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Abstract

In the recent past, major increases in the allocations to the provision for loan losses by several money center banks have focused the attention of the financial community and regulators on the implications of such decisions for the short term earnings of those banking firms versus their ability to weather major borrower defaults in the medium and long run.

This study examines the process by which a bank determines the size of its loan loss provision for any particular period. This decision is influenced by many internal bank factors as well as competition, regulation, and tax factors. This work approaches the bank's decision with respect to the provision for loan losses from a decision-theoretic standpoint. It is shown that a normative rule results which is not only consistent with the principle of expected utility maximization, but also intuitive and easy to implement.

Introduction

In the recent past, major increases in the allocations to the Provision for Loan Losses by several money center banks in the United States and the United Kingdom, led by Citicorp, have focused the attention of the financial community and regulators to the implications of such decisions for the short-term earnings of those banking firms versus their ability to weather major defaults by Third World sovereign borrowers in the medium and long run.

Little has been said about the process by which a bank arrives or should rationally arrive at a decision with respect to the Provision for Loan Losses. However, that decision in and of itself is of fundamental importance to the bank, not only because it has implications for its capital structure, but primarily due to the penalties associated with making a set of decisions over time which are either too conservative or too aggressive. A bank that is consistently "conservative" in its decision with respect to the Provision for Loan Losses (i.e., the provision consistently exceeds the actual losses by a large amount) will have reduced earnings, a lower leverage multiplier, and reduced growth rates. On the other hand, a bank that is consistently "aggressive" (i.e., the provision consistently falls short of actual losses) will experience increased regulatory attention, pressure to increase capital and, if loan losses are severe enough, ultimate bankruptcy. This point deserves further elaboration.

Consider the following argument. An increase in the riskiness of a commercial bank's loan portfolio has two effects: it increases earnings, but also increases the probability that losses will be incurred. Whatever losses emerge would be provided for by the Loan Loss Reserves (LLR); unanticipated losses over and above that level would result in write-offs of capital, with a consequent change in the capital structure of the banking firm in precisely the opposite direction of that desired by the regulatory authority. To see this, notice that when the banking firm decides on the Provision for Loan Losses in a given period, it does so based on some beliefs or anticipations of possible asset losses. Once this decision is made, three possibilities result. The first is uninteresting and represents the case where the provision exactly matches the losses of the period and no changes in the Loan Loss Reserve result.[1] The second represents the behavior of a "conservative" institution, where the losses turn out to be less than the amount of the provision; in this case, a net addition to the Loan Loss Reserve results. If we accept the inclusion of the Loan Loss Reserve in the broader definition of bank capital,[2] a change in the bank's capital structure also results, in the direction of a less leveraged position. The third possibility is the opposite of the second--an "aggressive" bank would experience actual losses greater than its provision in a particular period. There would be a net decline in the LLR and a consequent change in the capital structure towards a more leveraged position.[3]

This paper attempts to approach the bank's decision with respect to the Provision for Loan Losses from a decision-theoretic or Bayesian standpoint.[4] Figure 1 shows a schematic view of this approach. The three major building blocks which contribute to the bank's decision are its prior information, contemporaneous information (represented by a likelihood function), and a loss function. The prior distribution and the likelihood function, according to Bayesian decision theory, combine to form a posterior distribution for loan losses. Given the loss function, the resulting Provision for Loan Losses is such that minimizes expected posterior loss and therefore maximizes expected utility. The actual loan losses, in turn, are added to the information set represented by the prior distribution and the whole decision cycle starts again. This work argues that, given careful choices of those major building blocks for the model, particularly the loss function, it is possible to arrive at a normative rule for the Provision of Loan Losses which is both theoretically defensible, in the sense of being consistent with the principle of maximization of expected utility, and easy to use.[5]

The paper is organized as follows. Section 1 presents some definitions and the notation which will be used throughout this work. Section 2 discusses the loss function. Section 3 addresses the problem of the appropriate functional form for the prior distribution of loan losses. Sections 4 and 5 form the core of the model. Section 4 explains the application of Bayesian analysis to the problem at hand and in Section 5 the Bayes rule for the Provision for Loan Losses is derived. In Section 6 the important problems of admissibility and robustness of the resulting Bayes rule are considered. Finally, Section 7 presents some concluding observations and suggestions for the implementation of the statistical model suggested in this study.

