Regression Tests of the Present Value Model and Speculative Bubbles

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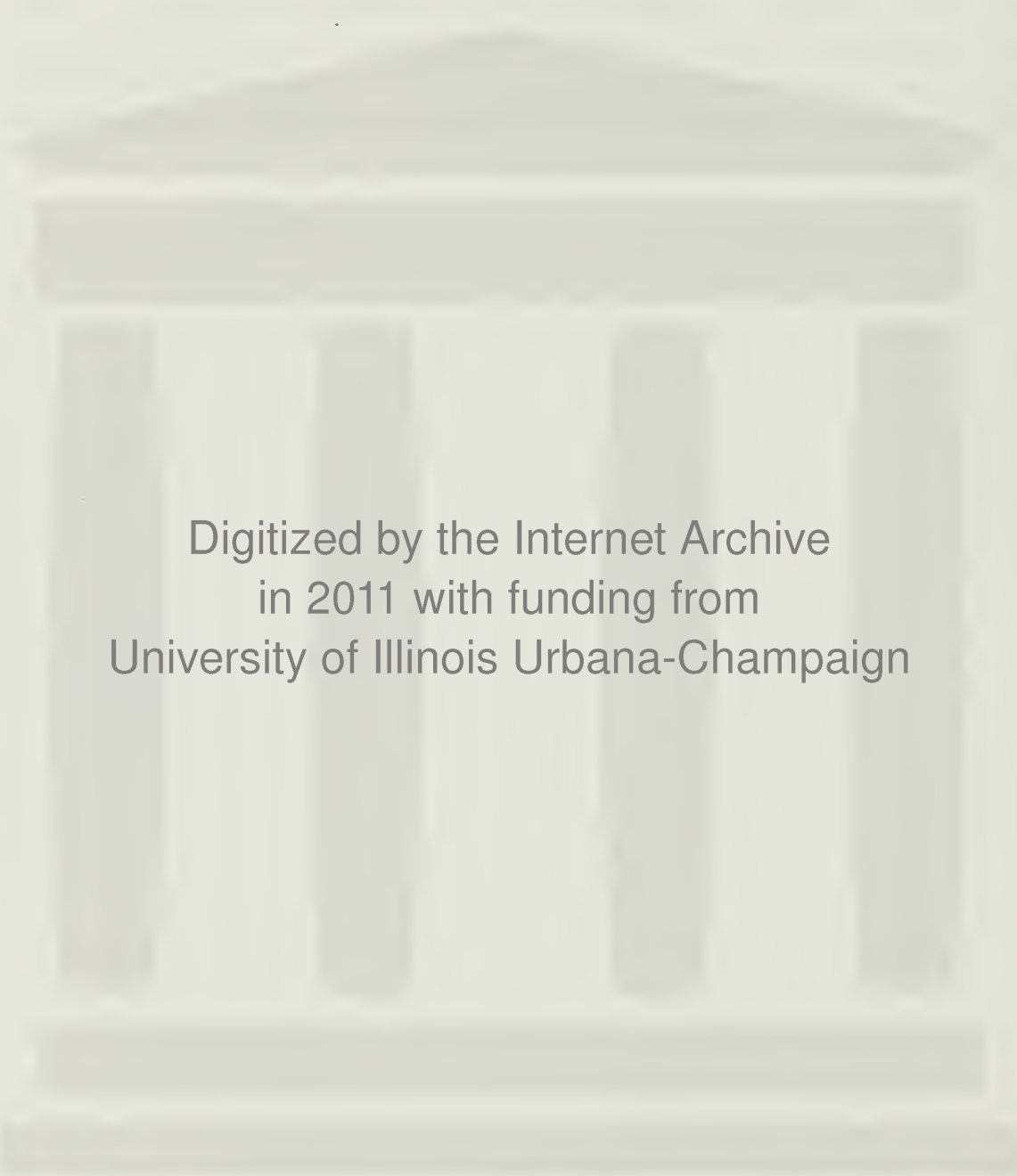
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Abstract

This paper develops a regression test of the present value model, which holds regardless of whether rational bubbles are present in stock prices. We test whether the forecast error of the stock price with respect to the *ex post* rational price is consistent with the present value model. The statistical results cannot reject the null hypothesis.

Introduction

The objective of this paper is two-fold. First, this paper demonstrates that the failure of some regression tests to accept the present value model cannot be attributed to the existence of rational speculative bubbles. Second, the paper proposes an alternative regression test of the present value model that is valid regardless of whether rational bubbles are present in stock prices.

