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Production with Quality Differentiated Inputs

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Production with Quality Differentiated Inputs

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ABSTRACT

The paper concerns production theory when some inputs are quality differentiated. Our approach is to marry hedonic theory and the duality theory of cost functions. We then apply the theory to the case of coal-fired electric power generation where fuel quality depends on sulfur and ash impurities. Environmental regulations induce a negative value on sulfur whereas ash impurities degrade performance and thus reduce production possibilities. A number of empirical results emerge including a fairly elastic demand for sulfur and significant rates of technical change that are sulfur and ash saving though capital using.

I. INTRODUCTION

Production theory typically involves a finite set of distinct and well defined inputs. Duality theory is well developed in this situation and involves a cost function with as arguments a finite set of prices corresponding to the inputs. This paper concerns the situation where some inputs are differentiated by quality; in essence there are an infinite set of possible inputs corresponding to different quality levels. Firms choose not only quantities of inputs but quality levels as well. This situation cannot be handled simply by making costs a function of quality since the price of all quality levels are simultaneously considered by the firm when choosing the optimal quality.

Examples of quality differentiated inputs into production are legion. Basic metal manufacturing chooses among different ore grades; electric power producers choose among different fuel quality levels; manufacturing industries face choices regarding the quality of the labor inputs. In fact an input into production that is <u>not</u> quality-differentiated would seem to be the exception rather than the rule.

The traditional approach to quality differentiation is to deal with an hedonic price function, parameterized by quality. The derivative of the hedonic price function with respect to the quality parameter gives the marginal valuation of quality, the "price" of quality. However, as pointed out by McConnell and Phipps (1987) among others, the appropriate parameter is not the "price" of quality but the entire price function; firms choose the optimal quality level taking into account the entire price function. The implication is that the cost function has as a "parameter" the entire price function, or some summary measure of that function.

This paper is divided into two parts. In the next section we modify conventional production theory to account for quality differentiated inputs. This involves two issues. One is the modification of production and cost functions to include differentiated inputs and development of the associated curvature and monotonicity conditions. The second issue concerns defining the price functions for these inputs. This entails an extension of hedonic price theory.

The second part of the paper concerns an application of the theory to estimating the technology of electric power generation. Coal is used as a power generation fuel and differs greatly in terms of quality. Fuel quality affects plant performance as well as emissions of regulated pollutants. We examine the coal-fired power plants licensed between 1971 and 1979 in the U.S.; these plants were subject to an emission limit as their only regulation of sulfur output. In this empirical analysis, we interpret the estimated cost function in terms of substitution between positive and negative inputs, scale effects and technical change.

II. THEORY

There are two basic situations that induces a firm to place different valuations on different quality levels of the same product. The most common situation is that a higher quality input reduces input requirements, <u>ceteris paribus</u>. For instance, higher quality labor, while more costly, allows less labor and/or other inputs to be used to obtain the same output level. Or, higher quality coal reduces expenditures on pollution control equipment. Alternately, higher input quality permits the same inputs to be used to yield more positively valued output.

A. Production

The basic situation we will consider is a production technology involving a vector of outputs, $\mathbf{y}$. These outputs may be desirable (e.g., electricity) or undesirable (e.g., smoke). Inputs will be assumed to be conventional goods, x, except for one input, q, which is available in a variety of qualities (possibly vector valued), $\mathbf{z}$. Without loss of generality, let x be a composite and thus a scalar. Letting t denote the state of technology, the production set can be expressed implicitly as

$$g(\boldsymbol{x}, q, \boldsymbol{z}, \boldsymbol{y}, t) \leq 0 \tag{1}$$

with the frontier defined when (1) holds with equality. Assume g is quasi-convex;[1] i.e., the level sets (defined when the right-hand side of (1) is replaced by any constant) are convex.

Producers face a single price for the inputs x, p_x. For the differentiated input q, producers face an entire nonlinear price function $\rho(\mathbf{z};\alpha)$ where $\mathbf{z}$ is the vector of qualities and α is a vector of parameters of the hedonic price function. The reason for parameterizing ρ by α is that if there are multiple markets, there will most likely be multiple distinct hedonic price functions. This is problematic. What

[1]It is well known (e.g., Starrett, 1971) that production functions involving externalities may involve nonconvexities. We assume all operations are in the convex region.

we want is a single function that yields these different hedonic price functions by varying the parameter α. In fact, if a single market is under consideration, then α is constant and can be suppressed. However, if multiple markets are of concern, with multiple equilibrium price functions, α allows us to distinguish among them (see McConnell and Phipps, 1987).

The producer's problem is to solve

$$\begin{aligned} C(p_x, \boldsymbol{\alpha}, \boldsymbol{y}, t) = \min_{q, \boldsymbol{z}, x} \; & q\rho(\boldsymbol{z}, \boldsymbol{\alpha}) + p_x x \\ s.t. \; & g(x, q, \boldsymbol{z}, \boldsymbol{y}, t) \le 0 \\ & q,\ \boldsymbol{z},\ x \ge 0. \end{aligned} \tag{2}$$

This minimization defines an optimal value function giving the minimum cost of producing **y**. Note that as is conventional, all quantities are non-negative. The constraint set for (2) is convex (since g is quasi-convex). If ρ is convex in $(\mathbf{z},\alpha)$, then the objective function in (2) is convex, and thus C is concave in prices and α over the region where solutions to (2) exist (Mangasarian and Rosen, 1964).

First-order conditions for the solution of (2) are

$$\rho(z, \alpha) = p_x \frac{g_q}{g_x} \tag{3a}$$

$$\nabla_z \rho = \frac{p_x}{q} \frac{\nabla_z g}{g_x} \tag{3b}$$

along with the constraints of (2). The left-hand side of (3a) is the hedonic price function and of (3b) the derivative of the hedonic price function. The right-hand side of (3a) is the technical rate of substitution between q and x while for (3b) it is the same with respect to z and x. Equation (3b) represents equality between the marginal prices of quality and the marginal rate of transformation. Effectively, x is the numeraire good as equation (3) is written. If q is exogenous (q is frequently assumed to be unity in hedonic models), then (3a) may be suppressed.

The conventional approach is to estimate (3), possibly in conjunction with the hedonic price function itself. Focusing on equation (3b), both sides of the equation involve endogenous variables. In fact, it has been pointed out by a number of authors that generally there may be problems in identifying all of the parameters in equation (3) (see McConnell and Phipps, 1987; Bartik, 1987; Epple, 1987).

Demand functions or their equivalent can be derived using the envelope theorem:

$$\frac{\partial C}{\partial P_x} = x* \tag{4a}$$

$$\nabla_\alpha C = q*\nabla_\alpha \rho(\mathbf{z}*, \alpha) \tag{4b}$$

where x*, q* and **z*** indicate the optimal choices of x, q and **z** derived from equation (2). Equation (4a) is a demand function as written. Equation (4b) implicitly defines the demand for q and **z** as a function of (p_x, α, y, t), although there may be some redundancy in (4b) if there are more α's than elements of (q,z).

Note that while the production function, g, does not contain any parameters, α, of the hedonic price function, the cost function (2) does. It is appropriate to ask why it is necessary to carry the "baggage" of the entire price function, in terms of α, in order to represent costs and demands. In his 1974 article on hedonic prices, Rosen argues that marginal prices ($\nabla_z \rho$) are sufficient to parameterize demand for a differentiated product. However, that discussion applied to a single equilibrium; i.e., a single market. Thus α is constant. It is easy to see that simply knowing marginal prices at an equilibrium is insufficient to recover costs and demand if α is not known. The only exception to this is when ρ is linear in which case $\nabla_z \rho$ does not depend on α.

B. The Hedonic Price Function

We now turn to the hedonic price function, representing the price of the differentiated input as a function of quality. While at one level this is a simple concept, by adding some structure to the problem, we can derive curvature and other restrictions on the price function (at least for some cases).

The general framework we consider is as outlined above where the differentiated input has quality $\mathbf{z}$ and the unit price of the differentiated input is given by $\rho(\mathbf{z};\alpha)$. As pointed out by Rosen (1974), in many cases nothing more can be said about $\rho(\mathbf{z};\alpha)$ except that it is monotonic in $\mathbf{z}$, provided $\mathbf{z}$ is properly defined. He argues that it is generally appropriate to exclude arbitrage among characteristics. It thus becomes impossible to impose restrictions on the curvature of the price function. Specifically, he excludes untying (two average quality employees cannot be untied to yield one high quality and one low quality employee). However, it turns out that repackaging is sometimes plausible. While it is not appropriate to repackage a low quality and a high quality employee as two medium quality employees, a pound of high sulfur coal can be repackaged with a pound of low sulfur coal to yield two pounds of medium sulfur coal.

This illustrates an entire class of quality differentiated inputs that arises when the quality of an input is derived from undesirable impurities. The basic problem is that the "good" aspects of the input are bundled with the "bad" aspects of the input and they cannot be costlessly unbundled. For instance, reflecting back to the example of sulfur in coal, the heat content of the coal (desirable) is bundled with the sulfur (undesirable). If these two products could have been economically unbundled, that would have been done.[2] A negative valuation of the externality (smoke) induces a negative valuation on the bundled bad (sulfur).

