RAILROAD FIELD GEOMETRY

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PREFACE

An apology for the appearance of another text on railroad field geometry may be demanded by some persons.

The author can only say that after teaching the subject for twenty-five years he has tried to supply his own need for a text prepared for classroom use. He hopes that his methods will be found useful by others as well.

In making the series of books of which this volume is one, an effort has been made to keep the idea in mind that a text-book should be neither a treatise nor a manual of practice.

The plan of the book is wholly different from that of any book covering the same field with which the author is familiar. An attempt has been made to depart from the usual brief formal statement of problem and solution, with the hope that the student will be led to reason for himself from a knowledge of how the problem arises, rather than to commit problems and solutions simply as exercises in geometry. Some of the less commonly used problems have been wholly worked out, but in general a considerable portion of the work has been left for the student to do by methods indicated.

The chapters on rerunning old lines and making right-of-way descriptions and maps are new, and may be thought by some to be out of place. The author has seen the time when he would have been glad of some such hints as are contained in these chapters.

The treatment of spirals follows Talbot, but contains some work of Mr. J. B. Jenkins and some original work in the development of short formulas for practical use.

Portions of the chapter on turnouts contain original methods that it is hoped will prove acceptable.

The earthwork chapters are essentially a revised reproduction of the author's "Notes on Railway Earthwork," published in 1894 by the Rensselaer Society of Engineers. The discussions of haul and mass diagram are fuller than any the author has seen, and he hopes that these discussions will lead to a clearing up of the troublesome question of overhaul, and a general adoption of some rational method of procedure.

The author wishes to acknowledge his indebtedness for advice and assistance to Prof. Shelby S. Roberts formerly of the University of Illinois, Prof. B. J. Dalton of the Kansas State University, Prof. O. V. P. Stout of the University of Nebraska, Prof. W. D. Pence of the University of Wisconsin, and Mr. Jenks B. Jenkins, Assistant Engineer of the Baltimore and Ohio Railroad. He also acknowledges the courtesy of William Wharton, Jr., & Co. for permission to use reproductions from their catalogue.

Although not intended for a field-book, this book is published in pocket-book form for the convenience of students in classroom and field.

In the near future the author hopes to bring out a field-book on a somewhat new plan, and containing some new matter, as a companion to this volume. Those who use this volume for study will use some one of the numerous field-books now published for tabular quantities needed in computations.

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STATE UNIVERSITY OF IOWA,
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Railroad Field Geometry

CHAPTER I.

INTRODUCTION.

The Railroad Line and Surveys. — The projection of the center line of a railroad on a horizontal plane consists of a series of straight lines joined by arcs of circular curves tangent to the straight lines. The straight lines are called tangents.

The object of a railroad survey is to seek a suitable route between two proposed termini, and to lay out — locate — the center line on the ground. When this is done subsequent surveying operations are necessary to "stake out" the work for construction, to determine when it is built to line and grade, to secure descriptions and maps of the necessary right of way and station grounds, for water supply, etc.

A suitable route is a route which, all things considered, is the cheapest that can be found between the termini. Were it possible to construct it, the cheapest line in first cost might be that one in which the center line would lie altogether on the surface of the ground, requiring only leveling across to prepare the roadbed for the track. It would be this one unless the detour made necessary to keep the line on the surface and within the maximum allowable rate of grade should so increase the length over that of a line requiring cutting into the hills and filling across the valleys, that the cost of this additional length would more than balance the saving in earthwork and bridging. This line of cheapest first cost is not always the cheapest line, and moreover it is practically impossible to keep the center line on the surface, as in ordinary wagon road building, because of the irregularity that would result. Hence the line that is run lies between the straight line joining successive traffic centers and the surface lines that might be run.

The work of the engineer — previous to construction — consists in going over the various possible routes on foot or on horseback and selecting that general route that he deems best,* conducting a preliminary survey over the route selected, and subsequently making a location survey.

The preliminary survey consists in establishing and properly connecting a series of consecutive lines forming one broken line, on the ground, the lines being chosen to lie, as nearly as may be judged by the engineer, where the final center line of the constructed road will lie. The lengths of these lines and the angles they make one with another are noted, and a profile is made considering the several lines as one continuous broken line. A grade line is drawn on the profile for the purpose of making a preliminary estimate of the quantity of earth to be moved, the height and length of bridges, etc. This preliminary line also serves as the base for a topographical map of a narrow belt on either side of the line, and on this map, more or less extensive and precise, the final line, consisting of tangents and curves, is drawn. The line is afterwards laid out on the ground from notes taken from the map, and this constitutes the location survey. Where the ground is comparatively flat and the position of the line evident, little topography is needed and the line may be located by a trial method, *i.e.*, the line is run ahead for a distance, a profile taken and examined to see what changes can be made that will lessen the earthwork or better the alinement, and, if any appear, the line may be rerun on better ground.

* The conditions affecting his decision are discussed in "Elements of Railroad Engineering."

The lengths of the straight lines and curves are usually expressed in stations of 100 feet. In land and other surveys a *station* is a corner or point in the survey occupied by the instrument; but in American railroad surveys it is 100 feet of any line that is being run. Stakes are placed not only at the points occupied by the instrument, as in land surveys, but at the end of each 100 feet from the beginning of the line. The stake at the beginning is numbered **0**; the stake at the end of the first 100 feet or station is numbered **1**, at the end of the second 100 feet, **2**, and so on. The numbering does not begin anew from each point where the direction of the line changes nor from each point occupied by the instrument, but is continuous, so that at any point the number on the stake indicates the number of 100-feet units from the beginning of the line. These stakes are often called stations, the term being applied indiscriminately to the 100-foot length and the stake that marks its end. The context will usually indicate what is intended. Frequently the point that must be occupied by the instrument is not at the end of a full station. When this occurs the stake marking this point is marked with the number of the preceding stake plus the number of feet from that point to the instrument point; thus, if the instrument must be set seventy-five feet ahead of stake number thirty-six the stake at the instrument would be marked 36 + 75. The next stake would be driven twenty-five feet ahead of the instrument point and numbered 37. An instrument point is marked by a stout hub driven almost flush with the ground, "centered" with a tack and referenced with a guard stake driven alongside. Stations which are not instrument points are marked only by stakes.

It was formerly customary to connect the curves directly with the tangents as indicated in the first paragraph of this chapter. It is now more usual to introduce a short piece of what is known as an easement curve or transition curve, or spiral, between the tangent and the main curve. It will be first assumed that the curves are simple circular arcs connected directly with the tangents, and the subject of spirals will be introduced later.

Curves are of three general classes: 1, Simple curves; 2, compound curves; 3, reverse curves.

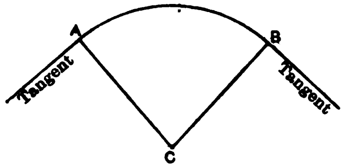

Fig. 1.

A simple curve is the arc of a circular circumference.

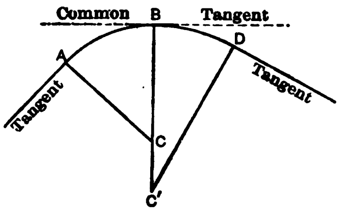

Fig. 2.

A compound curve consists of two or more contiguous simple curves of different radii which have a common direction of radius at the points of junction, and centers on the same side of the common tangent.

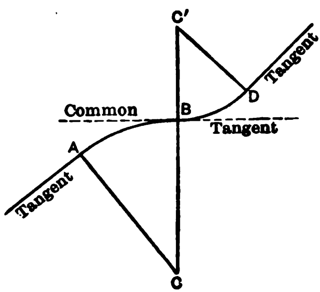

Fig. 3.

A reverse curve consists of two contiguous simple curves of the same or different radii having a common direction of radius at their point of junction and whose centers lie on opposite sides of the common tangent.

Thus, Fig. 1 is a simple curve, Fig. 2 is a compound curve, and Fig. 3 is a reverse curve.

Reverse curves are used only in laying out sidetracks and yards. Compound curves are used when in laying out a line around a hill or bend of a shore line of a river, lake, or sea, a series of arcs of different radii will fit the contour of the ground better than a single arc of one radius.

The methods of laying out curves on the ground and of solving the numerous problems that arise are all based on the principles of geometry and trigonometry, and the student or young surveyor familiar with these principles should find no difficulty in handling any problem likely to arise in his practice.

CHAPTER II.

SIMPLE CURVES.

1. Fundamental Formulas. — The elements of a simple curve useful to the railway surveyor are the following: The angle I, known as the intersection angle, or total deflection angle, or central angle. It is the deflection angle of the two tangents.

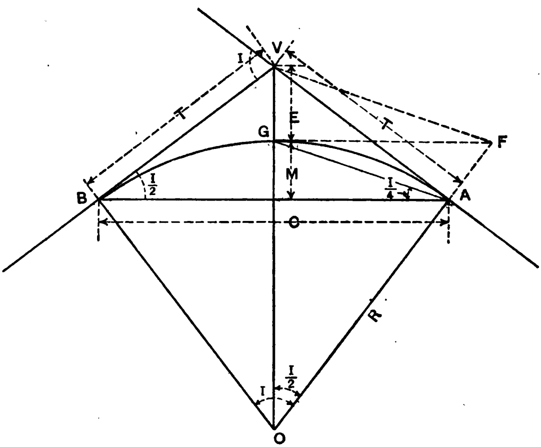

Fig. 4.

The "tangent distance" $T = R \tan \frac{1}{2} I$. (1)
The "long chord" $C = 2 R \sin \frac{1}{2} I$. (2)
The "middle ordinate" $M = R \text{ vers } \frac{1}{2} I$. (3)
The "external distance" $E = R \text{ ex sec } \frac{1}{2} I$. (4)

(The external secant is the secant -1, and it is tabulated like other trigonometric functions in railroad field books.)

The angle $VFE = GBC = \frac{1}{4} I$, and GF being drawn tangent at G, the middle point of the curve, equals AV; hence

$$E = T \tan \tfrac{1}{4} I. \tag{5}$$

Also

$$C = 2 M \cot \tfrac{1}{4} I, \tag{6}$$

$$C = 2 T \cos \tfrac{1}{2} I, \tag{7}$$

$$GA = \frac{C}{2} \sec \tfrac{1}{4} I. \tag{8}$$

For any portion of a curve subtending an angle Δ at the center, the same equations give the corresponding elements if Δ be substituted for I.

Curves are known by their "degrees." The degree of a curve is the angle subtended at the center by a chord of 100 feet. Thus, in a four-degree curve a chord of 100 feet subtends four degrees at the center of the curve; in a "four-thirty" curve a chord of 100 feet subtends four degrees and thirty minutes at the center.

From the above definition and equation (2), if D be the degree of any curve,

$$R = \frac{50}{\sin \frac{1}{2} D}. \tag{9}$$

Hence, when D is known R may be found, and, conversely, if R is known, D may be found. Values of R found by equation (9) and the corresponding logarithms are tabulated in most engineering field books.

In Spanish American states and countries the metric system is used and what are called metric curves. While the practice varies, it is most common to define the degree of curve as the number of degrees subtended by a chord of 20 meters, this being the ordinary length of chain or tape used. The chain or tape is divided into 100 links or parts each 0.2 m. long.

Any circular curve in which a chord of 100 units subtends 1° will have a radius of 5729.65 of those units, therefore a 1° metric curve will have a radius of 5729.65 × 0.2 m., or 1145.93 m. As in ordinary United States practice, the radii of other curves will be approximately inversely as the degrees, or exactly

$$R_m = \frac{50 \times 0.2}{\sin \frac{1}{2} D} = \frac{10}{\sin \frac{1}{2} D} \tag{9a}$$

In general, since the link unit of the metric tape is $\frac{2}{10}$ of the meter — the measuring unit — so any linear quantities of the first power computed in feet for the United States curve system can be converted into meters for metric curves of equal numerical degree by dividing by 5. But tables of these functions giving values directly in meters are more convenient and will be included in a succeeding volume.

Examples. 1. Find the radii, length, tangent distance, long chord, middle ordinate and external distance for 1°, 2°, 5°, 10° and 15° curves having central angles of 30°.

2. From the foregoing find the linear elements of metric curves of the same degrees.

2. Field Determination of *V* and *I*. It may be that the broken line *ABDEF* has been run on the ground and

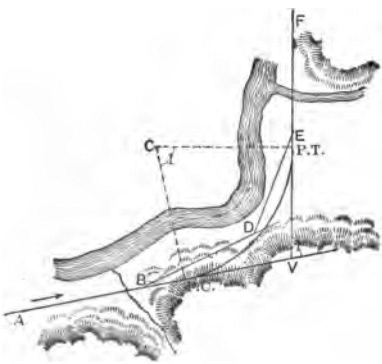

Fig. 5.

it is found that the straight lines *AB* and *EF* lie about right and should be adopted for two tangents to be connected with a curve. The two tangents may be produced in the field to intersection at *V*, or the position of *V* may be computed from the notes of the line $ABDEF$. With the transit at V the angle I may be measured. It is not necessary so to measure I, since it equals the sum of the deflection angles at B, D, and E.*

If V is determined by running the lines to intersection, the process is as follows: With the transit over a convenient point in the line AB, the line of sight is directed in AB produced, and two stakes are driven and centered close to but on opposite sides of the extension of the line FE. The approximate position of this extension is determined by eye from two flags set at two stakes respectively on the line EF. A string is now stretched between the two centered stakes in AB produced while the transit is being set over a convenient point in EF. With the transit pointing in the line FE produced, a stake is set and centered under the string for the point V.

If the point V is to be determined by computation, as is frequently necessary owing to inaccessibility, the process depends on the number of short lines between B and E. If there is but one, the line being straight from B to E,

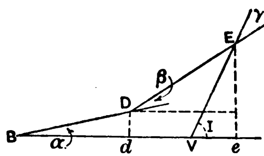

Fig. 6.

BEV is solved as a triangle for BV, which reduced to stations and added to the station of B gives the station of V. If there are several lines it will be better to compute by latitudes and longitudes, the line AB being assumed a meridian.

$$Be = BD \cos \alpha + DE \cos (\alpha + \beta),$$
$$Ee = BD \sin \alpha + DE \sin (\alpha + \beta),$$
$$eV = Ee \cot I,$$
$$BV = Be - eV.$$

* The student may prove this to fix it in his memory.

3. Determination of *D*. The degree of the curve is determined by judgment or by topography. If there is no topograghical limitation, the curve the engineer thinks best is used, generally the smallest degree curve that will fit in. If speeds of 50 miles an hour are not to be exceeded, a 3° 45′ curve may be freely used and will be cheaper for a given central angle than a less degree, since there will be less curved track to maintain, though actually more total feet of track. If 60 miles an hour is to be realized, the maximum degree should be about 2° 40′. This is because 60 miles an hour on a 2° 40′ curve requires the maximum advisable cant of the track, as does also 50 miles an hour on a 3° 45′ curve. The maximum cant allowed in the foregoing is 6 inches, which is sometimes exceeded, and when so exceeded the curve can be correspondingly sharpened.

If the topography fixes the radius, the speed must be adapted to the curves.

On high-speed, heavy-traffic lines, it may be profitable to disregard the topographical limitations and by heavy work in bridging, tunneling and earthwork, obtain the larger radius required for high speed.* Occasionally the degree is so fixed by physical conditions that it must be computed, as will appear in the problems, but it may be fairly said that as a rule it is determined by the judgment of the engineer based on many considerations of differences in cost of construction and operation which cannot be discussed here.

Example. The following data are taken from the notes of a preliminary survey.

Line straight from sta. 376 to sta. 397. At 397 deflect 12° left; at sta. 401 deflect 14° 30′ left; at sta. 406 deflect

* The Baltimore and Ohio Railroad is reducing (1909) all its curves sharper than 1° on its line between Washington and Philadelphia to 0° 30′ curves for the sole purpose of permitting fast running. One-degree curves are allowed to remain. Six inches elevation of the outer rail on a 1° curve would allow for a speed of 92+ miles an hour, which is not likely to be made in the near future.

13° left from where line is straight to sta. 421. Find a curve that will be tangent to the first and last lines, and approximately follow the line run. Hint: — To determine the approximate degree, note that 39° 30′ are turned in 9 stations with two short stretches of 4 and 5 stations, respectively, between the first and final tangents. Adding half of these gives a length of 13½ stations for an approximate length of curve which indicates about a 3° curve which may be tried. The line should be sketched to make the solution clear. The line may be carefully drawn to scale and a curve fitted by trial, the radius being measured and the nearest round-number degree or half degree selected.

Let the student put the solution of this and all following examples in a good note book. The results in the earlier examples of the book are referred to and used in the solution of the later examples.

4. Location by Deflection Angles. — The curve begins and ends at whole stations. Curves are usually laid out on the ground by "deflection angles." If the station V of intersection of tangents is known, and the angle I, T is computed by Eq. (1), and the station of A is obtained by subtracting the number of stations in T from the station of V. The beginning of the curve is known as the P. C. (point of curve) and the end as the P. T. (point of tangency). In this article it is assumed that A, the P. C., Fig. 7, falls at the end of a station. The transit is set at A and with zeros together is clamped with its telescope in the line AV; since by Geometry $VAd = \frac{1}{2} AOd$, the transit is turned to the right through an angle equal to $\frac{1}{2}D$, thus directing the transit telescope in line with the end of the first full station on the curve. Simi-

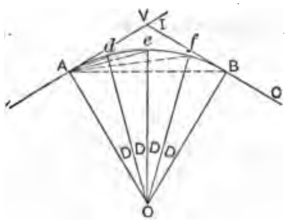

Fig. 7.

larly, if Ad, de, ef and fB are 100-feet chords, $dAe = eAf = fAB = dAV = \frac{1}{2} D$.

Hence, having $\frac{1}{2} D$ set off from AV, the chain or tape is stretched in the line Ad and a stake driven at d. Next, the angle $dAe = \frac{1}{2} D$ is turned from Ad and the chain stretched from d putting the forward end in the line Ae, and a stake is driven at e. The process is continued around the curve to B. When B is located the instrument is placed over it and with zeros together is clamped on A.* From the line BA the angle $VBA = \frac{1}{2} I$ is turned to the right and the line of sight should fall on V. This checks the work. When V is not established on the ground, the telescope is transited and should point in the line BC.

5. Location by Deflection Angles. — The curve begins and ends with fractional or sub-stations. Curves do not usually begin or end with a whole station.

For all curves under 10° it is usual to assume that any chord less than 100 feet will subtend an angle at the center proportional to its length; thus, a 50-foot chord will subtend an angle of $\frac{1}{2} D$, a 25-foot chord an angle of $\frac{1}{4} D$, a 1-foot chord an angle of $\frac{1}{100} D$, etc. Assuming this proportionality, the length in stations of a curve of degree D is approximately

$$L = \frac{I}{D}. \tag{10}$$

Equation (10) will usually give a mixed number.

The station of the P. T. is found by adding the length of curve in stations to the station of the P. C. The station numbering proceeds on the tangent to the P. C., thence around the curve to the P. T., thence on the forward tangent; it does not proceed on the tangent up to the point of intersection and from there on.

No matter whether a fractional or "sub-chord" is at the beginning of the curve at the end, or in the middle,

* This expression means that the instrument is clamped in azimuth with the telescope pointing toward A.

the deflection angle for it is $\frac{1}{2}d$, *i.e.*, one-half the angle it subtends at the center.

If it is found that V falls at sta. 30 + 10 and I is 23° 0′, and it is required to locate a 5° curve, then

$$T = \overset{(R)}{1146.28} \times \tan \overset{(\frac{1}{2}I)}{11^\circ 30'} = 233.2,$$

$$\text{and } 30 + 10 - (2 + 33.2) = 27 + 76.8,$$

which is the station of the P. C.

The first chord to sta. 28 will then be 23.2 feet and the deflection angle will be $\frac{23.2}{100} \times \overset{(\frac{1}{2}D)}{2\frac{1}{2}^\circ} = 0^\circ\ 34.8'$. The deflection to sta. 29 will be $0^\circ\ 34.8' + 2\frac{1}{2}^\circ = 3^\circ\ 04.8'$. The deflections to the succeeding stations will be

Station	Deflection.
30	5° 34.8′
31	8° 04.8′
32	10° 34.8′
32 + 36.8 P.T.	11° 30.0′

The length of the curve is found from Eq. (10) and added to the station of P. C. to determine the station of the P. T.

As a check on the work the computed deflection for the last sub-chord, 36.8 feet, is added to the deflection to sta. 32, and the work is correct if the sum is half the angle I. The field measurements may be checked as before by setting at B and turning the angle $\frac{1}{2}I$ from BA, when the line of sight should lie in the tangent BV.

Example. Compute and tabulate the deflection angles from the tangent at the P. C. to each station on the curve of the example of Art. 3.

6. The Whole Curve cannot be run from the P. C. — The instrument must be moved to the farthest stake that can be set from the P. C., — which stake, since it is to be an instrument point, should be a centered plug with a guard stake,— and the remainder of the curve run in from that point. Sometimes several points must be occupied.

There are two common methods of setting the vernier when the instrument is on an intermediate station. These are: 1. The vernier is set at zero when pointing to the back sight. 2. The vernier is set to read the deflection from the P. C. to the station used as a back sight. The second is the better method. Let it be supposed that in the last example, Fig. 8, it is necessary to occupy stations 29 and 31. Station 29 having been located with deflection angle of 3° 04.8′, the instrument is set there. The vernier is brought to read zero (the deflection for the P. C.) and a back sight taken on the P. C. If now the telescope be turned to the right till the vernier reads 0° 34.8′, it will point to sta. 28, and if turned to a reading of 3° 04.8′ it will be tangent to the curve at sta. 29, occupied. The telescope is now transited and the vernier brought to 5° 34.8′, when the telescope will point to sta. 30, which is now located by measuring one hundred feet from sta. 29. Another hundred feet and a vernier reading of 8° 04.8′ locates sta. 31. With the instrument at sta. 31 the vernier is brought to read to the right 3° 04.8′ (the deflection at the P. C. to sta. 29) and the telescope is set on sta. 29. If now the vernier be brought to read 0° 34.8′ or zero, the telescope will point to sta. 28 or the P. C. respectively. If brought to read 5° 34.8′ it will point to sta. 30, and at 8° 04.8′ it will be in the tangent at sta. 31, occupied. If the telescope is now transited and turned to a reading of 10° 34.8′, it will point to sta. 32, which may be located and the work completed as usual.

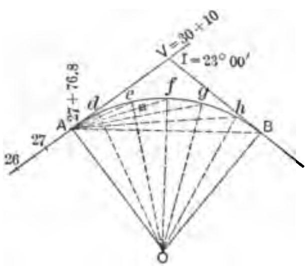

Fig. 8.

It will be observed that by this method the telescope is always pointing to any given stake when the vernier reads the deflection from the tangent at the P. C. to that stake.

Example. A 3° curve begins at sta. 368 + 75 and has a central angle of 22° 30′. In running in the curve it is necessary to set the instrument at stations 371 and 374. Determine the readings to all stations serving as back sights, or located from each setting of the instrument including those at the P. C. and P. T.

7. The Curve is of Short Radius, or Large Degree.— When it is desirable to locate a curve of comparatively small radius with considerable precision, requiring chords shorter than 100 feet, a convenient deflection angle may be assumed and the length of corresponding chord computed from $c = 2R \sin \frac{1}{2} d$, $\frac{1}{2} d$ being half the angle subtended at the center by the chord. Thus, if it is desired to locate a ten-degree curve with approximately twenty-five-foot chords, $\frac{1}{2} d$ is assumed to be $\frac{1}{4}$ of $\frac{1}{2}$ of D or $1\frac{1}{4}°$, and

$$c = 2 \times \overset{R}{573.69} \sin \overset{\frac{1}{2}d}{1° \, 15'} = 25.02 + \text{ feet.}$$

Since $\frac{2}{100}$ of a foot equals about $\frac{1}{4}$ of an inch, a 10° curve run in with actual 25-foot chords and deflection angles of $1\frac{1}{4}°$ would be out of position at its end by about one inch per station.

A convenient chord length could be assumed and the corresponding deflection angle computed from the same equation, but as it is easier to measure distances to fractions of a foot than to turn angles to fractions of a minute, it is better to assume the angle.

Examples. 1. Find the deflection angle necessary to locate a 12° curve by 25-foot chords.

2. Find the chord necessary to locate a 12° curve by deflection angles of $1\frac{1}{2}°$.

3. A curve is located with 25-foot chords and deflection angles of $1\frac{1}{4}°$. What is its radius? degree?

8. The Curve is through a Wood.—In locating a curve through woods, unless the transit is set at each station, the ordinary method of deflection angles requires a number of lines to be cleared, one from the instrument to each station as well as the line of the curve for the chainmen. To avoid so much work the curve is located by offsets from one or more long chords. The deflection angle for two, three, four, or more stations is turned off and the length of the long chord and distances on it to points opposite the intermediate stations, and lengths of offsets to those stations, are computed This may usually be done by the aid of tables while the line of the long chord is being cleared.

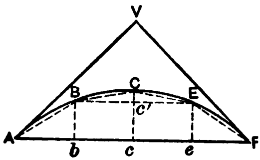

Fig. 9.

In Fig. 9 the curve is supposed to begin at a full station. The deflection angle $VAF = 4 \times \frac{1}{2} D$ is turned from AV and the following computations made:

$$\begin{aligned}
AF &= 2\,R \sin 4 \times \tfrac{1}{2} D,\\
Ab &= 100 \times \cos 3 \times \tfrac{1}{2} D \qquad (4 \times \tfrac{1}{2} D - \tfrac{1}{2} D),\\
Bb &= 100 \times \sin 3 \times \tfrac{1}{2} D,\\
bc &= Bc' = 100 \cos \tfrac{1}{2} D \qquad (4 \times \tfrac{1}{2} D - 3 \times \tfrac{1}{2} D),\\
Cc &= Bb + Cc' = Bb + 100 \sin \tfrac{1}{2} D,\\
ce &= bc, \qquad eF = Ab, \qquad eE = bB.
\end{aligned}$$

Care must be taken not to use a chord so long that the offsets become too long to permit close location of the station stakes. Two or three stations are about all that should be used. The principle is the same when the curve begins and ends with sub-chords, but the work is not symmetrical about the middle point for the first and last long chords.

When full stations are used the work of computation may be lessened by taking the values of long chords and middle ordinates for the necessary number of stations from tables of these quantities,* and using them as indicated below.

In the example given AF is the 'ong chord for four stations, BE is the long chord for two stations, and

$$\begin{aligned}
Ab = eF &= \tfrac{1}{2}\,(AF - BE),\\
bc = ce &= \tfrac{1}{2}\,BE.
\end{aligned}$$

* See any field book.

Similarly, Cc is the middle ordinate for four stations, Cc' the middle ordinate for two stations, and

$$bB = eE = Cc - Cc'.$$

Another method of computing long chords and ordinates is as follows: In Fig. 10, Bb' is ½ long chord for two stations, Cc' is ½ long chord for four stations, etc. Bb is the middle ordinate for two stations, Cc is the middle ordinate for four stations, etc.

$$\begin{aligned} Bb' &= R \sin D, \\ Cc' &= R \sin 2\,D, \\ Ee' &= R \sin 3\,D, \text{ etc.} \\ Bb &= R \text{ vers } D, \\ Cc &= R \text{ vers } 2\,D, \\ Ee &= R \text{ vers } 3\,D, \text{ etc.} \end{aligned}$$

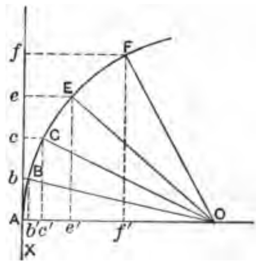

Fig. 10.

Because in the first method the multiplication is very simple (only the moving of the decimal point), the work is shorter for full stations than in the second method. The use of the tables is best.

Examples. 1. Find the necessary quantities to locate a 4° curve of 40° beginning at sta. 768, from 3 long chords. Let this example be solved by each of the three methods given.

2. A 3° 30′ curve begins at sta. 463 + 33⅓ and has a central angle of 30°. Find the necessary quantities, so far as practicable by each of the three methods, to locate the curve from 3 long chords.

APPROXIMATIONS AND SHORT METHODS.

9. Approximate Relation of *R* and *D*. If a curve were measured on the arc instead of by chords, then, since by geometry equal arcs of unequal circumferences subtend angles inversely as the radii, any two curves of degrees D and D' would have radii inversely proportional to their degrees, or

$$\frac{R}{R'} = \frac{D'}{D}. \tag{11}$$

But since the measurement is by chords, this proportion is not strictly true. It is so nearly true up to, say, a five-degree curve, that it is frequently used as correct for field work. If a ten-degree curve be considered as a curve in which two chords of 50 feet subtend 10° at the center, then the proportion given is still sufficiently exact for practical field use; and if a 14° or 16° curve be one in which four chords of 25 feet subtend 14° or 16° at the center, then the proportion is still sufficiently exact for field use. That is to say, if all curves between a five-degree curve and a ten-degree curve be measured with 50-foot chords subtending half the nominal degree of the curve, and if for curves of greater degree than ten, 25-foot chords subtending one-fourth the nominal degree be used, the proportion given is sufficiently precise for field use, but the curves run are not of the supposed degrees under the generally accepted definition.

By Eq. (9) the radius of a 1° curve is found to be 5729.65 feet. If this be taken as 5730 feet, Eq. (11) shows that the radius of any curve of D degrees may be said to be

$$R = \frac{5730}{D}. \tag{12}$$

This value is used in the derivation of a number of valuable approximate formulas. In almost all problems of an exact nature the correct value of R as determined by Eq. (9) is used.

The author firmly believes that American practice should be changed and the degree of curves defined as being the number of degrees subtended by an arc of 100 feet. Almost all computations would be greatly simplified. The one disadvantage would be that the chord lengths that must be used in location would be fractional. But a table of chord lengths which would not be extensive would largely offset the disadvantage, and many problems would be solved by simple arithmetic that are now solved by trigonometry.

10. Approximate General Functions. — It will be seen from Equations (1) to (4) that the principal curve functions all vary directly as R, and therefore, since R is assumed to vary inversely as D, these functions vary inversely as D. If, then, the several functions for a 1° curve be tabulated for all probable central angles, the values of the functions for any curve of degree D may be found by dividing the values for a 1° curve by D. Expressed as an equation:

$$\left.\begin{aligned} T_{D^\circ} &= \frac{T_{1^\circ}}{D}, \\ E_{D^\circ} &= \frac{E_{1^\circ}}{D}, \\ M_{D^\circ} &= \frac{M_{1^\circ}}{D}, \\ C_{D^\circ} &= \frac{C_{1^\circ}}{D}. \end{aligned}\right\} \qquad (13)$$

Conversely, if the function itself be known, the corresponding degree of curve may be found by dividing the given value by the value of the function for a 1° curve, as may be seen by transposing the several equations (13).

Examples. 1. Find the several functions of a 1° curve for a central angle of 30° by Equations (1) to (4); then for 2°, 5°, 10° and 15° curves by Equations (13), and compare the results with those of the examples of Art. 1. The close correspondence of M and E should be noticed. If the curve were a parabola they would be equal.

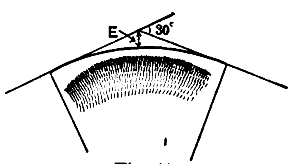

Fig. 11.

2. It is desired to join the two tangents o Fig. 11 by a curve that will lie about right for the hill if E is approximately 50 feet. What degree of curve should be used? Solve by Equation (13) and get the nearest whole degree.

11. Tangent Offsets.—It sometimes becomes convenient when no transit is at hand to locate a curve approximately by the use of the chain and flags. This may occur on construction work when an engineer out with a level, rod, and tape line, is asked by some foreman to reset some stakes he has lost on a curve. Stakes marking the tangent will usually exist or can be set by tape measurement from side reference points, and generally the P. C. or P. T. may be thus set. One method of setting the stakes on the curve is called the method by tangent offsets and is as follows:

Referring to Fig. 10, the tangent XA is produced by eye, using an extemporized flag pole or simply the stakes, and the points b, c, e, f, etc., are set, the distances being obtained as in Art. 8. The offsets bB, cC, etc., are obtained in the same way and measured in to establish the stations B, C, E, etc. The right angle is established at b, c, e, f, etc., either by eye alone if the work need not be very precise and the offsets are short, or by forming three lengths of tape into a right triangle of sides in the proportion of 3, 4, 5.

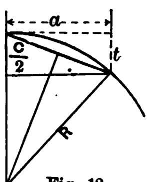

Fig. 12.

Another method of determining the tangent offsets for any chord length C from the point of tangency is as follows: In Fig. 12, the similarity of triangles will give

$$t = \frac{C^2}{2R}. \tag{14}$$

The t for a chord of 100 feet is the middle ordinate for two stations, and since in (14) t varies as the square of the chord length, the t for any sub-chord c is given by

$$t_c = t_{100}\frac{c^2}{10,000}. \tag{15}$$

For approximate values see Art. 13.

12. Offsets from a Chord Produced.—A second method for approximately locating the stakes on a curve is called the chord offset method and is as follows: As in the preceding article, if the curve begins at a whole station, the distances Ab and bB are computed and laid off for the station B; the chord AB is then produced 100 feet to c. If a tangent be drawn at B bisecting cC in c', cc' evidently equals bB, and cC is twice this; hence the chord offset is twice the tangent offset for one station. Therefore c being located, the chain is swung about B as a center to C, found by making cC equal to 2 bB, or twice the tangent offset for one station, or middle ordinate for two stations. BC is then produced to e and E located as was C.

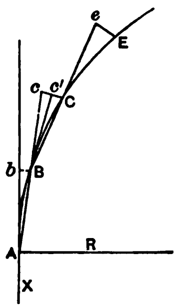

Fig. 13.

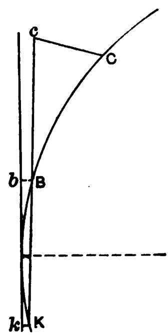

Fig. 14.

When the curve begins with a sub-chord the tangent offset, bB, for the sub-chord and its distance, Ab, are computed and B established; then the tangent offset and distance for a chord of 100 minus the sub-chord are computed and measured to k and K respectively. KB is then a full chord of 100 feet and may be produced to c and C established as before.

Example. A 3° 30′ curve of 35° begins at sta. 373 + 66⅔. Find the quantities necessary, and describe the procedure, to locate the curve by offsets from the chord produced. Let the full description of the procedure be entered in the note book.

13. Valuable Approximate Formulas.—The tangent offset at the end of n stations of any curve of degree D is approximately

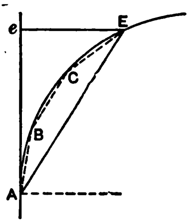

Fig. 15.

$$O = \tfrac{7}{8}\, n^2 D. \qquad (16)$$

Two arcs of different radii having a common tangent depart from each other approximately by the same law, if the *difference* of their degrees be substituted for D. This formula should be memorized. It is derived thus:

E being the end of the nth station on the curve,

$$\text{Angle } eAE = \tfrac{1}{2}\, nD.$$

$$eE = AE \sin \tfrac{1}{2}\, nD.$$

$$AE = 100\, n \text{ (approx.)}.$$

$$\therefore \quad eE = 100\, n \sin \tfrac{1}{2}\, nD.$$

Assuming that the sines of small angles are as the angles themselves,

$$\sin p^\circ = p \sin 1^\circ.$$

$$\therefore \quad \sin \tfrac{1}{2}\, nD^\circ = \tfrac{1}{2}\, nD \sin 1^\circ.$$

$$\therefore \quad eE = \tfrac{1}{2} \times 100 \times \sin 1^\circ \times n^2 D.$$

$$\sin 1^\circ = 0.017453.$$

$$100 \sin 1^\circ = 1\tfrac{3}{4} \text{ approx.}$$

$$\tfrac{1}{2} \times 100 \sin 1^\circ = \tfrac{7}{8} \text{ approx.}$$

$$\therefore \quad eE = O = \tfrac{7}{8}\, n^2 D.$$

The tangent offset for n stations of D° curve being $\tfrac{7}{8}\, n^2 D$ and for n stations of D_1° curve $\tfrac{7}{8}\, n^2 D_1$, the difference of these or approximate separation of two curves having a common P. C. after n stations have been run is

$$\tfrac{7}{8}\, n^2 (D - D_1). \qquad (16a)$$

It should be remembered that 100 times the sine of 1° is $1\tfrac{3}{4}$ approximately, and hence that two straight lines n stations long starting from one point and with an angle of Δ degrees between them, are apart approximately $\tfrac{7}{4}\, n\Delta$ feet.

It should also be remembered that 100 times the sine of

$0° 01'$ is $\frac{3}{100}$ of a foot, and that two lines diverging at an angle of m minutes separate at the rate of $\frac{3}{100}\,m$ feet per 100 feet of length.

Examples. 1. By Eq. (16) find the tangent offset at the third station of a 4° curve.

2. A 3° curve begins at sta. 373 + 20. Find the tangent offset at sta. 376.

3. What is the approximate distance at sta. 376 between the curve of example 2 and a 4° curve beginning at the same P.C.?

14. Deflection Angles for Sub-chords. — Assuming that the deflection angle for a sub-chord is proportional to the length of the chord, and letting l be the length of any sub-chord whose deflection angle is $\frac{1}{2}\,d$,

$$\tfrac{1}{2}\,d = \frac{l}{100} \times \tfrac{1}{2}\,D.$$

If D be 1° expressed as 60 minutes,

$$\tfrac{1}{2}\,d \text{ in minutes} = 0.3\,l,$$

and for any curve of degree D

$$\tfrac{1}{2}\,d \text{ in minutes} = 0.3\,lD. \tag{17}$$

This is a handy formula to remember.

Examples. Find the deflections for sub-chords of 20, 25, $33\frac{1}{3}$, 50, 62.5, 75 and $83\frac{1}{3}$ feet for curves of 1°, 3°, 5°, 10°.

15. Middle Ordinate in Terms of Chord and Radius. — While it is usually convenient to determine the middle ordinate for any chord of any curve of degree D or radius R by Eq. (3), since the central angle for a given chord of a given curve is readily found, still it is sometimes convenient to determine the middle ordinate from the chord and radius without reference to tables.

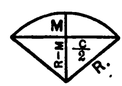

Fig. 16.

From Fig. 16 it may easily be shown that

$$M = R - \sqrt{R^2 - \left(\frac{c}{2}\right)^2}, \tag{18}$$

or remembering that the product of the sum and difference of two quantities is the difference of the squares of the quantities, Eq. (18) may be put in better form for logarithmic computation, thus:

$$M = R - \sqrt{\left(R + \frac{c}{2}\right)\left(R - \frac{c}{2}\right)}. \tag{19}$$

A useful approximate value is found thus:

$$(R - M)^2 = R^2 - \left(\frac{c}{2}\right)^2,$$

$$R^2 - 2\,RM + M^2 = R^2 - \left(\frac{c}{2}\right)^2.$$

Neglecting M^2 as small in comparison with the other quantities and reducing,

$$M = \frac{c^2}{8\,R}. \tag{20}$$

Equation (20) may also be derived from Eq. (18) by expanding the radical of that equation to two terms by the binomial formula. The geometrical significance of the approximation may be seen by the following derivation:

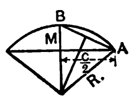

Fig. 17.

In Fig. 17,

$$\frac{M}{AB} = \frac{\frac{AB}{2}}{R},$$

$$M = \frac{\overline{AB}^2}{2\,R}.$$

If, now, AB be assumed equal to $\frac{c}{2}$, which it is very nearly for short chord lengths,

$$M = \frac{c^2}{4 \times 2\,R} = \frac{c^2}{8\,R}.$$

Hence the approximation of the formula consists in considering the chord of half the arc as equal to half the chord of the arc.

Equation (20) may be altered in form so as to make less work in computation, though the new form is not so easily memorized. If $\frac{5730}{D}$ be substituted for R in Eq. (20), and the fraction reduced to a decimal, the result is

$$\left.\begin{array}{ll} \text{For } c \text{ in stations of 100 feet} & M = 0.2182\, c^2D, \\ \text{For } c \text{ in feet} & M = 0.00002182\, c^2D, \end{array}\right\} \quad (21)$$

and the value from these equations is correct to within 0.001 ft. for any chord not exceeding one station of any curve not exceeding 10°. It is slightly in excess for curves under 6° and a little too small for sharper curves.

It is sufficiently precise for determining the middle ordinate of a 33-foot rail for any curve likely to be laid out, even in the yard, for a steam railroad. It is customary to bend the rails for curves sharper than from 2° to 4°, according to the custom of the particular road, before laying, and the sufficiency of the bending is determined by measuring the middle ordinate of the bent rail.

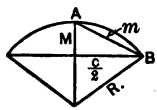

Fig. 18.

For all curves of 6° and under Eq. (21) is sufficiently precise for locating through woods by offsets from a long chord of not more than 4 stations. The error for a 6° curve is less than 1 inch.

Having the middle ordinate for 100 feet or other chord, the middle ordinate of the chord of half the arc is found to be approximately $\frac{M}{4}$.

$$M = \frac{c^2}{8R},$$

$$m = \frac{\overline{AB}^2}{8R}$$

$$AB = \frac{c}{2} \text{ approx.}$$

$$m = \frac{c^2}{4 \times 8R} = \frac{M}{4}. \quad (22)$$

Examples. 1. Using Equations (19) and (20), find the error of Eq. (20) for the following chords and curves:

(a) 10° curve, chords of 33 feet, 100 feet, 200 feet.
(b) 5° curve, chords of 33 feet, 100 feet, 200 feet.
(c) 1° curve, chords of 33 feet, 100 feet, 200 feet.

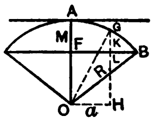

Fig. 19.

2. Find the error of Eq. (21) for a 20° curve and chords of 33 feet and 100 feet.

16. Ordinates at any Point of a Chord. — The student may show that at any point L distant from F by a distance a, on the known chord c,

$$K = \sqrt{(R+a)\,(R-a)} - \sqrt{\left(R+\frac{c}{2}\right)\left(R-\frac{c}{2}\right)}. \qquad (23)$$

An approximate value may be found by considering the curve to be the arc of a parabola in which the tangent offset (abscissa) varies as the square of the tangent length (ordinate).

M is the tangent offset for a tangent length of $\frac{c}{2}$ and, as has been shown, is given by

$$M = \frac{c^2}{8\,R}.$$

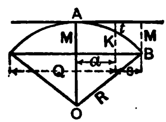

Fig. 20.

The tangent offset t for the tangent length a is then found, thus:

$$\frac{t}{M} = \frac{a^2}{\left(\frac{c}{2}\right)^2},$$

$$t = 4\,M\frac{a^2}{c^2},$$

$$K = M - t = M - 4\,M\frac{a^2}{c^2} = M\left(1 - 4\,\frac{a^2}{c^2}\right) \text{ approx.} \qquad (24)$$

If a be successively at the quarter and eighth points of the chord,

$$a = \tfrac{1}{8}\,c, \qquad K = \tfrac{15}{16}\,M \quad \text{approx.}$$
$$a = \tfrac{1}{4}\,c, \qquad K = \tfrac{3}{4}\,M \quad \text{approx.}$$
$$a = \tfrac{3}{8}\,c, \qquad K = \tfrac{7}{16}\,M \quad \text{approx.}$$

Again from Eq. (24),

$$K = \frac{4\,M}{c^2}\left(\frac{c^2}{4} - a^2\right) = \frac{4\,M}{c^2}\left(\frac{c}{2} + a\right)\left(\frac{c}{2} - a\right),$$

$$K = \frac{4\,M}{c^2}\,Q \times S.$$

And since $$M = \frac{c^2}{8\,R}\ \text{approx.},$$

$$K = \frac{Q \times S}{2\,R}\ \text{approx.} \tag{25}$$

Or, the ordinate at any point of a chord approximately equals the product of the segments of the chord on either side of the point divided by twice the radius of the curve.

Substituting $\dfrac{5730}{D}$ for R gives the value in terms of the degree of the curve,

$$K = \frac{872 \times Q \times S \times D}{10{,}000{,}000}. \tag{26}$$

CHAPTER III.

FIELD PROBLEMS IN SIMPLE CURVES.

17. The Point of Curve or Point of Tangent is Inaccessible. — Let it be supposed that the point of curve only is inaccessible, but that the line of the two tangents and the angle I have been obtained and the station of the P. C. has been computed. There are many ways of overcoming the difficulty

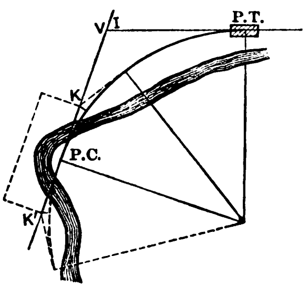

Fig. 21.

1. If the vertex V is readily accessible, the tangents may be run to intersection, the P. T located and the curve run backwards as far as possible.

2. If it is not convenient to run the tangents out, the ordinary surveying methods for passing an obstacle may be used to recover the tangent line beyond the obstruction at the nearest convenient point to the P. C., as K in Fig. 21. The distance from the P. C. will be known, and this distance is the tangent distance T_d for some central angle d which may be found from Eq. (1), p. 6. This angle d may be turned from the advance point on the tangent and the distance T_d run ahead to a point on the curve. The station of the point will be known from the station of the P. C. and the angles d and D, and the curve may be run backward and forward from this point as necessary.

3. If the obstacle to setting the instrument at the P. C. is not such as to prevent chaining, a point may be located

on what would be the curve produced back of the P. C. as in method 2, and the curve run ahead from this point.

In either method 2 or 3 a point on the curve could be located by tangent offsets at the subvertexes K or K', or, bisecting the angle at K or K', the external distances for the angle d might be set out to the curve, but these distances being short, some other method is better.

4. Such another method, using the tangent offsets, is to assume for the purpose of computation a distance on the curve that will clear the obstruction; to compute the tangent distance and offset; and to lay these off forward and back of the P. C. by the method of 2; then, with the transit on the forward point the long chord joining the two points is used for a back sight, and the curve run in for whatever stations may be required back and forward.

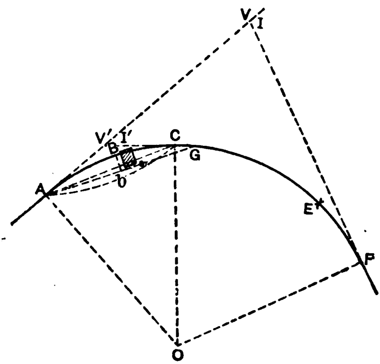

Fig. 22.

If the P. T. is inaccessible, the same methods are used as for the P. C., with such modifications as may be necessary.

Examples. 1. Let Fig. 21 represent a case in which the P. C. is at sta. 263 + 60, $D = 4° 00'$, $I = 33° 00'$. It is 50 ± feet from the P. C. to dry ground forward and 60 feet backward. Make the necessary computations and determine the steps to be taken to recover the curve at, say, sta. 265, and to locate from there sta. 264 + 50, 266, etc., to the P. T.

2. Determine what to do to pass the obstructed P. T. of the curve of Example 1, the obstruction being a small

building about 20 feet square at about the center of which the P. T. falls.

18. An Obstruction on the Curve. — If an obstruction occurs on the curve, Fig. 22, a distance *Bb* that will clear the obstruction may be measured normal to the tangent at a convenient station, *B*. Letting this distance be a middle ordinate, the corresponding arc and long chord may be computed and the half chord laid off at right angles to the offset to *G*. The direction of the tangent at *G* is obtained by turning an angle from the long chord equal to half the arc subtended. Any stations between *B* and *G* and beyond *G* may then be located in the ordinary way. Other methods will suggest themselves, as running out the tangents *AV′* and *VC*, or running an inverted curve from *A* to *C*.

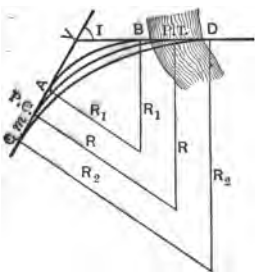

Fig. 23.

Example. Let the curve of Example 1, Art. 17, be clear except for a building 20 feet square covering sta. 266. Assume a method for passing the obstacle, make the necessary computations and determine the steps in the field work.

19. To Change the P. T. or P. C. — Let it be supposed that the curve has been run in or computed and found to end in a fairly wide stream, hence on a bridge, which is undesirable, and that the tangents lie about as they should. If Fig. 23 represents the condition, the radius may be shortened to R_1 giving the curve *AB*, or lengthened to R_2 giving the curve *CD*. If there is nothing but the alinement to consider, the longer radius is the better, since if the shorter is used the beginning of the spiral is very likely to fall on the bridge, and in general a longer radius curve is to be preferred.

