	1.3495	1.4470
9000	1.1103	1.1896	1.2749	1.3650	1.4631
10000	1.1247	1.2045	1.2903	1.3810	1.4797
11000	1.1396	1.2200	1.3064	1.3976	1.4969
12000	1.1550	1.2360	1.3229	1.4147	1.5146
Average Coefficient 0–12000 kg.	$+0.0_41292$	$+0.0_41242$	$+0.0_41210$	$+0.0_41172$	$+0.0_41137$

The final results for resistance as a function of pressure and temperature are given in Table VII. The striking and unexpected result, as in the case of lithium, is the positive coefficient. The coefficient is greatest at the lowest temperature, having the average value $+0.0_41292$ at 0°. Both the instantaneous and the average coefficient increase with rising pressure. In both these particulars the behavior is much like that of bismuth. The absolute value of the coefficient is of the order of half that of bismuth.

Bismuth is abnormal in so many other respects than its pressure coefficient, that it seemed of interest to find whether calcium has the same sort of abnormalities. The large increase of resistance of bismuth in a magnetic field is one of the well known abnormal effects. I tried the effect of a field of approximately 27000 Gauss on the resistance of a coil of calcium, the wires of the coil being everywhere at right angles to the field, and found a decrease of resistance of only 1/1600. It is evident, therefore, that any parallelism between the conduction mechanisms of calcium and bismuth cannot be very complete. A more exhaustive investigation of the various properties of calcium than has yet been made seems well worth while.

In a previous paper it has been shown that the abnormal positive pressure coefficients of resistance of bismuth and antimony are associated with values of the thermal expansion abnormal as compared with the other properties. The same comparison for calcium appeared of interest, but the thermal expansion of calcium has apparently not been previously measured, and I accordingly made special measurements of it.

The thermal expansion was determined by ordinary methods, using a glass dilatometer. The calcium was the same specimen as that from which the resistance sample was cut. It was turned over its entire surface in the lathe, and finished to a cylinder about 6 cm. long and 20 cm^3 volume. The surface was smooth, without blowholes. In the glass dilatometer the calcium was surrounded with Nujol. The calcium and Nujol had been previously heated together in another vessel to 100° to avoid as far as possible chemical action during the dilatometer readings. Discoloration of the surface of the calcium by this preliminary heating was very slight. Air bubbles in the dilatometer were avoided by filling in vacuum. The Nujol is so viscous that without special precaution error may be introduced by sticking of the Nujol to the walls of the capillary. This error was avoided by heating the capillary before the readings with a small gas flame, and by making readings with increasing temperature. A preliminary attempt was made to avoid the error by using petroleum ether as the medium, which is very much more fluid than the Nujol. This was unsuccessful because of chemical action between the petroleum ether and the calcium. Readings of the expansion of the calcium and Nujol were made at 0°, 25°, and 50°. At 50° a very few exceedingly minute bubbles appeared, probably the beginning of chemical action. Accordingly the readings at 50° were discarded. The auxiliary data needed in the computation were obtained from the dilatations when the dilatometer was entirely filled with Nujol and with mercury respectively. The volumes of the dilatometer and the capillary were obtained by weighing the mercury required to fill them. The bore of the capillary was calibrated for uniformity by conventional methods. The density of the calcium was obtained by weighing it in air and under Nujol. The densities given are corrected for vacuum.

The following values were obtained for calcium:

Density at 21° 1.5563
Coefficient of volume expansion, 0° to 21°, 0.000075.

The values for Nujol were obtained incidentally and are recorded.

Density at 21° 0.8786
Coefficient of volume expansion, 0° to 21°, 0.000717.

STRONTIUM. Particular interest attaches to this metal because of the fact that it is underneath calcium in the periodic table, and calcium is unusual in having a positive coefficient. The material I owe to Dr. G. E. Glascock, who very kindly placed at my disposal some of the material whose preparation and properties he has described.[12] The metal was prepared by the electrolysis of the fused salts. It was in the form of fused nodules of sometimes two or three cubic centimeters volume, and had been kept since preparation under kerosene. Some sort of action had taken place between the metal and the kerosene, under which most of the kerosene disappeared, and the metal became coated with a fine gray powder. The action of calcium on kerosene I have found to be very similar. On scraping off the gray powder, the coherent metal is found underneath. On cutting into the cleaned nodules with a cold chisel, slag-like inclusions are sometimes found. Lithium prepared by electrolysis shows the same appearance. By careful selection it was possible to find pieces large enough free from these inclusions.

The measurements were made on the metal in the form of wire approximately 0.020 inches in diameter, formed by extrusion from one of the selected pieces of clear metal. The extrusion is considerably easier than that of calcium, and may be successfully performed at a temperature of 230°. At room temperature the metal spits out of the die in small pieces. The wire is fairly soft and pliable and can be bent to a radius of perhaps ten times the diameter of the wire, but it is quite different in mechanical properties from the alkali metals, such as sodium, and shows brittleness if too sharp a bend is attempted.

Previous measurements on the electrical properties of strontium seem to have been published only by Matthiesen.[8] He extruded the metal to wire in much the same way as above. He gives for the specific resistance at 20° the value 25×10^{-6} ohms per cm. cube. I found at 0° for my specimen the value 30.7×10^{-6}. This specimen is presumably considerably purer than Matthiesen's, and this value would seem to be preferred. Matthiesen did not attempt to measure the temperature coefficient of resistance, probably because of resistance of the contacts. He did not use a potentiometer method, but had to correct as best he could for the resistance of his leads and contacts. With a potentiometer method as used here, there is no such difficulty. There is difficulty, however, in the chemical action accompanying changes of temperature, which produces permanent changes of resistance. Error from this effect was avoided by the same procedure as that used previously for the alkali metals. Four thermostated baths were kept simultaneously in operation, and the specimen transferred rapidly from one to another. The bare wire was immersed in a glass tube of Nujol for the measurements, the Nujol having been previously heated with sodium to remove all moisture and exhaust as far as possible all tendency to chemical action. There nevertheless seems to have been some specific action between the oil and the strontium. About fifteen minutes were required for the attainment of temperature equilibrium after the wire had been transferred from one bath to the next. Readings were made with ascending and descending temperature between 0° and 96°, starting with 0° and ending with 0°. The mean of the ascending and descending readings was taken as the true effect. The permanent change of zero after the excursion was 12% of the maximum effect. The mean coefficient between 0° and 100° found from these readings was 0.00383. The effect is not quite linear with temperature, but the change becomes more rapid at the higher temperatures, as is normal.

The value of the temperature coefficient is quite normal for pure metals, and in the absence of further information, makes it probable that this material was of satisfactory purity. It is not possible to obtain the analysis from the paper of Glascock, because the products of his separate electrolyses were not always of the same purity, and the different batches were indiscriminately mixed in the material as supplied to me. The least pure of any of the specimens of Glascock had about 2% impurity, and the best about 0.15%.

Pressure measurements were made in the regular way with the potentiometer. The wire was used bare, and the connections were made with spring clips. The resistance of the contacts increased during the runs, and sometimes became troublesomely large; it could then be reduced by passing a high tension current from a small magneto through the contacts. The wire was seasoned by a preliminary application of 3000 kg. at room temperature; there was no perceptible change of zero after this application. Three runs were made; at 0°, 50.5°, and 97°. At the highest temperature the zero of pressure was taken as 500 kg. in order to prevent chemical action, and the value at atmospheric pressure obtained by extrapolation. Considering the chemical activity of this material the readings showed a gratifying regularity. At 0° there was a permanent change of zero of 1.8% of the total effect, at 50° the change was 0.3%, and at 97° 2.1%. The maximum departure of any of the other points from the smooth curve was 1.6% at 0°, 1.2% at 50°, and except for one bad point, 1.5% at 97°. The observed resistances were smoothed and a table constructed for the resistance at regular intervals of pressure and temperature in the regular way.

The results are shown in Table VIII and Figure 7. The resistance increases under pressure, the same as for calcium. The increase is furthermore very large; it is five times as large as that of calcium, and three times that of bismuth, and is the largest positive coefficient yet found. The behavior is in other respects like that of other metals with a positive coefficient. When the resistance is plotted against pressure, the curve is concave upwards, the change becoming more rapid at the higher pressures. The coefficient becomes markedly smaller at the higher temperatures. The instantaneous coefficient becomes smaller at the higher pressures; this was not the case for bismuth. One may be puzzled at first by the Table which shows a smaller instantaneous coefficient at both 0 kg. and 12000 kg. than the average coefficient between 0 and 12000. The reason for this is that the instantaneous coefficient is calculated in terms of the resistance at the pressure in question, which becomes rapidly greater at the higher pressures, whereas the average coefficient is calculated in terms of the initial resistance at 0 kg. The resistance shows a regular drift

TABLE VIII.

STRONTIUM.

Pressure kg/cm²	Resistance 0°	50°	100°
0	1.0000	1.1768	1.3828
1000	1.0516	1.2320	1.4341
2000	1.1058	1.2906	1.4908
3000	1.1622	1.3519	1.5539
4000	1.2220	1.4166	1.6214
5000	1.2850	1.4842	1.6936
6000	1.3508	1.5544	1.7712
7000	1.4196	1.6280	1.8527
8000	1.4916	1.7051	1.9381
9000	1.5670	1.7852	2.0272
10000	1.6462	1.8684	2.1198
11000	1.7290	1.9550	2.2157
12000	1.8160	2.0446	2.3144
Average Coefficient 0–12000 kg.	$+0.0_4 6800$	$+0.0_4 6146$	$+0.0_4 5614$
Coefficient at 0 kg.	$0.0_4 502$	$0.0_4 456$	$0.0_4 351$
Coefficient at 12000 kg.	$0.0_4 492$	$0.0_4 451$	$0.0_4 432$

with increasing temperature. At 0° the third derivative of the resistance as a function of pressure is positive, at 50° it is zero, and at 100° it is negative.

MERCURY. I have already published results for the resistance of liquid mercury as a function of pressure and temperature,[13] but I have now found it possible to considerably extend the range of the previous work. Previous work on the liquid was between 0° and 50° and to 6500 kg., and a few very rough qualitative results were obtained for the solid. The results for the liquid have now been extended over the temperature range 0° to 100° and over the pressure range to 12000 kg. In addition, the resistance of the solid has been accurately measured at 0° between the solidifying pressure (7640) and 12000 kg., and the change of resistance on solidifying at 0° has been also determined.

The liquid was measured with the Carey Foster bridge, by the same method used for most metals in the form of fine wire. The mercury was contained in a U-shaped capillary, and connections were made to

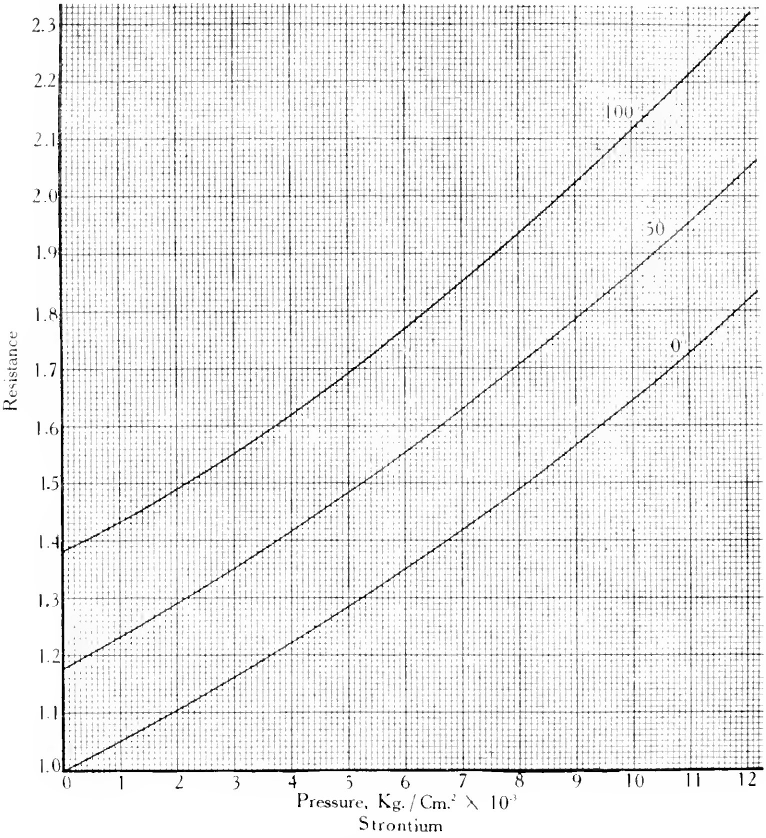


FIGURE 7. Relative resistances of strontium at several constant temperatures as a function of pressure.

the mercury by amalgamated copper wires dipping into cups at the upper ends of the U. The resistance was about 10 ohms. The capillary was of Jena glass No. 3880a, the same grade of glass as that used in the previous work. This particular capillary was blown in 1907, and had been last used in 1911, when it had been exposed to the freezing pressure of mercury at 0°, and had since been resting in a dust proof container. The measurements on the liquid recorded here were made in 1917. The technique of the previous work has been considerably improved in the interval between 1908 and 1917. The transmitting liquid used in 1908 was a mixture of water and glycerine, and elaborate precautions had to be taken to avoid short circuits. The liquid is now a hydrocarbon, and the insulation properties are perfect. The values now obtained for the change of resistance differ by about 1.5% from the previous ones. Part of this difference is perhaps to be ascribed to improvement in technique, part to change in the behavior of the glass, which has been resting ten years since blowing and annealing, and perhaps part to difference in the standard of pressure. My very earliest results, which reached to only 6500 kg., depended on a pressure gauge of smaller range and somewhat different construction from that used in all my subsequent work reaching to 12000 kg. or more. All this later work depends on the same gauge, and assumes as the fundamental constant that the freezing pressure of mercury at 0° is 7640 kg. Any discrepancy now found with the former values for the resistance of mercury cannot affect the validity of any of the later work up to a range of 12000 kg.

The glass was seasoned before the measurements by an application of 11300 kg. at room temperature, and then by raising temperature to 96° while still under pressure. Five runs were made on the liquid, in addition to the seasoning runs; these were at 0°, 26°, 50°, 76°, and 97°. The maximum zero displacement after a run at any of the temperatures was 0.46% of the total effect. The maximum departure from a smooth curve of any of the observed points at any temperature was 0.6% of the total effect, and the points usually lay on a smooth curve within the sensitiveness of the readings, which was about one part in 5000 of the maximum effect.

The observed results were smoothed for pressure and temperature and a table constructed for the resistance at uniform intervals of pressure and temperature in the regular way. The resistance of the liquid is shown in Table IX, and the coefficients are shown later in Table XX. The resistances given in this table are corrected for the compressibility and thermal expansion of the glass, using the values already given. The correction for temperature amounts to a change of observed resistance of 0.0007 for the range of 100°, which is about 0.74% of the measured effect, and the correction on the pressure effect is 0.0007 for 1000 kg., which is initially 2.2% of the pressure change. The values given in the Table are, therefore, relative values of the

TABLE IX.

Relative Specific Resistance of Liquid Mercury.

Pressure kg/cm²	Resistance 0°	25°	50°	75°	100°
0	1.0000	1.0240	1.0480	1.0719	1.0959
1000	.9689	.9911	1.0137	1.0360	1.0574
2000	.9400	.9609	.9823	1.0032	1.0224
3000	.9136	.9331	.9533	.9726	.9902
4000	.8894	.9076	.9268	.9449	.9611
5000	.8667	.8838	.9023	.9192	.9345
6000	.8459	.8615	.8793	.8951	.9096
7000	.8263	.8408	.8579	.8729	.8866
8000		.8214	.8377	.8516	.8650
9000		.8034	.8200	.8318	.8445
10000		.7863	.8010	.8133	.8254
11000		.7704	.7844	.7958	.8071
12000		.7554	.7684	.7790	.7895

specific resistance, that is, they are the relative values of the resistance which would be shown by a body of mercury of invariable dimensions, but with the total mass changing with pressure and temperature.

Certain differences between the new and the old results should be mentioned. At 25° the change of resistance under 6500 kg. is now found to be 0.1586, against the old value 0.1562. The previous formula given for the resistance up to 6500 kg. gives rather good results when used for extrapolation to 12000 kg. At 25° the observed decrement of resistance is now found to be 0.2622 against the value 0.2600 computed by the formula previously given. The difference is approximately the same as that between the previous and the present observed values at 6500 kg. Over the narrower temperature range of the previous work the effect of temperature was taken as linear at any constant pressure. Over the greater temperature range of the present work, this is found not to hold, but the effect of temperature becomes greater at the higher temperatures, and passes through a minimum near 25°. At atmospheric pressure, however, the relation between resistance and temperature is still found to be linear within the limits of error, which are about 5/10,000 of the total resistance.

The resistance of the solid under pressure was measured by filling a capillary of very thin glass with the liquid, exposing to a pressure sufficient to freeze it, and making measurements on the solid contained in the glass without allowing the pressure to fall low enough to melt the mercury. By the use of thin glass I hoped to eliminate error due to the constraining action of the container. It was necessary to use a capillary of rather large section, because of the mechanical difficulties of blowing and handling a very fine capillary with excessively thin walls, and the potentiometer method of measurement was therefore used. The leads were of fine platinum sealed through the glass. Under pressure the glass cracked around the seals, but of course this introduced no error in the measurements of the solid. Only one set of measurements was made on the resistance of the solid, at 0°. Measurements were made on the liquid up to the freezing pressure, pressure was then increased very cautiously beyond the freezing point so that freezing took place slowly, and the solid so formed was seasoned by an excursion to 12000 kg. and back nearly to the freezing pressure. Readings were now made in the domain of the solid to 12000 kg. These readings were exceedingly regular; they showed no departure from the smooth curve within the sensitiveness of measurement, which amounted to one part in 1500 on the total effect for the range 7640 to 12000 kg. This was gratifying, because it showed that the very thin glass exerted no perceptible constraining effect. In the previous work with the liquid in heavy glass capillaries very irregular results were found after the metal had frozen.

The relative values for the resistance of the solid are shown in Table X in terms of the resistance of the solid at 7640 kg. and 0° as unity. In comparing these values with those of the liquid it must be remembered that the values for the solid are relative values of the "observed" resistance, and must be corrected by a factor equal to the linear compressibility in order to give relative values of specific resistance. The pressure coefficient of "observed" resistance may be found from the table to be -0.0_4236, and within the sensitiveness of the measurements it is constant over the pressure range from 7640 to 12000. This value for the pressure coefficient is somewhat higher than the minimum value set in the previous work, which was -0.0_52. The accuracy of the previous work for the solid was so low that it was stated that the maximum value for the solid might not impossibly be ten times the minimum. The pressure coefficient of the solid is very nearly that of the liquid, which is -0.0_5224 at 6500 kg., when corrected by one third the volume compressibility of the liquid mercury so as to be strictly comparable with the value for the solid. It is surprising

TABLE X.

Relative Specific Resistances of Solid Mercury at 0° C.

Pressure kg/cm²	Resistance
7640	1.00000
8000	0.99185
9000	0.96870
10000	0.94615
11000	0.92405
12000	0.90250

The ratio of the resistance of the liquid to that of the solid at the freezing point at 0° and 7640 kg. is 3.345.

that the values for the solid and the liquid are so close, and still more surprising that the value for the solid is greater than that for the liquid. The former rough work suggested the opposite and more natural behavior.

The ratio of the resistance of the solid to that of the liquid at the freezing pressure may also be calculated from the measurements. In view of the extreme care taken to compel the freezing to go slowly, and the fact that when the apparatus was taken apart the capillary was found cracked only at the bend, it is probable that the observed ratio of resistance of solid to liquid refers to the relative resistances of material occupying space of the same dimensions in the liquid and solid states, that is, to relative values of the specific resistance. The value found for the ratio of resistance of liquid to solid at 0° and 7640 kg. (the equilibrium coördinates) was 3.345. This falls within the range of the very irregular values found in the previous work.

Gallium. The raw material from which this rare metal was prepared I owe to the kindness of Dr. K. Stock of the Bartlesville Zinc Co. The purification I owe to the kindness of Professor T. W. Richards. He was engaged in a redetermination of the atomic weight and certain other physical properties, and was kind enough to include some of my raw material with his, and let me have some of the purified product.

The actual work of purification was done by Mr. S. Boyer, under the direction of Professor Richards. The final product had less than 0.01% total impurity. Professor F. A. Saunders was kind enough to make a spectroscopic analysis for me, and was able to detect traces of zinc as the impurity. There may also have been some indium present.

There was available for my measurements about one gram. This was ample for a determination of the effect of pressure on resistance, and also for a determination of the variation of freezing temperature with pressure and an exploration for other allotropic modifications (which had not been previously done) but was not sufficient for a determination of the change of volume on freezing. The complete freezing data are, therefore, not yet determined.

The measurements here described include the specific resistance of the liquid, temperature coefficient of resistance of solid and liquid at atmospheric pressure, change of resistance when the solid melts to the liquid, effect of pressure on resistance of both solid and liquid, and variation of freezing pressure with temperature.

The determination of the freezing curve and the exploration for new modifications was the first task, in order to fix the range over which the resistance measurements were to be extended. Gallium is, of course, abnormal in that it expands when it freezes, and the freezing temperature is accordingly depressed under increased pressure. It would not be unnatural to expect new modifications at high pressures like the other modifications of ice. Because of the limited quantity of material available the method of exploration for new modifications had to be an electrical one. It was my original intention to form the material into wire by extrusion, and to measure the resistance of the solid as a function of pressure at different temperatures. A change from one modification to another would be shown by a discontinuous change of resistance, and melting by open circuiting. Unexpected difficulties were found in the extrusion. If the extrusion is performed at room temperature, the metal melts under the one-sided stress instead of extruding (the melting temperature is 29.85°), and if this effect is avoided by lowering the temperature, the metal becomes so exceedingly stiff and brittle that extrusion is very difficult. After an unsuccessful attempt at room temperature, I tried extrusion at the temperature of ice and salt. At this temperature the metal spit out of the die in short pieces. However, by careful work, I did get a few inches of wire at this extrusion temperature. Perhaps some intermediate temperature would be more successful.

Mechanically the extruded wire is very crystalline and brittle, and will support almost no bending. When enclosed in a glass capillary, however, and if the capillary breaks across without breaking the gallium, as sometimes happens when the capillary is ruptured by freezing, the metal may be pulled out by a tensile stress and appears as ductile as lead.

The extruded wire was used in an attempt to find the melting curve. Short lengths were layed horizontally between two copper wires, short-circuiting them. It was expected that when melting took place the system would open circuit. The surface tension of the gallium proved to be so high, however, that this method did not work. The metal, even when melted, only sagged between the supports, and no measurements could be made. Several modifications of this scheme were tried with indifferent success. These preliminary measurements made pretty evident, however, that there were no new modifications, and that above the normal melting point the metal remains liquid at all pressures below 12000 kg.

Measurements were now made on the resistance of the liquid as a function of pressure above the melting temperature. The liquid gallium was enclosed in a glass capillary, provided with four platinum terminals, and measurements made by the potentiometer method. The capillary was filled in high vacuum to avoid error from air sticking to the walls. From a comparison of the resistance of the capillary when filled with gallium and when filled with mercury it was possible to obtain the resistance of gallium in terms of that of mercury, and so the specific resistance of liquid gallium.

After completion of measurements on the liquid, the temperature of the apparatus was lowered into the region of the solid, and after some trouble, the liquid was induced to freeze. The melting curve was now determined by finding, as a function of temperature, the pressure at which the resistance began to change discontinuously. It was necessary to do this very cautiously. In changing pressure the thermal effects of compression might easily be sufficient to entirely melt the gallium, when long and tedious manipulation would be necessary to make it freeze again, because of the well known property of supporting great subcooling. In this way the melting curve was mapped out to 12000 kg.

Measurements were also made on the resistance of the solid as a function of pressure while enclosed in the glass envelope, but there were irregularities, and it was evident that the solid must be unconstrained in order to give reliable results. Accordingly a small rod of solid gallium was sculptured out with considerable difficulty with a warm wire as a sculptor's tool, four platinum terminals were set into it, and measurements made of the pressure coefficient of resistance by the potentiometer method. The temperature coefficient of the solid at atmospheric pressure was also determined with this free specimen. This seems to be the first time that this has been done. Previous measurements of the temperature coefficient of the solid have been on the solid in glass, and no correction has been applied for the constraining effect of the glass. After the completion of the resistance measurements on the free solid another point was found on the melting curve, which checked with the points found previously. The resistance of the solid showed no discontinuities within the errors of the measurements, and hence it is not likely that there are new modifications in the region of the measurements. This was rather a disappointment.

TABLE XI.

GALLIUM.

Melting Curve.

Pressure kg/cm²	Temperature °C.
0	29.85°
1000	27.8
2000	25.7
3000	23.55
4000	21.4
5000	19.2
6000	17.0
7000	14.8
8000	12.6
9000	10.35
10000	8.10
11000	5.85
12000	3.55

The smoothed coördinates of the melting curve are given in Table XI, and the observed points with the curve are shown in Figure 8. For the melting point at atmospheric pressure I used the value determined for this sample by Mr. Boyer, which is 29.85°. This is lower than the value originally given by deBoisbaudran,[14] 30.15°, but is higher than that given by Guntz and Broniewski,[15] 29°. The value of Boyer is the mean of a number of values determined with extreme care, and is seen to fit perfectly with the values which I have found at higher pressures. Considering the extreme purity of this sample, there is every reason to give this value the preference.

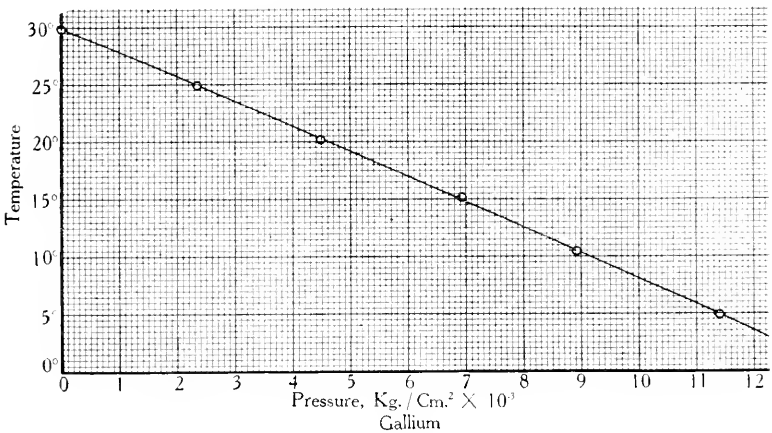


FIGURE 8. The melting curve of gallium.

The melting curve has the same form as that of water and bismuth, the only other substances I know with curves of this abnormal type. It is characteristic of these curves that the slope becomes numerically greater at the higher pressures. The slope of the melting curve of gallium increases numerically by about 15% in the pressure range of 12000 kg.

The slope of the melting curve may be combined with other data to give the latent heat on melting. Boyer's values for the densities of the solid and liquid at the melting point are 5.90 and 6.09 respectively. This gives 0.00529 for the change of volume per gm. on melting. Taking from the above curve the value 0.00203 for the initial slope of the melting curve, and substituting in Clapeyron's equation, we find 18.5 cal. per gm. as the latent heat of melting. Boyer has recently determined this to be 19.1. The agreement is probably within the error of the density determinations. The variation of the latent heat along the melting curve cannot be found until the variation of the change of volume along the melting curve is also known, and this will require a larger sample.

The ratio of the resistance of the liquid to that of an equal volume of mercury at the melting point was found to be 0.2683, against the value, 0.272 of Guntz and Broniewski.[14] Taking for the specific resistance of mercury at this temperature 96.59×10^{-6}, this gives 25.92×10^{-6} for the specific resistance of liquid gallium at the melting point.

Three runs were made on the resistance of the liquid as a function of pressure, at 34.2°, 62.5°, and 94.4°. Before the final measurements another run was made on another sample. This run had somewhat greater error, but agreed with the final results within the irregularities of the measurements. The resistance measurements of the liquid showed a distinct hysteresis, doubtless due to the action of the glass capillary. Any such effects would be expected to be especially large for gallium because the pressure coefficient of resistance is so low. The difference between readings with increasing and decreasing pressure might amount to 2.5% of the maximum effect. Except for this, the points lay regularly within the sensitiveness of the readings. The values obtained from these three runs were smoothed in the regular way, and the resistance tabulated at regular intervals of temperature and pressure. The results are shown in Table XII. In this table

TABLE XII.

GALLIUM.

Relative Specific Resistances of the Liquid.

Pressure kg/cm^2	Resistance 30°	65°	100°
0	0.6456	0.6647	0.6824
1000	.6415	.6605	.6783
2000	.6380	6564	.6743
3000	.6342	.6527	.6705
4000	.6308	.6488	.6667
5000	.6272	6452	.6629
6000	.6238	.6418	.6593
7000	.6205	.6383	.6558
8000	.6171	.6349	6522
9000	.6139	.6316	6486
10000	.6105	.6283	.6451
11000	.6075	.6252	.6418
12000	.6044	.6223	6386
Average Coefficient 0–12000	-0.0_5531	-0.0_5532	-0.0_5534

the unit of resistance is the resistance of the solid at atmospheric pressure and 0° C. There may be some question as to the accuracy of the value for the ratio of resistance of solid to liquid, but this cannot affect the relative accuracy of the values listed in the table for the resistance of the liquid.

The values listed in the table have been corrected for the thermal expansion and compressibility of the glass, so that the values are proportional to the specific resistances. In making this correction the thermal expansion of the glass was assumed to be $0.0_5 8$, and the linear compressibility $0.0_6 7$. The correction for thermal expansion is small, and any error in the value assumed for the coefficient cannot appreciably affect the results, but because of the smallness of the pressure coefficient of resistance, the correction for the compressibility of the glass is rather important. The correction for compressibility, as given in the table, amounts initially to 11% of the pressure coefficient. The compressibility of different varieties of glass varies a good deal; it is perhaps conceivable that the value assumed is as much as 20% different from the correct one, so that the possibility must be recognized that the values given in the table for the pressure coefficient of resistance may be in error by as much as 2%. However, any correction of this sort will not affect the relative curvature, since the compressibility of glass is sensibly linear, and it seems justified to retain all the significant figures of the table. The effect of the corrections for the glass is to increase both the observed temperature and pressure coefficients of resistance.

The average pressure coefficient of the liquid to 12000 kg. is seen to vary little with temperature. In absolute value it is somewhat less than one half that for bismuth, and one sixth that for liquid mercury. The curvature is in the normal direction, that is, the coefficient becomes less at the higher pressures.

The pressure coefficient of resistance of the solid was measured only at 0°; at this temperature the entire pressure range of 12000 kg. was available, whereas at higher temperatures the pressure range was restricted by the melting, and the accuracy was proportionally less. The absolute resistance of the specimen was low, so that measurements could not be made with as much accuracy as usual. Beside the final run from which the tabulated results were taken, several rougher runs were made with other samples; these agreed with the final results within the accuracy of the measurements. Within the limits of error the relation between resistance and pressure is linear at 0° to 12000 kg., and the coefficient is $-0.0_5 247$. The maximum departure of any of the observed points from a linear relation was 2.7% of the total effect.

The constraining effect of a glass capillary on the solid is shown by the low value, -0.0_5191, found for the coefficient of the solid in glass.

The pressure coefficient of the solid is seen to be negative, that is, normal. This was rather a surprise; I had anticipated because of the abnormal expansion on freezing and the fact that bismuth also expands on freezing and has a positive coefficient of resistance that the coefficient of gallium might be positive also. The numerical value of the coefficient of the solid is quite normal, when compared with other metals. It is the value characteristic of a hard metal, which in most other cases also means a metal with a high melting point. The coefficient of the solid is of the order of one half that of the liquid. This again is as one would expect, except for the abnormal volume relations on freezing. However, the solid is less compressible than the liquid in spite of its greater volume [15a]; so that from this point of view the relative magnitudes of the pressure coefficients of liquid and solid do not seem unnatural.

The temperature coefficient of resistance of the unconstrained solid was obtained from readings at 0° and 21.5°. The value for this range is 0.003963, an entirely normal value. Previous values for this coefficient are exceedingly uncertain. Guntz and Broniewski's readings were quite irregular, the effect even reversing in sign above 18.6°. This may have been due to the constraining effect of the glass; such an effect is to be expected, and in the observed direction.

It may be mentioned that I made measurements on the subcooled liquid at 0°, and found the resistance to lie on a regular prolongation of the curve for the resistance above the melting point. Guntz and Broniewski, on the other hand, found the resistance of the liquid to pass through a minimum and to increase again in the unstable region below the melting point.

The ratio of the resistance of the solid to that of the liquid at the freezing point was found from measurements of the resistance of the solid at 0° in the glass capillary and the resistance of the liquid in the same capillary. The resistance of the solid was extrapolated to the melting point with the coefficient found. This procedure may be open to some question, but it seemed as satisfactory as any other that presented itself. The specific resistance of the solid was found to be 1.733 times that of the liquid at the melting point. Notice that the relative magnitude of the volumes governs the relative magnitudes of the resistance; the solid with the larger volume also having the larger resistance. Guntz and Broniewski found 2.09 for the ratio of the two resistances.

Titanium. By the kindness of the Research Laboratory of the General Electric Co. I was enabled to make measurements on a filament of titanium deposited on tungsten, which had been used for experimental work with incandescent lamps. The dimensions of the tungsten core were such that the total impurity of tungsten was only 1.8%. The method of deposition of the titanium on the core is not known to me; the surface of the filament was distinctly crystalline in appearance, probably due to recrystallization after deposition. It had been glowed out in vacuum at high temperature after deposition in order to remove impurities of hydrides, nitrides, and oxides, all of which are readily formed with this substance. This glowing out must have produced alloying with the tungsten core, and the alloy so formed is evidently quite different in its properties from the pure metal. This may be stated with confidence because the General Electric Co. found for the specific resistance of this filament the value 350×10^{-6} ohms per cm. cube, which is higher than would be given by the tungsten core alone. The same thing is indicated by the low value of the temperature coefficient, which was 0.000221.

The difficulties of the pressure measurements were very great, and it was not possible to obtain results which were at all regular. It can be stated only that the pressure coefficient is exceedingly small, probably not greater than 10^{-7} per kg., and that the likelihood is that the resistance increases with pressure.

Zirconium. Two filaments deposited on tungsten in the same way as the titanium were made available through the kindness of the General Electric Co. The treatment of the filaments had been the same as that of titanium. It is probable that the temperature of glowing out had been sufficient to produce alloying with the tungsten core. This is strongly suggested by the low value for the temperature coefficient of resistance, which, between 0° and 100°, was 0.00004 for one specimen, and 0.00058 for the other. The dimensions of the wires on which these filaments were deposited would indicate a total impurity of tungsten of 1.8% and 0.6% for the two samples respectively. The impurer sample has the smaller coefficient, as is usual. The exceedingly low value of both coefficients indicates that the impurity has a specific effect, and that any results found for the pressure coefficient may not be very close to the values for the pure metal.

In view of the probably large effect of the impurity, and also of the difficulty of the measurements, a great deal of effort was not put on this substance. The potentiometer method of measurement was used. This substance cannot be soldered, so that it was necessary to make connections with fine springs; slipping of the springs was perhaps accountable for some of the irregularities. The surface layer has a very high resistance, which again introduced irregularities at the spring contacts.

Two runs were made on the impurer specimen, at 0° and 94°. The irregularities of the second run were so great that the results were not computed. Within the limits of error the relation between pressure and resistance is linear at 0°, and the coefficient is -0.0_65.

The results on the second and purer sample were much more regular. Readings were made at 0° and 95°. At both these temperatures the relation is linear to 12000 kg. The best value for the pressure coefficient is -0.0_6398 at 0°, and -0.0_6396 at 95°. The coefficient is seen to be very small; such small values have been found only for certain of the high resistance alloys. The specific resistance of these Zirconium filaments was also very high; 200×10^{-6} ohms/cm^3 is the value given me by the General Electric Co.

Arsenic. Considerable interest attaches to this element because of its position in the periodic table above bismuth and antimony and below black phosphorus, all of which are abnormal in behavior under pressure. The arsenic used in this experiment was furnished by Eimer and Amend. It had been distilled in vacuum, but was otherwise of ordinary commercial quality, and I have no way of knowing what the impurities might have been. I attempted to cast it in a mold of pyrex glass, supported on the outside with magnesia, and enclosed in an iron pipe with caps tightly screwed on the ends. The melting temperature of arsenic was high enough to melt the pyrex, however, and the arsenic was found after the heating in the form of a solid slug in the lower part of the magnesia powder. It may possibly have come in contact to a slight extent with the iron of the pipe while in the molten condition. A slender rod about 1 mm. square in section and 2 cm. long was worked out of the slug with a file and a hack saw and by grinding. Grooves were filed on the ends, connections made with spring clips, and measurements made with the potentiometer in the regular way. I was surprised to find after I had completed the measurements that Matthiesen [16] had soldered connections to arsenic, and I verified for myself that it is as easy to soft solder to this metal as to antimony, for example. In fact the completely metallic character of the massive casting is a surprise contrasted with the appearance of the sublimed material as ordinarily furnished. Of course if I ever repeat this work I shall make connections by soldering.

A measurement of the temperature coefficient of resistance showed a discouragingly high probable impurity; the average coefficient between 0° and 95° was 0.00076, whereas Matthiesen [15] had found 0.0038. The high probable impurity of this sample did not make it worth while to make any very extended pressure measurements. One run was made. There were considerable irregularities, but within the limits of error the relation between pressure and resistance is linear to 12000 kg., and the value of the coefficient is -0.0_5326.

This coefficient is similar to that of a number of metals both as regards magnitude and sign. Arsenic is seen therefore to acquire neither the abnormal sign of the coefficient of its neighbors bismuth and antimony on the one side, nor the abnormally high numerical value of the coefficient of black phosphorus on the other.

Liquid Bismuth. The pressure coefficient of resistance of solid bismuth is abnormal in being positive; it was of particular interest to find whether the same abnormal behavior holds for the liquid. The bismuth used for these measurements was from the same lot of electrolytic bismuth as that whose pressure coefficient was previously measured. It was melted into a U-shaped fine glass capillary provided with four sealed-in platinum terminals for use with the potentiometer method. Special precautions were necessary to prevent the bismuth from cracking the capillary on freezing; this was accomplished by very slow cooling from the bottom up after the capillary had been filled with liquid bismuth. In this way congealing ran upward from the bottom of the capillary toward the open top, and no liquid was entrapped by the solid to crack the glass by its expansion on freezing.

A special arrangement of the pressure apparatus was necessary to permit the electrical measurements. The same arrangement was also used with lithium, sodium, and potassium, but since the apparatus was first used with bismuth, it will be described here. The difficulty was with the insulating plug, which was packed with soft rubber. This would have been carbonized by the temperature of melting bismuth. The pressure apparatus was accordingly constructed in three parts, instead of the customary two. There was an upper cylinder, as usual, in which pressure was produced, and in which was located the measuring coil of manganin wire. This upper cylinder was connected by a stout tube with the cylinder below it, in which was placed the bismuth in the glass capillary. This second cylinder was surrounded with a bath of Crisco, by which the desired temperature was maintained by thermostatic regulation. Out of the bottom of the second pressure cylinder was led another piece of stout tubing, which passed through a stuffing box in the bottom of the Crisco bath, and below the bath connected with a third pressure cylinder. This third cylinder was kept cool by a bath of water at room temperature. This bath was stirred to maintain the temperature uniform, but it was not necessary to regulate the temperature thermostatically. In the lower pressure cylinder was situated the insulating plug, of the same design as used in all the work with the potentiometer method. The plug was connected with the bismuth in the second cylinder by four insulated leads brought down through the pipe connecting the second and third cylinders. The insulation of these wires was asbestos; asbestos covered copper wire is now a commercial product. In this way the insulating plug was kept cool, so that there was no danger of leakage or failure of insulation because of the high temperature. The only trouble to be anticipated was large parasitic e.m.f.'s because of the large differences of temperature, but the parts in which there were temperature gradients were composed of electrically homogeneous material, and no more trouble was found from this effect than at ordinary temperatures.

Runs were made on liquid bismuth at 274.6°, 260.0°, and 239.6°, in this order. For fear of damaging the capillary (fused in platinum leads almost always make trouble under pressure) the pressure was not raised to the maximum of this work, 12000 kg., until the last run, so that I did not obtain data for the resistance of the liquid over the entire possible range. At 239.6°, however, pressure was run to the maximum with no bad effects. After the measurements on the liquid, the bismuth was allowed to freeze under pressure, and measurements were attempted on the solid. The results for the solid were not good, however, probably because of strains introduced on freezing in the fine capillary. There was no way of controlling the freezing under pressure and making it take place from the bottom up as had been possible in initially setting up the apparatus. The effects of strains were apparent in two ways; the pressure coefficient of resistance of the solid was negative over part of the range below the solidifying point, whereas that of the unconstrained solid is positive, and the freezing point was depressed a couple of degrees, which is in the direction to be expected if there are internal strains. Irregularities introduced by these strains are of importance, however, only when it was desired to obtain the relative changes of resistance with changes of pressure, and it was possible to find a value for the change of resistance on solidifaction which should not be greatly in error.

The readings on the liquid went very smoothly. Within the limits of sensitiveness there was no difference between points obtained with ascending or descending pressure, and except for one point, all of the observed points lay on smooth curves within the sensitiveness of reading, which was about 0.2% of the total effect. The observed values, smoothed for temperature and pressure, are shown in Table XIII in terms of the resistance at the melting point at atmospheric pressure as unity. The values tabulated are "observed" values, that is, they have not been corrected for the thermal expansion or

TABLE XIII.

BISMUTH.

Relative Values of Observed Resistance of the Liquid in Glass Capillary.

Pressure kg/cm²	Resistance 275°	260°	240°
0	1.0019		
1000	.9900		
2000	.9789		
3000	.9684	.9617	
4000	.9584	.9520	
5000	.9490	.9426	
6000	.9400	.9336	.9253
7000	.9314	.9249	.9169
8000		.9167	.9088
9000			.9008
10000			.8931
11000			.8855
12000			.8783

The resistance of the liquid at atmospheric pressure and 271.0° is taken as unity.

compressibility of the glass capillary. It did not seem best to do this because of the uncertainty in the values for the glass at the temperatures and pressures of the measurements. The glass used for the capillary was an ordinary soft soda glass. The best value for the cubic compressibility is probably 2.7×10^{-6}, taking Amagat's values for the compressibility and temperature coefficient of compressibility, and for the linear thermal expansion the best value is probably 8×10^{-6}.

The most important result shown by the table is that the pressure coefficient of resistance of the liquid is negative like all normal metals.

The positive coefficient of the solid is therefore presumably due to its crystalline structure. The liquid behaves in other ways also like normal metals. When resistance is plotted against pressure, the curve is convex toward the pressure axis; that is, the pressure coefficient decreases relatively (and also absolutely) at the higher pressures. The pressure coefficient is little affected by temperature, within this range, and also the temperature coefficient is little affected by pressure. The initial pressure coefficient of the "observed" resistance at 275° is -0.0_4123, which corrects, using the constants above, to -0.0_4132 for the specific resistance. Both of these coefficients are to be distinguished from the pressure coefficient of the "observed" resistance of a solid. The pressure coefficient of the liquid is of the same magnitude as that shown by the softer solid metals, such as lead, and is also very nearly the same numerically, although of opposite sign, as that of solid bismuth.

The temperature coefficient of the "observed" resistance at 275° is 0.00047, which corrects to 0.00048 for the coefficient of the specific resistance. This is about five times less than the value for a normal solid at the same temperature. It is almost always true that the temperature coefficient of the liquid is materially less than that of the solid.

At 7000 kg. at the equilibrium point, the resistance of the liquid is approximately 45% of that of the solid. At atmospheric pressure Northrup and Sherwood [17] found for the ratio 43%. There was considerable preliminary rounding of their melting curve, so their results are probably not any more accurate than mine, but it is at any rate evident that this ratio does not suffer any large change with increasing pressure.

Tungsten. In the preceding paper [18] results were given for the pressure coefficient of resistance of tungsten, but the value of the temperature coefficient of resistance of the sample used was so low (0.00322) that it was probable that the tungsten was not very pure. Since the publication of my earlier paper Beckman [19] has measured the effect of pressure to 1600 kg. on the resistance of a sample of tungsten having a considerably higher temperature coefficient than my original piece, and has found a higher initial value of the pressure coefficient than I did.

The sample of tungsten on which I previously experimented was the purest which the General Electric Co. was at that time in a position to offer me. I have since learned that it was probably "doped," that is, thoriated, the impurity of thorium being 0.2 or 0.3%. Through the kindness of the manufacturers I have since been able to measure the resistance of two samples of "undoped" tungsten of high purity. The Westinghouse Lamp Works gave me a specimen which they estimated to contain less than 0.03% total impurity, and the Research Laboratory of the General Electric Co. placed at my disposal a specimen which they estimated to be even purer. Judging by the test of the temperature coefficient the General Electric sample was appreciably purer. The average temperature coefficient between 0° and 100° of the Westinghouse sample was 0.003925, and that of the General Electric sample 0.004209.

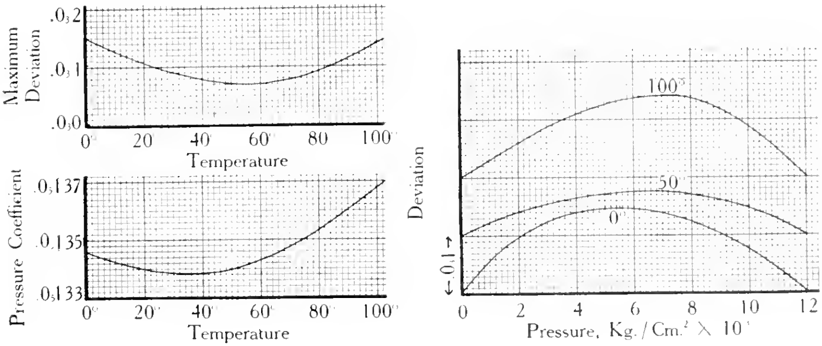
Pressure measurements were made on both samples, but only those on the purer are given in detail here. The treatment of both specimens was the same. The wire was 0.002 inches in diameter; it was wound bare on a bone core, and connections were made by fusing to it pure nickel wire with an arc in hydrogen. This method of making

TABLE XIV.

TUNGSTEN.

Temp. °C.	Resistance	Pressure Coefficient			Maximum Deviation from Linearity	Pressure of Maximum Deviation
		At 0 kg.	At 12000 kg.	Average 0–12000		
0	1.0000	-0.0_5143	-0.0_5131	-0.0_51346	.00015	5500
50	1.2084	137	133	1340	7	7000
100	1.4209	140	136	1368	14	7500

connections was taught me by the General Electric Co., and is simpler than the combination of gold and platinum which I formerly used, although the former connection was just as satisfactory electrically. The wires were seasoned by a long preliminary heating to 125° and by an application of 12000 kg. Three runs were made, at 0°, 50°, and 95°. Except for two bad points, the greatest departure of any of the observed points from a smooth curve at any of the three temperatures was 0.2% of the total effect, and the displacement of the zero after a run was not greater than the irregularity of the other points. The observed results were smoothed, and a table constructed for the resistance as a function of pressure and temperature by regular methods. The results are shown in Table XIV and Figure 9. The relation between pressure and resistance is nearly linear, and the departure is in the normal direction, that is, the coefficient becomes less at the higher pressures. The results are somewhat unusual in that the pressure coefficient does not advance regularly with increasing temperature, but is less at 50° than at either 0° or 100°. The same behavior was shown by the impurer sample also, and is doubtless real. The departure from linearity is also less at 50° than at either 0° or 100°.



Tungsten

FIGURE 9. Results for the measured resistance of tungsten. The deviations from linearity are given as fractions of the resistance at 0 kg. and 0°C. The pressure coefficient is the average coefficient between 0 and 12000 kg.

The results for the impurer sample were similar, but the numerical values were different. The average coefficients to 12000 at 0°, 50°, and 100° being respectively -0.0_51387, 1331, and 1362. These may be compared with the values of the Table; the differences are of the order of 0.5% at 50° and 100°, but the difference rises to 3% at 0°. At 0° the coefficient of the impurer is larger, and at the two other temperatures is smaller.

These results may be compared with those recently published by Beckman.[18] He finds for the average temperature coefficient of his sample 0.00399, which is a trifle higher than that of the impurer sample above. His initial pressure coefficient at 0° is -0.0_5151, reduced from atmospheres as the pressure unit to kg cm^2. This value is 2.7% higher than that of the impurer of the two samples above, and 5.6% higher than that of the purer. A comparison of the present results with my former ones shows that both of my new samples, which are purer than my former one, have a higher pressure coefficient. It is evident that the sign of the effect of impurity on the pressure coefficient cannot be predicted with probability, as can the effect of impurity on the temperature coefficient. At the same time the fact is to be emphasized, previously already found to hold in a number of cases, that the effect of impurity on the pressure coefficient is usually much less than on the temperature coefficient. Thus in the present case, a change in the temperature coefficient of 31% (present compared with former work) is accompanied by a change of pressure coefficient of only 9%.

LANTHANUM. This material I owe to the kindness of Professor Charles Baskerville, who had prepared it from the fused salts by electrolysis. No chemical analysis was available, but a spectroscopic analysis by Professor F. A. Saunders showed a large amount of Mg (possibly 10–20%) and a considerable amount of Si. There was a trace of Ca, no Ba, and none of the other rare earth metals were detected. The rare earths tested for were Ce, Pr, Nd, Er, Y, Yt, Dy, Lu. A nodule about one gram in amount was available for the measurements. A small homogeneous piece was cut from the nodule, and extruded to wire in a small die of special construction. It is necessary to heat to about 450° to extrude, and even then the extrusion is a matter of some difficulty. The wire so formed is exceedingly stiff; it is evident that its elastic constants and its elastic limit are both high. It is quite brittle, and can be bent only into a circle of large radius. I prepared two pieces of wire, one at a somewhat higher extrusion temperature than the other; the mechanical properties seemed unaffected by the temperature of extrusion. The wire on which measurements were made was only 1.7 cm. long, and was that prepared at the lower extrusion temperature. In order to attach the four terminals, spring clamps of special design had to be used; it is not possible to solder this metal. The clamps gave some difficulty with shifting of position, and the results were not so regular as usual.

The temperature coefficient of this material between 0° and 100° was only 0.001476. This is very low, and indicates that the material was not very pure. For this reason it did not seem worth while to spend a great deal of effort on the pressure measurements, although these could have been improved by repeating the measurements with a longer specimen, which was obtained after the easiest extrusion temperature was discovered.

Two runs were made for the pressure coefficient, at 0° and 50°. The ascending points of the run at 0° were entirely regular, but the descending points were irregular, probably because of slipping of the contacts. At 50° the ascending and descending points agreed more closely, but there were irregularities both ascending and descending, less in magnitude than at 0°, and averaging 2.7% of the maximum pressure effect.

The results are collected in Table XV and Figure 10; they are seen to be quite normal. The pressure coefficient is negative, and

TABLE XV.

LANTHANUM.

Temp. °C.	Resistance	Pressure Coefficient At 0 kg.	Pressure Coefficient At 12000 kg.	Pressure Coefficient Average 0–12000	Maximum Deviation from Linearity	Pressure of Maximum Deviation
0	1.0000	-0.0_539	-0.0_525	-0.0_5331	.0020	5800
50	1.0752	39	36	377	4	5800
100	1.1476					

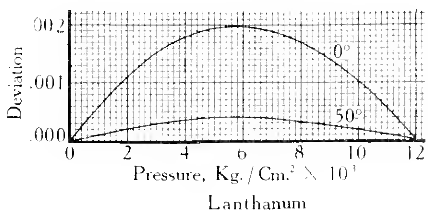


FIGURE 10. The deviations from linearity of the measured resistance of lanthanum in fractional parts of the resistance at 0 kg. and 0°C.

increases at the higher temperatures. The relation between pressure and resistance is also normal, the coefficient being smaller at the higher pressures. It is perhaps unusual that the departure from linearity is less at the higher temperatures. The deviations from linearity are so small and so nearly symmetrical about the mean pressure that it is not necessary to reproduce the deviation curve graphically.

I have been able to find no previous values for the specific resistance of this metal. The approximate value for the specimen above, obtained from micrometer measurements of its dimensions, was 59×10^{-6} ohms per cm. cube.

NEODYMIUM. This, as well as the lanthanum, I owe to the kindness of Professor Baskerville. Professor Saunders was kind enough to make a spectroscopic analysis of this also. He found a large amount of Mg, a little Si and La, a trace of Ca, no Ba, and nothing else recognizable. He tested for the rare earths Ce, Pr, Y, Yt, Lu, Dy, Er. The form and method of preparation of the specimen was essentially the same as that of lanthanum. It was extruded into wire 0.020 inches in diameter at 450°. The extrusion was materially easier than that of lanthanum. The wire is not so stiff, and may be straightened after extrusion without fear of breaking. Nevertheless it obviously belongs to the metals with high elastic constants and high elastic limit. The specimen used for the measurements was 7.1 cm. in length; the manner of attachment of the connections was the same as with lanthanum. The greater length of the specimen, and perhaps greater skill in handling it, led to much more regular results.

A preliminary measurement of the temperature coefficient was made at 0°, 50°, and 95°. Within this range the relation between temperature and resistance was found to be linear, and the coefficient was 0.000799. This is extraordinarily low, much lower than for lanthanum even, and it did not seem worth while to expend a great deal of time on the pressure measurements.

Two runs were made for the pressure coefficient, at 0° and 50°. The results were rather regular. There was no difference between readings with increasing and decreasing pressure, and the zero was well recovered. The maximum departure of any single point from a smooth curve was 2.6% of the total effect at both 0° and 50°. The numerical results are shown in Table XVI. The values are quite

TABLE XVI.

NEODYMIUM.

Temp. °C.	Resistance	Pressure Coefficient At 0 kg.	Pressure Coefficient At 12000	Pressure Coefficient Average 0–12000	Maximum Deviation from Linearity	Pressure of Maximum Deviation
0	1.0000	-0.0_5238	-0.0_5183	-0.0_5213	.00078	6000
50	1.0400	250	197	226	87	6000

normal. The coefficient is negative, and becomes greater numerically at the higher temperature. The departure from linearity with pressure is in the normal direction, the coefficient being less at the higher pressures. The departure from linearity is symmetrical, a second degree curve accurately reproducing the results, so that it is not necessary to draw the deviation curves. The deviation becomes greater at the higher temperatures, as is normal.

I can find no tabulated values for the specific resistance of neodymium. The value for the specimen above, obtained from micrometer measurements of its dimensions, was 107×10^{-6} ohms per cm. cube, about twice the value for lanthanum.

Carbon. These results on carbon must be regarded as of an entirely tentative and orienting character. Present technical methods are not yet sufficiently perfect to permit of the manufacture of carbon, either amorphous or graphitic, of specifiable or reproducible properties. Any massive form of carbon always contains at least a slight amount of binder of unknown properties, amorphous carbon is always impure with a slight amount of graphite, and the purest graphite contains a small and unknown amount of amorphous carbon.

Experiments were made on three samples of carbon. The first of these was supposedly amorphous carbon, an arc carbon made by an unknown German firm. The second was Acheson graphite cut from a piece of graphite furnished by the Acheson Co. for a resistance furnace, and presumably not made with any unusual precautions. The third specimen of graphite was also Acheson graphite, furnished by the Acheson Co., in response to a special request for graphite of the greatest obtainable purity. It was stated by them to contain the minimum of binder, and to have been graphitized with unusual thoroughness, but otherwise its properties were not known.

In view of the unreproducible character of the results it will not pay to give them in great detail.

Two sets of readings were made on the gas carbon, a complete run to 12000 kg. and back at 30°, and a few readings at 96°. This specimen was about 3.5 inches long, and 0.154 inches diameter. Measurements were made by the potentiometer method, using the three terminal plug as usual. Connections were made to the carbon with spiral springs snapped into grooves filed around the surface of the rod. A preliminary seasoning was made to 6000 kg., but there was very little permanent change of resistance. At 30°, the resistance decreases with rising pressure, and the direction of curvature is normal, that is, the proportional effect becomes smaller at the higher pressures. The percentage decrease of resistance was 6.68% at 6000 kg., and 12.07% at 12000 kg. At 96°, up to 3000 kg., the coefficient is about 4% greater. The resistance decreases linearly with temperature between 0° and 100°, the total decrease for 100° being 2.56% of the resistance at 0°. The readings on this carbon were entirely regular, showed little difference between ascending and descending values, and an almost perfect recovery of the zero.

The first specimen of Acheson graphite was cut from a rod of 1.5 inches diameter to about the same dimensions as the gas carbon. Measurements were made by the same method. Three complete series of readings were made, at 0°, 51°, and 97.4°. The readings were not so regular as with the gas carbon, showing large hysteresis effects, rising at the maximum to 12% of the total pressure effect, and there were also parasitic e.m.f.'s so large as to necessitate a special arrangement of the constants of the circuit. The sign of the effect is positive, the reverse of what it is for gas carbon, and there is very considerable departure from linearity with pressure, the coefficient becoming numerically less at the higher pressures. The total fractional increase of resistance under 12000 kg. decreases with rising temperature, being 4.75% at 0°, 4.23% at 50°, and 4.23% also at 100°. The large departure from linearity may be judged from the fact that at 6000 kg. at 0° the increase of resistance is 3.44%, which is 72% of the increase under 12000 kg. The resistance decreases with rising temperature at atmospheric pressure, and the change is not linear. At 0° the resistance on an arbitrary scale is 1.0000, at 50° 0.9135, and at 100° 0.8687.

The second specimen of graphite, supposed to be especially pure, was subjected to a special preliminary seasoning in order to eliminate as far as possible the pores. It was sealed into a lead tube and subjected to a fluid pressure of 12000 kg. on the outside of the tube. The diameter of the specimen was reduced by about 2%, but there were a great many pores still visible to the naked eye. It was further seasoned by heating to 125° in vacuum; this treatment should have removed all moisture, of which no traces, however, were evident. This specimen was cut to the same dimensions, and measurements made in the same way as on the two preceding samples. Only one run was made, at 50°. This was terminated by an explosion at 12000 kg. In view of the unreproducible character of the results it did not seem worth while to repeat the effort to obtain a complete set of readings. As with the other sample of Acheson graphite, the effect of pressure is to increase the resistance, but the change was much less numerically than for the other piece, and the departure from linearity was much greater, suggesting strongly that at a high enough pressure the resistance may pass through a maximum. The following increases of resistance were found at 2000, 4000, 6000, 8000, and 10000 kg. respectively; namely, 0.69%, 1.25%, 1.62%, 1.79%, and 1.86%. The temperature coefficient of resistance of this specimen was not measured.

Although the results obtained above are not of much accuracy and are not reproducible, two interesting facts stand out; the opposite signs of the pressure coefficient for carbon in the amorphous and graphitic states, and the large departure of the effect from linearity with graphite, indicating a maximum. No substance has yet been found in which a maximum or minimum of resistance has been actually reached at high pressures.

Silicon. It is well known that technical means are as far from perfect for producing a pure and reproducible silicon as they are for carbon. For instance, the temperature coefficient of resistance varies in sign with different pieces of apparently the same manufacture. In view of this situation it was worth while to make only a few pressure measurements in order to establish the general nature of the effects. Two specimens were used, both provided by the General Electric Co.

The first sample had not been manufactured by them, but had been obtained from the Carborundum Co. It was in the form of a cylinder about 5 mm. in diameter and 8 cm. long. Connections were made with spring clips, and measurements made by the potentiometer method, as usual. The effects were very irregular; I satisfied myself that the irregularities were inherent in the material itself. There were large seasoning effects on the first application of pressure, there were always permanent changes of zero after a run, and there were differences between the readings with increasing and decreasing pressure in a direction the reverse of hysteresis. Two series of runs were made, at 0° and 52°. The resistance decreases under pressure, as is normal. At 0° the total decrease under 12000 kg. was 14.0%, and at 52° 15.8%. The effect is not linear with pressure, but the coefficient becomes less at the higher pressure, as is normal. The average temperature coefficient of this sample between 0° and 52° was 0.000117.

The second sample was also furnished originally by the Carborundum Co., but it had been partially purified by the General Electric Co. by melting in vacuum. The purification was by no means complete, for it was possible to see with the naked eye small slag-like inclusions and there were numerous fairly large pores. (There were visible pores in the first sample also). It need not be anticipated that the pores cause any error in the pressure coefficient, for the transmitting liquid freely penetrates the pores and transmits pressure uniformly to all parts; there is never any permanent change of dimensions after an application of pressure. The second sample was of approximately the same dimensions and was treated in the same way as the first. Two sets of readings were made with this second sample, at 0° and 95°. The pressure coefficient is negative in sign, as it was for the first sample, but the numerical values are somewhat different. At 0° the resistance decreases by 10.1% under 12000 kg., and at 95° by 15.3%. The change is not linear with pressure, but the coefficient becomes larger at the higher pressures, which is the opposite of the normal behavior of the first sample. The temperature coefficient of this sample between 0° and 95° was + 0.0000615, about half as large as that of the first sample.

In spite of the very marked differences these two samples agree much more nearly in their pressure coefficients than they do in their temperature coefficients. This agrees with previous experience, that in general the temperature coefficient is much more susceptible to impurity than the pressure coefficient. We may expect, therefore, that the pressure coefficient of resistance of pure silicon will be found to be negative, and of the order of −0.000012, pressure being expressed in kg/cm^2. Compared with most metals, this coefficient is high, being about the same as that of lead.

Black Phosphorus. Runs were made on two samples of this substance. The first was from the same piece as that which gave the values for the specific resistance and temperature coefficient of resistance already published.[20] The method of formation and some of the other properties have also been described. During the six years since the previous measurements, this specimen has been kept in a glass bottle, closed with a cork stopper and sealed with paraffine. The protection from the action of the air was not perfect, however, because the phosphorus had become covered with a layer of moisture. This moisture is probably due to slow oxidation of the phosphorus in the air. The result of oxidation is the formation of phosphoric acid, which is well known to be very hygroscopic, and therefore rapidly absorbs moisture from the air. An attempt was made to remove the acid from the sample by boiling it with water for a number of hours, and then heating in vacuum for a number of hours in addition.

The specimen previously used was a cylinder about 0.5 inches in diameter. For the resistance measurements a square prism was cut from the center of this about 0.2 inches on a side. The resistance was measured by the potentiometer method, with the three terminal plug. The terminals were attached to the phosphorus by means of helical coils of very fine wire snapped over the prism in grooves filed on its surface. The distance between potential terminals was about 1.5 cm.

One run was made with this sample, at 0°. The points with increasing pressure ran smoothly, and on decreasing pressure the points with increasing pressure were repeated, except the final zero, where there developed a parasitic e.m.f. so large that further readings were impossible. The general character of the pressure effect was an enormous decrease of resistance under pressure. The results before the parasitic e.m.f. appeared were very nearly the same as those found later with the second sample. The parasitic e.m.f. was ascribed to the imperfect removal of the phosphoric acid, and the specimen was again treated for a number of hours with boiling water, but without success. It was evident that the acid permeated the material too deeply to be removed by surface treatment in this way. It was accordingly necessary to prepare a fresh specimen of phosphorus.

In preparing this fresh sample, advantage was taken of an observation made by Dr. A. Smits [21] in preparing the phosphorus for measurements of the vapor pressure. He found that the kerosene by which pressure had been transmitted to the phosphorus during formation was exceedingly difficult to remove. A chemical analysis by Professor Baxter had also shown some carbon as an impurity of the phosphorus; it is possible that some of this might also have been introduced by the kerosene. It was therefore indicated that the black phosphorus should be formed if possible without contact with kerosene. This was simply done by surrounding the yellow phosphorus with water in the lower cylinder, transmitting pressure to the water with kerosene as usual, but so choosing the dimensions that the kerosene should never come in contact with the phosphorus. This was entirely successful; the transition went essentially as before, when kerosene was used. In particular, occasion was taken to again measure the time rate of transition, and the same results found which have already been published,[22] and which make the explanation of the transition from yellow to black phosphorus so puzzling. That the phosphorus formed under water was purer than the phosphorus previously formed under kerosene was suggested by the absence of the peculiar odor, which had permeated the earlier product, characteristic of kerosene which has been exposed to high temperatures and pressures. It would be of interest if the vapor pressure measurements of Professor Smits could be repeated on this specimen.

The specimen so formed was dried in vacuum for a number of hours, at 125°, and sealed into an exhausted glass tube until ready for use. It was cut to the same dimensions, and mounted in the same way as the other specimen. Three runs were made with this, at 0°, 51°, and 95°. In addition to the temperature seasoning incidentally done when it was heated in vacuum, it was given a pressure seasoning by an application of 12000 kg. at 0°. The runs all went smoothly; parasitic

TABLE XVII.

Black Phosphorus.

Pressure kg/cm²	Resistance 0°	50°	100°
0	1.000	0.662	0.421
1000	.796	.521	.323
2000	.643	.406	.250
3000	.492	.313	.1950
4000	.372	.239	.1517
5000	.277	.1816	.1183
6000	.2042	.1371	.0910
7000	.1493	.1028	.0701
8000	.1079	.0766	.0542
9000	.0783	.0572	.0425
10000	.0565	.0427	.0337
11000	.0409	.0318	.0266
12000	.0297	.0238	.0209

e.m.f.'s were no larger than would be expected from the high thermal e.m.f. of this material, the behavior was perfectly reversible with ascending and descending pressure, and the alteration of zero after a run was no larger than the irregularities of any other of the observed points.

The outstanding feature of the results is the exceedingly large decrease of resistance brought about by pressure, much larger than for any other substance which I have measured. At 0° and 12000 kg. the resistance is only 3% of its value at atmospheric pressure. The ordinary method of plotting is not adapted to such a wide range of relative values, and accordingly in smoothing the results and making the interpolations and extrapolations involved in making a table of resistance at uniform intervals of pressure and temperature, the logarithm of the resistance was plotted against pressure instead of resistance itself.

Except for two bad points, the maximum departure of any point from a smooth curve at any temperature was 2% of the resistance at that point, and the agreement was usually much closer.

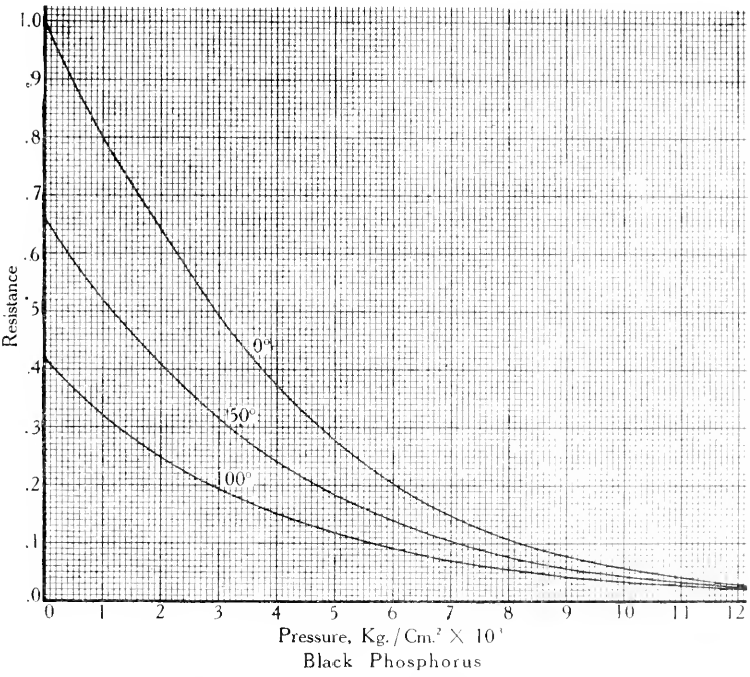


FIGURE 11. Relative resistances of black phosphorus at several constant temperatures as a function of pressure.

The smoothed results are shown in Table XVII, which gives resistance at 0°, 50°, and 100° at even 1000 kg. intervals, and in Figure 11. The use of significant figures in the table should be mentioned, the number of decimal places not being constant throughout the table. The accuracy of the results justifies only the places shown. The resistance itself was measured at every point with enough accuracy so that it would have been permissible to keep throughout the table a constant number of decimal places, but the pressure itself at the lower pressures is not known with a high enough accuracy to justify keeping more significant figures than shown.

When the logarithm of resistance is plotted against pressure a nearly linear relation is found at all three temperatures. This means that $\frac{1}{R}\frac{dR}{dp}$ is approximately constant at all pressures at constant temperature, where R is the instantaneous value of the resistance at the pressure in question. The instantaneous pressure coefficient is a function of temperature, however. The average value of the instantaneous coefficient between 0 and 12000 kg. is -0.000293 at 0°, -0.000277 at 50°, and -0.000250 at 100°. The deviation of the logarithm from exact linearity changes sign with rising temperature. At 0° and 50° the instantaneous coefficient becomes greater with increasing pressure, which is not what one might expect, whereas at 100° it becomes less. At the two lower temperatures the deviations from linearity run smoothly with the pressure, but at 100° the variations, although much less numerically, show one or two points of inflection with rising pressure. At 0° the initial value of the instantaneous coefficient is -0.000200 and at 12000 kg. it has risen to -0.000320; the corresponding values for 50° are -0.000231 and 0.000290, and for 100° -0.000262 and -0.000249.

The specific resistance was also determined. At 0° this was found to be exactly 1.000 ohms per cm. cube. This is higher than the value previously published for the other specimen, which was 0.711. The effect of temperature on the new specimen is also greater than on the previous one. The values for this specimen are shown in the table. The resistance decreases with increasing temperature, and the effect is not linear, as of course it cannot be, for otherwise the resistance would become zero at some finite temperature. The coefficient found for the other specimen was also negative, but smaller numerically and within the temperature range, the relation was linear. Previously the relative resistance was found to drop from 1.000 to 0.711 between 0° and 50°, whereas here the drop is found to be from 1.000 to 0.622 for the same temperature interval. In view of the greater precautions in the preparation of this sample, there can be no doubt but that the present values are to be preferred.

It was considered of sufficient interest to measure the thermal e.m.f. of this specimen of black phosphorus. The details of the method need not be described; they were sufficiently obvious. Asymmetry in the specimen was eliminated by making two sets of readings with the hot and cold ends reversed. The difference of readings in the two positions was only 3 %. The temperature interval was from 0° to 21°. The effect is very large. The thermal e.m.f. against copper in this interval was at the rate of 0.000413 volts per degree Centigrade, positive current flowing from copper to phosphorus at the hot junction.

Iodine. The measurements on iodine were part of the systematic attempt to measure the effect of pressure on the resistance of all the elements which could be handled with sufficient ease. The striking effects found for black phosphorus, and the nearness of phosphorus and iodine in the periodic table gave rise to the hope that a similar effect might be found with iodine.

The measurements proved of unexpected difficulty, because of the readiness with which iodine dissolves in most of the liquids by which pressure can be transmitted. It was found, for instance, that iodine dissolves in kerosene, or petroleum ether, or glycerine. It was a surprise to find that the solutions are fairly good conductors. Considerable effort was spent in devising a suitable method of transmitting pressure to the iodine and getting electrical connections into it, but without much success. In the arrangement finally adopted, the iodine was melted into an open glass cup, provided with two platinum electrodes connected with wires sealed through the base. The cup was placed in a second larger cup, and the wires led up between the inner and the outer cup, and bent over the edge of the outer cup. The upper part of the inner cup was filled with water to a sufficient depth to completely cover the iodine. The outer cup was filled with Nujol, which covered the iodine and water in the inner cup. Connections were made as usual to an insulating plug, this time one of the old single-terminal plugs. The object of the double arrangement of cups was to keep the iodine from contact with the oil, and to keep the water from contact with the insulating plug and any part of the leads, which would otherwise have been short circuited. The arrangement was not satisfactory, for the glass cracked around the platinum leads under pressure, allowing a slight amount of iodine to go into solution in the oil, and furthermore, because of unequal compressibility of the glass and iodine, some water crept between the surface of the glass and the iodine, thus making a short circuit possible. The iodine further dissolved to some extent in the water under pressure, and from the water it again diffused into the surrounding oil, so that there was a second possibility of short circuit. The initial resistance of the arrangement was of the order of one megohm. Pressure was not pushed higher than 8000 kg., in order not to freeze the water. At this pressure the resistance had dropped to 35000 ohms. There were large polarization effects, and on releasing pressure the resistance did not recover its initial high value.

These experiments can only justify the conclusion, therefore, that under high pressure iodine does not at any rate become metallic in its conductivity, but the specific resistance remains high. It is quite possible that the relative resistance may suffer large changes, but the probability is small that the change of relative resistance is as high as it is for black phosphorus.

The iodine used for this experiment was Kahlbaum's, previously dried in vacuum. The platinum electrodes were approximately 1 cm^2 each in area, and 3 mm. distance from each other. These dimensions, together with the value of the minimum resistance recorded above, allow a minimum value to be set for the specific resistance at 8000 kg. of about 100,000 ohms per cm. cube. The correct value is doubtless many fold greater. The specific resistance of iodine under ordinary conditions seems too much affected by impurities to allow of its accurate determination, and I have not been able to find a value anywhere recorded.

"CHROMEL A." This is an alloy for high temperature resistance units essentially similar to the alloys known more familiarly under the name of "Nichrome." "Chromel A" is made by the Hoskins Co. of Detroit, and has the composition 80% nickel and 20% chromium. It was furnished by the manufacturer in the form of a wire 0.005 inches diameter, and was double silk covered by the New England Electrical Works. I wound it for these measurements into a coreless toroid of 118 ohms resistance at 0°. It was seasoned for the measurements by keeping it at 135° for four hours, and by a preliminary application of 2000 kg.

The effect of pressure is in the normal direction, that is, the resistance decreases with increasing pressure, but the effect is very small, smaller than any which I have previously found. The maximum displacement of the slider of the Carey Foster bridge was 4 cm., so that the sensitiveness of the measurements was not greater than one part in 400. Within the limits of error the relation between pressure and resistance is linear to a maximum pressure of 12000 kg. At 0° the two points at the highest pressures were irregular, probably because of viscosity in the transmitting medium, and at 90° there were hysteresis effects amounting to 4% of the total pressure effect.

At 0° the pressure coefficient was -0.0_6134, and at 90° -0.0_6137. The temperature coefficient between 0° and 90° was 0.000163.

"CHROMEL B." This alloy is much similar to "Chromel A." It is made by the same concern, and has a composition of 85% nickel and 15% chromium. Like the previous material it was furnished in the form of wire 0.005 inches in diameter, and was double covered with silk insulation by the New England Electrical Works. It was also wound into a coreless toroid of approximately 100 ohms resistance at 0°. It was seasoned for temperature at the same time as the "Chromel A" by four hours at 135°, but was in addition seasoned for pressure by a single application of 12000 kg. at room temperature, and a single application of 2000 kg. after mounting ready for the measurements.

The general character of the results is the same as for "Chromel A." The coefficient is not quite so small, and the results were considerably more regular. This was in part due to the choice of a less viscous transmitting medium. The relation between pressure and resistance is linear within the limits of error. Two series of measurements were made, at 0° and 95°, to a maximum pressure of 12000 kg. At 0° the maximum departure of any single reading from the linear relation was 1.5% of the maximum effect, and at 95° it was 1.9%, except for the zero, which showed a displacement of 3.5%.

At 0° the average pressure coefficient was -0.0_6158, and at 95° -0.0_6169. The average temperature coefficient of resistance at atmospheric pressure between 0° and 95° was 0.000212.

"CHROMEL C." This alloy is also intended for high resistance heating units. It is made also by the Hoskins Co., but unlike Chromel A and B contains some iron in addition to nickel and chromium. The exact composition is Fe 25%, Ni 64%, and Cr 11%. The wire was 0.005 inches in diameter, double silk covered, and wound into a coreless toroid of such dimensions as to have at 0° a resistance of 178 ohms. It was seasoned by one preliminary application of 12000 kg., and after connecting to the pressure apparatus by three applications of 2000 kg. Measurements were made on the Carey Foster bridge, as usual with materials of high resistance.

Three runs were made, at 0°, 52.24°, and 95.88°. The variation of resistance is not throughout linear with pressure and temperature, but shows departures in abnormal directions. The departures from linearity do not run uniformly, so that it was not possible from the three series of readings to construct a table of resistance which could be used by interpolation to obtain the resistance to the limit of accuracy at any temperature and pressure within the range. This alloy would merit further study for its own sake, but an elaborate investigation did not fall within the present program, and the results are given as found.

At 0° the relation between pressure and resistance is linear within the limits of error. The maximum departure of any point from the linear relation was 0.3% of the total pressure effect, and the departures from linearity were distributed at random. The average pressure coefficient of resistance between 0 and 12000 kg. was $-0.0_6 4272$.

At 52.24° the relation between pressure and resistance was again linear within the limits of error, but there was sensible hysteresis. The maximum width of the hysteresis loop was 1% of the total effect. The ascending and descending points all lay smoothly on their respective branches of the hysteresis loop without departures of more than 0.06% of the effect. The average pressure coefficient between 0 and 12000 kg. was $-0.0_6 4194$, less than the value at 0°.

At 95.88° the relation between pressure and resistance was sensibly not linear, but could be represented within the limits of error by a second degree curve. The maximum departure of any observed point from the second degree curve was 0.35% of the maximum effect. The departure from linearity is in the abnormal direction, that is, the average coefficient between 0 and 6000 is less numerically than the average coefficient between 0 and 12000 kg. The average coefficient 0 to 12000 was $-0.0_6 4488$, and that between 0 and 6000 was $-0.0_6 4372$. It is to be noticed that somewhere between 0° and 100° the pressure coefficient of resistance has passed through a minimum.

The temperature coefficient of resistance at atmospheric pressure is normal in being positive, but the direction of curvature is abnormal. The average coefficient between 0° and 52° is 0.001076, and between 0° and 96° 0.001030.

"Comet" Alloy. This is an alloy of the following composition:

Cr	1.75%
Ni	31–32%
C	.20–.25%
Si	.20–.25%
Mn	1.8–2.0%
P and S	very low
Fe	balance

It is made by the Electrical Alloy Co. and was furnished by them in the form of wire 0.005 inches in diameter, and doubly covered with silk insulation. It was wound for the measurements into a coreless toroid of 283 ohms resistance at 0°. Readings were made on the Carey Foster bridge in the usual way at 0°, 51.22°, and 95.32°.

The wire was seasoned by a preliminary application of 12000 kg. at room temperature, and after soldering to the insulating plug, by four additional applications of 2000 kg. at room temperature. That the seasoning was adequate is shown by the fact that there was no further perceptible change of zero after the first excursion to 12000 and back.

The readings showed a small but distinct hysteresis, increasing at the higher temperatures. At 0° the width of the loop is 0.45% of the total pressure effect, at 51° 0.5% of the effect, and at 95° 0.67%. At 95° there was a displacement of the zero after the run of an amount equal to the width of the hysteresis loop. At the other temperatures there was no perceptible change of zero.

The results were computed in the usual way, and are shown in Table XVIII, and Figure 12. This alloy is unusual in that the pres-

TABLE XVIII.

"COMET" ALLOY.

Temp. °C.	Resistance	Pressure Coefficient At 0 kg.	At 12000	Average 0–12000	Pressure of Maximum Deviation	Maximum Deviation from Linearity
0	1.00000	-0.0_5263	-0.0_5222	-0.0_52413	5000	.00046
50	1.04644	224	206	2176	6500	25
100	1.09061	203	194	2008	6500	7

sure coefficient of resistance becomes less at the higher temperatures, although the resistance itself becomes greater. The behavior is normal in that the pressure coefficient becomes less at the higher pressures at constant temperature. The relation between pressure and resistance becomes more nearly linear at the higher temperatures, which would be unusual for a pure metal.

"THERLO." This is an alloy much like manganin in its properties, made by the Driver Harris Co. The composition is Cu 85%, Mn 13%, Al 2%. It has been used in the high pressure work at the Geophysical Laboratory as a substitute for manganin in high pressure gauges. The sample on which I made measurements was 0.005 inches in diameter, double silk covered, and wound into a coreless toroid of a resistance at 0° of 127 ohms. This was very nearly the resistance of the manganin pressure gauge, so that a very accurate comparison of the pressure coefficients of manganin and Therlo could be made by plotting on a large scale the difference of the readings with the two alloys. The Therlo was seasoned by one application of 12000 kg. at room temperature, and after soldering to the insulating plug, by four additional applications of 2000 kg.

Three runs were made, at 0°, 51.05°, and 94.80°. The variations with temperature were so slight that the readings could be reduced to regular temperature intervals by an interpolation or extrapolation so short that there was no possibility of error. The resistance of this sample of Therlo did not vary quite linearly with pressure, that is,

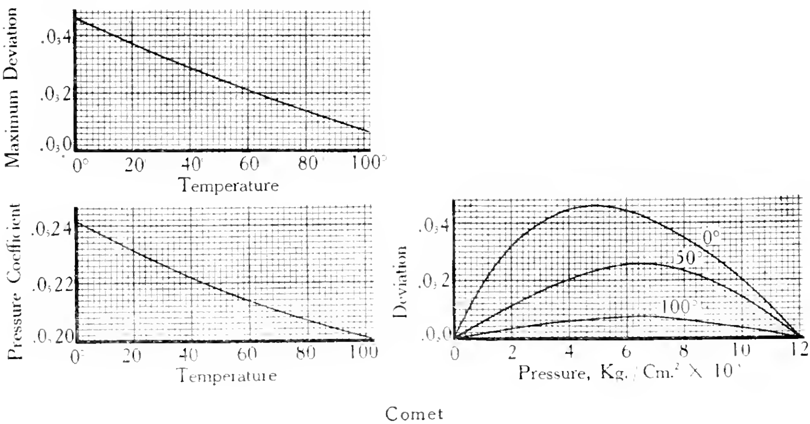


FIGURE 12. Results for the measured resistance of Comet alloy. The deviations from linearity are given as fractions of the resistance at 0 kg. and 0°C. The pressure coefficient is the average coefficient between 0 and 12000 kg.

it did not vary linearly with the manganin. (The manganin was originally calibrated against an absolute gauge and found linear within 0.1%). The deviations from linearity of the Therlo are greatest at the lower pressures and are not symmetrical.

