RADIO-FREQUENCY MEASUREMENTS BY BRIDGE AND RESONANCE METHODS

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BY

L. HARTSHORN
D.Sc., A.R.C.S., D.I.C.
Principal Scientific Officer, The National Physical Laboratory.
Vice-President of the Physical Society

NEW YORK
JOHN WILEY & SONS INC.
440 FOURTH AVENUE

PRINTED IN THE UNITED STATES OF AMERICA

AUTHOR'S PREFACE

A CELEBRATED experimentalist divides mankind into those who do things, and those who merely talk and write about the things that other people do. The distinction is perhaps not as clear-cut as it sounds, but most scientific men know well enough what it means, and will appreciate my point when I say that this book is intended for those who are setting out to *do* things by means of radio-frequency technique. The things referred to are those expressible in terms of resistance, capacitance, inductance, dielectric constant, and the like. The technique is of obvious importance to the radio engineer for the purposes of precision testing; it also provides research workers with a most powerful means of investigating many electrical, physico-chemical, and biological problems. With the idea of making the necessary information accessible to all such workers, the subject has been developed as far as possible from first principles; and the aim has been to present, not an encyclopædic account of everything that has been written on the subject, but a systematic account of the basic principles and general working ideas, that form the tools of the practising technician.

Radio-frequency measuring devices have a great range of application, but they have a habit of responding to several variables simultaneously; and it is often easier to measure a quantity than to know exactly what quantity has been measured. A knowledge of a circuit diagram and a few equations is not enough for a reliable measurement. Special attention has on this account been given to such matters as screening, earth connections, the physical nature of the quantities measured, and sources of error.

The methods described are for the most part those which my own experience suggests to be the most generally useful. Many others are on record, but they are largely of the nature of variations on a few themes. The aim has been to state the main themes, and the rules to be followed when making variations, with a sufficient number of examples to cover all present-day requirements.

It would be difficult and probably misleading to assign the methods to individual authors. They have been gradually evolved as the result of the work of many investigators, and when references to original papers are quoted, they are to be regarded merely as examples, selected as convenient sources of further information on special points.

The book was written to fill what appeared to be a definite gap in the literature of the subject; for although more general works are available, there appears to be no previous exposition from the point of view here adopted; no systematic treatment of either radio-frequency bridge methods, or stationary wave methods; and only very general discussions of resonance methods.

I am glad to acknowledge my indebtedness to several present and former colleagues, particularly Mr. N. F. Astbury, Mr. L. Essen, Dr. T. I. Jones, Mr. W. H. Ward and Mr. W. Wilson for helpful comments and information on particular points.

L. HARTSHORN.

February, 1940.

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LIST OF SYMBOLS

The symbols used are, for the most part, those recommended by the International Electrotechnical Commission, and the British Standards Institution. The chief ones are :—

i	Current, instantaneous.
$\hat{i}$	,, maximum.
I	,, effective (r.m.s.).
$\mathbf{I}$	,, vector.
v, $\hat{v}$, V, $\mathbf{V}$	Potential difference, voltage.
P	Power.
W	Energy.
ϕ	Phase difference, phase angle.
$\cos\phi$	Power factor.
δ	Loss angle.
$\tan\delta$	Loss tangent.
Z	Impedance $= \sqrt{R^2 + X^2}$.
$\mathbf{Z}$	Impedance operator $= R + jX$.
R	Resistance.
X	Reactance.
Y	Admittance $= \sqrt{G^2 + B^2}$.
$\mathbf{Y}$	Admittance operator $= G + jB$.
G	Conductance.
B	Susceptance.
L	Self-inductance.
M	Mutual inductance.
C	Capacitance.
L_s, R_s, C_s	Equivalent series inductance, resistance, capacitance.
L_p, R_p, C_p	Equivalent parallel ditto.
ω	Angular frequency, pulsation, $= 2\pi f$.
f	Frequency.
μ	Permeability.
ϵ	Permittivity, dielectric constant.
ρ	Resistivity.
σ	Conductivity.
γ	Propagation constant $= \alpha + j\beta$.
α	Attenuation constant.
β	Wavelength constant.
λ	Wavelength.
n	Refractive index.
c	Velocity of light.

For units :

A	Ampere.
V	Volt.

Ω Ohm.
F Farad.
H Henry.
c/s Cycles per second.

With the prefixes :

k Kilo- ($\times 10^{3}$).
M Mega- ($\times 10^{6}$).
m Milli- ($\times 10^{-3}$).
μ Micro- ($\times 10^{-6}$).
$\mu\mu$ Micro-micro- ($\times 10^{-12}$).

INTRODUCTORY NOTE

THE subject of this book is the measurement of certain quantities arising in the use of alternating electric currents of radio frequency, *i.e.*, of frequencies between, say, ten thousand and a few hundred million cycles per second. Three kinds of quantities are involved in radio-frequency technique: (1) those such as current, voltage and power, which measure in various ways the strength or intensity of the electrical oscillations; (2) those such as impedance, admittance, capacitance, inductance, resistance and phase, which depend on ratios of the quantities first named, and which are regarded as measuring in various ways the properties of the electrical circuit or the path in which the current flows, *i.e.*, the properties of the apparatus or materials employed; (3) frequency, which depends on time alone, and is not essentially electrical in character.

The accuracy at present attainable in measurements of the first group of quantities is comparatively low: it is very seldom as high as 0·1 per cent. The quantities depending on ratios can be measured with rather greater accuracy, in much the same way as it is usually possible to compare two quantities with greater accuracy than can be attained in absolute measurements of either. The measurement of frequency, which is essentially a time measurement, can now be made with an accuracy possibly greater than that attainable in any other kind of measurement.

The present volume is devoted to a consideration of the measurement of the second group of quantities only. The methods employed differ considerably from those employed in other branches of electrical measurement. They are obviously of the greatest importance to the radio engineer. They also provide the physicist and physical chemist with a most powerful means of investigating certain properties of materials, namely those associated with vibrations, electronic, ionic or molecular, which fall within the radio range of frequencies, *e.g.*, properties expressed in terms of dielectric constant, absorption, magnetic susceptibility, and electrolytic resistance.

CHAPTER I

IMPEDANCE AND RELATED QUANTITIES

1. Resistance and Inductance. Let an alternating current of sine wave form, of frequency f, and maximum value $\hat{\imath}$, be passed through a wire of resistance R. Then, provided the frequency is sufficiently low, Ohm's Law applies, and the potential difference between the terminals of the wire is given by

$$\begin{aligned} v &= Ri \\ &= R\hat{\imath} \sin 2\pi f t = R\hat{\imath} \sin \omega t, \qquad \text{say} \end{aligned}$$

and energy is dissipated in the wire at the rate $vi = R\hat{\imath}^2 \sin^2 \omega t$, a pulsating quantity, the mean value of which is

$$W = \tfrac{1}{2} R\hat{\imath}^2 = RI^2$$

where $I = \hat{\imath}/\sqrt{2}$ is the effective value of the current.

Referring to Fig. 1, the current $i = \hat{\imath} \sin \omega t$ may be represented by the projection ON on a fixed axis OY of a vector OP of length $\hat{\imath} = I\sqrt{2}$ rotating about the origin with uniform angular velocity ω. The potential difference v in the above example is clearly represented by the corresponding projection of a similar vector OP_1 of length $\hat{v} = R\hat{\imath}$ rotating in phase with that representing i.

If the frequency is steadily increased a stage is reached at which the p.d. becomes appreciably affected by the magnetic field of the current. The current is opposed by an e.m.f. of self-induction, given by $e = L\, di/dt$, where L denotes the self-inductance of the wire. The total p.d. is now given by

$$\begin{aligned} v &= Ri + L\frac{di}{dt} \\ &= R\hat{\imath} \sin \omega t + \omega L\hat{\imath} \cos \omega t. \\ &= R\hat{\imath} \sin \omega t + \omega L\hat{\imath} \sin \left(\omega t + \frac{\pi}{2}\right). \end{aligned}$$

Thus the p.d. is the sum of two components, the first associated with the resistance R and represented by the projection of the rotating vector OP_1 of length $R\hat{\imath}$ already described, and the second associated with the self-inductance L and represented by the corresponding projection of a vector OP_2 of length $\omega L\hat{\imath}$ rotating with the same angular velocity, but $\pi/2$ radians or 90° ahead in phase of the vectors representing $\hat{\imath}$ and $R\hat{\imath}$. It follows

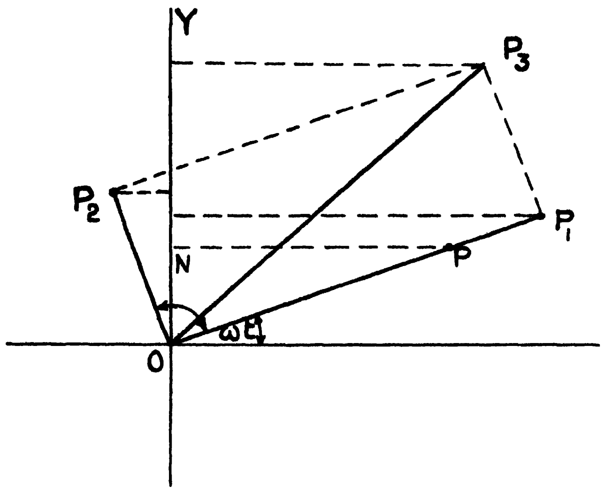


FIG. 1.

that v is represented by the corresponding projection of OP_3, the vector sum of OP_1 and OP_2.

As long as we are concerned only with the relations between maximum or effective values of current and voltage, we may ignore the projections of the vectors, and their motion, and consider only their magnitudes and relative orientations.

It is convenient to adopt the following notation. Rotating vectors representing alternating currents and voltages will be denoted by **I** and **V** respectively. The angular velocity of the vectors, $\omega = 2\pi f$, will be called the angular frequency of the current or voltage. Then we see that the voltage $\mathbf{V}_R$ required to send a current **I** through a resistance R is given by

$$\mathbf{V}_R = R\mathbf{I}$$

while the voltage $\mathbf{V}_L$ required to force the current $\mathbf{I}$ against the action of an inductance L is represented by

$$\mathbf{V}_L = j \, . \, \omega L\mathbf{I} = jX\mathbf{I}$$

where j symbolises the operation of rotating the vector to which it is attached through a right angle in the positive direction.

The quantity $X = L\omega$ is evidently somewhat analogous to the resistance R, and is called the reactance.

2. **Capacitance and Conductance.** Consider now an alternating voltage $\hat{v} \sin \omega t$ applied to two metal plates forming a condenser, of capacitance C. The charges on the two plates of the condenser at any instant are given by

$$q = \pm \, C \hat{v} \sin \omega t.$$

and therefore the currents flowing into these plates are given by

$$i = dq/dt = \pm \, \omega C \hat{v} \cos \omega t = \pm \, \omega C \hat{v} \sin \left(\omega t + \frac{\pi}{2} \right).$$

In vector notation we may say that the voltage $\mathbf{V}$ causes a current

$$\mathbf{I}_c = j\omega C\mathbf{V}$$

to flow into the high-potential plate and out of the low-potential plate. In effect the current $\mathbf{I}_c$ flows through the condenser, but as the plates are separated by a non-conductor, we must regard this current as of a special character. Such currents are called capacitance or displacement currents. It should be noticed that they lead the corresponding voltage in phase by the angle $\pi/2$.

If the medium between the plates is not a perfect non-conductor, but has a resistance R, an ordinary conduction current $\mathbf{I}_G$ will also flow, where

$$\mathbf{I}_G = \frac{1}{R} \, . \, \mathbf{V} = G\mathbf{V}.$$

The quantity $G = 1/R$ is called the conductance while the corresponding quantity ωC for the displacement current is the susceptance.

3. **Impedance and Admittance.** We have seen that the relation between the current $\mathbf{I}$ and voltage $\mathbf{V}$ in a coil of resistance R and reactance X may be represented by

$$\mathbf{V} = R\mathbf{I} + jX\mathbf{I} = (R + jX)\mathbf{I} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the vectors $R\mathbf{I}$ and $jX\mathbf{I}$ being added to form $\mathbf{V}$ by the laws of vector addition, as illustrated in Fig. 1. Their relative magnitudes and directions may therefore be represented by the simplified diagram of Fig. 2, from which we see that the voltage vector leads the current vector by an angle ϕ such that

$$\tan\phi = X/R \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and the amplitudes of the current and voltage obey the relation

$$\hat{v} = \hat{\imath}\sqrt{R^2 + X^2} = \hat{\imath}Z \qquad . \quad . \quad . \quad . \quad . \quad (3)$$

Dividing throughout by $\sqrt{2}$ we see also that the effective values V and I obey the same relation. The quantity Z which governs the relative values of current and voltage is called the impedance. It consists of two components, the resistance R and the reactance X, the relative values of which determine the phase angle ϕ.

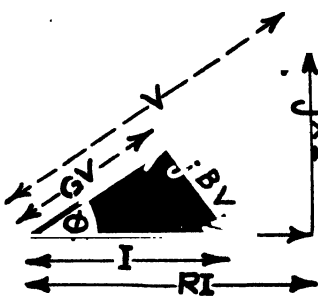


Fig. 2.

The above considerations show that if alternating currents and voltages are represented by rotating vectors, such an impedance may be represented by the operator $R + jX$, which acting on the vector $\mathbf{I}$, multiplies its length by $Z = \sqrt{R^2 + X^2}$, and rotates it through an angle $\phi = \tan^{-1}(X/R)$. In an alternative notation the same operator is represented by $Z\underline{/\phi}$.

It is to be noted that equation (1) is analogous to Ohm's Law, and it follows that this law may be applied to alternating currents and networks if we represent the currents and voltages by vectors and substitute for resistances impedance operators of the form $R + jX$.

Equation (1) may also be written

$$\mathbf{I} = \frac{1}{R + jX}\mathbf{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

from which it follows that the effect of the operator $1/(R + jX)$ on the vector $\mathbf{V}$ is to multiply its length by $1/Z = Y$ (say) and to rotate it through an angle $-\phi$. This operator may therefore also be represented by $1/Z\underline{/-\phi}$ or $Y\underline{/-\phi}$. The quantity Y which is the reciprocal of the impedance is called the admittance. The admittance operator $Y\underline{/-\phi}$ may evidently also be represented by $G+jB$ where $Y=\sqrt{(G^2+B^2)}$ and $\tan\phi=-B/G=X/R$. It follows that $B/Y=-X/Z$ and $G/Y=R/Z$ so that the relation between current and voltage may be written

$$\mathbf{I}=(G+jB)\mathbf{V}=\left(\frac{R}{Z^2}-j\frac{X}{Z^2}\right)\mathbf{V} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Comparing this equation with that obtained for the current between two plates separated by material of conductance G and capacitance C,

$$\mathbf{I}=(G+jC\omega)\mathbf{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

we obtain a physical interpretation of the two components of the admittance operator for this particular case: G represents the leakage conductance, and B is the susceptance $C\omega$.

It follows from the above relations that the distribution of alternating currents and voltages in networks may be determined in exactly the same way as in the corresponding direct current problem, *i.e.*, by application of Ohm's Law or Kirchhoff's Laws, but with impedance operators of the form $R+jX$ instead of simple resistances, and admittance operators $G+jB$ instead of conductances. Algebraic transformations of the operators follow the rules of ordinary algebra, if j^2 is interpreted as -1, which follows from the fact that, two successive rotations through a right angle (*i.e.*, $j\,.\,j$) merely reverse the sign of a vector, *i.e.*, multiply it by -1. Thus equation (5) immediately follows from (4) by multiplying both numerator and denominator by $R-jX$, and putting -1 for j^2 in the denominator.

4. **Effective Values of Inductance, Capacitance, Resistance, etc.** Let $\mathbf{I}$ be the current passing between any pair of terminals, and let $\mathbf{V}$ be the p.d. between them. Let ϕ be the phase angle between current and voltage, ϕ being taken as positive when the voltage leads, and negative when the current leads. Applying the ideas of the last paragraph, we see that such a current and voltage determine an impedance operator $R_S+jX_S=\mathbf{V}/\mathbf{I}$, and therefore a reactance X_s associated with

a resistance R_s, given by

$$X_s = \frac{V}{I} \sin \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$R_s = \frac{V}{I} \cos \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The same current and voltage also determine an admittance operator $G_p + jB_p = \mathbf{I}/\mathbf{V}$, and therefore a susceptance B_p associated with a conductance G_p, where

$$B_p = \frac{-I}{V} \sin \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$G_p = \frac{I}{V} \cos \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Now impedance operators, like d.c. resistances, are only additive for circuit-elements in series. Thus the resistance R_s and the reactance X_s must be regarded as in series. Admittance operators, on the other hand, like d.c. conductances, are only additive for circuit-elements in parallel, so that the conductance G_p must be regarded as in parallel with the susceptance B_p.

We have seen that an inductance L has a reactance $X = L\omega$, *i.e.*, an impedance operator $jL\omega$, and therefore an admittance operator $1/jL\omega = -j(1/L\omega)$ or a susceptance $B = -1/L\omega$. It follows that the reactance X_s is equivalent to an inductance L_s where

$$L_s = \frac{V}{I\omega} \sin \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and the susceptance B_p is equivalent to an inductance L_p where

$$L_p = \frac{V}{I\omega} \frac{1}{\sin \phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Similarly, since a capacitance C has a susceptance $B = C\omega$, an admittance operator $jB = jC\omega$ and therefore an impedance operator $1/jB = -j\left(\frac{1}{B}\right) = -j(1/C\omega)$ and a reactance $X = -\frac{1}{C\omega}$, the susceptance B_p is equivalent to a capacitance C_p where

$$C_p = \frac{-I}{V\omega} \sin \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and the reactance X_s is equivalent to a capacitance C_s where

$$C = \frac{-I}{V\omega}\frac{1}{\sin\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

The conductance G_p is obviously equivalent to a resistance $R_p = 1/G_p$.

These various pairs of quantities are shown diagrammatically in Figs. 3 and 4, the series connection being used for those

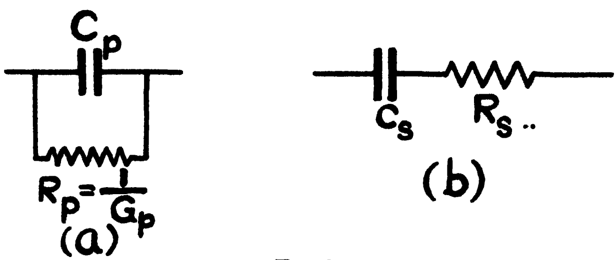


FIG. 3.

derived from impedances, and the parallel connection for those from admittances, for the reason already given. Thus all these diagrams may be used to represent the same circuit-element, though the inductances and capacitances may be positive or negative, depending on the sign of ϕ. In practice, it is found that

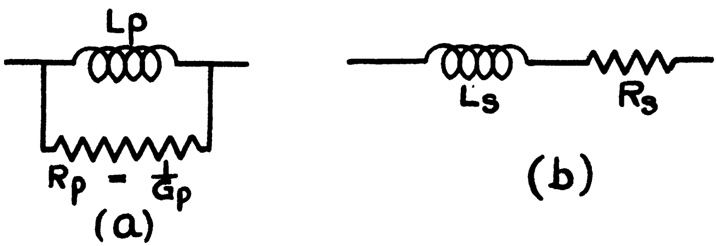


FIG. 4.

for circuits or material systems of any kind containing no source of e.m.f., the angle ϕ is never numerically greater than $\pi/2$, although it may be either positive or negative. It follows that for such systems the resistances R_s and R_p are always positive or zero, but that the quantities L_s, L_p, C_s, C_p may be of either sign, the inductances being positive and the capacitances negative when ϕ is positive, and these signs being reversed when ϕ is negative.

The following relations follow immediately from equations (7) to (14).

$$\tan\phi = \frac{L_s\omega}{R_s} = -R_pC_p\omega = -\frac{1}{R_sC_s\omega} = \frac{R_p}{L_p\omega} \quad . \quad . \quad . \quad . \quad (15)$$

$$\frac{R_s}{R_p} = \cos^2\phi \qquad \frac{L_s}{L_p} = \sin^2\phi \qquad \frac{C_p}{C_s} = \sin^2\phi \, . \quad (16)$$

Thus the two resistances R_s and R_p are only equal when $\phi = 0$, while the two inductances L_s and L_p only become equal when $\phi = \pi/2$, and the same holds for the capacitances C_s and C_p.

These quantities represent in various ways the relative values of the current and voltage in the circuit-element under consideration. They are called effective or equivalent values of resistance, inductance and capacitance. In order to distinguish between the members of the various pairs, they will be called the equivalent series resistance R_s, inductance L_s and capacitance C_s, and the equivalent parallel or shunt resistance R_p, inductance L_p and capacitance C_p. Hypothetical circuits representing these quantities as in Figs. 3 and 4 are called equivalent circuits.

5. **Power Factor, Decrement, and Loss Angle.** Consider now the energy which is supplied to a circuit when the current and voltage are given by $i = \hat{\imath}\sin\omega t$ and $v = \hat{v}\sin(\omega t - \phi)$. The power supplied is given by $P = vi$, which by a simple expansion becomes

$$P = \hat{v}\hat{\imath}\cos\phi\sin^2\omega t - \frac{\hat{v}\hat{\imath}}{2}\sin\phi\sin 2\omega t \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Thus the power may be regarded as consisting of two terms. The first is proportional to $\sin^2\omega t$ or to i^2, and therefore never becomes negative during the cycle. It represents a pulsating but unidirectional supply of energy. The second term is proportional to $\sin 2\omega t$, which becomes alternatively positive and negative twice during the cycle of current or voltage. This term may therefore correspond to a flow of energy into the circuit followed by an equal flow out of it, the transfer occurring twice per cycle.

Alternatively P may be written in the form :—

$$P = \frac{\hat{v}\hat{\imath}}{2}\cos\phi - \frac{\hat{v}\hat{\imath}}{2}\cos(2\omega t - \phi) \quad . \quad . \quad . \quad . \quad (18)$$

which corresponds to a supply of energy at the constant rate $\frac{1}{2}\hat{v}\hat{\imath}\cos\phi$, together with the supply and return of energy at a rate given by the alternating second term. In practice energy is seldom, if ever, dissipated by alternating currents at a constant rate. The expression $\frac{1}{2}\hat{\imath}\hat{v}\cos\phi$ always represents the mean rate of dissipation over any whole number of cycles or half-cycles, but in practice the energy is usually dissipated by resistance in accordance with Ohm's Law. When the energy is dissipated wholly by series resistance, *i.e.*, resistance carrying the whole current i, the power dissipation is proportional to i^2, and is given by the first term in (17). The remaining term $\frac{1}{2}\hat{v}\hat{\imath}\sin\phi\sin 2\omega t$ must then represent a reversible flow of energy into and out of the circuit. The amount of energy transferred can be found by integrating the term under discussion. Thus :—

$$W = \frac{\hat{v}\hat{\imath}\sin\phi}{2}\int_0^t \sin 2\omega t\,.\,dt = \frac{\hat{v}\hat{\imath}\sin\phi}{2\omega}\sin^2\omega t.$$

The amplitude of the expression is $(\hat{v}\hat{\imath}\sin\phi)/2\omega$, and we may regard the circuit as alternately storing and returning this amount of energy twice in each cycle.

This expression has been derived for the case of energy dissipation by series resistance. An equally important case is that in which it is dissipated by parallel resistance, *i.e.*, the resistive element experiences the full voltage instead of the total current. This case gives the same result, as may be seen by merely interchanging i and v in all the equations, the power dissipation being in this case proportional to v^2 instead of to i^2. The expression for the total reversibly stored energy is exactly the same as before ; it is only its phase relation to the current and voltage that is different.

It is interesting to notice in passing that if by the use of a suitable non-linear circuit-element the power dissipation could be made constant then equation (18) would apply, and the reversibly stored energy would amount to $\hat{v}\hat{\imath}/4\omega$ instead of $(\hat{v}\hat{\imath}\sin\phi)/2\omega$. This shows that observations of current, voltage and phase difference alone, give no certain indication of the magnitude of the energy stored in the magnetic fields of the inductors, or the electric fields of the condensers of a circuit. Information as to the law of energy dissipation is also required. It is on this account that capacitance and inductance as measured by alternating current methods, have been defined in terms of reactance and susceptance, instead of in terms of the energy stored in the magnetic and electric fields. The mean power dissipation is, however, always given by :—

$$\text{Mean Power Dissipation} = VI \cos \phi \quad . \quad . \quad . \quad (19)$$

where $I = \hat{\imath}/\sqrt{2}$ and $V = \hat{v}/\sqrt{2}$ denote the effective values of current and voltage. On account of this relation $\cos \phi$ is called the power factor of the circuit.

In the two important cases of energy dissipation due to series resistance, and parallel resistance, we have also :—

$$\text{Energy stored per half-cycle} = (VI \sin \phi)/\omega \quad . \quad . \quad (20)$$

Denote by Δ the ratio of the energy dissipated to the energy stored per half-cycle. Then we have :—

$$\Delta = \frac{\text{Energy dissipated}}{\text{Energy stored}} = \frac{VI \cos \phi}{2f} \cdot \frac{\omega}{VI \sin \phi}$$
$$= \pi \cot \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The quantity Δ is often called the logarithmic decrement of the circuit-element. It was originally defined by reference to the free oscillations of a system, which diminish in amplitude owing to the dissipation of its energy. Δ is equal to the Naperian logarithm of the ratio of two successive maximum displacements of the system in the same direction.

By making use of equations (7) to (14), equation (19) for the mean power dissipation may also be written

$$\begin{aligned} P &= I^2 R_s = V^2 G_p = V^2/R_p \\ &= I^2 L_s \omega \cot \phi = (V^2 \cot \phi)/L_p \omega \\ &= -\,V^2 C_p \omega \cot \phi = -\,I^2 \cot \phi / C_s \omega. \end{aligned}$$

It is evident from these equations that $\cot \phi$, when ϕ is positive (*e.g.*, for inductors) or $-\cot \phi$ when ϕ is negative (*e.g.*, for condensers) might also be regarded as a power factor, which becomes zero in the ideal cases when $\phi = \pm \pi/2$. It is, however, usually more convenient to consider the angle δ by which ϕ differs from the ideal values $\pm \pi/2$.

The above equations may then be written

$$P = I^2R_s = V^2G_p = V^2/R_p \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$P = I^2L_s\omega \tan \delta = (V^2 \tan \delta)/L_p\omega \quad . \quad . \quad (23)$$

$$P = V^2C_p\omega \tan \delta = (I^2 \tan \delta)/C_s\omega \quad . \quad . \quad . \quad (24)$$

Moreover δ is in practice often a very small angle so that to a first approximation $\delta = \tan \delta$, and the power dissipation is in such cases proportional to δ. The angle δ is called the loss angle, and $\tan \delta$ is called the loss tangent. Since $\tan \delta = \cot \phi$ we have, by (15)

$$\tan \delta = \frac{R_s}{L_s\omega} = \frac{L_p\omega}{R_p} = \frac{1}{R_pC_p\omega} = R_sC_s\omega \quad . \quad . \quad (25)$$

6. **Constants of Typical Circuit-Elements.** The relations established in paragraphs (4) and (5) are applicable to all circuits, circuit-elements, or current paths, whatever their nature, provided only that there are two terminals defining the points at which the current and voltage are to be measured. For the purposes of measurement it is necessary to have standard circuit-elements, the constants of which are accurately known. These consist of resistors, inductors and condensers, either alone or in combination. In actual practice it is impossible to separate completely resistance, inductance and capacitance ; *i.e.*, no material system is completely free from any one of these quantities, and all real, as distinct from ideal, standards must be regarded as possessing in some measure all these quantities linked together in various ways equivalent to series or parallel connection, or combinations of the two. The quantities measured by alternating current methods are the equivalent values L_s, R_s, C_s (series), or L_p, R_p, C_p (parallel), discussed in the previous paragraphs ; and it therefore becomes necessary to determine the relation between these quantities and the various possible combinations of real values L, R, C. The equivalent values for any combination may be calculated by the methods already outlined. Those required most frequently in practice are summarised in Table 1.

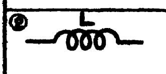
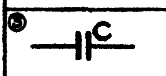
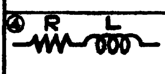
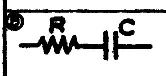
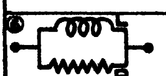
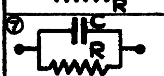
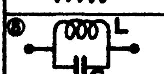
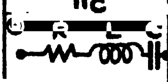
It will be obvious from this table that the measured values L , R_s, R_p, C_p, etc., can only be identified with true values of inductance, etc., L, R, C, when the nature of the current path is known, and it is possible to obtain information as to the values of α, β and γ. Such information is usually obtained by observa-

Table I. Useful Formulae for Equivalent Resistance, Inductance, Capacitance etc.

$$\alpha = \frac{L\omega}{R} \qquad \beta = RC\omega \qquad \gamma = LC\omega^2 = \alpha\beta$$

NOTE:- α, β, and γ are ratios of zero dimensions e.g.

$$\gamma = L\omega / \left(\frac{1}{C\omega}\right)$$

Circuit	R_S	L_S	$R_P = 1/G_P$	C_P	Tan ϕ
	R	0	R	0	0
	0	L	∞	$-\frac{1}{L\omega^2}$	∞
	0	$-\frac{1}{C\omega^2}$	∞	C	$-\infty$
	R	L	$R(1+\alpha^2)$	$-\frac{1}{L\omega^2}\cdot\frac{\alpha^2}{1+\alpha^2}$	α
	R	$-\frac{1}{C\omega^2}$	$R(1+\frac{1}{\beta^2})$	$\frac{C}{1+\beta^2}$	$-\frac{1}{\beta}$
	$R.\frac{\alpha^2}{1+\alpha^2}$	$\frac{L}{1+\alpha^2}$	R	$-\frac{1}{L\omega^2}$	$\frac{1}{\alpha}$
	$\frac{R}{1+\beta^2}$	$\frac{-CR^2}{1+\beta^2}$	R	C	$-\beta$
	0	$\frac{L}{1-\gamma}$	∞	$C-\frac{1}{L\omega^2}$	$\pm\infty$ or 0
	R	$L(1-\frac{1}{\gamma})$	$R\left[\frac{\beta^2+(1-\gamma)^2}{\beta^2}\right]$	$\frac{C(1-\gamma)}{\beta^2+(1-\gamma)^2}$	$\alpha-\frac{1}{\beta}$
	$\frac{R}{(1-\gamma)^2+\beta^2}$	$\frac{L(1-\gamma)-CR^2}{(1-\gamma)^2+\beta^2}$	$R(1+\alpha^2)$	$C-\frac{1}{L\omega^2}\frac{\alpha^2}{(1+\alpha^2)}$	$\alpha(1-\gamma)-\beta$
	$\frac{R\alpha^2}{\alpha^2+(1-\gamma)^2}$	$\frac{L(1-\gamma)}{\alpha^2+(1-\gamma)^2}$	R	$C-\frac{1}{L\omega^2}$	$\frac{1}{\alpha}-\beta$

tions at various frequencies, *i.e.*, from the variation of L_s, R_s, C_s, etc., with frequency.

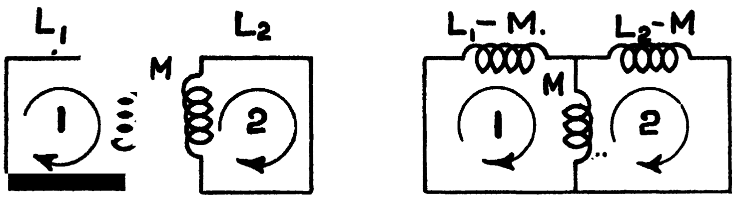
7. Mutual Impedance. We have so far considered only two-terminal systems, which may be regarded as single isolated circuits. It is also necessary to consider the actions between neighbouring circuits. The simplest case is that of the mutual inductance of two coils. It is well known that the passage of a current i in one coil induces an e.m.f. $M\,di/dt$ in any neighbouring circuit, where M_0 is the coefficient of mutual inductance for the pair of circuits in question. Thus an alternating current $i=\hat{\imath}\sin\omega t$ produces an e.m.f. $E=M_0\omega\hat{\imath}\cos\omega t=M_0\omega\hat{\imath}\sin(\omega t+\pi/2)$ in the other. This e.m.f. is clearly in quadrature with the primary current, and we may therefore represent it in vector notation by

$$\mathbf{E}=j\omega M_0\mathbf{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

In practice this e.m.f. can never be directly observed. We can only observe the p.d. V_s between the terminals of the secondary circuit. This would be equal to E if the secondary circuit were of infinite impedance so that no current could flow in it. In practice, however, there are always circulating currents in the secondary circuit, since it is always closed by the capacitance between its terminals and between neighbouring turns of the coil, even if there is no other current path. Thus the secondary p.d. V_s differs slightly from E and is usually not quite in quadrature with the primary current, and we must therefore write

$$\mathbf{V}_s=(\sigma+j\omega M)\mathbf{I}=(\sigma+jX_m)\mathbf{I} \quad . \quad . \quad . \quad . \quad (27)$$

which means that V_s possesses a small in-phase component $\sigma\mathbf{I}$ as well as the quadrature component $j\omega M\mathbf{I}$. M, which is not quite the same as M_0, is called the effective mutual inductance. This equation is analogous to (1) so that the quantity represented by $(\sigma+jM\omega)$ may be regarded as an impedance, the mutual impedance Z_m of the two circuits, consisting of two components, their mutual resistance σ, and their mutual reactance $X_m=M\omega$. Similarly, by taking the reciprocal of Z_m we may derive the mutual admittance, mutual conductance, etc., of the two circuits. Moreover, these considerations apply to any two circuits, not only those linked by mutual inductance, but also to those possessing current paths in common as in Figs. 6, 7 and 8. It is interesting to note that as far as the values of terminal alternating currents and voltages are concerned, the circuits of Figs. 5 (*a*) and 5 (*b*) are identical. Thus the effect of a mutual inductance M between two coils is exactly the same as if a portion M of their self-inductance was common to both. Similarly two circuits of impedance Z_1 and Z_2 possessing a mutual impedance Z_m may be



(*a*) (*b*)

FIG. 5. Circuits with magnetic coupling. (*a*) Indirect. (*b*) Direct.

represented by Fig. 6, although in practice the circuits may possess no common elements, but may be isolated as in Fig. 5 (*a*).

8. **Coupled Circuits.*** Circuits such as those described above which possess a finite mutual impedance are often described as coupled, since electrical oscillations of one of the circuits generate e.m.f.s in the other, and thereby tend to produce oscillations in this second circuit, so that provided the second circuit is closed the energy of one circuit is communicated in a greater or less degree to the other. When the circuits possess a common element as in Fig. 5 (*b*), the coupling is said to be direct, while circuits such as those of Fig. 5 (*a*), possessing no common point, are said to be indirectly coupled. In such cases the invisible connecting link between the two circuits is provided either by their magnetic fields, or by their electric fields, the one being called magnetic or inductive coupling, and the other electrostatic or capacitive coupling.

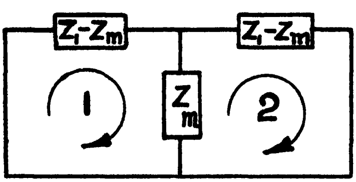


FIG. 6.

We have seen that in any circuit there is a reversible storage and discharge of energy in each half-cycle amounting to $VI \sin \phi/\omega$,

* G. W. O. HOWE. *Wireless Engineer*, 1932, Vol. 9, p. 485.

which, using equations (7) to (14), may be expressed in the following forms.

Energy stored per half-cycle =

$$\frac{I^2X_s}{\omega} = I^2L_s = \frac{V^2B_p}{\omega} = V^2C_p . \quad . \quad . \quad . \quad (28)$$

from which it is clear that when the reactance is purely inductive the energy is that of the magnetic field of the inductive coils ($\frac{1}{2}i^2L$), while when the reactance is purely capacitive, the energy is that of the electric field of the condensers ($\frac{1}{2}v^2C$) or of any metal parts which, being at different potentials, are equivalent to a condenser. This energy is to be regarded as located in the medium surrounding the circuits, and in so far as the electric and

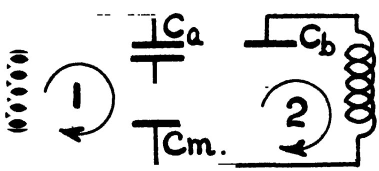


FIG. 7. Direct capacitive coupling.

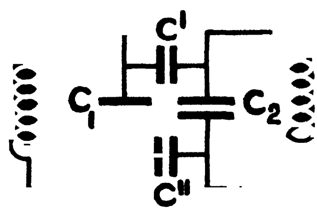


FIG. 8. Indirect capacitive coupling.

magnetic fields of the one circuit penetrate the other, a portion of this stored energy is as much linked with the second circuit as the first, and in suitable circumstances may be discharged in the second circuit. When only a very small fraction of the total stored energy may be transferred in this way from one circuit to the other, the coupling is said to be loose, while when the fraction which may be transferred is comparatively large the coupling is said to be tight. The magnitude of the coupling is measured by the coefficient of coupling, which may be defined as the geometric mean of the two numbers expressing the fraction of the stored energy of one circuit which is capable of transference to the other. Now, since for a given current and frequency stored energy is proportional to reactance, it will be clear from Fig. 5 (*b*) that the ratio of the common stored energy to the total stored energy of circuit 1 is X_m/X_1 where X_m is the mutual reactance, and X_1 the reactance of circuit 1. Similarly the ratio of common stored energy to the total stored energy of circuit 2 is X_m/X_2. The coefficient of coupling k is therefore given by

$$k = \sqrt{\frac{X_m}{X_1} \cdot \frac{X_m}{X_2}} = \frac{X_m}{\sqrt{X_1 X_2}} \quad . \quad . \quad . \quad . \quad . \quad (29)$$

This expression may be applied to either magnetic or capacitive coupling, it being understood that when X_m represents mutual inductance then X_1 and X_2 represent the inductive reactance of circuits 1 and 2, while when X_m represents capacitive linking such as those of Figs. 7 and 8, X_1 and X_2 represent the capacitive reactance of circuits 1 and 2. Thus it is easy to see that for the circuits of Fig. 5, $k = M/\sqrt{L_1 L_2}$, and for that of Fig. 7, $k = \sqrt{C_1 C_2}/C_m$ where C_1 is the capacitance of C_a and C_m in series,

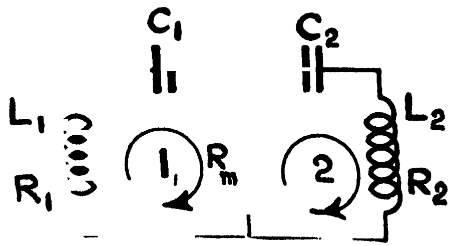


FIG. 9. Resistive coupling.

and C_2 that of C_b and C_m in series. Fig. 8 is more complicated but it can be shown that in this case $k = C_m/\sqrt{(C_m + C_1)(C_m + C_2)}$ where C_m is the capacitance of C' and C'' in series. This diagram might well represent the case in which the coupling between two circuits arises, not from condensers, but from the capacitance between their various components, or their "stray" electric field, which is represented by the equivalent condensers C' and C''.

It should be noticed that the coefficient of coupling is a pure number which is always less than unity. In general both magnetic and electric coupling exist between two circuits, and therefore they will possess a coefficient of electric coupling and one of magnetic coupling. Usually one of the two kinds of coupling is made very small compared with the other, and is then neglected. It will be seen later that electric or capacitive coupling can be eliminated by screening, *i.e.*, by completely enclosing one circuit by a conductor, so that the whole of the electric field of that circuit is confined within this conductor, and is therefore unable to supply energy to the other circuit.

Circuits such as those of Fig. 9 in which R_m is a small resistance are sometimes described as resistance-coupled, and by analogy with the previous cases the coefficient of coupling is given as $k = R_m/\sqrt{R_1 R_2}$. The analogy is perhaps formal rather than physical, and probably does no more than provide a convenient description for an alternative means of energising one circuit by the passage of a definite current in another.

CHAPTER II

GENERAL PRINCIPLES OF IMPEDANCE MEASUREMENT: RESONANCE METHODS

1. **General Features of High-frequency Measurements.** In measurements of impedance, or quantities derived from it, we are concerned with the relative values of current and voltage in an electric circuit, and most methods depend on an application of Ohm's Law, or some generalisation deduced from it, to a network carrying current. Resistance to direct current is usually measured by means of the well-known Wheatstone Bridge network, and most readers will know how easy it is to obtain very high accuracy in such measurements. When this method is applied to alternating currents many difficulties arise. The flow of current is no longer controlled by the single variable, resistance; e.m.fs. of self and mutual inductances influence the current in every conductor, and displacement currents (p. 3) flow through all the insulating materials. These effects may by suitable design of the circuits be made small or controlled but they can never be wholly eliminated. The e.m.fs. and displacement currents which are not localised or controlled are often called stray e.m.fs. and stray capacity-currents. It should be observed that for a given voltage the displacement current through the dielectric is proportional to the frequency, whereas the current through a conductor does not increase with rise of frequency. Thus the higher the frequency the more does the current tend to leave the conductors and flow across the intervening media, and therefore the more uncertain does the current distribution in a network become. The difficulties arising are analogous to those which would be produced by immersing a d.c. network into a more or less conducting fluid such as water. The stray e.m.fs. are also proportional to the frequency, so that at radio frequencies the current distribution is always somewhat uncertain owing to displacement currents, and the voltage distribution is always somewhat uncertain owing to stray e.m.fs. It is on this account that so many

radio frequency measurements take the form of simple substitution of one component for another (the standard) in a circuit of the simplest possible form, viz., the simple resonating circuit.

2. Bridge and Resonance Methods Compared. Consider the simple resonating circuit of Fig. 10. A coil of inductance L and resistance R is connected to a condenser of capacitance C. An e.m.f. $e = \hat{e} \sin \omega t$ is induced in the circuit and causes a current I to circulate in it. The impedance of the network to the circulating current may be written

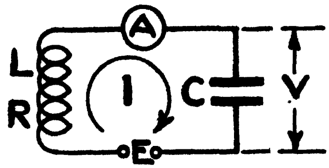


FIG. 10. Simple resonating circuit.

$$\mathbf{Z} = R + j\left(L\omega - \frac{1}{C\omega}\right) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The current may be represented

$$\mathbf{I} = \frac{\mathbf{E}}{R + j\left(L\omega - \frac{1}{C\omega}\right)} = \frac{\mathbf{E}\left[R - j\left(L\omega - \frac{1}{C\omega}\right)\right]}{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

The magnitude of the current vector is

$$I = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and if C, L or ω are adjusted until

$$L\omega = \frac{1}{C\omega} \text{ or } LC\omega^2 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the current then reaches a maximum value, and the circuit is said to be in resonance. The maximum value of the current is given by

$$I_r = \frac{E}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The earliest and still the most commonly used methods of measurement of capacitance, inductance and resistance at radio frequencies depend on the application of equations (3), (4) and (2) to observations made with this circuit. Thus if L is known and the frequency at which resonance occurs is measured, C may be deduced from (3). In the same way L may be measured if C is known, while the value of a resistance may be determined by adding it to the circuit and noting the change it produces in the value of the current at resonance, and then applying equation (4). Alternatively R may be measured by establishing resonance, and then varying L, C or ω by a known small amount, and measuring the change of current produced. A simple application of equations (2) and (4) then gives R. The details need not concern us here. The point to notice is that the same circuit is used for all the measurements, the reason being that it is the simplest possible form of circuit—one mesh with no branches. The current distribution is therefore the simplest possible, one circulating current only. It is true that displacement currents must be regarded as flowing from various points on the circuit to all neighbouring objects, but the circuit is always adjusted so that the circulating current has its maximum possible value, and the stray currents therefore tend to become as insignificant in comparison with the main current as it is possible to make them. When they are ignored, as they are when the above equations are applied, the errors introduced are not usually very large, yet it must be confessed that the method of measurement itself is a comparatively crude one, although well suited to difficult conditions.

It is interesting to note in passing that the same circuit is also used in direct current measurements when the difficulty of uncertainty of current distribution appears. For the measurement of a very high resistance we do not use the Wheatstone bridge, but a simple circuit including only a battery, galvanometer and the high resistance to be measured. In this case the total current is very small, and the effect of even minute leakage current is apt to become serious. The simplest possible circuit is therefore used, special attention being paid to the insulation of those points at which leakage would be most serious. The simple method is more satisfactory than the more elaborate bridge methods when the conditions are very difficult, and similarly when an investigation is being pushed into the region of the highest possible frequencies, resonance methods are likely to be more serviceable than bridge methods. However, the accuracy obtainable under suitable conditions by bridge methods is much better than the cruder methods will ever give. The general superiority of null-methods over deflectional methods is well known. It is therefore advisable to use bridge methods, when the conditions are favourable, as in this way accuracies otherwise unobtainable may be reached. Thus for the measurement of inductance, capacitance and resistance at low frequencies, bridge methods are almost universally used, and accuracies of the order of 0·01 per cent. are readily obtainable at audio frequencies, an accuracy which is probably unattainable by resonance methods. In theory, there is no reason why almost any one of these bridges should not also be applied to radio frequencies. The difficulties are the practical ones of constructing the apparatus so that capacitive currents are either confined to the arms of the network or else diverted into a course that does not affect the balance of the bridge; and further of disposing the various conductors so that undesired e.m.fs. are excluded or balanced.

Further consideration of bridge methods is reserved for Chapter III. It will now be necessary to discuss in more detail the phenomenon of resonance in electrical circuits. This phenomenon is probably the most characteristic phenomenon of radio-frequency circuits and is the basis of most applications of radio-frequency currents.

3. Series Resonance: Current Resonance. We shall now consider further the values of current and voltage in the simple resonance circuit of Fig. 10. Let the e.m.f. E be applied between the points marked E, and let the current I be measured by some form of ammeter at A. Keeping the applied e.m.f. constant let the value of the capacitance C be varied. The value of the current will be given by equation (2),

$$I^2 = \frac{E^2}{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

If now the value of the current is plotted against capacitance, we obtain the well-known resonance curves shown in Fig. 11. The current reaches a maximum value $I_r = E/R$ when the capacitance has the value C_r which satisfies the condition of resonance $LC_r\omega^2 = 1$. Thus the smaller the value of R the resistance of the circuit, the greater is the maximum value of the current and the sharper is the peak of the resonance curve. The curves *a*, *b*, *c* of Fig. 11 correspond to ascending values of resistance, other factors being the same. The sudden rise in the value of the current to a maximum value is frequently used in radio-frequency measurements as an indication that the condition of resonance has been established. It is important in such measurements that.. the resonance curve shall rise to a narrow peak.

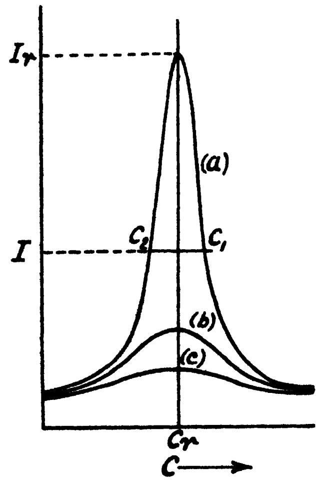


Fig. 11. Resonance curves.

It is instructive to derive a quantity which may be regarded as measuring the sharpness of resonance. Thus let I_r be the maximum value of the current at resonance when the value of capacitance is C_r and I the value of the current corresponding to the value C of capacitance at a point on the side of the curve. We have

$$I^2 = \frac{E^2}{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} = \frac{E^2}{R^2} \cdot \frac{1}{1 + \frac{1}{R^2}\left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$= I_r^2 \frac{1}{1 + \frac{1}{R^2}\left(\frac{1}{C_r\omega} - \frac{1}{C\omega}\right)^2}$$

Thus
$$\frac{I_r^2}{I^2} = 1 + \frac{1}{\omega^2 R^2}\left(\frac{1}{C_r} - \frac{1}{C}\right)^2$$

$$\frac{\sqrt{\frac{I_r^2 - I^2}{I^2}}}{\pm\left(\frac{C - C_r}{C}\right)} = \frac{1}{RC_r\omega} = \frac{L\omega}{R} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The quantity on the left-hand side of this equation is evidently a measure of the fractional fall in the value of I^2 from the resonance value corresponding to a given fractional change of capacitance. It is defined as the *sharpness of resonance.* It is to be noted that it is equal to the reciprocal of tan δ or to π/Δ, for the inductance L in series with the resistance R, or for the capacitance C_r in series with the resistance R, δ being the loss angle and Δ the decrement of the resonating circuit, *i.e.*, the ratio of energy dissipated to energy stored per half-cycle, the value being the same for the condition of resonance, whether the energy stored in the coil or the condenser is taken. Evidently determinations of loss angle, decrement and resistance can be made by a measurement of the sharpness of a resonance curve.

There are, of course, two values of C corresponding to any given value of I, one point being on either side of the resonance peak. Equation (5) may be written in the form

$$\pm \frac{(C - Cr)}{C} = \frac{\Delta}{\pi}\sqrt{q-1}$$

where q is the ratio I_r^2/I^2 and Δ is the decrement of the circuit at the resonance frequency $\omega/2\pi$. Thus the two values of C (say C_1 and C_2) corresponding to a given value of q, *i.e.*, of I, are given by

$$C_1 - C_r = \frac{\Delta}{\pi}\sqrt{q-1} \, . \, C_1 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and

$$-C_2 + C_r = \frac{\Delta}{\pi}\sqrt{q-1} \, . \, C_2 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Subtracting, we obtain

$$C_r = \frac{C_1 + C_2}{2} - \frac{\Delta}{\pi}\sqrt{q-1} \, . \, \frac{(C_1 - C_2)}{2} \quad . \quad . \quad . \quad (8)$$

and adding (6) and (7) we have

$$C_1 - C_2 = \frac{\Delta}{\pi}\sqrt{q-1} \, . \, (C_1 + C_2) \quad . \quad . \quad . \quad . \quad (9)$$

It is important to notice that the values of C_1 and C_2 may always be observed with greater accuracy than that of C_r, since C_1 and C_2 are on the sides of the resonance curve where its slope is steep, while C_r corresponds to the maximum point, where the slope is zero, so that a small change of capacitance produces a relatively small change of current. Equation (9) shows how the decrement Δ may be accurately determined from observations of C_1, C_2 and q. Equation (8) shows that the value of C_r is not equal to the mean value of C_1 and C_2, but differs from it by a quantity, which becomes small when Δ, q or $(C_1 - C_2)$ become small, *i.e.*, when the circuit is of low decrement, or when the points are taken near the peak of the curve. The resonance curve is therefore not strictly symmetrical but broadens out on the right-hand side, as follows from equations (6) and (7).

4. **Voltage Resonance.** The phenomenon of resonance in the simple circuit of Fig. 10 may also be observed by measuring the voltage between any two points of the circuit, by means of some form of voltmeter, instead of measuring the current in the manner discussed in the preceding paragraph. Suppose for example the voltage **V** across the condenser of Fig. 10 is observed. We have in vector notation

$$\mathbf{V} = \frac{\mathbf{E}}{R + j\left(L\omega - \dfrac{1}{C\omega}\right)} \cdot \frac{-j}{C\omega} = \frac{-j\mathbf{E}}{RC\omega + j(LC\omega^2 - 1)}.$$

Thus the magnitudes of **V** and **E** are given by

$$V^2 = \frac{E^2}{R^2C^2\omega^2 + (LC\omega^2 - 1)^2} \quad . \quad . \quad . \quad . \quad (10)$$

If now the e.m.f. E is kept constant while C varies, V will be found to rise sharply to a maximum value V_r for a certain value of C, in much the same way as the current I did. Indeed, it is obvious that the very sharp rise in the value of the current at the resonance point discussed in the last paragraph must be accompanied by a sharp rise in the voltage. We shall find however, that the maximum value of the voltage does not occur at the same value of C as the maximum value of the current, and that the curve showing the rise and fall of voltage as C is varied is not quite of the same shape as the curve showing the rise and fall of the current. We shall therefore distinguish the two cases by referring to them as cases of *current resonance* and *voltage resonance* respectively.

The value of C at which the voltage V reaches its maximum value may be found by differentiating with respect to C the denominator in equation (10) and equating to zero. This gives as the condition for voltage resonance

$$C_r = \frac{L}{R^2 + L^2\omega^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where C_r denotes the value of capacitance at which resonance occurs. Thus for voltage resonance, we no longer have the simple relation $LC\omega^2 = 1$. The resonance point is now affected by the value of the resistance R. We have

$$LC_r\omega^2 = \frac{L^2\omega^2}{R^2 + L^2\omega^2} = 1 - \frac{R^2}{R^2 + L^2\omega^2} = 1 - \cos^2 \phi$$
$$= 1 - \sin^2 \delta$$

where ϕ is the phase angle and δ is the loss angle of the resistance R in series with the inductance L.

The value of the voltage at resonance (V_r) is obtained by substituting (11) in (10). We find that

$$V_r^2 = \frac{E^2(R^2 + L^2\omega^2)}{R^2} = \frac{E^2}{\cos^2\phi} \quad . \quad . \quad . \quad . \quad (12)$$

As in the case of current resonance, to any other value of the voltage V, there will correspond two values of capacitance C, one on either side of the resonance peak. Combining equations (10) and (12), we see that these two values of capacitance are given by the equation

$$\frac{V_r^2}{V^2} = q = \frac{[R^2C^2\omega^2 + (LC\omega^2 - 1)^2]\,[R^2 + L^2\omega^2]}{R^2}$$

or $$C^2\omega^2(R^2 + L^2\omega^2) - 2L\omega \, . \, C\omega - q\frac{R^2}{R^2 + L^2\omega^2} + 1 = 0$$

which is a quadratic in $C\omega$, the two roots of which are

$$C\omega = \frac{L\omega \pm \sqrt{(q - 1)R^2}}{R^2 + L^2\omega^2} = C_r\omega \pm \sqrt{q - 1} \, . \, \frac{R}{R^2 + L^2\omega^2}.$$

Denote as before, the two values of C corresponding to a given ratio q ($= V_r^2/V^2$) by C_1 and C_2.

Then $$C_r = \tfrac{1}{2}(C_1 + C_2),$$

which means that in this case the curve is symmetrical. Also

$$C_1 - C_2 = \frac{2\sqrt{q-1}}{\omega} \cdot \frac{R}{R^2 + L^2\omega^2}$$

$$= \frac{2\sqrt{q-1}}{\omega} \frac{R}{-} . C$$

$$C_1 - C_2 = \sqrt{q-1} \cdot \frac{R}{L\omega} (C_1 + C_2). \; .$$

This last equation is exactly the same as was obtained for the corresponding case of current resonance (equation (9)). In other respects the two resonance curves are different as we have seen.

When dealing with voltage resonance it is important to specify between which two points of the circuit the voltage is measured, and also at what point of the circuit the e.m.f. is introduced. We have assumed that the voltage is measured between the condenser terminals, and that the exciting e.m.f. E is introduced at a point external to these terminals, *e.g.*, in the coil or the leads to the coil. If however this e.m.f. were introduced into the condenser leads, it would be included in the voltage measured, which would now be given by $V - E$ (*cf.* Fig. 10) so that the equations of resonance would be modified. It is unnecessary to consider this and other cases in greater detail, but the fact that coupling is not strictly localised and that e.m.fs. are frequently introduced into all points of a circuit should not be lost sight of in the practical application of these equations.

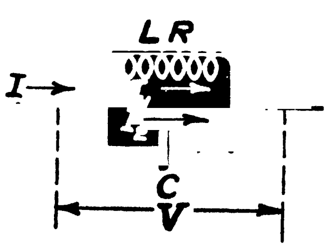


FIG. 12. Circuit for parallel resonance.

5. Parallel Resonance. A third method of observing electrical resonance remains to be considered. Let a coil of resistance R and inductance L be connected in parallel with a capacitance C across a source of alternating voltage of constant value V (Fig. 12). Consider the value of the current I flowing into the parallel circuit where it divides into the two components I_1 and I_2.

We have

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \frac{\mathbf{V}}{R + jL\omega} + j\mathbf{V}C\omega$$

$$= \mathbf{V}\left[\frac{R - jL\omega}{Z^2} + jC\omega\right] \text{ where } Z^2 = R^2 + L^2\omega^2$$

$$= \mathbf{V}\left[\frac{R}{Z^2} + j\omega\left(C - \frac{L}{Z^2}\right)\right]$$

Thus the magnitude of the current is given by

$$I^2 = V^2\left[\left(\frac{R}{Z^2}\right)^2 + \omega^2\left(C - \frac{L}{Z^2}\right)^2\right] \quad . \quad . \quad . \quad . \quad (13)$$

If the capacitance C is varied, the value of the current will evidently reach a minimum value when

$$C = \frac{L}{Z^2} = \frac{L}{R^2 + L^2\omega^2} = C_r \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The minimum value of the current is given by

$$I = V \cdot \frac{R}{Z^2} = V \cdot \frac{R}{R^2 + L^2\omega^2} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and it is in phase with the voltage $\mathbf{V}$. If the value of the current I is plotted against capacitance C we obtain a curve like that shown

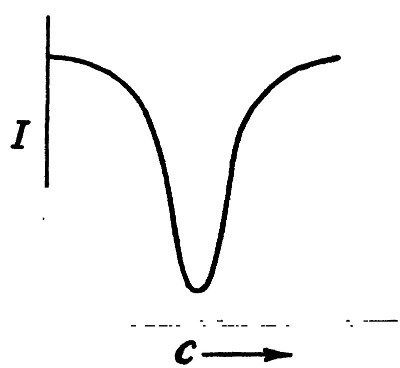


FIG. 13. Curve for parallel resonance.

in Fig. 13, which may be regarded as a resonance curve with a sharp dip instead of a sharp peak. The equation of the curve, *i.e.*, (13) may be written

$$I^2 = I_r^2 + V^2\omega^2\left(C - \frac{L}{Z^2}\right)^2$$

$$\therefore \qquad \frac{I^2 - I_r^2}{I_r^2} = \frac{\omega^2\left(C - \frac{L}{Z^2}\right)^2}{R^2} \cdot Z^4 = \frac{\omega^2(CZ^2 - L)^2}{R^2}$$

Thus the two values of C corresponding to a given ratio $I^2/I_r^2 = q$ are given by

$$\pm\sqrt{q-1}\,\frac{R}{\omega} = CZ^2 - L$$

$$\text{or} \quad C = \frac{L}{Z^2} \pm \frac{R}{Z^2}\,\frac{\sqrt{q-1}}{\omega} = C_r \pm \frac{R}{Z^2}\,\frac{\sqrt{q-1}}{\omega} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Evidently the resonance curve is symmetrical and we have for the difference of the two values of C

$$C_1 - C_2 = \frac{2R}{R^2 + L^2\omega^2} \cdot \frac{\sqrt{q-1}}{\omega} = \frac{\sqrt{q-1}}{\omega}\,\frac{R(C_1 + C_2)}{L} \quad . \quad (17)$$

This formula is the same as that obtained for the previous cases. The only difference is that q represents the ratio of different quantities in the three cases.

The circuit we have considered in each of these cases of resonance is the same, but it is to be noted that the value of the capacitance at which series current resonance occurs is different from that at which voltage resonance or parallel current resonance occurs. What we mean exactly by the condition of resonance is a matter of arbitrary definition, and it is important to recognise this when making precision measurements based on observations of resonance. We have assumed that resonance has been obtained by variation of capacitance in each case. This is the most important practical case, but it is also possible to obtain similar maxima and minima of current and voltage by variation of inductance or of frequency. Examination of the equations will show that the conditions of resonance, *i.e.*, maxima and minima of current or voltage, are different if these quantities are varied instead of capacitance, but when R is very small compared with $L\omega$, which is often the case in practice, the condition of resonance is practically the same whether it is obtained by variation of capacitance, inductance, or frequency and whether it is observed as current or voltage.

6. **The Effect of Leakage or Dielectric Loss.** In the above discussion we have assumed that the condenser C is perfect. A point of some practical importance is the effect of leakage in this condenser on the condition of resonance. We shall suppose the capacitance C to be shunted by a conductance G (Fig. 14) and consider its effect on the three cases of resonance considered above. The leaky condenser, *i.e.*, the combination of capacitance and conductance may be represented by the admittance operator $G + jC\omega$, and the vector equations previously obtained still apply if we replace $jC\omega$ by $G + jC\omega$. Thus considering the case of series current resonance, the current is given by

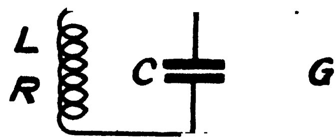


FIG. 14. Oscillating circuit with leakage or dielectric loss.

$$\mathbf{I} = \frac{\mathbf{E}}{R + jL\omega + \dfrac{1}{G + jC\omega}} = \frac{\mathbf{E}}{R + jL\omega + \dfrac{G - jC\omega}{G^2 + C^2\omega^2}}$$

$$= \frac{\mathbf{E}}{R + \dfrac{G}{Y^2} + j\omega\left(L - \dfrac{C}{Y^2}\right)}$$

where $$Y^2 = G^2 + C^2\omega^2.$$

Thus the magnitude of the current is now given by

$$I^2 = \frac{E^2}{\left(R + \dfrac{G}{Y^2}\right)^2 + \omega^2\left(L - \dfrac{C}{Y^2}\right)^2}$$

The condition for the maximum value of the current as C is varied is readily found by differentiating the denominator with respect to C and equating to zero. We find that it is

$$LC\omega^2 = 1 + 2RG + LG^2/C$$

or $$LC\omega^2(1 - \tan^2 \delta) = 1 + 2RG.$$

It is not necessary to pursue the equations further. It will be evident that the simple equations holding when the condenser is perfect are very considerably complicated by the presence of leakage.