There are two methods of attacking the problem: 1. A distance $PT - D$, sufficient to clear the stream, may be assumed to be added to the already known tangent distance for a new tangent distance by which, since I does not change, the new radius, R_2, may be found from Eq. (1), p. 6. 2. A new degree, D_2, greater than the original degree, D, may be assumed such that the increase in tangent distance will probably be sufficient, and the correctness of the assumpt on tested by finding the new tangent distance. The second method is probably the better, since it gives a round-number degree rather than a fractional degree, troublesome to lay out. It is true that a round-number degree may be found by the first method by assuming the nearest convenient number to the precise value found from the new tangent distance, but this would require the computation of a new tangent distance, hence the second method involves less labor. As the curve must be run in anew, the P. C., C, must be found. The movement of the P. C., m, is found from the difference in the two tangent distances, thus:

$$T = R \tan \tfrac{1}{2} I.$$
$$T_2 = R_2 \tan \tfrac{1}{2} I.$$
$$m = T_2 - T = (R_2 - R) \tan \tfrac{1}{2} I. \qquad (27)$$

The work is shortened by finding T and T_2 from a table of tangent distances for a 1° curve.

Examples. 1. A 3° curve begins at sta. 367 + 70 and ends at sta. 374 + 20 in a stream. If the P. T. were moved ahead about 80 feet the stream would be cleared; what curve shall be used?

Suggested solution, which the student should verify: From a table of tangents to a 1° curve it is found that a 3° curve for a central angle of 19° 30′ has a tangent distance of about 328 feet, while a 2° curve for the same central angle has a tangent distance of about 492 feet or 164 feet more than that of the 3° curve. The tangent distance being about proportional inversely to the degree, the tangent distance of a 2° 30′ curve would be about a mean of those of 3° and 2° curves, and hence would give the required 80 feet more or less. Therefore a 2° 30′ curve may be used. The student may complete the example, getting the exact change in station of P. C. and P. T. It should be noted that while the rough approximation to the required curve gives ample precision for most cases, the approximating must cease with the selection of the curve, the computations from this point on being carried to the limit of precision of measurement. It may be said here that in general all computations of quantities to be laid out, as angles or distances, must secure results of the degree of precision

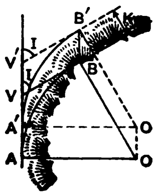

Fig. 24.

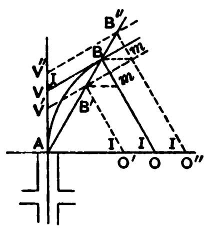

Fig. 25.

possible in the measurement. This means the nearest one-hundredth foot in lineal measurement and at least the nearest minute in angle work. The student must learn to distinguish between what may be approximated and what must be precisely determined.

2. In example 1, what is the difference in total length of line via the 3° curve and the 2° 30′ curve; and what is the change in position at the middle of the curve?

20. To move a Tangent of a Located Curve a Given Distance Parallel to Itself. — Reference to Figs. 24 to 26 will show that there may be three general cases: 1. Fig. 24, there are no conditions other than that the tangent is to be moved the given distance K; 2. Fig. 25, the tangent is to be moved through the distance K, and the P. C. is to remain fixed; 3. Fig. 26, the tangent is to be moved through the distance K, and the curve is to end at a point directly opposite the former ending. It will be noted that since the changing tangent remains parallel to its first position, there is no change in the angle I. In the first case the curve may be imagined moved along the first tangent until it becomes tangent to the second tangent in its new position, and since the degree does not change, it is only necessary to find the new stationing. The new P. C. is found by computing the distance $AA' = VV' = BB' = OO'$, through which the curve is moved. The student may show that this is given by

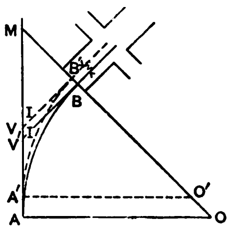

Fig. 26.

$$AA = \frac{K}{\sin I}. \tag{28}$$

In the second case it is to be noted that since neither the central angle nor the position of P. C. is changed, the long chords of whatever curves may be run will all lie in the same straight line; hence, if the original curve $AB - O$ and the new tangent be drawn, the new P. T. will be found at the intersection of the new tangent and the long chord produced if necessary. The new center will then be found by drawing a final radius through the new P. T. parallel to the original final radius, and noting the intersection with the radius from the P. C., produced, if necessary. It will be evident that the degree of curve is changed and the problem is to find the new radius (*i.e.*, the new degree). There are several methods of solution. Perhaps the simplest considers the triangle $BB'm$, the construction of which will be evident. The student may show that it is an isosceles triangle and that from it the relation between the radii is found to be, according as the tangent is moved in (toward the center) or out:

$$R' = R \mp \frac{K}{\text{vers } I}. \tag{29}$$

That is, the change in radius is

$$\frac{K}{\text{vers } I}.$$

The student should find at least one other method of solution.

In the third case the radius (or degree) also changes as does the position of the point of curve. Since the new final radius is to lie in the same line as the original final radius, and the first tangent is not changed in position, if the new tangent be drawn, the new vertex is found at V' and the tangent distance is fixed at $V'B'$. Since the tangent distances of a curve are equal, the distance $V'B'$ may be laid off from V' to A' and the new P. C. is established through which the new first radius may be drawn to an intersection with the final radius (produced if necessary) for a new center O'. The problem here, then, is to find the new radius and the movement AA' of the P. C. As before, there are several methods of solution. One of the simplest uses the external secant found by extending the final radius to M. The student may show that the relation of the radii is given by

$$R' = R \pm \frac{K}{\text{ex sec } I}, \tag{30}$$

according as the change in tangent is toward or from the center, and that the change in P. C. is given by

$$AA' = (R - R') \tan I \tag{31}$$

and also by

$$AA' = K \cot \tfrac{1}{2} I. \tag{32}$$

The P. C. is moved ahead when R' is less than R, which occurs when the change in tangent is outward, and back when the change in tangent is inward, and this will be shown by the sign of the parenthesis of Eq. (31).

The student should develop one other method of solution.

Examples. 1. A 5° curve has been run from sta. 367 + 80 to sta. 372, where it is found that the forward tangent will lie better if it be moved 15 feet toward the center. Find the new stations of P. C. and P. T. and the deflection angles to each full station of the curve.

2. In example 1 let it be supposed that the P. C. must remain fixed. Find the necessary quantities to relocate the curve.

3. In example 1 let it be supposed that the curve must end on the same radial line as before. Find the necessary quantities to locate the curve.

21. The Forward Tangent Changes its Direction.—(*a.*) Let it be supposed that a curve of degree D, radius R, had been run for a central angle of $I°$ from A to B, and that it is found desirable to change I by a given angle P, the vertex to remain fixed. There are two possibilities: (1.) There are no other conditions, (2.) The P. C. is to remain fixed. Under the first condition the curve is imagined moved along the first tangent until it becomes tangent to the new position of the final tangent. Since the angle has changed the tangent distance will change and the P. C. will be moved ahead or back by the difference in tangent distances (Eq. 1) for the two central angles. Under the second condition the tangent distance remains fixed and a new degree curve must be computed from this tangent distance and the new central angle.

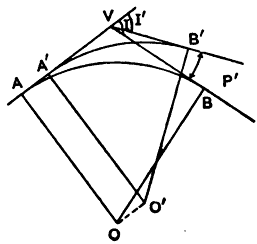

Fig. 27.

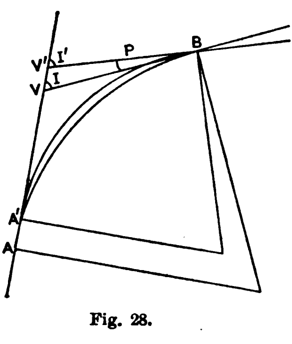

Fig. 28.

(*b.*) Let it be supposed that the forward tangent is to change its direction by the angle P at the P. T., which is to remain fixed in position. Reference to Fig. 28 will show that it is necessary to find a new degree (radius) and the new P. C., and that both curves have the same versed sine for the whole central angle, which furnishes the clue to the solution for the new radius. It will also be noted that the difference in the sines of the two curves is the change in the P. C.

The student may show that

$$R' = R\frac{\text{vers } I}{\text{vers } I'}. \tag{33}$$

$$AA' = \pm (R \sin I - R' \sin I'). \tag{34}$$

I' is known from I and P and the $\pm$ sign refers to the direction of change of P. C. forward or back.

Examples. 1. A 3° 00′ curve begins at sta. 746 + 25 and ends at sta. 752 + 75. It is found desirable to increase the deflection angle I at the vertex by 3°; what change is required in the P. C. and what will be the new station of P. T.?

2. In example 1 it is required to retain the original P. C. What is the new degree of curve and station of P. T ?

3. In example 1 the change of direction is to be made at the P. T., which is to be retained. Find the quantities necessary to relocate the curve.

22. Other Methods of Changing Tangents. — Let it be supposed that a curve of degree D has been run from A to B and that it is found that the line will lie on better ground if it be shifted over K feet (inside in the Figure). If B is the end of the curve and the tangent lies in the right direction, the change may be made by the methods of Art. 20. If the direction of the forward tangent is not fixed, or B is not the end of the curve, the change may be made by the approximate method of Art. 13, using Eq. (16a), the P. C. remaining fixed and the degree changing. Indeed, by this method the direction of the forward tangent may be preserved by putting in the same total angle I instead of the

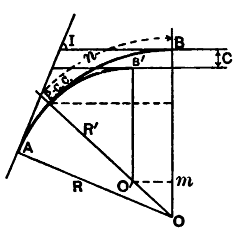

Fig. 29.

same number of stations. The resulting movement of the tangent will not be exactly K.

What is perhaps a better way, since it will not result in troublesome fractional degree of curve, is to assume a new degree, less or greater than the original degree, according as the curve is to be thrown out or in, and from this to determine by Eq. (16a) the number of stations of the curve to be changed. If the first assumption of a degree proves impracticable, a second can be made. This method would be used when the curve is long, the necessary change not great, and the first part of the curve lies well on the ground.

The problem may be exactly solved as follows: —

From Fig. 29 it is seen that $OO' = R - R'$ and that therefore $Om = (R - R') \cos \alpha$. It is also apparent from the figure that $R = R' + Om + K$. Therefore

$$\begin{aligned} R - R' &= Om + K \\ &= (R - R') \cos \alpha + K. \end{aligned}$$

Whence

$$\cos \alpha = 1 - \frac{K}{R - R'}. \tag{35}$$

The degree of curve D' is assumed, which gives R', and when α is determined, n, the number of stations to be changed, is given by $n = \frac{\alpha}{D}$. The result of these last two methods is a compound curve. The second branch is run in by establishing the P. C. C. (Point of Compound Curve) on the first simple curve, setting the transit at this point, and, after getting the line of collimation in the tangent at the P. C. C., running the second branch as a simple curve from the P. C. C. The method of setting the deflection angles is exactly as explained in Art. 6, but should the transit be set on the second branch at any other station than the P. C. C., the line of collimation will not point to stations of the first branch when the vernier is set to read the deflection from the P. C. to those stations.

Examples. 1. A 4° 30′ curve has been run from sta. 538 + 75 to sta. 546 + 25, and it is found desirable to shift the forward tangent about 25 feet outward and to keep it parallel with its present position without changing the first few stations of the curve. Using Eq. (16a) and assuming that a 2° 30′ curve will answer the purpose, find the stations of P. C. C. and P. T.

2. Solve example 1 by the exact method, using Eq. (35), and note the variation from the approximate results.

23. To pass a Curve Through a Given Point. — 1. The point is given by the offset K from a given station H on the known tangent ST, the forward tangent and angle I being undetermined. It will perhaps be evident without demonstration that an indefinite number of curves can be passed through P and tangent to AV. A curve determined from any other consideration, as convenient degree, or approxi-

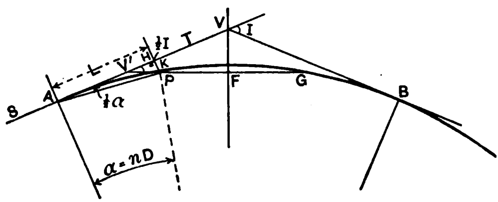

Fig. 30.

mate position of P. C., desired, may therefore be assumed and other quantities computed. If D is assumed, R is known, whence α and L may be computed, giving the station of P. C. If the P. C. is assumed, again α and R may be computed, knowing L and K. If approximate results are sufficient, Eq. (16) may be used. Again K is the middle ordinate of twice the number of stations from A to P, and a table of middle ordinates may be used to find D or AH (approx.) = n.

2. The point P is given as before, but I and V are determined. There is now but one curve that will join the two given tangents and pass through P. It will be

convenient first to find AV, the tangent distance, from which and I the radius is found. Let it be imagined that the curve is drawn in, as it may be by trial on the drawing board, and let a line be drawn through P, making an angle $\frac{1}{2}I$ with ST at V'. This line will be perpendicular at F to the bisecting line from V, and FG will equal PF. From the triangle $V'HP$, $V'P$ and $V'H$ may be found, giving $V'V$, and hence $V'F$ and PF and FG and PG. Then from the geometric relation of a tangent and secant drawn from a point without a circle, AV' may be found, giving the station of P. C. and the tangent distance AV from which, with I, R is determined. If the determined D is a troublesome fraction and P need not be exactly on the line, the nearest value of D to that determined that can be expressed by an even* number of minutes may be assumed, a new tangent distance, or the change in tangent distance, computed, from which the P. C. is found and the curve run in.

Examples. 1. Opposite sta. 367 and 31.3 feet distant is a point through which it is desired to pass a curve that shall begin approximately at sta. 364. What curve will answer and what will be the station of P. C.? Solve by three methods, one of which shall use a table of middle ordinates.

2. Opposite sta. 367 and 31.3 feet distant is a point through which it is desired to pass a curve that shall meet the two tangents that make an angle of 36° at sta. 368 + 67.7. Find the station of P. C., degree of curve, and station of P. T.

24. Miscellaneous Problems. — 1. A 4° curve to the right begins at sta. 752 and ends at sta. 760 in a tangent which it is found would lie better at sta. 765 if it were 17 feet to the right measured at right angles. It is required to determine how much to add to the 4° curve that the tangent may pass through the required point. The new tangent is not parallel to the old one.

* An even number is assumed that the deflection angle which is half the degree need not be fractional.

2. A 2° curve to the right begins at sta. 373 and ends at sta. 384, from where the line is tangent to sta. 400, where a 4° curve to the right begins and continues to sta. 408. It is found that the line will lie much better if, in the vicinity of the P. T. of the 2° curve, it can be thrown out to the left about 14 feet. Application of Eq. (16) shows that this may be accomplished if the 2° curve be compounded to a 1° curve at sta. 380. It is required to find the new total I of the 2° and 1° curves, and therefore the sta. of the P. T. that the new forward tangent may come tangent somewhere on the 4° curve, and the station of the point of tangency which will be the new P. C. The portion of line here indicated is part of a long located line, hence the stationing must not be disturbed further than necessary. It will therefore be advisable to introduce a long station (*i.e.*, more than 100 feet) just after the P. T. of the 1° curve or just preceding the new P. C. of the 4° curve.

It is suggested that a part of this problem be solved by the use of coördinates (latitudes and longitudes) and that the origin of coördinates be taken at the center of the 2° curve, with the meridian in the original final radius. It is also suggested that the solution be first by the use of letters for the several quantities, and that the problem be drawn by trial before an algebraic solution is attempted.

3. The following are notes of a part of a located line.

Sta. 367 + 50 P. C. 4° 30′ *R*. (4° 30′ curve to the right)
Sta. 373 P. T.
Sta. 376 P. C. 5° *R*.
Sta. 382 + 50 P. T.

This gives what is known as a broken-back curve, which is considered less desirable than a continuous curve, even though the middle portion be considerably flatter than the two ends. It is required to replace the three hundred feet of tangent by a 1° curve. The quantities to be found are the stations of junction of the 1° curve with the 4° 30′ and 5° curves. The length of line will be slightly changed, hence to retain the old station numbering a long station will be introduced at the end of the 1° curve. This problem introduces compound curves, but requires no further knowledge of them than that joining curves have common directions of tangents and radii at the points of junction. The best method for avoiding the broken-back curve of this problem is the introduction of spiral easement curves, but for this the student is not yet prepared.

4. An existing 3° 30′ curve is to be connected with a straight line which intersects the curve at a point determined in the field and at an angle of 20° with the tangent at the point of intersection. Reference to Fig. 31 will show that within limits any curve may be used for the

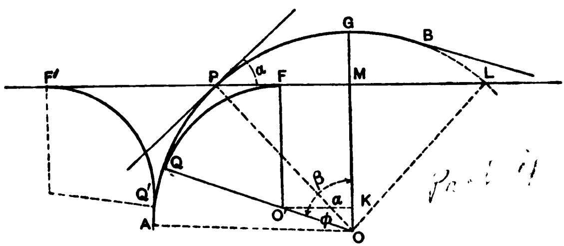

Fig. 31.

connection which may be inside or outside the existing curve as shown at QF and $Q'F'$, or in either of two other positions that the student may discover, for it will appear that any curve of not too great radius can be drawn on a piece of tracing paper and then moved until it becomes tangent to the line and curve. Let a radius be assumed for the connecting curve less than that of the 3° 30′ curve, say that of a 5° curve, and let the connection be inside. The problem is to find the distance PQ or, what is the same thing, the angle ϕ. The lines of the figure suggest the method of solution. The student may also solve an outside connection.

5. From a point P, Fig. 32, it is required to run a line that shall be tangent to the 4° 30′ curve of center O. In the field a line is run to intersect the curve, say at K, and

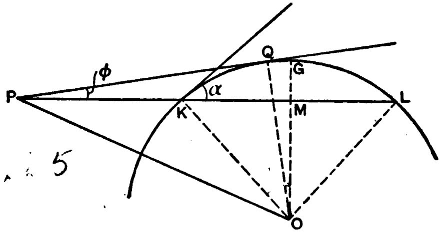

Fig. 32.

the angle α is measured between the line and a tangent at the point of intersection. Let the length of the line PK be 600 feet and the angle α be 3°. The problem is to find the length of curve KQ from K to point of tangency, the angle ϕ to be turned at P, and, if desired, the length PQ. The line PK is not run to L but KM and KL may be computed from the known data, and PQ determined from the geometrical relation of tangent and secant.

6. Let it be required to connect an existing straight track and curve. The degree and radius of the curve will be known. Setting a transit at any convenient point P on

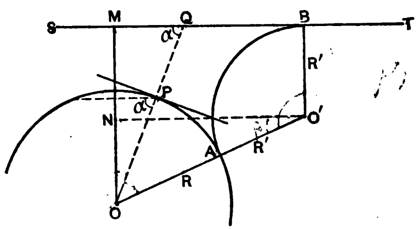

Fig. 33.

the curve, find the direction of the radius at that point and run a line in the radius extended to an intersection with the straight track, measuring the angle of intersection

α and the distance PQ. It will be evident that any number of curves (within limits) may be fitted in as a connection, therefore a radius for the connecting curve must be assumed. The problem is to find the distance PA or equivalent angle POA, the distance QB, if desired, and the angle $BO'A$.

7. Let it be required to connect a curve and external straight line as in Fig. 34, the existing curve being of center O, radius R; the connecting curve being AB of radius R' to be assumed.

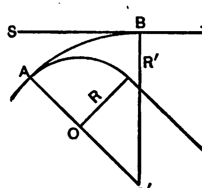

Fig. 34.

It should be noted in problems 4, 6, and 7 that the assumed radius of the connecting curve must be practicable. It is not possible to assume any radius at will, but in any given case the existing data will suggest a rational value for a radius.

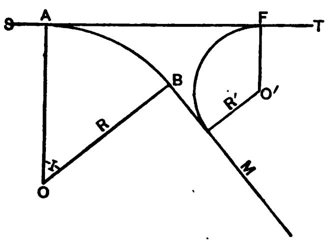

Fig. 35.

8. A curve AB of degree D and central angle I leaves a tangent ST at A and joins a tangent BM at B. It is required to find where a curve of degree D shall start on the tangent BM to join the tangent ST, where it will end on ST and its length.

9. In Fig. 36 let it be required to connect the curve ABM of given radius R with the tangent ST by a curve of given radius R'. The student should find all quantities necessary to make the connection.

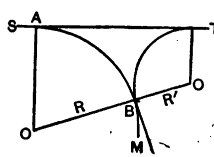

Fig. 36.

10. In Fig. 37 it is required to connect the two given lines ABM and AEF by a curve of given radius R''.

Problems 8, 9 and 10 are called Y problems, and arise when two tracks at a junction point are to be connected, or where a Y for reversing engines or trains is to be laid out

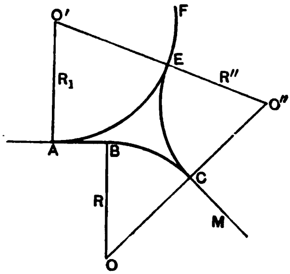

Fig. 37.

temporarily or permanently at small branch terminal stations in place of a turntable.

CHAPTER IV.

COMPOUND CURVES.

25. To Find the Elements of a Located Curve. — Compound curves are usually fitted to the ground by trial on the ground or on a topographical map. They may be of several branches, and when so, the problems that arise may usually be solved by an extension of the methods here suggested for a curve of two branches If a compound curve ABC (Fig. 38) has been located on the ground and it is desired to compute its tangent distances AV and VC (always unequal, with the longer tangent next the arc of longer radius), it may be done as follows: Draw the common tangent at B, the P. C. C., and note that $AV_1 = V_1B$ is the tangent distance for a simple curve of radius R_1, and central angle Δ_1, and that $BV_2 = V_2C$ is the tangent distance for a simple curve of radius R_2 and central angle Δ_2. These two radii and central angles are known. The sum of the two tangent distances, V_1B and BV_2, is V_1V_2, and the sum of Δ_1 and Δ_2 is I; therefore in the triangle VV_1V_2 there are known the side V_1V_2, the three angles Δ_1, Δ_2, and the supplement of I.

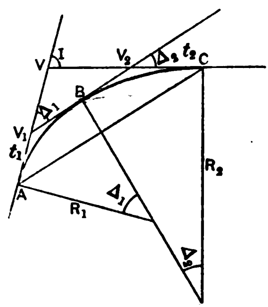

Fig. 38.

The triangle may be solved for V_1V and VV_2, which added to t_1 and t_2, respectively, give AV and VC, the required tangents.

If the long chord AC is required, it may be obtained by solving the triangle AVC, in which two sides and the included angle are known, and by the same solution the angles VAC and VCA may be had if desired.

Example. The notes of a compound curve are

P. C. at Sta. 346 + 50	4° 30′ R
P. C. C. at Sta. 351	7° 00′ R
P. T. at Sta. 356 + 50.	

Required the two tangent distances, the long chord and the angle between tangent and chord at the P. C. and P. T.

26. Some Limitations. — In the solution of the problem of the preceding article the following equations, which can be readily obtained by reference to Fig. 38, are used:

$$I = \Delta_1 + \Delta_2, \tag{36}$$

$$t_1 = R_1 \tan \tfrac{1}{2} \Delta_1,$$

$$t_2 = R_2 \tan \tfrac{1}{2} \Delta_2,$$

$$V_1V_2 = t_1 + t_2,$$

$$VV_1 = \frac{\sin \Delta_2 \,(R_1 \tan \frac{1}{2} \Delta_1 + R_2 \tan \frac{1}{2} \Delta_2)}{\sin I}, \tag{37}$$

$$VV_2 = \frac{\sin \Delta_1 \,(R_1 \tan \frac{1}{2} \Delta_1 + R_2 \tan \frac{1}{2} \Delta_2)}{\sin I}, \tag{38}$$

$$\left.\begin{aligned} T_1 &= t_1 + VV_1 \\ T_2 &= t_2 + VV_2 \end{aligned}\right\}. \tag{39}$$

Consideration of Eq. (36) shows that if the initial and final tangents are fixed in direction, *i.e.*, if I is fixed, only one of the Δ's may be assumed at will, since the sum of the two Δ's must equal I.

It is evident, therefore, that there are some limitations in the assumptions that may be made when fitting a curve to the ground by trial. For instance, considering Fig. 39, it is evident that a compound curve of a short radius followed by a longer radius will best fit the hill and join the two tangents VX, VY, which may be supposed to be fixed in direction and position. It has already appeared that the two Δ's may not be assumed at random or at will, because if they are they may not equal I. Just what elements may be assumed it is the purpose of this article to show.

It will be considered that there are seven elements of the curve, I, Δ_1, Δ_2, T_1, T_2, R_1, R_2. Four of these, including only two angles, may be assumed at will. This is true only mathematically. Practically four, including only two angles, may be assumed if the assumptions are within certain practical limits.

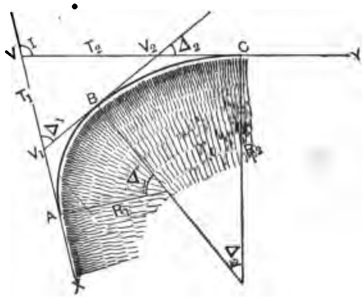

Fig. 39.

Considering the triangle VV_1V_2, if two angles are fixed, the third is fixed and therefore the form of the triangle. If one side is also fixed, the triangle is determined in all dimensions. If the two radii be assumed, then, since the two Δ's are determined, the two subtangent distances V_1B and BV_2 are determined, fixing V_1V_2 and the triangle VV_1V_2 and also T_1 and T_2, since these are respectively made up of sides of the triangle and the fixed subtangent distances. Four elements have been assumed, I, one Δ, and the two R's, and the other elements have followed. If T_1, R_1, Δ_1 and I be assumed, Δ_2 is at once fixed and the form of the triangle VV_1V_2 determined, the subtangent $V_1B = AV_1$ is determined and hence V_1V, and hence the triangle VV_1V_2, and hence BV_2, and hence R_2 and T_2. The student may make other assumptions. If R_1, R_2, Δ_1 and Δ_2 are assumed, giving the problem of Art. 25, I is fixed, but so long as the curve remains simply a mathematical conception it may be shifted in any direction to fit the ground.

In compound-curve problems other than those like that of Art. 25, the initial and final tangents are usually fixed in position, thus fixing I, when but three other elements of the curve may be assumed at will

Sometimes the tangent distances T_1 and T_2 are fixed, when but one more assumption, usually that of a radius, may be made. Not infrequently the first curve is assumed and run for some distance, thus fixing also the first tangent distance, and it is required to find what central angle and radius will complete the curve.

Some of these problems may be best solved by reference to Figs. 38 and 39 and the transposition and composition of equations (36) to (39) inclusive; others are better solved by means of the quantities considered in the next article.

27. A Fundamental Proposition. *If two or more circular arcs of equal angle begin at the same point on a common tangent, their chords lie in the same straight line.*

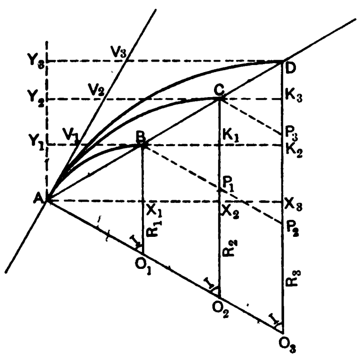

Fig. 40.

Since the angles between the chord of an arc and the tangents at its extremities are equal and each is half the angle subtended by the arc, the chords of the equiangular arcs, AB, AC, AD, make equal angles with the common tangent at A, and hence lie in the same straight line. If T_1, T_2 and T_3 be the respective tangent distances AV_1, AV_2, AV_3; R_1 R_2 and R_3 the respective radii, and if Bp_1p_2 and Cp_3 be drawn parallel to $AO_1O_2O_3$, and $Ax_1x_2x_3$ parallel to V_1B, V_2C, and V_3D and hence perpendicular to each of the three final radii, and if

$Ay_1y_2y_3$ be drawn parallel to the three final radii and hence perpendicular to the three final tangents, the following relations may be established, noting that Cp_3D, Bp_1C and Bp_2D are isosceles triangles — which the student may prove:

$$\begin{aligned}
x_1B &= R_1 \text{ vers } I = Ay_1 = T_1 \sin I,\\
x_2C &= R_2 \text{ vers } I = Ay_2 = T_2 \sin I, \qquad (40)\\
x_3D &= R_3 \text{ vers } I = Ay_3 = T_3 \sin I.
\end{aligned}$$

$$\begin{aligned}
k_1C &= Bp_1 \text{ vers } I = (R_2 - R_1) \text{ vers } I = y_1y_2 = (T_2 - T_1) \sin I,\\
k_2D &= (R_3 - R_1) \text{ vers } I = (T_3 - T_1) \sin I, \qquad (41)\\
k_3D &= (R_3 - R_2) \text{ vers } I = (T_3 - T_2) \sin I.
\end{aligned}$$

$$\left.\begin{aligned}
AB &= 2\,R_1 \sin \tfrac{1}{2} I\\
AC &= 2\,R_2 \sin \tfrac{1}{2} I\\
AD &= 2\,R_3 \sin \tfrac{1}{2} I
\end{aligned}\right\} \quad \therefore \quad \left\{\begin{aligned}
BC &= 2\,(R_2 - R_1) \sin \tfrac{1}{2} I,\\
BD &= 2\,(R_3 - R_1) \sin \tfrac{1}{2} I, \qquad (42)\\
CD &= 2\,(R_3 - R_2) \sin \tfrac{1}{2} I.
\end{aligned}\right.$$

$$\begin{aligned}
O_1x_1 &= R_1 \cos I,\\
O_2x_2 &= R_2 \cos I, \qquad (43)\\
O_3x_3 &= R_3 \cos I.
\end{aligned}$$

$$\begin{aligned}
y_1V_1 &= T_1 \cos I,\\
y_2V_2 &= T_2 \cos I, \qquad (44)\\
y_3V_3 &= T \cos I.
\end{aligned}$$

$$\begin{aligned}
Ax_1 &= R_1 \sin I = T_1 + T_1 \cos I = T_1\,(1 + \cos I),\\
Ax_2 &= R_2 \sin I = T_2 + T_2 \cos I = T_2\,(1 + \cos I), \qquad (45)\\
Ax_3 &= R_3 \sin I = T_3 + T_3 \cos I = T_3\,(1 + \cos I).
\end{aligned}$$

$$\begin{aligned}
x_1x_2 &= Bk_1 = (R_2 - R_1) \sin I = T_2 - T_1 + (T_2 - T_1) \cos I\\
&= (T_2 - T_1)\,(1 + \cos I),\\
x_1x_3 &= Bk_2 = (R_3 - R_1) \sin I = (T_3 - T_1)\,(1 + \cos I), \qquad (46)\\
x_2x_3 &= k_1k_2 = (R_3 - R_2) \sin I = (T_3 - T_2)\,(1 + \cos I).
\end{aligned}$$

In solving the problems that follow, a general figure of a compound curve should be drawn, and that branch about which the most is known (there will always be at least one more item known about one branch than is known about the other) should be produced till it ends in a tangent parallel to the ending tangent of the other branch. One or two construction lines will then suggest themselves and a solution will be readily obtained.

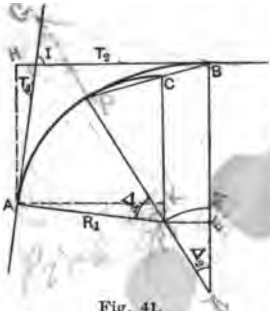

Fig. 41.

28. Problems.

I. If I, T_1 and T_2 are given, then to find the compound curve to connect A and B, Fig. 41, one other element must be assumed.

a. Let that element be R_1. Then R_2 and the two Δ's must be found. In this problem R_1 is produced to C, the construction lines shown are drawn, and it may be shown that

$$(R_2 - R_1)\sin\Delta_2 = T_2 + T_1\cos I - R_1\sin I, \tag{47}$$

$$(R_2 - R_1)\operatorname{vers}\Delta_2 = T_1\sin I - R_1\operatorname{vers} I, \tag{48}$$

Then, since $\dfrac{\sin A}{\operatorname{vers} A} = \cot \tfrac{1}{2} A$ (Trigonometry),

$$\cot \tfrac{1}{2}\Delta_2 = \frac{T_2 + T_1\cos I - R_1\sin I}{T_1\sin I - R_1\operatorname{vers} I}. \tag{49}$$

Having Δ_2, Δ_1 is known, and R_2 may be found by substitution in (47) or (48). Using (48),

$$R_2 = R_1 + \frac{T_1\sin I - R_1\operatorname{vers} I}{\operatorname{vers}\Delta_2}. \tag{50}$$

There are two practical limits in the choice of R_1: first, it must not be larger than that of a simple curve joining the tangents, at which limit R_2 becomes infinite; second, it must not be smaller than the allowable curvature of the road.

b. Let the assumed element be R_2. The student may show from a new figure, producing R_2, that

$$(R_2 - R_1)\sin\Delta_1 = R_2\sin I - T_1 - T_2\cos I, \tag{51}$$

$$(R_2 - R_1)\operatorname{vers}\Delta_1 = R_2\operatorname{vers} I - T_2\sin I, \tag{52}$$

$$\cot \tfrac{1}{2}\Delta_1 = \frac{R_2\sin I - T_1 - T_2\cos I}{R_2\operatorname{vers} I - T_2\sin I}, \tag{53}$$

$$R_1 = R_2 - \frac{R_2\operatorname{vers} I - T_2\sin I}{\operatorname{vers}\Delta_1}, \tag{54}$$

The practical limits of choice for R_2 lie between infinity and that value which can be used with the minimum practical value of R_1. The student should draw some figures showing the effect of varying the values of R_1 and R_2.

c. The assumption Δ_1 or Δ_2 is never made, and while the computation of the other quantities from this assumption is possible and not difficult, it is tedious and will not be given, since it has only mathematical interest.

Examples. 1. At station 763 + 62 a deflection angle of 54° 30′ is made to a new tangent. A six-degree curve begins at sta. 758 and the curve is expected to end on the new tangent 685 feet beyond the intersection point. What will be the station of the P. C. C. and what the radius of the second branch and the station of the P. T.?

2. *a*. If no radius is assumed in the foregoing example, how many possible curves can be located between the two tangent points? *b*. If the first curve be made less or more than six degrees, what will be the effect on the second curve and the two central angles? *c*. What is the smallest degree that can be used for the first curve? *d*. Neglecting the item of direction of motion, can the two tangents be connected by a compound curve when the first branch is of less degree than found above, and if so, what will be the form of the curve? *e*. By experimental diagrams find whether there are other ways that are possible geometrically, even if not practical, of connecting the two tangents when the first degree is smaller than the minimum computed.

II. If I, T_1 and R_1 are given, either Δ_1, Δ_2, R_2 or T_2 may be assumed. As the problem would most likely occur either Δ_1 or T_2 would be known or assumed. If the latter, the problem is the same as I, *a*; if the former, it remains to find Δ_2 and R_2. T_2 may also be found, but would not be needed for anything but a possible check or for platting, for which it may or may not be necessary, according to the method of platting.

From Fig. 41,

$$T_1 \sin I - R_1 \operatorname{vers} I = (R_2 - R_1) \operatorname{vers} \Delta_2.$$

Δ_2 is known, since $\Delta_1 + \Delta_2 = I$; therefore, Eq. (50),

$$R_2 = R_1 + \frac{T_1 \sin I - R_1 \operatorname{vers} I}{\operatorname{vers} \Delta_2},$$

solves for R_2. T_2 may be found from Eq. (47) to be

$$T_2 = (R_2 - R_1) \sin \Delta_2 + R_1 \sin I - T_1 \cos I, \qquad (55)$$

or T_2 may be found from Eq. (52) to be

$$T_2 = \frac{R_2 \operatorname{vers} I - (R_2 - R_1) \operatorname{vers} \Delta_1}{\sin I}, \qquad (56)$$

III. The student may show that if I, T_2, R_2 and Δ_2 are the given quantities, Eq. (54) gives

$$R_1 = R_2 - \frac{R_2 \operatorname{vers} I - T_2 \sin I}{\operatorname{vers} \Delta_1},$$

in which Δ_1 is known because $I = \Delta_1 + \Delta_2$; and that

$$T_1 = R_2 \sin I - T_2 \cos I - (R_2 - R_1) \sin \Delta_1, \qquad (57)$$

or

$$T_1 = \frac{R_2 \operatorname{vers} I - (R_2 - R_1) \operatorname{vers} \Delta_1}{\sin I}. \qquad (58)$$

Examples. 1. At station 782 + 50 a deflection of 60° to the right is made, and at station 777 a 6° 30′ curve begins and extends to station 782. Using station 782 as a P. C. C., what curve will complete the connection with the forward tangent and what will be the station of the P. T.?

2. The result of the foregoing computation will be a fractional degree. Assuming a curve for the second branch that shall approximate the required curve to the nearest half degree, what changes will result? Suggestion: This example means that the two R's are assumed with I and T_1 known, and the student may show from Fig. 41 that

$$\operatorname{vers} \Delta_2 = \frac{T_1 \sin \Delta - R_1 \operatorname{vers} I}{R_2 - R_1}, \qquad (59)$$

and from a new figure, producing curve R_2 instead of curve R_1, that

$$\text{vers } \Delta_1 = \frac{R_2 \text{ vers } I - T_2 \sin I}{R_2 - R_1}. \tag{59a}$$

IV. If a compound curve has been located and the forward tangent is later moved parallel to itself, either out (from the center) or in, the curve may be changed to meet the new tangent in any one of several ways. *a.* It may be moved along the first tangent as a simple curve is moved, a distance $AA' = \frac{k}{\sin I}$, when only the stationing will change, but the whole curve must be rerun on new ground. *b.* If the first branch of the curve lies well, the second branch only may be changed from the P. C. C. as if it were a simple curve, the new R_2 being found by Equation

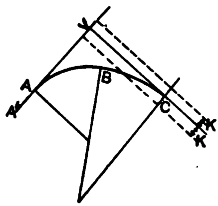

Fig. 42.

$$R'_2 = R_2 \pm \frac{k}{\text{vers } \Delta_2}$$

or

$$R'_1 = R_1 \pm \frac{k}{\text{vers } \Delta_1}$$

according as the curve ends with the larger or smaller radius. *c.* Approximately accomplishing the same result both radii may be retained and the two central angles changed, the P. C. C. being moved back or forward as the case may be. If the proposed tangent lies inside the located one and the curve ends with the longer radius, the P. C. C. will be advanced; if outside, the P. C. C. will be moved back. If the curve ends with the shorter radius, the movement of the P. C. C. will be reversed. The student may make a diagram showing this, and that the following equations give the new values for the final Δ for the several conditions:

$$\text{vers } \Delta'_2 = \text{vers } \Delta_2 \pm \frac{k}{R_2 - R_1} \tag{60}$$

$$\text{vers } \Delta'_1 = \text{vers } \Delta_1 \mp \frac{k}{R_2 - R_1} \tag{60a}$$

V. The problem here presented is not now common but formerly occurred frequently. It is to substitute a three-centered compound curve for a simple curve and between the same two extremities. The curve as shown in Fig. 43 will be symmetrical about a central axis OV, and is a double compound curve in which the two longer radii are equal, as are the two corresponding Δ's. The given quantities on either side of the center are one tangent distance, T_2, and I_1, which is $\frac{1}{2} I$. There being but two quantities given, two may be assumed. Two methods have prevailed in practice: *a*. The first radius R_2 and Δ_2 have been assumed, R_1 and Δ_1 being computed. *b*. The two radii have been assumed.

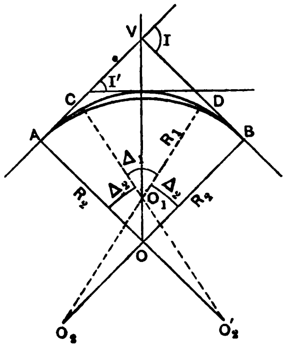

Fig. 43.

With the first method it has been usual to assume one or two stations of 0° 30′ or 1° 00′ curve at the beginning; with the second method a curve of approximately half the degree of the curve to be replaced is assumed for the end and one slightly in excess of the curve to be replaced for the central arc. It is probable that the first is the better method. By the first method R_1 is given by Eq. (54), and if the second method is used Δ_1 is given by Eq. (59). The rest of the solution will be evident.

An independent solution may be had from the triangle O_1OO_2 or $O_1OO'_2$, giving the following equation which the student may derive:

$$R_2 - R_1 = \frac{(R_2 - R) \sin \frac{1}{2} I}{\sin \frac{1}{2} \Delta_1}, \tag{61}$$

in which two of the three unknowns may be assumed and the other found.

The object of this three-centered curve was to ease the transition from tangent to curve, but this result is now much better obtained by introducing "transition" or "easement" curves, often called spirals, since they begin at the tangent with a radius of infinity, which radius is reduced in proportion to the length of the curve to that of the connecting central curve at the ends of the spirals.

Examples. 1. A 6° curve begins at station 762 + 50 and ends at station 770. It is desired to substitute a three-centered curve having one station of 1° 00′ curve at each end. Find the quantities necessary to locate the curve.

2. Let it be required to substitute for the curve of example 1 a three-centered curve of degrees 3 and 6½. Find the quantities necessary to locate the curve.

It should be noted in both examples that the length of line between P. C. and P. T. is changed, and hence not only the stations of P. C. C. are to be found, but also the new station of P. T. If the line has been already located, a "long" station would be introduced at the P. T., so that the station numbering forward of that point need not be disturbed. By a "long" station is meant that a note will be put on the map, on the profile and in the notes stating, for instance, "station 71 to 72 is 102.5 feet," or "From station 72 + 30 to station 73 is 73.36 feet."

VI. It may be found desirable to change the direction of the final tangent of a compound curve without disturbing the curve more than necessary. This is perhaps best done by changing the tangent at the P. T., keeping that point fixed. The angular change α, Fig. 44, will be known, and hence in the triangle $VV'C$ all the angles and one side are known and the new tangent distances AV' and $V'C$ and the angle I' are known for the new curve. The first radius may be retained and the new second radius and central angles computed by the equations already found.

A very neat graphical solution is possible and a discussion of this solution discloses some limitations of the problem.

Whatever compound curve is drawn, if the shorter radius be laid off on the final longer radius from the P. T., say to D in Fig. 45, the remainder of the longer radius will equal the distance between the two centers, and the triangle formed by joining the center O_1 with the point D, Fig. 45, will be isosceles. Therefore, if O_1D be bisected and a perpendicular erected at its mid point, that perpendicular will pass through the second center.

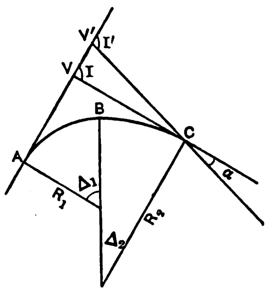

Fig. 44.

Therefore, if $V'C$ be the new tangent, the new center of the second branch of the revised curve must lie on CK drawn normal to $V'C$ at C; and if CD' be made equal to R_1, O_1D' connected, and the distance bisected and a perpendicular erected at its mid point, this perpendicular will pass through the new center O'_2 on CK. O'_2O_1 produced to B' gives the new P. C. C., and AO_1B' and $B'O'_2C$ are the new Δ's.

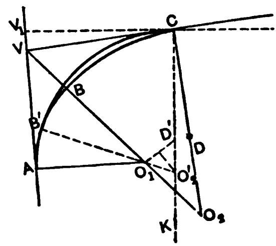

Fig. 45.

If in Fig. 45 the tangent be swung just enough to make the two tangent distances AV' and $V'C$ equal, a simple curve will result, Δ_2 will become I and Δ_1 will vanish. If the tangent is swung still further, R_1 cannot be retained, since it now becomes adjacent to the longer tangent, is consequently R_2 and is not long enough for a practical curve between the fixed points of curve and tangency.

The student should draw figures for such problems, and should also apply the same method of solution to a problem in which it is the tangent adjacent to the arc of shorter radius that is swung. In this case it is the longer radius that is retained, and this must be laid off on the line of the new shorter radius produced. The student may also discuss the limitations for this case.

Examples. 1. Let the final tangent of example 1, Art. 25, be changed so as to increase I by 1°. Retaining the 4° 30′ curve, find the new stations of the P. C. C. and P. T. and the new degree of the second branch of the curve.

2. Let the change in direction be in the beginning tangent and such as to decrease I by 1°.

CHAPTER V.

CANTING THE TRACK ON CURVES.

29. Central Deviating Force required to Cause a Body to move in a Circular Path. — When a body P is revolved about a center C, as when a ball attached to a string is swung, there is a force required to keep the body in its circular path, since when started the body tends at every instant to move straight ahead in the direction in which it is moving at the instant. It is the pull on the string attached to the ball that makes it move in the curve, and the faster the ball swings the harder the pull on the string must be, as may be easily demonstrated experimentally.

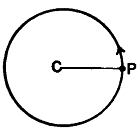

Fig. 46.

It may be shown by mechanics that the central deviating force (force acting toward the center and causing the body to deviate from a straight to a circular path) produces an acceleration toward the center given by $\frac{v^2}{R}$, in which v is the velocity of the moving body and R is the radius of the circular path in which it moves. The force of gravity acting on a falling body produces an acceleration of approximately 32.2 feet per second in each second, and it is also true (Newton's second law of motion) that forces producing varying accelerations on the same body are proportional to the accelerations they produce. Therefore, calling the central force C, the acceleration it produces $\frac{v^2}{R}$, the force of gravity W (the weight of the body) and the acceleration it gives, g, there results

$$\frac{C}{W} = \frac{\frac{v^2}{R}}{g},$$

or

$$C = \frac{Wv^2}{gR}.$$

The force necessary to hold the body in its circular path, therefore, varies with the square of the speed and the weight of the body, and inversely with the radius of the circle.

30. Application to a Train on a Curve. — When a train enters a curve there is no string to make it take the circular path, but if the track is level across, the train is constrained in this path by the pressure of the outer rails on the flanges of the outer wheels of the trucks.

If a train were to take a flat curve at some speed so that a considerable force would be required to hold it in its curved path, two dangers would result: 1. There would be danger that the rails would spread or the wheels climb over the rails because the pressure of the wheels on the rails (the reaction from the pressure of the rails on the wheels — Newton's law of motion) would become very great. 2. An overturning moment would be generated tending to throw the train from the track. This last is explained thus:

In Fig. 47, the car whose center of gravity is at C is supposed to be rounding a curve to the left going from the reader. A pressure P constrains the car to move in the curved path. The inertia of the car may be said to act against this in an equal amount P', and through the center of gravity of the car, which is a distance d above the track. There is therefore the force P' with the leverage d tending to overturn the car, or, stated in terms of mechanics, there is an overturning moment $P \times d$.

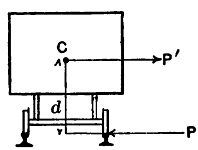

Fig. 47.

It is necessary for safety, therefore, that the deviating force be supplied in some other way than by the pressure of the rails on the wheel flanges, and it is provided by canting the track so that the resultant of the force of gravity acting vertically, and the resistance of the track acting normal to the plane of the track, shall be the necessary unbalanced horizontal deviating force. Thus, in Fig. 48 (car running on a curve to the right away from the reader), W is the weight of the car, force of gravity, R is the resistance of the track, and C the resultant of these two. The track is supposed to be tipped just enough so that C shall be the necessary deviating force. Perhaps it would be better to say that R is the resultant of W and a central force equal and opposite to C — commonly called the centrifugal force. This is the same as saying, as is sometimes said, that the resultant of the weight of the car and the centrifugal force must be normal to the surface through the tops of the rails.

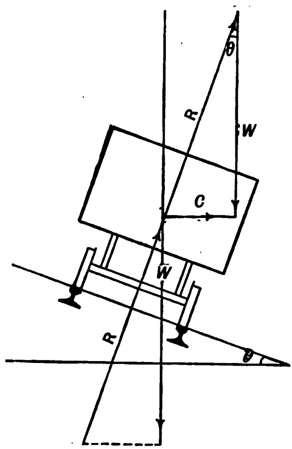

Fig. 48.

31. Derivation of Formula for Difference in Level of Two Rails. — From the relations of the lines in Fig. 48 it will be seen that if θ is the angle of cant of the track,

$$\tan \theta = \frac{C}{W} = \frac{Wv^2}{WgR} = \frac{v^2}{gR}.$$

This is the value of the tangent from the mechanics of the problem.