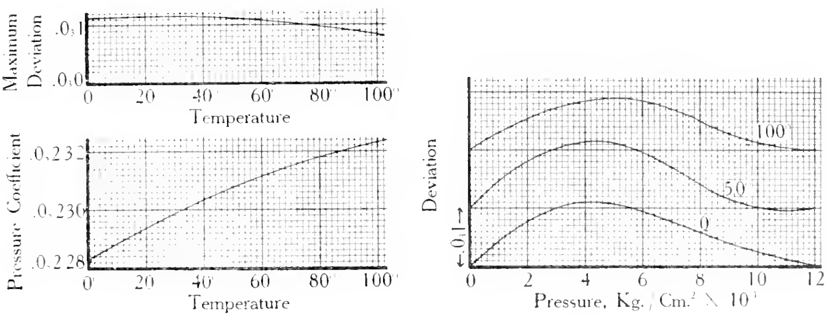
At 0° the maximum departure of any single observed point from a smooth curve was 0.09% of the total pressure effect, at 51° 0.14%, and at 95° 0.05%.

The results have been computed in the regular way, and are exhibited in Table XIX and Figure 13. The method of representation is the same as that used in the preceding paper on resistance under

TABLE XIX.

"Therlo."

Temp. °C.	Resistance	Pressure Coefficient At 0 kg.	Pressure Coefficient At 12000	Pressure Coefficient Average 0–12000	Maximum Deviation from Linearity	Pressure of Maximum Deviation
0	1.00000	$+0.0_52361$	$+0.0_52273$	$+0.0_52283$	.000112	4000
50	1.00120	2386	2318	2308	112	4500
100	1.00104	2367	2320	2323	83	5000



Therlo

Figure 13. Results for the measured resistance of Therlo alloy. The deviations from linearity are given as fractions of the resistance at 0 kg. and 0°C. The pressure coefficient is the average coefficient between 0 and 1200 kg.

pressure. It will be noticed that the average pressure coefficient shows greater variation with temperature than does that of manganin, and the variations are in fact greater than the variations of resistance itself. The resistance passes through a maximum in the neighborhood of 75°, the increase between 0° and 75° being 0.120%, and between 0° and 100° 0.104%, whereas the average pressure coefficient continues to increase over the entire range between 0° and 100°. The instantaneous coefficient at 0 kg., however, passes through a maximum between 0° and 100°.

"#193 Alloy." This is an alloy containing Fe 68%, Ni 30%, and

Cr 2%, made by the Driver Harris Co. for use in heating units. I was interested in the pressure and temperature coefficients because I had used it as the capillary for containing liquid lithium. The accuracy required in the coefficients was not high, so that measurements of the pressure coefficient at only one temperature and of the temperature coefficient between only two temperatures were sufficient.

The specimen was in the form of a capillary 0.045 inches outside diameter, and 0.032 inches inside diameter, about 6 cm. long. The resistance was too low to be measured by the Carey Foster method, and accordingly the potentiometer was used, as with other metals of low resistance. The temperature of the pressure readings was 94.2°. At this temperature the resistance decreases with increasing pressure, the relation is linear within the limits of error, and the average coefficient between 0 and 12000 kg. is -0.0_51790. Except for a single bad point, the maximum departure of any reading from the linear relation was 0.7% of the total effect, and the arithmetic mean of all the departures was 0.25%.

The average temperature coefficient of resistance at atmospheric pressure between 0° and 94° was 0.000684.

General Survey of Results.

We have in the first place to inquire whether these new results for elements somewhat unusual in their properties are the same in character as those previously obtained for the more common elements. In discussing the new data it will be convenient to discuss separately metals in the solid and liquid state, and also metals with positive or negative pressure coefficients of resistance. The previous results were almost entirely for solid metals; measurements for only one liquid metal, mercury, had been made at that time. Furthermore, the pressure coefficient of all solids, except bismuth and antimony, was negative. In the following a solid or liquid is called normal if its pressure coefficient of resistance is negative. The alloys will require separate discussion.

Normal Solids. The normal solids embraced in the present series of measurements are Na, K, Mg, Hg, Ga, Ti, Zr, As, W, La, Nd, Si, and black phosphorus. The special interest of these measurements attaches to those substances with large coefficients. Many of the above list do not belong in this category, and may be dismissed with a few words.

Mg and W were measured in the previous paper. Except for the improvement in the numerical values afforded by the new measurements, these substances require no further discussion. It is to be noticed that the revised values of the pressure coefficient are in such a direction as to make the pressure coefficients of atomic amplitude and resistance even more divergent than was found previously.[1]

La and Nd are the first metals of the rare earth group whose pressure coefficients of resistance have been measured. The coefficients of both these substances are not distinguished in any particular way over those of the elements of the previous paper, and do not require further discussion.

Ti and Zr also belong to a class of elements not previously measured. There was considerable impurity in these materials, and the results have no considerable accuracy. The results are chiefly remarkable for the smallness of the coefficients, which are smaller than for any other pure substances measured. It is even possible that Ti belongs to the abnormal metals, and that its resistance increases with increasing pressure, but the experimental accuracy was not high enough to allow this to be stated with certainty.

Arsenic is a substance which might be expected to show abnormal results because of its position in the periodic table, but it is actually found to be quite normal both in regard to the sign of the coefficient and its magnitude.

Gallium is another substance for which abnormal results were expected because of its anomalous property of expanding on freezing. The coefficient is however, normal in sign and magnitude. The accuracy of the measurements was not great enough to give the variation of the pressure or temperature coefficients over the range open to measurement.

Solid mercury has been here measured for the first time over a restricted range. It is quite normal with regard to sign and size of the coefficient.

Silicon and phosphorus are non-metallic in character, and will be discussed later. This leaves of the above list of normal metals only the alkali metals sodium and potassium as needing special comment because of the magnitude of their coefficients. Of the metals previously studied lead was found to have the greatest coefficient, the resistance under 12000 kg. being 14% less than under atmospheric pressure. Contrasted with this is a decrease of over 40% in the resistance of sodium and over 70% in that of potassium under a pressure of 12000 kg. The question is whether substances with such high coefficients show any change in the usual types of behavior formerly found. The principle facts found before for normal metals were: (1) The pressure coefficient is little affected by temperature, (2) The temperature coefficient is little affected by pressure within the range, and (3) the instantaneous pressure coefficient decreases with increasing pressure (the maximum change in the instantaneous coefficient was that of lead which changed about 30% under 12000 kg.).

Sodium and potassium show no such constancy of behavior, as might be expected from the high values of their compressibilities and pressure coefficients of resistance. Thus for sodium the values of the instantaneous pressure coefficients $\left[\frac{1}{w}\left(\frac{\partial w}{\partial p}\right)_T\right]$ at 0° at 0, 6000, and 12000 kg. respectively are -0.0_4663, 0.0_4435, and 0.0_4268, a total decrease by a factor of 2.49. At 80° the corresponding coefficients are -0.0_4786, 0.0_4466, and 0.0_4307, a total decrease by a factor of 2.56. This is a little larger than the factor of decrease at 0°, which is what one would expect. A comparison of corresponding pressure coefficients at 0° and 80° shows the pressure coefficient of sodium is by no means independent of temperature, but the change in the pressure coefficient with temperature is less than the change in resistance itself. The mean temperature coefficient of resistance may also be found from the table of resistance of sodium, and is 0.00475 at 0 kg., and 0.00408 at 12000 kg. The relative change is much larger than that found previously for any of the other metals, but still is not large compared with the variations of the pressure coefficient over the same range of pressure.

The same sort of phenomena are seen to characterize potassium, although the coefficients are not known over so wide a range as are those of sodium. At 25° the instantaneous pressure coefficients of resistance at 0, 6000, and 12000 kg. respectively are 0.0_3186, 0.0_4955, and 0.0_471, a decrease by a factor of 2.62. At 95° the pressure coefficients at 6000 and 12000 kg. are 0.0_31024 and 0.0_4885, and at 165° the coefficient at 12000 kg. is 0.0_31027. The pressure coefficient therefore increases markedly with increasing temperature. The mean temperature coefficients between 25° and 60° at 0, 6000, and 12000 kg. respectively are 0.00454, 0.00341, and 0.00184. The decrease is relatively much larger than for sodium, and is nearly as large as the relative decrease in the pressure coefficient over the same range.

The alkali metals sodium and potassium differ, therefore, in the following particulars from the metals previously measured. The instantaneous pressure coefficient increases with rising temperature and decreases with rising pressure by amounts which, for the range of this work, may amount to a factor between 2 and 3. The temperature coefficient of resistance decreases with increasing pressure by very perceptible amounts, and decreases much more for potassium than sodium.

The decrease in temperature coefficient at the higher pressures is especially significant. One might perhaps expect that at higher pressures the metal would be compressed into an approach toward its behavior at 0° Abs under atmospheric pressure, since the volume may be reduced by pressure to less than its value at 0° Abs. Now as the absolute zero is approached at atmospheric pressure the temperature coefficient of resistance becomes much greater than the reciprocal of the absolute temperature; this is the exact opposite of the behavior found above at high pressures, the temperature coefficient becoming less. The effect of increasing pressure is seen to be merely that of making the part played by temperature of less and less relative importance, which is after all not unnatural from a certain point of view. In the absence of specific information to the contrary it is natural to connect the unusual behavior of sodium and potassium with the large change of volume, and to expect that other metals will show the same sort of behavior under correspondingly increased pressures.

The non-metals Si and P would not be expected to agree in behavior with the metals, but it is interesting, nevertheless, to summarize their behavior. The magnitude of the mean coefficient of Si is about the same as that of lead. The coefficient may increase very largely with increasing temperature, however, and also may apparently increase with increasing pressure. This is quite contrary to expectations, and would seem to indicate an approach to some sort of instability at high pressures; perhaps as the atoms are pushed more closely together there is an approach to metallic conductivity. The variations of the temperature coefficient of silicon are also abnormal. Initially the coefficient is normal in sign, but small numerically; as pressure is increased it reverses in sign. This reversal in sign of the temperature coefficient, unlike the behavior of the pressure coefficient, does not indicate an approach to metallic conductivity. Too great weight should not be attached to these results, because the silicon was impure. However, it is evident that there are some interesting possibilities here, and the measurements should be repeated when it is possible to obtain purer material.

Black phosphorus is remarkable for the great magnitude of the coefficient, the resistance decreasing to only 3% of its initial value under 120000 kg. In spite of this abnormally large effect, the relative variation with pressure of the pressure coefficient is much less than that of sodium or potassium. The figures have already been given. There is a reversal of behavior with rising temperature. At 0° and 50° the instantaneous pressure coefficient increases with rising pressure, which is not what we would expect, but at 100° the coefficient falls with rising pressure. The temperature coefficient of black phosphorus is abnormal in sign, being negative. The coefficient decreases numerically with rising pressure, at first slowly, but more and more rapidly. From the table of resistance it may be found that the mean temperature coefficient between 0° and 100° are − 0.00579, 554, and 299 at 0, 6000, and 12000 kg. respectively. The readings at the higher pressures are not so accurate as the others, so that possibly the rate of fall of the coefficient at high pressures may be too rapid.

Abnormal solids. Previously there were measurements on only two abnormal solids, bismuth and antimony. The results for antimony were not sufficiently accurate to show the variation of pressure coefficient with pressure, but except for this the two metals agreed in that the pressure coefficient increases with increasing pressure and falls with increasing temperature, and the temperature coefficient falls with increasing pressure.

The instantaneous pressure coefficient of lithium increases with rising pressure, having the following values at 0, 6000, and 12000 kg. respectively; 0.0_568, 0.0_574, and 0.0_5796. The accuracy of the measurements was not sufficient to establish variations of pressure coefficient with temperature, or of temperature coefficient with pressure within the range. So far as the results are certain, however, the behavior of Li is like that of Bi and Sb.

For calcium the following values may be found from the table of resistance. The instantaneous pressure coefficients at 0° have at 0, 6000, and 12000 kg. the respective values 0.0_4106, 0.0_4121, and 0.0_4135. The corresponding values at 100° are 0.0_592, 0.0_4107, and 0.0_4119. The pressure coefficient therefore increases with increasing pressure, and decreases with rising temperature. The average temperature coefficients of resistance between 0° and 25° are 0.00299, 0.00291, and 0.00281 at 0, 6000, and 12000 kg. respectively, thus decreasing with rising pressure. Within the limits of error the temperature coefficients between 75° and 100° are the same as between 0° and 25°. It would be normal for the temperature coefficients to decrease with rising temperature. In all particulars of comparison, therefore, Ca is like Bi and Sb.

The pressure coefficient of strontium is abnormal. At 0° the instantaneous coefficient varies only little with increasing pressure, but what change there is is a decrease, which is abnormal. The range of values is from 0.0_4503 to 0.0_4492. At 50° the pressure coefficient at first increases with rising pressure, which is what we have come to regard as normal for this type of substances, but between 2000 and 3000 kg. passes through a flat maximum, and from there on decreases. The range is from 0.0_4469 to 0.0_4451. At 100° the behavior is like that at 50° except that the maximum with pressure is very much more pronounced, and the maximum occurs at 7000 kg. At 100° the initial value of the instantaneous coefficient is 0.0_4351, the maximum at 7000 is 0.0_4452, and at 12000 kg. it has dropped to 0.0_4432. There is, however, nothing abnormal in the temperature coefficient. The average temperature coefficients between 0° and 100° are 0.00383, 0.00311, and 0.00275 at 0, 6000, and 12000 kg. respectively. In respect therefore to the variation of pressure coefficient with temperature and temperature coefficient with pressure strontium is like the other metals with positive coefficient, but the variation of pressure coefficient with pressure is like that of the others over only a part of the range. It is to be remarked that the absolute value of the pressure coefficient of strontium is much higher than that of any other metal.

Summarizing, the behavior of the five abnormal metals, with the exception of the pressure variation of the pressure coefficient of strontium, is alike in that the instantaneous pressure coefficient increases with rising pressure and decreases with rising temperature, and the temperature coefficient falls with rising pressure.

Carbon, in the form of graphite, is the only other element at present known with a positive pressure coefficient of resistance. Since it is not metallic, comparisons are unprofitable. Furthermore, it was not possible to obtain results that were numerically reproducible. It may be worth mentioning, however, that graphite is like the metals above in that the pressure coefficient decreases with increasing temperature, but that it is different in that the pressure coefficient is very much less at the higher pressures.

Normal Liquids. The only liquid metal previously measured was mercury. It was found for it that the instantaneous pressure coefficient decreases with rising pressure and increases with rising temperature, and that the temperature coefficient decreases with rising pressure and rising temperature. The behavior is in all respects that which appeals to us as normal. It is worth while to give the numerical values for liquid mercury, since the range of the previous measurements has now been considerably extended. The results are shown in Table XX.

TABLE XX.

LIQUID MERCURY.

Temperature	$\frac{1}{w}\left(\frac{\partial w}{\partial p}\right)_T$		
	0 kg.	6000 kg.	12000 kg.
0°	0.0_4322	0.0_4239	
50°	340	253	
100°	367	227	
	Average Temperature Coefficient		
0–25°	0.00096	0.00074	
75–100°	90	65	0.00054

The measurements on liquid sodium did not cover so wide a range as those on liquid mercury, but within the range they show the same characteristics. At 200° the instantaneous pressure coefficients at 0, 6000, and 12000 kg. respectively are 0.0_4922, 0.0_4594, and 0.0_4396. The relative decrease with rising pressure is considerably greater than is the case with mercury, and furthermore, the coefficient itself is considerably greater. The mean temperature coefficient between 180° and 200° is 0.00325 at 0 kg., and 0.00244 at 12000 kg. This decrease is relatively not so large as that of mercury, although the coefficient itself is larger.

Potassium was liquid over a still smaller range than sodium, so that it is not possible to give as complete results. The instantaneous pressure coefficient decreases with rising pressure, the values at 165° being 0.000168 and 0.000136 at 0 and 6000 kg. respectively. The average temperature coefficient between 135° and 165° increases from 0.00322 at 0 kg. to 0.00463 at 5000 kg., which is the reverse of the behavior of liquid sodium and mercury. The variation with temperature of the pressure coefficient is also abnormal. The initial pressure coefficient of the liquid at 62.5° is $0.0_3 26$, and at 165° this has dropped to $0.0_3 17$.

Liquid gallium shows a rather large decrease of the instantaneous pressure coefficient with rising pressure. At 30° the values of the coefficient at 0, 6000, and 12000 kg. respectively are $0.0_5 640$, $0.0_5 535$, and $0.0_5 490$. At 100° the corresponding values are 634, 541, and 491. The effect of temperature on the pressure coefficient is therefore relatively slight; at the lower pressures the coefficient decreases with rising temperature, and at the higher pressures it decreases. The mean temperature coefficient of resistance between 30° and 100° changes relatively little, being 0.000815, 829, and 808 at 0, 6000, and 12000 kg. respectively. Compared with sodium and potassium the changes of all the coefficients of gallium are relatively small. It is to be remarked also that the pressure coefficient of gallium is of the same order of magnitude as that of many solid metals; we have come to expect relatively slight variations of the coefficients of those substances with small coefficients.

Liquid bismuth was measured over only part of its region of stability, so that again complete results are not at hand. At 275° the instantaneous pressure coefficient drops from $0.0_4 123$ at 0 kg. to $0.0_5 94$ at 6000 kg., and at 240° the coefficient is $0.0_5 92$ at 6000 kg., and $0.0_5 80$ at 12000. The temperature coefficient of resistance at 275° drops from 0.00047 at 0 kg. to 0.000453 at 6000 kg. Liquid bismuth is therefore entirely normal in all respects, that is, a falling pressure coefficient with rising pressure and falling temperature, and a falling temperature coefficient with rising pressure. This complete normality is in spite of the fact that solid bismuth is abnormal in having a positive pressure coefficient. The presumption is therefore very strong that the abnormality of the solid is mainly due to the crystalline structure. It is known of course that bismuth crystallizes in the hexagonal system which is not normal, nearly all the elements being cubic.

Summarizing, except for potassium, the behavior of all these liquid metals is of the same type; the pressure coefficient decreases with rising pressure and increases with rising temperature, and the temperature coefficient decreases with rising pressure.

Abnormal Liquids. Only one abnormal liquid, that is, a liquid with a positive pressure coefficient of resistance, is known, liquid lithium. For this the relation between pressure and resistance was linear within the limits of error and the coefficient was independent of temperature between 200° and 240°. A linear relation between resistance and pressure means a pressure coefficient becoming less at the higher pressures. This is what one might at first expect, but this is the first time that we have found it in a substance with positive coefficient. Since the pressure coefficient is independent of temperature, the temperature coefficient is independent of pressure over the range of the measurements.

Relative Behavior of the Same Metal in the Solid and the Liquid States. This is a matter of considerable importance as suggesting the relative parts played in the mechanism of conduction by the crystalline structure and the properties of the atoms as such. It will in the first place pay to recall the fact already well known that the direction in which the resistance changes when a metal melts is also the direction in which the volume changes. If the metal expands on melting, as is normal, the specific resistance increases on melting, and if the metal expands on freezing, the resistance of the liquid is less than that of the solid. This rule is without exception. Gallium and bismuth are the only two metals known at present in the second class; the data for antimony do not as yet seem well established. In the present work I was able to add lithium to the list of substances which obey this rule. This is of interest, because solid lithium is abnormal.

With regard to the magnitude of the change of resistance on melting there have been a number of theoretical proposals. The inaccuracy of the experimental results has allowed considerable latitude here. Thus theoretical considerations have been based on the assumption that the ratio of the resistance of the solid to that of the liquid is approximately an integer.[23] There is perhaps a tendency for the values to cluster about the figure 2, but it is now certain that within the limits of error the ratio is not integral. Attempts have been made to connect quantitatively the volumes of solid and liquid with the resistance, as would be suggested by the above general rule. Thus Professor Hall [24] has suggested that if the resistance of the solid is extrapolated to such a temperature that the volume expansion is sufficient to bring the volume of the solid up to the volume of the liquid at the melting temperature, the resistance of the solid will be found to be the same as that of the liquid. Of course any such long-range extrapolation must always be open to question, and it is probable that the numerical agreement found is no more significant than the general rule relating to volume already mentioned.

The above measurements under pressure bring out a fact that could not have been known before, namely that the ratio of the resistance of solid to liquid is approximately a constant characteristic of the particular substance, which does not change greatly as pressure and temperature are changed along the melting curve. We now have the figures for the ratio of the resistance of solid to liquid for six metals at different pressures and temperatures. For lithium the accuracy was not high enough to permit more than the statement that the ratio does not change greatly in a pressure range of 8000 kg. For sodium the ratio is 1.45 at atmospheric pressure, and has dropped to 1.36 on the melting curve at 12000 kg. The difference of volume between solid and liquid has dropped to half its initial value in the same pressure range, so that the ratio of resistances is evidently more constant than the difference of volume. For potassium the ratio of resistance of liquid to solid is 1.56 at 0 kg., and has dropped only to 1.55 at 9700 kg. Contrasted with this almost negligible change in the ratio of the resistances is a decrease under 9700 kg. of the difference of volume between solid and liquid to 0.31 of its initial value. For mercury, I determined the ratio of resistance of liquid to solid at the melting point at 0° and 7640 kg. to be 3.345. I did not make measurements at any other temperature but there are values by other observers. Onnes [25] finds 4.22, Bouty and Cailletet [26] 4.08, and Weber [27] gives 3.8 as the mean of six determinations, all for the ratio at the freezing point at atmospheric pressure. The error is so large that it is not possible to say more than that the change in the ratio along the melting curve is not large, and is in the direction of a decrease with increasing pressure. The change is probably greater than the change in the difference of volume between solid and liquid, which is abnormally constant for mercury, there being a decrease in the difference of only 1% over the pressure range of 7640 kg. The change in the resistance of the liquid over this range is, however, 19%, which is probably larger than the change in the ratio of the resistance of liquid to solid.

The behavior of the two abnormal metals gallium and bismuth is similar. At 7000 kg. I found the ratio of the resistance of liquid to solid bismuth to be 0.45, and at atmospheric pressure Northrup and Sherwood [16] found 0.43. The ratio is probably constant within the limits of error. For Gallium I found 0.58 for the ratio at atmospheric pressure, and calculated the value at 12000 kg. to be 0.61. This again is perhaps to be regarded as constant within the limits of error, but it is noteworthy that the little variation there is is in the same direction for both gallium and bismuth, and is toward an increase with rising pressure, whereas what variation there was for normal metals was always in the direction of a decrease with rising pressure.

We now compare the relative magnitudes of the pressure and temperature coefficients of solid and liquid. With regard to the temperature coefficients at atmospheric pressure it has long been known that the coefficient of the liquid is less than that of the solid. This is verified for all the metals measured here, except potassium.

With regard to the pressure coefficient of resistance it is natural to expect that of the liquid to be greater than that of the solid at the same temperature. This is true for sodium. At 120° the pressure coefficient of the liquid is about 7% greater than that of the solid extrapolated to the same temperature. It is however, perhaps surprising that the relative change of the pressure coefficient of solid sodium brought about by an increase of pressure of 12000 kg. is greater than that of the liquid under the same increase of pressure. The relative decrease of the temperature coefficient under 12000 kg. is greater for liquid sodium, however, than for the solid.

The behavior of liquid potassium is not as we would expect. At the melting point at atmospheric pressure the pressure coefficient of liquid potassium is greater than that of the solid. Because of the abnormal temperature coefficient of the pressure coefficient of the liquid, however, the coefficient of the solid would become greater than that of the liquid if the solid could be superheated sufficiently. The relative variation with pressure of the pressure coefficient is greater for the solid than the liquid. This again is not what we might expect. The data for potassium do not cover a sufficient range to permit a comparison of the variation with pressure of the temperature coefficients of solid and liquid.

The pressure coefficient of solid mercury has been found to be constant over the range from 7640 to 12000 kg. The coefficient of the liquid, on the other hand, decreases with rising pressure. It has already been mentioned as surprising that the coefficient of the solid is greater than that of the liquid at 6500 kg. This difference would become still more accentuated if the liquid could be carried in the metastable state into the region of stability of the solid; in this range its pressure coefficient would be found to vary considerably less than that of the solid. The measurements were not accurate enough to permit a comparison of the variations of the temperature coefficients of the solid and liquid. It is known, however, that at atmospheric pressure the temperature coefficient of the solid is normal, while that of the liquid is abnormally low even for a liquid.

The pressure coefficient of solid gallium is of the order of 2.5 less than that of the liquid. The coefficient of the solid is independent of the pressure, whereas that of the liquid decreases markedly with increasing pressure. Measurements were not made on the variation with pressure of the temperature coefficient of the solid.

Comparison cannot properly be made between liquid and solid bismuth, because the solid is abnormal and the liquid is normal. It is interesting, however, that numerically the coefficient of the solid is greater than that of the liquid. This may mean that some of the tendency to abnormality still persists in the liquid, making its coefficient lower than it would otherwise be.

Lithium is abnormal in both liquid and solid. If the data for the solid are extrapolated from 100° to the melting temperature at 180° the figures given would indicate a pressure coefficient of the solid numerically less than that of the liquid. The difference would be still further accentuated if the unknown correction for the compressibility of the solid is applied so as to make the coefficients of both solid and liquid the coefficients of specific resistance. Although the coefficient of the liquid is greater than that of the solid, its variation with pressure is much less, and in fact is opposite in sign, the coefficient of the liquid becoming smaller at higher pressures, and the coefficient of the solid becoming greater. The temperature coefficient of liquid lithium is independent of pressure to 12000 kg., as is that of the solid also.

Summarizing the relations between the coefficients of the liquid and the solid, except for the temperature coefficient of the liquid being less than that of the solid, there does not seem to be a tendency to any one type of behavior. It is noteworthy, however, that in many cases the resistance of the liquid responds more sluggishly to changes of pressure than does that of the solid, the coefficient of the liquid being actually less than that of the solid, or else the change of coefficient with pressure being less for the liquid.

Alloys. The above data on alloys are entirely unsystematic and fragmentary, so that it is not possible to draw any conclusions as to the behavior of alloys in general. It is interesting to notice, however, that the pressure coefficient of all the alloys, with the exception of that of "Comet," is less numerically than would be computed by the law of mixtures from the coefficients of its components, and in the case of "Therlo" this tendency to a lower value may go so far as to reverse the sign. In making this statement I have assumed that the pressure coefficient of pure Chromium and Manganese is negative, a conclusion which has not been checked by experiment, but which seems very probable from the behavior of similar metals.

Theoretical Bearings.

Since the purpose of this paper is primarily the presentation of new data, I cannot more than touch on two matters of theoretical interest suggested by considerations of the previous papers.

It has been known for some time that the temperature coefficient at constant volume of liquid mercury is negative instead of positive, as is the coefficient at constant pressure. In my previous theoretical paper [22] I suggested reasons for this. It is now of interest to find whether the other liquid metals have the same property.

The coefficient of resistance at constant volume is given by the relation

$$\left(\frac{\partial w}{\partial \tau}\right)_v = \left(\frac{\partial w}{\partial \tau}\right)_p - \left(\frac{\partial w}{\partial p}\right)_\tau \frac{\left(\frac{\partial v}{\partial \tau}\right)_p}{\left(\frac{\partial v}{\partial p}\right)_\tau}$$

Hence in addition to the pressure and temperature coefficients of resistance, which have been determined in the present work, values of the thermal expansion and compressibility are also needed. These have not been determined experimentally for any of the metals above, but in some cases an indirect estimate may be made with the help of various data from the melting curve. I have previously given an estimate of the difference of compressibility and thermal expansion between solid and liquid sodium, potassium, and bismuth.[28] With these data, the temperature coefficients at constant volume may be computed, as is shown in Table XXI. The fundamental data are

TABLE XXI.

The Temperature Coefficient at Constant Volume of Liquid Metals at their Melting Points.

Substance	$\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_\tau$	$\frac{1}{v}\left(\frac{\partial v}{\partial \tau}\right)_p$	$\frac{1}{w}\left(\frac{\partial w}{\partial p}\right)_\tau$	$\frac{1}{w}\left(\frac{\partial w}{\partial \tau}\right)_p$	$\frac{1}{w}\left(\frac{\partial w}{\partial \tau}\right)_v$
Sodium	-0.0_4186	$+0.0_3344$	-0.0_488	$+0.00325$	$+0.00170$
Potassium	-0.0_4358	$.0_3349$	-0.0_3204	0.0044	$+0.0025$
Bismuth	-0.0_532 (?)	$.0_312$ (?)	-0.0_412	0.000475	$+0.0_415$

exceedingly uncertain, because in addition to the uncertainties in the differences of compressibility and thermal expansion between solid and liquid, the compressibilities and thermal expansions of the solids themselves at the melting points are in doubt, the actual measurements having been made in most cases only at room temperature. I have had to guess what the temperature variation of the compressibility might be. However, the uncertainty cannot be so large as to change the sign of the effect for sodium and potassium, for which there can be no doubt that the temperature coefficient at constant volume, as well as the coefficient at constant pressure, is positive. This is the reverse of the behavior of mercury. The data for bismuth are in much more doubt, however. Assuming the figures shown, the coefficient at constant volume is also positive, but the uncertainty is so great that the sign might well be negative.

The coefficient at constant volume of liquid lithium is of course positive, since the pressure coefficient at constant temperature is abnormal in being positive. The data are not at present known for gallium, so that it is not possible to make any sort of an estimate as to the probable value of its coefficient at constant volume.

The outcome of this investigation, therefore, for the only two metals for which the results can be sure, is to reverse the behavior previously found for liquid mercury. In this connection it is to be remarked that the temperature coefficient at constant pressure of liquid mercury is abnormal in being very low, and the corresponding coefficients of liquid sodium and potassium are abnormal in being very high. It does not yet appear, therefore, what the probable value of the constant volume coefficient would be for the more usual metals, such as lead.

The second point of theoretical interest brought out in the previous discussion was the intimate connection between the changes of resistance and the amplitude of atomic vibration.[22] It appeared that the relative change of resistance, whether brought about by a change of pressure or of temperature, was approximately equal to twice the relative change of amplitude under the same change. The relation was by no means exact, there being failures by as much as a factor of two in some cases, but on the average the agreement was rather good for a large number of metals. The question is whether these new elements also show the same relation?

In making the computation the following formula for the change of amplitude with pressure was used

$$\frac{1}{a}\left(\frac{\partial a}{\partial p}\right)_\tau = -\frac{1}{C_p}\left(\frac{\partial v}{\partial \tau}\right)_p ,$$

where a is atomic amplitude, and C_v specific heat at constant volume per unit volume. It is therefore necessary to know the thermal expansion and specific heats of the new elements. Unfortunately the data are not known for a number of the metals of this work. The computation has been made for all those normal substances for which the data are available, and the results are collected in Table XXII.

TABLE XXII.

Comparison of the Changes under Pressure of Resistance and Amplitude of Atomic Vibration.

Substance	$\frac{1}{v}\left(\frac{\partial v}{\partial \tau}\right)_p$	C_v Kg. cm. cm.	$\frac{2}{\alpha}\left(\frac{\partial \alpha}{\partial p}\right)_\tau$	$\frac{1}{w}\left(\frac{\partial w}{\partial p}\right)_\tau$
Solid Na, 20°	0.0_3217	11.1	-0.0_439	-0.0_468
Liquid Na, 98°	$.0_3344$	10.6	-0.0_465	-0.0_491
Solid K, 20°	$.0_3248$	6.48	-0.0_476	-0.0_3186
Liquid K, 63°	$.0_3341$	6.64	-0.0_3102	-0.0_326
Liquid Hg, 0°	$.0_3181$	17.1	-0.0_4211	-0.0_4358
Arsenic	$.0_416$	26.7	-0.0_512	-0.0_533
Magnesium	$.0_478$	17.7	-0.0_588	-0.0_548
Tungsten	$.0_4101$	27.6	-0.0_674	-0.0_5143

In this table are included the recomputed values for Mg and W. The best agreement is for liquid bismuth. In nearly all the other cases the computed value is much lower than the observed. This is a reversal of the behavior shown by the previous substances, for which in the majority of cases the computed value was too high.

The table seems to show no essential difference between a solid and a liquid metal as far as the connection with amplitude goes.

There seems to be no reason to modify the previous conclusion, which was that in a large way the changes of amplitude of atomic vibration are an exceedingly important factor in affecting changes of resistance. Superposed on this large effect common to all metals, are specific effects, such as peculiarities of atomic structure or arrangement. In particular the factor of atomic arrangement is responsible for the difference between a solid and a liquid metal, and may be so important in some cases as to control the sign of the effect.

Summary.

In this paper results are given for the effect of pressure and temperature on the resistance of twenty elements and several alloys. Endeavor was made to choose elements from unusual places in the periodic table, and also to investigate more fully the behavior of liquid metals.

The resistance of the same metal in the liquid and the solid state has now been measured for six elements. The temperature coefficient of the liquid is less than that of the solid except for potassium. The change of resistance on melting invariably follows the direction of the change of volume. The ratio of resistance of liquid to solid is approximately constant along the melting curve, although the difference of volume may change greatly. The pressure coefficient of the liquid is in some cases less than that of the solid. Liquid bismuth has a negative pressure coefficient of resistance, and is normal, but liquid lithium has a positive coefficient, and is the only such liquid metal yet found. The new liquids do not show a negative temperature coefficient of resistance at contant volume, as did liquid mercury.

The alkali metals sodium and potassium are remarkable for the large changes of resistance under pressure. The pressure coefficient decreases greatly with increasing pressure, and decreasing temperature. The temperature coefficient may decrease greatly with increasing pressure. The variations of these coefficients for the metals investigated in the previous paper were always small.

Three more solid elements have been found with positive pressure coefficients of resistance; lithium, calcium, and strontium. Of these the pressure coefficient decreases with increasing temperature, the temperature coefficient decreases with increasing pressure, and, except for strontium, the pressure coefficient increases with increasing pressure.

Of the non-metallic elements, black phosphorus is remarkable for a very large negative coefficient, the resistance under 12000 kg. dropping to only 3% of its initial value; silicon has a negative coefficient which becomes numerically larger with increasing pressure, and carbon has a negative coefficient in the amorphous state, and a positive coefficient in the graphitic state, which decreases greatly with increasing pressure.

The additional evidence from these new materials still gives every reason to think that the amplitude of atomic vibration is the largest single factor in determining the changes of resistance under pressure or temperature.

The Jefferson Physical Laboratory,
Harvard University, Cambridge, Mass.

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I regret that through an inadvertence on my part I stated in the first of these papers that the previous work of Lisell and Beckman contained a source of error due to the permanent closing of the battery current. Dr. Beckman has been so kind as to call to my attention that this error was present only in the preliminary work of Lisell, and was not present at all in the final results of Lisell, or in any of his own results.

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By Joseph Lipka.

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MOTION ON A SURFACE FOR ANY POSITIONAL FIELD OF FORCE.

By Joseph Lipka.

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§ 1. Introduction.

The first part of the paper presents a study of the geometric properties of the system of trajectories generated by the motion of a particle on any constraining surface (spread of two dimensions) under any positional forces. The complete characteristic properties are derived.[1] Starting at any point on the surface in a given direction and with a given speed, a unique trajectory is generated, and the complete set of trajectories forms a triply infinite system of curves corresponding uniquely to a given field of force.

Through a given point O and in a given direction, there pass ∞^1 trajectories. We associate with these trajectories the ∞^1 curves obtained by orthogonal projection into the tangent plane to the surface at O. The first two properties derived deal with the bicircular quartic which is the locus of the foci of the osculating parabolas of the associated system.

A second set of geometric properties is derived by considering the ∞^2 trajectories through a point O on the surface, and the directions through O in which the trajectories hyperosculate (have 4-point contact with) their corresponding geodesic circles of curvature. It is found that the hyperosculation property holds for only one trajectory in each direction, and that the corresponding locus of the centers of geodesic curvature is a conic.

A final set of properties is then derived showing the relations existing at a point between the geodesic curvatures of the trajectories and the lines of force, and any isothermal net of curves on the surface. It is shown that the entire five properties are characteristic of the system

1 For a study of the corresponding problem in a plane and in ordinary 3-space, see Edward Kasner, "*The trajectories of dynamics*," Trans. Am. Math. Soc., **7** (1906), pp. 401–424; also "*Dynamical Trajectories: the motion of a particle in an arbitrary field of force*," Trans. Am. Math. Soc., **8** (1907), pp. 135–158. The results of these 2 papers are summarized in Professor Kasner's Princeton Colloquium Lectures on the differential geometric aspects of dynamics, chapt. I.

of trajectories, and that any triply infinite system of curves on a surface possessing these five properties may be considered as generated by the motion of a particle in a unique field of force.

We next point out how an additional property serves to characterize the motion when the field of force is conservative.

Another part of this paper presents briefly an analogous study for certain other classes of triply infinite systems of curves on a surface, in particular, brachistochrones, catenaries and velocity curves in a conservative field of force. For all such systems characteristic properties differing but slightly from those for trajectories are derived.

§ 2. Differential Equation of the Trajectories.

If we choose an isothermal net of curves as parameter curves on the surface

$$x = x(u, v), \quad y = y(u, v), \quad z = z(u, v), \tag{1}$$

the element of arc length may be written

$$ds^2 = \mu(u, v)\,[du^2 + dv^2]. \tag{2}$$

The motion of a particle on the surface may be most simply expressed by the Lagrangian equations [2]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot u}\right) - \frac{\partial T}{\partial u} = \phi, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot v}\right) - \frac{\partial T}{\partial v} = \psi \tag{3}$$

where T is the kinetic energy

$$2T = \mu(\dot u^2 + \dot v^2), \tag{4}$$

and φ and ψ are the components of the force given as functions of the coördinates u, v.[3] Introducing the value of T in (3), we get the explicit equations of motion

$$\left\{\begin{aligned} \ddot u &= \frac{1}{\mu}\left[\phi - \frac{1}{2}\left(\mu_u \dot u^2 + 2\mu_v \dot u\,\dot v - \mu_u \dot v^2\right)\right] \\ \ddot v &= \frac{1}{\mu}\left[\psi + \frac{1}{2}\left(\mu_v \dot u^2 - 2\mu_u \dot u\,\dot v - \mu_v \dot v^2\right)\right], \end{aligned}\right. \tag{5}$$

2 See E. T. Whittaker, *Analytical Dynamics*, p. 39.

3 Throughout the paper, dots refer to derivatives with respect to t (time), primes refer to total derivatives with respect to u, and literal subscripts to partial derivatives.

or if, for convenience, we write

(6) $$\lambda = \frac{1}{2}\log\mu, \text{ and } \mu = e^{2\lambda},$$

equations (5) become

(7) $$\begin{cases} \ddot{u} = \dfrac{\phi}{e^{2\lambda}} - (\lambda_u \dot{u}^2 + 2\lambda_v \dot{u}\,\dot{v} - \lambda_u \dot{v}^2) \\ \ddot{v} = \dfrac{\psi}{e^{2\lambda}} + (\lambda_v \dot{u}^2 - 2\lambda_u \dot{u}\,\dot{v} - \lambda_v \dot{v}^2). \end{cases}$$

To get the differential equations of the trajectories we must eliminate the time from equations (7). We evidently have

$$v' = \frac{\dot{v}}{\dot{u}}, \quad v'' = \frac{\dot{u}\,\ddot{v} - \dot{v}\,\ddot{u}}{\dot{u}^3},$$

and hence

(8) $$v'' = \frac{1}{e^{2\lambda}\dot{u}^2}(\psi - \phi\, v') + (\lambda_v - \lambda_u\, v')\,(1 + v'^2).$$

Using the abbreviations

(9) $$\begin{cases} G \equiv v'' - (\lambda_v - \lambda_u\, v')\,(1 + v'^2), \\ G' \equiv \dfrac{dG}{du} \equiv v''' + v''\,(\lambda_u - 2\lambda_v\, v' + 3\lambda_u\, v'^2) \\ \qquad\qquad - (\lambda_{uv} + \lambda_{vv}\, v' - \lambda_{uu}\, v' - \lambda_{uv}\, v'^2)\,(1 + v'^2), \end{cases}$$

where $G = 0$ is the differential equation of the geodesics on the surface, we may write (8) in the form

(10) $$\dot{u}^2 = \frac{\psi - \phi v'}{e^{2\lambda}\, G}.$$

Differentiating (10) with respect to u and using (7), we get

(11) $$(\psi - \phi v')\, G' = G\{(\psi_u + 2\lambda_v\phi) + (\psi_v - \phi_u + 2\lambda_v\psi - 2\lambda_u\phi)v' - (\phi_v + 2\lambda_u\psi)v'^2 - 3\phi v''\},$$

as the differential equation of the trajectories. If we replace G and G' by their values from (9), we get a differential equation of the form

(12) $$v''' = P + Qv'' + Rv''^2$$

where

(13) $$\begin{cases} P = (\alpha_0 + \alpha_1 v' + \alpha_2 v'^2 + \alpha_3 v'^3 + \alpha_4 v'^4 + \alpha_5 v'^5)/(\psi - \phi v'), \\ Q = (\beta_0 + \beta_1 v' + \beta_2 v'^2 + \beta_3 v'^3)/(\psi - \phi v'), \\ R = -3\phi/(\psi - \phi v'), \end{cases}$$

the α's and β's being functions of ϕ, ψ, and λ, and hence of u and v. In deriving the geometric properties of the trajectories we shall use the equation in the form (11) almost exclusively.

The triply infinite system of curves represented by (11) is thus uniquely determined by the force components ϕ, ψ. Two different fields of force ϕ, ψ and $\bar{\phi}$, $\bar{\psi}$ cannot give rise to the same system of trajectories. For if the two systems

$$(14)\quad G' = G(a + bv' + cv'^2 + dv'') \text{ and } G' = G(\bar{a} + \bar{b}v' + \bar{c}v'^2 + \bar{d}v'')$$

coincide, we must have

$$(15)\qquad a = \bar{a},\ b = \bar{b},\ c = \bar{c},\ d = \bar{d}.$$

The last of these equations, by comparison with (12) and (13), gives

$$(16)\qquad \bar{\psi}/\psi = \bar{\phi}/\phi,$$

so that we may write

$$(17)\qquad \bar{\psi} = \alpha\psi,\ \bar{\phi} = \alpha\phi,$$

where α is some function of u, v; substituting these values in the first three equations, we find

$$(18)\qquad \alpha_u = 0,\ \alpha_v = 0,$$

and hence α is simply a constant, k. Therefore the forces are the same except for a constant factor, which corresponds merely to a change in the unit of force. Hence we may state

THEOREM 1. *The system of trajectories corresponding to a field of force completely defines that field.*

We further note that $G = 0$ satisfies equation (11), so that the geodesics form part of every system of trajectories, i.e. for every field of force. Indeed, from (8), we note that the speed of the particle describing a geodesic is infinite.

§ 3. The Associate System and the Focal Locus.

Consider the ∞^1 trajectories passing through a point O in a direction v', i.e. having a common initial element (u, v, v'). Project these curves orthogonally into the tangent plane at O.[4] We shall call the

---

[4] For our purpose it is merely necessary to project the elements up to the third order, v', v'', v'''.

resulting curves the associate system corresponding to the given initial element. Our first query is: what is the locus of the foci of the osculating parabolas of the ∞^1 curves of the associate system? To assist us in answering this, we shall choose the tangent plane as the XOY plane, the point O as the origin, and the OZ axis as the normal to the surface. We shall further choose the x- and y- axes as the tangent lines to our isothermal v- and u- parameter curves, respectively.

Here $z = 0$. The trajectory determined by (o, o, v', v'', v''') is associated with the curve determined by $\left(o, o, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}\right)$. The latter derivatives may be expressed in terms of the former by using the equations of the surface (1). Thus we have

$$(19)\quad \begin{aligned} &x' = x_u + x_v v',\ x'' = x_{uu} + 2x_{uv}v' + x_{vv}v'^2 + x_v v'',\\ &x''' = x_{uuu} + 3x_{uuv}v' + 3x_{uvv}v'^2 + x_{vvv}v'^3 + 3x_{uv}v'' + 3x_{vv}v'v'' + x_v v''', \end{aligned}$$

with similar expressions for y', y'', y'''. Then,

$$(20)\qquad \frac{dy}{dx} = \frac{y_u + y_v v'}{x_u + x_v v'}.$$

By our choice of isothermal parameters, the tangent of the angle between the initial element and the v- parameter curve is v', so that

$$(21)\qquad x_v = 0,\ y_u = 0,\ x_u = y_v.$$

Differentiating (20) and using (21), we find

$$(22)\qquad \begin{cases} \dfrac{dy}{dx} = v', \\[2mm] \dfrac{d^2y}{dx^2} = \dfrac{1}{x_u}v'' + a, \\[2mm] \dfrac{d^3y}{dx^3} = \dfrac{1}{x_u^2}v''' + b\,v'' + c, \end{cases}$$

where a, b, and c are functions of u, v, v' only.

Now the coördinates (α, β) of the focus of the osculating parabola to the associate element $\left(o, o, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}\right)$ are given by

$$
(23)\quad \begin{cases} 2\alpha = \dfrac{-3\dfrac{d^2y}{dx^2}\left\{\dfrac{d^3y}{dx^3}\left[\left(\dfrac{dy}{dx}\right)^2-1\right]+2\dfrac{dy}{dx}\left[3\left(\dfrac{d^2y}{dx^2}\right)^2-\dfrac{dy}{dx}\dfrac{d^3y}{dx^3}\right]\right\}}{\left(\dfrac{d^3y}{dx^3}\right)^2+\left[3\left(\dfrac{d^2y}{dx^2}\right)^2-\dfrac{dy}{dx}\dfrac{d^3y}{dx^3}\right]^2}, \\[2ex] 2\beta = \dfrac{-3\dfrac{d^2y}{dx^2}\left\{\left[3\left(\dfrac{d^2y}{dx^2}\right)^2-\dfrac{dy}{dx}\dfrac{d^3y}{dx^3}\right]\left[\left(\dfrac{dy}{dx}\right)^2-1\right]-2\dfrac{dy}{dx}\dfrac{d^3y}{dx^3}\right\}}{\left(\dfrac{d^3y}{dx^3}\right)^2+\left[3\left(\dfrac{d^2y}{dx^2}\right)^2-\dfrac{dy}{dx}\dfrac{d^3y}{dx^3}\right]^2}. \end{cases}
$$

With the aid of (22) we can express these coördinates in terms of v', v'', v'''. We are considering the ∞^1 trajectories with the fixed initial element (o, o, v') but for which v'' and v''' are variables. Equations (23) and the equation of the trajectories (11) furnish us three equations from which we may eliminate v'' and v''', and thus get the equation of the focal locus. After considerable reductions we find for the equation of this locus

$$
(24)\quad A_0(\alpha^2+\beta^2)^2 + A_1(\alpha v'-\beta)(\alpha^2+\beta^2) + (\alpha v'-\beta)(A_2\alpha + A_3\beta) = 0,
$$

where A_0, A_1, A_2, A_3 are functions of u, v, v', the coördinates of the fixed element, only, and may thus be considered as constants. We find

$$
(25)\quad \begin{cases} A_2 = -3\,x_u^2\,(1+v'^2)\,(\psi v'^2 + 2\phi v' - \psi), \\ A_3 = -3\,x_u^2\,(1+v'^2)\,(\phi v'^2 - 2\psi v' - \phi). \end{cases}
$$

Now (24) is the equation of a bicircular quartic (a quartic possessing a pair of nodes at the circular points at infinity). The form of the equation indicates that the bicircular quartic (24) has the following properties:

(*i*) Its third double point is at the origin or initial point O and is a node, having two real and distinct tangents at O, viz.,

$$
(26)\quad \alpha v' - \beta = 0 \quad \text{and} \quad A_2\alpha + A_3\beta = 0,
$$

the first of which has the direction of the initial element v'.

(*ii*) The inverse of the quartic with respect to O is

$$
(27)\quad (\alpha v'-\beta)(A_2\alpha + A_3\beta + A_1) + A_0 = 0
$$

a hyperbola with $\alpha v' - \beta = 0$, the initial element, as asymptote.[5]

---

5 For a study of the bicircular quartic, see Basset, *Elementary treatise on cubic and quartic curves*, chapt. IX; also Loria, *Specielle algebraische und transscendente ebene Kurven*, pp. 102–108. It may easily be shown that property (*ii*) may be replaced by the following: the fundamental point of the bicircular quartic lies on a line through O perpendicular to the initial element.

We easily see that the equation of a bicircular quartic having properties (*i*) and (*ii*) is of the form (24). Hence we may state

THEOREM 2. *The ∞^1 trajectories passing through a given point in a given direction have associated with them their orthogonal projections in the tangent plane to the surface at the given point. The locus of the foci of the osculating parabolas of the associate system is a bicircular quartic having the given point as node and the given direction both as tangent line and also as one of the asymptotes of the hyperbola which is the inverse of the quartic with respect to the given point.*

Returning to our bicircular quartic (24), we have already noted that the given point O is a node and that the two tangents at O are given by (26). The first of these has the direction of the initial element v', and the second has the direction ξ given by

$$\xi = -\frac{A_2}{A_3} = -\frac{\psi v'^2 + 2\phi v' - \psi}{\phi v'^2 - 2\psi v' - \phi}. \tag{28}$$

Hence

$$\frac{\psi - \phi v'}{\phi + \psi v'} = \frac{v' - \xi}{1 + \xi v'}, \tag{29}$$

or

$$\tan\theta_1 = \tan\theta_2 \tag{30}$$

where θ_1 is the angle between the initial direction v' and the direction of the force vector ψ/ϕ, and θ_2 is the angle between the direction of the second tangent ξ and the initial direction v'. Hence we have

THEOREM 3. *The focal locus described in Theorem 1 has two distinct tangents at the given point. The initial element, which has the direction of one of these tangents, bisects the angle between the force vector and the second tangent.*

§ 4. Curves with Properties I and II.

Theorems 2 and 3 express geometric properties of the system of ∞^3 trajectories on the surface. The question arises whether these properties are characteristic of the system, i.e. whether the system of trajectories is the only system enjoying these properties. To answer this, let us now find all the systems of ∞^3 curves on a surface which possess

Property I. The ∞^1 curves passing through a given point in a given direction have associated with them their orthogonal projections in the tangent plane to the surface at the given point. The locus of the foci of the osculating parabolas of the associate system is a bicircular quartic with the given point as node, and the given direction both as tangent line and also as one of the asymptotes to the hyperbola which is the inverse of the quartic with respect to the given point.

Any triply infinite system of curves on a surface may be represented by a differential equation of the form

$$v''' = f(u, v, v', v''). \tag{31}$$

Using the same notation and same coördinate system as in §3, the equation of a bicircular quartic described in Property I has the form

$$B_0(\alpha^2 + \beta^2)^2 + B_1(\alpha v' - \beta)(\alpha^2 + \beta^2) + (\alpha v' - \beta)(B_2\alpha + B_3\beta) = 0 \tag{32}$$

where the B's are arbitrary functions of u, v, v'. If in (32) we substitute for α, β their values as given by (23), we find with the aid of (22) and after considerable reduction, that the differential equation (31) has the form

$$v''' = A + Bv'' + Cv''^2, \tag{33}$$

where A, B, C, are arbitrary functions of u, v, v'. It is evident that equation (33) is much more general than equation (11). Thus we may state

THEOREM 4. *The most general triply infinite system of curves on a surface possessing Property I, is defined by a differential equation of the form (33) involving three arbitrary functions of u, v, v'.*

Let us now convert Theorem 3. We must here replace the direction of the force vector through each point by a fixed direction through each point, but which may vary from point to point. We are to find the most general system of ∞^3 curves possessing Property I and also

Property II. The focal locus or bicircular quartic associated with each element (u, v, v') by Property I, is such that the initial element, which has the direction of one of the tangents, bisects the angle between a fixed direction through the initial point and the other tangent.

Let the fixed direction be given by $\omega(u, v)$. The system of curves possessing Property I is defined by the differential equation (33). For these curves the focal locus corresponding to the element (u, v, v') has for its second tangent the line

$$\alpha[Cv'(1 + v'^2) + 3(1 - v'^2)] - \beta[C(1 + v'^2) - 6v'] = 0 \tag{34}$$

whose direction is given by

$$\kappa = \frac{Cv'(1 + v'^2) + 3(1 - v'^2)}{C(1 + v'^2) - 6v'}. \tag{35}$$

For the bisection property we must have

$$\frac{\omega - v'}{1 + \omega v'} = \frac{v' - \kappa}{1 + \kappa v'}, \tag{36}$$

from which we find

$$C = \frac{3}{v' - \omega} \tag{37}$$

so that C is no longer an arbitrary function of u, v, v'. We may now state

THEOREM 5. *The most general triply infinite system of curves on a surface possessing Properties I and II is defined by a differential equation of the form*

$$v''' = A + Bv'' + \frac{3}{v' - \omega} v''^2, \tag{38}$$

involving two arbitrary functions A and B of u, v, v', and one arbitrary function ω of u, v.

Since properties I and II do not characterize our system of trajectories we shall seek further properties.

§ 5. Hyperosculation and the Central Locus.

Consider a point O on the surface. The geodesic curvature and center of geodesic curvature of a curve through O are respectively the curvature and center of curvature of the orthogonal projection of the curve on the tangent plane to the surface at O. Using the same coördinate system as in §4, we find that the geodesic curvature is

$$\frac{1}{\rho} = \frac{G}{e^{\lambda}(1 + v'^2)^{\frac{3}{2}}} = \frac{v'' - (\lambda_v - \lambda_u v')(1 + v'^2)}{e^{\lambda}(1 + v'^2)^{\frac{3}{2}}} \tag{39}$$

and that the coördinates of the center of geodesic curvature are

$$\xi = -\frac{v'\rho}{\sqrt{1 + v'^2}}, \quad \eta = \frac{\rho}{\sqrt{1 + v'^2}}. \tag{40}$$

Now, for each trajectory c which passes through O we may draw the curve g which osculates c and which has constant geodesic curvature (that of c at O) throughout. We call g the *osculating geodesic circle* of c. The question arises: how many of the ∞^1 trajectories which pass through O in a given direction v', will hyperosculate (have 4-point contact with) the corresponding geodesic circles? To answer this we need simply apply the condition

$$\frac{d\rho}{ds} = 0 \tag{41}$$

to the form (39); we get

$$(1 + v'^2)G' - G\{(\lambda_u + \lambda_v v')(1 + v'^2) + 3v'v''\} = 0. \tag{42}$$

Substituting in this the value of G' from (11), and solving for v'', we find

$$v'' = \frac{\{(\psi_u - \lambda_u\psi + 2\lambda_v\phi) + (\psi_v - \phi_u - \lambda_u\phi + \lambda_v\psi)v' - (\phi_v - \lambda_v\phi + 2\lambda_u\psi)v'^2\}\{1 + v'^2\}}{3(\phi + \psi v')} \tag{43}$$

where we have discarded the factor G whose vanishing leads to the geodesics; as has already been noted, these curves form part of every system of trajectories; for the geodesics ρ, ξ, η are infinite. In (43) we note that for a given value of u, v, v', there corresponds only one value of v'', so that we have

THEOREM 6. *Through every point and in every direction through that point on a surface, there passes one trajectory which hyperosculates its corresponding geodesic circle of curvature.*

If, now, we keep the point O fixed and vary the initial direction v', the center of geodesic curvature of the hyperosculating trajectories of Theorem 6, will describe a certain locus. We get the equation of this locus by eliminating ρ, v', v'' from (39), (40), and (43). From (39) and (40) we find

$$v' = -\frac{\xi}{\eta}; \quad v'' = \frac{(\lambda_u\xi + \lambda_v\eta + e^\lambda)(\xi^2 + \eta^2)}{\eta^3}, \tag{44}$$

and substituting these in (43), we get

$$\xi^2(\phi_v - \lambda_v\phi - \lambda_u\psi) + \xi\eta(\psi_v - \phi_u + 2\lambda_u\phi - 2\lambda_v\psi) - \eta^2(\psi_u - \lambda_u\psi - \lambda_v\phi) + 3e^\lambda(\phi\eta - \psi\xi) = 0, \tag{45}$$

a conic passing through the point O. The equation of the tangent to this conic at O is

$$\phi\eta - \psi\xi = 0, \tag{46}$$

whose direction is that of the force vector ψ/ϕ. Hence, we have

THEOREM 7. *The locus of the centers of geodesic curvature of the* ∞^1

hyperosculating trajectories which pass through any point on the surface, is a conic passing through the point in the direction of the force vector.

We shall call this conic the *central locus.*

§ 6. Curves with Properties I, II, and III.

We shall now find all triply infinite systems of curves on a surface possessing Property I and the following

Property III. Through every point on the surface and in every direction through that point there passes one curve of the system which hyperosculates its corresponding geodesic circle of curvature. The locus of the centers of geodesic curvature of the ∞^1 hyperosculating trajectories which pass through a point is a conic passing through the point in a fixed direction.

To find all such systems we shall evidently have to convert Theorems 2, 6, and 7, replacing the direction of the force vector at any point by a fixed direction ω (u, v) through that point. By Theorem 4, a triply infinite system of curves possessing Property I may be represented by a differential equation of the form

$$v''' = A + Bv'' + Cv''^2. \tag{33}$$

The condition for hyperosculation of the curve and the element (u, v, v') is given by

$$(1 + v'^2)G' - G\{(\lambda_u + \lambda_v v')(1 + v'^2) + 3v'v''\} = 0, \tag{42}$$

where G and G' are given by (9) and v''' is to be replaced by its value from (33). Now $G = 0$ must satisfy (42), since the geodesic with element (u, v, v') certainly hyperosculates (indeed coincides with) its corresponding geodesic circle of curvature. And if, excluding the geodesic, there is to be only one curve of the system which has the hyperosculation property, equation (42) must give only one value of v''. Now G is linear in v'', and G' is quadratic in v'', so that G' must have the form

$$G' = G(a + bv'') \tag{47}$$

where a and b are functions of u, v, v' only. Equation (47) is therefore a restriction on the forms of the quantities A, B, C appearing in (33). Introducing (47) into (42), we get for the value of v'' corresponding to the direction of hyperosculation,

$$v'' = \frac{[a - (\lambda_u + \lambda_v v')][1 + v'^2]}{3v' - b(1 + v'^2)}. \tag{48}$$

We may take for the equation of the conic of Property III lying in the tangent plane at O and passing through O in the fixed direction ω,

$$(49)\qquad \alpha_0\xi^2 + \alpha_1\xi\eta + \alpha_2\eta^2 + 3e^{\lambda}(\eta - \omega\xi) = 0,$$

where $\alpha_0, \alpha_1, \alpha_2, \omega$ are functions of the coördinates (u, v) of O only, and ξ, η are the coördinates of the center of geodesic curvature referred to O as origin. Substituting the values of ξ, η as given by (40), we find

$$(50)\qquad v'' = \frac{(\beta_0 + \beta_1 v' + \beta_2 v'^2)(1 + v'^2)}{3(1 + \omega v')},$$

where the β's are functions of u, v only. Comparing this with (48) we see that a and b are particular functions of u, v, v' satisfying the condition

$$(51)\qquad \frac{a - (\lambda_u + \lambda_v v')}{3v' - b(1 + v'^2)} = \frac{(\beta_0 + \beta_1 v' + \beta_2 v'^2)}{3(1 + \omega v')},$$

Finally, combining (47) and (51), we may state

THEOREM 8. *The most general triply infinite system of curves on a surface possessing Properties I and III, is defined by a differential equation of the form* (47), *in which*

$$a = \frac{[\beta_0 + \beta_1 v' + \beta_2 v'^2][3v' - b(1 + v'^2)]}{3(1 + \omega v')} + (\lambda_u + \lambda_v v').$$

The differential equation involves one arbitrary function of u, v, v' and four arbitrary functions of u, v.

Equation (47) expanded takes the form

$$(52)\qquad v''' = D_0 + D_1 v'' + b v''^2$$

where D_0 and D_1 are special functions of u, v, v'. From Theorem 5 we infer that all systems with Properties I and III will also possess Property II provided the function b has the form

$$(53)\qquad b = \frac{3}{v' - \omega}.$$

Substituting this in (51), we find

$$(54)\qquad a = -\frac{\gamma_0 + \gamma_1 v' + \gamma_2 v'^2}{v' - \omega},$$

where the γ's are arbitrary functions of u, v. The value of v'' in (48) corresponding to a hyperosculating curve is now given by

$$(55)\quad v'' = \frac{\{(\gamma_0 + \gamma_1 v' + \gamma_2 v'^2) + (\lambda_u + \lambda_v v')(v' - \omega)\}\{1 + v'^2\}}{3(1 + \omega v')}.$$

We may finally state

THEOREM 9. *The most general triply infinite system of curves on a surface possessing Properties I, II, and III is defined by a differential equation of the form*

$$(56)\qquad (\omega - v')G' = G(\gamma_0 + \gamma_1 v' + \gamma_2 v'^2 - 3v'')$$

involving four arbitrary functions $\gamma_0, \gamma_1, \gamma_2, \omega$ *of* u, v.

By comparison of (56) with the differential equation of the trajectories

$$(11)\quad (\psi - \phi v')G' = G\{(\psi_u + 2\lambda_v\phi) + (\psi_v - \phi_u + 2\lambda_v\psi - 2\lambda_u\phi)v' - (\phi_v + 2\lambda_u\psi)v'^2 - 3\phi v''\},$$

involving only two arbitrary functions of u, v, we note that Properties I, II, and III are not sufficient to characterize the system of trajectories. We may here note the similarity in form of equations (11) and (56).

§ 7. The Lines of Force. Curves with Property I, II, III, IV.

On the surface, a line of force is a curve such that its tangent line at any point has the direction of the force vector through that point. The lines of force thus form a simple system of ∞^1 curves defined by the differential equation

$$(57)\qquad v' = \psi/\phi.$$

Employing (39), we find for the geodesic curvature of the line of force passing through the point O,

$$(58)\quad \frac{1}{\rho} = \frac{\phi^2\psi_u - \psi^2\phi_v + \phi\psi(\psi_v - \phi_u) - (\lambda_v\phi - \lambda_u\psi)(\phi^2 + \psi^2)}{e^\lambda(\phi^2 + \psi^2)^{\frac{3}{2}}}.$$

How does this compare with the curvature of the unique hyperosculating trajectory passing through O in the direction of the force vector? The value of v'' corresponding to a hyperosculating trajectory is given by (43); introducing this and the direction $v' = \psi/\phi$ into (39), we find for the required geodesic curvature,

$$(59)\quad \frac{1}{R} = \frac{\phi^2\psi_u - \psi^2\phi_v + \phi\psi(\psi_v - \phi_u) - (\lambda_v\phi - \lambda_u\psi)(\phi^2 + \psi^2)}{3e^\lambda(\phi^2 + \psi^2)^{\frac{3}{2}}}$$

Comparing (58) and (59), we may state

THEOREM 10. *For any point on the surface, the geodesic curvature of the line of force is equal to three times the geodesic curvature of the hyperosculating trajectory which passes out in the direction of the force vector.*

Let us now find the systems possessing Properties I, II, III, with the additional property got by converting Theorem 10. We replace the force vector by the fixed direction ω, tangent to the central locus of Property III. We may now ask for all the triply infinite systems of curves on a surface which possess Properties I, II, III, and

Property IV. With each point on the surface O, Property III associates a direction through the point, viz., the tangent to the central locus or conic. The totality of all such directions on the surface, defines a simple system of ∞^1 curves, which may be called the tangential lines. The geodesic curvature of the tangential line through O is equal to three times the geodesic curvature of the hyperosculating trajectory which passes through O in the same direction.

The geodesic curvature of the tangential line $v' = \omega$ through O is

$$(60)\qquad \frac{1}{\rho} = \frac{(\omega_u + \omega\omega_v) - (\lambda_v - \omega\lambda_u)(1 + \omega^2)}{e^{\lambda}(1 + \omega^2)^{\frac{3}{2}}}.$$

The curves possessing Properties I, II, III are defined by the differential equation (56). For the hyperosculating trajectory in the direction $v' = \omega$, we have, by (55).

$$v'' = \frac{\gamma_0 + \gamma_1\omega + \gamma_2\omega^2}{3},$$

and for the geodesic curvature,

$$(61)\qquad \frac{1}{R} = \frac{(\gamma_0 + \gamma_1\omega + \gamma_2\omega^2) - 3(\lambda_v - \omega\lambda_u)(1 + \omega^2)}{3e^{\lambda}(1 + \omega^2)^{\frac{3}{2}}}.$$

Setting $1/\rho$ equal to three times $1/R$, we find

$$(62)\qquad \gamma_0 + \gamma_1\omega + \gamma_2\omega^2 = (\omega_u + \omega\omega_v) + 2(\lambda_v - \omega\lambda_u)(1 + \omega^2).$$

Hence we have

THEOREM 11. *The most general triply infinite system of curves possessing Properties I, II, III, IV, is defined by a differential equation of the form (56) together with the condition (62), thus involving three arbitrary functions of u, v.*

We must therefore seek one other geometric property which would reduce the number of arbitrary functions of u, v in (56) to two and thus reduce this equation to that of the trajectories (11); this fifth property would then complete the characterization.

§ 8. Curves with Properties I, II, III, IV, V. — Complete Characterization.

The analytical expression for the final property is most readily found by comparing the coefficients in equations (56) and (11). If (56) is to reduce to (11), we must evidently have

$$(63)\quad \omega = \frac{\psi}{\phi};\quad \gamma_0 = \frac{\psi_u}{\phi} + 2\lambda_v;\quad \gamma_1 = \frac{\psi_v}{\phi} - \frac{\phi_u}{\phi} + 2\lambda_v\frac{\psi}{\phi} - 2\lambda_u;$$
$$\gamma_2 = -\frac{\phi_v}{\phi} - 2\lambda_u\frac{\psi}{\phi}.$$

Substituting

$$\psi = \omega\phi;\quad \psi_u = \omega_u\phi + \omega\phi_u;\quad \psi_v = \omega_v\phi + \omega\phi_v,$$

we get from the second and fourth of equations (63),

$$(64)\quad \frac{\phi_u}{\phi} = \frac{\gamma_0 - 2\lambda_v - \omega_u}{\omega} = (\log\phi)_u;\ \frac{\phi_v}{\phi} = -\gamma_2 - 2\lambda_u\omega = (\log\phi)_v.$$

Substituting these values in the third equation (63), we have

$$(65)\qquad \gamma_0 + \gamma_1\omega + \gamma_2\omega^2 = (\omega_u + \omega\omega_v) + 2(\lambda_v - \omega\lambda_u)(1 + \omega^2).$$

Equations (64) may be combined into

$$(66)\qquad (\gamma_2 + 2\lambda_u\omega)_u + \left(\frac{\gamma_0 - 2\lambda_v - \omega_u}{\omega}\right)_v = 0.$$

If, then, (56) is to reduce to (11), the functions γ_0, γ_1, γ_2, ω must necessarily satisfy the relations (65) and (66). Conversely, if γ_0, γ_1, γ_2, ω satisfy (65) and (66), it is possible, by virtue of (66), to find a function $\log\phi$ (and hence ϕ) to satisfy both equations (64); and if we then choose $\psi = \omega\phi$, we shall have found a pair of functions ϕ, ψ, or a field of force, which satisfies all the equations (63). We may thus state

Theorem 12. *In order that an equation of the form* (56) *should represent a system of trajectories under some field of force, it is necessary and sufficient that the four arbitrary functions of u, v satisfy equations* (65) *and* (66).

Now (65) is the same condition as (62), and we have already interpreted this geometrically by Property IV. It remains therefore to interpret condition (66) geometrically and thus complete the characterization.

Consider, at a point O, the isothermal u and v parameter curves and the hyperosculating curves of the system in these directions. Noting that $v'' = 0$ for the isothermal curves, we have for the geodesic curvatures of the u and v parameter curves,

$$\frac{1}{\rho_1} = -\frac{\lambda_u}{e^\lambda}; \quad \frac{1}{\rho_2} = -\frac{\lambda_v}{e^\lambda}.$$

Again, for the hyperosculating curves, v'' is given by (55), and the geodesic curvatures of these curves in the directions of the parameter curves are

$$\frac{1}{R_1} = \frac{\gamma_2 + \lambda_v + 3\lambda_u\omega}{-3e^\lambda\omega}; \quad \frac{1}{R_2} = \frac{\gamma_0 - \lambda_u\omega - 3\lambda_v}{3e^\lambda}.$$

Now (66) may be written

$$(67)\quad (\gamma_2 + \lambda_v + 3\lambda_u\omega)_u - (\lambda_u\omega)_u + \left(\frac{\gamma_0 - 3\lambda_v - \lambda_u\omega}{\omega}\right)_v + \left(\frac{\lambda_v}{\omega}\right)_v - (\log \omega)_{uv} = 0.$$

Introducing the values of ρ_1, ρ_2, R_1, R_2, this becomes

$$(68)\quad \left[e^\lambda\omega\left(\frac{1}{\rho_1} - \frac{3}{R_1}\right)\right]_u - \left[\frac{e^\lambda}{\omega}\left(\frac{1}{\rho_2} - \frac{3}{R_2}\right)\right]_v - (\log \omega)_{uv} = 0.$$

Introducing the abbreviations

$$(69)\quad \frac{1}{\kappa_1} = \omega\left(\frac{1}{\rho_1} - \frac{3}{R_1}\right); \quad \frac{1}{\kappa_2} = \frac{1}{\omega}\left(\frac{1}{\rho_2} - \frac{3}{R_2}\right),$$

and expanding (68), we get

$$(70)\quad e^\lambda\left[\left(\frac{1}{\kappa_1}\right)_u - \left(\frac{1}{\kappa_2}\right)_v\right] + e^\lambda\left[\frac{\lambda_u}{\kappa_1} - \frac{\lambda_v}{\kappa_2}\right] - [\log \omega]_{uv} = 0.$$

Finally, expressing λ_u and λ_v in terms of ρ_1 and ρ_2, dividing by $e^{2\lambda}$, and remembering that the arc lengths along the u and v isothermal parameter curves are given by

$$ds_1 = e^\lambda dv, \quad ds_2 = e^\lambda du$$

we may write (70) in the form

$$(71)\quad \frac{\partial}{\partial s_2}\left(\frac{1}{\kappa_1}\right) - \frac{\partial}{\partial s_1}\left(\frac{1}{\kappa_2}\right) - \frac{1}{\rho_1\kappa_1} + \frac{1}{\rho_2\kappa_2} - \frac{\partial^2(\log \omega)}{\partial s_1\,\partial s_2} = 0.$$

The quantities ρ_1, ρ_2, R_1, R_2, ω entering (71) are all geometric quantities, and (71) expresses a relation connecting their rates of variation with respect to the arcs of the isothermal parameter curves as we move out on the surface from O. Furthermore, although the parameter curves seem to enter this relation, (71) is really an *intrinsic* property of our system, for it is evidently true for any and every set of orthogonal isothermal curves that may be chosen. We may now state

Property V. Construct any isothermal net on the surface. At any point O this net determines two orthogonal directions in which there pass two isothermal curves of the net and two hyperosculating curves of Property III. If ρ_1, ρ_2, R_1, R_2 are the radii of geodesic curvature of these four curves, s_1, s_2, the arc lengths along the isothermal curves, and ω, the tangent of the angle between the fixed direction of Property III and the isothermal curve with arc s_2, then, as we move along the surface from O, these quantities vary so as to satisfy the relation

$$\frac{\partial}{\partial s_2}\left(\frac{1}{\kappa_1}\right) - \frac{\partial}{\partial s_1}\left(\frac{1}{\kappa_2}\right) - \frac{1}{\rho_1\kappa_1} + \frac{1}{\rho_2\kappa_2} - \frac{\partial^2(\log \omega)}{\partial s_1\,\partial s_2} = 0,$$

where

$$\frac{1}{\kappa_1} = \omega\left(\frac{1}{\rho_1} - \frac{3}{R_1}\right), \quad \frac{1}{\kappa_2} = \frac{1}{\omega}\left(\frac{1}{\rho_2} - \frac{3}{R_2}\right).$$

Property V thus completes the characterization. We may now state

THEOREM 13. *In order that a triply infinite system of curves (∞^1 in each direction through each point) on a surface may be identified with a system of dynamical trajectories under any positional field of force, the given system must possess Properties I, II, III, IV, V.*

§ 9. Special Case.— Conservative Forces.

If the field of force is conservative, there exists a work function (negative potential) W of which the force components ϕ, ψ are the derivatives: hence

$$(72) \qquad \phi = W_u, \quad \psi = W_v; \quad \text{or} \quad \psi_u = \phi_v.$$

We may interpret this relation geometrically by noting that if the conic (45) is to be a rectangular hyperbola, the sum of the coefficients of ξ^2 and η^2 must be zero; hence $\psi_u = \phi_v$; and conversely. Therefore, we have

THEOREM 14. *If the field of force is conservative, the locus of the* *centers of geodesic curvature of the ∞^1 hyperosculating trajectories which pass through any point of the surface, is a rectangular hyperbola.*

Combining Theorems 13 and 14, we state

THEOREM 15. *In order that a triply infinite system of curves on a surface may be identified with a system of dynamical trajectories under a conservative field of force, the given system must possess Properties I, II, III, IV, V, and the additional property that the central locus of Property III is a rectangular hyperbola.*

§ 10. Brachistochrones, Catenaries, Dynamical Trajectories and Velocity Curves.

If the field of force is conservative, we may study certain types of ∞^3 curves on a surface other than dynamical trajectories. Among these, two cases of special interest are the systems of *brachistochrones* and *catenaries*. To get the equations of these systems we proceed as follows.

Consider the motion of a particle in a conservative field of force from one position P_0 to another P_1, with the sum of its kinetic and potential energies equal to a given constant. If T is the kinetic energy, W, the work function (negative potential), v, the velocity, and h, the constant of energy, we have

$$(73)\qquad T - W = h, \text{ or } \tfrac{1}{2}v^2 - W = h, \text{ or } v^2 = 2(W + h).$$

(*i*) If the motion takes place under the *principle of least action*, i.e., so that

(74) *Action* =

$$\int_{(P_0)}^{(P_1)} 2T\,dt = \int_{(P_0)}^{(P_1)} v^2 dt = \int_{(P_0)}^{(P_1)} v\,ds = \int_{(P_0)}^{(P_1)} \sqrt{2(W+h)}ds = minimum,$$

the paths are *dynamical trajectories*.

(*ii*) If the motion takes place so that the time elapsed is least, i.e., so that

$$(75)\qquad Time = \int_{(P_0)}^{(P_1)} dt = \int_{(P_0)}^{(P_1)} \frac{ds}{v} = \int_{(P_0)}^{(P_1)} \frac{ds}{\sqrt{2(W+h)}} = minimum,$$

the paths are *brachistochrones*.

(*iii*) If the motion takes place along the position of equilibrium of a homogeneous flexible inextensible string, then

$$(76)\qquad \int_{(P_0)}^{(P_1)} v^2\,ds = \int_{(P_0)}^{(P_1)} 2(W + h)ds = \mathit{minimum},$$

and the paths are *catenaries*.

For a given constant of energy, h, (74), (75), or (76) will give ∞^2 curves, one through each point in each direction on the surface.[7] If we allow h to vary, we shall get triply infinite systems of curves: complete systems of dynamical trajectories, brachistochrones, or catenaries. The systems defined by (74), (75), and (76) may be considered as special cases of the system defined by

$$(77)\qquad \int_{(P_0)}^{(P_1)} (W + h)^{\frac{m}{2}}\,ds = \mathit{minimum},$$

where we have trajectories, brachistochrones, or catenaries, according as $m = 1, -1$, or 2.

Replacing ds by $e^{\lambda}\sqrt{1 + v'^2}\,du$, and applying the Euler condition for the vanishing of the first variation, to

$$(78)\qquad \int (W + h)^{\frac{m}{2}} e^{\lambda}\sqrt{1 + v'^2}\,du = \int H\,du = \mathit{minimum},$$

viz.,

$$H_v - H_{v'u} - v'\,H_{v'v} - v''\,H_{v'v'} = 0,$$

we find

$$(79)\quad v'' = \left\{ \left[\log\,(W+h)^{\frac{m}{2}} + \lambda \right]_v - \left[\log\,(W+h)^{\frac{m}{2}} + \lambda \right]_u v' \right\} \left\{ 1 + v'^2 \right\},$$

as the differential equation of the system of ∞^2 curves. To find the differential equation of the system of ∞^3 curves, we must differentiate (79) and eliminate h. This is most readily done by writing (79) in the form

$$\frac{m}{2}\,\frac{W_v - W_u v'}{W + h} = \frac{v'' - (\lambda_v - \lambda_u v')\,(1 + v'^2)}{1 + v'^2} = \frac{G}{1 + v'^2},$$

---

7 These systems of ∞^2 curves form special cases of the extremals connected with a variation problem of the form $\int F ds =$ minimum, where F is a function of the coördinates. Such systems, termed "*Natural Systems*," have been characterized geometrically by the author in "*Natural families of curves in a general curved space of N dimensions*," Trans. Am. Math. Soc., vol. 13 (1912), pp. 77–95. The author has also characterized these curves in a different way in "*Some geometric investigations on the general problem of dynamics*," Proceedings of the Am. Academy of Arts and Sciences, Vol. 55 (1920), pp. 285–322.

where G is defined as in (9), or

$$\frac{2}{m}(W + h) = \frac{(W_v - W_u v')(1 + v'^2)}{G}.$$

Differentiating this last expression, we get

$$(80)\quad \frac{2}{m}(W_u + W_v v') = \frac{G\dfrac{d}{du}\left\{(W_v - W_u v')(1 + v'^2)\right\} - G'(W_v - W_u v')(1+v'^2)}{G^2}.$$

If we introduce the components of the force

$$(81)\qquad \phi = W_u, \quad \psi = W_v,$$

and solve for G', at the same time replacing $2/m$ by n, we find

$$(82)\quad (\psi - \phi v')G' = G\left\{(\psi_u + n\lambda_v\phi) + (\psi_v - \phi_u + n\lambda_v\psi - n\lambda_u\phi)v' - (\phi_v + n\lambda_u\psi)v'^2 + \left[(2-n)\frac{\phi + \psi v'}{1 + v'^2} - 3\phi\right]v''\right\}$$

for the required differential equation, where

$n = 2$ corresponds to dynamical trajectories
$n = -2$ " " brachistochrones
$n = 1$ " " catenaries,

and G and G' are defined by (9). We shall designate the curves defined by (82) as an "n" system. We may here note the similarity of equation (82) for the "n" system and equation (11) for the dynamical trajectories in any field of force.

We may also study certain other types of curves on a surface, termed "velocity" curves. They are defined dynamically as follows: A curve is a velocity curve corresponding to the speed $\dot{s}_0$, if a particle starting with that speed from any point of such a curve and in the direction of the curve, describes a trajectory osculating the curve.

To get the differential equation of such a system, we note that the differential equation (11) of the trajectories for any positional field of force was obtained by eliminating the variable component $\dot{u}$ of the velocity from equation (8). Using the relation

$$\dot{s}^2 = e^{2\lambda}(\dot{u}^2 + \dot{v}^2) = e^{2\lambda}\dot{u}^2(1 + v'^2),$$

we may write equation (8) in the form

$$(8') \qquad v'' = \frac{1}{\dot{s}^2}[(\psi + \lambda_v) - (\phi + \lambda_u)v'][1 + v'^2].$$

Equation (8′) holds for any trajectory and along this the velocity $\dot{s}$ varies from point to point. Now, if in (8′) we replace $1/\dot{s}^2$ by a constant c, we get

$$(8'') \qquad v'' = c[(\psi + \lambda_v) - (\phi + \lambda_u)v'][1 + v'^2],$$

a differential equation of the second order representing a system of ∞^2 curves, one through each point in each direction. Each of these curves, therefore, has the dynamical property used above in defining a velocity curve.[8]

For each constant value assigned to the speed $\dot{s}$, we get a velocity system, and the totality of ∞^1 systems obtained by varying $\dot{s}$ constitute a complete velocity system of ∞^3 curves on the surface. To find the differential equation of the complete velocity system, we, therefore, differentiate (8″) and eliminate the parameter c. Writing (8″) in the form

$$\frac{1}{c} = \frac{(\psi - \phi v')(1 + v'^2)}{G},$$

we get by direct differentiation,

$$(82') \quad (\psi - \phi v')G' = G\left\{\left[\psi_u + (\psi_v - \phi_u)v' - \phi_v v'^2\right] + \left[2\frac{\phi + \psi v'}{1 + v'^2} - 3\phi\right]v''\right\},$$

for the required differential equation of a complete velocity system. We now note that this is the form taken by equation (82) if $n = 0$. Hence, we may state that *an "n" system represents a velocity system when $n = 0$.* For a velocity system in a conservative field of force we merely add conditions (81).

