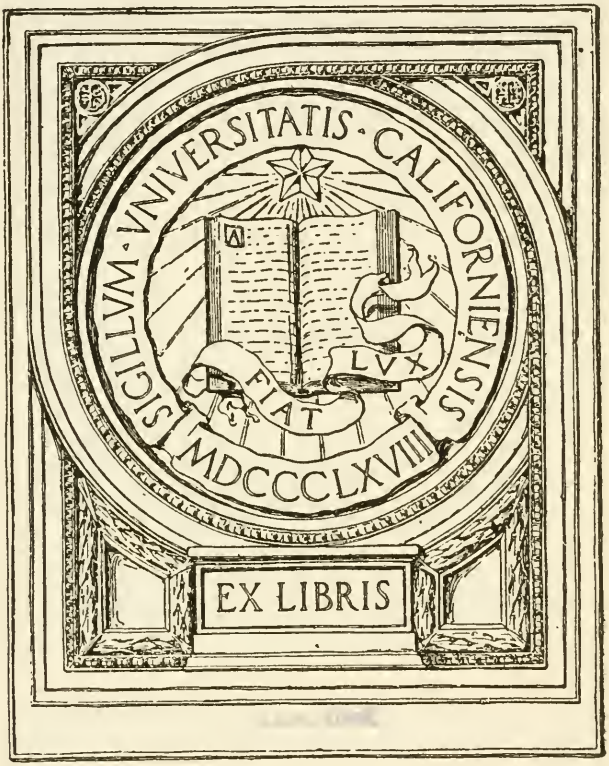
RATIONAL ARITHMETIC

By George R. Lord



RATIONAL ARITHMETIC

COMPLETE

BY

GEORGE P. LORD

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PREFACE

Rational Arithmetic is intended for use in business colleges, and in commercial high schools, by pupils who have completed the equivalent of the eighth or ninth grade in the public school system.

While deficiencies of early training may be remedied by its use, it is not intended as a textbook for those who are approaching the subject for the first time. Neither is it intended to take the place of any of the many excellent works now in use in the grades for the purpose of developing a general understanding of mathematical principles. Such books, while they have satisfactorily discharged this function, have failed to develop the accuracy and facility so vitally essential in commercial calculations.

Other commercial arithmetics have tried to overcome this weakness by following similar plans of instruction in abridged form. Rational Arithmetic follows a very different plan. It is purely a vocational work and aims to teach the "how" rather than the "why." It is a reference book of commercial operations, rather than a method of presentation, and should be so used.

Part One is a collection of practice exercises arranged along the lines of the generally accepted order of presentation.

Part Two contains illustrated solutions covering the entire range of commercial arithmetic as generally understood. The methods used are those of business. The explanations are expressed in language which may be understood easily, rather than in the more scholarly language usually employed.

References throughout the book are by paragraph numbers, which will allow the pupil to ascertain for himself the best method of solving any desired problem.

The aim has been to produce a book so elastic that the teacher may arrange a course of study to suit himself. The author has found it advisable, however, to start pupils with the subject of balancing accounts which arouses their interest and gives them something new and practical.

Drill on decimals should immediately follow this, for the purpose of developing accuracy in locating the decimal point.

The advisability of work on the subject of fractions depends entirely upon the attainments of the individual pupil. The writer has found that fully 75 per cent of his pupils are greatly benefited by taking up this subject before beginning the strictly commercial work which commences with the subject of aliquot parts.

It is suggested that the problems in addition at the beginning of Part One be used as drill problems throughout the course.

Teachers will not find it necessary to use all the problems provided for each subject. The aim has been to give enough problems to meet any demand that may arise.

No claim is made for originality in any of the methods presented. Every method that appears in this book may be found, in some form, elsewhere. To give credit to the sources from which the author has obtained assistance in the compilation of this book would be to name all the textbooks consulted by him in an experience of nearly thirty years as arithmetic teacher.

GEORGE P. LORD

SALEM, MASS.

CONTENTS

PART ONE

PART TWO

RATIONAL ARITHMETIC

PART ONE

The following problems are intended to afford sufficient practice to develop a thorough working knowledge of practical business arithmetic.

An effort has been made to confine the problems as far as possible to actual business conditions and to present only problems similar to those met in actual business experience.

References are by paragraphs to Part Two and are sufficiently copious to allow ready solution by the pupil of all problems.

PRELIMINARY PROBLEMS

ADDITION

The following exercises in addition afford opportunity for frequent drills. The pupil should practice upon them and similar problems provided by the teacher throughout the course, or until he is able to add in the time specified, or in less time.

Study carefully paragraphs 96 to 101 inclusive.

1. Practice until you are able to add each of the following in 15 seconds or less:

1	2	3	4	5	6
324	196	596	287	812	285
436	289	321	422	263	635
243	781	284	389	426	149
429	423	675	674	529	728
182	317	329	263	198	463
327	262	918	721	984	824
148	425	786	416	623	296
283	348	465	129	467	179

2. Practice until you are able to add each of the following in 25 seconds or less:

7	8	9	10	11	12
324	463	247	472	289	521
642	721	962	749	394	347
285	567	721	638	672	625
763	289	463	236	416	262
297	143	265	429	781	729
425	264	789	642	186	453
642	721	496	187	237	642
193	168	721	346	421	287
721	459	453	721	563	472
438	672	624	254	464	296
267	284	289	689	789	563
193	596	198	746	462	189

3. Practice until you are able to add each of the following in 45 seconds or less:

13	14	15	16	17
2864	3829	4962	8426	5479
4233	6471	2794	7195	6294
7185	1687	6171	3824	1781
1679	2762	2437	6271	3326
2763	4463	8263	2617	7182
4638	9174	2819	5409	4963
9162	2896	6279	6276	7126
2746	4789	2854	2830	4413
8427	6217	1962	3418	7824
1679	2834	7148	9016	1671
2634	9753	6523	2468	5607
4791	2891	3764	1695	2896

4. Practice until you are able to add each of the following in 60 seconds or less:

18	19	20	21	22
264118	528563	428137	252763	427183
428307	742896	298461	376329	284562
711695	478132	541672	167251	711456
386472	264389	832744	146327	378275
369143	146227	167182	421791	462871
642785	584296	322907	573619	146265
617192	817529	541891	287513	551681
548237	428127	851693	324671	287354
167589	362419	395816	271293	851762
294462	780962	724594	348162	718319
162781	278438	280790	912872	440892
146229	261971	642031	268047	632757
382716	446236	451682	634918	642819

5. Practice until you are able to add each of the following in 75 seconds or less:

23	24	25	26	27
416342	487902	284062	614385	812716
913457	226439	713345	422716	341675
296731	914362	167182	312814	472386
284562	167943	421671	567583	611743
811706	208209	284371	642217	296342
273468	613317	176327	551638	542138
296329	672438	420416	281954	617516
284672	719243	798296	371621	271642
542983	264738	146329	560932	182133
718296	194513	817043	174837	162904
287981	382761	241671	165329	816238
273468	280642	146329	241671	271609

28	29	30	31	32
24627	93281	27682	14632	91387
48231	44638	38225	71136	26785
62783	71289	16781	28483	54321
24167	45642	91483	43729	28654
85262	71483	82675	52847	43832
71843	19721	46721	37625	71819
29636	54163	53482	54783	44623
71083	27386	27624	29654	48729
27642	16721	14729	18729	17453
29827	28294	61453	47387	37529
16429	26783	27185	26475	62745
54540	54296	16291	83267	71258
68296	78287	54385	29453	54183

33	34	35	36
$6743.76	$3462.78	$4527.82	$7345.60
2846.75	5287.95	3675.18	2847.29
8421.62	6379.86	2643.89	5640.36
7329.44	7429.80	1796.97	7281.28
3780.50	5463.29	4238.54	1267.43
2894.62	7128.42	2879.36	3629.75
7481.13	3864.19	7481.29	4678.37
2563.27	7133.64	3279.81	8126.42
6247.16	4461.72	4782.63	3726.42
3729.42	9562.70	2871.54	4671.38
8427.16	4683.49	7627.18	6275.29
2945.71	7216.30	2375.62	5463.72
4453.75	3200.50	4671.16	2963.47

37	38	39	40
$3678.44	$5617.81	$2896.75	$8297.50
2917.53	2976.37	4429.36	6384.96
6279.45	3728.44	1785.29	7185.16
1468.71	1671.38	6271.38	4183.94
2783.94	5418.75	1862.45	5467.28
7162.85	2763.48	5483.92	4392.40
1671.36	6378.27	2861.57	5671.39
2768.29	5423.75	4675.38	2716.42
5419.75	1627.30	9863.75	3895.16
1683.27	4862.19	5728.50	5482.95
4462.91	7143.84	6275.81	7436.81
2789.65	3627.52	3829.62	2918.15
3601.82	5462.79	7426.85	4183.95
4618.79	2183.96	5387.09	6343.19
4844.60	4528.17	6425.70	2874.67

6. Practice until you are able to add each of the following in 90 seconds or less:

41	42	43	44	45
283	487	275	438	219
347	365	381	622	736
462	791	456	563	432
728	384	179	729	275
529	429	527	247	863
633	186	623	384	179
387	795	819	862	327
472	324	942	721	453
694	432	163	903	618
458	175	209	415	721
182	293	725	541	153
791	764	483	287	286
453	379	342	453	723
645	453	671	628	429
286	186	538	143	186
729	791	827	517	791
452	428	113	386	917
628	384	452	295	283
139	292	938	672	418
255	574	286	286	726
386	387	721	295	814
675	198	453	721	428
341	421	618	193	297
296	502	721	382	548
416	218	453	453	353
209	721	618	182	287
381	347	721	763	186

7. Pupils who have acquired proper facility in the preceding problems will have no difficulty in performing the following with sufficient rapidity:

$1283.64	$ 284.16	$ 94.43	$3728.54
785.30	1728.32	2168.75	6241.57
2721.83	4278.19	1413.80	287.63
1763.20	2674.19	287.60	729.42
487.32	905.16	782.19	98.17
9.05	728.40	3187.42	46.54
87.62	7.14	4163.27	19.72
44.33	287.64	973.29	83.71
1286.75	4239.17	4871.30	129.40
343.06	2476.28	2972.43	987.60
428.06	287.19	187.62	2763.42
71.37	458.16	98.43	1486.71
289.70	6724.13	642.19	4938.27
2729.40	8.60	3894.16	9132.38
2876.42	278.40	721.32	764.20
29.00	4291.62	193.64	381.65
427.40	71.85	4287.16	28.53
9.90	13 63	1328.72	9.19
727.62	2842.75	46.84	2.85
3478.24	4871.20	13.16	642.38
9288.75	389.45	9.68	94.73
2471.05	642.16	171.24	2652.81
287.63	71.00	283.95	453.17
274.28	9 80	90.60	287.29
297.62	2621.13	468.13	4182.19
48.39	38.79	71.24	78.50
184.19	178.63	457.62	1246.53

8. Complete the following statement by finding the totals of columns and the totals from left to right:

	Corn	Wheat	Oats	Apples	Beans	Flour	Totals
Mar. 1	284.65	487.62	125.30	285.64	697.60	48.60	
2	78.39	629.34	326.45	171.80	721.80	75.60	
3	164.70	587.19	217.16	219.17	450.67	308.00	
4	348.62	984.63	98.43	287.60	385.90	9.50	
5	175.29	729.40	454.20	58.09	287.40	24.72	
7	98.40	1180.68	90.12	171.23	458.62	317.80	
8	467.28	960.27	72.15	264.12	721.30	240.16	
9	298.70	450.18	248.00	175.11	318.90	421.75	
10	587.64	829.30	316.90	147.16	427.65	48.90	
11	298.63	486.90	109.09	209.04	386.37	97.50	
13	728.45	752.83	402.14	348.17	516.80	105.12	
14	687.44	647.92	48.30	250.60	428.54	215.70	
15	285.90	473.85	178.85	658.12	719.80	84.95	
16	429.18	738.24	148.16	347.60	579.60	198.60	
17	697.65	287.16	242.90	795.80	387.40	385.00	
18	597.67	587.90	371.42	721.80	473.20	95.16	
20	738.42	699.48	416.78	424.70	517.68	171.20	
21	914.16	826.16	212.13	368.40	429.90	384.60	
22	576.80	487.28	98.42	519.67	343.80	142.80	
23	894.48	745.60	248.16	295.48	624.39	218.70	
24	1098.27	952.75	500.00	487.29	387.26	140.68	
25	384.62	397.26	187.19	368.12	419.72	214.09	
27	725.30	678.40	238.16	563.27	611.41	190.54	
28	456.82	1238.50	71.45	448.12	516.75	138.20	
29	689.63	927.24	122.58	219.60	218.19	97.42	
30	521.16	627.42	108.12	348.75	365.40	164.83	
31	782.73	795.38	314.60	218.90	409.08	120.95	
Totals							

SUBTRACTION

While no practice seems necessary on simple subtraction of integers, the pupil should read carefully 102 to 109 inclusive, and then balance the following

accounts according to the method explained in 108 and 109.

9. Balance the following accounts:

1. *Dr.* QUAKER OATS CO. *Cr.*

$22.50	$76.50
21.40	16.75
1.79	39.75
79.90	23.

2. *Dr.* D. T. AMES & CO. *Cr.*

$185.25	$603.75
8.	215.50
2.10	73.94
	121.50
	24.

3. *Dr.* INTEREST *Cr.*

$1.	$ .68
7.70	9.21
2.52	10.
3.70	1.40
1.12	

4. *Dr.* F. B. SMITH *Cr.*

$ 6.75	$33.45
3.75	6.
14.50	10.25
12.25	5.40
	13.25

5. *Dr.* SALES *Cr.*

$12.20	$ 6.60
4.65	1143.75
5.10	843.19
4.85	6.
	8.
	5.50

6. *Dr.* PARK & STEWART *Cr.*

$1021.65	$589.05
964.41	56.40
558.72	236.25
280.	661.50

7. *Dr.* JAMES CARTER *Cr.*

$ 1.91	$278.60
716.	956.20
733.13	8.20
588.03	9.16
436.	833.70
	47.31

8. *Dr.* BLANCHARD & CO. *Cr.*

$ 16.50	$ 3.60
7.64	10.80
18.50	7.75
219.21	19.60
12.50	
4.23	

9. *Dr.* F. G. MILLER *Cr.*

$63.60	$ 4.17
25.	.60
14.50	140.
25.	207.50
21.15	55.
16.67	9.
	61.50

10. *Dr.* CHARLES SMITH *Cr.*

$ 76.	$ 51.01
98.55	60.03
83.02	171.
59.20	80.61
105.08	2.10
43.20	210.33
13.10	
29.76	

Dr. H. A. THOMAS *Cr.*

$1240.50	$ 500.
876.	750.
453.35	1250.
96.73	2575.
1000.	
354.56	

Dr. SAWLER & HASKINS *Cr.*

$1246.34	$2354.56
878.14	3143.42
2543.65	1245.15
3746.32	1873.94

DECIMALS

MULTIPLICATION

The object of these exercises is to acquire accuracy in locating the decimal line.

References 133, 134, 135.

10.

1. $.974 \times .35 =$
2. $8.2 \times 9.6 =$
3. $284 \times .75 =$
4. $346.5 \times 10.02 =$
5. $27 \times .38 =$
6. $.1655 \times 18 =$
7. $.4355 \times 16.66 =$
8. $844.5 \times 7.404 =$
9. $1263 \times 8.04 =$
10. $.44 \times 8.05 =$
11. $65.410 \times .585 =$
12. $75000 \times .0098 =$
13. $5.9 \times 26.7362 =$
14. $75 \times 6.0053 =$
15. $3.926 \times 464 =$
16. $4.872 \times .386 =$
17. $84672 \times 8.4 =$
18. $2.8973 \times .80806 =$
19. $.94 \times 100.82 =$
20. $446.8 \times 3.044 =$
21. $287 \times 9.0104 =$
22. $.9634 \times 58 =$
23. $283.86 \times .396 =$
24. $94.652 \times 4.87 =$
25. $84 \times .000238 =$

DIVISION

Study 136 to 141 inclusive.

In solving the following problems make strict application of the rules given in 137 and 138.

Paragraphs 139, 140, and 141 fully illustrate the method of solution.

11.

1. $14.875 \div 3.5 =$
2. $338.52 \div 8.4 =$
3. $1385.128 \div 21.6 =$
4. $3.456 \div 12 =$
5. $654.5 \div 11 =$
6. $2.464 \div 1100 =$
7. $43.4172 \div 74.6 =$
8. $.01581 \div .255 =$
9. $6.305 \div 3.25 =$
10. $8.63 \div 3.84 =$
11. $79.896 \div .53264 =$
12. $372.012 \div 58 =$
13. $14.157 \div 2.6 =$
14. $14.157 \div .26 =$
15. $1284.7 \div .3875 =$
16. $246.9 \div 23 =$
17. $1640.625 \div 1875 =$
18. $286.996 \div .914 =$
19. $17.408 \div 12.8 =$
20. $6264 \div .348 =$
21. $14.825 \div 8.29 =$
22. $286.327 \div 156 =$
23. $16.38 \div .284 =$
24. $284.62 \div 84 =$
25. $29.728 \div 8.4 =$
26. $1.75 \div 23.5765 =$
27. $437.8675 \div 23.8 =$
28. $23.183 \div 19.7463 =$
29. $246. \div 7.875 =$
30. $75 \div 125 =$
31. $1.8675 \div 5.75 =$
32. $124.56 \div 15.8 =$
33. $48567.75 \div 4.875 =$
34. $76.50 \div 1250 =$
35. $.5863 \div 12.52 =$
36. $.9875 \div .584 =$
37. $23.45675 \div 1.375 =$
38. $23. \div 27 =$
39. $125. \div 56 =$
40. $324. \div 678 =$
41. $12.875 \div 4.25 =$
42. $125.682 \div 7.75 =$
43. $415.875 \div .1275 =$
44. $234.15 \div 5.875 =$
45. $56.625 \div 128.50 =$
46. $153.8756 \div 53.962 =$
47. $346.4278 \div 8.4695 =$
48. $16.4584 \div 3.4565 =$
49. $125.4632 \div 18.4965 =$
50. $4356.4589 \div 27.4875 =$

FRACTIONS

Study carefully 160 to 180 inclusive.

References 181, 182, 183.

12. Reduce the following to lowest terms:

1. $\frac{720}{1200}$
2. $\frac{630}{2835}$
3. $\frac{910}{1040}$
4. $\frac{105}{231}$
5. $\frac{210}{455}$
6. $\frac{35}{45}$
7. $\frac{670}{1608}$
8. $\frac{675}{1200}$
9. $\frac{115}{782}$
10. $\frac{232}{551}$
11. $\frac{1\frac{1}{3}}{9}$
12. $\frac{4\frac{4}{5}}{12}$
13. $\frac{3\frac{2}{3}}{8}$
14. $\frac{7\frac{1}{5}}{25}$
15. $\frac{6\frac{4}{5}}{18}$

References 184, 185, 186.

13. Change the following fractions to the denominations designated:

1. $\frac{4}{5}$ to 125ths.
2. $\frac{7}{8}$ to 56ths.
3. $\frac{4}{9}$ to 72ds.
4. $\frac{8}{13}$ to 195ths.
5. $\frac{5}{7}$ to 84ths.
6. $\frac{5}{8}$ to 88ths.
7. $\frac{7}{12}$ to 1524ths.
8. $\frac{3}{7}$ to 126ths.
9. $\frac{8}{11}$ to 352ds.
10. $\frac{5}{9}$ to 27ths.

References 187, 188, 189.

14. Change the following to mixed numbers:

1. $\frac{27}{8}$
2. $\frac{123}{5}$
3. $\frac{76}{9}$
4. $\frac{124}{5}$
5. $\frac{515}{16}$
6. $\frac{323}{15}$
7. $\frac{427}{9}$
8. $\frac{125}{7}$

References 190, 191, 192.

15. Change the following to improper fractions:

1. $1\frac{2}{3}$	4. $9\frac{4}{7}$	7. $81\frac{7}{9}$	10. $214\frac{5}{8}$
2. $4\frac{1}{3}$	5. $23\frac{5}{8}$	8. $37\frac{5}{8}$	11. $34\frac{5}{16}$
3. $8\frac{5}{9}$	6. $46\frac{4}{9}$	9. $123\frac{4}{5}$	12. $43\frac{4}{5}$

References 193, 194, 195.

16. Change the following to common fractions or mixed numbers:

1. .375	5. .15625	9. $.66\frac{2}{3}$
2. .875	6. 8.25	10. $.272\frac{8}{11}$
3. .0625	7. 17.875	11. $1.77\frac{7}{9}$
4. .125	8. 9.28125	12. $.384\frac{8}{13}$

References 196, 197, 198, 199, 200.

17. Change the following to decimal equivalents:

1. $\frac{3}{4}$	5. $\frac{4}{9}$	9. $\frac{11}{17}$	13. $8\frac{7}{8}$
2. $\frac{4}{5}$	6. $\frac{5}{6}$	10. $\frac{9}{23}$	14. $9\frac{4}{13}$
3. $\frac{5}{8}$	7. $\frac{7}{8}$	11. $3\frac{5}{9}$	15. $12\frac{3}{8}$
4. $\frac{11}{13}$	8. $\frac{9}{13}$	12. $4\frac{5}{6}$	

ADDITION OF FRACTIONS

References 201, 202, 203, 204, 205.

18.

1. $\frac{3}{4}+\frac{5}{8}=$
2. $\frac{7}{8}+\frac{4}{5}+\frac{1}{2}=$
3. $\frac{1}{13}+\frac{3}{24}+\frac{5}{18}=$
4. $12\frac{5}{8}+8\frac{3}{4}+27\frac{2}{3}=$

5. $14\frac{7}{8}+5\frac{3}{4}+23\frac{7}{10}+81\frac{1}{12}=$
6. $21\frac{5}{8}+46\frac{17}{32}+29\frac{5}{9}=$
7. $628\frac{3}{16}+56\frac{5}{8}+16\frac{1}{8}+30\frac{7}{12}=$
8. $34\frac{1}{8}+26\frac{3}{4}+91\frac{8}{10}+63\frac{2}{3}=$
9. $4\frac{5}{9}+27\frac{4}{27}+33\frac{1}{3}+12\frac{17}{42}=$
10. $7\frac{1}{7}+9\frac{1}{9}+15\frac{2}{15}+5\frac{1}{5}=$

SUBTRACTION OF FRACTIONS

References 206, 207.

19. 1. $\frac{3}{4}-\frac{3}{8}=$
    2. $12\frac{5}{8}-7\frac{2}{3}=$
    3. $214\frac{13}{15}-185\frac{1}{3}=$
    4. $25\frac{1}{3}-5\frac{7}{8}=$
    5. $146\frac{7}{8}-79\frac{5}{9}=$
    6. $1246\frac{5}{16}-983\frac{1}{4}=$
    7. $25\frac{5}{9}-14\frac{2}{3}=$
    8. $346\frac{8}{9}-217\frac{5}{12}=$

MULTIPLICATION OF FRACTIONS

References 208 to 218 inclusive, and 135.

20. 1. $\frac{4}{5}\times\frac{5}{8}=$
    2. $12\frac{1}{2}\times 8=$
    3. $124\times 16\frac{2}{3}=$
    4. $218\times 29\frac{5}{9}=$
    5. $289\frac{4}{5}\times 126=$
    6. $12\frac{1}{2}\times 5\frac{2}{3}=$
    7. $124\frac{1}{5}\times 27\frac{3}{7}=$
    8. $128\frac{9}{11}\times 23\frac{1}{3}=$
    9. $512\frac{5}{27}\times 137=$
    10. $46\frac{5}{8}\times 43=$
    11. $319\frac{2}{3}\times 18\frac{5}{6}=$
    12. $2.15\frac{1}{5}\times 24=$
    13. $24\times .12\frac{1}{2}=$
    14. $246\times .16\frac{2}{3}=$
    15. $814\times .44\frac{4}{9}=$
    16. $31.62\frac{1}{2}\times 8\frac{5}{8}=$
    17. $312\times 24.08\frac{1}{3}=$
    18. $459\times 312\frac{7}{9}=$
    19. $1246\frac{2}{3}\times 34.52=$
    20. $4.16\frac{2}{3}\times .12\frac{1}{2}=$

DIVISION OF FRACTIONS

References 219 to 225 inclusive, and 139, 140, 141.

21. 1. $\frac{4}{5} \div \frac{5}{8} =$

2. $\frac{12}{19} \div \frac{8}{9} =$

3. $\frac{7}{8} \div \frac{5}{6} =$

4. $125 \div \frac{2}{3} =$

5. $24 \div \frac{5}{9} =$

6. $346 \div \frac{5}{8} =$

7. $1246 \div 15\frac{4}{5} =$

8. $482 \div 17\frac{2}{3} =$

9. $846\frac{2}{3} \div 26 =$

10. $532\frac{2}{3} \div 18\frac{7}{12} =$

11. $246\frac{5}{8} \div 13\frac{3}{4} =$

12. $1246\frac{5}{8} \div 4\frac{1}{2} =$

13. $418\frac{7}{8} \div 15\frac{2}{3} =$

14. $216\frac{2}{3} \div 19 =$

15. $1248\frac{3}{5} \div 27\frac{2}{3} =$

16. $456\frac{5}{17} \div 125 =$

17. $5.14\frac{2}{3} \div 23 =$

18. $4246 \div .17\frac{5}{6} =$

19. $128.57\frac{5}{8} \div .12\frac{1}{2} =$

20. $43.55\frac{5}{9} \div .16\frac{2}{3} =$

PRACTICE PROBLEMS INVOLVING THE USE OF FRACTIONS AND DECIMALS

The following problems are intended to show the application of the general principles of common fractions. Their proper solution involves a knowledge of paragraphs 160 to 225 inclusive.

22. 1. Four pieces of cloth measure respectively $31\frac{1}{4}$ yd., $43\frac{5}{16}$ yd., $56\frac{9}{16}$ yd., and $44\frac{1}{2}$ yd. What is the total length?

2. What is the sum of $23.8\frac{3}{16}$, $32.35\frac{5}{8}$, $56\frac{7}{8}$, 194, and $\frac{7}{8}$? Carry to the fourth decimal place.

3. I am about to ship a box containing $20\frac{3}{4}$ lb. coffee, $3\frac{7}{16}$ lb. tea, $23\frac{1}{2}$ lb. ham and $16\frac{2}{3}$ lb. bacon to my camp. If the box weighs $2\frac{3}{4}$ lb., what is the total weight of the shipment?

4. A grocer bought six bags of coffee, weighing respectively $132\frac{5}{8}$ lb., $128\frac{1}{4}$ lb., $127\frac{3}{4}$ lb., $136\frac{5}{8}$ lb., $134\frac{3}{8}$ lb., and $128\frac{3}{4}$ lb. Allowing $\frac{15}{16}$ lb. for the weight of each bag, what would be the total net weight of the coffee?

5. I bought 5 barrels of sugar. The net weight of each respectively was $275\frac{1}{5}$ lb., $283\frac{1}{3}$ lb., $271\frac{5}{8}$ lb., $293\frac{3}{5}$ lb., and $285\frac{2}{3}$ lb. Find the total net weight.

6. I bought a $\frac{7}{8}$ interest in a bowling alley, and sold my brother a $\frac{5}{16}$ interest. How much do I own?

7. An automobilist on a tour completes $\frac{3}{16}$ of the trip on the first day, $\frac{1}{8}$ on the second day, and $\frac{1}{4}$ on the third day. He then finds himself 350 miles from his destination. What is the total length of the trip and how far has he already advanced?

8. I bought a house for $7500. I paid $\frac{1}{4}$ of the purchase price in cash, $\frac{2}{3}$ of the remainder was paid in six months, and I am now ready to make the final payment. For what sum must my check be written?

9. I can do a piece of work in 5 days. My brother requires 7 days to do the same thing. If we work together how long will it take to complete the job?

10. If coffee loses $\frac{1}{16}$ of its weight in roasting, how many pounds of green coffee must be roasted to produce 375 lb.?

11. A farmer bought a cow for $\$52\frac{3}{4}$ and a ton of hay for $\$29\frac{4}{5}$. How much change would he receive out of a one-hundred-dollar bill?

12. A bookkeeper's pay envelope contains three \$10's, one \$5, one \$2, and one \$1. He paid for board

$\$11\frac{1}{2}$, a bill amounting to $\$8\frac{3}{4}$, and bought a hat for \$3, a pair of gloves for $\$1\frac{3}{4}$, and two pairs of stockings at \$.50 a pair. What part of his week's pay did he have left?

13. What will $17\frac{8}{12}$ dozen eggs cost at $\$.58\frac{3}{4}$ per dozen?

14. If $7\frac{2}{3}$ tons of hay cost $\$182\frac{1}{2}$, what will $11\frac{5}{8}$ tons cost?

15. I bought $4375\frac{2}{3}$ bushels of corn at $\$.80\frac{3}{4}$ a bushel, and $2350\frac{1}{4}$ bushels of oats at $\$.61\frac{5}{8}$ a bushel. What was the entire investment?

16. Find the total cost of the following: 350 lb. Rio coffee at $\$.47\frac{7}{8}$; 450 lb. Mocha coffee at $\$.41\frac{5}{8}$; 900 lb. white sugar at $\$.10\frac{2}{3}$; 900 lb. brown sugar at $\$.09\frac{3}{4}$; 970 lb. granulated sugar at $\$.08\frac{3}{4}$; 172 lb. butter at $\$.56\frac{4}{5}$.

17. A merchant sold 80 lb. of butter at $\$.57\frac{2}{3}$; 43 dozen of eggs at $\$.61\frac{2}{3}$ per dozen; $32\frac{1}{2}$ gallons of milk at \$.60 a gallon. What was the total amount of sales?

18. A piece of cloth containing $47\frac{3}{4}$ yd. was sold for $\$9.94\frac{7}{8}$. What was the price per yard?

19. One-third of a firm's capital is invested in merchandise, three-eighths in real estate, and the rest, \$18,200, is cash. What is the capital of the firm? How much is invested in merchandise, and how much in real estate?

20. A farm yields 96.08 bushels of potatoes to the acre, 36.625 bushels of oats per acre, 15.52 bushels of wheat per acre. 156 acres were planted in potatoes, 214 acres in oats, and 19.3 acres in wheat. What is the total number of bushels harvested?

21. I have withdrawn $\frac{7}{8}$ of my money from the bank and have $376.40 remaining. How much did I withdraw?

22. A partnership consists of three members who invest respectively $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$ of the capital, and agree to share losses and gains in the same proportion. How much will be each partner's share, if there is a profit of $13,416.75?

23. A business man finds himself unable to meet his entire obligations. He owes $12,360 and has $10,300 available with which to pay. What part of his liabilities can he meet? How many cents on the dollar is this?

24. A invested $\frac{1}{3}$ of the capital of a firm, B $\frac{1}{4}$, C $\frac{1}{5}$, and D the remainder. D's share is $1690. What was A's, B's, and C's investment?

25. A house and lot cost $6600. The house costs $\frac{1}{5}$ more than the land. What was the cost of each?

26. How many bushels is .75 of 640 bushels?

27. A merchant sold 162 barrels of flour which is $\frac{3}{8}$ of his stock of flour. How much flour had he at first?

28. A merchant sold 480 barrels of flour which is .625 of his entire stock. How many barrels had he at first?

29. A man, at his death, left $30,000 to his wife, son, and daughter; .5 of this sum went to his wife, .375 to his daughter, and .125 to his son. How much did each receive?

30. I have just learned that one of my customers has failed and is able to pay only $.525 on the dollar. My claim against him amounted to $134.40. How much will I receive?

DENOMINATE NUMBERS

Reference 229.

23. 1. Reduce £34, 8*s*, 7*d* to pence.
2. Reduce 4 T., 5 cwt., 85 lb. to pounds.
3. Reduce 14 gal., 3 qt., 1 pt. to pints.
4. Reduce 1 cwt., 24 lb., 3 oz. to ounces.
5. Reduce 1 da., 3 hr., 25 min. to seconds.
6. Reduce 14 yr., 5 mo., 3 wk. to days.
7. Reduce 1 m. 25 rd., 4 yd., $2\frac{1}{2}$ ft. to inches.
8. Reduce 2 hhd., 14 gal., 3 qt. to pints.
9. Reduce 14 bu., 3 pk. to pints.
10. Reduce 3 A., 2 sq. rd., 10 sq. yd. to sq. ft.

Reference 230.

24. 1. Reduce 3462 sq. in. to higher denominations.
2. Reduce 1468*d* to higher denominations.
3. Reduce 17696 lb. to higher denominations.
4. Reduce 32625 gr. to higher denominations.
5. Reduce 12760 in. to higher denominations.
6. Reduce 18428 sq. in. to higher denominations.
7. Reduce 3896 cu. ft. to higher denominations.
8. Reduce 4843*d* to higher denominations.
9. Reduce 120615 sec. to higher denominations.
10. Reduce 633 pt. to higher denominations.

Reference 231.

25. 1. Reduce .327 m. to lower denominations.
2. Reduce .35 hr. to lower denominations.
3. Reduce .875 yd. to lower denominations.
4. Reduce .135 yr. to lower denominations.
5. Reduce $\frac{5}{9}$ mo. to lower denominations.
6. Reduce .125 m. to lower denominations.
7. Reduce £2.3456 to lower denominations.
8. Reduce £12.456 to lower denominations.
9. Reduce $\frac{3}{5}$ m. to lower denominations.
10. Reduce $\frac{2}{11}$ yr. to lower denominations.

Reference 232.

26. 1. Reduce 4 yd., 2 ft. to a decimal of a rod.
2. Reduce 3 gal., 2 qt., 1 pt. to gallons.
3. Reduce 2 pk., 3 qt., 1 pt. to a decimal of a bushel.
4. Reduce 35 min., 18 sec. to a decimal of a day.
5. Reduce 18 rd., 4 yd., 2 ft. to rods.
6. Reduce 4 cwt., 85 lb. to a decimal of a ton.
7. Reduce 8*s*, 10*d*, 2 *f* to a decimal of a pound.
8. Reduce 18 sq. rd., 4 sq. yd. to a decimal of an acre.
9. Reduce 14 hr., 35 min., 10 sec. to a decimal of a day.
10. Reduce 185 lb., 12 oz. to a decimal of a ton.

ALIQUOT PARTS

Study paragraphs 240 and 241, memorizing the table and noting application as explained in note *a*.

Reference 241.

27. Find the cost of:

1. 720 lb. at 50¢; at $33\frac{1}{3}$¢; at 25¢.
2. 120 lb. at $33\frac{1}{3}$¢; at 25¢; at 20¢; at $12\frac{1}{2}$¢.
3. 360 lb. at $6\frac{1}{4}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
4. 840 yd. at 10¢; at $12\frac{1}{2}$¢; at $14\frac{2}{7}$¢; at 25¢.
5. 4800 lb. at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 10¢.
6. 240 yd. at $8\frac{1}{3}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
7. 2480 yd. at 25¢; at 50¢; at $33\frac{1}{3}$¢; at 20¢.
8. 480 yd. at $6\frac{1}{4}$¢; at $8\frac{1}{3}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
9. 560 yd. at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
10. 204 yd. at 50¢; at $33\frac{1}{3}$¢; at 25¢; at $16\frac{2}{3}$¢.
11. 4200 yd. at 10¢; at $12\frac{1}{2}$¢; at $14\frac{2}{7}$¢; at $16\frac{2}{3}$¢; at 25¢.

12. 1800 lb. at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 20¢; at 25¢; at $33\frac{1}{3}$¢.

13. 1500 yd. at \$1; at $12\frac{1}{2}$¢; at $14\frac{2}{7}$¢; at $16\frac{2}{3}$¢; at 25¢.

14. 490 doz. at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 20¢; at $6\frac{2}{3}$¢.

15. 960 yd. at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at 10¢; at $12\frac{1}{2}$¢; at $6\frac{2}{3}$¢.

Reference 241.

28. Find the total cost of:

1. 38 lb. at 25¢
84 lb. at $37\frac{1}{2}$¢
72 lb. at 75¢
48 lb. at $41\frac{2}{3}$¢
96 lb. at $33\frac{1}{3}$¢
24 lb. at $12\frac{1}{2}$¢

2. 63 yd. at $28\frac{4}{7}$¢
81 yd. at $33\frac{1}{3}$¢
$18\frac{3}{4}$ gr. at 52¢
28 doz. at 50¢
58 yd. at $14\frac{2}{7}$¢
235 yd. at 40¢

3. $6\frac{1}{4}$ lb. at 48¢
480 lb. at $16\frac{2}{3}$¢
25 lb. at 44¢
$16\frac{2}{3}$ lb. at 48¢
240 lb. at $18\frac{3}{4}$¢
72 lb. at $37\frac{1}{2}$¢

4. 25 bu. at 96¢
20 bu. at 88¢
$31\frac{1}{4}$ bu. at \$2
50 bu. at \$11.50
$12\frac{1}{2}$ bu. at \$3.60
25 bu. at \$1.64

5. 25 yd. at 76¢
$37\frac{1}{2}$ yd. at 96¢
750 yd. at $12\frac{1}{2}$¢
168 doz. at $12\frac{1}{2}$¢
420 yd. at $33\frac{1}{3}$¢
176 yd. at $31\frac{1}{4}$¢

6. $37\frac{1}{2}$ bu. at 72¢
75 bu. at \$3.20
$62\frac{1}{2}$ bu. at \$1.36
$14\frac{2}{7}$ bu. at \$1.54
50 bu. at \$5.85
$12\frac{1}{2}$ bu. at \$.64

Reference 241.

29. Find the cost of:

1. $6\frac{1}{4}$ A. land at \$192.
2. 125 lb. tea at 48¢.
3. 34 lb. tea at 50¢.
4. 25 lb. coffee at 44¢.
5. 25 T. coal at \$10.80.
6. 72 pieces lace at \$1.25.
7. 44 yd. velvet at \$2.50.
8. $2\frac{1}{2}$ bu. potatoes at \$1.48.
9. $12\frac{1}{2}$ bu. turnips at 74¢.
10. $12\frac{1}{2}$ yd. silk at \$1.04.
11. 84 tables at \$12.50.
12. 36 sets chairs at \$125.
13. $12\frac{1}{2}$ yd. linen at 56¢.
14. 25 pieces lace at \$6.60.
15. $62\frac{1}{2}$ T. coal at \$9.50.
16. 375 T. coal at \$11.50.
17. 264 A. land at \$37.50.
18. 320 bu. potatoes at \$2.12$\frac{1}{2}$.
19. 810 T. coal at \$12.50.
20. 1250 bbl. pork at \$24.
21. 1280 lb. rice at $12\frac{1}{2}$¢.
22. 366 yd. silk at \$1.66$\frac{2}{3}$.
23. $11\frac{1}{9}$ yd. duck at 36¢.
24. 474 gal. cider at $33\frac{1}{3}$¢.
25. 1680 qt. vinegar at $16\frac{2}{3}$¢.
26. 648 lb. sugar at 10¢.
27. 208 yd. tape at $2\frac{1}{2}$¢.
28. 176 lb. tea at 50¢.
29. 1742 yd. silk at \$1.50.
30. 560 gal. oil at $12\frac{1}{2}$¢.

Reference 241.

