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IN PORTFOLIO MANAGEMENT: A STATISTICAL
ANALYSIS

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REAL VS. NOMINAL RATES OF RETURN MATRICES IN
PORTFOLIO MANAGEMENT: A STATISTICAL ANALYSIS

by

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ABSTRACT

Possible methods for testing the statistical relationships be-
tween nominal and real matrices in portfolio managements are explored.
An exact relationship between real correlation coefficients and nominal
correlation coefficients is derived to show the importance of real
correlation coefficients for determining the diversification effect in
the portfolio management.

I.   Introduction

Kennedy (1960), Bodie (1976), Brian (1969), Johnson, Reilly and

Smith (1971) [JRS], Hendershott and Van Horne (1973), Ondet (1973),

Reilly, Smith and Johnson (1975) [RSJ] and others have shown that there

exist some important impacts of inflation on common stock values and

bond rates.   Therefore, the impact of inflation on portfolio analysis

is of interest to security analysts.   Under a Markowitz algorithm [see

Markowitz (1959)], information associated with both estimated covariance

matrix and estimated correlation matrix is generally used to select the

efficient portfolio.   To investigate the possible effect of inflation

on the portfolio selection, Sarnat (1973) and Biger (1975, 1976) have

done some empirical studies to demonstrate how the portfolio selection

can be affected by the inflation factor.   However, none of these studies

has statistically tested their empirical results.

The main purposes of this paper are: to examine the problem of test-

ing the equality between nominal and real covariance matrices, and to

investigate the possibility of testing the equality between nominal and

real correlation matrices.   The relationship between the real and the

nominal correlation coefficients is also derived under a relative general

condition.   In the second section, the existing method of testing the

equality between nominal and real covariance matrices is discussed; the

problem associated with the existing method is demonstrated.   In the third

section the statistical relationship between real and nominal correlation

coefficients are derived.   Some implications associated with this re-

lationship are explored.   Possible methods of testing the equality between

nominal and real correlation matrices are examined and criticized. In the fourth section an alternative method based upon the asymptotic distribution of a sample correlation coefficient is used to re-examine Biger's conclusions associated with testing the relationship between nominal and real correlation matrices. The relative advantage of Biger's (1975, 1976) method of investigating the impact of inflation on portfolio selection relative to that of Sarnat's (1973) method is explored. Finally, results of this paper are summarized. Some concluding remarks are indicated.

II. The Relationship Between Real and Nominal Covariance Matrices

Based upon multivariate log normal assumption, Biger (1975, 1976) defined the relationship between the nominal and real rates of return on security in terms of logarithmic transformation as:

$$\tilde{R}_j^* = \log(\tilde{R}_j) - \log(\tilde{P}) = \tilde{r}_j - \tilde{p} \tag{1}$$

Where $\tilde{R}_j$ = 1 + nominal rates of return on j^{th} security

$\tilde{P}$ = 1 + inflation rate

$\tilde{R}_j^*$ = real rates of return on j^{th} security.

Given the definition of the real rate of return and the nominal rate of return, the variance of $\tilde{R}_j^*$ and the conveniance between $\tilde{R}_j^*$ and $\tilde{R}_i^*$ can be defined as:

a) $\text{Var}(\tilde{R}_j^*) = \text{Var}(\tilde{r}_j) + \text{Var}(\tilde{p}) - 2\,\text{Cov}(\tilde{r}_j, \tilde{p})$

b) $\text{Cov}(\tilde{R}_j^*, \tilde{R}_i^*) = \text{Cov}(\tilde{r}_j, \tilde{r}_i) - \text{Cov}(\tilde{r}_j, \tilde{p}) - \text{Cov}(\tilde{r}_i, \tilde{p}) + \text{Var}(\tilde{p})$

$$\tag{2}$$

If the real rates of return is independent of the inflation,[1] then equation (2) can be rewritten as:

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[1] This kind of assumption has been used by Roll (1973), Hagerman and Kim (1976) and others.

a) $\text{Var}\ (\tilde{R}_j^*) = \text{Var}\ (\tilde{r}_j) - \text{Var}\ (\tilde{p})$

b) $\text{Cov}\ (\tilde{R}_j^*,\ \tilde{R}_i^*) = \text{Cov}\ (\tilde{r}_j,\ \tilde{r}_i) - \text{Var}\ (\tilde{p})$

(3)

This implies that the real covariance matrix (RCM) is generally not equal to the nominal covariance matrix (NCM) unless the variance of inflation is zero.