1. Definitions and Notation

This section presents some fundamental definitions, pertaining to the decision theoretic approach to inference, which will be referred to in several occasions. In addition, the notation which will be used in the several expressions introduced throughout the paper is also presented below.

a. Definitions[6]

(1) A decision problem is a problem in which the decision maker, without knowing the outcome of the experiment, must make a decision, the consequences of which will depend on the outcome of the experiment. The elements of a decision problem are a parameter space Ω, a decision space D, and a real-valued loss function L (the negative of the utility function) which is defined on the product space $\Omega \times D$.

(2) A statistical decision problem is a decision problem in which the decision maker, before choosing a decision from the set D, has the opportunity of observing the value of a random variable or random vector $\underset{\sim}{Y}$ that is related to the parameter W; the observation of $\underset{\sim}{Y}$ provides the decision maker with some information about the value of W which may be helpful in choosing a good decision. The elements of a statistical decision problem are the same as above plus a family of conditional p.d.f.s $\{f(\cdot|w), w\epsilon\Omega\}$ of an observation $\underset{\sim}{Y}$ whose value will be available when a decision is taken.

(3) An estimation problem is a statistical problem in which the decision is the estimate of the value of some parameter vector $\underset{\sim}{W} = (w_1, \ldots, w_k)'$ whose values belong to a subset Ω of $R^k (k \geq 1)$. The

analyst's decision $\underset{\sim}{d} = (d_1, \ldots, d_k)' \in R^k$ is his or her estimate of the value $\underset{\sim}{w} = (w_1, \ldots, w_k)'$ of W, and the loss $L(\underset{\sim}{w}, \underset{\sim}{d})$ which he or she incurs reflects the discrepancy between the value $\underset{\sim}{w}$ and his or her estimate $\underset{\sim}{d}$.

(4) A test of hypotheses is a decision problem in which the decision space D contains exactly two decisions $D = \{d_1, d_2\}$. Decision d_1 is appropriate if the parameter W lies in a certain subset Ω_1 of the parameter space Ω and decision d_2 is appropriate if W lies in the complementary subset $\Omega_2 = \Omega_1^c$. There may be some points in either Ω_1 or Ω_2 for which the decisions d_1 and d_2 are equally appropriate.

(5) Risk is defined as expected loss. A decision maker should choose, if possible, a decision which minimizes the risk (i.e., expected loss), for this decision is consistent with the expected utility hypothesis.

(6) The Bayes risk is defined as the greatest lower bound for the risks of all decisions. Any Bayes decision will be an optimal decision for the decision maker because the risk cannot be smaller for any other decision. It is possible, however, that no decision in the space D is a Bayes decision.

b. Notation[7]

In the discussion that follows the notation introduced below will apply. Some additional conventions may be introduced on occasion for the sake of clarification.

θ the parameter of interest, i.e., net losses in the bank's portfolio of loans and commitments;

Θ the parameter space;

$\mathcal{A}$ the set of all possible actions;

a a particular action; in this study, the action is the amount of the provision for loan losses;

$L(\theta,a)$ a loss function. We will assume it is defined for all $(\theta,a) \in \Theta \times$;

$\underset{\sim}{X} = (X_1, X_2, \ldots, X_n)'$ vector of independent observations from a common distribution, i.e., a random vector representing the outcome of a statistical investigation performed to obtain information about θ;

x a particular realization of X;

$\mathcal{X}$ the sample space, i.e., the set of all possible outcomes;

$\pi(\theta)$ the prior density for θ;

δ a nonrandomized decision rule;

$L(\theta,\delta)$ the loss function for a nonrandomized decision rule;

$R(\theta,\delta)$ the risk function (expected loss) of a decision rule;

$\mathcal{D}$ the class of nonrandomized decision rules with $R(\theta,\delta) < \infty$ for all θ;

$r(\pi,\delta)$ the Bayes risk of a decision rule;

δ^{π} a Bayes decision rule;

$r(\pi)$ the Bayes risk of π (i.e., $r(\pi) = r(\pi,\delta^{\pi})$);

$\ell(\theta)$ the likelihood function (i.e., $\ell(\theta) = f(x|\theta)$);

$m(x)$ the marginal density of X (i.e., $m(x) = \int_{\Theta} f(x|\theta)dF^{\pi}(\theta)$);

$\pi(\theta|x)$ the posterior distribution of θ given x (i.e., $\pi(\theta|x) = f(x|\theta)\pi(\theta)/m(x)$);

C or C(x) a confidence of credible region for θ;

$U(\theta,a)$ utility function;

H_0, H_1 null hypothesis, alternative hypothesis.

2. The Loss Function

A loss function is one of the basic components of a decision-theoretic statistical model. The equivalence of utility maximization and loss minimization is well established in the literature. This equivalence implies that expected loss is the proper measure of loss in a random situation. This fact, in turn, justifies the use of expected loss as a decision criterion when talking about risks as well as Bayes risks.