30. Find the total cost of the following:

1. 180 lb. at $33\frac{1}{3}$¢
760 lb. at 25¢
54 lb. at $37\frac{1}{2}$¢
144 lb. at $33\frac{1}{3}$¢
72 lb. at $37\frac{1}{2}$¢
150 lb. at $66\frac{2}{3}$¢

2. 138 lb. at $33\frac{1}{3}$¢
728 lb. at $62\frac{1}{2}$¢
224 lb. at 25¢
960 lb. at $66\frac{2}{3}$¢
72 lb. at $12\frac{1}{2}$¢
904 lb. at $87\frac{1}{2}$¢

3. 196 yd. at $16\frac{2}{3}$¢
180 yd. at $66\frac{2}{3}$¢
288 yd. at $33\frac{1}{3}$¢
459 yd. at $11\frac{1}{9}$¢
72 yd. at 25¢
48 yd. at $16\frac{2}{3}$¢
183 yd. at $33\frac{1}{3}$¢

4. 147 yd. at $55\frac{5}{9}$¢
24 yd. at $66\frac{2}{3}$¢
28 yd. at 75¢
84 yd. at 25¢
56 yd. at $12\frac{1}{2}$¢
48 yd. at 75¢
246 yd. at 25¢

5. 66 gal. at $33\frac{1}{3}$¢
64 gal. at $87\frac{1}{2}$¢
63 gal. at $11\frac{1}{9}$¢
144 gal. at $83\frac{1}{3}$¢
91 gal. at $71\frac{3}{7}$¢
96 gal. at $62\frac{1}{2}$¢
16 gal. at $87\frac{1}{2}$¢
945 gal. at $55\frac{5}{9}$¢

6. 441 gal. at $55\frac{5}{9}$¢
3248 gal. at $6\frac{1}{4}$¢
117 gal. at $22\frac{2}{9}$¢
266 gal. at $28\frac{4}{7}$¢
384 gal. at $62\frac{1}{2}$¢
368 gal. at $31\frac{1}{4}$¢
248 gal. at $87\frac{1}{2}$¢
366 gal. at $83\frac{1}{3}$¢

7. 750 yd. at $33\frac{1}{3}$¢
427 yd. at $42\frac{6}{7}$¢
$87\frac{1}{2}$ yd. at 50¢
52 yd. at $62\frac{1}{2}$¢
450 yd. at $6\frac{2}{3}$¢
2112 yd. at $8\frac{1}{3}$¢
240 yd. at $3\frac{1}{3}$¢
174 yd. at $16\frac{2}{3}$¢
249 yd. at 25¢

8. 648 yd. at $6\frac{1}{4}$¢
684 yd. at $33\frac{1}{3}$¢
496 yd. at 75¢
186 yd. at $83\frac{1}{3}$¢
125 yd. at 18¢
144 yd. at $37\frac{1}{2}$¢
297 yd. at $44\frac{4}{9}$¢
287 yd. at $14\frac{2}{7}$¢
918 yd. at $33\frac{1}{3}$¢

9. 144 lb. at $83\frac{1}{3}¢$
480 lb. at $87\frac{1}{2}¢$
282 lb. at $83\frac{1}{3}¢$
312 lb. at $33\frac{1}{3}¢$
427 lb. at $71\frac{3}{7}¢$
184 lb. at $12\frac{1}{2}¢$
940 lb. at $75¢$
462 lb. at $66\frac{2}{3}¢$
342 lb. at $16\frac{2}{3}¢$

10. 144 lb. at $16\frac{2}{3}¢$
216 lb. at $62\frac{1}{2}¢$
872 lb. at $12\frac{1}{2}¢$
348 lb. at $25¢$
72 lb. at $8\frac{1}{3}¢$
186 lb. at $87\frac{1}{2}¢$
138 lb. at $33\frac{1}{3}¢$
96 lb. at $6\frac{1}{4}¢$
384 lb. at $50¢$

11. 84 yd. at $91\frac{2}{3}¢$
288 yd. at $12\frac{1}{2}¢$
345 yd. at $66\frac{2}{3}¢$
192 yd. at $37\frac{1}{2}¢$
423 yd. at $33\frac{1}{3}¢$
280 yd. at $12\frac{1}{2}¢$
324 yd. at $41\frac{2}{3}¢$
284 yd. at $25¢$
396 yd. at $33\frac{1}{3}¢$
64 yd. at $56\frac{1}{4}¢$

12. 84 lb. at $58\frac{1}{3}¢$
960 lb. at $16\frac{2}{3}¢$
728 lb. at $62\frac{1}{2}¢$
36 lb. at $43\frac{3}{4}¢$
64 lb. at $41\frac{2}{3}¢$
96 lb. at $12\frac{1}{2}¢$
72 lb. at $41\frac{2}{3}¢$
348 lb. at $75¢$
246 lb. at $33\frac{1}{3}¢$
344 lb. at $37\frac{1}{2}¢$

13. $87\frac{1}{2}$ yd. at $2.48
192 yd. at $87\frac{1}{2}¢$
28 yd. at $75¢$
$11\frac{1}{9}$ yd. at $18¢$
144 yd. at $11\frac{1}{9}¢$
25 yd. at $44¢$
75 yd. at $24¢$
$87\frac{1}{2}$ yd. at $2.88
270 yd. at $11\frac{1}{9}¢$
24 yd. at $75¢$

14. 176 yd. at $1.12\frac{1}{2}$
75 yd. at $16¢$
27 yd. at $75¢$
56 yd. at $83\frac{1}{3}¢$
17 yd. at $25¢$
$12\frac{1}{2}$ yd. at $39¢$
72 yd. at $41\frac{2}{3}¢$
344 yd. at $37\frac{1}{2}¢$
24 yd. at $8\frac{1}{3}¢$
$87\frac{1}{2}$ yd. at $88¢$

15. $1151\frac{3}{4}$ lb. at 20¢
960 lb. at $16\frac{2}{3}$¢
728 lb. at $62\frac{1}{2}$¢
32 lb. at $43\frac{3}{4}$¢
64 lb. at $8\frac{1}{3}$¢
96 lb. at $12\frac{1}{2}$¢
72 lb. at $41\frac{2}{3}$¢
348 lb. at 75¢
246 lb. at $33\frac{1}{3}$¢
344 lb. at $37\frac{1}{2}$¢

16. 156 lb. at $66\frac{2}{3}$¢
284 lb. at 25¢
396 lb. at $33\frac{1}{3}$¢
64 lb. at $56\frac{1}{4}$¢
384 lb. at $87\frac{1}{2}$¢
84 lb. at $58\frac{1}{3}$¢
960 lb. at $16\frac{2}{3}$¢
728 lb. at $62\frac{1}{2}$¢
96 lb. at $43\frac{3}{4}$¢
98 lb. at $37\frac{1}{2}$¢

17. 96 yd. at $8\frac{1}{3}$¢
96 yd. at $12\frac{1}{2}$¢
348 yd. at 75¢
72 yd. at $41\frac{2}{3}$¢
246 yd. at $33\frac{1}{3}$¢
344 yd. at $37\frac{1}{2}$¢
156 yd. at $66\frac{2}{3}$¢
132 yd. at $91\frac{2}{3}$¢
84 yd. at 50¢
328 yd. at 25¢

18. 594 lb. at $66\frac{2}{3}$¢
963 lb. at $83\frac{1}{3}$¢
312 lb. at $37\frac{1}{2}$¢
251 lb. at 50¢
603 lb. at $11\frac{1}{9}$¢
552 lb. at $66\frac{2}{3}$¢
133 lb. at $14\frac{2}{7}$¢
528 lb. at $6\frac{1}{4}$¢
273 lb. at $83\frac{1}{3}$¢
368 lb. at $31\frac{1}{4}$¢

19. 146 gal. at $37\frac{1}{2}$¢
245 gal. at $42\frac{6}{7}$¢
672 gal. at $16\frac{2}{3}$¢
18 gal. at $87\frac{1}{2}$¢
162 gal. at $16\frac{2}{3}$¢
332 gal. at $18\frac{3}{4}$¢
369 gal. at $33\frac{1}{3}$¢
828 gal. at $31\frac{1}{4}$¢
693 gal. at $16\frac{2}{3}$¢
918 gal. at $44\frac{4}{9}$¢

20. 200 yd. at $37\frac{1}{2}$¢
384 yd. at $18\frac{3}{4}$¢
288 yd. at $83\frac{1}{3}$¢
294 yd. at $66\frac{2}{3}$¢
918 yd. at $44\frac{4}{9}$¢
459 yd. at $11\frac{1}{9}$¢
18 yd. at $12\frac{1}{2}$¢
111 yd. at $33\frac{1}{3}$¢
164 yd. at $62\frac{1}{2}$¢
8 yd. at $87\frac{1}{2}$¢

EXERCISES IN BILLING

Reference 241.

31. 1. Copy and extend the following bill:

Salem, Mass., July 20, 1919

Mr. J. A. Brown,
3 Leach St.,
City.

To S. S. Pierce & Co., *Dr.*

Terms Cash

8 bu. beans	@	$3.75				
108 lb. butter	@	.50				
84 lb. cheese	@	$.33\frac{1}{3}$				
129 doz. eggs	@	.50				
150 lb. lard	@	$.42\frac{6}{7}$				
25 bu. potatoes	@	2.16				
72 lb. rice	@	$.16\frac{2}{3}$				
12 lb. Japan tea	@	$.56\frac{1}{4}$				
360 lb. granulated sugar	@	.10				
128 lb. coffee	@	$.43\frac{3}{4}$				

2. Feb. 1, A. W. Smith & Co., Boston, Mass., sold to Jones & French, Marblehead, Mass., on 30 days' credit: 36 boxes oranges at $\$3.66\frac{2}{3}$; 12 chests T. H. tea, 840 lb., at $50¢$; 14 chests Japan tea, 980 lb., at $37\frac{1}{2}¢$; 12 bbl. St. Louis flour at $9.10; 4 bags coffee, 576 lb., at $33\frac{1}{3}¢$; 54 boxes lemons at $4.60; 14 bbl. pineapples at $7.50; 44 bunches bananas at $5.45. Write the bill.

3. R. C. Adams, Danvers, Mass., bought of Davis & Bicknell, Salem, Mass., on account 60 days: 25 bbl.

St. Louis flour at \$11.48; 35 boxes apricots, 25 lb. each, at 23¢; 20 boxes apples, 25 lb. each, at $11\frac{1}{9}$¢; 10 boxes peaches, 25 lb. each, at $37\frac{1}{2}$¢; 6 boxes raisins, 25 lb. each, at 25¢; 8 boxes prunes, 25 lb. each, at $16\frac{2}{3}$¢; 13 boxes currants, 25 lb. each, at 16¢; 15 cases Quaker Oats at \$3.75; 12 cases canned corn, 24 doz., at $\$2.12\frac{1}{2}$; 18 cases canned tomatoes, 36 doz., at $\$1.87\frac{1}{2}$; 15 chests Japan tea, 70 lb. each, at 54¢; 1600 lb. gunpowder tea at $43\frac{3}{4}$¢. Write the bill.

4. Jan. 31, K. R. Good & Co. sold to Lewis W. Sears, Middleton, Mass., on 60 days' time: 6 pairs men's kid gloves at \$2.48; 5 doz. napkins at \$5.50; 4 doz. children's hose at \$2.75; 9 pr. blankets at \$6.50; 2 pieces jeans, 80 yd., at 25¢; 2 pieces point, 80 yd., at $12\frac{1}{2}$¢; 5 doz. towels at \$3.50; 15 doz. spools thread at $87\frac{1}{2}$¢; 7 doz. ladies' collars at $\$2.12\frac{1}{2}$; 7 doz. ladies' cuffs at \$3.25; 14 robes at $\$2.33\frac{1}{3}$; 4 pieces Irish linen, 156 yd., at $66\frac{2}{3}$¢. Write the bill.

5. March 1, James K. Broderick & Co., Boston, Mass., sold to Henry T. Lewis, Peabody, Mass., on 30 days' time: 2 pieces gingham, 63 yd., at $37\frac{1}{2}$¢; 1 piece blue denim, 31 yd., at 28¢; 1 piece brown denim, 30 yd., at $28\frac{4}{7}$¢; 1 piece duck, 31 yd., at 50¢; 1 piece shirting, 35 yd., at $22\frac{2}{9}$¢; 1 piece sheeting, 29 yd., at $43\frac{3}{4}$¢; 2 pieces cottonnade, 76 yd., at $33\frac{1}{3}$¢; 1 piece jeans, 34 yd., at 30¢; 1 piece Irish linen, 40 yd., at $62\frac{1}{2}$¢; 2 pieces jaconet, 84 yd., at 25¢; 5 doz. children's hose at \$2.75; 8 pr. ladies' kid gloves at \$2.50; 4 doz. spools thread at $62\frac{1}{2}$¢. Write the bill.

PERCENTAGE

Study 242 to 254 inclusive.
Reference 244.

32. Express the following, both in decimal and common fractional forms:

1. $1\frac{1}{2}\%$.
2. $2\frac{1}{4}\%$.
3. $3\frac{1}{8}\%$.
4. $3\frac{3}{4}\%$.
5. $7\frac{1}{2}\%$.
6. $7\frac{3}{5}\%$.
7. $31\frac{3}{4}\%$.
8. $32\frac{1}{2}\%$.
9. 38%.
10. $43\frac{3}{4}\%$.
11. $68\frac{3}{4}\%$.
12. $92\frac{1}{2}\%$.
13. $93\frac{3}{4}\%$.
14. $112\frac{1}{2}\%$.
15. $118\frac{3}{4}\%$.
16. $131\frac{1}{4}\%$.
17. $137\frac{1}{2}\%$.
18. $143\frac{3}{4}\%$.
19. $156\frac{1}{4}\%$.
20. $162\frac{1}{2}\%$.
21. $\frac{3}{8}\%$.
22. $\frac{5}{16}\%$.
23. $\frac{16}{25}\%$.
24. $\frac{39}{40}\%$.
25. $\frac{15}{16}\%$.
26. $.2\%$.
27. $.4\%$.
28. $.5\%$.
29. $.12\%$.
30. $.25\%$.

References 255, 256, 257, 258.

33. Find:

1. 23% of 460 acres.
2. 34% of 60 cords.
3. 15% of $75.
4. 55% of 280 feet.
5. 45% of 360.
6. $62\frac{1}{2}\%$ of $.50.
7. 48% of 175 gallons.
8. 77% of 430 bushels.
9. 85% of 1270 pounds.
10. $166\frac{2}{3}\%$ of 480 tons.
11. 225% of 699 bushels.
12. 625% of $4560.
13. .2% of $1750.
14. .4% of $1825.

15. .6% of $156.25.
16. $\frac{1}{8}$% of $86,424.
17. 3.5% of $1250.
18. 4.4% of $875.
19. $7\frac{1}{2}$% of $1560.
20. $6\frac{1}{4}$% of $8\frac{8}{9}$.
21. $6\frac{2}{3}$% of $112\frac{1}{2}$.
22. $7\frac{1}{5}$% of $87\frac{1}{2}$.
23. $\frac{3}{4}$% of $1260.
24. $36\frac{4}{9}$% of $1214.
25. $\frac{3}{8}$% of $33\frac{1}{3}$ gallons.

26. A man having $1960 spent 23% of it. How much did he have left?

27. A gentleman having $25,565 invested 18% of it in city lots, 22% in railroad stock, 30% of it in bank stock, and the rest in a truck farm. How much did he invest in each kind of property?

28. 13% of a grocer's bill of $1665 was for coffee at 45¢ per pound. How many pounds did the grocer buy?

29. I bought 200 little pigs at $7.50 each; 25% of them died. At what price per head must I sell those that are left in order to incur no loss?

30. A farmer who raised 1880 bushels of corn sold $37\frac{1}{2}$% of it at $87\frac{1}{2}$¢ a bushel. How much did he receive for what was sold?

31. An agent collected $2430 for his client whom he charged a 5% fee for his services. How much did he receive and how much did he remit to his client?

32. C. E. Bates has failed owing me $1645. He is able to pay only 43% of his debts. At that rate, how much should I receive?

33. I bought 27 bales of cloth, 12 pieces to the bale averaging 47 yards to the piece, at $1.25 per yard. For what must I sell the entire quantity to gain $16\frac{2}{3}$%?

34. A private bank that has failed declared a dividend of $87\frac{1}{2}\%$. A's balance on deposit was \$6,437.50, B's \$3,856.56, C's \$872. How much did each lose?

35. A man, at his death, left an estate valued at \$150,000. He left 10% of it to organized charity, $12\frac{1}{2}\%$ of it to his college, and $4\frac{1}{2}\%$ to his church. He divided the remainder among his family so that his wife received $62\frac{1}{2}\%$ of it and his son and daughter each $18\frac{3}{4}\%$. What did each receive?

References 259, 260, 261.

34.

1. 18 is 9% of what?
2. 38 is 5% of what?
3. 54 is $12\frac{1}{2}\%$ of what?
4. 84 is 15% of what?
5. \$460 is 23% of what?
6. \$1143 is 35% of what?
7. \$650 is 42% of what?
8. \$9420 is 96% of what?
9. \$3150 is 3.5% of what?
10. \$48.60 is 7.5% of what?
11. \$148.20 is $16\frac{2}{3}\%$ of what?
12. \$1375.50 is $33\frac{1}{3}\%$ of what?
13. \$1198 is 55.5% of what?
14. \$6570 is 234% of what?
15. \$1254 is 675% of what?

16. Of my flock of pigeons $16\frac{2}{3}\%$ died. If 215 are left, how many pigeons were there in the original flock?

17. I have just paid a bill of $135.45, which represented $18\frac{3}{4}\%$ of my available cash. How much did I have before paying the bill? How much is left?

18. A broker received $26.25 as his fee for selling a certain piece of property on a commission of $2\frac{1}{2}\%$. What was the value of the property?

19. An analysis shows that a merchant's costs and fixed charges amounted to $87\frac{1}{2}\%$ of his gross sales for a certain year. If his total expenses and costs amounted to $246,400, what were his sales?

20. A merchant sold 15% of his stock of goods for $45,350. What was his entire stock worth before he sold any?

21. A farmer bought 87 acres of land which is $37\frac{1}{2}\%$ of what he previously owned. How much did he own after the purchase?

22. A sold a carriage at a profit of $16\frac{2}{3}\%$, thereby gaining $43.25. What did it cost and what did it sell for?

23. If it takes 60 days to complete $16\frac{2}{3}\%$ of a contract, how long will it take to finish the job?

References 262, 263.

35. 1. What per cent of 260 is 13?
2. What per cent of 480 is 72?
3. What per cent of $2.40 is $32?
4. What per cent of $188.50 is $22.62?
5. What per cent of $640 is $131.60?
6. What per cent of 28 bu. is 7 bu.?

7. What per cent is \$314.50 of \$1850?

8. What per cent of \$.95 is \$.70?

9. What per cent of \$2664 is \$826.04?

10. What per cent of $1\frac{1}{2}$ is $\frac{3}{8}$?

11. What per cent of $77\frac{7}{9}$ is $66\frac{2}{3}$?

12. What per cent of \$456.75 is \$219.24?

13. What per cent is \$1414.80 of \$5240?

14. What per cent of 324 is 64.8?

15. What per cent of \$1940 is \$9.70?

16. A man owing a debt of \$1680, paid \$940.80. What per cent remains unpaid?

17. In a school of 480 pupils, 24 were absent on a certain day. What was the percentage of absence?

18. A merchant purchased goods for \$425 and sold them for \$510. How many dollars did he gain? This was what per cent of the cost? What per cent of the selling price?

19. A lawyer charged \$14.19 for collecting a claim of \$473. What rate per cent did he charge?

20. I sell a house that cost me \$2500 for \$2125. The loss is what per cent of the cost?

21. Of an army of 45,000 men 5625 were killed in battle and 10,125 were wounded. What was the percentage of loss?

22. An insurance company with a capital stock of \$250,000 declared an annual dividend of \$21,250. The dividend was what percentage of the capital?

23. A miller keeps one quart out of every bushel he grinds. What is the percentage of his toll?

24. A merchant invested $34,395.30 in business. At the end of the year he finds he has gained $3821.70. This gain is what per cent of the money invested?

25. If I sell $\frac{5}{9}$ of a quantity of goods for what $\frac{2}{3}$ of them cost, what is the gain per cent?

26. John Brown, failing in business, owes $3650. His entire resources are $2920. What per cent of his indebtedness can he pay?

27. The assets of an insolvent concern are $23,450; its liabilities are $33,500. What per cent can it pay and what will A receive on a claim of $1350?

28. I paid $300 for apples bought at $5.40 a barrel. I sold 38 barrels for $220.40. What percentage did I gain on the quantity sold?

29. The enrollment in a certain High School is 846. 102 are enrolled in the Classical Course, 228, the General Course, 246, the Manual Training Course, and 270, the Commercial Course. What percentage of the school could each course claim? Carry the result to the fourth decimal place if necessary.

30. At the close of the baseball season of 1915, the first four teams in the American League stood as given below. Figure the percentage of each, carrying to the fourth decimal place.

	WON	LOST
Boston	101	50
Detroit	100	54
Chicago	93	61
Washington . . .	85	68

GENERAL PROBLEMS IN PERCENTAGE

The following problems cover the entire range of simple percentage. A thorough understanding of the matter covered by paragraphs 242 to 263 inclusive will enable the student to solve them with facility and accuracy.

These problems are not graded so as to present increasing difficulties. They are rather arranged so an average lesson of ten problems will offer the varying conditions of ease and difficulty that are usually found in actual experience.

Every problem should be solved in the easiest possible way.

36. 1. After experiencing a loss of $5000, a business man has $30,000 left. His loss was what per cent of his original capital?

2. The assets of a bankrupt were $27,179.38. His liabilities were $43,487. What per cent could he pay? How much would be due A whose claim is $3540.75?

3. One of our creditors who met with financial reverses agreed to pay our claim in annual installments of $16\frac{2}{3}\%$. He has made four payments. How much does he still owe, the original claim being $1847.38?

4. A owns $\frac{1}{2}$ interest in a business, B $\frac{3}{8}$, and C $\frac{1}{8}$. A sells $33\frac{1}{3}\%$ of his share for $1874. What is the entire value of the business? What is B's share? What is C's share?

5. A and B are partners. A's investment is $36,783.60 and B's is $26,636.40. To what percentage of the profits is each entitled?

6. Out of an inheritance of $17,500, I invested 45% in real estate, 25% in United States bonds, and put the rest into a mortgage. What sum does the mortgage represent?

7. A owns $\frac{1}{8}$ of the stock of a corporation, B 25%, C $\frac{1}{5}$, D $37\frac{1}{2}\%$, and E the remainder. B's investment is $3500. What is E's investment?

8. A and B engage in business as partners. A invests $6980 and B $5584. Each partner's share represents what per cent of the total investment?

9. A regiment went into battle with 938 men and came out with 804. What percentage was lost?

10. My holdings in real estate are worth $8500; my personal property is worth $4350. If, during the coming year, my real estate increases in value 23% and my personal property 9%, what will be the total value then and what will be the percentage of increase of both?

11. An operator bought a large tract of land and sold 40% of it to one customer, 20% of the remainder to a second customer, 25% of what still remained to a third customer. If 234 acres remain unsold, how many acres were there in the original tract of land?

12. In 1918 a merchant's sales amounted to $234,520. In 1919 they amounted to $213,413.20. If they increase during 1920 at the same rate that they decreased in 1919, what will be the sales for 1920?

13. On January 17, 1916, merchandise was bought for $1475.85 on three months' credit, subject to a discount of 3% if paid within ten days. What sum was required to settle the bill on January 27, 1916?

14. Two railroads, one 300 miles long and the other 500 miles long, carry 340 barrels of potatoes at a through rate of 23¢ a barrel. This freight is to be divided in proportion to each railroad's per cent of the total mileage. How much does each road receive?

15. An agent sold 160 bales of cotton, averaging 240 lb. each, at $32\frac{1}{4}$¢ a pound; 75 hhd. of tobacco, averaging 360 lb. each, at $26\frac{2}{3}$¢ a pound; 130 bbl. sugar, averaging 180 lb. each, at $10\frac{1}{2}$¢ a pound. He charges $2\frac{1}{2}$% of the amount received. What was his charge?

16. A speculator bought a farm of 175 acres of land for $11,900 and sold 64 acres for $5440. What per cent did he gain on the part sold?

17. By energetic effort, the sales department of a certain business was able to increase the sales 20% each year for three successive years. The total increase amounted to $36,400. What were the sales the year before the first increase was effected?

18. A 9% dividend on stock amounted to $873. What was the face value of the stock?

19. A second-hand dealer sold two automobiles for $640 each. On one he gained 20% and on the other he lost 20%. Did he gain or lose by the transaction? How much?

20. A speculator invested \$5340. He gained 10% the first year, 13% the second year, lost 18% the third year, and gained 5% the fourth year. What is his capital at the end of the fourth year?

21. Several years ago 176,834,300 pounds of fish passed through the Boston market. Of this quantity, Gloucester furnished 110,637,829 pounds. At a more recent date the total amounted to 215,643,330 pounds, of which Gloucester furnished 120,418,563 pounds. What per cent was furnished by Gloucester in each year?

22. An automobile manufacturer decided to reduce the price of his cars 10%, and called upon his sales department to increase the sales a sufficient amount to counteract the reduction in price. What per cent increase would the general manager of the sales department be obliged to show?

23. A certain piece of property having depreciated \$2355, is now worth \$3925. What was its original value? What per cent has it depreciated?

24. A leather manufacturer owning 75% of a factory building sold $16\frac{2}{3}\%$ of his share and received \$1555. Find the value of the factory.

25. Hoyt's stock of goods is worth \$9462, which is 15% more than Taylor's, and 15% less than Ashton's. What is the value of the stock carried by each?

PROFIT AND LOSS

Study 265 to 273 inclusive.
References 274, 275.

37. What is the profit or loss and selling price of goods costing:

1. \$43.42 and sold at a profit of 9%?
2. \$87.54 and sold at a profit of $12\frac{1}{2}$%?
3. \$175.89 and sold at a profit of 27%?
4. \$21.43 and sold at a loss of $8\frac{1}{4}$%?
5. \$312.51 and sold at a loss of $13\frac{2}{3}$%?
6. \$15.07 and sold at a profit of $37\frac{1}{2}$%?
7. \$102.73 and sold at a profit of $33\frac{1}{3}$%?
8. \$240.81 and sold at a loss of 15%?
9. \$181.03 and sold at a profit of $28\frac{3}{4}$%?
10. \$37.56 and sold at a profit of 20%?

11. Property valued at \$3750 increased $8\frac{1}{3}$% in value. It was then sold. What was the profit? What was the selling price?

12. If I buy cloth at \$5.40 and sell it at $16\frac{2}{3}$% loss, what is my selling price?

13. Bought an automobile for \$1145. It was then sold at a loss of $16\frac{2}{3}$%. What was the loss and what was the selling price?

14. Paid \$240 for a pair of horses and sold them at a profit of $31\frac{1}{4}$%. How much did I gain?

15. Bought hats for \$36 a dozen and sold them at a profit of $33\frac{1}{3}\%$. What was the selling price of each hat?

16. My balance sheet shows that advertising, rent, clerk hire, etc., commonly called *overhead charges*, amount to about $8\frac{1}{3}\%$ of the total amount of my purchases. To leave a proper margin of safety I have decided to figure these overhead charges as 10% of the cost. In order to provide for this, how much must we mark goods costing \$13.40 to clear a profit of 15%?

17. Allowing an overhead of 10%, what must the following goods be marked to show a profit of 10%? of 15%? of 20%?

Refrigerators	costing	\$ 24.50
Chamber sets	costing	125.80
Persian rugs	costing	163.49
Pianos	costing	560.
Buffets	costing	61.18
Dining-room sets	costing	242.30
Tea-wagons	costing	21.73
Couches	costing	75.50
Parlor sets	costing	407.92
Veranda sets	costing	195.60

References 276, 277, 278.

38. Find the cost:

1. Loss \$151.20, rate of loss 10%.
2. Loss \$107.91, rate of loss $2\frac{1}{2}\%$.
3. Loss \$205.78, rate of loss $1\frac{1}{4}\%$.

4. Loss \$456.38, rate of loss $2\frac{1}{2}\%$.
5. Loss \$220.15, rate of loss $6\frac{1}{4}\%$.
6. Loss \$117.50, rate of loss $8\frac{1}{3}\%$.
7. Gain \$34.23, rate of gain 21%.
8. Gain \$79.85, rate of gain 25%.
9. Gain \$12.73, rate of gain $33\frac{1}{2}\%$.
10. Gain \$19.05, rate of gain $22\frac{3}{4}\%$.
11. Gain \$22, rate of gain $16\frac{1}{2}\%$.
12. Gain \$17.98, rate of gain 20%.
13. Selling price \$1056.80, rate of gain 5%.
14. Selling price \$2435.28, rate of gain 20%.
15. Selling price \$3672.25, rate of gain $16\frac{2}{3}\%$.
16. Selling price \$1434.75, rate of gain $12\frac{1}{2}\%$.
17. Selling price \$1806.75, rate of gain $33\frac{1}{3}\%$.
18. Selling price \$2584.82, rate of gain 2%.
19. Selling price \$950.28, rate of loss 5%.
20. Selling price \$42.00, rate of loss 50%.
21. Selling price \$245.75, rate of loss $16\frac{2}{3}\%$.
22. Selling price \$5042.80, rate of loss $22\frac{2}{9}\%$.
23. Selling price \$550.25, rate of loss $6\frac{1}{4}\%$.
24. Selling price \$64.80, rate of loss $1\frac{2}{3}\%$.
25. By selling goods for \$140 I lose $12\frac{1}{2}\%$. At what price must I sell them to gain $12\frac{1}{2}\%$?
26. Sold a row boat that cost \$80 at a gain of $12\frac{1}{2}\%$, and with the proceeds I purchased another boat which I sold at a loss of 10%. How much did I gain by both transactions?
27. A cow was sold for \$67.50 which was 10% below cost. What was the cost and what was the loss?

28. I bought a safe for $118.80. This was 10% higher than the manufacturer's price. What was the retailer's profits?

29. A merchant marked his goods at 37½% above cost. What is the cost of an article that he marked at $156.64?

30. I sold goods to a customer at a price which would have netted me a profit of 25% had the customer paid his bill. He failed, however, and was able to pay me only 85¢ on the dollar. In spite of this, I netted a profit of $314 on the transaction. What was the amount of my bill?

31. A manufacturer gained 25%; the wholesaler made a profit of 20%; the retailer made a profit of 33⅓%. What was the actual manufacturing cost of an article the retail price of which was $100?

32. Suppose, in the above question, that the manufacturer should sell direct to the customer, thereby increasing the cost of the article 10%. At what price could he afford to sell at retail and still make the same per cent profit that he now enjoys?

References 279, 280.

39. Find the per cent of gain or loss:

1. Cost $105.50, gain $21.10.
2. Cost $165, gain $49.54.
3. Cost $40.75, gain $7.34.
4. Cost $140.75, gain $14.08.
5. Cost $200, gain $75.50.

6. Cost $103.40, loss $17.23.
7. Cost $755.90, loss $60.47.
8. Cost $64.80, loss $19.44.
9. Cost $21.70, loss $1.74.
10. Cost $75, loss $23.25.
11. Cost $220, gain $125.40.
12. Cost $115.20, gain $63.36.
13. Cost $1256.40, gain $527.68.
14. Cost $750.48, gain $202.63.
15. Cost $90, gain $14.85.
16. Cost $15.24, gain $8.08.
17. Cost $420.50, gain $50.47.
18. Cost $50.17, gain $9.03.
19. Cost $430.85, gain $12.93.
20. Cost $59.84, loss $17.95.
21. Cost $2412.50, loss $627.25.
22. Cost $650.75, loss $221.26.
23. Cost $108, loss $54.
24. Cost $542, loss $338.75.

25. A milliner bought hats at $27 a dozen and sold them for $3 each. What was the gain per cent?

26. What is gained by buying paper at $2 a ream and retailing it for 1¢ a sheet?

27. I bought a horse for $350 and sold him for $400. What was the gain per cent?

28. If a horse cost $400 and was sold for $350, what per cent was the loss?

29. A stationer bought 2 bundles of paper for $1.75 a ream and sold it at retail at the rate of 3 sheets for 2¢. What per cent did he gain and how much did he gain in all?

30. A dealer bought 150 crates of fruit for $1 a crate. He sold 35 crates at $1.25 a crate, 30 crates at $1.15 a crate, 60 crates at $1.20 a crate, 10 crates at cost, and threw the rest away as worthless. What did he gain or lose and what per cent?

31. What per cent profit will be realized from the sale of peaches at 3¢ each if they cost $1.25 a hundred and 10% of them are lost by decay?

32. I sold C. S. Chase goods that cost $430 so as to make a profit of 60%, on 30 days' credit. Before the account was due, Chase failed, paying only 37½¢ on the dollar. What was my per cent of loss?

33. A merchant bought gloves at $8 per dozen pairs and sold them at $1.25 a pair. What was the per cent gained?

GENERAL PROBLEMS IN PROFIT AND LOSS

References 274 to 280 inclusive.

40. 1. By selling goods at a profit of 37½% I made $215.45. What do the goods cost and what do they sell for?

2. Goods were sold at a profit of 20%. If the seller received $36.54, what was his profit?

3. I bought tea for 53¢ a pound. What price must I sell it for in order to gain 32%?

4. What is the loss on goods sold for $4348.50, which is 19% below cost?

5. Goods are sold for $436.28 at a loss of 15%. For what price should they be sold to gain 15%?

6. A dealer sold hats at retail for $3.50 each, and at wholesale for $33 a dozen. At retail, his profit was 40%. Does the wholesale price show a profit or loss? How much per hat? What per cent?

7. A barrel of flour sold for $9.45 nets a profit of 35%. At what price could we sell it if we were content with a profit of 20%?

8. A coal dealer buys coal for $9.75 by the long ton and sells it for $11.75 by the short ton. What per cent profit does he make?

9. A sold a factory building to B for $8621. By so doing he lost 10%. B expended $2300 installing a sprinkler system and then sold the factory for 20% more than A paid for it. How much did B gain and what per cent did he gain on his investment?

10. How much should be asked a pound for fish costing $6.50 a hundred to net a profit of 10% and allow for 10% waste?

11. A book agent sells two books for $5 each. On one he loses 20%, and on the other he gains 20%. Does he gain or lose, how much and what per cent?

12. Goods bought at $4 a gross and sold at 40¢ a dozen yield what per cent profit?

13. I sold goods to a retailer at a profit of 40%. Before settlement he failed, paying 25¢ on the dollar. What was my loss on goods costing $320?

14. Bought a horse from A at 20% less than it cost him. Sold it for 25% more than I paid for it. I gained $15 in the transaction. What did the horse cost A; what did it cost me; how much did I sell it for?

15. I buy oranges at $2.50 a hundred. What price must I mark them a dozen to gain 25%, allowing 10% for decay and 15% overhead expenses?

16. A dealer buys 6 bags Rio coffee, 218 lb. in a bag, at $41\frac{2}{3}$ ¢ a pound; 12 bags Java coffee, 75 lb. in a bag, at 25¢ a pound. After mixing the two kinds, he sells at a profit of 75%. What is the price a pound?

17. What price can I afford to pay for property that rents for $50 a month in order to make a net profit of 5% a year, allowing $200 annually for necessary repairs, taxes, etc.?

18. Find the gain or loss per cent of each of the following, carrying to the fourth decimal place:

A.	Purchases	$13,502.10
	Sales	12,786.50
	Inventory at closing	4,983.70
B.	Inventory at beginning	3,908.00
	Purchases	10,680.20
	Sales	10,450.50
	Inventory at closing	6,400.75
C.	Inventory at beginning	3,100.85
	Purchases	8,900.50
	Sales	11,500.00
	Returned to us	540.30
	Returned by us	560.50
	Inventory at closing	4,550.00

TRADE DISCOUNT

Study 284 to 290 inclusive.
References 291, 292.

41. Find the net amount of the following bills:

1. List price \$340.25, discount 35%.
2. List price \$1256.35, discount 27%.
3. List price \$438.40, discount 23%.
4. List price \$750.50, discount $33\frac{1}{3}$%.
5. List price \$755.75, discount $37\frac{1}{2}$%.
6. List price \$351.20, discount 20%.
7. List price \$1050.30, discount 16%.
8. List price \$978.80, discount 42%.
9. List price \$127.70, discount 21%.
10. List price \$2040.50, discount 2%.
11. List price \$1434.25, discounts 20% and 10%.
12. List price \$760.20, discounts $12\frac{1}{2}$ % and 10%.
13. List price \$126.34, discounts 20% and 25%.
14. List price \$285.40, discounts $27\frac{1}{2}$% and 20%.
15. List price \$1244.18, discounts 25% and 15%.
16. List price \$556.30, discounts 27% and 12%.
17. List price \$112.80, discounts 17% and 10%.
18. List price \$680.12, discounts $6\frac{1}{2}$% and 5%.
19. List price \$120.40, discounts 22% and 20%.

20. List price \$450.75, discounts 16% and 12%.

21. List price \$845.12, discounts 20%, 10%, and 5%.

22. List price \$360.70, discounts 20%, 25%, and 10%.

23. List price \$850.20, discounts 25%, 25%, and 10%.

24. List price \$351.65, discounts 10%, 10%, and 5%.

25. List price \$1201.30, discounts 18%, 7%, and 3%.

26. List price \$700.50, discounts $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, and $6\frac{1}{4}\%$.

27. List price \$325.40, discounts 12%, 10%, and 5%.

28. List price \$970.60, discounts 15%, 12%, and 4%.

29. List price \$1010.25, discounts 15%, 10%, and 8%.

30. List price \$242.50, discounts 25%, 25%, and 5%.

31. List price \$1068.20, discounts 50%, 50%, and 10%.

32. List price \$978.35, discounts 25%, 40%, and 35%.

33. List price \$326.30, discounts 50%, 30%, and 25%.

34. List price \$829.20, discounts 40%, 50%, and 10%.

35. List price \$2048.80, discounts 60%, 25%, and 15%.

36. List price \$1218.75, discounts $62\frac{1}{2}\%$, $37\frac{1}{2}\%$, and 10%.

37. List price \$650.50, discounts $66\frac{2}{3}\%$, 40%, and 12%.

38. List price \$450.20, discounts 50%, 30%, and 20%.

39. List price \$360.20, discounts 65%, 15%, and 10%.

40. List price \$520.80, discounts 30%, 50%, and 20%.

Reference 293.

42. What single discount is equal to a discount series of:

1. 20%, 10%, and 5%?
2. 30%, 10%, and 5%?
3. 10%, 5%, and 2%?
4. 12%, 5%, and 2%?
5. 20%, 8%, and 5%?
6. 15%, 10%, and 6%?
7. 10%, 10%, and 5%?
8. 20%, $12\frac{1}{2}$%, and 10%?
9. $33\frac{1}{3}$%, 10%, and 10%?
10. 50%, 20%, and 10%?
11. 25%, 15%, and 6%?
12. 50%, 25%, and 10%?
13. 35%, $37\frac{1}{2}$%, and $12\frac{1}{2}$%?
14. 40%, 25%, and 10%?
15. $16\frac{2}{3}$%, 5%, and 2%?
16. 15%, 12%, and 4%?
17. 30%, 10%, and 8%?
18. 10%, 5%, and 10%?
19. 10%, 20%, and 20%?
20. 25%, 25%, and 10%?
21. 50%, 50%, and 20%?
22. 35%, 25%, 20%, and 10%?
23. 50%, 20%, 20%, and 10%?
24. 60%, 40%, and 20%?

References 294, 295, 296.

43. Find the gross amount of the following bills:

1. Discount \$125.30, rate of discount 25%.
2. Discount \$104.15, rate of discount 20%.
3. Discount \$275.50, rate of discount $12\frac{1}{2}$%.
4. Discount \$340.75, rate of discount 5%.
5. Discount \$210.42, rate of discount $37\frac{1}{2}$%.
6. Net \$134.80, rate of discount 10%.
7. Net \$84.60, rate of discount 60%.
8. Net \$234.50, rate of discount 15%.

9. Net \$119, rate of discount $12\frac{1}{2}\%$.
10. Net \$218.26, rate of discount 30%.
11. Discount \$125, rate of discount 35%.
12. Discount \$215.50, rate of discount 25%.
13. Discount \$36.75, rate of discount $12\frac{1}{2}\%$.
14. Discount \$105.15, rate of discount 15%.
15. Discount \$341.70, rate of discount 35%.
16. Net \$85.50, rate of discount 25%, 10%.
17. Net \$128.40, rate of discount 40%, 25%, 10%.
18. Net \$275.75, rate of discount 25%, 10%, 10%.
19. Net \$187.26, rate of discount 50%, 10%, 5%.
20. Net \$150.50, rate of discount 10%, 5%, 2%.

Reference 297.

44. What rate of discount shall we allow on a bill of:

1. \$346.40 to net \$259.80?
2. \$780 to net \$530.40?
3. \$112.50 to net \$93.75?
4. \$218.40 to net \$136.50?
5. \$187.26 to net \$112.37?
6. \$360 to net \$288?
7. \$450 to net \$393.75?
8. \$234.50 to net \$195.42?
9. \$312.90 to net \$208.60?
10. \$520 to net \$374.40?
11. \$760 to net \$585.20?
12. \$365.75 to net \$219.45?
13. \$1600 to net \$1400?

14. \$2340 to net \$1521?
15. \$298 to net \$172.84?
16. \$169.75 to net \$105.25?
17. \$2359 to net \$2064.13?
18. \$250 to net \$182.50?
19. \$3296 to net \$2307.20?
20. \$265.50 to net \$199.13?

Reference 298.

45. At what price must the following goods be marked in order to allow the prescribed discount and still make the designated profit?

	Cost	Required Profit	Discount
1.	\$100	$37\frac{1}{2}\%$	20%
2.	\$250	24%	35%
3.	\$304.75	12%	20% and 10%
4.	\$480.20	35%	$37\frac{1}{2}\%$
5.	\$1050.50	40%	25%, 10%, and 5%
6.	\$785.40	$33\frac{1}{3}\%$	23%
7.	\$570	18%	32%
8.	\$921.20	27%	50%
9.	\$893.68	$12\frac{1}{2}\%$	25% and 10%
10.	\$723.85	18%	30%, 10%, and 2%
11.	\$5200	22%	21%
12.	\$225.20	50%	10%
13.	\$720.30	27%	5% and 2%
14.	\$850	25%	40%
15.	\$365.50	30%	17%

	Cost	Required Profit	Discount
16.	$180	40%	10%, 10%, and 5%
17.	$127.50	21%	32%
18.	$150.25	15%	$12\frac{1}{2}$%
19.	$900	10%	5%, 5%, and 10%
20.	$675.80	20%	23%

GENERAL PROBLEMS IN TRADE DISCOUNT

References 291 to 298 inclusive.

46. 1. I bought a bill of $546.30 at a trade discount of 20%, 10%, and 5% with an additional cash discount of 2%. I took advantage of the cash rate and then sold the goods at the original list price with a flat discount of 25%. How much did I gain? What per cent?

2. If I buy goods at a discount of 30% from the list price and sell at the list price, what per cent profit do I make?

3. Goods bought at $6 a gross at a discount of 20%, and sold at 75¢ a dozen yield what per cent profit?

4. Which is better and how much on a bill of $240, a discount of 20%, 10%, and 5%, or a discount of 35%?

5. By selling goods at $1.50 a yard I make 25% profit. What must I mark them in order to deduct 10% and still make the same profit?

6. Goods costing $345 are marked up 30% and are then sold at a discount of 20%. How much is gained and what is the gain per cent?

7. Goods are marked up 20%. What discount can the seller allow on this price and still net the cost?

Make out bills for the following:

8. Use current date, your own locality. Adams and Baxter bought of the Boston Hardware Company, terms net 30 days, 2% 10 days: 2 dozen chisels at \$3.20; $5\frac{7}{12}$ dozen 10-inch drawing knives at \$5.50; $1\frac{6}{12}$ dozen ratchet screw drivers at \$8.75, subject to a discount of 10%, 10%, and 5%; 12 steel shovels at $87\frac{1}{2}$¢; 9 spades at 70¢; 3 dozen garden rakes at \$5; 4 dozen trowels at \$1.35, subject to a discount of 20%, 10%, and 10%; 4 dozen cans prepared paint at \$3.15; 480 feet of $\frac{3}{4}$-inch garden hose at 10¢; 5 lawn mowers at \$4.50, subject to a discount of $16\frac{2}{3}$%, 5%, and 5%; 8 dozen boxes of 4-inch bolts at \$2.35. Make out bill showing the total due at the expiration of credit and the amount due on the cash terms.