Consider now the voltage resonance. The voltage across the leaky condenser is given by

$$\mathbf{V} = \frac{\mathbf{I}}{G + jC\omega} = \frac{\mathbf{E}}{\left(R + jL\omega + \frac{1}{G + jC\omega}\right)(G + jC\omega)}$$

$$= \frac{\mathbf{E}}{(R + jL\omega)(G + jC\omega) + 1} = \frac{\mathbf{E}}{R + jL\omega} \cdot \frac{1}{G + jC\omega + \frac{1}{R + jL\omega}}$$

$$= \frac{\mathbf{E}}{R + jL\omega} \cdot \frac{1}{G + \frac{R}{Z^2} + j\omega\left(C - \frac{L}{Z^2}\right)} = \frac{\mathbf{I}'}{G + \frac{R}{Z^2} + j\omega\left(C - \frac{L}{Z^2}\right)}$$

where $\mathbf{I}'$ represents the vector $\mathbf{E}/(R + jL\omega)$ *i.e.*, the current which would flow if the coil were short circuited. If now C alone is varied then $\mathbf{I}'$ is constant and evidently the voltage V will reach a maximum value when

$$C_r = L/Z^2.$$

This maximum value is given by

$$\mathbf{V}_r = \frac{\mathbf{E}}{R + j\omega L} \cdot \frac{1}{\left(G + \frac{R}{Z^2}\right)}$$

$$= \frac{\mathbf{E}(R - j\omega L)}{GZ^2 + R}.$$

Its magnitude is therefore given by

$$V_r = \frac{EZ}{GZ^2 + R} = \frac{E}{GZ + \frac{R}{Z}} = \frac{E}{G\sqrt{\frac{L}{C_r}} + R\sqrt{\frac{C_r}{L}}}$$

The voltage V corresponding to any other value of C is given in magnitude by

$$\frac{V_r^2}{V^2} = \frac{\left(G + \frac{R}{Z^2}\right)^2 + \omega^2\left(C - \frac{L}{Z^2}\right)^2}{\left(G + \frac{R}{Z^2}\right)^2}$$

or
$$\frac{V_r^2}{V^2} - 1 = \frac{\omega^2\left(C - \frac{L}{Z^2}\right)^2}{\left(G + \frac{R}{Z^2}\right)^2}.$$

Consider finally the case of parallel resonance. The total current I produced by a constant applied voltage V is now given by

$$\mathbf{I} = \mathbf{I}_L + \mathbf{I}_C = \mathbf{V}\left[\frac{1}{R + j\omega L} + G + jC\omega\right]$$
$$= \mathbf{V}\left[\frac{R}{Z^2} + G + j\omega\left(C - \frac{L}{Z^2}\right)\right].$$

If C alone is varied, the condition for the minimum value of the current is

$$C_r = \frac{L}{Z^2}$$

and the current is then in phase with the voltage V and has the value

$$I_r = V\left(G + \frac{R}{Z^2}\right).$$

The current I corresponding to any other value of C is given by

$$\frac{I^2}{I_r^2} \quad \frac{\left(G + \frac{R}{Z^2}\right)^2 + \omega^2\left(C - \frac{L}{Z^2}\right)^2}{\left(G + \frac{R}{Z^2}\right)^2}$$

or
$$\frac{I^2}{I_r^2} - 1 = \frac{\omega^2\left(C - \frac{L}{Z^2}\right)^2}{\left(G + \frac{R}{Z^2}\right)^2}$$

It should be noticed that L and R in Figs. 10 and 14 necessarily represent the equivalent series inductance L_s and resistance R_s of the whole circuit external to the condenser. If the equivalent parallel inductance L_p is used, the condition of voltage resonance becomes $L_pC_r\omega^2 = 1$, for $L_p = L_s/\sin^2\phi = Z^2/L_s\omega^2$ (*cf.* equations (15), (16), Chapter I).

An examination of these equations for the three cases of resonance reveals the following important points :—

1. The simple equation of resonance $L_s C\omega^2 = 1$ only applies to the case of series current resonance when the condenser is free from leakage (or dielectric losses which are equivalent to leakage). In this case only is the condition of resonance independent of the resistance of the circuit.
2. The condition of voltage resonance is the same as that of parallel current resonance, and is independent of conductance in the condenser, although it does depend on the series resistance of the coil. The equation is

$$C_r = L_s/Z^2 = L_s/(R_s^2 + L_s^2\omega^2).$$

   which may also be expressed in the form $L_p C_r \omega^2 = 1$.
3. When conductance is present in the condenser, the condition of series current resonance depends on both the resistance of the coil and the conductance of the condenser.
4. For voltage resonance and parallel current resonance the equation giving the two values of capacitance on the resonance curve corresponding to any given value of current or voltage may be written

$$\sqrt{q-1} = \frac{\pm\left(C - \frac{L_s}{Z^2}\right)\omega}{\left(G + \frac{R_s}{Z^2}\right)}$$

or $$C = \frac{L_s}{Z^2} \pm \left(G + \frac{R_s}{Z^2}\right)\frac{\sqrt{q-1}}{\omega}$$

or $$C = C_r \pm \left(G + \frac{R_s}{Z^2}\right)\frac{\sqrt{q-1}}{\omega} \quad . \quad . \quad (18)$$

   where q denotes the ratio V_r^2/V^2 for voltage resonance and I^2/I_r^2 for parallel current resonance. It follows that the resonance curves in these two cases are strictly symmetrical even when leakage is present, and that if C_1 and C_2 are the two values of capacitance

$$C_r = \tfrac{1}{2}(C_1 + C_2) \quad . \quad . \quad . \quad . \quad (19)$$

and $$C_1 - C_2 = \frac{2}{\omega}\left(G + \frac{R_s}{Z^2}\right)\sqrt{q-1}$$

It should further be noted that $\frac{R_s}{Z^2}$ $\frac{R_s}{R_s^2 + L_s^2\omega^2}$ is the equivalent shunt conductance of the coil, and therefore $G + R_s/Z^2$ is the total shunt conductance of the whole circuit, which may be written G_t. Thus we have the most useful equation for the measurement of conductance,

$$C_1 - C_2 = \frac{2}{\omega} \cdot G_t \cdot \sqrt{q-1} \quad . \quad . \quad . \quad (20)$$

5. The corresponding equations for the case of series current resonance when leakage is present are much more complicated and therefore less suitable for measurement purposes. When, however, the condenser is free from leakage the corresponding equation for series current resonance takes the equally useful form

$$C_1 - C_2 = \frac{R}{L\omega} \cdot (C_1 + C_2)\sqrt{q-1},$$

but it is to be remembered that in this case the capacitance C_r at resonance is not equal to $\frac{1}{2}(C_1 + C_2)$.

6. **Reaction from a Resonating Circuit.** We have so far confined our attention to the variation of the currents and voltages in the resonating circuit itself. It is also of interest to consider the reaction of the resonating circuit on the source of oscillations from which it is energised, for it will be shown that it is possible to deduce the properties of the circuit by observations of this reaction, and we may therefore have another group of methods for the measurement of impedance based on indirect observations of resonance.

FIG. 15. Primary circuit and tuned secondary circuit.

In Fig. 15 let the primary coil L_pR_p represent a coil in the output circuit of an oscillator, and let the secondary circuit $L_sR_sC_s$ be coupled to it magnetically, the only coupling being that represented by the mutual inductance M. The current I_p

in the primary circuit will induce a current I_s in the secondary circuit. Let $\mathbf{V}_p$ be the voltage across the primary coil. We shall consider the variation of these currents and voltages as the capacitance C_s is varied in the neighbourhood of resonance, *i.e.*, as the secondary circuit is tuned. We have for the primary coil

$$\mathbf{V}_p = \mathbf{I}_p(R_p + jL_p\omega) - jM\omega\mathbf{I}_s$$

and for the secondary circuit

$$0 = \mathbf{I}_s\left[R_s + j\left(L_s\omega - \frac{1}{C_s\omega}\right)\right] - jM\omega\mathbf{I}_p,$$

or if X_p denote the reactance of the primary coil, and X_s that of the secondary circuit,

$$\begin{aligned} V_p &= \mathbf{I}_p(R_p + jX_p) - jM\omega\mathbf{I}_s \\ 0 &= \mathbf{I}_s(R_s + jX_s) - jM\omega\mathbf{I}_p \end{aligned}$$

Substituting the value of $\mathbf{I}_s$ given by the second of these equations into the first we have

$$\mathbf{V}_p = \mathbf{I}_p\left[R_p + jX_p + \frac{M^2\omega^2}{R_s + jX_s}\right] \qquad (21)$$

$$= \mathbf{I}_p\left[R_p + \frac{M^2\omega^2}{R_s^2 + X_s^2} \cdot R_s + j\left(X_p - \frac{M^2\omega^2}{R_s^2 + X_s^2} \cdot X_s\right)\right].$$

Thus the change in resistance ΔR_p of the primary coil due to the coupling of the secondary coil is given by

$$\Delta R_p = \frac{M^2\omega^2}{R_s^2 + X_s^2} \cdot R_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

while the change in reactance is given by

$$\Delta X_p = -\frac{M^2\omega^2}{R_s^2 + X_s^2} \cdot X_s \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and

$$\frac{\Delta R_p}{\Delta X_p} = -\frac{R_s}{X_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

This relation is sometimes useful for determining R_s or X_s, although certain other relations to be derived are usually more convenient in practice.

Consider the variation of reactance, given by equation (23). Let the capacitance of the secondary circuit be varied, all the other quantities remaining constant, *i.e.*, X_s will vary and M, ω

and R_s remain constant. The resulting variation of X_p may be represented by Fig. 16, in which the continuous curve shows ΔX_p as a function of X_s, *i.e.*, the change in the reactance of the primary circuit measured from the value which it has in the absence of the secondary circuit, as a function of the total reactance of the secondary circuit. We see that when $X_s = 0$, *i.e.*, $L_s\omega - 1/C_s\omega = 0$ (the condition for series current resonance of the secondary circuit) the change in reactance of the primary circuit is zero. When $X_s = R_s$ the change of primary reactance reaches a minimum value, and when $X_s = -R_s$ it reaches

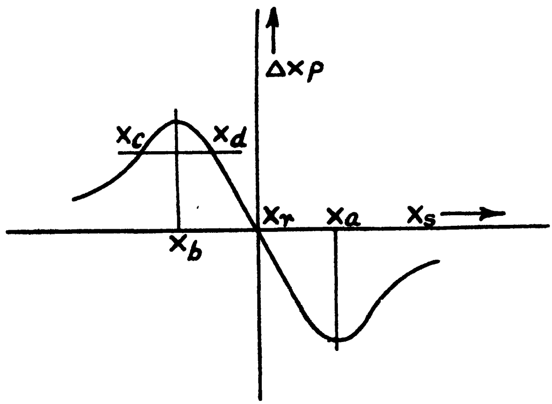


FIG. 16. Effect of secondary resonance on primary reactance.

a maximum value, these critical values being given by $\Delta X_p = \pm M^2\omega^2/2R_s$, so that they are greater the larger the mutual inductance and the lower the secondary resistance. When the resistance of the secondary circuit is small compared with the reactance of its condenser or inductance, which is usually the case at radio frequencies, the range of X_s from $+R_s$ to $-R_s$ in which the transition from maximum to minimum values occurs is small, and thus the curve of reactance variation is very steep in this range, which is as we have seen the region of resonance of the secondary circuit. Thus this rapid change of reactance may afford a very sensitive means of detection of resonance in a coupled circuit. The means of observing the change will be considered in a later chapter (p. 169).

The curve of Fig. 16 also provides means for the measurement of R_s, the resistance of the secondary circuit. Thus let X_a and X_b be the values of secondary reactance corresponding to the minimum and maximum respectively. Then

$$X_a - X_b = 2R_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The change from X_a to X_b has been produced by variation of capacitance alone. Let C_a and C_b be the values of capacitance corresponding to X_a and X_b, *i.e.*, to the maximum and minimum. Then

$$X_a - X_b = \frac{1}{C_a\omega} - \frac{1}{C_b\omega} = 2R_s$$

or

$$R_s = \frac{1}{2\omega}\left(\frac{C_b - C_a}{C_bC_a}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

It is to be noted that $(C_b - C_a)$ is the difference between two readings of a variable condenser. It may usually be determined with great accuracy, even when C_a and C_b are not separately known with very high accuracy.

An alternative method of working is as follows. Let X_c and X_d be the two values of secondary reactance corresponding to a given change ΔX_p of primary reactance. The values X_c and X_d are the two roots of the quadratic (23), which may be written

$$X_s^2 + \frac{M^2\omega^2}{\Delta X_p} . X_s + R_s^2 = 0.$$

The product of the roots is given by

$$X_cX_d = R_s^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Now the values of X_c and X_d are not usually known, but it is a simple matter to observe the corresponding values of capacitance C_c and C_d and also the value of capacitance C_r corresponding to resonance $\Delta X_p = 0$ and $X_r = 0$ (Fig. 16).

Then

$$X_c = X_c - X_r = \frac{1}{\omega C_c} - \frac{1}{\omega C_r}$$

and similarly

$$X_d = \frac{1}{\omega}\left(\frac{1}{C_d} - \frac{1}{C_r}\right).$$

Equation (27) therefore takes the form

$$R_s^2 = \frac{1}{\omega^2}\left(\frac{1}{C_c} - \frac{1}{C_r}\right)\left(\frac{1}{C_d} - \frac{1}{C_r}\right)$$

or

$$R_s^2 = \frac{1}{\omega^2}\frac{(C_r - C_c)(C_r - C_d)}{C_c C_d C_r^2} \quad . \quad . \quad . \quad . \quad (28)$$

The quantities on the right-hand side of this equation are all easily observed. The actual value of ΔX_p is not required; merely the two values of capacitance corresponding to the same value of ΔX_p. It will readily be understood that the values of capacitance corresponding to two points such as c and d which lie on steep portions of the curve may sometimes be determined with greater precision than the value corresponding to the maximum of the curve, and that equation (28) is therefore sometimes preferable to the simpler one (26).

We have so far considered only the effect on the primary coil of tuning the secondary circuit by varying the capacitance. The secondary circuit may also be brought into resonance by varying the frequency of the current in the primary coil, and equations (21) to (24) apply to this case also. As before, the change of reactance of the primary circuit will be zero when the secondary reactance X_s is zero, *i.e.*, when the secondary circuit is in resonance. As the frequency departs from the value for resonance, X_s will increase rapidly in the positive or negative direction, and the resulting variation of primary reactance plotted against X_s will resemble Fig. 16 although the maximum and minimum points will be slightly displaced from the values $X_s = \pm R_s$ owing to the variation of ω in this range of X_s (usually very small).

CHAPTER III

THE GENERAL PRINCIPLES OF SCREENING AND THE RADIO-FREQUENCY BRIDGE

1. The General Impedance Bridge. In this chapter the general principles governing the use of the Wheatstone-bridge principle at radio frequencies will be considered, and it will be found that this involves a discussion of the use of screens for electrical apparatus. A thorough understanding of this matter is of great importance in most precision measurements at radio frequencies and it is therefore treated in detail.

Most of the bridges which are suitable for use at radio frequencies may be represented by the general bridge diagram shown in Fig. 17 in which O_1D_1, O_1D_2, D_1O_2 and D_2O_2 are four

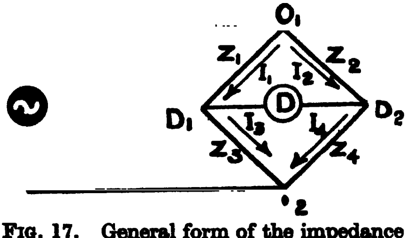


FIG. 17. General form of the impedance bridge.

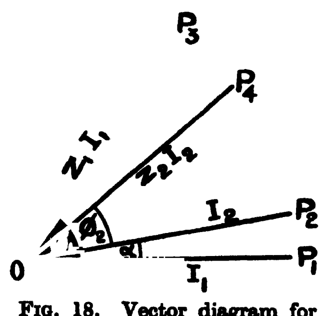


FIG. 18. Vector diagram for the impedance bridge.

current-paths of any form possessing impedances* of Z_1/ϕ_1, Z_2/ϕ_2, Z_3/ϕ_3 and Z_4/ϕ_4 respectively. A source of electrical oscillations is connected to the two opposite junction-points O_1 and O_2, and a detector of such oscillations is connected to the other junction-points D_1 and D_2. By means of suitable adjustments to the magnitudes and phase angles of the four impedances, the two

* For the sake of brevity it is often convenient to refer to a conducting path possessing an impedance operator Z/ϕ as an impedance Z/ϕ.

points D_1 and D_2 may be brought to the same potential, and the bridge is then said to be balanced. The detector D indicates zero current in this condition.

The conditions of balance may readily be derived as follows. Let the currents flowing through the four impedances be $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$ and $\mathbf{I}_4$, as shown in Fig. 17. Let the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ be represented by the rotating vectors OP_1 and OP_2 in Fig. 18 inclined at an angle α which represents the phase difference between the currents. The p.d. between O_1 and D_1 may be represented by OP_3, and that between O_1 and D_2 by OP_4, where

$$OP_3 = Z_1\mathbf{I}_1 \text{ and } \angle P_3OP_1 = \phi_1$$

and

$$OP_4 = Z_2\mathbf{I}_2 \qquad \angle P_4OP_2 = \phi_2.$$

When the potentials of D_1 and D_2 are the same, these two p.ds. must be identical and the vectors OP_3 and OP_4 must coincide. The conditions for this are

$$\phi_1 - \phi_2 = \alpha$$

and

$$Z_1\mathbf{I}_1 = Z_2\mathbf{I}_2 \qquad \text{or} \qquad \frac{Z_1}{Z_2} = \frac{\mathbf{I}_2}{\mathbf{I}_1}.$$

Similarly it follows that we must also have

$$\phi_3 - \phi_4 = \beta$$

and

$$Z_3\mathbf{I}_3 = Z_4\mathbf{I}_4 \qquad \text{or} \qquad \frac{Z_3}{Z_4} = \frac{\mathbf{I}_4}{\mathbf{I}_3}$$

where β is the phase difference between the currents $\mathbf{I}_3$ and $\mathbf{I}_4$. But since there is no current in the detector when the bridge is balanced, the currents $\mathbf{I}_1$ and $\mathbf{I}_3$ must be identical, as also must $\mathbf{I}_2$ and $\mathbf{I}_4$. Thus the two conditions of balance are

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\phi_1 - \phi_2 = \phi_3 - \phi_4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

These may also be written

$$Z_1 = \frac{Z_3}{Z_4} \cdot Z_2 = \rho Z_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\phi_1 = (\phi_3 - \phi_4) + \phi_2 = \phi_2 + \delta \quad . \quad . \quad . \quad (4)$$

where ρ is the ratio of the magnitudes of the two impedances Z_3 and Z_4, and δ is the difference between their phase angles.

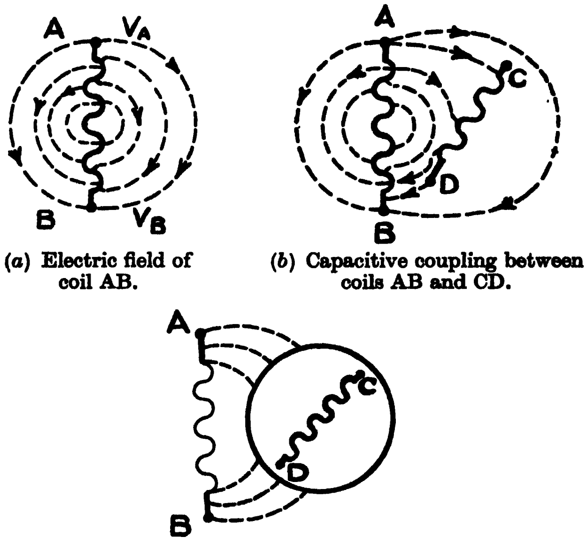
Thus if such a balance can be obtained, and we know the magnitude and phase angle of one impedance (the standard impedance) and also the ratio and phase difference of the other two (called the ratio-arms), we may readily calculate the magnitude and phase angle of the remaining impedance which is the quantity to be measured. In order to balance the bridge we must have two adjustments, one to vary the phase angle ϕ_2 or the phase difference δ so that equation (4) is satisfied; and the other to vary the magnitude Z_2 or the ratio ρ until (3) is satisfied. The two adjustments are usually operated alternately, always in the direction which diminishes the current indicated by the detector, until the condition of perfect balance is reached.

It is to be noted that the ideal bridge we have considered consists only of two meshes in addition to that required for supplying the current, and therefore comes next to the single resonating circuit in order of simplicity. Bridge circuits of more meshes, such as Anderson's bridge for the comparison of self-inductance and capacitance, may be worked successfully at low frequencies, but it is scarcely worth while considering such networks for radio frequency work, as with three meshes the difficulties of controlling the current distribution are greatly increased. Indeed, it is not often possible to use the above simpler network in its most general form ; it is usually advisable to use for the two ratio-arms Z_3 and Z_4, two exactly similar impedances, so that ρ is equal to unity and δ is zero. The bridge then becomes symmetrical, and under these conditions any stray currents or e.m.fs. which are also symmetrical will not affect the balance point. This property will be considered in more detail later.

2. **The Bridge Components.** It will now be clear that the complete bridge circuit consists of the following components. (1) A source of current of radio frequency. (2) A sensitive detector of radio-frequency current. (3) Two ratio-arms, consisting of impedances which are as nearly as possible identical. (4) A standard impedance, which is continuously adjustable for both magnitude and phase angle, and of which the calibration holds good at radio frequencies. In general the two quadrature components of this impedance are separately adjustable, *i.e.*, the resistance or the conductance, and the reactance or susceptance

(inductance or capacitance). Details of the construction of these components will be considered in subsequent chapters, but it will be convenient to consider at this stage certain general features which they must possess.

3. Screening.* We have seen that stray displacement currents in our measuring circuits are one of the chief difficulties we have to contend with at high frequencies. We must now consider how the flow of such currents may be controlled by the use



(*a*) Electric field of coil AB.

(*b*) Capacitive coupling between coils AB and CD.

(*c*) Screening of a coil CD.

Fig. 19. Capacitive coupling and screening.

of electric screens. Let AB (Fig. 19 (*a*)) represent a coil carrying radio-frequency current. There will be a potential gradient along the coil and therefore the coil will be surrounded by an electric field which may be represented by the broken lines. As the potentials are alternating these lines also represent the lines of flow of displacement current, the displacement current density at any point being proportional to the potential gradient, the frequency, and the value of the dielectric constant or permittivity

* Campbell, G. A. *Collected Papers*, p. 40, or *Electrical World*, 1904, Vol. 43, p. 647. Shackleton and Ferguson. *Bell Technical Journal*, 1928, Vol. 7 p. 70. Ferguson J. G. *Bell Technical Journal*, 1929, Vol. 8, p. 560.

at that point. Suppose now a second conductor, CD (Fig. 19 (*b*)) such as a second coil is placed near AB. The electric field becomes distorted as shown : in other words, the distribution of displacement current is altered. Some of it now flows from the upper half of AB to the upper part of CD, and from the lower part of CD to the lower part of AB. This current flows through a portion of CD as a conduction current, and thus creates a small potential difference in CD. In other words, there is capacitive coupling between AB and CD.

Consider now a closed hollow conductor placed near the coil AB as in Fig. 19 (*c*). The electric field of AB is again distorted and displacement current flows to and from the conductor as shown. However, since the potential at any point inside a closed hollow conductor containing no charge is the same, then although this potential may alternate as the current through AB alternates, there will never be a potential gradient inside the conductor, and therefore there will be no displacement current inside it. Thus displacement current cannot penetrate a closed hollow conductor which therefore acts as a displacement-current screen to bodies contained within it. In radio-frequency work such a screen is commonly referred to as an electrostatic screen, though the conditions are anything but static. Still as its action is explained by laws of electrostatics, the term is convenient. The above-mentioned facts may also be expressed by the statement that there is no capacitive coupling between two circuits, one of which is wholly inside, and the other wholly outside a closed hollow conductor.

Consider now our bridge with its oscillator and detector as shown in Fig. 17. We have assumed that current from the oscillator enters the bridge at the two points O_1 and O_2 only, and that current passes to the detector *viâ* the two points D_1 and D_2 only. But AB (Fig. 19 (*b*)) may be taken as representing any coil of the oscillator and CD any arm of the bridge, and we see therefore that unless steps are taken to prevent it, displacement current will pass from the oscillator to all four arms of the bridge, entering the bridge network, not only at the corners but also at points distributed along the arms. It is desirable to prevent this by surrounding the bridge by a closed hollow conductor (an electric screen), the oscillator remaining outside the conductor.

Similarly, it is desirable to place the detector in its own separate screen, so that it will receive no displacement current direct from the oscillator. It will, however, be readily seen that it is impossible to obtain perfect screening in this simple way, since the oscillator, bridge and detector must have points in common, and therefore they cannot be completely separated by screens. However, the arrangement shown in Fig. 20 may be regarded as an approximation to the ideal scheme, and it will simplify the distribution of displacement current to a considerable extent. Each of the three circuits, oscillator, bridge and detector is completely enclosed in a conducting screen (represented in Fig. 20 by the broken lines) and in order to permit the necessary separation of the circuits,

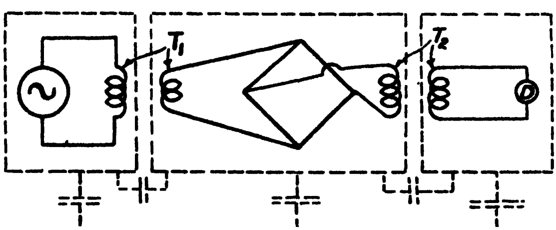


FIG. 20. Use of screens to eliminate capacitive coupling between oscillator, bridge circuit and detector.

the oscillator and detector are coupled to the bridge circuit by means of transformers T_1 and T_2. The primary of the oscillator transformer is inside the oscillator screen, while the secondary is inside the bridge screen, and similarly the windings of the detector transformer are separated by screens.

4. The Earth-Connection. The localisation of displacement currents by means of screens surrounding the various units of the system, as shown in Fig. 20, is in practice seldom perfect. The conditions required are (1) that the screens shall be equipotentials, and completely closed, and (2) that the whole equipment shall be contained within them. With regard to (1) a metal box is not a true equipotential, for the impedance between any two points on its walls, although very small, is not usually zero; both the resistance and inductance may be appreciable, and therefore a potential difference may be established by currents in the screen. The boxes, moreover, may not be completely closed, owing to the necessity of providing convenient access to various parts; but provided steps are taken to secure good electrical contact between all the walls. and between the walls and the cover-plates or lids covering any aperture, metal boxes form satisfactory screens for most purposes. It is, however, often by no means easy to satisfy the condition (2), for the complete equipment may include public power mains. Strictly speaking, the observer, and all other large objects in the neighbourhood form part of the electrical system, for they are connected to the various units by displacement-current paths, which as we have seen, are equivalent to condensers. Fig. 20 shows in broken lines examples of such condensers, linking the units together, and to a large neighbouring object *AB*. The most satisfactory way of avoiding difficulties due to such " stray capacitances " is to short-circuit them. This means that all the external screens, and all large neighbouring objects should be connected together by a conductor of zero impedance. The earth is certainly the largest object in the neighbourhood, and the earth-capacitances are apt to be the largest and most troublesome of these stray capacitances. It therefore becomes important to connect the outer screens and all large neighbouring objects to earth by means of a path of negligible impedance. Unfortunately the earth is poorly conducting in parts, and it is not always easy to make good contact with it. Connection to a network of water pipes covering a large area is usually the best that can be done, but at very high frequencies the impedance of a water pipe may be considerable, for its reactance will be proportional to the frequency. In these circumstances it is sometimes convenient to use as an earth terminal a large metal sheet above which the apparatus is supported, the appropriate points of the apparatus being connected to it by means of short lengths of copper strip. The capacitance between such a plate and earth may provide at very high frequencies a current path of a lower impedance than could be obtained by the use of any actual conductor, for capacitive reactance varies inversely as the frequency.

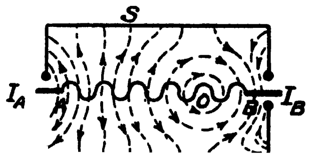
The most complete solution of the problem is a screened, *i.e.*, metal-lined, room containing the complete equipment, including the observer, but it is very expensive and usually unnecessary. In any actual case the efficacy of the earth connection can only be judged by the consistency of the measurements made.

5. **Internal Screens.** We have now shown that as far as possible each section of the measuring system should be completely enclosed in a conducting screen, which should be earth-connected. The effects of stray displacement currents are then confined entirely to the section in which they occur. Thus we now may ignore the oscillator and detector sections of Fig. 20, and confine our attention to the central section which contains the bridge-proper and the coupling coils for the oscillator and detector. Although the currents in this section are now almost completely unaffected by displacement currents outside, yet there will be displacement currents passing between all the conductors inside and in order to restrict the current paths to the simple bridge circuit of Fig. 17 a further system of internal screens will be necessary. A plan which immediately suggests itself is to separate all the components from one another by a number of conducting partitions, all of which are in metallic contact with the outer screen, and therefore earth-connected. This internal screening cannot of course be quite complete, since it will be necessary to make small holes in the screens, through which the connecting leads may pass, but small imperfections of this kind may usually be made negligible. If this plan is adopted each arm of the bridge is enclosed in its own screen, which is earth-connected. Displacement currents from any one component cannot then enter any other component which is one of the conditions required. We must, however, consider the effects of the displacement currents which will flow between each component and its earth-connected screen.

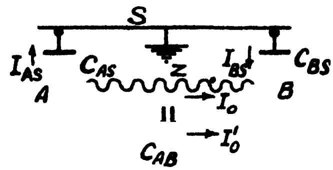
6. **Screened Impedance Standards.** Let AB (Fig. 21) represent some form of impedance standard surrounded by an earth-connected screen S, and forming one component of the bridge network. We may suppose current of radio frequency to pass through small holes in the screen, I_A being the current entering the terminal A, and I_B that passing through the terminal B. As we shall see, these two currents are not in general of the same value.

Let V be the alternating p.d. between the terminals A and B. Then V_A the potential of A will at one instant exceed that of B

by an amount V, and at an instant half a period later, it will be less than that of B by the same amount. Consider now the relation between these potentials and that of the screen S, which is permanently fixed at the value zero. The simple theory of the ideal bridge is completely independent of the absolute potential at any point on the network, *i.e.*, of the p.d. between the various points on the network and earth, but in practice we find that it is necessary to fix the potential of one point on the actual bridge, with reference to earth. (The methods adopted will be discussed later.) Thus we may imagine that by some means or other the potential of a point O on the impedance AB is fixed at a value



(*a*) Electric field and lines of flow of displacement current round screened coil AB.



(*b*) Equivalent network for the screened coil AB.

FIG. 21.

equal to that of the screen S, viz. zero. Thus when the maximum current I_A passes in the direction AB there is an alternating p.d. of value V_A between A and the screen, and one of value V_B between B and the screen. As we pass from A to O along the wire forming AB, the alternating p.d. between the wire and the screen steadily decreases to the value zero, probably also undergoing changes of phase. As we pass from O to B, the alternating p.d. between wire and screen increases from zero to the value V_B, this p.d. being 180° out of phase with respect to that between A and S. Thus inside the screen we have an alternating electric field which may be represented roughly by the broken lines shown in Fig. 21 (*a*). This field will tend to be greatest near the terminals A and B and will become small near O, the only component at this point being one due to the potential gradient in AB itself. The alternations of this field must be regarded as a flow of displacement current, and we may regard this current as flowing along lines somewhat similar in distribution to those of Fig. 21 (*a*). Thus during the half cycle when conduction current flows from A to B

along the wire, displacement current leaves it at all points between A and O, and flows to the screen and thence to earth, while at all points between O and B, displacement current from the screen, *i.e.*, earth, enters the wire. Thus the current passing along the wire AB itself steadily diminishes from the value I_A at the terminal A, to a minimum value I_O at O, and then increases as we pass from O to B, reaching the value I_B at the terminal B.

The question now arises: what do we mean by the impedance of the arm AB?

Impedance is defined as the ratio of p.d. to current, but it is obvious that in this case, not only does the current vary from point to point along the coil, but the value at any point depends on the two terminal potentials V_A and V_B as well as on the potential difference V. The problem is considerably simplified by confining our attention to the values of the current at the two terminals, a procedure which is justified by the fact that these are the only values of current that can be observed. It may then be shown that as far as terminal currents and voltages are concerned, the circuit of Fig. 21 (*b*) is equivalent to the system of Fig. 21 (*a*). Thus the terminal current I_A may be regarded as consisting of two components, one $I_O = \frac{V}{Z}$ say, which flows directly to B through the coil, and the other a displacement current $I_{AS} = j\omega C_{AS} V_A$ which flows to the screen. Similarly the terminal current I_B includes the displacement current $I_{BS} = j\omega C_{BS} V_B$ as well as the current I_O. This current I_O also consists in part of a displacement current, and this may be represented by that in the capacitance C_{AB}, the so-called self-capacitance of the coil. Alternatively the value of the impedance Z may be such as to include the effect of the shunt capacitance C_{AB}.

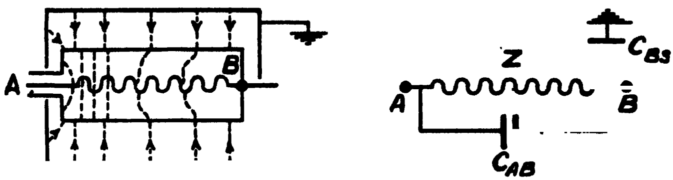
We shall call this value the *direct impedance* of the coil under the existing conditions, and it should be noted that it is a value which is independent of the potentials V_A and V_B and is quite definite in spite of the fact that the current in the coil varies from point to point along its length. There is one other value for the impedance of the coil which is equally definite, and that is the value, which would be deduced from the terminal currents and voltage of the coil if observations could be made with the screen S perfectly isolated, *i.e.*, at an infinite distance from all other conductors. In such a case all the current entering the terminal A must ultimately emerge from the terminal B so that $I_{AS} = I_{BS}$ and the impedance is simply the ratio of the voltage V_{AB} to the common terminal current, $I_O + I_{AS}$ or $I_O + I_{BS}$. The condition of perfect isolation can never be realised in practice, but it is evident that the same current distribution will be obtained if the potentials V_A and V_B are so adjusted with respect to the screen potential that $I_{AS} = I_{BS}$. The condition to be satisfied is

$$V_{AS} \cdot C_{AS} = - V_{BS} \cdot C_{BS}$$

and for this particular potential distribution the value of the impedance is also quite definite. We shall refer to this condition as the isolated condition and call this value of the impedance the *total impedance*. In the case of a symmetrical coil for which $C_{AS} = C_{BS}$, the required potential distribution is that in which $V_{AS} = - V_{BS}$, *i.e.*, the potential of one terminal is always as much above that of the screen (earth) as that of the other is below it. This is known as the balanced condition.

C_{AS} and C_{BS} are often called the screen-capacitances or earth-capacitances of the coil. In practice they must be regarded as associated with a certain amount of conductance for the currents to screen are not strictly in quadrature with V_A and V_B, and strictly speaking we should use the term screen-admittance. These two admittances are however largely capacitive in practice. It should be noted that for any given coil, the values of Z, C_{AB}, C_{AS} and C_{BS} will all depend on the size of the screen and the position of the coil within it. Similar considerations apply to resistors of any form, inductors, condensers and any combinations of these. Thus, if we place an unknown impedance in a bridge in order to determine its value, we shall obtain a value which will no longer hold when the impedance is removed from the bridge. Similarly, the calibration of our standard impedance and the ratio-arms will only hold good so long as they are in the bridge. This difficulty may be overcome as follows: Let the impedance AB be completely surrounded by a conducting screen, which is connected to one terminal, say B, as in Fig. 22 (*a*). The electric field and the distribution of displacement currents will be completely altered as shown. All the displacement current leaving AB must now pass to the internal screen, *i.e.*, to the terminal B.

Further, the electric field inside this screen will for a given p.d. between A and B, be independent of the outer screen. Thus although the value of C_{AB} may be greatly increased by the addition of the internal screen, both it and the value of Z will be made independent of the size, shape and position of the outer screen. Such a screened circuit-element of any kind may therefore be removed from the bridge without affecting the value of its direct impedance. This value is definite for all conditions



(*a*) Electric field and displacement currents.

(*b*) Equivalent network.

FIG. 22. Electric field and equivalent network for a coil AB with double screens.

of the screened instrument, and in this case we may speak simply of the impedance of the instrument without any ambiguity.

7. **The Elimination of Earth-Capacitances: Multiple Screens.** There is another consequence of the use of double screens in the manner described above, which is of great importance. Since all the displacement current leaving AB passes to the internal screen, *i.e.*, to the terminal B, none reaching S directly, the value of C_{AS} is reduced to zero by the presence of the internal screen. The capacitance C_{BS} will evidently be increased by the presence of the screen, and this capacitance will also vary with the position of the screened impedance AB inside the outer shield S. The equivalent network of the system is now that of Fig. 22 (*b*).

We now see that the radio-frequency bridge can never be represented by the simple network of Fig. 17. Provided that the components (the four arms and the oscillator and detector coupling coils) are each separately enclosed by earth-connected screens, we may represent them by equivalent networks of the form of Fig. 21 (*b*), and thus arrive at the equivalent network of Fig. 23 for the whole bridge, where the capacitances O_1E, D_1E, O_2E, D_2E are the sums of the various earth-capacitances connected to points O_1, O_2, D_1, D_2 respectively. If in addition each component of the bridge carries its own screen which is connected to one of its terminals, certain of the earth-capacitances may be reduced to zero, and in this way we may reduce the system to that shown in Fig. 24, in which only two of the earth-capacitances remain, these two coupling the terminals of one arm to earth. This network is the nearest approach to the ideal network of Fig. 17 that can in practice be obtained. It is quite impossible to eliminate all earth-impedances from the measuring circuit, although, as we shall see, it is possible to avoid errors due to their presence. Fig. 25 shows the arrangement of screens required to produce the network of Fig. 24. The screens of Z_1, Z_2 and the oscillator coupling coil O are all connected to their common terminal O_1, the sum of their capacitance to the external screen S, forming the earth-capacitance C_O of Fig. 24. The earth-capacitance C_D

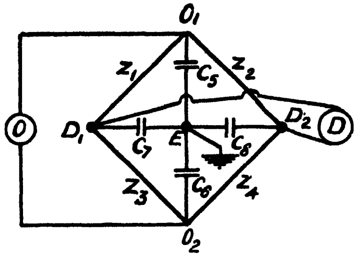


FIG. 23. Equivalent network for a bridge of four arms each with an earth-connected screen.

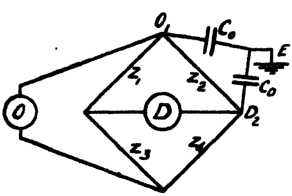


Fig. 24. Simplified equivalent network obtained by triple screening as in Fig. 25.

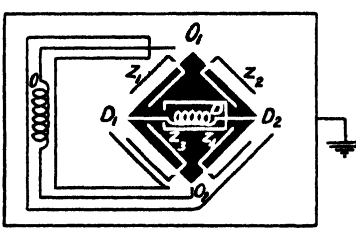


FIG. 25. Arrangement of screens giving simplified equivalent network of Fig. 24.

is that between the screen of the detector coupling-coil D (which is connected to the point D_2) and the external screen S. The screens of Z_3 and Z_4 are connected to the point O_2. They thus may be regarded as one terminal of the coil O, and in order to reduce their earth-capacitance to zero, the screen of O must be lengthened so as to surround the two screens. The capacitance between the screen of O and these two inner screens then merely increases the self capacitance of O, *i.e.*, it affects the impedance of O, but not the balance-point of the bridge. If the screen of O were extended still further so as to surround that of D, then C_D of Fig. 24 would also be reduced to zero, but the self-capacitance of Z_2 would be increased, *i.e.*, we should have the system of Fig. 26, which is an alternative to Fig. 24, that might in some circumstances possess advantages. In the arrangement of Fig. 25, the impedances Z_3 and Z_4 are sometimes said to have triple screens, but it should be noticed that the three screens serve different purposes. The innermost screens make the values of the impedances Z_3 and Z_4 definite, and reduce the earth capacitances of the other terminals of Z_3 and Z_4 to zero. The next screen reduces the earth-capacitance of the inner-screens to zero, while the outermost screen, which is earth-connected, eliminates any effects due to stray displacement currents.

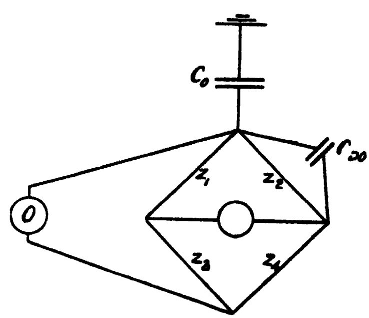


FIG. 26. Alternative form of simplified equivalent network obtainable by triple screening.

From these general considerations it will be clear that each component of a radio-frequency bridge should possess at least one metal screen, which should be connected to one of its terminals. Further, if the component is to be suitable for use in a variety of circuits, it should have two screens, the outer one insulated from the inner. The insulation of the screens should always be of good quality, otherwise the problem will be complicated by leakage currents, which will vary considerably, and in an unknown manner, with frequency.

8. **The Potential of the Bridge Relative to Earth : The Direct Earth-Connection.** We have seen that a bridge circuit which is screened as in Fig. 25 may be represented by Fig. 24, so that the bridge network of Fig. 17 must be regarded as linked to earth by the capacitances C_O and C_D. It is evident that the potentials of the bridge network with reference to earth will be determined by the values of these capacitances, and also that the series-combination of C_O and C_D acts as a shunt across the arm Z_2, so that the condition of balance of the bridge does not depend

simply on Z_1, Z_2, Z_3 and Z_4 in accordance with equations (1) to (4). Z_2 must be replaced by the impedance of Z_2 in parallel with the capacitance-combination. The conditions of balance are most simply expressed in terms of the admittance operators Y_1, Y_2, Y_3 and Y_4 of the shielded impedances forming the bridge network and the capacitance of C_O and C_D in series, *i.e.*, $C_O C_D/(C_O + C_D)$. We have for the required condition

$$\frac{Y_1}{Y_3} = \frac{Y_2 + j\omega C_O C_D/(C_O + C_D)}{Y_4} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The condition of balance may be simplified by connecting either O_1 or D_2 to earth, which has the effect of short-circuiting C_O or C_D. Thus if the terminal O_1 of the oscillator is connected to earth the condition of balance becomes

$$\frac{Y_1}{Y_3} = \frac{Y_2 + j\omega C_D}{Y_4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

while if one terminal D_2 of the detector is connected to earth the equation becomes

$$\frac{Y_1}{Y_3} = \frac{Y_2 + j\omega C_O}{Y_4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

It should be noted that the capacitance C_O is the capacitance between the earth-connected screen and all the screens connected to the point O_1, *i.e.*, it includes the earth-capacitance of the unknown impedance Z_1, and it will vary with the linear dimensions of the instrument under test. On the other hand, the capacitance C_D is merely the earth capacitance of the screened-detector coil and is a constant of the bridge network independent of the unknown impedance Z_1. Thus it is in general preferable to connect O_1 to earth and then use equation (6). This procedure is of course only possible when it is permissible to connect to earth the screen and one terminal of the instrument to be tested.

9. The Measurement of Balanced Impedance. It is sometimes necessary to measure the impedances of an instrument, the screen of which is not connected to one of its terminals; for example, we may wish to measure the total impedance in the isolated condition or in the balanced condition for a symmetrical instrument. In this case let the two terminals of the instrument be connected to O_1 and D_2 of Fig. 25 and let its screen be connected to the outer earth-connected screen of the bridge. Then again we have the system of Fig. 24, in which the earth-capacitances of the impedance to be measured (C_{AS}, C_{BS} of Fig. 21 (*b*)) now form part of C_O and C_D. Evidently the balanced condition will be satisfied if C_O is made equal to C_D (these capacitances are easily adjusted by means of small auxiliary condensers), while the isolated condition for an unsymmetrical instrument will be satisfied if we make the ratio $C_O : C_D$ (Fig. 24) equal to the ratio of the earth-capacitances of the instrument under test ($C_{AS} : C_{BS}$ of Fig. 21 (*b*)). The bridge readings in these cases give us the required impedance in parallel with the series combination of the parts of C_O and C_D which are not associated with the instrument under test. Due allowance must be made for this capacitance.

10. **The Measurement of Unscreened Instruments.** The instrument to be tested may be unscreened, and in this case the accuracy of the measurement is limited by the fact that the quantity to be measured is indefinite, as we have seen in paragraph 6. In order to make the bridge readings definite (independent of movements of the observer's hands, etc.) some sort of screen should be provided. The bridge usually possesses a shielded and earth-connected compartment, into which the instrument under test is inserted. The bridge readings then give the impedance under the particular conditions of the experiment, which may be varied in the manner discussed in the preceding paragraph.

11. **Measurement by Difference.** We have seen that the screened bridge is equivalent to the circuit of Fig. 24, and that whether we connect to earth O_1 or D_2, or adjust C_O and C_D to some special condition, there will always remain certain unwanted capacitances contributed by the screens in parallel with one arm of the bridge. We must now consider how these may be eliminated from the results. The most generally useful plan is to work by differences.

For example, in Fig. 25, let Z_1, Z_2, Z_3 and Z_4 be four shielded resistors, one of which is continuously variable, and let one of these, say Z_1, be shunted by a variable condenser C_1. If the four resistors are of the same phase angle, and if $Z_1 = Z_2$, it is obvious that at balance the value of C_1 will balance the resultant capacitance across Z_2. If now the admittance to be measured is connected in parallel with Z_2, and the bridge is again balanced by varying Z_1 (or Z_2) then the difference between the initial and final admittance values of Z_1 (or Z_2) will be equal to the added admittance. The screen-capacitances C_0 and C_D are present in both cases, and disappear on taking the difference.

The practice of working by differences also has the advantage of eliminating errors due to undesired phase differences in the bridge components. Thus referring again to equation (4), we find that at balance the phase angles of the bridge arms must obey the relation

$$\phi_1 = \phi_2 + \delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where δ is the unknown phase difference of the ratio arms Z_1 and Z_2. The equation for differences observed as described above becomes simply

$$\Delta\phi_1 = \Delta\phi_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

since δ remains constant. Thus a knowledge of the value of δ is not necessary when working by differences.

12. The Symmetrical Bridge: The Screened and Balanced Transformer. We have seen that owing to the importance of displacement currents at high frequencies and the impossibility of avoiding the presence of earth-connected bodies, the ideal bridge circuit of Fig. 17 can never be realised, and that in general a simple bridge circuit may be represented by Fig. 23. We have seen further, how by the use of multiple screens, this circuit may be simplified to that of Fig. 24, and the undesired capacitances then balanced by preliminary adjustment, so that difference-measurements give the "correct" result, *i.e.*, a result corresponding to equation (1).

There is an alternative method of working that is specially applicable to bridges in which the ratio-arms are equal ($Z_3 = Z_4$), so that for the correct condition of balance $Z_1 = Z_2$. It is obvious by inspection of Fig. 23 that if the two capacitances C_7 and C_8 are equal, then the whole bridge is symmetrical about O_1 and O_2, and therefore whatever be the values of the capacitances C_5 C_6, the balance point indicated by the detector D will be correct. Thus the condition to be satisfied is that the earth-capacitances at the two detector terminals D_1 and D_2 shall be equal. A simple method of obtaining an approximation to this condition consists in the use of a symmetrically constructed transformer for connecting the detector to the points D_1 and D_2, as is shown diagrammatically in Fig. 27. The two capacitances to earth at D_1 and D_2 are now those of the primary winding of this transformer, and we may ensure their equality by constructing this winding in two equal parts, wound on the core in positions which make the whole structure symmetrical. An electrostatic screen, shown by the broken line in Fig. 27, is used to shield the primary from the secondary winding. The screen also must be constructed so as to preserve the symmetry. It is to be remembered that the bridge-arms Z_1 and Z_2 may also contribute to the total earth-capacitance at D_1 and D_2, and it is important to make these capacitances also either zero, by shielding as in Fig. 25, or equal; otherwise the advantage obtained by the use of the *screened and balanced transformer*, as it is frequently called, is lost.

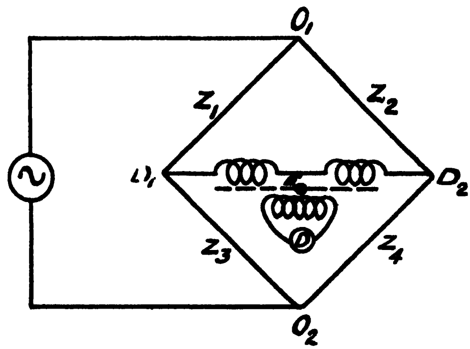


FIG. 27. Arrangement of a screened and balanced transformer in a symmetrical bridge circuit.

The balancing of the earth-capacitances may be checked, and adjusted if necessary by the following procedure. Referring to Figs. 23 and 27: disconnect the arms Z_1 and Z_2 from the bridge and connect the oscillator lead O_1 to E. If now the detector is silent, the bridge, $ED_1D_2O_2$ (Fig. 23) is balanced, which is the condition required. If it is not balanced, a small variable condenser may be connected across D_1E or D_2E and adjusted until balance is obtained. If leakage to earth is also significant, the capacitance adjustment may only give a minimum of sound in the detector. In order to get a true balance, the leakage must also be balanced. This may be done by introducing a suitable high resistance across D_1E or D_2E.

This system of balancing by adjustment may of course be applied to bridges with unequal ratio arms, but for reasons which are discussed later, these are seldom used at radio frequencies.

13. The Wagner Earth-Connection.* There is another general method of eliminating errors due to the effects of the earth-capacitances (C_5, C_6, C_7 and C_8 of Fig. 23) on the condition of balance of the bridge. This is the Wagner earth-connection, which is essentially a device for adjusting the potentials of the bridge circuit relative to earth, so that at balance the two points D_1 D_2, and therefore the whole detector system, are at earth-potential. In this condition, the capacitances C_7 and C_8 of Fig. 23 carry no current and may therefore be assumed to be absent. The capacitances C_5 and C_6 will carry current, but since none of this can reach D_1 or D_2, it may be considered as shunted outside the bridge, and may be ignored. The condition of balance is therefore given by the simple relation

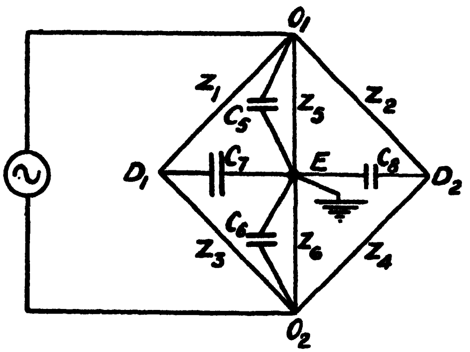


FIG. 28. Equivalent network of a bridge circuit with a Wagner earth-connection. The detector is omitted to avoid confusion.

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}.$$

It is important to notice that in this relation for the bridge with the Wagner earth-connection, Z_1 is the direct impedance between the terminals O_1 D_1, and so on for Z_2, Z_3 and Z_4. This arrangement is therefore peculiarly suitable for the measurements of direct impedance.

The necessary condition is achieved as follows. The points O_1 and O_2 are connected to earth E by two additional impedances Z_5 Z_6 one of which is variable, and the detector is so arranged that it may be transferred from D_1D_2 to D_1E or D_2E. We then have the network of Fig. 28, in which the earth-capacitances C_5 and C_6 are in parallel with Z_5 and Z_6 respectively. The detector is first placed across D_1D_2 and the bridge $Z_1Z_2Z_3Z_4$ balanced approximately by adjusting Z_2 say. This will be called for convenience the main balance. The detector is then transferred to D_1E and the bridge $Z_1Z_2Z_5Z_6$ balanced by adjusting Z_5 say. This will be

* WAGNER, K. W. *Elektrotechn. Zeits.*, 1911, Vol. 32, p. 1001.

called the Wagner balance. These two balancing operations are then repeated alternately, when it is usually found that a condition of simultaneous main and Wagner balance is approached by successive approximations. In this condition E, D_1 and D_2 must be at the same potential. It is therefore the condition required.

14. **The Conditions for Convergence.** It is important that the number of adjustments required to reach this condition shall not be unduly large, and it is worth while noting the conditions favourable to this state of affairs. Referring to Fig. 28, we see that if the two capacitances C_7 and C_8 are both zero, then the balance-point observed when the detector is connected to D_1D_2 (the main balance-point) is correct, whatever the values of Z_5 and Z_6, *i.e.*, however much the potential of D_1 and D_2 may differ from that of E; the current path Z_5Z_6 becomes merely a shunt across the whole bridge or the oscillator. Thus when constructing a bridge for use at high frequencies it is of the utmost importance to make the earth-capacitances at the detector-points D_1 and D_2 as small as possible. The first balance-point, when the earth-admittances at D_1 and D_2 are small quantities in comparison with the admittances of the arms of the bridge, will be nearly correct, even when D_1, D_2 and E are at different potentials. Therefore when D_1, D_2 and E are brought approximately to the same potential by the Wagner balance, the currents in D_1E and D_2E become small quantities of the second order, and the next main balance-point is the correct one. This condition is, broadly speaking, more easily satisfied the lower the frequency, and at telephonic frequencies it is very seldom that more than one adjustment of the Wagner balance is required.

When the earth-admittances are so high that they may not be regarded as small quantities, the conditions are apt to become very complicated. The detailed analysis is on record,* but it is a case where analysis is of no great practical value, and we shall therefore limit ourselves to a few general considerations of a bridge with equal ratio arms ($Z_3 = Z_4$), since it is found that at high frequencies unequal ratio arms are almost unworkable.

Consider then the symmetrical bridge. As we have noted previously, when the earth-impedances D_1E, D_2E are equal the whole network becomes symmetrical about O_1 and O_2, so that

* Ogawa, K. *Researches Electrotech. Lab. Tokyo*, Japan, 1929, No. 254.

the main balance is correct, whatever the potential of D_1D_2. Thus we should make the capacitances C_7 and C_8 not only small but also as nearly as possible equal. Then the first main balance will be approximately correct, whatever the potentials of D_1, D_2 and E; the first Wagner balance will diminish the error, and so on: the successive bridge readings will rapidly converge to the condition of simultaneous balance. The necessary symmetry may be obtained by the means discussed in (§ 12).

It is interesting to note that even when the bridge has unequal ratio arms, it is possible to adjust the earth-capacitances C_7 and C_8, so that they will cause no error, whatever the potential of D_1D_2. Thus if the balance is correct we must have

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} = \rho$$

and

$$Z_1I_1 = Z_2I_2$$

$$Z_3I_3 = Z_4I_4$$

where I_1 is the current through Z_1, and so on.

$$\therefore \frac{I_1}{I_3} = \frac{\dfrac{Z_2}{Z_1}I_2}{\dfrac{Z_4}{Z_3}.I_4} = \frac{I_2}{I_4}$$

$$\therefore \frac{I_1 - I_3}{I_3} = \frac{I_2 - I_4}{I_4}$$

$$\therefore \frac{I_7}{I_8} = \frac{I_1 - I_3}{I_2 - I_4} = \frac{I_3}{I_4},$$

and since the potentials across C_7 and C_8 must be equal,

$$\therefore \frac{C_7}{C_8} = \frac{I_3}{I_4} = \frac{Z_4}{Z_3} = \frac{Z_2}{Z_1},$$

which is the condition required. This condition may be obtained very simply by disconnecting Z_1 and Z_2, transferring the oscillator lead from O_1 to E, and adjusting the capacitance C_7 or C_8 until the detector indicates the balanced condition.

Consider finally a badly constructed bridge, viz. one in which the earth-capacitances D_1E and D_2E are of the same order as those of Z_1, Z_2, etc., and also different in magnitude, *e.g.*, in Fig. 28 let the capacitance C_7 be large and C_8 small.

For the first main balance the detector is connected to D_1D_2. Obviously the balance will be incorrect since it will be affected by the inequality of C_7 and C_8. For the first Wagner balance the end of the detector connected to D_2 is transferred to E, and this must diminish C_8, since part of the capacitance D_2E is supplied by the detector in the case of the first balance : C_7 however, will remain unchanged. Evidently the effect of the transference of the detector will be to upset the first balance, so that although the first Wagner balance will bring D_1 to the potential of E, it will leave D_2 at a different potential. Therefore current will pass through C_8 even when the first Wagner balance is obtained, and when the detector is transferred back to D_1D_2, the first Wagner balance will be upset. There is evidently no certainty that the second main balance will be more nearly correct than the first, and indeed it sometimes happens that the second main balance-point is farther removed from the correct balance-point than the first, so that the successive bridge readings do not converge to those corresponding to the condition of simultaneous balance, but get further and further away from these values. The deciding factor is the extent to which the first main balance and the first Wagner balance are controlled by the currents flowing from D_1 and D_2 to E. The successive adjustments only converge to the correct value when this control is relatively small. We have seen that two conditions favourable to rapid convergence are (1) smallness of C_7 and C_8, and (2) equality of C_7 and C_8. Another favourable condition is that for the first main balance Z_5 and Z_6 shall be such that the common potential of D_1D_2 is nearly equal to that of E, *i.e.*, Z_5 and Z_6 should be so set in the first instance that

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} = \frac{Z'_5}{Z'_6}$$

approximately, where Z'_5 and Z'_6 are the resultant impedances between O_1 and E, and O_2 and E respectively. This is not always so easy as may be imagined, because Z'_5 and Z'_6 must be influenced by the unknown earth-capacitances C_5 and C_6 which may be so large that they entirely control the values of Z'_5 and Z'_6, in which case the nominal values of the components used to construct the Wagner arms O_1E, O_2E, *i.e.*, Z_5 and Z_6, are totally different from the actual resultant values. For this reason it is desirable to make the earth-capacitances C_5, C_6 as small as possible, and to make the admittances of Z_5 and Z_6 fairly large, so that the effects of the unknown capacitances are relatively small. Then by setting the nominal values of the various bridge components before the first balance is made, so that $Z_1Z_6 = Z_3Z_5$, the required condition is immediately established. It is often convenient to make $Z_1 = Z_5$ and $Z_3 = Z_6$.

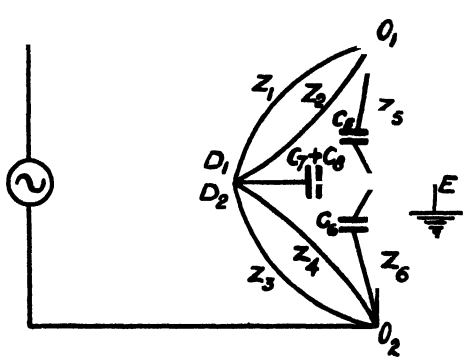


FIG. 29. Equivalent network for a modified Wagner balance in which C_7 and C_8 cause no error.

In difficult cases the following modification of the usual procedure possesses advantages. Having obtained the first main balance-point as usual, transfer the detector to D_1E, and also connect D_1 and D_2 together. We then obtain the network of Fig. 29. The earth-capacitance C_8 is merely added to C_7 to form a shunt across the detector, and it will therefore cause no error in this form of the Wagner balance. The first Wagner balance becomes free from error except that due to the incorrect adjustment of Z_2 in the first main balance. The errors in successive balances cannot therefore become cumulative, and the convergence must be more rapid.

15. **Stray Mutual Inductance: The Wiring of Bridge Circuits.** All the foregoing considerations of bridge circuits are based on the assumption that there exists no mutual inductance between the arms of the bridge, the source of current and the detector system. We have previously pointed out that the induced e.m.f. arising from the existence of mutual inductance is proportional to the frequency of the inducing current, and that therefore such e.m.fs. are apt to be of great importance at radio frequencies. We must therefore bear in mind the fact that the current in any

one arm of the bridge will induce e.m.fs. into every other arm, and that such e.m.fs. will invalidate the ordinary equations of balance, unless they can be made either negligibly small, or of such magnitudes as to balance one another. At low frequencies it is usually possible to make these e.m fs. negligibly small, but at radio frequencies it is desirable to arrange the connecting leads of the bridge circuit, so that the stray mutual inductances are not only small but also of constant value and balanced. It is on this account that the symmetrical bridge with equal ratio arms has such very great advantages for radio-frequency work. We have seen that it is convenient in other respects, but whereas it is possible to overcome the other difficulties connected with unequal ratio arms, the existence of unavoidable finite but unknown mutual inductances between the various circuit elements sets a rather low limit to the frequency at which any bridge except a symmetrical one may be successfully used. For any radio-frequency work therefore a symmetrical bridge should be used, and the wiring should satisfy the following conditions :

(*a*) The leads should be rigidly fixed so as to make the mutual inductances of the system definite.

(*b*) No arm of the bridge should be allowed to form an open loop, and similarly the meshes of the circuit should not be allowed to form open loops, as in Figs. 23 to 29. There would obviously be considerable mutual inductances between any such loops. The loops formed should be as small in area as possible, and they should as far as possible be set with their planes at right angles, positions in which the mutual inductance is zero.

(*c*) Where possible, go and return leads should form a concentric pair ; the e.m.fs. induced into such leads by currents in outside conductors will be practically equal but opposite, and will therefore balance. A pair of wires twisted together is almost equally good, but it must be remembered that their capacitance is usually greater. A pair of parallel wires, as close together as is permissible from considerations of capacitances comes next in order of preference.

(*d*) When inductive coils are employed they should wherever possible be of toroidal form, thereby ensuring that their mutual inductance with any external conductor is practically zero.

(*e*) The various arms of the bridge should be symmetrically disposed in pairs with respect to the source and detector circuits which should be placed at a sufficient distance apart to ensure that there is no mutual inductance between them.

Bearing these conditions in mind, we see that the wiring of a bridge to be used for measurements at high frequency should resemble Fig. 30 rather than the preceding diagrams. Thus the four bridge corners O_1, O_2, D_1, D_2 must be placed near together and the leads symmetrically disposed in pairs. The leads to the oscillator and detector may well be at right angles to the plane of the four arms Z_1, Z_2, Z_3, Z_4 (the horizontal, say), and these leads may be concentric or twisted pairs. The leads to the Wagner arms Z_5 and Z_6 should also be a twisted or concentric pair. It is easy to see that any e.m.f. induced in the detector leads would be liable to make the attainment of the condition of simultaneous main and Wagner balance very difficult, for the successive balance-points would always be false ones.