In this study we will consider two major types of "standard" losses: the squared error loss and the linear loss. The squared-error loss has the form

$$L(\theta,a) = (\theta-a)^2 \tag{1}$$

The use of this type of loss in decision analysis makes the calculations relatively simple, which explains its popularity. Problems, however, arise because one can reason that the loss function should usually be bonded and (at least for large errors) concave. The squared-error loss is neither of these. Moreover, in our problem the symmetry of the squared-error loss is disturbing. The penalties associated with an overestimation of the Provision for Loan Losses (i.e., a decreased growth rate of assets and lower earnings) are smaller than those associated with consistent underestimation (write-offs of capital and ultimate bankruptcy). Thus a generalization of squared-error loss, which is of interest, is

$$L(\theta,a) = w(\theta)(\theta-a)^2 \tag{2}$$

This loss, the <u>weighted squared-error loss</u>, has the attractive feature of allowing the squared error, $(\theta-a)^2$, to be weighted by a function of θ, reflecting the fact that the consequences of an estimation error often vary according to the magnitude of the loan losses.

The second major type of loss of interest for this research is the linear loss. But consider first the loss

$$L(\theta,a) = |\theta-a| \tag{3}$$

which is a particular case of linear loss called <u>absolute error loss</u>. The symmetry of this loss causes the same problems of the squared-error loss in this study. Note, however, that penalties are less severe for large errors.

The general case of <u>linear loss</u> is more interesting. We can write this type of loss as

$$L(\theta,a) = \begin{cases} K_0(\theta-a) & \text{if } \theta - a > 0, \\ K_1(a-\theta) & \text{if } \theta - a \leq 0. \end{cases} \tag{4}$$

Notice that the constants K_0 and K_1, which will usually be different, can be chosen to reflect the relative importance of underestimation and overestimation, a feature that fits well the needs of the problem under study.

The specific choice of a loss function will be discussed in the context of robustness, i.e., the sensitivity of the performance of the decision rule to assumptions with respect to the loss function. We now turn to the analysis of another fundamental component of this model, the prior distribution.

3. Prior Information

Among the techniques available for the subjective determination of a prior density, it seems appropriate for this study to use the matching of a given functional form. That is, we will assume that $\pi(\theta)$ is of a given functional form, and then choose the density of this given form (i.e., the parameters) which most clearly matches prior beliefs.

In this work, we have the relatively rare opportunity of observing the past values of θ, as opposed to having a knowledge of the data, x_i, arising from past θ_i, in which case the recovery of past information from the x_i can be difficult. If values θ_1, θ_2, ..., θ_n of θ (i.e., net loan losses in past periods) are available, it is clear that they should be used in the construction of $\Pi(\theta)$.

Moreover, if the past values of θ are the sole input, the problem of determining the prior distribution is the standard statistical problem of determining a density from a series of observations from that density. Unfortunately, this gain in simplicity comes at the expense of neglecting non-data based information, which is both existent and relevant for this research.[8]

Berger (1980) suggested two ad hoc procedures for combining past data and subjective (i.e., non-data based) prior information.[9] The first is to proceed as follows. Determine the subjective prior $\pi_S(\theta)$ (ignoring the past data) and the past data prior $\pi_D(\theta)$ (ignoring the subjective information). Then choose a number N for which the degree of confidence in π_S would be equivalent to the degree of confidence in a past data prior based on N past observations. If n is the actual number of past observations used in constructing π_D, a natural choice for the combined prior $\pi(\theta)$ is then

$$\pi(\theta) = \frac{N}{n + N} \pi_S(\theta) + \frac{n}{n + N} \pi_D(\theta) \qquad (5)$$

A second possible ad hoc procedure for determining $\pi(\theta)$ is to assume a given functional form for the prior, as suggested above, and then proceed to combine the past data with the subjective beliefs in estimating the parameters of the functional form. Although the ultimate choice of the functional form for the prior distribution of loan losses will rest on the particular loan loss experience of the banking firm when this model is applied, two continuous probability density functions are of special interest due to their wide application in classical statistical analysis, as well as some properties that they exhibit which contribute to a better understanding of this decision-theoretic model.