9. Use current date, your own locality. Naumkeag Company bought of Childs Provision Company, terms 2% for cash and net 10 days: 150 barrels Baldwin apples at \$5.50; 25 barrels greenings at \$4.50; 45 bushels of beans at \$4.75; 90 bushels of potatoes at \$1.95, subject to a discount of 10% and 10%; 4 firkins of butter, 65 pounds each, at 50¢; 5 firkins creamery butter, 50 pounds each, at 48¢; 6 boxes American cheese, 50 pounds each, at 30¢; 4 boxes Young America cheese, 33 pounds each, at 36¢, subject to a discount of 10%, 5%, and 2%; 4 sacks of Rio coffee, 140 pounds each, at 48¢; 2 bags Java coffee, 215 pounds each, at 46¢; 2 barrels rice, 320 pounds each, at 16¢; 1 barrel New Orleans molasses, 45 gallons, at 35¢; 1 barrel Porto Rico molasses, 45 gallons, at $33\frac{1}{3}$¢, subject to a

discount of 10%, 10%, and 5%. Make out bill showing the total due at the expiration of credit and the amount due on the cash terms.

10. Use current date, your own locality; seller, L. A. White, Wholesale Company; purchaser, Parker Clothing Company; terms, net 60 days, 2% 10 days: 45 girls' rain capes at $4; 75 girls' sweaters at $3.75; 125 children's rompers at $1.50; 75 boys' rubber coats at $4.25, subject to a discount of 15%, 10%, and 2%; 35 voile waists at $2.25; 40 organdie waists at $3.50; 70 pairs women's gloves at $1.60; 50 pairs of women's silk gloves at $1.25; 14 dozen women's handkerchiefs at $2; 12 dozen men's handkerchiefs at $2.25; 25 men's bath robes at $5; 75 men's shirts at $1.50; 120 white skirts at $3.24; 60 dress skirts at $5.75; 75 serge dresses at $14.50; 50 ladies' belts at 75¢, subject to a discount of 10%, 10%, and 10%. Make out bill showing the total due at the expiration of credit and the amount due on the cash terms.

11. Use current date; your own locality; you are the seller; your teacher the purchaser; terms, net 30 days, 2% off for cash: 3 #124 phonographs at $85; 2 #062 phonographs at $125; 4 #68a phonographs at $75; 1 #300 phonograph at $250; 2 doz. Operatic Records at $2 each; 2 doz. Standard Song Records at $1.25 each; 2 doz. Band and Orchestra Records at $1.50 each; 1 doz. Comic Monologue Records at 85¢ each; 1 doz. Records for Dancing at 85¢ each.

COMMISSION

Study 299 to 310 inclusive.
References 311, 312.

47. Find the commission and net proceeds:

	Sales	Commission	Charges
1.	\$4342.50	$2\frac{1}{2}\%$	\$214.30
2.	\$356.40	3%	\$85
3.	\$1256	5%	\$13.25
4.	\$8784.50	$3\frac{1}{2}\%$	\$43.50
5.	\$788.40	4%	\$38.56

Find the commission and gross cost:

	Purchases	Commission	Charges
6.	\$435.25	3%	\$48.56
7.	\$1840.50	$2\frac{1}{2}\%$	\$128.50
8.	\$843.35	3%	\$83.87
9.	\$1258.40	5%	\$324.50
10.	\$5643.50	8%	\$125.40

References 311, 312.

48. 1. An agent sold a farm for \$3690 at $2\frac{1}{2}\%$ commission. What was his commission?

2. I sold 128 barrels of sugar, each weighing 350 pounds, at $9\frac{3}{4}¢$ a pound. What was my commission at $2\frac{3}{4}\%$?

3. A commission merchant sold 250 barrels of sugar, each weighing 350 pounds, at $9\frac{1}{2}¢$ a pound and 158 barrels of molasses, each containing 48 gallons, at $77\frac{1}{2}¢$ a gallon. Find his commission at 2%.

4. A real estate agent sold a house for $4450 at $1\frac{1}{2}\%$ commission. What sum did he send the owner?

5. Rule a sheet of paper and copy the following account sales, making the necessary extensions:

ACCOUNT SALES

Albany, N. Y., August 14, 1919

Sold for the Account of

C. F. Adams, Troy, N. Y.
By W. H. Smith, Commission Merchant.

1919					
June	13	100 bbl. G. M. flour		$10.50	
	28	150 bbl. G. M. flour		10.75	
July	10	200 bbl. G. M. flour		10.25	
	15	100 bbl. G. M. flour		10.50	
		Charges			
June	10	Freight $125	Cartage	$27.00	
July	8	Storage 15.50	Insurance	5.20	
	12	Guaranty 1%	Commission	4%	
		Net Proceeds			

6. Prepare an account sales under date of January 10, for 5000 bushels of wheat, sold by J. J. Campbell & Co., Springfield, Mass., for the account of Walter Bros., North Adams, Mass., Sales, No. 25; 500 bushels at $2.08, December 30; the remainder at $2. Charges: freight, $91; cartage, $15; storage, $15.50; insurance, $\frac{1}{2}\%$; guaranty, 1%; commission, $2\frac{1}{2}\%$.

7. Prepare an account sales under date of October 15 for the account of J. C. Brown & Co., sold by Thomas Moody & Co., both of Chicago, Ill., October 3; 50 barrels of W. R. flour at $9.25; 100 barrels of K. A. flour at $9.80. Charges: freight, $50; cartage, $14.50, both under date of Oct. 1; Oct. 11, storage, 150 barrels at 4¢ a barrel; insurance, $\frac{1}{4}\%$; commission, $3\frac{3}{4}\%$.

8. Put the following in the form of an account sales: William C. Jones, St. Louis, Mo., sold for account of Charles W. Franklin, Chicago, Ill., the following goods: August 4, 7 pieces of summer silk, 284 yards, at $1.85; August 13, 5 pieces black silk, 216 yards, at $1.50; August 17, 16 pieces calico, 798 yards, at 19¢; September 3, 19 pieces alpaca, 548 yards, at 38¢; September 10, 25 pieces diagonals, 587 yards, at 75¢. Charges: August 1, freight and cartage, $65.48; insurance, $\frac{1}{2}\%$; commission, 4%. Find the net proceeds.

9. Arrange the following in the form of an account sales: E. P. Clark & Co., Peekskill, N. Y., sold for account of John Mason, the following: November 1, 300 bushels potatoes at $1.95; November 16, 200 bushels at $1.85; December 1, 240 bushels at $1.90. Charges: November 1, freight, $85.45; cartage, $2\frac{1}{2}$¢ per bushel; storage at 2¢ per bushel; commission, $4\frac{1}{2}\%$. Find the net proceeds.

10. Rule a sheet of paper and copy the following account purchase, making the necessary extensions.

ACCOUNT PURCHASE

UTICA, N. Y., MAY 10, 1919

Purchased by F. J. BOWEN & Co.

For the Account of E. L. GREEN, ROME, N. Y.

1919							
April	24	3 half-ch. J. tea, 165 lb.	48¢				
	29	4 half-ch. O. tea, 240 lb.	50¢				
May	7	5 half-ch. J. tea, 350 lb.	42¢				
	9	8 mats Rio coffee, 600 lb.	46¢				
		Charges					
		Cartage	$7.50				
		Commission 5%					
		Amount charged to your account					

11. In accordance with the foregoing form prepare an account purchase of tea purchased by W. L. Thomas, Feb. 21, for the account of Jones, White & Co., both of Boston, Mass.; 10 half-chests of J. tea, 600 lb., at 47¢; 5 half-chests O. tea, 250 lb., at 45¢; 5 cases C. tea, 250 lb., at 50¢; 8 half-chests E. B. tea, 480 lb., at 49¢. Charges: cartage, $8.80; commission, 4%.

12. Prepare an account purchase for the following: David Carey, New York City, bought of the account of Henry Grant & Co. of Newark, N. J., August 16, 1916, 68 yd. fancy prints at 25¢; 42 yd. colored silk at $1.25; 1 dozen ladies' felt hats, $30; 18 yd. black cassimere at $1.50; 3 suits boys' clothing at $10. Charges: packing and cartage, $2.40. Find entire cost, commission being 5%.

13. Prepare an account purchase for the following: March 18, 1916. A. B. Morse & Co., Trenton, N. J., bought for account of Harris & Price, of Philadelphia, Pa., March 18, 150 bbl. Dakota flour at \$9.75; 80 bbl. buckwheat flour at \$12.40; 480 bu. ground feed at 60¢; 500 bu. bran at 30¢; 20 bbl. G. M. flour at \$11. Cartage, \$7.90, commission, 4%. What is the entire cost?

14. A merchant in Boston shipped to his broker in New York a carload of potatoes, 967 bushels, which were sold at \$2.25 a bushel. What was realized on the sale if the broker charged 4½% for selling and the freight was \$67.96? How many pounds of Java coffee could be purchased with the proceeds of the sale of potatoes if coffee is 45¢ a pound and the broker charges 2% for buying?

References 313, 314, 315.

49. 1. A commission merchant working on a 4% commission earned \$240.50 for selling a consignment of flour. What did the flour sell for?

2. My commission for selling goods at 2% amounted to \$250.50. What was the selling price of the goods?

3. I bought goods, receiving \$75.30 as my 5% commission. What did I pay for the goods and what was the total cost to my principal?

4. A commission merchant whose charge is 1½% finds that his total receipts for commissions for 3 months amount to \$4856. What was the value of his sales for the same time?

5. A collector charges 5% for his services. In order that he may clear $3000 a year, what must his collections amount to?

6. I receive $736.96 as the net proceeds of the sale of goods through a commission merchant, the only charges being 2% for commission. What was the gross sales?

7. My lawyer sends me $77.80 as the proceeds of a claim which he has collected for me. What was the claim, his commission being 5%?

8. We have received a check for $344.75 as the net proceeds of a sale on which the commission was $1\frac{1}{2}\%$. What was the total sales and what was the commission?

9. I sent my commission merchant $138.65 to pay for goods purchased by him, including commission of $3\frac{1}{2}\%$. What did he pay for the goods?

10. I received $185.40 from J. C. Bryan to cover the amount of goods purchased and my commission of 3%. What was the amount of the purchase?

11. I received $564.20 with instructions to purchase certain supplies, my commission to be $1\frac{1}{2}\%$. What amount will I invest in supplies and what will my commission be?

12. A commission merchant received $1606 to invest, after deducting a commission of $\frac{3}{8}\%$. How much can he invest?

13. We received $145.50 as the net proceeds of a consignment. The rate of commission was 2% and other charges $2.50; what was the selling price of the goods?

14. A commission merchant remitted $704.29 after deducting his commission of $1\frac{3}{4}\%$ and other charges amounting to $3.20. What was the selling price of the goods and what was the commission?

15. My commission merchant has just sent me an account purchase showing gross cost to be $668.60. The commission was figured at 4%; the other charges amounted to $8.20. What was the prime cost of the purchase?

16. I paid a real estate broker $225 for selling a house and lot. This sum included his commission of 5% and other expenses amounting to $50. What sum did the house sell for?

17. A commission merchant received $12,500 to invest in cotton after deducting his commission of 5%. What sum does he invest? How many bales of 400 pounds can be bought at 35¢ a pound?

18. I shipped my agent in New York 950 tons of hay which he sold for me at $22.50 a ton. Charges were: for freight, $950; cartage, $327.50; storage, $.85 a ton; his commission, $2\frac{1}{2}\%$. I instructed him to invest the proceeds in wheat for me. If he charged me at the same rate for investing that he did for selling the hay, how much did he invest and what was his entire commission?

19. I bought a lot of apples for $6.50 a barrel on 3% commission. If my commission amounted to $81.51, how many barrels did I buy?

20. I received $893.03 with instructions to invest it in apples after deducting a commission of 5%. How many barrels can I buy at $5.50 a barrel?

Reference 316.

50. What is the rate of commission:

1. If the prime cost of merchandise is \$480 and the commission for buying is \$4.20?

2. If the net proceeds is \$944.40 and the commission is \$15.60?

3. If the first cost is \$3264 and the commission for buying is \$4.08?

4. If the gross proceeds is \$3200 and the commission for selling is \$128?

5. If a commission merchant receives \$37.02 for selling \$1234 worth of goods?

6. If a commission merchant receives \$85.75 for selling \$3430 worth of goods?

7. A real estate dealer bought a house for a client, paying \$8750 for it. His charges were \$262.50. What was his rate of commission?

8. A commission merchant bought 346 barrels of apples at \$5.75, receiving \$65.74 for his commission; other charges amounted to \$43.13. What is the gross purchase and what is his rate of commission?

9. A lawyer earned \$37.40 for collecting a claim on 5% commission. What was the claim?

10. A merchant sent his principal \$325.23 as the net proceeds of a consignment which he sold for \$343.43. What was the rate of commission?

GENERAL PROBLEMS IN COMMISSION

References 311 to 316 inclusive.

51. 1. My agent sells $1400 worth of goods for me at 3% commission. What amount must he remit?

2. An agent sells goods for $2468. His charges are: commission, $2\frac{1}{2}\%$; storage, insurance, etc., $125. What are the net proceeds?

3. I sold for a Chicago firm as follows: 950 bu. corn at 85¢ a bushel; 120 bbl. of pork at $15.50 per barrel. My commission on the corn was $1\frac{1}{4}$¢ a bushel and the commission on the pork was $2\frac{1}{2}\%$. Charges were: freight, $134; storage, $67; advertising, $63. What were the net proceeds due my employer?

4. I purchased for a South American firm goods valued at $3750. My commission was to be 2%. For what sum must I draw?

5. I have just received $4168 as the net proceeds of a consignment. The figures in the Account Sales are blurred and I am unable to read either the amount of the sales or the rate of commission, which is $145.84. Ascertain both.

6. I paid a real estate dealer $215 for selling a house and lot on 5% commission. Advertising and other expenses amounted to $50. What amount does the sale net me?

7. I have purchased for a Southern firm 45,620 feet of pine lumber at $16.25 a thousand; 34,257 feet of hemlock boards at $15 a thousand; 37,250 feet of spruce at $18.60 a thousand. My commission is

$3\frac{1}{2}\%$ and my other charges amounted to \$214. For what sum shall I draw on the firm to cover the cost of my purchase and charges?

8. A commission merchant received \$648.40 to invest in wool after deducting all his expenses. How much did he pay for the wool if his commission for buying was 2% and his other charges amounted to \$13.50?

9. A commission merchant received a consignment of 625 barrels of flour on which he paid \$84 for freight; \$12.75 for cooperage; \$22.50 for storage; \$19.50 for cartage. He sold 110 barrels at \$9.25 a barrel; 175 barrels at \$10.75; 123 barrels at \$11.25, and the remainder at \$10. His commission for selling was 2%. What was the net proceeds?

10. The gross cost of goods purchased through an agent was \$1221. If the commission was \$6 and the other charges \$15, what was the rate of commission?

11. I have received \$325 to invest in apples after deducting all expenses. How many barrels could I buy at \$5.75 a barrel; charges being 3% for commission; $2\frac{1}{2}$¢ a barrel for drayage; 12¢ a barrel for freight; \$5 for advertising? What was the unexpended balance, if any?

12. I sent \$1547 to my agent in Chicago with instructions to buy wheat after deducting his commission of 3%. How much did he invest in wheat?

13. A commission merchant received from a speculator \$2091 to invest in corn after deducting his commission of $2\frac{1}{2}\%$. He was instructed to hold the corn

subject to the purchaser's order. After an advance in value he was ordered to sell, and did so, obtaining $1.50 a bushel. After deducting his commission of $2\frac{1}{2}\%$ and $20 for storage, he paid the speculator $2220 as the balance due him. What did the commission agent pay a bushel for the corn?

14. A commission merchant's regular charges were 3% for selling and 2% for guaranteeing the purchase price. If he remitted $6842 to his principal as the net proceeds of the sale, what did the goods sell for?

15. A commission merchant sold 625 barrels of potatoes at $11.25 a barrel and invested the proceeds in wheat at 85¢ a bushel, first deducting a commission of 2% for buying and 2% for selling. How many bushels of wheat could he buy and what was the unexpended balance, if any?

Find the missing quantities in the following:

	Sale	Commission	Rate of Com.	Net Proceeds
16.	1246.50	———	5%	———
17.	———	234.40	$2\frac{1}{2}\%$	———
18.	5450.	272.50	——	———
19.	———	47.49	——	1535.51
20.	6254.50	———	——	6004.32
21.	———	———	4%	3317.76
22.	8564.50	———	6%	———
23.	1327.	79.62	——	———
24.	———	33.87	6%	———
25.	856.40	———	——	834.99

TIME

Study carefully 317 to 325 inclusive.
Reference 238.

52. By compound subtraction find the time from:

1. Jan. 3, 1910 to March 5, 1916.
2. Dec. 28, 1912 to June 11, 1914.
3. May 13, 1909 to Dec. 2, 1911.
4. June 28, 1910 to May 12, 1911.
5. April 3, 1912 to Oct. 14, 1913.
6. Feb. 28, 1912 to Jan. 1, 1914.
7. July 4, 1914 to April 14, 1916.
8. May 13, 1905 to Dec. 9, 1915.
9. March 9, 1908 to Feb. 6, 1912.
10. June 11, 1909 to April 3, 1916.
11. Dec. 6, 1914 to Aug. 7, 1917.
12. Nov. 28, 1913 to June 28, 1917.
13. Sept. 8, 1907 to Aug. 21, 1915.
14. May 30, 1905 to July 20, 1906.
15. Oct. 9, 1909 to June 11, 1916.
16. June 17, 1913 to May 16, 1916.
17. Feb. 6, 1908 to May 9, 1917.
18. July 8, 1911 to Aug. 2, 1912.
19. April 4, 1910 to March 17, 1913.

20. July 4, 1908 to May 22, 1916.
21. Dec. 13, 1909 to Nov. 12, 1917.
22. Feb. 3, 1907 to May 11, 1915.
23. June 18, 1911 to Aug. 14, 1917.
24. April 21, 1916 to Dec. 16, 1917.
25. Aug. 17, 1909 to June 11, 1916.
26. Jan. 30, 1912 to Feb. 18, 1912.
27. Oct. 27, 1908 to July 14, 1915.
28. Nov. 13, 1905 to Jan. 26, 1909.
29. April 5, 1910 to Oct. 9, 1917.
30. March 3, 1906 to June 8, 1916.
31. May 30, 1913 to March 12, 1914.
32. Sept. 24, 1906 to April 30, 1915.
33. Nov. 27, 1909 to Sept. 28, 1917.
34. Sept. 13, 1911 to May 11, 1914.
35. June 20, 1905 to Oct. 14, 1916.
36. Dec. 11, 1912 to Feb. 4, 1917.
37. Nov. 13, 1914 to Aug. 24, 1917.
38. May 11, 1908 to July 31, 1912.
39. Feb. 28, 1906 to June 30, 1913.
40. Nov. 8, 1913 to Sept. 3, 1917.
41. Jan. 1, 1908 to July 11, 1915.
42. Aug. 3, 1913 to June 11, 1916.
43. Nov. 21, 1908 to Oct. 9, 1917.
44. March 3, 1911 to June 16, 1912.
45. April 13, 1910 to Feb. 22, 1916.
46. Dec. 31, 1909 to Feb. 22, 1916.
47. Aug. 19, 1912 to April 3, 1917.

48. Oct. 4, 1913 to June 8, 1915.
49. June 11, 1914 to May 3, 1917.
50. Sept. 3, 1909 to June 21, 1916.

Reference 239.

53. Find the time in exact days from:

1. Jan. 13, 1908 to Nov. 12, 1908.
2. April 6, 1915 to June 29, 1915.
3. Dec. 14, 1908 to May 12, 1909.
4. May 8, 1916 to Nov. 30, 1916.
5. June 11, 1915 to Oct. 27, 1915.
6. Aug. 12, 1916 to July 3, 1917.
7. Nov. 16, 1915 to Jan. 18, 1916.
8. Aug. 17, 1914 to Dec. 31, 1914.
9. July 30, 1913 to Jan. 1, 1914.
10. May 27, 1906 to April 28, 1907.
11. Feb. 12, 1908 to Nov. 13, 1908.
12. Dec. 6, 1916 to Aug. 6, 1917.
13. May 12, 1916 to Oct. 28, 1916.
14. June 11, 1909 to Jan. 3, 1910.
15. Oct. 24, 1912 to May 16, 1913.
16. July 16, 1911 to Dec. 25, 1911.
17. Sept. 3, 1908 to May 27, 1909.
18. Aug. 18, 1915 to Nov. 26, 1915.
19. Sept. 27, 1908 to May 13, 1909.
20. Feb. 13, 1914 to Dec. 6, 1915.
21. June 17, 1915 to April 8, 1916.
22. July 4, 1916 to Dec. 25, 1916.

23. May 2, 1914 to April 18, 1915.
24. Dec. 16, 1913 to Sept. 29, 1914.
25. May 28, 1916 to Jan. 2, 1917.
26. Sept. 21, 1914 to Dec. 1, 1914.
27. April 13, 1915 to March 30, 1916.
28. Oct. 27, 1914 to June 3, 1915.
29. Nov. 28, 1906 to Dec. 13, 1906.
30. Jan. 31, 1914 to Jan. 3, 1915.
31. May 6, 1911 to April 14, 1912.
32. Dec. 25, 1915 to July 4, 1916.
33. June 22, 1909 to May 23, 1910.
34. April 28, 1916 to March 17, 1917.
35. Sept. 8, 1916 to March 17, 1917.
36. Dec. 4, 1913 to Feb. 12, 1914.
37. Dec. 25, 1916 to July 4, 1917.
38. Aug. 31, 1914 to May 12, 1915.
39. Dec. 30, 1915 to Sept. 9, 1916.
40. June 11, 1914 to Jan. 1, 1915.
41. April 3, 1915 to Jan. 4, 1916.
42. Dec. 28, 1914 to May 19, 1915.
43. Jan. 7, 1915 to Sept. 8, 1915.
44. Feb. 28, 1916 to Jan. 12, 1917.
45. June 3, 1912 to May 21, 1913.
46. May 14, 1911 to Nov. 28, 1911.
47. Sept. 9, 1912 to June 11, 1913.
48. Oct. 28, 1916 to July 14, 1917.
49. May 28, 1914 to April 7, 1915.
50. Dec. 8, 1915 to Sept. 29, 1916.

INTEREST

Study carefully 317 to 352 inclusive.
References 333, 334.

54. Find the interest on:

1. $462.40 for 2 mo. 6 da. at 6%.
2. $385.60 for 3 mo. 18 da. at 6%.
3. $460.75 for 1 mo. 8 da. at 6%.
4. $200 for 1 mo. 3 da. at 6%.
5. $260.70 for 72 da. at 6%.
6. $650 for 4 mo. 9 da. at 6%.
7. $450 for 6 mo. 15 da. at 6%.
8. $124 for 8 mo. 14 da. at 6%.
9. $285.50 for 93 da. at 6%.
10. $450.65 for 1 yr. 3 mo. 13 da. at 4%.
11. $562.30 for 5 mo. 13 da. at 4%.
12. $287.95 for 9 mo. 16 da. at 8%.
13. $396.40 for 1 yr. 3 mo. 14 da. at 8%.
14. $756 for 8 mo. 18 da. at 5%.
15. $468.40 for 2 yr. 5 mo. 6 da. at 7%.
16. $400 for 29 da. at $7\frac{1}{2}\%$.
17. $216.80 for 1 yr. 5 mo. 6 da. at $4\frac{1}{2}\%$.
18. $375.90 for 7 mo. 28 da. at $4\frac{1}{2}\%$.
19. $219.76 for 1 yr. 3 mo. 18 da. at 9%.

20. $468.75 for 10 mo. 13 da. at 3%.
21. $240 for 3 mo. 22 da. at 12%.
22. $375.80 for 1 mo. 10 da. at 4%.
23. $265.72 for 3 mo. 7 da. at $4\frac{1}{2}$%.
24. $486.85 for 4 mo. 3 da. at 6%.
25. $265.72 for 1 yr. 7 mo. 14 da. at 8%.
26. $380.60 for 8 mo. 15 da. at 7%.
27. $450 for 3 mo. 17 da. at $7\frac{1}{2}$%.
28. $264.40 for 5 mo. 11 da. at 9%.
29. $942.60 for 2 yr. 9 mo. 16 da. at 5%.
30. $384.32 for 3 mo. 12 da. at $4\frac{1}{2}$%.
31. $295.73 for 1 yr. 6 mo. 15 da. at 3%.
32. $389.60 for 11 mo. 18 da. at 6%.
33. $560.35 for 2 mo. 14 da. at 8%.
34. $387.60 for 6 mo. 5 da. at 4%.
35. $418.62 for 3 mo. 28 da. at $4\frac{1}{2}$%.
36. $560.32 for 7 mo. 19 da. at 6%.
37. $362.40 for 1 yr. 8 mo. 20 da. at $4\frac{1}{2}$%.
38. $271.35 for 4 mo. 24 da. at 7%.
39. $361.75 for 44 da. at 7%.
40. $285.60 for 2 yr. 8 mo. 24 da. at $7\frac{1}{2}$%.
41. $397.80 for 7 mo. 12 da. at 5%.
42. $184.25 for 8 mo. 21 da. at $4\frac{1}{2}$%.
43. $1495.60 for 1 yr. 2 mo. 13 da. at 6%.
44. $372.75 for 11 mo. 8 da. at 7%.
45. $175.43 for 2 yr. 7 mo. 14 da. at $7\frac{1}{2}$%.
46. $295.60 for 9 mo. 24 da. at 6%.
47. $362.70 for 8 mo. 21 da. at 6%.

48. $467.80 for 1 yr. 3 mo. 5 da. at 3%.
49. $284.60 for 9 mo. 13 da. at 8%.
50. $575.80 for 6 mo. 18 da. at $4\frac{1}{2}$%.

References 335, 336.

55. Find the interest on:

1. $600 from June 1, 1916 to Aug. 13, 1916 at 6%.
2. $360 from Oct. 3, 1914 to June 3, 1915 at 5%.
3. $180 from Dec. 6, 1915 to July 13, 1916 at 8%.
4. $840.75 from May 12, 1912 to Dec. 8, 1914 at 7%.
5. $454.54 from Jan. 16, 1916 to Oct. 28, 1916 at 5%.
6. $544.44 from April 5, 1916 to Jan. 1, 1917 at $7\frac{1}{2}$%.
7. $850 from July 18, 1914 to Dec. 31, 1916 at 9%.
8. $809 from Sept. 13, 1912 to June 11, 1915 at 5%.
9. $256 from Nov. 28, 1913 to Mar. 12, 1914 at $7\frac{1}{2}$%.
10. $660.80 from Aug. 17, 1914 to Jan. 3, 1916 at 7%.
11. $840 from April 1, 1914 to Jan. 3, 1916 at 7%.
12. $629 from Nov. 13, 1908 to Aug. 4, 1910 at $4\frac{1}{2}$%.
13. $548 from Jan. 30, 1912 to Sept. 28, 1914 at 7%.
14. $465.10 from Oct. 2, 1913 to Sept. 12, 1914 at 8%.
15. $654 from Feb. 12, 1914 to Dec. 21, 1916 at 6%.
16. $360 from June 16, 1909 to Sept. 30, 1914 at $4\frac{1}{2}$%.
17. $126 from Aug. 28, 1907 to Feb. 16, 1912 at 4%.
18. $480 from April 6, 1910 to June 30, 1914 at 5%.
19. $1000 from Oct. 13, 1914 to Nov. 28, 1915 at 6%.
20. $975 from May 12, 1916 to Nov. 11, 1916 at 9%.
21. $649.24 from Jan. 1, 1914 to April 3, 1916 at 6%.
22. $100 from Oct. 2, 1915 to Feb. 12, 1916 at 5%.

23. $654 from Dec. 12, 1915 to Aug. 7, 1916 at 5%.
24. $962 from March 8, 1915 to Aug. 7, 1916 at 4½%.
25. $269.05 from Dec. 3, 1909 to Sept. 8, 1915 at 4%.
26. $680 from April 21, 1914 to Nov. 2, 1914 at 6%.
27. $500 from Aug. 19, 1913 to May 28, 1915 at 8%.
28. $85 from Nov. 12, 1914 to April 3, 1915 at 4%.
29. $450 from Feb. 9, 1916 to Dec. 21, 1916 at 5%.
30. $240 from Jan. 8, 1913 to June 9, 1915 at 7%.
31. $400 from Sept. 12, 1909 to Oct. 9, 1914 at 7½%.
32. $560 from July 4, 1910 to Sept. 8, 1914 at 4½%.
33. $200 from March 2, 1913 to Sept. 27, 1913 at 9%.
34. $460 from Dec. 8, 1916 to Feb. 3, 1917 at 3%.
35. $296.50 from May 8, 1904 to June 11, 1913 at 8%.
36. $320.60 from Oct. 28, 1913 to July 7, 1915 at 4%.
37. $576 from Jan. 23, 1909 to Aug. 13, 1912 at 6%.
38. $320.60 from Dec. 6, 1914 to June 28, 1916 at 4½%.
39. $720.14 from July 11, 1912 to Aug. 29, 1916 at 8%.
40. $365.40 from Sept. 14, 1911 to Feb. 18, 1915 at 7%.
41. $428.60 from May 11, 1916 to Nov. 8, 1916 at 12%.
42. $576.80 from June 20, 1909 to Dec. 31, 1909 at 10%.
43. $162.38 from Oct. 27, 1914 to Dec. 3, 1916 at 8%.
44. $316.20 from July 26, 1913 to Oct. 19, 1914 at 5%.
45. $483.90 from Jan. 8, 1915 to July 4, 1916 at 4½%.
46. $265.70 from Sept. 4, 1915 to May 30, 1916 at 9%.
47. $456.75 from Dec. 9, 1914 to June 30, 1916 at 4½%.
48. $195 from Nov. 19, 1913 to April 19, 1914 at 4%.
49. $362.80 from Sept. 12, 1914 to Aug. 8, 1915 at 6%.
50. $195.64 from Aug. 14, 1915 to May 3, 1916 at 3%.

ACCURATE INTEREST

References 326, 327, 328; 337 to 343 inclusive.

While both methods are used in business, the one explained in 340 is better because of the infrequency with which accurate interest is used.

56. Find the accurate interest of:

1. \$1436 for 295 days at 6%; at 8%.
2. \$484.50 for 193 days at 6%; at $7\frac{1}{2}$%.
3. \$956.35 for 1 year 214 days at 6%; at $4\frac{1}{2}$%.
4. \$632 for 462 days at 6%; at $4\frac{1}{2}$%.
5. \$1284.50 from Aug. 8, 1916 to Jan. 12, 1917 at 8%.
6. \$543.32 from Apr. 7, 1916 to Feb. 25, 1917 at 5%.
7. \$246.50 from Jan. 5, 1915 to May 27, 1916 at $4\frac{1}{2}$%.
8. \$3432.40 from June 13, 1914 to Feb. 29, 1916 at $3\frac{1}{2}$%.
9. £120 9*s* 8*d* for 214 days at 8%.
10. £253 11*s* 10*d* for 313 days at 5%.
11. £586 14*s* 4*d* for 1 year 246 days at $4\frac{1}{2}$%.
12. £732 15*s* 9*d* for 2 years 97 days at 3%.

NOTE: Change English money to pounds; see 232, and then apply 343. Reduce resulting decimal to lower denomination, 231.

TO FIND TIME

References 346, 347, 348.

57. 1. In what time will \$417.40 produce \$7.43 at $3\frac{1}{2}$% interest?

2. How long will it take \$325.80 on interest at 10% to produce \$30.86?

3. $895.80 earned $16.50 at 7% interest. How long was the money at interest?

4. How long will it take $9500 to earn $1524.75 at 9%?

5. I loaned $592.25 to my brother at 5% interest. He paid me $13.74 interest. How long did he have the money?

6. $182.40 drawing interest at 4% earned $9.48. How long was it invested?

7. $4150.30 was invested at 8% long enough to earn $295.58. How long was it invested?

8. $318.60 at 3% would require how long to produce $7.38 interest?

9. I loaned $7500 at 7%. When the loan was paid I received $7702.71. For what time was the loan made?

10. I received a check for $533.85 to cancel a loan of $519.75 effected at $4\frac{1}{2}\%$. How long had the loan been standing?

TO FIND RATE

References 349, 350.

58. 1. At what rate will $1836 earn $23.46 in 115 days?

2. The interest on $852 for 18 days is $2.13. What is the rate?

3. In 93 days $75 increases at interest $1.55. What is the rate?

4. In 144 days $375 amounts to $381.75. What is the rate?

5. At what rate would $982 be placed at interest for 2 mo. 12 da. to earn $9.82?

6. A loaned $2160 for 5 yr. 9 mo. 1 da. It amounted to $3091.95. What rate did he charge?

7. $588 on interest for 2 yr. 3 mo. and 18 da. earns $67.62. What is the rate?

8. At what rate must $3500 be placed at interest for 4 mo. and 15 da. to amount to $3605?

9. $296 produced $3.70 in 45 days. At what rate was it invested?

10. $810 amounts to $829.44 in 6 months and 12 days. What was the rate?

11. $750 on interest from March 1, 1916 to August 7, 1916 earns $26.50. What is the rate?

12. From April 7, 1915 to October 2, 1915, $360 earns $16.02. What is the rate?

TO FIND PRINCIPAL

References 351 to 356.

59. 1. What principal will be required to earn $13.18 in 11 mo. 11 da.?

2. What principal will be required to earn $39.92 in 192 days at $3\frac{1}{2}\%$?

3. How much money invested at $5\frac{1}{2}\%$ for 86 days will earn $24.62?

4. The interest is \$16, time 5 months and 18 days, rate 8%. What is the principal?

5. At 9% the interest for 3 mo. 27 da. is \$150.07. What is the principal?

6. What principal will in 6 mo. 13 da. at 6% amount to \$720.55?

7. What principal will in 8 mo. 25 da. at 5% earn \$15.73?

8. In 4 mo. 9 da. at 4% what principal will amount to \$591.61?

9. At 6% what principal will in 1 yr. 9 mo. 28 da. yield \$74.99?

10. Money invested for 1 yr. 7 mo. 14 da. at $4\frac{1}{2}\%$ amounts to \$531.30. What was the principal?

11. A certain sum of money on interest at 3% from May 19, 1910 to July 15, 1915 earns \$392.35. What is the investment?

12. What sum was loaned on April 5, 1910 at 7% if it were paid by check for \$942.66 on Jan. 9, 1916?

13. What principal on interest from October 25, 1905 to May 21, 1912 at 12% would amount to \$566.44?

14. What sum on interest from July 15, 1915 to October 19, 1915 at 7% will earn \$14.90?

15. What sum on interest from May 13, 1910 to August 25, 1919 would amount to \$11,446.47?

GENERAL PROBLEMS IN INTEREST

Before attempting to solve the following problems, the student should be thoroughly familiar with the entire subject of interest as presented in paragraphs 317 to 356 inclusive. The following problems comprise a series of tests on the subject of interest and are not graded according to difficulty, but are arranged as such problems might present themselves in business. Solve each in the simplest possible way.

60. 1. A note for \$852 dated May 2, 1915 with interest at 5% was paid on Feb. 14, 1916 by certified check. What was the amount of the check, time being computed in exact days?

2. I have just received a legacy of \$5000. I have an obligation of \$395 which will be due in 1 yr. 6 mo. and 14 da. from to-day. I have decided to set aside enough of my legacy at 4% interest to pay the obligation when it is due. How much will I have to set aside?

3. Brown borrowed of me \$400 at 7%, Jones \$647 at 5%, and Smith \$398 at $4\frac{1}{2}$%. Brown's loan ran 1 year 8 months and 11 days, Jones' ran 6 months and 19 days, and Smith's ran 297 days. What were my total receipts for interest?

4. After a loss by a fire the insurance company has agreed to pay me \$4340 in full settlement of the claim. They will pay this amount in full at the end of 60 days or will make a cash settlement, deducting 2%. Which proposition should I accept and what will I gain by so doing, money being worth 6%?

5. On May 26, 1915 I gave my note for \$1000 for 6 months at 5%. November 26, 1915 I paid the note

and accumulated interest, reckoned on the basis of exact days. How much did I pay?

6. A bill of $780.14 due on March 3, 1909, was not paid until October 15, 1909, when it was settled with interest at 6%. What was the amount paid, computing time in exact days?

7. A house costing $7500 rents for $60 a month. What rate of interest does the investment pay if the annual expenses, including repairs, taxes, etc., amount to $250?

8. What will be the difference in the amount of interest involved on a claim for $85, running from May 12, 1909 to October 19, 1909 at 8%, between the amount due by computing the time in exact days and computing the time in months and days?

9. On May 1, 1915, I bought a bill of hides at $15,300, on 60 days' credit, 2% off for cash, and borrowed the money at 6% on the hides as security to accept the cash price, giving my note for 60 days. At the expiration of the note I had tanned the hides at an expense of $1500 and sold them at an advance of 25% on the price paid for them. After paying my note, what was my net profit?

10. I bought a bill of $1450 subject to a discount of 20%, 10%, and 5%, with an additional discount of 2% for cash, and borrowed the money to pay for it at 5%. After 45 days I sold the goods at the same list price, subject to a discount of 25%, 2% extra for cash, receiving cash settlement, and paid my loan. What was my profit?

PARTIAL PAYMENTS

Study carefully 357 to 370 inclusive, before attempting to work on partial payments.

References 371, 372, 373.

Use the United States Rule.

61. 1. What is the balance due on July 1, 1916 on a note for $1500 dated February 1, 1913, upon which the following payments were made: July 24, 1913, $250; Aug. 7, 1913, $100; March 9, 1915, $50; Jan. 1, 1916, $300; interest at the rate of 5%?

2. What is the balance due on December 31, 1917 on a note for $1400 dated May 2, 1915, bearing interest at 6% and having the following indorsements: July 1, 1915, $200: September 25, 1915, $90; February 28, 1916, $175; May 19, 1917, $475?

3. On April 21, 1913, I gave my note for $3550, payable in three years, interest at $4\frac{1}{2}\%$. I paid as follows: Oct. 15, 1913, $125; March 3, 1914, $125; July 20, 1914, $875. How much will be required to settle the note at maturity?

4. W. A. Jones gave a note on June 4, 1914 to Frank Brown for $1285.50 with interest at 7%. He made payments as follows: Dec. 15, 1914, $340; Feb. 17, 1915, $330; March 11, 1915, $400. What was due on May 15, 1915?

5. Find the value of a note for \$2500, given Oct. 10, 1915, with interest at $4\frac{1}{2}\%$, and on which the following payments have been indorsed: Jan. 5, 1916, \$815; June 18, 1916, \$350; Oct. 10, 1916, \$250; Jan. 17, 1917, \$150. Settlement was made Sept. 6, 1917.

6. On a note of \$3080, dated Oct. 1, 1915, the following payments have been made, interest being at 6%: Dec. 31, 1915, \$300; Feb. 29, 1916, \$50; June 5, 1916, \$500; Oct. 2, 1916, \$700. What will be due on Dec. 31, 1916?

7. Find the amount due Feb. 24, 1917 on a note for \$3000, dated March 12, 1915, with interest at 7%, upon which the following payments were made: Aug. 18, 1915, \$235; April 9, 1916, \$80; July 3, 1916, \$400; Dec. 5, 1916, \$175.

8. What is the balance due on Jan. 1, 1917 on a note for \$500, dated July 15, 1916, bearing interest at 5% and having the following indorsements: Aug. 20, 1916, \$27.50; Oct. 8, 1916, \$125; Nov. 12, 1916, \$110; Dec. 11, 1916, \$65?

References 374–375.

Use the Merchants' Rule.

62. 1. A note of \$850 was dated May 25, 1914, interest at 6%. It was indorsed Aug. 13, 1914, \$50; Nov. 7, 1914, \$324.95. What was due March 25, 1915?

2. A note for \$1250, dated Jan. 25, 1916, interest at 8%, bears the following indorsements: March 10, 1916, \$462.50; Aug. 4, 1916, \$100; May 22, 1917, \$556. What was due on Jan. 1, 1918 to settle the note in full?

3. On a note for $550, dated Feb. 5, 1913, interest at 6%, the following payments were made: Oct. 17, 1913, $66.10; March 5, 1914, $140. What was due Nov. 11, 1914?

4. A note for $2000 was dated Dec. 12, 1915, interest at 7%. It was indorsed as follows: June 19, 1916, $200; Dec. 6, 1916, $338; Aug. 21, 1917, $276.50; Sept. 12, 1917, $60. What was due Oct. 15, 1917?

5. I gave my note for $1080 with interest at 5% on Jan. 25, 1914. I made the following payments: Mar. 1, 1914, $364.40; May 13, 1914, $341.50; Sept. 1, 1914, $205. What was due on settlement, Jan. 25, 1915?

6. A note for $1500 was dated May 11, 1913, bearing interest at $7\frac{1}{2}\%$. The following payments were made: Feb. 14, 1914, $150; Sept. 23, 1914, $300; July 8, 1915, $100; May 29, 1916, $200. What was due September 4, 1916?

7. On a note for $1120, dated August 7, 1914, interest at 7%, payments were made as follows: Sept. 13, 1914, $80; Nov. 7, 1914, $200; Sept. 15, 1915, $450. What was due Aug. 7, 1916?

8. The following payments were made on a note for $580, dated Oct. 17, 1915, bearing interest at 5%: Aug. 5, 1916, $52.50; April 17, 1917, $49.30; Aug. 5, 1917, $250. What was due Sept. 9, 1917?

BANK DISCOUNT

Study 376 to 381 inclusive.
Reference 382.