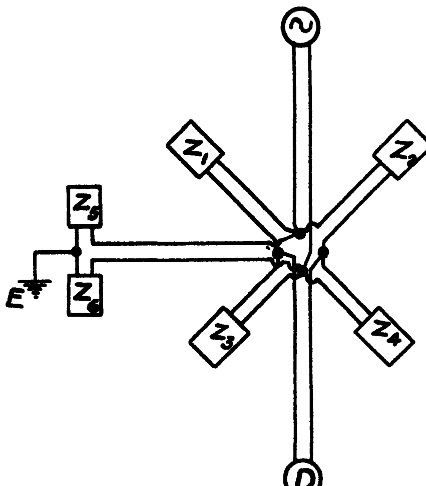


FIG. 30. Disposition of leads in order to avoid errors due to stray mutual inductance.

Mutual inductance between the coils in the oscillator system and those of the detector system should be minimised by placing the oscillator and detector at a considerable distance apart, their leads proceeding from the "bridge centre" O_1, O_2, O_3, O_4 in opposite directions. If toroidal coils are used the external magnetic field of the oscillator will probably be negligible even at points fairly near to it, but open coils are so much more easily constructed that it is often expedient to use them and to eliminate the stray mutual inductance associated with them by electromagnetic shielding.

16. **Electromagnetic Shielding.** An electromagnetic shield consists essentially of a metal wall separating an inductive circuit and a generator of alternating magnetic fields. In the absence of the shield an e.m.f. is induced in the inductive coils of the circuit by the alternating field, but when the shield is present eddy currents are induced in the metal in such a direction as to oppose the field producing them; and provided the metal shield is of such a form and thickness that the eddy currents are not unduly restricted, the resultant field on the side of the shield remote from the generator becomes negligible, and the mutual inductance between the generator and the coil is therefore reduced almost to zero by the metal shield. A shield in order to be efficient must obviously have a very low resistance in all directions. It should therefore completely surround the coil system to be shielded, and should be of metal of high conductivity and sufficiently thick. The eddy currents in the metal are produced by an induced e.m.f., which increases with rise of frequency, so that the eddy currents tend to increase in strength with an increase of frequency, and electromagnetic shielding therefore tends to become more efficient the higher the frequency.

The factors involved in electromagnetic shielding are best understood by considering the simple case of an infinite mass of metal bounded by a plane face. Let currents be induced in the metal by an external source near the plane face, and consider the simple case in which the current has the same value and direction in any plane parallel to the surface. Then application of the fundamental electromagnetic equations* to any element of volume of the metal shows that, if the current at the surface is $i_0 = \hat{i}_0 \cos \omega t$, then the current at a distance x from the surface (measured perpendicularly) is

$$i = \hat{i}_0 e^{-mx} \cos (\omega t - mx)$$

where $$m = \sqrt{2\pi\mu\omega\sigma} = 2\pi\sqrt{\mu f \sigma},$$

μ = magnetic permeability of the metal, f = frequency, σ = conductivity of the metal.

* Thomson, J. J. *Elements of Electricity and Magnetism*, 4th edition, p. 428. Cambridge University Press, 1909.

Also the currents are associated with a magnetic field, which in this case is parallel to the surface of the metal and perpendicular to the current, and is given by

$$H = \frac{2\sqrt{2\pi}}{m} i_0 e^{-mx} \cos\left(\omega t - mx - \frac{\pi}{4}\right).$$

Thus both the current and the magnetic field decay exponentially with distance from the surface, and with the coefficient m, which increases with frequency, permeability and conductivity. It follows that as the magnetic field is propagated through the metal its amplitude diminishes exponentially, a continuous wall of thickness x reducing the magnitude to a fraction e^{-mx} of its original value. For efficient shielding this fraction must be made equal to say 1 per cent., or perhaps 1 part per million, depending on the conditions of the experiment in hand. It is impossible to lay down any hard and fast rule and it must be remembered that in practice the magnetic field is not always parallel to the surface of the shield. However, experience shows that the above relations form a useful guide to the use of shields for the more complicated cases occurring in practice, and it may be said that in general the thickness of metal required to provide a given degree of shielding is roughly inversely proportional to the coefficient m. Calculation of m from the values of frequency, permeability and resistivity shows that at a frequency of 100 c/s a thickness of 4 cm. of copper is required to reduce the field to about 0·2 per cent. of its original value, while at 1 Mc/s a thickness of only 1 mm. of copper reduces the value to 1 part per 6,000,000. Iron has a resistivity about ten times that of copper, but its permeability for weak fields at high frequencies is about 100 as against 1 for copper. Thus m for iron is from 3 to 4 times that for copper, and the thickness of iron required for efficient shielding is therefore only about one-quarter of that of copper. For certain purposes, therefore, iron shields have considerable advantages at the lower radio frequencies. It must be remembered, however, that their high permeability is liable to affect the inductance of any coil in the neighbourhood. At high radio frequencies the thickness of either copper or aluminium required for adequate shielding is so small that it presents no difficulty. It should be remembered that any joints of relatively high resistance in the shield will disturb the flow of eddy currents, and therefore reduce the shielding action. It is sometimes necessary to solder all joints, or to seal them with mercury.*

It has been shown that a metal screen must be used in order to limit the flow of displacement currents, and this screen may also act as an electromagnetic shield in the way just described. The electrostatic screen usually only produces the required result when it is connected to some definite point of the system, as explained in previous paragraphs. The electromagnetic shielding is, on the other hand, quite independent of any such connection.

17. **Effects of Leads: Measurement by Substitution.** We have seen that when the conditions discussed in the previous paragraphs are satisfied, the relation $Z_1Z_4 = Z_2Z_3$ may be used to determine any one of these quantities in terms of the other three. It is however important to notice that in the equation Z_1 is the resultant direct impedance between the junctions O_1 and D_1, and so on for the other arms of the bridge. Then the value of the impedance must include the effects of any leads which may be used to connect the corresponding instrument to the terminals O_1 and D_1. It is often necessary to make separate measurements or calculations of the resistance, inductance and capacitance of such leads, and to calculate the correction which must be applied to any bridge reading in order to allow for them. When, however, a standard variable impedance of the same phase angle as the one under test is available, it is possible to avoid all errors due to connecting leads by using a substitution method of measurement. The instrument to be measured is connected to the bridge by means of suitable leads; the bridge is balanced and the readings noted. The variable standard is then substituted for the instrument under test in the bridge circuit (*i.e.*, it is connected to the same leads) and it is adjusted until balance is again obtained. The reading of the standard then gives the value for the instrument under test, quite free from errors due to leads. This procedure also has the great advantage that it eliminates any residual errors, due to imperfectly balanced mutual inductances, or earth-capacitances and leakage, or phase difference of the ratio-arms (*cf.* paragraph 11) for the conditions are exactly the same before and after the substitution, so that all the errors are the same for

* Smith-Rose, R. L. *Proc. Phys. Soc.*, 1922, Vol. 34, p. 127.

both measurements. The value of the instrument under test is therefore the same as that of the standard, and the fact that the finite impedance of the connecting leads is added to this value in each case is of no consequence for the calibration of the standard gives the value minus the effect of the added leads. In some cases the effect of the connecting leads varies with the impedances to earth as well as the direct impedance of the instrument to which they are connected, and in such cases the earth-impedances of the standard must be adjusted also if the substitution is to be perfect. When the instrument under test and the standard are perfectly screened and the screens are connected to one of the terminals, the most satisfactory plan is to connect both screens permanently to one of the leads, and to make the substitution by connecting the other lead, first to the remaining terminal of one instrument, and then to that of the other. The total earth-capacitance (that of both screens) then remains constant the whole time.

The method of substitution is applicable to all methods of measurement, provided a suitable standard is available, and it is advisable to use it for radio frequency measurements whenever possible; for it so frequently happens that all the sources of error have not been recognised, and this is the only method of avoiding errors of unknown origin.

CHAPTER IV

GENERATORS

1. Thermionic Valves and their Circuits. This chapter and the one following will be devoted to a discussion of generators and detectors of radio-frequency currents, suitable for measurement purposes. The generators consist exclusively of thermionic oscillators of various forms, and the detectors also consist very largely of thermionic valve circuits. It will therefore be convenient to outline the essential features of these devices.

Fig. 31 shows a typical triode circuit. The valve itself consists of (1) the filament F, which when heated by current from the battery B_1, becomes a source of free electrons, and acts as the cathode of the valve. (2) The anode A, which in virtue of its high potential, produced by the battery B_2, collects the electrons emitted by the cathode, the effect being to cause the passage of current from A to F through the valve.

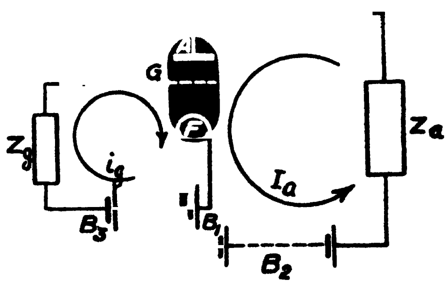


FIG. 31. The input and output circuits of a triode.

(3) The grid G situated between the cathode and anode, so that its potential controls the rate at which electrons pass from cathode to anode, *i.e.*, the anode current. This current will obviously depend on the potential of both G and A, but since G is the nearer to the cathode, variations of its potential have the larger effect. A variation V_g in the potential of G has the same effect as one of μV_g in that of A, where μ is the amplification factor of the valve. The grid is maintained at a suitable potential with reference to that of the cathode by means of the battery B_3. When this potential is negative the grid collects very few electrons, and the current I_g in the grid circuit is very small, while that in the anode circuit I_a

may be large. Confining our attention to the alternating components of these currents and voltages, the complete system may be seen to consist of (1) the input or grid circuit, including an external impedance Z_g, in series with the input impedance z_g of the valve, which is mainly due to the small capacitances between the electrodes; and (2) the output or anode circuit, including an external impedance Z_a (which may be described as the load on the valve) in series with the impedance of the valve z_a between F and A, which consists of a resistance r_a corresponding to that of the electron stream (usually of the order of a few thousand ohms), in parallel with a very small inter-electrode capacitance C_{af}.

The grid circuit being always of high impedance, a small voltage applied to the input terminals produces only a very small current I_g in the grid circuit, but a large current I_a in the anode circuit. The valve therefore acts as an amplifier. However, a voltage applied to the anode circuit causes practically no current in the grid circuit, so that these two circuits have practically no coupling. It should be noticed, however, that the very small capacitance C_{ga} between G and A does give rise to capacitive coupling, and although this is in many cases negligible, it is sometimes important; so much so that in certain types of valve an electrostatic screen in the form of an additional grid is inserted between G and A so that when this screen-grid is maintained at constant potential, C_{ga} is reduced almost to zero. Such valves are called screen-grid tetrodes. It should be noticed that the screen also shields the cathode from the effects of changes of anode voltage, with the result that these valves tend to have very high amplification factors and high resistances. Typical values are $\mu = 500$, $r_a = 300{,}000$ ohms, $C_{ga} = 0{\cdot}003\ \mu\mu F$. The screen-grid is always maintained at a positive potential, otherwise excessive anode voltages would be needed in order to obtain the emission current required for satisfactory operation. Valves of this type are excellent for the amplification of very small voltages, but for fairly large voltages their properties are complicated by the fact that when the anode voltage is increased beyond a certain point, the electrons acquire such a high velocity that their impact with the anode liberates other electrons from it (secondary emission) and these electrons being attracted by the positive screen-grid, are lost by the anode, so that the anode current decreases with increasing voltage over a certain range (a condition of negative resistance). The characteristic curve of the valve, representing I_a as a function of V_a, shows sharp bends and changes of slope in this region, and oscillations covering this range of voltage have distorted current waves. This effect may, however, be avoided by inserting yet another grid, the suppressor-grid, between the anode and the screen-grid, thereby forming a pentode. The suppressor-grid is connected to the cathode, so that the electric field near the anode is always in such a direction as to drive all the electrons in this region towards the anode. The secondary electrons are therefore driven back to the anode, and the curve connecting anode voltage and current is free from irregularities. Pentodes may have values such as $\mu = 100$, $r_a = 25{,}000$ ohms or even less, $C_{ga} = 1\ \mu\mu F$; or, on the other hand, values of the order of those given for tetrodes. They are very useful when a large power output is desired.

Some valves are "indirectly heated," *i.e.*, the cathode carries no heating current, but consists of a separate electrode k, insulated from the filament f by which it is heated (Fig. 34). Alternating current may then be used for the heating, and the cathode being free of connections to batteries, need not be connected to earth. The grid can then be given a negative potential with respect to that of the cathode without using a battery B_3. An example is given later (Fig. 38). The batteries shown may of course be replaced by any other convenient source of d.c. power, of constant voltage and low impedance.

2. **The Thermionic Oscillator.** Nearly all thermionic oscillators consist essentially of amplifying systems with a definite mutual impedance or coupling between their output and input circuits. Any small alternating current arising in the input circuit causes a larger one in the output circuit, and this, in virtue of their mutual impedance, introduces a p.d. into the input circuit. If now this p.d. is in such phase as to increase the initial current, the whole process is cumulative, and the currents steadily increase in value, until the power dissipated in the circuits just balances the power taken from the batteries. This stationary state persists as long as the batteries supply the necessary power, and continuous oscillations are therefore produced. The frequency

of the oscillations is fixed by including in one of the circuits a resonator of the required frequency. This resonator may take the form of a simple tuned circuit, or it may be a mechanical resonator, such as a tuning fork, a piezo-electric crystal, or a magneto striction resonator. It is merely necessary that its frequency of resonance should be well defined, and that it should be capable of electrical excitation. The electrical disturbance produced by switching on the amplifier will always set up minute oscillations at the resonant frequency, and these then build up as previously explained. Such oscillators take many forms, since the necessary coupling may be obtained in many ways, and the amplifier may include either a single valve or two or more valves in cascade. Perhaps the simplest oscillator is that obtained by using a single valve, with a simple tuned circuit (inductor L and condenser C in parallel) for Z_a (Fig. 31), and another coil for Z_g, the necessary coupling being provided by mutual inductance between the two coils, which, being reversible, can always be made to provide coupling of the appropriate phase. This is the well-known "tuned-anode" oscillator. Alternatively the "tuned-grid" oscillator has a tuned circuit for Z_g, and an additional coil for Z_a, with mutual inductance between them. Other oscillators avoid the use of two coils by employing a single-tuned circuit, a portion of which is made common to both grid and anode circuits so as to provide the necessary coupling between them, by making connection to a tapping on either the coil or the condenser. As these are perhaps the most generally useful oscillators for the purposes of measurement, they have been selected for more detailed consideration in the following paragraph.

3. **The Hartley and Colpitts Circuits.** Fig. 32 shows a simple oscillator of this type. The frequency is mainly controlled by the tuned circuit L_1C_1. The impedance of the grid circuit, and therefore the magnitude of the coupling, is controlled by the small variable condenser C_2. The tapping connection on the coil is usually made at the middle of the winding, but the position is not critically important. R_1 is a resistor of fairly high value (10,000 ohms to 1 megohm) which shunts the grid circuit and fixes the negative potential of the grid. This potential tends to become strongly negative owing to the passage of electrons to the grid in the course of the oscillations, and if the grid were perfectly

insulated, its potential would soon fall to a value which would so reduce the emission current that oscillations would cease. The "grid leak" resistor R_1 provides a leakage path by which the electrons may return to the cathode. Its value will evidently be one factor controlling the mean grid potential. The large condenser C_b shunting the battery B_2 acts as a by-pass of very low impedance for the high-frequency current, while the resistance R_s still further diminishes the dissipation of high-frequency power in the anode voltage supply.

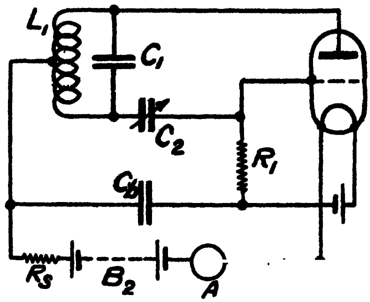


FIG. 32. Hartley oscillator; series fed. Typical values, $C_1 = 500\ \mu\mu F$ L_1 to tune to frequency required. $R_1 = 0{\cdot}5\ M\Omega$, $C_2 = 100\ \mu\mu F$, $C_b = 2\mu F$, $R_s = 2000\ \Omega$. Valve, power type, $\mu = 7$, $r_a = 2000\ \Omega$.

An alternative arrangement is shown in Fig. 33. Here the anode-voltage supply is in parallel with the output circuit, instead of in series with it; and in order to exclude the oscillations from this supply circuit, a choke coil *Ch* is included in it. A modification containing an indirectly heated valve is shown in Fig. 34.

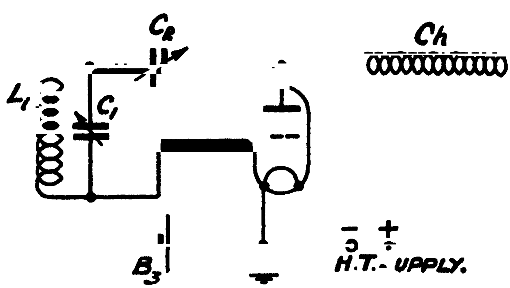


FIG. 33. Hartley circuit, shunt fed. Typical values, $C_1 = 500\ \mu\mu F$, L_1 to tune to frequency required. $C_2 = 100\ \mu\mu F$, Ch = choke of high impedance at working frequency.

It should be noticed that in this form one end of the coil and one bank of condenser plates is at earth potential. The tapping point on the coil should for this arrangement be rather nearer the earth-connected end, so as to minimise the effect of the capacitance between the cathode and its heater on the tuning of the circuit L_1C_1. The condenser C_b forms a connection of very low impedance between tuned circuit and anode, and also acts as a by-pass

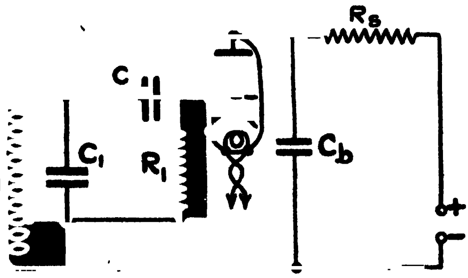


Fig. 34. Hartley circuit with indirectly heated valve and "cathode tap."

for the anode-voltage supply. Corresponding components are similarly lettered in the diagrams of these valve circuits.

When the tapping is on the condenser side of the tuned circuit we have the arrangement of Fig. 35 (*a*). It should be noticed, however, that the valve capacitances C_{ak} and C_{gk} in series form a

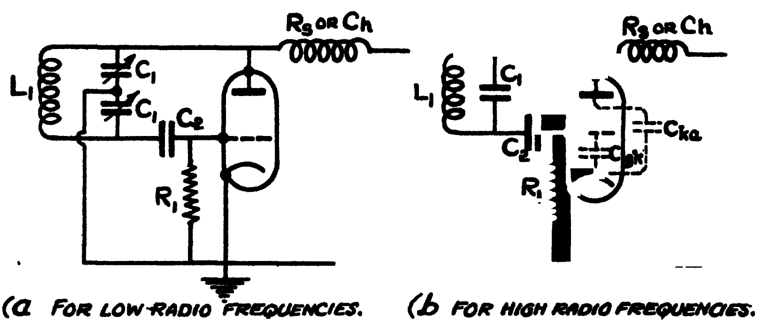


Fig. 35. The Colpitts oscillator.

shunt across the tuned circuit, and that the cathode necessarily forms a tapping connection to this shunt. At low frequencies the impedance of the shunt is so high that the tapping is ineffective, but for frequencies greater than, say, 10 Mc/s, it becomes so effective that no additional tapping is required, and the circuit takes the very simple form of Fig. 35 (*b*).

These simple circuits with an ordinary small valve and a suitable range of coils and condensers for the tuned circuit will generate oscillations of any frequency from a few hundred cycles per second to about 100 Mc/s.

4. **Push-Pull Oscillators.** In order to increase the amount of power available it is sometimes necessary to use two valves. These may be simply connected in parallel, but this doubles the valve capacitances, and for the higher frequencies it is preferable to use the connection of Fig. 36. If either valve is disconnected this circuit is simply that of Fig. 32. Each valve generates oscillations in the tuned circuit L_1C_1, and the cross connection of the grids and anodes ensures that the oscillations are in opposite phase in the valves themselves, and therefore in-phase in the tuned circuit. It is an advantage of the arrangement that there is no high-frequency current in the common battery leads, as in these leads the oscillations produced by the two valves are out of phase and annul one another. The two valves should have as far as possible the same characteristics. The oscillations of potential are all balanced with respect to earth, a feature which is desirable for some measurements. For this circuit a true centre tapping on the coil is important. If the tapping is made at a point which is not the electrical centre of the coil, an unsymmetrical arrangement leading to loss of power is obtained. This may be improved by including a small high-frequency choke in the lead to the tapping, thereby allowing the variation of potential at this point which is necessary for symmetrical operation.

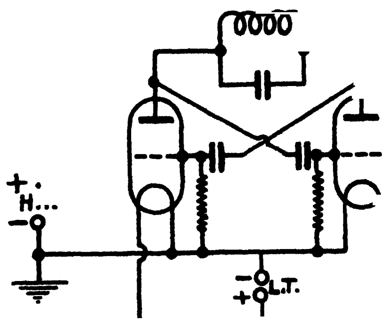


FIG. 36. Push-pull oscillator.

5. **The Stability of Oscillators.*** The properties of most importance in oscillators for measurement purposes are constancy of frequency and amplitude, and purity of wave-form. We shall

* COLEBROOK, F. M. *Valve Oscillators of Stable Frequency Radio Research Special Report, No.* 13. 1934.

now consider briefly the means by which these properties may be secured. We have seen that the frequency is controlled by that of the resonator, and therefore the first essential is that the resonator itself shall be of high stability. In the above examples the coil L_1 and condenser C_1 should be of rigid construction and low temperature coefficient. It is then desirable that the conditions of operation shall be such that the oscillations are maintained at a frequency as nearly as possible equal to the natural frequency of the resonator. The factors which cause the angular frequency of maintenance ω to depart from that of the resonator ω_r, are capacitance in the valves, phase-change in the amplifying system and distortion of the wave-form. Obviously capacitance between the electrodes of the valve is added to C_1, and as this valve capacitance is apt to vary with the operating conditions, it should be made small relative to C_1.

The effect of the amplifying system is best understood by considering the conditions for maintenance of oscillations. Let the amplifying system possess a voltage amplification factor $M(1 + j \tan \phi)$, and an output impedance z_0. Then ϕ represents the phase displacement associated with the amplification. Let the load Z on the output circuit consist of a tuned circuit with equivalent shunt values of L, C and R respectively. An initial input voltage V_i will produce an output current I_0 where

$$I_0 = \frac{M(1 + j \tan \phi)V_i}{z_0 + Z}$$

and an output voltage $V_0 = I_0 Z$. In virtue of the coupling, a fraction of this, say V_0/M_k, is supplied to the input circuit, and if this is equal to the initial value V_i, then the voltages are maintained even when the initial stimulus vanishes. Thus the condition of maintenance is

$$\frac{V_0}{M_k} = \frac{M(1 + j \tan \phi)V_i}{M_k} \cdot \frac{Z}{z_0 + Z} = V_i$$

or $$\frac{M}{M_k}(1 + j \tan \phi) = 1 + \frac{z_0}{Z} = 1 + z_0\left[\frac{1}{R} + j\left(C\omega - \frac{1}{L\omega}\right)\right]$$

If now the valve capacitance is included in C, z_0 becomes a pure resistance r_0, and equating real and imaginary terms we obtain the conditions

$$\frac{M}{M_k} = 1 + \frac{r_0}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and $$\frac{M}{M_k} \tan \phi = r_0 \left(C\omega - \frac{1}{L\omega} \right) = \frac{(LC\omega^2 - 1)}{L\omega} \quad . \quad . \quad . \quad (2)$$

By making use of the relation $LC\omega_r^2 = 1$, equation (2) may be written

$$\frac{M}{M_k} \tan \phi = \frac{r_0 LC(\omega^2 - \omega_r^2)}{L\omega} = r_0 C(\omega + \omega_r) \frac{(\omega - \omega_r)}{\omega} \quad . \quad . \quad (3)$$

Thus the fractional displacement of the frequency from ω_r, being usually small, is given by

$$\frac{\delta f}{f} = \frac{\omega - \omega_r}{\omega_r} = \frac{M}{M_k} \tan \phi \frac{1}{2r_0 C\omega_r} = \frac{R + r_0}{2r_0} \cdot \frac{1}{RC\omega_r} \cdot \tan \phi \, . \quad (4)$$

Now we have seen that $1/RC\omega_r = \Delta/\pi$ where Δ is the decrement of the resonator. Thus

$$\frac{\delta f}{f} = \frac{R + r_0}{2r_0} \cdot \frac{\Delta}{\pi} \cdot \tan \phi = \frac{1}{2} \tan \phi \left[\frac{\Delta}{\pi} + \frac{1}{r_0} \sqrt{\frac{L}{C}} \right] \quad . \quad . \quad (5)$$

It follows that the frequency displacement is zero when $\phi = 0$, or when there is no phase displacement in the amplifier; and if phase displacement occurs, the frequency displacement is minimised by using (*a*) a resonator of low decrement, (*b*) one of small L/C ratio, and (*c*) an amplifying system of high output resistance r_0. It should be noticed that this output resistance may be increased by inserting resistance between the anode of the output valve and the resonating circuit, and some oscillators are "stabilised" in this way.

The above argument takes no account of the existence of harmonics, which are another cause of frequency displacement. Oscillations consist essentially in the reversible transference of stored energy from the coil to the condenser of the resonator, and it is a condition of maintenance that the energy stored in these two components shall be equal. When no harmonics are present the required condition is satisfied when the reactance of the two components are equal, which occurs at the resonance frequency. At the harmonic frequencies, the reactance of the condenser is much the lower, and therefore harmonics are mainly confined to the condenser branch, and tend to increase its stored energy. The balance can only be restored by a fall in frequency, which reduces the reactance of the coil, and therefore increases the energy stored by it for a given applied voltage. Harmonics should therefore be avoided. They are obviously associated with curvature of the characteristic curves of the valves over the range of currents and voltages covered by the cycle of the oscillations ; and they can be minimised by choosing the mean electrode

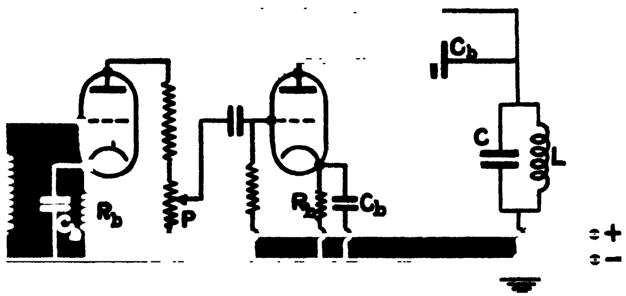


FIG. 37. The Yates-Fish oscillator: a two-stage amplifier with a tuned circuit LC common to both input and output circuits. The large condensers C_b provide connections of very low impedance, but high d.c. insulation resistance. The potentiometer P controls amplitude and therefore wave-form. The valves may be triodes or pentodes.

potentials so that the mean operating point occurs at the straightest part of the characteristic, and by reducing the amplitude of the oscillations to the smallest possible amount. This can be done by increasing the coupling factor M_k until the condition (1) for maintenance is satisfied for very small amplitudes. M_k is usually controlled by inserting an impedance of a suitable kind between the resonator and the valve. Many examples will be found in the literature of the subject. The condenser C_2 in the circuits of Figs. 32 and 33 provides the required adjustment for the Hartley oscillator. For other circuits it is usually not difficult to find a suitable adjustment by trial. Note that decreasing M ha the same effect as increasing M_k. Yates-Fish* has devised a very useful oscillator employing a two-stage amplifier, in which

* YATES-FISH, N. L. *Proc. Physical Soc.*, 1936, Vol. 48, p. 125

the amplitude is controlled by varying M, the overall voltage-amplification factor, by a potentiometer P controlling the input to the second valve. His circuit has the advantage of requiring no tapped coils or condensers (see Fig. 37). This circuit has the advantages of the more familiar dynatron with a wider range of operating conditions.

6. **The Crystal-Controlled Oscillator.** We have so far considered only oscillators, the frequency of which is controlled by a simple tuned circuit or electrical resonator. It is, however, possible to control the frequency of a valve circuit by means of mechanical resonators, such as tuning forks, stretched wires, vibrating reeds and the like; and as mechanical resonators usually possess decrements very much smaller than those of electrical resonators, they provide oscillators of greater stability. The most perfect type of mechanical resonator so far discovered is a piece of quartz. Plates, rods, or rings may be used, and they may be made to vibrate in many ways, *i.e.*, transversely, longitudinally or torsionally. The elastic properties of the quartz are so good that the decrement is usually of the order of 1×10^{-4}, whereas the decrement of a tuned circuit is often 100 times larger than this. As, moreover, quartz is a particularly stable material, with small temperature coefficients of length and elasticity, it possesses all the qualities required in a resonator to be used for frequency control. It is of course necessary to provide some form of coupling between the mechanical oscillator and the electrical circuits. With tuning forks magnetic coupling is the simplest, but with quartz crystals it is most convenient to make use of the piezo-electric properties. It is not necessary to consider these in detail, but it may be recalled that when a piece of quartz crystal is compressed or elongated in certain directions it develops electric charges on its surface, and conversely if an electric field is applied to the quartz in these directions, it contracts or elongates according to the direction of the field. Accordingly an alternating voltage applied to a quartz crystal will throw it into a state of mechanical vibration, and resonance will occur if the frequency of the applied voltage is equal to the natural frequency of the mechanical vibrations. Conversely, mechanical vibrations of a quartz crystal will produce an alternating voltage of the same frequency in electrodes suitably placed on or near the surface of the crystal.

Thus the necessary coupling is easily obtained. It is possible to cut pieces of quartz with natural frequencies of almost any value from 1,000 cycles to, say, 30 megacycles per second, so that for the production of stable radio frequencies, the combination of quartz resonator and amplifying valve is almost ideal. The combination is usually called a quartz oscillator.

Several forms of quartz oscillator are available, but we shall consider only one, which will be found to meet all ordinary laboratory requirements. The circuit is shown in the left-hand portion of Fig. 38. The crystal X is connected to the grid and filament of the valve and virtually forms the whole of the input

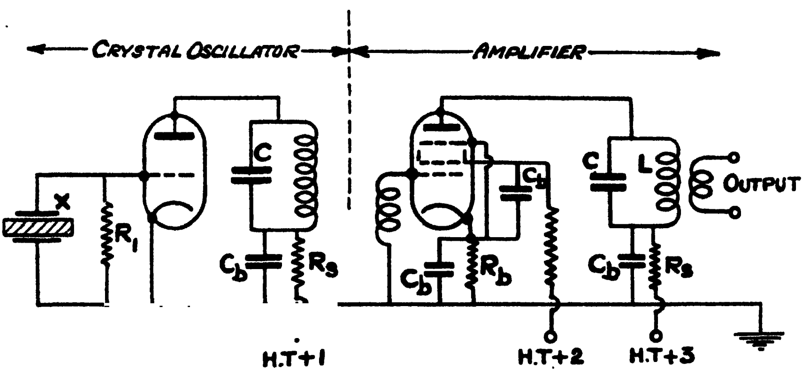


FIG. 38. Crystal-controlled oscillator with amplifier valve.

circuit outside the valve. In order to secure the maximum stability the grid is maintained at a definite negative potential. This may be done by means of a battery and choke connected between grid and cathode. Alternatively, a " grid leak " of high resistance may be used as shown in Fig. 38. Sometimes the leakage resistance of the grid circuit is sufficiently constant to serve this purpose. The output circuit of the valve is closed by a tuned circuit LC together with the usual arrangement for applying a high potential to the anode. The necessary coupling between the input and output circuits is provided by the grid-anode capacitance of the valve. Now this coupling is only regenerative so long as the load in the anode circuit is inductive, *i.e.*, so long as the capacitance C is smaller than the value required to tune the circuit LC to the frequency of the oscillations. Also the amplification and therefore the retroaction (for a given value of C_{ga}) is greater the larger the value of the impedance of this load. The consequence is that as the capacitance C is increased from a very small value to a value very near to that required to tune the circuit to the frequency of the resonator, the amplitude of the oscillations increases, slowly at first, and afterwards more and more rapidly, until the critical value is passed, when the oscillations cease abruptly as shown by the curves of Fig. 39. We have here ignored the resistance of the tuned circuit. It is obvious that the greater its decrement the smaller will be the amplitude. The upper curve of Fig. 39 corresponds to an anode circuit of very low decrement: the lower curve corresponds to a circuit of the same inductance but higher decrement. For the mathematical theory and further details of this circuit, the reader should consult the monograph of P. Vigoureux.* In practice the capacitance is adjusted to give oscillations of small amplitude corresponding to the condition of maximum stability. Oscillators of this type can be made to give a stability of frequency of the order of 1 part in 10^8. Oscillators controlled by tuned circuits are so much less stable that they are often described as "uncontrolled." Nevertheless they will remain constant in frequency to a few parts per million over periods of a few hours, and are adequate for most purposes. They have the great advantage of providing a continuous range of frequencies instead of, at the most, a few isolated frequencies.

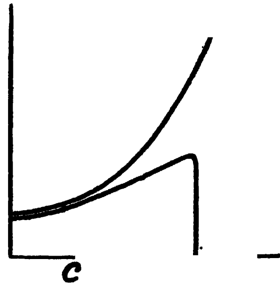


FIG. 39. Variation of output of crystal oscillator with capacitance in tuned circuit.

7. **Amplifier and Buffer Valves.** When oscillators such as those already described are used to energise a measuring circuit, *e.g.*, by arranging a coil in the measuring circuit so that it is magnetically coupled to one in the oscillatory circuit, the power dissipated in the measuring circuit virtually alters the constants of the oscillatory circuit, usually increasing its decrement, and therefore altering its frequency and amplitude. It follows that

* VIGOUREUX, P. *Quartz Resonators and Oscillators*. H.M. Stationery Office, London. 1931.

unless the coupling between the oscillator and measuring circuits is extremely loose, adjustments of the measuring circuit are likely to affect the stability of the oscillator in a manner which causes considerable errors in certain types of measurement. Sufficiently loose coupling may often be obtained by placing the oscillator and measuring circuits very far apart, or by using a very small coupling coil, but another plan which is very useful when the power generated by the oscillator is very small as it is in most "stabilised" circuits, is to use the oscillator merely to control the input circuit of an amplifying valve, and to take the power from the anode circuit of that valve. Changes in this anode circuit have usually no appreciable effect on its grid circuit, so that the frequency of the oscillator remains constant in spite of considerable changes in output power. Fig. 38 shows an amplifying valve of this kind coupled to the crystal oscillator. A large output, together with the minimum amount of coupling between the output and grid circuits is obtained by using a pentode for the amplifying valve. An indirectly heated valve is shown. Note that the grid is at earth potential, and that the p.d. between grid and cathode is that between the terminals of the resistor R_b, which is in series with the cathode, and therefore carries the whole emission current I_0. Thus the grid bias $= I_0 R_b$. The condenser C_b is a by-pass for the high-frequency current, and eliminates the resistive coupling between grid and anode circuits, which would otherwise be established by R_b. Arrangements of this kind may be used with any form of oscillator, and the additional valve may be desirable even when no amplification is called for. In such cases it is called a buffer valve. The oscillator itself, which controls the frequency but delivers almost no power, is called a master oscillator.

8. **Modulated Oscillators.** We have so far considered only circuits for the production of oscillations of constant amplitude. It is sometimes desirable to be able to vary the amplitude of the oscillations in a regular manner, *e.g.*, to make the variations of amplitude a periodic function of audible frequency. The oscillations are then said to be modulated. It is possible to produce such oscillations by means of a single valve, but the use of two valves, one for the radio-frequency oscillations and one for audio-frequency, is more straightforward, and also more flexible, and

therefore more generally useful. We shall therefore limit our considerations to one such circuit, shown in Fig. 40. The two oscillators are series-fed Hartley circuits. The output voltage from the audio-frequency oscillator is impressed on the anode of the radio-frequency oscillator by transformer coupling. Thus the anode voltage of the radio-frequency valve fluctuates at an audio-frequency and therefore the amplitude of the oscillations it generates, also vary at this frequency. The extent of the

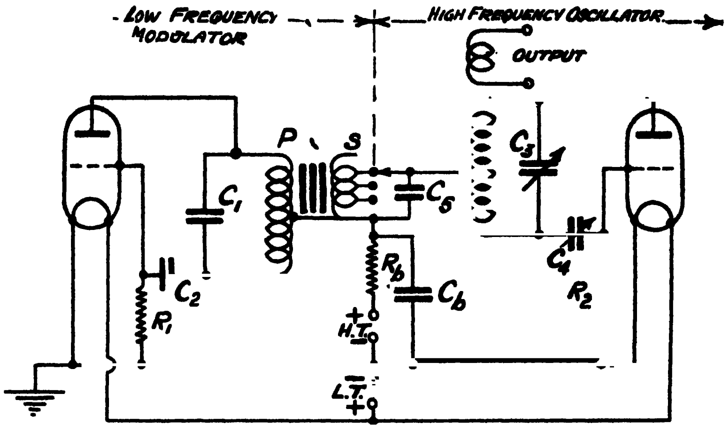


FIG. 40. Modulated oscillator. Typical values: $R_1 = 1\ M\Omega$, $R_2 = 0{\cdot}5\ M\Omega$, $C_1 = 0{\cdot}01\ \mu F$, $C_2 = 0{\cdot}005\ \mu F$, $C_3 = 500\ \mu\mu F$, $C_4 = 100\ \mu\mu F$, $C_5 = 0{\cdot}001\ \mu F$, $C_b = 1\ \mu F$, $R_b = 1{,}000\ \Omega$, $P = 4{,}000$ turns, say, centre-tapped, on small laminated iron core (*e.g.* audio-frequency inter-valve transformer type). $S = 50$ to 200 turns on same core. Valves: power type, low impedance.

fluctuations of amplitude, usually referred to as the depth of the modulation, may be readily controlled by varying the output of the audio-frequency oscillator.

9. **Miscellaneous Oscillators.** Many other oscillators have been devised, but the examples given illustrate sufficiently well for our purpose the most important principles. Further details of quartz oscillators are given by Vigoureux,* while the monograph of H. A. Thomas † contains a full account of con-

* VIGOUREUX, P. *Quartz Oscillators and their Applications.* H.M. Stationery Office, London. 1939.

† THOMAS, H. A. *Theory and Design of Valve Oscillators.* Chapman & Hall, London. 1939.

tinuously variable oscillators. Crystal-controlled oscillators serve as primary standards of frequency, while stable oscillators of the continuously variable type form convenient wavemeters. It is only necessary to calibrate the scale of the condenser in the tuned circuit by reference to standards of frequency. The oscillators described in this chapter will cover all ordinary requirements at frequencies up to about 150 Mc/s. Some account of oscillators for higher frequencies will be found in Chapter XII.

CHAPTER V

DETECTORS

1. **General.** In this chapter we shall consider instruments suitable for the detection or measurement of currents of radio frequency, such as are employed for measurement purposes. Two classes of detector are required: those suitable for use in null-methods (bridge methods) and which are therefore merely required to indicate the presence of the smallest possible current or voltage; and others for use in resonance methods, which must give an indication of the magnitude of the current or voltage. In these measurements it is not usually necessary to know the absolute value of the current or voltage; relative values only are required.

2. **Thermal Detectors.** It is obvious that any instrument which depends for its action on the thermal effect of a current will detect currents of radio frequency as well as currents of low frequency or direct currents, and there can be little doubt that the simplest possible detector for radio frequency measurements is the well-known combination of heater, thermojunction and d.c. galvanometer. The current to be measured passes through a heater consisting of a thin wire, the temperature of which is thereby raised. A thermopile or single pair of junctions is mounted on or near the heater, so that the temperature of one junction follows that of the heater, while the other remains constant. The junctions are connected directly to the d.c. galvanometer, the deflection of which is proportional to the thermo-e.m.f. developed, and is therefore a measure of the current in the heater. The combination of heater and thermojunction is often regarded as converting an alternating current into a direct one, and it is therefore called a thermal-converter. In order to obtain the highest possible sensitivity it is necessary to provide the thermal-converter with the best possible thermal insulation. It is therefore often mounted in an evacuated glass bulb, and is then called a vacuum thermojunction. These

instruments are made by several firms, and are very useful for radio-frequency measurements of many kinds. The active junction is generally attached to the heater by means of a minute bead of glass, which insulates the d.c. galvanometer circuit from the a.c. circuit, but at the same time provides the good thermal contact required for high sensitivity. Such instruments may be calibrated with direct current, and as the heater is made very thin, the change of resistance due to " skin-effect " only becomes important at very high frequencies, and the d.c. calibration therefore holds good up to these frequencies. Even when the " skin-effect " is appreciable, the same calibration may be used for relative values at any one frequency. Generally speaking the instrument possesses a " square law " scale, *i.e.*, the deflection of the d.c. galvanometer is approximately proportional to the mean square of the current through the heater. If the thermo-junction is not insulated from the heater it is necessary to calibrate with alternating current of low frequency, as in this case there is a p.d. across the junction, due to the resistance (small but not usually negligible) common to both heater and junction, and this p.d. will be of different sign with respect to the thermo-e.m.f. for the two directions of current in the heater. The magnitude of this effect is of course very simply checked by reversing the current in the heater when calibrating.

The thermal-converter and galvanometer is one of the most useful of all detectors. Its most serious disadvantage is its very small overload capacity. There is also usually a tendency for the deflection corresponding to a definite current to increase very slowly with time for several seconds before the final steady value is reached. This " creeping " action may limit the accuracy and rate of working. If the heaters are made thicker, not only is the sensitivity diminished, but the instrument becomes sluggish in its action, and measurements become tedious and less accurate.

Another form of thermal detector is the bolometer. This consists of a very fine platinum wire mounted in an evacuated glass bulb and arranged to form one arm of a Wheatstone bridge, which is balanced with direct current. The current of radio frequency is passed through the platinum wire, thereby heating it, and altering its resistance. The change of resistance, which is measured by means of the Wheatstone bridge, may be used as an indication of the magnitude of the radio-frequency current. The bolometer wire is connected to the bridge network by suitable chokes, which exclude the high-frequency current from the other arms of the bridge. The arrangement is obviously less simple in operation than the thermojunction, but is capable of giving higher sensitivity, and is therefore valuable in extreme conditions.

3. **The Crystal Rectifier.** Another very simple form of detector of high-frequency oscillations consists of a combination of crystal rectifier, condenser and galvanometer, as shown diagrammatically in Fig. 41. It is not necessary for our purpose to consider the mechanism of the action of the crystal rectifier: it will be sufficient to recall the fact that the contact between certain crystalline substances, *e.g.*, zincite (ZnO) and chalcopyrite (FeS . CuS); carborundum and steel, possesses the property of rectification, *i.e.*, of passing current more easily in one direction than the other, so that when an alternating p.d. is applied to such a contact the pulses of current in one direction are very much larger than those in the other. The resultant current is therefore of very unsymmetrical wave-form, and may be regarded as consisting of two components, one an ordinary direct current and the other an alternating current of symmetrical wave-form. The d.c. component may be measured by means of a d.c. galvanometer G (Fig. 41). The condenser C may be regarded as a by-pass for the a.c. component. It avoids any considerable high-frequency p.d. across the galvanometer, and therefore allows the full alternating p.d. to be applied to the crystal. The condenser may sometimes be omitted, as the self-capacitance of the galvanometer and its connecting leads are often large enough to provide the necessary low impedance. This arrangement may be very sensitive, much more so than the thermal-converter, but unfortunately the sensitivity is very variable, and the device is now only used in special circumstances, *e.g.*, for the detection of very short waves, owing to the large amount of time it demands for adjustment to high sensitivity and calibration.

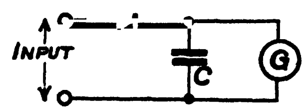


FIG. 41. Simple rectifier as detector.

There is, however, one form of crystal rectifier which is free from the trouble, and although it does not give a sensitivity equal to the best obtainable with crystals of the above types, it is quite reliable, and is very useful as a substitute for a thermal converter, being cheaper and very much more robust. It is the well-known copper-oxide rectifier, which consists of a plate of copper, the surface of which has been oxidised at high temperature to cuprous oxide. The contact between the cuprous oxide and the copper, which is quite permanent and requires no adjustment, possesses the property of rectification. Rectifiers of this type are sold commercially in various forms suitable for use at the lower radio frequencies, and a combination of such a rectifier with a small mica condenser and a d.c. microammeter, in the arrangement of Fig. 41, forms an extremely convenient portable detector of high-frequency currents.

4. **The Thermionic-Valve Rectifier.** It is a well-known fact that the thermionic valve is a rectifier, and indeed that in its simplest form, the diode, it was at first devised solely for use as a detector of high-frequency currents. Thus the rectifier shown in Fig. 41 may take the form of a simple diode, one electrode consisting of an electrically heated filament or cathode, which emits electrons, and the other, the anode, a cold plate which serves to collect these electrons. The rectifying action then arises from the fact that one only of the electrodes is a source of electrons, and that therefore the current can pass in one direction only. This form of rectifier may therefore be regarded as an alternative to the crystal rectifier already discussed. Diodes of very small linear dimensions are now obtainable and are valuable as indicators of voltage at very high frequencies.

The triode possesses the property of amplification, in addition to that of rectification, and the combination of these two properties gives it great advantages as a detector, so that it is probably the most widely used of all detectors for measurements requiring high sensitivity.

The rectifying action is not usually of the simple type described above, due to the passage of electrons in one direction only, but of the more general type which occurs whenever the relation between current and voltage is not linear. As a result of this condition a certain change of voltage in one direction causes a greater change of current than the same change in the opposite direction; so that when an alternating voltage of symmetrical wave form is applied, the resulting pulses of current in the one direction are greater than in the other. The resultant current therefore has a mean value which is not zero, *i.e.*, it may be divided into two components, one a d.c. component equal to this mean value, and the other an alternating component. The d.c. component is regarded as the rectified current. Its magnitude is controlled by that of the applied high-frequency voltage, and therefore measurements of this current serve to detect and measure the high-frequency oscillations. It is quite difficult, when using thermionic valves, to avoid a rectifying action of this kind, since the relation between current and voltage is never strictly linear, but it will be obvious that the rectifying action is greatest under conditions in which the valve-characteristic (the curve showing the relation between current and voltage) is most sharply bent, *i.e.*, the curvature and $\frac{\partial^2 I}{\partial V^2}$ are greatest. Rectification of this nature may occur in the anode circuit of a valve due to the non-linear character of the relation between anode current and voltage. It may also take place in the grid circuit due to the non-linear relation between grid current and grid voltage, when the conditions are such that the grid current is appreciable, *i.e.*, when the mean potential of the grid is positive. There are therefore several ways of arranging a triode as a detector of oscillations. We shall limit our discussion of the matter to two or three of the circuits which are found to be most generally useful in laboratory measurements. These will be found to cover almost any requirement.

5. **Single Triode with Anode-Circuit Rectification.** Fig. 42 shows a triode arranged for anode-circuit rectification. The grid potential is negative, so that there is practically no grid current, and the input impedance is very high: the arrangement behaves almost like an electrostatic voltmeter in this respect. The rectified current is measured by the d.c. indicating instrument G, which may be a microammeter or reflecting galvanometer, depending on the sensitivity required. With a sensitive galvanometer a full-scale reading may correspond to say 0·1 volt or even less, while a pointer microammeter may give full-scale reading for perhaps 1 volt. For the higher voltages it is

necessary to increase the grid bias in order to keep the grid of the valve always at a negative potential. The condenser C_b is a by-pass for the alternating component of the current. The potentiometer arrangement R_1, R_2, R_3 shown is not essential, but in its absence the instrument G will pass a current even when the input p.d. is zero, and it is the change of this current which measures the applied voltage. The potentiometer sends an adjustable current through the galvanometer in the opposite direction to the initial anode current, so that the galvanometer

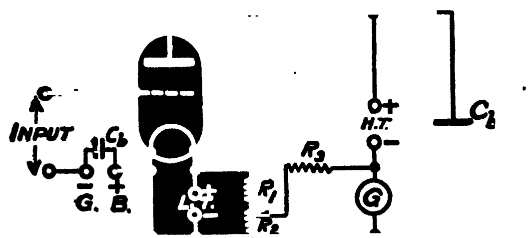


FIG. 42. Thermionic voltmeter with single triode working with anode-circuit rectification.

reading may be reduced to zero, for zero input p.d. and any subsequent reading is then a direct measure of the input p.d.

Many variations of this circuit are possible. Thus three batteries are not essential. It is possible to supply the three potentials from a potentiometer fed by a single battery of say 6 volts. The well-known Moullin* portable voltmeter is a simple version of this circuit. The reader should have no difficulty in adapting this circuit to any particular problem. It is to be noted that a conducting path between the input terminals is essential, otherwise the grid will not be maintained at the required negative mean potential. It is of course possible to provide a conducting path by connecting a high resistance (a " grid leak ") across the terminals, but since this diminishes the input resistance of the instrument it is not in general desirable.

6. **Triode arranged for Grid-Circuit Rectification.** The triode voltmeter described above may readily be arranged for grid-circuit rectification. It is only necessary to make the

* See Ref. 12 of Bibliography.

mean grid potential positive, and to provide a "grid leak" R and condenser C_1 as shown in Fig. 43. The anode current flowing when the input p.d. is zero will in this case be much larger than for the previous arrangement, and as the input p.d. increases, this current will be found to diminish, the actual diminution being a measure of the applied p.d. As before, the initial current through the galvanometer may be adjusted to zero by means of the potentiometer arrangement. The action may be explained as follows. The grid current is always appreciable and follows a curved characteristic. When an alternating p.d. is applied, the resulting pulses of current in the positive direction are therefore greater than those in the negative direction, and the mean grid

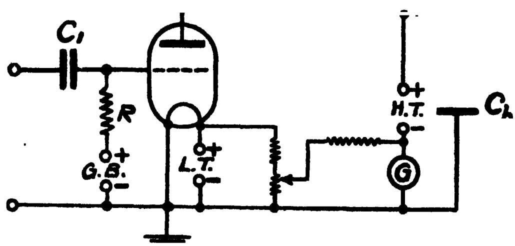


FIG. 43. Triode voltmeter arranged for grid-circuit rectification.

current is therefore increased. This current must flow from the battery GB through the high resistance R, and the mean p.d. across R therefore increases as the mean grid current increases. But an increased p.d. across R produces a corresponding fall of mean grid potential, which causes a diminution of anode current which is indicated by G. The condenser C_1 serves to insulate the grid from any source of potential (d.c.) other than GB and R, but at the same time transmits alternating potentials. It should be large enough to be of low impedance at the working frequency, and its leakage conductance should be negligibly small.

This form of voltmeter is sometimes found to be more sensitive than the form previously described, but it suffers from the disadvantage that its input resistance is much lower, *i.e.*, it absorbs more power from the circuit under test. It is also likely to be less stable in calibration, and is in general less satisfactory for measurement purposes than the anode-circuit rectifier. Under suitable conditions and for small applied voltages, the scale of both instruments will obey the " square law," *i.e.*, the deflection will be proportional to the square of the input voltage. The calibrations may be made at low frequency.

7. **Compensated Voltmeter of High Stability.** The sensitivity of the thermionic voltmeters described above may be increased by increasing the sensitivity of the d.c. galvanometer *G* of Figs. 42 and 43, but in practice a limit is set to the sensitivity obtainable, by the fact that the various battery voltages are

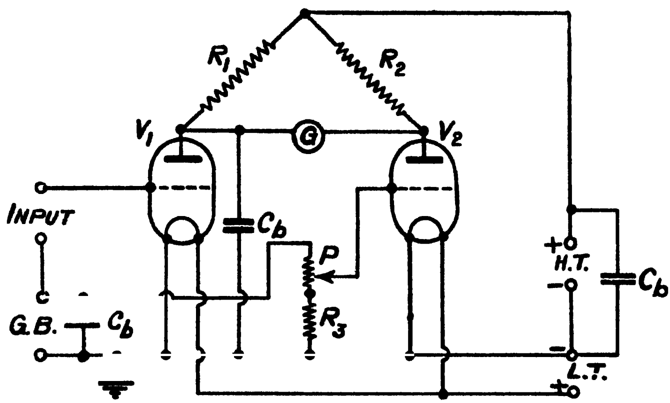


FIG. 44. Thermionic voltmeter compensated for battery fluctuations $R_1 = R_2 = 5{,}000\ \Omega$, $C_b = 2\ \mu F$, but lower values for very high frequencies. $P = 1{,}000\ \Omega$ potentiometer, $R_3 = 20{,}000\ \Omega$. Valves: small power type. It is sometimes better to transfer the earth-connection to the lower input terminal.

always subject to fluctuations, and the anode current is therefore also subject to corresponding fluctuations, which cause an " instability of zero " of the voltmeter. Clearly the p.d. to be measured must produce a deflection at least as large as the random variations of the zero, if measurement is to be possible. It follows that any increase of stability will permit the use of a more sensitive galvanometer, and therefore give a higher sensitivity. Some of the devices for the attainment of stability have been discussed by the author elsewhere.* Here we shall consider only one example of a voltmeter, " compensated " for battery fluctuations. This will be found to form an excellent general-purpose voltmeter of

* *Reports on Progress in Physics.* The Physical Society, 1934, Vol. 1, p. 325.

high sensitivity. The arrangement is shown in Fig. 44. It will be observed that two valves are used. Of these only V_1 is a rectifier. V_2 is a dummy, chosen to have as nearly as possible the same properties as V_1. The three batteries are common to the two valves, which are arranged in the form of a Wheatstone-bridge network, with two equal resistance ratio-arms R_1 and R_2, supplied with current by the H.T. battery, and having the galvanometer G as detector of balance. The p.d. to be measured is applied to the grid-circuit of valve V_1, but suppose in the first instance that this p.d. is zero, that the valves are accurately matched, and that the grid-potential of V_2 is equal to that of V_1. Then the anode-currents of the two valves will be equal, the bridge circuit will be balanced, and there will be no deflection of G. Further, although each of the three voltages H.T., L.T. and G.B. may fluctuate, the galvanometer will remain undeflected, for the anode currents of the two valves will always vary in exactly the same way, and the balance will hold good : fluctuations in the valve V_1 are always exactly compensated by similar ones in V_2. When, however, the applied p.d. is not zero, the mean anode current of V_1 is changed, the balance is upset, and the galvanometer is deflected. The arrangement constitutes a thermionic voltmeter of very high zero stability, so that a galvanometer of very high sensitivity may be used and great sensitivity attained, *e.g.*, a p.d. of 0·01 volt can be made to give a deflection which may be read with an accuracy of better than 1 per cent. In practice it is not possible to obtain valves which are perfectly matched, so that the compensation is never perfect. It is therefore necessary to provide a potentiometer P arranged so that the grid potential of V_2 is capable of fine adjustment over a small range. This potentiometer is adjusted until the deflection of G is zero when the input terminals are short-circuited.

It may be noted that any voltmeter with a condenser in series with it will measure alternating voltage only, and may therefore be used to measure an alternating voltage superimposed upon a direct voltage.

8. **Rectifier with Telephone as Null-Detector.** The detecting devices we have so far considered are most suitable as detectors of resonance, for in every case the deflection is approximately proportional to the square of the current or voltage, so

that a higher sensitivity is obtained, the larger the quantity to be measured; and the greatest sensitivity is obtained at the resonance peak. For the same reason these detectors are not very suitable for null-methods of measurement, for the sensitivity vanishes as the balance-point is approached. It is therefore necessary to consider by what means a higher sensitivity may be obtained.

The use of an amplifying valve in series with a thermionic voltmeter immediately suggests itself, and indeed a two-stage voltmeter of this kind may be found useful. We have, however, seen that fluctuations in the emission current of a valve set a limit to the sensitivity of any galvanometer, or detecting-device sensitive to d.c., that may follow the valve, and the sensitivity obtainable with any such combination is always limited in this way. This limitation may, however, be overcome to a very considerable extent by the use of a telephone as indicator, instead of a d.c. galvanometer. For the ear is able to discriminate between one sound and another, and is therefore often able to detect indications of one particular kind (notes of a characteristic frequency) in the presence of random disturbances. We therefore find that the combination of amplifier, rectifier, and telephone may be made to act as an extremely sensitive detector of radio frequency oscillations, and is used for most null-methods requiring very high sensitivity.

In order that a telephone may be employed in this way it is necessary that the current delivered to it shall be periodic and of audible frequency. A radio-frequency voltage of constant amplitude applied to a combination of amplifier, rectifier, and telephone, would of course only produce in the telephone a constant rectified current, which the ear would not be able to detect. It is therefore necessary to add some device for making the rectified current periodic, and easily recognisable by the ear. Perhaps the simplest possible arrangement consists of an interrupter, or contact vibrating at an audible frequency, in series with the telephone, but although this method was used to a considerable extent at one time, it has now given place to the heterodyne method, or to the use of modulated oscillations.

9. Modulation Detection. We have already described a modulated oscillator and have shown that the amplitude of the oscillations which it generates, varies at an audible frequency. When these oscillations are rectified, the rectified current must also vary at this audible frequency, and may therefore be detected by means of a telephone. Thus if for any measurements a modulated oscillator is employed, a rectifier-amplifier and telephone may be used as detector without modification or addition.

10. **Heterodyne Detection.** The essence of the heterodyne method of detection consists in the use of an auxiliary oscillation of a frequency differing from that of the oscillation to be observed by a value falling within the audible range. The two oscillations are combined and thereby give rise to beats analogous to those obtained from two musical notes. In both cases the beats are merely fluctuations of amplitude which occur at a frequency equal to the difference between those of the component oscillations, but whereas in the acoustic case the oscillations are audible and the beats therefore perceived by the ear, the electrical oscillations are inaudible, and therefore the electrical beats are also inaudible, and have no audible effect on a telephone. If, however, the electrical oscillations are rectified, the beats or fluctuations of amplitude, produce corresponding fluctuations of rectified current. The frequency of this current is that of the beats, viz. the difference of those of the component oscillations, an audible frequency; and therefore its effect on a telephone is to produce an audible note, often called a beat-note.

The following alternative method of considering the matter will be found instructive. Consider the mathematical identity

$$(A + B \sin pt) \sin \omega t \quad A \sin \omega t + \tfrac{1}{2}B \cos (\omega - p)t - \tfrac{1}{2}B \cos (\omega + p)t$$

The left-hand side may be regarded as representing an electrical oscillation of frequency $\omega/2\pi$, and of amplitude $A + B \sin pt$. Thus the amplitude fluctuates at a frequency $p/2\pi$, between a maximum value $A + B$ and a minimum value $A - B$, *i.e.*, it is a modulated oscillation, the depth or ratio of modulation being B/A. The right-hand side, on the other hand, represents three oscillations, each of constant amplitude and sine wave form. The frequency of the first is equal to that of the modulated oscillation, while those of the other two differ from this value by the modulation frequency, one being greater and the other less than the frequency of the modulated oscillation. It follows that a modulated oscillation may be regarded as consisting of three unmodulated components (often called the carrier wave and the two side bands); alternatively we may write

$$a \sin \omega t + b \cos (\omega - p)t \equiv (a + 2b \sin pt) \sin \omega t + b \cos (\omega + p)t$$

and

$$a \sin \omega t - b \cos (\omega + p)t \equiv (a + 2b \sin pt) \sin \omega t - b \cos (\omega - p)t$$

Let $a \sin \omega t$ represent a heterodyne oscillation acting on a receiver, and let $b \cos (\omega - p)t$ represent an incoming oscillation whose frequency $(\omega - p)/2\pi$ differs from that of the heterodyne oscillation $\omega/2\pi$ by the audio frequency $p/2\pi$. Evidently this combination is equivalent to a modulated oscillation, of a frequency equal to that of the heterodyne oscillation, and of a modulation-frequency equal to the frequency-difference $p/2\pi$, together with a second unmodulated oscillation of frequency slightly differing from those of heterodyne and received oscillations. Thus the heterodyne oscillation is another device for the production of a modulated oscillation, which as we have seen may be readily detected by a combination of rectifier, amplifier, and telephone. The additional unmodulated oscillation has no audible effect. It will be clear from the above two expressions that the frequency of the received oscillation may be either slightly greater or slightly less than that of the heterodyne oscillation. The resulting modulated component-oscillation is the same.

It may be useful to consider a few points concerning the relative sensitivity of the modulated-oscillator and heterodyne methods of detection. We have mentioned that all the rectifiers we have considered possess characteristics such that for small applied alternating voltages the rectified current is approximately proportional to the square of the applied voltage, and therefore that the sensitivity becomes smaller the smaller the voltage to be detected. When, however, the applied voltage is large, the rectified current usually becomes approximately proportional to the voltage, and this fact has some bearing on the present question. For with heterodyne detection, the modulated component of the oscillation applied to the rectifier, viz. $(a + 2b \sin pt)$ $\sin \omega t$ has a maximum amplitude $a + 2b$ and a minimum amplitude $(a - 2b)$, and since in null-methods of measurement a may be considerably greater than $2b$, the detector is always working with a comparatively large applied p.d., so that the rectified current is approximately proportional to the voltage, *i.e.*, proportional to $(a + 2b \sin pt)$. Thus the alternating component of the rectified current is proportional to $2b \sin pt$ and has an amplitude proportional to $2b$. If, on the other hand, the oscillator is modulated, the total voltage applied to the rectifier becomes smaller as the voltage diminishes, and the rectified current therefore becomes proportional to the square of the amplitude. The alternating component of this current will depend on the depth of modulation, but assuming this to be unity, the current in the telephone will be proportional to b^2, as against $2b$ for heterodyne detection. Evidently heterodyne detection will be the more sensitive in null methods of measurement.

