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Abstract

This shows that when public goods are intermediate inputs as well as final consumption goods, stationarity of aggregate property value under a marginal change in public spending implies satisfaction of a generalized Samuelson efficiency condition. This result shows that the empirical test for efficiency of Brueckner (1982) remains valid when public goods enter firm production functions.

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by

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1. Introduction

Several recent papers (Brueckner (1979, 1982)) showed that if a marginal increase in public spending has no effect on aggregate property value in an open community (holding the housing stock fixed), then the Samuelson condition for allocative efficiency in the provision of public goods is satisfied. Brueckner (1982) used this result to argue that if a cross-section regression relating aggregate property values to explanatory variables yields a zero public spending coefficient, this must be viewed as evidence of public sector efficiency in the sample (results from a Massachusetts sample did in fact show a zero spending coefficient).

This connection between property values and efficiency was demonstrated in a model where public goods benefit consumers without affecting the operation of firms. Given that public services such as police and fire protection are actually essential inputs for many businesses in the real world, the practical relevance of the above efficiency result might therefore appear questionable. The purpose of this letter is to show that, in fact, the main lessons of the previous analysis generalize to a more realistic setting where public goods enter firm production functions as well as consumer utility functions. We show that in such a world, local stationarity of aggregate property value implies satisfaction of a generalized Samuelson efficiency condition. Thus, even when public goods are intermediate inputs, a zero public spending coefficient in an aggregate property value regression is evidence of public sector efficiency.

In the next section of the letter, we derive the generalized Samuelson efficiency condition. The subsequent section investigates the behavior of property values.

2. Allocative Efficiency

For simplicity, we assume that the economy has a single pure public good,[1] denoted z, and two private goods, x and y. Consumer i's tastes are represented by the strictly quasi-concave utility function $u_i(x_i, y_i, z)$, i=1, ..., n. Commodity y is produced with inputs of x and the public good z by m individual firms according to the concave production functions $y^j = f^j(x^j, z)$, j=1, ..., m (superscripts denote firm variables). The public good is produced using only x as an input (the concave production function is $z = t(x_z)$). A Pareto-efficient allocation is found by differentiating the Lagrangean expression

$$\begin{aligned}
&\sum_{i=1}^{n} \lambda_i (u_i(x_i, y_i, z) - \bar{u}_i) \\
&\quad - \theta\Big(\sum_{i=1}^{n} y_i - \sum_{j=1}^{m} f^j(x^j, z) \Big) \qquad (1) \\
&\quad - \mu\Big(\sum_{i=1}^{n} x_i + \sum_{j=1}^{m} x^j + x_z - \bar{x} \Big) \\
&\quad - \gamma\big(z - t(x_z) \big),
\end{aligned}$$

where θ, μ, γ and λ_i, i=2, ... n, are multipliers (λ_1=1), $\overline{u}_i$, i=2, ..., n, are fixed utility levels ($\overline{u}_1$=0), and $\overline{x}$ is the economy's x-endowment. Differentiating (1) and simplifying yields the standard result

$$\frac{u_{iy}}{u_{ix}} = \frac{\theta}{\mu} = \frac{1}{f_x^j}, \text{ all } i,j \qquad (2)$$

(subscripts denote partial derivatives) together with the generalized Samuelson condition

$$\sum_{i=1}^{n} \frac{u_{iz}}{u_{ix}} + \sum_{j=1}^{m} \frac{f_z^j}{f_x^j} = \frac{1}{t'} \quad . \qquad (3)$$

Eq. (3) states that the public good level is efficient when the sum of the marginal rates of substitution between z and x plus the sum of the rates of technical substitution between z and x equals the marginal cost of the public good in terms of x.[2] Intuitive insight into condition (3) can be gained by noting that since $1/f_x^j$ equals the MRS between y and x by (2), the ΣRTS^j expression is just the sum over j of terms equal to the marginal product of z in firm j times the marginal value of y in terms of x. Thus, the LHS of (3) gives the marginal value to consumers of extra z, with both the direct consumption and intermediate input effects counted. Note finally that if $f_z^j \equiv 0$ for all j (if the public good is not an intermediate input), then (3) reduces to the ordinary Samuelson condition.

3. Property Values

In the discussion of property values, we introduce another private good, the services of real estate, into utility and production functions. For consumers, this commodity is simply housing services, while for firms, the commodity is the services of commercial building space. Given that a distortionary property tax is in fact levied on structures, the efficiency issue becomes more complicated when real estate services are introduced. As shown by Atkinson and Stern (1974), the Samuelson condition (or, here, its generalization) is inappropriate when public spending is financed by distortionary taxation. To circumvent this complication, Brueckner (1982) adopted the notion of public sector efficiency conditional on a non-optimal stock of structures. If real estate services are added to the model of section 2, and the levels of such services are held fixed at their (non-optimal) equilibrium values, then conditional efficiency is achieved by allocating the economy's remaining (non-structure) resources in a Pareto-optimal fashion. Clearly, the generalized Samuelson condition is a necessary condition for such an allocation. Note that the notion of conditional efficiency is sensible given the long lifespans of residential and business structures. With structures providing non-optimal service levels but essentially frozen in place, a pragmatic planning goal is to allocate society's remaining resources in an efficient manner.