To test whether the RCM is statitically significantly different from NCM, Anderson's (1958) approximate χ^2 statistic can be used to perform the test. To show the possible problem associated with using approximate χ^2 statistic in testing the equality between the nominal and real covariance matrices, the approximate χ^2 statistic is briefly described as follows.

Following Anderson (1958, Chapter 10), to test the equality of q covariance matrices, the null hypothesis can be defined as:

Ho: $\qquad \Sigma_1 = \ldots = \Sigma_i \ldots = \Sigma_q$

(4)

In equation (4), Σ_i (i = 1, 2, ..., q) indicates the covariance matrix associated with i^{th} population. The testing statistic with approximate χ^2 distribution for the null hypothesis test of (4) can be defined as:

$$\Upsilon = -2\rho \log\ W_1 \qquad\qquad (5)$$

Where ρ is the function of the sample size and the dimension of the covariance matrix; W_1 is the function of sample size, dimension of covariance matrices and the determinants of covariance matrices.[2]

The approximate χ^2 distribution of Υ is derived under the assumption that the covariance matrices are from q independent populations. Now we

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[2] See Anderson (1958, pp. 247-256) for detail.

will examine whether the nominal rates of return is independent of the real rates of return. Let $X = \tilde{R}^*$ and $Y = \tilde{P}$. If $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$ and X and Y are independent. Then the joint density function of X and Y is:

$$f(X, Y) = (2\pi)^{-1} (\sigma_1^2)^{-\frac{1}{2}} (\sigma_2^2)^{-\frac{1}{2}} \exp^{-[\frac{1}{2\sigma_1^2}(X-\mu_1)^2 + \frac{1}{2\sigma_2^2}(Y-\mu_2)^2]} \tag{6}$$

From equation (1), the nominal rates of return is defined as:

$$W = X + Y \tag{7}$$

Then the joint distribution of W and Y is:

$$g(W, y) = (2\pi)^{-1} (\sigma_1^2)^{-\frac{1}{2}} (\sigma_2^2)^{-\frac{1}{2}} \exp^{-[\frac{1}{2\sigma_1^2}(W-y-\mu_1)^2 - \frac{1}{2\sigma_2^2}(y-\mu_2)^2]} \tag{8}$$

Since $g(W/y) \alpha\, g(W, y)$ and the conditional distribution of W given y is $N(y + \mu_1, \sigma_1^2)$. But the marginal distribution of $W = X + Y$ is $N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

Hence:

$$g(W/y) \neq g(W)$$

Thus, it can be concluded that W is not independent of Y. In other words, the real rates of return is generally not independent of the nominal rates of return. Therefore, the approximate χ^2 method of (5) is not an appropriate method for testing the equality between the real covariance matrix and the nominal covariance matrix.

III. The Relationship Between Real and Nominal Correlation Matrices and its Implications

If the real rates of return are independent of the inflation as indicated in (3), then the nominal correlation coefficient can be defined as:

$$\rho_{i \cdot j} = \frac{\text{Cov}(\tilde{r}_i, \tilde{r}_j)}{\sqrt{\text{Var}(\tilde{r}_i) \cdot \text{Var}(\tilde{r}_j)}} \tag{9}$$

The real correlation coefficient can be defined as:

$$\rho^*_{i \cdot j} = \frac{\text{Cov } (\tilde{r}_i, \tilde{r}_j) - \text{Var } (\tilde{p})}{\sqrt{[\text{Var } (\tilde{r}_i) - \text{Var } (p)] \cdot [\text{Var } (\tilde{r}_j) - \text{Var } (p)]}} \qquad (10)$$

It can be shown that the relative magnitude between ρ^* and ρ can be determined by the magnitude of Var $(\tilde{p})$, i.e. [3]

$$\rho^*_{i \cdot j} \gtreqless \rho_{i \cdot j} \quad \text{when}$$

$$\text{Var } (\tilde{p}) \gtreqless \frac{2 \text{ Cov } (\tilde{r}_i, \tilde{r}_j) - \rho_{i \cdot j}^2 \text{ Var } (\tilde{r}_i) - \rho_{i \cdot j}^2 \text{ Var } (\tilde{r}_j)}{1 - \rho_{i \cdot j}^2} \qquad (11)$$

