College Entrance Examination Board

EXAMINATION QUESTIONS

IN

MATHEMATICS

THIRD SERIES

1911-1915

GINN AND COMPANY
BOSTON · NEW YORK · CHICAGO · LONDON
ATLANTA · DALLAS · COLUMBUS · SAN FRANCISCO

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PREFACE

While the annual volume of examination questions published by the College Entrance Examination Board has met the needs of many candidates for examination and their teachers, the Board is constantly in receipt of communications asking for the questions set in certain subjects in successive years. In order to meet this demand the Board has prepared pamphlets containing the questions in certain subjects from 1911 to 1915 inclusive. These pamphlets are as follows:

1. Examination questions in Latin and Greek, 1911–1915.
2. Examination questions in English and other modern languages, 1911–1915.
3. Examination questions in mathematics, 1911–1915.
4. Examination questions in history, 1911–1915.
5. Examination questions in the natural sciences and in drawing, 1911–1915.

Besides meeting the needs of candidates for examination and their teachers, these publications ought to have more widely a beneficial influence upon teaching for the reason that they illustrate in concrete form principles agreed upon by many leading teachers of the subjects represented.

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MATHEMATICS A

ELEMENTARY ALGEBRA COMPLETE

MATHEMATICS *AI and II*—ELEMENTARY ALGEBRA COMPLETE

Monday 9.30 a. m.–12.30 p. m.

Six questions are required; two from Group A, two from Group B, and both questions of Group C. No extra credit will be given for more than six questions.

Group A

1. (*a*) Factor each of the following:
   (1) $(a^2-4)^2-(a+2)^2$
   (2) $2(x^3-1)+7(x^2-1)$
   (3) $x^4-7x^2y^2+y^4$
   (*b*) Simplify $\dfrac{a+x}{2a-2x}+\dfrac{x-a}{4x+4a}+\dfrac{a^2+14ax+x^2}{8x^2-8a^2}$.

2. (*a*) Simplify $(\sqrt{7-3\sqrt{5}})\,(\sqrt{7+3\sqrt{5}})$.
   (*b*) Given $a^2(x^2-yz)^{-\frac{2}{3}}$. Introduce a^2 into the parenthesis without changing the value of the expression.
   (*c*) Simplify $\left\{\sqrt[5]{\dfrac{a^{\frac{1}{2}}x^{-2}}{x^{\frac{1}{2}}a^{-2}}}\div\sqrt{\dfrac{x^{-1}\sqrt{a}}{a\sqrt{x}}}\right\}^{-4}$.

3. A man starts from his home to catch a train, walking at the rate of one yard in one second, and arrives two minutes late. If he had walked at the rate of four yards in three seconds he would have arrived two and a half minutes early. Find distance from his home to the station.

Group B

4. (*a*) Solve $x^{\frac{4}{3}}+7x^{\frac{2}{3}}=8$.
   (*b*) Find the middle term of $\left(2a^3-\dfrac{3b^2}{4}\right)^8$.

5. (*a*) Solve $\sqrt{3x+10}=\sqrt{10x+16}+\sqrt{x+2}$. Which value of x satisfies the equation?
   (*b*) In the equation $x^2+mx+k=0$, what relation exists between m and k if one root of the equation is twice the other?

6. (*a*) Three numbers whose sum is 24 are in arithmetic progression. If 2, 6, and 17 are added to them respectively, the results are in geometric progression. Find the numbers.
   (*b*) (1) Plot the graphs of the following system of equations and from these graphs find the approximate values of x and y.
   $$\begin{cases} x^2+y^2=4 \\ 3x-2y=6 \end{cases}$$
   (2) Plot the graph of $y=2x^2-11x+5$ and from the graph determine the values of x which make $y=0$.

7. Solve $\begin{cases} 2x^2-1=y^2+3xy \\ 5-x^2=y^2 \end{cases}$

Write results so that with each value of x, the proper value of y is associated.

8. On the same day A, B, and C start to solve a certain number of problems. A solved 6 a day and finished them 4 days after B. C solved 3 more a day than B and finished 2 days before he did. Find the number of problems and the number of days each worked.

MATHEMATICS *AI and AII*—ELEMENTARY ALGEBRA COMPLETE

Monday 9.30 a.m.–12.30 p.m.

No extra credit will be given for more than six questions.

GROUP A. (*Omit one question of this group.*)

1. (*a*) Simplify $\frac{8c^3-1}{9c^2-12c+4}\cdot\left(1-\frac{4}{3c+2}\right)\div\left(\frac{2c-1}{9c^2-4}\right)$.

   (*b*) Solve for x, $\frac{b^2}{ax}-\frac{1}{a}-\frac{3b-3a}{x}=\frac{a^2}{bx}-\frac{1}{b}$.

2. (*a*) Simplify $\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)^2+\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)^2$.

   (*b*) Simplify $xy^{\frac{2}{3}}\left(\frac{x^{-\frac{1}{2}}}{y^{-\frac{2}{3}}}\right)^2\div\left(x^{\frac{1}{4}}y^{-3}\right)^{-\frac{2}{3}}$.

3. A man has \$1.80 in nickels, dimes, and quarters, 12 coins in all. If the number of dimes and quarters were interchanged he would have \$1.65. How many of each has he?

GROUP B. (*Omit one question of this group.*)

4. (*a*) Solve $\sqrt{7x+1}-\sqrt{3x+10}=1$.
   Test the values found and explain why one of them does not satisfy the equation.

   (*b*) Solve for x and y, $x-y=1$,

   $$\frac{x}{y}-\frac{y}{x}=\frac{5}{6}.$$ Check the results.

5. A man worked a number of days and earned \$75. If he had received 50 cents more per day, he would have earned the same amount in 5 days less. How many days did he work?

6. Construct with respect to the same axes of reference the graphs of

   $$y+2x^2-3x-9=0,$$
   and
   $$y+x-3=0.$$
   Estimate from the figure the values of x and y which satisfy both equations.

GROUP C. (*Answer both questions of this group.*)

7. (*a*) An elastic ball bounces to three-fourths the height from which it falls. If it is thrown up from the ground to a height of 15 feet, find the total distance traveled before it comes to rest.

   (*b*) Find the coefficient of x^4 in $\left(2x^2-\frac{1}{4x}\right)^8$.

8. (*a*) Solve $x^2+5x+d=0$. What is the least integer which, when substituted for d in the equation, makes the roots of the equation imaginary?

   (*b*) The sum of 3 numbers in geometrical progression is 70. If the first be multiplied by 4, the second by 5, and the third by 4, the resulting numbers will be in arithmetical progression. Find the three numbers.

MATHEMATICS *A*—ELEMENTARY ALGEBRA COMPLETE

Monday 9–11 a.m

No extra credit will be given for more than six questions.

GROUP A. (*Answer both questions of this group.*)

1. *a*) Factor

$$2mx+6ny-my-12nx,$$
$$6x^2+11x-10,$$
$$x^4-a^3x+bx^3-a^3b.$$

b) Simplify $1-\left\{\frac{c^3+y^3}{(c-y)^2}\div\left[\frac{c^4+c^2y^2+y^4}{c^3-y^3}\times\frac{(c+y)^2}{c^2-y^2}\right]\right\}.$

2. *a*) Simplify and combine $\frac{6}{\sqrt{3}}-18\sqrt{\tfrac{1}{3}}-\tfrac{1}{6}\sqrt{108}+12^{\frac{1}{2}}+3^{\frac{3}{2}}+\frac{\sqrt{3}}{5^{-1}}.$

b) Rationalize the denominator and simplify

$$\frac{\sqrt{3}-\sqrt{\tfrac{1}{2}}}{\sqrt{2}+\sqrt{\tfrac{1}{3}}}.$$

GROUP B. (*Answer both questions of this group.*)

3. *a*) Solve $\begin{cases}2x+y=-2,\\ 2xy-y^2+6x+9=0.\end{cases}$

Associate properly the values of x and y.

b) Solve $\sqrt{\frac{x}{2x+1}}+2\sqrt{\frac{2x+1}{x}}=3.$

4. *a*) Solve $\begin{cases}\frac{1}{x^2}+\frac{1}{y^2}=4c^2+d^2,\\ xy=-\frac{1}{2cd}.\end{cases}$

Associate properly the values of x and y.

b) If $b:c=5:3$ in the equation $x^2+bx+c^2=0$, are the roots of the equation real? Give the reason for your answer.

GROUP C. (*Omit one question of this group.*)

5. At his usual rate a man can row 15 miles downstream in 5 hours less time than it takes him to return. Could he double his rate, his time downstream would be only one hour less than his time up. What is his usual rate in still water and what is the rate of the current?

6. The second term of an arithmetic progression is $\frac{1}{3}$ of the 8th and the sum of 20 terms is 63. Find the progression.

7. *a*) Graph $y=1+3x^2$.

b) In the expansion of $\left(3x-\frac{1}{3x^{\frac{1}{2}}}\right)^7$ find the term which, when simplified, contains $x^{\frac{5}{2}}$.

MATHEMATICS *A*—ELEMENTARY ALGEBRA COMPLETE

Monday 9–11 a.m.

1. *a*) Factor

$$6x^2+37x-60,$$
$$x^4+x^3y-xy^3-y^4,$$
$$x(x+1)(4x-5)-6(x+1).$$

b) Simplify

$$\left\{\frac{x+y}{x-y}-\frac{x-y}{x+y}+\frac{4y^2}{y^2-x^2}\right\}\div\left(\frac{x-y}{x+y}-1\right).$$

2. *a*) Solve

$$a^2x^2-(a^2+ab)x=2a^2-5ab+2b^2.$$

b) Simplify

$$\sqrt{\frac{\sqrt{20}+\sqrt{12}}{\sqrt{5}-\sqrt{3}}}.$$

3. *a*) Simplify

$$2(8)^{\frac{2}{3}}-\sqrt{3}(12)^{\frac{1}{2}}-2(3)^0+(a^{\frac{1}{3}}b^{-1})^3b^3-\frac{1}{a^{-1}+b^{-1}}.$$

b) Solve

$$\begin{cases} 3x-4y=1, \\ 3x^2+2xy-x-2y=2. \end{cases}$$

Associate properly the values of x and y and check one pair.

