Faculty Working Papers

PROPERTY VALUE MAXIMIZATION AND PUBLIC SECTOR EFFICIENCY

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by

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I. Introduction

It has long been recognized that in an economy with local public goods, differences in public consumption levels among communities will be capitalized into property values.[1] Building on this nearly self-evident principle, a number of recent papers have explored subtler features of the relationship between property values and local fiscal variables. A striking proposition which emerges from several of these studies is that by choosing the public good output in a community to maximize aggregate property value, the local government can generate a Pareto-efficient consumption pattern for its constituents. This proposition appears to provide a solution to the public sector efficiency problem raised by Samuelson (1954); apparently, all that governments need do to guarantee optimality is search for a property value maximum. While these points were made most clearly by Sonstelie and Portney (1978), they emerge also in the analysis of Brueckner (1979b).

Unfortunately, the conclusions of both previous studies are not definitive because they emerge from models without a traditional housing market equilibrium. For example, Sonstelie and Portney's analysis relies heavily on a Rosen-style (1974) housing price function without characterizing the market equilibrium required to generate such a function. In Brueckner's analysis, the housing stock is fixed, but consideration of market equilibrium is legitimately avoided via the ("open city") assumption that consumer utility levels are exogenous.

By employing a general equilibrium model with a fully specified housing market, the present paper remedies the incompleteness of earlier studies and provides the first fully acceptable investigation of the normative implications of property value maximization. Focusing on a single closed community, it is shown that if the government levies a "house tax" (a head tax on each house owner) and chooses the public good level to maximize property value in a qualified sense spelled out below, then the resulting equilibrium is Pareto-efficient. It is shown that the same conclusion does not follow when revenue is raised by a property tax, a consequence of the familiar distortion introduced by such a tax. Later sections of the paper relate the dicussion to open communities and the Tiebout hypothesis and cite empirical evidence relating to property value maximization.[2]

2. Property Value Maximization with a House Tax

The analysis will focus on a single closed community with n residents, each of whom possesses an endowment of the numeraire commodity x and owns an exogenous share of the fixed land area belonging to the community. In addition to his consumption of x, each resident consumes the public good z and housing q. Housing is produced with a constant returns technology whose inputs are land and the numeraire commodity x, while x is the sole input into public production.

It is necessary first to consider the input choices of the housing producer. Letting r denote land rent per acre and ℓ and x^h denote housing inputs of land and x respectively, the Lagrangean expression for the producer's cost minimization problem is

$$x^h + r\ell - \lambda(H(x^h,\ell) - Q), \tag{1}$$

where H is the (constant returns) production function and Q is the specified level of output. The first-order conditions yield

$$\frac{H_2(x^h,\ell)}{H_1(x^h,\ell)} = \frac{H_2(x^h/\ell,1)}{H_1(x^h/\ell,1)} = r, \tag{2}$$

where the first equality follows from the first degree homogeneity of the production function. Letting S denote x^h/ℓ, (2) implies that S is a function of r, S(r). Recalling that the Lagrange multiplier λ is marginal (and average) production cost and noting the first-order condition $\lambda = 1/H_1(x^h,\ell)$, marginal cost may be written as

$$\frac{1}{H_1(S(r),1)} \equiv a(r). \tag{3}$$

The next step is to derive the rent for a house of size q which leaves the producer zero profit after payment of the "house tax" levied to finance the public good. Letting C(z) denote the (weakly) convex cost function for the public good, a house tax of C(z)/n levied on the owner of each dwelling in the community will allow provision of a uniform public consumption level of z.[3] Letting F denote house rent, zero profit for the producer requires $F - a(r)q - C(z)/n = 0$, implying that for given r, the required rental payment as a function of q and z may be written

$$F(q,z;r) \equiv a(r)q + C(z)/n. \tag{4}$$

Turning now to the consumer side of the market, it will be assumed that individual i has the strictly quasi-concave utility function $v_i(x_i,q_i,z)$. Since it will be necessary in the following analysis to treat consumer utility levels explicitly as endogenous variables, let consumer i enjoy a utility level u_i, so that $v_i(x_i,q_i,z) = u_i$. This expression may be inverted to yield $x_i = x_i(q_i,z,u_i)$, which gives the amount of x consumption required to generate utility u_i for the consumer when his housing and public good consumption are q_i and z respectively. Note that $\partial x_i/\partial q_i = -v_{i2}/v_{i1}$, $\partial x_i/\partial z = -v_{i3}/v_{i1}$, and $\partial x_i/\partial u_i = 1/v_{i1}$. Now the consumer's housing rental payment R_i must leave an amount of income sufficient to purchase just enough x to reach utility level u_i. That is, R_i must satisfy

$$y_i(r) - R_i = x_i(q_i,z,u_i), \tag{5}$$

where $y_i(r) \equiv w_i + r\ell_i$ is the value of consumer i's endowment (ℓ_i and w_i are i's endowments of land and the numeraire commodity respectively). Rearrangement yields the consumer's "bid-rent" function

$$R_i(q_i,z,u_i;r) \equiv y_i(r) - x_i(q_i,z,u_i), \tag{6}$$

which gives the rental payment consistent with the specified utility level as a function of house size, public good consumption, and land rent.

