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### **BEBR** FACULTY WORKING **PAPER NO. 90-1679**

# Protecting the Winner: Second-Price Versus Oral Auctions

Richard Engelbrecht-Wiggans Charles M. Kahn



College of Commerce and Business Administration Bureau of Economic and Business Research University of Illinois Urbana-Champaign



## BEBR

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Protecting the Winner: Second-Price Versus Oral Auctions

Richard Engelbrecht-Wiggans Department of Business Administration

> Charles M. Kahn Department of Economics

University of Illinois, Urbana-Champaign

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#### ABSTRACT

An important difference between oral auctions and second-price sealed bid auctions is that in the oral auction the winner never reveals his reservation price to anyone. We model a situation in which outside parties can exploit information revealed in the auction, and we investigate the effect on bidders' strategies in the two auctions. We show that when this exploitation is a significant problem, the oral auction is more efficient than the sealed-bid second price auction, and preferred by all participants in the auction.

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#### PROTECTING THE WINNER: SECOND-PRICE VERSUS ORAL AUCTIONS

A common perception in auction theory is that in simple circumstances, the oral auction is equivalent to Vickery's [1961] sealed-bid second-price auction. In each auction, a bidder's strategy can be described by a choice of a price (which in the oral auction is the price at which to drop out; in the second-price auction, the price to propose). In the case of private values, the optimal choice is a price equal to the bidder's reservation value for the good. The equivalence means that both bidders and auctioneer should be indifferent between running an auction as an oral auction or as a second-price auction. Nonetheless, oral auctions are common; while second-price auctions are extremely rare.<sup>1</sup>

Observers have proposed one important distinction between second-price and oral auctions: When bidders engage in a second-price auction, they all must make announcements (which in simple circumstances represent their true reservation value for the good). When bidders engage in an oral auction, the winner never reveals his reservation value; the process stops when the second highest bidder drops out. Instinctively, observers have argued that when a winning bidder has an interest in keeping his reservation value secret (for example, when other agents -- e.g., the bidder's unionized employees -- will be able to exploit this information) then bidders will prefer oral auctions to second-price auctions.<sup>2</sup>

This intuition, while suggestive, is unsatisfactory. Implicitly it is being asserted that in many circumstances a bidder's actions serve <u>two</u> roles -- first to give him a chance at receiving the good, and second, to signal to other agents the private information held by the bidder. But it is not clear

<u>a priori</u> that the bidders' behavior under the two auction rules will remain equivalent once this additional concern is taken into account -- or indeed, what the behavior of a bidder in such a situation should be.

Our paper examines this intuition. We consider an example in which an outside agent will sometimes extract concessions from the winning bidder based on information gained through observing the bid. We confirm the intuition that second-price auctions may reveal more information to outsiders than do oral auctions. We find that as the concession to the outsider becomes more probable, the oral auction dominates the second price auction. However, it is not the case that the bidder's strategies are identical under the two auctions. Nor is it the case that the outsider gains from the information that is revealed. Rather, bidders, in their attempts to avoid revealing information to outsiders, destroy the efficiency of the second price auction. In our example, outsiders will be <u>indifferent</u> between the two forms of auctions, but all participants in the auction <u>strictly prefer</u> the privacy of the oral auction.

It might be objected that if this were the sole disadvantage of second-price auctions, it could easily be rectified by prohibiting the auctioneer from making public the winning bidder's bid. There are two counter-objections. First in many circumstances it is a legal requirement that the bids in sealed bid auctions be made public. Second, and more fundamentally, in many applications it will be the auctioneer himself whom the bidder fears will exploit the information conveyed. This is particularly natural if the auctioneer is the government or if the bidder engages in repeated auctions. Even if the auctioneer is not able to exploit the information directly, bidders may fear the possibility of collusion between the auctioneer and those who can use the information to advantage. In the

circumstances we model below, third parties would have an incentive to pay the auctioneer to have the bid revealed to them ex post. In general then, the key difference between the two types of auctions is who bears responsibility for keeping the winner's information secret. Bidders may rationally feel that the only true secrets in the world are secrets revealed to nobody.

In outline, our example is as follows: An auctioneer (for example, the government) is interested in procuring a specially made object. The auctioneer auctions off the right to a contract to produce the object. The value of winning the contract depends on the bidder's cost parameter, which is private information. The value of winning also depends on what arrangements will be struck with the bidder's employees. The employees will, ex post, make a take-it-or-leave-it offer, exploiting any information revealed in the auction. Thus the bidder must take into account not only the auction but subsequent effects in determining his bid.