63. Find the bank discount and net proceeds:

	Face	Date	Time	Date of Disc.	Rate of Disc.
1.	$240	Jan. 3, 1916	60 da.	Jan. 4, 1916	6%.
2.	$300	Sept. 8, 1916	2 mo.	Oct. 1, 1916	7%.
3.	$1000	June 1, 1915	90 da.	June 7, 1915	8%.
4.	$400	May 1, 1916	3 mo.	June 1, 1916	5%.
5.	$250	June 1, 1916	3 mo.	Aug. 3, 1916	6%.
6.	$350	Dec. 30, 1914	4 mo.	Jan. 2, 1915	8%.
7.	$500	Apr. 3, 1915	6 mo.	May 15, 1915	6%.
8.	$850	June 1, 1916	6 mo.	Oct. 14, 1916	4%.
9.	$600	Mar. 3, 1915	90 da.	Apr. 4, 1915	$7\frac{1}{2}$%.
10.	$375	Feb. 3, 1916	60 da.	Feb. 20, 1916	$4\frac{1}{2}$%.
11.	$460	May 9, 1914	3 mo.	May 12, 1914	6%.
12.	$2500	July 1, 1915	6 mo.	Aug. 30, 1915	5%.
13.	$36500	Mar. 3, 1916	4 mo.	June 30, 1916	6%.
14.	$845	Jan. 3, 1916	90 da.	Feb. 8, 1916	5%.
15.	$280	Sept. 8, 1914	30 da.	Sept. 10, 1914	$7\frac{1}{2}$%.
16.	$430	Nov. 9, 1915	3 mo.	Nov. 20, 1915	8%.
17.	$375	Oct. 12, 1916	2 mo.	Nov. 13, 1916	5%.
18.	$3000	Jan. 10, 1916	4 mo.	Jan. 12, 1916	7%.
19.	$575	Dec. 4, 1915	3 mo.	Jan. 29, 1916	8%.
20.	$490	Aug. 8, 1915	60 da.	Sept. 18, 1915	7%.
21.	$450	Mar. 3, 1916	90 da.	Apr. 5, 1916	6%.
22.	$340	May 2, 1915	6 mo.	Aug. 3, 1915	$7\frac{1}{2}$%.
23.	$500	July 5, 1916	60 da.	July 10, 1916	4%.
24.	$1500	Dec. 3, 1914	4 mo.	Feb. 9, 1915	5%.
25.	$475	Jan. 4, 1916	3 mo.	Jan. 20, 1916	8%.

Reference 383.

64. Find the bank discount and proceeds of the following interest-bearing notes:

	Face	Date	Time	Rate	Date of Disc.	Rate of Disc.
1.	$800	Sept. 1, 1915	3 mo.	6%	Sept. 11, 1915	6%.
2.	$400	June 15, 1916	2 mo.	6%	June 30, 1916	5%.
3.	$2240	Jan. 10, 1916	90 da.	7%	Feb. 8, 1916	8%.
4.	$480	June 1, 1916	6 mo.	5%	Aug. 3, 1916	6%.
5.	$1530	May 4, 1916	60 da.	6%	May 4, 1916	8%.
6.	$285	Sept. 3, 1916	4 mo.	5%	Sept. 20, 1916	4%.
7.	$390	Feb. 6, 1916	30 da.	6%	Feb. 6, 1916	5%.
8.	$2500	Mar. 10, 1916	3 mo.	6%	Mar. 15, 1916	6%.
9.	$460	June 12, 1916	4 mo.	5%	Aug. 2, 1916	8%.
10.	$1400	Aug. 3, 1916	90 da.	8%	Sept. 1, 1916	8%.
11.	$2150	July 5, 1916	60 da.	7%	July 7, 1916	7%.
12.	$580	May 12, 1916	6 mo.	6%	May 12, 1916	8%.
13.	$490	June 6, 1916	30 da.	5%	June 8, 1916	6%.
14.	$3400	Dec. 2, 1916	3 mo.	4%	Jan. 3, 1917	5%.
15.	$1560	Mar. 3, 1916	5 mo.	6%	Mar. 15, 1916	6%.
16.	$780	Aug. 1, 1915	4 mo.	5%	Aug. 11, 1915	7%.
17.	$240	July 11, 1916	90 da.	6%	Aug. 1, 1916	5%.
18.	$1375	Mar. 18, 1916	30 da.	4%	Mar. 24, 1916	8%.
19.	$650	June 19, 1916	5 mo.	7%	Aug. 1, 1916	7%.
20.	$425	Jan. 31, 1916	1 mo.	5%	Feb. 4, 1916	7½%.
21.	$730	Oct. 6, 1916	2 mo.	7%	Oct. 6, 1916	7%.
22.	$260	Jan. 14, 1916	5 mo.	6%	Mar. 3, 1916	6%.
23.	$475	May 10, 1916	90 da.	10%	May 11, 1916	5%.
24.	$390	Apr. 5, 1916	30 da.	9%	Apr. 12, 1916	6%.
25.	$1575	July 1, 1916	1 mo.	7½%	July 5, 1916	8%.

References 384–385.

65. For what sum must I write my note in order to yield the following proceeds if discounted on the date of the note?

	Proceeds	Time	Rate
1.	$385	90 da.	6%
2.	$450	3 mo.	5%
3.	$1285	60 da.	8%
4.	$370	4 mo.	7%
5.	$260	90 da.	8%
6.	$580	2 mo.	$4\frac{1}{2}\%$
7.	$290	6 mo.	8%
8.	$365	4 mo.	7%
9.	$290	90 da.	6%
10.	$460	30 da.	8%
11.	$1340	2 mo.	7%
12.	$360	3 mo.	6%
13.	$1500	1 mo.	4%
14.	$2560	5 mo.	$7\frac{1}{2}\%$
15.	$775	60 da.	6%
16.	$550	30 da.	7%
17.	$1250	2 mo.	8%
18.	$875	4 mo.	8%
19.	$1360	90 da.	6%
20.	$728	60 da.	5%
21.	$450	5 mo.	7%
22.	$1385	30 da.	8%
23.	$3760	60 da.	6%
24.	$1485	3 mo.	5%
25.	$960	90 da.	8%

COMPOUND INTEREST

Study 386 to 388 inclusive.

Reference 388.

66. Find the compound interest:

	Principal	Rate	Time	Compounded
1.	$7800	6%	2 yr.	Annually
2.	$4600	5%	2 yr.	Semi-annually
3.	$8400	8%	3 yr.	Annually
4.	$9000	4%	3 yr.	Quarterly
5.	$3500	6%	4 yr. 5 mo.	Semi-annually
6.	$4650	6%	1 yr. 8 mo.	Quarterly
7.	$3865	4%	1 yr. 2 mo. 15 da.	Quarterly

8. A note for $495.60, dated June 10, 1912, and drawing interest at 6% per annum, compounded semi-annually, was paid March 22, 1916. What was the amount due, if no payments of either interest or principal had been made?

9. What amount will, on June 30, 1917, discharge a note of $3560, dated Dec. 1, 1914, and drawing interest at 8% per annum, compounded quarterly, no previous payments having been made?

10. What is the amount due April 1, 1916, upon a note for $480.50, dated May 10, 1911, and drawing interest at 8% per annum, compounded semi-annually, no previous payments having been made?

11. A young man deposited $200 in a savings bank which paid 4% per annum, compounded quarterly. If nothing was withdrawn, what amount was to his credit at the end of the third year?

12. For the benefit of his son who is 12 years old, Mr. A deposited in a savings bank $1000 at 4%, interest compounded semi-annually. How much should the son receive when he becomes 21 years old?

PERIODIC INTEREST

Study 389 to 390 inclusive.

67. Find the periodic interest:

	Principal	Rate per Annum	Time	Interest Due
1.	$5500	6%	4 yr.	Annually
2.	$450	5%	3 yr.	Annually
3.	$3000	8%	4 yr.	Semi-annually
4.	$2850	7%	5 yr. 3 mo.	Annually
5.	$4650	4%	1 yr.	Quarterly
6.	$956	6%	2 yr. 10 mo.	Quarterly
7.	$380	8%	4 yr. 1 mo.	Semi-annually

8. What amount will be due Feb. 1, 1922, on a note of $3000, dated Jan. 1, 1920, and drawing interest at 6% per annum, payable semi-annually, if the first four interest payments are paid when due, and no subsequent payments made?

9. What amount was due July 15, 1917, on a note of $4600, dated March 13, 1913, drawing interest at 5% per annum, payable semi-annually, no previous payments having been made?

10. No interest having been previously paid, what was the amount of a note of $1400 at 6%, interest payable quarterly, dated Jan. 1, 1914, and paid Feb. 1, 1916?

11. What sum was due Jan. 28, 1917, on a note of $4000, dated May 18, 1913, and drawing interest at 5% per annum, payable semi-annually; no payments having been made previous to that time?

12. A merchant bought a store building for $9000, giving his note without interest, payable 2 years from date, and 8 separate non-interest-bearing notes for the quarterly interest at 6% per annum. If nothing was paid until the maturity of the note, what was the amount then due?

13. I purchased a $1250 mortgage on which interest at 6% was due semi-annually on Jan. 15 and July 15. Owing to the fact that no interest had been paid since Jan. 15, 1917, I secured the mortgage at less than its face value. On Oct. 27, 1919, I made arrangements with the mortgagor whereby he paid the interest in full and $500 on the face of the mortgage. How much did I receive in all?

14. What interest is due Jan. 7, 1920, on $875 from Nov. 13, 1917, at 6%, interest due quarterly and none having been paid?

15. What is the total interest on $386.40 from Dec. 31, 1914, to Sept. 1, 1919, interest due annually and none having been paid? Rate $4\frac{1}{2}\%$.

16. What amount was due Jan. 8, 1920, on a note of $2340 dated Sept. 1, 1917, drawing interest at 6%, interest payable semi-annually, if the first two payments were made when due and no subsequent payments made?

AVERAGE ACCOUNTS

Study 391 to 396 inclusive.
Reference 397.

68. Average the following:

1. *Dr.* HAROLD CHUTE *Cr.*

1917			
Jan.	10	$316.20	
Feb.	19	415.23	
Mar.	24	99.	
May	10	271.	

2. *Dr.* FRED ELLIS *Cr.*

		1917	
		Apr. 18	$367.40
		May 6	572.
		May 23	923.
		June 2	134.50

3. *Dr.* BENJAMIN JONES *Cr.*

1916			
Oct.	12	$ 67.85	
Dec.	20	71.15	
1917			
Jan.	5	143.50	
Mar.	2	116.20	

4. *Dr.* HOWARD COLSON *Cr.*

1916		
Dec. 1	$540.	
Dec. 15	236.10	
1917		
Jan. 2	200.	
Jan. 31	150.	

5. *Dr.* JAMES MULLANEY *Cr.*

1917		
Feb. 5	$1050.10	
Mar. 10	826.	
May 1	924.	
May 31	186.	

Reference 398

69. Average the following:

1. Find cash balance on April 18, 1915.

Dr. PAUL DUNCANSON *Cr.*

1915			
Jan. 27	30 da.	$420.	
Feb. 17	10 da.	300.	
Mar. 1	20 da.	540.	
Apr. 12	30 da.	600.	

2. Find cash balance on Jan. 1, 1917.

Dr.	ARTHUR BENNETT		*Cr.*
1916			
Oct. 17	15 da.	$432.	
Nov. 20	2 mo.	864.	
Nov. 30	30 da.	286.	
Dec. 19	10 da.	627.	

3. Find cash balance on Dec. 31, 1916.

Dr.	C. D. ADAMS		*Cr.*
1916			
Aug. 9	30 da.	$234.	
Sept. 15	60 da.	562.	
Nov. 29	10 da.	52.96	
Dec. 21	15 da.	715.	

4. Find cash balance on Sept. 7, 1916.

Dr.	GEORGE DUNCAN		*Cr.*
1916			
May 6	30 da.	$128.	
June 30	20 da.	126.	
July 19	10 da.	213.20	
Sept. 3	30 da.	185.	

5. Find cash balance on May 1, 1915.

Dr.				PAUL JONES			*Cr.*
1915							
Jan.	5	10 da.	$400.				
Jan.	31	30 da.	90.60				
Mar.	8	10 da.	150.				
Apr.	25	2 mo.	86.12				

References 399–400.

70.

1.

Dr.				A. C. DAVIS			*Cr.*
1917				1917			
Jan.	1	2 mo.	$600.	Feb.	1	Cash	$200.
Feb.	2	30 da.	240.	Mar.	18	Cash	150.
Apr.	6	10 da.	360.	Apr.	3	Cash	75.

When is the above due by average?
What was the cash balance Apr. 15, 1917?

2.

Dr.				J. F. HOWARD			*Cr.*
1916				1916			
May	18	60 da.	$209.70	June	1	Cash	$100.
June	3	30 da.	180.	June	30	Cash	50.
July	10	15 da.	750.	July	19	Cash	300.
Aug.	1	10 da.	280.50				

When is the above due by average?
What was the cash balance Aug. 21, 1916?

3. *Dr.* GEORGE STEVENS *Cr.*

1915			1915		
Jan. 20	2 mo.	$219.50	Feb. 25	Cash	$ 50.
Feb. 25	30 da.	218.75	Mar. 31	Cash	75.
Mar. 28	10 da.	413.	Apr. 30	Cash	200.
June 30	10 da.	216.			

When is the above due by average?
What was the cash balance July 1, 1915?

4. *Dr.* FRED ELLIS *Cr.*

1917			1917		
Jan. 1	10 da.	$600.	Feb. 28	Cash	$400.
Feb. 2	10 da.	200.	Mar. 31	Cash	100.
Mar. 3	10 da.	350.	Apr. 30	Cash	150.

When is the above due by average?
What was the cash balance May 5, 1917?

5. *Dr.* WILLIAM WALKER *Cr.*

1915			1915		
Jan. 31	2 mo.	$540.	Feb. 15	Cash	$225.
Feb. 15	60 da.	450.	Mar. 1	Cash	345.
Mar. 30	10 da.	306.50	Mar. 10	Cash	295.

When is the above due by average?
What is the cash balance July 3, 1915?

6. *Dr.* BENJAMIN BROWN *Cr.*

1916				1916		
Apr.	2	10 da.	$150.	June 25	Cash	$300.
May	1	15 da.	540.	July 31	Cash	360.
June	3	10 da.	450.	Aug. 10	Cash	250.
July	2	10 da.	323.			

When is the above due by average?
What is the cash balance Aug. 29, 1916?

7. *Dr.* CHARLES SMITH *Cr.*

1916				1916		
Sept.	1	2 mo.	$315.60	Oct.	5	Cash	$200.
Oct.	25	60 da.	419.10	Nov.	1	Cash	150.
Nov.	16	10 da.	216.05	Dec.	2	Cash	375.

When is the above due by average?
What was the cash balance Mar. 5, 1917?

8. *Dr.* ROBERT BROWN *Cr.*

1915				1915		
Jan.	2	1 mo.	$1800.	Feb.	18	Cash	$300.
	30	10 da.	600.	Feb.	27	Cash	300.
				Mar.	5	Cash	300.

When is the above due by average?
What was the cash balance Mar. 10, 1915?

9. *Dr.* E. Bowdoin *Cr.*

1916				1916			
Oct.	1	30 da.	$350.	Oct.	21	Cash	$300.
Nov.	8	10 da.	340.	Nov.	24	Cash	300.
Dec.	9	15 da.	210.				
1917							
Jan.	20	10 da.	116.				

When is the above due by average?
What was the cash balance Feb. 1, 1917?

10. *Dr.* Sidney Berry *Cr.*

1915				1915			
June	1	60 da.	$410.	Aug.	1	Cash	$300.
July	5	30 da.	135.	Aug.	31	Cash	200.
Aug.	1	10 da.	216.39	Sept.	4	Cash	100.
Aug.	31	15 da.	162.54				

When is the above due by average?
What was the cash balance Oct. 6, 1915?

TAXES

The general principles of percentage are used in figuring taxes. Study 401–404 inclusive.

References 242–263 inclusive.

71. 1. What is the tax on property assessed for \$17,400, the rate of taxation being $\frac{3}{8}\%$?

2. What is the tax on property assessed for \$8500, rate of taxation being $16\frac{2}{3}$ mills on the dollar?

3. What is the tax on property assessed for \$23,500, the rate of taxation being \$19.20 on the thousand?

4. What is the tax on property assessed for \$7588, the rate of taxation being \$1.20 on the hundred?

5. I own real estate worth \$19,500 upon which I pay a tax at the rate of \$21.40 a thousand. I also pay an income tax of 6% on a net taxable income of \$1400 and a poll tax of \$2. What is my entire tax?

6. My real estate is assessed at \$6500, my personal property at \$1570; my net taxable income is \$2400. Tax on the tangible property is levied by the city at the rate of \$19.40 a thousand; an income tax is levied by the state at the rate of 3%; my poll tax is \$2. What is my entire tax?

7. The assessed value of real estate in a town is \$1,869,000; personal property is \$2,450,000. It is

necessary to raise by taxation $412,560. What would be the rate a thousand if there are 1742 polls at $2 each?

8. In a town whose valuation is $25,000,000, there is an increase in the budget to cover additional expenses of the public schools amounting to $40,000. How many cents a thousand is the tax increased thereby? How much will the improvement cost a citizen who is worth $30,000?

CUSTOMS AND DUTIES

Ad valorem duties are estimated according to the value of the goods in conformity to the principles of percentage. Study 405–422 inclusive.

References 242–263 inclusive.

Values of units of foreign currency expressed in United States money will be found in 468.

72. 1. What is the ad valorem duty upon an importation valued at £430, 8*s*, 9*d*, allowing 10% for breakage, duty being at 25%?

2. Find the ad valorem duty on an invoice of 15,834 marks at 23%.

3. Find the ad valorem duty on an invoice of 3446.18 francs, duty being 43%.

4. Find the ad valorem duty on a bill of 1475 pesos if the duty is 24%.

5. What is the specific duty on 13 tons of tan bark on which there is a duty of 3 cents a hundred pounds?

6. What is the specific duty on an invoice amounting to $760, allowing 10% for breakage, duty being at 35%?

7. Find the duty at 10 cents a square yard and 40% ad valorem on a rug 12′×18′, imported from England and invoiced at £14.

8. What is the total duty on 140 cases of plate glass, each containing 25 plates, 20″×48″ at 8¢ a square foot?

9. What is the duty on an invoice of 2300 yards of 27-inch goods, invoiced at 8*s* 9*d* a yard, subject to an ad valorem duty of 40% and a specific duty of 6¢ a square yard?

10. What is the duty at 60% on a bill amounting to £736 9*s* 8*d*?

11. What is the duty at 30% ad valorem on two bales of burlap, each bale containing 40 webs, each web being 48 yd. long and 30 in. wide, invoiced at 30¢ per square yard?

12. What is the duty at 20¢ per square yard and 35% ad valorem on 1750 yards of cloth invoiced at 7 francs per yard?

13. A merchant imported a lot of steel knives from England as follows: 75 doz. at 12*s* 6*d*; 50 doz. at 18*s* 6*d*; 30 doz. at £1 5*s* 6*d*; 20 doz. at £1 8*s* 6*d*; 12 doz. at £2 9*s* 6*d*; 10 doz. at £2 10*s* 6*d*. The charges in England amount to £7 12*s* 6*d*. The consul's fee was 12*s* 6*d*. Marine insurance was 20¢ per hundred on the value of the invoice. The cartage amounted to $2.50. The duty was 30% ad valorem and 30¢ per dozen. Find the total cost of the invoice.

INSURANCE

Study 423–434 inclusive.
References 242–263 inclusive.

73. 1. My house is insured for \$4500 for a period of five years at $2\frac{1}{2}\%$. What is the premium?

2. A merchant insured his stock of goods for \$5600 at the rate of $1\frac{1}{4}\%$ per annum. What annual premium does he pay?

3. A factory is insured for \$125,000 in four companies. A carries $\frac{1}{4}$ of the insurance, B carries $\frac{1}{4}$, C carries $\frac{1}{3}$, and D $\frac{1}{6}$. A fire occurs causing a damage of \$50,000. For how much will each company be responsible?

4. A stock of goods is insured in four companies as follows: \$1500 in A, \$2400 in B, \$3200 in C, and \$2500 in D. The goods are damaged to the extent of \$8000. How much should each company pay?

5. A building worth \$85,000 was insured for \$68,000, and afterwards damaged by fire to the extent of \$4500. The policy contains the average clause. What amount of insurance can be collected from the company?

6. A vessel worth \$50,000 is insured for \$20,000 in company A and \$18,000 in company B. The vessel is damaged to the extent of \$20,000. What amount is to be paid by each company?

7. I insured my building worth $80,000 for 80% of its value at $\frac{3}{4}\%$ premium with the Ætna Insurance Company. The Ætna Insurance Company later reinsured $20,000 in the Niagara Insurance Company and $18,000 in the Massachusetts Fire and Marine Insurance Company. The property is damaged to the extent of $30,000. What was the net loss to each of the companies?

8. I have a policy, containing the average clause, for $7500 on merchandise in stock worth $9000, upon which I have paid a premium of $\frac{3}{4}\%$. A fire occurs by which the goods are damaged to the extent of $4000. What was my total loss and the net loss to the company?

LIFE INSURANCE

Study 435–441 inclusive.
Reference 442.

74. 1. What would be the annual premium on a policy for $2500, premiums payable annually during life, at the age of 21 years? 26 years? 32 years? 38 years?

2. What would be the annual premium on a fifteen-year endowment policy for $5000, at the age of 23 years? 27 years? 32 years? 37 years?

3. What would be the annual premium on a policy for $4000, premiums due annually for a period of ten years, policy payable at death only, at the age of 20 years? 25 years? 35 years?

4. What would be the annual premium on a twenty-year endowment policy for $6500, age of insured at nearest birthday 23 years? 29 years? 37 years?

5. What would be the annual premium on an ordinary life policy for $3500, premiums to be paid annually for twenty years, policy to mature at death, age of insured at nearest birthday 28 years? 23 years? 38 years?

6. A man insured his life at the age of 23 years on a twenty-year endowment plan, payments to be paid annually, amount of policy $5000. He died at the age of 33 years. How much less would he have paid in premiums if he had been insured by the ordinary life plan?

7. A man at the age of 35 took out a fifteen-year endowment policy for $2000. What annual premium must he pay? He lives 20 years and receives the face of the policy. How much less will this amount to than it would have if he had invested the premium at 4% compound interest?

8. A man at the age of 25 took out a $3000 twenty payment life policy. He died after paying ten premiums. What was the annual premium? How much more did his family receive than the premiums amounted to, making no allowance for interest?

9. Three men, aged 24, take a policy for $1000 each. One takes an ordinary life policy, one a twenty-year life policy, and one a twenty-year endowment policy. At the end of five years how much had each paid in premiums?

EXCHANGE

DOMESTIC EXCHANGE

TO FIND THE VALUE OF A SIGHT DRAFT

Study 443–449 inclusive.
References 255–258 inclusive.

75. Find the value of the following drafts:

1. \$2300 bought at $\frac{1}{2}\%$ discount.
2. \$1400 bought at $1\frac{3}{4}\%$ premium.
3. \$1740 sold at $\frac{1}{2}\%$ premium.
4. \$3000 bought at $1\frac{1}{8}\%$ discount.
5. \$2450 sold at $1\frac{1}{8}\%$ premium.
6. \$4500 bought at \$1.50 premium.
7. \$1240 sold at \$1.25 discount.
8. \$1450 bought at \$.50 premium.
9. \$4300 sold at $\frac{2}{3}\%$ discount.
10. \$9000 sold at $\frac{1}{8}\%$ discount.

TO FIND THE VALUE OF A TIME DRAFT

76. To find the cost or selling price of a time draft: Find the net proceeds of the draft according to the principles of bank discount (381–382). From this *deduct* the exchange *discount*, or to it *add* the exchange *premium*, found as in 75.

77. What is the cost of a

1. 60-day draft for \$6000, $\frac{3}{4}\%$ premium, interest at 6%?

2. 30-day draft for \$2200, $\frac{1}{2}\%$ premium, interest at 7%?

3. 15-day draft for \$2500, $\frac{3}{4}\%$ discount, interest at 7%?

4. 30-day draft for \$750, $\frac{5}{8}\%$ premium, interest at 5%?

5. 30-day draft for \$5000 at $\frac{7}{8}\%$ discount, interest at 6%?

6. 90-day draft for \$2300 at $\frac{1}{2}\%$ premium, interest at $4\frac{1}{2}\%$?

7. 60-day draft for \$2500 at $\frac{2}{3}\%$ discount, interest at 4%?

8. 30-day draft for \$2350 at $\frac{5}{8}\%$ premium, interest at $4\frac{1}{2}\%$?

9. 60-day draft for \$1240 at \$1.25 premium, interest at 5%?

10. 30-day draft for \$2350 at \$1.50 discount, interest at 6%?

TO FIND THE FACE OF A DRAFT

78. Find the value of a draft of \$1 as explained in 75 and 76, and divide the given value by this.

79. What is the face of a sight draft which can be bought for

1. \$1207.50 if exchange is at $\frac{5}{8}\%$ premium?

2. \$1091.75 if exchange is at $\frac{3}{4}\%$ discount?

3. \$2453.28 if exchange is at $\frac{7}{8}\%$ premium?
4. \$1636.02 if exchange is at $\frac{2}{3}\%$ discount?
5. \$4234.62 if exchange is at $\frac{7}{8}\%$ discount?

80. What is the face value of a 30-day draft which can be bought for

1. \$1183.50 at $\frac{7}{8}\%$ discount, interest 6%?
2. \$1453.84 at $\frac{3}{4}\%$ premium, interest 5%?
3. \$2493.62 at \$1.20 premium, interest $4\frac{1}{2}\%$?
4. \$3977.33 at \$1.50 discount, interest 5%?
5. \$2843.55 at $\frac{5}{8}\%$ premium, interest 6%?

FOREIGN EXCHANGE

Study 450–452 inclusive.
Reference 468.

81. Find the exchange value of a bill for

1. £540 at $4.83\frac{1}{2}$.
2. £1476 at $4.85\frac{3}{4}$.
3. £250 9s 8d at $4.85\frac{1}{2}$.
4. £783 13s 11d at $4.84\frac{1}{4}$.
5. 15,642 francs at $5.18\frac{3}{4}$.
6. 8575.75 francs at 5.19.
7. 8462.73 francs at $5.20\frac{1}{4}$.
8. 2648.55 francs at $5.19\frac{3}{4}$.
9. 1284 marks at $94\frac{1}{4}$.
10. 2556 marks at $95\frac{1}{4}$.
11. 6742 marks at $94\frac{5}{8}$.
12. 1287.5 marks at $94\frac{7}{8}$.

13. 789.7 guilders at $40\frac{1}{4}$.
14. 2345 guilders at $40\frac{1}{2}$.
15. 1286 guilders at $40\frac{1}{4}$.
16. 1286.5 guilders at $39\frac{7}{8}$.

82. What is the face of an English bill of exchange that cost

1. \$2213.88 at $4.85\frac{1}{2}$?
2. \$6060.95 at $4.84\frac{7}{8}$?
3. \$585.37 at $4.84\frac{5}{8}$?
4. \$1209.38 at $4.83\frac{3}{4}$?

83. What is the face of a French bill of exchange that cost

5. \$819.29 at $5.19\frac{3}{8}$?
6. \$2316.04 at $5.18\frac{1}{2}$?
7. \$225.44 at 5.19?
8. \$88.42 at $5.20\frac{1}{4}$?

84. What is the face of a German bill of exchange that cost

9. \$348.60 at $95\frac{5}{8}$?
10. \$2945.31 at $94\frac{1}{4}$?
11. \$393.46 at $95\frac{1}{2}$?
12. \$13795.19 at $94\frac{7}{8}$?

85. What is the face of a Dutch bill of exchange that cost

13. \$226.10 at $40\frac{3}{8}$?
14. \$458.97 at $39\frac{7}{8}$?
15. \$235.52 at $40\frac{1}{2}$?
16. \$6415.60 at $40\frac{1}{4}$?

STOCKS AND BONDS

The general principles of percentage are involved in solving the following problems (242–263).

Study 453–464 inclusive.

86. 1. A railroad with a capital stock of $2,500,000 declared a dividend at the rate of 5%. What was the total amount of the dividend? How much did A, the owner of 350 shares, receive?

2. What will be the total dividend at 5%, declared by a $3,000,000 corporation?

3. A manufacturing corporation with a capital of $50,000 levies an assessment of 8% upon its stockholders. What is the total assessment, and what will B be called upon to pay, if he holds 230 shares of $100 each?

4. What dividend would I receive on 163 shares of $100 stock at the rate of 5%?

5. A corporation with a capital stock of $475,500, divided $38,040 among its stockholders. What was the rate of this dividend?

6. A corporation of which I am a stockholder declares a dividend of $4\frac{1}{2}\%$. My dividend check is $652.50. How many shares, par value of $100, do I own?

7. A mining corporation of which I am a stockholder declares a dividend of 10%. I receive \$125 as my dividend. How many shares of \$10 par value do I own?

8. A corporation with \$1,500,000 capital had a gross income of \$975,000. Its total expenses were \$785,000. Its directors set \$100,000 aside as a reserve fund; the rest was divided among the stockholders. What per cent dividend was declared?

9. What is the market value of 130 shares, par value \$100, of Q., O. & K. C. R. R. quoted at $113\frac{1}{4}$?

10. What is the market value of 640 shares, par value of \$10 each, of New England Manufacturing Company, quoted at $85\frac{1}{4}$?

11. How much must I pay for 75 shares (\$100 par value) B. & M. R. R. at $49\frac{1}{8}$, brokerage $\frac{1}{8}\%$?

12. What is the cost of 130 shares (\$100 par value) Bell Telephone at $418\frac{1}{4}$, brokerage $\frac{1}{8}\%$?

13. Find the total cost of \$1000 L. & E. R. R. 2d 4's at $102\frac{3}{4}$; \$4000 C. & N. W. 5's at $102\frac{3}{4}$; 40 shares (\$100 par value) of A. T. & S. F. R. R. at $43\frac{2}{3}$; 75 shares (\$100 par value) B. & M. R. R. at $34\frac{5}{8}$; brokerage on all $\frac{1}{8}\%$?

14. What is the net cost of 150 shares (\$100 par value) of B. & A. R. R. at $134\frac{5}{8}$; 75 shares (\$100 par value) M. C. R. R. at $104\frac{3}{8}$; \$5000 S. E. L. Co. 6's at $95\frac{1}{3}$; brokerage on all $\frac{1}{8}\%$?

15. What is the proceeds of 450 shares (\$50 par value) sold at $102\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

16. How much must I invest in U. S. 4's of 1932 to secure a quarterly income of \$450, bonds selling at $108\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

17. I invested \$3376.25 through my broker at $\frac{1}{8}\%$ commission, in U. S. 4% bonds at $115\frac{3}{4}$. What will be my annual income?

18. How much must I invest in U. S. 4's of 1935 to secure a quarterly income of \$600, bonds selling at $108\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

19. Sold 75 shares (\$100 par value) railroad stock through a broker and received \$7388 net proceeds. At what quotation did the broker sell the stock?

20. At what price may 6% stock be bought to receive 5% on the investment, brokerage $\frac{1}{8}\%$?

21. What price can I afford to pay for 7% bonds in order to realize 8% income on the investment, brokerage $\frac{1}{8}\%$?

22. What price will I pay for 5% bonds bought through a broker so as to bring in a net income of 4% on the investment?

23. At what quotation could 8% preferred stock be bought through a broker to realize 5% income on the investment?

24. What price can I afford to pay for 7% bonds bought through a broker so as to receive a net income of 6% on the investment?

25. What per cent income on the investment will be realized if 4% stock is bought at $79\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

26. What per cent is realized on the investment if 6% stock is bought at $74\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

27. 5% bonds bought at $124\frac{7}{8}$ would bring what per cent on the investment, brokerage $\frac{1}{8}\%$?

28. Stock bought at $79\frac{7}{8}$, brokerage $\frac{1}{8}\%$, yields 4% on the investment. What is the rate of dividend?

29. Which is the better investment and how much: stock paying 6% dividend, bought at $74\frac{7}{8}$, or stock paying 9% dividend, bought at $119\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

30. I bought, through a broker, 52 shares of stock at 84. I paid an assessment of 5% and then sold them at $99\frac{3}{4}$. How much did I gain, brokerage $\frac{1}{8}\%$?

31. I have $8000 to invest. I am offered bank stock at 375 yielding $3\frac{1}{2}\%$ each three months, or stock in a shoe manufacturing company at 150 paying 4% semi-annually. I have made up my mind to invest in the stock which will give me the greater dividend. Which shall I buy and what will be the total dividend each year?

32. By investing $18,750 in 150 shares of stock I am able to realize 4% on the investment. What rate of dividend does the stock pay?

33. What can I afford to pay for 8% stock to realize 5% on the investment?

34. I have been offered a block of $4\frac{1}{2}$ Liberty Bonds at 98. What per cent would they yield on the investment?

PART TWO

RATIONAL ARITHMETIC

PART TWO

87. **Arithmetic** is the measure of values or quantities expressed in figures.

All arithmetic consists of increasing or decreasing values or quantities.

88. **Addition** is the simple or basic operation of increasing values or quantities.

The sign of addition is $+$, read *plus*.

89. **Subtraction** is the simple or basic operation of decreasing values or quantities.

The sign of subtraction is $-$, read *minus*.

90. **Multiplication** is a short method of addition by which quantities or values are increased at a fixed ratio — by a given number.

The sign of multiplication is $\times$, read *times*.

91. **Division** is a short method of subtraction by which a certain quantity or value is reduced at a fixed ratio — by a given number.

The sign of division is $\div$, read *divided by*.

Inasmuch as these operations are quite different in their applications, they are treated separately, and are known as the four fundamental operations of arithmetic.

NOTATION

92. For a thorough understanding of arithmetic it is necessary to be familiar with the system of notation used in expressing values and quantities in figures.

Ten characters (figures) are used, nine of which have a positive or integral value. These are represented by the figures 1 2 3 4 5 6 7 8 9. The tenth figure, 0 (read *cipher*, *zero*, or *naught*), represents nothing and has no integral value. In other words, the figure 3 stands for three individual units; 5, for five individual units; 7, for seven; 9, for nine; while the cipher is used to visualize nothing.

These nine integral units, with their accompanying cipher, are given a distinct value according to their position in relation to a fixed line represented by the decimal point. Thus, one in the units column — the first column to the left of the line — means one whole unit.

Move this 1 to the next column, one place to the left; fill in the space from which it has been taken with a cipher to show that nothing is there; it then represents the value of " ten " and is so read.

Move it one more column to the left and it represents ten times ten, or one hundred, and so on.

Writing the 1 in the second, or tens column, and the figure 3 in the first, or units column, we show 10 units (in the tens column) and 3 units (in the units column), that is, 13, read *thirteen*.

93. It will readily be seen that *the removal of a figure one column to the left multiplies its value by ten.* It will

also be apparent that bringing it back *one place to the right divides its value by ten.*

This is the governing principle of notation and may be applied on either side of the decimal line. The figure 1 starting to the left of the line, in the units column, and moving one place to the right becomes $\frac{1}{10}$ of 1; moved another place to the right it becomes $\frac{1}{10}$ of $\frac{1}{10}$, which is $\frac{1}{100}$; moved another place to the right it becomes $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$, and so on without limit.

If a student has difficulty in learning the value of a figure to the right of a decimal point (which is merely the line of division between whole numbers and their fractional parts represented by tenths), it is suggested that he take an ordinary sheet of writing paper, turn it sidewise, write the names of the various places in the columns thus formed, draw a heavy line to represent the decimal line, write the decimal notation to the right of this line, and then place figures in such columns as may appeal to him, calling them by the names of the values written in the columns.

Trillions	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units	Decimal line	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths	Millionths	Ten Millionths	Hundred Millionths	
												1										Read one
											1	0										Read ten
											1	3										Read thirteen
										1	0	0										Read one hundred
									1	1	1	3										Read one thousand, one hundred thirteen
														1								Read one-tenth
														0	1							Read one one-hundredth
														0	0	1						Read one one-thousandth
1	6	3	4	8	9	5	6	3	2	4	8	3										Read one trillion, six hundred thirty-four billion, eight hundred ninety-five million, six hundred thirty-two thousand, four hundred eighty-three.

Exercises of this kind are very valuable for students whose minds are so constituted that they have difficulty in reading decimals. Difficulty in reading decimals should not be ascribed to arithmetical weakness, but rather to inability to use the imagination in representing figure pictures.

94. Figures to the left of the decimal line represent *whole numbers* and are called **Integers.**

95. Figures to the right of the decimal line represent *parts* (tenths, hundredths, etc.), and are called **Decimals.**

COMMON PROCESSES

ADDITION — INTEGERS

96. Addition is the process of combining several numbers into one quantity that shall equal the value of all.

(*a*) Only numbers representing like values or like quantities, or parts of like values or like quantities, can be added, thus, 5 cows can be added to 2 cows and the result will be 7 cows. 5 horses added to 5 cows would produce 10 things, but they would be neither horses nor cows — just 10 animals.

(*b*) Figures not used to measure value or quantity can be added as a mere matter of counting. Furthermore, unlike things may be added, if first reduced to common terms. The term "animal" is common to both horses and cows.

97. The name **Addend** is applied to any of the individual quantities or values that are added.

98. The name **Sum** is applied to the total value of the addends. It is the result of addition.

ILLUSTRATED SOLUTION

99. *Problem:* $\$324.23 + \$89.96 + \$742.05 + \$23 + \$1.95 + \$796.45 = ?$

$324	23
89	96
742	05
23	
1	95
796	45
$1977	64

Arrange the addends in a column, placing decimal point under decimal point so as to form the decimal line. Add either up or down. Beginning with cents (first right-hand column) reading up and combining values as we go, we have 5, 10, 15, 21, 24 cents. This is 2 tens-cents and 4 units-cents. Write the 4 units-cents under the first column. Carry 2 tens-cents to the next column. Adding as before the result is 26. Write 6, carry 2. Adding the next, or unit-dollars column, we have 27. Write the 7, carry 2. The sum of the next column is 27. Write the 7, carry 2. The total of the next column is 19. As this is the last column write both figures.

100. *To Check the Work:* Add each column separately, beginning with the first right-hand column. Write the results as partial sums, one under the other, each one place to the left of its predecessor, thus:

First column adds		24
Second column adds	2	4
Third column adds	25	
Fourth column adds	25	
Fifth column adds	17	
	1977.64	

or we may begin on the *left* and work in the opposite direction, adding, either up or down, thus:

First column adds	17	
Second column adds	25	
Third column adds	25	
Fourth column adds	2	4
Fifth column adds		24
	1977.64	

101. In billing it is sometimes desirable to add quantities written in a horizontal line, thus:

$$24+112+36+43+246+95=556$$

The secret of accurately adding in this way is to *add from left to right* — because we *read* from left to right and the eye naturally "picks up" the proper quantities in traveling this way, and does not become confused, as is often the case if we try to add from right to left.

Thus, in the above, beginning with units in the first left-hand quantity, we add with the following results, 4, 6, 12, 15, 21, 26: write 6 as units, carry 2. In the tens from left to right we add 2, 4, 5, 8, 12, 16, 25: write the 5 as tens, carry 2. In the hundreds from left to right we add 2, 3, 5: write the 5 as hundreds, giving 556 as the total.

NOTE. For practice problems in addition see pars. 1 to 8 inclusive.

SUBTRACTION — INTEGERS

102. **Subtraction** is the process of decreasing one value, or quantity, by taking from it a smaller value, or quantity.

103. The **Minuend** is the larger number; the one from which another number is taken.

104. The **Subtrahend** is the number that is deducted from the minuend; it is the smaller number which is taken out of the larger.

105. The **Difference**, or **Remainder**, is the number that is left after the subtrahend has been taken from the minuend.

ILLUSTRATED SOLUTIONS

106. *Problem:* $2294.18 − $346.23 = ?

1 83	
$2294	18
346	23
$1947.95	

Write the subtrahend under the minuend so that the decimal points will form the decimal line. Subtract: 8 − 3 = 5. Write 5 in the proper column. Since 2 cannot be taken from 1 we must bring over 1 from the next column to the left. We know that one in the third column is 10 of the second column (par. 93); hence we now have 11 in this column. 11 − 2 = 9. Write 9. In the third column we now have 3 − 6 which cannot be performed. Then take 1 from 9 in the next column, which would give us 13 − 6 = 7. Write the 7. In the fourth column we now have 8 − 4 = 4. Write the 4. In the fifth column we now have 2 − 3 which cannot be performed. Take one from the next column which gives us 12 − 3 = 9. Write the 9. In the last column we have 1 − 0 = 1. Write the 1.

107. *To Check the Work:* Add the subtrahend and the remainder.

The result should be the minuend, thus:

$$\$1947.95 + \$346.23 = \$2294.18$$

108. Another Method of Subtraction. From the work used in checking, another method for finding the difference between two quantities is derived. By it we simply write the figures that must be added to the subtrahend to make it equal the minuend, thus:

109. *Problem:* $2294.18 − $346.23 = ? Ans. $1947.95

Start with the subtrahend, $346.23. To the first right-hand figure, 3, add 5 to make 8, the right-hand figure of the minuend. Write 5 in the difference. To the second figure, 2, add 9, which makes 11; write 9 in the difference. Change the third figure, 6, to 7 by adding the 1 carried from 11. To the new third figure, 7,

add 7 to make 14. Write 7 in the difference. Change the fourth figure, 4, to 5, by adding the 1 carried from 14. To the new fourth figure, 5, add 4 to make 9. Write 4 in the difference. To the last figure, 3, add 19 to make 22. Write 19, making 1947.95 in all.

This process is especially valuable in balancing accounts, thus:

Dr.			CASH			Cr.	
		519	25			125	40
		136				123	32
	798.77	143	52		295.22	46	50
				Balance		503	55
		798	77			798	77

In the above, add each side of the account separately, setting results in small figures, on proper side. Then add to the smaller sum the figures required to make it equal the larger, inserting these figures, as Balance. Then add both sides as a check.

Note. For practice problems in subtraction see par. 9.

MULTIPLICATION — INTEGERS

110. Multiplication is the process of increasing a given value or quantity a given number of times.

111. Multiplicand is the name applied to the value or quantity that is increased.

112. Multiplier is the name applied to the number representing the number of times the multiplicand is increased.

113. Product is the name applied to the final result of increasing the multiplicand the number of times indicated by the multiplier.

114. **Factor** is a name applied to *either* the multiplicand or multiplier, because both are factors (or makers) of the product.

ILLUSTRATED SOLUTIONS

115. *Problem :* What is the value of 6 times $4682?

$$\begin{array}{r} \$4682 \\ 6 \\ \hline \$28092 \end{array}$$

Begin with the units' column, $6\times2=12$. 12 is 2 units and 1 ten. Write the 2 units in the units' column, carry 1 ten. $6\times8=48+1$ (carried) $=49$ tens, which is 9 tens and 4 hundreds. Write the 9 in the tens' column, carry 4. $6\times6=36+4=40$. Write 0, carry 4. $6\times4=24+4=28$. As this is the last column, write 28.