To be somewhat more precise, consider a single market in which bundled commodities (consisting of bads and goods) are available in a variety of bundlings. Let the vector of bads be denoted by **B** and the vector of goods by **G**. Note that **B** and **G** are not characteristics but

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[2]Several authors have considered production theory involving the generation of externalities. Pittman (1981, 1983) estimates production functions taking into account undesirable outputs and finds some striking differences from the case where these outputs are ignored. Tran and Smith (1983) estimate a joint output production function where outputs are of electricity and air and water pollutants. Gollop and Roberts (1983, 1985) come closest to the subject of this paper by estimating a cost function for electric power, taking into account the price of two grades of fuel and "regulatory intensity." While their choice of variables may have been adequate for measuring productivity change, they do not treat explicitly the tradeoffs between negative and positive inputs. Furthermore, their "regulatory intensity" variable is inadequate to induce the correct firm preferences regarding positive and negative inputs.

quantities of bads and goods. In the coal example, B and G would be scalers with B total quantity of sulfur and G total thermal content. The sulfur fraction (such as percent sulfur) would not qualify as appropriate for B since that would be a negative characteristic, not a quantity of the bad.

Denote the value of a transaction for the bundle $(\mathbf{B},\mathbf{G})$ by the function $V(\mathbf{B},\mathbf{G})$. There are two properties we would expect V to possess. We would expect it to be subadditive:

$$V(\hat{\boldsymbol{B}},\hat{\boldsymbol{G}}) + V(\tilde{\boldsymbol{B}},\tilde{\boldsymbol{G}}) \geq V(\hat{\boldsymbol{B}}+\tilde{\boldsymbol{B}},\hat{\boldsymbol{G}}+\tilde{\boldsymbol{G}}), \tag{5}$$

where $(\hat{\mathbf{B}},\hat{\mathbf{G}})$ and $(\tilde{\mathbf{B}},\tilde{\mathbf{G}})$ are two bundles. Equation (5) must hold since $(\hat{\mathbf{B}}+\tilde{\mathbf{B}},\hat{\mathbf{G}}+\tilde{\mathbf{G}})$ can trivially be assembled from $(\hat{\mathbf{B}},\hat{\mathbf{G}})$ and $(\tilde{\mathbf{B}},\tilde{\mathbf{G}})$.

Furthermore, we would expect an unbundled bad to be non-positively valued and an unbundled good to be positively valued. This implies V is monotone in each component of $(\mathbf{B},\mathbf{G})$.

If we assume that there are constant returns in providing the bundle, then V will be homogeneous of degree one; furthermore, from equation (5), V will be convex. Thus we can pick a numeraire good, G_N, and rewrite V as

$$V(\boldsymbol{B},\boldsymbol{G}) = G_N \rho(\boldsymbol{b},\boldsymbol{g}) \tag{6}$$

where

$$b_i = B_i/G_N, \qquad i = 1,\ldots,M$$

$$g_j = G_j/G_N, \qquad j = 1,\ldots,N-1.$$

This is the more conventional situation where the bundle has a unit price expressed in terms of one of the goods (silicon in dollars per pound; coal in dollars per million Btu). Thus the **b** and **g** are now characteristics of a unit of the commodity and $\rho(\mathbf{b},\mathbf{g})$ is a hedonic price function.

The possibility of repackaging follows from subadditivity and homogeneity of ρ. To see this, consider two bundles with characteristics $(\hat{\mathbf{b}},\hat{\mathbf{g}})$ and $(\tilde{\mathbf{b}},\tilde{\mathbf{g}})$ and an arbitrary $0 \leq \lambda \leq 1$; homogeneity and subadditivity imply

$$\begin{aligned}\lambda\rho(\hat{\mathbf{b}},\hat{\mathbf{g}}) + (1-\lambda)\rho(\tilde{\mathbf{b}},\tilde{\mathbf{g}}) &= V(\lambda\hat{\mathbf{b}},\lambda\hat{\mathbf{g}},\lambda) + V((1-\lambda)\tilde{\mathbf{b}},(1-\lambda)\tilde{\mathbf{g}},(1-\lambda)) \\ &\geq V(\lambda\hat{\mathbf{b}}+(1-\lambda)\tilde{\mathbf{b}},\lambda\hat{\mathbf{g}}+(1-\lambda)\tilde{\mathbf{g}},1) = \rho(\lambda\hat{\mathbf{b}}+(1-\lambda)\tilde{\mathbf{b}},\lambda\hat{\mathbf{g}}+(1-\lambda)\tilde{\mathbf{g}})\end{aligned} \tag{7}$$

where the inequality is from (5). This implies ρ is convex which is equivalent to permitting repackaging. Monotonicity of ρ follows from monotonicity of V.

It is useful to interpret the meaning of hedonic prices in the case of negative characteristics. If someone is purchasing a bundle consisting of a good and a bad, then the more of the bad purchased, the lower the bundle price. The marginal price on the bad represents a "bribe" or compensation for agreeing to take the bad along with the good. For instance, the thermal value of the coal may be \$2 per million Btu. But by agreeing to take one half a unit of sulfur along with a unit of heat, at a sulfur price of -50¢, one only pays \$1.75 per million Btu: \$2 for the heat less a 50¢ x 1/2 = 25¢ bribe to take the sulfur, to compensate for the difficulties associated with using the sulfur. In this case, convexity results in the absolute value of the price of the sulfur characteristic diminishing as the concentration of sulfur increases.

III. THE TECHNOLOGY OF COAL-FIRED ELECTRICITY GENERATION

We now turn to implementing the theory presented in the previous section. The production process we consider is that of coal-fired electricity generation in the U.S. Coal combustion is a major source of air pollution, including acid rain, and has been subject to relatively strict emissions regulation in the U.S. since at least 1970. These regulations have induced negative prices on emissions of sulfur dioxide and consequently on inputs of sulfur. A complicating factor is that regulations keep changing and different regulations apply to different vintages of technologies. We deal with this by restricting our attention to those coal-fired power plants permitted between 1970 and 1979--the period in which all new plants were only subject to an emission limit on sulfur.

An interesting characteristic of this industry and production technology is that producers generally can choose from a variety of different coals whose price varies inversely with the sulfur content. is price premium for low sulfur coal has been induced in large part by environmental regulations in coal combustion. In order to meet the emission regulation, a producer can choose costly low-sulfur coal or less expensive higher sulfur coal and use desulfurization capital (scrubbers) at the generating station. Thus we have a classic choice of paying for higher quality fuel or paying for desulfurization capital.

It is of significant policy and academic interest to quantify the tradeoffs that can be made between sulfur and capital. Certainly there have been engineering studies of the cost of scrubbers as add-ons. That is an oversimplification of the sulfur-capital tradeoff. As capital is substituted for sulfur, fuel costs drop and operating costs may increase due to efficiency losses. The appropriate way to measure the costs associated with desulfurization capital is to estimate a cost function for the technology based on actual firm-level experience.

Another issue which can only be addressed in a cost function framework is the effect on costs of ash in the coal. Ash is undesirable because of regulation on emissions of flyash, but probably more importantly, ash can degrade the performance and/or shorten the life of boilers, crushers and other coal-handling equipment at a generating unit. Thus ash increases production costs.

Finally, the extent to which technical change has reduced costs or been biased towards one input or another is also a germane question.

Our approach to estimating the production technology is to partition the eastern half of the U.S. into K distinct regions (states or groups of states) and estimate a hedonic price function for coal in each region on a yearly basis. We then estimate, over all regions and time periods simultaneously, a cost function in conjunction with factor demand equations for the generating technology.

As mentioned earlier, there have been several papers (Epple, 1987; Bartik, 1987) pointing out that when one is estimating a technical rate of substitution equation such as (3b), the marginal prices (left-hand side of (3b)) depend on $\mathbf{z}$ which appear on the right-hand side. Thus $\mathbf{z}$ is correlated with the error term, in which case OLS estimation of each equation is inappropriate. Furthermore, McConnell and Phipps (1987) show that the hedonic equation may not even be identified.

The sample we use to estimate our hedonic price functions includes all utility coal transactions; this is a set containing many more producers and consumers than we consider in our cost function estimation. Consequently, it is reasonable to assume in our case that the cost function errors are uncorrelated with the hedonic price function errors. Thus we use OLS for each of the hedonic price functions.

The question of ex-ante vs. ex-post technology has surfaced again and again in estimating electric power production functions (e.g., Cowing & Smith, 1978; Fuss and McFadden, 1978). When firms make their capital investment decisions, they make them on the basis of expected future input and output prices as well as uncertainty in those prices. Expected factor prices determine the tradeoff between capital and variable factors. Uncertainty in price expectations influences the flexibility built into the ex-ante technology. Unfortunately, one does not observe price expectations. Our approach is to adopt a rational expectations hypothesis regarding future factor prices and to ignore the flexibility-efficiency issue. We estimate input prices for the first full year of unit operation and assume all generating units in our sample make ex-ante investment decisions based on those realized factor prices.