Recalling the definition of the function x_i, it follows that

$$\frac{\partial R_i}{\partial q_i} = \frac{v_{i2}(x_i(q_i,z,u_i),q_i,z)}{v_{i1}(x_i(q_i,z,u_i),q_i,z)} > 0 \tag{7}$$

$$\frac{\partial R_i}{\partial z} = \frac{v_{i3}(\cdot)}{v_{i1}(\cdot)} > 0 \tag{8}$$

$$\frac{\partial R_i}{\partial u_i} = -\frac{1}{v_{i1}(\cdot)} < 0. \tag{9}$$

In addition, it is easy to see from a diagram that the strict quasi-concavity of the utility function means that the function x_i is strictly convex in q_i and z and hence that R_i is a strictly concave function of q_i and z.

Using the bid-rent function (6), the consumer's choice of an optimal house size may be illustrated in a somewhat unconventional fashion. Recalling that (6) defines a family of bid-rent functions parameterized by u_i, it is clear that for fixed z and r, the consumer's goal is to find a point on the market rent function (4) which lies on the lowest bid-rent curve (from (9), utility is inversely related to the level of the bid-rent curve). The solution requires tangency between a (concave) bid-rent curve and the market rent function, as illustrated in Figure 1 (asterisks indicate equilibrium values).[4] The equilibrium conditions, which for fixed z and r yield equilibrium values for both house size and utility, are[5]

$$R_i(q_i,z,u_i;r) = a(r)q_i + C(z)/n \tag{10}$$

$$\frac{\partial R_i(q_i,z,u_i;r)}{\partial q_i} = a(r). \tag{11}$$

To fully determine the housing market equilibrium, a market clearing condition must be added to (10) and (11). Given constant returns, the community's total housing output may be written $\bar{\ell}H(x^h/\bar{\ell},1) = \bar{\ell}H(S(r),1) \equiv \bar{\ell}h(S(r))$, where $\bar{\ell}$ is the fixed community land area. Market equilibrium then requires

$$\Sigma q_i = \bar{\ell}h(S(r)). \tag{12}$$

To explain how the local government chooses z, an expression for aggregate property value must first be derived. The value of a house is the price which the property will fetch on the open market once construction is complete. Since a buyer will be willing to offer at most an amount equal to the rent which the house commands minus the house tax liability, the value of the house inhabited by individual i is $R_i(q_i,z,u_i;r) - C(z)/n$.[6] Aggregate property value is then

$$\Sigma R_i(q_i,z,u_i;r) - C(z). \tag{13}$$

The crucial behavioral assumption of the present analysis is that the government chooses z to maximize (13) taking r, q_i, and u_i, i=1,...,n, _as parametric_. These variables are influenced by z as a result of (10)-(12), but the government, behaving like a perfect competitor, _ignores this dependence_ in choosing the public good level. The first-order condition for choice of z is consequently[7]

$$\Sigma \frac{\partial R_i(q_i,z,u_i;r)}{\partial z} = C'(z). \tag{14}$$

Conditions (10)-(12), together with (14), determine equilibrium values for the 2n + 2 variables z, r, q_i, u_i, i=1,...,n.[8]

It is useful to translate the equilibrium conditions into more familiar terms, making use of (7) and (8). First, the variables u_i are eliminated from the problem by evaluating the functions $x_i(q_i,z,u_i)$ and calling the result x_i, as before. Using (7), (11) then reduces to

$$\frac{v_{i2}(x_i,q_i,z)}{v_{i1}(x_i,q_i,z)} = a(r), \tag{15}$$

while using (8), (14) becomes

$$\Sigma \frac{v_{i3}(x_i,q_i,z)}{v_{i1}(x_i,q_i,z)} = C'(z). \tag{16}$$

Eq. (16) is, of course, the well-known Samuelson condition, which states that the sum of the marginal rates of substitution between the public good and the numeraire equals the marginal cost of the public good. It is this condition which emerges from competitive property value maximization by the local government. While (12) needs no simplification, (6) can be used to substitute for the bid-rent function in (10) to give

$$w_i + r\ell_i - x_i - a(r)q_i - C(z)/n = 0. \tag{17}$$

(recall $w_i + r\ell_i \equiv y_i(r)$).