11. **Amplifiers: General Considerations.** We have noted already that the sensitivity of any form of detector may be increased by the use of an amplifier. A detailed discussion of amplifiers is beyond the scope of this work, but we shall illustrate with a few examples the principles governing the construction of detector-amplifiers of high sensitivity.

In the last chapter we saw that an amplifier with regenerative coupling between the input and output circuits tends to generate oscillations. The higher the amplification, the smaller is the coupling between input and output necessary to produce oscillations, and it therefore follows that the more sensitive the amplifier, the greater the tendency to oscillation; and indeed the chief difficulty in the use of highly sensitive amplifiers is the prevention of oscillations, or the maintenance of stability. It will be immediately obvious that the important condition is the avoidance of all forms of coupling between the input and output circuits. If, however, there exists appreciable coupling, it will be a great advantage if the corresponding e.m.fs. are out of phase, in which case the coupling is not regenerative.

The coupling between input and output circuits may be of various types. We may have inductive or capacitive coupling between the parts of the circuits external to the valve, while inside the valve there may be coupling due to the capacitance between the grid and anode, or to secondary ionisation. We must now consider how these various forms of coupling may be eliminated. Capacitive coupling may, as we have already seen (p. 41), be eliminated by means of electrostatic screens. The components to be uncoupled should merely be separated by metal sheets or boxes, which are connected to a point at a fixed potential —either to earth, or to a point linked to earth by a very small impedance only, *e.g.*, a battery of low resistance, or a condenser of large capacitance.

External inductive coupling may be eliminated by using toroidal coils, by setting inductive coils in conjugate positions, separating them by distances as large as possible, avoiding the

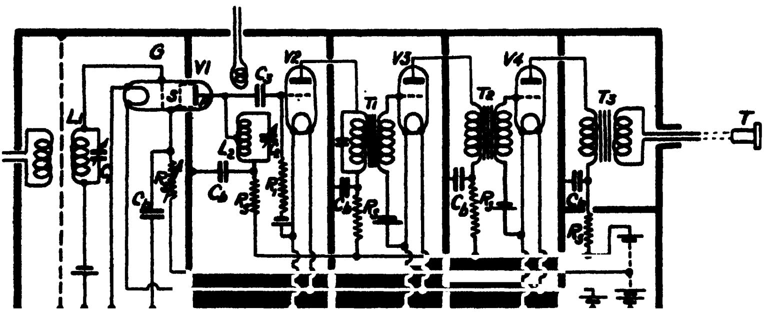


FIG. 45. Detector-amplifier for null methods. First stage, radio-frequency amplification with screen-grid tetrode. Second stage, triode rectifier. Third and fourth stages, audio-frequency amplification with transformer coupling. L_1C_1 and L_2C_2 to tune to working frequency, $C_3 = 200\ \mu\mu F$, $R_1 = 1\ M\Omega$, T_1 and T_2 inter-valve transformers ; T_3 output transformer. $C_b = 2\ \mu F$, $R_s = 10{,}000\ \Omega$, variable in first stage for controlling sensitivity.

use of coils with large external magnetic fields, *e.g.*, coils of large diameter. If this form of coupling is present it may sometimes be made innocuous by reversing one coil and so altering the phase of the induced e.m.f. It will be obvious that the input and output sides of the amplifier should be well separated, and it is often not desirable to aim at great compactness in the design.

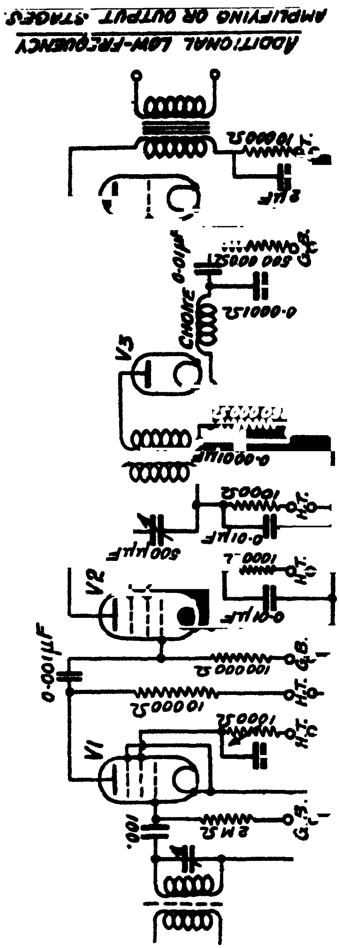
Another form of coupling which is often present is resistive coupling arising from the use of common batteries for the various valves of an amplifying system. The impedance of any such battery constitutes mutual impedance coupling all the circuits. The obvious remedy is to divert the alternating currents from the batteries. This is easily done by providing each lead to the common battery with a condenser of low impedance (C_b in

Figs. 45 and 46), which shunts any a.c. the lead may carry, outside the battery. A high resistance in series with the battery will further reduce the a.c. in the battery. For example, see R_s in Fig. 45. Such resistors and condensers are often called de-coupling resistors and condensers.

Coupling within the valves is minimised by the use of screen-grid tetrodes and pentodes, which have already been discussed.

The arrangement of both internal and external screens is shown in Fig. 45, which shows a detector-amplifier, suitable for radio-frequency bridge work. The first valve V_1 is a screen-grid tetrode. Both the input and output circuits are tuned to the frequency to be detected, and it is to be noted that with the arrangement of screens shown, they are almost completely screened from one another. The anode circuit is couplied to the triode V_2, which is arranged for anode-circuit rectification. The heterodyne oscillation (if any) is induced into the tuned circuit L_2C_2. The triodes V_3 and V_4 are arranged for audio-frequency amplification with transformer coupling, and the output is supplied to a telephone earpiece T through a suitable output transformer. The L.T. and H.T. batteries which are enclosed by the outer screen are common to all the valves. De-coupling resistors R_s and condensers C_b are therefore used as previously explained. It will be observed that the input is applied *viâ* a tuned high-frequency transformer. The desirability of such an arrangement for bridge measurements has been outlined in Chapter III (p. 43). The screen between the primary and secondary coils must of course be electrostatic only. It must therefore be constructed so as to avoid the formation of eddy currents, and has for this reason been shown dotted in the diagram. If the screen is of sheet metal, it must be slit in appropriate places. An additional winding of insulated strip or wire between the primary and secondary coils sometimes forms a useful screen. It must be left open-circuited, with one point connected to earth. The coils may be solenoids, or better, toroids wound one over another.

Fig. 46 shows the circuit for a very sensitive detector-amplifier employing pentodes V_1 and V_2 for radio-frequency amplification. These are followed by a diode rectifier V_3, which may be followed by one or more stages of amplification of the low-frequency current obtained as a result of the rectification. It will be observed that the load on V_1 is resistive, while that on V_2 is tuned. This is a particularly stable combination.



amplification at the radio frequency than
o-frequency amplification. V_3 is a diode
er, which serves to control the overall
cy amplification. The choke must be of
It prevents radio-frequency currents from
ming can be arranged as in Fig. 45.

It must be realised that the two amplifiers shown in Figs. 45 and 46 are to be regarded as typical only. Their various features may be combined in any desired way. Thus diode — or triode — rectification may be applied to any amplifier. This may be preceded by amplification at radio frequencies, by either screen-grid tetrodes, as in Fig. 45, or pentodes (Fig. 46), and followed by low-frequency amplification by triodes, or again by pentodes. The coupling in the low-frequency stages may be by transformers, as in Fig. 45, or resistance and capacitance (as that between V_1 and V_2 in Fig. 46). In every case it will be desirable to screen the various stages as in Fig. 45, although the actual form of the screens are capable of considerable variations. Thus H.F. pentodes are often provided with metal-coated bulbs, and therefore carry their own screens. Again, most transformers are also made with screens. The actual form of the screen can only be decided by reference to the actual components of the apparatus. It is often an advantage to use toroidal coils, since in this way stray magnetic coupling is avoided.

12. The Superheterodyne Receiver as Laboratory Detector. It is a comparatively simple matter to design a sensitive detector suitable for any particular investigation on the principles described in the preceding paragraphs. As a general-purpose instrument, however, the modern commercial radio receiver of the communication type is probably cheaper and better than one made under experimental conditions. Such an instrument will operate at all frequencies from say 100Kc/s to 30 Mc/s. It is well screened, mounted in a metal case, and the components are easily accessible. It is unnecessary to discuss the design of such receivers, but their more general features are worth notice. Fig. 47 shows a typical arrangement in outline. The first stage is an amplifier tuned to the working frequency. In the second stage, the received oscillation is combined with one produced by a local oscillator, which is tuned in such a manner that one of the resultant oscillations, formed by heterodyne action, is of a fixed supersonic frequency. The major part of the

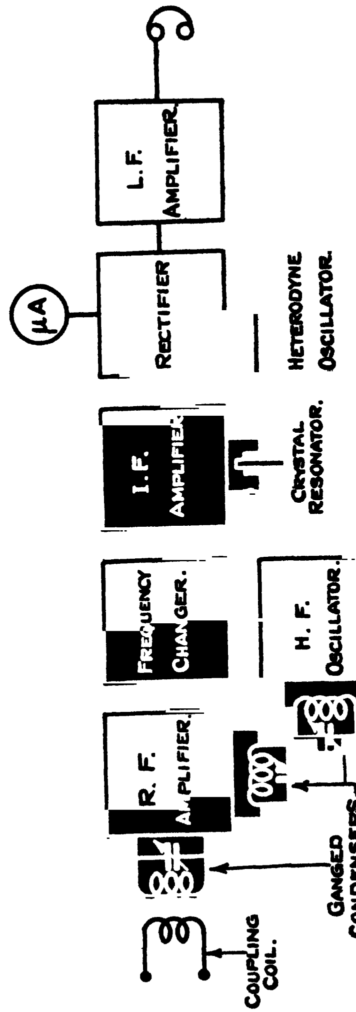


FIG. 47. Superheterodyne receiver for use as detector.

amplification occurs at this fixed frequency in the third stage. The frequency chosen is one at which amplification is most efficient, and since only one frequency has to be dealt with, it is possible to include a crystal resonator of very low decrement in this stage, and thus to obtain very high selectivity. A second local oscillator is provided at this stage for use when ordinary heterodyne detection is required. This is unnecessary if the received oscillation is modulated. A rectifier, audio-frequency amplifier, and telephones then follow as in Figs. 45 or 46. The rectified voltage is often used to provide automatic volume control in ordinary communication work, and a meter indicating the magnitude of the rectified current is often provided. Observation of this meter enables the instrument to be used as a sensitive voltmeter for resonance methods, while the telephones or a loud-speaker are used for null-detection. The complete equipment is necessarily somewhat complicated in construction, but it is very simple in operation, since only the initial stages require adjustment to correspond to any desired working frequency.

CHAPTER VI

STANDARDS OF CAPACITANCE

1. The Physical Nature of Capacitance. It follows from the considerations outlined in Chapter I, that capacitance as measured by alternating current methods is essentially a property of the displacement-current path between two conductors or equipotential surfaces. The capacitance of such a path may be regarded as the ratio of the displacement current to the rate of change of the voltage V, or as the ratio of the energy of the electric field to $\frac{1}{2}V^2$. A standard of capacitance (a capacitor, or condenser) is therefore an instrument in which the application of a definite voltage produces an electric field, the energy of which is accurately known. It follows that the ideal instrument consists of a portion of dielectric bounded and completely defined by two perfectly conducting metal plates.

It is in some respects unfortunate that capacitance is frequently regarded as a property of a conductor. From our point of view " the capacitance of a sphere " which is so often stated to be " equal to its radius " represents the electric field round the sphere rather than the sphere itself, which is only one boundary of the field. The value is only definite if the other boundary is also stated. It is, of course, usually assumed to be a second conductor or equipotential surface surrounding the first, and separated from it by an infinite distance in all directions. It is obvious, however, that such a system can never form part of a practical circuit, and the capacitance of a conductor in this sense is therefore of academic interest only. The practical value which approaches it most closely is the so-called " earth-capacitance " of the conductor, *i.e.*, the capacitance of the dielectric path between the conductor and all the surrounding earth-connected objects, which may include the walls of the room.

The point to be emphasised is that no capacitance is precisely defined unless it is related to two conductors. Further, unless one conductor completely surrounds the other, the field between them will be influenced by neighbouring objects. It follows that any standard of capacitance must consist of two or more insulated conductors, one of which completely surrounds the rest, as shown in Fig. 48, in which the conductor O completely surrounds 1, 2 and 3.

2. Component Capacitances. It will be obvious that there must be a displacement-current path between each pair of the conductors shown in Fig. 48 (*a*), and each of these paths can be represented by a definite capacitance. Thus, if C_{12} represents that between conductors 1 and 2, and so on, the system can be represented by the equivalent network of Fig. 48 (*b*). These capacitances are said to be the component capacitances of the system. The total displacement current entering any conductor when definite potentials are applied to such a system may obviously be found by adding together the current received through the various component capacitances linking that conductor to the others.

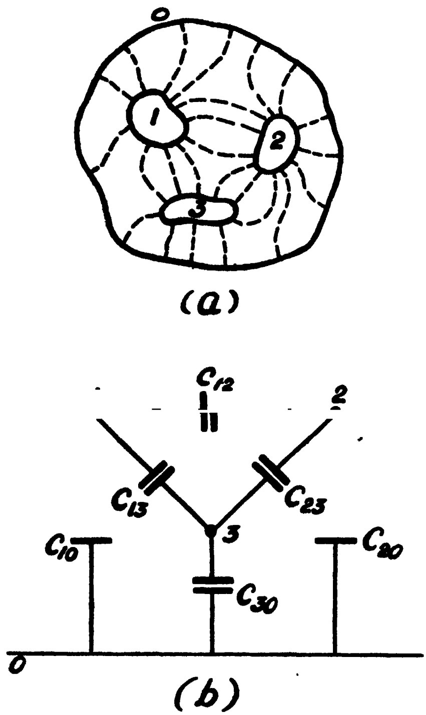


FIG. 48. Component capacitances of a system of four conductors.

As mentioned in Chapter III, the effect of interposing an electrostatic screen between two conductors is to reduce to zero their mutual component capacitance.

3. The Calculation of Capacitance. The capacitance corresponding to a given electric field depends on the size and shape of the field and the nature of the dielectric. The size and shape of the field may be defined either by means of the tubes of force, which are also tubes of flow of displacement current, or by means of the equipotential surfaces, *i.e.*, the terminal conductors. Values of capacitance are usually calculated from the dimensions of the conductors since these are the quantities directly observed, but it may be noted that the most general expression is obtained by considering the tubes of force. It takes the form *

$$C = \kappa \Sigma \frac{1}{\int_{l_1}^{l_2} \frac{dl}{\Delta a}} = \kappa G_0 \text{ say} \quad \ldots \ldots \quad (1)$$

where l denotes distance along an elementary tube of force, Δa denotes its cross section, 1 and 2 refer to the terminal equipotential surfaces (*cf.* Fig. 49), and Σ denotes the summation of the quantity following it for all the tubes of force of the field in question. Thus, the quantity denoted by G_0 is a purely geometrical quantity of unit dimension in length. For want of a better term it will be called the geometrical conductance.† The coefficient κ depends on the nature of the dielectric. It is obviously the capacitance of a field of unit geometrical conductance in the medium in question. It is analogous to conductivity σ, and might well be called the capacitivity § of the medium, and expressed in farads per unit length. It is obvious from these considerations that capacitance is not a geometrical quantity. It can only be calculated when the coefficient κ for the medium in question has been experimentally determined, and is on exactly the same footing as resistance which can be calculated when the resistivity is known. The experimental value of κ for a vacuum is $\kappa_0 = 0{\cdot}0885_5$ $\mu\mu F$ per cm. The value for any other dielectric is most conveniently specified by the ratio $\kappa/\kappa_0 = \epsilon_r$, which is called the dielectric constant or permittivity (relative) of the medium. Thus if lengths are measured in cm., capacitance C is given in $\mu\mu F$ by the formula

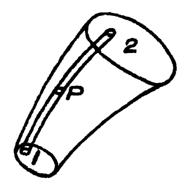


FIG. 49. Portion of tube of electric force terminated by equipotential surfaces 1 and 2.

$$C = 0{\cdot}0885_5\, \epsilon_r G_0 \quad \ldots \ldots \quad (2)$$

* RUSSELL, A. *Alternating Currents*, Vol. 1. Cambridge University Press. 1914.

† Since the conductivity of the same system is given by σG_0 where σ is the conductivity of the medium.

§ Capacitivity, κ, is not the same as permittivity (absolute), ϵ, but $\epsilon = 4\pi\kappa$.

Values of G_0 corresponding to various arrangements of terminal conductors have been calculated. The most useful of the formulæ for capacitance so obtained are given below. All dimensions are in cm., and capacitances in $\mu\mu F$.

(*a*) *Parallel Plates*, of area A, and distance apart l

$$C = 0{\cdot}0885_5\, \epsilon_r A/l \quad . \quad . \quad . \quad . \quad (3)$$

(*b*) *Concentric Spheres* of radii r_1 and r_2

$$C = 0{\cdot}0885_5\, \epsilon_r \,.\, 4\pi r_1 r_2/(r_2 - r_1) \quad . \quad . \quad . \quad . \quad (4)$$

(*c*) *Coaxial Cylinders* of radii r_1 and r_2; length x ($>> r_2$)

$$C = 0{\cdot}0885_5\, \epsilon_r 2\pi x/\log_e(r_2/r_1) \quad . \quad . \quad . \quad . \quad (5)$$

(*d*) *Parallel Discs* of radius r, thickness t, and distance apart b. If $b << r$ and $t << r$

$$C = 0{\cdot}0885_5\, \epsilon_r \left[\frac{\pi r^2}{b} + r\left\{\log_e \frac{16\pi(b+t)r}{b^2} + \frac{t}{b}\log_e \frac{b+t}{t} - 3\right\}\right] \text{ (Kirchhoff's formula)} \quad . \quad . \quad . \quad (6)$$

(*e*) *Parallel Cylinders*, *e.g.*, two wires, of radius r, and length l, distance between axes d. When $l >> r$ and $l >> d$

$$C = \frac{0{\cdot}0885_5\, \epsilon_r \pi l}{\log_e[\{d + (d^2 - 4r^2)^{\frac{1}{2}}\}/2r]} \text{ (Russell)* } \quad . \quad . \quad . \quad (7)$$

(*f*) *Wire Parallel to Plate*. For a length l of a cylinder of radius r, and a plate parallel to and at a distance d from its axis

$$C = 0{\cdot}0885_5\, \epsilon_r \frac{2\pi l}{\log_e\left(\dfrac{d + \sqrt{d^2 - r^2}}{r}\right)} \quad . \quad . \quad . \quad (8)$$

The formulæ (6), (7) and (8) above will be found useful for estimating the values of capacitance between parallel wires, wires and instrument cases, terminals, etc. Such values are frequently required when estimating errors in practical measurements due to stray capacitances, etc.

4. The Guard Ring. The above formula for parallel plates is based on the assumption of a uniform electric field between the plates, while the formula for coaxial cylinders applies so long as the field is purely radial. In practice, the field near the edges of plates or cylinders never satisfies the condition assumed,

* RUSSELL A. *Alternating Currents*, Vol. 1.

and the formulæ therefore only apply accurately to the central portions of the plates or cylinders. Sometimes a correction is applied to the simple formula to allow for the distortion of the field at the edges—the so-called "edge-correction" or "fringing correction." Kirchhoff's formula given above includes the edge-correction for two parallel discs. Alternatively, the measurements may be confined to the central portion of the plates or cylinders by the use of a guard-electrode.

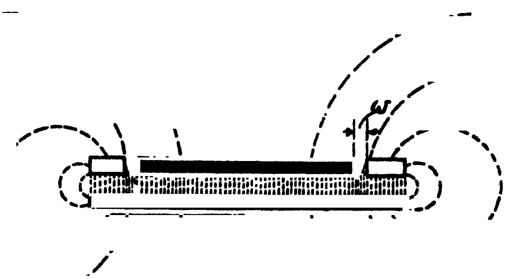


FIG. 50. Section of disc electrodes with guard-ring, showing lines of electric force (the broken lines).

The principle may be understood from Fig. 50. The central portion of the upper plate is isolated from the remainder, which is usually termed the guard-ring. Obviously, if the central electrode and guard-ring are at the same potential, and if the gap between the two is very small, the field between electrode and lower plate will be very nearly uniform and the component capacitance between the two will be calculable from equation (3), to a much closer approximation than could be obtained without the guard-ring. Fig. 51 shows in more detail the lines of force between a pair of electrodes and a guard-ring. It will be evident from this diagram that there must be a slight fringing effect even when a guard-ring is used. The lines of force from the central electrode spread outwards into the gap between it and the guard-ring, and from considerations of symmetry we see that they will extend about half-way across this gap as shown in Fig. 51. Thus if A is the area of the central electrode and δA the area of the

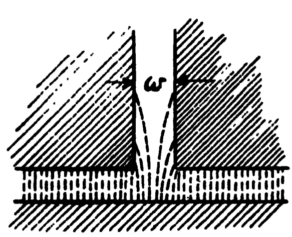


FIG. 51. Enlarged drawing of gap between central disc and guard-ring of Fig. 50.

gap between electrode and guard-ring, the area of cross section of the electric field between central electrode and opposite electrode is approximately $A + \frac{1}{2}\delta A$, and the geometrical conductance is approximately $(A + \frac{1}{2}\delta A)/l$. We may say that the effective area of the central electrode is increased by an amount corresponding to an additional strip of width $w/2$ applied all round its edge, w being the width of gap between electrode and guard-ring. The conditions assumed in the derivation of this correction are that the width of gap w is small compared with the dimensions of the central electrode; the thickness of central electrode and guard-ring is large; and the width of the guard-ring is large compared with the distance l between the two electrodes. Rosa * and Dorsey have shown that a more accurate formula for the width δ of the " additional strip " under these conditions is

$$\delta = \frac{w}{2} - \frac{w}{\pi}\sin^{-1}\frac{w}{\sqrt{4l^2 + w^2}} + \frac{l}{\pi}\log_e\frac{4l^2 + w^2}{4l^2} \quad . \quad . \quad (9)$$

Provided l is small compared with the dimensions of the electrodes this formula may be applied as an approximation to condensers consisting of coaxial cylinders as well as to flat plates. In the application of the guard-ring principle to cylindrical condensers, the central portion of one of the cylinders of length x is isolated from the two end portions by narrow gaps of width w cut in planes at right angles to the common axis. Evidently each gap will increase the effective length of the central portion by an amount δ which may be calculated from the above formula, l now being the distance between the inner and outer cylinders measured radially. Thus the geometrical conductance of a cylindrical condenser with guard cylinders may be calculated from equation (5) by taking the effective length as $x + 2\delta$, δ being calculated from (9). For the case of flat disc electrodes with a guard-ring we may use equation (3), taking the radius of the central electrode as $r + \delta$, where r is the actual radius and δ is calculated as above. For this case also Maxwell has given the simpler formula

$$G_0 = \frac{\pi r^2}{l} + \frac{\pi r w}{l + 0{\cdot}22w}\left(1 + \frac{w}{2r}\right) \quad . \quad . \quad . \quad . \quad (10)$$

* Rosa, E. B., and Dorsey, N. E. *Bulletin Bureau Standards*, 1907, Vol. 3, p. 517.

from which we see that the fringing correction is given approximately by

$$\frac{\Delta C}{C} \simeq \frac{w}{r}\left(\frac{l}{l + 0{\cdot}22w}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

As in all these formulæ for guard-rings, the electrodes are assumed to be thick, so that when high accuracy is required thick metal plates (or tubes for the cylindrical case) should be used, and the width of the gap w should be kept very small.

5. **Fixed Air Condensers.** Standards of capacitance which are required to have the greatest possible permanence of value

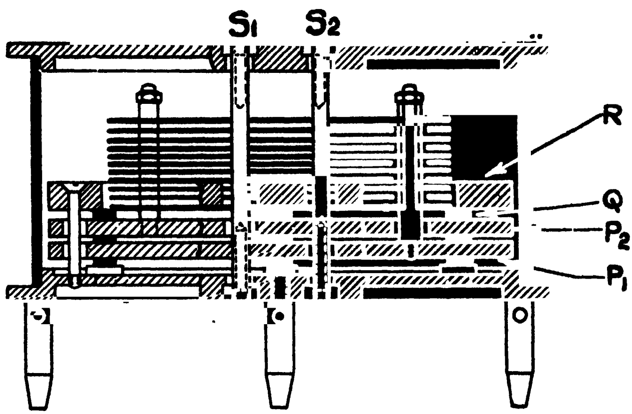


FIG. 52. Fixed air condenser (after Giebe and Zickner).

under all conditions usually take the form of air condensers of fixed value. We have seen that such an instrument must comprise two conductors, insulated from one another (by air); and in order that the value may be definite, one conductor should completely surround the other, or alternatively the two may be entirely surrounded by a third conductor, in which case the system will possess three values of capacitance. The conductors usually take the form of flat plates parallel to one another, or concentric cylinders, and when capacitances greater than, say, 100 $\mu\mu F$ are required, it is necessary to use several plates for each conductor, and to arrange them so that they are separated by a very small distance, the plates forming the two conductors being interleaved. Many types of mechanical construction have been employed for such instruments, but we shall consider in detail one design only, which may be regarded as typical of the best modern instruments. This design is that developed at the Physikalisch-Technische Reichsanstalt,* Berlin, where the properties of fixed air condensers have been studied in great detail. The arrangement may be understood by reference to Fig. 52. We have essentially two conductors insulated from each other, and from the metal shield which completely surrounds them and which forms the case of the instrument. Each conductor consists of a bank of flat metal plates of circular form. Each bank is supported on a massive base plate P_1 or P_2 by means of three brass pillars, the plates being maintained parallel by means of suitable spacing washers, which are slipped over the pillars, and the whole bank being made rigid by clamping nuts at the upper ends of the pillars. The pillars of one system pass through clearing holes in the plates forming the other, so that the two systems are interleaved. Only one pillar of each system is shown in the diagram, but it will be obvious that in order to obtain the maximum rigidity of the whole structure the pillars should be symmetrically distributed in the plan of the instrument, each pillar of one system falling midway between two of the other system. The base plates P_1 and P_2 of the two systems are separated by three small cylindrical pieces Q of fused quartz, which act as distance-pieces and insulators. The lower base plate P_1 is separated from the bottom of the instrument case or shield by three similar quartz pieces, while a third set of similar insulators separates the upper base-plate P_1 from a massive clamping-ring R, which is bolted to the base of the instrument by a number of uniformly spaced bolts, one of which is shown. These bolts pass through clearing holes in P_1 and P_2. Thus the clamping-ring and the case of the instrument form one conductor which acts as a shield for two other conductors—the two plate systems. The plate systems are therefore insulated from one another by three pieces of fused quartz, and each is insulated from the shield by three similar pieces of fused quartz. The whole of the quartz is in compression, so that it is very strong mechanically. The structure is designed for permanence of form and therefore of capacitance.

* GIEBE, E. and ZICKNER, G. *Zeits. f. Instrum.*, 1933 Vol. 53, p. 1.

The instrument is not provided with screw terminals but connection is made to the two plate systems by means of two brass pillars S_1 and S_2, which are screwed into the base-plates P_1 and P_2 respectively, and which pass through clearing holes in the clamping-ring and the top and bottom of the case, terminating in the form of sockets at each end. Thus we have access to the plate system by means of one pair of sockets flush with the top of the instrument, and also through a second pair flush with the bottom surface. The chief advantage of this arrangement is that a number of such condensers may be connected in parallel simply by piling them one on the top of another, double-ended plugs being inserted in the sockets, so as to make the necessary connections. These plugs are completely enclosed by the two condensers so that they do not affect the capacitance of the combination, which is simply the sum of the capacitances of the various condensers. The bottom condenser rests upon a special base-plate provided with screw terminals at the side, connected to a pair of the standard sockets on its upper face. This base-plate is first connected in circuit, and then any desired capacitance is added to it by placing the appropriate condenser-sections in position. Condensers of capacitance from 100 $\mu\mu$F. to 0·1 μF. are made to this general design, the number of plates, their diameter and distance apart being adjusted to give the required capacitance.

By inserting into either of the sockets S_1 or S_2 a special plug terminating in a metal disc which makes contact with the case, it is possible to connect either bank of plates to the case, thereby converting the condenser from a three-conductor instrument with three component-capacitances to a two-conductor instrument with only one value of capacitance.

6. **Dielectric Imperfections.*** It was shown in Chapter I that there is no net loss of power when an alternating voltage is applied to an ideal condenser, *i.e.*, when an alternating field is established in a perfect dielectric. Such a dielectric therefore would possess zero conductivity and would allow displacement currents only to pass through it. All known material bodies, however, possess a finite conductivity, usually of an electrolytic or ionic character, and the conductivity of their surface layers is

* HARTSHORN L. *J. Inst. Elec. Eng.*, 1926, Vol. 64, p. 1152.

apt to be greater than that of the interior owing to the presence of adsorbed films of water, or other contaminating material. We must therefore take into account conduction currents through and over the solid insulators of condensers as well as the displacement currents. The conduction currents may be regarded as a drift of ions in the electric field, while the displacement current corresponds to the elastic displacement of the elementary positive and negative charges constituting the molecules of the body. In addition, the body may contain ions or unsymmetrically charged molecules, not capable either of drifting, or of elastic displacement, but capable of displacement against the action of a frictional or viscous force. The motion of such charges constitutes an absorption current. It gives rise to the dissipation of power within the material, to an extent which increases with rise of frequency, and which is therefore usually of predominating importance at radio frequencies. Whatever its cause, the loss of power may be represented by an equivalent conductivity, or loss tangent, in the manner already discussed, and it follows that the loss tangent (tan δ) of any material is a valuable guide to its quality as a dielectric for the construction of condensers. Typical values are given in Table II, which is a collection of values representing approximately the more important properties of insulating materials under ordinary working conditions.

7. **Variable Air Condensers.** In its mechanical construction, a variable air condenser consists essentially of two conductors whose relative positions can be adjusted in such a way that the capacitance of the dielectric path (the air-gap) between them may be varied between certain limits. As in the case of the fixed condensers already described, the two conductors usually take the form of parallel flat plates, and when large capacitances are required, each conductor consists of a bank of several such plates assembled in some suitable way into the form of a rigid structure, the plates of the two conductors being interleaved, so that the dielectric path between them is short but of large total cross-section—the air-gaps between adjacent plates are in parallel. The variation of capacitance is usually made possible by mounting one bank of plates on a spindle, which is capable of rotation, and as the spindle is turned these plates move in their own plane in or out of the gaps between the plates of the other bank, which is

TABLE II.*—*Properties of Insulating Materials*

Material	Dielectric constant	tan δ at 10^n c/s.	n	Resistivity		Electric strength kv./mm.	Softening (or max.) temp. °C.	Mechanical Strength	
				Vol. Ω cm.	Surface Ω			Cross-breaking kg./cm.^{2}	Impact kg./cm.
Ebonite (pure)	3·0	0·006	3	10^{16}	10^{18} new	150	65	1,100	50
		0·009	6		10^{9} old				
Ebonite (with heavy mineral loading).	4·5	0·012	3	10^{14}	—	70	85	750	12
		0·03	6						
Ebonite (silica loaded)	3·5	0·006	3	—	—	—	80	700	10
		0·007	6						
Keramot	3·6	0·010	6	—	10^{13}	—	—	—	—
Mouldings:									
+P.f. resin (wood-filled)	5	0·04	—	10^{11}	10^{12}	10	110	800	2·5
†U.f. resin (paper-filled)	6	0·03	—	10^{12}	10^{12}	10	110	850	3·0
Laminated boards:									
+P.f. resin—paper	5	0·03	6	10^{12}	10^{12}	30	110	1,500	9
P.f. resin—fabric	—	0·05	—	10^{10}	10^{9}	12	110	1,500	80
Mycalex	6	0·006	3						
		0·003	6	10^{13}	10^{11}	14	400	1,500	6
Pure resins:—									
‡Polystyrene	2·5	0·0003	6	10^{17}	10^{19}	30	70	600	2
Cellulose acetate		0·03	3						
	6	0·06	6	10^{13}	—	30	60	550	5
Diakon	2·8	0·03	3	10^{16}	10^{14}	20	60	600	3
		0·02	6						
Amber	2·8	0·002	6	10^{17}	10^{14}	—	—	—	—
Mica	7	0·0002	—	10^{17}	10^{11}	50	—	—	—
Fused quartz	3·8	0·0002	—	10^{17}	10^{13}	20	600	—	—
Porcelain	5·5	0·008	6	—	—	—	—	—	—
Calan	6·5	0·0004	7	—	—	40	1,400	1,000	3
Frequentite	6	0·0008	7	—	—	50	1,400	1,200	4
Tempa S.	16	0·0005	3	—	—	—	—	—	—
		0·0001	6						
Kerafar	80	0·03	3	—	—	—	—	—	—
		0·001	6						
Paraffin wax	2·2	0·0001	6	10^{17}	10^{16}	20	50	—	—
Petroleum jelly	2·2	0·0001	2	—	—	20	—	—	—
Transformer oil	2·2	0·0001	3	—	—	—	—	—	—
Seekay wax R 93 C	5	0·001	2	10^{14}	—	—	93	—	—
(Chlorinated naphthalene).		0·006	6						
Permitol	5	0·01	3	—	—	—	—	—	—
(Chlorinated diphenyl)									

+ Phenol-formaldehyde, *e.g.*, Bakelite, etc. † Urea-formaldehyde. ‡ Distrene, Victron, Trolitul.

fixed. A typical construction is shown in Fig. 53. The two sets of plates are parallel so that the length of the dielectric path of the condenser remains constant, but its cross section, which is approximately equal to the area of the overlapping portion of the two plate-systems, multiplied by the number of air-gaps in parallel, may be varied over wide limits. The angular position of the moving conductor is indicated by means of a pointer rigidly attached to the spindle and moving over a circular scale, which may be either uniformly divided or calibrated to read micro-microfarads directly. Evidently the capacitance will be

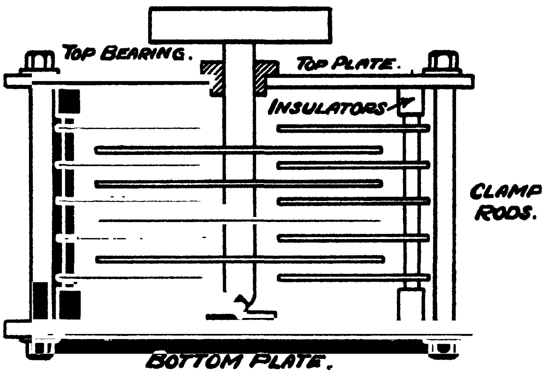


FIG. 53. Variable air condenser (schematic).

very nearly proportional to the overlapping area of the two plate systems, and by suitably shaping the plates, this area and therefore the capacitance may be made to vary in almost any desired way with the angular setting of the spindle. When the plates are semicircular, the axis of the spindle passing through the line of centres, the law of variation of capacitance with angular reading is linear,

$$C = a + b\theta.$$

This is perhaps the most generally useful type of scale for measurement purposes, but for some measurements the plates are shaped so as to give scales conforming to other laws.

8. **The Design of Variable Air Condensers.** Variable air condensers of the flat plate type differ considerably in the details of their mechanical design. Omitting all such details, the arrange-

ment shown diagrammatically in Fig. 53 may be regarded as typical. We shall now consider the qualities that are desirable in a precision condenser of this type with special reference to Fig. 53. Similar considerations may readily be applied to any other design.

The required conditions are as follows :—

(*a*) Each plate system must form a rigid conductor. In Fig. 53 the fixed plates are separated by circumferential spacing rings, and are held together by bolts passing through both plates and spacing rings, the whole being clamped by nuts. Such a system approximates to a single rigid conductor, but it cannot be regarded as perfect. It is important that the electrical contact between all the parts shall be good—both plates and spacing rings must be carefully cleaned before clamping up, and the plates should be accurately constructed and assembled so that clamping does not set up mechanical stresses in the system that are not likely to remain permanent. Plate systems cut from a solid metal block or even die-cast may be better, but are not practicable when many plates are required. It must be remembered that unequal thermal expansion of the components is likely to set up stresses in the plates, possibly causing distortion and changes of capacitance. It is therefore advisable to use the same metal throughout. The moving plate system is of similar construction except that instead of a series of bolts spaced round the circumference, there is a single large bolt—the spindle—in the centre, the plates being clamped together by large nuts screwing on the spindle. Again, good electrical contact and freedom from mechanical stress is important. Sometimes the plates are soldered together, and the improvement in electrical contact so obtained may be important at very high frequencies.

(*b*) The bearing must be true. The accuracy of most condensers is probably determined by the quality of the bearing. In order that a given setting of the instrument may correspond to a definite capacitance it is obvious that there must be no wear, and not the slightest trace of wobble in the spindle. Also in order that settings may be made with great precision the motion should be very smooth. Many condensers are provided with a reduction-gear or slow-motion device for making fine adjustments. This, of course, always gives a very open scale, but the accuracy of the condenser is not correspondingly increased unless the main bearing is of the necessary high quality.

(*c*) The dielectric path of the condenser should be completely free from power loss. Dry air at ordinary low voltages may be regarded as sensibly free from power loss. It is essential, however, for mechanical reasons to use a certain amount of solid insulating material in the construction of the condenser. Thus in Fig. 53 the fixed plate system is supported by six pillars of fused quartz. Since, as we have seen, no solid material is completely free from power loss, it is important that the solid material actually used shall introduce the minimum of power loss. We have seen that the power dissipation in any system is given by

$$W = V^2 C\omega \tan \delta,$$

C being the effective capacitance of the system and $\tan \delta$ the loss tangent. We may apply this equation to the solid insulators of a condenser, and it follows that in order to ensure that the power dissipation in the insulators has a minimum value, the product $C \tan \delta$ for those insulators must be as small as possible, *i.e.*, $G_0 \epsilon \tan \delta$ must be as small as possible where G_0 is the geometrical conductance of the insulators, and ϵ the dielectric constant of the material. Now G_0 depends solely on the linear dimensions of the insulators, and ϵ and $\tan \delta$ depend only on the nature of the material. It follows that in order to obtain the minimum power dissipation we must choose a material such that the product $\epsilon \tan \delta$ has the smallest possible value, and then make with this material insulators having the smallest possible value of G_0, which means a large ratio of length to cross section. In Fig. 53 the solid insulators take the form of hollow cylinders of fused quartz. It will be observed from Table II that this material has an extremely low value of $\epsilon \tan \delta$ at all frequencies, while the hollow cylinder has very small ratio of cross section to length, and at the same time provides great mechanical stability. It should be noted that it is not sufficient merely to reduce the amount of solid insulating material to a minimum. If, for example, the pillars of Fig. 53 were shortened to one-tenth of their length the power dissipation would be increased about tenfold, since G_0 would be increased by this amount. The power dissipation in the arrangement of Fig. 53 could be halved by omitting the upper three pillars, and sometimes arrangements of this kind are used. However, the symmetrical arrangement of Fig. 53 has several advantages. It is more robust, and in spite of unequal thermal expansion of the insulator and metal, the symmetry is unaltered by changing temperatures and the temperature coefficient of such condensers is usually very small, of the same order as the coefficient of linear expansion of the metal. Unequal thermal expansion of insulators and metal is apt to produce large temperature coefficients of capacitance in unsymmetrically constructed condensers. Sometimes the insulators take the form of loaded cantilevers. A long cantilever, while giving a low value of G_0, is usually less satisfactory than the arrangement of Fig. 53 from the point of view of thermal and mechanical stability.

(*d*) The connections between the plate systems and their terminals should possess the smallest possible resistance and inductance. The instrument of Fig. 53 is connected to any circuit by means of the terminals T_1 and T_2 (not shown), and in practice we are concerned with the total impedance or admittance of the current path from T_1 through the condenser to T_2. The terminal T_1 is connected to the fixed plate system by a short thick metal rod, while the terminal T_2 is essentially the metal case of the instrument, which is connected to the moving plate system through the bearing. It has already been shown that this bearing must be designed from the point of view of providing true turning, *i.e.*, one degree of freedom only. Good electrical contact between the rubbing surfaces may or may not be obtained, but usually the electrical resistance of the bearing depends appreciably on its state of lubrication, and is apt to vary in use. Accordingly it is usual to provide an additional electrical connection between the moving plates and the case of the instrument, *i.e.*, the "earthy" terminal. This connection must, of course, be flexible, and it often consists of a flat spiral of phosphor-bronze strip attached at one end to the spindle and at the other to the case of the instrument. This arrangement is free from the uncertainties of a rubbing contact, and is of low resistance, but it introduces a variable inductance, which may be troublesome at the highest frequencies. Sometimes the connection is made by some form of brush contact bearing on the spindle. This arrangement is of constant inductance and although the contact is a rubbing one, it may by suitable design be made much more certain than that of the bearing. Whatever form the connections to the terminals may take they will possess a finite resistance and inductance and therefore the instrument will only behave simply as a standard of capacitance if this resistance and inductance are made negligibly small.

9. **The Series-Gap Condenser.** The use of a flexible connector or rubbing contact, both of which are apt to be very troublesome at very high frequencies, may be avoided by the use of a condenser with three plate-systems, arranged as in Fig. 54, two fixed and one movable by means of a spindle. The terminals of the instrument are connected to the two fixed plate systems, while the moving system is insulated from both and so placed that on rotating the spindle some of the plates pass between those of one fixed plate system, while the remainder pass between those of the other fixed plate systems. Thus each of the fixed plate systems forms with the moving system a variable condenser, and these two condensers are connected in series between the two terminals of the instrument. Evidently the capacitance of such a condenser must be considerably smaller than that which

would be given by the same number of plates arranged as a condenser of the ordinary type (parallel gaps) and therefore the construction is more expensive except when only small capacitances are required. The law of the scale may, as in the previous case, be varied at will by using plates of a suitable shape, for it will be clear that the law is the same as that of the two constituent condensers. The potential of the moving system of such a condenser will be mid-way between those of the two terminals and thus the arrangement is peculiarly suitable for use in circuits which are symmetrical or balanced with respect to

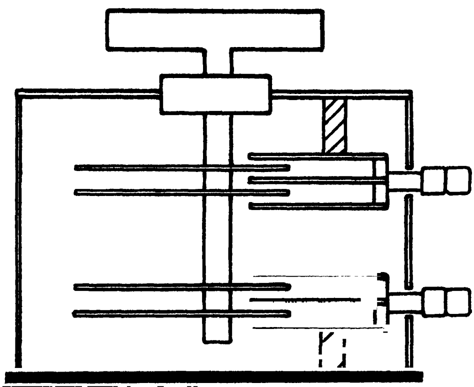


FIG. 54. Series-gap condenser (schematic).

earth. The moving system may then be in metallic connection with the case of the instrument and earth. If the condenser is required for use in a circuit in which one terminal is at earth potential, then since the moving system is not at earth potential it must be insulated from the handle and the case of the instrument.

10. Micrometer Condensers. For certain measurements a condenser of very low range is required. Condensers of the types previously discussed are readily made for all ranges of capacitance between, say, 50–2,000 $\mu\mu$F. and 10–50 $\mu\mu$F. The scale usually covers 180° and with good construction the capacitance reading may be reproduced and read to about 1 part in 5,000, so that with the smallest range changes of capacitance may be measured to the nearest 0·01 $\mu\mu$F. with instruments of this type. It is, however, sometimes necessary to measure at least to the nearest 0·001 $\mu\mu$F., and for this purpose a condenser of the cylindrical type is preferable. A simple design * which has been found very satisfactory is shown in Fig. 55. One electrode of the condenser consists of a hollow metal cylinder. The other consists of a solid cylinder coaxial with the first and capable of translatory motion along the common axis. Adjustments of the position of the moving cylinder are made by means of a micrometer screw. In the arrangement shown the stem of a micrometer of ordinary size forms the moving cylinder, and any range up to about 10 $\mu\mu$F. may be obtained in this way. The micrometer is in metallic connection with an outer metal cylinder

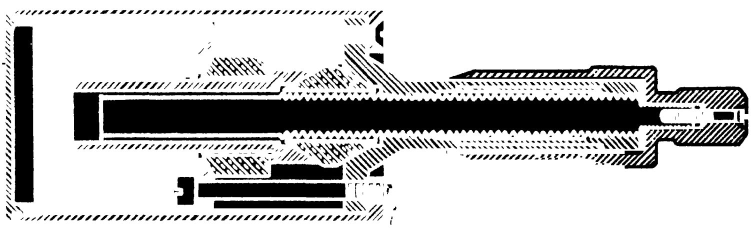


FIG. 55. Micrometer condenser (Ward and Pratt). There are three clamping bolts, only one of which is shown.

with acts as a shield for the instrument, the inner cylinder being insulated from it by specially shaped pieces of fused quartz. The scale of such a condenser will obviously be linear over the central portion where variations in the " end-effects " are negligible.

11. Effective Resistance, Power Factor, and Self-Inductance of Condensers. The current path between the terminals of an air condenser may be divided into three parts : (1) a conduction current in the metal electrodes or plate-systems, (2) a displacement current across the air-gaps, (3) a current, including displacement, absorption, and conduction components in the solid insulating supports. It follows that the electrical behaviour of the condenser is not merely that of a capacitance of definite value. The displacement current across the air-gap may certainly be represented without sensible error by a definite capacitance, say C_0 in Fig. 56, but in series with this we must

* WARD, W. H. and PRATT, E. J. *J. Sci. Instrum.*, 1939, Vol. 16, p. 192.

put a small but finite resistance r and self inductance l to represent the conduction current in the metal plates and pillars, while in parallel we must put an additional small condenser of capacitance C_i and conductance g to represent the current in the solid insulators. Thus the complete condenser may be expected to behave like the network of Fig. 56. Let C be the total capacitance of the dielectric path, including air and solid insulators, *i.e.*,

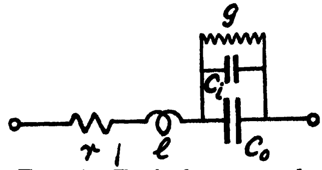


FIG. 56. Equivalent network of condenser.

$$C = C_0 + C_i.$$

The impedance operator of the network may be written

$$r + jl\omega + \frac{1}{g + jC\omega} = r + jl\omega + \frac{g - jC\omega}{g^2 + C^2\omega^2}$$
$$= r_s - \frac{j}{C_s\omega}$$

where r_s is the equivalent series resistance of the condenser, and C_s is its equivalent series capacitance. Now $g/C\omega$ for any well-designed condenser is small compared with unity. It follows that g^2 is negligible compared with $C^2\omega^2$, and, therefore, equating real and imaginary components of the above equations we have

$$r_s = r + \frac{g}{C^2\omega^2}$$

and $$C_s = \frac{C}{1 - lC\omega^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The loss tangent of the condenser is given by

$$\tan \delta = r_sC_s\omega = \frac{rC\omega}{(1 - lC\omega^2)} + \frac{g}{C\omega(1 - lC\omega^2)}.$$

The conductance g may be represented by the equation

$$g = g_s + C_i\omega \tan \delta_i$$

where g_s represents the surface conductance of the solid insulators and $\tan \delta_i$ the loss tangent of the material of which they are composed. Substituting this value of g in the above equation we have, for frequencies at which $lC\omega^2$ is small compared with unity

$$r_s = r + \frac{C_i \tan \delta_i}{C^2\omega} + \frac{g_s}{C^2\omega^2} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\tan \delta = rC\omega + \frac{C_i}{C} \tan \delta_i + \frac{g_s}{C\omega} \quad . \quad . \quad . \quad . \quad (14)$$

Equations (12) to (14) show the effect of the imperfections of the condenser on its electrical behaviour. Thus the effect of the self-inductance is to cause the effective capacitance to increase with rise of frequency, in accordance with equation (12), while the resistance of the plate systems and leakage and power loss in the insulators give the condenser a finite power factor and effective resistance in accordance with (13) and (14). When using these equations it must be remembered that l, r, C_i, $\tan \delta_i$ and g_s are not necessarily constant with varying frequency ($\omega/2\pi$) or capacitance C. Thus r and l may vary with frequency on account of eddy currents (skin-effect), while $\tan \delta_i$ may vary with frequency as shown by Table II, and g_s also often increases with frequency, since it represents the conductance of what is probably a discontinuous film. However, g_s, C_i and $\tan \delta_i$ in a variable condenser are independent of C, *i.e.*, of the angular settings of the plate systems, although r and l may vary with this setting since it alters the distribution of current in the plates. Further, r and l are in most condensers largely due to the comparatively thin supporting pillars and leads to terminals, and not to the plates themselves, which are of large cross section and surface area. It therefore follows that these quantities may often be regarded as constants of the instrument. A standard variable air condenser may therefore usually be said to have a definite self-inductance l, and its loss tangent is often found to obey an equation of the form

$$\tan \delta = rC\omega + \frac{a + b\omega}{C} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

i.e., its effective resistance is given by

$$r_s = r + \frac{a}{C^2\omega} + \frac{b}{C^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

A comparison of equations (13) and (16) suggests that when equation (16) is found to hold, r must be the value of the resistance of the plate systems, and the surface conductance g_s is proportional to the square of the frequency. From what has already been said, however, it will be clear that the deduction is by no means certain. For a laboratory standard air condenser of good quality with solid insulators of fused quartz, r is likely to be of the order of 0·01 ohm, while $\tan \delta$ at a frequency of about 1 Mc/sec. is likely to be about $0{\cdot}5 \times 10^{-4}$. The value of l is usually of the order of 0·03 μH., so that the term $lC\omega^2$ in equation (12) has the value 0·3 for a condenser of low-frequency value 100 $\mu\mu$F., at a frequency of 50 Mc/sec. Thus the effective value of capacitance at such a frequency would be about 30 per cent. greater than the nominal value.

We have considered only the equivalent series capacitance of our condenser. It may readily be shown by a consideration of the admittance operator corresponding to Fig. 56 that the equivalent parallel capacitance C_p is to a close approximation given by

$$C_p = \frac{C(1 - lC\omega^2)}{(1 - lC\omega^2)^2 + r^2C^2\omega^2} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and since $r^2C^2\omega^2$ is almost always negligible compared with unity, this expression is practically identical with that for C_s (equation (12)).

12. **The Permanence of Condensers.** It is important to remember that no condenser can be trusted to retain its calibration indefinitely and that the changes of capacitance are usually greater the greater the variations of temperature to which the condenser is subjected. The best mica condensers may vary by no more than 0·01 per cent. in a year under ordinary laboratory conditions, and may show no variations greater than $\pm$ 0·01 per cent. for several years, but many mica condensers will change by several times this amount, especially the smaller values of capacitance, less than 0·01 μF., say. It is sometimes assumed that fixed air condensers are perfectly stable, but this is by no means the case, especially for condensers of large capacitance made by clamping up banks of flat plates. The stresses in the system are apt to vary owing to temperature fluctuations and other causes, and the consequent bending of the plates may cause considerable changes of capacitance, especially when the plates are of large diameter, and the air-gap is small, conditions which necessarily

hold in condensers of large capacitances. Most clamped condensers increase in capacitance in the first year or two after their construction. For condensers of capacitance 0·01 μF. and upwards the change may amount to as much as 0·2 or 0·3 per cent., but for condensers of the order of 1,000 $\mu\mu$F. the increase may only amount to 2 or 3 parts in 10^4. After this initial period laboratory standard air condensers of the fixed type may remain constant to $\pm$ 2 parts in 10^4 and sometimes even less than this. The best variable air condensers show a constancy of the same order. A detailed discussion of the effects of changes of temperature on condensers is given by H. A. Thomas ("The Theory and Design of Valve Oscillators," Chapter VII. Chapman and Hall, London, 1939).

CHAPTER VII

RESISTORS

1. **General Considerations.** The ideal standard resistor would consist of a conductor which under all working conditions possessed a constant resistance, a phase angle of zero, and a power factor of unity. Since the existence of either a magnetic field or an electric field necessarily involves the storage of energy, and therefore a power factor less than unity, such a resistor would possess neither a magnetic field nor an electric field and therefore it cannot exist in practice. The properties of all real resistors depend in part on their magnetic fields, or their inductances, and on their electric fields, that is to say their capacitances, or the displacement currents associated with them. These quantities are all made as small as possible, but they are seldom completely negligible at radio frequencies. Moreover, since capacitances are only definite for a screened instrument, standard resistors should be completely enclosed in a conducting screen as explained in Chapter III. Figs. 21 and 22 may be regarded as representing such standards. Screens may only be dispensed with when the effects of capacitance can be proved to be negligible.

2. **Residual Inductance and Phase Angle.** Consider the simple case of a resistor for which the effects of earth-capacitance are negligible. We have seen that the resistor must possess a certain amount of inductance and capacitance as well as resistance. Moreover, the inductance and resistance will obviously be associated with the same current, viz., that in the actual conductor, and may therefore be represented by L and R, say, in series, while the capacitance must be associated with the potential drop on the resistor, and is therefore most simply represented by a capacitance C shunting its terminals. Thus the simplest equivalent network for a resistor is (10) in Table I. It will be seen from this table that the general expressions for the effective series resistance, inductance and phase angle of such a resistor

are somewhat complicated, but it is to be noted that in practice the quantities $\alpha = L\omega/R$, and $\beta = RC\omega$ are small compared with unity for any resistor, and their product $\alpha\beta = LC\omega^2$ is a small quantity of the second order and usually negligible. We therefore have the approximate relations :

$$R_s \simeq R(1 - R^2C^2\omega^2 + 2LC\omega^2) \quad . \quad . \quad . \quad . \quad (1)$$

$$L_s \simeq L(1 - R^2C^2\omega^2 + LC\omega^2) - CR^2 \quad . \quad . \quad (2)$$

and

$$\tan\phi \simeq \frac{L\omega}{R} - RC\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is to be noted that in equations (2) and (3) the terms representing the main effects of inductance and capacitance are of opposite sign, and that the phase angle may be either positive or negative, according as one effect or the other predominates. If the relation $L = CR^2$ is satisfied the phase angle has the value zero for all frequencies for which $LC\omega^2$ is negligible compared with unity. The effect of self-capacitance increases rapidly with the value of the resistance, and is therefore of special importance in resistors of high value. Such resistors usually have an effective residual inductance L_s of negative value. Both the residual inductance L_s, and effective resistance R_s, will be independent of frequency only so long as the effect of capacitance represented by the term $R^2C^2\omega^2$ is negligibly small.

3. The Effect of Distributed Self-Capacitance.* The above simple treatment of the inductance and capacitance of a resistor is sufficient for many purposes, but for work at very high frequencies it is necessary to consider in more detail the effect of the distribution of capacitance throughout a resistor. Consider first a conducting path AB (Fig. 57) of d.c. resistance R_0 and zero inductance. Let the resistance per unit length be constant. Suppose, for example, we have a straight wire or rod of negligible inductance. The lines of electric force connecting points on the resistor at different potentials must be symmetrical, and therefore may be represented by condensers, such as C_a connecting points which are equidistant from the centre of the rod. Consider the effect of C_a on the properties of the resistor. Let $R_a = aR_0$ be the portion of R_0 spanned by C_a. Then the

* Howe, G. W. O. *Wireless Engineer*, 1935, Vol. 12, p. 291.

remainder is $R_b = (1 - a)R_0$. The impedance operator is given by

$$\mathbf{Z} = \frac{R_a + R_b + jR_bR_aC_a\omega}{1 + jR_aC_a\omega} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Now suppose the resistor is represented by the equivalent network of Fig. 57 (*b*). Equating the admittance operator of this network

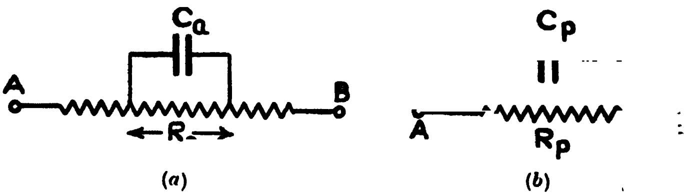


FIG. 57.

with the value of $1/\mathbf{Z}$ obtained above, and equating real and imaginary parts, we have for R_p the effective shunt resistance

$$\frac{1}{R_p} = \frac{1}{R_0}\frac{1 + R_a{}^2C_a{}^2\omega^2(1 - a)}{1 + R_a{}^2C_a{}^2\omega^2(1 - a)^2}$$

and for the effective self-capacitance, C_p,

$$C_p = \frac{a^2C_a}{1 + R_a{}^2C_a{}^2\omega^2(1 - a)^2}.$$

Now $R_a{}^2C_a{}^2\omega^2$ will in general be very small compared with unity and we may therefore obtain the approximations

$$R_p \simeq R_0[1 - R_a{}^2C_a{}^2\omega^2a(1 - a)] \simeq R_0[1 - R_0{}^2C_a{}^2\omega^2a^3(1-a)] \,. \quad (5)$$

and

$$C_p \simeq a^2C_a[1-R_a{}^2C_a{}^2\omega^2(1-a)^2] \simeq a^2C_a[1-R_0{}^2C_a{}^2\omega^2(1-a)^2a^2] \,. \quad (6)$$

In order to obtain the effect of the whole electric field or distributed capacitance of the resistor, we must consider a whole series of such capacitances as C_a with all values of " a " between 0 and 1. Provided $R_0{}^2C_a{}^2\omega^2$ is small compared with unity, we may simply add the various small terms in the expressions for R_p and C_p and obtain the approximation

$$R_p \simeq R_0\left[1 - \omega^2R_0{}^2\sum_0^1 C_a{}^2a^3(1 - a)\right] \quad . \quad . \quad . \quad . \quad (7)$$

$$C_p \simeq \sum_0^1 a^2C_a - \omega^2R_0{}^2\sum_0^1 C_a{}^3a^4(1 - a)^2 \quad . \quad . \quad . \quad . \quad (8)$$

Thus both the equivalent shunt resistance R_p and the equivalent shunt capacitance C_p (often simply referred to as the self-capacitance) of the resistor diminish with increase of frequency, the diminution being proportional to the square of the frequency, and to the square of the d.c. resistance for resistors of a given capacitance or of given size and shape. Moreover, for resistors of the same shape but different sizes the value of C_a is proportional to the linear dimensions, and hence the diminution of effective resistance will be proportional to the square of the linear dimensions. Thus resistors for use at very high frequencies should be of very small linear dimensions.

The variation of the component of self-capacitance C_a of the resistor with " a " will not in general be known. For the case of two parallel wires (a coil with bifilar winding) the distribution will be uniform, but for a straight wire or rod, C_a will diminish with increase of " a." Let us write as an approximation for the general case

$$C_a = \delta C_t = \frac{C_0}{a^r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where C_t denotes the total self-capacitance of the resistor, *i.e.*, the value which would be measured if it were cut into two parts at its electrical mid-point, and treated as a condenser. Then

$$C_t = \int_0^1 C_0 a^{-r} da = C_0/(1 - r) \quad . \quad . \quad . \quad . \quad (10)$$

$$C_p \simeq \int_0^1 a^2 C_a da = \int_0^1 C_0 a^{2-r} da = C_0/(3 - r) = \frac{(1 - r)}{(3 - r)} C_t \quad . \quad (11)$$

Also

$$\int_0^1 C_a^2 a^3 . (1 - a) da = \int_0^1 C_0^2 (a^{3-2r} - a^{4-2r}) da = C_0^2 \left[\frac{1}{2-2r} - \frac{1}{3-2r}\right]$$

$$= C_0^2/(2 - 2r)(3 - 2r) = C_t^2 . \frac{(1 - r)^2}{(2 - 2r)(3 - 2r)}$$

$$\therefore \; R_p \simeq R_0\left[1 - \frac{(1 - r)}{2(3 - 2r)} \omega^2 R_0^2 C_t^2\right] \quad . \quad . \quad . \quad (12)$$

The phase angle ϕ is evidently given by

$$\tan \phi = - \omega C_p R_p \simeq - \frac{(1 - r)}{(3 - r)} \omega C_t R_0 \quad . \quad . \quad . \quad (13)$$

For the case of uniformly distributed capacitance we may write $r = 0$. It is to be noted that the effective series resistance R_s of the resistor is different from the shunt value R_p given above. For the impedance operator of Fig. 57 (*a*) may be written

$$\mathbf{Z} = R_b + R_a \frac{(1 - jR_aC_a\omega)}{1 + R_a^2C_a^2\omega^2}$$

from which we obtain the relation

$$R_s = R_0(1 - a^3R_0^2C_a^2\omega^2) = R_0(1 - a^{3-2r}R_0^2C_0^2\omega^2)$$

and, integrating as before

$$R_s = R_0\left[1 - \frac{\omega^2R_0^2C_t^2(1-r)^2}{2(2-r)}\right] \quad . \quad . \quad . \quad . \quad (14)$$

Thus the equivalent series resistance R_s diminishes more rapidly with rise of frequency than the equivalent parallel resistance R_p. The value required will depend on the measurement to be undertaken.