A. Hedonic Prices of the Bundled Inputs

Our view of coal is that thermal content is the characteristic utilities desire. The two major impurities found in most coals are sulfur and ash. Sulfur is undesirable because its emissions are subject to control. Increased ash content tends to degrade boiler performance, lowering output.

1. The Sample. To estimate the hedonic price function we use all reported purchases of coal by regulated electric utilities in the 1976-85 period. Fuel data (see appendix) includes information on price and quantity as well as sulfur, ash and thermal content. Each transaction is reported to the Federal Energy Regulatory Commission.

2. The Price Function. We assume that the transaction function $V(F,S,A)$ in equation (5) is homogeneous of degree 1, so we work in terms of the price function $\rho(s,a;\alpha)$. F is fuel in millions of Btu, S is sulfur content in pounds and A is ash in pounds. Sulfur content, s, equals S/F and ash content, a, equals A/F. As argued earlier, the hedonic price function is monotone and convex in s and a. Monotonicity assures us that the implicit prices always have the same sign (e.g., sulfur is always a bad and thermal content is always a good). Convexity yields downward sloping implicit demand functions (among other things). We have chosen a quadratic for the price function so that convexity can be imposed globally. It is difficult to impose monotonicity on a quadratic over a specific region and is of course impossible to impose globally without reducing prices to a linear function. Thus the price function has the form

$$\rho(s,a;\alpha) = \alpha_F + \alpha_s s + \alpha_a a + 1/2\begin{pmatrix} s \\ a \end{pmatrix}' \begin{pmatrix} \alpha_{ss} & \alpha_{sa} \\ \alpha_{sa} & \alpha_{aa} \end{pmatrix} \begin{pmatrix} s \\ a \end{pmatrix} \tag{8}$$

where a and s are the ash and sulfur levels (per thermal unit of coal). We require the matrix of α_{ij} to be symmetric and positive semi-definite.

Marginal prices for sulfur (S), ash (A) and thermal content (F) can be easily computed from (8):

$$u_S = \frac{\partial\left[F\rho\left(\frac{S}{F}, \frac{A}{F}; \alpha\right)\right]}{\partial S} = \alpha_s + \alpha_{ss}s + \alpha_{sa}a \tag{9a}$$

$$u_A = \frac{\partial\left[F\rho\left(\frac{S}{F}, \frac{A}{F}; \alpha\right)\right]}{\partial A} = \alpha_a + \alpha_{aa}a + \alpha_{sa}a \tag{9b}$$

$$u_F = \frac{\partial\left[F\rho\left(\frac{S}{F}, \frac{A}{F}; \alpha\right)\right]}{\partial F} = \rho(s,a;\alpha) - u_s s - u_a a \tag{9c}$$

Note that convexity of ρ gives $\alpha_{ii} \geq 0$. Thus for i = S,A, even if $\alpha_i \leq 0$, it is still possible for $u_i > 0$ (we would expect u_i to be non-positive for i = S,A).

3. The Estimation. One of the key distinguishing characteristics of the coal market is that coal prices vary over space. The closer one is to a low sulfur deposit, the lower the price of low sulfur coal. Thus it would be inappropriate to estimate a single hedonic price function for the whole U.S. Rather we estimate a series of functions for regions of the country. The sample of generating units discussed in the previous section determines the regions of the country for which we are interested in hedonic price functions. We have estimated a separate hedonic price function for each of the states where a sample generating unit is located.

Equation (8) was estimated separately for each state and each year so that a price equation was available for the first full year of operation of each generating unit. Thus in general a different function was estimated for each generating unit in the sample. Convexity was imposed heuristically and the function was restricted to be downward sloping with respect to s and a at the origin (i.e., $\alpha_s \le 0$, $\alpha_a \le 0$). Results of the estimation are given in the appendix. Although the adjusted R^2 was generally low, the coefficients were quite significant due to the large sample size. The α so estimated were used as exogenous variables in the cost function, described in the following section.

B. The Electricity Production Technology

1. The Sample. The goal is to estimate the technology of coal-fired power generation including a representation of the possible tradeoffs between fuel quality and the use of other factors (such as capital) induced by environmental regulations. The difficulty of this task is compounded by the fact that current emissions regulations for sulfur dioxide dictate technology, giving the firm little leeway in choice of fuel or technology. Fortunately, during the period 1971-1979, the new source performance standards in the U.S. specified a limit on sulfur emissions of 1.2 pounds per million Btu of fuel burned. The regulation left it completely up to the firm as to how this emission limit should be met. The regulation was applicable to all generating units whose initial license was sought during this period. Because of the long-lead times involved in plant construction, most units that have become operational in the late 1970s through the mid-1980s fall under this regulation.

Another compounding factor is that most data are at the plant level, with each plant made up of several generating units of potentially different vintages. And it is the generating units to which environmental regulations apply. A single plant can have some units subject to no new source regulations, some units subject to the original new source performance standard, and some units subject to the current new source performance standard. As a consequence of this we must further restrict our sample to plants where all the units are subject to the same emission regulation so that the necessary data are available.

A final consideration is that generating units in much of the western U.S. were essentially unconstrained by the original new source performance standard. Local low-sulfur coal was not only the cheapest, but also was capable of meeting the 1.2 pounds per million Btu limit without any additional costs. Thus western power plants were eliminated from our sample.

We are left with 51 different generating units spread over the eastern half of the U.S. These constitute our sample.[3]

2. The Cost Function. As was discussed earlier, our goal is to estimate a cost function, equation (2). Three problems with adopting a flexible functional form for this cost function are a) some variables are negative; b) some variables are positive and negative; and c) sometimes variables go to zero. Thus any function involving logarithms or square roots of prices and/or parameters (such as the translog or generalized Leontieff) is unacceptable. We have chosen the Generalized

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[3]Publicly and cooperatively owned units are excluded due to a lack of data.

McFadden Cost Function (GM) as discussed by Diewert and Wales (1987). This functional form has the advantage of allowing variables to take on positive or negative values and, with the exception of the numeraire good, zero values. Furthermore, we can impose convexity globally.

For notational simplicity, let $\mathbf{w} = (\boldsymbol{\alpha}, \mathbf{p_x})$, with W the dimension of w. Thus the cost function in (2) can be written, following Diewert and Wales (1987),

$$C(\mathbf{p_x}, \alpha, Y, t) \equiv C(\mathbf{w}, Y, t) = g(\mathbf{w})\,Y + \sum_{i=1}^{W} b_{ii} w_i Y$$
$$+ \sum_{i=1}^{W} b_i w_i + \sum_{i=1}^{W} b_{it} w_i t Y + b_t\left(\sum_{i=1}^{W} \delta_i w_i\right) t \tag{10a}$$
$$+ b_{YY}\left(\sum_{i=1}^{W} \beta_i w_i\right) Y^2 + b_{tt}\left(\sum_{i=1}^{W} \gamma_i w_i\right) t^2 Y$$

with

$$g(\mathbf{w}) = (1/2)\; w_1^{-1} \sum_{i=2}^{W} \sum_{j=2}^{W} c_{ij} w_i w_j \qquad \text{with } c_{ij} = c_{ji} \quad \text{for } 2 \le i,j \le W \tag{10b}$$

and with the parameters $\boldsymbol{\delta}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ set arbitrarily in advance to scale the problem. There are W(W-1)/2 different c_{ij} in equation (10b) and 3W+3 additional **b** parameters in equation (10a). Constant returns to scale imply the restrictions that $b_i = b_t = b_{YY} = 0$ for i = 1,...,W. Let $\tilde{C}$ be the (W-1)x(W-1) square matrix consisting of c_{ij}, $2 \le i,j \le W$. Diewert and Wales (1987) show that the cost function C is concave in w if and only if $\tilde{C}$ is negative semidefinite.

3. <u>The Estimation</u>. In our application, there are four inputs into production. Three are bundled together in coal: sulfur (S), ash (A), both of which are bads, and heat (F). One input, capital (K), is unbundled. We have neglected labor because of its modest role in generation costs,[4] as well as our desire to restrict the number of exogenous variables, given our small sample size. Data sources are discussed in the appendix.