§ 11. Geometric Characterization of "n" Systems.

Let us now find the geometric properties of the system defined by (82). Since this equation has the form of equation

---

8 A velocity system of ∞^2 curves is characterized geometrically by the fact that the locus of the centers of geodesic curvature of the ∞^1 curves which pass through a given point, is a straight line. See the author's paper "*Geometric characterization of isogonal trajectories on a surface*," Annals of Math., 2d series, **15** (1913), pp. 71–77. For further discussion of velocity systems see the author's paper "*Note on velocity systems in curved space of n-dimensions*," Bull. Am. Math. Soc. 2d series, **27** (1920), pp. 71–77.

(33) $$v''' = A + Bv'' + Cv''^2,$$

Theorem 4 is applicable here, hence we have

THEOREM 16. *The "n" system possesses Property I.*

For the "n" system,

(83) $$C = \frac{1}{\psi - \phi v'}\left[(2 - n)\frac{\phi + \psi v'}{1 + v'^2} - 3\phi\right];$$

hence the bisection property II does not hold unless $n = 2$, i.e., for dynamical trajectories. For the general "n" system the bicircular quartic of Property I has a node at the given point, one of the tangents having the direction of the initial element v', and the other having the direction.

(84) $$\xi = \frac{Cv'(1 + v'^2) + 3(1 - v'^2)}{C(1 + v'^2) - 6v'},$$

where C is given by (83). Substituting this value and introducing the direction of the force vector $\psi/\phi = \omega$, we may write (84) as

(85) $$\frac{\omega - v'}{1 + \omega v'} = \frac{n + 1}{3}\,\frac{v' - \xi}{1 + v'\xi},$$

or

(86) $$\frac{\tan\theta_1}{\tan\theta_2} = \frac{n + 1}{3},$$

where θ_1 is the angle between the initial direction and the force vector, and θ_2 is the angle between the direction of the second tangent and the initial direction. Conversely, we may easily show that if the angles θ_1 and θ_2 are related as in (86), C must have the value given in (83). We thus have

THEOREM 17. **Property II′.** *For an "n" system, the focal locus described by Property I has two distinct tangent lines at the initial point. The tangent of the angle which the initial element makes with the force vector is to the tangent of the angle which the second tangent line makes with the initial element as $n + 1$ is to 3.*

We may easily show that the most general triply infinite system of curves possessing Properties I and II′ has an equation of the form

$$v''' = A + Bv'' + \frac{1}{\omega - v'}\left\{(2 - n)\frac{1 + \omega v'}{1 + v'^2} - 3\right\}v''^2$$

where a fixed direction $\omega(u, v)$ replaces the direction of the force vector.

We further find that through every point and in every direction through that point there passes one trajectory which hyperosculates its corresponding geodesic circle of curvature. This trajectory is given by

$$(87)\quad \frac{v''}{1+v'^2} = \frac{[\psi_u-\lambda_u\psi+n\lambda_v\phi]+[\psi_v-\phi_u+(1-n)(\lambda_u\phi-\lambda_v\psi)]v'-[\phi_v-\lambda_v\phi+n\lambda_u\psi]v'^2}{(1+n)(\phi+\psi v')},$$

and the central locus, or locus of centers of geodesic curvature of the ∞^1 hyperosculating trajectories which pass through any point on the surface, is a conic passing through the given point in the direction of the force vector. The equation of this conic is

$$(88)\quad \xi^2(\phi_v-\lambda_v\phi-\lambda_u\psi)+\xi\eta(\psi_v-\phi_u+2\lambda_u\phi-2\lambda_v\psi) - \eta^2(\psi_u-\lambda_u\psi-\lambda_v\phi)+(1+n)e^\lambda(\phi\eta-\psi\xi)=0.$$

Since, by (81), $\phi_v=\psi_u$, this conic is a rectangular hyperbola. Hence

THEOREM 18. *The "n" system possesses Property III. The conic described in this property is a rectangular hyperbola.*

We may now show that the most general triply infinite system of curves possessing Properties I, II′, III, has an equation of the form

$$(89)\quad (\omega-v')G' = G\left\{\gamma_0+\gamma_1 v'+\gamma_2 v'^2+\left[(2-n)\frac{1+\omega v'}{1+v'^2}-3\right]v''\right\},$$

where $\gamma_0, \gamma_1, \gamma_2, \omega$ are arbitrary functions of u, v.

From (88) we conclude that at any point O the asymptotes of the rectangular hyperbolas associated with all "n" systems are parallel, and that the locus of the centers of these hyperbolas is a straight line through O.

We further find that the geodesic curvature of the hyperosculating curve (87) which passes out in the direction of the force vector ψ/ϕ is

$$(90)\quad \frac{1}{R}=\frac{\phi^2\psi_u-\psi^2\phi_v+\phi\psi(\psi_v-\phi_u)-(\lambda_v\phi-\lambda_u\psi)(\phi^2+\psi^2)}{(1+n)e^\lambda(\phi^2+\psi^2)^{\frac{3}{2}}}.$$

Comparing this with the geodesic curvature $1/\rho$ of the line of force $v'=\psi/\phi$ given by (58), we conclude that

$$(91)\quad \frac{1}{\rho}=\frac{1+n}{R}.$$

Hence,

THEOREM 19. **Property IV′.** *For an "n" system, at any point on the surface, the geodesic curvature of the line of force is equal to $(n+1)$*

times the geodesic curvature of the hyperosculating curve which passes through the point in the direction of the force vector.

We may now show that the most general triply infinite system of curves possessing Properties I, II′, III, defined by the differential equation (89), will also possess Property IV′, where a fixed direction ω replaces the direction of the force vector, provided the condition

$$(92)\qquad \gamma_0 + \gamma_1\omega + \gamma_2\omega^2 = (\omega_u + \omega\omega_v) + n(\lambda_v - \lambda_u\omega)(1+\omega^2)$$

is satisfied.

It is evident that Properties I, II′, III, IV′ do not completely characterize an "n" system. To complete the characterization, let us compare the differential equations (89) and (82); we evidently have

$$(93)\qquad \omega = \frac{\psi}{\phi};\quad \gamma_0 = \frac{\psi_u}{\phi} + n\lambda_v;\quad \gamma_1 = \frac{\psi_v}{\phi} - \frac{\phi_u}{\phi} + n\left(\lambda_v\frac{\psi}{\phi} - \lambda_u\right);$$
$$\gamma_2 = -\frac{\phi_v}{\phi} - n\lambda_u\frac{\psi}{\phi}.$$

As in §8, conditions (93) may be reduced to (92) and

$$(94)\qquad \left(\gamma_2 + n\lambda_u\omega\right)_u + \left(\frac{\gamma_0 - n\lambda_v - \omega_u}{\omega}\right)_v = 0.$$

In order that an equation of the form (89) should represent an "n" system, it is, therefore, necessary and sufficient that the four arbitrary functions $\gamma_0, \gamma_1, \gamma_2, \omega$ satisfy (92) and (94).

To interpret (94) geometrically, we may write it in the form

$$(95)\qquad \left[\gamma_2 + \lambda_v + (n+1)\lambda_u\omega\right]_u - \left[\lambda_u\omega\right]_u + \left[\frac{\gamma_0 - (n+1)\lambda_v - \lambda_u\omega}{\omega}\right]_v$$
$$+ \left[\frac{\lambda_v}{\omega}\right]_v - \left[\log\omega\right]_{uv} = 0.$$

Introducing here the geodesic curvatures

$$\frac{1}{\rho_1} = -\frac{\lambda_u}{e^\lambda},\quad \frac{1}{\rho_2} = -\frac{\lambda_v}{e^\lambda}$$

of the isothermal u and v parameter curves, and the geodesic curvatures

$$\frac{1}{R_1} = \frac{\gamma_2 + \lambda_v + (n+1)\lambda_u\omega}{-(1+n)e^\lambda\omega},\quad \frac{1}{R_2} = \frac{\gamma_0 - (n+1)\lambda_v - \lambda_u\omega}{(1+n)e^\lambda},$$

of the hyperosculating curves of Property III which have the directions of the u and v parameter curves, (95) becomes

$$(96)\quad \left[e^{\lambda}\omega\left(\frac{1}{\rho_1} - \frac{n+1}{R_1}\right)\right]_u - \left[\frac{e^{\lambda}}{\omega}\left(\frac{1}{\rho_2} - \frac{n+1}{R_2}\right)\right]_v - \left[\log \omega\right]_{uv} = 0.$$

Introducing the abbreviations

$$(97)\qquad \frac{1}{\kappa_1} = \omega\left(\frac{1}{\rho_1} - \frac{n+1}{R_1}\right),\ \frac{1}{\kappa_2} = \frac{1}{\omega}\left(\frac{1}{\rho_2} - \frac{n+1}{R_2}\right),$$

and expanding (96), we get

$$(98)\qquad e^{\lambda}\left[\left(\frac{1}{\kappa_1}\right)_u - \left(\frac{1}{\kappa_2}\right)_v\right] + e^{\lambda}\left[\frac{\lambda_u}{\kappa_1} - \frac{\lambda_v}{\kappa_2}\right] - \left[\log \omega\right]_{uv} = 0.$$

Finally, expressing λ_u and λ_v in terms of ρ_1 and ρ_2, dividing by $e^{2\lambda}$, and employing the arc lengths

$$ds_1 = e^{\lambda}\, dv, \quad ds_2 = e^{\lambda}\, du$$

along the u and v isothermal parameter curves, (98) becomes

$$(99)\qquad \frac{\partial}{\partial s_2}\left(\frac{1}{\kappa_1}\right) - \frac{\partial}{\partial s_1}\left(\frac{1}{\kappa_2}\right) - \frac{1}{\rho_1\kappa_1} + \frac{1}{\rho_2\kappa_2} - \frac{\partial^2(\log \omega)}{\partial s_1\, \partial s_2} = 0.$$

The quantities ρ_1, ρ_2, R_1, R_2, ω in (99) are all geometric quantities, and this equation expresses an intrinsic property of our "n" system, for it is evidently true for any and every set of orthogonal isothermal curves that may be chosen. Hence,

THEOREM 20. **Property V′.** *Construct any isothermal net on the surface. At any point O, this net determines two orthogonal directions in which there pass two isothermal curves of the net and two hyperosculating curves of Property III. If ρ_1, ρ_2, R_1, R_2 are the radii of geodesic curvature of these four curves, s_1, s_2, the arc lengths along the isothermal curves, and ω, the tangent of the angle between the fixed direction of Property III and the isothermal curve with arc s_2, then, as we move along the surface from O, these quantities vary so as to satisfy the relation*

$$\frac{\partial}{\partial s_2}\left(\frac{1}{\kappa_1}\right) - \frac{\partial}{\partial s_1}\left(\frac{1}{\kappa_2}\right) - \frac{1}{\rho_1\kappa_1} + \frac{1}{\rho_2\kappa_2} - \frac{\partial^2 (\log \omega)}{\partial s_1\, \partial s_2} = 0,$$

where

$$\frac{1}{\kappa_1} = \omega\left(\frac{1}{\rho_1} - \frac{n+1}{R_1}\right),\ \frac{1}{\kappa_2} = \frac{1}{\omega}\left(\frac{1}{\rho_2} - \frac{n+1}{R_2}\right).$$

Property V′ thus completes the characterization of an "n" system, and we may finally state

THEOREM 21. *In order that a triply infinite system of curves on a surface may be identified with an "n" system, the given system must possess Properties I, II′, III, IV′, V′.*

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ARCTIC COPEPODA IN PASSAMAQUODDY BAY.

By A. Willey (McGill University, Montreal).

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By A. Willey (McGill University, Montreal).
Presented by Samuel Henshaw.

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The effect of the heavy tides in the Bay of Fundy upon the distribution of fishes in that region has been discussed by Dr. A. G. Huntsman (1918). He found that certain species of migratory fishes, whilst not being excluded from the Bay of Fundy, are unable to breed there successfully. All divisions of the marine fauna will naturally be exposed to the same influence and it only remains to follow up the question with reference to other orders. More than one circumstance lends interest to the investigation. Through the work of the Biological Stations at Woods Hole, Mass., a very complete knowledge has been obtained of the fauna of the Vineyard Sound Region to the south of Cape Cod which marks the position of the "Great Divide" between the boreal and temperate zones on the Atlantic coast of America. We have therefore an excellent standard of comparison at our disposal, and I have considered it worth while to emphasize the Arctic elements in our local marine fauna.

For some years, under the direction of Dr. Huntsman, plankton gatherings have been taken methodically at stations situated in and around Passamaquoddy Bay, in addition to outlying stations farther afield in the Gulf of St. Lawrence. Of these stations, one known as "Prince" Station 6 lies a little above the actual mouth of the St. Croix River, in the deep channel between the Biological Station near St. Andrews, N. B., and the settlement of Robbinston on the coast of Maine. In 1916–17, the plankton at this station showed exceptional features, particularly in the winter months, when there was a great demonstration of red oily *Calanus* constituting what I have elsewhere named a red macrocalanoid plankton. The result of the examination of this plankton, to which my attention was drawn by Dr. Huntsman, is the subject of the present note. In 1919–20 there was no such indraught of large calanoids, the winter plankton being sparse and microcalanoid.

The four species whose presence gives a special significance to the material comprise the largest of the free-living Copepods, namely, *Calanus finmarchicus, Calanus hyperboreus, Euchaeta norvegica,* and

Metridia longa. They are all more or less common constituents of the plankton in the Atlantic water where it mixes with the Labrador and Cabot currents to the southeast of Nova Scotia. Here the material obtained by the Canadian Fisheries Expedition under the direction of Dr. Johan Hjort (1914–15) contained many young and a fair admixture of adult females, both of *Euchaeta norvegica* and of *C. hyperboreus*, but very few adult males of *Euchaeta* and none of *C. hyperboreus*. Their zoogeographical character has been defined by C. W. S. Aurivillius (1898), *C. finmarchicus* being arctic and antarctic in the wide sense with far reaching adaptability to thermal changes, while the other three are arctic in the strict sense. Only the first-named (*C. finmarchicus*) figures in the list from Woods Hole. In the winter plankton of Passamaquoddy Bay (1914–15), McMurrich observed *C. hyperboreus* only in one gathering and then only as a single individual.

Passamaquoddy Bay is a branch of the Bay of Fundy which in its turn may be regarded, in its present configuration, as an arm of the Gulf of Maine. It is therefore necessary to refer briefly to the occurrence of the above-named species in the Gulf of Maine as determined by the explorations of the U. S. F. S. "Grampus" under the direction of Dr. Henry B. Bigelow. In the 1912 cruise of the "Grampus," *Metridia longa* did not appear in any gathering. Of 50 stations where *C. finmarchicus* occurred, *C. hyperboreus* was present twice only, one individual at station 40 and six examples amongst thousands of *C. finmarchicus* at station 23 (20–0 m.; Bigelow, 1914, p. 102). *Euchaeta norvegica*, a typical constituent of the deep water plankton of the Gulf of Maine, occurred in greatest abundance at station 43, where it formed the bulk of the haul in the closing net at 85 to 60 fathoms, its numbers equalling those of *C. finmarchicus*. At this depth (85–60 fathoms) the temperature was 42° F., and the salinity about 33.5‰.

In the 1913 cruise of the "Grampus," *Metridia longa* was recorded from several stations in the Gulf of Maine in small numbers. At station 10100, between Penobscot Bay and Cape Sable (*i. e.* opposite to the mouth of the Bay of Fundy), the quantitative haul (90–0 fathoms) registered 270 *C. hyperboreus* to 5400 *C. finmarchicus*. At this station *Euchaeta norvegica* was likewise abundant (Bigelow, 1915, p. 292).

In order to appreciate the contrast between the seasons 1916–17 and 1919–20, in Passamaquoddy Bay, it will be enough to consider two typical hauls at "Prince" station 6 in 1920, and then to present the exhibits at the same station in 1916–17. On January 16, 1920 at 3.15 P.M., towing a net of No. 5 mesh at about 5 metres below the surface yielded 4 cc. of a microcalanoid plankton consisting mostly of young to submature copepods. So conspicuous was the paucity in adult individuals and so predominant was the genus *Acartia* that it might fittingly be designated a nepionic Acartia plankton. The following percentages were obtained, only the submature and mature Acartia being counted:—

Acartia clausi	68 per cent.	
" longiremis	13 "	
Pseudocalanus elongatus	16 "	
Temora longicornis	3 "	(juniors).

About the same date (January 27) in 1915, Professor McMurrich recorded a maximum of *Temora* at this station. Such fluctuations in the plankton are possibly correlated to some extent with the weir-catches of young herring, *Temora* being herring food *par excellence*, but there are no data at hand for proving this to be the case. On March 25, 1920, with the tow-net (No. 5) at about 7 metres, 6 cubic centimeters of an Acartia-Balanus plankton were obtained, Balanus larvae and Copepods occurring in subequal quantity. In addition there were a few *C. finmarchicus* from stage IV to adults of both sexes, two *C. hyperboreus* of stage IV, some *Pseudocalanus*, and a single example of *Monstrilla canadensis* McMurrich, dark brown in colour after preservation.

Turning to the winter of 1916–17, the catches are not strictly comparable with those of 1919–20, as they were taken at a different depth with a closing net (mesh No. 0) towed at 18–23 metres. On November 2, 1916, 64 cc. of a red macrocalanoid plankton were obtained, several Sagittae being present, but no fish eggs. One *Calanus hyperboreus*, stage V, was seen; one *Eurytemora herdmani* with ovisac, and one male *Metridia longa*. The remaining percentage content indicated a Calanus-Tortanus demonstration:—

Calanus finmarchicus V	46 per cent.
" " ♀	2 "
Pseudocalanus elongatus	5 "
Temora longicornis	1 "
Tortanus discaudatus (♂ and ♀)	46 "

On December 8, 1916, the plankton had the same general appearance and was still more copious, amounting to 87 cc., with Sagittae fairly plentiful. The species of Sagitta is *S. elegans* Verrill (Huntsman, 1919, p. 445). It should be mentioned that *Acartia* was present, but as it apparently passes readily through the meshes of the net employed, it does not appear in the records. One *Metridia longa* ♀, one *Temora longicornis* ♀, and one *C. finmarchicus* ♂ were seen. The percentage count showed predominance of *C. finmarchicus*, stage V:—

C. finmarchicus V	73 per cent.
" VI (adult)	9 "
Pseudocalanus elongatus	9 "
Tortanus discaudatus	9 "

The climax of this macrocalanoid intrusion was reached on February 23, 1917, though the quantity, 85 cc., was a little below that of December 8. Many large Sagittae were present. There was a sprinkling of large copepods, conspicuous amongst the rank and file composed of *C. finmarchicus*, but not in sufficient numerical strength to appear in the percentage estimation, so that in such a gathering as this, the count alone would fail to convey its most distinctive features. One characteristic however is not lost in the table, namely, the large number of male *C. finmarchicus*:—

C. finmarchus V	36 per cent.
" ♀	27 "
" ♂	20 "
Pseudocalanus elongatus (♂ and ♀)	13 "
Tortanus discaudatus (♂ and ♀)	4 "

[There was a solitary larva of the Greenland sculpin, 8 mm. in length, identified by Dr. Huntsman].

In this setting of *C. finmarchicus* there was a scattering of *Metridia longa*, male and female, and, standing out boldly from the mass, *C. hyperboreus* and *Euchaeta norvegica*. The sample denotes the winter Calanus maximum for 1917 in Passamaquoddy Bay in the tidal channel of the St. Croix River.

On April 7 following, the mass of plankton (51 cc.) at station 6 consisted of a nearly pure culture of Balanus nauplii with *C. finmarchicus* (males, females and juniors) sprinkled sparsely through it. On May 1st, *Sagitta*, *Calanus*, and the comb-medusa (*Pleurobrachia*) were present, but the bulk (70 cc.) was pure Balanus, nauplii and cypris-larvae in subequal numbers, the latter somewhat more numerous and, being heavier, tending to collect in a layer at the bottom, the whole sample presenting the appearance of millet seed. This was the spring Balanus maximum for 1917 at the same station where the Calanus maximum was celebrated earlier in the year. Finally on May 17, the surface tow at the same station yielded 7 cc. of plankton which, apart from some lighter material and few Balanus nauplii, consisted of cypris-larvae of Balanus. It should be added that this remarkable material was well known to Dr. Huntsman who kindly permitted me to examine it. A similar efflorescence has been noted by Professor Herdman in the Irish Sea. In 1907 the nauplii first appeared in the Bay at Port Erin on February 22, attaining their maximum on April 15, and disappearing on April 26. The cypris-larvae were first taken on April 6, rose to the maximum on the same day with the nauplii, and were last caught on May 24.

The males of *C. finmarchicus* formerly passed for extremely rare because they only appear in the upper layers at the epoch of reproduction in the spring (Damas 1905). In the spawning centres of *C. finmarchicus*, the eggs are stated by Damas to be sometimes so abundant that they constitute one of the principal elements in the plankton in certain regions, tens of millions in a sample, even to the exclusion of other forms. They float at variable depths down to 200 metres, but accumulate in the upper layers. Such spawning *en masse* has not been observed in Canadian waters nor in the Gulf of Maine which would seem to be the southern headquarters of *C. finmarchicus* in the northwestern Atlantic. O. Paulsen (1906) found *C. finmarchicus* propagating in Icelandic waters to the south of Iceland from March to June. C. W. S. Aurivillius (1898) states that males of *C. finmarchicus* were observed fairly generally in the deep water of the Gullmarfjord in the Skagerak in August and September 1897. It is to be presumed that they were derived from the spring spawning of that year in other waters.

From the examination of rich material collected by Norwegian and Danish vessels, Damas found that the distribution of the great mass of eggs of *C. finmarchicus* coincides with that of the maxima of frequency of the adults. The abundance is at its height in May and June in the waters of the Gulf Stream around the Faeroë Islands. The meeting of the waters which descend from the north, passing to the east of Jan Mayen island in the Greenland Sea, with the warm and salt Atlantic water, is the new factor which brings these individuals to the "ponte." The stations which traverse the Faeroë-Shetland Channel present a profusion of eggs, whilst a five minute surface tow with a metre net may collect more than a litre of Copepods,— the best example of a monotypic plankton. Damas adds very pertinently that the determination of areas of reproduction is the first sure step in the study of marine productivity.

The males of *C. hyperboreus* have only been met with in comparatively recent years and there was no expectation that they would ever be found in the waters of the maritime provinces of Canada, because their proper province lies in high latitudes. According to Damas, the Copepod plankton of the blue water of the Polar Current to the northeast of Iceland is a *C. hyperboreus* plankton; that of the central part of the Norwegian sea is a *Pseudocalanus* plankton; that over the coastal banks of Norway is a *Temora* plankton; and the bathypelagic plankton of the Norwegian sea is a *Euchaeta* plankton. In the *C. hyperboreus* plankton of the Norwegian stations northeast of Iceland in May and June 1904, there was a majority of juniors, some females, but not one male. The absence of large eggs in the oviducts and of larvae in the sea, as well as the absence of males, showed that the species was not reproducing there at that season. In gatherings taken by Captain Amundsen near Francis Joseph Land, the great nauplius and the metanauplius of this species were found by Damas to abound in company with opaque females gravid with eggs and with some males which, as mentioned, were totally absent from the Icelandic plankton.

Nordgaard (1905) found males of *C. hyperboreus* in deep water in the Ofoten Fjord and vicinity (between 68° and 69° N. lat.) and concluded that the spawning season for this species in the northern fjords lies in the months of February, March and April. If this is so there must be two broods in the year. The captures were the following:

Date	Depth (m.)	Bottom (m.)	Females	Males
7 II, '99	300–350	360	74	8
"	200–250	258	25	1
17 II, '99	450–550	630	17	4
"	550–620	630	2	2
16 III, '99	400–500	640	6	2

Damas and Koefoed (1907) state that the spawning of *C. hyperboreus* was observed by Vanhöffen (1897) off the west coast of Greenland, but I have not had access to Vanhöffen's article and do not know whether he recorded the males. In the cruise of the "Belgica" promoted by the Duc d'Orléans in the Greenland Sea (1905), Damas and Koefoed reported that the samples from stations 46, 47 and 48 were remarkable for the abundance of eggs and nauplii of *C. hyperboreus*, together with females whose oviducts were full of eggs, as well as some males which are elsewhere so rare. The following are the positions of the stations in questions:—

Station 46. Lat. 77° 29′ N., Long. 18° 31′ W.
Depth 265 metres; vertical haul 13–0 m.; Aug. 4, 1905.
Eggs and nauplii of *C. hyperboreus* in great quantity.

Station 47. Lat. 76° 47′ N., Long. 15° 21′ W.
Depth 180 metres; vertical haul 20–0 m.; Aug. 8, 1905.
Nauplii of *C. hyperboreus* in great quantity.

Station 48. Lat. 71° 22′.5 N., Long. 18° 58′ W.
Depth 1130 metres; vertical haul 10–0 m.; Aug. 15, 1905.
Oviducts full of eggs; nauplii in great quantity.

It follows from Damas' researches that the two species, *C. finmarchicus* and *C. hyperboreus* reproduce principally at two extreme points of the basin of the Arctic Ocean, the former in the south, the latter in the north. Whilst they are often mixed in planktonic samples, the conditions of their reproduction are quite different.

From the February material of "Prince" station 6 (1917), I picked out 31 examples of *C. hyperboreus* distributed as follows:—

C. hyperboreus	IV	6
"	V	14
"	♀	5 (7.25 mm. long)
"	♂	6 (6.1 mm. long)

The adults of both sexes were new, transparent, freshly exuviated and turgid with pink oil. Whereas the many males of *C. finmarchicus* averaged a little over 3 mm. in length, the male *C. hyperboreus* somewhat exceeded 6 mm. and were proportionately bulky so that the contrast in size was most striking. The lateral corners of the last thoracic segment (*th 5*) of these new males was nearly rounded, at most very obtusely pointed, less pointed that it sometimes appears as a variation in *C. finmarchicus,* but the spinous armature of the outer branch of the left fifth foot and the coxal teeth of the fifth feet, together with the large size, proved beyond question that they were the males of *C. hyperboreus.* The large females which accompanied the males and were equally new and oily, showed the specific point at the lateral edge of *th 5;* and the coxal teeth of their fifth pair of feet were likewise those of *C. hyperboreus.*

Of the great red oily *Euchaeta norvegica*, the mature females of which measured up to 8 mm. in length, and the males a little more than 6 mm., 46 examples were observed, distributed as under:—

Euchaeta norvegica ♀	37	}	"Prince" Station 6, February 23, 1917.
" " ♂	4		
" " ♂ (jun.)	4		
" " ♀ (jun.)	1		

Some of the females were carrying an ovisac laden with eggs, others had a spermatophore attached to the genital segment, and two of the males each held a spermatophore by the cheliform extremity of the left fifth foot. The relatively large proportion of males is noticeable. During the Canadian Fisheries Expedition (1914–15) only six males were observed altogether at the stations of the C. G. S. "Acadia," southeast of Nova Scotia and only in one vertical haul as many as two were seen; the remainder were found singly.

A still greater proportion of *Euchaeta* males appeared in the stomach contents of the coal-fish (*Gadus* or *Pollachius virens*), locally known as the "pollack," taken off Wilson's Beach, Campobello Island, at the entrance to Passamaquoddy Bay on August 2–4, 1916. In the first one which I examined, the bulk of the contents consisted of the Schizopod, *Meganyctiphanes norvegica;* there were also some of the smaller species, *Rhoda* or *Thysanoëssa inermis*, some Caprellids, fish remains, several Sagittae, and 56 *Euchaeta*, namely:—

Euchaeta norvegica ♀	27
" " ♂	7
" " V	22

Of the females, 4 retained the egg-sac.

Another "pollack" stomach was full of "red feed" consisting of a densely packed mass of *Euchaeta* mixed with an approximately equal quantity of Schizopod remains half digested. Euchaeta composed a moiety of the total content. One intact *Meganyctiphanes* was present. The following percentage of the Copepod content was made out very clearly:—

Calanus finmarchicus V	1 per cent.
" " ♀	1 "
" " ♂	1 "
" hyperboreus V	2 "
" " ♀	3 "

Euchaeta norvegica	IV	1	per cent.
" "	V	56	"
" "	♀	20	"
" "	♂	14	"
Metridia longa		1	"

The high percentage of males of Euchaeta is a proportion not hitherto met with in the free plankton of this region, possibly owing to the fact that bottom-tows, *i.e.* with the net a little over the bottom in deep water, are not usually attempted at our stations. A tow at ten fathoms at an adjacent station ("Prince" station 1) off Eastport at the same time (August 2, 1916) yielded 54 cc. of plankton material from which the following Copepod count was made:—

Calanus finmarchicus	II	2
" "	III	8
" "	IV	17
" "	V	80
" "	♀	8
" "	♂	22
" hyperboreus	IV	9
" "	V	8
" "	♀	3
Pseudocalanus elongatus		50
Euchaeta norvegica	V	5
" "	♀	2
Metridia longa		12
" lucens		4
Total counted		230

The excess of male *C. finmarchicus* in this gathering has no particular significance because the same station three weeks earlier (July 10th) yielded males and females in equal numbers. The individuals of stage V varied in length from 2.6 to 4.6 mm., and would be continually replenishing the stock of females.

The mere presence of the three Arctic species (*C. hyperboreus*, *E. norvegica*, and *M. longa*) in Passamaquoddy Bay is not unexpected but in our experience it is exceptional to find them extending in winter so far up the bay as to penetrate well within the mouth of the St. Croix river. Moreover the association of males and females of *C. hyperboreus*, although in a nascent state, adult in form but not fully mature in sex, came as a surprise. I am indebted to Dr. Huntsman for the following hydrographic data relating to "Prince" station 6. It will be seen that they throw no fresh light upon the special biological features of the plankton:—

Date.	Depth (metres).	Temperature (C.).	Salinity (‰).
Nov. 2, 1916	20	8.69	31.62
Dec. 4, 1916	20	5.30	32.12
Feb. 23, 1917	20	−0.77	31 91
Mar. 28, 1917	20	1.32	31.64
May 1, 1917	20	2.50	30.05
Jan. 16, 1920	10	0.46	30.84
" "	30	0.67	31.44

Notwithstanding the negative evidence afforded by the hydrographic determinations, there is still a possibility that the somewhat extraordinary nature of the plankton samples taken off Robbinston at "Prince" station 6 during the winter of 1916–17 can be attributed ultimately to a remote flooding of Arctic water in the Labrador current from Baffin Bay, acting in conjunction with the Cabot current issuing from the Gulf of St. Lawrence and with the tides of the Bay of Fundy. Its proximate causation may have had a more local agency. It is certain that the individuals of *C. hyperboreus* and *Metridia longa* were not produced nor would they reproduce where they were found in the estuary of the St. Croix river, although the exuviation from stage V to stage VI may easily have occurred within our waters. If we may judge from their store of oil, they had thriven amazingly during their wanderings and would have had a good chance to avail themselves of reversed currents to regain their spawning area in the north. Failing this, they would presumably perish as stragglers without leaving descendants. Not only do they occur in the Baffin Bay region to the west of Greenland, but they attain their largest dimensions there (Aurivillius 1896). Both species were taken in the Dolphin and Union Strait by the Canadian Arctic Expedition.

In conclusion it cannot be pretended that our knowledge of the great northern currents, which is so essential in connection with fishery problems, is materially advanced by the present contribution. Its chief claim to consideration is the biological interest appertaining to the identification of the rare males of the hyperborean Calanus in comparatively shallow water a little above the forty-fifth parallel of latitude.

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Proceedings of the American Academy of Arts and Sciences.

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THE POTENTIAL OF THE THALLIUM ELECTRODE AND THE FREE ENERGY OF FORMATION OF THALLOUS IODIDE.

The Conductivity and Ionization of Thallous Nitrate Solution; the Solubility of Thallous Chloride and of Thallous Iodide; the Normal Potentials of the Thallium and of the Iodine Electrodes.

By Grinnell Jones and Walter Cecil Schumb.

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In a paper by Grinnell Jones and M. L. Hartmann [1] it was demonstrated that the free energy of formation of silver iodide is greater at 25°C. than at 0°C., thus justifying the inference that the mutual atomic attraction of silver and iodine increases with a rise of temperature, and furnishing, with the aid of Richards' [2] theory of compressible atoms, a plausible explanation of the hitherto inexplicable fact that silver iodide, when heated, contracts — an almost unique property. But the results, when interpreted with the existing data on the specific heats and heat of formation of the substances involved, are not quantitatively in accord with the requirements of the so-called "Nernst Heat Theorem."

The present paper records the results of an attempt to test the generality of the relations observed by the investigation of another case.

Thallous iodide was selected as the first case to be studied on account of the marked similarity between the iodides of thallium and of silver.

It is true that neither the essential specific heat data nor measurements of the coefficient of expansion of thallous iodide are as yet available, but it is hoped to measure these properties of thallous iodide at the earliest opportunity. The present paper records measurements of the free energy of formation of thallous iodide at 25°C. and at 0°C., together with the results of some subsidiary measurements which were required to interpret these observations. At first an attempt was made to measure directly the free energy of formation of thallous iodide by a method analogous to that used successfully with silver

[1] Grinnell Jones and Miner L. Hartmann, Jour. Amer. Chem. Soc., **37**, 752 (1915).
[2] T. W. Richards, Jour. Amer. Chem. Soc., **36**, 2417 (1914). This paper gives a summary of the theory and a bibliography of earlier papers.

iodide. An electrolytic cell was prepared, containing as one electrode, metallic thallium coated with thallous iodide immersed in a solution of potassium iodide, and as the other electrode, an iodine electrode. This type of cell, however, proved to be lacking in constancy and reproducibility. After many attempts to overcome its defects, it was reluctantly abandoned. The direct method having failed to give satisfactory results, an indirect method was developed. An electrode of metallic thallium in a saturated solution of thallous chloride was found to give a constant and reproducible potential when atmospheric oxygen was rigorously excluded. From determinations of the conductivity of saturated solutions of thallous chloride and of thallous iodide, conclusions can be drawn in regard to the relative concentration of the thallous ion in these solutions, and hence the potential of thallium in a saturated solution of thallous iodide can be computed. These computations require a knowledge of the mobility of the thallous ion, and therefore the conductivity of a series of solutions of thallous nitrate of progressively increasing dilution was determined. Finally measurements of the potential of an iodine electrode in solutions of potassium iodide of known strength were needed. All the measurements were carried out at both 25°C. and 0°C., these temperatures being selected partly because they are convenient working temperatures and partly because the other data needed are available in the existing literature for these temperatures with sufficient accuracy.

Section 1. The Conductivity of Solutions of Thallous Nitrate from 0.001 N to 0.1 N and the Equivalent Conductance of the Thallous Ion at 25°C. and 0°C.

In 1895 E. Franke [3] published measurements of the equivalent conductance at 25°C. of solutions of thallous nitrate and of several other thallous salts. Franke calculated the equivalent conductance of these salts at infinite dilution by extrapolation by Ostwald's "valence and dilution rule." He concluded that the equivalent conductance of the thallous ion at 25° is 71.2 Siemens units. His data, when converted into modern units and extrapolated by Noyes' method, give 75.9 mhos as the equivalent conductance of the thallous ion at 25°C.

Hunt [4] has measured the equivalent conductance of solutions of thallous sulphate at 18° and 25°C. At 25°C. his data, when extrapolated by Noyes' method, give 156.0 as the equivalent conductance of thallous sulphate, and if the equivalent conductance of the sulphate

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[3] E. Franke, Z. phys. Chem., **16**, 463 (1895).
[4] Franklin L. Hunt, Jour. Amer. Chem. Soc., **33**, 795 (1911).

ion be taken as 80, according to Noyes and Falk,[5] the equivalent conductance of the thallous ion becomes 76.0.

At 18°C. there are measurements on thallous nitrate by Kohlrausch[6] and von Steinwehr and on thallous sulphate by Hunt. The value 65.9 for the equivalent conductance of the thallous ion at 18°C. has been computed from these measurements[7] by Noyes and Falk.

We have been unable to find any data from which the equivalent conductance of the thallous ion at 0°C. can be computed.

For the purpose of determining the mobility of the thallous ion the nitrate was chosen as the most suitable salt on account of its ready solubility and the ease with which it can be purified and dried.

The thallous nitrate used for this work was prepared from some thallous sulphate which had been carefully purified by Dr. L. T. Fairhall for other work. To a dilute solution of this salt was added a dilute solution of barium nitrate (free from chloride) in equivalent amounts. The solution was filtered from the barium sulphate, evaporated, the thallous nitrate recovered by crystallization and purified by several recrystallizations with centrifugal drainage. Finally the thallous nitrate was melted in platinum. The melt was clear and colorless and there was no indication of decomposition. Careful qualitative testing gave negative results for barium, sulphate, thallic salts, and halogens. A positive test for nitrite was given by the very delicate alpha-naphthylamine-sulphanilic acid reaction. A repetition of this test quantitatively showed that the nitrite present was not over 0.01%—an amount which could not influence the conductivity by a significant amount. A solution of the salt was neutral to litmus and to phenolphthalein. Spectroscopic examination of the purified thallous nitrate revealed only the characteristic green thallium line $\lambda = 535$ and very weak lines of sodium.

The conductivity cells were of the pipette type and were four in number with cell constants 3.058, 0.5610, 0.05316, and 0.0104 at 25°C. The cell constants were determined independently by two observers in accordance with the method of Kohlrausch, using potassium chloride at both 25°C. and 0°C. A small Leeds and Northrup motor-generator

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5 A. A. Noyes and K. G. Falk, Jour. Amer. Chem. Soc., **34**, 479 (1912).

6 F. Kohlrausch and H. von Steinwehr, Sitzungsber. kgl. pr. Akad. Wiss. Berlin, **26**, 581 (1902).

7 J. F. Spencer, Z. phys. Chem., **76**, 360 (1911), has computed the conductance of thallous nitrate solution at 25°C. from the data of Kohlrausch and von Steinwehr at 18°C. using Kohlrausch's temperature coefficient, obtaining 147 as the equivalent conductance of $TlNO_3$ and hence 76.4 for the thallous ion. Incidentally it may be worth while to call attention to the fact that Spencer's figures for the 0.02 N and 0.05 N solution are incorrect. Spencer attributed the value for the 0.05 N solution to the 0.02 N solution and at the same time introduced a wholly extraneous figure for the 0.05 N solution.

served as the source of alternating current. The bridge, by the same manufacturer, was made with extension coils, by which the wire could be lengthened ten-fold in accordance with the design of Washburn.[8] The bridge wire and resistance box were carefully calibrated. An electrostatic condenser was used to balance the electrical capacity in the arms of the bridge and was a great help in securing a sharp minimum of sound in the telephone. Measurements were made at both 25.00° ± 01° and at 0.00°. The 0.1 N solutions were made up quantitatively by weight and the more dilute solutions made from these. All dilutions were made by weight rather than volume on account of the greater accuracy of this method. The densities of the 0.1 N and 0.05 N solutions were determined at both temperatures and the densities at lower concentrations computed on the assumption that the density is a linear function of the concentration.

Two completely independent series were carried out. The following table shows the results of these measurements.

TABLE I.

CONDUCTIVITY OF THALLOUS NITRATE SOLUTIONS AT 25° C.

Density	Wt. Normality	Vol. Normality	Specific conductivity κ observed	Water correction $\kappa \times 10^6$	κ corrected	Λ
First Series						
1.01926	0.097704	0.09959	0.011620	0.96	0.011619	116.67
1.00823	0.04944	0.04985	0.0061988	0.91	0.0061979	124.34
	0.04944	0.04985	0.0061993	0.91	0.0061984	124.35
(1.00156)	0.019909	0.01994	0.0026348	1.17	0.0026336	132.08
(0.99932)	0.009978	0.009971	0.0013610	0.965	0.0013600	136.39
(0.99820)	0.004995	0.004986	0.00069635	0.78	0.00069557	139.51
(0.99752)	0.001999	0.001994	0.00028498	0.66	0.00028432	142.56
(0.99730)	0.0009999	0.0009972	0.00014460	0.76	0.00014384	144.24
Second Series						
	0.09771	0.09959	0.011618	0.79	0.011617	116.65
	0.04942	0.04983	0.0061960	0.83	0.0061952	124.32
	0.019909	0.01994	0.0026353	0.86	0.0026344	132.12
	0.009978	0.009971	0.0013611	0.79	0.0013603	136.42
	0.004993	0.004984	0.00069597	0.72	0.00069525	139.50
	0.001999	0.001994	0.00028493	0.665	0.00028426	142.53
	0.0009999	0.0009972	0.00014465	0.80	0.00014385	144.25

[8] E. W. Washburn, Jour. Amer. Chem. Soc., **38**, 2431 (1916).

TABLE II.

CONDUCTIVITY OF THALLOUS NITRATE SOLUTIONS AT 0°C.

Density	Wt. Normality	Vol. Normality	Specific conductivity κ observed	Water correction κ x 10⁶	κ corrected	Λ
First Series						
1.02344	0.097704	0.09999	0.0064801	0.47	0.0064796	64.80
1.01165	0.04944	0.05002	0.0034680	0.44	0.0034676	69.33
	0.04944	0.05002	0.0034683	0.44	0.0034679	69.335
(1.00458)	0.019909	0.02000	0.0014781	0.59	0.0014775	73.875
(1.00222)	0.009978	0.01000	0.00076408	0.48	0.00076360	76.36
(1.00104)	0.004995	0.005000	0.00039125	0.35	0.00039090	78.18
(1.00034)	0.001999	0.002000	0.00015991	0.31	0.00015960	79.80
(1.00010)	0.0009999	0.001000	0.000081008	0.36	0.000080647	80.65
Second Series						
	0.09771	0.10000	0.0064791	0.37	0.0064787	64.79
	0.04942	0.05000	0.0034663	0.39	0.0034659	69.32
	0.019909	0.02000	0.0014783	0.40	0.0014779	73.90
	0.009978	0.01000	0.00076421	0.38	0.00076383	76.38
	0.004993	0.004998	0.00039109	0.36	0.00039073	78.18
	0.001999	0.002000	0.00015991	0.31	0.00015960	79.80
	0.0009999	0.001000	0.000081049	0.40	0.000080648	80.65

Extrapolation to infinite dilution, using the method of Noyes [9] both graphically and algebraically (method of least squares), gave the following results:

	0°	25°
1st series	82.486	148.974
2nd series	82.527	148.938
Mean	82.51	148.96

According to Noyes and Falk [10] the mobility of the nitrate ion at 25° is 70.6 mhos and at 0° is 40.7 mhos; hence the mobility of the thallous ion at 25°C. is 78.4 and at 0°C. is 41.8.

[9] See G. A. Abbott and W. C. Bray, Jour. Amer. Chem. Soc., **31**, 745, and John Johnston, *ibid.*, 1010 (1909); and A. A. Noyes, Carnegie Institution Publication, No. 63 (1907).

[10] A. A. Noyes and K. G. Falk, Jour. Amer. Chem. Soc., **34**, 479 (1912).

The equivalent conductances and degrees of dissociation of thallous nitrate solutions at round concentrations deduced from the above data by interpolation over small intervals are shown in the following table:

TABLE III.

EQUIVALENT CONDUCTANCE AND DEGREE OF DISSOCIATION OF THALLOUS NITRATE SOLUTIONS.

Normality	0° C. Equivalent Conductance (mhos)	0° C. Degree of Dissociation (%)	25° C. Equivalent Conductance (mhos)	25° C. Degree of Dissociation (%)
0.1000	64.79	78.52	116.60	78.28
0.0500	69.32	84.01	124.29	83.44
0.0200	73.88	89.55	132.06	88.66
0.0100	76.37	92.56	136.37	91.55
0.00500	78.18	94.75	139.50	93.65
0.00200	79.80	96.72	142.53	95.68
0.00100	80.65	97.74	144.24	96.83
0.00000	82.51	100.00	148.96	100.00

Section 2. The Conductivity of Saturated Solutions of Thallous Chloride and of Thallous Iodide at 25°C. and 0°C. and the Solubility of these Salts.

For the interpretation of the electromotive force measurements to be described in the next section a knowledge of the concentration of the ions in saturated solutions of thallous iodide and of thallous chloride at 0°C. and at 25°C. is required. Measurements were therefore made of the conductivity of the saturated solutions, from which the concentrations of the ions have been computed by the aid of the mobility of the ions which had been determined for the purpose (see Section 1).

The total solubility of thallous chloride at 25° has been determined by several experimenters with good agreement in results. Berkeley's [11] results over a range of temperature extending from 0° to 100° have been plotted in a curve, from which the values for 25° and 0° may be determined by interpolation. His values for the two temperatures,

[11] Earl of Berkeley, Phil. Trans., **203A**, 208 (1904).

respectively, are 0.0161 and 0.00696 mol per liter. At 25°, Noyes [12] and Geffcken [13] likewise found 0.0161 mol per liter and Hill [14] found 0.01629. Lewis [15] gives the concentration of the ionized portion of this solution at 25° as 0.0143 mol per liter.

Abegg [16] gives the solubility product of thallous iodide at 25° as 5.8×10^{-8}, corresponding to a solubility of 2.4×10^{-4} mol per liter. At 0° we have found no data for the solubility of thallous iodide.

The technique of the determination of the solubility of a slightly soluble salt has been thoroughly worked out, in particular by Kohlrausch, and later by Böttger.[17] The only innovation adopted in the present work was in the design of the cell, which was a modification of Böttger's [17] and is shown in Figure 1. Two platinum-iridium electrodes were sealed in through one end of the receptacle, in a manner designed to render their position rigidly fixed. The stout sealed-in platinum wires were welded to short copper leads, which extended several cm. into the tube at one end of the cell. These wires were insulated from one another, and at the same time connected with the bridge leads, by surrounding them with two narrow glass tubes and filling the space about these tubes with melted paraffine, which when solidified kept the tubes in position and sealed the openings at their lower ends. Mercury could now be poured into the tubes and the lead wires from the bridge inserted in the usual way. Another method of making connection between the cell and the outer circuit is shown in Figure 1 at A. Here the copper leads passed entirely outside the cell and were bent into hooks as in Kohlrausch's method, the terminating tube of the cell being completely filled with paraffine as before. This modification was not suited for work at 0°, as during the rotation of the cell in the ice-bath the wires became much bent. The other form could be used at both temperatures.

FIGURE 1.

The stopper and neck of the cell were covered with a thin rubber cap during its immersion in the constant temperature bath, and a similar protection was provided for the other end during rotation.

12 A. A. Noyes, Z. Phys. Chem., **6**, 249 (1890).
13 G. Geffcken, Z. Phys. Chem., **49**, 296 (1904).
14 A. E. Hill, Jour. Amer. Chem. Soc., **32**, 1189 (1910).
15 G. N. Lewis, and C. L. von Ende, Jour. Amer. Chem. Soc., **32**, 732 (1910).
16 R. Abegg, Handb. Anorg. Chem., **III**, (1), 424.
17 W. Böttger, Z. Phys. Chem., **46**, 551 (1903).

The grinding of the stopper was of course made carefully enough so that no grease was needed to make it tight. The material of the cell was sufficiently insoluble so that the specific conductance of the solutions measured did not increase by a significant amount during the period of the experiment.

Two cells were used in these determinations, one for thallous chloride, the other for the iodide. In the first cell the electrodes were lightly coated with platinum black; while in the latter cell the electrodes were of bright, burnished metal, and placed not over 1 mm. apart. The cell constants of these cells were 0.14658 and 0.02969. The rest of the conductivity apparatus was essentially the same as that already described. Temperature control was very important in the matter of obtaining concordant results; at 0°, especially, it was found necessary that the conductivity cell be deeply immersed in the bath of cracked ice and distilled water. The cell was caused to rotate end-over-end by being attached at right angles to a horizontal shaft. The solid within the cell was thus brought into intimate contact with all parts of the solution undergoing saturation. Only a small bubble of air was allowed to remain within the cell on filling.

Thallous chloride for these determinations was prepared from pure thallous nitrate, prepared as described on page 201, by precipitating by means of redistilled hydrochloric acid, followed by repeated washing with conductivity water. The precipitate was not dried, but kept under water in the dark until used. Thallous iodide was prepared from carefully purified thallous nitrate by precipitating with dilute potassium iodide solution. The precipitate is at first flocculent and orange, but on washing and agitation it becomes granular and light yellow. On long exposure to light it takes on a somewhat greenish tinge, but in these experiments the salt was protected against such exposure by a suitable shield above the thermostat and the use of a red heating lamp in the bath. The precipitated thallous iodide was washed with conductivity water many times by decantation, the flask being shaken violently so as to break up the larger particles. Like the thallous chloride, it was allowed to soak in conductivity water in the dark for four months before the rinsing was completed. A small quantity of the moist salt was introduced — more than enough for a number of saturations, however — and rinsed several times with conductivity water; and finally the cell was filled with water, the specific conductance of which was determined at the same time in the "water cell." The rubber caps were affixed to both ends of the cell, and the latter rotated in the constant temperature bath for such a length of time that further rotation did not alter the specific conductance. This usually did not occupy more than a few minutes, but a considerably longer period was generally employed to insure the attainment of equilibrium. The experimental results follow.

TABLE IV.

CONDUCTIVITY OF SATURATED THALLOUS CHLORIDE SOLUTION.

25°C.			
R cell	Specific conductivity observed $\kappa \times 10^6$	Water correction $\kappa \times 10^6$	Specific conductivity corrected $\kappa \times 10^6$
67.43	2173.8	1.18	2172.6
67.42	2174.0_7	1.27	2172.8
67.42	2174.1	1.41	2172.7
Mean			2172.7
0°C.			
(288.63)	......	0.62	
288.88	507.31	0.62	506.69
288.98	507.12	0.70	506.42
288.98	507.14	0.70	506.44
288.93	507.22	0.70	506.52
Mean			506.5

SOLUBILITY OF IONIZED THALLOUS CHLORIDE.

At 25°, if $\Lambda_{Tl^+} = 78.4$

$\Lambda_{Cl^-} = 75.8$

$$c = \frac{1000\kappa}{\Lambda_{Tl^+} + \Lambda_{Cl^-}} = \frac{2.1727}{154.2} = 0.014094$$

At 0°, $\Lambda_{Tl^+} = 41.8$

$\Lambda_{Cl^-} = 41.3$

$$c = \frac{0.5065}{83.1} = 0.006095$$

TABLE V.

Conductivity of Saturated Thallous Iodide Solution.

R cell	Specific conductivity observed $\kappa \times 10^6$	Water correction $\kappa \times 10^6$	Specific conductivity corrected $\kappa \times 10^6$
25°C.			
796.02	37.30	0.89	36.41
793.46	37.42	0.96	36.46
791.98	37.49	1.08	36.41
792.12	37.48	1.08	36.40
794.20	37.39	0.97	36.42
Mean			36.42
0°C.			
5480.3	5.418	0.414	5.00
5407.4	5.491	0.484	5.01
5484.3	5.414	0.415	5.00
5493.4	5.405	0.484	4.92
5356.7	5.543	0.552	4.99
5353.2	5.546	0.480	5.07
Mean			5.00

Solubility of Ionized Thallous Iodide.

At 25°, $\Lambda_{Tl^+} = 78.36$

$\Lambda_{I^-} = 76.5$

$$c = \frac{0.03642}{154.86} = 0.000235_2$$

At 0°, $\Lambda_{Tl^+} = 41.8$

$\Lambda_{I^-} = 43.4$

$$c = \frac{0.00500}{85.2} = 0.0000587$$

Section 3. The Potential of the Thallium Electrode at 25°C. and at 0°C.

The earliest work on the thallium electrode is probably that of J. Regnauld,[18] who studied the cells: Tl, Tl_2SO_4, $ZnSO_4$, Zn; and Tl, Tl_2SO_4, $CdSO_4$, Cd; and Tl, Tl_2SO_4, $TlHg_x$ (unsaturated thallium amalgam made up from 2.04 grams of Tl and 10 grams of Hg). He found that the metallic thallium is electro-positive to cadmium by an amount equal to 8 times the thermo-electric force between copper and bismuth with the junctions at 0° and 100°C. (or about 57 millivolts). Goodwin [19] studied several cells containing thallous chloride or bromide as a depolarizer with electrodes of saturated thallium amalgam. These results, although showing the behavior of these sparingly soluble salts as depolarizers, do not permit the calculation of the normal potential of thallium against any standard electrodes, and need not be discussed in detail here.

The first measurements of the thallium electrode against the normal calomel electrode are those of Neumann [20] in 1894, who measured both metallic and amalgamated electrodes in saturated solutions of thallous sulphate, thallous chloride, and thallous nitrate against the calomel electrode. For the combination in which we are specially interested, namely, the cell: Tl, TlCl sat., 1.N KCl, Hg_2Cl_2, Hg; Neumann obtained the value +0.711 volt at 17°.[21] No special precautions to prevent oxidation were taken. Lewis and von Ende [22] have shown that when his results are plotted against the logarithm of the ion concentration they show no regularity and are, therefore, apparently subject to some fortuitous error.

Abegg and Spencer [23] studied the effect of the thallous ion concentration on the potential of the thallium electrode, but their values like those of Neumann show marked deviations from the requirements of the Nernst equation. They used amalgamated thallium electrodes on platinum points. They found for $TlHg_x$, TlCl sat., 0.1 N KCl, Hg_2Cl_2, Hg; E = +0.7752 at 25°. Later work has shown this value to be too low.

Spencer [24] has measured the potential of a series of thallium amalgams, varying in composition from 0.001831% thallium up to 55.68% thallium, against metallic thallium at 18°C. For all of the saturated

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18 J. Regnauld, Compt. rend., **64**, 611 (1867).
19 H. M. Goodwin, Z. physik, Chem., **13**, 577 (1894).
20 B. Neumann, Z. physik. Chem., **14**, 219 (1894).
21 Throughout this paper a positive sign of the potential indicates that the positive current flows through the cell as written from left to right.
22 G. N. Lewis and C. L. von Ende, Jour. Amer. Chem. Soc., **32**, 732 (1910).
23 R. Abegg and J. F. Spencer, Z. anorg. Chem., **46**, 408 (1905).
24 J. F. Spencer, Z. Elektrochemie, **11**, 681 (1905).

amalgams (varying from 44.6% to 55.68% Tl) he finds the amalgam 0.0008 volt more positive than the metal itself.

Shukoff [25] measured (incidentally to some work on thallic diethyl chloride) the potential of a saturated thallium amalgam in a saturated thallous chloride solution against the normal calomel electrode at 25° and obtained +0.727 volt. This result differs only by about one millivolt from the later measurements discussed below.

Kurnakow and Puschin [26] have investigated the freezing points of a series of thallium amalgams. They find a maximum in the liquidus curve at 15.0°C. with 33.33 atomic per cent. of thallium and two eutectic points at −60°C. with 8.34% thallium and +3.5°C. with 40% thallium. These authors conclude that thallium forms a solid compound, $TlHg_2$. As will be pointed out below, this work of Kurnakow and Puschin has been discredited by later investigators.

Sucheni [27] has measured the potential of a series of thallium amalgams in contact with 0.1 N potassium chloride saturated with thallium chloride, against the decinormal calomel electrode. He was interested principally in unsaturated amalgams but states that saturated two-phase amalgams have the same potential as metallic thallium. Contrary to his expectations based upon Kurnakow's work, he failed to find a break in the concentration-potential curves at the point corresponding to the compound $TlHg_2$. Accepting the existence of the compound $TlHg_2$ as proved by the work of Kurnakow, he concludes that Tl and $TlHg_2$ form no solid solutions but that Hg and $TlHg_2$ form solid solutions. (It will be shown below that this conclusion is incorrect, because based upon the erroneous work of Kurnakow.) Sucheni found the potential of the cell: $TlHg_x$, TlCl sat., 0.1 N KCl, 0.1 N KCl, Hg_2Cl_2, Hg; containing a saturated two-phase amalgam, to be +0.830 volt at 37° and +0.820 volt at 0°. The interpolated value for 25° is +0.827 volt.

Some measurements (unpublished) were made by R. W. Kent (1906) in this laboratory upon the thallium metal and amalgam electrodes in solutions of N/10 thallous sulphate at 25°. His cell consisted of: Tl or $TlHg_x$, 0.1 N Tl_2SO_4, 0.1 N $NaNO_3$, 1. N KCl, Hg_2Cl_2, Hg. His corrected measurements, recorded to millivolts, for the metal electrodes — electrolytically prepared metal was used — varied from 0.685 to 0.690 v., with an average of 0.689 v. The degree of dissociation of thallous sulphate is here 61.5% (data of Hunt [28]), from

25 J. Shukoff, Ber., **38**, 2691 (1905).
26 N. S. Kurnakow and N. A. Puschin, Z. anorg. Chem., **30**, 86 (1902).
27 A. Sucheni, Z. Elektrochemie, **12**, 726 (1906).
28 F. L. Hunt, Jour. Amer. Chem. Soc., **33**, 795 (1911).

which the normal potential of thallium is calculated to be 0.617 volt. The average value for the amalgam electrode was 0.690 volt, corresponding to a normal potential of 0.618 v. The variations in potential in Kent's measurements were, however, of the order of millivolts; and, furthermore, no especial precautions appear to have been taken to avoid oxidation.

Brislee,[29] working at room temperature, approximately 17°, but without a thermostat, measured the potential of thallium metal bars in solutions of thallous chloride, thallous nitrate, and thallous hydroxide, separated from the tenth-normal calomel electrode by saturated solutions of ammonium nitrate. He bubbled hydrogen through the solution about the thallium electrodes during measurements. For the cell; Tl, 0.01 N TlCl, NH_4NO_3 sat., 0.1 N KCl, Hg_2Cl_2, Hg; he obtained 0.773 v. This corresponds to a normal potential of 0.602 volt. Using the temperature coefficient of the normal potential of the thallium electrode, as determined by our experiments, the interpolated value for the normal potential at 25°, according to Brislee, would be 0.612 v.

Much more important than any of the work referred to above is the investigation of G. N. Lewis and C. L. von Ende.[30] First, they investigated the influence of changing concentration of thallous ion on the potential of an unsaturated thallium amalgam. They found that, when adequate precautions were taken to prevent solution of thallium from the amalgam by atmospheric oxidation, the observed potentials, in the case of dilute solutions (0.002 N to 0.0333 N), are in almost perfect accord with the Nernst equation. However, with 0.1 N thallous nitrate solution they find a deviation of about one and one-half millivolts, as is to be expected by analogy with many other similar cases. In other words, in dilute solutions the electromotive "activity" of the ions is the same as its concentration as computed from the conductivity. "Instead then of finding any anomaly in the electromotive behavior of thallous ions, we may assert that the Nernst equation holds for the thallium electrode over a large range of concentration with greater accuracy than it has as yet been shown to do in the case of any other electrode." The conflicting observations of Neumann and of Abegg and Spencer are ascribed to insufficient experimental precautions to prevent oxidation. Lewis and von Ende then measured the normal potential of the thallium electrode. They say, "In order to determine the absolute electrode potential of

29 F. J. Brislee, Trans. Faraday Soc., **4**, 159 (1909).
30 G. N. Lewis and C. L. von Ende, Jour. Amer. Chem. Soc., **32**, 732 (1910).

thallium, we must use an electrode of pure metallic thallium or some electrode that is its equivalent. It was shown by Kurnakov and Puschkin [31] [*sic*] that thallium and mercury form no solid compound above 15°. Sucheni [32] showed that mercury does not dissolve to any extent in solid thallium, and that a saturated solution of thallium in mercury therefore has the same potential as pure thallium. This fact, which we have also corroborated,[33] enables us to use a paste of thallium and thallium amalgam in place of solid thallium, which like all solid metals gives an inconstant potential owing to uncontrollable surface variations." They found the potential of a saturated two-phase amalgam in contact with a saturated thallous chloride solution measured against the normal calomel electrode to be 0.7257 volt at 25°C. The normal potential of the thallium electrode is computed to be 0.6170 volt.

However, since this paper of Lewis and von Ende was published, the work of Kurnakow and Puschin, on which they relied, has been discredited by three independent investigations. P. Pavlovich, [34] in a repetition of the work of Kurnakow and Puschin, finds, as did the earlier workers, a eutectic at —60°, corresponding to 8 at. per cent. thallium; another eutectic at +2°, with 40% thallium; but his dystectic occurs at 14.8°, covering a range of from 28.7 to 29.7 at. per cent. thallium. Pavlovich postulates the existence of the compound $TlHg_3$, and notes that solid solutions separate between the limits 21 to 31% Tl, as well as between 86–100 at. per cent. Tl.

G. D. Roos [35] has also repeated the work of Kurnakow and Puschin on the freezing points of thallium amalgams and obtains results of a quite different character. He finds that the solid compound of thallium and mercury has the formula Tl_2Hg_5 instead of $TlHg_2$. This compound forms solid solutions with both excess of mercury and of thallium within the limits 20% Tl and 31.3% [36] Tl. Of more importance for the present purpose, however, is the proof by measurements of eutectic halts in the cooling curves, that metallic thallium forms solid solutions with mercury, which may contain as much as 18% of mercury at the eutectic temperature, which was found to be +0.6°C. instead of 3.5°C. as observed by Kurnakow and Puschin. The work

31 N. S. Kurnakow and N. A. Puschin, Z. anorg. Chem., **30**, 86 (1902).
32 A. Sucheni, Z. Elektrochemie, **12**, 726 (1906).
33 No details of this corroboration are given.
34 P. Pavlovich, Jour. Russ. Phys. Chem. Soc., **47**, 29 (1915); Bull. Soc. Chim., **20**, 2 (1916).
35 G. D. Roos, Z. anorg. Chem., **94**, 369 (1916).
36 *Atomic* percentages; in the present case nearly equal to percentages *by weight*.

of Roos shows that the two phases present in the eutectic mixture are Tl_2Hg_5, containing sufficient excess thallium in solid solution to bring its composition up to 31.3% Tl, and solid thallium, containing 18% of mercury in solid solution.

Richards and Daniels,[37] in the course of an extensive investigation of the thermodynamic properties of unsaturated thallium amalgams, have also measured the freezing points and electromotive force of a series of thallium amalgams, and their results are entirely in accord with the later independent work of Roos, but not in accord with those of Kurnakow and Puschin. Although Richards and Daniels did not measure the cooling curves of these amalgams and therefore do not confirm this part of Roos' work directly, they have shown by direct comparison that the electromotive force of metallic thallium is not the same as the electromotive force of a two-phase thallium amalgam. At 20°, Richards and Daniels find the potential of metallic thallium to be 2.49 millivolts more negative than the potential of the two-phase amalgam.

Sucheni's results, on which Lewis and von Ende relied in part, do not justify the conclusions which Sucheni drew from them. He measured the potential of a series of thallium amalgams in contact with a 0.1 N potassium chloride solution saturated with thallium chloride against the decinormal calomel electrode. "The potential values, which are given later, represent the mean of several series in which measurements were made through the entire series of amalgams from 0% Tl to 100% Tl. The individual values for the same thallium content differ from each other in the different series by from 0.001 to 0.004 volt. Up to 100% Tl the value remains constant at 0.830," (i.e., from a Tl content of 49.17%, which is shown in his table of results — for 37°C — to have a potential of 0.830 volt, whereas an amalgam of 43.59% and all weaker amalgams are shown to have a lower potential). Therefore his statement that the potential of thallium metal and that of a two-phase amalgam at 37°C. agree, means only that they are within four millivolts of each other. "The electromotive force, as one perceives from the measurements, remains constant from the point where the potential curve becomes horizontal up to pure thallium. From this it follows that in all these amalgams which contain over 43 atomic per cent. of thallium, thallium is present as an independent phase and that thallium does not appreciably dissolve the compound $TlHg_2$."[38]

37 T. W. Richards and F. Daniels, Jour. Amer. Chem. Soc., **41**, 1765 (1919). This work was completed in 1914, but the publication was delayed by the war.

38 A. Sucheni, Z. Elektrochemie, **12**, 729 (1906).

In his measurements at 0°C. he finds a constant potential of 0.820 to 0.8195 volt from 49.66% thallium down to amalgams containing 31.81% thallium, whereas an amalgam containing 28.44% thallium shows a potential of 0.816 volt followed by a rapid drop to 0.782 volt for a solution containing 21.22% thallium.

In the interpretation of these results his reasoning is based on the premise that the compound $TlHg_2$ exists (as indicated by the results of Kurnakow and Puschin). The absence of any break in the concentration-potential curve at $33\frac{1}{3}\%$ thallium (corresponding to $TlHg_2$) he regards as proof that all mixtures containing more than $33\frac{1}{3}\%$ thallium at 0° are mixtures of pure $TlHg_2$ and pure thallium and that no mixed crystals are produced, whereas $TlHg_2$ and Hg form solid solutions in all proportions. However, since his premise based upon Kurnakow's result has been shown to be in error, this reasoning has no force. His experimental results are, however, entirely in accord with the data of Roos and of Richards and Daniels. According to these experimenters the solid compound is Tl_2Hg_5 (28.57% Tl) which can (according to Roos) form solid solutions with thallium up to the limit 31.3% thallium. Similarly solid thallium can form solid solutions with mercury containing up to 18% mercury or 82% thallium. The eutectic mixture which freezes at +0.6°C. would therefore consist of a mixture of two solid solutions containing respectively 31.3% and 82% of thallium. Any amalgam whose total analytical composition fell between these limits would at 0° be made up of these same two phases in proportions which depend upon the composition and would, therefore, be expected to show a constant potential. On the other hand any amalgam containing less than 31.3% thallium but more than 20% thallium would be a single solid phase which would have a potential dependent on the composition. The experimental results of Sucheni are thus entirely in accord with the data and interpretation of Roos, though not in accord with his own interpretation thereof.

This argument will be made clear by an inspection of Figure 2. In the uppermost curves are plotted the results of Kurnakow and Puschin, of Roos, and of Richards and Daniels, on the freezing-points of thallium amalgams. It is very plain that the results of the first mentioned investigators are seriously at variance with those of the more recent workers, and that the wholly independent determinations of Roos and of Richards and Daniels agree very closely over the range covered by the latter.

The curve of eutectic halts, plotted from the results of Roos, shows two intervals in which the halt is vanishingly small; indicating the existence of solid solutions, formed, on the one hand, by Tl_2Hg_5 with both excess of thallium and excess of mercury, and on the other hand by thallium and mercury.

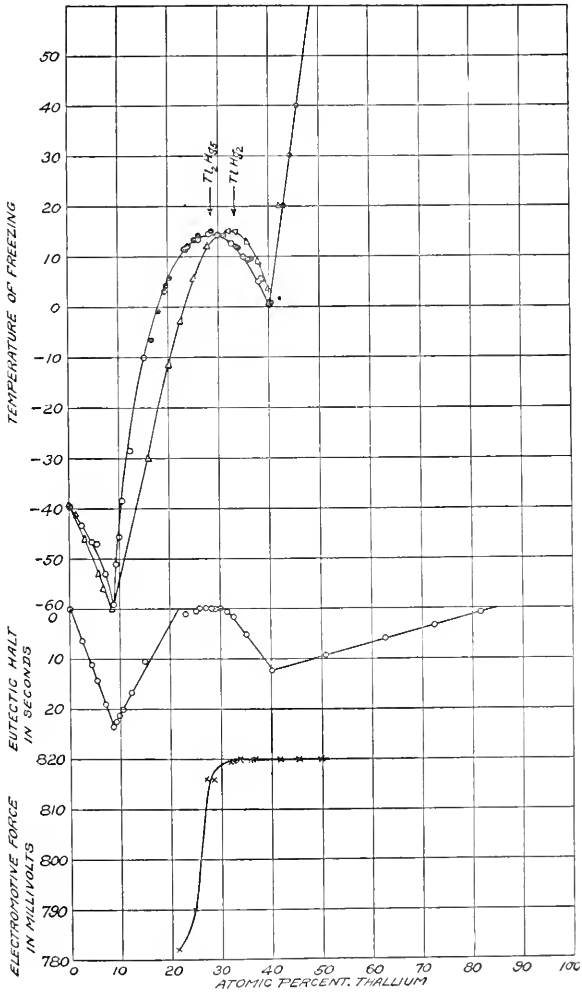

FIGURE 2. Triangles indicate values of Kurnakow and Puschin; black circles, those of Richards and Daniels; white circles, those of Roos; crosses, those of Sucheni.

The lowest curve shows the results of Sucheni on the potential of thallium amalgam electrodes at 0° (measured against the decinormal calomel electrode). The sharp break in the curve occurs very close to the point on the horizontal axis indicated by the composition of the compound, Tl_2Hg_5. The evidence cited strongly supports the following conclusions: (a) mercury and thallium form a solid compound Tl_2Hg_5 (not $TlHg_2$); (b) this compound can dissolve excess of either mercury or thallium within definite limits; (c) solid thallium forms a solid solution with mercury up to a limit roughly indicated to be 18% of mercury; (d) the eutectic mixture which is stable at 0°C. cannot contain pure solid thallium as one phase; (e) a two-phase solid amalgam would not be expected to have the same potential as pure metallic thallium.

In our investigation we determined to avoid the use of amalgam electrodes, if metallic electrodes could be obtained which would be constant, reproducible, and reversible. Although the metallic electrodes first prepared did not meet these tests satisfactorily, the difficulty was traced to atmospheric oxidation. After apparatus was developed which permitted the rigid exclusion of atmospheric oxygen from all contact with electrodes or solution the metallic electrodes proved to be satisfactory. Many of them were measured at intervals for several days and did not vary more than 0.2 millivolt. Each of the measurements recorded below refer to an independent experiment. Between each experiment the cells were completely dismantled and set up again with fresh thallium electrodes, new solutions, and new calomel electrodes. As will be seen, variations of more than 0.2 millivolt are rare. Most of the cells contained two thallium electrodes side by side in the same solution. This arrangement made it possible to pass a small current between the two electrodes, thus polarizing one of them cathodically and the other anodically, thereby causing a difference of potential of several millivolts between them. This polarization, however, disappeared completely on standing or on rinsing the electrodes with a fresh solution, thus showing the electrodes to be reversible. In our first series of experiments no amalgam electrodes were used but two metallic thallium electrodes in contact with a saturated solution of thallous chloride were measured against a tenth-normal calomel electrode. These cells gave a value for the normal potential 2.6 millivolts higher than the result of Lewis and von Ende. We, therefore, determined to make a direct comparison of a metallic electrode and an amalgam electrode prepared according to the method of Lewis and von Ende. As will be seen in the table below the metallic electrode at 25°C. proved to be 2.8 millivolts more negative than the saturated amalgam electrode in the same solution. At 0°C. the difference was 1.8 millivolts in a single measurement. Since the amalgam electrode was discredited by the measurements at 25°C. we were content with a single measurement at 0°C. It may be pointed out that these results are quite in accord with the measurements of Richards and Daniels, who found a difference of 2.49 millivolts at 20°C.

Our value for the potential of the two-phase thallium amalgam electrode at 25°C. is 0.7762 volt, or 0.5 millivolt higher than that of Lewis and von Ende. This difference may perhaps be due to a more

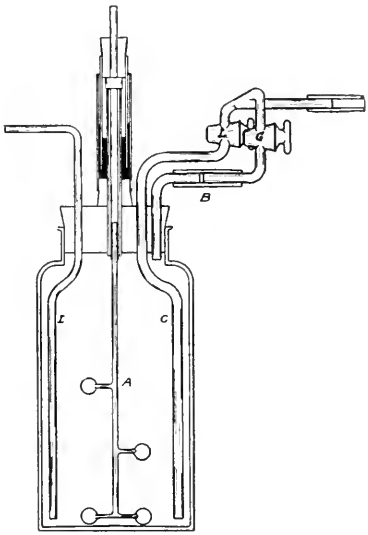

FIGURE 3.

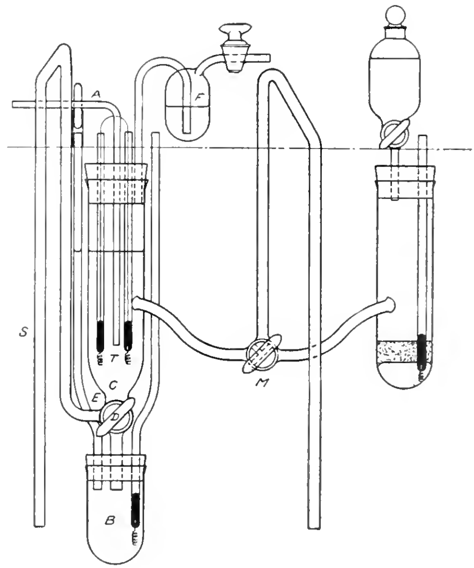

FIGURE 4.

complete exclusion of oxygen in our experiments, or a difference in the purity of the thallium. Our metal was prepared electrolytically, from carefully purified thallous nitrate, whereas Lewis and von Ende used the purest metal they could buy.

Experimental Details.— For the measurement of the potential of thallium against the calomel electrode the apparatus shown in Figures 3 and 4 was devised. The bottle in Figure 3 was employed for the preparation of a saturated solution of thallous chloride at a definite temperature, in an atmosphere of nitrogen. A wide-mouthed bottle holding about 800 cc., was provided with a rubber stopper, perforated by four holes. Through the central hole passed a glass bearing, containing a closely-fitting brass tube, into which was affixed a glass stirrer (A), which was rotated by a small motor, the vertical shaft of which was attached to the stirrer by several inches of flexible, stout rubber tubing. The bearing was made gas-tight by means of a mercury seal. Through a second hole in the stopper passed an inlet tube for nitrogen gas. Two other tubes, serving respectively for the outlet of the gas or of the solution, passed through the remaining holes of the stopper, and were combined through a T-joint into a single tube, which entered the cell itself. This bottle was completely immersed in a thermostat at 25.00° during the saturation, and pure atmospheric nitrogen gas was permitted to bubble through the solution during this process. For the work at 0°, the bottle and cell were buried in a large tub of cracked ice and water. The stopper of the bottle was thoroughly coated with paraffine before immersion, to render it water-tight. The stirrer was usually allowed to run over night, while a slow current of nitrogen swept through both the bottle and the cell in order to ensure saturation of the solution and absence of oxygen.

The nitrogen generator was kindly loaned us by Dr. A. T. Larson. The removal of the atmospheric oxygen was accomplished in the cold by cuprous ammonium carbonate solution, which was circulated by means of air-lifts through two towers filled with copper chips and gauze. The removal of oxygen is practically complete after passage of the air through the first column; as shown by the fact that the absorbing liquid in the second column remained practically colorless at all times. This generator would deliver a slow stream of gas for many hours, and was frequently allowed to run over night without any attention.

The cell was of special design, shown in Figure 4. The purpose of the apparatus was to permit the plating out of metallic thallium electrodes from a solution of thallous chloride, in the absence of oxygen, and also to permit the rinsing of the cell without access of air, and the refilling of the electrode vessel with thallous chloride solution of definite concentration, as prepared in the saturation bottle. A is the inlet tube for either nitrogen or the solution. If the three-way cock, D, is properly set, and the stopcock of F is open, both the chamber, B, which is used as the anode chamber during the plating out of the thallium electrodes, T, and C, the cathode chamber in this operation, may be filled at the same time. The arrangement shown avoids the use of a siphon in the plating out process. The thallous chloride solution in B becomes turbid in the vicinity of the platinum anode, but the liquid in C is not contaminated. The siphon-tube, S, is provided so that the cell may be rinsed out with fresh liquid from the saturation bottle without the passage of the liquid through the anode chamber. The rubber stopper of chamber C has four holes, one for the inlet tube, A, one for the outlet tube and trap, F, and two for the thallium electrodes.

The chamber, C, is connected to the calomel electrode chamber by a side-arm, which contains a three-way cock, M. One limb of this cock consists of a siphon-tube, through which, by the adjustment of the stopcock, liquid may be run out from either half of the cell, thus making a fresh junction. The calomel electrode chamber was provided with a small separatory funnel, which served as a reservoir of potassium chloride solution saturated with calomel. The cell when in use was connected to the saturation bottle by means of a short piece of rubber tubing, and could be immersed to the line indicated in the figure, either in the thermostat at 25°, or in a tub of equal size filled with cracked ice and distilled water, for the 0° measurements.

The electrodes themselves consisted of platinum wire, wound into small spirals, and sealed through glass tubes, electrical connection being made as usual with mercury.

Thallous chloride was prepared as already described; see page 206.

Mercury was purified by passing it 20 to 25 times through a five-foot tower of mercurous nitrate solution acidified with dilute nitric acid, followed by redistillation at reduced pressure, allowing a very small current of air to bubble through the liquid during distillation. Part of the mercury thus purified was converted into calomel, by dissolving it in dilute, redistilled nitric acid in the presence of excess mercury, and precipitation of the mercurous chloride by means of dilute, pure hydrochloric acid. The precipitate was washed by shaking up with pure water in a glass-stoppered bottle at least twelve times, decanting the supernatant liquid each time after the precipitate had settled. Then a portion of the solid was placed in a smaller (250–cc.) bottle and the washing continued. The precipitate was now filtered off on a Büchner funnel with the aid of suction, and after a further washing on the funnel, as much of the adhering water as would drain off was allowed to do so, and the moist solid was transferred to a Jena bottle and shaken up several times with approximately 0.1 N potassium chloride solution. Finally it was shaken up with a carefully prepared 0.1 N solution of potassium chloride for about an hour, a few cc. of purified mercury being added to reduce any mercuric chloride present.

Several series of cells were investigated, in the first of which metallic thallium electrodes in saturated thallous chloride solution were measured against the decinormal calomel electrode, at 25° and at 0°; in the second series, the normal calomel electrode was used, at the same temperatures. The difference in potential between the two series was compared with the difference observed between the decinormal and normal calomel electrodes, at the two temperatures. In the third series of cells, one of the two metallic thallium electrodes was replaced by a saturated thallium amalgam electrode, for the purpose of testing the validity of the assumption that the two have identical potentials under the same conditions.

The measurements were made on a Wolff potentiometer of 20,000 ohms resistance, on the dials of which the e.m.f. could be read off directly to 0.00001 v. The galvanometer was a sensitive instrument of Leeds & Northrup manufacture (type H), and the standard Weston cell had been certified by the Bureau of Standards. The potentiometer was occasionally tested for parasitic and thermoelectric disturbances.

The filling of the calomel chamber involved no new principles. The calomel layer was at least 1 cm. high, and the potassium chloride solution was shaken up at the temperature of the experiment with a calomel-mercury paste for about an hour previous to the filling of the cell. The thallium side of the cell was filled as follows. Referring to Figures 3 and 4: the cock, G, (Fig. 3) was closed and L opened. The pressure of nitrogen thus forced over some of the thallous chloride solution from the saturation bottle into the cell; and if the vent, E, cock, D, and trap, F, (Fig. 4) were opened, the liquid would fill both the lower and upper compartments, B and C, to the desired height — a little below the side-arm. The cock, L, was now closed and G opened, whereupon nitrogen bubbled through both the saturation bottle and the cell, escaping through trap F into the air. The two well-cleaned and ignited platinum electrodes, T, were now connected in parallel to the negative pole of a 16-volt storage battery. The positive lead was introduced into the long tube, through the bottom of which was sealed the platinum anode in chamber B. The stopcock, D, (Fig. 4) was adjusted so that the desired current — as registered by a milliammeter — was obtained for plating out the thallium electrodes. This plating out was continued for a shorter or longer period, according to the strength of the current. Frequently a fraction of a milliampere was employed and the process allowed to run all night, with a stream of nitrogen passing through the apparatus. When the desired coating had been obtained, the electrolysis was stopped, and stopcock, D, turned so as to shut off communication between chambers B and C. If desired, the anode chamber, B, with its stopper and tubes, could be detached from the rest of the apparatus at this point; but this was not essential.

The rinsing of the thallium electrode chamber was effected by alternately running out the liquid in C through siphon, S, and refilling from the saturation bottle. This necessitated the proper manipulation of the stopcocks, G and L (Fig. 3), and D and F (Fig. 4). When the chamber, C, had been rinsed sufficiently (about six or seven times), it was finally filled almost completely full, together with the side-arm and the bore of the stopcock, M. The cell was now ready for measurement. The test applied to determine whether chamber C had been sufficiently rinsed was to note whether successive rinsings caused any change in the e. m. f. of the cell. An indication of the completeness of removal of oxygen from the chamber was the constancy of e. m. f. after nitrogen had been permitted to bubble through the cell for a short time. The difference between any two thallium electrodes in the same cell rarely exceeded a tenth of a millivolt, and usually was not greater than a few hundredths. This indicates that, when carefully prepared, the metallic thallium electrode is both constant and reproducible.

The amalgam electrodes were of two designs, one of which was described by Sucheni.[39] This consisted of a long glass tube, drawn down to a point and sealed at the lower end, and partly filled with a saturated thallium amalgam under water (or paraffine). A long platinum wire was embedded in the amalgam at one end, and sealed through the top of the glass tube at the other. When the electrode was ready for measurement, the tip was broken off and the amalgam exposed. The other form consisted of a J-shaped tube with a small cup at its lower end, containing the amalgam (Fig. 5). This form of electrode permitted more thorough rinsing of the surface of the amalgam. A short platinum wire was sealed through the bottom of the cup in order to make electrical connection between the amalgam and the mercury in the longer

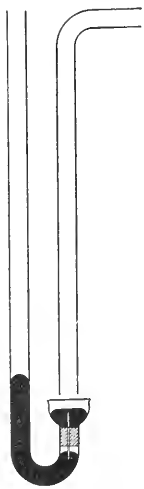

FIGURE 5.