It is easily shown that (15)-(17) and (12) characterize a Pareto-efficient allocation. The Lagrangean for the Pareto problem is

$$v_1(x_1,q_1,z_1) - \sum_{i=2}^{n} \delta_i(v_i(x_i,q_i,z) - \bar{u}_i)$$

$$- \gamma(\Sigma w_i - \Sigma x_i - x^h - C(z)) \qquad (18)$$

$$- \theta(\Sigma q_i - H(x^h,\bar{\ell}))$$

(note that there is no housing land cost to society since the land is internally owned). It is easy to see that the first-order conditions from (18) for choice of x_i, q_i, and z reduce to (15) and (16) (recall that a(r) equals the marginal cost of housing, $1/H_1(x^h,\bar{\ell})$). In addition, summing (17) over i yields $\Sigma w_i - \Sigma x_i - (a(r)\Sigma q_i - r\bar{\ell}) - C(z) = 0$, which is equivalent (noting $a(r)\Sigma q_i - r\bar{\ell} = x^h$) to the second-to-last constraint in (18). Given the equivalence of the last constraint and (12), it follows finally that the equilibrium characterized by (15)-(17) and (12) satisfies all the necessary conditions for Pareto-optimality.[9]

It is important to realize that the competitive nature of the local government's property-value-maximizing behavior is responsible for the efficiency of equilibrium. Were the government to take account of the influence of z on q_i, u_i, and r in maximizing (13), the Samuelson condition would not emerge and the equilibrium would be inefficient. The local government therefore cannot generate an efficient equilibrium by searching for an actual property value maximum; it needs to know the form of consumer bid-rent functions (and hence the form of utility functions) to pursue the competitive type of property value maximization which generates an efficient equilibrium. Since the information needed to pursue the correct policy is therefore the same as that required to directly compute a Pareto-optimum, property value maximization is not an operational method for achieving efficiency. This conclusion clearly contradicts the view of property value maximization as a practical policy found in some earlier studies (see especially Sonstelie and Portney).

3. The Effect of a Property Tax

Suppose that instead of levying a house tax, the government raises revenue via a property tax. Letting F denote rent, house value V is determined by the relationship $V = F - \tau V$, where τ is the property tax rate (τV is the property tax liability). Solving for V yields $V = F/(1+\tau)$, and the zero profit condition $F - \tau F/(1+\tau) - a(r)q = 0$ ($\tau F/(1+\tau)$ is the tax liability) implies that the rental payment required for zero profit is

$$F(q,\tau;r) = (1+\tau)a(r)q. \tag{19}$$

Using (19), consumer equilibrium requires

$$R_i(q_i,z,u_i;r) = (1+\tau)a(r)q_i \tag{20}$$

$$\frac{\partial R_i(q_i,z,u_i;r)}{\partial q_i} = (1+\tau)a(r), \tag{21}$$

and the housing market clearing condition is once again

$$\Sigma q_i = \bar{\ell}h(S(r)). \tag{22}$$

The government's budget constraint, which relates z and τ via aggregate property value $\Sigma R_i/(1+\tau)$, is

$$\frac{\tau}{1+\tau} \Sigma R_i(q_i,z,u_i;r) = C(z). \tag{23}$$

Under property taxation, the government will choose z and τ in a competitive fashion to maximize aggregate property value subject to (23). The Lagrangean is

$$\frac{1}{1+\tau} \Sigma R_i(q_i,z,u_i;r) - \mu(\frac{\tau}{1+\tau} \Sigma R_i(q_i,z,u_i;r) - C(z)). \tag{24}$$

Differentiating (24) with respect to z and τ and combining the first-order conditions yields the requirement

$$\Sigma \frac{\partial R_i(q_i,z,u_i;r)}{\partial z} = C'(z), \tag{25}$$

which is the same as (14).

Equations (20)-(23) and (25) determine equilibrium values for the variables z, τ, r, q_i, u_i, i=1,...,n. Eq. (25) yields the Samuelson condition, and combination of (22) and (20) gives back the economy's aggregate resource constraint. The equilibrium characterized by (20)-(23) and (25) is not Pareto-efficient, however, because of the distortion introduced by the property tax. From (21), the marginal rate of substitution between housing and the numeraire is not equal to the marginal cost of housing in equilibrium, as required by efficiency, but instead equals (1+τ) times marginal cost. In spite of this distortion of consumer choice engendered by the property tax, a limited efficiency result may be stated. In particular, it is easy to see that the equilibrium defined by (20)-(23) and (25) is Pareto-efficient <u>conditional on the (inefficient) equilibrium housing stock</u>. That is, if the q_i are fixed at their equilibrium values and a Pareto optimum is characterized (see (18)), the necessary conditions will be fulfilled by equilibrium conditions (20), (22), and (25). Thus, while consumption of z and the numeraire is non-optimal in general, non-housing consumption is efficient taking as given the (non-optimal) housing stock.