4. *a*) Reduce

$$\sqrt{(x-4)^2+y^2}+\sqrt{(x+4)^2+y^2}=10$$

to an equation free from radicals and as compact as possible.

b) The sum of the first eight terms of a geometric progression is seventeen times the sum of the first four terms. Find the value of the common ratio.

5. If a number of two digits is divided by the sum of its digits the quotient is 2 and the remainder 2. If it is multiplied by the sum of its digits the product is 112. Find the number.

6. A certain number of bolts can be bought for a dollar. If 10 more could be bought for a dollar, the price would be half a cent less per dozen. What is the price per dozen?

MATHEMATICS A 1

ALGEBRA TO QUADRATICS

MATHEMATICS AI—ALGEBRA TO QUADRATICS

Monday 9.30–11.30 a. m.

Six questions are required; both questions of Group A, two from Group B, and two from Group C. No extra credit will be given for more than six questions.

Group A

1. (a) Find prime factors of each of the following expressions:
   (1) $12a^5b-58a^3b+40ab$.
   (2) $2(x^3-1)+7(x^2-1)$.
   (b) Find the H.C.F. and the L.C.M. of x^3-125, $5x^3-125x$, and $x^2-10x+25$. Leave the L.C.M. in factored form.

2. (a) Simplify $\dfrac{2x-y}{a-x}-\dfrac{3a(y-x)}{x^2-a^2}+\dfrac{x-2y}{a+x}$.
   (b) Solve $\dfrac{x}{4}(x-1)-\left(\dfrac{x+1}{2}\right)^2=\frac{2}{3}(x-\frac{1}{2})$ for x.

Group B

3. (a) Simplify $\left(\sqrt{7-3\sqrt{5}}\right)\left(\sqrt{7+3\sqrt{5}}\right)$.
   (b) Given $a^2(x^2-yz)^{-\frac{2}{3}}$. Introduce a^2 into the parenthesis without changing the value of the expression.
   (c) Solve $\sqrt{x}-\sqrt{x-8}=\dfrac{2}{\sqrt{x-8}}$ for x.

4. (a) When $a=2$, $b=8$, $c=4$ find the value of

$$\sqrt{\frac{a^2b^{-\frac{2}{3}}}{c^0}+\sqrt{\frac{b}{ac^{-1}}}}$$

to two decimal places.

   (b) Divide $1-\sqrt{p}-q\sqrt{p}+pq$ by $q\sqrt{p}-q$.

5. (a) Solve for x and y:

$$\begin{cases}\dfrac{5}{3x}+\dfrac{2}{5y}=7\\[2ex]\dfrac{7}{6x}-\dfrac{1}{10y}=3\end{cases}$$

   (b) Solve for b:

$$\frac{\dfrac{bc+d}{a}}{\dfrac{bc}{d}}=\frac{2d}{a}$$

6. \$12,000 is divided among A, B, C, and D. B gets half as much as A; the excess of C's share over D's share is equal to $\frac{1}{5}$ of A's share, and if B's share were increased \$2,000 he would get as much as C and D together. How much did each receive?

7. A and B together can do a certain piece of work in 10 days; but at the end of 7 days A stops working and B finishes the piece by working alone for 5 days. How long would it take each man to do the entire piece working alone?

8. Given three metals of the following composition by weight: The first, 5 parts gold, 2 silver, 1 lead; the second, 2 parts gold, 5 silver, 1 lead; the third, 3 parts gold, 1 silver, 4 lead. To obtain 9 ounces of a metal containing equal quantities by weight of gold, silver, and lead, how many ounces of the first, second, and third, must be taken and melted together?

MATHEMATICS *AI*—ALGEBRA TO QUADRATICS

Monday 9.30–11.30 a.m.

No extra credit will be given for more than six questions.

GROUP A. (*Answer both questions of this group.*)

1. (*a*) Simplify $\frac{8c^3-1}{9c^2-12c+4}\cdot\left(1-\frac{4}{3c+2}\right)\div\left(\frac{2c-1}{9c^2-4}\right)$.

(*b*) Express in simplest form the value of $\frac{a}{b}-\frac{c}{d}$ when $a=2x-1$, $b=2x^2-7x+6$, $c=2x+1$, $d=2x^2+x-6$.

2. (*a*) Solve $\frac{2x+1}{2x-1}+\frac{8}{1-4x^2}=\frac{2x-1}{2x+1}$. Check the result.

(*b*) Solve for x, $\frac{b^2}{ax}-\frac{1}{a}-\frac{3b-3a}{x}=\frac{a^2}{bx}-\frac{1}{b}$.

GROUP B. (*Omit one question of this group.*)

3. Solve for x and y,
$$3x+2y=12\tfrac{1}{4},$$
$$\frac{5}{x+3}:\frac{4}{3y+\frac{1}{2}}=3:\tfrac{2}{5}.$$

4. (*a*) Solve $\sqrt{x+20}-\sqrt{x-1}=3$.

(*b*) Find to two decimal places the value of $\left[\frac{(s-b)(s-c)}{s(s-a)}\right]^{\frac{1}{2}}$, when $a=16.5$, $b=10$, $c=14$, $s=18$.

5. (*a*) Simplify $\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)^2+\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)^2$.

(*b*) Simplify $xy^{\frac{2}{3}}\left(\frac{x^{-\frac{1}{2}}}{y^{-\frac{2}{3}}}\right)^2\div\left(x^{\frac{1}{4}}y^{-3}\right)^{-\frac{2}{3}}$.

GROUP C. (*Omit one question of this group.*)

6. A man has \$1.80 in nickels, dimes, and quarters, 12 coins in all. If the number of dimes and quarters were interchanged he would have \$1.65. How many of each has he?

7. An alloy of gold and silver weighing 40 pounds, loses $2\frac{1}{2}$ pounds when placed in water. How many pounds of each does it contain if gold when placed in water loses $\frac{1}{19}$ of its weight, and silver $\frac{1}{10}$ of its weight?

8. A wholesale shoe dealer sold on the average 3,600 pairs of shoes a day. He reduced his prices 10 per cent, and found that his daily cash receipts from sales were increased 20 per cent. How many pairs of shoes did he sell daily at the reduced prices?

MATHEMATICS *AI*—ALGEBRA TO QUADRATICS

Monday 9–11 a. m.

No extra credit will be given for more than six questions.

GROUP A. (*Answer both questions of this group.*)

1. (*a*) Find the Highest Common Factor and the Lowest Common Multiple of $12x^2+x-6$, $6x^2-19x+10$, $3x^3-2x^2-12x+8$.

   (*b*) Combine and simplify

$$\frac{3}{2c+h}-\frac{8c^2+7h^2}{8c^3+h^3}-\frac{2c-6h}{4c^2-2ch+h^2}.$$

   Check the result by substituting $c=0$, $h=1$ in the original fractions and in the answer.

2. (*a*) Solve $$\frac{3x-1}{x+3}-\frac{2x+3}{1-x}=\frac{5x^2-2x-1}{x^2+2x-3}.$$

   (*b*) Solve $$\frac{x+a-b}{a-b}+\frac{x-a-b}{a+b}=\frac{2a(2c-x)}{a^2-b^2}.$$

GROUP B. (*Omit one question of this group.*)

3. Solve

$$\begin{aligned} 2x-3y+\ z&=-2,\\ 4x-4y-3z&=2,\\ 6x+\ y-4z&=6. \end{aligned}$$

4. (*a*) Solve $$(3x-11)^{\frac{1}{2}}-2=\tfrac{1}{3}(27x-243)^{\frac{1}{2}}.$$

   (*b*) Find the value of x^3-2x+1 when $x=\dfrac{-1+\sqrt{5}}{2}$.

5. (*a*) Simplify $$\frac{1}{a^{-1}-b^{-1}}+\frac{\sqrt{a^5b^{-1}}(a^{-\frac{1}{2}}b^{\frac{1}{2}})^3}{a-b}.$$

   (*b*) Find the value of $8^{-\frac{2}{3}}\times 16^{\frac{3}{4}}\times 2^0$.

GROUP C. (*Omit one question of this group.*)

6. If the digits of a certain number of two figures are interchanged, the result is 6 less than twice the original number. The sum of the digits is $\frac{1}{4}$ of the original number. Find the number.

7. A passenger train traveling *m* miles an hour starts *t* hours later than a freight train whose rate is *r* miles an hour. In how many hours will the passenger train overtake the freight train?

8. At an election there are two candidates, A and B. A's supporters are taken to the polls in carriages holding 8 each, and B's are taken in carriages holding 12 each. If the voters, 740 in all, just fill 75 carriages, find which man wins the election and by what majority.

MATHEMATICS *AI*—ALGEBRA TO QUADRATICS

Monday 9–11 a. m.

No extra credit will be given for more than six questions.