39 A. Sucheni, Zeit. f. Elektrochemie, **12**, 726 (1906).

limb of the electrode. The amalgams used contained approximately 55% of thallium. They were heated under water until liquid and then allowed to cool, forming a two-phase amalgam.

The results of these measurements are tabulated below. Each thallium electrode and each calomel electrode were prepared separately, the cell being entirely dismantled between each two determinations. The thallous chloride solutions were saturated anew each time, and fresh materials were also used in making up each calomel electrode; so that the values obtained were entirely independent.

In the work at 0°, the concentrations of the potassium chloride solutions used were adjusted for the change in temperature. The saturation of the thallous chloride solutions at 0° was attained from above as well as below the solubility of the salt at that temperature.

The check measurements at 0° upon the normal and decinormal electrodes were carried out in a cell of the type shown below in Figure 6. The 25° value has been fixed with accuracy by a number of experimenters.[40]

In the following tables each line gives the measurements on an independent cell, the cells being completely dismantled each time and set up again with new electrodes and fresh solutions. The two measurements on each line refer to the two different thallium or thallium amalgam electrodes in each cell measured against the same calomel electrode. The figures given are actual measured potentials without any corrections, the calomel electrode being always positive.

From this table the normal potential of the thallium electrode is calculated to have the following values, which refer to the metal electrode only:

$$\left.\begin{array}{l}\text{At } 25°\text{: } E_n = +0.6188 \text{ v.}\\ \text{At } \ 0°\text{: } E_n = +0.5885\end{array}\right\}\text{against N/1 Electrode.}$$

Lewis, Brighton, and Sebastian [41] in a study of the calomel electrode conclude that the value of the electromotive force between the tenth-normal and the normal calomel electrode at 25° is −0.0529 volt. In

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40 This enables us to determine the temperature coefficient of the potential difference between the decinormal and normal calomel electrodes over the range 0°–25°; assuming the temperature coefficient to be independent of the temperature in this interval we have: $\frac{-0.0533-(-0.0489)}{25} = -0.00018$ volt per degree. (See p. 225.)

41 G. N. Lewis, T. B. Brighton and R. L. Sebastian, Jour. Amer. Chem. Soc., **39**, 2245 (1917).

TABLE VI.

E. M. F. of Cell: Tl, TlCl sat., 0.1 N KCl, Hg_2Cl_2, Hg.

25°C.

	(0.7825)	(0.7825)	rejected in computing mean
	0.7822	0.7822	
	0.7820	0.7820	
	0.7820	0.7821	
	0.7821	0.7821	
Mean	0.7821 volt		

0°C.

	0.7583	0.7583
	0.7583	0.7582
	0.7582	0.7582
Mean	0.75825 volt	

TABLE VII.

E. M. F. of Cell: Tl, TlCl sat., 1.0 N KCl, Hg_2Cl_2, Hg and $TlHg_x$ (sat. amalgam), TlCl sat., 1.0 N KCl, Hg_2Cl_2, Hg.

25°C.

	Tl Amalg.	Tl Metal
	0.7264	0.7289
	0.7261	0.7290
	0.7257	0.7290
	0.7262	0.7289
	0.7264	0.7291
Mean	0.7262 volt	0.7290 volt

0°C.

	Tl. Amalg.	Tl Metal		
	0.7084		0.7102	
		(0.7107)	(0.7109)	rejected in computing mean
		0.7102	0.7102	
		0.7102	0.7102	
		0.7102	0.7102	
Mean	0.7084 volt	Mean 0.7102 volt		

TABLE VIII.

E. M. F. of Cell: Hg, Hg_2Cl_2, 0.1 N KCl, 1.0 N KCl, Hg_2Cl_2, Hg.

	0°C.
	−0.0482
	−0.0480
	−0.0481
	−0.04805
Mean	−0.0481

SUMMARY.

TABLE IX.

Temp.	Cell Measured	No. of determin's	Average deviation from mean (volt)	Mean obs'd E. M. F. (volt)	L. J. Potent. calc. (volt)	E. M. F. corr'd (volt)
25°	Tl...TlCl...0.1 N.E. (sat.)	8	.00006	0.7821	.0005	0.7816
	Tl...TlCl...1. N.E. (sat.)	5	.00006	0.7290	.0007	0.7283
	Difference:					0.0533
	0.1 N.E....1. N.E.[42]			0.0529	.0004	0.0533
0°	Tl...TlCl...0.1 N.E. (sat.)	6	.00005	0.75825	.0010	0.75725
	Tl...TlCl...1. N.E. (sat.)	7	.00006	0.7102	.0017	0.7085
	Difference:					0.04875
	0.1 N.E....1. N.E.	4	.00006	0.0481	.0008	0.0489
25°	Tl Hg_x..TlCl...1. N.E. (sat.)	5	.00020	0.7262	.0007	0.7255
0°	Tl Hg_x..TlCl...1. N.E.	1	———	0.7084	.0017	0.7067

42 The value, 0.0529 volt, is taken from G. N. Lewis, T. B. Brighton and R. L. Sebastian, Jour. Amer. Chem. Soc., **39**, 2245 (1917).

1914, Lewis and Randall[43] state that numerous investigations in their laboratory had fixed the value of this e. m. f. as −0.0530 v. at the same temperature. This value is uncorrected for the liquid junction potential. While the potential between these two liquids cannot be calculated with accuracy, still, inasmuch as it is not very large, the data at hand for its calculation should be good enough to furnish us with a good check upon our measurements. Calculation according to the Henderson equation gives the value 0.0004− volt at 25° and 0.0008− volt at 0°, for this potential. The corrected potential between the two calomel electrodes at 25° thus becomes 0.0533− v. It will be seen that the difference between the first two cells in Table IX. is exactly the same: 0.0533 v.

Slade[44] also measured the potential of the normal calomel electrode against the decinormal electrode. Three electrodes of each kind were made up according to the directions given in the handbook by Ostwald-Luther, and measured against one another from time to time. The observed value at 25° was −0.0534 ± 0.0002 volt.

At 0°, we measured by independent experiment the potential between the two calomel electrodes, the normal and decinormal, in a bath of cracked ice and water. The mean of four determinations gave as the value for this combination (see Table VIII) at 0°: −0.0481 volt; so that the corrected value becomes −0.0489 volt, when we take into account the liquid junction as above mentioned. The corrected difference between our two cells at 0° is seen to be 0.0488 volt.

A further check upon the values given in the last column of Table IX is obtained from the following considerations. Richards[45] found the temperature coefficients of the normal and tenth-normal calomel electrodes to be these:

	Calc.	Found	Diff.
N/1	.00057	+.00061	+.00004
N/10	.00075	+.00079	+.00004

The difference between the temperature coefficients of the N/10 and N/1 electrodes is seen from these figures to be .00079 − .00061 = .00018 volt. This value should be equal to the temperature coefficient of the potential difference observed between the two electrodes as seen in the last column of Table IX, and in turn equal to the temperature coefficient of the difference between the two pairs of cells measured at 25° and 0°. In each case the observed difference is .0045 v.; the

43 G. N. Lewis and M. Randall, Jour. Amer. Chem. Soc., **36**, 1974 (1914).
44 R. E. Slade, Jour. Chem. Soc., **99**, 1977 (1911).
45 T. W. Richards, Proc. Amer. Acad., **33**, 1 (1897).

temperature coefficient of the difference is therefore $-.0045/25 = -.00018$ volt per degree.

Our experiments, therefore, corroborate the estimate made by Lewis and Randall in 1914: —

"The temperature coefficient of the e. m. f. between the decinormal and normal calomel electrodes should be determined more accurately. At present we can obtain only an approximate result as follows: The e. m. f. at 25° we have shown to be −0.0530 v., and at 18°, Sauer has found for the same combination −0.0514. Hence, for the temperature coefficient we find −0.00023. If, instead of using Sauer's result, we take 0.0530 at 25°, and assume the e. m. f. of the combination to be proportional to the absolute temperature, then we find −0.00018 for the coefficient. For want of better information we shall average these two values and assume the temperature coefficient to be independent of the temperature."

Following is a summary of values of the normal potential of thallium, including only those results with which the values obtained in the present research may fairly be compared. Starred values are calculated results based on determinations under somewhat different experimental conditions:

TABLE X.

Year	Name	Temp. °C.	Cell	Obs. E.M.F.	E_n 25°	E_n 0°
1894	Neumann	17	Tl, TlCl, ind. salt, N/1 cal.	.711*	.615	
1905	Abegg & Spencer	25	$TlHg_x$, TlCl, N/10 cal.	.7752*	.613	
1905	Shukoff	25	$TlHg_x$, TlCl N/1 cal.	.727	.618	
1906	Sucheni	0	$TlHg_x$, TlCl, KCl (N/10), N/10 cal.	.820*		.588
1906	Kent	25	Tl, Tl_2SO_4, $NaNO_3$, N/1 cal.	.689	.617–8	
1910	Brislee	17	Tl, (.01N)TlCl, NH_4NO_3, N/10 cal.	.773*	.612	
1910	Lewis & v. Ende	25	$TlHg_x$, TlCl, N/1 cal.	.7257	.6170	
1920	Present paper	25	Tl, TlCl, N/1 cal.	.7283	.6188	
		0		.7085		.5885

Section 4. The Potential of the Iodine Electrode at 25°C. and 0°C.

The normal potential of the iodine electrode is the only remaining quantity required for the calculation of the free energy of formation of thallous iodide. An examination of the existing literature shows that apparently trustworthy measurements of this quantity are available at 25°C., but we were unable to find any measurements at 0°C.

The early potential measurements of Beetz, Doat, Regnauld, Peirce, Laurie, Bancroft, and Smale[46] on various cells containing an iodine electrode need not be discussed in detail, since they either do not permit the computation of the normal potential or when judged by modern standards are very crude and inaccurate. In this early work the influence of ion concentration on electrode potential was not sufficiently appreciated. Moreover, these earlier measurements were carried out before it had been shown by Le Blanc and Noyes[47] by means of freezing point measurements that iodine dissolves in iodide solutions with the formation of some complex ion, and before Jakovkin[48] had demonstrated by distribution measurements that the complex ion is a triiodide ion (I_3^-). These conclusions of Jakovkin were confirmed and extended by Noyes and Seidensticker,[49] who found that in solutions saturated with iodine at 25°C. the amount of triiodide formed is almost exactly half of the original iodide present or equal to the remaining iodide.

Küster and Crotogino[50] were the first to carry out a systematic study of the potential of the iodine electrode with some degree of accuracy. They measured the potential of a bright platinum electrode immersed in solutions of potassium iodide varying in concentration from 1 N to about 0.001 N containing varying amounts of free iodine against a normal calomel electrode at 25°C.

These measurements were repeated in part by Sammet,[51] who measured the potential of a series of cells containing a platinum electrode immersed in solutions of potassium iodide varying from 1-normal to $\frac{1}{16}$-normal and saturated with iodine at 25° against a normal calomel electrode at 18°C. This procedure involves an un-

46 W. Beetz, Pogg Ann., **90**, 42 (1853).
V. Doat, Compt. rend., **42**, 855 (1856).
J. Regnauld, Compt. rend., **43**, 47 (1856).
B. O. Peirce, Wied. Ann., **8**, 98 (1879).
A. P. Laurie, Phil. Mag., (5) **21**, 409; Jour. Chem. Soc., **49**, 700 (1886).
W. D. Bancroft, Z. physik. Chem., **10**, 387 (1892).
F. J. Smale, Z. physik. Chem., **14**, 577 (1894).

47 M. Le Blanc and A. A. Noyes, Z. physik Chem., **6**, 401 (1890).

48 A. A. Jakovkin, Z. physik. Chem., **13**, 539 (1894); **20**, 19 (1896).

49 A. A. Noyes and J. Seidensticker, Z. physik. Chem., **27**, 357 (1898).

50 F. W. Küster and F. Crotogino, Z. anorg. Chem., **23**, 87 (1900) and F. Crotogino, Z. anorg. Chem., **24**, 247 (1900).

51 V. Sammet, Z. physik. Chem., **53**, 674 (1905).

certain Thomson effect in the solution as well as the temperature effect on the calomel electrode, and is therefore objectionable. Sammet considers that a correction of −0.004 volts should be applied to his results to make them equivalent to direct measurement against a normal calomel electrode at 25°C. When this correction is applied, his results agree with those of Küster and Crotogino within one millivolt. Sammet then calculated the normal potential of the iodine electrode. In making this calculation, he assumed on the basis of the investigations of Le Blanc and Noyes, Jakovkin, and Noyes and Seidensticker already referred to, that in a solution saturated with iodine, one-half of the original iodide is converted into triiodide (the later work of Bray and MacKay shows that these assumptions are only approximately true). He found a fairly constant value for the normal potential in concentrated solutions but a rapid rise in the dilute solutions, and therefore neglected the results obtained with the dilute solutions. The measurements obtained with the stronger solutions gave a value of −0.341 for the normal potential of the iodine electrode, but the more dilute solutions gave higher values rising to −0.363 with the 0.001 N solution.

Maitland [52] measured the potential of cells containing less iodine than sufficient to cause saturation, the amount of free iodine being determined by distribution experiments. His results with 0.1 N KI gave a value for the normal potential of −0.3409, but show a variation with concentration of potassium iodide in the opposite direction to the measurements of Küster and Crotogino. Unfortunately, Maitland's measurements did not extend below 0.1 N potassium iodide.

Lewis and Randall [53] report that the value −0.3407 was found by P. V. Farragher, but no details are given beyond the bare statement that this value was obtained with dilute solutions.

So far as we have been able to find, no measurements of this potential at any other temperature have been published. Since the work described above was published, the equilibrium between iodine and dilute (0.001 N to 0.1 N) potassium iodide solutions has been studied more carefully by Bray and MacKay [54] at 25°C., and later a similar investigation was carried out at 0°C. by Jones and Hartmann.[55] These investigations show that the assumptions made by Sammet in his calculation of the normal potential are only approximately true.

52 W. Maitland, Z. Elektrochem., **12**, 263 (1906).

53 G. N. Lewis and M. Randall, Jour. Amer. Chem. Soc., **36**, 2264 (1914).

54 W. C. Bray and G. M. J. MacKay, Jour. Amer. Chem. Soc., **32**, 914 (1910); and W. C. Bray, *ibid.*, **32**, 932 (1910).

55 Grinnell Jones and M. L. Hartmann, Jour. Amer. Chem. Soc., **37**, 241 (1915).

Although the existing data on the potential of iodine electrode at 25° appeared to be unusually reliable, it was necessary for our purpose to measure this potential at 0°C. Since measurements could also be made at 25°C. with little extra effort and since we were primarily interested in the temperature coefficient of this potential, we determined to make the measurements at both temperatures. The temperature coefficient thus obtained may be expected to be more

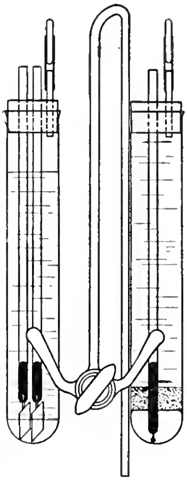

FIGURE 6.

reliable than one deduced by comparison of measurements made by different experimenters, since as far as possible all the conditions except the temperature remain unchanged.

Experimental Details: — The iodine and potassium iodide used were prepared by the method used by Jones and Hartmann.[56] The solutions were saturated with iodine by long continued shaking with excess of solid iodine at the proper temperature before being added to the cells for the potential measurements. The cells used are shown in Figure 6. The three way cock between the two halves of the cell was closed during the filling of the cell, thus preventing mixing, and also facilitating the formation of a fresh junction by removing solution from both halves of the cell through the siphon tube. The platinum electrodes were about 1. × 1.5 cm. in dimension, and were ignited in a blast lamp immediately before being placed in the cells. A considerable excess of solid iodine was used to ensure saturation. Two

56 Grinnell Jones and M. L. Hartmann, *loc. cit.*

electrodes were placed in each cell, thus permitting a test of the reversibility of the electrodes, with results even more satisfactory than in the case of the thallium electrodes. The difference between any pair of electrodes in the same solution in no case exceeded a few hundredths of a millivolt. The measurements were made against a tenth normal calomel electrode instead of a normal electrode, because it was believed that computation of the liquid junction correction would be more reliable than if normal calomel electrodes were used. The following tables show the experimental results:

TABLE XI.

E. M. F. of Cell: Pt, I_2 (solid), 0.1 N KI, 0.1 N KCl, Hg_2Cl_2, Hg.

OBSERVED E. M. F.

25°C.		0°C.	
Pt Elec. No. 1	Pt Elec. No. 2	Pt Elec. No. 1	Pt Elec. No. 2
−0.2804	−0.2804	−0.2755	−0.2755
−0.2803	−0.2803	−0.2752	−0.2752
−0.2805	−0.2805	−0.2753	−0.2753
−0.2805	-0.2805_5	−0.2753	−0.2753
Mean:	−0.2804	Mean:	−0.2753

TABLE XII.

E. M. F. of Cell: Pt, I_2 (solid), 0.05 N KI, 0.1 N KCl, Hg_2Cl_2, Hg.

OBSERVED E. M. F.

25°C.		0°C.	
Pt Elec. No. 1	Pt Elec. No. 2	Pt Elec. No. 1	Pt Elec. No. 2
−0.2975	−0.2975	−0.2911	-0.2911_5
−0.2977	−0.2977	−0.2912	−0.2912
−0.2977	−0.2977	−0.2907	−0.2906
−0.2977	−0.2977	−0.2912	−0.2912
−0.2975	−0.2975	−0.2907	−0.2907
Mean:	−0.2976	−0.2907	−0.2907
		Mean:	−0.2909

TABLE XIII.

E. M. F. of Cell: Pt, I_2 (solid), 0.02 N KI, 0.1 N KCl, Hg_2Cl_2, Hg.

OBSERVED E. M. F.

25°C.		0°C.	
Pt Elec. No. 1	Pt Elec. No. 2	Pt Elec. No. 1	Pt Elec. No. 2
−0.3199	−0.3199	−0.3114	−0.3113
−0.3200	−0.3200	−0.3113	−0.3113
−0.3199	−0.3199	−0.3114	−0.3114
−0.3201	−0.3201−	−0.3114	−0.3114
Mean:	−0.3200	Mean:	−0.3114

TABLE XIV.

Summary of Potential Measurements on Iodine Cells.

Cells Measured: Pt, $\left\{\begin{array}{l}0.1\ N \\ 0.05\ N \\ 0.02\ N\end{array}\right\}$ KI solution saturated with iodine, 0.1 N KCl, Hg_2Cl_2, Hg.

	25°C.			0°C.		
	0.1 N KI	0.05 N KI	0.02 N KI	0.1 N KI	0.05 N KI	0.02 N KI
I^- conc.	0.04267	0.02261	0.00937	0.04498	0.02348	0.00977
I_2 conc.	0.00132	0.00132	0.00132	0.000638	0.000638	0.000638
Observed potential	−0.2804	−0.2976	−0.3200	−0.2753	−0.2909	−0.3114
Liquid junction correction	−0.0031	−0.0022	−0.0015	−0.0024	−0.0019	−0.0015
Concentration correction	−0.0041	+0.0122	+0.0348	−0.0136	+0.0017	+0.0224
0.1 N calomel electrode: 1.0 N calomel electrode	−0.0533	−0.0533	−0.0533	−0.0489	−0.0489	−0.0489
	−0.3409	−0.3409	−0.3400	−0.3402	−0.3400	−0.3394

Mean value for cell;

$$\text{Pt, } I_2 \text{ molal, } I^- \text{ normal, N. E. at} \left\{\begin{array}{l} 25°;\ E = -0.3406 \\ 0°;\ E = -0.3399 \end{array}\right.$$

Or, if we consider cells saturated with solid iodine, we have:

$$\text{Pt, } I_2 \text{ sat., } I^- \text{ normal, N. E.} \left\{\begin{array}{l} \text{at } 25°;\ E = -0.2555 \\ \text{at } 0°;\ E = -0.2525 \end{array}\right.$$

The normal potential of the iodine electrode shown in table XIV was computed from the observed results by applying three corrections:

(1) The correction for the potential at the junction of the potassium iodide and the potassium chloride solution calculated by the Henderson formula,

(2) the correction for the influence of varying concentration of the iodide ion [57] and of the free iodine (I_2) according to the Nernst equation: $+\frac{RT}{2F} \ln \frac{(I^-)^2}{I_2}$, thus giving the hypothetical potential which

57 W. C. Bray and G. M. J. MacKay, Jour. Amer. Chem. Soc., **32**, 914 (1910) and W. C. Bray, **32**, 932 (1910). A small correction has been applied to these results (see Jones' and Hartmann's Table IV, page 251, which gives the figures actually used).

would be observed in a solution in which the concentration of the iodide ion and free iodine is unity, on the assumption that the Nernst equation is valid from the concentration of the measurement up to unity,

(3) a correction for the difference between the tenth-normal and the normal calomel electrode. (See pages 224–225.)

The calculation of the first and of the second corrections requires a knowledge of the concentration and mobilities of the ions present. For the potassium iodide solutions saturated with iodine at 25° this information is supplied by the work of Bray and MacKay[57] and at 0° by the work of Jones and Hartmann.[58] Tenth-normal potassium chloride[59] was assumed to be 85.2% dissociated at 25° and 86.0% at 0°. The mobilities of the ions used in these calculations are as follows:

	25°	0°
K^+	74.8	40.1
Cl^-	75.8	41.3
I^-	76.5	43.4
I_3^-	41.5	22.8

Summarizing the results of four different experimenters for the potential of solid iodine in a solution normal with respect to iodide ion, calculated for 25°, and referred to the normal calomel electrode, we have:

Küster and Crotogino:	−0.256 v.
Sammet:	−0.256
Maitland: (1) avg. of all solns.	−0.2566
(2) avg. of results with N/10 KI only	−0.2560
Present paper:	−0.2555

In addition, the corresponding value at 0° is determined in the present work as −0.2525 volt.

Section 5. The Free Energy of Formation of Thallous Iodide.

By the aid of the data which we have accumulated in the work described in the foregoing pages, we are now able to carry out the calculation of the free energy of formation of thallous iodide at 25° and at 0°.

This quantity, expressed in terms of electromotive force, will be seen to equal the algebraic sum of the e. m. f.'s of the following combinations:

---

58 Grinnell Jones and M. L. Hartmann, Jour. Amer. Chem. Soc., **37**, 250, 251, 253, 255 (1915).

59 A. A. Noyes and K. G. Falk, *ibid.*, **34**, 468 (1912).

(1) Tl, TlI sat., TlCl sat., Tl
(2) Tl, TlCl, sat., 0.1 N KCl, Hg_2Cl_2, Hg
(3) Hg, Hg_2Cl_2, 0.1 N KCl, x N KI, I_2 sat., Pt
(4) Pt, I_2 sat., x N KI, TlI sat., I_2 sat., Pt

Adding: Tl, TlI sat., I_2 sat., Pt

The potential (1) is calculated by the Nernst equation from the measured concentration of the thallous ion in thallous chloride and in thallous iodide.

The potentials (2) and (3) are the directly measured values after applying the liquid junction corrections. The potential (4) is calculated by the Nernst equation from the known concentration of the iodide ion in saturated thallous iodide and in the potassium iodide solutions saturated with iodine.

The free energy of the reaction Tl (solid) + 1/2 I_2 (solid) = TlI (solid) is evidently F (= 96,500 coulombs) multiplied by the sum of these four potentials.

TABLE XV.

25°C.

	volts.		
(1) Tl, TlI sat., TlCl sat., Tl			
$E = \frac{RT}{F} \ln \frac{0.01409}{0.000235} =$	+0.1051	0.1051	0.1051
(2) Tl, TlCl sat., 0.1 N KCl, Hg_2Cl_2, Hg	+0.7816	0.7816	0.7816
(3) Hg, Hg_2Cl_2, 0.1 N KCl, 0.1 N KI, I_2 sat., Pt =	+0.2835		
Hg, Hg_2Cl_2, 0.1 N KCl, 0.05 N KI, I_2 sat., Pt =		0.2998	
Hg, Hg_2Cl_2, 0.1 N KCl, 0.02 N KI, I_2 sat., Pt =			0.3215
(4) Pt, I_2 sat., 0.1 N KI, TlI sat., I_2, Pt			
$\frac{RT}{F} \ln \frac{0.04267}{0.000235} =$	+0.1336		
	1.3038		
do., (0.05 N KI) $\frac{RT}{F} \ln \frac{0.02261}{0.000235} =$		0.1173	
		1.3038	
do., (0.02 N KI) $\frac{RT}{F} \ln \frac{0.00937}{0.000235} =$			0.0947
			1.3029
MEAN		1.3035	

TABLE XVI.

0°C.

	volts.		
(1) Tl, TlI sat., TlCl sat., Tl $E = \frac{RT}{F} \ln \frac{0.006095}{0.0000587}$	0.1093	0.1093	0.1093
(2) Tl, TlCl sat., 0.1 N KCl, Hg_2Cl_2, Hg	0.7573	0.7573	0.7573
(3) Hg, Hg_2Cl_2, 0.1 N KCl, 0.1 N KI, I_2 sat., Pt	0.2777		
Hg, Hg_2Cl_2, 0.1 N KCl, 0.05 N KI, I_2 sat., Pt		0.2928	
Hg, Hg_2Cl_2, 0.1 N KCl, 0.02 N KI, I_2 sat., Pt			0.3129
(4) Pt, I_2 sat., 0.1 N KI, TlI sat., I_2, Pt $\frac{RT}{F} \ln \frac{0.04498}{0.0000587}$	0.1563 1.3006		
do., (0.05 N KI) $\frac{RT}{F} \ln \frac{0.02348}{0.0000587}$		0.1410 1.3004	
do., (0.02 N KI) $\frac{RT}{F} \ln \frac{0.00977}{0.0000587}$			0.1204 1.2999
MEAN	1.3003		

The free energy (A) of the reaction: Tl (solid) + 1/2 I_2 (solid) = TlI (solid), is obtained from these potentials by multiplying by F (96,500 coulombs); and the heat of formation may be calculated by the Helmholtz equation. The results may be summarized as follows:

	25°	0°
Electromotive force (E)	1.3035 volts	1.3003 volts
$\frac{dE}{dT} =$	+0.000128 volts per degree	
Free Energy of Formation (A)	125.79 kilojoules	125.48 kilojoules
" " " " (A)	30.078 kg. calories	30.005 kg. calories
$\frac{dA}{dT} =$	+12.4 joules per degree	
$\frac{dA}{dT} =$	+2.95 calories per degree	
Heat of Formation (U)	122.11 kilojoules	
" " " (U)	29.198 kg. calories	

Thomsen [60] found 30,180 cals. for the heat of formation of thallous iodide.

Summary.

1. The equivalent conductance of a series of solutions of thallous nitrate from 0.001 N to 0.1 N was measured at 25°C. and at 0°C. By extrapolation to infinite dilution by Noyes' method the equivalent conductance of the thallous ion at 25°C. was found to be 78.36 mhos, and at 0°C., 41.8 mhos.

2. The conductivity of saturated solutions of thallous chloride and of thallous iodide was measured at 25°C. and at 0°C. The equivalent concentrations of the ionized fraction of these salts, calculated from these results, are as follows:

	25°	0°
TlCl	0.014094	0.006095
TlI	0.000235	0.0000587

3. The potential of the metallic thallium electrode in a saturated solution of thallous chloride was measured against the normal and decinormal calomel electrodes at both 25°C. and 0°C. with the following observed results (uncorrected for liquid junctions):

		25°	0°
Tl, TlCl sat., 0.1 N KCl, Hg_2Cl_2, Hg;	E =	+0.7821 v.	+0.7582 v.
Tl, TlCl sat., 1.0 N KCl, Hg_2Cl_2, Hg;	E =	0.7290	0.7102

From these results the normal potential of the thallium electrode is computed to be +0.6188 volt at 25°, and 0.5885 volt at 0°. It was found by direct comparison that a metallic thallium electrode is more negative than a saturated two-phase amalgam electrode by 2.8 millivolts at 25°C. and 1.8 millivolts at 0°C. This is contrary to the assumption commonly made by earlier investigators of this electrode.

4. The decinormal calomel electrode was measured against the normal calomel electrode at 0°C. with the following results:

Hg, Hg_2Cl_2, 0.1 N KCl, 1. N KCl, Hg_2Cl_2, Hg; E = −0.0481 volt; or, corrected for the liquid junction potential, the difference is −0.0489 volt.

5. The potential of an iodine electrode with solutions of 0.1 N,

60 J. Thomsen, Jour. Prak. Chem., **12**, 116 (1875).

0.05 N and 0.02 N potassium iodide, saturated with iodine, was measured against the decinormal calomel electrode at 25° and 0° with the following results (uncorrected for liquid junctions):

	25°	0°
Pt, I_2 sat., 0.1 N KI, 0.1 N KCl, Hg_2Cl_2, Hg; E =	−0.2804 v.	−0.2753 v.
Pt, I_2 sat., 0.05 N KI, 0.1 N KCl, Hg_2Cl_2, Hg; E =	−0.2976	−0.2909
Pt, I_2 sat., 0.02 N KI, 0.1 N KCl, Hg_2Cl_2, Hg; E =	−0.3200	−0.3114

From these results the normal potential of the iodine electrode is computed to be:

	25°	0°
Pt, 1 molal I_2, 1. N I^-, 1.0 N KCl, Hg_2Cl_2, Hg; E =	−0.3406 v.	−0.3399 v.

6. From the foregoing results the following results are computed for the reaction: Tl (solid) + 1/2 I_2 (solid) = TlI (solid):

	25°	0°
Free Energy of Formation (A)	125.79 kj.	125.48 kj.
Heat of Formation (U)	122.11 kj.	

This investigation has been aided by a grant from the Cyrus M. Warren Fund of the American Academy of Arts and Sciences, for which we wish to express our appreciation.

Harvard University,
Cambridge, Mass.

VOLUME 55.

(Continued on page 2 of cover.)

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ANAXIMANDER'S BOOK, THE EARLIEST KNOWN GEOGRAPHICAL TREATISE.

By William Arthur Heidel,

Professor of Greek in Wesleyan University.

ANAXIMANDER'S BOOK, THE EARLIEST KNOWN GEOGRAPHICAL TREATISE.[1]

By William Arthur Heidel.

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Anaximander of Miletus is admittedly one of the foremost figures in the history of thought, and much has been written about him. He remains, however, somewhat enigmatical, and the obscurity which invests his character involves in some measure the entire line of early Milesian thinkers. When one considers the commonly accepted view regarding Anaximander one can hardly escape the impression that it is somewhat incongruous if not wholly inconsistent.

That he wrote a book is not called in question, though one is not sure whether he or Pherecydes is to be credited with writing the earliest prose treatise in Greek. Anaximander is generally called a 'philosopher' and his book is supposed to be properly described by its traditional title *On Nature*. No disposition has been shown to doubt that he was sufficiently interested in geography to prepare a map of the earth, which made a deep and lasting impression, insomuch that it may be said to have fixed the type, preserved with successive enlargements, refinements, and modifications, to be sure, but essentially the same, until in the Alexandrian Age various attempts were made to adjust its outlines in conformity with the newer conception of the earth as a spheroid. Duly considered this admitted fact would of itself suggest the questions whether Anaximander was not a geographer rather than a philosopher, and whether he must not be presumed to have written a geography in addition to drawing a map. When one finds that he is expressly credited with a geographical treatise in the biographical and bibliographical tradition of the Greeks, one asks why this has been called in question. We have, then, first to canvass the evidence regarding his book and his map.

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1 This essay does not aim to discuss the opinions of Anaximander in detail. So far as his 'philosophy' is concerned, I undertook to do that in Class. Philology, **7** (1912), pp. 212–234. A few minor points I should now judge somewhat differently. With regard to Anaximander's geography and sundry other opinions I believe it is possible to ascertain more than has been suspected; but this cannot be done until the position of Hecataeus shall have been more fully and clearly established. On this I have been at work for ten years with results which I hope sometime to publish. Then, *si dis placet*, I may take up Anaximander anew.

I.

The question regarding Anaximander's book cannot be divorced from that concerning his map. This is not equivalent to saying that the existence of either proves the existence of the other; for maps did exist without accompanying texts, presumably before Anaximander's time; and geographical treatises might, and did in fact, exist without maps. In the case of Anaximander, however, quite apart from general considerations, of which more may be said hereafter, the literary evidence for the geographical treatise is so closely interwoven with that for the chart, that it must all be considered together.

Themistius [2] says that Anaximander was 'the first of the Greeks to our knowledge who ventured to publish a treatise *On Nature.*' [3] This statement, except so far as it bears witness to Anaximander's authorship, deserves no credence; for it belongs to a class of data peculiarly untrustworthy. From early times the Greeks amused themselves by investigating the historical beginnings of various activities and contrivances. These studies gave rise in time to treatises *On Inventions,* [4] and undoubtedly contained much information of value; but pronouncements on matters of this sort are obviously relative to the knowledge of the investigator, and, where the author of a dictum and the sources and limitations of his information are alike unknown, we have no right to accept it as truth. In this instance we may at most conclude that Themistius, or rather his unknown source, did not credit Thales with a treatise *On Nature.*

The entry of Suidas runs thus: "Anaximander of Miletus, son of Praxiades, a philosopher; kinsman, disciple and successor to Thales. He first discovered the equinox, the solstices, and dials to tell the hours, and that the earth lies midmost. He introduced the gnomon (sundial) and, speaking generally, set forth the essential outlines of geometry. He wrote *On Nature, Tour of the Earth, On the Fixed Stars, Sphere,* and some other treatises." For the moment we may pass over all but the bibliographical data. [5] The title *On Nature* we have already

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2 Diels, *Die Fragmente der Vorsokratiker,* 3d edition (hereafter abbreviated V^3) I., p. 15, 16.

3 Περὶ φύσεως. My reasons for retaining this rendering may be found in my essay, Περὶ Φύσεως. *A Study of the Conception of Nature among the Pre-Socratics,* Proceed. Amer. Acad. of Arts and Sciences, **45** (1910), pp. 79–133, hereafter abbreviated Περὶ Φύσεως.

4 Εὑρήματα. No satisfactory discussion of this literary kind as a whole exists.

5 V^3, I. 14, 23 ἔγραψε Περὶ φύσεως, Γῆς περίοδον καὶ Περὶ τῶν ἀπλανῶν καὶ Σφαῖραν καὶ ἄλλα τινά.

met. *Tour of the Earth* was one of the accepted names for a geographical treatise. *On the Fixed Stars* and *Sphere* would be suitable titles for works dealing with astronomy. What account shall we take of this bibliographical index?

In the times of Anaximander and for long thereafter it was not customary for authors to prefix titles to their writings. Such indication as the writer vouchsafed to give of the contents of his book would ordinarily be contained, along with his name, in the introductory sentence, as was done, for example, by Herodotus, more than a century after Anaximander. Ephorus was, apparently the first geographer and historian who divided his work into 'books.' However reasonable the earlier practice may have been for the writer, it was extremely inconvenient for a librarian. The latter required a convenient ticket to attach to the scroll, and hence invented titles where they were not furnished by use and wont. The librarians at Alexandria thus found in current use not only such general titles as *Iliad* and *Odyssey*, but also certain sub-titles referring to episodes or distinct divisions of larger wholes. It is obvious that in the catalogues of the Alexandrian libraries these titles recognized by usage and found in the *testimonia*, particularly regarding rare books or *desiderata*, were duly listed.[6] Hence it might well happen, and demonstrably did repeatedly happen, that one and the same book was represented in the catalogues by various titles.

Now as regards Anaximander we need not pause at present to inquire whether, assuming the essential truth of the bibliographical data furnished by Suidas, he is to be credited with more than one treatise.[7] The title *On Nature*, though in no sense original or really authentic, is admitted by all to apply to a genuine work of Anaximander from which, it is assumed, derive in the last resort the reports of his 'philosophical' opinions, excerpted by Theophrastus and preserved in the form of tablets triturate in the doxographic tradition. Even the most superficial knowledge of such things must suffice to justify the application of such a title as *On the Fixed Stars* to at least a portion of this treatise. As for *Sphere*, it is true that it is not strictly applicable, because there is no adequate ground for thinking that Anaximander spoke of celestial spheres, the luminaries being according to his teaching annular bodies; but from Aristotle onwards the Pythagorean

6 This was suggested by Diels, *Herodot und Hekataios*, Hermes, **22**, p. 414, n. 1, and is made so evident by a study of bibliographical data regarding ancient authors that it cannot reasonably be denied.

7 It was apparently the multiplicity of titles, assumed to imply an equal number of treatises, that led Zeller, *Phil. der Gr.* I⁵, p. 197 n., to suspect that the bibliography of Suidas arose from a misunderstanding.

astronomy and cosmology so established itself that even the best writers spoke of 'spheres' where really 'circles' more accurately described the facts.[8] Hence it would be pressing too far a current expression to object to *Sphere* as a possible sub-title of the treatise *On Nature*. Of the *Tour of the Earth* we need for the movement to remark only that it is included in the list and refers beyond question to a geographical treatise, or in any case to a portion of a work, attributed to Anaximander, supposed to contain matters germane to geography. Whether other evidence of the existence of such a work in antiquity can be discovered elsewhere, we shall have presently to inquire.

Now this bibliographical index has been lightly, perhaps too lightly, set aside as valueless by modern scholars. Generally this is done without even a word of explanation; where anything is said, it is apt to be suggested that there is a mistake or that the titles were read out of references to the map and the celestial globe attributed to Anaximander.[9] As has been already stated, no exception is taken to the datum regarding the title *On Nature*, though it is now agreed to have been of later origin. To Zeller, apparently, it was the multiplicity of titles that occasioned surprise and doubt. Why it should do so, he did not indicate; but one may surmise that he had in mind the probability that Anaximander was the first prose writer and assumed that all beginnings are modest. We may later recur to the question whether Anaximander may with great probability be regarded as the originator of Greek prose; meanwhile it may suffice to say that this is not so certain as to justify *à priori* deductions from the hypothesis. To add to the difficulty, apparently, Suidas, after

8 This has led to grave misunderstandings, from Aristotle to our own day, even among the most critical historians of Greek thought.

9 For Zeller, see above, n. 7. Diels (V^3 I. 14, 23 note) suggests that the titles were concocted from such statements as that of Diogenes Laertius (ibid. 1.9) *καὶ γῆς καὶ θαλάσσης περίμετρον πρῶτος ἔγραψεν, ἀλλὰ καὶ σφαῖραν κατεσκεύασε*, and from the fact that Anaximander first distinguished the 'sphere' (!) of the fixed stars. The use of *κατεσκεύασε* in regard to the 'sphere' shows that Suidas did not merely mistake the statement of Diogenes regarding the stars, as does the duplication *Περὶ τῶν ἀπλανῶν καὶ Σφαῖραν*. It is true that a confusion might have arisen in regard to *ἔγραψε . . . Γῆς περίοδον*, for it is conceded that *γῆς περίοδος* might mean a 'map,' and *γράφειν* with it might mean 'draw' as well as 'write'; but the entire list of Suidas is so patently bibliographical, and there is so little except *à priori* considerations to be urged against it, that one cannot take these suggestions seriously. Of *Περὶ τῶν ἀπλανῶν* and *Σφαῖρα* we have already said that on such a view the duplication is difficult to explain; even more so is *καὶ ἄλλα τινά*, which is hardly due to Suidas or his immediate source (Hesychius?), implying as it does the existence of other titles, as indeed we hear of one more from another source.

enumerating the four titles already considered, adds that Anaximander wrote 'some other treatises' not specified by name. Now we shall subsequently find another title, not included in the list of Suidas, which may with probability be referred to our Anaximander.

This circumstance might be considered as aggravating the difficulty to the point of rendering what was before an improbability a sheer impossibility. But this also does not necessarily follow; for it may be that we shall have to revise our notions concerning Anaximander and what is possible or probable in regard to him. It so happens that the title elsewhere cited and not specifically mentioned in the list of Suidas is identical with a title cited in reference to Hecataeus of Miletus. Now Hecataeus, as we shall presently see, was only a trifle over a generation younger than his fellow townsman Anaximander, and pursued studies in good part at least identical with his, taking up and perfecting his map and writing a geographical treatise which enjoyed a great and well deserved reputation. If it should prove that Anaximander was in intention primarily a geographer, the work of these two eminent Milesians would in fact lie quite in the same plane and run in part parallel, though each extended his line in one direction beyond the other's. Of this we shall have to speak more at length presently: what for the moment concerns us is to point out the fact that there is a striking similarity between these almost contemporary authors in regard to the titles ascribed to them. Hecataeus seems to have written one work, or at most two, but the recorded titles are quite numerous.[10] That these bibliographical data derived in part from the catalogues of the Alexandrian libraries admits of no doubt, and is in fact not questioned. Why we should not assume the same source for the bibliography of Anaximander does not appear. We have, therefore, thus far found no good reason for rejecting the testimony of Suidas, subject of course to the limitations which apply to all early titles, including that *On Nature*. The only apparently good reason will be presently found on closer examination to confirm the record that Anaximander wrote a geographical treatise.

We have seen that Suidas attributes the introduction of the gnomon or sun-dial to Anaximander. Diogenes Laertius reports[11] that "he first invented the gnomon and, according to the *Miscellany* of Favori-

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10 See Jacoby, 'Hekataios' in Pauly-Wissowa's *Real-Encyc.*, 7 (hereafter abbreviated Jacoby), col. 2671 sq. This elaborate essay is at present by far the best study of Hecataeus, and must be read in connection with the same scholar's essay on Herodotus in the same work.

11 V^3 I. 14, 7.

nus, set up at Sparta, in the place called the Dial, one that showed the solstices and equinoxes, and contrived a sun-dial to tell the hours. And he first drew an outline of land and sea, and moreover constructed a (celestial) sphere." Critics have taken exception to certain details of this statement. While Suidas, Diogenes Laertius, and Eusebius[12] agree in attributing the invention of the sun-dial to Anaximander, the Greeks, according to Herodotus,[13] learned the use of the dial and the twelve-fold division of the day from the Babylonians. In view of what we said above regarding the ancient reports of inventions we may well concede that Anaximander did not invent, but merely introduced the instrument. Perhaps even the mere introduction was not due to him; for it is quite possible that dials had been brought to Ionia either from Babylon or from Egypt before his time. We have, however, no reason to doubt that Anaximander was one of the earliest known Greeks to make a scientific use of the instrument.[14] The dial which, according to Favorinus, he set up at Sparta, showed the equinoxes and solstices, and, according to Pliny,[15] Anaximander discovered the obliquity of the zodiac, which, together with the beginnings of the seasons as marked by the rising and setting of certain constellations, could be, and at least in later times were actually, marked on the dial, as connected with seasonal changes in the position of the sun. Such observations are manifestly related to astronomy, with which Anaximander is acknowledged to have greatly concerned himself. The heliacal setting of the Pleiades, long before observed, could with the aid of the dial be definitely dated with reference to the autumnal equinox.[16] But there is evidence that the risings and settings of the sun at the solstices and equinoxes were in early times used for geographical as well as for astronomical purposes. It is significant that there is no certain reference to the height of the sun at midday until the discovery was made in the time of Eratosthenes that the sun at the summer solstice was vertical over Syene.[17]

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12 V^3 I. 14, 28. 13 Hdt. 2. 109.

14 There are grounds for attributing the use of the gnomon to Thales; but they are doubtful, and may here be disregarded.

15 V^3 I. 15, 1. 16 V^3 I. 19, 17.

17 Hdt. 2.25 is no evidence to the contrary. Eudoxus of Cnidus and (probably) Dicaearchus attempted to reckon latitudes, but the former at least seems to have limited his observations to the star Canopus. The alleged observations of Pytheas were questioned in antiquity and are even now in dispute. Observations of the sun were probably made earlier than Eratosthenes, but must have been very uncertain, as the calculations of the circumference of the earth and the determinations of latitude show. Even later the positions of the sun at rising and setting were observed along with its meridian height, as is proved by Strabo 2.5 (C 109) δύσεις καὶ ἀνατολὰς καὶ

By that time the mathematical theory of the globe-earth was fully worked out [18] and the value of the observation could be seen and the necessary conclusions drawn from it, which resulted in the geographical location of the tropics and the equator. But long before that we learn of the use of three equatorials, the central one passing through the straits of Gibraltar and defined with reference to the equinoxial rising and setting of the sun, and two other lines related respectively to the summer and to the winter sunrise and sunset, the former running from the Pyrenees to the Caucasus along the course of the Ister, the latter running parallel along the line supposed to be described by the upper course of the Nile from the Atlas Mountains to the upper Cataracts.[19] This scheme, known from Herodotus, is obviously a geographical projection of the lines of a flat disk sun-dial, which originally concerned itself with the positions of the sun, not at the meridian, but at the rising and setting, where its variations were far more conspicuous. Since this geographical scheme is unquestionably derived from the early Ionians, we naturally think of it as going back either to Hecataeus or to Anaximander; and of the two Anaximander surely has the better claim to it.

But it has been said that the sun-dial at Sparta cannot be attributed to Anaximander, since Pliny [20] says that it was Anaximenes, the disciple of Anaximander, who set it up. It is surprising that this objection should have been seriously considered; for in the same breath Pliny attributes the invention of the science of the gnomon to Anaximenes. The latter statement no one accepts, and with good reason; for Anaximenes is in comparison with Anaximander a figure of hardly secondary importance. The natural inference is that in this case Pliny misunderstood a statement in his source, which may well have been superficially ambiguous, since the clause regarding the discovery of the science of the gnomon and the erection of a dial at Sparta, might well have been introduced by a demonstrative susceptible of reference either to Anaximander or to Anaximenes, both of whom were mentioned. In this connection it is well to remark that while there is nothing else in the literary tradition associating

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μεσουρανήσεις. This can hardly be explained except as a survival from earlier practice. Traditionally *μεσημβρία* merely meant 'noonday' or 'south,' as was natural to any people living north of the tropic of Cancer.

18 As is clear from Aristotle.

19 See, *e. g.*, Ephorus *fr.* 38 (Müller *Hist. Gr. Frag.* I, 243). I hope to discuss this more fully on another occasion. I do not, of course, refer to the alleged 'zones,' of Parmenides: they are either a gross misinterpretation of something quite different or a pure invention of later writers.

20 V^3 I. 24, 40.

Anaximenes with Sparta, it is reported [21] that Anaximander warned the Spartans to abandon their city and houses and live in the open because he anticipated the earthquake which destroyed the entire city. We cannot, unfortunately, rely implicitly on these statements; but if they were true we should have the more reason to suspect that the bronze map of the earth which Aristagoras of Miletus brought to Sparta [22] in his effort to persuade that State to aid the Ionians in overthrowing the Persian power was in fact the map of Anaximander who was commended to the Spartans by personal relations. [23] But whether this map was that made by Anaximander or, as some prefer to think, the revised and perfected map of Hecataeus, who was conspicuously prominent in the Ionian revolt, makes very little difference for our purposes. The presence of a sun-dial at Sparta is as intelligible as that of the map; for both were related to geography, and Anaximander no less than Hecataeus was a geographer. Nor should it cause surprise that Anaximander should thus be supposed to have visited Sparta. It is true that we have no other record of his travels, but in view of the scantiness of the reports regarding him, this is not significant. One can hardly think of a geographer, especially of a pioneer in the field, as confining his studies to his native city and to such information as he could there obtain at second hand. His successor Hecataeus, we are told, had travelled widely.

Of the 'sphere' which Anaximander is said to have constructed we cannot say much. Its relation to his astronomical or cosmological studies is sufficiently obvious. It was as natural that he should attempt a graphic or plastic representation of the heavens as of the earth. Whatever its form, it would serve to visualize the obliquity of the zodiac, which he discovered, and to relate the constellations, in their risings and settings, to the seasons and the changing position of the sun. Anaximander presumably never realized how much this attempt was destined to contribute to the final overthrow of his conception of a disk earth and to the eventual revision of his map; for, once the heavens came to be clearly visualized as a sphere, the advance of geometry, which he is said to have cultivated, and the

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21 V^3 I. 15, 5. 22 Hdt. 5.49.

23 It is quite possible that Anaximander may have been sent by Croesus as one of his ambassadors to Sparta when he sought an alliance with the leading power in Greece (Hdt. 1.69); though the Lydians were by this time pretty well Hellenized, diplomacy would suggest that on such an errand he should send Greeks, and none would have been more suitable, it would seem, than a man of prominence among the Milesians, his subject allies. At any rate, Lydia and Miletus clearly had relations with Sparta at that time which would readily account for Anaximander going there.

detailed observation of the stars, to which his sun-dial and 'sphere' must have added impetus, led inevitably to the postulate of a spheroidal earth. This postulate, as is well known, came not from geography, but from the study of the heavens. Indeed, long after the newer conception of the earth had sprung from the speculations of the Pythagoreans, who were geometers and not at all practical geographers, the maps of the geographers, though gradually modified, preserved the impress of their Ionian originators and of their conception of the disk earth.

As to Anaximander's map, we do not know through what intermediaries the notice regarding it came to the handbook of Diogenes Laertius. His testimony does not, however, stand alone, but is supported by the geographical tradition. The filiation in detail of the works on geography in which his map is mentioned need not detain us here. Leaving aside several unimportant notices [24] which add nothing to our enlightenment, we may confine ourselves to the statements of Agathemerus and Strabo. The former says [25] "Anaximander of Miletus, who 'heard' Thales, first had the hardihood to depict the inhabited earth on a tablet. After him Hecataeus of Miletus, a man who had travelled widely, refined his work to the point of admiration. Hellanicus of Lesbos, indeed, a man of wide learning, handed down the fruits of his research unaccompanied with a formal representation. [26] Then Damastes of Sige, borrowing chiefly from the works of Hecataeus, wrote a geographical work; in due order Democritus and Ephorus and certain others composed systematic *Tours of the Earth* and *Geographies*." Of this statement it need only be said that it contains a selected list of geographers, whether they prepared maps or not, extending roughly down to the time of Eratosthenes. The list is carelessly drawn and the phrasing is at more than one point ambiguous; but the difficulties which it presents do not specially concern us here.

To Strabo we are indebted for two passages relating to Anaximander as a geographer. Beginning his geographical treatise he says, "Geography, which I have now chosen to consider, I hold as much the pursuit of the philosopher as any other science. That my opinion is sound is clear from many considerations. For not only were the

24 Eustathius, *Comm. in Dionys.* (Müller, *Geogr. Gr. Minor.* II. 208, 14) and *Schol. in Dionys.* (*ibid.* II. 428, 7).

25 V^3 I. 15, 9 quotes the first sentence only. For full text see Müller, *o.c.* II. 471.

26 This is the accepted meaning of the phrase ἀπλάστως παρέδωκε τὴν ἱστορίαν, but it is not certain: it is possibly a criticism of the literary style of Hellanicus.

first who boldly essayed the subject men of this sort,— Homer, and Anaximander, and Hecataeus, (as Eratosthenes also says) his townsman, and Democritus also and Eudoxus and Dicaearchus and Ephorus and others more; and besides, after their time, Eratosthenes and Polybius and Posidonius, philosophers all. But, what is more, wide and varied learning, by which alone it is possible to achieve this task, belongs peculiarly to the man who contemplates all things divine and human, the science of which we call philosophy." Here Anaximander is presented as a geographer who, like numerous other worthies, including Homer, is regarded as a philosopher. The two-fold fact that Eratosthenes is cited as authority for the citizenship of Hecataeus and that the list of worthies falls into two groups of which Eratosthenes heads the second, suggests that that eminent geographer had something to do with drawing up the roll of geographers. His contribution does not, of course, extend beyond the first division; and even there we must except Homer, whom Eratosthenes declined to recognize either as a philosopher or as a geographer, insisting that he was to be regarded solely as a poet bent on entertaining his readers.[27] Nor can we credit Eratosthenes with rating the others as philosophers; for, aside from the vague conception of what constitutes a philosopher, which is characteristic of Strabo's cast of thought,[28] it is to be noted that Hecataeus, for reasons which will engage our attention later on, was never seriously regarded as a philosopher and hence does not figure in the doxographic tradition.[29] The same is true of Ephorus.[30]

The reference of Strabo, however, to the 'wide and varied learning' (polymathy) required of the geographer recalls that Heraclitus rebuked

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27 See Strabo I. 2, 3 C.15. It is probable that Strabo in his judgment of Homer was merely echoing Ephorus.

28 If Ephorus was his model, we might think of Isocrates and his conception of philosophy. Strabo however was a Stoic; and in late Stoics a similar conception of philosophy may be found.

29 In *Dox. Gr.*, p. 681 Diels referred Aetius II. 20, 16 to him, but has since corrected the error and assigns the notion to Hecataeus of Abdera (V^3 II. 153, 3).

30 Aetius IV. 1, 6 is only an apparent exception. It refers to the inundation of the Nile and probably comes from the 'Posidonian Areskonta.' The theme properly belongs to geography and was a favorite subject of speculation. Diodorus Sic.1. 37, 1 says *μεγάλης δ'οὔσης ἀπορίας περὶ τῆς τοῦ ποταμοῦ πληρώσεως, ἐπικεχειρήκασι πολλοὶ τῶν τε φιλοσόφων καὶ τῶν ἱστορικῶν ἀποδοῦναι τὰς ταύτης αἰτίας.* That Herodotus (Aetius IV. 1, 5) appears in the list along with Thales, Euthymenes, Democritus, Ephorus and Eudoxus, is sufficiently significant. All but Thales (and he is not certainly an exception) were geographers. Anaxagoras likewise is included in the list, but possibly by confusion; for the explanation attributed to him is older than he, as is plain from Aeschylus *fr.* 300. Possibly Anaximander may have been the originator of the theory that the flood of the Nile was due to the melting of snows in Ethiopia.

it, saying[31] "Polymathy does not teach one to have understanding, else would it have taught Hesiod and Pythagoras, and again Xenophanes and Hecataeus." It was apparently the historical and geographical interest[32] of these men that invited the rebuke of the Ephesian recluse. Such breadth of interest was of course characteristic of the whole line of historians and geographers. Pythagoras alone seems strange in such company. Why he should have been decried as a polymath is not clear;[33] but we must recall that he came from Samos, where he may well have imbibed some of the varied knowledge of the Milesian circle, and that we have no authentic account of the range of his interests. What is commonly attributed to him is for the most part true of his school only. He may have been interested in geography, but neither the nature of the reports to that effect nor the record of his school would warrant one in affirming that he was. Heraclitus is said[34] to have referred to Thales' prediction of the eclipse: in what terms he may have done so, we do not know. There is nothing to show what he may have thought of Anaximander.

But to return to Strabo. A few pages after the passage above quoted he resumes:[35] "Let this suffice to justify the statement that Homer was the first geographer; but those also who succeeded him are known as noteworthy men and of the kindred of philosophy; the first of whom after Homer, Eratosthenes says, were two, Anaximander, an acquaintance and fellow citizen of Thales, and Hecataeus the Milesian; the one, he says, first gave out a geographical tablet (map), the other, Hecataeus, left a treatise attested as his by his other writing."[35a] The importance of this statement, coming from Eratosthenes the renowned geographer and chronologist, who served as head of the great library at Alexandria, is at once apparent. Just what it

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31 *Fr.* 49 Diels.

32 Cp. Philo. Jud. *De congressu eruditionis gratia*, 15, p. 521 M. *γραμματικὴ μὲν γὰρ ἱστορίαν τὴν παρὰ ποιηταῖς καὶ συγγραφεῦσιν ἀναδιδάξασα πολυμάθειαν ἐργάζεται.* For Xenophanes one may cite (quite apart from his supposed poem *Περὶ φύσεως*) his observations of natural phenomena in various places, his obvious interest in ethnology, and his poems regarding the founding of Colophon and Elea. Why one should doubt the report that he wrote these poems does not appear. Hecataeus (fr. 352) refers to the colonization of Sinope.

33 One might think of his 'borrowing' his 'symbols' from Egypt, if Heraclitus was of the opinion of Herodotus; but Heraclitus himself may be charged with borrowing from Egypt, or at least with utilizing notions which the Ionians thought to discover in Egypt.

34 *Fr.* 38 Diels.

35 I. 1, 11 C. 7.

35a For the use of *γραφῇ* here cf. Dionys. Hal. *De Thucyd.* 1 *ἐδήλωσα καὶ περὶ Θουκυδίδου τὰ δοκοῦντά μοι, συντόμῳ τε καὶ κεφαλαιώδει γραφῇ περιλαβών.*

signifies is perhaps not quite so clear. Let us consider it somewhat more at length.

This passage bears out the conclusion we reached above in regard to the list of geographers drawn up by Eratosthenes. It was not for their supposed connection with philosophy, but solely as geographers that he mentioned Anaximander and Hecataeus, and Homer was not entered in the roll of honor. As regards the contribution of these pioneers of the science it hardly needs to be said that the testimony of so great an authority as Eratosthenes to the fact of Anaximander drawing a map has been accepted as conclusive evidence by all modern scholars. This conclusion is justified, however, not because Eratosthenes made no mistakes in regard to the authenticity of works, but rather by the circumstances, (1) that we cannot in this instance go farther and check his conclusion by better evidence from other sources, and (2) that in this case his decision is positive and not negative. The first of these principles must always hold in historical inquiries when there is no sufficient reason for impugning the testimony of a generally trustworthy and competent witness. The second is of importance in relation to the judgments of the Alexandrian librarians, because they assumed a critical attitude and erred in general, when they erred, in refusing to admit rather than in affirming the genuineness of works entered in their catalogues. For us, therefore, there remains no alternative but to accept the map as an historical fact.

But what of the geographical treatise attributed to Anaximander? It will perhaps be urged that the statement of Eratosthenes reproduced by Strabo confirms the judgment of those who would reject the report of Suidas, to the effect that Anaximander wrote a *Tour of the Earth*. If this be true, we are on dangerous ground when we refer the list of his works preserved by Suidas to the catalogues of the Alexandrian libraries. But what is affirmed, and what is implied, in the statement of Eratosthenes? The genuineness of Anaximander's map and of Hecataeus' geographical treatise is unquestionably affirmed. One may, if one will, insist that the word 'first' be taken with both statements, so that Eratosthenes shall be made to affirm not only that Anaximander first gave out a map but that Hecataeus first left a geographical treatise. Though possible, the construction is extremely improbable and forced. Yet, even if so much were granted, what is implied in the sentence as a whole? It is not stated that a geographical treatise attributed to Anaximander did not exist or had not existed; rather the affirmation that the claims of such a treatise, attributed to Hecataeus, to be regarded as genuine were confirmed by his other writing, when considered in relation to the sentence as a whole, would seem to imply that Eratosthenes had knowledge of a treatise attributed to Anaximander, which, however, was not so or otherwise sufficiently authenticated.[36] If this exegesis be sound, and I believe it is, we discover in the very text which, superficially viewed, seems to discredit the bibliography of Suidas, a confirmation of our thesis that the geographical treatise of Anaximander was entered in the Alexandrian catalogues. We may, however, infer from the statement of Eratosthenes that in his time, at least, it was noted as subject to question.[37]

We must now inquire how much weight we should assign to the doubt of Eratosthenes regarding the genuineness of the work attributed to Anaximander. At first sight it would appear that he was as competent a judge in such matters as one could readily find; for one recalls that he was alike eminent as a geographer and as a student of chronology, the former interest seeming to qualify him in a special way to speak with authority on matters connected with the history of geography and in particular with geographical literature, the latter bespeaking for him uncommon credit in regard to the moot questions concerning 'inventions.' But upon closer examination one discovers that these pretensions vanish in thin air. Eratosthenes was, indeed, a geographer and a chronologist of deservedly high repute; but in both fields it was not the antiquarian details, but the scientific principles involved, that chiefly engaged his attention and owed to his efforts a noteworthy contribution. It might well happen, in consequence, that he should err in judgment in regard to matters which lay outside his proper field of study and less invited his interest. We should not be surprised, therefore, if others more directly interested and in such matters more competent should prove to have abandoned his doubts regarding the geographical treatise of Anaximander. That such was in fact the case we shall now try to show with such degree of certainty

36 *Per contra*, the existence of a map drawn by Hecataeus is not denied, though it likewise is not affirmed. The geographical treatise of Anaximander and the map of Hecataeus are on the same footing, except in so far as the authenticity of the treatise of Hecataeus, as confirmed by his other writing, suggests a want of such authentication for the treatise of his predecessor. It is sometimes said that the remark about Hellanicus implies that Hecataeus made a chart; but that remark, as was pointed out above (n. 26), is by no means unambiguous.

37 One may apply to Eratosthenes what Jacoby (col. 2673) has said of Callimachus in regard to the geographical treatise of Hecataeus, to wit, that it is not probable that he had gone carefully into the question of the authenticity of the document. There being for some reason occasion for doubt or question, it was prudent not to affirm it.

as is possible in such matters. As is generally the case in historical questions where a conclusive text is not to be produced, the evidence in this instance must be cumulative and must be presented piecemeal with a running commentary. The judicial reader will follow and weigh the arguments in detail, suspending sentence until the whole case has been presented.

We may sum up the conclusions which we have thus far reached in the following propositions. (a) Anaximander wrote a treatise, current in antiquity and accepted as genuine, which was commonly entitled *On Nature*. (b) There were recorded in the catalogues of the Alexandrian libraries certain other titles purporting to belong to him, such as *Tour of the Earth, On the Fixed Stars, Sphere,* and others not specified. (c) These titles were one and all of later origin, and, being quite possibly at least in part subtitles, indicate at most the scope of his writing without in any way revealing his predominant interest or creating a reasonable presumption in regard to the number of treatises, one or more, which he may be thought to have written. (d) He was credited with the invention or introduction of the sun-dial, which he employed for scientific purposes, certainly as regards cosmology or astronomy, probably in the interest of geography. (e) He drew a map of the earth, which was believed to be the first of its kind and to entitle him to be considered the first scientific geographer. (f) He is credibly reported to have constructed a 'sphere' or representation of the heavens. (g) Eratosthenes had knowledge of a geographical treatise attributed to Anaximander, which he did not consider sufficiently authenticated to justify him in crediting the father of scientific geography with more than the drawing of a map.

So far, then, as the geographical treatise is concerned, down to the time of Eratosthenes the verdict must be *non liquet,* though the admitted interest of Anaximander in the science of geography may be said to favor the presumption that he did not in his book, the existence of which is acknowledged, forego treating a subject so certainly in his thoughts. But Eratosthenes does not mark the close of scientific and antiquarian studies at the Alexandrian libraries in the fields of geography and chronology. The later advances in the science of geography do not concern us here; but it is necessary to direct attention to another scholar whose chosen pursuits especially qualified him to carry forward and revise in detail the historical studies of Eratosthenes. Apollodorus of Athens, the most illustrious disciple of the great Alexandrian critic Aristarchus, devoted himself with zeal and learning especially to the antiquarian aspects of geography and chronology. Attentively reading a vast number of books he published the results of his historical studies chiefly in two works,—his versified *Chronicles*, in which the dates of writers were as precisely as possible fixed, and *The Catalogue of Ships*, in which he gathered about the Homeric *Catalogue* the fruits of his researches in geography. Connected with this learned commentary was his work *On the Earth*, like the *Chronicles*, versified. He wrote, besides, on mythology, prompted by an interest in literary history. Apollodorus is, therefore, of all ancient critics the one whose testimony regarding the book of Anaximander we should most wish to learn.

By a singular good fortune we are in fact in a position to ascertain at least in part what Apollodorus knew of Anaximander's writings. Diogenes Laertius says [38] that Anaximander "gave a summary exposition of his opinions, on which Apollodorus of Athens somewhere chanced, who in his *Chronicles* reports that he was sixty-four years of age in the second year of the fifty-eighth Olympiad and died shortly thereafter." In the light of our foregoing discussion, this most interesting statement deserves somewhat fuller consideration than it has hitherto received.

Whence Diogenes derived this datum is not certain; but one may conjecture from the phraseology that it came mediately or immediately from the late doxographic document which Diels has called the Posidonian Areskonta. It is known that Posidonius followed closely in the footsteps of Apollodorus, whose geographical studies he took up and prosecuted to the best of his ability. But in any case Apollodorus made the statement in question in his *Chronicles*. By 'opinions' Diogenes of course means 'philosophical' opinions, and therefore, though Apollodorus does not seem to have cited the title of the 'summary exposition,' it is commonly assumed that it was the treatise appropriately called *On Nature* to which he referred. Diels has cited [39] as a parallel the chronological datum given by Democritus in his *Brief Cosmology*, and suggests [40] that the year designated was that of the publication of the treatise, which contained autobiographical references capable of astronomical determination. This is of course conceivable; but, it must be pointed out, this hypothesis, while possibly accounting for the given year, fails to explain

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38 *V*³ I. 14, 11.

39 *Chronologische Untersuchungen über Apollodors Chronika*, Rhein. Museum, N. F. XXI, p. 24. Cf. Jacoby, *Apollodors Chronik*, p. 53 sq.

40 *V*³ I. 14, 13 (note). It is worth mentioning that we have no record of any remarkable astronomical phenomenon in that year.

the further statement of Apollodorus that Anaximander died shortly afterwards. If, as we are bound to do, we confine ourselves to such information as Apollodorus might derive from the book he had met with, without assuming either data from other sources or unwarranted inferences on the part of the chronologist, we must frame a better theory. Such an hypothesis is in fact not far to seek, and has indeed been already in part suggested by Professor Burnet. [41]

Diogenes dates Anaximander by the second year of the fifty-eighth Olympiad. This statement does not derive directly from Apollodorus, who used the year of the Athenian Archon Eponymus instead of the Olympiad. But the chronological practice of Apollodorus suggests not only that the datum really comes from his work but also that it was based upon information of a peculiarly definite sort. Ordinarily Apollodorus contented himself (perforce, no doubt) with determining the *floruit* of a man, taken as forty years of age, with reference to some epoch or the beginning of a king's reign, if a relation could be established. One sees at once the exceptional character of the datum regarding Anaximander; for the age is not forty, but sixty-four, and the year (B.C. 547/6) is not one of his regular epochs. The following year, however (546/5), is one of the important epochs of Apollodorus, being that of the fall of Sardis. The preceding year, moreover, though not marking an epoch for Apollodorus, was one of fateful consequences to the Ionians of Miletus, among whom Anaximander was a man of great prominence; for the march of the Persians under Cyrus against Croesus, whose subject allies the Milesians were, and the defeat of Croesus at the Halys, must have filled Anaximander with dismay. Nothing would be more natural than for him to mention these events,[42] if he dealt at all with geographical or historical matters; for they were obviously of great potential significance from either point of view. If he was personally active in this campaign, as he may well have been, he might properly give his age. Thus we should have a reasonable hypothesis to account for the report

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41 *Early Greek Philosophy*³, p. 51. He refers to the circumstance that the date in question is the year before the fall of Sardis, and to the question (which, to judge by Xenophanes, *fr.* 22, was considered interesting in those days), 'How old were you when the Mede appeared?' *Ibid*, n. 3 Burnet adds, "The statement that he 'died soon after' seems to mean that Apollodorus made him die in the year of Sardeis (546/5), one of his regular epochs."

42 It is by no means certain that 'the common report of the Greeks' (rejected by Hdt. 1.75) that Thales served Croesus as engineer on this occasion, turning the Halys River, was false. If Thales was in the army of Croesus, Anaximander his 'comrade and kinsman' quite probably also was there: sufficient reason, in either case, for mentioning the event.

of Apollodorus regarding his age in this precise year. The additional statement that he died not long thereafter would be adequately explained if Anaximander's book made no mention of the fall of Sardis, which soon followed.

It will be seen that this hypothesis carries us beyond the natural scope of a cosmological treatise, such as would necessarily be entitled *On Nature*, into a field more closely connected with history or geography. It may be said, however, that, while this assumption would meet the necessary conditions, others equally as good might be made to fit the requirements. Granted that an hypothesis, however satisfactory, is not to be accepted as proof, we are justified in saying that this particular hypothesis has more probability in its favor than any other that has been proposed, and is at least in a measure supported by the evidence of Anaximander's interest in geography.

But we have not yet exhausted the possible sources of information regarding Anaximander's book. Aside from a fragment preserved by Simplicius, to which we shall later return, there are mentioned certain opinions and passages attributed in our sources to 'Anaximander.' Their status, however, is uncertain, because we hear of another Anaximander of Miletus, to whom they have generally been referred. We are bound, therefore, to consider the new claimant and the validity of his claims.

What we may be said to know about the other Anaximander is little enough, being contained in two brief notices. Diogenes Laertius reports[43] that "there was also another Anaximander, an historian, he too being a Milesian, who wrote in Ionic." The entry of Suidas regarding him runs thus: "Anaximander the Younger, son of Anaximander, of Miletus—historian. He lived in the time of Artaxerxes Mnemon; wrote *Interpretation of Pythagorean Symbols*, e.g. 'not to step over a cross-bar,' 'not to poke the fire with a poniard,' 'not to eat from a whole loaf,' and the rest." The first gives nothing omitted in the second notice except that he wrote in Ionic, which was natural if he wrote before Attic became the recognized medium for prose, as it did in the fourth century B.C. As Artaxerxes Mnemon reigned 405–359 B.C., and the chronological datum 'he lived in the time of such and such a king' regularly refers to the date of the king's accession (405), this information was not material. Suidas, then, alone gives us significant data. We gather, then, that Anaximander was a fairly frequent name at Miletus, as this notice acquaints us with

43 V^3 I. 14, 18.

two, father and son. If they were not related to the great Anaximander, his fame may account for the perpetuation of the name in his city. Anaximander the Younger is called an historian, which might be significant if we were not immediately informed what sort of history he wrote without getting the least intimation that he produced anything beyond the *Interpretation of Pythagorean Symbols.* Though Suidas gives us no specimen of his interpretations and we have no evidence in regard to them from other sources, we fortunately are not without instructive examples of the same literary kind from other hands.[44] At their best they are reports of an antiquarian character regarding curious practices in Greece or foreign lands; at their worst they are stupid attempts at symbolical interpretation. They cannot predispose us to think of Anaximander the Younger as a serious rival of his great eponym; and in any case the fact that he is called an historian, with such a work and such only to his credit, offers no justification for assigning to him every fragment or notice which would be appropriate to an historian.

Having seen what manner of man Anaximander the Younger was according to the only certain information we have regarding him, we must now canvass the data which Greek tradition attributes simply to 'Anaximander' and modern scholars commonly credit to the account of the author of the *Interpretation of Pythagorean Symbols.* A scholium on Dionysius Thrax[45] reports, "Ephorus among others in his second book says that Cadmus was the inventor of the alphabet; others say that he was not the inventor but the transmitter to us of the invention of the Phoenicians, as Herodotus also in his *History* and Aristotle report, for they say that Phoenicians invented the alphabet and Cadmus brought it to Greece. Pythodorus, however, in his treatise *On the Alphabet* and Phillis of Delos, in his treatise *On Chronology,* say that before the time of Cadmus Danaus imported it; and they are confirmed by the Milesian writers Anaximander and Dionysius and Hecataeus, who are cited in this connection by Apollodorus also in his treatise *On the Catalogue of Ships.*" The word vaguely rendered 'writers' in the foregoing version should probably be translated 'historians,' its usual meaning in later Greek. Possibly it was this consideration that prevailed with modern scholars, leading them

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44 See V^3 I. 357, 13 sq. When one considers Heraclitus *fr.* 129 Diels and Hdt. 2.49, 2.81, Plut. *Qu. Conviv.* 728, 729A, where Hdt. 2.37 is quoted, one understands the reason for studying the derivation of the Pythagorean symbols.

45 P. 183. 1 Hilgard.

to assign this datum to Anaximander the Younger. Diels alone, apparently, has latterly had misgivings; for after ignoring this passage in the first two editions of his invaluable *Vorsokratiker* he included it in the third, but marked it as dubious under the heading of the elder Anaximander.[46]

Intrinsically there can be no valid objection to the assumption that the early Milesians Anaximander, Dionysius and Hecataeus concerned themselves with the question regarding the source of the Greek alphabet. The absurd notions regarding the late beginnings of literature in Greece, based on a few ignorant utterances of late Greeks and fostered by the incomprehensible influence of Wolf's *Prolegomena* might account for the hesitation of some modern scholars to credit such a report, which obviously implies that to the Ionians of the time of Pisistratus writing was so familiar a fact that they must seek its origins in the distant mythical past; but, apart from such preconceptions, there is no ground for calling it in question. It is rather just what the intelligent student should have expected, not only from a reading of the Homeric poems, whose literary perfection and contents are incomprehensible except on the supposition of long literary practice, but also from a critical reading of the extant remains of early Greek historical writings. Herodotus, whose dependence on early Ionian, especially Milesian, writers is unquestionable, has no doubt of the derivation of all higher elements of Greek civilization from Egypt, and constantly presupposes two lines of transmission, one direct, mediated by Danaus, from Egypt to Argos, the other indirect, in which the Phoenicians play the rôle of intermediaries.[47] In the latter line Cadmus, who is supposed to have come from Tyre to Thebes, is not the only link. It is not necessary here to go into details. Suffice it to say that on the ground of intrinsic probability no objection can be urged against the assumption that Anaximander, the contemporary of Croesus, held the opinion that Danaus brought the alphabet direct from Egypt.

Nor can there be a reasonable doubt that the elder, and not the younger, Anaximander is intended. The datum furnished by the scholiast is referred to Apollodorus of Athens, who had cited 'the Milesian historians Anaximander, Dionysius and Hecataeus' for the opinion in question, in his great historico-geographical treatise *On*

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46 V³ I. 21, 24.

47 See Hdt. 5. 58; 2. 37, 44, 49, 54, 79. For the views of Hecataeus in regard to the derivation of Greek civilization from the Orient, see Jacoby, col. 2678, 2697, 2741.

the Catalogue of Ships. He it was, we recall, who somewhere met with Anaximander's book, from which he must have derived the information enabling him to date the old Milesian with such singular precision. The character of that information, as we have pointed out, was presumably historical and geographical. The datum regarding the origin of the Greek alphabet, being of the same character, naturally found its way into a work belonging to the same line of tradition. But the date of Anaximander was, as we have seen, properly given in Apollodorus' *Chronicles*. Since he was especially interested in chronology, we should expect Apollodorus to give the names in chronological order.[48] In the versified *Chronicles* he might for metrical reasons depart from this natural order, but not in his prose treatise *On the Catalogue of Ships.* He observed, therefore, the order — right or wrong — in which these writers appeared in his *Chronicles*, barring metrical difficulties not likely to occur except in the event that they had to be mentioned in the same clause.

Now it happens that the names Anaximander, Dionysius, Hecataeus actually follow one another in alphabetical order. In the case of Anaximander, acknowledged to be older than the other members of the group, nobody would deny that chronological considerations might nevertheless have determined his position at the head of the list; but in regard to the other two question will at once arise, because Dionysius is commonly considered junior to Hecataeus. We have then to canvass the question of their chronology, especially in so far as Apollodorus may be supposed to have been concerned in fixing it. The datum unquestionably from his treatise *On the Catalogue of Ships* we have seen, as well as the reasons for regarding his list as arranged in chronological order. Aside from this we have several other statements. Heraclitus, as we have observed, names his polymaths in the following sequence: Hesiod, Pythagoras, Xenophanes, Hecataeus. That this order is roughly chronological will not be denied, though a question might arise regarding the relative ages of Pythagoras and Xenophanes. They were in any case roughly contemporary, and Xenophanes referred to Pythagoras.[49] Hence we may disregard this

48 In careful prose writers this was the common practice; cf. Hecataeus *fr.* 143 and 365; but it was of course not invariable. Sometimes, as *e.g.* in scholia carelessly composed or resulting from the abbreviation of longer discussions, the rule is not observed, cf. Hecataeus *fr.* 334. In the case of Apollodorus it is almost a foregone conclusion that he followed this practice, natural to one who was so deeply interested in determining the chronology of authors.

49 *Fr.* 7 Diels. This does not, of course, prove that Pythagoras was the older; but it might be so taken by Heraclitus.

nice question of chronology, which cannot be settled. Hecataeus, at all events, was younger than both and earlier than Heraclitus. We have, moreover, the evidence of Herodotus,[50] who represents Hecataeus as prominent in the councils of Miletus in the Ionian Revolt (499 and 497/6). The rôle which he plays in these events proves that he was an 'elder statesman.' In the article of Suidas about Hellanicus there is clearly a confusion, which others have sought to correct.[51] In view of this corruption of the text, one cannot of course adduce it in evidence. Even worse is the entry of Suidas in regard to Dionysius. "Dionysius of Miletus, historian: *Events after Darius* in five books, *Geography, Persian History* in the Ionic dialect, *Trojan History* in three books, *Mythical History, Historic Cycle* in seven [52] books." Here, as all acknowledge, we have a hopeless jumble arising from the confusion of an indeterminate number of the almost innumerable writers who bore the name of Dionysius. This being so, we must clearly rule out this datum also as incapable of yielding a date; for the only item in this bibliographical farrago possibly serviceable for chronological purposes, the *Events after Darius*, not only shares the general doubt attaching to the list as a whole, but is in itself ambiguous.[53] That Dionysius of Miletus was an historian we must grant, and that he wrote on Persian history in the time of Cambyses and Darius I. is exceedingly probable.[54] Beyond that we have thus far been unable to go.

There is, however, another chronological datum to be found in the entry of Suidas regarding Hecataeus: "Hecataeus, son of Hegesimander, of Miletus; he lived in the time of Darius who was king after Cambyses, when Dionysius of Miletus also lived, in the sixty-fifth Olympiad: a writer of history." This chronological notice, obviously

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50 Hdt. 5. 36, 125.

51 καὶ Ἑκαταίῳ τῷ Μιλησίῳ ἐπέβαλε γεγονότι κατὰ τὰ Περσικὰ καὶ μικρῷ πρός (πρόσθεν?). Here various scholars have proposed to read γεγονὼς (for γεγονότι), referring the date to Hellanicus, who would thus be regarded as forty years of age ca.480 B.C., and consequently forty years younger than Hecataeus. That the text is corrupt is evident: the proposed emendation is doubtful.

52 Eudocia says '*six* books.'

53 Not to speak of 'Darius the Mede,' *i.e.* Astyages (according to Syncellus), there were three Persian kings of that name, between whom it is impossible to choose without knowing which Dionysius is supposed to have written the history.

54 Certain parts of the story of Cambyses as told by Herodotus, such as the embassy of the Ichthyophagi to 'the long-lived Ethiopians' (3.20 sq.), certainly are steeped in Milesian science and almost certainly come from a Milesian author; but there is nothing apparently to suggest Hecataeus. Here one might well conjecture that Herodotus was borrowing from Dionysius. Other similar matters occur in the third book of his *History*.

derived from some chronicle, was it would seem, somewhat awkwardly combined with a bibliography, which has in consequence been lost.[55] It admits of no doubt, however, that the chronologist was himself quite clear in regard to the Milesian writers and their date; and I do not entertain the least doubt that the chronological datum comes ultimately from Apollodorus. In form it agrees perfectly with numerous others derived from his *Chronicles:* it is obviously intended to fix the *floruit* of Hecataeus and Dionysius at the year 520 B.C., the year in which Darius, the last Persian king with whom the chronologist could establish a connection, became King of Babylon.[56] Apollodorus no doubt dated by the Athenian Archon Eponymus, but some later author converted the date, using the corresponding Olympiad. We thus see that Apollodorus might perfectly well name the two Milesian historians in either order, Dionysius and Hecataeus, or Hecataeus and Dionysius, since he fixed their *floruit* in the same year. The two statements therefore complement and confirm one another. Regarding Hecataeus, at any rate, the date, which would make him over sixty years of age at the time of the Ionian Revolt, cannot be far wrong: and respecting Dionysius we have no information that in the least justifies us in questioning the correctness of Apollodorus' calculation.

Having disposed of the objections to the acceptance of the statement attributed to Apollodorus and having justified our reference of it to the elder Anaximander, it remains for us to signalize the importance of the fact reported as evidence of the character and contents of his book. So much at least we may confidently affirm: that it did not confine itself to cosmological and other cognate matters which might justify the title *On Nature* to the exclusion of such a title as *Tour of the Earth;* for a glance at the list of authorities whom the scholiast on Dionysius Thrax cites in regard to the origin of the

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55 The statement that Herodotus, his junior, profited by the work of Hecataeus, which here occurs, recurs elsewhere; when it is asserted that Hecataeus was a 'hearer' of Protagoras, there is obviously a confusion, most probably due to the dropping out of a sentence or two. Protagoras probably came into the passage as going with Herodotus to Thurii; beyond that one is at a loss for reasonable conjecture. The further statement of the entry regarding Hecataeus, that he was the first to publish a prose history, while Pherecydes published the first prose treatise, a farrago of literary odds and ends belonging to the class of *εὑρήματα*, shows the general character of the source from which Suidas derived this part of his notice. It somehow crowded out the data regarding Hecataeus' own book derived from the catalogues of the Alexandrian libraries. These we are able in a manner to recover from other sources.

56 This is precisely in the manner of Apollodorus. Why Jacoby should deny that this date comes from him I cannot comprehend.

Greek alphabet, as well as a consideration of the logical context of such an inquiry, must suggest that the book of Anaximander, like those of Hecataeus, Dionysius, and Herodotus, belonged at least in part to the historico-geographical line of tradition. Regarding Hecataeus and Dionysius there can be no question; but Anaximander also is classed with these admittedly 'historical' writers.