116. *To Check the Work:* Multiply each figure separately beginning with the *left* column. Write the products under one another for addition, but removing each *one place to the right*, thus:

$$\begin{array}{ll} 6\times4= & 24 \\ 6\times6= & 36 \\ 6\times8= & 48 \\ 6\times2= & 12 \\ & \hline & 28092 \end{array}$$

117. *Problem :* $239\times\$8461 = ?$

$$\begin{array}{r} \$8461 \\ 239 \\ \hline 76149 \\ 25383 \\ 16922 \\ \hline \$2022179 \end{array}$$

First multiply 8461 by 9 units as explained above. The result is 76149. Write this as units. Then multiply by 3 tens, which is 25383 tens. Then multiply by 2 hundreds, which is 16922 hundreds. These are called partial products. Write the partial products each one place to the left of the previous one, and add. The result is the total product.

118. *To Check the Work:* Let the multiplier and multiplicand change places and proceed as before, thus:

$$\begin{array}{r} 239 \\ 8461 \\ \hline 239 \\ 1434 \\ 956 \\ 1912 \\ \hline 2022179 \end{array}$$

DIVISION — INTEGERS

119. **Division** is the process of decreasing a given value or quantity a given number of times.

120. **Dividend** is the name applied to the value or quantity that is decreased or divided.

121. **Divisor** is the name applied to the number by which the dividend is decreased.

122. **Quotient** is the number of times the divisor is contained in the dividend; the result of division.

123. Division is the reverse of multiplication. Having the product and either of the factors given, the other factor may be found by dividing the product by the given factor. Division may, therefore, be used as a method of checking multiplication, and vice versa.

ILLUSTRATED SOLUTIONS

124. *Problem :* $1736 \div 7 = ?$

$$\begin{array}{r} 248 \\ 7)\overline{1736} \end{array}$$

Or,

$$\begin{array}{r} 7)\overline{1736} \\ 248 \end{array}$$

7 is contained in 17 twice, with 3 remaining. Write the figure 2 over (or under) the 7 as the first figure of the quotient. The 3 hundreds left over, combined with the next figure, 3, makes 33 tens. 7 is contained in 33 four times, and 5 are left. Write the quotient figure 4, placing it over (or under) the 3 of the dividend. The 5 tens left from this operation, combined with the 6 units, makes 56 units. 7 is contained 8 times in 56. Place the 8 over (or under) the 6 of the dividend and the final quotient is complete, 248.

125. *To Check the Work:* Multiply the quotient by the divisor. The result should be the dividend. Thus, $248 \times 7 = 1736$.

126. *Problem :* $248963 \div 139 = ?$

$$\begin{array}{r} 1791\frac{14}{139} \\ 139)\overline{248963} \\ 139 \\ \hline 1099 \\ 973 \\ \hline 1266 \\ 1251 \\ \hline 153 \\ 139 \\ \hline 14 \end{array}$$

Write the problem in proper form for division, thus, $139)\overline{248963}$.

139 is contained in 248 once. Write 1 over the 8 to show this. $139 \times 1 = 139$. 139 subtracted from 248 leaves 109. To this annex the next figure in the dividend, 9. 139 is contained in 1099 seven times. Write 7 over the 9. $7 \times 139 = 973$. $1099 - 973 = 126$. Bring down 6 and proceed as before, so continuing until all the figures of the dividend have been used. After the last figure is used, the remainder, if any, should be written above a line with the divisor below, thus, $\frac{14}{139}$.

127. *To Check the Work:* Multiply the whole number of the quotient, 1791, by the divisor, 139, and add the remainder, 14, thus:

$$\begin{array}{r} 1791 \\ 139 \\ \hline 16119 \\ 5373 \\ 1791 \\ \hline 248963 \end{array} \qquad \begin{array}{r} \\ \\ \\ \\ 14 \\ \\ \end{array}$$

DECIMALS

128. All figures to the right of the decimal line represent parts of a unit. Each removal of the figure one place from the line, to the right, divides its value by ten. From the Latin *decem*, meaning *ten*, we derive the name decimal, which is applied to all values or quantities represented by figures written to the right of the decimal line.

From the fact that figures representing decimal values are written in exactly the same way as to represent integral values, except that they appear to the right of the decimal line, the actual processes of addition, subtraction, multiplication, and division are the same as for integers.

129. The only matter requiring special attention is the *decimal line* or *decimal point.*

ADDITION AND SUBTRACTION — DECIMALS

130. Decimals are added and subtracted in exactly the same manner as integers. The essential thing is to place the decimal points under one another so as to form the decimal line.

ILLUSTRATED SOLUTIONS

131. *Problem :* $248.15+.43642+12.05+124.3096+85.03752+100.075+2.12465=?$

248	15
	43642
12	05
124	3096
85	03752
100	075
2	12465
572	18319

After arranging the figures so as to form the decimal line, add as explained in par. 99.

132. *Problem :* $246.75-4.88625=?$

246	75000
4	88625
241	86375

After arranging the figures so as to form the decimal line, subtract as explained in par. 106.

MULTIPLICATION — DECIMALS

133. The actual work of multiplying decimals is exactly the same as for multiplying integers. It is necessary, however, to be able to locate the *decimal line*, or *decimal point*, in the product with unfailing accuracy.

One tenth (.1) multiplied by five tenths (.5) equals five hundredths (.05). Then *one* decimal place multiplied by one decimal place produces *two* decimal places in the product.

Five hundredths (.05) multiplied by three tenths (.3) equals fifteen thousandths (.015). That is, two decimal places multiplied by one decimal place produces three decimal places in the product.

From this we see that the product contains as many decimal places as the multiplicand and multiplier together contain.

134. *To Locate the Decimal Point in the Product:* Count the number of decimal figures in both factors together. Place the decimal line, or point, that number of places from the right-hand end of the product.

ILLUSTRATED SOLUTION

135. *Problem:* Multiply .875 by .63.

.875
.63
2625
5250
.55125

Multiply as explained in par. 117, regardless of the point.

Since there are three decimals in one factor and two decimals in the other, *counted together* there will be five decimals in the product. Count five figures beginning with the right figure of the product. Place the decimal line, or point, to the left of the fifth figure, thus, .55125

NOTE. For practice problems in the multiplication of decimals see par. 10.

DIVISION — DECIMALS

136. The actual work of division of decimals is the same as for division of integers, but it is necessary to be able to place the decimal point in the quotient with *absolute accuracy.*

Since the process of division is the direct opposite of that of multiplication; since the dividend of division is the product of multiplication; since the quotient of division is one of the factors of multiplication; and since the divisor is the other factor; if we take the number of decimals in the divisor (one factor) from the number of decimals in the dividend (the product) it will give us the number of decimals in the quotient (the other factor).

With this knowledge it is easy to understand the following rule, which should be *memorized* and carefully followed under all circumstances.

137. *To Locate the Decimal Point in the Quotient:* Imagine the divisor placed upon the dividend so that the decimal line of the divisor covers the decimal line of the dividend. Count the number of decimal places "covered" in the dividend. Place the new decimal line at this point, extending it up into the quotient. Proceed as in ordinary division.

138. *To be absolutely sure that the decimal line, or point, is in the right place, it should be placed in the quotient before any figures are written there.*

ILLUSTRATED SOLUTIONS

139. *Problem:* $12.435 \div 1.5 = ?$

```
      8 | 29
1.5)12.4 | 35
   12 0
    4 3
    3 0
    1 35
    1 35
```

Apply the above rule (137): Imagine 1.5 placed upon 12.435 in the expression $1.5\overline{)12.435}$ so that decimal point covers decimal point. One figure, 4, in the dividend is covered.

Place the decimal line between 4 and 3 extending it into quotient, thus $1.5\overline{)12.4|35}$ Now proceed as for integers (par. 126).

Ans. 8.29.

140. *Problem:* $12.875 \div 14 = ?$

```
      |919 9/14
14.)12|875
   12 6
    27
    14
    135
    126
      9
```

Apply the same rule: Imagine 14. placed on 12.875 in the expression $14.\overline{)12.875}$ so that decimal point covers decimal point. No decimal figures in the dividend are covered. Therefore, the decimal line remains unchanged, thus, $14.\overline{)12|875}$ Now proceed as for integers and the result will be $.919\frac{9}{14}$.

Ans. $.919\frac{9}{14}$.

141. *Problem :* $13.5 \div .875 = ?$

$$
\begin{array}{r}
15|428^{+} \\
.875\overline{)13.500|000} \\
8\ 75 \\
\hline
4\ 750 \\
4\ 375 \\
\hline
375\ 0 \\
350\ 0 \\
\hline
25\ 00 \\
17\ 50 \\
\hline
7\ 500 \\
7\ 000 \\
\hline
500
\end{array}
$$

Apply the same rule: Imagine .875 placed on 13.5 in the expression $.875\overline{)13.5}$ so that decimal point covers decimal point. Three decimal places in the dividend are covered. Two of these places must be visualized with 0's. Therefore, the decimal line falls between the third and fourth places, thus, $.875\overline{)13.500|}$ Now proceed as for integers and the result will be 15.428^{+}.

Ans. 15.428^{+}.

In this use $^{+}$ means "and more."

NOTE. For practice problems in the division of decimals see par. 11.

FACTORING

142. Every number can be divided by 1 and by itself without leaving a remainder.

143. Numbers that can be divided exactly *only* by themselves and by 1 are **Prime Numbers.** 1, 2, 3, 5, 7, 11, 13, 17, 19, etc., are prime numbers.

144. Numbers that can be divided exactly by numbers other than themselves and 1 are **Composite Numbers.**

145. Composite numbers are made up of two or more prime numbers multiplied together. These are called **Prime Factors.** It is sometimes necessary to find what prime factors go to make up a number.

146. *Problem :* What are the prime factors of 420?

ILLUSTRATED SOLUTION

$$\begin{array}{r} 2)\underline{420} \\ 2)\underline{210} \\ 3)\underline{105} \\ 5)\underline{\ 35} \\ 7 \end{array}$$

The smallest prime number contained in 420 is 2. It is contained 210 times. The smallest prime number contained in 210 is 2. It is contained 105 times. The smallest prime number contained in 105 is 3. It is contained 35 times. The smallest prime number contained in 35 is 5. It is contained 7 times. 7 is itself prime. Therefore, the prime factors of 420 are 2, 2, 3, 5, 7.

Ans. 2, 2, 3, 5, 7.

147. *To Check the Work :* Multiply the prime factors. The result will be the original quantity, thus, $2\times2\times3\times5\times7=420$.

LEAST COMMON MULTIPLE

148. A **Common Multiple** of several numbers is a quantity that contains all of them.

(*a*) Thus: 48 is a common multiple of 6, 8, 4, 3, and 2, because it contains all of them.

(*b*) The product of the several numbers is always a common multiple of them. That is, $8\times5\times4$ (160) is a common multiple of 8, 5, and 4.

149. The **Least Common Multiple** of several numbers is the *least* quantity that contains all of them.

(*a*) Thus: 24 is the *least* common multiple of 6, 8, 4, 3, and 2, because it is the least number that contains all of them.

(*b*) The product of the *prime factors* of several numbers is the least common multiple of those numbers.

150. *To Find the Least Common Multiple of Several Numbers:* Find the prime factors of the numbers. Multiply these factors together. The result will be the least common multiple of the original numbers.

ILLUSTRATED SOLUTION

151. *Problem:* Find the least common multiple of 24, 15, 36, and 9.

2)24	15	36	9
2)12	15	18	9
3) 6	15	9	9
3) 2	5	3	3
2	5	1	1

$2\times2\times3\times3\times2\times5=360.$

Write the numbers in line. 2 is the smallest prime factor contained in any two or more. Divide by 2 where possible, bringing down numbers of which 2 is not a factor. The result is 12, 15, 18, 9. 2 is a factor of 12 and 18. Proceed as before. The result is 6, 15, 9, 9. 3 is a prime factor of 15 and 9. Proceed as before. The result of this is 2, 5, 3, 3. 3 is a factor of 3 and 3. Proceed as before. The result is 2, 5, 1, 1, each of which is a prime number. The prime factors then are 2, 2, 3, 3, 2, 5. Multiply these and we obtain 360, which is the least common multiple.

152. *To Check the Work:* Divide the least common multiple by each of the original numbers, thus:

$360\div24=15$ $\qquad$ $360\div15=24$

$360\div36=10$ $\qquad$ $360\div\ 9=40$

GREATEST COMMON DIVISOR

153. A **Common Divisor** of two or more numbers is any number that will exactly divide each of them.

154. The **Greatest Common Divisor** is the greatest number that will exactly divide each of them.

7 is a common divisor of 21, 35, and 14. It will divide each of them exactly. Some numbers have no common divisor. No number will divide both 9 and 8 exactly. Such numbers are said to be prime to each other.

155. *To Find the Greatest Common Divisor :* Divide the greater number by the lesser. Then divide the first divisor by the remainder, and so continue until the division is exact, or there is a remainder of one. If exact, the last divisor is the greatest common divisor.

If the remainder is one, there is no common divisor. The numbers are prime to each other.

ILLUSTRATED SOLUTIONS

156. *Problem :* What is the greatest common divisor of 351 and 459?

$$
\begin{array}{r}
1 \\
351\overline{)459} \\
351 \quad 3 \\
\hline
108\overline{)351} \\
324 \quad 4 \\
\hline
27\overline{)108} \\
108 \\
\hline
\end{array}
$$

Divide 459 by 351 (par. 126). The remainder is 108. Divide 351 by 108 and the remainder is 27. Divide 108 by 27. There is no remainder. Therefore, 27 is the G. C. D. of 459 and 351.

G. C. D. = 27.

157. *To Check the Work :*

$459 \div 27 = 17$ $\qquad$ $351 \div 27 = 13$

158. *Problem :* Find the G. C. D. of 209, 247, and 456.

```
      1
209)247
   209   5
   38)209
      190  2
      19)38
         38
```

Find the G. C. D. of 209 and 247, as in the previous problem. Result, 19.

```
      24
19)456
   38
   76
   76
    0
```

Find the G. C. D. of 19 and 456, which is 19. Therefore, 19 is the G. C. D. of all.

G. C. D. = 19.

159. *Problem :* Find the G. C. D. of 943 and 35.

```
     26
35)943
   70
   243
   210  1
    33)35
       33 16
        2)33
          2
          13
          12
           1
```

Proceeding as above, we derive a final remainder of 1. Therefore, the numbers are prime to each other.

No G. C. D.

FRACTIONS

160. A **Fraction** is a part of an integer.

(*a*) Any quantity or value may be divided into any number of equal parts. One or more of these equal parts constitutes a fraction.

(*b*) If a quantity be divided into four equal parts, each of them will be known as one-fourth ($\frac{1}{4}$) and three of them would be three-fourths ($\frac{3}{4}$). If the whole were divided into a thousand equal parts, each part would be called one one-thousandth ($\frac{1}{1000}$) and any number of these parts could be made to form the fractional value, as 748 thousandths ($\frac{748}{1000}$).

161. A fraction is composed of two terms. One term shows the number of parts into which the original integer was divided. The other term shows the number of these equal parts taken to make the fraction.

162. **Denominator** is the name applied to the term showing into *how many parts* the integer was divided.

163. **Numerator** is the name applied to the term showing the *number* of these equal parts used in the fraction.

164. There are two methods of fractional notation, common fractions and decimal fractions.

165. A **Common Fraction** is expressed by writing the numerator above the denominator with a line between, thus, $\frac{4}{5}$.

166. A Decimal Fraction is any fraction whose denominator is 10 or any multiple of 10.

It is not necessary to write the denominators of decimal fractions. All that is required is to write the numerator and place it in the proper decimal column to show what denominator is intended, thus, six-tenths ($\frac{6}{10}$) may be written .6. Six-thousandths may be expressed by placing the figure six in the proper decimal column, thus, .006, and so on.

167. Every fractional value may be expressed either as a common fraction or as a decimal fraction.

In some cases it is easier to use the common fraction form; while in others it is easier to use the decimal form expressing the same value.

168. A Proper Fraction is a common fraction whose numerator is smaller than its denominator. Its value is *less* than one.

169. An **Improper Fraction** is a common fraction whose numerator is larger than its denominator. Its value is *more* than one.

170. A Mixed Number is an integral quantity and a fraction taken together, as, $1\frac{1}{4}$.

171. In its decimal form this quantity would be called a **Mixed Decimal** and would be written 1.25.

172. It is not always possible to reduce a fractional value to an exact decimal. In such cases it is necessary to retain a common fraction as part of a decimal, thus, $.16\frac{2}{3}$, read sixteen and two-thirds hundredths.

173. A Complex Decimal is a decimal whose right-hand place is a common fraction.

174. A **Complex Fraction** is a common fraction, the right-hand place of whose numerator is itself a fraction, thus, $\frac{16\frac{2}{3}}{100}$.

175. Every common fraction represents an unperformed division.

$\frac{8}{9}$ is the result of performing the operation $8 \div 9$. It may be read "eight divided by 9" or it may be read "eight-ninths." $\frac{2}{3}$ and also $2 \div 3$ may each be read "2 divided by 3."

176. **Reduction of Fractions** is the process of changing the form of a fraction without changing its value.

177. Reduction of fractions comprises:

Changing to lower terms.
Changing to higher terms.
Changing improper fractions to mixed numbers.
Changing mixed numbers to improper fractions.
Changing decimals to common fractions.
Changing common fractions to decimals.

CHANGE TO LOWER TERMS

178. Common fractions are in their lowest terms when the numerator and denominator are prime to each other; that is, when they have no common factors, no common divisor.

179. Decimal fractions, when written in the form of common fractions, may be reduced to lower terms when their numerator and denominator are not prime to each other.

The decimal fraction, four-tenths, if written in its common fraction form $\frac{4}{10}$, shows at once that its real or lowest value is $\frac{2}{5}$ because 2 is a common factor of both 4 and 10. In other words, if an integer is divided into ten parts and four of these parts are taken, the value of the fraction would be exactly the same as if the integer had been divided into five parts and two of these parts taken.

180. *To Change to Lower Terms:* Strike out all factors common to both the numerator and the denominator; or, divide both the numerator and the denominator by their G. C. D.

ILLUSTRATED SOLUTIONS

181. *Problem:* Reduce $\frac{420}{525}$ to lowest terms.

4
~~12~~
~~60~~
~~420~~
~~525~~
~~75~~
~~15~~
5

Ans. $\frac{4}{5}$.

Seven is a factor of both 420 and 525. Dividing the numerator, 420, by 7 gives a new numerator of 60. Dividing the denominator, 525, by 7 gives a new denominator of 75. Five is a common factor of both 60 and 75. Dividing 60 by 5 gives a new numerator of 12. Dividing 75 by 5 gives a new denominator of 15. Three is a common factor of 12 and 15. Divide 12 by 3 and we have a new numerator of 4. Dividing 15 by 3 gives a new denominator of 5. No factor is common to 4 and 5. Therefore, $\frac{4}{5}$ is the lowest equivalent of $\frac{420}{525}$

Or,

$$\begin{array}{r} 1 \\ 420\overline{)525} \\ 420 \\ \hline 105\overline{)525} \\ 525 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 4 \\ 105\overline{)420} \\ 420 \\ \hline 105\overline{)525} \\ 525 \\ \hline 5 \end{array} = \frac{4}{5}$$

Find the G. C. D. of 420 and 525 (pars. 155 and 156). The G. C. D. is 105. $420 \div 105 = 4$. Therefore, the new numerator is 4. $525 \div 105 = 5$. Therefore, the new denominator is 5, making the fraction $\frac{4}{5}$.

182. *Problem:* Change $\frac{1\frac{2}{3}}{9}$ to lowest terms.

$\frac{1\frac{2}{3}}{9} = \frac{5}{27}$

Ans. $\frac{5}{27}$.

This is a complex fraction. The first step is to change the numerator and the denominator to the same kind of parts, thirds. One unit $= \frac{3}{3}$; then $1\frac{2}{3} = \frac{5}{3}$; 9 units $= \frac{27}{3}$. Then the numerator equals 5 thirds and the denominator equals 27 thirds; 5 and 27 being prime to each other, $\frac{5}{27}$ is in its lowest terms.

183. *Problem:* Change $\frac{2\frac{4}{5}}{8}$ to lowest terms.

$\frac{2\frac{4}{5}}{8} = \frac{14}{40}$

$\frac{\cancel{14}^{7}}{\cancel{40}_{20}} = \frac{7}{20}$

Ans. $\frac{7}{20}$.

Proceeding as before, we find that the complex fraction is equal to $\frac{14}{40}$. Both terms may be divided by 2, producing $\frac{7}{20}$; 7 and 20 being prime to each other, $\frac{7}{20}$ is in its lowest terms.

NOTE. For practice problems in reducing fractions to lowest terms see par. 12.

CHANGING TO HIGHER TERMS

184. In handling common fractions in business problems, it is sometimes necessary to change a common fraction from its lowest terms to an equivalent common fraction of some higher denomination.

To do this it is necessary to introduce such factors into both the denominator and numerator as will change the value of the denominator from the given figure to that required.

185. *To Change a Common Fraction to a Required Denominator:* Divide the required denominator by

the given denominator. Multiply *both* the numerator and the denominator of the given fraction by the quotient thus obtained.

ILLUSTRATED SOLUTION

186. *Problem:* Change $\frac{4}{5}$ to 525ths.

$$5\overline{)525}\quad 105$$

$4\times105=420$

$5\times105=525$

Ans. $\frac{420}{525}$.

Five is contained 105 times in 525; therefore, 5 must be multiplied by 105 to produce 525. Multiplying 4 by 105 gives 420 for the new numerator. Multiplying 5 by 105 gives 525, the required denominator.

NOTE. For practice problems in changing fractions to higher terms see par. 13.

CHANGING AN IMPROPER FRACTION TO A MIXED NUMBER

187. The denominator shows into how many parts the original integer has been divided. If the number of parts taken is greater than the number into which the integer was divided, as is the case with every improper fraction, then the value of every improper fraction must be greater than the value of the original integer. It will be as many times the original integer as the denominator is contained in the numerator. If the denominator is not contained in the numerator an even number of times, the remaining number of parts would constitute the number of fractional units that are left.

188. *To Change an Improper Fraction to a Mixed Number:* Divide the numerator by the denominator.

ILLUSTRATED SOLUTION

189. *Problem :* Change $\frac{32}{9}$ to a mixed number.

$$\begin{array}{r} 3\frac{5}{9} \\ 9\overline{)32} \\ \underline{27} \\ 5 \end{array}$$

The denominator, 9, is contained $3\frac{5}{9}$ times in the numerator, 32. Therefore, $\frac{32}{9} = 3\frac{5}{9}$.

Ans. $3\frac{5}{9}$.

NOTE. For practice problems in changing improper fractions to mixed numbers see par. 14.

CHANGING A MIXED NUMBER TO AN IMPROPER FRACTION

190. The denominator of a fraction shows the number of parts into which the unit has been divided. Every unit of a mixed number then will contain as many of these parts as are expressed by the denominator. The fractional value of the whole then may be found by multiplying the integral number by the denominator of the fraction and adding the numerator of the fraction to this result.

191. *To Change a Mixed Number to an Improper Fraction:* Multiply the integer by the denominator of the fraction. To the result add the numerator of the fraction. The total should be written as the numerator of the new fraction. The denominator remains unchanged.

ILLUSTRATED SOLUTION

192. *Problem :* Change $3\frac{5}{8}$ to an improper fraction.

$3\times8=24$
$24+5=29$
Ans. $\frac{29}{8}$.

The value of three units is twenty-four eighths. Twenty-four eighths plus five eighths equals twenty-nine eighths. Therefore, $3\frac{5}{8}$ equals $\frac{29}{8}$.

NOTE. For practice problems in changing mixed numbers to improper fractions see par. 15.

CHANGING A DECIMAL FRACTION TO A COMMON FRACTION

193. *To Change a Decimal Fraction to a Common Fraction:* Write the decimal in its common fraction form and then reduce to lowest terms.

ILLUSTRATED SOLUTIONS

194. *Problem:* Change .125 to a common fraction.

$\frac{\cancel{125}}{\cancel{1000}}=\frac{1}{8}$
Ans. $\frac{1}{8}$.

Write .125 in its fraction form. It is apparent that 125 is a common divisor of both the numerator and the denominator. 125 is contained once in the numerator. 125 is contained eight times in the denominator.

195. *Problem:* Change $.16\frac{2}{3}$ to a common fraction.

$16\frac{2}{3}=50$
$100=300$
$\frac{\cancel{50}}{\cancel{300}}=\frac{1}{6}$
Ans. $\frac{1}{6}$.

Writing $.16\frac{2}{3}$ as a common fraction produces a complex fraction. Reduce this complex fraction to its lowest terms (par. 182). The numerator contains fifty thirds, the denominator contains three hundred thirds, making $\frac{50}{300}$ or $\frac{1}{6}$.

NOTE. For practice problems in changing decimal fractions to common fractions see par. 16.

CHANGING A COMMON FRACTION TO A DECIMAL FRACTION

196. Every common fraction is the statement of an unperformed division (par. 175). The result of performing this division is a decimal.

(*a*) The principle involved in changing a common fraction to a decimal is practically the same as that for changing an improper fraction to a mixed number.

(*b*) In changing from an improper fraction to a mixed number all the work is on the integral, or left, side of the decimal line; while in changing from a common fraction to a decimal fraction the work is all to the right of the decimal line.

(*c*) In changing from mixed numbers to mixed decimals, the work is on both sides of the line.

197. *To Change a Common Fraction to a Decimal Fraction:* Divide the numerator by the denominator.

Decimal values are seldom carried beyond the sixth decimal place. Any fraction remaining at this point is usually disregarded, although when absolute accuracy is desired the decimal should be carried out until exact, or the fraction should be retained, making a complex decimal.

ILLUSTRATED SOLUTIONS

198. *Problem:* Change $\frac{1}{8}$ to a decimal fraction.

```
  |125
8)1|000
Ans. .125.
```

Divide the numerator by the denominator, first placing the decimal point in the quotient, as shown in pars. 139, 140, 141. Three decimal places will be used, making the result .125.

199. *Problem :* What is the decimal value of $\frac{13}{15}$?

$$
\begin{array}{r|l}
 & .866\frac{2}{3} \\
\hline
15 & 13|000 \\
 & 12\ 0 \\
\hline
 & 1\ 00 \\
 & 90 \\
\hline
 & 10
\end{array}
$$

Ans. $.866\frac{2}{3}$.

Solve this problem in the same way as the previous one. After two decimal places have been used, we find that the remainder will continue to repeat itself. This shows that the division will never be exact. No matter how far carried, the decimal figure will be 6. We may, therefore, stop at any decimal place, retain the fraction $\frac{2}{3}$, and make the result a complex decimal, $.866\frac{2}{3}$, $.8666\frac{2}{3}$, or $.86\frac{2}{3}$, etc.

200. *Problem :* Change $2\frac{4}{5}$ to its decimal form.

$$
\begin{array}{r|l}
 & .8 \\
\hline
5 & 4|0
\end{array}
$$

Ans. 2.8.

Reducing the common fraction $\frac{4}{5}$ as shown above, the result is .8. The integer 2 remains the same and the result is, therefore, 2.8.

Or,

$2\frac{4}{5} = \frac{14}{5}$

$$
\begin{array}{r|l}
 & 2|8 \\
\hline
5 & 14|0
\end{array}
$$

Ans. 2.8.

Changing $2\frac{4}{5}$ to fifths, we have $\frac{14}{5}$, which, when reduced according to rule, gives 2.8.

NOTE. For practice problems in changing common fractions to decimal fractions see par. 17.

ADDITION OF FRACTIONS

201. Only numbers representing like values or like quantities or like parts of such values and quantities can be added (par. 96, *a*, *b*).

(*a*) In adding *decimals* all that is necessary is to carefully arrange the numbers so that the decimal points form a decimal line. In

this way tenths come over tenths, hundredths over hundredths, and so on, thus making it possible to add like parts.

(*b*) In adding *common fractions* it is necessary to change all fractions to equivalent fractions having the same denominator.

202. *To Add Common Fractions :* Find the L. C. M. of all the denominators (par. 150). Use this L. C. M. as a common denominator. Change each given fraction to a fraction having this denominator (par. 185). Add the numerators of the new fractions. The result is the numerator of the sum. The L. C. M. of the given denominators is the denominator of the sum. Reduce the sum-fraction to its lowest terms.

ILLUSTRATED SOLUTIONS

203. *Problem :* $\frac{2}{3}+\frac{5}{8}+\frac{7}{12}+\frac{5}{6}=?$

$2)3-8-12-6$
$2)3-4-\ 6-3$
$3)3-2-\ 3-3$
$1-2-\ 1-1$
$2\times2\times3\times2=24$ *

$\frac{2}{3}-16$
$\frac{5}{8}-15$
$\frac{7}{12}-14$
$\frac{5}{6}-20$

$\frac{65}{24}=2\frac{17}{24}$ *Ans.* $2\frac{17}{24}$.

We find the L. C. M. of the given denominators, 3, 8, 12, 6, to be 24 (pars. 150, 151). Change each of the given fractions to twenty-fourths. $\frac{2}{3}=\frac{16}{24}$, $\frac{5}{8}=\frac{15}{24}$, $\frac{7}{12}=\frac{14}{24}$ and $\frac{5}{6}=\frac{20}{24}$.

As the denominators are all the same and we are to add only the numerators, it will save time and confusion if we simply write the numerators 16, 15, 14, 20.

Added, we have $\frac{65}{24}$, which reduced to a mixed number (pars. 188, 189) equals $2\frac{17}{24}$.

* This L. C. M. can be determined mentally.

204. *Problem :* $12\frac{4}{5}+9\frac{1}{2}+23\frac{3}{8}+19\frac{5}{6}=?$

$2)5-2-8-6$
$2)5-1-4-3$
$5-1-2-3$
$2\times2\times5\times2\times3=120$
$12\frac{4}{5}-\ 96$
$9\frac{1}{2}-\ 60$
$23\frac{3}{8}-\ 45$
$19\frac{5}{6}-100$
$2\quad\frac{301}{120}=2\frac{61}{120}$
$65\frac{61}{120}$
Ans. $65\frac{61}{120}$.

Arrange the mixed numbers in a column. Add the fractions as explained in the previous solution. The result will be $\frac{301}{120}=2\frac{61}{120}$.

The $\frac{61}{120}$ will be the fraction of the final sum.

Add the 2 with the given integers. The total sum is $65\frac{61}{120}$.

205. *Problem :* $12.8+19\frac{1}{2}+14.875+5.8\frac{1}{3}=?$

12.8
19.5
14.875
$5.833\frac{1}{3}$
$53.008\frac{1}{3}$
Ans. $53.008\frac{1}{3}$.

Mixed numbers, mixed decimals and complex decimals can be added, but it is necessary first to change the mixed numbers to mixed decimals; that is, $19\frac{1}{2}$ must be changed to 19.5. The complex decimals must be reduced to the same order as the longest decimal; that is, $5.8\frac{1}{3}$ must be changed to $5.833\frac{1}{3}$. After these reductions have been made, add as explained in the previous solutions.

NOTE. For practice problems in addition of fractions see par. 18.

SUBTRACTION OF FRACTIONS

206. Subtraction of fractional values is performed in the same way as subtraction of integral quantities.

As in addition, it is necessary to change the given fractions to fractions having a common denominator.

ILLUSTRATED SOLUTION

207. *Problem :* $412\frac{1}{2}-204\frac{5}{9}=?$

1	27
$41\not{2}\frac{1}{2}$	$\not{9}$
$204\frac{5}{9}$	10
207	$\frac{17}{18}$

Ans. $207\frac{17}{18}$.

The given denominators are 9 and 2. The common denominator is 18. $\frac{1}{2}=\frac{9}{18}$, $\frac{5}{9}=\frac{10}{18}$. Ten cannot be taken from 9. Take one from the units column of the integers. $1=\frac{18}{18}$. $\frac{9}{18}+\frac{18}{18}=\frac{27}{18}$. $\frac{27}{18}-\frac{10}{18}=\frac{17}{18}$. The difference between the integers is 207. The total difference is $207\frac{17}{18}$.

NOTE. For practice problems in subtraction of fractions see par. 19.

MULTIPLICATION OF FRACTIONS

208. When an integer is multiplied by an integer, the product shows a value greater than either.

209. When an integer is multiplied by a fraction, the product shows a value less than the integer.

(*a*) $5\times5=25$. Both factors are integral numbers and the product, 25, shows an increase in value.

(*b*) $\frac{1}{3}\times2=\frac{2}{3}$. $\frac{2}{3}$ is a fraction and the product shows a value less than 2.

(*c*) The expression $\frac{1}{3}\times2$ may be read $\frac{1}{3}$ of 2. $\frac{1}{3}\times2$, and $\frac{1}{3}$ of 2 represent the same arithmetical operation.

(*d*) The sign "$\times$" and the word "of" are interchangeable in fractional computation.

210. *To Multiply a Fraction by a Fraction :* Multiply the numerators. The result will be the numerator of the product. Multiply the denominators. The result will be the denominator of the product. Reduce the new fraction to its lowest terms.

The work may be simplified by first casting out factors that may be common to one of the numerators and one of the denominators.

ILLUSTRATED SOLUTIONS

211. *Problem :* Multiply $\frac{2}{3}$ by $\frac{4}{9}$.

$\frac{2}{3}\times\frac{4}{9}=\frac{8}{27}$

Ans. $\frac{8}{27}$.

Multiply the numerators, $2\times4=8$. This is the numerator of the product. Multiply the denominators, $3\times9=27$. This is the denominator of the product.

212. *Problem :* $\frac{5}{8}$ of $\frac{2}{3}$ of $\frac{7}{10}$ = ?

$$\frac{5}{8}\text{ of }\frac{2}{3}\text{ of }\frac{7}{10}=\frac{70}{240}$$

$$\frac{7\cancel{0}}{24\cancel{0}}=\frac{7}{24}$$

Ans. $\frac{7}{24}$.

Multiply the numerators, $5\times2\times7=70$. This is the numerator of the new fraction. $8\times3\times10=240$. This is the denominator of the new fraction. $\frac{70}{240}$ reduced to the lowest terms equals $\frac{7}{24}$.

Or,

$$\frac{1}{\cancel{5}}\text{ of }\frac{1}{\cancel{2}}\text{ of }\frac{7}{\cancel{10}}=\frac{7}{24}$$

$\cancel{2}$

1

Ans. $\frac{7}{24}$.

The solution may be simplified by casting out factors as follows: Five is contained in the numerator of the fraction $\frac{5}{8}$ and in the denominator of the fraction $\frac{7}{10}$. Casting out makes the first fraction $\frac{1}{8}$ and the last $\frac{7}{2}$. Two is a common factor of the numerator of $\frac{2}{3}$ and the denominator of $\frac{7}{2}$ and may be cast out of each. Then multiply $1\times1\times7=7$ for the numerator of the product. $8\times3=24$ for the denominator of the product.

213. *To Multiply an Integer by a Fraction :* Multiply the integer by the numerator and divide the product thus obtained by the denominator of the fraction.

ILLUSTRATED SOLUTION

214. *Problem :* Multiply 324 by $\frac{4}{5}$.

$$\begin{array}{r} 324 \\ \frac{4}{5} \\ \hline 5)1296 \\ \hline 259\frac{1}{5} \end{array}$$

Applying the above rule, $4 \times 324 = 1296$.
$1296 \div 5 = 259\frac{1}{5}$.

Ans. $259\frac{1}{5}$.

215. *To Multiply Mixed Numbers :* Multiply the fractions; multiply the integers; combine the results.

ILLUSTRATED SOLUTIONS

216. *Problem :* Multiply 246 by $23\frac{4}{5}$.

$$\begin{array}{r} 246 \\ 23\frac{4}{5} \\ \hline 196\frac{4}{5} \\ 738 \\ 492 \\ \hline 5854\frac{4}{5} \end{array} \qquad \begin{array}{r} 196\frac{4}{5} \\ 5)984 \end{array}$$

First multiply the integer by the fraction. The product of 246 by the numerator, 4, is 984, divided by 5 gives $196\frac{4}{5}$ as the partial product by $\frac{4}{5}$.

Multiply 246 by 3 and by 2 as explained in par. 117. Add the results. The answer is $5854\frac{4}{5}$.

Ans. $5854\frac{4}{5}$.

217. *Problem :* Multiply $68\frac{2}{3}$ by 24.

$$\begin{array}{r} 68\frac{2}{3} \\ 24 \\ \hline 16 \\ 272 \\ 136 \\ \hline 1648 \end{array} \qquad \begin{array}{r} 16 \\ 3)48 \end{array}$$

Multiply as in the previous problem. Multiplying the integer 24 by the numerator, 2, of the fraction $\frac{2}{3}$, gives 48. Dividing this by 3 gives 16. This is the partial product by $\frac{2}{3}$. Multiply 68 by 24 (par. 117). Add, and the result is 1648.

Ans. 1648.

218. *Problem :* Multiply $243\frac{2}{3}$ by $42\frac{4}{5}$.

$243\frac{2}{3}$

$42\frac{4}{5}$

$\frac{8}{15}$ 8

$194\frac{2}{5}$ 6

28

486

972

10428 $\frac{14}{15}$

Ans. $10428\frac{14}{15}$.

$\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$

5)972

$194\frac{2}{5}$

3)84

28

Multiply the fraction of the multiplicand by the fraction of the multiplier. The product of the numerators, 4 and 2, equals 8. The product of the denominators, 5 and 3, equals 15. The first partial product is $\frac{8}{15}$.

Multiply the integral part of the multiplicand by the fraction of the multiplier. Four times 243 equals 972 (par. 214). 972 divided by 5 equals $194\frac{2}{5}$, which is the second partial product.

Multiply the fractional part of the multiplicand by the integral part of the multiplier (par. 214). 42×2 equals 84. 84 divided by 3 equals 28, which is the third partial product.

Multiply the integer of the multiplicand by the integer of the multiplier (par. 117). Add the partial products and the total is $10428\frac{14}{15}$.

NOTE. For practice problems in multiplication of fractions see par. 20.

DIVISION OF FRACTIONS

219. Division of fractions, like division of integers, is the direct opposite of multiplication.

220. When an integer is divided by an integer, the result shows a decrease. When an integer is divided by a fraction the result shows an increase.

Ten divided by 5 equals 2. That is, one integer divided by another gives a decrease. Ten divided by one-half equals 20, for in 10 units there are 20 halves.

221. *To Divide Fractions by Fractions :* Multiply the numerator of the dividend by the denominator of the divisor. The result is the numerator of the quotient. Multiply the denominator of the dividend by the numerator of the divisor. The result is the denominator of the quotient. Reduce the quotient to its lowest terms.

Or: Invert the divisor and proceed as in multiplication.

ILLUSTRATED SOLUTIONS

222. *Problem :* Divide $\frac{4}{5}$ by $\frac{2}{3}$.

$\frac{4}{5} \div \frac{2}{3} = \frac{12}{10} = 1\frac{1}{5}$

Ans. $1\frac{1}{5}$.

The numerator of the dividend is 4 and the denominator of the divisor is 3. $4 \times 3 = 12$, which is the numerator of the quotient. The denominator of the dividend is 5 and the numerator of the divisor is 2. $2 \times 5 = 10$. The denominator of the quotient is 10. $\frac{12}{10}$ is an improper fraction, reduced to a mixed number equals $1\frac{1}{5}$.

Or,

$\frac{4}{5} \div \frac{2}{3} = ?$

$\frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = 1\frac{1}{5}$

Ans. $1\frac{1}{5}$.

In the problem $\frac{4}{5} \div \frac{2}{3}$, the dividend is $\frac{4}{5}$ and the divisor $\frac{2}{3}$. Inverting $\frac{2}{3}$ gives $\frac{3}{2}$. $4 \times 3 = 12$. $5 \times 2 = 10$. $\frac{12}{10} = 1\frac{1}{5}$.

223. *To Divide Mixed Numbers :* Change both the divisor and the dividend to improper fractions having a common denominator. Divide the numerator of the dividend by the numerator of the divisor. The result will be the quotient required.

ILLUSTRATED SOLUTIONS

224. *Problem :* Divide \$124.50 by $23\frac{2}{3}$.

$$
\begin{array}{r}
5\,|\,26 \\
71)\overline{373\,|\,50} \\
355 \\
\hline
18\,5 \\
14\,2 \\
\hline
4\,30 \\
4\,26 \\
\hline
4 \\
\hline
71
\end{array}
$$

In this problem, both the dividend and the divisor must be changed to thirds.

\$124.50 should be changed to thirds by multiplying by three, which gives 373.50 thirds.

$23\frac{2}{3}=\frac{71}{3}$ (pars. 191, 192). Divide 373.50 by 71 (par. 140). The result is $\$5.26\frac{4}{71}$.

Ans. $\$5.26\frac{4}{71}$.

Problem : Divide $248\frac{4}{5}$ by 34.

$$
\begin{array}{r}
7\frac{27}{85} \\
170)\overline{1244} \\
1190 \\
\hline
54 \\
\hline
170
\end{array}
=\frac{27}{85}
$$

Changing the dividend, $248\frac{4}{5}$, to fifths equals 1244 fifths. Changing the divisor, 34, to fifths equals 170 fifths. Divide the numerator of the dividend, 1244, by the numerator of the divisor, 170. The result is $7\frac{27}{85}$.

Ans. $7\frac{27}{85}$.

225. *Problem :* Divide $433\frac{1}{2}$ by $18\frac{2}{3}$.

$$
\begin{array}{r}
23 \\
112)\overline{2601} \\
224 \\
\hline
361 \\
336 \\
\hline
25 \\
\hline
112
\end{array}
$$

Change the fraction in the dividend and the fraction in the divisor to fractions having a common denominator. $433\frac{1}{2}=433\frac{3}{6}$ and $18\frac{2}{3}=18\frac{4}{6}$. Then $433\frac{1}{2}\div 18\frac{2}{3}$ is the same as $433\frac{3}{6}$ divided by $18\frac{4}{6}$. Changing the dividend to sixths, we have 2601 sixths. Changing the divisor to sixths, we have 112 sixths. Dividing 2601 by 112, gives $23\frac{25}{112}$.