The model is of interest from a technical standpoint as well, since it in effect combines the signalling and the auction theory literatures.

#### <u>The model</u>

There are the following agents: one <u>auctioneer</u>, N <u>bidders</u> and a number of <u>employees</u>. All agents are risk neutral. The auctioneer solicits bids from the bidders to take on a project, whose completion is worth \$X to the auctioneer. The project will require the bidder to purchase a fixed amount of materials and, perhaps, to hire additional labor. The non-labor costs for any bidder are known only by that bidder.

If the project requires additional labor this is determined by the successful bidder only after the work has begun. The probability of the need for additional labor will be denoted by  $\alpha$ . If additional labor turns out to

be necessary, the successful bidder will then bargain with an employee over the wage to be paid. For simplicity assume bargaining takes the following form: The employee makes a take-it-or-leave it offer of w. The bidder decides whether or not to accept. Thus if a bidder with costs m wins the project at price p and does not need additional labor, his profits are p - m. If he needs additional labor, and reaches an agreement to pay his employee an amount w, his profits are p - m - w.

Obviously, the demand made by laborers ex post will depend on what they think they can extract from the successful bidder; and this will depend on what the process of the auction has revealed about the bidder's costs.

In a world in which the value of winning depends on ex post realizations of costs, it becomes very important to specify exactly what the terms of the contract are. The exact form of the procurement contract is a matter of interest in the literature<sup>3</sup> but it is tangential to the point we wish to make in this paper. Therefore we stick with the following simple assumption: The terms of the contract specify a payment p if the project is completed, and a penalty h for non completion. (The fixed amount penalty can be regarded as a bankrupcy cost or the damages assessed for non fulfillment of the contract). The penalty is fixed; bidders in the auction compete on the dimension of payment p.

We assume that the bidders' non-labor costs are independent random variables, taking on one of two values H or L, with H > L. Let q be the probability that a bidder realizes a low cost.

#### Full Information

First, suppose that the winning bidder's type were ex-post observable by the employee. Then the sealed bid and ascending auction are equivalent. Define w(p,m) to be the wage demanded of a bidder with costs m who will receive p for the contract. Such a bidder will accept the employee's take-it-or-leave-it offer as long as

 $p - w(p,m) - m \geq -h.$ 

Thus the employee demands a wage

A bidder facing such a take-it-or-leave-it offer is reduced to indifference between accepting the wage demand and reneging on the contract. This means that the expected profits of the winning bidder if he has cost m are

 $(1-\alpha)(p-m) - \alpha h$ 

By the standard analysis for second price auctions, the dominant strategy for an individual of type m is to make a bid  ${\rm b_m}$  where  $^4$ 

$$b_m \equiv m + \alpha h/(1-\alpha)$$

To see that this is an equilibrium, note a higher bid runs the risk of missing profitable opportunities; a lower bid risks winning when expected profits are less than zero. Note that expected profits are zero for each type: profits if the employee does not make a wage demand balance the losses when he does.

From the auctioneer's point of view, this situation has the following characteristics: The lowest cost player always wins the contract. He never reneges on the contract. The auctioneer bears the entire cost of the hold-up by employees.

#### Employee Strategy When the Winning Bidder Type Is Private Information

A bidder's optimal strategy depends on the use to which future employees may put the information they gain from the bidding. The employee's subjective beliefs place weight only on the two points  $\{H,L\}$  in the objective probability distribution. Therefore there are only two values which are candidates for the employee's choice of w -- namely w(p,H), the maximum wage which will be acceptable to high cost employers or w(p,L), the maximum wage acceptable to low cost employers. Since low cost employers will be willing to accept high cost wages but not vice versa, we can calculate the employee's expected profits as a function of his subjective assessment of the probability that the employer is a low cost type. Let  $\pi$  denote this subjective probability. If

$$\pi w(\mathbf{p}, \mathbf{L}) > w(\mathbf{p}, \mathbf{H}) \tag{1}$$

the employee will demand wage w(p,L), which we will term an <u>aggressive</u> demand. If the inequality is reversed, he will demand wage w(p,H) a <u>non-agressive</u> demand.

#### An auction with publicly revealed bids.

Assume the auction is run according to the following rules:

1. Each bidder i submits a bid b.

2. The bids are publicly revealed.

3. The low bidder wins the contract at the second lowest bid. (In the case of ties the contract is randomly allocated among low bidders.)

We proceed to demonstrate that this game generically has a unique symmetric equilibrium and to investigate the properties of this equilibrium.