4. Open Communities and the Tiebout Model

While the previous analysis has focused on a single closed community, an important question is whether competitive property value maximization in a system of open communities levying house taxes leads to a Pareto-efficient equilibrium. It is fairly easy to see that the answer to this question is an unfortunate no. The argument establishing this fact is similar to that used by Brueckner (1979a) to show the possibility of inefficient inter-community equilibria in a public goods model without housing consumption where head taxes finance public spending. In the model, the public good is congested, so that the public good cost function is $C(z,n)$, $C_2 > 0$, yielding a head tax of $C(z,n)/n$. Certain types of congestion imply the existence of a finite optimal community size for each taste group, a value of n where the per capita cost of the (optimally-chosen) public output reaches a minimum.[10] If the total population of each taste group is fortuitously equal to a multiple of the optimal community size for that group, then an efficient Tiebout-style equilibrium of homogeneous optimal-size communities exists. Brueckner (1979a) shows, however, that when public outputs are chosen by majority vote, it is possible to construct examples of inefficient equilibria with heterogeneous communities, even when taste group sizes are "right" and an efficient Tiebout equilibrium exists.

A similar argument can be made in the present model. Although competitive property value maximization yields a Pareto-efficient consumption pattern for the current residents of a community, it does not rule out an inefficient allocation of individuals across communities. For example, consider an economy with two taste groups where each group is able to fill exactly one optimal size community. Although a configuration with two homogeneous communities is clearly the optimum *optimorum*, it is possible to construct an example of a globally inefficient equilibrium with two heterogeneous communities, each of which is, however, internally Pareto-efficient as a result of competitive property value maximization.[11] Such an example is presented in the appendix. Clearly, internal community efficiency is a necessary but not a sufficient condition for overall efficiency; competitive property value maximization need not guide the economy to a global optimum.

5. Empirical Evidence

Empirical evidence relating to the present analysis is provided by Brueckner (1979b). The empirical model in that study portrays a system of communities where consumers with identical tastes but differing incomes reach exogenously specified utility levels (these are uniform across communities for a given income group). Housing stocks are also viewed as exogenous, as is the matching of consumers to houses, and a property tax finances public spending. Under these circumstances, aggregate property value P in a community may be written $P = f(Q,Y,z)$, where Q is the vector of house sizes and Y is the vector of consumer income levels. A final assumption is that all communities in the sample are inefficient in the same direction in providing the public good. That is, given their housing stocks and the exogenous consumer utility levels, all communities simultaneously provide the public good at a level below or above the relevant property-value-maximizing level (formally, $z_k < z_k^*$ holds for all observations k, where z_k^* maximizes $f(Q_k, Y_k, z)$, or the reverse inequality holds for all k). Under these circumstances, it is clear that a regression plane fitted to the data (with P as the dependent variable) will indicate by the sign of its z coefficient whether communities operate generally below or above their property value maxima. Note that instead of yielding an estimate of the entire hypersurface corresponding to $P = f(Q,Y,z)$, this procedure yields a hyperplane which is approximately "tangent" to the surface.[12] The empirical results show a negative z coefficient, indicating that communities operate on the "downhill" (in the z direction) portion of the property value hypersurface. That is, holding utilities and house sizes fixed, a decrease in z in a sample community would increase its aggregate property value. This means that the Samuelson condition is not satisfied in the sample communities ($\sum_{i \varepsilon k} MRS_i < C'(z_k)$ holds for all communities k), indicating that public good outputs in the sample are inefficient conditional on community housing stocks.[13] What this means is that holding its housing stock fixed, a lower public good output in a sample community could have allowed some consumer utility levels to increase. Of course, since local governments have no reason to behave like competitive property value maximizers, empirical results showing real-world inefficiency should not be surprising.

6. Conclusion

The purpose of this paper has been to precisely delineate the connection between property value maximization and public sector efficiency. It has been shown that under a house tax regime, competitive property value maximization (in which the government ignores its influence on the local economy) leads to a Pareto-efficient equilibrium in a closed community. Non-competitive property value maximization (where the government searches for an actual property value maximum) is, by contrast, inefficient. The inefficiency of equilibrium when behavior is non-competitive is of course a familiar result, and its emergence in the present context seems natural.

The particular kind of competitive government behavior required for efficiency unfortunately renders property value maximization non-operational as real-world policy. While the government must view the community's housing stock and land rent as parametric, it must also act as if consumer utility levels are fixed. As a result, the government needs to know each consumer's utility function to pursue the correct policy. This information, however, facilitates direct computation of a Pareto-optimum, obviating the need for a separate approach.