In estimating the cost function (10), it is appropriate to utilize the fact that it results from cost minimization, namely estimating the cost function simultaneously with optimality conditions (4). Let the **w** vector in (10) be $\mathbf{w} = (P_K, \alpha_F, \alpha_s, \alpha_a, \alpha_{ss}, \alpha_{sa}, \alpha_{aa})$. Then using (8) we can translate (4) into

$$K* = \frac{\partial C}{\partial P_K} \tag{11a}$$

$$F* = \frac{\partial C}{\partial \alpha_F} \tag{11b}$$

$$S* = \frac{\partial C}{\partial \alpha_s} \tag{11c}$$

$$A* = \frac{\partial C}{\partial \alpha_a} \tag{11d}$$

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[4]In 1983, variable costs for producing power (from all fuel sources) in the U.S. were $56 billion of which 6 percent was non-fuel expenses excluding returns to capital; thus at most 6 percent of variable costs are labor. In our sample, the value share of capital in total generating costs ranges roughly between 40 percent to 75 percent. Thus labor plays a very small role in total costs.

$$\frac{S*^2}{2F*} = \frac{\partial C}{\partial \alpha_{ss}} \tag{12a}$$

$$\frac{A*S*}{F*} = \frac{\partial C}{\partial \alpha_{sa}} \tag{12b}$$

$$\frac{A*^2}{2F*} = \frac{\partial C}{\partial \alpha_{aa}}. \tag{12c}$$

This system is overdetermined in that there are five endogenous variables, C, K*, F*, S* and A* and eight equations, including the cost function. Thus, any five equations give sufficient information to determine the endogenous variables. We focus on (10-11), discarding (12) as redundant and assuming there is additive error associated with each endogenous variable, related to errors in optimization. We then estimate the five equations using a nonlinear iterative Zellner approach as implemented in TSP4.1. Table I presents results of the estimation with t-statistics in parentheses. As Diewert and Wales (1987) show, concavity of the cost function over strictly positive prices and outputs is equivalent to the matrix of c_{ij} in equation (10b) being negative semidefinite. Lau (1978) shows that any negative semidefinite matrix can be written as LDL′ where L is lower triangular with 1's on the diagonal and D is a nonpositive diagonal matrix. The model was estimated using these Cholesky factors of the c_{ij} matrix to impose concavity. Comparing the unrestricted and the restricted model using a likelihood ratio test, concavity could be rejected at the 95% level but not at the 96% level.

IV. DISCUSSION OF RESULTS

There are a number of interesting results that emerge from the estimation. One set of results concerns the hedonic price function for coal. As an equilibrium concept, the hedonic price function tells us how the market values differ in the sulfur and ash content in fuel. Another set of results concerns the nature of the generating technology and the extent of technical change. We address each of these two issues separately.

A. The Sulfur/Ash Penalty

Figure 1a is a plot of the average estimated hedonic price function (i.e., $\rho(s,a;\bar{\alpha})$ where $\bar{\alpha}$ is the mean value of α over the sample). Shown in the figure is price as a function of sulfur content, with ash content held at the generating unit sample mean (9.01 lb/10^6 Btu). As sulfur <u>content</u> decreases, the price of coal in terms of the numeraire good, million Btu, increases. If sulfur content is reduced from 1.5% to 0.5%, the price of coal rises about 20¢ per million Btu. Note that for large sulfur content, monotonicity starts to break down, with price increasing with increases in sulfur content.

Figure 1b shows the implicit price of sulfur (equation (9a)), as a function of sulfur content, using the same sample average hedonic price function parameter values and ash level. There are several things to note from the figure. First, as with Figure 1a, for large values of sulfur content, price goes anomalously positive. This is due to the limitations of the quadratic hedonic price function which yields linear marginal prices. Secondly, for lower levels of sulfur content, the "price" of sulfur is highest in absolute value and declines as sulfur content increases. Remembering that the hedonic price function is an equilibrium concept, this reflects the fact that it is more and more costly to reduce sulfur content to lower and lower levels. To interpret this price further, consider prices from the average price function at the sample average sulfur content, 1.366 lb. $S/10^6$ Btu. At this sulfur content, the price of coal is $1.60 per million Btu, with the price of heat, sulfur and ash respectively $1.81 per million Btu, $-0.071 per pound of sulfur and $-0.121 per pound of ash (from equation (9)). Thus, when a customer buys a million Btu of heat energy in coal for $1.82, he or she accepts the sulfur and ash impurities for compensation of 10¢ ($\alpha_s \times s = 0.071 \times 1.366$) and 11¢ ($\alpha_a \times a = 0.121 \times .901$), resulting in a total price for the coal bundle of $1.60. The computed price of coal ranges from 89¢ to $2.83 over the sample with an average of $1.61 per million Btu. The price of heat ranges from 99¢ to $2.49 per million Btu. The price of sulfur and ash range from -$.38 and -12¢ per pound, respectively, to positive values.[5] The sulfur and ash "bribes" are as high as 74¢ and 73¢ per million Btu, averaging 18¢ and 12¢, respectively.

The Federal government and others are currently spending large sums to improve the technology for reducing sulfur level in coal. Our model can be used to infer upper limits on the cost and performance of such technologies. Any new desulfurization technology would change the

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[5]The positive values reflect, as in Figure 1b, the restrictions imposed by a quadratic hedonic price function--namely that marginal prices are linear. Positive prices on sulfur and ash explain why the highest price of coal (bundled with impurities) is higher than the highest price of the good thermal content.

marginal valuation on sulfur, although it is difficult to say in what direction. Prices of low-sulfur coal would be reduced, prices of high-sulfur coal might increase from increased demand or might stay constant. Figure 1 does give an indication of the maximum desulfurization cost the market might currently support, on average. For instance, to reduce the sulfur content of coal from the generating unit sample mean to 0.6 lb. $S/10^6$ Btu (formerly known as compliance coal), producers could currently expect a price premium of 23¢ per million Btu or about $5 per ton (assuming 22 million Btu per ton coal). This would be the upper limit on the cost of achieving that reduction in sulfur, assuming ash content does not change. This of course is on average. Specific geographic markets may support a greater premium.

B. Factor Substitution and Technical Change in Electricity Generation

In Section III of the paper we discussed the estimation of a cost function for coal-fired electricity generation. The typical approach to understanding the economic characteristics of such a cost function is to lcok at the price elasticities of factor demand, elasticities of substitution among factors and the bias and level of technical change.

Factor demand is relatively easily obtained from the cost function, as in equation (11). Sensitivity of factor demand with respect to the price of capital is straightforward. How such demand changes with respect to the price of coal is more ambiguous since the price of coal is a function (of heat, sulfur and ash), not a scalar. For instance, consider a coal with the price function given in Figure 1a. A change in the hedonic price function could involve the price function in the figure shifting up or down, corresponding to a change in α_F; or the function could rotate, changing the price of sulfur and ash. Furthermore, the function could remain unchanged locally while changing substantially elsewhere. A non-local change in the price function could induce a consumer to substantially shift a consumption bundle.

We will consider three shifts in the hedonic price function for coal: basically an upward or downward shift in the price of sulfur, ash or heat; i.e., changes in the parameters α_F, α_s and α_a. An upward change in α_s would cause the sulfur price line in Figure 1b to shift upwards; the hedonic price function in Figure 1a would tend to flatten (since its slope is the price of sulfur).

As a consequence, using equation (11), we can define the following own- and cross-price elasticities of demand:

$$\epsilon_{ij} = \frac{\partial^2 C}{\partial P_i \partial P_j} P_j \div \frac{\partial C}{\partial P_i} \tag{13}$$

where

$$P_i = \begin{cases} P_K & for\ i = K \\ \alpha_i & for\ i = F, S, A. \end{cases}$$

What sign do we expect for these elasticities? The law of demand generally calls for negative own-price elasticities. Concavity of costs implies $\frac{\partial^2 C}{\partial P_i \partial P_j} \le 0$, using the notation in equation (13). Furthermore, $\alpha_i \le 0$ for i = S,A. As an example, consider the case of sulfur. As α_s increases, the price line (Figure 1b) shifts upward making S less desirable to accept. Thus, one would expect consumption of S to decrease. However, α_s is negative, thus we expect $\epsilon_{SS} \ge 0$. Similar logic yields positive own-price elasticities for ash. Similarly, we can define the Allen elasticities of substitution:

$$\sigma_{ij} = \frac{C \dfrac{\partial^2 C}{\partial P_i \partial P_j}}{\dfrac{\partial C}{\partial P_i} \dfrac{\partial C}{\partial P_j}} = \frac{\epsilon_{ij}}{\theta_j} \tag{14}$$

where $\theta_j = \dfrac{P_j X_j}{C}$, $X_j = K, F, S, A$

and $P_i = \begin{cases} P_K & for\ i = K \\ \alpha_i & for\ i = F, S, A. \end{cases}$

In contrast to price elasticities, we expect own substitution elasticities to always be negative, since θ_A and θ_S are negative.

Table II presents price and substitution elasticities evaluated at the mean (over the generating unit sample) of the exogenous variables. The t-statistics are computed from a second-order expansion of equations (15)-(16). As can be seen, he signs of own elasticities are as expected: own-price elasticity for sulfur and ash should be non-negative. The ash price elasticity is significantly positive although the other own-price elasticities are not significantly non-zero.

The results in Table II suggest that the demands for fuel and ash are relatively inelastic and the elasticity of demand for capital close to unity. Both price elasticities between capital and fuel suggest substitution; however, the elasticity of substitution is positive suggesting complementarity. As the price of capital goes up, fuel use goes up but capital use goes down even more resulting in the positive substitution elasticity. In contrast, the price elasticities between ash and capital both suggest substitutability in the sense that as capital becomes more expensive, ash is increased. We would expect complementarity.

During the decade covered by our sample, significant advances were being made in generating technologies although other factors such as regulatory problems have been suggested as adding to costs. Table III shows the estimates of measures of technical change for our sample. It is clear that technical progress tended to be capital-using at a significant rate of 3% per year and basically neutral with respect to fuel use. Interestingly, technical change is sulfur and ash saving at very substantial rates. This suggests that there has been a substantial shift to higher quality fuel, induced by technical change.