GROUP A. (*Answer both questions of this group.*)

1. *a*) Factor

$$2mx+6ny-my-12nx,$$
$$6x^2+11x-10,$$
$$x^4-a^3x+bx^3-a^3b.$$

b) Simplify

$$1-\left\{\frac{c^3+y^3}{(c-y)^2}\div\left[\frac{c^4+c^2y^2+y^4}{c^3-y^3}\times\frac{(c+y)^2}{c^2-y^2}\right]\right\}.$$

2. *a*) Solve

$$\tfrac{1}{3}\{x-\tfrac{1}{3}(x+1)\}+\tfrac{1}{4}(4x-5)=\frac{27}{4}-\tfrac{1}{3}(x+1).$$

b) Solve

$$\frac{7a-x}{b-3a}+4=\frac{3x-5a}{3b-a}.$$

GROUP B. (*Answer both questions of this group.*)

3. *a*) Solve for x and y

$$\begin{cases} 1.6x-2.05y=0.39\,, \\ 5.2x+4.1y\ =3.42\,. \end{cases}$$

Verify your answers.

b) Simplify and combine:

$$\frac{6}{\sqrt{3}}-18\sqrt{\tfrac{1}{3}}-\tfrac{1}{6}\sqrt{108}+12^{\frac{1}{2}}+3^{\frac{3}{2}}+\frac{\sqrt{3}}{5^{-1}}.$$

4. *a*) Solve

$$\frac{1}{\sqrt{x+1}}-\frac{1}{\sqrt{x-1}}+\frac{1}{\sqrt{x^2-1}}=0.$$

b) Rationalize the denominator and simplify

$$\frac{\sqrt{3}-\sqrt{\frac{1}{2}}}{\sqrt{2}+\sqrt{\frac{1}{3}}}.$$

GROUP C. (*Omit one question of this group.*)

5. A mixture of alcohol and water contains 10 gals. A certain amount of water is added and the alcohol is then 30 per cent of the total. Had double the amount of water been added, the alcohol would have been 20 per cent of the whole. How much water was actually added and how much alcohol is there?

6. Two points move at constant rates along the circumference of a circle whose length is 150 ft. When they move in opposite directions, they meet every 5 seconds, and when they move in the same direction, they are together every 25 seconds. What are their rates?

7. 146 francs are worth as much as 117 shillings. A dollar and 4 francs are together worth 32 cents more than 6 shillings. Find the values in cents of a franc and a shilling.

MATHEMATICS *A1*—ALGEBRA TO QUADRATICS

Monday 9–11 a. m.

1. *a*) Factor

$$6x^2+37x-60,$$
$$x^4+x^3y-xy^3-y^4,$$
$$x(x+1)(4x-5)-6(x+1).$$

b) Simplify

$$\left\{\frac{x+y}{x-y}-\frac{x-y}{x+y}+\frac{4y^2}{y^2-x^2}\right\}\div\left(\frac{x-y}{x+y}-1\right).$$

2. *a*) Solve

$$3.4\,x-0.17(x-2)=51\left(\frac{x}{5}-3\right).$$

b) Solve

$$\frac{2x-2a}{a-b}+2\frac{(a-b)x-a^2}{a^2-b^2}+\frac{a}{a-b}=0.$$

3. *a*) Solve

$$\begin{cases} x+2y-z=2\,, \\ 3x-2y+2z=0\,, \\ 5x-4y+3z=1\,. \end{cases}$$

b) Simplify

$$\sqrt{\frac{\sqrt{20}+\sqrt{12}}{\sqrt{5}-\sqrt{3}}}\,.$$

4. *a*) Find the square root of

$$a^{-2}+9b^{\frac{4}{3}}+16c^{-\frac{1}{2}}+6a^{-1}b^{\frac{2}{3}}-8a^{-1}c^{-\frac{1}{4}}-24b^{\frac{2}{3}}c^{-\frac{1}{4}}.$$

b) Simplify

$$2(8)^{\frac{2}{3}}-\sqrt{3}(12)^{\frac{1}{2}}-2(3)^0+(a^{\frac{1}{3}}b^{-1})^3b^3-\frac{1}{a^{-1}+b^{-1}}.$$

5. A photographer has two bottles of diluted developer. In one bottle 10 per cent of the contents is developer and the rest water; in the other, the mixture is half and half. How much must he draw from each bottle to make 8 ounces of a mixture in which 25 per cent is developer?

6. A and B find a purse with dimes in it. A takes out 2 and $\frac{1}{6}$ of what is left. Afterward B takes out 3 and $\frac{1}{6}$ of what then remains. They find they have taken out equal amounts. How many dimes were there?

MATHEMATICS A 2

QUADRATICS AND BEYOND

MATHEMATICS *A2*—QUADRATICS AND BEYOND

Monday 9.30–11.30 a. m.

Six questions are required; two from Group A, two from Group B, and both questions of Group C. No extra credit will be given for more than six questions.

Group A

1. (*a*) Solve by factoring $acx^2-bcx+adx=bd$.
   (*b*) Solve $x^{\frac{4}{3}}+7x^{\frac{2}{3}}=8$.

2. Solve $\sqrt{3x+10}=\sqrt{10x+16}+\sqrt{x+2}$. Which value of x satisfies the equation?

3. Solve $\begin{cases} 2x^2-1=y^2+3xy \\ 5-x^2=y^2 \end{cases}$
   Write results so that with each value of x, the proper value of y is associated.

Group B

4. The arithmetic mean between two numbers is $42\frac{1}{2}$ and their geometric mean is 42. Find the numbers.

5. Three numbers whose sum is 24 are in A. P. If 2, 6, and 17 be added to them respectively, the results are in G. P. Find the numbers.

6. (*a*) Find the middle term of $\left(2a^3-\frac{3b^2}{4}\right)^8$.
   (*b*) In the equation $x^2+mx+k=0$, what relation exists between m and k if one root of the equation is twice the other?

Group C

7. (*a*) Draw the graphs of the two equations
$$\left.\begin{aligned} x^2+y^2&=9 \\ 6x-4y&=24 \end{aligned}\right\}$$
   and tell the algebraic meaning of the fact that the two graphs do not intersect.
   (*b*) Solve graphically
$$2x^2-11x+5=0.$$

8. On the same day, A, B, and C start to solve a certain number of problems. A solved 6 a day and finished them 4 days after B. C solved 3 more a day than B and finished 2 days before he did. Find the number of problems and the number of days each worked.

MATHEMATICS A2—QUADRATICS AND BEYOND

Monday 9.30–11.30 a.m.

No extra credit will be given for more than six questions.

GROUP A. (*Answer both questions of this group.*)

1. (*a*) Solve $\frac{3x}{x-2}-\frac{2}{x+3}+\frac{2}{2-x}=0$.

   (*b*) Solve for x and y, $x-y=1$,

   $$\frac{x}{y}-\frac{y}{x}=\frac{5}{6}.$$ Check the results.

2. (*a*) Solve $\sqrt{7x+1}-\sqrt{3x+10}=1$.
   Test the values found and explain why one of them will not satisfy the equation.

   (*b*) Solve $3(x^2+3x+1)^2-7(x^2+3x+1)+4=0$.

GROUP B. (*Omit one question of this group.*)

3. Two men, A and B, start at the same time from a certain point and walk east and south, respectively. At the end of 5 hours A has walked 5 miles farther than B, and they are 25 miles apart. Find the rate of each.

4. A man worked a number of days and earned \$75. If he had received 50 cents more per day, he would have earned the same amount in 5 days less. How many days did he work?

5. Construct with respect to the same axes of reference the graphs of

   $$y+2x^2-3x-9=0.$$

   and $$y+x-3=0.$$

   Estimate from the figure the values of x and y which satisfy both equations.

GROUP C. (*Omit one question of this group.*)

6. An elastic ball bounces to three-fourths of the height from which it falls. If it is thrown up from the ground to a height of 15 feet, find the total distance traveled before it comes to rest.

7. (*a*) Find the coefficient of x^4 in $\left(2x^2-\frac{1}{4x}\right)^8$.

   (*b*) Solve $x^2+5x+d=0$. What is the least integer which, when substituted for d in the equation, makes the roots of the equation imaginary?

8. The sum of 3 numbers in geometrical progression is 70. If the first be multiplied by 4, the second by 5, and the third by 4, the resulting numbers will be in arithmetical progression. Find the three numbers.

MATHEMATICS *A2*—QUADRATICS AND BEYOND

Monday 11.15 a.m.–1 p.m.

No extra credit will be given for more than six questions.

GROUP A. (*Answer both questions of this group.*)

1. (*a*) Solve $$(x+2)(2x-3)=6.$$

(*b*) Solve $$3x=x^2+y^2-1,$$ $$4y=x^2+y^2-6.$$

Check one pair of answers.

2. (*a*) Solve $$\sqrt{x+3}+\frac{2}{\sqrt{x-3}}=3\sqrt{x-3}.$$

Do both answers satisfy the equation?

(*b*) Solve $$\frac{2x-a}{b}+3=\frac{4a}{2x-b}.$$

GROUP B. (*Omit one question of this group.*)

3. A square is surrounded by a border whose width lacks one inch of being $\frac{1}{4}$ the length of a side of the square. The ratio of the area of the border to the area of the square is 7:9. Find the dimensions of the square and of the border.

4. A line 16 inches long is divided into two parts such that the longer part is a mean proportional between the whole line and the shorter part. Find the length of the shorter part to two places of decimals.

5. Construct with respect to the same axes of reference the graphs of

$$x-4y=4,$$

and

$$x^2+y^2=16.$$

Estimate from the figure the values of x and y which satisfy both equations.

GROUP C. (*Omit one question of this group.*)

6. Twelve potatoes are placed in line at distances 6, 12, 18, ... feet from a basket. A boy, starting from the basket picks up the potatoes and carries them back, one at a time, to the basket. How far must he run to complete the potato race?

7. (*a*) Write in its simplest form the fourth term of $\left(\sqrt[3]{x}-\frac{1}{\sqrt{x}}\right)^{18}$.

(*b*) Write and expand the quadratic equation whose roots are

$$-3+2\sqrt{5}, \text{ and } -3-2\sqrt{5}.$$

8. There are four numbers of which the first three are in arithmetical progression, and the first, the second, and the fourth are in geometrical progression. The sum of the first and the third is 8. The sum of the second and the fourth is 36. Find the numbers.

MATHEMATICS *A2*—QUADRATICS AND BEYOND

Monday 11.15 a.m.–1 p.m.

No extra credit will be given for more than six questions.