We may be pardoned for insisting that we are thus abundantly justified in holding fast our faith in the essential correctness of the bibliography of Anaximander preserved by Suidas and in the derivation of the list of titles from the catalogues of the Alexandrian libraries. It was there in all probability that Apollodorus found the book, which, as we have seen, was known by report, if not by sight, to Eratosthenes. It is possible, indeed, that Apollodorus discovered the book of Anaximander at Pergamum, where he was active in later life, and where his interest in antiquities must have made him welcome; but the book must have been rare in his day, and Alexandria is surely the place where we should most expect to find it. If we assume that the book was there in the time of Eratosthenes we shall not violate probabilities; for the case, far from being unexampled, would find a close parallel in the same field. We know from Athenaeus[57] that Callimachus, presumably in his catalogue, attributed either the *Tour of Asia*, or, more probably, the entire *Tour of the Earth* of Hecataeus, of which the *Tour of Asia* was a part, to one Nesiotes, though we do not know on what grounds.[58] Conjecture is easy but profitless. But Eratosthenes, his successor, as we learn from Strabo, vindicated the authorship of Hecataeus as confirmed by the *Genealogies*, which he apparently regarded as a separate work and above suspicion. When, therefore, Apollodorus set aside the doubts of his predecessor regarding the book of Anaximander he was merely doing what other scholars, ancient and modern, have done. It was only natural that such indications of doubtful authorship as may have been noted in the Alexandrian catalogues should be expunged, once the doubts of the librarians were resolved.

There still remain to be considered a number of passages before we can be sure that we have garnered in all the notices of Anaximander's treatise. Athenaeus,[59] expatiating on the *skyphos*, a cup or goblet, remarks that the name occurs as both a masculine and a neuter noun and is sometimes written *skyphos*, sometimes *skypphos*. In this con-

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57 70 B; cf. 410 E.
58 See note 37 above.
59 Athen. 498 A sq.

nection he says, "Similarly Anaximander in his *Heroölogy*, in these terms: 'Amphitryon having parcelled out the booty among the allies and keeping the skypphos (masculine) which he chose for himself,' and again: 'Poseidon gave the skypphos to his son Teleboas, and Teleboas to Pterelaus; this he took and sailed away.'" Athenaeus, you observe, is concerned solely with a grammatical point; it is worthy of remark, however, that he cites as using the form skypphos, besides Anaximander, the poets Hesiod and Anacreon, who employ it, as does Anaximander, as a masculine noun of the second declension, and the epic poet Panyassis, a (somewhat older) relative of the historian Herodotus, who treats it as a neuter noun of the third declension. The agreement at this point of the Anaximander in question with Hesiod, who was older, and with Anacreon, who was but slightly younger, than the elder Anaximander, and the disagreement of Panyassis, junior by over two generations, constitute a point, of no great weight perhaps, but taken for what it is worth, in favor of the elder rather than of the younger Anaximander. But this is not the only possible clew; for Athenaeus cites the title of the work from which the quotations derive. They come, he says, from the *Heroölogy* of Anaximander. This title occurs neither in the bibliography of Anaximander the Elder nor, of course, in that of the Younger, which contains but the one title, *Interpretation of Pythagorean Symbols;* but, as we have seen, whereas in the case of the latter Suidas does not intimate the existence of other titles, in that of the elder Anaximander he prepares us for more by adding to his list "and certain others." If we attach any weight to the bibliographies of Suidas, the finding of a new title attributed to the elder Anaximander can occasion no surprise; but with his namesake the case is quite different. More important, however, than either of these considerations is the circumstance that the title quoted by Athenaeus is identical with one of the titles of Hecataeus. Respecting the latter we do not know whether the *Heroölogy* is another name for his *Genealogies* or *Histories*, or a subtitle of that treatise. It can hardly be doubted, however, that it is one or the other. Bearing in mind the close relation which certainly existed between Hecataeus and his predecessor and the established fact that the elder Anaximander included matters of history connected with the mythical past in the book found by Apollodorus, one cannot reasonably question his claim to these passages from Athenaeus. To assign them to Anaximander the Younger, of whom we know nothing except that he lived more than a century after Hecataeus and wrote an *Interpretation of Pythagorean Symbols* is not critical scholarship, but the renunciation of it.

Now if we attribute the words quoted by Athenaeus, as apparently we are bound to attribute them, to the elder Anaximander, we see both by the title and the contents that he dealt with the heroic genealogies which were studied, as we know from Hecataeus, Pherecydes, and Hellanicus, for clews in regard to early history and geography.[60] The scope of his work or works is thus shown, as indeed we were led to infer from his reference to Danaus and the importation of the alphabet from Egypt, to have included not only the beginnings of the cosmos but also the legendary history of Greek lands. In all such cases, whether known from Greek or Hebrew sources, the geographical and ethnographical status at the time of the would-be historian furnished the starting point which was to be explained by reconstructing the past.[61] Whether it be the story of Creation and the genealogical tables of the descendants of Noah, in the *Book of Genesis*, or the Hesiodic *Theogony* and *Catalogues*, the purpose and the method are everywhere essentially the same: it is the conception of a universal history. One may call it *Genesis*, if one will, or *Physis*, if one prefers;[62] but the central interest of the men who thus set themselves the task of reading the beginnings is always in the earth and its inhabitants as they found them. The description of the earth and its peoples is what constitutes geography, and has always constituted the science. If, as is certain, Anaximander sketched the beginnings of the cosmos and the early history of the Greeks as reflected in the genealogical tables of the heroes, there is no reason why we should doubt that he who constructed a map of the earth actually wrote, as he is reported to have written, a geographical work, which gave the disposition of the peoples and the boundaries of continents and lands in agreement with the pictorial representation of his chart. That was at once the logical thing to do and the thing which by all the tokens we must infer he was most concerned to do.

There is one more reference to an Anaximander which calls for consideration. In Xenophon's *Symposium*[63] the guests are asked to

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60 The reference to the goblet, which descended as an heirloom from father to son and then fell into the hands of Amphitryon, the putative father of Heracles, served no doubt as a means of fixing the date of the great hero from whom many royal families, Greek and foreign, claimed descent. The date of Heracles thus became an important datum for history and historical geography. Cp. Hdt. 6.53.

61 See Περὶ Φύσεως, p. 85 sq., especially n. 32.

62 The Greek said not only αἰτιολογεῖν and φυσιολογεῖν, but also in essentially the same sense — γενεαλογεῖν: cp. Hippocr. Περὶ ἑπταμήνου, 4 (7.442 L.) and Aelian, *V. H.* 4,17 καὶ τὸν σεισμὸν ἐγενεαλόγει (Pythagoras) οὐδὲν ἄλλο εἶναι ἢ σύνοδον τῶν τεθνεώτων.

63 C. 3.

[illegible] themselves, and [illegible] father had compelled him to [illegible] of H[illegible] and that he would still [illegible] with the [illegible] as much and that [illegible] greater simplicity in the [illegible] circumstance and present a [illegible] that the simplicity [illegible] out. Again, it is not necessary to assume that all the 'teachers' of Niceratus practised their hyponoetic interpretation on Homer. Stesimbrotus, who wrote about moot questions in Homer, may be acknowledge to have done so; Anaximander the Younger, who interpreted the Pythagorean symbols, may be said to have fulfilled all that the words of Xenophon require us to suppose if, as is altogether probable, his method of exegesis was essentially the same as that applied to the poets.

Of the elder Anaximander we have not spoken in this connection because very little can be said in favor of him in relation to the hyponoetic interpretation presupposed by the passage from Xenophon's *Symposium*. It is true that indications are not wanting of somewhat allegorical interpretations of Homer at the close of the sixth century B.C.; but neither is there any evidence that the practice dates back to the middle of that century, nor is there anywhere a hint that the great Milesian would have concerned himself with it, if it did. It was rather in the latter half of the fifth century that the method which was destined in time to enjoy great favor and an extended application won its first laurels. Hence we may with tolerable confidence identify the Anaximander of Xenophon's *Symposium* with Anaximander the Younger, the interpreter of Pythagorean symbols.

If our results are assured we have added at least two brief verbatim fragments to the one previously allowed to have come from the book of Anaximander, the reputed originator of Greek prose. Of these fragments it is not necessary now to speak at length; it is perhaps deserving of mention, however, that in point of style they closely resemble not only the fragment communicated by Simplicius but likewise the fragments of Hecataeus. Of the three the sentence quoted by Simplicius is the most complex; for the most part their structure is exceedingly simple, no elaborate periods being attempted; but there is no want of precision or lucidity. Hecataeus, as far as we can judge, made no advance in this respect; even Herodotus in general constructed his sentences on the same model, only occasionally betraying the influence of fifth century sophists. It would therefore be as proper to speak of Herodotus as primitive as it would of Anaximander. That Simplicius should think the terms used by Anaximander rather 'poetic' is only a compliment to the imaginative vigor of his speech.[64]

64 V³ I. 15, 29. There is nothing really 'poetic' about the phraseology of Anaximander; the style is imaginative and more or less elevated, but less so than that of Heraclitus. It was probably due to the theory that prose in Anaximander's time was new, being recently introduced to supplant verse,

Though Anaximander and Hecataeus of course used Ionic forms, the distinctive differences from the Attic have almost entirely vanished in the course of transmission.

II.

Our survey of the tradition regarding Anaximander's book shows that it touched on matters not germane to a philosophical treatise, properly so called. This fact raises a variety of questions which are of importance in relation to the history of Greek philosophy; for though Thales is commonly regarded as the originator of this kind of inquiry, we know so little of his scientific work that it is only with his successor Anaximander that we begin with a certain assurance to discern the character and direction of thought cultivated at Miletus, the city which bred the new interests destined to have an illustrious career in the western world.

Were it profitable to express a feeling of regret, perhaps the greatest loss to history might be said to be that which virtually blotted out the story of this ancient city of Miletus, the pride of Ionia. Even the excavations conducted there in recent years have produced almost nothing: one gratefully excepts the headless statue erected by appreciative contemporaries to Anaximander, showing that they valued him even in his lifetime. What we need to remember is that Miletus even in his day had passed the zenith of its power: it is always in the afternoon or evening of a civilization that man turns to reflection on the world, its origin and its destiny.[65] Of the earlier period, in which Miletus ran its feverish course of unexampled activity, there are no written records extant: like the good mother she was, she lived on in her almost countless children, the colonies she had planted. Among the last of these was Apollonia, which Anaximander served as founder.

Thales seems to have lived to an advanced age and to have died about the same time as Anaximander. There is every indication

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that Simplicius added the reference to the poetic style. See Strabo 1.2,6 C. 18. In reality Anaximander may quite possibly have derived his form of expression and the suggestion of the cosmic process in question from a curious practice observed in his home, which Strabo reports (12.8,19 C 580): *φασὶ δὲ καὶ δίκας εἶναι τῷ Μαιάνδρῳ, μεταφέροντι τὰς χώρας, ὅταν περικρουσθῶσιν οἱ ἀγκῶνες, ἁλόντος δὲ τὰς ζημίας ἐκ τῶν πορθμικῶν διαλύεσθαι τελῶν.*

65 It is well to recall that Theophrastus, the first formal historian of philosophy, was not of the opinion that it really began with Thales. Simplicius (V^3 1.12,34) says, *Θαλῆς δὲ πρῶτος παραδέδοται τὴν περὶ φύσεως ἱστορίαν τοῖς Ἕλλησιν ἐκφῆναι, πολλῶν μὲν καὶ ἄλλων προγεγονότων, ὡς καὶ τῷ* [probably we should read *αὐτῷ*] *Θεοφράστῳ δοκεῖ, αὐτὸς δὲ πολὺ διενεγκὼν ἐκείνων ὡς ἀποκρύψαι πάντας τοὺς πρὸ αὐτοῦ.*

that they formed the nucleus of a group of men, their fellow-citizens, among whom certain intellectual interests were cultivated. It is perhaps misnaming this circle to call it a school; but it was destined to become the parent of all the 'schools.' One thinks of the circle formed by Franklin at Philadelphia, which eventually became a university: but Franklin had models to copy, while Thales and his fellows pretty certainly had no predecessors who worked in the same spirit.

What, we naturally ask, were the interests which inspired the goodly fellowship at Miletus to organize this first 'college?' They must have been related to the life of the group and of the city; but there is no indication that in the beginning political questions, questions that is concerning the government of Miletus, engaged the attention of the group. They were citizens, of course, and shared the life of the city; but it is as statesmen concerned with larger issues that we hear of them. Thales proposed the unification of Ionia into a federal state; what Anaximander may have done to win the honor bestowed on him we can only conjecture; Hecataeus, one of the last of the 'school,' was prominent as an elder statesman at the time of the Ionian Revolt. But of such political activity, looking to the control of the government of their city, as appears in the circle of Pythagoras, we have not a hint; though it is more than likely that Pythagoras, in this as in other respects, was inspired by the example of the Milesian 'college.' The discussion of the principles to be followed in founding colonies, in which Anaximander at least participated, inevitably led to the consideration of the ideal city; that it was a topic discussed by the Milesian 'school' goes without saying, but is abundantly clear upon reflection, for city-planning and the utopian schemes of the Greek historico-geographical tradition everywhere lead back to the Milesians of the sixth century. Such ideals quite naturally arose in a city with far-flung colonies and with relations, through Naucratis, with Egypt, which in the entire Greek tradition appears as the utopia *par excellence*.[66]

Aside from the direct reports regarding Thales, Anaximander and Anaximenes, who for us at least constitute the Milesian 'school of philosophy,' the best evidence we have regarding that memorable circle comes from our knowledge of the men most influenced by this triad. Leaving out of account those who are said to have had relations with 'the philosophy of Anaximenes,' because their debt is too special, we think necessarily of Pythagoras, Xenophanes and Hecataeus,

66 Of this I hope to treat at length on another occasion.

the three whom, with Hesiod, Heraclitus rebuked for their polymathy, which failed to teach them understanding. This list is not the product of chance; for a clear line, running from Hesiod through Thales and Anaximander, leads us to Pythagoras, the mathematician, to Xenophanes, the rationalist interested in ethnography and history, and to Hecataeus the geographer, historian, and naturalist. Heraclitus is said to have testified to the interest of Thales in astronomy, and Eudemus called him the father of geometry. The latter also, apparently, is responsible for the data we have for Anaximander's ideas regarding the magnitudes and intervals of the heavenly bodies and for the statement that he outlined the subject of geometry.[67]

How these several intellectual interests were cultivated in the Milesian circle, we do not know; but it is not difficult to discover a relation between their studies and the problems which crowded upon the intelligent citizens of Miletus. Thus Thales was credited with a work on nautical astronomy [68] and with various nautical devices natural in the busiest trading center of the Levant, whose sailors went everywhere. Of similar interest was presumably his study of the calendar. As for the pursuit of geometry, its relation to city-planning and to the allotment of lands was well recognized in antiquity; the relation of both astronomy and geometry to geography was no less distinctly seen.[69] The schematic geometrical treatment of the early Ionian (Milesian) maps is known to every student of ancient geography: hence we need not suppose that Eratosthenes first brought geometry to the service of cartography.

In the busy streets of Miletus there met men who had voyaged to Egypt and seen the Ethiopians, snub-nosed and dark of skin, and to Thrace, and knew its blue-eyed and red-haired inhabitants: [70] one could gather there, even without travel, to which every Milesian must have been tempted, the most varied lore about all sorts of strange peoples and their customs. Pretty nearly everything we learn of the 'barbarians' before the close of the fifth century comes ultimately from early Ionian writers. They interested themselves also in the progress of civilization and the steps and 'inventions' whereby it was advanced.[71]

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67 V^3 I. 19, 12, I. 14, 22.

68 Whether it really belongs to Thales or not — and this is not clear — the work was undoubtedly quite ancient.

69 See Strabo 2.5 and Ar. *Nub.* 201 sq.

70 Xenophanes *fr.* 16 Diels.

71 Xenophanes *fr.* 18 Diels; cp. the interest shown by Herodotus in the contributions of various foreign lands, especially Egypt and Babylonia, to Greek civilization. Strabo 2.5,18 expresses the thought which underlies the historico-geographical tradition from the beginning.

But all this store of information was in the true Greek manner to be somehow set in order and brought into relation to the time and place in which they found themselves. How consciously these early Milesians worked we do not know; but the result of their labors is writ large in the physiognomy of Greek science.

Hesiod occupies a strangely anomalous position in Greek literature. The Homeric epic, at least in its finished form, is acknowledged to be the product of Ionia. After their childhood days, in which the Ionians gave themselves to telling tales of the long ago merely for the delight they took in heroic adventure, ensues an age of almost total eclipse of this extraordinary people, from which it emerges in the sixth century, past its prime and in a measure decadent. When it thus again comes to view it is engaged in writing treatises in prose on scientific themes. Meanwhile, for us, the connecting link between the heroic epic and the sober prose of science appears in Hesiod, not in Ionia, but in the Mother Land, which was, intellectually considered, centuries behind the Greeks of Asia Minor.[72] It is perhaps idle to speculate, but one cannot refrain from asking how this fact is to be explained. Can we believe that there was really a lapse of consciousness in Ionia and that when the people suddenly awoke, they took the cues of their intellectual life from Greece Proper? Would it not be more reasonable to suppose that the beginnings of Ionian prose lie in that dark age, the general substance of the attempts of the Ionians at reconstructing their past being worked up by Hesiodic poets who borrowed their ideas from contemporary Ionia and the form from the Homeric epic?[73] If we could assume that the first 'theogonies' and 'catalogues' were in prose, we should be able to account both for their disappearance and for certain characteristics of the Hesiodic poetry itself.

Be that as it may, Miletus in the sixth century has definitively broken with the Hesiodic and Homeric past. It is not in a mood for poetry: the gods have melted away in the cosmos, and genealogies are useful solely as materials from which one may by criticism and combination extract history.[74] Miletus is at a certain point on a map of the earth, which has its assignable place in the cosmos. The

72 The importance of Boeotia, for example, in the early development of Greek religion cannot be denied; but this very fact, especially in view of the character of this religious movement, is really the best evidence of the essentially different attitude of the motherland. Ionia tended to secularize and rationalize everything.

73 Tradition said that Hesiod's father came from Cyme in Aeolis, on the border of Ionia; if this is legend, it is at least *ben trovato*.

74 One must be dull indeed to miss the fine irony of Hecataeus' reference to his descent from a god in the sixteenth generation (Hdt. 2.143).

Milesian who walks its streets feels himself likewise at a definite point of time, and he sets about reaching backward to fixed points in the past in order to reconstruct a chronology. His immediate data are furnished by family traditions, and an approximate, limited scale is constructed by using the generation as a unit. The absolute scale was not to be found in Greece: Hecataeus discovered it in the immemorial civilization of Egypt with it records running back thousands of years. The connection between the Greek and Egyptian scales, and therefore the control and placing of Greek dates in relation to the absolute scale, were brought about by certain identifications, no matter how arbitrary they may have been. These are the methods of a thoroughly rational and conscious science: they have been refined in the course of time, and the admissibility and validity of certain data have been more sharply scrutinized; but historical science in all its essentials was achieved in Miletus before the close of the sixth century.[75] This, as historians recognize, is the dawn of the historical period in Greece; and here we first find really historical dates. Dates for a generation or two farther back are approximately correct, as one might expect; beyond that we have, so far as Greek sources are concerned, nothing intrinsically different from the data with which the Milesians themselves had to deal.

This was the crowning achievement of the Milesians, and we are justified in regarding it as the expression of the dominant interest of the Milesian 'school.' To this result the several members of that epoch-making circle made their contributions. Anaximander, as we have seen, concerned himself with geography and history as well as with cosmology. We should like very much to know more about his book; but everything beyond what we have already said must be learned by inference.

There are two classes of facts from which permissible inferences may be drawn regarding the character of Anaximander's book. One consists of the testimony of the ancients regarding it and the opinions of its author. Of a part of these data we have already taken cognizance; others we have still to consider. When we shall have completed the review of the contents of the book, and drawn such inferences from them as appear to be justified, we shall be in position to institute a new inquiry: to wit, whether other works of like character existed, or still exist, from which one may draw further inferences.

Anaximander is generally regarded as a philosopher: hence we will begin with such opinions as seem to justify that title. He dealt, we

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75 This I expect to show more fully on another occasion.

are told, with the origin of the cosmos. Out of the Infinite it came: into the Infinite it will be resolved. The cosmos is described, and its constituent members are placed in due order with their intervals noted, the earth being at the centre. This world is but one of an infinite number, past, present and still to come. These worlds may be called gods. Certain phenomena of the cosmos are noted and explained. The earth does not lie in the plane of the zodiac, but the latter runs obliquely about it. The nature of sun, moon and stars is set forth, and eclipses are explained. He explains also the origin of the sea and the reason why it is salt, and offers explanations of various meteorological phenomena. Aristotle treats most of these topics in *De caelo*, *De generatione et corruptione*, and *Meteorologica*.[76]

One recurrent note cannot fail to arrest the attention: it is the quest after origins, and the glance ahead, seeking to divine the end. Whence and how the cosmos arose, where it will vanish at last; how the sea originated and became salt, and how in the end it will quite dry up:— these are, one may say, the all-inclusive questions, giving the setting of the whole and providing for matters in detail of lesser importance. Judging by our records, the outlook of Anaximander is in the last analysis historical. It is in keeping with this point of view that, as we have seen in tracing the record of his book, Anaximander displayed an interest in heroic genealogies and in the origin of writing, a question then as now of vital concern to the historian, because the use of writing is the precondition of the existence of records serviceable for history. Of like character are the questions, to which we know Anaximander addressed himself, concerning the origin of land animals and especially of the human species.

That Anaximander concerned himself with geography we know. We have tried to point out the actual, or at any rate the possible, relation to his geographical studies of his pursuit of geometry and his use of the gnomon. But this instrument had obvious uses also in determining the calendar, which is the basis of chronology and of history; for history without definite measures of time is impossible. History and geography go necessarily together: in Greece they formed from the beginning a unit, geography being generally treated in excursus in the historical narrative. It was only by Ephorus and Polybius that geography came to be set apart as distinct books, though embodied, as were the earlier geographical excursus, in works by inten-

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76 The importance of Aristotle's *Meteorologica* for the spirit and contents of Milesian science does not seem to be fully recognized,— probably because he does not often refer to his predecessors by name.

tion and predominant interest historical. This statement would be paradoxical, were it not for our knowledge of Hecataeus and Herodotus: even where the predominant interest is apparently geographical, the geographical account falls within the historical scheme. In describing countries which furnished no historical clues the *status quo* of course alone was given.

When one tries to form a conception of Anaximander's book, one thinks inevitably of Hecataeus, his successor, who can hardly have remained uninfluenced by his example. As we have before remarked, the traditional titles applied to the work of Hecataeus are numerous; two, however, are clearly inclusive of the rest, (a) *Histories* (or *Genealogies*) and (b) *Tour of the Earth.* In later times these were regarded as separate treatises, but it is far from certain that they were originally and to the mind of their author distinct. Two facts point rather to the conclusion that they were conceived as a unity. First, we know of only one introduction, in which the author gave his name, and that was prefatory to the *Genealogies*. Secondly, the *Tour of the Earth*, being evidently without express mention of its author, was mistakenly assigned to Nesiotes, and therefore the claims of Hecataeus were for a time disputed. The statement sometimes made[77] that the *Tour of the Earth* was composed before the *Histories*, is wholly without foundation. Logically the *Tour of the Earth*, having regard primarily for the *status quo* in its author's time, follows the *Genealogies*, which treated of the past. If we assume that Hecataeus composed his treatise as a single work, which was subsequently divided for convenience, the facts known about it are most readily explained.[77a] To be sure, we are thus led to postulate in the sixth century an historico-geographical treatise in size comparable perhaps to the *Histories* of Herodotus. To some this may seem incredible: perhaps they shall have to revise their preconceived notions regarding what was possible in sixth century Miletus.

If we ask where in such a treatise, as we suppose Anaximander's

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77 Jacoby, col. 2741–2.

77a That the Γενεαλογίαι and the Γῆς περίοδος were not really distinct but merely separated, either arbitrarily or for practical purposes, is indeed not susceptible of strict proof, but is both in itself probable and suggested by analogy. Strabo 7.3.9 C 302 quotes from the (geographical) fourth book of Ephorus [*fr.* 76 Müller] the statement that Phineus was borne by the Harpies

Γλακτοφάγων εἰς γαῖαν, ἀπήναις οἰκί' ἐχόντων

as derived from Hesiod's Γῆς περίοδος, a passage which Rzach [*fr.* 54], following Schoemann, *Diss. de comp. Theogoniae Hesiod.* p. 20, is certainly right in assigning to the Hesiodic *Catalogue.* In the same way it is quite impossible to dissociate Anaximander's Γῆς περίοδος from his Ἡρωολογία.

to have been, certain questions on which he held opinions may have been discussed, we again look to the models which have come down to us intact. Let us take Herodotus and Strabo as examples. The debt of Herodotus to Hecataeus is acknowledged. We may well suppose that it was the *Tour of the Earth* to which the fifth century historian was chiefly indebted,[78] though he unquestionably used also the *Genealogies*. Now Hecataeus clearly gave no cosmology: so far as he interested himself in matters pertaining to meteorology and kindred fields of science, he brought forth his opinions in connection with the description of particular lands, especially of Egypt. After Anaximander the 'school' of Miletus seems to have had a fate similar to that of Aristotle: where the master, in the true encyclopedic fashion, sought to cover the whole field of science, his successors divided the field. There is no evidence that Anaximenes gave any thought to geography and history, and the same is true of those who are brought by tradition in relation to 'the philosophy of Anaximenes'; while Hecataeus, as has already been remarked, neglected cosmology, and devoted himself to history and geography. Herodotus, though with inadequate comprehension of the spirit and the achievements of the Ionians, was inspired by the example of Hecataeus and Dionysius, thus falling roughly in line with their branch of the tradition originating at Miletus.

Another branch of the tradition can be traced through Eratosthenes to Strabo: it is largely indebted, however, to the collateral line which is concerned primarily with history. Geography, as we have previously remarked, had come to occupy a place apart as an independent science. Whether Democritus was at least in part responsible for this innovation we cannot say; certainly Ephorus, by separating his geographical books distinctly from the historical, contributed not a little to this result. Eudoxus certainly conceived his geography as related to his astronomy. In Eratosthenes, as in Strabo, the discussion of the earth, as the subject of geography, is made to follow that of the cosmos.[79] Anaximander had done the same, describing the circles of the celestial bodies with the earth at their centre. Here, then, we have those parts of his treatise to which could be given the titles *Sphere* or *On the Fixed Stars*. This, we may be sure, formed the

78 See Hecataeus *fr.* 292. Jacoby tries to determine in detail the debt of Herodotus to him.

79 See Strabo 1.4,1; 2.5,1 sq. Strabo is himself so much occupied with his criticism of Eratosthenes that he cannot describe the cosmos, though he recognizes that the beginning of the descriptive part of geography is the proper place for the astronomical setting of the earth. Strabo 1.3,3 C 48 sq. is important for the relation of the discussions περὶ φύσεως to geography.

beginning of his treatise. It is a matter of no small interest, however, that we find not only in Strabo but in Diodorus also a second cosmology introduced in their accounts of Egypt.[80] This fact can hardly be due to anything else but the force of literary tradition. There were those who had discussed the earth both in relation to the heavens and to the geological history of our planet in connection with Egypt, with which they must have made a beginning. One may plainly see the results of this practice in Herodotus, traces of it being visible throughout the entire geographical tradition. Hecataeus, as we have already remarked, set forth his own geological opinions chiefly in his account of Egypt; but the observations upon which his opinions were based, such as the presence of fossils in the stones of the pyramids, were not new in his day, as we chance to know that Xenophanes had previously made similar observations in other places. There can hardly be a doubt regarding the source of the latter's interest in such evidence, since his relation to the earlier Milesians is unquestionable. How significant these facts are may be seen when one considers another point. Later geographers regularly began their description of the *orbis terrarum* with the Pillars of Hercules, passing clock-wise round the Mediterranean. Now, if it be true, as has been plausibly maintained,[81] that Hecataeus of Miletus began his *Tour of the Earth* with the Pillars of Hercules, proceeding clock-wise about the *orbis terrarum*, the whole of Europe and Asia (proper) had been traversed before he reached Egypt. That under these circumstances he should have paused at this point to give a detailed account of the earth and its formation would seem to call for an explanation, the need of which is further emphasized by the fact, already mentioned, that even the

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80 Strabo 17.1,36 sq., referring back to 1.3,4. Much that in the latter passage is referred to Eratosthenes and Strato can be shown, by reference to the second book of Herodotus, to come from the early Ionians. In Diodorus the situation is especially significant, because he begins his history with Egypt. The result is that the same themes are twice discussed (1.6–8 and 1.10 sq.) in close succession with a clear break between. The matter has been discussed, though with little intelligence, by K. Reinhardt (*Hekataios von Abdera und Demokrit*, Hermes XLVII, 496 sq.), who tries to get rid of the break. The duplication is clearly due to the amalgamation of two lines of tradition, one of which began with a cosmology proper while the other, though not wholly ignoring cosmology, treated of the φύσις γῆς as introductory to the study of Egypt, with which history and geography began. Both lines of tradition began with the ancient Milesians, and for a time were separated, only to be reunited at the last with the signs of the imperfect combination left to tell the tale.

81 See Jacoby, col. 2691.

late historians like Diodorus, and geographers, like Strabo, at this point insert a cosmology.[82]

This phenomenon, hardly to be explained except by the conservatism of literary tradition, raises the question whether Hecataeus, like his successors, was not herein betraying the influence of his predecessors. But of predecessors of Hecataeus in this field we know Anaximander only, unless we should assume that Thales also was in some sense a geographer. For Thales, however, there is no direct evidence whatever. Nevertheless there is a point that deserves consideration in this connection. Almost the sole doctrine attributed to Thales is that all things come from water: which Aristotle and Theophrastus interpret with reference to the origin of life. Now Anaximander likewise expressed opinions regarding the origin of life, animal and human; and he also held that it originated in water, as with the gradual progress of evaporation of the once all-engulfing sea dry land emerged. But not only Herodotus, but the entire Greek tradition, where reference is not had to such myths as that of the creation of man by Prometheus, represent the 'scientific' theory of the origin of life as brought into relation with the swamps of the Nile Delta. There fishes were spontaneously generated; there existed the ideal conditions for the beginnings of life; there was the cradle of the human race and the fountain head of civilization. There, we may with reason assume, Anaximander (and perhaps Thales) laid the scene of the early life history of the earth.[83] But if this were true, the work in which Anaximander set forth his theory of the origin of life would bear a definite relation to the later known works of the historico-geographical tradition. The derivation of the alphabet from Egypt through Danaus proves that Anaximander shared the view which sought in that land of many wonders the beginnings of civilization.

It is quite possible also that many of the explanations of meteorolog-

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82 The 'cosmology' is in any case embryonic, and may more properly be called a brief treatise Περὶ φύσεως. What signifies is not so much the content of the account (although here also tradition was strong), since the explanation of the world and the formation of the earth, of the *origines* in general, necessarily varied with the philosophical school to which the writer belonged, as the bare fact that an explanation of any sort was given at the particular place in the general economy of the treatise; for this, as one clearly sees from Strabo, was a matter of constant discussion, in which tradition ruled. Strabo 2. 5 is here invaluable as showing the topics properly treated at the beginning of a descriptive geography.

83 On another occasion I hope to show in considerable detail how much of early Milesian science was concerned with Egypt. A critical study of Egypt as it appears in the Greek tradition will serve to elucidate at many points the thought of the Milesians, including Anaximander.

ical and other phenomena which Theophrastus and his successors found in Anaximander's book were scattered through it and were offered in connection with various lands or places. Not only the second book of Aristotle's *Meteorologica*, but the various historical and geographical treatises also which survive from antiquity in whole or in part, afford sufficient examples to justify such a theory. It is not necessary to assert that this was true of Anaximander's book; it suffices for our purposes that it may be true. For a consideration of the character of the doxographic record of his opinions will readily show that it affords no presumption whatever regarding the form of his treatise. Through the genius of Professor Diels [84] we are enabled to gather from the welter of late extracts a view of the *Opinions of the Physical Philosophers* of Theophrastus, the disciple and successor of Aristotle, from which ultimately derive most of the statements regarding the doctrines of the early thinkers. That work, as we see, was a systematic account of Greek philosophy arranged under heads following closely the order which Aristotle himself had used in his treatises. Thus the arrangement is that of the Peripatetic historian and bears no necessary relation to the order in which the early 'philosophers' set forth their opinions. What we know, moreover, of the method of Theophrastus, who was wholly under the influence of his master, suggests caution in respect to the 'philosophical,' that is to say especially the metaphysical or ontological, doctrines which he reports; for both Aristotle and Theophrastus were prone to discover in statements intended as descriptions of physical processes a deeper meaning which would bear a metaphysical interpretation.

Now it happens that Theophrastus, following Aristotle, allowed no place in his scheme for matters pertaining directly to history and geography. We cannot, therefore, be surprised that the doxographic tradition contains no hint of Anaximander's services in these directions: they were not germane to the 'physical philosopher' with whom alone Theophrastus was concerned. Strictly speaking, to be sure, this statement is at best a half-truth. Theophrastus, like Aristotle, speaks not of physical *philosophers*, but of *physikoi* or *physi-*

84 In his *Doxographi Graeci*, Berlin, 1879. A serviceable summary is given by Professor Burnet, *Early Greek Philosophy*,[3] pp. 31 sq. On p. 181 sq. Diels gives a table of the contents of the 'Vetusta Placita' or the 'Posidonian Areskonta,' which most fully represents the original Φυσικῶν δόξαι of Theophrastus. One sees that it includes, besides the topics treated in Aristotle's *Metaphysica*, *De caelo*, *De generatione et corruptione*, and *Meteorologica*, the subjects which belong to his *De anima* and the lesser psychological treatises and also a large group of questions discussed by medical writers who wrote Περὶ φύσεως.

ologoi, meaning thereby those who discoursed about *physis*. Having elsewhere treated of the latter term I have no intention of taking it up afresh. Suffice it for the present to say that men of the most diverse interests in detail had in the fifth century much to say about *physis* who never found their way into the group recognized as physikoi or physiologoi by the doxographic tradition. When one considers the members of the group in detail, and the questions which were noted as falling within the purview of the historian of their opinions, one sees that the classification, though intelligible from the point of view of Aristotle's conception of philosophy, is altogether arbitrary as regards the fields of knowledge cultivated by men of science in that day. Opinions relating, or supposed to relate, to 'material' and 'efficient' causes, to cosmology, to the more important meteorological and terrestrial phenomena, to the gods, to the soul and its faculties, and to the more striking physiological functions, were included; such as concerned the biological sciences, zoölogy and botany, and the fields of history and geography, were ignored. It is the same process of selection which, at a later stage, determined the choice of those among the writings of Aristotle that should have the honor of a commentary; only in the latter instance the interest of the schools in logic led to the inclusion of the *Organon*.

The same inconsistency is found in regard to the title *On Nature* (Περὶ φύσεως). Probably the earliest extant reference to this catchword as a title occurs in the Hippocratic treatise *On the Old School of Medicine*:[85] "Certain persons, physicians and sophists, assert that no one can know medicine who doesn't know what man is, how he originated, and whereof he was framed at the start: that is what he must fully learn who would properly treat men. But their proposition relates to philosophy, like Empedocles or others who have written *On Nature*." Earlier references in the fifth century are to be found in Euripides and elsewhere, but hardly so pointedly or so clearly to titles of treatises. Finally, as in Galen,[86] we find it the generally

85 Περὶ ἀρχαίης ἰητρικῆς, 20.

86 V^3 I. 131, 18. Besides the persons specifically mentioned by Galen the following are elsewhere cited as having written Περὶ φύσεως: Anaximander, Xenophanes, Heraclitus, Orpheus, Anaxagoras, Diogenes of Apollonia, Metrodorus, and Zeno of Citium. In my Περὶ Φύσεως I spoke somewhat at length regarding the dominant interests of these treatises. A more intensive study of the historico-geographical tradition since I wrote that essay has not only confirmed my general view but has taught me much besides, especially in regard to the mutual relations of the early scientists in their several fields. If I ever set forth my conclusions as a whole it will have to be done in a history of early Greek science, which I have contemplated for many years.

accepted name: "All the treatises of the ancients were entitled Περὶ φύσεως,— those of Melissus, Parmenides, Empedocles, Alcmaeon, Gorgias and Prodicus, and all the rest." This is no rational classification: it is an omnibus label which gives no guaranty of the contents of any particular work to which it may be applied.

Hence we attach no importance to the fact that Themistius calls Anaximander the first Greek who to our certain knowledge ventured to write a treatise entitled Περὶ φύσεως, and that Suidas reports the same title alongside the others, which we have already mentioned, — *Tour of the Earth*, *On the Fixed Stars*, and *Sphere*. One is as well attested as the other; but all the other titles have this advantage over Περὶ φύσεώς that they suggest something definite, and likewise true, of the contents of Anaximander's book. Regarding this work we may say with confidence that it was conceived as a single whole and was presumably brief in comparison with the treatise of Hecataeus. The peculiar way in which Eratosthenes speaks of Anaximander in contrast with Hecataeus does, however, suggest the question whether the distinctly historico-geographical part of his book, which might properly be called *Tour of the Earth*, had not, like the similar portion of Hecataeus' work, become detached and had so come to lead for a time a separate existence until reclaimed for its author by Apollodorus, who discovered in it historical data calculated both to fix its author's date and to place his identity beyond question.

III.

We have spoken of the doxographic tradition and noted briefly its scope. Deriving immediately from Theophrastus, who was concerned with the φυσικοί, it revealed the conception he had of them and of the interests which properly characterized them. He does not expressly call them philosophers, but as such of course he regards them. We have already pointed out how closely the scheme of the Φυσικῶν δόξαι followed the order of topics in the systematic works of Aristotle and have remarked that in substance also, that is to say in his interpretation of the data regarding the early thinkers, Theophrastus (except possibly in a few instances) adhered to the views of his master. This was the more natural because Aristotle himself was wont to report briefly the opinions of his predecessors as furnishing the proper basis for a discussion of the several questions with which he had to deal. It is almost as if Aristotle had had at hand for the purposes of his lectures a brief digest of the earlier history of philosophy. What

Theophrastus did was in effect to expand these pronouncements of his teacher, using where he could the original texts for fuller statements, and criticising the views reported at greater length. Thus in a real sense the doxographic tradition is essentially Aristotelian. Now with the advance in the critical study of early Greek thought it has become increasingly apparent that Aristotle, with all his advantages and excellences, was not a safe guide for the interpretation of texts. Great as a systematizer and keen as a critic, he lacked the true historical sense; hence the manifest need of going, so to speak, behind the returns, of checking the Aristotelian account wherever possible by other data. Besides the authentic fragments of the early thinkers, to which the historian will have recourse, other sources of information exist, which it is not now necessary to discuss in detail; but it is of some importance to trace this tradition, which we may rightly call Aristotelian, somewhat farther back.

Behind Aristotle stands Plato, to whom we owe relatively few but precious statements regarding the opinions of his predecessors. Possessing perhaps the keenest intellect known to history, he was endowed likewise with that rarest of all gifts, the faculty of entering sympathetically into the point of view of men from whom he radically differed. This is what made him the greatest master of the philosophical dialogue, and might have made him the greatest dramatic genius, had he not fallen under the spell of Socrates. Not a slave to system, but following the argument wherever it might lead, two interests — perhaps, rather instincts — alone seemed to govern his thinking and writing, the love of truth and an innate sense of form. From such a man we may expect, as indeed we receive, reports which, when scrutinized with reference to his intention, are transparently true.

Before Plato we do not find in the works of philosophers references to one another which are of such a character as to yield appreciable assistance to the student of their opinions. There is, however, even here something that may be called a tradition: it concerns the very fundamental matter of the conception of the philosopher and of philosophy. We cannot here pause to review the evidence as to the employment of such terms as 'sage,' 'sophist' and 'philosopher'; but so much may be said to be beyond question: whether the old Milesians did or did not use the terms philosopher and philosophy, a change at least came in their employment by Heraclitus and the Pythagoreans. However much indebted both Heraclitus and Pythagoras were to the Milesians, they cannot have failed to perceive that they were making a distinct departure in their points of view and in the matters which chiefly claimed their attention. Of Pythagoras, indeed, we are able to judge solely by later Pythagoreans; as for Heraclitus, he has in his own words so clearly expressed himself as to leave no room for doubt. For him the philosopher, though he must of course know many things, does not attain wisdom by much learning; the wisdom of the philosopher lies in the understanding of the unifying and governing principle. Pythagoras might have said that it lay in learning the mathematical formula for the law. This new interest is metaphysical, or at all events akin to metaphysics. It does not, so far as I can discover, appear anywhere in the Milesians; but through Socrates, who was equally influenced by Heracliteans and Pythagoreans, this conception of what marks the philosopher and philosophy descended to Plato and to Aristotle, and so shaped the doxographic tradition.

The interests of the Milesians — chronology, descriptive geography, ethnography, ethnic and biological history — could not be included in the scope of philosophy proper. However scientific their methods and aims, they were then and still remain essentially empirical; however much they may in subordinate matters employ mathematics, principles, such as Heraclitus and the Pythagoreans sought, are not to be discovered there. Only in cosmology might one perhaps think to find an exception; but even here we pass from would-be history among the Milesians to mathematical and descriptive astronomy in the Pythogoreans, of whom Aristotle quaintly says that "they intend to construct a cosmos and to speak in the manner of the *φυσικοί*," [87] when in fact they were merely offering a mathematical construction of it.

From this survey of the doxographic tradition, it becomes apparent that the early Milesians, so far as they appear in it, are out of their element, and in any case could not hope to be represented sympathetically, especially from Aristotle onwards. If they appear at all, it must be in a capacity at least doubtful. Aristotle regarded Thales, Anaximander and Anaximenes as proposing doctrines concerned with ontological principles and with processes of change having metaphysi-

87 *Met.* 1091ᵃ 18 *κοσμοποιοῦσι καὶ φυσικῶς βούλονται λέγειν.* Actually, of course, they were interested chiefly in very different things; such resemblance as their language had to that of the *φυσιολόγοι* was probably due to one or other of two influences, either to the influence of the Ionians who were really *φυσιολόγοι*, or to that of our common way of thinking, which generally gives even to purely abstract and mathematical constructions a time-form. Hence the debate regarding the question whether Plato ascribed to the cosmos an origin in time.

cal implications. We have no intention of debating here the question whether in any or all of these matters he was right or wrong: for our purposes it is sufficient to say that in no instance is he certainly right, since the opinions in question are quite susceptible of interpretation with reference to purely physical facts or processes.

The influences which we have so cursorily reviewed gave rise to a body of statements of supposed fact and critical judgments, constituting and perpetuating a literary tradition. Properly it is only from Theophrastus onwards that one may call it doxographic; but for our present purposes we may apply this name to the earlier stages also, in which were framed the conceptions which dominate it to the end. Thanks to the intensive study of Greek philosophy and especially to the illuminating analysis of Professor Diels this particular literary tradition can be traced in the main with great precision. The phenomenon is, however, not at all isolated; for every literary kind has its traditions more or less clearly defined. This becomes at once apparent to the student who surveys any series of books on a given subject: the innovations appear trivial in comparison with the mass of purely traditionary matter and opinion. Mathematics, for obvious reasons, presents perhaps the best examples; as Sir Thomas Heath has recently said,[88] "elementary geometry *is* Euclid, however much editors of text-books may try to obscure the fact." These special literary traditions deserve far more attention and critical study than scholars have accorded them.

Now, as we have seen, a considerable field of early scientific thought and interest lies partly or wholly without the scope of the doxographic tradition. Considering the development which can be discerned in Greece from the fifth century onwards we are justified in speaking of it as that of history and geography together, because they were not really separate. To our view this fact is apt to be obscured by the solitary eminence of Thucydides, who breaks the line of continuity and with those whom he immediately inspired forms a group apart. Dealing with a circumscribed area well known to his readers he had no need to digress into geographical descriptions or considerable ethnographic details.[89] But if we disregard Thucydides and his kind, the continuity of the historico-geographical tradition in Greece is palpable. So evident is the wholesale appropriation of matter from predecessors

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88 *Euclid in Greek, Bk. I.*, Preface, p. v.

89 The same is of course true of chronology; for, as Thucydides was concerned solely with the Peloponnesian War, which lasted a few years only, he had no need to discourse on chronology in general.

by successors that an eminent student of ancient history mournfully denounces the procedure as plagiarism.[90] Without concerning ourselves with moral judgments we may content ourselves with signalizing the solid basis of what we call the tradition. Unfortunately there exists for this literary kind no such exhaustive treatise as we possess for the doxographers; but the complexity and the generally fragmentary state of the materials amply explain its absence.[91] Of the historico-geographical tradition we do, however, know enough to be able in most cases to tell whether a given writer belongs to it or not.

Now a survey of our sources of information regarding the earlier Greek thinkers, with whom we are at present chiefly concerned, is worth making for many reasons. We may disregard as sufficiently appraised those which fall to the doxographic tradition; but for Thales and Anaximander we must clearly take account of others. Thus Herodotus refers to the prediction of an eclipse of the sun by Thales, to the report current among the Greeks that he diverted the waters of the Halys and so enabled Croesus to cross, and to his advice to the Ionians to form a federal state in order to maintain their independence; he mentions, besides, the explanation of the Nile floods as due to the Etesian winds, which later writers of the historico-geographical tradition attribute to Thales.[92] We know, then, not to go farther into details nor to pass final judgment in disputable matters, that Thales figured as a man of science in the historico-geographical tradition even before Herodotus. Duly considered this certain fact is of prime importance. It is unfortunate on all accounts that Herodotus

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90 Hermann Peter, *Wahrheit und Kunst, Geschichtschreibung und Plagiat im klassischen Altertum*, 1911. He nowhere shows the least comprehension of the facts of human nature which underlie the creation of a tradition in art and literature.

91 F. Jacoby, *Über die Entwicklung der griechischen Historiographie*, Klio, vol. IX, has attempted a sketch which is the best available, but requires much revision and, in particular, an extension to include the connections of history with other scientific interests. Berger's *Geschichte der wissenschaftlichen Erdkunde der Griechen*[2], 1903, is even less satisfactory for similar reasons, though it has great merits in certain respects.

92 Hdt. 1.74; 1.75; 1.170; 2.20. For the various explanations of the inundations of the Nile see Diels, *Dox. Gr.*, pp. 384–6. Diels (*ibid.* p. 228 sq.) suggests that Pseudo-Plutarch derived these data from Theophrastus; this is possible, but by no means certain. In any case they derive ultimately from works belonging to the geographical tradition, including Hecataeus. Anaximander does not figure in the list, but this is doubtless due either to an oversight or possibly to confusion between him and Anaxagoras, who is credited with an explanation almost certainly older than he. A study of the entire tradition regarding Thales will be found instructive, but would here be out of place.

is the only extant representative of this branch of literature from the sixth and fifth centuries,— in fact down to the time of Diodorus; all the really representative authors in this kind being known solely through detached quotations or fragmentary reports at second or third hand. For Herodotus, though an accomplished writer and an indispensable source for the Persian Wars, comprehended little of Ionian science, and was never deemed worthy of being accounted a geographer until Strabo, almost as superficial as himself, did him the honor of so regarding him. Furthermore, whether from literary affectation or for other reasons, Herodotus often suppresses names and facts which were obviously known to him. Thus it happens that he is for our purposes as little serviceable as one who dealt with the same subject-matter in the fifth century could possibly be. What can be learned from his book has usually to be wrung from him as from an unwilling witness. Anaximander, who certainly played an important rôle, he does not even mention: Hecataeus, to whom he owed the whole of his second book, he names a few times only, and then (in the part borrowed from him) in rather clumsy ridicule. Anaximenes and his 'school' are, as we said above, not mentioned at all except in the doxographical tradition, which clearly points to the conclusion that he did not share the historical and geographical interests of the other Milesians.

If now we turn to Anaximander we find a considerable number of sources regarding him which have no real or original connection with the doxographers. Thus, when Aelian [93] reports that Anaximander led in the colonization of Apollonia in Thrace from Miletus, it is obvious that he derived this datum from some work pertaining to history, whether general in character or dealing with biographies of philosophers. There is no reason to connect this source with Theophrastus or any of his kind. To a similar treatise we must look for the source of the statement, already mentioned, and reported by Favorinus, that he set up a dial at Sparta.[94] From the history of astronomy prepared by Eudemus, who like Theophrastus was a disciple of Aristotle, derive in all probability the data given by Pliny as to Anaximander's discovery of the obliquity of the ecliptic and the date he assigned for the heliacal setting of the Pleiades.[95] Certainly Simplicius cites Eudemus as authority for the statement that Anaxi-

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93 V^3 I. 14, 25. Cp. Bilabel, Die Ionische Kolonisation (Philologus, Supplb. XIV, I (1920), p. 13 sq.

94 V^3 I. 14, 7.

95 V^3 I. 15, 1; 19, 17.

mander was the first to set forth an opinion regarding the sizes and intervals of the planets.[96] Eudemus of course regarded Anaximander solely from the point of view of the mathematician and astronomer. But we know that such matters engaged the attention of historians before Eudemus. Herodotus speaks not only of Egyptian mathematic, but also of Thales' prediction of an eclipse; and Hippias of Elis about the same time interested himself in the earlier history of mathematics. To what source Plutarch[97] owed his reference to Anaximander's theory of the origin of man from fishes it is not possible to say. He as well as Censorinus[98] may have derived their information from the doxographers; but there is reason to believe that there was an even earlier tradition on this question among the historians who treated of Egypt.

The data regarding Anaximander's map we have already traced back to Eratosthenes and his geographical treatise. From him derive the statements of Agathemerus and Strabo as well as the other extant references. But one naturally asks whether Eratosthenes had access to a map unquestionably belonging to Anaximander. This possibility cannot be denied;[99] but in view of the evidence that even in Herodotus' day there existed numerous Ionian maps, and of the great probability that charts were both multiplied and modified almost at will, it cannot be regarded as very likely. But, under such circumstances, one must ask, Where did Eratosthenes find the information which justified his statements regarding the charts of Anaximander and Hecataeus? The reasonable answer is surely that they were referred to by historical and geographical writers now lost to us but accessible to him. This is made all but certain by the confident tone in which the charts of Anaximander, Hecataeus and Damastes are mentioned. Statements such as we find in Agathemerus, who depends on Eratosthenes, presuppose a critical discussion of maps and accompanying texts with clear distinctions drawn between the several contributions and indebtedness of geographers in chronological order; and these geographers cannot, as in Herodotus, have been left unnamed or thrown together under the collective title of 'Ionians.' Thus we have every reason for postulating even before

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96 V^3 I. 19, 10. Whether the questionable report of Theon Smyrnaeus (V^2 I. 20, 20) derives from Eudemus is doubtful: for Theon is known to have drawn on two other sources also, Dercyllides and Adrastus.

97 V^3 I. 21, 13.

98 V^3 I. 21, 9.

99 For a remark on the sources at Eratosthenes' command are Strabo 2.1, 5 C. 69.

Eratosthenes a literature dealing with the progress of geography, even if, as is perhaps most likely, it did not take the form of disinterested learned comparison, but consisted of the works of successive geographers who took cognizance of the opinions and charts of their predecessors. Just such criticism is in fact abundant in Strabo, drawn in good part (with express citations) from his predecessors. In other words, there must have existed before Eratosthenes a geographical tradition in all essentials like that which we know from Strabo existed after his time.

It is to this tradition, then, that we owe the record of Anaximander's map. As we have seen, Eratosthenes did not know enough of Anaximander's book to be sure that it was genuine. His own treatise marked the beginning of a new epoch; for it was the first in which a serious, if somewhat too confident, effort was made to describe and chart the inhabited earth on the basis of the newly established shape and dimensions of the earth. We know from Strabo that Hipparchus at many points recurred to the old maps for things which Eratosthenes had discarded. This is hardly explicable except on the hypothesis that the procedure of Eratosthenes was characterized by a radical departure from tradition, which his keen critic could not justify. Such being the case, we can readily believe that Eratosthenes was not an altogether sympathetic student of the earlier geographers. In the first flush of enthusiasm over the new geography the old was naturally neglected. It was only after a generation or two had passed that the texts of Anaximander and Hecataeus were again brought forth from the archives and studied with intelligent interest by Demetrius of Scepsis and Apollodorus,[100] while Hipparchus, like Eratosthenes but apparently in greater measure, displayed a keen interest in the ancient charts. Thus the geographical tradition was assured of a continuity, which for a moment seemed threatened. This continuous tradition makes it possible even now in part to prove, in part to divine, the character of Anaximander's book.

However much prose literature may or may not have existed in Ionia before the days of Anaximander, his book is for us at once the earliest known prose treatise and the earliest known literary document, whether in verse or prose, of the scientific interests of Ionia. As such it naturally possesses a peculiar fascination for us, and we could

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100 We know that Demetrius used Hecataeus' book, and was in turn used by Apollodorus. This comports well with the well-known antiquarian interests which experienced a marked revival at Alexandria and Pergamum in the third and second centuries B.C.

wish to know far more about it than the grudging record vouchsafes. It is perhaps possible even yet by a close scrutiny of the whole early tradition to gather certain further data in detail regarding his opinions and the structure of his chart; but of the economy and spirit of Anaximander's book we seem with our present resources to be able to learn no more than we have above set forth. A part even of this is of course not susceptible of strict proof; but we have endeavored to conduct our inquiry with due regard to the evidence and the principles which must be observed in historical studies. Though Anaximander was apparently a name not uncommon in Miletus, we do not meet it elsewhere; and the literary tradition seems to have recorded two persons only of that name as authors of books.[101] We have therefore to choose between them when it is a question of assigning a datum attributed simply to 'Anaximander'; and the result of our inquiry is what one might reasonably have expected. The name 'Anaximander' must have suggested to the Greek the great Milesian of the sixth century as naturally as the name 'Jefferson' or 'Washington' suggests to an American the well known personages of our own history. That another writer of the same name and born in the same city was known is indeed clear from the record; but the sole reference to him outside the entries of a biographical and bibliographical nature is of a sort to lead naturally to his identification, even though he is not expressly called 'Anaximander the Younger.'

Anaximander's book must be seen in its true perspective, that is to say, in relation to the tradition of which it was a part. Whether it stood at the head of the series or itself had predecessors, we do not certainly know; but of its successors we may discover enough to discern in part the lines of connection. From Anaximander onward we can trace several streams of tradition growing in volume and progressively differentiating themselves until they give rise to quite distinct and special sciences. One takes on the form of cosmogony or cosmology and a study of the microcosm, the latter developing into scientific medicine;[102] another begins as history with a geographical appendix, which in time constitutes a science apart. Mathematics and cosmography, fructified by a new interest born partly of Orphic and other religious speculations, give rise to new points of view and to

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101 This is certainly implied in the name 'Anaximander the Younger.'

102 In this study I have chosen for various reasons to say little or nothing of the medical tradition. One reason is that in my Περὶ Φύσεως I treated of it at some length, the other, and more important, is that no intelligent study of the *Corpus Hippocrateum* in relation to the earlier history of medicine exists. The clue, I believe, lies in the treatise Περὶ ἀρχαίης ἰητρικῆς; but I have not found the leisure to follow it up.

questions which in their development eventuate in metaphysics or ontology,[103] and determine the history of philosophy, as they shape the doxographic tradition. The latter, though dominated by alien interests, depends perforce on the historico-geographic tradition for the necessary data regarding the beginnings of Ionian science, since Aristotle clearly did not possess a copy of a book by Thales, if such a book ever existed. Regarding Anaximander's book Aristotle seems to have known little or nothing at first hand: his opinions regarding the Infinite and the reason why the earth keeps its central position he clearly did not understand, and in the latter case he certainly attributed to the Milesian an explanation utterly alien to his thought.[104] That Theophrastus perpetuated the blunder is certain and significant. Meanwhile the central interest of the early Milesians was ruled out as not germane to philosophy, and the best record of their thought derives from other branches of the tradition.

In conclusion let us attempt to frame a picture of this ancient book. In compass it cannot have been large, if the statement of Diogenes Laertius,[105] which in this particular is probably drawn from either Apollodorus or Posidonius, is true; for he reports that Anaximander gave "a summary exposition of his opinions." In spirit and intention it was historical, purporting to sketch the life-history of the cosmos from the moment of its emergence from infinitude to the author's own time, and looking forward to the death and dissolution not only of the earth and its inhabitants but also of this and all particular worlds. This being so the exposition naturally followed the order of chronological sequence, recounting first the origin of the world and of the earth, proceeding with the origin of life and the evolution of species capable of living on land as the once all engulfing sea gradually allowed dry land to appear, the origin of human life, probably in Egypt, and the spread of the race and its civilization. Heroic genealogies bridged the interval between the beginnings and the disposition of the peoples and their habitats in Anaximander's time, which were, however briefly, sketched in his book as well as figured on his chart. In this portion of his treatise, presumably, occurred some at least of the explanations which he gave of certain outstanding natural phe-

103 This is a thesis which I have long maintained and hope sometime to develop, that metaphysics begins with Pythagoras, Heraclitus and Parmenides. Its roots in mathematics and in the conception of the soul, which finds expression in the κύκλος γενέσεως, I have at various times pointed out, but not at length.

104 Aristotle, thinking in Pythagorean terms, speaks of spheres, whereas Anaximander's cosmography operates wholly with circles.

105 V^3 I. 14, 11.

nomena, such as earthquakes, and such strictly historical data as the old Milesian saw fit to give.[106] They would most naturally concern the royal houses, not improbably linked up with Heracles, of the great powers of Asia, the Lydians and the Medes.

Such a book, however significant to one whose antiquarian or broadly historical interest enabled him to detect in it the germs of future great developments, was of course destined to be speedily antiquated and thus ignored by the vulgar. The greatest wonder is that it did not disappear without a trace, as Theophrastus believed, probably with good reason, that many still earlier books had done; for until institutional libraries began to be formed in the days of Plato and Aristotle books, except such great favorites as the major poets, must have had an extremely precarious existence. The schools of philosophy took an interest in Anaximander's book in virtue of a part of its contents and because it was the earliest of its kind that came to their knowledge; geographers sought out, if not his authentic map, at least such information regarding it as might be gleaned from later writers in the same field; finally, at the very height of Alexandrian criticism, Apollodorus, qualified as no other was by training and interest to assess its worth, had the good fortune to retrieve it from obscurity, and the grace to use it in a way to reveal its true scope and character. The doxographic tradition, no doubt, called it a treatise *On Nature*, which sufficiently characterized it in part; someone possessed of a truer perspective, and regarding the whole book in the light of its conclusion called it a *Tour of the Earth*, unless — as is indeed possible — the distinctly geographical portion of the book had become detached from the beginning and so led for a time a life divorced, in which case the latter title may have been originally given to the separate part. In any case, the fortunes of Anaximander's book would seem to have been strikingly similar to the fortunes experienced by the work of his successor, Hecataeus.

MIDDLETOWN, CONN.,
Jan. 14, 1921.

106 Hecataeus also, it seems, gave his historical (as distinguished from mythical and prehistoric) data in his Περιήγησις or Γῆς περίοδος; this at least is the opinion of Jacoby, and it is altogether probable, since the Γενεαλογίαι served to reconstruct the prehistoric period, and the Γῆς περίοδος, though concerned chiefly with describing the *status quo*, could not well, even if its author was primarily interested in geography, avoid touching on historical occurrences which affected the map. How intimately history and geography are bound up with one another must be clear to any thoughtful person, especially at the present moment.

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By William Morton Wheeler.

(Continued from page 3 of cover.)

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By William Morton Wheeler.

OBSERVATIONS ON ARMY ANTS IN BRITISH GUIANA.[1]

By William Morton Wheeler.

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Although the progress of myrmecology during the past twenty years has been so rapid that more seems to have been written on the subject during that period than throughout the nineteenth century, there are several problems that yield to solution very slowly and reluctantly. One of these is the ethology of the nomadic legionary, or army ants of the subfamily Dorylinæ, which possess worker, male and female forms so peculiarly specialized and diverse that they can be correlated with certainty only when captured in the same colony. The workers are rarely encountered, except when foraging and at seasons of the year when the brood is not well developed, the males are seldom seen, except at lights, and the huge wingless females are among the rarest of insects. Notwithstanding all these obstacles, considerable information has been gradually accumulated concerning the Ethiopian and Indian species of the genera *Dorylus* and *Ænictus* by Emery, Forel, Santschi, Brauns, Vosseler and others. Less progress has been made in the study of our American species, which belong to the genera *Eciton* and *Cheliomyrmex*.

Emery divides the former genus, which comprises more than a hundred described species, and ranges from Argentina to North Carolina, Missouri and Colorado, into three subgenera: *Eciton* sens. str., *Labidus* and *Acamatus*. The genus *Cheliomyrmex* contains only a few rare species, and ranges only from Brazil and British Guiana to tropical Mexico. While the females of certain species of *Labidus* and *Acamatus* are known, those of *Eciton* s. str., have been sought in vain for over half a century. During the summer of 1920, while I was working at the Tropical Laboratory of the New York Zoölogical Society at Kartabo, British Guiana, I was able to secure the female of *E. burchelli*, its males and those of *E.* (*A.*) *pilosum* Smith, and to make observations on the habits of these and several other species. As my first myrmecological paper, on some Texan Ecitons, was published just twenty years ago, and as I have not since had occasion to con-

[1] Contributions from the Entomological Laboratory of the Bussey Institution, Harvard University, No. 183.

tribute much to our knowledge of the group, it seems advisable to begin with a brief resumé of what has been accomplished in correlating the various phases of *Eciton*, both in South and North America.

The following table gives a conspectus of the number of described species of the genus and the various subgenera and the number of known phases. It will be seen that of the 104 species described up to date, 38 are known only from worker specimens, 52 only from males and that we know both the worker and male of 8 and all three phases of only 6 species.

Subgenera	Total Number of Species	☿ Alone Known	♂ Alone Known	☿ and ♂ Alone Known	☿ ♀ and ♂ Known
Eciton sens. str.	12	5	3	3	1
Labidus	17	5	10	—	2
Acamatus	75	28	39	5	3
Total	104	38	52	8	6

The first female *Eciton* to be discovered was that of *E.* (*L.*) *coecum* Latr., and was described by Ernest André in 1885 from Mexico under the name of *Pseudodichthadia incerta*. The insect measured 25 mm. (gaster alone 20 mm.). Although *coecum* is a common species from Argentina to Texas a second female has not since been captured. Nine years later the Rev. Jerome Schmitt, O. S. B., took a female of *E.* (*A.*) *opacithorax* Emery at Belmont, North Carolina, and permitted me to publish a figure and description of it (1901). In 1900 and 1901 and during more recent years I have taken several females of this species and of *E.* (*A.*) *schmitti* Emery in the Southwestern States. Forel, while on a visit to North Carolina in 1899 captured the female of *E.* (*A.*) *carolinense* Emery, and Mr. W. T. Davis captured both the female and male of this species from a colony at Clayton, Georgia, in June 1909. I give below descriptions and figures of these specimens, which he generously presented to me. In 1918 Luederwaldt described and figured the female of *E.* (*L.*) *praedator* F. Smith from São Paulo, Brazil. It measured 33 mm. (head, thorax and petiole 7 mm., gaster 26 mm.). The female of *E. burchelli*, captured during the past summer, completes the list of known female Ecitons.

Although a number of male Ecitons were long ago described as species of *Labidus* by Jurine (1807), Lepeletier (1838), Shuckard (1840), Westwood (1842), Haldeman (1852), F. Smith (1859) and Cresson (1872), the first to find a male in the nest with the workers was Wilhelm Müller (1886). He identified the species which he studied at Blumenau, in the Province of Santa Catharina, Brazil, as *E. hamatum* Fabr., but it proved to be *burchelli* Westwood. Mayr in the same year described both the male and worker of *E.* (*A.*) *hetschkoi*, taken by Hetschko from a nest in the Province of Paraná, Brazil, and Emery, in 1896, described the males of *E. hamatum* and *quadriglume*. The former he seems to have determined by a process of exclusion, the latter was taken from the nest by Schmalz's sons in the Province of Santa Catharina. In 1900 Emery was able to recognize the males of *E.* (*A.*) *legionis* and *E.* (*L.*) *coecum*. In the same year and in 1901 I described the males of our North American *E.* (*A.*) *schmitti* Emery and *opacithorax* Emery. Forel described the male of *E.* (*L.*) *praedator* Smith in 1906, and in 1912 I took the male of *E. vagans* Olivier near San José, Costa Rica, accompanying a file of workers. Recently Gallardo (1915) has found that the workers of the Argentinian *E.* (*A.*) *spegazzinii* Emery are cospecific with the male previously described as *spinolæ* by Westwood, and Bruch (1916) has proved that the worker *E.* (*A.*) *nitens* Mayr is cospecific with the same author's male described as *strobeli* on a preceding page of his paper published in 1868. The male of *E.* (*A.*) *pilosum* Smith, described below, which proves to be the same as *mexicanum* Smith, and the male of *E.* (*A.*) *carolinense* also described in this paper, complete the list of Ecitons in which the males have been definitively correlated with cospecific workers. Forel (1897) believes that his *E.* (*A.*) *antillarum* from Grenada may be the worker of *klugi* Shuckard from that island and St. Vincent, and it is very probable, as I have stated in a former paper (1908, p. 410) that *E.* (*L.*) *crassicorne* Smith is the worker of *esenbecki*, previously described by Westwood from a male specimen. I shall also give reasons for regarding *Labidus morosus* Smith as the male of *Cheliomyrmex nortoni* Mayr.

Eciton burchelli Westwood.

(Figs. 1–4, 5c, 6a–d.)

Among the various Ecitons which I observed at Kartabo, this was the most abundant and the most aggressive, and therefore the dominant species throughout the jungle. Scarcely a day passed that I did not witness a foray of one of its enormous armies, which often came close to the laboratory and on one occasion actually entered the kitchen and would have overrun the building, had not the cook placed live coals in the path of the oncoming hosts. During the summer of 1919 an army actually took possession of the laboratory and bivouacked for several days in a corner of the storeroom. It is to this species that Beebe's two fascinating articles (1917, 1919) in the Atlantic Monthly refer. The forays seem to be most frequent during the rainy season. At any rate, during the latter part of August 1920, when the rains were becoming somewhat less frequent and copious, fewer armies were encountered in the jungle.

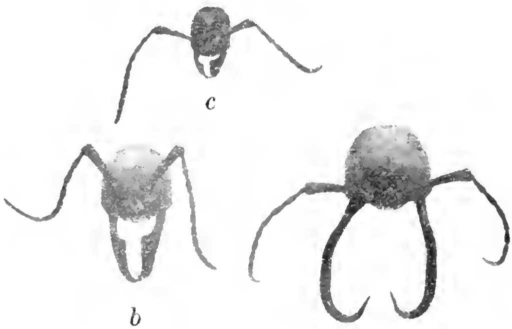

FIGURE 1. *Eciton burchelli* Westw. Heads of soldier (*a*), large worker (*b*) and small worker (*c*).

So many accurate and detailed accounts of the behavior of *burchelli* have been published by such accomplished observers as Bates (1863), Sumichrast (1868), Belt (1874) and Wilhelm Müller (1886) that I find little of interest to add. Three occurrences, however, mentioned by some of these authors are so striking and so regularly connected with the forays as to deserve long and careful study by some future student at the Tropical Laboratory. These are, first, the behavior of the chirping flock of ant-thrushes which accompany the fan-shaped van of the army and feed on the numerous insects and spiders, driven into the open by the feverishly ferreting workers; second, the swarms of small flies of the Conopid genus *Stylogaster* and of the family Tachinidæ, which hover over the van, and third, the plundering of the larvæ and pupæ from the nests of the most diverse ants. The army that attempted to enter the kitchen had plundered all the nests of *Pheidole fallax* and of an allied species in the sandy yard of the laboratory and had temporarily stored the larvæ, pupæ and young callows in piles like handfuls of rice around the bases of the tall Javanese bamboo clumps. This army also attacked a flourishing colony of yellow wasps (*Polybia* sp.) in the wall of the kitchen, expelled the workers and carried off the brood.

The swarms of small flies above mentioned were regularly seen hovering from one to two feet above the advancing columns of ants. Bates (1863), who seems to have been the first to observe this singular phenomenon, says: "The armies of all Ecitons are accompanied by small swarms of a kind of two-winged fly, the females of which have a very long ovipositor, and which belongs to the genus *Stylogaster* (family Conopsidæ). These swarms hover with rapidly vibrating wings, at a height of a foot or less from the soil, and occasionally one of the flies darts with great quickness towards the ground. I found they were not occupied in transfixing ants, although they have a long needle-shaped proboscis, which suggests that conclusion, but most probably in depositing their eggs in the soft bodies of insects, which the ants were driving away from their hiding-places. These eggs would hatch after the ants had placed their booty in their hive as food for their young. If this supposition be correct, the *Stylogaster* would offer a case of parasitism of quite a novel kind." Townsend also gives an account of *Stylogaster*, which he observed in 1897 in the State of Vera Cruz: "Fifty-one specimens of this interesting genus were taken hovering over the front ranks of a moving army of ants, in a cafetal at Paso de Telayo, during the last hour or two of daylight on March 29. In company with them were numerous specimens of *Hyalomyia* and some other small tachinids. The ants have been determined by Mr. Theo. Pergande as *Eciton foreli* Mayr [= *burchelli* Westw.]...The column of ants was about 15 feet wide and 25 feet long, and moved slowly but surely through the cafetal, swarming rapidly over the thick covering of dead leaves, branches and other obstructions that strewed the ground under the coffee-trees. The specimens of *Stylogaster* hovered continually over the ants, now and again darting at them, without doubt for the purpose of ovipositing in their bodies. During the whole three months of my collecting in this locality, I saw not a single specimen of *Stylogaster* at any other time, but on this occasion, during the short time that I had before dark overtook me, I succeeded in capturing fifty-five specimens, by sweeping closely with the net over the front ranks of the ants." Although I saw these flies on several occasions, accompanying the advancing armies of *burchelli* and darting at the ants or even at vacant parts of the ground, I could see nothing that convinced me that they were ovipositing. On one occasion I came upon a swarm of both sexes of *Stylogaster* hovering over a spot where there were no Ecitons, although a few workers of *Gigantiops destructor* and *Ectatomma ruidum* were running about in the vicinity. This observation and the fact that some species of *Stylogaster* occur in North America north of the range of *Eciton*, make it seem doubtful whether these flies are as intimately attached to the ants as some authors have supposed. They are, perhaps, attracted by the rank odor of the Ecitons.

On several occasions I followed prey-laden files of *burchelli* workers to their temporary nests under great logs, but was so severely stung and bitten when I attempted to make closer observations that I had to desist. On the morning of July 19 my son Ralph discovered a colony in a more favorable situation for study only a few hundred yards from the laboratory. The ants had selected a dead tree trunk about a yard in diameter, hollow but still standing. At its base there was a long narrow hole, nearly two feet high covered, except for a small opening near the ground, with a huge, inert mass of workers, dark brown and punctuated here and there with the ivory-white heads of the soldiers. Into the small opening below the cluster a dense file of workers was pouring, laden with prey of all kinds, including many larvæ of alien ants. After Mr. Tee Van had photographed the pendent cluster, I stirred it up with a stick, in the hope of finding the queen, but was attacked so viciously that I had to leave the premises. Returning the following day I found that the ants had all withdrawn into the roomy cavity of the tree trunk, leaving the long opening fully exposed to view. Through it the colony could be dimly seen in great masses draped on the walls of the cavity. Early in the morning of July 21st Mr. Alfred Emerson and I decided to smoke the ants out and, if possible, to secure their queen. We ringed the legs of two chairs with carbolated vaseline, planted them in front of the opening, crouched on their seats and with long tweezers placed in the bottom of the cavity a lot of moist bamboo leaves and paper. A match was applied and soon a dense smudge filled the cavity and even issued from cracks in the old wood at a height of nearly twenty feet from the ground. The ants remained quiet for some time, but when the smoke

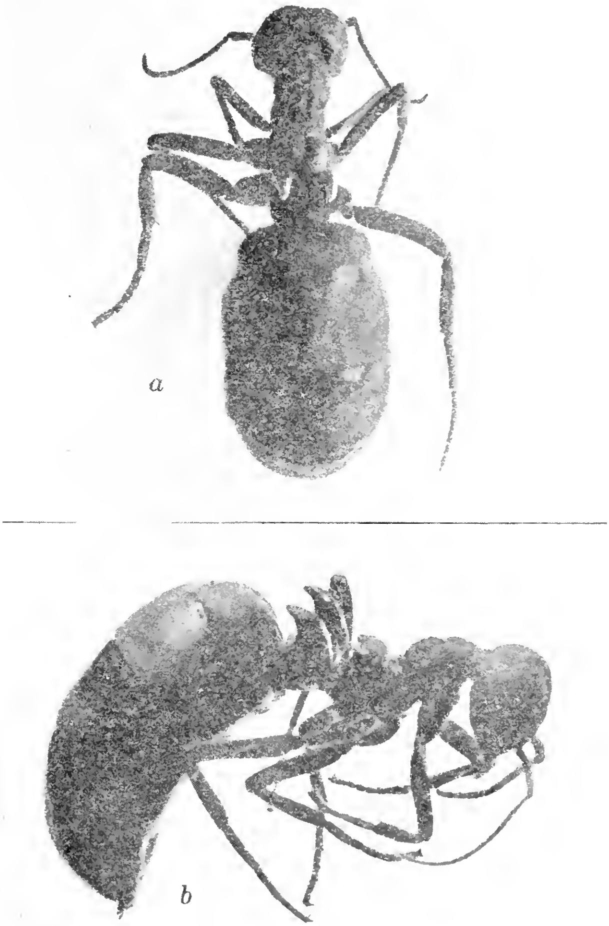

Figure 2. *Eciton burchelli* Westw. Female; (*a*) dorsal, (*b*) lateral view.

grew denser decided to move, and columns of workers and soldiers began to emerge from the top of the long orifice, crawled out over the bark and descended to the ground. In these columns balls of ants appeared from time to time, each more or less completely enveloping an elongate object, which proved to be a large cocoon, 20 mm. long, of a rich brown color and tough consistency. I supposed at first that these cocoons belonged to some large *Camponotus* or *Pachycondyla* whose nest had been plundered on one of the preceding days, but on opening one of them I was delighted to find that it contained a nearly mature male *Eciton* pupa (Fig. 3b and c)! In all more than a hundred of these cocoons were brought out and each was moved along with the greatest solicitude by an ant cluster as big as a hen's egg. There were also a few young or half grown *burchelli* larvæ, but no worker pupæ.

We now discovered that a large column of the ants was leaving the cavity by a small opening, which we had overlooked, on the opposite side of the trunk, and was descending to the ground and assembling in masses on the dead leaves. After we had moved our chairs to this new scene of activity, Mr. Emerson observed a large halting mass of workers in the column and on thrusting his tweezers into it drew forth a young queen (Fig. 2), which was being very slowly piloted along by a dense cloud of attendants. We had spent the whole morning crouching on our chairs in an uncomfortable position, though out of reach of the ants, and being elated by the capture of the queen returned to the laboratory for lunch. At three o'clock we found masses of ants still resting on the dead leaves a few yards from the tree. A second queen, precisely like the first, was discovered in one of these masses. I infer that this was the only remaining female in the colony, for after her removal a perceptible apathy or dejection seemed to fall on the whole body of ants. They became much less active and aggressive and with some hesitation formed a single dense and rather slowly moving column that made off through the jungle, attended by a small swarm of hovering Stylogasters, which had somehow made their appearance during the lunch hour. We searched the departing army for ecitophiles and succeeded in capturing several specimens of three species of Staphylinids (*Mimeciton*). They very closely resembled the smallest workers both in color and behavior, but were more difficult to catch and all seemed to belong to portions of the colony that had formed the immediate *entourage* of the queens. At any rate, none was taken from the masses of ants enveloping the male cocoons, although I had thrown all of these masses together with

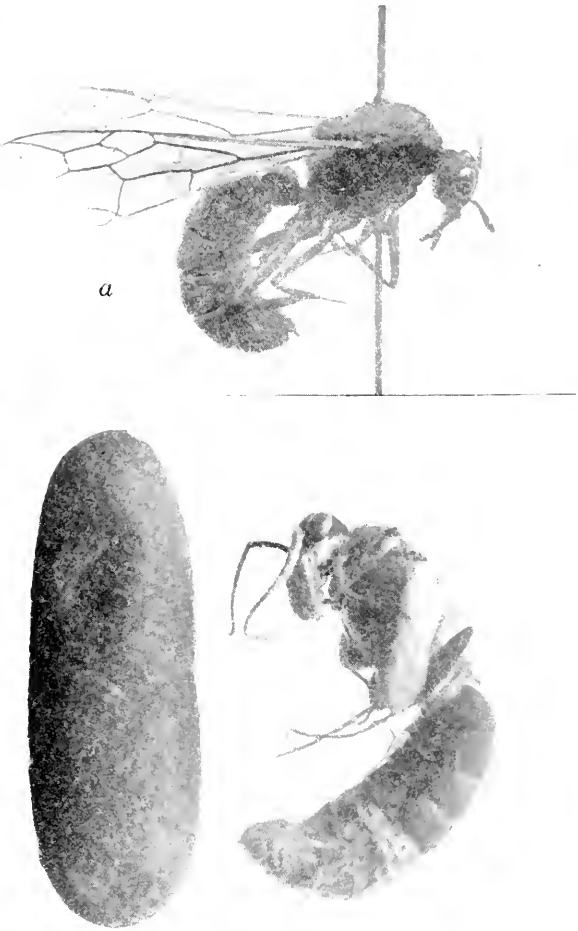

FIGURE 3. *Eciton burchelli* Westw. (*a*) Male; lower figures, mature pupa (*b*) and cocoon (*c*).

the cocoons into jars of formalin and was therefore able to go through the material at my leisure in the laboratory.

I had collected every one of the cocoons in the hope that some of them might contain pupal females, but all proved to be males in precisely the same stage and very nearly ready to hatch. The two females had evidently recently emerged for their colors were very brilliant and the delicate golden pile on their bodies was intact. Moreover, their ovaries were undeveloped as shown by the relatively small size of the gaster. That this part of the body must later become greatly distended, with the maturation of the ovaries, and must have its sclerites separated by the extension of the white intersegmental membranes, may be inferred from Luederwaldt's figures of an old queen of *E.* (*L.*) *praedator* and from my observations on aged queens of the North American species of *Acamatus* (1900, 1901). I believe, therefore, that the large *burchelli* colony which Mr. Emerson and I were able to investigate, had already completed the production of its annual brood of workers and soldiers and that the sexual forms constituted a later or, at any rate, a retarded brood, consisting of a large number of males, all in the same stage and destined to hatch before the end of July, or before the incidence of the dry season, and a very few females, which had hatched before any of the males. The old female, or mother of the colony, had probably died recently. Hence it would seem that this *burchelli* colony was proterogynic, and destined later to separate into two colonies, each with its own young queen, for it is practically certain that new *Eciton* colonies must be thus formed by fission of an old colony and not, as in most ants, by isolated, recently fecundated females. The fecundation of the *Eciton* queens is still an unsolved problem. Perhaps the two *burchelli* queens would have been fecundated by some of their brothers about to hatch, i. e. adelphogamically, though we cannot exclude the possibility of fecundation by males from other colonies of the same species. Such males might be temporarily adopted or might hastily fecundate the young queens while they are being moved along during one of the frequent migrations of the colony.

I was astonished to find the male *burchelli* pupæ in cocoons as all the pupæ I had seen in the Texan Ecitons (subgen. *Acamatus*) were nude. But these were all worker pupæ. On reading Beebe's account (1919) of the *burchelli* colony that had bivouacked in the store-room of the Kartabo laboratory, I learned that he had given an interesting account of the method of spinning the cocoon by a lot of larvæ which had been assembled by the workers on the surface of an old board. He says:

"On the flat board were several thousand ants and a dozen or more groups of full-grown larvæ. Workers of all sizes were searching everywhere for some covering for the tender immature creatures. They had chewed up all available loose splinters of wood, and near the rotten, termite-eaten ends, the sound of dozens of jaws gnawing all at once was plainly audible. This unaccustomed, unmilitary labor produced a quantity of fine sawdust which was sprinkled over the larvæ. I had made a partition of a bit of a British officer's tent which I had used in India and China, made of several layers of colored canvas and

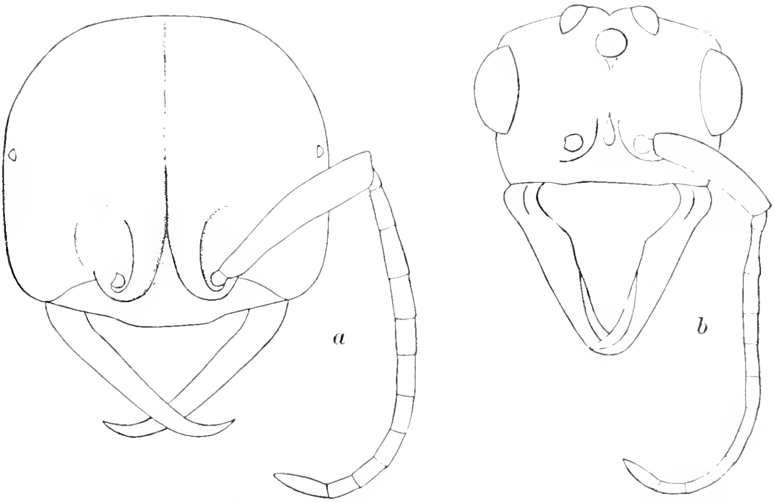

FIGURE 4. *Eciton burchelli* Westw. (*a*) Head of female, dorsal view; (*b*) head of male.

cloth. The ants found a loose end of this, teased it out, and unraveled it, so that all the larvæ near by were blanketed with a gray particolored covering of fuzz.