The track level placed on the rails to determine when the cant is right rests on points approximately the gage plus one rail head apart measured in the inclined plane of the track. It would be better if track levels were so made that the distance determining the difference in level should always be the gage — a fixed quantity — rather than the distance between bearing points on rails, which is a variable depending on the weight, pattern, and wear of the rails. Substituting G for the gage + one rail head of Fig. 49,

$$\tan\theta = \frac{e}{\sqrt{G^2 - e^2}}.$$

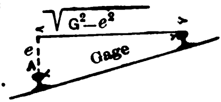

Fig. 49.

This may be called the value of the tangent from the construction of the problem. These two values of the tangent placed equal to one another give

$$\frac{v^2}{gR} = \frac{e}{\sqrt{G^2 - e^2}}.$$

The difference in level of the two rails, or what is commonly called the "elevation of the outer rail" e, is then found by solving the foregoing equation for e, getting

$$e = \frac{v^2 G}{\sqrt{v^4 + g^2 R^2}}.$$

But in this equation v is in feet per second, and it is desirable to speak of railroad speeds in miles per hour, S.

$$v = \frac{5280\,S}{3600},$$

$$v^2 = \left(\frac{5280\,S}{3600}\right)^2.$$

Substituting this value, using 32.16 for g and 4.9 for G = gage + one rail head,

$$e = \frac{4.9}{\sqrt{1 + 223.5\,\frac{R^2}{S^4}}}. \qquad (62)$$

If it be considered that $\sqrt{G^2 - e^2} = G$, which will not be more than $\frac{1}{10}$ of 1 per cent out of the way for an average elevation,

$$\frac{v^2}{gR} = \frac{e}{G},$$

$$e = \frac{v^2 G}{gR}.$$

Converting v into speed in miles per hour gives

$$e = \frac{5280^2 S^2 G}{32.16 \times 3600^2 R}$$

$$= \frac{.3278\, S^2}{R}. \tag{63}$$

The American Railway Engineering and Maintenance of Way Association uses $G = 4.708$, or the exact gage of the track, for the formulas from which its tables are computed. This gives a formula

$$e = \frac{.3149\, S^2}{R}, \tag{64}$$

or an elevation about 4 per cent less than should theoretically be used with usual track methods, but the revised manual states that this is the elevation at the gage line.

In these formulas the elevation e is in feet. Trackmen usually work in feet and inches, and in inches for all measurements less than a foot. Letting $e = \frac{E}{12}$ and substituting $\frac{5730}{D}$ for R, Equations (63) and (64) become

$$E \text{ inches} = 0.000686\, S^2 D, \tag{65}$$
$$E \text{ inches} = 0.00066\, S^2 D, \tag{66}$$

which may be put into words as a

RULE: *The difference of level in inches of the two rails of a curved track of standard gage is from two-thirds to seven-tenths of one-thousandth of the square of the speed of passing trains multiplied by the degree of the curve.*

Which of the two formulas, (65) or (66), should be used depends on the form of the track level used. All level boards known to the author require (65), while accepted tables for the use of trackmen are based on formula (66). When there is nothing to be gained for simplicity or convenience by the use of inconsistent quantities, even though the error be considered practically negligible, it would seem more reasonable to use consistent quantities. Trackmen's tables of elevation of the outer rail are evaluated to the nearest $\frac{1}{8}$ inch, and the difference between the two formulas is as much as $\frac{1}{4}$ inch for the higher speeds requiring maximum allowable elevation. Hence the difference in this case is not negligible and the formula more rigidly conforming to the work, formula (65), should be used. But better still, track levels should be designed to give the difference in level at gage lines and then Equation (66), which is better than (65), may be used.

32. The Practice. — 1. *Speed Assumed.* On single-track roads difficulty is experienced in applying the rule because of the considerable variation in speed of the trains using the track. Freight trains move much slower than passenger trains, neither class of trains moves with the same speed at all points in both directions because of the difference in grades. The difficulty of variation in speed with direction is overcome by double-tracking, and of variation in class of trains largely by four-tracking. There are still some difficulties owing to some differences in speed of various trains of the same class, but these are not so considerable.

On single-track roads recommended practice is to cant the track for the fast passenger trains as a measure of safety. On double-track roads allowance can be made for the probable usual differences in speed at a given curve due to its position with reference to a grade — whether near the top of a steep hill, where it would be taken slower, or near the bottom, where it would probably be taken faster, than if on a level or light grade. Near a station that is a stopping place for all trains the speed will be slower than out on the road, and the canting of the track should be determined accordingly. On four-track roads the best possible arrangement can be made, and the plan that should be followed is to determine the proper cant for each curve on the line and make a table of these for the trackmen. Probably the best way on an operating road is to find the speed of the various fast trains at the several curves and determine the cant accordingly.

2. *Maximum Cant.*— The practice as to the maximum cant that will be used is not uniform, but the American Railway Engineering and Maintenance of Way Association advises an ordinary maximum of 8 inches and that the speed of trains should be regulated accordingly. Substituting 8 for E in formulas (65) and (66), and solving for S, will give the maximum speeds allowable on various curves. Higher speeds are dangerous.

$$S = \frac{108}{\sqrt{D}}, \tag{67}$$

$$S = \frac{110}{\sqrt{D}}. \tag{68}$$

3. *Outer Rail versus Both Rails.* — The foreign practice is to raise the outer rail and lower the inner rail equal amounts, thus keeping the center line at grade. While it has been customary in America to follow this practice on new construction, it has also been customary to keep the inner rail at grade on maintenance, elevating the outer rail through the entire required difference in elevation. This latter practice is recommended by the American Railway Engineering and Maintenance of Way Association, but the author has never been convinced that this is the better way. The fact that it introduces a slight grade at the beginning of the curve may interfere seriously with the movement of heavy freight trains near the top of a maximum grade and makes a longer spiral necessary to comfortable riding of passenger trains. The author knows of no argument for it except convenience of track surfacing, which he thinks is a small matter after the track has been once surfaced. If curve compensation * be increased by the amount of the grade introduced by

* Curve compensation means reducing the grade on which a curve occurs by a determined amount to make the resistance on the curve no greater than on straight track. The subject is discussed in "Elements of Railroad Engineering."

keeping the inner rail at grade, this would be the better method.

33. The Pressure on the Rails. — From Fig. 48 it will be seen that the normal pressure on the rails (which is equal to the resistance of the rails) is more than the weight of the train. The excess is not much, varying from nothing plus to a probable maximum of a little more than one per cent on sharp curves taken at high speed.

Providing the central deviating force by canting the track does not remove all the pressure of the outer rail on the flanges of the outer wheels. Not only must the train be constrained to move in the circular path, but also the direction of its axis must be changed by the central angle of the curve. This is accomplished for each car axis by the trucks acting through their center pins, and for the trucks by the pressure of the outer rail on the flanges of the front outer wheels. The trucks revolving about their center pins must slide across the tops of the rails, and as the contact between wheels and rails is not frictionless, a very considerable pressure between rail and front outer wheel flange is necessary to twist the truck. This pressure is largely independent of speed or degree of curve, and is just that necessary to slide the wheels on the rails. Because the coefficient of friction is less for rapid than for slow motion, and because the rate of sliding is greater for a given speed over a sharp curve than over a flat curve, it is probable that the pressure is less on a sharp than on a flat curve. The wear on the rail is more, because the distance slid is greater per unit of length, and the angle between wheel and rail is greater. This means also that the work necessary to slide the wheels and wear the rails is greater per unit of length but probably less per degree of curve on a sharp curve than on a flat curve. This work is what causes curve resistance, which must be overcome by the locomotive, which, however, is not concerned in producing the central deviating force which is obtained by canting the track.

34. Connecting with the Tangent. — It will be evident that the full deviating force must be provided the instant the train enters the curve; hence at the point of tangency theory requires two different conditions of surface, one for the tangent — level — and one for the curve — inclined. This being practically impossible, it was formerly the custom to begin the canting some distance back on the tangent, bringing it up to its full amount at the beginning of the curve. This practice was not satisfactory, and with medium high speeds there is but one satisfactory method, namely, the introduction of a spiral curve between tangent and main curve, which spiral curve begins on the tangent with an infinite radius (zero degree), the radius varying exactly, or practically, inversely as the length to the radius of the main curve at the point of junction. The track canting may then begin with nothing at the beginning of the spiral, and be increased uniformly with the length of the spiral till the full amount is reached at the junction with the main curve, and be everywhere as nearly as practicable theoretically correct in amount. The run-off from curve to spiral and tangent is of course equally uniform and correct.

Flat curves taken at slow speed are not spiraled, and for these curves the elevation is "run off" on the tangent, experience seeming to show that it may be run out safely at a rate of 1 in 100 to 1 in 150, or 1 inch in from 10 to 12½ feet.

CHAPTER VI.

SPIRALS.

35. Object and Forms of Spirals.— The spiral is introduced between a curve and tangent to provide easy change from straight to circular motion of the train and to provide a correct method for introducing the necessary cant of the track for the curve.

The spiral is of several mathematical forms, but all result in practically the same curve, none of them varying more than a few inches from any of the others, except possibly in the extreme cases rarely met with.

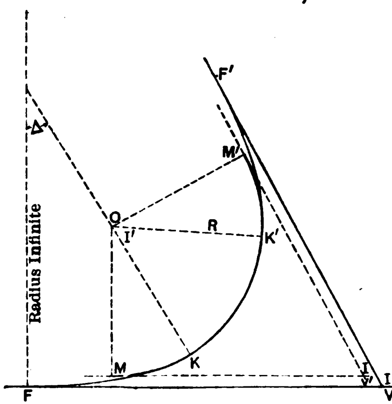

Fig. 50.

The theoretically proper curve is one which begins (say at F, Fig. 50) with an infinite radius (zero degree) which is reduced gradually and uniformly with the increasing length of the curve (*i. e.*, varies inversely as the length of the curve) to that of the main curve at the junction of spiral and curve (say at K, Fig. 50). The discussion of this spiral, though not difficult, involves infinite series, and requires somewhat awkward formulas for determining deflections to points on it when precision is required and the angle consumed by the spiral is fairly large. Very simple approximate formulas may be derived that are sufficiently precise for probably ninety per cent of the cases arising in practice. But the spiral must be laid out by chords as is any simple curve, and when the central angle is large, the chord measurement near the curve end of the spiral does not agree with the curve as precisely as some practice demands, and hence a curve which is developed by chords instead of as a continuous curve has been favored by some engineers. This curve has taken two forms, the compound transition curve, most fully developed by Searles, and the *n*-chord spiral. The compound transition curve of Searles consists of a curve laid out as successive equal chords of curves of uniformly changing radii, *i.e.*, the first chord may be that of a 0° 30′ curve, the second of a 1° curve, etc., each succeeding chord being of a curve of 0° 30′ greater degree than that of the preceding chord. These curves may be made to change by varying amounts per chord, as 0° 30′, 1° 00′, 2° 30′, which are the commonest rates.

The 10-chord spiral has been developed by Mr. Jenks B. Jenkins, of the Baltimore and Ohio Railroad, for the American Railway Engineering and Maintenance of Way Association.

The 10-chord spiral is the locus of a point at the end of a series of consecutive equal lines (chords) making angles with another line (the tangent), varying in the proportion of 1, 7, 19, 37, 61, 91, 127, 169, 217 and 271. This curve approximates closely to the true spiral, but is considered to have the advantage that its ends may be definitely fixed by coördinates having definite finite values. The reason for the numerical proportion given above will appear later.

No attempt will be made to go into the history of the development of the Railroad Spiral, but a few names that have become well known in connection with it may be mentioned. Rankine described the curve briefly and ascribed it to Froude. Wellington developed simple approximate formulas for it that are sufficiently exact for much work. Crandall, Holbrook, Searles, Kellogg, and Talbot are perhaps the American names best known in connection with the spiral, though several others have published discussions of it.

36. Conditions Determining the Spiral. — The particular spiral to be used with any curve may depend on, 1. Topography, *i.e.*, the way the curve and tangents fit the ground. 2. Possible shift in an existing unspiraled track. 3. The speed of trains. The last named condition usually governs and determines the length of the spiral. The length being fixed, the other elements are computed from special spiral equations or tables using the known elements of the curve to be spiraled.

The rate at which it is possible to cant a track, or develop a central deviating force, without discomfort to the passenger, has been the controlling consideration. This rate varies, some persons being much more sensitive to slight variations of normal conditions than others. On the New York Central Railroad, it is estimated that the track can be canted on a tangent approach to a circular curve so that a train will be canted at a rate of about $\frac{3}{8}$ in. per second. It is estimated by the officers of this same road that on a spiral approach, the canting may be much faster. It is entirely probable that the precise rate of cant on a spiral, if the spiral is taken at the speed for which it is designed, is comparatively unimportant, because at this speed the resultant forces are normal to the surface of the track, and if the passenger were at the level of the rails he would have no peculiar sensation. But since he is not at this level, he is raised or lowered or thrown to one side, depending on his position and the method of canting the track, and the car is being twisted.

It is thought that a deficiency in canting produces a more disagreeable sensation than an excess. It is probably true that a continued deficiency of cant on a circular curve produces a much more disagreeable sensation — in that it is lasting — than the momentary sensation due to too rapid canting on a short spiral. This question is an unsettled one and one may follow his own ideas.

From an investigation conducted for the Track Committee of the American Railway Engineering and Maintenance of Way Association, Mr. Jenkins has concluded that a perfectly safe practice, and one that will result in no discomfort to passengers, even if the speed for which the track is canted be exceeded by not more than 15 per cent, is to make the length in feet of spiral per inch of cant not less than $\frac{2}{3}$ of the speed in miles per hour for which the track is canted. Whether or not one agrees with the method of analysis, the conclusion seems to be in accord with the practice on the Pittsburgh and Lake Erie Railroad, the officers of which, including Mr. Holbrook, have studied the matter experimentally for many years.

If r be the allowable rate of canting per second in inches, and E be the inches of total cant to be given, then $\frac{E}{r}$ will be the number of seconds that must be consumed in passing the spiral on which the cant is attained.

If L be the length of the spiral in feet, and V the speed of the train in feet per second, $\frac{L}{V}$ will be the seconds required to pass the spiral which must equal $\frac{E}{r}$, therefore,

$$\frac{L}{V} = \frac{E}{r},$$

$$L = \frac{VE}{r}.$$

Substituting for V its equivalent in miles per hour, S,
$V = \frac{5280\,S}{3600}$.

$$L = \frac{5280}{3600} S \frac{E}{r},$$

and

$$\frac{L}{E} = \frac{5280}{3600} \frac{S}{r} = \frac{22}{15} \frac{S}{r}.$$

$\frac{L}{E}$ is the length of spiral per inch of cant.

To make $\frac{L}{E} = \frac{2}{3}S$, r must be 2.2 inches per second, which is probably entirely allowable on a spiral approach.

It is considered unnecessary to put in a spiral approach where the shifting of the track from where it would be without a spiral would amount to an insignificant fraction of an inch. A practical limit is said to be reached when a curve, by reason of long radius or slow speed of trains over it, requires not more than 2 inches of cant. Curves requiring less cant are frequently spiraled.

Following the rule that makes $\frac{L}{E} = \frac{2}{3}S$ gives shorter spirals for sharp curves than are considered good practice, even though they may be perfectly safe and without discomforting effect. For instance, the maximum cant assumed at 8 inches gives an allowable speed of from 27 to 27.5 miles an hour on a 16° curve, according to the formula used. (See Art. 32.) Two-thirds of 27 is 18, and a cant of 1 inch in 18 feet is considered too rapid, the highest limit considered as good practice being 1 inch in 30 feet. This low limit is said to be not so much on account of the speed with which the curve is canted as because of the rigidity of some cars which prevent the equal distribution of their weight to wheels resting on a warped surface. Three halves of 30 are 45 miles, which, with 8 inches cant, corresponds to a curve of about 6°. Hence, for curves sharper than about 6°, which are to be taken at full speed, the spiral should be not less than $8 \times 30 = 240$ feet long, to be in accord with recommended practice.

If the spirals are to be a part of the permanent alinement, not to be changed with ordinary changes in traffic conditions, then, since as increasing speed requires the lengthening of spirals originally designed for slow speed, they should be designed in the first instance for the fastest speed ever likely to regularly occur on them. Therefore, those curves of less than 6° which may sometime limit the speed of trains, and which therefore may sometime require the maximum cant of 8 inches, should have spirals designed for 8 inches cant though they be longer than necessary for present conditions. For such curves

$$\frac{L}{8} = \frac{2}{3} S,$$

$$L = 5\tfrac{1}{3} S.$$

Substituting for S from equations (67) and (68)

$$L = \frac{576}{\sqrt{D}}, \tag{69}$$

or

$$L = \frac{587}{\sqrt{D}}, \tag{70}$$

according as Eq. (67) or (68) be used for S. If the round number 600 be used for the numerator, the approximation will be on the safe side and will not add materially to the length of the spiral, therefore the value

$$L = \frac{600}{\sqrt{D}}, \tag{71}$$

is suggested for all curves less than 6° likely ever to limit the speed of trains.

For all minor curves not likely to limit the speed of trains, the minimum length of spiral required may be found by

$$L = \tfrac{4}{3} SE \tag{72}$$

or

$$L = 30 E. \tag{73}$$

in which E is in inches, S, miles per hour, and L, in feet. The minimum lengths of spirals are therefore given by the following rules:

For curves of 6° or over, on which the track is canted 8 inches, $L = 240$ feet.

For curves flatter than 6° likely to limit speed, $L = \dfrac{600}{\sqrt{D}}$.

For minor curves not likely to limit speed, $L = \frac{2}{3} SE$ or 30 E.

For a maximum cant of 6 inches,

$$\frac{L}{6} = \frac{2}{3} SD,$$

$$L = 4S,$$

$$L = \frac{432}{\sqrt{D}},$$

or

$$L = \frac{440}{\sqrt{D}}.$$

Six inches is perhaps more generally adopted for maximum cant than is eight inches. If six inches is the maximum cant the minimum spiral length for curves of degrees of over 4° 30′ would be 180 feet.

37. Fundamental Relations. — Referring to Fig. 51, it will be seen that, since the spiral flattens the main curve at its ends, longer tangents will be required for the spiraled curve than for the simple curve, and the radius of the main curve must be shortened or the whole curve thrown in along its middle radius to provide room for the flattening. The space between the tangents and parallel tangents to the smaller radius curve or offsetted curve is designated by O in Fig. 51.

Let the spiral $F\,S\,K$ be a curve, of length L, whose radius of curvature is inversely as its length. Let it connect the tangent FV with the circular curve of radius R and degree $D = \frac{5730}{R}$, which relation of D and R will be considered exact.

By the assumed property of the curve the radius r at any point distant l from F is

$$r = \frac{L}{l} R. \tag{74}$$

The average degree of the spiral is $\frac{D'}{2}$.

The central angle consumed will, therefore, be

$$\Delta = \frac{L}{100} \times \frac{D}{2}. \tag{75}$$

The degree, d, at a distance l from F, is

$$d = \frac{l}{L} D. \tag{76}$$

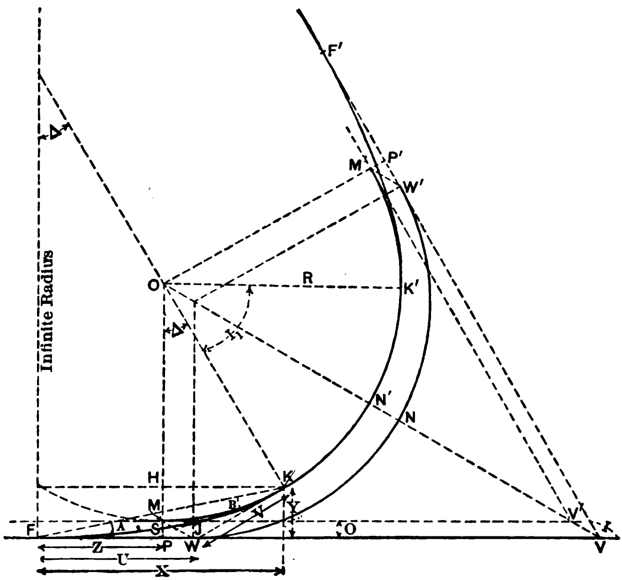

Fig. 51.

The central angle consumed by the length l is

$$\delta = \frac{l}{100} \times \frac{d}{2}. \tag{77}$$

Substituting the value of d from (76),

$$\delta = \frac{l^2 D}{200\, L}, \tag{78}$$

or the central angle varies as the square of the length, and therefore

$$\delta = \frac{l^2}{L^2} \Delta. \tag{79}$$

It is not necessary that the two spirals of any central curve be equal. In general, they will be, and always when the length is determined by speed. When unequal they will have unequal central angles. Therefore, the central angle I' of the spiraled curve will be given by one of the two following equations:

$$I' = I - 2\Delta, \tag{80}$$

$$I' = I - \Delta_1 - \Delta_2. \tag{81}$$

The only reason for making the spirals unlike will be a better fitting of the ground in new location, or in revising alinement under difficult limitations as to movement of tracks. That is to say, the central curve may be shifted around within the tangents to lie most advantageously, and a spiral then fitted to each end, the spiral depending on the offset O. It is desirable to know what this offset is in moving existing unspiraled track so that the shift of the track on the roadbed may be found. It is frequently more expeditious to locate the offsetted simple curve for the full angle I and put in the spiral by offsets from this and the tangents than to locate the spiral by deflection angles.

It is therefore necessary to find expressions for O, L, Δ, deflection angles to the spiral, tangent distances of the spiraled curve, coördinates of the point of curve of the offsetted simple curve referred to axes through the beginning of the spiral, and intermediate offsets from tangent and curve to spiral, in terms of the determined beginning quantity (usually L, sometimes O), and the radius or degree of the connecting central curve.

Examples: Find the proper lengths of spirals and the values of Δ for

1. A 1° curve taken at 60 miles an hour.
2. A 3° curve taken at 60 miles an hour.

3. A 4° curve taken at the highest proper speed for 8 in. cant.

4. A 6° curve taken at the highest proper speed for 8 in. cant.

5. A 10° curve taken at the highest proper speed for 8 in. cant.

6. A 20° curve taken at the highest proper speed for 8 in. cant.

38. The Coördinates of the Spiral. — Let the tangent FV be the axis of X, and the infinite radius at F be the axis of Y. From calculus:

$$dx = dl \cos \delta, \tag{a}$$

$$dy = dl \sin \delta. \tag{b}$$

Expanding cos δ and sin δ in series,

$$\cos \delta = 1 - \frac{\delta^2}{2} + \frac{\delta^4}{24} - \text{etc.}, \tag{c}$$

$$\sin \delta = \delta - \frac{\delta^3}{6} + \frac{\delta^5}{120} - \text{etc.}, \tag{d}$$

in which δ is in arc and equals $\frac{\delta^\circ}{57.3}$.

But $$\delta^\circ = \frac{l^2 D}{200\,L} \quad \text{and} \quad D = \frac{5730}{R}.$$

Substituting these values,

$$\delta \text{ arc} = \frac{l^2}{2\,RL}.$$

Substituting this value in (c) and (d) and placing the results in (a) and (b) and integrating,

$$x = l - \frac{l^5}{40\,R^2L^2} + \frac{l^9}{3456\,R^4L^4} - \text{etc.}, \tag{82}$$

$$y = \frac{l^3}{6\,RL} - \frac{l^7}{336\,R^3L^3} + \frac{l^{11}}{42240\,R^5L^5} - \text{etc.} \tag{83}$$

If X and Y be the coördinates of the point K, l becomes L and

$$X = L - \frac{L^3}{40\,R^2} + \frac{L^5}{3456\,R^4} - \text{etc.,} \tag{84}$$

$$Y = \frac{L^2}{6\,R} - \frac{L^4}{336\,R^3} + \frac{L^6}{42240\,R^5} - \text{etc.} \tag{85}$$

Examples. 1. Find X and Y for the examples of Art. 37, computing each term separately. Tabulate them, using Δ for the argument, and note what terms of each formula are necessary to give the coördinates to a precision of 1 in 100,000.

2. Plat curves on cross-section paper, with values of Δ for abscissas and *corrections* to the *first terms* of the formulas necessary to make those terms correct within 1 in 100,000 for ordinates. Study the results.

39. The Deflection Angle for the Spiral. — The deflection angle from the tangent at F to any point on the spiral is given by

$$\tan a = \frac{y}{x},$$

and substituting in this the values of y and x, in equations (82) and (83), gives

$$\tan a = \frac{l^2}{6\,RL} + \frac{l^6}{840\,R^3L^3} + \frac{13\,l^{10}}{2494800\,R^5L^5} + \text{etc.} \tag{a}$$

Tan $\frac{1}{3}\,\delta$ expressed in a series, giving $\delta°$ its value $\frac{l^2}{2\,RL}$, is

$$\tan \tfrac{1}{3}\,\delta = \frac{l^2}{6\,RL} + \frac{l^6}{648\,R^3L^3} + \frac{l^{10}}{58320\,R^5L^5} + \text{etc.} \tag{b}$$

Comparing (a) and (b) it will be seen that a is almost exactly $\frac{1}{3}\,\delta$, and since δ varies as the square of l, Eq. (78), a varies approximately as the square of l, and if A be the deflection for the whole spiral,

$$\left.\begin{aligned} A &= \tfrac{1}{3}\,\Delta. && (86)\\ a &= \frac{l^2}{L^2}\,A. && (87)\\ a &= \tfrac{1}{3}\,\delta. && (88) \end{aligned}\right\}\ \text{(Approx.)}$$

The relation of (86) is so nearly true that a correction of $.00005\,\Delta^3$ (Δ in degrees, the correction in minutes) will give a result correct to a very few seconds when Δ is 40°, a practically unknown case.

Still more exact expressions are given in the following equations:

$$\left.\begin{array}{lll} & A = \frac{1}{3}\,\Delta - .00005\,\Delta^3 & \text{(correction in minutes).} \\ \text{or} & A = \frac{1}{3}\,\Delta - .003\,\Delta^3 & \text{(correction in seconds).} \\ & \text{more exactly.} & \\ & A = \frac{1}{3}\,\Delta - .00297\,\Delta^3 & \text{(correction in seconds).} \end{array}\right\} \quad (89)$$

$$\left.\begin{array}{lll} & a = \frac{1}{3}\,\delta - .00005\,\delta^3 & \text{(correction in minutes).} \\ \text{or} & a = \frac{1}{3}\,\delta - .003\,\delta^3 & \text{(correction in seconds).} \\ & \text{or more exactly} & \\ & a = \frac{1}{3}\,\delta - .00297\,\delta^3 & \text{(correction in seconds).} \end{array}\right\} \quad (90)$$

Except for the first two examples following, it will be considered in all subsequent discussions that $A = \frac{1}{3}\,\Delta$ and $a = \frac{1}{3}\,\delta$.

Examples. 1. Find the values of A for the examples of Art. 37.

2. From these values of A, find the angles B between long chord of spiral and common tangent at the junction of spiral and central curve.

3. Assuming $A = \frac{1}{3}\,\Delta$, what is B in terms of Δ?

4. Find the numerical values of a to the middle points of the spirals of examples of Art. 37, assuming A and a to be $\frac{1}{3}\,\Delta$ and $\frac{1}{3}\,\delta$, respectively.

5. Suppose a spiral to be divided into 6 equal parts (they will be measured as chords), and that $a = \frac{1}{3}\,\delta$, what is the deflection angle a in seconds expressed in terms of l and D, to the end of the first chord? From the result, and the known law for a, formulate a rule for deflection angles when the spiral is divided into (1) six parts, (2) twelve parts.

6. Let the work of example 5 be performed assuming the spiral divided into ten equal chords, the first deflec-

tion being required in minutes. Note the simplicity of the expression if the chord length, c, is given in stations $= \frac{c}{100} = m$.

40. Coördinates of the P. C. of the Offsetted Simple Curve. The coördinates of the point M of Fig. 51 are required. They are

$$FP = Z = FG - HK,$$

$$Z = X - R \sin \Delta. \tag{91}$$

$$PM = O = KG - HM,$$

$$O = Y - R \operatorname{vers} \Delta \tag{92}$$

Example. Find the values of Z and O for the spirals of Art. 37.

41. Approximate Expressions for *O*. — Since the average degree of the spiral is $\frac{D}{2}$, the length of the spiral to cover an angle Δ must be twice that of the simple curve (measured in chords) to cover the same angle; therefore, it is very nearly true that KM of Fig. 51 equals $\frac{L}{2}$, and KS closely approximates $\frac{L}{2}$, and hence SF.

The spiral is supposed to separate from the simple curve KM at the same rate that it separates from the tangent FP, and hence, since S is approximately at the middle of the spiral, MS is approximately equal to SP, or, approximately, the offset O at P bisects the spiral and is bisected by it. For a large proportion of work this approximation is sufficiently precise for field or office use.

Using only the first term of Eq. (83), it is seen that the tangent offset to the spiral varies as the cube of the length, hence if S is at the middle point, $PS = \frac{1}{8} Y$ and $O = \frac{1}{4} Y$.

A correction found by inspection and experiment gives a sufficiently exact formula for O for any probable value of Δ. The corrected expression is,

$$O = \frac{Y}{4} + \frac{\Delta^3}{160{,}000}. \tag{93}$$

Since the deflection angles vary approximately as l^2 and are nearly $\frac{1}{3}\delta$, the deflection angle at F to S may be considered as $a_s = \frac{1}{4} \times \frac{1}{3}\Delta$, and assuming $FS = \frac{1}{2}L$ and straight, $PS = \frac{1}{2}L \sin \frac{1}{12}\Delta$, and since O is approximately twice PS, $O = L \sin \frac{1}{12}\Delta$. This is not close enough for a wide range of practice, but by experiment and inspection, the following empirical formulas that are sufficiently precise are found:

$$O = \tfrac{1}{6} L \sin \tfrac{1}{2}\Delta \tag{94}$$

$$O = \frac{11}{60} L \sin \frac{5}{11}\Delta. \tag{95}$$

$$O = \frac{L}{10}(\sin \tfrac{1}{2}\Delta + \sin \tfrac{1}{3}\Delta). \tag{96}$$

Equations (94) and (95) are the simplest for logarithmic computation, (94) being sufficiently precise where Δ does not exceed 20°, and (95) being precise enough if Δ does not exceed 40°. Equation (96), devised by Mr. Jenkins, is simplest for use with natural functions, is very readily handled, and is sufficiently precise for all probable values of Δ.

Examples. Find values for O for the examples of Art. 37, by the approximate formulas (94) to (96), and compare with the values found by Eq. (92).

42. Approximate Expressions for Z. For very many cases it is sufficiently exact to consider

$$Z = \frac{L}{2}. \tag{97}$$

When not sufficiently exact, one of the following empirical formulas found by trial will answer:

$$Z = \frac{L}{2}\cos\left(\frac{\Delta}{4} + 0.4\,\Delta \text{ minutes}\right). \tag{98}$$

$$Z = L\left(\frac{1}{2} - \frac{1}{30}\operatorname{vers}\Delta\right). \tag{99}$$

Examples. Find Z for the examples of Art. 37, and compare with the values found from Eq. (91).

43. The Tangent Distance and External Secant. — From Fig. 51 the tangent distance for the spiraled curve is $VF = VW + WP + PF$. $VP = V'M$ = tangent distance for the simple curve with central angle I, or, $R \tan \frac{1}{2} I$. The angle WMP is $\frac{1}{2} I$ and $WP = O \tan \frac{1}{2} I$. PF is Z. Therefore, calling the tangent distance T_s,

$$T_s = (R + O) \tan \tfrac{1}{2} I + Z. \tag{100}$$

The external distance $VN' = VN + NN' = VN + MW$. But VN is the external distance for the simple curve for angle I and MW is $O \sec \frac{1}{2} I$ or $O \operatorname{ex\,sec} \frac{1}{2} I + O$. Therefore, calling the external distance E_s,

$$E_s = (R + O) \operatorname{ex\,sec} \tfrac{1}{2} I + O, \tag{101}$$

or

$$E_s = R \operatorname{ex\,sec} \tfrac{1}{2} I + O \sec \tfrac{1}{2} I. \tag{102}$$

Example. Find T_s for the spirals of the example of Art. 37.

44. Other Functions. – The long chord FK, Fig. 51, is evidently

$$C = \frac{Y}{\sin A}, \tag{103}$$

and an approximate formula derived by Mr. Jenkins and correct to 1 in 1,000,000 is

$$C = L \left(\cos \tfrac{8}{10} \Delta + .004 \operatorname{ex\,sec} \tfrac{3}{4} \Delta\right), \tag{104}$$

which is much simpler than it looks when used with natural functions or even with logarithms.

The spiral tangents may sometimes be desired, though neither the long chord nor these tangents are usually required. The spiral tangents $KJ = V$ and $FJ = U$ are given respectively by

$$V = \frac{Y}{\sin \Delta}, \tag{105}$$

$$V = \frac{C \sin A}{\sin \Delta}, \tag{106}$$

$$U = X - Y \cot \Delta, \tag{107}$$

or
$$U = Y(\cot A - \cot \Delta), \tag{108}$$
$$U = C\frac{\sin(\Delta - A)}{\sin \Delta}. \tag{109}$$

Also, in terms of C, when it is known,
$$X = C \cos A, \tag{110}$$
$$Y = C \sin A, \tag{111}$$
and from (93)
$$Y = 4\,O - \frac{\Delta^3}{40{,}000}. \tag{112}$$

45. Laying Out the Spiraled Curve. — The following nomenclature has been used (referring to Fig. 50 and considering the curve to be located from F to F'):

T. S. (Tangent-Spiral) for the point F.
S. C. (Spiral-Curve) for the point K.
C. S. (Curve-Spiral) for the point K'.
S. T. (Spiral-Tangent) for the point F'.
P. C. for the point M.*
P. T. for the point M'.

The spiral may be laid out by offsets. When this method is used, the procedure is as follows: The point P, Fig. 51, is found, the offset O measured to M, the instrument set over M and with a fore- or back-sight on a point previously set a distance O from the tangent FV, the simple curve is run in for the full angle I, to M', where the operation at PM is reversed to get on to the tangent VF'. Afterwards, when staking out for construction begins, offsets as many as are desired are measured from the tangent to the spiral between F and P and P' and F', and from the curve to the spiral between M and K and K' and M'. It is considered that the spiral departs from the simple curve for its half length adjacent to the curve exactly as it

* Mr. Richard Mansfield Merriman suggests leaving the "P" for "point" out of all notation. The author received this suggestion too late for general use in this volume, but thinks it a good one. P. C. becomes T. C. (tangent curve), P. T. becomes C. T. (curve tangent), and P. C. C. and P. R. C. become C. C. (curve to curve), the context showing whether the curve is compound or reversed. S. S. is used where two spirals join, as they sometimes do.

does from the tangent for the half length adjacent to the tangent, and therefore offsets computed for distance l on the half length FP are used for the half length MK, measuring l from K toward M. It is usually considered that S is midway between P and M, and a stake may be placed at S. The offsets at the quarter points are taken as $\frac{1}{8}$ $P\ S$ or $\frac{1}{16}$ O. (Ordinates vary approximately as l^3.) If it is desired to set a particular station that occurs on the spiral, its offset from tangent or curve is determined by noting its distance from T. S. or S. C. and applying the rule that the offsets vary as l^3. The points T. S., S. C., C. S., and S. T. should be monumented.

The spiraled curve may be laid out by deflection angles, and there are several procedures. The simple curve may be run as described in the last paragraph, the previously determined S. C. and C. S. being set on the curve and the spirals run in by computed deflection angles for equal or unequal chord lengths, the transit being set at the S. C. and the spiral run backward from the T. S., or the transit set at the T. S. and the spiral run backward from the S. C., or the transit set at the T. S. and the spiral run forward to the S. C. Or the spiral may be run from T. S. to S. C. by deflection angles, the central curve from S. C. to C. S., and the second spiral from C. S. to S. T. by deflection angles. This last is the more direct method, but the author's experience leads him to favor the location by offsets as likely to prove quicker and freer from accidental errors carried beyond the curve.

Another method is to establish the P. I. of the spirals at the intersection of the tangents U and V, set the instrument at the P. I., turn the angle Δ from the tangent, and establish the S. C. and C. S. by measuring V. Spirals and simple curves may then be run by deflection angles with the instrument at the S. C. and C. S. The student may make a program for an assumed case.

When the deflection angle method is used, it is advised that the simple curve be run in as for the offset method, the S. C. and C. S. being set, and the spirals subsequently run in from T. S. to S. C., with the transit at the S. C. To know how to do this, it is necessary to discuss deflection angles measured at the S. C. to points on the spiral.

46. Deflection Angles at the S. C. — The deflection angle from the tangent for a length l measured from F, Fig. 52, is $\frac{l^2D}{600\,L}$, and since the spiral is assumed to depart from the central curve at K just as it does from the tangent at F, the deflection angle from the central curve to the spiral, for a length l measured from K, is also $\frac{l^2D}{600\,L}$.

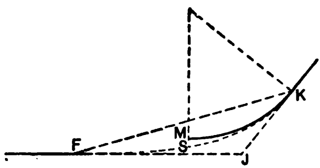

Fig. 52.

The deflection angle from the tangent KJ to a point on the simple curve distant l from K is $\frac{lD}{200}$; therefore, the deflection angle from the tangent KJ to a point on the spiral distant l from K is approximately

$$\beta = \frac{lD}{200} - \frac{l^2D}{600\,L}. \qquad (113)$$

It has been already shown in the examples that the angle B, tangent to long chord at the S. C., is

$$B = \tfrac{2}{3}\,\Delta \text{ (approx.)},$$

and substituting L for l in Eq. (113) gives, simply for illustration,

$$B = \frac{LD}{200} - \frac{LD}{600} = \frac{2\,LD}{600},$$

or

$$B = \frac{2}{3}\,\frac{LD}{200} = \frac{2}{3}\,\Delta.$$

The deflection angle from the tangent at the S. C. to any point on the spiral distant l from the S. C. may be found, in practically all cases likely to arise, from Eq. (113).

But when the spiral is divided into a number of equal chords, it is simpler to compute the deflection angle to the chord points from the long chord, and approximate expressions for these angles will be developed.

If the spiral be divided into n equal chords of length c, then $n = \frac{L}{c}$ and the deflections to the several chord points from the tangent at the T. S. are given by

$$\left.\begin{aligned} a_1 &= \frac{c^2D}{600\,L} = \frac{cD}{600\,n}, \\ a_2 &= \frac{4\,cD}{600\,n}, \\ a_3 &= \frac{9\,cD}{600\,n}, \\ &\cdots\cdots \\ a_n &= \frac{n^2cD}{600\,n} = \frac{ncD}{600} = \frac{LD}{600} = \frac{\Delta}{3}. \end{aligned}\right\} \qquad (114)$$

Referring now to Eqs. (113) and (114), the deflection from the tangent at the S. C. to the first chord point on the spiral from the T. S. would be, since the distance from the S. C. will be $(n-1)\,c$,

$$\begin{aligned} \beta_1 &= \frac{(n-1)\,cD}{200} - \frac{(n-1)^2\,cD}{600\,n} \\ &= \frac{3\,n\,(n-1)\,cD}{600\,n} - \frac{(n-1)^2\,cD}{600\,n} \\ &= a_1\,(3\,n\,(n-1) - (n-1)^2) \\ &= a_1\,(2\,n^2 - n - 1). \end{aligned}$$

But
$$\begin{aligned} \beta_0 = B \text{ is } & \frac{ncD}{200} - \frac{n^2cD}{600\,n} \\ &= \frac{3\,n^2cD}{600\,n} - \frac{n^2cD}{600\,n} \\ &= \frac{2\,n^2LD}{600\,n} \\ &= 2\,n^2a_1. \end{aligned}$$

Subtracting β_1 from β_0 gives

$$\begin{aligned} \beta_0 - \beta_1 &= a_1\,(2\,n^2 - (2\,n^2 - n - 1)), \\ &= a_1\,(n+1). \end{aligned}$$

Proceeding similarly for the angle $\beta_1 - \beta_2$, etc., we find for the series

$$\left.\begin{array}{l} \beta_0 - \beta_1 = (n+1)\,a_1, \\ \beta_1 - \beta_2 = (n+3)\,a_1, \\ \beta_2 - \beta_3 = (n+5)\,a_1, \text{ etc.} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \beta_{n-1} - \beta_n = (3n-1)\,a_1, \end{array}\right\} \qquad (115)$$

and adding these to get the deflection angle from the long chord to the successive chord points,

$$\left.\begin{array}{l} b_1 = (n+1)\,a_1, \\ b_2 = (2n+4)\,a_1, \\ b_3 = (3n+9)\,a_1, \text{ etc.} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ b_n = B = 2n^2a_1. \end{array}\right\} \qquad (116)$$

Example. Find the deflection angles in terms of a_1, from the long chord at the S. C. to the chord points on the spiral, assuming a division into, *a*, 6 equal chords; *b*, 10 equal chords.

47. Deflection Angles at any Point on the Spiral. — It may be necessary to locate the spiral in more than one section, setting the instrument on one or more intermediate points. The following discussion will indicate how to proceed.

The degree, d, of the spiral at any point distant l from the T. S. is $\frac{lD}{L}$, and from this point the spiral departs from the curve of degree $\frac{l}{L}D$, just as it does from the tangent or from the simple curve at the S. C. Between the point and the T. S. the spiral lies outside the curve $\frac{l}{L}D$, and between the point and the S. C. the spiral lies inside the curve. Therefore, assuming the transit at the point l from the T. S. back-sighted on the T. S., the deflection to the tangent at the point is $2a$, a being the deflection from tangent at the T. S. to the point, and the deflection to any point l' beyond the point would be found by adding the spiral tangent deflection for a length l' to the deflection for the same length of curve $\frac{l}{L} D$,

$$a' = \frac{l'}{100} \frac{l}{L} \frac{D}{2} + \frac{l'^2 D}{600 L}. \tag{117}$$

If the spiral be divided into n equal chords of c feet each the degree of the spiral at the end of the several chords will be respectively (approximately)

$$\left.\begin{aligned} d_1 &= \frac{D}{n}, \\ d_2 &= \frac{2D}{n}, \quad \text{etc.} \\ &\cdot \quad \cdot \quad \cdot \\ d_n &= D, \end{aligned}\right\} \tag{118}$$

If the transit be set at the end of the sth chord, the degree of spiral at that point is $\frac{sD}{n}$, the deflection at the point from the long chord for that point to the tangent at the point is $\frac{2 s^2 cD}{600 n}$ or $2 s^2 a_1$, and from this tangent to the following chord points in succession,

$$\left.\begin{aligned} a'_{s+1} &= \frac{sDc}{2n\,100} + \frac{cD}{600 n} \\ &= \frac{sDc}{2n\,100} + a_1 = 3 s a_1 + a_1 = a_1 (3s + 1) \\ a'_{s+2} &= \frac{2 sDc}{2n\,100} + 4a_1 = a_1 (6s + 4) = 2 a_1 (3s + 2) \\ a'_{s+3} &= \frac{3 sDc}{2n\,100} + 9a_1 = a_1 (9s + 9) = 3 a_1 (3s + 3) \\ a'_{s+4} &= \qquad 4 a_1 (3s + 4), \text{etc.} \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ a'_n &= \frac{(n-s) sDc}{2n\,100} + (n-s)^2 a_1 \\ &= (n-s)(3s + n - s) a_1. \end{aligned}\right\} \tag{119}$$

The nth point is the S. C.

Deflections from the tangent at the point occupied to points on the spiral between the T. S. and the point occupied are found by subtracting the second terms of the foregoing formulas, thus:

$$\left.\begin{aligned} a'_{s-1} &= \frac{sDc}{2n\,100} - a_1 = a_1(3s-1) \\ a'_{s-2} &= \frac{2\,sDc}{2n\,100} - 4\,a_1 = 2\,a_1(3s-2) \\ a'_{s-s} &= \frac{s^2Dc}{2n\,100} - s^2a_1 = 2\,s^2a_1. \end{aligned}\right\} \quad (120)$$

Examples. 1. Assuming $D = 4°$, $L = 300$ ft., and the T. S. at Sta. 364 + 57.35, find the deflections to stations 365, 366, and 367.

2. Assume the spiral of Example 1 to be divided into 6 chords. (*a*) Find the deflection angle to each chord point; (*b*) Suppose the transit set on the middle point; find the deflection to the following points up to the S. C.; (*c*) Find the deflections to the preceding points.

For the two following examples the deflection angles are to be taken as proportional to the square of the chord number, and the angles are to be determined in terms of the first chord angle a_1, which is to be found in n and Δ. It must be remembered that the assumption here made is an approximation.

3. Assuming any spiral divided into 6 chords, find the angles made by each chord (produced where necessary) with the main tangent to the spiraled curve.

4. Assuming any spiral divided into 10 chords, find the angles made by each chord (produced when necessary) with the main tangent to the spiraled curve.

5. Construct a table showing multiples of the first chord deflection that are to be used with the transit at each successive chord point. Make the table for 20 chords and arrange it as follows:

Instrument at	Factors by which first chord deflections are multiplied to give deflections to chord points numbered							
	0	1	2	3	4	5	6	etc.
0	0	1	4	9	16	25	36	etc.
1	2	0	4	10	18	28	40	etc.
2	8	5	0	7	16	27	40	
3	18	14	8	0				
4	32	27	20	etc.				
5	50	44	36					
6	72	65	56					
etc.								

After the table is started note the simple arithmetical work necessary to continue it.

48. General Examples in the Use of the Spiral. — 1. The 4° curve of example 4 of Art. 37 is to be located between tangents, making an angle of 54° at temporary station 476. Find all the quantities necessary to lay out the curve by offsets, locating any stations that may come on the spirals.

For the following examples, use the data of Ex. 1.

2. Find the quantities necessary to locate the curve, using deflection angles and dividing the spiral into 6 equal chords.

3. Find the quantities necessary to locate the curve, using deflection angles, and dividing the spiral into 10 equal chords.

4. Suppose it necessary to set the transit at the fourth chord point, find the necessary deflection angles to the points beyond, using (*a*) 6 chords, and (*b*) 10 chords.

5. Find the deflection angles supposing the spirals to be wholly located from the S. C. and C. S.

49. The Chord Spiral. — As the spiral is laid out by chords, when deflection angles are used, and as the theory has been developed for the curve measured on the arc, it has been suggested that the development would better accord with the practice if it were by chords, and Mr. Jenkins' 10-chord spiral has been developed in this way. The development is such that practically all the equations except those for x and y and tangent A that have been developed for the spiral may be used for the 10-chord spiral, with perhaps even greater precision in some cases than for the true spiral, because they are really approximate expressions for the true spiral based on chord measurements. Finding that the angles of successive equal chords of the true spiral with the tangent are approximately as found in examples 3 and 4, Art. 47, Mr. Jenkins conceived a curve which is the locus of the end of 10 equal chord lines which make exactly the angles with the tangent found to be approximate angles for the true spiral.

By the examples mentioned, it will already have appeared that a series of equal chords closely approximating the spiral will make angles with the tangent of, respectively, beginning at the T. S., 1, 7, 19, 37, 61, 91, 127, 169, 217, 271, etc. times a_1. But

$$a_1 \text{ is } \frac{1}{n^2} \times \frac{\Delta}{3} \text{ and for } n = 10.$$

$$a_1 = \frac{\Delta}{300}.$$

Therefore,

$$X = \frac{L}{10}\left(\cos\frac{\Delta}{300} + \cos\frac{7\,\Delta}{300} + \text{etc.} \ldots \cos\frac{271\,\Delta}{300}\right). \quad (121)$$

$$Y = \frac{L}{10}\left(\sin\frac{\Delta}{300} + \sin\frac{7\,\Delta}{300} + \text{etc.} \ldots \frac{271\,\Delta}{300}\right). \quad (122)$$

$$\tan A = \frac{Y}{X}. \quad (123)$$

The foregoing assumes that the degree of curve of the spiral at any chord point s from the S. T. is $\frac{s}{n} D$. As the curve that would be laid would be continuous and pass through these chord points, it would not be theoretically a curve of uniformly varying radius or degree, but it would approach as near to it as it is possible to have it by any practicable method, and for probably ninety-nine per cent of the cases in practice the difference between chord and arc measurements for the spiral is negligible.

The advantage of dividing the spiral into 10 or 20 chords, rather than 6, 12, or 18, is not apparent to the author.