The first is the univariate normal distribution, which can be written as $\mathcal{N}(\mu,\sigma^2)$: $\mathcal{X} = R^1$, $-\infty < \mu < \infty$, $\sigma^2 > 0$, and

$$f(x|\mu,\sigma^2) = \frac{1}{(2\pi)^{1/2}\sigma} e^{-(x-\mu)^2/2\sigma^2}. \qquad (6)$$

The normal distribution is especially useful to demonstrate the use of conjugate distributions, a procedure that simplifies considerably the calculations leading to the posterior distribution. The second p.d.f. of interest is the Student t distribution. This interest arises mainly from robustness considerations which will be considered below. At this point it suffices to say that, given its "flat" tail, the Student t distribution is a good choice in order to minimize the influence of the tail of the prior on the optimal decision rule. We can write a t distribution with α degrees of freedom as

$$\tau(\alpha,\mu,\sigma^2)\colon \; \mathcal{X} = R^1, \; \alpha > 0, \; -\infty < \mu < \infty, \; \sigma^2 > 0$$

and

$$f(x|\alpha,\mu,\sigma^2) = \frac{\Gamma[(\alpha+1)/2]}{\sigma(\alpha\pi)^{1/2}\Gamma(\alpha/2)} \left(1 + \frac{(x-\mu)^2}{\alpha\sigma^2}\right)^{-(\alpha+1)/2} \qquad (7)$$

where Γ is the gamma function, i.e., for any positive integer n, $\Gamma(n) = (n-1)!$.

Summing up, in the problem under study the fundamental elements for the construction of the prior distribution will be, first, a time series of past loan losses (data-based prior information),[10] as well as tax and regulatory considerations (non-data based prior information). We now proceed to the implementation and evaluation of this decision-theoretic analysis based on the Bayes principle.

4. The Posterior Distribution of Loan Losses: Applying the Bayes Theorem

Bayesian analysis is performed by combining the prior information ($\pi(\theta)$) and the sample information ($\underset{\sim}{x}$) into what is called the posterior distribution of θ given $\underset{\sim}{x}$, from which all decisions and inferences are made. Thus, the posterior distribution $\pi(\theta|x)$ reflects the updated beliefs about θ after observing the sample $\underset{\sim}{x}$.

This assertion can be demonstrated as follows. The joint (subjective) density of θ and X can be written as

$$h(x,\theta) = \pi(\theta)\ f(x|\theta). \tag{8}$$

In addition, the marginal density of the observations X, m(x), is

$$m(x) = \int_{\Theta} f(x|\theta)\pi(\theta)dF^{\pi}(\theta). \tag{9}$$

Substituting equation (8) into equation (9), we obtain

$$m(x) = \int_{\Theta} h(x,\theta)d\theta. \tag{10}$$

It follows that, provided that $m(x) \neq 0$,

$$\pi(\theta|x) = \frac{h(x,\theta)}{m(x)}\ . \tag{11}$$

That is, the posterior distribution, by definition, is the conditional distribution of θ given the sample observation $\underset{\sim}{x}$.

In this study, the sample information is composed by the most recent loan loss experience of a cross-section of commercial banks whose characteristics are as close as possible to the institution under analysis.[11] This procedure may be justified on the grounds that it gives a better perspective of the bank within a group of similar institutions, instead of focusing only on the present experience of the bank under study. The criteria for the selection of this sample would include size, location, and similarities of the structure of the loans and securities portfolio (i.e., this is a proxy for "risk class").[12]

An important objection to this approach is that the institution under study might be an outlier in this sample, in the sense of having a particular loan loss experience far above or below some measure of location (e.g., the sample mean). In this case, the sample information would be essentially irrelevant. A counterargument to this objection is that the posterior distribution follows not only from a combination of the prior distribution and the sample information, but its parameters also reflects the degree of confidence (or quality) that the analyst has in the prior and in the sample.

In order to illustrate this point, consider the simple case of a sample $\underset{\sim}{X} = (X_1, \ldots, X_n)$ from a $\mathcal{N}(\theta,\sigma^2)$ distribution (σ^2 known). In addition, assume that $\pi(\theta)$ has a $\mathcal{N}(\mu,\tau^2)$ density. Nothing that $\overline{X} \sim \mathcal{N}(\theta,\sigma^2/n)$ it follows that the posterior distribution of θ, given $\underset{\sim}{x} = (x_1, \ldots, x_n)$, is $\mathcal{N}(\mu(\underset{\sim}{x}),\rho^{-1})$, where

$$\mu(\underset{\sim}{x}) = \frac{\sigma^2/n}{(\tau^2+\sigma^2/n)}\,\mu + \frac{\tau^2}{(\tau^2+\sigma^2/n)}\,\overline{x} \tag{12}$$

and

$$\rho = (n\tau^2+\sigma^2)/\tau^2\sigma^2. \tag{13}$$

The upshot of this example is to show that the precision measures of the prior and the sample information will function as weights when computing the parameters of the posterior distribution.