"All this strange work was hurried and carried on under great excitement. The scores of big soldiers on guard appeared rather ill at ease, as if they had wandered by mistake into the wrong department. They sauntered about, bumped into larvæ, turned and fled. A constant stream of workers from the nest brought hundreds more larvæ, and no sooner had they been planted and débris of sorts sifted over them, than they began spinning. A few had already swathed themselves in cocoons — exceedingly thin coverings of pinkish silk. As this took place out of the nest, in the jungle, they must be covered with wood and leaves. The vital necessity of this was not apparent, for none of this débris was incorporated into the silk of the cocoons, which were clean and homogeneous. Yet the hundreds of ants gnawed and tore and labored to gather this little dust, as if their very lives depended upon it. . . . When first brought from the nest, the larvæ lay quite straight and still, but almost at once they bent far over in the spinning position. Then some officious worker would come along, and the unfortunate larva would be snatched up, carried off, and jammed down in some neighboring empty space, like a bolt of cloth rearranged upon a shelf. Then another ant would approach, antenna the larva, disapprove, and again shift its position. It was a real survival of the lucky, as to who should avoid being exhausted by kindness and over-solicitude. . . . . There was no order of packing. The larvæ were fitted together anyway, and meagerly covered with dust of wood and shreds of cloth. One big tissue of wood nearly an inch square was too great a temptation to be left alone, and during the course of my observation it covered in turn almost every group of larvæ in sight, ending by being accidentally shunted over the edge and killing a worker near the kitchen middens. There was only a single layer of larvæ; in no case were they piled up, and when the platform became crowded, a new column was formed and hundreds taken outside. To the casual eye there was no difference between these legionaries and a column bringing in booty of insects, eggs and pupæ; yet here all was solicitude, never a bite too severe, or a blunder of undue force." These observations show that in thus covering their larvæ with foreign particles just before pupation, *Eciton burchelli* behaves exactly like many other ants, a fact which Beebe did not know, for legless larvæ, like those of ants, cannot, of course, spin their cocoons without a temporary covering of earth or débris to which they can attach their silk. Unfortunately he fails to tell us anything about the castes to which the cocoon-spinning larvæ belonged, and we are left to infer that they were probably, in great part at least, workers and soldiers.

When I looked up the earlier literature on *Eciton* on my return to Boston I found that Wilhelm Müller (1886) had found pupæ of *burchelli* enclosed in cocoons. March 1st, 1885, he found a little to one side of an *Eciton* file a deälated male *Labidus burchelli* which was being partly dragged and partly pushed along by a couple of workers. As the insect, when placed among a lot of workers and soldiers, remained unharmed, he naturally inferred that it belonged to the species, which he incorrectly called *E. hamatum*. March 14th he captured and anaesthetized a large mass of workers and brood taken from the colony while it was bivouacking in a hollow tree, and found

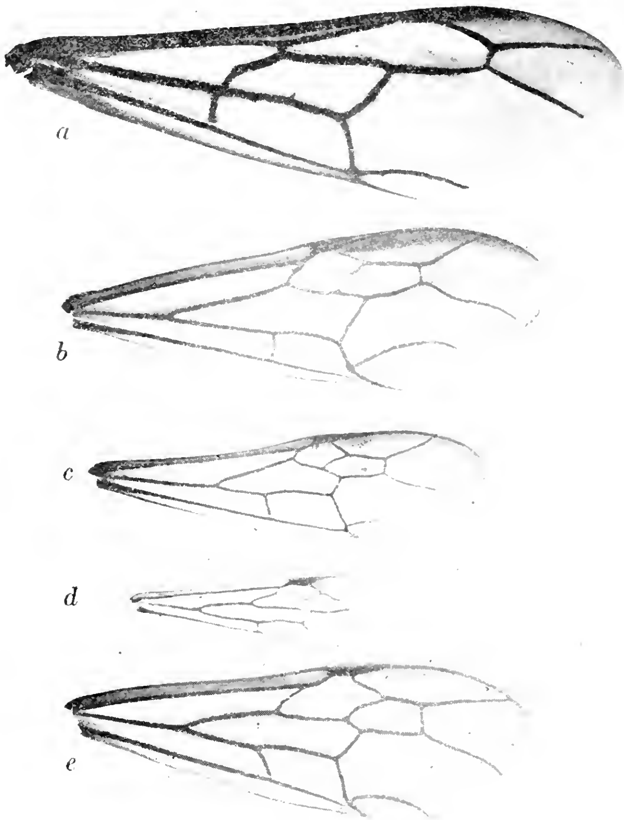

FIGURE 5. Fore wings of (*a*) *Dorylus* (*Typhlopone*) *fulvus* Westw. subsp. *badius* Gerst.; (*b*) *Cheliomyrmex megalonyx* n. sp.; (*c*) *Eciton burchelli* Westw.; (*d*) *Eciton* (*Acamatus*) *schmitti* Emery; (*e*) *Eciton* (*Labidus*) *coecum* Latr.

in the lot a single large male cocoon and numerous cocoons containing soldier pupæ. The latter were easily identified by their long, hook-shaped mandibles. His account of the worker larvæ is not very clear, but I infer that they also regularly spin cocoons. He states, however, that one of the smallest workers was seen to pupate without a covering. Some confusion was introduced into the account by Müller's finding among the *Eciton* brood a number of small cocoons which Forel interpreted as containing pupæ of "substitution males," but which Emery (1900) later interpreted as kidnapped Ponerinæ. From what is now known of ant-larvæ it can be positively asserted that Müller's description and Fig. 2 refer to larvæ of the Ponerine genus *Pachycondyla* and very probably to one of the common species, *harpax* Fabr. or *striata* F. Smith. While it thus appears that the larvæ of at least one species of *Eciton* sens. str. spin cocoons, the habit is probably not widespread in the subfamily Dorylinæ. Both the worker and soldier pupæ of various Congolese species of *Dorylus* of the subgenus *Anomma* in my collection are all nude, Forel (1912) figures the pupæ of *A. nigricans* as nude and, according to Emery (1901) neither the worker nor the male pupa of *Dorylus* (s. str.) *affinis* Shuckard is enclosed in a cocoon.

I insert here technical descriptions of the female *E. burchelli* and of the male, which was not described in sufficient detail by Westwood.

Female (Fig. 2, 4*a*). Length 21–23 mm.; head, thorax and petiole 9–10 mm., gaster 12–13 mm.

Head as broad as long, distinctly broader in front than behind, with straight sides, feebly impressed in the ocular regions and with convex, rounded posterior border. A pronounced groove runs down the middle, deepest on the anterior half and expanding in the region of the frontal area, feebler behind and obsolescent near the occipital border. There are small obtuse projections at the inferior occipital corners of the head, corresponding to the acute, recurved spines in the soldier. Eyes at the middle of the sides of the head, in the form of small, convex, ocellus-like structures, of the same size as in the soldier. Ocelli absent. Mandibles long, slender, falcate and toothless, straight except at their tips. Clypeus short and broad, slightly impressed in the middle, its anterior border feebly and evenly arcuate in the middle and very feebly sinuate on the sides. Antennal foveæ not carinate externally as in the worker, the frontal carinæ farther apart, each forming a thick welt, which suddenly narrows anteriorly to curve around the front of the antennal insertion. Antennæ long; scapes robust, about half as long as the head and clypeus together; funiculi slender, their first joint as long as broad, the remaining joints growing gradually shorter and narrower to the penultimate; joints 2–4 twice as long as broad, terminal joint shorter than the two preceding taken

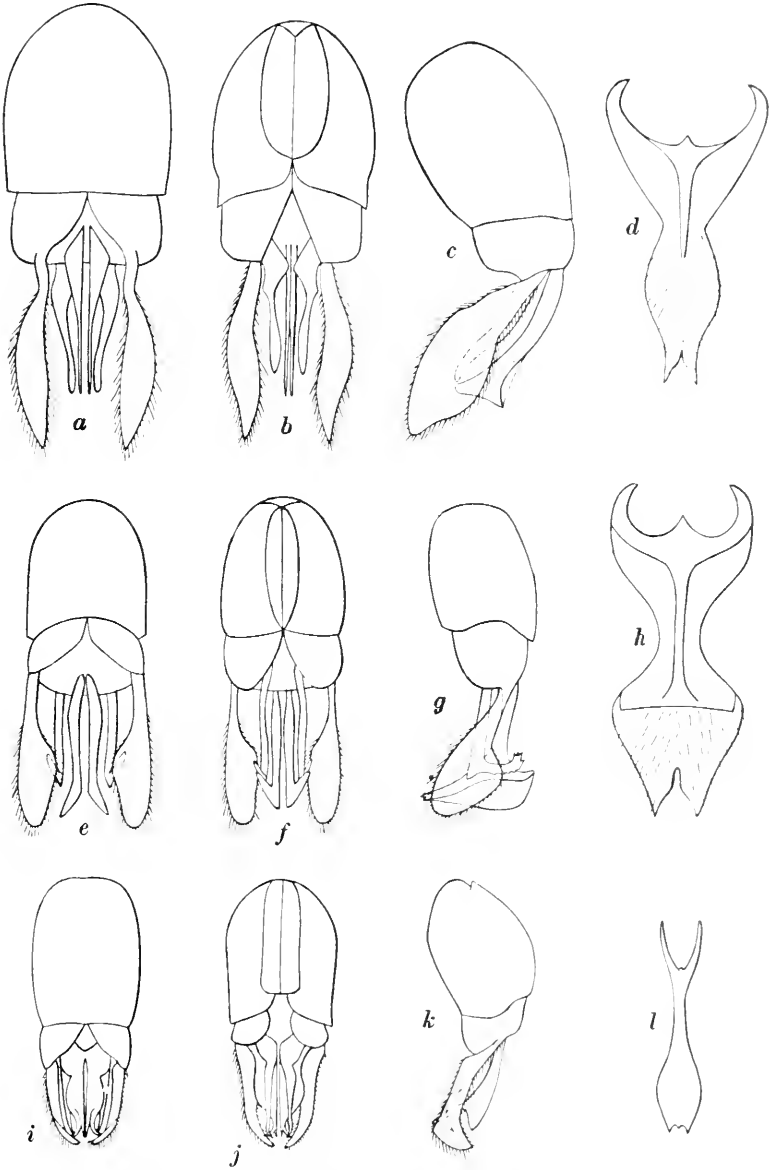

FIGURE 6. Male genitalia of *Eciton burchelli* Westw., (*a*) dorsal, (*b*) ventral, (*c*) lateral view; (*d*) subgenital plate of same; (*e*), (*f*), (*g*), and (*h*) corresponding parts of male *E.* (*Labidus*) *coecum* Latr.; (*i*), (*j*), (*k*), (*l*) corresponding parts of male *E.* (*Acamatus*) *schmitti* Emery.

together. Thorax small and short, only a little longer than the head with the mandibles and much narrower; seen from above as broad through the pronotum as through the epinotum, but distinctly narrowed at the mesonotum. Pronotum as broad as long, feebly convex and rounded above, somewhat flattened on the sides, but expanded ventrally. Promesonotal suture obsolete, but represented by a pronounced, lunate groove. Mesonotal stigmata elevated as strong, rounded tubercles. Mesonotum a little longer than broad, its dorsal surface in the form of two longitudinal welts, separated by a groove which does not extend to the lunate promesonotal groove. In profile the dorsal outline of the mesonotum is slightly convex in front, then straight and sloping backward, and finally abruptly vertical to the low metanotum, which is distinct and transverse, short and bounded by distinct, straight mesometanotal and metaëpinotal sutures. Metanotal stigmata small, tubercular. Epinotum broader than long, from above transversely oblong, its base in profile convex and rounded, rising high above the metanotum, its declivity concave. Seen from above the base is divided by a deep longitudinal furrow into two parts, each of which is very convex and terminates behind in a strong, blunt projection. Sides of epinotum somewhat flattened, with very large, slit-shaped stigmata. Petiole very large, much higher than long, broader than the epinotum and nearly twice as broad as long, repeating the shape of the epinotum in a more exaggerated form. It bears above two large, erect horns, which curve upward and backward and are as high as the pronotum and terminate in blunt points. Between these horns the dorsal surface of the petiole is deeply and broadly concave. Ventral surface of petiole smooth and nearly straight in profile, without a tooth or projection. Gaster very large, in life fully 6–7 mm. broad, very convex above and concave beneath, in profile elongate elliptical, the first segment somewhat truncated anteriorly and with a pair of feeble impressions to receive the tips of the petiolar horns. Stigmata on the various segments very large, slit-shaped. Hypopygium terminating in two broad triangular points separated by an angular excision; pygidium convex, its posterior border evenly and broadly rounded, entire. Sting short and robust, but exserted. Legs long and stout; claws well-developed, dentate.

Surface nearly opaque and very finely, densely and evenly punctate, or granular, except the mesopleuræ, sides and declivity of epinotum, the whole of the petiole, coxæ, venter and anterior portions of the gastric segments, which are shining and very finely and superficially shagreened. Head and dorsal portion of thorax covered with sparser, coarser piligerous punctures. These punctures are more scattered on the legs, mandibles and antennal scapes, but otherwise their sculpture is like that of the head.

Hairs bright golden yellow, in some lights almost ruby red, short, erect or reclinate, very short and most abundant on the head, longer and of uneven length and thickness on the thoracic dorsum, sparse and reclinate on the petiole and gaster. The longer hairs on the scapes and legs are also more reclinate and somewhat flexuous. Venter and sides of petiole, the meso- and metapleuræ and the external surfaces of the coxæ almost hairless. On the clypeus and gula the hairs are long and many of them flexuous. Pubescence absent, except on the antennal funiculi.

Rich reddish brown; mesopleuræ, declivity of epinotum, coxæ and femora, except at their articulations, castaneous brown; epinotum, except its dorsal surface, petiole and gaster black; apical halves of petiolar horns on their dorsal surface bright orange yellow as are also the posterior borders of the ventral gastric segments and a broad apical band, almost interrupted in the middle, on each of the dorsal segments.

Male (Fig. 3*a*, Fig. 4*b*, Fig. 5*c*, Fig. 6*a–d*). Length nearly 20 mm.

Head large, rather flat in front, through the eyes somewhat less than twice as broad as long. Eyes small, only a little more than three times as long as the cheeks and not very convex. Ocelli small, the distance from each lateral ocellus to the eye being fully twice the greatest diameter of the former. Frontal carinæ very short, not prolonged backward and curved outward behind. Frontal area distinct, impressed, elongate. Antennal scapes about as long as the sides of the head, slightly curved, moderately stout; funiculi long; first joint longer than broad, joints 2–4 twice as long as broad, remaining joints shorter, except the last, which is slender and pointed. Clypeus rather flat, its anterior border very broadly, feebly and arcuately concave. Mandibles long, narrow, straight, and flattened, very abruptly bent downward at the extreme base, their tips curved, blunt; their blades broadest at the basal third where the inner border has a broad, very blunt tooth. Mesonotum not broader than the head including the eyes, not very convex in front and not concealing the pronotum when the insect is viewed from above, somewhat longer than broad, somewhat narrowed in front, with a median longitudinal groove on its anterior half. Sides of pronotum concave. Epinotum in profile with very short base and long, sloping, concave declivity, the lateral corners between the two surfaces forming blunt protuberances. Epinotal stigmata large and slit-shaped as in the female. Petiole broadly and deeply concave above, its sides and posterior corners produced backward as a pair of large flattened, bluntly pointed projections. Gaster rather short, rather strongly curved, very convex above, concave below. Subgenital plate lanceolate, with a sharp median longitudinal carina and terminating in two slightly diverging, acuminate points. Stipites subtrapezoidal, with bluntly angular tips; sagittæ slender, of even diameter except at the tip which is abruptly expanded and truncated, longer than the volsellæ which are slender, tapering, with simple, blunt tips. Legs rather long and slender; claws toothed. Wings only moderately long (13 mm.).

Opaque; very densely and finely punctate, the head, thorax and petiole also with larger, coarser, rather evenly distributed, piligerous punctures. Such punctures are also present, but much sparser, on the mandibles, legs and sides of gaster.

Hairs fulvous, long, suberect, moderately abundant, absent on the sides of the thorax and mid-dorsal portions of the more posterior gastric segments, very long and conspicuous on the clypeus, gula, mandibles, front, scutellum, epinotum, petiole, base, sides and tip of the gaster, more reclinate and of uneven length on the pro- and mesonotum. There are also a few long hairs on the antennal scapes and tips of the basal funicular joints on the extensor surface. Pubescence fulvous, short, dense, appressed, confined to the antennal funiculi and to the dorsal surface of the gaster, which has a velvety appearance.

Dark brown or blackish; petiole, tibiæ, articulations of thorax and legs and tips of mandibles somewhat paler and more reddish; gaster, tarsi and tips of petiolar projections much paler, brownish red. Wings slightly yellowish, with dark brown veins.

Described from two females taken at Kartabo, British Guiana, July 21st, 1920, two males taken at light in the same locality July 20; two males taken by Mr. Wm. Beebe, also at Kartabo June 20, 1919, three males from Pará, Brazil (C. F. Baker), and numerous mature pupal males taken from the colony containing the two females.

The female *burchelli* exhibits a very peculiar development of the thorax compared with the known females of *Acamatus*, which are all much simpler in structure, as may be seen by comparing Figure 2 with the figure of the *carolinense* female (Fig. 8). Judging from the descriptions and figures of André and Luederwaldt, the known *Labidus* females are intermediate between those of the two other subgenera, as would be expected from a study of the workers and males. Undoubtedly the old *burchelli* queen, with fully expanded gaster, must be a much larger insect than those described above, as large as or even larger than the *E.* (*L.*) *praedator* queen, which, according to Luederwaldt, measures 33 mm. The great development of the thoracic and abdominal spiracles in the females and males of the Dorylinæ seems to me to be an adaptation to the peculiar conditions in which the sexual phases of these insects live, for when the colony is resting these phases are enveloped in such a dense mass of ill-smelling workers that some provision would seem to be necessary to increase the supply of oxygen. Emery calls attention to the fact that all the known female specimens of *Dorylus* have their appendages more or less mutilated and surmises that this may result from their being dragged along during the migrations by the workers over the rough ground and pebbles. All the *Eciton* females I have seen, however, had perfect legs and antennæ. The longer and more powerful legs and proportionally smaller gaster in these insects, as compared with the *Dorylus* queens, suggest that the former are not so much dragged as piloted or urged along by the workers when the colony is migrating. On the other hand, the male Ecitons found in columns of workers are often more or less mutilated, i. e. deälated. I have observed this in *E.* (*A.*) *pilosum* (*vide infra*) and Wilh. Müller noticed it in *E. burchelli*. This deälation is due not to a tearing of the wing membranes but to their tendency to weaken and break off at the base, precisely as in the recently fecundated female ants of other subfamilies. Even cabinet specimens of *Eciton* males are sometimes liable to lose their wings at the slightest touch. This, too, is probably an adaptation, enabling the workers readily to deprive their own or adopted males of their wings, thus preventing them from escaping from the colony and facilitating the fecundation of the young queens.

Eciton hamatum Fabricius.

At Kartabo this species, though common, is less abundant than *burchelli*. Its columns are also clearly less populous and aggressive, though the soldiers and larger workers can sting and bite severely. Mann noticed in Brazil that *hamatum* "is a timid species in comparison with some of the others, such as *E. vagans*." Both *burchelli* and *hamatum* have white-headed soldiers, with very similar long, hook-shaped mandibles (Fig. 1*a*), but the head in the latter is smooth and shining, instead of opaque, as in the former species.

Eciton (Labidus) coecum Latreille.

(Fig. 5*c*, Fig. 6*e–h*).

This widely distributed, hypogaeic species, so common in southern and central Texas and in many parts of Central and South America, seems to be rare at Kartabo, where I found it only once, foraging under some large logs in a damp spot near the laboratory. I saw it also in the Botanical Garden at Georgetown. Although several varieties of the male have been recognized (*biloba* Emery of Ecuador, *jurinei* Shuckard of Brazil, *servillei* Westwood of Central and South America, *kulowi* Forel of Mexico and *hostilis* Santschi of French Guiana), only one variety, *selysii* Forel, has been recognized among worker specimens. The validity of this form seems to me to be doubtful, as it was based on rather small workers. All the soldiers and workers in my collection, representing numerous localities from Texas to Paraguay show very little variation, with the exception of a single soldier taken by Mann at Pará. This evidently represents a distinct variety which may be called **opacifrons** var. nov. It differs from the soldier of the typical form in having the head broader in front and less excised in the middle of the posterior border, the tips of the mandibles are coarser and much more decidedly incurved, with scarcely a trace of the subapical tooth. The front, vertex and gula are opaque and densely shagreened, instead of very shining as in the typical form and the whole pronotum is opaque and sharply shagreened, instead of being more or less smooth and shining on the sides. The hairs on the body are shorter, more abundant, and of more even length, especially on the dorsal surface of the head and thorax and on the legs. The surface of the mandibles is very strongly, arcuately rugose. The legs, coxæ, petiole, postpetiole and gaster are distinctly yellowish, the remainder of the body as dark red as in the typical *coecum*. Of course, this form may be the unknown soldier of one of the varieties described from male specimens.

Eciton (Labidus) praedator F. Smith.

Like *coecum*, this species has a very wide range, from Argentina to tropical Mexico, but it is not known from the Sonoran Region or from our Southern States. In habits it is somewhat intermediate between the species of *Eciton* s. str. and *E. coecum*, making its forays above ground when it is not convenient to keep under the dead leaves. It also constructs galleries or cause-ways of particles of soil. When foraging or migrating its columns are conspicuously broader and denser than those of other army ants. Bates (1863) and Mann (1916) have described its habits in Brazil. The latter, who observed it along the Upper Madeira, says: "Houses along the railroad were frequently raided at night by *E. praedator*, which is well-known to the Brazilians and called by them "cazadoro" (hunter). I had the opportunity of observing one hut while the ants were in possession. The ground was covered with the ants, which swarmed also in the cracks and on the few pieces of furniture, while the owner of the place, a Barbados negress, not accustomed to such intrusions, stood for safety in a puddle of soapy water with which she had attempted to drive the ants away, and begged me to tell her what to do to get rid of them."

E. praedator is not common at Kartabo, and the few foraging colonies I encountered were much smaller than those described by Bates and Mann. Moreover, they belonged to an undescribed variety which I shall call **guianense** var. nov. The soldier of this form is smaller than the type and measures only 7.5–8.5 mm.; head 3–3.5 mm. (typical form 8.5–11.5 mm.). The upper surface of the head is decidedly more opaque, very finely, densely and evenly punctate or reticulate, with the larger, sparser, piligerous punctures more indistinct. The body is paler, being dull ferruginous, with pale brown legs and blackish gaster. The worker, however, is darker and colored more like the typical form, but the head is somewhat less shining above. Though considerably darker, *guianense* approaches the subspecies *emiliæ* Mann in color, but is much closer to the type. In the soldier *emiliæ* the head is more finely and more indistinctly sculptured and somewhat glossy and the gaster and sides of the thorax are less shining. The var. *ferruginea* Norton from Mexico is probably closely allied to *guianense* but was inadequately described. Santschi has based a subsp. *auropubens* on male specimens from French Guiana and this may prove to be covarietal with my soldiers and workers from Kartabo. In my collection there are a few soldiers and workers taken by Forel at Esperanza, Colombia, which also belong to the var. *guianense*.

Eciton (**Acamatus**) **angustinode** Emery subsp. **emersoni** subsp. nov.

This small form, which I dedicate to Mr. Alfred Emerson, was seen only on two occasions at Kartabo, once rapidly running over the logs of an old stelling and once migrating with larvæ across the sandy soil of the laboratory yard. On both occasions the workers formed a straggling file and were extremely timid, turning back on their trail at the slightest disturbance. The typical form occurs in Southern Brazil and Paraguay; the Kartabo form has the postpetiole as broad as the petiole (narrower in the type), and only $1\frac{1}{3}$ times as long as broad ($1\frac{1}{2}$ times in the type), the sides of the petiole are longitudinally rugulose, the dorsal surface of its node shining, with a few coarse punctures. The antennæ, head, thorax and petiole are deep red, the remainder of the body and appendages yellow, the mandibles infuscated. I have compared the specimens with a small cotype of the typical *angustinode* from Rio Grande do Sul in my collection.

Eciton (**Acamatus**) **pilosum** F. Smith var. **beebei** var. nov.

(Fig. 7*b*).

Only two foraging colonies of this ant were seen in the jungle about Kartabo. July 20, Mr. Beebe found one of them near the *burchelli* nest above described, running in a long file from which he took two partially deälated males that were being conducted along by the workers. On going to the spot somewhat later I captured two more males, also partially deälated. The ants had their nest in a huge log where it could not be reached. On July 21, another foray of the same colony was observed but no males were seen. The workers were ascending and descending a large liana and carrying to their nest dozens of cocoons of some small *Camponotus* which apparently nested in the epiphytes on one of the trees, at a considerable distance from the ground. Thus laden with its prey the column resembled a party of *Polyergus* returning from a slave-raid on *Formica fusca*. The second colony, observed July 28, was foraging on the ground in a long column.

On examining the worker and male specimens taken July 20, I find that the former represent a variety of *pilosum* very close to var. *angustius* Forel, originally described from specimens taken by K. Fiebrig at San Barnardino, Paraguay. Comparison with a dozen cotypes of this variety in my collection shows that the Kartabo form, which I call *beebei*, differs only in the following particulars: The color of the body is even darker, being almost jet black and much darker than the typical *pilosum*; the mandibles, cheeks and the antennal funiculi are light brown throughout, whereas in *angustius* the upper surface of the funiculi is black, the mandibles and cheeks are fuscous or piceous and the legs are somewhat paler than in *beebei*. In both varieties the polymorphism of the worker is less pronounced than in the typical *pilosum*.

Turning to the male we find that it is merely a variety of *Eciton mexicanum* Smith, a form easily recognized by the peculiar shape of its mandibles (Fig. 7*a*). Smith described the worker as *Eciton pilosum* from Brazil in 1858, and the male as *Labidus mexicanus* from Orizaba, Mexico in 1859. Of the male Forel has described a var. *aztecum* from Guatemala and a subsp. *rosenbergi* from Northwestern Ecuador.

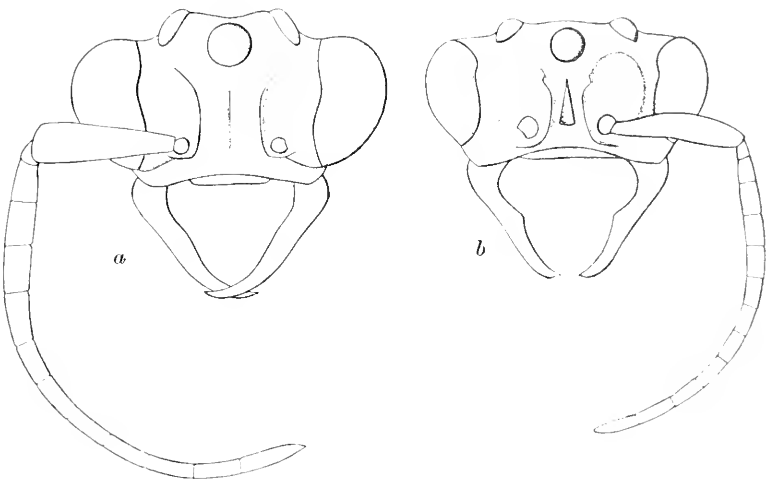

FIGURE 7. (*a*) Head of male *E.* (*A.*) *pilosum* F. Sm. (in sp.); (*b*) head of male *Eciton* (*Acamatus*) *pilosum* F. Sm. var. *beebei* var. nov.

That the typical *mexicanum* is the male of the typical *pilosum* (worker) is evident from the fact that both of these forms, and only these, occur in Texas. The male of the var. *beebei* measures 10.5–11 mm. and is therefore decidedly smaller than the male of the typical form of the species, which measures nearly 13 mm., and exhibits also the following differences: Ocelli smaller, head seen from above proportionally longer; frontal carinæ distinctly more acute, with their posterior ends more sharply dentate. Antennæ shorter. Thorax proportionally longer and more slender, with less elevated scutellum. Color more yellowish, with the posterior part of the head jet black; subgenital plate, posterior border of each gastric segment and the funicular articulations fuscous. In some specimens the last gastric segment is infuscated and the mesonotum has a fuscous streak on each parapsidal furrow and a broader, paler brown, anteromedian streak. The pilosity, especially on the thorax, is shorter than in the typical *pilosum*. Wings less yellowish, slightly infuscated, with brown, instead of resin yellow veins.

I regard as typical *pilosum* males several specimens in my collection from Frontera, Tabasco, Mexico; Nogales, Arizona (Oslar), Texas (Belfrage), Wharton, Texas (Cornell Univ.) and Austin, Texas (Wheeler). Forel's var. *aztecum* is extremely close to some of these. His subsp. *rosenbergi* is scarcely smaller (13 mm.), the head behind the eyes is described as feebly convex, instead of concave, the mesonotum as not so far advanced anteriorly over the pronotum, the tibiæ as less enlarged apically, the color as darker and the wings as tinged with blackish brown. I am inclined to believe that this is not a form of *pilosum*, but a distinct species, as Forel himself surmised.

Eciton (Acamatus) carolinense Emery.

Female (Fig. 8*a* and *b*). Length nearly 14 mm.; gaster 8.6 mm.

Head from the front as long as broad, scarcely broader anteriorly than behind, its sides rather rounded, its posterior border slightly emarginate, with a median longitudinal groove, deep and very distinct on the anterior half of the head, very faint on the posterior half. At the sudden transition between the two halves of the groove the surface is distinctly impressed. Seen from behind the head is subpentagonal, narrowed above, with concave occipital border and bluntly angular inferior occipital corners. Ocelli lacking; eyes reduced to minute white dots, just in front of the posterior third of the head. Mandibles narrow, edentate, straight, their pointed tips not incurved. Clypeus broad, somewhat impressed in the middle, the anterior border nearly straight, very feebly arcuate. Frontal carinæ short, welt-like, not encircling the antennal foveæ in front. Antennal scapes robust, about half as long as the head, excluding the clypeus; funiculi nearly three times as long as the scapes and more slender; first joint small, as broad as long; joints 2–8 a little longer than broad; remaining joints longer; last joint nearly as long as the two preceding taken together. Thorax long, narrow through the pronotum, broader through the mesonotum and broadest through the epinotum, which does not, however, equal the width of the head. In profile the thorax is fully three times as long as high, its dorsal outline straight, its dorsal surface flattened. Promesonotal, mesometanotal and meta-ëpinotal sutures indicated by impressed lines. Pronotum as long as broad, its anterior border marginate at the neck; mesonotum slightly longer than broad, its anterior border strongly arcuate, its posterior border less so; metanotum in the form of a very short, transverse, arcuate band; epinotum broader than long, with rounded rectangular posterior corners, its surface broadly impressed in the middle and with a distinct, median, longitudinal groove on its anterior half; the base in profile longer than the straight declivity with which it forms an obtuse angle. Petiole as high as the epinotum, but much narrower, from above transversely oblong, with rounded sides, about $1\frac{1}{4}$ times as broad as long, broadly impressed in the middle; the ventral portion protruding downward and forward as a thick convexity. Gaster long and rather slender, though more than twice as broad as the petiole, its anterior segment rectangular from above; pygidium large, convex, bluntly pointed and entire behind; hypopygium angularly excised in the middle and terminating in two membranous, rather blunt points. Sting small, partly exserted. Legs rather short; claws simple. Stigmata of the thorax, petiole, postpetiole and gaster large.

Head, thorax and petiole nearly opaque, very finely shagreened, densely and rather coarsely punctate; gaster shining, with much smaller, shallower and more scattered punctures, except on the first segment (postpetiole), which approaches the petiole in sculpture.

Hairs yellowish, short, erect, abundant on the head, thorax, petiole and appendages, much shorter and sparser on the gaster; mandibles, clypeus, gula and scapes also with numerous long, flexuous hairs of uneven length.

Rich ferruginous red; antennæ and legs slightly paler and more yellowish; pygidium brown.

Male (Fig. 8*c*). Length 9.3 mm.

Closely resembling the male of *opacithorax* Emery but slightly smaller and distinctly more slender. Eyes and ocelli even smaller. Mandibles much narrower and less expanded in the middle. Antennæ more slender. Thorax shorter; petiole shorter in proportion to its width. Terminal points of subgenital plate longer and less curved. Sculpture, pilosity and color much as in *opacithorax*, except that the thorax is not shining but opaque and more densely and more coarsely punctate. The scapes and mandibles are black, not red as in *opacithorax*.

Described from single specimens taken by Mr. W. T. Davis at Clayton, Georgia (2000–3700 ft.) during June, 1909.

The female can be readily distinguished from that of *opacithorax* by the following characters: The eyes are even smaller, the anterior clypeal border is much less arcuate, the epinotum is broader and more angular behind, its declivity shorter compared with the base and the angle between the two surfaces more pronounced. The pet-

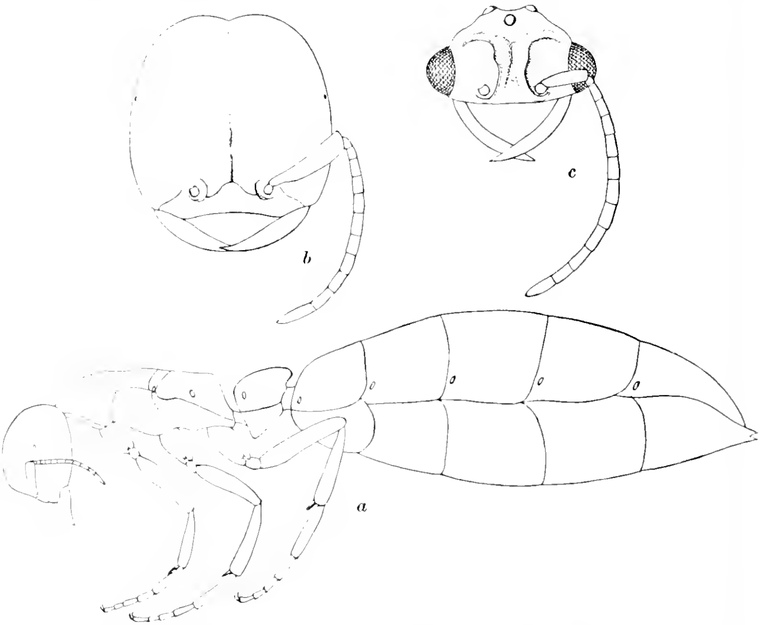

FIGURE 8. (*a*) Female of *Eciton* (*Acamatus*) *carolinense* Emery, lateral view; (*b*) head of same more enlarged, dorsal view; (*c*) head of male.

iole is larger, with more pronounced anterior angles and broader and deeper dorsal concavity. The surface of the head, thorax and petiole and especially of the head, is much more opaque and more coarsely punctate and beset with longer and more abundant hairs.

Cheliomyrmex Mayr.

(Fig. 5*b*, Figs. 9 and 10.)

Mayr established the genus *Cheliomyrmex* in 1870 on worker specimens of a species which he called *nortoni*, because he had received them from Edward Norton. They were evidently collected by Sumichrast

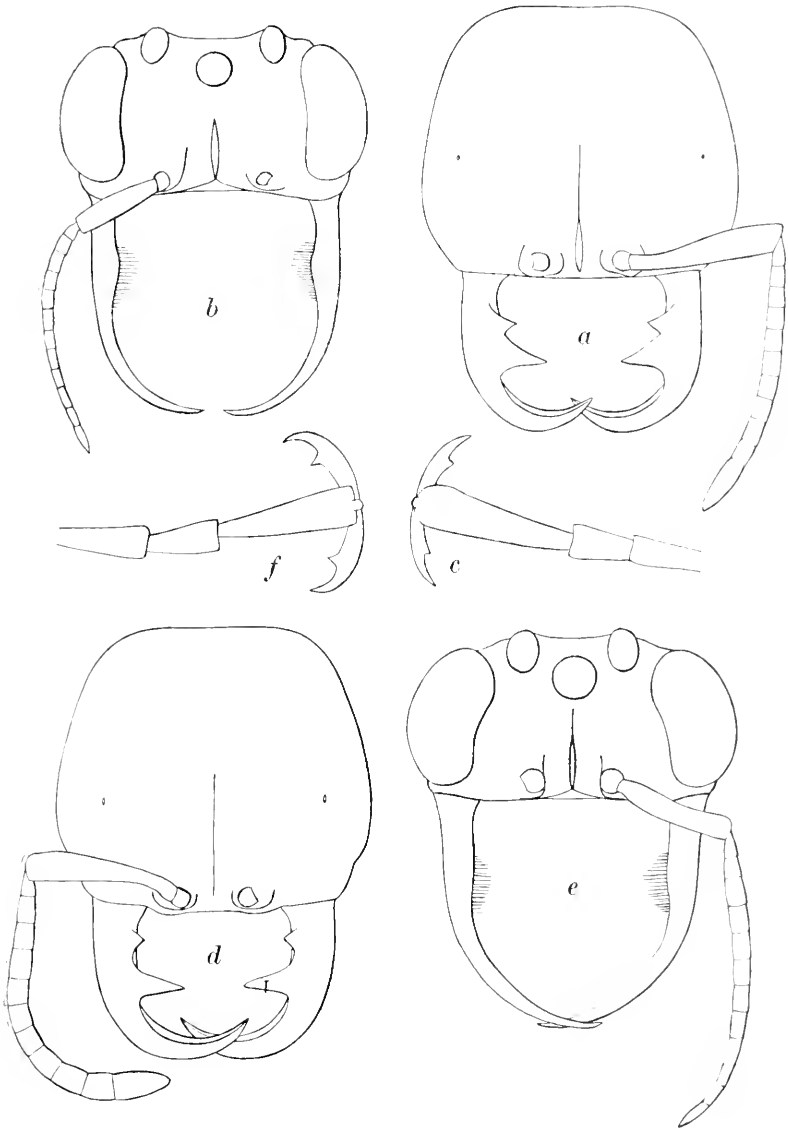

FIGURE 9. *a*, Head of soldier *Cheliomyrmex morosus* F. Sm.; *b*, head of male; *c*, terminal tarsal joints of worker; *d*, head of soldier *C. megalonyx* sp. nov.; *e*, head of male; *f*, terminal tarsal joints of worker.

near Mount Orizaba, in the state of Vera Cruz, Mexico. Mayr was struck by the peculiarities of the insect, in which he detected resemblances to the African *Dorylus* (*Typhlopone* and *Anomma*) on the one hand and to the Ponerinæ on the other. Since its description, *C. nortoni* has been recorded only from Mexico and British Honduras. Very recently Mann has taken a few specimens of it in Spanish Honduras, but it seems to be rare and sporadic, and no observations have been published on its habits. In 1894, Emery described an opaque form from Peru as the subsp. *andicola*, but Forel, after examining numerous specimens from the highlands of Colombia, raised it to specific rank.

In his comparative studies of the Dorylinæ Emery made the interesting suggestion that the male of *C. nortoni* might be the form previously described from Mexico as *Labidus morosus* by F. Smith (1859) and provisionally cited in the Genera Insectorum as *Eciton* (*Labidus*?) *morosum*. Forel described from Honduras and Mexico a variety of this male as *payarum*, which seems to differ from the type only in a few insignificant characters, and Emery has also described a male specimen as subsp. *ursinum* from Brazil. It will be seen that *nortoni* and *morosus* have precisely the same distribution. This is also evident from the specimens in my collection, which comprise worker topotypes taken by F. Silvestri at Orizaba, and males taken by Fred. Knab in the same locality, also workers taken at Manatee, British Honduras by J. D. Johnson and a male taken in the same country by Prof. C. H. Fernald. These and more cogent considerations to be cited below so thoroughly convince me that Mayr's *nortoni* is merely the worker of Smith's *morosus* that I propose to change the name of the species to *Cheliomyrmex morosus* (F. Smith). In British Guiana I found the workers of a *Cheliomyrmex* which I at first took to be *morosus* and also secured the males, which I am sure had escaped from its colonies, but on closer examination I am inclined to regard this form as a distinct species and describe it below as *C. megalonyx* sp. nov. Since the range of the genus has thus been extended to British Guiana, it is probable that Emery's *ursinum* really represents a fourth species, peculiar to Brazil.

Emery has made the further interesting suggestion that *Cheliomyrmex* is an archaic genus, and this is borne out by a study both of the worker and the male, for the characters of both show a mixture of the characters of *Dorylus* and of *Eciton*, together with certain peculiarities observable in no other ants of the subfamily. Thus in the worker, the structure of the thorax and pedicel, the feeble carina-tion of the cheeks and the distinctly advanced clypeus in the smaller workers remind one of *Dorylus*, and the simple pygidium, the toothed claws and the antennæ are Ecitine, whereas the structure of the mandibles of the soldier and the scale-like transverse petiolar node are peculiar to the genus. In the male the venation (Fig. 5*b*) and toothed claws conform to the *Eciton* type and the flattened femora, especially the hind pair, and the shape of the antennal funiculi are as strongly reminiscent of *Dorylus*, while the structure of the mandibles, though sickle-shaped and toothless, and more like those of certain Ecitons, are nevertheless very singular. The genitalia (Fig. 10) conform in the main to the *Eciton* type (Fig. 6) in having the stipites articulated and not fused with the basal annulus, but their shape is more like those of certain *Dorylus*. The structure of the subgenital plate (Fig. 10*d*), however, with its four terminal teeth is peculiar. Emery (1910, p. 16 nota) believes that *Cheliomyrmex* is "very close to *Eciton*, subgen. *Labidus* and especially to *E. coecum*," and therefore includes it in the tribe Ecitini. I am unable to follow him in this procedure since I believe that the resemblance to *coecum* is superficial and illusory and due to similarity of habits (convergence) and that the genus *Cheliomyrmex* should constitute an independent tribe, the Cheliomyrmicini. It is, in fact, the most archaic and generalized of all the tribes of the Doryline subfamily, just as the Leptanillii constitute the most specialized, or degenerate tribe. Undoubtedly *Cheliomyrmex* is a very ancient group, very near the ancestral stem from which both the Dorylini and the Ecitini sprang and diverged, possibly during the late Cretaceous. The discovery of the female will probably yield additional arguments in favor of this contention.

C. megalonyx was first found at Kartabo July 20, on the edge of the jungle and just back of the laboratory, where it was foraging in columns under prostrate logs. At first sight this species may be mistaken for *E. coecum*, but the columns are denser and more populous and the soldiers and workers exhibit much less variation in stature. *Cheliomyrmex* seems also to be even more photophobic than *coecum*, disappearing at once into the soil when the light is let into its galleries. It stings much more painfully and will attack fiercely when intercepted in its movements. On several successive days I saw detachments of the same army under logs in the same locality. There were no larvæ nor pupæ and very probably what I saw were merely hunting columns of a huge colony which I failed to locate till a month later. August 21, on visiting the taxidermist's hut, behind the laboratory and less than a hundred yards from the spot in which I first found the species, I came upon a great army moving its larvæ. Sam, the negro laboratory attendant, informed me that this army had been living for many days under a pile of large logs about forty feet from the hut. He had disturbed the pile on the preceding day and the *Cheliomyrmex* had begun to move. They were running along in dense, orderly columns under leaves, sticks or boards, wherever such cover was available, but where they had to cross open spaces, they had built covered galleries about four-fifths of an inch wide, of small particles of earth. The column kept in the shade and crossed the earthen floor of the shed diagonally, disappearing in the dense grass and weeds behind it. There were also numerous openings in the soil, usually circular and about the size of a cent-piece, and from these files of ants, after having proceeded long distances beneath the surface, were emerging to join the columns in the surface galleries. These holes and all the openings in the galleries presented an extraordinary appearance for both the circumferences of the former and the edges of the latter, wherever their ceiling had caved in — and this had occurred in places for distances varying from a few inches to a foot — had a regular guard of soldiers, standing close together, side by side, on extended legs, with their heads directed upward, their mandibles wide open and their antennæ waving about in the air. Each round hole presented a beautiful rosette of these guards and each open surface gallery two parallel rows, between which the workers were hurrying along in a dense procession, the smallest carrying the larvæ tucked under their bodies. Sam was offered a substantial reward for the queen, but although he devoted most of the day to watching the ants, the only unusual object he found in their moving columns was a fine red myrmecophilous Staphylinid allied to *Xenocephalus*. The extraordinary behavior of the soldiers of this army is of considerable interest as indicating certain ethological affinities of *Cheliomyrmex* with the African species of *Dorylus* of the subgenus *Anomma*, for very similar behavior has been repeatedly observed in these ants by Savage, Vosseler, Lang and others, but, to my knowledge, has never been seen in any species of *Eciton*. The *Cheliomyrmex* larva closely resembles other Doryline larvæ which I have examined (*Eciton* s. str., *Acamatus* and *Anomma*) but has no rudiments of antennæ. In this respect it also agrees with *Anomma* and differs from the Ecitini.

On many nights, from July 26 to August 31, a few male Dorylines were observed — usually from one to five or six — flying to the lights in the laboratory. I collected all of these specimens, 41 in number, and on examining them found them to comprise two males of *E.*

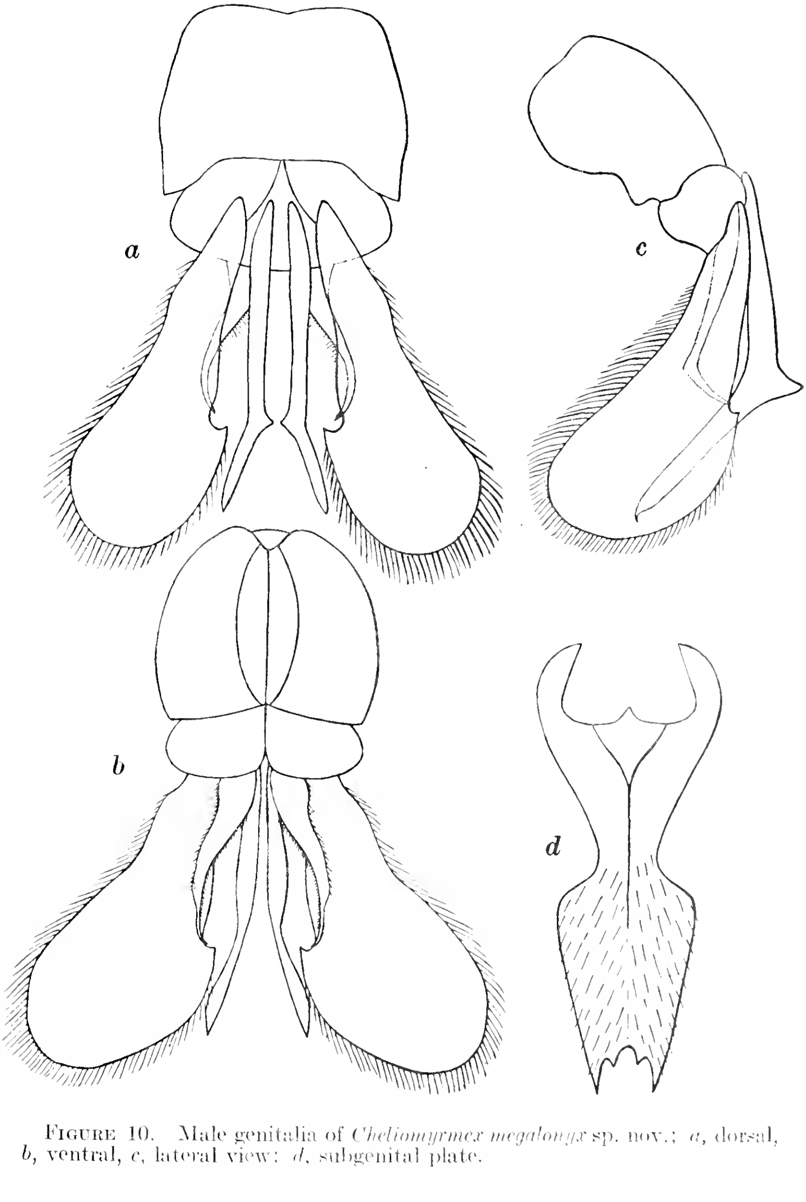

FIGURE 10. Male genitalia of *Cheliomyrmex megalonyx* sp. nov.; *a*, dorsal, *b*, ventral, *c*, lateral view; *d*, subgenital plate.

burchelli and 39 of a form very close to *E. morosus* but of a paler, more reddish brown color and with somewhat longer mandibles. No other Dorylines appeared at the lights during my stay at Kartabo. I conclude that these 39 males had escaped from the *Cheliomyrmex* colony just back of the laboratory for the following reasons: First, their flight coincided with the sojourn of the large *Cheliomyrmex* colony behind the laboratory. Second, the laboratory is on a point of land at the confluence of two great rivers, the Mazaruni and Cuyuni, which cannot be crossed by such feeble flyers as these male Dorylines. Furthermore, the building is screened on the sides from the rivers by huge clumps of bamboo. Hence the males must have come from the immediate vicinity behind the laboratory. Third, my days of intensive search for ants about the laboratory convinced me that the only Doryline with dentate claws and unknown male in the immediate vicinity was the *Cheliomyrmex*, the males of the other species encountered (*E. burchelli*, *hamatum* and *coecum*) being known. Fourth, both the males taken at lights and the soldiers and workers of the *Cheliomyrmex* from the colony behind the laboratory differ specifically, or at least subspecifically from *morosus*. I therefore regard all of these phases as belonging to the same species, which may be distinguished from the Mexican and Honduran form as follows:

Cheliomyrmex megalonyx sp. nov.

Soldier (Fig. 9*d, f*). Very similar to *morosus* (Fig. 9*a, c*) but differing as follows: Color of body more deeply ferruginous red; head and mandibles proportionally somewhat longer, the latter narrower in the region of the median tooth, which is longer and narrower at its base. The basal tooth is more acute and separated by a distinct diastema from the median tooth, not arising from its base as in *morosus*. Antennæ somewhat longer, the median joints especially. Occipital border of the head, seen from above more deeply excised and sharply marginate, the margination continuing down the inferior occipital angle on each side as a sharp ridge on to the gular surface. In *morosus* these lateral ridges are absent and the median margination of the occipital border is feeble. Epinotum less swollen and convex, somewhat lower and more sloping than in *morosus*, its base and declivity more distinct, subequal and nearly straight in profile. Petiole slightly more compressed anteroposteriorly, the anterior and posterior surfaces of the node more flattened, the sides and dorsal surface less rounded; postpetiole distinctly shorter, more than twice as broad as long. Claws, especially on the hind legs, conspicuously larger and coarser than in *morosus*.

Mandibles more distinctly striolate, their punctures much smaller, less foveolate and less numerous. Surface of head, thorax, and petiole with small, sparse, piligerous punctures instead of the large, shallow, more or less oblique piligerous foveolæ of variable size of *morosus*, so that the surface is smoother and more even. Gaster somewhat more shining, with finer, scattered piligerous punctures. Pilosity very similar in the two species.

Worker. Very much like the corresponding phase of *morosus*, but with the occipital margination and ridges as in the soldier. Sculpture also like that of the soldier and differing in the same way from that of the *morosus* worker. In the large worker the basal tooth of the mandibles seems to be more frequently developed in the Guiana form.

Male (Fig. 5*b*, 9*c*, 10). Length about 19–20 mm.

Eyes and ocelli somewhat larger and more convex than in *morosus* (Fig. 9*b*), the inner orbits of the former more strongly sinuous; antennæ and mandibles longer. Petiole with more distinct posterior corners, its sides more depressed and with sharper borders, its middorsal surface much less concave. Genitalia large, retracted; stipites large, expanded, subelliptical, articulated with the basal annulus as in *Eciton*, volsellæ slender, apically geniculate and tapering; sagittæ longer and stouter, geniculate at the middle and there furnished with an acute dorsal process. Subgenital plate large, trowel-shaped, terminating in two large, acute lateral and two small acute median teeth.

Color much paler than *morosus*, being rich, reddish brown, usually with a narrow transverse band near the posterior border of each gastric segment, the base of the last segment, the posterior part of the head, two broad parapsidal streaks and an even broader anteromedian streak on the mesonotum, dark brown. These darker markings seem to vary considerably in different individuals. Wings much paler than in *morosus*, resin yellow, darker at the anterior border of the radial cell; veins also resin yellow. Pilosity on the mesonotum and dorsal surface of the gaster distinctly shorter than in *morosus*.

Described from numerous specimens taken at Kartabo Point, British Guiana.

In conclusion I list the four species of *Cheliomyrmex* and their synonymy as I understand it at the present time.

Cheliomyrmex morosus (F. Smith).

Labidus morosus F. Smith, Cat. Hym. Brit. Mus. 7, 1859, p. 6, ♂.

Eciton morosus Mayr, Wien. Ent. Zeitg. 5, 1885, p. 33, ♂.

Eciton morosum Dalla Torre, Cat. Hymen. 7, 1893, p. 4, ♂; Forel, Biol. Centr. Amer. Hym. 3, 1899, p. 27, ♂; Forel, Ann. Soc. Ent. Belg. 56, 1912, p. 43, ♂.

Eciton (*Labidus?*) *morosum* Emery, Gen. Insect. Dorylinæ 1910, p. 23, ♂.

Cheliomyrmex nortoni Mayr, Verh. zool. bot. Gesel. Wien. 20, 1870, p. 968, ♃; Ern. André, Rev. d'Ent. 6, 1887, p. 294, ♃ ☿; Dalla Torre, Cat. Hym. 7, 1893, p. 7, ♃ ☿; Forel, Biol. Centr. Amer. Hym. 3, 1899, p. 30, ☿; Ashmead, Proc. Ent. Soc. Wash. 8, 1906, p. 27, ☿; Wheeler, Bull. Amer. Mus. Nat. Hist. 23, 1907, p. 271, Pl. 11, figs. 4–9, ♃ ☿; Wheeler, Ants, etc. 1910, p. 255, Fig. 148, ♃ ☿; Wheeler, Boll. Lab. Zool. Gen. Agrar. Portici 3, 1909, p. 231, ♃ ☿; Emery, Gen. Insect. Dorylinæ, 1910, p. 17, ♃ ☿.

Cheliomyrmex morosus Wheeler. *Ante*, p. 318, ♃ ☿ ♂.

Type locality: Mexico.

Mexico: Orizaba, Vera Cruz, ☿ (Sumichrast); Atoyac, Vera Cruz, ♂ (Schumann); Orizaba, ♃ ☿ (F. Silvestri); Cordoba, Vera Cruz, ♂ (Fred. Knab); Santa Rosa ♂ (Wm. Schaus).

British Honduras: ♂ (C. H. Fernald); Manatee, ♃ ☿ (J. D. Johnson).

Honduras: La Ceiba, ♃ ☿ (W. M. Mann).

C. morosus var. **payarum** (Forel).

Eciton morosum var. *payarum* Forel, Biol. Centr. Amer. Hym. 3, 1899, p. 27, ♂; Forel, Mitteil. Naturh. Mus. Hamburg 18, 1901, p. 49, ♂.

Eciton (*Labidus?*) *morosum* var. *payarum* Emery, Gen. Insect. Dorylinæ, 1910, p. 23, ♂.

Type locality: Honduras (Staudinger).

Mexico.

Cheliomyrmex ursinus Emery.

Eciton morosum subsp. *ursinum* Emery, Bull. Soc. Ent. Ital. 33, 1901, p. 52, ♂.

Eciton (*Labidus?*) *morosum* subsp. *ursina* Emery, Gen. Insect. Dorylinæ, 1910, p. 23, ♂.

Cheliomyrmex ursinus Wheeler, *ante* p. 318, ♂.

Type locality: Brazil (André).

Cheliomyrmex andicola Emery.

Cheliomyrmex nortoni subsp. *andicola* Emery, Bull. Soc. Ent. Ital. 26, 1894, p. 185, ♃; Emery, Gen. Insect. Dorylinæ, 1910, p. 17, ♃.

Cheliomyrmex andicola Forel, Mem. Soc. Neuchat. Sc. Nat. 5, 1912, p. 10, ♃ ☿; Wheeler, *ante* p. 318, ♃ ☿.

Type-locality: Panamarca, Peru.

Colombia: Cafetal Camelia, near Angelopolis, 1819 m.; Aguacatal, Dep. Tolima, 1515 m.; Facatativa, Dep. Cundinamarca, 2588 m. (O. Fuhrmann).

Cheliomyrmex megalonyx Wheeler.

Ante p. 322, ♃ ☿ ♂.

Type-locality: Kartabo Point, British Guiana (Wheeler).

Postscript.

After the manuscript of this paper was completed Mr. Frederick M. Gaige wrote me that he had taken a female of *Eciton vagans* Olivier in the Arroyo de Arena, at an altitude of 400 ft., in the Santa Marta Mts. of Colombia, on August 25, 1920. Dr. Carlos Bruch also sends me word by Prof. J. C. Bradley that he has recently captured a female of *Eciton* (*Acamatus*) *strobeli* Mayr in the Argentine. Thus the number of species of which all three phases are known is increased to eight.

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THE AXES OF A QUADRATIC VECTOR.

By Frank L. Hitchcock.

(Continued from page 3 of cover.)

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THE AXES OF A QUADRATIC VECTOR.

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CONTENTS.

1. Introduction.

The well-known *linear vector function* [1] permits of several kinds of extension or generalization. We may, for example, have a function linear in each of two vectors, as $\varphi(\rho, \sigma)$; or we may have $F[\rho]$ quadratic in a single vector ρ. These two concepts evidently merge into one another, for $\varphi(\rho, \sigma)$ becomes a quadratic function of ρ when we let σ equal ρ.

In a former paper [2] a study was made of various types of vectors quadratic in a single vector ρ. The following results will be fundamental to the present discussion:

(A) An axis of $F\rho$ is a direction β such that $F\beta$ is parallel to β or else null. A quadratic vector has in general seven axes. If six axes lie on a quadric cone there are an infinite number of axes, and conversely; the vector function is then *reducible*. The number of *distinct* axes may be less than seven, since an axis may be of multiple order.

(B) In general any quadratic vector may be written as the sum of two terms

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[1] Hamilton, Elements of Quaternions, Chap. II, Section 6; Gibbs-Wilson, Vector Analysis, Chap. V.

[2] Proc. Amer. Acad. Arts & Sci., **52**, No. 7 (Jan. 1917), pp. 369–454.

$$F\rho = V\varphi\rho\theta\rho + \rho S\delta\rho \tag{1}$$

where φ and θ are linear vector functions and δ is a vector. Exception can only occur when one of the axes is of order at least five.

(C) The term $V\varphi\rho\theta\rho$ may in general be expanded as

$$V\varphi\rho\theta\rho = \alpha_1 x_2 x_3 + \alpha_2 x_3 x_1 + \alpha_3 x_1 x_2 \tag{2}$$

where α_1, α_2, and α_3 are constant vectors, and where the x's are given by

$$x_1 S\beta_1\beta_2\beta_3 = S\beta_2\beta_3\rho, \quad x_2 S\beta_1\beta_2\beta_3 = S\beta_3\beta_1\rho, \quad x_3 S\beta_1\beta_2\beta_3 = S\beta_1\beta_2\rho \tag{3}$$

and where the α's depend on nine scalars by the scheme

$$\begin{aligned} \alpha_1 &= A_{11}\beta_1 + A_{21}\beta_2 + A_{31}\beta_3 \\ \alpha_2 &= A_{12}\beta_1 + A_{22}\beta_2 + A_{32}\beta_3 \\ \alpha_3 &= A_{13}\beta_1 + A_{23}\beta_2 + A_{33}\beta_3 \end{aligned} \tag{4}$$

(D) In general the quadratic vector is fully determined when its axes are given, aside from the term $\rho S\delta\rho$ and a multiplicative scalar.

I shall refer to this former paper as C. Q. V.

2. The A's as Functions of the Axes.

It follows from (C) and (D) that the nine A's are determinate as functions of the axes, aside from a common scalar multiplier. A knowledge of these functions facilitates our attack on a variety of problems. I propose to express these A's in terms of the axes β_1, $\beta_2, \cdots \beta_7$ and to illustrate the utility of the results by some applications.

With the notation and results of pp. 377–384 of C. Q. V. we may write, omitting mere constant multipliers.

$$V\varphi\rho\theta\rho = \beta_4 \cdot \frac{(567)\,(P_5P_6P\rho)}{(P_5P_6P_7)} + \beta_5 \cdot \frac{(647)\,(P_6P_4P\rho)}{(P_6P_4P_7)} + \beta_6 \cdot \frac{(457)\,(P_4P_5P\rho)}{(P_4P_5P_7)} \tag{5}$$

Now it is evident from (2) that if we let $x_1 = 0$, $x_2 = x_3 = 1$, we shall reduce $V\varphi\rho\theta\rho$ to α_1; but by p. 381 of C. Q. V. we shall then reduce $P\rho$ to i. We thus obtain

$$\alpha_1 = \beta_4 \cdot \frac{(567)\,(P_5P_6i)}{(P_5P_6P_7)} + \beta_5 \cdot \frac{(647)\,(P_6P_4i)}{(P_6P_4P_7)} + \beta_6 \cdot \frac{(457)\,(P_4P_5i)}{(P_4P_5P_7)} \tag{6}$$

The factor (P_4P_5i) is one of the minors of the determinant (22) p. 381 of C. Q. V., which, developed by the method there given, yields

$$(P_4P_5i) = (123)\,(234)\,(235)\,(415) \tag{7}$$

Expanding the factors (P_5P_6i) and (P_6P_4i) in the same way and dropping the common factor (123) we have from (6)

$$\alpha_1 = \beta_4 \frac{(567)\,(235)\,(236)\,(516)}{(P_5P_6P_7)} + \beta_5 \frac{(647)\,(236)\,(234)\,(614)}{(P_6P_4P_7)} + \beta_6 \frac{(457)\,(234)\,(235)\,(415)}{(P_4P_5P_7)} \tag{8}$$

From (4) we see that $A_{11} = (\beta_2\beta_3\alpha_1)$ if we neglect the factor (123). We therefore have from (8)

$$A_{11} = (234)\,(235)\,(236) \left\{ \frac{(567)\,(516)}{(P_5P_6P_7)} + \frac{(647)\,(614)}{(P_6P_4P_7)} + \frac{(457)\,(415)}{(P_4P_5P_7)} \right\} \tag{9}$$

Similarly $A_{21} = (\beta_3\beta_1\alpha_1)$ giving

$$A_{21} = \sum_{456} \frac{(314)\,(567)\,(235)\,(236)\,(516)}{(P_5P_6P_7)} \tag{10}$$

where the terms of the sum are obtained from one another by cyclic advancement of the numbers 4, 5, 6. Also $A_{31} = (\beta_1\beta_2\alpha_1)$, giving

$$A_{31} = \sum_{456} \frac{(124)\,(567)\,(235)\,(236)\,(516)}{(P_5P_6P_7)} \tag{11}$$

3. Identities on which Depends the Simplification of the A's.

The other six A's are at once obtained from the first three by cyclic advancement of the numbers 1, 2, 3. Before doing this however, we may simplify the expressions just obtained, by means of identical relations connecting the factors which enter the numerators with the determinants which occur in the denominators.

The denominator $(P_4P_5P_7)$ is the determinant of the coefficients of the three vectors P_4, P_5, and P_7, and aside from a factor $(123)^2$ is equal, by C. Q. V. page 384, to the expression $C_{1234}(5, 7)$, which vanishes when the six vectors lie on a quadric cone. For our present purpose we may use a simpler notation and write

$$(P_4P_5P_7) = (123)^2\, C_6 \tag{12}$$

by designating the omitted vector. The other denominators may be transformed in a similar manner.

Consider now the identity

$$F\beta_1 C_1 + F\beta_2 C_2 + \cdots + F\beta_7 C_7 = 0 \qquad (13)$$

where $F\beta_1$ is any quadratic function of β_1, $F\beta_2$ is the same function of β_2, etc.[3] If we let $F\lambda = (\lambda\beta_1\mu)\,(\lambda\beta_1\nu)$ we shall have $F\beta_1 = 0$ and the identity becomes

$$(\beta_2\beta_1\mu)\,(\beta_2\beta_1\nu)C_2 + (\beta_3\beta_1\mu)\,(\beta_3\beta_1\nu)C_3 + \cdots + (\beta_7\beta_1\mu)\,(\beta_7\beta_1\nu)C_7 = 0 \quad (14)$$

which by the notation already employed may be written

$$\cdots + (21\mu)\,(21\nu)C_2 + (31\mu)\,(31\nu)C_3 + \cdots (71\mu)\,(71\nu)C_7 = 0 \quad (15_1)$$

and similarly by letting $F\lambda = (\lambda\beta_2\mu)\,(\lambda\beta_2\nu)$,

$$(12\mu)\,(12\nu)C_1 + \cdots + \quad (32\mu)\,(32\nu)C_3 + \cdots (72\mu)\,(72\nu)C_7 = 0 \quad (15_2)$$

Proceeding thus we obtain a set of seven equations of which the last is

$$(17\mu)\,(17\nu)C_1 + (27\mu)\,(27\nu)C_2 + \cdots + (67\mu)\,(67\nu)C_6 + \cdots \qquad (15_7)$$

If we multiply these equations respectively by $C_1, C_2 \cdots C_7$ and add, we note that each term of the sum is of the form $(12\mu)\,(12\nu)\,C_1C_2$ and that each such term occurs twice, since $(12\mu)\,(12\nu) = (21\mu)\,(21\nu)$. Cancelling the factor 2 we thus have the new identity

$$\Sigma(12\mu)\,(12\nu)C_1C_2 = 0 \qquad (16)$$

where the left side contains as many terms as pairs can be chosen from the numbers one to seven, that is 21 terms.

So far μ and ν are any vectors whatever. Now let $\mu = \beta_1$ and $\nu = \beta_7$ causing all terms containing C_1 or C_7 to vanish. The remaining ten terms may be arranged as follows,

$$\begin{aligned}
&(561\,(567)C_5C_6 + (641)\,(647)C_6C_4 + (451)\,(457)C_4C_5 \\
&+ C_2[(241)\,(247)C_4 + (251)\,(257)C_5 + (261)\,(267)C_6] \\
&+ C_3[(341)\,(347)C_4 + (351)\,(357)C_5 + (361)\,(367)C_6] \\
&\qquad\qquad\qquad + (231)\,(237)C_2C_3 = 0 \qquad (17)
\end{aligned}$$

Returning to (13) and putting $F\lambda = (2\lambda 1)\,(2\lambda 7)$ we have, since the first, second, and seventh terms vanish,

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[3] This identity may be proved by noting that the left side is quadratic in β_7 and vanishes when β_7 coincides with any one of the other six vectors, hence vanishes identically. For a more detailed consideration of the C's see "An identity connecting seven vectors," Proc. Royal Soc. Edinburgh, **40**, Part II (No. 14), June 1920.

$$(231)\,(237)C_3 + (241)\,(247)C_4 + (257)\,(251)C_5 + (261)\,(267)C_6 = 0 \quad (18)$$

That is, the expression in brackets in the second line of (17) is equal to $-(231)\ (237)C_3$. In a similar manner, the expression in brackets in the third line of (17) is equal to $-(231)\ (237)C_2$. Therefore (17) becomes

$$(561)\,(567)C_5C_6 + (647)\,(641)C_6C_4 + (451)\,(457)C_4C_5 = (231)\,(237)C_2C_3 \quad (19)$$

4. Simplification of the A's.

Return now to our expression (9) for A_{11}, transform the denominators as in (12), and simplify,

$$A_{11} = (234)\,(235)\,(236)\frac{[-(561)\,(567)C_5C_6-(641)\,(647)C_6C_4-(451)\,(457)C_4C_5]}{(123)^2C_4C_5C_6}$$

$$= -(234)\,(235)\,(236)\,(237)\frac{C_2C_3}{(123)C_4C_5C_6}, \text{ by (19)}. \quad (20)$$

Since it is evident that all the A's will have a common denominator this may be rejected, and we may write as the value of A_{11}

$$A_{11} = (234)\,(235)\,(236)\,(237)C_2C_3 \quad (21)$$

It thus appears that A_{11} vanishes when either of the four axes β_4, β_5, β_6, or β_7 lies in the plane of β_2, β_3. The quadratic vector is assumed irreducible, hence C_2 and C_3 do not vanish, as was proved in C. Q. V.

Considering next the simplification of A_{21}, we may first add the three terms, and reject the same factor as above. This gives

$$A_{21} = -(123)^{-1}\sum_{456}(314)\,(567)\,(561)\,(235)\,(236)C_5C_6 \quad (22)$$

From the fundamental identity (13), letting $F\lambda = (\lambda 67)\,(23\lambda)$,

$$(567)\,(235)C_5 + (467)\,(234)C_4 + (167)\,(231)C_1 = 0 \quad (23)$$

and by letting $F\lambda = (4\lambda 7)\,(23\lambda)$,

$$(457)\,(235)C_5 + (467)\,(236)C_6 + (417)\,(231)C_1 = 0 \quad (24)$$

It is now easy to eliminate C_5 from the last three identities. The terms in C_6C_4 cancel at the same time, giving

$$A_{21} = + (314)(561)(236)(167)C_1C_6 + (316)(451)(234)(417)C_1C_4 \quad (25)$$

For A_{31} in a precisely similar manner,

$$A_{31} = + (124)(561)(236)(167)C_1C_6 + (126)(451)(234)(417)C_1C_4 \quad (26)$$

It is evident that expressions of similar form might have been found by eliminating either C_4 or C_6 instead of C_5.

The other six A's may be found from (21), (25), and (26) by advancing the numbers 1, 2, 3, cyclically.

It is also evident that, by means of identities of the same form as (23) and (24), which connect any three C's with one another, we may express any one of the A's in terms of any pair of C's we wish, the coefficients being composed of scalar products of three axes.

5. Application to Cases of Coplanarity of Axes.

As a first illustration of the utility of the A's, we may inquire what simplification takes place in the form of a quadratic vector when a set of three axes become coplanar,— a question intimately related to the problem of finding particular solutions of differential equations.[4]

One obvious answer is that if the three coplanar axes be numbered 4, 5, and 7, the general expression (5) loses its third term and becomes a binomial vector; but the form (5) implies a knowledge of all the axes, while the data of the problem frequently furnish only three axes,— for example, quadratic point transformations are often specified in terms of the singular points, which are our three axes β_1, β_2, β_3 such that $F\beta$ is null. Finding the remaining four axes and constructing the general expression (5) would then require, in general, the solution of an equation of the fourth degree. I propose to show that, if we have the quadratic vector in the form (1), or its equivalent (2), it is never necessary to find the other four axes in order to detect the existence of coplanarity among the axes; and, conversely, we may, by giving proper values to the A's, immediately construct a quadratic vector possessing any required relations of coplanarity among the axes.

Taking the latter problem first, as being the simpler, if it be merely required *that one set of axes be coplanar*, we may let either of the three scalars A_{11}, A_{22}, or A_{33} be zero. This is at once evident from the form of (21). For example, if A_{11} is zero, one of the four axes β_4, β_5, β_6, β_7 must lie in the plane β_2, β_3. By a proper choice of the vector δ in (1),

4 See note to C. Q. V. page 385.

the quadratic vector can then be reduced to a binomial in the vectors β_2, β_3, the four diplanar axes becoming zeros, the value of δ being unique.

If it be required to have *two sets of three coplanar axes* one way is to let A_{11} and A_{22} both vanish. But we have another equally easy way; if we let A_{21} and A_{32} both vanish, we shall have two sets of coplanar axes, β_1 being common to both sets. To prove this, note that, by (25) and (26), if $A_{21} = A_{31} = 0$, we have two linear equations in the quantities

$$(561)\ (236)\ (167) \quad \text{and} \quad (451)\ (234)\ (417) \tag{27}$$

the determinant of the coefficients being $(314)\ (126) - (316)\ (124)$, which is the same as $(123)\ (146)$. Now (123) is different from zero by hypothesis, for in assuming the form (2) we assume β_1, β_2, and β_3 to be diplanar;[5] on the other hand (146) may well be zero; if so we may write $\beta_1 = m\beta_4 + n\beta_6$ and either of the two equations reduces at once to

$$m\ (236)C_6 - n(234)C_4 = 0$$

by cancelling factors which cannot vanish if the quadratic vector is irreducible. This is the same as

$$(176)\ (236)C_6 + (174)\ (234)C_4 = 0$$

which by an identity of the form (23) gives

$$(175)\ (235) = 0.$$

Now (235) cannot vanish along with (146) hence $(175) = 0$, that is, we have two sets of coplanar axes having β_1, in common.

If the determinant in question does not vanish, the quantities (27) must both vanish. Remembering that we cannot have two distinct sets of coplanar axes nor four coplanar axes, if the quadratic vector is irreducible, we see by inspection that neither (234) nor (236) can vanish. Hence we must have the same case as above, viz. two sets including β_1.

If it be required to have *three sets of coplanar axes* it is now easy to pick out two quite different cases:

1°. We may let A_{11}, A_{22}, and A_{33} all vanish. Three of the four axes β_4, β_5, β_6, β_7, lie, respectively, in the faces of the triedron whose edges are β_1, β_2, β_3.

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[5] It was shown in C. Q. V. that a quadratic vector possesses two distinct sets of three diplanar axes except in certain special cases.

2°. We may let A_{22} and A_{33} vanish and let $A_{23} = A_{32}$. The three sets of coplanar axes have β_1 in common.

The differential equations corresponding to these two cases are of very unlike character. In 1° the existence of the coplanar sets is evident from the form of the A's. To prove it for 2° we may let the vector δ of (1) be expanded thus,

$$\delta S\beta_1\beta_2\beta_3 = a_1 V\beta_2\beta_3 + a_2 V\beta_3\beta_1 + a_3 V\beta_1\beta_2 \tag{28}$$

and since we have identically

$$\rho = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \tag{29}$$

we shall have

$$S\delta\rho = a_1 x_1 + a_2 x_2 + a_3 x_3 \tag{30}$$

By expanding $V\varphi\rho\theta\rho$ as in (2), using the values of the a's from (4), and $S\delta\rho$ from (30), the fundamental equation (1) may be expanded in the frequently useful form

$$\begin{aligned} F\rho = \; & \beta_1[A_{11}x_2x_3 + (a_3 + A_{12})x_3x_1 + (a_2 + A_{13})x_1x_2 + a_1x_1^2] \\ & + \beta_2[(a_3 + A_{21})x_2x_3 + A_{22}x_3x_1 + (a_1 + A_{23})x_1x_2 + a_2x_2^2] \\ & + \beta_3[(a_2 + A_{31})x_2x_3 + (a_1 + A_{32})x_3x_1 + A_{33}x_1x_2 + a_3x_3^2] \end{aligned} \tag{31}$$

This is a way of expressing a quadratic vector which is always possible except in the very special cases examined in C. Q. V. where a set of three diplanar axes cannot be found; this expression depends on the twelve constants which occur explicitly, and on the three directions β_1, β_2, β_3 of the diplanar axes, equivalent to six more scalars. If now in addition to the three conditions $A_{22} = A_{33} = 0$, $A_{23} = A_{32}$ just assumed, we take

$$a_1 = -A_{32}, \qquad a_2 = 0, \qquad a_3 = 0$$

the quadratic vector takes the form

$$F\rho = \beta_1[A_{11}x_2x_3 + A_{12}x_3x_1 + A_{13}x_1x_2 - A_{23}x_1^2] + (\beta_2 A_{21} + \beta_3 A_{31})x_2x_3 \tag{32}$$

But this is a binomial; it must, therefore, by the reasoning of C. Q. V. page 384, possess three axes in the plane of the vector coefficients β_1 and $(\beta_2 A_{21} + \beta_3 A_{31})$. But we already know it possesses three axes in the planes β_1, β_2 and β_1, β_3, respectively. It therefore has three coplanar sets with β_1 in common. For brevity an axis common to three coplanar sets may be called a *central axis*.

6. Properties of a Quadratic Vector Possessing a Central Axis.

The differential equations determined by this type of vector have been studied by Darboux[6]; the connection between his viewpoint and that of vectorial algebra is established by certain theorems now to be proved.

Theorem 1. If a quadratic vector has a central axis, it may be thrown into binominal form in three distinct ways, a proper choice of the vector δ being made in each case.

One way has already been shown in (32), namely for the value of δ given by $a_1 = -A_{32}$, $a_2 = a_3 = 0$.

A second way, by inspection of (31), is to take $a_3 = 0$, $a_1 = -A_{32}$, and $a_2 = -A_{31}$. Since A_{33} is zero by hypothesis, the component along β_3 vanishes.

Similarly, if δ is determined by making $a_2 = 0$, $a_1 = -A_{23}$, $a_3 = -A_{21}$, the component along β_2 vanishes. The theorem is thus proved.

Theorem 2. If a quadratic vector has a central axis, the three values of δ by virtue of which we may pass from one binomial form to another are all perpendicular to the central axis.

Proof. These values of δ are the respective differences of the values occurring in the proof of theorem 1. They must therefore be coplanar. To determine their values explicitly, let the binomial forms be F_1, F_2, and F_3 in the order above given. Let the values of δ which changes F_1 into F_2, F_2 into F_3, and F_3 into F_1 be, in order, δ_1, δ_2, and δ_3. Subtracting values of the a's as above found we have, for δ_1, $a_1 = 0$, $a_2 = -A_{31}$, $a_3 = 0$; for δ_2, $a_1 = 0$, $a_2 = +A_{31}$, $a_3 = -A_{21}$; for δ_3, $a_1 = 0$, $a_2 = 0$, $a_3 = +A_{21}$. Since a_1 is zero in all three cases, the theorem is proved.

Theorem 3. If a quadratic vector having a central axis be thrown into either of the three binomial forms, a set of *rectangular* components X, Y, Z, can be found in terms of rectangular coördinates x, y, z, such that both X and Y are independent of z.

Proof. With the notation used in the proof of theorem 2, let the quadratic vector be in the binomial form F_2. Let $\beta_1 = k$, and let i and j be any two unit vectors such that i, j, k forms a rectangular unit system. Since δ_2 is perpendicular to i we may write $\delta_2 = ci + c'j$. Also $\rho = ix + jy + kz$. Hence $\rho S\delta_2\rho = -i(cx^2 + c'xy) - j(cxy + c'y^2) +$ terms in k. It was shown that $F_3 = F_2 + \rho S\delta_2\rho$. Now F_2 contained no component along β_3. By adding $\rho S\delta_2\rho$ we removed the component along β_2. But $\rho S\delta_2\rho$ does not contain z. Therefore

6 See note to C. Q. V. page 375.

F_3 contains a component along β_1 or k, and terms free from z. Similarly it may be shown that F_1 and F_2 may be expressed according to theorem 3.

Theorem 4. Conversely, if a quadratic vector be expressed in rectangular coördinates, and if its components X and Y are free from z, it has a central axis.

Proof. The two-dimensional quadratic vector $iX + jY$ has three axes. Call them γ_1, γ_2, and γ_3. Take i along γ_1 (by a rotation of axes in the xy plane if necessary, which cannot alter the fact that X and Y are free from Z). We may then write

$$iX + jY = i(ax^2 + bxy) + j(a'x^2 + b'xy + c''y^2)$$

It is evident from the form of the term $\rho S\delta\rho$ that we may now, by a proper choice of δ, remove the component along i, namely by taking $\delta = ai + bj$. The given quadratic vector will then have the form $jY + kZ$, hence possesses three axes in the plane perpendicular to γ_1. Similarly we may show that the given quadratic vector has three axes in the plane perpendicular to γ_2 and similarly for γ_3. But k is the common line of intersection of these three planes, and is itself an axis, since X and Y vanish when x and y are both zero. That is, k is a central axis.

Theorem 5. The vectors δ_1, δ_2, δ_3, which convert, respectively, F_1 into F_2, F_2 into F_3 and F_3 into F_1, are axes of the two-dimensional vector $iX + jY$.

Proof. Inspection of the three δ's obtained in proving theorem 2 shows that they are perpendicular, respectively, to F_3, F_1, and F_2. But in the proof of theorem 4 it appeared that the three γ's have the same property. Hence the directions of the δ's coincide with the respective γ's.

Theorem 6. If a quadratic vector $V\varphi\rho\theta\rho + \rho S\delta\rho$ has a central axis, and if a proper choice of rectangular coördinates be made, a value can be found for δ (which does not alter the axes), such that the differential equation $\dfrac{dy}{dx} = \dfrac{Y}{X}$ takes the Riccati form.

Proof. As in the proof of theorem 4 we may obtain the quadratic vector in the form

$$i(ax^2 + bxy) + j(a'x^2 + b'xy + c'y^2) + kZ$$

By taking $\delta = bj$ and adding the term $\rho S\delta\rho$ this becomes

$$iax^2 + j[a'x^2 + b'xy + (c' - b)y^2] + k[Z - byz]$$

which proves the theorem.

The connection between the theory of quadratic vectors and Darboux's treatment of differential equations is now fully established. To continue the study of the above differential equation would be merely to repeat Darboux's work.