Finally we turn to the question of scale economies. As was indicated earlier, we are able to reject the hypothesis of constant returns to scale in generating. In fact, scale economies seem to be very substantial. Scale economies as defined by Christensen and Greene (1970) are 0.1952 (constant returns implies a zero scale elasticity), suggesting that at the mean of the exogenous variables ($Y = 3.23 \times 10^9$ kwh/yr $\approx$ 550 MW), there are still scale economies to be realized, at least in generation, ignoring reliability.

V. CONCLUSIONS

This paper has developed a new method for estimating a production technology involving bundled inputs, some of which are negatively valued. We have seen that production theory can be "directly" applied o this problem although because negative inputs are necessarily bundled with positive inputs, a hedonic analysis is necessary to infer implicit prices to use in estimating the production technology. It is hoped that future work can build on this base, generating better interpretations of substitution and price elasticities, among other things.

DATA APPENDIX

All the generating unit data are constructed for the first <u>full</u> year of unit operation. This is assumed to be one year after the published date at which the unit enters commercial operation since that published date may correspond to an incomplete year. The first <u>full</u> year of operation is referred to "the first year of operation plus one." Data used for estimating the cost function are given in Tables A-I through A-III.

A. Quantity of Capital

The basic source at the unit level is "Construction Costs of U.S. Steam Electric Plants 1970-1985," Utility Data Institute, Inc. The costs have to be adjusted because they are calculated as the sum of the yearly capital expenditures during construction. A method proposed by Joskow and Rose (1985) permits us to correct for the effect of changes in prices and interest rates during the construction period (assumed to be five years):

$$\begin{matrix}\textit{Total costs in 1st} \\ \textit{year of operation}\end{matrix} = \frac{\textit{nominal costs}}{\sum_{t=1}^{5} S_t \left[\prod_{i=1}^{t} (1+p(i)) \cdot \prod_{j=t}^{5} (1+r(j))\right]} \qquad \text{(A-1)}$$

where S_t is the share of actual construction expenses in year t with t = 6 being the year of first operation (source: Personal Communication, Nancy Rose, 1987): $S_1 = .10$, $S_2 = .32$, $S_3 = .39$, $S_4 = .16$, and $S_5 = .03$; p(i) is the percentage change in input prices in year i (from the Handy-Whitman Construction Cost Index for all Steam

Plant by regions, taken from the Moody Utility Manual); r(j) is the average allowance for funds used during construction in year j (from Financial Statistics of Selected Electric Utilities, Energy Information Administration, Table 16: Electric Utility Plant Construction Work in Progress and Table 17: Net Income and Allowance for Funds Used During Construction). Costs so calculated are then adjusted to 1976 using the regional Handy-Whitman construction cost index to obtain the quantity of capital.

B. Price of Capital Services

The price of capital services is calculated following the Christensen and Jorgenson (1969) approach for the first year of operation plus one:

$$P_i = \frac{a-uz_i-k}{1-u}\ [q_{i,t-1}r + q_{it}d - (q_{i,t}-q_{i,t-1})] + q_iT \tag{A-2}$$

where u is the Effective Corporate Tax rate from the "U.S. Long Term Review," Data Resources, Inc; z is the present value of $1 of depreciation for tax purposes,

$$z = \frac{1}{rt}\ [1-(\frac{1}{1+r})^t], \tag{A-3}$$

and t is the life of the utility for tax purposes which is assumed to be 28 years (based on estimates in Christensen et al., 1980). Note that the price of capital services is in nominal terms and will escalate with the price level.

k is the investment tax credit rate. Two series are available in the DRI "U.S. Long Term Review." One is for equipment and the other for public utilities structures. As the power plant cost data published by the Utility Data Institute are broken down into land, structures and equipment, it is possible to construct, for each unit, a weighted average of the structures series and equipment series using the ratio of equipment over total costs and of structures over total costs as weights.

q is the price of capital measured with the Handy Whitman index and broken down into six main regions (table entitled: Costs Trends of Electric Light and Power Construction from Moody's Nation Wide Survey of Public Utility Progress). The July 1st observation for the relevant region is chosen with July 1, 1976, as the base year.

r is the rate of return. It is calculated as a weighted average of the rate of return on common equity, r_e, and the rate of return on long term debt, r_d. The weights are constructed with the help of the capitalization ratios presented in the "Statistics of Privately Owned Electric Utilities in the United States (DOE/EIA-0044, Table 37: Selected Financial Indicators). The rate of return of common equity, r_e, is also available in the above table. r_d is the coupon rate of the long term bonds issued in the first year of operation plus one. It is taken from the Moody's Public Utility Manual. Finally, in the case of a publicly owned utility, the rate of return is assumed to be the 30-year Treasury bonds rate. (Note that these financial data are only available by utility, they are not specific to the generating unit.)

d is the economic rate of depreciation. Following Christensen et al. (1980), it was chosen using the 1.5 declining balance method and using their engineering estimates of 33 years for the average service life; hence d = 1.5/33 = .045.

T is the effective rate of property taxation in the relevant county. The sources are the 1977 and the 1982 Census of Government, Volume II, entitled, Taxable Property Values and Assessment Sales Price Ratios. The year closest to the first year of operation plus one was chosen and the rates were adjusted in order to apply to land and structures only. (Note that the data were not very specific but it is likely that the error introduced by totally ignoring T would be greater.)

C. Quantity of Output

The quantity of output is constructed as quantity of coal purchased divided by the heat rate (thermal efficiency); this assumes that the quantity of coal purchased is approximately equal to the quantity of coal used. The reason for using this measure of output is that some units are multi-fuel. The quantity of coal is reported on FERC Form 423 and is also available in "Cost and Quality of Fuels for Electric Utility Plants (various years). The heat rate for each plant is published in Thermal-Electric Plant Construction and Annual Production Expenses (DOE/EIA-0323).

D. Sulfur Ash and Fuel in the Hedonic Price Functions

All electric utility transactions for coal are reported in FERC Form 423. Quality characteristics for these transactions include price, thermal content and sulfur and ash content in percent by weight. These data were used to estimate the hedonic price functions, converting sulfur into pounds per million Btu, ash into pounds per 100,000 Btu and price in $ per million Btu in order to estimate the price functions; the quantity data for fuel are converted into million Btu using the reported Btu per pound conversion factor. All transactions in a state during a year were used to estimate the individual hedonic price functions. In a few cases, neighboring small states were aggregated to increase sample size. Table A-III contains the results of the estimation of the hedonic price function for each generating unit.

E. Total Costs

Total costs for the cost function are calculated for the first year of operation plus one. They are constructed as the sum of the value of capital services calculated above and the value of coal for the first year of operation plus one. The value of coal is obtained by multiplying price per short ton by quantity in short tons, with the data from FERC Form 423, discussed above.

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TABLE I

Cost Function Parameter Estimates, With and Without Imposition of Convexity

Parameter	Unrestricted Model Estimate	Unrestricted Model Asymptotic t-Statistic	Concavity Imposed Estimate	Concavity Imposed Asymptotic t-Statistic
D_F	-0.51992E-03	-0.91708	-0.45598E-03	-0.79222
D_S	0.34822E-01	1.1269	-0.44712E-02	-0.30738
D_A	-0.40026E-01	-1.9314	-0.43419E-02	-0.16927
D_{S2}	0.22688E-02	0.59855	0	
D_{AS}	-31.551	-0.24854	0	
D_{A2}	29.306	0.27261	0	
L_{S-F}	-2.8327	-0.43943	-0.28535	-0.61237E-01
L_{A-F}	4.3082	0.87582	7.8370	0.82138
L_{S2-F}	-0.59148	-0.28143	-0.12389	-0.62996E-01
L_{AS-F}	-1.3519	-0.12777	8.9134	0.58354
L_{A2-F}	7.0469	0.82556	9.0313	0.73985
L_{A-S}	0.13881	0.24555	-1.5009	-0.40203
L_{S2-S}	0.22112	0.52843	0.35424	0.20673
L_{AS-S}	3.4908	0.92461	-4.1757	-0.44703
L_{A2-S}	-1.6362	-1.1615	0.65619	0.97960E-01
L_{S2-A}	-0.43145E-01	-0.42966	0.54014E-01	0.18636E-01
L_{AS-A}	2.0333	2.0222	-0.85650	-0.64347E-01
L_{A2-A}	0.73139	1.1548	2.7360	0.29728
L_{AS-S2}	121.18	0.41997	0	
L_{A2-S2}	-472.75	-0.57531	0	
L_{A2-AS}	-4.1236	-0.53235	0	
b_{Kt}	0.61250E-01	2.5274	0.48166E-01	1.9334
b_{Ft}	-0.90545E-02	-2.4651	-0.87936E-02	-2.3998

TABLE I (continued)