4. The Effect of Capacitance to Screen (or Earth). We have already pointed out, that in order to make the effects of

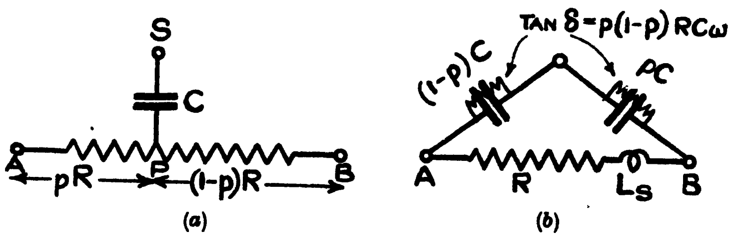


FIG. 58.

earth-capacitance definite it is necessary for precision work to enclose each standard resistor in a conducting screen, and that the screened resistor may be completely represented by an equivalent network of three impedances or admittances. Some assistance towards the understanding of the significance of these impedances may be obtained by a consideration of the following simple example. Let AB (Fig. 58 (*a*)) represent the conducting path of a resistor. Let us represent a portion of its capacitance to earth, or to screens, by the capacitance C between S and the point P which divides the conducting path into two portions of resistance pR and $(1-p)R$. Now it can be shown that three admittances, a, b, c, radiating from a common point may be replaced in any network by a mesh of three admittances $Y_1 Y_2 Y_3$ joining the open ends of the star abc, the currents and voltages of the network external to the star being unaffected provided that

$$\mathbf{Y}_1 = \frac{ab}{a+b+c}; \mathbf{Y}_2 = \frac{bc}{a+b+c}; \mathbf{Y}_3 = \frac{ca}{a+b+c},$$

$\mathbf{Y}_1$ connecting the open ends of a and b, and so on. In this way we obtain for the admittances in Fig. 58 (*b*) :

$$\mathbf{Y}_{AS} = \frac{(1-p)jC\omega}{1+jp(1-p)RC\omega}$$

$$\mathbf{Y}_{BS} = \frac{.jC\omega}{1+jp(1-p)RC\omega} \text{ (similarly)}$$

$$\mathbf{Y}_{AB} = \frac{\frac{1}{R}}{1+jp(1-p)RC\omega}.$$

Now it is clear that $\mathbf{Y}_{AS}$ represents a condenser of capacitance $(1-p)C$ and of loss tangent $p(1-p)RC\omega$, while $\mathbf{Y}_{BS}$ represents a condenser of capacitance pC and the same loss tangent, while $\mathbf{Y}_{AB}$ is equivalent to an impedance

$$\mathbf{Z}_{AB} = 1/\mathbf{Y}_{AB} = R[1+p(1-p)jRC\omega] \quad . \quad . \quad . \quad (15)$$

which corresponds to a resistance R, with a phase angle which is positive, and equal in magnitude to the loss angle of the above-mentioned condensers. The positive phase angle is such as would be caused by a self-inductance L_s in series with R and such that

$$L_s = p(1-p)CR^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Thus the direct impedance of a screened resistor may be inductive even though the current path possesses no true self-inductance. For a standard resistor $RC\omega$ should be small compared with unity, and in such a case we may calculate the effect of a distributed capacitance by dividing it into a number of components such as C in the above example, but with values of p varying from 0 to 1. If we have a total capacitance to screen C_s uniformly distributed, then evidently since the mean value of p and $1-p$ is $\frac{1}{2}$, each of the capacitances AS and BS will be $\frac{1}{2}C_s$ and since the mean value of $p(1-p)=p-p^2$ is $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$, the resultant value of L_s will be

$$L_s = \tfrac{1}{6}C_sR^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and the phase angle due to C_s will be given by

$$\tan\phi = \tfrac{1}{6}RC_s\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

It is, however, to be remembered that the distributed self-capacitance also affects the phase angle, and always in the negative direction. It follows that by suitable adjustment of self-capacitance, and capacitance to screen, the phase angle can be reduced to zero. Thus if we have a total self-capacitance C_t, and a total screen capacitance C_s, we have by (13) and (17), for frequencies at which the effects considered are small a resultant phase angle given by

$$\tan\phi = \tfrac{1}{6}\,.\,R\,.\,C_s\omega - \left(\frac{1-r}{3-r}\right).\,\omega C_tR \quad . \quad . \quad . \quad (19)$$

If in addition the resistor had a true self-inductance L, we should have an additional positive term, giving approximately

$$\tan\phi = \frac{\omega L}{R} + \frac{RC_s\omega}{6} - \left(\frac{1-r}{3-r}\right)RC_t\omega \quad . \quad . \quad . \quad (20)$$

It is to be noted that equations (15) to (20) apply only to the "direct impedance" of the resistor, and are therefore only applicable to measurements of the bridge type in which the effects of the capacitances to screen are eliminated. If one terminal is connected to the screen, the capacitance between screen and the other terminal is thereby connected in parallel with Z_{AB}, and the resultant impedance is correspondingly modified.

5. **Eddy Current Effects : Skin Effect**. When a conductor or coil carries alternating current of high frequency, the pulsations of its magnetic field set up eddy currents in all the metallic parts used in its construction, including the wire, terminals, screen and any metallic screws or pillars. Such currents must dissipate energy, and therefore increase the resistance values of the coil. They also tend to annul the magnetic field producing them, and therefore to diminish the inductance. It is obviously important that we should be able to estimate the magnitude of such effects in standard resistors and inductors, but owing to the complicated geometry of many coils, the general problem is very complex,

and it will only be possible here to consider very briefly the more salient points.

Consider first the eddy currents produced in the wire itself. The simplest case is that of a long straight wire, the magnetic field of which consists of concentric circles of magnetic flux, the planes of the circles being at right angles to the axis of the wire and the centres of the circles lying on this axis. The eddy currents tend to annul that portion of this circular field lying within the wire itself, and it is easy to see that they must therefore oppose the main current in the vicinity of the axis of the wire, but flow in the same direction as the main current near the circumference. The resultant current is therefore concentrated in the outer skin of the wire. It may be regarded as distributed over an annular cross-section, the outer boundary of which is the circumference of the wire, and the radial thickness of which diminishes with rise of frequency. The corresponding increase in the resistance of the wire is given by the equation

$$R = R_0(1 + F) \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and the diminution of inductance by

$$L = L_0(1 - U) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where R_0 is the d.c. resistance of the wire, and L_0 is its internal d.c. inductance, *i.e.*, that portion of its total inductance which represents the magnetic flux in the wire itself. F and U are two functions which increase with rise of frequency. The eddy currents must depend on the resistivity ρ or conductivity σ of the wire, its permeability μ_w, and diameter d, as well as the frequency f, or angular frequency ω; and since both F and U must be mere numbers (of zero dimensions), it is easy to show that they must be functions of the variable $z = d\sqrt{\pi\sigma\omega\mu_w}$. It is not possible to express these functions in simple finite terms valid for all values of z. Simple approximate forms may, however, be used for special cases. Thus, if z is small, say less than 1, we have

$$F \simeq \frac{1}{12}\left(\frac{z}{2}\right)^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$U \simeq \frac{1}{24}\left(\frac{z}{2}\right)^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

It therefore follows that at low frequencies both the increase of resistance and the decrease of inductance are proportional to the square of the frequency.

On the other hand, if z is very large, the following approximations may be used:

$$1 + F \simeq z/2\sqrt{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$1 - U \simeq 4/z\sqrt{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Thus at very high frequencies the resistance increases as the square root of the frequency, approaching infinity as the frequency becomes infinite. The internal inductance becomes zero at infinite frequencies ($z = \infty$). This is obvious since the current at such frequencies is entirely concentrated round the circumference of the wire, and there is therefore no magnetic field within the wire. The above expression may be used for values of z greater than say 5. For intermediate values of z no simple forms are available. The values of F and U have, however, been tabulated. See Table III and Fig. 63.

These values of F and U, strictly speaking, apply only to long straight wires. They may, however, also be applied approximately to all cases in which the magnetic field in any element of the wire is almost entirely due to the current in that element, and not to that in neighbouring turns of the coil, *i.e.*, to coils in which the turns are widely spaced. The complications arising with closely spaced windings will be considered in connection with inductors. For resistors, closely wound coils are only permissible at frequencies so low that eddy current effects are negligible, which is not often the case at radio frequencies. Resistors should obviously be constructed so that the skin-effect is small and accurately calculable. It is therefore necessary to ensure that the variable z is as small as possible for the conductor used, which means that the conductor should be very thin, and the material of low permeability and high resistivity. Resistors to be used at the highest frequencies should preferably take the form of single straight wires or rods, for which the skin-effect is accurately calculable. In some cases it is necessary to take account of the variation of residual inductance with frequency, given by equation (22).

TABLE III

Values of the functions F, U and G

$$z^* = d\sqrt{\pi\mu\sigma\omega} = \pi d\sqrt{2\mu\sigma f}$$

For copper $z = 0{\cdot}1078 d\sqrt{f}$ (d in cm., f in cycles/sec.).

z	F	U	G	z	F	U	G
0	0	0	0	10	2·799	0·718	1·641
Small	$z^4/192$	$z^4/384$	$z^4/64$	12	3·504	0·765	1·995
1·0	0·005	0·0026	0·0152	14	4·21	0·798	2·35
1·2	0·011	0·0053	0·0306	16	4·92	0·824	2·70
1·4	0·020	0·0098	0·0541	18	5·62	0·843	3·06
1·6	0·033	0·0166	0·0863	20	6·33	0·859	3·41
1·8	0·052	0·0261	0·1265	25	8·09	0·887	4·29
2·0	0·078	0·0389	0·1724	30	9·86	0·906	5·18
2·5	0·175	0·0865	0·2949	40	13·4	0·929	6·95
3·0	0·318	0·1548	0·405	50	16·9	0·943	8·71
3·5	0·492	0·235	0·499	60	20·5	0·953	10·5
4·0	0·678	0·314	0·584	70	24·0	0·960	12·3
5·0	1·043	0·444	0·755	80	27·5	0·965	14·0
6·0	1·394	0·535	0·932	100	34·6	0·972	17·6
8·0	2·094	0·649	1·287	Large	$(z\sqrt{2}-3)/4$	1·00	$(z\sqrt{2}-1)/8$
10·0	2·799	0·718	1·641				

6. Fixed Resistors. We have seen that resistors for high-frequency work should be of small linear dimensions, should not consist of coils of many turns, and that the conductor should be of very small cross-section to reduce "skin-effect." Further, any insulating material used in the construction should be of low power loss, otherwise the effective conductance of this material, which will certainly increase rapidly with rise of frequency, may appreciably affect the value of the resistor. A short straight thin wire of a resistance alloy such as manganin or eureka satisfies the above requirements to a considerable extent, and a very convenient form of resistor for high-frequency work is that shown in Fig. 59. It consists of a straight wire of length about 6 cm. or less, terminated by copper links which dip into mercury cups. The inductance of such a wire is not usually negligible, but if a series of such resistors is made, identical in every respect except diameter of wire, it is possible to change the resistance of a

* When calculating the value of z, the values of d, σ and μ must all be in the same system of units, *e.g.*, d in cm., σ in ohm^{-1} cm^{-1}, and $\mu_0 = 10^{-9}$ Henry/cm.

circuit without considerably affecting its inductance, merely by including the mercury cups in the circuit, and substituting one resistor for another. Since the inductance of a wire varies with its diameter it may be preferable to use wires of the same diameter and different resistivities, but even so, there may be differences of internal inductance due to skin effect. The range of variation obtainable in this way is somewhat limited, but it is always desirable to have one or two copper wires of diameters equal to those of the resistors, so that any resistance may be reduced nearly to zero without change of inductance, by the insertion of the appropriate copper wire. These thin wire resistors are apt to become appreciably heated by the passage of the current through them. They should, therefore, be made of eureka or manganin as far as possible, and the calibration should be checked with various values of current. Resistances up to 10 or 20 ohms are easily obtained with resistors of this design, but higher values demand the use of excessively thin wires, which are not very practicable.

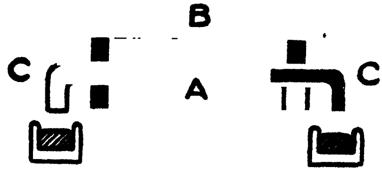


FIG. 59. Straight-wire resistor.

In order to obtain compact wire resistors giving values of 10 to 10,000 ohms, it is necessary to employ a considerable length of wire and to wind it in the form of a coil. For the lower values a single loop of thin wire is satisfactory, and it may well be immersed in oil to increase the current-carrying capacity. For higher values the wire is wound on a thin card of mica, bakelite or similar material. The coil so formed will of course possess appreciable inductance and capacitance, and the design should be such that the effects of the two on the phase-angle balance one another in accordance with equation (2). However, the capacitance must in no case be large or the effective resistance will vary in accordance with equation (1). A coil wound on a thin card necessarily has a fairly small inductance, and it will usually be found that with coils of resistance greater than say 100 ohms, the capacitance effect predominates. In the smaller coils the inductance predominates, and it is often minimised by winding two similar coils in opposite directions on the same card and connecting them in parallel. Obviously their magnetic fields will cancel one another to a considerable extent, and therefore their combined inductance will be very small. Several other types of non-inductive winding are possible, but in all cases it is necessary to balance inductance and capacitance by trial and error.

Resistors of high value, say 1,000 ohms to 10 megohms or even more, are now easily obtained in the form of thin films of graphite or metal, deposited on short rods of glass or other insulating material. Sometimes solid rods of a graphitic compound are employed. The disadvantage of such resistors for measurement purposes is their comparatively large temperature coefficient, but they may be made of very small linear dimensions, and therefore of very small inductance and capacitance, and are particularly useful at very high frequencies.

7. **The Coaxial Shielded Resistor.** In the above descriptions of resistors no mention has been made of electrostatic screens although it was shown in previous paragraphs that the constants of a resistor must be somewhat indefinite unless it is completely enclosed in a screen, since its associated capacitances will be apt to vary. It has been shown that the effects of capacitance are least on resistors of low value, and it is therefore often permissible to use straight wire resistors of low value without a screen. Again, short rod resistors also, being of low capacitance, seldom require screens. Indeed, it must be remembered that the presence of the screen generally increases the capacitances associated with the resistor, and therefore augments the changes of value due to capacitance. It follows that it is sometimes preferable to use such small resistors without screens. However, for the highest precision, screening is essential. The calculation of the properties of a screened resistor at radio frequencies is a complicated matter ; so much so that it may be said that there is only one type of resistor, the performance of which may be predicted with precision, for work at very high frequencies, viz., a straight cylindrical conductor (wire or rod), with a coaxial conductor of negligible resistance, as a screen.

We have already given approximate calculations showing the effects of distributed capacitance on such resistors when these effects are small. But for frequencies at which the effects are large, more accurate formulæ are required. The resistor and its screen may be regarded as the two conductors of a transmission line with uniformly distributed resistance, inductance, capacitance, and leakage conductance, R, L, C, and G being the values of these quantities per unit length. At any instant the current and voltage vary from point to point along such a line or resistor in accordance with well-known formulæ * involving $\mathbf{Z}_0$ the characteristic impedance of the line, and the hyperbolic functions $\sinh \gamma x$, $\cosh \gamma x$, etc. where x denotes distance measured along the line, and γ is its propagation constant

$$\mathbf{Z}_0 = \sqrt{(R + jL\omega)/(G + jC\omega)}$$
$$\gamma = \sqrt{(R + jL\omega)(G + jC\omega)}.$$

For a standard resistor we are only concerned with the terminal

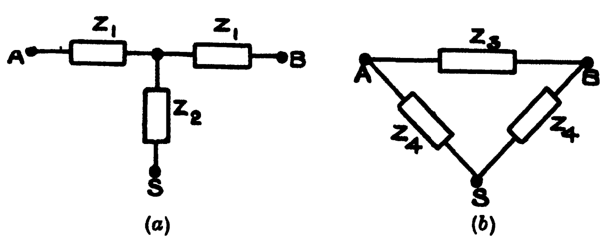


Fig. 60. Equivalent networks for a shielded resistor.

values of current and voltage, and with this limitation it may be shown that such a standard may be completely represented by the equivalent networks of Fig. 60 in which A and B are the terminals of the resistor proper, and S is the screen. In the equivalent T network

$$\mathbf{Z}_1 = \mathbf{Z}_0 (\cosh \gamma l - 1)/\sinh \gamma l = \mathbf{Z}_0 \tanh \frac{\gamma l}{2}$$
$$\mathbf{Z}_2 = \mathbf{Z}_0/\sinh \gamma l$$

where l is the length of the resistor or line.

For the equivalent ∇ network

$$\mathbf{Z}_3 = \mathbf{Z}_0 \sinh \gamma l$$
$$\mathbf{Z}_4 = \mathbf{Z}_0 (\cosh \gamma l + 1)/\sinh \gamma l = \mathbf{Z}_0 \coth \frac{\gamma l}{2}.$$

* See, for example, Mallett, E. *Telegraphy and Telephony*, p. 185. Chapman & Hall. 1929.

With these formulæ, the impedance of the standard at any frequency and for all working conditions can be calculated. For example, either of these networks represents the standard used as a three-terminal instrument for bridge work as explained in Chapter III.

When a Wagner earth-connection is used, the effective value of the impedance is $\mathbf{Z}_3$, while the value when the potential of the resistor is balanced with respect to earth is $2\mathbf{Z}_1$. When the screen is connected to one terminal, so as to form a two-terminal standard, the impedance takes the value $\mathbf{Z}_3\mathbf{Z}_4/(\mathbf{Z}_3 + \mathbf{Z}_4) = \mathbf{Z}_0 \tanh \gamma l$.

In order to determine these three working values for any given resistor, it is necessary to know the values of l, L, R, C, and G, for the working frequency $\omega/2\pi$. The value of R may be determined from the d.c. resistance, and the skin-effect correction, already discussed. The value of L may be calculated from the linear dimensions of the resistor (see Chapter VIII). The value of C may also be determined by calculation (Chapter VI), while G may usually be reduced to zero by the use of good insulating material. Alternatively, C and G may be measured experimentally (Chapters IX to XI). It is to be remembered that L, R, C and G all refer to unit length of the resistor, and that uniform distribution of these quantities has been assumed. Also the self-capacitance of the resistor is assumed to be negligible in comparison with capacitance to screen. For a thin wire, this approximation will almost certainly hold, but the limitation must not be overlooked. The assumption of uniform distribution also implies that end-effects are negligible. The approximation therefore tends to become closer as the ratio of length to diameter of the resistor increases. End-effects are almost invariably appreciable on account of the capacitances of terminal fittings, but these may usually be measured separately and treated as additional capacitances, external to the resistor.

8. **Resistance Boxes.** The ordinary resistance box consisting of one or more decades of fixed resistors connected in series, with switch contacts for controlling the number of resistors actually in circuit is well known as a satisfactory standard for d.c. or low-frequency work. Its success depends on the fact that d.c. resistances in series are additive, and it will be obvious from

the preceding paragraphs that the effective values at high frequencies will not in general be additive, for the self-capacitance of the system must vary in an irregular manner as the switches are operated, and the effective value is a function of this capacitance, as well as the d.c. resistance with which it is associated. Thus it is not to be expected that resistance boxes will be satisfactory except at the lower radio frequencies, and moreover, the higher values of resistance will be less satisfactory than the lower, since the effect of capacitance increases with the value of the resistance. The remarks on the construction of fixed resistors for high-frequency work apply also to resistors used in the construction of resistance boxes. The switches themselves should be of small linear dimensions, so as to keep the value of their capacitance as small as possible. For precision work, the box must be completely enclosed by a metal screen so as to make the capacitance definite, and for bridge work it is sometimes desirable to have double screens, the inner one being connected to one terminal of the resistor and the other to earth (*cf.* Chapter III). Resistance boxes of constant self-capacitance have been made by mounting on a rotating drum a series of resistors varying in resistance but of the same linear dimensions. As the drum rotates each resistor in turn is brought into contact with two fixed brushes, which connect to the terminals of the box. This arrangement is specially valuable for high values of resistance. For low values the effect of capacitance is often negligible, so that ordinary switches may be used.

9. **Continuously Variable Resistors.** Resistance boxes as described above provide resistances which are adjustable in discontinuous steps. When continuous adjustment is required, it is necessary to use a resistor of the slide-wire type. It has not been possible to produce such a resistor with zero phase angle at all settings, but it is possible to make variable resistors of constant inductance, and these are valuable for measurements of several kinds. Since the effect of capacitance on the effective inductance varies with the value of the resistance, constancy of effective inductance in spite of variations of resistance, can only be achieved when the effect of capacitance is negligible, *i.e.*, for low values of resistance, say values up to 1 ohm. It is then merely necessary to ensure that the configuration of the circuit remains

unaltered as the resistance is varied. This is accomplished by the use of two metals one of high and one of low resistivity, the circuit being arranged so that when the resistance is increased a length of conductor of high resistance replaces an equal length of the conductor of low resistance. The first resistor of this type was the constant-inductance rheostat of A. Campbell,* which consists of two circles of wire placed side by side, one of manganin and one of copper, connected together by a sliding brush-contact, which links corresponding points on the circles. The current flows round the manganin wire as far as the brush-contact, then across to the copper wire, finally completing the circle along the copper wire. Thus the inductance is always that of a single circle, while the resistance varies as the brush-contact is moved. Wenner and Dellinger devised a resistor of this type consisting of a column of mercury in a glass tube. A rod or wire of copper pushed into the tube, short-circuits more or less of the mercury, and therefore alters the resistance without appreciably affecting the inductance. Another type used by R. M. Wilmotte † consists of similar strips of copper, and an alloy of high resistivity, connected in series, and wound on a cylinder of bakelite in the form of a spiral. Two fixed brushes make contact with the strip so that there is always a constant length of the spiral in circuit. Rotation of the drum causes it to travel axially, so that the proportion of the copper and manganin strip between the brushes is varied, and thus the resistance is varied while the inductance remains constant. As in Campbell's instrument, the inductance of the spiral is reduced as far as possible, by connecting in series with it similar turns of copper wire wound in the opposite direction and placed as close to it as possible. Wilmotte's instrument is shown in Fig. 61.

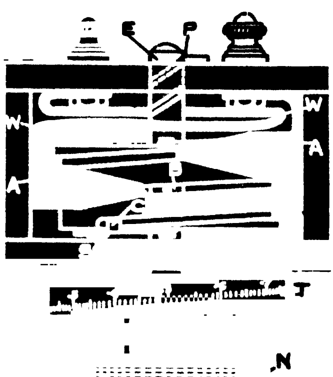


FIG. 61. Slide-wire resistor of constant inductance (Wilmotte's pattern).

* See Ref. 1 of Bibliography.

† WILMOTTE, R. M. *J. Sci. Instrum.*, 1928, Vol. 5, p. 369.

Variable resistors of high value may be obtained on the lines of the Wenner and Dellinger resistor described above, by using a column of an electrolytic solution of high resistivity, into which a glass rod may be pushed, thereby increasing the resistance of the column.

10. **The Properties of Typical Resistors.*** The quality of a coil as regards phase-angle is best judged by its time-constant $\frac{L_s}{R_s}$ which may be expressed in microhenries per ohm, or microseconds. The values given in Table IV were obtained for the

TABLE IV. *Time-constants and Phase-angles of Resistance Coils*

Resistance, R. (ohms,)	Time Constant L/R. (μH/ohm)	Tangent of Phase-Angle at 1,000 kc/s.
0·1	0·2	1·2
1·0	0·1	0·6
10	0·03	0·2
100	−0·003	−0·02
1,000	−0·01	−0·06
10,000	−0·01	−0·06

separate coils which formed parts of a resistance box of the "non-reactive" type. The negative sign before certain of the values means that the effective self-inductance L_s is negative; in other words, the self-capacitance more than compensates the self-inductance, and the net reactance of the coil is capacitive. This usually occurs for the higher values of resistance. For the lower values, the inductive effect predominates, and it is not easy to obtain a coil of low resistance and small phase-angle. For example, the 0·1 ohm coil of Table IV at 1,000 kcs. has a reactance larger than its resistance, and a phase angle greater than 45°. Of course it would be possible to obtain a very low inductance by using a very short length of thin wire, but even so, the inductance of the necessary leads to terminals is apt to predominate and make the time-constant considerable. It might be thought that small resistances are therefore quite impracticable

* HARTSHORN, L. *World Power*, 1927, Vol. 8, pp. 171 and 234.

for precision radio-frequency work, but it is to be remembered that in some methods of measurement, the reactance of the resistor is compensated by the tuning of the circuit to resonance, and it becomes possible to discriminate between the effects of the resistance and reactance of the resistor, on the rest of the circuit.

The above values of time-constant are for single coils. When such coils are built into the form of decade dials, the values are modified by the inductance of the wiring between switches and terminals, and by the capacitance associated with the switches. The inductance added by the wiring usually amounts to 0·1 or 0·2 μH per decade, depending on the linear dimensions of the box. This inductance will obviously greatly increase the time constant of the box for low values of resistance. Switch capacitances may vary from 5 to 50 $\mu\mu$F. Their effect is greatest on the resistors of high value.

We have seen that the effective resistance of a resistor tends to increase with frequency on account of skin-effect, and to diminish with frequency on account of distributed capacitance. The effect of capacitance usually predominates for high values of resistance, but is often negligible for low values, in which case the frequency variations are those due to skin-effect alone, and increase with the diameter or thickness of the conductor. As long as the corrections due to skin-effect and capacitance are small each is proportional to the square of the frequency, and when the two opposing effects are approximately balanced, the net correction is negligible for quite high values of frequency.

For a well-designed resistance box these corrections are negligible (less than 0·1 per cent.) at frequencies up to 50 kcs. For the sizes of wire commonly used, the skin-effect amounts to about 5 per cent. at a frequency of 1,000 kc/s., and this is therefore the order of the correction on resistors of values less than 1 ohm. On resistors of the order of 10 and 100 ohms, there is usually approximate compensation of skin-effect and capacitance effect, and the net correction is of the order of 1 per cent. or perhaps less, at a frequency of 1,000 kc/s. On resistors of 1,000 ohms and over the correction is usually negative, and may vary from 5 to 20 per cent. at a frequency of 1,000 kc/s. At frequencies of 6 Mc/sec. and over, the correction on wire-wound resistors of all values may amount to 20 or 30 per cent.

Resistors of the short-rod type * may, however, be used successfully at frequencies as high as 100 Mc/sec. The equivalent shunt capacitance of such a resistor, of length say 1 cm., is likely to be of the order of 0·1 to 0·5 $\mu\mu$F., and the equivalent shunt resistance will therefore probably differ from the d.c. value by less than 5 per cent. at a frequency of 100 Mc/sec. for resistors of 1,000 ohms, although the difference is likely to be of the order of 50 per cent. on resistors of value 1 megohm.

* Hartshorn, L. and Ward, W. H. *J. Sci. Instrum.*, 1937, Vol. 14, p. 132. Hartshorn, L., *Wireless Engineer*, 1938, Vol. 15, p. 363.

CHAPTER VIII

STANDARD INDUCTORS

1. Inductance, Self-Capacitance and Resistance. A standard inductor is essentially a conductor possessing a definite value of inductance under all working conditions. The significance of the quantity inductance in alternating current measurements was discussed in Chapter I, where it was pointed out that in practice it is always associated with resistance or power loss, and that what we measure is not the true inductance characteristic of the actual magnetic field of a coil, but the equivalent series value L_s defined by the reactance, or the equivalent shunt value L_p defined by the susceptance, these two values being related by the equation

$$L_s = L_p \sin^2 \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ϕ is the phase-angle of the coil. In practice L_s and L_p are both found to increase with rise of frequency, the law of variation usually corresponding to that for a pure inductance shunted by a pure capacitance, *i.e.*, it is of the form

$$L_s = L_1/(1 - L_1C_1\omega^2)\,; \qquad L_p = L_2/(1 - L_2C_2\omega^2) \quad . \quad (2)$$

where L_1, L_2, C_1 and C_2 are independent of ω. It follows that a coil may usually be represented by the equivalent networks of Fig. 62 (*a*) and (*b*), and the quantities L_1 and L_2 being independent of frequency are regarded as true values of inductance, the variations of L_s and L_p being ascribed to the distributed capacitance of the coil, which is represented by the "self-capacitance" C_1 or C_2. It should be noted that the variations of L_s and L_p may be in part due to eddy currents, and that their effect is therefore included in the values of C_1 and C_2. Moreover, it is obvious that the values of inductance L_1 and L_2, and those of capacitance C_1 and C_2 are not identical, but are related by the equations

$$\frac{L_1}{L_2} = \frac{C_2}{C_1} = \frac{L_s}{L_p} = \sin^2 \phi \simeq 1 - \frac{R_s^2}{L_s^2\omega^2} \quad . \quad . \quad . \quad . \quad (3)$$

We may replace the R_2 and L_2 in parallel of Fig. 62 (*b*) by the equivalent series combination R_3L_3 of Fig. 62 (*c*), which is often preferred as the equivalent network for an inductor, since it corresponds more closely to the physical facts : for the resistance and inductance of a coil are mainly associated with the same conductor, and therefore should carry the same current, as do R_3 and L_3, while the capacitance currents in a coil are proportional to the terminal voltage, which fact corresponds to the

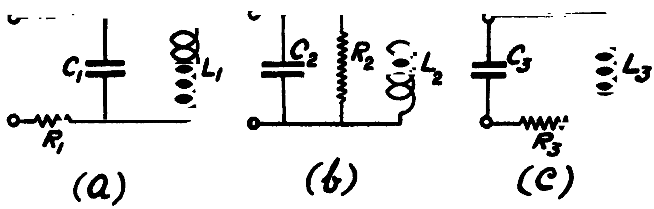


FIG. 62. Equivalent circuits of an inductor.

connection of C_3 to the terminals of the coil. The relations between the various quantities in Fig. 62 (*a*), (*b*) and (*c*), are readily obtained by the methods of Chapter I. We find

$$L_2 = L_3\left(1 + \frac{R_3^2}{L_3^2\omega^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$R_2 = R_3\left(1 + \frac{L_3^2\omega^2}{R_3^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$L_s = \frac{L_3(1 - L_3C_3\omega^2) - C_3R_3^2}{(1 - L_3C_3\omega^2)^2 + R_3^2C_3^2\omega^2} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$R_s = \frac{R_3}{(1 - L_3C_3\omega^2)^2 + R_3^2C_3^2\omega^2} \quad . \quad . \quad . \quad . \quad (7)$$

In precision work it is important to specify which of these quantities is being measured. Most bridge methods measure L_s and R_s. Methods based on current resonance often give L_3, while those based on voltage resonance give L_2. Standard inductors should be designed so that the resistances R_s and R_3, and the self-capacitances C_1, C_2 and C_3 are as small as possible. The quantities $R_3/L_3\omega$, $R_1/L_s\omega$, $L_1C_1\omega^2$, $L_2C_2\omega^2$, $L_3C_3\omega^2$ are then small compared with unity, while $R_3C_3\omega$ and $C_3R_3^2/L_3$ are still smaller, and the values of inductance L_1, L_2 and L_3 differ only by small amounts. Also R_s is nearly equal to R_3, but R_p is very large, since $R_p/L_p\omega = L_s\omega/R_s$.

2. **Magnification or Q value.** The ideal inductor, like the ideal capacitor, would have zero power factor; and the quality of a coil may well be judged like that of a condenser by the smallness of its power factor $\cos\phi$, or of its loss tangent $\tan\delta$. Many engineers, however, seem to prefer to deal with the reciprocal of these quantities, and to judge the quality of an inductor by the largeness of $1/\cos\phi$, which is usually denoted by Q, and called the magnification. This term appears to be derived from considerations of voltage resonance. It is shown in Chapter II (equation 12), that if an inductor L_sR_s is tuned by a pure capacitance to voltage resonance, the ratio of the maximum voltage across the coil, to the induced e.m.f. producing it, is $1/\cos\phi = Q$. Thus the coil may be considered as capable of magnifying a voltage by the factor Q. It should be noticed that

$$Q = \frac{1}{\cos\phi} = \frac{1}{\sin\delta} = \frac{Z}{R_s} = \frac{\sqrt{(R_s^2 + L_s^2\omega^2)}}{R_s} \quad . \quad . \quad . \quad (8)$$

But $\sin\delta$ is usually very small, and the following simple approximation is often used:

$$Q = \frac{1}{\tan\delta} = \frac{X_s}{R_s} = \frac{L_s\omega}{R_s} = \frac{R_p}{L_p\omega} \quad . \quad . \quad . \quad . \quad (9)$$

3. **Calculation of Inductance.** It is well known that the inductance of a coil made of non-magnetic materials can be calculated from its linear dimensions, and a few of the most useful formulæ will be noted here. It should be remembered that the calculation is essentially that of the energy of the magnetic field of a circuit, and that we can only calculate the quantity for the complete circuit. If we speak of the inductance of a wire or any portion of a circuit, we are assuming some particular distribution of the energy round the circuit, *e.g.*, uniform distribution. Precision standards of inductance must always for this reason consist of complete circuits, as far as this is practicable, and their terminals must therefore always be placed close together. The formulæ necessarily require a knowledge of the inductivity or magnetic permeability of the materials. For all non-magnetic media the value is 1 c.g.s. unit, or 10^{-9} Henries per cm. For magnetic materials this value must be multiplied by μ the relative

permeability of the material. In the following formulæ μ denotes the relative permeability of the wire, and the surrounding material is assumed to be non-magnetic. The portion of the magnetic field within the conductor itself varies with frequency owing to the skin-effect discussed in the previous chapter. It will be noticed that some of the formulæ contain a term involving μ and the skin-effect function $(1 - U)$. This term represents the internal inductance of the wire, which is a function of the variable z (Table III). When $z = 0$, $U = 0$, and when $z = \infty$, $U = 1$, *i.e.*, the flux in the wire vanishes at infinite frequency.

In the formulæ all lengths are to be expressed in cm. and the inductances are given in henries.

(*a*) *Circular coils*, of outside diameter D, length of winding b, depth of winding t.

$$L = L_0 N^2 D \times 10^{-9}\,* \quad \ldots \ldots \quad (10)$$

where L_0 is a shape factor given in Table V.

TABLE V. *Values of* L_0

b/D	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$
t/D					
0·0	∞	18·68	14·43	12·02	10·37
0·1	17·46	12·92	10·52	8·93	7·78
0·2	11·51	9·10	7·58	6·49	5·68
0·3	7·82	6·33	5·31	4·57	4·00
0·4	5·26	4·27	3·59	3·08	2·69
0·5	3·48	2·82	2·37	2·03	1·78

(*b*) *Single Layer Solenoid* of N turns, radius R, and length l.

$$L = 4\pi^2 \times 10^{-9} K N^2 R^2 / l \quad \ldots \ldots \quad (11)$$

where K is a factor depending on R/l. (See Table VI).

(*c*) *Toroid* of mean radius R, and N turns, each of radius r

$$L = 4\pi \times 10^{-9} N^2 [R - \sqrt{R^2 - r^2}] \quad \ldots \ldots \quad (12)$$

(*d*) *Toroidal Coil of Rectangular Cross-section.* Number of turns N, external radius R_1, internal radius R_2, axial length l,

$$L = 2 \times 10^{-9} N^2 l \log_e (R_1/R_2) \quad \ldots \ldots \quad (13)$$

* BUTTERWORTH, S. *Wireless Engineer*, 1926, Vol. 3, p. 421.

TABLE VI. *Values of K*

R/l	K	R/l	K	R/l	K	R/l	K
0·0	1·000	0·4	0·735	0·8	0·580	1·2	0·482
0·1	0·920	0·5	0·668	0·9	0·551	1·3	0·463
0·2	0·850	0·6	0·648	1·0	0·526	1·4	0·445
0·3	0·789	0·7	0·612	1·1	0·503	1·5	0·429

(*e*) *Single Turn of Wire.* Mean radius of turn $= R$, diameter of wire d. Provided that $d/2R < 0{\cdot}2$

$$L = 4\pi \times 10^{-9} R[\log_e (16R/d) - 2 + \tfrac{1}{4}\mu(1 - U)] \quad . \quad (14)$$

(*f*) *Straight Wire* of diameter d. For a length l

$$L = 2 \times 10^{-9} l[\log_e (4l/d) - 1 + \tfrac{1}{4}\mu(1 - U)] \quad . \quad . \quad . \quad (15)$$

(*g*) *Parallel Wires* (go and return) of diameter d, distance between axes D. For a length l of the pair

$$L = 4 \times 10^{-9} l[\log_e (2D/d) - D/l + \tfrac{1}{4}\mu(1 - U)] \quad . \quad . \quad (16)$$

Mutual inductance of the same two wires:

$$M \simeq 2 \times 10^{-9} l[\log_e (2l/D) - 1 + D/l] \quad . \quad . \quad . \quad (17)$$

(*h*) *Wires in Line.* Wires of lengths l and m, placed end to end. Mutual inductance M:

$$M = 10^{-9}\left[l \,.\, \log_e \left(\frac{l + m}{l}\right) + m \,.\, \log_e \left(\frac{l + m}{m}\right)\right] \quad . \quad (18)$$

(*i*) *Coaxial Circular Coils* of turns N_1 and N_2, mean radii R_1 and R_2, distance between their planes D. Mutual inductance, M:

$$M = 4\pi \times 10^{-9} N_1 N_2 \sqrt{R_1 R_2}\left\{\left(\frac{2}{k} - k\right)F - \frac{2}{k}E\right\} \quad . \quad (19)$$

where $k = \dfrac{2\sqrt{R_1 R_2}}{\sqrt{(R_1 + R_2)^2 + D^2}}$ and F and E are complete elliptic integrals of the first and second kinds respectively to modulus k. Values of F and E for any value of k may be obtained from tables. The quantity $\phi(k) = \left\{\left(\frac{2}{k} - k\right) F - \frac{2}{k}E\right\}$ has also been tabulated.*

* NAGAOKA, H. and SAKURAI, S. Scientific Papers, *Inst. Phys. and Chem. Res. Tokyo.* 1922, Vol. 1 and 1927, Vol. 2.

(*j*) *Coaxial Cable.* Length l large. Inner conductor of diameter d_1 and self-inductance L_1. Outer conductor thin, of internal diameter d_2, and self-inductance L_2. Mutual inductance M_{12}.

L_1 is given by equation (15).

$$L_2 = M_{12} = 2 \times 10^{-9}l[\log_e(4l/d_2) - 1]$$

$$L = L_1 + L_2 - 2M_{12} = L_1 - L_2$$

$$= 2 \times 10^{-9}l[\log_e(d_2/d_1) + \tfrac{1}{4}\mu(1 - U)]$$

At very high frequencies the second term vanishes and the simple formula

$$L = 2 \times 10^{-9}l \log_e(d_2/d_1)$$

holds even when the outer conductor is thick.

4. Resistance due to Eddy Currents.* The discussion of the skin-effect given in the previous chapter applies to inductors as well as to resistors, and the formulæ given enable us to calculate the variation of resistance and inductance with frequency for any coil in which the spacing of the turns is very large compared with the diameter of the wire.

When we come to deal with an ordinary inductor of many turns, we find that the magnetic field, and therefore the eddy currents in any turn of the coil, are very considerably affected by the currents in neighbouring turns. Thus the field will vary considerably from one turn to another, and the resulting system of eddy currents becomes very complicated. In circular coils in which the turns are well spaced (which is normally done for radio-frequency work in order to reduce the self-capacitance of the coil), the field in any turn due to the remaining turns may usually be regarded as uniform over the cross section of the wire and perpendicular to its axis. The resultant field in every turn therefore may be divided into two components, (1) the circular field due to the current in that turn, and (2) a uniform field perpendicular to the axis of the wire, but varying in magnitude and direction from turn to turn. Each of these two components may be regarded as producing its own system of eddy currents, and it can be shown that if we calculate the power dissipation in any one turn due to each of these two systems, the sum of the two gives the actual power dissipation in the turn in question.

* BUTTERWORTH, S. *Wireless Engineer*, 1926, Vol. 3, p. 203.

It follows that each of these two systems of eddy currents makes a definite contribution to the resistance of each turn, and the resultant resistance is the sum of these two contributions. Now the effect of component (1) has already been discussed. It is that which would hold for a long straight wire. We may therefore say that the total resistance of the coil may be expressed

$$R = R_s + R_h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where R_s is the resistance of the same wire pulled out straight, and R_h is the component of resistance due to the coiling. Further, R_h is the sum of a number of terms, one for each turn of the coil, each representing the power dissipation in a cylindrical wire due to alternations of a uniform magnetic field perpendicular to its axis. It can be shown that the power loss in a cylinder of diameter d, and d.c. resistance r_0, in an alternating transverse field of strength H (maximum value) is given by

$$\begin{aligned} W_h &= r_0 d^2 H^2 G/8 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21) \\ &= r_h I^2, \text{ say.} \end{aligned}$$

where G is a function of the variable z already discussed. Values of the function G are given in Table III and Fig. 63. At low frequencies we have the approximation

$$G \simeq z^4/64 \text{ for } z < 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and at very high frequencies ($z > 5$)

$$G \simeq (z\sqrt{2} - 1)/8 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

It follows from (21) that R_h for the whole coil is the sum of a number of terms r_h of the form $r_0 d^2 (H/I)^2 \,.\, G/8$. For a coil of many turns the result is somewhat complicated, but it is obvious that H/I will be proportional to the total number of turns N (other things being equal), and also that for coils of a given number of turns and shape, it will be inversely proportional to the linear dimensions of the coil (*e.g.*, the external diameter D). It follows that we may write

$$R_h = \frac{R_0 d^2 N^2}{D^2} \,.\, \frac{G}{4} \times K^2 = \frac{1}{4}\left(\frac{KNd}{D}\right)^2 G \,.\, R_0 \quad . \quad . \quad (24)$$

where K is a factor which depends only on the shape of the coil and R_0 is its *d.c.* resistance. Butterworth has calculated the value of K for most of the coil-shapes used in practice, and typical values are given in Table VIII. We now have for the total effective resistance of the coil

$$R = R_s + R_h = R_0\left[1 + F + \frac{1}{4}\left(\frac{KNd}{D}\right)^2 G\right] \quad . \quad . \quad (25)$$

It should be remarked here that this value of R refers to R_3 in

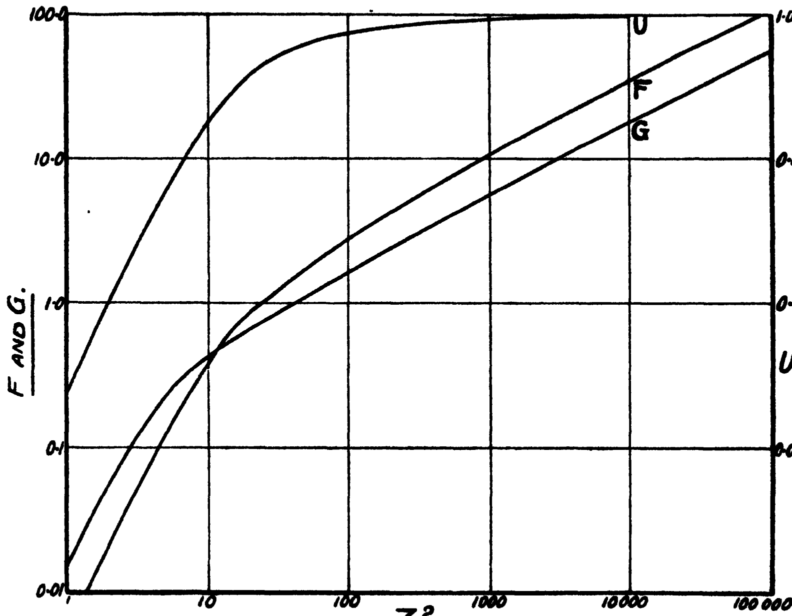


FIG. 63. The functions F, G, and U, representing the effects of eddy currents. The independent variable Z^2 is proportional to frequency.

Fig. 62 (*c*). Equation (25) shows how the effective resistance of coils wound with solid wire varies with the frequency, resistivity and diameter of wire, etc. Equations (25) and (10) are most useful for designing coils which shall have a large value of Q over a given range of frequency. It should be noticed that the component of resistance R_h always increases with increase of diameter of wire d, for R_0d^2 is independent of d, while G increases with d. Also R_h is apt to become large when N is large, and it is therefore advisable to wind coils of many turns with stranded wire, the strands being separately insulated and twisted so as to bring each in turn to the surface of the cable. Alternatively the strands may be woven into the form of a tape. For coils wound with stranded wire Butterworth has given the equations

$$R_s = R_0\{1 + F + k(nd/d_0)^2G\} \quad \ldots \quad (26)$$

$$R_h = \tfrac{1}{4}R_0(KNnd/D)^2G \quad \ldots \quad (27)$$

$$R = R_s + R_h = R_0\left\{1 + F + \left(\frac{k}{d_0^2} + \frac{K^2N^2}{4D^2}\right)n^2d^2G\right\} \quad . \quad (28)$$

where n = number of strands, d = diameter of each strand, d_0 = overall diameter of conductor, and k is a function of n as follows :—

TABLE VII

n	3	9	27	large
k	1·55	1·84	1·92	2

F and G have the same significance as before, but the variable z is now to be calculated from the diameter of a single strand. The diameters d, d_0 and D should of course all be measured in the same units.

TABLE VIII. *Values of the Shape Factor K (Butterworth)*

b/D	0		$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$
t/D						
0	—	—	30·1*	15·6*	10·7*	8·3*
0	—	∞	41·7	21·2	14·4	11·0
0·1	37·8†	52·4	23·3	15·4	11·6	9·5
0·2	20·6†	27·4	16·2	12·4	9·9	8·2
0·3	15·4†	19·6	13·7	10·7	8·8	7·5
0·4	13·2†	16·0	12·0	9·5	8·0	6·9
0·5	11·7†	13·8	10·4	8·4	7·0	6·0

* Single layer solenoid. † Single layer disc. Other values refer to many-layer coils.

5. The Design of Self-Inductors. When designing a standard inductor we have to decide the size and shape of the coil, the number of turns, their spacing, and the order in which they are to be wound, the diameter of the wire (or more generally the cross-section of the conductor), and the mechanical construction of the supports or coil-former. In general the most important condition to be satisfied is that the resistance of the coil at the working frequency shall be as small as possible, but it often happens that other considerations determine the size and shape of the coil. If for example we wish to induce an e.m.f. into the coil from a distant source a large turn area may be desirable, while if we wish to avoid picking up any stray e.m.f. a toroid may be necessary. Again, the length and diameter of conductor to be used may be limited by the question of cost, or the space into which the coil must fit may be limited. However, some of these factors are usually at our disposal, and it becomes important to choose them so as to obtain the minimum resistance at the working frequency. The general question has been discussed in some detail by Butterworth. We shall consider only a few points of general interest. Confining ourselves to circular coils with well-spaced turns, equation (10) gives the relation between the inductance, the number of turns and the size and shape of the coil, while (25) and (28) give the relation between resistance, the above quantities, the properties of the conductor, and the frequency. It follows that the condition for minimum resistance may be deduced from these equations. It should, however, be noted that the resistance in these formulæ is that due to the conductor only. Power losses in the insulating supports may add to this value, but with modern insulating materials and good design, these losses are almost negligible in most cases. Neglecting these losses for the moment, we may obtain the following results from the above equations.

6. **Best Shape of Coil.** If we have a given length of wire of given diameter and wish to wind a coil of a definite inductance, the formulæ show that the shape of coil that will give the minimum resistance is the single layer solenoid having a winding length equal to one-third of its diameter. A single-layer-disc coil with a winding depth equal to one-quarter of its diameter is almost as good. It is obvious, however, that when the inductance is large, these single layer coils will become very bulky, and in order to obtain reasonably compact coils it is necessary to use several layers. In such cases b/D and t/D may be varied considerably

without affecting the resistance to any great extent, and provided that

$$5t + 3b = D,$$

the resistance will not be greatly different from the minimum value for multi-layer coils. This value is about 30 per cent. greater than that for the single-layer coils.

7. **Best Diameter of Wire.** Having fixed the shape of the coil and the total length of wire, it is obvious that the diameter of wire which will give the lowest resistance will depend on the frequency. At low frequencies, F is negligible compared with unity and $R_s = R_0$ which varies inversely as d^2. At these frequencies, G and therefore R_h varies as d^4. Thus

$$R = R_s + R_h = \frac{A}{d^2} + Bd^4$$

where A and B are independent of d. Differentiating and equating to zero, we find that the value of d corresponding to the minimum value of R, is given by the condition

$$R_s = 2R_h.$$

At very high frequencies F and G are both large, and both proportional to d. Thus we have $R = R_s + R_h \simeq A'/d + B'd$, and the condition for best diameter is

$$R_s = R_h.$$

It follows from these results that the best size of wire will vary considerably with the number of turns and shape of coil, as well as the frequency and resistivity. Coils of few turns should be wound with thick wire, and indeed for coils of a very few turns such as are required at very high frequencies, it is advisable to use copper rod or tube several millimetres in diameter rather than wire.

8. **Effect of Size of Coil.** We have seen that the best shape of coil for a fixed total length of wire may lead to a very bulky coil in some cases. It is therefore advisable to consider the best shape to adopt when the bulk of the coil, which may be taken as proportional to its overall surface, is fixed. The overall surface being fixed, the shape determines D, and the inductance then determines N, so that the length of wire l is fixed. Thus the shape of coil now determines the length of wire. The best diameter of wire must be found as indicated above, and this will vary with

the frequency. The resistance now depends on length, diameter, shape and frequency. Calculation for various shapes shows that the best shape in these circumstances varies somewhat with frequency, and that it indicates a rather longer coil than that for a fixed length of wire. The best coil for all frequencies is still a single layer solenoid, but the winding length should be between $0{\cdot}5D$ and $0{\cdot}75D$. Next comes the single-layer-disc coil with a winding depth of $0{\cdot}3D$. The best multi-layer coils have a resistance which is higher than these by about 30 per cent. They should have a length of winding b between $\frac{1}{8}D$ and $\frac{1}{2}D$, and the depth of winding t should be such that

$$4\frac{t}{D} + 1{\cdot}5\,\frac{b}{D} = 1.$$

Within these limits the resistances of well-designed coils are nearly independent of the shape. The resistance is found to diminish with an increase in the size of the coil. For low frequencies the resistance varies inversely as $\sqrt{D}$, and for high frequencies inversely as D, provided the above conditions are satisfied for each size of coil.

9. **Use of Stranded Wire.** The above results show that the improvement obtained by using large coils is less marked at low frequencies than at high. This refers only to coils wound with solid wire, and it arises from the fact that at low frequencies R_h increases extremely rapidly with diameter of wire ($\propto d^4$) so that the diameter of wire can only be increased relatively slowly as the size of coil is increased. However, by using stranded wire, R_h can be greatly diminished, while retaining the same total cross-section of copper, and therefore the same R_s at low frequencies. Thus at low frequencies, large coils can be greatly improved by the use of stranded wire. The considerations outlined above apply also to the design of stranded-wire coils provided that equations (26) to (28) are used for R_s and R_h, the value of z being calculated for a single strand. Calculation shows that if the best diameter of strand is chosen for each case, then at low frequencies the resistance varies inversely as $n^{\frac{1}{3}}$, but that at very high frequencies, such that z becomes large even for the finest strands available, nothing is gained by the use of stranded wire, the resistance being almost independent of n.

In addition to the resistance arising from power losses in the conductor itself, we must also consider that due to losses in the coil-former.

10. **The Coil-Former and Terminals.** We have already noted that the coil possesses an electric field which may be represented by its self-capacitance C. If the turns of the coil are separated by air only this capacitance will be sensibly perfect, but if the coil is wound on a solid former, such as an ebonite cylinder, part of the electric field will be located in the solid dielectric, and if this portion of the electric field be represented by αC, where α is a fraction, depending on the proportion of the electric field occupied by solid dielectric, it will be associated with a power loss equivalent to a shunt resistance R_f where

$$1/R_f = \alpha C\omega \tan \delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

tan δ being the loss tangent of the dielectric at the angular frequency ω. This shunt resistance diminishes the value of R_2 (Fig. 62 (b)), and therefore diminishes the magnification $L_3\omega/R_3$ (equation (9)), and increases the resistance R_3. The conditions under which this portion of the resistance may be reduced to a minimum follow immediately from (29). The solid dielectric should have a minimum value of tan δ; the self-capacitance C should be made as small as possible, *i.e.*, the turns of the coil should be widely spaced; the fraction α of this capacitance that is located in the solid dielectric should be small, *i.e.*, the amount of solid dielectric should be reduced to a minimum, especially in the strongest part of the electric field. When these conditions are satisfied the part of the resistance due to dielectric losses is usually negligible compared with the conductor resistance. It should, however, be noted that the dielectric losses are proportional to the frequency, if, as is usually the case, tan δ is nearly constant. Thus the dielectric losses may become appreciable at very high frequencies, even when they are negligible at lower frequencies.

If any metal screws are used in the construction of the former, it is also necessary to take into account the power losses due to eddy currents in these screws, and to these should be added those in the terminals of the coil. Such losses are determined by an equation similar to (21). It follows that care should be taken to place such screws or terminals only where the magnetic field H

is small, and that their diameter d should be as small as is practicable. Furthermore, it may be noticed that whereas r_0 is always proportional to the resistivity ρ of the metal, at low frequencies $G \propto 1/\rho^2$, but at high frequencies $G \propto 1/\rho^{\frac{1}{2}}$. It follows that for low frequencies $(z < 1)$ $W_h \propto 1/\rho$, and it is therefore advantageous to use a metal of high resistivity, but for high frequencies $W_h \propto \rho^{\frac{1}{2}}$, and it is therefore better to use a metal of the lowest possible resistivity. Similar considerations apply to the metal to be used for the construction of screens. For work at high frequencies it is important that metal of very low resistivity shall be used.

11. **Self-Capacitance.** It is well known that capacitance is of unit dimension in length, and it therefore follows that, other things being equal, the self-capacitance of a coil will be proportional to its linear dimensions. Capacitance is also proportional to the dielectric constant of the dielectric. Thus in order to obtain low values of self-capacitance, coils should be made small, and the formers should be made of material of low dielectric constant, the minimum amount of solid material being used. The method of winding should be such that turns which are near together should never differ widely in potential, and the separation between turns should increase as the potential difference between them increases. Such an arrangement will obviously produce an electric field of minimum energy for a given applied voltage, *i.e.*, a minimum capacitance. The single-layer solenoid and single-layer disc coil satisfy this condition, and are therefore admirable in every respect (except bulk). When multi-layer coils are necessary, the winding may conveniently take the form of either a number of disc coils in series and suitably spaced, or else a number of coaxial single-layer solenoids, suitably spaced, the cross-sections of the winding being as shown in Fig. 64 (*a*) and (*b*). For the disc arrangement (axial spacing), the turns are wound in slots in the coil-former. For the solenoid arrangement (radial spacing), the turns must be wound on spacing bars, which are added to the former in turn as the various layers are completed. Examples of formers are shown in Fig. 64.

It is possible to estimate by calculation the self-capacitance of coils of certain simple forms. Thus Howe * has shown by an

* Howe, G. W. O. *Jour. I.E.E.*, 1921, Vol. 60, p. 67.

approximate method that the self-capacitance of a single-layer solenoid in $\mu\mu F$ is approximately equal to $0{\cdot}6R$ where R is the radius of the coil in cm. Variations in the length l of the coil have little effect on self-capacitance over the range $l/R = 1$ to 4.

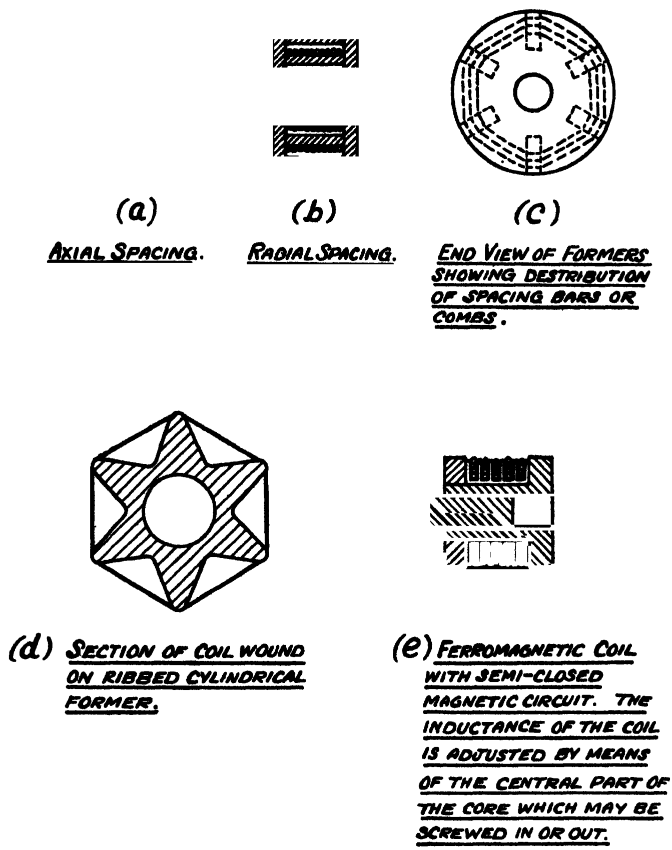


FIG. 64. Inductors of various types.

The calculation assumes air insulation only. The use of any solid dielectric will of course increase the value to some extent.

12. Mechanical Design : Temperature Coefficient. It is of obvious importance that the self-inductance of a standard coil shall be constant under all working conditions. It follows that the structure should be rigid, and should vary as little as possible with changing temperatures. Coils for use at very high frequencies may with advantage consist of only one or two turns of thick copper tube, and in such coils the conductor itself may be sufficiently rigid. In general, however, the rigidity can only be obtained by the use of a former constructed of a solid dielectric. Probably the best material from the point of view of rigidity and dielectric losses is fused quartz, and single-layer coils made by winding copper wire under tension on to fused quartz tubing make excellent standards. If the wire is under tension the thermal expansion of the coil is governed entirely by the quartz tube, and as the coefficient of expansion of this material is very small, the self-inductance of such a coil varies little with temperature. The thermal behaviour of most coils is determined partly by the former and partly by the copper winding. The coefficient of linear expansion of the copper is usually the greater, and moreover, as the wire is usually stressed during winding, and the stresses vary as the coil expands and contracts, the changes of inductance with temperature may be greater than those calculated on the basis of pure linear expansion. Hence any construction in which the geometrical form of the coil is entirely controlled by a rigid former has considerable advantages. This question has been investigated in detail by Thomas,* who has given a design for a standard coil compensated for temperature changes. The solid dielectric he employed was marble, which is also rigid and of low temperature coefficient. Griffiths † had previously devised a form of temperature-compensated coil more suitable for general use, though not capable of the same precision. In Griffiths' design, the turns are wound in either one layer, or several layers spaced from one another, on bars of ebonite which are virtually segments cut from an ebonite cylinder. The wires are definitely located in grooves cut in these bars, and therefore the axial expansion of the coil is determined by the linear expansion of the ebonite bars. This skeleton ebonite cylinder on which the coil is wound is held together by cheeks of bakelite to which the bars are fixed at either end. Thus the radial expansion of the coil is determined by the expansion of the bakelite. The radial expan-

* Thomas, H. A. *Jour. I.E.E.*, 1935, Vol. 77, p. 702.
† Griffiths, W. H. F. *Jour. Sci. Instrum.*, 1929, Vol. 6, p. 354.

sion increases the self-inductance of the coil, while the axial expansion diminishes it. Thus by a suitable choice of the two materials, bakelite and ebonite or similar materials, the coil may be compensated for temperature changes.

The ceramic insulating materials of the steatite class, having great rigidity, and low dielectric losses, are excellent for coil formers. Tubes having a spiral thread moulded on their surfaces are available in certain small sizes, and good coils may be made by winding wire under tension in these screw threads. Coils of relatively few turns may also be made by fusing spiral films of metal on to the surface of these materials. These films can then be thickened by electro-plating, or spraying, and thus a coil of low resistance formed. The dimensions of such coils will obviously be controlled entirely by those of the ceramic materials, which have low coefficients of expansion. Such coils are limited to small sizes. Formers for large coils are usually built up from materials of the bakelite, ebonite or mycalex types. Ribbed cylinders of the cross-section shown in Fig. 64 (*d*) are obtainable in various materials, and are valuable for coil construction in that they provide rigidity with the minimum amount of solid dielectric in the immediate neighbourhood of the wire. Thus they give coils of low self-capacitance and low dielectric loss. The ribs may be slotted in order to locate the turns of the coil.

Toroidal coils (of rectangular cross-section) may be wound on rings turned from sheets of insulating material; or if a large cross-section is required, a former may be built from two such rings separated by suitable spacing pieces. Toroidal formers of ceramic materials are also obtainable. Toroidal coils in which the winding proceeds continuously round the toroid usually have a rather large self-capacitance, since the initial and final turns are adjacent, but if the winding is divided into sections which are connected together so that turns between which there is a high p.d. are widely separated, the self-capacitance may be reduced.

13. Screens. It is often necessary to provide a coil with a screen, sometimes purely electrostatic, often both electrostatic and electromagnetic. A wire cage, so constructed that it provides no closed paths for eddy currents, may form a good purely electrostatic screen, but for complete screening it is usual to enclose the coil in a copper or aluminium can. Toroidal coils have no external magnetic field (provided a single turn passing round the ring in a direction opposite to that of the main winding is added in order to compensate for the spiral effect of the winding) and therefore the screen of a toroid may fit the coil fairly closely without introducing appreciable losses due to eddy currents. The screens of coils of other types must however be kept well away from the windings in order to minimise eddy current losses. Eddy currents in the screen will of course cause variations in effective inductance and resistance, and standard coils are therefore seldom provided with screens, but are placed when necessary inside separate screens of large size. It is obvious that coils which are efficient and at the same time well screened tend to be rather bulky. There are, however, many applications for which a compact coil, which is well screened, is required; and for such purposes coils with ferromagnetic cores have been developed.