If the spiral is divided into 6 equal chords, the deflection angle for the first chord has been shown by a previous example to be as many seconds as there are feet in the chord times the degree of the central curve, and the deflections to succeeding chord points are 4, 9, 16, 25, and 36 times the first deflection. This rule is easy to remember, and nothing but mental computations are required for the deflections. If 12 chords are used, the first deflection is $\frac{1}{4}$ that when 6 chords are used, and when 18 chords are necessary, the deflection is $\frac{1}{9}$ that for the first of 6 chords.

For this simplicity, when deflection angles are to be used, the author advises the selection of L to be the length divisible by 6, 12, or 18, that most nearly corresponds to the requirements of Eq. (71), (72), or (73), whichever is used.

No matter what the number of chords, it is always approximately true that the first deflection angle, a_1, is $\frac{\Delta}{3 n^2}$, and the several angles made by the chords with the tangent are 1, 7, 19, 37, 61, 91, 127, 169, 217, 271, etc., times a_1 as may be shown thus:

The angle between the long chord to the sth point and the tangent at that point is twice the angle at the P. S. The degree of curve at the sth point is $\frac{s}{n}D$ and the deflection angle for a length c of this curve is $\frac{c}{100} \times \frac{s}{n} \times \frac{D}{2}$, or, since $c = \frac{L}{n}$ and $\frac{LD}{200} = \Delta$ and $a_1 = \frac{\Delta}{3n^2}$, the deflection angle is $3sa_1$. The angle θ between the tangent and sth chord is the difference between $3sa_1$ and a_1, hence the angle between the long chord and the sth chord is

$$2s^2a_1 - 3sa_1 + a_1$$

and the angle ϕ is the sum of s^2a_1 and this last angle,

or
$$\phi = 3s^2a_1 - 3sa_1 + a_1$$
$$= a_1(3s^2 - 3s + 1).$$

This is an equation of the second degree and the second difference is a constant and is $6a_1$. Therefore by adding second differences we have

$s =$	1,	2,	3,	4,	5,	6,	etc.
$\phi =$	a_1,	$7a_1$,	$19a_1$,	$37a_1$,	$61a_1$,	$91a_1$,	etc.
1st diff. =		6	12	18	24	30	36 etc.
2d diff. =			6	6	6	6	6 etc.

If n is 6, n^2 is 36, and reducing Δ to seconds gives

$$a_{1\text{ seconds}} = \frac{3600\Delta}{3 \times 36} = \frac{100\Delta}{3} = \frac{100\,LD}{3 \times 200} = \frac{LD}{6}.$$

But $\frac{L}{6} = c$, the length of the chord, hence, in seconds,

$$a_1 = 100A \text{ or } cD, \tag{124}$$

either of which is easily remembered and computed.

When the spiral is divided into 10 chords and the chord length is reduced to stations, m, the first deflection has been shown by previous examples to be $a_1 = MD$ in minutes, which is as easy to remember as $a_1 = cD$. As the transitman deals with minutes on his instrument the division into 10 chords is also convenient.

As Δ is readily found when L and D are known, it is desirable that tables of spiral quantities be given in terms of Δ and L.*

50. Spirals for Compound Curves.—Spirals uniting the different branches of a compound curve are computed practically as those for simple curves. They will be required when the change in degree at the P.C.C. is such as to make a change of 2 inches or more in the cant. When the change is less than 2 inches, it is run out on the arc of longer radius just as it is on the tangent of a simple curve. When the change is more than 2 inches the length of spiral is computed by Eq. (71), (72), or (73), as desired, using the *difference* in degrees for the D of Eq. (71), and the *change* of elevation for the E of Eqs. (72) and (73).

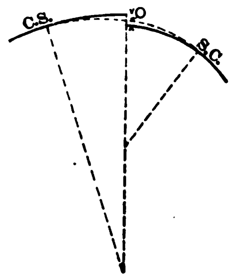

Fig. 53.

Assuming that the spiral runs from the C. S. to the S. C. in Fig. 53, it may be considered a portion of a spiral beginning on a tangent somewhere back of the C. S., the S. C. being the S. C. of such a spiral ending with degree D_1, that of the shorter radius curve. If so considered, and if

L = length of spiral C. S. to S. C.,
n = number of chords into which it is divided,
c = length of each chord,
L_1 = length of spiral from S. C. to T. S.,
n_1 = number of chords into which L_1 is divided, then

$$\frac{L_1}{L} = \frac{D_1}{D_1 - D} \quad \text{or} \quad L_1 = \frac{D_1 L}{D_1 - D},$$

$$\begin{aligned} n_1 c &= L_1 \\ nc &= L \end{aligned} \qquad \text{Dividing} \qquad \frac{n_1}{n} = \frac{L_1}{L}.$$

$$n_1 = \frac{nL_1}{L}.$$

The point C. S. would then be the sth chord point of

* Such tables will be found in the field book to follow this volume.

the spiral, T. S. – S. C., and s is $n_1 - n$. Therefore, from the tangent at C. S. the deflection to the several chord points on the spiral may be computed by the methods of Art. 47. If this spiral is considered, the functions would all be computed for D_1, L_1, c and n_1, the laying out of the spiral beginning at the sth chord point, *i.e.*, the C. S. of Fig. 53. The field procedure would then be to locate the arc of longer radius to the C. S., then the spiral by deflection angles to the S. C., and then the second branch to the C. S. of the ending spiral, or to the P. T. of the second branch.

But a simpler method is to consider the spiral between the two branches as a complete spiral connecting a tangent with a curve of degree $D_1 - D$, computing all functions accordingly. The distance Z (measured on the arc of greater radius), the offset O, and other elements will be thus given with no error of consequence. The two branches of the curve may then be located up to the P.C.C. with the offset O between them, and the spiral laid out by any method desired, the method by offsets being recommended.

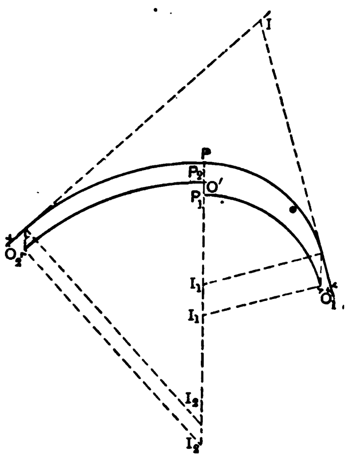

Fig. 54.

The change in degree at the P.C.C. of a compound curve is never as great as the change from tangent to curve of maximum degree and is rarely such as to require more than the simplest of the approximate formulas that have been developed.

Considering the whole spiraled curve, and referring to Fig. 54, the compound curve is supposed connected first directly to the tangents. To introduce the spirals, the arc of shortest radius must be thrown in along the common radius line, a distance PP_1 necessary to produce the required offset O_1 for the proper length of spiral L_1.

$$PP_1 = O_1 \sec I_1.$$

The arc of longer radius must be thrown in along the same line a distance PP_2 necessary to produce the required O_2 for the proper length of spiral L_2.

$$PP_2 = O_2 \sec I_2.$$

$PP_1 - PP_2$ must at least equal O', the proper offset for the connecting spiral L'. Therefore, after computing tangent distance for the curve connected directly to the tangents, L_2 is found and from this O_2 and PP_2; to the latter O', previously found, is added for PP_1, and from this a trial value of O_1 is determined. If found to be larger than necessary, no harm is done, and it cannot well be reduced. If found too small, O_2 and PP_2 may be increased, or O' may be increased enough to make O_1 correspond to the computed proper quantity. If O_2 is increased, a new L_2 will result longer than required, but not, therefore, undesirable, unless the shift in the line is too much, in which case other methods may be employed, as indicated in Art. 51.

Example. The following are notes of a temporary compound curve:

Sta. 768 + 50	P. C.	8° *R.*
Sta. 775	P. C. C.	4° *R.*
Sta. 783 + 50	P. T.	

Change the notes to conform to a spiraled curve for a maximum speed for the 8° curve, determining the stations of T. S., S. C_1., C_1. S., S. C_2., C_2 S. and S. T.

Formulate the procedure for laying out the curve.

51. Spiraling Existing Track. — There are several ways of converting an existing simple curve and tangents into a spiraled curve and tangents.

1. The radius may be shortened by the computed O for the determined length L, keeping the same center. The shift of the track will then be O throughout the central curve, and from this to nothing at the T.S. and S.T.

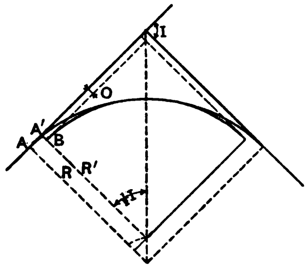

Fig. 55.

This method usually causes too much shift, and shortens the track, requiring the cutting of a rail when O is a considerable quantity. The cutting of a rail is not a serious matter and can rarely be avoided if a spiral of much length is introduced.

2. A better way is to maintain the same vertex of the curve, and shorten the radius by

$$R - R' = \frac{O}{\text{vers } \frac{1}{2} I}. \tag{125}$$

The greatest shift of track will be a little less than $\frac{1}{2}O$ and the tangent points will be moved toward the P. I. by a distance AA' Fig. 55.

$$t = (R - R') \sin \tfrac{1}{2} I. \tag{126}$$

3. Still less maximum shift at one point will occur if the middle point of the curve is moved outward along the central line about $\frac{1}{4}O$ before the radius is changed. The student may show that the new radius must then be

$$R' = R - \frac{\frac{1}{4} O \cos \frac{1}{2} I + O}{\text{vers } \frac{1}{2} I}, \tag{127}$$

and that the movement of the tangent points toward the P. I. is

$$t = (R - R' + \tfrac{1}{4} O) \sin \tfrac{1}{2} I. \tag{128}$$

When the existing curve is a compound curve, and the track is to be shifted as little as possible, the following method may be used: Referring to Fig. 56, find the O' of the connecting spiral; conceive the arc of larger radius shifted outward along the common radius line, and the arc of smaller radius inward along the same line, each by half O'. The tangent point of the larger radius curve will be moved outside its tangent by an amount $\frac{1}{2} O' \cos I_2$,

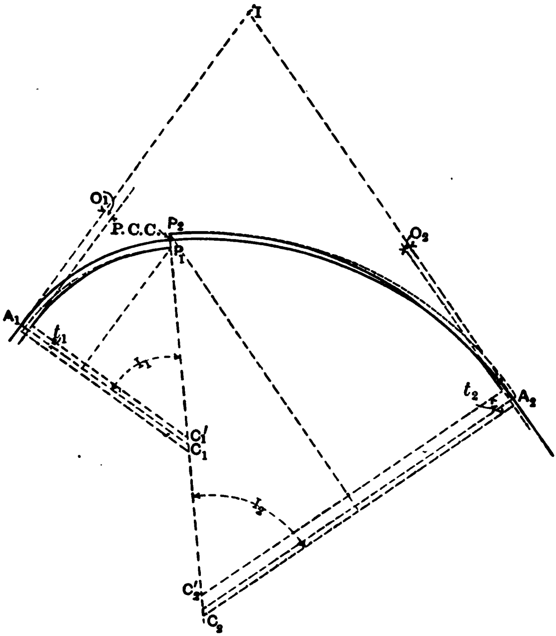

Fig. 56.

and the smaller radius curve will be moved inside its tangent by $\frac{1}{2} O' \cos I_1$.

Find a new radius for R_2 such that

$$R'_2 = R_2 - \frac{\frac{1}{2} O' \cos I_2 + O_2}{\text{vers } I_2}, \tag{129}$$

and for R_1 a new value,

$$R'_1 = R_1 - \frac{O_1 - \frac{1}{2} O' \cos I_1}{\text{vers } I_1}. \tag{130}$$

The movements of the tangent points toward the vertex are

$$t_2 = (R_2 - R'_2 + \tfrac{1}{2}\,O')\sin I_2, \qquad (131)$$

$$t_1 = (R_1 - R_1' - \tfrac{1}{2}\,O')\sin I_1. \qquad (132)$$

The extremities of the two arcs being thus located, the curve may be run in by any method already given.

Slightly less maximum shift of track at any one point may be had by shifting the P. C. C. a small amount outward along the common radius line, say, $\frac{1}{2}\,O'$. The student may work out the remaining procedure.

CHAPTER VII.

RIGHT OF WAY DESCRIPTION.

52. General Statement. — Right of way descriptions are frequently difficult to prepare and the surveys may be difficult to make. Many of the older lines have had very imperfect descriptions and have suffered from many encroachments of adjoining owners, who by long undisturbed possession have gained title to property that later becomes necessary to the purposes of the railroad company, which may find itself obliged to purchase a part of its property a second time.

The surveys and the writing of the descriptions require the utmost care, and descriptions should never be prepared by a right of way agent, unless he has a good knowledge of surveying, never by an attorney, but always by a careful and methodical engineer or surveyor.

The student being supposed to be familiar with ordinary surveying methods, this matter will be treated by giving a few illustrative examples.

53. The Taking is a Parallelogram. — Let it be supposed that the located center line passes through the lands of George Brown which lie between those of Calvin Jones and Peter Smith, and that the line is straight. Let it be supposed that the right of way is 100 feet wide, 50 feet on either side of the center line. For a single track road this is usually the case, but it may occur that a single track is first located with the expectation that a second track will soon follow, when it is well to have an equal space on the two sides of the center line between the two tracks, the first located line lying about 6½ feet (for 13 feet center to center) to one side of the center line of the right of way strip. Let it be first supposed that the line lies in that part of the United States covered by the public land surveys, and is in Section 11, of Township 6 N. R. 3 W. of the 5th Principal Meridian. A loose description very likely to have been written some years ago would be:

" A strip of land 100 feet wide, 50 feet on either side of the located line of the A.B.C. R.R. where the same passes through the land of the party of the first part, containing —— acres, more or less."

This is a wholly bad description. Some engineers would not say wholly bad because they believe the located line

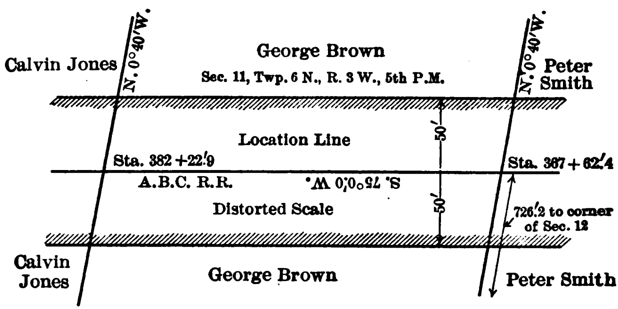

Fig. 57.

to be a more permanent and certain monument and mark than any ordinary land monument or line likely to be found. And when the property is farm land of little value, perhaps this contention is correct. Others would say, "the line will vary a little from time to time"; this is a good description, precisely because it is indefinite and the company may shift the line a little from time to time, the right of way shifting with the shift of the track. But it may be fairly said that it is desirable from the standpoint of both parties that the description be definite and such that the property line can be re-run at any future time with no uncertainty. It is of particular value to have these records complete and definite near and through cities where the land already is or is becoming of considerable value. A better description would be as follows:

A strip of land 100 feet wide, 50 feet on either side of the located center line of the A. B. C. R. R. where the same passes through the lands of the party of the first part; the said located center line being described as follows: to wit — Beginning at the intersection of the located center line of the A.B.C. R.R. with the easterly line of section 11, Twp. 6 N. R. 3 W. 5th Prin. Merid., distant northerly on said section line 726.2 feet from the S.E. corner of said section 11, and running thence S. 75° W., true bearing, 1460.5 feet more or less to the intersection of said located center line with the westerly line of the S. E. ¼ of the S. E. ¼ of the said section 11; containing —— acres more or less.

This description gives a permanent starting point fixed by a reference to an established public survey line.

If the property does not lie in that part of the country covered by the public land surveys, then, adopting the same figures so far as they are applicable, the description might be as follows:

A strip of land 100 feet wide, 50 feet on either side of the located center line of the A.B.C. R.R. where the same crosses the lands of the party of the first part, the description of said center line being as follows, to wit: Beginning at a point on the line between the lands of Peter Smith and George Brown, distant on said line northerly 726.2 feet from the southeast corner of the lands of the said George Brown, which southeast corner is marked by a stone set in the ground and from which a hard maple tree 12 inches in diameter bears N. 26° W. 25 feet, and a chestnut tree 15 inches in diameter bears S. 42° W. 53 feet; and running thence S. 75° W., true bearing, 1460.5 feet more or less to the line between the lands of George Brown and Calvin Jones, intersecting said line at a point distant southerly thereon 478.3 feet from a corner stone in said line, from which corner stone the N. E. corner of the house of the said George Brown bears S. 36° 40′ E. 176.4 feet.

(This corner would be better defined if there was another reference point.) The said strip of land contains —— acres more or less.

54. The Taking is Irregular. — If the line had crossed at a somewhat sharper angle in the preceding case the strip would not be quite regular. The damage to the small triangular piece of section 11 cut off from the rest of the section may be considered so great that the company may purchase it with the rest, in which case the description

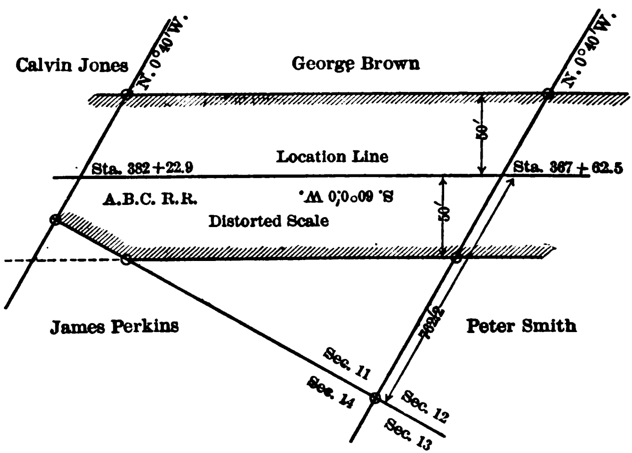

Fig. 58.

would be fairly simple, being: all that portion of the S. E. ¼ of the S.E. ¼ of Sect. 11, Twp. 6 N. R. 3 W., etc., lying south of a line parallel with and 50 feet measured at right angles northerly from the located center line, etc., . . . followed by a description of the line and a statement of acreage.

If the triangular piece is not to be purchased the description will be by " metes and bounds," thus: Beginning at a point on the easterly line of Sec. 11, Twp. 6 N. R. 3 W. of the 5th Principal Meridian, where the said section line is crossed by the located center line of the A.B.C. Railroad, which point is distant northerly on said section line 762.2 feet from the southeast corner of said section 11, and running thence S. 0° 40′ E. along said section line 57.3 feet more or less to a point 50 feet distant southerly and at right angles from the said located center line of said A.B.C. Railroad; thence S. 60° W. parallel with said located center line 1409.7 feet more or less to the southerly line of said section 11; thence due west along said southerly line of said section 11, 44.2 feet more or less to the westerly line of the S.E. ¼ of the S.E. ¼ of the said section 11, being the westerly line of the lands of the party of the first part; thence N. 0° 40′ W. along said westerly line 89.3 feet more or less to a point distant 50 feet northerly at a right angle from the said located center line of the said A.B.C. Railroad; thence N. 60° E. parallel with said located center line 1460.4 feet more or less to the easterly line of said section 11; thence S. 0° 40′ E. along said easterly section line 57.3 feet more or less to the point of beginning, containing —— acres more or less.

A description of an irregular piece of land not a part of the surveyed public lands would be similar to the foregoing, the boundary lines between individual owners, and the corners in those boundaries, taking the place of section lines, fractional section lines, and section and fractional section corners.

55. The Description involves a Curve. — A. *The strip is of uniform width, both sides being parallel with the center line of the railroad.* If the strip has parallel sides it may be described as such a strip with a description of the center line as a base tied to some existing monument. Thus in the case shown in the figure: A strip of land 100 feet wide 50 feet on either side of the located center line of the A.B.C. Railroad where the same crosses the lands of the party of the first part, and bounded on its westerly and southerly ends by the westerly and southerly boundary lines respectively of the lands of the party of the first part, containing

—— acres more or less. Said center line through said property is described as follows, to wit: — Beginning at a point in the easterly line of the lands of George Brown distant northerly on said easterly line 362.3 feet from the southeasterly corner of said property, and running thence N. 89° 42′ W. 337.5 feet; thence by a spiral curve to the left 300 feet through an angle of 12°; thence by a curve to the left, of 716.78 feet radius, 262.4 feet more or less measured by chords of 78.3 feet, 100 feet, and 84.1 feet more or less to the intersection of said located center line with the southern boundary line of the lands of the said George Brown.

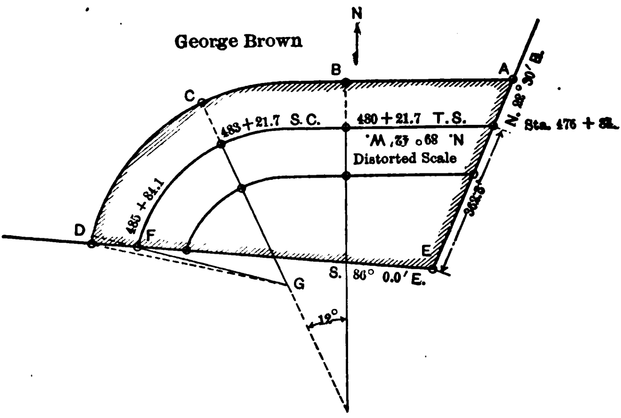

Fig. 59.

B. *The strip is not of uniform width.* One side may be parallel with the railroad line or neither side may be. In the foregoing case let it be supposed that the entire corner cut off from Mr. Brown's property by the railroad is to be taken as right of way or for station grounds or other purposes. The description will be by metes and bounds. The quantities to be determined before the description can be written will be: 1. The cross half width on the westerly boundary line, which is 50 sec. 22° 12′ *; 2. The length AB, which is $337.5 + 50 \tan 22° 12'$; 3. The length $B.C.$, the ratio of which to the length of center line spiral may be assumed to be the same as the ratio of the average radius of the spiral plus fifty feet to the average radius. In this case the average radius is that of a 4° curve. 4. The length of CD or its central angle. This can be determined in the field, or by solution of the triangle DFG — an ill-conditioned triangle. The solution would give the angle DGF, which added to the known angle FGC gives CGD; 5. DE, which may be measured in the field or computed as a defective course whose length is wanting, after the other quantities have been obtained.

Another way of describing the taking would be as follows: All that certain piece or parcel of land situated in the town of —— County of —— and State of ——, and bounded on the west by the westerly line of the lands of George Brown, on the south by the southerly line of the said lands of George Brown, and on the north and east by a line parallel with and 50 feet distant northerly and easterly from the located center line of the A.B.C. Railroad, which center line is described as follows, to wit: — (Here follows the description of the center line as already given.) Containing —— acres more or less.

56. A Practical Example. — One method of describing a boundary by metes and bounds where the line runs parallel to a curved and spiraled center line is shown in Fig. 60. Considering the 7° curve to the right of the middle on the side away from the river, the right of way line at the spiral and curve is not strictly parallel with the center line, but is made up of a 3 center compound curve having P. C. and P. T. opposite the T. S. and S. T. of the center line. The end curves are of a length $\frac{2}{3}$ the spiral, and central angle $\frac{1}{3}$ the spiral angle. The radius used is about $\frac{1}{4}$ that of the central curve.

* Let the student show the correctness or incorrectness of the figures given.

Another method suggested by Mr. Jenkins who furnished the description in Fig. 60 is to make the right of way lines parallel and concentric with an imaginary center line consisting of two tangents each approximately $\frac{1}{3}$ the length of the spiral it in part replaces, two simple curves, each of $\frac{2}{3}$ the length of the spiral it in part replaces, and a simple curve of the degree of the main curve. The two simple curves at the ends will each have a central angle equal to that of the spiral, and the radius of the simple curves will be approximately $\frac{4}{3}$ that of the central curve.

Example: Let the quantities mentioned as necessary to a description by metes and bounds be found for the case shown in Fig. 58 and the description written beginning at *E*.

Problem. Show that a tangent of $\frac{1}{3}$ the length of a given spiral, and a simple curve of $\frac{2}{3}$ the length of the spiral covering the same angle as the spiral, will have approximately the same coördinates at the end of the simple curve as the S. C., and show also that the radius of the simple curve will be approximately $\frac{4}{3}$ that of the central curve with which the spiral connects, or its degree will be $\frac{3}{4}$ of the degree of the central curve. It will be simpler to find the relation of the degrees first using the fact that the spiral and the simple curves subtend the same angle.

57. City Property. — In going through city property great care is required in making the necessary surveys, the measurements being always to hundredths of a foot and the angles being read to the least count of the vernier. It practically always happens that the company takes the whole lot or all of one end or side or corner of any lot crossed or lying within the necessary width of the right of way. When the line is a curve, it is usual to draw a straight line across each lot of which a part is taken, approximately where the curved line would lie, when the piece taken is easily described by metes and bounds, the corners of the lot, which will be numbered in a numbered or lettered block, being with the street lines sufficient reference points or lines.

In the case shown in Fig. 61 it is probable that the whole of lot 1 and perhaps lot 2 would be taken, although the company might again sell the unused portions of the lots; the portion, *A*, making an irregular lot facing First avenue and the small triangle, *B*, may be wished by someone for a small stand. The back portion of lot 3 may be sold to the owner of lot 10 if there is no alley, or if there is, it may be used for warehouse or barns with access from the alley. It may even be necessary to purchase lot 4 and dispose of the back portion. Lot 5 need not be touched unless there is not sufficient clearance at the S. W. corner. In any event the surveys will establish the points of intersection, and angles of the railroad line with the lot lines, when the description becomes simple. Bearings are not given but angles with established lines, as street lines or lot lines are stated.

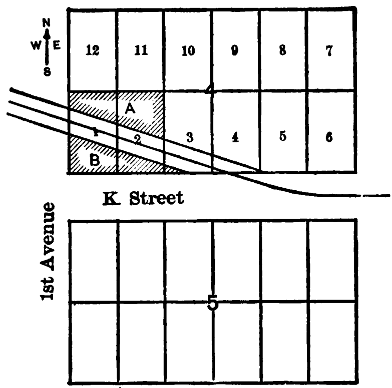

Fig. 61.

58. Suggestions. — When one line of a plot is supposed to extend to another line, point, or object, its length should be stated " more or less " " *to* " the described line, point or object.

It is well to monument the intersections of right of way line with private property lines by some sort of permanent marks as soon as possible after the line is located, and while the lines are still known and agreed to by adjoining owners.

It is well to prepare a careful map of each piece purchased, as, for instance, Fig. 60, binding the map into the deed. The form shown in Fig. 60 has a stub containing the description worked up by the engineering department. When this has served its purpose, the descriptions being written into the deed, the stub is taken off and the map, which is made on a sheet the width of legal paper and of whatever length necessary, is attached to the deed.

The right of way through cities is usually only as wide as is absolutely necessary and is often as little as 30 feet, or even less, for single track.

When embankments are so high or excavations so deep that the usual width of right of way is insufficient, the bounding line is surveyed as any irregular field to take in the necessary area, which will vary in width with the variation in width of embankments and excavations.

The bearings of the boundaries on maps should be written to read from left to right along the several lines in the direction of the description. Fig. 60 is a good example.

CHAPTER VIII.

SWITCHES AND FROGS.

59. Occurrence and Forms of Switches and Frogs. — When one track leaves another, it is by means of a switch and frog. The switch serves to turn a train from one track to the other, and the frog is a device permitting the car wheels to cross one rail of either track. In Fig. 62 the switch is shown at S and

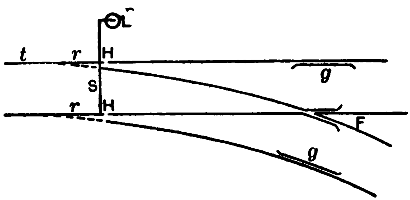

Fig. 62.

the frog at F. The form of switch shown is a stub switch not now much used. The rails r are spiked to the ties near one end only, and the other end is thrown at H from main line rail to turnout rail by the lever L. The

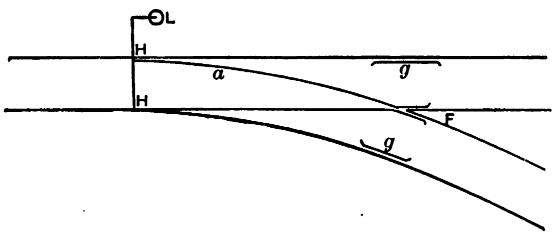

Fig. 63.

rails bend to an elastic curve which is considered for purposes of computation to be a part of the turnout curve tangent to the main line at t where the spiking begins. The rails g and g are guard rails to hold the wheel flanges away from the point of the frog, thus lessening danger of derailment.

Fig. 63 shows a split switch, the usual form. In this, one main line rail and one turnout rail are movable about ends *a*, the rails in Fig. 63 being set for the main line. The two switch rails are beveled off to a long blunt point at the ends *H* and lie up close against the fixed rails next them, making with those rails a small angle of approximately 1½ degrees, more or less. This angle is called the switch angle. The switch rail of the turnout track is placed so as to be in the tangent to the outer rail of the turnout curve at *a* when set for the turnout hard against the main line rail. The opposite main line rail is bent near *H* to the switch angle, and is tangent to the inner rail of the turnout curve where they connect.

The construction of a split switch is shown in Fig. 64. The plates 1, 2, 3, 4, 5, and 6 are called friction plates, and the four pieces at and opposite 1 and 3 are rail braces.

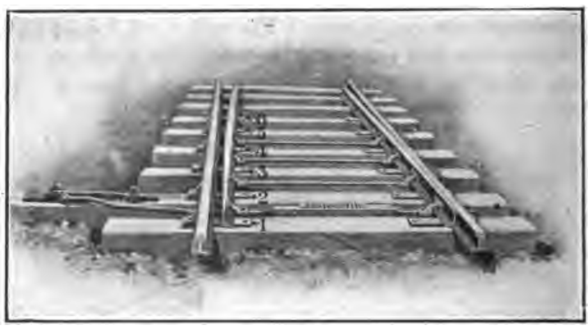

Fig. 64.

The rods connecting the rails are connecting rods, the one nearest the movable end is the main rod or head rod, to which the switch rod and lever are connected. The spring (not always included) attached to the head rod makes the switch partly automatic in that a train may enter the main line from the siding when the switch is set for the main line, the switch rails being thrown over against the spring by the wheels, and returned to position by the spring.

The outline form of an ordinary stiff frog is shown in Fig. 65, the different parts being named. There are also

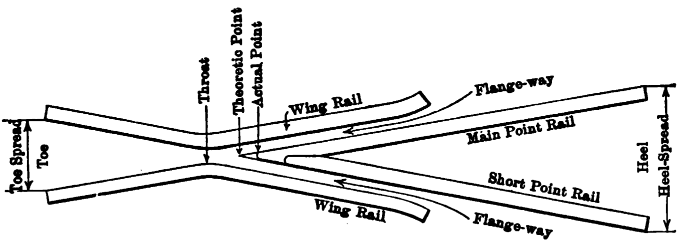

Fig. 65.

spring rail frogs; one is shown in Fig. 66. The main line wing is held against the frog point by the spring S, which returns the wing rail to position after the passing flanges

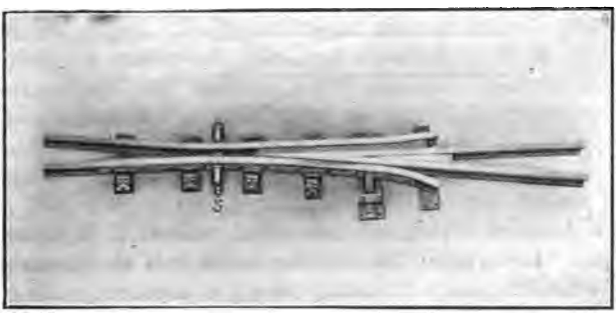

Fig. 66.

of the wheels of a train entering or coming from the siding have thrown it over.

Double turnouts or three-throw switches are used when two turnouts leave the main track at the same point. They may leave on opposite sides or on the same side of the main track. Both stub and split switch forms are shown in Fig. 67. The points of the switch rails in the split switch are separated by about two feet, but all are thrown by the same switch rod by the aid of suitable connecting rods. Perhaps more frequently the second switch is advanced beyond the end of the first one far enough to get good clearance, when one or two levers may be used to throw them, but usually one. The frog at C is known as the crotch frog. In England, and sometimes

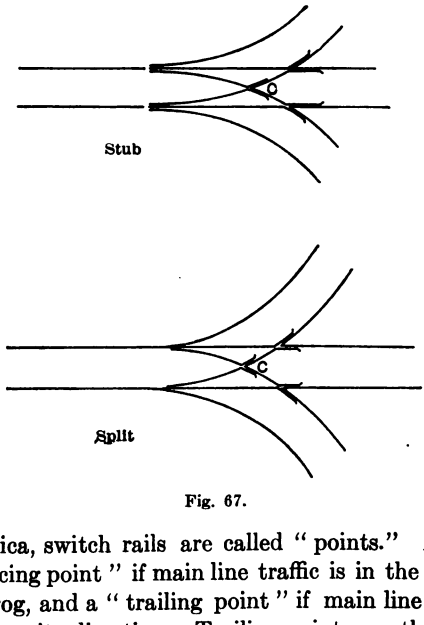

Fig. 67.

in America, switch rails are called "points." A switch is a "facing point" if main line traffic is in the direction switch-frog, and a "trailing point" if main line traffic is in the opposite direction. Trailing points are the safer.

The frog number is a quantity now less frequently used than formerly in formulas, but still used to distinguish frogs of different angles. The number of a frog is the length for a spread of one unit; thus a No. 10 frog spreads 1 foot in 10 feet, or 1 inch in 10 inches. The length is sometimes measured on the gage side of the rails, and sometimes along the middle line or bisector of the angle.

The latter method is that adopted by the American Railway Engineering and Maintenance of Way Association. By it, if F is the frog angle and N the frog number,

$$\tan \tfrac{1}{2} F = \frac{\text{half spread}}{\text{length}} = \frac{1}{2N}.$$

Whence $$N = \frac{1}{2 \tan \frac{1}{2} F} = \tfrac{1}{2} \cot \tfrac{1}{2} F. \qquad (133)$$

If the measurement of length be along the gage line

$$N = \frac{1}{2 \sin \frac{1}{2} F} = \tfrac{1}{2} \operatorname{cosec} \tfrac{1}{2} F. \qquad (134)$$

Certain makers of frogs use Eq. (134) because it is the length along the gage line that is used in calculating lengths of connecting rails.

The frog has a blunt point $\frac{3}{8}$ in. to $\frac{1}{2}$ in. thick. The switch rail of a split switch has also a blunt end at the point of switch usually $\frac{1}{4}$ in. thick. In Figs. 62 and 63 H is called the point of switch, and t and a respectively heel of switch. The distance from point of switch to point of frog is usually called the lead of switch, or simply lead, sometimes frog distance. The distance apart of the two rails at a, Fig. 63, is called the spread or heel spread, and the distance apart of rail heads at H in Fig. 62, the throw.

The sine of the switch angle of the split switch is obtained by dividing the spread less the thickness of the point by the length of the switch rail.

The spread and length of rail depend on the pattern and weight of the rail, the heavier rails requiring more spread, and consequently greater length to keep the angle small. The track leaving the main track is called a turnout as far as the frog, and may be a turnout to a branch road track, to a siding, to a yard, or to a crossover between two tracks.

In laying out a turnout, the quantities that will be known will be.

1. The number (or angle) of the frog, its point thickness, and its length, both toe and heel, *i.e.*, distances from point of frog along the rails to toe and heel.

2. The length, spread, and point thickness of the switch rail.

3. The gage of the track, and its radius if a curve.

The quantities to be found are, 1. The lead, or frog distance. 2. The radius of the turnout curve. 3. The point at which to bend the rail that is part main line and part turnout. It is desirable to arrange the turnout so as to make the least possible wasteful cutting of rails.

With the stub switch the length of switch rail free to move is not known, but is computed from the known throw and other elements to be developed.

Examples. 1. Find the angles of frogs numbered 4, 8, 11, 16.

2. If the point thickness is ½ in., find the distance from the theoretic to the actual point for the frogs of example 1.

60. Frog Distance. — Let F be the frog angle, f the toe length from theoretic point of frog, T the heel spread, and t the point thickness of the switch rail, and G the gage of the track. Then referring to Fig. 68,

$$\sin S = \frac{T - t}{l}. \tag{135}$$

$$P'U = G - (T + f \sin F).$$

Considering the chord $P'E$, the angle $P'EU$ is $\frac{1}{2}(F + S)$ because $P'CE = F - S$, the angle between a tangent at E and EP' would be $\frac{1}{2}(F - S)$, and between that tangent and EU is F; therefore,

$$P'EU = F - \tfrac{1}{2}(F - S) = \tfrac{1}{2}(F + S).$$

The chord $P'E$ is then

$$P'E = \frac{P'U}{\sin \frac{1}{2}(F + S)} = \frac{G - (T + f \sin F)}{\sin \frac{1}{2}(F + S)}. \tag{136}$$

$$UE = P'U \cot \tfrac{1}{2}(F+S)$$
$$= [G - (T + f \sin F)] \cot \tfrac{1}{2}(F+S). \qquad (137)$$

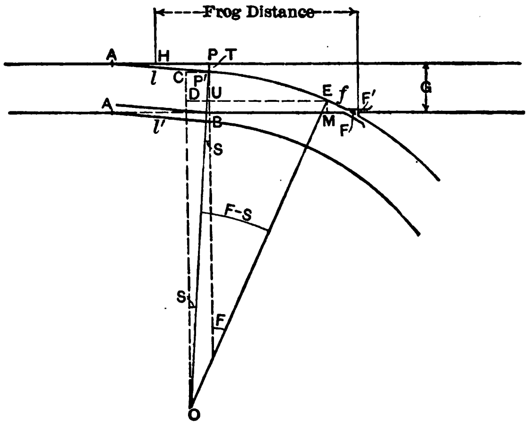

Fig. 68.

The chord length of $P'E$ will usually be sufficiently precise, but for very sharp turnouts it may be necessary to compute the length of the arc, which may be easily done when the angle and radius are known.

From actual point of switch to actual point of frog,

$$\text{Frog distance} = l + UE + MF + FF'$$
$$= l + [G - (T + f \sin F)] \cot \tfrac{1}{2}(F+S)$$
$$+ f \cos F + tN. \qquad (138)$$

The term tN is an approximation (the student may tell why), but the quantity is small and the difference between true and approximate values is negligible.

Examples. 1. The frog number, toe length to theoretic point, and point thickness are respectively 8, 4 ft. 9 in., and $\frac{1}{2}$ in.; the switch rail has a length of 16 ft. 6 in., a heel spread of $6\frac{1}{4}$ inches and a point thickness of $\frac{1}{4}$ inch. The gage is 4 ft. $8\frac{1}{2}$ in. Find the frog distance.

2. The frog number, toe length, and point thickness

are respectively 11, 6 ft. 0 in., and $\frac{1}{2}$ in.; the switch rail has a length of 22 ft., a heel spread of $6\frac{1}{4}$ in., and a point thickness of $\frac{1}{4}$ in. The gage is 4 ft. $8\frac{1}{2}$ in. Find the frog distance.

3. The frog number, toe length, and the point thickness are respectively 16, 8 ft. 0 in., and $\frac{1}{2}$ in.; the switch rail has a length of 33 feet, a heel spread of $6\frac{1}{4}$ in., and a point thickness of $\frac{1}{4}$ in. The gage is 4 ft. $8\frac{1}{2}$ in. Find the frog distance.

61. The Radius of the Turnout Curve. — Considering that the curve is tangent to the switch rail at P' and to the frog at E,

$$2\left(R+\tfrac{1}{2}G\right)\sin\tfrac{1}{2}(F-S) = P'E = \frac{G-(T+f\sin F)}{\sin\frac{1}{2}(F+S)},$$

$$R+\tfrac{1}{2}G = \frac{G-(T+f\sin F)}{2\sin\frac{1}{2}(F-S)\sin\frac{1}{2}(F+S)}, \qquad (139)$$

$$R+\tfrac{1}{2}G = \frac{G-(T+f\sin F)}{\cos S-\cos F}. \qquad (140)$$

Equation (140) may be derived directly from the figure,

since $CD = \left(R+\tfrac{1}{2}G\right)\cos S - \left(R+\tfrac{1}{2}G\right)\cos F = P'U.$

But equation (140) is not likely to give quite such consistent results as (139). The difference is not ordinarily of moment, particularly when it is remembered that frogs are rarely precisely of the angle intended. $(R+\frac{1}{2}G)$ and $(R-\frac{1}{2}G)$ are more frequently required than R, which may be had from (139) or (140) when desired.

The formulas may be somewhat simplified by considering the length, f, curved and part of the turnout curve. The student may find these simplified formulas.

Examples. 1, 2, and 3. Find the radius of the turnout curve for each of the examples 1, 2, and 3 of Art. 60.

62. Length of Bent Main Line Rail.—The length l' of main line rail that is to be bent to the turnout is found by considering similar triangles, thus:

$$\frac{l'}{T} = \frac{l}{T-t},$$

$$l' = \frac{lT}{T-t}. \qquad (141)$$

Examples. These examples are for the application of the discussions of Arts. 59–62 inclusive.

Given frog number 11, toe length 6 ft. 0 in., thickness of point ½ in., length of switch rail 22 ft., thickness of point ¼ in., heel spread 6¼ in., gage 4 ft. 8½ in.

1. Find the length of curved connecting rail.

2. Considering rails 28 to 33 feet long varying by whole feet, lay out the turnout on the drawing board, using ⅛ in. spacing between ends of rails, so as to make the least wasteful cutting of rails. Remember that rail pieces of good length, or proper length for another turnout, are not wasted.

3. If some cutting of small pieces seems necessary in example 2, try whether lengthening the lead a little, say 1 to 3 feet, and extending the switch rail and frog toe tangents, will make a better fit, and if so find the best lead, and corresponding lengthening of switch rail and frog toe tangents, and the degree of the connecting curve.

This does not mean that the switch rail or frog is lengthened.

63. Crotch Frog Number and Distance.—In a three-throw switch the crotch frog will be considered part of the turnout curves. This is not necessary, but as the number is usually small and the frog consequently short, the approximation is sufficiently close.

The frog distance, radius of turnout curve, length of lead rail (including length of crotch frog) for the side frogs, are computed as already explained for a simple turnout, the given quantities being the same as there noted and the two side frog angles being usually equal. It remains to find the number or angle, and the frog distance, of the crotch frog, and the length of the connecting rails $P'f''$ and f_1f' (Fig. 69).

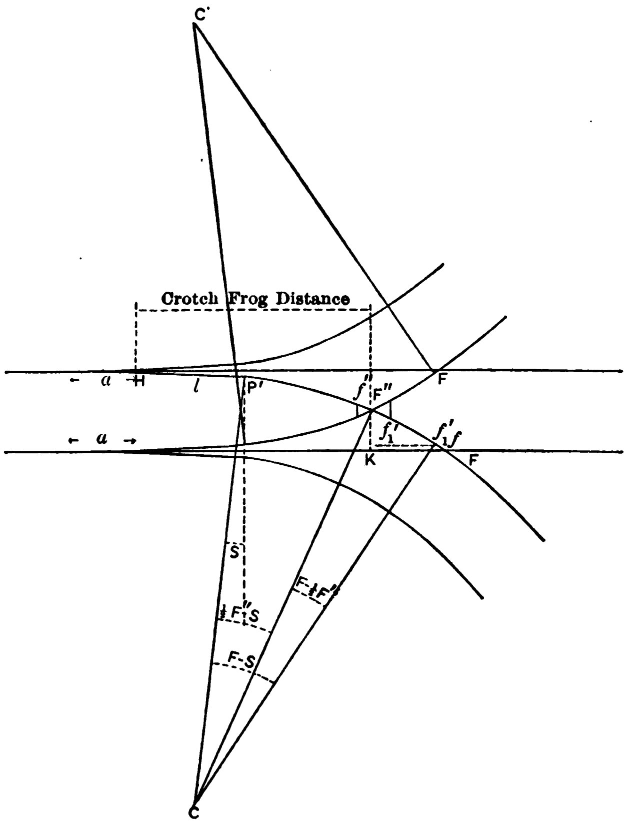

Fig. 69.

The angle $F''Cf' = (F - \frac{1}{2}F'')$.
Therefore the angle

$$F''f'K = \frac{1}{2}(F + \frac{1}{2}F'').$$

Hence

$$F''K = \tfrac{1}{2} G - f \sin F$$
$$= 2\left(r + \tfrac{1}{2} G\right) \sin \tfrac{1}{2}\left(F - \tfrac{1}{2} F''\right) \sin \tfrac{1}{2}\left(F + \tfrac{1}{2} F''\right),$$

or, from trigonometry,

$$\tfrac{1}{2} G - f \sin F = \left(r + \tfrac{1}{2} G\right)\left(\cos \tfrac{1}{2} F'' - \cos F\right).$$

Whence

$$\cos \tfrac{1}{2} F'' = \cos F + \frac{\tfrac{1}{2} G - f \sin F}{r + \tfrac{1}{2} G}. \qquad (142)$$

Having the angle, the number may be obtained.

The angle $P'CF'' = \tfrac{1}{2} F'' - S$.
Whence the chord $P'F''$ is given by

$$L'' = P'F'' = 2\left(r + \tfrac{1}{2} G\right) \sin \tfrac{1}{2}\left(\tfrac{1}{2} F'' - S\right). \qquad (143)$$

If necessary to compute the actual length of the arc the method is obvious.

In this switch the turnout switch points are advanced a distance a (usually about 2 feet) from the head block carrying the main line points. The frog distance HF'' from turnout points is given by

$$\text{Frog distance} = l + L'' \cos \tfrac{1}{2}\left(\tfrac{1}{2} F'' + S\right) + \frac{N}{32}. \qquad (144)$$

The length of connecting rail may be taken as $P'F''$ less the toe length of the frog, or the arc $P'F''$ may be computed if the turnout is so sharp as to require it. The length of connecting rail f_1f' equals the length of connecting rail for simple turnout less that for crotch frog, less the length of crotch frog.

When one set of points is advanced farther beyond the other, the crotch frog does not fall in the center line of the main track, its angle varies a trifle from that of the crotch frog of a symmetrical double turnout and the computations of angle and lead are a little more laborious. Letting Fig. 70 represent a part of such turnout and letting K be

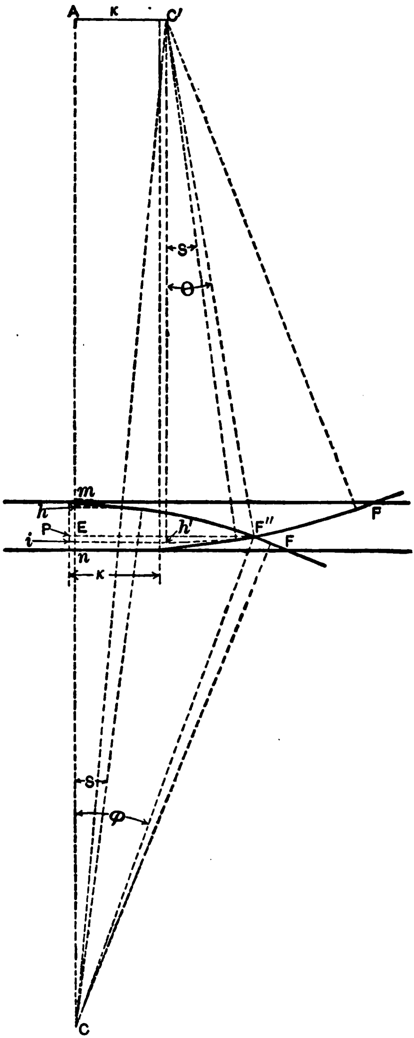

Fig. 70.

the distance between points, it will be seen that moving one point ahead a distance K moves the center of that curve ahead the same distance, so that

$$AC' = K,$$

also $$AC = 2\,(R + \tfrac{1}{2}\,G) \cos F + G - 2f \sin F.$$

Another method of deriving AC is as follows: Consider the two turnout arcs produced back to end in tangents parallel to the main track at h and h' respectively.

Then $$hm = T - (R + \tfrac{1}{2}\,G) \text{ vers } S = in.$$

Since this quantity can be used to advantage in other problems, and since for a given frog and switch angle it is always the same, it will be given a character, O, by which it will be hereafter known.