Parameter	Unrestricted Model: Estimate	Unrestricted Model: Asymptotic t-Statistic	Concavity Imposed: Estimate	Concavity Imposed: Asymptotic t-Statistic
b_{St}	-0.57384E-01	-4.3377	-0.59112E-01	-4.4780
b_{At}	-0.22051E-01	-6.3406	-0.24323E-01	-7.0601
b_{S^2t}	-0.96125E-01	-1.3972	-0.12665	-1.8594
b_{ASt}	0.37400	1.0680	-0.17668E-01	-0.72358E-01
b_{A^2t}	-0.83813E-01	-2.2931	-0.50897E-01	-1.7789
b_t	0.78528E-04	0.10422	0.21619E-04	0.28730E-01
b_{FF}	0.36547	26.288	0.36590	26.334
b_{SS}	0.69487	5.9888	0.70369	6.0279
b_{AA}	0.43161	12.495	0.45831	13.430
$b_{S^2S^2}$	0.90480	1.2292	1.2482	1.7236
b_{ASAS}	3.0128	1.2721	3.9809	1.9859
$b_{A^2A^2}$	0.48960	1.5974	0.65471	2.6174
b_{KK}	0.51169	2.6866	0.61853	3.1549
b_{YY}	-0.41869E-02	-2.0338	-0.48127E-02	-2.3337
b_{tt}	0.31983E-03	2.3737	0.31996E-03	2.3704
b_K	0.72182	4.8588	0.65698	4.3206
b_F	0.12453E-01	1.1717	0.86410E-02	0.81717
b_S	0.14049	1.9212	0.81958E-01	1.1479
b_A	0.17841E-01	0.92401	0.19003E-01	0.99086
b_{S^2}	0.75980E-01	0.37769	0.59163E-01	0.30694
b_{AS}	-2.8000	-2.3830	-2.0672	-2.5521
b_{A^2}	0.35923	1.8805	0.47417E-03	0.55399E-02

Note: Sample size = 51; $\tilde{C}$ in (12b) factored into LDL'; concavity imposed by requiring $D \leq 0$.

TABLE II

Elasticities Model with Concavity Imposed

Factors	Price Elasticities Value	Price Elasticities Asymptotic t-Statistic	Substitution Elasticities Value	Substitution Elasticities Asymptotic t-Statistic
K-K	-0.27815E-01	-0.58826	-0.79308E-01	-0.60191
K-F	0.23399E-01	0.84390	0.31501E-01	0.84856
K-S	-0.59749E-02	-0.26534	0.33588E-01	0.26364
K-A	-0.21202E-01	-1.6117	0.25279	1.5782
F-K	0.11049E-01	0.84977		
F-F	-0.14045E-01	-0.79654	-0.18908E-01	-0.80090
F-S	-0.80129E-03	-0.64568E-01	0.45045E-02	0.64527E-01
F-A	0.14103E-01	2.1358	-0.16815	-2.1219
S-K	0.11781E-01	0.26398		
S-F	0.33459E-02	0.64539E-01		
S-S	0.23178E-01	0.32531	-0.13030	-0.32265
S-A	-0.25470E-01	-1.1309	0.30368	1.1211
A-K	0.88662E-01	1.6234		
A-F	-0.12490	-2.1617		
A-S	-0.54019E-01	-1.1076		
A-A	0.18997	4.7420	-2.2649	-4.4514

Note: Price elasticities are of the first member of the factor pairs with respect to the price of the second; elasticities evaluated at means of exogenous variables; t-statistics computed using second order expansions about the means.

TABLE III

Measures of Technical Change and Scale Elasticity,
Model with Concavity Imposed

Parameter	Value	Asymptotic t-Statistic
$\frac{\partial \ell nK}{\partial t}$	0.0305	1.97
$\frac{\partial \ell nF}{\partial t}$	-0.000017	-0.006
$\frac{\partial \ell nS}{\partial t}$	-0.1528	-5.12
$\frac{\partial \ell nA}{\partial t}$	-0.0868	-8.26
$1 - \frac{\partial \ell nC}{\partial \ell nY}$	0.1952	3.292

Note: Elasticities evaluated at means of exogenous variables; t-statistics computed using second order expansions about the means.

Figure 1a

Coal Price for Mean Parameter Values

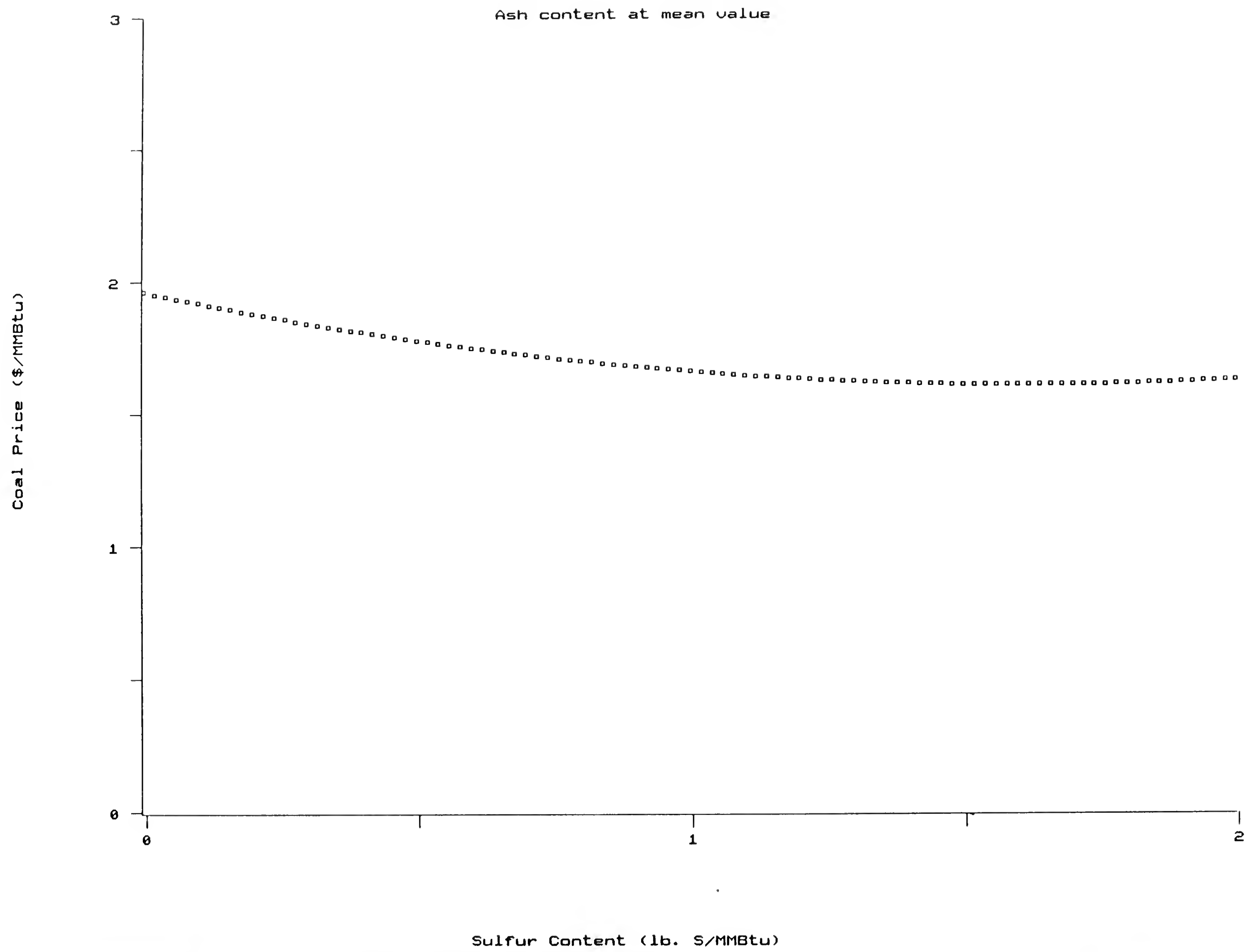
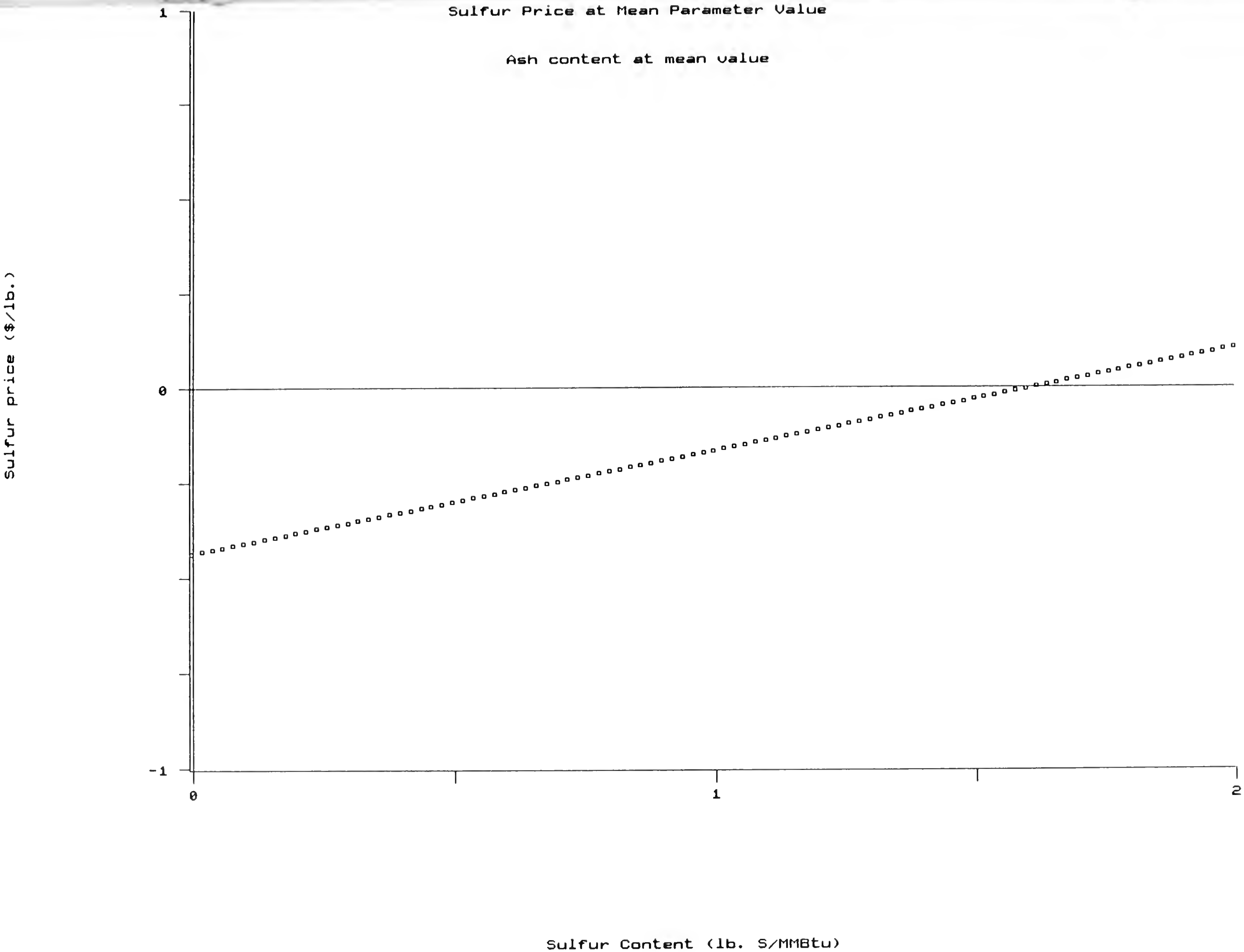