Although in light of the previous observations, it might be tempting to view the efficiency of competitive property value maximization as nothing more than a mathematical curiosum, it must be remembered that the analysis underlying this result also has empirical significance. As was shown in the last section of the paper, regression results relating property values to local public spending can indicate (under suitable assumptions) whether or not the local public sector is efficient in the conditional sense discussed in section 3. Therefore, the analysis in this paper offers more than purely theoretical illumination.

Appendix

This appendix provides an example of a globally inefficient two-community equilibrium in which each community is internally Pareto-efficient. It is assumed that there are two taste groups whose members have identical x-endowments w and utility functions $v_i(x_i, q_i, z) = x_i + t(q_i) + m_i(z)$, with $t', m_i' > 0$ and $t'', m_i'' < 0$, i=1,2. Also, it is assumed that community land areas are equal (communities could be viewed as equal sized "islands") and that land rent is divided equally among a community's current residents.

Since the function t does not depend on i, housing consumption in a given community is the same for all individuals regardless of taste. Eq. (15) gives

$$t'(q) = a(r), \tag{A1}$$

where q is the uniform level of housing consumption. Consumption of x is also uniform within a community. From (17),

$$x = w + r\bar{\ell}/n - a(r)q - C(z,n)/n, \tag{A2}$$

where C(z,n) is the cost function for the (congested) public good. Finally, since housing consumption is uniform, the market clearing condition (12) becomes

$$nq = \bar{\ell}h(S(r)). \tag{A3}$$

Eqs. (A1) and (A3) determine q and r as functions of n and (A2) then determines x as a function of z and n. Note that the solutions do not depend on the relative proportions of the two taste groups in the community population.

Suppose that n_1 type-one consumers and n_2 type-two consumers reside in a community. Then the Samuelson condition (16), which follows from competitive property value maximization, requires $n_1 m_1'(z) + n_2 m_2'(z) = C_1(z, n_1+n_2)$ or

$$\theta m_1'(z) + (1-\theta)m_2'(z) = C_1(z,n)/n, \tag{A4}$$

where $n = n_1 + n_2$ and $\theta = n_1/n$. Eqs. (A1)-(A4) fully determine a (Pareto-efficient) community equilibrium for the given group sizes. Note that setting $\theta = 1$ or $\theta = 0$ in (A4) gives the equilibrium for a homogeneous type-one or type-two community. It is easy to show that these homogeneous equilibria maximize the utility of the relevant group subject to the community resource constraint. That is, (A1)-(A3) and (A4) with $\theta = 1$ are equivalent to the optimality conditions for the problem max $x + t(q) + m_1(z)$ subject to $nw - nx - x^h - C(z,n) = 0$ and $nq - H(x^h, \bar{\ell}) = 0$, with an analogous statement holding for group 2.

Before proceeding to construct the example, it will be useful to calculate the increase in group 1's equilibrium utility level which follows from an increase in θ. Differentiating $v_1 = w + r\bar{\ell}/n - a(r)q - C(z,n)/n + t(q) + m_1(z)$ with respect to θ, recalling that r and q are independent of z and hence independent of θ, it follows that

$$\frac{dv_1}{d\theta} = (m_1'(z) - C_1(z,n)/n)\,\frac{\partial z}{\partial \theta}\,. \tag{A-5}$$

Now, using (A4),

$$\frac{\partial z}{\partial \theta} = - \frac{m_1'(z) - m_2'(z)}{\theta m_1''(z) + (1-\theta)m_2''(z) - C_{11}(z,n)/n}, \qquad (A6)$$

and it follows from $C_{11} \geq 0$ and $m_i'' < 0$, $i=1,2$, that $\partial z/\partial\theta$ has the sign of $m_1' - m_2'$. Furthermore, it is easy to show that when (A4) holds, $m_1' - m_2' > 0$ implies $m_1' - C_1/n > 0$ and $m_1' - m_2' < 0$ implies $m_1' - C_1'/n < 0$. This means $\frac{\partial z}{\partial \theta}$ and $m_1' - C_1/n$ have the same sign, implying from (A5) that $dv_1/d\theta > 0$. This result is perfectly intuitive: when the proportion of type-ones in the community rises holding n fixed, type-one tastes receive more weight in the Samuelson condition (A4), leading to a higher type-one utility level. Note that $dv_1/d\theta > 0$ means that for given n, type-one individuals reach the lowest utility level in a community where z is chosen to maximize the utility of a type-two consumer (this occurs when $\theta = 0$), with utility increasing monotonically as θ increases from zero to one. A similar discussion holds for group 2.