This example is also helpful to demonstrate the use of conjugate families. The calculation of the posterior distribution can be greatly simplified by finding a conjugate prior. The usual procedure is to examine the likelihood function $\ell_x(\theta) = f(x|\theta)$ and choose, as a conjugate family, the class of distributions $\mathcal{F}$ with the same functional form as the likelihood function.

The use of conjugate priors is appealing because it allows one to start with a prior of a given functional form and end up with a posterior distribution of the same functional form, but with parameters updated by the sample information. A note of caution should be added here, however. The basic question when choosing a priori distribution is whether or not a conjugate prior can be chosen which gives an approximation to the true prior, for it is this latter quality of the prior that is central to the accuracy of the Bayesian approach.

The logical sequence to the above line of reasoning is to perform the Bayesian inference based on the posterior distribution. Since the posterior distribution supposedly contains all the available information about θ (both sample and prior information), any inferences concerning θ should be made solely through this distribution. To estimate θ, a number of classical techniques can be applied to the posterior distribution, the most common being maximum likelihood estimation (MLE).

In this work, however, we have a major concern not only with the role of the prior information, but with the influence of the loss function as well. We face, in this work, a so-called "true decision problem." In other words, we are interested in deriving optimal statistical decision rules for the choice of the Provision for Loan Losses in commercial banks, in examining the admissibility of these rules and their robustness with respect to changes in the prior p.d.f. as well as the loss function, and in comparing these optimal rules with the actual choices (provisions) made by banks. Moreover, we are ultimately interested in how capital structure and asset portfolio regulations affect these decision rules and, a fortiori, the capital structure and solvency of banking firms. This analysis can be performed with the use of Bayesian decision theory, which forms the core of this statistical model. The Bayes rule, given the structure of this model, is derived in the next section.

5. Derivation of the Bayes Rule: The Optimal Decision Regarding Loan Losses

Two important assumptions will be introduced at this point. These assumptions are necessary to carry out the analysis below. We will assume, first, that the prior p.d.f. is proper. Second, we will assume that the problem has a finite Bayes risk. The method that will be used in this study for determining a Bayes rule is known as the extensive form of Bayesian analysis. This method may be developed as follows. Write the Bayes risk of a decision rule as

$$r(\pi,\delta) = \int_{\Theta} R(\theta,\delta)dF^{\pi}(\theta)$$

$$= \int_{\Theta}\int_{\mathfrak{X}} L(\theta,\delta(x))dF^{x|\theta}dF^{\pi}(\theta)$$

$$= \int_{\mathfrak{X}} [\int_{\Theta} L(\theta,\delta(x))f(x|\theta)dF^{\pi}(\theta)]dx, \quad (14)$$

in the case of continuous distributions. In order to minimize Bayes risk, i.e., the quantity in the right-hand side of equation (14), $\delta(x)$ should be chosen to minimize the expression inside the brackets, that is,

$$\int_{\Theta} L(\theta,\delta(x))f(x|\theta)dF^{\pi}(\theta)$$

for each $x \in \mathfrak{X}$. But note that if an action a minimizes

$$\int_{\Theta} L(\theta,a)f(x|\theta)dF^{\pi}(\theta)$$

then the same action a minimizes

$$[m(x)]^{-1}\int_{\Theta} L(\theta,a)f(x|\theta)dF^{\pi}(\theta) = \int_{\Theta} L(\theta,a)dF^{\pi(\theta|x)}(\theta). \quad (15)$$

The quantity in the r.h.s. of equation (15), i.e., the expected loss with respect to $\pi(\theta|x)$, the posterior distribution of θ given x, is called the posterior expected loss of the action a. This quantity is the same as the one which is called (somewhat loosely) "average loss" in Figure 1.

This result is summarized by Berger (1980) as follows: "A Bayes rule can be found by choosing, for each x, an action which minimizes the posterior expected loss, or equivalently, which minimizes

$$\int_{\Theta} L(\theta,a)f(x|\theta)dF^{\pi}(\theta)." \quad (16)$$

[13]

In order to obtain specific Bayes rules, we need to spell out the loss function that applies to the problem under study. This is one of the crucial points in the application of this model. The particular features of the problem under analysis, as well as robustness considerations (to be discussed below), are the major factors to be taken into consideration in the choice of the loss function. As discussed before, for this study two types of losses are of interest: the weighted squared-error loss and the linear loss. It can be demonstrated that the Bayes rules for these loss functions are the following.[14]