Figure 1b



Sulfur Price at Mean Parameter Value
Ash content at mean value
1
0
-1
Sulfur price ($/lb.)
0
1
2
Sulfur Content (lb. S/MMBtu)

TABLE A-I: Generating Unit Sample Description

Sample Pt. #	Plant	Unit #	State	1st Year	MW
1	AB BROWN	1	IN	1980	250
2	AMES TWO	1	IA	1983	65
3	BELLE RIVER	1	MI	1985	655
4	BELLE RIVER	2	MI	1986	655
5	BRANDON SHORES	1	MD	1985	620
6	BRUCE MANSFIELD	1	PA	1977	780
7	BRUCE MANSFIELD	2	PA	1978	780
8	BRUCE MANSFIELD	3	PA	1981	780
9	COUNCIL BLUFFS	1	IA	1979	700
10	CRYSTAL RIVER	1	FL	1983	685
11	CRYSTAL RIVER	2	FL	1985	685
12	DEERHAVEN	1	FL	1982	235
13	DUCK CREEK	1	IL	1977	380
14	EAST BEND	1	KY	1982	600
15	GREEN	1	KY	1980	263
16	GREEN	2	KY	1982	263
17	HAVANA	1	IL	1979	426
18	HOMER CITY	1	PA	1978	650
19	IATAN	1	MO	1981	670
20	INDEPENDENCE	2	AR	1985	815
21	JH CAMPBELL	1	MI	1981	770
22	KILLEN	1	OH	1983	600
23	LANSING	1	IA	1978	260
24	LOUISA	1	IA	1984	650
25	MADGETT	1	WI	1980	349
26	MARION	1	IL	1979	170
27	MAYO	1	NC	1984	705
28	MCINTOSH	1	FL	1983	334
29	MEROM	1	IN	1984	450
30	MEROM	2	IN	1983	450
31	MILLER	1	AL	1979	634
32	MILLER	2	AL	1986	634
33	MOUNTAINEER	1	WV	1981	1300
34	NEWTON	1	IL	1978	550
35	NEWTON	2	IL	1983	562
36	OTTUMWA	1	IA	1982	675
37	PETERSBURG	1	IN	1978	515
38	PLEASANT PRAIRE	1	WI	1981	580
39	PLEASANT PRAIRE	2	WI	1986	580
40	PLEASANTS	1	WV	1980	626
41	PLEASANTS	2	WV	1981	626
42	ROCKPORT	1	IN	1985	1300
43	SHERER	1	GA	1983	808
44	SHERER	2	GA	1985	808
45	SHERBURNE CO	1	MN	1977	700
46	SHERBURNE CO	2	MN	1978	700
47	SOUTHWEST	1	MO	1977	194
48	THOMAS HILL	1	MO	1983	630
49	VJ DANIEL	1	MS	1978	505
50	VJ DANIEL	2	MS	1982	505
51	WESTON	1	WI	1982	321

TABLE A-II: Key data for estimating cost and hedonic functions

Sample Pt	COST	P_K	K	P	F	s	a	BTU	Y
1	0.324	0.134	0.641	1.581	0.151	3.114	0.841	11337	0.439
2	0.098	0.296	0.189	1.882	0.023	0.746	0.861	8174	0.067
3	2.256	0.314	4.032	1.835	0.539	0.375	0.448	9593	1.553
4	1.824	0.358	3.354	1.726	0.362	0.387	0.440	9564	1.042
5	1.078	0.285	2.047	1.888	0.262	0.554	0.743	12817	0.779
6	0.840	0.096	2.928	1.028	0.544	2.589	1.408	11511	1.627
7	0.618	0.122	2.015	1.240	0.300	2.654	1.319	11642	0.895
8	1.155	0.110	2.658	2.194	0.393	2.620	1.094	11830	1.173
9	0.447	0.092	1.584	0.706	0.427	0.444	0.604	8333	1.239
10	2.835	0.311	1.993	2.599	0.852	0.841	0.780	12365	2.721
11	1.939	0.375	1.530	2.275	0.600	0.627	0.747	12596	1.915
12	0.512	0.265	0.716	2.382	0.135	0.549	0.604	12743	0.398
13	0.371	0.083	1.398	1.587	0.160	0.593	0.838	11303	0.476
14	0.898	0.241	1.872	1.463	0.306	2.473	0.999	11161	0.924
15	0.258	0.090	0.732	1.105	0.174	3.928	1.635	10311	0.446
16	0.361	0.241	0.730	1.175	0.158	3.378	1.455	10540	0.405
17	0.381	0.090	0.946	1.993	0.149	0.432	0.497	12506	0.392
18	1.029	0.133	1.575	1.310	0.626	1.809	1.733	11555	1.949
19	0.783	0.190	1.670	1.079	0.432	0.420	0.556	8816	1.355
20	1.331	0.345	1.749	1.513	0.481	0.253	0.590	8689	1.439
21	1.921	0.173	2.915	1.963	0.722	1.222	0.792	12028	2.423
22	0.922	0.297	2.430	1.927	0.104	0.489	0.921	12468	0.282
23	0.247	0.140	0.558	1.256	0.135	1.346	0.904	8840	0.363
24	1.031	0.342	2.290	1.591	0.155	0.420	0.713	8343	0.436
25	0.374	0.096	0.818	1.425	0.207	1.149	0.764	8787	0.609
26	0.194	0.054	0.416	0.970	0.176	2.971	1.442	10266	0.518
27	1.400	0.345	1.946	2.032	0.358	0.544	0.838	12322	1.105
28	0.632	0.177	0.978	2.144	0.214	1.705	1.205	11730	0.628
29	0.614	0.289	1.218	1.284	0.203	2.550	1.028	10901	0.598
30	0.976	0.295	2.015	1.485	0.257	2.469	0.908	11096	0.755
31	0.390	0.037	1.676	1.763	0.186	0.522	0.652	12847	0.594
32	1.374	0.335	1.641	2.706	0.305	0.464	0.884	12491	0.970
33	1.980	0.150	3.122	1.862	0.812	0.502	0.830	12559	2.701
34	0.614	0.135	1.378	1.226	0.349	2.161	1.102	10921	1.029
35	0.720	0.286	1.333	1.903	0.178	1.916	0.848	11588	0.526
36	0.672	0.211	1.474	1.364	0.264	0.355	0.632	8455	0.789
37	0.850	0.128	1.725	1.015	0.619	2.471	0.974	10888	1.856
38	0.874	0.156	2.020	1.726	0.323	0.435	0.698	8280	0.853
39	0.944	0.426	1.269	1.341	0.301	0.373	0.597	8307	0.796
40	0.735	0.107	2.043	1.307	0.394	2.352	0.983	12371	1.194
41	0.564	0.065	1.460	1.470	0.319	2.275	1.124	12310	0.968
42	2.560	0.288	4.251	2.503	0.533	0.456	0.695	7895	1.495
43	1.681	0.312	2.649	2.966	0.288	0.510	0.555	13131	0.845
44	1.621	0.426	1.581	2.940	0.322	0.537	0.711	12658	0.947
45	0.594	0.112	1.394	0.616	0.711	0.939	1.114	8736	2.079
46	0.415	0.159	0.848	0.691	0.405	0.883	1.082	8722	1.185
47	0.122	0.066	0.727	1.012	0.073	2.927	1.371	11410	0.204
48	1.388	0.206	1.929	1.570	0.631	4.145	1.266	10181	1.855
49	0.220	0.126	0.816	2.043	0.057	0.476	0.825	11558	0.172
50	0.780	0.270	0.790	2.963	0.192	0.482	0.705	11826	0.577
51	0.558	0.228	0.969	2.149	0.157	0.818	0.620	9286	0.413