The key to the following example is the dependence of utility on community size n. Consider the equilibrium type-one utility level in homogeneous communities of various sizes. This is computed by setting $\theta = 1$ and solving (A1)-(A4) for x, q, r, and z as functions of n and substituting the solutions into the utility function. This gives an "indirect" utility function which depends only on n: $\phi_1(n)$. An analogous procedure yields $\phi_2(n)$, the type-two utility level in a homogeneous community of size n. The following example assumes that $\phi_i(n)$, i=1,2, are single-peaked functions of n. While this need not always be the case, it can be guaranteed in the present example by assuming $v_i(x_i,q_i,z) \equiv x_i + q_i^{\alpha} + z^{\beta_i}$, $H(x^h,\ell) \equiv (x^h)^{\rho}\ell^{1-\rho}$, $C(z,n) \equiv z^{\tau}n^{\sigma}$ and choosing σ to be sufficiently smaller than unity.

Figure 2 shows the curves $\phi_1(n)$ and $\phi_2(n)$, with the optimal homogeneous community sizes for the groups denoted N_1^* and N_2^*. The Figure also shows curves $\phi_1^2(n)$ and $\phi_2^1(n)$, which are defined as follows: $\phi_i^j(n)$ gives the type i utility when z is chosen to satisfy type j tastes (for example, $\phi_1^2(n)$ is given by solving (A1)-(A4) with $\theta = 0$ and substituting the solutions into the type-one utility function). From the previous discussion, it is clear that when $0 < \theta < 1$, the utility level of a type-one consumer lies between the ϕ_1 and ϕ_1^2 curves at the appropriate value of n, with a similar conclusion holding for group 2.[14]

With this background, it is possible to present an example of a globally inefficient equilibrium. First, suppose that the total populations of the groups equal N_1^* and N_2^* respectively. This means it is possible to create two optimal-size homogeneous communities, leading to the highest possible utility levels for the two groups. However, consider a two-community configuration where one community (A) has a homogeneous type-one population of $P_A \equiv N_1^*-\delta$ while the other community has population P_B composed of N_2^* type-twos and δ type-ones. Figure 2 shows that this configuration can be an equilibrium. What is required is that no individual has an incentive to change communities. In the Figure, $\phi_1(N_1^*-\delta)$, the utility of a (type-one) resident of community A, equals the utility he would enjoy by moving to the (efficient) mixed community B (recall that when $0 < \theta < 1$, type-one utility is intermediate between the heights of the ϕ_1 and ϕ_1^2 curves). In addition, a type-two resident of community B reaches a utility level equal to what he would achieve in community A ($\phi_2^1(N_1^*-\delta)$).[15] Note that in contemplating a move to the other community, a consumer does not take account of his effect on θ.

This example shows the possibility of a globally inefficient equilibrium where each community is internally Pareto-efficient. Although a major community reorganization (moving δ type-ones from B to A) would increase everyone's utility, no single individual has an incentive to alter his residence. The example clearly illustrates the proposition that internal community efficiency is not a sufficient condition for global optimality.