Then $$AC = Ai + Ch - hi.$$

But $$hi = G - 2\,O.$$

Hence $$AC = (R + \tfrac{1}{2}\,G) + (R + \tfrac{1}{2}\,G) - (G - 2\,O),$$

$$AC = 2\,(R + O).$$

Since O must be computed, there is no value in this expression over that of the preceding paragraph for a single computation in which O is not wanted for any other purpose, but it is the more convenient for use where a number of problems are to be solved, or a table computed in which O is a factor.

From AC' and AC, CC' is computed, and in the isosceles triangle $CC'F''$ the angles are readily obtained, since the triangle may be divided into two equal right triangles with base and hypotenuse known. The angle at F'' is the supplement of the angle F'' of the crotch frog.

The crotch frog distance PF'' is $PE + EF''$, or, since $PE = l - (R + \tfrac{1}{2}\,G) \sin S$, and $EF'' = (R + \tfrac{1}{2}\,G) \sin ECF''$, then calling ECF'' ϕ,

Crotch frog distance from first point

$$= (R + \tfrac{1}{2}\,G)\,(\sin \phi - \sin S) + l, \qquad (145)$$

which holds whether l or $(R + \tfrac{1}{2}\,G) \sin S$ is the larger.

For logarithmic computation,

Crotch frog distance

$$= 2\,(R + \tfrac{1}{2}\,G)\cos\tfrac{1}{2}\,(\phi + S) + \cos\tfrac{1}{2}\,(\phi - S) + l. \quad (146)$$

From the second point the lead is this quantity less K. The angle ϕ is readily obtained from $C'CF''$ and $C'CA$ obtained from the triangles $CF''C'$ and $AC'C$ respectively.

The angle $F''CF = F - \phi$ and $F''C'F = F - \theta$.

The student may show how θ is obtained.

Examples. 1. Find the angle of crotch frog and its lead for a double, opposite, symmetrical turnout from a straight main track, using a number 11 frog and a switch rail of 22 ft. with other elements as given or determined in the examples of Arts. 53 and 54.

2. Find the several quantities necessary to lay out the turnout with one pair of points advanced 28 feet beyond the other. This means leads, crotch frog angle, and lengths of all separate pieces of connecting rail. The theoretic toe length of the crotch frog is 3 ft. 9 in. and the heel length 6 ft. 8 in. Compare the frog angles found in examples 1 and 2.

64. Crossovers. — A crossover is a track connecting two other tracks usually parallel. It may be laid out in one of two common forms: (*a*) straight between frog points, or (*b*) as a reversed curve.

(*a*) The track between frog points is straight. To lay out such a crossover the point of switch will be fixed, the number and dimensions of frog — the two frogs will be alike — and the length, heel spread, and point thickness of the switch rail will be known, and hence all the elements of each turnout will be determined. The perpendicular distance between track centers and the gage will be known. The point of one switch being fixed and the elements of the turnout determined, the point of frog is fixed, and it remains only to determine the distance along the track, BD, Fig. 71, to the second frog from which the second turnout may be laid out. The distance BD — theoretic point of frog to theoretic point of frog — may be readily shown to be

$$BD = (W - G) \cot F - \frac{G}{\sin F}. \tag{147}$$

From actual point of frog to actual point of frog the distance is that of Eq. (147) less 2 KN, in which K is the point thickness and N the number of the frog. The lengths of connecting rails depend on the heel lengths of

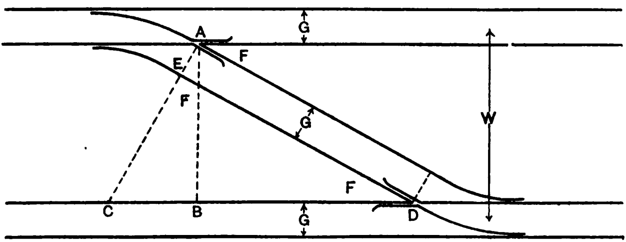

Fig. 71.

the frogs and the arrangements of the inner rails of the two turnouts. The length ED may be shown to be

$$ED = \frac{(W - G)}{\sin F} - G \cot F. \tag{148}$$

The total length of crossover measured along the straight track is evidently

$$L = BD + 2 \times \text{frog distance to theoretic point.}$$

(b) The crossover is a reversed curve. In this case a trifle of distance is saved, but the practice is not advised. The radius of the turnout curve is used for the reverse curve. The frog may be considered part of the curve or straight. If the frog is considered straight, the following discussion results:

In Fig. 72

$$O + R \operatorname{vers}(F + \theta) + d \sin F = \frac{w}{2}.$$

Whence $$\operatorname{vers}(F + \theta) = \frac{\frac{w}{2} - d \sin F - O}{R}. \qquad (149)$$

F being known θ is now known.

It will be remembered that O is the heel spread of the switch rail less $(R + \frac{1}{2} G)$ vers switch angle.

If p represent the difference between $(R + \frac{1}{2} G) \sin S$ and the length of the switch rail, then the distance along the straight track from point of switch to point of switch will be $L =$ Length of crossover

$$= 2\left\{R \sin(F + \theta) + d \cos F \mp p\right\} \qquad (150)$$

and the distance between theoretic points of frogs will be $L - 2 \times$ frog distance to theoretic points. And as before, this less $2\,KN$ is the distance between actual points. In the foregoing p will usually be negative, sometimes positive. It is negative when $(R + \frac{1}{2} G) \sin S > 1$. The length of rail between frogs is

$$\text{Length of connecting rail} = 2\,R \frac{\theta}{57.3}. \qquad (151)$$

If the frog is considered part of the curve some of the expressions are simplified. The student may discuss this case.

Example. Using a No. 8 frog, a switch point of 16 ft. 6 in., a heel spread of $6\frac{1}{4}$ in., toe length of frog 4 ft. 9 in., heel length 8 ft. 9 in., point thickness of frog $\frac{1}{2}$ in., and of switch rail $\frac{1}{4}$ in., find all the elements necessary to plan and lay out a crossover between parallel tracks 13 feet apart center to center, the gage being 4 ft. $8\frac{1}{2}$ in. Let the track between frog points be a, straight; b, a reverse curve.

65. Connecting a Turnout with a Parallel Side Track. — If a turnout is for a siding the radius and length of the

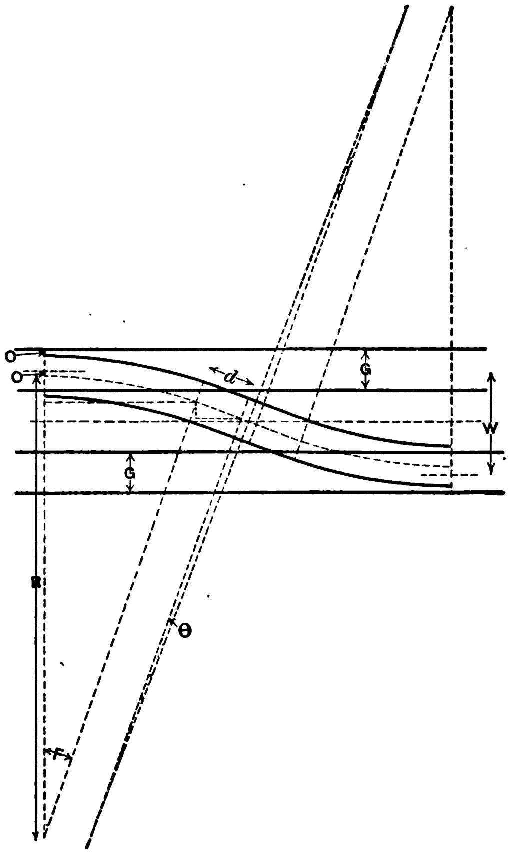

Fig. 72.

connecting curve must be determined. From Fig. 73 it may be readily shown that the central angle must be F, and

$$(R - \tfrac{1}{2} G) \operatorname{vers} F = W - G - f' \sin F,$$

in which f' is the theoretic heel length of the frog. From this relation

$$R = \frac{W - G - f' \sin F}{\operatorname{vers} F} + \tfrac{1}{2} G. \qquad (152)$$

The length of the curve is $R \frac{F}{57.3}$ and the distance along the straight track from point of frog to a point opposite the beginning of the straight portion of the siding is $f' \cos F + (R - \frac{1}{2} G) \sin F$. A high degree of precision in laying out ordinary sidings is not usual, and the frog may be considered a part of the curve, when the expression will be somewhat simplified. The student may discuss this case.

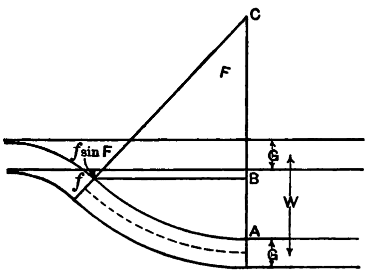

Fig. 73.

Example. Using the turnout elements of the example, of Art. 64, find the quantities necessary to lay out a curve connecting the turnout with a parallel siding 13 feet distant center to center from the main track.

66. Lead or Ladder Tracks. — At the beginning of a yard or cluster of tracks, one track leads out from the main track and the several tracks of the yard or cluster branch from this lead or ladder track. Sometimes the main lead-out track will branch into two lead or ladder tracks, and the two lead tracks may in turn branch again into two ladder tracks, as in Fig. 74.

These yard tracks are usually spaced from 12 to 13 feet center to center, the spacing between ladder track and thoroughfare track being wider. Where tracks are yard tracks for loading from or unloading to wagons, they are in pairs, each pair having its tracks from 12 to 13 feet apart

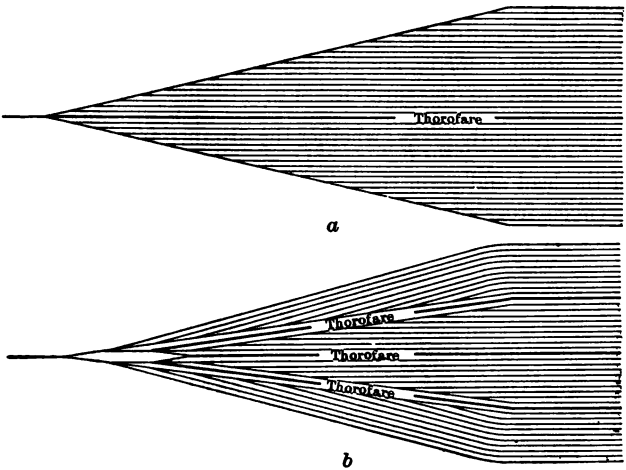

Fig. 74.

and the pairs separated by about 24 feet between centers of nearest tracks. Stub tracks, for wagon loading and un-

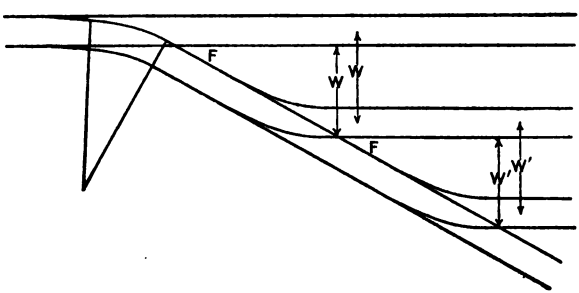

Fig. 75.

loading where wagons must be turned, may be 39 feet between centers of nearest tracks of adjoining pairs.

The usual problem is to find the distance from point of frog of the first ladder to point of switch of the first yard track, or from point of frog to point of frog, the distance to point of switch being this less the lead of the turnout. The known elements are the distance between tracks and the elements of the turnout. The distance from frog to frog is evidently $\frac{W}{\sin F}$, whether the distance from the frog of the lead and main track to the first yard track frog or between frogs of successive yard tracks from the ladder track is required. The distance from switch point to switch point along the ladder track is the same.

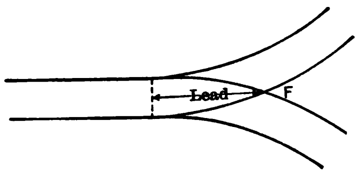

Fig. 76.

67. A Branching Track. — When one track branches into two symmetrically, as at the beginning of a yard cluster, the lead is given by the following expression which it is thought the student can verify:

$$\text{Lead} = l + (\tfrac{1}{2} G - T) \cot \tfrac{1}{2} (\tfrac{1}{2} F + S) + f \cos \tfrac{1}{2} F, \quad (153)$$

in which l is the length of the switch rail; G, the gage; T, the heel spread of the switch rail; F, the frog angle; S, the switch angle; and f the toe length of the frog.

The radius of the curves to be used is found to be

$$R = \frac{\tfrac{1}{2} G - f \sin \tfrac{1}{2} F - T}{\sin \tfrac{1}{2} (\tfrac{1}{2} F + S) \sin \tfrac{1}{2} (\tfrac{1}{2} F - S)}. \quad (154)$$

Example. Using the turnout elements of the example, Art. 60, and a No. 4 frog for the branching tracks, lay out the beginning end of a 40-track cluster with 4 ladder tracks and three thoroughfare tracks in the shortest practicable distance. Make the body tracks 12 feet center to center, the thoroughfare tracks 15 feet center to center of adjacent tracks. The layout should be essentially as in Fig. 74. Note that a No. 4 frog has not exactly twice the angle of a No. 8 frog and that hence a fudging or adjustment of the body tracks parallel to the central thoroughfare will be necessary. This will amount to only about 1½ minutes. If thought of sufficient importance, a special frog of just twice the main frog angle could be made for the branching tracks.

68. Turnouts from Curved Tracks. — ***A. Stub switch formulas.***

1. *Turnout on the inside:* When a turnout leaves a curved track a mathematically exact solution of the frog distance is not simple if the frog is considered straight and connected to a rail tangent at its toe. Moreover, it would have to be considered straight on its main line side as well. Turnouts from curves are unsatisfactory, but sometimes necessary. When in important places, as at stations, the switch rails and frogs should be curved to secure smoothness of track; but when in comparatively unimportant places and little used, there is no need of great refinement and approximate methods of laying out will answer. Considering both frog and switch rail curved and the latter tangent at the point of switch, as it would be in a stub switch at the heel, the following comparatively simple expressions may be found for a turnout on the inside of the main track.

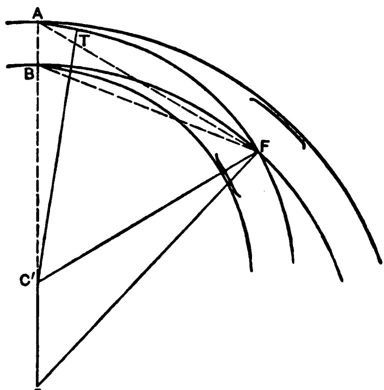

Fig. 77.

In Fig. 77 the lead or frog distance is BF measured either as a chord or along the rail, to get which the angle C must be determined. Considering the triangle AFC and calling R the radius of the main track, and G the gage,

$$AC = R + \tfrac{1}{2} G,$$
$$CF = R - \tfrac{1}{2} G,$$

and since in the isosceles triangle $C'AF$ the angle at A equals the angle at F, and since $F = CFA - C'FA$,

$$CFA - CAF = F.$$

Solving for the angle C by the tangent formula, since

$$CFA + CAF = 180° - C,$$

$$\frac{\text{cotan } \tfrac{1}{2} C}{\tan \tfrac{1}{2} F} = \frac{2R}{G},$$

and since $\cot \tfrac{1}{2} F =$ twice the frog number, N, (Eq. 133), inverting the expression and substituting $2N$ for $\cot \tfrac{1}{2} F$ gives

$$\tan \tfrac{1}{2} C = \frac{GN}{R}. \qquad (155)$$

The chord $$BF = 2\,(R - \tfrac{1}{2} G) \sin \tfrac{1}{2} C. \qquad (156)$$

The arc

$$BF = (R - \tfrac{1}{2} G)\frac{C}{57.3} = .0175\, C\, (R - \tfrac{1}{2} G) \text{ approx.} \qquad (157)$$

The radius r of the turnout may be obtained from either of the triangles BFC' or $C'FC$. From the former, using the tangent formula and noting that $C' = C + F$,

$$r = \frac{GN}{\tan \tfrac{1}{2}(F + C)}. \qquad (158)$$

From the latter

$$r + \tfrac{1}{2} G = \frac{(R - \tfrac{1}{2} G) \sin C}{\sin (F + C)}. \qquad (159)$$

If in (156) $\tfrac{1}{2} G$ be neglected as small in comparison with R, and if $\sin \tfrac{1}{2} C$ be considered equal to $\tan \tfrac{1}{2} C$, which is nearly true for small angles, then approximately

$$\text{Chord } BF = 2R \tan \tfrac{1}{2} C,$$

and substituting the value of tan $\frac{1}{2}C$ from (155),

$$\text{Chord } BF = 2\,GN.$$

It will be found later that this is the same as the frog distance of a stub switch turnout from a straight track. It is the expression generally used for field work, but not for carefully worked out values in the drawing room of the frog maker. The expression is, however, a closer approximation to the arc BF than to the chord, since both factors $(R - \frac{1}{2}G)$ and sin $\frac{1}{2}C$ are slightly increased in calling them respectively R and tan $\frac{1}{2}C$.

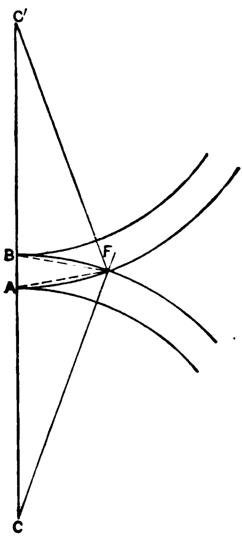

Fig. 78.

Moreover, it has been shown that a curve departs from its tangent at the approximate rate of $\frac{7}{8}n^2D$, in which n is the distance in stations from point of tangency and D is the degree of the curve, and that two curves of degrees D_1 and D_2 depart from each other by the same approximate law if the difference of degrees be substituted for D of the formula. Therefore, since a stub switch turnout departs from its tangent an amount G in the frog distance, and since the frog distance for a turnout from a curved main track is closely approximate to that from a straight main track, and since a curved main track and turnout separate G feet in the frog distance, therefore the difference of degrees of main and turnout tracks is the same as the degree of the turnout curve from a straight main track, or, letting D_c, D_s, and D stand for degree of curved main track, turnout from straight track, and turnout from curved track, respectively,

$$D = D_c + D_s. \text{ (approx.)} \qquad (160)$$

2. *Turnout on the outside:* When the turnout is to the outside of the main track, as in Fig. 78, in which C is the center of the main track and C' that of the turnout curve, the frog distance is BF measured on the outer rail instead of the inner, and solutions like the preceding, using the triangles CAF, BCF, $BC'F$, AFC, and $CC'F$, give

$$\tan \tfrac{1}{2} C = \frac{GN}{R} \text{ as before,} \tag{161}$$

$$\text{chord } BF = 2\,(R + \tfrac{1}{2} G) \sin \tfrac{1}{2} C, \tag{162}$$

$$\text{arc } BF = .0175\, C\,(R + \tfrac{1}{2} G), \tag{163}$$

$$r = \frac{GN}{\tan \tfrac{1}{2}(F - C)}, \tag{164}$$

$$r + \tfrac{1}{2} G = \frac{(R + \tfrac{1}{2} G) \sin C}{\sin (F - C)}, \tag{165}$$

and approximately

$$BF = 2\,GN, \tag{166}$$

$$D = D_s - D_c. \tag{167}$$

There is a second case for this turnout which occurs when the center of the turnout on the outside lies on the inside of the curve of the main track. Let the student construct a figure and by reasoning similar to the foregoing discover the following equations:

$$\tan \tfrac{1}{2} C = \frac{GN}{R}, \tag{168}$$

$$\text{chord } BF = 2\,(R + \tfrac{1}{2} G) \sin \tfrac{1}{2} C, \tag{169}$$

$$\text{arc } BF = .0175\, C\,(R + \tfrac{1}{2} G), \tag{170}$$

$$r = \frac{GN}{\tan \tfrac{1}{2}(C - F)}, \tag{171}$$

$$(r - \tfrac{1}{2} G) = \frac{(R + \tfrac{1}{2} G) \sin C}{\sin (C - F)}, \tag{172}$$

and approximately

$$BF = 2\,GN,$$

$$D = D_c - D_s. \tag{173}$$

B. Split Switches.

Since the frog distances for stub switches are essentially the same whether the main track be straight or curved, it may be assumed (and demonstrated by trial) that split switch frog distances obey the same law. And it is sufficiently exact for practically all unimportant cases of split switch turnouts from curved track to assume that the frog distance is the same as that for the same frog and switch rail from a straight track, and that the degree of the turnout curve is the sum or difference of the degrees of the main curve and the turnout from straight track.

Moreover the O and p of Arts. 63 and 64 may be taken as equal to the corresponding quantities for a split switch turnout from a curved track.

But it must be remembered that the frogs have been considered curved, and if they are straight, this introduces a small but measurable error, so that when the positions of head block or point of switch and frog have been determined, it will usually be necessary to fit in the connecting rails on the ground, unless they have been determined from large scale drawings.

The following discussion of one case of split switch turnout from curved track will indicate how these problems may be handled when the switch rail is made straight and the frog curved. The practice varies, but when the curves are anything but flat, it is perhaps the most usual practice to make the frogs curved and the switch rail also, except the planed portion, which remains straight. The heel spread is therefore increased, but that to be used in the computations is diminished, since it must be that at the end of the planed portion of the rail. This planed portion must also be used for the switch rail length in the computations. In important complicated curved work switches and frogs are always made curved, so that computations are made comparatively simple, and quantities not possible of computation are scaled from a large scale drawing.

The lead will be found to the theoretic point of frog, and the frog must be placed so that the actual point is the proper distance from the computed theoretic point.

In Fig. 79, C is the center of the main curved track and C' that of a turnout on the outside. The switch rail AH, being planed to the switch angle for straight track, will fit close against the curved rail only by putting the planed edge in the line of a tangent to the main curve at A. The spread BH will then be that usual for a turnout from a straight track (if we consider the whole rail straight) plus

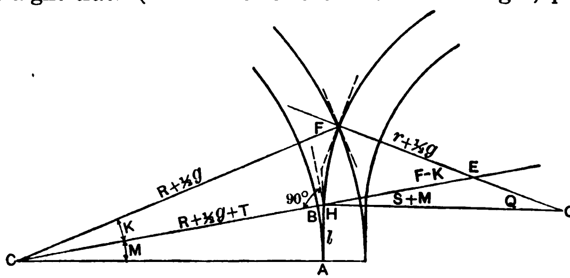

Fig. 79.

the tangent offset for the main curve for the angle M covered by the length of switch rail. This is an approximation practically exact. If only the planed portion of the switch rail is straight, the spread to be used will be less in proportion to the lessened length of rail used for computation.

M is given with sufficient exactness by

$$M = 57.3° \frac{l}{R}, \tag{174}$$

$$T' = T + R \text{ vers } M. \tag{175}$$

To find the lead HF, the auxiliary angle K is first found. From the triangle FCH

$$\frac{CF + CH}{CF - CH} = \frac{\tan \frac{1}{2}(CHF + CFH)}{\tan \frac{1}{2}(CHF - CFH)}, \tag{176}$$

but $$CHF + CFH = 180° - K,$$

and $$\tan \tfrac{1}{2}(180° - K) = \cot \tfrac{1}{2} K.$$

Also CHF is $90° + (S + M) + \frac{1}{2} Q$

and CFH is $90° - F + \frac{1}{2} Q$.

Hence $CHF - CFH = F + S + M$, all known.

Therefore, from (176)

$$\frac{2R + T'}{G - T'} = \frac{\cot \frac{1}{2} K}{\tan \frac{1}{2}(F + S + M)}$$

and
$$\tan \tfrac{1}{2} K = \frac{G - T'}{(2R + T') \tan \frac{1}{2}(F + S + M)}. \tag{177}$$

From the triangle EHC'

$$Q = (F - K) - (S + M) = F - (K + S + M). \tag{178}$$

Therefore
$$CHF = 90° + (S + M) + \tfrac{1}{2}[(F - K) - (S + M)]$$
$$= 90° + \tfrac{1}{2}(F - K + S + M).$$

In the triangle CFH

$$FH = \frac{CF \sin K}{\sin [90° + \frac{1}{2}(F - K + S + M)]}$$
$$= \frac{(R + \frac{1}{2} G) \sin K}{\cos \frac{1}{2}(F - K + S + M)}. \tag{179}$$

Then
$$r + \tfrac{1}{2} G = \frac{FH}{2 \sin (F - K - S - M)}. \tag{180}$$

69. To Connect a Curved Main Track with a Parallel Siding. — In Fig. 80 is shown a side track parallel to and W feet distant from the main track of center C, which is left by a turnout on the outside with frog angle F. To connect the turnout with the side track it is necessary to know the angle C giving the position of B and A, the end of the connecting curve, and the radius and central angle C' of that curve. By a method of analysis similar to that of the preceding

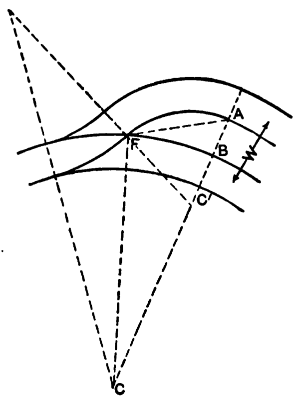

Fig. 80.

article, and using first the triangle CFA, the following results can be obtained:

$$CA = R + W - \tfrac{1}{2}G, \quad CF = R + \tfrac{1}{2}G.$$

The angle $A = C'FA$, and therefore

$$CFA - A = F \quad \text{and} \quad CFA + A = 180° - C.$$

Therefore, as the student may determine,

$$\tan C = \frac{(W - G)}{2R + W}, \quad \cot \tfrac{1}{2} F = \frac{(W - G) N}{R + \frac{W}{2}}. \tag{181}$$

From the triangle FBC, the radius r' of the connecting curve is

$$r' - \frac{W}{2} = \frac{(W - G) N}{\tan \frac{1}{2}(F + C)}. \tag{182}$$

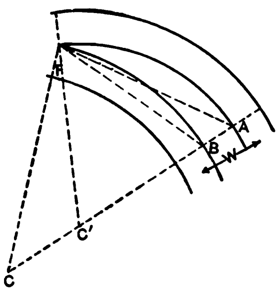

Fig. 81.

The length of FB, chord or arc, may be found when the angle C is known, and since $C' = F + C$, the length of curve FA may be determined when its radius or degree is known.

But the centers of the turnout curve and the main track may lie on the same side of the main track. Then from the triangles shown in Fig. 81, let it be demonstrated that

$$\tan \tfrac{1}{2} C = \frac{(W - G) N}{R + \frac{W}{2}}, \tag{183}$$

and

$$r' - \frac{G}{2} = \frac{(W - G) N}{\tan \frac{1}{2}(F + C)}. \tag{184}$$

That is, if the siding is on the outside, the connecting track is the same on whichever side of the main track the center of the turnout curve may lie, which is perhaps plain without demonstration, since the frog angle is the same.

If the siding is on the inside, it may be shown from the triangle of Fig. 82 that

$$\tan \tfrac{1}{2} C = \frac{(W - G) N}{R - \frac{W}{2}}, \qquad (185)$$

$$r' - \frac{W}{2} = \frac{W(- G) N}{\tan \frac{1}{2} (F - C)}, \qquad (186)$$

and FB and FA may be determined as before. It is approximately true in all these cases that the length of connecting curve is the same as for straight track sidings with the same spacing, and the degree of the connecting curve is the sum or difference of the straight track connecting curve and the curved main track degrees.

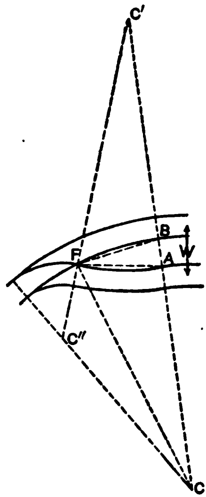

Fig. 82.

70. A Crossover between Curved Tracks. — If a crossover is to be laid as a reversed curve between two parallel curved tracks (Fig. 83), and the radius R, or degree D, of the main track, the track spacing W, and the frog angles F and F' or radii of turnout curves r' and r'' are known, the crossover may be laid out. If F and F',

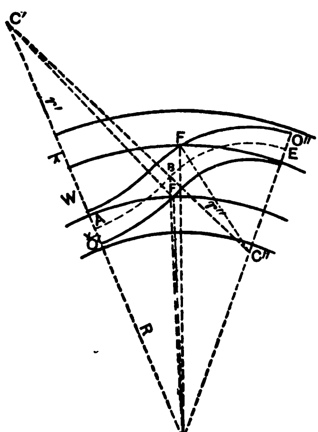

Fig. 83.

equal or unequal, are known, their respective radii may be found. To lay out the crossover it will be necessary to know the angles C' and C'', and a beginning point A or E and the angle C, which will locate the other extremity, when the two branches of known radius may be run from these extremities to connect at B. Or if the angle FCF' is determined, the spacing of the two frogs along the main track is found, and with their respective frog distances the points A and E may be found, and the crossover laid in by eye, as is quite common.

If the turnouts are assumed tangent at E and A as in a stub switch,

$$C'A = r', \quad AC = R, \quad C'B = r', \quad BC'' = r'', \quad EC'' = r'',$$
$$EC = R + W, \quad \text{and} \quad C''C = R + W - r''.$$

Then in the triangle $C'C''C$ the sides are known and the angles may be found. With the angle C known, the angles FCA and FCE may be had by the methods of previous articles and thus FCF' determined, which fixes the frog spacing along the main track.

If split switches be considered, and great precision required, and A and E be the points where the turnout curve produced backward comes parallel to the main track, the offsets O' and O'' must be considered. Thus

$$C'C = r' + R + O' \quad \text{and} \quad CC'' = R + W - O - r''.$$

And the quantities p' and p'' must also be considered, since the points of switch are not at A and E but p' and p'' from these points.

In making assumptions about such a crossover, it is necessary to know that B falls between F' and F.

71. Stub Switch. — In the stub switch (Fig. 84) the switch rail, l, is alternately part of main line and siding. It is spiked down to the left of A and when thrown over from main line to curve forms an elastic curve considered to be part of the turnout curve. The frog is considered curved, though this is not necessary except to the simple formulas that are derived. It will be evident that

$$\text{Frog distance} = BF = G \cot \tfrac{1}{2} F = 2\,GN, \qquad (187)$$

$$(R + \tfrac{1}{2} G)^2 - 2\,GN^2 = (R - \tfrac{1}{2} G)^2,$$

whence

$$R = 2\,GN^2. \qquad (188)$$

The throw t is usually given, and the corresponding length of switch rail unspiked is computed by the approximate relation that the offsets vary as the squares of the distances along the tangent (true of the parabola). Thus

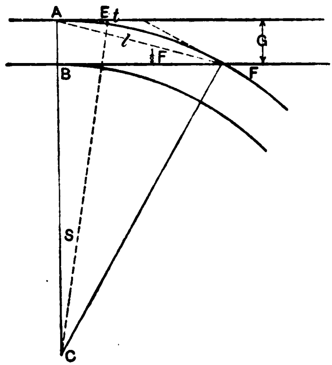

Fig. 84.

$$\frac{l^2}{BF^2} = \frac{t}{G},$$

$$l^2 = 4\,GN^2 t,$$

$$l = 2\,N\sqrt{Gt}. \qquad (189)$$

More exactly

$$\text{vers } S = \frac{t}{R + \tfrac{1}{2} G},$$

$$l = (R + \tfrac{1}{2} G) \sin S.$$

Or since both rails are made the same length, while the radius of one is $R + \frac{1}{2} G$ and of the other $R - \frac{1}{2} G$,

$$l = R \sin S. \qquad (190)$$

Simple formulas may also be had for the crotch frog number, namely,

$$N_c = \frac{N}{\sqrt{2}} = .707\,N \text{ (approx.)}, \qquad (191)$$

and crotch frog distance, namely,

Crotch frog distance $= R \tan \frac{1}{2} F_c,$ (192)

$$= \frac{R}{2 N_c}. \quad (193)$$

72. Crossings. — When two tracks cross, there are four points of intersection, hence four frogs. The angle of the crossing is obtained, the crossing constructed in the shop and brought to the site for placing. One form is shown in Fig. 85. There are two acute-angled frogs and

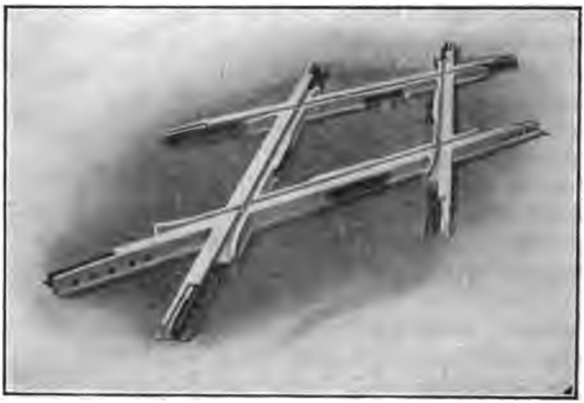

Fig. 85.

two obtuse-angled, being in angle supplementary. When the crossing is at a right angle the frogs are all alike in angle. The crossing is sometimes made in special manganese or nickel steel, or the frog points may be so made.

When the crossing is of a straight and a curved track, or of two curved tracks, the tangent angles at the center line intersections are determined, and from these and the radii the frog angles and spacing can be had as follows (Fig. 86):

$$GO = R \cos I,$$
$$OH = R \cos I + \tfrac{1}{2} G,$$
$$OK = R \cos I - \tfrac{1}{2} G.$$

The frog angles at B, C, D, and E are found by

$$\cos B = \frac{OH}{R + \frac{1}{2}G},$$

$$\cos C = \frac{OH}{R - \frac{1}{2}G},$$

$$\cos D = \frac{OK}{R + \frac{1}{2}G},$$

$$\cos E = \frac{OK}{R - \frac{1}{2}G}.$$

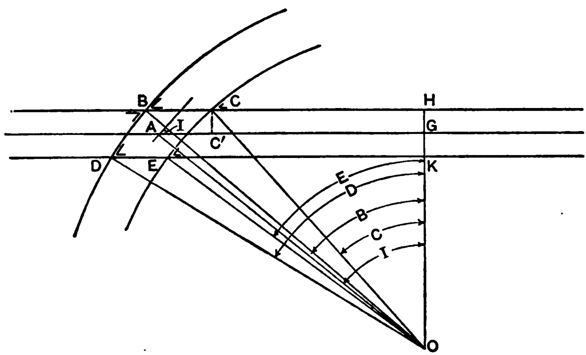

Fig. 86.

The length $BC = BH - CH$

$$= (R + \tfrac{1}{2}G)\sin B - (R - \tfrac{1}{2}G)\sin C.$$

The length DE is obtained similarly.

The length $DB = (R + \frac{1}{2}G)\dfrac{D - B}{57.3}$,

and the length EC is similarly obtained. Out of these lengths are taken the slots or flangeways for passing wheel flanges.

When two curves cross, the angle of intersection of center lines is found and, referring to Fig. 87, the triangle OAO', in which are known two sides and the included angle, is solved for the side OO'. Then in the triangles $OO'B$, $OO'C$, $OO'D$ and $OO'E$, the three sides are known, to find the angles at B, C, D and E, which are the frog angles, and the angles at O and O'.

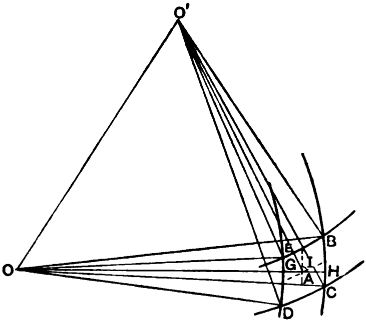

Fig. 87.

The length of rail

$$ED = (R - \tfrac{1}{2}G)\frac{EOD}{57.3}.$$

The length

$$DC = (R' + \tfrac{1}{2}G)\frac{CO'D}{57.3}.$$

The other lengths are found similarly. Out of these lengths are taken the slots or flangeways for the passing wheel flanges.

When crossings are at very small angles or are on curves, the movable-point pattern shown in Fig. 88 may be used

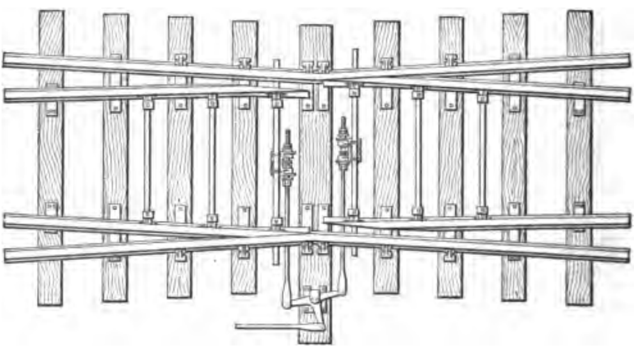

Fig. 88.

to advantage. It is also a good design to use in place of a stiff double-pointed frog with slip switches.

73. Slip Switches. — Slip switches are switches introduced in a crossing to permit a train to pass from one track to the other. They can be introduced only when the crossing angle is between about 4° and 9.30°, or when the frogs are between numbers 15 and 6. The slip switch is shown in Fig. 89.

A. *Straight track.* So much depends on getting clearances in the construction of slip switches that formulas

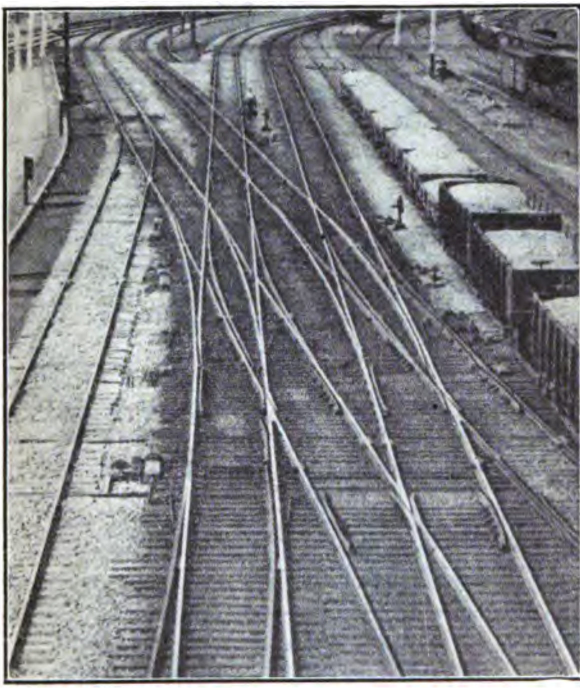

Fig. 89.

are not very satisfactory. A general method of computing them is as follows:

In Fig. 90, I is the angle of intersection of the straight tracks. CD and AB are assumed as short as the construction will allow. The switch rails are assumed curved, but they may be straight. It is desired to find the radius and length of the curved rails.

$$VC = VB = VD - CD = \frac{G}{\sin I} - CD,$$

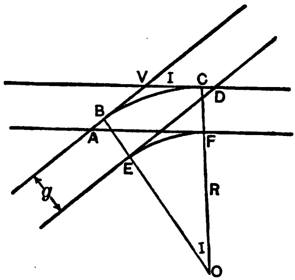

Fig. 90.

$$VC = (r + \tfrac{1}{2} G) \tan \tfrac{1}{2} I,$$

$$(r + \tfrac{1}{2} G) = VC \cot \tfrac{1}{2} I,$$

$$BC = \frac{I}{57.3}(r + \tfrac{1}{2} G),$$

$$EF \doteq \frac{I}{57.3}(r - \tfrac{1}{2} G).$$

B. *Curved and straight track.* Fig. 91.

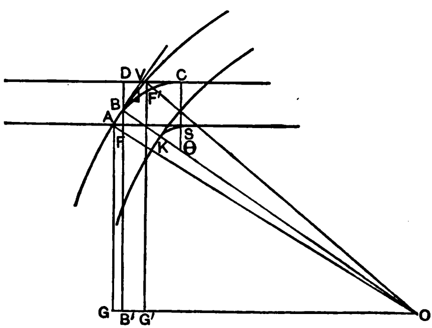

Fig. 91.

$$F'G' = (R + \tfrac{1}{2} G) \cos F',$$

$$AG = (R + \tfrac{1}{2} G) \cos F.$$

$$AOB = \frac{57.3\,AB}{R + \frac{1}{2}G} = \theta,$$

$$BB' = (R + \tfrac{1}{2}G)\cos(F - \theta),$$

$$BV = \frac{(R + \frac{1}{2}G)(\cos F' - \cos(F - \theta))}{\sin(F - \theta)},$$

$$BV \cot \tfrac{1}{2}(F - \theta) = r + \tfrac{1}{2}G,$$

$$BC = \frac{F - \theta}{57.3}(r + \tfrac{1}{2}G),$$

$$KS = \frac{F - \theta}{57.3}(r - \tfrac{1}{2}G).$$

C. *Curved tracks.* Fig. 92. Let the frog angle D and the radii of the two curves be known and the clearance distance DF assumed. Then in the triangle O_1DO_2 are

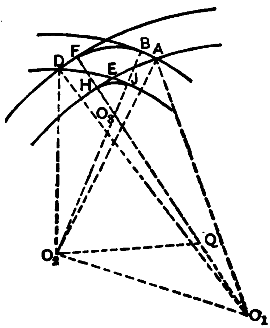

Fig. 92.

found O_1O_2 and the angle at O_1. The angle

$$FO_1D = \frac{FD}{R_1 + \frac{1}{2}G}\,57.3.$$

This gives the angle FO_1O_2. Since FO_3 and BO_3 must be equal, $O_3O_1 - O_3O_2 = R_1 - R_2$. Imagine O_2Q drawn to make $O_3O_2 = O_3Q$; then $O_1Q = R_1 - R_2$, and in the triangle QO_1O_2 are known two sides and the included angle. Solve for QO_2 and O_1QO_2. $O_2QO_3 = 180° - O_1QO_2$, and $$O_3Q = \frac{\frac{1}{2}\,O_2Q}{\cos O_2QO_3}.$$

The radius FO_3 is then given by

$$FO_3 = BO_3 = r = R_2 + \tfrac{1}{2}\,G - O_3Q.$$

The radius $$HO_3 = r - G.$$

The angle $$BO_3F = \theta = 180^\circ - 2 \times O_3QO_2$$

and the arc $$BF = \frac{\theta}{57.3}\,r.$$

$$JH = \frac{\theta}{57.3}\,r - G.$$

It must be known that a proper value for AB remains. Such computations are not of great value, and a much better method of getting these quantities is to make a drawing to large scale.

CHAPTER IX.

RERUNNING OLD LINES.

74. The Problem. — The tracks of many railroads that have been long built are badly out of original line and very frequently the records have been lost. When it is proposed to spiral such track or even simply to improve the alinement, or make records, or establish permanent centers for future use, it becomes necessary to rerun the old line to find either what the curves were and where they were, or new curves of uniform or regular curvature that may be established on the present roadbed with or without spirals, avoiding all obstacles, as signal poles, etc.

The problem is likely to be not so much to determine and rerun the old curve as to find a new curve that may be located with proper spirals, but it may be necessary to rerun the old curve first, or at least to compute its elements to have a working basis for determining the new curve with spirals.

The straight lines are not difficult to establish, but it is difficult and generally impossible to establish by eye the exact position of beginning or end of curves, and it is sometimes difficult to determine just what a given curve was even with the help of instrumental work.

There are in general two cases: 1. The P. I. is readily accessible and may be established by extending tangents at both ends of the curve to an intersection. 2. The P. I. is not readily accessible.

75. The P. I. is Readily Accessible. — When the P. I. may be readily found, the central angle I and the external distance may be measured. If the track is in good alinement, these will give the degree of curve and therefore the length. The curve may be run both ways from the center to its P. C. and P. T. Referring to Fig. 93 the P. I. at V is occupied, the angle $\frac{180 - I}{2}$ turned from the tangent and VE measured to the center of the track. The point E is then occupied, a tangent obtained by turning 90° from EV, and the curve run in toward the P. C. and P. T. for the distance $\frac{L}{2} = \frac{I}{2D}$. If the point E is in proper position the P. C. and P. T. will fall on the tangent lines already established; if not, they will fall inside or outside these tangents. Again, if the curve was originally spiraled, the measured E will be too large for the simple curve that was run originally, and as the new line is run around it will gradually leave the center of the constructed line, moving outward from that line. If this result is found and the variation seems fairly regular, the best way to proceed is to measure the deflection from the tangent at E to successive station points along the curve for two or three stations either side of E, if the curve is so long. By averaging the results a close determination of the original curve may be made. It may then be run for $\frac{1}{2}$ L both ways, L being found by dividing I by the degree found by trial. The two ends will then fall inside the tangents. The offsets to the tangents may be measured and the spiral probably used determined, when the method of rerunning the curve will be clear. The central portion should be rerun from E, establishing the SC and CS. The spiral may be located by offsets or deflection angles. The track will not conform exactly to the new line and should be relined.

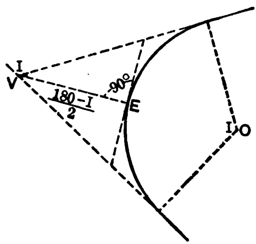

Fig. 93.

If in figuring or measuring the degree of curve it is found to be 4° 03′ or 3° 28′ or otherwise close to some round number, it is probable that the round number was the original degree. *VE* may then be computed for the supposed degree, the transit moved over the required amount and the curve run in. If necessary to keep the curve fairly on the roadbed, a curve of fractional degree may be used.

76. The P. I. is not Readily Accessible. — When the P. I. is not easily accessible, an application of the methods of Art. 2 may be used. Thus in Fig. 94, beginning well back on the tangent at *A* a random line *ABCD* is run to the final tangent, all the angles and distances being measured. Then if there are few points, as in Fig. 94, *AV* and *DV* may be computed through the triangles *ABG*, *GCH* and *HDV* (the student may show how); the degree of the curve may be determined by setting the instrument at random on the curve, measuring a few deflections to successive 100-foot points on the curve, and averaging the results; the tangent distances may be computed from the determined *D* and *I*, *I* being the sum of the deflections turned on the random line from *A* to *D* inclusive; and the distance of P. C. and P. T. from *A* and *D* respectively are found by subtracting the tangent distance from *VA* and *VD*. The curve may then be run in and the track relined.

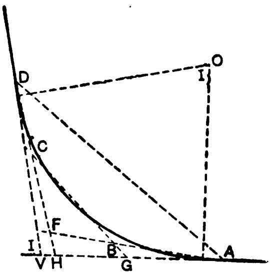

Fig. 94.

If the number of courses in the random line is more than two or three it is better to work with latitude and longitude differences to find *VA* and *DV*, as in Art. 2.

When the curve is a compound curve, probably the best method of procedure is to run a line around the curve, by deflection angles and measurements, taking measurements also to the edges of the roadbed or other objects that will control the final position of the track, as signal posts, platforms, buildings, etc., plot the survey on a large scale and apply curve templets to the plot to find by trial what curves most nearly fit the present track and governing objects. When the curves and their compounding point have been determined, the four elements R_1, R_2, Δ_1 or Δ_2, and I are known, the tangent distances may be computed and reckoned back or forward from the P. I. of the extreme tangents, found as in the preceding cases, to find the P. C. and P. T. Δ_2 or Δ_1 is known from I, found as before, and a definite assumption of one of the Δ's approximating that found by trial with the templets. The templet may be made by drawing a series of concentric arcs of varying radii on tracing paper or cloth to the same scale as the plot. This templet is probably better than fixed wooden, rubber, or celluloid curves which are often used. When the P. C. has been found the curve may be run in from the additional data now known, and the track relined.

In running the random line from tangent to tangent around the curve the following method has some advantages: — extend the tangent a convenient distance, keeping well on the roadbed and using the outer edge of the outer rail for the line instead of the center line of the track; set up over the end of the extended tangent and run a line that shall pass just tangent to the outer rail some distance ahead; measure the deflection angle, and extend the line to a convenient point ahead, where the operation is repeated; continue the procedure until a tangent is run that intersects the final tangent on the roadbed, when the point of intersection is established and the angle there measured. The method has the advantage of saving the time required to measure carefully to the center of the track for each point required on the curve, and helps to locate approximately a compounding point. Whether much time is saved by this method may be a question; it is preferred by some engineers.

77. An Approximate Determination of Degree. — A convenient method of roughly determining the degree of a curve is the following:

From A, Fig. 95, at a rail joint, sight is taken along a line just tangent to the inner rail — as at B — and the point of intersection, C, with the outer rail is noted; the number of rail lengths, whole and fractional, in ADC is determined and used in one of the following formulas, according as the rails are 30 feet or 33 feet long, D being the degree of curve and N the number of rails in ADC.