Equation (11) implies that the real correlation coefficient is not necessarily equal to the nominal correlation coefficient. To test the equality between ρ^* and ρ, it is shown that Anderson's (1958) Z approximate statistic for testing the equality of two simple correlation coefficients as defined in (12) can be used to test the statistical relationship between $\rho^*_{i \cdot j}$ and $\rho_{i \cdot j}$.

$$Z = \frac{(Z_1 - Z_2)}{\sqrt{\frac{1}{(n_1 - 3)} + \frac{1}{(n_2 - 3)}}} \qquad (12)$$

Where

$$Z_1 = \frac{1}{2} \log \left(\frac{1 + \hat{\rho}^*_{i \cdot j}}{1 - \hat{\rho}^*_{i \cdot j}} \right); \quad Z_2 = \frac{1}{2} \log \left(\frac{1 + \hat{\rho}^*_{i \cdot j}}{1 - \hat{\rho}^*_{i \cdot j}} \right)$$

and n_1 and n_2 are the sample sizes associated with $\hat{\rho}^*_{i \cdot j}$ and $\rho_{i \cdot j}$ respectively.

Now, it is shown that the approximate Variance of $(Z_1 - Z_2)$ is $\left(\frac{1}{n_1 - 3} + \frac{1}{n_2 - 3} \right)$. According to the definition of variance,

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[3] See Appendix (A) for the derivation of this relationship.

$$\text{Var } (Z_1 - Z_2) = \text{Var } (Z_1) + \text{Var } (Z_2) - 2 \text{ Cov } (Z_1, Z_2) \tag{13}$$

Following Anderson (1958, Chapter 2), equation (13) can be rewritten as:

$$\text{Var } (Z_1 - Z_2) = \frac{1}{n_1 - 3} + \frac{1}{n_2 - 3} - 2 \text{ Corr } (Z_1, Z_2) \left(\frac{1}{n_1 - 3} \right) \left(\frac{1}{n_2 - 3} \right) \tag{13}'$$

As Corr (Z_1, Z_2) is between -1 and 1, the second term of (13)' is of order $O(n^{-2})$ and hence asymptotically negligible. Therefore, the approximate Variance of $(Z_1 - Z_2)$ is $\left(\frac{1}{n_1 - 3} + \frac{1}{n_2 - 3} \right)$.

A generalized multivariate distribution method for testing the equality of two correlation matrices is still not available. However, if two sample statistics are obtained from two independent populations, we can test the equality of two correlation matrices by testing the equality of two covariance matrices. Besides the approximate χ^2 test as discussed in the previous section, Lee, Chang and Krishnaiah (1976) have derived the likelihood ratio test tables for testing the equality of two covariance matrices. As the real covariance matrix and nominal covariance matrix are not from two independent populations. The method of testing of covariance matrices can not be used to test the equality of correlation matrices. Futhermore, it can be shown that the equality of covariance matrix implies the equality of correlation matrices. However, the inequality of covariance matrices does not necessarily imply the inequality of correlation matrices.[4]

Now, the implications of the relationship between real and nominal correlation coefficients are discussed. The magnitude and the sign of the correlation coefficient among securities are generally used to determine the degree of deversification associated with a portfolio analysis. If the

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[4] See Appendix (B) for the proof.

real rates of return instead of the nominal rates of return are concerned
by the investors, then the real correlation coefficient instead of nominal
correlation coefficient should be used to analyze the diversification effect.
If the nominal correlation coefficient is not equal to the real correlation
coefficient, then the diversification effect estimated by the nominal cor-
relation coefficient will be biased. Therefore, it is of importance to
test whether the nominal correlation coefficient is equal to the real cor-
relation coefficient in the portfolio management. A portfolio manager can
use the relationship defined either in equation (11)or in equation (12) to
perform the empirical test.