Consider the weighted squared-error loss. If $L(\theta,a) = w(\theta)(\theta-a)^2$, the Bayes rule is

$$\delta^{\pi}(x) = \frac{E^{\pi(\theta|x)}[\theta w(\theta)]}{E^{\pi(\theta|x)}[w(\theta)]}$$

$$= \frac{\int \theta w(\theta) f(x|\theta) dF^{\pi}(\theta)}{\int w(\theta) f(x|\theta) dF^{\pi}(\theta)} . \tag{17}$$

Thus, the Bayes rule is a ratio of weighted averages of the posterior distribution. While the weight function plays a role similar to that of the prior $\pi(\theta)$, an interesting fact given robustness concerns, on the whole this decision rule does not seem attractive given the objectives of this study, for two reasons: first, because it does not suggest intuitively any particular location parameter or fractile of the posterior distribution; second, because of the disturbing presence of the (unknown) parameter of interest in the weight function.

The second type of standard loss function considered in this study--the linear loss--offers a more attractive result. If we rewrite the linear loss as

$$L(\theta,a) = \begin{cases} K_0(\theta-a) \text{ if } \theta - a \geq 0 \\ K_1(a-\theta) \text{ if } \theta - a < 0, \end{cases} \tag{4}$$

then <u>any</u> $(K_0/(K_0+K_1))$ fractile of $\pi(\theta|x)$ is a Bayes estimate of θ.

This result has several appealing features given the problem of estimating loan losses in a bank's portfolio. First, it is intuitive: it is relatively easy to conceptualize a fractile of a p.d.f. Second, it allows for the asymmetric effects of overestimation and underestimation to be reflected in the optimal decision rule. Third, and perhaps most important, the weights K_0 and K_1 can be used to represent the impact of different regulatory regimes. Thus, when robustness considerations with respect to the choice of the prior distribution and the loss function arise, this choice of loss function allows the impact of the regulator to be felt on both.

To sum up our progress thus far, we have been able to show that, under reasonable assumptions with respect to the choice of a loss function and a prior distribution of loan losses, a Bayes decision rule for the choice of the Provision for Loan Losses emerges which is both theoretically sound and intuitive. The remaining question to be dealt with in this study pertains to the admissibility of the Bayes rule and its robustness with respect to changes in the prior distribution and the loss function.

6. Assessing the Degree of Confidence on the Estimate of Loan Losses: Admissibility and Robustness of the Bayes Rule

Under the two basic assumptions introduced in the previous section, namely, that the prior p.d.f. is proper and the problem has a finite Bayes risk, there is little need for concern with respect to admissibility for the Bayes rules.

With proper priors, Bayes rules are virtually always admissible. The basic reason for this virtual certainty is that, if a rule with better $R(\theta,\delta)$ existed, that rule would also have better Bayes risk, since

$$r(\pi,\delta) = E^{\pi}[R(\theta,\delta)] \tag{18}$$

given the assumption that the Bayes risk of the problem is finite. As in the case of the assumption of proper priors, formal Bayes rules need not be admissible if their Bayes risks are infinite.

The issue of robustness deserves a more careful investigation. The robustness of a decision rule may be defined as the sensitivity of the rule to changes in the model's assumptions. In particular, in the case of decision-theoretic models, we are interested in robustness with respect to the sample density, the loss function, and the prior density. In the model formulated in this study, we will be especially concerned with the loss function and the prior density. The sample density, as discussed above, comes from a cross-section of banks with characteristics similar to the banking firm under study. Problems of sample selection bias are likely to arise, but these are beyond the scope of this work.

Next, in an increasing order of importance with respect to robustness, comes the robustness of a decision rule with respect to the loss function. The feature of a loss which can cause the most serious robustness difficulties is a weighting factor $w(\theta)$. However, decision rules are usually robust with respect to the specification of large errors. In the case of this study, since a linear loss (i.e., a loss of the form $L(\theta-a)$) is primarily used, the decision rule is usually robust with respect to the form of L for large $(\theta-a)$.[15]

This point is both reassuring and important. Since we are essentially free of robustness concerns with respect to our chosen loss function, we can conceptualize changes in its parameters K_0 and K_1 as effects of the regulatory regime on the perceived consequences of overestimation and underestimation of loan losses. In other words, we can vary K_0 and K_1 until the Bayes decision coincides with the actual decision and, when that happens, observe the values of K_0 and K_1 and evaluate the relative emphasis placed on perceived losses due to underestimation or overestimation. Alternatively, it is possible to choose values for K_0 and K_1 and examine the difference between the Bayes decision and the actual decision for several groups of banking firms (by size, region, etc.). We now can turn to robustness with respect to the prior p.d.f., which seems to be the major cause for concern in the case of this study.