To indicate a quite different application of the present theory we may note the following,—

Theorem 7. If a quadratic 1 : 1 point transformation be defined by the equations $x_1^1 = X_1(x_1, x_2, x_3)$, $x_2^1 = X_2(x_1, x_2, x_3)$, $x_3^1 = X_3(x_1, x_2, x_3)$, and if the four fixed points of the transformation be situated as follows: the singular points being A, B, C, two fixed points lie on a straight line through A, the other two lie respectively on AB and AC;—the transformation can be written in the form

$$x' = tx, \quad y' = Y(x, y,) + ty, \quad z' = Z(x, y, z)$$

where t is a linear function of x, y, z.

This theorem is, of course, an immediate consequence of theorems 2 and 3, stated in the language of point transformations.

7. Vectors with Four Sets of Coplanar Axes.

Continuing the study of the simplifications which occur in the form of a quadratic vector when sets of coplanar axes exist, let it be required to have four such sets. Here, again, we shall evidently have two cases, according as we have a central axis or not.

If there is to be no central axis, we may begin with case 1° of Art. 5, letting the six quantities A_{11}, A_{22}, A_{33}, and a_1, a_2, a_3, all vanish. The quadratic vector then takes the form

$$F\rho = \beta_1(A_{12}x_3x_1 + A_{13}x_1x_2) + \beta_2(A_{21}x_2x_3 + A_{23}x_1x_2) + \beta_3(A_{31}x_2x_3 + A_{32}x_3x_1); \qquad (33)$$

it is evident that three of the axes may be taken as follows,—

$$\beta_4 = \beta_2A_{21} + \beta_3A_{31}; \quad \beta_5 = \beta_3A_{32} + \beta_1A_{12}; \quad \beta_6 = \beta_1A_{13} + \beta_2A_{23}. \qquad (34)$$

To find β_7 we note that, by C. Q. V. page 385, this axis is perpendicular to the three values of δ by which we may pass from one binomial form of $F\rho$ to another. Thus by an easy calculation

$$\beta_7 = \sum_{123} [\beta_1(A_{12}A_{31} + A_{21}A_{13} - A_{21}A_{31})] \qquad (35)$$

We now have three coplanar sets given by (234) = (315) = (126) = 0. We may not include β_7 in a fourth set if the quadratic vector is to be irreducible and have no central axis. We must therefore take (456) = 0 which evidently implies the vanishing of the determinant

$$\begin{vmatrix} 0, & A_{21}, & A_{31} \\ A_{12}, & 0, & A_{32} \\ A_{13}, & A_{23}, & 0 \end{vmatrix} \tag{36}$$

Six axes of the quadratic vector will now be along the lines of intersection of four planes. This fact suggests an expression for the vector $F\rho$ which shall be symmetrical in these six axes: for consider the vector $V\gamma_1\gamma_2 S\gamma_3\rho S\gamma_4\rho$, where the γ's are taken at right angles to the four planes respectively; all six lines of intersection of the four planes with each other are axes of this vector term, and the same will be true no matter in what order we write the four subscripts. If, then, a_{12}, a_{13}, a_{23}, a_{14}, a_{24}, and a_{34} be any six scalars, the vector sum

$$F\rho = a_{12}V\gamma_1\gamma_2 S\gamma_3\rho S\gamma_4\rho + a_{13}V\gamma_1\gamma_3 S\gamma_2\rho S\gamma_4\rho + \cdots + a_{34}V\gamma_3\gamma_4 S\gamma_1\rho S\gamma_2\rho \tag{37}$$

will be a quadratic vector of the type under consideration, namely it will have six axes in the directions $V\gamma_1\gamma_2$, $V\gamma_1\gamma_3, \cdots V\gamma_3\gamma_4$. This form of $F\rho$ has a number of interesting properties easily proved by methods already exemplified. As instances,—

1. If $\rho S\delta\rho$ be added, there are four values of δ which render $F\rho$ a binomial. One value is

$$\delta_1 = -\,[a_{23}(123)\gamma_4 + a_{24}(124)\gamma_3 + a_{34}(134)\gamma_2] \tag{38}$$

and the others are of similar form.

2. If δ_1, δ_2, δ_3, δ_4 be the values of δ just mentioned, the differences $\delta_1 - \delta_2$, $\delta_1 - \delta_3$, $\delta_2 - \delta_3$ etc. are all perpendicular to the seventh axis. Hence the direction of this axis is easily calculated.

3. The quadratic vector (37) possess some properties closely analogous to those of the general linear vector function; for the latter may be written

$$g_{23}V\lambda_2\lambda_3 S\lambda_1\rho + g_{31}V\lambda_3\lambda_1 S\lambda_2\rho + g_{12}V\lambda_1\lambda_2 S\lambda_3\rho \tag{39}$$

and has for axes $V\lambda_2\lambda_3$, etc.

4. If $F\rho$ has the form (37) the cubic vector $V\rho F\rho$ takes the form $\Sigma b_1\gamma_1 S\gamma_2\rho S\gamma_3\rho S\gamma_4\rho$ where $b_1 = -a_{12} - a_{13} - a_{14}$ and b_2, b_3, b_4 are easily found. The sum of the b's is zero.

5. If $V\rho F\rho$ be divided by the product $S\gamma_1\rho S\gamma_2\rho S\gamma_3\rho S\gamma_4\rho$ the quotient is an irrotational vector; it is normal to the family of cones $(S\gamma_1\rho)^{b_1}(S\gamma_2\rho)^{b_2}(S\gamma_3\rho)^{b_3}(S\gamma_4\rho)^{b_4} = \text{const.}$

If, on the other hand, $F\rho$ *is to have a central axis,* and four coplanar sets, we may begin with case 2° of Art. 5 and let A_{11} be zero. The only new property introduced, so far as I am aware, is that the system $\frac{dx}{X} = \frac{dy}{Y} = \frac{dz}{Z}$ can now be fully solved by quadratures. The proof may be carried out by the aid of the theorems already established.

8. Other Special Types.

There are but two remaining types depending on coplanarity of axes. These are first, a vector sharing the properties of the two vectors of the last article, possessing, therefore, five coplanar sets, and having two central axes; second, a vector having six coplanar sets and four central axes. They are easily obtained from the types which precede. The vector having four central axes bears an interesting resemblance to the *self-conjugate* linear vector function. The investigation may be left to the reader.

9. The Converse Problem.

In what precedes it has been shown vectors of assigned type may be written down by giving proper values to the A's. I now propose to examine the converse problem: *given any values of the nine A's, to test whether the vector possesses three coplanar axes.*

Evidently, if the vector should happen to fall under one of the forms which have been explicitly written out, we would at once recognize its type; but these vectors have been merely "normal forms" of their respective types, hence not sufficient to serve as tests for comparison.

The necessary and sufficient condition for coplanarity of axes of the vector (1) may be obtained as follows. A set of three coplanar axes exists if, and only if, a value of δ can be found which makes the right side of (1) a binomial, (proved C. Q. V. p. 384). Let γ be normal to the plane of this binomial. Then

$$S\gamma F\rho = 0 \tag{40}$$

identically. Let γ be expanded as

$$\gamma = p_1 V\beta_2\beta_3 + p_2 V\beta_3\beta_1 + p_3 V\beta_1\beta_2 \tag{41}$$

Let this value of γ, and the expanded form of $F\rho$ from (31) be substituted in (40). By equating to zero the coefficients of x_1^2, x_1x_2, etc. we obtain six equations,

$$\begin{aligned} p_1a_1 &= 0, & p_1A_{11} + p_2(a_3 + A_{21}) + p_3(a_2 + A_{31}) &= 0 \\ p_2a_2 &= 0, & p_1(a_3 + A_{12}) + p_2A_{22} + p_3(a_1 + A_{32}) &= 0 \\ p_3a_3 &= 0, & p_1(a_2 + A_{13}) + p_2(a_1 + A_{23}) + p_3A_{33} &= 0 \end{aligned} \tag{42}$$

In order that γ may exist, not all the p's can vanish. We shall then have three cases, according as three, two, or one, of the a's shall be zero.

Case 1°. $a_1 = a_2 = a_3 = 0$. Values can be found for the p's if, and only if, the determinant of the A's vanishes. The vectors α of the right side of (2) are then coplanar, and three of the four axes β_4, β_5, β_6. β_7 lie in this plane.

Case 2°. Two only of the a's are zero. Suppose $a_2 = a_3 = 0$, with a_1 not zero. Then $p_1 = 0$. The six linear equations reduce to

$$p_2A_{21} + p_3A_{31} = 0 \tag{43}$$

$$p_2A_{22} + p_3(a_1 + A_{32}) = 0 \tag{44}$$

$$p_2(a_1 + A_{23}) + p_3A_{33} = 0 \tag{45}$$

Now p_2 and p_3 cannot both vanish if γ exists. Hence

$$a_1A_{21} + A_{21}A_{32} - A_{31}A_{22} = 0 \tag{46}$$

$$a_1A_{31} + A_{31}A_{23} - A_{21}A_{33} = 0 \tag{47}$$

$$a_1^2 + a_1(A_{23} + A_{32}) + A_{32}A_{23} - A_{22}A_{33} = 0 \tag{48}$$

From (46) and (47) we have

$$\begin{vmatrix} 0, & A_{21}, & A_{31} \\ -A_{21}, & A_{22}, & A_{32} \\ A_{31}, & A_{23}, & A_{33} \end{vmatrix} = 0 \tag{49}$$

If A_{21} and A_{31} are not both zero, a_1 can be found from (46) or from (47). Also if A_{21} and A_{31} are not both zero (48) is a consequence of (46) and (47). Since p_1 is zero, the plane of the coplanar axes contains β_1.

If A_{21} and A_{31} are both zero, two values of a_1 are found from (48), in general distinct, and there are two coplanar sets with β_1 in common.

In any case, if the determinant (49), or either of the similar determinants obtained from it by advancing subscripts, is zero, there is at least one set of coplanar axes.

Case 3°. Only one of the a's is zero. Suppose $a_3 = 0$, with a_1 and a_2 not zero. Then $p_1 = p_2 = 0$. The six equations reduce to

$$\begin{aligned} a_2 + A_{31} &= 0 \\ a_1 + A_{32} &= 0 \\ A_{33} &= 0 \end{aligned}$$

Hence if $A_{33} = 0$ the a's can be determined and a coplanar set exists. Similarly for A_{11} and A_{22}, in agreement with a former result.

Summary of tests for coplanarity of axes.

The quadratic vector (31) *possesses three axes in the same plane when, and only when, one of these seven conditions holds: the vectors* α_1, α_2, α_3 *defined by* (4) *are coplanar; one of the three determinants of the form* (49) *vanishes; or one of the constants* A_{11}, A_{22}, A_{33} *vanishes.*

10. Application to Irrotational Vectors.

As a second illustration of the utility of the constants A_{11} etc., I propose to examine the properties of a quadratic vector under the requirement that it be irrotational, i.e. its "curl" shall be zero or

$$V\nabla F\rho = 0 \tag{50}$$

The significance of this equation on the physical side is well known. To see the algebraic aspect of the problem we may recall that, as was shown by Hamilton,[7] any *linear* vector function $\varphi\rho$ may be written as the sum of two terms thus,

$$\varphi\rho = \omega\rho + V\epsilon\rho \tag{51}$$

where ϵ is a vector; ω is self-conjugate, irrotational, and has its three axes at right angles to each other. It is natural to enquire what restriction is imposed on the axes in the case of an irrotational quadratic vector.

The scope of the enquiry will appear from the following:

Theorem 8. If a quadratic vector can be made irrotational without altering its axes, its curl is of the form $V\delta\rho$.

The proof is evident from the identity

$$V\nabla(F\rho + \rho S\delta\rho) = V\nabla F\rho + V\rho\delta \tag{52}$$

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[7] *Elements*, Art. 349; 2nd Ed. p. 492.

for if $F\rho$ can be made irrotational by adding $\rho S\delta\rho$, (the only form of term which leaves the axes unaltered), the right member must vanish, and we have $V\nabla F\rho = V\delta\rho$; hence the theorem. The converse is equally obvious,—

Theorem 9. If a quadratic vector has its curl of the form $V\delta\rho$ it can be made irrotational by adding the term $\rho S\delta\rho$.

11. Conditions that the Curl shall be of the Form $V\delta\rho$.

I shall now show that, in general, a set of five scalar equations exists which are necessary and sufficient that the curl of a quadratic vector be of the form $V\delta\rho$. These will appear as equations connecting the nine A's, and involving also the axes β_1, β_2, β_3. Since the A's have been obtained as functions of the axes, these equations impose restrictions on the axes of an irrotational quadratic vector.

Taking (31) we let $a_1 = a_2 = a_3 = 0$, which is equivalent to neglecting the term $\rho S\delta\rho$. We then operate by $V\nabla$. Now $\nabla x_1 \cdot S\beta_1\beta_2\beta_3 = - V\beta_2\beta_3$ and similarly for x_2 and x_3. Hence easily (using the α's from (4))

$$V\nabla F\rho \cdot S\beta_1\beta_2\beta_3 = \sum_{123} V\alpha_1(x_2 V\beta_1\beta_2 + x_3 V\beta_3\beta_1) \tag{53}$$

The right side of this equation is a linear vector function of ρ, which we may call $\theta\rho$. Putting for the x's their values, and arranging the order of terms we may write

$$\theta\rho = \sum_{123} (V\alpha_2 V\beta_1\beta_2 + V\alpha_3 V\beta_3\beta_1) S\rho\beta_2\beta_3 \tag{54}$$

which must be of the form $V\delta\rho$. This is the same as saying that the self-conjugate part of θ must vanish, or that $\theta + \theta' = 0$, or again that $S\rho\theta\rho$ must vanish for all values of ρ. In general the vanishing of a self-conjugate linear vector function is equivalent to six scalar equations; but in this case we note that $S\nabla\theta\rho = \sum_{123} S(V\alpha_2 V\beta_1\beta_2 + V\alpha_3 V\beta_3\beta_1) V\beta_2\beta_3 = 0$ identically, hence the six equations are not independent.

A simple way to set up the six equations in explicit form is $S\beta_1\theta\beta_1 = 0$, $S\beta_2\theta\beta_2 = 0$, $S\beta_3\theta\beta_3 = 0$, $S\beta_2\theta\beta_3 + S\beta_3\theta\beta_2 = 0$, $S\beta_3\theta\beta_1 + S\beta_1\theta\beta_3 = 0$, and $S\beta_1\theta\beta_2 + S\beta_2\theta\beta_1 = 0$. Since by hypothesis β_1, β_2, and β_3 are not coplanar these are sufficient to make $\theta + \theta' = 0$. By (54) the first and fourth of these equations are

$$S \cdot V\beta_1\alpha_2 V\beta_1\beta_2 + S\beta_1\alpha_3 V\beta_3\beta_1 = 0 \tag{55}$$

$$S \cdot V\beta_2\alpha_1 V\beta_3\beta_1 + S \cdot V\beta_2\alpha_2 V\beta_2\beta_3 + S \cdot V\beta_3\alpha_3 V\beta_2\beta_3 + S \cdot V\beta_3\alpha_1 V\beta_1\beta_2 = 0 \tag{56}$$

and the others are obtained by advancing subscripts.

Inspection of the scalar products which occur in these equations shows that irrotationality of the quadratic vector $F\rho$ is dependent on the relation of the α's to the two systems of vectors β_1, β_2, β_3 and $V\beta_2\beta_3$, $V\beta_3\beta_1$, $V\beta_1\beta_2$. The most natural procedure is to expand the latter system thus

$$\left.\begin{aligned} V\beta_2\beta_3 \cdot S\beta_1\beta_2\beta_3 &= b_{11}\beta_1 + b_{21}\beta_2 + b_{31}\beta_3 \\ V\beta_3\beta_1 \cdot S\beta_1\beta_2\beta_3 &= b_{12}\beta_1 + b_{22}\beta_2 + b_{32}\beta_3 \\ V\beta_1\beta_2 \cdot S\beta_1\beta_2\beta_3 &= b_{13}\beta_1 + b_{23}\beta_2 + b_{33}\beta_3 \end{aligned}\right\} \tag{57}$$

whence we have

$$b_{11} = S \cdot V\beta_2\beta_3 V\beta_2\beta_3 = S^2\beta_2\beta_3 - \beta_2^2\beta_3^2; \quad b_{21} = b_{12} = S \cdot V\beta_2\beta_3 V\beta_3\beta_1$$
$$= \beta_3^2 S\beta_1\beta_2 - S\beta_2\beta_3 S\beta_3\beta_1 \quad (57)$$

and similarly for the other b's. We may now introduce the expansions of the α's from (4) and of $V\beta_2\beta_3$ etc. from (57) and the six equations of form (55) and (56) become

$$\begin{aligned}
b_{33}A_{22} - b_{22}A_{33} + b_{23}(A_{23} - A_{32}) &= 0 && (58_1)\\
b_{11}A_{33} - b_{33}A_{11} + b_{31}(A_{31} - A_{13}) &= 0 && (58_2)\\
b_{22}A_{11} - b_{11}A_{22} + b_{12}(A_{12} - A_{21}) &= 0 && (58_3)\\
- b_{11}(A_{23} - A_{32}) + b_{12}(A_{31} + A_{13}) - b_{13}(A_{12} + A_{21}) &= 0 && (58_4)\\
- b_{22}(A_{31} - A_{13}) + b_{23}(A_{12} + A_{21}) - b_{12}(A_{23} + A_{32}) &= 0 && (58_5)\\
- b_{33}'(A_{12} - A_{21}) + b_{13}(A_{23} + A_{32}) - b_{23}(A_{31} + A_{13}) &= 0 && (58_6)
\end{aligned}$$

Here we note that if the six equations be multipled, in order, by b_{11}, b_{22}, b_{33}, b_{23}, b_{31}, b_{12}, and the results added, the sum of the left members is identically zero; the six equations are not independent.

Since the A's have already been determined as functions of the axes, the six equations (58) are necessary conditions which the axes must satisfy when the quadratic vector is irrotational.

We note further that, assuming β_1, β_2, and β_3 to be real, the scalars b_{11}, b_{22}, and b_{33} are different from zero. These three scalars could all vanish only if $V\beta_2\beta_3$, $V\beta_5\beta_1$, and $V\beta_1\beta_2$ were all minimal vectors, i. e. imaginaries of null tensor; thus in general we may assume b_{11} different from zero; and except in this very special case, therefore, (58_1) is a consequence of the other five equations which, with the exception noted, are sufficient that the curl shall be of the form $V\delta\rho$.

12. Interpretation of the Equations (58).

It is manifest that the equations (58) are bilinear: and that the set of six b's are determined by the choice of three axes β_1, β_2 b_3, while the set of nine A's govern the other four axes. I shall first show how various simple solutions of these equations may be written down, and shall then discuss the general solution as a linear function of these simple solutions.

One solution is seen by inspection to be: A_{11}, A_{22}, A_{33}, proportional to b_{11}, b_{22}, and b_{33}, with the other six A's all zero. The resulting value of $F\rho$, which we may call $F_1\rho$, is, by (31),

$$F_1\rho = \beta_1 b_{11} x_2 x_2 + \beta_2 b_{22} x_3 x_1 + \beta_3 b_{33} x_1 x_2 \tag{59_1}$$

which, of course, may be multiplied by a scalar constant. It is easy to check the result by operating with ∇, using the values of the b's from (57_1), and showing that the expression is of the form $V\rho\delta$.

A second solution is equally obvious: let $A_{23} + A_{32}$, $A_{31} + A_{13}$, and $A_{12} + A_{21}$ be proportional to b_{23}, b_{31} and b_{12} with $A_{11} = A_{22} = A_{33} = 0$ and $A_{23} = A_{32}$, $A_{31} = A_{13}$, $A_{12} = A_{21}$, giving the solution

$$F_2\rho = \beta_1(b_{12}x_3x_1 + b_{13}x_1x_2) + \beta_2(b_{21}x_2x_3 + b_{23}x_1x_2) + \beta_3(b_{31}x_2x_3 + b_{32}x_3x_1) \tag{59_2}$$

We note that this solution is null if b_{23}, b_{31}, and b_{12} are all zero, that is, if the vectors β_1, β_2, and β_3 form a rectangular system. Let us suppose for the moment that one of the b's, as b_{23}, is not zero.

These two solutions are linear in the b's, and are not altered by advancing subscripts. A third solution, is found by assigning arbitrarily $A_{11} = A_{22} = A_{23} + A_{32} = 0$ and solving for the A's in terms of the b's. The result is

$$A_{33} = -2b_{31}b_{23},\ A_{31} = b_{23}b_{11},\ A_{13} = -b_{23}b_{11},\ A_{12} = b_{22}b_{11},$$
$$A_{21} = b_{22}b_{11},\ A_{23} = -b_{22}b_{31},\ A_{32} = +b_{22}b_{31},$$

which may be checked by substituting in (58). Whence by (31)

$$F_3\rho = \beta_1(b_{22}b_{11}x_3x_1 - b_{23}b_{11}x_1x_2) + \beta_2(b_{22}b_{11}x_2x_3 - b_{22}b_{31}x_1x_2) + \beta_3(b_{23}b_{11}x_2x_3 + b_{22}b_{31}x_3x_1 - 2b_{31}b_{23}x_1x_2) \tag{59_3}$$

quadratic and unsymmetrical in the b's.

A fourth solution may now be obtained by advancing subscripts, as also, of course, a fifth, which might be used in place of $F_2\rho$ in case all b's with unequal subscripts vanish, but a simpler treatment of this case will be given below. (It is evident that we cannot in general have more than four linearly independent solutions for the A's when the b's are assigned.) Thus

$$F_4\rho = \beta_1(-2b_{11}b_{31}x_2x_3 + b_{31}b_{22}x_3x_1 + b_{33}b_{12}x_1x_2) + \beta_2(-b_{31}b_{22}x_2x_3 + b_{33}b_{22}x_1x_2) + \beta_3(-b_{33}b_{12}x_2x_3 + b_{33}b_{22}x_3x_1) \quad (59_4)$$

It is furthermore clear, because the equations (58) are linear in the A's, that any linear function of the solutions already obtained will be a solution, the b's being taken as known constants, and, we will thus obtain the most general solution. For from the four solutions already written we may pick out the matrix of the four sets of values of A_{11}, A_{22}, A_{33}, namely

$$\begin{matrix} b_{11}, & b_{22}, & b_{33} \\ 0, & 0, & 0 \\ 0, & 0, & -2b_{31}b_{23} \\ -2b_{12}b_{31}, & 0, & 0 \end{matrix}$$

which by inspection are linearly independent sets. Hence the four solutions F_1, F_2, F_3, F_4 are linearly independent.

Consider now the vector $F_1\rho$. Remembering that the axes are the vector solutions of the equation $V\rho F\rho = 0$, we expand ρ as $\beta_1x_1 + \beta_2x_2 + \beta_3x_3$ and have these three equations to determine the axes of $F_1\rho$,

$$b_{33}x_1x_2^2 - b_{22}x_1x_3^3 = 0, \quad b_{11}x_2x_3^2 - b_{33}x_2x_1^2 = 0, \quad b_{22}x_3x_1^2 - b_{11}x_3x_2^2 = 0$$

Writing for brevity $c_1 = \sqrt{b_{11}}$, $c_2 = \sqrt{b_{22}}$, and $c_3 = \sqrt{b_{33}}$ we find easily the matrix of the coefficients of β_1, β_2, β_3 for the seven axes to be

$$\begin{matrix} \beta_1: & 1 & 0 & 0 \\ \beta_2: & 0 & 1 & 0 \\ \beta_3: & 0 & 0 & 1 \\ \beta_4: & c_1 & c_2 & c_3 \\ \beta_5: & -c_1 & c_2 & c_3 \\ \beta_6: & c_1 & -c_2 & c_3 \\ \beta_7: & c_1 & c_2 & -c_3 \end{matrix}$$

that is, the fourth axis is given by $\beta_4 = c_1\beta_1 + c_2\beta_2 + c_3\beta_3$ etc. By inspection of this matrix we see the following relations of coplanarity among the axes: $(145) = (167) = (246) = (257) = (356) = (347) = 0$. The vector $F_1\rho$ therefore is of the type mentioned in Art. 8 having four central axes, namely β_4, β_5, β_6, β_7.

Finding the axes of $F_2\rho$ in a similar manner we have the matrix

β_1:	1	0	0
β_2:	0	1	0
β_3:	0	0	1
β_4:	0	b_{12}	b_{13}
β_5:	b_{12}	0	b_{23}
β_6:	b_{13}	b_{23}	0
β_7:	$b_{12}b_{13}$	$b_{12}b_{23}$	$b_{23}b_{31}$

whence the relations of coplanarity $(126) = (234) = (315) = (147) = (257) = (367) = 0$, so that $F_2\rho$ belongs to the same type as $F_1\rho$, with central axes β_1, β_2, β_3, and β_7.

The vector $F_3\rho$ is of different type. The plane $x_3 = 0$ contains only the two axes β_1 and β_2. Besides β_3 we have the axes $(0, b_{22}, b_{23})$ and $(b_{11}, 0, b_{31})$ together with two imaginary axes in the plane

$$b_{22}b_{31}x_1 = b_{23}b_{11}x_2$$

where x_2 and x_3 satisfy the quadratic

$$b_{22}x_3^2 - b_{23}b_{22}x_2x_3 + 2b_{23}^2x_2^2 = 0.$$

We see that $F\rho$ is not necessarily of so restricted a type as $F_1\rho$.

13. Case where Three Given Axes form a Rectangular System.

Of special interest is the case where the three assigned axes β_1, β_2, β_3 are mutually perpendicular both because the four solutions (59) are no longer linearly independent, and because we might suspect here some greater analogy with linear vector functions. We now have $b_{12} = b_{23} = b_{31} = 0$ and the equations (58) have the evident solutions, (letting i, j, k replace β_1, β_2, β_3, so that $b_{11} = b_{22} = b_{33} = 1$),

$$\begin{aligned} F_1\rho &= iyz + jzx + kxy \\ F_2\rho &= jxy + kxz \\ F_3\rho &= kyz + ixy \\ F_4\rho &= ixz + jyz \end{aligned}$$

which are linearly independent. The last three solutions are, to be sure, reducible, but linear combinations of them will not in general be so. Thus in all cases the general solution may be expressed as the sum of four simpler solutions.

14. Application to Consecutive Chemical Processes.

As another illustration of the utility of the coefficients A_{11} etc., it may be noted that an important class of chemical processes, namely, the type known as "consecutive," leads to a pair of differential equations of the form

$$\frac{dx_1}{dt} = P_1(x_1, x_2), \qquad \frac{dx_2}{dt} = P_2(x_1, x_2)$$

where P_1 and P_2 are quadratic polynomials, (not in general homogeneous), in the dependent variables x_1, x_2, but are not functions of t.

These equations cannot in general be solved by quadratures, but by a proper choice of the conditions of the process they may often be made integrable in this way, and the labor of solving by series avoided. We have merely to render the polynomials P_1 and P_2 homogeneous by introducing a third variable x_3 and write down the nine A's for the quadratic vector

$$\beta_1 P_1 + \beta_2 P_2$$

which will always have at least one set of coplanar axes since it is a binomial. If then we can so choose our conditions that, according to the tests of Art. 9, there are three other sets of coplanar axes, the equations can be solved by quadratures.

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6. Thaxter, R.— Contribution toward a Monograph of the Laboulbeniaceæ. Part II. pp. 217–469. pls. xxviii–lxxi. June, 1908. $7.00.

Vol. 14. 1. Lowell, Percival. The Origin of the Planets. pp. 1–16. pls. i–iv. June, 1913. 60c.

2. Fernald, W. E., Southard, E. E., and Taft, A. E.— Waverley Researches in the Pathology of the Feeble-Minded. (Research Series, Cases I to X.) pp. 17–128. 20 pls. May, 1918. $6.00.

PROCEEDINGS. Vols. 1–53, $5 each. Discount to booksellers 25%; to Fellows 50%, or for whole sets 60%.

The individual articles may be obtained separately. A price list of recent articles is printed on the inside pages of the cover of the Proceeding.

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Memoir of Sir Benjamin Thompson, Count Rumford, with Notices of his Daughter. By George E. Ellis. $5.00.

[illegible] of the Life and Works of Rumford. 5 vols., $25.00; to Fellows, $5.00.

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Proceedings of the American Academy of Arts and Sciences.

Vol. 56. No. 10.—July, 1921.

THE RUMFORD FUND.

Awards of the Premium and Grants for Research in Light and Heat.

Compiled by Charles R. Cross.

(Continued from page 3 of cover.)

VOLUME 56.

1. Kennelly, A. E., and Kurokawa, K.— Acoustic Impedance and its Measurement. pp. 1–42. February, 1921. $1.25.
2. Bell, Louis.— Ghosts and Oculars. pp. 43–58. February, 1921. $.85.
3. Bridgman, P. W.— Electrical Resistance under Pressure, including certain liquid Metals. pp. 59–154. February, 1921. $1.25.
4. Lipka, Joseph.— Motion on a Surface for any Positional Field of Force. pp. 155–182. March, 1921. $1.00.
5. Willey, A.— Arctic Copepoda in Passamaquoddy Bay. pp. 183–196. May, 1921. $.75.
6. Jones, Grinnell, and Schumb, W. C.— The Potential of the Thallium Electrode and the Free Energy of Formation of Thallous Iodide. pp. 197–236. April, 1921. $1.10.
7. Heidel, W. A.— Anaximander's Book, The Earliest Known Geographical Treatise. pp. 237–288. April, 1921. $1.00.
8. Wheeler, W. M.— Observations on Army Ants in British Guiana. pp. 289–328. June, 1921. $1.25.
9. Hitchcock, Frank L.— The Axes of a Quadratic Vector. pp. 329–351. June, 1921. $.75.
10. Cross, Charles R. — The Rumford Fund. Awards of the Premium and Grants for Research in Light and Heat. pp. 353–373. July, 1921. $.45.

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Received April 29, 1921. Presented May 11, 1921.

A number of years since the Rumford Committee came to realize that there was a great lack of knowledge in the educated community in general and to some extent even within the Academy itself regarding the Rumford Fund and the part it had played in stimulating scientific research in this country. For this reason it was decided to issue a somewhat detailed printed statement which should give an account of the origin of the Rumford Fund and the circumstances under which it came to assume its present important position. Of still greater moment was the fact that no collected statement had ever been published of the grants for research which had been made during the previous years, of the papers which had been published with aid from the Fund, or even of the Rumford Medals which had been awarded by the Academy. To remove this deficiency the Rumford Committee prepared a comprehensive paper which was printed by the Academy in 1905, making a pamphlet of thirty-two pages, entitled "The Rumford Fund." Together with a historical sketch were given a list of the awards of the Rumford Premium by the American Academy of Arts and Sciences, 1839 to 1904, a list of the awards of the Rumford Premium of the Royal Society of London, 1802 to 1904, a list of the grants for research made from the Rumford Fund, 1832 to May 1905, and a list of papers published with aid from the Rumford Fund in the Memoirs or Proceedings of the Academy, 1833 to 1905.

A Supplement of nineteen pages to this publication was issued in 1912, which continued the list of awards of the Rumford Premium of the Academy up to and including 1912, and gave data regarding the individual medals of the complete series which had not been ascertained previously. It also continued the lists of the Royal Society's awards of that Premium, from 1905 to 1910, and the Grants for Research made by the Rumford Committee from October, 1905 to May, 1912 together with a list continued up to that date of the papers published by the Academy from 1905 to 1912 with aid from the Rumford

Fund. In addition to these there was given a list as reported by their authors of all published researches which had received aid from the Rumford Fund but which had been published elsewhere than in the publications of the Academy from 1892 to 1912. Prior to 1892 it had been understood that researches thus aided should be presented for publication to the Academy.

At the time of issue of the 'Supplement' it was felt by the Committee that it would be well regularly, perhaps every five years, to continue the lists of grants and published papers up to date. 1917, however, was not a favorable time for doing this, and it was evident that in any event such regular publication would most conveniently begin at the end of the second decade of the century. It seemed best also to print future publications of the kind in the Proceedings of the Academy.

The list of grants now published includes all from the beginning down to and including December, 1920. They have also been numbered, every entry being included for convenience of reference, although it may not indicate actual appropriation of funds. A complete list of the awards of the Rumford Premium is also given.

It is intended shortly to publish a continuation of the lists of published papers up to and including December, 1920.

Awards of the Rumford Premium of the American Academy.

1839. Robert Hare, of Philadelphia, for his invention of the compound or oxyhydrogen blowpipe.

1862. John Ericsson, of New York, for his improvements in the management of heat, particularly as shown in his caloric engine of 1858.

1865. Daniel Treadwell, of Cambridge, for improvements in the management of heat, embodied in his investigations and inventions relating to the construction of cannon of large calibre, and of great strength and endurance.

1866. Alvan Clark, of Cambridge, for his improvements in the manufacture of refracting telescopes, as exhibited in his method of local correction.

1869. George Henry Corliss, of Providence, for his improvement in the steam-engine.

1871. Joseph Harrison, Jr., of Philadelphia, for his mode of constructing steam-boilers, by which great safety has been secured.

1873. Lewis Morris Rutherfurd, of New York, for his improvements in the processes and methods of astronomical photography.
1875. John William Draper, of New York, for his researches on radiant energy.
1880. Josiah Willard Gibbs, of New Haven, for his researches in thermodynamics.
1883. Henry Augustus Rowland, of Baltimore, for his researches in light and heat.
1886. Samuel Pierpont Langley, of Allegheny, for his researches in radiant energy.
1888. Albert Abraham Michelson, of Cleveland, for his determination of the velocity of light, for his researches upon the motion of the luminiferous ether, and for his work on the absolute determination of the wave-lengths of light.
1891. Edward Charles Pickering, of Cambridge, for his work on the photometry of the stars and upon stellar spectra.
1895. Thomas Alva Edison, of Orange, N. J., for his investigations in electric lighting.
1898. James Edward Keeler, of Allegheny, for his application of the spectroscope to astronomical problems, and especially for his investigations of the proper motions of the nebulae, and the physical constitution of the rings of the planet Saturn, by the use of that instrument.
1899. Charles Francis Brush, of Cleveland, for the practical development of electric arc-lighting.
1900. Carl Barus, of Providence, for his various researches in heat.
1901. Elihu Thomson, of Lynn, for his inventions in electric welding and lighting.
1902. George Ellery Hale, of Chicago, for his investigations in solar and stellar physics and in particular for the invention and perfection of the spectro-heliograph.
1904. Ernest Fox Nichols, of New York, for his researches on radiation, particularly on the pressure due to radiation, the heat of the stars, and the infra-red spectrum.
1907. Edward Goodrich Acheson, of Niagara Falls, for the application of heat in the electric furnace to the industrial production of carborundum, graphite, and other new and useful substances.
1909. Robert Williams Wood, of Baltimore, for his discoveries in light, and particularly for his researches on the optical properties of sodium and other metallic vapors.

1910. Charles Gordon Curtis, of New York, for his improvements in the utilization of heat as work in the steam turbine.

1911. James Mason Crafts, of Boston, for his researches in high-temperature thermometry and the exact determination of new fixed points on the thermometric scale.

1912. Frederic Eugene Ives, of Woodcliff-on-Hudson, for his optical inventions, particularly in color photography and photo-engraving.

1913. Joel Stebbins, of Urbana, for his development of the selenium photometer and its application to astronomical problems.

1914. William David Coolidge, of Schenectady for his invention of ductile tungsten and its application in the production of radiation.

1915. Charles Greeley Abbot, of Washington, for his researches on solar radiation.

1917. Percy Williams Bridgman, of Cambridge, for his thermo-dynamical researches at extremely high pressures.

1918. Theodore Lyman, of Cambridge, for his researches on light of very short wave-length.

1920. Irving Langmuir, of Schenectady, for his researches on thermionic and allied phenomena.

Grants for Research from the Rumford Fund.

1832–1862.	1.	Observatory at Cambridge. For telescope and other apparatus	$3776
	2.	Enoch Hale. For rain gauges and sundry expenses for experiments and investigations relating to the fall of rain	1697
1862.	3.	Philander Shaw. Experiments relating to air-engines	600
1863.	4.	Ogden N. Rood. Physical relations of iodized plate to light. (Appropriation subsequently transferred to another research, viz., photometry, 7.)	300
1864.	5.	Wolcott Gibbs. For purchase of a Meyerstein spectrometer and Regnault's apparatus for measuring vapor tension	600
1865.	6.	Josiah P. Cooke, Jr. For purchase of glass prisms to be used in an investigation of metallic spectra. (These prisms were purchased from the Academy by Professor Cooke in 1871.)	200

1866.	7.	Ogden N. Rood. Photometry. (Appropriation 4, for relations of iodized plate to light, $300, transferred to this purpose.)	
1867.	8.	Wolcott Gibbs. For repairing Meyerstein spectrometer belonging to the Academy. (Additional to 5.)	$100
1869.	9.	Joseph Winlock. For purchase of spectroscopic instruments for observations of the solar eclipse of August, 1869	300
1870.	10.	Benjamin Apthorp Gould. For photometric and spectroscopic apparatus for the Observatory at Cordova. (Apparatus subsequently purchased by the Argentine Government.)	500
1875.	11.	John Trowbridge. Improvement of magneto-electric machine and induction coil	500
1876.	12.	Henry A. Rowland. New determination of mechanical equivalent of heat	600
	13.	Samuel P. Langley. Researches on radiant energy	600
1877.	14.	Benjamin O. Peirce, Jr. Investigation of the conduction of heat in the interior of bodies. ($60. only, called for.)	200
	15.	Edward C. Pickering. Atmospheric refraction .	520
1878.	16.	Wolcott Gibbs, John Trowbridge, Edward C. Pickering. Experiments on photometry and polarimetry. (A small portion only of this appropriation was called for.)	500
	17.	Charles A. Young. In aid of observations on solar eclipse of July 29, 1878. (Appropriation not called for.)	300
	18.	Nathaniel S. Shaler. Investigation on loss of internal heat of earth in the neighborhood of Boston. (Appropriation not called for.) . . .	200
	19.	William W. Jacques. Experiments on the distribution of heat in the spectrum	100
	20.	Wolcott Gibbs, Edward C. Pickering, John Trowbridge. Determination of indices of refraction. (A small portion only of this appropriation was called for.)	500
1879.	21.	John Trowbridge. Heat developed by magnetization and demagnitization of magnetic metals .	200
	22.	William W. Jacques. Radiation at high temperatures. (Additional to 19.)	200

	23.	William A. Rogers. To procure a metric standard of length	$350
1880.	24.	Silas W. Holman. Viscosity of gases	250
	25.	Wolcott Gibbs. Construction of dynamo-electric machine of a new plan	150
	26.	Samuel P. Langley. Distribution of heat in diffraction spectrum. (Additional to 13.)	300
1882.	27.	Edward C. Pickering. Stellar photography, with a view of obtaining a method of estimating the brightness of stars	500
	28.	John Trowbridge. Thomson effect and allied subjects	250
1883.	29.	John Trowbridge. Addition to last preceding appropriation	100
	30.	Frank N. Cole. Experiments on Maxwell's theory of light	50
1884.	31.	Rumford Committee. For purchase of Rowland grating	40
	32.	William H. Pickering. Experiments in photography	200
	33.	John Trowbridge, Edward C. Pickering, Charles R. Cross. Experiments on standard of light	300
	34.	Edward C. Pickering. Photometry. (Additional to 27.)	200
	35.	William A. Rogers. Production of constant temperatures	100
	36.	John Trowbridge. Effect of changes of temperature on magnetism	100
1885.	37.	William A. Rogers. For Construction of constant temperature room. (Additional to 35.)	82
	38.	Edward C. Pickering. Photometry. (Additional to 34.)	300
	39.	William H. Pickering. Photography and new standard of light. (Additional to 32.)	300
1886.	40.	William H. Pickering. Observations of Solar Corona, Eclipse of August, 1886	500
	41.	Henry P. Bowditch. Calorimetric observations on the heat of the human body. ($100, only, called for.)	500
	42.	John Trowbridge. Standard of light. (Appropriation subsequently transferred to another research, viz., radiant energy, 44.)	250

	43.	Charles R. Cross. Thermo-electric effect in Munich shunt method. (Appropriation not called for.) .	$75
1887.	44.	John Trowbridge. Investigations on radiant energy. (Appropriation 42, for Standard of light, $250, transferred to this purpose.)	
	45.	Charles R. Cross and Silas W. Holman. Thermometry	250
	46.	Erasmus D. Leavitt, Jr. Investigations upon a pyrometer. (Appropriation not called for.) . .	250
	47.	John Trowbridge. Metallic spectra	250
1888.	48.	John Trowbridge. Metallic spectra. (Additional to 47.)	500
	49.	William H. Pickering. For observations on solar eclipse of Jan., 1889	500
1889.	50.	Charles C. Hutchins. Investigation on lunar radiation	250
	51.	Edwin H. Hall. Investigations on cylinder temperature	100
	52.	Henry A. Rowland. Metallic spectra	500
1890.	53.	Edwin H. Hall. Investigations on cylinder temperature. (Additional to 51.)	100
	54.	Benjamin O. Peirce. Temperature changes in interior of solids. (Appropriation not called for.)	200
1892.	55.	Daniel W. Shea. Velocity of light in magnetic field	250
	56.	Benjamin O. Peirce. Propagation of heat within certain solid bodies. (Reappropriation of 54.) .	200
	57.	Henry A. Rowland. Investigations on solar spectrum. (Additional to 52.)	250
1893.	58.	William A. Rogers. Investigation on the pulsation of thermometers	175
	59.	William H. Pickering. Observations in Arizona on transparency and steadiness of the air and on the changes in temperature on the planet Mars. (Appropriation not called for.)	500
1894.	60.	Frank A. Laws. Thermal conductivity of metals.	300
	61.	Edward L. Nichols. Radiation from carbon at different temperatures	250
1895.	62.	Edwin H. Hall. Thermal conductivity of metals.	250
	63.	Arthur G. Webster. Velocity of electric waves.	250
	64.	Benjamin O. Peirce. Thermal conductivities of poor conductors. (Additional to 56.)	250

1896.	65.	Henry Crew. Electric, chemical, and thermal effects of electric arc	$400
	66.	Robert O. King. Thomson effect in metals . .	100
1897.	67.	Arthur G. Webster. Velocity of light. (Appropriation not called for.)	500
	68.	George E. Hale. For the construction of spectroheliograph	400
	69.	Arthur G. Webster. For the construction of revolving mirror. (Additional to 67. Appropriation returned.)	250
	70.	Arthur G. Webster and Robert R. Tatnall. The Zeeman effect. (Appropriation not called for.)	100
1898.	71.	Wallace C. Sabine. Researches on ultra-violet radiation	400
	72.	Albert A. Michelson. New form of diffraction grating. (Echelon spectroscope.)	500
	73.	Theodore W. Richards. For the construction of a microkinetoscope, to be applied to a study of the birth and growth of crystals	200
1899.	74.	Wallace C. Sabine. Further researches on ultra-violet wave-length. (Additional to 71.) . . .	200
	75.	Henry Crew. Spectrum of the electric arc. (Additional to 65.)	200
	76.	Arthur G. Webster. Distribution of energy in various spectra studied by means of the Michelson interferometer and the radiometer. (Appropriation not called for.)	200
	77.	Edwin B. Frost. To aid in the construction of a spectrograph especially designed for the measurement of stellar velocities in the line of sight . .	500
1900.	78.	Edward C. Pickering. For constructing a new type of photometer to be used in an investigation on the brightness of faint stars, to be carried out by coöperation with certain observatories possessing large telescopes. (Additional to 38.)	500
	79.	Theodore W. Richards. Transition temperatures of crystallized salts	100
	80.	Arthur L. Clark. Molecular properties of vapors in the neighborhood of the critical point	250
	81.	Charles E. Mendenhall. Investigations on a hollow bolometer. ($100 only, called for.) . .	200

	82.	George E. Hale. Application of the radiometer to the study of the infra-red spectrum of the chromosphere	$500
	83.	Arthur A. Noyes. Effect of high temperatures on the electrical conductivity of salt solutions . .	300
1901.	84.	Theodore W. Richards. Research on the expansion of gases	500
	85.	Henry Crew. Order of appearance of the different lines of the spark spectrum. (Additional to 75.)	100
	86.	Robert W. Wood. Anomalous dispersion of sodium vapor	350
	87.	Arthur G. Webster. For purchase of fluorite plates	65
1902.	88.	Ernest F. Nichols. For the purchase of a spectrometer, in furtherance of a research on resonance in connection with heat radiations	300
	89.	Theodore W. Richards. For the construction of a mercurial compression pump to be used in a research on the Joule-Thomson effect. (Appropriation subsequently transferred to another research, viz., the experimental study of chemical thermodynamics, 92.)	750
	90.	Arthur A. Noyes. Effect of high temperatures on the electrical conductivity of aqueous solutions. (Additional to 83.)	300
	91.	Ralph S. Minor. Dispersion and absorption of substances for ultra-violet radiation	150
1903.	92.	Theodore W. Richards. Experimental study of chemical thermodynamics. (Appropriation 89 for compression pump, $750, transferred to this purpose.)	
	93.	Sidney D. Townley. For the construction of a stellar photometer	100
	94.	Edwin B. Frost. For the construction of a special lens for use in connection with the stellar spectrograph of the Yerkes Observatory for the study of radial velocities of faint stars. (Additional to 77.)	200
	95.	Ernest F. Nichols and Gordon F. Hull. In aid of the investigation of the relative motion of the earth and the ether by the method of "Fizeau's	

polarization experiment." (Appropriation transferred to another research, viz., effect of motion of earth on intensity of radiation, 98) $250

96. George E. Hale. For the purchase of a Rowland concave diffraction grating to be used in the photographic study of the brighter stars . . . 300

97. Edward C. Pickering. For the construction of two stellar photometers to be placed at the disposal of the Rumford Committee. (Additional to 78.) . 150

98. Ernest F. Nichols and Gordon F. Hull. Effect of the motion of the earth on the intensity of radiation. (Appropriation 95 for Fizeau's polarization experiment, $250, transferred to this purpose.)

99. Frederic L. Bishop. Thermal conductivity of lead 75

100. Frederick A. Saunders. Characteristics of spectra produced under varying conditions 200

101. William J. Humphreys. Shift of spectrum lines due to pressure 300

102. Norton A. Kent. Circuit conditions influencing electric spark lines 250

103. Edward W. Morley. Nature and effects of ether drift 500

1904. 104. John A. Dunne. Fluctuations in solar activity as evinced by changes in the difference between maximum and minimum temperatures . . . 200

105. Carl Barus. Optical method of study of radioactively produced condensation nuclei. (Appropriation not called for.) 200

106. Dewitt B. Brace. Double refraction in gases in an electrical field 200

107. Robert W. Wood. Optical and other physical properties of sodium vapor. (Additional to 86.) 350

108. Norton A. Kent. (Additional to 102.) Circuit conditions influencing electric spark lines . . 100

109. Arthur L. Clark. Molecular properties of vapors in the neighborhood of the critical point. (Additional to 80.) 150

1905. 110. Dewitt B. Brace. Double refraction in gases in an electrical field. (Additional to 106.) . . . 200

	111.	Charles B. Thwing. Thermo-electric power of metals and alloys.	$150
	112.	Harry W. Morse. Fluorescence	500
	113.	John Trowbridge. Electric double refraction of light	200
	114.	Edwin H. Hall. Thermal and thermo-electric properties of iron and other metals. (Additional to 62.)	200
	115.	Arthur B. Lamb. Specific heat of salt solutions	200
	116.	John A. Parkhurst. For the purchase of a Hartmann photometer	225
	117.	Charles B. Thwing. Thermo-electric power of metals. (Additional to 111.)	400
1906.	118.	Edwin H. Hall. Thermo-electric properties of metals. (Additional to 114.)	100
	119	Frederick E. Kester. Joule-Thomson effect in gases	50
	120.	Edwin H. Hall. Thermo-electric properties of metals. (Additional to 118.)	25
	121.	Sidney D. Townley. Appropriation of $100 for a stellar photometer, 93, returned.	
	122.	Arthur A. Noyes. For the construction of a calorimeter for the determination of heats of reaction at high temperatures. (Additional to 90.).	300
	123.	Robert W. Wood. For the purchase of quartz mercury lamps. (Additional to 107.)	200
	124.	Norton A. Kent. Spectral lines. (Additional to 108.)	75
	125.	Leonard R. Ingersoll. Kerr effect in the infrared rays	200
	126.	Frederick E. Kester. Thermal properties of gases flowing through porous plug. (Additional to 119.)	315
1907.	127.	Harry W. Morse. Fluorescence. (Additional to 112.)	400
	128.	Percy W. Bridgman. Optical and thermal properties of bodies under extreme pressures	400
	129.	Percy W. Bridgman. Optical and thermal properties of bodies under extreme pressures. (Additional to 128.)	400
1908.	130.	Lawrence J. Henderson. New method for the	

		direct determination of physiological heats of reaction. (Balance of appropriation, $100, returned.)	$200
	131.	Joel Stebbins. Use of selenium in photometry	100
	132.	Willard J. Fisher. Viscosity of gases. (Balance of appropriation, $41, subsequently transferred to Edward L. Nichols. See 175.)	100
	133.	Norton A. Kent. For the purchase of a set of echelon plates. (Additional to 124.)	400
	134.	Joel Stebbins. Use of selenium in stellar photometry. (Additional to 131.)	100
1909.	135.	William W. Campbell. For the purchase of a Hartmann photometer to be used in the measurement of polarigraphic images of the solar corona	250
	136.	Robert W. Wood. Optical properties of mercury vapor. (Additional to 123.)	150
	137.	Martin A. Rosanoff. Fractional distillation of binary mixtures	300
	138.	Charles E. Mendenhall. Free expansion of gases	300
	139.	William W. Campbell. For the purchase of certain parts of a quartz spectrograph	300
	140.	Martin A. Rosanoff. Fractional distillation of binary mixtures. (Additional to 137.)	200
	141.	Leonard R. Ingersoll. Optical constants of metals	300
	142.	Joel Stebbins. Researches with the selenium photometer. (Additional to 134.)	350
	143.	William W. Campbell. Polariscopic study of the solar corona by means of a Hartmann photometer. (Additional to 135.)	125
1910.	144.	Charles E. Mendenhall and Augustus Trowbridge. Influence of ether drift upon the intensity of radiation	250
	145.	Charles E. Mendenhall. Free expansion of gases. (Additional to 138.)	250
	146.	Frank W. Very. For the purchase of photographic glass plates of the spectrum by George Higgs	50
	147.	Maurice DeK. Thompson. The high temperature equilibrium of the system of materials employed industrially in the carbide process for the fixation of atmospheric nitrogen	100

	148.	Percy W. Bridgman. Thermal and optical properties of bodies under extreme pressures. (Additional to 129.)	$400
	149.	Charles L. Norton. Thermal insulation	400
1911.	150.	Joel Stebbins. Researches with the selenium photometer. (Additional to 142.)	200
	151.	Martin A. Rosanoff. Fractional distillation of binary mixtures. (Additional to 140.)	300
	152.	Daniel F. Comstock. Possible effect of the motion of the source on the velocity of light	100
	153.	Gilbert N. Lewis. Free energy changes in chemical reactions	150
	154.	Robert W. Wood. Optical properties of vapors. (Additional to 136.)	150
	155.	Daniel F. Comstock. Possible effect of the motion of the source on the velocity of light. (Additional to 152.)	150
	156.	Frank W. Very. Intensity of spectrum lines. (Additional to 146.)	150
	157.	John Trowbridge. For research of Harvey C. Hayes on thermo-electricity	300
	158.	Robert W. Wood. Optical properties of vapors; long heat-waves. (Additional to 154.)	150
	159.	Arthur L. Clark. Physical properties of vapors in the neighborhood of the critical point. (Additional to 109.)	250
1912.	160.	Gilbert N. Lewis. Free energy changes in chemical reactions. (Additional to 153.)	250
	161.	Norton A. Kent. Purchase of a lens for magneto-spectroscopic researches. (Additional to 133.)	375
	162.	Frederick A. Saunders. Spectroscopic studies in the ultra-violet. (Additional to 100.)	100
	163.	William O. Sawtelle. Spectra of light from oscillatory discharge	250
	164.	George W. Ritchey. Construction of reflecting telescope employing mirrors with new forms of curves	500
1913.	165.	Edward L. Nichols. For research of W. P. Roop on effect of temperature on the magnetic susceptibility of gases	250
	166.	Frederick G. Keyes. For payment of computa-	

		tion expenses of thermodynamic tables for ammonia	$300
	167.	In aid of publication of Marie's Annual International Tables of Constants (at the request of the Council) through Theodore W. Richards	100
	168.	Gilbert N. Lewis. Free energy changes in chemical reactions. (Additional to 160.)	300
	169.	William O. Sawtelle. Spectra of the light from spark in an oscillatory discharge. (Additional to 163.)	300
	170.	Harvey N. Davis. Thermodynamical researches	200
	171.	Louis V. King. To defray expenses of computation for research on scattering and absorption of solar radiation in the earth's atmosphere	250
1914.	172.	Alpheus W. Smith. Hall and Nernst effects in the rare metals	100
	173.	Charles G. Abbot. Applications of solar heat to domestic purposes	150
	174.	Percy W. Bridgman. Thermodynamical researches at high pressures. (Additional to 148.)	250
	175.	Edward L. Nichols. Hall effect and allied phenomena in tellurium and selenium. (Balances of 132 and 165, $282, transferred to this Research.)	
	176.	Percy W. Bridgman. Thermal effects of high pressures. (Additional to 174.)	150
	177.	Frederick A. Saunders. On the spectra of metallic vapors. (Additional to 162.)	100
	178.	Frederic Palmer, Jr. Properties of light of extremely short wave-length	200
	179.	Henry Crew. Specific heat of liquids	200
	180.	Charles A. Kraus. Solutions in liquid ammonia; for purchase of a refrigerating apparatus	300
	181.	Herbert P. Hollnagel. Extreme infra-red spectrum; for purchase of motor-generator	300
1915.	182.	Joel Stebbins. Research with improved photo-electric-cell photometer upon variable stars. (Additional to 150.)	140
	183.	Farrington Daniels. Specific heats; for purchase of calorimetric apparatus	330
	184.	Raymond T. Birge. Comparator for spectroscopic researches	200

	185.	Percy W. Bridgman. Thermal phenomena at high pressures. (Additional to 176.)	$400
	186.	Arthur L. Clark. Physical properties of vapors near critical point. (Additional to 159.) . . .	300
	187.	Gilbert N. Lewis. Free energy. (Additional to 168.)	300
1916.	188.	Harrison M. Randall. Infra-red spectrum. (For salary of assistant.)	200
	189.	Raymond T. Birge. For purchase of comparator. (Additional to 184.)	175
	190.	Louis V. King. Molecular constants of gases from 25° K to 1273° K. (Research discontinued, appropriation returned.)	250
	191.	Frederic Palmer, Jr. Light of extremely short wave length. (Additional to 178.)	100
	192.	Robert A. Millikan. Photo-electric properties of metals in extreme vacua	500
	193.	John A. Parkhurst. Photometric scale of stellar magnitudes	300
	194.	Everett T. King. Color of pigments	25
	195.	Edward Kremers. Chemical action of light on organic compounds	300
1917.	196.	Floyd K. Richtmyer. Optical properties of thin films	500
	197.	Norton A. Kent. Spectral lines. (Additional to 161.)	400
	198.	Ancel St. John. Spectra of X-rays	200
	199.	David L. Webster. Intensity of lines in X-ray spectra. (For payment of assistant.) . . .	100
	200.	Frederic Palmer, Jr. Light of very short wave length. (Additional to 191.)	100
	201.	Bartholomew J. Spence. Color intensity photometer	75
	202.	Bartholomew J. Spence. New form of radiometer	150
	203.	Roswell C. Gibbs. Absorption of organic and other solutions for ultra-violet, visible and infra-red rays	500
	204.	Wesley M. Baldwin. Sensitization of animal tissues for X-rays by chemical means	125
	205.	Raymond T. Birge. Structure of series spectra. (Additional to 189.)	150

	206.	Ancel. St. John. For the purchase of refrigerating machine for research on crystal structure by X-rays	$500
	207.	In aid of Publication of Marie's Annual International Tables of Constants, through Theodore W. Richards. (Additional to 167.)	250
1918.	208.	Floyd K. Richtmyer. Optical properties of thin films. (Additional to 196.)	500
	209.	Arthur L. Foley. Photography of phases of electric discharge	150
	210.	Orin Tugman. Conductivity of thin metallic films when exposed to ultra-violet light	100
	211.	Roswell C. Gibbs. Absorption of organic and silver solutions for ultra-violet and infra-red rays. (Additional to 203.)	250
	212.	Louis T. E. Thompson. Development of a gun-sight for anti-aircraft guns	250
1919.	213.	Harrison M. Randall. Infra-red spectrum. (Additional to 188.)	200
	214.	Alpheus W. Smith. Hall effect and allied phenomena in rare metals and their alloys. (Additional to 172.)	100
	215.	In aid of publication of Marie's Annual International Tables of Constants, through Julius Stieglitz. (Additional to 207.)	250
	216.	Arthur G. Webster. Researches on pyrodynamics and practical interior ballistics	500
	217.	Percy W. Bridgman. Effect of temperature and pressure on physical properties of materials, particularly thermal conductivity. (Additional to 185.)	400
	218.	Horace L. Howes. Effect of temperature on luminescence and selective radiation of rare earths	500
	219.	Frances G. Wick. Phosphorescence of hexagonite and fluorite, at ordinary and low temperatures	300
	220.	Robert W. Wood. Optical researches. (Additional to 158.)	350
	221.	Frederick G. Keyes. Heats of neutralization at different temperatures. (Additional to 166.)	300

1920.	222.	Frederick A. Saunders. Spectral lines. (Additional to 177.)	$150
	223.	David L. Webster. X-ray spectra. (Additional to 199.)	350
	224.	In aid of publication of Marie's Annual International Tables of Constants, through Julius Stieglitz. (Additional to 215.)	250
	225.	Leonard R. Ingersoll. Polarizing effect of diffraction gratings	150
	226.	Harrison M. Randall. Structure of spectra, in infra-red. (Additional to 213.)	500
	227.	Arthur G. Webster. Pyrodynamics and interior ballistics. (Additional to 216.)	500
	228.	Norton A. Kent. Spectral lines. (Additional to 197.)	200
	229.	William W. Campbell. For the purchase of a special photographic lens. (Additional to 139.)	360
	230.	Horace L. Howes. Researches in luminescence. (Additional to 218.)	90
	231.	Percy W. Bridgman. Thermal and optical properties of bodies under high pressures. (Additional to 217.)	400

LIST OF GRANTEES.

VOLUME 55.

(Continued on page 2 of cover.)

PUBLICATIONS

OF THE

AMERICAN ACADEMY OF ARTS AND SCIENCES.

MEMOIRS. Old Series, Vols. 1–4; New Series, Vols. 1–13. 16 volumes, $10 each. Half volumes, $5 each. Discount to booksellers 25%; to Fellows 50%, or for whole sets 60%.

Vol. 11. Part 1. Centennial Celebration. 1880. pp. 1–104. 1882. $2.00.

Part 2. No. 1. Agassiz, A.— The Tortugas and Florida Reefs. pp. 105–134. 12 pls. June, 1885. (Author's copies, June, 1883.) $3.00.

Part 3. Nos. 2–3. Searle, A.— The Apparent Position of the Zodiacal Light. pp. 135–157 *and* Chandler, S. C.— On the Square Bar Micrometer. pp. 158–178. October, 1885. $1.00.

Part 4. No. 4. Pickering, E. C.— Stellar Photography. pp. 179–226. 2 pls. March, 1886. $1.00.

Part 4. No. 5. Rogers, W. A., and Winlock, Anna.— A Catalogue of 130 Polar Stars for the Epoch of 1875.0, resulting from the available Observations made between 1860 and 1885, and reduced to the System of the Catalogue of Publication XIV of the Astronomische Gesellschaft. pp. 227–300. June, 1886. 75c.

Part 5. No. 6. Langley, S. P., Young, C. A., and Pickering, E. C.— Pritchard's Wedge Photometer. pp. 301–324. November, 1886. 25c.

Part 6. No. 7. Wyman, M.— Memoir of Daniel Treadwell. pp. 325–523. October, 1887. $2.00.

Vol. 12. 1. Sawyer, E. F.— Catalogue of the Magnitudes of Southern Stars from 0° to —30° Declination, to the Magnitude 7.0 inclusive. pp. 1–100. May, 1892. $1.50.

2. Rowland, H. A.— On a Table of Standard Wave Lengths of the Spectral Lines. pp. 101–186. December, 1896. $2.00.

3. Thaxter, R.— Contribution towards a Monograph of the Laboulbeniaceæ. pp. 187–430. 26 pls. December, 1896. $6.00.

4. Lowell, P.— New observations of the Planet Mercury. pp. 431–466. 8 pls. June, 1898. $1.25.

5. Sedgwick, W. T., and Winslow, C. E. A.— (I.) Experiments on the Effect of Freezing and other low Temperatures upon the Viability of the Bacillus of Typhoid Fever, with Considerations regarding Ice as a Vehicle of Infectious Disease. (II.) Statistical Studies on the Seasonal Prevalence of Typhoid Fever in various Countries and its Relation to Seasonal Temperature. pp. 467–579. 8 pls. August, 1902. $2.50.

Vol. 13. 1. Curtiss, D. R.— Binary Families in a Triply connected Region with Especial Reference to Hypergeometric Families. pp. 1–60. January, 1904. $1.00.

2. Tonks, O. S.— Brygos: his Characteristics. pp. 61–119. 2 pls. November, 1904. $1.50.

3. Lyman, T.— The Spectrum of Hydrogen in the Region of Extremely Short Wave-Length. pp. 121–148. pls. iii–viii. February, 1906 75c.

4. Pickering, W. H.— Lunar and Hawaiian Physical Features Compared. pp. 149–179. pls. ix–xxiv. November, 1906. $1.10.

5. Trowbridge, J.— High Electro-motive Force. pp. 181–215. pls. xxv–xxvii. May, 1907. 75c.

6. Thaxter, R.— Contribution toward a Monograph of the Laboulbeniaceæ. Part II. pp. 217–469. pls. xxviii-lxxi. June, 1908. $7.00.

Vol. 14. 1. Lowell, Percival.— The Origin of the Planets. pp. 1–16. pls. i–iv. June, 1913. 60c.

2. Fernald, W. E., Southard, E. E., and Taft, A. E.— Waverley Researches in the Pathology of the Feeble-Minded. (Research Series, Cases I to X.) pp. 17–128. 20 pls. May, 1918. $6.00.

PROCEEDINGS. Vols. 1–53, $5 each. Discount to booksellers 25%; to Fellows 50%, or for whole sets 60%.

The individual articles may be obtained separately. A price list of recent articles is printed on the inside pages of the cover of the Proceedings.

Complete Works of Count Rumford. 4 vols., $5.00 each.

Memoir of Sir Benjamin Thompson, Count Rumford, with Notices of his Daughter. By George E. Ellis. $5.00.

Complete sets of the Life and Works of Rumford. 5 vols., $25.00; to Fellows, $5.00.

For sale at the Library of The American Academy of Arts and Sciences, 28 Newbury Street, Boston, Massachusetts.

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Proceedings of the American Academy of Arts and Sciences.

Vol. 56. No. 11.—September, 1921.

RECORDS OF MEETINGS, 1920–21.

BIOGRAPHICAL NOTICES.

OFFICERS AND COMMITTEES FOR 1921–22.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM

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VOLUME 56.

Proceedings of the American Academy of Arts and Sciences.

Vol. 56. No. 11. — September, 1921.

RECORDS OF MEETINGS, 1920–21.

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RUMFORD PREMIUM

INDEX.

(Title Page and Table of Contents.)

RECORDS OF MEETINGS.

One thousand and ninety-third Meeting.

OCTOBER 13, 1920.— STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

Eighty-two Fellows and four guests were present:

The Transactions of the last Meeting were read and approved.

The Corresponding Secretary presented the following letters:— from W. T. Bovie, B. P. Clark, W. E. Clark, T. H. Dillon, C. K. Drinker, Franklin Edgerton, J. E. Lodge, C. H. McIlwain, F. L. Olmsted, Harlow Shapley, H. M. Smith, W. L. Underwood, and Clark Wissler, accepting Fellowship; from Maurice Caullery and J. S. Hadamard, accepting Foreign Honorary Membership; from Cecil H. Peabody resigning Fellowship.

The Corresponding Secretary reported the receipt of a biographical notice of Charles H. Williams by Louis Bell.

The President announced the following deaths: Fellows, Frank Shipley Collins, Class II., Section 2; William Crawford Gorgas, Class II., Section 4; Bernadotte Perrin Class III., Section 2; Henry Morse Stephens, Class III., Section 3; John Nelson Stockwell, Class I., Section 1; Foreign Honorary Members, Joseph Norman Lockyer, Class I., Section 1; Wilhelm Friedrich Philipp Pfeffer, Class II., Section 2; Adam Politzer, Class II., Section 4; Augusto Righi, Class I., Section 2.

The newly elected Fellows present were then introduced by the President to the Academy.

The following Communications were presented:

Professor C. L. E. Moore. "Einstein's First Theory of Relativity."

Professor George D. Birkhoff. "Einstein's Generalized Relativity or Theory of Gravitation."

Mr. George R. Agassiz. "Astronomical Footnotes to the Einstein Theories."

Discussion followed by Elihu Thomson, E. H. Hall, W. E. Story, A. G. Webster, W. S. Franklin, E. B. Wilson, E. V. Huntington, and A. C. Lane.

The Meeting was then dissolved.

One thousand and ninety-fourth Meeting

November 10, 1920.—Stated Meeting.

The Academy met at its House at 8.15 P.M.

The President in the Chair.

Thirty-nine Fellows and two guests were present.

The Transactions of the last Meeting were read and approved.

On the recommendation of the Council, it was

Voted, That from the income of the General Fund an appropriation of two hundred dollars ($200) be made for Expense of Library.

The Corresponding Secretary reported that to fill a vacancy in the Council, the Council had elected Desmond FitzGerald, Class I., Section 4.

The President announced the death of the following Fellows: Harmon Northrop Morse, Class I., Section 3 (Chemistry); Arthur Searle, Class I., Section 1 (Mathematics and Astronomy).

The President announced that he had appointed E. B. Wilson, C. H. Warren, and F. N. Robinson as the Committee of three to consider the whole question of elections to the Academy.

The following Communication was presented:

Mr. Dows Dunham. "The Sudanese Pyramid," with lantern slide illustrations from Dr. Reisner's excavations.

The following paper was presented by title:

"The Motion of a Particle on a Surface for any positional Field of Force." By Joseph Lipka.

The Meeting was then dissolved.

One thousand and ninety-fifth Meeting.

DECEMBER 8, 1920.—STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

Sixty-four Fellows and several guests were present:

The Transactions of the last Meeting were read and approved.

The Corresponding Secretary reported the resignation of Prof. H. P. Talbot as chairman of the C. M. Warren Committee and the appointment of Prof. J. F. Norris as chairman in his place.

The Corresponding Secretary reported that the Council had authorized the Special Committee on Elections to the Academy to enlarge its membership and to serve for the current year as a Committee on Nominations.

The following Communication was presented:

Professor W. J. V. Osterhout. "Control of Life Phenomena."

The following paper was presented by title:

"The Potential of the Thallium Electrode and the Free Energy of Formation of Thallous Iodide." By Grinnell Jones and Walter C. Schumb.

The Meeting was then dissolved.

One thousand and ninety-sixth Meeting.

DECEMBER 15, 1920.—OPEN MEETING.

An Open Meeting was held at the House of the Academy from four to six o'clock.

The PRESIDENT in the Chair.

There were about two hundred Fellows and guests, including ladies, present.

The President of the Academy gave an address on "Alchemy, Ancient and Modern," with illustrations from lantern slides.

Tea was served at five o'clock in the Reception Room on the third floor.

One thousand and ninety-seventh Meeting.

JANUARY 12, 1921.— STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

Sixty-three Fellows and one guest were present:

The Transactions of the Meeting of December 8th were read and approved.

The Corresponding Secretary stated that the Council had appointed C. B. Gulick to be Acting Recording Secretary during the absence of the Recording Secretary; and W. J. V. Osterhout to be a member of the Committee on Meetings during the absence of G. H. Parker.

The President announced the death of John Elliott Pillsbury, Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe).

Voted, That the following Report of the Special Committee on Elections to the Academy be printed and sent to every Fellow, and that each Fellow be requested to send in writing to the Secretary his opinion with regard to the third topic, and that the Report be made a special order for the next meeting of the Academy.

REPORT OF THE COMMITTEE ON ELECTIONS.

Mr. President:

Your Committee was appointed to consider in a broad way questions concerning election to the Academy, and in particular,

I. General methods of improving the interest of the Academy in elections.

II Professional requirements for election, particularly for the younger men.

III. The question of the election of women.

I. On the matter of general interest in the election your Committee believes that the present plan, which has been in operation only a few years, of having the Council elect upon the basis of a preferential vote of the Academy at large, has maintained quite as much interest in the elections as the older method. We believe that with the responsibility definitely upon the Council the different nominees are discussed more freely and frankly than would be possible with the election by the Academy. We believe that the method of electing by the Council should be continued for a few years longer, at any rate, so that it may have ample trial before the Academy takes up the question of any change in the plan.

The weakness in respect to the elections, whether on the old method or the new, has been lack of interest by the Academy. This lack of interest is sometimes so great that only a small and perhaps indifferent list of nominees is presented to the Council from which the election may be held. The prime requisite at present is to insure a reasonably large and well-selected list of nominees. The American Philosophical Society, at its election last Spring, had seventy distinguished names upon its preliminary preferential ballot from which only fifteen were to be elected to the Society. The American Academy, under its present rules for Fellowship, is limited to six hundred Fellows. This number is not yet reached and when it has been reached the number of persons disappearing annually from the list will probably exceed twenty. The real problem of the Academy, therefore, is to maintain annually a list of nominees of high merit sufficiently large so that the Council may select twenty to thirty, and perhaps in the next few years even more. That such a list can be prepared is amply demonstrated by the experience of the American Philosophical Society.

Your Committee recommends that the Council appoint annually, in December, a nominating committee whose duty it shall be to coöperate with the Fellows of the Academy in seeing that a well-selected list of nominees goes to the Academy on the Preferential Ballot. Your Committee particularly recommends that the Nominating Committee be appointed annually in order that each year the Council may use its best judgment in putting upon the Committee those persons most likely to take a keen interest in the nominations for that year.

II. With respect to the professional requirements for election, particularly as concerns younger men, your Committee believes that it is entirely proper to continue for the present the distinction between resident Fellows and non-resident Fellows, to the effect that the professional requirements for election of resident Fellows are somewhat less than for election of non-resident Fellows. We believe this, because it is the duty of the Academy, and its privilege, to encourage work in the Arts and Sciences and to be, through its meetings, a center for getting together once a month a large number of persons interested in learning here in Boston. Reasonably prompt election of young men in this vicinity will aid in the accomplishment of both these purposes. We believe, however, that no one should be elected to the Academy who has not in his own name, and by his own determination, already given good evidence, through his publications, of accomplishment, and further evidence of the promise of accomplishment in the future. Mere promise for the future, unsupported by past accomplishment, should not entitle a young man to election. We believe that this has been the attitude in the recent past, and is sound for the present.

In this connection we desire to call your attention to the class of Resident Associates. We firmly believe that the Academy will raise its own standard for full membership and at the same time will encourage promising young men and aid in gathering together once a month an interested group of persons if the Academy will elect as Resident Associates those promising young persons whose present accomplishments would certainly not admit them to full Fellowship in the Academy, and if the Council is willing to take this point of view we recommend that the Nominating Committee be charged with the duty of furnishing a list of nominees for Resident Associateship in all Sections.

III. Your Committee was, perhaps, appointed primarily to report upon the admission of women. And it may be that you will regard us as recreant to our duty if we do not make a definite recommendation advising you to elect or to refuse to elect women. Nevertheless, your Committee, realizing that an innovation [1] is here involved and that there are a great many Fellows of the Academy who are violently opposed to the admission of women, while others are insistent on such elections, are unwilling to recommend to the Council any other action in this matter than to put upon the call of some Stated meeting, as regular business to be discussed, the question of the possible Fellow-

1 It should be observed that Maria Mitchell, of Nantucket, was elected an Honorary Member in 1848, and that a few years later, when the members of the Academy were divided into Fellows, Associate Fellows, and Foreign Honorary Members, Miss Mitchell's name was placed with the Fellows.

ship of women in the Academy. We believe that not less than a three-quarters vote of the Fellows present at such a meeting should be required as a necessary indication to the Council that the Academy is ready to elect women.

Associated with this question is the matter of how many negative votes in the Council should be sufficient to prevent the election of any nominee, whether man or woman. Your Committee believes that this matter can safely be left to the Council to determine. It would seem that a two-thirds vote of those members of the Council present and voting should be sufficient. In case of real doubt as to professional qualifications the dissenters could easily muster one-third; whereas in the very few cases in which opposition to an election might seem to two-thirds of the Council as emanating from undue prejudice, the dissenters should themselves feel willing to abide by a decision based on a two-thirds vote.

Probably most Fellows of the Academy and members of its Council realize fully that the matter of election to the Academy is a matter of excellence in Arts and Sciences and is not to be determined in any way by personal preference or personal prejudice, as would be entirely justifiable in the case of a social club.

Respectfully submitted,

E. B. Wilson,
C. H. Warren,
F. N. Robinson.

January 12, 1921.

The Rumford Medals were presented by the President of the Academy to Dr. Irving Langmuir of Schenectady, N. Y., for his researches in thermionic and allied phenomena, after Professor Cross, Chairman of the Rumford Committee, had stated the grounds on which the award was made.

The following Communication was presented:

Dr. Irving Langmuir. "Phenomena connected with the Passage of Electricity through High Vacua."

The following paper was presented by title:—

"Arctic Copepoda in Passamaquoddy Bay," by A. Willey. Presented by Samuel Henshaw.

The Meeting was then dissolved.

One thousand and ninety-eighth Meeting.

February 9, 1921.— Stated Meeting.

The Academy met at its House at 8.15 p.m.

The President in the Chair.

Fifty-seven Fellows were present.

The Transactions of the last Meeting were read and approved.

The Corresponding Secretary presented an invitation from the University of Virginia to the Academy, requesting the presence of a delegate to the one hundredth anniversary of the founding of the University to be held May 31 to June 3, 1921.

The President announced the death of the following Fellows: Henry Andrews Bumstead, Class I., Section 2 (Physics); Lincoln Ware Riddle, Class II., Section 2 (Botany); William Thompson Sedgwick, Class II., Section 3 (Zoölogy and Physiology); Barrett Wendell, Class III., Section 4 (Literature and the Fine Arts).

Voted, That $400 be appropriated from the General Fund for General-and-Meeting Expenses.

Voted, That a committee be appointed to consider a change in the number of Fellows for any one class.

The report of the Committee on Elections with regard to the election of women was read, and an informal ballot of those present at the meeting was taken with the result of 39–15 against the admission of women.

Voted, That the Council be instructed that the election of women is inadvisable.

Voted, To lay on the table the question of Resident Associates.

The following Communication was presented:

Professor George H. Chase. "Excavations at Sardis, 1910–1914."

The following paper was presented by title:

"Observations on Army Ants in British Guiana." By W. M. Wheeler.

The Meeting was then dissolved.

One thousand and ninety-ninth Meeting.

MARCH 9, 1921.—STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

Thirty-nine Fellows were present.

The Transactions of the last Meeting were read and approved.

The President appointed the Nominating Committee as follows:

Henry Lefavour, of Class I.

C. H. Warren, of Class II.

F. N. Robinson, of Class III.

On recommendation of the Council, the following appropriations were made for the ensuing year:

From the income of the General Fund, $8,300, to be used as follows:

for General and Meeting expenses	$1,300.00
for Library expenses	2,900.00
for Books, periodicals and binding	1,000.00
for House expenses	2,200.00
for Treasurer's expenses	900.00

From the income of the Publication Fund, $3,559.11, to be used for publication.

From the income of the Rumford Fund, $3,550.88, to be used as follows:

for Research	$1,000.00
for Books, periodicals, and binding	100.00
for Publication	600.00
for use at the discretion of the Committee	1,850.88

From the income of the C. M. Warren Fund, $990.17, to be used at the discretion of the Committee.

The President announced that he had appointed as the Committee on Biographical Notices, C. R. Cross, A. C. Lane and M. A. DeW. Howe.

The President announced the appointment of Dr. R. S. Woodward to represent the Academy at the one hundredth anniversary of the University of Virginia.

The following Communication was presented:

Professor William Duane, "Some Recent Researches in X-Radiation."

The following paper was presented by title:

"Axes of a Quadratic Vector." By F. L. Hitchcock.

The Meeting was then dissolved.

One thousand and one hundredth Meeting.

MARCH 23, 1921.— OPEN MEETING.

An Open Meeting was held at the House of the Academy from four to six o'clock.

The PRESIDENT in the Chair.

There were about two hundred Fellows and guests, including ladies, present.

Mr. LeRoy Jeffers, Librarian of the American Alpine Club, exhibited by lantern slides a series of photographs of "The Natural Wonders of the United States."

Tea was served at five o'clock in the Reception Room on the third floor.

One thousand one hundred and first Meeting.

APRIL 13, 1921.— STATED MEETING.

The Academy met at its House at 8.15 P.M.

VICE-PRESIDENT Thomson in the Chair.

Thirty-nine Fellows and one guest were present.

The Transactions of the Meeting of March 9th were read and approved.

The Corresponding Secretary announced the receipt of a Biographical Notice of C. E. Norton by M. A. DeW. Howe.

The Chair announced the death of the following Fellows:—

Sherburne Wesley Burnham, Class I., Section I (Mathematics and Astronomy); John Winthrop Platner, Class III., Section 1 (Philosophy and Jurisprudence).

The following Communication was presented:

Professor Irving Fisher, "Index Numbers."

The following paper was presented by title:

"On the Problem of Steering an Automobile round a Corner." By A. G. Webster.

The Meeting was then dissolved.

One thousand one hundred and second Meeting.

APRIL 29, 1921.— OPEN MEETING.

An Open Meeting was held at the House of the Academy from four to six o'clock.

The PRESIDENT in the Chair.

There were about eighty Fellows and guests, including ladies, present.

Professor Francis G. Allinson of Brown University spoke on "The Lady Legislators, and Others, from Aristophanes."

Tea was served at five o'clock in the Reception Room on the third floor.

One thousand one hundred and third Meeting.

MAY 11, 1921.— ANNUAL MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

Thirty-six Fellows and two guests were present.

In the absence of the Recording Secretary *pro tem*, the Corresponding Secretary was requested to assume his duties.

The Transactions of the last two Meetings were read and approved.

The following report of the Council was presented:

Since the last report of the Council, there have been reported the deaths of sixteen Fellows: James Phinney Baxter, Melville

Madison Bigelow, Henry Andrews Bumstead, Sherburne Wesley Burnham, Frank Shipley Col ins, William Crawford Gorgas, Harmon Northrop Morse, Bernadotte Perrin, John Elliott Pillsbury, John Winthrop Platner, Lincoln Ware Riddle, Arthur Searle, William Thompson Sedgwick, Henry Morse Stephens, John Nelson Stockwell, Barrett Wendell; and four Foreign Honorary Members: Joseph Norman Lockyer, Wilhelm Friedrich Philipp Pfeffer, Adam Politzer, Augusto Righi.

Thirteen Fellows and two Foreign Honorary Members were elected by the Council and announced to the Academy in May 1920. One Fellow has resigned.

The roll now includes 517 Fellows and 66 Foreign Honorary Members (not including those elected in April 1921).

The annual report of the Treasurer, Henry H. Edes, was read, of which the following is an abstract:

General Fund.

Receipts.

Balance, April 1, 1920	$8,669.46	
Investments	5,827.93	
Assessments	3,340.00	
Admissions	90.00	
Sundries	274.15	$18,201.54

Expenditures.

Expense of Library	$3,934.20	
Expense of House	2,253.83	
Treasurer	1,360.35	
Assistant Treasurer	250.00	
General Expense of Society	1,472.65	
President's Expenses	95.70	
Income transferred to principal	339.52	$9,706.25
Balance, April 1, 1921		8,495.29
		$18,201.54

Rumford Fund.

Receipts.

Balance, April 1, 1920	$4,558.08	
Investments	4,154.78	
Grant returned	217.11	$8,929.97

Expenditures.

Research	$2,997.10	
Books, periodicals and binding	355.32	
Publications	1,104.69	
Medals	347.29	
Sundries	10.97	
Income transferred to principal	175.96	$4,991.33
Balance, April 1, 1921		3,938.64
		$8,929.97

C. M. Warren Fund.

Receipts.

Balance, April 1, 1920	$4,925.05	
Investments	1,280.00	
Grant returned	250.00	$6,455.05

Expenditures.

Research	$2,172.25	
Vault rent, part	3.00	
Income transferred to principal	53.18	$2,228.43
Balance, April 1, 1921		4,226.62
		$6,455.05

Publication Fund.

Receipts.

Balance, April 1, 1920	$3,474.61	
Appleton Fund investments	1,939.48	

Centennial Fund investments	$2,751.17	
Author's Reprints	183.34	
Sale of Publications	722.53	$9,071.13

Expenditures.