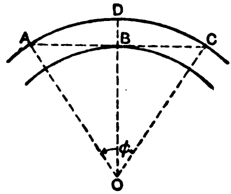

Fig. 95.

For 30-foot rails

$$D = \frac{240}{N^2}. \tag{194}$$

For 33-foot rails

$$D = \frac{198}{N^2}. \tag{195}$$

The demonstration is simple. Equation (20), Art. 15, gives the middle ordinate as

$$M = \frac{C^2}{8R}.$$

The middle ordinate of the arc ADC is the gage of the track, 4.708 feet, and C may be taken equal to ADC and will hence be $30N$ or $33N$. Assuming $R = \dfrac{5730}{D}$ and substituting in equation (20) gives the round number values of equations (194) and (195). It is the degree of the outer rail curve that is obtained by these expressions. This is slightly less than the degree of the center line, and the C is taken larger than it is, apparently increasing the error. But equation (20) is approximate and the approximation consists in assuming that the chord of an arc is twice the chord of half the arc, or smaller than a true formula would require. The result is a partial compensation, and the equations give results as close as any requirement warranting so crude a method of measurement demands. If the rails are not 30 feet or 33 feet, the equivalent number of 30-foot rails may be used in (194).

78. Rerunning Straight Lines. — It has been said that there is not much trouble with straight lines, but not infrequently these give more trouble than curves. Long tangents are frequently not straight, but the track must be kept on the roadbed in readjusting. Probably the best procedure is to rerun the entire tangent between points of adjacent curves or points near these before rerunning the curves. To do this two points in the center of the straight track and within clear seeing distance are selected and the line so determined may be plugged through.* If it lies practically in the center of the track throughout the length of the tangent, the tangent may be called straight and centers may be set as often as desired for track lining. If the line does not lie practically in the center of the track throughout, the beginning line may still be plugged through if it lies wholly on the roadbed. The discrepancy at the end of the tangent may be noted, the length measured, and an angle computed which turned from the beginning line at the beginning point will give a line that will lie in the center of the track at both ends and nearly so throughout. This line may be plugged through and track centers set as often as desired for lining.

If the first line follows the track for some distance and then departs from it, an angle in the track is indicated. Two points may be determined in the apparently straight line ahead, giving a line which may be run to an intersection with the first line where the angle will be measured. This may be done as many times as necessary. The dis-

* By "plugging through" is meant establishing transit points on hubs at convenient seeing distances without setting intermediate station stakes and with or without measurement.

tances must be measured and lateral measurements taken at frequent measured intervals to determine the lateral variation of the track from the traverse line being run. When the complete line has been run it may be platted with a much exaggerated lateral scale, and an attempt made to draw a straight line that will average the discrepancies and lie well enough on the roadbed to be established as the adopted line. The angle that it makes with any one of the courses run cannot be measured on the drawing but may be computed from measured lateral discrepancies and known lengths. This line may then be established on the roadbed and track centers set.

It may be that a satisfactory average line cannot be found. If so, then an attempt will be made to draw two such lines and establish their intersection point and angle. These lines would be so established that they would intersect enough off center of the track to permit running a short flat curve to connect them that will lie on the roadbed. Very flat curves of a degree expressed by a small number of minutes are considered undesirable by many trackmen, but are sometimes necessary. For this reason the discrepancies of a long tangent that must be met by the introduction of curves should be concentrated at one point if possible.

CHAPTER X.

STAKING OUT.

79. Preliminary Statement. — When a railroad center line has been laid out on the ground, levels are taken at all stations and points of change of slope between stations, and a profile is constructed from the level notes. On this profile a "grade line" is drawn. This line shows the level to which the low places are to be filled and the high places cut down in order that the roadbed may be practicable for the running of trains. The principles governing the choice of the steepest allowable rate of grade — known as the ruling grade — are discussed in "Elements of Railroad Engineering." For the present it is sufficient to say that a grade line is drawn that will provide a safe roadway, with the lightest practicable grades and with the minimum possible expense for grading. This frequently means that the grade lines will be so placed as to make equal quantities of excavation and embankment. The elevation of the grade will be written at each point of change of rate of grade, and the rate of the grade will be written along each stretch. The profile is drawn on profile paper, Plate *A* being generally preferred. Fig. 96 shows a small portion of a profile and grade line much reduced in scale, and Fig. 97 shows the appearance of the same ground after the work indicated on the profile has been done. Fig. 98 shows the design of Plates *A* and *B* profile paper. Plate *B* may be used for condensed profiles or when the country is rough. Plate *A* having a greater vertical scale will show small irregularities better than Plate *B*. The grade line is best fitted to the profile by stretching a piece of black thread and shifting it so as

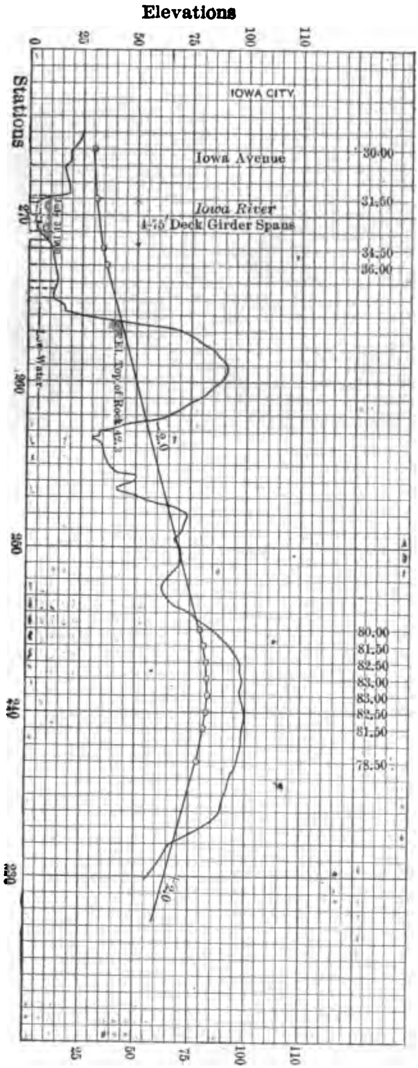
Elevations
0
25
50
75
100
110
Stations
IOWA CITY
Iowa Avenue
Iowa River
4-75' Deck Girder Spans
Low Water
Top of Rock
270
260
250
240
230
30.00
31.50
34.50
36.00
80.00
81.50
82.50
83.00
83.00
82.50
81.50
78.50
-2.0
-2.0

Fig 96.

Fig. 97

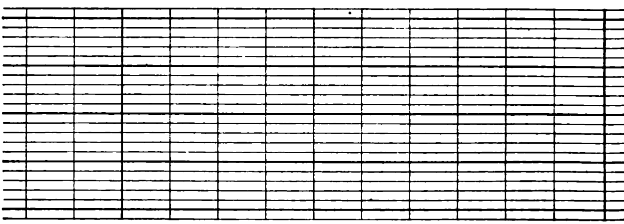

Plate A, 4 × 20 to the inch.

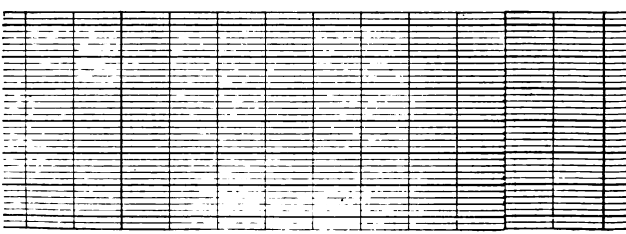

Plate B, 4 × 30 to the inch.

Fig. 98.

to get the longest and lightest rate grade lines possible that will somewhere nearly, as judged by eye, equalize the cut and fill. Long level stretches should always be raised by embankment in order to drain the roadbed properly, and cuts should be as few as possible, because they are troublesome to drain and are likely to collect snow. Hence the rule to equalize cut and fill may be departed from as occasion warrants. This rule usually results, after the location of the line is fixed, in a minimum of work, and hence is likely to give the cheapest grade line.

The work of making the excavations and embankments is usually paid for at an agreed rate per cubic yard of earth or rock moved. In addition to this, if the earth is moved farther than a certain distance specified in the agreement between the railway company and the contractor who does the work, an additional sum per cubic yard per station of "overhaul" is paid. The limit of free haul is usually five hundred feet, but is sometimes made one thousand feet. Hence it is usually necessary to measure the earth excavated and the distance it is hauled. This is done from the notes that are taken when the work is staked out for construction.

80. Form of Cross Section. — The usual cross-sectional form of a railroad cutting is one of the two shown in Figs.

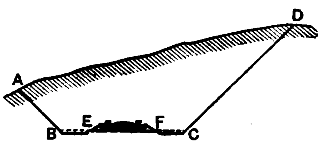

Fig. 99.

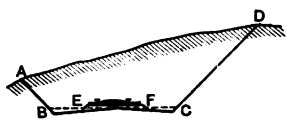

Fig. 100.

99 and 100. Ballast is placed between E and F, and the ties are bedded in the ballast.

The dimensions of cross sections are determined at each station and oftener as may be deemed necessary. The area determined is $ABCD$. The excess for ditches is a uniform quantity per station, and may be added to each station volume or may be added in a lump for the entire length of each cut.

The slopes of the sides of the cutting depend on the material through which the cutting is made. The following are common:

Solid Rock, vertical or ¼ horizontal to 1 vertical.

Loose Rock, ¼ horizontal to 1 vertical or ½ to 1 or ¾ to 1.

Compact Clay or Earth, 1 horizontal to 1 vertical at first; but the sides will not stand long at this slope, and in some cases the slopes are made 1½ to 1, and this is considered the best practice when money is sufficient.

Loose Earth or Sand, from 1½ to 1 to 3 to 1.

In embankments, earth slopes are usually made 1½ to 1, and with light materials sometimes flatter. Rock slopes may be made somewhat steeper. There are two general classes of railroad cutting — "through" or "thorough" cut, Fig. 101, and "side-hill" cut, Fig. 102.

Since embankments may be treated, so far as laying out and computing are concerned, as inverted cutting, the method of cross-sectioning and determining areas and volumes will be given for cutting alone. The volume of earthwork moved is determined by finding the areas of successive cross sections, and using them, with the lengths between them, as elements of prisms, wedges, or prismoids (prismatoids).

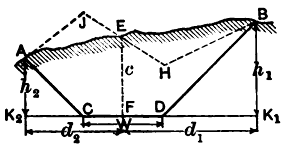

Fig. 101.

It is necessary to mark the edges of cuttings and embankments to guide the graders, and it is necessary to know the heights of these edge points above or below grade, and their distances from the center in order to calculate the area of the section. If the section is quite irregular in surface slope across the line of the road as indicated by the dotted line $AJHB$, it is necessary to know the heights of the points J and H, and any others where the cross slope changes, in order to approximate the cross-sectional area closely. The area *ACFE* could be considered that of a trapezoid of altitude c and h_2, and base K_2F, less the triangle ACK_2, and the right-hand side could be treated similarly. With c, h, h_2, d, d_2, and w known the computation is possible. If the cross slope is absolutely uniform the whole may be considered as one trapezoid of altitudes h and h_2 and base K_1K_2, less the two exterior triangles. But the slope is not usually uniform and the methods of computation do not assume it to be. If the cross slope is irregular the area is considered to be a series of trapezoids, the sum of which, less the two exterior triangles, gives the area. Sections having cross slopes sufficiently regular to require height at the center and two side points only are called "three-level" sections and are computed as triangles rather than as trapezoids, as will appear later. The same data already indicated to be necessary are used, and these data are found in the course of staking out or cross-sectioning. If the section is level across it is called a "level" section. *CD* or *W* is the width of roadbed, commonly about 20 feet for excavation and 14 feet for embankment for light single track first construction, but frequently in standard construction 20 feet for embankment and the same plus the width of ditches for excavation.

81. Method of Cross-sectioning. — Cross-sectioning, or staking out, consists in finding the points *A* and *B* where the surface is intersected by the planes of the sides and placing stakes at these points, on which stakes are marked the station and vertical distance above the bottom of the cut, or "grade," with the letter "*C*" to indicate "cut." For embankment sections the stake is marked "*F*" for "fill," with the distance of the point below grade. The "center cut" *EF* is also determined and in the case of side-hill work the "grade point," *G*, Fig. 102, or point where the surface intersects the roadbed.

The cut or fill is always marked on the center stake in feet and tenths, and at the grade point a "grade plug" is sometimes driven, the top of which is driven to be "at grade" and then chalked with keel or marking chalk.

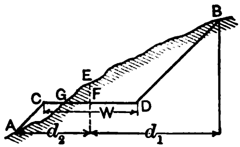

Fig. 102.

The heights above or below grade of all the points in the cross-section where stakes are driven or where changes in slope occur are found together with the distances of the points from the center, and both heights and distances are recorded in the notebook.

The work is all done to the nearest one-tenth of a foot, except the setting of grade plugs, which are driven to be within a few thousandths of a foot of "grade."

The method of determining the points A, B and G is as follows:

The elevation of grade for each station is known from the profile of the center line on which the grade line has been drawn.

The slope of the sides of the cut and the width of the roadbed CD are known from the general instructions of the chief engineer of the road and the nature of the ground.

The height of a level, so set as to command the station or stations whose section or sections may be desired, is determined from the nearest bench mark, and a rod then read on the center line at the required station, as at E. The elevation of the station is thus determined.

The difference between the elevation of the station and the "grade" is the cut or fill at that point, and this cut or fill is marked on the stake and recorded in the book.

If s represent the slope of the side $= \dfrac{\text{hor.}}{\text{vert.}}$,

then
$$d_1 = \frac{w}{2} + sh_1.$$

Hence to find the point B, the rodman, knowing the center cut, estimates the value of h_1, mentally computes d_1, and holds his rod at the computed distance from E. The leveler reads it and determines the value of *h* where the rod is held. If it is the same as has been estimated by the rodman, the stake is driven and marked, the record of height and distance made in the notebook, and the process repeated on the other side of the center. If the *h* is found to differ from the rodman's estimate, it is known at once whether the rodman is too far out or not far enough, according as the *h* proves larger or smaller than his estimate, and he tries again. An expert rodman will not make more than one change in ordinary ground, and frequently places his rod right at the first trial. After the point *B* is found, the rodman goes to *A* and the operation is repeated.

The rodman is usually assisted by an axman, or an axman and tapeman; and sometimes the engineer carries the rod and keeps the notes, the rodman merely manipulating the level. Sometimes the work is done with a cross-section rod from level notes of the center line previously taken. The author prefers the method given. Convenient or necessary modifications will suggest themselves to the practicing engineer.

One detail should be mentioned. It is better to carry the level work from station to station by heights above or below grade rather than by heights above some datum

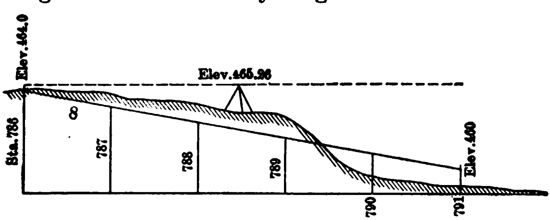

Fig. 103.

surface. Thus: An instrument set up as in Fig. 103 with line of sight at elevation 465.26 (465.3 for ordinary cross-section work) is 1.3 above grade at sta. 786, and this being determined, the difference between 1.3 and a rod reading obtained at sta. 786 is at once the cut at the station. The elevation 465.26 is forgotten for the time, and when the work is completed at sta. 786, the rod is carried to 787, which, as shown by the profile, is to be 0.8 lower than sta. 786, therefore the H. I. above grade at 787 is 1.3 + 0.8 = 2.1, and so on. At 790 the H. I. above grade is found to be 4.5, and as the profile indicates, the rod-reading will be more than this, hence the ground is lower than grade and there is a depth of fill of the difference between the rod and 4.5. When it becomes necessary to change the position of the instrument, return is made to the datum elevations for the new elevation of the instrument, from which is found a new H. I. above grade at the first station to be cross-sectioned after the change, and the work proceeds as before.

In case the ground be irregular as shown by the dotted line *BHJA*, Fig. 101, the heights *H* and *J* above grade, together with their distances from *E*, will also be determined and recorded. For a side-hill section the process is the same as for a thorough-cut section, but the section may have different slopes on the two sides, and different widths of half roadbeds. The grade point is always found and its distance from the center recorded.

82. Notes. There are many forms for keeping notes. Two forms are shown:

Sta.	Center Line.		Left.	C.	Right.
	Elev.	Grade.			
18	564.2	560.0	$\frac{-5\,6}{18.4}$	−4.2	$\frac{-3\,.6}{15.4}$
19	568.4	561.0	$\frac{-9.2}{23.8}$ $\frac{-8.6}{12.0}$	−7.4	$\frac{-6.0}{11.0}$ $\frac{-5\,2}{17.8}$
.....	....	....	....	.....	....
86	564.2	570.0	$\frac{+6.4}{16.6}$	+5.8	$\frac{+4.2}{13.3}$
.....	....	....	....	.....	....
102	572.4	570.0	$\frac{+1.2}{2.8}$ $\frac{0.0}{5.0}$	−2.4	$\frac{-3.4}{15.1}$

Sta.	Center Line.		Left.	C.	Right.
	Elev.	Grade.			
18	564.2	560.0	$\frac{5.6}{18.4}$	-4.2	$\frac{3.6}{15.4}$
19	568.4	561.0	$\frac{9.2}{23.8}$ $\frac{8.6}{12.0}$	-7.4	$\frac{6.0}{11.0}$ $\frac{5.2}{17.8}$
. . . . . .	. . . .	. . . .	. . . .	. . . . . .	. . . .
86	564.2	570.0	$\frac{6.4}{16.6}$	$+5.8$	$\frac{4.2}{13.3}$
. . . . . .	. . . .	. . . .	. . . .	. . . . . .	. . . .
102	572.4	570.0	$\frac{1.2}{8.8}$ $\frac{0.0}{3.0}$	-2.4	$\frac{3.4}{15.1}$

The first form is the more common and makes rather the neater notebook. In the second the character of the work — whether cut or fill — is indicated by the inclination of the lines separating the *h*'s and *d*'s; a slope downward *toward* the center indicating a cut, downward *from* the center a fill. Station 18 is in cut, Station 19 an irregular section in cut, Station 86 is in fill, Station 102 is a side-hill section.

In both forms the *h*'s are above the dividing lines and the *d*'s below. Some engineers reverse this order. In both forms the minus sign indicates a cut (something to be taken away — an order) and the plus sign a fill (something to be added). Some engineers reverse these signs, considering the minus to indicate ground too low — a statement rather than an order — and the plus sign ground too high.

The foregoing forms represent the left page; the right page is used for remarks or for computations to be shown hereafter.

The character of material, rock, earth, or hardpan, should be written along the station column.

83. Sections Required at a Grade Point. — At the place where a fill and a cut join, a surface grade line runs across the roadbed and usually diagonally. A plan of such a junction with the resulting cross-sections is shown in Figs. 104 and 105.

In Fig. 104, *K* is the nearest station in fill to the grade point *G*, and *L* the nearest station in cut, the distance *LK* being one station length. The center line is "at grade" at *G*. The fill runs to grade at *B* and *C*, and the cut at *A* and *D*. The volumes to be considered are a prismoid of fill from *K* to *BF*, the last point where a full section of fill can be had, the pyramid of fill *BFC* having a base of the area at *BF* and an altitude *MC*, a pyramid of cut *AED* having a base of area at *ED* and altitude *AN*, and a prismoid of cut from *ED* to *L*. The notes required, therefore, are those of a cross-section at *K*, a cross-section of the fill at *BF*, the distance *MC*, the distance *AN*, the cut cross-section at *ED*, and the cross-section at *L*.

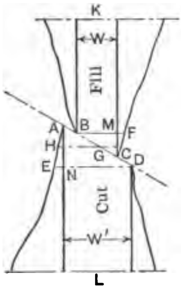

Fig. 104.

The point *G* is usually found and a grade plug driven, but it is not needed in computing.

Assuming a 14-foot roadbed in fill and a 20-foot roadbed in cut with slopes of $1\frac{1}{2}$ to 1, the notes for the cross-sections of Fig. 105 might be as in table on opposite page.

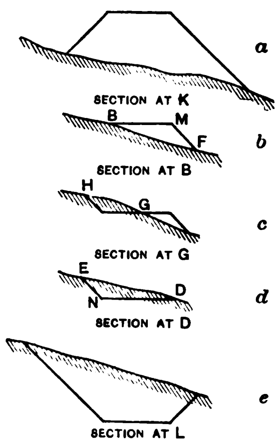

Fig. 105.

It is not uncommon when the surface grade line is nearly at right angles to the axis of the roadbed, and the work not too heavy, to assume that both fill and cut have a zero area at *G*, and the points *A B C D* and sections *BF* and *ED* are ignored.

Sta.	Center Line.		Left.	C.	Right.
	Elev.	Grade.			
21	462.2	465.0	$\frac{+3.4}{12.1}$	+2.8	$\frac{+2.0}{10.0}$
+40		464.6			$\frac{0.0}{10.0}$
+50	463.5	464.5	$\frac{+1.6}{9.4}$	+1.0	$\frac{0.0}{7.0}$
+70		464.3	$\frac{0.0}{7.0}$		
+75	465.4	464 25	$\frac{0.0}{10.0}$	−1.2	$\frac{-2.0}{13.0}$
22	467.0	464.0	$\frac{-2.0}{13.0}$	−3.0	$\frac{-4.0}{16.0}$

84. Vertical Curves. — Where two grades of different rates join, the angle is rounded off by introducing a vertical curve, which because of its convenient characteristics is a parabola. The length of this curve depends on the difference in rate of joining grades and on the length of train passing the curve.

Mr. Wellington has developed the statement that there will be no danger of bunching cars and pulling out suddenly when a sag is passed, if the difference in grades on which the two ends of the train may be at any moment is not more than the rate of the grade of repose, which is probably from 0.3% to 0.4%. To follow this rule would make very long curves where the grade rates differ much and would probably increase the earthwork more than would be considered desirable. A much greater difference may be assumed with reasonable safety, and the recommended practice of the American Railway Engineering and Maintenance of Way Association is: For first-class railways, curves changing by a rate not greater than 0.1 ft. per station on summits and 0.05 ft. per station in sags. For second-class roads, a rate not more than 0.2 per station on summits nor more than 0.1 per station in sags.

Very simple formulas can be devised that may be followed as a rule, but as these require rather blind following and close attention to signs of the terms, it is thought best to illustrate the determination of vertical curve points by working four examples covering the usual cases, and stating a general rule that will be apparent from the examples worked.

In Fig. 106 two grades of rates, 1.0% and 0.8%, meet at a summit at sta. 376, where the elevation of the meeting

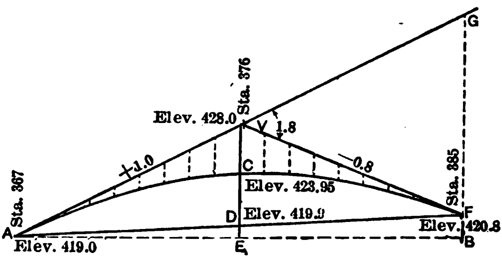

Fig. 106

point is 428.0. The change in the rate in passing the summit is 1.8%, or 1.8 feet per station, since it is from an up 1.0% to a down 0.8%. If the curve connecting these two grade tangents, as they may be called, is to change its rate not more than 0.1 per station, it must be $\frac{18}{1} = 18$ stations long, nine on either side of the summit. This puts the beginning of the curve at $376 - 9 = 367$ and the end at $376 + 9 = 385$. The elevation at 367 is $428.0 - 9.0 = 419.0$ and at 385 is $428.0 - (9 \times 0.8) = 428.0 - 7.2 = 420.8$. A curve properly computed should then begin at sta. 367 at an elevation of 419.0 and end at sta. 385 with an elevation of 420.8, and a computation station by station that does so begin and end is checked by the coincidence of the ending quantities. If the curve is to be a parabola its chords will be bisected by diameters drawn from the intersection of tangents at the extremities of the chords. Since the measurements are made on the horizontal, the projection of $AV =$ that of VF or $AE = EB$. Also, from the parallelism of the lines, $AD = DF$, or the chord is bisected by VD, which is thus a diameter. Moreover the diameter is bisected by the curve, so that $VC = CD$. The elevation at D is a mean of the elevations at A and F, or $\frac{419.0 + 420.8}{2} = 419.9$. The vertical distance from D to V is, then, $428.0 - 419.9 = 8.1$, which divided by 2 is 4.05, which added to 419.9 or subtracted from 428.0 gives 423.95 for the elevation of C, the middle of the curve.

By another property of the parabola the offsets from the tangent parallel to the diameter are as the squares of the distances along the tangent. Therefore the offset from tangent to curve at station 369 ($\frac{1}{9}$ the distance from A to V) is $\frac{1}{81} \times 4.05$, or 0.05. The offset at sta. 369 is $4 \times 0.05 = 0.2$; at 370 is $9 \times .05 = 0.45$; at 371, 0.8; at 372, 1.25; at 373, 1.8; at 374, 2.45; at 375, 3.2; at 376, 4.05; at 377 is 5.0 from the same tangent; and so on to the end, where it is $18^2 \times 0.05 = 16.2$ from G. The elevation of G is $419 + 18 \times 1.0 = 437.0$. $437.0 - 16.2 = 420.8$ as before. The elevations of intermediate stations are obtained by subtracting the offsets from the tangent elevations for the several stations.

But this method is used only when the curve is short — not more than four stations. When it is long the following procedure is used: It will be noted that the change in rate for the first station of the curve is only half of the assumed station change of 0.1, being the change from tangent to chord instead of the change from chord to chord. That is to say, the rate of the first station of the curve is $\frac{1}{2} \times 0.1$, or 0.05 less than the tangent grade. The rate of the second station should be 0.1 less than that of the first and so on. Thus:

The rate of the 1st station is	$+1.00 - 0.05$	$= +0.95$
" " " " 2d " "	$+0.95 - 0.1$	$= +0.85$
" " " " 3d " "	$+0.85 - 0.1$	$= +0.75$
" " " " 4th " "	$+0.75 - 0.1$	$= +0.65$

and so on.

The elevations of several stations would be found as follows:

The elevation of sta.	367	is 419.00	= 419.00
" " " "	368	" 419.00 + 0.95	= 419.95
" " " "	369	" 419.95 + 0.85	= 420.80
" " " "	370	" 420.80 + 0.75	= 421.55
" " " "	371	" 421.55 + 0.65	= 422.20
" " " "	372	" 422.20 + 0.55	= 422.75
" " " "	373	" 422.75 + 0.45	= 423.20
" " " "	374	" 423.20 + 0.35	= 423.55
" " " "	375	" 423.55 + 0.25	= 423.80
" " " "	376	" 423.80 + 0.15	= 423.95
" " " "	377	" 423.95 + 0.05	= 424.00
" " " "	378	" 424.00 − 0.05	= 423.95
" " " "	379	" 423.95 − 0.15	= 423.80
" " " "	380	" 423.80 − 0.25	= 423.55
" " " "	381	" 423.55 − 0.35	= 423.20
" " " "	382	" 423.20 − 0.45	= 422.75
" " " "	383	" 422.75 − 0.55	= 422.20
" " " "	384	" 422.20 − 0.65	= 421.55
" " " "	385	" 421.55 − 0.75	= 420.80

as before.

It should be noted that after the summit is passed the elevations are repeated in inverse order. It should also be noted that the summit is not sta. 376. Perhaps the rule can be formulated now and applied to three other examples.

To determine the elevations of a vertical curve:

1. Assume a rate of change of grade per station.

2. Divide the total change of rate at the apex of the intersecting straight grades by the assumed rate of change, the quotient being the length of the curve in stations.

3. Half the curve being on either side of the apex, find the station and elevation of the beginning and end of the curve.

4. Find the rate of grade of each station of the curve by subtracting (or adding) half the assumed station change from (or to) the tangent grade for the rate of the first station, and the whole assumed station change from (or to) the rate of the first station and from (or to) that of each succeeding station to the end of the curve.

5. Add (or subtract) the determined rate for any station to (or from) the elevation of the preceding station point for the elevation of the station point in question. The final elevation should agree with that previously determined.

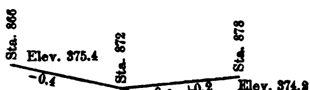

Fig. 107.

In Fig. 107 the total grade change is 0.6. Let the rate of change be 0.05. The curve will be $\frac{0.6}{0.05} = 12$ stations long. Six stations being on each side of the vertex, the station of beginning is 866 and of ending 878. The elevation of 866 is $373.00 + (6 \times 0.4) = 375.4$ and of 878 is

$$373.00 + (6 \times 0.2) = 374.2.$$

The rate of the first station is	-0.4	$+0.025$	$= -0.375$
" " " " second " "	-0.375	$+0.05$	$= -0.325$
" " " " third " "	-0.325	$+0.05$	$= -0.275$
" " " " fourth " "	-0.275	$+0.05$	$= -0.225$

and so on; the rates can be carried mentally.

The elevation	of	sta.	866	is		375.400
"	"	"	867	"	375.400 − 0.375 =	375.025
"	"	"	868	"	375.025 − 0.325 =	374.700
"	"	"	869	"	374.700 − 0.275 =	374.425
"	"	"	870	"	374.425 − 0.225 =	374.200
"	"	"	871	"	374.200 − 0.175 =	374.025
"	"	"	872	"	374.025 − 0.125 =	373.900
"	"	"	873	"	373.900 − 0.075 =	373.825
"	"	"	874	"	373.825 − 0.025 =	373.800
"	"	"	875	"	373.800 + 0.025 =	373.825
"	"	"	876	"	Repeating in	373.900
"	"	"	877	"	reverse order	374.025
"	"	"	878	"	beyond the vertex	374.20

which checks with the elevation previously found. It is thought the student can work the two other cases which are included in the examples at the end of the article.

If the length determined is an odd number of stations the same principles hold, but the numerical work is a trifle more complex, and such a result may be frequently avoided by selecting a rate of change that will give an even number for the station length of the curve. If the curve is short the first method of offsets proportional to the squares of the distances may be the simplest.

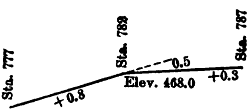

Fig. 108.

Examples. 1. The third case mentioned—two ascending grades. Let the rate of change be assumed at 0.05, giving an even number for curve length. Having made the computations, let 0.1 be assumed as the rate of change giving a short curve of an odd number of stations. This will be best handled by the first method.

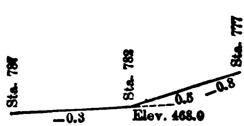

Fig. 109.

2. The fourth case mentioned—two descending grades. Let the same assumptions and methods be used as in example 1.

3. A descending grade of 0.5 is followed by a descending grade of 0.8 at sta. 746, the elevation of which is 572.0. Find the elevations of the stations of a vertical curve, (a) three stations long; (b) changing at the rate of 0.05 per station.

4. An ascending grade of 0.5 is followed by an ascending grade of 0.8 at sta. 496, the elevation of which is 853.0. Find the elevations of the stations on a vertical curve, (a) three stations long; (b) changing at the rate of 0.05 per station.

NOTE. — Examples 2 and 4 are essentially sags in the grade line and should have the minimum change rate, while examples 1 and 3 are essentially summits and the greater change rate may be used.

CHAPTER XI.

COMPUTING THE QUANTITIES.

AREA.

THE greatest work involved in computing the volumes lies in getting the areas of the cross sections. Some methods will be indicated. One that is not infrequently used with very irregular areas and occasionally with simpler areas is to draw the area to scale on cross-section paper and measure it with a planimeter. The method is not advised as economical.

85. Level Section. — If s be the slope ratio $= \frac{\text{hor.}}{\text{vert.}}$, let it be shown that

$$A = cw + sc^2.$$

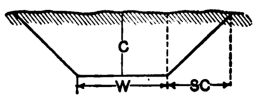

Fig. 110.

Examples. 1. Compute the area of the sections shown in the following notes.

Sta.	Left.	C.	Right.	
167	$\frac{+5.6}{15.4}$	+5.6	$\frac{+5.6}{15.4}$	
168	$\frac{+4.2}{13.3}$	+4.2	$\frac{+4.2}{13.3}$	Roadbed 14 feet. Slope $1\frac{1}{2}$ to 1.
169	$\frac{+0.6}{7.9}$	0.6	$\frac{+0.6}{7.9}$	

2. From the notes of the three middle columns show,

by inspection and computation, that the roadbed is 14 feet and the slope $1\frac{1}{2}$ to 1.

86. Three-Level Section. — The area is evidently equal to the sum of four triangles, indicated by the dotted lines, and may be shown to be

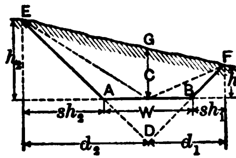

Fig. 111.

$$A = \frac{c}{2}(d_1 + d_2) + \frac{w}{4}(h_1 + h_2). \qquad (197)$$

Since
$$h_1 = \frac{d_1 - \frac{w}{2}}{s};$$

and
$$h_2 = \frac{d_2 - \frac{w}{2}}{s},$$

we may, by substituting these values and placing $(d_1 + d_2) = D$, get

$$A = \frac{D}{2}\left(c + \frac{w}{2s}\right) - \frac{w^2}{4s}. \qquad (198)$$

$\frac{w^2}{4s}$ is the area of the triangle ABD, Fig. 111, and the first term is the area of the figure $EDFGE$.

Since $\frac{w}{2s}$ and $\frac{w^2}{4s}$ are constant quantities for any given length of cut or fill, equation (198) will give results with less work than equation (197), but the latter is most frequently used and the mental effort is probably not more with it than with equation (198). Equation (198) is best for use in diagrams and tables, and may be obtained directly by considering EDF to be two triangles with common base $GD = c + \frac{w}{2s}$ and altitudes d_1 and d_2 respectively. From their combined areas is to be taken the area of the triangle ABD, which should be shown to equal $\frac{w^2}{4s}$.

Examples. 1. Compute the areas of the following sections by both formulas and compare the time required.

Sta.	Left.	C.	Right.	
378	$\frac{+2.4}{10.6}$	+3.6	$\frac{+4.8}{14.2}$	
379	$\frac{+3.1}{11.7}$	+4.2	$\frac{+5.3}{15.0}$	Roadbed 14 feet. Slope $1\frac{1}{2}$ to 1.
380	$\frac{+4.6}{13.9}$	+5.1	$\frac{+6.5}{16.8}$	

2. Show from the notes that the roadbed is 14 feet and the slope $1\frac{1}{2}$ to 1.

87. Irregular Section. — The area equals the sum of the areas of the trapezoids (Fig. 112),

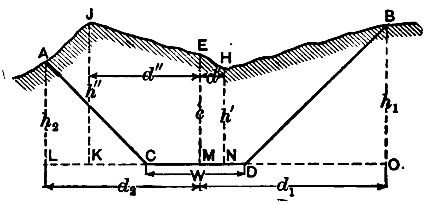

Fig. 112

$$ALKJ, JKME, EMNH, HNOB,$$

less the triangles ALC and DOB.

This gives for the right side

$$A_1 = \left(\frac{c + h'}{2}\right)d' + \left(\frac{h' + h_1}{2}\right)(d_1 - d') - \left(d_1 - \frac{w}{2}\right)\frac{h_1}{2} \quad \ldots$$

and for the left side

$$A_2 = \left(\frac{c + h''}{2}\right)d'' + \left(\frac{h'' + h_2}{2}\right)(d_2 - d'') - \left(d_2 - \frac{w}{2}\right)\frac{h_2}{2} \quad \ldots .$$

The principle applies to any irregular section however many points there may be.

These equations may be written as follows:

$$A_1 = \frac{1}{2}(c + h')\,d' + (h' + h_1)(d_1 - d') - \left(d_1 - \frac{w}{2}\right)h_1,$$

$$A_2 = \frac{1}{2}(c + h'')d'' + (h'' + h_2)(d_2 - d'') - \left(d_2 - \frac{w}{2}\right)h_2.$$

By multiplying out the parentheses and arranging the terms, the following may be obtained:

$$A_1 = \frac{1}{2}\left(cd' + h'd_1 - h_1d' + h_1\frac{w}{2}\right), \qquad \text{(a)}$$

$$A_2 = \frac{1}{2}\left(cd'' + h''d_2 - h_2d'' + h_2\frac{w}{2}\right). \qquad \text{(b)}$$

The above equations indicate that this is merely a special case of areas by coördinates; thus, if the origin for the right half be at M, the coördinates for the points

		Ordinates		Abscissas
M		0		0
E		c		0
H	are	h'	and	d'
B		h_1		d_1
D		0		$\frac{w}{2}$.

The field may be considered to be the polygon $MEHBDM$ and the following rule* may be used to determine the area of either side:

RULE: *From the sum of the products obtained by multiplying the ordinate of each point into the abscissa of the following point, subtract the sum of the products obtained by multiplying the ordinate of each point into the abscissa of the preceding point, and divide the result by 2.*

The rule is particularly applicable to this kind of work, because the field notes have the ordinates and abscissas

* See Raymond's Plane Surveying, Arts. 144–146.

conveniently arranged for the process. Thus, for the right side the field notes would be

C	R	
c	$\dfrac{h'}{d'}$	$\dfrac{h_1}{d_1}$

It is easy to imagine a zero under the c and to supply the last term $\dfrac{0}{\frac{w}{2}}$. Writing these as they may be mentally seen, they appear as follows:

$$\frac{c}{o} \times \frac{h'}{d'} \times \frac{h_1}{d_1} \times \frac{o}{\frac{w}{2}}$$

The rule is applied thus: multiply each numerator by the succeeding denominator and add the products; multiply each numerator by the preceding denominator and add the products. Subtract the second sum from the first and divide by 2. It should be shown that this operation produces equations (a) and (b).

Example. Compute the areas of the sections shown in the following notes by both systems and compare the time required. It will be well to make a rough sketch of the first section at least to bring the trapezoids out clearly. After one plotting the sections should be seen mentally. To make the time test fair the problems should be gotten clearly in mind before a start is made in the computations.

Sta.	*L.*				*C.*	*R.*			
836	$\frac{-2.6}{13.9}$	$\frac{-3.5}{10.4}$	$\frac{-2.4}{8.5}$		-3.6	$\frac{-3.8}{6.0}$	$\frac{-4.6}{11.4}$	$\frac{-4.0}{14.0}$	$\frac{-5.2}{17.8}$
837	$\frac{-4.0}{16.0}$	$\frac{-4.4}{12.6}$	$\frac{-3.4}{12.6}$	$\frac{-5.2}{9.0}$	-6.4	$\frac{-6.2}{7.5}$	$\frac{-7.5}{12.2}$	$\frac{-6.6}{15.4}$	$\frac{-7.6}{21.6}$
838	$\frac{-9.2}{23.8}$	$\frac{-6.6}{20.0}$	$\frac{-5.5}{15.4}$	$\frac{-6.4}{2.6}$	-8.2	$\frac{-8.0}{11.6}$	$\frac{-9.6}{21.0}$	$\frac{-10.8}{21.4}$	$\frac{-11.2}{26.8}$

Roadbed 20 feet. Slope $1\frac{1}{2}$ to 1.

88. Side-hill Section. — Reference to Fig. 102 will show that the area may be considered as two triangles, *AGC* in fill and *GDB* in cut, or the right side may be considered as one side of a three-level section and the remainder two triangles, *GFE* in cut and *ACG* in fill. It is unnecessary to write here the equations for these areas. The area may be irregular, when it will be computed by the method of the preceding article. If the section is such that the grade point would be found on one side only a little beyond the roadbed width of a cut, a slope stake would not be set marking a cut on that side, but the grade point would be found and the roadbed graded off level to the intersection with the hill surface. Likewise if the grade point falls only a little inside the edge of a fill roadbed, no small addition of fill would be staked or constructed.

Example.

Sta.	*L.*	*C.*	*R.*	
164	$\frac{+2.4}{10.6}$	0.0	$\frac{-3.2}{13.2}$	Roadbed 14 feet in fill, 20 feet in cut. Slope $1\frac{1}{2}$ to 1 in fill, 1 to 1 in cut.
165	$\frac{+1.2}{8.8}$ $\frac{0.0}{3.4}$	− 0.6	$\frac{-1.8}{10.2}$ $\frac{-2.0}{12.0}$	
166	$\frac{0.0}{7.4}$	− 1.4	$\frac{-4.6}{14.6}$	

Find the areas in cut and fill.

VOLUMES.

89. General Methods. — It is frequently convenient to know the amount of cut or fill in each one hundred feet or station, and hence a cut or fill is usually computed one station at a time. When, in any particular case, there is no reason for this, considerable time and labor may be saved by computing the entire cut or fill at once.

Having computed the areas at the ends of each station and at each cross section, the volume of material in cubic yards in the length between sections may be obtained by the method of average end areas from

$$V_e = \frac{A_1 + A_2}{2} \times \frac{l}{27}, \tag{199}$$

in which A_1 and A_2 are the areas at the two ends of the volume, and l is the distance between sections. When l is 100 feet,

$$V_e = \frac{50}{27}(A_1 + A_2). \tag{200}$$

By using the means of the dimensions of the end areas a middle area may be computed, and the volume by the method of mean areas is, for a whole station,

$$V_m = A_m \frac{100}{27}, \tag{201}$$

or, by the prismoidal formula the volume for a whole station is

$$V_p = (A_1 + A_2 + 4\,A_m)\frac{100}{6 \cdot 27}. \tag{202}$$

If it is desired to compute by either equation (201) or (202) and the sections are irregular, the middle section should be taken in the field; or the center height of level sections having the same areas as the end areas may be determined and the middle area assumed to be equal to a level area having a mean of these two heights for its center height. This might be called a method of equivalent mean heights.

In irregular earthwork, and in fact in any earthwork, it may be said that any one of the methods assumed gives only an approximate result, and in any special case that method should be employed which, with the least expenditure of time, will secure a sufficiently precise result.

The method of mean areas, entailing almost as much work as the prismoidal formula and being less accurate in result, is rarely used.

The method of average end areas, being the simplest and accurate — on light work, or heavy work having succeeding areas differing by small amounts — to within probably less than one per cent, is generally used. It is the legal method in some states.

On very heavy mountain work where slopes are steep, the error of this method may probably reach or exceed two per cent, and in particular instances much more, and in such work the prismoidal formula should be used.

By what is known as the graphical method (see Chapter XIII), the application of this formula is attended with very little labor, and indeed the use of properly prepared tables makes the work comparatively easy.

There are other methods for approximating to the volume in special cases, but those given are considered sufficient for practical use.*

Example. Find the volumes between successive cross sections as they appear in the examples of Arts. 84 to 88 inclusive, noting that there are pyramids involved in the notes of Art. 84. The cut and fill should be determined separately in the notes of Arts. 84 and 88, and in each case computations should be by both the method of average end areas and the prismoidal formula, and the actual error and percentage of error noted for each station.

It must also be remembered that the middle area used in the prismoïdal formula is not an average of the two end areas but has linear dimensions that are averages of the two corresponding end dimensions.

90. Computation by Average End Areas and Prismoidal Correction. — When the prismoidal formula is to be used it is customary and best to compute by average end areas first and apply a correction which is never large, the total effort being less than for computations by the prismoidal formula direct.

To determine the correction it is necessary to find an expression for the difference between the average end

* For an interesting discussion of various methods and their relative accuracy reference may be made to Wellington's "Computation from Diagrams of Railway Earthwork."

area volume and the prismoidal volume of the same solid.

The following expressions give the volumes by average end area and prismoidal formulas respectively.

$$V_e = \frac{l}{27} \frac{(A_1 + A_2)}{2}, \tag{203}$$

$$V_p = \frac{l}{6 \times 27} (A_1 + A_2 + 4 A_m). \tag{204}$$

The difference is

$$V_e - V_p = \frac{l}{3 \times 27} (A_1 + A_2 - 2 A_m). \tag{205}$$

A. *Application to three-level solids.*

When the sections are three-level sections, and the dimensions of the middle area are not measured, they are assumed to be means of the corresponding end dimensions. Therefore if for A_1, A_2, and A_m in Eq. (205) there be substituted

$$\frac{D_1}{2}\left(c_1 + \frac{w}{2s}\right) - \frac{w^2}{4s},$$

$$\frac{D_2}{2}\left(c_2 + \frac{w}{2s}\right) - \frac{w^2}{4s},$$

and

$$\frac{D_1 + D_2}{2 \times 2}\left(\frac{c_1 + c_2}{2} + \frac{w}{2s}\right) - \frac{w^2}{4s},$$

respectively, equation (205) may be reduced to

$$V_e - V_p = \frac{l}{12 \times 27} (c_1 - c_2)(D_1 - D_2). \tag{206}$$

When l is 100 feet, equation (206), which is the prismoidal correction to average end area volumes, becomes

$$C_p \text{ for three-level sections} = \frac{1}{3.24} (c_1 - c_2)(D_1 - D_2). \tag{207}$$

It will be noticed that neither w nor s appear in the formula except as they are included in D; hence the formula may be used for any values of w and s.

Example. 1. By the prismoidal correction method find the volumes indicated by the notes in Arts. 85 and 86.

2. After solving the foregoing example recompute these volumes by the prismoidal formula and then by the prismoidal correction method and note the time required for each. Also note by what percentage the time of the average end area method is increased by the application of the prismoidal correction.

B. *Application to level section.*

The level section areas may be represented by

$$A_1 = wc_1 + sc_1^2,$$
$$A_2 = wc_2 + sc_2^2,$$
$$A_m = w\frac{c_1 + c_2}{2} + s\left(\frac{c_1 + c_2}{2}\right)^2.$$

And these substituted in (205) give

$$V_e - V_p = \frac{ls}{6 \times 27}(c_1 - c_2)^2, \qquad (208)$$

or for a length of one station

$$C_p \text{ for level section } = \frac{s}{1.62}(c_1 - c_2)^2. \qquad (209)$$

C. *Application to any triangular pyramid frustum.*

If corresponding bases and altitudes of the triangular end sections of the frustum of a triangular pyramid be b_1, b_2, h_1, and h_2, respectively, the areas of the two ends and at the mid-section respectively are

$$\frac{b_1h_1}{2},\ \frac{b_2h_2}{2},\ \text{and}\ \ \frac{1}{2}\left(\frac{b_1 + b_2}{2} \times \frac{h_1 + h_2}{2}\right).$$

If these be substituted for A_1, A_2, and A_m in Eq. (205) there results

$$V_e - V_p = \frac{l}{12 \times 27}(b_1 - b_2)(h_1 - h_2). \qquad (210)$$

If l is 100 feet, then for the frustum of a triangular pyramid

$$C_p = \frac{1}{3.24}(b_1 - b_2)\ (h_1 - h_2). \qquad (211)$$

It is apparent from this that tables of prismoidal corrections for three-level sections and for frustums of triangular pyramids would be identical, since in each case the correction is the product of two variables (the two parentheses) and a constant which is the same for both cases. The same table would answer for level sections, since these are simply special cases of three-level sections and the field notes are the same as for three-level sections, thus giving the same data for computation.

SPECIAL FORMS.

91. Embankment Toe. — When an embankment ends at a trestle bridge the end of the bank is sloped as at the sides and the corners naturally round off as shown in Fig. 113. The stakes would be set at C and G and at H and D or only at the middle point between them, the notes should show the height at A and B, the height or distance below grade at G, C, D, H, and the distances EC, ED, FG, and FH. It will then be sufficiently precise to compute the corners as the quarters of cones whose altitudes are BE and AF respectively, or averages of the three heights at E, C and D and F, G and H respectively, and whose circular bases have for radii averages of the distances EC and ED and FG and FA, respectively; and the portions between the corner quarter cones as a triangular prism or frustum of a pyramid of height w and end bases BED and AFH. The result is not mathematically exact, neither is the work of construction. The method is sufficiently exact for use.

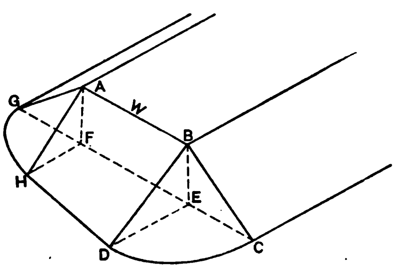

Fig. 113.

92. Widening Earthwork. — When a roadway is to be widened for a siding or second track, the area of any section in excavation would be of the general form shown in

Fig. 114. A stake would be set at *B* and the heights at *A* and *B* determined. Then if *w* is the width to be added

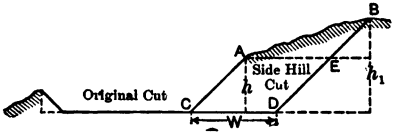

Fig. 114.

to the existing excavation or embankment — if the work is in embankment — the area will be

$$A = \frac{h + h_1}{2} w, \tag{212}$$

which the student should demonstrate.