14. **Ferromagnetic Cores.** Since the inductance of a given coil is proportional to the magnetic permeability of the surrounding medium, a given inductance can be obtained with a coil of fewer turns and smaller linear dimensions, if the wire is wound on a core of material of high magnetic permeability. Unfortunately the high permeability facilitates the production of eddy currents in the core, and these not only diminish the magnetic flux, and therefore the effective permeability of the material; but they also dissipate energy, and therefore increase the effective resistance of the coil to alternating current. For work at power frequencies laminated iron is generally employed as core material. The very high permeability of the iron increases the inductance enormously, while the high resistance between the laminations is usually sufficient to prevent an undue increase of resistance due to eddy currents. As the frequency increases, thinner laminations become necessary; and at telephonic frequencies even the thinnest available laminations may be unable to prevent excessive eddy currents. Very thin iron wire may then be used, but for telephone loading coils, it is more usual to use a powdered ferromagnetic alloy compressed into a solid mass with an insulating binding material. The same principle may be employed at radio frequencies, but it is obvious that the higher the frequency the finer must be the subdivision of the material in order to suppress

eddy currents. By the use of particles of colloidal size, ferromagnetic cores may be used with advantage at frequencies up to 10 Mc/s., and perhaps even higher. Ferromagnetic nickel-iron alloys such as mumetal are now available with permeabilities up to 10,000 or more, but the permeabilities obtainable by the use of such material in the form of fine powder are only of the order of 10. If we suppose for simplicity that the particles are regularly packed cubes of permeability μ_1, embedded in a medium of unit permeability, and if α is the fraction of the total volume occupied by the insulating medium, calculation * shows that the effective permeability is

$$\mu = \frac{\mu_1 - \frac{2}{3}\alpha(\mu_1 - 1)}{1 + \frac{\alpha}{3}(\mu_1 - 1)}$$

which as μ_1 approaches an infinite value, approaches the value $(3 - 2\alpha)/\alpha$. Thus if $\alpha = 0{\cdot}1$, $\mu = 28$, or a core containing 90 per cent. of iron of infinite permeability would have an effective permeability of the order of 30 only. Nevertheless permeabilities of the order of 10 are found to be well worth while when very compact coils are required. It is to be remembered that power losses in the core due to eddy currents, hysteresis, and also dielectric losses, will add to the resistance of the coil. On the other hand, the increased permeability reduces the length of wire required, and therefore the resistance. The hysteresis loss will depend only on the amount of iron, the eddy current loss will vary with frequency, the size of the particles, their resistivity, and the flux density. The dielectric losses are complicated by the fact that the material is a complex dielectric, consisting of a cross-connected network of resistances and capacitances in series and parallel. The dielectric losses in such composite dielectrics vary considerably with the shape of the particles, their resistivity, and the frequency. The whole problem is very complex, but it is easy to see that any one material is likely to suit only one particular range of frequencies.

It is important to remember that when ferromagnetic materials are used, the inductance may vary with the amplitude of the current or voltage.

* Howe, G. W. O. *Wireless Engineer*, 1933, Vol. 10, p. 1.

Rings moulded in these core materials are obtainable, and toroidal coils are very simply made by winding the turns directly on to the ring. Alternatively, more or less closed magnetic circuits are built up from strips or rods of the material, and former-wound coils are slipped over them. When the magnetic circuits of the coils are closed, the coils may be mounted in fairly closely fitting screens, and are therefore very compact. Coils of maximum linear dimensions of the order of 3 cm., made in this way, may have magnifications Q of the order of 300 over a range of radio frequencies.

15. **Mutual Inductors.** A mutual inductor consists essentially of a pair of coils arranged so that there is a definite mutual inductance between them. The passage of a definite current I_p through one coil (the primary) produces a definite potential difference V_s at the terminals of the other coil (the secondary); and the essential function of such an instrument is to provide a voltage (V_s), the magnitude and phase of which is accurately known in terms of a current I_p. In an ideal instrument, the voltage V_s would be exactly equal and opposite to the e.m.f. induced in the coil, and therefore V_s and I_p would be exactly in quadrature and we could write

$$V_s = j\omega M I_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where M is the value of the mutual inductance of the instrument. The above equation would hold if there were no eddy currents and displacement or capacity currents in the system, but in practice these always exist, and V_s and I_p are not strictly in quadrature, and we must write

$$V_s = [\sigma + j\omega(M + \Delta M)]I_p \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where σ is a small correcting term, usually called the "impurity," and ΔM is a correction, which must be applied to the theoretical value of mutual inductance. ΔM varies with frequency, and is often called the frequency correction. Unless the coils are carefully designed so as to minimise eddy currents and capacity currents, these corrections may become very large at radio frequencies, and it is therefore important to design the coils of a mutual inductor on the same lines as have been discussed for self-inductors. We must also remember that not only does each coil possess self-capacitance, but that the two coils are linked by distributed mutual capacitance. The mutual inductor is thus even more complicated than the self-inductor, and should be used with caution at radio frequencies. The general case is very complicated, and it is usually advisable to connect the two coils together at one point, in order to make their relative potentials definite. The simplest case is obtained when one terminal is made common to both. The system may then be represented by Fig. 65. It may be shown* by straightforward applications of Kirchhoff's Laws for networks, that this system possesses the following properties :—

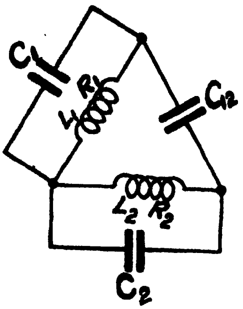


FIG. 65. Equivalent circuit of mutual inductor.

(*a*) Impurity. The impurity σ is given approximately by

$$\sigma = \omega^2[C_1R_1M + C_2R_2M + C_{12}\{R_1(L_2 + M) + R_2(L_1 + M)\}] \quad (32)$$

(*b*) Frequency Correction. ΔM is given approximately by

$$\Delta M = \omega^2[C_1L_1M + C_2L_2M + C_{12}(L_1 + M)(L_2 + M)] \quad (33)$$

It should be noticed that both σ and ΔM are approximately proportional to the square of the frequency. The phase defect $\tan \delta = \frac{\sigma}{M\omega}$ is therefore approximately proportional to the frequency.

In these and the following formulæ, M is to be taken as positive when the mutual inductance of the coils adds to the self-inductance, and negative when it diminishes the self-inductance, of the two coils in series.

(*c*) The effective series resistance R_α and inductance L_α of coil 1 measured with coil 2 attached are approximately given by the formulæ

$$R_\alpha = R_1[1 + 2\omega^2\{C_1L_1 + C_{12}(L_1 + M)\}] \quad . \quad . \quad . \quad . \quad (34)$$

$$L_\alpha = L_1 + \omega^2\{C_1L_1^2 + C_2M^2 + C_{12}(L_1 + M)^2\} \quad . \quad . \quad (35)$$

and similar formulæ give the corresponding values R_β and L_β for coil 2.

* BUTTERWORTH, S. *Proc. Phys. Soc.*, 1921, Vol. 33, p. 313. HARTSHORN, L. *Proc. Phys. Soc.*, 1926, Vol. 38, p. 303.

(*d*) The effective series resistance R_γ, and inductance L_γ, of the two coils connected in series, are given approximately by

$$\begin{aligned} R_\gamma &= R_\alpha + R_\beta + 2\sigma \\ &= R_1 + R_2 + 2\omega^2[R_1C_1(L_1 + M) + R_2C_2(L_2 + M) \\ &\qquad + C_{12}(R_1 + R_2)(L_1 + L_2 + 2M)] \qquad (36) \end{aligned}$$

$$\begin{aligned} L_\gamma &= L_\alpha + L_\beta + 2(M + \Delta M) \\ &= L_1 + L_2 + 2M + \omega^2\{C_1(L_1 + M)^2 + C_2(L_2 + M)^2 + \\ &\qquad C_{12}(L_1 + L_2 + 2M)^2\} \qquad (37) \end{aligned}$$

Notice that the resistance of the two coils in series is not equal to the sum of the resistances measured separately.

(*e*) The effective self-capacitance of coil 1 measured in the presence of the other is given approximately by

$$C_\alpha = C_1 + C_2 \frac{M^2}{L_1^2} + C_{12} \frac{(L_1 + M)^2}{L_1^2} \quad . \quad . \quad . \quad . \quad (38)$$

and the effective self-capacitance of the two in series is given by

$$C_\gamma = C_{12} + C_1 \frac{(L_1 + M)^2}{(L_1 + L_2 + 2M)^2} + \frac{C_2(L_2 + M)^2}{(L_1+L_2+2M)^2} \quad . \quad (39)$$

The system may be further simplified by introducing an electrostatic screen between the two coils, and connecting it to the common terminal. Evidently a metal box or wire screen completely surrounding one of the coils would be necessary. Such a screen must increase the self-capacitance of the two coils, but it reduces the mutual capacitance C_{12} to zero. This simplification is sometimes useful.

16. **Variometers.** Variable self and mutual inductors, sometimes called variometers, usually consist of two coils, one of which can be rotated about an axis, or displaced, so that their mutual inductance is varied. A pointer arranged to indicate the position of the moving coil moves over a scale which may be graduated to indicate either the mutual inductance of the coils, or their total self-inductance when connected in series or parallel. It will be obvious from the preceding discussion that at high frequencies such instruments may be subject to corrections which will vary considerably with both scale reading and frequency, since the capacitances of the system will change with the motion of the coils. They are therefore of little use as standards except at relatively low frequencies.

CHAPTER IX

THE MEASUREMENT OF CAPACITANCE AND INDUCTANCE BY RESONANCE METHODS

1. **General.** The general principles and formulæ utilised in resonance methods of measurement have been outlined in Chapter II, and the apparatus used has been discussed in subsequent chapters. It now remains to consider the details of the measurements.

2. **Measurement of the Effective Self-inductance and Self-capacitance of a Coil by Voltage Resonance.** The

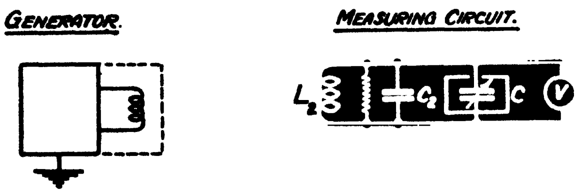


FIG. 66. Circuit for voltage resonance method.

measuring circuit, Fig. 66, consists of the coil under test of self-inductance * L_2, self-capacitance C_2, and conductance G, connected in parallel with a standard variable condenser C and a sensitive thermionic voltmeter V. The circuit is magnetically coupled to a generator, so that a constant e.m.f. E is induced in the coil, so long as the amplitude and frequency of the oscillator remain constant. The capacitance of the standard condenser is varied until the reading of the voltmeter passes through a maximum value corresponding to voltage resonance. The capacitance value corresponding to resonance is determined by taking the mean of two readings, one on either side of the resonance peak, giving the same voltmeter reading. The frequency of the generator is then changed and the process repeated, and in this way a number of corresponding readings of capacitance C and angular frequency ω are obtained. In Chapter II it is shown that the condition of

* *Cf.* Fig. 62 (*b*), p. 143.

voltage resonance is independent of the conductance of the resonating circuit, and that it is given by, (p. 32)

$$L_p C_r \omega^2 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where C_r is the total shunt-capacitance of the circuit at resonance, and L_p is the total self-inductance in parallel with C_r and the voltmeter. If C_v is the capacitance of the voltmeter, and C_l the capacitance of the connecting leads, we have $C_r = C + C_2 + C_v + C_l$ and the condition of resonance may be written :—

$$\frac{1}{\omega^2} = L_2(C + C_2 + C_v + C_l) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If now $\frac{1}{\omega^2}$ is plotted against C, we should obtain a straight line, the slope of which is L_2 (Fig. 67). This value of L_2 includes the

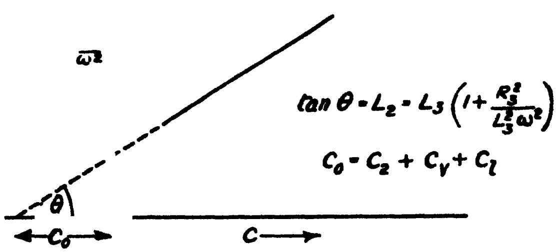


FIG. 67. Curve of observations for measurement of self-inductance and self-capacitance.

self-inductance of the leads connecting the coil to the condenser. This inductance must be separately determined either by measurement or calculation, and deducted from the measured value of L_2. Moreover, when $\frac{1}{\omega^2} = 0$, $C = -(C_2 + C_v + C_l)$. Thus the negative intercept of the line on the C axis gives the value of $C_2 + C_v + C_l$. The value of $C_v + C_l$ may sometimes be made negligibly small, or it may be estimated by calculation, or measured separately by a bridge or resonance method. It may be regarded as a correction which must be applied to the apparent value of self-capacitance determined by the above intercept.

Thus both the self-inductance and self-capacitance of the coil can be deduced from the observations.

In order to obtain accurate results it is necessary to make sure that the conditions assumed in deriving the equations have been satisfied. As we have seen, power losses or leakage in the voltmeter, condenser, etc., have no effect and may be ignored, but in order to ensure that the e.m.f. is induced in the coil only, and that it remains constant as the condenser is adjusted, several precautions must be observed. The measuring circuit must be very loosely coupled to the oscillator so as to avoid changes in the frequency or amplitude of the oscillator due to reaction of the resonating circuit. Provided a sensitive thermionic voltmeter is used, this condition is easily satisfied. The two circuits may be kept one or two metres apart. The coupling should be entirely magnetic ; if it is partly capacitive the induced e.m.f. may vary as the condenser is adjusted. Capacitive coupling is avoided by enclosing the generator in an electrostatic screen. Electromagnetic screening of the coil must of course be avoided. The screen round the coil therefore consists of a Faraday cage, the wires of which are connected together at one end only so as to minimise eddy currents. The screens of the generator and of the condenser and other components of the measuring circuit are connected to earth.* The voltmeter must be screened both electrostatically and electromagnetically, its screen being connected to that of the condenser, and the leads between voltmeter and condenser must be the shortest possible, as it is essential that no e.m.f. shall be induced in the voltmeter itself. The coil must not however be placed near the condenser or any other massive object, otherwise the eddy currents induced in the metal of the neighbouring object may affect its properties. Thus the leads between coil and condenser should be fairly long, say, 20 to 50 cm., and they should preferably consist of a coaxial wire and tube. Such leads have a definite capacitance and inductance, which may be determined once and for all, and they are free from stray mutual inductance. Thus the self-inductances of coil and leads are in this case additive, and subtraction of the leads-inductance is justified. The wire may with advantage be insulated from the tube by means of discs of polystyrene. The tube should of course

* But see Chapter III, §4 on Earth-connections.

be connected to the screen of the condenser and to the screen terminal of the coil (if any). The whole measuring circuit then becomes effectively screened. If the coil has no screen the value obtained for its self-capacitance may change if the connections to its terminals are reversed. The capacitance is smaller when the outer turns of the coil are connected to the screen side of the condenser, voltmeter, etc., for then the capacitance between these turns and the screen is short-circuited : when the connections are reversed this capacitance is thrown into the circuit. The apparent self-capacitance of an unscreened coil obviously depends to some extent on the rest of the circuit, and therefore can never be determined with very high precision unless the circuit is also defined.

It should be noted that the inductance determined by the above procedure is the equivalent self-inductance of the coil L_2, which is related to the equivalent series inductance L_3 by the relation $L_2 = L_3 (1 + R_3^2/L_3^2\omega^2)$. (See Fig. 62, p. 143).

3. **The Method of Current-Resonance.** The above measurements may also be made by observations of current-resonance. The circuit for this method is shown in Fig. 68. The coil under test is connected to a second condenser by co-axial leads as before, but instead of the voltmeter we have a current indicating instrument. Two possible arrangements are shown in the diagram.. A thermojunction T and galvanometer G_T may be included in the circuit, or, alternatively, a crystal or other rectifier X, condenser, and galvanometer G_X are connected to a coil of one or more turns of wire, coupled magnetically to the circuit. The current induced in this coil and measured by the rectifier is evidently proportional to the current in the coil L_3, while the current measured by the thermojunction is that flowing out of the coil and into the standard condenser. We have seen in Chapter II that the condition for maximum current in the coil is $L_3C_r\omega^2 = 1$ where C_r is the total capacitance of the circuit. Thus $C_r = C_3 + C_l + C$. This condition is of exactly the same

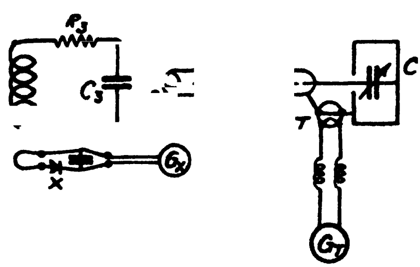


FIG. 68. Circuit for current resonance method.

form as that for voltage resonance; and therefore the inductance L_s and the self-capacitance C_s can be determined by the same procedure as before, the only difference being that the readings taken are those corresponding to maximum current instead of maximum voltage.

If the thermojunction is used the condition observed is that of maximum current in the condenser and this is not quite the same as that of maximum total current, but provided that the self-capacitance of the coil is small compared with the capacitance of the condenser the difference is small and the same procedure can be employed.

In order to obtain accurate results the same precautions as before must be observed about coupling and screening. On the whole the method is less satisfactory than that of voltage resonance, for the proximity of the small coil connected to the crystal rectifier may modify the properties of the coil under test, while the thermojunction does not measure the total current but only that part of it which flows into the external condenser. The junction is shown connected to the terminal of the coil which is at earth-potential; otherwise the leads to the galvanometer G_T may take up a high potential and therefore affect the capacitance of the system. Chokes are shown connected in these leads in order to prevent high-frequency current from reaching G_T. If the crystal rectifier is used it is of course essential that the detector-circuit shall not be oscillatory with a frequency of the order of that used for the test, otherwise an appreciable current might be induced in this circuit by the generator itself. This condition can easily be checked by removing the coil under test, when the galvanometer G_X should show no deflection, even when the coupling of the rectifier circuit to the generator is considerably increased.

4. Resonance-Detection by Reaction on the Generator. It is also possible to detect current-resonance in the measuring circuit by observations made on the generator, and this has the great advantage that it is not then necessary to attach any detecting instrument to the measuring circuit, and there is a corresponding freedom from unwanted capacitance and inductance. The theory of the reaction of the measuring circuit on the generator has been discussed in Chapter II (p. 33), where it is shown that the reactance, and therefore the impedance, of a primary coil undergo characteristic changes as the secondary coil is made to pass through the condition of resonance. Resonance in the measuring circuit may therefore be detected by observations of these changes in the inducing coil of the generator. The reactance changes are usually measured by the beat-frequency method; the impedance changes by the voltmeter and ammeter methods.

(*a*) *Beat-Frequency Method.** For this method the generator must be of a type in which the frequency is determined by a tuned circuit, and the coupling coil of this circuit is used as the inducing or primary coil. The measuring circuit is coupled to this coil and its capacitance varied. When the condition of resonance is approached the reactance of the inducing coil varies, as shown by Fig. 16. The frequency of the oscillator suffers corresponding changes, and if a heterodyne oscillator is also used the changes in the beat-note provide a sensitive indication of the changes of frequency. If the two oscillators are initially adjusted to exact synchronism (the measuring circuit being removed or opened), then the condition of resonance is indicated by the point of silence between two beat notes, as will be evident from Fig. 16. When the highest possible sensitivity is required the two oscillators are adjusted to give initially an audible beat-note, and this note is made to beat with a standard audible frequency (the so-called double-beat method). The variations of these beats serve to detect the changes of frequency with the utmost sensitivity. This method is ideal inasmuch as the measuring circuit is free from unwanted leads and components: the resonance frequency of a coil with no attachments may be measured in this way. But it is to be remembered that it is assumed that the frequency of the oscillator depends only on the reactance of its tuned circuit. The tuning of the measuring circuit also affects the resistance of the primary circuit, and these resistance changes are liable to cause small changes of frequency and therefore to introduce small errors. The use of the Hartley oscillator is recommended for this method, and it is often possible by varying the grid-coupling condenser of this oscillator to make its frequency practically independent of small resistance changes in the tuned circuit.

* Colebrook, F. M., and Wilmotte, R. M. *J. Inst. Elec. Eng.*, 1931, Vol. 69, p. 497. Colebrook, F. M. *Wireless Engineer*, 1931, Vol. 8, p. 639.

(*b*) *Voltmeter and Ammeter Methods.* The variation of the impedance of the inducing coil as the secondary circuit passes through the condition of resonance follows a curve very similar to that of Fig. 16, although the resonance point is not in this case equidistant from the maximum and minimum values. This variation may be observed by maintaining a constant current in the inducing coil and observing the voltage across it, which is proportional to the impedance. Mallett and Blumlein * have given a graphical construction for determining with precision the condi tion of resonance from such a curve, but when the resonating circuit is of low decrement the central portion of the curve is so steep that the departure of the resonance point from its middle point is negligible. An alternative procedure is to maintain a constant voltage across the inducing coil and to observe the variations of current, but the method is perhaps most widely used in the approximate form in which the secondary circuit is loosely coupled to the tuned circuit of a simple oscillator, the anode current of which is observed, as the secondary circuit is smoothly adjusted. On passing through the condition of resonance the sharply defined changes of impedance corresponding to Fig. 16 cause similar changes in the anode current, and the sudden change corresponding to the middle part of the curve indicates the condition of resonance.

5. Variation of Self-Inductance and Self-Capacitance with Frequency. The resonance methods for the determination of self-inductance L and self-capacitance C_0, all depend on an equation of the form :—

$$\frac{1}{\omega^2} = L(C + C_0)$$

and they only give definite results when the experiments show that this equation is linear (*cf.* Fig. 67). We have however seen that the self-inductance of a coil is apt to vary with frequency owing to eddy currents, so that even though C_0 may be constant the linear relation will not in general be exact. As explained in Chapter VIII, the inductance L diminishes with rise of frequency, and it follows that the curve connecting $1/\omega^2$ and C will curve upwards as in Fig. 69. The self-inductance tends to approach a

* Mallett, E., and Blumlein, A. D. *Jour. I.E.E.*, 1925, Vol. 63, p. 397.

constant value as the frequency becomes very high (*cf.* Fig. 63) so that the portion A B, say, of the experimental curve, is sensibly straight and yields the correct value of C_0 (Fig. 69). As the frequency diminishes from ω_1 corresponding to B to ω_2 corresponding to D, say, the inductance increases from the limiting high-frequency value L to the low-frequency value L_0, and therefore the slope of the curve gradually increases in the region B D reaching a constant limiting value at D. The variation of inductance with frequency can be determined by carrying out the measurements already described over various frequency ranges and measuring the slope of the experimental curve for each range. However the value of the true self-capacitance cannot be obtained by extrapolating the various curves. As shown in Fig. 69 the increase of self-inductance causes a comparatively large diminution of the intercept on the C axis, or the apparent self-capacitance, which may even be negative in some cases. These smaller values are useful constants representing the behaviour of the coil over the corresponding range of frequencies, and they may for convenience be referred to as the effective self-capacitance over the appropriate range of frequency, but they obviously cannot be identified with the true self-capacitance of the coil. That can only be obtained if it is possible to make measurements at frequencies so high that the variations of L are negligible.

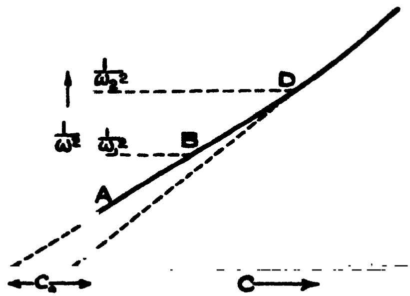
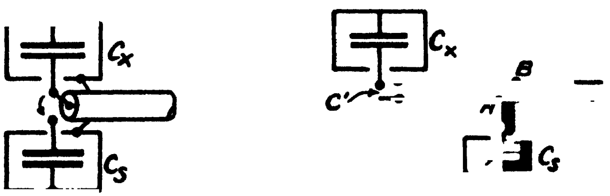


FIG. 69. Curve showing apparent variation of self-capacitance with frequency.

The self-capacitance of a coil having no former can also be measured by tuning it to resonance with a condenser connected in parallel, then immersing the coil in a medium of known dielectric constant ϵ, and finding the diminution of capacitance ΔC necessary to restore the condition of resonance. Then we have :—

$$\Delta C = (\epsilon - 1)C_0$$

6. Measurement of the Capacitance of a Condenser. If the condenser is placed in parallel with a coil of known self inductance and self-capacitance, and the frequency of resonance is determined by one of the methods outlined above, the capacitance of the condenser can obviously be deduced from the equation of resonance (2). It is however usually much more convenient in practice to use a standard variable condenser and to adopt the substitution method, which is as follows. The condenser under test C_x and the standard condenser C_s are placed side by side with their insulated terminals quite near together as in Fig. 70. A pair of coaxial leads is arranged as shown. The outer conductor being connected to the screen terminals of both condensers, and the inner wire being arranged so that a very short portion projecting from the outer tube may be connected at will to the insulated terminal of either condenser. The inner wire is con-



(a) SIMPLE SUBSTITUTION. (b) DIFFERENCE-SUBSTITUTION.

FIG. 70. Connections for measurement of capacitance by substitution.

nected first to C_x and the condition of, say, voltage resonance established. The wire is then transferred to C_s, thereby substituting C_s for C_x in the circuit, and the condition of resonance again established by varying C_s. When the substitution causes no change in the resonance frequency the capacitance of the two condensers must be equal, and the value of C_x is given by the reading of C_s. The following points should be noted. The screens of both condensers are connected in circuit the whole time so that the earth-capacitance of the circuit remains constant. As far as possible the leads are rigid and symmetrically arranged. The only movable part of the circuit is made very small and moves through the shortest distance possible. Thus the changes of stray capacitance by the motion are as small as possible, and their effects on the two condensers are similar and the result of the substitution measurement is not affected by them.

The above procedure is only possible when there is a point on the scale of C_s at which its capacitance is equal to that to be measured. An alternative scheme which is of more general application is as follows. The standard condenser C_s is connected permanently in circuit, while C_x is arranged exactly as before, *i.e.*, screen-terminal permanently connected to the screen lead, and insulated terminal capable of connection or disconnection by a slight displacement of a short length of the insulated lead, which acts as a switch of very low capacitance. The condenser C_s is set at a low value, which is noted, C_x is connected in circuit and resonance is again established. The difference ΔC_s between the two values of C_s is, apart from small corrections, equal to C_x. A correction may be necessary, since the displacement of the insulated lead does not completely remove C_x from the circuit; for if C' is the small capacitance between the insulated terminal of C_x and the insulated lead after disconnection, the series combination of C' and C_x is still in circuit. If C_x is large compared with C', the capacitance of the series combination is practically equal to C'. Thus the measured capacitance is too small by this amount, *i.e.*, $C_x = \Delta C_s + C'$. The correction C' is generally of the order of $0{\cdot}1\ \mu\mu F$.

It is not always convenient to arrange the circuit symmetrically as in Fig. 70 (*a*). If the arrangement is unsymmetrical, as in Fig. 70 (*b*), there may be an error due to the inductance of the leads. If, for example, the inductance of the leads from the points A B to C_s is l_s, and that of the leads from A B to C_x is l_x, then the effective capacitance between the points A B are respectively $C_s/(1 - l_sC_s\omega^2)$ and $C_x/(1 - l_xC_x\omega^2)$ instead of C_s and C_x. (*cf.* Chapter I, p. 12). The corrections for the inductance of the leads are apt to become very important at very high frequencies, and in such cases the appropriate corrections must be applied to each capacitance reading C_s before calculating ΔC_s, and then the apparent value of C_x must be corrected for l_x. These corrections must also be applied when the procedure described in the previous paragraph is adopted; they only cancel out when $l_s = l_x$ and $C_s = C_x$, as in the strictly symmetrical version of Fig. 70 (*a*). It is advisable to use coaxial leads for the arrangement of Fig. 70 (*b*) as well as Fig. 70 (*a*), for not only have such leads a definite and calculable inductance, but they are also free from stray mutual inductance to the rest of the circuit and the generator. Any such mutual inductance may introduce undesired e.m.f.s into the corresponding portions of the circuit and so introduce errors.

It should be noticed that when measuring capacitance by substitution the condenser under test is not removed or disconnected from both leads. Such a procedure would not only remove C_x from the circuit; it would also remove the capacitance between the insulated lead and the screen terminal of the condenser, and this extraneous capacitance would be included in the quantity measured.

The method of substitution as here described is applicable not only to all resonance methods, but also to bridge and other methods. When choosing a method it should be remembered that the voltage-resonance method is free from error due to the conductance of the condenser and is therefore a good method when leakage is suspected.

The measurement of three-terminal condensers will be considered under bridge methods.

7. Measurement of the Self-Capacitance of a Resistor. Resistors of low value are usually inductive, but in those of high value the distributed capacitance usually outweighs the inductance, and the resultant susceptance is capacitive.

In order to measure the self-capacitance of such a resistor the method of voltage resonance is used. The measuring circuit having been tuned, the resistor is connected in parallel with the tuning condenser and resonance again established. The difference between the two readings of the tuning condenser gives the self-capacitance of the resistor. The capacitance of the leads employed for the connection of the resistor can be measured separately in the same way and allowed for if necessary. The conductance of the resistor has no effect on the condition of voltage resonance, but it may destroy the sharpness of resonance if the frequency is not sufficiently high. The resistor must be disposed so that no e.m.f. is induced in it by the generator; otherwise the conditions for simple substitution, which have been assumed to hold, are not satisfied.

8. Measurement of Self-Inductance of a Choke. Self-inductances of high value such as those of chokes can be measured by a method very similar to that of the preceding paragraph.

The procedure is exactly the same: the reading of the tuning condenser C_1 corresponding to voltage resonance is first noted. The choke is then connected in parallel with the condenser and resonance again established by adjusting the condenser to a new value C_2. The additional capacitance $C_2 - C_1$ is obviously that required to tune the choke itself to resonance, and the inductance of the choke L_x is therefore given approximately by the relation

$$L_x = \frac{1}{(C_2 - C_1)\omega^2}.$$

The method applies only to frequencies at which the choke behaves approximately as a pure reactance.

9. **Methods of Energising the Measuring Circuit: Couplings.** The method of energising the measuring circuit described in paragraph 1 is that of pure magnetic coupling

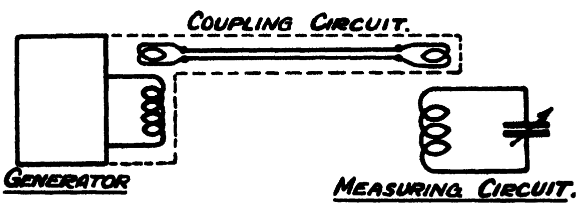


FIG. 71. Arrangement of intermediate coupling circuit.

between the generator and measuring circuits. If neither the inducing coil of the generator nor the measuring circuit is electrostatically screened, the electrostatic coupling is usually at least comparable with the magnetic coupling, and it is then found that the current in the measuring circuit cannot be reduced to zero by adjustment of the position of the two circuits. The electrostatic coupling is apt to be distributed over the whole measuring circuit, and therefore to be varied when any adjustment of the circuit is made. This state of affairs is obviously undesirable, and capacitance coupling is therefore best avoided by electrostatic screening.

When direct magnetic coupling is employed, the generator and measuring circuit are frequently about a metre or more apart, and it is therefore difficult to localise the induced e.m.f. in the measuring circuit. When this is essential, an intermediate coupling circuit, as shown in Fig. 71, may be used. An inducing coil of one or two small turns is placed quite near the coil into which the e.m.f. is to be induced. This is connected by long screened leads to a corresponding coil near the oscillator. When it is necessary that the circuit shall be balanced with respect to earth, the coils of the intermediate circuit may be connected to earth at their middle points. Such a potential distribution is not usually possible when direct coupling is employed.

Another means of applying a definite e.m.f. to a circuit is the so-called resistance coupling shown in Fig. 72. A measured current I is passed through a resistance r very low compared with that of the measuring circuit of which it forms a part. The voltage rI is therefore "injected" into the circuit at the place occupied by r. If when the circuit is tuned to resonance the voltage across the coil is V, the voltage magnification Q of the coil is given approximately by V/Ir. This is a good example of the kind of measurement for which the method of coupling is most useful.

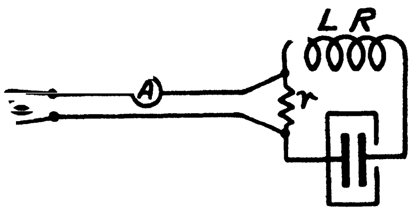


FIG. 72. Resistance coupling.

10. Measurement of the Residual Inductance of a Condenser.* The significance of the inductance of a condenser has been explained in Chapter VI, p. 118. The simplest method of measuring this quantity is undoubtedly to short-circuit the terminals of the condenser by means of a copper strip or wire of large diameter, and to determine the resonance frequency of the oscillatory circuit so formed. This measurement may conveniently be made by the simple method described in paragraph 4 (*b*). The condition of resonance is

$$(L_c + L_0)C\omega^2 = 1,$$

where L_c is the inductance to be measured, L_0 that of the short-circuiting link, which is very small, and can be estimated by calculation, C is the capacitance of the dielectric path of the condenser, which is practically equal to the capacitance of the condenser at low frequencies. Thus C and L_0 are known, and ω is

* WARD, W. H. *J. Sci. Instrum.*, 1936, Vol. 13, p. 251.

observed; the inductance L_c can therefore be calculated from the above equation.

It is sometimes impossible to determine the resonance frequency of the condenser as described above, either because the frequency is too high or because it is not possible to induce an adequate e.m.f. into the circuit. In such cases the following method can be employed. Two parallel wires which can be short-circuited by a movable bridge are connected to the terminals of the condenser as in Fig. 73. The arrangement forms an oscillatory circuit with a rectangular coil of variable length but constant breadth. The resonance frequency of this circuit is observed for various positions of the bridge by the method previously mentioned. The condition of resonance is

FIG. 73. Circuit for measurement of residual inductance of a condenser.

$$(L_c + L)(C + C_L)\omega^2 = 1$$

where L is the inductance of the rectangular coil, C_L is its self-capacitance, C is the capacitance of the condenser, and L_c its inductance, which is to be measured. Thus for a constant value of C (a constant setting of the condenser), $1/\omega^2$ is a linear function of L. If therefore $1/\omega^2$ is plotted against L a straight line is obtained, the slope of which gives $C + C_L$, while the intercept on the L axis gives L_c. Alternatively, if $1/\omega^2$ is plotted against C, as in Fig. 67, a straight line is obtained, the slope of which gives $L_c + L$, while the intercept on the C axis gives C_L. The value of L can be obtained by calculation or by a bridge method at low frequency. The adjustable rectangular coil described above is not of course essential; a series of circular coils may be used, but the arrangement of parallel wires is convenient since it gives a series of values of L, which are a linear function of the length of the rectangular loop. Thus if $1/\omega^2$ is plotted against the length of the loop, l, a straight line is obtained and its intercept on the l axis gives the length of loop to which the inductance of the condenser is equivalent. The calculations are therefore simplified.

11. Measurement of the Variation of the Capacitance of a Condenser with Frequency. It is explained in Chapter VI that the capacitance of a condenser may diminish with rise of frequency owing to dielectric absorption in any solid dielectric which may be present, and that it may increase with rise of frequency owing to the inductance of the plate systems and internal leads. The latter effect usually predominates and the inductance of a condenser can therefore be deduced from the variation of capacitance with frequency. Such measurements can only be made if a standard condenser of known or negligible inductance is available. A good condenser for such purposes can be made from two coaxial metal tubes, which can be insulated from one another by small wedges of fused quartz or polystyrene, or any material in which dielectric absorption is negligible. It is obvious that the inductance of the tubes will diminish as their diameter increases and as the radial spacing diminishes, and if tubes of about 7 cm. diameter, with a radial gap between them of about 1 mm. are employed, the inductance will be found to be less than 0·001 μH and therefore negligible for lengths of, say, 10 cm. The terminals should be placed on the tubes themselves at one end; their distance apart can then be adjusted by a rotation of one tube.

The procedure is as follows. The standard coaxial condenser is connected to an inductance coil and thermionic voltmeter, and the circuit so formed adjusted for voltage resonance as previously described. The coil, voltmeter, and connecting leads should all be electrostatically screened and all the screens connected together and to the screen (outer tube) of the condenser, so that the capacitances between all the screens are short-circuited and have no effect on the resonance point. The standard condenser is then removed and the variable condenser under test substituted for it, the leads used being the same in the two cases. The setting C of the variable condenser which restores the condition of resonance is observed. This procedure is repeated at various frequencies using different coils. It will be found that for all fairly low frequencies the value of C is constant and equal to the value of the standard condenser C_0, the effect of inductance being negligible. At any high frequency $\omega/2\pi$ the effective capacitance of the variable condenser will be increased by its self-inductance, and the setting C will correspond to a value $C/(1 - L_cC\omega^2)$ (Chapter VI, equations 12 and 17). It follows that the setting C of the variable condenser will diminish with rise of frequency in accordance with the equation

$$C_0 = \frac{C}{(1 - LC\omega^2)}.$$

Thus the inductance L can be calculated from the value C_0 obtained at lower frequencies (say, 1 Mc/s). The above equation has been obtained on the assumption that the effects of dielectric absorption and resistance are negligible, which is usually the case. Even if they are not negligible it is convenient to represent the observed variation of capacitance with frequency by the same equation, the value of L then being the "equivalent inductance" of the condenser. For standard variable air condensers the value is commonly of the order 0·03 μH. Experimental details and values are given by Ward.*

12. **Measurement of Very Small Inductances.** Very small inductances of the order of, say, 0·1 μH or less can sometimes be measured by the methods described for the measurement of the residual inductance of a condenser. Thus the small inductance may be connected to the terminals of a condenser of known capacitance and inductance, and the resonance frequency of the circuit determined. The total self-inductance of the circuit can then be calculated from the equation of resonance and this value, diminished by the inductance of the condenser, gives the required added inductance. This calculation assumes that the mutual inductance between condenser and added inductor is negligible, and such values are therefore always subject to a slight uncertainty on this account. It is however to be remembered that inductance values only apply with precision to complete circuits, and that the values for small inductors can never be more than approximate owing to the nature of the quantity to be measured.

If the coaxial tubular condenser is used for these measurements the inductance of the condenser is negligible. If the resonance frequency is then too high for measurement it can be reduced by adding a suitable length of coaxial lead between the condenser and the inductor to be measured. The inductance of

* Ward, W. H. *J. Sci. Instrum.*, 1936, Vol. 13, p. 251.

this lead can readily be calculated and allowed for, and it will also be practically free from mutual inductance.

Fortescue* has described a substitution method for very small inductances. His variable standard inductance consists of a straight wire in a coaxial brass tube, a movable plunger making contact between wire and tube. The change of self-inductance due to a given displacement of the plunger is readily calculated. This standard is included in a resonance circuit: the inductance to be measured is added in series with the circuit, and the amount by which the plunger must be displaced in order to restore the condition of resonance is measured. Neglecting small corrections,

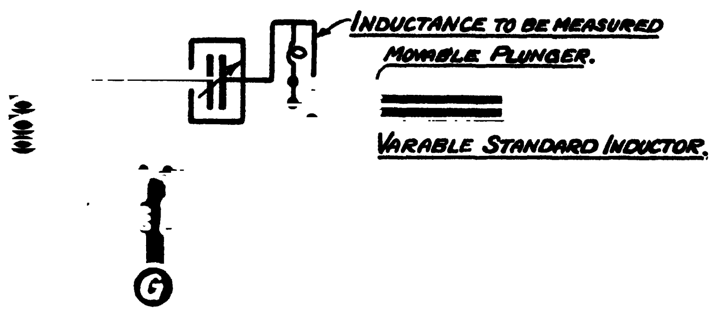


FIG. 74. Arrangement for measurement of very small inductances by substitution. (Fortescue.)

the corresponding change in the inductance of the standard is equal to the added inductance. It is to be noted that the resistance of the circuit changes during the measurement and therefore current resonance should be employed, since this is independent of series resistance. The circuit is shown in Fig. 74. It is essential that the current in the inductor under test and the variable standard shall be the same: their insulated terminal should therefore be connected together by the shortest possible link, as in the diagram. Any capacitance between this link and the screens of the various instruments will lead to stray displacement currents, which may cause errors. The inductance under test should be removed from the circuit when necessary by short-circuiting, so that the various screen capacitances remain unaltered. Fortescue has described a special form of mercury switch for this purpose,

* FORTESCUE, C. L. *J. Sci. Instrum.*, 1933, Vol. 10, p. 301.

and has also given formulæ for the corrections which must be applied when the departures from the above-mentioned conditions are appreciable. At very high frequencies it is of course necessary to take into account the distributed capacitance of the coaxial inductance (*cf.* Chapter VII, § 7, and Chapter X, § 13).

13. Measurement of Very Small Capacitances, e.g. of Valves. Very small capacitances, such as those of valves, can readily be measured by resonance methods. The conditions necessary are a very sensitive indicator of resonance and a standard condenser capable of very fine adjustment. A micrometer condenser (p. 117) serves admirably for the standard. If one is not available, a very fine adjustment of capacitance can be obtained by using a small fixed condenser c in series with a variable condenser C of the ordinary type. The capacitance of the combination is given by $Cc/(C + c)$, and it follows that a small change of capacitance ΔC of the large condenser produces a change in that of the combination of approximately $\Delta C \,.\, c^2/C^2$, and since the ratio c/C can be made very small, it is possible to produce extremely small known changes of capacitance in this way.

Perhaps the most sensitive method of resonance detection is the reaction beat-frequency method (p. 169), and this may be used provided there is no danger of the changes of capacitance being accompanied by changes of resistance of sufficient magnitude to affect the beat-frequency on their own account.

When such changes of resistance occur, *e.g.*, when the small capacitance is associated with considerable conductance, voltage resonance with a thermionic voltmeter as detector may with advantage be used. The resonance peak itself is necessarily not very sharp and the mean of two capacitance readings, one on either side of the peak, giving the same deflection is usually taken. This however considerably lengthens the number of observations. Sometimes an additional small fixed capacitance C_a is arranged so that it can be switched into the circuit at will, and the adjustment for resonance is then made by varying the main condenser until a point is reached at which the voltmeter reading is unchanged on switching in the small condenser C_a. The setting then corresponds to the point A in Fig. 75, and is evidently much sharper than that corresponding to the resonance peak itself. It is obvious that for substitution measurements this setting is just as satisfactory as the resonance peak itself. It is to be remembered that the addition of C_a must not cause any change in the e.m.f. induced in the circuit. The leads connecting this condenser to the main condenser must therefore not form an open loop into which an appreciable e.m.f. might be induced by the generator.

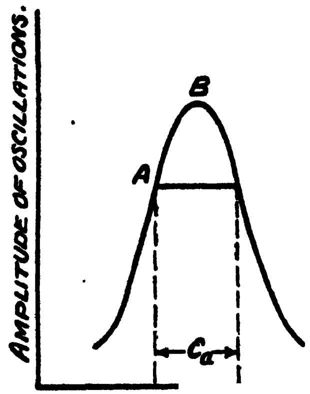


FIG. 75. Curve illustrating the use of a small fixed additional condenser C_a to increase the sharpness of the resonance adjustment. Setting to the point A is obviously sharper than to B.

Small capacitances can also be conveniently measured by their direct effect on the frequency of an oscillator of, say, the Hartley or the dynatron types. The capacitance to be measured is added to the tuned circuit of the oscillator, and the change of frequency compensated by the adjustment of a suitable standard condenser included in this circuit. Thus the measurement is again one of substitution. The changes of frequency are detected by the heterodyne method, a crystal-controlled oscillator of constant frequency being used as the auxiliary oscillator for the production of the beat-note. When the highest sensitivity is required the "double-beat" method (p. 169) of detection may be employed.

Generally speaking, however, the accuracy of measurement of very small capacities is limited not so much by lack of sensitivity of the detector as lack of stability of the circuits. Mechanical rigidity, smoothness of adjustments, and perfection of screening of the apparatus are usually more important than devices for magnifying the response of the detector to a given small change of capacitance.

CHAPTER X

RESISTANCE, POWER FACTOR, DECREMENT, ETC., BY RESONANCE METHODS

1. The Resistance-Variation Method. The most direct method of measuring the resistance of a circuit at high frequencies is that known as the resistance-variation method, which is based on the fact that the current I in a circuit which is adjusted for current resonance, is governed by the simple relation $I = E/R$, where E is the e.m.f. induced in the circuit, and R is its resistance. The circuit used is that shown in Fig. 68, the thermal converter being used as detector, with the addition of a variable resistor immediately adjacent to the current-measuring instrument, which must be calibrated to read relative values of current. If a thermojunction with sensitive reflecting galvanometer is used, it is usually sufficient to assume that the deflection θ is proportional to the value of I^2. The circuit is first adjusted for current resonance with no added resistance, and the maximum deflection θ_m, or value of current I_m noted. A known resistance R_x is then added to the circuit, the resonance adjustment checked, and the new value of current I or deflection θ noted. We now have the relation

$$\frac{R + R_x}{R} = \frac{I_m}{I} = \sqrt{\frac{\theta_m}{\theta}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

from which R the resistance of the circuit is easily obtained.

It is advisable to take several values of R_x and to note the corresponding values of I or θ. From the relation

$$R + R_x = \frac{E}{I} = \frac{E}{k\sqrt{\theta}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

it follows that if $1/\sqrt{\theta}$ is plotted against R_x, a straight line should be obtained, its intercept on the resistance axis giving R. The deviations of the observed points from the straight line provide a valuable indication of the probable accuracy of the result.

It should be observed that the method is only valid so long as the induced e.m.f. remains constant, and in order to ensure this constancy, not only must the coupling conditions discussed in the previous chapter be satisfied, but the changes of resistance must be made in such a manner that the configuration of the circuit remains unaltered throughout. Various resistors satisfying this condition to a greater or less degree of accuracy are described in Chapter VII. The slide-wire of constant inductance is very convenient when high accuracy is not essential, and at the lower frequencies it is possible to use a resistance box with dial switches, but for the highest accuracy it is almost essential to use straight-wire resistors. Two mercury cups are included in the circuit immediately adjacent to the thermo-element, and straight-wire resistors of constant length are connected in turn between these cups. Using straight wires of length, say, 2 cm., it is possible to obtain resistances from about 0·1 ohm to 100 ohms with wires varying in diameter and material. At very high frequencies the variations in self-inductance due to variation in diameter of wire may not be permissible. It is then necessary to use wires of the same diameter but different material, and even then the variations of internal inductance among the various wires must sometimes be considered. The desirability of using several values of R_x and checking the linear law will therefore be easily understood. Since fine wire resistors are not usually very stable in value, their d.c. resistance values should be checked frequently.

2. **The Reactance-Variation Method.** In the resistance-variation method discussed in the last section, the resistance of a resonating circuit is deduced from the reduction of current produced by the addition of a known resistance. The resultant reactance of the circuit is always reduced to zero, and therefore does not affect the measurement. A change of reactance of the circuit also causes a reduction of current and voltage, and since a series of known changes of reactance are usually more easily made than corresponding changes of resistance, it is important to consider a method whereby the resistance, conductance, power factor, etc., of a circuit can be deduced from the effect of known changes of reactance. The method is often called the reactance-variation method, or since the changes of reactance necessarily put the circuit somewhat out of tune, the de-tuning method, or

sharpness of resonance method. The changes of reactance may be made by changes of frequency or of capacitance, but since the method of capacitance-variation is the more satisfactory, we shall confine our attention to it. The governing equation was derived in Chapter II (p. 26). For a circuit in which the condenser (and voltmeter, if any) are free from leakage and dielectric losses, it may be written

$$\tan \delta = \frac{R}{L\omega} \doteq \frac{C_1 - C_2}{(C_1 + C_2)\sqrt{q-1}} \quad . \quad . \quad . \quad . \quad (3)$$

This equation was derived for both current and voltage resonance, and either may be used provided the condition of negligible conductance in the condenser, etc., is satisfied. In this equation C_1 and C_2 are the two readings of capacitance (one on either side of the resonance peak) which correspond to a mean square current or voltage of one qth of the value at resonance, and for voltage resonance $C_1 + C_2 = 2C_r$. Thus the loss tangent of the coil, or its reciprocal the magnification factor Q, may be deduced from these readings, and if the inductance is also measured by the methods already discussed, the resistance can be obtained.

It has also been shown in Chapter II that when leakage, etc., is not negligible, the equation of voltage resonance is simpler than that of current resonance, and this case alone will therefore be considered in detail. The circuit may be represented by Fig. 66, in which, in practice, V is a sensitive thermionic voltmeter, whose deflection is proportional to the square of the voltage. The procedure is as follows. The capacitance is varied until resonance is obtained and the maximum deflection θ_m and the corresponding capacitance C_r are noted. The capacitance is then increased until the deflection is reduced to $\theta_m/2$ (*i.e.*, $q = 2$) and the corresponding reading C_1 noted. The resonance readings θ_m, C_r are then checked, and the capacitance reduced until the deflection is again halved, and the corresponding reading C_2 noted. We may then apply equation (20) (p. 33),

$$\Delta C_q = C_1 - C_2 = \frac{2}{\omega} \cdot G_t \cdot \sqrt{q-1} \text{ or } G_t = \frac{\omega \cdot \Delta C_q}{2\sqrt{q-1}}, \quad . \quad (4)$$

which gives the equivalent conductance of the whole circuit in terms of the capacitance-difference $C_1 - C_2 = \Delta C_q$, say. The value of ΔC_q corresponding to other values of q, say 3 and 4, are also observed, and in this way a few independent values of G_t are obtained. The deviation of the values from the mean gives a valuable indication of the accuracy of the result.

It is to be noted that this method is essentially a measurement of the total conductance (or power loss/V^2) of the circuit, just as the resistance-variation method is a measurement of the total resistance. Values for individual components can only be obtained if the corresponding values for the remainder of the circuit are either already known, or determinable by an additional measurement of the same kind. We have already noted that if the condenser and voltmeter are known to be free from power losses, the observations give the magnification factor and loss tangent of the coil. The losses in coils are never negligible, so that measurements on condensers, dielectrics, resistors, etc., must always be made as a difference measurement, *i.e.*, by a determination of the change in total conductance produced by the addition of the component in question to the circuit. In all cases such as this where two values of conductance are measured, a valuable additional check on the results may be obtained by observations of the voltmeter deflections at resonance for the two conditions of the circuit. For in Chapter II it is shown that the maximum voltage is given by

$$V_r = \frac{E}{R + j\omega L} \cdot \frac{1}{G_t},$$

where E is the induced e.m.f. and $R + j\omega L$ is the impedance operator of the coil, and as long as both of these remain constant the voltage at resonance is proportional to the total conductance of the circuit. Thus when a square-law voltmeter is used, the ratio of the two conductance values G_1 and G_2 should be equal to the square root of the ratio of the two maximum deflections θ_1 and θ_2, the working equation being

$$\frac{G_2}{G_1} = \frac{V_1}{V_2} = \sqrt{\frac{\theta_1}{\theta_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It should also be noticed that when one value of G_t and the corresponding maximum deflection have been measured, any other value of G_t can be immediately obtained from a single reading of the deflection at resonance. As in the previous method care must be taken to ensure the constancy of the induced e.m.f. E.

3. Measurement of the Effective Resistance, Conductance, and Power Factor of a Condenser. This measurement as previously explained must be made by a measurement of the substitution type. The condenser under test is included in a circuit which can be tuned to the required frequency, and the total resistance or conductance of the circuit is measured by one of the above methods. The condenser is then removed from the circuit by a suitable disconnection, and replaced by a standard condenser of either negligible or known resistance and conductance. The resistance or conductance of the circuit is then again measured. The difference between the first and second measurements obviously gives the required value for the condenser. If the value for the standard condenser is known, it must be added to the measured difference. It is essential that the losses in the remainder of the circuit shall not be affected by the change of condenser, *e.g.*, the resistance of the leads must not be changed. The arrangement of Fig. 70 (a) may be used, care being taken to ensure that the leads to the two condensers C_x and C_s are exactly similar, and therefore of the same resistance.

We have shown in Chapter VI that ordinary standard condensers are never quite free from losses, and therefore their resistances and conductances are always appreciable. At the lower radio frequencies the losses are mainly due to the solid insulation, and may therefore be represented by a constant conductance, which is independent of the setting of the condenser. This loss may be eliminated from the measurements by the use of a difference-substitution method (Fig. 70b). In this case the standard condenser remains in circuit for both measurements, but its capacitance is increased for the second measurement. The increase of capacitance may be regarded as completely free from loss, unless the effect of the resistance of the plate systems, internal leads, and terminals is appreciable. This is likely to be the case at the higher radio frequencies, and therefore for the most accurate measurements it is necessary to use condensers specially designed to have zero plate resistance * as well as constant conductance, with varying capacitance. An

* Dye, D. W. *Proc. Phys. Soc.*, 1928, Vol. 40, p. 285.

arrangement used at the National Physical Laboratory is shown in Fig. 76. It consists essentially of a pair of coaxial copper tubes of fairly large diameter (7 to 10 cm.) and small radial separation (say 1 mm.) forming a cylindrical air condenser. The solid insulation is provided by three hollow quartz pillars which are clamped between lugs on the bases of the cylinders. The copper cylinders can be unscrewed from the bases, thereby removing an almost perfectly pure capacitance from the system, for the resistance of any length of the copper cylinders is negligible at almost all frequencies at which capacitances of the magnitude corresponding to the particular length of cylinder, are likely to be used. Several pairs of copper cylinders are available, so that capacitance of various values may be obtained. For measurements of the type under discussion, it is essential that the leads connecting together the standard and test condenser shall be of negligible resistance. The standard coaxial condenser is clamped to the terminals of the condenser under test, the connection being made by heavy copper lugs projecting from the base of the standard. (These are omitted from Fig. 76 in order to avoid confusion.) The outer tube which acts as a screen for the standard is of course connected to the screen of the condenser under test. The reactance-variation method is, in theory at least, the simplest in this connection.

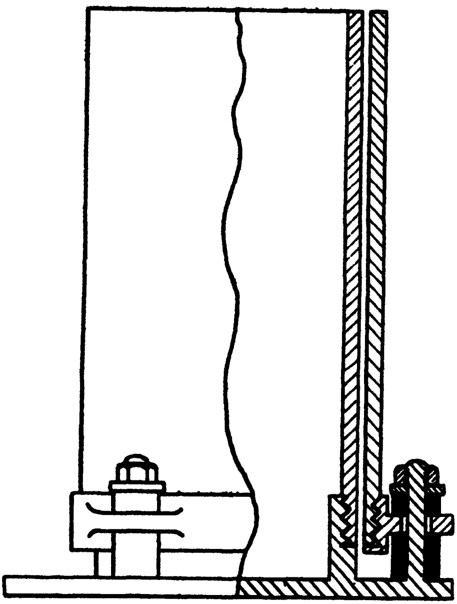


FIG. 76. Special condenser providing a capacitance-difference free from power loss.

The procedure is as follows. The base of the standard, which is permanently connected to the measuring circuit, is clamped to the terminals of the condenser under test, and the total equivalent conductance of the circuit containing the condenser is measured. This may be denoted by $G_L + G_v + G_b + G_x$, the four terms representing the conductances of the coil, voltmeter, base of the standard, and test-condenser respectively. The insulated terminal of the condenser C_x is then disconnected, and tubes of a length required to restore the capacitance to its original value are screwed on to the base of the standard. The total conductance of the circuit is then again measured. It is now equal to $G_L + G_v + G_b$ and therefore the difference of the two values gives G_x. It will be clear that this procedure is only applicable to condensers having capacitances equal to those of the standard, but if a variable air condenser is calibrated in this way at the points on its scale corresponding to the capacitance values which are available, the conductance or loss tangent may be plotted against scale reading, and their value for any other setting determined by interpolation. A variable standard calibrated in this way may then be employed for measurements on fixed condensers by simple substitution.

When the resistance-variation method is employed for difference-substitution measurements, it is necessary to make a correction on account of the difference between the total capacitance of the circuit C, and the capacitance C_x of the condenser under test. For the resistance of the circuit is measured with respect to the current in the thermo-element, *i.e.*, the current through the total capacitance C, which includes C_b, that of the base of the standard, as well as C_x. In other words, the resistance measured is that of the two condensers C_x and C_b in parallel, plus that of the rest of the circuit, R. The value is approximately given by

$$R + \frac{G_b}{C^2\omega^2} + r_x \frac{C_x^2}{C^2}$$

where r_x is the value for the condenser under test. For the second measurement the resistance measured is

$$R + \frac{G_b}{C'^2\omega^2}$$

The difference between the first and second measurements is therefore

$$r_x \frac{C_x^2}{C^2} = r_x \left(\frac{C_x}{C_b + C_x}\right)^2 \simeq r_x \left(1 - \frac{C_b}{C_x}\right)^2 . \quad . \quad . \quad (6)$$

The value of the power factor of the condenser is of course easily calculated from either r_x or G_x provided that C_x is also measured. Whatever the method of measurement, the results can be expressed in terms of series resistance, shunt conductance, power factor, loss tangent, loss angle, or decrement, at will.

4. Measurement of the Effective Resistance, Conductance, and Magnification Factor of Coils. Having obtained the values of the resistance and conductance of a standard condenser by the above procedure, it is a simple matter to obtain the values for a coil. It is only necessary to form a resonating circuit with the coil in question and the condenser and to measure the total resistance or conductance of the circuit by the methods of paragraphs 1 and 2. On subtracting the values for the condenser from the total value for the circuit, the values for the coil are obtained. The conductance of the voltmeter can be measured separately by the method described for a condenser, provided a second voltmeter is available as detector, or the values for the condenser and voltmeter in parallel may be measured at the same time. If, however, a very small capacitance (say 2 to $5 \mu\mu F.$) is placed in series with the voltmeter, its effective conductance often becomes negligible. The resistance of the thermo-element may similarly be measured together with that of the standard condenser, but it is often sufficiently accurate to use the d.c. value of its resistance.

Particular attention should be paid to the fact that the resistance-variation method measures the resistance of the coil with respect to the current in the condenser: the simple theory assumes that this is the only current circulating in the circuit. We imagine the coil to be replaced by its equivalent series resistance R_s, and inductance L_s, and it is the quantity R_s which is measured by the method. On the other hand the reactance-variation method with voltmeter detection measures the conductance and capacitance with respect to the total voltage. In other words, the quantity measured is the conductance

$G_p = 1/R_p = 1/R_2$ [Figs. 3 and 62]. The relations between these quantities are derived in Chapter VIII. Thus

$$R_s = \frac{R_3}{(1 - L_3C_3\omega^2)^2 + R_3^2C_3^2\omega^2} \quad . \quad . \quad . \quad . \quad (7)$$

$$R_p = R_3\left[1 + \frac{L_3^2\omega^2}{R_3^2}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Methods for the measurement of L_2 and L_3 are discussed in Chapter IX. In practice the two values are usually indistinguishable, as are also the various values of self-capacitance C_1, C_2 and C_3. The above equations may also usually be replaced by the approximations

$$R_s \simeq \frac{R_3}{(1 - L_3C_3\omega^2)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and
$$R_p \simeq \frac{L_3^2\omega^2}{R_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and thus the value of R_3 calculated from R_s or R_p. The error due to the approximations may then be estimated from the value so obtained, and a correction applied if necessary. The value of the magnification Q can be calculated from the exact equation

$$\frac{R_p}{R_3} = Q^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

or the approximation

$$Q \simeq \frac{L_3\omega}{R_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

5. **Dielectric Constant and Power Factor of Dielectrics.** The process of measuring the dielectric constant (permittivity) and power factor of a dielectric, consists essentially of three parts, (i) the construction of a condenser with the dielectric in question as the insulating medium, (ii) the measurement of the capacitance C_s and power factor of this condenser, (iii) the measurement or calculation of the capacitance C_0 of a condenser of the same linear dimensions, but with air or a vacuum as the insulating medium.

The ratio C_s/C_0 is the required dielectric constant, while the measured value of power factor is that of the dielectric, provided that the condenser is so constructed that there are no power losses in the electrodes. Thus the effect of the resistance of the electrodes must be negligible, there must be nothing of the nature of a " contact resistance " between electrodes and dielectric, and there must be no leakage between the electrodes except that through the dielectric. The actual measurements of both capacitance and power factor of the condenser can be made by the methods which have already been described. If the condenser is of simple geometrical form, and its linear dimensions accurately measurable, the " air-capacitance " C_0 can be calculated by the formulæ given in Chapter VI. It is therefore only necessary to consider here the forms of test-condenser suitable for various classes of dielectrics. The provision of electrodes which, when applied to a given sample, satisfy the condition stated above, is often one of the most difficult parts of the measurement.

6. **Electrodes for Solid Dielectrics.** Solids are usually tested in the form of flat sheets, and perhaps the most generally useful form of electrode for such samples consists of sheets of tinfoil stuck to the faces of the sample with a very thin film of petroleum jelly used as an adhesive. Tinfoil alone does not make a sufficiently intimate contact with the surface unless applied under very great mechanical pressure, and this is seldom permissible. In a typical experiment the sample consists of a disc of say, 10 cm., diameter and 3 mm. thickness. The surfaces are cleaned, and very lightly coated with petroleum jelly ; a mere trace is sufficient. Thin tinfoil is then applied and carefully pressed with the fingers into contact over the whole surface. The tinfoil is cut to a definite diameter with a sharp knife, the diameter of the electrodes being made say, 1 cm., less than that of the sample. Leakage between the two electrodes over the surface of the sample is thereby usually reduced to a negligible proportion. Copper discs of the same diameter as the foils are finally placed over the foil electrodes, and pressed into contact (by weighting if necessary). The condenser so formed is satisfactory for measurements at all frequencies up to about 10 Mc/s. The copper discs must of course be regarded as forming part of the electrodes. The resistance of the foils is only negligibly small to current passing through it transversely : to currents in its own plane its resistance may be relatively large. The copper discs must therefore make contact with the foil over the whole area, so that they act as

terminals which distribute the current over the whole area and prevent current from flowing in the plane of the foil. The power loss in the film of petroleum jelly is negligible, for not only is the quantity used very small, but the power factor of pure petroleum is very small at all practical frequencies.

The air-capacitance of such a test-condenser can be calculated from Kirchhoff's formula for parallel discs (p. 104), but it should be noted that the edge-capacitance is not located in the solid dielectric, except to a small incalculable extent. It is usually best to regard the calculated "edge-capacitance" in air, as an extraneous capacitance, which is unavoidably included with the capacitance C_s of the sample. Thus this edge-capacitance is deducted from the measured capacitance in order to obtain C_s, and is not included in the corresponding calculated value of C_0.

The surfaces of solids which can be strongly heated (*e.g.*, mica, glass and ceramic materials) can be silvered by a process used in the ceramic industry, and such silver films in intimate contact with the dielectric, may be used as electrodes instead of the tinfoil described above. The required area is coated with a "silvering solution" and the material then heated to a temperature of the order of 300° to 600° C. which results in the deposition of a metallic film, very firmly attached to the surface. The process is repeated if necessary until a continuous metallic film is obtained, and this is then backed with copper plates as before.