Equation (11) indicates that the magnitude of variance associated with
inflation is an important factor in determining the relationship between
real correlation coefficient and nominal correlation coefficient. It is
well-known that the variance associated with inflation is essentially de-
termined by the economic condition and the stage of business cycle. Hence,
the portfolio management based upon real instead of nominal rates of re-
turn can more explicitly take impact of business cycle on portfolio man-
agement into account. In investigating the causal relationship between
stock market index and other economic indicators--GNP, inflation rate and
money supply. Kraft and Kraft (1977) found that only inflation rate has
direct causal impact on the change of stock market index. This founding
has reinforced the importance of using real instead of nominal rates of
return in the portfolio management.

IV. Re-examine Biger and Sarnat's Studies

In accordance with the definition of correlation coefficient, both Biger
(1975, 1976) and Sarnat (1973) have calculated nominal correlation coefficient

matrix, N, and real correlation coefficient matrix, R. To test the relationship between these two correlation matrices, Biger postulated a 16 x 16 transformation T_1 and defined a relationship as:[5]

$$N \times T_1 = R \tag{14}$$

Using the relationship of equation (14), he posited two alternative hypotheses for T_1, i.e. (i) T_1 is either an identity matrix or a scale matrix, and, (ii) T_1 is neither an identity matrix nor a scale matrix. If the first hypothesis is true, then he would conclude that the inflation will have no impact on the portfolio selection. However, he found that the second hypothesis was true. Therefore, he concluded that inflation generally affects the portfolio selection. Now we will examine the possible problems faced by Biger in testing the relationship between N and R. To test his two alternative hypotheses, he calculated the transformation matrix by using the relationship as defined in equation (15).

$$T_1 = N^{-1}R \tag{15}$$

After comparing the elements in T_1, he concluded that the transformation matrix T_1 is far from being close to either identity matrix or constant matrix.[6] Biger's method of testing the relationship between N and R is subject to following three criticisms, i.e. (i) his method is a mathematical instead of a statistical method, (ii) his results are sensitive to the degree of singularity of N matrix, and (iii) T_1 is not a

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[5] In his *JFQA* article, Biger used the rates of return of 16 industries to calculate N and R. However, he used only 15 industries in his _JF_ paper.

[6] In his _JFQA_ study, he found that the transformation matrix contains values as high as 648.6 and as low as -559.6.

symmetric matrix. The last criticism can be proved by transposing T_1 matrix as follows. From (15) we know that:

$$T_1' = (N^{-1}R)' = R'(N^{-1})' = R'N^{-1} \qquad (16)$$

As $N^{-1}R$ is generally not equal to $R'N^{-1}$, therefore, the T_1' is not a symmetric matrix.

Using the formula given by equation (12), the z statistics are calculated for the pairwise nominal and real correlation coefficients, and the results are listed in table 1.[7] From the z statistics listed in table 1, it is found that there exists no nominal correlation coefficient which is statistically different from the real correlation coefficient. This implies that the nominal correlation matrix is not significantly different from the real correlation matrix.[8] This finding is similar to Sarnat's (1973) finding.

Besides testing the equality of two correlation matrices, Biger (1975, 1976) also derived the efficient frontiers by using both nominal and real rates of return. He has concluded that the portfolio constructed by using real rates of return is more efficient than that obtained from nominal rates of return. It would be noted that both nominal and real monthly T-Bill rates are included in Biger's portfolio selection. It has been shown that the impacts of inflation on rates of return on equity are generally different from that on the rates of return on Treasury Bill.

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[7] These z statistics are calculated by using Biger's JFQA data. Z statistics also are calculated by using his JF data, the conclusions are identical with what we have here.

[8] As the individual elements of nominal correlation matrix are pairwisely equal to the individual elements of real correlation matrix, the nominal correlation matrix will be equal to the real correlation matrix.