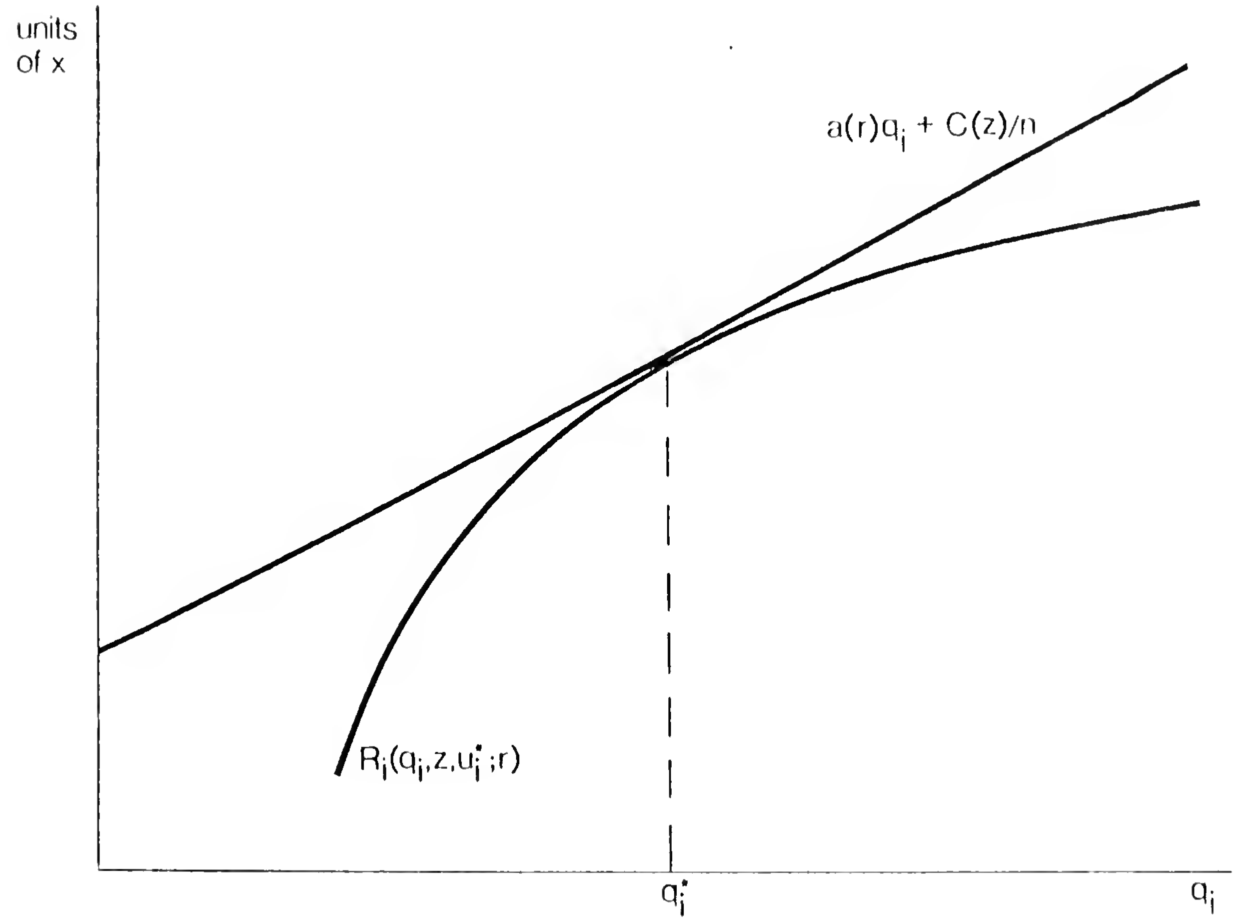

Fig. 1

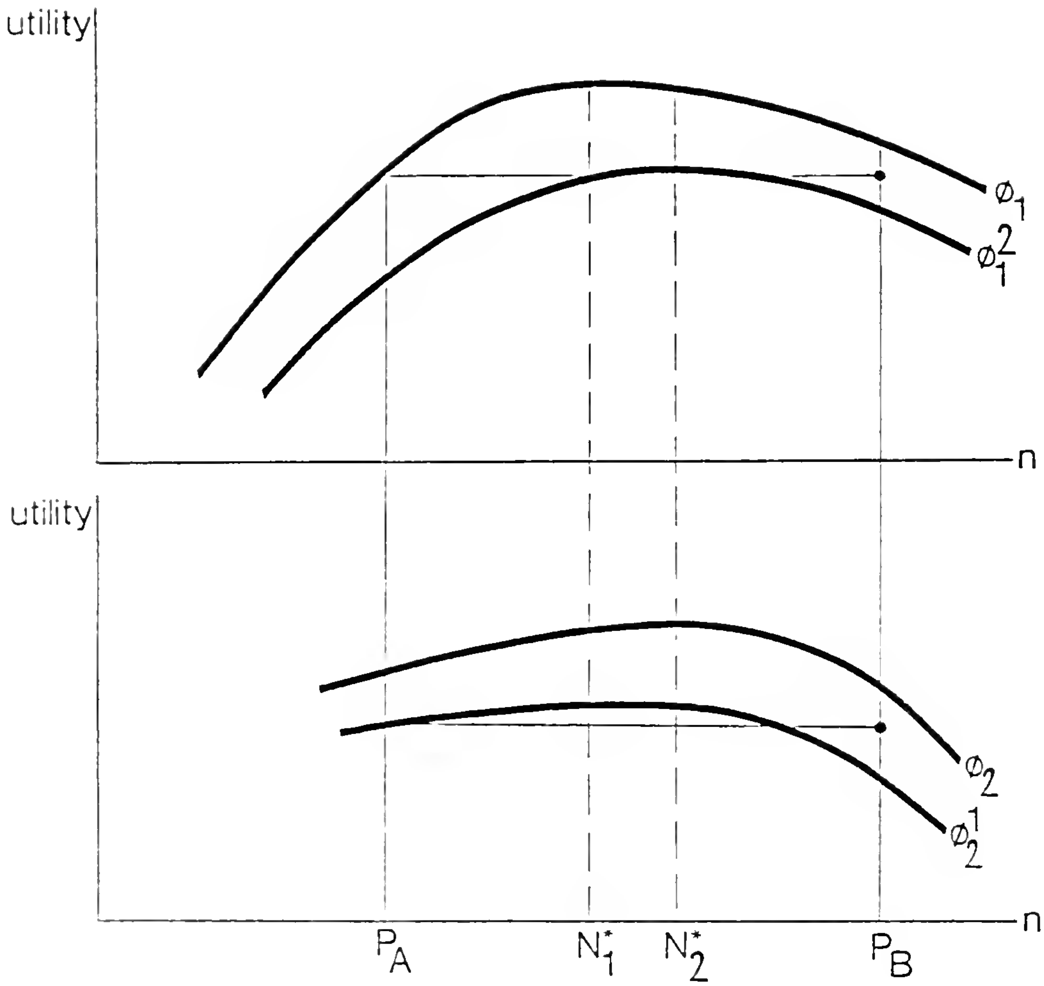

Figure 2

Footnotes

*I wish to thank Jon Sonstelie and David Wildasin for comments. Any errors, however, are my own.

[1]The first empirical evidence on capitalization was provided by Oates (1969).

[2]It should be noted that Edelson (1976) and Wildasin (1979) consider models where individual residents of a community vote for the public good level which maximizes the value of their own property. While Edelson was concerned with showing the circumstances under which voters agree on the optimal public good level, he mentions in passing (and without proof) that aggregate property value maximization is Pareto-optimal. Wildasin's discussion, which also yields an efficiency result, suffers from the same defect as Sonstelie and Portney's: a Rosen-style housing price function is used without characterization of the housing market equilibrium which generates it. In another study, Negishi (1972) shows the efficiency of an extremely restrictive type of land value maximization in a model based on very special assumptions. Finally, Starrett (1977) considers the effect of changes in the public good output on aggregate land rent in a circular city without considering the normative implications of property value maximization.

[3]Although the fact that n does not appear in the public cost function implies that z is a pure public good, the analysis is no different when the public good is congested and the cost function is $C(z,n)$, $C_2 > 0$. This is discussed further below.

[4]Note the similarity between Figure 1 and Rosen's (1974) diagrams depicting hedonic price determination. It is important to realize that while the above analysis could be couched in conventional supply and demand terms since housing is a homogeneous commodity in the model, Rosen's analysis cannot avoid use of a diagram such as Figure 1 since his commodities are differentiated by quality.

[5]Mathematically, the consumer's optimization problem is to maximize u_i subject to $R_i(q_i,z,u_i;r) = a(r)q_i + C(z)/n$. The first-order conditions for this problem reduce to (10) and (11).

[6]In equilibrium, of course, value equals production cost $a(r)q_i$ (see (10)). Note that while the discussion so far has implicitly assumed that producers rent out their completed houses, so that no active market in houses actually exists, it is easy to see that analysis of an economy in which producers sell their completed properties to landlords who rent them to consumers is identical to the above. Note also that if the analysis had been carried out in a multiperiod model with durable houses, house value would equal the discounted present value of the difference between rent and taxes.