Near the beginning and end of a siding the tracks will not be parallel and the width *w* will vary, and if the work is of sufficient magnitude the corrections for curvature discussed in article 94 must be applied. Usually this will be unnecessary, and the volumes will be computed as prisms (average end area methods) or as prismoids.

93. Borrow Pits. — When an embankment is built from material taken alongside, as it is when adjacent excavation is insufficient or too far away to be used economically, the material is excavated from what are called borrow pits. These are wide shallow trenches excavated within the right of way if this is wide enough and by arrangement with adjacent owners if the right of way be not wide enough. The general form of the cross profile of the right of way through a completed bank and accompanying

Fig. 115.

borrow pits is as shown in Fig. 115. The pit should begin not less than 6 feet — the usually specified distance — from the toe of the bank, — the space between the toe of the bank and the edge of the pit being called a berm — and should be sloped down at the slope used for the bank. The material should be excavated so that the pit will have a regular form.

When the specifications call for payment to be made for quantities measured in excavation only, it is necessary to measure the borrow pits, although sometimes the bank measure affected by an agreed per-cent factor is used. Sometimes borrow pits are made in near-by hills of gravel or other suitable material excavated by steam shovel in such irregular fashion as to make precise measurement almost impossible. In such cases bank measurement affected by a per-cent factor is probably the best method to follow. Borrow pits may be measured by taking cross profiles before and after excavation at as frequent intervals as may be judged necessary to secure good results, and as the pits are usually fairly regular the average end area method for volumes may be used.

When the material is to be measured by bank measurement and a per-cent factor, the following facts will be helpful:

Ordinary earth when excavated and placed in embankment first swells and afterward shrinks, so that from 4 per cent to 10 per cent more excavation is needed to form a given bank than is called for by the bank measurement. The excess depends on the character of the material and the method of making. Made with carts or wheel scrapers an embankment will measure from 3 per cent to 10 per cent less than the excavation from which its material was taken, immediately upon the completion of the work, and the bank may shrink slightly afterward. If the embankment is made with wagons or dump cars, rapidly in dry weather without water, it may shrink from 3 per cent to 10 per cent in the year following its construction and not much afterward.*

Some bottom land and banks of cemented gravel or

* See Gillette's "Earthwork and its Cost."

hardpan may swell somewhat unless rolled, and the latter even if rolled.

If one has doubt as to the allowance to be made he should make some experiments to determine the allowance by excavating known volumes and subjecting the excavation to packing as nearly similar as possible to that of the constructed embankments, and noting the results.

Rock swells permanently, though there is some settlement of a rock bank with age. The amount of the swelling depends on the sizes of the fragments into which the rock is broken and the care used in building the bank. The material is usually dumped from carts over the end of the growing bank, and when so dumped swells a maximum, but at the same time the pieces are very irregular in size, a condition that tends to make the bank more solid by decreasing the voids. It is probable that a solid yard of rock will make not less than from $1\frac{1}{2}$ to $1\frac{3}{4}$ yards of embankment. If broken as fine as for macadam or concrete it may make nearly two yards, particularly if the fragments are nearly uniform in size.

94. Correction on Curves. — The methods thus far given when properly applied are sufficient for the computation of all earthwork executed in straight stretches in connection with the construction of roads, railroads, or canals.

When a road is built on a curve, a correction must be applied to the quantities obtained by the preceding methods, in order to approximate the truth more closely.

This correction, called correction for curvature, arises from the fact that cross sections are taken in a plane perpendicular to the *tangent* to the curve at each station, while the computations suppose them taken in planes perpendicular to the *chord* between them. The effect of this is shown in Fig. 116.

The cross section will be taken on the line QB (plan), while in computing it will be considered as taken on the line JI for the station PP', and on the line HG for the station PP''. This results in omitting a doubly truncated prism GIP (plan) or $GIPC$ (section) of right base $PCDB$ (section) and in including twice a doubly truncated prism JHP (plan) of right base $QPCA$ (section).

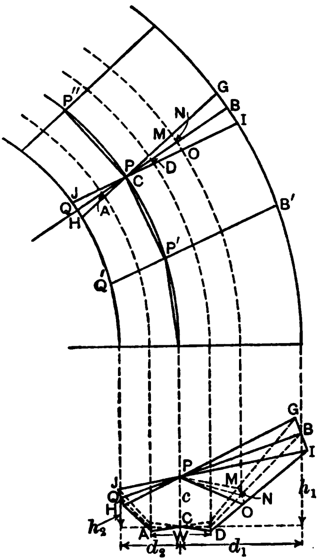

Fig. 116.

In the figure the ground slopes towards the center of the curve; hence the omitted prism is greater than the twice included prism by a doubly truncated triangular prism of right base BNP (plan or section) and whose altitudes are—referring to the plan or the section—GI at B, MO at N, and 0 at P. In the case shown, then, there should be added to each station a quantity equivalent to the volume of this prism. Half the volume will be added if volumes are required by stations, each station being increased by half the correction prism at each end.

This volume may be said to be

$$V_c = \text{area } BNP \times \frac{GI + MO + 0}{3}. \qquad (213)$$

$$\text{Area } PNB = PCDB - PCDN$$
$$= PCDB - PCAQ,$$ from which the student may show

$$\text{Area } PNB = \tfrac{1}{2} c\,(d_1 - d_2) + \tfrac{1}{4} w\,(h_1 - h_2).$$

Letting D be the degree of the curve, and considering full stations,

$$GI = 2\, d_1 \sin \tfrac{1}{2} D \text{ (approx.)}.$$
$$MO = 2\, d_2 \sin \tfrac{1}{2} D \text{ (approx.)}.$$

Substituting these values for *PNB*, *GI*, and *MO*, in Eq. (213) and remembering that $\sin \frac{1}{2} D = \frac{50}{R}$, R being the radius of the curve, there results after a slight reduction,

$$V_c = \frac{100}{3R} \left\{ \tfrac{1}{2} c \left(d_1^2 - d_2^2\right) + \tfrac{1}{4} w \left(h_1 - h_2\right)\left(d_1 + d_2\right) \right\}$$

Substituting for $h_1 - h_2$ its value $\frac{d_1 - d_2}{2s}$, there results

$$V_c = \frac{100}{6R} \left(d_1^2 - d_2^2\right)\left(c + \frac{w}{2s}\right). \tag{214}$$

Substituting for R its approximate value $\frac{5730}{D}$,

$$V_c = .00291\, D \left(d_1^2 - d_2^2\right)\left(c + \frac{w}{2s}\right), \tag{215}$$

or reducing to cubic yards,

$$C_c = .00011\, D \left(d_1^2 - d_2^2\right)\left(c + \frac{w}{2s}\right). \tag{216}$$

The altitudes of the prisms considered being proportional to the sines of the angles, and the sines of small angles being approximately proportional to those angles, it will be sufficiently accurate when less than full stations are involved to use that percentage of C_c that the sum of the two chords meeting at the point of cross section is of two stations; thus if one of the chords is 50 feet and the other 75 feet, the correction would be $\frac{125}{200} C_c$ as given by Eq. (216).

This correction is additive when the higher ground is on the convex side of the curve and subtractive when the higher ground is on the concave side.

The correction will be practically nothing in level sections, will increase as the slope of the ground across the road increases, and will be relatively of most importance in side-hill work, sidings, and double track.

The three forms of side-hill sections that may arise are as shown in Figs. 117, 118, 119.

Since it is usual to measure only excavation, the correction prism for Fig. 117 will have a base CDB, for Fig. 118 BDN, and for Fig. 119 the base may be taken as BDN with altitude at N negative.

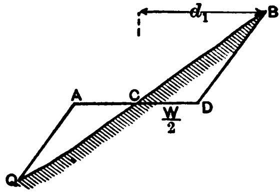

Fig. 117.

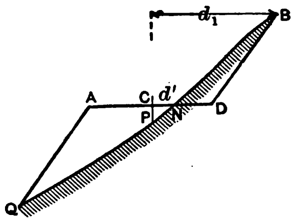

Fig. 118.

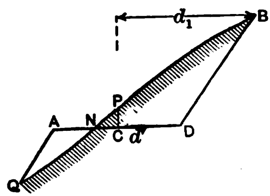

Fig. 119.

The several altitudes may be taken as:

Fig. 117, $.0175\, Dd_1,\quad .0175\, \frac{w}{2} D,\quad 0.$

$$\text{Average } .0058\, D\left(d_1 + \frac{w}{2}\right).$$

Fig. 118, $.0175\, Dd_1,\quad .0175\, \frac{w}{2} D,\quad .0175\, d'D.$

$$\text{Average } .0058\, D\left(d_1 + \frac{w}{2} + d'\right).$$

Fig. 119, $.0175\, Dd_1,\quad .0175\, \frac{w}{2} D,\quad -.0175\, d'D.$

$$\text{Average } .0058\, D\left(d_1 + \frac{w}{2} - d'\right).$$

These values are found by assuming the sine of 1° to be .0175 and that the sines of small angles are proportional to the angles. The student may try to get the results given.

Again, if the chords meeting at the cross section are less than full stations the fraction of the correction to use is found as in the preceding paragraph.

Example. Find the correction for curvature at the cross-section points that gave the following notes, on a 10° curve to the right. The roadbed is 20 feet in excavation, the side slopes 1½ to 1.

Sta.	Left.	C.	Right.	
146	$\frac{-4.6}{16.9}$	-3.6	$\frac{-2.6}{13.9}$	
147	$\frac{-10.8}{26.4}$	-4.2	$\frac{-1.4}{12.1}$	Full station on the curve preceding 146.
148	$\frac{-22.6}{43.9}$	-12.2	$\frac{-3.6}{15.4}$	
+60	$\frac{-18.2}{37.1}$	-2.1	$\frac{0.0}{4.0}$ $\frac{+4.2}{13.3}$	
149	$\frac{-10.0}{25.0}$ $\frac{0.0}{6.0}$	$+3.2$	$\frac{+7.6}{18.4}$	

CHAPTER XII.

EARTHWORK TABLES.

95. Tables for Level Section Volumes. — Tables of earthwork quantities have been made and published to lessen the work of computation. Mention may be made of Allen's, Crandall's, Johnson's, Pullen and Chandler's, Hudson's, Trautwine's, and Rice's. These tables are for computation of level section volumes, or triangular prism volumes used for three-level sections, and are based on both average end area and prismoidal methods of computation. If one finds himself in the field without such tables and is required to make a considerable number of computations, a saving of time will result if tables are computed covering the range of work to be estimated. This is a simple and rapid process, for the most part involving only addition.

By the average end area method

$$V = \frac{A_1 + A_2}{2}\frac{l}{27} = \frac{l}{54}A_1 + \frac{l}{54}A_2. \qquad (217)$$

Referring now to Fig. 120, A, B, C, D and E are station points on the profile shown. When estimates are being

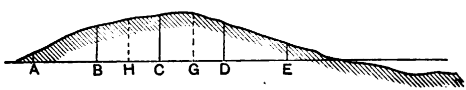

Fig. 120.

made from a preliminary or location profile before the work is staked out for construction, it is customary to measure the center height on the profile midway between two stations as at H and G, and compute the volumes for the full stations, as BC or CD, from areas based on the measured center heights at the middle of the stations, thus assuming in effect the mean area method rather than the average end area method, and giving an equation

$$V = A_m \frac{l}{27}. \tag{218}$$

For such a table l should be 100, the full station length, and Hudson's and Rice's tables were so made. When computations are made from cross-section notes, the center heights at the station points are used, and since the volume between two station points is generally required, the l used in making the tables should be 50. Thus from (217) and (218) the volume for the station is given by either

$$V = A_1 \frac{50}{27} + A_2 \frac{50}{27} \quad \text{or} \quad V = A_m \frac{100}{27}.$$

It will be understood that the volumes obtained by these two methods will not be identical, even if the center heights are equally well obtained and the slope from station to station is uniform. The mean area method always gives volumes less than the prismoidal formula,* while, as has been shown, the average end area method gives results greater than the prismoidal formula.

To make a table for level sections, values are determined and tabulated for the general equation $V = \frac{l}{27} A$, using 100 or 50 for l according to the purpose of the table, and using the center height as the argument — since for a given piece of earthwork it is the only variable on which V depends — thus: for level sections $A = cw + sc^2$ in which for a given piece of work — excavation or embankment — w and s are constants.

Since for level sections $V = \frac{l}{27} A$ and $A = cw + sc^2$,

$$V = \frac{l}{27}(cw + sc^2).$$

* By the method of Art. 177, the student may show that the mean area method gives results less than the prismoidal formula, and that the difference is half as great as that between the prismoidal and average end area methods.

Those familiar with the calculus will recognize this as an equation of the second degree which has a constant second difference. This may be shown by a figure, Fig. 121. Since $\frac{l}{27}$ is a constant factor, only the expression for area will be considered.

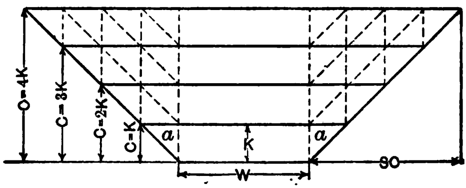

Fig. 121.

$$\text{When} \begin{cases} c = 0 & A_0 = 0, \\ c = k, & A_1 = kw + 2a, \\ c = 2k, & A_2 = 2kw + 8a, \\ c = 3k, & A_3 = 3kw + 18a, \\ c = 4k, & A_4 = 4kw + 32a. \end{cases}$$

Subtracting A_0 from A_1, A_1 from A_2, etc., gives a series of quantities called first differences, which are seen to be

$$\begin{array}{lc} \text{1st Difference.} & \text{2nd Difference.} \\ A_1 - A_0 = kw + 2a & \\ & 4a \\ A_2 - A_1 = kw + 6a & \\ & 4a \\ A_3 - A_2 = kw + 10a & \\ & 4a \\ A_4 - A_3 = kw + 14a & \\ & 4a \\ \text{and so forth} & \end{array} \tag{219}$$

and are variable in value, each being the constant quantity $4a$ greater than the preceding. Thus the second difference, being the difference between successive first differences, is a constant when c, the variable of the area expression, varies uniformly. From any one of the expressions for A and the formula for level section areas it is seen that a of Fig. 121 is

$$a = \tfrac{1}{2} sk^2 \tag{220}$$

and $4a$, the constant second difference, is

$$\text{Area 2nd Diff.} = 4a = 2sk^2,$$

$$\text{Volume 2nd Diff.} = \frac{l}{27}4a = \frac{2l}{27}sk^2. \qquad (221)$$

From (219) it is seen that if the constant second difference be added to any first difference a new first difference is obtained which added to the preceding area or volume gives the next succeeding area or volume. It is usual to tabulate volumes with c as the argument varying by tenths of a foot, so that k of 219, 220, etc., will be 0.1. The first first difference obtained from (219) and (220) is

$$kw + sk^2 = (0.1\,w + .01\,s)\frac{l}{27}.$$

The second difference from (220) is $\frac{2l}{27} \times sk^2 = 2\frac{ls}{2700}$.

Values for w and s may then be assumed and separate tables made for each combination of w and s desired. To begin such a table let $w = 20$, $s = \frac{3}{2}$, and $l = 50$. The first first difference, which is also the volume for $c = 0.1$, is

$$(0.1\,w + .01\,s)\frac{l}{27} = \left(2 + \frac{3}{200}\right)\frac{50}{27} = 3.73 \text{ cu. yd.}$$

The second difference is

$$2\frac{l}{27} \times \frac{s}{100} = \frac{2}{100}\cdot\frac{3}{2}\cdot\frac{50}{27} = \frac{1}{18} = .0555 \text{ cu. yd.}$$

Computations will then be arranged as follows:

$c = 0$	$v = 0.000$		
0.1	$v = 3.73$	3.73	1st 1st diff.
	3.79	.06	2nd diff.
0.2	$v = 7.52$	3.79	2nd 1st diff.
	3.84	.05	2nd diff.
0.3	$v = 11.36$	3.84	3rd 1st diff.
	3.90	.06	2nd diff.
0.4	$v = 15.26$	3.90	4th 1st diff.

and so forth.

Nothing but the bare numerical work need be recorded and much of that may be done mentally. It will be necessary occasionally to check by computation from the full volume formula to see that no cumulative error due to the omission of the final places of the endless decimal has reached a figure worth considering. Every 2 feet 0.01 yd. should be added for this particular table and at 20 feet an additional .01 yd. This is not a case where work to any particular number of significant figures is required, but rather the results should be correct to the nearest $\frac{1}{10}$ cu. yd., that in summing up, the final results of an estimate may appear to be accurate to the nearest whole yard. The final quantities will be arranged in a table as follows:

TABLE OF VOLUMES IN CUBIC YARDS FOR LEVEL SECTIONS WITH VARYING CENTER HEIGHTS.

Roadbed 20 feet. Side Slopes 3/2. Sections 50 feet long.

C	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	*C*
0	0.0	3.7	7.5	11.4	15.3	19.2	23.2	27.3	31.4	35.6	0
1	39.8	44.1	48.4	52.8	57.3	61.8	66.4	71.0	75.7	80.4	1
2	85.2	90.0	94.9	99.9	104.9	110.0	115.1	120.2	125.5	130.8	2
and so forth											

The student may verify the values given.

Level section volumes are used only in preliminary estimates, but are a good deal used for this purpose and hence are valuable.

Since this table is made for a length of 50 feet, it must be entered twice to get the volumes for a full station, once with the center height for one end and again for the center height at the other end. If the volume for a fractional station of l feet is required, the volume for 100 feet is found and multiplied by $\frac{l}{100}\cdot$ Since level sections are used mostly for preliminary estimates, tables of level section volumes are more useful if made directly for 100-foot lengths.

96. Tables for Three-Level Sections.—As before, by average end areas the volume for a whole station is

$$V = \frac{50}{27} A_1 + \frac{50}{27} A_2,$$

$$A = \frac{1}{2} cD + \frac{w}{4}(h_1 + h_2),$$

D being the total surface width, c the center height, and the h's the respective side heights.

Also $$A = \left(c + \frac{w}{2s}\right)\frac{D}{2} - \frac{w^2}{4s}.$$

All four terms of these two equations represent areas of triangles. In general the area of a triangle of base and altitude x and y is $\frac{1}{2}xy$, and the volume of a triangular prism of length l is, in cubic yards,

$$\frac{l}{27} \times \frac{xy}{2} = \frac{l}{54} xy. \tag{222}$$

In the first equation above given c may be x, and D may be y for the first term while $\frac{w}{4}$ is x, and $(h_1 + h_2)$ is y for the second term. Similarly $\left(c + \frac{w}{2s}\right)$ is x for the first term of the second equation, and D is y, while $\frac{w^2}{4s}$ is a constant quantity and need not be tabulated, as it may be quickly computed once for all for any given piece of work. Since $\frac{w}{2s}$ in the parenthesis of the first term is also constant, the use of the second equation may involve less work than the use of the first. For either equation a table of volumes of triangular prisms fifty feet long will suffice. It will be arranged thus;

VOLUMES OF TRIANGULAR PRISMS 50 FEET LONG WHOSE RIGHT SECTIONS HAVE BASES AND HEIGHTS x AND y.

x \ y	0	1	2	3	4	5	6	7	8	9	y / x
0	....	....	....	....	....	....	....	....	....	....	......
0.1	....	....	....	....	....	....	....	....	....	....	......
0.2	....	....	....	....	....	....	....	....	....	....	......
0.3	....	....	....	....	....	....	....	....	....	....	......
0.4	....	....	....	....	....	....	....	....	....	....	......
0.5	....	....	....	....	....	....	....	....	....	....	......
0.6	....	....	....	....	....	....	....	....	....	....	......
0.7	....	....	....	....	....	....	....	....	....	....	......
0.8	....	....	....	....	....	....	....	....	....	....	......
0.9	....	....	....	....	....	....	....	....	....	....	......
1.0	....	....	....	....	....	....	....	....	....	....	......
1.1	....	....	....	....	....	....	....	....	....	....	......

Since such a table must be gotten on the page of a book, it will be convenient to use only whole units for one dimension, while tenths may be used for the other.

If in a given prism one dimension is 7.3 and the other is 24.6, two quantities would be sought both opposite x = 24.6, one under column 7 and the other under column 3. The whole of the first and $\frac{1}{10}$ of the second would be added for the full volume. This gives $24.6\,(7 + 0.3)\,\frac{50}{54}$. If one has not such a table it is very rapidly made by simple addition. Such a table is found in Allen's "Field and Office Tables" and will be in the "Field Book" to follow this volume. One must enter such a table as many times as may be necessary to get the volume for both parts of each 100-foot station. If the station is fractional, of length l, the volume is first obtained for 100 feet and then multiplied by $\frac{l}{100}$.

Examples. 1. Compute a table of triangular prisms for values of x from 6.0 to 12.0 with y from 0 to 9, the length being 50 feet. Let x vary by tenths.

2. If the student has either Allen's or the author's tables, let him, remembering that the volumes are those

of triangular prisms, compute the volumes from the irregular section notes of the example of Art. 87. Compare the result with that obtained by average end areas in the example of Art. 89.

96 a. Allen publishes another table for three-level sections based on the following discussion.

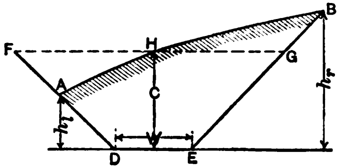

Fig. 122.

The area $ABED$ is the level area $FGED$, with center height that of the three-level section, less the triangle FHA and plus the triangle HBG. The area can thus be shown to be

$$(wc + sc^2) + \frac{1}{2}\left(\frac{w}{2} + sc\right)(h_l + h_r - 2c).$$

Considering a prism of 50 feet length,

$$V = \frac{50}{27}(wc + sc^2) + \frac{25}{27}\left(\frac{w}{2} + sc\right)(h_l + h_r - 2c).$$

The first term is the volume of a level section of height c, and is evaluated as in the table for level sections; the numerical coefficient and first parenthesis of the second term is evaluated for varying values of c and tabulated with the level section volumes as a factor by which the second parenthesis is to be multiplied. Calling this factor K, the table would take the following form:

VOLUMES IN CUBIC YARDS FOR 50–FOOT PRISMS

c	L. Volume level section.	K. Factor in $K(h_l + h_r - 2c)$	L	K	Etc.
0	$\frac{50}{27}(wc + sc^2)$	$\frac{25}{27}\left(\frac{w}{2} + sc\right)$			
1.0					
etc.					

The table would be entered with a given c, L and K found, K multiplied by $(h_l + h_r - 2c)$, usually computed mentally, and the product added to L for the volume of one end of the station. The process is repeated at the next section for the other end and the two sums added for the full station. If the volume is less than a station long, the volume for a full station is first obtained, then multiplied by the factor $\frac{l}{100}$, l being the length of the volume in feet. For use as a table of level sections this table is not so convenient as one for 100-foot prisms, and it is doubtful if it excels a table of triangular prisms for three-level sections.

97. Tables for Prismoidal Correction. — The prismoidal correction formula may be considered to represent the volume of a triangular prism whose right section has base and altitude $(c_1 - c_2)$ and $(D_1 - D_2)$ respectively, the correction in cubic yards being

$$C_p = \frac{1}{3.24}(c_1 - c_2)(D_1 - D_2),$$

when the volume is 100 feet long. This cannot be taken from the table already described for three-level sections since the numerical factor of the xy product is not the same. A special table must be made for it.

CHAPTER XIII.

DIAGRAMS.

98. Diagrams in General. — Every equation of the first or second degree may be represented by a curve in one plane, and this curve may be platted so that values of one of the variables of the equation may be scaled from the drawing for given values of the other variable. Many equations that seem to involve several variables and to be of higher than the second degree may be separated into parts that will permit representation on a single sheet of paper. Thus all of the equations thus far given for volumes may be represented by lines on paper, and the required volumes may be scaled from the drawings when the usual field notes are given. Before discussing these, some general diagrams of use in making various estimates will be shown.

Let the equation $y = ax$ be considered. Y may be simply the product of two numbers, one a constant and the other a variable, or it may be the area of each of a series of rectangles of equal bases a and varying altitudes x, or it may be the area of each of a series of triangles of constant base or altitude, 2 a and variable altitude or base x. The equation is of the first degree and is represented by a straight line through the origin when drawn with reference to two coördinate axes X and Y intersecting at the origin O. If various values of x be laid off from O on the axis X and corresponding values of y be computed and laid off at right angles to X at the points marking

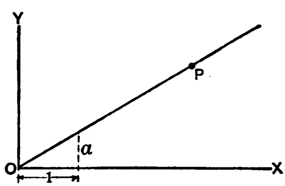

Fig. 123.

the values of x, the extremities of these lines will all lie in the straight line OP, which will pass through O, because when x is 0, y is 0. When x is 1, y is a, and from the considerations of trigonometry a is therefore the tangent of the angle that the line OP makes with the axis of x. Having drawn the line on, say, a piece of cross-section paper, a required value of y for a given value of x is found by measuring the perpendicular from OX to the line at the point on X representing x.

Let it be required to make a diagram for computing acres of right of way taken from various owners. The usual width is 100 feet, and a strip one station long would contain $\frac{100 \times 100}{43560} = 0.2295$ acre. If there are n stations on the land of one owner the acreage taken from him is $A = 0.2295\, n$. To construct a diagram for rapid estimation of a considerable number of takings, a point of origin is selected near one corner of a piece of cross-section paper; a distance along one intersecting line is laid off to represent n to scale — say $\frac{1}{2}$ inch to the station — as long as is likely to be needed to represent the greatest number of stations taken from one owner—say 40 stations, or 20 inches. On the perpendicular at this point to a scale of, say, 1 inch per acre (which will permit estimation to about the nearest $\frac{1}{100}$ acre), sufficiently close for preliminary estimates, the corresponding acreage, from $A = 0.2295 \times 40 = 9.18$ acres or inches, is laid off and a line drawn through the point and the origin.

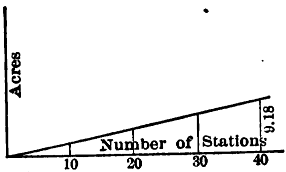

Fig. 124.

An ordinate to this line at any given point x on the axis of n gives the acreage due to x stations when the ordinate is measured to the scale used in laying off acres at the extreme end of the diagram. Thus in Fig. 124 the ordinate at $n = 10$ is one-quarter that at $n = 40$, or 2.295

inches or acres. The scale will not always be unit for unit or half unit for unit as in this case. The scales will be so selected that within the limits of the sheet as many calculations as possible may be made to the required degree of precision, whatever that may be.

An equation of the form $y = axz$, while an equation between three variables, requiring three axes for its representation, can be conceived to be made up of a number of equations of the form $y = abz$, b having different values in the several equations. Any one of these equations may be represented by a straight line as in the preceding case and the series of equations by a series of straight lines. Thus in Fig. 125, if one equation be $y = abz$ and b be zero, the value of y will be zero whatever the value of z or a, and the line representing the equation will be the axis of z. If $b = x = 1$, a line representing $y = az$ will be the diagram. If $b = x = 2$, a line representing $y = 2\ az$ will be the diagram, and so on.

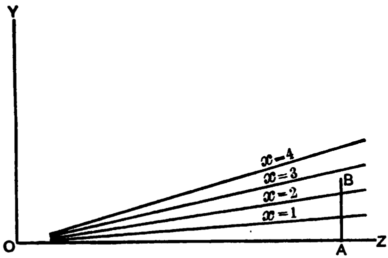

Fig. 125.

Therefore, a diagram for the equation $y = axz$ may be constructed as a series of straight lines with varying values of x, and when a given value of x is fractional, falling between two values that have been assumed in constructing the diagram, the reading must be made by interpolation. For a value of z represented by OA, Fig. 125, and of $x = 2.6$ the value of y would be the ordinate AB. Such a diagram in which a is 1 may be made to give the areas of a series of rectangles of varying sides.

Let a diagram be desired for reading the volumes of cubical stones delivered for some mason work. The dimensions may be taken in feet and decimals, and the volume in cubic yards of any stone will be $V = \frac{xyz}{27}$. First, let $u = xz$ and let a diagram of areas of rectangles be constructed using convenient values for x and z. A little inspection will show that $x = 10$, $z = 9$, and $y = 6$ will give a volume of 20 cubic yards, which will scale conveniently on cross-section paper. Let these values be assumed as the maximum values for constructing the diagram — larger than is likely to be needed. Let 10

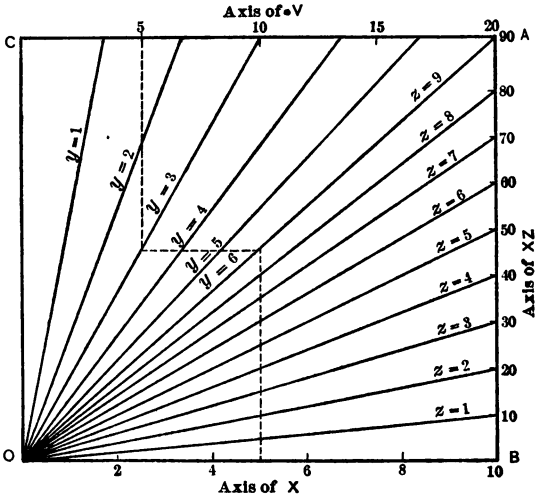

Fig. 126.

units be laid off from O to B in Fig. 126. For $z = 1$, $u = 10$, and so on. From the points thus found on BA, lines are drawn to O, and ordinates to these lines at points on OB representing varying values of x give the values of u corresponding to the given values of z and x when read to the scale used in laying off u on BA. Convenient scales to use will be 1 foot per inch for x and 10 square feet per inch for u. Now $V = \frac{uy}{27}$ and this is diagramed on the axis of u as a base. When u is 90 and y is 6, the volume is 20 cubic yards and the space CA which is 10 inches is called 20 cubic yards, giving 2 yards to the inch for the scale. Since the volume varies directly as y, the space CA may be divided into 6 equal parts and lines drawn from the points of division to O. These lines will represent the several values of y and ordinates to them from the axis of u will represent cubic yards when read to the scale 2 yards to the inch. Let a stone $8.2 \times 5.9 \times 3.4$ be measured. Its cubic content is determined by finding 8.2 on the axis of x following up the perpendicular line to approximately 5.9 as determined by interpolating between z lines 5 and 6, following horizontally to the left to a point $y = 3.4$ found by interpolating between the y lines 3 and 4, and then reading the volumes vertically above on the line CA.

Examples. 1. Let a diagram for estimating acres be drawn on cross-section paper to a scale that will permit reading results directly to $\frac{1}{10}$ and by estimation to $\frac{1}{100}$ acre.

2. Let a diagram for cubic contents of rectangular solids be made to a scale that will permit reading by estimation to $\frac{2}{100}$ cubic yard, and directly to 0.2 cubic yard. Before making the diagram read Art. 105.

99. Diagram for Level Section Volumes. — The equation for level section volumes may be diagramed in two ways. The first that will be given being the less convenient is given only to show how an equation may be divided.

$$V = \frac{50}{27}(cw + sc^2) = \frac{50}{27}cw + \frac{50}{27}sc^2.$$

The whole equation is that of a parabola of axes V and C through the origin and with its vertex at

$$c = -\frac{w}{2s} \quad \text{and} \quad V = -\frac{50}{27}\frac{w^2}{4s}.$$

But splitting it into two, $V = \frac{50}{27}\, cw$ is a straight line and $V = \frac{50}{27}\, sc^2$ is a parabola with vertex at the origin. If the two can be so arranged that the corresponding ordinates measuring V are automatically or graphically added, the diagram will be reasonably convenient. In Fig. 127 let the axes be as shown, with positive values of V measured up for the parabola and down for the straight line. There would be as many straight lines as there are different widths of roadbed to be used, and as many parabolas as there are values of s. The straight line is laid out as in the previously described diagrams. The greatest probable value of c is laid out along OB to B and a perpendicular erected and made equal — at some scale of cubic yards per inch of diagram — to the corresponding cubic yards, using one of the values of w, as for instance BF. The line FO is drawn, constituting the diagram for the one assumed value of w. For the same value of c and one value of s, the term $\frac{50}{27}\, sc^2$ is computed and laid off from B to, say, G, to the same scale used for BF. A parabola is then drawn through G and O constituting the diagram for the parabola term with one value of s. The total ordinate GF is then the total volume for the given c, s, and w; and for any other value of c, as OD, and the same values of s and w, the double ordinate EE' is the volume. There are several ways of drawing the parabola, but probably the best way is to compute volumes for several different values of c, plot these in their proper positions,

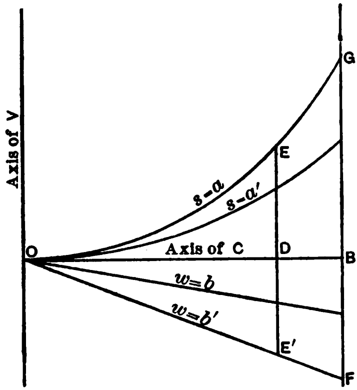

Fig. 127.

and draw the curve through the plotted points by the aid of ship curves or other irregular curves.

But the better way to diagram this equation is to treat

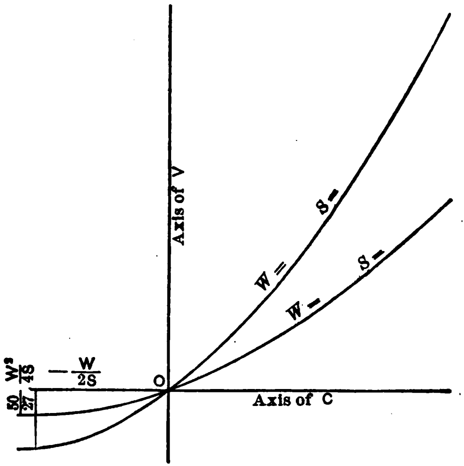

Fig. 128.

it as one parabola as in Fig. 128. The vertex need not be found. The curve can be plotted by points as in the last case. As many curves must be drawn as there are combinations of w and s. The diagram is rarely made by itself, but is usually drawn on a diagram of three-level sections.

Example. Let a diagram be drawn on cross-section paper for $s = \frac{3}{2}$, $w = 20$, using a maximum value of c of 10 feet, and making the volume scale such that single yards may be obtained by estimation and 10 yards read directly.

100. Diagram for Three-Level Sections. — When the prism has a three-level section the expression for volume is

$$V = \frac{50}{27}\left\{\frac{D}{2}\left(c + \frac{w}{2\,s}\right) - \frac{w^2}{4\,s}\right\}.$$

This may be divided into three equations,

$$V_1 = \frac{50}{27}\frac{D}{2}c, \tag{A}$$

$$V_2 = \frac{50}{27}\frac{D}{2}\frac{w}{2s} \tag{B}$$

$$V_3 = -\frac{50}{27}\frac{w^2}{4s}. \tag{C}$$

and each diagramed separately so as to make the summation of $V_1 + V_2 + V_3 = V$ automatic.

A complete separate diagram is required for every combination of s and w. For any one combination terms containing only numerals and s and w are constants.

V_3 is a constant negative quantity, and no matter what the value of c or D it is the same. It may be represented by the constant ordinates to a straight line parallel to the axis of D.

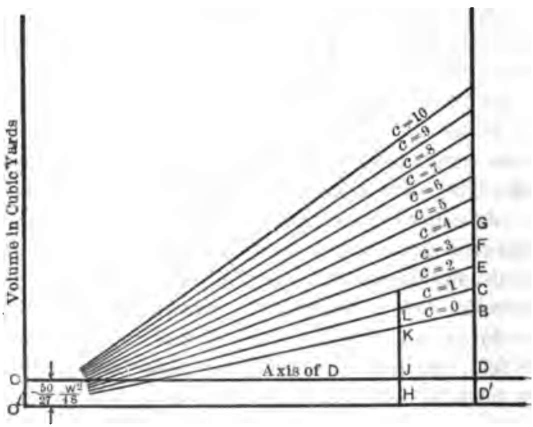

Fig. 129.

The term V_2 is a single straight line, and for the purpose of making the subtraction of V_3 graphically the origin will be shifted temporarily to O'. At D', the largest likely value of D, a perpendicular is erected to B equal to $\frac{50}{27}\frac{D}{2}\frac{w}{2s}$. Then ordinates from the axis OD to the line $O'B$ give the net volume $\frac{50}{27}\frac{D}{2}\frac{w}{2s} - \frac{50}{27}\frac{w^2}{4s}$. The term V_1, having three variables, must be diagramed as a series of straight lines, one for each assumed value of c. If c be 1, $V_1 = \frac{50}{27}\frac{D}{2}$, and this is laid off from B to C. If now a line be drawn from C to O', the part of the ordinate lying between the lines $O'C$ and $O'B$ at any plotted value of D, as OJ, gives the term V_1 for that value of D, and $c=1$. And the whole ordinate JL gives V, the algebraic sum of the three terms, as follows:

$$\begin{array}{cccc} - & JH & + HK & + KL \\ & -\frac{50}{27}\frac{w^2}{4s} & +\frac{50}{27}\frac{D}{2}\frac{w}{2s} & +\frac{50}{27}\frac{D}{2}c. \end{array}$$

For $c = 2$, BE is twice BC, for $c = 3$, BF is three times BC, etc., and from each of these plotted points lines are drawn and numbered with the respective values of c. It should be clear that the ordinate at any point from the original axis of D, OD, to any inclined line gives the volume for the corresponding D and c. In all these diagrams there will be more lines drawn than those corresponding to whole units. Between these lines, corresponding to whole unit values, additional lines will be drawn corresponding to tenths of units, or two-tenths, as the scale and size of the diagram may permit. The diagram is used just as a table to determine quantities for fifty-foot prisms, and each quantity is used twice, once on one side of the section and oncc on the other. The values from two adjacent sections are added to get the volume for a station. For a substation of length l, $\frac{l}{100}$ of the diagram quantity is used and the factor $\frac{l}{100}$ is applied either separately to both quantities obtained for the sections at the two ends, or once to the sum of these two quantities. The latter way makes less work.

To be useful, a diagram like that of Fig. 129 must be made to so large a scale as to be unwieldy, and, moreover, only a portion of the diagram, that portion lying near the curve of level sections, if it were drawn, would ever be used. That is to say, there would rarely be a large D with a small c or *vice versa*, and the volumes of three-level sections do not ordinarily differ widely from volumes of level sections of the same center heights. Therefore this diagram is cut up into pieces which are shifted, each piece being given sufficient numbers to indicate what part of the diagram it is and how it is to be used. Thus if Fig. 130 represents a three-level section diagram on which a curve of level sections for the same s and w has been drawn, only

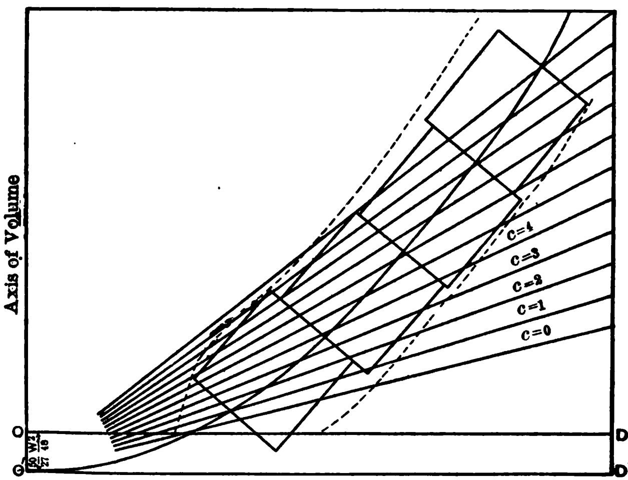

Fig. 130.

that portion of the diagram within the dotted curves would ordinarily be used. This may be approximately marked off into blocks and a new diagram made containing only these blocks arranged one over the other on a single sheet or each one on a page by itself, or two or more together on an inset double page in a book. Such are Wellington's diagrams. Diagrams not too unwieldy can be made to read directly to the nearest 2 cubic yards and by estimation to the nearest cubic yard, which in general is as close as the staking out or the excavation of earthwork warrants.

Example. Let a diagram be made to read volumes, for fifty-foot prisms with three-level sections having side slopes of 3/2, roadbed width of 20 feet and center heights varying from 0 to 10 feet and total widths from 20 feet to 60 feet. (It is suggested that metric paper having the centimeter divided into 5 parts be used, that the scale for D be 1 foot per centimeter, and for cubic yards 10 cubic yards per centimeter. The curve of level sections should be drawn on, as it may be without computation, by noting the points of intersection of the several c lines and corresponding level section D lines. The useful part of the diagram should be blocked out and the blocks traced on tracing paper, cloth, or section paper, one block above or beside another so as to reduce the diagram to its smallest practicable size. It will be noted that the numbering of the coördinate lines of the section paper will be different on the different blocks. Inclined lines drawn for the various values of c should be drawn for each 0.2 foot variation, the whole number lines being heavier than the intermediate lines.)

101. Diagram for Prismoidal Computation. — No convenient diagram for direct computation by the prismoidal formula has been devised as yet. The diagram for three-level sections is used in connection with a diagram for the prismoidal correction. The diagram for prismoidal correction is like that of Fig. 125, since the equation

$$C_p = \frac{1}{3.24}(c_1 - c_2)(D_1 - D_2)$$

is of the form $y = axy$,

a being $\frac{1}{3.24}$; x, $(c_1 - c_2)$; and y, $(D_1 - D_2)$.

Example. Let a diagram be made for the prismoidal correction with $(c_1 - c_2)$ a maximum of 10 feet and $(D_1 - D_2)$ of 40 feet. The paper may be the same as for the example in Art. 98, and the scale may be much enlarged, to, say, 1 cubic yard per centimeter and $\frac{1}{2}$ foot per centimeter for the difference of the D's. The lines for the various values of $(c_1 - c_2)$ should be drawn for each 0.2 foot, the whole foot lines being heavier than the intermediate lines.

102. Diagram for Triangular Prisms.—The formula for the volume of a triangular prism 50 feet long is

$$V = \frac{50}{27}\,\frac{xy}{2},$$

in which x and y are base and altitude of the section. Three-level sections may be computed from a diagram of such an equation just as from a table of such quantities. The diagram would be exactly like that for prismoidal corrections except that the inclination of the lines would be different because of the different numerical coefficients and the larger values of the variables. The scale must be reduced to, say, that used for the three-level section diagram.

One axis would represent $D = y$ of equation (222), p. 195, for three-level section, and separate inclined lines would be plotted for varying values of $x = c + \frac{w}{2s}$. The constant quantity $\frac{50}{27}\,\frac{w^2}{4s}$ must be computed separately and subtracted from the diagram quantities. Since it is the grade prism it may as well be doubled to $\frac{100}{27}\,\frac{w^2}{4s}$ and subtracted from the sum of the two volumes that give the whole station volume, and $\frac{l}{100}$ of it subtracted from the diagram volumes for any substation.

In the equation to be diagramed the quantities are as follows:

$$V = \quad a \quad \cdot \qquad x \quad \cdot \; y.$$

$$V = \frac{50}{27 \cdot 2} \cdot \left(c + \frac{w}{2\,s}\right) \cdot D.$$

$\frac{w}{2\,s}$ is a constant to be added mentally to c when entering the diagram. The diagram is made by assuming the maximum value for D, and at this point on the axis of y erecting a perpendicular, to an adopted scale of volumes, equal to $\frac{50}{27 \cdot 2} \times D$. To make the diagram perfectly general, $c + \frac{w}{2\,s}$ must be treated as a single quantity, and it is the line corresponding to this value and not to c alone that must be found in using the diagram. The diagram may be used for the other three-level equation

$$V = \frac{50}{27 \cdot 2}\,cD + \frac{50}{27 \cdot 2} \cdot \frac{w}{2}\,(h_1 + h_2),$$

entering the diagram twice, once for each term.

Example. Let a diagram be constructed for volumes of triangular prisms to a scale that will permit entering with units given to tenths of feet and reading to 2 yards directly and to 1 yard by estimation.

103. Diagram for Correction for Curvature. The equation for the correction for curvature expressed in cubic yards per station is

$$C_c = .00011\,D\,(d_1^2 - d_2^2)\left(c + \frac{w}{2\,s}\right).$$

For a 1° curve this is

$$= .00011\,(d_1^2 - d_2^2)\left(c + \frac{w}{2\,s}\right),$$

which may be divided into two parts each a parabola with vertex at the origin, thus:

$$C_1 = .00011\left(c + \frac{w}{2\,s}\right)d_1^2,$$

$$C_2 = -.00011\left(c+\frac{w}{2s}\right)d_2^2.$$

A separate diagram must be constructed for each combination of $\frac{w}{2s}$ and will consist of a series of parabolas on axes C_c and d, one parabola for each assumed value of c. The diagram would be entered twice, once for each d, and the difference of the results taken. It must be remembered that the sign has no significance and that the correction is to be added when the higher ground is on the convex side of the center and subtracted when the higher ground is on the concave side of the center. The diagram may be perfectly general, good for any combination of w and s if $\frac{w}{2s}$ be always mentally added to c before entering the diagram and if the parabolas be drawn simply for varying whole-number values of $\left(c+\frac{w}{2s}\right)$ as 1, 2, 3, etc. Thus when $\left(c+\frac{w}{2s}\right)=1$ there is an equation

$$C = .00011\,d^2.$$

When $\left(c+\frac{w}{2s}\right) = 2,$

$$C = .00022\,d^2, \text{ etc.}$$

Each of these represents a parabola. This is the better way to make the diagram. Since the correction is diagramed for a 1° curve the diagram quantities must be multiplied by the degree of curve in any case.

Example. Let a diagram be drawn for correction for curvature for a 1° curve and one for a 10° curve to determine which gives the better combination of scales and reasonable correctness of reading. When the diagram gives the correction for a 10° curve its quantities must be multiplied by that number that expresses the decimal part of 10 that a given curve degree is of 10. Thus for an 8° curve the factor is 0.8 and for a 14° curve the factor is 1.4.

But a simpler diagram may be constructed. Thus the term

$$C_1 = .00011\left(c + \frac{w}{2s}\right)d_1^2$$

may be considered a straight line if d be assumed to be a constant and either c or $c + \dfrac{w}{2s}$ be assumed to be the vari-

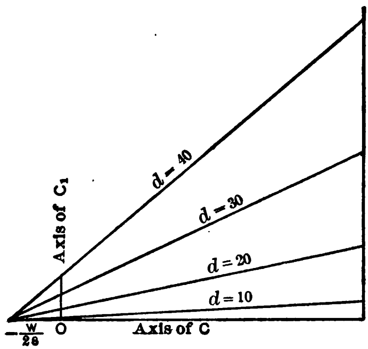

Fig. 131.

able. If c is the variable then the diagram is constructed on axes C_1 and c and separate lines are drawn for all different values of d. When $C_1 = 0$ $c = -\dfrac{w}{2s}$, and all the lines pass through this point. For $d = 1$ a line will be drawn through the points

$$\begin{cases} 0 \\ -\dfrac{w}{2s} \end{cases} \text{ and } \begin{cases} 0.00011\left(c + \dfrac{w}{2s}\right), \\ c \end{cases}$$

c being assumed the largest probable value. The d lines will not be separated by uniform intervals, since for a given c the volume varies as the square of d. In this diagram the length of the ordinate at a given c between the two inclined lines corresponding to d_1 and d_2 is the correction. This may be taken in a pair of dividers or on a piece of paper and applied to the axis of volumes, or a piece of cross-section paper marked on its edge to the scale of volumes may be applied to the diagram and the volume between two d lines read at once.

Example. Let such a diagram be made with $c = 30$ feet as a maximum.

If the horizontal axis be that of $c + \frac{w}{2s}$, the lines will all pass through the origin, one diagram will do for all combinations of w and s and the diagram may be kept a convenient size because $c + \frac{w}{2s}$ need not be assumed at more than say 20 feet, or even 10 feet, since the result varies directly with this quantity and the correction for $c + \frac{w}{2s} = 20$ feet is twice that for $c + \frac{w}{2s} = 10$ feet and for $c + \frac{w}{2s} = 36$ feet the correction is 3.6 times that for 10 feet or 4 times that for 9 feet, etc.

When the correction is to be applied to side-hill work, the diagram of triangular prisms may be used, the average length being found as in Art. 94.