In this game each bidder's strategy is a function  $b_i(m)$ . Consider the strategy chosen by a high-cost bidder. Let  $b_2$  be the lowest bid among the

competitors. The high-cost bidder knows that the <u>minimum</u> wage demand an employee might make will be so high as to make the bidder indifferent between accepting and reneging on the contract. Thus the high-cost bidder knows that his profits are

$$(1-\alpha)(b_2 - H) - \alpha h$$
<sup>(2)</sup>

if he enters a bid below  $b_2$  and zero otherwise. It is therefore a dominant strategy for the high cost bidder to enter a bid of  $b_H$  (provided this is less than any maximum imposed by the auctioneer; if the auctioneer's maximum binds, the high cost bidder will not participate.<sup>5</sup>)

Given this dominant strategy, the employee can only believe that a bidder making a bid of less than  $b_{\rm H}$  is certainly a low cost bidder. Thus there are only two possible strategies for a low cost bidder: Either imitate the high cost type by bidding  $b_{\rm H}$  or bid what would be optimal if the type were publicly known, that is,  $b_{\rm L}$ . Any other strategies are dominated by these two possibilities.

We now calculate the expected profits from each of these two bids. If the low cost type bids  $b_L$  and other bidders also bid  $b_L$ , then the expected profits are zero automatically. Thus the profits from a low bid depend on his being the only bidder at that price. Let d be the probability that no other bidder bids low. Then the expected profits from a bid of  $b_L$  are

$$d[(1-\alpha)(b_{T}-L) - \alpha h]$$
(3)

The second term in the brackets indicates that with probability  $\alpha$  the employee will make a wage demand which pushes the winning bidder to the brink of reneging.

Now consider the profits from a bid of  $b_{H}$ . If anyone else bids  $b_{L}$ , the bidder does not receive the contract, so his profits are zero. He receives

the contract only if everybody else bids  $b_H^{H}$  -- and even then, only l/Nth of the time. Thus the expected profits from to a low cost bidder from a high bid

$$(d/N)[(1-\alpha)(b_{H} - L) + \alpha(b_{H} - L - w)]$$

where w is either  $w(b_{H}^{},H)$  or  $w(b_{H}^{},L)$ . Suppose g is the probability that an employee makes an aggressive demand. Then the expected profit from a high bid can be rewritten:

$$(d/N)[(1-\alpha)(b_{H} - L) + \alpha(b_{H} - L - gw(b_{H}, H) - (1-g)w(b_{H}, L)]$$
  
=  $(d/N)[(1-\alpha)(b_{H} - L) + \alpha g(-h) + \alpha (1-g)[H - L - h]]$  (4)

Since d is positive<sup>6</sup> a comparison of (3) and (4) demonstrates that a low cost type will bid  $b_{I}$  if

 $(1-\alpha) > (1-\alpha g)/N$ 

and will bid  $\boldsymbol{b}_{H}^{}$  if the inequality is reversed.

The employee's choice of aggressive or non-aggressive demand depends on the probability that someone making a high bid is in fact a low cost type. Recall that we denote this probability by  $\pi$ . Substituting the functions w(), and the formulas  $b_{H}$  and  $b_{L}$  into (1) we find that the employee makes an aggressive wage demand if

 $\pi$  (H-L) > (1- $\pi$ ) h /(1- $\alpha$ )

and a passive wage demand if the inequality is reversed.

The parameters  $\pi$  and d in the above analysis depend on the distribution of high and low cost bidders and the probability that low-cost bidders will imitate high-cost bidders. Suppose the probability of such imitation is t. Then the probability that an individual makes a low bid is q(1-t) and

 $d = (1-q(1-t))^{N-1}$ .

The probability that someone is a low-cost type given that he has made a high bid, is

$$\pi = qt / [1-q+qt].$$
 (5)

Thus a symmetric equilibrium is characterized by a pair of numbers (g,t) in the interval [0,1] satisfying the following conditions:

 $t = 0 \text{ if } (1-\alpha) > (1-\alpha g)/N$   $t = 1 \text{ if } (1-\alpha) < (1-\alpha g)/N$   $g = 1 \text{ if } \pi (H-L) > (1-\pi) h / (1-\alpha)$  $g = 0 \text{ if } \pi (H-L) < (1-\pi) h / (1-\alpha).$ 

Recall that t denotes the probability of a low type imitating a high cost type and g denotes the probability that an employee makes an aggressive demand. The quantity  $\pi$  is defined by (5). Define the parameters

$$G = [1-N(1-\alpha)]/\alpha$$
  
T = h (1-q)/ [q (1-\alpha)(H-L)]

Then the characterization of equilibrium simplifies as follows:

Figure 1 illustrates these requirements. It shows the optimal response correspondences g(t) and t(g) for fixed values of the parameters G and T. Note that G < 1 and T > 0. The set of pairs that satisfy the requirements form a manifold in the parameter space; for any parameter values the equilibria form a non-empty connected set; generically the equilibrium is unique.