One may suppose that the stock price can be decomposed into a fundamental value and a rational bubble and that the fundamental value is determined by the present value model:

$$
\begin{aligned}
P_t &= F_t + B_t \\
&= \sum_{i=1}^{\infty} \frac{E_t D_{t+i}}{(1+k)^i} + B_t
\end{aligned}
\tag{1}
$$

where P_t is the stock price at the *beginning* of period $t+1$ (or at time t), F_t is the fundamental value at time t, B_t is the bubble at time t, D_{t+i} is the dividend paid *during* period $t+i$ (or from time $t+i-1$ through time $t+i$), k is the discount rate, which is assumed to be constant, and E_t is the investor's expectations operator conditional upon information available at time t. Flood and Garber (1980) show that when bubbles satisfy the Euler equation,[1] i.e., when bubbles are rational, we have

$$
B_t = \frac{1}{1+k} E_t B_{t+1}.
\tag{2}
$$

Following LeRoy and Porter (1981) and Shiller (1981), we define the

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[1]The Euler equation is the first-order condition for a representative consumer's lifetime expected utility maximization.

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"*ex post* rational" price, P_t^*, as

$$P_t^* = \sum_{i=1}^{\infty} \frac{D_{t+i}}{(1+k)^i}.$$ (3)

Since $E_t P_t^* = F_t$, we have

$$P_t^* = P_t - B_t + \eta_t$$ (4)

where η_t is a rational forecast error, in that it is uncorrelated with P_t and B_t. In the absence of bubbles (i.e., $B_t = 0$), we have

$$P_t^* = P_t + \eta_t$$ (5)

and

$$\text{var}(P_t^*) \geq \text{var}(P_t).$$ (6)

The failure of the statistical results of various studies to accept inequality condition (6) has ignited heated debates among researchers. Some researchers suggest that the failure of variance bounds tests to accept the present value model can be attributed to the existence of speculative bubbles. However, Flood and Hodrick (1986) demonstrate that most variance bounds tests preclude bubbles as an explanation. Others doubt, for various reasons, the usefulness of variance bounds tests. For example, Marsh and Merton (1986) suggest that since stock prices are non-stationary, imposing an upper bound on stock price volatility is not meaningful for testing the present value model.

From these debates, an alternative approach for testing the present value model has emerged. Scott (1985) suggests that ordinary least squares (OLS)

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regression tests of equation (7) are more powerful than variance bounds tests:[2]

$$P_t^* = a + bP_t + \eta_t \qquad (7)$$

where trends are removed from the data, and $a = 0$ and $b = 1$ under the null hypothesis.

Scott suggests that the existence of bubbles could cause his regression test to reject the present value model. As will be shown later, Scott's failure to accept the null hypothesis cannot be attributed to the existence of rational bubbles. Also, application of OLS to equation (7) yields *inefficient* coefficient estimates because the rational forecast error η_t is serially correlated. To show this, we utilize the definition of P_t^* such that

$$P_t^* = \frac{P_{t+1}^* + D_{t+1}}{1 + k}. \qquad (8)$$

From equations (1) and (2), we describe the stock price as

$$P_t = \frac{E_t(P_{t+1} + D_{t+1})}{1 + k}. \qquad (9)$$

Let

$$P_{t+1} + D_{t+1} = E_t(P_{t+1} + D_{t+1}) + \mu_{t+1} \qquad (10)$$

where μ_t is, by definition, a rational forecast error, which is serially *uncorrelated*. Equation (9) becomes

$$P_t = \frac{P_{t+1} + D_{t+1} - \mu_{t+1}}{1 + k}. \qquad (11)$$

Subtracting equation (11) from equation (8) yields

$$P_t^* - P_t = \frac{1}{1 + k}\left(P_{t+1}^* - P_{t+1}\right) + \frac{1}{1 + k}\mu_{t+1}. \qquad (12)$$

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[2]See also Shiller (1990).

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In the absence of bubbles, $\eta_t = P_t^* - P_t$, and η_t is the present value of future forecast errors (i.e., $\sum_{i=1}^{\infty} \frac{\mu_{t+i}}{(1+k)^i}$). OLS estimation of equation (7) thus yields inefficient coefficient estimates under the null hypothesis.

Applying generalized least squares (GLS) or maximum likelihood estimation (MLE) to equation (5) is equivalent to estimating (note that $D_t \equiv (1+k)P_{t-1}^* - P_t^*$) equation (13):

$$P_t = (1+k)P_{t-1} - D_t + \mu_t. \tag{13}$$

Chow (1989) finds that the data are not consistent with equation (13). However, Dokko (1991) suggests that without a theory of why and how dividends are paid, one may not test the present value model in the form of equation (13) using the observed stock price-dividend relation. For example, as discussed in Dokko, if we incorporate the informational effect of the dividend and dividend smoothing behavior into the present value model, we see that the relationships of the current stock price with the lagged stock price and the current dividend are determined by several forces, such as the capitalization of an unexpected dividend, the dividend smoothing policy, and the discount rate.