We now proceed to show that stationarity of aggregate property value in an open community implies satisfaction of the generalized Samuelson condition (and thus conditional efficiency). Focusing first on consumers, we note that the assumption that the community is open means that consumers of each type must enjoy the prevailing economy-wide utility level for their type. Let w_i and I_i denote the prevailing utility level and income for the consumer type to which i belongs (note that consumers i and k may belong to the same type, in which case they have identical utility functions and values of w and I). Then, letting p denote the price of good y and q_i denote the housing services consumed by individual i (as measured by the size of his house), the consumer's rental payment R_i must satisfy

$$\max_{y_i} u_i(I_i - R_i - py_i, y_i, q_i, z) = w_i \qquad (4)$$

(note that x, which is numeraire, has been eliminated via the budget constraint). Eq. (4) implicitly defines the consumer's bid-rent function $R_i(q_i, z_i; w_i, I_i)$, which gives the rental payment consistent with utility w_i as a function of q_i and z. Differentiating (4) and using $u_{iy}/u_{ix} = p$, it follows that $\partial R_i/\partial z = u_{iz}/u_{ix}$, indicating that rent must increase at a rate equal to the MRS between z and x to keep utility constant (for fixed q_i) as z increases.

Turning to firms, we assume in contrast to the discussion of Section 2 that production functions are identical. This allows us to consistently impose a zero-profit requirement on all firms. Letting R^j denote the rent paid by firm j and q^j denote its consumption of building services, zero profit requires

$$\max_{x^j}(pf(x^j, q^j, z) - x^j - R^j) = 0. \qquad (5)$$

This equation implicity defines the firm's bid-rent function $R^j(q^j,z)$, which, using $pf_x^j = 1$, satisfies $\partial R^j/\partial z = f_z^j/f_x^j$ (superscripts reflect the fact that q will be different across firms due to the fixity of structures).

Having derived bid-rent functions, we are now in a position to prove our principal result. First, note that the value v of a structure earning rent R must satisfy the relationship $v = (R-\tau v)/r$, where τ is the property tax rate and r is the discount rate. Summing across residential and commercial structures, aggregate property value $V \equiv \sum_i v_i + \sum_j v^j$ therefore satisfies

$$V = \frac{1}{r}\left(\sum_i R_i + \sum_j R^j - C(z)\right), \qquad (6)$$

where $C(z) \equiv t^{-1}(z)$ is the cost function for the public good (the government budget constraint $\tau V = C$ has been used). Differentiating (6) using the above results, we find

$$\frac{\partial V}{\partial z} = \frac{1}{r}\left(\sum_i \frac{u_{iz}}{u_{ix}} + \sum_j \frac{f_z^j}{f_x^j} - C'\right). \qquad (7)$$

Eq. (7) tells how aggregate property value must vary with z, holding the stock of structures (and the allocation of tenants within it)[3] fixed, in order to maintain profits at zero and utilities at prevailing external levels. Comparing (7) and (3), the main result of the analysis emerges: stationarity of aggregate property value ($\partial V/\partial z=0$) implies satisfaction of the generalized Samuelson condition.[4] Eq. (7) also yields the efficiency result of Brueckner (1982) as a special case. Inspection of the equation shows that when public goods are not intermediate inputs, $\partial V/\partial z = 0$ implies satisfaction of the ordinary Samuelson condition.

The above discussion shows that the efficiency test of Brueckner (1982) remains valid in a world where public goods are intermediate inputs as well as final consumption goods. If an aggregate property value regression yields a zero public spending coefficient in such a world, then, as in the case where consumers alone care about public goods, it must be true that the relevant efficiency condition is satisfied on average in the sample communities. As before, this means that public outputs in the sample are typically efficient conditional on existing stocks of structures.

Footnotes

[1]Public good congestion could be introduced without changing the argument. Also, the simple production framework below could be made more general (see Sandmo (1972)) without affecting the conclusions of the analysis.

[2]Sandmo (1972), who analyzed the case where the public good is solely an intermediate input, derived a condition analogous to (3) (his condition results from setting $u_{iz}=0$, $i=1, \ldots, n$, in (3)).

[3]Under the bid-rent framework, properties are allocated to the highest bidder. A complication ignored in Brueckner (1982) is that as z changes, the identity of the highest bidder for a given house may change. Thus, while small changes in z will not alter the allocation of people to houses, so that differentiation of (6) gives the change in V, large changes may alter the identities of the various occupants, so that movement along expression (6), with identities held fixed, incorrectly measures the change in V. Since the behavior of V for small changes in z is the key to the argument, however, this complication has no substantive effect.

[4]Since it may be shown that V is a strictly concave function of z under the maintained assumptions, it follows that the stationary point given by (7) in fact represents a property value maximum with the q's held fixed.

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