GROUP A. (*Answer both questions of this group.*)

1. *a*) Approximate to two decimal places the roots of

$$\frac{2}{3x-3}+1+\frac{4}{2x-3}=0\,.$$

b) Solve

$$\frac{1}{6x-5a}-\frac{2}{a}=\frac{5}{a-6x}\,.$$

2. *a*) Solve

$$\begin{cases} 2x+y-2=0\,, \\ 4x^2+6xy+2x-6y+1=0\,. \end{cases}$$

Associate properly the values of x and y.

b) Solve

$$x^{\frac{1}{2}}-12x^{-\frac{1}{2}}=-1.$$

Do both answers satisfy the equation?

GROUP B. (*Omit one question of this group.*)

3. A man walked 12 miles at a certain rate and then 6 miles farther at a rate $\frac{1}{2}$ mile an hour faster. Had he walked the whole distance at the faster rate, his time would have been 20 minutes less. Find his rate.

4. The circumference of the rear wheel of a carriage is 2 feet greater than the circumference of the front wheel. The front wheel makes 64 more revolutions than the rear wheel in traveling 3,496 feet. What is the circumference of each wheel?

5. Three men A, B, and C can do a piece of work together in 1 hour and 20 minutes. To do the work alone C would take twice as long as A and 2 hours longer than B. How long would it take each to do the work alone?

GROUP C. (*Omit one question of this group.*)

6. In the expansion of $\left(\sqrt{x}+\frac{1}{2\sqrt{x}}\right)^8$ the ratio of the 4th term to the 5th term is 4 : 1. Find x.

7. Find two numbers x and y such that x, y, and xy are in geometric progression, and x, y, and $4x+3$ are in arithmetic progression.

8. Plot $2x-3y=6$ and $2y+1=4x-4x^2$, using the same axes, and estimate from the graphs the solutions of the equations.

MATHEMATICS A2—QUADRATICS AND BEYOND

Monday 11.15 a. m.-1 p. m.

1. *a*) Find the value of x correct to two decimal places for

$$(2x-1)(x-3)=2.$$

b) Solve

$$x^2+\frac{1}{x^2}+x+\frac{1}{x}=4.$$

2. *a*) Solve

$$\frac{\sqrt{x+2a}-\sqrt{x-2a}}{\sqrt{x+2a}+\sqrt{x-2a}}=\frac{x}{2a}.$$

Check the results.

b) For what values of k are the roots of

$$9x^2+(1+k)x+4=0$$

equal?

3. Solve

$$\begin{cases} x^2-3xy-y^2=9, \\ 2x^2+2xy+3y^2=7. \end{cases}$$

Associate properly the values of x and y and check one pair.

4. *a*) Write the first term of

$$(x^{\frac{1}{2}}-x^{-\frac{2}{3}})^8$$

which, in its simplified form, has a negative exponent.

b) Arithmetic means are inserted between 1 and 21 so that the sum of these means is 132. Find the first two of them.

5. A principal of $2,500 put at simple interest at a certain rate and for a certain time amounts to $2,800. If the rate of interest had been one per cent higher and the time two years longer the amount would have been $3,200. Required the time and the rate.

6. A goes from P to Q in 14 hours. B starts at the same time from a point 10 miles behind P and arrives at Q at the same time as A. B finds that he takes a half hour less to go 20 miles than A does. Find the distance between P and Q.

MATHEMATICS B

ADVANCED ALGEBRA

MATHEMATICS *B*—ADVANCED ALGEBRA

Saturday 9–11 a. m.

Six questions are required. No extra credit will be given for more than six questions.

1. (*a*) How many different words can be indicated by 9 flags, of which 2 are red, 3 white, and the rest blue, if when all are hoisted together one above another each different display represents a word?

   (*b*) Find the value of r that the number of combinations of 10 things taken r at a time may be a maximum.

2. Reduce $\frac{11-3i}{1-3i}$ and $\frac{2i-9}{4+i}$ to simplest form and construct their difference graphically. $i=\sqrt{-1}$.

3. (*a*) Evaluate the determinant:

$$\begin{vmatrix} a & b & b & a \\ b & a & a & b \\ a & a & b & b \\ o & a & b & b \end{vmatrix}$$

   (*b*) Solve by determinants the following equations for y only: $4x-5y+3z=5$, $3x+2y-z=2$, and $5x-6y+3z=8$.

4. Determine the character of the roots of the equation

$$x^3+bx+c=0$$

   (1) when b and c are both positive, (2) when b is negative and c positive.

5. Construct the graph of the function $y=x^3-5x-3$ from $x=-3$ to $x=+3$. Estimate from the graph: (1) the value of the negative root of $x^3-5x-3=0$, (2) the value of x that will make $y=4$, (3) the positive value of x that will give y its smallest value.

6. Show if $a+ib$ is a root of an equation with real coefficients that $a-ib$ is also a root.

7. One root of $x^4-3x^2-42x-40=0$ is $-\frac{1}{2}(3-\sqrt{-31})$; find the other roots.

MATHEMATICS *B*—ADVANCED ALGEBRA

Saturday 9–11 a.m.

Six questions required. No extra credit will be given for more than six questions.

1. One root of $4x^4-14x^3+16x^2-9x+2=0$ is an integer; a second root is the reciprocal of this first root. Find all the roots of the equation.

2. (*a*) What information does Descartes's rule of signs give with respect to the roots of the equation $x^5+2x^3-5x^2+x+11=0$?

   (*b*) The three roots of $x^3-6x^2+kx+10=0$ are in the form $a-d$, a, and $a+d$, respectively. Solve the equation, and find the value of k.

3. Graph $y=x^3+3x^2-2x-5$ between the values $x=-4$ and $x=2$. From the figure determine between what consecutive integers the roots of the equation lie.

4. Find to two decimal places the positive root of

$$x^3+3x^2-2x-5=0.$$

5. Evaluate the determinant

$$\begin{vmatrix} 1 & 3 & -2 & 2 \\ 2 & 2 & -3 & 4 \\ -1 & 1 & 1 & -3 \\ 4 & 3 & -2 & 1 \end{vmatrix}$$

6. (*a*) Given
$$\begin{aligned} 2x-\ \ y+\ z&=1, \\ x-7y-8z&=1, \\ 7x+14y+2z&=7. \end{aligned}$$
Write the value of y as the quotient of two determinants. Do not expand the result.

   (*b*) Simplify $\dfrac{3-2i}{1+i}$, and represent the resulting complex number graphically.

7. From 14 men how many committees of four can be formed? Of these how many contain one particular man A? How many include A but do not include B?

MATHEMATICS *B*—ADVANCED ALGEBRA

Friday 4.15–6 p. m.

Six questions required. No extra credit will be given for more than six questions.

1. (*a*) For what values of k will 2 be a root of the equation

$$4x^3-3x^2-kx-4k^2=0?$$

(*b*) One root of $x^4+7x^3+12x^2+19x-15=0$ is $-1+2\sqrt{-1}$. Find the other roots of the equation.

2. (*a*) What information can be obtained by use of Descartes's Rule of Signs in regard to the roots of the equation $x^4+ax+3=0$, if a represents a real positive or negative number not zero?

(*b*) If k is a root of the equation $f(x)=0$, where $f(x)$ is a polynomial, prove that $x-k$ is a factor of $f(x)$.

3. Find the positive root of the equation $x^3+2x^2-16x-16=0$ to two places of decimals.

4. A rectangle whose perimeter is 26 inches is rotated on the line joining the middle points of two opposite sides as an axis, forming a cylinder of revolution whose volume is 198 cubic inches. Find the length of the sides of the rectangle. Assume $\pi=\frac{22}{7}$.

5. (*a*) Find the value of the determinant

$$\begin{vmatrix} 2 & -1 & 10 & -5 \\ -1 & 4 & 6 & -2 \\ 3 & 5 & -1 & 9 \\ -4 & -4 & 2 & 0 \end{vmatrix}$$

(*b*) Write the original determinant as the sum of four determinants.

6. How many different numbers, all told, can be formed from the digits 2, 3, 4, 5, 6, no digit being repeated in any number? How many of the numbers obtained are even?

7. (*a*) Prove that $\frac{(1+i)^2}{\sqrt{3}+i}=\frac{\sqrt{3}-i}{(1-i)^2}$, where $i=\sqrt{-1}$.

(*b*) Solve $x^3+27=0$ and represent the complex roots graphically.

MATHEMATICS *B*—ADVANCED ALGEBRA

Friday 4.15–6 p. m.

Six questions required. No extra credit will be given for more than six questions.

1. *a*) How many triangles can be drawn with each vertex in one of twenty given points, no three of which are in the same straight line?

   b) How many such triangles can be drawn if four of the given points lie in a straight line?

2. Without expanding the determinants, prove the following relations:

(*a*)
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1-x_2 & y_1-y_2 \\ x_2-x_3 & y_2-y_3 \end{vmatrix}.$$

(*b*)
$$\begin{vmatrix} -7 & -5 & 3 \\ 9 & 1 & 7 \\ 6 & 4 & -2 \end{vmatrix} = 0.$$

3. Solve the following equations by the use of determinants:

$$\begin{cases} x-3y=1, \\ y+4z=2, \\ 2x-z=3. \end{cases}$$

4. *a*) Find the value of $\dfrac{(2+3i)^2}{1+2i}$ where $i=\sqrt{-1}$.

   b) Find the sum of the reciprocals of the roots of $5x^3-3x^2-2x+7=0$.

5. *a*) By studying the signs of $x^4-2x^3-5x^2+6x-1=0$, what can you say about the signs and the reality of its roots?

   b) Graph $y=x^3-4x+2$ and determine between what consecutive integers lie the roots of the equation $x^3-4x+2=0$.

6. Find to two decimal places the root of $x^4-x^3-9x^2+4x+6=0$ which lies between 3 and 4.

7. Find all the roots, real and complex, of $8x^4+24x^3-x-3=0$. Plot the complex roots.

MATHEMATICS *B*—ADVANCED ALGEBRA

Friday 4.15–6.15 p. m.