Publications	$4,210.69	
Vault rent, part	10.00	
Interest on bonds bought	204.12	
Income transferred to principal	176.31	$4,601.12
Balance, April 1, 1921		4,470.01
		$9,071.13

FRANCIS AMORY FUND.

Receipts.

Investments		$1,445.70

Expenditures.

Publishing statement	$79.30	
Interest on bonds bought	11.08	
Income transferred to principal	1,355.32	$1,445.70

May 11, 1921.

The following Reports were also presented:—

REPORT OF THE LIBRARY COMMITTEE.

The Librarian begs to report for the year 1920–21, as follows:

During the year 77 books have been borrowed by 34 persons, including 21 Fellows and 4 libraries. As usual many books have been consulted, though not taken from the library. All books taken out have been returned or satisfactorily accounted for, except five.

The number of books on the shelves at the time of the last report was 37,033. 510 volumes have been added, making the number now on the shelves 37,543. This includes 39 purchased from the income of the General Fund, 37 from that of the Rumford Fund, and 434 received by gift or exchange.

The expenses charged to the Library during the financial year are:

Salaries	$2,864.90
Binding —	
General Fund	578.50
Rumford Fund	42.75
Purchase of periodicals and books:—	
General Fund	426.19
Rumford Fund	312.57
Miscellaneous	64.61
	$2,899.52

The Library has had the very efficient services of Mrs. Austin Holden, who has been with us for many years, and this year completes a term, reckoned together with that of her late husband, of fifty years. Mrs. Holden is now in poor health and may be obliged to retire. It is only proper that we put on record our appreciation of her great efficiency and assiduity.

ARTHUR G. WEBSTER, *Librarian.*

May 11, 1921.

REPORT OF THE RUMFORD COMMITTEE.

The Committee organized on October 13, 1920, by electing Charles R. Cross to be Chairman and Arthur G. Webster, Secretary.

The following grants in aid of researches in light or heat have been made during the academic year 1920–21.

October 13, 1920. To Professor Wm. W. Campbell of the Lick Observatory, for the purchase of a special photographic lens; Additional to former appropriation (229)	$360
To Professor Horace L. Howes of New Hampshire State College for his researches on luminescence; Additional to former appropriation (230)	90

December 8, 1920. To Professor Percy W. Bridgman of the Jefferson Physical Laboratory, for his researches on the Thermal and Optical Properties of Bodies under high pressures; Additional to former appropriation (231) $400

March 9, 1921. To Professor Paul F. Gaehr of Wells College for his research on the specific heat of tungsten (232) 250

April 13, 1921. To Professor Alpheus W. Smith of Ohio State University for his researches on the Hall, Nernst and allied Effects; Additional to former appropriation (233) 200

May 11, 1921. To Professor Leonard R. Ingersoll, of the University of Wisconsin, for his research on Magnetic-Rotation Dispersion, particularly of invisible Radiations; Additional to former appropriation (234) 200

The grants made by the Rumford Committee from the beginning have been numbered consecutively. These numbers are given for the above-mentioned.

Reports of Progress in their respective researches have been received from the following persons: C. G. Abbot (research finished), W. M. Baldwin (research finished), R. T. Birge, P. W. Bridgman, W. W. Campbell, A. L. Clark, H. Crew (research finished), F. Daniels, P. F. Gaehr, R. C. Gibbs, H. P. Hollnagel, H. L. Howes, L. R. Ingersoll, N. A. Kent, F. E. Kester, F. G. Keyes, C. A. Kraus, E. Kremers, R. A. Millikan, C. L. Norton, F. Palmer, Jr., J. A. Parkhurst, H. M. Randall, T. W. Richards, F. K. Richtmyer, F. A. Saunders, W. O. Sawtelle, A. W. Smith, B. J. Spence, L. S. E. Thompson, O. Tugman, F. W. Very, A. G. Webster, D. L. Webster, F. G. Wick, R. W. Wood.

The following papers in the Proceedings have been published with aid from the Rumford Fund since the presentation of the last Report of this Committee.

Specific Heat of Ammonia. H. A. Babcock, Vol. 55, No. 6.

Ghosts and Oculars. Louis Bell, Vol. 56, No. 2.

Electrical Resistance under Pressure including certain liquid metals. P. W. Bridgman, Vol. 56, No. 3.

Researches aided from the Rumford Fund. 1832–1920. Compiled by Charles R. Cross will appear shortly.

Charles R. Cross, *Chairman.*

May 11, 1921.

Report of the C. M. Warren Committee.

In a letter dated December 13, 1920, the Corresponding Secretary of the Academy announced to the Committee that Professor H. P. Talbot had been obliged to present his resignation of the chairmanship and that Professor J. F. Norris had been appointed Chairman by the Council.

The Committee had at its disposal at the end of the fiscal year in March 1920, $5,683.11. During the year ending March 31, 1921, the Treasurer paid out $1,450 on grants previously made by the Committee, and new grants amounting to $1,022.25 were made. The unexpended balance at the close of the fiscal year was $3,210.86.

Since March 31, 1921, grants amounting to $750 have been awarded. The Committee has under consideration at present five applications for grants which amount to $1,300.

The renewed activity in research which followed the war is evident from the large number of men seeking financial assistance in their work, and it is evident that the Committee will be able to use its resources in assisting research of the first importance.

Since the last annual report the following awards have been made:

July 1920, $400 to Professor C. James of the New Hampshire Agricultural College for an investigation of the atomic weight of yttrium.

December 1920, $222.25 to Professor J. B. Conant of Harvard University for a study of reversible oxidation processes in organic chemistry.

December 1920, $250 to Professor H. Hibbert of Yale University for an investigation of the constitution of cellulose and the reduction of unsaturated aldehydes.

April 1921, $500 to Professor J. M. Bell of the University of North Carolina for a research on the heats of fusion of the nitrotoluenes.

Reports of progress have been received from Professors James, Harkins, Kraus, and Conant.

James F. Norris, *Chairman.*

May 11, 1921.

Report of the Publication Committee.

The Committee of Publication reports as follows, for the period from April 1, 1920 to March 31, 1921:

During the above twelve months there have been issued Nos. 5 to 10 of Vol. 55 of the Proceedings, and Nos. 1 to 4 of Vol. 56. The very high cost of printing the Academy's difficult texts at present rates, and the corresponding cost of all materials used in printing, are shown in the item of expenses; these and the highly authoritative character of the papers printed are partially reflected in the unusually large amount received from sales.

Financial Statement.

Balance April 1, 1920	$4,531.68	
Appropriations, 1920–21	4,655.24	
Sales of Publications	722.53	
Received for authors' reprints	183.34	$10,092.79
Expenses		4,210.69
Balance, March 31, 1921		$5,882.10

The above statement does not include payment of $1,053.69 for the publication of papers at the expense of the Rumford Fund.

Respectfully submitted,

Louis Derr, *Chairman.*

May 11, 1921.

The following report, because of the absence of the chairman of the House Committee, was not presented.

Report of the House Committee.

The House Committee submits the following report for 1920–21:

With the balance of $66.18 left from last year, an appropriation of $2,200, and $64 received from other societies for the use of the rooms, the Committee has had at its disposal the sum of $2,330.18. The total expenditure has been $2,317.83, leaving an unexpended balance on April 1, 1921, of $12.35. The expenditure has been as follows:

Janitor		$925.00
Electricity	A. Light	168.44
	B. Power	88.23
Coal	Furnace	735.70
	Water Heater	26.50
Care of the elevator		91.45
Gas		54.92
Water		12.88
Telephone		77.91
Ice		23.27
Janitor's materials		25.84
Upkeep		68.68
Ash tickets		21.01
Total expenditure		$2,317.83

The amount of $64 contributed by other societies for the use of the building leaves the net expense of the House $2,253.83.

Meetings have been held as follows:

The Academy	
Regular meetings	8
Open meetings	3
Physics Section	1
American Antiquarian Society	2
Archeological Institute	1
Colonial Dames	1
Colonial Society	3
Geological Society of Boston	5
Harvard-Technology Chemical Club	7
Research Laboratory of Physical Chemistry	1
Special Libraries Association	1
Society of Landscape Architects	1
	34

The rooms on the first floor have been used for Council and Committee meetings.

Respectfully submitted,

John Osborne Sumner, *Chairman.*

May 11, 1921.

On the recommendation of the Treasurer, it was

Voted, That the Annual Assessment be $10.00.

The Corresponding Secretary reported that the Council had voted not to elect Resident Associates.

The Librarian having referred to the serious illness of the Assistant Librarian and to the long and honorable services of Mrs. Holden and the late Dr. Holden in that capacity, it was

Voted, That the Academy convey to the Assistant Librarian an expression of its appreciation and sympathy, and of its hope for her speedy recovery.

The annual election resulted in the choice of the following officers and committees:

George F. Moore, *President.*
Elihu Thomson, *Vice-President for Class I.*
Harvey Cushing, *Vice-President for Class II.*
Arthur P. Rugg, *Vice-President for Class III.*
Harry W. Tyler, *Corresponding Secretary.*
James H. Ropes, *Recording-Secretary.*
Henry H. Edes, *Treasurer.*
Arthur G. Webster, *Librarian.*

Councillors for Four Years.

William S. Franklin, *of Class I.*
Walter B. Cannon, *of Class II.*
Albert Matthews, *of Class III.*

Finance Committee.

Henry P. Walcott, John Trowbridge,
Harold Murdock.

Rumford Committee.

CHARLES R. CROSS,
ARTHUR G. WEBSTER,
ELIHU THOMSON,
THEODORE LYMAN,
LOUIS BELL,
PERCY W. BRIDGMAN,
HARRY M. GOODWIN.

C. M. Warren Committee.

JAMES F. NORRIS,
HENRY P. TALBOT,
CHARLES L. JACKSON,
GREGORY P. BAXTER,
WALTER L. JENNINGS,
WILLIAM H. WALKER,
ARTHUR D. LITTLE.

Publication Committee.

LOUIS DERR, *of Class I.*
HERBERT V. NEAL, *of Class II.*
ALBERT A. HOWARD, *of Class III.*

Library Committee.

HARRY M. GOODWIN, *of Class I.*
THOMAS BARBOUR, *of Class II.*
WILLIAM C. LANE, *of Class III.*

House Committee.

JOHN O. SUMNER,
WM. STURGIS BIGELOW,
ROBERT P. BIGELOW.

Committee on Meetings.

THE PRESIDENT,
THE RECORDING SECRETARY,
GEORGE H. PARKER,
EDWIN B. WILSON,
EDWARD K. RAND.

Auditing Committee.

GEORGE R. AGASSIZ,
JOHN E. THAYER.

The Council reported that the following gentlemen were elected members of the Academy:—

Class I., Section 1 (Mathematics and Astronomy):
Charles Greeley Abbot, of Washington, as Fellow.
Florian Cajori, of Berkeley, California, as Fellow.
Godfrey Harold Hardy, of Oxford, as Foreign Honorary Member.
Oliver Dimon Kellogg, of Cambridge, as Fellow.
Henry Norris Russell, of Princeton, as Fellow.
Frank Schlesinger, of New Haven, as Fellow.
Joel Stebbins, of Urbana, as Fellow.
Class I., Section 2 (Physics):
Samuel Jackson Barnett, of Washington, as Fellow.
Leslie Lyle Campbell, of Cambridge, as Fellow.
Frederic Eugene Ives, of Philadelphia, as Fellow.
Class I., Section 3 (Chemistry):
Edward Curtis Franklin, of Palo Ato, as Fellow.
Class I., Section 4 (Technology and Engineering):
Arthur Powell Davis, of Washington, as Fellow.
William Frederick Durand, of Palo Alto, as Fellow.
William Emerson, of Cambridge, as Fellow.
Charles Thomas Main, of Boston, as Fellow
Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe):
Norman Levi Bowen, of Washington, as Fellow.
John Casper Branner, of Palo Alto, as Fellow.
William Jackson Humphreys, of Washington, as Fellow.
Arthur Keith, of Washington, as Fellow.
James Furman Kemp, of New York as Fellow.
John Campbell Merriam, of Washington, as Fellow.
Gustaf Adolf Frederik Molengraaff, of Delft, as Foreign Honorary Member.
David White, of Washington, as Fellow.
Arthur Winslow, of Boston, as Fellow.
Class II., Section 2 (Botany):
Edward Wilber Berry, of Baltimore, as Fellow.
Hugo de Vries, of Holland, as Foreign Honorary Member.
Rollins Adams Emerson, of Ithaca, as Fellow.
Jacob Goodale Lipman, of New Brunswick, as Fellow.

Elmer Drew Merrill, of Manila, as Fellow.
Charles Vancouver Piper, of Washington, as Fellow.
Class II., Section 3 (Zoölogy and Physiology):
Gerritt Smith Miller, of Washington, as Fellow.
William Patten, of Hanover, as Fellow.
Henry Augustus Pilsbry, of Philadelphia, as Fellow.
Class II., Section 4 (Medicine and Surgery):
Charles Macfie Campbell, of Cambridge, as Fellow.
Rufus Cole, of New York, as Fellow.
Ross Granville Harrison, of New Haven, as Fellow.
William Henry Howell, of Baltimore, as Fellow.
William James Mayo, of Rochester, Minn., as Fellow.
Francis Weld Peabody, of Boston, as Fellow.
Charles Wardell Stiles, of Washington, as Fellow.
William Sydney Thayer, of Baltimore, as Fellow.
John Warren, of Boston, as Fellow.
Class III., Section 1 (Philosophy and Jurisprudence):
Paul Revere Frothingham, of Boston, as Fellow
William Earnest Hocking, of Cambridge, as Fellow.
Charles Francis Jenney, of Boston, as Fellow.
Frederick Lawton, of Boston, as Fellow.
Austin Wakeman Scott, of Cambridge, as Fellow.
Class III., Section 2 (Philology and Archaeology):
Carl Darling Buck, of Chicago, as Fellow.
Eugene Xavier Louis Henry Hyvernat, of Washington, as Fellow.
John Livingston Lowes, of Cambridge, as Fellow.
Chandler Rathfon Post, of Cambridge, as Fellow.
Class III., Section 3 (Political Economy and History):
Wilbur Cortez Abbott, of Cambridge, as Fellow.
John Spencer Bassett, of Northampton, as Fellow.
Clive Day, of New Haven as Fellow.
Max Farrand, of New York, as Fellow.
William Scott Ferguson, of Cambridge, as Fellow.
Allyn Abbott Young, of Cambridge, as Fellow.
Class III., Section 4 (Literature and the Fine Arts):
Irving Babbitt, of Cambridge, as Fellow.
Frederick Shepherd Converse, of Westwood, as Fellow.

Frank Edgar Farley, of Middletown, as Fellow.
Charles Martin Tornov Loeffler, of Medfield, as Fellow.
Charles Donagh Maginnis, of Boston, as Fellow.
William Lyon Phelps, of New Haven, as Fellow.
Charles Howard Walker, of Boston, as Fellow.

The following Communication was presented:

Professors W. M. Wheeler and I. W. Bailey, "Some Entomological and Botanical Investigations in British Guiana," with lantern illustrations.

The Meeting was then dissolved.

One thousand one hundred and fourth Meeting.

MAY 18, 1921.—SPECIAL MEETING.

A Special Meeting was held at the House of the Academy at 4.00 o'clock in honor of Professor Albert Einstein, of the Academy of Sciences in Berlin.

The PRESIDENT in the Chair.

There were about two hundred Fellows and guests, including ladies, present.

President Moore opened the meeting, addressing Professor Einstein in German, giving a brief history of the Academy and its work, and, continuing in English, spoke of the peculiarly international character of science, and of its advances, and welcomed Professor Einstein to the Academy.

Professor Einstein addressed the Academy in German, and gave a lucid exposition of the theory of general relativity which is associated with his name.

BIOGRAPHICAL NOTICES

WALTER GOULD DAVIS (1851–1919).

Fellow in Class II, Section 1, 1894.

The meteorological service of the Argentine Republic will be the enduring monument of Walter Gould Davis, whose death on April 30, 1919, at his old homestead in Danville, Vt., removed one of the world's best-known and most highly respected meteorologists.

Mr. Davis was born on September 28, 1851, in the house at Danville in which he died. His father, Walter Davis, was a successful farmer, and his mother, Achsa Gould Davis, who was a school teacher before her marriage, had strong literary and mathematical tastes. Walter Gould Davis was an only child. A "Wanderlust" was in him from his early years, and his desire to see other countries was as keen as it was unusual in the quiet farming community in which his youth was spent. He started his professional life as a constructing engineer on the railroad between St. Johnsbury and Cambridge, Vermont (1870–1872), and as the result of his own efforts he was able, by reading and study, to fit himself for the position of Chief Engineer of the railroad which, between 1872 and 1876, was being constructed through the famous Crawford Notch. This responsible post he held until the work was completed, when, in 1876, he went to South America, to satisfy his desire to see other lands and also with the purpose of putting his engineering experience to use in new countries.

It was not, however, as an engineer but as a meteorologist that Walter Gould Davis became known to scientific men the world over. On his arrival in Argentina he was given a three months' trial as a computer in the Astronomical Observatory, and was later made Second Assistant to Dr. Benjamin Apthorp Gould, who founded the Astronomical Observatory at Cordoba, and, in 1873, established the Argentine Meteorological Service, which was installed in the Astronomical Observatory, the two organizations being independent of one another, although under the same director. Dr. Gould continued in charge of this service until towards the end of 1884, when he left Argentina. In 1885, Mr. Davis succeeded Dr. Gould as director, continuing in that position until his retirement in May, 1915, after thirty years of active work.

Four years after he became Director, Mr. Davis was married in Boston (December 4, 1889) to Mabel Quincy, who was his constant companion during the remainder of his life in an unusually happy marriage.

Under Mr. Davis's able leadership, the Argentine Meteorological Service attained a position in the very front rank of government meteorological organizations. When he resigned his post, to secure well-deserved rest and to seek to regain his health in his own country, the Argentine service extended over an area of nearly 3,000 miles in a north-and-south line, its southernmost station being in the South Orkney Islands, in latitude 60°43′ south. Over 2,000 stations were then coöperating in the work of taking meteorological and magnetic observations. The morning and evening observations from nearly 200 stations were being used in the construction of the daily weather map, in addition to the daily rainfall records from about 1,350 rainfall stations.

The development of meteorological work under Mr. Davis was rapid and many-sided. In 1885, the year in which he became director, the Meteorological Office (Oficina Meteorológica Argentina) was made a separate organization, and its headquarters were moved from the Astronomical Observatory to a larger and better building, especially constructed for the purpose on the grounds immediately adjoining. In 1901 the central office was moved to Buenos Aires, where the telegraphic and other facilities for the preparation of a daily weather map, publication of which was begun on February 21, 1902, were much greater than at Cordoba. A hydrometric section was established in 1902; a magnetic section and a forecasting service in 1904; a rainfall service in 1912, and a system of weekly, or longer, forecasts in 1915. The section of climatic statistics has continued to have its headquarters at Cordoba, where it collects and compiles climatological data, maintains a first-class observatory, and is carrying on researches in agricultural meteorology.

Mr. Davis was a tremendously keen, active and progressive director. He was not only an unusually efficient executive officer, but he was also a man of wide learning and of a great variety of interests. Both as director, and as a man, he had the respect and loyal devotion of all his associates and employees. He was always well abreast of the times, and often was a pioneer in keeping ahead of the times. Not content with covering the mainland of his great district with meteorological stations, he extended his service into the Antarctic province to the south. An illustration of his desire to have the organization under his control contribute in every possible way to the advancement of meteorological knowledge was his acquirement, in 1904, of the meteorological and magnetic station at Laurie Island, in the South Orkneys, which had originally been established by the Scottish Antarctic Expedition. Since 1904, this remote southern station has been operated, without a break in its records, as a part of the Argentine Meteorological Service. The personnel of this lonely outpost is relieved only once each year, when supplies are sent for the coming twelve months. The men are then completely isolated, without (at last accounts) any mail or cable communication, until the relief vessel returns the following year. Under these conditions of extreme loneliness and hardship, the observers at Laurie Island have maintained their observations for over fifteen years. This is a remarkable record of scientific work of the greatest importance in the study of world meteorology. In his Laurie Island station Mr. Davis always took great pride, and well he might do so.

Fully alive to all the needs of his service, Mr. Davis called to help him in his scientific work the best meteorologists whom he could find. From this country, he secured Professor F. H. Bigelow, formerly of the Weather Bureau, who has had charge of the magnetic work in Argentina since September, 1915; Mr. H. H. Clayton, formerly of Blue Hill Observatory, and since 1913 chief of the Department of Forecasts in Buenos Aires; Mr. L. G. Schultz, chief of the magnetic section until 1915, and others. Mr. George O. Wiggin, the present director of the Argentine Meteorological Office, is also a native of the United States.

The high quality of Mr. Davis's work was fully appreciated by his meteorological colleagues everywhere. His reputation as a meteorologist and as the successful administrative head of a large and remarkably efficient organization won for him a position on the International Meteorological Committee, the highest international authority on meteorology. This was a well-deserved recognition of the importance of his contributions to meteorology, and of his sound judgment on scientific matters.

The many publications of the Argentine Meteorological Service which were issued under Mr. Davis's direction constitute an inspiring record of splendid work, well planned, thoroughly organized, and ably carried out. For comparatively few countries are there available such excellent meteorological and climatological publications, some of them in English, as the Argentine Meteorological Service has sent out.

By the death of Walter Gould Davis the world lost one of its most eminent meteorologists, and those of his colleagues who had the privilege of knowing him lost a warm-hearted, sympathetic and helpful friend.

Robert DeC. Ward.

CHARLES ELIOT NORTON (1827–1908)

Fellow in Class III, Section 4, 1860

The two volumes of "Letters of Charles Eliot Norton," published, "with biographical comment," in 1913, have rendered so accessible the record of the life and work of this Fellow of the Academy that it would be superfluous to supplement it here with an extensive memoir.

He was born, November 16, 1827, at Shady Hill, Cambridge, the house of his father Professor Andrews Norton, (1787–1853), also a Fellow of the Academy. In this house, the home of his lifetime, he died, October 21, 1908. The distinction of beginning and ending one's days under the same roof is not one to which many Americans can lay claim; yet Charles Eliot Norton was by descent from a long and distinguished New England ancestry an American of Americans. He was exceptional among his contemporaries, however, for a background of cosmopolitan experience in friendships and intellectual pursuits which made him, more than most New Englanders, a citizen of the world.

Graduating at Harvard College with the Class of 1846, he began his active life in the counting-house of a Boston firm of East India merchants. This afforded him the opportunity to sail for the Far East as supercargo of a ship in 1849. Before his return to Boston in 1851 he had seen much of India and its people, and, returning by way of Europe, had made many stimulating acquaintances in Paris and London, and one friendship — with George William Curtis — which played an important part in all his later life. In the years that immediately followed he began his career as a man of letters, publishing in 1853 his first and second books, "Five Christmas Hymns," of which he was the anonymous editor, and "Considerations of Some Recent Social Theories," written by himself. From 1855 to 1857 he was again in Europe, widely extending his circle of friends, of whom from this time he counted John Ruskin and Mrs. Gaskell among the most intimate, and beginning in Italy his serious, life-long studies of Dante. The nine ensuing years were passed in America, largely in editorial and patriotic labors. Through his intimate friendship with James Russell Lowell he bore a close relation to the *Atlantic Monthly* in the earliest years of its existence, and with the *North American Review*, of which for a time he was the editor. Through the period of the Civil War he rendered his country a valuable service by means of his editorial work for the New England Loyal Publication Society, an agency for distributing to the American press the best editorial expressions on behalf of the Northern cause. In 1862 he married Susan Ridley Sedgwick of Stockbridge and New York, and in 1868 returned to Europe with his wife and young children. During the five years that followed, old friendships were renewed and new and vital intimacies were begun: his letters and journals abound in illuminating records of intercourse with Carlyle, Ruskin, Leslie Stephen, Burne-Jones, and many others who made the England of the period what it was. But this period of Norton's life had its overwhelming shadow in the death of his wife at Dresden in 1872. When he returned to America in the following year, it was to take up his life on new terms.

It is not often that a man begins at nearly fifty years of age the work which makes him a distinguished figure in his generation. The opportunity to effect this remarkable achievement came to him when his cousin, President Eliot, had the foresight, growing from the knowledge of Norton's training in a field hitherto but slightly tilled in America, to offer him in 1874 a lectureship at Harvard College on the History of the Fine Arts as Connected with Literature, and in 1875 a professorship in the History of Art — a post which he held for the remainder of his active life, until 1898, and thereafter, until his death ten years later, as Professor Emeritus. His influence on a long series of college generations was incalculable. Into American society, dominated more and more by material things, he brought a sense of appreciation of beauty as expressed in all the arts which must be counted among the great, if imponderable, influences of his time. He was unsparing in his criticism of American tendencies which seemed to him at war with the highest development of his country, and consequently became himself the subject of much criticism. But the finest spirits of his generation, Lowell, Curtis, Godkin in America, and a kindred group in England, held his friendship as dear as he held theirs, and both upon his gifts and upon his use of them placed a high and distinctive value, now generally accepted. To the literature of Dante, of architectural history, of biography and criticism, he contributed an important series of works, all related closely to his own contacts with life. In his relations with friends of all ages he held a place which has become a treasured memory.

He was elected a Fellow of the Academy, November 14, 1860. His single contribution to its Proceedings was a paper on "The Dimensions and Proportions of the Temple of Zeus at Olympia," embodying many measurements based on those of French and German archaeological expeditions. This was presented, October 10, 1877, and printed in volume 13.

M. A. DeWolfe Howe.

American Academy of Arts and Sciences

OFFICERS AND COMMITTEES FOR 1921–22

PRESIDENT.

George F. Moore.

VICE-PRESIDENTS.

Class I.	Class II.	Class III.
Elihu Thomson,	Harvey Cushing,	Arthur P. Rugg.

CORRESPONDING SECRETARY.

Harry W. Tyler.

RECORDING SECRETARY.

James H. Ropes.

TREASURER.

Henry H. Edes.

LIBRARIAN.

Arthur G. Webster.

COUNCILLORS.

Class I.	Class II.	Class III.
George D. Birkhoff,	Charles H. Warren,	Frederic Dodge.
	Terms expire 1922.	
Cecil H. Peabody,	Edward M. East,	Charles B. Gulick.
	Terms expire 1923.	
Forris J. Moore,	Reid Hunt,	Ephraim Emerton.
	Terms expire 1924.	
William S. Franklin,	Walter B. Cannon,	Albert Matthews.
	Terms expire 1925.	

COMMITTEE OF FINANCE.

Henry P. Walcott, John Trowbridge, Harold Murdock.

RUMFORD COMMITTEE.

Charles R. Cross, *Chairman*,

Arthur G. Webster, Theodore Lyman, Percy W. Bridgman,
Elihu Thomson, Louis Bell, Harry M. Goodwin.

C. M. WARREN COMMITTEE.

James F. Norris, *Chairman*,

Henry P. Talbot, Gregory P. Baxter, William H. Walker,
Charles L. Jackson, Walter L. Jennings, Arthur D. Little.

COMMITTEE OF PUBLICATION.

Louis Derr, of Class I, *Chairman*,

Herbert V. Neal, of Class II, Albert A. Howard, of Class III.

COMMITTEE ON THE LIBRARY.

Arthur G. Webster, *Chairman*,

Harry M. Goodwin, of Class I, Thomas Barbour, of Class II,
William C. Lane, of Class III.

AUDITING COMMITTEE.

George R. Agassiz, John E. Thayer.

HOUSE COMMITTEE.

John O. Sumner, *Chairman*, Wm. Sturgis Bigelow, Robert P. Bigelow.

COMMITTEE ON MEETINGS.

The President,
The Recording Secretary,
George H. Parker, Edwin B. Wilson, Edward K. Rand.

LIST

OF THE

FELLOWS AND FOREIGN HONORARY MEMBERS.

(Corrected to September 1, 1921.)

FELLOWS.— 567.

(Number limited to six hundred.)

Class I.— *Mathematical and Physical Sciences.*— 197.

Section I.— *Mathematics and Astronomy.*— 46.

Charles Greeley Abbot	Washington, D. C.
George Russell Agassiz	Boston
Raymond Clare Archibald	Providence, R. I.
Solon Irving Bailey	Cambridge
Edward Emerson Barnard	Williams Bay, Wis.
George David Birkhoff	Cambridge
Charles Leonard Bouton	Cambridge
Ernest William Brown	New Haven, Conn.
William Elwood Byerly	Cambridge
Florian Cajori	Berkeley, Cal.
William Wallace Campbell	Mt. Hamilton, Cal.
Julian Lowell Coolidge	Cambridge
George Cary Comstock	Madison, Wis.
Leonard Eugene Dickson	Chicago, Ill.
Philip Fox	Evanston, Ill.
Fabian Franklin	New York, N.Y.
Edwin Brant Frost	Williams Bay, Wis.
Frank Lauren Hitchcock	Belmont
Edward Vermilye Huntington	Cambridge
Dunham Jackson	Minneapolis, Minn.

Oliver Dimon Kellogg	Cambridge
Edward Skinner King	Cambridge
Carl Otto Lampland	Flagstaff, Ariz.
Joseph Lipka	Cambridge
Joel Hastings Metcalf	Portland, Me.
George Abram Miller	Urbana, Ill.
Clarence Lemuel Elisha Moore	Watertown
Eliakim Hastings Moore	Chicago, Ill.
Frank Morley	Baltimore, Md.
Forest Ray Moulton	Chicago, Ill.
Henry Bayard Phillips	Boston
William Henry Pickering	Cambridge
Charles Lane Poor	New York, N. Y.
Roland George Dwight Richardson	Middletown, Conn.
Henry Norris Russell	Princeton, N. J.
Frank Schlesinger	New Haven, Conn.
Harlow Shapley	Cambridge
Vesto Melvin Slipher	Flagstaff, Ariz.
Frederick Slocum	Middletown, Conn.
Virgil Snyder	Ithaca, N. Y.
Joel Stebbins	Urbana, Ill.
William Edward Story	Worcester
Henry Taber	Worcester
Harry Walter Tyler	Newton
Robert Wheeler Willson	Cambridge
Frederick Shenstone Woods	Newton

Class I., Section II.— *Physics.*— 57.

Joseph Sweetman Ames	Baltimore, Md.
Samuel Jackson Barnett	Washington, D. C.
Carl Barus	Providence, R. I.
Louis Agricola Bauer	Washington, D. C.
Alexander Graham Bell	Washington, D. C.
Louis Bell	Newton
Percy Williams Bridgman	Cambridge
George Ashley Campbell	New York, N. Y.
Leslie Lyle Campbell	Boston

Emory Leon Chaffee	Belmont
Daniel Frost Comstock	Cambridge
William David Coolidge	Schenectady, N. Y.
Henry Crew	Evanston, Ill.
Charles Robert Cross	Brookline
Harvey Nathaniel Davis	Cambridge
Arthur Louis Day	Corning, N. Y.
Louis Derr	Brookline
William Johnson Drisko	Winchester
William Duane	Boston
Alexander Wilmer Duff	Worcester
Arthur Woolsey Ewell	Worcester
William Suddards Franklin	Cambridge
Harry Manley Goodwin	Brookline
George Ellery Hale	Pasadena, Cal.
Edwin Herbert Hall	Cambridge
Hammond Vinton Hayes	Boston
John Charles Hubbard	New York, N. Y.
Gordon Ferrie Hull	Hanover, N. H.
Charles Clifford Hutchins	Brunswick, Me.
Frederic Eugene Ives	Philadelphia, Pa.
James Edmund Ives	Worcester
William White Jacques	Boston
Norton Adams Kent	Cambridge
Frank Arthur Laws	Boston
Henry Lefavour	Boston
Theodore Lyman	Brookline
Thomas Corwin Mendenhall	Ravenna, O.
Ernest George Merritt	Ithaca, N. Y.
Albert Abraham Michelson	Chicago, Ill.
Dayton Clarence Miller	Cleveland, O.
Robert Andrews Millikan	Chicago, Ill.
Harry Wheeler Morse	Los Angeles, Cal.
Edward Leamington Nichols	Ithaca, N. Y.
Ernest Fox Nichols	New Haven, Conn.
Charles Ladd Norton	Boston
George Washington Pierce	Cambridge
Michael Idvorsky Pupin	New York, N. Y.

Frederick Albert Saunders . . . . Cambridge
John Stone Stone . . . . New York, N. Y.
Maurice deKay Thompson . . . . Brookline
Elihu Thomson . . . . Swampscott
John Trowbridge . . . . Cambridge
Arthur Gordon Webster . . . . Worcester
David Locke Webster . . . . Palo Alto, Cal.
Edwin Bidwell Wilson . . . . Brookline
Robert Williams Wood . . . . Baltimore, Md.
John Zeleny . . . . New Haven, Conn.

Class I., Section III.— *Chemistry.*— 51.

Wilder Dwight Bancroft . . . . Ithaca, N. Y.
Gregory Paul Baxter . . . . Cambridge
Arthur Alphonzo Blanchard . . . . Cambridge
Marston Taylor Bogert . . . . New York, N. Y.
Bertram Borden Boltwood . . . . New Haven, Conn.
William Crowell Bray . . . . Berkeley, Cal.
Russell Henry Chittenden . . . . New Haven, Conn.
Arthur Messinger Comey . . . . Chester, Pa.
Charles William Eliot . . . . Cambridge
Henry Fay . . . . Boston
George Shannon Forbes . . . . Cambridge
Edward Curtis Franklin . . . . Palo Alto, Cal.
Frank Austin Gooch . . . . New Haven, Conn.
Lawrence Joseph Henderson . . . . Cambridge
Charles Loring Jackson . . . . Boston
Walter Louis Jennings . . . . Worcester
Grinnell Jones . . . . Cambridge
Frederick George Keyes . . . . Cambridge
Elmer Peter Kohler . . . . Cambridge
Charles August Kraus . . . . Worcester
Arthur Becket Lamb . . . . Cambridge
Irving Langmuir . . . . Schenectady, N. Y.
Gilbert Newton Lewis . . . . Berkeley, Cal.
Warren Kendall Lewis . . . . Boston

Arthur Dehon Little	Brookline
Charles Frederic Mabery	Cleveland, O.
Forris Jewett Moore	Boston
George Dunning Moore	Worcester
Edward Williams Morley	West Hartford, Conn.
Edward Mueller	Cambridge
Samuel Parsons Mulliken	Boston
Charles Edward Munroe	Forest Glen, Md.
James Flack Norris	Boston
Arthur Amos Noyes	Pasadena, Cal.
William Albert Noyes	Urbana, Ill.
Thomas Burr Osborne	New Haven, Conn.
Samuel Cate Prescott	Brookline
Ira Remsen	Baltimore, Md.
Robert Hallowell Richards	Jamaica Plain
Theodore William Richards	Cambridge
Martin André Rosanoff	Pittsburgh, Pa.
Stephen Paschall Sharples	Cambridge
Miles Standish Sherrill	Brookline
Alexander Smith	New York, N. Y.
Harry Monmouth Smith	Brookline
Julius Oscar Stieglitz	Chicago, Ill.
Henry Paul Talbot	Newton
William Hultz Walker	Boston
Willis Rodney Whitney	Schenectady, N. Y.
Robert Seaton Williams	Cambridge
Alpheus Grant Woodman	Watertown

CLASS I., SECTION IV.— *Technology and Engineering.*— 43.

Henry Larcom Abbot	Cambridge
Comfort Avery Adams	Cambridge
Bernard Arthur Behrend	Boston
William Herbert Bixby	Chicago, Ill.
Francis Tiffany Bowles	Boston
Charles Francis Brush	Cleveland, O.

William Hubert Burr . . . . New Canaan, Conn.
John Joseph Carty . . . . New York, N.Y.
Eliot Channing Clarke . . . . Boston
Harry Ellsworth Clifford . . . . Newton
Theodore Harwood Dillon . . . . Cambridge
William Frederick Durand . . . . Palo Alto, Cal.
Frederic Harold Fay . . . . Boston
Desmond FitzGerald . . . . Brookline
John Ripley Freeman . . . . Providence, R. I.
George Washington Goethals . . . . New York, N. Y.
John Hays Hammond . . . . New York, N. Y.
Rudolph Hering . . . . New York, N. Y.
Ira Nelson Hollis . . . . Worcester
Henry Marion Howe . . . . New York, N. Y.
Hector James Hughes . . . . Cambridge
Alexander Crombie Humphreys . . . . New York, N. Y.
Dugald Caleb Jackson . . . . Cambridge
Lewis Jerome Johnson . . . . Cambridge
Arthur Edwin Kennelly . . . . Cambridge
Gaetano Lanza . . . . Philadelphia, Pa.
Charles Thomas Main . . . . Winchester
Lionel Simeon Marks . . . . Cambridge
Edward Furber Miller . . . . Newton
Hiram Francis Mills . . . . South Hingham
Frederick Law Olmsted . . . . Brookline
Charles Francis Park . . . . Boston
William Barclay Parsons . . . . New York, N.Y.
Harold Pender . . . . Philadelphia, Pa.
Albert Sauveur . . . . Cambridge
Peter Schwamb . . . . Arlington
Henry Lloyd Smyth . . . . Cambridge
Charles Milton Spofford . . . . Brookline
Charles Proteus Steinmetz . . . . Schenectady, N. Y.
George Fillmore Swain . . . . Cambridge
George Chandler Whipple . . . . Cambridge
Robert Simpson Woodward . . . . Washington, D. C.
Joseph Ruggles Worcester . . . . Boston

Class II.— *Natural and Physiological Sciences.*— 180.

Section I.— *Geology, Mineralogy, and Physics of the Globe.*— 54.

Wallace Walter Atwood	Cambridge
George Hunt Barton	Cambridge
Norman Levi Bowen	Washington, D. C.
Isaiah Bowman	Washington, D. C.
John Casper Branner	Palo Alto, Cal.
Thomas Chrowder Chamberlin	Chicago, Ill.
John Mason Clarke	Albany, N. Y.
Henry Helm Clayton	Canton
Herdman Fitzgerald Cleland	Williamstown
William Otis Crosby	Jamaica Plain
Reginald Aldworth Daly	Cambridge
Edward Salisbury Dana	New Haven, Conn.
William Morris Davis	Cambridge
Benjamin Kendall Emerson	Amherst
William Ebenezer Ford	New Haven, Conn.
James Walter Goldthwait	Hanover, N. H.
Louis Caryl Graton	Cambridge
Herbert Ernest Gregory	New Haven, Conn.
William Jackson Humphreys	Washinton, D. C.
Ellsworth Huntington	Milton
Oliver Whipple Huntington	Newport, R. I.
Robert Tracy Jackson	Peterborough, N. H.
Thomas Augustus Jaggar	Honolulu, H. I.
Douglas Wilson Johnson	New York, N. Y.
Arthur Keith	Washington, D. C.
James Furman Kemp	New York, N. Y.
Alfred Church Lane	Cambridge
Andrew Cowper Lawson	Berkeley, Cal.
Charles Kenneth Leith	Madison, Wis.
Waldemar Lindgren	Brookline
Frederic Brewster Loomis	Amherst
Alexander George McAdie	Readville
John Campbell Merriam	Washington, D. C.
William John Miller	Northampton

Charles Palache . . . . Cambridge
Raphael Pumpelly . . . . Newport, R. I.
Percy Edward Raymond . . . . Cambridge
William North Rice . . . . Middletown, Conn.
Robert Wilcox Sayles . . . . Cambridge
Waldemar Theodore Schaller . . . . Washington, D. C.
Charles Schuchert . . . . New Haven, Conn.
William Berryman Scott . . . . Princeton, N. J.
Hervey Woodburn Shimer . . . . Watertown
Thomas Wayland Vaughan . . . . Washington, D. C.
Charles Doolittle Walcott . . . . Washington, D. C.
Robert DeCourcy Ward . . . . Cambridge
Charles Hyde Warren . . . . Auburndale
David White . . . . Washington, D. C.
Herbert Percy Whitlock . . . . New York, N. Y.
Bailey Willis . . . . Palo Alto, Cal.
Arthur Winslow . . . . Boston
John Eliot Wolff . . . . Cambridge
Jay Backus Woodworth . . . . Cambridge
Frederick Eugene Wright . . . . Washington, D. C.

Class II., Section II.—*Botany.*—33

Oakes Ames . . . . North Easton
Irving Widmer Bailey . . . . Cambridge
Liberty Hyde Bailey . . . . Ithaca, N. Y.
Edward Wilber Berry . . . . Baltimore, Md.
Douglas Houghton Campbell . . . . Palo Alto, Cal.
George Perkins Clinton . . . . New Haven, Conn.
John Merle Coulter . . . . Chicago, Ill.
Bradley Moore Davis . . . . Ann Arbor, Mich.
Edward Murray East . . . . Jamaica Plain
Rollins Adams Emerson . . . . Ithaca, N. Y.
Alexander William Evans . . . . New Haven, Conn.
Merritt Lyndon Fernald . . . . Cambridge
George Lincoln Goodale . . . . Cambridge
Robert Almer Harper . . . . New York, N. Y.

John George Jack	East Walpole
Edward Charles Jeffrey	Cambridge
Fred Dayton Lambert	Tufts College
Jacob Goodale Lipman	New Brunswick, N. J.
Burton Edward Livingston	Baltimore, Md.
George Richard Lyman	Washington, D. C.
Elmer Drew Merrill	Manila, P. I.
Winthrop John Vanleuven Osterhout	Cambridge
Charles Vancouver Piper	Washington, D. C.
Alfred Rehder	Jamaica Plain
Benjamin Lincoln Robinson	Cambridge
Charles Sprague Sargent	Brookline
William Albert Setchell	Berkeley, Cal.
Arthur Bliss Seymour	Cambridge
Erwin Frink Smith	Washington, D. C.
John Donnell Smith	Baltimore, Md.
William Codman Sturgis	New York, N. Y.
Roland Thaxter	Cambridge
William Trelease	Urbana, Ill.

Class II., Section III.— *Zoölogy and Physiology.*— 54.

Thomas Barbour	Cambridge
Francis Gano Benedict	Boston
Henry Bryant Bigelow	Concord
Robert Payne Bigelow	Brookline
William T. Bovie	Milton
John Lewis Bremer	Boston
Charles Thomas Brues	Boston
Hermon Carey Bumpus	Providence, R. I.
Walter Bradford Cannon	Cambridge
William Ernest Castle	Belmont
Charles Value Chapin	Providence, R. I.
Benjamin Preston Clark	Boston
Samuel Fessenden Clarke	Williamstown
Edwin Grant Conklin	Princeton, N. J.
William Thomas Councilman	Boston
Joseph Augustine Cushman	Sharon

William Healey Dall	Washington, D. C.
Charles Benedict Davenport	Cold Spring Harbor, N. Y.
Gilman Arthur Drew	Woods Hole
Cecil Kent Drinker	Boston
Alexander Forbes	Milton
Samuel Henshaw	Cambridge
Leland Ossian Howard	Washington, D. C.
Herbert Spencer Jennings	Baltimore, Md.
Charles Willison Johnson	Brookline
Charles Atwood Kofoid	Berkeley, Cal.
Frederic Thomas Lewis	Waban
Ralph Stayner Lillie	Worcester
Jacques Loeb	New York, N. Y.
Richard Swann Lull	New Haven, Conn.
Edward Laurens Mark	Cambridge
Ernest Gale Martin	Palo Alto, Cal.
Albert Davis Mead	Providenee, R. I.
Gerrit Smith Miller	Washington, D. C.
Edward Sylvester Morse	Salem
Herbert Vincent Neal	Tufts College
Henry Fairfield Osborn	New York, N. Y.
George Howard Parker	Cambridge
William Patten	Hanover, N. H.
Raymond Pearl	Baltimore, Md.
John Charles Phillips	Wenham
Herbert Wilbur Rand	Cambridge
William Emerson Ritter	La Jolla, Cal.
Percy Goldthwait Stiles	Newtonville
John Eliot Thayer	Laneaster
William Lyman Underwood	Belmont
Addison Emory Verrill	Whitneyville, Conn.
John Broadus Watson	Washington, D. C.
Arthur Wisswald Weysse	Boston
William Morton Wheeler	Boston
Harris Hawthorne Wilder	Northampton
Edmund Beecher Wilson	New York, N. Y.
Frederick Adams Woods	Brookline
Robert Mearns Yerkes	Washington, D. C.

CLASS II., SECTION IV.— *Medicine and Surgery.*—39.

Edward Hickling Bradford	Boston
Charles Macfie Campbell	Cambridge
Alexis Carrel	New York, N. Y.
Henry Asbury Christian	Boston
Rufus Cole	New York, N. Y.
Harvey Cushing	Boston
David Linn Edsall	Cambridge
Harold Clarence Ernst	Jamaica Plain
Simon Flexner	New York, N. Y.
Robert Battey Greenough	Boston
William Stewart Halsted	Baltimore, Md.
Ross Granville Harrison	New Haven, Conn.
William Henry Howell	Baltimore, Md.
Reid Hunt	Brookline
Henry Jackson	Boston
Elliott Proctor Joslin	Boston
William Williams Keen	Philadelphia, Pa.
Frank Burr Mallory	Brookline
William James Mayo	Rochester, Minn.
Samuel Jason Mixter	Boston
Edward Hall Nichols	Boston
Francis Weld Peabody	Brookline
Theophil Mitchell Prudden	New York, N. Y.
William Lambert Richardson	Boston
Milton Joseph Rosenau	Boston
Frederick Cheever Shattuck	Boston
Theobald Smith	Princeton, N. J.
Charles Wardell Stiles	Washington, D. C.
Richard Pearson Strong	Boston
William Sydney Thayer	Baltimore, Md.
Ernest Edward Tyzzer	Wakefield
Frederick Herman Verhoeff	Boston
Henry Pickering Walcott	Cambridge
John Warren	Boston
John Collins Warren	Boston
William Henry Welch	Baltimore, Md.

Francis Henry Williams . . . . Boston
Simeon Burt Wolbach . . . . Boston
Horatio Curtis Wood . . . . Philadelphia, Pa.

Class III.— *Moral and Political Sciences.*— 190.

Section I.— *Theology, Philosophy and Jurisprudence.*— 52.

Thomas Willing Balch . . . . Philadelphia, Pa.
Simeon Eben Baldwin . . . . New Haven, Conn.
Willard Bartlett . . . . Brooklyn, N. Y.
Joseph Henry Beale . . . . Cambridge
Charles Henry Brent . . . . Buffalo, N. Y.
Howard Nicholson Brown . . . . Boston
Charles Warren Clifford . . . . New Bedford
Edmund Burke Delabarre . . . . Providence, R. I.
James De Normandie . . . . Roxbury
Frederic Dodge . . . . Belmont
Edward Staples Drown . . . . Cambridge
William Harrison Dunbar . . . . Cambridge
William Herbert Perry Faunce . . . . Providence, R. I.
William Wallace Fenn . . . . Cambridge
Frederick Perry Fish . . . . Brookline
George Angier Gordon . . . . Boston
John Wilkes Hammond . . . . Cambridge
Alfred Hemenway . . . . Boston
William Ernest Hocking . . . . Cambridge
Charles Evans Hughes . . . . New York, N. Y.
Frederick John Foakes Jackson . . . . . . New York, N. Y.
Charles Francis Jenney . . . . Hyde Park
William Lawrence . . . . Boston
Frederick Lawton . . . . Boston
Arthur Lord . . . . Plymouth
William Caleb Loring . . . . Boston
Nathan Matthews . . . . Boston
Samuel Walker McCall . . . . Winchester
Edward Caldwell Moore . . . . Cambridge
John Bassett Moore . . . . New York, N. Y.

James Madison Morton . . . . Fall River
George Herbert Palmer . . . . Cambridge
Charles Edwards Park . . . . Boston
Leighton Parks . . . . New York, N. Y.
Francis Greenwood Peabody . . . . Cambridge
George Wharton Pepper . . . . Philadelphia, Pa.
Roscoe Pound . . . . Belmont
Elihu Root . . . . New York, N. Y.
James Hardy Ropes . . . . Cambridge
Arthur Prentice Rugg . . . . Worcester
Austin Wakeman Scott . . . . Cambridge
Henry Newton Sheldon . . . . Boston
Moorfield Storey . . . . Boston
William Howard Taft . . . . New Haven, Conn.
William Jewett Tucker . . . . Hanover, N. H.
William Cushing Wait . . . . Medford
Williston Walker . . . . New Haven, Conn.
Eugene Wambaugh . . . . Cambridge
Edward Henry Warren . . . . Brookline
Winslow Warren . . . . Dedham
Samuel Williston . . . . Belmont
Woodrow Wilson . . . . Washington, D. C.

Class III., Section II.— *Philology and Archæology.*— 56.

Francis Greenleaf Allinson . . . . Providence, R. I.
William Rosenzweig Arnold . . . . Cambridge
Maurice Bloomfield . . . . Baltimore, Md.
Franz Boas . . . . New York, N. Y.
Carl Darling Buck . . . . Chicago, Ill.
Eugene Watson Burlingame . . . . Albany, N. Y.
Edward Capps . . . . Princeton, N. J.
George Henry Chase . . . . Cambridge
Walter Eugene Clark . . . . Chicago, Ill.
Roland Burrage Dixon . . . . Cambridge
Franklin Edgerton . . . . Philadelphia, Pa.
William Curtis Farabee . . . . Philadelphia, Pa.
Jesse Walter Fewkes . . . . Washington, D. C.

Jeremiah Denis Mathias Ford . . . . Cambridge
Basil Lanneau Gildersleeve . . . . Baltimore, Md.
Pliny Earle Goddard . . . . New York, N. Y.
Charles Hall Grandgent . . . . Cambridge
Louis Herbert Gray . . . . New York, N. Y.
Charles Burton Gulick . . . . Cambridge
Roy Kenneth Hack . . . . Cambridge
William Arthur Heidel . . . . Middletown, Conn.
George Lincoln Hendrickson . . . . New Haven, Conn.
Bert Hodge Hill . . . . Athens, Greece
Elijah Clarence Hills . . . . Bloomington, Ind.
William Henry Holmes . . . . Washington, D. C.
Edward Washburn Hopkins . . . . New Haven, Conn.
Joseph Clark Hoppin . . . . Pomfret, Conn.
Albert Andrew Howard . . . . Cambridge
William Guild Howard . . . . Cambridge
Aleš Hrdlička . . . . Washington, D. C.
Eugene Xavier Louis Henry Hyvernat . . . . Washington, D. C.
Carl Newell Jackson . . . . Cambridge
Hans Carl Gunther von Jagemann . . . . Cambridge
James Richard Jewett . . . . Cambridge
Alfred Louis Kroeber . . . . Berkeley, Cal.
Kirsopp Lake . . . . Cambridge
Henry Roseman Lang . . . . New Haven, Conn.
Charles Rockwell Lanman . . . . Cambridge
John Livingston Lowes . . . . Cambridge
David Gordon Lyon . . . . Cambridge
Clifford Herschel Moore . . . . Cambridge
George Foot Moore . . . . Cambridge
Hanns Oertel . . . . New Haven, Conn.
Chandler Rathfon Post . . . . Cambridge
Edward Kennard Rand . . . . Cambridge
George Andrew Reisner . . . . Cambridge
Edward Robinson . . . . New York, N. Y.
Fred Norris Robinson . . . . Cambridge
Rudolph Schevill . . . . Berkeley, Cal.
Edward Stevens Sheldon . . . . Cambridge
Herbert Weir Smyth . . . . Cambridge

Franklin Bache Stephenson	Claremont, Cal.
Charles Cutler Torrey	New Haven, Conn.
Alfred Marston Tozzer	Cambridge
Clark Wissler	New York, N. Y.
James Haughton Woods	Cambridge

Class III., Section III.— *Political Economy and History.*— 41.

Wilbur Cortez Abbott	Cambridge
Brooks Adams	Quincy
George Burton Adams	New Haven, Conn.
Charles McLean Andrews	New Haven, Conn.
John Spencer Bassett	Northampton
Charles Jesse Bullock	Cambridge
Thomas Nixon Carver	Cambridge
John Bates Clark	New York, N. Y.
Archibald Cary Coolidge	Boston
Richard Henry Dana	Cambridge
Clive Day	New Haven, Conn.
Davis Rich Dewey	Cambridge
Ephraim Emerton	Cambridge
Henry Walcott Farnam	New Haven, Conn.
Max Farrand	New Haven, Conn.
William Scott Ferguson	Cambridge
Irving Fisher	New Haven, Conn.
Worthington Chauncey Ford	Cambridge
Edwin Francis Gay	New York, N. Y.
Frank Johnson Goodnow	Baltimore, Md.
Evarts Boutell Greene	Champaign, Ill.
Arthur Twining Hadley	New Haven, Conn.
Albert Bushnell Hart	Cambridge
Charles Homer Haskins	Cambridge
Isaac Minis Hays	Philadelphia, Pa.
Charles Downer Hazen	New York, N. Y.
Henry Cabot Lodge	Nahant
Abbott Lawrence Lowell	Cambridge
William MacDonald	New York, N. Y.
Charles Howard McIlwain	Cambridge

Roger Bigelow Merriman	Cambridge
Samuel Eliot Morison	Boston
William Bennett Munro	Cambridge
Charles Lemuel Nichols	Worcester
James Ford Rhodes	Boston
William Milligan Sloane	New York, N. Y.
John Osborne Sumner	Boston
Frank William Taussig	Cambridge
William Roscoe Thayer	Cambridge
Frederick Jackson Turner	Cambridge
George Grafton Wilson	Cambridge
George Parker Winship	Cambridge

Class III., Section IV.— *Literature and the Fine Arts.*— 41.

Irving Babbitt	Cambridge
George Pierce Baker	Cambridge
William Sturgis Bigelow	Boston
Le Baron Russell Briggs	Cambridge
Charles Allerton Coolidge	Boston
Ralph Adams Cram	Boston
Samuel McChord Crothers	Cambridge
Wilberforce Eames	New York, N. Y.
Henry Herbert Edes	Cambridge
Edward Waldo Emerson	Concord
Arthur Fairbanks	Cambridge
Frank Edgar Farley	Middleton, Conn.
Arthur Foote	Brookline
Edward Waldo Forbes	Cambridge
Kuno Francke	Gilbertsville, N. Y.
Daniel Chester French	New York, N. Y.
Horace Howard Furness	Philadelphia, Pa.
Robert Grant	Boston
Morris Gray	Boston
Chester Noyes Greenough	Cambridge
James Kendall Hosmer	Minneapolis, Minn.
Mark Antony DeWolfe Howe	Boston
Archer Milton Huntington	New York, N. Y.

George Lyman Kittredge . . . . Cambridge
William Coolidge Lane . . . . Cambridge
John Ellerton Lodge . . . . Boston
Charles Donagh Maginnis . . . . Brookline
Allan Marquand . . . . Princeton, N. J.
Albert Matthews . . . . Boston
Harold Murdock . . . . Brookline
William Allan Neilson . . . . Northampton
Thomas Nelson Page . . . . Washington, D. C.
William Lyon Phelps . . . . New Haven, Conn.
Herbert Putnam . . . . Washington, D. C.
Denman Waldo Ross . . . . Cambridge
John Singer Sargent . . . . London, Eng.
Ellery Sedgwick . . . . Boston
Henry Dwight Sedgwick . . . . Cambridge
Richard Clipston Sturgis . . . . Boston
Charles Howard Walker . . . . Boston
Owen Wister . . . . Philadelphia, Pa.
George Edward Woodberry . . . . Beverly

FOREIGN HONORARY MEMBERS.—69.

(Number limited to seventy-five.)

Class I.— *Mathematical and Physical Sciences.*— 23.

Section I.— *Mathematics and Astronomy.*— 7.

Johann Oskar Backlund	Petrograd
Godfrey Harold Hardy	Oxford
Jacques Salomon Hadamard	Paris
Felix Klein	Göttingen
Tullio Levi-Civita	Rome
Charles Emile Picard	Paris
Charles Jean de la Vallée Poussin	Louvain

Class I., Section II.— *Physics.*— 7.

Svante August Arrhenius	Stockholm
Oliver Heaviside	Torquay
Sir Joseph Larmor	Cambridge
Hendrik Antoon Lorentz	Leyden
Max Planck	Berlin
Sir Ernest Rutherford	Manchester
Sir Joseph John Thomson	Cambridge

Class I., Section III.— *Chemistry.*— 4.

Fritz Haber	Berlin
Henri Louis Le Chatelier	Paris
Wilhelm Ostwald	Leipsic
William Henry Perkin	Oxford

Class I.— Section IV.— *Technology and Engineering.*— 5.

Heinrich Müller Breslau	Berlin
Ferdinand Foch	Paris
Joseph Jacques Césaire Joffre	Paris
Vsevolod Jevgenjevic Timonoff	Petrograd
William Cawthorne Unwin	London

Class II.— *Natural and Physiological Sciences.*— 21.

Section I.— *Geology, Mineralogy, and Physics of the Globe.*— 10.

Frank Dawson Adams . . . . Montreal
Charles Barrois . . . . Lille
Waldemar Christofer Brögger . . . . Christiania
Sir Archibald Geikie . . . . Haslemere, Surrey
Viktor Goldschmidt . . . . Heidelberg
Julius Hann . . . . Vienna
Albert Heim . . . . Zürich
Gustaf Adolf Frederik Molengraaff . . . . Delft
Sir William Napier Shaw . . . . London
Johan Herman Lie Vogt . . . . Trondhjem

Class II, Section II.— *Botany.*— 5.

John Briquet . . . . Geneva
Hugo de Vries . . . . Lunteren
Adolf Engler . . . . Berlin
Ignatz Urban . . . . Berlin
Eugene Warming . . . . Copenhagen

Class II.— Section III.— *Zoölogy and Physiology.*— 3.

Maurice Caullery . . . . Paris
Sir Edwin Ray Lankester . . . . London
George Henry Falkiner Nuttall . . . . Cambridge

Class II., Section IV.— *Medicine and Surgery.*— 3.

Sir Thomas Barlow, Bart . . . . London
Francis John Shepherd . . . . Montreal
Charles Scott Sherrington . . . . Oxford

Class III.— *Moral and Political Sciences.*— 25.

Section I.— *Theology, Philosophy and Jurisprudence.*— 5.

Rt. Hon Arthur James Balfour . . . . Prestonkirk
Heinrich Brunner . . . . Berlin

Albert Venn Dicey . . . . Oxford
Raymond Poincaré . . . . Paris
Rt. Hon. Sir Frederick Pollock, Bart. . . . . London

SECTION II.— *Philology and Archæology.*— 9.

Friedrich Delitzsch . . . . Berlin
Hermann Diels . . . . Berlin
Wilhelm Dörpfeld . . . . Athens
Henry Jackson . . . . Cambridge
Hermann Georg Jacobi . . . . Bonn
Arthur Anthony Macdonell . . . . Oxford
Alfred Percival Maudslay . . . . Hereford
Ramon Menendez Pidal . . . . Madrid
Eduard Seler . . . . Berlin

SECTION III.— *Political Economy and History.*— 6.

Rt. Hon. James Bryce, Viscount Bryce . . . . London
Adolf Harnack . . . . Berlin
Alfred Marshall . . . . Cambridge
Rt. Hon. John Morley, Viscount Morley of Blackburn . . . London
George Walter Prothero . . . . London
Rt. Hon. Sir George Otto Trevelyan, Bart. . . . . London

SECTION IV.— *Literature and the Fine Arts.*— 5.

Georg Brandes . . . . Copenhagen
Thomas Hardy . . . . Dorchester
Jean Adrien Antoine Jules Jusserand . . . . Paris
Rudyard Kipling . . . . Burwash
Sir Sidney Lee . . . . London

STATUTES AND STANDING VOTES

STATUTES

Adopted November 8, 1911: *amended May* 8, 1912, *January* 8, *and May* 14, 1913, *April* 14, 1915, *April* 12, 1916, *April* 10, 1918, *May* 14, 1919.

CHAPTER I

The Corporate Seal

Article 1. The Corporate Seal of the Academy shall be as here depicted:

Article 2. The Recording Secretary shall have the custody of the Corporate Seal.

See Chap. v. art. 3; chap. vi. art. 2.

CHAPTER II

Fellows and Foreign Honorary Members and Dues

Article 1. The Academy consists of Fellows, who are either citizens or residents of the United States of America, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, and each Class is divided into four Sections, namely:

Class I. *The Mathematical and Physical Sciences*
- Section 1. Mathematics and Astronomy
- Section 2. Physics
- Section 3. Chemistry
- Section 4. Technology and Engineering

Class II. *The Natural and Physiological Sciences*
- Section 1. Geology, Mineralogy, and Physics of the Globe
- Section 2. Botany
- Section 3. Zoölogy and Physiology
- Section 4. Medicine and Surgery

Class III. *The Moral and Political Sciences*
- Section 1. Theology, Philosophy, and Jurisprudence
- Section 2. Philology and Archaeology
- Section 3. Political Economy and History
- Section 4. Literature and the Fine Arts

Article 2. The number of Fellows shall not exceed Six hundred, of whom not more than Four hundred shall be residents of Massachusetts, nor shall there be more than Two hundred in any one Class.

Article 3. The number of Foreign Honorary Members shall not exceed Seventy-five. They shall be chosen from among citizens of foreign countries most eminent for their discoveries and attainments in any of the Classes above enumerated. There shall not be more than Twenty-five in any one Class.

Article 4. If any person, after being notified of his election as Fellow or Resident Associate, shall neglect for six months to accept in writing, or, if a Fellow resident within fifty miles of Boston shall neglect to pay his Admission Fee, his election shall be void; and if any Fellow resident within fifty miles of Boston or any Resident Associate shall neglect to pay his Annual Dues for six months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow or Resident Associate respectively; but the Council may suspend the provisions of this Article for a reasonable time.

With the previous consent of the Council, the Treasurer may dispense (*sub silentio*) with the payment of the Admission Fee or of the Annual Dues or both whenever he shall deem it advisable. In the case of officers of the Army or Navy who are out of the Commonwealth on duty, payment of the Annual Dues may be waived during such absence if continued during the whole financial year and if notification of such expected absence be sent to the Treasurer. Upon similar notification to the Treasurer, similar exemption may be accorded to Fellows or Resident Associates subject to Annual Dues, who may temporarily remove their residence for at least two years to a place more than fifty miles from Boston.

If any person elected a Foreign Honorary Member shall neglect for six months after being notified of his election to accept in writing, his election shall be void.

See Chap. vii. art. 2.

ARTICLE 5. Every Fellow resident within fifty miles of Boston hereafter elected shall pay an Admission Fee of Ten dollars.

Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding Fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when they shall become due; but any Fellow or Resident Associate shall be exempt from the annual payment if, at any time after his admission, he shall pay into the treasury Two hundred dollars in addition to his previous payments.

All Commutations of the Annual Dues shall be and remain permanently funded, the interest only to be used for current expenses.

Any Fellow not previously subject to Annual Dues who takes up his residence within fifty miles of Boston, shall pay to the Treasurer within three months thereafter Annual Dues for the current year, failing which his Fellowship shall cease; but the Council may suspend the provisions of this Article for a reasonable time.

Only Fellows who pay Annual Dues or have commuted them may hold office in the Academy or serve on the Standing Committees or vote at meetings.

Article 6. Fellows who pay or have commuted the Annual Dues and Foreign Honorary Members shall be entitled to receive gratis one copy of all Publications of the Academy issued after their election.

See Chap. x, art. 2.

Article 7. Diplomas signed by the President and the Vice-President of the Class to which the member belongs, and countersigned by the Secretaries, shall be given to Foreign Honorary Members and to Fellows on request.

Article 8. If, in the opinion of a majority of the entire Council, any Fellow or Foreign Honorary Member or Resident Associate shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his membership; and if three fourths of the Fellows present, out of a total attendance of not less than fifty at a Stated Meeting, or at a Special Meeting called for the purpose, shall adopt this recommendation, his name shall be stricken from the Roll.

See Chap. iii.; chap. vi. art. 1; chap. ix, art. 1, 7; chap. x. art. 2.

CHAPTER III

Election of Fellows and Foreign Honorary Members

Article 1. Elections of Fellows and Foreign Honorary Members shall be made by the Council in April of each year, and announced at the Annual Meeting in May.

Article 2. Nominations to Fellowship or Foreign Honorary Membership in any Section must be signed by two Fellows of that Section or by three voting Fellows of any Sections; but in any one year no Fellow may nominate more than four persons. These nominations, with statements of qualifications and brief biographical data, shall be sent to the Corresponding Secretary.

All nominations thus received prior to February 15 shall be forthwith sent in printed form to every Fellow having the right to vote, with the names of the proposers in each case and a brief account of each nominee, and with the request that the list be returned before March 15, marked to indicate preferences of the voter in such manner as the Council may direct.

All the nominations, with any comments thereon and with the results of the preferential indications of the Fellows, received by March 15, shall be referred at once to the appropriate Class Committees, which shall report their decisions to the Council, which shall thereupon have power to elect.

Persons nominated in any year, but not elected, may be placed on the preferential ballot of the next year at the discretion of the Council, but shall not further be continued on the list of nominees unless renominated.

Notice shall be sent to every Fellow having the right to vote, not later than the fifteenth of January, of each year, calling attention to the fact that the limit of time for sending nominations to the Corresponding Secretary will expire on the fifteenth of February.

See Chap. ii.; chap. vi. art. 1; chap. ix. art. 1.

CHAPTER IV

Officers

Article 1. The Officers of the Academy shall be a President (who shall be Chairman of the Council), three Vice-Presidents (one from each Class), a Corresponding Secretary (who shall be Secretary of the Council), a Recording Secretary, a Treasurer, and a Librarian, all of whom shall be elected by ballot at the Annual Meeting, and shall hold their respective offices for one year, and until others are duly chosen and installed.

There shall be also twelve Councillors, one from each Section of each Class. At each Annual Meeting three Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms.

The Councillors, with the other officers previously named, and the Chairman of the House Committee, *ex officio*, shall constitute the Council.

See Chap. x, art. 1.

Article 2. If any officer be unable, through death, absence, or disability, to fulfil the duties of his office, or if he shall resign, his place may be filled by the Council in its discretion for any part or the whole of the unexpired term.

Article 3. At the Stated Meeting in March, the President shall appoint a Nominating Committee of three Fellows having the right to vote, one from each Class. This Committee shall prepare a list of nominees for the several offices to be filled, and for the Standing Committees, and file it with the Recording Secretary not later than four weeks before the Annual Meeting.

See Chap. vi. art. 2.

Article 4. Independent nominations for any office, if signed by at least twenty Fellows having the right to vote, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call therefor, and shall be mailed to all the Fellows having the right to vote.

See Chap. vi. art. 2.

Article 5. The Recording Secretary shall prepare for use in voting at the Annual Meeting a ballot containing the names of all persons duly nominated for office.

CHAPTER V

The President

Article 1. The President, or in his absence the senior Vice-President present (seniority to be determined by length of continuous fellowship in the Academy), shall preside at all meetings of the Academy. In the absence of all these officers, a Chairman of the meeting shall be chosen by ballot.

ARTICLE 2. Unless otherwise ordered, all Committees which are not elected by ballot shall be appointed by the presiding officer.

ARTICLE 3. Any deed or writing to which the Corporate Seal is to be affixed, except leases of real estate, shall be executed in the name of the Academy by the President or, in the event of his death, absence, or inability, by one of the Vice-Presidents, when thereto duly authorized.

See Chap. ii. art. 7; chap. iv. art. 1, 3; chap. vi. art. 2; chap. vii. art. 1; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1.

CHAPTER VI

THE SECRETARIES

ARTICLE 1. The Corresponding Secretary shall conduct the correspondence of the Academy and of the Council, recording or making an entry of all letters written in its name, and preserving for the files all official papers which may be received. At each meeting of the Council he shall present the communications addressed to the Academy which have been received since the previous meeting, and at the next meeting of the Academy he shall present such as the Council may determine.

He shall notify all persons who may be elected Fellows or Foreign Honorary Members, or Resident Associates, send to each a copy of the Statutes, and on their acceptance issue the proper Diploma. He shall also notify all meetings of the Council; and in case of the death, absence, or inability of the Recording Secretary he shall notify all meetings of the Academy.

Under the direction of the Council, he shall keep a List of the Fellows, Foreign Honorary Members, and Resident Associates, arranged in their several Classes and Sections. It shall be printed annually and issued as of the first day of July.

See Chap. ii. art. 7; chap. iii. art. 2, 3; chap. iv. art. 1; chap. ix. art. 6; chap. x. art. 1; chap. xi. art. 1.

ARTICLE 2. The Recording Secretary shall have the custody of the Charter, Corporate Seal, Archives, Statute-Book, Journals, and all literary papers belonging to the Academy.

Fellows or Resident Associates borrowing such papers or documents shall receipt for them to their custodian.

The Recording Secretary shall attend the meetings of the Academy and keep a faithful record of the proceedings with the names of the Fellows and Resident Associates present; and after each meeting is duly opened, he shall read the record of the preceding meeting.

He shall notify the meetings of the Academy to each Fellow and Resident Associate by mail at least seven days beforehand, and in his discretion may also cause the meetings to be advertised; he shall apprise Officers and Committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy.

After all elections, he shall insert in the Records the names of the Fellows by whom the successful nominees were proposed.

He shall send the Report of the Nominating Committee in print to every Fellow having the right to vote at least three weeks before the Annual Meeting.

See Chap. iv. art. 3.

In the absence of the President and of the Vice-Presidents he shall, if present, call the meeting to order, and preside until a Chairman is chosen.

See Chap. i.; chap. ii. art. 7; chap. iv. art. 3, 4, 5; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1, 3.

Article 3. The Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them likely to promote its interests.

CHAPTER VII

The Treasurer and the Treasury

Article 1. The Treasurer shall collect all money due or payable to the Academy, and all gifts and bequests made to it. He shall pay all bills due by the Academy, when approved by the proper officers, except those of the Treasurer's office, which may be paid without such approval; in the name of the Academy he shall sign all leases of real estate; and, with the written consent of a member of the Committee on Finance, he shall make all transfers of stocks, bonds, and other securities belonging to the Academy, all of which shall be in his official custody.

He shall keep a faithful account of all receipts and expenditures, submit his accounts annually to the Auditing Committee, and render them at the expiration of his term of office, or whenever required to do so by the Academy or the Council.

He shall keep separate accounts of the income of the Rumford Fund, and of all other special Funds, and of the appropriation thereof, and render them annually.

His accounts shall always be open to the inspection of the Council.

ARTICLE 2. He shall report annually to the Council at its March meeting on the expected income of the various Funds and from all other sources during the ensuing financial year. He shall also report the names of all Fellows and Resident Associates who may be then delinquent in the payment of their Annual Dues.

ARTICLE 3. He shall give such security for the trust reposed in him as the Academy may require.

ARTICLE 4. With the approval of a majority of the Committee on Finance, he may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, he shall be responsible; or, with like approval and responsibility, he may employ any Trust Company doing business in Boston as his agent for the same purpose, the compensation of such Assistant Treasurer or agent to be fixed by the Committee on Finance and paid from the funds of the Academy.

ARTICLE 5. At the Annual Meeting he shall report in print all his official doings for the preceding year, stating the amount and condition of all the property of the Academy entrusted to him, and the character of the investments.

ARTICLE 6. The Financial Year of the Academy shall begin with the first day of April.

ARTICLE 7. No person or committee shall incur any debt or liability in the name of the Academy, unless in accordance with a previous vote and appropriation therefor by the Academy or the Council, or sell or otherwise dispose of any property of the Academy, except cash or invested funds, without the previous consent and approval of the Council.

See Chap. ii. art. 4, 5; chap. vi. art. 2; chap. ix. art. 6; chap. x. art. 1, 2, 3; chap. xi. art. 1.

CHAPTER VIII

The Librarian and the Library.

Article 1. The Librarian shall have charge of the printed books, keep a correct catalogue thereof, and provide for their delivery from the Library.

At the Annual Meeting, as Chairman of the Committee on the Library, he shall make a Report on its condition.

Article 2. In conjunction with the Committee on the Library he shall have authority to expend such sums as may be appropriated by the Academy for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

Article 3. All books procured from the income of the Rumford Fund or of other special Funds shall contain a book-plate expressing the fact.

Article 4. Books taken from the Library shall be receipted for to the Librarian or his assistant.

Article 5. Books shall be returned in good order, regard being had to necessary wear with good usage. If any book shall be lost or injured, the Fellow or Resident Associate to whom it stands charged shall replace it by a new volume or by a new set, if it belongs to a set, or pay the current price thereof to the Librarian, whereupon the remainder of the set, if any, shall be delivered to the Fellow or Resident Associate so paying, unless such remainder be valuable by reason of association.

Article 6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

Article 7. The Librarian shall have the custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

See Chap. ii. art. 6; chap. x. art. 1, 2.

CHAPTER IX

The Council

Article 1. The Council shall exercise a discreet supervision over all nominations and elections to membership, and in general supervise all the affairs of the Academy not explicitly reserved to the Academy as a whole or entrusted by it or by the Statutes to standing or special committees.

It shall consider all nominations duly sent to it by any Class Committee, and act upon them in accordance with the provisions of Chapter III.

With the consent of the Fellow interested, it shall have power to make transfers between the several Sections of the same Class, reporting its action to the Academy.

See Chap. iii. art. 2, 3; chap. x, art. 1.

Article 2. Seven members shall constitute a quorum.

Article 3. It shall establish rules and regulations for the transaction of its business, and provide all printed and engraved blanks and books of record.

Article 4. It shall act upon all resignations of officers, and all resignations and forfeitures of Fellowship or Resident Associateship; and cause the Statutes to be faithfully executed.

It shall appoint all agents and subordinates not otherwise provided for by the Statutes, prescribe their duties, and fix their compensation. They shall hold their respective positions during the pleasure of the Council.

Article 5. It may appoint, for terms not exceeding one year, and prescribe the functions of, such committees of its number, or of the Fellows of the Academy, as it may deem expedient, to facilitate the administration of the affairs of the Academy or to promote its interests.

Article 6. At its March meeting it shall receive reports from the President, the Secretaries, the Treasurer, and the Standing Committees, on the appropriations severally needed for the ensuing financial year. At the same meeting the Treasurer shall report on the expected income of the various Funds and from all other sources during the same year.

A report from the Council shall be submitted to the Academy, for action, at the March meeting, recommending the appropriation which in the opinion of the Council should be made.

On the recommendation of the Council, special appropriations may be made at any Stated Meeting of the Academy, or at a Special Meeting called for the purpose.

See Chap. x. art. 3.

Article 7. After the death of a Fellow or Foreign Honorary Member, it shall appoint a member of the Academy to prepare a biographical notice for publication in the Proceedings.

Article 8. It shall report at every meeting of the Academy such business as it may deem advisable to present.

See Chap. ii. art. 4, 5, 8; chap. iv. art. 1, 2; chap. vi. art. 1; chap. vii. art. 1; chap. xi. art. 1, 4.

CHAPTER X.

Standing Committees

Article 1. The Class Committee of each Class shall consist of the Vice-President, who shall be chairman, and the four Councillors of the Class, together with such other officer or officers annually elected as may belong to the Class. It shall consider nominations to Fellowship in its own Class, and report in writing to the Council such as may receive at a Class Committee Meeting a majority of the votes cast, provided at least three shall have been in the affirmative.

See Chap. iii. art. 2.

Article 2. At the Annual Meeting the following Standing Committees shall be elected by ballot to serve for the ensuing year:

(i) *The Committee on Finance,* to consist of three Fellows, who, through the Treasurer, shall have full control and management of the funds and trusts of the Academy, with the power of investing the funds and of changing the investments thereof in their discretion.

See Chap. iv. art. 3; chap. vii. art. 1, 4; chap. ix. art. 6.

(ii) *The Rumford Committee,* to consist of seven Fellows, who shall report to the Academy on all applications and claims for the

Rumford Premium. It alone shall authorize the purchase of books publications and apparatus at the charge of the income from the Rumford Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iii) *The Cyrus Moors Warren Committee,* to consist of seven Fellows, who shall consider all applications for appropriations from the income of the Cyrus Moors Warren Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iv) *The Committee of Publication,* to consist of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and the Memoirs shall be entrusted.

It shall fix the price at which the Publications shall be sold; but Fellows may be supplied at half price with volumes which may be needed to complete their sets, but which they are not entitled to receive gratis.

Two hundred extra copies of each paper accepted for publication in the Proceedings or the Memoirs shall be placed at the disposal of the author without charge.

See Chap. iv. art. 3; chap. vi. art. 1, 3; chap. ix. art. 6.

(v) *The Committee on the Library,* to consist of the Librarian, *ex officio,* as Chairman, and three other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

See Chap. iv. art. 3; chap. viii. art. 1, 2,; chap. ix. art. 6.

(vi) *The House Committee,* to consist of three Fellows, who shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to the care of other Committees or Officers.

See Chap. iv. art. 1, 3; chap. ix. art. 6.

(vii) *The Committee on Meetings,* to consist of the President, the Recording Secretary, and three other Fellows, who shall have charge of plans for meetings of the Academy.

See Chap. iv. art. 3; chap. ix. art. 6.

(viii) *The Auditing Committee,* to consist of two Fellows, who shall audit the accounts of the Treasurer, with power to employ an expert and to approve his bill.

See Chap. iv. art. 3; chap. vii. art. 1; chap. ix. art. 6.

ARTICLE 3. The Standing Committees shall report annually to the Council in March on the appropriations severally needed for the ensuing financial year; and all bills incurred on account of these Committees, within the limits of the several appropriations made by the Academy, shall be approved by their respective Chairmen.

In the absence of the Chairman of any Committee, bills may be approved by any member of the Committee whom he shall designate for the purpose.

See Chap. vii. art. 1, 7; chap. ix. art. 6.

CHAPTER XI

MEETINGS, COMMUNICATIONS, AND AMENDMENTS

ARTICLE 1. There shall be annually eight Stated Meetings of the Academy, namely, on the second Wednesday of October, November, December, January, February, March, April and May. Only at these meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made or amendments of the Statutes or Standing Votes be effected.

The Stated Meeting in May shall be the Annual Meeting of the Corporation.

Special Meetings shall be called by either of the Secretaries at the request of the President, of a Vice-President, of the Council, or of ten Fellows having the right to vote; and notifications thereof shall state the purpose for which the meeting is called.

A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August, and September; but no business shall be transacted at said meetings.

ARTICLE 2. Twenty Fellows having the right to vote shall constitute a quorum for the transaction of business at Stated or Special

Meetings. Fifteen Fellows shall be sufficient to constitute a meeting for literary or scientific communications and discussions.

Article 3. Upon the request of the presiding officer or the Recording Secretary, any motion or resolution offered at any meeting shall be submitted in writing.

Article 4. No report of any paper presented at a meeting of the Academy shall be published by any Fellow or Resident Associate without the consent of the author; and no report shall in any case be published by any Fellow or Resident Associate in a newspaper as an account of the proceedings of the Academy without the previous consent and approval of the Council. The Council, in its discretion, by a duly recorded vote, may delegate its authority in this regard to one or more of its members.

Article 5. No Fellow or Resident Associate shall introduce a guest at any meeting of the Academy until after the business has been transacted, and especially until after the result of the balloting upon nominations has been declared.

Article 6. The Academy shall not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its Publications.

Article 7. All proposed Amendments of the Statutes shall be referred to a committee, and on its report, at a subsequent Stated Meeting or at a Special Meeting called for the purpose, two thirds of the ballot cast, and not less than twenty, must be affirmative to effect enactment.

Article 8. Standing Votes may be passed, amended, or rescinded at a Stated Meeting, or at a Special Meeting called for the purpose, by a vote of two thirds of the members present. They may be suspended by a unanimous vote.

See Chap. ii. art. 5, 8; chap. iii.; chap. iv. art. 3, 4, 5; chap. v. art. 1; chap. vi. art. 1, 2; chap. ix. art. 8.

STANDING VOTES

1. Communications of which notice has been given to either of the Secretaries shall take precedence of those not so notified.

2. Fellows or Resident Associates may take from the Library six volumes at any one time, and may retain them for three months, and no longer. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

3. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy without the leave of the Librarian.

4. There may be chosen by the Academy, under such rules as the Council may determine, one hundred Resident Associates. Not more than forty Resident Associates shall be chosen in any one Class.

Resident Associates shall be entitled to the same privileges as Fellows, in the use of the Academy building, may attend meetings and present papers, but they shall not have the right to vote. They shall pay no Admission Fee, and their Annual Dues shall be the same as those of Fellows residing within fifty miles of Boston.

The Council and Committees of the Academy may ask one or more Resident Associates to act with them in an advisory or assistant capacity.

5. Communications offered for publication in the Proceedings or Memoirs of the Academy shall not be accepted for publication before the author shall have informed the Committee on Meetings of his readiness, either himself or through some agent, to use such time as the Committee may assign him at such meeting as may be convenient both to him and to the Committee, for the purpose of presenting to the Academy a general statement of the nature and significance of the results contained in his communication.

RUMFORD PREMIUM

In conformity with the terms of the gift of Sir Benjamin Thompson, Count Rumford, of a certain Fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court of Massachusetts for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of the Rumford Fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a Premium to the author of any important discovery or useful improvement in light or heat, which shall have been made and pubished by printing, or in any way made known to the public, in any part of the continent of America, or any of the American Islands; preference always being given to such discoveries as, in the opinion of the Academy, shall tend most to promote the good of mankind; and, if the Academy sees fit, to add to such medals, as a further Premium for such discovery and improvement, a sum of money not exceeding three hundred dollars.

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