We may summarize the problems with testing the present value model in the following ways: First, the variance bounds test may not be useful if the stock price is non-stationary. Second, OLS estimation of equation (7) is inefficient since the rational forecast error η_t is serially correlated under the null hypothesis. This inefficiency will be exacerbated by measurement errors in the estimated P_t^*. Finally, without a theory of dividends, it would be difficult to be sure that the observed stock price-dividend relation is capable of testing the present value model, as opposed to inadvertently

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estimating another relation.

The rest of the paper is organized as follows: Section I proves that the Scott-type regression tests of the present value model cannot detect the existence of rational bubbles. Section II presents an alternative regression test. Our approach does not require a theory of dividends, and the statistical results are valid regardless of whether speculative bubbles are present. We find that the data appear to be consistent with the present value model. Section III contains a brief conclusion.

I. On Testing for Rational Bubbles

Since P_t^* is not observed, researchers usually assume that the observed market price at the end of the sample period, P_T, is the same as the *ex post* perfect foresight price at that time, P_T^*. The estimated *ex post* rational price $\hat{P}_t^*$ is obtained in the following way:

$$\hat{P}_T^* = P_T = P_T^* + B_T - \eta_T$$

$$\hat{P}_{T-1}^* = \frac{1}{1+k}(\hat{P}_T^* + D_T) = P_{T-1}^* + \frac{1}{1+k}(B_T - \eta_T)$$

$$\vdots$$

$$\hat{P}_t^* = \frac{1}{1+k}(\hat{P}_{t+1}^* + D_{t+1}) = P_t^* + \frac{1}{(1+k)^{T-t}}(B_T - \eta_T). \qquad (14)$$

Using $\hat{P}_t^*$ and P_t, the least squares estimate $\hat{b}$ in regression (7), with the null hypothesis of $b = 1$, is

$$\hat{b} = \operatorname{cov}\left(\hat{P}_t^*, P_t\right)/\operatorname{var}(P_t)$$

$$= \operatorname{cov}\left(P_t^* + \frac{1}{(1+k)^{T-t}}(B_T - \eta_T), P_t\right)/\operatorname{var}(P_t)$$

$$= \operatorname{cov}\left(P_t + \eta_t - B_t + \frac{1}{(1+k)^{T-t}}B_T - \frac{1}{(1+k)^{T-t}}\eta_T, P_t\right)/\operatorname{var}(P_t)$$

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$$= \text{cov}\left(P_t - B_t + \frac{1}{(1+k)^{T-t}}B_T, P_t\right) / \text{var}(P_t). \tag{15}$$

The last equality holds because $\text{cov}(\eta_t, P_t) = \text{cov}(\eta_T, P_t) = 0$. Since bubbles evolve over time, the bubble at time T (B_T) is correlated with the bubble at time t $(B_t, t < T)$ and thus with P_t. In other words, we can express B_T as

$$
\begin{aligned}
B_T &= (1+k)B_{T-1} + e_T \\
&= (1+k)^2 B_{T-2} + (1+k)e_{T-1} + e_T \\
&\vdots \\
&= (1+k)^{T-t}B_t + (1+k)^{T-t-1}e_{t+1} + \cdots + e_T
\end{aligned}
\tag{16}
$$

where e_t is, by definition, white noise and uncorrelated with information available at time t. Since $\text{cov}(e_{t+i}, P_t) = 0$ for all $i \geq 1$, it follows that

$$\hat{b} = 1 \tag{17}$$

This proves that failure to accept the null hypothesis of $b = 1$ in regression (7), using $\hat{P}_t^*$ and P_t, cannot be attributed to the presence of rational bubbles.

II. An Alternative Regression Test of the Present Value Model

A. Methodology

The empirical analysis will be directed to examining whether the present value relation in equation (12) holds. The test of the present value model in the form of equation (12) has several advantages over the test of the present value model in the form of $P_t^* = P_t + \eta_t$. First, as Flood and Hodrick (1986)