Mercury also forms an admirable electrode material, since it will make intimate contact with a surface in spite of irregularities. Many forms of mercury electrodes have been developed. An example will be discussed later in connection with bridge measurements (p. 227), but such electrodes will not be discussed in any detail here, since they are not very convenient for work at the highest frequencies and are therefore less general in their application than the electrodes which have been described. For measurements at frequencies up to, say, 5 Mc/s. they are probably superior to any other electrode. For such measurements a flat disc of the dielectric may be floated on the surface of mercury contained in a vessel of slightly larger diameter. The mercury then forms an electrode covering the whole of the lower face of the sample. A pool of mercury on the upper face of the sample can be used for the upper electrode. The pool can be confined to a definite area by a rim of wax, or by a metal ring cut from tubing of a suitable diameter and laid on the surface. The edge-capacitance of such a system is not easily calculated, and it should only be used when the total capacitance is large enough to make the edge-capacitance of comparatively little importance. Connection to the circuit is made by wires dipping into the mercury. It is the limitations arising from the inductance and resistance of such wires which restrict the use of mercury to frequencies below a certain value.

Material in the form of tube can also be provided with mercury electrodes. The material is placed so as to be coaxial with two steel tubes one of larger and the other of smaller diameter. One end of the system is closed by immersion in molten paraffin wax, which is then allowed to set. Mercury poured into the annular spaces between the metal and dielectric tubes forms the inner and outer electrodes. Edge effects, including that of the wax, may be eliminated by observing the difference between two measurements made with the mercury standing at two different heights in the tubes. The dielectric constant is in this case calculated from the formula for a cylindrical condenser (p. 104).

Solids which can be melted, *e.g.*, waxes, may be tested in two ways. Flat plates of the material may be prepared by casting on a mercury surface: the molten material is poured on to the surface of mercury contained in a large dish and allowed to set. Plates of almost any desired thickness can be obtained in this way and then tested with tinfoil or mercury electrodes. Alternatively, the molten material may be poured into an air condenser (previously calibrated) and allowed to set, thereby forming the required test condenser. It should, however, be borne in mind that most materials contract on solidifying, and that there is a danger that the contraction may give rise to air films between the electrodes and the material. Such films may cause large errors in the measurements. It is clearly advisable to freeze such liquids from the bottom upwards, so as to allow the contraction to occur at the free surface.

7. Test Condensers for Liquids. Vessels for the testing of liquids consist essentially of small fixed air condensers which can be filled with the liquid. A very simple form consists of two coaxial tubes of copper, brass, nickel, etc. (depending on the chemical activity of the liquid under test), separated by a small radial gap, and fixed relative to one another by six small wedges of quartz or glass, inserted in the radial gap between the tubes, three at each end. A strip cut round the rim of each tube and bent upwards is used for connection to the circuit. Measurements are first made on this condenser in air, and it is then dipped into any desired liquid which may be contained in a test tube, etc.

A somewhat more elaborate form used at the National Physical Laboratory* is shown in Fig. 77. Here the insulation is provided by a tapered piece of fused quartz tubing, which is ground to fit the upper parts of the two electrodes, and thus also acts as a stopper for the vessel. Note also that the leads to the terminals of this condenser are very short and thick and therefore of very small inductance. This condenser is therefore suitable for use at high frequencies.

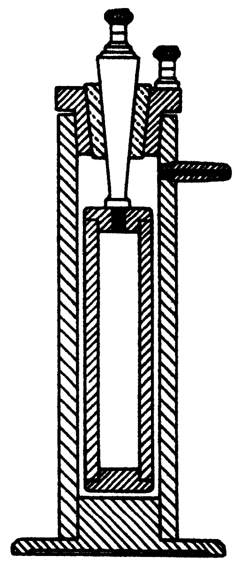


FIG. 77. Condenser for testing liquid dielectrics.

When making measurements with such condensers it is necessary to make allowance for the capacitance and conductance of the solid insulators. Their conductance is not affected by the liquid, so that the difference between the conductances measured when the cell is full and empty gives the net conductance of the liquid. The capacitance correction is determined as follows. The cell may be considered as consisting of two parts: one which may be filled with liquid, and which has the capacitance C_0 when filled with air, and ϵC_0 when filled with liquid of dielectric constant ϵ; and a second part, including the solid insulators, which is never filled with liquid, and which has a constant capacitance C_i. If C is the total capacitance,

$$C = \epsilon C_0 + C_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Evidently measurements of C for two different values of ϵ are sufficient to determine both C_0 and C_i. In practice a measurement is usually made with air and a standard liquid such as

* HARTSHORN, L. and RUSHTON, E. *J. Sci. Instrum.*, 1939, Vol. 16, p. 366.

benzene, then C_0 and C_t being known, a measurement of C with any other liquid is sufficient to determine ϵ for that liquid. The loss tangent for the liquid is then given by $G_l/\epsilon C_0 \omega$, where G_l, its conductance, is obtained by the difference measurement already mentioned.

8. **Test-cells for Electrolytic Resistance.** It should be noticed that the cells described above are also suitable for measurements of electrolytic resistance or conductivity. Indeed from the measurement point of view there is no essential difference between the conductivities of an electrolytic solution and a dielectric. The conductance G_l of the liquid is determined as before by the difference between the measured conductivities of the cell when full and when empty. The conductivity σ of the liquid is then given by

$$G_l = \sigma G_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where G_0 is the geometrical conductance (p. 103) of the portion of the cell which is filled with liquid. G_0 may obviously be determined by a measurement with a standard liquid. Alternatively, it may be deduced from the capacitance value C_0, for

$$C_0 = 0{\cdot}0885_5 G_0$$

when C_0 is in $\mu\mu F$ and G_0 in cm. This method is only applicable when the liquid occupies the whole electric field between the electrodes, *i.e.*, for electrodes which like the concentric system have a negligible stray field.

9. **Apparatus for Very High Frequencies.** The circuits required for the resonance methods which have been discussed may be assembled without undue difficulty from ordinary standard coils, condensers, etc., provided the working frequency is not greater than, say, 10 Mc/s. When, however, such circuits are required for use at frequencies of the order of 50 Mc/s., difficulties arise owing to the fact that even the shortest possible length of wire which may be used to connect two components in series or parallel, has an appreciable impedance. It becomes necessary to make due allowance for the residual inductance, capacitance and resistance of every piece of connecting lead in the measuring circuit, and the equations are apt to become very complicated. For example, a condenser of capacitance C and conductance G in series with leads of residual inductance L' and

resistance R' may be shown to be equivalent to a capacitance C_p and conductance G_p, where

$$C_p = \frac{C\left(1 - L'C\omega^2 - L'C\omega^2 \cdot \frac{G^2}{C^2\omega^2}\right)}{(1 + R'G - L'C\omega^2)^2 + (R'C\omega + L'G\omega)^2} \quad . \quad . \quad (15)$$

and

$$G_p = \frac{G(1 + R'G) + R'C^2\omega^2}{(1 + R'G - L'C\omega^2)^2 + (R'C\omega + L'G\omega)^2} . \quad . \quad (16)$$

and it becomes necessary to consider the magnitude of all the various correcting terms in these equations (most of which increase rapidly with rise of frequency) before they can be neglected. These difficulties are best overcome by the use of circuits, specially built as self-contained units without connecting leads. An instrument of this type, devised by the author and W. H. Ward * for use at frequencies up to 100 Mc/s., is shown in Fig. 78. It is essentially the circuit for voltage resonance with capacitance variation. The main tuning condenser consists of two parallel copper plates A and B, the upper one being adjustable by means of a micrometer head M_1. This plate is connected to the copper housing H by means of a cylindrical copper bellows of large diameter (and therefore extremely low resistance and inductance). The condenser for the fine adjustment is formed by making the stem of a second micrometer head M_2 project into a hole bored into the copper plate A. These condensers are mounted on the top of a rigid copper box containing the thermionic voltmeter. The rectifying valve V_1 is mounted immediately below the plate A, and is connected to it by a short copper rod J integral with A. The end of this rod rests on a thin sheet of mica which forms the small series condenser C_1, used to reduce the effective conductance of the voltmeter. The voltmeter is of the type shown in Fig. 44, and it is to be noticed that the whole of the high-frequency circuit is compactly mounted immediately underneath A. The housing H is connected to the screen of the voltmeter by three bolts F, one of which is shown, these bolts also serving to clamp the insulating pillars, which are of fused silica. These three bolts in parallel also have an extremely low resistance and inductance.

* HARTSHORN, L. and WARD, W. H. *J. Inst. Elect. Eng.*, 1936, 79, 597.

The coil for very high frequencies consists of a single turn of stout copper rod, inside a coaxial tube attached to one end to serve as a screen for the coil. The coil is connected to two terminals as shown, and two similar terminals on the other side of the apparatus allow any other component under test to be connected in parallel

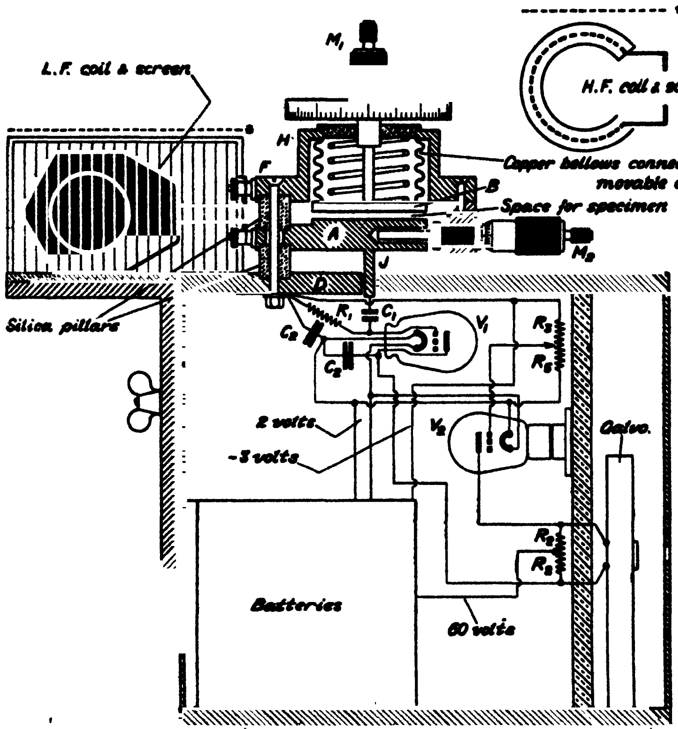


FIG. 78. Hartshorn and Ward's apparatus for frequencies up to 100 Mc/s.

with the circuit. The two micrometer condensers M_1 and M_2 are calibrated at low frequencies, and it is found that with this construction the same calibration may be used with errors that are only just detectable at frequencies up to 100 Mc/s.

10. Measurements on Dielectrics at Very High Frequencies. The apparatus described above reduces to very small proportions the errors due to the residual impedance of connecting leads, and will therefore correctly measure the constants of any resistor, condensers, etc., which may be inserted in the circuit. But in order to measure the power factors of dielectrics it is not only necessary to measure the power loss in a test condenser, it is also necessary to allow for the losses in the electrodes. The resistance of the electrode increases with rise of frequency, and, moreover, the power factor due to a given resistance also increases with frequency. It therefore follows that the power losses due to electrode resistance are apt to become the dominating factor at very high frequencies, even in condensers in which they are negligible at moderate radio frequencies. Thus at very high frequencies, measurements on dielectrics must be made by a difference method, in which the losses due to electrode resistance cancel out.

Hartshorn and Ward's apparatus described above was designed for this purpose. The dielectric sample, in the form of a flat disc, is inserted between the copper plates A and B, and the micrometer M_1 is screwed up until the plates make contact with the disc. The measurement of conductance is then made by operation of M_2. The sample is then withdrawn and the micrometer M_1 screwed up until the same capacitance is produced with air between the plates A and B. Then since the electrodes are the same, and the capacitance is the same, the losses due to electrode resistance are the same, and are included in the second measurement of conductance, which follows. Thus the difference between the two values of conductance gives that due to the dielectric only Silver film or tinfoil electrodes are applied to the sample in order to obtain a more intimate contact than copper plates alone will give, and these films are of course removed from the apparatus with the sample, so that their resistance is not allowed for. It is, however, possible to find out whether the film has an appreciable effect by making a second measurement with the film removed. The capacitance is then usually smaller owing to the imperfect contact, which may be considered to be due to the presence of an air film of zero power factor in series with the specimen. It follows that the apparent power factor should also be smaller in the same proportion as the capacitance, and if this is so the values with the films are correct. If not, the true capacitance value, C_s, can be obtained from a measurement with the film electrodes (at a somewhat lower frequency if necessary);

the apparent power factor $\tan \delta_0$ and capacitance C_0 are then measured without the films at the higher frequency, and the true power factor $\tan \delta_s$ then obtained from the relation

$$\tan \delta_s = \frac{C_s}{C_0} \tan \delta_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Measurements on liquid dielectrics at frequencies up to 100 Mc/s. may be made in the same way, but in this case the flat plates A and B are replaced by plates with curved surfaces. The upper surface of A is concave, and contains the liquid, while the lower surface of B is convex, the two surfaces fitting so that the air gap between them is of uniform width.

11. Measurement of the Power Factor of a Ferromagnetic Material. The power factor of a ferromagnetic material such as is used in the construction of inductors for radio frequency work, can be found by a procedure in some respects analogous to that employed for the measurement of the corresponding quantity for a dielectric. A toroidal inductor is constructed on a core of the material in question, and its equivalent shunt conductance G_p and inductance L_p are measured by the methods already discussed. Then by equation (25) (p. 11) the loss tangent of the coil is given by $\tan \delta = G_p L_p \omega$. The conductance G_p may be regarded as the sum of two components, one G_w equivalent to the power loss in the coil winding, and the other G_c equivalent to the loss in the ferromagnetic core. Thus the loss tangent of the ferromagnetic material itself is given by $G_c L\omega$, or $(G_p - G_w)L\omega$. The conductance of the winding G_w may be obtained approximately by a measurement on a similar coil wound on a non-magnetic former of, say, paraffin wax or trolitul. The loss tangent finally obtained is only characteristic of the material itself if the whole magnetic field of the inductor is located in the material. The toroidal coil approximately satisfies this condition: in this case G_c varies inversely, and L_p directly, as the linear dimensions of the core, so that $\tan \delta = G_c L_p \omega$ is independent of the size of the core. It must be remembered that G_w is only likely to be approximately the same for the coil and the dummy: it should therefore be made small compared with G_p, by the choice of a suitable wire for the winding, by the methods explained in Chapter VIII.

12. **Measurements on Resistors, Chokes,* etc.** The methods and apparatus already described can be used for the measurement of impedance of almost any kind and magnitude. Thus with the apparatus shown in Fig. 78 measurements can be made on resistors and chokes as well as on condensers and dielectrics at all radio frequencies up to about 100 Mc/s. The component to be measured, if of fairly high impedance, is connected in parallel with the condensers of the measuring circuit and the total conductance measured in the manner already described. The component is then disconnected and the measurement repeated. The difference between the two measured conductances is that of the component, while the increase of capacitance reading of M_2 (or M_1) required in order to restore the condition of resonance after removing the component, is equal to the equivalent shunt capacitance of the component, from which its susceptance is easily calculated. The conductance and susceptance together give the complete admittance operator of the component, from which its impedance operator, and any other constant such as inductance, resistance, phase angle, is easily calculable by the methods indicated in Chapter I. Both positive and negative susceptances can be measured, since the tuning capacitance can be either increased or diminished when the component is removed.

The above procedure fails when the impedance to be measured is small compared with that of the tuning condenser of the measuring circuit, for in this case the measuring circuit becomes effectively short-circuited by the addition of the component. In such cases the component may be placed in series with a small condenser, which has previously been measured, and the constants of the combination measured by the same procedure. The constants of the component can then be calculated from the results for the condenser and the series combination.

Resistors, conductors and condensers of very low impedance are best connected in series with the tuning coil and condenser and the usual circuit constants measured. The changes in these constants on removing the small impedance permit the value of this impedance to be determined.

* Hartshorn, L. and Ward, W. H. *J. Sci. Instrum.*, 1937, 14, 132.

Obviously the same general principles may also be applied when the resistance-variation method is employed.

13. Measurements on High-Frequency Cables. The method described in the preceding section for the determination of the impedance operator of a choke or condenser is equally applicable to a length of cable, or indeed anything else (such as an aerial), which may be connected to the measuring circuit at two terminal points. The impedance operator measured is of course in all cases that corresponding to the current and voltage at the terminal points in question. Consider, for example, the case of a high-frequency cable of the coaxial type, consisting of a central conductor, and a coaxial screen which forms the return conductor, suitably insulated from one another. Let a length l of such a cable be connected to the measuring circuit, the screen conductor and central conductor at one end being connected to the screen and insulated terminals, respectively, of the measuring circuit. The measurements already described will determine the input impedance of the cable. If this impedance is measured, first with the cable open at the far end and secondly with that end short-circuited, the results obtained are sufficient to determine all the constants of the cable (attenuation, characteristic impedance, etc) and therefore its performance in practice.

Formulæ for the terminal impedances of a coaxial pair of conductors carrying high-frequency currents have been given in Chapter VII in the discussion of the coaxial resistor. The formulæ apply also to cables and it follows that if $\mathbf{Z}_f$ is the impedance with the distant end free or open, and $\mathbf{Z}_g$ that with the distant end closed or grounded, we have

$$\mathbf{Z}_f = \mathbf{Z}_0 \,.\, \coth \gamma l = Z_f \underline{/\phi_f} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$\mathbf{Z}_g = \mathbf{Z}_0 \,.\, \tanh \gamma l = Z_g \underline{/\phi_g} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where Z_0 is the characteristic impedance of the cable, and γ the propagation constant. It follows that $\mathbf{Z}_0$ and $\tanh \gamma l$ can be calculated from the observed values of $\mathbf{Z}_f$ and $\mathbf{Z}_g$, using the relations

$$\mathbf{Z}_0 = \sqrt{\mathbf{Z}_f \,.\, \mathbf{Z}_g} = \sqrt{Z_f Z_g} \underline{/\tfrac{1}{2}(\phi_f + \phi_g)} \quad . \quad . \quad . \quad . \quad (20)$$

$$\tanh \gamma l = \sqrt{\frac{\mathbf{Z}_g}{\mathbf{Z}_f}} = \sqrt{\frac{Z_g}{Z_f}} \underline{/\tfrac{1}{2}(\phi_g - \phi_f)} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Tanh γl can now be expressed in the form $A + jB$, where

$$A = \sqrt{\frac{Z_g}{Z_f}} \times \cos \tfrac{1}{2}(\phi_g - \phi_f)$$

$$B = \sqrt{\frac{Z_g}{Z_f}} \times \sin \tfrac{1}{2}(\phi_g - \phi_f)$$

and γl is then obtained from the equation

$$\gamma l = \tfrac{1}{4} \log \frac{B^2 + (1 + A)^2}{B^2 + (1 - A)^2} + j \cdot \tfrac{1}{2} \left[\tan^{-1} \frac{B}{1 + A} + \tan^{-1} \frac{B}{1 - A} \right]$$

$$= \alpha l + j\beta l, \text{ say.}$$

It should be remembered that the ratio of the voltages at two points separated by a distance l on a cable of infinite length is given by,

$$e^{-\gamma l} = e^{-(\alpha l + j\beta l)} = e^{-\alpha l} \,.\, e^{-j\beta l} = e^{-\alpha l}\underline{/- \beta l}.$$

Thus, in passing along the length of cable the electric wave suffers a voltage amplitude reduction in the ratio $e^{-\alpha l}$, or a voltage reduction of αl nepers, and a change of phase of βl radians. It follows that the attenuation constant α is expressed in nepers per unit length, and the phase constant β in radians per unit length. Obviously, the unit of length may be the kilometre, mile, or any other convenient unit: it is merely necessary to express the measured length l in the appropriate unit. A voltage reduction in the ratio $e^{-\alpha l}$ corresponds to a power reduction in the ratio $e^{-2\alpha l} = 1/N$ say. The power attenuation in decibels is therefore given by $10 \log_{10} N = 20\alpha l/\log_e 10 = 8{\cdot}686\alpha l$, or $8{\cdot}686\alpha$ *db*, per unit length.

Having determined $\mathbf{Z}_0$ and γ in the manner described above, it is possible to calculate the resistance R, self-inductance L, capacitance C, and leakance G, all per unit length of cable, from the equation

$$R + jL\omega = \mathbf{Z}_0\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$G + jC\omega = \frac{\gamma}{\mathbf{Z}_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The complete set of calculations is somewhat involved, but the electrical observations consist essentially of two measurements of impedance (or admittance), and these are measured by exactly the same methods as have been described for condensers, chokes, and resistors. For example, the method of voltage resonance with capacitance variation may be employed, and for radio frequencies up to, say, 5 Mc/s. the circuit may be assembled from ordinary components. In such cases a length of 10 to 15 metres of cable would probably be employed, with a capacitance of the order of, say, 1,000 $\mu\mu F$. At higher frequencies, up to say 100 Mc/s., Hartshorn and Ward's apparatus can be used, with a length of, say, 2 or 3 metres of cable having a capacitance of the order of 50 $\mu\mu F$. At such frequencies it becomes important to allow for the effective resistance and inductance of the standard condenser, measured between the two terminals to which the cable is connected. The methods already described can be adapted for this purpose. Alternatively a short length of standard cable of calculable performance may be constructed in the form of a coaxial wire and tube, with practically perfect insulation (discs of fused silica or polystyrene), and the corrections for the circuit deduced from the results of measurements made on the standard. For the standard the value of L is obtained by calculation; that of R from d.c. measurements and the calculated "skin-effect"; G is negligible (the actual value may be obtained from separate measurements on the insulating discs); and C can be obtained by measurements at a lower frequency.

Nergaard* has described a standard transmission line, and a variable air condenser of very small linear dimensions, by means of which measurements of this kind can be made at frequencies of the order of 200 Mc/s.

It may be noted that the attenuation or loss of power in the cable arises from the resistance R and leakance G, and that the two loss tangents $R/L\omega$ and $G/C\omega$ are always made small compared with unity. In such cases the usual expression for γ can be simplified. Thus

$$\begin{aligned}\gamma = \alpha + j\beta &= \sqrt{(R + jL\omega)(G + jC\omega)} \\ &= j\omega\sqrt{LC}\left[\left(1 - j\frac{R}{L\omega}\right)\left(1 - j\frac{G}{C\omega}\right)\right]^{\frac{1}{2}} \quad . \quad (24) \\ &\simeq j\omega\sqrt{LC}\left[1 - j\tfrac{1}{2}\left(\frac{R}{L\omega} + \frac{G}{C\omega}\right)\right]\end{aligned}$$

* Nergaard, L. S. *R.C.A. Review*, 1938, Vol. 3, p. 156.

Hence we have the approximate relation

$$\beta = \omega\sqrt{LC} \quad \ldots\ldots\ldots\ldots \quad (25)$$

$$\alpha = \tfrac{1}{2}\omega\sqrt{LC}\left[\frac{R}{L\omega} + \frac{G}{C\omega}\right] \quad \ldots\ldots \quad (26)$$

The last equation shows at a glance how the attenuation constant of the cable is related to the two loss tangents $R/L\omega$ and $G/C\omega$. Evidently the ratio of the losses in the cable due to conductor resistance and dielectric losses respectively is given by the ratio of these two loss tangents.

14. Measurements of Capacitance and Impedance on Thermionic Valves. The resonance methods which have been

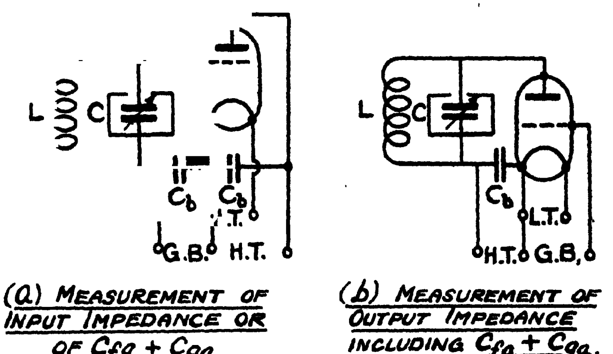


FIG. 79. Circuits for measurements on thermionic valves. (Note, $f \equiv k$).

discussed can also be applied to measurements on thermionic valves. The inter-electrode capacitances C_{kg}, C_{ka}, C_{ga} (where k, g and a denote cathode, grid, and anode, respectively), and also the impedances, may be measured in this way under working conditions. Circuit arrangements suitable for the measurement of both input and output impedances are shown in Fig. 79. It will obviously be advisable to choose a method in which the resonance condition is not affected by conductance, and since the capacitances to be measured are generally small, a very sensitive capacitance adjustment is necessary. The apparatus of Hartshorn and Ward satisfies both requirements. The input impedance is usually very high, and therefore the corresponding capacitance $(C_{kg} + C_{ga})$ can be measured without difficulty at almost any desired frequency. The output impedance is often low, and thus the arrangement of Fig. 79 (*b*) is only workable at very high frequencies. At the lower frequencies the circuit is so highly damped that the setting for resonance cannot be made with sufficient precision. The capacitance measured with this arrangement is $C_{ka} + C_{ga}$. If the measurements of capacitance are repeated with the filament cold, the corresponding electrostatic values are obtained, and in this way it is possible to study the effect of the emission current on the various capacitances under working conditions.

CHAPTER XI

BRIDGE METHODS

1. The Circuits Available. The general principles governing the construction of radio-frequency bridges have been discussed in Chapter III, which should be consulted before proceeding to the details given in the present chapter. It is to be understood that all the bridge circuits described in this chapter must be constructed in accordance with the principles laid down in Chapter III.

Bridge circuits of many types are available for use at low frequencies. Theoretically they can also be used at radio frequencies, but at these frequencies, errors due to imperfections of the standards and circuit components generally, are apt to become so great that in practice very few circuits are workable. We have already seen the advantages of symmetrical bridges, and it follows that two of the most satisfactory bridges for high frequency work are the symmetrical capacitance-conductance bridge of Fig. 80 (*a*), and the symmetrical inductance-resistance bridge of Fig. 80 (*b*). In each case the ratio arms Z_3 and Z_4 are equal, and it is obvious that the equations of balance are simply, for (*a*),

$$C_1 = C_2 \qquad G_1 = G_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and for (*b*),

$$L_1 = L_2 \qquad R_1 = R_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We have seen that as standards for high-frequency work, condensers are generally speaking more satisfactory than inductors, and it follows that the capacitance bridge is the better of the two. The fact that each of its arms consists of a single component rather than two in series simplifies the shielding (*cf.* Chapter III), and is itself a great advantage. In either bridge the ratio-arms Z_3, Z_4 may be condensers, resistors or inductors (chokes, say). The choice will depend on the circumstances of the measurement. Very high impedances are not desirable, for it is important that the admittance of each arm shall be large enough to be not unduly influenced by any stray admittances which may arise from inevitable slight imperfections in the screening and wiring of the circuit. Condensers and coaxial resistors of, say, 10 ohms, are excellent even at very high frequencies, but coil resistors are more easily adjusted in many cases, while chokes are equally easily

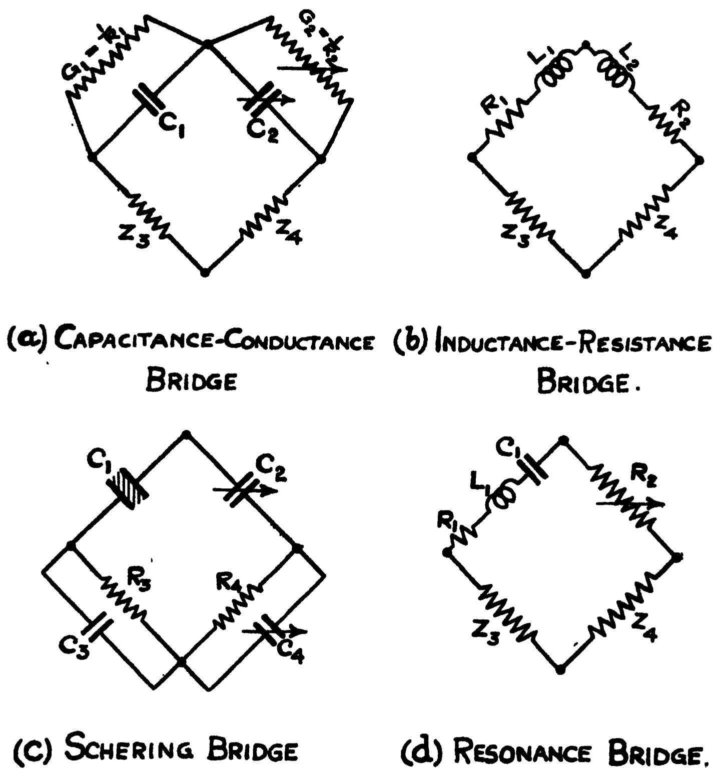


Fig. 80. Bridges for radio-frequency work.

adjusted, but absorb less power than resistors of the same impedance, a property which is occasionally useful, *e.g.*, when the available power is very small.

The capacitance-conductance bridge requires a variable resistor and a variable condenser in order that the two conditions of balance may be satisfied. Variable condensers have the great advantage that their variation is perfectly smooth : the variation of a resistor is always more or less discontinuous and the moving contact is always apt to give trouble in the course of time. On this account the Schering capacitance bridge of Fig. 80 (*c*) is often preferred to the conductance bridge of Fig. 80 (*a*). In both bridges, arm 1 is a condenser of capacitance C_1, loss angle δ_1, and conductance $G_1 = C_1\omega \tan \delta_1$, but in the Schering bridge, instead of balancing the conductance by a variable conductance G_2, the difference between the loss angles of arms 1 and 2 is balanced by varying the phase angle of one of the ratio-arms, by using for these arms resistors, one of which is shunted by a variable condenser, so that both adjustments are made by means of variable condensers. The Schering bridge is not strictly symmetrical owing to the difference in phase angle between the ratio-arms, but in practice the bridge is only used in conditions in which this phase angle, and therefore the departure from symmetry, is small. If the loss angles of the arms 1 and 2 are δ_1 and δ_2, and the phase angles of arms 3 and 4 are ϕ_3 and ϕ_4, the exact equations of balance are

$$\delta_1 - \delta_2 = \phi_3 - \phi_4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\frac{C_1(1 + \tan^2 \delta_1)^{\frac{1}{2}}}{C_2(1 + \tan^2 \delta_2)^{\frac{1}{2}}} = \frac{R_4}{R_3} \cdot \frac{(1 + \tan^2 \phi_4)^{\frac{1}{2}}}{(1 + \tan^2 \phi_3)^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which provided the δ's and ϕ's are small and $R_3 = R_4$, may be written with sufficient accuracy

$$C_1 = C_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$\begin{aligned} \tan \delta_1 - \tan \delta_2 &= \tan \phi_3 - \tan \phi_4 \\ &= R_4 C_4 \omega - R_3 C_3 \omega + \delta_{34} \quad . \quad . \quad . \quad (6) \end{aligned}$$

where δ_{34} denotes any residual phase difference of the two nominally equal resistors R_3 and R_4. Equations (5) and (6) are those used in ordinary practice.

The only other bridge which is specially suitable for high-frequency work is the resonance bridge shown in Fig. 80 (*d*). Here arm 1 consists of a resistance, inductance and capacitance in series, or any device which may be tuned to resonance at some particular frequency, which is the working frequency of the bridge. At this frequency the arrangement is equivalent to a pure resistance, and can therefore be balanced by a single variable resistor, as shown. In this bridge, the phase adjustment is that which controls the tuning: it may be variation of the condenser C or of the frequency. This bridge is also used with any equal ratio-arms Z_3 and Z_4, but is not really symmetrical, as may be seen by a consideration of the potential distribution. The potential of the junction of the coil and condenser will rise to a relatively very high value when resonance is established, and on this account the bridge must be used with caution, as stray capacitances at this point may cause large errors. Nevertheless on account of its simplicity the bridge has great advantages for measurements on systems which are actually resonant in their working condition. When $Z_3 = Z_4$ the working equations are simply

$$R_1 = R_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$L_1C_1\omega^2 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

It is possible to provide all the above adjustments in a single instrument, which may be described as a universal bridge, but such instruments are seldom suitable for accurate work at frequencies much greater than 1 Mc/sec. For the higher frequencies it is important to avoid all complications and to choose the minimum number of components.

It would be unprofitable to attempt to describe in detail all the bridge arrangements that have been used. Any individual worker is guided by the nature of his problem and the components at his command. The details are therefore seldom of general interest except as examples of the way in which the necessary conditions have been satisfied in particular instances. The following examples are to be regarded from this point of view.

2. **The Dye-Jones Bridge.*** This is a form of the Schering bridge network with a Wagner earth-connection, which has been used at the National Physical Laboratory for some years for work at frequencies up to about 1·5 Mc/s. Fig. 81 shows diagrammatically the arrangement of the bridge proper *KRC*, and Wagner earth-connection R_WC_W, screens being shown dotted. Heterodyne detection is employed, the auxiliary oscillator being

* Dye and Jones. *Jour. Inst. Elec. Eng.*, 1933, Vol. 72, p. 169.

similar to the supply oscillator. The condenser arms K consist of external standard variable air condensers, which are changed as required to suit the work in hand. They are linked to the bridge centre by means of rigid leads enclosed in brass tubes which serve as screens. The connections between condensers and leads are often made by means of short thick copper wires dipping into mercury cups, for it is essential to avoid changes of resistance, which would have the effect of altering the power factor of the condenser. The ratio-arms are connected to the bridge-centre by similar rigid screened leads, the screen being of large diameter to avoid excessive capacitance to earth. In

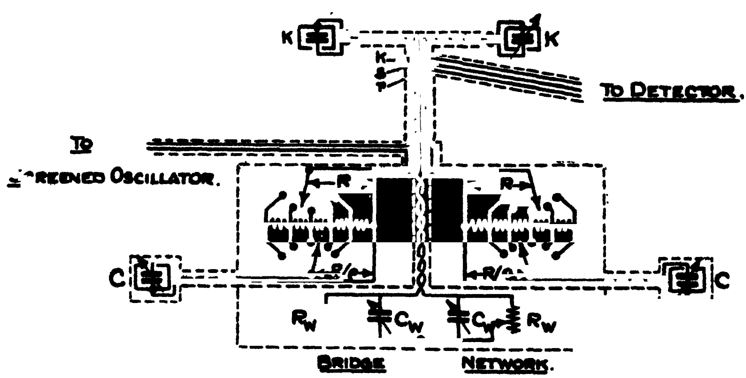


Fig. 81. Schering bridge for radio-frequencies. (Dye-Jones.)

Fig. 81 the ratio-arms consist of subdivided resistors of the coil type, but for work at the higher frequencies coaxial resistors are preferred. The arrangement of Fig. 81 is, however, convenient for general work at the lower radio frequencies, for it permits the range of the phase angle adjustment to be changed merely by altering the tapping point on R, to which C is connected. When the condenser C shunts the whole of the resistance R, the phase angle of the arm is $-\tan^{-1} RC\omega$. When, however, C shunts only the portion R/ρ the phase angle is only $-\tan^{-1} RC\omega/\rho^2$. In practice the tapping points are made such that $\rho^2 = 10$, 100, 1,000, etc. These resistors, and the components of the Wagner arms, are mounted in boxes lined with copper foil. The inductive coils in the oscillator and detector are all toroids mounted in

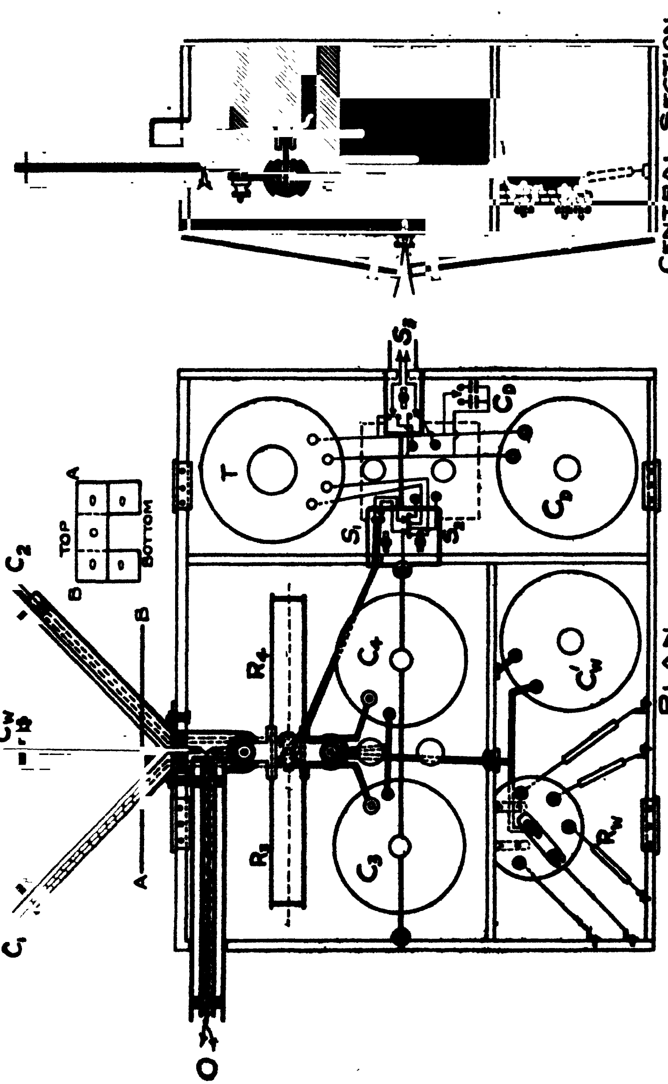


FIG. 82. Another arrangement of Schering bridge for frequencies from 50 c/s. to 10 Mc/s. R_3 and R_4 are resistors of the coaxial type forming the ratio arms. T = input transformer for detector. C_D = condenser to tune T. S_1 = selector switch for main and Wagner balances. S_2 and S_3 = selector switches for low- and high-frequency detectors. Commercial telephone keys are excellent for these switches. (T. I. Jones.)

copper screens, and separated by internal screens. An alternative arrangement in which the resistors are of a coaxial type and the oscillator and detector are interchanged is shown in Fig. 82.

3. The Campbell-Shackleton Bridge.* It should be recognised that almost all high-frequency bridges are based on the pioneer work of G. A. Campbell, so that his name might with justice be associated with every bridge of this type. However, it is convenient to adopt the name Campbell-Shackleton bridge for an arrangement devised at the Bell Laboratories, and applied by Shackleton, Ferguson and others to a variety of purposes. The form described by Shackleton for the measurement of inductive impedances is shown in Fig. 83, and that described by Ferguson for

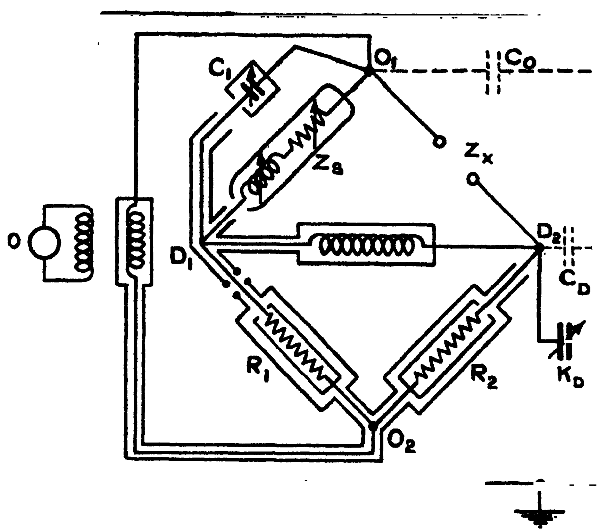


FIG. 83. Shackleton's bridge for inductive impedances.

capacitive impedances is shown in Fig. 84. A comparison of the two diagrams will show that the system of screening is the same in both cases, and applying the principles of Chapter III it may be seen that each bridge is equivalent to the network of Fig. 24, in which Z_1 represents the arm containing Z_s, the standard inductor in Fig. 83, and the standard capacitor in Fig. 84; Z_2 represents the unknown impedance Z_x, and Z_3 and Z_4 represent the equal ratio-arms R_1R_2. In each case there is an additional variable air condenser C_1 which is in parallel with the standard impedance, or Z_1 in Fig. 24. This condenser is made to balance

* See References to Chapter III, § 3.

the earth-capacitances shunting the opposite arm by a preliminary adjustment, after which the condition $Z_1 = Z_2$ may be applied. For this preliminary adjustment the standard impedance, and of course the unknown impedance, must be disconnected from the bridge.

Measurements of two kinds are made, (*a*) those in which one terminal of the impedance to be measured may be connected to screen and to earth, and (*b*) those in which the impedance to be measured must be balanced to earth. In case (*a*) the terminal O_1

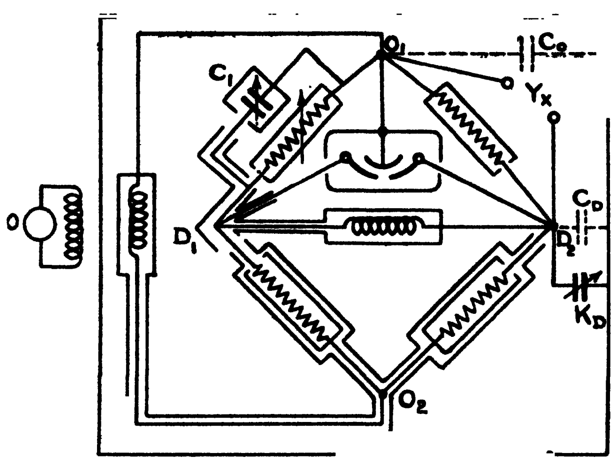


FIG. 84. Shackleton-Ferguson arrangement for capacitive admittances.

of the bridge is connected to the earth-connected screen, thereby short-circuiting C_0 Fig. 24, and the condenser C_1 is then varied until its capacitance balances the earth-capacitance C_D. It is an advantage if the arms O_1D_1, O_2D_2, include shunt resistors of high value, one of which is variable as in Fig. 84, as in this case the conductance component of the earth-admittance can also be balanced by the preliminary adjustment. The unknown impedance is then connected in the arm O_1D_2, and the standard in the arm O_1D_1, and the bridge rebalanced. The value of the unknown impedance is then given by the reading of the standard impedance, or if the standard impedance is not completely disconnected for the preliminary balance, from the change in its value for the first and second balances.

In case (*b*), in which the unknown impedance must be balanced to ground, the preliminary adjustment is made in accordance with the following considerations. The potential difference between the points O_1 and D_2 depends on the output of the oscillator O and the relative values of Z_1, Z_2, Z_3, Z_4, but the actual potentials of O_1 and D_2 relative to earth are controlled by the values of the earth-capacitances C_O and C_D, which together may be regarded as a capacitance potential-divider shunting the arm Z_2. If C_O and C_D are equal, this potential-divider is earth-connected at its middle point, and the potential of O_1 is always equal in magnitude but opposite in sign to that of D_2, in other words the required balanced condition is obtained. In order to obtain the highest accuracy the conductances associated with C_O and C_D must also be equalised. The procedure is as follows. The standard impedance and the one to be measured are first disconnected from the bridge. The point D_2 is then connected to earth, thereby short-circuiting C_D and connecting the capacitance C_O into the arm O_1D_2 of the bridge, which is then balanced by adjusting the condenser C_1. The point D_2 is then disconnected from earth, and O_1 is connected to earth thereby short-circuiting C_O, *i.e.*, removing it from the bridge, and replacing it by C_D which may be varied by adjustment of K_D. This adjustment is made, leaving the other arms of the bridge unaltered, until balance is restored. The two capacitances C_O and C_D are thereby made equal, and if now both O_1 and D_2 are disconnected from earth, the " balanced to ground " condition is established. The bridge must now be balanced once again, adjusting C_1 until it balances the series combination of C_O and C_D. The impedance to be measured, and the standard, are then connected to the bridge, balance again obtained, and the required impedance value obtained by difference, as before. Conductance to earth can obviously be balanced at the same time as capacitance provided suitable variable resistors are connected in series or in parallel with the adjustable condensers.

This system of the preliminary balance of earth-admittances may be regarded as an alternative to the Wagner earth-connection. The Wagner scheme in general permits greater accuracy, since the required condition is checked every time the reading of the bridge is taken. This procedure, however, involves a great deal of extra work which is not always justified, and provided care is taken that the necessary connections and disconnections do not alter the earth-capacitances that are being balanced, the simpler scheme will be found adequate for many purposes. It is especially valuable for routine measurements, but it is advisable to make a check measurement with a Wagner system for each of the types of measurement to be undertaken, *e.g.*, a measurement on an impedance of the required order at the highest frequency required. The standards of impedance used in bridges of this type may take the form of inductors, condensers and resistors, such as have been discussed in Chapters VI, VII and VIII. The bridge of Fig. 84 employs a differential condenser as standard, *i.e.*, essentially two variable condensers with a common rotor, the movement of which increases the capacitance in one arm and diminishes that in the other arm simultaneously. The instrument is calibrated to read the difference of the two capacitances directly. This is sometimes convenient but not of course essential. The standard itself must be calibrated by one of the methods discussed in previous chapters.

4. **The Fortescue-Mole Resonance Bridge.*** As an example of a bridge especially designed for work at very high frequencies, we may consider one of the resonance type developed by Fortescue and his collaborators and used by them at frequencies of 10 Mc/sec. and even higher. The bridge which is shown diagrammatically in Fig. 85 is in principle the same as Fig. 80 (*d*), but the resistors are all of the coaxial type, and the variable resistor is of the potentiometer type. The sliding contact, being connected to the detector, carries no current when the bridge is balanced, so that variations in contact-resistance have no effect on the measurements, a most important feature. It does, however, mean that the impedance under measurement is always in series with a portion of this resistor in one arm of the bridge, so that although a Wagner earth-connection is employed, the earth-capacitance at one terminal of the unknown impedance may affect the measurement; for this terminal does not occupy a corner of the bridge. This earth-capacitance is made as small as possible and is allowed for when necessary. The ratio-arms

* FORTESCUE, C. L. and MOLE, G. *Jour. Inst. Elec. Eng.*, 1938, Vol. 82, p. 687.

being equal, and the impedances of end-connections being small compared with those of the wire resistors themselves, the equations of balance may to a close approximation be written

$$R = \frac{R_4 - R_3}{(1 - L_3C_g\omega^2)^2} \quad . \quad . \quad . \quad . \qquad (9)$$

$$L\omega - \frac{1}{C\omega} = \frac{(L_4 - L_3)\omega}{(1 - L_3C_g\omega^2)^2} \qquad (10)$$

where the various quantities are as shown in Fig. 85, C_g being the earth-capacitance mentioned above. The variable resistor can be calibrated so as to give $R_4 - R_3$ directly, and since

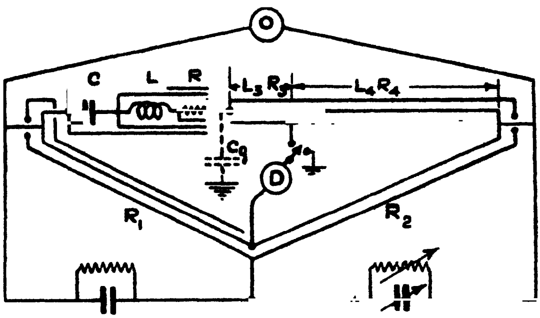


FIG. 85. Fortescue and Mole's bridge (schematic).

$L_4 - L_3$ is approximately proportional to this reading, the small correcting term which forms the right-hand side of equation (10) is very easily obtained. The term involving C_g is usually negligible in both (9) and (10).

The bridge is of course only applicable when the impedance to be measured can be tuned nearly to resonance, the slight deviation from true series resonance being indicated by the term on the right-hand side of (10). A coil may be tested by being placed in series with a condenser and *vice versâ*, but it is important to remember that the junction of such a series combination may reach a very high potential and that any stray earth-capacitance at this point may cause large errors. Such capacitance can be eliminated by the system of screening shown in Fig. 85. A resonance bridge should on this account be used with extreme caution for measurements on unscreened apparatus.

These bridges serve primarily for the measurement of resistance. They can, of course, in virtue of equation (10), be used for the measurement of reactance, if a standard coil or condenser is available, but such measurements are essentially resonance methods, the bridge merely serving as a special form of detector of resonance.

The form of variable coaxial resistor used by Fortescue and Mole is shown in Fig. 86, which is taken from their paper. The slide wire, which is of constantan, of diameter 0·2 mm., is kept taut by small springs, which are short-circuited by thin flexible copper strips. The connection to the moving contact passes through a longitudinal slot in the inner screen surrounding the

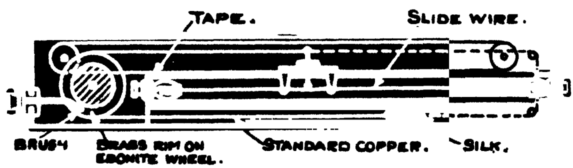


FIG. 86. Fortescue and Mole's slide-wire resistor.

resistance wire. The contact is actuated by a steel tape passing over pulleys, the graduations of the tape indicating the position of the contact, and the tape itself closing the longitudinal slot previously mentioned.

5. Measurement of Capacitance and Power Factor of Two-Terminal Condensers. We have seen that condensers are on the whole the most satisfactory standards for radio frequency work, and that substitution methods are the most accurate methods of measurement. It follows that whenever possible, measurements of the capacitance and power factor of condensers should be made by substitution against a standard condenser. The simplest case is that in which the screen of the condenser is connected to one terminal, sc that we have a two-terminal instrument. Substitution measurements on such a condenser by means of resonance methods have already been described (pp. 171, 187) and exactly the same considerations should be applied to bridge methods. The arrangements of leads shown in Fig. 70 (*a*) and (*b*) should be adopted and the same procedure should be followed as regards connections and capacitance readings, the only difference being that the standard condition of the circuit is that of balance instead of resonance, and that the readings which determine conductance, resistance or power factor are those appropriate to the particular bridge used. Thus if r, G and δ are the series resistance, shunt conductance, and loss angle of the condenser under test, and r_s, G_s, δ_s are the corresponding known values of the standard, the equations for the bridges of Fig. 80 (*a*), (*b*), (*c*) are

$$G - G_s = \Delta\left(\frac{1}{R_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$r - r_s = \Delta R_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

$$\delta - \delta_s = \tan^{-1}(R_4 \,.\, \Delta C_4 \,.\, \omega) \quad . \quad . \quad . \quad . \quad (c)$$

where ΔR_2 denotes the increment in the reading of R_2 produced by replacing the standard condenser by that under test in the bridge of Fig. 80 (*d*), and $\Delta(1/R_2)$ and ΔC_4 have similar meanings for the other two cases. This method is of course only practicable when a standard condenser of the appropriate range is available. This method, when carried out with a bridge of, say, the Dye-Jones type, is capable of very high accuracy. Capacitances may be compared with an accuracy better than 1 part in 10^5, and phase angles may be compared with an accuracy of 0·00001 radian. Indeed, the use of amplifiers in the detector circuit enables almost any desired sensitivity to be obtained; the stability of the bridge components is the limiting factor.

Condensers of larger value than the standard may be measured by connecting them in series with the standard, noting the change of capacitance and loss angle or resistance produced, and applying the formulæ for two condensers in series* (p. 7). The accuracy obtainable is, however, very much less than that obtainable by direct substitution.

Condensers of very low value may also be measured by a series method. A condenser of low value c is placed in series with a variable standard of ordinary range C. The capacitance of the combination being $cC/(C + c)$, a small change ΔC in the standard produces a change in the combination of approximately $\Delta C \,.\, c^2/(C + c)^2$, which is, for example, 100 times smaller than ΔC if $C/c = 9$. Thus if the small condenser to be measured is

* The values of R_s and $1/C_s$ are simply additive for a series combination.

added in parallel with such a combination, and the standard condenser is then diminished until the resultant capacitance is restored to its original value, the capacitance added is equal to the change in the reading of the standard divided by 100. This method is convenient when no standard of low range is available, but when a standard of the micrometer type is available the substitution method should be employed, as there is always some uncertainty in any estimation of the effect of the connecting lead in a series combination, however small the lead may be.

6. The Measurement of Self-Inductance and Effective Resistance of Coils. If a standard of self-inductance is available, the self-inductance of any coil within its range can be measured by substitution on exactly the same lines as the corresponding method for capacitance measurements. A bridge of the type of Fig. 80 (*b*) is the most suitable, and the measurement can be made at any frequency within the limits imposed by the construction of the bridge and the standard. As in the case of capacitance measurement, it is important that the coil and standard shall be connected in turn to the same leads, for it is essential that both the capacitance and inductance of the leads shall be the same for both readings of the bridge. It is also important that the mutual inductance between leads and coil shall be negligible in both cases, for this quantity is included in the quantity measured by the bridge. The advantages of standards of toroidal form for such measurements will be obvious in this connection. The effective resistance of the coil is determined by the change in the reading of the resistor in series with the inductors on making the substitution, together with the known value of the standard. It is important to remember that the effective resistance of an open coil may be affected by the eddy currents which it induces in neighbouring apparatus. Such coils must be well separated from other instruments of metallic construction during the measurements.

It is also important to remember that the quantities measured by the bridge are the effective series values L_s R_s, which are functions of self-capacitance and frequency as well as inductance proper (p. 143). The value of the self-capacitance can be obtained from measurements of L_s at various frequencies by application of the formula $L_s = L_1/(1 - L_1C_1\omega^2)$ (p. 142).

7. Very Small Self-Inductances or other Low Impedances. The substitution method is applicable to very small inductances if a standard of very low range, such as that of Fortescue (p. 180), is available, and adequate sensitivity is easily obtained at higher frequencies. Such inductances can, however, also be measured in terms of an ordinary standard condenser using a bridge of the Schering type at high frequencies. The small inductor is connected in series with the standard condenser. The effective capacitance of such a combination is given by $C/(1 - LC\omega^2)$ (p. 119). Thus on short-circuiting the inductor by a link of negligible inductance, say a short copper strip, the effective capacitance diminishes, but can be restored to its original value by adjusting the standard condenser from its original value C_1 to a larger value C_2. Then we have

$$C_1/(1 - LC_1\omega^2) = C_2$$

which reduces to

$$L = (C_2 - C_1)/C_1C_2\omega^2$$

The values of C and ω are chosen so as to obtain a value for $(C_2 - C_1)$ which can be read with the necessary accuracy on the scale of the standard condenser. The capacitance due to leads must of course be added to the condenser reading to obtain C_1 and C_2 for the product C_1C_2, but this correction cancels out when obtaining the difference $(C_2 - C_1)$.

The resistance of the inductor increases the apparent power factor of the condenser, and the value of the resistance R_L may be deduced from the value of this change, which is given by the bridge readings in the usual way. In each case the effective capacitance has the same value C_2, but the series resistance has the value $R_0 + R_{C1} + R_L$ when the inductor is in circuit, and $R_0 + R_{C2}$ when it is out of circuit, where R_0 is the resistance of the leads, and R_{C1}, R_{C2} are the values for the standard condenser. Thus the value of R_L is obtained by the approximate equation

$$\tan \delta_1 - \tan \delta_2 = (R_L + R_{C1} - R_{C2})C_2\omega.$$

For most standard condensers the difference $R_{C1} - R_{C2}$ is negligible. The value of $\tan \delta_1 - \tan \delta_2$ is of course given by the ordinary equation for the particular type of bridge used (*cf.* equations (1) and (6)).

It will be obvious that this method of measurement is applicable to any form of impedance of low value.

8. Large Self-Inductance, Chokes, and High Impedances Generally. Chokes, and high impedances generally, may be measured by a somewhat similar process. The high impedance is placed in parallel with a standard condenser, resistor, or any standard impedance of convenient value, and the change in resultant impedance measured by application of the usual bridge equations.

Thus let a bridge be balanced with a high impedance $R + jL\omega$ connected in parallel with a standard condenser. Now let the high impedance be removed and the bridge be restored to balance by adjusting the condenser, and whatever component controls the phase angle of the bridge. The second reading of the capacitance, C_2 say, evidently measures the resultant capacitance in both cases. Thus we have (see p. 12, Table 1, No. 10)

$$C_2 = C_1 - \frac{L}{Z^2} \text{ where } Z^2 = R^2 + L^2\omega^2$$

The effective conductance of the choke is given by R/Z^2, so that the change of loss angle δ produced by its disconnection is given by

$$\tan \delta = R/Z^2C_2\omega.$$

Thus the constants of the choke are easily determined from the bridge readings C_1 C_2 and $\tan \delta$ by means of the equations,

$$C_1 - C_2 = L/Z^2$$
$$C_2\omega \tan \delta = R/Z^2$$
$$Z^2 = R^2 + L^2\omega^2.$$

9. Three-Terminal Condensers. In the measurements so far discussed it has been assumed that each condenser is completely surrounded by a conducting screen which is connected to one of its terminals, so that the instrument has two terminals only and one value of capacitance at any given frequency. Some condensers, however, have their plate systems insulated from their screens, so that the instrument has three terminals and three component capacitances, as shown in Fig. 87. We have seen that when simultaneous balance of a bridge and a Wagner earth connection are obtained, the conditions of balance of the main bridge depend only on the direct impedances between the terminals connected to its corners. It follows that the capacitance between any two electrodes of a three-terminal condenser can be measured by connecting those two terminals to one arm of a suitable bridge and connecting the third terminal to earth. The Wagner connection then eliminates the effects of capacitances to this earth-connected terminal. The measurement itself is made in exactly the same way as for a two-terminal condenser, and the conductance or power factor associated with the component capacitance can of course be measured at the same time by the usual method.

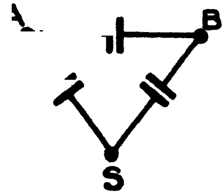


FIG. 87. Three-terminal condenser.

Similar considerations apply to any multiple electrode system, *e.g.*, a thermionic valve or a multiple cable with a conducting sheath. To measure the direct impedance between any two electrodes, it is only necessary to connect these two electrodes to one arm of the appropriate bridge with a Wagner earth-connection and to connect all the other electrodes to earth.

* When measuring three-electrode condensers in this way by the substitution method at high frequencies, it is very important to use leads of the lowest possible inductance and resistance to connect the condenser to the bridge, for such leads carry the current flowing through all the capacitances, and not only that in the one under measurement. Thus the effect of the leads is only eliminated by the substitution, if the standard condenser has all its capacitances made equal to those of the condenser under test, a condition which is not easily satisfied. The arrangement is shown in Fig. 88, in which O is the corner of the bridge connected to the oscillator, D that to the

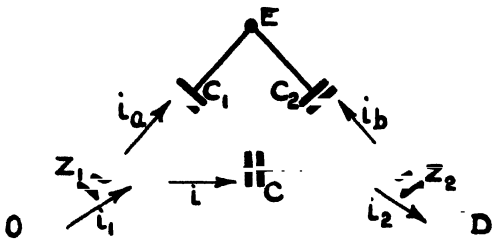


FIG. 88. Illustrating the effect of leads on measurements of a three-terminal condenser.

* JONES, T. I. *Jour. Inst. Elec. Eng.*, 1934, Vol. 74, p. 185.

detector, and E the earth connection. The leads are represented by the small impedances Z_1 and Z_2. Denoting the potentials by V_O, V_D, etc., and remembering that $V_D = V_E$ when the Wagner balance is obtained, it is easy to show that when Z_1 and Z_2 are small quantities we have approximately

$$i_b = jZ_2C_2\omega i_2$$
$$i = i_2 + i_b = i_2(1 + jZ_2C_2\omega)$$
$$i_1 = i_a + i = \frac{C_1 + C}{C} . (1 + jZ_2C_2\omega)i_2$$

whence

$$V_O - V_D = Z_1i_1 - \frac{j}{C\omega}i + Z_2i_2$$

which neglecting small terms involving the product Z_1Z_2 becomes

$$V_O - V_D = i_2\left[Z_1\left(\frac{C_1 + C}{C}\right) + Z_2\left(\frac{C + C_2}{C}\right) - \frac{j}{C\omega}\right]$$

Now the current leaving the arm OD of Fig. 88 and entering the adjacent arm of the bridge is i_2, and therefore the impedance measured is $(V_O - V_D)/i_2 = Z$ where

$$Z = Z_1\left(\frac{C + C_1}{C}\right) + Z_2\left(\frac{C + C_2}{C}\right) - \frac{j}{C\omega}$$

Thus, in effect, the impedances of the leads Z_1 and Z_2 are increased to $Z_1(C + C_1)/C$ and $Z_2(C + C_2)/C$. If the resistances of the leads are denoted by r_1 r_2 and the inductances by l_1 l_2 it follows that the apparent loss tangent is given by

$$\tan\delta = r_1(C + C_1)\omega + r_2(C + C_2)\omega.$$

and the apparent capacitance by

$$C_a = C/[1 - l_1(C_1 + C)\omega^2 - l_2(C + C_2)\omega^2].$$

Thus the effects of leads on both capacitance and phase angle will only be correctly allowed for by the substitution, if the quantities C, C_1 and C_2 all remain unaltered. It is obvious that when possible, the earth-capacitances C_1 and C_2 should be made small.

When no Wagner earth-connection is available the component admittances of a three-terminal system can be measured by short-circuiting each component in turn, and measuring the three two-terminal admittances so formed in the ordinary way. If the three components are Y_1, Y_2, Y_3, the three values measured are obviously $Y_1 + Y_2$, $Y_1 + Y_3$ and $Y_2 + Y_3$, and from these the separate values are very easily calculated. Another method which consists virtually of a measurement of the difference $(Y_1 + Y_2) - (Y_1 - Y_2) = 2Y_2$ is described in paragraph 10 (p. 229).