Ans. $23\frac{25}{112}$.

NOTE. For practice problems in division of fractions see par. 21.

For practice problems involving the use of fractions and decimals see par. 22.

DENOMINATE NUMBERS

226. A **Denominate Number** is a number expressed in units of weight, measure, or value.

227. A **Simple Denominate Number** is a quantity expressed in a single denomination.

4 pounds, 5 bushels, 2 quarts are simple denominate numbers.

228. A **Compound Denominate Number** (usually called a compound number) is a quantity expressed in two or more different denominations.

(*a*) 1 year, 9 months, and 9 days; 3 pounds, 9 shillings, and 4 pence are compound numbers.

(*b*) Tables of weight, measure, distance, values, etc., will be found on pages 130 to 146.

(*c*) In the following illustrated solutions, English money is used throughout for the purpose of uniformity. The solutions are equally applicable to any or all the tables.

REDUCTION OF DENOMINATE NUMBERS

ILLUSTRATED SOLUTION

229. *To Reduce a Compound Denominate Number to a Simple Denominate Number of Equivalent Value:*

Problem : Reduce £5 8*s* 9*d* to pence.

5 = £
20
100 = *s*
8
108
12
1296 = *d*
9
1305

Since there are 20*s* in £1, in £5 there are 5 times 20 or 100*s*. £5 8*s* equals 108*s*. Since there are 12*d* in 1*s*, in 108*s* there are 12 times 108, which is 1296*d*. If there are 1296*d* in £5 8*s*, in £5 8*s* 9*d* there are 1305*d*.

Ans. 1305*d*.

NOTE. For practice problems in reducing compound denominate numbers to simple denominate numbers of equivalent value see par. 23.

ILLUSTRATED SOLUTION

230. *To Change to Higher Denomination:*

Problem : Reduce 1305 pence to higher denomination.

12)1305*d*
20) 108*s* 9*d*
£5 8*s*

Ans. £5 8*s* 9*d*.

Since there are 12*d* in 1*s*, in 1305*d* there are as many shillings as 12 is contained in 1305*d* which is 108*s* and 9*d* left. Since there are 20*s* in £1, in 108*s* there are as many pounds as 20 is contained in 108*s* which is £5 and 8*s* left. Therefore, 1305*d* equals £5 8*s* 9*d*.

NOTE. For practice problems in changing simple denominate numbers to higher denominations see par. 24.

ILLUSTRATED SOLUTION

231. *To Change to Lower Denomination:*

Problem : Reduce £.575 to lower denomination.

.575£
20
11|5ØØ*s*
12
6|0*d*

Since there are 20*s* in £1, in £.575 there are 20 times £.575, which is 11.5*s*. Since there are 12*d* in 1*s*, in .5*s* there are 12 times .5 which is 6*d*.

Ans. 11*s* 6*d*.

NOTE. For practice problems in changing denominate numbers to lower denominations see par. 25.

ILLUSTRATED SOLUTION

232. *To Change a Compound Denominate Number to a Simple Denominate Number:*

Problem : Reduce 11*s* 6*d* to a decimal of a pound.

|5*s*
12)6|0*d*

£|575
20)11|500*s*

Ans. £.575

Since there are 12*d* in 1*s*, in 6*d* there are as many shillings as 12 is contained times in 6 which is .5*s*. 11*s* 6*d* equals 11.5*s*. Since there are 20*s* in £1, in 11.5*s* there are as many pounds as 20 is contained in 11.5*s* which is £.575.

NOTE. For practice problems in changing compound denominate numbers to simple denominate numbers see par. 26.

ADDITION OF COMPOUND NUMBERS

ILLUSTRATED SOLUTION

233. *Problem :* £125 13*s* 9*d*+£23 8*s* 7*d*+£13 5*d*= ?

£	*s*	*d*
125	13	9
23	8	7
13	0	5
162	2	9

Ans. £162 2*s* 9*d*.

Arrange the compound numbers so that similar denominations fall in the same column. Adding the column representing the lowest denomination, we have 21*d*. 21*d* equals 1*s* and 9*d*. Write the 9*d* under the proper column. Carry 1*s* to the column of shillings. Adding we obtain 22*s* which equals £1 and 2*s*. Write 2*s*, carry £1 to the pound column. Adding, we have £162.

SUBTRACTION OF COMPOUND NUMBERS

ILLUSTRATED SOLUTION

234. *Problem :* Subtract £15 12*s* 8*d* from £48 7*s* 6*d*.

7	6	
£4~~8~~	~~7~~*s*	6*d*
15	12	8
32	14	10

Ans. £32 14*s* 10*d*.

Beginning with the column representing the lowest denomination, we subtract 8*d* from 6*d*, which cannot be performed. Take 1*s* from 7*s*. One shilling equals 12*d*, making 18*d* in all. 18 minus 8 leaves 10*d*. 12*s* from 6*s* cannot be performed. Take £1 from £48. £1 equals 20*s*, plus 6 equals 26*s*. 26*s* minus 12*s* equals 14*s*, £47 minus £15 equals £32. The remainder then is £32 14*s* 10*d*.

NOTE. For practice problems in subtraction of compound numbers see pars. 52, 53.

MULTIPLICATION OF COMPOUND NUMBERS

ILLUSTRATED SOLUTION

235. *Problem :* Multiply £26 16*s* 9*d* by 23.

£26	16*s*	9*d*
		23
£598	368*s*	207*d*
£617	5*s*	3*d*

Ans. £617 5*s* 3*d*.

Beginning with the lowest denomination, multiply 9*d* by 23. This is 207*d*. 23 times 16*s* equals 368*s*. 23 times £26 equals £598.

Reduce 207*d* to shillings (par. 230) making 17*s* 3*d*. Write 3*d* as part of the final product.

17*s*+368*s*=385*s*. Reduce 385*s* to pounds (par. 230). Result, 19*d* 5*s*. Write 5*s* as part of the final product.

Add £19 to £598. Result, £617. Write £617 as the last of the final product.

DIVISION OF COMPOUND NUMBERS

ILLUSTRATED SOLUTION

236. *Problem :* Divide £617 5*s* 3*d* by 23.

£26	16*s*	9*d*
23)617	23)385*s*	23)207
46	23	207
157	155	
138	138	
£19	17*s*	
20	12	
380*s*	204*d*	
5	3	
385*s*	207*d*	

Ans. £26 16*s* 9*d*.

Commencing with the highest denomination, divide £617 by 23. This gives £26 with an undivided remainder of £19. Write the £26 as part of the final quotient.

Reduce £19 to shillings by multiplying by 20 (par. 229). This equals 380*s*. 380*s* plus 5*s* equals 385*s*. Divide 385*s* by 23, which gives 16*s*, with an undivided remainder of 17*s*. Write the 16*s* as part of the final quotient.

Reduce the 17*s* to pence (par. 229).

This equals 204*d*, plus 3*d* is 207*d*. 207*d* divided by 23 equals 9*d*. Write 9*d* as part of the final quotient, making the complete quotient £26 16*s* 9*d*.

COMPUTING TIME

237. In business it is often necessary to compute the interval of time between two given dates.

Two methods are followed: Compound Subtraction and Exact Days.

238. *To Find the Time by Compound Subtraction:*

ILLUSTRATED SOLUTIONS

Problem: Find the time between October 15, 1908, and July 1, 1916.

1916	7	1
1908	10	15
7	8	16

Ans. 7 yr. 8 mo. 16 da.

Use the later date as the minuend and the earlier date as the subtrahend. The minuend will be the 1916th year, 7th month, and 1st day. The subtrahend will be the 1908th year, 10th month, and 15th day. Write the subtrahend beneath the minuend and proceed as for compound subtraction (par. 234).

NOTE. For practice problems in finding the time by compound subtraction see par. 52.

239. *To Find the Time in Exact Days:*

Problem : How many days between January 8, 1916, and May 19, 1916?

Jan.	23	
Feb.	29	Leap Year
Mar.	31	
Apr.	30	
May	19	
	132 days	

Ans. 132.

Subtract the date (8) from the full number of days in the first month (31 days), leaving 23 days in January. February has 29 days (Leap Year), March 31, April 30, and May 19. Write these in a column and add.

NOTE. For practice problems in finding the time in exact days see par. 53.

ALIQUOT PARTS

240. The term **Aliquot Part** is applied to any number that is contained an even number of times in a given value or quantity.

The aliquot parts of 100 may be used to great advantage in many of the arithmetical operations involved in business transactions. Therefore, the term aliquot parts usually means the aliquot parts of 100.

241. The following table shows the aliquot parts of one hundred that are of practical use. They are expressed as common fractions, as decimals, and as cents, and should be memorized and used whenever practicable.

Fraction	Decimal	Cents	Fraction	Decimal	Cents
$\frac{1}{2}$	.5	50	$\frac{4}{9}$	$.44\frac{4}{9}$	$44\frac{4}{9}$
			$\frac{5}{9}$	$.55\frac{5}{9}$	$55\frac{5}{9}$
$\frac{1}{3}$	$.33\frac{1}{3}$	$33\frac{1}{3}$	$\frac{7}{9}$	$.77\frac{7}{9}$	$77\frac{7}{9}$
$\frac{2}{3}$	$.66\frac{2}{3}$	$66\frac{2}{3}$	$\frac{8}{9}$	$.88\frac{8}{9}$	$88\frac{8}{9}$
			$\frac{1}{10}$	.1	10
$\frac{1}{4}$	.25	25	$\frac{3}{10}$	.3	30
$\frac{3}{4}$	.75	75	$\frac{7}{10}$	.7	70
$\frac{1}{5}$	.2	20	$\frac{9}{10}$	.9	90
$\frac{2}{5}$	.4	40			
$\frac{3}{5}$	.6	60	$\frac{1}{11}$	$.09\frac{1}{11}$	$9\frac{1}{11}$
			$\frac{2}{11}$	$.18\frac{2}{11}$	$18\frac{2}{11}$
$\frac{4}{5}$	.8	80	$\frac{3}{11}$	$.27\frac{3}{11}$	$27\frac{3}{11}$
$\frac{1}{6}$	$.16\frac{2}{3}$	$16\frac{2}{3}$	$\frac{4}{11}$	$.36\frac{4}{11}$	$36\frac{4}{11}$
$\frac{5}{6}$	$.83\frac{1}{3}$	$83\frac{1}{3}$	$\frac{5}{11}$	$.45\frac{5}{11}$	$45\frac{5}{11}$
			$\frac{6}{11}$	$.54\frac{6}{11}$	$54\frac{6}{11}$
$\frac{1}{7}$	$.14\frac{2}{7}$	$14\frac{2}{7}$	$\frac{7}{11}$	$.63\frac{7}{11}$	$63\frac{7}{11}$
$\frac{2}{7}$	$.28\frac{4}{7}$	$28\frac{4}{7}$	$\frac{8}{11}$	$.72\frac{8}{11}$	$72\frac{8}{11}$
$\frac{3}{7}$	$.42\frac{6}{7}$	$42\frac{6}{7}$	$\frac{9}{11}$	$.81\frac{9}{11}$	$81\frac{9}{11}$
$\frac{4}{7}$	$.57\frac{1}{7}$	$57\frac{1}{7}$	$\frac{10}{11}$	$.90\frac{10}{11}$	$90\frac{10}{11}$
$\frac{5}{7}$	$.71\frac{3}{7}$	$71\frac{3}{7}$			
$\frac{6}{7}$	$.85\frac{5}{7}$	$85\frac{5}{7}$	$\frac{1}{12}$	$.8\frac{1}{3}$	$8\frac{1}{3}$
			$\frac{5}{12}$	$.41\frac{2}{3}$	$41\frac{2}{3}$
$\frac{1}{8}$	.125	$12\frac{1}{2}$	$\frac{7}{12}$	$.58\frac{1}{3}$	$58\frac{1}{3}$
$\frac{3}{8}$	.375	$37\frac{1}{2}$	$\frac{11}{12}$	$.91\frac{2}{3}$	$91\frac{2}{3}$
$\frac{5}{8}$	.625	$62\frac{1}{2}$			
$\frac{7}{8}$	.875	$87\frac{1}{2}$	$\frac{1}{16}$	$.06\frac{1}{4}$	$6\frac{1}{4}$
			$\frac{3}{16}$	$.18\frac{3}{4}$	$18\frac{3}{4}$
$\frac{1}{9}$	$.11\frac{1}{9}$	$11\frac{1}{9}$	$\frac{5}{16}$	$.31\frac{1}{4}$	$31\frac{1}{4}$
$\frac{2}{9}$	$.22\frac{2}{9}$	$22\frac{2}{9}$	$\frac{7}{16}$	$.43\frac{3}{4}$	$43\frac{3}{4}$

Fraction	Decimal	Cents	Fraction	Decimal	Cents
$\frac{9}{16}$	$.56\frac{1}{4}$	$56\frac{1}{4}$	$\frac{1}{20}$	.05	5
$\frac{11}{16}$	$.68\frac{3}{4}$	$68\frac{3}{4}$	$\frac{3}{20}$	.15	15
$\frac{13}{16}$	$.81\frac{1}{4}$	$81\frac{1}{4}$	$\frac{7}{20}$	.35	35
$\frac{15}{16}$	$.93\frac{3}{4}$	$93\frac{3}{4}$	$\frac{9}{20}$	.45	45
			$\frac{11}{20}$	.55	55
			$\frac{13}{20}$	.65	65
			$\frac{17}{20}$	.85	85
			$\frac{19}{20}$	.95	95

ILLUSTRATED SOLUTION

Application of Aliquot Parts

Problem: Find the cost of 640 lb. of tea at $37\frac{1}{2}$¢.

$37\frac{1}{2}¢ = \frac{3}{8}$ of \$1

$\frac{3}{8}$ of $640 = 240$

640 lb. at \$1 a pound would cost \$640. At $\frac{3}{8}$ of a dollar a pound, 640 lb. would cost $\frac{3}{8}$ of \$640, which is \$240.

To avoid fractions, first multiply by the numerator and then divide by the denominator.

Note. For practice problems in aliquot parts see par. 27 to 31 inclusive.

PERCENTAGE

242. The words *per cent* and the name *percentage* are derived from two Latin words, *per centum*, meaning *by the hundred.*

243. Percentage is a system of measurement in which 100 is used as the standard of comparison.

Every unit contains 100 one-hundredths or 100 per cent. The words *per cent* are used instead of the denominator, one-hundredths. Thus: Instead of saying five one-hundredths, we say 5 per cent. Nine per cent would be $\frac{9}{100}$ or .09. It could be used in either its fractional or decimal form.

244. The per cent sign (%) is generally used *instead of the words "per cent"* and *instead of the decimal point*, just as the cent sign (¢) is used *instead of the word "cents"* and *instead of the decimal point* in expressing United States money, which is really another use of the percentage system.

(*a*) Every per cent may be expressed in four different ways; as a common fraction with 100 for a denominator; as an equivalent common fraction in its lowest terms; as a decimal; and by the use of the sign.

(*b*) The sign and the decimal should never be used together except to designate a part of one per cent.

(*c*) $25\ \% = .25\ = \frac{25}{100} = \frac{1}{4}$

$16\frac{2}{3}\% = .16\frac{2}{3} = \frac{\frac{2}{3}}{100} = \frac{1}{6}$

$33\frac{1}{3}\% = .33\frac{1}{3} = \frac{\frac{1}{3}}{100} = \frac{1}{3}$

$23\ \% = .23\ = \frac{23}{100}$

$.8\ \% = \frac{8}{10}\% = .008 = \frac{\frac{8}{10}}{100} = \frac{8}{1000}$

245. *In solving problems in percentage that form should be used which makes the solution easiest.*

246. Several special terms are used in the subject of percentage. These are Base, Rate, Percentage, Amount, and Difference.

247. The **Base** is the value, or quantity, represented by 100%. It is the basis of comparison.

248. The **Rate** is the number of one-hundredths used in the comparison.

249. The **Percentage** is the value of the rate. It is the part of the base equal to the number of hundredths represented by the rate. It is the product of the base by the rate.

250. The **Amount** is the base plus the percentage.

251. The **Difference** is the base minus the percentage.

252. In the subject of aliquot parts (pars. 240, 241) a table showing the aliquot parts of one hundred is given. The aliquot parts of 100% are the same.

253. In the following table the fractional values of the easier rates are given. It will be well to use the common fraction form for any of these rates.

$2\frac{1}{2}\% = \frac{1}{40}$	$10\% = \frac{1}{10}$	$33\frac{1}{3}\% = \frac{1}{3}$	$62\frac{1}{2}\% = \frac{5}{8}$
$5\% = \frac{1}{20}$	$12\frac{1}{2}\% = \frac{1}{8}$	$37\frac{1}{2}\% = \frac{3}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$
$6\frac{1}{4}\% = \frac{1}{16}$	$16\frac{2}{3}\% = \frac{1}{6}$	$40\% = \frac{2}{5}$	$75\% = \frac{3}{4}$
$6\frac{2}{3}\% = \frac{1}{15}$	$20\% = \frac{1}{5}$	$50\% = \frac{1}{2}$	$80\% = \frac{4}{5}$
$8\frac{1}{3}\% = \frac{1}{12}$	$25\% = \frac{1}{4}$	$60\% = \frac{3}{5}$	$87\frac{1}{2}\% = \frac{7}{8}$

254. All operations in percentage are solved in accordance with the general principles of multiplication and division. They may be expressed as follows:

(*a*) Base × Rate = Percentage.
Percentage ÷ Rate = Base.
Percentage ÷ Base = Rate.

(*b*) Because the smallest coin used in the United States is one cent, *final answers* should be expressed in the nearest cent; that is, 5 mills (.005) or more would be called another cent, less than 5 mills would be disregarded.

255. *To Find the Percentage:*

In solving problems in this case always use the easiest method.

ILLUSTRATED SOLUTIONS

Problem: Find $37\frac{1}{2}\%$ of \$342.

\$342
3.42
$37\frac{1}{2}$
1 71
23 94
102 6
\$128.25

\$342 is the standard quantity or 100%. One per cent would be $\frac{1}{100}$ of this which is 3.42 and $37\frac{1}{2}\%$ would then be found by multiplying 3.42 by $37\frac{1}{2}$ (par. 135). The result is \$128.25.

Ans. \$128.25.

Or,

$$\begin{array}{r} \$342 \\ .37\frac{1}{2} \\ \hline 1\ 71 \\ 23\ 94 \\ 102\ 6 \\ \hline \$128.25 \end{array}$$

Thirty-seven and one-half per cent of $342 is the same as $.37\frac{1}{2}$ of 342, which means $.37\frac{1}{2} \times 342$. Performed as explained in par. 135, the result is $128.25.

Ans. $128.25.

Or,

$.37\frac{1}{2} = \frac{3}{8}$

$$\begin{array}{r} \$342 \\ 3 \\ \hline 8)1026 \\ \hline 128.25 \end{array}$$

Thirty-seven and one-half per cent equals $\frac{3}{8}$. $\frac{3}{8}$ of $342 (par. 214) is $128.25.

Ans. $128.25.

All of these methods will be found to apply to any problem in which it is necessary to find the percentage. In some cases one method will be easier than another. Always use the easiest method.

256. *Problem :* A man who is worth $8465 has 65% of his property in real estate, 15% of it in a mortgage, and the remainder in cash. How much cash has he?

$100\% = \$8465$
$65\% + 15\% = 80\%$
$100\% - 80\% = 20\%$
$20\% = \frac{1}{5}$
$\frac{1}{5}$ of $\$8465 = \$1693$

Ans. $1693.

For purpose of comparison in the above problem it will be seen that the man's entire property is 100%. Sixty-five per cent and 15% (80%) are known to be invested in real estate and mortgage. The remainder of the 100%, or 20%, is in cash. Our problem then is to find 20% of the property, $8465, by any of the above explained methods. Since 20% is one-fifth, the easiest way is to find one-fifth of $8465, which is $1693.

257. *Problem:* I invested \$7460 in business. During the first year I gained 25%. How much did I have invested in the business at the end of that time?

4) 7460
1865
9325

Twenty-five per cent is one-fourth. One-fourth of \$7469 is \$1865. If I invested \$7460 and gained \$1865, I must have now \$9325.

Ans. \$9325.

Or,

\$7460
1.25
37300
14920
7460
\$9325.00

The original investment was 100% of itself. If it is increased 25%, then the new capital would be 125% (1.25) of the original investment. Find 125% of \$7460 by multiplying by 1.25. Result, \$9325.

Ans. \$9325.

It is sometimes easier to combine the various methods illustrated in the above solutions.

258. *Problem:* Find 27% of \$6240.

4) 6240
1560 25%
124.80 2%
\$1684.80

Twenty-seven per cent is made up of two easy rates, 25% and 2%. Twenty-five per cent is $\frac{1}{4}$ (1560). Two per cent is twice 1%. One per cent is \$62.40 and 2% is \$124.80; adding this amount to \$1560, we have \$1684.80.

Ans. \$1684.80.

NOTE. For practice problems in finding the percentage see par. 33.

259. *To Find the Base:*

In solving problems, always use the easiest method.

ILLUSTRATED SOLUTIONS

Problem : \$1172.34 is 27% of what?

```
      $43|42
27) 1172|34
    108
     92
     81
     113
     108
       54
       54
```

\$43.42 = 1%

\$4342 = 100%

Ans. \$4342.

Since \$1172.34 is 27%, 1% would be $\frac{1}{27}$ of \$1172.34. Divide \$1172.34 by 27, which is \$43.42. This would be 1% of the original value. One hundred per cent, or the whole of the original value, would be 100 times 1%, which can be found by moving the figures in \$43.42 two places to the left. Therefore, 100% would be \$4342.

Or,

```
      $4342|
.27) 1172.34|
    108
     92
     81
     113
     108
       54
       54
```

Ans. \$4342.

\$1172.34 is .27 of some number and was found by multiplying the original number by .27. In other words, \$1172.34 is the product of one factor by .27. If we divide \$1172.34 (the product) by .27 (one factor), the result will be the other factor. Perform this division. The result is \$4342.

260. *Problem :* \$94.65 is $62\frac{1}{2}\%$ of what?

$5)\ 94|65 = \frac{5}{8}$
$18|93 = \frac{1}{8}$
8
$\$151.44 = \frac{8}{8}$

Ans. \$151.44.

Sixty-two and one-half per cent is $\frac{5}{8}$ of the required quantity. Since 94.65 is $\frac{5}{8}$, we will get $\frac{1}{8}$ of the quantity by dividing 94.65 by 5, which is 18.93. If 18.93 is $\frac{1}{8}$, the whole quantity, or $\frac{8}{8}$, would be 8 times 18.93, which is 151.44, the value of the original quantity or 100%.

The problem given above could be solved according to either of the preceding methods.

261. *Problem :* \$235.64 is 8% more than what?

```
      $218|185
1.08)235.64|000
     216
     196
     108
      884
      864
       200
       108
        920
        864
         560
         540
          20
```

Ans. \$218.19.

The original quantity was 100%. \$235.64 equals this and 8% more. Therefore, \$235.64 is 108% of what? Solving this by any of the above methods, we find the original number to be \$218.185, which, expressed in the nearest equivalent cent, is \$218.19.

NOTE. For practice problems in finding the base see par. 34.

262. *To Find the Rate:*

In solving problems, always use the easiest method.

ILLUSTRATED SOLUTIONS

Problem : \$128.25 is what per cent of \$342?

$\frac{128.25}{342.00} = \frac{3}{8}$

$\frac{3}{8} = .37\frac{1}{2}$

Ans. $37\frac{1}{2}\%$.

In the above problem we simply want to know what part \$128.25 is of \$342.

\$128.25 is $\frac{128.25}{342.00}$ of \$342. Reduced to lowest terms $\frac{128.25}{342.00}$ equals $\frac{3}{8}$. Then \$128.25 is $\frac{3}{8}$ of \$342. $\frac{3}{8}$ equals $37\frac{1}{2}\%$. Therefore, \$128.25 is $37\frac{1}{2}\%$ of \$342.

Or,

$$
\begin{array}{r|l}
 & 37\frac{1}{2} \\
\hline
342 & 128.25 \\
 & 102\ 6 \\
\hline
 & 25\ 65 \\
 & 23\ 94 \\
\hline
 & 171 \\
 & 342
\end{array}
= \frac{1}{2}
$$

Ans. $.37\frac{1}{2}$ or $37\frac{1}{2}\%$.

Inasmuch as \$128.25 is a certain per cent of \$342, it (\$128.25) is the product obtained by multiplying one factor (the desired rate) by another factor (\$342). Therefore, if we divide \$128.25 (the product) by 342 (one factor), we shall obtain the other factor (par. 123). Perform this operation as described in par. 140. The result is $.37\frac{1}{2}$, which equals $37\frac{1}{2}\%$.

263. *Problem :* \$9072 is what per cent of \$7560?

$$
\begin{array}{r|l}
 & 1.20 \\
\hline
7560 & 9072.00 \\
 & 7560 \\
\hline
 & 1512\ 0 \\
 & 1512\ 0 \\
\hline
 & \\
 &
\end{array}
$$

Ans. 1.20 or 120%.

\$9072 is the product obtained by multiplying one factor (7560) by another factor (the desired rate). If we divide 9072 (the product) by 7560 (one factor), we shall obtain the other factor (par. 123). The result, 1.20, equals 120%.

NOTE. For practice problems in finding the rate see par. 35.
For general problems in percentage see par. 36.

PROFITS AND LOSSES

264. In measuring and comparing profits and losses, it has been found best to do so by means of percentage.

265. For this purpose it is necessary to understand the exact meaning of the special terms used in connection with this subject.

266. **Cost** is the value of the investment.

267. **Prime Cost** of an article is the amount *actually paid* for it. The prime cost is sometimes called the *net cost* and also the *first cost.*

268. The **Gross Cost** of an article is the total amount invested in it, and includes the prime cost and the incidental expenses, such as freight, cartage, insurance, etc. The simple term "cost" usually means the gross cost.

269. The **Gross Selling Price** is the total amount received for the goods sold.

270. The **Net Selling Price** is the amount of the gross selling price left after incidental expenses of the sale have been deducted, such as freight, commission, insurance, etc.

271. **Profit** is the difference between the net selling price and the gross cost, *when the selling price exceeds the cost.*

272. **Loss** is the difference between the net selling price and the gross cost, *when the cost exceeds the selling price.*

273. In the subject of profit and loss:

$$Base = Gross\ Cost$$
$$Rate = Rate$$
$$Percentage = Profit\ or\ Loss$$
$$Cost = 100\%$$

In solving problems in this subject, always use the easiest method.

274. *To Find Profit, Loss, or Selling Price:*

ILLUSTRATED SOLUTIONS

Problem: A farm costing \$4200 increased in value $8\frac{1}{3}\%$ when it was sold. Find the profit and the selling price.

\$4200
$.08\frac{1}{3}$
1400
33600
\$350.00 Profit
4200.00
\$4550.00 Selling Price

Ans. \$350.
\$4550.

The cost is the standard of value, 100%. Then \$4200 is 100%. Eight and one-third per cent may be found, as explained in par. 255, to be \$350. If the cost is \$4200 and the profit is \$350, the selling price must be the sum of the two, \$4550.

Or,

$8\frac{1}{3}\% = \frac{1}{12}$

12)4200

350 Profit

4550 Selling Price

Ans. { $350. / $4550.

Since $8\frac{1}{3}\%$ is $\frac{1}{12}$, the profit is $\frac{1}{12}$ of the cost. One-twelfth of $4200 is $350. Added to the cost, the amount is $4550.

275. *Problem:* 1200 bushels of wheat were purchased at 75 cents per bushel and later sold at a loss of 17%. What was lost, and for what was the wheat sold?

1200 bu. @ $.75 = $900

$900

.17

$153.00 Loss

1200 bushels of wheat at $.75 cost $900 (found by the use of aliquot parts, par. 241). 17% of $900 (par. 255) equals $153.

$900

153

$747 Selling Price

Ans. { $153. / $747.

Since the goods cost $900 and are sold for $153 less than cost, they will be sold for $900 minus $153, which is $747.

NOTE. For practice problems in finding the profit or loss and selling price see par. 37.

276. *To Find Cost:*

ILLUSTRATED SOLUTIONS

Problem: My profit on a certain transaction, figured at $12\frac{1}{2}\%$, would be $720. What was the cost of the transaction?

```
      5 760|
.125)720.000|
     625
      95 0
      87 5
       7 50
       7 50   Ans. $5760.
```

In this problem, the rate is $12\frac{1}{2}\%$ and the percentage \$720. The base equals the cost, which may be found as explained in par. 259 or 260.

Or,

$12\frac{1}{2}\% = \frac{1}{8}$
\$720
8
\$5760 Cost

Ans. \$5760.

Twelve and one-half per cent equals $\frac{1}{8}$. Since $\frac{1}{8}$ is gained and the gain is \$720, then \$720 is $\frac{1}{8}$ of the cost. Therefore, the cost would be 8 times \$720.

277. *Problem :* I bought goods and afterwards sold them at a loss of 23%, receiving \$412.80 for them. What did they cost?

100%
23%
77%

```
      5 36|103
.77)412.80|000
    385
     27 8
     23 1
      4 70
      4 62
        8 0
        7 7
          300
          231
           69   Ans. $536.10.
```

The base, or cost, is 100%, from which 23% is lost, leaving 77%, the measure of the value for which the goods were sold. Then \$412.80 equals 77% of the cost, which may be found as explained in pars. 259, 260, 261.

278. *Problem:* Goods are sold for \$126, which shows a loss of $11\frac{1}{9}\%$. What did they cost?

$$\begin{array}{r} 100 \\ \underline{11\frac{1}{9}} \\ 88\frac{8}{9} \end{array}$$

$126 = .88\frac{8}{9}$

$$\begin{array}{r} \$141|75 \\ 8.\not{0}\not{0})\overline{1134|00} \end{array}$$

Ans. \$141.75.

One hundred per cent is the standard of measure, or cost; $11\frac{1}{9}\%$ has been lost. The goods then sold for $88\frac{8}{9}\%$ of the cost. The cost may be found as explained in par. 261.

Or,

$$\begin{array}{r} 8)\underline{126.00} \\ 15.75 \\ \underline{9} \\ 141.75 \end{array}$$

Ans. \$141.75.

Eleven and one-ninth per cent is equal to the common fraction $\frac{1}{9}$. If $\frac{1}{9}$ is lost, the goods are sold for $\frac{8}{9}$. Therefore, \$126 is $\frac{8}{9}$ of the cost. If \$126 is $\frac{8}{9}$ of the cost, $\frac{1}{9}$ of the cost would be found by dividing \$126 by 8, which is \$15.57. If \$15.75 is $\frac{1}{9}$ of the cost, the whole cost would be 9 times \$15.75, which is \$141.75.

NOTE. For practice problems in finding the cost see par. 38.

279. *To Find Rate of Profit or Loss:*

ILLUSTRATED SOLUTIONS

Problem: Goods costing \$723.45 are sold so as to gain \$241.15. What is the gain per cent?

$$\frac{\overset{1}{\cancel{241.15}}}{\underset{3}{\cancel{723.45}}} = \frac{1}{3} = 33\tfrac{1}{3}\%$$

Ans. $33\frac{1}{3}\%$.

The cost, or base, is \$723.45. The gain, or percentage, is \$241.15. Find the rate as explained in par. 262.

Or,

$$\begin{array}{r} |33\frac{1}{3} \\ 723.45)\overline{241.15|00} \\ \underline{217\ 03\ 5} \\ 24\ 11\ 50 \\ \underline{21\ 70\ 35} \\ 2\ 41\ 15 \end{array}$$

Ans. $33\frac{1}{3}\%$.

Use the fractional method when its reduction may be determined at a glance; use the decimal method when this is not the case.

280. *Problem :* Goods that cost \$414 are sold for \$492.66. What per cent is gained?

\$492.66
414.
\$ 78.66

|19
414)78|66
41 4
37 26
37 26

If the goods cost \$414 and sell for \$492.66, the difference, \$78.66, must be gain or profit. The problem then is: \$78.66 is what per cent of \$414, the cost? This can be ascertained as explained in par. 262.

Ans. 19%.

NOTE. For practice problems in finding the per cent of gain or loss see par. 39.

For general problems in profit and loss see par. 40.

DISCOUNT

281. A Discount is an amount deducted from a sum owed by one person to another. In measuring discounts the principles of percentage are used. There are two kinds of discount, trade discount and time discount.

282. Trade Discount is the discount allowed by a manufacturer or jobber to a retail dealer.

283. Time Discount is a discount allowed as a consideration for paying an amount during a certain time.

TRADE DISCOUNT

284. Manufacturers, wholesalers, and others doing business of a similar nature, and handling goods the value of which is likely to fluctuate from time to time, have a fixed list price for their goods. These list prices remain fixed, and fluctuations in market rates are met by allowing different discounts from time to time; that is, if the value drops, the discount is made larger, and if the price of the goods rises, the discount is made smaller.

285. The **List Price** of goods is the price at which they are listed in the catalogue and from which discounts are allowed.

Goods are always billed at the list price and then the discount is deducted from the total.

286. The **Gross Amount** of the bill is the total amount before any discounts have been deducted.

287. The **Net Amount** is the amount to be paid after all discounts have been deducted.

288. The **Discount** is the sum deducted from the gross amount.

289. **A Discount Series** is several discounts deducted one after another; as, 25%, 10%, and 5%.

The first discount is deducted from the gross amount; the second from the remainder, and so on; the final remainder being the net amount.

290. The principles of percentage are used in performing all operations in discount.

$$\begin{aligned} Base &= Gross\ Amount \\ Rate &= Rate \\ Percentage &= Discount \\ Difference &= Net\ Amount \\ Gross\ Amount &= 100\% \end{aligned}$$

291. *To Find the Discount or Net Amount:*

Always use the easiest possible method.

ILLUSTRATED SOLUTIONS

Problem: Goods listed at \$420.34 are sold at a discount of 28%. What is the discount and what is the net amount of the bill?

$420.34
.28
3362 72
8406 8
$117.69 52 Discount
$420.34
117.70
$302.64 Net Amount

Ans. { $117.70. $302.64.

The gross amount of the bill, $420.34, is 100%. Find 28% of this by the easiest method. This will give $117.70. The difference will equal the net amount, $302.64.

292. *Problem :* A certain line of goods is sold at a discount of 25%, 20%, 10%, and 5%. What would be the net amount of a bill of $1214.43, purchased under these terms? What would the discount amount to?

$1214.43
303.60¾ 25%
910.82¼
182.16¼ 20%
728.66
72.87 10%
655.79
32.78 5%
$623.01 Net Amount
$1214.43 Gross Amount
623.01 Net Amount
$ 591.42 Discount

Ans. { $623.01. $591.42.

This problem involves a discount series. The first discount is deducted from the gross amount. Twenty-five per cent of $1214.43 is $303.61; deducted from the gross amount leaves $910.82. Twenty per cent of this, found in the easiest way, is $182.16; deducted, leaves $728.66. Proceeding in the same way, deducting 10% and then 5%, the net amount of the bill is $623.01. If the net amount of the bill is $623.01 and the gross amount is $1214.43, the difference must be the discount, $591.42.

NOTE. For practice problems in finding the net amount see par. 41.

293. *To Find a Single Rate of Discount Equal to a Discount Series:*

ILLUSTRATED SOLUTION

Problem: What single discount is equal to a discount series of 25%, 20%, 10%, and 5%?

4)100%
25
5) 75
15
10) 60
6
20) 54
2.7
51.3
100
51.3
48.7%

Ans. 48.7%.

One hundred per cent represents the gross amount of the bill. The first discount is 25% or $\frac{1}{4}$ of this. One-fourth of 100% equals 25%. Deduct the first discount (25%) from 100%. This leaves 75% from which to deduct the second discount. The second discount is 20% or one-fifth. One-fifth of 75% equals 15%. Deduct this from 75%. This leaves 60%. The next discount in the series is 10% or one-tenth. One-tenth of 60% equals 6%. 60% minus 6% equals 54%. The last discount must then be deducted from 54%. The last discount is 5% or one-twentieth. One-twentieth of 54% equals 2.7%. 54% minus 2.7% equals 51.3%. This is the net amount to be paid. Then the discount is the difference between the gross amount (100%) and the net amount (51.3%). This is 48.7%.

NOTE. For practice problems in finding a single rate of discount equal to a discount series see par. 42.

294. *To Find the Gross Amount or the List Price:*

ILLUSTRATED SOLUTIONS

Problem: On a bill of goods sold at a discount of $33\frac{1}{3}$% the discount equals $55.40. What is the gross amount of the bill?

$33\frac{1}{3}\%$ of ? = \$55.40

\$55.40 $33\frac{1}{3}\%$
3
\$166.20 100%

The discount, $33\frac{1}{3}\%$, is \$55.40. If \$55.40 is $33\frac{1}{3}\%$, 100% found as explained in par. 259, is \$166.20.

Ans. \$166.20.

295. *Problem :* A check for \$953.80 was given in full payment for a bill of goods bought at 24% discount. What would the gross amount of the bill be?

\$12 55
.76)953.80
76
193
152
41 8
38 0
3 80
3 80

The gross amount of the bill is 100% and the discount 24%. The net amount of the bill must be 76%. Therefore, \$952.80 equals 76% of the gross amount. Find the gross amount as explained in par. 259.

Ans. \$1255.

296. *Problem :* The net amount of a bill of goods sold at a discount of 40%, 30%, and 20% was \$114.24. Find the gross amount of the bill.

100%
40
60
18
42
8.4
33.6%

Goods sold at a discount of 40%, 30%, and 20% are sold at a net price which equals 33.6% of the original bill (par. 293). If \$114.24 equals 33.6%, 100%, the gross amount, may be found, as explained in par. 259, to be \$340.

$$
\begin{array}{r}
\$340 \\
.336\overline{)114.240} \\
100\ 8 \\
\hline
13\ 44 \\
13\ 44 \\
\hline
\end{array}
$$

Ans. $340.

NOTE. For practice problems in finding the gross amount or the list price see par. 43.

297. *To Find the Rate of Discount:*

ILLUSTRATED SOLUTIONS

Problem: The gross amount of a bill of goods is $346.40. The net amount is $259.80. What is the rate of discount?

$$
\begin{array}{r}
\$346.40 \\
259.80 \\
\hline
\$86.60
\end{array}
$$

$$
\begin{array}{r|l}
 & 25 \\
346.40\overline{)86.60} & 00 \\
69\ 28\ 0 & \\
\hline
17\ 32\ 00 & \\
17\ 32\ 00 & \\
\hline
\end{array}
$$

The difference between the gross amount and the net amount equals the discount, which is $86.60. The problem then is: $86.60 is what per cent of $346.40? This is found to be 25% (par. 262).

or

$$\frac{\cancel{8660}^{\,1}}{\cancel{34640}_{\,4}} = \frac{1}{4} \text{ or } 25\%.$$

Ans. 25%.

NOTE. For practice problems in finding the rate of discount see par. 44.

298. *To Find What Price to Mark Goods in Order to Allow a Certain Discount and Still Make a Certain Profit:*

ILLUSTRATED SOLUTION

Problem: What price must we mark goods costing $214 in order that we may allow a discount series of 20%, 10%, and 5% and still make a profit of 25%?

$$\begin{array}{r} \$214 \\ 53.50 \\ \hline \$267.50 \end{array}$$

$$\begin{array}{r} 100\% \\ 20 \\ \hline 80 \\ 8 \\ \hline 72 \\ 3.6 \\ \hline 68.4\% \end{array}$$

$$\begin{array}{r} \$391\,|\,08 \\ .684\overline{)267.500\,|\,00} \\ 205\ 2 \\ \hline 62\ 30 \\ 61\ 56 \\ \hline 740 \\ 684 \\ \hline 56\ 00 \\ 54\ 72 \\ \hline 1\ 28 \end{array}$$

By the principles of profit and loss, we see that to make a profit of 25%, goods costing $214 must be sold for $267.50 (par. 274). Then we must sell the goods for a net amount of $267.50. If a discount series of 20%, 10%, and 5% is to be allowed from the gross amount of the bill, the net amount of the bill will be 68.4% of the gross amount (par. 293). Therefore, $267.50 equals 68.4% of the gross amount, or asking price, which will be found, as explained in par. 259, to be $391.08.

Ans. $391.08.

NOTE. For practice problems in finding what price to mark goods in order to allow a certain discount and still make a certain profit see par. 45.

For general problems in trade discount see par. 46.

COMMISSION AND BROKERAGE

299. A **Commission Merchant** is a person or firm who buys or sells merchandise for another person or firm.

A commission merchant actually handles the goods and buys and sells them as if for himself, but in reality for another person or firm.

300. A **Broker** is a person or firm who arranges transactions between other persons.

A broker does not handle the merchandise himself, but simply brings the buyer and seller together in the interest of one or the other.

301. The **Principal** is the person or firm for whom the business is transacted.

302. The **Commission** is the compensation allowed the commission merchant or the broker.

303. The **Gross Proceeds** of a sale or collection is the entire amount received from the purchaser or debtor by the commission merchant.

304. The **Charges** are the incidental expenses of the sale or purchase.

305. The **Net Proceeds** is the amount remaining after the charges have been deducted from the gross proceeds. It is the amount to be returned by the commission merchant to his principal.

306. **Prime Cost** is the first cost of goods purchased by the commission merchant in the interest of his principal.

307. The **Gross Cost** is the prime cost plus the charges incidental to the purchase.

308. **Account Sales** is an itemized statement of sales of merchandise by a commission merchant. It shows the amount for which the goods were sold, the charges, and the net proceeds of the sale.

309. **Account Purchase** is an itemized statement covering the merchandise purchased by a commission merchant and shows the prime cost plus the charges.