1. *a*) Solve $3x^4+20x^3+46x^2+41x+10=0.$

   b) Determine the number of positive, negative, and complex roots of

$$x^4+2x^2+3x-7=0.$$

2. *a*) Locate the roots of $x^3+x^2-4x-2=0.$

   b) Find the positive root of

$$x^3+x^2-4x-2=0$$

   to two decimal places.

3. In the equation

$$x^4+x^3+3x^2+4x+6=0$$

   the sum of two roots is -2 and the product of the other two roots is 3. Solve the equation.

4. *a*) Find the value of

$$\begin{vmatrix} 1 & 15 & 14 & 4 \\ 12 & 6 & 7 & 9 \\ 8 & 10 & 11 & 5 \\ 13 & 3 & 2 & 16 \end{vmatrix}$$

   b) Write the value of y in the equations

$$\begin{cases} x+2y+z+17=0, \\ 2x+y-z+1=0, \\ 3x+2z=y+2, \end{cases}$$

   leaving the answer as the quotient of two determinants.

5. *a*) Reduce $\frac{8+i}{1+2i}$ and $\sqrt{5+12i}$ to the form $a+bi$ and then represent their sum graphically.

   b) Prove that if $a+bi$ is a root of an equation with real coefficients, then $a-bi$ is also a root.

6. *a*) In how many ways can 8 men be arranged in a row so that neither of two given men can be at either end of the row?

   b) Prove ${}_nC_r+{}_nC_{r+1}={}_{n+1}C_{r+1}$, where ${}_nC_r$ means the number of combinations of n things taken r at a time.

MATHEMATICS C

PLANE GEOMETRY

MATHEMATICS *C*—PLANE GEOMETRY

Tuesday 9–11 a. m.

Six questions are required; four from Group A and two from Group B. No extra credit will be given for more than six questions.

Group A

The candidate is requested to state on cover of answer-book what text-book of Geometry he used in preparation.

1. Prove that the areas of two similar triangles are to each other as the squares of their corresponding sides, or as the squares of their corresponding altitudes.

2. Prove that if from a point outside a circle any two lines are drawn cutting the circle, the product of one secant and its external segment is equal to the product of the other secant and its external segment.
   If the point is taken inside the circle, state the corresponding theorem.

3. Define *locus*. What is the locus of the centers of all the circles which pass through two given points.
   Give a complete proof.

4. State and prove the converse of the proposition that the square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides.
   State, without proof, a proposition whose converse is not true.

5. Find the area of a square inscribed in a circle whose area is A. What is the numerical ratio of the area of the circle to that of the square?

6. (*a*) Divide geometrically a line six inches long into segments proportional to 2, 3, and 5.
   (*b*) Following the suggestion of 6 (*a*), show how to construct a triangle with a given perimeter which shall be similar to a given triangle.

Group B

7. A circular grass plot, 12 feet in diameter, is cut by a straight gravel path 3 feet wide, one edge of which passes through the center of the plot. What is the area of the remaining grass plot?

8. ABC is an equilateral triangle. AO and BO bisect the base angles. OD and OE are drawn parallel to CA and CB respectively. Prove that $AD = DE = EB$.
   Make an accurate construction of the figure, showing all arcs.

9. Show that the area of a regular hexagon inscribed in a circle is the mean proportional between the areas of the inscribed and circumscribed equilateral triangles.

MATHEMATICS C—PLANE GEOMETRY

Tuesday 9–11 a.m.

The candidate is requested to state on the cover of the answer-book what text-book of Geometry was used in preparation.

GROUP A. (*Answer four questions from this group.*)

1. If two circles intersect, the line of centers bisects their common chord at right angles.

2. Prove that the bisector of an angle of a triangle divides the opposite side into segments which are proportional to the other sides.

3. The areas of two triangles which have an angle of one equal to an angle of the other are to each other as the products of the sides including those angles.

4. Define *locus*. Find the locus of the center of a circle passing through two given points.

5. Given any angle and any point P within it. Describe a method of drawing a line through P meeting the sides of the angle in two points M, N, such that $MP=2PN$.

6. Construct a triangle ABC; given $AB=2''$, angle $B=75^\circ$, and the median to $AB=2\frac{1}{4}''$.

GROUP B. (*Answer two questions from this group.*)

7. Show how to construct a circle whose area is half that of a given circle. Explain all the steps.

8. Calculate the length of a side of a regular octagon inscribed in a circle of radius R.

9. A circular arch of masonry, radius 25 feet, rests on two stone piers which are 40 feet apart. Find the height of the center of the arch above the level of the top of the piers.

MATHEMATICS C—PLANE GEOMETRY

Tuesday 9–11 a.m.

The candidate is requested to state on the cover of the answer-book what textbook of Geometry was used in preparation.

GROUP A. (*Answer four questions from this group.*)

1. Two triangles are equal if the three sides of the one are equal respectively to the three sides of the other.

2. Construct a parallelogram, given a side $1\frac{1}{2}$ inches, the length of a diagona 2 inches, and the altitude upon the given side $\frac{3}{4}$ inch.

3. With each vertex of an equilateral triangle as a center, an arc is described within the triangle with a radius equal to half a side of the triangle and terminating in the sides. If the side of the triangle is 5 inches, compute to two decimal places the perimeter and the area of the figure enclosed by the three arcs.

4. (*a*) Prove that the opposite angles of an inscribed quadrilateral are supplementary.
   (*b*) If *ABCDE* is an inscribed pentagon and arc *DE* is 70 degrees, find the number of degrees contained in the sum of the angles *A* and *C*.

5. (*a*) In any obtuse triangle, the square of the side opposite the obtuse angle is equal to the sum of the squares of the other two sides, plus twice the product of one of the sides by the projection of the other upon it.
   (*b*) In the triangle whose sides are 5, 18, 20, compute the length of the projection of the longest side upon the shortest side.

6. If two polygons are similar, they can be separated into the same number of triangles, similar each to each, and similarly placed.

GROUP B. (*Answer two questions from this group.*)

7. A circle, radius 5 inches, contains a moving chord *AB*, length 8 inches, which is divided into four equal parts by the points *P*, *Q*, *R*. Determine the loci of *P*, *Q*, *R*.

8. Divide the entire circumference of a circle into three parts that shall be in the ratio of 1 to 2 to 3. Prove the correctness of your construction.

9. (*a*) Two secants are drawn from a point outside a circle. The chord segment of one of the secants is 35, and its outer segment is 5. The chord segment of the second is equal to its outer segment. Find the length of the second secant.
   (*b*) Prove the theorem which you used in the above solution.

MATHEMATICS *C*—PLANE GEOMETRY

Tuesday 9–11 a. m.

The candidate is requested to state on the cover of the answer-book what textbook of Geometry was used in preparation.

GROUP A. (*Answer four questions from this group.*)

1. Complete this theorem and prove: If two sides of a triangle are unequal, the angles opposite are unequal, and the greater . . . . .

   State the hypothesis of the above theorem.

   Point out where it is needed and used in your proof.

   State the converse of the theorem.

2. *a*) In the triangle ABC, $AB = 3$ inches, $A = 60°$, $B = 45°$. Accurately construct the triangle and its altitudes AD, BE, CF; show all necessary construction lines and arcs.

   b) Which of the angles of the figure thus drawn are equal to the angles A, B, C, of the original triangle? Prove one such equality.

3. Complete and prove: The angle between two secants intersecting without the circle is measured by . . . . .

4. Prove: If three sides of a trapezoid are equal, the diagonals mutually divide each other into segments which are in the ratio of one of the equal sides to the fourth side of the trapezoid.

5. Prove: If two triangles have their sides respectively proportional, they are similar.

6. *a*) Construct the locus of the center of a circle, radius one-half inch, which rolls around an equilateral triangle, altitude two inches.

   b) Compute to two decimals the area inclosed by the locus and the perimeter of the locus.

GROUP B. (*Answer two questions from this group.*)

7. Show how to construct an equilateral triangle equivalent to a given square. (Actual construction not required.)

8. A sloping embankment rises from a level field. One end of a prop, 20 feet long, rests on the ground 16 feet from the foot of the embankment, and the other end rests 9 feet up the embankment, measured along its sloping side. How high is the upper end of the prop above the level field? (Result in feet to one decimal.)

9. It is desired to construct a half-mile track. The start and finish are to be straight-ways intersecting at right angles at the goal. The rest of the track is to be an arc of a circle tangent to the two straight-ways. Find the radius of the arc and the length of the arc in feet; also the area inclosed by the track in acres. (Results to be correct to two decimals.)

MATHEMATICS C—PLANE GEOMETRY

Tuesday 9–11 a. m.

The candidate is requested to state on the cover of the answer-book what textbook of Geometry was used in preparation.

GROUP A. (*Answer four questions from this group.*)

1. Prove: If the sides of an angle are perpendicular respectively to the sides of another angle, the two angles are equal or supplementary.

2. Which of the following propositions are true and which false?
   a) If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.
   b) If the diagonals of a quadrilateral are equal, the quadrilateral is an isosceles trapezoid.
   c) If the diagonals of a parallelogram are perpendicular to each other, the parallelogram is a rhombus.

   State the converse of each of these propositions. Which of the converses are true?

3. Prove: The areas of two similar polygons are to each other as the squares of any two homologous sides.
   State the hypothesis of this theorem.
   Point out where in your proof you made use of the hypothesis.

4. *a*) Prove: If from a point P without a circle a secant and a tangent are drawn to the circle, the tangent is a mean proportional between the whole secant and its external segment.
   b) If the diameter of the circle is d inches and the distance of P from the center of the circle is a inches, express the length l of the tangent from P to the circle in terms of d and a. If $d=6$ and $a=5$, find the length of this tangent.