10. **Dielectric Constant and Power Factor.** Measurements of the dielectric constant and power factor of solid and liquid dielectrics have already been discussed in general in Chapter X, in which several forms of electrode suitable for resonance methods have been described. Such arrangements are of course also applicable to bridge methods. The condenser formed by the dielectric sample and its electrodes is treated just like any other two-terminal condenser and measured by substitution against a standard air condenser. Bridge methods have, however, the great advantage that they are applicable, as we have seen, to three-terminal systems; and it therefore becomes possible to use electrode systems including guard-rings in addition to the two electrodes required for the actual measurement. The way in which guard-rings give rise to electric fields of accurately calculated capacitance has already been explained (p. 104). No other practicable forms of capacitance can be calculated with the same precision, and it follows that bridge methods of measuring dielectric constant and phase angle are, at their best, capable of higher accuracy than other methods. It must however be remembered that the extra complications of the bridge circuit introduce errors at high frequencies, and that no advantage is gained by the use of bridge methods at frequencies at which such errors are as great as those due to the absence of guard rings.

As the measurement of the capacitance and phase angle of material in the form of flat sheets is one of considerable importance, we shall consider the bridge method including a guard-ring in some detail. Fig. 89 shows diagrammatically the way in which a flat sample of a solid dielectric A, provided with electrodes H and L and a guard-ring G, is connected to a bridge of the Schering type. The sample and its electrodes are completely surrounded by a screen S, which it will be observed, is connected to the guard-ring and to earth. The guarded electrode L is connected to a detector-corner of the bridge, and is therefore brought to earth potential when simultaneous balance is obtained. Thus G and L are at the same potential when the measurement is made, so that the condition necessary for the production of a uniform electric field within the portion of the sample covered by the inner electrode is established. Moreover, with the connections shown, the bridge measures the constants of this portion of the sample only, for it is only the current from the inner electrode which flows into the ratio-arm of the bridge. Thus the " air-capacitance " corresponding to the measured value of the " sample-capacitance " is given with high accuracy by the formula. In the absence of the screen S the electrodes H and L might be linked by lines of force

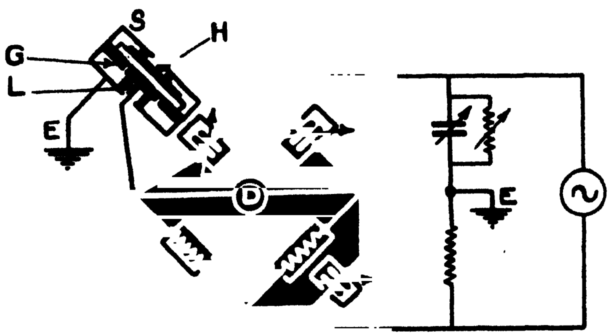


FIG. 89. Bridge circuit for measurements on a sample of solid dielectric with a guard-ring.

passing outside the sample and guard-ring, and this extraneous fringing-capacitance, and any power loss associated with it would then be included in the quantity measured. Some form of screen is therefore essential. It should also be noticed that the guard ring eliminates from the quantity measured, not only the edge-capacitance, but also surface leakage. The surface leakage current passes from the electrode H round the edge of the sample to the guard-ring G, and, provided this makes good contact with the material, the current passes into G and so to earth, and not into the bridge circuit. Moreover, since G and L are at the same potential there is no leakage current from G to L. The method therefore measures the true permittivity and power factor of the material, independent of surface leakage and edge corrections, and indeed the only precise method of evaluating such corrections is to make a measurement by this method, and also by the method for which the corrections are desired, on the same sample, and to deduce the corrections from the difference between the results obtained.

It should be noticed that the construction of the electrode system necessitates the use of solid insulating material between *H* and the screen, and between *L* and the screen. The power losses in this material merely form part of the earth-admittances from the corners of the bridge, and are therefore eliminated from the bridge circuit along with the corresponding earth-capacitances, by the Wagner balance. We have seen, however, that on account of the finite impedance of the leads there are slight departures

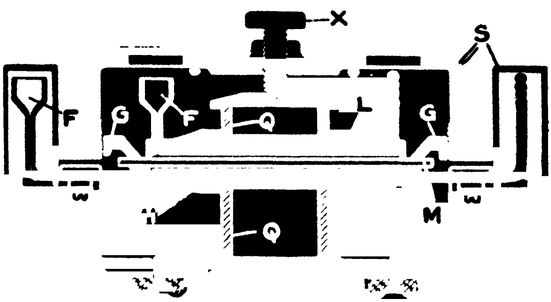


FIG. 90. Mercury electrodes for measurements on solid dielectrics in sheet form. (T. I. Jones.)

from the ideal conditions, and that it is therefore important to make the earth-capacitances small, especially for work at high frequencies. When designing electrodes for such work it is therefore important to make the capacitance between *H* and *S*, *L* and *S*, and *L* and *G* small, and to use material of low dielectric constant and low power factor for the solid insulating supports.

A mercury electrode system satisfying these requirements is shown in Fig. 90.* Here the solid insulation consists of fused quartz tubing *Q*; there is wide spacing between screen *S* and electrodes *H* and *L*, and the section of the guard ring *G* is shaped so as to reduce the capacitance between *G* and *L*, while leaving only a narrow gap between them on the actual surface of the material to be measured. The whole apparatus is made of iron,

* JONES, T. I. *Loc. cit.*, 1934.

since this is not attacked by mercury. Metals like brass and copper cause contamination of the mercury, which often leads to defective contact with the sample and a high apparent power factor. Mercury is poured into the funnels *F* and so enters the electrodes *H* and *L*, the displaced air escaping through the outlet tubes. For this operation the apparatus should be tilted slightly so that the mercury enters the electrode at its lowest point and gradually flows up to the outlet at the topmost point, leaving no air bubbles. The guard-ring contains no mercury. It is carried by the upper half of the case *S*, and is pressed into contact with the sample when this is bolted to the lower half. The bolts, which are distributed round the rim, are not shown in the diagram. The screw *X* applies pressure between the electrodes *H* and *L* and the sample and so makes the electrodes mercury-tight. It also serves to centralise the electrode *L*. Connection to the bridge is made by leads dipping into the mercury in the funnels. Since these funnels and leads are separately screened as shown, their capacitances are eliminated from the bridge by the Wagner balance.

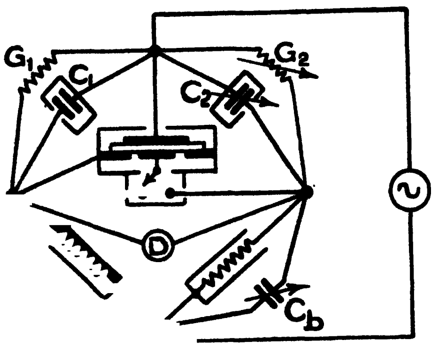


FIG. 91. Arrangement for testing dielectric specimen with guard-ring, without using a Wagner earth-connection.

Many other forms of mercury electrode have been devised, but that described above can be taken as typical. Tinfoil may also be used to make guard-ring systems. The foil is applied in the manner previously described (p. 192) and the narrow annular gap between *L* and *G* cut out with dividers, or a sharp blade fitted into compasses. Backing with copper plates is required as before. The method of measurement is of course the same as for mercury electrodes, and it is obvious that the same method is also applicable to cylindrical guard systems used for tubular specimens.

Guard-ring systems can also be measured by means of a simple bridge with a direct earth connection by the following procedure. The connections are arranged as shown in Fig. 91, a bridge with a shunt resistance or conductance adjustment for phase angle being shown as an example. We will suppose that the capacitance C_2 and conductance $G_2 = 1/R_2$ alone are varied. The guard-ring is permanently connected to one detector terminal, while the guarded electrode L is connected, first to this same terminal, and secondly to the other detector terminal. In both cases G and L are at the same potential when the bridge is balanced, so that the guard-ring condition is satisfied. If C_S and G_S are the capacitance and conductance required, and C_g, G_g those between the upper electrode and guard system, then the ratio-arms being equal, the first (C'_2, G'_2) and second (C''_2, G''_2) balance readings are given by

$$C_1 + C_g + C_S = C'_2$$
$$C_1 + C_g = C''_2 + C_S$$

and exactly similar equations for conductance, from which we have

$$C'_2 - C''_2 = 2C_S$$

and

$$G'_2 - G''_2 = 2G_S$$

whence

$$\tan \delta = (G'_2 - G''_2)/(C'_2 - C''_2)\omega.$$

It should be noted that symmetry of the bridge is essential, so that the earth-capacitance of the screen and guard system must be balanced with a small condenser C_b, by a preliminary adjustment. It is also important that the moving link, which is used to make and break the necessary connections, shall be effectively screened; so that the changes of capacitance caused by the motion are only capacitances to screen, and therefore merely a shunt across the detector, having no effect on the balance-point.

The same principle can obviously be applied to any four-arm bridge, and measurements of direct impedances of all kinds can therefore be made in a similar way. The two electrodes and guard ring can be replaced by any other three-terminal system. The admittance required, Y_1, is first added to arm 1 and then transferred to arm 2, and the change compensated by adjustments to one arm of the bridge. Of the remaining admittances, one, Y_2, is a constant shunt on arm 1, and the other, Y_3, is either short-circuited or else a shunt on the detector: in both cases it carries no current and may be ignored. The first bridge reading is proportional to $Y_1 + Y_2$ and the second to $-Y_1 + Y_2$, so that the difference between the two gives $2Y_1$.

11. **Miscellaneous Measurements.** The bridge methods which have been described can be applied to the measurement of any quantity which is of the nature of an impedance. It is only necessary that two or more terminals shall be accessible for connection to the bridge, and that the system to be measured

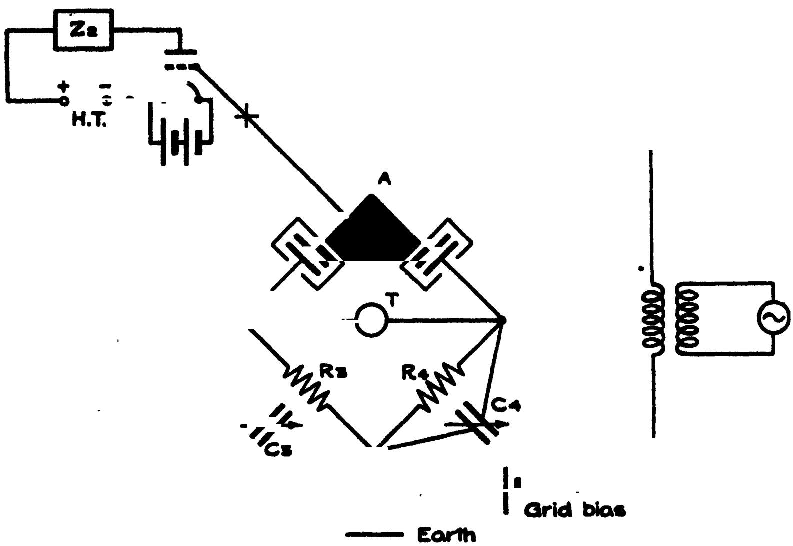


Fig. 92. Bridge for measurement of the input impedance of a valve circuit. (For the highest accuracy a Wagner earth-connection should be used.)

shall be passive during the measurement, *i.e.*, shall not itself pass into the bridge any current of the frequency at which the test is to be made. The result of the measurement will be expressed as a resistance, capacitance, or inductance according to the phase angle between terminal current and voltage. Thus measurements can be made on transformers, antennas, filter circuits, etc., as well as on the components which have actually been considered. It goes without saying that bridge methods can also be applied to measurements on thermionic valves similar to those described in the last Chapter. Suitable circuit arrangements * are shown in Figs. 92 and 93. The inter-electrode capacitances of valves with the filament cold can be measured just like any other small capacitances, and if a Wagner system is employed, each of the component capacitances can be separately measured. These circuits are suitable for capacitances down to about 0·01 $\mu\mu$F. For still smaller values the use of a capacitance potential-divider, as described by Astbury and Jones,† is more convenient.

The use of the bridge for the study of dielectric properties has

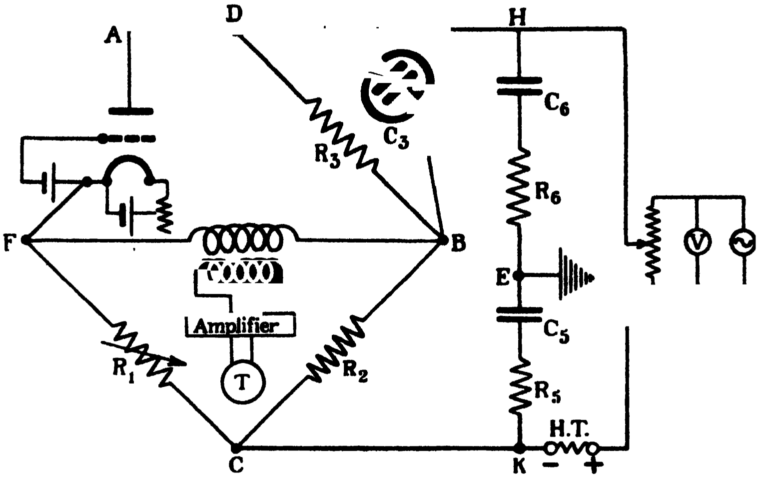


FIG. 93. Bridge for measurement of the anode circuit impedance of a valve.

been discussed in some detail. Similar considerations can be applied to the investigation of magnetic properties. The only difference is that in the one case a test condenser is constructed and measured, and in the other a test inductor must be prepared and measured. Thus if a toroidal inductor is wound on a core of magnetic material, and an exactly similar winding is wound on a non-magnetic core, the ratio of the two inductances gives the effective permeability of the magnetic material, while the difference R_m, of the equivalent series resistances of the two inductors,

* HARTSHORN, L. *Proc. Phys. Soc.*, 1927, Vol. 39, p. 108, and 1929, Vol. 41. p. 113. JONES, T. I. *Jour. Inst. Elec. Eng.*, 1937, Vol. 81, p. 658.

† ASTBURY, N. F., and JONES, T. I. *Jour. Sci. Instrum.*, 1936, Vol. 13, p. 407.

serves to measure that of the magnetic material. The loss tangent of the material is then given by $R_m/L_m\omega$, where L_m is the inductance of the coil on the magnetic core.

The application of the radio-frequency method to measurements of electrolytic conductivity, or indeed any form of conductivity whatever, requires no special discussion, for the electrical technique involved is essentially the same as that already described for resistors in general or for dielectrics.

CHAPTER XII

METHODS FOR VERY SHORT WAVES

1. **Frequency Limitation of Circuit Methods.** All the methods so far discussed are based on the assumption that the apparatus consists of circuits in which the application of a definite p.d. produces a definite current, this current having the same value throughout the whole of its path. It is however known that electric currents are to be regarded as manifestations of electromagnetic waves, which are propagated through dielectrics with a velocity of the order of that of light. Ordinary circuits are merely the boundaries of the media of propagation, and the current in any part of a conductor is that value which is characteristic of the magnetic field of the wave in its neighbourhood. It follows that the current at any instant, like the magnetic field, varies from point to point in the path of the wave, a variation from a maximum value to zero occurring in the course of a quarter wavelength ($\lambda/4$). Thus the current can only be regarded as having the same value throughout a circuit, if the linear dimensions of that circuit are small compared with $\lambda/4$. It is impracticable to make apparatus of dimensions less than a few cm. and it follows that the methods so far described cannot be employed for wavelengths less than, say, 2 metres, *i.e.*, frequencies greater than 150 Mc/s. Since waves in the region 200 to 1,000 Mc/s are steadily increasing in importance both scientifically and technically, the use of resonance methods in this region will be briefly considered. Bridge methods are scarcely practicable.

2. **Reflection and Stationary Waves in Transmission Lines.** The complications arising when it becomes necessary to take into account the variations of current and voltage from point to point in the current path, are so considerable that most methods of measurement are based on the simplest possible case, viz., that of a plane wave travelling along a dielectric path of uniform cross section. In practice such a wave is realised by

the use of either a pair of parallel wires (the familiar Lecher system) or a length of cable used as a "transmission line." The two conductors of the cable, or the two wires, are boundaries of the path of the wave, and direct its course. The passage of an electric wave along such a line is accompanied by currents in the two conductors, the value at any point depending on the magnetic field at that point, and being the same but of opposite sign for the two conductors. The electric field at any point of the path establishes a p.d. between the two conductors at that point, and the ratio of the p.d. to the current at any point defines an impedance. It can be shown that the impedance at the input end of an infinite length of such a uniform line has a finite value, the characteristic impedance Z_0 of the line, which can be calculated from the capacitance and inductance per unit length of line, quantities characteristic of the electric and magnetic fields respectively. If a short length of such a line is terminated by its characteristic impedance, the current-voltage relation is evidently the same as if the line were infinitely long, and any wave passing along the line simply disappears into the termination. If, however, the impedance of the termination differs from Z_0, the wave is not wholly absorbed, but at least partially reflected, a secondary wave travelling back along the line. If no energy is dissipated in the termination, which will be the case whenever the termination is free from resistance and conductance, *e.g.*, an open circuit, a short circuit, or any pure reactance, the amplitude of the reflected wave will be equal to that of the incident wave. If also the loss of energy (attenuation) in the line itself may be neglected, there will be two trains of waves of equal and constant amplitude moving in opposite directions along the line and we may represent them :—

Incident wave $E_1 = E_0 \sin(\omega t - 2\pi x/\lambda)$.

Reflected wave $E_2 = E_0 \sin(\omega t + 2\pi x/\lambda + \phi)$ where ϕ represents the change of phase on reflection, and x is the distance measured from the point of reflection.

Resultant

$$E = E_1 + E_2 = 2E_0 \cos\left(\frac{2\pi x}{\lambda} + \frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right).$$

The resultant is a system of stationary waves, the electric field having the same phase at every point along the line, but the amplitude varying with distance according to a cosine law. The amplitude is zero at points for which $2\pi x/\lambda + \phi/2 = (2n + 1)\pi/2$ or $x = \frac{(2n+1)\lambda}{4} - \frac{\phi\lambda}{4\pi}$. These are the voltage nodes, which occur at regular intervals of $\lambda/2$ along the line. At points midway between them (the antinodes) the amplitude reaches the maximum value of $2E_0$.

The above argument can also be applied to the magnetic field, or the current to which it is proportional, the only difference being that the current nodes coincide with the voltage antinodes, and *vice versâ*. It is to be noted that the distance of the nodes from the reflecting termination depends on the change of phase on reflection. At an open-circuit termination, the current must obviously become zero, and at such a termination there must therefore be a current node, and a voltage antinode. At a short-circuit termination the voltage must be zero, and we must therefore have a voltage node and current antinode. These two cases are those of zero and infinite reactance, and the change from one to the other shifts the nodes through a distance of $\lambda/4$. Terminations of reactance of a finite value give nodes in an intermediate position, and the reactance of a termination may be measured by the position of the nodes it produces by reflection, or what amounts to the same thing, the change of phase on reflection. If the terminating impedance contains a resistive component, energy is dissipated, and the amplitude of the reflected wave is correspondingly reduced. If reflection reduces the amplitude by a factor ρ and advances the phase by an angle ϕ, the operator $\rho\underline{/\phi} = K$ is called the reflection coefficient, and is given by :—

$$K_t = \frac{\mathbf{Z}_0 - \mathbf{Z}_t}{\mathbf{Z}_0 + \mathbf{Z}_t} = -\frac{\mathbf{V}_r}{\mathbf{V}_i} = \frac{\mathbf{I}_r}{\mathbf{I}_i} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$\mathbf{Z}_t$ being the impedance operator of the termination, $\mathbf{V}_i$ and $\mathbf{I}_i$ being the vectors representing voltage and current in the incident waves, and $\mathbf{V}_r$ and $\mathbf{I}_r$ the corresponding quantities for the reflected wave. Rearranging and putting K in the form $\rho \cos \phi + j\rho \sin \phi$, the relation may be written :—

$$\frac{\mathbf{Z}_t}{\mathbf{Z}_0} = \frac{1 - K_t}{1 + K_t} = \frac{1 - \rho^2 - j2\rho \sin \phi}{1 + \rho^2 + 2\rho \cos \phi} \quad . \quad . \quad . \quad . \quad (2)$$

It is possible to construct lines so that at very high frequencies $\mathbf{Z}_0$ is to the accuracy of measurement a pure resistance, Z_0, and in this case if $\mathbf{Z}_t = R_t + jX_t$ we have :—

$$\frac{R_t}{Z_0} = \frac{1 - \rho^2}{1 + \rho^2 + 2\rho \cos \phi} \text{ and } \frac{X_t}{Z_0} = \frac{-2\rho \sin \phi}{1 + \rho^2 + 2\rho \cos \phi} \quad (3)$$

Thus when $\mathbf{Z}_t$ is a pure reactance $R_t = 0$ and $\rho = 1$.
The expression for X_t then reduces to

$$X_t = -Z_0 \tan \frac{\phi}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It follows that ϕ is of opposite signs for positive (inductive) and negative (capacitive) reactances. Fig. 94 represents the voltage

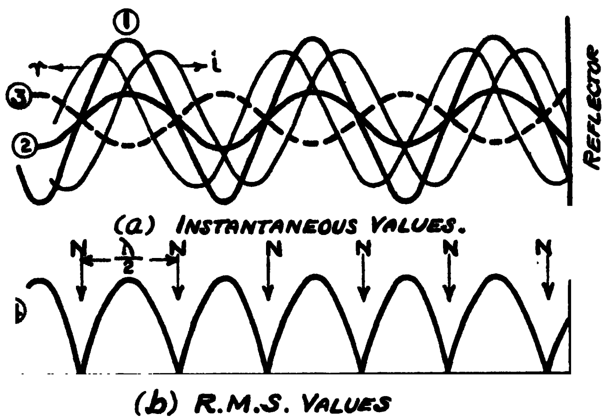


FIG. 94. Waves on line of zero attenuation terminated by a pure reactance. i = incident wave, r = reflected wave, (1) = their resultant, (2) and (3) = resultants at later instants when i and r have travelled equal distances to right and left respectively. N = node.

distribution along such a line. Curves (1) and (2) represent the instantaneous voltage at two instants. Curve (4) represents R.M.S. voltage, and therefore might be observed by a voltmeter travelling along the line.

When $\mathbf{Z}_t$ is a pure resistance, $X_t = 0$ and therefore $\sin \phi = 0$, $\cos \phi = \pm 1$, and

$$\frac{R_t}{Z_0} = \frac{1 - \rho^2}{(1 \pm \rho)^2} = \frac{1 + \rho}{1 - \rho} \text{ or } \frac{1 - \rho}{1 + \rho}. \quad (5)$$

If $R_t > Z_0$, the $-$ sign must be taken, and $\phi = \pi$, while if $R_t < Z_0$ the $+$ sign must be taken and $\phi = 0$. Thus variations of R_t confined to one side of the critical value Z_0 affect only the amplitude and not the phase of the reflected wave.

It should be noted that when the amplitude of the reflected wave is less than that of the incident wave there are no true nodes, although the interference of reflected and incident waves

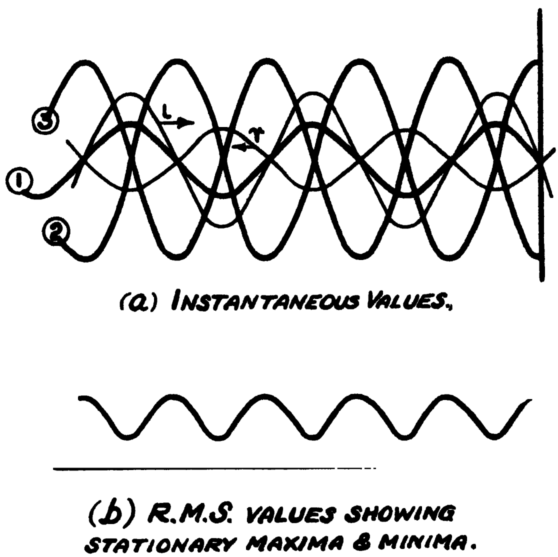


FIG. 95. Waves on line of zero attenuation terminated by a pure resistance, giving no change of phase on reflection. i = incident wave, r = reflected wave, (1) = their resultant, (2) and (3) = resultants at instants later by ¼ period, and ¾ period, respectively.

produces maxima and minima of voltage, more or less as shown in Fig. 95. When the change of phase on reflection is 0 or π, the maxima will obviously occur when the two waves are in phase, the resultant amplitude being $E_0 + \rho E_0 = E_0(1 + \rho)$, while the minima will occur when the two waves are in opposition, the minimum amplitude being $E_0(1 - \rho)$. Thus, in this case (termination, a pure resistance), the ratio of the maxima and minima of voltage $V_{\text{max.}}/V_{\text{min.}} = (1 + \rho)/(1 - \rho) = R_t/Z_0$ or Z_0/R_t by equation (5), can be used to determine R_t. The general case is more complicated, but the method has been developed, the general scheme being shown in Fig. 96.* We shall not enter into details, as other methods, which do not involve moving the voltmeter or other detector, are available. It may be noted, however, that in

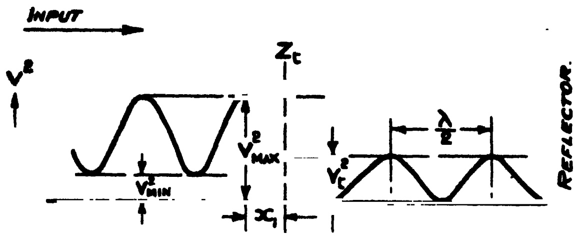


FIG. 96. Scheme for determination of impedance Z_t by voltage measurement. (Hempel.) Note that Z_t is placed at an antinode of the stationary waves produced by the reflector. The impedance of the line to the right of Z_t is therefore infinite, and Z_t is in effect the terminating impedance giving rise to the maxima and minima $V_{max.}$ and $V_{min.}$

the general case the magnitude of the terminal impedance is given by :—

$$\frac{Z_t}{Z_0} = b^2 = \frac{V_t^2}{V_{t+\lambda/4}^2} = \frac{V_t^2}{V_{max.}^2 + V_{min.}^2 - V_t^2}$$

and the reactance by

$$X_t = \tfrac{1}{2}Z_0(1 - b^2)\tan(4\pi x_1/\lambda).$$

3. Resonance in Lines. Consider a line on which stationary waves have been established. The ratio of voltage to current, *i.e.*, the impedance, varies from point to point along the line, having a maximum value, infinite in the ideal case of zero power loss, at the current nodes, and a minimum value, zero in the ideal case, at the voltage nodes. Let the point at which the measurement is to be made (the measurement terminals) be fixed. The position of the nodes with respect to this point may evidently be varied by adjusting the length of the line, the frequency and therefore λ, or the terminal reactance and therefore ϕ. Thus by varying any one of these quantities the measurement terminals may be made a voltage antinode, or a current antinode. In the first case, if the current passed into the line is kept constant, the measured voltage rises sharply to a maximum value, and the

* HEMPEL, W. *Elektronachrichtentech*, 1937, Vol. 14, p. 33.

condition is that of voltage resonance as described in Chapter II. The second case is that of current resonance. It is evident that a line may be tuned to resonance by variation of length, frequency or terminal reactance, and it will be found that the conditions of resonance, and the sharpness of the resonance curves provide methods of measurement analogous to those of Chapters IX and X, but applicable to much higher frequencies. The general equations are apt to become too complicated for practical use, and some approximation is inevitable. As a result several more or less alternative versions of these methods have been published. Those described below are given as typical. They will probably cover all present-day requirements.

4. Measurement of Reactance and Resistance by Voltage Resonance. A useful and instructive arrangement for

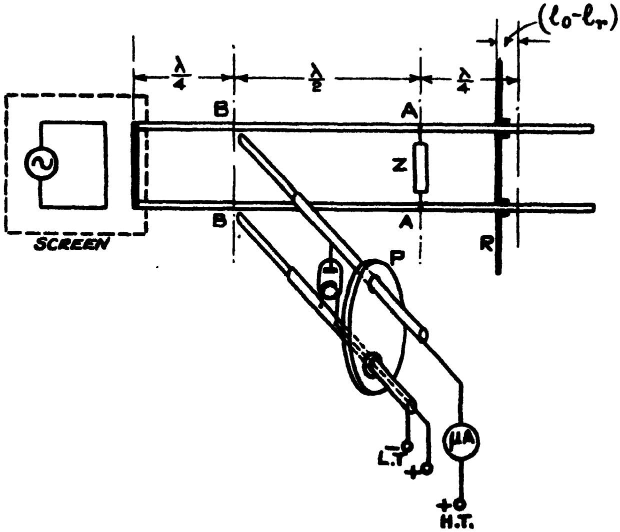


FIG. 97. Kaufmann's arrangement for measurements by voltage resonance. R = reflector, P = parallel plates forming a condenser (reactance termination) on the diode voltmeter system.

such measurements is that described by Kaufmann,* and shown in Fig. 97. A transmission line consisting of two brass rods *A B* is short-circuited at its input end, and loosely coupled, inductively to a shielded oscillator. At the far end of the line is a copper

* KAUFMANN, H. *Hochfrequenztechn. u. Elektroak*, 1939, Vol. 53, p. 61.

disc which acts as a termination of zero impedance, or perfect reflector. The copper disc is moved along the line until stationary waves are established. The detector used for making this adjustment and for locating the nodes and antinodes is a diode voltmeter capacitively coupled to the line as shown in the diagram. The diode is connected across an auxiliary Lecher system of length $\lambda/4$, perpendicular to the main system, the open end of the detector system being placed near but not touching the main system. The sensitivity of the complete voltmeter is easily controlled by varying the distance of the diode from the open end, or by using as terminal bridge a parallel plate condenser of variable reactance. The object under test is connected across the line at a voltage antinode A.

The measuring circuit proper is the portion of the line between A and R. It follows from a general principle of networks that the whole system to the left of $A\,A$ is equivalent to a generator of e.m.f. equal to the open-circuit voltage at $A\,A$, and of impedance equal to that measured at the terminals $A\,A$. Now $A\,A$ being an antinode, this impedance approaches infinity if the losses in the line are negligible. The impedance is therefore so large that variations in the impedance of the measuring circuit itself do not appreciably affect the total impedance, and in effect, a constant current is applied to the measuring circuit, and the voltage at $A\,A$ will therefore be proportional to the input impedance of this circuit. Now the input impedance $\mathbf{Z}_i$ of a line of length l, characteristic impedance $\mathbf{Z}_0$, and propagation constant $\gamma=\alpha+j\beta$, closed by a link of zero impedance at the end, is given by :—

$$\mathbf{Z}_i = \mathbf{Z}_0 \tanh \gamma l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If the admittance of the test object is $G + jB$, the total admittance $\mathbf{Y}$ of the measuring circuit is :—

$$\mathbf{Y} = G + jB + \frac{1}{\mathbf{Z}_0} \coth \gamma l.$$

If the attenuation in the line is negligible, we may put $\alpha = 0$, and $\gamma = j\beta$ and obtain :—

$$\mathbf{Y} = G + j\left(B - \frac{1}{Z_0}\cot \beta l\right). \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The voltage across $A\,A$ is inversely proportional to $\mathbf{Y}$, and will therefore reach a maximum value $\mathbf{V}_m$ as l is varied, when l takes the value l_r such that $B = \frac{1}{Z_0} \cot \beta l_r$ . . . . (8)

Moreover, if $\mathbf{V}$ and l are any other corresponding values we have :

$$\frac{\mathbf{V}_m}{\mathbf{V}} = \frac{G + j\left(B - \frac{1}{Z_0} \cot \beta l\right)}{G} = \frac{G + j\,\frac{1}{Z_0}\,(\cot \beta l_r - \cot \beta l)}{G}$$

This equation represents the resonance curve obtained by varying l and observing $\mathbf{V}$. The magnitudes of the voltages are evidently given by :—

$$\frac{V_m^2}{V^2} = \frac{G^2 + \frac{1}{Z_0^2}\,(\cot \beta l_r - \cot \beta l)^2}{G^2} = q \quad . \quad . \quad . \quad (9)$$

from which we have :—

$$G\sqrt{q - 1} = \frac{1}{Z_0}\,(\cot \beta l_r - \cot \beta l) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

It should be observed that these equations correspond closely to those given in Chapter III for voltage resonance in concentrated circuits. Notice also that at resonance the impedance of the measuring circuit reaches the maximum value $1/G$, and if V_m' and V_m'' are the maximum voltages obtained with two different test objects, their conductances G' and G'' are given by:—

$$\frac{V_m'}{V_m''} = \frac{G''}{G'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Equation (8) provides a method for measuring susceptance, and therefore capacitance or inductance. First, the value of l is adjusted until resonance is obtained with the test object removed, *i.e.*, $B = 0$. Let l_0 represent the length of the measuring circuit for this condition. Then $\cot \beta l_0 = 0$, $\beta l_0 = \pi/2$ and $l_0 = \lambda/4$. The test object is now added, and resonance restored by adjusting l to l_r. Then the equivalent shunt capacitance C_s of the test object is given by :—

$$C_s\omega = \frac{1}{Z_0} \cot \beta l_r = \frac{1}{Z_0} \tan\left(\frac{\pi}{2} - \beta l_r\right)$$

or $$C_s\omega = \frac{1}{Z_0} \tan \beta(l_0 - l_r) = \frac{1}{Z_0} \tan \frac{2\pi}{\lambda}\,(l_0 - l_r) \quad . \quad . \quad (12)$$

The reflecting plate R being provided with a micrometer adjustment the difference $l_0 - l_r$ can be measured with high accuracy. Z_0 is calculated from the linear dimensions of the line. Resistance and conductance in the line being made negligible we have $Z_0 = \sqrt{L/C}$,* where L and C are the inductance and capacitance per unit length of line, quantities which can be calculated from the formulæ in Chapters VI and VIII. When the susceptance is positive $l_0 > l_r$, *i.e.*, the addition of capacitance shortens the resonance length. Conversely, the addition of inductance increases the resonance length. The method will evidently measure either quantity.

Conductance or equivalent shunt resistance can be obtained from the width of the resonance curve, using equation (10), which can be put in the form :—

$$G\sqrt{q-1} = \frac{1}{Z_0}\left[\tan\frac{2\pi}{\lambda}(l_0 - l_r) - \tan\frac{2\pi}{\lambda}(l_0 - l_r - \tfrac{1}{2}\Delta)\right]. \quad (13)$$

where Δ is the width of the resonance curve, or $l_1 - l_2$, where l_1 and l_2 are the two lengths, one on either side of l_r for which the voltage V is such that $V_m^2/V^2 = q$.

Alternatively, conductance can be determined from observations of resonance voltage, using equation (11), provided one conductance is known. For all these measurements the procedure described for the circuit methods should be followed as far as it is applicable. It must be emphasised that the equations given only apply when the power losses in the measuring circuit apart from the test object are negligible. The observations with no added conductance will indicate the order of error involved. Kaufmann has given equations for the more general case, but they become more elaborate than is desirable for general working

* For a pair of twin parallel wires of diameter d, and distance between axes D, ($D \gg d$), this formula becomes for very high frequencies,

$$Z_0 \simeq 4\sqrt{\frac{\mu_0}{\epsilon_0}}\log_e\left(\frac{2D}{d}\right)$$

where μ_0 is the permeability, and ϵ_0 the permittivity of the medium surrounding the wires. Inserting the numerical values in practical units we obtain for a line in air,

$$Z_0 \simeq 276 \log_{10}(2D/d) \text{ ohms.}$$

The corresponding equation for a coaxial line is

$$Z_0 \simeq 138 \log_{10}(r_2/r_1) \text{ ohms.}$$

where r_1 is the radius of the inner conductor, and r_2 the internal radius of the outer conductor.

equations. In the above discussion it is assumed that the voltage measured is that across $A\ A$. If, however, the line is free from loss the voltage at the other antinodes will have the same value, so that the voltmeter may be coupled to the points $B\ B$ as shown in Fig. 97, the line of length $\lambda/2$ and negligible resistance acting as a 1/1 transformer. This arrangement avoids any change in the impedance of the test object due to the proximity of the voltmeter. The voltmeter must obviously be of the highest practicable impedance. The capacitive coupling and the use of the auxiliary line of length $\lambda/4$ make it possible to satisfy this condition in a convenient manner.

It should be remembered that measurements of impedance and admittance on resonant lines by voltage resonance can also be measured by the methods of Chapters IX and X, provided a suitable standard condenser is available. Nergaard * has described a variable air condenser with electrodes of linear dimensions not exceeding 1 cm., and such condensers can be used as standards of capacitance at wavelengths down to 1 metre. The standard condenser and a voltmeter are connected to the end of the line at which the impedance is to be measured. The line is loosely coupled to an oscillator, and voltage resonance established by varying the condenser. Since the length of line remains unaltered its effect on the voltmeter and condenser is merely that of a constant impedance which is measured exactly as described in Chapter X. Condensers and transmission lines suitable for such measurements at wavelengths of the order of 1 to 2 m. have been described by Nergaard.*

5. Measurement of Reflection Coefficients, Impedance, etc. by Current Resonance.* The scheme of measurements by current resonance to be considered is that described by R. A. Chipman. His treatment of the problem is of special value in bringing out clearly the physical factors involved and in providing a more general solution in a fairly workable form. As in the previous case the measuring circuit consists of a line of length l and propagation constant γ terminated by two bridges. The one at the input end is the test object, and the other is a thermal

* LAVILLE, G. *Annales de Physique*, Paris, 1924, Vol. 2, p. 328 ; NERGAARD, L. S. *R.C.A. Review*, 1938, Vol. 3, p. 156 ; CHIPMAN, R. A. *Jour. Applied Physics*, 1939, Vol. 10, p. 27.

current indicator (Fig. 98). The line is loosely coupled to an oscillator. It is obvious that waves passing along the line will be reflected at both terminations, and that the resultant is expressible in the form of a series of waves obtained by successive reflections at the two ends alternately. Thus using the notation of § 2, and using the suffixes 1 and 2 for the two ends, an incident wave $I_0e^{-\gamma l}$, produces at termination 2 a reflected wave $K_2I_0e^{-\gamma l}$, which on reaching termination 1 has the value $K_2I_0e^{-\gamma 2l}$,

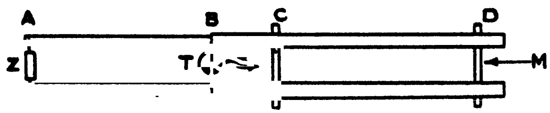


FIG. 98. Chipman's arrangement for measurement by current resonance. AB = measuring circuit terminated by impedance Z to be measured, and thermo-element T, C and D are tandem bridges, and M is a micrometer adjustment for varying the length of the measuring circuit.

produces a reflected wave $K_1K_2I_0e^{-2\gamma l}$, which on reaching 2 has the value $K_1K_2I_0e^{-3\gamma l}$, and so on. The resultant wave at termination 2 (the detector) has the value :—

$$\mathbf{I} = I_0e^{-\gamma l} + K_2I_0e^{-\gamma l} + K_1K_2I_0e^{-3\gamma l} + K_1K_2^2I_0e^{-3\gamma l} + K_1^2K_2^2I_0e^{-5\gamma l}$$

$$= I_0\frac{(1 + K_2)e^{-\gamma l}}{1 - K_1K_2e^{-2\gamma l}}$$

The effect of the loosely coupled oscillator on the line can be represented by a constant e.m.f. $\mathbf{E}$ in series with the impedance $\mathbf{Z}_1$, and the current in the detector can be expressed :—

$$\mathbf{I}_l = \frac{\mathbf{E}(1 + K_2)}{\mathbf{Z}_1 + \mathbf{Z}_0}\cdot\frac{e^{-\gamma l}}{1 - K_1K_2e^{-2\gamma l}}$$

But by (1), $(\mathbf{Z}_1 + \mathbf{Z}_0)/\mathbf{Z}_0 = 2/(1 + K_1)$, so that

$$\mathbf{I}_l = \frac{\mathbf{E}(1 + K_1)(1 + K_2)}{2\mathbf{Z}_0}\cdot\left[\frac{e^{-\gamma l}}{1 - K_1Ke^{-2\gamma l}}\right] \quad . \quad . \quad (14)$$

Now the term in square brackets alone varies with l, so that the conditions for current resonance with variation of l are determined by the relative values of this term alone, which will be denoted $f(l)$. Putting K_1 and K_2 in the form $\rho/\underline{\phi}$ or $\rho e^{j\phi}$ we have :—

$$K_1K_2 = \rho_1\rho_2 e^{j(\phi_1+\phi_2)} = e^{-2r}, \text{ say}$$

where

$$r = s + jt, \; \rho_1\rho_2 = e^{-2s}, \; \phi_1 + \phi_2 = -2t$$

then

$$f(l) = e^{r} \cdot \frac{e^{-(\gamma l + r)}}{1 - e^{-2(\gamma l + r)}} \quad \frac{1}{(K_1K_2)^{\frac{1}{2}}} \cdot \frac{1}{2 \sinh(\gamma l + r)}$$

$$\frac{1}{(K_1K_2)^{\frac{1}{2}}} \frac{1}{2 \sinh\{\alpha l + s + j(\beta l + t)\}}$$

Thus the magnitude of $f(l)$ is given by

$$|f(l)| = \frac{1}{2(\rho_1\rho_2)^{\frac{1}{2}}\{\sinh^2(\alpha l + s) + \sin^2(\beta l + t)\}^{\frac{1}{2}}} \quad . \quad (15)$$

Since α is very small compared with β, the denominator will pass through a minimum value when $\sin^2(\beta l + t) = 0$. Thus if l_r is the length of line at which resonance occurs, we have :—

$$\beta l_r + t = n\pi$$

or

$$\tfrac{1}{2}(\phi_1 + \phi_2) = -t = \frac{2\pi l_r}{\lambda} - n\pi \quad . \quad . \quad . \quad . \quad (16)$$

Thus the length of line at which resonance occurs determines the change of phase on reflection at the two terminations.

With given terminations the measured current I is proportional to $|f(l)|$, from which it follows that if I_r is the current at resonance, *i.e.*, for the length l_r, and I the current for the length l, we have

$$\frac{I_r^2}{I^2} = \frac{\sinh^2(\alpha l + s) + \sin^2(\beta l + t)}{\sinh^2(\alpha l_r + s)} = q \quad . \quad . \quad . \quad (17)$$

which is the equation of the resonance curve. Let δl_1 be the amount by which l_r must be increased to reduce the current from I_r to I, and δl_2 the amount by which l_r must be decreased to obtain the same value, so that $\Delta = \delta l_1 + \delta l_2$ is the width of the resonance curve at the height corresponding to q.
Then

$$\sinh^2[\alpha(l_r + \delta l_1) + s] + \sin^2\beta\delta l_1 = q \cdot \sinh^2(\alpha l_r + s) \quad . \quad (18)$$

and

$$\sinh^2[\alpha(l_r - \delta l_2) + s] + \sin^2\beta\delta l_2 = q \cdot \sinh^2(\alpha l_r + s) \quad . \quad . \quad (19)$$

When α/β is very small these two equations are very nearly the same; and $\delta l_1 = \delta l_2 = \frac{1}{2}\Delta$, and the resonance curve is symmetrical. The equation then becomes :—

$$\sqrt{q-1} \simeq \frac{\sin \beta\delta l}{\sinh(\alpha l_r + s)}$$

which, since αl_r is very small, may be written

$$\sqrt{q-1} \simeq \frac{\sin\beta\delta l}{\sinh s + \alpha l_r \cosh s} \quad . \quad . \quad . \quad . \quad (20)$$

When the reflection is practically perfect we have $\rho_1\rho_2 = e^{-2s} = 1$ and therefore $s = 0$, and

$$\sqrt{q-1} \simeq \frac{\sin \beta\delta l}{\alpha l_r} \simeq \frac{\beta\delta l}{\alpha l_r}$$

from which it follows that

$$\Delta = 2\delta l = 2\sqrt{(q-1)} \, . \frac{\alpha l_r}{\beta}$$

If also both bridges are of zero reactance, $\phi_1 = \phi_2 = 0$ and $l_r = n\lambda/2$, and we have :—

$$\Delta = \frac{\alpha n \lambda^2}{2\pi} \cdot \sqrt{q-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

This equation * gives α in terms of the width of the resonance curve when both bridges are of zero impedance. This value of α may be used in the general equations (18) and (19), which can then be used to determine s, and therefore $\rho_1\rho_2$ for any terminal impedance. Using the known value of α and a fixed value of q, the equations may be used to plot a curve giving $\rho_1\rho_2$ in terms of $\beta(\delta l_1 + \delta l_2)$ or $2\pi\Delta/\lambda$. Alternatively, when the resonance curve is symmetrical, the approximate equation (20) can be used.

The procedure for the measurement of impedance is as follows. The factor $\rho_2/\underline{\phi_2}$ is that of the current detector, which may be unknown but is kept constant. A short-circuit is first used for the input bridge, so that $\rho_1 = 1$ and $\phi_1 = 0$, and the product of the reflection factors $\rho'/\underline{\phi'}$ measured as described above. The unknown impedance is then substituted for the short-circuit input bridge, and the measurement repeated, the result being, say, $\rho''/\underline{\phi''}$. Then since $\rho'/\underline{\phi'} = \rho_2/\underline{\phi_2}$ and $\rho''/\underline{\phi''} = \rho_1\rho_2/\underline{\phi_1 + \phi_2}$ the reflection factor of the unknown impedance is given by

* To avoid misunderstanding it should perhaps be stated that equation (21) was derived by Laville, Kaufmann, and others, and is not given by Chipman.

$K_1 = \rho_1/\phi_1 = \frac{\rho''}{\rho'}/\phi'' - \phi'$. The value of the impedance can then be calculated from equation (2). Chipman has plotted curves from which values of resistance in the form R_i/Z_0, and reactance in the form X_i/Z_0, corresponding to any reflection factor, may be read directly.

6. **Applications of Methods using Waves on Lines.** The use of wave methods for the measurement of the resistance, and inductance or capacitance, of any component which can be connected across a line is sufficiently obvious from the above discussion. The methods are also valuable in investigation of the electric and magnetic properties of materials of all kinds at very high frequencies. The following examples * will indicate their scope.

7. **Magnetic Permeability.** A transmission line is constructed of twin wires of the magnetic material to be investigated. The magnetic permeability of the material at any frequency is then deduced from observations of the wavelength (λ) along the wires, and the attenuation constant (α), by application of the theoretical equations of G. Mie. Mie has shown that when α is small the wavelength (λ) along the wires is smaller than that (λ_0) in free space, the difference $\Delta\lambda$ being given by the equations

$$\frac{1}{\lambda^2} - \frac{1}{\lambda_0^2} = \frac{\alpha}{\pi\lambda} \quad \text{or} \quad \Delta\lambda \simeq \frac{\alpha\lambda^2}{2\pi} \quad . \quad . \quad . \quad . \quad (22)$$

Also that the relation between α and the permeability μ_ω, resistivity ρ, diameter d, and spacing D of the wires, is when $D \gg d$.

$$\alpha = \frac{\rho}{\mu_0 c} \frac{(\mu_\omega f/\rho)^{\frac{1}{2}}}{2d\log_e(2D/d)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

c being the velocity of light, and μ_0 the permeability of the space between the wires.† When α and λ are known μ_ω can be

* For detailed references, see HARTSHORN, L., *Reports on Progress in Physics*, The Physical Society, 1939, Vol. 6, p. 378.

† Equation (23) can also be written in the form

$$\alpha = R_0 \frac{z}{2\sqrt{2}} \frac{1}{8\sqrt{\mu_0/\epsilon_0}\log_e(2D/d)}$$

where z is the eddy-current variable for the wire (p. 132), ϵ_0 is the permittivity of the surrounding medium, and R_0 is the d.c. resistance per unit length of line (two wires in series). It is interesting to notice that this can be expressed in the form $R_f/2Z_0$, where R_f is the resistance per unit length of line, corrected for skin-effect by equation (25), Chapter VII, and Z_0 is the characteristic impedance of the line. The same result can also be obtained from equation (26), Chapter X, if the conductance G is assumed to be negligible.

calculated from these equations. λ is determined directly from observations of the distance between successive nodes or antinodes. α can be obtained from the width of the resonance curve as already explained. For copper wires of diameter 1 mm. or more, α is usually so small that $\Delta\lambda$ is too small to be observed, but for thinner wires of magnetic material $\Delta\lambda$ may be large enough to be observed directly as the difference between the wavelength along a pair of copper wires, and the wires under investigation, both coupled to the same oscillator. Details of experiments along these lines have been published by several workers.

8. **Radiation Resistance.** When measurements such as those described above are made with copper wires, the value of α deduced from the width of the resonance curve is found to be very small, but it is usually appreciably greater than the value obtained by calculation using the formula quoted, which represents the effect of resistivity of the wire and the skin effect. The difference $\Delta\alpha$ is found to increase with diminution of λ, and with increasing separation of the wires, and is to be explained by a loss of energy by radiation. There is some evidence that $\Delta\alpha$ diminishes as the length of line increases, which suggests that the radiation occurs largely at the ends, and can therefore be represented as a "radiation resistance" in series with the normal impedance of the terminations. It may also be necessary to regard the line as possessing a distributed "radiation resistance" in addition to its ordinary resistance. Differences in radiation resistance produced by changes which leave the normal impedances unaltered can obviously be determined by observations of the changes in α. It should be noted that radiation is a source of error in the measurements of magnetic permeability, but control experiments on non-magnetic wires will give an idea of the magnitude of the error.

9. **Dielectric Constant, Conductivity, Absorption and Dispersion in Dielectrics.** If the line is situated in a medium other than free space the wavelength and attenuation constant will depend on the dielectric properties of the medium, and these properties can also be measured by the above-mentioned methods. Consider plane waves travelling along a line formed by two parallel conductors of negligible resistance in free space. The

attenuation constant is zero, and we have for the propagation constant γ_0, and wavelength constant $\beta_0 = 2\pi/\lambda_0$,

$$\gamma_0 = j\beta_0 = \sqrt{(jL_0\omega)(jC_0\omega)},$$

where L_0 is the inductance, and C_0 the capacitance per unit length of the line. Now let the medium be changed to one of dielectric constant ϵ_r and loss tangent $\tan\delta$. The admittance per unit length of line now becomes $j\epsilon_r C_0\omega[1 - j\tan\delta]$, which can be written in the form :—

$$jC_0\omega[\epsilon_r - j\epsilon_r \tan\delta] \text{ or } jC_0\omega[\epsilon_r - j\epsilon'_r] \text{ where } \epsilon'_r = \epsilon_r \tan\delta,$$

showing that the effect of conductance or power loss in the medium may be represented by a complex dielectric constant, $\epsilon_r - j\epsilon_r' = \epsilon_A\underline{/-\delta}$, where $\epsilon_A = \sqrt{(\epsilon_r^2 + \epsilon_r'^2)}$. If the medium is non-magnetic the value of inductance L_0 still holds good, and we have for the propagation constant γ

$$\frac{\gamma}{\gamma_0} = \frac{\alpha + j\beta}{j\beta_0} = \sqrt{\epsilon_r - j\epsilon_r'}$$

or

$$\frac{\beta}{\beta_0} - \frac{j\alpha}{\beta_0} = \sqrt{\frac{\epsilon_A + \epsilon_r}{2}} - j\sqrt{\frac{\epsilon_A - \epsilon_r}{2}}$$

Hence

$$\frac{\beta}{\beta_0} = \frac{\lambda_0}{\lambda} = \sqrt{\frac{\epsilon_A + \epsilon_r}{2}} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and

$$\frac{\alpha}{\beta_0} = \frac{\alpha\lambda_0}{2\pi} = \sqrt{\frac{\epsilon_A - \epsilon_r}{2}} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The quantities λ_0, λ and α can be measured in the manner already described. The values of the various electrical constants of the medium are then easily calculated by means of the above equations. Squaring and subtracting (24) and (25) we obtain the dielectric constant ϵ_r,

$$\epsilon_r = \left(\frac{\lambda_0}{\lambda}\right)^2 - \left(\frac{\alpha\lambda_0}{2\pi}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Also, multiplying (24) and (25), we obtain ϵ_r',

$$' = \frac{\alpha\lambda_0^2}{\pi\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Then the loss angle is obtained from the relation

$$\tan \delta = \epsilon_r'/\epsilon_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

and the conductivity from the relation

$$\sigma = \kappa_0 \epsilon_r' \omega = \kappa_0 \cdot \frac{\alpha \lambda_0^2}{\pi \lambda} \cdot 2\pi f = 0{\cdot}1771 \frac{\alpha \lambda_0^2 f}{\lambda} \cdot 10^{-12} \, (\text{ohm}^{-1}\,\text{cm}^{-1}) \quad (29)$$

where κ_0 is the capacitivity of free space as defined in Chapter VI, the value ($0{\cdot}08855 \times 10^{-12}$ F/cm.) for which has been substituted in deriving the last form of the expression, so that the centimetre should be used as the unit of length throughout the measurements.

Measurements of this kind are most easily made on liquids. The parallel conductors may take the form of twin wires, passing through a horizontal trough or vertical tube, which can be filled with the liquid. Alternatively coaxial conductors may be used, the outer conductor also forming the container for the liquid. This arrangement has the advantage that the waves are entirely confined to the space within the tubular conductor, so that external bodies do not affect the measurements. The method is also applicable to powders and has been used for measurements on soils.

10. **Index of Refraction and Absorption by Reflection.*** Measurements of dielectric properties may also be made by observations depending on the reflection of waves at the surface of a material. Suppose, for example, a line is terminated by a slab of the material. The reflection coefficient $\rho/\underline{\phi}$ of this termination can be measured by the method of § 5. By means of Fresnel's formula this reflection coefficient can be expressed in terms of the complex refractive index $n(1 - jk)$, which is analogous to the complex dielectric constant $\epsilon_r - j\epsilon_r'$, n being the ordinary refractive index, and k the absorption index, we have :—

$$\rho \epsilon^{j\phi} = \frac{1 - n(1 - jk)}{1 + n(1 - jk)} = \frac{(1 - n) + jnk}{(1 + n) - jnk},$$

whence

$$\rho^2 = \frac{(n - 1)^2 + n^2k^2}{(n + 1)^2 + n^2k^2}$$

and

$$\tan \phi = \frac{2nk}{1 - n^2(1 + k^2)}.$$

* MALSCH. *Ann. Phys. Lpz.*, 1934, Vol. 20, p. 33.

Also

$$\epsilon_r - j\epsilon_r' = n^2(1 - jk)^2,$$

whence

$$\epsilon_r = n^2(1 - k^2)$$
$$\epsilon_r' = 2n^2k.$$

Thus it is possible to calculate n, k, ϵ_r and ϵ_r' from the measured values of ρ and ϕ. Research on these lines is obviously capable of giving valuable information on the behaviour of many materials with respect to very short waves. The layer of material must be so thick that the transmitted wave is completely absorbed, otherwise subsequent multiple reflection within the material will complicate the measurements.

11. **Experimental Details.** The apparatus used for wave methods is apt to vary considerably with the type of measurement to be made, and the subject has not yet reached a stage where some arrangements can be recommended to the exclusion of others. We shall conclude by noting a few of the practical points raised in the recent literature * of the subject.

(*a*) *Generators.* Ordinary valve oscillators form satisfactory generators for very high frequencies, provided suitable components are used. The upper limiting frequency is determined by the following factors : the finite time of transit of electrons from cathode to anode, the inductance of the leads to the electrodes, and the inter-electrode capacitance. Small transit times are obtained by the use of valves with electrodes separated by very small distances, with a high anode voltage, conditions which require an anode capable of high dissipation. The inductance and capacitance of the valve is minimised by using special types, in which the leads to the electrodes consist of short straight wires well spaced. The resonator tor the oscillator is most conveniently obtained by the use of a line of length $n\lambda/4$ closed at one end. Fig. 99 shows a push-pull oscillator of this type, which may be compared with that of Fig. 36. The tuned anode circuit is replaced by a tuned line : coupling to the grid circuit is provided by the inter-electrode capacitances of the valves : and the grid circuit also consists of an adjustable line, the length of which is adjusted so that the grid potentials have the suitable values,

* For references, see *Reports on Progress in Physics*, The Physical Society, 1939, Vol. 6, p. 388.

e.g., the grid line may be of length $3\lambda/4$, and the anode line of length $5\lambda/4$, as used by Chipman. High-frequency chokes are shown in the leads to all the power supplies. In a perfectly symmetrical system these would not be necessary, but owing to inevitable departures from the condition of perfect symmetry it is necessary to make provision for variations of potential at these points, otherwise power will be lost. Wavelengths down to about 40 cm. are obtainable with this technique. For still shorter wavelengths the magnetron is available. This consists of a diode with a straight wire cathode surrounded by a cylindrical anode divided axially into two halves, which are directly connected to the open end of a tuned line. The diode is mounted between the poles of a magnet so as to establish an axial magnetic field in the space between the electrodes. Thus the electrons proceeding towards the anode are deflected by the magnetic field, and when the field strength reaches a certain value, they travel round and round the cathode and do not reach the anode, so that the mean anode current falls almost to zero. Within a certain range, a decrease of voltage on one anode is accompanied by an increase of current, *i.e.*, a condition of negative resistance is obtained; and oscillations are established in the tuned line connected to the anodes at a frequency determined by the length of the line. Oscillations of wavelengths down to 1 cm. have been obtained with magnetrons.

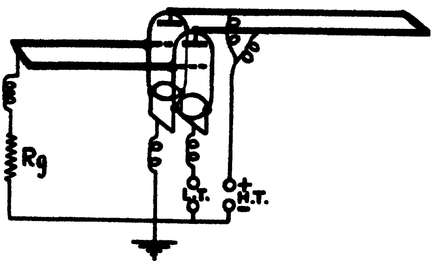


FIG. 99. Oscillator with tuned lines for frequencies of a few hundred Mc/s. It is sometimes advantageous to use tuned lines for the chokes (Chipman).

(*b*) *Detectors*. Bulky detectors are obviously inadmissible, but provided that the small types of the special valves mentioned above are used, the simple thermionic voltmeter can be used at wavelengths down to 50 cm., and for low-range instruments employing either a diode or a triode, the deflection is proportional to the square of the voltage, as at lower frequencies. For wavelengths of the order of a few cm. the crystal rectifier, *e.g.*, an iron pyrites and phosphor bronze contact, appears to be the most convenient detector. Thermo-elements * form very convenient detectors of current, and these also obey the square law, a most important consideration when applying equations (9) and (17).

(c) *Adjustable Lines and Bridges.* The commonest form of standard line consists of a pair of brass rods or tubes. Solid insulating supports are obviously necessary for mechanical reasons, and these should be placed at the voltage nodes, where they will cause the minimum of disturbance. The effective length of line can be varied either by the use of telescopic tubing, and fixed bridges, or by moving bridges along fixed conductors. A copper disc sliding along the conductors, which pass through closely fitting holes in the disc, is often used as a perfect reflector or bridge of zero impedance. Conducting bridges of small cross-section are less effective, and are therefore sometimes followed by additional or "tandem" bridges (Fig. 98) placed in the position which would be occupied by any voltage antinode that might tend to form on the portion of the wire beyond the bridge, as a result of incomplete reflection by the latter, *i.e.*, distance between bridges $\simeq \lambda/4$. Further details about particular applications of the methods will be found in the original papers quoted, but each experimenter will probably find it necessary to arrive at satisfactory conditions by a process of trial and error, using as a check the symmetry of the resonance curves, the consistency of the results obtained for various values of q, and the values obtained for impedances of calculable value, *e.g.*, thin straight wires and small condensers.

* Strutt and Knol. *Proc. Inst. Radio Eng.*, 1939, Vol. 27, p. 783.

BIBLIOGRAPHY

"Read less, good people, and observe more; and above all, leave us in peace."—"Hills and the Sea," H. BELLOC.

THE bibliography, says the editor, is to guide the student in his further reading. The author, knowing that the subject is not to be mastered by reading alone, would urge the greater importance of experimental work, and therefore gives pride of place in this bibliography (with apologies for one word) to Mr. Belloc. Extending the quotation, nothing "fills one with humility and right vision" so much as attempts to measure the same quantity by two entirely different methods.

References to certain original papers will be found in the text. Many others cover the same field, and the ones given are not necessarily considered to be those showing the greatest originality. They are merely quoted as convenient sources of more detailed information on special points. The author's opinion is that, provided the required information is given, the fewer the number of references the better. For complete bibliographical information the student is referred to the abstracts published in *The Wireless Engineer* or to *Science Abstracts*.

The following works bearing on the subject may be found useful:

Bridge Circuits and Standards

For detailed information on the varieties of bridge circuits, and the construction of standards, mainly for the lower frequencies.

1. A. CAMPBELL and E. C. CHILDS: *The Measurement of Inductance, Capacitance and Frequency*, Macmillan, London, 1935.
2. B. HAGUE: *Alternating Current Bridge Methods*, Fourth Edition, Pitman, London, 1938.

Waves on Lines

For information concerning the propagation of waves on lines :—

3. E. MALLETT: *Telegraphy and Telephony including Wireless Communication*, Chapman & Hall; London, 1929.
4. W. L. EVERITT: *Communication Engineering*, McGraw-Hill, New York and London, Second Edition, 1937.

Constructional Details

For practical information concerning the actual construction of oscillators, amplifiers, etc. :—

5. *The Radio Amateur's Handbook*, by the American Radio Relay League, 1940 Edition.

The Calculation of Inductance and Capacitance.

6. W. H. Nottage: *Calculation and Measurement of Inductance and Capacity*, Wireless Press, London, Second Edition, 1924.
7. E. B. Rosa and F. W. Grover: *Formulas and Tables for the Calculation of Mutual and Self-Induction*, Bulletin of Bureau of Standards, Vol. 8, No. 1, 1911.
8. A. Russell: *Treatise on the Theory of Alternating Currents*, Vol. I, Cambridge, 1914.
See also 10, below.

General Radio-Frequency Measurements

9. Bureau of Standards Circular, No. 74, Radio Instruments and Measurements, 1920.
10. R. T. Glazebrook: *Dictionary of Applied Physics*, Vol. 2, Electricity, Macmillan, London, 1922. Articles on Radio Frequency Measurements and the Calculation of Inductance.
11. A Hund: *High-Frequency Measurements*, McGraw-Hill, London, 1933.
12. E. B. Moullin: *Theory and Practice of Radio Frequency Measurements*, Griffin, London, Second Edition, 1931.
13. F. E. Terman: *Measurements in Radio Engineering*, McGraw-Hill, New York and London, 1935.

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