TABLE 1 z STATISTICS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1.	0															
2.	.04945	0														
3.	.06827	.06187	0													
4.	.06410	.06558	.04818	0												
5.	.01195	.04321	.07839	.07383	0											
6.	.02138	.02715	.09841	.10965	.06317	0										
7.	.08974	.08227	.04829	.06460	.09314	.01422	0									
8.	.04976	.04632	.01661	.02125	.03835	.03295	.04966	0								
9.	.00754	.02486	.03019	.01197	.01338	.00902	.05165	.02423	0							
0.	.02722	.00365	.04179	.03616	.00164	.01474	.05921	.01301	.03330	0						
1.	.29866	.58730	.04359	1.36763	.02624	.97307	.06995	.01474	.56636	1.56107	0					
2.	.06815	.08180	.06461	.06624	.77569	.07530	.05921	.02904	.02464	.98454	.04684	0				
3.	.06285	.06559	.02922	.04088	.06180	.20306	.03690	.00260	.03553	.04200	.02464	.04684	0			
4.	.00068	.02182	.04820	.03998	.00592	.02197	.05112	.02213	.00249	.01619	.03176	.03957	.04079	0		
5.	.01823	.02249	.05552	.04783	.01624	.06911	.06876	.03577	.00230	.02611	.60477	.04469	.05527	.00782	0	
6.	.06888	.12177	.08082	.11139	.04033	.00795	.05252	.11274	.02371	.04033	.10484	.10689	.01786	.03355	.00351	0

This argument is essentially based upon the theory that the "Fisher Effect" is applicable to analyze the impact of inflation on rates of return on Treasury Bill and is not necessarily applicable to analyze the impact of inflation on rates of return on common equity.[9] In sum, the different conclusions drawn by Biger and Sarnat on the relationship between inflation and portfolio selection may well be due to the fact that Biger (1975, 1976) instead of Sarnat (1973) treated Treasury Bill rates of return as one of the risky assets.

Now, the advantage of including Treasury Bills in the portfolio is analyzed. If one assumes that index-linked T-Bills are available, and that their return is r_f^*, then it can be shown that the yield on nominal T-Bills is:[10]

$$r_{TB} = r_f^* + \beta_f \, [E \, (r_m^*) - (r_f^*)] \tag{17}$$

Where r_m^* is the real market rate of return; β_f is the systematic risk of the nominal T-Bills, or their non-diversifiable purchasing power risk. This coefficient is equal to

$$\beta_f = \frac{\text{Var} \, (\tilde{p}) - \text{Cov} \, (\tilde{r}_m^*, \, \tilde{p})}{\text{Var} \, (\tilde{r}_m) + \text{Var} \, (\tilde{p}) - 2 \, \text{Cov} \, (r_m^*, \, p)} \tag{18}$$

Equation (18) indicates that $\beta_f \neq 0$ even if $\text{Cov} \, (\tilde{r}_m^*, \, p) = 0$, hence the yield on nominal T-Bills would include risk primium even if $E \, (\tilde{p}) = 0$. Thus, in markets which are dominated by expected utility maximizers who have no "money illusion", then nominal Treasury Bills are risky. Under this circumstance, the Treasury Bill should be treated as one of the risky assets in selecting efficient portfolio. Thus, Biger's (1975, 1976)

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[9]See Jaffe and Mandelker (1976) for detail.

[10]This relationship is based upon the capital asset pricing model developed by Sharpe (1964), Lintner (1965) and Mossin (1966).

proach in investigating the impact of inflation on portfolio selecting

more acceptable relative to that used by Sarnat (1973).

Summary and Concluding Remarks

This paper has investigated the possible problems of testing real

d nominal matrices in portfolio analysis. It is shown that real rates

return are generally not independent of nominal rates of return and the

proximate χ^2 technique is not suitable for testing the relationship

minal and real matrices. To resolve this problem, the exact relation-

ip between the nominal correlation coefficient and real correlation

efficients is derived. In addition, it is also shown that the approximate

statistic can be used to test the equality between real and nominal

rrelation coefficients. Furthermore the relative advantage of including

easury Bill rates of return in the portfolio selection is also analyzed

accordance with the capital asset pricing theory and the effect of in-

tion on rates of return on common equity and Treasury bond.

The degree of stability of portfolio components over time based

on real rates of return relative to those based upon nominal rates of

turn should be investigated in the future research in the portfolio

nagements. In addition, it is very worthwhile to note that a gen-

alized method for testing the statistical relationship between real and

minal matrices should be developed to improve the portfolio management

der a economic environment with non-negligible unexpected inflation.