5. Point out the fallacy in the following reasoning: Let AB be the base of an isosceles triangle, and P any point on AB. Draw PC. Then, in the triangles APC and BPC, $\angle A=\angle B$, $AC=BC$, $PC=PC$. Triangles APC and BPC are, therefore, congruent (equal in all respects), having two sides and an angle of one equal to the homologous parts of the other. Hence, $AP=BP$, i.e., any point of AB bisects AB.

GROUP B. (*Answer two questions from this group.*)

6. Construct a triangle having given an angle and a side adjacent and a side opposite to the angle. Discuss all possible cases.

7. A circle of radius 2 inches rolls around a square whose side measures 4 inches. Construct the locus of the center of the circle, and find to two decimal places both the length of the locus and the area inclosed by it.

8. Three equal circles, each tangent to the other two, have an acre of ground between them. How many rods in the perimeter of this acre? (An acre contains 160 square rods.)

MATHEMATICS D

SOLID GEOMETRY

MATHEMATICS *D*—SOLID GEOMETRY

Tuesday 9–11 a. m.

Six questions are required. No extra credit will be given for more than six questions.

The candidate is requested to state on cover of answer-book what text-book of Geometry he used in preparation.

1. Prove that two diedral angles are equal if their plane angles are equal.

2. Prove that the plane passing through two opposite edges of a parallelopiped divides it into two equivalent triangular prisms.

3. (*a*) Prove that the sum of the angles of a spherical triangle is greater than two, and less than six, right angles.
   (*b*) On a sphere whose radius is 4, given a triangle whose angles are 60°, 65°, and 75° respectively. Find the area of the triangle.

4. Prove that a sphere can be circumscribed about any tetraedron.

5. Prove that if any plane is passed through two opposite vertices of a parallelogram, the perpendiculars to it from the other vertices are equal.

6. Find the area of the zone on a sphere of radius r which is illuminated by a lamp placed at a distance a from the surface of the sphere.

7. A cone of wood has its vertical angle equal to 60°, and the radius of its base equal to 2 inches. A cylindrical hole of one inch radius is bored through the entire cone, the axis of the hole coinciding with that of the cone. How much of the cone goes into chips?

MATHEMATICS *D*—SOLID GEOMETRY

Tuesday 9–11 a.m.

Six questions are required. No extra credit will be given for more than six questions.

The candidate is requested to stated on the cover of the answer-book what text-book of Geometry was used in preparation.

1. (*a*) Prove that if one of two parallel lines is perpendicular to a plane, then is the other also.
   (*b*) If two straight lines a and c are parallel to a third line b, they are parallel to each other.
2. What is the locus of points in space equally distant from two given points? From three given points not on a straight line? From four given points not in a plane? Omit demonstration.
3. (*a*) Prove that every plane section of a sphere is a circle.
   (*b*) Find the area of a plane section of a sphere of radius 10, which passes 6 units from the center.
4. The sum of two face angles of a trihedral angle is greater than the third face angle.
5. Find the surface and volume of a regular octahedron having an edge 4 units long.
6. Given a solid having the shape of a cylinder with a hemispherical end. The diameter and length of the cylindrical portion are each 6 inches. Find the total surface and volume of the solid.
7. If the diagonal of a cube is 24 inches, what is the surface of the cube? The volume?

MATHEMATICS *D*—SOLID GEOMETRY

Tuesday 11.15 a.m. to 1 p.m.

Five questions are required. No extra credit will be given for more than five questions.

The candidate is requested to state on the cover of the answer-book what textbook of Geometry was used in preparation.

1. If any two lines in space are cut by three parallel planes, the corresponding segments are proportional.

2. (*a*) Every point in the bisecting plane of a dihedral angle is equally distant from the faces.
   (*b*) Show that the three planes bisecting the dihedral angles of a trihedral angle all pass through the same line.

3. (*a*) The sum of the face angles of any convex polyhedral angle is less than four right angles.
   (*b*) Find the sum of the face angles of the polyhedral angle at any vertex of a regular octahedron.

4. (*a*) Define: small circle on a sphere; pole; polar distance; lune; zone.
   (*b*) In a zone of one base, the diameter of the base is 16 inches and the altitude is 4 inches. Find the radius of the sphere and the area of the zone.

5. (*a*) Prove that the section of a circular cone made by a plane parallel to the base is a circle.
   (*b*) Given a circular cone of altitude h and with a base of radius r. Find the volume of the cone cut off by passing a plane one-third the distance from the vertex to the base and parallel to the base.

6. A square whose side is 3 inches is revolved about one of its diagonals, forming a solid. Find (*a*) the surface, and (*b*) the volume of this solid.

7. A wooden solid consists of two hemispherical caps fitting exactly on the bases of a cylinder. The length of the cylindrical portion is half the total length of the solid. Express the total surface and the volume in terms of the total length of the solid.

MATHEMATICS *D*—SOLID GEOMETRY

Tuesday 11.15 a. m. to 1 p. m.

Six questions are required. No extra credit will be given for more than six questions. The candidate is requested to state on the cover of the answer-book what textbook of Geometry was used in preparation.

1. The segments intercepted by three parallel planes on all straight lines meeting them are in the same proportion.

2. If a plane contains one element of a cylinder and meets the cylinder in one other point, then it contains another element also, and the section is a parallelogram.

3. Complete and prove the theorem: The sum of the angles of a spherical triangle is greater than ......, and is less than ....

4. The area of a spherical triangle is 100 square inches, and its angles are 100°, 64°, 200°. What is the radius of the sphere on which the triangle lies?

5. A glass vessel made in the form of a right circular cylinder contains a certain amount of water. The diameter of the base of the vessel is 5 inches. When an irregular mass of gold is dropped into the vessel it is entirely covered by the water and the level of the water rises 3 inches. What is the weight in ounces of the lump of gold if gold weighs 11 ounces per cubic inch?

6. The stone cap of a gate post is in the form of a regular square pyramid whose base measures 4 inches on a side and whose altitude is 15 inches. If the top of the cap is cut off by a plane parallel to its base and 5 inches above it, what is the volume of the piece cut off?

7. Define prism, pyramid, parallelopiped, conical surface, spherical triangle, lune. Give mensuration formulas for the volume of a prism, cylinder, cone, pyramid, sphere; and for the lateral area of a prism, regular pyramid, cone of revolution, surface of a sphere.

MATHEMATICS *D*—SOLID GEOMETRY

Tuesday 11.15 a. m.–1 p. m.

The candidate is requested to state on the cover of the answer-book what textbook of Geometry was used in preparation.

Answer six questions. No extra credit will be given for more than six questions.

1. Prove: If a straight line is perpendicular to two intersecting straight lines at their point of intersection, it is perpendicular to the plane of those lines.

2. *a*) Prove: If two intersecting straight lines are parallel to a plane, their plane is parallel to the given plane.

   b) Prove: If two planes are parallel to a third plane, they are parallel to each other.

3. Prove: A plane passed through the diagonally opposite edges of a parallelopiped divides it into two equivalent triangular prisms.

4. *a*) Define: Sphere, small circle on a sphere, polar distance, spherical angle, spherical polygon.

   b) State and prove a theorem from which the area of a spherical triangle may be found, given the radius of the sphere and the angles of the triangle.

5. *a*) Find the length of the diagonal of a rectangular parallelopiped whose dimensions are 8, 9, and 12.

   b) A rectangle is rotated about one of its sides as an axis. What is the ratio of the volumes generated by the triangles into which the rectangle is divided by one of its diagonals?

6. If the temperate zones were between the 30° and 60° parallels of latitude, what proportion of the earth's surface would they comprise? Give the details of the computation.

7. A plane is to be passed through a right circular cone parallel to its base and so that the lateral area of the small cone cut off by the plane shall be equivalent to the lateral area plus one base of a right circular cylinder. The altitude of the cone is 12 inches and the radius of the base is 4 inches. The altitude of the cylinder is 4 inches and the radius of its base is 2 inches. How far from the vertex of the cone must the plane be passed? The use of radicals is permissible in the calculation and in the final result.

MATHEMATICS CD

PLANE AND SOLID GEOMETRY

MATHEMATICS *CD*—PLANE AND SOLID GEOMETRY

Tuesday 9 a. m.–12 m.

This paper will be rated as a whole; separate credits will not be given on this paper for Plane Geometry and Solid Geometry.

Eight questions are required; four from Group A and four from Group B. No extra credit will be given for more than eight questions.

Group A

The candidate is requested to state on cover of answer-book what text-book of Geometry he used in preparation.

1. Prove that the areas of two similar triangles are to each other as the squares of their corresponding sides, or as the squares of their corresponding altitudes.

2. Prove that if from a point outside a circle any two lines are drawn cutting the circle, the product of one secant and its external segment is equal to the product of the other secant and its external segment.

   If the point is taken inside the circle, state the corresponding theorem.

3. Define *locus*. What is the locus of the centers of all circles which pass through two given points?

   Give the complete proof.

4. (*a*) Divide geometrically a line six inches long into segments proportional to 2, 3, and 5.

   (*b*) Following the suggestion of 4 (*a*), show how to construct a triangle with a given perimeter which shall be similar to a given triangle.

5. A circular grass plot, 12 feet in diameter, is cut by a straight gravel path 3 feet wide, one edge of which passes through the center of the plot. What is the area of the remaining grass plot?

Group B

6. Prove that two diedral angles are equal if their plane angles are equal.

7. (*a*) Prove that the sum of the angles of a spherical triangle is greater than two, and less than six, right angles.

   (*b*) On a sphere of radius 4, given a triangle whose angles are $60°$, $65°$, and $75°$ respectively. Find the area of the triangle.

8. Prove that if any plane is passed through two opposite vertices of a parallelogram, the perpendiculars to it from the other vertices are equal.

9. Find the area of the zone on a sphere of radius r which is illuminated by a lamp placed at a distance a from the surface of the sphere.

10. A cone of wood has its vertical angle equal to $60°$, and the radius of its base equal to 2 inches. A cylindrical hole of one inch radius is bored through the entire cone, the axis of the hole coinciding with that of the cone. How much of the cone goes into chips?