Generically there are three possibilities:

Case I: G < 0. In this case, t = 0 and g = 0. That is, the unique equilibrium is separating; low cost types always bid  $b_L$  and the employees make non-aggressive demands when they observe a bid of  $b_H$ . In this case, the outcome is identical with the full information base line.

Case II: G > 0 and T > 1. In this case t = 1 and g = 0. That is, the unique equilibrium is pooling; everybody always bids  $b_{H}$  and the employees make non-aggressive demands.

Case III: G > 0 and T < 1. In this case t - T and g - G. Low cost bidders randomize between low and high bids, and the employees randomize between aggressive and non-aggressive demands.

Note that it is never an equilibrium for the employee always to behave aggressively. Moreover, if the employee is sometimes aggressive, the low cost bidders must bid  $b_u$  with positive probability.

The cases may be understood intuitively as follows: If the probability of an employee making a demand is sufficiently small, the auction works exactly as the second price auction without the threat of employee entering. However, as the threat of employee demands becomes sufficiently great, low cost types will want to pretend to be high cost types. The pooling will be complete if complete pooling is insufficient to make employees behave aggressively, for example, if the ex ante probability of being a low cost type is small, or the difference in their costs is small. If the employees would behave aggressively were all low cost types to pool with high cost types, then low cost types will separate sufficiently often to eliminate the benefit; employees will make aggressive demands sufficiently often to eliminate the desirability of pooling.

The auctioneer finds this situation undesirable in comparison with the full information case in two respects. First imitation increases the price

the auctioneer must pay for the contract: sometimes the high price will be paid even though there are two bidders with low costs. Second, the aggressive behavior of the employee means that reneging will sometimes occur. The employees are sometimes hurt by the lack of information, since they will sometimes drive a bidder to renege. High cost types are unaffected by the difference in information; low cost types are ambiguously affected. There are two sources of efficiency loss: the reneging and the fact that the contract does not always go to the lowest cost bidder.

#### An auction with winning bid witheld

We model the oral auction by the following rules:

- 1. Each bidder chooses a bid b<sub>i</sub>.
- 2. All bids except the lowest bid are publicly revealed.
- 3. The low bidder wins the contract at the second lowest bid.

As before, the dominant strategy for a high cost bidder is to bid  $b_{H}$ . Now the employee no longer observes the winning bid, nonetheless, he can infer something about the winning bid from what he does observe -- namely that it must be at least as low as the observed bids. Thus the employee's expectation must be that if ever a bid is observed which is below  $b_{H}$  then the winner (whose bid must be at least as low) must be a low cost type.

Thus by a slightly more complicated argument than in the previous section, a low cost type will choose to bid either  $b_H$  or  $b_L$ . If he bids  $b_H$  he will be pooled with high cost types. If he bids  $b_L$  he will also be pooled with high cost types, provided no one else bids  $b_L$  (because the employees will not be able to deduce his bid unless <u>two</u> bidders make the bid). If he bids  $b_H$ , equation (4) continues to describe his expected profits. If he bids  $b_L$ ,

his expected profits are zero if someone else bids  $b_L$ , but if no one does then his benefits are the same as they would be if he won using  $b_H$ . The sole difference is that if he bids  $b_L$  he gets these profits with probability d; if he bids  $b_H$  he gets them with probability d/N. Thus the low type bidder always prefers to separate.

Although the bids are always separate, an employee may still prefer to make an aggressive wage demand on observing a payment  $b_{H}$ . A high price can be paid either because all bids were high <u>or</u> because exactly one bid was low. Thus, conditional on observing a high bid, the probability of a low bidder winning is

$$N(1-q)^{N-1}q / [(1-q)^{N} + N(1-q)^{N-1}q] = Nq / (1-q+Nq)$$
(6)

Using this expression for  $\pi$ , and substituting into the employee's calculation we conclude that the employee acts aggressively if

 $(H-L)Nq > h(1-q)/(1-\alpha)$ 

and acts non-aggressively if the inequality is reversed. Using the previous definition of T, and letting  $g^*$  denote the probability of an agressive bid

$$g^* = 1$$
 if N > T  
 $g^* = 0$  if N < T

Intuitively this result can be understood as follows: Given the observation of a high price in the auction, the odds of the winner being a low cost type depend on two things: the probability of any one bidder being low cost (q) and the number of bidders. The greater the number of bidders, the less likely that all are high cost, and the more likely that exactly one is high cost.