The concern about robustness with respect to the specification of the prior distribution comes from the fact that, in a Bayesian analysis, one could be led into making a poor decision because of an inadequate description of prior beliefs. Given our assumption that the prior p.d.f. is proper, the basic issue is to determine the degree of accuracy of the prior specification needed for the analysis.

In the case of this study, since we are dealing with typical subjectively chosen priors, it is necessary to distinguish between the "central" portion of the prior (i.e., the part that corresponds to, say, 90 percent or 95 percent of the a priori credible region of θ) and the "tail" (the extreme regions of small probability). This is so because, first, as noted by Berger (1980), Bayes procedures will usually be robust with respect to small changes in the central portion of the prior, but only rarely will be robust with respect to large changes. Thus, it is important to try to accurately specify the central portion of the prior.[16] The tail of the prior, in contrast, is hard to specify, so robustness with respect to this tail is desirable. One way to minimize the influence of the tail of the prior is to use a prior with a "flat" tail. In particular, when the observation x is extreme,[17] in the sense that the likelihood function $\ell(\theta) = f(x|\theta)$ gives considerable weight to the tail of the prior, the posterior distribution will be significantly affected by the type of prior tail chosen. This, in turn, will cause a lack of robustness.

The upshot of this argument is that the use of conjugate priors, and in particular conjugate normal priors, while very convenient, can be dangerous if x is extreme. That is why, in the previous discussion of the possible choices for prior distributions, the use of the Student _t_ prior has been suggested as an interesting alternative to the normal prior. Its use seems to be quite adequate if, as in this study, one uses the functional form approach to develop the prior. Some concluding observations are presented next.

7. Concluding Observations

This paper has attempted to approach the bank's choice of the Provision for Loan Losses from a decision-theoretic standpoint. We have argued that this approach produces a decision rule which is theoretically defensible, intuitive, and easy to implement. The statistical model leading to such rule is a Bayesian model. Most of our discussion concerned the examination of the three major building blocks of the model: the loss function, the prior distribution, and the likelihood function. A final word concerning the application of this model is now in order.

This model is not only superior to an arbitrary choice of the Provision for Loan Losses, but addresses several objectives. First, it may be used as a normative model in order to provide guidance for the optimal choice of the Provision for Loan Losses in commercial banks. Second, it provides a rigorous way to evaluate the actual decisions made by banking firms and to compare them with the optimal Bayes decisions. Third, it allows us to investigate the impact of regulatory constraints on the decision, both through the prior distribution and the loss function, and to obtain meaningful conclusions with respect to the effectiveness and the desirability of regulatory actions, given their impact on the Loan Loss Reserve and ultimately on the capital structure of the banking firm.

NOTES

[1]We abstract here from possible intuitional complications and accounting practices that might allow the bank to make such a decision ex post for practical purposes.

[2]For a detailed discussion of the new bank capital standards and the "nine percent capital rule," see R. Alton Gilbert, Courtenay C. Stone, and Michael E. Trebing (1985, May). The new bank capital adequacy standards, Monthly Review, Federal Reserve Bank of St. Louis 67 (5), pp. 12-20.

[3]The labels "conservative" and aggressive" should not be interpreted strictly. Conservative banks may make insufficient provisions in certain periods and aggressive banks may do the reverse. The idea is to capture a consistent or overall behavioral pattern. Also, we are abstracting here from recoveries occurring during the period, which would also be added to the LLR and change the bank's capital structure.

[4]For an alternative perspective, see, for example, David C. Cates (1985, March), What's an adequate loan loss reserve? ABA Banking Journal LXXVII (3), p. 42.

[5]The intellectual debt that this research owes to George Vojta's works is an important one. In the development of the statistical model below, what is essentially proposed is to provide a sound theoretical justification for his views, with the use of statistical decision theory and Bayesian methods. See Vojta (1973a, 1973b).

[6]Most of the definitions are taken from Morris H. DeGroot (1970), Optimal Statistical Decisions (New York: McGraw-Hill), several chapters.

[7]This notation follows James O. Berger (1980), Statistical decision theory: Foundations, concepts, and methods (New York: Springer-Verlag).

[8]Non-data based prior information takes primarily the form of tax and regulatory considerations. See Figure 1.

[9]Berger (1980), pp. 83-84.

[10]The length of this time series is arguable. It should be long enough to allow an approximation of a continuous p.d.f. It seems reasonable to say that 50-60 data points would suffice.

[11]See Figure 1.

[12]The size of this sample is also arguable. In practice, we would be inclined to accept the largest sample that meets the criteria mentioned in the text.