Units: P_K (price of capital services)--Nominal \$ per \$ of capital; K (capital)--10^8 1976\$;
P (price of coal)--Nominal \$/10^6 Btu; F (coal consumption)--10^{14} Btu;
s (sulfur content)--lb. sulfur/10^6 Btu; a (ash content)--lb. ash/10^5 Btu;
Btu (coal heat content)--Btu/lb.; Y (output)--3.2278 x 10^9 kWh (normalized to yield mean of 1.000).

TABLE A-III: Estimated Coefficients for Hedonic Price Function (Standard errors in parentheses)

Sample Pt #	α_F	α_s	α_a	α_{ss}	α_{aa}	α_{as}	# Obs	Adjusted R-sqrd
1	2.1158 (0.0542)	-0.4473 (0.0338)	-0.3152 (0.0508)	0.132 (0.0113)	0	0	945	0.32
2	2.1389 (0.0695)	-0.7799 (0.1123)	0.4649 (0.0624)	0	0	0	287	0.17
3	2.1698 (0.398)	-0.0901 (0.0282)	-0.228 (0.0585)	0	0	0	822	0.04
4	2.1268 (0.0645)	-0.282 (0.1163)	-0.109 (0.0594)	0.124 (0.0762)	0	0	949	0.02
5	2.459 (0.0879)	-1.2901 (0.1747)	-0.2267 (0.0565)	1.2903 (0.1656)	0	0	403	0.17
6	1.1999 (0.01957)	-0.0319 (0.0080)	-0.1604 (0.0079)	0	0	0	2613	0.14
7	1.4633 (0.0230)	-0.0742 (0.0078)	-0.219 (0.0180)	0.0136 (0.0064)	0	0	2420	0.24
8	1.9233 (0.0441)	-0.1539 (0.0342)	-0.2816 (0.0335)	0.0401 (0.0181)	0.0177 (0.0104)	0.0367 (0.0104)	2371	0.14
9	1.6255 (0.0729)	-0.0451 (0.0107)	-0.2026 (0.1178)	0	0.1827 (0.0212)	0	664	0.04
10	2.5388 (0.0332)	-0.2621 (0.0205)	0	0	0	0	362	0.31
11	2.3027 (0.0229)	-0.1754 (0.0145)	0	0	0	0	541	0.21
12	2.5722 (0.0345)	-0.2945 (0.0195)	0	0	0	0	283	0.45
13	1.7394 (0.0281)	-0.3837 (0.0278)	-0.2234 (0.0175)	0.1245 (0.0132)	0	0	1171	0.43
14	2.2131 (0.0365)	-0.4749 (0.1261)	-0.1647 (0.0303)	0.1407 (0.0120)	0	0	1189	0.44
15	1.8739 (0.0302)	-0.3912 (0.0238)	-0.1421 (0.0249)	0.115 (0.0107)	0	0	1218	0.41
16	2.2131 (0.0365)	-0.4749 (0.1261)	-0.1647 (0.0303)	0.1407 (0.0120)	0	0	1189	0.44
17	2.3057 (0.0263)	-0.6402 (0.0310)	-0.3086 (0.0197)	0.2264 (0.0160)	0	0	1001	0.68
18	1.4633 (0.0230)	-0.0742 (0.0078)	-0.219 (0.0180)	0.0136 (0.0064)	0	0	2420	0.24
19	1.6983 (0.0373)	-0.1321 (0.0279)	0	0.021 (0.0093)	0	0	511	0.12
20	1.8687 (2.3679)	-2.1465 (0.9203)	0	6.537 (2.3679)	0	0	47	0.26
21	2.2668 (0.0788)	-0.1669 (0.0136)	-0.455 (0.0188)	0	0.5509 (0.2160)	0	1228	0.12
22	2.7667 (0.0708)	-0.3562 (0.344)	-1.7891 (0.1346)	0.1674 (0.0152)	1.8078 (0.1397)	-0.084 (0.0315)	2139	0.22
23	1.5556 (0.0298)	-0.0318 (0.0083)	-0.1585 (0.0306)	0	0	0	688	0.11
24	1.5516 (0.4114)	0.0547 (0.1632)	-0.3583 (0.9310)	0.6881 (0.0854)	2.7477 (1.0754)	-1.3459 (0.1898)	318	0.22
25	1.6545 (0.0468)	0	-0.2293 (0.1159)	0	0.1209 (0.1387)	0	1130	0.02
26	2.3057 (0.0263)	-0.6402 (0.0310)	-0.3086 (0.0197)	0.2264 (0.0160)	0	0	1001	0.68
27	2.4637 (0.0690)	-0.8035 (0.1451)	-0.1845 (0.0295)	0.4845 (0.1580)	0	0	1198	0.18
28	2.5388 (0.0332)	-0.2621 (0.0205)	0	0	0	0	362	0.31
29	3.1833 (0.0626)	-0.9996 (0.0522)	-0.5713 (0.0569)	0.3696 (0.0234)	0	0	1119	0.43
30	3.4669 (0.1210)	-1.0344 (0.1185)	-0.8042 (0.1362)	0.3756 (0.0548)	0	0	612	0.34
31	2.3382 (0.0839)	-0.555 (0.0575)	-1.013 (0.1325)	0.1257 (0.0246)	0.5712 (0.1163)	0.2637 (0.0519)	1236	0.12
32	1.8325 (0.0286)	-0.1556 (0.0200)	0	0	0	0	724	0.08
33	1.905 (0.0303)	-0.1812 (0.0126)	-0.0417 (0.0236)	0	0	0	1464	0.13
34	1.8952 (0.0287)	-0.3959 (0.0314)	-0.1086 (0.0194)	0.11 (0.0156)	0	0	1172	0.38
35	3.6558 (0.0575)	-1.4002 (0.0718)	-0.4313 (0.0388)	0.5289 (0.0371)	0	0	684	0.66
36	1.9497 (0.0656)	-0.2572 (0.0839)	0	0.1451 (0.0429)	0	0	328	0.03
37	1.6257 (0.0453)	-0.2901 (0.0236)	-0.0951 (0.0526)	0.0591 (0.0097)	0.0417 (0.0247)	0.0272 (0.0144)	1222	0.25
38	1.7596 (0.0415)	-0.1514 (0.0494)	0	0	0	0	732	0.01
39	1.8308 (0.1878)	0	-0.8238 (0.5265)	0	1.35 (0.7132)	0	644	0.01
40	1.683 (0.0137)	-0.1871 (0.0109)	0	0	0	0	1438	0.17
41	1.905 (0.0303)	-0.1812 (0.0126)	-0.0417 (0.0236)	0	0	0	1464	0.13
42	3.2084 (0.0809)	-0.9806 (0.0611)	-0.5441 (0.0909)	0.3542 (0.0285)	0	0	747	0.47
43	3.391 (0.1768)	-0.5198 (0.1183)	-2.4073 (0.4929)	0.1048 (0.0692)	1.8759 (0.7973)	0.3452 (0.2049)	729	0.17
44	3.0196 (0.0730)	-1.028 (0.0802)	-0.4085 (0.0769)	0	0.498	0	913	0.27
45	1.2055 (0.0629)	-0.0371 (0.0155)	-0.1926 (0.0502)	0	0	0	362	0.04
46	1.1663 (0.0582)	-0.0426 (0.0597)	0	0	0	0	360	0.08
47	1.2358 (0.0327)	-0.147 (0.0239)	0	0.02 (0.0083)	0	0	752	0.17
48	1.6537 (0.0310)	-0.0168 (0.0110)	-0.0626 (0.0398)	0	0	0	449	0.03
49	1.9918 (0.0796)	-0.4797 (0.0916)	0	0.1639 (0.0491)	0	0	146	0.32
50	3.6035 (0.1128)	-1.8347 (0.2095)	0	0.9405 (0.1508)	0	0	125	0.75
51	2.0422 (0.0594)	0	-0.3938 (0.0726)	0	0	0	910	0.03

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