310. In solving problems in commission, the general principles of percentage are used.

Base = *Gross Sales or Prime Cost*
Rate = *Rate of Commission*
Percentage = *Commission*
Net Proceeds = *Difference*
Gross Cost = *Amount*
Gross Sales or Prime Cost = 100%

311. *To Find the Commission and Net Proceeds or Gross Amount of Purchase:*

ILLUSTRATED SOLUTIONS

Problem: A commission merchant sold goods for $2346.40. His commission was $2\frac{1}{2}\%$. Charges for insurance, freight, etc., amounted to $98.40. Find the commission and net proceeds.

$2346.40	$2\frac{1}{2}\%$ Commission
58.66	
98.40	Charges
157.06	
$2189.34	Net Proceeds

Ans. { $58.66. / $2189.34.

Gross sales, 100%, is $2346.40. Two and one-half per cent of this, the commission, is $58.66. Since the commission merchant spent $98.40 for expenses and kept $58.66 for commission, the sum of these, $157.06, must be deducted from $2346.40, leaving a net proceeds of $2189.34.

312. *Problem :* A commission merchant bought 500 barrels of apples at $2.75 on a commission of 5% and paid $15 for cartage and $52.50 for cooperage. For what sum must his principal write a check to cover the entire transaction?

500 bbl. @ $2.75 = $1375	
$1375	
68.75	5%
15.	
52.50	
$1511.25	

Ans. $1511.25.

Five hundred barrels of apples at $2.75 cost $1375. The commission, 5%, equals $68.75. The principal will, therefore, have to send the commission merchant $1375 to pay for the goods, $68.75 for his commission, $15 to pay for cartage, and $52.50 for cooperage, or $1511.25 in all.

NOTE. For practice problems in finding the commission and net proceeds or gross amount of purchase, see pars. 47 and 48.

313. *To Find the Gross Sales or Net Purchase Price:*

ILLUSTRATED SOLUTIONS

Problem : A commission merchant working on $2\frac{1}{2}\%$ commission received $134.50 for selling a consignment of flour. What did the flour sell for?

$\$134.50 = 2\frac{1}{2}\%$

$\$5\,380$

$.025)\overline{134.500}$

125

9 5

7 5

2 00

2 00 *Ans.* $5380.

$134.50 is $2\frac{1}{2}\%$. Find 100% as explained in par. 259.

Or,

$2\frac{1}{2}\% = \frac{1}{40}$

$134.50 = \frac{1}{40}$

40

$5380.00 *Ans.* $5380.

314. *Problem :* The net proceeds is $568.40. The charges for freight, insurance, etc., are $27.40. The commission is 3%. For what were the goods sold?

$568.40

27.40

$595.80

6 14|22

$.97)\overline{595.80|00}$

582

13 8

9 7

4 10

3 88

22 0

19 4

2 60

1 94 *Ans.* $614.22.

The net proceeds is $568.40. Charges for freight, insurance, etc., are $27.40; added to $568.40 equals $595.80. This is the amount remaining from the sales after the commission alone has been deducted. If the commission is 3%, then $595.80 is 97%. Find the total amount of the sales as explained in par. 259, which is $614.22.

315. *Problem:* I sent a commission merchant $927 to invest in apples. How many barrels at $3.75 can be purchased after deducting a commission of 3%?

```
        9 00|
1.03)927.00|
      927
     ------

        2 40|
3.75)900.00|
      750
      -----
      150 0
      150 0
      -----
```

Ans. 240.

$927 includes the amount of purchase and the commission of 3%. Therefore, $927 is 103%. 100% may be found, as explained in par. 261, to be $900.

If one barrel of apples cost $3.75, for $900 we can buy as many barrels as $3.75 is contained in $900, which is 240.

Or,

3% of $3.75 = .1125, commission on 1 bbl.

$3.75	Cost of 1 bbl.
.1125	Commission on 1 bbl.
$3.8625	Gross cost of 1 bbl.

```
              240|
3.8625)927.0000|
        772 50
        -------
        154 500
        154 500
        -------
```

Ans. 240.

Since the market price of 1 bbl. is $3.75, the commission on one barrel would be 3% of $3.75 which is .1125, and the gross cost of one barrel would be $3.8625. For $927 the commission merchant could buy as many barrels as $3.8625 is contained in $927, which is 240.

NOTE. For practice problems in finding the gross sales or net purchase price see par. 49.

316. *To Find the Rate of Commission:*

ILLUSTRATED SOLUTION

Problem: A commission merchant charges $80.16 for selling a bill of goods for $1336. What is his rate of commission?

```
        |06
1336)80|16
     80 16
```

Ans. 6%.

Since the commission is figured as a certain per cent of the amount of sales, this problem really is: $80.16 is what per cent of $1336, which, solved as explained in par. 262, is .06 or 6%.

NOTE. For practice problems in finding the rate of commission see par. 50.

For general problems in commission see par. 51.

INTEREST

317. Interest is the amount paid for the use of money.

(*a*) When the use of real estate is allowed to someone other than the owner, the compensation is called *rent;* the compensation for the use of personal property is called *hire;* the compensation for the use of manual labor is called *wages;* the compensation for the use of mental labor and time is called *salary.*

(*b*) The amount of interest to be paid depends upon the time that the money is used, the sum that is used, and the way it is used (risk involved).

(*c*) The first two are self-fixing. The third is subject to agreement between the parties. If there is danger of the original sum being lost, a larger rate should be paid for its use. Also when money is plentiful and easy to hire, the rate should be lower than if it were scarce and hard to hire.

(*d*) It has been found that the best way of figuring interest is on a percentage basis. Therefore, the principles of percentage with another element, time, govern the subject of interest.

318. Principal is the sum for the use of which interest is charged.

319. Rate is the per cent of the principal charged for the use of the principal for one year.

320. The Legal Rate is the rate fixed by law to be understood when no rate is mentioned by the parties. It differs in different states. The legal rate in a majority of the states is 6%.

Charging more than a reasonable compensation is called usury. Some of the states name a definite maximum rate; in such states to charge more is usury. Where no maximum rate is fixed by law, it is a question for the courts to decide whether or not usury is charged in a given case.

321. **Time** is the period for which the principal is used.

322. **The Amount** is the sum of the principal and interest.

323. The interest on \$1 for one year at 6% would be 6% of \$1, which is \$.06. The interest on \$1240 for one year at 6% would be 6% of \$1240, which is \$74.40. The interest for one year, then, is always equal to the percentage of the principal represented by the rate.

324. The general principle on which all interest is based is:

$$\text{Principal} \times \text{Rate} \times \text{Time} = \text{Interest}$$

In figuring the interest for part of a year, two different methods arise: Accurate Method and Ordinary Method.

ACCURATE INTEREST

325. **Accurate Interest** gives a year, or any part of a year, its exact value.

(*a*) In computing the accurate interest for any part of a year the time is counted in days, and each day given its actual value, $\frac{1}{365}$ of a year.

(*b*) This method is used only in figuring interest on United States bonds, on foreign moneys, and by special agreement.

326. **Accurate Interest** is found by applying the general principle of interest with absolute accuracy.

ILLUSTRATED SOLUTIONS

327. *Problem :* Find the accurate interest of $1140 for 93 days at 6%.

$$\begin{array}{r} \$1140 \\ .06 \\ \hline \$68.40 \end{array}$$

Six per cent of $1140 equals $68.40. The interest on $1140 for one year is $68.40. The interest for 93 days equals $\frac{93}{365}$ of $68.40, which, found as explained in par. 214, is $17.43.

Find $\frac{93}{365}$ of $68.40.

$$\begin{array}{r} \$68.40 \\ 93 \\ \hline 205\ 20 \\ 6156\ 0 \\ \hline \$6361.20 \end{array}$$

$$\begin{array}{r|l} & \$17|427 \\ \hline 365) & 6361|200 \\ & 365 \\ \hline & 2711 \\ & 2555 \\ \hline & 156\ 2 \\ & 146\ 0 \\ \hline & 10\ 20 \\ & 7\ 30 \\ \hline & 2\ 900 \\ & 2\ 555 \\ \hline & 345 \end{array}$$

Ans. $17.43.

328. *Problem :* Find the accurate interest of $1140 for 458 days.

458 days = 1 year 93 days.

$$\begin{array}{r} \$68.40 \\ 17.43 \\ \hline \$85.83 \end{array}$$

Ans. $85.83.

This problem differs from the preceding one only in the matter of time. The interest for one year is $68.40. For 93 days the interest, found as in the previous problem, is $17.43. The sum of the two will equal the interest for 1 year and 93 days.

ORDINARY INTEREST

329. Ordinary Interest Method is that used by business men under ordinary circumstances.

It differs from exact interest simply in that one day is roughly considered one three-hundred-sixtieth of a year.

This is arrived at in this way: one-twelfth of a year is called one month, and one-thirtieth of a month is called a day. So that one day is one-thirtieth of one-twelfth, or one three-hundred-sixtieth of a year.

330. To figure the ordinary interest for parts of a year, business men sometimes count the time in months and days, considering each month to have thirty days; sometimes the time is figured in exact days.

The latter plan is always followed by banks. Some authorities divide ordinary interest into two classes, *Common* and *Bankers'*.

331. The best method of calculating ordinary interest is by what is known as the *Sixty-day Method.*

By this method the interest is *always found at* 6% *first* and then changed to the rate desired.

EXPLANATION

At 6%, the interest on any sum for one year would be six one-hundredths of the principal. One one-hundredth of the principal, then, would be the interest at 6% for one-sixth of a year. One-sixth of a year is 2 months, or 60 days. Then the interest on any principal for 60 days, or 2 months, at 6%, would be one one-hundredth of itself. Thus the interest at 6% on $2420 for 60 days is $24.20. For 12 days it would be one-fifth of the interest for 60 days, or $4.84, and

so on. The interest at 1% would be one-sixth of the interest at 6%, or $4.0333 for 60 days. The interest at $3\frac{1}{2}\%$ would be $3\frac{1}{2}$ times $4.0333, which is $14.11 for 60 days.

SIXTY-DAY METHOD — ORDINARY INTEREST RULE

332. Write the principal. Set off the interest for 2 months, or 60 days, at 6%. Using the interest for 60 days as a basis, the interest at 6% for any other period may be easily ascertained. When the interest for the desired period has been found at 6%, change to the rate desired.

Interest in partial results should always be carried to the fourth decimal place.

ILLUSTRATED SOLUTIONS

333. *Problem :* Find the interest on $1246.40 for 1 year 9 months 16 days at 6%; at 4%.

$1246.40	1 yr. 9 mo. 16 da. at 6%; at 4%
12.4640	Int. for 60 da. at 6%
74.7840	Int. for 1 yr. at 6%
49.8560	Int. for 8 mo. at 6%
6.2320	Int. for 1 mo. at 6%
2.0773	Int. for 10 da. at 6%
1.2464	Int. for 6 da. at 6%
6)134.1957	Int. for 1 yr. 9 mo. 16 da. at 6%
22.3659	Int. for 1 yr. 9 mo. 16 da. at 1%
4	
$89.4636	Int. for 1 yr. 9 mo. 16 da. at 4%

Set off the interest for 2 months or 60 days at 6%, \$12.4640. We now find the interest for 1 year, 8 months and 1 month, 10 days and 6 days. One year is 6 times 2 months. Eight months is 4 times 2 months. One month is one-half of 60 days. Ten days is one-sixth of 60 days. Six days is one-tenth of 60 days.

12.4640 × 6 = \$74.7840, which is interest for 1 yr. at 6%
12.4640 × 4 = \$49.8560, which is interest for 8 mo. at 6%
12.4640 ÷ 2 = \$6.2320, which is interest for 1 mo. at 6%
12.4640 ÷ 6 = \$2.0773, which is interest for 10 da. at 6%
12.4640 ÷ 10 = \$1.2464, which is interest for 6 da. at 6%

Adding these partial results, we have a total of \$134.1957, which is the interest for 1 year 9 months 16 days at 6%. Dividing this by 6 gives us the interest at 1%, which is \$22.3659. Multiplying this by 4 gives \$89.4636, which is the interest for 1 year 9 months 16 days at 4%.

Or,

\$134.1957 at 6%
44.7319 at 2%
\$89.4638 at 4%

Divide the interest at 6% by 3, which gives \$44.7319, which is the interest at 2%. Subtract this from the interest at 6%. The difference is \$89.4638, the interest at 4%.

334. *Problem :* What is the interest on \$428.75 for 214 days at 6%?

\$428.75	214 da. at 6%
4.2875	Int. for 60 da. at 6%
12.8625	Int. for 180 da. at 6%
2.1437	Int. for 30 da. at 6%
.2143	Int. for 3 da. at 6%
.0714	Int. for 1 da. at 6%
\$15.2919	Int. for 214 da. at 6%

Ans. \$15.29.

In this problem, the most convenient periods of time would be 180-30-3-1 days. Set off the interest for 60 days, which is \$4.2875. One hundred eighty days is 3 times 60. Thirty days is one-half of 60. Three days is one-tenth of 30. One day is one-third of 3 days. Adding, gives us \$15.2919, the interest for 214 days at 6%.

NOTE. For practice problems in finding the interest see par. 54.

335. *Problem :* Find the interest on \$1234.28 from January 3, 1915, to August 1, 1916, at 6%.

1916	8	1	\$1234.28	1 yr. 6 mo. 28 da. at 6%
1915	1	3	12.3428	Int. for 60 da. at 6%
1	6	28	74.0568	Int. for 1 yr. at 6%
			37.0284	Int. for 6 mo. at 6%
			4.9368	Int. for 24 da. at 6%
			.8228	Int. for 4 da. at 6%
			\$116.8448	Int. for 1 yr. 6 mo. 28 da. at 6%

Ans. \$116.8448.

In this problem it is necessary to find the time between January 3, 1915, and August 1, 1916. This will be found (par. 238) to be 1 year 6 months and 28 days.

One year is 6 times 2 months, 6 months is 3 times 2 months, 24 days is 4 times 6 days (6 days being one-tenth of 60 days). 4 days is one-sixth of 24. The total is \$116.8448, which is the interest at 6% for 1 year 6 months and 28 days.

336. *Problem :* Find the interest on \$514.95 from January 18, 1915, to June 7, 1915, at $4\frac{1}{2}$%. Time computed in exact days.

Jan.	13	$514.95	140 da. at $4\frac{1}{2}\%$
Feb.	28	5.1495	Int. for 60 da. at 6%
Mar.	31	10.2990	Int. for 120 da. at 6%
Apr.	30	1.7165	Int. for 20 da. at 6%
May	31	4)12.0155	Int. for 140 da. at 6%
June	7	3.0038	Int. for 140 da. at $1\frac{1}{2}\%$
	140 days	$9.0117	Int. for 140 da. at $4\frac{1}{2}\%$

Ans. $9.0117.

Find the exact number of days between January 18, 1915, and June 7, 1915 (par. 239), which is 140 days. This consists of 120 days and 20 days. One hundred twenty days is twice 60 days. Twenty days is one-third of 60 days, making the total interest $12.0155 for 140 days at 6%. The difference between 6% and $4\frac{1}{2}\%$ is $1\frac{1}{2}\%$. One and one-half per cent is one-fourth of 6%. Divide $12.0155, the interest at 6%, by 4, which gives us $3.0038, the interest at $1\frac{1}{2}\%$. Subtract this from the interest at 6%, which leaves $9.0117, the interest at $4\frac{1}{2}\%$.

NOTE. For practice problems in finding the interest when the time has to be found either in exact days or by compound subtraction see par. 55.

SIXTY-DAY METHOD — ACCURATE INTEREST

337. The difference between ordinary interest and accurate interest for *any part* of a year is one seventy-third of the ordinary interest.

338. The interest for one year or any number of years is the percentage value of the rate and is the same for ordinary and accurate because it is the standard for both methods.

339. Ordinary interest for any part of a year is greater than accurate interest for the same time.

340. To change the ordinary interest for any part of a year to the accurate interest for the same time, divide the ordinary interest by 73. Subtract this from the ordinary interest.

341. *Problem :* Find the accurate interest on $246 for 73 days at 6%.

$240. 73 da. at 6%
2.40 Int. for 60 da. at 6%
.48 Int. for 12 da. at 6%
.04 Int. for 1 da. at 6%
73)2.92 Ordinary interest
.04
2.88 Accurate interest
Ans. $2.88.

First find the ordinary interest, as explained in par. 334, on $240 for 73 days at 6%, which is $2.92.

One seventy-third of $2.92 is $.04. The ordinary interest, therefore, is $.04 larger than the accurate interest, which is found by subtracting $.04 from $2.92.

342. *Problem :* Find the accurate interest for 419 days at $4\frac{1}{2}\%$ on $425.

419 days = 1 year and 54 days
$425. 1 yr. 54 da. at $4\frac{1}{2}\%$
4.25 Int. for 60 da. at 6%
2.125 Int. for 30 da. at 6%
1.700 Int. for 24 da. at 6%
73)3.825 Ordinary interest for 54 da.
.0523
3.7727 Accurate interest for 54 da. at 6%
25.50 Accurate interest for 1 yr. at 6%
4)29.2727
7.3181
$21.9546
Ans. $21.95.

This problem differs from the previous one in that the time is more than a year. Three hundred sixty-five days should be subtracted from 419, leaving 54 days more than a year, so that the full time is 1 year and 54 days.

Find the interest by the ordinary interest method for 54 days (par. 334). This is \$3.825 at 6%. Change this to accurate interest as explained above. The accurate interest for 54 days at 6% is \$3.7727.

Six per cent of \$425 gives the interest for one year (both accurate and ordinary). Six per cent of \$425 is \$25.50. Add this to the accurate interest for 54 days and we have the accurate interest for 1 year and 54 days, which is \$29.2727.

Dividing this by 4 gives the interest at $1\frac{1}{2}$%, which is \$7.3181. Deduct this from the accurate interest at 6% and we have \$21.9546, the accurate interest for 1 year and 54 days at $4\frac{1}{2}$%.

343. *Problem :* Find the accurate interest on £214 11*s* 9*d* for 115 days at 5%.

12)9	*d*	
	75*s*	
11		
20)11	75*s*	
	5875£	
£214.5875		115 da. at 5%
2.1458		Int. for 60 da. at 6%
2.1458		Int. for 60 da. at 6%
1.0729		Int. for 30 da. at 6%
.7152		Int. for 20 da. at 6%
.1788		Int. for 5 da. at 6%
73)£4.1127		Ordinary interest at 6%
.0563		
4.0564		Accurate interest at 6%
.6760		Accurate interest at 1%
£3.3804		Accurate interest at 5%

£3804	.608
20	12
7.6080*s*	7.296*d*

Ans. £3 7*s* 7*d*.

Reduce £214 11*s* 9*d* to pounds (par. 232), which is £214.5875. Find the ordinary interest (par. 334). Change to accurate interest by subtracting one seventy-third of itself (par. 341). This equals £4.0564, which is the accurate interest at 6%. Subtract one-sixth (par. 333). This equals accurate interest at 5%.

NOTE. For practice problems in accurate interest see par. 56.

344. The best combinations for finding the interest by the sixty-day method for from one to thirty days:

1 day $= \frac{1}{60}$ of 60, or $\frac{1}{6}$ of 6

2 days $= \frac{1}{30}$ of 60, or $\frac{1}{3}$ of 6

3 days $= \frac{1}{20}$ of 60, or $\frac{1}{2}$ of 6

4 days = 3 days + 1 day

5 days $= \frac{1}{12}$ of 60 days

6 days $= \frac{1}{10}$ of 60 days

7 days = 6 days + 1 day

8 days = 6 days + 2 days

9 days = 6 days + 3 days

10 days $= \frac{1}{6}$ of 60 days

11 days = 10 days + 1 day

12 days $= \frac{1}{5}$ of 60 days

13 days = 10 days + 3 days

14 days = 12 days + 2 days

15 days $= \frac{1}{4}$ of 60 days or $\frac{1}{2}$ of 30

16 days = 10 days + 6 days

17 days = 15 days + 2 days

18 days = 12 days + 6 days or 3 × 6

19 days = 15 days + 3 days + 1 day

20 days = $\frac{1}{3}$ of 60 days

21 days = 20 days + 1 day

22 days = 20 + 2

23 days = 20 + 3

24 days = 20 + 4 or 4 × 6

25 days = 20 + 5

26 days = 20 + 6

27 days = 24 + 3 ($\frac{1}{8}$ of 24)

28 days = 24 + 4 ($\frac{1}{6}$ of 24)

29 days = 24 + 5

30 days = $\frac{1}{2}$ of 60

345. *To Find the Interest at Common Rates Other Than 6%:*

$\frac{1}{2}\% = \frac{1}{12}$ of 6%	$2\% = \frac{1}{3}$ of 6%	$5\% = 6\% - 1\%$
$\frac{1}{4}\% = \frac{1}{2}$ of $\frac{1}{2}\%$	$2\frac{1}{2}\% = 2\% + \frac{1}{2}\%$	$7\% = 6\% + 1\%$
$\frac{3}{4}\% = \frac{1}{2} + \frac{1}{4}\%$	$3\% = \frac{1}{2}$ of 6%	$8\% = 6\% + 2\%$
$1\% = \frac{1}{6}$ of 6%	$4\% = 6\% - 2\%$	$9\% = 6\% + 3\%$
$1\frac{1}{2}\% = \frac{1}{4}$ of 6%	$4\frac{1}{2}\% = 6\% - 1\frac{1}{2}\%$	$10\% = 10 \times 1\%$

346. *To Find the Time:* Divide the given interest by the interest on the principal for one year, at the given rate.

(*a*) The divisor should be carried to the fourth decimal place if necessary.

(*b*) The quotient should be carried to the fourth decimal place.

(*c*) In the final result, more than half should be considered another day.

ILLUSTRATED SOLUTIONS

347. *Problem:* In what time will $532.56 produce $48, interest at 5%?

```
         $532.56
             .05
        $26.6280

                1|8026
  26.628)48.000|0000
          26 628|
          21 372 0
          21 302 4
              69 600
              53 256
              16 3440
              15 9768
                 3672

   1.8026 yr.
       12
   9.6312 mo.
       30
   18.9360 da.
```

Ans. 1 yr. 9 mo. 19 da.

Five per cent of $532.56 is the interest on $532.56 for one year. It is $26.6280.

If $532.56 earns $26.6280 in one year, it will take as many years to earn $48 as $26.6280 is contained in $48, which is 1.8026 years.

Reducing 1.8026 years to years, months, and days (par. 231), we have 1 year 9 months and 19 days.

348. *Problem :* In what time will \$560 amount to \$625.71 at 6% interest?

\$625.71 — 560. = \$ 65.71

\$560 × .06 = \$33.60

```
                1|9556
33.60)65.71|0000
      33 60|
      32 11 0
      30 24 0
       1 87 00
       1 68 00
         19 000
         16 800
          2 2000
          2 0160
            1840
```

1.9556 yr.
× 12
11.4672 mo.
× 30
14.0160 da.

First find the interest by deducting the principal, \$560, from the amount, \$625.71. This shows that the interest is \$65.71.

The interest at 6% on \$560 for one year is \$33.60.

Proceeding as in the previous illustrated problem, we find the time required to be 1.9556 years. Reduced to years, months, and days, this equals 1 year 11 months and 14 days (par. 231).

Ans. 1 yr. 11 mo. 14 da.

NOTE. For practice problems in finding the time see par. 57.

349. *To Find the Rate:* Divide the interest by the interest on the principal for the given time at 1%.

The divisor should be carried to the fourth decimal place if necessary.

ILLUSTRATED SOLUTIONS

350. *Problem :* At what rate will \$475 earn \$8.84 in 134 days?

\$475.	134 da.
4.75	Int. for 60 da. at 6%
9.50	Int. for 120 da. at 6%
.95	Int. for 12 da. at 6%
.1583	Int. for 2 da. at 6%
6)10.6083	Int. for 134 da. at 6%
\$1.7680	Int. for 134 da. at 1%

5
1.768)8.840
8.840

Ans. 5%.

The interest on \$475 for 134 days at 1% is \$1.7680. The interest at 1% is contained in the given interest 5 times; therefore the interest must be reckoned at 5% in order to produce \$8.84 in 134 days.

NOTE. For practice problems in finding the rate see par. 58.

351. *To Find the Principal:* Divide the *Given Interest* by the *Interest* on one dollar for the given time at the given rate.

352. Divide the *Given Amount* by the *Amount* of one dollar for the given time at the given rate.

The divisor must be absolutely correct. The interest on \$1 should be carried to three places and *all fractions must be retained.*

ILLUSTRATED SOLUTIONS

353. *Problem:* What principal will be required to earn \$152.50 in 1 year 7 months 20 days at 8%?

1.

.01	Int. for 2 mo. at 6%
.06	Int. for 1 yr. at 6%
.03	Int. for 6 mo. at 6%
.005	Int. for 1 mo. at 6%
.003$\frac{1}{3}$	Int. for 20 da. at 6%
3).098$\frac{1}{3}$	Int. for 1 yr. 7 mo. 20 da. at 6%
.032$\frac{7}{9}$	Int. for 1 yr. 7 mo. 20 da. at 2%
\$.131$\frac{1}{9}$	Int. for 1 yr. 7 mo. 20 da. at 8%

.131$\frac{1}{9}$)152.50

\$11 63|135

1.18Ø)1372.50|000

118

192

118

74 5

70 8

3 70

3 54

16 0

11 8

4 20

3 54

660

590

70

First find the interest on \$1 for 1 year 7 months and 20 days at 8% by the sixty-day method, *retaining all fractions* (par. 333). This is \$.131$\frac{1}{9}$.

If \$1 produces \$.131$\frac{1}{9}$ in 1 year 7 months and 20 days at 8%, it will take as many dollars to produce \$152.50 as \$.131$\frac{1}{9}$ is contained in \$152.50. Performing this division (par. 224), we find that \$1163.14 is the principal required.

Ans. \$1163.14.

354. *Problem :* What principal will amount to \$1250 in 287 days at 6%?

1.

.01	Int. for 60 da. at 6%
.04	Int. for 240 da. at 6%
.005	Int. for 30 da. at 6%
.002½	Int. for 15 da. at 6%
.000⅓	Int. for 2 da. at 6%
1.047⅚	Amount on \$1 for 287 da. at 6%

1.047⅚)1250.

\$1 192|937
6.287)7500.000|000
6287
1213 0
628 7
584 30
565 83
18 470
12 574
5 896 0
5 658 3
237 70
188 61
49 090
44 009
5 081

First find the interest on \$1 for 287 days at 6%. This is \$.047⅚. Add this to \$1 and we find that \$1 will amount to \$1.047⅚.

It will take as many dollars to amount to \$1250 as \$1.047⅚ is contained in \$1250, which is \$1192.94.

Ans. \$1192.94.

355. The work of finding the interest on \$1 at 6% may be simplified by using the following rule:

Multiply the number of years by 6. Call the result cents.

Divide the number of months by 2. Call the result cents.

Divide the number of days by 6. Call the result mills, or tenths of a cent.

356. Applying this rule in the above illustrated solutions we would have:

In the first: Find the interest on \$1 for 1 year 7 months 20 days at 6%, thus:

$6 \times 1 = 6$ written .06
$7 \div 2 = 3\frac{1}{2}$ written .035
$20 \div 6 = 3\frac{1}{3}$ written $\underline{.033\frac{1}{3}}$
$.128\frac{1}{3}$ = Int. on \$1 at 6%

In the second: To find the interest on \$1 for 287 days at 6%, thus:

$287 \div 6 = 47\frac{5}{6}$ written $.047\frac{5}{6}$ = Int. on \$1 at 6%

NOTE. For practice problems in finding the principal see par. 59. For general problems in interest see par. 60.

COMMERCIAL PAPERS

357. **Commercial Papers** comprise notes, drafts, and checks.

358. A **Note** is a written promise of one party to pay a second party a certain sum at a certain time.

There are two parties to a note.

359. The **Maker** is the party who promises to pay.

360. The **Payee** is the party to whom, or to whose order, payment is to be made.

361. A **Draft** is a written order of one party telling a second party to pay a third party a certain sum at a certain time.

There are three parties to a draft.

362. The **Drawer** is the party requesting payment.

363. The **Drawee** is the party to whom this request is addressed: the party told to pay.

364. The **Payee** is the party to whom, or to whose order, payment is to be made.

365. A **Sight Draft** is a draft which by its terms is to be paid by the drawee immediately upon its presentation to him.

(*a*) In some states, three days, called *days of grace*, are allowed on sight drafts.

366. A **Time Draft** is a draft payable at a certain time after presentation to the drawee, or after date.

367. The **Date of Maturity** of either a note or a draft is the date upon which the payment is due.

(*a*) To fix the maturity of a time draft due "after sight," it is necessary to present the draft to the drawee to see if he is willing to pay it. If he is willing to pay it, he so indicates by writing the word "accepted," together with the date, over his signature, across the face.

(*b*) The maturity of a note is ascertained by reckoning the specified time from date of the note.

(*c*) The maturity of a draft drawn "after sight" is ascertained by reckoning the specified time from the date of acceptance.

(*d*) The maturity of a draft drawn "after date" is ascertained by reckoning the specified time from the date of the draft.

368. The **Face** of a note or draft is the amount of money mentioned in it.

369. The **Amount** due at maturity is the sum that is to be paid. This may be the face, or it may be the face plus interest.

370. A **Check** is an order by one who has funds on deposit in a bank, telling the bank to pay a certain sum from this deposit to a certain party. Checks are treated as cash.

(*a*) The drawer of a check is the depositor.

(*b*) The drawee of a check is the bank where the funds are on deposit.

(*c*) The payee of a check is the party in whose favor the check is made: The party to whom funds from the deposit are to be paid by the bank.

PARTIAL PAYMENTS

371. It is sometimes necessary to make a part payment on a promissory note or other obligation. It is usually customary in such cases to cancel the original note and issue another for the reduced amount. When it is not feasible to do this, the amount of the part payment, together with the date, is indorsed on the back of the original instrument.

Various rules are in use for finding the balance due on obligations upon which part payments have been indorsed.

The more important of these are the United States Rule and the Merchants' Rule. The United States Rule has been sanctioned by the Supreme Court of the United States. The Merchants' Rule is used by most bankers and business men because of its simplicity.

THE UNITED STATES RULE FOR PARTIAL PAYMENTS

372. *To Find the Balance Due at a Given Time:* Find the interest on the principal from the date of the instrument to the date of the first payment. If this interest is *less* than the first payment, add the interest to the original principal and subtract the payment from this amount. Treat the remainder as

a new principal and proceed as before, so continuing until the date of settlement is reached.

If at any time the interest is *greater* than the payment, the interest should be disregarded and the interest found to such date as the sum of the payments exceeds the interest.

ILLUSTRATED SOLUTION

373. *Problem :* What is the balance due July 1, 1916, on a note of $1200, dated January 1, 1914, upon which the following payments have been made: June 24, 1914, $250; August 16, 1914, $100; July 8, 1915, $40; January 1, 1916, $300?

$1200	Face Jan. 1, 1914
34.60	Int. to June 24, 1914, 5 mo. 23 da.
$1234.60	
250.	1st payment
984.60	New principal June 24, 1914
8.53	Int. to Aug. 16, 1914, 1 mo. 22 da.
993.13	
100.	2d payment
893.13	New principal Aug. 16, 1914
73.68	Int. to Jan. 1, 1916, 1 yr. 4 mo. 15 da.
	(Third payment not equal to interest)
966.81	
340.	3d and 4th payments
626.81	New principal Jan. 1, 1916
18.80	Int. to July 1, 1916, 6 mo.
$645.61	

Ans. $645.61.

$1200 draws interest from January 1, 1914, to June 24, 1914. We find this time (par. 238) to be 5 months and 23 days. The interest on $1200 for 5 months and 23 days is $34.60 (par. 335), making the amount due on June 24, 1914, $1234.60, upon which $250 is paid, leaving the balance of $984.60. This is on interest from June 24, 1914, to August 16, 1914, a period of 1 month and 22 days. The interest on $984.60 for 1 month and 22 days is $8.53, making $993.13 due August 16, 1914. On this $100 is paid, leaving a balance of $893.13 to draw interest from August 16, 1914. The next payment is made on July 8, 1915. The interest on $893.13 from August 16, 1914, to July 8, 1915, is greater than the payment; therefore we disregard the interest at this time and find the interest to the date of the next payment, January 1, 1916. The time from August 16, 1914, to January 1, 1916, is 1 year 4 months and 15 days. The interest on $893.13 for 1 year 4 months and 15 days is $73.68, making $966.81 due. On this $40 was paid on July 6, 1915, and $300 on January 1, 1916, which is $340 in all, leaving $626.81 to draw interest from January 1, 1916, to the date of settlement, July 1, 1916, 6 months. The interest on $626.81 for 6 months is $18.80, making $645.61 due on July 1, 1916.

NOTE. For practice problems in partial payments (United States Rule) see par. 61.

MERCHANTS' RULE

374. *To Find the Balance Due at a Given Time:* Find the interest on the face of the obligation from the date at which it begins to draw interest to the date of final settlement. Add this interest to the face of the debt. Find the interest on each payment from the date of the payment to the date of final settlement. Add the payments and the interest on the payments. Subtract this sum from the amount of the principal and interest. The difference will be the balance due.

ILLUSTRATED SOLUTION

375. *Problem :* What is the balance due July 1, 1916, on a note of $1200, dated January 1, 1914, upon which the following payments have been made: June 24, 1914, $250; August 16, 1914, $100; July 8, 1915, $40; January 1, 1916, $300?

$1200	Face Jan. 1, 1914
180	Int. to July 1, 1916, 2 yr. 6 mo.
$1380	
$250.	Paid June 24, 1914
30.29	Int. to July 1, 1916, 2 yr. 7 da.
100.	Paid Aug. 16, 1914
11.25	Int. to July 1, 1916, 1 yr. 10 mo. 15 da.
40.	Paid July 8, 1915
2.35	Int. to July 1, 1916, 11 mo. 23 da.
300.	Paid Jan. 1, 1916
9.	Int. to July 1, 1916, 6 mo.
$742.89	
$1380	
742.89	
$637.11	Balance due July 1, 1919.

A $1200 note given January 1, 1914, would earn $180 interest to July 1, 1916, making its value at maturity $1380. $250 paid June 24, 1914, would earn $30.29, interest to July 1, 1916, a period of 2 years 7 days. $100 paid August 16, 1914, would earn $11.25 to July 1, 1916. $40 paid on July 8, 1915, would accrue $2.35 interest to July 16, 1916. $300 would accrue $9 interest to July 1, 1916. The payments and accrued interest amount to $742.89. The value of the note at maturity, $1380, minus the payments and accrued interest $742.89, leaves $637.11 due on July 1, 1916.

NOTE. For practice problems in partial payments (Merchants' Rule) see par. 62.

BANK DISCOUNT

376. Bank Discount is the charge made by a bank for cashing an obligation before it is legally due. It is the interest on the amount due at maturity for the unexpired time.

377. The **Maturity** of a debt is the date upon which it becomes legally due.

(*a*) A few states allow three days in addition to the time mentioned in a note or draft. These are called *days of grace.*

(*b*) Most states allow *days of grace* on sight drafts only.

378. The **Term of Discount** is the number of days between the date of discount and the date of maturity.

379. The **Bank Discount** is the interest on the amount due at maturity for the term of discount.

380. The **Proceeds** is the difference between the amount due at maturity and the bank discount. It is the cash value of the debt on the date of discount.

381. *To Find the Proceeds:* Find the date of maturity. Ascertain the amount due at maturity. Find the time in exact days from the date of discount to the date of maturity. Compute the interest on the amount due at maturity for this time. The result will be the bank discount. Deduct the bank discount from the amount due at maturity. The result will be the proceeds.

ILLUSTRATED SOLUTIONS

382. *Problem :* Find the bank discount and the proceeds of a note for 60 days for $5000, dated March 3, 1916, discounted April 1, 1916, at 5%.

March 3, 1916+60 days = May 2, 1916 = date of maturity.

April 1, 1916, to May 2, 1916 = 31 days = term of discount

$5000.	
50.	
25.	30 days
.8333	1 day
6)25.8333	6%
4.3055	1%
21.5278	5% Bank Discount

$5000.	
21.53	
$4978.47	Proceeds

Ans. $4978.47.

A note given March 3, 1916, for 60 days would fall due on May 2, 1916, which would be the date of maturity. The time from April 1, 1916, to May 2, 1916, is 31 days. This is the term of discount. The interest on $5000 for 31 days at 5% is $21.53. This is the bank discount. The face of the note was $5000, the bank discount $21.53; the net proceeds would be the difference, $4978.47.

NOTE. For practice problems in finding the bank discount and proceeds of non-interest-bearing notes see par. 63.

383. *Problem :* Find the bank discount and proceeds of a sixty-day note for $5000, dated January 1, 1915, bearing interest at 6%, discounted February 6, 1915, at 5%.

January 1, 1915+60 days = March 2, 1915

February 6, 1915, to March 2, 1915 = 24 days

$5000	Face
50	Interest
$5050	Due at Maturity
50.50	60 days
6)20.20	24 days at 6%
3.3666	
$16.8334	Bank Discount
$5050	
16.83	
$5033.17	Proceeds

Ans. $5033.17.

A note, dated January 1, 1915, to run 60 days would fall due on March 2, 1915. If it were discounted on February 6, 1915, it would then have 24 days to run. The term of discount, therefore, is 24 days. As this note is given with interest, its face value, plus 60 days' interest on $5000, is $5050. As the amount due at maturity is $5050, the bank discount would be figured on this amount for 24 days at 5%, which is $16.83. The amount due at maturity being $5050 and the bank discount $16.83, the net proceeds would be the difference, or $5033.17.

NOTE. For practice problems in finding the bank discount and proceeds of interest-bearing notes see par. 64.

384. In making a loan at a bank when a definite amount is desired, the note must be made for a sum that, when discounted, will leave as the net proceeds the amount of loan desired.

To Find the Sum for Which a Note Must Be Drawn So That, if Discounted at Date, the Proceeds Will Be a Given Sum: Find the proceeds of a note for $1 for the given time at the given rate. Divide the given proceeds by this. The quotient will be the face required.

The divisor must be absolutely correct. Carry the discount on $1 to the third place and *retain all fractions.*

ILLUSTRATED SOLUTION

385. *Problem:* For what sum must a ninety-day note be drawn so that if discounted on its date at $4\frac{1}{2}\%$ the proceeds may be \$1875?

\$1.00	
.01	60 days
.005	30 days
4).015	6%
$.003\frac{3}{4}$	$1\frac{1}{2}\%$
$.011\frac{1}{4}$	Bank Discount on \$1
$.988\frac{3}{4}$	Proceeds on \$1

$\$1875 \div .988\frac{3}{4}$

```
      1 896|333
3.955)7500.000|000
      3955
      3545 0
      3164 0
       381 00
       355 95
        25 050
        23 730
         1 320 0
         1 186 5
           133 50
           118 65
            14 850
            11 865
             2 985
```

The bank discount on a note of \$1 for 90 days at $4\frac{1}{2}\%$ would be $\$.011\frac{1}{4}$. The net proceeds of \$1 would be $\$.988\frac{3}{4}$. If \$1 yields proceeds of $\$.988\frac{3}{4}$, it would take as many dollars to yield \$1875 as $\$.988\frac{3}{4}$ is contained in \$1875, which is 1896.33, the face of the note required.

Ans. \$1896.33.

NOTE. For practice problems in finding the sum for which a note must be drawn so that if discounted at date the proceeds will be a given sum, see par. 65.

COMPOUND INTEREST

386. Compound Interest is interest on the principal and interest combined, as fast as the interest falls due.

(*a*) Compound interest can only be charged by special agreement, and then care must be exercised that the laws of usury are not violated.

(*b*) Interest is usually compounded annually, semi-annually, or quarterly.

(*c*) Compound interest is little used except in savings banks.

387. *To Find Compound Interest:* Find the amount of the principal and interest at the end of the first interest period. Use this amount as a new principal for the next period, and so on. Deduct the original principal from the final amount. The difference will be the compound interest.

ILLUSTRATED SOLUTION

388. *Problem:* Find the compound interest on $1400 from March 8, 1912, to June 15, 1916, at 5%, interest compounded annually.

1916	6	15
1912	3	8
4	3	7 = Four full periods and 3 mo. 7 da. extra

$1400	Original Principal
70	Int. at 5% for 1st period
$1470	New Principal
73.50	Int. at 5% for 2d period
$1543.50	New Principal
77.18	Int. at 5% for 3d period
$1620.68	New Principal
81.03	Int. at 5% for 4th period
$1701.71	New Principal
17.0171	Int. for 2 mo. at 6%
17.0171	Int. for 2 mo. at 6%
8.5085	Int. for 1 mo. at 6%
1.7017	Int. for 6 da. at 6%
.2836	Int. for 1 da. at 6%
6)27.5109	Int. for 3 mo. 7 da. at 6%
4.5851	Int. for 3 mo. 7 da. at 1%
22.9258	Int. for 3 mo. 7 da. at 5%
$1724.6358	Final Amount
1400	
$324.6358	Compound Int. *Ans.* $324.64.

NOTE. For practice problems in compound interest see par. 66.

PERIODIC OR ANNUAL INTEREST

389. Periodic Interest, often called *annual interest*, is interest on the principal and interest on each overdue payment of interest.

(*a*) It is the result of a business custom in certain lines, and is merely an application of the general principles of simple interest.

(*b*) It is not entitled to be considered as a separate arithmetical subdivision, and is only so treated in this book because many authors have seen fit to introduce it as a separate kind of interest.

(*c*) It has no legal status.

ILLUSTRATED SOLUTIONS

390. *Problem :* $1400 was loaned on January 8, 1915, for 2 years at 6%, interest payable semi-annually; each installment of interest to draw interest at 6% from its due date until paid. What sum would be required to cancel the debt and all interest on January 8, 1917, nothing having been paid previously?

$1400.	
14.	Int. for 2 mo. at 6%
$42.	Int. for 6 mo. at 6%
4.	
$168.	Int. for 2 yr. at 6%

1 yr.	6 mo.	— 1st installment overdue
1 yr.		— 2d " "
	6 mo.	— 3d " "
2 yr.	12 mo.	= 3 yr.

$42.	
.42	Int. on interest for 2 mo. at 6%
2.52	Int. on interest for 1 yr. at 6%
5.04	Int. on interest for 2 yr. at 6%
7.56	Int. on interest for 3 yr. at 6%
168.	Int. on the Principal
1400.	Principal
$1575.56	Amount due Jan. 8, 1917

Ans. $1575.56.