MATHEMATICS *CD*—PLANE AND SOLID GEOMETRY

Tuesday 9 a.m.–12 m.

This paper will be rated as a whole; separate credits will not be given on this paper for Plane Geometry and Solid Geometry.

The candidate is requested to state on the cover of the answer-book what text-book of Geometry was used in preparation.

GROUP A. (*Answer four questions from this group.*)

1. If two circles intersect, the line of centers bisects their common chord at right angles.

2. Define *locus*. Find the locus of the center of a circle passing through two given points.

3. The areas of two triangles which have an angle of one equal to an angle of the other are to each other as the products of the side including those angles.

4. Construct a triangle ABC; given $AB=2$ in., angle $B=75°$, and the median to $AB=2\frac{1}{4}$ in.

5. A circular arch of masonry, radius 25 feet, rests on two stone piers which are 40 feet apart. Find the height of the center of the arch above the level of the top of the piers.

GROUP B. (*Answer four questions from this group.*)

6. (*a*) Prove that if one of two parallel lines is perpendicular to a plane, then is the other also.
   (*b*) If two straight lines a and c are parallel to a third line b, they are parallel to each other.

7. What is the locus of points in space equally distant from two given points? Three given points not on a straight line? Four given points not in a plane?

8. (*a*) Prove that every plane section of a sphere is a circle.
   (*b*) Find the area of the plane section of a sphere of radius 10, which passes 6 units from the center.

9. Given a solid having the shape of a cylinder with a hemispherical end. The diameter and length of the cylindrical portion are each 6 inches. Find the total surface and volume of the solid.

10. If the diagonal of a cube is 24 inches, what is the surface of the cube? The volume?

MATHEMATICS E

TRIGONOMETRY

MATHEMATICS *E*—TRIGONOMETRY (PLANE AND SPHERICAL)

Saturday 1.30–3.30 p. m.

Six questions are required; three from Group A and three from Group B. No extra credit will be given for more than six questions.

Group A

1. (*a*) Express the six functions of $(90^\circ + A)$ in terms of functions of the acute angle A.

   (*b*) Find by logarithms a value of the angle A, between 0° and 90°, from the following:

$$\cos \tfrac{1}{2}A = \sqrt{\frac{42.165 \times 0.032628}{1.6465}}.$$

2. (*a*) Prove by means of a figure the formula $\cos(A-B) = \cos A \cos B + \sin A \sin B$ in which A and B are angles in the first quadrant.

   (*b*) Derive the formula $2 \sin^2 \frac{x}{2} = 1 - \cos x$.

   (*c*) Show that $\dfrac{\sin 2A + \sin 4A}{\cos 2A + \cos 4A} = \tan 3A$.

3. Solve the equation $3 \cos x - 2 \sin x = 2$, and obtain all values of x between 0° and 360° which satisfy the equation.

4. The sides of a plane triangle are 71.57, 54.15, and 36.84 feet respectively; find the greatest angle.

Group B

5. In a right spherical triangle. $C = 90^\circ$, prove geometrically any two of the following formulas:

$$\cos c = \cos a \cos b$$
$$\tan a = \cos B \tan c$$
$$\sin b = \sin B \sin c.$$

6. In a right spherical triangle, ABC, $a = 42^\circ\ 53'$, $B = 53^\circ\ 47'$, $C = 90^\circ$, find c and A.

7. Find the angle of an equilateral spherical triangle in which each side is $67^\circ\ 36'$.

8. Given $L = 40^\circ\ 42'$, $z = 57^\circ\ 49'$, $A = 283^\circ\ 5'$, find t and d by means of the equations:

$$\tan M = \tan z \cos A$$
$$\tan t = \frac{\tan A \sin M}{\cos(L-M)}$$
$$\tan d = \tan(L-M) \cos t$$

MATHEMATICS *E*—TRIGONOMETRY (PLANE AND SPHERICAL)

Saturday 1.30–3.30 p.m.

GROUP A. (*Answer three questions from this group.*)

1. Describe the variation in $\sin x$ as x varies from 0° to 360°.
 Show that $\sin x = \sin (180° - x)$; $\sin (-x) = -\sin x$.

2. (*a*) By means of ruler and compasses show how to construct the angles A, B, x, given:

$$\tan A = \tfrac{3}{4};\quad \cos B = \tfrac{1}{2};\quad \cot x = \sqrt{3}.$$

 (*b*) If $\tan A = \dfrac{2mn}{m^2 - n^2}$, find sin A.

3. (*a*) Derive the formula $\sin (A+B) = \sin A \cos B + \cos A \sin B$, where A, B, $A+B$, are all acute angles.
 (*b*) Prove the identity

$$\tan x + \cot x = 2 \csc 2x.$$

4. Find all the values of the angle x expressed in degrees and minutes, not greater than 360°, which will satisfy the following equations:
 (*a*) $\cos 2x + \sin x = 4 \sin^2 x$;
 (*b*) $\tan \left(\frac{\pi}{4} + x\right) = 2$.

5. The diagonals of a parallelogram are 12.5 feet and 12.8 feet, respectively, and their included angle is 52° 16′. Find the sides and the area of the parallelogram.

GROUP B. (*Answer three questions from this group.*)

6. By means of a figure derive the formulas necessary to solve the right spherical triangle, given the two sides a and b, both acute.

7. By means of Napier's rules determine whether the unknown parts are acute or obtuse, given:
 (*a*) $a = 40°$, $B = 30°$, $C = 90°$.
 (*b*) $A = 70°$, $B = 110°$, $C = 90°$.

8. In a right spherical triangle prove either (*a*) or (*b*).
 (*a*) $\sin b = \cos c \tan a \tan B$.
 (*b*) $\sin^2 A + \sin^2 B = 1 + \sin^2 a \sin^2 B$.

9. Solve the right spherical triangle, given $c = 80° 28'$; $A = 33° 21'$; $C = 90°$.

MATHEMATICS *E*—TRIGONOMETRY (PLANE AND SPHERICAL)

Saturday 2–4 p. m.

Group A. (*Omit one question from this group.*)

1. (*a*) Construct each of the following angles, given:

$$\cos A = \tfrac{1}{2}; \quad \cot B = -2; \quad \sin C = \tfrac{3}{5}.$$

(*b*) Explain the variation in $\tan x$ when x increases from 45° to 225°.

2. (*a*) Find the value of $\sqrt[3]{0.00034}$ by means of logarithms.

(*b*) Derive the formula:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

3. Prove the following identities:

(*a*) $$\sin(45^\circ + x) + \cos(45^\circ + x) = \sqrt{2}\cos x.$$

(*b*) $$\tan\frac{x}{2} = \frac{\sin x}{1 + \cos x}.$$

Group B. (*Omit one question from this group.*)

4. In a right spherical triangle, prove either (*a*) or (*b*).

(*a*) $$\sin b = \cos c \tan a \tan B.$$

(*b*) $$\sin(b + c) = 2\cos^2\frac{A}{2}\cos b \sin c.$$

5. Show how to solve a quadrantal triangle when the quadrantal side and any other two parts are given.

6. Write the formulas necessary to solve the spherical triangle, given

$$a = 60^\circ, \quad b = 70^\circ, \quad C = 90^\circ.$$

Group C. (*Answer both questions.*)

7. When an airship is 1050 feet high above a level plane, find the distance of an object on the plane, the angle of depression being 72°.

8. Given A = 70°, B = 60°, C = 90°. Find *a*, *b*, *c*.

MATHEMATICS *E*—TRIGONOMETRY (PLANE AND SPHERICAL)

Saturday 2–4 p. m.

GROUP A. (*Omit one question from this group.*)

1. *a*) Express the six functions of $(180°+x)$ in terms of functions of the acute angle x. Prove one of these relations.

   b) Describe the variation in $\cos x$ as x varies from $90°$ to $270°$. Illustrate by means of figures.

2. *a*) Find the value of x, given

$$\cos x=\sqrt{\frac{43.2\times 0.074}{17.234}}$$

   b) Prove $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=45°$.

3. Find all the values of x between $0°$ and $360°$ that satisfy the equation

$$\sin x+\sin 2x=1.$$

GROUP B. (*Omit one question from this group.*)

4. In a right spherical triangle, prove

$$\cot A\ \cot B=\cos c.$$

5. *a*) Define polar triangles; quadrantal triangles; spherical excess.

   b) In a right spherical triangle, not biquadrantal, the side opposite the right angle is nearer a right angle than is either oblique side.

6. Write the formulas necessary to solve the quadrantal triangle given

$$a=90°,\ b=60°,\ C=70°.$$

GROUP C. (*Answer both questions.*)

7. Each of two ships A and B, 415 yards apart, measures the horizontal angle subtended by a cliff and the other ship; the angles are $48°\ 17'$ and $110°\ 10'$ respectively. If the angle of elevation of the cliff from A is $15°\ 24'$ what is the height of the cliff?

8. Solve the spherical triangle, given

$$a=55°,\quad b=65°,\quad C=90°.$$

MATHEMATICS *E*—TRIGONOMETRY (PLANE AND SPHERICAL)

Saturday 2–4 p. m.

GROUP A. (*Answer two questions from this group.*)

1. Given that $\tan x = \frac{7}{24}$ and that x is in the third quadrant; also that $\sec y = -\frac{13}{5}$, y being in the second quadrant; find the values of $\cos(x-2y)$, $\cot 2x$, $\sin \frac{1}{2}y$.

2. Using logarithms, find the value of

$$\left[\frac{3.416 \times \sqrt{25.9} \times \sqrt[3]{-0.046}}{2\sqrt[3]{\frac{4}{3}}}\right]^{\frac{1}{3}}.$$

3. Find all the angles between 0° and 360° which satisfy the equation:

$$2\cos^2 x + 3\sin x - 3 = 0.$$

GROUP B. (*Answer two questions from this group.*)

4. Prove that in a right spherical triangle ABC, with right angle at C, the tangent of B is equal to the tangent of the side b divided by the sine of the side a, the sides b and a being the sides opposite the angles B and A respectively.