[7]The second-order condition is satisfied since the R_i are concave in z and C(z) is convex.

[8]It should be noted that conditions (10)-(14) characterize the Nash equilibrium of a game where the players are the government, consumers, and producers. In addition to normal competitive behavior in the housing market, market participants view z as fixed in making consumption and production decisions. Similarly, the government views the decision variables of housing market participants as fixed in choosing z.

[9]If $\bar{u}_i$ is set equal to u_i (the equilibrium value of i's utility from (15)-(17) and (12)) for i=2,...,n, then the maximized value of individual 1's utility from (18) will equal u_1, the equilibrium utility value. Thus the equilibrium is Pareto-efficient; holding n-1 utilities fixed at their equilibrium values, the equilibrium value of the remaining utility is as high as possible given society's resource constraints.

[10]Letting $v_i(x_i,z)$ be the utility function and denoting income by y, the consumer maximizes $v_i(x_i,z)$ subject to $y - x_i - C(z,n)/n$. The first-order conditions are $v_{i2}/v_{i1} = C_1/n$ and $C_2 = C/n$. The latter condition may not be satisfied for any (z,n) if congestion is weak; infinitely large communities are then optimal.

[11]Note that when communities are open, the local government must view its community's population as parametric in pursuing competitive property value maximization.

[12]Since data on aggregate property value are not available, median house value was used as the dependent variable in the regression. Note that maximizing aggregate property value is approximately the same as maximizing median house value. For details of the empirical approach, see Brueckner (1979b).

[13]Since the empirical model realistically assumes the existence of a property tax, housing stocks must be viewed as non-optimal.

[14]Note that without the invariance of the equilibrium values of x, q, and r to the composition of the community population (to θ), it would not be possible to express utilities simply as functions of n.

[15]Since all that is required is that a type-two individual in B has no incentive to move to A, the configuration is an equilibrium as long as the utility of type two individual in B is equal to <u>or greater than</u> the utility he would achieve in A.

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