#### Comparison of the two auctions

Figure 2 divides the parameter space into regions according to the equilibrium to be found in each of the games. The region of greatest interest is the one where T is high and G is high. This region corresponds to letting  $\alpha$  approach 1, i.e., to increasing the likelihood that the employee makes a wage demand.<sup>7</sup> As this happens the prices bid increase to compensate for the expected holdup. This increase in prices makes it more and more possible for the employee to grab a bigger and bigger amount when he does enter, and this means it is not worth the employee's while to risk losing these large amounts by playing aggressively.

In this region, given the non-aggressive play by the employee, the low cost types prefer not to have their type revealed, so that they can retain some benefit even if the employee makes a demand. If their bids are made public, they will therefore imitate high cost types. If the winning bid is not made public, they will be willing to bid truthfully.

The low cost types prefer the auction which does not reveal their bid. In this auction the allocation is efficient: the winner is always the lowest cost bidder. The auctioneer also prefers this auction: it gives him lower bids. Surprisingly, the employee is indifferent: the gains he receives from knowing that a bidder is a low cost type are offset exactly by the lower bid that the low cost type makes.

Therefore this is the parameter region which conforms to the initial intuition: As the costs of revealing the winning bid increase, the second price auction is dominated by the oral auction.

The other regions of the parameter space also merit attention. The region with T and G both low corresponds to a low value of  $\alpha$ . As the hold up problem dissappears, bidders behave identically in the two games, separating

in both. The employees are more likely to behave aggressively when they cannot tell if a winner is really high or low type; since they will make errors, they prefer the publicly observed bid. The low cost type is indifferent between the two auctions (as is the high cost type). Although bidding and payments are identical in the two games, the auctioneer prefers the one in which the winner's bid is publicly revealed, if reneging is costly to the auctioneer as well.

If T is large and G is small the games are identical and identical to the full information case. If T is small and G is large, preferences are ambiguous: there is an inefficiency in the public bid case from partial pooling; there is an inefficiency in the private bid case from more aggressive employee demands.

#### Summary and Extensions

We have described a simple example showing that an oral auction and a second price auction are not identical when the winning bidder is worried about the ex post consequences of revealing his true valuation. If this fear is a dominant consideration, bidders will not give a truthful account of their valuation and as a consequence, the auction will not be efficient. All parties to the auction will prefer an auction which keeps the winner's valuation private. In such circumstances an ascending-bid auction will be preferred to a second-price auction.

The example we have presented is extemely specialized and therefore ripe for generalization. It is natural to consider auctions intermediate between the two we have examined: Suppose, for example, the winner's bid is revealed with probability between zero and one. We have investigated this family of auctions as well; the equilibrium correspondence is continuous in this

dimension, and the equilibria we describe in the paper are the limits as the probability of revelation approaches zero or one.

It is also of interest to investigate the consequences of more complex penalties for reneging. In the case where reneging is costly to the auctioneer, other forms of contracts might be desirable. We have briefly examined the case of penalties proportional to the bid; the results appear similar. Expectational damages correspond to a linear function; it would be worthwhile to examine these as well. More generally, a complete analysis would investigate the question of <u>optimal</u> stipulated penalty in the presence of ex post bargains.



FIGURE 1



FIGURE 2

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<sup>1</sup>Contrast Cassady [1967], chapter 5, with Rothkopf et al., [1990]. For investigations of the equivalences among various forms of auctions, see Milgrom and Weber [1982], Myerson [1981] and Riley and Samuelson [1981].

<sup>2</sup>Rothkopf et al. [1990].

<sup>3</sup>See for example, Tirole [1986].

<sup>4</sup>Provided this bid is less than any maximum imposed by the auctioneer. Bidders of type m will not participate if  $b_m$  exceeds the auctioneer's maximum. <sup>5</sup>Henceforth we therefore assume that the auctioneer's maximum M satisfies the following equation:

 $M > H + \alpha h/(1-\alpha)$ 

If the inequality is reversed the high cost types do not participate, and all low cost types bid  $b_L$ , provided  $b_L$  does not exceed the maximum. (If it does then there is no participation.) When the inequality is reversed there is no distinction between the two types of auctions.

 $^{6}$ Provided b<sub>H</sub> is less than the maximum permitted bid, high cost types will always bid b<sub>H</sub>.

 $^{7}$ As  $\alpha$  increases we must ensure that the bid b<sub>H</sub> continues to lie below the maximum bid permitted.



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