5. *a*) State Napier's Rules. What use is made of them?
   b) What relations exist between the angles of a spherical triangle and certain parts of the polar triangle?

6. Give the formulas necessary for the complete solution of an isosceles spherical triangle, given its base and one of the equal angles. Write out a blank form for the logarithmic solution of such a triangle.

GROUP C. (*Answer both questions of this group.*)

7. Brown's farm lies between two straight roads which intersect at an angle of 32° 15′. He wills to his oldest son a triangular plot whose area is ten acres and which is situated in the angle of the roads. If this piece is to have a frontage of 1,000 feet on one road, what must be its frontage on the other? (One acre=43,560 sq. ft.)

8. Solve the spherical triangle, given

$$a = 80°\ 30', \quad b = 61°\ 20', \quad c = 114°\ 45'.$$

MATHEMATICS F

PLANE TRIGONOMETRY

MATHEMATICS *F*—PLANE TRIGONOMETRY

Saturday 1.30–3.30 p. m.

Six questions are required. No extra credit will be given for more than six questions.

1. (*a*) Express the six functions of $(90^\circ+A)$ in terms of functions of the acute angle A.

   (*b*) Find by logarithms a value of angle A, between 0° and 90°, from the following:

$$\cos \tfrac{1}{2}A=\sqrt{\frac{42.165\times 0.032628}{1.6465}}.$$

2. (*a*) Prove by means of a figure the formula $\cos(A-B)=\cos A \cos B+\sin A \sin B$ in which A and B are angles in the first quadrant.

   (*b*) Derive the formula $2\sin^2\frac{x}{2}=1-\cos x$.

   (*c*) Show that $\dfrac{\sin 2A+\sin 4A}{\cos 2A+\cos 4A}=\tan 3A$.

3. (*a*) If $\tan A=m+1$ and $\tan B=m-1$ show that $2\cot(A-B)=m^2$.

   (*b*) Demonstrate the identity $\dfrac{1+\tan x}{1-\tan x}=\sec 2x+\tan 2x$.

4. Solve the equation $3\cos x-2\sin x=2$ and obtain all values of x between 0° and 360° which satisfy the equation.

5. (*a*) Find the length of the chord of an arc of 30° in a circle whose radius is 20 feet.

   (*b*) Find the area of the segment between the above chord and its arc.

6. The sides of a triangle are 71.57, 54.15, and 36.84 feet respectively; find the greatest angle.

7. In the oblique triangle ABC, given $a=91.068$, $b=58.405$, $A=51^\circ\ 9'$, find B and c.

MATHEMATICS *F*—PLANE TRIGONOMETRY

Saturday 1.30–3.30 p.m.

Six questions are required. No extra credit will be given for more than six questions

1. Describe the variation in $\sin x$ as x varies from 0° to 360°.
Show that $\sin x = \sin(180° - x)$; $\sin(-x) = -\sin x$.

2. (*a*) By means of ruler and compasses show how to construct the angles A, B, x, given:
$$\tan A = \frac{3}{4}; \quad \cos B = \frac{1}{5}; \quad \cot x = \sqrt{3}.$$
(*b*) If $\tan A = \dfrac{2mn}{m^2 - n^2}$, find $\sin A$.

3. (*a*) By means of logarithms calculate to four places of decimals the value of
$$\frac{10.056\sqrt[3]{0.87}}{1.827(3.106)^2}$$
(*b*) Prove that the logarithm of the product is equal to the sum of the logarithms of the factors.

4. (*a*) Derive the formula $\sin(A+B) = \sin A \cos B + \cos A \sin B$, where A, B, $A+B$ are all acute angles.
(*b*) Prove the identity
$$\tan x + \cot x = 2 \csc 2x.$$

5. Find all the values of the angle x expressed in degrees and minutes not greater than 360°, which will satisfy the following equations:
(*a*) $\cos 2x + \sin x = 4 \sin^2 x$;
(*b*) $\tan\left(\frac{\pi}{4} + x\right) = 2$.

6. In a circle whose radius is 111.3 feet, find the area included between a chord whose length is 129.3 feet, and a diameter parallel to it.

7. The diagonals of a parallelogram are 12.5 feet and 12.8 feet, respectively, and their included angle is 52° 16′. Find the sides and the area of the parallelogram.

MATHEMATICS *F*—PLANE TRIGONOMETRY

Saturday **2–4 p. m.**

GROUP A. (*Omit one question from this group.*)

1. (*a*) Construct each of the following angles, given:

$$\cos A = \tfrac{1}{2}; \quad \cot B = -2; \quad \sin C = \tfrac{3}{5}.$$

(*b*) Explain the variation in $\tan x$ when x increases from 45° to 225°.

2. (*a*) Find the value of $\sqrt[3]{0.00034}$ by means of logarithms.

(*b*) Derive the formula $a^2 = b^2 + c^2 - 2bc \cos A$.

3. Prove the following identities.

(*a*) $\sin(45^\circ + x) + \cos(45^\circ + x) = \sqrt{2} \cos x$.

(*b*)
$$\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}.$$

4. Find all the values of x less than 2π that will satisfy the equation

$$\tan x + 2 \cot x = 3.$$

GROUP B. (*Omit one question from this group.*)

5. Derive the formulas:

(*a*)
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

(*b*)
$$a^2 = b^2 + c^2 - 2bc \cos A.$$

6. Given $a = 16.832$, $b = 22.146$, $C = 33^\circ 19'$, find the remaining parts.

7. When an airship is 1050 feet high above a level plane, find the distance of an object on the plane, the angle of depression being 72°.

8. To find the distance between two points *A* and *B* on opposite sides of a river, a line *BC* 100 feet long was measured off and then the angles *ABC* and *ACB* were measured, with the result that angle $ABC = 67^\circ$ and angle $ACB = 58^\circ$. What is the distance from *A* to *B*?

MATHEMATICS *F*—PLANE TRIGONOMETRY

Saturday 2–4 p. m.

GROUP A. (*Omit one question from this group.*)

1. *a*) Express the six functions of $(180°+x)$ in terms of functions of the acute angle x. Prove one of these relations.

   b) Describe the variation in $\cos x$ as x varies from 90° to 270°. Illustrate by means of figures.

2. *a*) By means of the ruler and compasses show how to construct the angles x and y, given:

$$\cos x=\tfrac{2}{3};\quad \tan y=-3.$$

   b) Assuming the formula for the cosine of the sum of two angles, prove:

$$\cos 3x=\cos x\,(1-4\sin^2 x),$$

   and verify for $x=30°, 45°, 60°$.

3. *a*) Find the value of x, given

$$\cos x=\sqrt{\frac{43.2\times 0.074}{17.234}}$$

   b) Prove $\tan^{-1}\tfrac{1}{2}+\tan^{-1}\tfrac{1}{3}=45°$.

4. Find all the values of x between 0° and 360° that will satisfy the equation

$$\sin x+\sin 2x=1.$$

GROUP B. (*Omit one question from this group.*)

5. Prove the following theorem:
   In any plane triangle

$$\frac{a+b}{a-b}=\frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}.$$

6. The sides of a triangle are 3, 4, $\sqrt{39}$; show, without solving, that the largest angle is greater than 120°.

7. In the oblique plane triangle ABC, given $a=91.12$, $b=72.43$, $C=41°\ 10'$, find the remaining parts.

8. Each of two ships A and B, 415 yards apart, measures the horizontal angle subtended by a cliff and the other ship; the angles are 48° 17′ and 110° 10′ respectively. If the angle of elevation of the cliff from A is 15°24′ what is the height of the cliff?

MATHEMATICS F—PLANE TRIGONOMETRY

Saturday **2–4 p. m.**

Answer six questions, including the last.

1. *a*) Give definitions of the sine, cosine, and tangent of an angle which will apply to an angle in any quadrant.

   b) Prove the relation connecting the squares of the sine and cosine of an angle, taking account in your proof of the algebraic signs of these functions.

   c) Which is the greater in absolute value, sin 134° or cos 134°? Explain your answer without reference to the tables.

2. Without using tables find the value of the following:

   a) $\cos 180°+\sqrt{3}\sin 60°-\sec^2 45°+\sin 270° \operatorname{cosec} 90°-4\sin 30°$.

   b) $\tan 75°+\cos(-1230°)$.

3. Given that $\tan x=\frac{7}{24}$ and that x is in the third quadrant; also that $\sec y=-\frac{13}{5}$ and that y is in the second quadrant; find the values of $\cos(x-2y)$, $\cot 2x$, $\sin\frac{y}{2}$.

4. Assuming the formulas for $\sin(x+y)$ and $\sin(x-y)$, prove that

$$\sin^2 x-\sin^2 y=\sin(x+y)\sin(x-y).$$

   Which of the two members of this equality is better adapted to logarithmic computation. Why?

5. Brown's farm lies between two straight roads which intersect at an angle of 32° 15′. He wills to his son a triangular plot whose area is ten acres and which is located in the angle of the roads. If the field is to have a frontage of 1,000 feet on one road, what must be its frontage on the other road? (One acre contains 43,560 sq. ft.)

6. At the top of an observation tower which is 200 feet high and whose base is at sea level, the angles of depression of two ships (i.e., the angles between the lines of sight and the horizontal) are observed to be 20° 32′ and 18° 40′. At the bottom of the tower the angle subtended by the line joining the two ships is found to be 40° 20′. What is the distance between the ships to the nearest foot?

7. Using logarithms, find the value of

$$\left[\frac{3.416\times\sqrt{25.9}\times\sqrt[3]{-0.046}}{2\sqrt[3]{\frac{4}{3}}}\right]^{\frac{1}{3}}.$$