APPENDIX A

From equations (9) and (10), the relationship between $\rho^*_{i \cdot j}$ and $\rho_{i \cdot j}$ can be written as:

$$\rho^*_{i \cdot j} = \rho_{i \cdot j} \frac{1 - \dfrac{\text{Var}(\tilde{p})}{\text{Cov}(\tilde{r}_i, \tilde{r}_j)}}{\sqrt{\left[1 - \dfrac{\text{Var}(\tilde{p})}{\text{Var}(\tilde{r}_i)}\right]\left[1 - \dfrac{\text{Var}(\tilde{p})}{\text{Var}(\tilde{r}_j)}\right]}} \qquad [A]$$

From equation [A], the relationship between $\rho^*_{i \cdot j}$ and $\rho_{i \cdot j}$ can be analyzed as follows:

$\rho^*_{i \cdot j} \gtreqless \rho_{i \cdot j}$ implies that

$$\frac{1 - \dfrac{\text{Var}(\tilde{p})}{\text{Cov}(\tilde{r}_i, \tilde{r}_j)}}{\sqrt{\left[1 - \dfrac{\text{Var}(\tilde{p})}{\text{Var}(\tilde{r}_i)}\right]\left[1 - \dfrac{\text{Var}(\tilde{p})}{\text{Var}(\tilde{r}_j)}\right]}} \gtreqless 1 \qquad [B]$$

Equation [B] implies that:

$$1 - 2\frac{\sigma_p^2}{\rho_{i \cdot j}\,\sigma_i\,\sigma_j} + \frac{\sigma_p^4}{\rho_{i \cdot j}^2\,\sigma_i^2\,\sigma_j^2} \gtreqless 1 - \frac{\sigma_p^2}{\sigma_i^2} - \frac{\sigma_p^2}{\sigma_j^2} + \frac{\sigma_p^4}{\sigma_i^2\,\sigma_j^2} \qquad [C]$$

Where $\sigma_p = \text{Var}(\tilde{p})$, $\sigma_i = [\text{Var}(\tilde{r}_i)]^{\frac{1}{2}}$ and $\sigma_j = [\text{Var}(\tilde{r}_j)]^{\frac{1}{2}}$. Equation [C] implies that:

$$\sigma_p^2 \left[\frac{\sigma_p^2 - 2\,\rho_{i \cdot j}\,\sigma_i\,\sigma_j}{\rho_{i \cdot j}\,\sigma_i^2\,\sigma_j^2} \right] \gtreqless \sigma_p^2 \left[\frac{\sigma_p^2 - \sigma_i^2 - \sigma_j^2}{\sigma_i^2\,\sigma_j^2} \right] \qquad [D]$$

After some arrangements, we know that equation [D] implies that:

$$\sigma_p^2 \gtreqless \frac{2\,\rho_{i \cdot j}\,\sigma_i\,\sigma_j - \rho_{i \cdot j}^2\,\sigma_i^2 - \rho_{i \cdot j}^2\,\sigma_j^2}{1 - \rho_{i \cdot j}} \qquad [E]$$

Equation [E] can be used to determine the relationship between real correlation coefficient and nominal correlation coefficient.

APPENDIX B

If two by two covariance matrices V and V' are defined as:

$$V = \begin{pmatrix} \sigma_1^2 & \rho\,\sigma_1\,\sigma_2 \\ \rho\,\sigma_1\,\sigma_2 & \sigma_2^2 \end{pmatrix}, \qquad V' = \begin{pmatrix} \sigma_1'^2 & \rho'\sigma_1'\sigma_2' \\ \rho'\sigma_1'\sigma_2' & \sigma_2'^2 \end{pmatrix}$$

and the correlation matrices associated with C_1 and C_2 are defined as:

$$R = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \qquad R' = \begin{pmatrix} 1 & \rho' \\ \rho' & 1 \end{pmatrix}$$

If $V = V'$, then $\sigma_1^2 = \sigma_1'^2$, $\sigma_2^2 = \sigma_2'^2$, and $\rho\,\sigma_1\,\sigma_2 = \rho'\,\sigma_1'\,\sigma_2'$.

These equalities imply that $\rho = \rho'$. Hence $R = R'$. However, if $R = R'$,

then V will not equal to V' unless $\sigma_1 = \sigma_1'$ and $\sigma_2 = \sigma_2'$.

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