[13]Berger (1980), Result 1, p. 109, emphasis mine. It should be noted that the expected posterior loss might have more than one minimizing action, so there might be more than one Bayes rule. Also, refer to Figure 1 for an overall view of the logic of this method.

[14]For a derivation of these results see, for example, Berger (1980), pp. 111-112.

[15]For a more detailed discussion of this point, see Berger (1980), pp. 128-129.

[16]Berger (1980), p. 140.

[17]Recall that the likelihood function is formed by a cross-section of similar banks and represents contemporaneous loan losses. In this regard, x can be taken as the location parameter of that p.d.f.

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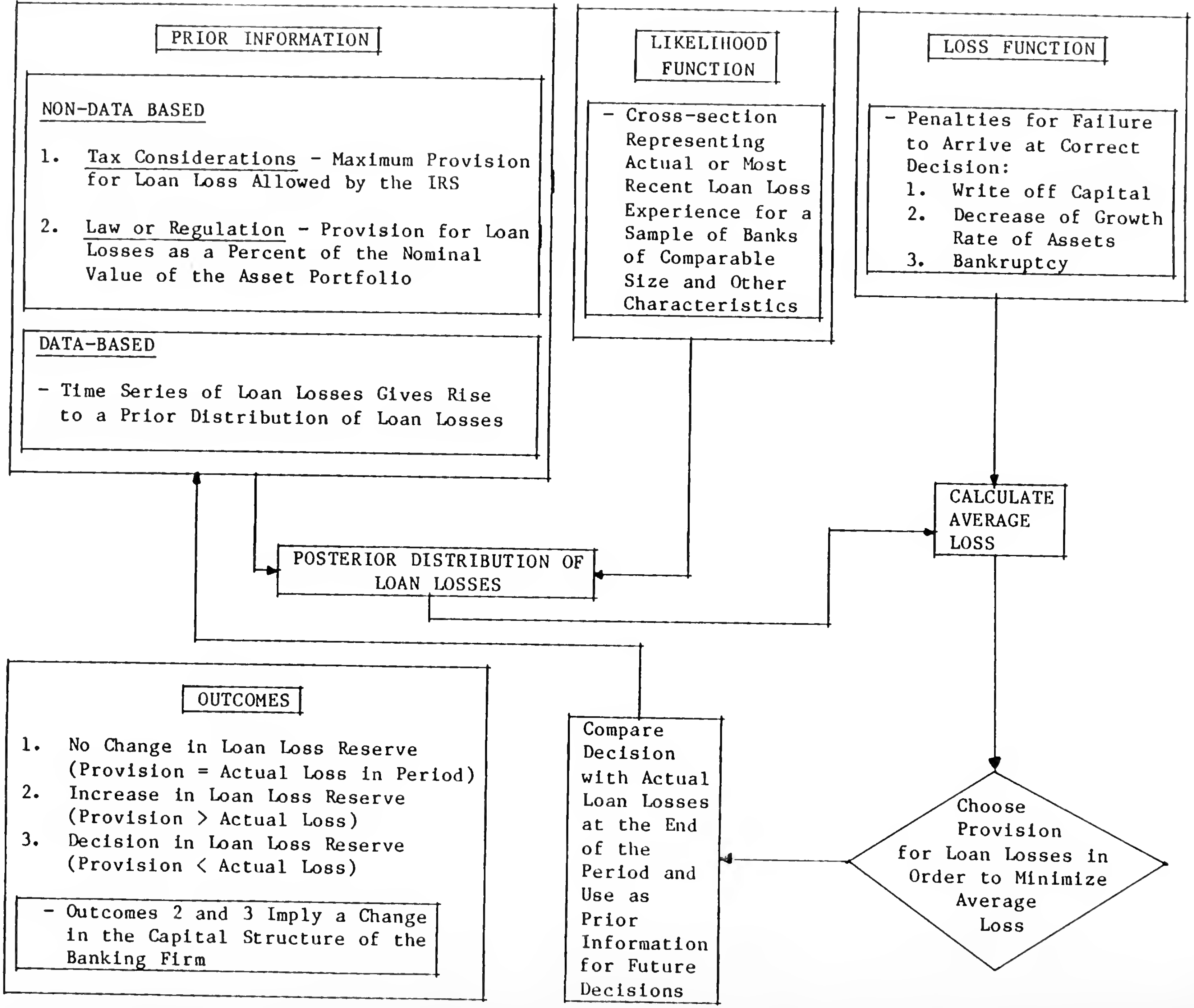

Figure 1. The Decision of the Regulated Banking Firm.
(Parameters of interest are the losses on the loans and leases portfolio.)