104. Diagram for Preliminary Estimates. — One of the most useful of earthwork formulas has not yet been derived in this book. It is a formula for estimating volumes from center heights and cross-surface slopes rather than from center heights alone — considering the section level across — or from cross-section notes, which are not available for preliminary estimates since the work is not cross-sectioned until the line is located and construction is about to begin.

On preliminary surveys it is often customary for the rodman to carry a clinometer and to lay his rod down across the line at each station or point where the rod is held, place the clinometer on it, read the cross slope in angle or fall per unit of length, and call this slope to the levelman, who records it in his book. If this is not done, the cross slope should be known approximately from the topographer's contour map and will then be known by the distance required to fall a contour interval — usually 5 feet.

It is desirable, therefore, to have an expression that will give areas of sections and therefore volumes when only the center height, which may be scaled from the profile,

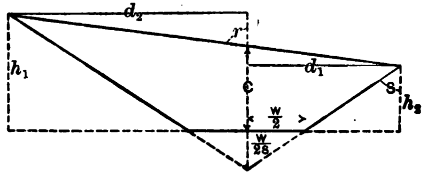

Fig. 132.

and the cross-surface slope are known. In Fig. 132 if s be the slope ratio of the sides and r that of the surface expressed in the same way, $\frac{\text{hor.}}{\text{vert.}}$,

$$c + \frac{d_2}{r} = h_2 = \frac{d_2 - \frac{w}{2}}{s}.$$

Whence
$$d_2 = \frac{\left(c + \frac{w}{2\,s}\right) rs}{r - s}.$$

Similarly
$$d_1 = \frac{\left(c + \frac{w}{2\,s}\right) rs}{r + s}.$$

These may be substituted for $D = (d_1 + d_2)$ in the equation for three-level areas and there results

$$A = \left(c + \frac{w}{2\,s}\right)\left(c + \frac{w}{2\,s}\right)\left(\frac{1}{r - s} + \frac{1}{r + s}\right)\frac{rs}{2} - \frac{w^2}{4\,s}.$$

Letting $c + \frac{w}{2\,s} = C$ and reducing there is obtained

$$A = \frac{C^2 r^2 s}{r^2 - s^2} - \frac{w^2}{4\,s},$$

$$V = \frac{l}{27}\left(\frac{C^2 r^2 s}{r^2 - s^2}\right) - \frac{l}{27} \cdot \frac{w^2}{4\,s}. \qquad (223)$$

If both numerator and denominator of the first term of the right-hand member of equation (223) be divided by r^2s,

$$V = \frac{l}{27}\frac{C^2}{\left(\frac{1}{s} - \frac{s}{r^2}\right)} - \frac{l}{27}\frac{w^2}{4\,s}.$$

And if R be the surface slope angle with the horizontal and S be the side slope angle, then since r and s are respectively the cot R and cot S,

$$V = \frac{l}{27}\frac{C^2}{\tan S - s\tan^2 R} - \frac{l}{27}\frac{w^2}{4\,s}. \qquad (224)$$

Equation (224) is as Mr. Wellington develops it.* Either (223) or (224) may be used for computing the quantities necessary to make a diagram. Choosing (223), a separate diagram must be made for each different value of s, since the first term is all that is diagramed. For a given value of s the first term may be written,

$$V_1 = \frac{ls}{27}\left(\frac{C^2r^2}{r^2 - s^2}\right).$$

If l be 100 and s be $\frac{3}{2}$,

$$V_1 = \frac{50}{9}\left(\frac{C^2r^2}{r^2 - s^2}\right) = \frac{200}{9}\frac{C^2r^2}{4\,r^2 - 9}.$$

If r be assumed a constant, this is the equation of a parabola with vertex at the origin, on axes V and C. Assuming several values for C and one value for r such a parabola is drawn. With the same values of C and another value of r a second parabola is drawn, and so on for as many values of r as are desired. One of the values of C will be the maximum required. It must be remembered that C is $c + \frac{w}{2\,s}$.

The r's may be marked on the several parabolas in degrees as $r = 1°, 2°$, etc., if the slopes are to be taken in degrees, and in this case perhaps it would be better to

* "Computation from Diagrams of Railway Earthwork," A. M. Wellington.

compute the several values for plotting from equation (224). If the slopes are read as slopes, so many feet for a fall of 5 feet or 1 foot, the parabolas may be marked 5 in 100, 5 in 40, etc., or 1 in 20, 1 in 8, etc. As contours are usually drawn at 5-foot intervals and the distance from one to another may be scaled from a map, 5 in x units would be the better marking where such maps are to be used.

In using the diagram it must be remembered that the quantities are too great by the volume of the grade prism. If, as is usual, the quantities are not wanted for each station separately but only for each whole cut or fill, the volume for each station is taken from the diagram, the quantities added for the whole cut or fill, and the grade prism for the entire length of cut or fill subtracted at once. It would be $\frac{100}{27} \cdot \frac{w^2}{4\,s}$ multiplied by the number, whole or fractional, expressing the number of stations in the cut or fill. If it is desired to have the volumes of the stations separately, as it may be in preparing a preliminary mass diagram,* the diagram should be made for 50-foot lengths instead of 100-foot lengths, and each volume taken from the diagram would be used twice, once on one side of the station and once on the other side. From the two parts making the whole station the grade prism for a station would be subtracted.

When only the sum of a series of volumes, as a whole cut or fill, is required, the best way to use this or any other of the diagrams is to lay off the several volume ordinates in succession on a strip of paper, the total length marked being then applied to a long strip on which there has been drawn a scale of volumes like that of the diagram.

Example. Let a diagram of the first term of equation (223) be made for $s = \frac{3}{2}$, maximum $C = 20$ feet and values of $r = 5$ in 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, and let a curve of level section volumes be drawn on it,

* See Chapter XV.

remembering that the expression for level section may be written,

$$V = \frac{l}{27}\left(c + \frac{w}{2s}\right)^2 s - \frac{l}{27}\frac{w^2}{4s}.$$

Only the first term of the right-hand member will be drawn.

105. Suggestions for Making Diagrams. — Diagrams drawn on cross-section paper can be made by ordinary geometrical methods for drawing the lines or curves. Thus, if the diagram is a series of straight lines passing through an origin, but one other point need be found for each line. If the lines are parabolas and the common vertex and one point on each are found, the parabolas may be drawn. But these are not the best ways. The cross-section paper is never uniform in ruling and it is best to compute values for the quantities so that independent plottings can be made at intervals of from 2½ to 4 inches. The lines for whole values of the variable should be drawn first and the intermediate lines interpolated afterward, the work being almost always mechanical. Successive equal values should not be stepped in with dividers, as the result is a cumulative error. Each point should be plotted independently. Owing to the unevenness of the paper, points that should lie in a straight line may not do so, and the drawing-pen must be swayed a trifle back and forth to make the drawn line pass through the required points. The quantities that are to be used in plotting should be first computed and arranged in a table before beginning the diagram. Consideration must be given to the scale of the diagram before beginning it, to the end that on the available paper the quantities with which the diagram is entered and those to be read from it may be found with sufficient precision for the purpose in hand. The diagram must not be too bulky, and two or three sheets of moderate size are better than one sheet of awkward dimensions.

The making of all diagrams may be much simplified by a choice of values; thus the equation for triangular prisms is

$$V = \frac{50}{54} cD.$$

If this is made on a base axis of D, values should be chosen for D that are multiples of 54 to lessen computation. If D is taken as 54, $V = 50\,c$, and nothing but easy mental effort is required to find the values for $c = 1$, 2, 3, etc. The results will always be whole numbers, which may be more accurately plotted than fractional numbers. In the case given, if the volume scale is 100 cubic yards to the inch and cross-section paper divided into inches and tenths is used, values for c varying by single units will be 5 divisions apart, and intermediate values varying by 0.2 of a unit will fall on the graduation points of the paper. Other series of points may be computed by making D successively 5.4 10.8, 16.2, 21.6, 27.0, 32.4, 37.8, 43.2, 48.6, or as many of these values as may be desired. Similarly for the diagram for correction for curvature, if $(d - d)$ is made a multiple of 3.24, the computation is much facilitated. The student will note what values to use for other diagrams to make the computation mechanical, or easy mental work.

CHAPTER XIV.

HAUL.

106. Overhaul Defined. — Railroad earthwork is usually paid for in excavation only, and good practice requires that the fills be made from the adjacent cuts as far as possible. Should the adjacent cuts contain more material than is required in the fill, the latter may be widened with the waste material. The surplus material is called "waste" and the irregular heaps in which it is sometimes piled alongside the cuts are called "spoil banks." It is better to widen adjacent banks than to waste in spoil banks.

When the distance that the material must be hauled from the cut to its new position in the bank exceeds a certain limit (varying with the material, road over which it is hauled, and the kind of vehicle in which it is hauled) it becomes cheaper for the contractor to waste from the cut and build the bank from adjacent or near-by borrow pits. But this practice is not usually permitted by the specifications, which generally require that all the material in the cuts shall be placed in the adjacent banks.

Evidently it would be unfair to company and contractor alike that a stipulated sum per cubic yard should be paid for all material of a given class regardless of the distance it must be hauled, and so it is customary to specify that the price paid for excavation shall include payment for placing the material in the adjacent bank wherever the distance hauled is less than a certain specified limit — varying from 200 feet to 500 feet or even 1000 feet — and that for all material hauled beyond this distance a specified sum per cubic yard per station (100 feet), hauled beyond the limit of free haul, shall be paid in addition to the regular price per cubic yard for excavation.

The quantity on which extra payment is based is called overhaul. It may be explained by the aid of Fig. 133,

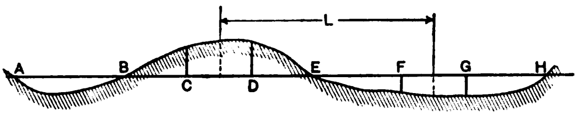

Fig. 133.

which represents a portion of a railroad profile, the straight line being the grade line and the irregular line the surface along the center line of the road. It may be supposed that the portion of cut DE will make the portion of bank EF and that the distance DF is the limit of free haul. It may also be supposed that the portion of cut BC is placed in the bank to the left and that CD is equivalent to FG. Very likely not all of CD gets into the space between F and G and quite likely some of DE does get into this space, but the computations are made on the basis suggested. The positions of the transverse planes D and F are found in various ways to be mentioned hereafter. If V_1 be any small portion of the volume DC, V_2, V_3, etc., other equal small portions, and if l_1, l_2, l_3, etc., be the several distances these portions are hauled from their respective positions in CD to their respective positions in FG, then the distance in stations that V_1 is overhauled is $\frac{l_1 - DF}{100}$; V_2 is

$$\text{overhauled } \frac{l_2 - DF}{100} \text{ stations, etc.}$$

If there be n such small portions in CD the total overhaul is the sum of all the partial products,

$$V\frac{l - \overline{DF}}{100} = V\frac{l_1 + l_2 \text{ etc.} \ldots ln - n\overline{DF}}{100} = V\frac{\Sigma l - n\overline{DF}}{100}$$

and this quantity multiplied by the specified price per cubic yard per station overhauled will give the extra sum due the contractor for moving CD to FG. But it is practically impossible to determine the distance any particular portion is moved, and it is unnecessary, for the same result may be obtained by multiplying the average distance that the whole quantity *CD* is overhauled by its volume in cubic yards. This average distance is the distance *L*, Fig. 133, from the transverse plane containing the center of gravity of the volume *CD* to the transverse plane containing the center of gravity of the volume *FG*, less *DF*, the free haul limit.

Some specifications allow no overhaul until the *average* haul exceeds the haul limit. Thus in Fig. 133, if the center of volume of the portion from *C* to *E* is not more than the limit of free haul from the center of volume of *EG*, there will be no overhaul even though some material has been hauled beyond the specified limit. When this specification obtains, the free haul limit should be shorter than when the method first given is used. Overhaul is somewhat more easily computed by the second method than by the first. The specifications should make clear which method is to obtain.

107. Algebraic Method of Computation. — The only difficulty involved is that of determining the positions of the transverse planes containing the centers of gravity of the two solids. This is readily done when it is known that the distance from one end of any prismoid to the transverse section containing its center of gravity is expressed by the following simple formula:

$$X = \frac{l}{2} + \frac{l^2}{12}\frac{(A_2 - A_1)}{V},* \qquad (225)$$

in which l is the length of the prismoid, A_2 the area of the end farthest from that assumed as the origin, A_1 the area of the end assumed as the origin, and V the volume

* This valuable formula applicable to all prismoids, including all regular solids of revolution, was first developed by J. Woodbridge Davis, C. E. A demonstration may be found in the author's "Notes on Railway Earthwork," published in 1894, and in Allen's "Railway Curves and Earthwork."

It may be demonstrated in the following manner for a frustrum of a triangular pyramid, and the demonstration may be extended if desired by of the prismoid. If the volume is given in cubic yards, the second term, $\frac{l_2}{12}\left(\frac{A_2 - A_1}{V}\right)$, must be divided by 27.

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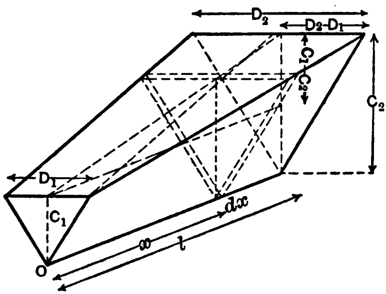
using the expressions for areas of three-level sections instead of those for triangular sections.

The moment of the volume about an axis through O perpendicular to l is VX, in which V is the volume of the solid and X is the distance of its center of gravity from O. The moment of any thin slice at a distance x from O is xA_xdx, in which A_x is the sectional area at x, dx is the small thickness of the slice and x is the distance from O. The sum of the



moments of all of such slices makes the moment of the whole solid, and by calculus this is

$$M = \int_0^l xA_xd_x = VX,$$

$$\text{and } X = \frac{M}{V}.$$

The sign $\int_0^l x$ means "the sum between zero and l of." An expression or A_x must be found to use in the equation for M.

$$C_x = C_1 + \frac{x}{l}(C_2 - C_1).$$

$$D_x = D_1 + \frac{x}{l}(D_2 - D_1).$$

$$A_x = \frac{1}{2}C_xD_x.$$

$$A_x = \frac{1}{2}\left\{C_1 + \frac{x}{l}(C_2 - C_1)\right\}\left\{D_1 + \frac{x}{l}(D_2 - D_1)\right\}.$$

Expanding this, multiplying by xdx and summing between the limits 0 and l by the methods of the calculus gives

$$M = \int_0^l x\,A_xd_x = \frac{l^2}{12}\left(\frac{C_1D_1}{2} + \frac{C_2D_1}{2} + \frac{C_1D_2}{2} + \frac{3\,C_2D_2}{2}\right)$$

Probably the volume CD will be made up of several stations and possibly some substations, each of which is a prismoid. The center of gravity of a series is found by the principle of moments. Referring to Fig. 134 let it be

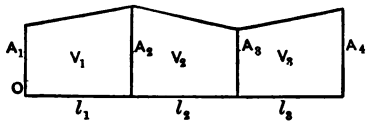


Fig. 134.

supposed first that the center of gravity of each prism is at its mid-section, then the moment of V_1 about O will be $V_1 \frac{l_1}{2}$, that of V_2 about O will be $V_2\left(l_1 + \frac{l_2}{2}\right)$, and of V_3, $V_3\left(l_1 + l_2 + \frac{l_3}{2}\right)$. The sum of these will give the total moment about O, and this divided by the sum of the

---

and this divided by V gives X.

If the center of gravity is assumed to be at the mid length, the erroneous moment would be $V \times \frac{l}{2}$. But

$$V = \frac{l}{6}(A_1 + A_2 + 4\,A_m),$$

hence

$$M_{\frac{l}{2}} = \frac{l^2}{12}(A_1 + A_2 + 4\,A_m),$$

$$A_1 = \frac{C_1D_1}{2},\quad A_2 = \frac{C_2D_2}{2},\quad A_m = \frac{1}{2}\left(\frac{C_1 + C_2}{2} \times \frac{D_1 + D_2}{2}\right).$$

Substituting these values

$$M_{\frac{l}{2}} = \frac{l^2}{12}\left(\frac{2\,C_1D_1}{2} + \frac{2\,C_2D_2}{2} + \frac{C_2D_1}{2} + \frac{C_1D_2}{2}\right),$$

and this divided by V of course gives $\frac{l}{2}$. Therefore the distance of the center of gravity from the mid point is

$$\frac{M - M_{\frac{l}{2}}}{V},$$

which is readily shown to be

$$\frac{\frac{l^2}{12}\left(\frac{C_2D_2}{2} - \frac{C_1D_1}{2}\right)}{V},$$

or

$$\frac{l^2}{12}\left(\frac{A_2 - A_1}{V}\right).$$

If V is in cubic yards the expression must be divided by 27.

volumes will give the average lever arm or distance of the center of gravity of the series from O. If the volumes are all of equal length the sum of the moments is

$$\frac{l}{2}(V_1 + 3\,V_2 + 5\,V_3, \text{ etc.}),$$

and the average lever arm or distance of the center of gravity from O is given by

$$X_{\frac{l}{2}} = \frac{l}{2}\left(\frac{V_1 + 3\,V_2 + 5\,V_3}{\Sigma V}\right),$$

and it may be shown that the distance from this erroneous center of gravity to the real center of gravity is given by an expression like (225), namely,

$$X - X'_{\frac{l}{2}} = \frac{l^2}{12}\left(\frac{A_4 - A_1}{\Sigma V}\right).* \qquad (226)$$

which shows that the same correction holds good for a series of volumes as for a single volume *provided the lengths are equal.* This gives the following rule for finding the center of gravity of a prismoid or a series of prismoids of equal length.

Determine the distance of the center of gravity from one end

* For the series of volumes of Fig. 134 the center of gravity of the whole found by assuming the center of each volume to be at its mid-section is

$$X_{\frac{l}{2}} = \frac{l}{2}\left(\frac{V_1 + 3\,V_2 + 5\,V_3}{\Sigma V}\right).$$

The true center of gravity is at a section whose distance from O is

$$X = \frac{V_1\left(\frac{l}{2} + \frac{l^2}{12}\,\frac{(A_2 - A_1)}{V_1}\right) + V_2\left(\frac{3\,l}{2} + \frac{l_2}{12}\,\frac{(A_3 - A_2)}{V_2}\right) + V_3\left(\frac{5\,l}{2} + \frac{l^2}{12}\,\frac{(A_4 - A_3)}{V_3}\right)}{\Sigma V}$$

and

$$X - X_{\frac{l}{2}} = \frac{V_1\,\frac{l^2}{12}\,\frac{(A_2 - A_1)}{V_1} + V_2\,\frac{l^2}{12}\,\frac{(A_3 - A_2)}{V_2} + V_3\,\frac{l^2}{12}\,\frac{(A_4 - A_3)}{V_3}}{\Sigma V}$$

$$= \frac{l^2}{12}\,\frac{(A_4 - A_1)}{\Sigma V},$$

which was to be shown.

assuming that the center of gravity of each prismoid is in the mid-section. To the result thus found add the product, one-twelfth the square of the length of a single prismoid times the difference of the extreme end areas (always subtracting the first from the last) divided by the total volume of all the prismoids. If the volumes are given in cubic yards divide again by 27.

If the sign of the correction is minus, the real center is nearer the origin than the false center.

The formulas developed involve the areas of the cross sections. Where tables or diagrams are used these areas are not computed and the formulas must be modified to avoid the computation. For a single prismoid of a full station computed by average end areas as prisms

$$V_1 = \frac{50}{27} A_1,$$

$$V_2 = \frac{50}{27} A_2,$$

$$A_1 = \frac{27}{50} V_1,$$

$$A_2 = \frac{27}{50} V_2,$$

$$V_1 + V_2 = V.$$

Then substituting in

$$X = \frac{l}{2} + \frac{l^2}{12}\left(\frac{A_2 - A_1}{27\,V}\right)$$

and making $l = 100$, a full station, there results

$$X = 50 + \frac{100}{6} \cdot \frac{V_2 - V_1}{V},$$

and in general for length l

$$X = \frac{l}{2} + \frac{l}{6} \cdot \frac{V_2 - V_1}{V}, \qquad (225a)$$

in which the V's are the full values for half and for whole lengths respectively. Stated otherwise, the correction to be applied to the half length is

$$C_{100} = \frac{100}{6}\,\frac{V_2 - V_1}{V},$$

$$C_l = \frac{l}{100} \cdot C_{100}$$

in which V_2 and V_1 are tabular or diagram values for 50-foot prisms and V is their sum.

For a series of volumes of equal length it may also be shown that if V_1 is the volume for the first half length and V_n that for the last half length, l is the length of a single volume and ΣV the sum of the volumes, the correction to the center of gravity distance found by assuming each volume to be centered at its mid-section is

$$X - X_{\frac{l}{2}} = \frac{l}{6}\,\frac{(V_n - V_1)}{\Sigma V} \text{ or } X = X_{\frac{l}{2}} + \frac{l}{6}\,\frac{(V_n - V_1)}{\Sigma V}. \quad (226a)$$

In the series of prismoids shown in Fig. 134, if the lengths are not equal and the position of the plane containing the true center of gravity is wanted, the position of this plane for each prismoid must be found, the moment of each volume about one end, as O, determined by multiplying the volume by the distance of its center of gravity from the end assumed as the center of moments, and the several moments must be summed and divided by the sum of the volumes, the quotient being the distance of the center of gravity from the end assumed as the center of moments.

The positions of the planes D and F, Fig. 133, are found by trial. Two points distant apart the limit of free haul are chosen and the quantities of cut and fill between them computed. They will probably not balance, and a second choice of points is made, and a third if necessary until the two points are found between which the cut and fill balance. If the cut is rock, allowance must be made for its swelling, a solid yard of rock excavation making probably $1\frac{2}{3}$ yards of embankment.

Examples. 1. Apply equation (225) to a hemisphere of radius r, getting the distance of the center of gravity from the center of the sphere.

2. Apply the same equation to a cone of altitude h and radius of base r.

3. Find the position of the center of gravity of any of the volumes in the examples of articles 85 and 86.

108. Practical Methods of Computation. — A skilled computer can tell very nearly where the centers of gravity should be by inspection of the quantities, and a rough computation generally suffices, since great precision is not requisite, the price paid being usually from 1 cent to 2 cents a cubic yard per station (100 feet) overhauled. An error of 10 feet in locating the center of gravity of a 1000-yard volume would involve an error of from \$1.00 to \$2.00 in the total payment, and it is doubtful if such a volume is ever executed or measured to the nearest dollar's worth.

One practical way is as follows:

Find the free haul limiting points, and the point bounding the portion of cut to be overhauled and the portion of bank it is assumed to make; make separate profiles of quantities for the overhauled cut and the portion of bank it makes. To do this the volume of each station is laid off on a piece of profile paper as an ordinate at the middle of the station, and a curved line drawn through the extremities of the ordinates. It may be better to erect the ordinates through what appear to be the centers of gravity of the several stations. Cut out each profile thus made and balance it on a knife-edge to find the line through the center of gravity. The positions of the verticals through the centers of gravity of the volume in its two positions being determined, the problem is solved. Somewhat greater precision will result if the scale of distances is considerably enlarged, say to 200 feet to the inch or even 100 feet to the inch according to the length of the volume to be estimated.

109. A Graphical Method. — On the profile the extremities of the portion of cut for which the center of gravity is wanted are marked, as C and D in Fig. 135. Vertically above or below points C and D two profiles are begun, the ordinates to them being measured at each station point

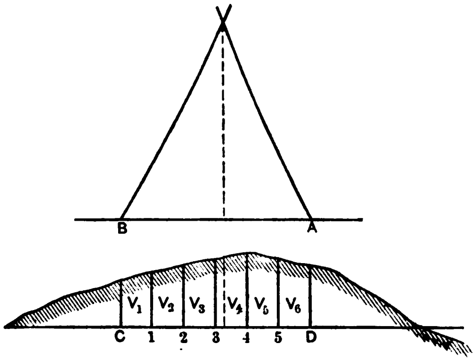


Fig. 135.

proportional to the sum of the volumes to that point; thus at 5 the ordinate is proportional to V_6, at 4 to $V_6 + V_5$, at 3 to $V_6 + V_5 + V_4$, etc.; at 1 the ordinate is proportional to V_1, at 2 to $V_1 + V_2$, etc. Curves are drawn through the tops of these ordinates and their point of intersection will be approximately vertically over or under the center of gravity of the portion of cut plotted. The same thing may be done for the fill.

CHAPTER XV.

MASS DIAGRAMS.

110. General Statement. — It is very desirable that before a contract is let the engineer shall determine the quantities of earth to be moved in each cut and fill, the disposition to be made of the excavation, and the source of the necessary "borrow." This is not always possible, since it seems frequently necessary for a contract to be let covering several hundred miles. This is done under general specifications to secure rapid construction, often following close on the location, by competent contractors of large means and ample equipment.

But even if not determined before the letting of the contract, it is desirable to study the effect of changes of the grade line on the relative quantities of excavation and embankment, and the amount of overhaul, that the most economical arrangement may result. As material is rarely hauled *very* far beyond the free haul distance, and as material from an excavation on one side of a moderate stream crossing is rarely carried to the other side, examination of the profile for quantities and overhaul is made on comparatively short stretches at a time. Such a study is best made by the use of what is usually called the mass diagram. The mass diagram is a profile of volumes made as was the profile of Art. 109 except that it is made continuous from one end of a given section of work to the other covering both cuts and fills. An ordinate at any given point from the base line on which the profile is constructed to the profile is proportional to the total net volume (excavation being considered plus and embankment minus, or *vice versa*) from the beginning of the section to the point in question.

111. Construction of Mass Diagram. — The method of constructing a mass diagram is best shown by explaining the construction for a special case. In Fig. 136 is shown a portion of the profile of a constructed railroad, the surface and grade lines shown being those of a real survey and construction. Above the profile is shown a mass diagram for a portion of the profile. It was constructed as follows: The station quantities were taken from a table of level sections for the center heights shown on the profile and tabulated in the table shown on p. 241, the excavation being considered plus and the embankment minus. (This is what would be done for a preliminary estimate, or an estimate would be made by the method of Art. 104. For a final estimate the notebook quantities from the cross sections would be used and the ditch quantities included.) These station quantities were then summed algebraically and the sums appear in the fourth and fifth columns. Beginning at station 146 these sums were laid off as ordinates to a base line at the points marking the respective stations and plusses, and the irregular line constituting the mass diagram was drawn through the plotted points (only the extremities of the ordinates being plotted). The curve is plotted just as a profile is plotted, using quantities instead of elevations. The curves are drawn on the same paper as the profile, and this scale will be sufficient for a general study of the earthwork; but if overhaul is to be computed from it as explained later, the scale should be enlarged to secure high precision, though in many cases this is not necessary. While tenths of yards are shown in the table, they are useless, since the plotting cannot be done to the nearest yard, and with the scale used in Fig. 136 hardly to the nearest 10 yards.

112. Interpretation of the Mass Diagram. — A knowledge of the method of construction and an inspection of the diagram and corresponding profile will make evident the following characteristics of the mass diagram.

1. The ordinate at any point from the base line on which

the curve is constructed to the curve is to scale the net total volume either plus or minus from the beginning station to the point where the ordinate is taken.

2. Downward slope in the direction of progress of profile construction indicates embankment, and upward slope indicates excavation.

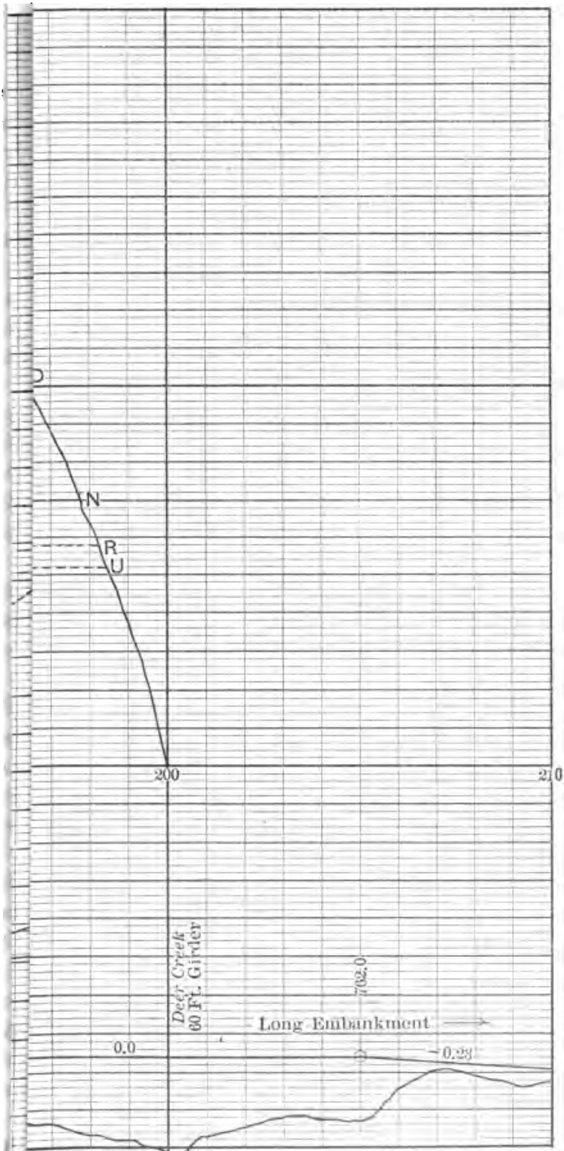
3. Grade points of the profile correspond with maximum and minimum points of the mass curve, the end of an embankment (referring to the direction of progress) and beginning of an excavation always being a minimum point, as *KK′*, Fig. 136, while the end of an excavation and beginning of an embankment is always a maximum point, as *LL′*, Fig. 136.

4. On a given slope — excavation or embankment — the algebraic difference of any two ordinates represents the volume between them.

5. A line parallel to the base line (practically always horizontal) intersecting the mass curve in two points is a balancing line, there being between the two points of intersection equal volumes of excavation and embankment.

When the excavation is rock, which when placed in embankment swells to from 150% to 167% of its original volume, the balancing line is not parallel to the base line, but its direction may be found by making the ordinate from the level of the maximum or minimum points between its extremities to the embankment intersection $1\frac{1}{2}$ to $1\frac{2}{3}$ that to the excavation intersection. The balancing line is not then a line balancing volumes as computed but does balance volumes as constructed. Thus the 1350 ± yards of cut between *A* and *J*, Fig. 136, would make about 2160 yards of bank if the material were rock. The balancing line from *A*, then, should be drawn to a point between *B* and *D* such that the distance below *J* is 2160 yards to the scale of the drawing.

6. The embankment is not always made from the excavation between the two extremities of any balancing line that may be drawn, because the haul may be too



N
R
U
200
210
Deer Creek
60 Ft. Girder
762.0
Long Embankment
0.0
-0.23

great for economy, or a stream too wide to be temporarily bridged may occur. But when the embankment is made from the excavation the haul will be forward — referring to the direction in which the profile proceeds — when a "summit" or "hill" or maximum point is cut off and lies above the balancing line, and backward if a "valley," "sag," or minimum point is cut off and lies below the balancing line. For instance, the haul is forward from *A* to *B* and backward from *T* to *S* if the lines *AB* and *ST* are used as balancing lines.

7. The extreme haul of any particle in a section cut off by a balancing line is the station length between extremities, though no particle may be hauled so far, since the portion of excavation next the embankment may be hauled some distance into the bank, leaving space in the bank near the cut for the most distant part of the cut.

Whether this is done or not the average haul is the same, and the total haul — the total quantity times the average haul — is given by the area of the diagram cut off by the balancing line.

This area must be taken with respect to the scale of the drawing; thus, if the distance scale is 4 stations (400 ft.) per inch and the volume scale is 2000 cubic yards per inch, a square inch of paper represents 8000 cubic yards hauled 100 feet or 1 station, or 8000 yard-stations of haul. This is not *over*haul. It is *total* haul.

113. Computation of Overhaul by Mass Diagram. — Referring to Fig. 136, let it be assumed that the cut from station 177 + 50 is hauled into the bank to about station 196 + 50. As indicated by the two intersections of the base line, which may be considered for the present as a balancing line, the cut and fill (without allowance for shrinkage) are equal between these points; and let it be assumed that under the specifications the limit of free haul is 1000 feet. It is required to find the overhaul.

A balancing line is found, as *AB*, such that its length is 1000 feet. This indicates that the cut from station 184 + 25 makes the bank to station 194 + 25, and as this distance is the limit of free haul, all the material between station 177 + 50 and 184 + 25, and which is represented by the ordinate to 184 + 25, is overhauled. The total haul for this material may be had by measuring the area *ABDC*. The average haul is found by dividing the area by the volume hauled, and this less 1000 feet is the average length of overhaul, which multiplied by the volume hauled is the overhaul to be paid for. More simply stated, the area representing total haul less 1000 times the volume hauled is the overhaul. The area may be estimated by dividing it into small trapezoids or better by the use of a planimeter.

Again, the total area *CJD* represents total haul, the area *EAJBF* free haul, leaving the overhaul in two pieces *CEA* and *FBD*, which may be measured directly with the planimeter, or by division into trapezoids, and added. An approximate method of getting the haul is to multiply the yards hauled, scaled from the diagram, by the average distance hauled, assumed to be the length of a horizontal line drawn through the middle of the yard ordinate to intersection with the mass curve on either side.

Example. Let the student compute the overhaul on the given diagram with the given assumption of free haul and balancing line. It will be shown in the next paragraph that this is not the best arrangement for the work.

114. Other Uses of the Mass Diagram. — There are two principal uses of the mass diagram beside the computation of overhaul: A. The study of the most economical disposition of excavated material when the grade line is fixed; B. The study of the details of the grade line.

A. Let it be supposed that excavation costs the "Company" 21 cents and the free haul is 1000 feet with an overhaul price of 1½ cents; then it is cheaper for the "Company" to move all material from necessary excavations into embankments so long as the maximum haul does not exceed 24 stations, since when the extreme haul equals 24 stations the extreme overhaul is 14 stations, and a yard hauled 14 stations costs as much for hauling as the excavation of a yard "borrowed" within 1000 feet of its place in the bank. Therefore when the extreme overhaul is more than 14 stations, or the total extreme haul is more than 24 stations, it is cheaper to waste surplus material from an excavation, if it can be wasted within 1000 feet, and borrow the necessary extra earth for the bank, than to haul the surplus earth of the necessary excavation to its place in the distant bank. For ultimate economy, therefore, balancing lines should not be longer than the limit of free haul plus the limit of economical overhaul. As has been seen the limit of economical overhaul depends on the relation of the prices for excavation and overhaul. It is true, too, that if some of the necessary excavation that would be wasted by this rule must itself be hauled more than the limit of free haul, the limit of economical overhaul is increased because the price of excavation is in effect increased, for the amount of bank equivalent to the waste, by the cost of the overhaul on the waste. Again, extra right of way may be required from which to excavate "borrow." If this is true, the justifiable length for overhaul is increased up to the point where the cost per yard overhauled equals the cost within the free haul limit plus the cost of the purchased material, which is the cost of the extra property, divided by the number of yards obtained from it.

From the contractor's standpoint that arrangement is most economical that limits the maximum haul to that equaling the *cost* (not price) of excavation. General specifications made for a contract to be let before the disposition of the material is determined should contain a provision that the engineer is to determine the disposition.

Let Fig. 136 be examined for the most advantageous disposition of material. It will be assumed that no material will be hauled across the creek at sta. 150 +. Therefore a mark may be made on the mass curve at, say, 150 + 40, and the computation will begin there. Let a balancing line be drawn from there to the right clear through to sta. 197+75 (about). It will pass above the two small hills at stas. 158+ and 168+ and will intersect the next cut at sta. 176+25 (about). The excavation and embankment between 150 + 40 and 176 + 25 are equal, but the maximum haul is more than 2400 feet, while to the right of 176 + 25 the cut and fill balance and there is no excess over the economical haul limit. If the balancing line is dropped about 600 yards till it just touches the top of the hill at sta. 168+, there will be no excess over economical haul on either end and there will be about 600 yards of borrow at the left end. If the line is dropped still farther till it touches the top of the hill at 158+, there will be no overhaul at all to the left of the long cut intersection at 175 + 50 (about), and the limit of economical haul will not be quite reached to the right, while there will be about 900 yards of borrow at the left end. The whole of the first cut will be moved to the left, backward, and most of the second cut. Only a little (about 1050 yards) of the long cut will be moved to the left. With change of the balancing line downward, the borrow on the left has been increased and that on the extreme right diminished by equal amounts, and the overhaul apparently reduced, first, to a limit within the maximum economical haul, and, second, to none at all on the left of the long cut.

But let the disposition of material and the computation of overhaul be further considered. Let the balancing line *OPQR* be adopted. In general a contractor wishes to open a cut at both ends and work both ways. Assuming that he will do so, some of the cut *VP* will go to the right and some to the left, although the cut *WQ* is sufficient to make the bank *PW* and all of *VP* and *K'L'* is needed to make the fills to the left to *O*. This will necessitate some borrow at the left end near *O*. And if more than 300 yards of *VP* — say *MP* — are put in *PW* there must be more borrow for *L'V* or *OK'* or both, according to the method of disposing of $K'L'$. It is a reasonable specification that the engineer may direct the disposition of material, and may require it to be hauled through or around intermediate work or over streams if the hauling is reasonably favorable and the distance not greater than that making the overhaul price equal the excavation price. Therefore, since there is no haul beyond the economic limit, it will be assumed that the excavation between O and P will make the embankment between the same two points; that WQ will be hauled into PW and QJ into JR. The overhaul from Q to R is easily determined by the methods already given, but there are several possible ways of considering the overhaul to the left of P. The first suggestion would perhaps be that, since MV will make VL' and $L'K'$ will make $K'I$, these two cuts will be so disposed of and MP will be hauled into OI, making 300 yards overhauled about 700 feet, or 2100 yard-stations of overhaul.

But this is not what would be done. $L'K'$ would probably be hauled into OK', not quite filling it, and VP would be hauled into VL' and $K'O$. Considering that the fill to the left of O is to be borrowed, it is reasonable to require that $L'K'$ be put into $K'O$; that MV be put into VK', filling it up for a road on which MP may be hauled to $K'O$ and distributed over the length $K'O$. This would give 300 yards overhauled an average distance of about 500 feet, or 1500 yard-stations of overhaul.

Another assumption is also reasonable.

The fill $L'V$ may not be quite completed from VM, but VP may be drawn partly into VL' and partly into $K'O$. This is what would be most likely to happen, and it is reasonable to assume that the three hundred yards excess of OK' over $K'L'$ will be taken from no particular part of VP but from all along VP, making its center of mass at about sta. 165 + 60 and its center in the bank OK' at about sta. 153 + 25, or a distance apart of centers of 1235 feet, apparently giving 300 yards overhauled $2\frac{35}{100}$ stations or 705 yard-stations of overhaul. But as a matter of fact not all this 300 yards is hauled 1000 feet. That near to V is only about 700 feet from K'. It would be useless to attempt to locate the 300 yards more precisely.

But $K'L'$ could be hauled so as to complete the bank near O, leaving the 300 yards excess to be built near K', say between sta. 153+ and K'. If this were done, then the 300 yards from an average point of sta. 165 + 60 would be deposited in a space with an average point of about sta. 154 + 50, or a total distance of 1110 feet or 300 yards overhauled $1\frac{1}{10}$ stations, or 330 yard-stations of overhaul. But this last method is not quite reasonable, since it would be unnecessarily difficult to haul $K'L'$ to the left and complete the bank from O to 153+ with an abrupt steep end to climb at 153+. The best that could be done would be to make a long incline with its middle point at about 153+, which would somewhat increase the overhaul.

No fixed method of estimating overhaul under such conditions has been established, and there is a large difference between 330 yard-stations and 2100 yard-stations for a 300-yard volume. The excavation may cost \$66.00; 2100 yard-stations at $1\frac{1}{2}$ cents will increase the cost nearly 50 %, while 330 yard-stations will add only about $7\frac{1}{2}$ %.

From the method of estimating already given for adjoining cut and fill by which is found the free-haul limit, within which computed volumes of excavation and embankment are equal, and overhaul is computed for the volumes lying outside the free-haul limit, it might be inferred that a similar procedure should be had here, and that $K'L'$ and VM should be assumed to make the fills next them as far as they would reach, namely, to I and L' respectively, and that MP should be assumed to make OI. There is another consideration that seems to point to this solution. Since $L'K'$ will make IK' and MV will make $L'V$, MP would seem to be essentially "borrow" for OI, particularly as it naturally belongs in PS. Of course the average haul is the same, whatever method is used for estimating the overhaul, and while the contractor would seek to have that method used that would make a maximum pay overhaul, the author thinks that method should be used that corresponds most nearly with what is actually done, and in this case it would probably be the method by which 705 yard-stations are estimated. For the earth taken from a distant cut is essentially borrow, and when borrow is taken from a borrow pit and placed in an embankment the distance from the center of volume in the borrow pit, as nearly as it can be determined, to the center of volume in the bank is found, the free-haul limit subtracted, and the excess multiplied by the volume, for the yard-stations of pay haul. This is what should be done in the case that has been studied.

This case has been considered to show that when the work between the two extremities of a single balancing line includes more than one cut or fill, and possibly some borrow or waste, each portion of excavation must be considered by itself, as will appear more fully in the following.

Measuring roughly from the diagram, the average haul between *O* and *U* is 0.27 stations less for the balancing line *OR* than for *IU*, and the overhaul yard-stations in *IU* are about 5800 more than in *QR*, the total yardage from *O* to *U* being of course unchanged by any change of balancing lines. The borrow is simply changed in position. Therefore what seemed the better balancing line may not be the better. If 705 yard-stations be added for the overhaul in *OP*, then the line *OR* seems to be better for the company by 5095 yard-stations, or about $75.00, than the line *IU*. Whether it is better for the contractor depends on the *cost* of hauling. The average haul on 9090 yards, being 0.27 stations less by *OR* than *IU*, makes 2454 yard-stations less of haul. Even if 2100 yard-stations be allowed in *OR*, *OR* seems to be still the better line.

These rough measurements were determined as follows: Assuming the line *IU*, *OI*, 300 yards, was assumed to be hauled one station — being borrowed probably near by; the ordinate at K', 600 yards, was multiplied by the horizontal line through its mid-point for the yard-stations of $IK'L'$; $L'VM$, MPS, SWT, and TJU were similarly treated; the results were added and divided by the sum of the volume ordinates, 9090 yards, for the average haul. The same method was followed for the line OR, the process giving approximately the areas of the several figures cut off by the balancing line, which areas measure the total haul in yard-stations. This method will in general be sufficiently exact for first estimates; final estimates should be made with a knowledge of where a given lot of material was moved from and to. The volume in the ditches must not be forgotten.

This whole discussion, which brings out the possible differences that may arise between contractor and company, shows clearly that whenever possible the disposition of material and the average haul of each lot should be plainly indicated on the profile submitted to contractors for bids; that there should be no provision for overhaul except for material borrowed outside the right of way from places not possible to determine in advance; and that the specification should provide that the contract price is to include payment for excavating, hauling, and placing in embankment, as indicated on the profile.

Examples. 1. Let the student determine the most economical position for the balancing line and mark on the profile the division lines between material to be hauled in different directions from the excavations, indicating it thus:

2. Let the limit of free haul be only 500 feet and let the most economical position of the balancing line be found. This should show some waste on the left end of the long cut. Let the student determine how much.

B. Let the student study the effect of a slight alteration in the grade line, for instance, making it + 0.5 from sta.

152 to 172, or an even greater change. To do this he will need to recompute the volumes throughout the change and correct the totals beyond the change, and construct a new mass curve. By doing this he will see more clearly what is meant by a study of grade details than he will through any general explanation. If the free haul is only 500 feet a change of the grade line on the right may be advisable to avoid waste.

TABLE OF VOLUMES FOR PLOTTING MASS CURVES.

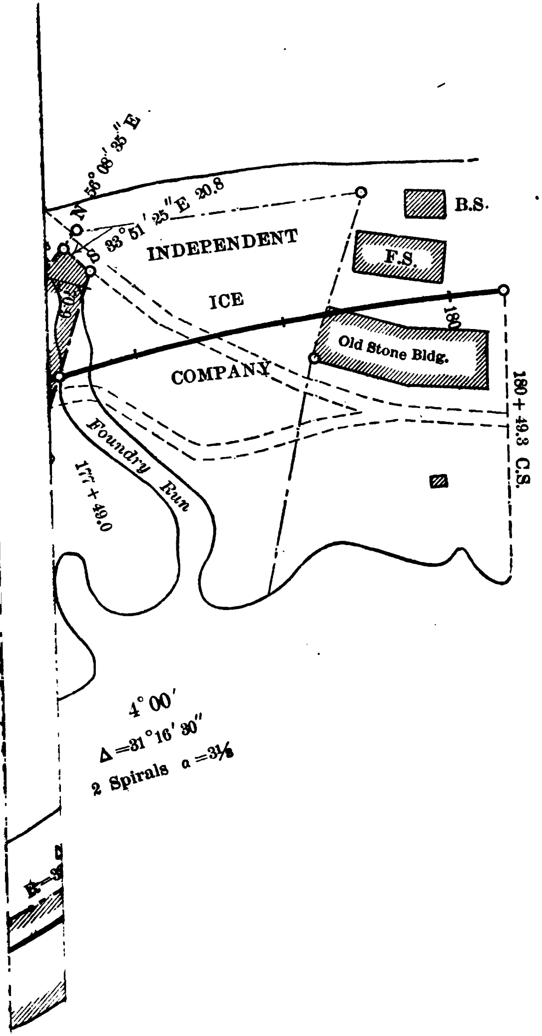
Taken from a portion of railroad profile, Stations 146 to 200.
+ indicates Excavation; –, Embankment.

Station.	Station volumes from notebook or profile.		Sums of station volumes, ordinates to mass curves.	
	+	–	+	–
146		0		0
7		71.2		71.2
8		105.6		176.8
9		294.0		470.8
+80		298.4		769.2
150		192.1		961.3
+25		414.4		1375.7
+85		834.1		2209.8
1		98.9		2308.7
2		294.0		2602.7
3		96.8		2699.5
4		105.6		2805.1
5		151.4		2956.5
+60		52.9		3009.4
6	37.0			2972.4
7	312.4			2660.0
8	244.4			2415.6
+10	3.4			2412.6
9		87.1		2499.3
160		142.0		2641.3
1		272.2		2913.5
2		361.8		3275.3
+80		98.9		3374.2
3	20.8			3353.4
4	57.0			3296.4
5	28.5			3267.9
6	166.4			3101.5
7	354.6			2746.9
8	472.0			2274.1
+50	156.2			2118.7

TABLE OF VOLUMES FOR PLOTTING MASS CURVES.

(Continued.)

Station.	Station volumes from notebook or profile.		Sums of station volumes, ordinates to mass curves.	
	+	−	+	−
9	....	152.5	....	2271.2
170	....	361.8	....	2633.0
1	....	338.6	....	2971.6
2	....	272.2	....	3243.8
3	....	170.4	....	3414.2
+40	....	30.2	....	3444.4
4	92.3	....	....	3352.1
5	257.0	....	....	3095.1
6	1244.4	....	....	1850.7
7	1345.8	....	....	504.9
8	1126.4	....	621.5	....
179	711.2	....	1332.7	....
180	231.2	....	1563.9	....
1	57.1	....	1621.0	....
2	192.0	....	1813.0	....
3	166.4	....	1979.4	....
4	33.8	....	2013.2	....
5	104.5	....	2117.6	....
6	192.0	....	2309.6	....
7	354.6	....	2664.2	....
8	457.0	....	3121.2	....
9	298.6	....	3419.8	....
+25	5.6	....	3425.4	....
190	....	47.1	3378.3	....
1	....	114.6	3263.7	....
2	....	96.8	3166.9	....
3	....	294.0	2872.9	....
4	....	644.4	2228.5	....
5	....	806.0	1422.5	....
6	....	948.2	474.3	....
7	....	1116.6	....	642.3
8	....	1205.0	....	1847.3
9	....	1390.2	....	3237.5
200	....	1688.8	....	4926.3


N 56° 08' 35" E
S 33° 51' 25" E 20.8
INDEPENDENT
ICE
COMPANY
B.S.
F.S.
180
Old Stone Bldg.
180 + 49.3 C.S.
Foundry Run
177 + 49.0
4° 00'
Δ =31° 16' 30"
2 Spirals a =3⅛

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