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warned, model specification for testing rational bubbles, using *ex post* data, is not an easy task. It is desirable to develop a model, such as equation (12), which holds irrespective of the presence of rational bubbles. Second, since μ_t is serially uncorrelated under the null hypothesis, OLS estimation of equation (12) is efficient. Third, even though P_t^* is unobserved, estimation of equation (12) is robust with respect to measurement errors in $\hat{P}_t^*$. This can be seen as follows:

$$
\begin{aligned}
\hat{P}_t^* - P_t &= P_t^* - P_t + \frac{1}{(1+k)^{T-t}}(B_T - \eta_T) \\
&= \frac{1}{1+k}(P_{t+1}^* - P_{t+1}) + \frac{1}{1+k}\mu_{t+1} + \frac{1}{(1+k)^{T-t}}(B_T - \eta_T)
\end{aligned}
$$

$$(18\text{-}a)$$

$$
\hat{P}_{t+1}^* - P_{t+1} = P_{t+1}^* - P_{t+1} + \frac{1}{(1+k)^{T-(t+1)}}(B_T - \eta_T). \qquad (18\text{-}b)
$$

Substituting $\hat{P}_{t+1}^* - P_{t+1} - \frac{1}{(1+k)^{T-(t+1)}}(B_T - \eta_T)$ for $P_{t+1}^* - P_{t+1}$ in equation (18-a) yields

$$
\hat{P}_t^* - P_t = \frac{1}{1+k}(\hat{P}_{t+1}^* - P_{t+1}) + \frac{1}{1+k}\mu_{t+1}. \qquad (19)
$$

We use equation (19) to test the present value model.

B. The Data Base

The data base is the same as that used in Campbell and Shiller (1987). P_t is the stock price for January of each year $t+1$ from 1872 through 1987, deflated by the price deflator for January of that year. D_t is the dividend for each year t from 1872 through 1986, deflated by the average price deflator for the corresponding year.[3] The assumed discount rate is 8.35%, which

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[3]If P_t is the stock price for January 1987, the corresponding D_t is the dividend paid during the year of 1986.

is the average annual real rate of return on common stocks for the sample period.

C. Statistical Results

If $\hat{P}_t^* - P_t$ is stationary, the testing equation is

$$\hat{P}_t^* - P_t \;=\; \beta_0 + \beta_1 \left(\hat{P}_{t+1}^* - P_{t+1} \right) \tag{20}$$

where β's are regression coefficients to be estimated, and the null hypothesis is that $\beta_0 = 0$ and $\beta_1 = 1/(1+k)$.

If $\hat{P}_t^* - P_t$ is non-stationary,[4] the empirical model analog of equation (19) is (following Scott)

$$\frac{\hat{P}_t^* - P_t}{D_t} \;=\; \beta_0 + \beta_1 \left(\frac{\hat{P}_{t+1}^* - P_{t+1}}{D_t} \right). \tag{21}$$

For the 1872 through 1986 period, we obtain OLS results

$$
\begin{aligned}
\hat{P}_t^* - P_t \;=\; & \underset{(0.007)}{-0.007} \;+\; \underset{(0.039)}{0.908} \left(\hat{P}_{t+1}^* - P_{t+1} \right) \\
& \text{Adj. } R^2 = 0.82, \;\; \rho = \underset{(0.09)}{0.12}, \;\; F = 0.55
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\frac{\hat{P}_t^* - P_t}{D_t} \;=\; & \underset{(0.404)}{-0.624} \;+\; \underset{(0.046)}{0.869} \left(\frac{\hat{P}_{t+1}^* - P_{t+1}}{D_t} \right) \\
& \text{Adj. } R^2 = 0.76, \;\; \rho = \underset{(0.09)}{0.06}, \;\; F = 1.26
\end{aligned} \tag{23}
$$

where the numbers in parentheses are standard errors, ρ is the first-order autocorrelation of the regression residual, and F is the F-statistic with 2

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[4]One may suggest that the absolute magnitude of μ_t grows over time as the stock price and the dividend grow over time.

and 113 degrees of freedom for testing the joint hypothesis of $\beta_0 = 0$ and $\beta_1 = 0.923$. ($0.923 = 1/1.0835$)

The statistical results appear to be consistent with the present value model with a constant discount rate. We reject none of the following: (i) $\beta_0 = 0$; (ii) $\beta_1 = 0.923$; (iii) the joint hypotheses of $\beta_0 = 0$ and $\beta_1 = 0.923$. The regression residual, which is the estimate of the present value of the forecast error ($\frac{\mu_{t+1}}{1+k}$), is not serially correlated. This is also consistent with a rational forecast error. The latter result should be interpreted cautiously. As argued by Summers (1986), tests for autocorrelations of single period returns could have extremely low power against the inefficient market hypothesis.

III. Concluding Remarks

This paper has shown that the regression of $\hat{P}_t^*$ onto P_t cannot detect speculative bubbles and has proposed an alternative test of the present value model. The paper has tested whether the present value relation holds for the rational forecast error η_t. This approach has several advantages over the regression of $\hat{P}_t^*$ onto P_t. The statistical results could not reject the present value model.

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