The interest on $1400 for 6 months at 6% is $42. Then $42 should be paid every 6 months. There would be four such payments due in 2 years. $4 \times 42 = \$168$, which must be paid as interest

on the principal. The first installment of this, amounting to $42, was due on July 8, 1915. This is overdue 1 year 6 months. The next installment of $42 is overdue 1 year; the next, 6 months; and the last is just due. Besides the interest on the principal, then, there is interest on the interest due. This last is the interest on $42 for 1 year 6 months, and for 1 year, and for 6 months. In other words, the interest on the interest equals the interest on $42 for 3 years, which is $7.56. Adding interest on interest, interest on principal, and principal gives $1575.56, the amount due at maturity.

NOTE. For practice problems in periodic interest see par. 67.

AVERAGING ACCOUNTS

391. Averaging of Accounts is the process of ascertaining the date on which an account may be paid without loss of interest to either the debtor or the creditor.

Averaging of accounts has been abandoned by most lines of business. It is now the general custom to settle each item, separately, at maturity. The subject, however, is not entirely obsolete.

392. Cash Balance is the amount of cash required to settle an account without loss of interest to either party on any date other than the average due date.

GENERAL PRINCIPLES OF AVERAGE

393. If an account is paid before it is due, the payer loses the interest on the sum paid, and the receiver gains it.

394. If an account is paid after it is due, the payer gains the interest on the money paid and the receiver loses it.

395. The average due date of several items, due at different dates, is the date when the payer's losses of interest and gains of interest would be equal, or within, one half day's interest of being equal.

396. *To Average an Account:* Assume a date of settlement. Find the net gain or loss of interest by paying on that date. Find how long it will take the amount to be paid to earn this interest. Count that time forward or backward from the assumed date according to whether the payer would lose or gain.

(*a*) The assumed date is called the focal date.

(*b*) The focal date may be any date.

(*c*) The easiest focal date to use is the *zero date* of the earliest month in which any one of the items is due; thus, if items are due June 8, July 15, and August 9, the easiest focal date to use would be June 0, which is, in reality, May 31.

ILLUSTRATED SOLUTIONS

397. *Problem :* Average the following:

CHARLES S. CHASE

1916			
Jan.	5	$434.	
	27	123.50	
Feb.	8	215.	
Apr.	9	310.65	

Focal date Jan. 0.

Jan.	5	434.	5 da.	.3616
	27	123.50	27 da	.5557
Feb.	8	215.	39 da.	1.3975
Apr.	9	310.65	99 da.	5.1256
		1083.15		7.4404

6)1.0831	Int. for 6 da.	.1805)7.4404	41
.1805	Int. for 1 da.	7 220	
		2204	
		1805	
		399	

Ans. Jan. 0+41 = Feb. 10.

Assuming January 0 as the focal date, on the first item, $434, Chase would lose 5 days' interest, because he would pay it 5 days before it was due. The interest on $434 for 5 days is $.3616. By paying $123.50 on January 0, the interest for 27 days would be lost, which would be $.5557. By paying $215, due February 8, on January 0, the interest for 39 days would be lost, which would equal $1.3975. If $310.65 is paid 99 days before it is due, the interest lost would be $5.1256. Then by paying $1083.15 on January 0, $7.4404 interest would be lost. The interest on $1083.15 for one day is $.1805. It will take as many days to earn $7.4404 as $.1805 is contained in $7.4404, which is 41 days. Then Chase should pay the money 41 days later than January 0, which is February 10.

NOTE. For practice problems in averaging accounts see par. 68.

398. *Problem :* Average the following:

Find cash balance on Jan. 1, 1916.

1915			
Sept. 4	$625	for 2 mo.	Credit
Oct. 23	350	for 30 da.	Credit
Nov. 18	215	for 10 da.	Credit
Dec. 8	643	for 60 da.	Credit

Sept. 4	2 mo.	Nov. 4	$ 625	4 da.	$ .4166
Oct. 23	30 da.	Nov. 22	350	22 da.	1.2832
Nov. 18	10 da.	Nov. 28	215	28 da.	1.0032
Dec. 8	60 da.	Feb. 6	643	98 da.	10.5023
			$1833		$13.2053

Focal date Nov. 0.

6)1.833	Int. for 6 da.	.3055)13.2053	43
.3055	Int. for 1 da.	12 220	
		9853	
		9165	
		688	

Nov. 0+43 da. = Dec. 13.

Dec. 18	1833.	19 days at 6%	
Jan. 1	18.33	Int. for 60 da. at 6%	
19 days	4.5825	Int. for 15 da. at 6%	
	.9165	Int. for 3 da. at 6%	
	.3055	Int. for 1 da. at 6%	
	5.8045	Int. for 19 da. at 6%	
	1833		
	$1838.80	Cash Bal. Jan. 1, 1916	

Ans. { Dec. 13, 1915.
$1838.80.

Goods billed on September 4 for 2 months' credit would be due on November 4. A bill bought October 23 on 30 days' credit would be due November 22. A bill bought November 18 on 10 days' credit would be due November 28. A bill bought December 8 on 60 days' credit would be due on February 6. Assuming November 0 as the focal date and averaging as in par. 397, we find the average due date to be December 13, 1915.

If the debtor should pay $1833 on December 13, but did not do

so until January 1, he would owe $1833 plus 19 days' interest. The interest on $1833 for 19 days is $5.8045, making the amount due January 1, $1838.80.

NOTE. For practice problems in averaging accounts and finding the cash balance see par. 69.

399. *Problem :*

Dr.		C. E. BATCHELOR			*Cr.*
1915			1915		
May 8	30 days	$525.30	May 15	Cash	$300.
June 7	60 days	415.40	June 5	Cash	425.50

When is the above due by average?

What was the cash balance December 15, 1915?

May 8+30 da. = June 7	$525.30	38	$3.3269
June 7+60 da. = Aug. 6	415.40	98	6.7848
	940.70		10.1117
	725.50		3.303
	$215.20		$6.8087

May 15	$300	15	$.75
June 5	425.50	36	2.553
	$725.50		$3.303

Focal date May 0.

			190
6).2152	Int. for 6 da. at 6%	.0358)	6.8087
.0358	Int. for 1 da. at 6%		3 58
			3 228
			3 222
			67

May 0+190 days = Nov. 6, 1915.
Nov. 6 to Dec. 15 is 39 days.

$215.20	
2.15	Int. for 60 da. at 6%
1.076	Int. for 30 da. at 6%
.2152	Int. for 6 da. at 6%
.1076	Int. for 3 da. at 6%
1.3988	Int. for 39 da. at 6%
215.20	
$216.60	Cash Bal. Dec. 15, 1915

Ans. { Nov. 6, 1915.
$216.60.

In the above account C. E. Batchelor owes the items on the debit side of the account and we owe him, theoretically, the items on the credit side. The debit side being the larger, he owes us the balance of the account. First find the due dates of all the items. On the debit side $525.30 is due June 7. $415.40 is due August 6. On the credit side $300 is due May 15. $425.50 was due June 5. Assuming the zero date of the earliest month as the focal date, this would be May 0. Batchelor, by paying $525.30 on May 0, would lose 38 days' interest, which is $3.3269, and by paying $415.40 on May 0 he would lose 98 days' interest, which is $6.7848. If the account were settled on May 0, Batchelor would gain the interest on $300 for 15 days, which is $.75 and he would gain the interest on $425.50 for 36 days, which is $2.553. This would amount to $3.303. The balance of the account is $215.20. If Batchelor paid this on May 0, he would lose $10.1117 and gain $3.303, or he would make a net loss of $6.8087. The interest on $215.20 for one day is $.0358. To make up $6.8087, it would take as many days as $.0358 is contained in $6.8087, which is 190 times, or 190 days. Since by settling the account on May 0 the interest for 190 days is lost, it should be settled 190 days later, which would be November 6.

Whenever the balance of interest and balance of account fall on the same side, the payer will lose the interest and the time should then be counted *forward.*

400. *Problem :*

Dr. F. H. Bray *Cr.*

1916			1916		
June 6	2 mo.	$2200	Aug. 12	Cash	$ 108
Aug. 12	12 da.	1400	Sept. 5	Cash	2892

Find the cash balance for Sept. 12, 1916, at 6%. Focal date August 0.

Aug.	6	$2200	6	2.20	Aug.	12	$ 108	12	.216
	24	1400	24	5.60	Sept.	5	2892	36	17.352
		$3600		7.80			$3000		17.568
		3000							7.80
		$ 600							9.768

6)$.600$ Int. for 6 da. at 6%
$.10$ Int. for 1 da. at 6%

$$.10\overline{)9.76}\,\Big|\,\begin{matrix}97\\6\\8\end{matrix}$$

9 0
76
70
6 8

97.6 days = 98 days.

98 days counted backward from Aug. 0 = April 24, Average Date.

April 24 to Sept. 12 = 141 days.

600		$600.	Balance of account
6.		14.10	
12.	= 120	$614.10	Cash Bal. Sept. 12
2.	= 20		
.10	= 1		
14.10	= Int. for 141 da.		

Ans. $614.10.

The above problem is similar to 399, except that the balance of account and the balance of interest fall on opposite sides. This amount shows that F. H. Bray owes the balance of $600. By paying this balance on August 0 he would lose the interest on $2200 for 6 days and the interest on $1400 for 24 days, or $7.80 on both items, because he would be paying before due. However, his loss of interest on the items on the debit side of the account is more than offset by his gain of interest on the items on the credit side (the interest on $108 for 12 days and the interest on $2892 for 26 days) which is $17.568. The difference between $17.568 and $7.80 is $9.768, which is the amount of interest that Bray would gain by paying the balance on August 0. The balance, $600, requires 98 days to earn $9.768. Then Bray, in order to neither gain nor lose, should settle the account 98 days before August 0, which is April 24.

When the balance of account and the balance of interest fall on opposite sides, count the time *backward* from the focal date.

NOTE. For practice problems in finding the amount due by average and the cash balance of two-sided accounts see par. 70.

TAXES

401. A **Tax** is a sum of money levied upon a citizen or his property to meet the expenses of maintaining the Government.

(*a*) Taxes are levied to pay the expenses of the city, county, state, and United States.

(*b*) The first three are levied directly upon the person, property, and, in some cases, the income of the individual.

(*c*) In case of the United States, the tax is levied through duties and customs and by a tax on incomes.

402. A **Poll Tax** is a tax levied on the person, and in most states is assessed upon all male citizens of 20 years of age or more.

403. A **Property Tax** is a tax assessed on property, either real or personal, and is levied upon all persons owning taxable property, irrespective of age or sex.

404. An **Income Tax** is a tax upon incomes, and is levied alike on all citizens receiving certain incomes, regardless of age or sex.

Income taxes are levied by the United States Government and by some states, in accordance with laws passed by Congress or State Legislatures.

NOTE. For practice problems in taxes see par. 71.

DUTIES AND CUSTOMS

405. Duties and Customs are taxes assessed by the United States Government on imported merchandise.

406. An **Ad Valorem Duty** is a certain percentage of the net cost (value) of the importation.

407. A **Specific Duty** is a specified sum levied on each article, or on each unit of measure, regardless of the value.

(*a*) Ad valorem duties are not computed on fractions of a dollar. Cents are disregarded for less than 50 and are considered another dollar for more than 50.

(*b*) Some articles are subject to both ad valorem and specific duties.

(*c*) Specific duties are not computed on fractions of a unit. The long ton, or 2240 pounds, is used in computing specific duties.

408. A **Tariff** is a schedule showing the different rates of duties imposed by Congress on different articles.

409. A **Free List** is a schedule of articles upon which no duties are to be levied.

410. A **Customhouse** is a branch office of the Treasury Department of the United States Government.

Customhouses are established at various ports; each customhouse has jurisdiction over certain territory.

411. A **Port of Entry** is a port where a customhouse is established.

412. All ports, whether of entry or otherwise, are called *ports of delivery*.

413. The **Customhouse Business** is distributed among three departments.

414. The **Collector's Office** takes charge of entries and papers, issues permits, and collects the duties.

415. The **Surveyor's Office** takes charge of the vessels and cargoes, receives the permits, ascertains the quantities, and delivers the merchandise to the importer.

416. The **Appraiser's Office** examines the merchandise and determines the value and rate of duty on the goods.

417. **Internal Revenue** is a revenue raised by the Government by placing duties on such articles of luxury as may be determined by Congress. These duties are collected by the Treasury Department and vary from time to time according to the needs of the country.

418. A **Manifest** is a memorandum, signed by the master of the vessel, showing the name of the vessel, its cargo, and the names and addresses of the consignors and consignees.

419. An **Invoice** is a detailed statement showing the items and value of the goods imported and is made out in the weights and measures of the country of export.

The values of foreign moneys are periodically proclaimed by the Secretary of the Treasury, and these values must be taken in estimating duties. See par. 468.

420. A Bonded Warehouse is a warehouse provided for the storage of goods upon which duties have not yet been paid.

(*a*) Any importer may deposit goods in the warehouse by giving bond for the payment of duties on the goods thus stored.

(*b*) On goods remaining in bond more than a year, 10% additional duty is charged.

(*c*) Goods left in a government warehouse for three years are forfeited to the Government and sold at auction.

(*d*) Goods may be withdrawn from a warehouse for export without payment of duty.

421. An Excise Duty is a tax levied upon goods produced and consumed in the United States.

In this class come taxes upon tobacco and such articles of luxury as Congress may from time to time prescribe.

422. If goods on which either excise or import duties have been paid are exported, the amount of duty is refunded. This is called a *drawback*.

NOTE. For practice problems in duties and customs see par. 72.

INSURANCE

423. **Insurance** is a contract by which one party (the insurer) agrees to reimburse another party (the insured) in case of damage or loss to the latter's property or person.

424. The insurance business is usually conducted by corporations called Insurance Companies, which limit their operations to certain classes of risks. Some companies handle *fire insurance*, others *marine insurance*, others *accident insurance*, others *life insurance*, etc.

425. There are two kinds of insurance companies, *stock companies* and *mutual companies*.

426. A **Stock Company** is one whose capital is owned by stockholders who share the profits and who are liable for the losses.

427. A **Mutual Insurance Company** is one in which there are no stockholders, but in which the parties insured share the profits and losses.

428. A **Policy** is the contract of insurance.

429. A **Premium** is the sum paid for the insurance. It is the consideration.

(*a*) Sometimes the rate is expressed as a certain percentage of the value insured and sometimes at so much per hundred dollars of insurance.

(*b*) The rate of premium depends upon the amount and the nature of the risk and the length of time for which the risk is taken.

430. Fire Insurance is insurance against loss or damage by fire, or from the means employed for extinguishing it, or to prevent its spread.

431. Owners of property may insure in one or more companies. When the risk is placed in several companies, care should be taken to have the policies uniform in every particular. *Each company will then pay such part of the total loss as its risk is of the total risk.*

432. The **Average Clause**, contained in many policies, is to the effect that the liability of the company in case of a partial loss shall be *such part of the loss as the insured value is of the actual value of the property.*

Thus, a building worth $20,000 is insured for $18,000, which is nine-tenths or 90% of the value. In case of loss the company would pay, under the average clause, 90% of the loss.

433. Short Rate is the rate for less than a year.

434. Marine Insurance is insurance against loss or damage to a vessel or her cargo by storm or other dangers of the sea.

Marine policies always contain the average clause.

NOTE. For practice problems in insurance see par. 73.

LIFE INSURANCE

435. Under this head are considered *life insurance*, *accident insurance*, and *health insurance*.

436. **Life Insurance** is indemnity for loss of life.

437. **Accident Insurance** is indemnity for loss or disability caused by accident.

438. **Health Insurance** is indemnity for loss occasioned by sickness.

439. There are two kinds of life insurance policies, *life policies* and *endowment policies*.

(*a*) Under the ordinary life policy, premiums are paid annually during the life of the insured.

(*b*) There is another kind of life policy known as the Limited Payment Life Policy. Under this policy the premiums are paid during a certain number of years only.

440. A **Life Policy** is a contract on the part of the insurance company to pay the beneficiary a designated sum upon the death of the insured.

441. An **Endowment Policy** is a contract on the part of the insurance company to pay the beneficiary at the death of the insured, or after the lapse of an agreed period of time, if the insured is then alive.

442. The following table shows the rates charged by an insurance company :

LIFE PREMIUMS *Insurance of $1000, payable at death only*				ENDOWMENT PREMIUMS *Insurance of $1000, payable as specified or on prior decease*		
Age	Annual Premiums during			Age	Annual Payments	
	Life	10 Years	20 Years		In 15 Years	In 20 Years
20	$18.95	$43.85	$27.65	20	$68.10	$49.45
21	19.35	44.55	28.10	21	68.20	49.55
22	19.75	45.25	28.55	22	68.25	49.65
23	20.20	46.00	29.00	23	68.35	49.75
24	20.65	46.75	29.55	24	68.45	49.85
25	21.15	47.55	30.05	25	68.55	50.00
26	21.65	48.40	30.60	26	68.70	50.10
27	22.20	49.25	31.15	27	68.80	50.25
28	22.75	50.15	31.75	28	68.95	50.40
29	23.35	51.10	32.35	29	69.10	50.55
30	23.95	52.05	33.00	30	69.25	50.75
31	24.60	53.05	33.65	31	69.40	50.95
32	25.30	54.10	34.35	32	69.55	51.15
33	26.05	55.20	35.05	33	69.75	51.35
34	26.80	56.30	35.80	34	69.95	51.60
35	27.65	57.45	36.60	35	70.20	51.90
36	28.50	58.65	37.45	36	70.40	52.15
37	29.40	59.95	38.30	37	70.70	52.50
38	30.35	61.25	39.20	38	71.00	52.85
39	31.40	62.60	40.15	39	71.30	53.25
40	32.50	64.00	41.20	40	71.65	53.70

By this table the premium on an ordinary life policy of $1000 at the age of 20 would be $18.95, for a $5000 policy the premium would be $5 \times \$18.95$, or $94.75.

At the same age, a $1000 ten-payment life policy would cost $43.85 per year, and a $5000 policy $5 \times \$43.85$, or $219.25 per year.

A $1000 fifteen-year endowment policy would cost $68.10 a year, and a $5000 policy would cost $340.50.

Note. For practice problems in life insurance see par. 74.

EXCHANGE

443. **Exchange** is a system of paying debts in distant places by means of drafts.

444. A **Draft**, or **Bill of Exchange**, is an order of one party directing a second party to pay a third party a certain sum of money at a certain time.

445. There are two kinds of bills of exchange, *domestic* and *foreign*.

446. A **Domestic Bill of Exchange**, sometimes called an *inland bill of exchange*, is one drawn and payable in the same state or country.

447. **Foreign Bills of Exchange** are those drawn in one state or country and payable in another state or country.

448. The **Face** or **Par Value** of a bill of exchange is the sum of money for which it is written.

Bills of exchange are always written in the coinage of the country in which they are to be paid.

DOMESTIC EXCHANGE

449. Domestic Exchange quoted at a premium is worth the given percentage of the face more than the face.

Domestic Exchange quoted at a discount is worth the quoted percentage of the face less than the face.

(*a*) Thus, $\frac{1}{2}\%$ premium means that \$1 would cost \$1.005. Exchange quoted at $\frac{1}{2}\%$ discount would mean that \$1 would cost \$.995.

(*b*) When Boston owes New York the same that New York owes Boston, exchange will be at par in both places. When Boston owes New York more than New York owes Boston, exchange on New York will be at a premium in Boston, since there will be more buyers of New York exchange than sellers; and when New York owes Boston more money than Boston owes New York, exchange on New York will sell at a discount in Boston and exchange on Boston will sell at a premium in New York.

The general principles of percentage (pars. 242–254) are used in solving problems in exchange.

NOTE. For practice problems in domestic exchange see pars. 75 to 80 inclusive.

FOREIGN EXCHANGE

450. Foreign Exchange is exchange drawn in one country and payable in another country.

Foreign bills are always made in the coinage of the country where they are to be paid.

451. The **Intrinsic Par of Exchange** is the actual value of the money of one country expressed in the money of another.

Intrinsic value of foreign money expressed in the money of the United States will be found in the table, par. 468.

452. Commercial Rate of Exchange is the market value of the money of one country expressed in the money of another.

(*a*) This value changes from time to time, according to the demand that may exist and according to the different conditions of commerce that may arise.

(*b*) The rate of exchange on Great Britain is expressed by giving the market value of a pound in United States money.

(*c*) On France, Belgium, and Switzerland the rate of exchange is expressed by giving the number of francs that may be secured for $1.

(*d*) On Germany the rate of exchange is expressed by giving the market value of 4 reichsmarks in United States money.

(*e*) On Holland the rate of exchange is expressed by giving the value of 1 guilder in United States money.

(*f*) Gold is exported at a profit when the cost of foreign exchange is enough greater than the intrinsic value of the bill to pay the cost of safe shipment and yet leave a margin; and gold is imported at a profit when the cost of exchange is enough less than the intrinsic value of the bill to pay the same expenses and leave a margin. Thus under normal conditions, the commercial rate is not allowed to vary from the intrinsic par by more than enough to pay the expense of shipping gold.

NOTE. For practice problems in foreign exchange see pars. 81 to 85 inclusive.

STOCKS AND BONDS

453. A **Corporation** is an association of persons authorized by law to act as one person.

454. The **Capital Stock** of a corporation is the value of its investment. This is divided into equal parts called *shares*.

455. The **Par Value** of a share of stock is the value placed upon each share at the time of the original division of its capital stock.

The usual par value of one share of stock is $100. Stock is frequently issued in other sized shares, however, usually $50, $25, $10, $5, or $1.

456. A **Stock Certificate** is a document issued by the company to the shareholder specifying the number and par value of the shares to which he is entitled.

457. The **Market Value** of a share is the sum for which it will sell in the open market.

Sometimes stock is worth more than par and sometimes less. This depends upon the condition of the business.

458. A **Dividend** is that part of the net earnings of a corporation that is divided among its stockholders.

459. An **Assessment** is a sum levied upon the stockholders to make up losses.

There are two kinds of stock, *common* and *preferred*.

460. **Common Stock** participates in the net earnings of the company, after all other expenses have been met, in such proportion as the directors of the corporation may determine.

461. **Preferred Stock** participates in the net earnings of the corporation at a fixed rate before any dividend may be declared on the common stock.

462. A **Bond** is an obligation of a corporation to pay money on a long term of credit.

(*a*) Bonds are usually secured by deeds of trust and mortgages. They are generally issued as securities for loans. They are similar to promissory notes, but are more formal and are also made under seal.

(*b*) Bonds are usually issued in $500, $1000, or multiples thereof.

(*c*) Quotations on bonds are given on $100 par value.

(*d*) Bonds are issued in two classes, *registered* and *coupon*.

463. **Registered Bonds** are those payable to the order of the owner and can be transferred only by acknowledged assignment.

Interest on registered bonds is paid by check from the corporation made to the holder of record.

464. A **Coupon Bond** is one made payable to the bearer, and has interest certificates attached. These certificates, called coupons, are to be cut off as they

become due and presented at the designated place for payment.

(*a*) Bonds are named from the nature of the security; the name of the corporation issuing them; the date on which they are payable; the rate of interest they bear; or the purpose for which they are issued.

(*b*) Both stocks and bonds are quoted at some per cent of par value.

(*c*) The regular commission allowed to brokers for buying or selling either stocks or bonds is $\frac{1}{8}\%$ of the par value. There is a minimum charge for small transactions, however.

(*d*) Dividends and assessments are always figured on the par value of the stock.

NOTE. For practice problems in stocks and bonds see par. 86.

TABLES

UNITED STATES MONEYS

465. United States Money consists of gold coins, silver coins, United States Treasury notes and certificates, and national bank notes.

The unit of measure is the gold dollar of 25.8 grains.

10 mills = 1 cent (¢)	$ .01
10 cents = 1 dime (d.)	$ .10
10 dimes = 1 dollar ($)	$ 1.
10 dollars = 1 eagle (e.)	$10.
20 dollars = 1 double eagle (d. e.)	$20.

ENGLISH MONEYS

466. English or Sterling Money is the legal currency of Great Britain.

The unit of measure is the pound, worth $4.8665 in United States money.

4 farthings = 1 penny (*d*)
12 pence = 1 shilling (*s*)
20 shillings = 1 pound (£)

NOTE. 21 shillings = 1 guinea (used in the retail trade).

FOREIGN MONEYS

467. Once each year the Director of the United States Mint is required to compare the values of foreign coins with the United States Gold Dollar and certify the result of his comparison to the Secretary of the Treasury, who then proclaims the value of foreign money thus found to be the value to be used in estimating the worth of all foreign merchandise imported. Values thus found are called *intrinsic* or *real values* and should be distinguished from *commercial* or *exchange values.*

468. The table of values on pages 132 and 133 was proclaimed Oct. 1, 1918.

WEIGHT

Troy Weight

469. *Troy Weight* is used in weighing precious metals.

TABLE

24 grains (gr.) = 1 pennyweight (dwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

Diamond Weight

470. *Diamond Weight* is used in weighing precious stones.

The unit is $3\frac{1}{6}$ Troy grains and is called a *carat.*

This carat is not the same as that used in estimating the relative purity of gold in coins and jewelry. Pure gold is 24 carats fine; 18 carats fine means $\frac{18}{24}$ pure gold and $\frac{6}{24}$ alloy.

VALUES OF FOREIGN COINS

Country	Legal Standard	Monetary Unit		Value in Terms of U. S. Money	Remarks[1]
Argentine Republic	Gold	Peso		$0.9648	Currency: Depreciated paper, convertible at 44 per cent of face value; exchange rate about $.42½.
Austria-Hungary	Gold	Crown		.2026	Greatly depreciated.
Belgium	Gold and silver	Franc		.1930	Member of Latin Union; gold is the actual standard.
Bolivia	Gold	Boliviano		.3893	12½ bolivianos equal 1 pound sterling.
Brazil	Gold	Milreis		.5462	Currency: Government paper; exchange rate 23 cents to the milreis.
British Colonies in Australasia and Africa	Gold	Pound sterling		4.8665	
Canada	Gold	Dollar		1.0000	
Central American States:					
Costa Rica	Gold	Colon		.4653	Exchange rate $.25 = 1 colon.
British Honduras	Gold	Dollar		1.0000	
Nicaragua	Gold	Cordoba		1.0000	Exchange rate $1.00 = 1.01 cordobas.
Guatemala	Silver	Peso		.7234	Guatemala: Currency, inconvertible paper; exchange rate about 40 pesos = $1.00.
Honduras	Silver	Peso		.7234	Honduras: Currency, bank notes; exchange rate about $.55.
Salvador	Silver	Peso		.7234	Salvador: Currency, convertible into silver on demand: exchange rate $.40.
Chile	Gold	Peso		.3650	Currency: Inconvertible paper; exchange rate $.32.
China	Silver	Tael	Haikwan. (customs)	1.2066	The tael is a unit of weight; not a coin. The customs unit is the Haikwan tael. The values of other taels are based on their relation to the value of the Haikwan tael.
		Dollar	Yuan	.7771	The Yuan silver dollar of 100 cents is the monetary unit of the Chinese Republic; it is equivalent to .644+ of the Haikwan tael.
		Dollar	British	.7800	
		Dollar	Mexican	.7857	

[1] The exchange rates shown under this heading are not to take the place of the consular certificate where it is available.

Colombia	Gold	Dollar	.9733	Currency: Government paper and gold; exchange
Cuba	Gold	Peso	1.0000	[rate, approximately, 1.04 pesos to $1 gold.
Denmark	Gold	Crown	.2680	
Ecuador	Gold	Sucre	.4867	
Egypt	Gold	Pound (100 piasters)	4.9431	The actual standard is the British pound sterling,
Finland	Gold	Mark	.1930	[which is legal tender for 97½ piasters.
France	Gold and silver	Franc	.1930	Member of Latin Union; gold is the actual standard. Exchange value $9.1828.
German Empire	Gold	Mark	.2382	Greatly depreciated.
Great Britain	Gold	Pound sterling	4.8665	
Greece	Gold and silver	Drachma	.1930	Member of Latin Union; gold is the actual standard.
Haiti	Gold	Gourde	.2500	Currency: Inconvertible paper; exchange rate, $.183.
India [British]	Gold	Rupee	.3244	(15 rupees equal 1 pound sterling.)
Italy	Gold and silver	Lira	.1930	Member of Latin Union; gold is the actual standard.
Japan	Gold	Yen	.4985	
Liberia	Gold	Dollar	1.0000	Currency: Depreciated silver coins. Customs duties
Mexico	Gold	Peso	.4985	Exchange value $.535. [are collected in gold.
Netherlands	Gold	Florin	.4020	Exchange value $.4825.
Newfoundland	Gold	Dollar	1.0000	
Norway	Gold	Crown	.2680	Exchange rate $.308 = 1 crown.
Panama	Gold	Balboa	1.0000	
Paraguay	Silver	Peso	.7648	Currency: Depreciated paper; exchange rate 1,550 per cent.
Persia	Gold	Achrefi	.0959	Silver circulating above its metallic value; exchange
	Silver	Kran	.1332	value of silver kran, approximately, $.179.
Peru	Gold	Libra	4.8665	
Philippine Islands	Gold	Peso	.5000	
Portugal	Gold	Escudo	1.0805	Currency: Inconvertible paper; exchange rate $.62.
Roumania	Gold	Leu	.1930	
Russia	Gold	Ruble	.5146	Exchange rate $.12 = 1 ruble.
Santo Domingo	Gold	Dollar	1.0000	
Serbia	Gold	Dinar	.1930	
Siam	Gold	Tical	.3709	
Spain	Gold and silver	Peseta	.1930	Valuation is for the gold peseta; currency, notes of Bank of Spain; exchange value $.23.
Sweden	Gold	Crown	.2680	Exchange rate $.335 = 1 crown.
Switzerland	Gold	Franc	.1930	Member of Latin Union; gold is the actual standard. Exchange value $.225.
Turkey	Gold	Piaster	.0440	100 piasters equal to the Turkish £.
Uruguay	Gold	Peso	1.0342	Exchange rate $1.00 = .805 peso.
Venezuela	Gold	Bolivar	.1930	

Apothecaries' Weight

471. *Apothecaries' Weight* is used by physicians and apothecaries in writing and preparing prescriptions for dry medicines.

TABLE

20 grains (gr.)	= 1 scruple (sc. or ℈)
3 scruples	= 1 dram (dr. or ʒ)
8 drams	= 1 ounce (oz. or ℥)
12 ounces	= 1 pound (℔.)

Avoirdupois Weight

472. *Avoirdupois Weight* is used in commerce in all cases excepting those requiring Troy or Apothecaries' weight.

TABLE

16 ounces	= 1 pound (lb.)
25 pounds	= 1 quarter (qr.)
4 quarters	= 1 hundredweight (cwt.)
20 hundredweights	= 1 ton (T.)
2240 pounds	= 1 long ton

473. *Comparison of Troy and Avoirdupois Weights.*

1 pound Troy	= 5760 grains
1 pound Avoirdupois	= 7000 grains
1 ounce Troy	= 427 grains
1 ounce Avoirdupois	= 480 grains

474. The following table shows the weight of a bushel used commercially in measuring grain and other farm products:

Barley	48 lb.	Oats	32 lb
Beans	60 "	Onions	57 "
Buckwheat	48 "	Peas	60 "
Clover Seed	60 "	Potatoes	60 "
Corn, shelled	56 "	Timothy Seed	45 "
Corn, in the ear	70 "	Rye	56 "
Corn Meal	50 "	Rye Meal	50 "
Flaxseed	56 "	Wheat	60 "
Hemp Seed	44 "	Wheat Bran	20 "
Malt	34 "		

Liquid Measure

475. *Liquid Measure* is used in measuring liquids.

TABLE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

476. Standard liquid gallon contains 231 cubic inches.

There are various kinds of casks for containing liquids. In commerce each is gauged and its capacity marked upon it. The various kinds of casks are:

Tierce	about	42 gal.
Puncheon	"	84 "
Pipe	"	126 "
Butt	"	126 "
Tun	"	252 "
Hogshead (hhd.)	"	63 "

Apothecaries' Liquid Measure

477. *Apothecaries' Liquid Measure* is used in prescribing and compounding liquid medicines.

TABLE

60 minims (℥)	= 1 fluid drachm (f3)
8 fluid drachms	= 1 fluid ounce (f$\bar{3}$)
16 fluid ounces	= 1 pint (O)
8 pints	= 1 gallon (Cong.)

The gallon of this measure is the same as the gallon of the liquid measure.

Dry Measure

478. *Dry Measure* is used in measuring grain, fruits, vegetables, etc., which are not sold by weight.

TABLE

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

Long Measure

479. *Long Measure* is used in measuring lengths, or distances.

TABLE

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards ($16\frac{1}{2}$ ft.)	= 1 rod (rd.)
40 rods	= 1 furlong (fur.)
8 furlongs (320 rods)	= 1 mile (mi.)

Surveyors' Long Measure

480. *Surveyors' Long Measure* is used by surveyors in measuring distances.

TABLE

7.92 inches = 1 link (l.)
100 links = 1 chain (ch.)
80 chains = 1 mile (mi.)

Square Measure

481. *Square Measure* is used in measuring extent of surfaces.

TABLE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
40 square rods = 1 rood (R.)
4 roods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Surveyors' Square Measure

482. *Surveyors' Square Measure* is used by surveyors in finding the area of land.

TABLE

625 square links (sq. l.) = 1 square rod or pole (sq. rd. or p.)
16 poles = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

483. *Cubic Measure* is used in measuring the contents of anything which has length, breadth, and thickness.

TABLE

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

Wood Measure

484. *Wood Measure* is used in measuring wood.

TABLE

16 cubic feet	= 1 cord foot (cd. ft.)
8 cord feet (128 cu. ft.)	= 1 cord (cd.)

A cord of wood is a pile 8 feet long, 4 feet wide, and 4 feet high, or its equivalent.

TIME

485.

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
30 days	= 1 month (mo.)
52 weeks	= 1 year (yr.)
12 months	= 1 year (yr.)
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

486. The day is the time during which the earth makes one revolution on its own axis.

487. The *Solar Year* is the time the earth requires to make one complete revolution around the sun. It actually takes the earth $365\frac{1}{4}$ days to make this revolution. Therefore, every fourth year is given 366 days. This extra day is added to the month of February, the shortest month, and the year is called *Leap Year.*

These figures are not absolutely accurate but are practically so.

488. Any year whose number can be divided by 4 is a leap year, except that a century year must be divisible by 400.

(*a*) The year 1916 could be divided by 4 and was a leap year, while 1915 could not be divided by 4 and was an ordinary year.

(*b*) The year 1900 was divisible by 4 but not by 400 and was not, therefore, a leap year. The year 2000 being divisible by 4 and 400 will be a leap year.

489. *Months of the Year, and Days in Each :*

1. January	31	7. July	31
2. February	28 or 29	8. August	31
3. March	31	9. September	30
4. April	30	10. October	31
5. May	31	11. November	30
6. June	30	12. December	31

MISCELLANEOUS

490. Some articles are sold by quantity according to the following table:

TABLE

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gr.)
12 gross = 1 great gross (g. gr.)
20 units = 1 score

Paper Measure

491. Paper is measured according to the following table:

TABLE

24 sheets = 1 quire (qu.)
20 quires = 1 ream (rm.)
2 reams = 1 bundle (bdl.)
5 bundles = 1 bale (bl.)

THE METRIC SYSTEM

492. The **Metric System** is a decimal system of weights and measures, similar to the decimal system used in measuring United States money. It was originated in France early in the nineteenth century, and has been adopted by nearly all the commercial nations except United States and England.

The Metric System was made legal in the United States in 1866, but is not generally used except in scientific work.

493. The **Meter.** The basic unit is the meter. The other units, those of weight and of capacity, are based on it.

494. The length of the meter was originally determined by taking one ten-millionth of the distance from the equator to the pole. This length is 39.37 inches.

495. The primary units are:

For length — *meter*
For capacity — *liter*
For weight — *gram*

496. The desired integral multiples of these are formed by using the following Greek prefixes:

Deca = 10 (decameter = 10 meters)
Hecto = 100 (hectometer = 100 meters)
Kilo = 1000 (kilometer = 1000 meters)
Myria = 10,000 (myriameter = 10,000 meters)

497. To designate decimals of a meter, the following Latin prefixes are used:

Deci $= \frac{1}{10}$ (decimeter $= \frac{1}{10}$ meter)
Centi $= \frac{1}{100}$ (centimeter $= \frac{1}{100}$ meter)
Milli $= \frac{1}{1000}$ (millimeter $= \frac{1}{1000}$ meter)

The most commonly used denominations in the following tables are indicated by heavy-faced type.

Linear Measure

498. The unit of *Linear Measure* is the meter.

TABLE

10 **millimeters** (mm.)	= 1 **centimeter** (cm.)
10 **centimeters**	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
10 **meters**	= 1 decameter (dm.)
10 decameters	= 1 hectometer (hm.)
10 hectometers	= 1 **kilometer** (km.)
10 **kilometers**	= 1 myriameter (mm.)

Square Measure

499. The unit of *Square Measure* is the square meter.

TABLE

100 square millimeters	= 1 **square centimeter** (cmq.)
100 **square centimeters**	= 1 square decimeter (dmq.)
100 square decimeters	= 1 **square meter** (mq.)
100 **square meters**	= 1 square decameter (dcmq.)
100 square decameters	= 1 square hectometer (sq. hm.)
100 square hectometers	= 1 square kilometer (sq. km.)

Land Measure

500. The unit of *Land Measure* is the are.

TABLE

100 centiares (ca.)	= 1 **are** (a.)	= 100 sq. m.
100 **ares**	= 1 **hectare** (ha.)	= 10,000 mq.

Cubic Measure

501. The unit of volume is the cubic meter.

TABLE

100 cubic millimeters (cmm.) = 1 **cubic centimeter** (cmc.)
100 **cubic centimeters** = 1 **cubic decimeter** (dmc.)
100 **cubic decimeters** = 1 **cubic meter** (mc.)

Wood Measure

502. The unit of wood measure is the stere.

TABLE

10 decisteres (dst.) = 1 **stere** (st.) = 1 cu. m.
10 **steres** = 1 decastere (dast.) = 10 cu. m.

Measure of Capacity

503. The unit of capacity for either solids or liquids is the liter, which is equal in volume to 1 cu. dm.

TABLE

10 milliliters (ml.) = 1 **centiliter** (cl.)
10 **centiliters** = 1 deciliter (dl.)
10 deciliters = 1 **liter** (l.)
10 **liters** = 1 decaliter (dl.)
10 decaliters = 1 **hectoliter** (hl.)
10 **hectoliters** = 1 kiloliter (kl.)

Measure of Weight

504. The unit of weight is the gram, which is the weight of 1 dmc. of distilled water in a vacuum, at its greatest density. It weighs 15.4324 gr.

TABLE

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
10 grams	= 1 decagram (dg.)
10 decagrams	= 1 hectogram (hg.)
10 hectograms	= 1 kilogram (kg.)
10 kilograms	= 1 myriagram (mg.)
10 myriagrams	= 1 quintal (q.)
10 quintals	= 1 tonneau (t.)

TABLES OF EQUIVALENTS

505.

Convenient Equivalent Values

1 cu. cm. of water = 1 ml. of water, and weighs 1 gram
= 15.432 gr.

1 cu. dm. of water = 1 l. of water, and weighs 1 kg.
= 2.2046 lb.

1 cu. m. of water = 1 kl. of water, and weighs 1 tonneau
= 2204.6 lb.

506.

Measures of Weight

1 grain, Troy	= .0648 of a gram
1 ounce, Troy	= 31.104 grams
1 ounce, Avoir.	= 28.35 grams
1 lb. Troy	= .3732 of a kilogram
1 lb. Avoir.	= .4536 of a kilogram
1 ton (short)	= .9072 of a tonneau or ton

1 gram	= 15.432 grains, Troy
1 gram	= .03215 of an oz. Troy
1 gram	= .03527 of an oz. Avoir.
1 kilogram	= 2.679 lb. Troy
1 kilogram	= 2.2046 lb. Avoir.
1 tonneau	= 1.1023 tons (short)

507.

Measures of Capacity

1 dry quart	= 1.101 liters
1 liquid quart	= .9463 of a liter
1 liquid gallon	= .3785 of a decaliter
1 peck	= .881 of a decaliter
1 bushel	= .3524 of a hectoliter

1 liter	= .908 of a dry quart
1 liter	= 1.0567 liquid quarts
1 decaliter	= 2.6417 liquid gal.
1 decaliter	= 1.135 pecks
1 hectoliter	= 2.8377 bushels

508.

Linear Measure

1 inch	= 2.54 centimeters
1 foot	= .3048 of a meter
1 yard	= .9144 of a meter
1 rod	= 5.029 meters
1 mile	= 1.6093 kilometers

1 centimeter	= .3937 of an inch
1 decimeter	= .328 of a foot
1 meter	= 1.0936 yards
1 dekameter	= 1.9884 rods
1 kilometer	= .62137 of a mile

509.

Surface Measure

1 sq. inch	= 6.452 sq. centimeters
1 sq. foot	= .0929 of a sq. meter
1 sq. yard	= .8361 of a sq. meter
1 sq. rod	= 25.293 sq. meters
1 acre	= 40.47 ares
1 sq. mile	= 259 hectares

1 sq. centimeter	= .155 of a sq. inch
1 sq. decimeter	= .1076 of a sq. foot
1 sq. meter	= 1.196 sq. yards
1 are	= 3.954 sq. rods
1 hectare	= 2.471 acres
1 sq. kilometer	= .3861 of a sq. mile

510.

Cubic Measure

1 cu. inch	= 16.387 cu. centimeters
1 cu. foot	= 28.317 cu. decimeters
1 cu. yard	= .7646 of a cu. meter
1 cord	= 3.624 steres

1 cu. centimeter	= .061 of a cu. inch
1 cu. decimeter	= .0353 of a cu. foot
1 cu. meter	= 1.308 cu. yards
1 stere	= .2759 of a cord

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(Figures refer to paragraph numbers.)